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Acoustic black holes and their applications for vibration damping and sound absorption

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Abstract

During the last decade, new interesting physical objects have been invented and investigated - 'acoustic black holes', whereby it is possible to achieve almost 100% absorption of the incident wave energy. The main principle of the 'acoustic black hole effect' is based on a gradual power-law-type decrease in velocity of the incident wave with propagation distance, linear or faster, to almost zero, which should be accompanied by efficient energy absorption in the area of low velocity via inserted highly absorbing materials. So far, this effect has been investigated mainly for flexural waves in thin plates for which the required gradual reduction in wave velocity with distance can be easily achieved by changing the plate local thickness according to a power law, with the power-law exponent being equal or larger than two. The present paper provides a brief review of the theory of acoustic black holes, including their comparison with 'optic black holes' invented about three years ago. It is shown in particular that optic black holes are based on the same principle that governs the behaviour of acoustic black holes. Review is also given of the recent, mainly experimental, work carried out at Loughborough University on damping structural vibrations based on the acoustic black hole effect. This is followed by the discussion on potential applications of the acoustic black hole effect for sound absorption in air.

1 Introduction

During the last decade, the present author and his co-workers have developed the theory and suggested a number of practical applications of the new phenomenon that has become known as 'acoustic black hole effect', whereby it is possible to achieve almost 100% absorption of the incident wave energy irrespective of the incidence angle (see e.g. [1-6]). The main principle of this effect is based on a linear or higher order power-law-type decrease in velocity of the incident wave with propagation distance to almost zero accompanied by efficient energy absorption in the area of low velocity via small pieces of inserted absorbing materials. So far, this effect has been investigated mainly for flexural waves in thin plates for which the required gradual reduction in wave velocity with distance can be easily achieved by changing the plate local thickness according to a power law, with the power-law exponent being equal or larger than two. This principle has been applied to achieve efficient damping of flexural waves in plate-like structures using both one-dimensional 'acoustic black holes' (power-law wedges with their sharp edges covered by narrow strips of absorbing materials) and two-dimensional 'acoustic black holes' (power-law-profiled pits with small pieces of absorbing materials attached in the middle).

Recently, the above-mentioned basic principle of power-law reduction in velocity accompanied by efficient energy absorption in the area of low wave velocity has been re-invented in optics to create the so-called 'optic black holes' [7, 8]. In particular, it has been shown that a spherical or a cylindrical layer with dielectric constant altering with distance according to the inverse power law can capture incident light and dissipate it efficiently in the central absorbing core.

In the present paper, the theory of acoustic black holes is briefly discussed, and a comparative analysis of acoustic and optic black holes is given. The theory of acoustic and optic black holes is based on
geometrical acoustics and geometrical optics approximations respectively, and it is shown in this paper that it is largely the same. The public attention to acoustic and optic black holes though differs significantly. Optic black holes have immediately attracted attention of the optics research community as effective means of harvesting solar energy and its conversion into heat [9]. However, their acoustic counterparts, in spite of the earlier start, are still limited mainly to vibration damping applications in plate-like structures [5, 6].

Some recent examples of such vibration damping applications are discussed in the present paper. Experimental investigations have been carried out on a variety of plate-like and beam-like structures. Such structures included plates or beams bounded by the attached power-law wedges, with the addition of small amounts of absorbing material at the sharp edges. The above-mentioned wedge-like structures at the edges materialise one-dimensional acoustic black holes. Other structures that have been investigated are plates with tapered pits of power-law profile drilled inside the plates. In the case of quadratic or higher-order profiles, such pits materialise two-dimensional acoustic black holes for flexural waves. To make them even more efficient, ensembles of several (up to six) two-dimensional black holes have been used for experimental investigations. Among other promising types of acoustic black-hole geometry that have been studied were power-law slots machined inside plates. Some specific structures considered were composite plates and panels with one- and two-dimensional acoustic black holes and turbo-fan blades with trailing edges machined according to a power-law profile to form one-dimensional acoustic black holes.

