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INVERTER CONTROL OF LINEAR INDUCTION MOTORS
WITH DIFFERENT REACTION PLATES

by

SALAH HAMIED AL-DABBAS, B.Sc.

A Master's Thesis
Submitted in partial fulfilment of the requirements
for the award of
Master of Science Degree of the
Loughborough University of Technology

September, 1980

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Department of Electronics and Electrical
Engineering

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I dedicate this thesis to
my Mother
ACKNOWLEDGEMENTS

I am deeply indebted to my supervisor, Dr J K Hall for his guidance and constant encouragement throughout the course of this research. I am also grateful for his valuable suggestions during the preparation of this thesis.

My heartfelt thanks and great appreciation are extended to my mother, my elder brother Mr Sabah Al-Dabbas for the financial and moral support they have readily given to me throughout the course of my study.

Finally, I wish to thank my colleagues of the Power Electronics Laboratory with whom I often held useful discussions.
SUMMARY

In recent years, attempts to develop new means of high speed efficient transportation have led to considerable worldwide interest in high speed trains. This in turn has generated interest in the linear induction motor, which is considered to be one of the most suitable propulsion systems for super-high-speed trains.

The operation of linear induction motors is based upon the same natural principles as the cylindrical rotor form of the machine, and this implies that the same methods of speed control by static frequency changer can be used. Linear motor speed control using a variable frequency inverter, which has been used for some years with rotary induction motors, is investigated in this work.

The steady-state performance of a motor fitted, in turn, with an aluminium reaction plate and aluminium cladded steel reaction plate, is described.

A 3-phase variable frequency d.c. link-fed inverter has been constructed to provide speed control for the motor. The d.c. link is supplied from a voltage source. The inverter has a single a.c. side commutation and operates with three-thyristor triggering, that is with 180° thyristor conduction, giving a quasi-square wave output voltage.

The logic control and thyristor switching circuits have been designed and built and are described in detail.

From no-load and locked rotor tests, the equivalent circuit parameter for the linear induction motor with the two types of reaction plates have been measured for different frequencies with a sinusoidal supply. These are used to predict the motor steady-state performance.
The measured performance of the motor with each type of rotor with a sinusoidal supply is compared with the results predicted for various frequencies (50, 40, 30, 20 and 10 Hz), allowing for the effect of the variation of the equivalent circuit rotor resistance parameter with slip frequency.

A comparison of the motor performance is made when fitted with both types of reaction plate, when excited both sinusoidally and from the inverter.
LIST OF PRINCIPAL SYMBOLS

a - Deceleration
C₁, C₂ - Commutating capacitors
D₁-D₆ - Free wheeling or return diodes
δ - Skin depth
E - Linear motor rotor voltage referred to stator
f - Frequency
I₁ - Linear motor stator current
I₂ - Linear motor rotor current referred to stator
Iₘ - Linear motor magnetising current
J - Moment of inertia
L₁, L₂, L₃ - Commutating inductors
P₁ - Resistance
P₂ - Volume resistivity of the plate material
Q₁ - Inductive reactance
R₁ - Linear motor stator resistance
R₂ - Linear motor rotor resistance referred to stator
S - Per unit slip
T - Thrust
Tₕ - Mechanical loss torque
Tₐ-tₖ - Invertor main power thyristors
Tₐₜ-tₖₙ - Invertor auxiliary thyristors
t - Time
V - Linear motor stator applied voltage
Vₛ - Synchronous linear speed
Vₗ - Rotor linear speed
Vₛ - Invertor d.c. link instantaneous voltage
Vₗₚ - Invertor auxiliary d.c. supply voltage
Vₗₚ - Peak point voltage
Vₗₚ - Valley voltage
ω - Final angular speed
ω₀ - Initial angular speed
Xₗ - Linear motor stator leakage reactance
$X_m$ - Linear motor magnetising reactance
$Z_{in}$ - Linear motor input impedance

* All other symbols are defined as they appear.
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CHAPTER 1

INTRODUCTION

1.1 General Background

The history of linear induction motors extends as far back as the 19th century. During this period, many attempts at commercially applying the linear motor were made, but owing to the problems of control, excessive heat generation and retention, very few developments were pursued to commercial achievement. During the first half of the 20th century further progress was made and workable machines were developed. Again, commercial exploitation did not follow due to excessive material costs and problems in fully controlling the machines.

The linear form of the induction motor is believed to have been invented in 1890. However, these motors were almost forgotten for 30 or 40 years before ideas for practical application began to emerge, and it was not until after World War II that developments of major significance were announced\(^1\).

Nevertheless, from time to time, certain engineers have tried to revive interest in linear motion and as early as 1908, the famous French engineer Boucherôt expressed interest\(^2\).

In 1905 proposals were made by Alfred Zehden to use linear motors for railway traction. He suggested a double-sided motor primary mounted on the vehicle. His aims were the reduction of track cost to its lowest possible value and the elimination of 'magnetic pull' between the pole faces of what are effectively opposite sets of electromagnetics in the conventionally arranged primary and secondary windings of an induction motor\(^3,9\).

In the same year, another proposal was made to use linear induction motors for railway transit, but the project was not developed due to the cost and the availability of other forms of propulsion\(^8\).
In 1923, a linear motor moving walkway in New York was proposed and test track was constructed. However, the full scale project was abandoned\(^{(2)}\).

In 1946 the Westinghouse Company of America successfully applied the linear motor to the concept of the aircraft accelerator known as the "Electropult". A speed of 185 km per hour was achieved in 4.2 seconds. The system was abandoned because of the initial cost, and cheaper systems such as the steam catapult became accepted\(^{(2,4)}\).

During the 1950's Professor Eric Laithwaite at Manchester University became interested in various "exotic" types of machines and became quite enthusiastic about the linear induction motor. Since then, he has published widely\(^{(5,6,7,8)}\), attracting the attention of many engineers to the subject. Investigators started to be convinced of the practicality of linear motors\(^{(4)}\).

During the 1950's the workers at Manchester University, in collaboration with British Rail, re-invented the double sided sandwich arrangement for the same reason as did Zehden, without the knowledge of the latter's work\(^{(9)}\).

In 1968 the first commercial linear machine in the United Kingdom was designed as an accelerator for crash testing cars at the Motor Industry Research Association Laboratories at Nuneaton\(^{(9)}\).

The thyristor was first introduced in 1957. Since then it has grown in popularity and now finds use in widely differing fields. Nowhere has its effect been felt more than in machine control. Owing to its small size, short turn-off time, and low forward voltage drop, ruggedness and relatively low cost, the thyristor has given practical reality to systems which were previously only an experimenter's dream\(^{(11)}\).
The invention of the thyristor has opened up new vistas in the field of motor control and thyristor drives have been widely used throughout industry. The static variable-frequency a.c. drive uses the robust squirrel cage induction motor, or a synchronous reluctance motor, powered by a static frequency convertor. This gives a versatile and robust variable-speed machine which has the advantage over thyristor-controlled d.c. motor drives.

The main objection to the static a.c. drive has been on economic grounds, since the cost of the static frequency convertor has often been considered excessive. However, power semiconductor prices are steadily decreasing as production volume grows and manufacturing techniques improve, and the future of the solid-state a.c. drive is assured.

There are two basic types of static frequency convertors; firstly, there is the cycloconvertor in which alternating supply is converted directly into a.c. of variable frequency. In this case the thyristors are used to selectively connect the load to the supply source, so that the low-frequency output voltage waveform is fabricated from segments of the supply voltage waveform. The disadvantage of this type of convertor is that the highest output frequency is limited to not more than 1/3 of the supply frequency. The second type of convertor is the d.c. link 3-phase bridge invertor, in which the a.c. supply is first rectified to d.c. and then inverted to a.c. at variable frequency. The main thyristors of the invertor are triggered sequentially such that a rectangular or stepped wave voltage is generated at the output. In contrast to the cycloconvertor, the output frequency of the d.c. link invertor can range from a few hertz to several hundred hertz. Mainly for this reason, d.c. link invertors have found wide application (16).

The linear induction motor has all the properties of a cylindrical machine to a varying degree, and this implies the possibility
of speed control by the similar control techniques.

The linear motor has several advantages over the conventional squirrel cage motor:

- It is smaller than its counterpart for a given rating.
- It is a compact unit of regular shape, usually rectangular or tubular.
- It is lighter in weight, vibration free, costs less to acquire and maintain.
- It is silent in operation.
- It does not require direct conventional gearing and therefore needs no moving parts. This reduces maintenance costs.
- Once constructed a linear motor can be encapsulated in an impregnable protective resin coating and insulated from the external environment.
- The thrust is continuous and stepless as no gears are required.

Because of these advantages, over its rotating counterparts, it has the following attractions for railway traction (10):

1. The absence of sliding electrical contacts, rotating electromagnetic parts and gears.
2. Freedom from limitations, imposed by adhesion.
3. Reduction of the weight of the motor carried on the vehicle because the rotor element is fixed to the track.
4. Reduced cost of the electromagnetic part of the motor because of its simple construction.
5. Improvement in thermal performance because the reaction losses are left behind as the motor proceeds.
6. Freedom from restrictions imposed by the peripheral surface speed of rotating motors.

Use of a linear motor has corresponding disadvantages. These are as follows (10):
i) Loading gauge restrictions.
ii) Cost of reaction plate.
iii) Low efficiency and power factor.
iv) Need for a lateral guidance system.
v) Difficulties on curved track at points and crossings.
vi) Three-phase supply with variable voltage and frequency.

Research on linear induction motors is actively being pursued in a number of countries such as Canada, Japan, West Germany and the United Kingdom. The last few years have seen intensive development and a number of companies are manufacturing linear motors for such diverse applications as conveyors, sliding door gear, overhead cranes, rapid transit vehicles, super high-speed trains.

1.2 Material of Investigation

A three-phase, thyristorised bridge-connected invertor was built for the investigation. The logic circuit was designed and built to control the power circuit of the invertor with a $180^\circ$ thyristor conduction pattern and for output frequencies of 10 Hz to 50 Hz. This is described in the following chapter.

A three-phase diode bridge connected rectifier supplied from a variable-amplitude a.c. source was built to provide a voltage fed d.c. link supply for the invertor. A similar rectifier was built to provide the d.c. commutating supply.

The linear induction motor used for the investigation was of the rotating disc-type. This gives a much more compact system than with pure linear motion, with less difficulty in loading and its measurement. The term stator and reaction plate are used in this work for the primary and secondary circuit. Two discs were available, one aluminium and the other aluminium cladded steel, and both have been used.
1.3 **Object of the Investigation**

The objectives of the investigation are as follows:

1. To design and build a voltage-fed inverter to drive a linear induction motor fitted with two types of reaction plate, i.e. aluminium and aluminium cladded steel.

2. To obtain the equivalent circuit parameters with a 50 Hz sinusoidal supply for the 3-phase linear induction motor fitted with both types of reaction plate.

3. To measure the performance of the two types of linear motor at varying frequencies 50, 40, 30, 20, 10 Hz on sinusoidal supply and compare with predicted results obtained from the equivalent circuit.

4. To measure the performance of the two types of linear motor when fed with a quasi-square wave voltage obtained from the variable frequency inverter.

5. To compare the performance of the motor when fitted with both types of reaction plate when fed from both sinusoidal and quasi-square wave supplies.

1.4 **Method of Approach**

So far, little published work exists concerning speed control of linear induction motors from variable frequency invertors\(^{(12,13)}\).

Previous work in this department on the linear induction motor investigated the linear motor's behaviour when supplied from an inverter compared with a variable frequency sinusoidal excitation. The work also considered its interaction with the inverter. The two thyristor triggering mode was used i.e. two of the six main
thyristors were triggered simultaneously\(^{(14)}\). However, Allin, Creighton and Hall showed the disadvantages of the two thyristor \((120^\circ)\) triggering mode through their analysis of the invertor-motor system. This gives rise to discontinuities in the voltage waveform during an invertor period\(^{(12)}\).

Recent work\(^{(15)}\) compared the performance of an invertor fed induction-motor system operating in the \(180^\circ\) and \(120^\circ\) conduction modes. It was found that the \(180^\circ\) conduction mode gave the better performance, in particularly smaller torque pulsations, and this mode, which requires three thyristors to conduct simultaneously, has been adopted for this investigation.

It was intended that the work should fall into two main categories:

a) Selection of a suitable variable frequency invertor and its construction.

b) A comparison between the operation of the linear induction motors on both a variable frequency sinusoidal supply, and the invertor supply.

The first parts of the thesis deal with the design and construction of the invertor switching and logic circuits. The later chapters compare the performance of the linear motor, when fitted with both aluminium and aluminium clad steel reaction plates, and when excited sinusoidally or from the invertor.

