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WEDGE ACOUSTIC WAVES:
NEW THEORETICAL AND EXPERIMENTAL RESULTS

V.V. Krylov

Faculty of Physics, Moscow M.V. Lomonosov State University, Moscow 117234, USSR

ABSTRACT

New theoretical and experimental results on wedge acoustic waves are reported with emphasis to the wedges, which physical and geometrical properties differ from them in the idealized wedges. The geometrical-acoustic theory of localized waves in sharp-angle wedges is described with applications to truncated and curved wedges, to wedges with arbitrary cross-section, and anisotropic wedges. The symmetric wedge modes in sharp-angle and in obtuse-angle wedges are also considered, as well as the influence of surface effects on wedge waves. The conclusive problem under consideration is the scattering of wedge acoustic waves.

1. INTRODUCTION

The localized modes of vibration propagating along the tips of elastic wedges, i.e., wedge acoustic waves, were predicted by numerical calculations more than 15 years ago 1,2). Since that time these waves are widely investigated both theoretically and experimentally 3-5) due to the fact they have such remarkable features as strong energy concentration near the tip, absence of dispersion and diffraction loss, relatively low phase velocity. These waves as well as surface phonons give their contribution to low-temperature specific heat of a bounded solid body, representing the fundamental interest for solid state physics. Their practical applications may cover the acoustoelectronic devices, especially nonlinear devices, and nondestruc-
tive testing of some special technical constructions, e.g., trailing edges of airfoils.

In present time, however, the use of wedge acoustic waves for practical purposes is limited by the absence of clear understanding of their behavior in real solid structures having any imperfections. In this work we shall describe some new theoretical and experimental results on wedge waves, concerning mainly to the wedges, which physical and geometrical properties differ from them in the idealized wedges.

2. GEOMETRICAL-ACOUSTIC THEORY OF WEDGE WAVES

2.1 Geometrical Acoustics of Flexural Waves in a Sharp-Angle Wedge

As it is well known 1-5), in the case of arbitrary wedge angles $\theta$ the wedge waves allow only numerical consideration. However, for sharp-angle wedges (Fig. 1) the analytical approach is possible 5), because in this case the antisymmetric wedge waves, the most important type of wedge waves, are physically connected with flexural waves in thin plates (the lowest antisymmetric Lamb mode). The complexity of the nature of wedge waves makes it very important to develop approximate approaches for analytical treatment of the problem. Geometrical-acoustic or, in other words, ray approach to the theory of wedge waves 6,7) seems to be very perspective in this way. Such an approach, being applicable for wedges with enough small wedge angles $\theta$, is based on the solution of the vibration equation for a thin plate with variable thickness $h$ in geometrical-acoustic approximation 7). Similar approach, combining ray ideas with the solution of certain model Helmholtz wave equation, was independently proposed in Ref. 8. Earlier the geometrical acoustic methods were successfully applied to the problems of Rayleigh wave reflection and transmission in sharp-
angle wedges \(^9,10\).

We shall start from the well known two-fold equation of flexural vibrations of a thin plate with variable thickness in \(x\) direction (see Fig. 1)

\[
\frac{\partial^2}{\partial x^2} \left[ D(x) \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \right] + 2(1-\sigma) \frac{\partial^2}{\partial x \partial y} \left[ D(x) \frac{\partial^2 w}{\partial x \partial y} \right] + \\
\frac{\partial^2}{\partial y^2} \left[ D(x) \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \right] - \omega^2 \rho h(x) w = 0 .
\]

(1)

Here \(w\) is the normal displacements of a middle of a plate (the plane of symmetry of the wedge), \(D(x) = Eh^3(x)\) ·
· \([12(1-\sigma^2)]^{-1}\) is a local flexural rigidity, \(h(x)\) is a local thickness of a wedge, \(E\) and \(\sigma\) are Young's modulus and Poisson ratio of a wedge material. Supposing the wedge angle \(\Theta\) to be sharp enough, we simplify the expression for local thickness \(h(x) = 2xtg(\Theta/2) \approx \Theta x\) and introduce the notation \(D(x) = \hat{A}h^3(x) \approx \hat{A}\Theta^3 x^3\), where \(\hat{A} = E/12(1-\sigma^2) = \rho c_p^2/12\). Here \(c_p = 2c_l(1-c_t^2/c_l^2)^{1/2}\) is the velocity of a longitudinal wave in a thin plate (the lowest symmetric Lamb mode), \(c_l\) and \(c_t\) are the velocities of longitudinal and shear bulk waves in infinite medium.

