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BEHAVIOUR OF METALS AS A FUNCTION OF STRAIN-RATE AND TEMPERATURE

by

MARK ASHTON

A Doctoral Thesis
submitted in partial fulfilment of the requirements
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ABSTRACT

Five materials, copper (two versions), iron, and armour plate steel (two versions) have been tested at different strain-rates and temperatures. All tests were in compression. The materials were studied to provide experimental data for input into hydrocode models of armour behaviour by the Defence Research Agency, Fort Halstead. A wide selection of metals was examined so that comparisons could be drawn between modelling the behaviour of face centred and body centred cubic metals, and to carry out a broader investigation into how the results obtained were affected by the test methods.

Experiments were performed at temperatures from -100 °C to 20 °C and mean plastic strain-rates from $10^{-3}$ to $10^{3}$ s$^{-1}$, using a Split Hopkinson Pressure Bar (SHPB) system for high strain-rates and a Hounsfield 50 kN machine for quasistatic conditions. The stress-strain behaviour of the materials as a function of temperature and strain-rate was then determined.

The effects of interfacial friction on the measured compressive properties of copper and the armour plate steels have been investigated. Since the coefficient of friction was the critical parameter, ring tests were carried out and the Avitzur analysis applied. In general, the coefficient of friction decreased with increasing strain-rate and temperature. The tested specimen's appearance indicated the same friction trends.

Hydrocode modelling of the SHPB system produced corrections to the flow stress, to compensate for interfacial friction, that agree well with those predicted by the Avitzur analysis. Deformed finite element mesh plots analysed in conjunction with barrelled specimens have given a clearer insight into the mechanisms of interfacial friction.

The Armstrong-Zerilli constitutive models have been applied to copper, iron and armour plate steel results corrected for thermal softening and specimen-platen interfacial friction. These models have been shown to provide a reasonable description of the materials' behaviour.

The research investigation has shown that in order to obtain fundamental stress-strain behaviour of the materials, then corrections must be applied, which can be quite significant. These corrections must take into account the effects of material thermal softening and the specimen-platen interfacial friction.
CONTENTS

CHAPTER 1
1 INTRODUCTION
   1.1 MATERIALS USED IN THIS STUDY 2
   1.2 OBJECTIVES OF THIS WORK 3

CHAPTER 2
2 LITERATURE REVIEW
   2.1 TESTING TECHNIQUES 4
       2.1.1 STANDARD TESTING TECHNIQUES 4
           SHPB 4
           Drop Forge 5
           Expanding Ring 5
           Hydraulic Testing Machines 6
           Cam Plastometer 6
           Dynamic Indentation 6
           Metal Working Operations 7
           Taylor Test 7
           Plate Impact Test 8
           Flying Wedge 8
       2.1.2 SHPB DEVELOPMENTS/VARIANTS 8
       2.1.3 PROBLEMS ASSOCIATED WITH THE SHPB 11
           Friction 11
           Adiabatic Softening 22
           Inertia 24
           Wave Dispersion 28
           Multiple Wave Reflection within the Specimen 33
   2.2 MECHANICAL BEHAVIOUR OF METALS 34
       2.2.1 ELASTIC MATERIAL RESPONSE 35
2.2.2 PLASTIC MATERIAL RESPONSE

- Perfect Crystal Structure
- Dislocations
- Mathematical Description of Dislocation Motion
- Yield Stress
- Yield Points
- Work Hardening
- Grain Boundaries
- Strengthening of Steel

2.3 CONSTITUTIVE MODELS

- Components of Flow Stress
- Temperature and Strain-Rate Dependence of Flow Stress

2.3.1 PROMINENT CONSTITUTIVE MODELS

- Armstrong-Zerilli Model
- Variants of the Armstrong-Zerilli Model
- Johnson-Cook Model
- Klepaczko Model
- Mechanical Threshold Stress Model

CHAPTER 3

3 TESTING TECHNIQUES

3.1 TEST MATERIALS

3.2 QUASISTATIC STRAIN-RATE TESTING

3.3 DYNAMIC STRAIN-RATE TESTING - THE SHPB SYSTEM

- 3.3.1 SHPB STRAIN GAUGES
- SHPB Strain Gauge Circuitry and Formulae
- 3.3.2 SHPB THEORY
- 3.3.3 SHPB EXPERIMENTAL PROCEDURE
- 3.3.4 PROBLEMS ASSOCIATED WITH THE SHPB
  - Inertia
  - Multiple Wave Reflection within the Specimen
  - Wave Dispersion
Friction: Avitzur Ring Tests 84
Friction: Deformed Specimen Observations 88
Friction: Specimen Surface Finish 90
Adiabatic Softening 90
3.3.5 SHPB DATA ANALYSIS 92
3.3.6 FITTING THE ARMSTRONG-ZERILLI MODELS 96
FCC Metals 97
BCC Metals 97

CHAPTER 4
4 RESULTS 99
4.1 TEST MATERIALS 99
Nomenclature 99
General Information 102
4.2 RESULTS: COPPER 103
Armstrong-Zerilli Model for Copper 118
4.3 RESULTS: RHA/UK100 ARMOUR PLATE STEEL 121
Armstrong-Zerilli Model for RHA/UK100 139
4.4 RESULTS: ARP ARMOUR PLATE STEEL 142
4.5 RESULTS: PURE IRON 152
Armstrong-Zerilli Model for Iron 162

CHAPTER 5
5 HYDROCODE MODELLING 165
5.1 HYDROCODE SOFTWARE 165
INGRID 165
DYNA2D 165
ORION 166
5.2 CONSTITUTIVE EQUATIONS 166
5.3 SIMULATION RESULTS 167
Simulation A 168
Simulation B 176
CHAPTER 6
6 ANALYSIS AND DISCUSSION 184
  6.1 TESTING TECHNIQUE 184
  6.2 FRICTION 185
  6.3 TEMPERATURE SENSITIVITY AND ADIABATIC SOFTENING 188
  6.4 STRAIN-RATE SENSITIVITY 194
  6.5 ARMSTRONG-ZERILLI MODELS 199
  6.6 ACTIVATION VOLUME 203
    Activation Volume for BCC Metals 204
    Activation Volume for FCC Metals 207

CHAPTER 7
7 CONCLUSIONS AND RECOMMENDATIONS 211
  7.1 CONCLUSIONS 211
  7.2 RECOMMENDATIONS FOR FUTURE WORK 212

REFERENCES 214

APPENDIX 1: AVITZUR FRICTION ANALYSIS PROGRAMS
APPENDIX 2: SHPB ANALYSIS PROGRAM
APPENDIX 3: HYDROCODE MODELLING: TYPICAL INGRID FILE
CHAPTER 1

1 INTRODUCTION

For centuries the strength of metals has been known to depend upon their temperature and processing. The drive for stronger tools and weapons led to the development of material processes such as work hardening, surface hardening and tempering. The development of these processes was mainly through trial and error rather than through scientific deduction. The rapid development of science and instrumentation in the 19th and 20th centuries has enabled the scientific basis of these processes to be discovered. During this period of rapid discovery it was also found that the strength of a metal depends upon the rate at which it is loaded.

Strain-rate is defined as the rate of application of strain:

\[ \dot{\varepsilon} = \frac{d\varepsilon}{dt} \]  \hspace{1cm} (1.1)

In general the higher the strain-rate the higher the flow stress. Therefore to accurately describe a metal flow stress response mathematically, the effects of strain-rate must be taken into consideration. A general constitutive equation describing the flow stress of a metal can be expressed as:

\[ \sigma = f(\varepsilon, \dot{\varepsilon}, T) \]  \hspace{1cm} (1.2)

where:

\( \sigma \) = flow stress
\( \varepsilon \) = strain
\( \dot{\varepsilon} \) = strain-rate
\( T \) = temperature.

From equation (1.2) the components of flow stress can be expressed more clearly:

\[ d\sigma = \left( \frac{\partial \sigma}{\partial \varepsilon} \right)_{\varepsilon, T} d\varepsilon + \left( \frac{\partial \sigma}{\partial \dot{\varepsilon}} \right)_{\varepsilon, T} d\dot{\varepsilon} + \left( \frac{\partial \sigma}{\partial T} \right)_{\varepsilon, \dot{\varepsilon}} dT \]  \hspace{1cm} (1.3)
The flow stress is now expressed in terms of work hardening, strain-rate hardening and thermal softening components.

Equation (1.2) implies that the flow stress of a metal is only dependent upon strain, strain-rate and temperature. In reality the flow stress depends upon the dislocation structure which in turn depends on the metallurgical history of strain, strain-rate and temperature. Therefore equation (1.3) expresses the instantaneous flow stress in terms of its instantaneous sensitivities to strain, strain-rate and temperature.

Broadly speaking strain-rates can be divided into five regimes:

- $10^{-8}$ to $10^{-5}$ s$^{-1}$ Creep
- $10^{-5}$ to $10^{-1}$ s$^{-1}$ Quasistatic
- $10^{-1}$ to $10^{2}$ s$^{-1}$ Dynamic
- $10^{2}$ to $10^{4}$ s$^{-1}$ Dynamic with wave propagation effects
- $>10^{4}$ s$^{-1}$ Hypervelocity (shock-wave propagation)

At "high" strain-rates ($>10^{2}$ s$^{-1}$) the load is applied so fast that part of the body is stressed whilst part of it is not. The stress propagates through the body as a wave until all the body is stressed. Most of the tests within this study fall within this high strain-rate regime.

1.1 MATERIALS USED IN THIS STUDY

Two coppers, two armour plate steels and a pure iron have been supplied for study by Mr. B. D. Goldthorpe of the Defence Research Agency (DRA), Fort Halstead. DRA Fort Halstead is involved in the development of materials for both the civilian and military sectors. The main objective of this study is to determine constitutive models for these materials. These constitutive models can then be fed into computer hydrocodes developed at DRA Fort Halstead and used to predict the material response in particular applications (described below).

The two armour plate steels are both forms of the standard rolled homogeneous armour (RHA) steel. RHA steel has been used for many years as a benchmark against which other newer forms of armour can be compared. It is based upon AISI 4340
composition quenched and tempered steel. RHA steel is used primarily as heavy armour for tanks. It also serves as the tank chassis.

Copper is used in shaped charges. A shaped charge consists of an explosive charge in contact with a hollow conical metal (often copper) liner. On detonation the metal liner is accelerated into the axis of the cone. Part of the liner is accelerated forward and forms a "jet" and part is accelerated backwards at a lower rate forming the slug. This jet is accelerated up to very high velocities and can penetrate deeply into armour.

Pure iron is used in explosively formed (or forged) projectiles (EFP). These are very similar to shaped charges except the metal (often pure iron) liner is saucer shaped. On detonation the liner is deformed into a rod shaped slug that is accelerated towards the armour.

Further details of the materials used in this study are given in the experimental methods chapter.

1.2 OBJECTIVES OF THIS WORK

1. To determine the mechanical properties of the supplied materials over a range of strain-rates and temperatures.
2. To determine the possible effects of friction, adiabatic softening, inertia, wave dispersion and multiple wave reflection on the flow stress measured using the Loughborough University Split Hopkinson Pressure Bar system.
3. To correct the measured flow stress if required for any of the problems listed in objective 2 to produce isothermal flow stress results.
4. To use corrected results to determine constitutive models for the supplied materials.
CHAPTER 2

2 LITERATURE REVIEW

In writing this literature survey it is assumed that the reader has a good knowledge of materials science. Attention is focused on theory relating to high strain-rate testing and constitutive relationships for metals. Avitzur ring theory and the Armstrong-Zerilli models are described in detail as they are applied extensively to the results.

2.1 TESTING TECHNIQUES

The pioneering work of Davies in 1948 [cited in Meyers 1994] and Kolsky (1949) has lead to an explosion of interest in the field of dynamic plasticity of metals over the past five decades. Coupled with this new field of research has been the evolution of the original Split-Hopkinson Pressure Bar (SHPB) system and the development of new testing techniques.

2.1.1 STANDARD TESTING TECHNIQUES

At strain-rates greater than those employed in quasistatic testing it is necessary to use some kind of energy storage device to obtain the required loading conditions. The wide variety of possible energy storage devices that could possibly be used has resulted in the development of many different forms of high strain-rate testing machines. Reviews of high strain-rate testing machines are given in Holzer [1979], Malvern [1984], Field et al. [1994], and Meyers [1994]. The most successful techniques are described below:

SHPB

The SHPB is the most commonly used high strain-rate testing technique. A small cylindrical specimen is inserted between the ends of adjacent steel bars. A stress pulse initiated in one of the steel bars loads the specimen (usually in compression). Determination of the stress-strain behaviour of the specimen is carried out either by direct measurement from the specimen, or by measurement of strains/stresses in the
adjacent steel bars. Other forms of the SHPB exist including those designed for testing in tension, shear and torsion. Problems associated with the SHPB include multiple wave reflection within the specimen, wave dispersion effects, specimen inertia, heating and friction effects. Detailed descriptions of the SHPB testing technique can be found in Kolsky [1949], Davies and Hunter [1963], and Wasley [1973]. For a more detailed description of the Loughborough University SHPB facility please refer to Chapter 3.

