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Effect of Tunnel Diameter on Ground Vibrations Generated by Underground Trains

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ABSTRACT
A theoretical model has been developed of generating ground vibrations by underground trains travelling in idealised circular tunnels of finite diameter. By means of the reciprocity principle, the displacement field radiated by a point force applied to the bottom of the tunnel, i.e., the Green’s function of the problem, has been derived in zero and first approximations versus tunnel diameter. This more precise Green’s function has been applied to carry out calculations of railway-generated ground vibrations using earlier developed methods. The results show that the velocities of generated low-frequency ground vibrations increase with the increase in tunnel diameter. It is also shown that zero approximation is accurate only at very low frequencies.

1. INTRODUCTION
Although the development of railways has undergone rise and fall, facing the challenge from road transport and aviation industry, they have become attractive again due to the appearance of high-speed trains making railway communications fast and convenient[1]. After French TGV and German ICE trains, more and more high-speed trains are being put into operation within Europe. These are Italian Pendolino, Swedish X2000, Spanish AVE, Belgium Thalys and the Eurostar.

Despite the fact that modern trains have the advantage of high speeds, energy saving, safety and reliability, they cause serious concern regarding the associated environment hazards. In particular, ground vibrations generated by underground trains is one of the major problems related to noise and vibration pollution in urban areas since many big cities have trains running underground. In the light of this, it is important to develop more precise models of generation and propagation of ground vibrations for existing and new underground railway lines, thus assisting in engineering design minimising noise and vibration levels.

Although a number of experimental investigations of ground vibrations from underground railways have been carried out[2-4], very few theoretical works based on analytical solution of the elastic field equations have been undertaken. Probably, the first publication considering an analytical approach to the problem of generating ground vibrations by underground trains was the paper of one of the authors[5]. However, the paper[5], being limited to very low-frequency vibrations, used an idealised model neglecting the influence of the tunnel diameter which was assumed to be very small in comparison with characteristic wave lengths of generated ground vibration spectra. In practice, the tunnel diameter may not be that small and may have a significant effect on
generation and propagation of ground vibrations. The purpose of this paper is to develop a more precise model of generating ground vibrations by underground trains that would include the influence of tunnel diameter on amplitudes and radiation patterns of generated vibrations.

2. THEORETICAL DEVELOPMENT

2.1 Formulation of the problem

Generally speaking, one can construct a rigorous analytical solution to the problem under consideration using the expansion of radiated fields into series of cylindrical functions. However, this way is too time consuming and the results are too complicated to be used in engineering practice. Therefore, in this paper we have adopted an approximate analytical approach to the problem based on the use of the reciprocity principle. Also, the following obvious assumptions have been made: a) an individual sleeper may be regarded as a point force; b) the characteristic wavelengths of generated elastic waves are essentially larger than the tunnel diameter. Under such assumptions, the general expression for the vertical component of vibration velocity associated with ground vibrations generated by underground trains can be written as [5]:

\[ v(y,z,\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x',y',z',\omega) G_{\omega}(r,\omega) dx'dy'dz' \] (1)

Here \( G_{\omega}(r,\omega) \) is the correspondent component of the Green's function which describes the vertical component of the ground vibration velocity due to a unit vertical point force applied to the bottom of the tunnel; \( P(x',y',z',\omega) \) is the Fourier transform of the total distribution of train loads along the tracks; \( \omega = 2\pi f \) is the circular frequency; and \( r = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \) is the distance from the current elementary source \( (x',y',z') \) to the observation point \( (x,y,z) \).

To derive the Green's function of the problem, i.e., the displacement field generated by a point force, we consider the tunnel as being ideally circular and located at the depth \( H \) below the surface (see Fig. 1).

Here \( F \) is a point force of magnitude \( F \) and time dependence \( f(t) \); \( r, \phi, \theta \) are spherical co-ordinates; \( X, Y, Z \) are rectangular co-ordinates; \( a \) is the tunnel diameter; \( H \) is the tunnel depth below the ground surface; \( U \) is the vertical displacement of the ground caused by the applied point force \( F \).

The point force \( F \) can in turn be considered as a superposition of two pairs of forces (Fig. 2): the first pair [2.2] is two identically directed point forces acting respectively on the top and on the bottom of the tunnel, and another pair [2.3] is two conversely directed point forces, also applied to the top and to the bottom of the tunnel. Here \( d \) and \( b \) indicate the top and the bottom points of the tunnel wall.