We shall seek the solution of Eq. (1) in the form, usual for geometrical acoustics (optics) \(^{11,12}\):

\[
w = A(x) \exp \left[ ik_p S(x,y) \right] ,
\]

(2)

where \(A(x)\) and \(S(x,y) = S(x) + (k/k_p)y\) are slowly varying amplitude and eiconal of the wave, \(k\) is a projection of the wavevector of a propagating quasi-plane wave on the \(y\) axis, \(k_p = \omega / c_p\) is the wavenumber of a symmetric plate wave.

Substitution of Eq. (2) into Eq. (1) and equalization of the real part to zero give in principal approximation
the corresponding eiconal equation

\[ |\nabla S(x,y)|^4 = \frac{\omega^2 \rho}{\omega^2 \pi^2 k_p^4} = \frac{k_a^4(x)}{k_p^4} = n_a^4(x), \quad (3) \]

where \( k_a(x) = \left(\frac{\omega^2 \rho}{\omega^2 \pi^2 h^2(x)}\right)^{1/4} = 12^{1/4} k_p^{1/2} / \theta x \) is a local wavenumber of a flexural wave in plates, \( n_a(x) \) is a corresponding coefficient of refraction. The solution of Eq. (3), corresponding to propagating waves, takes the form

\[ S(x) = (1/k_p) \int \left[ k_a^2(x) - k^2 \right]^{1/2} dx. \quad (4) \]

The equalization of the imaginary part to zero gives the so called transfer equation for the case under consideration, which is not written here due to it's cumbrousity. The solution of this equation together with Eq. (4) gives the amplitude

\[ A(x) = \frac{G}{\sqrt{\frac{k_a^2(x) - k^2}{x}}} = \frac{G}{\sqrt{\frac{2 \cdot 3^{1/2} k_p / \theta x - k^2}{x}}}^{1/4}, \quad (5) \]

where \( G \) is an arbitrary constant.

The evaluation of omited terms shows that geometrical-acoustic solution (2), (4), (5) is valid for

\[ k_p x / \theta \gg 1, \quad (6) \]

i.e., it loses the sense near the wedge tip (for small \( k_p x \)) and/or for large wedge angles \( \theta \). Besides this, directly from Eq. (5) the usual ray-acoustic restriction follows, which forbids the localization of observation points near the ray turning-points, i.e., near the caustics.

2.2 Dispersion Equation for Ideal Sharp-Angle Wedge

To develope the ray theory of localized wedge vibrations, i.e., wedge acoustic waves, we shall use the geomet-
tical-acoustic solution (2), (4), (5), supposing the quantity \( k \) in this solution to be the desired wavenumber of wedge wave. Using the well known methods of geometrical-acoustic calculation of guided modes in open waveguide \(^{11}\) and accounting that the phase shift of flexural wave after it's reflection from the tip is equal to \( \pi/2 \) \(^{9}, 10, 13\) and the phase shift after it's reflection from simple caustics is equal to \(-\pi/2 \) \(^{10}, 11\), we can write immediately the dispersion equation of antisymmetric wedge waves in the form of Bohr - Sommerfeld condition:

\[
\int_{0}^{x_t} [k_a^2(x) - k^2]^{1/2} dx = \pi n ,
\]  
\( (7) \)

where \( x_t = 2 \cdot 3^{1/2} \frac{k_p}{\theta k^2} \) is the coordinate of a ray turning point, \( n = 1, 2, 3... \) Analytical calculation of the integral in Eq. (7) gives the simple expression for the velocities of wedge modes \( c = \omega/k \) \(^7\):

\[
c = \frac{c_p n \theta}{3^{1/2}} .
\]  
\( (8) \)

Note, that the expression (8) for the velocities of wedge modes is close to the "exact" solution of Ref. 5 and tends to it asymptotically with the increase of mode number \( n \). If we take into attention that \( c_p/3^{1/2} \approx c_R \) for \( \delta \approx 1/3 \), where \( c_R \) is the velocity of Rayleigh surface wave, then for small wedge angles \( \theta \) the geometrical-acoustic expression (8) coincides with the well known empirical formula for wedge acoustic wave velocities \( c = c_R \sin(n\theta) \), which was obtained in Ref. 3 as a result of analytical approximations of numerically calculated dispersion curves for aluminum wedges ( \( \delta \approx 1/3 \)).

