Mathematical optimisation
and signal processing
techniques in wireless relay
networks

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- Sri Krishna
2.47 Bhagavad Gita
Mathematical Optimisation and Signal Processing Techniques in Wireless Relay Networks

Thesis submitted to Loughborough University in candidature for the degree of Doctor of Philosophy.

Ranaji Krishna

Advanced Signal Processing Group,
Department of Electronic and Electrical Engineering,
Loughborough University
2009
Abstract

With the growth of wireless networks such as sensor networks and mesh networks, the challenges of sustaining higher data rates and coverage, coupled with requirement for high quality of services, need to be addressed. The use of spatial diversity proves to be an attractive option due to its ability to significantly enhance network performance without additional bandwidth or transmission power. This thesis proposes the use of cooperative wireless relays to improvise spatial diversity in wireless sensor networks and wireless mesh networks. Cooperation in this context implies that the signals are exchanged between relays for optimal performance. The network gains realised using the proposed cooperative relays for signal forwarding are significantly large, advocating the utilisation of cooperation amongst relays.

The work begins with proposing a minimum mean square error (MMSE) based relaying strategy that provides improvement in bit error rate. A simplified algorithm has been developed to calculate the roots of a polynomial equation. Following this work, a novel signal forwarding technique based on convex optimisation techniques is proposed which attains specific quality of services for end users with minimal transmission power at the relays. Quantisation of signals passed between relays has been considered in the optimisation framework. Finally, a reduced complexity scheme together with a more realistic algorithm incorporating per relay node power constraints is proposed. This optimisation framework is extended to a cognitive radio environment where relays in a secondary network forward signals without causing harmful interferences to primary network users.
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Statement of Originality

The novelty in Chapter 4 is in the design of an MMSE based signal forwarding algorithm using cooperation amongst relays. A novel technique for searching the roots of the Lagrangian polynomial equation is also developed in the chapter and a lemma showing the decomposition of the optimum rank one matrix into a product of receiver and transmitter beamformers in the single source-destination scenario is proposed.

A semidefinite programming based signal forwarding scheme has been developed in Chapter 5. The scheme ensures that each user achieves the specified target SINR with minimal use of power at the relays. A lemma proving that the relaxed semidefinite program will always yield a rank one solution is also given in the chapter.

The novelty in Chapter 6 is in developing a reduced complexity signal forwarding scheme that has identical performance to the semidefinite scheme proposed in Chapter 5. The scheme is also modified to include a, more realistic, per relay antenna power constraint. A novel relaying scheme with reduced complexity for applications in underlay cognitive radio scenario is also developed in the chapter. The work is supported by the following publications:


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I wish to take this opportunity to thank my sister Lila for her love and support throughout my studies.
Statement of Originality

I hereby declare that the work described in this thesis was carried out entirely by the author in the Advanced Signal Processing Group, within the Department of Electronic and Electrical Engineering, Loughborough University. The thesis does not incorporate, without acknowledgement, any material previously submitted for a degree or diploma in any university. And that to the best of my knowledge, it does not contain any materials previously published or written by any other individual except where due reference is made in the text.
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<tr>
<th>Acronym</th>
<th>Definition</th>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
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<tr>
<td>LAN</td>
<td>Local Area Network</td>
</tr>
<tr>
<td>LMI</td>
<td>Linear Matrix Inequality</td>
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<tr>
<td>LOS</td>
<td>Line-Of-Sight</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Program/Programming</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximal-Ratio Combiner</td>
</tr>
<tr>
<td>QCQP</td>
<td>Quadratically Constrained Quadratic Program/Programming</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>QP</td>
<td>Quadratic Program/Programming</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase-Shift Keying</td>
</tr>
<tr>
<td>SDP</td>
<td>Semidefinite Program/Programming</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
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<tr>
<td>SDR</td>
<td>Semidefinite Relaxation</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Interference plus Noise Ratio</td>
</tr>
<tr>
<td>SN</td>
<td>Sensor Network</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SOCP</td>
<td>Second Order Cone Program/Programming</td>
</tr>
<tr>
<td>VAA</td>
<td>Virtual Antenna Array</td>
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<tr>
<td>WAN</td>
<td>Wide Area Network</td>
</tr>
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<td>WMN</td>
<td>Wireless Mesh Network</td>
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List of Symbols

\( N \) Block length of the transmitted signal

\( M \) Number of source-destination pairs of nodes

\( \otimes \) Kronecker product

\( I_K \) \( K \times K \) Identity matrix

\( F \) Relay transceiver matrix

\( v_s \) Noise vector at the relay nodes

\( v_t \) Noise vector at the destination nodes

\( \gamma_m \) Target signal-to-interference plus noise ratio at the \( m^{th} \) destination node

\( s \) Signal vector transmitted by the source nodes

\( r \) Signal vector received by the relay nodes

\( x \) Signal vector transmitted by the relay nodes

\( y \) Signal vector received by the destination nodes

\( F_{opt} \) Optimum signal forwarding matrix

\( H_s \) Channel convolution matrix between source nodes and relay nodes
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$H_t$</td>
<td>Channel convolution matrix between relay nodes and destination nodes</td>
</tr>
<tr>
<td>$E{\cdot}$</td>
<td>Expectation operator</td>
</tr>
<tr>
<td>$|\cdot|_p$</td>
<td>$l_p$ norm</td>
</tr>
<tr>
<td>$|\cdot|$</td>
<td>Euclidean norm</td>
</tr>
<tr>
<td>$(\cdot)^H$</td>
<td>Hermitian transpose operator</td>
</tr>
<tr>
<td>$(\cdot)^T$</td>
<td>Transpose operator</td>
</tr>
<tr>
<td>$\text{diag}(b)$</td>
<td>Diagonal matrix composed of diagonal elements from a vector $b$</td>
</tr>
<tr>
<td>$\nabla_{w^*}$</td>
<td>Partial differentiation with respect to $w^*$</td>
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Chapter 1

INTRODUCTION

Wireless communications is the fastest growing segment of the communications industry. A wireless network can be broadly defined as a collection of nodes that communicate with each other over wireless links. This definition encompasses a variety of networks such as sensor networks (SNs) and wireless mesh networks (WMNs). These networks have been emerging from envisioned ideas to concrete technologies meeting their application specific tasks. It is predicted that their growth will be as rapid as cellular communications, which in the past five decades, has witnessed an exponential increase in users currently amounting to two billion worldwide [1]. Both SNs and WMNs can be established as local area networks (LANs) and wide area networks (WANs). The interest of this thesis lies in SNs and WMNs that serve as WANs. This chapter provides a brief overview of the developments and applications of SNs and WMNs in the WAN domain, and addresses the challenges of providing higher data rates, larger network coverage and increased network capacity due to ever growing use demands.

1.1 Wireless Sensor Networks

A sensor network comprises of a large number of sensor nodes. The sensor nodes are widely used for tracking and monitoring purposes. Their applications
Section 1.1. Wireless Sensor Networks

Figure 1.1: A wireless sensor network environment.

are wide ranged from environmental, medical and home to military. Sensors come in different shapes and sizes, specific to their application. Their wireless capability is one of the features that sensors have in common. This feature enables ease of deployment either inside the phenomenon to be monitored or very close to it [2]. As shown in Figure 1.1, networks are formed by sensor nodes communicating to pass information they sense, via multihops, to a central processing unit for a decision to be made. The small sized sensors can only broadcast to short distances due to limited power and propagation losses between the transmitting and receiving sensors. In environments that require extensive area coverage, the use of sensors may be expensive. Therefore, whilst increasing the number sensors to achieve a larger network coverage and higher capacity with lower transmission power seems feasible, it is economically unattractive. One way to deal with this problem is to
pass the transmitted signal through one or more relay nodes. This option is attractive for at least two reasons. First, relay channels add spatial diversity, which helps combat the fading effect of wireless links, increases capacity and provides higher data rates. Secondly, relays can help overcome obstacles such as buildings in urban environments as well as terrain obstacles such as mountains in rural areas where SNs could be in deployment. Therefore the use of relays to improve the performance of SNs in WANs is considered.

1.2 Wireless Mesh Networks

WMNs are self organizing and self configuring, with the nodes automatically establishing an ad hoc network that maintains the mesh connectivity. It is a fully wireless network that employs multihop communications to forward traffic en route to and from wired internet entry points (also known as access points). As shown in Figure 1.2 the network architecture employs wireless
Section 1.3. Cooperative Relay Networks

mesh routers that communicate with each other providing wireless transport services (or a wireless backbone) to data traveling from users to either other users or access points. Data emerging from the access points (connected to the internet) are directed to the nomadic user nodes through wireless mesh routers. The routers also move data from the nomadic users to the access points, thereby providing a bridge between the access points and the users. With increasingly congested frequency bands and an ever-growing demand for higher data rates, the challenges are to increase the spectral efficiency over wireless links, that have to be met with optimal use of transmission power. Higher data rates, increased network capacity and network coverage translate to improved quality of services (QoS) such as bit error rate (BER) and signal-to-interference plus noise ratios (SINRs). This thesis proposes the use of relays and develops novel algorithms to exploit their ability to provide spatial diversity to enhance such QoS.

1.3 Cooperative Relay Networks

Relays forward information received from the source nodes to destination nodes. The relay channel was introduced by van der Mulen [3] and its analysis from an information theory point of view has been studied by Thomas Cover and Abbas A. El Gamal [4]. Relays can be used to resolve the problem of attenuation faced by power limited sensors in SNs, and mesh routers in WMNs, and thereby increase network coverage with minimal sensor and mesh router usage. For example, to reduce the number of sensors while combating channel fading, their deployment could be restricted to sensing purposes only. Relays dedicated to signal forwarding can be introduced into the network for this purpose. This would prevent sensors from being used for relaying purposes, thereby reducing the number of sensors being
deployed. Furthermore, since the data rate reduces with transmission distance, to meet the challenges of higher data rates in WMNs, relaying networks can be formed by adding dedicated relays that have the potential to provide spatial diversity in wireless links [5–7]. In a WMN, relays also prove to be beneficial in providing access to mesh router terminals that are hidden in valleys created by buildings in metropolitan environments, or are being shadowed by buildings in their vicinity. The low cost and low power relays can be readily deployed within existing WMNs because of the absence of a wired connection to interface with the existing infrastructure.

The thesis studies the application of relay networks within a WMN and SN framework, and proposes the use of cooperative relays to exploit spatial diversity to forward signals in SNs and WMNs. The utilisation of cooperation in relay networks to provide enhanced QoS with higher power efficiency is advocated. Cooperation, in this sense, implies exchange of signals between cooperating relays to forward data. Relays could cooperate amongst themselves to forward data between source and destination nodes, thereby reducing the number of sensors and mesh routers required to increase capacity and network coverage, and provide better QoS with minimal power consumption. Novel signal forwarding schemes have to be developed to achieve this. Alternatively these signal forwarding schemes can be used by sensor nodes and mesh routers themselves to relay signals from their neighboring nodes with efficient power consumption. In either case, algorithms need to be developed to relay information from source to destination nodes with minimum power consumption.
1.4 Thesis Outline

The thesis proposes various signal forwarding techniques using cooperative relays for SNs and WMNs. To begin with, a literature survey on the existing diversity techniques has been provided in Chapter 2. The aim is to lay a foundation justifying the work in the preceding chapters. The problem of channel fading and the various techniques developed in the literature to overcome the fading problem has been discussed.

A comprehensive background theory on convex optimisation, covering the essential concepts used in the development of algorithms in Chapters 5 and 6, is provided in Chapter 3. Chapter 4 develops a signal forwarding scheme for a sensor network using a minimum mean square error (MMSE) approach. The algorithm is designed to achieve MMSE at the destination nodes for a given power budget at the relays. The optimisation problem is formulated such that it yields a closed form solution that is solved using Lagrangian optimisation techniques. A novel method is proposed to reduce the complexity of solving this problem. In Chapter 5 an algorithm that is capable of specifying QoS, in terms of SINR at the destination nodes, using a set of cooperative relays has been developed. The scheme minimises total transmission power at the relaying terminals whilst satisfying the constraints on target SINR at the destination nodes. The optimisation problem is configured into a semidefinite program form that is solved using interior point methods. Since the cooperating relays exchange information amongst themselves to forward signals from the source nodes to destination nodes, quantisation of signals passed between relays is considered. A robust relaying scheme is proposed to counteract uncertainties in the channel estimates at the relays. A reduced complexity scheme is proposed in Chapter 6 based on uplink-downlink
beamforming decomposition techniques. It is shown that the proposed algorithm provides an identical performance to the semidefinite programming scheme proposed in Chapter 5. An extension is made so that relays can use their beamforming capability to steer signals towards secondary users and minimise interference leakage to primary users as in a cognitive radio network.

Conclusions are drawn and the future of cooperative relaying techniques is discussed in Chapter 7.
Signal fading in multipath propagation is a prevalent phenomenon in wireless networks. Propagation losses due to signal attenuation and shadowing lead to severe channel impairments that can be mitigated through the use of diversity. The underlying idea behind diversity is to provide multiple copies of the same signal at the receiver so that the probability of multiple copies experiencing deep fade is low. Diversity can be provided in domains of time, frequency and space, which define the type of diversity scheme employed. Diversity techniques can be grouped into two categories, depending on the type of propagation losses they aim to overcome. Macrodiversity techniques aim to mitigate losses caused by signal attenuation due to shadowing from buildings and objects. Microdiversity techniques aim to mitigate losses due to multipath fading.

The structure of this chapter is as follows. Initially we make the reader familiar with the types of existing diversity techniques paving the way for cooperative diversity techniques using relays. To begin with the types of fading experienced in a communication channel are discussed and a list of fading models is provided. Having done this, the three types of diversity schemes are covered with an emphasis on spatial diversity. Since work in
this thesis does not exploit diversity in time and frequency, details on coding techniques that exploit these diversity schemes are not provided. In tune with the contribution of the thesis, the chapter instead discusses techniques providing spatial diversity. Discussions regarding spatial diversity schemes begin with introducing the multiple-input multiple-output (MIMO) model. Receiver and transmitter diversity schemes are then discussed and techniques implementing them are covered. The section following this discussion is on user cooperation diversity. At this instance it is envisaged that the ongoing analysis will provide adequate grounds for employing relays in wireless networks and motivate the utilisation of cooperative relay networks for performance enhancement in SNs and WMNs. This will be discussed in the final section of the chapter.

2.1 Fading Channels

In wireless communications, the power of the transmitted signal decreases as the signal propagates over a certain medium. The deviation or attenuation can be modeled as a random process that may vary with time, frequency or geographical location. This attenuation may be due to multipath propagation, referred to as multipath induced fading, or due to shadowing (diffraction) from obstacles affecting the wave propagation, sometimes referred to as shadow fading.

Due to obstacles in the medium of propagation that tend to reflect a transmitted signal, multiple signals from multiple paths are received at the receiver. The receiver therefore receives a superposition of multiple copies of the transmitted signal, each traveling on a different path, experiencing different attenuation, delay and phase shift. The result can be either constructive or destructive and it is the destructive interference, that is frequently referred to
as a deep fade, which may result in temporary failure of communication due to a severe drop in the channel signal-to-noise ratio (SNR) at the receiver. The rate at which the magnitude and phase of the signal changes describes its slow or fast fading nature. In slow fading channels, the time required by the magnitude change in the channel to become decorrelated from its previous value is large compared to the symbol duration. In a fast fading channel, this time is relatively small compared to the symbol duration. The amplitude and phase change imposed by the channel in a slow fading channel can be considered roughly constant over the period of use and the channels are often modeled as time-invariant over a number of symbol intervals.

If the bandwidth of the transmitted signal is small compared with the coherence bandwidth (the range of frequencies over which the channel has a constant magnitude), then all frequency components of the signal would roughly undergo the same degree of fading. The channel is then classified as frequency non-selective (also called flat fading). Since the relationship between coherence bandwidth and coherence time is reciprocal, similar as the relationship between bandwidth and symbol duration, in a frequency non-selective channel, the symbol duration is large compared with coherence time. In this case, delays between different paths are relatively small with respect to the symbol duration. Therefore the receipt of only one copy of the signal, whose gain and phase are actually determined by the superposition of all those copies that come within its duration, is assumed. On the other hand, if the bandwidth of the transmitted signal is large compared with the coherence bandwidth, then different frequency components of the signal (that differ by more than coherence bandwidth) would undergo different degrees of fading. The channel is then classified as frequency selective. Due to the reciprocal relationships, the symbol duration is small compared with the coherence time. Therefore
delays between different paths can be relatively large with respect to the symbol duration and the receipt of multiple copies of the signal is assumed [8].

Various fading models have been developed to accurately characterize the channel fading [9]. These include:

- **Rayleigh fading**
  
  This fading model is most applicable when there is no dominant propagation along the line-of-sight (LOS) between the transmitter and the receiver. A reasonable model that can be used in heavily built up environments, where there is no LOS as well as in environments where tropospheric and ionospheric scattering take place. In such environments, the scatter is sufficient to model the channel response by the Gaussian process. The channel fading amplitude, denoted \( \alpha \), has a probability distribution function that can be expressed as

  \[
  p(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right), \quad \alpha \geq 0 \tag{2.1.1}
  \]

  where \( \Omega \) is the average fading power [9].

- **Rician fading**

  When one of the paths, typically a LOS signal, is much stronger than the others, the Rice distribution model is used. The distribution is also known as Nakagami-\( n \) distribution, and can be expressed as

  \[
  p(\alpha) = \frac{2(1 + n^2)}{\Omega} \exp\left(-\frac{(1 + n^2)\alpha^2}{\Omega}\right) \times \left| \frac{1 + n^2}{\Omega} \right|^{\frac{(1 + n^2)}{2}} I_0\left(2n\alpha\sqrt{\frac{1 + n^2}{\Omega}}\right) \quad \alpha \geq 0 \tag{2.1.2}
  \]

  where \( I_0 \) is the modified Bessel function of the first kind and \( n \) is the
Nakagami-\(n\) fading parameter, which ranges from 0 to \(\infty\). The factor \(n^2\) corresponds to the ratio of the power of the LOS component to the average power of the scattered component. When this factor is zero, the Rician distribution is identical to the Rayleigh distribution [9].

- Log-normal shadow fading

Empirical measurements reveal that shadowing can be modeled by a log-normal distribution for various outdoor and indoors measurements environments. The probability distribution function can be expressed by the standard log-normal expression as

\[
p(\alpha) = \frac{4.34}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(10\log_{10}\alpha - \mu)^2}{2\sigma^2}\right]
\]  

(2.1.3)

where \(\mu\) and \(\sigma\) are mean and standard deviation of \(10\log_{10}\alpha\) respectively [9], [10].

- Nakagami-\(m\) fading

This distribution closely models land and indoor mobile multipath propagation as well as ionospheric radio links. The distribution can be expressed as

\[
p(\alpha) = \frac{2m^m\alpha^{2m-1}}{\Omega^m\Gamma(m)} \exp\left(-\frac{m\alpha}{\Omega}\right), \quad \alpha \geq 0
\]  

(2.1.4)

where \(\Gamma(\cdot)\) is the gamma function and \(m\) is the Nakagami-\(m\) fading parameter ranging from \(1/2\) to \(\infty\). The Nakagami-\(m\) distribution includes the Rayleigh distribution (\(m = 1\)) and converges to the AWGN channel as \(m \rightarrow \infty\) [9].

