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NEW TECHNIQUES FOR THE DETERMINATION OF SYNCHRONOUS MACHINE PARAMETERS

by

Ali Mohammed Baki, B.Sc.

A Master's Thesis

Submitted for the Award of the Degree of

Master of Science

of

Loughborough University of Technology

March 1979

Supervisor:

Professor J R Smith, B.Sc., Ph.D., D.Sc., C.Eng., F.I.E.E.

C by Ali Mohammed Baki
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SYNOPSIS

The ever-increasing complexity of power systems is leading to more severe problems of system stability, and considerations of this are requiring a more detailed modelling of the system generators than has hitherto been undertaken. The induced eddy currents in the iron body of the machine, and the considerable active resistance of this iron, have previously been regarded as fully accounted for by a single damper circuit on each axis of the machine. However, recent work has shown that this is not an adequate representation, and that calculations on this basis may exhibit considerable discrepancies with practical measurements of the transient performance of the machine. Measurements of the generator parameters has been based on a single damper representation, and it is now necessary to re-examine carefully the assumptions underlying these tests and to determine if they can be used to provide an improved machine model.

The main aim of this thesis is to investigate the possibility of using step-response tests to obtain the parameters of a machine model with two damper windings on each axis. From an investigation of this form of equivalent circuit, expressions in the form of transfer functions are developed for the operational inductance of the machine, and these are correlated with measured results to provide the necessary numerical data. Some of the results required can also be obtained from well-established test procedures, and these are therefore used to provide further confirmation of the accuracy of the new machine representation.
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NOMENCLATURE

Subscript notation:

The subscript notation is as follows:

- \(a, b, c\) = armature windings of a, b and c phases respectively
- \(F\) = field winding
- \(1D, 1Q\) = damper windings of 1D, 1Q axes respectively
- \(2D, 2Q\) = damper windings (representing iron effects) of 2D, 2Q axes respectively

The quantities given in the following are per-unit values (unless in bold-type as defined in Appendix 7.1, which are in SI units).

- \(E_{fd}\) = main field winding voltage
- \(I_d, I_q\) = armature current in d, q axes, respectively
- \(I_{1d}, I_{2d}\) = currents in damper circuits
- \(I_{1q}, I_{2q}\) = currents in damper circuits
- \(r_d, r_q\) = armature resistances in the d and q-axes respectively
- \(R_{fd}, R_{1d}\) = armature resistances in the d and q-axes respectively
- \(R_{2d}, R_{1q}, R_{2q}\) = resistances of F, 1D, 2D, 1Q and 2Q respectively
- \(L_{afd}\) = mutual inductance between armature and field
- \(L_{a1d}, L_{a2d}\) = mutual inductances between armature and 1D and 2D, respectively
- \(L_{a1q}, L_{a2q}\) = mutual inductances between armature and 1Q and 2Q, respectively
- \(L_{11d}, L_{22d}\) = self-inductances of 1D and 2D, respectively
- \(L_{11q}, L_{22q}\) = self-inductances of 1Q and 2Q, respectively
- \(L_{ffd}\) = self-inductance of the main field
- \(L_{f1d}, L_{f2d}\) = mutual inductances between the main field and 1D, 2D, respectively
- \(L_{21d}\) = mutual inductance between 2D and 1D
- \(L_{21q}\) = mutual inductance between 2Q and 1Q
- \(\psi_d\) = d-axis flux linkages with the armature winding
- \(\psi_q\) = q-axis flux linkages with the armature winding
\[ \psi_{fd} \] = flux linkages with the field winding

\[ \psi_{1d}, \psi_{2d} \] = flux linkages with 1D and 2D, respectively

\[ \psi_{1q}, \psi_{2q} \] = flux linkages with 1Q and 2Q, respectively

\[ L_d, L_d', L_d'' \] = d-axis synchronous, transient, subtransient and additional subtransient inductances, respectively

\[ L_q, L_q', L_q'' \] = q-axis synchronous, subtransient and additional subtransient inductances, respectively

\[ L_d(p) \] = operational inductances of the d and q-axes, respectively

\[ L_q(p) \] = operational inductances of the d and q-axes, respectively

\[ G(p) \] = an operator which relates field voltage with armature current and linkages in the d-axis

\[ X_d, X_d', X_d'' \] = d-axis synchronous, transient, subtransient and additional subtransient reactances, respectively

\[ X_q, X_q', X_q'' \] = q-axis synchronous, subtransient and additional subtransient reactances, respectively

\[ X_d(p)' \] = operational impedances of the d and q-axes, respectively

\[ X_q(p)' \] = operational impedances of the d and q-axes, respectively

\[ K_{af1d}, K_{a1d} \] = coupling coefficients between a and F, a and 1D

\[ K_{a2d}, K_{f1d} \] = a and 2D, F and 1D, F and 2D, 2D and 1D, a and 1Q

\[ K_{f2d}, K_{21d} \] = a and 2Q, 2Q and 1Q, respectively

\[ K_{a1q}, K_{a2q} \]

\[ \sigma_{a1q} \] = q-axis total coefficient of dispersion (defined by equation (2.14))

\[ \sigma_{21q} \] = coefficient of dispersion between 2Q and 1Q (defined by equation (2.15))

\[ \sigma_{af12d} \] = d-axis total coefficient of dispersion (defined by equation 2.38)

\[ \sigma_{f12d} \] = coefficient of dispersion between F, 2D and 1D (defined by equation 2.39)
\( \theta \) = electric angle of rotor

\( \omega t \) = time in electrical radians

\( p \) = \( d/dt \)

\( u \) = unit step function

\( \mathcal{L} \) = Laplace transform of

\( s \) = Laplace transform variable

E = applied d.c. voltage on a-phase

\( I_{Ao} \) = steady-state current of a-phase (defined by equations 3.11 and 3.53)

\( I_{A1}, I_{A2} \) = decaying components of \( I_q \) and \( I_d \) (with the field winding open for the latter)

\( I_{A3} \)

\( I_{A1s}, I_{A2s} \) = decaying components of \( I_d \) with the field winding closed

\( I_{A3s}, I_{A4s} \)

\( T_{A1}, T_{A2} \) = time constants of \( I_{A1}, I_{A2} \) and \( I_{A3} \), respectively

\( T_{A3} \)

\( T_{A1s}, T_{A2s} \) = time constants of \( I_{A1s}, I_{A2s}, I_{A3s} \) and \( I_{A4s} \), respectively

\( T_{A3s}, T_{A4s} \)

\( T_{ad}, T_{aq} \) = d and q-axes armature open circuit time constants

\( T_{fd}, T_{1d} \)

\( T_{2d}, T_{1q} \) = time constants of \( F, 1D, 2D, 1Q \) and \( 2Q \), respectively

\( T_{2q} \)

\( T_{d0}, T_{d0} \) = d-axis transient, subtransient and additional subtransient open-circuit time constants

\( T_{d0}' \)

\( T_{q0}, T_{q0} \) = q-axis subtransient and additional subtransient open circuit time constants
\( T'_{d}, T''_{d} \) = d-axis transient, subtransient and additional subtransient short circuit time constants

\( T''_{d} \) = subtransient short circuit time constants

\( T'_{q}, T''_{q} \) = q-axis subtransient and additional subtransient short circuit time constants
INTRODUCTION

The basis for modern analysis of salient-pole synchronous machines was established about 40 years ago, when Doherty and Nickle extended the 2-reaction theory of Blondel, and developed methods for determining the performance under certain steady-state and transient conditions. Subsequently, Park generalized the approach by using Heaviside's operational calculus to analyze the behaviour, and he introduced the now familiar characteristic operational impedances.

Waring and Crery explained this impedance in terms of the machine constants and, by deriving formulae for the important reactances and time constants, threw new light upon the performance of these machines. Following this early work, some investigators have presented different techniques for machine analysis and others have developed methods for determining the machine parameters from test. Summaries of the test procedures are contained in references (3) and (5).

Comparatively recent experience has shown that the performance of a modern salient-pole machine may differ in certain important respects from that predicted by the idealized theory based on Park's equations, and studies of more accurate machine representations have been made by several investigators. In recent years, interest has been growing in the use of static methods to obtain machine data, replacing some of the more familiar tests requiring rotation of the machine. A careful appraisal of the results obtained from these tests is clearly important, since the conditions under which the measurements are taken are artificial and reproduce only to a
limited degree the normal-operating conditions of the machine.

Methods 10,12 have been developed to obtain the synchronous, transient and subtransient reactances from the analysis of current transients caused by the creation or cessation of direct current in the armature. The presence of the eddy currents produces a damping action which is difficult to calculate, but which is sufficiently important to cause the behaviour of the machine to differ appreciably from that predicted by the usual simplified theory.

Despite the enormous quantity of work reported on measurement of the parameters of salient-pole machines, no experimentally sound technique has been developed which accounts for the induced eddy currents in the rotor iron. Indeed, a careful review of the literature reveals that several authors employing static tests 10,12 have disregarded this effect entirely.

The indicial response method 12 proposed by Kaminosono and Uyeda has been used by Yao-nan Yu and H.A.M. Moussa 6, but their tests have failed to give reasonable results.

In this thesis, step-response methods are applied to a 5 kVA salient-pole laboratory generator, and the results obtained indicate more than one exponential component in the transient response curve. The experimental work was repeated for different levels of current, and each time it was found that this extra component was significant both for the quadrature axis and for the direct axis. Neglecting this component gives results which are far from accurate when compared with other tests 11,14. It is thus necessary to re-examine the assumptions on which the equations for those measurements are based.
each rotor axis is inadequate to calculate the transient conditions, since the actual characteristic differs considerably from that often assumed when examining the transient processes. The question then naturally arises of how to represent more exactly the performance of a salient-pole generator, and hence how to determine more accurately the circuit parameters using a step-response test. To obtain the best results from this type of test requires that the effect of saturation, the frequency dependence of the parameters and the complex influence of the rotor iron are taken into account. However, to attempt to do this fully would unreasonably complicate an initial investigation of the principal effects involved. With an indicial response method, it is possible to include saturation to a small extent by additional or bias magnetization on the stator or the field. By ignoring the effects of frequency and by replacing the rotor damping action by two closed damper circuits on each axis, a similar but more elaborate version of the primitive machine of Park is obtained.

Scope of the Thesis

The object of this thesis is to show how Park's theory for the salient-pole synchronous generator can be modified to include the damping effect of the rotor iron, and to determine how the machine's parameters of the new mathematical model can easily be calculated from step response test.

The first step when examining the response of the machine is to express the machine equations in operational form. In the first part of this thesis, operational inductances are derived in terms of the machine inductances and resistances, following the Kirschbaum p.u.
system. In this thesis, each axis is analyzed in the same way as a multi-winding transformer, and the derivation of these new operational inductances is clearly now cumbersome, because of the two damper circuits on each axis.

In deriving the parameters stated in Chapters (2) & (3), the direct and quadrature axis equations are represented in terms of the synchronous inductances, the coupling coefficients and the time constants of the individual circuits. This enables a comparison to be made between the present matrix forms and those derived in similar form in a previous investigation, and identifies the inaccuracy in this previous work. To show the damping effect of the additional rotor circuit and to determine the parameters of the machine, measurements were made using the indicial response method on the 5 kVA, 240V, 50 Hz salient-pole synchronous generator. Measurements were made with armature currents up to 14A. A least squares curve fitting technique was used to show the inaccuracy of disregarding the damping effect of the rotor iron. The measurement technique required a step voltage to be applied to one armature phase with the armature stationary, with the resulting transient current being measured by a U.V. recorder. The curves obtained consisted of a multi-exponential decay, and their analysis consisted of resolving them into the sum of three exponential functions for the quadrature axis and three or four exponential functions for the direct axis when the field winding is open and short circuited respectively. It is demonstrated in the thesis that the magnitudes and time constants of the exponential components are linked by simple mathematical relations to the parameters of the machine.
CHAPTER 1

EARLY ANALYSES OF THE PARAMETERS OF SALIENT-POLE MACHINES

The damping effect of the rotor iron plays a considerable part in determining the performance of a synchronous machine, as discussed in the Introduction. Although this effect has been ignored in many previous investigations, it is becoming increasingly necessary to include it when a rigorous analysis of the performance is attempted. Nevertheless, since the previous studies form the basis from which techniques described later in the thesis were developed, the following sections are included to provide a summary of two methods\textsuperscript{10,12} used to determine the parameters of a salient-pole machine with the effect of the rotor iron disregarded. Although these methods involve slightly different tests these differ only in detail, and since they both originate from the same basic concept the information provided is fundamentally the same.

The first test determines the parameters of the machine from the decay of a direct current through two series phases of the armature winding, and the second uses a similar approach in determining the parameters from a test involving a step increase of voltage to one phase of the armature. The basic principle involved in the two tests requires knowledge of the operational impedances of an idealised salient-pole machine, as established by Waring and Crary\textsuperscript{4} and discussed below.
1.1 Operational Impedances of an Idealised Salient-Pole Synchronous Machine

Early attempts to derive the operational impedances of a salient-pole synchronous machine, in terms of the machine reactances and resistances, were based on considerations of an idealised machine with one damper circuit on each of the direct (d) and quadrature (q) axes, and with a field winding on the d-axis. Based on the 3-winding transformer analogue shown in Fig. 1.1, the authors obtained expressions for the d- and q-axis operational impedances from consideration of the rotor circuit voltages and flux linkages as:

\[
X_d(p) = X_d - \frac{p^2(X_{11d}a_d^2 - 2X_{1d}X_{11d}a_d + X_{11d}f_dX_{11d}f_d) + p(X^2R_d + X^2R_{d_f})}{p^2(X_{11d}f_d^2 + X_{11d}f_d^2) + p(X_{11d}R_d + X_{11d}R_{d_f}) + R_{1d}R_{d_f}}
\]

and

\[
X_q(p) = X_q - \frac{pX_{11q}^2}{pX_{11q}^2 + R_{1q}}
\]

respectively. In the above equations, subscripts d and q refer to the d and q axes of the machine, and subscripts 1, a, f to the damper, armature and field circuits respectively. Thus \(X_d\) is the synchronous reactance of the d-axis armature circuit, \(X_{11d}\) is the self-reactance of the d-axis damper circuit, and \(X_{afd}\) is the mutual reactance between the armature and the field circuit along the d-axis. The other resistance and reactance terms are similarly defined. The d and q axis subtransient reactances are then found by letting \(p \to \infty\) in the
above equations, when, from equation (1.1)

\[ X'' = X_d(\infty) = X_d - \frac{X_{11d}X^2 - 2X_{11d}X_{a1d}X_{a1d}X_{a1d}X_{ffd}X_{a1d}X_{ffd}X_{a1d}}{X_{11d}X_{ffd}X_{ffd}} \]  

(1.3a)

Alternatively, in terms of the various coupling coefficients, equation (1.3a) becomes:

\[ X'' = X_d \left[ 1 - \frac{K^2_{a1d}X_{ffd}X_{a1d}X_{a1d}X_{ffd}X_{a1d}}{1 - K^2_{f1d}} \right] \]  

(1.3b)

where: \( K^2_{a1d} = \frac{X_{a1d}}{X_{a1d}} \), \( K_{f1d} = \frac{X_{ffd}}{X_{ffd}} \), etc.

Similarly, from equation (1.2):

\[ X'' = X_q(\infty) = X_q - \frac{X^2_{a1q}}{X_{11q}} \]  

(1.4a)

or in terms of the coupling coefficient:

\[ X'' = X_q \left( 1 - K^2_{a1q} \right) \]  

(1.4b)

The d-axis transient reactance is found by letting \( p \to \infty \) and

\[ X_{11d} = \infty \] in equation (1.1), when

\[ X' = X_d - \frac{X_{a1d}}{X_{ffd}} \]  

(1.4c)

or in terms of the coupling coefficient

\[ X' = X_d \left( 1 - K^2_{a1d} \right) \]  

(1.4d)
1.2 **Determination of the Machine Parameters from a Current-Decay Test**

In 1962, M.K. Pawluk\(^{10}\) described a further method for determining the parameters of a salient-pole machine from the decay of a direct-current through two phases in series with the armature stationary. The resulting transient current is measured by an oscillograph, and its analysis consists of resolving it into the sum of two exponential functions for the q-axis and the sum of two and three exponential functions for the d-axis when the field is respectively open and short circuited. From the magnitudes and time constants of the individual components of the measuring transient current, the synchronous, transient and subtransient reactances included in the machine equations are found.

On the basis of an idealised salient-pole machine, and from the flux linkages and voltage equations\(^{10}\), the operational equations obtained for the q-axis were:

\[
\begin{bmatrix}
R_q + pX_q & pX_{qQ} \\
pX_{qQ} & R_Q + pX_q
\end{bmatrix}
\begin{bmatrix}
i_q' \\
i_Q'
\end{bmatrix}
= 
\begin{bmatrix}
X_q & X_{qQ} \\
X_{qQ} & X_Q
\end{bmatrix}
\begin{bmatrix}
i_q \\
i_Q
\end{bmatrix}
+ 
\begin{bmatrix}
i_{qQ}'
\end{bmatrix}
\]

(1.5)

where in the original terminology \(R_q\) and \(R_Q\) are the resistances of the armature and the q-axis damper circuit respectively; \(X_q\) and \(X_Q\) are the q-axis reactances of the armature and damper circuits respectively; \(X_{qQ}\) is the q-axis mutual reactance between the armature and the respective damper circuit; \(i_q'\) and \(i_Q'\) are the q-axis currents in the armature and damper circuits, and \(i_{qQ}'\) is the initial
value of $i_q$. After solving equation (1.5) for the current $i'_q$, and introducing the time constants $T'_q$ and $T'_{qq}$ and the dispersion coefficient $\sigma_{qq}$ in place of the reactances, and resistances, the following equation is obtained:

$$ i'_q = \frac{p^2 + p \frac{1}{T'_q}}{p^2 + p \left( \frac{1}{T'_{qq}} + \frac{1}{T'_q} \right) + \frac{\sigma_{qq}}{T'_{qq}}} i_{q0} $$

where:

$$ \sigma_{qq} = \text{total coefficient of dispersion on the q-axis} $$

$$ \sigma_{qq} = \frac{X_q X_{qq} - X_q^2}{X_q X_{qq}} $$

$$ T'_{qq} = \text{time constant for the armature on the q-axis, with the damper circuit closed and its resistance } R_q \text{ assumed negligible.} $$

$$ = T_q \sigma_{qq} \quad (1.6c) $$

$$ T_q = \text{time constant for the q-axis armature circuit with the damper circuit open.} $$

and $T'_{qq} = T_q \sigma_{qq} \quad (1.6d)$. 

The time constants $T_q$ and $T'_{qq}$ have similar interpretations relative to the damper winding $Q$. 
The denominator of equation (1.6a) is a 2nd-order polynomial in \( p \), with the two roots

\[
P_{q1}, P_{q2} = \frac{1}{2} \left[ -\left( \frac{1}{T_{qQ}} + \frac{1}{T_{Qq}} \right) \pm \sqrt{\left( \frac{1}{T_{qQ}} + \frac{1}{T_{Qq}} \right)^2 - 4 \sigma_{QQ}} \right]
\] (1.7a)

As \( 0 < \sigma_{QQ} < 1 \), the term under the root of equation (1.7a) is always positive and the denominator of equation (1.6a) has therefore two real and negative roots. Consequently, the time variation is the sum of two exponential functions with time constants

\[
T_{q1} = -\frac{1}{P_{q1}}
\] (1.7b)

and \( T_{q2} = -\frac{1}{P_{q2}} \)

From the known relationships between the sum and the products of the roots of 2nd-order algebraic equations, the following relations are obtained:

\[
T_q + T_Q = T_{q1} + T_{q2}
\] (1.8)

\[
\frac{1}{T_{qQ}} + \frac{1}{T_{Qq}} = \frac{1}{T_{q1}} + \frac{1}{T_{q2}}
\] (1.9)

and \( \sigma_{QQ} = \frac{T_{q1} T_{q2}}{T_q T_Q} = \frac{T_{qQ} T_{Qq}}{T_{q1} T_{q2}} \) (1.10)

By applying the Heaviside theorem, the time variation of the current \( i_q \) is obtained as:
After putting \( i_{q1} = \frac{i_{q1}}{i_{q0}} \) and \( i_{q2} = \frac{i_{q2}}{i_{q0}} \), and solving equations (1.8) and (1.10):

\[
T'_{qQ} = \frac{1}{\frac{i_{q1}}{t_{q1}} + \frac{i_{q2}}{t_{q2}}}
\]

and

\[
T'_{qQ} = \frac{1}{\frac{i_{q1}}{t_{q2}} + \frac{i_{q2}}{t_{q1}}}
\]

In practice, the test procedure required the recording of the transient current to be transferred to semilog paper, from which the two decaying components and their corresponding time constants could be calculated by the well-known means\(^{14}\). These values were then substituted in equation (1.12), and when the values of \( T'_{qQ} \) and \( T'_{qQ} \) thus found were substituted into equation (1.10), the coefficient of dispersion \( \sigma_{qQ} \) was obtained.

The time constant \( T_q \) follows simply from equation (1.6c) and by measuring the q-axis armature resistance per phase, the q-axis synchronous reactance is given by:

\[
X_q = T_q R_q
\]

The subtransient reactance is:
The d-axis synchronous reactance \(X_d\) may be similarly obtained, merely by replacing subscript \(q\) by subscript \(d\).

The operational equations which apply for the d-axis with the field circuit closed are:

\[
\begin{vmatrix}
R_d + pX_d & pX_{df} & pX_{dD} & 1_d & X_d & X_{df} & X_{dD} & 1_{d0} \\
pX_{df} & R_f + pX_f & pX_{fD} & 1_f & X_{df} & X_f & X_{fD} & 0 \\
pX_{dD} & pX_{fD} & R_D + pX_D & 1_D & X_{dD} & X_{fD} & X_D & 0 \\
\end{vmatrix}
\]

\(1.15\)

where, again, in the original terminology, \(i_d\), \(i_f\) and \(i_D\) are respectively the d-axis armature, field and damper currents; \(X_f\) and \(X_D\) the self reactances of the field and damper windings; \(R_f\) and \(R_D\) the field and damper resistances; \(X_{df}\) and \(X_{dD}\) the mutual reactance between the armature/field and field/damper circuits respectively; \(X_{fD}\) is the mutual reactance between the field and the damper winding.

On solving equation (1.15) for \(i_d\):

\[
i_d = \frac{p^3 + p^2 \left( \frac{1}{T_{dfD}} + \frac{1}{T_f} \right) + p \frac{1}{\sigma_{dfD} T_f T_D}}{p^3 + p^2 \left( \frac{1}{T_{dfD}} + \frac{1}{T_f} + \frac{1}{T_D} \right) + p \frac{T_d + T_f + T_D}{\sigma_{dfD} T_d T_f T_D} + \frac{1}{\sigma_{dfD} T_d T_f T_D}} \cdot i_{d0}
\]

\(1.16a\)
where:

\[ \sigma_{dfD} = \text{total coefficient of dispersion on the d-axis} \]

\[ \sigma_{dfD} = \frac{X_d X_f X_D + 2X_d X_f X_D - X_d X_f X_D - X_d X_f X_D}{X_d X_f X_D} \]

\[ T_f = \text{field time constant} \]

\[ T_D = \text{time constant of the d-axis damper winding} \]

and \[ T_f^* = \frac{\sigma_{dfD}}{\sigma_{fD}} T_f \]

\[ \sigma_{fD} = \text{coefficient of dispersion between the field and the damper winding} \]

\[ \sigma_{fD} = \frac{X_f X_D - X^2_{fD}}{X_f X_D} \]

and \[ T_f^* = \frac{\sigma_{dfD}}{\sigma_{fD}} T_f \]

\[ \sigma_{dD} = \frac{X_d X_D - X^2_{dD}}{X_d X_D} \]

and \[ T_D^* = \frac{\sigma_{dfD}}{\sigma_{df}} T_D \]

\[ \sigma_{df} = \frac{X_d X_f X_D - X^2_{df}}{X_d X_f} \]
From the relationship applying for a 3rd-order polynomial equation, the relation between the machine time constants and the three time constants read from an oscillographic recording are:

\[ T_d + T_f + T_D = T_{d3} + T_{d4} + T_{d5} \]  \hfill (1.17)

\[ \frac{1}{T_{dfD}} + \frac{1}{T_f} + \frac{1}{T_D} = \frac{1}{T_{d3}} + \frac{1}{T_{d4}} + \frac{1}{T_{d5}} \]  \hfill (1.18)

and \( \sigma_{dfD} = \frac{T_{d3} T_{d4} T_{d5}}{T_d T_f T_D} \)  \hfill (1.19)

where \( T_{d3}, T_{d4} \), and \( T_{d5} \) are the time constants found from replotting the recording transient current on semilog graph paper. \( T_d \) and \( T_D \) were found previously from the d-axis decay test with the field winding open circuited, and \( T_{fD} \) and \( \sigma_{dfD} \) are found from equation (1.17) and (1.19) respectively.