**Drop Forge**

The drop forge (or drop weight) testing technique involves impacting a small cylindrical specimen with a large weight that is driven by either gravity, compressed gas or other means. In most experimental set-ups the surface ("anvil") upon which the specimen is supported is assumed to be rigid, - however this is not usually the case. Theoretical calculations (similar to those that can be employed for the SHPB) can be used to help correct for anvil movement, specimen inertia, heating and friction effects. Another major problem associated with drop forges is that the strain-rates are not constant, - particularly if large strains are involved [Physics Dept.].

**Expanding Ring**

The sample in this case is in the form of a narrow ring that is wrapped around a hollow cylinder. This hollow cylinder is then subjected to a very fast internal pressure rise and rapidly expands loading the outer ring sample. There are three widely used techniques for loading the ring, - 1) tube expansion using a driving fluid (e.g. mercury), 2) explosive expansion, e.g. vaporisation of copper wire, and 3) electromagnetic expansion, e.g. opposing currents in coils. Measurements are usually taken using very fast optical equipment e.g. IMACON camera. Very high strain-rate and temperature increases are associated with this form of testing, - hence the results obtained provide very stringent tests for the validity of new constitutive relationships. A typical expanding ring system is described in detail in Al-Maliky and Parry [1994, 1996].
Hydraulic Testing Machines

These are useful for both testing in compression and tension at low and medium strain rates (quasistatic to $10^2$ s$^{-1}$). The main errors are due to friction effects and displacement/strain measurement problems. The compliance of the testing machine is different in tension compared to compression, and becomes critical when very stiff samples are tested (i.e. very small strains are involved). Sample strain is usually measured by some form of displacement transducer clipped between the faces of the testing platens loading the sample, or by strain gauges on the surface of the specimen. Force (and hence stress) is measured by the load cell [Holzer 1979].

Cam Plastometer

The cam plastometer uses a logarithmic shaped cam to deform the specimen at a constant strain-rate. The energy required for the deformation is supplied by a rotating flywheel. Strain-rates of the order of $10^2$ s$^{-1}$ are obtainable. The constant strain-rate achieved in this form of testing is very desirable but perhaps does not warrant the complex mechanical set-up required [Meyer 1994].

Dynamic Indentation

Tirupataiah and Sundararajan [1991] have developed a dynamic indentation technique suitable for the determination of high strain-rate flow behaviour of ductile metals and alloys. This technique involves dropping or firing (using compressed gas) a 4.76 mm diameter tungsten carbide ball at the specimen. The incident and reflected ball speeds are measured using two photo-diodes and a digital timer. The diameter of the crater caused by the impact on the specimen is measured with an optical microscope. The calculated hardness values for different impact velocities are related to the stress-strain characteristics of the material by empirically derived equations. Tests for copper and iron were performed for strain-rates up to $10^4$ s$^{-1}$.

For many years dynamic hardness tests were considered of little use as a research tool because the stress system is non-uniform, triaxial and confined to a limited volume.
of material [Davies and Hunter 1963]. However, in general the results obtained by the
dynamic indentation technique (described above) compare well with data obtained on
similar materials using the SHPB technique.

**Metal Working Operations**

Attempts to measure the stress-strain behaviour using metal working operations
have resulted only in average stress-strain values being quoted (at strain-rates of
$\approx 10^4 \text{ s}^{-1}$). Techniques such as orthogonal machining where the work passes
continuously through the deformation zone in a steady condition involve many
complicating factors that must be taken into account. The temperature rise due to plastic
work is considerable. Unusual friction conditions involved in metal machining are hard to
estimate or model. Finally, the formation of microcracks and voids in the deformation
zone must affect the stress-strain characteristics. The possibility of extracting material
behaviour from the complex behaviour of the chip (material removed during machining)
is difficult, and is made harder by stronger chip inertia effects at higher machining rates
[Holzer 1979].

**Taylor Test**

This test involves the impact of a short cylindrical rod at a stiff planar interface.
Stress-strain relationships can be calculated from the shape of the deformed rod. The
strain-rate variation along the axis of the rod is great and can provide a useful array of
data for different strain-rates from one test. Microstructural examination and hardness
testing along the length of the deformed rod are the most commonly employed analysis
tools. The wealth of information obtained from one test outweighs the initial expensive
experimental set-up costs and makes this test one of the most popular today [Field et al.
1994].
**Plate Impact Test**

A flyer plate is propelled usually at speeds of up to $2 \text{ km s}^{-1}$ to produce a planar impact on a target plate. The planar impact is ensured by the carriage of the flyer plate down a gun barrel at the front of a precision built sabot. On impact a uniaxial plane compressive shockwave is induced in the target plate. The centre of the target plate is uniaxially strained in the direction of the shockwave until release waves from the edges of the target plate reach the centre. The experiment is finished within a few microseconds. The stress time history within the target plate is measured usually with piezoresistive stress gauges or velocity interferometry. Strain-rates of the order of $10^5 \text{ s}^{-1}$ are achievable in this test [Meyer 1994].

**Flying Wedge**

Barton et al. [1994] have used a novel tensile testing technique that can achieve strain-rates in excess of $10^4 \text{ s}^{-1}$. The tensile specimen is held at either end by sliders that are positioned within guide rails restricting their movement to the longitudinal axis of the specimen. A "flying wedge" is launched at speed forcing the sliders apart and hence straining the specimen.

**2.1.2 SHPB DEVELOPMENTS/VARIANTS**

There are many minor differences that exist between set-ups of the SHPB belonging to different authors. In this section only modifications that have had an important effect on the apparatus' capabilities will be reported.

In most SHPB systems the projectile is driven down a narrow bore barrel using a high pressure gas. Parry and Griffiths [1979] have devised a system in which the projectile is driven under atmospheric pressure along a wide-bore evacuated tube. This system is simple and safe to use in comparison to the usual more complex and expensive high pressure gas driven systems.

The strain-rate can vary considerable throughout a test in the standard SHPB. This variation together with the Pochhammer-Chree oscillations introduced by the
projectile impact can cause problems in the interpretation of results. Ellwood, Griffiths
and Parry [1982] have used a three maraging steel bar SHPB. A pre-loading bar is used
-together with a pulse-shaping dummy specimen (made from the same material as the test
specimen) sandwiched between the pre-loading bar and the incident bar. The new
incident pulse (the transmitted pulse through the dummy specimen) has vastly reduced
Pochhammer-Chree oscillations (none at high projectile impact velocities) and enables
tests to be performed at virtually constant strain-rates.

If a constant strain-rate is not critical to result interpretation for a given material,
then the above system can be modified to remove most of the Pochhammer-Chree
oscillations without the necessity of using an extra dummy specimen each test. Parry,
Walker and Dixon [1995] replaced the maraging steel pre-loading bar with a lower
strength 431 steel bar. The lower strength bar causes substantial damping of the high
frequency Pochhammer-Chree oscillations, especially at higher projectile velocities. This
system together with the atmospheric pressure projectile driving technique described
above is used in this study and is described in more detail in Chapter 3.

The relatively long rise-time ($\approx 10 \mu s$) of the loading pulse in a typical SHPB
system limits the maximum strain-rates achievable to approximately $10^3 \text{s}^{-1}$. Lindholm
(see Mentha, Pope and Field 1984] has used 5 mm diameter bars to reduce the effects of
dispersion and inertia thereby achieving strain-rates of $10^4 \text{s}^{-1}$. Mentha, Pope and Field
[1984] have used a miniature direct impact Hopkinson bar that achieves a rise-time of
1 $\mu$s and strain-rates of $10^4 \text{s}^{-1}$.

Gorham, Pope and Field [1992] have developed the miniature Hopkinson bar
system of Mentha, Pope and Field [1984] for compressive stress-strain measurements of
high strength metals at very high strain-rates (up to $\approx 10^5 \text{s}^{-1}$). This direct impact system
uses 3 mm diameter pressure bars $\approx 150$ mm long made from titanium 6Al4V alloy, high-
strength tungsten alloy or tungsten carbide. The specimen is $\approx 1-2$ mm in diameter and
0.5-1.0 mm long. The specimen strain is calculated using the data from two silicon
semiconductor strain gauges on the output bar and a simple model of the deformation
process:

$$h(t) = h_0 - v_0 t + \frac{(Z_p + Z_B)}{Z_p Z_B} \int_0^t f(t) dt$$

(2.1)
where:

$h(t) = \text{length of specimen at time } t \text{ (m)}$

$h_0 = \text{initial specimen length (m)}$

$v_0 = \text{impact velocity of projectile (m s}^{-1}\text{)}$

$f(t) = \text{force history (N)}$

$Z_A = \text{acoustic impedance of projectile (kg m}^{-2}\text{ s}^{-1}\text{)}$

$Z_B = \text{acoustic impedance of pressure bar (kg m}^{-2}\text{ s}^{-1}\text{)}$

$A = \text{non-strained cross-sectional area of pressure bar (m}^2\text{)}$.

Strain values are also checked and the whole system calibrated using high-speed photography. Total experimental error is estimated to be $\approx 2\%$.

Maximum stresses in excess of 2 GPa are possible using this system. The specimen diameter must be kept less than $\approx 0.5$ times the pressure bar diameter to ensure that the pressure bars will remain elastic (even if the specimen material is the same as the pressure bar material). The pressure bars will supposedly remain elastic because plastic flow inside them is constrained by the material surrounding the stressed area.

Shioiri, Sakino and Santoh [1994] have used a modified direct impact Hopkinson bar to allow instantaneous reductions in strain-rate. During the test the projectile collides with a decelerator bar reducing the velocity of the front of the projectile, hence reducing the strain-rate of the specimen.

Clifton (see Gorham, Pope and Field [1992]) has developed a pressure-shear testing technique for very high strain-rates in the region $10^4$ to $10^7$ s$^{-1}$. Very thin specimens machined or deposited as film are compressed between the inclined end faces of two axially colliding elastic bars. Two major problems associated with this technique are 1) the stress-state developed in the sample is not comparable to that developed in a SHPB, and 2) very sophisticated instrumentation is required to work at such high strain-rates.

Zhao and Gary [1997] have taken measurements over a much larger time duration than is usually possible with a conventional SHPB. Recording the signals at two different points on the incident and transmitter bars together with an iterative shifting process taking into account wave dispersion effects has increased the measurement
duration by up to 100 times. Tests were performed to much larger strains than originally possible for foam materials and aluminium square tubing.

Noble and Harding [1994a] have used a thermal scanning camera to measure the temperature rise of Remco iron specimens tested in a tensile SHPB. This technique incurred large errors (+150 °C to -30 °C) due to changes in specimen movement/orientation to the detector and specimen emissivity changes during the test. Craig et al. [1994] used a similar high speed radiometry technique to measure the temperature rise of aluminium 2024 tested in a compressive SHPB. Better results were achieved by applying a layer of soot over the specimen to eliminate the errors associated in assessing the specimen's emissivity. Because the detection area on the specimen was small, the range of temperatures measured (100 °C to 140 °C) for the given test condition might be affected by temperature localisation in shear banding.

2.1.3 PROBLEMS ASSOCIATED WITH THE SHPB

Due to the very nature of high strain-rate testing it is natural to expect a specific set of experimental problems. These problems can be subdivided into fundamental characteristics inherent in the SHPB test, i.e. friction, adiabatic softening, inertia, wave dispersion and multiple wave reflection within the specimen.

Friction

When a solid cylindrical specimen is compressed between parallel platens, the mode of specimen deformation is greatly influenced by the friction conditions between the specimen faces and loading platens. Schroeder and Webster [1949] identified three distinct cases of friction: 1) relative sliding motion between specimen faces and platens except at the geometric centre of the specimen face (low or zero friction condition), 2) no relative sliding motion between surfaces where the specimen deformation is completely in shear (high friction), and 3) a combination of these two conditions where sticking occurs in a central zone and sliding in an annular zone near the edge (medium friction).
The effects of friction on the quasistatic compression of solid cylindrical specimens have been modelled by Mescall, Papimo and McLaughlin [1983] using the HEMP finite difference code. The code clearly simulated specimen deformation mechanisms observed in a 4340 steel specimen. Frictional forces opposed the outward flow of metal in close contact with the platen faces, whereas material at specimen mid-height flowed out relatively undisturbed. This caused specimen barrelling and the formation of a relatively less strained conical region near each loading platen surface known as a "dead zone". The borders of the dead zones were heavily deformed in shear. A small central region of the specimen deformed under an axially compressive strain. Material originally located on the cylindrical surface of the specimen had "rolled over" and become part of the flat end faces of the specimen in contact with the loading platens.

The specimen dimensions can also greatly affect the role friction plays in the deformation mechanism. As the dead zones approach and overlap, they cause an increase in force for a given increment of strain and the load-deformation curve bends upward [Dieter 1986]. To reduce this effect a low specimen diameter/height ratio can be used, however below about 0.5 for this ratio the specimen might buckle instead of barrel. The variation of the flow stress with different specimen diameter/height ratios is the basis of the Cooke and Larke [1945] technique described later in this section. Under dynamic loading friction problems tend to be greater with larger specimens. Larger specimens deformed at the same strain-rate as smaller specimens involve longer radial displacements at higher velocities. Larger displacements and velocities means that there is a greater chance of lubrication breakdown [Pearsall and Backofen, cited in Gorham 1991a].