Obviously, the first pair of forces [2.2] acting in the same direction and having the amplitudes \( F/2 \) reduces to a single force \( F \) which was considered in the earlier investigated case [5] for a very small tunnel diameter. The second pair of forces [2.3] forms an acoustic dipole and describes the effect of tunnel diameter.

It would be rather difficult to directly solve the problem [2.3]. However, it can be simplified using the reciprocity principle for elastic waves which allows to calculate radiation of bulk and surface elastic waves from a variety of complex sources. According to the reciprocity principle, the problem [2.3] can be transformed to finding the corresponding radial displacements of the tunnel wall caused by a point force acting on the ground surface [2.5] (see Fig. 2).
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Figure 1  To the derivation of the Green’s function

Thus, the initial problem has now been resolved into two individual parts: with and without taking into account the effect of tunnel diameter. Keeping this in mind, we may rewrite Eq.(1) as

\[ v(x,y,z,\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x',y',z',\omega)[G_{x1}(r,\omega) + G_{x2}(r,\omega)] \, dx' \, dy' \, dz', \quad (2) \]

where \( G_{x1}(r,\omega) \) is a tunnel diameter independent part of the Green’s function, which has been used in [5]; and \( G_{x2}(r,\omega) \) is a tunnel diameter dependent part of the Green’s function, which will be derived below.
2.2 Part of the Green’s function independent of the tunnel diameter (zero approximation)

As was mentioned above, under the assumption that the tunnel diameter is relatively small and the observation distance is large in comparison with wavelength of radiated waves, the vertical displacement field radiated by two identically directed point forces \( F/2 \) is equivalent to the field generated by a single point force \( F \) and can be written as \[ U_1 = \frac{F}{4\pi \rho r} \left( \frac{\cos^2 \phi}{C_t^2} e^{i \frac{m r}{C_t}} + \frac{\sin^2 \phi}{C_t^2} e^{i \frac{m r}{C_t'}} \right). \] (3)

where \( \rho \) is mass density, \( C_t \) and \( C_t' \) are compressional shear wave velocities, and \( \phi \) is an observation angle.

This gives the following expression for the tunnel diameter independent part of the Green’s function for the vertical component of particle vibration velocity which had been used in \[ G_{\alpha \beta} = \frac{i \omega}{4\pi \rho r} \left( \frac{\cos^2 \phi}{C_t^2} e^{i \frac{m r}{C_t}} + \frac{\sin^2 \phi}{C_t^2} e^{i \frac{m r}{C_t'}} \right). \] (4)

Here terms \( e^{i \frac{m r}{C_t}} \) and \( e^{i \frac{m r}{C_t'}} \) describe contributions of the radiated compressional and shear waves.

2.3 Part of the Green’s function dependent on the tunnel diameter (first approximation)

Projecting the stress components of longitudinal and shear waves generated by the force \( F/2 \) along the tunnel axis and in two perpendicular directions, calculating the upper and bottom radial displacements of the tunnel wall in quasi-static approximation \[ U_2 = \frac{i \omega a F}{4\pi E r} \left[ \frac{A(\phi, \theta)}{C_t} e^{i \frac{m r}{C_t}} + \frac{B(\phi, \theta)}{C_t'} e^{i \frac{m r}{C_t'}} \right]. \] (5)

Here \( a \) is a tunnel diameter, \( E \) is the Young’s modulus and \( A(\phi, \theta) \) and \( B(\phi, \theta) \) are defined as directivity functions which have the following forms:

\[
A(\phi, \theta) = \cos \phi \left[ (3\cos^2 \phi - \sin^2 \phi \sin^2 \theta) + \frac{v}{1-v} (3\sin^2 \phi - \cos^2 \phi \sin^2 \theta - \sin^2 \phi \cos^2 \theta) \right] - \frac{v^2}{1-v} \left( \sin^2 \theta + \cos^2 \phi \cos^2 \phi - \sin^2 \phi \cos^2 \theta \right).
\]

\[
B(\phi, \theta) = \sin \phi \sin 2\phi [4 - (1-v) \cos^2 \theta].
\]
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Here \( v \) is the Poisson's ratio of the ground. From Eq. (5), one can obtain the tunnel diameter dependent part of Green's function, which describes the vertical component of the particle vibration velocity produced by two oppositely directed point forces acting on the tunnel wall. It can be written as

\[
G_{\alpha\alpha} = \frac{(i\omega)^2 a}{4\pi\rho r} \left[ \frac{A(q,\theta)}{C_i} e^{im\theta} + \frac{B(q,\theta)}{C_j} e^{im\theta} \right]
\]  