The d-axis subtransient reactance is given by:

\[ X_d'' = X_d \sigma_{dfD} = R_d \left( T_d \frac{\sigma_{dfD}}{\sigma_{fD}} \right) = R_d \frac{T_{f1}}{T_D} \]  \hfill (1.20)

where \( \sigma_{fD} \) is found from the current decay test as applied to the d-axis field winding with the armature winding open circuit (similar to the d-axis armature current decay test with the field winding open) as:

\[ \sigma_{fD} = \frac{T_{f1} T_{f2}}{T_f T_D} \]  \hfill (1.21)

= equation (1.16d).
where $T_{f1}$ and $T_{f2}$ are the time constants obtained from transferring the recorded transient field current to semilog paper.

By substituting for $\sigma_{fd}$, together with $X_d$ and $\sigma_{dfD}$, into the equation (1.20), $X''_d$ can be found.

If the values of $\sigma_{dfD}$ and $\sigma_{fd}$ given by equations (1.16b) and (1.16d) are written in terms of the coupling coefficients of the machine, equation (1.20) becomes:

$$X''_d = \frac{X_d (1 + 2K_{df}^2 K_{fd} K_{dD} - K_{df}^2)}{(1 - K_{fd}^2)}$$

or

$$X''_d = X_d [1 - \frac{K_{df}^2 - 2K_{df} K_{fd} K_{dD} + K_{dD}^2}{1 - K_{fd}^2}]$$

(1.22)

where:

$$K_{df}^2 = \frac{X_{df}^2}{X_d X_f}, \quad K_{fd} = \frac{X_{fd}}{X_f X_D} \quad \text{etc.}$$

Equation (1.22) is similar to that derived by Waring and Crary\textsuperscript{4}, and given by equation (1.3b).

The d-axis transient reactance is given by:

$$X'_{d} = X'_{df} = X_{d} \sigma_{df}$$

(1.23)

where $\sigma_{df}$ is the coefficient of dispersion between the d-axis armature winding with the field winding. Since this cannot be found (since the damper winding cannot be opened) the following procedure is required. By application of a current decay test on
the d-axis armature, with the field winding open, \( \sigma_{dD} \) can be
found. By substituting \( \sigma_{dD} \) together with \( \sigma_{dfD} \) and \( T_f \) into equation
\( (1.16e) \), \( T_f^m \) can be found. \( T_f^m \) then follows from equation \( (1.18) \),
and by substituting into the equation \( (1.16g) \), \( \sigma_{df} \) can be found.
Finally, the d-axis transient reactance is given by:

\[
X'_d = X_d \sigma_{df} \quad (1.23)
\]

1.3 Determination of Parameters from Indicial Response Tests

Kaminsono and Uyeda\(^ {12} \) performed indicial response tests, in
which a step increase of voltage was applied to one phase of an
armature winding. Their analysis follows that of the previous
section, and they obtained the components of the transient phenomena
in exactly the same manner. Using terminology as defined in the
nomenclature, the p.u. differential equations which apply when
their test is performed on a phase aligned with the d-axis are,
with the field winding open circuit:

\[
\begin{bmatrix}
E_1 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
\frac{pL_d + r_d}{T_{1d}} & -pK_{a1d} & I_d \\
-pK_{a1d} & p + \frac{1}{T_{1d}} & I_{1d} \\
-pK_{a1d} & p + \frac{1}{T_{1d}} & I_{1d}
\end{bmatrix}
\]

(1.24)

and with the field winding short circuit:

\[
\begin{bmatrix}
E_1 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
\frac{pL_d + r_d}{T_{fd}} & -pK_{a1d} & -pK_{afd} & I_d \\
-pK_{a1d} & p + \frac{1}{T_{1d}} & -pK_{f1d} & I_{1d} \\
-pK_{afd} & -pK_{f1d} & p + \frac{1}{T_{fd}} & I_{fd}
\end{bmatrix}
\]

(1.25)
The transient current through the armature phase winding is obtained from the Laplace transform of equation (1.24) as:

\[
I_d = I_{A0} - I_{A1}e^{-t/T_{A1}} - I_{A2}e^{-t/T_{A2}}
\]  

(1.26)

where from equation (1.24) and (1.26):

\[
L_d = r_d T_{A1} \left(1 + \frac{I_{A2} T_{A2}}{T_{A1}}\right)/\left(1 + \frac{I_{A2}}{I_{A1}}\right)
\]

(1.27)

and

\[
K_{f1d}^2 = \frac{I_{A2}}{T_{A1}} \left(1 - \frac{T_{A2}}{T_{A1}}\right)^2 \left(\frac{I_{A2}}{T_{A1}} + \frac{T_{A2}}{T_{A1}}\right)(1 + \frac{I_{A2}}{I_{A1}} T_{A2}/T_{A1})
\]

(1.28)

The transient current with the field winding closed follows similarly from equation (1.25) as:

\[
I_d = I_{A0} - I_{A1s}e^{-t/T_{A1s}} - I_{A2s}e^{-t/T_{A2s}} - I_{A3s}e^{-t/T_{A3s}}
\]

(1.29)

while from solution of equations (1.25) and (1.29), the following relations were obtained:

\[
\frac{1}{T_{fd}} = \frac{1}{T_{A1s}} + \frac{1}{T_{A2s}} + \frac{1}{T_{A3s}} - \frac{1}{T_{ad}} + \frac{1}{T_{1d}}
\]

(1.30)

\[
K_{f1d}^2 = \frac{I_{A1s}}{E} \left(\frac{1}{T_{A1s}} - \frac{I_{A2s}}{T_{A2s}} \frac{I_{A3s}}{T_{A3s}} - \frac{I_{A1s}}{T_{1d}} + \frac{T_{A1s}}{T_{1d}} \frac{T_{A1s}}{T_{f1d}} - 1\right)
\]

(1.31)
The \( q \)-axis quantities may be obtained by a similar procedure, simply by replacing subscript \( d \) by subscript \( q \) in equations (1.24), (1.26), (1.27) and (1.28).

The per unit synchronous reactances of the machine are calculated from:

\[
X_d = \frac{3}{2} L_d \tag{1.33}
\]

and

\[
X_q = \frac{3}{2} L_q
\]

where equation (1.33) shows the relation between the p.u. synchronous inductances introduced by Kaminosono and the equivalent impedance conventionally used.

The p.u. subtransient reactances of the machine are:

\[
X_d'' = \frac{3}{2} L_d \left[ 1 - \frac{K_{afd}^2 + K_{a1d}^2 - 2K_{f1d} K_{a1d} K_{afd}}{1 - K_{f1d}^2} \right] \tag{1.34}
\]

and

\[
X_q'' = \frac{3}{2} L_q (1 - K_{a1q}^2) \tag{1.35}
\]

Again, equations (1.34) and (1.35) show the relation between quantities introduced by Kaminosono and their familiar equivalent.
impedance (for more details see reference 28).

An examination of the matrix equations (1.24) and (1.25), from the point of view of the p.u. values of the quantities involved, shows that these p.u. equations were incorrectly derived by the original authors, and their solution therefore provides a set of equations which differ from equations (1.27), (1.28), (1.30), (1.31) and (1.32).

Equations (3.1) and (3.41) in Chapter 3 represent the p.u. operational equations derived for the modified salient-pole machine where the damping effect of the rotor iron has been taken into consideration. If this effect is ignored, and subscript q is replaced by subscript d in equation (3.1), then eqns. (3.1) and (3.41) reduce to:

With the field winding open circuit:

\[
\begin{bmatrix}
E_1 \\
0
\end{bmatrix} = \begin{bmatrix}
pL_d + r_d & -pL_d & I_d \\
-pK_{a1d} & p + \frac{1}{T_{1d}} & I_{1d}
\end{bmatrix}
\]  

(1.36)

and with the field winding short circuit:

\[
\begin{bmatrix}
E_1 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
pL_d + r_d & -pL_d K_{a1d} & -pL_d K_{afd} & I_d \\
-pK_{a1d} & p + \frac{1}{T_{1d}} & -pK_{f1d} & I_{1d}
\end{bmatrix}
\]  

(1.37)
Equations (1.36) and (1.37) represent the correct p.u. operational equations of the idealised salient-pole synchronous machine and from their solution we obtain:

\[
T_{fd} = T_{A1s} + T_{A2s} + T_{A3s} - T_{ad} - T_{1d}
\]

\[
K_{f1d}^2 = 1 - \frac{I_{A1s} T_{A2s} T_{A3s} - I_{A2s} T_{A1s} T_{A3s} + I_{A3s} T_{A1s} T_{A2s}}{T_{1d} T_{fd} T_{A0s}}
\]

and

\[
K_{adf}^2 = 1 + \frac{1}{T_{ad} T_{fd}} [T_{1d} T_{fd} (1 - K_{f1d}^2) + T_{ad} T_{1d} (1 - K_{adf}^2) - T A1s T A2s - T A2s T A3s - T A3s T A1s]
\]

Comparison of the equations (1.36) and (1.37) with equations (1.24) and (1.25) respectively, shows the necessity of correcting the latter two equations by multiplying the second term of the first row of equation (1.24), and the second and the third terms of the first row of equation (1.25) all by \(L_d\).

Comparison of equations (1.38), (1.39) and (1.40) with the previously derived equations (1.30), (1.31) and (1.32) shows the necessary corrections. Equations (1.36), (1.38), (1.39) and (1.40) may also be obtained by replacing the damping circuit of the rotor iron by the field winding (i.e. replacing suffix 2 by suffix f) and subscript q by subscript d in equations (3.1), (3.12), (3.39) and (3.40), derived later in Chapter 3 (with suffix s added to each current component and its respective time constant). It
is necessary to make clear that equation (1.38) was also obtained by Pawluk\(^{10}\) and given by equation (1.17). Equation (1.37) may be checked by comparison with equation (3.41), when the damping circuit of the rotor iron is ignored.

### 1.4 Summary

As has been described in this Chapter, two similar step response tests have been used to determine the parameters of a synchronous machine. The first method\(^{10}\) determines the synchronous, transient and subtransient reactances from the decay of a direct current through two armature phases in series, and requires a high resistance \(R \gg r\) (the armature phase resistance) to be used as shown in Fig. 1.2 to protect the supply after the armature terminals have been short circuited. The condition that \(R \gg r\) will also minimise the voltage drop across the contacts of the switch \(S\), and thereby reduce the associated measurement errors.

The second method\(^{12}\) determines the synchronous and subtransient reactances of a machine from tests requiring the application of a d.c. step voltage to one phase of the armature as shown in Fig. 1.3 and provided that the impedance of the d.c. source is negligible, it will provide results identical with those given by the first method.
CHAPTER 2

OPERATIONAL INDUCTANCES OF A MODIFIED SALIENT-POLE SYNCHRONOUS GENERATOR

2.1 The Modified Salient-Pole Synchronous Generator

As explained in Chapter 1, the performance of a salient-pole synchronous generator differs in some important respects from that predicted from an idealised theory based on Park's equations, and to reduce this difference it is necessary to improve the representation by re-examining the assumptions on which the derivation of the performance equations is based.

The presence of the considerable mass of the (possibly) solid rotor iron within the magnetic circuits of the machine, needs to be taken into account when calculating accurately the transient processes. To alleviate the complexity of the equations derived, attempts have been made to adopt an improved generator representation, based on modified Park's equations and with two damper circuits employed on each axis. Figure 2.1 shows such a modified d,q circuit model, with the armature phase windings represented by coils a, b and c (mutually separated by 120° electrical), the field winding by coil f, the damper windings by two closed circuits 10 and 10, and the damping effect of the rotor iron by coils 20 and 20 respectively. In analysing this model, the following assumptions are involved.

1. The flux distribution in the air-gap is sinusoidal, with slot effects being neglected.
2. Saturation and hysteresis effect are neglected; this includes local saturation caused by the skin effect associated with the eddy currents in the rotor iron.

3. The effect of frequency on all parameters is neglected. All the eight circuits represented in Fig. 2.1 have individual resistances and self-inductances, and a mutual inductance with respect to all other circuits on the same axis.

4. No mutual resistance between the damper circuits on the same machine axis is considered. This assumption is valid because there are no current paths common to both damper circuits.

5. In the 2-axis representation of the machine shown in Fig. 2.1, it is assumed that the fluxes in the two axes act independently, and that there is no mutual inductance between windings on the d and the q axes.

2.2 Operational Inductances

In many important operating problems, it is necessary to determine results viewed from the machine armature terminals as, for example, when computing the short-circuit currents or any other kind of transient change of current and voltage. Early attempts to derive the operational impedances of a synchronous machine were based on simplified considerations of an idealized machine. It is now regarded as necessary to show that the damping effect of the eddy currents in the rotor iron has an influence on the transient processes, and hence that the familiar equivalent arrange-
ment with one damper circuit on each rotor axis, is insufficient. In consequence, the previous theory\textsuperscript{4} is sometimes not adequate and requires modification. It is possible to represent the whole group of \(d\) or \(q\) axis circuits by an equivalent static circuit. Since no mutual resistance exists between these circuits, each axis may be analysed in exactly the same manner as a multiwinding transformer\textsuperscript{4}, with the number of windings being the same as the number of circuits symmetrical about the axis being considered. Reciprocity of the mutual inductance coefficients is an essential feature of a static equivalent circuit, and this introduces a very helpful simplification in calculating the operational inductances, which become increasingly cumbersome as additional damping circuits are introduced.

For the reason mentioned above, the new operational inductances are derived following H.S. Kirschebaum\textsuperscript{21}, using the same basic equations. In deriving the operational inductances for both groups of symmetrical coils, the \(q\)-axis is represented by the 3-winding transformer of Figure 2.2, and the \(d\)-axis by the 4-winding transformer of Figure 2.3.

### 2.2.1 \(q\)-axis operational inductance

Using the terminology defined in the nomenclature, the voltage equations for the \(q\)-axis rotor circuits may be written directly from Figure 2.2 as:

\[
I_{1q} R_{1q} + p \psi_{1q} = 0
\]  
(2.1)

and

\[
I_{2q} R_{2q} + p \psi_{2q} = 0
\]  
(2.2)
FIG. 2.1. MODIFIED CIRCUIT MODEL OF A 3-PHASE SALIENT-POLE SYNCHRONOUS GENERATOR

FIG. 2.2. A 3-WINDING TRANSFORMER ANALOGOUS TO THE CIRCUITS SYMMETRICAL ABOUT THE q-AXIS

FIG. 2.3. A 4-WINDING TRANSFORMER ANALOGOUS TO THE CIRCUITS SYMMETRICAL ABOUT THE d-AXIS
The p.u. equations follow Kirschbaum's\textsuperscript{21} p.u. system (see equations 7.1.8 - 7.1.10 in Appendix 7.1.1).

i.e.

\[ \psi_{1q} = L_q I_{1q} + L_q K_{12q} I_{2q} + L_q K_{a1q} I_q \] \hspace{1cm} (2.3)

\[ \psi_{2q} = L_q K_{21q} I_{1q} + L_q I_{2q} + L_q K_{a2q} I_q \] \hspace{1cm} (2.4)

and the q-axis flux linkage with the armature is:

\[ \psi_q = L_q K_{a1q} I_{1q} + L_q K_{a2q} I_{2q} + L_q I_q \] \hspace{1cm} (2.5)

Solving equations (2.1) and (2.2) and noting that \( K_{12q} = K_{21q} \), we obtain:

\[
I_{1q} = \frac{[p^2L_q^2 (K_{21q} K_{a2q} - K_{a1q}) - pL_q K_{a1q} R_{2q}]}{[p^2L_q^2 (1 - K_{21q}^2) + pL_q (R_{1q} + R_{2q}) + R_{1q} R_{2q}]} I_q \] \hspace{1cm} (2.6)

and

\[
I_{2q} = \frac{[p^2L_q^2 (K_{21q} K_{a1q} - K_{a2q}) - pL_q K_{a2q} R_{1q}]}{[p^2L_q^2 (1 - K_{21q}^2) + pL_q (R_{1q} + R_{2q}) + R_{1q} R_{2q}]} I_q \] \hspace{1cm} (2.7)

After substitution of equations (2.6) and (2.7) in equation (2.5)

\[
\psi_q = \frac{K_{a2q}^2 R_{1q} - R_{1q} R_{2q} + R_{1q} R_{2q}}{[p^2L_q^2 (1 - K_{21q}^2) + pL_q (R_{1q} + R_{2q}) + R_{1q} R_{2q}]} \] \hspace{1cm} (2.8)
The general equation, derived by Park \(^2\) for the q-axis flux linkages with the armatures can be written as:

\[
\psi_q = L_q(p) I_q
\]  

(2.9)

where \(L_q(p)\) = operational inductance of the q-axis circuit.

Comparing equations (2.8) and (2.9), it is evident that with two rotor circuits on the q-axis:

\[
L_q(p) = \frac{L_q^2 [p^2 L_q^2 (1 - K_{21q}^2 - K_{a1q}^2 - K_{a2q}^2 + 2 K_{a1q} K_{a2q} K_{21q}) - p L_q (K_{a1q} R_{2q} + K_{a2q} R_{1q}) + K_{a2q}^2 R_{1q} - R_{1q} R_{2q} + R_{1q} R_{2q}]}{[p^2 L_q^2 (1 - K_{21q}^2) + p L_q (R_{1q} + R_{2q}) + R_{1q} R_{2q}]} \]  

(2.10)

The direction of \(I_q\) has been defined as positive when an m.m.f. is produced in the same direction as the m.m.f. produced by the damper circuit, and follows that used by Kirschbaum.\(^2\). If the reference direction of \(I_q\) is reversed, the sign of the last terms on the right-hand side of equations (2.3)-(2.5) becomes negative. The signs of \(I_{1q}\) and \(I_{2q}\) in equations (2.6) and (2.7) become negative, but since the positive signs of \(I_q\) in equations (2.8) and (2.9) become negative, the signs of all the terms in equation (2.10) remain unchanged. In other words, reversing the direction of the reference current \(I_q\) has no effect on any of the terms of equation (2.10).
2.2.2 q-axis subtransient and additional subtransient inductances

Having deduced the operational form of $L_q(p)$ in equation (2.10), it is easy to find the q-axis additional subtransient inductance $L''_q$ of the machine. (This new definition has been given because of the damping effect of the rotor iron, and differs from that conventionally used and given by $L_d''$ when an idealised machine is considered).

The operator $L_q(p)$ represents the inductance of the q-axis circuits, and its value at $t = 0$ is found by letting $p \to \infty$, which in the notation of the operational calculus corresponds to taking the initial value of a function. Thus, when neglecting the rotor resistance, we get for $t = 0$, and $p \to \infty$, the additional subtransient inductance as:

$$L''_q = L_q(\infty) = L_q \left[ 1 - \frac{K^2_{a1q} + K^2_{a2q} - 2K_{a1q}K_{a2q}K_{12q}}{1 - K^2_{21q}} \right] \quad (2.11)$$

or in terms of the corresponding inductances:

$$L''_q = L_q \frac{L_{22q}^2 L_{a1q}^2 + L_{11q}^2 L_{a2q}^2 - 2L_{a1q}L_{a2q}L_{21q}}{L_{11q} L_{22q} - L_{12q}^2} \quad (2.12)$$

Equation (2.11) can be written in terms of the coefficients of dispersion, in the same manner as described in reference [15], i.e.
where:

\[
\sigma_{a21q} = 1 - (K_{a1q}^2 + K_{a2q}^2 + K_{21q}^2 - 2K_{a1q}K_{a2q}K_{21q})
\]  

(2.14)

\[
\sigma_{21q} = 1 - K_{21q}^2
\]  

(2.15)

and \(\sigma_{21q}\) = coefficient of dispersion between the first and second damper circuits

The \(q\)-axis subtransient inductance of an idealised machine \(L''_q\) (with the damping effect of the rotor iron neglected) may be found from equation (2.10), by letting \(L_{22q} = \infty\), and \(p \rightarrow \infty\) as

\[
L''_q = \frac{L_{q}}{1 - K_{a1q}^2}
\]  

(2.16)

The relation between the quantities shown by equations (2.11), (2.12), (2.13) & (2.16) and the conventional equivalent impedances are:

\[
X_q = \frac{3}{2} L_q
\]

\[
X''_q = \frac{3}{2} L''_q
\]  

(2.17)

and \(X''''_q \approx \frac{3}{2} L''''_q\)
Multiplying equation (2.16) by a factor \( \frac{3}{2} \) thus leads to the familiar q-axis subtransient reactance \( X''_q \) derived by Waring and Crery.

### 2.2.3 d-axis operational inductance

Using the terminology defined in the nomenclature, the voltage equations of the d-axis rotor circuits may be written directly from Figure 2.3 as:

\[
E_{fd} = I_{fd} R_{fd} + p \psi_{fd} \quad (2.18)
\]

\[
0 = I_{1d} R_{1d} + p \psi_{1d} \quad (2.19)
\]

\[
0 = I_{2d} R_{2d} + p \psi_{2d} \quad (2.20)
\]

The p.u. equations follow those used by Kirschbaum\(^{21}\)

(see Appendix 7.1.2)

i.e.

\[
\psi_{fd} = L_d K_{f1d} I_{1d} + L_d K_{f2d} I_{2d} + L_d I_{fd} + L_d K_{afd} I_d \quad (2.21)
\]

\[
\psi_{1d} = L_d I_{1d} + L_d K_{12d} I_{2d} + L_d K_{1fd} I_{fd} + L_d K_{a1d} I_d \quad (2.22)
\]

\[
\psi_{2d} = L_d K_{21d} I_{1d} + L_d I_{2d} + L_d K_{2fd} I_{fd} + L_d K_{a2d} I_d \quad (2.23)
\]

and the d-axis flux linkages with the armature is:

\[
\psi_d = L_d K_{a1d} I_{1d} + L_d K_{a2d} I_{2d} + L_d K_{afd} I_{fd} + L_d I_d \quad (2.24)
\]
Solving equations (2.18)-(2.20) simultaneously, and noting that

\[ K_{1f} = K_{f1} \quad K_{2f} = K_{f2} \quad \text{and} \quad K_{12} = K_{21} \]

results in:

\[ \left[ p^2 L_d^2 (1-K_{21}^2) + pL_d (R_{1d} + R_{2d}) + R_{1d}R_{2d} \right] I_{fd} + \]

\[ \left\{ pK^3_d \left( \frac{K_{ef} a_{2d} K_{a1d} + K_{f2d} a_{2d} K_{ef} a_{1d} + K_{a2d} K_{21} K_{a2d}}{K_{ef} a_{2d} K_{a1d} + K_{f2d} a_{2d} K_{ef} a_{1d} + K_{a2d} K_{21} K_{a2d}} \right) - K_{f2d} K_{21d} K_{a1d} \right\} I_{fd} - p^2 L_d^2 (K_{f2d} K_{a2d} R_{1d} + K_{f1d} K_{a1d} R_{2d}) \]

\[ I_{fd} = \frac{K_{ef} (R_{1d} + R_{2d}) - pL_d (K_{ef} R_{1d} R_{2d})}{A(p)} \quad (2.25) \]

\[ \left[ p^2 L_d^2 (K_{f2d} K_{21d} - K_{f1d}) - pL_d (K_{f1d} R_{2d}) \right] I_{fd} + \]

\[ \left\{ p^3 L_d^3 \left( \frac{K_{f2d} a_{1d} K_{ef} a_{2d} K_{21d} K_{a2d} + K_{f2d} a_{2d} K_{21d} K_{a2d}}{K_{f2d} a_{1d} K_{ef} a_{2d} K_{21d} K_{a2d} + K_{f2d} a_{2d} K_{21d} K_{a2d}} \right) + p^2 L_d^2 (K_{f2d} K_{a2d} R_{2d} + K_{a2d} K_{21d} R_{fd}) \]

\[ I_{1d} = \frac{a_{1d} R_{2d} K_{a1d} R_{fd}}{A(p)} \quad (2.26) \]
\[
[p^2L_d^2(K_{f1d}K_{21d} - K_{f2d}^2) - pL_d(K_{f2d}R_{1d})] E_{fd} + \\
[p^3L_d^3(K^2_{f1d}K_{a2d} - K_{f1d}K_{a1d} - K_{f1d}K_{21d}K_{21d} + K_{21d}K_{a1d} \\
+ K_{f2d}K_{a2d} - K_{a2d}) + p^2L_d^2(K_{f2d}K_{a2d}R_{1d}^2 + K_{a1d}K_{21d}R_{fd}^2 \\
- K_{a2d}R_{1d}K_{a2d}R_{fd}) - pL_d(K_{a2d}R_{1d}^2R_{fd})] I_d \\
(A(p))' \] (2.27)

where:

\[
A(p)' = p^3L_d^3 \left( 1 - K_{f1d}^2 - K_{a2d}^2 - K_{21d}^2 + 2K_{f1d}K_{a2d}K_{21d} \right) \\
+ p^2L_d^2 \left( R_{1d}R_{2d}^2 + R_{a2d}^2 - K_{f2d}^2 - R_{21d}^2R_{a2d}^2 - K_{21d}^2R_{fd}^2 \right) \\
+ pL_d \left( R_{1d}R_{fd} + R_{2d}R_{fd} + R_{1d}R_{2d} + R_{2d}R_{fd} \right) + R_{1d}R_{2d}R_{fd} \] (2.26)

After substitution of equations (2.27) in equation (2.24), we obtain:

\[
\psi_d = L_d \left[ p^2L_d^2(K_{21d}K_{a2d} - K_{f1d}K_{21d} - K_{f2d}K_{a1d}K_{21d} + K_{f1d}K_{a1d} \right. \\
+ K_{f2d}K_{a2d} - K_{a2d}) + \\

pL_d \left( K_{f1d}K_{a1d}R_{2d} + K_{f2d}K_{a2d}R_{1d} - K_{a2d}R_{1d} - K_{a2d}R_{2d} \right) \\
\left. - (K_{a2d}R_{1d}R_{2d}) \right] E_{fd}/A(p)' \]
The general relation between $\psi_d$, $E_{fd}$ and $I_d$ in terms of the operators $G(p)$ and $l_d(p)$ is:

$$\psi_d = G(p) E_{fd} + L_d(p) I_d$$  \hspace{1cm} (2.30)

By comparing equations (2.29) and (2.30) it is evident that:

$$G(p) = L_d[p^3 L_d^2(K_{21d} R_{1d} R_f + K_{1d} R_{1d} R_f + K_{1d} R_{1d} R_f)] / A(p)'$$  \hspace{1cm} (2.31)
The direction of $I_d$ in equations (2.21)-(2.24) again follows Kirschbaum. If this direction is reversed, the sign of the last terms on the right hand side of equations (2.21)-(2.24) become negative, and the sign of the terms multiplied by $I_d$ become negative in equations (2.25)-(2.27). Consequently, the positive signs of $I_d$ in equations (2.29) and (2.30) become negative, so that equations (2.31) and (2.32) remain unchanged.

