NOx formation in gas-fired pulse combustors

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NOx Formation in Gas-Fired Pulse Combustors

by
Hok Wang AU-YEUNG
BSc, MSc, DIC

A Doctoral Thesis
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Supervisors:
Dr. C P Garner
Department of Mechanical Engineering

Prof. V I Hanby
Department of Civil and Building Engineering

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Abstract

The main focus of this investigation was to get a greater understanding of the effect of combustion frequency, positive pressure amplitude, relative air:fuel ratio ($\lambda$), water jacket temperature and input firing rates on the emissions of NO from pulse combustors. This study was carried out by a programme of experimental work combined with the development of a one-dimensional model.

Results obtained in this study from experimental measurement, revealed evidence that a Schmidt tube has the ability to operate over a wide range of parameters (such as operating frequency, positive pressure amplitude, relative air:fuel ratio, water jacket temperature and input firing rates) with variable NO emissions. It was found that the level of NO emissions became lower with increasing operating frequency and positive pressure amplitude. As an example, when the rig was operated at input firing rate 25 kW and a positive pressure amplitude of 0.12 bar, increasing the frequency from 35 Hz to 73 Hz produced a monotonic reduction in NO emissions from 61 ppm to 29 ppm (dry, 3% $O_2$). An increase in positive pressure amplitude from 0.05 to 0.12 bar produced a change in NO emissions from 46 ppm to 34 ppm. It was also found that the values of NO emissions fell with increasing excess air for $\lambda$ > 1.1. However, NO emissions increased with increasing water jacket temperature ($T_w$) along the length of tailpipe and with increasing input firing rates.

Experimental results showed that the positive pressure amplitude was not dependent on the wall jacket temperature. However, the operating range of stable pressure oscillation could be extended from $1 \leq \lambda \leq 1.3$ to $1 \leq \lambda \leq 1.5$ when the controlled wall temperature along the length of tailpipe was altered from 55 °C to 75 °C.

A model was developed to study NO formation in valveless Schmidt type pulse combustors. The computer model was based on a one-dimensional approach, combined with sub-models of an inlet valve and a two-step kinetic combustion model with rate equations for the formation of prompt and thermal NO. Convective and radiative heat transfer to the tube walls was also incorporated in the model. The inlet valve was represented by a quadratic pressure drop/flow rate equation with a high discharge coefficient in the reverse flow direction; hence inward flow of fresh fuel:air mixture could only occur during the negative pressure part of the cycle. The frequency of operation was defined by the length of the tube, which was considered to be resonating in a quarter-wavelength longitudinal mode.

The model was run over the same envelope as defined in experimental programme. It predicted that an increased in frequency of operation produced a reduction in total NO formation. The effect of higher combustion frequency in producing lower NO formation rates, due to shorter residence time combined with the reduced volumetric efficiency, was strongly evident. Calculation suggested that prompt NO contributed approximately 6-19% to the total predicted NO formation over a range of input firing rates.

The model also predicted a reduction in NO formation with increasing positive pressure amplitude. Results of the detailed calculations were found to be reasonable agreement with the experimental observations.
Publications from this work


2. Poster presentation at the Institute of Physics Current Research in Combustion Physics: A forum for Research Students and Young Researchers, which was held at the BG Technology Centre, Loughborough, September 1997.


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Nomenclature

- \( f \)  Combustion frequency (Hz)
- \( L \)  Tailpipe length (m)
- \( +\Delta P \)  Combustion positive pressure amplitude (Pa)
- \( \dot{Q}_{\text{fuel}} \)  Input firing rate (kW)
- \( T_w \)  Water jacket temperature (°C)
- \( \lambda \)  Relative air:fuel ratio (-)
- \( \tau \)  Cycle time (s)
- \( R \)  Universal gas constant (J/mole-K)
- \( \text{NOth} \)  Thermal NO (ppm)
- \( \text{NOpr} \)  Prompt NO (ppm)
- \( \frac{d(\text{NOth})}{dt} \)  Thermal NO formation rates (mole/sec-cm³)
- \( \frac{d(\text{NOpr})}{dt} \)  Prompt NO formation rates (mole/sec-cm³)
- \( \text{P}_{\text{mean}} \)  Mean pressure (bar)
- \( \text{P}_{\text{amp}} \)  Oscillation pressure across combustion chamber (Pa)
- \( \rho_{\text{mix}} \)  Density of the mixtures (kg/m³)
- \( \text{cd} \)  Volumetric flow coefficient (-)
- \( \frac{d[\text{CH}_4]}{dt} \)  The rate of fuel burning (mole/sec-cm³)
- \( [\text{CH}_4] \)  Concentration of fuel (mole/cm³)
- \( \frac{d[\text{CO}_2]}{dt} \)  The rate of carbon dioxide formation (mole/sec-cm³)
- \( [\text{CO}] \)  Concentration of carbon monoxide (mole/cm³)
- \( [\text{O}_2] \)  Concentration of oxygen (mole/cm³)
- \( [\text{O}_2]_e \)  Equilibrium concentration of oxygen (mole/cm³)
- \( [\text{N}_2]_e \)  Equilibrium concentration of nitrogen (mole/cm³)
- \( T \)  Gas temperature (K)
- \( c_v \)  The net specific heat of the fuel (J/kg)
- \( H_R \)  The enthalpy of fuel and air (J)
- \( H_P \)  The enthalpy of products (J)
- \( h \)  Heat transfer coefficient (W/m²K)
Pulse combustion

D  Dimeter of tube (m)
Cp  Specific heat of gas at constant pressure (J/kg-K)
μ  Viscosity of combustion gas (N-s/m²)
ν  Kinematic viscosity of gas (m²s⁻¹)
k  Thermal conductivity of gas (W/mK)
V  Gas particle velocity (m/s)
Vₘ  Steady flow particle velocity (m/s)

\( \bar{h} \)  Time averaged heat transfer coefficient (W/m²K)
B  Dimensionless pulsation velocity (-)
\( h₀ \)  Heat transfer coefficient without oscillation (W/m²K)
hr  Effective radiation coefficient (W/m²K)
qr  Radiation heat transfer (W)
σₐ  Stefan-Boltzmann constant (W/m²K⁴)
αₐ  Absorptivity of gas (-)
εₐ  Emissivity of gas (-)
T wi  Inner wall temperature (K)
α  Oxygen reaction order (-)
β  Fuel reaction order (-)
σ  Standard deviation of peak pressure amplitude (bar)
ω  Angular frequency (s⁻¹)
1 Introduction

1.1 Background of this work

Pulse combustion heating devices have many advantages over conventional systems. These include high thermal efficiency, high heat transfer and the ability to self-aspirate. Another major advantage is their lower NO emissions e.g. 34-36 ppm NO emissions in pulsating devices compared to 58-138 ppm in conventional boilers [1.1][1.2]. These advantages have encouraged the development of a number of pulse combustion devices such as space and water heaters. However, the development of such devices has been based mainly upon trial-and-error experiments since knowledge of the underlying physical mechanisms is limited. The present research was aimed at improving the understanding of the mechanisms of pulse combustion, both theoretically and experimentally, particularly with regards to NO formation and emissions.

The fundamental processes found in pulse combustion are both physical and chemical and include chemical kinetics, fluid mechanics and transport processes. Once these are understood, the characteristics of pulse combustors with regards to operation pressure, combustion frequency, temperature, fuel-air ratio and gas velocity etc. may be determined. Greater fundamental knowledge would provide the ability to improve the performance of existing pulse combustors and reduce the development time of new systems. The knowledge can also encourage the development of new applications of pulse combustion.
In this first chapter, we consider the development of pulse combustors during this century. The thesis objectives and overview of its structure are then presented and finally the achievements of the present work are summarised.

1.2 The development of pulse combustors

The exact details of how a pulse combustor operates will be discussed in chapter 2. Here, the important milestones leading to the current study are discussed.

Initial experimental investigations concerning the use of pulse combustion in gas turbine and jet propulsion engines began at the start of the 20th century [1.3]. This work led to the pulse jet, a device which was used as the propulsion unit for the German V-1 flying bombs in the 1939-45 war. Despite the success of the V-1 bombs, interest in the pulse jet engine halted at the end of the war and remained essentially dormant for approximately two decades.

In the early 1960's the potential commercial benefit of pulsation techniques in the energy generation industry was highlighted and interest in the combustion process was renewed. It was at this time that Francis et al [1.4] initiated a series of investigations into the working mechanisms of pulsating devices and their potential in industrial applications. A large number of experiments, such as functional tests to measure the effect of burner geometry on combustion frequency and stability, were conducted at the Midland Research Station. Francis et al found the combustion frequency to be inversely proportional to the length of the exhaust tube. Ideally the exhaust tube length should be such that it is able to resonate as a quarter-wave tube. This would set up a standing pressure wave whereby a pressure antinode would exist near the flame and node at the burner exit. Francis et al [1.4] also reported a sudden frequency “jump” from low to higher values (a “hop mode”) when the exhaust tube length was increased beyond a certain length. It was thought that the combustion frequency increase corresponded to a higher harmonic of oscillation.
In 1969, Hanby [1.5] reported that velocity fluctuation produced by combustion-driven oscillations directly affects convective heat transfer processes in pulse combustion devices. In systems containing a stationary wave (organ pipe), the maximum and minimum heat transfer coefficient values are at the velocity antinode (i.e. at the open end of the tube) and the velocity node (the closed end) respectively. Hanby [1.5], reported high combustion intensity and superior heat transfer rates in these pulsating devices. Such devices can achieve heat transfer coefficient values 2 to 3 times greater than those from a conventional, non-pulsating burners. These findings have encouraged further development of pulse combustion systems and its industrial applications. However, the chemical and physical mechanisms inside the combustion zone are complex and fundamental parameters governing such processes are still some way from being fully understood.

In the 1980's, an experimental research program aiming to enhance the understanding of fundamental processes in pulse combustion was initiated at Sandia National Laboratories, USA. This project was focused primarily on factors which may affect the amplitude of pressure oscillation in combustion chambers and flue gas emissions. A range of laser techniques such as Laser Doppler Velocimetry and two line atomic fluorescence were used to make temporally and spatially resolved gas velocity and gas temperature measurements throughout the combustion chamber and tailpipe. Results revealed no evidence of sufficient reverse flow due to the dynamic velocity of gases from the tailpipe back into the combustion region of the chamber, and this eliminated the hypothesis that mixing of cool tailpipe gases with combustion gases lowers the product temperature and hence the formation rate of NO during combustion. Profiles of product gas velocity in the tailpipe, obtained by John and Keller [1.6], presented here in Figure 1.1, were found to be divided into three regions: region one had a flat velocity profile across the centre line which was essentially the bulk velocity behaviour. The second region exhibited a symmetric profile, while the third region revealed earlier reverse in flow direction, in the flow adjacent to the chamber wall when compared to the core flow of gas (normalized cycle time 0.25 to 0.7). Full profiles of velocity at various cycle times are discussed in greater depth in reference [1.6].
Figure 1.1. Time-resolved velocity profiles across the tailpipe. $t'$ is time normalized by the period of a cycle. The direction of the arrows indicates the direction of the temporal velocity gradient. From [1.6].

The temperature profiles in the combustion chamber produced by Keller [1.7] also depicted three distinct regimes. Initially between a cycle time of zero to approximately 0.6 of cycle time, the gas temperature showed a steady decline due to the injection of cold reactants into the combustion chamber. As the reactant become adequately mixed with hot products in the chamber, ignition occurred and the temperature rose sharply. This took place within 0.6-0.8 of the cycle time. Beyond this period of rapid energy release, the temperature decreased during the final stages of the combustion cycle (normalized cycle time of 0.8-1.0) due to the mixing of the warmed gases with cooler residual products of the previous cycle. These profiles not only increased the understanding of fluid mechanical processes during pulsation, but provided an insight into the mechanisms responsible for the enhanced scalar transport processes and also reinforced the motivation to understand how pulse combustion devices exhibit low levels of NO emission.
Further Sandia [1.7] work concerning the fundamental mechanisms responsible for low NO production in a Helmholtz type pulse combustor concluded that lower residence time at high temperatures and hence low thermal NO levels, is the result of augmented heat transfer rates. The work also suggested that strain rate effects, upon the mixing of reactants with residual products, may also contribute to a lower production rate [1.2]. Studies investigating the effects of fuel composition variation on NO formation have been reported in [1.8]. However, to date, there appears to be no reported work on the effects of combustion frequency, pressure amplitude, relative air:fuel ratio, input firing rates and wall temperature distribution along the tailpipe on NO emissions over a significant operating range.

1.3 Thesis objective

The main purpose of this research was to use a simple quarter wave Schmidt tube pulse combustor to explore the relationship between combustion frequency, pressure amplitude, relative air:fuel ratio, input firing rates, wall temperature distribution along the tailpipe and NO emissions. The work was carried out in two major stages. The first stage was to design a natural gas Schmidt type pulse combustor. The work included the design and construction of the rig and verification of its ability to operate over a wide range of parameters (such as air:fuel ratios etc.) with variable NO emissions. The experimental results obtained in this work provide information to help explain the operating characteristics of quarter wave type combustors and hence can lead to improved performance in terms of efficiency and emissions. The second stage of the work was to develop a one-dimensional mathematical model to predict the NO formation in Schmidt type pulse combustor. The objective here was to simulate the relationship between combustion frequency, pressure amplitude, relative air:fuel ratio, input firing rates, wall temperature distribution along the tailpipe and NO formation. Computer simulated results were then compared with the experimental data.
1.4 Contributions of this work

The significant achievements of this research are summarised as follows:

1. The design, construction and operation of an experimental Schmidt tube pulse combustion system that operates controllably over a wide range of frequencies, amplitudes, air:fuel ratios, firing rates and water jacket temperatures.

2. A large experimental data-set showing the independent effects of combustion pressure amplitude, combustion frequency, tube length, air:fuel ratio, firing rates and wall temperature on NO emissions.

3. A mathematical model incorporating two-step combustion, prompt and thermal NO formation and dynamic heat transfer. The results compare favourably with the experimental findings and provide valuable new insights to the combustion, emission and heat transfer characteristics of pulse combustion.

4. A new understanding of the dominant mechanism promoting low NO emissions from pulse combustors.

This work has addressed a number of important questions raised in the scientific and technical literature. The achievements are considered to be a significant and timely contribution to the understanding of pulse combustors.

1.5 Outline of thesis

The remaining chapters of this thesis are summarised as follows:

Chapter 2. An introduction to pulse combustion and its operating principle is given. Applications of industrial pulse combustors are also discussed.
Chapter 3. The design and construction of a natural gas Schmidt type pulse combustor is described. Each experimental instrument is also individually discussed. In addition, the results of a set of preliminary tests to establish the operation of the unit are reported.

Chapter 4. A series of systematic experimental results are presented. The results from this work provides information to explore the relationship between combustion frequency, pressure amplitude, relative air:fuel ratio, input firing rates and water jacket temperature along the tailpipe on NO emissions.

Chapter 5. A one-dimension mathematical model, incorporating two-step combustion, prompt and thermal NO formation and dynamic heat transfer, is developed. Predicted results are compared with available experimental data.

Chapter 6. In this final chapter, the overall conclusions and some recommendations for future work are given.

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2 Characteristics of pulse combustion and its applications

2.1 Introduction

The phenomenon of combustion driven oscillation was first observed in 1777 by Higgin [2.1] and was explained by Lord Rayleigh in 1878 [2.2], who proposed the hypothesis of driving oscillations by periodic heat release in a gaseous medium. This hypothesis states that for a combustion driven oscillation to occur, a varying rate of heat release in phase with the variation of pressure must be present. When heat release is totally in phase with the pressure oscillation, the pressure amplitude will be driven to high value. This is known as the Rayleigh Criterion. Using these basic ideas, designs have been produced for pulsating combustion in aeronautical engines (e.g. V-1 pulse jet), space heaters, boilers and even for a musical instruments (e.g. the pyrophone). This musical instrument, based upon combustion driven oscillation, consists of a small gas jet in tubes of varying length. When a key is pressed, the jet is ignited and produces a clear note. The pyrophone is on display at the Science Museum, London.

In order to obtain an understanding of the combustion-driven acoustic oscillation phenomenon, different types of pulse combustion burner and more recently its development in industrial applications, are discussed in the following sections.
2.2 Pulse combustion cycles mechanism

Lord Rayleigh [2.2] suggested that combustion-driven acoustic oscillation may be classified into three basic types as summarised below:

1. Longitudinal: The gases move back and forth along the axis of the chamber as like as organ pipe oscillations.
2. Radial: The gases oscillate between the axis of the chamber and the wall.
3. Tangential: The gases oscillate around the perimeter of the chamber.

A combination of these three modes is also possible. The acoustic combustion instability has been found to be in the longitudinal mode in the majority of previous experimental investigations. The field of longitudinal oscillation has been reviewed recently by Putnam [2.4] and Beale [2.5] and further information is found in [2.1], [2.2] and [2.6]. Generally, acoustic driven-oscillatory combustion in the organ pipe behaves as open cycles which is comparable to processes such as intake, compression, power, and exhaust strokes in internal combustion engines. The main stages of such combustion are as follows:

1) During initial firing (i.e. start-up), fuel and air are admitted into the combustion chamber through an inlet system. The initial charge is then ignited by means of a spark plug. This process is illustrated in Figure 2.1:a.

2) As the gas mixture burns, a positive pressure builds up and closes (either acoustically or mechanically) the inlet valve system (Figure 2.1:b). This expansion process in the chamber drives the exhaust gases out through the resonance tube.

3) As time progresses, the inertia of the moving gas column in the exhaust tubes creates a low pressure in the combustion chamber. This acoustically or mechanically opens the inlet valve and thus, more air and fuel are drawn into the combustion zone (Figure 2.1:c).
4) Re-ignition occurs automatically by the hot gases residing in the chamber and the cycle is repeated. The spark plug is no longer required.

Several types of pulse combustion devices such as the quarter standing wave, Rijke, Helmholtz resonator and Reynst tubes are available. The operating processes of all these devices are governed by the same controlling principle of periodic addition of heat in phase with periodic pressure oscillations.

Figure 2.1: a. Starting cycle: the mixture is drawn into combustion chamber and then ignited by means of a spark plug.
Figure 2.1: b. Positive pressure cycle: during combustion, the amplitude of the combustion-generated pressure rise within the combustion zone which acoustically closes the inlet system and drives the exhaust gases out through the resonance tube.

Figure 2.1: c. Negative pressure cycle: the inertia of the tailpipe outflow eventually lowers the pressure in the combustion zone below that of the surroundings with the result that the inlet valve acoustically re-opens and a new charge of mixtures are drawn into the combustion zone.