The results of the experimental investigations have demonstrated that in all of the above-mentioned cases the efficiency of vibration damping based on the acoustic black hole effect is substantially higher than that achieved by traditional methods. The key advantage of using the acoustic black hole effect for damping structural vibrations is that it requires very small amounts of added damping materials, which is especially important for damping vibrations in light-weight structures used in aeronautical and automotive applications.

There are still very few investigations of acoustic black holes for absorption of sound in air. The possibilities of construction of such acoustic black holes that could be used for traditional noise control are considered, and the associated difficulties and limitations are discussed.

# 2 Outline of the theory

## 2.1 Effect of zero reflection: general case

The ‘effect of zero reflection’ is the wave phenomenon that can be described using geometrical acoustics (optics) solutions for wave propagation in inhomogeneous media with some specific types of variation of wave velocity with propagation distance. Considering for simplicity a one-dimensional wave propagation characterised by the distance \( x \) in an ideal medium with power-law dependence of wave velocity \( c \) on \( x \) as \( c = ax^n \), where \( n \) is a positive rational number and \( a \) is a constant, one can express the geometrical acoustics solution for the complex amplitude \( U(x) \) of a wave propagating from any arbitrary point \( x \) towards zero point (where \( c = 0 \)) as

\[
U(x) = A(x)e^{i\Phi(x)},
\]

where

\[
\Phi = \int_{0}^{x} k(x)dx
\]

is a total accumulated phase. Since \( k(x) = \omega/c(x) = \omega/ax^n \), one can see from Eqn (2) that the phase \( \Phi \) becomes infinite if \( n \geq 1 \). This means that under these circumstances the wave never reaches the edge. Therefore, it never reflects back either, i.e. the wave becomes trapped, thus indicating that the above
mentioned ideal medium with a linear or higher power-law profile of wave velocity can be considered as ‘acoustic black hole’ for the wave under consideration.

For the first time this phenomenon has been described in 1946 by Pekeris [10] for sound waves in a stratified ocean, for a layer with sound velocity profile linearly decreasing to zero with increasing depth. Later on, several other authors have predicted the possibility of the effects of zero reflection for wave phenomena of different physical nature: in particular, for internal water waves in a horizontally inhomogeneous stratified fluid [11, 12], as well as for particle scattering in quasi-classical approximation of quantum mechanics [13]. Mironov [14] has predicted a practically important possibility of zero reflection of flexural waves from a tip of an ideal quadratic wedge. Note that a quadratic wedge provides the above-mentioned linear decrease in flexural wave velocity towards a sharp edge. And, whereas the conditions providing a linear or higher-order decrease in wave velocity can be rarely found in a real ocean environment, elastic wedges of arbitrary power-law profile are relatively easy to manufacture. Thus, elastic solid wedges give a unique and very convenient opportunity to materialise the above-mentioned zero-reflection effects associated with ‘black holes’ and to use them for practical purposes.

2.2 One-dimensional acoustic black holes for flexural waves

To understand the phenomenon of acoustic black holes for the case of flexural waves one can consider the simplest one-dimensional case of plane flexural wave propagation in the normal direction towards the edge of a free elastic wedge described by a power-law relationship $h(x) = \varepsilon x^m$, where $m$ is a positive rational number and $\varepsilon$ is a constant (Figure 1).

![Figure 1: Geometry of an elastic wedge of power-law profile](image)

Since flexural wave propagation in such wedges can be described in the geometrical acoustics approximation (see [15, 16] for more detail), the integrated wave phase $\Phi$ resulting from the wave propagation from an arbitrary point $x$ located in the wedge medium plane to the wedge tip ($x = 0$) can be expressed by the above-mentioned Eqn (2). In this case though $k(x)$ is a local wavenumber of a flexural wave for a wedge in contact with vacuum: $k(x) = 12^{1/4} k_p^{1/2}(\varepsilon x^m)^{1/2}$, where $k_p = \omega/c_p$ is the wavenumber of a symmetrical plate wave, $c_p = 2c_s(1-c_s^2/c_t^2)^{1/2}$ is its phase velocity, and $c_l$ and $c_s$ are longitudinal and shear wave velocities in a wedge material, and $\omega = 2\pi f$ is circular frequency. Again, one can easily see that the integral in Eqn (2) diverges for $m \geq 2$. This means that the phase $\Phi$ becomes infinite under these circumstances and the wave never reaches the edge. Therefore, it never reflects back either, i.e. the wave becomes trapped, thus indicating that the above mentioned ideal wedges represent acoustic ‘black holes’ for incident flexural waves.