### 2.2.4 d-axis reactances

The $d$-axis additional subtransient inductance $L_d''$ at time $t = 0$, is found from equation (2.32), by neglecting the rotor resistances and letting $p \to \infty$. Hence the $d$-axis additional sub-
transient inductance of a salient pole synchronous machine with

two damper circuits is:

\[
L''_d = L_d(a) = L_d - \frac{2K_{a1d}K_{a2d}K_{afd} - K_{a1d}K_{a2d}K_{afd} + 2K_{a1d}K_{a2d}K_{afd}}{1 - K_{f1d}K_{f2d}K_{21d}}
\]

(2.33)

where: \( K_{a1d} = \frac{L_{a1d}}{L_d L_{a1d}} \), \( K_{21d} = \frac{L_{21d}}{L_{11d} L_{22d}} \), etc.

or when equation (2.33) is written in terms of the corresponding inductances:

\[
L''_d = L_d - \frac{2L_{f1d}L_{f2d}L_{a1d}L_{a2d}L_{afd}L_{21d} + 2L_{f1d}L_{f2d}L_{a1d}L_{afd}L_{21d}}{L_{11d}L_{22d}L_{afd} + L_{f2d}L_{a2d}L_{afd}L_{11d} - 2L_{f2d}L_{a1d}L_{a2d}L_{21d}}
\]

\[
+ \frac{L_{11d}L_{22d}L_{afd}L_{22d}L_{afd}L_{22d}L_{21d} + L_{11d}L_{22d}L_{afd}L_{22d}L_{21d}}{L_{11d}L_{22d}L_{afd} + L_{f2d}L_{a2d}L_{afd}L_{11d} - 2L_{f2d}L_{a1d}L_{a2d}L_{21d}}
\]

(2.34)
By putting $p \to \infty$ in equation (2.32), the d-axis subtransient inductance $L''_d$ is obtained as:

$$L''_d = L_d \left(1 - \frac{K_{afd}^2 + K_{a1d}^2 - 2K_{f1d}K_{a1d}K_{afd}}{1 - K_{f1d}^2}\right) \quad (2.35)$$

or in terms of the corresponding inductances

$$L''_d = L_d - \frac{L_{afd}^2 - 2L_{f1d}L_{a1d}L_{afd} + L_{fdd}L_{a1d}^2}{L_{f1d}L_{fdd} - L_{a1d}^2} \quad (2.36)$$

The d-axis transient inductance $L'_d$ may be found by letting $p \to \infty$ with $L_{11d} = L_{22d} = \infty$ in equation (2.32), i.e. by considering no circuits on the d-axis other than the main field. Equation (2.32) thus leads to:

$$L'_d = L_d \left(1 - K_{afd}^2\right)$$

$$= L_d - \frac{L_{afd}^2}{L_{fdd}} \quad (2.37)$$

The corresponding reactances are obtained when each inductance of equations (2.33)-(2.37) is multiplied by a factor of $\frac{3}{2}$. Thus equations (2.35) or (2.36) and (2.37) lead to the familiar two equations (18) and (20) of reference [4].
The d-axis additional subtransient inductance may also be written in terms of the coefficient of dispersion. If all four circuits (the armature, field and two damper circuits) are considered, the total coefficient of dispersion is defined by:

\[
\sigma_{af12d} = 1 - K_{f1d}^2 - K_{f2d}^2 - K_{21d}^2 - a_{1d}^2 - a_{2d}^2 - a_{afd}^2 - a_{1d}^2 - a_{2d}^2
\]

\[
+K_{a2d}^2 - K_{f1d}^2 - K_{dfd}^2 + 2K_{f1d}^2 - K_{f2d}^2 + 2K_{k1d}^2 - K_{k2d}^2 + 2K_{k1d}^2 - K_{k2d}^2
\]

\[
+2K_{a2d}^2 - K_{f2d}^2 + 2K_{a1d}^2 - 2K_{f2d}^2 - K_{a2d}^2 + 2K_{a1d}^2 - K_{a2d}^2 - 2K_{f1d}^2 + 2K_{f2d}^2 - 2K_{21d}^2
\]

(2.38)

The coefficient of dispersion for the case when three coils are considered (the field and the two damper) is:

\[
\sigma_{f12d} = 1 - K_{f1d}^2 - K_{f2d}^2 - K_{21d}^2 + 2K_{f1d}^2 - K_{f2d}^2 - K_{21d}^2
\]

(2.39)

hence \( L_d^m = \frac{\sigma_{af12d}}{\sigma_{f12d}} \cdot L_d \) (2.40)

It will be seen later that these operational inductances are helpful in developing simple mathematical forms of the machine reactances, that is in terms of the transient current decaying components and their respective time constants.
These characteristic operators will also be helpful in investigations of various transient processes, i.e. the determination of new relations for the resulting armature and field currents following a sudden 3-phase short circuit.
CHAPTER 3

ANALYSIS OF A MODIFIED SALIENT-POLE SYNCHRONOUS GENERATOR

When analysing the behaviour of the modified salient-pole generator, it is assumed that it can be represented by the model of Figure 2.1, and that the various reactances involved can be obtained from the various expressions developed from this model.

3.1 Analysis of the q-axis Circuits

When deriving the voltage equations, it is convenient to use the time rate-of-change of flux linkages, and this leads to expressions for the self and mutual inductances of the machine windings. The voltage in any circuit is given by the algebraic sum of the resistive drop and the rate-of-change of the total flux linkages with the circuit.

The q-axis representation of phase a, together with the first and the second q-axis damper windings is shown in Figure 2.1. The second damper winding accounts for the damping effect of the induced eddy currents in the body of the rotor.

Using terminology as defined in the nomenclature, the per unit equations, as used by Kirschbaum 21 (see Appendix 7.1) are:

\[
\begin{align*}
E_1 & = pL_{q,a1} + r_q + pL_{q,a2} + pL_{q,a1q} I_q + pL_{q,a2q} I_q \\
0 & = pK_{a1q} + p + \frac{1}{T_{1q}} + pK_{21q} I_{1q} \\
0 & = pK_{a2q} + pK_{21q} + p + \frac{1}{T_{2q}} I_{2q}
\end{align*}
\]

(3.1)
If $I_q$, $I_{1q}$ and $I_{2q}$ are zero at $t = 0$, then by taking the Laplace transform of (3.1) and solving by Cramer's rule we obtain:

$$\mathcal{L} \, I_q = \frac{E[s^2(1-K_{21q}^2)] + s \left( \frac{1}{T_{1q}} + \frac{1}{T_{2q}} \right) + \frac{1}{T_{1q}T_{2q}}}{s \left[ L_q[(1-K_{21q}^2) - K_{a1q}^2 - K_{a2q}^2 + 2K_{a1q}K_{a2q}21q]s^3 + \left[ \frac{1}{T_{1q}} + \frac{1}{T_{2q}} \right] \right] + \frac{1}{T_{1q}T_{2q}}}$$

$$+ \frac{r_q}{T_{1q}T_{2q}}$$

(3.2)

Since the measured $q$-axis transient armature current contains three distinct decaying components as is shown in Chapter 4, the armature transient current ($I_q$) can be written as:

$$I_q = I_{A0} - \frac{-t}{T_{A1}} I_{A1} e^{-\frac{-t}{T_{A1}}} - \frac{-t}{T_{A2}} I_{A2} e^{-\frac{-t}{T_{A2}}} - \frac{-t}{T_{A3}} I_{A3} e^{-\frac{-t}{T_{A3}}}$$

(3.3)

where $I_{A0}$, $I_{A1}$, $I_{A2}$, $I_{A3}$, $T_{A1}$, $T_{A2}$ and $T_{A3}$ are defined in the nomenclature.

Taking the Laplace transform of equation (3.3) gives:

$$\mathcal{L} \, I_q = \frac{\left( \frac{I_{A1}}{T_{A1}} + \frac{I_{A2}}{T_{A2}} + \frac{I_{A3}}{T_{A3}} \right) s^2 + \left[ \frac{1}{T_{A1}T_{A2}} \right] - \frac{I_{A1}}{T_{A2}T_{A3}}}{s \left[ s^3 + \left( \frac{1}{T_{A1}} + \frac{1}{T_{A2}} + \frac{1}{T_{A3}} \right) s^2 + \left( \frac{1}{T_{A1}T_{A2}} + \frac{1}{T_{A2}T_{A3}} + \frac{1}{T_{A3}T_{A1}} \right) \right] + \frac{1}{T_{A1}T_{A2}T_{A3}}$$

(3.4)
Dividing both the numerator and the denominator of equation (3.2) by \( K = L_q \left[ (1-K^2) - K^2a_{1q} - K^2a_{2q} + 2K_{q1}a_{2q}K_{q21} \right] \), and comparing the coefficients of the powers of \( S \) in equations (3.2) and (3.4) we obtain:

From the numerator:

\[
\frac{E}{K} \left( 1-k^2 \right) \bigg|_{21q} = \frac{I_{A1}}{TA1} + \frac{I_{A2}}{TA2} + \frac{I_{A3}}{TA3}
\]

\[ (3.5) \]

\[
\frac{E}{K} \left( \frac{1}{Tq1q} + \frac{1}{Tq2q} \right) = \frac{I_{A0}}{TA1} + \frac{I_{A0}}{TA2} + \frac{I_{A0}}{TA3}
\]

\[ (3.6) \]

and from the denominator:

\[
\frac{L_q}{K} \left( \frac{1}{Tq1q} + \frac{1}{Tq2q} \right) - \frac{L_q}{K} \left( K^2a_{1q} + K^2a_{2q} + r_q(1-k^2) \right) = \frac{1}{TA1} + \frac{1}{TA2} + \frac{1}{TA3}
\]

\[ (3.8) \]

\[
\frac{L_q}{K} \frac{1}{Tq1q} + \frac{r_q}{K} \left( \frac{1}{Tq1q} + \frac{1}{Tq2q} \right) = \frac{1}{TA1} + \frac{1}{TA2} + \frac{1}{TA3}
\]

\[ (3.9) \]

\[
\frac{r_q}{K} \frac{1}{Tq1q} = \frac{1}{TA1} + \frac{1}{TA2} + \frac{1}{TA3}
\]

\[ (3.10) \]
Dividing equation (3.7) by equation (3.10) yields:

\[ \frac{E}{\ell_q} = I_{A0} \]  \[ (3.11) \]

### 3.1.1 q-axis synchronous and additional subtransient reactances

Dividing equation (3.9) by equation (3.10) yields:

\[ T_{aq} = T_{A1} + T_{A2} + T_{A3} - T_{1q} - T_{2q} \]  \[ (3.12) \]

and by dividing equation (3.6) by equation (3.7), and subtracting this result from equation (3.12), we obtain the q-axis open circuit time constant of the armature winding as:

\[ T_{aq} = T_{A1} \frac{1 + AB + CD}{1 + A + C} \]  \[ (3.13) \]

where:

\[ A = I_{A2}/I_{A1}; \quad B = T_{A2}/T_{A1}; \quad C = I_{A3}/I_{A1} \quad \text{and} \quad D = T_{A3}/T_{A1} \]

Since the self-inductance of the armature winding is \( r_q T_{aq} \), therefore:

\[ L_q = \frac{r_q T_{A1} (1 + AB + CD)}{1 + A + C} \]  \[ (3.14) \]

From equations (2.17) and (3.14)

\[ X_q = \frac{3}{2} \frac{r_q T_{A1} (1 + AB + CD)}{1 + A + C} \]  \[ (3.15) \]
The q-axis additional subtransient inductance $L'_q$ derived in Chapter 2 and given in equation (2.11) is:

$$L'_q = L_q \left(1 - \frac{K^2_{e1q} + K^2_{e2q} - 2K_{e1q}K_{e2q}K_{21q}}{1 - K^2_{21q}}\right)$$

Solving equation (3.5) for \(1 - \frac{K^2_{e1q} + K^2_{e2q} - 2K_{e1q}K_{e2q}K_{21q}}{1 - K^2_{21q}}\), and substituting the result in the above equation yields:

$$L'_q = \frac{r_q \cdot T_{A1} (1 + A + C)}{1 + \frac{A}{B} + \frac{C}{D}}$$  \(3.16\)

From equations (2.17) and (3.14):

$$X''_{q'} = \frac{3}{2} \frac{r_q T_{A1} (1 + A + C)}{1 + \frac{A}{B} + \frac{C}{D}}$$  \(3.17\)

### 3.1.2 Derivation of the q-axis reactances from the q-axis open and short circuit time constants

The value of $L'_q(p)$ is given in equation (2.10). The negative reciprocals of the roots of the numerator gives the short-circuit time constants $T'_q$ and $T''_q$. Similarly, the negative reciprocals of the roots of the denominator provides the open circuit time constants $T'_q$ and $T''_q$. Hence, the q-axis operational inductance of equation (2.10) can be written as:
\[
L_q(p) = \frac{(1 + p T'') (1 + p T''')}{(1 + p T'' q_0) (1 + p T'''' q_0)} L_q
\]

Multiplying \( L_q \) in equation (3.18a) by \( \frac{3}{2} \), and rewriting as Laplace transform yields:

\[
X_q(s) = \frac{(1 + s T'') (1 + s T''')}{(1 + s T'' q_0) (1 + s T'''' q_0)} X_q
\]

If \( s \) tends to infinity, the limiting value of \( X_q(s) \) defined as \( X''_q \) is found from equation (3.18b) (similar procedure is applied in reference 17) as:

\[
X''_q \approx \frac{T''}{T'' q_0} X_q
\]

which is the \( q \)-axis reactance effective during the first instants of a transient condition.

In the absence of any iron damping, the high frequency value of \( X_q(s) \), defined by the high frequency limit of \( X''_q \), follows from (3.18b) as:

\[
X''_q \approx \frac{T''}{T'' q_0} \cdot X_q
\]

During the steady-state, when \( s = 0 \), the \( q \)-axis operational impedance \( X_q(s) \) is approximately equal to the \( q \)-axis synchronous reactance \( X_q \). From equations (3.19) and (3.20):
3.1.3 q-axis open and short circuit time constants

The open and short circuit time constants can be found from the magnitudes and time constants of the decaying components. Thus the transient current of equation (3.4) is:

\[
I_q(s) = \frac{I_{AQ}}{s} - \frac{I_{AQ} T_A [\frac{(1+s T_A^2)(1+s T_A^3)+AB(1+s T_A^4)(1+s T_A^3)+CD(1+s T_A^4)(1+s T_A^2)}{(1+A+C)(1+s T_A^2)(1+s T_A^3)}]}{1 + A + C - 2 A_1}
\]  

(3.22)

or

\[
I_q(s) = \frac{I_{AQ}}{s} \frac{[\frac{(1+s T_A^2)(1+s T_A^3)+A(1+s T_A^4)(1+s T_A^3)+C(1+s T_A^4)(1+s T_A^2)}{(1+A+C)(1+s T_A^2)(1+s T_A^3)}]}{1 + A + C - 2 A_1}
\]  

(3.23)

where constants A, B, C and D are given in section 3.1.1. The first of equations (3.1) can also be written in terms of the operational impedance \( X_q(p) \) as:

\[
E_1 = I_q r + p \psi = I_q r + p I X_q(p)
\]  

(3.24)

and if the Laplace transform of this equation is obtained, the transient current is:

\[
I_q(s) = E/s \left[ r_q + s X_q(s) \right]
\]  

(3.25)

By substituting for E in equation (3.11) we obtain:
If \( X_q(s) \) is replaced by \( \frac{q}{D_q(s)} \), the two \( q \)-axis open circuit time constants \( T_q^o \) and \( T_q^m \) may be obtained by equating equations (3.23) and (3.26) to zero. Thus

\[
(1+sT_A2)(1+sT_A3) + A(1+sT_A1)(1+sT_A3) + C(1+sT_A1)(1+sT_A2) = 0
\]

(3.28)

Where the negative reciprocals of the roots \( \alpha_{1q} \) and \( \alpha_{2q} \) of equation (3.28) are the \( q \)-axis open circuit time constants, i.e.

\[
T_q^o = \frac{1}{\alpha_{1q}}
\]

(3.29)

and

\[
T_q^m = -\frac{1}{\alpha_{2q}}
\]

Similarly, the \( q \)-axis short circuit time constants \( T_q^s \) and \( T_q^m \) are obtained by equating equations (3.22) and (3.27) to zero. Thus

\[
(1+sT_A2)(1+sT_A3) + AB(1+sT_A1)(1+sT_A3) + CD(1+sT_A1)(1+sT_A2) = 0
\]

(3.30)
where the negative reciprocals of the roots \( \beta_{1q} \) and \( \beta_{2q} \) of equation (3.30) give the q-axis short circuit time constants i.e.

\[
T''_q = -\frac{1}{\beta_{1q}}
\]

\[
T''_q = -\frac{1}{\beta_{2q}}
\]

(3.31)

and

The roots of the denominator of \( L_q(p) \) in equation (2.10) are

\[
\alpha_{1q}, \alpha_{2q} = \frac{-(T_{1q} + T_{2q}) \pm \sqrt{(T_{1q} + T_{2q})^2 - 4 T_{1q} T_{2q} (1-K_{2q})}}{2 T_{1q} T_{2q} (1-K_{2q})}
\]

(3.32)

Using equation (3.29), the algebraic sum and product of the roots of equation (3.32) are respectively:

\[
\frac{T'''_q + T'''_q}{T''_q T''_q} = \frac{T_{1q} + T_{2q}}{T_{1q} T_{2q} (1-K_{2q})}
\]

(3.33)

\[
\text{and} \quad \frac{T'''_q T'''_q}{T''_q} = T_{1q} T_{2q} (1-K_{2q})
\]

(3.34)

from equations (3.33) and (3.34)

\[
\frac{T'''_q + T'''_q}{T''_q} = T_{1q} + T_{2q}
\]

(3.35)

Similarly, from the roots of the numerator of equation (2.10), and by using equation (3.31), the algebraic sum and product of the roots, are respectively:
\[
\frac{T''}{T' \cdot T'''} = \frac{T_{1q} (1 - K_{a1q}^2) + T_{2q} (1 - K_{a2q}^2)} {T_{1q} T_{2q} (1 - K_{a1q}^2 - K_{a2q}^2 - K_{21q}^2 + 2 K_{a1q} K_{a2q} K_{21q})}
\]

(3.36)

and

\[
T'' \cdot T''' = T_{1q} T_{2q} (1 - K_{a1q}^2 - K_{a2q}^2 - K_{21q}^2 + 2 K_{a1q} K_{a2q} K_{21q})
\]

(3.37)

hence

\[
T'' + T''' = T_{1q} (1 - K_{a1q}^2) + T_{2q} (1 - K_{a2q}^2)
\]

(3.38)

3.1.4 q-axis magnetic coupling coefficients

The coupling coefficients \(K_{21q}\) and \(K_{a2q}\) may be found from equations (3.5), (3.8) and (3.10). On dividing equation (3.5) by equation (3.10) we obtain:

\[
K_{21q}^2 = \left[1 - \frac{(I_{A1} T_{A2} T_{A3} + I_{A2} T_{A1} T_{A3} + I_{A3} T_{A1} T_{A2})}{T_{1q} T_{2q} T_{A0}}\right]
\]

(3.39)

and on dividing equation (3.6) by equation (3.10):

\[
K_{a2q}^2 = 1 + \frac{1}{T_{aq} T_{2q}} \left[ T_{1q} T_{2q} (1 - K_{a1q}^2) + T_{aq} T_{1q} (1 - K_{a1q}^2) - (T_{A1} T_{A2} + T_{A2} T_{A3} + T_{A3} T_{A1}) \right]
\]

(3.40)

3.2 Analysis of the d-axis Circuits

The d-axis representation of phase a, the field, and the first and second d-axis damper windings is shown in Figure 2.3.
The d-axis quantities with the field circuit open may be obtained simply by replacing subscript q by subscript d in equations (3.1 - 3.40).