An equivalent circuit for the linear motor was determined to give the motor parameters at various frequencies with a sinusoidal supply for both types of reaction plate. The equivalent circuit was used to predict the motor steady-state performance with sinusoidal supply. A comparison was made between the predicted and measured performance, allowing for the effect of the variation of the equivalent circuit rotor resistance with slip frequency. The conclusions and suggestions for further work then follow.
CHAPTER 2

THE DC-LINK INVERTOR

2.1 Introduction

The invertor used in the investigation belongs to that class of controlled convertors that provide variable frequency 3-phase supplies from d.c. main; it is of the type often called d.c. link invertors. The main components are thyristors acting as switching devices and basically the operation is one of sequentially switching the d.c. supply to a poly-phase load. Variable output voltage is needed obtained by varying d.c. link voltage with a fixed invertor conduction pattern. The invertor is of the voltage fed type. Forced commutation utilising auxiliary components to turn off the main thyristors, is required as the load will not develop its own e.m.f. to provide natural commutation, as does a synchronous machine.

Various types of commutation methods and their relative merits which can be employed in these invertors are discussed in Section 2.3. The 3-phase bridge invertor used is a single a.c. side commutated type. In this the commutation is forced by the auxiliary circuit which causes the turn-off of all thyristors in either the top or the bottom half of the bridge alternately at the end of each invertor period i.e. 1/6 of the operating cycle. The operation is explained in Section 2.3. This invertor requires only two commutating thyristors, thus providing a very economic proposition.

2.2 Invertor Conduction Patterns

The sequence by which the main thyristors are turned on and off is known as the conduction pattern, and this determines the motor phase voltage waveform and has an influence on the input power to the motor. The two usual conduction patterns are the
two-thyristor triggering mode (i.e. $120^\circ$ thyristor conduction) and the three-thyristor triggering mode (i.e. $180^\circ$ thyristor conduction). With reference to the inverter circuit of Figure 2.1, the former conduction patterns may be explained as follows.

A single thyristor in the top row (1, 3 or 5) conducts, connecting R, S or T of the load to the d.c. positive whilst another thyristor in the bottom row (2, 4 or 6) returns current to the d.c. negative. In the latter method, conduction is via any two thyristors in the top row, with a return path through one in the bottom row or vice versa. The two modes of the thyristor triggering sequences and their idealised output voltage waveforms are shown in Figure 2.2.

Both modes of operation have been investigated elsewhere\(^{(15,18)}\) and are in wide use.

The advantages and disadvantages of the $120^\circ$ and $180^\circ$ conduction patterns are:

i) $120^\circ$ conduction pattern

a) For the $120^\circ$ mode of operation, only two thyristors conduct at any instant (i.e. each thyristor conducts for $120^\circ$ of $360^\circ$ cycle).

b) The logic and trigger circuit in this case is simpler than for $180^\circ$ conduction since a thyristor is not required to conduct until $60^\circ$ after turn-off signal is applied to the complementary thyristor in the opposite half of a given phase.

c) The a.c. line currents become discontinuous for high power factor load. During the zero current interval the phase voltage appearing across the motor is not zero but is an e.m.f. arising from the mutual coupling between the winding and the other rotor and stator windings. Hence for the $120^\circ$ mode of operation the inverter output phase voltage is a
FIG. 2.1.— THE GENERAL D.C. LINK TYPE INVERTOR CIRCUIT.
THREE — Thyristor Sequence
(Resistive and inductive loads).

TWO — Thyristor Sequence
(Only inductive load).

FIG. 2.2. — INVERTOR VOLTAGE WAVEFORMS.
function of the motor load during that part of the inverter operation for which the phase is disconnected from the supply.

d) When the motor is fed from 120° inverter the open circuit does not occur on light load. However at rated and heavy load the open circuit usually occurs giving an increment in the r.m.s. value of stator-current, therefore the motor losses are increased. This results in a reduction of the torque available at a given speed.

e) In order to produce a given value of torque using two-thyristor triggering and a fixed value of the d.c. link voltage, the value of slip will be greater than that when three-thyristor triggering is employed (15,18).

f) With the 120° mode, the commutation current is increased implying the need for larger commutation components and indicating higher inverter switching losses.

g) With the 120° mode, the inverter voltage does not have a well-defined waveform and its shape varies considerably during transient load conditions.

ii) 180° conduction pattern

a) For the 180° mode of operation, three thyristors are always triggered at any instant of time (i.e. each thyristor conducts for 180° of the cycle) and this provides continuous conduction in all three motor lines.

b) A problem with the 180° mode is that the outgoing thyristors must be completely turned off before firing the new thyristor group, since an incoming thyristor will be of the same phase as an outgoing one (e.g. 1 and 4). If a commutation failure occurs, it results in the short circuit of the d.c. supply. However this is overcome by providing a delay between initia-
tion of turn-off of one thyristor and the initiation of turn-on of the next thyristor so that the first thyristor has fully turned off before the second thyristor commences conduction.

c) Logic design for 180° thyristor conduction inverter is relatively complex compared with 120° conduction pattern.

d) With the 180° mode, the open circuit does not occur in both cases, i.e. light and heavy load, since all motor phases are connected to the inverter during a conduction period.

With due consideration to the above advantages and disadvantages, the 180° conduction mode has been adopted for this investigation.

2.3 Inverter Commutation Arrangements

In a three-phase bridge inverter there are many possible circuit arrangements in which auxiliary thyristors are used for forced commutation. A main thyristor in the upper half of the bridge may be turned-off by reducing the anode below the cathode potential, or alternatively by raising momentarily the cathode potential above the anode potential for an interval longer than the turn off time. These processes are called d.c. and a.c. side commutation respectively, since the commutation pulse is applied to the d.c. or a.c. terminals of the inverter by the auxiliary circuit. The auxiliary commutation circuits can be arranged:

i) to reverse bias all the inverter thyristors simultaneously (fully commutated).

ii) to reverse bias the upper or lower group of thyristors (half commutated).
iii) to reverse bias only one (individually commutated).

A half a.c. side commutation technique is considered in this work since it has the advantage that the number of commutating thyristors is only two.

The basic requirements in this inverter are:

i) To provide a capacitor which is always charged to the correct level and polarity immediately before it is required to reverse bias an outgoing thyristor.

ii) To provide alternative paths for the decaying currents directed from the outgoing thyristor.

iii) The commutating thyristors must be fired in the correct sequences to provide suitable commutation.

The complete circuit diagram of the single a.c. side commutated d.c. link inverter is shown in Figure 2.3.

2.4 Inverter Operation

The following description of the inverter operation applies to the three-thyristor triggering-mode and all voltages quoted are with reference to the negative side of the inverter supply $V_s$ whilst feeding a 3-phase linear induction motor load. Each side of the inverter is fed with the commutation pulses alternately. An auxiliary voltage supply $V_r$ is provided to ensure adequate recharging of the commutating capacitors ready for the next commutation.

The two sides of the inverter are separated by three centre tapped chokes which allow the cathodes of the three positive side thyristors to be raised in voltage while any of the negative side thyristors are conducting and vice versa.
FIG. 2.3.— THE COMPLETE INVERTOR INCLUDING THE COMMUTATING CIRCUIT.
All pulses for the main thyristors are inhibited for the duration of each pulse to either auxiliary thyristor A or B in order to prevent pulses appearing at the gate of any thyristor which is reverse biased. This is not shown in Figure 2.4 but is fully described in Section 2.6.2.

Referring to Figure 2.3, suppose thyristors 6, 1 and 2 are conducting and the voltage across capacitor $C_1$ is $-V_r$ volts (upper plate negative). In order to move into the next inverter period (one sixth of a cycle), thyristor 6 must be turned off and thyristor 3 turned on. This commutation is achieved by triggering thyristor B which connects capacitor $C_1$ across main thyristors 4, 6 and 2 causing them to turn off. The motor currents carried by the main thyristors in the bottom half of the bridge are transferred to diodes $D_6$ and $D_2$ respectively, and through thyristor B, to the commutating capacitors $C_{1'}$ and $C_2$ through the $V_r$ source. In addition voltages are impressed across each set of the two series-connected coils ($L_R$, $L_Y$, $L_B$) in the three legs of the inverter bridge giving rise to currents which circulate in the inverter. These currents, together with the motor currents, change the voltage across both capacitors. After a period of time, which should be in excess of the turn-off time for the main thyristors, the voltage across $C_1$ reduces to zero and reverses.

The main thyristor in the bottom half of the bridge becomes forward biased and thyristor $T_2$ is turned on by applying a train of pulses to the gate.

This action completes a loop within the inverter which causes a reduction of the charging current to the commutating capacitors. The voltage across both capacitors continues to charge until the cathode potential of thyristor B rises above the potential of its anode. The device therefore is turned off by current starvation as $D_B$ conducts. At this time the voltage across $C_1$ is $V_s$ and that across $C_2$ is $-V_r$. 

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Thyristor T₃ is fired at the same time as T₂, thyristor T₁ having remained conducting. The commutation is now complete with capacitor C₂ correctly charged to force the next commutation of T₁ and T₃. Thyristor T₄ is gated to initiate this commutation which proceeds in a similar way. For the next inverter period, thyristors T₂, T₃ and T₄ conduct.

The process of switching between thyristors in opposite sides of the bridge continues in correct sequence and the m.m.f. in the air-gap of the machine travels at a speed determined by the frequency selected.

2.5 Thyristor Protection

The protection circuits have been omitted for simplicity from Figure 2.3.

Each thyristor and diode in the bridge inverter has a high speed fuse connected in series to provide over-current protection against the fast rising short circuit current. In addition, each thyristor is protected from the effect of the rate of rise of forward blocking voltage by an R-C snubber connected between anode and cathode of the thyristor to operate in conjunction with the series inductors Lₐ, L₇ and L₉, which give protection against spurious triggering when adjacent thyristors are turned on. This is particularly important with the main thyristors and with

\[ C = 0.5 \mu \text{f}, \quad R = 70\Omega \] in series between the anode and cathode to limit \( \frac{dv}{dt} \) to a value not approaching 200 V/\mu S.

The commutating thyristors require additional protection from the rate of rise of current as they take a high instantaneous load current at turn-on. Air cored chokes of value 50 \( \mu \text{H} \) are connected in series with these thyristors to limit \( \frac{di}{dt} \) to a value below the limit of 400 A/\mu S. These chokes together with R-C snubbers connected between the anodes and cathodes of these thyristors also provide protection against the rate of rise of
forward blocking voltage. Values of 21Ω and 0.5 μf R-C snubbers provide limitation of $\frac{dv}{dt}$ to a value not approaching 200 V/μS.

2.6 The Invertor Control Circuits

2.6.1 Requirements

Operation of thyristor bridges requires the following conditions to be fulfilled.

a) Sequential pulsing to the six main and two commutation thyristors.

b) Gate pulses must be maintained on the main thyristors until the anode current through them has reached the latching current value.

c) In this mode of operation, there must be a delay between initiation of turn-off of one thyristor group and the initiation of turn-on of the next thyristor group.

d) A pulse train is preferred to a d.c. block signal of the equivalent period in order to cut down on the power dissipated at the gate without sacrificing signal amplitude as well as to restrict the size of the gate-drive transformer.

e) The gate pulses must come within the constraints of power, voltage and current which are determined by the thyristor gate characteristics. The pulse must also have sufficient power and short duration rise time in order to turn on the thyristor quickly, thus minimising localised heating of the thyristor gate region at turn-on.

Figure 2.4 shows both main and commutating thyristor trigger pulses; the gate pulses to the main thyristors in numerical sequence should appear at 60° intervals. Single pulses are insufficient however as the linear induction motor presents an
FIG. 24.—
THYRISTOR GATE PULSES FOR 180° CONDUCTION.
inductive load to the invertor. This requires a train of gate pulses to each main thyristor until the anode current reaches its latching value.

For the commutating thyristor a single turn-on pulse is sufficient since no starting problem arises, and therefore these thyristors immediately conduct the supply current away from the main thyristors and rapidly reach the latching value.

The pulse transformer is required to isolate the power circuitry, with its high voltage levels, from the logic circuitry (i.e. to obtain electrical isolation between the two circuits).

The gates were provided with greater current and voltage pulse than the minimum value in order to achieve successful device operation (care was taken to avoid exceeding the maximum average power rating).

Finally the requirement of the gate signals is that they should be able to provide variable operating frequency of the invertor. This is obtained by variation of the output frequency of the master oscillator used to initiate the firing sequence.

2.6.2 Logic Circuit

The detailed circuitry of the transistor-transistor logic used to provide the triggering pulses to the main and commutating thyristors is given in Appendix III. The following section is intended to give a description of the controlling circuitry in outline only.