Real fabricated wedges, however, always have truncated edges. And this adversely affects their performance as ‘black holes’. If for ideal wedges of power-law shape (with $m \geq 2$) it follows from Eqn (2) that even an infinitely small material attenuation, described by the imaginary part of $k(x)$, would be sufficient for the total wave energy to be absorbed, this is not so for truncated wedges. Indeed, for truncated wedges the lower integration limit in Eqn (2) must be changed from $0$ to a certain value $x_0$.
describing the length of truncation. Therefore, as was first noticed in [14], for typical values of attenuation in such materials as steel, even very small truncations $x_0$ result in the reflection coefficients $R_0$ becoming as large as 50-70%, which makes it impossible to use such wedges as practical vibration dampers.

However, it was proposed by the present author [1-4] that the situation for real wedges (with truncations) can be drastically improved via increasing wave energy dissipation in the area of slow wave velocity (near the sharp edges) by covering wedge surfaces near the edges by thin absorbing layers (films), e.g. by polymeric films. The simplest way of approaching this problem is to use the already known solutions for plates covered by absorbing layers of arbitrary thickness obtained by different authors with regard to the description of damped vibrations in such sandwich plates. Using this approach (see [3, 4] for more detail), one can derive the corresponding analytical expressions for the reflection coefficients of flexural waves from the edges of truncated wedges covered by absorbing layers.

For example, for a wedge of quadratic shape, i.e. with $h(x) = \varepsilon x^2$, covered by thin absorbing layers on both surfaces the following analytical expression for the resulting reflection coefficient $R_0$ can be derived [3, 4]:

$$R_0 = \exp(-2\mu_1 - 2\mu_2),$$

where

$$\mu_1 = \frac{12^{1/4}k_{p}^{1/2}\eta}{4E_{1}^{1/2}} \ln \left( \frac{x}{x_0} \right),$$

$$\mu_2 = \frac{3 \cdot 12^{1/4}k_{p}^{1/2}\nu\delta}{4E_{2}^{1/2}} \frac{E_2}{E_1} \frac{1}{x_0^2} \left( 1 - \frac{x_0^2}{x^2} \right).$$

Here $\nu$ is the loss factor of the material of the absorbing layer and $\delta$ is its thickness, $\eta$ is the loss factor of the wedge material, $x_0$ is the wedge truncation length, $E_1$ and $E_2$ are respectively the Young’s moduli of the plate and of the absorbing layer, $x$ is the coordinate of the point of observation taken at a sufficiently large distance from the wedge tip. In the case of a wedge of quadratic shape covered by damping layers on one surface only, Eqns (3) and (4) remain unchanged, whereas Eqn (5) is replaced by

$$\mu_2 = \frac{3 \cdot 12^{1/4}k_{p}^{1/2}\nu\delta}{8E_{2}^{1/2}} \frac{E_2}{E_1} \frac{1}{x_0^2} \left( 1 - \frac{x_0^2}{x^2} \right).$$

In deriving Eqns. (3)-(6), the effect of absorbing layers on flexural wave velocity has been neglected, which can be done for very thin absorbing layers. The extension of the analysis to the case of absorbing layers of arbitrary thickness has been made in the paper [4]. Note that geometrical acoustics approximation for the above-mentioned quadratic wedges ($m = 2$) is valid for all $x$ provided that the following applicability condition is satisfied:

$$\frac{\omega}{c_t \varepsilon} >> 1,$$

where $c_t$ is shear wave velocity in the wedge material. For the majority of practical situations this condition can be easily satisfied even at very low frequencies.