Under stationary conditions, and with the field circuit closed, the equations for the circuits of Figure 2.3 are (see Appendix 7.1.2):

\[
\begin{align*}
E_1 & = \begin{bmatrix} pL_d r_d & pL_d K_{a1d} & pL_d K_{a2d} & pL_d K_{af1d} \\ 0 & pK_{a1d} & p + \frac{1}{L_{1d}} & pK_{21d} & pK_{f1d} \\ 0 & pK_{a2d} & pK_{21d} & p + \frac{1}{L_{2d}} & pK_{f2d} \\ 0 & pK_{af1d} & pK_{f1d} & pK_{f2d} & p + \frac{1}{L_{fd}} \end{bmatrix} \begin{bmatrix} I_d \\ I_{1d} \\ I_{2d} \\ I_{fd} \end{bmatrix} \\
\end{align*}
\]

By taking Laplace transforms of (3.41), and solving by Cramer's rule for \( I_d \) we obtain:

\[
L_d = \frac{\frac{K_{f1d}^2}{T_{1d}} - \frac{K_{f2d}^2}{T_{fd}}}{s \left[ L_d s^2 \left( 1 - \frac{K_{f1d}^2}{T_{1d}} - \frac{K_{f2d}^2}{T_{2d}} - \frac{K_{a1d}^2}{T_{1d}} - \frac{K_{af1d}^2}{T_{fd}} + 2K_{f1d} K_{f2d} K_{21d} \right) \right] + \frac{1}{T_{1d}} + \frac{1}{T_{2d}} + \frac{1}{T_{fd}} - \frac{K_{f1d}^2}{T_{2d}}}
\]

\[
(3.42)
\]
\[ 2K_{a1d}K_{a2d}K_{21d} + 2K_{a1d}K_{f1d}K_{afd} + 2K_{a2d}K_{f2d}K_{afd} + K_{a1d}^2K_{f2d}^2 \]
\[ + K_{afd}^2K_{21d}^2 - 2K_{a1d}K_{21d}K_{f2d}K_{afd} - 2K_{f1d}K_{2d}K_{a1d}K_{a2d} \]
\[ - 2K_{f1d}K_{afd}K_{21d}K_{a2d} \] + \( s^3 \left[ L_d \left( \frac{1}{T_{1d}} + \frac{1}{T_{2d}} + \frac{1}{T_{f1d}} - \frac{K_{f1d}^2}{T_{1d}} - \frac{K_{f2d}^2}{T_{1d}} \right) - \frac{K_{a2d}^2}{T_{1d}T_{f1d}} \right] \]
\[ + \frac{2K_{f1d}K_{f2d}K_{a1d}K_{21d}^2}{T_{2d}} + \frac{2K_{f2d}K_{afd}K_{a2d}}{T_{1d}} \] + \( r_d \left[ 1 - K_{f1d}^2 - K_{f2d}^2 - K_{21d}^2 \right] \)
\[ + 2K_{f1d}K_{f2d}K_{21d} \] + \( s^2 \left[ L_d \left( \frac{1}{T_{1d}T_{2d}} + \frac{1}{T_{f1d}T_{2d}} + \frac{1}{T_{1d}T_{f1d}} - \frac{K_{a2d}^2}{T_{1d}T_{2d}} - \frac{K_{afd}^2}{T_{1d}T_{2d}} - \frac{K_{f1d}^2}{T_{1d}} - \frac{K_{f2d}^2}{T_{1d}} - \frac{K_{a1d}^2}{T_{1d}} \right) \right] \]
\[ + \frac{K_{a2d}^2}{T_{1d}T_{2d}T_{f1d}} \]
\[ + \frac{K_{afd}^2}{T_{1d}T_{f1d}T_{2d}} \]
\[ + \frac{1}{T_{1d}T_{2d}T_{f1d}} \]
\[ + \frac{1}{T_{2d}T_{f1d}T_{2d}} \]
\[ + \frac{1}{T_{2d}T_{1d}T_{f1d}} \]
\[ + \frac{1}{T_{2d}T_{1d}T_{2d}} \]

Since the measured d-axis armature transient current with the field winding closed contains four distinct decaying components, as will be shown in Chapter 4, the armature transient current \( I_d \) can be written as:

\[ I_d = I_{A0} - \frac{t}{T_{A1s}} - \frac{t}{T_{A2s}} - \frac{t}{T_{A3s}} - \frac{t}{T_{A4s}} \]

(3.43)
By taking $\mathcal{L}$-place transforms of equation (3.43) we obtain:

\[
\begin{align*}
\{s^3 (\frac{I_{A1s}}{A1s} + \frac{I_{A2s}}{A2s} + \frac{I_{A3s}}{A3s} + \frac{I_{A4s}}{A4s}) + s^2 (\frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s}) \\
+ \frac{I_{A2s}}{A1s} + \frac{1}{A2s} + \frac{1}{A4s} + \frac{I_{A3s}}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \\
+ \frac{I_{A4s}}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} + \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \\
+ \frac{1}{A1s} + \frac{I_{A2s}}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} + \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \\
+ \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} + \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \\
\} \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A4s} + \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \\
\end{align*}
\]

\[
\mathcal{L}I_d = \frac{s \{s^3 + s^2 (\frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s}) + s^2 (\frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s}) \\
+ \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} + \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \\
+ \frac{1}{A3s} + \frac{1}{A4s} + \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \} \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A4s} + \frac{1}{A1s} + \frac{1}{A2s} + \frac{1}{A3s} + \frac{1}{A4s} \\
\} 
\]
Let $K_s = L_d\left(1-K^2_{f1d} - K^2_{f2d} - K^2_{a1d} - K^2_{a2d} - K^2_{afd} - 2K_{f1d}K_{f2d}K_{21d} + 2K_{a1d}K_{a2d}K_{21d}\right) + 2K_{a1d}K_{a2d}K_{21d} + 2K_{afd}K_{f1d}K_{a2d} + 2K_{afd}K_{f2d}K_{a1d} + K^2_{a1d}K^2_{f2d} + K^2_{a2d}K^2_{f1d} + K^2_{afd}K^2_{21d} - 2K_{a1d}K_{21d}K_{f2d}K_{afd} - 2K_{f1d}K_{f2d}K_{a1d}K_{a2d} - 2K_{afd}K_{21d}K_{a1d}K_{a2d})$

Dividing the numerator and the denominator of equation (3.42) by $K_s$, and comparing the coefficients of powers of $s$ with those in equation (3.44), we obtain:

from the numerator

\[ \frac{E}{K_s} \left(1 - K^2_{f1d} - K^2_{f2d} - K^2_{a1d} - K^2_{a2d} - K^2_{afd} - 2K_{f1d}K_{f2d}K_{21d}\right) = \frac{I_{A1s}}{T_{A1s}} + \frac{I_{A2s}}{T_{A2s}} + \frac{I_{A3s}}{T_{A3s}} + \frac{I_{A4s}}{T_{A4s}} \]

(3.45)
\[
\frac{I_{A3s}}{T_{A3s}} \left( \frac{1}{T_{A1s}} + \frac{1}{T_{A2s}} + \frac{1}{T_{A4s}} \right) + \\
\frac{I_{A4s}}{T_{A4s}} \left( \frac{1}{T_{A1s}} + \frac{1}{T_{A2s}} + \frac{1}{T_{A3s}} \right) 
\]

\[(3.46)\]

\[
\frac{E}{K_s} \left( \frac{1}{T_{1d}T_{2d}T_{fd}} + \frac{1}{T_{1d}T_{fd}} + \frac{1}{T_{2d}T_{fd}} \right) = \frac{I_{A1s}}{T_{A1s}} \left( \frac{1}{T_{A2s}T_{A3s}} + \frac{1}{T_{A3s}T_{A4s}} + \frac{1}{T_{A2s}T_{A4s}} \right) + \\
\frac{I_{A2s}}{T_{A2s}} \left( \frac{1}{T_{A1s}T_{A3s}} + \frac{1}{T_{A3s}T_{A4s}} + \frac{1}{T_{A4s}T_{A1s}} \right) + \\
\frac{I_{A3s}}{T_{A3s}} \left( \frac{1}{T_{A1s}T_{A2s}} + \frac{1}{T_{A2s}T_{A4s}} + \frac{1}{T_{A1s}T_{A4s}} \right) + \\
\frac{I_{A4s}}{T_{A4s}} \left( \frac{1}{T_{A2s}T_{A3s}} + \frac{1}{T_{A3s}T_{A1s}} + \frac{1}{T_{A1s}T_{A2s}} \right) 
\]

\[(3.47)\]

\[
\frac{E}{K_s} \left( \frac{1}{T_{1d}T_{2d}T_{fd}} \right) = \frac{I_{A0}}{T_{A1s}T_{A2s}T_{A3s}T_{A4s}} 
\]

\[(3.48)\]

and from the denominator
\[
\frac{1}{K_s} \left[ L_d \left( \frac{1}{T_{1d}^2} + \frac{1}{T_{2d}^2} + \frac{1}{T_{fd}^2} + \frac{K_{a1d}^2}{T_{1d}^2} - \frac{K_{a2d}^2}{T_{1d}^2} - \frac{K_{21d}^2}{T_{1d}^2} - \frac{K_{a1d}^2}{T_{fd}^2} - \frac{K_{a2d}^2}{T_{fd}^2} - \frac{K_{21d}^2}{T_{fd}^2} \right) \right]
\]

\[
- \frac{K_{a2d}^2}{T_{1d}^2} - \frac{K_{a1d}^2}{T_{1d}^2} + \frac{2K_{a1d}^2K_{a2d}^2}{T_{fd}^2} + \frac{2K_{f1d}^2K_{a1d}^2}{T_{fd}^2} + \frac{2K_{a2d}^2K_{a1d}^2}{T_{fd}^2} + r_d \left[ \frac{1}{T_{1d}^2} - \frac{K_{a1d}^2}{T_{1d}^2} - \frac{K_{a2d}^2}{T_{1d}^2} + \frac{2K_{f1d}^2K_{a1d}^2}{T_{fd}^2} \right]
\]

\[
= \frac{1}{T_{A1s}} + \frac{1}{T_{A2s}} + \frac{1}{T_{A3s}} + \frac{1}{T_{A4s}} \quad (3.49)
\]

\[
\frac{1}{K_s} \left[ L_d \left( \frac{1}{T_{1d}^2} + \frac{1}{T_{2d}^2} + \frac{1}{T_{fd}^2} \right) - \frac{K_{a1d}^2}{T_{1d}^2} - \frac{K_{a2d}^2}{T_{2d}^2} - \frac{K_{21d}^2}{T_{1d}^2} - \frac{K_{a2d}^2}{T_{1d}^2} - \frac{K_{a1d}^2}{T_{1d}^2} + \frac{2K_{a1d}^2K_{a2d}^2}{T_{fd}^2} + \frac{2K_{a1d}^2K_{a2d}^2}{T_{fd}^2} \right]
\]

\[
+ r_d \left[ \frac{1}{T_{1d}} - \frac{K_{a1d}^2}{T_{1d}} - \frac{K_{a2d}^2}{T_{1d}} + \frac{2K_{f1d}^2K_{a1d}^2}{T_{fd}} \right] = \frac{1}{T_{A1s}} + \frac{1}{T_{A2s}} + \frac{1}{T_{A3s}} + \frac{1}{T_{A4s}} \quad (3.50)
\]
Dividing equation (3.48) by equation (3.52), we obtain the steady state current $I_{AD}$:

Hence \[
\frac{E}{r_d} = I_{AD} \tag{3.53}
\]

which is the same as given by equation (3.11). Dividing equation (3.51) by equation (3.52) we obtain:

\[
T_{fd} = T_{A1s} + T_{A2s} + T_{A3s} + T_{A4s} - T_{ad} - T_{1d} - T_{2d} \tag{3.54}
\]

3.2.1 \textit{d-axis synchronous and additional subtransient reactances}

Dividing equation (3.47) by equation (3.52), and by using equation (3.54) we obtain:

\[
T_{ad} = \frac{T_{A1s} \left( 1 + A_s \cdot B_s + C_s \cdot D_s + E_F \right)}{1 + A_s + C_s + E_s} \tag{3.55}
\]
where:

\[ A_s = \frac{I_{A2s}}{I_{A1s}}; \quad B_s = \frac{T_{A2s}}{T_{A1s}}; \quad C_s = \frac{I_{A3s}}{I_{A1s}}; \]

\[ D_s = \frac{T_{A3s}}{T_{A1s}}; \quad E_s = \frac{I_{A4s}}{I_{A1s}} \quad \text{and} \quad F_s = \frac{T_{A4s}}{T_{A1s}}. \]

hence the d-axis synchronous reactance is:

\[ L_d = \frac{r_d T_{A1s} (1 + A B_s + C D_s + E F_s)}{1 + A_s^2 + C_s^2 + E_s^2}. \] (3.56)

From equations (2.17) and (3.56) (replacing subscript q by subscript d in the former):

\[ X_d \approx \frac{3}{2} \frac{r_d T_{A1s} (1 + A B_s + C D_s + E F_s)}{1 + A_s^2 + C_s^2 + E_s^2}. \] (3.57)

Solving equation (3.45) and substituting from equation (2.33) yields:

\[ L''_d = \frac{r_d T_{A1s} (1 + A_s + C_s + E_s)}{1 + \frac{A_s}{B_s} + \frac{C_s}{D_s} + \frac{E_s}{F_s}}. \] (3.58)

Since \( X_d \approx \frac{3}{2} L_d \)

hence \[ X''_d \approx \frac{3}{2} \frac{r_d T_{A1s} (1 + A_s + C_s + E_s)}{1 + \frac{A_s}{B_s} + \frac{C_s}{D_s} + \frac{E_s}{F_s}}. \] (3.59)
3.2.2 Derivation of the d-axis reactances from the d-axis open and short circuit time constants

By a procedure similar to that of Section (3.1.2), the d-axis operational inductance \( L_d(p) \) given by equation (2.32) in Chapter 2 is:

\[
L_d(p) = \frac{(1 + pT'_d)(1 + pT''_d)(1 + pT''''_d)}{(1 + pT'_d)(1 + pT''_d)(1 + pT''''_d)} L_d
\]  

(3.60)

multiplying \( L_d(p) \) by \( \frac{3}{2} \), and rewriting in Laplace transform notation:

\[
X_d(s) = \frac{(1 + sT'_d)(1 + sT''_d)(1 + sT''''_d)}{(1 + sT'_d)(1 + sT''_d)(1 + sT''''_d)} X_d
\]  

(3.61)

If \( s \to \infty \), the limiting value of \( X_d(s) \) defined as \( X'_d \) and found from equation (3.61) is:

\[
X'_d = \frac{T'_d}{T'_d T''_d T''''_d} X_d
\]  

(3.62)

which is the d-axis reactance effective during the first instants of a transient condition.

In the absence of any iron damping, the high-frequency limit of \( X_d(s) \) defined by the high-frequency value of \( X''_d \), follows from equation (3.61) as:

\[
X''_d = \frac{T'_d}{T'_d T''_d T''''_d} X_d
\]  

(3.63)
In the absence of the two damper circuits, i.e. by considering only the main field on the d-axis, the value of $X_d(\delta)$ defined by $X'_d$ and found from equation (3.61) is:

$$X'_d = \frac{T'_d}{T_d^0} X_d$$

\[(3.64)\]

### 3.2.3 d-axis open and short circuit time constants

The transient current of equation (3.44) can be rewritten:

$$I_d(\delta) = \frac{I_{d0}}{s} = I_{d0} A_1 s^2 \left[ (1+8T_{A1}\delta) (1+8T_{A2}\delta) (1+8T_{A3}\delta) s^3 + A B (1+8T_{A1}\delta) (1+8T_{A2}\delta) (1+8T_{A3}\delta) \right]$$

\[(3.64)\]

or

$$I_d(\delta) = \frac{I_{d0}}{s} \left[ (1+8T_{A1}\delta) (1+8T_{A2}\delta) (1+8T_{A3}\delta) s^3 \right]$$

\[(3.65)\]
where constants $A$, $B$, $C$, $D$, $E$, and $F$ are given in section (3.2.1).

The $d$-axis open circuit time constants are obtained by equating equations (3.66) and (3.28) (with subscript $q$ in the latter replaced by subscript $d$, and then $X_d(s)$ by $\frac{N_d(s)}{D_d(s)}$) to zero. Thus:

\[
(1 + sT_A2s)(1 + sT_A3s)(1 + sT_A4s) + A_s(1 + sT_A1s)(1 + sT_A3s)(1 + sT_A4s)
\]

\[
+ C_s(1 + sT_A1s)(1 + sT_A2s)(1 + sT_A4s) + E_s(1 + sT_A1s)(1 + sT_A2s)(1 + sT_A3s) = 0
\]

(3.67)

where the negative reciprocal of the roots ($\alpha_{1d}$, $\alpha_{2d}$ and $\alpha_{3d}$) of equation (3.67) are the $d$-axis open circuit time constants, i.e.

\[
T'_{do} = -\frac{1}{\alpha_{1d}}
\]

\[
T''_{do} = -\frac{1}{\alpha_{2d}}
\]

and

\[
T'''_{do} = -\frac{1}{\alpha_{3d}}
\]

(3.68)

Similarly, the $d$-axis short circuit time constants are obtained by equation equations (3.65) and (3.27) (with subscript $q$ in the latter replaced by subscript $d$, and then $X_d(s)$ by $\frac{N_d(s)}{D_d(s)}$) to zero. Thus:
\[
\left(1 + s T A_{2s}\right)\left(1 + s T A_{3s}\right)\left(1 + s T A_{4s}\right) + A_B S_5 \left(1 + s T A_{1s}\right)\left(1 + s T A_{3s}\right)\left(1 + s T A_{4s}\right) + \\
C_D S_5 \left(1 + s T A_{1s}\right)\left(1 + s T A_{2s}\right)\left(1 + s T A_{4s}\right) + E_F S_5 \left(1 + s T A_{1s}\right)\left(1 + s T A_{2s}\right)\left(1 + s T A_{3s}\right) = 0
\]

(3.69)

where the negative reciprocal of the roots \(\left(\beta_{1d}, \beta_{2d}, \text{and } \beta_{3d}\right)\) of equation (3.69) give the d-axis short circuit time constants, i.e.

\[
T_{d'} = -\frac{1}{\beta_{1d}}
\]

\[
T_{d''} = -\frac{1}{\beta_{2d}}
\]

(3.70)

\[
T_{d'''} = -\frac{1}{\beta_{3d}}
\]

Using equation (3.68), the algebraic sum and product of the roots of the denominator in equation (2.32) (with the numerator and denominator being factorised) are respectively:

\[
T_{d0} + T_{d0}'' + T_{d0}''' = T_{1d} + T_{2d} + T_{3d}
\]

(3.71)
Replacing subscript $q$ for subscript $d$ in equation (3.34) and substituting into equation (3.72c) yields:

$$T'_{d0} = T_{fd} \left[ 1 - \frac{K_f^2 d + K_{f2d}^2 - 2K_{f1d} f2d K_{21d}^2}{1 - K_{21d}^2} \right]$$  (3.73)

Similarly, from the roots of the numerator in equation (2.32), and by using equation (3.70), the algebraic sum of the roots is:

$$T'_{d0} + T''_{d0} + T'''_{d0} = T_{fd} (1 - K_{a1d}^2) + T_{1d} (1 - K_{a2d}^2) + T_{2d} (1 - K_{af2d}^2)$$  (3.74)

Again replacing subscript $q$ by subscript $d$ in equation (3.38) and subtracting from equation (3.74) yields:

$$T'_{d} = T_{fd} (1 - K_{a2d}^2)$$  (3.75)
3.2.4 d-axis magnetic coupling coefficients

Dividing equation (3.50) by equation (3.52) yields

\[ K_{afd}^2 = 1 + \frac{1}{T_{ad} T_{fd}} \left[ T_{1d} T_{fd} (1-K_{f1d}^2) + T_{2d} T_{fd} (1-K_{f2d}^2) + T_{1d} T_{2d} (1-K_{21d}^2) + T_{1d} T_{ad} (1-K_{a1d}^2) + T_{2d} T_{ad} (1-K_{a2d}^2) - (T_{A1s} T_{A2s} + T_{A1s} T_{A3s} + T_{A2s} T_{A4s} + T_{A3s} T_{A4s}) \right] \]

Dividing equation (3.46) by equation (3.52) yields

\[ T_{1d} T_{fd} (1-K_{f1d}^2) + T_{2d} T_{fd} (1-K_{f2d}^2) + T_{1d} T_{2d} (1-K_{21d}^2) = \frac{1}{T_{A0}} \left[ I_{A1s} (T_{A2s} T_{A3s} + T_{A2s} T_{A4s}) + I_{A2s} (T_{A1s} T_{A3s} + T_{A1s} T_{A4s}) + I_{A3s} (T_{A1s} T_{A2s} + T_{A1s} T_{A4s}) + I_{A4s} (T_{A1s} T_{A2s} + T_{A1s} T_{A3s}) \right] \]

\[ (3.76) \]
It has been shown that the parameters of the modified machine can be found in simple mathematical forms in terms of the decaying components and their respective time constants; these simple relations are used in the next chapter.
CHAPTER 4

METHODS OF MEASUREMENT OF PARAMETERS

The methods of measurement used for testing the 5 kVA experimental salient-pole machine can be divided into two groups. The first uses standstill measurements and the second measurements with the machine rotating. The standstill tests employed an indicial response method, and it will be shown that this produces an accurate prediction of the machine's parameters, based on the theory developed in Chapter 3. The other conventional methods of measurement were used to validate the values obtained from the indicial response test (e.g. open and short circuit tests, slip test, Dalton and Cameron method, maximum lagging current method and reluctance motor method).

4.1 Indicial Response Test

The indicial-response test, as illustrated by Figure (4.1) required the application of a step function of voltage to one of the armature phases, and the measurement of the corresponding transient current in that phase, the remaining two armature phases being open circuited.

For each test the variation of the armature current is obtained via an ultra-violet recorder, and an analysis of the curves obtained yields the sum of three and four exponential functions for the q and d axes respectively (with the field circuit closed for the latter).
From the magnitudes and time constants of the individual components of the transient current, the reactances included in the modified Park's equation may be determined.

A program was run on a 1900 IC computer, using the magnitudes and time constants of the individual components to confirm the measured values of the transient current and a comparison between the two sets of results for different currents are given in Appendix 7.3. The q- and a-axis positions of the rotor were obtained by the standard procedure of connecting a voltmeter to the terminals of the rotor, with an a.c. supply to the a-phase of the armature. The q-axis position corresponds to minimum (near) zero voltage induced in the field, and the d-axis position is obtained when the voltage becomes a maximum. The indicial-response test requires a constant voltage source. This is provided by a heavy-duty battery with very low internal resistance. (a 12V, 91 AH car battery was used).

The applied d.c. voltage, the transient current in the armature phase winding and the induced voltage in the field winding, were recorded directly on a multi channel ultra-violet recorder.

High speed recordings (up to 2 m/s) were made to expand the time scale of the readings and thereby to increase the accuracy of the results obtained.

The total resistance of the circuit shown in Figure 4.1, including that of the ammeter, the shunt and the connecting leads was minimised, as any external resistance decreases the time constants and tends to reduce the accuracy with which the parameters are measured.
4.2 Determination of the q-axis Synchronous, Subtransient and Additional Subtransient Reactances by the Indicial Response Method

Indicial-response tests were made on a 3-phase salient-pole synchronous generator, this machine has armature winding on the rotor, and the power supply is connected via slip rings and brushes. The field and damper windings are on the stator. The name plate data of the star connected machine is:

- Output kVA = 5
- Line voltage = 240V
- Current = 12A
- Frequency = 50 Hz
- Speed = 1500 r.p.m.
- Number of Poles = 4

As explained in Section 4.1, the rotor of the machine was aligned with the q-axis, and a step of d.c. voltage was applied to the a-phase armature winding. Oscillograms of the applied d.c. voltage, the transient current in the armature phase winding and the induced voltage in the field winding were recorded, as shown in Figure 4.2. The small induced field voltage shown is due to the difficulty in aligning an exact q-axis position, but since the field winding has a large number of turns, this small induced voltage can be reasonably assumed to be negligible. Figure 4.3 shows the induced voltage in the other two phase windings.

The exact value of the steady-state current in the armature a-phase winding was measured by the ammeter (A), a few seconds after closure of the switch (S).

To prevent the armature winding from over-heating, the current was only allowed to flow for the shortest period needed
to obtain the measurements required, this being especially necessary when the current was near the rated value.

The decaying components were obtained by plotting the difference of the transient current and the steady-state value on semilog paper. Consideration of the results of the test, show that there are three distinct decaying components $I_{A1}$, $I_{A2}$ and $I_{A3}$ in the transient current, as shown in Figure 4.4, implying clearly that two damper circuits are required on each axis, as described in Section 2.2. The time constants $T_{A1}$, $T_{A2}$ and $T_{A3}$ were found by multiplying the magnitude of each decaying component by $1/e$. The magnitudes and time constants of the decaying components obtained with a test current of 9.72A are:

\[ I_{A1} = 2.0A; \quad I_{A2} = 4.2A; \quad I_{A3} = 3.52A \]
\[ T_{A1} = 0.026s; \quad T_{A2} = 0.0045s; \quad T_{A3} = 0.0016s. \]

i.e. \[ A = 210 \times 10^{-2} \quad \text{and} \quad B = 17.3 \times 10^{-2} \]
\[ C = 176 \times 10^{-2} \quad \text{and} \quad D = 6.153 \times 10^{-2} \]

where: $A = I_{A2}/I_{A1}$; $B = T_{A2}/T_{A1}$; $C = I_{A3}/I_{A1}$ and $D = T_{A3}/T_{A1}$.