Figure 2.5 shows a schematic diagram of the complete trigger circuits used for both main and commutating thyristors.

The master oscillator is used to initiate and time all the stages following. The sequencing unit is the section of the logic which determines the duration and the phasing of the main pulse.
FIG. 2.5.—SCHEMATIC DIAGRAM OF THE INVERTOR
LOGIC CIRCUIT.
trains. In addition it controls the timing pulses fed to the gates of the commutating thyristors. This unit is formed from the suitable inter-connection of three D-type bistable flip-flops.

Figure 2.6 gives the truth table for D-type flip-flops as well as the layout and the truth table of the sequencing unit.

The on states of Q and Q output from the sequencing unit provide the envelopes of the main pulse trains which are modulated by the pulses produced by a break-up oscillator operating at fixed frequency, Figure 2.5. Each pulse train is derived from the output of an AND gate which has two inputs: one the envelope from the sequencing unit and the other the output from the break-up oscillator operating at 8.25 KHz. However these do not have enough energy to trigger the thyristors, therefore a power amplifier is introduced between the TTL and the gates of the main thyristors. Circuit diagrams of the main thyristors gate drive are shown in Figure 4, Appendix III.

The production of pulses required to control the commutating thyristors has several stages. Since one commutating pulse is required at the end of each inverter period, negative-edge triggered monostables are used, receiving their inputs from Q and Q lines of the sequencing unit to produce the six pulses required per cycle.

Since Q outputs from the sequencing unit provide the pulse trains for the main thyristors T1, T3 and T5, the monostable outputs derived from the same Q lines are grouped together by routing via an inverter preceding a 3-input NAND to a power amplifier to thyristor A. Thyristor B is triggered by pulses similarly produced from the Q output derived from sequencing unit.

A problem that arises from the general arrangement of these gate signals is that gate signals are applied to main thyristors
CLOCK PULSES

a - SEQUENCING UNIT AND TRUTH TABLE

<table>
<thead>
<tr>
<th>CL</th>
<th>D</th>
<th>Q</th>
<th>\overline{Q}</th>
<th>D</th>
<th>Q</th>
<th>\overline{Q}</th>
<th>D</th>
<th>Q</th>
<th>\overline{Q}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>1</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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</tr>
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<td>1</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

THYRISTOR  | 6  | 3  | 4  | 1  | 1  | 5  |

b - TRUTH TABLE FOR D-TYPE FLIP-FLOP

<table>
<thead>
<tr>
<th>D</th>
<th>CLOCK</th>
<th>OUTPUT Q</th>
<th>OUTPUT \overline{Q}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

FIG. 2.6.

23
that are reverse biased. However this is avoided by inhibiting the output from the break-up oscillator, as shown in Figure 2.5 when either commutating thyristor $T_A$ or $T_B$ is driven.

Spurious operation of all firing circuits was a serious problem initially. This was found to be caused by a series of faults. All were due to some form of interference, both radiated and conducted. Interference within the logic circuitry was eliminated by connecting 7000 $\mu$F capacitors between $V_{CC}$ rail and earth to decouple the logic from any incoming voltage spikes. All leads to the driver stages were screened against radiated interference and the screen grounded. However, it was necessary to use twisted pair leads from the pulse transformer, secondaries to the thyristor gate to minimise mutual coupling effects which could cause spurious firing. Capacitors $0.01 \mu$F, were placed between the gate and cathode of all thyristors to reduce the pick up effect of the low energy signals. A delta-star connected transformer was inserted between the main supply $V_S$ and the commutating supply $V_r$ to electrically isolate the two ends and to avoid any harmonics which may be caused by the switching of main-thyristors feeding back via the main inverter rectifier through the commutation circuit rectifier causing false triggering to the commutating thyristors. Large smoothing capacitors across the d.c. outputs were also effective in this respect. The master oscillator was separated from the break-up oscillator component card to avoid interference between the two.

Satisfactory operation was achieved when all integrated circuits were screened, a 0.01 $\mu$F ceramic decoupling capacitor was connected between $V_{CC}$ and the earth point of each I.C. to decouple the logic from high frequency noise. However due to sensitivity of the monostable to the noise, a 200 $nF$ capacitor was connected between pin (11) (i.e. $R_x/C_x$ timing pin) and the earth in order to avoid a false triggering of the device which may be happening if the input waveforms are slow causing the device to trigger on the wrong edges as well as the correct ones.
Practical triggering gate pulses of the main and commutating thyristors are shown in Figure 2.7.

2.6.3 The Master Oscillator

i) Basic requirements

The master oscillator must have the following characteristics:

a) It must be simple, compact and have a low power consumption.
b) It must be thermally stable.

The unijunction transistor U.J.T. relaxation oscillator adequately fulfils these requirements. It also has a stable trigger voltage and a low trigger current, therefore there is less interference with the supply. The characteristics of the U.J.T. together with circuit design are given in Appendix II.

ii) The basic U.J.T. oscillator

The master oscillator is a development of the basic U.J.T. oscillator.

In Figure 2.8 the capacitor C charges exponentially through resistor R. When the capacitor voltage reaches the peak point voltage $V_p$ of the U.J.T. the emitter commences to conduct.

The ohmic value of resistor $R_{bl}$ is small, therefore C is rapidly discharged through it. The discharged current causes an output voltage spike across $R_{bl}$.

The capacitor continues to discharge through $R_{bl}$ until its voltage has dropped to the valley voltage $V_v$ of the U.J.T. (This is usually about 2 volts) after which it reverts to its blocking state. If the value of $R$ is varied this alters the C-R time constant which in turn alters the time taken for the
Scale: \hspace{1em} \text{time} \hspace{0.5em} 5\text{ms/\text{div.}}
\hspace{1em} \text{voltage} \hspace{0.5em} 2\text{V/\text{div.}}

Upper: \hspace{1em} \text{waveform of main thyristor pulse trains applied}
\hspace{1em} \text{to the gate of the thyristor (T₁) at 50Hz.}

Lower: \hspace{1em} \text{waveform of the commutation thyristor pulses}
\hspace{1em} \text{applied to the gate of the thyristor (Tₐ) at 50Hz.}

\textbf{FIG. 2.7.} - \textsc{practical waveform of thyristor}
\hspace{1em} \text{gate signals.}
FIG. 2.8.— THE BASE U.J.T. RELAXATION OSCILLATOR.

FIG. 2.9.— BASIC U.J.T. OSCILLATOR PULSES.
capacitor to charge up to the peak point voltage. This gives variable output frequency of the pulses.

Figure 2.9 shows the U.J.T. oscillator pulses.

iii) **The constant charging current U.J.T. oscillator**

The circuit described above does not give a linear output frequency/resistance $R$ characteristic. This is due to the non-linearity of the charging of $C$ especially for long time constants i.e. for low frequencies.

A circuit which overcomes this problem is shown in Figure 2.10. Diode $D$ is used to compensate the voltage drop across $T_{R1}$ P-N junction. The operation of this circuit depends upon the $I_C/V_{ce}$ characteristics of transistor $T_{R1}$ (see Figure 2.11). If potentiometer $R_m$ is varied the base emitter voltage $V_{be}$ of $T_{R1}$ alters. This changes the collector current which charges the capacitor. The $I_C/V_{ce}$ characteristics are parallel and equally spaced for multiple values of $V_{be}$. This means that the charging rate of the capacitor is directly proportional to the value of $V_{be}$ at any instant, hence the output frequency/resistance $R$ characteristic is linear, as shown below:

$$T = \frac{1}{f} = R.C. = \frac{C.\Delta V}{I_C}$$

$$f = \frac{I_C}{C.\Delta V}$$

$$I_C = \frac{(15 - 0.7)}{10^3 \text{ KΩ}} \cdot \frac{r}{R}$$

$$f = \frac{14.3V}{C.\Delta V} \cdot \frac{r}{10^3 \text{ KΩ}.R}$$

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FIG. 2.10. — CONSTANT CHARGING CURRENT U.J.T. OSCILLATOR.

FIG. 2.11. — TYPICAL COMMON EMITTER CHARACTERISTICS.
i.e. far

Figure 2.12 shows the master oscillator circuit and Figure 2.13 shows the master oscillator calibration curve, i.e. the variation of the main inverter frequency with $R_m$. 
$T_1 = T_2 = \text{Transistor, Ferranti type ZTX502}$

$T_3 = \text{U-J-T, GE type 2N1671A}$

$T_4 = \text{Transistor, Ferranti type ZTX302}$

**FIG. 2.12. — MASTER OSCILLATOR AND AMPLIFIER.**
Fig. 2.13.— Inverter Frequency Calibration.
CHAPTER 3
LINEAR MOTOR WITH SINUSOIDAL SUPPLY

3.1 Description of the Motor

The linear induction motor can be described as a singly-fed machine consisting of two parts, a stator and a reaction plate which corresponds to the rotor of a normal rotating induction motor. The stator is an arrangement of coils which when connected to a three phase supply generates a magnetic field which travels from one end of the stator to the other. This travelling field induces eddy currents in the secondary (rotor) plate which react with the flux to produce a linear thrust on the plate in the same direction as the travelling field. Additionally, if the plate itself is of ferromagnetic material, a heavy attractive side pull is exerted towards the stator block. A double-sided motor, with exactly equal air gaps between the plate and the stator block at each side, provides cancellation of these side pull effects. They are present to only a negligible extent with a plate of paramagnetic material such as aluminium. A double sided motor has been used for the study.

The linear motor differs in its electrical characteristics from its rotating counterpart due to two reasons:

a) The practical mechanical clearance requirement between rotor plate and stators blocks, together with the fact that the rotor plate may very well be of non-ferrous conductor, leads to an effectively large airgap giving a heavy magnetising current demand in order to produce reasonable working flux densities in the machine.

b) The transient effects at leading and trailing edges of the stators can lead to some electrical unbalance between
the stators phases. The mechanical performance is slightly affected by the transients and also those at the stator transverse edges.

Experiments on linear induction motors involve some difficulty because the dimensional needs of the linear motor are large compared with those of the rotating motor. This increases the cost of experimental equipment, takes up vastly more space, and makes loading and its measurement more difficult. In order to avoid these difficulties the rotating disc type was employed for experimental purposes. In this type of motor a large-diameter rotating conductive disc takes the place of the secondary conductive plate, with the two stator blocks positioned at the sides near the periphery (Figures 3.1 and 3.2). The motion of the disc between the stators blocks is approximately linear, making the machine almost equivalent to a true linear motor. In this project two types of reaction disc were investigated, one being of aluminium and the other a composite construction of aluminium-cladded steel. The double sided linear motor produces increased flux per pole for the same thickness of rotor, by connecting opposite windings in the two stator blocks so that the polarities assist each other in driving flux through the rotor plate.

The motor specification as well as motor dimension are given in Appendix V.

3.2 Equivalent Circuit

3.2.1 Form of the equivalent circuit

The equivalent circuit for the linear induction motor was set up to predict the performance of the machine.

The conventional equivalent circuit for rotating machines was used (Figure 3.3). It has been shown elsewhere that for machines having a sheet secondary of paramagnetic material,
FIG. 3.2.—LINEAR MOTOR STATOR BLOCK.
FIG. 3.3.—LINEAR MOTOR EQUIVALENT CIRCUIT.
typically aluminium alloy, the secondary leakage inductance can be assumed to be negligible\(^8\), leading to the simpler equivalent circuit given. Furthermore, it is assumed that the friction and windage, edge effect and primary (stator) core losses are negligible.

3.2.2 Measurement of equivalent circuit parameters

The stator d.c. resistance was measured at working temperature using the voltmeter-ammeter method and was found to be 3Ω per phase with the two stators connected in parallel.

A no-load test was performed with the machine sinusoidally excited at its rated frequency (50 Hz) and voltage (220V) and the disc speed externally boosted to its synchronous speed. The stator voltage, line current and input power were noted and from the results, the input impedance \(Z_{in}\) obtained where:

\[
Z_{in} = R_1 + jX_0
\]

giving \(X_0\), \(X_0 = X_1 + X_m\).

From the locked rotor test at the same frequency, the stator voltage, line current, and input power were noted. The values of \(R_2\), \(X_1\), and \(X_m\) were obtained (Appendix VI) using the method of reference 12. This was repeated to give the equivalent circuit parameters at various frequencies (50, 40, 30, 20, 10 Hz) for the motor fitted with both aluminium reaction plate and aluminium cladded steel reaction plate.

During the tests, the applied voltage/frequency ratio was kept constant (4.5 V/Hz) in order to keep constant flux over the whole frequency range. Speeds were measured in (rev/min)
at the periphery of the disc using a digital tachometer, and converted to linear speed (m/s) at the mid-point of the stator blocks.