2.3 Amplitude Distributions of Wedge Modes

By means of superposition of quasi-plane waves (2) with accounting Eqs. (4), (5), (8) and the phase shifts after
the reflections one can construct by standard methods \(^{11}\)
the amplitude distributions of wedge acoustic modes in the
direction, perpendicular to the direction of wave propaga-
tion: \(W(x)\). The final expression for the amplitudes of the
displacements of antisymmetric wedge waves \(w(x,y) = W(x)\cdot\exp(iky)\) for \(x < x_t\) takes the form

\[
w(x,y) = \frac{W_0}{(k_p x/\theta)L^{1/2}} \cos [Lk_p x/\theta - \text{ncrsinM} - \frac{\pi}{4} \exp(13^{1/2}k_p y/n\theta)],
\]

where \(L = [3^{1/2}(2\theta/k_p x - 3^{1/2}/n^2)]^{1/2}, M = (1 - 3^{1/2}k_p x/n^2\theta)\).

The amplitude distributions \(W(x)\), calculated for the
first three modes according to Eq. (9), show that in agree-
ment with the condition of applicability of ray approxima-
tion (6) the better work of the approach occurs for modes
with higher numbers \(n\) (\(n = 2\) and \(3\)). Significant parts of
energies of these modes are concentrated in the regions
with relatively large values of \(k_p x/\theta\). Evident failure of
geometrical-acoustic approximation occurs for the values of
\(k_p x/\theta\) corresponding to the regions near the wedge tip and
near the caustics, where the field tends to infinity.

3. WAVES IN TRUNCATED WEDGES

The above considered geometrical-acoustic approach allo-
wes to analyse easily the different cases of wedge waves pro-
pagation in structures, which geometry differ from geometry
of idealized wedge. For example, if the wave is truncated
(see Fig. 1), then the phase velocity of wedge waves can be
determined from the basic relation (7). However, integrati-
on with respect to \(x\) is now to perform not from zero, but
from the value of truncation height \(l\). After the integrati-
on we receive the next algebraic equation relative to the
velocity \(c\):
\[
\frac{c_{\text{hn}}}{c_{o}} = \pi n + \left[ \frac{2 \cdot 3^{1/2} k_{p} l}{\theta} \left( 1 - \frac{c_{o}^{2}}{c_{p}^{2}} \frac{3^{1/2} k_{p} l}{2 n^{2} \theta} \right) \right]^{1/2} + \\
+ \frac{c_{v} \cos(1 - \frac{c_{o}^{2}}{c_{p}^{2}} \frac{3^{1/2} k_{p} l}{2 n^{2} \theta})}{c_{o}}
\]

(10)

where \( c_{o} = c_{p} n^{3/2} \) is the velocity in idealized wedge (see Eq. (8)). The plots of \( c/c_{o} \) on \( 3^{1/2} k_{p} l \theta \) for the first three modes, obtained as a result of numerical solution of Eq. (10), demonstrate that truncation causes the dispersion of wedge waves. For the first wedge mode the approximate solution, obtained from Eq. (10), differs significantly from the corresponding "exact" solution (as one could await). But for the second and third modes the agreement for large values of \( 3^{1/2} k_{p} l \theta \) is quite satisfactory.

4. WAVES IN CURVED WEDGES

The geometrical-acoustic approach allows to analyze easily such an important problem as the propagation of localized flexural vibrations along the tip of a wedge curved in its plane \( \theta \). Let the radius of curvature be positive (convex tip). Then, using the solution of the eiconal equation (3) in cylindrical coordinates \( 11 \), in which the tip of the wedge is described by the relation \( r = R \), we rewrite the basic integral relation (7) in the form

\[
- \int_{R}^{R_{t}} [k_{a}^{2}(r - R) - k^{2}(R^{2}/r^{2})]^{1/2} dr = \pi n \ .
\]

(11)

Supposing the value of \( R \) to be large enough \( (R \gg |R - r_{t}|) \) and introducing the notation \( v = R - r \), we shall transfer from Eq. (11) to approximate relation

\[
\int_{0}^{v_{t}} [k_{a}^{2}(v)(1 - \frac{2 v}{R}) - k^{2}]^{1/2} dv = \pi n ,
\]