From equation (3.15), the q-axis synchronous reactance is:

\[
X_q \approx 1.5 \frac{r_q}{\left(1 + A + B\right)}
\]

\[
= 1.5(61.728 \times 10^{-2}) \times 0.028 (1 + 210 \times 10^{-2} + 17.3 \times 10^{-2} + 176.10^{-2} \times 6.153 \times 10^{-2}) \Omega
\]

\[
= 11.547(1 + 210 \times 10^{-2} + 176 \times 10^{-2})
\]

or \[ X_q \approx 19.825 \times 10^{-2} \text{ p.u.} \]
where:

\[ \omega = 2\pi f = 314 \text{ radians/second}; \quad \text{rated impedance/phase} = \frac{V_{\text{ph}}}{I} = 11.547 \Omega \]

\[ r_q = \text{q-axis armature circuit resistance} \]

\[ = \text{armature resistance/phase + brush resistance (which is a function of current) + connection lead resistance} \]

\[ + \text{ammeter internal resistance} \]

\[ + \text{shunt resistance + internal battery resistance} \]

\[ = 61.728 \times 10^{-2} \Omega \]

The change in resistance of the brushes during the transient is assumed negligible compared with the total resistance of the armature circuit. Substituting the data into equation (3.28) yields:

\[ s^2(6.01 \times 10^{-4}) + s(2.354 \times 10^{-1}) + 9.72 = 0 \]

The roots of this equation are:

\[ \alpha_{1q} = -46.888 \]

and \[ \alpha_{2q} = -344.951 \]

From equation (3.29)

\[ T_{q0}'' = -\frac{1}{\alpha_{1q}} = \frac{1}{46.888} = 2.132 \times 10^{-2} \text{ sec} \]

and \[ T_{q0}'' = -\frac{1}{\alpha_{2q}} = \frac{1}{344.951} = 2.699 \times 10^{-3} \text{ sec} \]
where: \( T_{q0} \) = q-axis subtransient open circuit time constant

and \( T'_{q0} \) = q-axis additional subtransient open circuit time constant.

Similarly by substituting the data into equation (3.30) yields:

\[
\begin{align*}
\sigma \left( 1.819 \times 10^{-6} \right) + s(1.010 \times 10^{-3}) + 7.653 \times 10^{-2} &= 0
\end{align*}
\]

The roots of this equation are:

\[
\beta_{1q} = -90.533
\]

and \( \beta_{2q} = -464.716 \)

From equation (3.31)

\[
T_q'' = -\frac{1}{\beta_{1q}} = \frac{1}{90.533} = 1.105 \times 10^{-2} \text{ sec}
\]

and \( T_q''' = -\frac{1}{\beta_{2q}} = \frac{1}{464.716} = 2.151 \times 10^{-3} \text{ sec} \)

where: \( T_q'' = q\)-axis subtransient short circuit time constant

and \( T_q''' = q\)-axis additional subtransient short circuit time constant.
The q-axis subtransient reactance obtained from equation (3.20) is:

\[ X''_q = \frac{X''_q}{X'_q} \times X_q \]

\[ = \frac{1.105 \times 10^{-2}}{2.132 \times 10^{-2}} \times 19.825 \times 10^{-2} \]

i.e. \[ X''_q = 10.275 \times 10^{-2} \text{ p.u.} \]

The q-axis additional subtransient reactance is obtained from equation (3.17):

\[ X''_q = 1.5 \frac{r_q T_{A1} (1 + A + C)}{1 + \frac{A}{B} + \frac{C}{D}} \]

\[ X''_q = 1.5(61.728 \times 10^{-2}) \times \frac{0.028 \times 314(1+210 \times 10^{-2}+178 \times 10^{-2})}{11.547(1+ \frac{210 \times 10^{-2}}{17.3 \times 10^{-2}} + \frac{176 \times 10^{-2}}{6.153 \times 10^{-2}})} \text{ p.u.} \]

\[ = 7.621 \times 10^{-2} \text{ p.u.} \]

The q-axis additional subtransient reactance can also be obtained from equation (3.19):

\[ X'''_q = \frac{T''_q}{T''_q \times X_q} \]

\[ = \frac{(1.105 \times 10^{-2})(2.131 \times 10^{-2})}{(2.132 \times 10^{-2})(2.899 \times 10^{-2})} \times 19.825 \times 10^{-2} \]

\[ = 7.623 \times 10^{-2} \text{ p.u.} \]
The result obtained from this equation is useful, as it enables the accuracy of the results obtained from equation (3.17) to be determined, and in consequence the accuracy of the subtransient reactance obtained from equation (3.20). The procedure was repeated for a low value of test current of 2.62A. The corresponding components of the current and their time constants are as shown in Figure 4.5. A comparison of the open and short circuit time constants with the corresponding reactances at the two different currents are given below in Table 4.1.

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>T''qo (Sec)</th>
<th>T'''qo (Sec)</th>
<th>T''q (Sec)</th>
<th>T'''q (Sec)</th>
<th>X_q (p.u)</th>
<th>X''_q (p.u)</th>
<th>X'''_q (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.72</td>
<td>2.132x 10^{-2}</td>
<td>2.899x 10^{-3}</td>
<td>1.05x 10^{-2}</td>
<td>2.151x 10^{-3}</td>
<td>19.825x 10^{-2}</td>
<td>10.275x 10^{-2}</td>
<td>7.621x 10^{-2}</td>
</tr>
<tr>
<td>2.62</td>
<td>1.828x 10^{-2}</td>
<td>2.863x 10^{-3}</td>
<td>1.077x 10^{-2}</td>
<td>1.543x 10^{-3}</td>
<td>24.2x 10^{-2}</td>
<td>14.273x 10^{-2}</td>
<td>7.696x 10^{-2}</td>
</tr>
</tbody>
</table>

**TABLE 4.1**
4.3 *d-axis Indicial Response Test; Field Circuit Open*

This test was performed with the a-phase of the armature aligned with the d-axis, using the method described in Section 4.1, and the test procedure of Section 4.2.

Figure 4.6 shows typical oscillograms of the applied voltage, the transient current in the a-phase armature winding and the induced voltage in the field winding. An analysis of the transient current shows three decaying components as indicated in Figure 4.7. The magnitudes and time constants of the decaying components obtained on resolution of the transient current of 11.09A are:

\[ I_{A1} = 3.31A; \quad I_{A2} = 2.4A; \quad I_{A3} = 5.38A \]
\[ T_{A1} = 0.0627s; \quad T_{A2} = 0.007s; \quad T_{A3} = 0.0018s. \]

i.e. \( A = 72.507 \times 10^{-2} \)
\( B = 11.164 \times 10^{-2} \)
\( C = 162.537 \times 10^{-2} \)
and \( D = 2.870 \times 10^{-2} \)

The d-axis synchronous and additional subtransient reactances, open and short circuit time constants, and subtransient reactance are simply obtained by replacing subscript q by subscript d in equations (3.15) and (3.17), (3.28) and (3.29), (3.30) and (3.31), respectively. On substituting the above data into equations (3.15) and (3.17) yields:

\[ X_d = 1.5 \times \frac{r_d T_{A1} (1+AB+CD)}{1+A+C} = 1.5 \times 0.541 \times 6.27 \times 10^{-2} \times 314(1+72.507 \times 11.164 \times 10^{-6} + 162.537 \times 2.870 \times 10^{-2}) \]

\[ = 46.566 \times 10^{-2} \text{ p.u.} \]
and \( X''_d = 1.5 f_d T_A \frac{(1+A+C)}{1-A+\frac{C}{B}} \) \( \approx \frac{1.5 \times 0.541 \times 6.27 \times 10^{-2} \times 314 (1+72.507 \times 10^{-2} + 162.537 \times 10^{-2})}{11.547 \left( \frac{1+72.507 \times 10^{-2}}{11.164 \times 10^{-2}} + \frac{162.537 \times 10^{-2}}{2.870 \times 10^{-2}} \right)} \)
\[= 7.228 \times 10^{-2} \text{ p.u.} \]

where: \( X_d = \) d-axis synchronous reactance

and \( X''_d = \) d-axis additional subtransient reactance

On substituting the data into equation (3.28), and by using equation (3.29) yields:

\[ T''_{d0} = 4.504 \times 10^{-2} \text{ sec} \]

and \( T''_{d0} = 5.352 \times 10^{-3} \text{ sec} \).

where: \( T''_{d0} = \) d-axis open-circuit subtransient time constant

and \( T''_{d0} = \) d-axis open-circuit additional subtransient time constant.

Similarly from equations (3.30) and (3.31)

\[ T''_d = 1.228 \times 10^{-2} \text{ sec} \]

and \( T''_d = 3.052 \times 10^{-3} \text{ sec} \).

where: \( T''_d = \) d-axis short-circuit subtransient time constant

and \( T''_d = \) d-axis short-circuit additional subtransient time constant.
The d-axis subtransient reactance \( X_d'' \) is obtained from equation (3.20)

\[
X_d'' = \frac{T_d''}{T_d''} \cdot X_d
\]

\[
= \frac{1.226 \times 10^{-2}}{4.504 \times 10^{-2}} \times 46.566 \times 10^{-2}
\]

\[
= 12.675 \times 10^{-2} \text{ p.u.}
\]

The procedure was repeated for a test current of 6.8 A, the corresponding components of this current and their time constants are shown in Figure 4.8, and a comparison between the two values of d-axis time constants and reactances are given in Table (4.2).

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>( T_d'' ) (sec)</th>
<th>( T_d''' ) (sec)</th>
<th>( T_d'''' ) (sec)</th>
<th>( X_d' ) (pu)</th>
<th>( X_d'' ) (pu)</th>
<th>( X_d''' ) (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.09</td>
<td>4.504x10^{-2}</td>
<td>5.352x10^{-3}</td>
<td>1.226x10^{-2}</td>
<td>3.052x10^{-3}</td>
<td>46.566x10^{-2}</td>
<td>12.675x10^{-2}</td>
</tr>
<tr>
<td>6.8</td>
<td>5.381x10^{-2}</td>
<td>4.586x10^{-3}</td>
<td>1.436x10^{-2}</td>
<td>2.719x10^{-3}</td>
<td>47.505x10^{-2}</td>
<td>12.677x10^{-2}</td>
</tr>
</tbody>
</table>

TABLE 4.2
4.4 **d-axis Indicial Response Test; Field Circuit Closed**

The test procedure of Section (4.3) was repeated with the field circuit closed, and Figure 4.9 shows typical oscillograms of the applied d.c. voltage and the transient current through the phase winding. It was not possible to obtain an accurate record of the current in the short-circuited field winding, since inserting the high resistance shunt needed had an appreciable effect on the time constants involved.

The analysis of the transient armature current given in Section (3.2) showed the four decaying components indicated in Figures (4.10) and (4.11), and the magnitudes and time constants of these for a final current of 11.09A are:

\[
\begin{align*}
I_{A1s} &= 0.86A; & I_{A2s} &= 1.2A; & I_{A3s} &= 4.0A; & I_{A4s} &= 5.03A \\
T_{A1s} &= 0.20s; & T_{A2s} &= 0.0215s; & T_{A3s} &= 0.0043s; & T_{A4s} &= 0.0018s.
\end{align*}
\]

\[
\text{a.s. } A_s = 139.534 \times 10^{-2} \\
B_s = 10.750 \times 10^{-2} \\
C_s = 465.116 \times 10^{-2} \\
D_s = 2.150 \times 10^{-2}
\]

The d-axis synchronous reactance \(X_d\) is obtained from equation (3.57):

\[
X_d = 1.5 \times 10^{-2} \frac{(1 + A_B s + C_D s + E_F s)}{1 + A_G s + E_G s}
\]

\[
as \frac{1.5 \times 0.541 \times 0.2 \times 314 (1 + 139.534 \times 10^{-2} + 10.750 \times 10^{-4} + 465.116 \times 2.150 \times 10^{-4})}{584.883 \times 0.9 \times 10^{-4}} \times \frac{11.547 (1 + 139.534 \times 10^{-2} + 465.116 \times 10^{-4} + 584.883 \times 10^{-2})}{584.883 \times 0.9 \times 10^{-4}}
\]

\[= 44.583 \times 10^{-2} \text{ p.u.}\]
The d-axis additional subtransient reactance is obtained from equation (3.59):

\[ X''_d = 1.5 \cdot r_d \cdot T_{\text{A1s}} \cdot \frac{(1 + A_s + C_s + E_s)}{(1 + B_s + D_s + F_s)} \]

\[ = \frac{1.5 \cdot 0.541 \cdot 0.2 \cdot 314}{1.547} \left(1 + 139.534 + 10.750 + 465.116 + 584.883\right) \]
\[ = 6.466 \times 10^{-2} \text{ p.u.} \]

Substituting the data into equation (3.67) yields:

\[ s^3(1.259 \times 10^{-4}) + s^2(4.681 \times 10^{-2}) + s(2.3) + 11.09 = 0 \]

Using a Newton and Raphson method, the approximate roots of this equation are found as:

\[ \alpha_{1d} = -5.4288 \]

\[ \alpha_{2d} = -49.751 \]

and \[ \alpha_{3d} = -326.051 \]

From equation (3.68)

\[ T''_{d0} = -\frac{1}{\alpha_{1d}} = \frac{1}{5.4288} = 1.842 \times 10^{-1} \text{ sec} \]

\[ T''_{d0} = -\frac{1}{\alpha_{2d}} = \frac{1}{49.751} = 2.01 \times 10^{-2} \text{ sec} \]
and $T''_{d0} = \frac{1}{\alpha_{3d}} = \frac{1}{326.051} = 3.067 \times 10^{-3}$ sec.

where: $T'_{d0} = d$-axis open circuit transient time constant

Substituting the data into equation (3.69) yields:

$$s^3 (3.691 \times 10^{-7}) + s^2 (1.839 \times 10^{-4}) + s (1.595 \times 10^{-2}) + 2.241 \times 10^{-3} = 0$$

The roots of this equation are:

$$\beta_{1d} = -17.513$$

$$\beta_{2d} = -86.206$$

and $\beta_{3d} = -402.068$

From equation (3.70)

$$T'_d = -\frac{1}{\beta_{1d}} = \frac{1}{17.513} = 0.571 \times 10^{-1}$$

$$T''_d = -\frac{1}{\beta_{2d}} = \frac{1}{86.206} = 1.160 \times 10^{-2}$$

and $T'''_d = -\frac{1}{\beta_{3d}} = \frac{1}{402.068} = 2.487 \times 10^{-3}$

where: $T'_d = d$-axis short circuit transient time constant.
The d-axis subtransient reactance \( X''_d \) is obtained from equation (3.63)

\[
X''_d = \frac{T'_d}{T''_d} \cdot X_d
\]

\[
\approx \frac{5.71 \times 1.16 \times 10^{-1}}{1.164 \times 2.01} \times 44.583 \times 10^{-2}
\]

\[
\approx 7.975 \times 10^{-2} \text{ p.u.}
\]

and the d-axis transient reactance \( X'_d \) is obtained from equation (3.64)

\[
X'_d = \frac{T'_d}{T_d} \cdot X_d
\]

\[
\approx \frac{5.71 \times 10^{-1}}{1.842} \times 44.583 \times 10^{-2}
\]

\[
\approx 13.82 \times 10^{-2} \text{ p.u.}
\]

The values of the open and short circuit time constants, and the corresponding reactances obtained from Figures (4.10) and (4.11) are shown in Table 4.3.

It is necessary to plot the curve accurately to obtain accurate values of the various transient components and their respective time constants from the ultra-violet readings.
There is a small discrepancy between values obtained for $T''_{d0}$, $T''_{do}$ and $X''_d$ from tests with the field winding open and short-circuited, but this unfortunately still remains unexplained.

### TABLE 4.3

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>$T'_{d0}$ (S)</th>
<th>$T''_{d0}$ (S)</th>
<th>$T'''_{d0}$ (S)</th>
<th>$T'_{d}$ (S)</th>
<th>$T''_{d}$ (S)</th>
<th>$T'''_{d}$ (S)</th>
<th>$X'_{d}$ (pu)</th>
<th>$X''_{d}$ (pu)</th>
<th>$X'''_{d}$ (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.09</td>
<td>1.842 x 10^{-1}</td>
<td>3.087 x 10^{-3}</td>
<td>0.571 x 10^{-1}</td>
<td>1.160 x 10^{-2}</td>
<td>2.487 x 10^{-2}</td>
<td>44.583 x 10^{-2}</td>
<td>13.82 x 10^{-2}</td>
<td>7.207 x 10^{-2}</td>
<td>6.464 x 10^{-2}</td>
</tr>
<tr>
<td>6.8</td>
<td>18.514 x 10^{-2}</td>
<td>1.67 x 10^{-2}</td>
<td>2.951 x 10^{-2}</td>
<td>5.54 x 10^{-2}</td>
<td>2.357 x 10^{-2}</td>
<td>46.17 x 10^{-2}</td>
<td>13.82 x 10^{-2}</td>
<td>7.207 x 10^{-2}</td>
<td>6.464 x 10^{-2}</td>
</tr>
</tbody>
</table>

4.5 Determination of the Field Time Constant from the d-axis Field Transient Current with Armature Windings Open

As with the previous test, the present test was performed with the a-phase winding aligned with the d-axis, and with the armature phase winding open. A low d.c. voltage was applied to the field winding and oscillograms of this and the corresponding field transient current were recorded as shown in Figure 4.12. The difference between the steady state and the transient currents was resolved by plotting on semilog graph paper.

The test indicated three distinct decaying components, as shown in Figure 4.13, implying that for each axis, two damper circuits need to be considered.

This test is similar to that described in Section 4.3, with the supply voltage applied to the field rather than the armature phase winding. Formulae similar to those of equations (3.12) and
(3.13) will apply (with suffix a replaced by suffix f and subscript q replaced by subscript d).

Thus, for the d-axis circuits comprising the f, 10 and 20 windings only:

\[ T_{fd} = T_{F1} + T_{F2} + T_{F3} - T_{1d} - T_{2d} \]  \hspace{1cm} (4.1)

and

\[ T_{fd} = T_{F1} \left(1 + A_F B_F + C_F D_F \right)/\left(1 + A_F + C_F \right) \] \hspace{1cm} (4.2)

where:

\[ A_F = \frac{T_{F2}}{T_{F1}}; \quad B_F = \frac{T_{F2}}{T_{F1}}; \quad C_F = \frac{T_{F3}}{T_{F1}} \]  \quad and \quad \[ D_F = \frac{T_{F3}}{T_{F1}} \]

Applying equation (4.2), the field time constant \( T_f \) obtained from Figure 4.15 is

\[ T_f = 0.19 \text{ seconds} \]

4.6 Conventional Methods of Measurement of Parameters

4.6.1 Determination of the d-axis synchronous reactance from the no-load saturation and sustained 3-phase short circuit characteristics

The open-circuit characteristic of the test machine was obtained from the variation of the no load voltage with the excitation current at rated speed. Due to a residual voltage of 2.8V (Figure 4.14), the characteristic does not pass through the origin, and a correction was therefore introduced by extending the straight portion of the no-load curve to the point of intersection with the abscissa (point C in Figure 4.14), the length of the abscissa cut by the projected curve (OC) representing the correction value. Following this technique, a value of 0.02A was added to all measured values of the excitation current, and the corrected no-load saturated curve is
shown in Figure 4.15.

The short-circuit characteristic was obtained by driving the test machine at rated speed, and recording the armature current as the excitation current was varied. The resulting straight-line curve is shown in Figure 4.15.

The d-axis synchronous reactance \(X_d\) is given\(^3,14\) from this test by:

\[
X_d = \frac{O_a}{O_b} = \frac{0.5}{0.967} = 0.517 \text{ p.u. (according to AIEE definition}^{14})
\]

or

\[
X_d = \frac{C_d}{d} = \frac{138.56}{23.2} = 5.972\Omega
\]

\[
= 0.517 \text{ p.u. (according to reference 3).}
\]
4.6.2 Determination of d- and q-axis additional subtransient reactances from Dalton and Cameron method

The test is performed with the rotor stationary in any arbitrary position, and the field winding short-circuited through an ammeter. A low single-phase voltage (0-24V) of rated frequency is applied to two armature terminals a and b, with the third terminal c left open (see Figure 4.16). Following measurement of the armature voltage and current, the test voltage is transferred to terminals b and c (with terminal a open), and thereafter to terminals c and a (with terminal b open). The armature voltage and current are measured and recorded for both of the re-connections.

The results of the three measurements gives three stator voltage/current ratios, that is three additional subtransient reactances between the stator terminals, corresponding to three different positions of the rotor. The voltage/current ratios may be designated\(^{11,14}\) as \(A_{ss}, B_{ss}\) and \(C_{ss}\) respectively. It is frequently considered that the angular variation of the reactance of a single-phase rotor winding comprises a constant term plus a sinusoidal function varying at twice the frequency of the rotor displacement. The constant term is given by\(^{11,14}\):

\[
K_{ss} = \frac{(A_{ss} + B_{ss} + C_{ss})}{3} \quad (4.3)
\]

while the amplitude of the sinusoidal component is:

\[
M_{ss} = \sqrt{\left(\frac{(B_{ss} - K_{ss})^2}{3}\right) + \frac{(C_{ss} - A_{ss})^2}{3}} \quad (4.4)
\]
The ohmic values of the per phase d- and q-axis additional sub-
transient reactances are:

\[ X''_d = \frac{(K - M)}{\text{ss ss}} \]  \hspace{1cm} (4.5)

and \[ X''_q = \frac{(K + M)}{\text{ss ss}} \]  \hspace{1cm} (4.6)

and the p.u. values are obtained by dividing these values by the
base impedance/phase.

At a given voltage of 6.63V and a current of 2.9A, the
additional subtransient reactances were calculated as:

\[ A_{ss} = \frac{6.63}{2.9} = 2.286\Omega \]
\[ B_{ss} = \frac{6.5}{3} = 2.167\Omega \]

and \[ C_{ss} = \frac{6.71}{2.94} = 2.283\Omega \]

so that from equation (4.3) \[ K_{ss} = 2.245\Omega \]

and from equation (4.4) \[ M_{ss} = 0.078\Omega \]

Hence: \[ X''_d = 1.083\Omega = 1.083/11.547 \text{ p.u.} \]

\[ X''_d = 9.39 \times 10^{-2} \text{ p.u.} \]

and \[ X''_q = 1.161\Omega = 0.100 \text{ p.u.} \]

When the test was repeated with 12.1V and 6.02A, the constants
obtained were:

\[ A_{ss} = \frac{12.1}{6.02} = 2.010\Omega \]
\[ B_{ss} = \frac{10.95}{5.97} = 1.834\Omega \]

and \[ C_{ss} = \frac{12.15}{6.08} = 1.998\Omega \]
from which \( K_{ss} = 1.947\Omega \) and \( M_{ss} = 0.114\Omega \)

hence: \( X''_d = 7.95 \times 10^{-2} \) p.u.

and \( X''_q = 8.93 \times 10^{-2} \) p.u.

The values of the \( d\)- and \( q\)-axis additional subtransient reactances obtained for different currents are given in Table 4.4.

<table>
<thead>
<tr>
<th>Armature current (A)</th>
<th>2.9</th>
<th>6.02</th>
<th>8.9</th>
<th>12.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X''_d ) (p.u)</td>
<td>9.39 \times 10^{-2}</td>
<td>7.95 \times 10^{-2}</td>
<td>7.5 \times 10^{-2}</td>
<td>7.11 \times 10^{-2}</td>
</tr>
<tr>
<td>( X''_q ) (p.u)</td>
<td>10 \times 10^{-2}</td>
<td>8.93 \times 10^{-2}</td>
<td>8.64 \times 10^{-2}</td>
<td>8.41 \times 10^{-2}</td>
</tr>
</tbody>
</table>

**TABLE 4.4**

**4.6.3 Determination of \( d\)- and \( q\)-axis synchronous reactance from a slip test**

A slip test was performed by driving the test generator by a d.c. machine and applying to the armature (with the field open circuit) 3-phase positive sequence voltages of rated frequency. The armature voltage was sufficiently low for the machine to be working below the knee of the open-circuit characteristic.

The test was made at a slip sufficiently small for the induced currents in the damper winding and in the rotor iron not to become excessive.

Figure (4.17) shows oscillograms of the armature line voltage and current, together with the e.m.f. induced in the field winding at a slip of 0.34%. Although the armature was energised by symmetrical 3-phase voltages from a 3-phase transformer, the oscillogram
of field induced voltage exhibits a series of harmonics, which make it difficult to determine the actual fundamental component of the induced e.m.f. Further, as Figure (4.17) shows, the maximum and minimum armature voltages obtained do not coincide with the minimum and maximum armature currents, and in calculating the reactances the minimum and maximum armature currents were divided into the voltages at the corresponding points on the oscillogram of the terminal voltage.