Calculations according to Eqns (3)-(6) show that, if an absorbing layer is present ($\delta \neq 0$ and $\nu \neq 0$), it brings a very substantial reduction of the reflection coefficient, sometimes down to 1-3% [3, 4], which constitutes the ‘acoustic black hole effect’. This means that the combined effect of power-law geometry of a wedge and of a thin absorbing layer makes such systems viable and attractive for practical applications.
The same principle of combining power-law geometry with thin absorbing layers can be applied also to beams of power-law profile [6, 17].

The first experimental observation of the ‘acoustic black hole effect’ for a wedge of quadratic profile has been described in the paper [5]. The system under investigation consisted of a steel wedge of quadratic shape covered on one side by a strip of absorbing layer located at the sharp edge. The wedge dimensions were: 280 mm (length) and 200 mm (width). Its thickness at the thick end was 4.5 mm, and the value of the quadratic wedge parameter $\varepsilon$ was $5 \times 10^{-5}$ mm$^{-1}$. The mass density of the wedge material was 7900 kg/m$^3$, with the velocities of longitudinal and shear elastic waves being equal to 5900 m/s and 3200 m/s respectively. The mass density of the absorbing layer was 1160 kg/m$^3$, with the velocity of longitudinal waves equal to 1900 m/s. Measurements of point mobility (also known as driving point admittance) have been carried out in the frequency range 100-6500 Hz for a free wedge, for a wedge covered by narrow strips of thin absorbing layers (e.g. isolating tapes), and for a free and covered reference plates of constant thickness $h = 4.5$ mm having the same length and width as the above-mentioned quadratic wedge.

![Figure 2: Measured point mobility of a quadratic steel wedge: free wedge (solid curve) and wedge covered by a thin adhesive strip of thickness $\delta = 0.2$ mm (dashed curve) [5]](image)

The results of the measurements of point mobility are shown in Figure 2. It can be seen that, in agreement with the theory, a very significant reduction of resonant peaks (up to 20 dB) can be observed in a wedge covered by an absorbing layer, in comparison with the uncovered wedge or with the reference plates of constant thickness. This demonstrates that vibration damping systems utilising the acoustic black hole effect are efficient and suitable for practical applications.

### 2.3 Two-dimensional acoustic black holes for flexural waves

Two-dimensional acoustic black holes for flexural waves, such as protruding cylindrically symmetrical indentations (pits) drilled in a regular thin plate of constant thickness (see Figure 3), have been first proposed and considered by the present author [6] using geometrical acoustics approach in Hamiltonian formulation. Earlier, a similar approach was applied to analysing Rayleigh surface wave propagation across smooth large-scale surface irregularities [18, 19]. The analysis shows that, in the case of symmetrical pits of power-law profile with $m \geq 2$, a number of rays that are close enough to a direct ray,
including a direct ray itself, will deflect towards the centre of the pit, approaching it almost in the normal direction (Figure 4). Since the central area of the pit covered by absorbing material (not shown in Figure 3) acts as an efficient absorber for flexural waves, all such ‘captured’ rays can be considered as fully absorbed rays that have taken away part of the energy of the incident wave.

![Diagram of a cylindrical pit](image1)

**Figure 3:** A protruding cylindrically symmetrical pit of power-law profile materialising a two-dimensional acoustic black hole [6]

Note that the above-mentioned two-dimensional acoustic black holes (power-law pits) can be placed at any point of a plate or any other plate-like or shell-like structure. The effect of such black hole is in eliminating some rays, intersecting with the black hole, from contributing to the overall frequency response function of a structure, which may result in substantial damping of some resonant peaks in the frequency response function. To amplify the vibration damping effect of two-dimensional acoustic black holes one can place ensembles of several black holes distributed over the structure (e.g. periodic arrays of black holes), if this does not compromise its main functions, e.g. its rigidity.

![Diagram of ray trajectories](image2)

**Figure 4:** Typical ray trajectories illustrating propagation of bending waves over a pit of power-law profile; the ray below is trapped by the black hole, which means that its reflection (scattering) from it can be negligible [6]
One of the most important advantages of the above-mentioned one-dimensional and two-dimensional acoustic black holes as dampers of structural vibrations is that they are efficient even for relatively thin and narrow strips of attached absorbing layers. The reason for this is that wave energy dissipation takes place mainly in a very narrow area near sharp edges. This is in contrast with the traditional techniques employing covering the whole surfaces of structures by relatively thick layers of absorbing materials [20, 21]. And this important feature of the acoustic black holes can be very attractive for many practical applications, especially for those involving light-weight structures used in aeronautical and automotive engineering.