Siebel [cited by Gorham et al. 1991a, 1992] found the average pressure needed to deform a material of yield strength $\sigma_y$ is:

$$P = (1 + 2\mu a/3h)\sigma_y$$  \hspace{1cm} (2.2)

where:

- $\mu$ = coefficient of friction
- $a$ = specimen radius (m)
- $h$ = specimen height (m).
Inserting popular values for the aspect ratio of specimens used in the compression SHPB gives values for the coefficient of friction between 0.02-0.06. The corresponding minimum error in the yield strength is ≈ 4% even when the lubrication can be considered fully effective.

To try and minimise the effects of friction Lovato and Stout [1992] have investigated large strain (ε ≈ 1), quasistatic tests using a range of specimen materials, specimen end face geometries, different lubricants, test temperatures and platens. In general, materials with a work hardening coefficient > 0.15 and high strain-rate sensitivity exhibited more uniform deformation. Up to strains of ≈ 0.5 accurate stress-strain data were obtained for these materials regardless of lubrication conditions employed. Deltort, Neme and Tanguy [1997] have also tried to reduce friction effects by redesigning the specimen. A dumbbell shaped specimen allowed deformation in the central region that was not restricted by friction acting at the ends. This specimen shape did reduce errors introduced by friction (and elastic platen punching) but introduced errors due to bending in the specimen.

Specimens specially designed to reduce friction effects can be costly to manufacture so the vast majority of compression testing is performed using solid cylindrical specimens. This standardisation on one specimen design has led to a vast amount of literature describing possible friction correction techniques applicable to the compression of solid cylindrical specimens. The ring test is the most popular method in the metal working industry of establishing the frictional conditions present in the compression of solid cylindrical specimens [Kalpakjian 1991].

When a ring (a hollow cylinder) is compressed between rigid, parallel platens its outer radius increases. The inner radius might increase or decrease depending on the friction conditions prevalent. Avitzur [1964] has analysed the deformation of a ring specimen assuming the ring material obeys Mises' stress-strain-rate laws, and that there is a constant shear factor m between the disc and the platen faces.

The constant shear factor m is defined as:

\[
m = \frac{\tau}{k} = \frac{\text{interface shear strength (Pa)}}{\text{yield stress in shear (Pa)}}
\]  

(2.3)
i.e. the interface shear strength is taken as a constant fraction $m$ of the yield strength in shear of the softer material in a sliding pair (i.e. of the specimen). The shearing stress at the interface never exceeds the yield stress in shear of the specimen. For $m = 1$ (sticking friction) deformation proceeds by subsurface shearing in the specimen. The definition of the constant shear factor $m$ is preferred to the Coulomb definition of friction given below:

$$\mu = \frac{\tau}{P}$$  \hspace{1cm} (2.4)

where:

- $\mu$ = Coulomb coefficient of friction
- $\tau$ = shear stress at the interface (Pa)
- $P$ = stress normal to the interface (Pa).

Since the yield strength of the specimen in shear is not affected by normal stress, the Coulomb coefficient of friction increases with decreasing normal stress, contrary to physical reality. The constant shear factor $m$ is independent of the normal stress at the interface [Dieter 1986].

For a ring specimen Avitzur [1964] derived the following equation for $R_n \leq R_i$:

$$\frac{P_{seq}}{\sigma_0} = \frac{1}{1 - \left(\frac{R_n}{R_0}\right)^2} \left\{ \sqrt{1 + \frac{1}{3} \left(\frac{R_n}{R_0}\right)^4} - \sqrt{\left(\frac{R_n}{R_0}\right)^4 + \frac{2}{3\sqrt{3}} m \frac{R_0}{T} \left[ 1 - \left(\frac{R_n}{R_0}\right)^3 \right]} \right\}$$  \hspace{1cm} (2.5)

where:

$$\left(\frac{R_n}{R_0}\right)^2 = \frac{\sqrt{3}}{2} \frac{1 - \left(\frac{R_n}{R_o}\right)^4}{\sqrt{x(x - 1) \left[ 1 - \left(\frac{R_n}{R_0}\right)^4 \right]}}$$  \hspace{1cm} (2.5a)
and $x$ in equation (2.5a) is given by:

$$x = \left\{ \frac{R_o}{R_i} \exp \left[ -m \frac{R_o}{T} \left( 1 - \frac{R_i}{R_o} \right) \right] \right\}^2$$

And for $R_i \leq R_n < R_o$:

$$\frac{P_{ave}}{\sigma_0} = \frac{1}{1 - \left( \frac{R_i}{R_o} \right)^2} \left\{ \sqrt{1 + \frac{1}{3} \left( \frac{R_n}{R_o} \right)^4} - \sqrt{\left( \frac{R_i}{R_o} \right)^4 + \frac{1}{3} \left( \frac{R_n}{R_o} \right)^4} + \frac{2}{3\sqrt{3}} \frac{R_n}{T} \left[ 1 + \left( \frac{R_i}{R_o} \right)^3 - 2 \left( \frac{R_n}{R_o} \right)^3 \right] \right\}$$

(2.6)

where:

$$\frac{R_n}{R_o} \approx \frac{2\sqrt{3} \left( \frac{m R_o}{T} \right)}{\left( \frac{R_o}{R_i} \right)^2 - 1} \left\{ \sqrt{1 + \left( \frac{R_i}{R_o} \right)^4 \left( \frac{R_o}{R_i} \right)^{2 - 1}} - 1 \right\}$$

(2.6a)

Where:

- $P_{ave}$ = average stress applied (Pa)
- $\sigma_0$ = effective flow stress (Pa)
- $R_i$ = internal radius (m)
- $R_o$ = original radius, outer radius (m)
- $R_n$ = neutral radius (m)
- $T$ = thickness (m).
Equation (2.5) is valid when:

\[ m \frac{R_0}{T} \leq \frac{1}{2 \left(1 - \frac{R_i}{R_0}\right)} \ln \left( \frac{3 \left(\frac{R_0}{R_i}\right)^2}{1 + 1 + 3 \left(\frac{R_0}{R_i}\right)^4} \right) \]  

(2.7)

and equation (2.6) is valid when:

\[ m \frac{R_0}{T} \geq \frac{1}{2 \left(1 - \frac{R_i}{R_0}\right)} \ln \left( \frac{3 \left(\frac{R_0}{R_i}\right)^2}{1 + 1 + 3 \left(\frac{R_0}{R_i}\right)^4} \right) \]  

(2.8)

The neutral radius \( R_n \) defines the radius of the ring that remains unchanged during the test. It can be visualised as a boundary dividing the metal flow in two directions, toward and away from the centre of the ring [Martorell 1983]. However, under low friction conditions \( R_n \) can be smaller than the internal radius \( R_i \), effectively not within the metal of the deforming ring [Avitzur, van Tyne and Umana 1978].

According to the von Mises yield criterion \( k \) in equation (2.3) can be replaced by:

\[ k = \frac{\sigma_0}{\sqrt{3}} \]  

(2.9)

Therefore the constant shear factor \( m \) can be replaced by an average Coulomb coefficient of friction, \( \mu_{\text{ave}} \) [Avitzur 1964]:

\[ \tau = \mu_{\text{ave}} P_{\text{ave}} \]  

(2.10)

where:
\[ \mu = \mu_{ave} = \frac{m \sigma_0 / \sqrt{3}}{P_{ave}} = \frac{m / \sqrt{3}}{P_{ave} / \sigma_0} \]  

(2.11)

Using equations (2.5)-(2.8) and the iterative numerical procedure described below, it is possible to calculate the value of \( P_{ave} / \sigma_0 \) with strain. The outer and inner radii velocities are described by:

\[ \dot{U}_R \bigg|_{R=R_0} = -\frac{\dot{U} R_0}{2T} \left[ 1 - \left( \frac{R_0}{R_0} \right)^2 \right] \]

(2.12)

\[ \dot{U}_R \bigg|_{R=R_i} = -\frac{\dot{U} R_i}{2T} \left[ 1 - \left( \frac{R_i}{R_i} \right)^2 \right] \]

where:
\[ \dot{U}_R \] = outer radial velocity (m s\(^{-1}\))
\[ \dot{U} \] = platen velocity (m s\(^{-1}\)).

The incremental changes in \( T, R_0, \) and \( R_i \) for a given time step \( \Delta t \) (s) are given by:

\[ \Delta T = \dot{U} \Delta t \]

\[ \Delta R_0 = \dot{U}_R \bigg|_{R=R_0} \Delta t \]

\[ \Delta R_i = \dot{U}_R \bigg|_{R=R_i} \Delta t \]  

(2.13)

The data required for the Avitzur analysis are the constant shear factor, the platen velocity, the time interval for the iteration process, the original specimen inner and outer radii and the original and final specimen lengths. If the specimen is a solid disc \( (R_i = R_o = 0) \) then either equation (2.5) or (2.6) can be used to predict \( P_{ave} / \sigma_0 \). If \( R_i \neq 0 \) (i.e. a ring specimen) then the conditions described by equations (2.7) and (2.8) are used to determine if equation (2.5) or (2.6) is used to predict \( P_{ave} / \sigma_0 \) for the current iteration. \( R_n \) in equation (2.5) is calculated using equation (2.5a). \( R_n \) in equation (2.6) is calculated using equation (2.6a). The incremental changes in the specimen's thickness and inner and
outer radii are calculated using equations (2.12) and (2.13). The new values of $R_0$, $R_i$, and $T$ are then used in the next iteration repeating the procedure described above. This cycle is repeated until the specimen thickness calculated by the Avitzur analysis is less than or equal to that measured from the real test specimen.

Therefore for a given Avitzur ring analysis, knowing the initial specimen dimensions and the final specimen thickness, a range of values for $m$ can be processed using the numerical method described above. For each value of $m$, a different set of final specimen inner and outer diameter dimensions can be obtained. The final inner and outer diameter dimensions predicted by the Avitzur theory can then be matched with the real final ring specimen dimensions, and hence a value for $m$ can be determined. Once $m$ is determined by a ring test for a given set of test conditions, the above procedure can be repeated to predict the correction for a solid disc tested under the same conditions by putting $R_i = R_n = 0$. Using equation (2.11) it is possible to visualise how $\mu$ varies with strain. Further details describing the application of the Avitzur technique can be found in Section 3.3.4.

Avitzur, van Tyne and Umana [1978] further analysed the ring test and showed that changes in $R_i$ are more sensitive to friction than changes in $R_0$. The rate of change of $R_i$ with respect to $m$ is always greater than that of $R_0$. Therefore, even though the absolute change in $R_0$ is greater than that in $R_i$, $R_i$ may increase for low values of friction and decrease for high values of friction, whereas $R_0$ increases for all values of friction with a mildly slower increase for higher friction values.

The sensitivity of $R_i$ to friction also depends greatly on the ring geometry. Avitzur, van Tyne and Umana [1978] found that thin rings are always superior to thick rings as long as the initial $R_i$ is chosen dependent on the expected level of friction. In general, for better sensitivity from thin rings, a large $R_i$ must be used when friction is low and a small $R_i$ when friction is high.

Measurement of the internal diameter of an axially compressed ring gives sensitive and realistic indications of the frictional constraints present [Gorham et al. 1992]. Gorham [1984] performed Avitzur ring tests using aluminium specimens with different surface finishes and lubricants. The most reproducible friction conditions were obtained using a graphite based lubricant and 3 µm diamond polished aluminium sample surfaces. Consistent values for $\mu \approx 0.024$ were obtained, - which using equation (2.2)
translated into a reduction of \( \approx 1\% \) in the stress values. The lowest value of \( \mu \approx 0.017 \) was obtained using a 600 grade silicon carbide abraded surface finish and Rocol J166 lubricant (includes lead, copper, graphite and molybdenum disulphide solid additives).

Male and Cockcroft [1964] have also developed a method of calculating \( \mu \) using ring specimens. Specimen dimensions for sticking friction were determined by testing an aluminium ring at 600 °C. Specimen dimensions for near zero friction conditions were determined by testing a warmed wax ring specimen between heated platens. Intermediate values of \( \mu \) were determined by testing solid cylindrical specimens and applying the analysis of Schroeder and Webster [cited in Male and Cockcroft 1964a]. Ring specimens were then tested under the same conditions as the solid specimens to establish a relationship between ring deformed dimensions and \( \mu \). Tests were performed on aluminium ring specimens at room temperature at strain-rates of \( 10^{-2} \), 10 and 1200 s\(^{-1} \) using paraffin as a lubricant. For quasistatic rates \( \mu \approx 0.06 \) and increased with increasing strain. At the intermediate strain-rate \( \mu \approx 0.022 \) and at the high strain-rate \( \mu \approx 0.02 \). \( \mu \) was not affected by the level of strain until at \( \approx 30\% \) strain for the intermediate strain-rate and at \( \approx 40\% \) strain for the high strain-rate, \( \mu \) began to increase rapidly. This increase in \( \mu \) at higher strains for intermediate and high strain-rates was interpreted as an indication that the lubricant film had become too thin to support hydrodynamic lubrication.

The effect of strain-rate on the aluminium specimens lubricated with paraffin was discussed by Male [1966]. Male [1966] suggests that there is an effective lubricant time after which the lubricant breaks-down. During quasistatic testing, when the time to achieve bulk yielding in the specimen is greater than the effective lubricant time, i.e. after most of the lubricant has squeezed out from in between the specimen/platen interface, then friction will be high. At higher strain-rates when bulk yielding can occur before lubricant breakdown, then \( \mu \) is considered to be relatively constant. Increasing the strain-rate further will not alter the effective value of \( \mu \) but will allow greater deformation before lubricant breakdown. Photographs of the aluminium ring specimens' faces show reduced bright areas (areas of lubricant breakdown) with increasing strain-rate, giving support to the above theory. Ring specimens tested quasistatically had bright rings on their faces around both the outer edge and the hole. Ring specimens tested at the highest
strain-rate had a clearly defined bright ring only around the outer edge of the specimen face. Devenpeck and Rigo [1983] have made the same observations of bright rings formed on steel specimen faces under high friction conditions.