In standing quarter wave mode, pulse combustion produces pressure and velocity antinodes at the combustion chamber and at the open end of the tube respectively. According to the principle of sound theory [2.2], operation of acoustic-combustion driven oscillations in the standing wave mode, is based on the fundamental
combustion frequency. However, upon increasing the length of a quarter wave tube, (since the frequency is controlled by the length of the tube [2.2][2.7]), the frequency may suddenly jump to a higher value at a critical tube length. The frequency at this length represents the next-possible harmonic of combustion frequency which the pulsation device may be operated (i.e. a "Hop mode"). The longitudinal distribution of pressure and velocity amplitudes, occurring at the fundamental and first harmonic frequencies, is shown in Figures 2.2:a and b respectively.

![Diagram of Quarter Wave Combustor](image)

Figure 2.2:a. Acoustic pressure amplitude and velocity along the quarter wave pulse combustor at fundamental frequency.

![Diagram of Third-Quarter Wave Combustor](image)

Figure 2.2:b. Acoustic pressure amplitude and velocity along the third-quarter wave pulse combustor at first harmonic frequency.

Cheung and Zinn [2.8] reported that this phenomenon is mainly due to the timing and duration of heat release in the pulse combustion process. At relatively high
frequencies (short cycle duration), heat release is primarily in phase with the fundamental pressure oscillation. However, as the frequency of pulsation is decreased, (i.e. a longer cycle time), heat release becomes more in phase with the first harmonic of pressure fluctuation. The system now is excited and therefore the oscillation frequency jumps from fundamental value to first harmonic value. The experimental results are presented in the Figures 2.3:a and b respectively.

![Diagram 1](image1.png)

Figure 2.3:a. Relationship between heat release ($Q'$) and relative timing of pressure oscillation ($P'$) at high frequencies. From [2.8].

![Diagram 2](image2.png)

Figure 2.3: b. Relationship between heat release ($Q'$) and relative timing of pressure oscillation ($P'$) at low frequencies. From [2.8].

In some pulse combustors, different types of inlet valve are fitted in the combustion chamber. If the mechanical inlet non-return valve is omitted, combustors are usually termed valveless or aerodynamically valved pulse combustors. The advantages and disadvantages of these discussed in the next section.
2.3 Type of inlet valve fitted in pulse combustors

Past research on pulse combustors has paid much attention to the design of the combustion chambers and tailpipes in an attempt to increase combustion intensity and heat transfer rates that are associated with the oscillatory behaviour. The outcome of such research has led to the design of, for example, the Helmholtz and Schmidt type burners.

The Schmidt type combustor has a combustion chamber with same diameter as its resonance tube. This type combustor is also known as a quarter wave pulse combustor. In the Schmidt burner, a cylindrical pulse combustion chamber fitted with various valves, is located near the air-inlet. The design of the valves used in such devices defines the basic type of Schmidt burner:

   (1) Mechanical valve Schmidt combustor
   (2) Aerodynamic valve Schmidt combustor
   (3) Valveless Schmidt combustor

Each type of valve has its own advantages and disadvantages in practical applications. These are discussed in the following sub-sections with the aim of selecting the best arrangement for the present work.

2.3.1 Mechanical valve Schmidt combustor

Two types of mechanical valve are shown in Figures 2.4 and 2.5. A single reed valve (Figure 2.4) consists of one or more air holes covered by strips of a thin synthetic rubberised material which are clamped along the inner tube edge. This enables the device to bend freely under the action of pressure differences across the valve. The circular reed which is fixed in the centre of the apparatus, has a large diameter and has
a limited lift restricted by a rounded or flat face reversible back-stop. According to Francis et al [2.7], paper-gaskets, thin metallic and rubber sheets have been tested for construction of valve reeds.

Francis et al [2.7] reported that the design of a mechanical air-inlet valve with a suitable reed is a major problem with pulse combustor design. Paper gasket material reeds vibrating at 50 Hz caused the material to develop preferential deformation under prolonged usage. This leads to reduced efficiency and eventual failure. Noise caused by the operation of metallic discs are an additional disadvantage. Reeds in the form of vanes are damaged by long-term usage and gradually led to the formation of cracks which caused air leakage. Natural neoprene rubbers possess preferential frequencies and are prone to deformation when used for a long periods. In addition, the function of the gas volume between the air-inlet and valve plate may affect combustion phase oscillations and the duration of the valve open time by producing a cushioning effect on the back flow of combustion products. This could eventually destroy the pulsating combustion mechanism.

The multiple-reed air-inlet mechanical valve has been used for the majority of experimental work to-date. When the fluctuation pressure in the conical volume in this design rise above atmospheric, the reeds press against the plate and cause the valve to shut. However, when pressure in the chamber becomes lower than atmospheric, the reeds lift off the plate re-opening the valve and allowing air to be admitted through the gap. Although this type of valve is sufficient for experimental purposes, the reeds tend to fail by fracture and may cause the unit to function with an incorrect ratio of air and gas. Thus, such devices are not suitable for continuous practical applications.

In order to overcome material fatigue, deformation and incorrect air-fuel ratios, further types of Schmidt burners are now considered.
2.3.2 Aerodynamic valve Schmidt combustor

Aerodynamic valves may be used to eliminate the fatigue problem associated with reed valves. Advantageous features of aerodynamic valves includes their capability to operate at elevated temperatures, their tolerance to erosion from particles in the inlet.
Pulse combustion

air and the absence of moving parts. Thus the device eliminates the inherent defects common in all mechanical valves.

One kind of aerodynamic valve is known as a Borda mouth which is used to measure the influence of air-inlet lengths on burner frequencies. The Borda mouth consists of a short length of open, straight pipe with a smaller diameter than the resonance tube, as shown in Figure 2.6.

![Figure 2.6. Aerodynamic valve Schmidt combustor.](image)

The small diameter inlet forces the flame to remain enclosed in the combustion chamber at low gas rates. However, when the gas flow rate is increased, a large amount of backflow of combustion products into inlet passage presents a problem in which the flame emerges from the air inlet and destroying the pulsating mechanism. Although aerovalves have been designed successfully to emulate a non-return valve, the air inlet passage design causes difficulties for experimental measurement purposes and for practical applications.
2.3.3 Valveless Schmidt combustor

This valveless burner does not employ mechanical or aerodynamic valves, but does require a steady supply of fuel and air. The basic geometry of such a combustion device is shown in Figure 2.7.

![Diagram of Valveless Schmidt Combustor]

Figure 2.7. Valveless Schmidt combustor.

To date, work with this type of burner has been used to investigate pressure amplitude oscillations and other parameters with steady fuel-air supplies. This type of inlet-arrangement was chosen for the present study because its design contains the following features:

(a) this rig can be operated at different fuel-air ratios without changing parameters such as the air inlet geometry

(b) the combustion frequency and positive pressure amplitude can be varied independently (see later)

(c) the fuel injector can be moved axially, thus altering the location of the heat release region

(d) the combustor will function without mechanical valves

(e) the simple geometry eases mathematical modelling
2.4 Industrial application of pulse combustors

Due to the practical advantages of pulse combustors (high intensity, high heat transfer, self-aspiration and low emissions), pulsating combustors have been considered for commercial application in several areas such as propulsion [2.9], power generation, heating and drying [2.10]. The following subsection briefly discusses industrial applications of pulse combustors. More detail on the practical applications of pulse combustion can be found in references [2.1], [2.10] and [2.11].

2.4.1 Water heaters and steam boilers

The majority of pulse combustors employed in industry are used in space and water heating machines. Such applications generally require pulse combustion devices to be immersed in heated media, allowing heat to transfer through the walls of the combustor. Examples include the storage water heater and a steam boiler shown in Figures 2.8 and 2.9 respectively. Zinn [2.1], [2.10] stated that heating efficiencies of 95% in storage water heaters and 98% in steam boilers were obtained. When compared with non-pulsation combustors, efficiencies of pulse burners have been found to be high.

Fulton Boiler Ltd. produce a Helmholtz pulse combustor boiler that has both contained high efficiency and low NOx emissions. For example, NOx as low as 35 ppm meets latest and projected European and International standards for NOx emissions. More details about Fulton Boiler information is presented in Appendix A3.3.
Figure 2.8. Structure of a typical storage water heater. From [2.10].

Figure 2.9. Structure of a steam boiler. From [2.10].
The design of pulsating devices in water heaters is based on earlier designs of the Canadian "Pulsamatic" and "Swing Fire" pulse heaters; the latter was developed in Germany in the early 1950's. The German "Swing Fire" was used in the rapid heating of fire trucks, small buildings, tents, water and de-icing. When pulse combustors such as the "Swing Fire" are operated under full power, large amplitude pressure oscillations inside their combustors are produced. This leads to a large exhaust flow velocities at the units exit. These velocities have been used to drive small turbines which in turn propel fans in heat exchangers to circulate air through the device when the unit was used for space heating.

2.4.2 Baking oven

In recent years a growing interest in the application of pulse combustors in commercial equipment has led to the development of a pulse burning oven in Japan. A schematic of such a device is shown in Figure 2.10.

![Figure 2.10. Structure of a gas oven. From [2.10].](image_url)
Periodic combustion processes in the baking oven generates a hot exhaust flow which passes through a mult-heat exchange pipe. This leads to a large amount of heat transfer from heat-exchanger-pipe to the oven space. As a result, this system is capable of cooking food rapidly.

### 2.4.3 Fogging generator

The larger Pulse-combustor fogging units are usually vehicle mounted, the smaller ones are normally hand-held. A small hand-held pulse-combustor fogger is illustrated in Figure 2.11. Usually the pulse combustors are gasoline fuelled with flapper-valve-controlled air inlets. The fogging liquid is normally introduced into the pulse-combustor tailpipe which in turn vapourises due to the heat transfer along the tailpipe. This vapourised liquid is then injected through the tailpipe to kill insects, pests, fungus diseases and bacteria.

![Figure 2.11. A “Swingfog” pulse combustor, hand-held, fogging unit manufactured by Motan Gmbh (courtesy Motan Gmbh). From Kentfield [2.11].](image-url)
2.4.4 Air heater

Figure 2.12. A Lennox domestic, pulse combustor, warm-air, house heater. From [2.11].

Figure 2.12 shows, diagrammatically, the main components of a domestic air heaters pulse combustor which is manufactured by the Lennox company. Such applications require an electric motor fan to drive the cool air passes through a finned, condensing, heat exchanger. This leads to a large amount of heat transfer from heat-exchanger to the cool air. As a result, this cool air is heated and subsequently warms the building.

Kentfield [2.11] stated that the average energy conversion efficiency in this type of air heating system is in the region of 94 to 96 percent. Such high efficiencies are sufficient to condense much of the water vapour contained in the products of combustion. This situation allows the exhaust pipe carrying the products of combustion out of the building to be made from PVC tubing without risk of it melting.
2.5 Conclusion

A literature survey into characteristics of pulse combustion has been conducted. Past studies have revealed that the acoustically driven combustion oscillations are governed by the magnitude of heat release in phase with the pressure fluctuation (the Rayleigh Criterion [2.2]). In most cases, such acoustic combustion was found to be a longitudinal mode.

Three different types of valve have been reviewed. Due to potential material fatigue, deformation and problems related with incorrect air:fuel ratio, mechanical and aerodynamic valves were considered to be not suitable for the present experimental purposes. Hence, a valveless Schmidt type pulse combustor was chosen for the present study.

The high commercial value of pulsating devices for industrial applications, due to its high heating efficiencies such as 95% in storage water heater and 98% in steam boilers, is strongly evident. This has encouraged further investigations, such as the present work, into the operation of this type of combustion system.

In the next chapter, the design and construction of a new pulse combustor and the associated experimental apparatus will be discussed in detail.

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3 Design of a premixed natural gas Schmidt type pulse combustor

3.1 Introduction

The ability to measure variables affected by combustion oscillation was a primary consideration in the design and construction of the Schmidt type pulse combustor. Other factors affecting the design included the need to facilitate mathematical modelling to allow for comparison between the computer data and experimental results. Thus, the following parameters were considered during the design and construction of combustor:

1. It was decided that the Schmidt tube would have an internal diameter of 50 mm. This dimension conforms with previously reported designs which had operated satisfactorily and would enable the comparison of results (e.g. Hanby [3.1]).

2. To obtain an operational frequency range of 75 Hz to 30 Hz, the tailpipe length needed to be variable from 1.5m to 3m. This is based on a quarter-wave “organ pipe” principle as discussed later in section 3.2.3.

3. A water cooling jacket along the length of the tailpipe would ensure accurate control of temperatures and enable energy balance calculations to be conducted.
4. Oscillation pressure amplitude in the combustion chamber would be varied by damping tubes (see later) independently of other variables such as air-fuel ratio and oscillation frequency.

5. The ratio of air and fuel flow rates to be manually controlled.

6. Pressure oscillations and gas temperature to be measured along the tube.

7. A computer controlled exhaust gas analyser to be used to measure the combustion products (e.g. NO, O₂ etc.).

Information regarding this design was obtained through a survey of previous studies which included both experimental finding and related theories. Factors concerned with such information are discussed in the following sub-section.

3.2 Factors that affect the pulse combustor design

Twelve basic operational parameters, as listed below, are of interest in the design of the Schmidt type pulse combustor.

(1) Gas input rate
(2) Geometry of combustion chamber
(3) Operation combustion frequency
(4) Oscillation pressure amplitude
(5) Damping tube design
(6) Water jacket design
(7) Dimension of air line
(8) Dimension of nozzle
(9) Variable length of flanged section design
(10) Material selection
3.2.1 Gas input rate

The heat output of the pulse combustor may be controlled by the input gas flow rate. The design of the pulse combustor should enable the burner to function with a gas input rate range of 5 kW to 25 kW in order to compare experimental results with Hanby [3.1][3.2]. Details from [3.3][3.4] show that many commercial appliances nominally run in the order of 15 kW. For example, [3.5] reported that a space heater application required a heat output of 5.8 kW while a residential furnace application needs heat output within the range of 17 kW and 30 kW. Once the gas input rate has been decided, the sizes of the basic combustor elements (e.g. combustion chamber) can be determined.

3.2.2 Geometry of combustion chamber

Reference [3.5] reported that the range of input gas flow rates may be used as an approximate guide in the identification of a potential basic element size. Unfortunately to-date, precise guidelines and operational data for the selection of combustion chamber sizes in the Schmidt type valveless, pulse combustors are not available. However, experimental measurements from a simple pulse combustor were obtained. Hanby [3.1][3.2] reported results for a quarter wavelength organ pipe of internal diameter 50.8mm with a tailpipe length of approximately 2m. He showed that a positive amplitude pressure oscillation (+ΔP) of 0.3 bar was produced inside the combustion chamber when 17 kW of input propane was supplied. Using these results as a basic guideline, the dimension of the combustion chamber in this present design was decided to be 50.8 mm.
3.2.3 Operating frequency

Papers [3.1][3.2][3.6][3.7][3.8][3.9] and [3.10] reported that the frequency of the fundamental mode of the organ pipe is given by simple acoustic theory:

\[ f = \frac{C}{4L} \]

Where:

- \( f \) \hspace{1cm} \text{Operating frequency, (Hz)}
- \( L \) \hspace{1cm} \text{Tailpipe length, (m)}
- \( C = \sqrt{\gamma RT} \) \hspace{1cm} \text{Speed of sound, (m/s)}
- \( T \) \hspace{1cm} \text{Gas temperature, (K)}
- \( R \) \hspace{1cm} \text{Gas constant, (J/kgK)}
- \( \gamma = \frac{c_p}{c_v} \) \hspace{1cm} \text{Ratio of specific heats, (-)}

Reference [3.1] showed that the operation frequency may be estimated to within an accuracy of 5% when the root mean gas temperature is taken as a basis for computing the speed of sound. For the basic requirement of the rig design, the operating frequency should be altered between 30 to 75 Hz. Thus, the minimum and maximum length of the tailpipe may be calculated from this equation which were found to be 1.5 to 3m respectively.

3.2.4 Prediction of oscillation amplitude of pressure in combustion chamber

Prediction of the pressure oscillation amplitude is dependent upon the dimensions of the pulse combustion chamber, tailpipe and gas input flow rate [3.5]. Data supplied by Hanby [3.1][3.2] reveal that the operating pressure amplitude in the combustion
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chamber of an organ pipe, 50.8 mm in diameter, to be approximately +/- 0.3 bar (gauge value) when operating at approximately 17 kW. These data suggest that the operating pressure in the present rig design is predictable within a firing range of 5 kW to 25 kW gas input flow rate.

3.2.5 Damping tube design

Damping tubes may be mounted on the combustion chamber as shown in Figure 3.1 to act as acoustic filters. They consist of tubes closed at one end with an adjustable piston. By altering the piston position, the combustion pressure amplitude may be altered. Two tubes are used so that both the oscillation pressure amplitude at the fundamental frequency and a further higher harmonic may be attenuated. Thus this system allowed for the Schmidt tube to be operated at any pressure amplitude from quiescent to maximum without altering the basic geometry of the combustor or the frequency of operation. Hanby [3.2] showed that the damping tube length needs to be approximately 1.2 m if 0.3 bar of oscillation amplitude pressure is to be eliminated. Using Hanby's data [3.2], the maximum necessary damping tube lengths in the present design were calculated to be 1.5 m. The internal diameter of the damping tube was designed to be 14 mm with a wall thickness of 5.7 mm.

Figure 3.1. Quarter wave pulse combustor with two damping tubes.
3.2.6 Water jacket design

The role of the water jacket is to keep a constant temperature distribution along the tailpipe. Heywood [3.11] reported that the rate of NO formation is strongly influenced by burnt gas temperature. High temperature and oxygen concentrations lead to high NO formation rates. These results are showed in Figure 3.2.

Figure 3.2. Effect of gas temperature on NO formation rate. From [3.11].

Hanby [3.2] also reported the direct effect of amplitude on the heat transfer coefficient. He found the heat transfer coefficient at the open end of the tailpipe to be approximately twice the value of that at the closed end due to the increase in velocity along the tube length.
The design of the water jacket in the present pulse combustor has been based upon these experimental findings [3.2]. This design allows for both a parallel and counter-flow arrangement. The flow of water in a parallel flow set up is in the same direction as the exhaust gases. The flow of water and net exhaust gas flow arranged to be in the counter-flow. This arrangement allows for the testing of gas temperature distribution along the tailpipe at various intervals. Thus the effect of varying gas temperature along the tailpipe can be found.

The water jacket dimension is dependent on water flow rate, initial water temperature and the amount of heat transferred into the water from the input gas. The minimum water jacket diameter in the present design was calculated to be approximately 76 mm. Full of calculation details are shown on Appendix A3.1.

3.2.7 Dimension of air tube

The size of the air supply tube is dependent on the diameter of the combustion chamber [3.10]. Experimental results from Francis [3.10] reveal that the maximum air inlet diameter must not be greater than half of the combustion chamber diameter in a Schmidt tube. Above this limit, both ends of the tube effectively become open and cease to function as a quarter wave tube. Using these data as a guideline, the diameter of the air inlet pipe was chosen to be 19 mm.