However, as shown in Figure (4.17) the maximum and minimum armature voltages differ only by a small amount, and the above considerations do not much affect the actual results. From Figure (4.17) the corresponding reactances are calculated as:

\[ I_{\text{min}} = 0.06 \times 0.1/0.015 = 5.733 \text{A (peak to peak)} \]
\[ V_{\text{max}} = 2.2 \times 25 = 55 \text{V (peak to peak)} \]
\[ I_{\text{max}} = 2.13 \times 0.1/0.015 = 14.2 \text{A (peak to peak)} \]
and \[ V_{\text{min}} = 2.12 \times 25 = 53 \text{V (peak to peak)} \]

where \( 0.015 \Omega \) is the shunt resistance in the armature circuit.

According to the standard test\(^{14}\),

\[ X_d = \left( \frac{V_{\text{max}}}{I_{\text{min}}} \right) = \frac{55}{5.733} \Omega = \frac{55}{5.733 \times 20} \text{ p.u.} = 0.479 \text{ p.u.} \]
\[ X_q = \left( \frac{V_{\text{min}}}{I_{\text{max}}} \right) = \frac{53}{14.2} \Omega = \frac{53}{14.2 \times 20} \text{ p.u.} = 0.187 \text{ p.u.} \]

where:

\[ 20 \Omega = \frac{V}{I} = \text{base impedance in ohms.} \]

\( X_q \) can also be found\(^{14}\) from:
\[
X_q = X_d \left( \frac{V_{\text{min}}}{V_{\text{max}}} \right) \left( \frac{I_{\text{min}}}{I_{\text{max}}} \right)
\]  \hspace{1cm} (4.7)

where \( V_{\text{min}} \) and \( V_{\text{max}} \), \( I_{\text{min}} \) and \( I_{\text{max}} \) are the minimum and maximum values of voltage and current obtained from Fig. (4.17), and \( X_d \) is found \(^{14}\) from Figure (4.15).

Hence:

\[
X_q = 0.517 \left( \frac{53.25}{58.25} \right) \left( \frac{5.733}{14.2} \right) = 0.198 \text{ p.u.}
\]

It will be seen later that these results are very close to those obtained from indicial response methods.

4.6.4 Determination of q-axis synchronous reactance from maximum lagging current method

The test was performed \(^{14}\) with the machine operating as a synchronous motor with an initial applied test voltage of 107V/phase and a corresponding excitation of 0.802A. (It is recommended that the test should be carried out at a voltage not greater than about 75% of rated voltage, with approximately normal no load excitation). The excitation current was gradually reduced to zero, following which its polarity was reversed and then gradually increased until instability occurred (i.e. the machine began to pole slip). The supply voltage and current and the excitation current were recorded up to the maximum stable negative excitation, as given in Table 4.5a. It can be seen that the line voltage and current corresponding to the maximum stable negative excitation of 0.865A are 161V and 38.9A respectively. The q-axis synchronous reactance is

\[
X_q = \frac{V}{I}
\]  \hspace{1cm} (4.8)
<table>
<thead>
<tr>
<th>Line Voltage (Volts)</th>
<th>185.3</th>
<th>184.4</th>
<th>183.5</th>
<th>181.8</th>
<th>180.4</th>
<th>179.7</th>
<th>178.0</th>
<th>177.0</th>
<th>175.6</th>
<th>173.3</th>
<th>172.5</th>
<th>170.8</th>
<th>169.7</th>
<th>168.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Power (Watts)</td>
<td>1.14</td>
<td>1.75</td>
<td>3.44</td>
<td>5.48</td>
<td>7.55</td>
<td>9.57</td>
<td>11.74</td>
<td>14.00</td>
<td>16.95</td>
<td>19.32</td>
<td>21.63</td>
<td>24.22</td>
<td>26.35</td>
<td>28.00</td>
</tr>
<tr>
<td>Excitation Current</td>
<td>0.802</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.094</td>
<td>0</td>
<td>-0.1</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-0.48</td>
</tr>
<tr>
<td>(Amperes)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line Voltage (Volts)</th>
<th>168.3</th>
<th>168.0</th>
<th>167.9</th>
<th>167.1</th>
<th>166.6</th>
<th>166.4</th>
<th>165.9</th>
<th>165.4</th>
<th>164.1</th>
<th>163.1</th>
<th>*161</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Power (Watts)</td>
<td>28.6</td>
<td>28.75</td>
<td>29.4</td>
<td>30.0</td>
<td>30.76</td>
<td>31.5</td>
<td>32.55</td>
<td>33.6</td>
<td>34.65</td>
<td>36.75</td>
<td>*39.9</td>
</tr>
<tr>
<td>Excitation Current</td>
<td>-0.48</td>
<td>-0.494</td>
<td>-0.514</td>
<td>-0.538</td>
<td>-0.565</td>
<td>-0.6</td>
<td>-0.642</td>
<td>-0.675</td>
<td>-0.715</td>
<td>-0.715</td>
<td>*0.885</td>
</tr>
<tr>
<td>(Amperes)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line Voltage (Volts)</th>
<th>165.5</th>
<th>153.4</th>
<th>151.7</th>
<th>149.9</th>
<th>148.6</th>
<th>146.8</th>
<th>146.1</th>
<th>145.4</th>
<th>144.6</th>
<th>144.0</th>
<th>142.0</th>
<th>141.1</th>
<th>139.4</th>
<th>*138.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Power (Watts)</td>
<td>2</td>
<td>14.07</td>
<td>16.59</td>
<td>19.2</td>
<td>21.5</td>
<td>23.9</td>
<td>24.57</td>
<td>25.46</td>
<td>26.25</td>
<td>28.9</td>
<td>30.45</td>
<td>31.6</td>
<td>*33.6</td>
<td></td>
</tr>
<tr>
<td>Excitation Current</td>
<td>0.7</td>
<td>0</td>
<td>-0.1</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.36</td>
<td>-0.4</td>
<td>-0.44</td>
<td>-0.454</td>
<td>-0.488</td>
<td>-0.58</td>
<td>-0.63</td>
<td>-0.655</td>
<td>*-0.86</td>
</tr>
<tr>
<td>(Amperes)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Point at which the machine begins to pole slip*
where \( V \) = terminal voltage, p.u.

\( I_1 \) = armature current at stability limit, p.u.

Hence:

\[
X_q = \frac{161/240}{39.9/12} = 0.202 \text{ p.u.}
\]

The test was repeated at an initial line voltage of 165.5V and an initial excitation of 0.7A. As shown in Table 4.5b, the supply voltage and current corresponding to the maximum negative stable excitation of 0.66A are 138.5V and 33.6A, so that:

\[
X_q = \frac{138.5/240}{33.6/12} = 0.206 \text{ p.u.}
\]

It is clear that the tests described provide results which agree very closely.

4.6.5 Determination of \( q \)-axis synchronous reactance from reluctance-motor method

This test was performed with the machine operating on no-load and as a reluctance motor\(^2\) with the terminal voltage being gradually reduced until the machine fell out of step. The no-load input power and the terminal voltage up to the moment at which loss of synchronism occurred were measured and recorded, as listed in Table 4.6a.

From the final stable readings, the \( q \)-axis synchronous reactance can be calculated\(^2\) as:

\[
X_q = \frac{V^2 (X_d + 2r)}{V^2 + 2pX_d}
\]  

(4.9)
### Table 4.6a

<table>
<thead>
<tr>
<th>Line Voltage (Volts)</th>
<th>240</th>
<th>235.7</th>
<th>225.1</th>
<th>207.8</th>
<th>190.5</th>
<th>173.2</th>
<th>155.9</th>
<th>138.5</th>
<th>121.3</th>
<th>103.9</th>
<th>86.6</th>
<th>76.2</th>
<th>72.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Power (Watts)</td>
<td>402</td>
<td>1386</td>
<td>1128</td>
<td>888</td>
<td>816</td>
<td>744</td>
<td>699</td>
<td>687</td>
<td>660</td>
<td>666</td>
<td>636</td>
<td>660</td>
<td>666</td>
</tr>
<tr>
<td>Excitation Current (Ampere)</td>
<td>0.97</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 4.6b

<table>
<thead>
<tr>
<th>Line Voltage (Volts)</th>
<th>69.2</th>
<th>66.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Power (Watts)</td>
<td>696</td>
<td>702</td>
</tr>
<tr>
<td>Excitation Current (Ampere)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE 4.6a**

**TABLE 4.6b**

*Point at which loss of synchronism occurred.*
where:

\[ V = \text{terminal line voltage, } V \]

\[ X_d = \text{d-axis synchronous reactance found from Figure 4.17} \]

by well known methods\(^3\),\(^4\), \(\Omega\)

\[ p = \text{total no-load input power, } W \]

\[ r = \text{resistance per phase, } \Omega \]

hence:

\[
X_q = \frac{(66.68)^2 (5.97 + 2 \times 0.5)}{(66.68)^2 + (2 \times 702 \times 5.97)} = 2.410 \Omega
\]

i.e. \(X_q = \frac{2.41}{11.547} = 0.209 \text{ p.u.}\)

The test was repeated at a voltage below the normal level, with the results shown in Table 4.6b. From the readings at the stable limit of the no-load input power, the q-axis synchronous reactance is:

\[
X_q = \frac{(65.81)^2 (6.97)}{(65.81)^2 + (2 \times 714 \times 5.97)} = 2.348 \Omega = 2.348/11.547 \text{ p.u.}
\]

i.e. \(X_q = 0.203 \text{ p.u.}\)

It is evident from these two sets of measurements, that pull-out occurred at a terminal voltage of approximately 0.25 of rated voltage. The magnetic circuit of the machine was therefore unsaturated, and \(X_q\) obtained by this method is an unsaturated value.
4.7 Effect of the Brushes on the Parameter Measurement

The tests of Sections 4.2, 4.3, 4.4 and 4.6.2 were repeated with a direct connection to the armature winding, i.e. to the slip rings, so that current did not flow through the brushes, these tests are classified as follows.

4.7.1 Measurements of q-axis synchronous, subtransient and additional subtransient reactances (without brushes)

The tests were carried out using the procedure of Section 4.2. Figure 4.18 shows typical oscillograms of the applied voltage, the transient current through the a-phase of the armature winding, and the induced field voltage for a test current of 9.22 A. Figures 4.19 and 4.20 present analyses of transient armature currents, which have a steady-state value of 9.22 A and 4.71 A respectively, and the corresponding time constants and reactances are shown in Table 4.7.

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>T' qO (Sec)</th>
<th>T&quot; qO (Sec)</th>
<th>T' q (Sec)</th>
<th>T&quot; q (Sec)</th>
<th>X' q (pu)</th>
<th>X&quot; q (pu)</th>
<th>X‴ q (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.22</td>
<td>2.63x10⁻²</td>
<td>6.57x10⁻³</td>
<td>1.574x10⁻²</td>
<td>4.23x10⁻³</td>
<td>18.7x10⁻²</td>
<td>11.2x10⁻²</td>
<td>7.20x10⁻²</td>
</tr>
<tr>
<td>4.71</td>
<td>2.31x10⁻²</td>
<td>4.27x10⁻³</td>
<td>1.377x10⁻²</td>
<td>2.7x10⁻³</td>
<td>19.8x10⁻²</td>
<td>11.8x10⁻²</td>
<td>7.46x10⁻²</td>
</tr>
</tbody>
</table>

TABLE 4.7
4.7.2 Measurements of d-axis synchronous, subtransient and additional subtransient reactances; with the field winding open (without brushes)

The test procedures of Section 4.3 were repeated with the phase a aligned with the d-axis, and the field winding open-circuited. Figure 4.21 shows typical oscillograms for a test current of 13.52 A. Figures 4.22 and 4.23 show the development of the transient components for test currents of 13.52 and 9.15 amperes respectively. The corresponding time constants and reactances are shown in Table 4.8, from which it can be seen that the reactances are affected slightly by iron saturation, which is dependent on the flux density which is a function of the armature current.

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>$T'_d$ (s)</th>
<th>$T''_d$ (s)</th>
<th>$T'_o$ (s)</th>
<th>$T''_o$ (s)</th>
<th>$X'_d$ (pu)</th>
<th>$X''_d$ (pu)</th>
<th>$X'''_d$ (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.52</td>
<td>3.865 x10^{-2}</td>
<td>4.452 x10^{-3}</td>
<td>1.096 x10^{-2}</td>
<td>2.484 x10^{-3}</td>
<td>43.258 x10^{-2}</td>
<td>12.227 x10^{-2}</td>
<td>6.823 x10^{-2}</td>
</tr>
<tr>
<td>9.15</td>
<td>3.935 x10^{-2}</td>
<td>4.457 x10^{-3}</td>
<td>1.096 x10^{-2}</td>
<td>2.585 x10^{-3}</td>
<td>44.549 x10^{-2}</td>
<td>12.406 x10^{-2}</td>
<td>7.194 x10^{-2}</td>
</tr>
</tbody>
</table>

TABLE 4.8
4.7.3 Measurements of d-axis synchronous, transient, sub-transient and additional subtransient reactances, with the field circuit closed (without brushes)

The test procedures of Section 4.4 were again carried out with the a-phase of the rotor aligned with the d-axis, and the field winding short circuited. Figure 4.24 shows typical oscillograms of the applied voltage and the transient armature current. The short-circuited field current was not recorded for the reason explained previously in Section 4.4. The analyses of two different transient current tests are shown in Figures 4.25 and 4.26 respectively, and the corresponding time constants and reactances are shown in Table 4.9. The effect of saturation due to armature current is to reduce the value of reactances, as is apparent from a comparison of the values obtained for armature currents of 13.52A and 9.15A.

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>$T'_d$ (s)</th>
<th>$T''_d$ (s)</th>
<th>$T'''_d$ (s)</th>
<th>$T'_d$ (s)</th>
<th>$T''_d$ (s)</th>
<th>$T'''_d$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.52</td>
<td>18.22 x10^{-2}</td>
<td>2.020 x10^{-2}</td>
<td>3.238 x10^{-3}</td>
<td>5.735 x10^{-2}</td>
<td>1.095 x10^{-2}</td>
<td>2.512 x10^{-3}</td>
</tr>
<tr>
<td>9.15</td>
<td>18.41 x10^{-2}</td>
<td>1.832 x10^{-2}</td>
<td>2.812 x10^{-3}</td>
<td>5.79 x10^{-2}</td>
<td>1.792 x10^{-2}</td>
<td>2.213 x10^{-3}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>$X'_d$ (p.u.)</th>
<th>$X''_d$ (p.u.)</th>
<th>$X'''_d$ (p.u.)</th>
<th>$X''''_d$ (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.52</td>
<td>42.90 x10^{-2}</td>
<td>13.49 x10^{-2}</td>
<td>7.311 x10^{-2}</td>
<td>5.669 x10^{-2}</td>
</tr>
<tr>
<td>9.15</td>
<td>43.96 x10^{-2}</td>
<td>13.825 x10^{-2}</td>
<td>7.546 x10^{-2}</td>
<td>5.94 x10^{-2}</td>
</tr>
</tbody>
</table>

TABLE 4.9
4.7.4 Determination of d and q-axis additional sub-transient reactances by Dalton and Cameron method (without brushes)

The test procedure described in Section 4.6.2 was performed, and the following values were obtained for a test current of 5.46 A:

\[
A_{sc} = 9.39 \text{ V}/5.46 \text{ A} = 1.720\Omega \\
B_{sc} = 9.26 \text{ V}/5.92 \text{ A} = 1.564\Omega \\
C_{sc} = 9.36 \text{ V}/5.47 \text{ A} = 1.710\Omega
\]

hence from equations 4.3 - 4.6:

\[
K_{sc} = 1.665\Omega \\
M_{sc} = 0.100\Omega
\]

Thus:

\[
X''_d = (K_{sc} - M_{sc})/2 \quad \Omega = \frac{(1.665 - 0.100)}{2 \times 11.547} = 6.78 \times 10^{-2} \text{ p.u.}
\]

and

\[
X''_q = (K_{sc} + M_{sc})/2 \quad \Omega = \frac{(1.665 + 0.100)}{2 \times 11.547} = 7.642 \times 10^{-2} \text{ p.u.}
\]

For a test current of 8.91 A, the values obtained were:

\[
A_{sc} = 15.05 \text{ V}/8.91 \text{ A} = 1.690\Omega \\
B_{sc} = 14.9 \text{ V}/9.89 \text{ A} = 1.510 \Omega \\
C_{sc} = 15.04 \text{ V}/8.96 \text{ A} = 1.680\Omega
\]

hence:

\[
K_{sc} = 1.624\Omega \quad \text{and} \quad M_{sc} = 0.118\Omega
\]

i.e.

\[
X''_d = 6.55 \times 10^{-2} \text{ p.u.}
\]

and

\[
X''_q = 7.55 \times 10^{-2} \text{ p.u.}
\]
The test was repeated for test currents of 10.91 A, and 13.29 A, the corresponding additional subtransient reactances are as shown in Table 4.10

<table>
<thead>
<tr>
<th>Armature Current (A)</th>
<th>5.46</th>
<th>8.91</th>
<th>10.91</th>
<th>13.29</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X''_q$ (p.u)</td>
<td>$7.64 \times 10^{-2}$</td>
<td>$7.55 \times 10^{-2}$</td>
<td>$7.38 \times 10^{-2}$</td>
<td>$7.11 \times 10^{-2}$</td>
</tr>
<tr>
<td>$X''_d$ (p.u)</td>
<td>$6.76 \times 10^{-2}$</td>
<td>$6.55 \times 10^{-2}$</td>
<td>$6.49 \times 10^{-2}$</td>
<td>$6.25 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

TABLE 4.10

4.8 Decay Test

Although a heavy duty battery was used as the source for the indicial response tests, a decay test was carried out on the q-axis windings to ensure that a good step of voltage had been provided by the battery. With the phase-a aligned with the q-axis, the d.c. supply voltage from an external source was applied directly to the phase winding (i.e. not through the brushes), through an L-C filter circuit to remove any ripple from the voltage, and a high resistance $R_1$, as shown in Figure 4.27. The phase winding was short circuited by closing switch $S_1$ and the decay current recorded as shown in Figure 4.28.

The high resistance $R_1$ avoids errors due to an increase in the current from the supply as $R_1$ is very much larger than the resistance of winding plus shunt (i.e. an increase in the current
would tend to increase the voltage drop across the terminals of the armature short circuit switch \( S_1 \) which introduces an appreciable error in the decay current), and also protects the supply from the effect of the short circuit current. To determine the exact starting point of the decaying current, the voltage applied to the phase winding was recorded across the terminals of the switch \( S_1 \).

Figure 4.29 shows the analysis of the transient decay of the armature current. The q-axis synchronous, subtransient and additional subtransient reactances are given by equations (3.15), (3.20) and (3.17) respectively. (The formula for the transient decay of the armature current is the same as that of equations (3.15), (3.20) and (3.17)).

The corresponding time constants and reactances obtained from Figure 4.29 are shown in Table 4.11. It can be seen that the results obtained agree well with those from the indicial-response tests of Table 4.7, it is considered therefore that the heavy-duty battery provides an acceptable step function of voltage.

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>( T_{q0}^\pi ) (Sec)</th>
<th>( T_{q0}^\pi ) (Sec)</th>
<th>( T_{q}^\pi ) (Sec)</th>
<th>( T_{q}^\pi ) (Sec)</th>
<th>( X_{q} ) (pu)</th>
<th>( X_{q} ) (pu)</th>
<th>( X_{q} ) (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.9</td>
<td>2.88x10^{-2}</td>
<td>4.969x10^{-3}</td>
<td>1.437x10^{-2}</td>
<td>3.516x10^{-3}</td>
<td>20.25x10^{-2}</td>
<td>10.10x10^{-2}</td>
<td>7.151x10^{-2}</td>
</tr>
</tbody>
</table>
4.9 Linear Regression (Method of Least Squares)

In order to show the influence of ignoring the damping effect of the iron on the machine parameters, the linear regression method is applied, using one of the two techniques of Appendix 7.2.

1. Figure 4.4

Method (a): On applying method (a) of Appendix 7.2 (see Figure 7.2.1) to Figure 4.4, the second curve is replaced by a straight line, starting from 7.72A and passing through one point only.

For the first ten points on the 2nd curve, i.e. from 0.001s to 0.01s:

\[
\begin{align*}
\sum t_j &= 5.5 \times 10^{-2} \text{ s} \\
\sum t_j^2 &= 3.85 \times 10^{-4} \text{ s}^2 \\
\sum i_j &= 18.617 \text{ A} \\
\sum i_j t_j &= 6.37 \times 10^{-2} \text{ A.s}
\end{align*}
\]

where: 
- \(i_j, t_j\) = the coordinates of the point on the curve
- \(\sum i_j\) = the algebraic sum of the instantaneous values of the current at the first ten points
- \(\sum t_j\) = the algebraic sum of the time intervals at the first ten points.

From equation (7.2.1):
\[
\frac{1}{m} = \frac{\sum t_j^2}{\sum \left( t_j - 0.632 \right) t_j} = \frac{3.85 \times 10^{-4}}{6.37 \times 10^{-2} - 7.72 \times 5.5 \times 10^{-2}} = -1.06 \times 10^{-3}
\]

where \( m \) is the slope of the line.

The time constant of the line \( (T) \) is obtained from equation (7.2.2) as:

\[
T = \frac{\frac{1}{m} \left( \frac{1}{e} - 1 \right)}{\frac{1}{e}} = 7.72 \left( -0.632 \right)( -1.06 \times 10^{-3}) = 5.204 \times 10^{-3} \text{ s}
\]

i.e. the second curve of Figure 4.4 is replaced by a straight line, starting from 7.72A and with a time constant of 5.204 \times 10^{-3} \text{ s}.

According to this technique, Figure 4.4 consists of two decaying components, their magnitudes and the corresponding time constants being:

\[
\begin{align*}
I_{A1} &= 2A & \text{and} & I_{A2L} &= 7.72A \\
T_{A1} &= 2.6 \times 10^{-2} \text{ s} & T_{A2L} &= 5.204 \times 10^{-3} \text{ s}
\end{align*}
\]

i.e.

\[
A_L = \frac{I_{A2L}}{I_{A1}} = 7.72 \times 2 = 3.86
\]

\[
B_L = \frac{T_{A2L}}{T_{A1}} = \frac{5.204 \times 10^{-3}}{2.6 \times 10^{-2}} = 2.001 \times 10^{-1}
\]

The new suffix \( L \) is used here and is to indicate that a least square method has been used, and to be distinguished from those used in Section 4.2 and 4.3. According to the two decaying components and their respective time constants given above, equation (3.15) thus reduces to:
The value of q-axis reactances measured in Section 4.2 and given in Table 4.1 are:

\[ X_q = 19.825 \times 10^{-2} \text{ p.u.} \]

\[ X''_q = 10.275 \times 10^{-2} \text{ p.u.} \]

\[ X'''_q = 7.621 \times 10^{-2} \text{ p.u.} \]
and the value of q-axis additional subtransient reactance measured of a test current of 12.05A by the static test (Dalton and Cameron) in Section 4.6.2 and given in Table 4.4 is:

\[ X'''_q = 8.41 \times 10^{-2} \]

Comparison of these values with the corresponding values obtained by using method (a), shows the considerable difference especially with subtransient reactance values.