(12)

where the coordinate of ray turning-point is now determi-
ned as \( v_t = v_{to}(1 - 2v_{to}/R); \) \( v_{to} = 2 \cdot 3^{1/2}k_p/\theta k^2 \) is the ray turning-point in the absence of curvature. Calculating the integral in Eq. (12), we receive the expression for phase velocities of wedge waves propagating along the convex curved tip:

\[
c = \frac{c_o n \theta}{3^{1/2}} \left( 1 + \frac{3^{1/2}}{2} \frac{n^2 \theta}{k_p R} \right)
\]

(13)
i.e., in this case the wedge waves also have a dispersion. For wedge tips with negative curvature (concave tips) the expression (13) remains valid if \( R \) is replaced by \(-R\).

If the wedge is curved perpendicular to its plane, it follows from symmetry conditions that the additions to the unperturbed velocities are proportional to \( R^{-2} \).

5. WEDGES WITH ARBITRARY CROSS-SECTION AND ANISOTROPIC WEDGES

The approach under consideration can help to solve also some other practically important problems, on which we shall not stay here in detail. One of these problems is the derivation of phase velocities for wedge waves in structures, which cross-sections are bounded by arbitrary intersecting curves, in contrast to straight lines in the case of ideal wedge. Such structures may have certain importance in solid state physics and in nondestructive testing. The analysis of these structures evidently can be based on the relation (7) for corresponding laws of \( k_a(x) \).

The relation (7) can be easily generalized to the case of wedges manufactured from anisotropic crystals. To do this, one must replace the wavenumber \( k_a(x) \) in Eq. (7) by the quantity \( k_a(x, \alpha) \), which takes into account the orientation dependence of flexural wave velocity in crystalline plates (see, e.g., Ref. 14), and express the angle of sliding \( \alpha \) through \( k_a(x, \alpha) \) and \( k \).

6. ON SYMMETRIC WEDGE MODES
6.1 Symmetric Mode of Truncated Sharp-Angle Wedge

As we seen above, the geometrical-acoustic approach allows to study the antisymmetric localized wave propagation in truncated sharp-angle wedges. In the last case the ray approach predicts also the existence of symmetric wedge mode, which can not propagate in ideal sharp-angle wedges. To obtain this result, we may start from the system of coupled differential equations of elasticity, which describe the displacements $u_x$ and $u_y$ in the plane of wedge symmetry in the plane-stress approximation (see, e.g. Ref. 15). As it is well known, these equations coincide with the elastic equations for unbounded medium, if the usual Poisson ratio $\delta$ is replaced by modified Poisson ratio $\delta = \delta/(1 + \delta)$. The role of longitudinal waves is now played by the symmetric mode of thin plate (the lowest symmetric Lamb mode), which propagates with the above mentioned "plate velocity" $c_p$, and the role of shear waves is played by SH-mode of thin plate propagating with the velocity $c_t$. If the geometrical-acoustic approach is used, these modes become uncoupled and propagate independently in a sharp-angle wedge.

In what follows we shall use the geometrical-acoustic solutions for the displacements of longitudinal $u_{li}$ and shear $u_{ti}$ modes:

$$u_{li} = \frac{A}{x^{1/2}(k_p^2 - k^2)^{1/4}} \exp(iky - v_1x),$$
$$u_{ti} = \frac{B}{x^{1/2}(k_t^2 - k^2)^{1/4}} \exp(iky - v_tx),$$

where $v_{1,t} = (k^2 - k_{p,t}^2)^{1/2}$, $A$ and $B$ are constants, $i$ takes the values $x$ and $y$. An analysis shows that geometrical-acoustic expressions (14) are valid for $|k_{p,t}^2 - k^2|^{1/2} > 1$. Therefore, the height of wedge truncation $l$ must
be large enough. On the truncated tip (for x = 1) the expressions (14) must satisfy the stress-free boundary conditions $\delta_{xx} = \delta_{xy} = 0$, which (in terms of deformations) have the form

$$\delta u_{yy} + (1-\delta) u_{xx} = 0 ,$$

$$u_{xy} = 0 .$$

(15)

Besides these, the well known conditions div$\mathbf{u}$ = 0 and rot$\mathbf{u}$ = 0 must be satisfied.