3.2.8 Dimension of fuel nozzle

The main factor when choosing the nozzle size is the gas input flow rate (refs.[3.2][3.5][3.7][3.10]). Francis [3.10] reported that the diameter of the nozzle must be kept small (up to 2 mm) in order to produce an even distribution of gas and air. This arrangement has been used for all past experimental burners and prototype units [3.10].
3.2.9 Variable length of flanged section design

Flanged sections of tubes with water jackets were incorporated in the pulse combustor design to allow changes in the tailpipe length and therefore operating frequency. Ten specially selected tube section lengths enabled the tailpipe length to be increased from 0.6 to 3m in 50mm increments by connecting them together in different combinations [see Appendix A3.2]. Various sampling ports were provided along each of the flanged tube sections. Pressure transducer or thermocouple adapters, would be housed on these sampling ports.

3.2.10 Material selection

Ahrens [3.12] [3.13] showed experimentally that materials used in the construction of pulse combustors containing water jackets, must be resistant to general corrosion. This is because corrosion products can build up inside water jacket cooling channels and directly affect the flow of water and gas temperature along the tailpipe. This could then lead to a series of measurement problems.

Despite the high cost of 316L type stainless steel, it was decided that this alloy grade was a suitable construction material for the present burner. Information supplied by Fulton company Ltd (Appendix A3.3) show that this material possess a high resistance to corrosion attack, low thermal resistance and has a high temperature operating range. This material has been used in the past in the construction of various types of pulsating machines such as the Helmholtz burner. These combustors have proved to be a success and are now used in many types of heating applications such as boilers.
3.2.11 Design of the end section of tailpipe length

.. Rayleigh [3.14] reported that sound vibrations from an organ pipe produce a nominal antinode outside the end of the tube, near to the pipe opening. The distance between the pipe opening and the effective antinode is known as the "end correction" [3.14]. Ref. [3.14] also shows that a constricted opening at the end of a pipe can lower the natural vibration frequency inside the tailpipe and affect the expansion wave propagation in the tube. Thus, it is necessary to include an appropriate end-correction distance at the flanged tailpipe section.

Experimental data from [3.14], show that the minimum end correction distance for a cylindrical pipe, flanged normal to the axis of the pipe, to be 0.82r, where r is the radius of the pipe in inches. The minimum tailpipe end section length extension is based upon this data [3.14]. The end section of the tailpipe length in the present design is 100 mm (> 0.82 r). The assembly drawing is shown in Appendix A3. 2.

3.2.12 Design of the wall thickness of tailpipe and water jacket

In order to minimise the time required for the temperature along the tailpipe to reach equilibrium, the tailpipe was designed with walls of minimum thickness. The tailpipe components were welded together. For stainless steel, welding is best achieved in tubes with a minimum thickness of 5 mm. Thus it was decided that a wall thickness 6.36 mm was a suitable choice for this burner.
3.3 Experimental apparatus set up

Following a year of literature survey, analysis and design, the construction of the valveless Schmidt type pulse combustor was successfully completed and used for a series of experimental measurements. In particular, this included amplitude of pressure oscillation measurement throughout the system, determination of temperature along the tailpipe and the amount of NO emissions, etc. A schematic of the Schmidt type pulse combustor and associated instrumentation is shown in Figure 3.3. The assembly drawing is presented in Appendix A3.4.

![Schematic of experimental apparatus](image)

Figure 3.3. Schematic of experimental apparatus.

There are several basic operational parameters, as listed below, which are of interest in the design of the pulse combustion burner.

3.3.1 Frequency control

The tube length could be varied in the range 0.6m to 3.0m by inserting different combinations of length tube sub-sections as described earlier. This enabled the
combustion frequency to be altered in the range of approximately 30 to 120 Hz if desired.

3.3.2 Pressure amplitude control

As discussed earlier, two adjustable water-cooled “damping tubes” containing a moveable pistons were connected to the pressure anti-node region of the Schmidt tube so that the amplitude of oscillation could thus be controlled independently of other variables such as air-fuel ratio, combustion chamber geometry or oscillation frequency.

3.3.3 Water temperature control

Temperature control along the Schmidt tube was made possible by a water-cooling jacket of internal diameter 88.9 mm and thickness 6.36 mm which extended along the length of the pipe. A rotameter was used to measure the water flow rate inside the jacket while the temperature of the cooling water was controlled by a closed circuit arrangement. (Appendix A3.5).

3.3.4 Air and fuel flow rates control

The pulse combustor was fuelled by commercial grade methane (99.5% pure CH₄) fed from a pressurised gas bottle. Combustion air was supplied via a pressurised system. The flow rate of air was measured upstream of a choked nozzle so that measurements were not affected by the combustion oscillations.
3.4 Experimental instrumentation

The combustion pressure amplitude was measured at the sampling ports using a quartz crystal piezoresistive transducer of Kistler model type 4045A which were housed in a water-cooled adapter. The transducer was connected to the combustion chamber perpendicular to the direction of gas flow. The dynamic pressure signal from the transducer was displayed on an oscilloscope and also digitally recorded by using DT 2805 series software package. (More details of the DT2805 series software-package is presented in Appendix A3.6).

The NO concentration data were recorded using a Signal NO analyser. Sample gases at 10 ppm NO were used for calibration. The NO sample line was heated for half an hour to prevent the water in the combustion products from condensing before reaching the detector. The samples were all extracted via a sampling port 1 m along the tailpipe from the burner inlet. It is important to note that the NO data presented here have been corrected to a dry, 3% O₂ basis and the instrument is accurate to within ±0.1 ppm.

A rotameter was employed for measuring of the air flow rates up to 45.6 m³/h and was calibrated by the manufacturers to an accuracy of +/-2.5% of full scale deflection. Combustion air was supplied via a pressurised system. The air flow rates were measured using a choked nozzle in a similar fashion to the fuel delivery, again to isolate the measurements from the combination oscillations.

The flow rate of the fuel was measured by a rotameter over the range from 0.01 to 4.2 m³/h (at 15°C and 101.3 kPa abs.). The rotameter was calibrated by the manufacturers to an accuracy of +/-2.0% (F.S.D) of indicated flow. This error was considered to have an acceptably small effect on measurements. A large pressure drop was produced in the fuel supply line at the combustion chamber inlet by flow across a nozzle which prevented flow oscillations from entering the supply line.

The air:fuel ratio of the combustion mixtures was calculated by two methods. The first was by measuring the air and fuel rates with rotameters. The second involved
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using the Spindt's method [3.16] to calculate air:fuel ratio from the exhaust gas composition. Comparing these two methods, the difference was found to be a maximum of 5% (typically < 2%) over a range of relative air:fuel ratio from $\lambda=1$ to 1.5. This error was considered to be acceptable. The air:fuel ratios quoted in the rest of the thesis are all based on the rotameters readings.

3.5 Preliminary test results

Sustained operation was achieved by adjusting the gas supply valves which in turn adjusted air:fuel ratios of combustion mixtures. Ideal ignition conditions were obtained when the air supply was set at 5.7 m$^3$/hr, gas supply at approximately 0.75 m$^3$/hr and the gauge pressures at 1.5 bar and 1.03 bar respectively (the gauge pressure acting on the air and the gas were controlled by regulators). The critical range of gas supply for the burner when operating with an air supply of 5.7 m$^3$/hr, (operating on gauge pressure 1.5 bar) was measured using narrow scales. Continuous pulse combustion was replaced by a random pulsating action when the gas supply exceeded 0.75 m$^3$/hr (operating at a gauge pressure of 1.03 bar). This problem was solved by slowly and carefully opening the gas flow valves.

Stable operation within a large range of heat input, between 0.48 m$^3$/hr and 3.0 m$^3$/hr, required optimum values of fuel and air ratios. Gradual opening of gas flow rate valves caused equivalence air:fuel ratios to increase towards the optimum value and thus allowed the stable pulse combustion process to continue at higher heat inputs. Successful pulsating combustion was achieved for a maximum gas flow rate of 3.0 m$^3$/hr. Some preliminary experimental results obtained are discussed in the following section.

3.5.1 Pressure measurement

The presence of a standing quarter wave in the combustor is shown by the pressure distribution curve presented in Figure 3.4. This graph indicates the pressure antinode at the closed end of the tube and pressure node at the open end.
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Relative air:fuel ratio, $\lambda=1$, $L=2.0\text{m}$ and $Tw=55^\circ\text{C}$

Figure 3.4. Positive pressure amplitude (+$\Delta P$) along the tailpipe.

At 10 kW and 15 kW, the acoustic maximum positive pressure (+$\Delta P$) was found to be approximately 0.12 and 0.14 bar respectively inside the combustion chamber. Typical combustion chamber pressure traces are presented in Figures 3.5:a and b.

Figure 3.5:a. Stable pressure trace signal (from DT2805 software package) (tested at 10 kW).

Figure 3.5:b. Stable pressure trace signal (from DT2805 software package) (tested at 15 kW).
A comparison between operation at 10 kW and 15 kW firing rates is illustrated in Figure 3.4. This shows the presence of a higher maximum positive pressure in the combustion chamber at higher input firing rates. This observation implies that the operating pressure is dependent on the input firing rate.

The relationship between the positive pressure amplitude (\( +\Delta P \)) in the combustion chamber and the relative air:fuel ratios is illustrated in Figure 3.6. Curves shown in Figure 3.6, indicate that the highest positive pressure amplitude (measured in the combustion chamber near to the acoustically closed end of the tube) was obtained at relative air:fuel ratios of approximately \( \lambda=1.0 \) and 1.1 for operations at 10 kW and 15 kW respectively. Positive pressure amplitude gradually decreased with increasing relative air:fuel ratio values.

![Graph](image)

Figure 3.6. Positive pressure amplitude (\( +\Delta P \)) in the combustion chamber variation with relative air:fuel ratio.

Figure 3.7 shows that the positive pressure amplitude could be reduced to almost quiescent operation with a damping tube piston position of 160mm. This observation was also reported by Hanby [3.2].
3.5.2 Frequency measurement

Operating frequencies generated by oscillating combustion showed that decreasing the length of tailpipe resulted in higher fundamental frequencies, as illustrated in Figure 3.8. As expected, the combustion frequency was found to be dependent on the overall length of the tailpipe. This observation is supported by basic theory and was also in good agreement with the experimental studies by Francis [3.10] and Hanby [3.15].
A plot of frequency against input firing rates (Figure 3.9) shows that increasing input firing rates from 10 to 25 kW resulted in an increase in combustion frequency. According to the Rayleigh principle [3.14], the frequency of the vibration in a pipe is proportional to the velocity of sound propagation in the gas. Gases burning with higher input firing rates would lead to a higher speed of sound and thus an increase in frequency. The trend is therefore reasonable.

Figure 3.9. Observed change of combustion frequency with input firing rates.

3.5.3 Mean gas temperature measurement

The temporal mean gas temperature in the tailpipe at the centre line was measured by a thermocouple. The results are shown in Figure 3.10 which reveal a drop in temperature along the tailpipe which for the first part is approximately an exponential reduction. However, the slope of the curves becomes steeper towards the ends of the tailpipe which is an indication of a high rate of heat loss at the end of pipe and the introduction of excess air into tailpipe during the reverse flow part of the cycle.
3.6 Conclusions

Preliminary test results obtained from the experimental measurements showed evidence that the Schmidt tube rig has the ability to operate over a wide range of parameters. These preliminary test results also confirm that the design and construction of Schmidt type pulsation combustor was successful.

As expected, the pressure antinode was found in the combustion chamber and pressure node was found at the open end of the tube. Alteration of operating frequency, between 30 Hz to 75 Hz, was achieved by adjusting the tailpipe length. The pressure amplitude in the combustion chamber would be altered by varying the position of the piston inside each damping tube.

In the next chapter, a series of experimental measurements focusing on NO emissions, will be presented. Relationship between the operational parameters such as combustion frequency and oscillation pressure etc., and NO emission will inturn be discussed in detail.

Figure 3.10. Mean gas temperature along the tailpipe of Schmidt combustor.
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4 Systematic experimental measurements

4.1 Introduction

The inherent characteristic of pulse combustors of producing low NO could be due to the rapid combustion process, which allows little time for the NO to form [4.4]. Past work suggests that strain rate effects upon the mixing of reactants with residual products may also contribute to a lower NO production rate [4.1]. However, to-date data and insights on the role of combustion frequency, pressure amplitude, relative air:fuel ratio, input firing rates and wall temperature distribution along the tailpipe on NO emissions are not readily available and the exact mechanisms are not understand.

The main purpose of this section is to provide an overall picture of the NO formation in a gas-fired Schmidt type pulse combustor. The primary means of achieving this aim was by carrying out an experimental programme which isolated the effect of individual factors (such as operating frequency and amplitude etc.) on the formation of NO and hence yielded an indication of such factors which are important in the development of low-emission combustion units.

4.2 NO emissions measurement

A series of experimental measurements which involved independently varying the key operational parameters such as operational frequency, pressure amplitude, air-fuel ratio, input firing rates and water jacket temperature were conducted. The overall test envelope is shown in Table 4.1.
Table 4.1: Experimental Test Envelope

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative air:fuel ratio, $\lambda$</td>
<td>1.0 to 1.5</td>
</tr>
<tr>
<td>Input firing rate, $\dot{Q}_{\text{fuel}}$</td>
<td>10 kW to 25 kW</td>
</tr>
<tr>
<td>Tailpipe length, $L$</td>
<td>1.55, 2.00, 2.55 and 3.00 m</td>
</tr>
<tr>
<td>Combustion frequency, $f$</td>
<td>30 to 73 Hz</td>
</tr>
<tr>
<td>Combustion positive pressure amplitude, $\Delta P$</td>
<td>0.05 to 0.15 bar</td>
</tr>
<tr>
<td>Water jacket temperature, $T_w$</td>
<td>55°C, 65°C and 75°C</td>
</tr>
</tbody>
</table>

The results of the experimental investigation are discussed below.

4.2.1 Relationship between combustion frequency and NO emissions

Measured NO concentrations, (at $\lambda = 1$ and $T_w = 55 ^\circ C$), as a function of frequency are shown in Figures 4.1:a-b. These data show that NO emissions are reduced when the tailpipe length was reduced from 3.0 to 1.55 m (i.e. the combustion frequency was increased). At an input firing rate of 10 kW, NO emissions decreased from 36 ppm to 30 ppm when the frequency was increased from 30 Hz to 59 Hz. Similar behaviour was observed when the rig was operated on 25 kW input firing rates although higher absolute values of NO emissions were obtained.
When the pulse combustor rig was operated at 65 °C and 75 °C respectively, a similar pattern was obtained. The results are presented in Figures 4.2:a-d and Figures 4.3:a-d. The frequency effect shown in these figures may be explained by considering the effect of cycle time during pulsating combustion: At constant air and fuel flow rates, an increase in frequency will result in reduction of the volumetric efficiency in the combustor which leads to a smaller admittance of fresh mixture during each cycle. This may lead to faster mixing with the products of the previous cycle and hence reduce the gas residence time in regions of high temperature and oxygen concentration.
Figures 4.2:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1$ and Tw=65 $^\circ$C).
Figures 4.3a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1$ and $T_w=75$ °C).
Pulse combustion

The effect of frequency on NO emissions is significant over the range of relative air:fuel ratios between $\lambda = 1.0$ and $\lambda = 1.5$ in these experiments (tested at $T_w = 55 \, ^\circ C$). As shown in Figures A4.1.1:a-d, Figures A4.1.2:a-d, Figures A4.1.3:a-d, Figures A4.1.4:a-d and Figures A4.1.5:a-b (in Appendix A4.1), NO emissions are also strongly dependent on the combustion frequency at lean burn conditions, although lower absolute values of NO emissions were obtained at higher relative air:fuel ratio values. A similar behaviour was observed when the water jacket temperature distribution along the tail pipe was operated at 65 °C and 75 °C respectively. These results are presented in Appendix A4.2 (see Figures A4.2.1:a-d to Figures A4.2.8:a-b).

4.2.2 Relationship between combustion pressure amplitude and NO emissions

The relationship between the NO emissions and pressure amplitude is illustrated in Figures 4.4:a-b (tested condition at $\lambda = 1$ and $T_w = 55 \, ^\circ C$). These results show that lower NO emissions were obtained when the pressure amplitude was increased. At 10 kW input firing rate along a 2m long tailpipe, NO emissions decreased from 29 ppm to 26 ppm when the operating pressure amplitude was increased from 0.05 bar to 0.12 bar. A similar behaviour was observed when the rig was operated with various tailpipe lengths and an input firing rate of 25 kW. However, higher levels of NO emissions were obtained at higher firing rates.
Figures 4.4:a-b. Relationship between NO emissions and positive pressure amplitude (+ΔP) at different input firing rates (tested at $\lambda=1$ and $T_w=55$ °C).

When the rig was operated at 65 °C and 75 °C respectively, the same pattern of trends was obtained. The results are graphically summarised in Figures 4.5:a-b and Figures 4.6:a-b respectively.
Figures 4.5:a-b. Relationship between NO emissions and positive pressure amplitude (+ΔP) at different input firing rates (tested at $\lambda=1$ and $T_w=65^\circC$).
Figures 4.6:a-b. Relationship between NO emissions and positive pressure amplitude (+\Delta P) at different input firing rates (tested at $\lambda=1$ and $T_w=75^\circ C$).

At lean burn conditions, a similar behaviour was observed when the rig was operated with different input firing rates and different water jacket temperatures. The results shown in Appendix A4.3 (Figures A4.3.1:a-d to Figures A4.3.14:a-d), are as expected due to the following reasons:

a) It is known [4.5] that the convective heat transfer coefficient is related to the pressure amplitude in the combustor: a higher pressure amplitude produces a higher velocity amplitude in the tailpipe. If flow reversal takes place then the convective film coefficient may be several times the equivalent steady-state value. This will produce a higher rate of temperature drop in the tube and hence inhibit the formation of thermal NO.
b) At higher input rates, the thermal capacity rate of the combustion products is increased and would lead to a lower heat exchanger effectiveness for the tube. The resulting higher gas temperatures would produce a rise in NO emissions.

Results from experimental measurement concluded that NO emissions are inversely dependent on the pressure amplitude. Although these trends were not as marked as the variation with frequency, it is consistent with the known effects of amplitude on the operational characteristics of the Schmidt tube.