Male [1965] has considered the effects of temperature and adsorbed contaminant films on the low temperature friction behaviour of a variety of metals including copper, lead and various steels. Tests were performed under "dry" conditions, i.e. no lubricant was used. \( \mu \) was reasonable constant below 120 °C but increases above this temperature. Contaminant films on the surface of the specimen reduced \( \mu \). Above 120 °C the partial removal of these contaminant films increased \( \mu \).

In order to calculate \( \mu \) in a SHPB test, Walley et al. [1997] have employed a method that combines high-speed photography with numerical modelling. A simulation of a drop-weight system with glass platens was performed using a DERA modified form of the Lagrange hydrocode DYNA2D. XM copper, RHA and EN24T steel ring specimens were tested using the drop-weight system and a variety of lubricants. A high-speed photographic record of the ring deformation was taken and compared with that predicted by the numerical simulation. Any differences between the deformation mechanisms between the SHPB and the drop-weight system could be recognised during the simulation. Close agreement was found between the simulation and experiment yielding a value for \( \mu = 0.1 \) for a MoS\(_2\) lubricated specimen. Bright rings (or "foldover" zones) were observed on the faces of the ring specimens around both the outer edge and the hole. Marks on the specimen faces referred to as "machining rings" were observed on recovered test specimens and were thought to have been created as part of the foldover process. The simulations clearly showed a non-uniaxial stress state within the specimen due to shear stresses introduced by the frictional restraints at the specimen faces. This stress state could be clearly seen in the typical diamond shaped localisation pattern seen in the deformed meshes. In the simulation the glass platens were seen to deform around the edges of the specimen. This elastic platen punching affects the load distribution on the specimen which in turn greatly affects the variation with strain of the radii of curvature at the inner and outer radii.

The application of the ring test directly to the SHPB is very rare. Lichtenberger, Lach and Bohmann [1994] have applied a ring analysis technique developed by Burgdorf
for static testing to dynamic ring tests using a SHPB. In general, for tests on a steel and copper \( \mu \) was found to decrease with increasing strain-rate.

Singh and Padmanabhan [1991a, 1991b] have compared the ring test technique of Male and Cockcroft [1964] with the Cooke and Larke [1945] techniques. The Cooke and Larke technique requires that specimens of different initial diameter-to-height ratios, \( D_0/H_0 \), are compressed by fixed amounts. The observed stress, \( \sigma_{\text{obs}} \), is plotted against \( D_0/H_0 \). Extrapolation to \( D_0/H_0 = 0 \) gives the compressive yield stress for a given degree of compression. The modified Cooke and Larke technique (due to Watts and Ford, see Singh and Padmanabhan [1991a]) uses a plot of the percentage reduction against \( D_0/H_0 \) to obtain the percentage compression for the frictionless case by extrapolation to \( D_0/H_0 = 0 \). Under conditions of sliding friction, both the Cooke and Larke techniques and the ring compression test gave similar values for the coefficient of friction. (Coefficient of friction for Cooke and Larke techniques was calculated using equation (2.2)). For conditions of sticking, or near sticking friction, the modified Cooke and Larke technique is more accurate because the ring compression test ignores the strain-rate sensitivity of the flow stress. At strain rates of \( \approx 10 \text{ s}^{-1} \) and above the ring tests were taken to be more reliable. For a Cooke and Larke technique to be valid, at a fixed strain the specimen temperature for all \( D/H \) ratios should be the same. This was not true for the steel specimens tested due to the temperature rise caused by plastic deformation. It should also be noted that the shape of the specimen affects the heat loss characteristics and hence the flow stress characteristics. Greater heat loss results from an increase in the \( D/H \) ratio at larger strains (time of deformation) (surface area to volume ratio varies as \( 2\pi r(h + r)/(\pi r^2 h) \)).

Malinowski and Klepaczko [1986] have used a unified approach to inertia and friction in the SHPB through the consideration of energy balance. A technique similar to that of Cooke and Larke involving the testing of ten different height to diameter ratio aluminium specimens was used to determine friction levels. Friction and to a lesser extent inertia were found as the main errors in determining the flow stress at high strain-rates.

Carden and Kim [1986] have derived a mathematical equation that allows the coefficient of friction to be calculated from the difference in flow stress between an incremental and a monotonic loading. Unfortunately this approach has severe problems when applied to the SHPB. The flow stress for the incrementally loaded specimen does
not necessarily represent a zero friction case as assumed. The monotonically loaded specimen may suffer thermal softening effects. Matching the strain-rates between the monotonic and incremental tests is experimentally difficult.

All the papers cited above have assumed/treated the friction conditions between the two faces of the specimen and the loading platens as equal. Hashmi [1978] has considered the deformation between platens with unequal frictional properties. A lumped mass model was used for a solid cylindrical specimen compressed between a platen travelling at up to 100 ft s\(^{-1}\) and a stationary anvil. For no interfacial friction, the cylindrical surface profile was slightly tapered, the thicker end of the specimen closest to the moving platen. For different values of friction for each specimen face, the specimen barrelled asymmetrically about the midsection. The radius of the specimen face with the higher friction was always less than that of the lower friction face (as observed experimentally in Hashmi [1978] and Hashmi and Hamouda [1994]). The radius of the midsection was always greater than those measured at the specimen faces.

**Adiabatic Softening**

Adiabatic thermal softening has been shown to account for the converging [Ellwood, Griffiths and Parry 1984, Parry and Walker 1989] and overlapping [Dixon and Parry 1991] of stress strain curves obtained from high strain-rate tests (adiabatic) with those obtained from quasistatic tests (isothermal).

Dixon and Parry [1991] have applied corrections to adiabatic data obtained from a SHPB to produce isothermal flow stress curves. A test was considered adiabatic above a given critical strain-rate:

\[
\dot{\varepsilon}_A = \frac{4\varepsilon K}{C_p R^2} \tag{2.14}
\]

where:

- \(\dot{\varepsilon}_A\) = critical strain-rate above which a compression test may be considered adiabatic (s\(^{-1}\))
- \(K\) = thermal conductivity of specimen (W m\(^{-1}\) K\(^{-1}\))
\( C_p = \) volume specific heat of specimen at constant pressure (J m\(^3\) K\(^{-1}\))

\( R = \) radius of solid cylindrical specimen (m)

\( \varepsilon_c = \) critical strain above which adiabatic shear may occur (s\(^{-1}\)), given by:

\[
\varepsilon_c = -\frac{n C_p}{\frac{\partial \sigma}{\partial T}} \bigg|_{\varepsilon_c}
\]  

(2.15)

where:

\( n = \) work hardening exponent

\( \sigma = \) flow stress (Pa)

\( \varepsilon = \) strain

\( T = \) temperature (K).

Assuming all the mechanical work of deformation is converted into heat, the temperature rise corresponding to a strain \( \varepsilon \) is given by:

\[
\Delta T = \frac{1}{C_p} \int_0^\varepsilon \sigma(\varepsilon) d\varepsilon
\]  

(2.16)

Measurement of the stress strain behaviour allows \( \Delta T \) to be evaluated. The reduction in flow stress (\( \Delta \sigma \)) due to adiabatic heating effects can be calculated from:

\[
\Delta \sigma = \left. \frac{\partial \sigma}{\partial T} \right|_{\varepsilon=5\%} \times \Delta T
\]  

(2.17)

where \( \left. \frac{\partial \sigma}{\partial T} \right|_{\varepsilon=5\%} \) is obtained from stress versus temperature curves. The gradient at 5 \% strain was used since 5 \% true strain is large enough for the flow stress at this point to be undisturbed by yield point effects, and small enough to ensure that no significant temperature rise will have yet taken place during compression.
Inertia

The possibility of specimen inertia influencing the measured stress-strain characteristics has been recognised for a considerable period of time. Kolsky [1949] originally tried to take specimen inertia effects into account using an energy balance argument to predict inertial stresses. Effects due to inertia in the axial direction were minimised by using thin specimens. A correction for radial inertia predicted a reduction in the flow stress of a few percent. This correction was only significant when the strain-rate was changing rapidly, i.e. within the first few microseconds of the test.

In general, as the rate of deformation increases, the force required to accelerate the specimen material increases. If this inertia force is significant compared to the deformation loads, then deformation can no longer be considered uniform and the measured load history is not accurate. Samanta [cited by Gorham 1991a, 1991b] derived the following expression to calculate the inertial stress for a specimen with a Poisson's ratio of 0.5 deforming on a rigid anvil:

\[
\sigma_i = \rho \left( \frac{a^2}{16} + \frac{h^2}{6} \right) \dot{\varepsilon}^2 + \rho \left( \frac{h^2}{6} - \frac{a^2}{8} \right) \ddot{\varepsilon}.
\]  

(2.18)

where:

- \(\sigma_i\) = inertial stress measured at the anvil (Pa)
- \(\rho\) = density of specimen (kg m\(^{-3}\))
- \(a\) = specimen radius (m)
- \(h\) = specimen height (m)
- \(\dot{\varepsilon}\) = strain-rate (s\(^{-1}\))
- \(\ddot{\varepsilon}\) = strain-rate “acceleration” (s\(^{-2}\)).

Davies and Hunter [1963] also derived equation (2.18), however neither a Poisson's ratio of 0.5 was assumed nor a strain-rate term included. Specimens were tested with an aspect ratio that reduced the strain-rate acceleration term to zero at the start of the test. The strain-rate acceleration term's contribution to an inertia correction is usually greatest during the first few microseconds of the test (see below).
Gorham [1991a, 1991b] has made extensive studies into the effects of specimen size in high strain-rate compression tests. It is well known that the flow stress of many metals increases sharply above a critical strain-rate. The deformation in the region below this critical strain-rate is controlled by thermal activation mechanisms. Above the critical strain-rate it was originally thought that velocity-dependent drag on dislocation motion caused the additional limitation to deformation. However, it is now thought that this change is more likely due to structure evolution (twinning) in copper and other F.C.C. materials. More severe mechanical twinning in copper has been shown to be favoured above a critical strain-rate [Haque, Pickering and Hashmi 1985]. Gorham [1991a, 1991b] has also related this critical strain-rate to the effect of inertia. Assuming that a ≈ h, and that the inertial stress effects will become particularly noticeable in test results at approximately 10% of the measured flow stress (i.e. $\sigma_t = 25$ MPa in equation (2.18)), results for copper give $d\dot{\varepsilon} \approx 1.6 \times 10^2$ m s$^{-1}$ (where $d$ is the specimen diameter). This relationship plotted on a graph of critical strain-rate versus specimen diameter lies outside the experimental data region. This implies that inertia effects arising from the assumption of steady and uniform deformation are not likely to be significant.

Gorham [1991a, 1991b] also considered the effects of inertia during the initial rise-time of the loading pulse. Assuming the rise-time in microseconds is approximately numerically equal to the bar diameter (D) in millimetres, and that the strain-rate is fully established within this rise-time:

$$\dot{\varepsilon} \approx \frac{\dot{\varepsilon} \times 1000}{D}$$  \hspace{1cm} (2.19)

Using the assumptions in the previous paragraph and assuming that the specimen diameter is equal to 0.8D gives $d\dot{\varepsilon} \approx 42$ m s$^{-1}$. This relationship plotted on a graph of critical strain-rate versus specimen diameter lies very close to the experimental data region. This implies that transient inertial stresses are significant at very high strain-rates and must be accounted for.

Bertholf and Kerns [1975] have performed a two-dimensional analysis of the SHPB using an elastic-plastic wave propagation computer code. If $d = 0.8D$ as above, then $d\dot{\varepsilon} < 40$ m s$^{-1}$ can be derived from Bertholf and Kerns' analysis as a condition for
minimal inertia operation. This condition is slightly more stringent than Gorham’s above because the rise-time of the loading pulse in the Bertholf and Karnes analysis is longer (1.6 μs per bar diameter millimetre).

Bertholf and Karnes [1975] also simulated the variation of inertia with strain-rate, pulse rise-time and specimen size. The strain-rate was increased by increasing the size of the loading pulse (and keeping the rise-time constant). At higher strain-rates oscillations were introduced into the stress-strain curves caused by radial waves resulting from the short pulse rise-time. Oscillations in the stress-strain curves at the highest average strain-rate (3300 s\(^{-1}\)) caused a large overestimation of the flow stress. Variations in stress and strain increased with strain-rate throughout the specimen. Increasing the pulse rise-time from approximately 10 μs to 40 μs eliminated the oscillations seen at higher strain-rates on the stress-strain curves without seriously reducing the average strain-rate. Specimens with three length/diameter ratios were simulated. The size of the loading pulse was increased for larger length/diameter ratio specimens to ensure a constant strain-rate. Consequently inertia effects increased with increasing length/diameter specimen ratios.