Substitution of Eq.(14) into Eq.(15) with accounting the last conditions gives the dispersion equation for the symmetric mode of truncated sharp-angle wedge

$$(2k^2 - k_t^2)^2 - 4k^2v_1v_t = 0 ,$$

(16)

which coincides with the well known Rayleigh equation for quasi-Rayleigh wave propagating along the tip of a thin plate. However, in contrast to quasi-Rayleigh wave, the symmetric mode of truncated wedge has an additional decay of amplitudes, proportional to $x^{-1/2}$.

6.2 The Case of Obtuse-Angle Wedge

Let us consider an obtuse-angle wedge (Fig. 2) and analyse the guided modes of such a structure. Note, that the tip of this truncated obtuse-angle wedge is bounded by two ribs of ideal obtuse-angle wedges. If we know the reflection coefficients of Rayleigh waves from the ribs $\tilde{R}$, we can derive the dispersion equation for modes, guided along the tip, from general relation $^{16)}$

$$\exp(2ikR \cos\alpha)R^2 = 1 ,$$

(17)

where $k_R = \omega/c_R$ is the Rayleigh wavenumber, $\alpha$ is the angle of incidence of Rayleigh wave onto the rib AB or CD. We shall use now the expression for $R = R(\alpha)$ in Born approximation $^{17)}$. For small sliding angles $Q = \pi/2 - \alpha$ this
expression takes the form

$$\hat{R} = i\hat{\beta}Q^{-1},$$

(18)

where $\hat{\beta} = (\pi - \theta)/2$ are the additional wedge angles (see Fig. 2), $\hat{\lambda} = -2k_t^2(k_R^2 - k_t^2)^{1/2}/F'(k_R)$ is the positive dimensionless quantity depending on Poisson ratio, $F'(k_R)$ is the value of derivative of Rayleigh determinant at $k = k_R$.

Substituting Eq. (18) into Eq. (17), we receive in the same approximation the solution for $Q$: $Q = i\hat{\beta}(1 - k_Ra\hat{\lambda}\hat{\beta})$.

Since the propagation constant of guided wave is determined by the relation $k = k_R\cos\theta \approx k_R(1 - Q^2/2)$, we shall have for phase velocity of the lowest symmetric mode $c = \omega/k:\n
$$c = c_R\left[1 - \frac{\hat{\lambda}^2(\pi - \theta)^2}{8}(1 - k_Ra\hat{\lambda}(\pi - \theta))\right].$$

(19)

In the limiting transition $a \rightarrow 0$, corresponding to an ideal obtuse-angle wedge, the dispersion law (19) goes over to the relation

$$c = c_R\left[1 - \frac{\hat{\lambda}^2(\pi - \theta)^2}{8}\right],$$

(20)

which describes satisfactorily the velocity of a well known dispersion-free symmetric mode of an ideal obtuse-angle wedge $^4)$. Note, that formula of the type (20), but differing from it by the value of coefficient before $(\pi - \theta)^2$, earlier was obtained by V.G. Mozhaev with the help of other method. The amplitude decay of the above mode in the approximation considered is described by the proportionality factor $\hat{C} = \exp(ik_Rx\cos\alpha)$, where $x$ is the coordinate measured along the surface in the direction, perpendicular to the wedge tip. Since $\cos\alpha = \sin\theta \approx Q = i\hat{\lambda}(\pi - \theta)/2$, we receive $\hat{C} = \exp[-i\hat{\lambda}k_Rx(\pi - \theta)/2]$.  

6.3 Experimental Investigation of Symmetric Mode of $90^0$-angle Wedge
In our experiments 6, 18) the symmetric mode was excited in 90°-angle aluminum wedge by means of transversally polarized piezokeramic plate with vibrations oriented in a wedge symmetry plane 19). We have measured the velocities of symmetric and antisymmetric modes as well as their attenuation. The measurements have shown that the relative velocity change for antisymmetric mode \( \Delta c/c_R = (c - c_R)/c_R = -0.028 \) and for symmetric mode \( \Delta c/c_R = (c - c_R)/c_R = 0.014 \). Within the accuracy of measurements (1\%) these values agree with corresponding numerically calculated values 4, 20). The measurements of attenuation have shown that amplitudes of antisymmetric mode practically is not changed with distance along the tip, whereas the symmetric mode is characterized by a strong attenuation (\( \sim 1.5 \text{ dB/cm} \)). This indicates that the symmetric mode of 90°-angle wedge is leaky-mode, in agreement with the numerical calculations of Ref. 4.