4.2.3 Relationship between air:fuel ratio and NO emissions

As shown in Figures 4.7:a-d, Figures 4.8:a-d and Figures 4.9:a-d, NO emissions vary with both the relative air:fuel ratio and input firing rates at various tail pipe lengths. The highest NO emissions were obtained at relative air:fuel ratios of $\lambda = 1.0$ to 1.1. Similar results were observed with higher input firing rates (25 kW). This observation is due to the following factors:

a) Since the NO formation has been proved to be strongly temperature dependent in pulse combustors [4.1], gases burning close to $\lambda = 1.0$ give the highest burned gas temperature and this increases the rate of NO emissions.

b) At higher relative air:fuel ratio ($\lambda > 1.1$), excess air cools the flame which leads to a drop in the burned gas temperature and thus reducing NO formation and emissions.

The observed values show that the thermal NO emissions are strongly dependent on both temperature and oxygen concentrations. The interdependence of these two factors generally produces maximum NO just on the lean side of stoichiometric.
Figures 4.7:a-d. Relationship between NO emissions and relative air:fuel ratio at input firing rates of 10kW and 25kW (tested at Tw=55 °C and possible maximum positive pressure amplitude).
Figures 4.8:a-d. Relationship between NO emissions and relative air:fuel ratio at input firing rates of 10 kW and 25 kW (tested at Tw=65 °C and possible maximum positive pressure amplitude).
Figures 4.9:a-d. Relationship between NO emissions and relative air:fuel ratio at input firing rates of 10 kW and 25 kW (tested at Tw=75 °C and possible maximum positive pressure amplitude).
4.2.4 Relationship between water jacket temperature and NO emissions

The effect of the water jacket temperature on NO emissions are shown in Figures 4.10:a-b to Figures 4.13:a-b. The discrepancy between the values at either end of the curves is an indication that higher levels of NO emissions occur only at higher water jacket temperatures. During operation the temperature distributions along the 2 m tailpipe and between 55 °C to 75 °C, (at 10 kW input firing rate at $\lambda = 1$ and 0.05 bar positive pressure amplitude), the NO emissions increased from 30 ppm to 34 ppm. A similar set of results were obtained when the rig was operated at higher input firing rates. The trends were more clearly marked along a 3m tailpipe length (see Figures 4.13:a-b).

**Figures 4.10:a-b.** Relationship between NO emissions and water jacket temperature at different input firing rates (tested at $\lambda = 1$ and tailpipe length =1.55m).
Figures 4.11:a-b. Relationship between NO emissions and water jacket temperature at different input firing rates (tested at $\lambda=1$ and tailpipe length = 2.0m).
Figures 4.12:a-b. Relationship between NO emissions and water jacket temperature at different input firing rates (tested at $\lambda=1$ and tailpipe length = 2.55m).
Figures 4.13:a-b. Relationship between NO emissions and water jacket temperature at different input firing rates (tested at $\lambda = 1$ and tailpipe length = 3.0m).

This phenomenon may be explained by considering the effect of the heat transfer along the tailpipe: by considering the convection heat transfer equation, $\dot{Q}_{\text{Heat transfer}} = hA(T_g - T_w)$, which states that a higher wall temperature would lead to a lower heat transfer. This in turn would lead to higher product gas temperatures along the tailpipe, causing higher levels of NO emissions.

Measured positive pressure amplitude as a function of water jacket temperature are shown in Figures 4.14:a-b, Figures 4.15:a-b, and Figures 4.16:a-b respectively.
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**Figures 4.14:** a-b. Effect of water jacket temperature on positive pressure amplitude (\(+\Delta P\)) at input firing rates of 10kW and 25kW respectively (tested at \(\lambda=1-1.5\) and tailpipe length =1.55m).

**Figures 4.15:** a-b. Effect of water jacket temperature on positive pressure amplitude (\(+\Delta P\)) at input firing rates of 10kW and 25kW respectively (tested at \(\lambda=1-1.5\) and tailpipe length =2.0mm).
As expected, the positive pressure amplitude is not dependent on the wall temperature. However, the operating range of stable pressure traces could be extended from $1 \leq \lambda \leq 1.3$ to $1 \leq \lambda \leq 1.5$ when the well temperature distribution along the tailpipe was controlled from 55 °C to 75 °C (see Figure 4.17:a-c to Figure 4.19:a-c).

These observations may be examined by the Coefficient of Variation value (COV) which indicates the stability of pressure oscillation in the combustion chamber. The COV value can be expressed as:

$$\text{COV} = \frac{\sigma}{+\Delta P}$$

Where:

$+\Delta P$ Mean peak pressure amplitude (bar)

$\sigma$ Standard deviation of peak pressure amplitude (bar)
Pulse combustion

This is similar to the Coefficient of Variation used to measure cyclic variability of peak pressure in internal combustion engines. In this case, more than 85 cyclic variation of peak pressure amplitude as samples were used to calculate the Coefficient of Variation value in pulse combustion chamber. The method of calculation is presented in Appendix A4.4. The results of COV value are presented in Figures 4.17:a-c to Figures 4.19:a-c.

Figure 4.17:a-c. Effect of water jacket temperature on Coefficient of Variation with relative air:fuel ratio from $\lambda=1$ to $\lambda=1.5$ (tested at $T_w=55$ °C and input firing rates from 10 kW to 25 kW).
Figures 4.18:a-c. Effect of water jacket temperature on Coefficient of Variation with relative air:fuel ratio from $\lambda=1$ to $\lambda=1.5$ (tested at $T_w=65\, ^\circ C$ and input firing rates from 10 kW to 25 kW).
Figures 4.19:a-c. Effect of water jacket temperature on Coefficient of Variation with relative air:fuel ratio from $\lambda = 1$ to $\lambda = 1.5$ (tested at $T_w = 75$ °C and input firing rates from 10 kW to 25 kW).
For example, by increasing the range of the input firing rates from 10 kW to 25 kW, in a 2.0m tailpipe at a water jacket temperature of 55 °C, COV values remain fairly constant between 0.05 to 0.15 when the rig was operated between $\lambda=1$ and $\lambda=1.3$. At $\lambda > 1.3$, the value of COV gradually increased from 0.15 to 0.3. This is indication of unstable pressure signals with irregular sinusoidal wave forms. Similar results were obtained when the wall temperature distribution along the tailpipe was set at 75 °C. However, the value of COV remained on 0.05 to 0.2 when the rig was operated between $\lambda=1$ and $\lambda=1.5$. The oscillation of pressure traces are presented in Appendix A4.5 (see Figures A4.5.1:a-f and Figures A4.5.2:a-f).

It was also found that a similar behaviour was observed when the rig was operated at lengths of 1.55m and 2.55m. Thus, in conclusion, the water jacket temperature not only has effect on the NO emission, but also influences the stability of oscillations when the rig is operated in lean burn condition.

4.2.5 Relationship between input firing rate and NO emissions

Graphs of NO emissions against input firing rates in Figures 4.20:a-d show that NO emissions increased with increasing input firing rates. With 2m long tailpipe with 0.05 bar positive pressure amplitude (tested condition $\lambda=1$ and $T_w=55$ °C), NO emissions increased from 29 ppm to 36 ppm when input firing rates were increased from 10 kW to 25 kW. A similar result was obtained with different tailpipe lengths (see Figures 4.20:a-d), positive pressure amplitude (Figures 4.20:a-d), relative air:fuel ratios (Figures 4.21:a-d, tested at $\lambda=1.2$) and water jacket temperatures (Figures 4.22:a-d and Figures 4.23: a-d, tested at $\lambda=1$).
Figures 4.20:a-d. Relationship between NO emissions and input firing rates over a range of positive pressure amplitude and tailpipe lengths (tested at $\lambda=1$ and $T_w=55^\circ$C).
Figures 4.21:a-d. Relationship between NO emissions and input firing rates over a range of positive pressure amplitude and tailpipe lengths (tested at $\lambda=1.2$ and $T_w=55^\circ C$).
Figures 4.22:a-d. Relationship between NO emissions and input firing rates over a range of positive pressure amplitude and tailpipe lengths (tested at $\lambda=1$ and $Tw=65^\circ$C).
Figures 4.23:a-d. Relationship between NO emissions and input firing rates over a range of positive pressure amplitude and tailpipe lengths (tested at \( \lambda=1 \) and \( T_{w}=75 \, ^{\circ}C \)).
Pulse combustion

This observation may be explained by considering the effect of the fresh mixture volume and the thermal capacity of combustion products: As the tailpipe length is maintained (i.e. constant combustion frequency), the effect of increasing input firing rates will be to increase the volumetric mixture during each cycle, i.e. a larger fresh mixture volume will be admitted into the combustion chamber. This may cause the length of flame to expand leading to an increase in the thermal capacity rate of the combustion products and eventually to lower heat exchanger effectiveness for the tubes. The resulting higher gas temperatures would then produce a rise in NO emissions.

4.3 Experimental results conclusions

Results of experimental investigations concerning the effects of combustion frequency, pressure amplitude, air:fuel ratios, input firing rates and water jacket temperature on NO emissions, were obtained in this study. A large data set was obtained and presented individually in each section. Using this experimental information, several important findings are made. These are summarised as below:

1. The NO emissions from a Schmidt type pulse combustor were found to be inversely related to combustion frequency over a range of input firing rates. At an input firing rate of 25 kW, NO emissions decreased from 56 ppm to 42 ppm when the frequency was increased from 36 Hz to 72 Hz. An indication of further reduction in NO emissions is possible if higher operation frequency can be achieved.

2. Only slight variations in NO emissions were observed with increasing positive pressure amplitude. Results show that lower values of NO emissions occur more readily in high maximum positive pressure amplitudes, although the reduction was not as marked as the variation with frequency.
3. Highest NO emissions were found at relative air:fuel ratios of $\lambda=1.0$ to $\lambda=1.1$ and at highest input firing rates. At input firing rates of 10 kW and 25 kW, the maximum NO emissions were found to be 27 ppm and 34 ppm respectively when the rig was operated on a 2.0 m long tailpipe.

4. The NO emissions from pulse combustors of this type are proportionally related to water jacket temperature distribution along the tailpipe. i.e. NO emissions increase with increasing water jacket temperature.

5. The operating range of stable pressure oscillation traces could be extended from $1 \leq \lambda \leq 1.3$ to $1 \leq \lambda \leq 1.5$ when the wall temperature distribution along the tailpipe was controlled from 55 °C to 75 °C.

In the next chapter, a model is developed which helps to interpret the experimental findings further.

4.4 Chapter 4 references


4.4 J.O. Keller and I. Hongo,
The mechanisms of NOx production, *Combustion and Flame* 80, 1990, pp. 219-237.

4.5 V.I. Hanby,
Convective heat transfer in a gas-fired pulsation combustor,
5 Numerical modelling of pulse combustion

5.1 Introduction

The aim of this chapter is to present the development of a one-dimensional computer model which seeks to simulate the relationship between combustion frequency, positive pressure amplitude, relative air:fuel ratios, water jacket temperature, input firing rates and NO emissions. This mathematical model was combined with a valve sub-model, a two-step kinetic combustion model together with rate equations for the formation of prompt and thermal NO. Convective and radiative heat transfer to the tube walls was also incorporated in the model. More details of individual sub-model are presented and discussed in following sub-section respectively.

5.2 Mathematical modelling

Computer simulation used in this current study was based upon an existing experimental set-up, as shown previously in Figure 3.3. The relevant technical data is listed below:

a) Geometry
Inner tube internal diameter $D_1=50.8\text{mm}$;
Inner tube wall thickness $WT_1=6.35\text{mm}$;
Outer tube internal diameter $D_2=88.9\text{mm}$;
Outer tube wall thickness $WT_2=6.35\text{mm}$;
Tube lengths $L=1.55\text{m}, 2\text{m}, 2.55\text{m}$ and $3\text{m}$.
b) Air and fuel

Air supply temperature = 25°C

Fuel supply temperature = 25°C

The one-dimensional computer simulation used a cylindrical co-ordinate system whereby the geometry around the inlet end was simplified as shown in Figure 5.1. The total grid space and time steps used for this study were 100 and 4000 respectively.

![Diagram](image)

Figure 5.1. 1-D geometry used for computer simulation.

5.2.1 Boundary conditions

a) Pressure

Variation of the measured pressure amplitude inside the pulse combustor with time was shown to be approximately sinusoidal in shape [5.1]. Thus, in the model, the pressure amplitude oscillation at the point of combustion is expressed as:

\[ P = P_{\text{mean}} + (\Delta P)\sin(\omega t) \]
Where \( \omega = 2\pi f \) and \( f \) is the frequency of oscillation. The mean pressure \( P_{\text{mean}} \) based on experimental data was assumed to be 1 bar and the positive pressure amplitude of \( \pm \Delta P \) was adjusted to reproduce the experimentally measured values. Since this is a standing wave in the tailpipe, the spatial variation of pressure (i.e. in the x direction) is considered to be a single cosine function.

b) Fuel and air inlet

The mathematical description of the volume flow rate of mixtures entering the combustion chamber was governed by the pressure drop correlation in the form of

\[
\dot{V} = J \sqrt{\frac{2P_{\text{amp}}}{\rho_{\text{mix}}}}
\]

where:

\[
J = (C_d)A_{\text{orifice}} \quad (m^2)
\]

\( \dot{V} \) Instantaneous flow rates \( (m^3/s) \)

\( \rho_{\text{mix}} \) Density of the mixtures \( (kg/m^3) \)

\( C_d \) Volumetric flow coefficient \((-)\)

\( A_{\text{orifice}} \) Area \( (m^2) \)

\( P_{\text{amp}} \) Oscillation pressure across the combustion chamber \( (Pa) \)

When \( P_{\text{amp}} < 0 \), the valve was assumed to be acoustically open and the inlet of mixtures with flow boundary conditions should be applied. When \( P_{\text{amp}} > 0 \), the valve was assumed acoustically closed and the boundary should be treated as a solid wall, thus \( V \) equals zero.
The value of $J$ was determined by the flow rate ($V$) entering the combustion chamber, density of the mixture ($\rho_{\text{mix}}$) and the value of $P_{\text{amp}}$. The method of calculation is presented in Appendix A5.1.

c) Wall boundary

The combustion chamber end wall of the tube was assumed to be adiabatic and the temperature distributions along the tailpipe was set at a constant value (e.g. 55°C, 65°C or 75 °C). The pressure at the open end of the tailpipe was assumed to be 1 bar.

d) Further assumptions

1. Combustion was assumed to take place at maximum positive pressure amplitude.
2. The overall heat-transfer coefficient $U_o$, was based on the outside surface of inner tube.
3. The energy changes associated with NO formation were small and were therefore not evaluated.

5.2.2 Kinetic combustion model

The computer modelling was primarily focused on the prediction of NO formation. The prediction was achieved by adopting a kinetic combustion model to simulate experimental data. The two reaction schemes were based on the following two-equations [5.2][5.3]:

$$\frac{\partial [CH_4]}{\partial t} = 10^{10} \left( \frac{RT}{P} \right) \exp \left( \frac{-12019}{T} \right) [CH_4][O_2]$$ \hspace{1cm} (1)

$$\frac{\partial [CO_2]}{\partial t} = 10^{10} \left( \frac{RT}{P} \right) \exp \left( \frac{-12019}{T} \right) [CO][O_2]$$ \hspace{1cm} (2)

Where:
Pulse combustion

\[ \frac{d[CH_4]}{dt} \] The rate of fuel burning (mole/cm³ s)

R Universal gas constant (J/mole K)

T Gas temperature (K)

[CH₄] Concentration of fuel (mole/cm³)

P Pressure (Pa)

[CO] Concentration of carbon monoxide (mole/cm³)

[O₂] Concentration of oxygen (mole/cm³)

\[ \frac{d[CO_2]}{dt} \] The rate of carbon dioxide formation at each time step (mole/cm³ s)

The combustion chamber was discretized in the direction of gas flow. Combustion was considered to take place in an infinitesimally thin region at the tube inlet. No flame propagation was considered. This method has been successfully used by Hanby [5.2] to simulate NO formation in boilers.

During combustion, mole fractions of the burned chemical species, CH₄ and CO₂ field, represented the reactants of the two-step reaction mechanisms for methane at each time step and thus were used to update the gas temperature and to calculate the amount of NO formation. A description of the gas temperature calculation, heat transfer along the tailpipe and NO formation are presented in following sub-sections.

5.2.3 Gas temperature sub-model

The gas temperature sub-model provided a flame temperature routine which corresponded to the energy release of tailpipe gases after combustion in each time step. As proposed by several investigators such as Hanby [5.4] and Heywood [5.10], the simulation of adiabatic flame temperature may be calculated by an energy balance. The method of adiabatic flame temperature calculation is discussed in Appendix A5.2.
5.2.4 Heat transfer model

5.2.4.1 Convection sub-model

Heat transfer in a pulsating burner was due to the effects of convection and radiation. Prediction of convective heat transfer occurring between a fluid and a surface was based on empirical correlations for the heat transfer coefficient \( h \).

Modification of convective heat transfer coefficients in pulsating flows principally arises from non-linear relationships between heat transfer coefficients and stream velocities. This treatment assumes that the heat transfer coefficient at a given instant can be calculated from instantaneous values of velocity by means of an appropriate steady-state relationship. If the velocity wave form is known, then the derived instantaneous heat flux can be integrated over one cycle to give a time-averaged heat flux prediction. Data from Mcadams [5.11], yields a convective heat transfer coefficient for turbulent flow in pipes as:

\[
\frac{hD}{k} = 0.0214 \left( \frac{C_p \mu}{k} \right)^{0.4} \left( \frac{VD}{\nu} \right)^{0.8}
\]

Where:

- \( h \) Heat transfer coefficient (W/m\(^2\)K)
- \( D \) Diameter of tube (m)
- \( C_p \) Specific heat of gas at constant pressure (J/kgK)
- \( \mu \) Dynamic viscosity of combustion gas (Ns/m\(^2\))
- \( \nu \) Kinematics viscosity of the gas (m\(^2\)/s)
- \( k \) Thermal conductivity of gas (W/mK)
- \( V \) Gas particle velocity (m/s)
According to Hanby [5.5], the superposition of the particle velocity of a gas in an acoustic wave on to a steady flow rate can be represented by 

\[ V = V_m (1 + B \cos \omega t) \]

where the dimensionless pulsation velocity \( B = \omega A / V_m \), where \( A \) is the gas displacement amplitude. Substituting this in the heat transfer coefficient, equation yields:

\[
h = 0.0214 \frac{k}{D} \left( \frac{C_p \mu}{k} \right)^{0.4} \left( \frac{D}{V} \right)^{0.8} \left( V_m |1 + B \cos \omega t| \right)^{0.8}
\]

The modulus of the term in brackets accounts for the situation when reversal flow occurs while the heat transfer coefficient remains positive, i.e. when \( B > 1 \). In this case, the mean value over 1 cycle was given by:

\[
\bar{h} = 0.0214 \frac{k}{2\pi D} \left( \frac{C_p \mu}{k} \right)^{0.4} \left( \frac{V_m D}{V} \right)^{0.8} \frac{2\pi}{\int_{0}^{2\pi} \left( |1 + B \cos \omega t| \right)^{0.8} d(\omega t)}
\]

Where:

- \( h \) Instantaneous heat transfer coefficient (\( \frac{W}{m^2K} \))
- \( \bar{h} \) Time averaged heat transfer coefficient (\( \frac{W}{m^2K} \))

By dividing the above equation by \( h_0 \) (heat transfer coefficient without oscillation) into the above equation, the heat transfer coefficient improvement ratio becomes:

\[
\frac{\bar{h}}{h_0} = \frac{1}{2\pi} \int_{0}^{2\pi} \left( |1 + B \cos \omega t| \right)^{0.8} d(\omega t)
\]
The result of a numerical integration of this equation, when considering a step of $\pi/50$ is shown in Figure 5.2. The initial heat transfer coefficient improvement ratio was found to decrease before flow reversal was obtained. As the value of $B$ increased beyond the point or initial flow reversal, a linear increase in improvement ratio was predicted.