The first experimental investigation of two-dimensional acoustic black holes has been described in [22] (see also [23]) for a power-law pit located in one of the foci of an elliptical plate, with an electromagnetic shaker being located in another focus. In such a plate, all radiated flexural wave rays were focused in the area of black hole, which amplified the damping efficiency. Experimental investigation of two-dimensional acoustic black holes placed in arbitrary locations of rectangular plates was reported in the paper [24].

### 2.4 Optic black holes and their comparison with acoustic black holes

Recently, the above-mentioned basic principle of power-law reduction in velocity accompanied by efficient energy absorption in the ‘slow’ area has been re-invented in optics to create the so-called ‘optic black holes’ [7, 8]. In particular, it has been shown in [7] that a spherical or a cylindrical layer with dielectric constant \( \varepsilon = \varepsilon(r) \) altering with distance \( r \) according to the inverse power law: \( \varepsilon \sim 1/r^n \), with \( n \geq 2 \), can capture incident light and direct it towards the absorbing core. Since the velocity of light is inversely proportional to \( (\varepsilon)^{1/2} \), the condition \( \varepsilon \sim 1/r^n \), where \( n \geq 2 \), implies that the velocity of light should experience linear or higher order power-law type decrease towards the centre. This is exactly the same type of velocity reduction that has been considered above for the general case of wave propagation, regardless of the physical nature of the wave (see Section 2.1). Note that the method of analysis used in [7], mentioned by the authors as ‘semiclassical approach’ and ‘Hamiltonian mechanics’, is in fact equivalent to the geometrical acoustics (optics) approach in Hamiltonian formulation used in [6] for the analysis of two-dimensional acoustic black holes for flexural waves. Apparently, the authors of [7, 8] were unaware of the above-mentioned earlier papers on acoustic black holes [3-6], let alone the papers on zero reflection (‘wave capturing’) properties of layers and systems with wave velocities varying with distance according to power-law profiles [10-14], starting from the work of Pekeris published in 1946.

It is quite obvious that the principles and theories of the above-mentioned acoustic and optic black holes are almost identical. The public attention to these objects though differs significantly. If optic black holes have immediately attracted attention of the optics research community and practitioners as effective means of harvesting solar energy and its conversion into heat [9], their acoustic counterparts, in spite of the earlier start, are still limited mainly to a few vibration damping applications. Some of these applications and their investigations will be briefly discussed in the following section.

### 3 Some recent investigations of the acoustic black hole effect

#### 3.1 Rectangular plates with attached wedges of power-law profile

One of the most important types of structures that have been recently investigated at Loughborough included plates or beams bounded by the attached power-law wedges, with the addition of small amounts of absorbing material at their sharp edges [25, 26]. The above-mentioned wedges-like structures (see Figure 5) materialise one-dimensional acoustic black holes for flexural waves placed at the edges of basic rectangular plates vibrations of which are to be damped. The experiments have demonstrated that power-law wedges covered by narrow strips of thin absorbing layers, when attached to edges of rectangular
plates, are much more efficient dampers of structural vibrations than traditional rather thick layers of absorbing materials covering entire plate surfaces.

Figure 5: Photo showing part of a rectangular plate bounded by a power-law profiled wedge (with \( m = 2.2 \)) [26]