(6)

Eq. (6) may be regarded as a first order correction to the Green's function in zero approximation due to the consideration of the tunnel diameter. By superimposing Eq. (4) and Eq. (6), the resulting more precise Green's function can finally be obtained:

\[
G_{\alpha\alpha} = \frac{i\omega}{4\pi r} \left[ \frac{\cos^2q}{\rho C_i^2} + \frac{i\omega a}{EC_i} A(q,\theta) e^{im\theta} + \frac{\sin^2q}{\rho C_i^2} + \frac{i\omega a}{EC_j} B(q,\theta) e^{im\theta} \right]
\]  

(7)

This Green's function takes into account the effect of tunnel diameter and describes the vertical component of ground vibration velocity due to a unit point force acting in a certain depth onto the bottom of the tunnel.

2.4 Train-induced excitation forces

When a train is moving at speed \( v \) on the track with sleeper periodicity \( d \), each wheel axle represents a load force \( T \) applied to the track, as shown on Fig. 3. These forces cause downward deflections of the track which produce a wave-like motion along the track and over all the sleepers involved into the deflection distance. Each sleeper, in turn, acts as a vertical force \( P \) applied to the underneath foundation during the time necessary for the deflection curve to pass through the sleeper. Thus, an individual sleeper may be regarded as a point source in the low frequency domain which can radiate elastic waves. Some of the radiated elastic waves travel through the tunnel structure towards the ground surface. As a result, the ground vibrations are generated on the surface. The ground vibration velocity \( V_z \) at a designated observation point on the ground surface is a superposition of the contributions from all sleepers.

The excitation force distribution over all sleepers caused by a complete train moving at speed \( v \) along an underground track can be written in the form [5,9]

\[
P(t,x',y',z') = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n \left[ P(t - \frac{z'+nL}{v}) + P(t - \frac{z'+nL+M}{v}) \delta(x'-H)\delta(y')\delta(z'-md) \right]
\]  

(8)

Here \( m \) is a number of a current sleeper, \( d \) is the distance between adjacent sleepers, and Dirac's delta-function takes the discreteness and periodic distribution of sleepers into account, other parameters are explained on Fig. 3.

Taking the Fourier transform of Eq. (8), we obtain the force frequency spectrum:

\[
P(x',y',z',\omega) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} P_0(\omega) \left[ e^{i\omega(z'-md)/v} + e^{i\omega(z'+nL+M)/v} \right] \delta(x'-H)\delta(y')\delta(z'-md),
\]  

(9)
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where

\[ P_0(\omega) = \frac{8Td}{\pi^2 v} \left( \frac{\omega^4}{\beta^4 v^4} \right)^{-1}. \]  

(10)

Figure 3  Sketch of a train, track and tunnel

Substituting Eqs. (7) and (9) into Eq. (1), with account of Eq. (10), we can obtain the following expression for the frequency spectra of vertical ground vibration velocity generated by an underground train:

\[ V_x = \frac{\text{ino} \Phi_0(\omega)}{4\pi} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} \left( 1 + e^{\frac{\pi M}{L}} \right) e^{\frac{\pi m}{L} (md+nd)} \frac{1}{r_m} \left[ \frac{\cos^2 \varphi_m}{\rho C_r^2} \frac{\text{inoa}}{A_m(\gamma_m, \theta_m)} \right] e^{\frac{\text{inoa}}{C_r^2}} \left\{ \begin{array}{l} \frac{\sin^2 \varphi_m}{\rho C_r^2} + \frac{\text{inoa}}{E C_i} B_m(\gamma_m, \theta_m) \left[ \frac{\gamma_m}{C_r^2} \right] \end{array} \right\} \]  

(11)

Here \( r_m = \sqrt{y^2 + (md)^2 + H^2} \), \( \varphi_m = \arccos \frac{H}{r_m} \), and \( \theta_m = \arccos \frac{md}{\sqrt{y^2 + (md)^2}} \).