![Figure 5.2](image.png)

Figure 5.2. Theoretical improvement of heat transfer coefficient by the effect of flow oscillation. From [5.5].

5.2.4.2 Radiative heat transfer sub-model

Development of thermal radiation model in pulse combustion was based on temperature difference, emissivity and absorptivity of the product gases along the
Pulse combustion
tailpipe. A brief description of the radiation heat transfer calculation is given on
following equations:

\[ \text{hr} = \frac{qr}{(T - T_{wi})} \]

\[ qr = \sigma_s (e_g T^4 - \alpha_g T_{wi}^4) \]

where:

\( \sigma_s \) Stefan-Boltzmann constant (W/m\(^2\)K\(^4\))

\( \text{hr} \) Effective radiation coefficient (W/m\(^2\)K)

\( T_{wi} \) Inner wall temperature (K)

\( T \) Mean gas temperature (K)

\( qr \) Radiation heat transfer (W)

\( \alpha_g \) Absorptivity of gas (-)

\( \varepsilon_g \) Emissivity of gas (-)

5.2.5 NO formation model

Development of an NO model for pulse combustor incorporated two types of primary
mechanism of nitric oxide formation: (i) the prompt NO (Fenimore mechanism) and
(ii) the thermal NO (Zeldovich mechanism) were considered.

The mechanism of prompt-NO formation was first proposed by Fenimore, who
concluded that NO formation in the reaction zone involves highly reactive
hydrocarbon radicals. Hence, this reaction applies only to high gas temperatures. The
equation below has been modified for natural gas with low and high temperature
flames [5.2][5.3][5.6].

\[ \frac{\partial [NO] \partial \alpha}{\partial A} = f_c A_T^* \left[ O_2 \right]^{\alpha} \left[ N_2 \right]^{\beta} \left[ CH_4 \right]^{\gamma} \text{EXP} \left( \frac{-E}{RT} \right) \]
Pulse combustion

Where:

\[ \begin{align*}
A & \quad \text{Pre-exponential factor (cm}^3 / \text{mole})^{n-\alpha}\beta \\
n & \quad \text{Constant (-)} \\
\alpha & \quad \text{Oxygen reaction order (-)} \\
\beta & \quad \text{Fuel reaction order (-)} \\
[CH_4] & \quad \text{Concentration of fuel (mole/cm}^3\) \\
[O_2] & \quad \text{Concentration of oxygen (mole/cm}^3\) \\
\bar{R} & \quad \text{Universal gas constant (kJ/moleK)} \\
T & \quad \text{Gas temperature (K)} \\
E & \quad \text{Overall activation energy (kJ/mole)} \\
[N_2] & \quad \text{Concentration of nitrogen (mole/cm}^3\) \\
\frac{\partial [NO]_p}{\partial t} & \quad \text{The rate of nitric oxide formation at each time step (mole/cm}^3\text{s)} \\
f_c & \quad 4.75 + C_1n_c - C_2\phi + C_3\phi^2 - C_4\phi^3 \\
C_1 & \quad 8.18e^{-2} \\
C_2 & \quad 23.2 \\
C_3 & \quad 32.0 \\
C_4 & \quad 12.2 \\
\phi & \quad \text{Fuel:air equivalence ratio (-)} \\
n_c & \quad \text{Number of carbon atoms in the hydrocarbon fuel (-)}
\end{align*} \]

There are two possible values of activation energy \( E \) depending on the gas temperature. If the gas temperature is less than 1920 K, the value of \( E \) was taken as 178 kJ/mol. For gases temperatures greater than 1920 K, the value of \( E \) used was 303 kJ/mol. These values of \( E \) are based on experimental data [5.3][5.7]. The correction factor \( f_c \) is applicable for all aliphatic alkane hydrocarbon fuels and for relative
Pulse combustion

air:fuel ratios of 0.65 to 1.33. The correlation equation of $f_c$ is shown above and this
is also empirically-based.

$T^n$ represents the non-Arrhenius behaviour of equation 3 at experimental
measurement conditions. In this investigation, the constant, $n=0$ was used for normal
air-natural gas combustion [5.7]. The pre-exponential factor $A$ has a value of
$6.4 \times 10^6 \left(\frac{RT}{P}\right)^{a+\beta}$. Dupont [5.3] reported that the reaction orders, $\alpha$ and $\beta$ are
dependent on the gas temperature. $\alpha$ and $\beta$ can take values from 0 to 1 for $O_2$ and
0.2 to 1 for $CH_4$ respectively. At low temperature $(T<1000 \text{ K})$, the reactivity of $CH_4$
with $O_2$ is very low and the reaction order of $O_2$ is taken to be zero. As the
temperature rises, the values of reaction order of $O_2$ will gradually increase. At low
temperature, the concentration of the fuel $CH_4$ plays an important role in the formation
of prompt-NO and the value of $\beta$ can be taken as 1.

In addition to prompt NO, thermal-NO formation also occurs during combustion
processes. Thermal-NO is formed in the high-temperature region through the attack
on nitrogen molecules by oxygen atoms. Zeldovich [5.3][5.8] proposed the following
mechanism:

\[ O + N_2 \rightarrow NO + N \]
\[ K_3 = 1.9 \times 10^{14} EXP \left( \frac{-38379}{T} \right) \]

\[ N + O_2 \rightarrow NO + O \]
\[ K_4 = 6.4 \times 10^9 (T) EXP \left( \frac{-3161}{T} \right) \]

\[ N + OH \rightarrow NO + H \]
\[ K_5 = 3.8 \times 10^{13} \]
Here $K$, is the specific reaction rate constant and $T$ is the temperature in K. By assuming a steady-state concentration for the N-atom and assuming the O-atom concentration is in equilibrium; the thermal-maximum-NO formation due to Zeldovich mechanism can be expressed as:

$$\frac{\partial [NO]_{th}}{\partial \alpha} = 6 \times 10^{10} T^{-1} \left[ O_2 \right]_e \left[ N_2 \right]_e \exp \left( \frac{-69090}{T} \right)$$

where:

- $[O_2]_e$: Equilibrium concentration of oxygen (mole/cm$^3$)
- $[N_2]_e$: Equilibrium concentration of nitrogen (mole/cm$^3$)
- $\frac{\partial [NO]_{th}}{\partial \alpha}$: The rate of nitric oxide formation at each time step (mole/cm$^3$s)
- $T$: Gas temperature (K)

The above equations were solved using a finite difference scheme where the energy changes were evaluated at each time step for equation (1) and (2). The energy changes associated with NO formation were small and were therefore not evaluated.

Since the heat transfer from the product gases to the combustion chamber and along tailpipe had been simulated by both radiation and convection, the formation of thermal and prompt NO in the post-flame gases was modelled in each segment by solving equation (3) and (4) simultaneously at each time step.
5.3 Predicted results discussion and validation

A series of simulations were carried-out over a range of operational parameters such as relative air:fuel ratio ($\lambda$), positive pressure amplitude ($+\Delta P$), combustion frequencies ($f$), water jacket temperature distribution ($T_w$), input firing rates to investigate their affect on nitric oxide (NO) formation. The overall test envelope is shown in Table 5.1 which is the same as for the experimental study discussed in chapter 4. Note that, in the same manner as the experimental data reported in chapter 4, all the NO emissions presented here are corrected to 3% $O_2$ dry.

Table 5.1. Modelling test envelope

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative air:fuel ratio, $\lambda$</td>
<td>1.0 to 1.3</td>
</tr>
<tr>
<td>Input firing rate, $Q_{fuel}$</td>
<td>10 to 25 kW</td>
</tr>
<tr>
<td>Tailpipe length, $L$</td>
<td>1.55, 2.00, 2.55 and 3.00m</td>
</tr>
<tr>
<td>Combustion frequency, $f$</td>
<td>30 to 73 Hz</td>
</tr>
<tr>
<td>Combustion positive pressure amplitude, +$\Delta P$</td>
<td>0.05 to 0.15 bar</td>
</tr>
<tr>
<td>Water jacket temperature, $T_w$</td>
<td>55°C, 65°C and 75°C</td>
</tr>
</tbody>
</table>

5.3.1 Relationship between combustion frequency and predicted NO formation

Figures 5.3:a-d present the predicted thermal, prompt and total-NO formations (at $\lambda=1$ and $T_w=55^\circ$C). Predictions indicate that the thermal and prompt NO formation is strongly dependent on the combustion frequency. The predicted NO formation decreased monotonically with increasing the operating frequencies from 34 to 65 Hz when the tube length was decreased from 3.0 to 1.55 m. The values of prompt NO were calculated to contribute approximately 6-19% to the total predicted NO formation. These results are shown in Table 5.2. The data clearly indicate that the prompt NO contribution decreased with increasing the input firing rate. This confirms that more thermal NO is produced at higher input firing rates.
Figures 5.3:a-d. Relationship between predicted NO formation and combustion frequency at different input firing rates (at $\lambda=1$, $T_w=55^\circ C$ and $+\Delta P=0.05$ bar).
Pulse combustion

Tested condition:
Positive pressure amplitude $+\Delta P = 0.05$ bar
Relative air/fuel ratio, $\lambda = 1.0$
Water Jacket temperature, $T_w = 55^\circ C$
Input frequency, $f = C/4L$

Where:

- \( C \) Sound velocity = 400 (m/s$^1$)
- \( L \) Length of tailpipe (m)

<table>
<thead>
<tr>
<th>Length of tailpipe, L in m</th>
<th>Input firing rate, kW</th>
<th>Frequency, Hz</th>
<th>Predicted total NO, ppm</th>
<th>NOpr, ppm</th>
<th>Percentage of NOpr contributions, %</th>
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<th>Frequency, Hz</th>
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<th>NOpr, ppm</th>
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<th>NOpr, ppm</th>
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<th>Frequency, Hz</th>
<th>Predicted total NO, ppm</th>
<th>NOpr, ppm</th>
<th>Percentage of NOpr contributions, %</th>
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Table 5.2. Predicted NO formation per a cycle at different input firing rates and different tailpipe lengths.
Similar predicted results were obtained when higher water jacket temperatures were applied, although higher predicted values of NO formation were obtained. The results are graphically presented in Figures A5.3.1:a-d and Figures A5.3.2:a-d (See Appendix A5.3).

Two potential causes for the reduction of NO levels with increasing frequency are changes in bulk heat transfer rates from the reacting mixture and the effect on the formation rate kinetics of the reduced cycle time. Consideration of existing knowledge of heat transfer in the Schmidt tube suggests that changes in heat transfer rates with frequency will be minimal.

It is known that convective heat transfer is strongly related to the local value of the ratio of the maximum oscillatory particle velocity to the mean flow velocity ($B$) (Hanby [5.5]). In a standing wave this is a function of pressure amplitude hence the product of displacement amplitude and angular frequency ($\omega A$) is not affected by a frequency change. In addition, the value of the dimensionless pulsation velocity ($B = \omega A / V_m$) is always small at the closed (velocity node) end of the tube where the NO is formed.

To examine the possible effects of cycle time on the kinetics of NO formation, temporally resolved NO formation rates were investigated. Predicted NO formation rates as a function of cycle time are shown in Figure 5.4 ($L = 1.55m$) and Figure 5.5 ($L = 3.0m$).
Figure 5.4. Temporally resolved thermal and prompt NO formation rates in combustion chamber and tailpipe length 1.55m.

Figure 5.5. Temporally resolved thermal and prompt NO formation rates in combustion chamber and tailpipe length 3.0m.
The results of these calculations show clearly that firstly, the rate of formation of both thermal and prompt NO reaches zero well before the end of the cycle and that the peak rate of prompt NO is much higher at the lower frequency. The consequences of these effects is better illustrated by considering the comparative formation rates of thermal and prompt NO in real time, as shown in Figures 5.6 and 5.7 respectively (note the different time axes used).

Figure 5.6. Time-resolved rate of formation of thermal NO for frequencies 30 and 60 Hz.

Figure 5.7. Time-resolved rate of formation of prompt NO for frequencies 30 and 60 Hz.
Given the dominant contribution of thermal NO on the total NO produced, Figure 5.6 gives a strong indication that reduced NO would be result from a higher operational frequency due to the higher quenching-action which results from the lower mixtures volume combusting in the cycle.

Calculation of the areas under the curves in Figure 5.6, it was found that 55 ppm and 22 ppm of thermal NO were obtained in 30 Hz and 60 Hz respectively. These data show that thermal NO formation in 30 Hz is more than double the formation of thermal NO in 60 Hz. It is therefore evident that operation frequency does strongly effect the NO formation.

When the model was based on a 3.0m tailpipe length, at spatial locations of 20mm from combustion chamber, thermal NO formation rate changed from essentially zero to a peak of \(3.406 \times 10^{-8}\) moles/sec-cm\(^3\) in 0.0065 cycle (217 \(\mu s\)). Beyond the peak value, the formation rate of NO declined very rapidly before the end of the cycle. Further examination of the predicted results of 1.55m tailpipe length shows a similar profile of thermal NO formation rates. However, the volumetric efficiency is lower than that the value predicted at 3.0m. This phenomenon is due to a rapid decline in the peak temperature at higher operating frequency (shorter residence time), thus, quenching the formation of NO and eventually reducing the efficiency of thermal NO formation rates.

The effect of combustion frequency on prompt NO formation rates is also illustrated in Figure 5.7 which shows the time-resolved rate of formation of prompt NO for frequencies 30 Hz and 60 Hz. The steeper quenching rate associated with the shorter residence time combined with the reduced volumetric efficiency is very evident. Calculating the areas under the curves in Figure 5.7, indicated that prompt NO formation are 10 ppm and 1.5 ppm at 30 Hz and 60 Hz respectively. Predicted result suggested that prompt NO approximately 6-15% to the total predicted NO formation. Although small amount of prompt NO was obtained, this demonstrated the relative importance of the prompt NO formation route in pulse combustion.
The relationship between the maximum value of NO formation rates and combustion frequency, is illustrated in Figure 5.8. Curves shown in Figure 5.8 indicate that the maximum of prompt NO formation rates decreased gradually with increasing the operating frequencies from 30 Hz to 60 Hz when the tube length was decreased from 3.0m to 1.55m.

![Graph showing relationship between maximum value of NO formation rates and combustion frequency.]

Figure 5.8. Relationship between maximum value of NO formation rates and combustion frequency.

This implies that the maximum value of prompt NO formation rate is dependent on combustion frequencies. However, the maximum value of thermal NO formation rate remains fairly constant over a range of operating frequencies.

The predicted and measured exhaust NO formation and emission are plotted in Figures 5.9:a-d to Figure 5.12.
Figures 5.9:a-d. Predicted and measured NO concentration against combustion frequency (at $\lambda=1$, $T_w =55^\circ C$ and $+\Delta P=0.05$ bar).
Figure 5.10:a-c. Predicted and measured NO concentration against combustion frequency (at $\lambda=1$, $T_w=55 \, ^\circ C$ and $+\Delta P=0.09$ bar).
Figures 5.11:a-d. Predicted and measured NO concentration against combustion frequency (at $\lambda=1$, $T_w=55^\circ C$ and $+\Delta P=0.12$ bar).
In simulating NO formation with tailpipe length of 1.55m, the total predicted NO emissions were approximately 7-16 ppm lower than experiment results at different input firing rates. However, 1-3 ppm and 13-17 ppm higher predicted results were obtained at 2 and 2.55m long tailpipes with different input firing rates respectively. Further examination of the data at 3m long tailpipe, however, shows predicted results 30-40 ppm higher than the experimental value. In essence, the input combustion frequency for simulation were underestimated at higher input firing rates (e.g. 20kW and 25kW). Referring to the Figure 5.13, the combustion frequency was found to increase with the input firing rate. According to Rayleigh principle [5.9], the frequency of the vibration in a pipe is proportional to the velocity of propagation of sound in the gas. Gases burning with higher input firing rates lead to higher gas temperatures and hence higher speed of sound and thus increasing in frequency.
Figure 5.13. Relationship between combustion frequency and tailpipe length and input firing rates.

Similar predicted results were obtained when the model was run using higher water jacket temperatures although higher values of NO formation were obtained. These graphs are considered too cumbersome to include in the body of this chapter: they are therefore presented in Appendix A5.4 as Figures A5.4.1:a-d and Figure A5.4.2:a-d respectively.

In general, predicted NO concentrations were found to be in good agreement with experimental results, exhibiting the same trends.

5.3.2 Relationship between positive pressure amplitude and predicted NO formation

A plot of predicted NO formation against positive pressure amplitude (Figure 5.14:a-b) is an indication that higher levels of NO concentration occur at lower positive pressure...
amplitudes. At 10kW of input firing rate with 2m tailpipe, predicted NO formation progressively reduced from 30ppm to 27ppm when the positive pressure amplitude increased from 0.05 bar to 0.15 bar.

![Graph](image.png)

Figures 5.14:a-b. Predicted NO formation against positive pressure amplitude at different input firing rates (at $\lambda=1$, $T_{w}=55^\circ C$).

A similar result was obtained when the computer model was run over a range of input firing rates, tailpipe lengths and water jacket temperature. However, a higher level of NO concentration was observed with higher input firing rates and higher water jacket temperature distribution along the length of tailpipe. The effect of water jacket temperature on predicted NO formation is also presented in Figures A5.5.1:a-d and Figures A5.5.2:a-d in Appendix A5.5.

The predicted and measured results with different tailpipe lengths and an input firing rates are presented individually in Appendix A5.6 (Figures A5.6.1:a-d to Figures A5.6.4:a-d). Comparison of predicted and measured results at 2.0m long of tailpipe
Pulse combustion

(Figures A5.6.2:a-d), the predicted NO concentrations are approximately 0.4ppm to 7ppm higher with different input firing rates. This result indicates good agreement with the measured values. However, simulation results show that the calculations were 2-16ppm under-estimated at 1.55m long of tailpipe (see Figures A5.6.1:a-d). It is due to the fact that the heat transfer along the tailpipe pipe is over-estimated (lower temperature) as indicated by the exit gas temperature which is 65 °C lower than the measured mean value of approximately 304 °C at 25 kW input firing rate.