Method (b): On applying method (b) of Appendix 7.2, (see Figure 7.2.2) to the second curve of Figure 4.4, the slope of the line is given by equation (7.2.3):

\[
\begin{vmatrix}
\Sigma t_j^2 & \Sigma t_j t_j & \Sigma t_j \\
\Sigma t_j & 10 & 0 \\
\Sigma t_j & 10 & 0
\end{vmatrix} = \begin{vmatrix}
6.37 \times 10^{-2} & 5.5 \times 10^{-2} \\
18.617 & 10 \\
3.85 \times 10^{-4} & 5.5 \times 10^{-2} \\
5.5 \times 10^{-2} & 10
\end{vmatrix} = \frac{3669.35 \times 10^{-4}}{6.25 \times 10^{-4}}
\]

i.e. \( m = \frac{\Sigma t_j^2}{\Sigma t_j} = \frac{6.37 \times 10^{-2} - 5.5 \times 10^{-2}}{3.85 \times 10^{-4} - 5.5 \times 10^{-2}} = 8.25 \times 10^{-3} \)

the initial value of the line is given by equation (7.2.4):

\[
\begin{vmatrix}
\Sigma t_j^2 & \Sigma t_j^2 & \Sigma t_j \\
\Sigma t_j & \Sigma t_j & 0 \\
\Sigma t_j & 10 & 0
\end{vmatrix} = \begin{vmatrix}
3.85 \times 10^{-4} & 6.37 \times 10^{-2} \\
5.5 \times 10^{-2} & 18.617 \\
8.25 \times 10^{-4} & 8.25 \times 10^{-4}
\end{vmatrix} = \frac{3.664 \times 10^{-3}}{8.25 \times 10^{-4}}
\]

\[ C_1 = 3.664 \times 10^{-3} \]

\[ = 4.441A \]
and from equation (7.2.5), the time constant of the line is

\[ T = \frac{C_1}{\theta - 1} \cdot \frac{1}{m} = \frac{4.441 (-0.632) \times 8.25}{-3869.35} \]

\[ = 5.984 \times 10^{-3} \text{s}. \]

Here the second curve of Figure 4.4 is replaced by a straight line with an initial value of 4.441A and a time constant of 5.984 \times 10^{-3} \text{s}. According to this technique, Figure 4.4 also has two components, but their magnitudes and the corresponding time constants are:

\[ I_{A_1} = 2A \quad \text{and} \quad I_{A_2L} = 4.441A \]

\[ T_{A_1} = 2.6 \times 10^{-2} \text{s} \quad \text{and} \quad T_{A_2L} = 5.984 \times 10^{-3} \text{s} \]

hence

\[ A_L = \frac{4.441}{2} = 2.220 \]

\[ B_L = \frac{5.984 \times 10^{-3}}{2.6 \times 10^{-2}} = 2.301 \times 10^{-1} \]

From equation (4.10):

\[ X_q = \frac{1.5 \times 0.617 \times 0.026 \times 314 (1 + 2.22 \times 2.301 \times 10^{-1})}{11.547 (1 + 2.22)} \]

\[ = 30.702 \times 10^{-2} \text{ p.u.} \]
and from equation (4.11):

\[ X'' = \frac{1.5 \times 0.617 \times 0.026 \times 314 \times (1 + 2.22)}{11.547 \times (1 + \frac{2.22}{2.301 \times 10^{-1}})} \]

\[ = 19.787 \times 10^{-2} \text{ p.u.} \]

It can be seen that the results obtained from both methods (a) and (b), especially the subtransient reactance value, do not agree with those given in Tables 4.1 and 4.4. However method (a) is more appropriate than method (b), and so method (a) will be applied to the following figures.

2. **Figure 4.7**

By applying the method (a) to the second curve of Figure 4.7 we obtain:

\[ \frac{1}{m} = \frac{3.85 \times 10^{-6}}{6.142 \times 10^{-2} - 7.78 \times 0.055} = -1.050 \times 10^{-3} \]

and \( T = 7.78 \times (0.632) \times 1.050 \times 10^{-3} = 5.163 \times 10^{-3} \text{ s} \)

Hence the second curve is replaced by a straight line, with an initial value of 7.78A and a time constant of 5.163 \( \times 10^{-3} \) s.

i.e. \( A_L = \frac{7.78}{3.31} = 2.350 \)

\[ B_L = \frac{5.163 \times 10^{-3}}{62.7 \times 10^{-2}} = 8.235 \times 10^{-2} \]
The d-axis synchronous ($X_d$) and subtransient reactances ($X''_d$) are obtained from equations (4.10) and (4.11) (with subscript $q$ replaced by subscript $d$) as:

$$X_d = \frac{1.5 \times 0.541 \times 0.0627 \times 314 \left(1 + 2.350 \times 8.235 \times 10^{-2}\right)}{11.547 \left(1 + 2.350\right)}$$

$$\approx 49.294 \times 10^{-2} \text{ p.u.}$$

and

$$X''_d = \frac{1.5 \times 0.541 \times 0.0627 \times 314 \left(1 + 2.350\right)}{11.547 \left(1 + \frac{2.350}{8.235 \times 10^{-2}}\right)}$$

$$\approx 15.692 \times 10^{-2} \text{ p.u.}$$

The value of d-axis reactances measured in Section 4.3 and given in Table 4.2 are:

$$X_d = 46.566 \times 10^{-2} \text{ p.u.}$$

$$X''_d = 12.675 \times 10^{-2} \text{ p.u.}$$

$$X''''_d = 7.228 \times 10^{-2} \text{ p.u.}$$

and the value of d-axis additional subtransient reactance, measured at a test current of 12.05A by the static test described in Section 4.6.2, and given in Table 4.4 is:

$$X''''_d = 7.11 \times 10^{-2} \text{ p.u.}$$
Comparison of these values with the corresponding values obtained by using method (a), shows the considerable difference, especially with the value of subtransient reactance.

3. Figure 4.22

By applying the method to the Figure 4.22, the following results are obtained:

\[
\frac{1}{m} = -9.7502 \times 10^{-4}
\]

\[T = 5.287 \times 10^{-3} \text{s}\]

\[A_L = 1.737\]

\[B_L = 8.961 \times 10^{-2}\]

From equations (4.10) and (4.11)

\[X_d = 45.117 \times 10^{-2} \text{ p.u.}\]

and \[X''_d = 14.347 \times 10^{-2} \text{ p.u.}\]

The value of the d-axis reactances measured in Section 4.7.2 and given in Table 4.8 are:

\[X_d = 43.258 \times 10^{-2} \text{ p.u.}\]

\[X''_d = 12.227 \times 10^{-2} \text{ p.u.}\]

\[X'''_d = 6.823 \times 10^{-2} \text{ p.u.}\]
The d-axis additional subtransient reactance measured at a test current of 13.29A in Section 4.7.4 and given in Table 4.10 is:

\[ X_d'' = 6.25 \times 10^{-2} \text{ p.u.} \]

Again it can be seen that the value of subtransient reactance obtained by using method (a), is much different from those obtained in Sections 4.7.2. and 4.7.4.

4. Figure 4.25

Applying method (a) to this figure, the following results are obtained:

\[ \sum_j = 5.5 \times 10^{-2} \text{ s}^2 \]
\[ \sum_j^2 = 3.85 \times 10^{-4} \text{ s}^2 \]
\[ \sum_i = 24.84 \text{ A} \]
\[ \sum_i \cdot \sum_j = 8.158 \times 10^{-2} \text{ A.s} \]

Hence:

\[ \frac{1}{m} = \frac{3.85 \times 10^{-4}}{8.158 \times 10^{-2} - 10.58 \times 5.5 \times 10^{-2}} \]

\[ = -7.695 \times 10^{-4} \]

\[ T = 10.58 (-0.632) (-7.695 \times 10^{-4}) \]

\[ = 5.145 \times 10^{-3} \text{ s} \]
i.e. the third curve of Figure 4.25 is replaced by a straight line, with an initial value of 10.58A and a time constant of $5.145 \times 10^{-3}$ s. According to this technique, Figure 4.25, consists of three decaying components, their magnitudes and the corresponding time constants being:

\[
\begin{align*}
I_{A_{1s}} &= 1.24A, \\
I_{A_{2s}} &= 1.7A, & \text{and } I_{A_{3sL}} &= 10.58A \\
T_{A_{1s}} &= 0.2S, & T_{A_{2s}} &= 0.023S & \text{and } T_{A_{3sL}} &= 5.145 \times 10^{-3} \\
\text{i.e. } A_s &= \frac{I_{A_{2s}}}{I_{A_{1s}}} = 1.37 \\
B_s &= \frac{T_{A_{2s}}}{T_{A_{1s}}} = 1.15 \times 10^{-1} \\
C_{sL} &= \frac{I_{A_{3sL}}}{I_{A_{1s}}} = 8.532 \\
D_{sL} &= \frac{T_{A_{3sL}}}{T_{A_{1s}}} = 2.572 \times 10^{-2}
\end{align*}
\]

Again the new suffix $L$ is used here to indicate that a least square method has been used, and to be distinguished from those used in Section 4.4. According to the three decaying components and their corresponding time constants given above, eqns. (3.57), (3.59), (3.62), (3.67), (3.68), (3.69) and (3.70) thus reduce, respectively, to:

\[
X_d = \frac{1.5 r_d T_{A_{1s}} (1 + A_s B_s + C_{sL} D_{sL})}{1 + A_s + C_{sL}} \quad (4.12)
\]
\[ x''_d = \frac{1.5 r_d T_{A_{1S}} (1 + A_s + C_{sL})}{(1 + \frac{A_s}{B_s} + \frac{C_{sL}}{D_{sL}})} \] (4.13)

\[ x''_d \approx \frac{T'_d T''_d}{T'_d T''_d} \cdot x_d \] (4.14)

from which:

\[ x'_d \approx \frac{T'_d}{T'_d} \cdot x_d \] (4.15)

\[(1+sTA_{2S})(1+sTA_{3S}) + A_s (1+sTA_{1S})(1+sTA_{3S}) + C_{sL} (1+sTA_{1S})(1+sTA_{2S}) = 0 \] (4.16)

\[ T'_{d0} = -\frac{1}{a_{1d}} \] (4.17)

\[ T''_{d0} = -\frac{1}{a_{2d}} \] (4.18)

\[(1+sTA_{2S})(1+sTA_{3S}) + A_s B_s (1+sTA_{1S})(1+sTA_{3S}) + C_{sL} D_{sL} = 0 \] (4.19)

and

\[ T'_d = -\frac{1}{B_{1d}} \]

\[ T''_d = -\frac{1}{B_{2d}} \] (4.19)
On substituting $A_{sL}$, $B_{sL}$, $C_{sL}$ and $D_{sL}$ into equations (4.12), (4.13), (4.16) and (4.18) respectively, we obtain:

\[ X_d = 45.728 \times 10^{-2} \text{ p.u.} \]

\[ X''_d = 11.456 \times 10^{-2} \text{ p.u.} \]

\[ \alpha_{1d} = -5.485 \]

\[ \alpha_{2d} = -48.780 \]

\[ \beta_{1d} = -15.385 \]

\[ \beta_{2d} = -69.930 \]

From equations (4.17) and (4.19):

\[ T'_{d0} = 0.1823 \text{ s} \]

\[ T''_{d0} = 0.0205 \text{ s} \]

\[ T'_{d} = 0.085 \text{ s} \]

\[ T''_{d} = 0.0143 \text{ s} \]

Hence from equation (4.15):

\[ X'_d = 0.163 \]

The value of the reactances found in Section 4.7.3 and given in Table 4.9 are:
The value of additional subtransient reactance measured at a test current of 13.2A in Section 4.7.4 and given in Table 4.10 is:

\[ X''_d = 6.25 \times 10^{-2} \text{ p.u.} \]

Comparison of these reactances with those determined by using method (a) above, shows that ignoring the damping effect of the rotor iron has:

1. had no considerable influence on the measured value of synchronous reactance.
2. a small influence on the measured value of transient reactance.
3. a big influence on the measured value of subtransient reactance.

From what is explained and proved in this section, it can be said that ignoring the damping effect of the rotor iron mainly introduces a large error into the value of the measured subtransient reactance. Therefore the damping effect of the rotor iron cannot be neglected.
4.10 Comparison of the Results

In order to show the accuracy of this approach, the measured parameters are compared with those measured by static test and conventional methods as given in Tables 4.12a and 4.12b. From these two tables it will be seen clearly that the new approach produces satisfactory results, more so when the brushes are removed from the armature circuit (Table 4.12b).
<table>
<thead>
<tr>
<th>Methods</th>
<th>$X_q$ (p.u)</th>
<th>$X''_q$ (p.u)</th>
<th>$X''''_q$ (p.u)</th>
<th>$X_d$ (p.u)</th>
<th>$X'_d$ (p.u)</th>
<th>$X''_d$ (p.u)</th>
<th>$X''''_d$ (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No load and sustained short-circuit test</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>51.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A.C. supply to two armature phases with the rotor stationary (Delton &amp; Cameron)</td>
<td>-</td>
<td>-</td>
<td>8.41 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.11 x10^{-2}</td>
</tr>
<tr>
<td>q-axis indicial-response test on one armature phase I = 9.72A</td>
<td>19.825 x10^{-2}</td>
<td>10.275 x10^{-2}</td>
<td>7.621 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>d-axis indicial-response test on one armature phase (field open) I = 11.09A</td>
<td>-</td>
<td>-</td>
<td>46.586 x10^{-2}</td>
<td>-</td>
<td>12.675 x10^{-2}</td>
<td>7.228 x10^{-2}</td>
<td>-</td>
</tr>
<tr>
<td>d-axis indicial-response test on one armature phase (field closed) I = 11.09A</td>
<td>-</td>
<td>-</td>
<td>44.583 x10^{-2}</td>
<td>13.82 x10^{-2}</td>
<td>7.975 x10^{-2}</td>
<td>6.464 x10^{-2}</td>
<td>-</td>
</tr>
<tr>
<td>Slip test (at V = 62.5 volts)</td>
<td>19.8 x10^{-2}</td>
<td>-</td>
<td>47.9 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Maximum lagging current method (at V = 0.75 V normal)</td>
<td>20.4 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Reluctance motor method</td>
<td>20.8 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE 4.12a**
Measurement of Parameters with Brushes in the Armature Circuit
<table>
<thead>
<tr>
<th>Methods</th>
<th>$X_q$ (p.u)</th>
<th>$X''_q$ (p.u)</th>
<th>$X''_q$ (p.u)</th>
<th>$X_d$ (p.u)</th>
<th>$X'_d$ (p.u)</th>
<th>$X''_d$ (p.u)</th>
<th>$X''_d$ (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>q-axis indicial-response test on one armature phase</td>
<td>18.7 x10^{-2}</td>
<td>11.2 x10^{-2}</td>
<td>7.20 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>I = 9.22A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d-axis indicial-response test on one armature phase</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>43.258 x10^{-2}</td>
<td>-</td>
<td>12.227 x10^{-2}</td>
<td>6.823 x10^{-2}</td>
</tr>
<tr>
<td>(field open) I = 13.52A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d-axis indicial-response test on one armature phase</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>42.90 x10^{-2}</td>
<td>13.492 x10^{-2}</td>
<td>7.311 x10^{-2}</td>
<td>5.669 x10^{-2}</td>
</tr>
<tr>
<td>(field closed) I = 13.52A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A.C. supply to two armature phases with the rotor</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.11 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>6.25 x10^{-2}</td>
</tr>
<tr>
<td>stationary (Dalton and Cameron) I = 13.29A</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>q-axis decay test I = 4.9A</td>
<td>20.25 x10^{-2}</td>
<td>10.10 x10^{-2}</td>
<td>7.15 x10^{-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
</tbody>
</table>

**TABLE 4.12b**

Measurement of Parameters with No Brushes in the Armature Circuit
FIG. 4.1. SCHEME OF MEASUREMENT OF THE TRANSIENT CURRENT
\[ E = \text{Applied voltage of armature winding phase } a. \]
\[ I_q = \text{Transient current of armature winding phase } a. \]
\[ V_F = \text{Induced voltage of field winding.} \]

**FIG. 4.2. OSCILLOGRAM FOR INDICIAL RESPONSE METHOD AT } \theta = 90 \text{ DEGREES**
$2.5 \text{ V/cm}$

$0.5 \text{ V/cm}$

$V_b$

$V_c$

$0.04 \text{ s}$

FIG. 4.3.
E = Applied voltage of armature winding phase a
I_d = Transient current of armature winding phase a
V_F = Induced voltage of field winding

FIG. 4.6.- OSCILLOGRAM FOR INDICIAL RESPONSE
METHOD θ = 0 DEGREES (FIELD OPEN)
$I_{A_0} = 11.09$ AMPS

$r_2 = 0.541 \Omega$

$T_{A_0} = 0.0018$

FIG. 4.7. ANALYSIS OF TRANSIENT CURRENT OF INDICIAL RESPONSE METHOD AT 0.0 DEGREES (FIELD OPEN)
Figure 4.8

Transient current (amps)

Time (secs)

$T_1 = 0.4997$

$T_2 = 0.6895$

$T_3 = 0.9912$

$T_4 = 1.4997$

$T_5 = 2.0017$
$E = \text{Applied voltage of armature winding phase a}$

$I_d = \text{Transient current of armature winding phase a}$

FIG. 4.9 - OSCILLOGRAM FOR INDICIAL RESPONSE

METHOD $\theta = 0$ DEGREES (FIELD CLOSED)
\[ I_{dc} = 11.09 \text{ AMPS} \]
\[ r_d = 0.541 \Omega \]
$I_{A0} = 6.8 \text{ AMPS}$

$R_d = 0.588 \Omega$
Applied voltage to the field winding

FIG. 4.12.
FIG. 4.15.
FIG. 4.16. MEASURING CIRCUIT OF d & q AXIS ADDITIONAL SUBTRANSIENT REACTANCES.
FIG. 4.17.
FIG. 4.18.
$I_{A_0} = 4.71\text{ AMP}$

$r_0 = 0.425\,\Omega$
FIG. 4.21
$I_{A_d} = 13.52$ AMPS

$\tau_d = 0.444 \, \Omega$
$I_{A_0} = 13.52 \, \text{amps}$

$r_d = 0.444 \, \Omega$
FIG. 4.26.
Fig. 4.28.

1.0 V/cm

0.01 V/cm

0.005 secs
The analyses of the stability of a power system and the study of its short-circuit behaviour are very important, more so as the system gets bigger and more complex.

Previous attempts to establish operational impedances and hence the parameters of the machine have been entirely based on elementary analyses using very simple machine models. 10,12

In practice a salient-pole synchronous machine has a significant part of its rotor made of solid iron, which exercises a major influence on the operational performance. The results obtained may differ greatly from the theoretical results if this influence is neglected. In order to include this effect and to provide a more accurate representation, an additional rotor circuit in the machine model is required. Accordingly, a new approach has been attempted, with the damping action of the iron represented by an additional damper winding along each axis. This naturally leads to an important modification to the familiar form of Park's equations.

In this study a step-by-step derivation of the operational inductances and parameters for the new model of a salient-pole synchronous generator were developed from fundamental principles. As shown in Chapter 3, the parameters are derived from the basic equations applied to the new model, with two damper windings in each of the d- and q-axes.

From the experimental work presented in this thesis, the following conclusions can be drawn:
1. It suffices to consider both the q- and d-axes as each having two damper windings.

2. The results obtained from the new analysis agree closely with those obtained from static and conventional methods as shown in Tables 4.12a and 4.12b.

3. Saturation effects are small as is clear from Tables 4.1-4.3 and 4.7-4.9.

4. By removing the brushes from the armature circuit, the measured parameters change considerably, as is clear from the comparison of the tables 4.1 and 4.7, 4.2 and 4.8, 4.3 and 4.9, and finally 4.4 and 4.10.

The effect of the brushes is to introduce a non-linear resistance in series with the measuring circuit. This introduces errors in the measured time constants during the step response test. The result is that the measured values of reactances are different when measurements are made with and without brushes. Hence to obtain accurate and constant values of parameters, it is necessary to make measurements, if possible, without brushes. (This applies particularly to step-response tests as the non-linear effect of brush resistance affects the transient current).

5. The damping effect of the iron cannot be neglected, otherwise the derived parameters will be considerably in error.
REFERENCES


7.1.1 D-axis voltage equations with the field circuit open

Under stationary condition, and with the field circuit open as in Figure 2.3, the voltage equations in terms of the time rate-of-change of the flux linkages are:

\[ E_1 = (pL_d + r_d)I_d + pL_{a1d}I_{1d} + pL_{a2d}I_{2d} \]  \hspace{1cm} (7.1.1)

\[ 0 = pL_{1ad}I_d + (pL_{11d} + R_{1d})I_{1d} + pL_{12d}I_{2d} \]  \hspace{1cm} (7.1.2)

\[ 0 = pL_{2ad}I_d + pL_{21d}I_{1d} + (pL_{22d} + R_{2d})I_{2d} \]  \hspace{1cm} (7.1.3)

where the bold letters indicate quantities in normal (SI) units.

To transform these equations to the p.u. system, and at the same time to preserve the reciprocity of the mutual inductances, the d-axis equivalent self-inductance \( L_d \) must be multiplied by \( 2/3 \) and both sides of each equation must be divided by the base stator d-axis voltage (i.e. \( e_{d0} \), where \( e_{d0} = \omega_0 i_{d0} L_d \), \( i_{d0} = \) base stator d-axis current).

The relation between the stator and rotor base currents are:

\[ i_{d0} = f_{d0} \sqrt{\frac{L_{idd}}{2/3 L_d}}, \quad \text{and} \quad i_{d0} = f_{nd0} \sqrt{\frac{L_{nnd}}{2/3 L_d}} \]

where:

\( f_{d0} = \) base field current in the d-axis
\( f_{nd0} = \) base damper current in the d-axis
\[ L_{\text{ffd}} = \text{self inductance of field} \]

and \[ L_{\text{nd}} = \text{self inductance of any direct-axis damper circuit.} \]

Hence equation (7.1.1) becomes:

\[
E_1 = r_d I_d + p(L_d I_d + L_{a1d} I_{a1d} + L_{a2d} I_{a2d}) \tag{7.1.4a}
\]

or

\[
E_1 = (pL_d + r_d) I_d + pL_{a1d} I_{a1d} + pL_{a2d} I_{a2d} \tag{7.1.4b}
\]

where

\[
L_d = \frac{2}{3} \frac{L_d}{L_{d0}}, \quad K_{a1d} = \sqrt{\frac{L_{a1d}}{L_{1ld}^2 \frac{2}{3} L_d}} \quad \text{and} \quad K_{a2d} = \sqrt{\frac{L_{a2d}}{L_{22d}^2 \frac{2}{3} L_d}}
\]

\[ L_{d0} = \text{base stator inductance of d-axis circuit.} \]

Similarly, on dividing equation (7.1.2) by \( e_d \) and substituting for the corresponding rotor currents, and multiplying by

\[
\frac{L_{d0}}{\sqrt{L_{1ld}^2 \frac{2}{3} L_d}}
\]

we obtain:

\[
0 = I_{1d} R_{1d} + p(L_{d} K_{a1d} I_{a1d} + L_{d} I_{1d} + L_{d} K_{a2d} I_{a2d}) \tag{7.1.5a}
\]

or

\[
0 = pK_{a1d} I_{a1d} + (p + \frac{1}{I_{1d}}) I_{1d} + pK_{a2d} I_{a2d} \tag{7.1.5b}
\]

Similarly, by multiplying equation (7.1.3) by \( \frac{L_{d0}}{e_d} \) \( L_{22d} \) \( \frac{1}{3} L_d \)

and substituting for the corresponding damper currents, we obtain:
Hence from equations (7.1.4b), (7.1.5b) and (7.1.6b), the p.u. matrix form of these equations is:

\[
\begin{bmatrix}
E_1 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
pL_d + r_d & pL_d K_a1 & pL_d K_a2 \\
pK_a1 & p + \frac{1}{T_1d} & pK_{21} \\
pK_a2 & pK_{21} & p + \frac{1}{T_2d}
\end{bmatrix} \begin{bmatrix}
I_d \\
I_{1d} \\
I_{2d}
\end{bmatrix}
\] (7.1.7)

Similar expressions for the q-axis circuits can be obtained by replacing subscript \(d\) in equations (7.1.1-7.1.7) by subscript \(q\).