7. INFLUENCE OF SURFACE EFFECTS ON WEDGE WAVES

It is well known that elastic properties of a body near any real surface differ from them in the volume of the medium. Influence of this difference (the surface effects) on Rayleigh waves is usually small enough 21). However, it may be strongly increased for flexural waves in thin plates 22). Since the ray theory of antisymmetric wedge waves in sharp-angle wedge uses the dispersion equation for flexural waves in thin plates, the influence of surface effects on wedge waves can be investigated analytically. The analysis shows that the expression for phase velocities of the modes in an isotropic sharp-angle wedge takes the form

\[
c = \frac{c_p n\theta}{3^{1/2}} \left(1 - \frac{\rho^S}{\rho} \frac{k_p}{n^2 \theta^2}\right),
\]

where \( \rho^S \) is the so called surface mass density 21), and \( q \sim 1 \) is a positive dimensionless constant.
Thus, the surface effects also cause the dispersion of wedge waves. It is seen from Eq. (21) that the contribution of surface effects is magnified strongly for wedges with small wedge angles $\theta$ and for modes with small numbers $n$.

8. SCATTERING OF WEDGE WAVES

8.1 Theoretical Approach

Any inhomogeneities on the path of propagation of wedge waves cause their scattering into wedge modes of different numbers as well as into bulk and surface acoustic waves. An analysis of wedge wave scattering on a small or large and smooth tip inhomogeneities in a sharp-angle wedge may be performed by perturbation methods or by means of ray approach, respectively. In particular, the perturbational approach may be developed on the basis of the exact solution of the equation (1). In the case of topographic inhomogeneities of the tip, e.g., shallow grooves, the Rayleigh hypothesis may be applied to obtain the equivalent boundary conditions on the wedge tip with a truncation:

$$M_x = f_1(y), \quad \frac{\partial M_x}{\partial x} - 2\frac{\partial M_{xy}}{\partial y} = f_2(y), \quad (22)$$

where $M_x$ and $M_{xy}$ are bending and twisting moments, $f_1(y)$ and $f_2(y)$ are functions describing the inhomogeneity. Note, that the values of $f_1(y)$ and $f_2(y)$ are proportional to the depth $h$ of the defect. The convolution of $f_1(y)$ and $f_2(y)$ with corresponding Green's functions for sharp-angle wedge allows to obtain the formal integral expression for the general scattered acoustic field. The contribution of the poles in the integrand describes the scattering into other wedge modes including the reflection into initial mode. Preliminary results show that scattering in a truncated wedge has much common with the scattering of Rayleigh waves. In particular, the amplitudes of scattered waves are proportional to $h/\lambda$ for small $h/\lambda$. 
where $\lambda$ is the wedge wavelength.

### 8.2 Experimental Investigations With $90^\circ$-angle Wedge

Experimental investigations of wedge waves scattering were carried with the fundamental antisymmetric mode of $90^\circ$-angle aluminum wedge \(^6, 18\) (Fig. 3). Two main types of wedge topographic inhomogeneities were investigated: the precipice of the wedge and the rectangular groove on the wedge tip. The coefficients of reflection $|R|$ and transmission $|T|$ were measured as well as the angular characteristics of wedge wave scattering into Rayleigh surface waves. In particular, for the case of precipice the measurements have shown that $|R| = 0.5 \pm 0.05$ and $|T| = 0.4 \pm 0.06$ (due to the symmetry of the problem the values of $T$ have opposite signs for two diverging ribs).

For the case of rectangular groove, which depth $h$ varied from 0.2 mm to 2 mm, the measurements at the frequency 2.1 MHz ($\lambda = 1.32$ mm) have shown that for small $h/\lambda$ a linear dependence of $|R|$ versus $h/\lambda$ occurs, like in the case of Rayleigh surface waves. If one writes $|R|$ in the form $|R| = 2C(h/\lambda)\sin(2\pi d/\lambda)$, where $d$ is the width of the groove, then, according to the experiments, the value of the constant $C$ is equal to 0.72. This value is of the same order as the corresponding constant for Rayleigh waves in aluminum ($C^{(R)} = 0.31$). Thus, the scattering of wedge waves in the case under consideration is similar with the scattering of Rayleigh surface waves.

### REFERENCES

Fig. 1. Sharp-angle wedge (with possible truncation - dashed lines)

Fig. 2. Truncated obtuse-angle wedge

Fig. 3. Scattering of wedge acoustic waves