An example for the equivalent circuit parameter calculation is given in Appendix VI.

Tables 3.1 and 3.2 represent equivalent circuit parameters at various frequencies with both motor discs.

<table>
<thead>
<tr>
<th>$f$/Hz</th>
<th>$R_1/\Omega$</th>
<th>$R_2/\Omega$</th>
<th>$X_1/\Omega$</th>
<th>$X_m/\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>3</td>
<td>5.34</td>
<td>7.25</td>
<td>6.84</td>
</tr>
<tr>
<td>40</td>
<td>3</td>
<td>5.33</td>
<td>5.3</td>
<td>6.1</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>4.7</td>
<td>4.45</td>
<td>4.5</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>5.07</td>
<td>2.5</td>
<td>3.54</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>4.59</td>
<td>0.89</td>
<td>2.39</td>
</tr>
</tbody>
</table>

**TABLE 3.1** Equivalent Circuit Parameters with Aluminium Reaction Plate

<table>
<thead>
<tr>
<th>$f$/Hz</th>
<th>$R_1/\Omega$</th>
<th>$R_2/\Omega$</th>
<th>$X_1/\Omega$</th>
<th>$X_m/\Omega$</th>
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<tbody>
<tr>
<td>50</td>
<td>3</td>
<td>9.81</td>
<td>7.69</td>
<td>7.73</td>
</tr>
<tr>
<td>40</td>
<td>3</td>
<td>9.68</td>
<td>6.01</td>
<td>7.73</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>9.96</td>
<td>3.82</td>
<td>6.57</td>
</tr>
</tbody>
</table>

**TABLE 3.2** Equivalent Circuit Parameters with Aluminium Cladded Steel Reaction Plate
3.2.3 Temperature rise

During the tests the stator temperature increased greatly due to the stator/rotor winding having a short-term current rating. Forced cooling was used with fans arranged on each side of the disc to blow air horizontally towards the gap between the stator and the disc. However this was not very successful in limiting the temperature for continuous work because the stator blocks were fully encapsulated.

3.2.4 Voltage/frequency operational characteristic

As already stated, it is usual to vary the supply voltage of an induction motor approximately in proportion to frequency in order to maintain constant flux/pole over the controlled speed range. More precisely, for constant flux the motor induced e.m.f. must be varied in proportion to the frequency. This is clear by writing the standard e.m.f. equation in the form:

\[ E/f = 4.44 K_f N \]  \hspace{1cm} (3.1)

Relating this to the equivalent circuit, the e.m.f. is developed by the magnetising current flowing through the magnetising reactance and the magnetising current will remain constant if the E/f ratio is constant. For the equivalent circuit of the linear motor, in contrast to the rotating type, the values given in Tables 3.1 and 3.2 indicate that \( X_l \) and \( X_m \) (Figure 3.3) are similar and about double that of \( R_1 \) at 50 Hz. For normal low values of slip,

\[ \frac{R_2}{S} \gg j X_m; \]
thus variation of slip does not influence the supply current \( I_1 \), very much.

If \( R_1 \) is neglected,

\[
E = V \cdot \frac{X_m}{X_1 + X_m}
\]

(3.2)

that is, \( E \) is a fixed proportion of \( V \). So, maintaining a constant \( V/f \) ratio gives approximately a constant \( E/f \) ratio, and constant flux. This approximation becomes less accurate as the supply frequency is decreased, when the stator resistance \( R_1 \) has more influence since the reactances fall in proportion to frequency.

Now for the motor,

\[ V_S = \text{synchronous linear speed} \]

\[ = 2fS_p \]

(3.3)

where \( S_p \) = pole pitch.

\[ \text{Slip} \; S = \frac{V_S - V}{V_S} \]

(3.4)

Thrust/phase \[ T = \frac{\text{rotor copper loss}}{\text{slip speed}} \]

\[ = \frac{I_2^2 R_2}{S \cdot V_S} \]

\[ = \frac{E^2}{R_2^2 \cdot \left( \frac{R_2}{S} \cdot \frac{1}{V_S} \right)} \]
\[
\frac{dT}{dV} = \frac{-E^2}{R_2 \cdot V_s^2} = \frac{-1}{4 \cdot S_2 \cdot R_2} \cdot \frac{E^2}{f^2}
\]

Thus the thrust-speed characteristic slope is constant if \( E/f \) ratio is maintained constant, neglecting any change in \( R_2 \).

This equation shows that the linear motor can produce a series of similar thrust speed characteristics at any frequency providing the \( V/f \) ratio is maintained constant, in a similar way to the rotating induction motor.

As the frequency is decreased below 20 Hz, while \( V/f \) is at a fixed value, the above reasoning becomes less valid owing to stator winding resistance. An increased \( V/f \) ratio is then necessary to maintain motor performance, (See Fig. 4.2).

### 3.2.5 Calculation of the rotor resistance

The rotor resistance \( R_2 \) was calculated from the equivalent circuit (Figure 3.3) at various frequencies.

From the no-load test, the value of the total stator reactance \( X_0 \) was obtained, \( Z_{in} \) being calculated from the results and \( R_1 \) being known from previous measurements.
From the locked rotor test, the value of $R_2$, $X_m$, $X_1$, were obtained as follows:

Let the impedance of one phase of the linear motor with secondary (rotor) plate locked be

$$Z_{in} = R_1 + jX_o$$  \hspace{1cm} (3.7)$$

$$Z_{in} = (R_1 + P_1) + jQ_1$$  \hspace{1cm} (3.8)$$

where

$$\frac{V}{I} \cos\phi = R_1 + P_1$$

$$\frac{V}{I} \sin\phi = Q_1$$

$$R_2 = P_1 \left[1 + \frac{P_1^2}{(X_o - Q_1)^2}\right]$$  \hspace{1cm} (3.9)$$

$$X_m = X_o - Q_1 + \frac{P_1^2}{X_o - Q_1}$$  \hspace{1cm} (3.10)$$

$$X_o = X_1 + X_m$$  \hspace{1cm} (3.11)$$

Then the variation of rotor resistance with slip frequency is shown in Figures 3.4 and 3.5 when fitted first with the aluminium reaction plate and then with aluminium-cladded steel reaction plate.
FIG. 3.4.—VARIATION OF ROTOR RESISTANCE WITH FREQUENCY FOR THE ALUMINIUM DISC ROTOR.
FIG. 3.5.—VARIATION OF ROTOR RESISTANCE WITH FREQUENCY FOR THE ALUMINIUM CLADDED STEEL DISC ROTOR

10 20 30 40 50
ROTOR FREQUENCY / Hz

$R_2 \Omega$

0 1 2 3 4 5 6 7 8 9 10
With the aluminium cladded steel reaction plate, the variation of the rotor resistance with slip was small from 50 Hz to 30 Hz, but below this, the results were unrealistically high, perhaps due to the method of measurement. However, rotor current penetration depth is important as explained later. Hence it is not possible to predict the steady-state performance of the aluminium cladded steel reaction plate motor over the full frequency range. Therefore, in this chapter the comparison between measured and predicted steady state performance will be restricted to only the aluminium reaction plate motor.

Figure 3.4 shows that the measured values of \( R_2 \) were fairly constant down to 30 Hz but dropped gradually at lower frequencies by about 14%. This was due to the following reasons:

i) Increased stator temperature gives an increased value of \( R_1 \) which was taken as a constant d.c. value and subtracted from the measured total to give \( R_2 \).

ii) The effect of transverse end effect on the effective secondary resistance was neglected although it is necessary to take into account the above effect when an attempt is made to determine the effective rotor resistance of the secondary sheet(20) by three-dimensional analysis. However this is very complicated and is outside the scope of this work. The above effect with the edge effect may give a reduced path resistance at lower frequencies.

iii) Linear stator blocks were used during tests. Whilst representing a considerable saving in cost compared with arc-shaped blocks, they unfortunately introduce some adverse operational effects. The first is due to variation in the rotor linear speed across the stator blocks. The second is opposite the centre of the stator blocks, is not quite in the same
linear direction as the travelling field. These features give noticeable error in performance calculations as explained later.

iv) As the supply frequency is reduced in the locked rotor test, the skin depth of the secondary plate eddy current increases, thus increasing the effective path area and reducing the resistance.

For the composite rotor, at high frequencies (50 Hz) the skin effect constrains the rotor currents to flow mainly in the aluminium cladding. As the frequency falls, the penetration depth increases so the currents flow increasingly in the higher resistivity steel, giving an increasing value of $R_2$ as the frequency is reduced, to give some idea of the relevance of skin effect, consider the following.

If the airgap between stators and disc rotor are neglected, the skin depth $\delta$ is given by (21):

$$\delta = \sqrt{\frac{2 \rho_2}{W\mu_0\mu_r}}$$

at standstill and 50 Hz supply.

A correction factor to the stator plate resistance per phase must be used to increase the apparent resistance to allow for skin effect.

This factor is $K = \frac{\text{plate thickness}}{\delta}$

and must be greater than unity to be used.
For aluminium

Plate thickness $t = 1.1$ cm

Resistivity $P_2 = 2.74 \mu \Omega$ cm, giving

Skin depth $\delta = \frac{1.18}{\sqrt{5}}$ cm at 50 Hz supply frequency

$K = 0.465 \sqrt{5}$

For 50 Hz supply with the machine at standstill, $S = 1$, and $K = 0.465 \ i.e. < 1$

That means no skin effect occurs in the aluminium disc.

For steel

$t = 1.1$ cm, $P_2 = 13 \mu \Omega$ cm, $\mu_r = 800$ approximately, neglecting any saturation when $\mu_r$ decreases.

Skin depth $\delta = \frac{0.091}{\sqrt{5}}$ cm.

and $K = 6.1 \sqrt{5}$

For $S = 1 \ K = 6.1 \ \text{at standstill}.$

That means skin effect occurs in the steel, the effective rotor resistance being increased by a factor of six in a steel rotor plate.

For the composite plate, the aluminium cladding thickness is 0.2 cm per side. Hence, the eddy current penetration will be through the aluminium and into the steel. Here, skin effect does occur, giving an effective increase in rotor resistance compared with the plain aluminium disc (Figures 3.4 and 3.5).
The resistivity of steel is also effective in this.

The latter point also affects conditions under load where the rotor plate currents are at slip frequency. This has been ignored in later calculations.

Better accuracy may be achieved for measurement of the stator leakage reactance by using a search coil placed on the surface of the primary iron core with the disc removed from its position. The open circuit test can be done without the secondary plate, i.e. without end effect, and the flux density distribution can then be measured by the search coil wound around each tooth\(^{(20)}\). The value of \(X_1, X_m\) can be obtained easily for the aluminium rotor. This method would not apply for the composite rotor because part of the flux path, namely the steel centre of the reaction plate, is removed for the test, thus altering the flux pattern to some extent.

3.3 Steady-State Performance with a Sinusoidal Supply

3.3.1 Prediction of the steady-state performance

The equivalent circuit of the linear induction motor with aluminium reaction plate using appropriate parameters was used to give the performance of the motor with a sinusoidal supply at chosen frequencies of 10, 20, 30, 40, 50 Hz.

Calculations were carried out for selected slip values as shown below.

From the equivalent circuit (Figure 3.3)

\[
Z_{in} = \left[ R_1 + \frac{(R_2/S) X_m^2}{(R_2/S)^2 + X_m^2} \right] + j \left[ X_1 + \frac{X_m (R_2/S)^2}{(R_2/S)^2 + X_m^2} \right]
\]

\[ (3.12) \]
Knowing the values of $R_1$, $R_2$, $X_m$, $X_1$, $Z_{in}$ can be obtained:

\[ I_1 = \frac{V}{Z_{in}} \]  \hspace{1cm} (3.13)

\[ I_1 = (A - jB) \]  \hspace{1cm} (3.14)

\[ \cos \phi = \frac{A}{\sqrt{A^2 + B^2}} \text{ lagging} \]  \hspace{1cm} (3.15)

Input power = $3|I_1|V \cos \phi$ \hspace{1cm} (3.16)

\[ I_2 = |I_1| \cdot \frac{j X_m}{\left| \frac{R_2}{S} + j X_m \right|} \]  \hspace{1cm} (3.17)

Output power = $3 \left( \frac{R_2}{S} - R_2 \right) I_2^2$ \hspace{1cm} (3.18)

\[ P_{out} = 3 \left( \frac{1 - S}{S} \right) R_2 \cdot I_2^2 \]  \hspace{1cm} (3.18)

Efficiency = $\frac{\text{output}}{\text{input}} \times 100\%$ \hspace{1cm} (3.19)

Linear speed = $V_s (1 - S)$ \hspace{1cm} (3.20)

\[ \text{Thrust } T = \frac{P_{out}}{V_s (1 - S)} \]  \hspace{1cm} (3.21)
Similarly the performance of the motor with aluminium cladded steel reaction plate was calculated.