Eq. (11) describes the contributions of different carriages, axles and sleepers with time and space differences being taken into account. It also considers the effect of tunnel diameter. As can be seen from Eq. 11, its first part, with terms in square brackets before the exponent \( e^{\frac{\text{inoa}}{C_r^2}} \), describes the contribution of compressional waves, while the second part, with terms in square brackets before the exponent \( e^{\frac{\gamma_m}{C_r^2}} \), describes the contribution of the shear waves.

3. NUMERICAL CALCULATIONS AND DISCUSSION

Numerical calculations of ground vibration velocities generated on the surface by underground heavy-freight trains have been carried out according to Eq. (11) for the following chosen parameters:

For the train – the number of carriages \( N=5 \), the train speed \( v= 13.89 \text{m/s} \) (50 km/h), the total carriage length \( L=8.3 \text{m} \), the distance between bogies of each carriage \( M=4.88 \text{m} \), the distance between two axles \( c=2.2 \text{m} \), the axle load \( T=10 \text{kN} \); for the track – the distance between two sleepers \( d=0.7 \text{m} \), the number of sleepers \( m=150...150 \); for the soil – the inverse track deflection distance
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\( \beta = 1.28 \, m^{-1} \), the Poisson’s ratio \( \nu = 0.25 \), the Young’s modulus \( E = 3.7 \times 10^8 \, kg/m^2 \), the compressional speed \( C_t = 471 \, m/s \), the shear speed \( C_s = 272 \, m/s \), the mass density \( \rho = 2000 \, kg/m^3 \); for the tunnel - the tunnel diameter \( a = 2 \) or \( 4 \) m.

To make comparison between the two approximations more explicit, one can rewrite Eq. (11) in the following form:

\[
V_n = \frac{i \omega \rho^2(\omega) }{4 \pi} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} \left( 1 + e^{i \pi} \right) e^{i \pi (m+\pi/2)} r_m \left[ \frac{1}{r_m \rho C_t^2} \left( \cos^2 \theta_m + \sin^2 \theta_m \right) \right]
\]

\[
+ \frac{i \omega}{E} \left[ A_m(\theta_m, \phi_m) e^{imn} C_s + B_m(\theta_m, \phi_m) e^{imn} C_t \right]
\]

\[
= V_{x1} + V_{x2}
\]

\[ (12) \]

Figure 4 Comparison of the contributions of \( V_{x1} \) and \( V_{x2} \) to the total ground vibration field \( V_x \) for \( a = 4 \) m and \( \gamma = 0 \): a) \( f = 4 \) Hz, and b) \( f = 8 \) Hz
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It can be seen that the terms in Eq.(12) are regrouped in two parts: the terms inside the first square brackets give the Green's function in zero approximation, while the terms inside the second square brackets represent the Green's function in the first approximation. The corresponding ground vibration velocities are referred to as $V_{x1}$ and $V_{x2}$, i.e., the notation $V_{x1}$ stands for a part of the vibration velocity without taking the influence of the tunnel diameter into account, while $V_{x2}$ stands for the part of the vibration velocity associated with the effect of tunnel diameter.

Contributions of $V_{x1}$ and $V_{x2}$ into $V_s$ are displayed on Fig. 4 as functions of the tunnel depth $H$ (in dB versus the reference level of $10^{-9} \text{ m/s}$) for different spectral components $f$. We can see clearly that zero approximation is accurate only at very low frequencies, for example at 1-4Hz (Fig.4a), where the resultant vibration velocity is mainly due to the contribution from $V_{x1}$. However, as frequency increases, the effect of the tunnel diameter increases as well. One can notice that already at frequency of 8 Hz, the contribution from $V_{x2}$ starts to dominate (Fig.4b). Thus, one can conclude that the influence of tunnel diameter should be taken into account to describe generated vibrations with higher accuracy.

CONCLUSIONS

Using the reciprocity principle, the displacement field radiated by a point force applied to the bottom of the tunnel, i.e., the Green's function of the problem, has been derived in zero and first approximations versus the tunnel diameter.

This more precise Green's function has been applied to the calculations of ground vibrations generated by underground trains. The results show that the velocities of generated low-frequency ground vibrations increase with the increase of tunnel diameter. It is also shown that zero approximation is accurate only at very low frequencies.

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