In the paper [27], the effects of deviations of real manufactured wedge-like structures from ideal elastic wedges of power-law profile on damping flexural vibrations have been investigated experimentally. Namely, the effect of mechanical damage to wedge tips has been studied, including tip curling and early truncation, as well as the placement of absorbing layers on different wedge surfaces. Also, the effects of welded and glued bonding of wedge attachments to basic rectangular plates (strips) have been studied. In particular, it has been found in [27] that a tip damage (curling), resulting in a wedge with an extended sharp edge, is not detrimental for its damping performance. On the contrary, the extended wedge provided a very efficient damping. Note that similar experimental results have been obtained independently by Bayod [28] who investigated vibration damping in a power-law wedge extended at the sharp edge to form a thin plate of constant thickness, which was made specifically to overcome difficulties associated with manufacturing of very sharp wedges. It also has been demonstrated in [27] that attaching power-law profiled wedges to a rectangular plate (strip) by welding or via glue results in damping performance that generally is not worse than the performance of a homogeneous sample containing the same wedge at its edge. The main and very important conclusion that can be drawn from these investigations is as follows. Although the above-mentioned geometrical and material imperfections generally reduce the damping efficiency to various degrees, the method of damping utilising the acoustic black hole effect is robust enough and can be used widely without the need of high precision manufacturing.

### 3.2 Plates containing two-dimensional acoustic black holes

Other important structures that have been investigated at Loughborough were rectangular plates with tapered indentations (pits) of power-law profile drilled inside the plates [24, 25]. In the case of quadratic or higher-order profiles, such pits materialise two-dimensional acoustic black holes for flexural waves. To make them even more efficient, ensembles of several (up to six) two-dimensional black holes have been used. Note that two-dimensional black holes and their ensembles offer an important advantage in comparison with the case of one-dimensional acoustic black holes (wedges of power-law profile). Namely, the potentially dangerous sharp edges of power-law wedges can be eliminated.

It has been demonstrated in [24] that basic power-law indentations that are just protruding over the opposite plate surface cause very small reduction in resonant peak amplitudes, which may be due to their relatively small absorption cross-section. Introduction of a 2 mm central hole improved the situation and increased damping. To increase damping even more, the absorption cross-section has been enlarged by increasing the size of the central hole in the indentation up to 14 mm, while keeping the edges sharp. As
expected, such pits, becoming in fact curved power-law wedges, resulted in substantially increased
damping that was comparable with that achieved by one-dimensional wedges of power-law profile.

Theoretical and experimental investigations have been carried out also for a specific case of circular
indentations of power-law profile drilled in the centre of a circular plate [29, 30]. The advantage of this
geometrical configuration is that it facilitates theoretical calculations of the frequency response function.
Such calculations have been carried out, and they have shown good agreement with the experimental
measurements. In the paper [31], further research has been conducted into two-dimensional black holes in
elliptical plates in the case of focusing of radiated flexural waves (rays). One-dimensional theoretical
model has been developed in this case that has shown a good agreement with the measurements.

3.3 Some other acoustic black hole configurations and applications

Among other promising types of acoustic black-hole geometry that have been recently studied at
Loughborough were slots of power-law profile made inside rectangular plates [32]. As was mentioned
above, one of the problems faced by one-dimensional black holes formed by power-law wedges attached
to edges of structures is having the sharp wedge tip exposed on the outer edge. One of the solutions to this
problem is the two-dimensional black holes described in the previous section. Slots of power-law profile
placed inside structures represent another possible solution that moves the power-law wedges inside a
plate, so that they form edges of power-law slots within the plate. Different configurations of such slots in
plates have been manufactured and tested experimentally. It has been demonstrated that slots of power-
law profile located within plates materialise a specific type of quasi-one-dimensional acoustic black holes
for flexural waves and represent an effective method of damping structural vibrations. The maximum
damping achieved on a steel plate with a slot was about 11 dB.

Following the idea first proposed in [5], experimental investigations into damping of flexural vibrations in
turbofan blades with trailing edges tapered according to a power-law profile have been carried out [33].
Four samples of model fan blades have been manufactured. Two of them were then twisted, so that a more
realistic fan blade could be considered. All model blades were excited by an electromagnetic shaker, and
the corresponding frequency response functions have been measured. The results have shown that the fan
blades with power-law tapered edges have the same pattern of damping that can be seen for plates with
attached wedges of power-law profile, when compared to their respective reference samples. The resonant
peaks are reduced substantially once a power-law tapering is introduced to the sample. The obtained
results demonstrate that power-law tapering of trailing edges of turbofan blades can be a viable method of
reduction of blade vibrations.