An example of predicted performance calculations is given in Appendix VII.

3.3.2 Test procedure

The aim is to determine the performance of the linear induction motor for a sinusoidal applied voltage waveform when fitted either with aluminium reaction plate or with aluminium cladded steel reaction plate.

Tests were performed at the same excitation frequencies (10, 20, 30, 40, 50 Hz) as were adopted for the predicted performance.

The circuit connections for the measurements are shown in Figure 3.6. The following quantities were calculated from the results:

a) Power factor.
b) Force.
c) Line current.
d) Efficiency.
e) Input power.
f) Slip.

The machine was loaded by a disc brake (Figure 3.1) with the braking force indicated in Newtons at a radius of 0.31 m. The radius to the centre of the stator blocks was 0.82 m hence the force developed by the machine is given by:

\[ \text{Force} = \frac{0.31 \times \text{indicated force}}{0.82} \]

During the tests the stator line voltage/frequency ratio was varied in accordance with Fig. 4.2 to maintain constant flux.
FIG. 3.6.—CIRCUIT CONNECTIONS FOR PERFORMANCE MEASUREMENT.
The effective airgap between the two stator faces was (1.52 cm) when aluminium reaction plate was used, while the effective airgap between the two stator faces was (1.02 cm) when an aluminium cladded steel reaction plate was employed.

3.3.3 Discussion of results

a) Performance characteristics:

Experimental results of the performance of the linear induction motor when fitted with both reaction plates in turn are shown in Figures 3.7 to 3.15.

The graphs show the measured results for the quantities listed in Section 3.3 to a base of slip at the same frequencies of 10, 20, 30, 40, 50 Hz, as were used for the predicted performance.

It was observed that with both aluminium and aluminium cladded steel reaction plates, the stator primary currents are unbalanced (Figure 3.16) due primarily to the end and edge effects; since each phase of the three-phase winding is located in a different position on the stator iron core with respect to the two ends of the block, i.e. each of the three phases has slightly different reactances. Throughout the equivalent circuit prediction average values have been used.

It is to be noted that power factor and efficiency are generally low, due to the large effective airgap of the aluminium reaction plate. The high magnetising current drawn increases the stator losses without contributing to the output power.

The power factor increases as the frequency is reduced, from a peak value of 0.44 at 50 Hz to 0.76 at 10 Hz for the same value of slip 0.35. This is due to the reducing value of $X_l$ and $X_m$ with frequency, giving a more resistive equivalent circuit.
FIG. 3.7.— LOAD PERFORMANCE WITH ALUMINIUM REACTION PLATE AT 50 Hz.
FIG. 3.8. — LOAD PERFORMANCE WITH ALUMINIUM REACTION PLATE AT 40Hz.
FIG. 3.9.—LOAD PERFORMANCE WITH ALUMINIUM REACTION PLATE AT 30 Hz.
(a) = Power Factor
(b) = Linear Force
(c) = Line Current
(d) = Efficiency
(e) = Input Power

FIG. 3.10.-LOAD PERFORMANCE WITH ALUMINIUM REACTION PLATE AT 20Hz
FIG. 3.11 - LOAD PERFORMANCE WITH ALUMINIUM REACTION PLATE AT 10 Hz
FIG. 3.12.—LOAD TEST WITH ALUMINIUM CLADDED STEEL REACTION PLATE AT 50Hz.
Fig. 3.13.— Load Test with Aluminium Cladded Steel Reaction Plate at 40Hz

(a) = Power Factor
(b) = Linear Force
(c) = Line Current
(d) = Efficiency
(e) = Input Power
FIG. 3.14—LOAD TEST WITH ALUMINIUM CLADDED STEEL
REACTION PLATES AT 30 Hz.
FIG. 3.15.—LOAD TEST WITH ALUMINIUM CLADDED STEEL REACTION PLATE AT 20Hz.
Aluminium reaction plate

Aluminium cladded steel reaction plate

FIG. 3.16—STATOR SUPPLY LINECURRENTS AT 50Hz.
The efficiency decreases as frequency decreases due to the fall of magnetising reactance giving an increasing stator loss without a corresponding increase in output. It is shown in Appendix VIII that approximately the efficiency is proportional to the square of the supply frequency for a given slip.

From Figure 3.7, the efficiency is 19% at a slip of 0.25. It is to be expected that at 20 Hz, the corresponding efficiency would be \((20/50)^2 \times 19 = 3.1\%\). From Figure 3.10, a value of 3.8% is indicated.

The line current decreases somewhat as the frequency is decreased more so than at 10 Hz; this is due to the very low voltage level at low speed which produces a more marked fall in flux owing to the high value of stator voltage drop. The magnetising component of current through \(X_m\) falls, giving a reduced input current.

An important observation from the load tests is that the machine runs at much higher slip values as the frequency is decreased to 10 Hz. Again, this is caused by reducing \(E/f\) ratio and flux, and lower force developed.

b) Comparison between predicted and experimental results

It is to be noted that the predicted primary currents are almost constant at 50, 40, 30 Hz respectively over a wider slip range, while at 20 and 10 Hz, the primary currents decrease as the slip approaches unity, this is mainly due to high value of stator voltage drop. However there is good agreement between the predicted and measured primary currents, the discrepancy being between 1.14 and 2.23% as shown in the graphs for aluminium disc motor.
In all cases, the predicted power factor is slightly lower than that measured. This is due to the equivalent circuit not allowing for all electrical losses in the machine.

The reasons for the difference between predicted and measured force values is due to the following effects:

a) Friction and windage losses.
b) Core losses.
c) End and edge effects at the stators.
d) Measurement errors.

It is also observed that the difference between predicted and measured efficiencies were between (7-15)%; this is due to the reasons mentioned above giving a lower output power than could be predicted by the simplified analysis used.

Tests were carried out to measure the friction and windage losses of both motors and these show that friction and windage losses in the aluminium reaction plate motor is 10 watts, while windage and friction losses in the aluminium cladded steel reaction plate motor is 7.4 watts. See Appendix IX. It was difficult to obtain precise results from a graphical method, but it is demonstrated that the mechanical losses are very low in comparison with the input power, which is of the order of 1 kW.

It is considered that stator core losses and stator end and edge effects contribute mainly to the higher predicted performance. The somewhat circular motion of the disc between the stator blocks, instead of purely linear motion, contributes additional losses that are not accounted for in the equivalent circuit. The effect of varying rotor resistance with frequency was incorporated.
c) **Comparison between aluminium and composite reaction plate machines**

The linear induction motor with aluminium sheet secondary, i.e. paramagnetic material, has inherently a large airgap leading to a heavy magnetising current demand in order to produce a reasonable flux in the machine. Hence the power factor, thrust and efficiency of the motor are respectively low. However, the steel core, aluminium cladded rotor plate provides a lower reluctance path for the magnetic flux between the stator surfaces. This reduces the effective airgap and hence the excitation currents of the stator windings. As a result, the power factor, thrust and efficiency of the composite rotor linear induction motor are better than for aluminium rotor in general.

The graphs show Figures 3.12 to 3.15 that the value of line currents are almost constant over a wide range of frequency for the composite disc. These currents are less than those measured with aluminium discs by about (7.69 - 14.77)% over the frequency range (20 - 50) Hz.

It is seen that at the same airgap flux density, that is the same V/f ratio, the power output of the machine with aluminium reaction plate is higher than that of the machine with a composite reaction plate. However, the losses in the former machine are also considerably higher since larger values of stator current are required to maintain the same airgap flux density. As a result, the efficiency of the machine with composite reaction plate is higher. However, this is not pronounced.
CHAPTER 4
LINEAR MOTORS WITH A QUASI-SQUARE WAVE SUPPLY VOLTAGE

4.1 Introduction

For normal constant speed applications, a cylindrical rotor induction motor is supplied by a sinusoidal voltage source. However with inverter-fed machines, the supply voltage is not sinusoidal and effects due to supply harmonics usually result. The harmonics present in the voltage waveform produce harmonic components in the magnetising current. The rotating m.m.f. wave produced has components which have the same number of poles as the fundamental synchronously rotating field, but which rotate at speeds which are higher multiples of it. These rotating m.m.f's produce losses and torques in the same way as the fundamental m.m.f. which produces the desired motor performance.

Work on rotating induction motors shows that the 5th, 11th, and 17th harmonics of the phase voltage result in m.m.f's which are contra-rotational with respect to the fundamental and therefore produce negative sequence torque components i.e. which oppose the fundamental torque. The 7th, 13th, 19th etc. harmonics of the phase voltage produce a positive sequence torque component i.e. which assist the fundamental torque (23). The harmonic torques are of two types.

a) Steady harmonic torques.
b) Pulsating harmonic torques.

a) Steady harmonic torques are developed by the reaction of harmonic air-gap flux with harmonic rotor currents of the same order. However, these steady harmonic torques are a very small fraction of the rated torque and being contra-rotating, tend to cancel each other. They thus have only a small effect on the motor operation.
b) Pulsating torques arise from the interaction between the fundamental rotating flux and the harmonic rotor currents. The 5th harmonic stator currents, for example, have negative sequence and produce a space m.m.f. wave which rotates at five times the fundamental synchronous speed in the opposite direction to the fundamental field. The rotor m.m.f's induced by this harmonic field react with the fundamental rotating field, the phase relationship between the two varying at a rate which is the difference between the two rotating vector speeds. This produces a pulsating torque at six times fundamental frequency. The 7th harmonic m.m.f. has forward rotation, and hence the two pulsating torques at six times fundamental frequency combine to produce a fluctuation in the electromagnetic torque developed by the motor. Similarly for the 11th and 13th harmonics. However in general there is no alteration in the steady-state torque of the motor since the pulsating torques have zero average value, but their presence causes the angular velocity of the rotor to vary during a revolution. At very low speeds the motor rotation takes place in a series of jerks or steps, and the presence of pulsating torques may set a lower limit to the useful speed range of the motor\(^{(16,24)}\).

The additional losses due to the harmonic currents in the machine increases the losses and heating and can lead to a reduction of its rated output.

With linear induction motors, it is to be expected that similar torque and loss effects will occur when a non-sinusoidal supply is used. In addition, end and edge effects produce additional losses which are very difficult to analyse, and may be significant in comparison with those due to the harmonics in the supply.
4.2 Experimental Performance on a Quasi-Square Wave Supply

4.2.1 Supply Voltage/Operating Frequency Ratio

It has been explained in Section 3.2.4 that it is necessary with variable frequency control, to vary the supply voltage approximately in proportion to the supply frequency in order to maintain the motor airgap flux constant. This was considered in relation to a sinusoidal supply. For tests with a quasi-square wave supply, it is desirable to still maintain the required V/f relationship, with V representing the fundamental component of the waveform, since it is the fundamental component that is producing substantially the useful power. Hence a relationship is required between the r.m.s. value of the waveform which can easily be measured on a meter, and the r.m.s. value of the fundamental component, to aid adjustment of the voltage setting for the appropriate fundamental voltage to frequency ratio.

The idealised phase and line voltage waveforms are shown in Figure 4.1.

R.M.S. values are as follows:

Phase voltage,

\[ V_p \text{ r.m.s.} = \frac{2}{3} \sqrt{\frac{1}{\pi} \left[ 2 \int_0^{\pi/3} (\frac{V}{2})^2 \, dt + \int_0^{\pi/3} V^2 \, dt \right]} \]  \hspace{1cm} (4.1)

\[ V_p \text{ r.m.s.} = \frac{2}{3} \frac{V}{\sqrt{2}} = \frac{\sqrt{2}V}{3} \]  \hspace{1cm} (4.2)

Line voltage,

\[ V_L \text{ r.m.s.} = \sqrt{\frac{2}{3}} V. \]  \hspace{1cm} (4.3)
a) phase (line-neutral) voltage.

b) line voltage

FIG. 4.1--IDEALISED VOLTAGE WAVEFORM APPLIED TO THE MOTOR.
Fundamental components are:

The line voltage harmonic series is

\[ V_L = \frac{2\sqrt{3}}{\pi} V \left[ \cos \omega t - \frac{1}{5} \cos 5\omega t \right] \]  \hspace{1cm} (4.4)

Fundamental (r.m.s) \( V_{1L} = \frac{2}{\sqrt{2}} \frac{\sqrt{3}}{\pi} V = \frac{\sqrt{6}}{\pi} V \)  \hspace{1cm} (4.5)

The phase voltage harmonic series is:

\[ V_p = \frac{3}{\pi} \left( \frac{2}{3} V \right) \left[ \cos \omega t + \frac{1}{5} \cos 5\omega t + \cdots \right] \]  \hspace{1cm} (4.6)

Fundamental (r.m.s) \( V_{1p} = \frac{3}{\pi \sqrt{2}} \frac{2}{3} V = \frac{\sqrt{6}}{\pi} V \).  \hspace{1cm} (4.7)

Hence for the phase voltage,

\[ \frac{V_p \text{ r.m.s.}}{V_{1p}} = \frac{\sqrt{2/3} V}{\pi \sqrt{2/\pi} V} = 1.05 \]

and for the line voltage,

\[ \frac{V_L \text{ r.m.s.}}{V_{1L}} = \frac{\sqrt{2/3} V}{\pi \sqrt{2/\pi} V} = 1.05 \]

From the above, the line/phase voltage ratio is \( \sqrt{3} \), for both r.m.s. and fundamental values.