Simulations in Chapters 4, 5 and 6 of this thesis have been drawn for Rayleigh fading channels. The algorithms developed can be used in other fading
environments as well with appropriate modifications. Generally, to mitigate channel fading, various diversity techniques can be used as discussed in the following sections.

2.2 Types of Diversity

As mentioned earlier, diversity relies on the ability of generating multiple, independent, streams of data. There are many ways to accomplish this in terms of time, frequency and space. The section to follow gives the reader a brief insight into time and frequency diversity techniques and a detailed description of spatial diversity techniques.

2.3 Time Diversity

Time diversity is used in digital communication systems to combat error bursts due to time-varying channel conditions. The scheme involves transmission of the same information over the same channel at different times. The idea behind successive transmissions is that the probability of successive bursts experiencing deep fade will be low. Error bursts may be caused by fading in combination with a moving receiver, transmitter or obstacle, or by intermittent electromagnetic interference, for example from crosstalk in a cable, or co-channel interference from radio transmitters. A redundant error code may also be added to overcome error bursts as shown in Figure 2.1. However, since most codes designed for AWGN channels cannot correct for long bursts of errors exhibited in fading channels, coding is typically combined with bit interleaving to minimise the effect of error bursts. The interleaver spreads the error bursts due to deep fades over large code words. This reduces the number of simultaneous symbol errors which, after interleaving,
Section 2.4 Frequency Diversity

Fading Channel noise
Retrieved signal

Figure 2.1: Diagram showing time diversity scheme.

Research into mitigating fading have led to the development of various coding techniques that utilise time diversity \([1, 11, 12]\). Popular coding techniques for wireless communications include block coding, convolution coding, Trellis coding and Turbo coding and each of these coding techniques has its specific interleaver. Details of interleaving techniques and coding techniques have been discussed in \([8]\) and \([13]\).

2.4 Frequency Diversity

Frequency diversity can be used to combat frequency selective fading. Transmission of signals in frequency diversity schemes take place over varying frequencies. That is, the same information bearing signal is transmitted on different carrier frequencies, where the separation between successive carriers equals or exceeds the coherence bandwidth of the channel. This is done so that different copies of the signal experience independent fading. The transmission of signal over multiple frequency bands requires additional power, however, since the likelihood that the signals will suffer the same level of attenuation at different frequencies is low, the BER of the system is lowered. At the receiver, the independent fading copies are optimally combined to yield a decision of the transmitted symbol. The optimal combiner at the receiver is the maximum ratio combiner, which will be discussed later in the chapter. Frequency diversity can be exploited using orthogonal frequency division multiplexing access \([14]\) and frequency-hopping spread spectrum techniques.
Section 2.5. Space Diversity—the MIMO model

Space or antenna diversity relies on multiple transmit or receive antennas, also called antenna arrays, to achieve independent fading paths in wireless systems. The technique generates independent fading signal paths exploiting the low probability of different paths experiencing deep fades simultaneously. It is therefore especially effective in mitigating multipath fading. Antenna diversity can be realized in several ways. Depending on the environment and the expected interference, designers can employ one or more of these methods to improve the signal quality. Spatial diversity techniques in wireless systems are discussed in this section.

In spatial diversity, multiple antennas are used to increase data rates through multiplexing or to improve radio link layer performance through diversity. Initial interest in multiple antennas at the transmitter and receiver, commonly referred as MIMO, was sparked by Winters [15], Foschini [16], Foschini and Gans [17] and Telatar [18,19]. The technology has attracted attention in wireless communications, since it offers significant increases in data throughput and link range without additional bandwidth or transmit power.

A point-to-point MIMO system is shown in Figure 2.2. The system can be represented by the following discrete-time model

\[
\begin{bmatrix}
    y_1(n) \\
    \vdots \\
    y_M(n)
\end{bmatrix}
= 
\begin{bmatrix}
    h_{11} & \cdots & h_{1M_r} \\
    \vdots & \ddots & \vdots \\
    h_{M_t1} & \cdots & h_{M_tM_r}
\end{bmatrix}
\begin{bmatrix}
    s_1(n) \\
    \vdots \\
    s_{M_t}(n)
\end{bmatrix}
+ 
\begin{bmatrix}
    v_1(n) \\
    \vdots \\
    v_{M_r}(n)
\end{bmatrix}
\]

(2.5.1)
or simply as \( y(n) = \mathbf{H}s(n) + \mathbf{v}(n) \). In the equation shown, \( \mathbf{H} \) is the \( M_r \times M_t \) matrix of channel gains where \( h_{i,j} \) represents the gain from the \( i^{th} \) transmitter antenna to the \( j^{th} \) receiver antenna. The transmitted symbol vector is denoted by \( s(n) \), \( y(n) \) is the received signal vector and \( \mathbf{v}(n) \) is the noise vector at the receiver antenna. Provided \( \mathbf{H} \) is non-singular the transmitted symbol can be obtained at the receiver nodes by inverting the channel matrix

\[
\mathbf{s}(n) = \mathbf{H}^{-1}\mathbf{y}(n) = \mathbf{H}^{-1}(\mathbf{H}\mathbf{y}(n) + \mathbf{v}(n)).
\]

(2.5.2)

However, methods based on MMSE receiver and maximum likelihood estimator would provide a better performance. Knowledge of the channel gains can be assumed at the transmitter or receiver. To facilitate this, a pilot signal can be sent from the transmitter to estimate the channels at the receiver. For channels to be known at the transmitter, a feedback loop should exist between the receiver and the transmitter. A mismatch between presumed
and the actual channel state information (CSI) could arise due to feedback errors and the non-stationarity of the environment, i.e. by the time the CSI is used at the relays, the true channels could have changed. A discrepancy between the known and the actual CSI yields to loss in system performance. Robust schemes that are insensitive to such errors in channel estimates have been developed, for example in [20–23], to provide better system performance despite the presence of errors in channel estimates.

### 2.5.1 Receiver Diversity Techniques

In receiver space diversity, the independent fading paths at multiple receive antennas are combined to obtain a signal that is passed into a demodulator. The combining can be performed in several ways that entail various tradeoffs in complexity and overall performance. Most combining methods are linear where the output of the combiner is a weighted sum of different branches of the receiver as shown in Figure 2.3. In this figure, \( r_i e^{j \theta_i} s(n) \) denotes the signal received at the \( i^{th} \) antenna, where \( s(t) \) is the transmitted source signal, \( \theta_i \) is the phase angle and \( r_i \) is the gain of the signal. The noise component is not shown. To enhance the signal power, the signals from all the branches need to be added coherently. Combining more than one path requires co-phasing. This is where the phase of the \( i^{th} \) branch is nulled through multiplication by an appropriate complex coefficient \( \alpha_i = a_i e^{-j \theta_i} \) for some real valued \( a_i \) depending on combining techniques used. The signal output from the combiner is the product of the original transmitted signal and the complex amplitude term \( \alpha \Sigma = \sum_i a_i r_i \), i.e.

\[
\sum_{i=1}^{M_T} \alpha_i r_i e^{j \theta_i} s(n) = \alpha \Sigma s(n).
\]
At the combiner output, the SNR achieved has a distribution that is a function of the number of diversity paths, the fading distribution on each path and the combining technique. The combining technique yielding the highest array gain is the maximal-ratio combiner (MRC).

Maximal-Ratio Combining

Assuming the same noise power, $\sigma_n^2$, on each branch, the total noise at the combiner output will be given by $N_{\text{tot}} = \sum_{i=1}^{M} a_i^2 \sigma_n^2$. Therefore, the SNR at the output of the combiner will be

$$
\gamma_b = \frac{(\sum_{i=1}^{M} a_i r_i)^2 \sigma_s^2}{\sum_{i=1}^{M} a_i^2 \sigma_n^2} \tag{2.5.4}
$$

where $\sigma_s^2$ is the source signal power. The weights of the combiner are designed such that the resulting SNR is maximised. Using partial derivatives
and the Cauchy-Schwarz inequality, the optimal weights turn out as $a_i = r_i$, so that the resulting SNR at the combiner is given by

$$\gamma_S = \sum_{i=1}^{M} r_i^2 \frac{\sigma_i^2}{\sigma_n^2}.$$  \hspace{1cm} (2.5.5)

From the above equation, the average combiner SNR increases linearly with the number of diversity branches $M$ [1].

**Selection Combining**

In selection combining, the receiver selects the branch with the largest SNR, given as $r_i^2 \frac{\sigma_i^2}{\sigma_n^2}$. The output of the combiner therefore has an SNR that is the maximum of the SNR of all the branches, i.e. $\max_i r_i^2 \frac{\sigma_i^2}{\sigma_n^2}$. Since the use of only one branch is made at a time, often only one receiver is required to switch into an active antenna. Simultaneous monitoring of SNR however requires a dedicated receiver at each antenna. In his case also the average SNR gain increases with $M$. This increase is not linear and in general, increasing $M$ yields diminishing returns in terms of the array gain [1].

**Threshold Combining**

For systems that transmit continuously, selection combining may require a dedicated receiver on each branch to continuously monitor the branch SNR. To avoid the use of a dedicated receiver for each branch, threshold combining sequentially scans each branch and outputs the first signal whose SNR is above a given threshold, $\gamma_T$. Since only one branch output is used at one time, co-phasing is not required.
2.5.2 Transmit Diversity Techniques

The multiple antennas in a transmit diversity scheme are co-located at the transmitter. It is generally more difficult to exploit transmitter diversity than receiver diversity. This is partly due to the transmitter having no or partial knowledge about the channel. The design is similar to receiver diversity when the channels are known at the transmitter. Without the knowledge of the channels, transmit diversity requires combination of space and time diversity via a novel technique called the Alamouti scheme [24]. A comprehensive information-theoretic treatment for many of the transmit diversity schemes in frequency, time and space domains that have been studied is presented in [25]. The channel information known at the transmitter is considered first.

Channel Known at the Transmitter

A transmit diversity system with $M_t$ transmit antennas and one receive antenna is considered. Assuming a path gain of $r_{m}e^{-j\theta_{m}}$ from the $m^{th}$ transmitter antenna is known at the transmitter, the signal transmitted from this antenna is co-phased by the multiplication with a complex gain $\alpha_{m} = a_{m}e^{j\theta_{m}}$ prior to transmission. To restrict the average total transmission energy, the weights must satisfy $\sum_{m=1}^{M} a_{m}^{2} = 1$. At the receiver the signals add up to yield

$$r(n) = \sum_{m=1}^{M} a_{m}r_{m}s(n)$$

(2.5.6)

where $s(n)$ is the transmitted signal at the $n^{th}$ time index. To maximise the received SNR, the weights $a_{m}$ can be expressed as

$$a_{m} = \frac{r_{m}}{\sqrt{\sum_{m=1}^{M}r_{m}^{2}}}$$

(2.5.7)
resulting in an SNR of

\[ \gamma_\sigma = \frac{\sigma_s^2}{\sigma_n^2} \sum_{m=1}^{M} r_m^2 = \sum_{m=1}^{M} \gamma_m \]  

(2.5.8)

where \( \gamma_m = r_m^2 \frac{\sigma_s^2}{\sigma_n^2} \) is the SNR between the \( m^{th} \) transmit antenna and the receive antenna, \( \sigma_s^2 \) and \( \sigma_n^2 \) are the source signal power and noise power respectively. Hence it is noticed that the received SNR is the sum of SNRs on each of the individual branches. On the occasion of all antennas having the same gain, \( r_m = r \), the SNR is given as, \( \gamma_\sigma = M r^2 \frac{\sigma_s^2}{\sigma_n^2} \), thereby an array gain of \( M \)-fold increase in SNR over a single antenna transmitting with the full power is achieved.

The Alamouti scheme

When the channel is not known at the transmitter, a simple but ingenious technique- the Alamouti scheme- can be used to attain full diversity [24]. As shown in Figure 2.4, in the first phase two different symbols \( s_1 \) and \( s_2 \) are transmitted simultaneously from antennas 1 and 2 respectively. In the next symbol period \(-s_2^*\) and \( s_1^* \) are transmitted from antennas 1 and 2 respectively. Assuming the channel is frequency flat and remains constant
over the two symbol periods, the signals $y_1$ and $y_2$ received over the two symbols, expressed in a vector form $\mathbf{y}$, can be expressed as

$$\mathbf{y} = \mathbf{H}_{\text{eff}} \mathbf{s} + \mathbf{v}, \quad (2.5.9)$$

where $\mathbf{s} = [s_1 \ s_2]^T$ and $\mathbf{v} = [v_1 \ v_2]$ is the AWGN vector at the receiver associated with the transmitted symbol. The effective channel matrix

$$\mathbf{H}_{\text{eff}} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}, \quad (2.5.10)$$

is orthogonal (i.e., $\mathbf{H}_{\text{eff}}^H \mathbf{H}_{\text{eff}} = ||\mathbf{h}||_2^2 \mathbf{I}_2$). If $\mathbf{z} = \mathbf{H}_{\text{eff}}^H \mathbf{y}$, then

$$\mathbf{z} = ||\mathbf{h}||_2^2 \mathbf{I}_2 \mathbf{s} + \tilde{\mathbf{v}}, \quad (2.5.11)$$

is obtained, where $E\{\tilde{\mathbf{v}}\} = \mathbf{0}_{2,1}$ and $E\{\tilde{\mathbf{v}}\tilde{\mathbf{v}}^H\} = ||\mathbf{h}||_2^2 \sigma_n^2 \mathbf{I}_2$. Hence, the effective channel for symbols $s_i$ ($i = 1, 2$) becomes

$$z_i = ||\mathbf{h}||_2^2 s_i + \tilde{v}_i, \quad i = 1, 2, \quad (2.5.12)$$

and the receiver SNR, per symbol, is given by

$$\gamma_{\text{SNR}} = \frac{||\mathbf{h}||_2^2 \sigma_s^2}{\sigma_n^2}, \quad (2.5.13)$$

The received SNR is therefore equal to the sum of SNRs on each branch. From (2.5.12), the Alamouti scheme achieves a diversity order of two, the maximum possible for a two-antenna transmit system, despite the fact that channel knowledge is not available at the transmitter.
**2.5.3 Multiuser Beamforming Techniques**

Beamforming is a classical method of processing temporal sensor array measurements for signal estimation, interference cancelation, source direction, and spectrum estimation. It has ubiquitously been applied in areas such as radar, sonar, wireless communications, speech processing, and medical imaging (see, for example, [26–30] and the references therein.) The basic principle in the use of an antenna array is to multiply the signal at the different antenna branches with complex weight factors, either before the signal is transmitted or before the received signals are summed. The setup can be regarded as a spatial filter, the signals at different antenna branches representing spatial samples of radio channels, with complex weight factors as filter coefficients. Where as receiver diversity techniques increase the received SNR and mitigate deep fades, beamformers are applied when there is signal coherence between the signals at the antenna elements, so that a narrow beam is created towards the desired user [31]. The standard data-independent beamformers include the delay-and-sum approach as well as methods to control sidelobes based on weight vectors. Adaptive or data-dependent beamformers have a better interference rejection capability since the beamformers select the weight vectors as a function of data to optimise the performance subject to various constraints [20].

To obtain a better understanding of the works in Chapters 5 and 6, receiver and transmitter beamforming methods are discussed in the subsections to follow. Here the concepts of uplink and downlink beamforming is introduced in the context of multiuser wireless communications, and appropriate references for a detailed understanding of their development are cited.
Figure 2.5: Uplink beamforming in a mobile communication network.

**Uplink Beamforming**

Figure 2.5 depicts the uplink beamforming in a mobile communication network. These are $K$ users transmitting independent information from spatially different locations. $K$ number of beamformers are required at the basestation to retrieve each source. The output signal from the $k^{th}$ beamformer can be expressed as

$$y_k(n) = w_k^H x(n)$$  \hspace{1cm} (2.5.14)

where $n$ is the time index, $x(n) = [x_1(n), \ldots, x_M(n)]^T \in \mathbb{C}^{M \times 1}$ is the complex vector of array observations, $w_k = [w_{k,1}, \ldots, w_{k,M}]^T \in \mathbb{C}^{M \times 1}$ is the complex vector beamformer weights for the $k^{th}$ user and $M$ is the number of antennas at the receiver. The array observation vector received
from the transmitting sources is given by

\[ x(n) = h_k s_k(n) + \sum_{l=1, l \neq k}^{K} h_l s_l(n) + v(n) \]  \hspace{1cm} (2.5.15)

where \( h_k \) and \( h_l \) are the respective channels from the desired (assumed to be the \( k^{th} \) source) and interfering users to the array receiver respectively and \( v(n) \) is the noise present at the receiving array. The optimal beamforming weight vectors can be obtained by maximizing the received SINR of each user. This can be expressed as

\[ \text{SINR}_k = \frac{\mathbf{w}_k^H \mathbf{R}_s \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{R}_{i+n} \mathbf{w}_k} \]  \hspace{1cm} (2.5.16)

where \( \mathbf{R}_s \triangleq \mathbb{E}\{(h_k s_k)(h_k s_k)^H\} \) and \( \mathbf{R}_{i+n} \triangleq \mathbb{E}\{(h_l s_l + v)(h_l s_l + v)^H\} \) are \( M \times M \) signal and interference-plus-noise matrices respectively, and \( \mathbb{E}\{\cdot\} \) denotes the statistical expectation. The maximisation of the SINR in equation (2.5.16) is equivalent to maintaining a distortionless response to the desired signal while minimizing the output interference-plus-noise power, i.e.

\[ \min_{\mathbf{w}_k} \quad \mathbf{w}_k^H \mathbf{R}_{i+n} \mathbf{w}_k \]  \hspace{1cm} (2.5.17)

\[ \text{s.t.} \quad \mathbf{w}_k^H \mathbf{R}_s \mathbf{w}_k = 1. \]

The solution to the problem (2.5.17) can be found by maximizing the infimum of the Lagrangian function

\[ L(\mathbf{w}_k, \lambda) = \mathbf{w}_k^H \mathbf{R}_{i+n} \mathbf{w}_k + \lambda(1 - \mathbf{w}_k^H \mathbf{R}_s \mathbf{w}_k). \]  \hspace{1cm} (2.5.18)

The infimum is given by

\[ \nabla_{\mathbf{w}} L(\mathbf{w}_k, \lambda) = \mathbf{R}_{i+n} \mathbf{w}_k - \lambda \mathbf{R}_s \mathbf{w}_k = 0 \]  \hspace{1cm} (2.5.19)
which can be expressed as

\[ R_{-1}^{-1} R_s w_k = \frac{1}{\lambda} w_k. \tag{2.5.20} \]

The function is maximised by the eigenvector corresponding to the largest eigenvalue of matrix \( R_{-1}^{-1} R_s \). The resulting weight has to be normalized to satisfy the constraint \( w^H_{\text{opt}} R_s w_{\text{opt}} = 1 \) [32]. This solution is also known as solution of generalized eigenvalue problem.