Gorham [1991a] derived some empirical relationships between specimen size and the critical strain-rate (\(\dot{\varepsilon}_c\)) from published data for copper. Good agreement with experimental data was found with \(\dot{\varepsilon}_c \propto d^{-1.4}\) and \(\dot{\varepsilon}_c \propto (hd)^{-0.84}\). Friction, strain-rate and wave-propagation effects, together with inertia effects, will all play a role in the variation of the critical strain-rate with specimen size.

Most inertia models (e.g. equation (2.18)) assume a moving loading face and a stationary anvil. Although this type of analysis gives a “feel” for inertia effects in the SHPB, it does not accurately reflect the real situation. Gorham [1989] has analysed the effects of inertia in the SHPB assuming the faces of the pressure bars in contact with the specimen are moving at different velocities, giving:

\[
P_i = \sigma_y + \rho \left( \frac{a^2}{8} + \frac{h^2}{3} \right) \dot{\varepsilon} + \rho \left( \frac{a^2}{16} - \frac{h^2}{3} \right) \dot{\varepsilon}^2 + \frac{\rho h \dot{v}}{2}
\]  

(2.20)

\[
\frac{P_1 + P_2}{2} = \sigma_y - \rho \left( \frac{a^2}{8h} + \frac{h^2}{12} \right) \dot{\varepsilon} + \rho \left( \frac{a^2}{16} - \frac{h^2}{12} \right) \dot{\varepsilon}^2
\]  

(2.21)
where:

\[ P_1 = \text{pressure on the specimen face in contact with the incident bar (Pa)} \]

\[ P_2 = \text{pressure on the specimen face in contact with the transmitter bar (Pa)} \]

\[ \sigma_y = \text{compressive yield stress (Pa)} \]

\[ v = \text{velocity of specimen face in contact with transmitter bar (m s}^{-1}) \]

Equations (2.20)-(2.22) have terms in strain-rate and strain-rate acceleration which are very difficult to accurately establish from experimental records due to noise and dispersion/wave propagation effects. Therefore Gorham [1989] used a simple numerical procedure to evaluate the inertia terms in equation (2.22) for a 13 mm diameter copper specimen \((h/a = 1)\) compressed between steel bars at an average strain-rate of \(10^3 \text{ s}^{-1}\). A maximum flow stress correction due to inertia of approximately 1.2% was predicted for the start of the test, reducing to approximately 0.5% 25 \(\mu\text{s}\) into the test. Kornev [1992] has also derived an inertia correction for the SHPB including second degree differential terms, but gives no indication as to the magnitude of the correction.

Wu and Gorham [1997] measured the difference in the forces acting on the faces of copper specimens tested using a SHPB with a smoothed loading pulse. Three specimen length to diameter ratios were used (0.76 mm, 1.5 mm and 3.9 mm long and all 4.7 mm in diameter). Each of these specimens was loaded with a nominal projectile impact velocity of 22.3 m \(\text{s}^{-1}\). The stress on the front face of the specimen was found to be higher than that on the rear face and this stress difference was found to increase in magnitude and duration with increasing specimen length. For the 4.7 mm long specimen the stress on the rear face never reached the level of stress on the front face. Wu and Gorham [1997] combined equations (2.20) and (2.22) ignoring terms in \(\dot{\varepsilon}^2\) and \(\ddot{\varepsilon}\) to calculate the difference between stresses on the front and rear faces. This calculated difference in stress due to inertia effects is a large proportion of the experimentally measured difference. Effects due to multiple wave reflection within the specimen (please see below) are also thought to be responsible for the difference between the levels of stress on the front and rear faces, especially as inertia effects are only significant within

\[
P_2 = \sigma_y - \rho \left( \frac{a^2 - h^2}{8} \right) \ddot{\varepsilon} + \rho \left( \frac{a^2 - h^2}{16} \right) \dot{\varepsilon}^2 - \frac{\rho \dot{v}}{2}
\]
the early stages of the test. If the flow stress is calculated using only the transmitted pulse, it will be significantly lower during the early stages of the test than if it was calculated using either just the stress on the front face or an average of the front and rear face stresses.

Wu and Gorham [1997] suggested that due to the higher stress on the front face of the specimen compared to that on the rear, the specimen may become slightly tapered towards the rear face. Then the increases in stress and strain measured at the front face later in the test would be a function of specimen shape as well as plastic wave propagation effects. Wu and Gorham did not observe any tapering of their test specimens. However, Nguyen [1994] observed severe tapering towards the rear face of the specimen for a variety of metals tested using a SHPB at slightly higher strain-rates than those of Wu and Gorham [1997].

Malatynski [cited in Malinowski and Klepaczk 1986] notes that the use of a ring specimen as opposed to a solid disc specimen dramatically reduces the effects of radial inertia in a high strain-rate compression test. Therefore if the flow stress during the early stages of a ring test is significantly lower than that of a solid specimen tested under the same conditions radial inertia is likely to be the cause.

**Wave Dispersion**

If a trapezoidal elastic pulse travels along a metal bar then its shape will slowly change due to dispersion effects. High frequency components with a wavelength comparable to the radius of the bar are generated by the short pulse rise-time. As the pulse propagates radial inertia due to these high frequency components generates release waves at the surface of the metal bar. These release waves interact with the longitudinal pulse causing continuous fluctuations of particle velocity near the surface of the bar. Fluctuations in particle velocity cause fluctuations in stress and strain resulting in a non-uniform distribution of stress and strain across any section of the bar. These fluctuations result in the commonly observed Pochhammer-Chree oscillations [Meyers 1994]. Interaction of the longitudinal pulse with the metal bar surface effectively causes a continual partial conversion of shear waves to longitudinal waves and vice versa [Wasely 1973]. Because the velocity of a particular high frequency component depends upon the
amount of radial inertia associated with that frequency, bars made from a material with a low Poisson's ratio would produce little dispersion. Unfortunately most materials with a low Poisson's ratio are either too brittle or poisonous [Field et al. 1994].

Higher frequency components generated by the short rise-time of the pulse travel at lower velocities than lower frequency components. Therefore sudden changes of shape in the pulse are smoothed (rise-time increases), oscillations appear along the total length of the crest of the pulse, and the whole pulse broadens [Johnson, 1972]. Pochhammer and Chree independently derived the same equations describing elastic wave propagation along an infinitely long bar [see Kolsky 1963, Wasely 1973 or Johnson 1972]. The application of these Pochhammer-Chree equations to a finite length bar results in a very complex equation. Rayleigh derived an approximate solution for a finite length bar for the case when the bar radius to wavelength ratio is relatively small. Rayleigh's equations describe the group velocity of the packet of component waves, the rate at which the energy of the longitudinal component waves is propagated:

\[
\frac{C_g}{C_0} = 1 - 3\nu^2 \pi^2 \left(\frac{a}{\Lambda}\right)^2
\]  

(2.23)

and the phase velocity [Kolsky 1963]:

\[
\frac{C_p}{C_0} = 1 - \nu^2 \pi^2 \left(\frac{a}{\Lambda}\right)^2
\]  

(2.24)

where:

- \(C_g\) = group velocity (m s\(^{-1}\))
- \(C_0\) = longitudinal wave velocity in a bar (m s\(^{-1}\))
- \(C_p\) = phase velocity (m s\(^{-1}\))
- \(\nu\) = Poisson's ratio
- \(a\) = bar radius (m)
- \(\Lambda\) = wavelength (m).
If \( a/\Lambda < 0.1 \) in equations (2.23) and (2.24), i.e. if the length of the pulse is long compared to the radius of the metal bar, then the differences between \( C_g \) and \( C_0 \) and between \( C_p \) and \( C_0 \) are less than 5\%, i.e. \( C_p \approx C_g \). Equations (2.23) and (2.24) agree well with Pochhammer-Chree theory for \( a/\Lambda < 0.1 \), however for larger values of \( a/\Lambda \) the Pochhammer-Chree theory gives more accurate predictions. Equations (2.23) and (2.24) unrealistically predict that \( C_g \) and \( C_p \) tend to zero as \( \Lambda \) decreases. In reality a plateau forms for \( C_g \) and \( C_p \) at approximately \( 0.58C_0 \) - equivalent to the Rayleigh surface wave velocity. Equation (2.24) demonstrates that high frequency, short wavelength oscillations introduced by a short pulse rise-time travel at velocities much lower than \( C_0 \), i.e. the pulse will be dispersed.

The few microsecond rise time on the loading pulse in a conventional SHPB system means that the strain-rate in the sample rises smoothly from zero. A nominal value of strain-rate is reached only after significant microstructural deformation has occurred. This initial deformation will have significant effects on the following deformation behaviour. Direct impact systems do not suffer from this problem [Pope and Field 1984]. Another advantage of direct impact systems is that dispersion is only significant at higher frequencies due to the smaller bar diameter [Gorham, Pope and Field 1992].

As mentioned in Section 2.1.2 Parry, Walker and Dixon [1995] have produced a smooth loading pulse in a modified SHPB by using a low strength 431 steel pre-loading bar. This pulse is virtually oscillation free at low projectile velocities and is completely oscillation free at high projectile velocities. Consequently the reflected and transmitted pulses are also virtually oscillation free. Analysis of these pulses results in stress-strain curves with no oscillations. Therefore it is much easier to accurately determine material characteristics such as yield points in this system compared to a conventional SHPB. Another major advantage of this system is that because the loading, reflected and transmitter pulses have fewer high frequency components, their shapes are only minimally distorted by dispersion effects during propagation. The average flow stress was fractionally lower for the first 4\% strain for the copper (\( \approx 3\% \) reduction) and 224 carbon steel (\( \approx 1\% \) reduction) specimens tested with the smoothed loading pulse compared with the old non-smoothed loading pulse. Above 4\% strain the average flow stress curve for each material was independent of the loading pulse (smoothed or non-
smoothed). This SHPB system is used in this study, please refer to Section 3 for more details.

The flow stress determined by a standard SHPB system should always be calculated from the transmitted pulse. Bertholf and Karnes [1975] have shown from their two-dimensional analysis of the SHPB that Pochhammer-Chree oscillations on the loading pulse are mostly reflected back at the first loading bar/specimen interface. This is clearly demonstrated by the SHPB pulse records in Parry, Walker and Dixon [1995].

In some cases the length of the rise-time of the loading pulse in a SHPB may alter the measured stress-strain properties or oscillations due to dispersion cannot be removed by an experimental adaptation (e.g. the direct impact test). In these cases a numerical correction for dispersion can be applied.

Gorham [1983] used a Fast Fourier Transform technique together with phase velocities calculated using a numerical solution to the Pochhammer-Chree equations by Bancroft [cited by Gorham 1983]. Appropriate phase shifts were applied to each frequency component to deduce the original pulse shape in a direct impact test. This correction removed almost all the oscillations from the pulse and reduced the pulse rise-time by a factor of two. This method of dispersion correction is inaccurate for large amounts of dispersion due to differences between theoretical and experimental conditions (i.e. load distribution and errors in empirically determined material constants).

Gorham and Wu [1997] have developed another dispersion correction technique that can also correct for higher levels of dispersion. The dispersion characteristics of a metal bar were derived from measurements of stress pulses in the metal bar caused by the elastic impact of small steel spheres at one end face. This method makes no theoretical assumptions about wave propagation in the metal bar. A test was performed on a copper specimen using a standard SHPB system where the loading pulse was deliberately smoothed by placing a double layer of paper over the impact face of the incident bar. The loading pulse was virtually oscillation free. Application of the dispersion correction technique resulted in a significant reduction (approximately 15 %) in the flow stress over the first 1 % strain. Therefore dispersion can still have a significant effect on the first 1 to 2 % strain of a test even if oscillations are not apparent in the measured pulses. It is interesting to note that when Parry, Walker and Dixon [1995] reduced the effects of dispersion by using a smoothed loading pulse with a longer rise-time, only very small
reductions in flow stress for the first few percent strain were evident. This implies that even with oscillations present on the original non-smoothed loading pulse, the effects of dispersion (identified by loading pulse oscillations) on the flow stress were only fractional different. It could be argued that a large dispersion correction still needs to be applied. However, it is more reasonable to argue that dispersion effects with either the smoothed (rise-time ≈ 14 μs) or non-smoothed (rise-time ≈ 7 μs) pulses used by Parry, Walker and Dixon [1995] will have significantly less dispersion than the smoothed pulse of Gorham and Wu [1997] (rise-time ≈ 2 μs). The SHPB system employed by Parry, Walker and Dixon [1995] also has a higher pulse width to bar diameter ratio than that of Gorham and Wu [1997], further reducing the effects of dispersion.

Many other mathematical techniques exist in the literature to correct for the effects of dispersion, some of these most applicable to SHPB testing are mentioned here. Zhao and Gary [1995] have derived a three dimensional analytical solution of the longitudinal wave propagation in an infinite linear viscoelastic bar. This theory was applied to correct for dispersion effects found when testing low impedance materials such as foams in a SHPB with low impedance viscoelastic bars. Watson Jr. [1972] investigated the effects of measuring the time history of a passing dispersive wave with a finite length strain gauge. Time and amplitude errors between the actual values (obtained using very small strain gauges) and the measured values (obtained using a strain gauge with a long gauge length) of the wave function were related to the gauge length, properties of the materials and spectral content of the incident wave. Bacon, Carlsson and Lataillade [1991] took into account wave dispersion effects to evaluate the force and particle velocity at the heated end of a rod subjected to impact loading. Gong, Malvern and Jenkins [1990] made corrections for dispersion effects by measuring two consecutive pulses in the transmitter bar of a SHPB arrangement. They assumed that the dispersive high frequency oscillatory components riding on the top of the main pulse originated from the first mode vibration. The dispersion was corrected by using the Fast Fourier Transform and Fourier series expansion numerical schemes. Thurston [1992] has written a review paper providing mathematical descriptions of many different aspects of elastic wave propagation in rods, including dispersion effects.