Therefore, to ensure a required fundamental phase voltage for V/f control, the phase and line r.m.s. voltages must be increased by a factor of 1.05 on the required value of the fundamental voltage. In practice, the d.c. link voltage was adjusted to give an r.m.s.
line voltage at the motor terminals of 1.05 times the required fundamental value.

However at lower frequencies the stator resistive voltage drops are significant and the induced e.m.f. becomes a smaller percentage of the applied voltage. This requires an increase in the value of the applied voltage if the level of the airgap flux is to be maintained. It follows that the relationship between the voltage applied to the motor and the frequency of this supply must follow a law of the form defined in Figure 4.2. If the required fundamental-flux level is to be obtained for the operating frequencies.

A similar method can be applied to the linear induction motor. In this case, the rated motor voltage at 50 Hz is 220 V, sinusoidal. This requires $1.05 \times 220 = 231$ V r.m.s. with quasi-square wave at 50 Hz (fundamental). This defines point A in Figure 4.2.

A second point B at (say) 10 Hz, is obtained from equivalent circuit by establishing the fundamental phase voltage required to produce an e.m.f. across the magnetising reactance/ at 50 Hz. This is 36.5V per phase which gives a required r.m.s. line voltage on quasi-square waves for point B of $36.5 \times \sqrt{3} \times 1.05V$ (= 66.4 V).

4.2.2 Measurement of Motor Performance when Invertor-Fed

The linear motor performance was investigated with both aluminium and aluminium cladded steel rotors when supplied with the quasi-square wave invertor voltage. Tests were performed at different excitation frequencies (50, 40, 30, 20, 10) Hz respectively.

In all cases the following quantities were calculated from results:

i) Input power and power factor.
ii) Shaft output.
FIG. 4.2.—PRACTICAL FORM OF RELATIONSHIP BETWEEN THE INVERTOR r.m.s. OUTPUT VOLTAGE AND THE FREQUENCY.
iii) Efficiency.
iv) Linear force.
v) Slip.
vi) Line current.

During tests the r.m.s. line voltage/frequency ratio was adjusted in accordance with the characteristic of Figure 4.2.

Waveforms of motor terminal voltage and supply current were obtained.

The instrumentation used for the measurement is shown in Figure 4.3. The voltmeter and ammeter and wattmeter were integral.

Two types of digital a.c. power meters were used to measure the voltage, current and input power applied to the motor. The meters are specially designed to measure distorted waveforms of the types produced by static convertors with superlative noise immunity and stability and negligible instrument loss. The two universal meters specifications are given below.

i) 100 mW - 5 kW, Clarke-Hess Communication Research Corporation, Model 255, was used to measure current, voltage and power at frequency range 10 Hz to 30 Hz.

ii) 300 mW - 18 kW, Yokogawa Electric Work Ltd., Type 2503 was used to measure current, voltage and power at frequency range 40 Hz to 50 Hz.

Current waveforms were measured by a bifilar strip shunt of resistance 0.1Ω. This has a very low inductance due to the magnetic fields from the strip conductors tending to cancel. There is always, however, a residual inductance which results from the flux in the area enclosed within a thin loop formed by the double strip.
**FIG. 4.3. — LOAD TEST CIRCUIT.**
Only a little evidence of distortion of the current waveforms with high rates of change due to this low inductance were observable in the waveforms.

4.2.3 Voltage and Current Waveforms

The voltage waveform is generally similar to the idealised shape (Figure 4.1) but exhibits an interruption during a commutation midway through each 120° conduction interval.

Figures (4.4 - 4.7) show the oscillograms of line voltage and current for the aluminium plate motor under both no-load and full-load conditions.

Figures (4.8 - 4.11) show the oscillograms of line voltage and current for the composite motor. There appears to be a slight change in voltage and current waveforms with loading at 50 Hz.

Instead of the waveform being flat-topped, it falls due to the loading of the inverter producing a back e.m.f. across the inductors in series with conducting thyristors and also, to the inability of the capacitors across the d.c. input to the inverter to maintain perfect smoothing, certain waveforms (e.g. Figure 4.8) apparently demonstrate a commutation pause at the end of the 120° conduction period. Since the three thyristors triggering method connects all motor phases to the inverter during a conduction period, much variation of voltage wave shape with load or changing frequency is not to be expected. The same is true for Figures 4.8 - 4.11 for the composite secondary motor, although the commutation transients on full load (Figure 4.9a) appear to be more severe.

The current waveforms are fairly typical of those for induction motors fed with such a quasi-square wave supply. They consist of a series of transients, these being six per cycle, produced
FIG. 4.4. — OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR ALUMINIUM PLATE MOTOR UNDER NO LOAD AT 50Hz.
FIG. 4.5.—OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR ALUMINIUM PLATE MOTOR UNDER FULL LOAD AT 50Hz.
FIG. 4.6. — OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR ALUMINIUM PLATE MOTOR UNDER NO LOAD AT 10 Hz.
FIG. 4.7.— OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR ALUMINIUM PLATE MOTOR UNDER FULL LOAD AT 10Hz.
No-Load

Line Voltage

Scale: time 5ms/div
voltage 250V/div

No-Load

Stator Current

Scale: time 5ms/div
current 10A/div

FIG 4.8.- OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR COMPOSITE PLATE MOTOR UNDER NO LOAD AT 50Hz.
(a) Full-Load Line Voltage
Scale: time 5ms/div voltage 250V/div

(b) Full-Load Stator Current
Scale: time 5ms/div current 10A/div

FIG. 4.9. — OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR COMPOSITE PLATE MOTOR UNDER FULL LOAD AT 50Hz.
No-Load Line Voltage
Scale: time 20ms/div voltage 50V/div

No-Load Stator Current
Scale: time 20 ms/div current 10A/div

FIG. 4.10.— OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR COMPOSITE PLATE MOTOR UNDER NO LOAD AT 10Hz.
FIG. 4.11.– OSCILLOGRAMS OF LINE VOLTAGE AND CURRENT FOR COMPOSITE PLATE MOTOR UNDER FULL LOAD AT 10 Hz.
by the phase voltage applied across the input impedance of
the motor phase, which is variable with load, and with the
type of secondary plate used. For the aluminium secondary,
there is a little variation of wave shape between no-load and
full-load at either 50 Hz and 10 Hz, but some difference is
apparent between the waveforms at the two frequencies due to
the factor of five in the time interval of the one sixth of
the waveform period.

The variation of current waveforms with frequency is far
more pronounced with the aluminium-cladded steel rotor plate
machine, where the equivalent circuit is more resistive due to
the somewhat higher magnetising reactance and considerably
higher motor resistance (Tables 3.1 and 3.2). The amplitude
of the currents with the composite rotor plate are slightly
lower than those of the aluminium plate due to the higher phase
input impedance of the former.

The spikes in the oscillograms of the current waveforms,
which occur at the commutation points, are due to the measuring
shunt not being completely non-inductive. These are more appa­
rent in the composite rotor machine, particularly in the 50 Hz
waveforms, where the commutation transients are not compressed
by the CRO timebase setting, as they are at 10 Hz.

4.2.4 Discussion on Motor Performance

The performance curves for the aluminium and composite motors
are shown in Figures 4.12 - 4.20. The equivalent characteristics
obtained with a sinusoidal supply are again given to aid compari­
son.

For both types of rotor plate, the stator supply currents
are fairly constant over the load range at any set frequency owing
FIG. 4.12.—LOAD PERFORMANCE WITH ALUMINIUM ROTOR AT 50Hz.
FIG. 4.13. — LOAD PERFORMANCE WITH ALUMINIUM ROTOR AT 40Hz.
(a) = Power Factor
(b) = Force
(c) = Line Current
(d) = Efficiency
(e) = Input Power

FIG. 4.14. - LOAD PERFORMANCE WITH ALUMINIUM ROTOR
AT 30 Hz.
(a) = Power Factor
(b) = Force
(c) = Line Current
(d) = Efficiency
(e) = Input Power

FIG. 4.15.-LOAD PERFORMANCE WITH ALUMINIUM ROTOR
AT 20Hz.
(a) = Power Factor
(b) = Force
(c) = Line Current
(d) = Efficiency
(e) = Input Power

**FIG. 4.16.** LOAD PERFORMANCE WITH ALUMINIUM ROTOR AT 10 Hz.
(a) = Power Factor
(b) = Force
(c) = Line Current.
(d) = Efficiency
(e) = Input Power

FIG. 4.17.—LOAD PERFORMANCE WITH ALUMINIUM CLADDED STEEL ROTOR AT 50Hz.
(a) = Power Factor
(b) = Force
(c) = Line Current
(d) = Efficiency
(e) = Input Power

FIG. 4.18.—LOAD PERFORMANCE WITH ALUMINIUM CLADDED STEEL ROTOR AT 40Hz.
FIG 4.19.—LOAD PERFORMANCE WITH ALUMINIUM CLADDED STEEL ROTOR AT 30Hz.
(a) = Power Factor
(b) = Force
(c) = Line Current
(d) = Efficiency
(e) = Input Power

FIG. 4.20.-LOAD PERFORMANCE WITH ALUMINIUM CLADDED STEEL ROTOR AT 20Hz.
to the high magnetising currents. However, the currents at the higher frequencies are generally higher than at low frequencies, probably due to the $V/f$ relationship used not quite compensating for the effect of stator resistance. The poorer performance generally at low frequencies reflects this. The currents for the composite rotor machine were slightly lower than for the other owing to the higher equivalent circuit impedance. Little difference is apparent between the current levels with inverter and and sinusoidal supply. No trend can be discussed owing to the measurement inaccuracy, although it is to be expected that the r.m.s. value of the inverter supplied current will be the larger assuming that the slip is a function purely of the fundamental component.

The general shape of the force-slip characteristics is similar for inverter and sinusoidal supply, in all cases namely a fairly linear rise of force with slip, as is usual for induction motors. There is a general trend towards the inverter-fed machine developing a lower force than the sinusoidally fed one, by up to about 20%, though inconsistency of results prevents precise figures being inferred. The previous discussion (Section 4.2) regarding the effect of torque components developed by the harmonics in supply waveforms, is verified clearly here.

The reduced torque for given slip values leads to lower efficiencies with a quasi-square wave supply. The trend in this respect is apparent at all frequencies. Additional iron and copper losses from the harmonics give efficiency values reduced by a few percent from their values with sinusoidal excitation\(^{(25)}\). The efficiency is slightly higher with the composite plate motor, for both types of supply, due to its better magnetic circuit.

The power factor is consistently lower with quasi-square wave supply, than with the sinusoidal supply, in general by about 10%. Power factor is defined as a ratio of real power/apparent power. The real power is substantially produced by the fundamental components of the voltage and current waveforms whereas
the apparent power is the product of the r.m.s. values of the voltage $V_L$ and current $I_L$, i.e.

$$\text{Power factor} = \frac{\sqrt{3} V_1 I_1 \cos \phi}{\sqrt{3} V_L I_L}$$

Since, for a quasi-square wave, $V_L$ and $I_L$ include the harmonic components, but $V_1$ and $I_1$ do not, a lower power factor is to be expected in this case.

Generally, the effects of the quasi-square wave supply on the motor performance are similar to those with rotating induction motors.
CHAPTER 5

CONCLUSIONS

5.1 Summary of the Work Undertaken

The investigation can be summarised as follows:

a) A three-phase thyristor bridge invertor has been designed and built to operate over the frequency range 10 Hz to 50 Hz. The invertor was used to drive a 3-phase linear induction motor with either an aluminium or an aluminium cladded steel reaction plate.

b) The control circuit for the invertor, operating with a $180^\circ$ thyristor conduction mode, was designed and built.

c) An equivalent circuit for the linear induction motor was derived and the motor parameters were determined at the frequencies 50, 40, 30, 20 and 10 Hz, with a sinusoidal excitation and for both types of reaction plate.

d) The equivalent circuit was used to predict the steady-state performance of the motor with a sinusoidal supply and a comparison was made between the predicted and measured results.

e) The steady-state performance of both types of linear motor was measured for a quasi-square wave supply obtained from the invertor.

f) A comparison was made between the motor performances for both sinusoidal and quasi-square wave supplies and with both reaction plates.
5.2 Comments

The steady-state performance of the linear motor, when supplied by the invertor, is close to the performance obtained when using a sinusoidal supply. This is true throughout the frequency range and with both types of disc rotor. It is shown that the efficiency of the motor is generally very low, particularly at low frequencies. The reasons for this have been discussed.