When the computer model was run with 2.55m and 3.0m length tailpipe, the predicted results are 11-23ppm and 27-43 ppm higher than experimental measurements at 10 to 25 kW input firing rates respectively (see Figures A5.6.3:a-d and Figures A5.6.4:a-d in Appendix A5.6). It may be seen that the combustion frequency is under-estimated at higher input firing rates and the heat transfer may be under-estimated as indicated by the exit gas temperature which is 32 °C higher than the measured value at the open end section of the tailpipe.

Generally, the relationships between predicted NO formation and positive pressure amplitude were found to be in good agreement with the observation from experiments.

5.3.3 Relationship between relative air:fuel ratio and predicted NO formation

As shown in Figures 5.15:a-b, predicted NO formation varies with relative air:fuel ratio and input firing rates with 1.55m long tailpipe. The maximum predicted total NO concentration was obtained at a relative air:fuel ratio of 1. At leaner mixtures, the calculated total NO formation gradually decreased with increasing relative air:fuel ratio. This observation is due to excess air leading to a reduction in flame temperature and therefore in NO formation. Computer predictions also indicated prompt NO formation remains fairly constant with different relative air:fuel ratios. The pattern shows that thermal NO formation is strongly dependent on product gas temperature.
Pulse combustion

Figures 5.15:a-b. Predicted NO formation varies with relative air:fuel ratio from 1 to 1.3 at different input firing rates (tested at Tw=55 °C and tailpipe length 1.55m).

A similar predicted result was obtained when the model was simulated with a range of tailpipe lengths. The predicted results are plotted in Figure A5.7.1:a-d to Figure A5.7.3:a-d and presented in Appendix A5.7.

5.4 Predicted results conclusions

A model study of NO formation in a valveless Schmidt type combustor has been developed. The model incorporated an imposed pressure fluctuation, a gas temperature model and convection and radiation heat transfer along the tailpipe with two step kinetic model for the treatment of combustion and gave satisfactory results. In summary, these were:
Pulse combustion

1. The predicted NO formation decreased monotonically with increasing the operating frequency. There was found to be in good agreement with experimental results. The data clearly indicated that calculation and measurement of NO concentrations both produced the same trends.

2. Calculation suggested that prompt-NO approximately 6-19% to the total predicted NO formation over a range of input firing rates. This demonstrated the relative importance of the prompt NO formation route in pulse combustion.

3. When the model was run with a range of tailpipe lengths from 1.55 to 3.0m, the predicted results showed that prompt NO decreased with increased input firing rates. This clearly indicated that more thermal NO is formed at higher input firing rates.

4. The predicted data suggested that NO formation is generally reduced by increasing the maximum positive pressure amplitude. Results of the calculation were found to be in good agreement with observation from experiments.

5. Maximum NO formation was predicted at relative air/fuel ratios of $\lambda=1$. At higher values of $\lambda$, the calculated total NO formation gradually decreased with increasing relative air:fuel ratio. Predictions also confirmed that thermal NO formation is strongly dependent on temperature.

6. Computer predictions indicated that prompt NO formation remains fairly constant with varying relative air:fuel ratios.

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6 Overall conclusions and suggestions for further work

6.1 Overall conclusions

The primary objective of this study was to understand the effect of combustion frequency, positive pressure amplitude, relative air:fuel ratio, water jacket temperature and input firing rate on NO emissions. This has been achieved by the design and construction of a new Schmidt type pulse combustor to operate over a wide test envelope. Extensive experimental work has been conducted and has produced a large data set.

The main part of the combustor consisted of water-cooled sections that the overall length of the tube could be varied in 50 mm increments. This enabled the operating frequency (in quarter-wave mode) to be varied in the range of approximately 30 to 75 Hz. Independent control of the pressure amplitude of oscillation was obtained by the use of two water-cooled damping tubes located at the acoustically-closed end of the tube. The amplitude of oscillation could thus be controlled independently of other variables such as relative air:fuel ratio or combustion frequency. Exhaust gas composition was measured through a heated sample line with the probe located 1.0m along the tailpipe from the burner inlet. This was to ensure independence with tailpipe length and to avoid any possibility of dilution by reverse flow of ambient air. Pressure measurements were made by a pressure transducer and were digitally recorded.
In general, it was found that the lowest emissions of NO occurred under conditions of high resonant frequency, high maximum pressure amplitude and lean air:fuel ratios. This is probably due to the following factors:

1. If the flow rates of air and fuel are maintained, the effect of an increase in frequency will be to reduce the volumetric efficiency of combustor: therefore, a smaller volume of fresh mixture will be admitted into chamber during each cycle facilitating rapid mixing with the products of the previous cycle. This reduces the residence times under conditions of high temperature and high oxygen concentration and hence lead to lower rates of NO formation.

2. It is known that pressure amplitude is the significant parameter affecting the convective heat transfer coefficient: a higher pressure amplitude produces a higher velocity amplitude along the tailpipe. If flow reversal takes place then the convective film coefficient may be several times the equivalent steady-state value. This will produce higher rate of temperature drop in the tube and hence inhibit the formation of thermal NO.

3. At higher relative air:fuel ratio, excess air cools the flame which leads to a drop in the burned gas temperature and thus reducing NO formation and emissions.

NO emissions were found to increase by raising the input firing rates and water jacket temperature along the length of tailpipe. This phenomenon may be explained by the following:

1. At higher input firing rates, the thermal capacity rate of the combustion products are increased, leading to a lower heat exchanger effectiveness for the tube. This resulting higher gas temperatures would produce a rise in NO formation.

2. It is known that temperature is the significant parameter affecting convection heat transfer: At higher water jacket temperatures, a higher wall temperature distribution
Pules combustion

leads to lower the heat transfer and higher product gas temperature along the tailpipe, therefore higher levels of NO emissions are formed.

A model was developed to predict NO emissions in pulse combustor. The basis of the modelling work was a one-dimensional approach, combined with sub-model of inlet valve, a two step kinetic combustion model together with rate equations for the formation of prompt and thermal NO. Convective and radiative heat transfer to the tube wall was also incorporated in the model.

The model was run over the same envelope as defined in the experimental programme. It confirmed that an increase in frequency of operation produced a reduction in total NO production and that prompt NO formed a comparatively small (6-19%) proportion of the total NO emissions. This phenomenon is mainly due to a rapid decline in the peak temperature at higher operating frequency (shorter residence time), thus, quenching the formation rate of NO and hence reducing NO emissions. In addition, the model allowed new insight into the time-resolved formation of NO during the combustion cycle, which proved particularly valuable in understanding the effect of frequency on NO formation.

6.2 Suggestions for further work

The results of experimental measurement indicated that the present Schmidt type pulse combustor can afford a high degree of flexibility in term of defining a performance envelope. One of the immediate tasks for further investigation is to operate the present pulse combustor with different fuels over a range of test envelope. The main focus of this investigation would be to obtain a closer understanding the relationship between combustion frequency, positive pressure amplitude and NO emissions with different type of gaseous fuels.
Further examination of the measurements in three-quarter wavelength operation is suggested—This investigation would provide a better understanding of behavior of NO emissions in “hop mode” conditions.

The present one-dimensional model could be further extended and run the same test envelope as defined in the experimental programmes. The extension of this model could include further development of a combustion and gas-dynamic model for the prediction of the NO formation. In order to achieve this prediction, a good understanding the chemical information of each product gas species will be required. Chemical kinetic combustion modelling techniques should be employed in the calculation of heat releases and mass-fraction of chemical species in the combustion chamber. The chemical kinetics of flame temperatures would then be determined from these calculated results.

Changes in temperature distribution may also influence the viscosity and density of the combustion mixtures, and hence, a better understanding of dynamic flow properties are also necessary. These wave dynamic transport processes can be modelled by an unsteady one-dimensional methods which are capable of resolving non-linear wave formation. For example, predictions of the influence of system geometries and operating parameters of wave structures in pulse combustors, will involve the use of a basic numerical model which requires one-dimensional non-linear, gas dynamic equations. These equations can model variable area geometries which are influenced by heat-transfer, wall-friction and heat release. These factors can be investigated by parametric modelling, a method which involves systematic variations of each of these parameters.

These wave dynamic transport equations may be solved by using MacCormack explicit finite-difference method [6.1]. This method combines first order accurate forward and backward finite differences, and produces second order accurate algorithms which assures the propagation of non-linear pressure waves at correct amplitudes and speeds. The example of explicit finite-difference scheme is briefly described in Appendix A6.1.
Computer simulated results will be used to compare with available experimental data. It is hoped that the model together with the experimental results will help to explain further the mechanism for the observed NO formation and emissions trends.

6.3 Chapter 6 references

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*Hyperbolic equations and waves*, pp. 96-109, Publisher cup, 1958.
Appendix A3.1: Design of internal diameter of water jacket

Assumptions:
1. The water temperature difference between the inlet and outlet of the tailpipe is ($\Delta T$) 15°C.
2. The average velocity of water flow is ($\dot{U}$) 1 m/s.
3. Heat absorbed into water ($\dot{Q}$) is about 80% from input gas flow rate into combustion chamber.

In case (1):-Assuming the input gas flow rate is 15 kW, the diameter of the internal tube can be calculated by the following procedure.

By energy balance equation,

$$\dot{Q} = \dot{m}_w C_{pw} \Delta T \hspace{1cm} \text{(A3.1)}$$

Where:

- $\dot{Q}$: Heat absorbed into water, J/s
- $\dot{m}_w$: Mass flow rate of water, kg/s
- $C_{pw}$: Specific heat capacity of water 4180, J/kg-K
- $\Delta T$: Temperature difference between the inlet and outlet of water in the tailpipe, K

From the above data, the mass flow rate of water can be calculated by substitution this data into equation (A3.1).
In calculation,

$$15000 \times 0.8 = m_w \times 4180 \times 15$$

$$m_w = 0.1914 \text{ kg/s}.$$ 

According to equation \( \dot{V}_w = \frac{m_w}{\rho} \), the volume of water can expressed as:

$$\dot{V}_w = 0.1914/1000 \text{ m}^3/\text{s}$$

The velocity of water flow in the tailpipe is guessed as 1 m/s, the time taken for the water flow through a length of the tailpipe (1.9 m) is 1.9 second. The volume of water requiring for the water travel in 1.9 sec. is

$$V_{WN} = 1.914 \times 10^{-4} \times 1.9 = 3.6366 \text{ m}^3$$

According to these data, the total volume of water jacket within the length of tailpipe (1.9 m) can be expressed as:

$$V_J = V_{WN} + V_T \quad \textbf{-----------------------------------------------(A3.2)}$$

Where

- \( V_J \quad \text{Total volume of water jacket, m}^3 \)
- \( V_T \quad \text{The volume of inter tube,} \quad \frac{\pi D_T^2}{4} \text{ L} \text{ m}^3 \)
- \( D_T \quad \text{The inter tube diameter (50 mm), m} \)
- \( L \quad \text{The length of tailpipe (guessed at 1.9 m), m} \)

From equation (A3.2), the total volume of water jacket is

$$V_J = 3.6366 \times 10^{-4} + 3.731 \times 10^{-3} \text{ m}^3$$
\[ V_j = 4.1 \times 10^3 \text{ m}^3 \]  \hspace{1cm} (A3.3)

Since the water jacket is cylindrical, the diameter of the water jacket can be calculated by the following step.

\[ V_j = \frac{\pi D_j^2}{4} L \]  \hspace{1cm} (A3.4)

Substitution equation (A3.3) into (A3.4), the diameter of the water jacket is

\[ 1.9 \times \frac{\pi D_j^2}{4} = 4.1 \times 10^3 \]

\[ D_j = 54.7 \text{ mm} \]

Repeating the calculation with different guessed values, (e.g. input gas flow rate and water flow rate), a series values of water jacket diameter is obtained. The results from the calculation is shown in table A3.1.

<table>
<thead>
<tr>
<th>Heat input J/s</th>
<th>15 kW ((L=1.9 \text{ m}))</th>
<th>30 kW ((L=1.9 \text{ m}))</th>
<th>15 kW ((L=3.0 \text{ m}))</th>
<th>30 kW ((L=3.0 \text{ m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of water flow, m/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>D=54.7 mm</td>
<td>D=59.0 mm</td>
<td>D=54.6 mm</td>
<td>D=65.0 mm</td>
</tr>
<tr>
<td>1.0</td>
<td>D=52.4 mm</td>
<td>D=54.9 mm</td>
<td>D=52.4 mm</td>
<td>D=60.6 mm</td>
</tr>
</tbody>
</table>

Table A3.1. The diameter of water jacket minimum required for different value of water flow velocity and input firing rates.
Appendix A3.2: Variable length of flanged tube section design

Ten tailpipe sections of variable lengths, designed with flanged ends, are shown below, which can be used in a number of combinations so that it can used to alter the frequency of the Schmidt tube.

$L_0 = 500$ mm

$L_1 = 200$ mm

$L_2 = 200$ mm
Pulse combustion

L3 = 150 mm

L4 = 300 mm

L5 = 350 mm
L6 = 400 mm

L7 = 400 mm

L8 = 450 mm
Various tailpipe lengths are produced by joining together different flanged tube sections. For example:

$L_0 + L_10 = 600$ mm
$L_0 + L_5 + L_7 + L_8 + L_1 + L_10 = 2000$ mm
$L_0 + L_8 + L_7 + L_3 + L_1 + L_6 + L_5 + L_2 + L_4 + L_10 = 3050$ mm etc.

Combination of tailpipes of different lengths can be used to length in 50 mm steps. For example, it is possible to join different length of tailpipes to produce 2000 mm and 2050 mm variable length of the tailpipe respectively. The length of this extended tube, could then be increased by joining it with another tube. In corrected combination, the distance between two sample ports along the tailpipe is always 200 mm.
Appendix A3.3: Fulton Boiler Works Ltd. information
Fulton Boiler Works, (Great Britain) Ltd.

LoNOx Pulse Heating Boiler

80 - 260kW

- 98% efficiency
- 55 - 55 ppm NOx

Independent verification

Models:
PHB 80 • PHB 135
PHB 200 • PHB 260
Fulton LōNOx Pulse Boilers

Setting new efficiency and environmental standards for commercial and industrial heating

Pulse Boilers combining the simplicity of pulse combustion operation within a rugged steel pressure vessel. Fulton's exclusive 'Thermallx' 316L Stainless Steel pipe design minimises thermal stress caused by varying temperatures to which the boiler may be exposed. These boilers can be condensing or non-condensing, depending on return water temperature, and offer the highest efficiency available. When operating in the condensing mode the boilers extract latent heat from the combustion products, resulting in efficiencies up to 96%.

Advantages of LōNOx Pulse Combustion Boilers

Operating efficiency up to 96%. Low NOX as low as 35ppm meets latest and projected European & International standards for NOX emissions. Facility to operate in condensing mode when site conditions demand. Simplified Flue Venting - no chimney required. No boiler room make up air required. Simple, reliable operation - no burner adjustments necessary. Low maintenance costs - one service per annum only. Fully packaged - assembled and tested prior to delivery. Unique microprocessor based intelligent combustion control. Facility to interface with B.M.S.

The Fulton Pulse Control 7865

A Smart One

Programmed to provide a high level of safety and reliability, it has functional capability and features beyond the capacity of conventional controls. The Fulton Pulse Control 7865 is a microprocessor based integrated burner control for application with the Fulton Gas Pulse Combustion Boilers.

Functions provided by the 7865 include automatic burner sequencing, system status indication, system and self-diagnostics and trouble shooting.

Standard Features

- Application flexibility.
- Access for external electrical voltage checks.
- Dependable, long term operation provided by micro-computer technology.
- Five Light Emitting Diodes (LED's) for sequence information and fault detection.
- Non volatile memory: 7865 retains history, files and sequencing status after loss of power.
- Selectable re-try attempts for ignition.

Safety Features

- Closed loop logic test.
- Dynamic input check.
- Dynamic safety relay test.
- Dynamic self-check logic.
- Expanded safe-start check.
- Internal hardware status monitoring.
- Tamper resistant timing and logic.

Options

- Keyboard Display Module.
- Remote Display Module.
- First-Out Expanded Annunciator.
- Remote Reset.
- Personal Computer Interface capability.

Boiler Efficiency/Return Boiler Water Temperature

The boiler goes into the High Efficiency condensing mode, when the exhaust gas temperature drops below 70°C. The amount of gas condensed is directly related to the return water temperature and flow rate, which farther determine the flue gas temperature.
Fulton Pulse Combustion
Design Technology with manufacturing durability and high performance operation.

Compact
These boilers are suitable for modular application where space is restricted in new and replacement applications.

Spark Ignition
For initial lighting only

Insulation
Casing losses less than 0.5%.

Air Metering Flapper Valve

High Quality Insulated Casing
Attractive high gloss paint finish with control console and facia panel.

Purge Fan
Only operational during starting sequence and as a post purge device: shuts off during combustion cycle.

Internal Control Panel
Incorporating the unique Fulton 7865 microprocessor based intelligent combustion control.

Exhaust Dcoupler
Scaled chamber collecting exhaust gases.

Compact and Fully Packaged Boiler
Small footprint
PHB 80 & 135 700 x 927mm
PHB 200 & 260 850 x 1270 mm

Anti Vibration Mountings and Flexible Connections
For ultra quiet applications, the boiler is available with four matched anti-vibration mounting feet; three flexible pipe connections for the boiler flow and return and gas supply pipe.

Heat Exchanger Tube Bank
'Thermaflex' 316L stainless steel heavy duty giving flexibility and reliability in operation.

Exhaust Outlet
75mm diameter for PHB 80 & 135; 110mm diameter for PHB 200 & 260.

Conductal Drain
Boiler Exhaust Vent & Combustion Air Inlet

as never been easier!

One of the unique design features of boiler is that no conventional flue is required. The boiler can be vented through a side wall, using 75mm or 110mm diameter exhaust duct. Similarly, combustion air is introduced directly to the boiler through the same size vents.

This allows flexibility in selecting boiler locations and on site costs in time and money are considerably reduced compared to conventional boilers.

The exhaust vent pipework can be supplied in single skin stainless steel or high temperature plastic pipework. Full details are available from Fulton Technical Department and in the Fulton 40X Pulse Boiler Application Manual.

Site Requirements

The boiler does not require a special base. It can be positioned on a level non-combustible floor.

The boiler will pass through a 1 metre wide opening.

Clearances of 1 metre front, 400mm rear and 600mm side and top.

The noise level of the boiler is approximately 75dBA under full load conditions. For further details contact Fulton Technical Department.