**Downlink Beamforming**

In the downlink beamforming scenario, if the signal transmitted at the antenna array is \( x(n) \), the received signal at the \( k^{th} \) mobile is given by

\[ r_k(n) = h_k^T x(n) + v_k(n) \tag{2.5.21} \]

where \( h_k \) is the complex Gaussian channel between the basestation arrays and the \( k^{th} \) receiver with \( x \in N(0, R_x) \) and \( v_k(n) \) is the AWGN at the \( k^{th} \) receiver with variance of \( \sigma_{v_k}^2 \). Beamforming weight vectors can be designed to achieve pre-determined SINRs, \( \gamma_n = w_k^H R_s w_k / w_k^H R_{-1} + n w_k \), at each receiver for a minimum possible transmission power at the basestation. This can be implemented by imposing constraints that require the SINR to be above a certain threshold at each receiver, while minimizing the total power transmitted, \( P_{\text{tot.}} = \sum_{k=1}^{K} w_k^H w_k \), from the basestation, where \( w_k \in \mathbb{C}^{M \times 1} \) is the downlink beamforming weight vector for the \( k^{th} \) user and \( K \) is the total number of users [20] [33]. The optimisation framework can be
expressed as

\[
\min_w \sum_{k=1}^{K} w_k^H w_k
\]

s.t. \( \gamma_n (w_k^H R_{k+n} w_k) - w_k^H R_n w_k \leq 0, \forall k = 1, \ldots, K \)  

(2.5.22)

can be solved using convex optimisation techniques by applying interior point method algorithms [34]. Alternatively an SINR maximisation scenario similar to uplink beamforming can be derived to design beamforming weight vectors that achieve maximum signal power at desired users while keeping the total power transmitted to all the users below a certain threshold. The details of this optimisation framework can be found in [35].

The discussion on spatial diversity techniques is concluded by stressing that colocated antennas at transmitters and receivers are used to achieve gains in diversity. The application of uplink and downlink beamforming techniques in cooperative relay networks is covered in Chapter 6 of the thesis.

Prior to discussing cooperative relay networks, a brief introduction to cooperative user diversity is provided. The aim of the section is to draw the reader to the concept of user assisted spatial diversity, from which the idea of utilizing cooperation amongst relays in a relay network to forward signals from source to destination nodes emerged.

### 2.6 User Cooperation Diversity

It is predicted that the fourth generation of wireless mobile communication network would offer a vast range and diversity of converged devices, services and networks revolutionizing the communication industry. The requirement of ultra-high data rates and efficient utilisation of frequency bands would be essential for such networks to exist. This opens the challenge of providing
Section 2.6. User Cooperation Diversity

The imminent need for spectral efficiency in communication networks [36, 37]. The exploitation of spatial diversity for this purpose seems an attractive opportunity, however it faces an inherent problem of having only one receiving antennas in a mobile handset. The feature of using colocated antennas for spatial diversity, in the uplink scenario, poses a problem since the transmitting devices are not capable of accommodating multiple antenna without avoiding large correlations of signals that are transmitted or received due to their small size. Therefore, despite multiple antenna being desirable in providing spatial diversity, their use becomes impractical. In order to overcome this limitation, yet still emulate diversity techniques such as uplink-downlink beamforming and transmit-receive diversity techniques, a new form of spatial diversity was proposed where users cooperate to achieve diversity gains [38].

The idea was to let spatially adjacent mobile terminals communicate with each other pooling their resources to form a virtual antenna array (VAA) that realize spatial diversity gains in a distributed fashion. This introduced the concept of VAA that emulates the MIMO channel transmitter and receiver, providing spatial diversity to the intended user. In the cellular communication network, the information is encoded at the base station as if there was a MIMO channel available with multiple antennas at the receiver. Information is then passed to the intended user by the mobile units in its vicinity. These received mobiles play the role of VAAs providing multiple paths to the intended receiver [39]. Therefore an increase in link QoS in terms of BER versus SNR, similarly to multiple colocated antenna in spatial diversity techniques discussed above, is achieved.

In user cooperation diversity, the users tend to relay each others information thereby providing multiple paths from source to the destination. Various
cooperative schemes have been presented in literature. Examples of cooperative communications include distributed beamforming where relay nodes cooperate to direct a beam towards the receiver under individual relay power constraints [40]. The work in [41] considers the problem of distributive beamforming under the assumption of second order statistics of the channel coefficients being available at the relays. This assumption allows a system that is robust to the uncertainties in the channel model to be designed. 

The use of distributed antennas in lieu of colocated ones opens scope for research into networks utilizing multihop signal forwarding protocols. A three-node network is considered in [42] where one node relays the message of another node towards the third one. The ergodic and outage capacity of various protocols are studied and their spatial diversity performance is analyzed. In [43], low complexity, half-duplex relaying protocols such as fixed, selection and incremental relaying protocols are discussed. Under the fixed relaying scheme the amplify-and-forward scheme, inspired by [4, 44, 45], and the decode-and-forward scheme, introduced by [38] are examined. It is also shown that for channels with multiple relays, cooperative diversity with appropriately designed codes realizes full spatial diversity gain. Various decode-and-forward and compress-and-forward relaying topologies are studied in [46]. Strategies for multi access relay channels and broadcast relay channels are developed and bounds on their capacity and achievable rates are drawn. The capacity of the relay networks are studied from an information theoretic point of view in [4, 47, 48]. The implementation aspects of user cooperation diversity is investigated in [49]. 

Based on the user cooperation diversity network the utilisation of cooperation amongst relays, in the sense that the relays exchange signals amongst themselves, to forward signals from the source to destination points is proposed. The
final section of the chapter is on such cooperative relay networks, which is the central idea of the thesis.

### 2.7 Cooperative Relay Networks

User cooperation diversity using virtual antennas to improvise spatial diversity in cellular networks is considered. This has potential applications in other networks apart from cellular communication. Such cooperation is envisaged in SNs and WMNs where limited communication resources, such as battery lifetime of devices and scarce bandwidth, impose constraints on the QoS provided to the users and restrict network coverage. The limited size of wireless sensors and mesh routers refrain multiple antennas from being colocated, therefore providing antennas in form of relays appears as a good solution for implementing spatial diversity in such networks. In addition to this relays provide multihops between source and destination nodes, which combats path loss due to attenuation.

As mentioned in Chapter 1, relaying could be achieved either by dedicated signal forwarding relays or by other users in the vicinity that cooperate to forward signals of neighboring nodes. Cooperation among relays on a level where relays exchange information between themselves to forward the signal from the source to destination nodes is proposed. Therefore non-cooperative relaying, in this context, implies that relays forward signals without sharing information between themselves.

The use of cooperative relaying networks with emphasis on the savings on transmitted power to achieve enhanced QoS at the receiving terminals is proposed. While non cooperative relaying schemes provide such savings, the use of cooperation considerably enhances the reduction in power consumption. Therefore while it is feasible to do so, utilizing cooperation is an attractive
option to achieve performance gains within a tight power budget. Furthermore, the implementation of cooperative relays within an already existing wireless network, where information is transmitted over long distances between transmitting and receiving devices (such as in SNs and WMNs), can be readily achieved. The use of cooperative relays for signal forwarding based on these ideologies is proposed.

The thesis looks at the design of half-duplex, two-hop, amplify-and-forward relaying schemes to achieve QoS at the destination nodes with minimum power consumption by the relays. As it shall be seen in the preceding contribution chapters, a signal forwarding matrix is designed for this purpose. In non-cooperative relaying schemes this matrix is diagonal, where the diagonal elements of the matrix represent optimal relaying weights that the signals received by the relays are multiplied with prior to transmission to the destination nodes. The design of optimal, non-diagonal, cooperative relaying matrix requires each relay to have knowledge of the received signal at the other relays in the cooperative framework. The sharing of this information by each relay with its cooperating partners involves overheads in terms of power consumption. Chapter 5 takes into account the overheads involved and draws comparisons showing the superiority of cooperative relaying architectures over non-cooperative ones.
Chapter 3

CONVEX OPTIMISATION

THEORY

Unconstrained optimisation, such as the least squares method, and constrained optimisation such as linear programming have been around from many years and have been widely exploited. A new general class of optimisation, known as convex optimisation, has emerged in the last decade or so. Convex optimisation can be described as the fusion of three disciplines: optimisation, convex analysis and numerical computation. In recent years this computational tool has played a vital role in engineering optimisation due to its ability to solve large practical problems faced in engineering disciplines reliably and efficiently. Much of this solving ability is owed to the development of computational algorithms such as interior point methods [50]. In some sense, convex optimisation is providing new computational tools, thereby extending the ability to solve much larger and richer classes of problems [51]. Since most engineering problems are not convex when directly formulated, the challenge involved is in expressing them in a convex form that can be solved using an optimisation tool.
3.1 Why Convex?

In engineering a vast number of the design problems may be cast as a constrained optimisation problem of the form [52]

$$\begin{align*}
\min_x & \quad f_0(x) \\
\text{s.t.} & \quad f_i(x) \leq 0 \quad i = 1, \ldots, m \\
& \quad h_i(x) = 0 \quad i = 1, \ldots, p
\end{align*}$$

(3.1.1)

where $x$ is the optimisation variable of the problem, and functions $f_0, f_i$ and $h_i$ are the cost, the inequality constraints, and equality constraints respectively. There are several potential hurdles, which effectively make the problem (3.1.1) quite tedious to solve. The following impediments may arise: [52]

1. The dimension $n$ of the optimisation variable may be very large.
2. The domain of the problem may have local optima.
3. The problem might not be feasible.
4. Stopping criteria available may be arbitrary.
5. The algorithms might have poor convergence rates.

The first three problems could be easily dealt with provided $f_i$ are convex and $h_i$ are affine [53]. If this is the case, the problem is convex and any local optimum, is in fact, a global optimum; feasibility can be determined unambiguously, and an accurate stopping criterion can be obtained using the principle of duality [52]. Convergence and numerical sensitivity were potential problems, until it was shown that if $f_i$, in addition to being convex, satisfied the property of self-concordance, the problems of convergence and numerical sensitivity may be dealt with using interior point methods.
Interestingly, a large number of functions used in engineering satisfy the self-concordance property, hence they can now be solved with great efficiency.

### 3.2 Basic Optimisation Concepts

The section introduces some basic optimisation concepts that are widely used throughout the remainder of this thesis.

#### 3.2.1 Convex Sets

A set $S$ is convex if for any two points $x, y \in S$, the line segment between these two points is also in $S$, i.e.

$$
\theta x + (1 - \theta)y \in S, \quad \forall \theta \in [0, 1] \text{ and } x, y \in S. \quad (3.2.1)
$$

For example, the ball $S = \{x \mid \|x\| \leq \varepsilon\}$ is convex, however a sphere $S = \{x \mid \|x\| = \varepsilon\}$ is not, since the line segment joining any two points is no longer on the sphere. In general, convex sets have non-empty interior i.e. they must have solid body with no holes. Other examples of convex sets include ellipsoids and polyhedra.

#### 3.2.2 Convex Functions

A function $f(x) : \mathbb{R}^n \to \mathbb{R}$ is said to be convex if for any two points $x, y \in \mathbb{R}^n$ [52]

$$
f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y), \quad \forall \theta \in [0, 1]. \quad (3.2.2)
$$
The function $f$ is concave, if $-f$ is convex. The convexity of a differentiable function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ can also be characterized by its differential, $\nabla f$ and Hessian $\nabla^2 f$. The use of the following first-order condition is made: $f$ is convex if and only if for all $x, x_0 \in \text{dom} f$, $f(x) \geq f(x_0) + \nabla f(x_0)^T(x - x_0)$ i.e., the first order approximation of $f$ is a global under estimator [52]. Recalling that the Hessian of $f$, $\nabla^2 f$, yields a second order Taylor series expansion around $x_0$

$$f(x) \approx f(x_0) + \nabla f(x_0)^T(x - x_0) + \frac{1}{2}(x - x_0)^T\nabla^2 f(x_0)(x - x_0) \quad (3.2.3)$$

the following necessary and sufficient second order condition is used: a twice differentiable function $f$ is convex if and only if for all $x \in \text{dom} f$, $\nabla^2 f(x) \succeq 0$, i.e. its Hessian is positive semidefinite on its domain [52]. Thus, for example a linear function is always convex, while a quadratic function $x^T P x + a^T x + b$ is convex if and only if $P \succeq 0$.

### 3.3 Convex Optimisation Problems

A convex problem can be written in standard form as [52, 53]

$$\min_{x} \quad f_0(x)$$

s.t. $f_i(x) \leq 0 \quad i = 1, \ldots, m$ \quad (3.3.1)

$$h_i(x) = 0 \quad i = 1, \ldots, p$$

where the vector $x \in \mathbb{R}^n$ is the optimisation variable of the problem and the function $f_0$ is the objective function or cost function. The functions $f_i$, $i = 1, 2, \ldots, m$ are convex functions and the functions $h_i$, $i = 1, 2, \ldots, p$ are
linear functions. The inequalities \( f_i(x) \leq 0 \) are called the inequality constraints and equalities \( h_i(x) = 0 \) are termed as the equality constraints.

The domain of the optimisation problem (3.3.1) is defined as the set of points that satisfy the condition

\[
D = \bigcap_{i=0}^{m} \text{dom} f_i \cap \bigcap_{i=0}^{p} \text{dom} h_i.
\]

Problem (3.3.1) is said to be feasible if there exists a point \( x \in D \) that satisfies all the constraints \( f_i(x) \leq 0 \) and \( h_i(x) \), the problem is said to be non-feasible otherwise. The optimal value or the solution of the optimisation problem is achieved at the optimal point \( x^* \) if and only if it has the smallest objective among all feasible points, i.e. for any feasible point \( z \in D \), \( f_0(z) \geq f_0(x^*) \).

### 3.3.1 The Art of Using Convex Optimisation

The ability of converting non convex problems into convex ones is key to using convex optimisation for problem solving. Unfortunately, a systematic method to achieve this does not exist. There are two main ways to formulate problems into a convex form: [52, 55]

- Firstly, by using change of variables, a non-convex problem may be formulated into a convex problem which is equivalent to the original problem. For example considering the minimisation of the \( \ell_2 \)-norm of a vector, i.e. \( \min \|w\|_2 \), a change of variable, \( W = ww^H \) formulates the problem into minimizing the trace of the new variable \( W \), i.e. \( \min \text{trace}\{W\} \). This is equivalent to minimizing the \( \ell_2 \)-norm of \( w \).

---

1A function \( h : \mathbb{R}^n \rightarrow \mathbb{R} \) is linear if it satisfies \( h(\alpha x + \beta y) = \alpha h(x) + \beta h(y) \) for all \( x, y \in \mathbb{R}^n \) and all \( \alpha, \beta \in \mathbb{R} \).
• Secondly, by removing some of the constraints, a non convex problem could be relaxed into a convex one. This technique is sufficient as long as both the non-convex problem and its formulated convex problem are equivalent, i.e. they share the set of optimal solutions (related by some mapping). An example of this technique is semidefinite relaxation (SDR), where a non-convex constraint restricting the rank of the optimisation variable matrix may be dropped. As shown in the above example, the trace of matrix $W$ could be minimised, which is equivalent of minimizing the $\ell_2$-norm of the vector $w$. However, the change of variable introduces an additional non-convex constraint, $\text{rank}\{W\} = 1$, making the whole optimisation problem non-convex. Later, in the thesis, it is seen that, for the problem in consideration, despite dropping this constraint the resulting convex optimisation problem yields a rank-1 matrix. However, it should be said that this may not necessary hold for all problems.

### 3.4 Convex Optimisation Problems

In this section, the most general forms of convex optimisation problem formulations, which are extremely useful in practice are provided. Once a problem is cast into one of these forms, it can be solved using efficient software packages [56].
3.4.1 Linear Program

The simplest of these is a linear program (LP), where the objective and the constraint functions are affine. A general LP has the form

\[
\begin{align*}
\min_x & \quad c^T x + d \\
\text{s.t.} & \quad Gx \leq h \\
& \quad Ax = b
\end{align*}
\]  

(3.4.1)

where \(G \in \mathbb{R}^{m \times n}\) and \(A \in \mathbb{R}^{p \times n}\).

3.4.2 Quadratic Programming

A quadratic program (QP) is where the objective function is quadratic, and the constraint functions are affine. It has the form of

\[
\begin{align*}
\min_x & \quad x^T P x + q^T x + r \\
\text{s.t.} & \quad Gx \leq h \\
& \quad Ax = b
\end{align*}
\]  

(3.4.2)

where \(P \in \mathbb{S}_+^n\), \(G \in \mathbb{R}^{m \times n}\), and \(A \in \mathbb{R}^{p \times n}\). A QP minimises a convex quadratic function over a polyhedron. It includes the LP as a special case; which may be obtained by setting \(P = 0\) in the objective function of problem (3.4.2). A variation of QP is a quadratically constrained quadratic program (QCQP) where both the objective and the constraints are quadratic. This has the form

\[
\begin{align*}
\min_x & \quad x^T P_0 x + q_0^T x + r_0 \\
\text{s.t.} & \quad x^T P_i x + q_i^T x + r_i \leq 0, \quad i = 1, 2, \ldots, m \\
& \quad Ax = b
\end{align*}
\]  

(3.4.3)
where $P_i \in S^n_+$, $\forall i = 1, 2, \cdots, m$. In a QCQP a convex quadratic function over a feasible region obtained from the intersection of ellipsoids is minimised. Similarly to a QP, an LP is obtained by setting $P_i = 0$, $\forall i = 1, 2, \cdots, m$ in the constraints of (3.4.3).

### 3.4.3 Second Order Cone Programming

A second order cone program (SOCP) can be written as

$$
\begin{align*}
\min_{x} & \quad f^T x \\
\text{s.t.} & \quad \|A_i x + b_i\|_2 \leq c_i^T x + d_i \quad i = 1, 2, \ldots, m \\
& \quad F x = g
\end{align*}
$$

(3.4.4)

where $x \in \mathbb{R}^n$ is the optimisation variable, $A_i \in \mathbb{R}^{n_i \times n}$, and $F \in \mathbb{R}^{p \times n}$. The first constraint in (3.4.4) is known as a second order cone constraint since it requires the affine function $(Ax + b, c^T x + d)$ to lie in the second-order cone in $\mathbb{R}^{k+1}$. Setting $c_i = 0$, $\forall i = 1, 2, \ldots, m$, and squaring both sides of the constraints, a QCQP is obtained [52]. Similarly, if $A_i = 0$, $\forall i = 1, 2, \cdots, m$, the SOCP reduces to a LP [55]. SOCPs are more general than both QCQPs and LP's.

### 3.4.4 Semidefinite Programming

The most general of all the forms is a semidefinite program (SDP). This subsumes linear, quadratic and second-order cone programming. An SDP
can be written as,

$$
\min_x \quad c^T x \\
\text{s.t.} \quad x_1 F_1 + x_2 F_2 + \ldots + x_n F_n + G \preceq 0
$$

(3.4.5)

$$
Ax = b
$$

where $x \in \mathbb{R}^n$ is the optimisation variable and $G, F_0, F_1, \ldots, F_n \in \mathbb{S}^{k \times k}$ are symmetric matrices, and $A \in \mathbb{R}^{p \times n}$. The inequality constraints in (3.4.5) are also known as linear matrix inequalities (LMIs). An SDP simplifies to an LP if the matrices $G, F_1, \ldots, F_n$ are diagonal.

So far the basic structure of the most commonly used form of the canonical optimisation problem has been outlined. However, it must be noted that not all optimisation problems will have one of the above structures, namely an LP, QP, QCQP, SOCP or an SDP. This effectively means the readily available software for solving convex problems might not be useful and custom code (software) might be needed to solve the problem. In this case, the ellipsoid, subgradient or cutting plane methods, which offer exact stopping criteria and only need gradient information maybe employed. On the other hand, if the Hessian information is also available, the interior-point methods which offers faster a convergence rate maybe employed [52].