The object of introducing the composite secondary motor is to allow the ferrous material to provide a low reluctance to the magnetic flux, and the non-ferrous material to provide a low resistance path for the secondary currents, giving improvement in the performance of the motor. The practical results show this to be achieved although there is no improvement in the power factor. This is probably due to skin effects as well as end effects (20), but since these phenomena need special investigation, it is outside the scope of this thesis.

The use of TTL integrated circuits for firing circuits give difficulties regarding interference. The interference encountered was both radiated and conducted. Logic malfunctioning was reduced to an acceptable level by screening and the use of decoupling capacitors. Reliability of the logic circuit would be improved by the use of CMOS instead of TTL due to its greater immunity to spurious noise.

In conclusion, the motor with a composite disc rotor is better than the aluminium disc motor in terms of overall power requirement and efficiency. However, it suffers to some extent from unbalanced magnetic pull and is more expensive to manufacture and, for a long track, would probably be uneconomical.
5.3 Further Work

The system can be further developed by work in the following areas:

1. Closed loop control of the inverter-linear motor system.

2. Operation of the linear motor from an inverter incorporating pulse-width modulation techniques.

3. Operation of the linear motor from the current source inverter which possesses certain advantages over the voltage fed inverter. These are:
   a) Commutation is relatively slow because of the large commutation capacitors. Therefore non inverter-grade thyristors may be used.
   b) The commutation voltage is a function of the load current, so that as the load current increases, commutation capability increases.
   c) Inherent ability to regenerate back to the a.c. supply through a full wave bridge rectifier as a result of the d.c. link current which is unidirectional under all operating conditions.
   d) The high impedance current source concept is very rugged and self protecting under abnormal operating conditions as compared to conventional low impedance voltage source invertors.

4. A more detailed investigation of the equivalent circuit is required to allow for end and edge effects of the stator blocks and the varying rotor circuit resistance with slip frequency.
REFERENCES


APPENDIX I

INVERTER DETAILS

a) Main thyristors (Thyristors 1-6 of Figure 2.3)

Manufacturer: Westcode Semiconductors Type (P027QH08FJO) (fast turn-off).

Maximum repetitive peak reverse voltage: 800V,
Average on state current: 27A
I²t: 810 A²S.

Fusing: International Rectifier Ltd. Type B1000/20.

\( \frac{dv}{dt} \) protection: R-C shunt between anode and cathode of 70Ω and 0.5 μF.

Gate circuit: Series resistor 10Ω
Gate-cathode resistor 100Ω.

b) Commutating Thyristors (Thyristors A and B of Figure 2.3)

Manufacturer: Westcode Semiconductors Type (P015UH08FJO) (fast turn-off).

Maximum repetitive peak reverse voltage: 800V,
Average on state current: 15A,
I²t: 265 A²S.

Fusing: International Rectifier Ltd. Type B1000/20.

\( \frac{dv}{dt} \) protection: R-C shunt between anode and cathode of 21Ω and 0.5 μF.

Gate circuit: Series resistor 10Ω
Gate-cathode resistor 100Ω.

c) Return and Commutating Diodes: (D₁-D₆, Dₐ and Dₐ of Figure 2.3)

Manufacturer: AEI Type MF20 (fast-recovery)

Average on state current: 20A,
Maximum repetitive peak reverse voltage: 600V,
Reverse recovery time: 350 ns
Fusing: International Rectifier Ltd. Type B 1000/20.

d) **Main Inductors:** (Inductors $L_R$, $L_Y$ and $L_B$ of Figure 2.3)

Made at Loughborough University. Two series coils each of 120 turns of 16 S.W.G. on a square cross-sectional wood former, 6 cm wide and 10 cm long air-cored.

**Inductance:** 1 mH (2 x 0.5 mH).

e) **Secondary Inductors:** (Each inductor connected in series with each commutating thyristor providing $\frac{dI}{dt}$ protection).

Made at Loughborough University. 47 turns of 16 S.W.G. on the square cross-sectional wood former 4 cm wide and 7 cm long - air cored.

**Inductance:** 50 $\mu$H

f) **Commutating Capacitors:** (Capacitors $C_1$ and $C_2$ of Figure 2.3)

**Manufacturer:** TCC
**Capacitance:** 8 $\mu$F - 1000V

g) **Smoothing Capacitors:**

**Manufacturer:** Plessey Ltd.
**Capacitance:** 5000 $\mu$F - 325V.
(made up from five 1000 $\mu$F - 325V units).
APPENDIX II

U.J.T. CHARACTERISTIC AND OSCILLATOR DESIGN

The U.J.T. has three terminals which are called the emitter (e), base-one (b₁) and base-two (b₂). Between b₁ and b₂, the unijunction has the characteristic of ordinary resistance. This is called interbase resistance and has a value between 4.7 KΩ and 9.1 KΩ at 25°C. The emitter-base has a diode characteristic.

Figure II-1 shows the equivalent circuit of a U.J.T. in the off state, where (V_D) is the diode forward voltage drop.

For conduction to commence through the emitter/base-one diode, the diode has to be forward biased. The voltage required to forward bias the diode is the peak point voltage (V_p) and is given by the following equation:

\[ V_p = n V_{BB} + V_D \]

where:

- \( V_p \) = is the peak point voltage
- \( n \) = is the intrinsic stand-off ratio and lies between 0.51 and 0.82.
- \( V_{BB} \) = is the base-to-base voltage
- \( V_D \) = is the diode forward voltage drop (usually about 0.5V at 25°C).

The oscillator output waveform in Figure II-2 correspond to the circuit diagram Figure II-3, from these an expression for the periodic circuit parameter may be obtained.

\[ V_Y = V_{BB} \left(1-e^{-t/\tau}\right) \] (where \( \tau \) is the CR time constant)
FIG. II.1.— EQUIVALENT CIRCUIT OF U.J.T.
FIG. II. 2. — CAPACITOR VOLTAGE $V_c$.

FIG. II. 3. — BASIC RELAXATION OSCILLATOR CIRCUIT.
\[ V_p = V_{BB} (1 - e^{-(t/\tau + T/\tau)}) \]

\[ (V_{BB} - V_p)/(V_{BB} - V_p) = e^{(T/\tau)} \]

\[ \therefore \quad T = \tau \ln (V_{BB} - V_p)/(V_{BB} - V_p) \]

\[ = \tau \ln \left( \frac{1}{1 - V_p/V_{BB}} \right) \quad \text{Assuming } V_{BB} \gg V_p \]

but \[ V_p = \eta V_{BB} + V_D = \eta V_{BB} \]

\[ \therefore \quad T = \tau \ln \left( \frac{1}{1 - \eta} \right) = RC \ln \left( \frac{1}{1 - \eta} \right) \quad (II-1) \]

**Temperature Compensation**

The value of \( V_p \) is the temperature dependent, decreasing for increase in the temperature. However, it is possible to compensate for temperature drift by utilising the positive temperature coefficient of the interbase resistance. By placing \( R_{b2} \) in series with base two as shown in Figure II-3, increase of the interbase resistance due to temperature causes the interbase voltage to rise. If \( R_{b2} \) is correctly chosen, the increase in the interbase voltage will compensate for decrease in \( V_p \). An approximate formula for the determination of \( R_{b2} \) is given below:

\[ R_{b2} = \frac{0.40}{\eta} \frac{R_{BB}}{V_1} + \frac{(1 - \eta)}{\eta} \frac{R_{b1}}{V_1} \quad (II-2) \]

where: \( V_1 \) = is the supply voltage.
\( R_{BB} \) = is the interbase resistance at 25°C and has a value in the range from 4.7 KΩ to 9.1 KΩ.
The parameters of the circuit can be determined as follows:

Required output frequency = 300 Hz giving a periodic time $T = 3.33$ msec. Assuming a value of $R = 5.6$ kΩ and using equation II-1:

$$3.33 \times 10^{-3} = C \cdot 5.6 \times 10^3 \ln \left(\frac{1}{1-0.6}\right)$$

\[\therefore C = 0.65 \ \mu F\]

A 1 $\mu$F capacitor was chosen since it is the closest standard value.

Using equation II-2:

$$R_{b2} = \frac{0.40 \times 6 \times 10^3}{0.6 \times 15} + \frac{(1 - 0.6) \times 100}{0.6}$$

$$R_{b2} = 333.3 \Omega$$

A 330Ω resistance was chosen since it is the closest standard value.

The choice of $R_{b1}$ is arbitrary. However a value of 100Ω was chosen as this would give a sharp edged pulse.
APPENDIX III

THE TTL UNITS OF THE INVERTOR CONTROL SYSTEM

The detail of the various sections of the circuit used for controlling the invertor is illustrated in Figures III.1 to III.5.

Further details of the associated equipment used:

a) 15V power supply type (Gardners PU42) - 500 mA.
b) 5V power supply type (Gardners PU02) - 1000 mA.

<table>
<thead>
<tr>
<th>Type</th>
<th>Quantity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>75451</td>
<td>8</td>
<td>Dual peripheral positive AND driver</td>
</tr>
<tr>
<td>7474</td>
<td>2</td>
<td>Dual D-type positive-edge triggered bi-stable flip-slop</td>
</tr>
<tr>
<td>74121</td>
<td>7</td>
<td>Variable-period negative-edge triggered monostable</td>
</tr>
<tr>
<td>7405</td>
<td>3</td>
<td>Hexagonal invertor with open-collector output</td>
</tr>
<tr>
<td>7410</td>
<td>1</td>
<td>Triple 3 input positive NAND gate.</td>
</tr>
<tr>
<td>7400</td>
<td>1</td>
<td>Quadruple two input positive NAND gate.</td>
</tr>
<tr>
<td>7413</td>
<td>2</td>
<td>Dual four-input positive NAND with Schmitt input.</td>
</tr>
</tbody>
</table>
FIG. III. 1.—BREAK UP OSCILLATOR.

FIG III 2.—VARIABLE MONOSTABLE
(NEGATIVE EDGE TRIGGERED).
FIG. III.3.- PRODUCTION OF PULSE TRAIN INHIBITED DURING COMMUTATION.
FIG. III. 4.— MAIN THYRISTOR DRIVE CIRCUIT.

$T_1$ - Transistor, Ferranti type ZTX 451
$T_2$ - Transistor, Motorola type MJE 340
$T$ - Transformer, Mullard ferrite E-core type FX 1239
$N_1 = N_2 = 80$ turns (30 s.w.g.)
PULSES FROM OR GATE

T₁ - Transistor Ferranti type ZTX 451
T₂ - Transistor Motorola type MJE 340
T₃ - Transformer Mullard ferrite E core type FX 1239
N₁ = N₂ = 80 turns (30 s.w.g.)

FIG. III. 5. - COMMUTATING THYRISTOR DRIVE CIRCUIT.
Satisfactory operation can be achieved as follows.

i) Set the master oscillator to the appropriate frequency which is six times the invertor frequency.

ii) Switch on the master oscillator.

iii) Switch on the +5V and +15V power supplies to the logic circuit.

iv) Set $V_r$ double-pole isolator to the on position.

v) Switch on $V_r$ rectifier and increase to the required starting voltage.

vi) Set $V_s$ double-pole isolator to the on position.

vii) Switch on $V_s$ rectifier and increase to the required voltage.

For switch off

i) Decrease $V_s$ voltage to zero.

ii) Decrease $V_r$ voltage to zero.

iii) Close $V_s$ isolator.

iv) Close $V_r$ isolator.

v) Switch off the main supply.

vi) Switch off the +5V and +15V power supplies.

vii) Switch off the master oscillator.
APPENDIX V

LINEAR MOTOR DETAIL

Machine data
Manufacturer: Linear Motors Ltd., Stator blocks type A/50/C/25/S/C2.
Number of coils per phase per stator = 14.
Number of turns per coil = 136.
Number of slots per pole per phase = 1.
Slot pitch = 19 mm.
Number of poles per stator (2 stators) = 14.
Pole pitch = 57 mm.
Stator length = 290 mm.
Stator vertical height = 50 mm.
Stator depth = 62 mm.
Maximum current = 5.7 A/phase/stator, short time rated.
Rotor diameter = 1.83 m.
Radius to stator mid-point = 0.82 m.
Rotor thickness (aluminium) = 1.1 cm.
Slot vertical = 32 mm.
APPENDIX VI

EQUIVALENT CIRCUIT PARAMETERS CALCULATIONS

(Aluminium Rotor Disc)

At 50 Hz:

a) **Open Circuit Test:**

<table>
<thead>
<tr>
<th>$I_1$/amps</th>
<th>$I_2$/amps</th>
<th>$I_3$/amps</th>
<th>$W_1$/watts</th>
<th>$W_2$/watts</th>
<th>V/Volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.9</td>
<td>8.3</td>
<td>9.25</td>
<td>-330</td>
<td>+1300</td>
<td>220</td>
</tr>
</tbody>
</table>

Average current = 8.81 amps/ph.
Phase voltage $= \frac{220}{\sqrt{3}} = 127$ Volts/ph.