Fulton Boiler Works, (Great Britain) Ltd.

Factory and Export Sales
Hawthorn Road, Hirst H54 4TU.
Telephone: (0272) 723112
Sales and Service: (0272) 72561
Fax: (0272) 721358

In the U.S.A.
Fulton Boiler Works, Inc, Fort & Jefferson St., Box 257, Pulaski, New York, USA 13142.
Telephone: (315) 298-5121
Telex: 16461 FULVUJUS
Tel: 1400/1400

Every effort has been made to ensure accuracy at time of going to press. However, as part of our policy of continual product improvement, we reserve the right to alter specifications without prior notice.
Appendix A3.4: Technical assembly drawing
Appendix A3.5: Water closed circuit

A closed water circuit is used to control the temperature distribution along the Schmidt combustor. This device consists of several components and is attached to the combustor as shown in Figure A3.5.1. The function of each of the components is explained in this section.

Water flowing through the water jacket from the burner to the end section of the tailpipe was provided via a pump. A alternating supply voltage between 0-240 volts was used to power the pump. During normal pump operation, the supply voltage was set at 220 volts.

Temperature variations along the tailpipe were measured with type-K, mineral insulated thermocouple probe. Two thermocouples were initially installed at the inlet and outlet line of the water closed circuit respectively. The recorded temperatures were displayed on a digital thermometer.

The water temperature inside the closed circuit is governed by a shell-and-tube heat exchanger. This type of exchanger consists of round tubes mounted on a cylindrical shell with their axes parallel to that of the shell. Figure A3.5.2 illiterates the main features of a shell-and-tube exchanger a fluid flows within the tubes while another fluid flows outside
the tubes. Both fluids are pumped through the exchanger; hence heat transfer on both the tube and the shell side is by forced convection.

Figure A3.5.2. A shell and tube heat exchanger.

Hot water expansion inside the circuit is restricted by the utilisation of an expansion tank.
Appendix A3.6: DT 2805 series software package

The DT-2805 model software package, can be used as an analog to digital (A/D) converter. An (A/D) converter changes analog voltages (comparable to the output signal of a pressure sensor or a thermocouple) into a digital signals. The following procedure is an example of A/D conversion (refer to the block diagram of an A/D converter in Figure A3.6.1).

1. The analog multiplexer chooses an input channel from those connected to the board.

2. The programmable gain amplifier buffers the analog input, and may increase its voltage level. Since the analog input signal from the transducer may be only +20mV maximum on the DT2805, an amplifier is used to boost the signal's voltage to the 0 to +10 volt level required by the board's A/D converter.

3. A sample and hold circuit acquires the selected analog signal from the multiplexer and stores it on a capacitor, keeping the voltage of the signal constant so an accurate A/D conversion can occur.
4. Finally, the A/D converter translates the analog held by the sample and hold into a digital signal.

The DT 2805 model A/D system has 12 bits of resolution. Once a signal is received from the pressure sensor, the analog signal is converted into a binary number. The binary number is resolved into 12 bits long, and can assume 4096 different states (4096 equals 2 raised to the twelfth power). A 12-bits converter can therefore resolve differences on an analog input, of minimal magnitude 0.024% (0.024 equals 100x1/4096) of the selected analog input range.
Appendix A4.1.

Effect of frequency on NO emissions over a range of relative air:fuel ratios between \( \lambda = 1.1 \) to \( \lambda = 1.5 \) (tested at \( T_w = 55 \, {}^\circ\text{C} \))
Figures A4.1.1:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.1$ and $T_w=55^\circ C$).
Figures A4.1.2:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.2$ and $T_{w}=55$ °C).
Figures A4.1.3:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.3$ and $T_w=55^\circ\text{C}$).
Pulse combustion

\[ \lambda = 1.4, \ Tw = 55^\circ C \text{ and input firing rate } 10kW \]

\[ \lambda = 1.4, \ Tw = 55^\circ C \text{ and input firing rate } 15kW \]

\[ \lambda = 1.4, \ Tw = 55^\circ C \text{ and input firing rate } 20kW \]

\[ \lambda = 1.4, \ Tw = 55^\circ C \text{ and input firing rate } 25kW \]

Figures A4.1.4:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at \( \lambda = 1.4 \) and \( Tw = 55^\circ C \)).
Figures A4.1.5:a-b. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.5$ and $T_w=55^\circ C$).
Appendix A4.2.

Effect of frequency on NO emissions over a range of relative air:fuel ratios between $\lambda=1.1$ to $\lambda=1.5$ (tested at $Tw=65^\circ C$ and $75^\circ C$)
Figures A4.2.1:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.1$ and $T_w=65^\circ C$).
Figures A4.2.2:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at \( \lambda=1.2 \) and \( T_w=65^\circ C \)).
Figures A4.2.3:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.3$ and $T_w=65^\circ C$).
Figure A4.2.4. Relationship between NO emissions and combustion frequency at input firing rate of 20 kW (tested at $\lambda=1.4$ and $T_w=65^\circ$C).
Figures A4.2.5:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.1$ and $T_w=75^\circ C$).
Pulse combustion

\[ \lambda = 1.2, \, Tw = 75^\circ C \text{ and input firing rate } 10kW \]

<table>
<thead>
<tr>
<th>NO (ppm) @% CO</th>
<th>Combustion frequency, Hz</th>
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<tbody>
<tr>
<td></td>
<td>20</td>
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<tr>
<td>0% CO</td>
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<tr>
<td>60% CO</td>
<td></td>
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<tr>
<td>70% CO</td>
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</tbody>
</table>

- Positive pressure amplitude = 0.05 bar
- Positive pressure amplitude = 0.09 bar
- Positive pressure amplitude = 0.12 bar

\[ \lambda = 1.2, \, Tw = 75^\circ C \text{ and input firing rate } 15kW \]

<table>
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<th>NO (ppm) @% CO</th>
<th>Combustion frequency, Hz</th>
</tr>
</thead>
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<tr>
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<td>20</td>
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<tr>
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<td>60% CO</td>
<td></td>
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<tr>
<td>70% CO</td>
<td></td>
</tr>
</tbody>
</table>

- Positive pressure amplitude = 0.05 bar
- Positive pressure amplitude = 0.09 bar
- Positive pressure amplitude = 0.12 bar

\[ \lambda = 1.2, \, Tw = 75^\circ C \text{ and input firing rate } 20kW \]

<table>
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<th>NO (ppm) @% CO</th>
<th>Combustion frequency, Hz</th>
</tr>
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<tbody>
<tr>
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<td>60% CO</td>
<td></td>
</tr>
<tr>
<td>70% CO</td>
<td></td>
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</tbody>
</table>

- Positive pressure amplitude = 0.05 bar
- Positive pressure amplitude = 0.09 bar
- Positive pressure amplitude = 0.12 bar
- Positive pressure amplitude = 0.15 bar

\[ \lambda = 1.2, \, Tw = 75^\circ C \text{ and input firing rate } 25kW \]

<table>
<thead>
<tr>
<th>NO (ppm) @% CO</th>
<th>Combustion frequency, Hz</th>
</tr>
</thead>
<tbody>
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<tr>
<td>0% CO</td>
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<td>10% CO</td>
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<tr>
<td>60% CO</td>
<td></td>
</tr>
<tr>
<td>70% CO</td>
<td></td>
</tr>
</tbody>
</table>

- Positive pressure amplitude = 0.05 bar
- Positive pressure amplitude = 0.12 bar

Figures A4.2.6:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at \( \lambda = 1.2 \) and \( Tw = 75 ^\circ C \)).
Figures A4.2.7:a-d. Relationship between NO emissions and combustion frequency at different input firing rates (tested at \( \lambda=1.3 \) and Tw=75 °C).
Figures A4.2.8:a-b. Relationship between NO emissions and combustion frequency at different input firing rates (tested at $\lambda=1.4$ and $T_w=75^\circ C$).
Appendix A4.3.

Relationship between NO emissions and positive pressure amplitude at different input firing rates over a range of relative air:fuel ratios between $\lambda=1.1$ to $\lambda=1.5$ (tested at $T_w=55 \, ^\circ C, 65 \, ^\circ C$ and $75 \, ^\circ C$)
Pulse combustion

Figures A4.3.1:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.1$ and $T_w=55 ^\circ C$).
Figures A4.3.2:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.2$ and $T_w=55^\circ C$).
Figures A4.3.3:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.3$ and $T_w=55^\circ$C).

\begin{align*}
\lambda=1.3, \; T_w=55^\circ$C and input firing rate 10kW
\end{align*}
Pulse combustion

\[ \lambda = 1.4, \, T_w = 55^\circ C \text{ and input firing rate 15kW} \]

\[ \lambda = 1.4, \, T_w = 55^\circ C \text{ and input firing rate 20kW} \]

\[ \lambda = 1.4, \, T_w = 55^\circ C \text{ and input firing rate 25kW} \]

Figures A4.3.4:a-c. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at \( \lambda = 1.4 \) and \( T_w = 55 \, ^\circ C \)).
Figure A4.3.5. Relationship between NO emissions and positive pressure amplitude at input firing rate of 25 kW (tested at $\lambda=1.5$ and $T_w=55$ °C).
Figures A4.3.6:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.1$ and $T_w=65$ °C).
Figures A4.3.7:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.2$ and $Tw=65^\circ C$).
Pulse combustion

\[ \lambda = 1.3, \text{Tw}=65^\circ \text{C} \text{ and input firing rate } 10\text{kW} \]

\[ \lambda = 1.3, \text{Tw}=65^\circ \text{C} \text{ and input firing rate } 15\text{kW} \]

\[ \lambda = 1.3, \text{Tw}=65^\circ \text{C} \text{ and input firing rate } 20\text{kW} \]

\[ \lambda = 1.3, \text{Tw}=65^\circ \text{C} \text{ and input firing rate } 25\text{kW} \]

Figures A4.3.8:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at \( \lambda = 1.3 \) and Tw=65 °C).
Figures A4.3.9a-b. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.4$ and $T_w=65^\circ C$).
Figures A4.3.10:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.1$ and $T_w=75$ °C).
Figures A4.3.11:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.2$ and $T_w=75\, ^\circ\mathrm{C}$).
Pulse combustion

\[ \lambda = 1.3, \: T_w = 75^\circ C \text{ and input firing rate 10kW} \]

\[ \lambda = 1.3, \: T_w = 75^\circ C \text{ and input firing rate 15kW} \]

\[ \lambda = 1.3, \: T_w = 75^\circ C \text{ and input firing rate 20kW} \]

\[ \lambda = 1.3, \: T_w = 75^\circ C \text{ and input firing rate 25kW} \]

Figures A4.3.12:a-d. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at \( \lambda = 1.3 \) and \( T_w = 75^\circ C \)).
Pulse combustion

\[ \lambda = 1.4, T_w = 75^\circ C \text{ and input firing rate } 10\text{kW} \]

\[ \lambda = 1.4, \ T_w = 75^\circ C \text{ and input firing rate } 15\text{kW} \]

\[ \lambda = 1.4, \ T_w = 75^\circ C \text{ and input firing rate } 20\text{kW} \]

Figures A4.3.13:a-c. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at \( \lambda = 1.4 \) and \( T_w = 75 \ ^\circ C \)).
Figures A4.3.14:a-b. Relationship between NO emissions and positive pressure amplitude at different input firing rates (tested at $\lambda=1.5$ and $Tw=75\,^\circ$C).
Appendix A4.4: Coefficient of Variation (COV)

The COV value can be expressed as:

\[
\text{COV} = \frac{\sigma}{+\Delta P}
\]

Where:

+\Delta P \quad \text{Mean peak pressure amplitude (bar)}

\sigma \quad \text{Standard deviation of peak pressure amplitude (bar)}

In experimental measurement, a mean peak pressure amplitude (+\Delta P) was calculated more than 85 cyclic variation of peak pressure amplitude such as +\Delta P_1, +\Delta P_2, +\Delta P_3, ......., +\Delta P_{85}. It’s definition can be expressed as:

\[
+\Delta P = \frac{(+\Delta P_1) + (+\Delta P_2) + (+\Delta P_3) + ......... + (+\Delta P_n)}{n}
\]

Where:

+\Delta P \quad \text{Mean peak pressure amplitude (bar)}

+\Delta P_1 \quad 1\text{st of cyclic variation of peak pressure amplitude (bar)}

+\Delta P_2 \quad 2\text{nd of cyclic variation of peak pressure amplitude (bar)}

+\Delta P_3 \quad 3\text{rd of cyclic variation of peak pressure amplitude (bar)}

+\Delta P_n \quad n\text{th of cyclic variation of peak pressure amplitude (bar)}

n \quad \text{Number of cyclic variation of peak pressure amplitude (-)}
Pulse combustion

Results show that the value of mean peak pressure oscillation (+ΔP) was generally to fairly constant when 60 cyclic variations of peak pressure amplitude were examined. For example, in a 2.55m tailpipe at input firing rate 25 kW, by increasing the number of cyclic variations of peak pressure amplitude from 60 to 70, the values of mean peak pressure amplitude (+ΔP) obtained were 0.157 bar and 0.1572 bar respectively. Using these results as a basic guideline, the minimum number of cyclic variations of peak pressure amplitude was decided to be 85.

The value of standard deviation (σ) is based on the number of cyclic variation of peak pressure amplitude. It is used to measure of how widely values are dispersed from the mean peak pressure amplitude (+ΔP). By definition, standard deviation can be expressed as:

\[ \sigma = \sqrt{\frac{n \sum (+\Delta P)^2 - (\sum +\Delta P)^2}{n(n - 1)}} \]

Where:

∑ Sum of peak pressure amplitude (-)

n Number of cyclic variation of peak pressure amplitude (-)

To obtain a better understanding of the Coefficient of Variation (COV) value responsible for the stability of pressure oscillation found in combustion chamber, the value of standard deviation (σ) were investigated. Calculated values of standard deviation (σ) and mean peak pressure amplitude are indicated in Figure A4.4.1 and Figure A4.4.2.
Pulse combustion

\( \lambda = 1, L = 2.55 \text{ m}, T_w = 55^\circ \text{C} \) and input firing rate 25 kW

Figure A4.4.1. Standard deviation of peak pressure amplitude in combustion chamber, at relative air: fuel ratio 1 and input firing rate 25 kW (stable oscillation).

\( \lambda = 1.4, L = 2.55 \text{ m}, T_w = 55^\circ \text{C} \) and input firing rate 25 kW

Figure A4.4.2. Standard deviation of peak pressure amplitude in combustion chamber, at relative air: fuel ratio 1.4 and input firing rate 25 kW (unstable oscillation).
Pulse combustion

The oscillation of pressure traces, under the same test condition, are also presented in Figure A4.4.3 and Figure A4.4.4.

\[ \lambda = 1, L = 2.55\text{m}, T_w = 55^\circ\text{C} \text{ and input firing rate } 25\text{ kW} \]

Figure A4.4.3. Stability of pressure oscillation traces vs time at relative air:fuel ratio 1, input firing rate 25 kW and tailpipe length 2.55m (stable oscillation).

\[ \lambda = 1.4, L = 2.55\text{m}, T_w = 55^\circ\text{C} \text{ and input firing rate } 25\text{ kW} \]

Figure A4.4.4. Stability of pressure oscillation traces vs time at relative air:fuel ratio 1.4, input firing rate 25 kW and tailpipe length 2.55m (unstable oscillation).
The data show that the values of standard deviation became higher with increasing the unstable pressure oscillation. As an example, when the stability of pressure oscillation was operated from stable ($\lambda=1$) to unstable condition ($\lambda=1.4$), the value of standard deviation increased from 0.01 bar to 0.06 bar. Since the Coefficient of Variation (COV) value are directly related to the value of standard deviation, thus, COV would be used to indicate the stability of pressure oscillations in combustion chamber.
Appendix A4.5: Oscillation of pressure traces
Pulse combustion

Figure A4.5.1: a.

Figure A4.5.1: b.

Figure A4.5.1: c.
Figures A4.5.1:a-f. Stability of pressure oscillation traces vs time, at input firing rate 15kW and over a range of relative air:fuel ratios from 1 to 1.5 (at Tw=55 °C).
Pulse combustion

L = 2.0 m, $T_w = 75^\circ C$, $\lambda = 1$ and input firing rate 15 kW

Figure A4.5.2: a.

L = 2.0 m, $T_w = 75^\circ C$, $\lambda = 1.1$ and input firing rate 15 kW

Figure A4.5.2: b.

L = 2.0 m, $T_w = 75^\circ C$, $\lambda = 1.2$ and input firing rate 15 kW

Figure A4.5.2: c.
Pulse combustion

L=2.0m, Tw=75°C, $\lambda=1.3$ and input firing rate 15kW

Figure A4.5.2: d.

L=2.0m, Tw=75°C, $\lambda=1.4$ and input firing rate 15kW

Figure A4.5.2: e.

L=2.0m, Tw=75°C, $\lambda=1.5$ and input firing rate 15kW

Figure A4.5.2: f.

Figures A4.5.2:a-f. Stability of pressure oscillation traces vs time, at input firing rate 15kW, and over a range of relative air:fuel ratios from 1 to 1.5 ( at Tw=75 °C).
Appendix A5.1: Mathematical description of flow rates entering the combustion chamber

The instantaneous volume flow rate (m³/s) of gas into the combustion chamber can be described by an orifice flow, thus:

\[ V = J \sqrt{\frac{2P_{\text{amp}}}{\rho_{\text{mix}}}} \]

Where \( J = C_d A_o \), the products of the discharge coefficient and the orifice area. The value of \( J \) can be calculated from the mean flow, by integration over a whole cycle period as shown below.

Equation (A5.1) can be integrated thus:

\[ \int_0^T V dt = J \int_0^T \sqrt{\frac{2(\Delta P)}{\rho_{\text{mix}}}} \int_0^T \sqrt{\sin(\omega t)} dt \]

Letting \( I_1 = \int_0^T \sqrt{\sin(\omega t)} dt \)

\[ U = \omega T \text{ therefore } dU = \omega dT \]

and \( dT = \frac{1}{\omega} dU \)

When \( T=0, U=0 \) and \( T=(2\pi)^{-1}, U=\pi \), hence,

\[ I_1 = \int_0^\pi \sqrt{\sin(U)} dU \]

Equation (A5.4) can be integrated by the gamma function in the form of

\[ I_1 = \frac{1}{\omega} \frac{\Gamma \left( \frac{3}{4} \right)}{\Gamma \left( \frac{5}{4} \right)} \sqrt{\pi} \]
Pulse combustion

Where,

\[ \Gamma(\frac{3}{4}) = \frac{0.9191005}{0.75} \quad \text{and} \quad \Gamma(\frac{5}{4}) = 0.910813 \]

Equation A5.5 can therefore be simplified as:

\[ \Xi = \frac{\sqrt{\pi}}{\omega} \cdot 1.34546532 \]

The mean value of \( \bar{V} \) can be calculated by integration over a cycle period giving,

\[ \bar{V} = \frac{1}{T} \int_{0}^{T} (V_{\text{air}} + V_{\text{fuel}}) \, dt = (\bar{V}_{\text{air}} + \bar{V}_{\text{fuel}}) \]

Where \( \bar{V}_{\text{air}} \) and \( \bar{V}_{\text{fuel}} \) are mean volume flow rates of the mixtures into combustion chamber. These values are dependent on the input firing rate and the relative air:fuel ratio.