### 3.5 Duality and KKT Conditions

The Lagrangian $L : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \rightarrow \mathbb{R}$ for the original (or primal) problem in (3.3.1) is defined as the objective function augmented with a weighted sum of the constraint functions. This can be expressed as

$$
L(x, \lambda, \nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x),
$$

(3.5.1)
where \( \lambda_i \) and \( \nu_i \) are the *Lagrange multipliers* associated with the \( i \)th inequality, \( f_i(x) \leq 0 \), and equality, \( h_i(x) = 0 \), constraints respectively. The objective \( f_0(x) \) in (3.5.1) is known as the *primal objective* and the optimisation variable \( x \) is termed the *primal variable*. Lagrange multiplier vectors \( \lambda \) and \( \nu \) associated with problem (3.5.1) are known as the *dual variables*, and the *dual objective* or the *dual function* \( g(\lambda, \nu) \) defined as the minimum value of the Lagrangian over \( x \) for \( \lambda \in \mathbb{R}^m \) and \( \nu \in \mathbb{R}^p \), can be expressed as

\[
\begin{align*}
g(\lambda, \nu) &= \inf_{x \in \Omega} \left( f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x) \right). 
\end{align*}
\]

Since it is a pointwise infimum of a family of affine functions of \( (\lambda, \nu) \), the dual function is concave even when the original problem is not convex [52]. The term *dual feasible* is used for \( \lambda \) and \( \nu \) if \( \lambda \geq 0 \) and \( g(\lambda, \nu) \) is finite i.e. \( g(\lambda, \nu) > -\infty \). The dual function \( g(\lambda, \nu) \) serves as a lower bound on the optimal value \( f^* \) of the problem (3.3.1) [52]. For any feasible set \((x, \lambda, \nu)\):

\[
\begin{align*}
f_0(x) \geq f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x) \\
\geq \inf_{z \in \Omega} \left( f_0(z) + \sum_{i=1}^{m} \lambda_i f_i(z) + \sum_{i=1}^{p} \nu_i h_i(z) \right) \\
= g(\lambda, \nu).
\end{align*}
\]

This follows from \( f_i(x) \leq 0 \) and \( h_i(x) = 0 \) for any feasible \( x \), and \( \lambda_i \geq 0 \) for any feasible \( \lambda_i \) in the first inequality. Therefore, for a feasible set \((x, \lambda, \nu)\),

\[
\min_x f_0(x) \geq \max_{\lambda, \nu} g(\lambda, \nu).
\]
**Duality gap** is the measure of the difference between the primal objective \( f_0(x) \) and the dual objective \( g(\lambda, \nu) \). If (3.5.4) is satisfied with strict inequality the problem is said to have **weak duality**. When (3.5.4) is satisfied with equality, **strong duality** holds. The best lower bound on the original problem may be obtained by solving the following optimisation problem,

\[
\max_{\lambda, \nu} \quad g(\lambda, \nu) \\
\text{s.t.} \quad \lambda \geq 0.
\]  

(3.5.5)

Problem (3.5.5) is commonly known as the **Lagrange dual problem** and is a convex optimisation problem since the objective to be maximised \( g(\lambda, \nu) \) is always concave and the constraint is convex. This holds regardless of whether or not the primal problem (3.3.1) is convex [52].

As mentioned earlier, mathematical optimisation problems normally suffer from having arbitrary stopping criterion. However the above results from the Lagrange dual problem provide a non-heuristic stopping criterion. This is because a primal-dual feasible point \((x, (\lambda, \nu))\) localizes the optimum solution in the interval defined by the duality gap, \( f^* \in [g(\lambda, \nu), f_0(x)] \).

If \( g(\lambda, \nu) = f_0(x) \), the duality gap is zero, and both the primal and the dual variables are at the optimal solution. The primal optimum variable is denoted as \( x \) and dual optimum variable as \((\lambda^*, \nu^*)\). Since \( x^* \) minimises \( L(x, \lambda^*, \nu^*) \) over \( x \), the gradient of \( L(x, \lambda^*, \nu^*) \) vanishes at \( x^* \), i.e.,

\[
\nabla f_0(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla f_i(x^*) + \sum_{i=1}^{p} \nu_i^* \nabla h_i(x^*) = 0.
\]  

(3.5.6)
Therefore,

\[ f_i(x^*) \leq 0 \quad i = 1, 2, \ldots, m, \quad (3.5.7) \]
\[ h_i(x^*) = 0 \quad i = 1, 2, \ldots, p, \quad (3.5.8) \]
\[ \lambda_i^* \geq 0 \quad i = 1, 2, \ldots, m, \quad (3.5.9) \]
\[ \lambda_i^* f_i(x^*) = 0 \quad i = 1, 2, \ldots, m. \quad (3.5.10) \]

Conditions (3.5.6) - (3.5.10) are collectively known as the Karush-Kuhn-Tucker (KKT) conditions for optimality [56]. Conditions (3.5.7) and (3.5.8) represent primal feasibility of \( x^* \). Condition (3.5.9) represents dual feasibility and condition (3.5.10) signifies the complementary slackness for the primal and dual inequality constraint pairs: \( f_i(x) \leq 0 \) and \( \lambda_i \geq 0 \).

KKT conditions are necessary but not sufficient for optimality. However, for convex optimisation problems, KKT conditions are also sufficient [52]. In the absence of constraints, KKT conditions reduce to the well known stationary conditions \( \nabla f_i(x^*) = 0 \), i.e. a minimum must be obtained at the point where the gradient of \( f_0 \) vanishes. However, in the presence of constraints, the optimal solution is attained at a KKT point \( x^* \), which, together with some dual feasible vector \((\lambda^*, \nu^*)\), satisfies the KKT conditions (3.5.6) - (3.5.10). KKT conditions have proven to be very useful in practice to obtain solutions analytically (when possible).

### 3.6 Robust Convex Optimisation

Robust optimisation models in mathematical programming have received much attention recently [57–59]. Some of these models and their extensions
are reviewed. Considering a convex optimisation of the form

\[
\min_x f_0(x) \quad \text{s.t.} \quad f_i(x) \leq 0, \quad i = 1, 2, \ldots, m
\]  

(3.6.1)

where \( f_i \) are convex, the data defining the constraint and the objective functions in many engineering design applications may be inexact, corrupted by noise, or may fluctuate around with time around a nominal value. In the application of beamforming such errors may arise due to imperfect channel information at the transmitter, receiver or both. The modeling of these uncertainties in the CSI, in the channel covariance estimate, classifies the robust techniques into two categories, *Bayesian* (or stochastic) and the *Minimax* (or worst-case) approach [52, 55].

- In cases where the Bayesian approach considers the statistics of the error to be known, the traditional approach simply solves (3.6.1) by using the nominal (or mean) value of the data. An example of such an approach can be found in [60, 61], where a multi-antenna transmitter was designed to maximise the mean SNR and mean BER assuming errors due to Gaussian noise and quantisation errors.

- The minimax approach considers the errors belonging to a predefined uncertainty region, and the final objective involves the optimisation of the worst system performance for any error in this region. In practice the assumption of errors being bounded is met with a high probability. Denoting the set of perturbed functions parameterized by \( \delta \) : \( f_i(x; \delta) \), with \( \delta \) taken from an uncertainty set \( S \), the feasible robust solution \( x \) is one that satisfies \( f_i(x; \delta) \leq 0, \quad \forall \ \delta \in S \), or equivalently,
The optimisation problem can be written as

$$\max_{\delta \in \mathcal{S}} f_i(x; \delta) \leq 0.$$ \hspace{1cm} \text{(3.6.2)}

Unlike the Bayesian approach, this approach guarantees a minimum instantaneous performance for any errors modeled in the uncertainty region. An example where such a design is used for multi-access MIMO channels is given in [62]. Examples of design of receiver beamformers based on worst-case performance optimisation can be found in [63–65]. The work in [33,66,67] show the downlink beamforming problem under the worst-case performance optimisation framework.

The thesis develops algorithms using convex optimisation concepts explained in this chapter.

3.7 Interior Point Methods

This section is included here for completeness of convex optimisation theory and the details of Interior point methods are outside the scope of this thesis. Convex optimisation problems may be solved analytically, using the duality theorem and KKT conditions. However this does not hold for all optimisation problems. In general, iterative methods are required to solve the convex optimisation problems [52,53]. Several advanced iterative methods exist, that efficiently solve convex optimisation problems. These include the interior-point method, which was initially proposed for linear programming, and may be generalized to all convex optimisation problems [54]. A very general framework for solving convex optimisation problems was developed in [54]. Interior-point methods solve
the original problem (3.3.1) by applying Newton's method to a sequence of equality constrained problems, or to a sequence of modified versions of KKT conditions. The duality theorem also plays a key role in these schemes, since at each iteration, the difference between the objective and the optimum solution can be upper-bounded. This, as mentioned earlier, allows for non-heuristic stopping criteria, where a solution can be reached within a specific resolution. Examples of other popular methods include Barrier methods, cutting-plane methods and the ellipsoids methods. Work in [50,52] and references therein provide a thorough overview of interior-point methods and their implementation issues.

3.8 Summary

A brief overview of convex optimisation theory has been provided in this chapter. Basic concepts and tools of convex optimisation that are readily used in this thesis were introduced. The most generic forms of canonical optimisation problems namely LPs, QPs, SOCPs and SDPs were presented. The concepts of Lagrange duality and KKT conditions were also discussed. Robust convex optimisation, has also been discussed with special emphasis on worst-case performance optimisation. Interior point methods, although out of the scope of this thesis, were included for the completeness of convex optimisation theory. Finally, robust techniques based on convex optimisation theory were discussed, where the CSI available at the transmitter is assumed to be in error.
In this chapter an MMSE based signal forwarding technique for a cooperative relay network is proposed. Transmission of information between multiple source-destination pairs through a set of relays is considered. A general model for relay cooperation has been considered, however, for a single source-destination scenario, the proposed MMSE relaying strategy has been shown to be a product of receiver and transmitter beamformers at the relay layer. Cooperation between relays has been shown to improve substantially the BER performance as compared to non cooperative relays under a total power constraint.

4.1 Relays in Wireless Sensor Networks

A wireless sensor network consists of small sensors that are geographically distributed and they are required to sense, compute and communicate [5], [68]. The small sized sensors can only broadcast over short distances due to limited power and propagation loss between the transmitting and receiving
sensors. A way to overcome this limitation is to employ one or more layers of relays in the network. Relay schemes can be categorized into three different groups: amplify-forward [43], compress-forward [46] and decode-forward [69], [6]. In the amplify-forward scheme, the relay nodes amplify the received signal and broadcasts it towards the destination nodes. This scheme can be used to resolve the problem of attenuation faced by small sensors with limited power. In addition to this, relay channels have the potential to provide spatial diversity to combat fading effect in wireless links [5], [6], [7]. The relay scheme proposed in this paper employs cooperation between relay nodes to forward the signal. A constraint on the global power usage by the relays is imposed. Under this constraint it is shown that the proposed scheme outperforms schemes that do not cooperate. Relay cooperation is however possible when the relays are clustered together with short range local links between them so that signals can be transferred with negligible errors. A similar approach for basestation cooperation has been used for interference cancelation in wireless networks in [70] where signals are transferred between basestations. A scenario where multiple sensors communicate with multiple destinations simultaneously through a set of relays is considered. An MMSE based transceiver for cooperative relays is proposed where relays attempt to perform source separation and spatial multiplexing. This scheme will be shown to de-generalize into a product of receiver-transmitter beamformers for the single source-destination scenario.

4.2 Problem Statement

Transmission between $M$ pairs of source-destination sensors through a set of $N$ cooperative relay nodes is shown in Figure 4.1. A two phase protocol is employed for transmission of data where the relays in the network observe
Section 4.2. Problem Statement

Figure 4.1: A relay network of M sources, M destinations and N cooperating relay nodes.

M source sensors broadcasting a signal vector $s$ in the first phase (broadcasting phase), and the relay sensors transmitting their signals to the destination sensors during the second phase (relaying phase). Let $H_s \in \mathbb{C}^{N \times M}$ denote the channel matrix consisting of complex channel coefficients between the sensors (sources) and the relay nodes, $H_s = [h_{s,1}, h_{s,2}, \ldots, h_{s,M}]$ where $h_{s,m} = [h_{s,m,1}, h_{s,m,2}, \ldots, h_{s,m,N}]^T$ for $m = 1, \ldots, M$, is a column vector consisting of channel coefficients between the $m^{th}$ source and the relays.

The received signal vector at the relay nodes is given as

$$r = H_s \, s + v_s$$

where $v_s \in \mathbb{C}^{N \times 1}$ is zero-mean circularly symmetric complex additive white Gaussian (AWGN) noise vector with covariance matrix $\sigma_v^2 I$. The channel is assumed to be quasi-static fading so that the channel realisations
stay fixed for the duration of a number of frames. The cooperating relay
nodes are assumed to be in close proximity with high SNR links between
them so that received signals and the channel coefficients can be passed
between relays with negligible error. It is also assumed that the relays
have a perfect knowledge of the forward channels through feedbacks from
the destination sensors. In the second phase of transmission, the relays
rebroadcast a transformed signal vector as

$$x = Fr$$  \hspace{1cm} (4.2.2)$$

where $F \in \mathbb{C}^{N \times N}$ is a linear transformation matrix (relay transceiver) to be
determined in order to optimise receiver performance. In [7], the authors
considered a single source-destination scenario and imposed a condition
that $F$ is a diagonal matrix, i.e. the relays do not cooperate. Even though
this model is attractive due to its simplicity of no cooperation amongst
relays, it is demonstrated that relay cooperation enhances the performance
substantially and it is well suited for a multiple source-destination scenario.

The received signal at the destination sensors can be written as

$$t = H_t x + v_t$$  \hspace{1cm} (4.2.3)$$

where $H_t \in \mathbb{C}^{M \times N}$ denotes the channel matrix consisting of complex
channel coefficients between the relay nodes and the destination sensors,
$H_t = [h_{t,1}, h_{t,2}, \ldots, h_{t,M}]^T$ where $h_{t,m} = [h_{t,m,1}, h_{t,m,2}, \ldots, h_{t,m,N}]$ for
$m = 1, \ldots, M$ and $v_t \in \mathbb{C}^{M \times 1}$ is a zero-mean circularly symmetric AWGN
vector with covariance matrix $\sigma_{v_t}^2 I$. Substituting (4.2.2) into (4.2.3) and
using (4.2.1)

$$t = H_t F s + H_t F v_s + v_t$$  \hspace{1cm} (4.2.4)$$
4.3 MMSE Cooperative Relay Strategy

The aim is to determine $F$ in order to minimize the mean-square error (MMSE) between the received signal $H_t x$ and the transmitted signal $s$, i.e.

$$\hat{F} = \arg \min_F J(F)$$

where the cost function $J(F)$ is expressed as

$$J(F) = \sum_{m=1}^{M} E \{|h_{t,m} x - s_m|^2\}.$$ 

(4.3.1)

The cost function can be expanded as

$$J(F) = \text{tr} \left( E \left\{ (H_t x - s)(H_t x - s)^H \right\} \right)$$

$$= \text{tr} \left( H_t F R_r F^H H_t^H - H_t F H_s \sigma_s^2 - H_s^H F^H H_t^H \sigma_s^2 + \sigma_s^2 I \right)$$

(4.3.2)

where $\text{tr}(\cdot)$ is a trace operator, and

$$R_r = H_s H_s^H \sigma_s^2 + \sigma_s^2 I.$$ 

A constraint on the power usage of the relays in the network is imposed as follows

$$E\{x^H x\} = \text{tr} \left( F R_r F^H \right) = p$$

(4.3.3)

where $p$ is the total power available to the relays in the network and it controls the ability of the relays to amplify the signal before forwarding it to the destination node. Hence the unconstrained optimisation problem in
(4.3.1) is converted to a constrained optimisation problem using Lagrangian multiplier $\tilde{\lambda}$ as

$$J(F) = \text{tr}(H_t F R_s F^H H_t^H - H_t F H_s \sigma_s^2 - H_s^H F H_t^H \sigma_s^2 + \sigma_s^2 I)$$

$$+ \tilde{\lambda} (\text{tr}(F R_s F^H) - p).$$

(4.3.4)

Differentiating with respect to $F^*$,

$$\frac{\partial J}{\partial F^*} = H_t^H H_t F R_s - H_t^H H_s \sigma_s^2 + \tilde{\lambda} F R_s = 0$$

(4.3.5)

is obtained. Hence optimum $F$ is determined in terms of $\tilde{\lambda}$ as

$$F_{\text{opt}} = (H_t^H H_t + \tilde{\lambda} I)^{-1} H_t^H H_s R_s^{-1} \sigma_s^2.$$  (4.3.6)

Substituting (4.3.6) into (4.3.3),

$$p = \text{tr}[(H_t^H H_t + \tilde{\lambda} I)^{-1} H_t^H H_s R_s^{-1} H_s H_t (H_t^H H_t + \tilde{\lambda} I)^{-1}] \sigma_s^4$$

(4.3.7)

is obtained. The Lagrangian multiplier $\tilde{\lambda}$ needs to be determined using (4.3.7). This can be obtained using exhaustive search to satisfy (4.3.7), however, to avoid matrix inversion at every stage, (4.3.7) is simplified using eigendecomposition, and a polynomial equation of order $2M$ in $\tilde{\lambda}$ is obtained. Hence

$$H_t^H H_t = QAQ^H,$$

$$\tilde{\lambda} I = \tilde{Q} \tilde{\Lambda} Q^H$$

(4.3.8)
where $\Lambda = \text{diag} \{ \lambda_1, \cdots, \lambda_M, 0, \cdots, 0 \}$ consists of eigenvalues of rank $M$ matrix $H^H_t H_t$, and $\tilde{\Lambda} = \text{diag} \{ \tilde{\lambda}, \cdots, \tilde{\lambda} \}$. The matrix $(H^H_t H_t + \tilde{\lambda} I)^{-1}$ in equation (4.3.7) can be expressed as,

\[
(H^H_t H_t + \tilde{\lambda} I)^{-1} = Q(\Lambda + \tilde{\Lambda})^{-1} Q^H
\]

(4.3.9)

where $(\Lambda + \tilde{\Lambda})^{-1}$ is an $N \times N$ diagonal matrix

\[
(\Lambda + \tilde{\Lambda})^{-1} = \text{diag} \{ (\lambda_1 + \tilde{\lambda})^{-1}, (\lambda_2 + \tilde{\lambda})^{-1}, \cdots, (\lambda_M + \tilde{\lambda})^{-1}, \tilde{\lambda}^{-1}, \cdots, \tilde{\lambda}^{-1} \}.
\]

Therefore, (4.3.7) can be expressed as

\[
\text{tr} \left( Q(\Lambda + \tilde{\Lambda})^{-1} Q^H B Q(\Lambda + \tilde{\Lambda})^{-1} Q^H \right) \sigma_s^2 = p
\]

(4.3.10)

where $B = H^H_t H^H_s R_r^{-1} H_s H_t$. Using properties of the trace operation, (4.3.10) can be written as

\[
\text{tr} \left( Q(\Lambda + \tilde{\Lambda})^{-2} Q^H B \right) \sigma_s^4 = p, \quad \text{or} \quad \text{tr} \left( (\Lambda + \tilde{\Lambda})^{-2} Q^H B Q \right) \sigma_s^4 = p.
\]

(4.3.11)

Defining $C = Q^H B Q = Q^H H^H_t H^H_s R_r^{-1} H_s H_t Q$, (4.3.11) can be written as

\[
\sum_{i=1}^N C_{i,i} (\lambda_i + \tilde{\lambda})^{-2} \sigma_s^4 = p
\]

(4.3.12)

Since $H^H_t H_t$ is a rank $M$ (at most) matrix, only the first $M$ columns of $Q$ span the column space of $H^H_t$. Hence the last $(N - M)$ columns of $H_t Q$ are zero vectors. Therefore the last $(N - M)$ diagonal elements of $C$ are zero. Hence a $2M^{th}$ order polynomial in (4.3.12) is obtained.
4.4 De-generalisation to Single Source-Destination Scenario

Based on the derivations for the generalized case, the optimum $F$ matrix for the single source-destination case can be written as

$$F_{opt} = (h_t^H h_t + \bar{\lambda} I)^{-1} h_t^H h_s^H R_x^{-1} \sigma_s^2$$

(4.4.1)

where $R_x = h_s h_s^H \sigma_s^2 + \sigma_v^2 I$, and

$$h_s = [h_{s,1}, h_{s,2}, \ldots, h_{s,N}]^T \in \mathbb{C}^{N \times 1}$$
$$h_t = [h_{t,1}, h_{t,2}, \ldots, h_{t,N}] \in \mathbb{C}^{1 \times N}$$

are column and row vectors consisting of channel coefficients from source to relays and relays to destination sensor respectively. The Lagrangian multiplier $\bar{\lambda}$ in this case is obtained as a solution of the second order polynomial

$$C_{1,1} (\lambda_1 + \bar{\lambda})^{-2} \sigma_s^4 = p.$$  

(4.4.2)

Both the roots were observed to equally minimise $J$ and yield identical BER performance. The optimum $F$ is a rank one, non-diagonal, matrix. This is in contrary to the $F$ matrix considered in the non-cooperative scheme developed in [7]. However, as shown in the following lemma, for this single source-destination scenario, the optimum $F$ matrix decomposes into a product of receiver and transmitter beamformers at the relay layer. This cooperative scheme provides a better BER performance than the non-cooperative scheme in [7].