In the paper [34], experimental investigations of one- and two-dimensional acoustic black holes made in
composite plates and panels have been carried out. The addition of acoustic black holes resulted in further
increase in damping of resonant vibrations, in addition to the already substantial inherent damping due to
large values of the loss factor for composites (0.1 - 0.2). Note that, due to large values of the loss factor for
composite materials used, no additional layers of absorbing material were required, as expected.

The results of these and earlier mentioned experimental investigations have demonstrated that in all of the
above-mentioned cases the efficiency of vibration damping based on the acoustic black hole effect is
substantially higher than that achieved by traditional methods. The key advantage of using the acoustic
black hole effect for damping structural vibrations is that it requires very small amounts of added damping
materials, which is especially important for damping vibrations in light-weight structures used in
aeronautical and automotive applications.

4 On acoustic black holes for sound absorption in air

It would be natural to extend the above-mentioned successful implementations of acoustic black holes for
flexural waves (as well as optic (electromagnetic) black holes, see Section 2.4) to the case of acoustic
black holes designed for sound absorption in air. If successful, such black holes could be applied widely for noise control both in open areas and inside enclosed spaces. There are still no convincing working prototypes of such black holes, and there are only two published papers where such objects have been considered. In the first theoretical paper [35], it was proposed to use an inhomogeneous waveguide with walls of variable impedance to achieve the required linear decrease in acoustic wave velocity. No absorbing core was assumed in this theoretical paper. Therefore, one could expect that in a real manufactured device there would be a substantial wave reflection, making this device unworkable. Adding an absorbing core though, as it was done in the above-mentioned cases of acoustic black holes for flexural waves, as well as in the case of optic black holes, could have made such a devise practical.

Another possibility of creating acoustic black holes for absorption of sound in air is to use specially designed graded metamaterials as wave retarding structures, as it was done in the case of optic black holes [7-9]. The first consideration of this possibility has been made recently [36] using graded sonic crystals formed by circular arrays of small scatterers with their filling fractions varying along the radius. In an array of small rigid circular cylinders, the effective density varies with filling fraction. This means that by varying the filling fraction the desired dependence of the effective density, and hence the required dependence of sound velocity on \( r \), can be achieved. This principle of wave velocity reduction has been implemented, and an absorbing core (a porous material) has been added to form the acoustic black hole [36]. The above-mentioned two types of acoustic black holes for sound absorption in air require further investigations, both theoretical and experimental. The expected outcome is to develop efficient sound absorption devices that would require much smaller amounts of absorbing material and thus would be lighter and more convenient than the existing sound absorbing panels.

In contrast to the cases of acoustic black holes for flexural waves and optic black holes, there is a serious problem associated with the development of acoustic black holes for sound absorption in air. This is the fact that at typical acoustic frequencies, the acoustic wavelengths are comparable with expected geometrical dimensions of practical black holes, which can make them non-operational at low and medium frequencies. The use of quicker varying power-law profiles could help to make black holes operational at medium frequencies. In this connection, it also seems beneficial to consider new types of geometry of acoustic black holes, in addition to circular cylinders and spheres [36]. In particular, similarly to elastic wedges of power-law profile implementing one-dimensional black holes for flexural waves, one can propose quasi-one-dimensional (flat) acoustic black holes for sound absorption in air (with the absorbing core placed on one side). Such devices could be attached to walls of an enclosure to reduce the overall noise level. Also, following the geometry of power-law slots within plates (see Section 3.3), representing quasi-one-dimensional double-sided black holes for flexural waves, one can propose double-sided quasi-one-dimensional acoustic black holes (with the absorbing core placed in the middle). Such black holes could be suspended on cables to absorb noise on both sides. To amplify the effect of acoustic black holes, ensembles or periodic arrays of black holes can be used.

5 Conclusion

As it follows from the above discussion, research into acoustic black holes for flexural waves now enters the age of maturity. Several useful designs and applications for efficient vibration damping have been proposed and tested. Research into acoustic black holes for sound absorption in air though is still in its infancy. Further investigations are needed to explore their full potential and test their viability.

References