Substitution equation (A5.6) and (A5.7) into equation (A5.2), yield the expression for \( J \) form of

\[ J = \frac{\bar{V}}{\Xi \sqrt{\frac{2(\Delta P)}{\rho_{\text{mix}}}}} \]

Substitution of the \( J \) value into equation (A5.1), gives the instantaneous mixture flow rate into combustion chamber.
Appendix A5.2: Adiabatic flame temperature calculation

The procedures of simulation of adiabatic flame temperature may be calculated by executing the following steps.

1. Energy balance equation may be evaluated as $H_p = cv + H_R$.

Where:

- $cv$ The net specific heat of the fuel (J/kg).
- $H_R$ The enthalpy of the fuel and air (ref. 25 °C) and may be calculated from $H_R = (t_i - 25) \sum (mc)_R$. The summation is carried out for each of the species present in the reactants and $t_i$ is initial temperature of fuel and air.
- $H_p$ The enthalpy of the products and is expressed as $H_p = (t_f - 25) \sum (mc)_P$.

2. Since $H_p$ is expressed as $H_p = (t_f - 25) \sum (mc)_P$, the value of $t_f$ is not solved explicitly. However, it was suggested by Hanby [5.4] that $t_f$ may be estimated in calculation of specific heats of combustion products at the average between the flame and the reference temperature, $\left(\frac{t_f + 25}{2}\right)°C$. The evaluated specific heat value of species may then be used to solve energy balance equation.

3. A substantial discrepancy exits between this new value of $t_f$ and the original value. The new-value of $t_f$ would be used to re-evaluate specific heats values of combustion species corresponding to step 2. Thus, steps (2) and (3) would be repeated until convergence is achieved.

This method of predicting the flame temperature is used in many engineering calculations. Hanby [5.4] reported that the value of $t_f$ generally converges to within 5 °C in four iterations.
Appendix A5.3

Relationship between predicted NO formation and combustion frequency at different input firing rates (at $\lambda=1$, $T_w=65^\circ C$ and $75^\circ C$, and $+\Delta P=0.05$ bar)
Figures A5.3.1:a-d. Relationship between predicted NO formation and combustion frequency at different input firing rates (at $\lambda=1$, $T_w=65 \, ^{\circ}C$ and $+\Delta P=0.05 \, \text{bar}$).
Figures A5.3.2:a-d. Relationship between predicted NO formation and combustion frequency at different input firing rates (at $\lambda=1$, $Tw=75^\circ C$ and $+\Delta P=0.05$ bar).
Appendix A5.4

Predicted and measured NO concentration against combustion frequency at different input firing rates (at $\lambda=1$, Tw=65 °C and 75 °C, and $+\Delta P=0.05$ bar)
Pulse combustion

Figures A5.4.1:a-d. Predicted and measured NO concentration against combustion frequency (at $\lambda=1$, $Tw=65^\circ C$ and $\Delta P=0.05$ bar).
Figures A5.4.2:a-d. Predicted and measured NO concentration against combustion frequency (at $\lambda=1$, $T_w=75^\circ$C and $+p=0.05$ bar).
Appendix A5.5

Predicted NO formation against positive pressure amplitude at different input firing rates (at $\lambda=1$, $T_w=65 \degree C$ and $75 \degree C$)
Figures A5.5.1:a-d. Predicted NO formation against positive pressure amplitude at different input firing rates (at $\lambda=1$, $T_w=65^\circ C$).
Figures A5.5.2:a-d. Predicted NO formation against positive pressure amplitude at different input firing rates (at $\lambda=1$, $Tw=75^\circ C$).
Appendix A5.6

Predicted and measured NO concentration against positive pressure amplitude at different input firing rates (at $\lambda=1$, $T_w=55^\circ C$ and $L=1.55 \text{ m to 3.0m}$)
Figures A5.6.1:a-d. Predicted and measured NO concentration at different input firing rates (at $\lambda=1$, $T_w=55 ^\circ C$ and tailpipe length 1.55m).
Figures A5.6.2:a-d. Predicted and measured NO concentration at different input firing rates (at $\lambda=1$, $T_w=55^\circ$C and tailpipe length 2.0m).
Figures A5.6.3:a-d. Predicted and measured NO concentration at different input firing rates (at $\lambda=1$, $T_w=55{^\circ}C$ and tailpipe length 2.55m).
Pulse combustion

\[ \lambda = 1, \ Tw = 55^\circ C, \ L = 3.0 \text{m and input firing rate 10kW} \]

\begin{center}
\begin{tabular}{c}
\begin{tikzpicture}
\begin{axis}[
    width=\textwidth,
    height=0.5\textwidth,
    xlabel=Positive pressure amplitude $+\Delta P$, bar,
    ylabel=\text{NO (ppm) @3\% O}_2, \text{(dry)},
    xmin=0,
    xmax=0.15,
    ymin=0,
    ymax=120,
    xtick={0,0.05,0.1,0.15},
    ytick={0,20,40,60,80,100,120},
    xticklabels={0,0.05,0.1,0.15},
    yticklabels={0,20,40,60,80,100,120},
    legend pos=north west,
]
\addplot [solid,mark=square] table [y index=1] {data1.dat};
\addlegendentry{Measured NO emissions}
\addplot [dashed,mark=dot] table [y index=1] {data2.dat};
\addlegendentry{Predicted total NO formation}
\end{axis}
\end{tikzpicture}
\end{tabular}
\end{center}

\[
\lambda = 1, \ Tw = 55^\circ C, \ L = 3.0 \text{m and input firing rate 15kW} \]

\begin{center}
\begin{tabular}{c}
\begin{tikzpicture}
\begin{axis}[
    width=\textwidth,
    height=0.5\textwidth,
    xlabel=Positive pressure amplitude $+\Delta P$, bar,
    ylabel=\text{NO (ppm) @3\% O}_2, \text{(dry)},
    xmin=0,
    xmax=0.15,
    ymin=0,
    ymax=120,
    xtick={0,0.05,0.1,0.15},
    ytick={0,20,40,60,80,100,120},
    xticklabels={0,0.05,0.1,0.15},
    yticklabels={0,20,40,60,80,100,120},
    legend pos=north west,
]
\addplot [solid,mark=square] table [y index=1] {data1.dat};
\addlegendentry{Measured NO emissions}
\addplot [dashed,mark=dot] table [y index=1] {data2.dat};
\addlegendentry{Predicted total NO formation}
\end{axis}
\end{tikzpicture}
\end{tabular}
\end{center}

\[
\lambda = 1, \ Tw = 55^\circ C, \ L = 3.0 \text{m and input firing rate 20kW} \]

\begin{center}
\begin{tabular}{c}
\begin{tikzpicture}
\begin{axis}[
    width=\textwidth,
    height=0.5\textwidth,
    xlabel=Positive pressure amplitude $+\Delta P$, bar,
    ylabel=\text{NO (ppm) @3\% O}_2, \text{(dry)},
    xmin=0,
    xmax=0.15,
    ymin=0,
    ymax=120,
    xtick={0,0.05,0.1,0.15},
    ytick={0,20,40,60,80,100,120},
    xticklabels={0,0.05,0.1,0.15},
    yticklabels={0,20,40,60,80,100,120},
    legend pos=north west,
]
\addplot [solid,mark=square] table [y index=1] {data1.dat};
\addlegendentry{Measured NO emissions}
\addplot [dashed,mark=dot] table [y index=1] {data2.dat};
\addlegendentry{Predicted total NO formation}
\end{axis}
\end{tikzpicture}
\end{tabular}
\end{center}

\[
\lambda = 1, \ Tw = 55^\circ C, \ L = 3.0 \text{m and input firing rate 25kW} \]

\begin{center}
\begin{tabular}{c}
\begin{tikzpicture}
\begin{axis}[
    width=\textwidth,
    height=0.5\textwidth,
    xlabel=Positive pressure amplitude $+\Delta P$, bar,
    ylabel=\text{NO (ppm) @3\% O}_2, \text{(dry)},
    xmin=0,
    xmax=0.15,
    ymin=0,
    ymax=120,
    xtick={0,0.05,0.1,0.15},
    ytick={0,20,40,60,80,100,120},
    xticklabels={0,0.05,0.1,0.15},
    yticklabels={0,20,40,60,80,100,120},
    legend pos=north west,
]
\addplot [solid,mark=square] table [y index=1] {data1.dat};
\addlegendentry{Measured NO emissions}
\addplot [dashed,mark=dot] table [y index=1] {data2.dat};
\addlegendentry{Predicted total NO formation}
\end{axis}
\end{tikzpicture}
\end{tabular}
\end{center}

Figures A5.6.4:a-d. Predicted and measured NO concentration at different input firing rates (at $\lambda = 1$, $Tw = 55^\circ C$ and tailpipe length 3.0m).
Appendix A5.7

Predicted NO formation varies air:fuel ratio from 1 to 1.3 at different input firing rates (at Tw=55 °C and L=2.0 m, 2.55m and 3.0m)
Figures A5.7.1:a-d. Predicted NO formation varies with relative air:fuel ratio from 1 to 1.3 at different input firing rates (at $T_w=55^\circ C$ and tailpipe length 2.0m).
Figures A5.7.2:a-d. Predicted NO formation varies with relative air: fuel ratio from 1 to 1.3 at different input firing rates (at Tw=55 °C and tailpipe length 2.55m).
Figures A5.7.3:a-d. Predicted NO formation varies with relative air:fuel ratio from 1 to 1.3 at different input firing rates (at Tw=55 °C and tailpipe length 3.0m).
Appendix A6.1: Explicit Finite-Difference Scheme

The transition of differential to finite-difference equations (FDE) can be accomplished by several methods \([6.1][6.2]\). For example, explicit and implicit finite-difference schemes, are related by concepts of accuracy, order of precision, convergence, stability of numerical schemes, and truncation errors. A simple scalar equation such as the diffusion equation is used here to illustrate these concepts. The parabolic linear partial differential diffusion equation can be written as a one space variable as shown below:

\[
\frac{\partial u}{\partial t} = C \frac{\partial^2 u}{\partial x^2} \quad 0 < x < 1, \ t > 0
\]

Where:

- \(u\) Dependent variable
- \(x\) Spatial variable
- \(t\) Temporal variable
- \(C\) Positive constant

The effect of this equation is expressed in Figure A6.1.1:

![Figure A6.1.1. Initial and boundary condition of this equation.](image)
Initial condition:

\[ u(x,0) = u_0(x), \quad 0 \leq x \leq 1 \]

Boundary conditions:

\[ u(0,t) = g_0(t) \quad t > 0 \]
\[ u(1,t) = g_1(t) \quad t > 0 \]

A sufficiently smooth function \( f(s) \) stating the distance travelled by gases can be expressed by the Taylor series about a given parameter, "s" as shown below:

\[
\begin{align*}
 f(s+h) &= f(s) + hf(s) + \left( \frac{h^2}{2} \right)f'(s) + \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (A6.1) \\
 f(s-h) &= f(s) - hf(s) + \left( \frac{h^2}{2} \right)f'(s) - \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (A6.2)
\end{align*}
\]

Where \( h \) is the incremental spacing along the s-direction as illustrated in Figure A6.1.2, the prime indicates differentiation with respect to s.

Figure A6.1.2. Variation of the function \( f \) along the s-coordinate.

Forward differences of the derivative of \( f(s) \) are defined as:
\[ \frac{f(s + h) - f(s)}{h} = \left( \frac{1}{h} \right) \left[ f(s) + hf'(s) + \frac{h^2}{2!} f''(s) + \ldots \right] - f(s) \]

\[ = f'(s) + \frac{h}{2!} f''(s) + \frac{h^2}{3!} f'''(s) + \ldots \quad \text{(A6.3)} \]

Simplifying this equation gives:

\[ f'(s) = \left( \frac{f(s + h) - f(s)}{h} \right) + O(h) \quad \text{(A6.4)} \]

Where \( O(h) \) represents the first "order of h". The appearance of higher derivation terms of the equations, known as truncation errors (TE), are due to the rounding off of higher-order terms in the Taylor series. A truncation at the second term \( \frac{h}{2!} f''(s) \), portrays the accuracy of the order \( h \) while that at the third term \( \frac{h^2}{3!} f'''(s) \) illustrates the accuracy of \( h^2 \) etc. When the number of the grid points are large, \( h \) is small and the truncation error is reduced.

Backward differences of derivatives are defined as:

\[ \frac{f(s + h) - f(s - h)}{2h} = \frac{1}{2h} \left[ f(s) + hf'(s) + \frac{h^2}{2} f''(s) + \ldots - f(s) + hf'(s) - \frac{h^2}{2} f''(s) - \ldots \right] \]

\[ \frac{f(s) - f(s - h)}{h} = \frac{1}{h} \left[ f(s) - f(s) + hf'(s) - \frac{h^2}{2} f''(s) - \ldots \right] \]

\[ = f'(s) - \frac{h}{2} f''(s) + \frac{h^2}{3!} f'''(s) - \ldots \quad \text{(A6.5)} \]
Equation (A6.5) can also be expressed in other forms:

\[ f'(s) = \frac{f(s) - f(s-h)}{h} + 0(h) \] .......................... (A6.6)

Combination of equations (A6.4) and (A6.6) gives the centered-difference form of first derivatives, an average of the difference between forward and backward formats.

\[ \frac{f(s+h) - f(s-h)}{2h} = \frac{1}{2h} \left[ f(s) + hf'(s) + \frac{h^2}{2} f''(s) + \cdots - f(s) + hf'(s) - \frac{h^2}{2} f''(s) + \cdots \right] \]

This equation can also be written as:

\[ \frac{f(s+h) - f(s-h)}{2h} = f'(s) + \frac{h^2}{3!} f''(s) + \cdots \] .......................... (A6.7)

A truncation at the second term of centered difference forms gives a precision of the order \((h^2)\) as opposed to that of \((h)\) for both forward and the backward difference schemes.

This means that halving the step length \(h\), approximately quarters the cause of error as the problem is treated in a series of finite steps.

It is also possible to define the second terms of first-order differences; with reference to Figure A6.1.3, an approximation of the second derivative can be written as:
\[ f''(s) = \frac{1}{h} \left[ \frac{f(s+h) - f(s)}{h} - \frac{f(s) - f(s-h)}{h} \right] + O(h^2) \] ................................(A6.9)

\[ f(s) - f(s-h) \quad f(s+h) - f(s) \]

\[ \frac{h}{h} \quad \frac{h}{h} \]

Figure A6.1.3: Alternative diagram for a finite-difference scheme.

A truncation at the second term of second-order differences also gives a precision of \( O(h^2) \), as illustrated in equation (A6.9). Second-order differences can also be defined in terms of first-order differences. According to Figure A6.1.3, approximations of second derivatives can be represented by:

\[ f''(s) = \frac{1}{h} \left[ \frac{f(s+h) - f(s)}{h} - \frac{f(s) - f(s-h)}{h} \right] + O(h^2) \]

or

\[ f''(s) = \frac{f(s+h) - 2f(s) + f(s-h)}{h^2} + O(h^2) \] ................................(A6.10)

Explicit finite-difference schemes for diffusion equations can be characterized by describing two derivatives, one in the t-direction and the other along the x-direction. Using a forward-difference approximation for time derivatives and a central-difference approximation for spatial derivatives, the diffusion equation can be simplified to:

\[ \frac{\partial u}{\partial t} = C \frac{\partial^2 u}{\partial x^2} \]
This equation can be written as:

\[
\frac{u_{j+1}^n - u_j^n}{\Delta t} = C \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}
\] (A6.11)

Figure A6.1.4. illustrates the numbering of grid points with spatial co-ordinates j-1, j, j+1 and temporal co-ordinates n and n+1.

Figure A6.1.4: Diagram for the explicit finite-difference scheme.

In order to solve this explicit method, the unknown function \( u_{j+1}^{n+1} \) for new time levels \((n+1)\) must be expressed specifically in terms of known values of \( u \) at locations j-1, j and j+1 on previously calculated time levels \((n)\). This calculation proceeds directly from one time increment to the next until the desired time lapse \((\bullet) t = n\Delta t\), is reached. The resulting solution gives the value of \( u \) at discrete points in space and time.

The truncation error may be reduced by selecting a finer grid, i.e, let smaller size grids represent time and space increments. This allows diffusion equations (PDE) to be written in numerical forms (FDE) and truncation errors to be defined as:
Pulse combustion

\[
\frac{\partial u}{\partial t} - C \frac{\partial^2 u}{\partial x^2} = \frac{\partial u_{j+1}^{n+1}}{\partial t} - \frac{C}{(\Delta x)^2} \left( u_{j+1}^n - 2u_j^n + u_{j-1}^n \right) + \left[ \left( -\frac{\partial^2 u}{\partial t^2} \right)_{n,j} \frac{\Delta t}{2} + C \left( \frac{\partial^4 u}{\partial x^4} \right)_{n,j} \frac{(\Delta x)^2}{12} + \ldots \right]
\]

Where:

\[
PDE = \frac{\partial u}{\partial t} - C \frac{\partial^2 u}{\partial x^2}
\]

\[
FDE = \frac{\partial u_{j+1}^{n+1}}{\partial t} - \frac{C}{(\Delta x)^2} \left( u_{j+1}^n - 2u_j^n + u_{j-1}^n \right)
\]

\[
TE = \left[ \left( -\frac{\partial^2 u}{\partial t^2} \right)_{n,j} \frac{\Delta t}{2} + C \left( \frac{\partial^4 u}{\partial x^4} \right)_{n,j} \frac{(\Delta x)^2}{12} + \ldots \right]
\]

The function \( u_{j+1}^{n+1} \) can be approximated to:

\[
u_{j+1}^{n+1} = u_j^n + \frac{C\Delta t}{(\Delta x)^2} \left( u_{j+1}^n - 2u_j^n + u_{j-1}^n \right) \ldots \ldots \ldots \ldots (A6.12)
\]

In the explicit scheme, the solution \( u_j^{n+1} \) can be computed numerically in a straightforward way as shown in Figure A6.1.5, where the symbol (●) indicates points at which initial and boundary conditions are given.
Figure A6.1.5: Grid points for the explicit finite-difference scheme.

(•) in Figure A6.1.5 indicates points of known solutions and (*) illustrates points of required solutions.