**Lemma.** The optimum rank one matrix $F$ in (4.4.1) decomposes into a
product of receiver and transmitter beamformers:

\[ F_{opt} = \frac{k \, h_t^H h_s^H}{\lambda^{-1} \sigma_v^2} \]

where, \( k = \frac{\lambda^{-1} \sigma_v^2}{(1 + \lambda^{-1} \|h_t\|^2)(\sigma_s^{-2} + \sigma_v^{-2} \|h_s\|^2)}. \) \( \quad (4.4.4) \)

**Proof of Lemma.**

\[ F_{opt} = (h_t^H h_t + \lambda)^{-1} h_t^H h_s^H \left( h_s h_s^H \sigma_s^{-2} + \sigma_v^{-2} \right)^{-1} \sigma_s^2. \] \( \quad (4.4.5) \)

Using Woodbury’s identity,

\[ (A + UCV)^{-1} = A^{-1} - A^{-1} U (C^{-1} + VA^{-1} U)^{-1} VA^{-1} \]

the terms in (4.4.6) can be expressed as

\[ (h_t^H h_t + \lambda I)^{-1} = \lambda^{-1} I - \frac{\lambda^{-1} h_t^H h_t \lambda^{-1}}{1 + \lambda^{-1} \|h_t\|^2} \]

\[ (h_s h_s^H \sigma_s^2 + \sigma_v^2 I)^{-1} = \frac{\sigma_v^{-2} h_s h_s^H \sigma_v^{-2}}{\sigma_s^{-2} + \sigma_v^{-2} \|h_s\|^2}. \]
The matrix $F_{opt}$ is written as

$$
F_{opt} = \left( \frac{\lambda^{-1}I - \tilde{\lambda}^{-1}h_t^H h_t \tilde{\lambda}^{-1}}{1 + \lambda^{-1}||h_t||^2} \right) h_t^H h_s^H \left( \sigma_{vs}^{-2}I - \frac{\sigma_{vs}^{-2} h_s h_s^H \sigma_{vs}^{-2} \sigma_{vs}^{-2}}{\sigma_{vs}^{-2} + \sigma_{vs}^{-2} ||h_s||^2} \right) \sigma_s^2 I
$$

$$
= \left( \frac{\lambda^{-1} - \tilde{\lambda}^{-1} \sigma_{vs}^{-2} ||h_t||^2}{\sigma_{vs}^{-2} + \sigma_{vs}^{-2} ||h_s||^2} - \frac{\tilde{\lambda}^{-2} \sigma_{vs}^{-2} ||h_t||^2}{1 + \lambda^{-1}||h_t||^2} \right) h_t^H h_s^H \sigma_s^2
$$

$$
+ \left( \frac{\lambda^{-1} \sigma_{vs}^{-2} \sigma_{vs}^{-2}}{\sigma_{vs}^{-2} + \sigma_{vs}^{-2} ||h_s||^2} \right) h_t^H h_s^H
$$

$$
= \left( \frac{\lambda^{-1} \sigma_{vs}^{-2}}{(1 + \lambda^{-1}||h_t||^2)(\sigma_{s}^{-2} + \sigma_{vs}^{-2} ||h_s||^2)} \right) h_t^H h_s^H
$$

$$
\lambda^{-1} \sigma_{vs}^{-2}
$$

$$
= \left( \frac{\lambda^{-1} \sigma_{vs}^{-2}}{(1 + \lambda^{-1}||h_t||^2)(\sigma_{s}^{-2} + \sigma_{vs}^{-2} ||h_s||^2)} \right) h_t^H h_s^H
$$

4.5 Simulation Results

The performance of the proposed scheme is investigated for a relay network with one source, one destination and a relay layer comprising of two cooperative relays, as shown in Figure 4.2. The transmitted signal from the source sensor is assumed to be Quadrature phase-shift keying (QPSK) with unity power. The channels $h_s$ and $h_t$ have been generated using zero-mean unity variance complex Gaussian noise. The total power consumption is set to unity for both cooperating and non-cooperating relay schemes. In order to set the total power constraint to unity, the algorithm in [7] was modified so that the $i^{th}$ relay coefficient is

$$
f_i = \frac{1}{\sqrt{\sum_{i=1}^{N} \frac{|h_{s,i}|^2}{|h_{t,i}|^2} \left( |h_{s,i}|^2 \sigma_s^{-2} + \sigma_{vs}^{-2} \right)}} \frac{h_{s,i}^* h_{t,i}^*}{|h_{t,i}|^2}. \tag{4.5.1}
$$

All the sensors have identical noise power $\sigma_{vs}^{-2} = \sigma_{vs}^{-2}$. Under identical total power constraint, the proposed cooperative relay scheme provides a better
BER performance than the non cooperative scheme as shown in Figure 4.3. An improvement of approximately $4\, dB$ in SNR is observed. When the scheme is extended to $N = 4$ cooperating relays an $8\, dB$ improvement in performance is noticed at $BER = 10^{-2}$. To characterize the performance of the MMSE relay transceiver for multiple source-destination scenario, $M = 2$ and $N = 4$ and 8 is considered. The BER performance is depicted in Figure 4.4. In this case, even though sensors at the source and destination have single antenna, the cooperative relays attempt to perform source separation and spatial multiplexing.
Section 4.5. Simulation Results

Figure 4.3: BER performance curve of a relay network with one source and one destination node. $N = 2$ and 4 relay nodes have been considered.

Figure 4.4: BER performance curve of a relay network with two source-destination pairs of sensors and $N$ cooperating relays.
4.6 Summary

A signal forwarding algorithm for cooperative relays in a sensor or mesh network has been proposed. It has been shown that the cooperation between relays enhances BER performance at the destination as compared to non-cooperative relays. Even though source and destination nodes employ only single antenna, relay cooperation has the ability to generate multiple spatial links between sources and destination nodes.
The MMSE scheme developed in the previous chapter does not allow the systems from specifying different QoS at different destination nodes. The ability to discriminate, in terms of QoS provided to different destination nodes, could be beneficial since it allows the QoS provided to be determined by other variables. For example, from an economic point of view, in a WMN, tariffs could be setup for different QoS, broadening the subscription option available to customers. In a SN environment, relay networks can be used to divide an area into sub sections, with sensor nodes in different sub sections receiving different QoS. Therefore the ability of relay networks to provide such directionality can be advantageous.

In this chapter a cooperative signal forwarding scheme for wireless sensor and mesh networks using semidefinite constraints is proposed. A multiple
source-destination scenario where a set of relays assists forwarding signals from sources to destinations is considered. The work assumes cooperation between relays. The semidefinite programming framework allows us to impose various quality of services (QoS) for each source-destination pairs. The proposed scheme outperforms an MMSE based cooperative relaying strategy. The design also considers quantisation of information passed between cooperating relays and possible errors on the CSI at the relays using a worst-case performance optimisation approach. The proposed scheme, even in the presence of signal quantisation noise, outperforms a non-cooperative relaying scheme.

5.1 Introduction

Multi-hop wireless networks employ relays to transmit information from source to destination nodes in order to optimise coverage, resource utilisation and capacity. Such schemes arise in WMN, including wireless backhaul network for broadband networks, metropolitan area mobile networks and city wide surveillance systems [71], [72]. Relays are employed to overcome severe fading, due to low power and low height of communicating nodes. In addition, relays could also exploit spatial diversity by forming multiple paths between source and destination nodes [5]. A scenario where multiple source nodes communicate with multiple destination nodes simultaneously through a set of cooperative relays is considered. In this chapter, the definition of cooperative relays is that the signals received from the source by the relays are passed between the relays after quantisation. Therefore the definition of non-cooperative relay, in this context, is that the received signals are not exchanged between relays. Cooperation is possible when the cooperating relays are in close vicinity with high SNR links between them so that the
quantized signals and channel estimates can be shared between them. This scenario has been considered in the previous chapter where a MMSE based relay transceiver design has been proposed, without signal quantisation, to minimise the mean square error (MSE) at the destination nodes subject to a constraint on the total transmit power at the relay nodes [73].

Similar but independent works in [74] and [75] considered convex optimisation based optimal signal forwarding schemes, but they assumed no cooperation between relays, i.e. signals are not passed between relays. The work in [74] considered minimisation of total transmitted power at the relays to achieve target SINRs at the destination nodes using a semidefinite programming framework, where as the work in [75] considered minimisation of interference power, using a second order cone programming framework, while setting signal power at the destination to unity. The cooperative relaying scheme considered in this chapter uses SDP framework to minimise transmit power at the relay layer while satisfying target SINR constraints at the destination nodes. The MMSE based relay transceiver design proposed in [73] attempts to minimise mean square error at the destination nodes for a given total transmitted power at the relays. Therefore, in the MMSE method, required SINR targets for the destination nodes cannot be set as a condition, whereas the semidefinite programming framework proposed here has the ability to attain specific SINR targets at the destination nodes for minimum possible transmission power at the relays.

Cooperative schemes require signals to be exchanged amongst the relays. In practice these signals need to be quantized before being passed to cooperating relay nodes. Therefore signal quantisation noise is included in the design and it is demonstrated that the proposed semidefinite programming based cooperative scheme, even in the presence of quantisation noise, outperforms
the non-cooperative scheme. The relay diversity scheme also requires CSI of the forward and backward channels. A mismatch between the actual and the presumed CSI could arise due to the non-stationarity of the channel environment, feedback delays and CSI quantisation. Therefore a robust scheme based on worst-case performance optimisation is considered.

5.2 Problem Statement

A two-hop relaying protocol is employed between $M$ pairs of source-destination nodes through a set of $N$ cooperative relays. In the first phase (broadcasting phase) a signal vector $s \in \mathbb{C}^{M \times 1}$ is transmitted by the source nodes and is received by a set of relay nodes. The relay nodes then process the received signal and transmit the data to the destination nodes in the second phase (relaying phase). Let $H_s \in \mathbb{C}^{N \times M}$ denote the channel matrix consisting of complex channel coefficients between the source nodes and the relay nodes,

\[
H_s = [h_{s,1}, h_{s,2}, \ldots , h_{s,M}],
\]

where $h_{s,m} = [h_{s,m,1}, h_{s,m,2}, \ldots , h_{s,m,N}]^T$ for $m = 1, \ldots , M$ is a column vector consisting of channel coefficients between the $m^{th}$ source and the observing relays. The received signal vector at the relay nodes is given as

\[
r = H_s s + v_s
\]

where $s = [s_1(n), s_2(n), \ldots , s_M(n)]^T$ is a vector consisting of signal components from $M$ source nodes and $v_s \in \mathbb{C}^{N \times 1}$ is a zero-mean circularly symmetric complex additive white Gaussian (AWGN) noise vector with covariance matrix $\sigma_v^2 I$. In the absence of information quantisation, the relays rebroadcast a transformed signal vector to the destination nodes as $x = Fr$, where $F \in \mathbb{C}^{N \times N}$ is a linear transformation matrix (relay transceiver) to be determined in order to optimise the performance at the destination nodes. The signal
transmitted by the \(i^{th}\) relay to the destination node (i.e. the \(i^{th}\) element of \(x\)) is
\[
x_i = F_{i,i}r_i + \sum_{j \neq i,j=1}^{N} F_{i,j}r_j,
\]
where \(F_{i,j}\) is the \((i,j)^{th}\) element of \(F\) and \(r_j\) is the signal received by the \(j^{th}\) relay node. Therefore, relays need to pass the signal received from the source nodes between them. In practice, this information needs to be quantized before being passed between relays due to finite bandwidth requirement of their available local links. Therefore, in the presence of information quantisation, the \(i^{th}\) relay will transmit
\[
x_i = F_{i,i}r_i + \sum_{j \neq i,j=1}^{N} F_{i,j}\hat{r}_j,
\]
where \(\hat{r}_j = r_j + v^q_j\) is the quantized information passed by the \(j^{th}\) relay and \(v^q_j\) is the quantisation noise. The variance of the quantisation noise can be determined from the signal variance and the number of bits available to quantize the signal received by the relay from the source. In principle, the signal \(r_i\) multiplied by \(F_{i,i}\) will also have quantisation noise due to analogue-to-digital conversion at the relays, but compared to the AWGN at the relay terminal and the quantisation noise introduced in \(\hat{r}_j\) (to pass the signal between relays), the quantisation noise in \(r_i\) can be ignored. In any case, this can also be absorbed into the thermal noise term (if the probability density function of the noise term is ignored).

The signal transmitted by the relays can be written in a vector form as,
\[
x = F_d r + F_q r_q
\]
(5.2.2)

where \(F_d\) is a diagonal matrix having the same diagonal elements as \(F\), \(F_q\) consists of all the elements of \(F\) except the diagonal elements, i.e. \(F_q = F - F_d\). The term \(r_q\) in (5.2.2) is the quantized signal passed between the cooperating relays. It is given as \(r_q = r + v_q\), where the vector \(v_q \in \mathbb{C}^{N \times 1}\) accounts for the quantisation noise \([v^q_1, v^q_2, \ldots, v^q_N]^T\). From here onwards, for convenience, the AWGN term \(v_s\) (in \(r\)) is absorbed into \(v_q\) and \(r_q\) is
expressed as \( r_q = H_s + v_q \). The covariance matrix of \( v_q \) is assumed to be \( \sigma^2_{v_q}I \). The cooperating relay nodes are assumed to be in close proximity with high SNR links so that the information and the channel estimates are shared amongst them with negligible error. A quasi-static fading channel is assumed where the channel realisations stay fixed for the duration of a number of frames. Initially it is assumed that the relays have perfect knowledge of the forward channels through feedback from the destination nodes. However, imperfect CSI will be considered later in the chapter and a robust scheme that is insensitive to such uncertainties will be developed.

The received signal vector can be written as

\[
\mathbf{t} = H_x + v_t
\]  

(5.2.3)

where \( H_t \in \mathbb{C}^{M \times N} \) denotes the channel matrix consisting of complex channel coefficients between the relay nodes and the destination nodes, 

\[ H_t = [h_{t,1}, h_{t,2}, \ldots, h_{t,M}]^T \]

where \( h_{t,m} = [h_{t,m,1}, h_{t,m,2}, \ldots, h_{t,m,N}] \) for \( m = 1, \ldots, M \) and \( \mathbf{v_t} \in \mathbb{C}^{M \times 1} \) is a zero-mean circularly symmetric AWGN vector with covariance matrix \( \sigma^2_{v_t}I \). Substituting (5.2.1) into (5.2.2) and using (5.2.3)

\[
\mathbf{t} = H_tF_s + H_tF_qv_s + H_tF_qv_q + v_t
\]  

(5.2.4)

is obtained.

### 5.3 Formulation of the SINR Cost Function

The aim is to determine an optimum \( F \) that minimises the total transmit power at the relay nodes, \( p_{\text{pow}} \), subject to a set of QoS constraints, \( \text{SINR}_m \geq \)
\( \gamma_m \) for \( m = 1, \cdots, M \), imposed at each destination node \( m \) (where \( \gamma_m \) is the minimum required target SINR for the \( m^{th} \) destination). The total transmitted power at the relay nodes can be written as

\[
E \{ x^H x \} = \text{tr}(F_d H_s^H F_d^H) \sigma_s^2 + \text{tr}(F_d F_d^H) \sigma_{v_d}^2 + \text{tr}(F_q F_q^H) \sigma_{v_q}^2 = P_{\text{pow}}. \tag{5.3.1}
\]

The received signal power at the \( m^{th} \) destination node can be written as

\[
p_{\text{sig,m}} = E \{ (h_{t,m} F h_{s,m} s_m(n))(h_{t,m} F h_{s,m} s_m(n))^H \}
= h_{t,m} F h_{s,m} h_{s,m}^H F^H h_{t,m}^H \sigma_s^2. \tag{5.3.2}
\]

Similarly the received interference power at the \( m^{th} \) destination node due to signal contribution from all other source nodes (except the \( m^{th} \)) is given as

\[
p_{\text{int,m}} = \sum_{k=1, k \neq m}^M E \{ (h_{t,m} F h_{s,k} s_k(n))(h_{t,m} F h_{s,k} s_k(n))^H \}
= \sum_{k=1, k \neq m}^M (h_{t,m} F h_{s,k} h_{s,k}^H F^H h_{t,m}^H) \sigma_s^2. \tag{5.3.3}
\]

The noise power transferred to the \( m^{th} \) destination node from the relay nodes can be written as

\[
p_{\text{noise,m}} = E \{ (h_{t,m} F v_s)(h_{t,m} F v_s)^H \} = h_{t,m} F F^H h_{t,m}^H \sigma_{v_s}^2. \tag{5.3.4}
\]

and the noise power introduced to the \( m^{th} \) destination due to the relay signal quantisation noise \( v_q \) is expressed as

\[
p_{\text{quant,m}} = E \{ (h_{t,m} F_q v_q)(h_{t,m} F_q v_q)^H \} = h_{t,m} F_q F_q^H h_{t,m}^H \sigma_{v_q}^2. \tag{5.3.5}
\]
Therefore the optimisation problem is formulated as

\[
\min_{\mathbf{F}} \cdot \quad P_{\text{pow}}
\]

\[
\text{s.t.} \quad \frac{P_{\text{sig},m}}{P_{\text{int},m} + P_{\text{acc},m} + P_{\text{opt},m} + \sigma_v^2} \geq \gamma_m, \quad m = 1, \ldots, M
\]

where \(\sigma_v^2\) is the variance of the noise present at the destination nodes. It is noted that the objective function in (5.3.6) is convex. The constraint set however is not convex but it can be converted to a convex form using semidefinite relaxation (SDR), and solved using interior point methods [34].