Dispersion also takes place within the specimen, not just within the SHP bars. Multiple wave reflections within the specimen cause dispersion of the loading pulse so
waves transmitted through the specimen to the transmitter bar will have a longer rise-time [Al-Maliky, 1997]. Rodriguez et al. [1995] noted that using smaller diameter specimens in a SHPB test resulted in more oscillations in the stress-strain curve.

Multiple Wave Reflection within the Specimen

An equilibrium stress state is built up within a specimen tested with a conventional SHPB by multiple reflections of elastic and plastic waves. Parry et al. [1994] have developed a computer simulation based upon one dimensional elastic wave propagation theory that graphically illustrates the development of the incident, reflected and transmitted pulses. The SHP bars in the simulation were defined as steel. Stress equilibrium was said to be reached when the transmitted pulse equalled the sum of the incident and reflected pulses. A material with a high acoustic impedance (Duralumin) reached stress equilibrium in a shorter time than one with a low acoustic impedance (high density polyethylene (HDPE)). Stress equilibrium was achieved faster using a ramped as opposed to a rectangular loading pulse, although this effect was more marked for the lower acoustic impedance material. Using the ramped loading pulse (rise-time = 10 μs), the Duralumin reached 90% of its equilibrium state in about 4.0 μs, while the HDPE reached the same level in about 7.5 μs. The Young's modulus calculated from the simulated pulses agreed exactly with that calculated from elastic SHPB tests if the transmitted stress was shifted forward by half the elastic wave specimen traverse time.

Briscoe and Nosker [1984] have performed a similar analysis to that of Parry et al. [1994]. However the usefulness of this analysis is greatly restricted due to over-simplifications: a rectangular loading pulse and a specimen with the same cross-section as the loading bars.

Wu and Gorham [1997] have suggested that the difference in the measured stresses on the front and rear faces of a SHPB specimen could in part be due to elastic and plastic multiple wave reflection within the specimen. A considerable elastic impedance mismatch existed between the copper specimens and the titanium SHP bars used by Wu and Gorham. This impedance mismatch would delay the formation of the stress equilibrium within the specimen. In the latter stages of the test, when effects such as inertia are not so significant, it was suggested that plastic multiple wave reflection
within the specimen could be responsible for the difference in the stresses on the specimen's faces.

Couque and Walker [1994] have investigated the strain-rate variation along the length of a SHPB specimen by applying strain gauges along the length of the specimen and by numerical simulation using the HEMP hydrocode. Both methods showed that during the early stages of loading the specimen material closest to the front face was loaded at a much higher strain-rate than the material at the rear end. For the iron, copper and tungsten alloy specimens tested a state of uniaxial stress did not exist for strains less than 0.1. The long length of the specimens (12.7 mm) used in this study made multiple wave reflection effects significant.

The size of the specimen can also affect the time required for an equilibrium stress field to be established in a SHPB specimen. The elastic and plastic wave specimen traverse times will be significantly longer in longer specimens. In general, to reach a given strain-rate, the stress history in a shorter specimen reaches equilibrium much sooner than in a longer specimen, and involves smaller fluctuations around the mean value. This is very important because the nucleation of dislocations and the formation of loops and twins is very sensitive to the magnitude and time scale of the imposed stress. The initial dislocation structures formed control the future behaviour of the material [Gorham 1991a].

### 2.2 MECHANICAL BEHAVIOUR OF METALS

The mechanical behaviour of metals is a very large subject. A summary of dislocation theory is given below which explains the basic plastic material responses of face-centred cubic (FCC) (e.g. copper) and body-centred cubic (BCC) materials (e.g. iron and steel) used in this study. Constitutive equations are then discussed that model the response of metals under different test conditions. Finally some well known constitutive equations are described, including the Armstrong-Zerilli model which is applied extensively throughout this study.

If the reader requires highly detailed descriptions of dislocation theory and the plastic response of metals then reference should be made to two excellent books by Hull and Bacon [1984] and Honeycombe [1984] respectively.
2.2.1 ELASTIC MATERIAL RESPONSE

The elastic response of a metal is usually described by the modulus of elasticity or Young's modulus given by the gradient of the elastic portion of the stress-strain curve. The Young's modulus is directly related to the slope of the force-distance curve describing atomic attraction and repulsion. Therefore metals with high binding forces between their atoms have a high Young's modulus [Askeland 1996]. The binding forces between atoms are strongly affected by the distance between the atoms, therefore the elastic modulus will vary with direction within the crystal lattice. In general the elastic modulus for anisotropic metals (or single crystals) is a maximum in the ⟨111⟩ (the closest packing direction in FCC and BCC materials) and a minimum in the ⟨100⟩ directions. Therefore in a polycrystalline isotropic material the Young's modulus is independent of direction and is an average for all directions [Smallman and Bishop 1995].

Stress is usually assumed to be directly proportional to strain for an elastic solid. However if an elastic stress is applied to a specimen and released, the strain will decrease until a small portion remains, the anelastic strain, that slowly decreases with time. Also if a metal is tested at a higher strain-rate then the elastic modulus is higher. Both these effects show that elastic strain is time dependent. This time dependence can be explained in terms of "internal friction". Internal friction is a generic heading that groups together various energy dissipating effects such as the migration of atoms, lattice defects and thermal energy barriers. All these causes of internal friction are themselves time-dependent hence the time-dependence of strain. The concept of internal friction is described in more detail below when the stress required to overcome the lattice resistance to movement of a dislocation, the Peierls-Nabarro stress, is discussed [Smallman and Bishop 1995].

If a region of metal is heated, the amplitude of atomic vibrations in the heated region increases. An atom with a large vibrational amplitude can interact with a neighbouring atom with a smaller vibrational amplitude, hence energy is transferred from one atom to another. This energy transfer between atoms creates phonons - an elastic wave that transfers energy through a material. The number of phonons is a measure of the material heat content. Although heat transfer occurs through a material by phonons, in metals about 95% of heat transfer at room temperature is by the motion of free
electrons. When the whole material is at the same higher temperature then the amplitude of atomic vibrations of all the atoms will be greater. These atoms will behave as though they have a larger atomic radius and the average distance between them will increase. The attractive force between atoms further apart is weaker hence the elastic modulus will be lower. Also the internal friction effects caused by thermal energy barriers will be reduced at higher temperatures, again reducing the elastic modulus [de Podesta 1996].

2.2.2 PLASTIC MATERIAL RESPONSE

Plastic deformation in metals is caused by two mechanisms: slip and twinning. Slip can be envisaged as the sliding of one plane of atoms over another plane of atoms along a certain crystallographic plane known as a slip plane. Twinning occurs by the complex motion of partial dislocations. A crystal is twinned when one portion of the lattice is a mirror image of a neighbouring portion. The sheared region in twinning occurs over many atomic planes, whereas in slip the sheared region is across one slip plane only. Therefore slip usually occurs at much lower applied stresses than twinning [Dieter 1986].

Perfect Crystal Structure

In a perfect crystal, if it is assumed that a periodic shearing force is applied to move one plane of atoms over another plane of atoms, then the applied shear stress, $\tau$ (Pa), is:

$$\tau = \frac{Gb}{2\pi a} \sin \left( \frac{2\pi x}{b} \right)$$  \hspace{1cm} (2.25)

where:

- $G$ = shear modulus (Pa)
- $b$ = atomic spacing in direction of shear stress (m)
- $a$ = spacing of planes of atoms (m)
- $x$ = displacement in the slip direction between the planes of atoms (m).
It is interesting to note that for small strains equation (2.25) reduces to Hooke's Law. The right hand side of equation (2.25) is periodic in b, therefore the theoretical critical shear strength, \( \tau_{th} \) (Pa), is given by:

\[
\tau_{th} = \frac{Gb}{2na}
\]  

(2.26)

If as a rough approximation \( a = b \), then:

\[
\tau_{th} = \frac{G}{2\pi}
\]  

(2.27)

The shear modulus for most metals is in the range 20 to 150 GPa. Therefore the theoretical shear strength of a perfect metallic crystal is in the range 3 to 25 GPa. This theoretical shear strength is at least 100 times greater than the observed shear strength of metal crystals. Therefore another mechanism other than the shearing of whole planes of atoms must be responsible for slip. Slip by dislocation motion is responsible for shear strengths much lower than the theoretical value [Hull and Bacon 1984].

**Dislocations**

The crystal structure of a metal is not perfect. There are a variety of imperfections (line defects e.g. dislocations, planar defects e.g. grain boundaries, stacking faults and twins, point defects e.g. vacancies, interstitial defects and substitutional defects) that affect the deformation behaviour of a metal. There are two basic types of dislocation, the screw dislocation and the edge dislocation. A screw dislocation can be visualised by cutting part way through a perfect crystal lattice. If all the planes above the cut are skewed by one atomic spacing with all the planes below the cut, then the line along which the shearing occurs (i.e. the line along the bottom of the cut) is a screw dislocation. Again visualising a cut in a perfect crystal lattice, if this cut is forced apart and an extra plane of atoms inserted, the line at the bottom of the cut represents an edge dislocation. A Burgers vector describes the magnitude and direction
of the slip resulting from the motion of a single dislocation. The Burgers vector is parallel to the screw dislocation and perpendicular to the edge dislocation. Near the dislocation line some atoms are not in their minimum energy positions and only a small movement is required to cause one atom to slide over another. Therefore plastic deformation by dislocation motion can take place at much lower shear stresses than plastic deformation of perfect crystals [Hull and Bacon 1984].

Both the edge and screw dislocations can move by glide. Glide is the movement of a dislocation in a plane (the slip plane) that contains the dislocation line and Burgers vector. Glide is a conservative motion, i.e. motion without the diffusion of atoms. Glide of many dislocations results in slip. The dislocation line and the Burgers vector define a single plane for edge dislocations but not for screw dislocations. Therefore motion by glide is restricted to a specific plane for edge dislocations but not for screw dislocations. Edge dislocations can move into parallel adjacent slip planes, i.e. move out of the glide surface normal to the Burgers vector by climb. Climb is a non-conservative motion, i.e. it involves the diffusion of atoms and is therefore thermally activated. Climb does not apply to screw dislocations. Climb of a short section of a dislocation line results in the formation of two steps (known as jogs) in the dislocation line from one atomic slip plane to an adjacent parallel slip plane. Steps in the dislocation line on the same plane are called kinks. A jog on a screw dislocation only has edge character and therefore can only glide along the dislocation line. Movement of the jog into adjacent slip planes can be accomplished by climb. Therefore a jog on a screw dislocation severely impedes its gliding motion. Hence screw dislocations travel slower than edge dislocations [Hull and Bacon 1984].

The Peierls-Nabarro stress, $\tau_p$, is the shear stress required for dislocation glide in a particular direction through the crystal lattice:

$$\tau_p = \frac{2G}{1-v} \exp \left( \frac{-2\pi \omega}{b} \right)$$

(2.28)

$\omega = \frac{a}{1-v}$ for an edge dislocation

$\omega = a$ for a screw dislocation
where:
\[ \tau_r = \text{resolved shear stress (Pa)} \]
\[ G = \text{modulus of elasticity in shear (Pa)} \]
\[ \nu = \text{Poisson's ratio} \]
\[ w = \text{dislocation width (m)} \]
\[ a = \text{distance between slip planes (m)} \]
\[ b = \text{distance between atoms in the slip direction (Burgers vector) (m)}. \]

Equation (2.28) assumes a sinusoidal force-distance law for atomic interaction so it cannot be used to accurately predict applied resolved shear stresses. However this equation does predict applied resolved shear stresses of the same order of magnitude as those seen in real metals. Clearly the resolved shear stress depends on the width of the dislocation, the distance between the slip planes and the distance between the atoms in the slip direction. The closer the packing of atoms in the slip direction the lower the resolved shear stress. The wider the dislocation and the greater the distance between the slip planes the lower the resolved shear stress. Planes with a relatively large inter-planar spacing also have a high atomic density. Therefore in metals the slip plane is the plane of greatest atomic density and the slip direction is the closest-packed direction within the slip plane. A slip plane and a slip direction constitute a slip system [Hull and Bacon 1984].

In the FCC structure the \((110)\) directions are close-packed and therefore the slip directions. The \((110)\) directions lie in the close-packed \{111\} planes. There are four sets of parallel \{111\} planes, each \{111\} plane containing three \((110)\) directions. Therefore the FCC structure has twelve slip systems. In the BCC structure the \((110)\) directions are close-packed as in the FCC structure. However there are no close-packed planes. There are several planes \((110), \{112\}, \{123\}\) which have equally high packing density. Therefore slip occurs in the \((110)\) directions in these planes giving a possible 48 slip systems. Since the slip planes in the BCC structure are not as close-packed as those in the FCC structure, higher shearing stresses are required to cause slip in BCC structures. Dislocations in a BCC structure can move from one plane to another by cross slip. Indeed iron often slips on all the slip planes at once. Therefore the slip line (the line of
intersection of the slip plane with the outer surface of the crystal) has a wavy appearance [Dieter 1986].