Input impedance $= \frac{1.27}{8.81} = 14.41 \Omega$/ph

Stator resistance = 3Ω/ph from d.c. measurements.

$Z_{in} = R_l + jX_o$

$(14.41)^2 = 3^2 + X_o^2$

$X_o = 14.09 \Omega$/ph

b) **Locked Rotor Test:**

<table>
<thead>
<tr>
<th>$I_1$/amps</th>
<th>$I_2$/amps</th>
<th>$I_3$/amps</th>
<th>$W_1$/watts</th>
<th>$W_2$/watts</th>
<th>V/Volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.15</td>
<td>10.45</td>
<td>10.9</td>
<td>+200</td>
<td>+2030</td>
<td>220</td>
</tr>
</tbody>
</table>

Phase voltage = 127 Volts/ph
Phase current = 10.83 amps/ph.
Power = 743.33 Watts/ph

$Z = 11.72 \Omega$/ph.
\[ \cos \phi = \frac{W}{V_I} = \frac{743.33}{127 \times 10.83} = 0.54 \]

\[ \phi = 57.31 \]

\[ \sin \phi = 0.84 \]

\[ \frac{V}{I} \cos \phi = R_1 + P_1 \]

\[ 11.72 \times 0.54 = 3 + P_1 \]

\[ \therefore P_1 = 3.32 \]

\[ 11.72 \times 0.84 = Q_1 \]

\[ \therefore Q_1 = 9.84 \]

\[ R_2 = P_1 \left[ 1 + \frac{P_1^2}{(X_0 - Q_1)^2} \right] \]

\[ = 3.32 \left[ 1 + \frac{(3.32)^2}{(14.09 - 9.84)^2} \right] \]

\[ = 5.34 \ \Omega/\text{ph} \]

\[ X_m = X_0 - Q_1 + \frac{P_1^2}{X_0 - Q_1} \]

\[ X_m = 14.09 - 9.84 + \frac{(3.32)^2}{(14.09 - 9.84)} \]

\[ \therefore X_m = 6.84 \ \Omega/\text{ph} \]
\[ x_0 = x_1 + x_m \]

\[ 14.09 = x_1 + 6.84 \]

\[ x_1 = 7.25 \, \Omega/\text{ph.} \]
APPENDIX VII

CALCULATION OF THE STEADY-STATE PERFORMANCE

Sample calculation for motor with the aluminium disc.
For a supply frequency of 50 Hz.

\[ R_1 = 3 \Omega/\text{ph}, \quad X_1 = 7.25 \Omega/\text{ph}, \quad X_m = 6.84 \Omega/\text{ph}, \quad R_2 = 4.6 \Omega/\text{ph} \]

At an assumed slip of 0.22:

Rotor frequency = 11 Hz

\[
Z_{in} = \left[ R_1 + \frac{(R_2/S).X_m^2}{(R_2/S)^2 + X_m^2} \right] + j \left[ X_1 + \frac{X_m(R_2/S)^2}{(R_2/S)^2 + X_m^2} \right]
\]

\[
Z_{in} = [3 + \frac{(20.9)(46.78)}{436.68 + 46.78}] + j [7.25 + \frac{(6.84)(436.68)}{436.68 + 46.78}]
\]

\[
Z_{in} = (5.02 + j13.42)
\]

\[
I_1 = \frac{127}{(5.02 + j13.42)}
\]

\[
I_1 = (3.1 - j8.3) \quad \text{where } I_1 = (A - jB)
\]

\[
|I_1| = \sqrt{(3.1)^2 + (8.3)^2}
\]

\[
|I_1| = 8.86 \text{ amps/ph}
\]
\[ \cos \phi = \frac{A}{\sqrt{A^2 + B^2}} \text{ lagging} \]

\[ \cos \phi = 0.35 \]

Input power = 3 \(|I_1|\cdot V \cdot \cos \phi \]

Input power = 1181.46 watts

\[ |I_2| = |I_1| \cdot \frac{X_m}{\frac{R_2}{S} + jX_m} \]

\[ |I_2| = 8.86 \cdot \frac{6.84}{|21.48 + j6.84|} \]

\[ |I_2| = 2.72 \text{ amps/ph} \]

Output power = 3 \( \left( \frac{R_2}{S} - R_2 \right) \cdot I_2^2 \)

\[ = 367.99 \text{ watts} \]

Efficiency = \( \frac{\text{output}}{\text{input}} \times 100\% \)

Thrust \( T = \frac{P_{out}}{\sqrt{s} (1 - S)} \)

\[ = 81.63 \text{ Newtons} \]
APPENDIX VIII

RELATIONSHIP BETWEEN EFFICIENCY AND OPERATING FREQUENCY

Referring to the equivalent circuit shown in Figure (3.3):

\[ Z_{in} = \left[ \frac{j X_m R_2 / S}{(R_2 / S + j X_m)} \right] + j X_1 + R_1 \]

\[ Z_{in} = \left[ R_1 + \frac{X_m^2 (R_2 / S)}{(R_2 / S)^2 + X_m^2} \right] + j \left[ X_1 + \frac{X_m^2 (R_2 / S)^2}{(R_2 / S)^2 + X_m^2} \right] \]  \( \text{(VIII.1)} \)

Stator power losses = \( I_1^2 R_1 \)

\[ |I_1|^2 R_1 = \frac{V^2 R_1}{\left[ R_1 + \frac{X_m^2 (R_2 / S)}{(R_2 / S)^2 + X_m^2} \right]^2 + \left[ X_1 + \frac{X_m^2 (R_2 / S)^2}{(R_2 / S)^2 + X_m^2} \right]^2} \]  \( \text{(VIII.2)} \)

At low frequencies:

\( X_m < (R_2 / S) \quad X_m^2 \ll (R_2 / S)^2 \)

Approximately therefore:

\[ |I_1|^2 R_1 = \frac{V^2 R_1}{\left[ R_1 + \frac{X_m^2}{(R_2 / S)^2} \right]^2 + \left[ X_1 + X_m \right]^2} \]
\[ I_2 = \frac{V^2 \cdot R_1}{R_1^2 + (X_1 + X_m)^2} \]  

(VIII.3)

Assuming \( X_m^2 / R_2/S \) to be small as compared with \( R_1 \).

\[ |I_2| = |I_1| \cdot \frac{X_m}{\sqrt{(R_2/S)^2 + X_m^2}} \]  

(VIII.4)

Using equation (VIII.3) to give \( I_1^2 \),

\[ I_1^2 = \frac{X_m^2}{X_m^2 + (R_2/S)^2} \cdot \frac{V^2}{R_1^2 + (X_1 + X_m)^2} \]

Since \( X_m^2 \ll (R_2/S)^2 \)

\[ I_1^2 = \frac{X_m^2}{(R_2/S)^2} \cdot \frac{V^2}{R_1^2 + (X_1 + X_m)^2} \]  

(VIII.5)

Rotor outpower = \( I_2^2 \cdot R_2 \cdot (1/S - 1) = I_1^2 \cdot R_2/S \cdot (1 - S) \)

\[ = \frac{V^2 \cdot X_m^2 \cdot R_2/S}{(R_2/S)^2 \cdot [R_1^2 + (X_1 + X_m)^2]} \cdot (1 - S) \]

(VIII.6)

\[ = \frac{V^2 \cdot X_m^2}{R_1^2 + (X_1 + X_m)^2} \cdot \left[ \frac{1}{(R_2/S)} - \frac{S}{(R_2/S)} \right] \]
Rotor input power = $I_2^2 \cdot \frac{R_2}{S}$

$$\frac{V^2 \cdot X_m^2}{R_1^2 + (X_1 + X_m)^2} \cdot \frac{1}{(R_2/S)} \quad (\text{VIII.7})$$

Stator input = rotor input + stator copper loss.

$$= \frac{V^2}{R_1^2 + (X_1 + X_m)^2} \cdot \frac{X_m^2}{R_1 + \frac{X_m^2}{R_2/S}} \quad (\text{VIII.8})$$

Efficiency = \frac{\text{rotor output}}{\text{stator input}}

$$\frac{X_m^2 \left[ \frac{1}{(R_2/S)} - \frac{S}{(R_2/S)} \right]}{R_1 + \frac{X_m^2}{(R_2/S)}}$$

Efficiency = \frac{X_m^2 \left[ \frac{R_2/S - R_2}{(R_2/S)^2 \cdot R_1} \right]}{X_m^2 \ll (\frac{X_3}{S})^2}

Efficiency = $X_m^2 \cdot R_2 \left[ \frac{1 - S}{S \cdot \frac{R_2^2}{S^2} \cdot R_1} \right]$

$$= \frac{S \cdot X_m^2}{R_1 R_2} \left[ 1 - S \right]$$

$$= \frac{S}{R_1 R_2} \left( 1 - S \right) \cdot 4 \pi^2 f^2 L_m^2 \quad (\text{VIII.9})$$
At a constant slip, the efficiency falls with operating frequency approximately in the relationship:

Efficiency $\propto (\text{frequency})^2$
Test (1):

The test was performed generally as in reference (22). The motor with aluminium disc was sinusoidally excited and run on no load with normal frequency and voltage until power input stabilised. The voltage was then gradually reduced to 20 volts with rated frequency maintained and the following measurements were taken for the decreasing voltage.

1. Input power.
2. Line voltage.
3. Line current.

The power input at rated voltage is the sum of friction and windage, core loss, no-load primary resistance loss and harmonic losses. Subtracting the calculated primary resistance loss at the temperature of the test from input gives the sum of the other losses. The separation of these other loss components to give the friction and windage losses at the power measurement stage is not necessary with this test.

The input power minus the stator resistance loss was then plotted as the ordinate with the $(\text{Input voltage})^2$ as the abscissa. Extrapolating the lower part of the curve in a straight line to intercept the zero-voltage axis determines the friction and windage losses as shown on the curve, Figure IX-1. From this, the friction and windage losses for the aluminium disc motor are about 10 watts. While for the aluminium cladded steel motor, the losses are about 7.4 watts as shown in Figure IX-2.
FIG. IX. 1.- GRAPHICAL DETERMINATION OF FRICTION AND WINDAGE LOSSES FOR ALUMINIUM MOTOR.
Fig. IX. 2.—Graphical Determination of Friction and Windage Losses for Aluminium Cladded Steel Motor.
Test (2):

For the sake of accuracy, another test was performed as follows, using calculated moment of inertia.

Rated voltage at rated frequency was supplied to the unloaded motor and the rotor speed was measured. The supply was disconnected and the speed was again measured after 3 minutes. Then the friction and windage losses were determined as shown below:

Initial angular speed $\omega_0 = 6.93$ rad s$^{-1}$
After 3 minutes:
Final angular speed $\omega = 2.09$ rad s$^{-1}$
Now $\omega_0 = \omega - a$
Hence deceleration $a = 2.68 \times 10^{-2}$ rad s$^{-2}$

Alternatively, using the number of counted revolutions in the 3 minute period (= 118 revolutions):

$\omega^2 = \omega_0^2 - 2a\theta$ where $\theta = \text{revolution in radians}$.

$a = 3.01 \times 10^{-2}$ rad s$^{-2}$

The calculated polar moment of inertia $J = 47.37$ Kg.m$^2$.
The mechanical loss torque is given by:

$T_f = J.a$

$\therefore T_f = 47.37 \times 3.01 \times 10^{-2} = 1.43$ Nm.

Friction and windage losses at full speed (6.93 rad s$^{-1}$)

$= 1.43 \times 6.93$

$= 9.9$ W
which agrees well with the results obtained from the previous test (10W).

Using the same method, the results for the composite disc were:

From test 1, mechanical losses = 7.4W.

From test 2, $T_f = 47.37 \times 2.25 \times 10^{-2} = 1.06$ Nm.

Mechanical losses at full speed (6.67 rad s$^{-1}$) = $1.06 \times 6.67$

= 7.07W

which also agrees well with the results obtained from the previous test (7.4W).