### 5.4 Convex Formulation Using Semidefinite Programming

To this end, it is aimed to solve the optimisation problem by converting it to a SDP form that can be solved using interior-point methods. Defining \(\text{vec}(\cdot)\) as vectorisation operator that forms a vector by stacking columns of a matrix, and using the Kronecker identity \(\text{vec}(AXB) = (B^T \otimes A)\text{vec}(X)\), the first part of (5.3.1) can be expressed as

\[
\operatorname{tr}(\mathbf{F}^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{F}^H) \sigma_v^2 = \operatorname{tr}(\mathbf{I} \mathbf{F}^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{F}^H \mathbf{I}^H) \sigma_v^2
\]

\[
= \text{vec}(\mathbf{I} \mathbf{F} \mathbf{H}_s)^H \text{vec}(\mathbf{I} \mathbf{F} \mathbf{H}_s) \sigma_v^2
\]

\[
= \mathbf{f}_d^H (\mathbf{H}_s^T \otimes \mathbf{I})^H (\mathbf{H}_s^T \otimes \mathbf{I}) \mathbf{f}_d \sigma_v^2. \quad (5.4.1)
\]

The second and third parts of equation (5.3.1) can be written as \(\mathbf{f}_d^H \mathbf{f}_d \sigma_v^2\) and \(\mathbf{f}_q^H \mathbf{f}_q \sigma_v^2\) respectively, where \(\mathbf{f}_d\) and \(\mathbf{f}_q\) are the vectorized form of the transceiver matrix \(\mathbf{F}_d\) and \(\mathbf{F}_q\) to be optimised. A matrix \(\mathbf{Q} \in \mathbb{C}^{N_s^2 \times N_s^2}\) is introduced such that \(\mathbf{f}_d = \mathbf{Q} \mathbf{f}\). The matrix \(\mathbf{Q}\) has been constructed to select the elements in vector \(\mathbf{f}\) that correspond to the diagonal elements of \(\mathbf{F}\) (i.e. elements that multiply with received signals without quantisation, \(r\)). These
elements appear in the sequence \( j(N + 1) + 1 \), for \( j = 0, 1, \cdots, (N - 1) \),
in the vector \( \mathbf{f} \) so that all elements of \( \mathbf{Q} \) are zero except \( \mathbf{Q}_{x,x} = 1 \), where
\( x = j(N+1)+1 \), for \( j = 0, 1, \cdots, (N-1) \). Similarly, \( \mathbf{f}_q = (\mathbf{I} - \mathbf{Q}) \mathbf{f} \), where
the matrix \( (\mathbf{I} - \mathbf{Q}) \) selects the elements in \( \mathbf{f} \) that appear in the off-diagonal
of \( \mathbf{F} \) (i.e. elements that multiply with the quantized received signal, \( r_q \)).
Using matrix \( \mathbf{Q} \) and the properties of trace operation, power usage by the
relay nodes (5.3.1) can be expressed as

\[
P_{\text{pow}} = \mathbf{f}^H (\mathbf{H}^T_g \otimes \mathbf{I})^H (\mathbf{H}^T_s \otimes \mathbf{I}) \mathbf{f} \sigma_s^2 + \mathbf{f}_d^H \mathbf{f}_d \sigma_{v_s}^2 + \mathbf{f}_q^H \mathbf{f}_q \sigma_{v_q}^2
\]

\[
= \mathbf{f}^H \mathbf{P}_{\text{pow}} \mathbf{f} = \text{tr}(\mathbf{P}_{\text{pow}} \mathbf{D}) \quad (5.4.2)
\]

where \( \mathbf{P}_{\text{pow}} = (\mathbf{H}^T_g \otimes \mathbf{I})^H (\mathbf{H}^T_s \otimes \mathbf{I}) \mathbf{f} \sigma_s^2 + \mathbf{Q} \mathbf{f}_d \mathbf{f}_d \sigma_{v_s}^2 + (\mathbf{I} - \mathbf{Q}) \mathbf{f}_q \mathbf{f}_q \sigma_{v_q}^2 \), \( \mathbf{D} = \mathbf{f} \mathbf{f}^H \),
\( \mathbf{P}_{\text{pow}}, \mathbf{D} \in \mathbb{C}^{N^2 \times N^2} \) and \( \mathbf{D} \) is a rank-one positive semidefinite (PSD) matrix.
Since \( (\mathbf{h}_{t,m} \mathbf{F} \mathbf{h}_{s,m}) \) in (6.3.3) is a scalar, \( \text{vec}(\mathbf{h}_{t,m} \mathbf{F} \mathbf{h}_{s,m}) = \mathbf{h}_{t,m} \mathbf{F} \mathbf{h}_{s,m} \), the
received signal power at the \( m \text{th} \) destination node can be expressed in the
Kronecker product form as

\[
P_{\text{sig},m} = \mathbf{f}^H (\mathbf{h}_{s,m}^T \otimes \mathbf{h}_{t,m})^H (\mathbf{h}_{s,m}^T \otimes \mathbf{h}_{t,m}) \mathbf{f} \sigma_s^2
\]

\[
= \mathbf{f}^H \mathbf{P}_{\text{sig},m} \mathbf{f} = \text{tr}(\mathbf{P}_{\text{sig},m} \mathbf{D}) \quad (5.4.3)
\]

where \( \mathbf{P}_{\text{sig},m} = (\mathbf{h}_{s,m}^T \otimes \mathbf{h}_{t,m})^H (\mathbf{h}_{s,m}^T \otimes \mathbf{h}_{t,m}) \sigma_s^2 \in \mathbb{C}^{N^2 \times N^2} \) is a PSD matrix.
The total interference power at the \( m \text{th} \) destination node from all other
source nodes, shown in (6.3.4), can be expressed in terms of Kronecker
product as

\[
P_{\text{int},m} = \sum_{\substack{k=1, \ k \neq m}}^{M} \mathbf{f}^H (\mathbf{h}_{s,k}^T \otimes \mathbf{h}_{t,m})^H (\mathbf{h}_{s,k}^T \otimes \mathbf{h}_{t,m}) \mathbf{f} \sigma_s^2
\]

\[
= \mathbf{f}^H \mathbf{P}_{\text{int},m} \mathbf{f} = \text{tr}(\mathbf{P}_{\text{int},m} \mathbf{D}) \quad (5.4.4)
\]
where \( P_{\text{int},m} = \sum_{k=1,k\neq m}^{M} (h_{t,k}^T \otimes h_{t,m})^H (h_{e,k}^T \otimes h_{t,m}) \sigma_v^2 \in \mathbb{C}^{N^2 \times N^2} \) is a PSD matrix. The noise power \( \sigma_v^2 \) (due to thermal noise) transferred to the \( m \)-th destination node from the relay nodes described in (6.3.5) can be written as

\[
P_{\text{nse},m} = h_{t,m} F_d I^H F_d^H h_{t,m}^H \sigma_v^2 = f^H P_{\text{nse},m} f = \text{tr}(P_{\text{nse},m} D) \tag{5.4.5}
\]

where \( P_{\text{nse},m} = Q(I \otimes h_{t,m})^H (I \otimes h_{t,m}) Q^H \sigma_v^2 \in \mathbb{C}^{N^2 \times N^2} \) is a PSD matrix. Similarly the total quantisation noise power \( \sigma_v^2 \) transferred to the destination node \( m \), shown in (5.3.5), can be expressed as

\[
P_{\text{qnt},m} = h_{t,m} F_q I^H F_q^H h_{t,m}^H \sigma_v^2 = f^H P_{\text{qnt},m} f = \text{tr}(P_{\text{qnt},m} D) \tag{5.4.6}
\]

where \( P_{\text{qnt},m} = (I - Q)(I \otimes h_{t,m})^H (I \otimes h_{t,m})(I - Q)^H \sigma_v^2 \in \mathbb{C}^{N^2 \times N^2} \) is a PSD matrix. The optimisation framework in (5.3.6) can now be expressed as

\[
\begin{align*}
\min \limits_f &\quad f^H P_{\text{pow}} f \\
\text{s.t.} &\quad f^H P_{\text{int,m}} f + f^H P_{\text{nse},m} f + f^H P_{\text{qnt,m}} f + \sigma_v^2 \geq \gamma_m, \\
&\quad m = 1, \ldots, M.
\end{align*} \tag{5.4.7}
\]

It is noted that the SINR constraints in the optimisation problem (5.4.7) can be expressed as \( f^H Z_m f + \gamma_m \sigma_v^2 \leq 0 \), where \( Z_m = \gamma_m P_{\text{int},m} + \gamma_m P_{\text{nse},m} + \gamma_m P_{\text{qnt},m} - P_{\text{int},m} \). Using properties of the trace operator the above optimisation problem can be expressed as

\[
\begin{align*}
\min \limits_D &\quad \text{tr}(P_{\text{pow}} D) \\
\text{s.t.} &\quad \text{tr}(P_{\text{int,m}} D) + \text{tr}(P_{\text{nse},m} D) + \text{tr}(P_{\text{qnt,m}} D) + \sigma_v^2 \geq \gamma_m, \\
&\quad \text{rank}(D) = 1, D \succeq 0, D = D^*, \quad m = 1, \ldots, M
\end{align*} \tag{5.4.8}
\]
where \( D = \mathbf{f}^H \mathbf{f} \) and \( D \succeq 0 \) denotes \( D \) is a PSD matrix. Problem (5.4.8) is not convex due to the constraint on rank, i.e. \( \text{rank}(D) = 1 \). It can however be relaxed into a convex form using the standard techniques of SDR by dropping the constraint \( \text{rank}(D) = 1 \) [76], so that the optimisation problem is formulated as

\[
\begin{align*}
\min_{D} & \quad \text{tr}(P_{\text{pow}} D) \\
\text{s.t.} & \quad \text{tr}(Z_m D) + \gamma_m \sigma_n^2 \leq 0, \ m = 1, \ldots, M, \\
& \quad D \succeq 0, \ D = D^*.
\end{align*}
\]  

(5.4.9)

The objective function and constraints in (5.4.9) are convex and can be solved using interior point methods [34].

**Lemma.** Provided the problem in (5.4.9) is feasible, the relaxation provides a rank-one matrix \( D \) which achieves the same global minimum as the original problem.

**Proof of Lemma.** According to the work in [76] the Lagrangian dual functions of the original optimisation problem (OP) in (5.4.7) and the relaxed problem (RP) in (5.4.9) are the same and can be written as

\[
\begin{align*}
\max_{\lambda} & \quad \sum_{m=1}^{M} \lambda_m \gamma_m \sigma_n^2 \\
\text{s.t.} & \quad P_{\text{pow}} + \sum_{m=1}^{M} \lambda_m Z_m \succeq 0, \ \lambda \succeq 0, \ m = 1, \ldots, M.
\end{align*}
\]  

(5.4.10)

However since (RP) is convex and satisfies Slates condition, there exists strong duality between (RP) and its Lagrangian dual (or simply dual) [52]. The Lagrangian of (OP) could be written as

\[
L(\mathbf{f}; \lambda_m) = \mathbf{f}^H (P_{\text{pow}} + \sum_{m=1}^{M} \lambda_m Z_m) \mathbf{f} + \sum_{m=1}^{M} \lambda_m \gamma_m \sigma_n^2
\]  

(5.4.11)
where $Z_m = \gamma_m P_{\text{int},m} + \gamma_m P_{\text{nse},m} + \gamma_m P_{\text{qnt},m} - P_{\text{sig},m}$ and $\lambda_m$ are non-negative Lagrangian multipliers, and obtain the corresponding Hessian matrix as

$$
\nabla^2_{ff} L(f; \lambda_m) = P_{\text{pow}} + \sum_{m=1}^{M} \lambda_m Z_m.
$$

(5.4.12)

According to the work in [77], provided the Hessian is PSD, there is strong duality between (OP) and its dual problem. It is apparent, provided (RP) is feasible, that the constraint $P_{\text{pow}} + \sum_{m=1}^{M} \lambda_m Z_m \geq 0$ in (5.4.10) is satisfied. This implies that there is strong duality between (OP) and its dual. Since the dual functions of (OP) and (RP) are the same, the minimum of (RP) is the global minimum of (OP) which completes the proof.

The signal space can be extracted from matrix $D$ to form the vector $f$, and the optimum relaying matrix $F$ can be formed by reversing the vectorisation operation on $f$.

5.5 Robust Formulation and Convex Optimisation using Semidefinite programming

The cooperative scheme considered so far requires exact CSI of the forward and backward channels. In practice this is not possible due to the dynamics of the channels, feedback delays and the quantisation of channel coefficients. Therefore robustness is introduced into the design by considering errors in CSI. Due to errors between the true channel and the estimates, the matrices $P_{\text{pow}}, P_{\text{sig},m}, P_{\text{int},m}, P_{\text{nse},m}$ and $P_{\text{qnt},m}$ (which have been constructed using the forward and backward erroneous channels) can be written as $\tilde{P}_{\text{pow}} = P_{\text{pow}} + \Delta_1,m, \tilde{P}_{\text{sig},m} = P_{\text{sig},m} + \Delta_2,m, \tilde{P}_{\text{int},m} = P_{\text{int},m} + \Delta_3,m, \tilde{P}_{\text{nse},m} = P_{\text{nse},m} + \Delta_4,m$, and $\tilde{P}_{\text{qnt},m} = P_{\text{qnt},m} + \Delta_5,m$. Matrices $\tilde{P}_{\text{pow}}, \tilde{P}_{\text{sig},m}, \tilde{P}_{\text{int},m}, \tilde{P}_{\text{nse},m}$ and $\tilde{P}_{\text{qnt},m}$ denote the true matrices and $\Delta_1,m, \Delta_2,m, \Delta_3,m, \Delta_4,m$ and $\Delta_5,m$. 
are their respective unknown mismatch matrices, bounded above by some known constants as $||\Delta_{1,m}||_F \leq \epsilon_{1,m}$, $||\Delta_{2,m}||_F \leq \epsilon_{2,m}$, $||\Delta_{3,m}||_F \leq \epsilon_{3,m}$, $||\Delta_{4,m}||_F \leq \epsilon_{4,m}$ and $||\Delta_{5,m}||_F \leq \epsilon_{5,m}$ (where $|| \cdot ||_F$ denotes the Frobenius norm). To provide robustness against such norm-bounded mismatches, the optimum relaying matrix $F$ is designed by minimizing the worst-case transmit power at the relaying layer to achieve a set of worst-case target SINRs at the destination nodes. The worst-case objective and constraint functions can be expressed as [32],

$$\max_{\|\Delta_{1,m}\|_F \leq \epsilon_{1,m}} f^H \tilde{P}_{\text{pow}} f = f^H (P_{\text{pow}} + \epsilon_{1,m} I) f,$$

$$\min_{\|\Delta_{2,m}\|_F \leq \epsilon_{2,m}} f^H \tilde{P}_{\text{sig,m}} f = f^H (P_{\text{sig,m}} - \epsilon_{2,m} I) f,$$

$$\max_{\|\Delta_{3,m}\|_F \leq \epsilon_{3,m}} f^H \tilde{P}_{\text{int,m}} f = f^H (P_{\text{int,m}} + \epsilon_{3,m} I) f,$$

$$\max_{\|\Delta_{4,m}\|_F \leq \epsilon_{4,m}} f^H \tilde{P}_{\text{nse,m}} f = f^H (P_{\text{nse,m}} + \epsilon_{4,m} I) f,$$

$$\max_{\|\Delta_{5,m}\|_F \leq \epsilon_{5,m}} f^H \tilde{P}_{\text{qnt,m}} f = f^H (P_{\text{qnt,m}} + \epsilon_{5,m} I) f.$$  \hfill (5.5.1)$$

The robust optimisation scheme can now be formulated as

$$\min_{f} \quad f^H \tilde{P}_{\text{pow}} f$$

s.t. \quad \frac{f^H \tilde{P}_{\text{sig,m}} f}{f^H \tilde{P}_{\text{int,m}} f + f^H \tilde{P}_{\text{nse,m}} f + f^H \tilde{P}_{\text{qnt,m}} f + \sigma_v^2} \geq \gamma_m,$$ \hfill (5.5.2)

$$m = 1, \ldots, M$$

where $\tilde{P}_{\text{pow}} = P_{\text{pow}} + \epsilon_{1,m} I$, $\tilde{P}_{\text{sig,m}} = P_{\text{sig,m}} - \epsilon_{2,m} I$, $\tilde{P}_{\text{int,m}} = P_{\text{int,m}} + \epsilon_{3,m} I$, $\tilde{P}_{\text{nse,m}} = P_{\text{nse,m}} + \epsilon_{4,m} I$ and $\tilde{P}_{\text{qnt,m}} = P_{\text{qnt,m}} + \epsilon_{5,m} I$. Using the properties of trace operation and dropping the constraint on rank, (5.5.2) can be expressed as

$$\min_{D} \quad \text{tr}(\tilde{P}D)$$

s.t. \quad \text{tr}(\tilde{Z}_m D) + \gamma_m \sigma_v^2 \leq 0, \quad m = 1, \ldots, M$$  \hfill (5.5.3)
where $\hat{z}_m = \gamma_m \hat{p}_{int,m} + \gamma_m \hat{p}_{mc,m} + \gamma_m \hat{p}_{qut,m} - \hat{p}_{sig,m}$. The objective function and the constraints in (5.5.3) are convex and can be solved using interior point methods [50, 52, 56]. Similar to (5.4.9), the algorithm yields a rank-one matrix $D$ from which the signal space is extracted to form vector $f$, and the optimum relaying matrix $F$ can be formed.

5.6 Simulation Results

The performance of the proposed scheme is investigated for a relay network with two sources, two destinations and a relay layer comprising of four cooperating relays, $N = 4$, as shown in Figure 5.1. The transmitted signal from the source node is assumed to be QPSK with unity power. The channels $h_s$ and $h_t$ have been generated using zero-mean unity variance complex Gaussian noise. Comparison of the cooperative relaying scheme is made with a scheme which performs matched filtering at the relay layer, and a scheme that employs an MMSE based relay transceiver [73], for forwarding signals to the destination nodes. The received signal vector in the matched filtering based scheme (denoted scheme 1) can be written as $q = \eta H_s G H_s s + \eta H_s G v_s + v_t$, where $G = H_t^H H_s^H$ is the relay transceiver matched to the forward and backward channels and $\eta$ is a scaling factor introduced to control the transmit power at the relay nodes which is given as $p = \text{tr}(G H_s H_s^H G H_s^H \sigma_s^2 + G G^H \sigma_v^2) \eta^2$. For the MMSE based relay transceiver scheme (denoted scheme 2), the optimum linear transformation matrix $F_{opt}$ is obtained as [73], $F_{opt} = (H_t^H H_t + \bar{\lambda} I)^{-1} H_t^H H_s^H R_s^{-1} \sigma_v^2$, where the Lagrangian multiplier $\bar{\lambda}$ is determined by the power usage of the relay nodes which is expressed as $\text{tr}[(H_t^H H_t + \bar{\lambda} I)^{-1} H_t^H H_s^H (H_s H_s^H \sigma_s^2 + \sigma_v^2 I)^{-1} H_s H_s^H (H_t^H H_t + \bar{\lambda} I)^{-1}] \sigma_s^2$. The performance is measured by observing the bit error rate (BER) averaged over both users for different target SINRs, while keeping
Figure 5.1: A relay network of 2 sources, 2 destinations and 4 cooperating relay nodes.