**Mathematical Description of Dislocation Motion**

A simple mathematical examination of dislocation motion elucidates the dependence of plastic strain on many factors such as temperature, strain, strain-rate and crystal structure. The glide force per unit length of dislocation, \( F \) (N m\(^{-1}\)) is given by:

\[
F = \tau b 
\]  

where:

\( \tau \) = shear stress in the glide plane resolved in the direction of \( b \) (Pa)

\( b \) = Burgers vector (m).

The glide of many dislocations under this applied force will result in a shear strain, \( \gamma \), given by:

\[
\gamma = \rho bl
\]  

where:

\( \rho \) = mobile dislocation density (number per unit area)

\( b \) = Burgers vector (m)

\( l \) = average distance moved by dislocation (m).

It must be emphasised that \( \rho \) is the mobile dislocation density. Dislocations that are immobile do not contribute to the plastic strain. From equation (2.30) the shear strain rate, \( \dot{\gamma} \) (s\(^{-1}\)), is given by:

\[
\dot{\gamma} = \rho bv
\]  

(2.31)
where:
\( v \) = average dislocation velocity (m s\(^{-1}\)).

In equation (2.31) the plastic behaviour is dependent on \( p \), \( b \) and \( v \). \( p \) and \( v \) are themselves dependent on stress, time, temperature and thermomechanical history. \( b \) depends on crystal structure.

The dislocation velocity in a number of metal whisker crystals is described by:

\[
v = K\tau^m
\]

(2.32)

where:
\( \tau \) = applied resolved shear stress (Pa)
\( K, m \) = material constants.

\( m \) is approximately 2 at room temperature for FCC metal whisker crystals and increases with increasing strain-rate, increasing alloying or decreasing temperature. \( m \) increases as the temperature is reduced because there are less dislocation drag forces due to phonon scattering (less phonons at lower temperatures). From equation (2.32) once the shear stress required to start dislocation glide has been reached, small increases in the shear stress result in large increases in the dislocation velocity [Hull and Bacon 1984].

The same empirical equation (2.32) applicable to metal whisker crystals can be used to describe the dislocation velocity in polycrystalline metals:

\[
v = Ct^n
\]

(2.33)

where:
\( C, n \) = material constants.

\( n \) for BCC metals is \( \approx 40 \) and for FCC metals is \( \approx 200 \). \( n \) varies in a similar manner to \( m \) above, i.e. decreasing the temperature or increasing alloying or strain-rate increases the stress level required to produce a given dislocation velocity [Smallman and Bishop 1995].
Dislocation motion is examined in more detail in Section 2.3.

**Yield Stress**

A high Peierls-Nabarro stress is associated with materials with narrow dislocations (see equation (2.28)). Hence the Peierls-Nabarro stress is a short-range barrier to dislocation motion effective over only a few inter-atomic spaces. Thermal activation can assist the applied stress in overcoming short range barriers. A thermally assisted portion of a dislocation can cross the short range barrier and glide by the sideways movement of kinks. Materials with a high Peierls-Nabarro stress are therefore temperature sensitive, losing their strength with increasing temperature [Smallman and Bishop 1995]. The time a dislocation is obstructed by a barrier depends upon the probability that the dislocation will receive a large enough thermal fluctuation of the free activation energy to overcome the barrier [Meyers 1994]. Therefore thermal activation is less effective at higher strain-rates. This also implies that materials that are temperature sensitive are also strain-rate sensitive [Smallman and Bishop 1995].

As stated earlier the slip planes in BCC metals are not as close-packed as those in FCC metals, therefore higher shearing stresses are required to cause slip in BCC metals. Also a screw dislocation in a BCC metal can dissociate on three equivalent slip planes. This dislocation must be constricted before it can glide in any one of the slip planes. This constriction will be more difficult to make at lower temperatures or higher strain-rates. Therefore the yield stress in BCC metals is higher at lower temperatures and higher strain-rates. Also the yield stress is higher in BCC metals compared with FCC metals due to the intrinsically higher Peierls-Nabarro stress [Smallman and Bishop 1995].

In FCC metals the dislocation can only dissociate on the \{111\} planes. There is no direction in these slip planes along which the dislocation could dissociate on other planes. Therefore the temperature and strain-rate dependence of the yield stress in FCC metals is small [Smallman and Bishop 1995].
Yield Points

The existence of yield points in BCC and FCC metals can be explained in terms of dislocation mobility. For metals with a low initial mobile dislocation density, the only way the right hand side of equation (2.31) can equal the imposed shear strain-rate is for the dislocation velocity to be high. In BCC metals the Peierls-Nabarro stress is quite large. Therefore dislocation motion at high velocities can only occur under large applied shear stresses (see equation (2.33)). This gives rise to the upper yield point in BCC metals. It must be noted that the dislocation density of annealed iron is at least $10^6 \text{ cm}^{-2}$. However many of these dislocations are pinned by solute-dislocation interaction or by precipitation of fine carbides or nitrides along the dislocation [Dieter 1986].

In FCC metals the Peierls-Nabarro stress is quite small. Therefore the shear stress in a FCC metal required to move a dislocation even at high velocities is much lower than in a BCC metal (see equation (2.33)). Therefore to achieve the high dislocation velocities and high shear stresses associated with the upper yield point, the mobile dislocation density (equation (2.31)) in FCC metals must be reduced to virtually zero. FCC metal whisker crystals have virtually no dislocations and the upper yield strength approaches the theoretical yield strength. Solute atoms in FCC metals can lock dislocations that have dissociated into partial dislocations with an edge component. Reduction of mobile dislocations by solute atom locking can result in an upper yield point in FCC metals [Smallman and Bishop 1995].

At the upper yield stress dislocations move and multiply. Dislocation multiplication occurs by Frank-Read sources, multiple cross-slip mechanisms, climb (Bardeen-Herring sources), grain boundary sources and at stress concentrators [Hull and Bacon 1984]. The increased dislocation density causes some strain hardening. However because of the higher dislocation density, a lower average mobile dislocation velocity is required to maintain a constant strain-rate (equation (2.31)). This drop in the average mobile dislocation velocity outweighs the effects of strain hardening. Therefore the applied shear stress required for dislocation glide is lower (the yield drop to the lower yield point). As deformation continues the increasing dislocation density causes greater strain hardening and the flow stress begins to increase with strain [Smallman and Bishop 1995].
The stresses for the upper and lower yield points therefore depend on the mobile dislocation density and the exponent in equation (2.33) describing the stress dependence of the dislocation velocity. Combining equations (2.31) and (2.33) gives:

\[
\frac{\tau_U}{\tau_L} = \left(\frac{\rho_U}{\rho_L}\right)^{1/n}
\]

(2.34)

where:

- \(\tau_U\) = upper yield point shear stress (Pa)
- \(\tau_L\) = lower yield point shear stress (Pa)
- \(\rho_L\) = mobile dislocation density at upper yield point (number per unit area)
- \(\rho_U\) = mobile dislocation density at lower yield point (number per unit area)
- \(n\) = material constant.

For small values of \(n\) (BCC metals) the ratio of the upper to lower yield point stresses will be large and there will be a strong yield drop. Conversely for large values of \(n\) (FCC metals) there is no significant difference in the upper and lower yield point stresses and hence no yield drop [Dieter 1986].

**Work Hardening**

Work-hardening (or strain-hardening) of a metal results from increasing dislocation motion restriction with increasing plastic strain. The most important strain-hardening mechanism in FCC metals is the intersection of dislocations moving in the slip plane with other dislocations intersecting the slip plane. Dislocations threading through the slip plane are called "forest dislocations". As plastic deformation continues the slip plane of one slip system can intersect the slip plane of another slip system, thus increasing the number of forest dislocations. Intersection of one dislocation with another results in a jog in the dislocation line. Jogs on a screw dislocation restrict its motion and hence contribute to work-hardening [Hull and Bacon 1984].
Jogs can also form by the cross-slip of screw dislocations or mixed dislocations with screw components. Cross-slip is a useful mechanism for dislocations to detour barriers and obstacles (including other dislocations). Screw dislocations can also be annihilated during cross-slip by interaction with screw dislocations of opposite sign. The annihilated dislocation may be replaced from its own original source resulting in slip-band formation. If cross-slip of screw dislocations did not occur then dislocation motion would be restricted early in the deformation process resulting in very large work-hardening rates. In BCC metals which have a large number of slip systems cross-slip is much easier than in FCC metals with fewer slip systems. Therefore work-hardening due to forest dislocation intersection in BCC metals is very much lower than in FCC metals [Dieter 1986].

Strain-hardening due to the intersection of dislocations arises from forces acting over a few inter-atomic distances. Therefore strain-hardening in FCC metals is a short-range thermally activated process that is temperature and strain-rate dependent. In BCC strain-hardening is virtually independent of temperature and strain-rate. Strain-hardening in BCC is due mainly to long range internal stresses caused by dislocation pile ups [Dieter 1986]. New dislocation pinning at existing dislocation pile ups (created at lower levels of strain) results in "cell" structures of high dislocation density [Dieter 1986, Hughes 1992].

Tirupataiah and Sundararajan [1991] showed for copper that the strain-hardening rate of hardness under dynamic indentation conditions is higher than that under static conditions. Thus, the strain-hardening rate of hardness (or equivalent flow stress) is itself strain-rate sensitive. Follansbee [1989] has also observed a similar behaviour in his experiments with copper. The rate sensitivity of strain-hardening rate was attributed to the fact that at very high strain-rates the dislocation immobilisation distance was shortened. This distance could be the distance the dislocation can move during the imposed time of duration of deformation, rather than the much larger distances between the dislocations.

Results obtained for iron showed that the strain-hardening rate of hardness under dynamic indentation conditions is lower than that under static indentation conditions, implying a negative strain-rate sensitivity of the strain-hardening rate. A probable reason for such behaviour could be the relatively lower strain hardening rate exhibited by the
thermally activated component of the flow stress as compared to the athermal component. Since the proportion of the thermally activated component of the flow stress should increase with increasing strain-rate, such a postulate would lead to a decreasing strain hardening rate with increasing strain-rate [Tirupataiah and Sundararajan 1991].

Work hardening due to grain boundaries is considered in the next section.

**Grain Boundaries**

A grain boundary is a planar defect representing the interface between crystal lattices of different orientation. A sub-grain boundary or low angle boundary is an array of simple dislocations that produces a small mis-orientation (one or two degrees) between adjoining crystal lattices. A large angle grain boundary causes a mis-orientation of ten degrees or more [Dieter 1986].

Grain boundaries are known to affect the yield stress and strain-hardening rate in metals. The Hall-Petch equation describes the relationship between the yield stress and grain size:

\[
\sigma_0 = \sigma_i + kD^{\alpha/2}
\]  

(2.35)

where:
- \(\sigma_0\) = yield stress (Pa)
- \(\sigma_i\) = friction stress, i.e. resistance of crystal lattice to dislocation motion (Pa)
- \(k\) = "locking parameter" (Pa m\(^{3/2}\))
- \(D\) = grain diameter (m).

The original model for the Hall-Petch equation assumed that grain boundaries acted as barriers to dislocation motion. If dislocations pile-up at the grain boundary, then the stress concentration at the pile-up must reach a critical value before dislocation sources in neighbouring grains can become active [Smallman and Bishop 1995]. The greater the grain boundary mis-orientation angle the greater the density of dislocations at the grain boundary. Grain boundaries are also an effective source of dislocations. The constraints
imposed by grain boundaries can cause slip near the grain boundary on several slip systems including non-close-packed planes. Therefore since more slip systems than usual operate in a region close to the grain boundary, the hardness of the material next to the grain boundary will be greater than that at the centre of the grain. The smaller the grain size the more the effects of grain boundary hardening will be felt near the centre of the grain. Thus the strain-hardening of a fine grained metal is greater than that of a coarse grained metal. Therefore $k$ in equation (2.35) is a measure of the hardening contribution of the grain boundaries [Dieter 1986]. Lassila [1989], Hansen and Ralph [1982], Rigney, Divakar and Kuo [1992], Schmidt et al. [1991] and Parker [1991] have all confirmed the general Hall-Petch relationship for many metals.

Other models for grain boundary strengthening do not consider the stress-concentration due to the pile-up of dislocations. Instead grain boundary strengthening is thought to depend on the variation of dislocation density with grain size [Dieter 1986].

**Strengthening of Steel**

At temperatures below 910 °C pure iron takes up a BCC structure called alpha iron. Between 910 °C and 1,492 °C the atoms of pure iron can rearrange themselves by a diffusion mechanism into a FCC structure called gamma iron. A carbon atom is less than half the size of an iron atom. Therefore a carbon atom dissolved in iron can take up an interstitial position forming an interstitial solid solution. There is more interstitial space in the FCC structure than there is in the BCC structure. Therefore carbon atoms take up interstitial positions far more easily in the iron FCC structure. The maximum solubility of carbon atoms in gamma iron (austenite) is 2.11 % compared with only 0.0218 % in alpha iron (ferrite) [Askeland 1996].