Hence from equations (7.1.4a), (7.1.5a) and (7.1.6a) the q-axis flux-linkages in p.u. are:

\[
\psi_q = L_{q a1} I_{1q} + L_{q a2} I_{2q} + L_{q} I_{q}
\] (7.1.8)

\[
\psi_{1q} = L_{q} I_{1q} + L_{q 12} I_{2q} + L_{q a1} I_{q}
\] (7.1.9)

\[
\psi_{2q} = L_{q 21} I_{1q} + L_{q} I_{2q} + L_{q a2} I_{q}
\] (7.1.10)

and from equations (7.1.4b), (7.1.5b) and (7.1.6b), the p.u. matrix form of the q-axis voltage equations is:

\[
0 = I_{2d} R_{2d} + p(L_d K_{a2} I_{1d} + L_{d 21} I_{1d} + L_d I_{2d})
\] (7.1.6a)

or

\[
0 = pK_{a2} I_{d} + pK_{21d} I_{1d} + (p + \frac{1}{T_{2d}}) I_{2d}
\] (7.1.6b)
7.1.2 \( d \)-axis voltage equations with the field circuit closed

Under stationary condition, and with the field circuit of Fig. 2.3 closed, the voltage equations are:

\[
E_1 = (pL_{d} + r_d)I_d + pL_{a1}I_{a1} + pL_{a2}I_{a2} + pL_{af}I_{fd} \tag{7.1.12}
\]

\[
0 = pL_{1d}I_d + (pL_{11d} + r_{1d})I_{1d} + pL_{12d}I_{2d} + pL_{1fd}I_{fd} \tag{7.1.13}
\]

\[
0 = pL_{2ad}I_d + pL_{21d}I_{1d} + (pL_{22d} + r_{2d})I_{2d} + pL_{2fd}I_{fd} \tag{7.1.14}
\]

\[
0 = pL_{fad}I_d + pL_{f1d}I_{1d} + pL_{f2d}I_{2d} + (pL_{ffd} + r_{fd})I_{fd} \tag{7.1.15}
\]

By dividing each equation by \( e_{d0} \) and multiplying equations (7.1.13), (7.1.14) and (7.1.15) by \( \frac{L_{d0}}{\sqrt{L_{1d}^2}}, \frac{L_{d0}}{\sqrt{L_{2d}^2}}, \text{ and } \frac{L_{d0}}{\sqrt{L_{fd}^2}} \) respectively, we obtain

the p.u. matrix form of these equations:

\[
\begin{bmatrix}
E_1 \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
pL_{q} & pL_{K}a1q & pL_{K}a2q \\
pK_{a1} & p + \frac{1}{T_{1q}} & pK_{21q} \\
pK_{a2} & pK_{21q} & p + \frac{1}{T_{2q}}
\end{bmatrix}
\begin{bmatrix}
I_q \\
I_{1q} \\
I_{2q}
\end{bmatrix}
\tag{7.1.11}
\]
From equation (7.1.16) the p.u. flux linkage equations are:

$$
\psi_d = L_d^K a1d i_1d + L_d^K a2d i_2d + L_d^K afd i_fd + L_d^I i_d
$$  \hspace{1cm} (7.1.17)

$$
\psi_{1d} = L_d^I i_{1d} + L_d^K a2d i_2d + L_d^K afd i_fd + L_d^K a1d i_d
$$  \hspace{1cm} (7.1.18)

$$
\psi_{2d} = L_d^K a2d i_2d + L_d^I i_2d + L_d^K afd i_fd + L_d^K a2d i_d
$$  \hspace{1cm} (7.1.19)

and

$$
\psi_{fd} = L_d^K afd i_fd + L_d^K f2d i_2d + L_d^I i_fd + L_d^K afd i_d
$$  \hspace{1cm} (7.1.20)

where:

$$
K_{f1d} = \frac{L_{f1d}}{\sqrt{L_{ffd} L_{11d}}} , \quad K_{afd} = \frac{L_{afd}}{\sqrt{L_{ffd} L_d / 3}} \quad \text{etc.}
$$
APPENDIX 7.2

LINEAR REGRESSION (METHOD OF LEAST SQUARES)

The decrement curve of Fig. 7.2.1 can be replaced by a straight line, by either of the following methods:

a) A straight line starting from point $p$ and passing through one point on the curve, such as $C_A$, the coordinates of which are $t_j, i_j$. If the equation for the curve is:

$$i_{C_A} = Op + mt < 0$$

where $m$ is a slope, we must minimise the function

$$\phi(m) = \sum_{j=1}^{n} d_j^2$$

where $n$ is the number of first points on the decay curve.

Hence for the first ten points:

$$\phi = \sum_{j=1}^{10} [(Op + mt_j) - i_j]^2$$

For $\phi$ to be a minimum $\frac{d\phi}{dm} = 0$, or

$$m \sum t_j^2 = \sum i_j t_j - Op \sum t_j$$

i.e.  \[
\frac{1}{m} = \frac{\sum t_j^2}{\sum i_j t_j - Op \sum t_j} \quad (7.2.1)
\]

where:

$$\sum i_j = \text{the algebraic sum of the currents at the first ten points on the curve.}$$

$$\sum t_j = \text{the algebraic sum of the time intervals at the first ten points.}$$
The time constant of the line can be found from:

\[
T = \frac{0 \rho (1/e - 1)}{m}
\]  

(7.2.2)

b) The method of least squares.

In this method the curve is represented by a straight line which does not need to start from \( p \), but from some other point like \( C_1 \) as shown in Fig. 7.2.2. Hence:

\[
i = mt + C_1
\]

where \( m \) is a slope, and \( C_1 \) a constant.

We must minimise the function \( \phi = \sum d_j^2 \)

\[
\phi = \sum_{j=1}^{n} (mt_j + C_1 - i_j)^2
\]

\[
\therefore \frac{\partial \phi}{\partial m} = 0 = \sum 2 (mt_j + C_1 - i_j)(t_j)
\]

and

\[
\frac{\partial \phi}{\partial C_1} = 0 = \sum (mt_j + C_1 - i_j)
\]

\[
\therefore m \sum t_j^2 + \sum C_1 t_j - \sum i_j t_j = 0
\]

and \( m \sum t_j + \sum C_1 - \sum i_j = 0 \)

i.e. \( m \) and \( C_1 \) are given from these equations.

Hence, for the first ten points:

\[
m \sum t_j^2 + C_1 \sum t_j = \sum i_j t_j
\]
and \( m \sum t_j + 10 C_1 = \sum i_j \)

i.e.

\[
m = \frac{\begin{vmatrix} \sum i_j & \sum t_j \\ \sum i_j & 10 \end{vmatrix}}{\begin{vmatrix} \sum t_j^2 & \sum t_j \\ \sum t_j & 10 \end{vmatrix}}\]  \quad (7.2.3)

and \( C_1 = \frac{\begin{vmatrix} \sum t_j^2 & \sum i_j t_j \\ \sum t_j & \sum i_j \end{vmatrix}}{\begin{vmatrix} \sum t_j^2 & \sum t_j \\ \sum t_j & 10 \end{vmatrix}} \)  \quad (7.2.4)

and the time constant of the line is:

\[
T = \frac{C_1 (1/e - 1)}{m} \]  \quad (7.2.5)
FIG. 7.1.

FIG. 7.2.
A computer program for calculating the transient currents using measured values of time constants and their corresponding magnitude values of current is given in the next page. The program reads the values of time constants and their magnitudes of the current corresponding to the test under consideration. The X- and Y-scales for plotting the results are adjusted to correspond with the actual speed of the U.V. paper and the deflection of the current trace (depending on shunt resistance used and the sensitivity of the galvanometer). This enables direct comparison of calculated plot of transient current and the measured trace obtained on the U.V. recording paper. The shunt resistances used were 0.01 and 0.001Ω in series. The variation in contact resistances between tests was between two limits (0.0005 to 0.0015) resulting in the effective shunt resistance variation from 0.0115 to 0.0125Ω.
LIBRARY(ED, SUBGROUPGINO)
PROGRAM(ALIB)
OUTPUT 2=LPU
INPUT 1=CR
COMPRESS INTEGER AND LOGICAL
TRACE 0
END
MASTER(ALIB)
THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION(3,3)
THE CALCULATED CURVES ARE SHOWN IN FIGURES 7.3 AND 7.4
RESPECTIVELY
REAL IA1, IA2, IA3, IA0
DIMENSION X(1000), Y(1000)
STP=0.0001
CALL LU1984
CALL UNITS(10,
CALL DEVPAD(30, 25, 1)
CALL AXIPOS(1.5, 3, 20, 1)
CALL AXIPOS(1.5, 3, 18, 2)
CALL AXISCA(3, 20, 0, 1.1)
CALL AXISCA(3, 18, 0, 0, 18, 2)
DO 10 K=1, 2
5 T=0.
READ(1, 5) TA1, TA2, TA3, IA1, IA2, IA3, IA0
WRITE(2, 7) TA1, TA2, TA3, IA1, IA2, IA3, IA0
7 FORMAT(9(9F0.9))
DO 20, J=1, 1000
20 T=T+STP
I=IA0-IA1*EXP(-T/TA1)-IA2*EXP(-T/TA2)-IA3*EXP(-T/TA3)
X(J)=I*X0.0125
Y(J)=I+X(T)
DO 20 CONTINUE
CALL AXIDRA(1, 1, 1)
CALL AXIDRA(-1, -1, 2)
CALL GRAPOL(X, Y, 1000)
CALL PICCLE
10 CONTINUE
CALL DEVEN
STOP
END
FINISH

****

DOCUMENT DATA

0.026 0.005 0.0016 2.0 4.2 3.52 9.72
0.0276 0.0052 0.0011 0.6 1.12 0.9 2.62

****
LIBRARY(FD, SUBGROUPINO)
PROGRAM(ALIB)
OUTPUT 2=LP0
INPUT 1=CR0
COMPRESS INTEGER AND LOGICAL
TRACE 0
END
MASTER(ALIB)

FOR THE TEST CURRENT OF 9.15 A, THE TEST IS CARRIED OUT WITH
A DIRECT CONNECTION TO THE SLIP RINGS WITH OUT BRUSHES)
THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION (3:3)
(WITH SUBSCRIPT q BEING REPLACED BY SUBSCRIPT c)
THE CALCULATED CURVES ARE SHOWN IN FIGURES (7.5) AND (7.12)

REAL IA1, IA2, IA3, IA0, I
DIMENSION X(1000), Y(1000)
STP=0.0001
CALL LU1934
CALL UNITS(10.)
CALL DEVPAP(30., 25., 1)
CALL AXIPOS(1, 3., 20., 1)
CALL AXIPOS(1, 3, 3., 18., 2)
CALL AXISCA(3, 20, 0., 1, 1)
CALL AXISCA(3, 18, 0., 0, 18., 2)
DO 10 K=1, 2
T=0
READ(1, 5) TA1, TA2, TA3, IA1, IA2, IA3, IA0
5 FORMAT(9F0.0)
WRITE(2, 7) TA1, TA2, TA3, IA1, IA2, IA3, IA0
7 FORMAT(1X, 9G11.3)
DO 20 J=1, 1000
T=T+STP
I=IA0-IA1*EXP(-T/TA1)-IA2*EXP(-T/TA2)-IA3*EXP(-T/TA3)
X(J)=T
Y(J)=I*0.0115
20 CONTINUE
CALL AXIDRA(-1.1, 1)
CALL AXIDRA(-1.01, -1.2)
CALL GRAPOL(X, Y, 1000)
CALL PICCLE
10 CONTINUE
CALL DEVEN
STOP
END
FINISH

****
DOCUMENT DATA
0.0627 0.007 0.0018 3.31 2.4 5.41 11.02
0.0605 0.0065 0.0018 3.42 2.42 3.31 9.15
****
LIBRARY('ED, SUBGROUP, PGINO')
PROGRAM('ALIB')
OUTPUT 2 = LP0
INPUT 1 = CRO
COMPRESS INTEGER AND LOGICAL
TRACE 0.
END

MASTER('ALIB')
C THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION (3.3)
C (WITH SUBSCRIPT q BEING REPLACED BY SUBSCRIPT d)
C THE CALCULATED CURVE IS SHOWN IN FIGURE 7.6
REAL IA1, IA2, IA3, IAn, 1
STP=0.0001
CALL L1934
CALL UNITS(10.)
CALL DEV PAP(30., 25., 1)
CALL AXIPOS(1, 3, 3, 20., 1)
CALL AXIPOS(1, 5., 3, 18., 2)
CALL AXISCA(3, 20., 0...2; 1)
CALL AXISCA(3, 18., 0...0, 18., 2)
T=0.
READ(1,5)TA1, TA2, TA3, IA1, IA2, IA3, IA0
5 FORMAT(0F0.0)
WRITE(2,7)TA1, TA2, TA3, IA1, IA2, IA3, IA0
7 FORMAT(1X, 9G11.3)
DO 20 J=1, 2000
T=T+STP
I=IAN-IA1*EXP(-T/TA1)-IA2*EXP(-T/TA2)-IA3*EXP(-T/TA3
X(J)=1*O.012
Y(J)=1*0.012
CONTINUE
CALL AXIDRA(1, 1, 1)
CALL AXIDRA(-1, -1, 2)
CALL GRAPOL(X, Y, 2000)
CALL DEVEND
STOP
END

FINISH

***
DOCUMENT DATA
0.07 0.0065 0.0017 1.66 2.02 3.12 6.8
***
LIBRARY(ED, SUBGROUP GINO)
PROGRAM(ALIB)
OUTPUT 2 = LP0
INPUT 1 = CP0
COMPRESSION INTEGER AND LOGICAL
TRACE 0
END
MASTER(ALIB)

C THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION (3.43)
C FOR THE TEST CURRENT OF 9.15 A, THE TEST IS CARRIED OUT WITH
C A DIRECT CONNECTION TO THE SLIP RINGS (WITH OUT BRUSHES)
C THE CALCULATED CURVES ARE SHOWN IN FIGURES (7.7) AND (7.14)
REAL IA1S, IA2S, IA3S, IA4S, IA0, I
DIMENSION X(1000), Y(1000)
STP=0.0001
CALL L01934
CALL UNITS(10,)
CALL DEV PAP(30, 25, 1)
CALL AIXPOS(1, 3, 3, 20, 1)
CALL AIXPOS(1, 3, 3, 18, 2)
CALL AIXSCA(3, 20, 0, 1, 1)
CALL AIXSCA(3, 18, 0, 0, 1, 2)
Do 10 K=1, 2
T=0.
READ(7, 5) TA1S, TA2S, TA3S, TA4S, IA1S, IA2S, IA3S, IA4S, IA0
5 FORMAT(9F9.0)
WRITE(2, 7) TA1S, TA2S, TA3S, TA4S, IA1S, IA2S, IA3S, IA4S, IA0
7 FORMAT(1X, 9G11.3)
DO 20 J=1, 1000
T=T+STP
I=IA0+IA1S*EXP(-T/TA1S)+IA2S*EXP(-T/TA2S)+IA3S*EXP(-T/TA3S)-
2*IA4S*EXP(-T/TA4S)
X(J)=T
Y(J)=1+0.0115
20 CONTINUE
CALL AXIDRA(1, 1, 1)
CALL AXIDRA(-1, -1, 2)
CALL GRAPOL(X, Y, 1000)
CALL PICCLE
10 CONTINUE
CALL DEVEND
STOP
END
FINISH

*** DOCUMENT DATA ***

0.2 0.0215 0.0043 0.0018 0.86 1.2 4.0 5.03 11.09
0.202 0.023 0.004 0.0017 0.88 1.1 4.5 2.67 9.15

***
LIBRARY(ED, SUBGROUPGINO)
PROGRAM(ALIB)
OUTPUT 2=LP0
INPUT 1=CRU
COMpress INTEGER AND LOGICAL
TRACE 0
END
MASTER(MHOS)
THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION(3.43)
THE CALCULATED CURVE IS SHOWN IN FIGURE(7.8)
REAL IA1S, IA2S, IA3S, IA4S, IAO, 1
STP=0.0001
CALL LU1934
CALL UNITS(10., )
CALL DEVPAP(30., 25., 1)
CALL AXIPOS(1,3, 3, 20., 1)
CALL AXIPOS(1,3, 3, 18., 2)
CALL AXISCA(3, 20, 0, .2, 1)
CALL AXISCA(3, 18, 0, 0.18, 2)
T=0.
READ(1,5) TA1S, TA2S, TA3S, TA4S, IA1S, IA2S, IA3S, IA4S, IAO
5 FORMAT(9F0.0)
WRITE(6, 7) TA1S, TA2S, TA3S, TA4S, IA1S, IA2S, IA3S, IA4S, IAO
7 FORMAT(1X, 2G11.3)
DO 20 J=1, 2000
T=T+STP
I=IAO-IA1S*EXP(-T/TA1S)-IA2S*EXP(-T/TA2S)-IA3S*EXP(-T/TA3S)-
2IA4S*EXP(-T/TA4S)
X(J)=T
Y(J)=I*0.012
20 CONTINUE
CALL AXIDRA(1,1,1)
CALL AXIDRA(-1, -1, 2)
CALL GRAPOL(X, Y, 2000)
CALL DEVEND
STOP
END

****
DOCUMENT DATA
0.2 0.017 0.0045 0.0017 0.5 0.75 3.1 2.45 6.8
****
LIBRARY(ED, SUBGROUPINO)
PROGRAM(ALIB)
OUTPUT 2=LPO
INPUT 1=CRO
COMPRESS INTEGER AND LOGICAL
TRACE 0
END
MASTER(ALIB)

C
THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION (3.3)
C
THE CALCULATED CURVES ARE SHOWN IN FIGURES 7.9 AND 7.10
C
RESPECTIVELY
REAL IA1, IA2, IA3, IA0, I
DIMENSION X(1000), Y(1000)
STP=0.0001
CALL LU1934
CALL UNITS(10.)
CALL DEV PAP(30., 25., 1)
CALL AXIPOS(1, 5., 3., 20., 1)
CALL AXIPOS(1, 5., 3., 18., 2)
CALL AXISCA(3, 20., 0., 1., 1)
CALL AXISCA(3, 18., 0., 0., 18., 2)
DO 10 J=1, 1000
T=0.
READ(1, 5) TA1, TA2, TA3, IA1, IA2, IA3, IA0
FORMAT(9F0.0)
WRITE(2, 7) TA1, TA2, TA3, IA1, IA2, IA3, IA0
FORMAT(1X, 9F11.3)
DO 20 J=1, 1000
T=T+STP
I=IA0-IA1*EXP(-T/TA1)-IA2*EXP(-T/TA2)-IA3*EXP(-T/TA3)
X(J)=T
Y(J)=I*0.012
20 CONTINUE
CALL AXIDRA(1, 1, 1)
CALL AXIDRA(-1., -1., 2)
CALL GRAPOL(X, Y, 1000)
CALL PICCLE
10 CONTINUE
CALL DEVEND
STOP
END
FINISH

****
DOCUMENT DATA
0.03225 0.00875 0.0025 2.0 2.4 4.82 9.22
0.0295 0.00725 0.002 1.25 1.88 1.58 4.71
****
LIBRARY(ED, SUBGROUP, GDNO)
PROGRAM (ALIB)
OUTPUT 2=LP0
INPUT 1=CR0
COMPRESS INTEGER AND LOGICAL
TRACE 0
END
MAST E(ALIB)
C THE TEST IS CARRIED OUT WITH A DIRECT CONNECTION
C TO THE SLIP RINGS (WITHOUT BRUSHES)
C THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION (3.3)
C (WITH SUBSCRIPT q BEING REPLACED BY SUBSCRIPT 0)
C THE CALCULATED CURVE IS SHOWN IN FIGURE (7.11)
REAL IA1, IA2, IA3, IA0, I
DIMENSION X(1000), Y(1000)
STP=0.0001
CALL LU1934
CALL UNITS(10.)
CALL DEV PAP(30., 25., 1)
CALL AXIPOS(1., 3., 3., 20., 1)
CALL AXIPOS(1., 3., 3., 18., 2)
CALL AXISCA(3., 20., 0., 1, 1)
CALL AXISCA(3., 18., 0., 0.45, 2)
T=0.
READ (1, 5) TA1, TA2, TA3, IA1, IA2, IA3, IA0
5 FORMAT (5F8.0)
WRITE (2, 7) TA1, TA2, TA3, IA1, IA2, IA3, IA0
7 FORMAT (1X, 9G11.3)
DO 20 J=1, 1000
T=T+STP
I=IA0-IA1*EXP(-T/TA1)-IA2*EXP(-T/TA2)-IA3*EXP(-T/TA3)
X(J)=T
Y(J)=I*0.0124
20 CONTINUE
CALL AXIDRA(1, 1, 1)
CALL AXIDRA(-1,-1, 2)
CALL GRAPOL(X, Y, 1000)
CALL DEVEND
STOP
END
FINISH

****
DOCUMENT DATA
0.059 0.0065 0.0017 4.94 3.55 5.03 13.52
****
LIBRARY(ED, SUBGROUPINO)
PROGRAM(ALIB)
OUTPUT 2=LPO
INPUT 1=CRO
COMPRESSION INTEGER AND LOGICAL
TRACE 0
END

MASTER(SAMB)

THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION(3.43)
THE TEST IS CARRIED OUT WITH A DIRECT CONNECTION
TO THE SLIP RINGS (WITH OUT BRUSHES)
THE CALCULATED CURVE IS SHOWN IN FIGURE(7.13)

REAL IA1S,IA2S,IA3S,IA4S,IA0,1
DIMENSION X(1000), Y(1000)
STP=0.0001
CALL LUVHS4
CALL UNITS(10, )
CALL DEV PAP(30, 25, 1)
CALL AXIPOS(1, 0.3, 1.3, 20, 1)
CALL AXIPOS(1, 0.3, 1.3, 18, 2)
CALL AXISA(3, 20, 9, 1, 4)
CALL AXISCA(3, 18, 0, 0, 45, 2)
T=0.
READ(1, 5) TA1S, TA2S, TA3S, TA4S, IA1S, IA2S, IA3S, IA4S, IA0
5 FORMAT(9F0.0)
WRITE(2, 7) TA1S, TA2S, TA3S, TA4S, IA1S, IA2S, IA3S, IA4S, IA0
7 FORMAT(1X, 9G11.3)
DO 20 J = 1, 1000
T=T+STP
I=IA0-IA1S*EXP(-T/TA1S)-IA2S*EXP(-T/TA2S)-IA3S*EXP(-T/TA3S)
2IA4S*EXP(-T/TA4S)
X(J)=T
Y(J)=I*0.0124
20 CONTINUE
CALL AXIDRA(1, 1, 1)
CALL AXIDRA(-1, -1, 2)
CALL GRAPOL(X, Y, 1000)
CALL DEVEND
STOP
END

***

DOCUMENT DATA

0.2 0.023 0.0045 0.0018 1.24 1.7 4.9 5.68 13.52

***
LIBRARY(ED, SUBGROUPRNO)
PROGRAM(ALIB)
OUTPUT. Z=LP0
INPUT I=CR0
COMPRESS INTEGER AND LOGICAL
TRACE 0
END
MASTER(GPSB)
THE DOCUMENT DATA ARE SUBSTITUTED INTO EQUATION(3.3),
WITH IAO REPLACED BY ZERO AND ALL THE MINUS SIGNS CHANGED
TO POSITIVE SIGNS
THE TEST IS CARRIED OUT WITH A DIRECT CONNECTION
TO THE SLIP RINGS (WITH OUT-INTENSITIES)
THE CALCULATED CURVE IS SHOWN IN FIGURE(7.15)
REAL IA1, IA2, IA3, IAO, I
DIMENSION X(1000), Y(1000)
STP = 0.0001
CALL LUP 1934
CALL UNITS(10.)
CALL DEVS(30. , 29. , 1)
CALL AXPOS(1.3. , 3. , 20. , 1)
CALL AXPOS(1.3. , 3. , 18. , 2)
CALL AXISCA(3.20. , 0. . . 1. , 1)
CALL AXISCA(3.18. , 0. . . 0.18. , 2)
T=0.
READ(1,5) TA1, TA2, TA3, IA1, IA2, IA3, IAO
5 FORMAT(9F0.0)
WRITE(2,7) TA1, TA2, TA3, IA1, IA2, IA3, IAO
7 FORMAT(1X, 9G11.5)
DO 20 J=1, 1000
T=T+STP
I=IAO-IA1*EXP(-T/TA1)-IA2*EXP(-T/TA2)-IA3*EXP(-T/TA3)
X(J)=T
Y(J)=I*0.0115
20 CONTINUE
CALL AXIDRA(1,1,1)
CALL AXIDRA(-1,-1,2)
CALL GRAPOL(X, Y, 1000)
CALL DEVS
STOP
END

****
DOCUMENT DATA
0.03 6 0.007 0.0074 -1.18 -1.6 -2.12 0.0
****
MEASURED BY U.V.R.

CALCULATED BY EQUATION 3.3.

FIG. 7.2

\[ x = 10^{-7} \]
--- MEASURED BY U.V.R.
- - - CALCULATED BY EQUATION 3.3

FIG. 7.4.
CALCULATED BY EQUATION 3.3.

MEASURED BY U.V.R.

FIG. 7.6.
FIG 7.7

MEASURED BY U.V.R.

CALCULATED BY EQUATION 3.43.
MEASURED BY U.V.R.

CALCULATED BY EQUATION 3.43.
MEASURED BY U.V.R.

CALCULATED BY EQUATION 3.3.
FIG. 7.10.

- - - MEASURED BY U.V.R.

--- CALCULATED BY EQUATION 3.3.
CALCULATED BY EQUATION 3.3

MEASURED BY U.V.R.
MEASURED BY U.V.R.

CALCULATED BY EQUATION 3.3.
MEASURED BY U.V.R.

CALCULATED BY EQUATION 3.43.