The power of the AWGN present at the receivers and relays is fixed at 0.01 (−20dB). The total power used by the relay nodes to achieve a target SINR in the proposed scheme is computed, and the same power has been used for schemes 1 and 2, i.e. total power used at the relays for all three schemes is kept identical. Occasionally, at low SNR, the problem becomes infeasible and the corresponding outage probability has been summarized in Table (5.1). For noise powers less than −10dB (i.e. 0.1) the outage probability appears as zero for one million simulation runs. As shown in Figure 5.2, under identical total power at the relays, the proposed scheme provides a better BER performance as compared to the matched filter based scheme and the MMSE based scheme. An improvement of 2dB in SINR is noticed at 10^{−3} BER as compared to the MMSE scheme. The reason for the difference
Figure 5.2: BER performance curves at various power levels. The x-axis shows target SINRs achieved by the proposed scheme. The power required to achieve the target SINRs in the proposed scheme is allocated to the other two schemes and the BER achieved by them is shown.

<table>
<thead>
<tr>
<th>Noise power (dB)</th>
<th>-2</th>
<th>-5</th>
<th>-8</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP.</td>
<td>0.46</td>
<td>0.12</td>
<td>0.02</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.1: Table of outage probability (Op.) at varying SNRs.

In performance between the MMSE and the SDP schemes can be stated as follows. The MMSE scheme minimises the total MSE at the destination nodes for a given transmitted power at the relays. Hence different MMSEs (therefore different SINRs) are achieved at different destination terminals. However, in the SDP scheme, a balanced SINR is achieved if identical target SINRs are set for all destination terminals. Hence a better performance in BER is observed. The BER does not even approach $10^{-2}$ for the matched filtering based scheme. Moreover, the proposed scheme has the advantage of setting various QoS (target SINRs) explicitly for different destination nodes.
Section 5.6. Simulation Results

Figure 5.3: Power utilisation at the relay nodes for various target SINRs and signal quantisation noise power.

The performance of the cooperative scheme with signal quantisation noise is compared with a non-cooperative relay scheme proposed in [74]. The optimum relaying matrix in the non-cooperative relay scheme is a diagonal matrix. The power transmitted by the relay nodes, to achieve the same target SINRs at destination nodes, is used as a performance index for both schemes. The power of the AWGN present at the relays and destination nodes is fixed to 0.01 for both schemes. Comparison is made for three different quantisation noise powers: 0.13, 0.50 and 1.00. In the simulations carried out, the received signal power at each relay is two. Hence signal-to-quantisation noise power ratio (SQNR) considered are 12dB, 6dB and 3dB respectively. Assuming the signal is uniformly distributed, a B-bit quantiser would divide the dynamic range of the quantisation noise power into $2^B$ levels, producing an SQNR of $6.02B$. Therefore the considered quantisation noise powers of
Table 5.2: Outage probabilities (Op.) for cooperative and non-cooperative schemes for various target SINRs.

<table>
<thead>
<tr>
<th>Target SINR (dB)</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP:Non Cooperative</td>
<td>0.024</td>
<td>0.166</td>
<td>0.482</td>
<td>0.839</td>
</tr>
<tr>
<td>OP:Cooperative</td>
<td>0</td>
<td>0</td>
<td>0.081</td>
<td>0.426</td>
</tr>
</tbody>
</table>

0.50 and 0.13 correspond to one-level and two-level quantisers respectively.

The results depicted in Figure 5.3 confirm that the proposed cooperative relay scheme, even with large quantisation noises such as 3dB, 6dB and 9dB, outperforms the non-cooperative relay schemes to achieve identical target SINRs in terms of power utilisation at the relay nodes. Even at very high quantisation noise, a two fold decrease in power consumption is observed as compared to a non-cooperative scheme. For instance the power utilised by the cooperating relays to achieve an 8dB target SINR is $7.5e^{-2}$dBW when the quantisation noise is 12dB, $8e^{-2}$dBW when the quantisation noise is 6dB and $9.5e^{-2}$dBW when a 3dB quantisation noise is present. The non cooperating relays use a power of $2.5e^{-1}$dBW for the same target SINR to be achieved under identical channel and noise values.

Finally, the BER performance of the robust and non-robust schemes is provided for various target SINRs. The power of the AWGN present at the destination nodes and relays is fixed for both schemes to 0.01. The norm bounds on the mismatch matrices have been obtained by plotting a histogram of the Frobenius norms of the mismatch matrices, generated from the difference of actual and known matrices, and selecting a value that provides a specific worst-case outage. For a 75% worst-case outage, the norm bounds have been determined as $\epsilon_1 = 0.04$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.60$, $\epsilon_4 = 0.15$ and $\epsilon_5 = 0.006$. Figure 5.4 depicts the BER performance of the robust and the non-robust schemes. The proposed robust scheme outperforms the non-robust scheme. A histogram is plotted to show the
Section 5.6. Simulation Results

Figure 5.4: A relay network with two sources and two destination nodes. \( N = 4 \) relay nodes have been used.

Figure 5.5: Distribution of achieved SINR for a particular user. Target SINR is 8dB.
distribution of the SINR achieved by the robust and the non-robust schemes for a required target SINR of 8dB. The distribution of the achieved SINR for a particular user is shown in Figure 5.5. The robust scheme, having been optimised for the worst-case scenario, always attains the target SINR. However, the non-robust scheme attains the target SINR of 8dB only 50% of the time.

5.7 Summary

A relaying strategy for multiple source-destinations based wireless networks using semidefinite constraints has been proposed. The scheme is based on minimizing the total transmit power subject to attaining specific quality of services for different users. It requires cooperation between relays, hence signal quantisation noise is considered. The proposed cooperative relaying scheme, even with large signal quantisation noise power, a two fold decrease in power consumption is noticed compared to a non-cooperative relaying scheme.
Chapter 6

COMPLEXITY REDUCTION THROUGH UPLINK-DOWNLINK BEAMFORMER DECOMPOSITION

6.1 Introduction

A scenario where multiple source nodes communicate with multiple destination nodes through a set of relays between is considered. The source nodes could be user access nodes in wireless mesh networks that communicate with other hosts (destination nodes), via relays or even through set of hosts that act as relays, forming multipath between source-destination pairs. Chapter 5 of the thesis developed a scheme that minimised the total power at the relays while it met the QoS (in terms of target SINRs) specified at the destination nodes. It did so by formulating the optimization problem into a semidefinite optimization form, the optimum of which was obtained using interior point methods algorithms. The formulation into a semidefinite form
involved the use of Kronecker product identity the vectorisation operator, yielding an optimisation variable of dimension $N^2 \times N^2$, where $N$ is the number of relays in the network. The work in this chapter looks to reduce the complexity of this design by reducing the dimension of the optimising variable. In this chapter the application of uplink and downlink beamforming techniques is proposed to achieve this. A complexity analysis of the two schemes is provided. A modification in the scheme is also proposed to restrict the power emitted by each relay node below a threshold. With the incorporation of such power constraints, the modified scheme is more realistic since the power at each relay is limited in a practical environment.

The chapter finally looks to extend the work to an underlay cognitive radio network. Cooperative relays are used to improvise spectrum sharing, and their ability to direct beams with varied strengths to different receivers in the same frequency spectrum is utilised. This allows relays to be readily used for exploiting white spaces in the spectrum, thereby increasing spectral usage efficiency in wireless networks where multiple antennas cannot be collocated at the transmitting devices.

6.2 Problem Statement

As done in previous chapters, $M$ pairs of source-destination nodes and a set of $N$ distributed relays between sources and destinations nodes are considered. In the first phase (broadcasting phase) a signal vector $s$ is transmitted by the source nodes and is received by a set of relay nodes. The relay nodes then transmit the data to the destination nodes in the second phase (relaying phase). Let $H_s \in \mathbb{C}^{N \times M}$ denote the known channel matrix consisting of complex channel coefficients between source nodes and relay nodes, $H_s = [h_{s,1}, h_{s,2}, \ldots, h_{s,M}]$ where $h_{s,m} = [h_{s,m,1}, h_{s,m,2}, \ldots, h_{s,m,N}]^T$ for $m =$
1, \cdots, M$, is a column vector consisting of channel coefficients between the $m^{th}$ source and the observing relays. The received signal vector at the relay nodes is given as

$$r = H_s s + v_s$$  \hspace{1cm} \text{(6.2.1)}$$

where $s = [s_1(n), s_2(n), \cdots, s_M(n)]^T$ is a vector consisting of signal components from $M$ source nodes and $v_s \in \mathbb{C}^{N \times 1}$ is zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_v^2 I$. The channel is assumed to be quasi static, with channel realisations staying fixed for the duration of a number of frames. In this chapter, instead of directly optimizing the relay transceiver matrix, a two stage design process, first by obtaining soft estimate of the transmitted signal through receiver beamformer and then designing transmitter beamformers is considered. The uplink beamforming weight vectors are obtained by maximizing the received $\text{SINR}$ at the relay nodes. This can performed using generalized eigenvector decomposition technique. Assuming received signals are passed between relays, the received $\text{SINR}$ at the relay nodes can be expressed as

$$\text{SINR}_m = \frac{w_m^H(h_m h_m^H) w_m \sigma_s^2}{w_m^H \left( \sum_{k=1, k\neq m}^{M} (h_k h_k^H) + \sigma_v^2 I \right) w_m} \quad \forall \ m = 1, \cdots, M$$  \hspace{1cm} \text{(6.2.2)}$$

where $w_m$ is the beamforming weight vector that maximises the received $\text{SINR}$ from the $m^{th}$ source node. The relays rebroadcast a transformed signal vector as

$$y = Gx$$  \hspace{1cm} \text{(6.2.3)}$$

where $G \in \mathbb{C}^{N \times M}$ is a linear transformation matrix (relay transceiver) to be determined to optimise the performance at the destination nodes and $x$ is
The weight matrix $W \in \mathbb{C}^{N \times M}$ consists of uplink beamforming weight vectors $[w_1, w_2, \ldots, w_M]$ at the relays obtained using (6.2.2). The received signal vector at the destination nodes can be written as

$$t = H_t y + w_t$$  \hfill (6.2.5)$$

where $H_t \in \mathbb{C}^{M \times N}$ denotes the channel matrix consisting of complex channel coefficients between the relay nodes and the destination nodes, $H_t = [h_{t,1}, h_{t,2}, \ldots, h_{t,M}]^T$ where $h_{t,m} = [h_{t,m,1}, h_{t,m,2}, \ldots, h_{t,m,N}]$ for $m = 1, \ldots, M$ and $w_t \in \mathbb{C}^{M \times 1}$ is a zero-mean circularly symmetric AWGN vector with covariance matrix $\sigma_w^2 I$. Substituting (6.2.3) into (6.2.5) and using (6.2.1),

$$t = H_t G W^H H_s s + H_t G W^H v_s + w_t$$  \hfill (6.2.6)$$

is obtained.

### 6.3 Formulation of the SINR Cost Function for Downlink Beamforming

The downlink beamforming weight vectors at the relays is designed using the optimisation framework that minimises the total power dissipated by the relay nodes, $p_{\text{pow,in}}$, in order to achieve a prescribed target SINR at the
destination nodes. This can be written as

$$
\text{min}_D P_{\text{pow}},
$$

\text{s.t. } \text{SINR}_m \geq \gamma_m, \quad m = 1, \ldots, M

(6.3.1)

where $\gamma_m$ is the minimal acceptable SINR for the $m^{th}$ destination. The total power dissipated by the relays can be computed as

$$
E\{y^H y\} = \text{tr} (GR_rG^H) = P_{\text{pow}}.
$$

(6.3.2)

where $\text{tr}(\cdot)$ is a trace operator, and $R_r = W^H H_s H_s^H W \sigma_s^2 + W^H W \sigma_{v_r}^2$.

The signal power at the $m^{th}$ destination nodes can be derived from the product of the transfer function $H_s G W^H H_s$ with signal vector $s$ in (6.2.6) as

$$
p_{\text{sig},m} = E \{(h_{t,m} G W^H h_{s,m} s) (h_{t,m} G W^H h_{s,m} s)^H\}
= h_{t,m} G W^H h_{s,m} h_{s,m}^H W G^H h_{t,m}^H \sigma_s^2.
$$

(6.3.3)

Similarly, the received interference power at the $m^{th}$ destination access node due to signal contribution from all other source nodes can be expressed as

$$
p_{\text{int},m} = \sum_{m=1, m \neq k}^{M} E \{(h_{t,m} G W^H h_{s,k} s)(h_{t,m} G W^H h_{s,k}s)^H\}
= \sum_{m=1, m \neq k}^{M} (h_{t,m} G W^H h_{s,k} h_{s,k}^H W G^H h_{t,m}^H \sigma_s^2.
$$

(6.3.4)
The noise power transferred to the $m^{th}$ destination access node from the relay nodes can be written as

$$p_{\text{int},m} = E \left\{ (h_{t,m} G W^H v_s) (h_{t,m} G W^H v_s)^H \right\}$$

$$= h_{t,m} G W^H W G^H h_{t,m} \sigma_{v_s}^2.$$  \hspace{1cm} (6.3.5)

Therefore the optimisation problem is formulated as

$$\begin{align*}
\min_{F} \quad & p_{\text{pow.}} \\
\text{s.t.} \quad & \frac{p_{\text{sig},m}}{p_{\text{int},m} + p_{\text{noise,m}} + \sigma_{v_s}^2} \geq \gamma_m, \ m = 1, \ldots, M
\end{align*}$$  \hspace{1cm} (6.3.6)

where $\sigma_{v_s}^2$ is the variance of the noise present at the destination. It is noted that the objective function in (6.3.6) is convex. The constraint set however is not convex but it can be converted to a convex form using SDR, and solved using interior point method algorithms.

### 6.4 Formulation of Objective and Constraint Functions using SDP

To this end, the optimisation problem is solved by converting it to a SDP form that can be solved using interior-point algorithms. From (6.3.2) power usage by the relay nodes can be written as

$$E\{y^{H}y\} = \text{tr}(G W^H H_s H_s^H W G^H) \sigma_{v_s}^2 + \text{tr}(G W^H W G^H) \sigma_{v_s}^2 = p_{\text{pow.}}.$$  \hspace{1cm} (6.4.1)

The following Kronecker identity is used,

$$\text{vec}(AXB) = (B^T \otimes A) \text{vec}(X)$$  \hspace{1cm} (6.4.2)
where $A$, $X$ and $B$ are matrices of conformable dimensions and $\text{vec}(\cdot)$ is a vectorisation operator that forms a vector by stacking columns of a matrix. Using (6.4.2), the first part of (6.4.1) can be expressed as

$$
\text{tr}(GW^H H_s H_s^H WG^H) \sigma_s^2 = \text{tr}(IGQQ^H G^H I) \sigma_s^2
= \text{vec}(IGQ)^H \text{vec}(IGQ) \sigma_s^2
= g^H (Q^T \otimes I)^H (Q^T \otimes I) g \sigma_s^2
$$

(6.4.3)

where $Q = WHs \in \mathbb{C}^{M \times M}$ and $g = \text{vec}(G) \in \mathbb{C}^{MN \times 1}$ is formed by the vectorisation operation. Expressing the second part in a similar form and using properties of trace operation, power usage by the relay nodes can be expressed as

$$
p_{\text{pow.}} = g^H ((Q^T \otimes I)^H (Q^T \otimes I) \sigma_s^2 + (W^* \otimes I)^H (W^* \otimes I) \sigma_s^2) g
= \text{tr}(PD)
$$

(6.4.4)

where $P = ((Q^T \otimes I)^H (Q^T \otimes I) \sigma_s^2 + (W^* \otimes I)^H (W^* \otimes I) \sigma_s^2) \in \mathbb{C}^{MN \times MN}$, $D = gg^H \in \mathbb{C}^{MN \times MN}$ and $D$ is rank one. Since $(ht,mGWHh_s,m)$ in (6.3.3) is a scalar, $\text{vec}(ht,mGWHh_s,m) = ht,mGWHh_s,m$, and the received signal power at the $m$th destination access node can be expressed in the form of Kronecker product as

$$
p_{\text{sig.,m}} = (g^H (qt,m \otimes ht,m)^H (qt,m \otimes ht,m) g) \sigma_s^2
= \text{tr}(g^Hp_{\text{sig.,m}}g) = \text{tr}(P_{\text{sig.,m}}D)
$$

(6.4.5)

where $P_{\text{sig.,m}} = (qt,m \otimes ht,m)^H (qt,m \otimes ht,m) \sigma_s^2 \in \mathbb{C}^{MN \times MN}$ and $q_m$ a vector corresponding to the $m$th column of $Q$. The total interference power at the $m$th destination access node from all other source access nodes, shown in
(6.3.4), can be expressed in terms of Kronecker product as
\[
P_{\text{int},m} = \sum_{m=1,m\neq k}^{M} (g^H (q_{ik}^T \otimes h_{t,m}) h_{t,m}^H g) \sigma_s^2
\]
\[
= \text{tr} \left( g^H P_{\text{int},m} g \right) = \text{tr} (P_{\text{int},m} D)
\] (6.4.6)

where \(\sum_{m=1,m\neq k}^{M} (q_{ik}^T \otimes h_{t,m}) h_{t,m}^H g \sigma_s^2 \in \mathbb{C}^{MN \times MN}\) is a PSD matrix.

The total noise power transferred to the \(m^{th}\) destination access node from the relay nodes described by (6.3.5) can be written in terms of Kronecker product as
\[
P_{\text{nce},m} = (h_{t,m} G W^H W G^H h_{t,m}^H)
\]
\[
= (g^H (W^* \otimes h_{t,m}) h_{t,m}^H g) \sigma_s^2
\]
\[
= \text{tr} (g^H P_{\text{nce},m} g) = \text{tr} (P_{\text{nce},m} D)
\] (6.4.7)

where \(P_{\text{nce},m} = (W^* \otimes h_{t,m}) h_{t,m}^H g \sigma_s^2 \in \mathbb{C}^{MN \times MN}\) is a PSD matrix. The optimisation problem in (6.3.6) can now be expressed as
\[
\min_{D} \quad \text{tr} (PD)
\]
\[
s.t. \quad \frac{\text{tr} (P_{\text{sig},m} D)}{\text{tr} (P_{\text{int},m} D) + \text{tr} (P_{\text{nce},m} D) + \sigma_s^2} \geq \gamma_m
\]
\[
\text{rank}(D) = 1, \quad D \succeq 0, \quad D = D^*
\]
\[
m = 1, \ldots, M.
\] (6.4.8)

The above scheme is not convex due to the constraint on rank, i.e. \(\text{rank}(D) = 1\). The problem can however be relaxed into a convex problem by using the standard techniques of SDR by dropping the constraint, \(\text{rank}(D) = 1\), so
that the optimisation problem is formulated as

$$\begin{align*}
\min_{D} & \quad \text{tr}(PD) \\
\text{s.t.} & \quad \text{tr}(P_{\text{sig,m}}D) - \gamma_m \text{tr}(U_mD) \geq \gamma_m \sigma_t^2 \\
& \quad D \succeq 0, \quad D = D^* \\
& \quad m = 1, \ldots, M
\end{align*} 
$$

(6.4.9)

where $U_m = P_{\text{int,m}} + P_{\text{noise,m}}$. The objective function and constraints in (6.7.4) are convex and can be solved using interior point methods [34]. The algorithm generally yields $D$ as a rank one matrix from which the signal space is extracted to form the vector $g$. When the rank of $D$ turns out to be greater than one, a randomisation approach could be used to obtain $g$ from $D$ [77], [78].

### 6.5 Complexity Analysis

The complexity of solving the optimisation problem proposed in Chapter 5 equation (5.4.9), denoted scheme 1 is analyzed. The optimisation problem consists of one matrix variable of size $N^2 \times N^2$ and $M$ linear constraints, where $M$ is the number of communicating pairs of source-destination nodes and $N$ is the number of relays involved. The interior point methods will require $O[\sqrt{N^2}\log(1/\epsilon)]$ iterations to converge with a solution accuracy of $\epsilon$ at the termination of the algorithm. Each iteration would require $O[(N^2)^6 + MN^2]$ arithmetic operations in the worst-case bound.

The complexity of the optimisation scheme proposed in this chapter (denoted scheme 2) can be analyzed using similar arguments as above. Similar to the scheme analyzed above, the optimisation problem in (6.7.4) consists
of one matrix variable. However unlike above, the size of this variable is related to the number of communicating source-destination pairs and the number of relays, and is given as $MN \times MN$ (see equation (6.7.4)). Similar to the above problem, there are $M$ linear constraints in the optimisation problem. To solve this problem, the interior point methods will require $\mathcal{O}(\sqrt{MN}\log(1/\epsilon))$ iterations to converge with a similar solution accuracy of $\epsilon$ as above, at the termination of the algorithm. Each iteration would require $\mathcal{O}((MN)^6 + M(MN)^2)$ arithmetic operations in the worst-case bound. Therefore for $M < N$ scheme 2 has a reduced order of complexity compared to scheme 1. It can be noticed that the complexity of the proposed scheme will approach that of its predecessor as the number of source-destination pairs approaches the number of relays involved in signal forwarding. When $M = N$, the two schemes have a similar order of complexity. For scenarios where $M > N$, scheme 1 has a lower order of complexity than scheme 2.

### 6.6 Per-antenna Constraints

The total power constraint in the above optimisation problem is modified and a per relay node power constraints are introduced. This constraints is more realistic in terms of setting a limitation on power consumption at each relay, which is limited in a practical environment. This scheme is expected to draw a higher total power than the previous scheme with a constraint on the total power consumption. This is because the domain of the optimisation problem is shrunk. Subtle changes to the existing scheme are made and the signal to be transmitted by the relays is expressed as $y = x^T K \in \mathbb{C}^{1 \times N}$ where $K = G^T$. The transmitted signal can then be expressed in Kronecker
product form as

\[ y = s^T(n) (I \otimes Z) k + v_s^T (I \otimes W^*) k \quad (6.6.1) \]

where \( Z = H_s^T W^* \) and \( k = \text{vec}(K) \in \mathbb{C}^{MN \times 1} \). The total power transmitted can then be expressed as

\[ P_{\text{tot.}} = E\{y^H y\} = ((I \otimes Z)^H (I \otimes Z) + (I \otimes W^*)^H (I \otimes W^*)) D \quad (6.6.2) \]

where \( D = kk^H \in \mathbb{C}^{MN \times MN} \) is the rank-one optimum relaying matrix to be determined using semidefinite programming. The per relay node power constraint can now be expressed as

\[ p_{\text{pow},i} = \text{tr}(P_{\text{pow},i} D) \quad \forall \ i = 1, \ldots, N \quad (6.6.3) \]

where \( P_{\text{pow},i} = P_{\text{tot.}} T_i \). Matrix \( T_i = \text{diag}\{a_i\} \), where \( a_i \in \mathbb{C}^{MN \times 1} \) is a vector with 1 at rows \((Mi, Mi - 1, \ldots, M(i - 1) + 1)\) for \( i = 1, \ldots, N\) and 0 elsewhere. The optimisation problem can now be expressed as

\[
\begin{align*}
\min_D & \quad \text{tr}(P_{\text{tot.}} D) \\
\text{s.t.} & \quad \text{tr}(P_{\text{sig},m} D) - \gamma_m \text{tr}(U_m D) \geq \gamma_m \sigma_v^2, \ \forall \ m = 1, \ldots, M \\
& \quad \text{tr}(P_{\text{pow},i} D) \leq \beta_i, \ \forall \ i = 1, \ldots, N \\
& \quad D \succeq 0, \ D = D^* 
\end{align*}
\]

where \( \beta_i \) is the power constraint at the \( i^{th} \) relay node. The matrices \( P_{\text{sig},m} \) and \( U_m \) follow from equations (6.4.5) and (6.7.4) respectively.
Section 6.7. Interference Mitigation using Relays

The scheme is extended to incorporate a spectrum sharing network scenario for example, an underlay cognitive radio network, Figure 6.1. The relays mitigate the interference leaked to the primary users and achieve prescribed target SINRs at the secondary users, whilst minimizing the total power consumption at the relays. This is performed by introducing the constraint, $P_{Pr,int,p} + P_{Pr,noise,p} \leq \epsilon$ where $P_{Pr,int,p}$ is the interference power leaked to the $p^{th}$ primary user from the source nodes and $P_{Pr,noise,p}$ is the noise power leaked from the relay nodes to the $p^{th}$ primary user. To distinguish between channels between secondary and primary users, the notation $H_o$ for channels to primary users and $H_u$ for channels to secondary users is introduced. The interference leakage to the $p^{th}$ primary user from the source nodes can be...
Section 6.7. Interference Mitigation using Relays

expressed as

\[ p_{Pr.int,p} = \sum_{m=1}^{M} (h_{v,p}GW^Hh_{s,m}h_{s,m}WG^Hh_{v,p}) \sigma_s^2 \]

\[ = \sum_{m=1}^{M} (g^H(q_m^T \otimes h_{v,p})^H(q_m^T \otimes h_{v,p})g) \sigma_s^2 \]

\[ = \text{tr}(P_{Pr.int,p}D) \]  \hspace{1cm} (6.7.1)

where \( P_{Pr.int,p} = (q_m^T \otimes h_{v,p})^H(q_m^T \otimes h_{v,p})g \in \mathbb{C}^{MN \times MN} \), \( g \) and \( q_m \) are the same variables as defined in the previous sections. The noise power leaked to the primary receivers from the relays is written as

\[ p_{Pr.nse,p} = (h_{v,p}GW^HWG^Hh_{v,p}) \sigma_v^2 \]

\[ = (g^H(W^* \otimes h_{v,m})^H(W^* \otimes h_{v,m})g) \sigma_v^2 \]

\[ = \text{tr}(P_{Pr.nse,p}D) \]  \hspace{1cm} (6.7.2)

where \( P_{Pr.nse,p} = (W^* \otimes h_{v,m})^H(W^* \otimes h_{v,m}) \in \mathbb{C}^{MN \times MN} \) and \( W \) is the same as defined above. Therefore the optimisation scheme can be stated as

\[
\min_{D} f^H P_{tot} f
\]

s.t. \( f^H P_{sig,m} f - \gamma_m f^HZmf \geq \gamma_m \sigma_v^2, \ \forall \ m = 1, \ldots, M \) \hspace{1cm} (6.7.3)

\[ f^H P_{pow,i} f \leq \beta_i, \ \forall \ i = 1, \ldots, N \]

\[ f^H (P_{Pr.nse,p} + P_{Pr.int,p}) f \leq e_p, \ \forall \ p = 1, \ldots, P \]

where \( e_p \) is the acceptable interference limit for the \( p^{th} \) primary user and \( P \) denotes the number of primary users. The problem can be formulated into a semidefinite form and relaxed into a convex problem by dropping the rank
one constraint. The relaxed problem can be expressed as

\[
\begin{align*}
\text{min.} & \quad \text{tr}(\mathbf{P}_{\text{tot}} \mathbf{D}) \\
\text{s.t.} & \quad \text{tr}(\mathbf{P}_{\text{sig,}m} \mathbf{D}) - \gamma_m \text{tr}(\mathbf{U}_m \mathbf{D}) \geq \gamma_m \sigma^2_{v_m}, \quad \forall \ m = 1, \ldots, M \\
& \quad \text{tr}(\mathbf{P}_{\text{pow,i}} \mathbf{D}) \leq \beta_i, \quad \forall \ i = 1, \ldots, N \\
& \quad \text{tr}((\mathbf{P}_{Pr,\text{nse},p} + \mathbf{P}_{Pr,\text{int},p}) \mathbf{D}) \leq \epsilon_p, \quad \forall \ p = 1, \ldots, P \\
& \quad \mathbf{D} \succeq 0, \ D = D^*.
\end{align*}
\]

where \( \mathbf{U}_m \) is defined as before. This rank relaxed SDP can be solved using interior point methods.

**Lemma.** Provided the problem in (6.7.4) is feasible, the relaxation provides a rank-one matrix \( \mathbf{D} \) which achieves the same global minimum as the original problem.

**Proof of Lemma.** The proof of the lemma follows from the proof derived in the Appendix of Chapter 5. It is required to show that Hessian of the Lagrangian is positive semidefinite [77] for a rank one solution to exist when the relaxed problem \( \text{(RP)} \) (6.7.4) is feasible. The Lagrangian of the original optimisation problem \( \text{(OP)} \) (6.7.3) can be expressed as

\[
L(f, \lambda_m, \mu_i, \alpha_p) = f^H \mathbf{P}_{\text{tot}} f + \sum_{m=1}^{M} \lambda_m [\gamma_m f^H \mathbf{Z}_m f - f^H \mathbf{P}_{\text{sig,}m} f + \gamma_m \sigma^2_{v_m}] \\
+ \sum_{i=1}^{N} \mu_i [f^H \mathbf{P}_{\text{pow}} f - \beta_i] \\
+ \sum_{p=1}^{P} \alpha_p [f^H \mathbf{P}_{Pr,\text{nse},p} f + f^H \mathbf{P}_{Pr,\text{int},p} f - \epsilon_p]
\]

\[(6.7.5)\]
where $\lambda_m, \mu_i, \alpha_p$ are non negative Lagrangian multipliers. The Hessian follows from this and can be expressed as

$$
\nabla^2, L(f; \lambda_m; {\mu_i}; \alpha_p) = P_{\text{tot.}} + \sum_{m=1}^{M} \lambda_m [\gamma_m Z_m - P_{\text{sig,m}}] + \sum_{i=1}^{N} \mu_i [P_{\text{pow}}]
$$

$$
+ \sum_{p=1}^{P} \alpha_p [P_{Pr.nse,p} + P_{Pr.int,p}].
$$

(6.7.6)

To show that the Hessian is positive semidefinite the dual of (OR) is derived and is expressed as

$$
\max_{\lambda_m, \mu_i, \alpha_p} \sum_{m=1}^{M} \lambda_m [\gamma_m Z_m - P_{\text{sig,m}}] + \sum_{i=1}^{N} \mu_i \beta_i - \sum_{p=1}^{P} \alpha_p e_p
$$

s.t. $P_{\text{tot.}} + \sum_{m=1}^{M} \lambda_m [\gamma_m Z_m - P_{\text{sig,m}}] + \sum_{i=1}^{N} \mu_i [P_{\text{pow}}]
$$

$$
+ \sum_{p=1}^{P} \alpha_p [P_{Pr.nse,p} + P_{Pr.int,p}] \geq 0.
$$

(6.7.7)

Since (RP) satisfies Slater’s constraint qualification condition [76] it has a strong duality with its dual. This implies that when (RP) is feasible, so is the dual problem. Since the constraint of the dual problem (6.7.7) is the Hessian of the Lagrangian, when (RP) is feasible, the Hessian is positive semidefinite and a strong duality exists between (OP) and its dual. Since the dual of (OP) and (RP) are the same, it follows that both (OP) and (RP) achieve the same global minimum implying that a rank one solutions exists.

As mentioned earlier in Chapter 5, the signal space can be extracted from matrix $D$ to form the vector $f$, and the optimum relaying matrix $F$ can be formed by reversing the vectorisation operation on $f$. 


6.8 Simulation Results

The performance of the proposed scheme is investigated for network with a relay layer comprised of four relays. The channels are generated using zero-mean unity variance complex Gaussian variables. The power of the information symbols is set to unity and the variance of AWGN at the relays and all the users is kept to $-15$ dB.

The performance of the reduced complexity scheme is compared with its predecessor, i.e. the semidefinite scheme proposed in Chapter 5. Identical network parameters, in terms of noise variance and channel coefficients, are set in both the schemes. The total power drawn by the relays in both the schemes for identical target SINRs is computed. Figure 6.3 shows this comparison for two, three and four pairs of source-destination nodes. From the simulations drawn, it is evident that the reduced complexity scheme has an identical performance to the scheme proposed in Chapter 5.

The performance of relays in a cognitive radio scenario is investigated for a relay network comprising of four relays, two source nodes, two secondary and one primary destination node as shown in Figure 6.2. The power used by the per relay antenna was restricted to 1W. The variance of AWGN is fixed to $-15 dB$. Simulations are drawn at various target SINRs at secondary destination nodes, for different upper bounds on the interference power leaked to the primary destination nodes. Figure 6.4 shows the performance of the proposed scheme.

6.9 Summary

The chapter proposed a reduced complexity multiple relaying strategy for wireless networks with user discretion. The performances of the proposed
Figure 6.2: A relay network of 2 sources, 2 secondary user nodes, 1 Primary user node and 4 cooperating relays.
Figure 6.3: Power utilisation at the relay nodes at various target SINRs, for 2-, 3- and 4-pairs of source destination nodes.

scheme and its predecessor, proposed in Chapter 5, were shown to be identical through simulation results. The proposed scheme was shown to have a reduced complexity for scenarios where $M < N$. A more realistic per relay antenna power constraint algorithm was developed. The scheme was further modified to be applied in an underlay cognitive network. It achieved a set of target SINRs at the secondary user destination nodes while limiting the interference and noise power leaked to the primary destination terminals below a pre-defined threshold. Semidefinite programming was used to obtain optimal weight vectors to satisfy the required criterion. Simulations showing trade offs between power consumption at the relays, varying target SINRs and threshold of interference and noise powers leakage to the primary users were drawn.
Figure 6.4: Trade off between power consumption, target SINR and threshold of interference and noise power leaked to the primary destination node.
Wireless relays prove to be beneficial in providing essential diversity gains in networks where multiple antennas cannot be colocated at source-destination nodes, responding to their demands of higher data rates, increased capacity and broader network coverage. Their application is also favoured by their ease of implementation due to the absence of wired infrastructure in interfacing with the existing network. Wireless relay networks can be created by merely deploying relays into the existing wireless network framework. Relays can also add new dimensions to the network. For instance, relays can be used to vary QoS provided to end users, creating competitive price versus QoS packages for end users. In a cognitive radio scenario, relays can be applied to utilise white spaces (spectrum holes), enhancing the efficiency of spectrum utility while controlling the interference leakage to primary user destination nodes. The realisation of this potential calls for creative algorithms to be developed. This thesis addressed this need and proposed novel algorithms for smart use of cooperative relay networks. Cooperation, on a signal exchange at the relay level, is supported by drawing results showing the performance enhancement against non cooperative schemes in terms of power consumption and outage probability. The thesis has three
Section 7.1. Conclusions

contributing chapters and the conclusion of each chapter is summarized below.

7.1 Conclusions

An MMSE based optimisation algorithm for relay networks was developed in Chapter 4. The scheme exploits transmit-receive diversity by employing cooperation amongst relays in the sense that relays pass signals between themselves to design an optimum relaying matrix. With a constraint on the total power consumption at the relay nodes, a closed form optimisation problem was formulated and solved using Lagrangian optimisation. A novel technique to reduce the complexity in searching the roots of the polynomial equation was proposed. A comparison was made to study the performance enhancement using the proposed cooperative scheme over a non cooperative, MMSE based signal forwarding scheme, proposed in [7]. This analysis was based on the BER performance of both the schemes and the superiority of the proposed scheme was realised with a 2dB gain in SNR at a BER of $1 \times 10^{-1}$, using two relays in a single source-destination scenario. At a BER of $2 \times 10^{-7}$, a gain of 7dB was achieved using four relays. Increasing the number of relays from four to eight improved the BER performance by 3dB at a BER of $2 \times 10^{-2}$ in a two source-destination scenario. A lemma was proposed to show the decomposition of the optimum relaying matrix into a product of receiver and transmitter beamformers, for a single source-destination scenario at the relay layer.

A convex optimisation based algorithm was proposed in Chapter 5. The algorithm enabled QoS to be set at the destination nodes while minimizing the total power required to achieve them. Since cooperation involved signal
Section 7.1. Conclusions

to be exchanged amongst relays, the overheads involved in terms of quantisation noise powers (incurred as a result of quantisation of signals at the relay nodes before being exchanged between them) was accounted for. A BER comparison was drawn to demonstrate an improvement of 2dB SINR at a BER of $2 \times 10^{-2}$ over the MMSE scheme proposed in Chapter 4, under identical power consumption at the relays. This gain is attributed to the ability of the semidefinite scheme to impose target SINR at destination nodes, which controls the power assigned to them. Table 5.1 showed the gains in terms of outage probabilities, i.e. the probability that the scheme does not achieve the target SINR, of the cooperative scheme over the non cooperative scheme. At a target SINR of 12dB the non cooperative scheme fails to achieve the target twice as many times as the cooperative scheme.

The power consumption of the cooperative scheme, whilst accounting for the quantisation noise using one, two and three bit quantisers. The robust scheme developed was shown to achieve the target SINR 50% of the time yielding a better BER performance compared to the non robust scheme. A lemma was provided to prove that the rank relaxed semidefinite optimisation problem, when feasible, always yielded a rank one solution. From this chapter, the main conclusion to be drawn is that cooperative schemes provide a better power economy than the non cooperative schemes.

The final contributing chapter, Chapter 6, was on complexity reduction of the relaying scheme developed in Chapter 5. This was performed using uplink-downlink beamforming techniques, which were formulated into a two stage signal forwarding scheme. A more realistic per relay antenna power constraint was also implemented and an algorithm for the application of cooperative relay networks in an underlay cognitive network scenario was developed. A complexity analysis was carried out to show the reduced
complexity of the proposed scheme over the scheme developed in Chapter 5 when the number of relays is more than the number of source-destination pairs. Simulations showed that the reduced complexity algorithm provides identical performance gains to the scheme proposed in Chapter 5. Simulations were carried out to show the tradeoffs between power consumption, target SINR and threshold of interference and noise power leaked to the primary destination node.

7.2 Future work

Much work remains to be done to completely realise the potential of cooperative relay networks. Radically new algorithms have to be developed for this purpose. Two possible areas of extension are envisaged.

7.2.1 Admission Control Techniques

In a given environment sustaining relays, the ability of the relay network to achieve desired QoS using the least number of relays would prove beneficial. Such a network would be capable of selectively admitting relays based on the magnitude of task it is required to execute and the resources available to the relay nodes. Such a system would be feasible only in a cooperative framework. It is envisaged that the suboptimal solutions to such a problem can be obtained using combinatorial optimisation techniques. The key to solving this problem is in overcoming the issue of nonconvexity of the objective function by narrowing the domain of the optimisation problem to suffice convexity. The outcome to this solution would propel research opportunities in the direction of convex optimisation using integer programming techniques.
7.2.2 Maximizing the QoS

The full potential of an individual relay nodes can be exploited when they operate at the maximum of their power budget. A scheme that ensures this would be maximizing the QoS that the relays could provide. Such a scheme would be ideal in SN applications where user discretion is not important and destination nodes can only benefit from higher received SINRs. Iterative solutions to this problem exist, but deem impractical and rather trivial. The main problem in obtaining an optimal solution to this problem is in formulating an appropriate objective function that would maximise the QoS over a domain that is larger than the power budget of each of the relays. Further research into this problem would lead to smarter relay networks.
References


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References

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References