If iron containing for example 0.3 % carbon is heated over 910 °C to form a FCC structure all the carbon will be dissolved. If this alloy is cooled slowly, the carbon atoms have time to move out of the way of the developing BCC structure. These surplus carbon atoms form iron carbide $\text{Fe}_3\text{C}$ (cementite) which is very strong and hard. Cementite precipitates act as barriers to dislocation motion. A dislocation has to bow around the cementite precipitates (by the Orowan mechanism). The steel is said to be dispersion strengthened. If the alloy is cooled quickly, there is not enough time for the
iron atoms to rearrange into a BCC structure or for the carbon atoms to form iron carbide. The new structure is called martensite. Martensite can be thought of as a highly distorted BCC structure (body centred tetragonal) due to the presence of unwelcome interstitial carbon atoms. This body centred tetragonal structure has no close-packed slip planes in which dislocations can easily move. Martensite also has a fine grain structure with even finer substructure within the grains. Martensitic steel is said to be solid solution strengthened [Anderson et al. 1985].

Martensite is hard and brittle and so the properties are usually modified by re-heating or tempering. When martensite is heated below the eutectoid temperature, BCC alpha iron and iron carbide precipitate. Tempering increases the ductility whilst decreasing the strength and hardness. At low tempering temperatures a low carbon martensitic phase and an epsilon carbide Fe$_2$C phase are possible. At higher tempering temperatures the Fe$_3$C can become very coarse and reduce the dispersion strengthening effect greatly [Smallman and Bishop 1995].

The hardness of a steel after tempering is directly related to the tempering temperature that determined its microstructure. Speich [cited in Reed-Hill and Abbaschian 1992] has related the Vickers hardness to the likely microstructure resultant from tempering a martensitic steel for one hour. For example, a 0.3 % carbon martensitic steel tempered for one hour, with a resulting Vickers hardness of less than about 350 HV, will have developed a BCC structure with large coalesced cementite precipitates. These cementite precipitates may appear as round particles (spheroidal iron carbide), especially with decreasing hardness.

2.3 CONSTITUTIVE MODELS

The last few years has seen a great upsurge of interest in the development and application of constitutive models. A number of approaches have been taken ranging from the purely empirical fitting of experimental data, to purely theoretical models based on the micro-mechanical processes. Engineers are more likely to go for an empirical (data) based solution because they only need to apply this solution to a particular problem. However, the theoretical approach is still of great importance in that it provides a justification for particular aspects of a given empirical relationship and can also indicate
its overall limitations. A purely empirical approach requires a large (and potentially very expensive) testing scheme. A theoretically based approach usually requires less testing to establish the required equation parameters. A "good" constitutive model from a practical point of view should be based on both theoretical micro-mechanical processes and available experimental data (macroscopic behaviour) [Harding 1989].

From the previous sections describing dislocation behaviour, the flow stress characteristics of FCC and BCC metals can be represented by Figure 2.1. The following sections describe mathematically the temperature and strain-rate dependence of the flow stress. This mathematical analysis is the fundamental basis for many constitutive models. Finally descriptions are given of some of the more prominent constitutive models in the literature today.

**Components of Flow Stress**

The flow stress for a typical metal can be represented by Figure 2.2. The flow stress can be split into three different regions each related to a different deformation mechanism defined by the test temperature and strain-rate.
Region 1 describes the athermal flow stress, i.e. the component of flow stress that is essentially not thermally activated. It is the stress required to overcome long-range barriers to dislocation motion such as grain boundaries, other dislocations and solute interactions. Therefore the athermal stress is more prominent in metals with increasing alloy content, and so highly alloyed metals are less strain-rate sensitive. The only temperature sensitivity of the athermal stress is due to the temperature sensitivity of the shear modulus. This has an effect on the shear stress required for dislocation motion as described by the Peierls-Nabarro equation (equation (2.28)). This region corresponds to low strain-rates and high temperatures [Harding 1987, EI-Magd 1994].

The flow stress in region 2 consists of an athermal and a thermal component. At lower temperatures and higher strain-rates the short-range, thermally activated barriers to dislocation motion become more important. The main short-range barrier for BCC metals is the Peierls-Nabarro stress. The main short-range barrier for FCC metals is the intersection of dislocations on the slip plane with dislocation forests [Harding 1987, EI-Magd 1994].
At very high strain-rates (region 3), i.e. above approximately $10^4 \, \text{s}^{-1}$, the flow stress increases rapidly showing a linear dependence on strain-rate. This material response can be interpreted as a viscous flow stress counteracting the motion of high speed dislocations generated at these higher stresses (see equation (2.33)). Therefore if the metal is assumed to act as a Newtonian viscous material, the opposing force per unit length, $f_v$ (N m$^{-1}$), on the dislocation is given by:

$$f_v = Bv \quad (2.36)$$

where:

- $B$ = viscous damping coefficient (Pa s)
- $v$ = dislocation velocity (m s$^{-1}$).

Combining equations (2.29) and (2.36) gives:

$$\tau b = Bv \quad (2.37)$$

where:

- $\tau$ = shear stress in the glide plane resolved in the direction of $b$ (Pa)
- $b$ = Burgers vector (m).

Combining equation (2.31) and (2.37) gives:

$$\tau b = \frac{BM\dot{\varepsilon}}{\rho b} \quad (2.38)$$

where:

- $M$ = orientation factor to convert shear strain into longitudinal strain
- $\dot{\varepsilon}$ = longitudinal strain rate (s$^{-1}$)
- $\rho$ = mobile dislocation density (number per unit area).
Making $\tau = \frac{1}{2}\sigma$, where $\sigma =$ longitudinal flow stress (Pa) gives:

$$\sigma = \frac{2BM}{\rho b^2\dot{\varepsilon}} \tag{2.39}$$

Equation (2.39) clearly demonstrates that if the flow stress is directly proportional to the strain-rate viscous dislocation drag mechanisms could be operating. If the viscous drag mechanism is non-Newtonian then the viscous damping coefficient could increase with increasing dislocation velocity [Meyers 1994].

There are many dislocation drag mechanisms including phonon viscosity, phonon scattering, the thermoelastic effect, electron viscosity and anharmonic radiation. Some of these mechanisms are thermally activated [Meyers 1994].

Many authors agree that the flow stress in region 3 can be explained in terms of a viscous drag mechanism [Kumar and Kumble 1969, Regazzoni and Montheillet 1984]. This region has also been explained using conventional thermal activation theory incorporating the observed increase in the mobile dislocation density with stress [Follansbee, Regazzoni and Kocks 1984, Zerilli and Armstrong 1992, Zerilli and Armstrong 1997]. The rapid increase in flow stress in this region has also been explained in terms of mechanical twinning [Gorham 1991a, Ostwaldt, Klepaczko and Klimanek 1997].

Generally most constitutive models are applied to mechanisms operating in regions 1 and 2 in Figure 2.2. The flow stress is then considered to be composed of two components, the athermal and the thermal flow stresses [Meyers 1994]:

$$\sigma = \sigma_a + \sigma^* \tag{2.40}$$

where:

$\sigma =$ flow stress (Pa)

$\sigma_a =$ athermal (component of) flow stress (Pa)

$\sigma^* =$ thermal (component of) flow stress (Pa).
Temperature and Strain-Rate Dependence of Flow Stress

When a dislocation encounters a barrier, a force has to be applied to the dislocation for it to overcome the barrier. If the barrier is short-range in nature then thermal vibrations of the atom can assist in overcoming the barrier. The force required to overcome a given barrier is shown schematically in Figure 2.3a.

\[ F_0, \sigma_0 \]
\[ \Delta G_0, \Delta G_1, \Delta G_2, \Delta G_3 \]
\[ T_c > T_2 > T_1 \]

**Figure 2.3** Barriers to Dislocation Motion: (a) Short-Range Barrier, (b) Short-Range Rectangular Barrier Superimposed on Long-Range Athermal Barrier.

The area under the force-distance profile represents the isothermal energy change (the Helmholtz free energy change) required for the dislocation to overcome the barrier. This
energy ($\Delta G_o$) can be divided into the work done by the applied resolved shear stress and thermal energy:

$$\Delta G = \Delta G_o - \int_{0}^{F^*} \lambda(F) dF$$  \hspace{1cm} (2.41)$$

where:

$\Delta G$ = free energy of activation (Gibbs free energy) (eV or J)

$\Delta G_o$ = Helmholtz free energy change (eV or J)

$F^*$ = force applied to dislocation by resolved shear stress (N)

$\lambda(F)$ = barrier width (m) as a function of force.

The Gibbs free energy is the thermal energy change associated with the dislocation overcoming the barrier at the same temperature and applied resolved shear stress. The probability, $P_B$, that the Gibbs free energy will be equalled or exceeded by thermal fluctuations at a given temperature is given by the Maxwell-Boltzmann distribution law or Arrhenius equation:

$$p_B = \exp\left(-\frac{\Delta G}{kT}\right)$$  \hspace{1cm} (2.42)$$

where:

$k$ = Boltzmann's constant (J K$^{-1}$)

$T$ = absolute temperature (K).

The frequency $f_B$ (s$^{-1}$) at which the dislocation overcomes the barriers divided by the number of attempts per second, given by the vibrational frequency of the dislocation $f_D$ (s$^{-1}$), equals the probability $p_B$:

$$f_B = f_D \exp\left(-\frac{\Delta G}{kT}\right)$$  \hspace{1cm} (2.43)$$
The time taken by a dislocation to travel a distance \( d \) (m) between two barriers can be divided into a waiting time \( t_w \) (s) in front of the barrier (given by the reciprocal of equation (2.43)), and a running time \( t_r \) between the barriers. The running time is controlled by viscous drag mechanisms which are very weak in regions 1 and 2 of Figure 2.2. Therefore \( t_w \) is very much greater than \( t_r \), and the dislocation velocity \( v \) (m s\(^{-1}\)) is assumed to be \( d/t_w \) in:

\[
v = df_0 \exp\left(-\frac{\Delta G}{kT}\right)
\]  

(2.44)

Combining equations (2.31) and (2.44) gives:

\[
\dot{\varepsilon} = \dot{\varepsilon}_0 \exp\left(-\frac{\Delta G}{kT}\right)
\]

(2.45)

\[
\dot{\varepsilon}_0 = \frac{f_0 \rho bd}{M}
\]

where:

\( \dot{\varepsilon} \) = longitudinal strain-rate (s\(^{-1}\))

\( \rho \) = mobile dislocation density (number per unit area)

\( b \) = Burgers vector (m)

\( M \) = orientation factor to convert shear strain into longitudinal strain.

Manipulating equation (2.45) gives:

\[
\Delta G = kT \ln\left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}}\right)
\]

(2.46)

Equation (2.46) shows that the Gibbs free energy increases with temperature and decreases with increasing strain-rate.
Combining equations (2.41) and (2.46) gives an equation that is the foundation for many constitutive relationships [Meyers 1994]:

\[ kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}} \right) = \Delta G_0 - \int_0^{\tau^*} \lambda(F) \, dF \]  \hspace{1cm} (2.47)

The shape of the barrier will greatly affect the material response predicted by the constitutive model. For example consider the rectangular barrier of width \( \lambda \) (m) shown schematically in Figure 2.3b. From equation (2.29) the force (N) on the dislocation per barrier can be expressed as:

\[ F = \tau b d^* \]  \hspace{1cm} (2.48)

where:

- \( \tau \) = shear stress in the glide plane resolved in the direction of \( b \) (Pa)
- \( d^* \) = barrier spacing along the dislocation line (m).

Therefore the mechanical work done by the applied force equal to the last term in equation (2.47) is given by:

\[ \int_0^{\tau^*} \lambda(F) \, dF = \int_0^{\tau^*} \lambda(\tau) b d^* \, d\tau = \tau^* b d^* \lambda = V \tau^* \]  \hspace{1cm} (2.49)

where:

- \( \tau^* \) = thermal shear stress (Pa)
- \( V \) = activation volume (m³).

The activation volume can be visualised as a physical volume of material in which a stress field exists generated by a short range (thermal) barrier to dislocation motion.

From Figure 2.3b the base stress level is given by \( \sigma_0 \). Therefore \( \tau^* \) in equation (2.49) can be expressed as \( \sigma - \sigma_0 \) (flow stress - athermal stress = \( \sigma^* \)). Now substituting equation (2.49) into equation (2.47) gives:
Equation (2.50) is a basic constitutive model.

Let us consider the variation of the flow stress components with temperature as predicted by the constitutive model for a rectangular barrier. At 0 K the Gibbs free energy is zero from equation (2.46). Therefore from equation (2.50):

\[
\Delta G_0 = V\sigma_0 \quad (2.52)
\]

where:
\[
\sigma_0 = \text{thermal component of stress at 0 K (Pa)}.
\]

At a critical temperature \( T_c \) (K) for a given strain-rate, the Gibbs free energy of activation will be great enough to overcome the barrier. The thermal component of the flow stress for this situation will then be zero. Therefore from equation (2.50):

\[
kT_c \ln\left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}}\right) = \Delta G_0 \quad (2.53)
\]

Combining equation (2.50) (with \( \sigma^* = \text{thermal component of flow stress} \)) with equation (2.53) gives:

\[
\frac{V\sigma^*}{\Delta G_0} = 1 - \frac{T}{T_c} \quad (2.54)
\]
Combining equations (2.52) and (2.54) gives:

\[ \frac{\sigma^*}{\sigma_0} = 1 - \frac{T}{T_c} \]  \hspace{1cm} (2.55)

Using equations (2.40) and (2.55) and considering different strain-rates, a graphical description of flow stress as a function of temperature can be constructed as given in Figure 2.4. The precise dependence of the flow stress on temperature obviously depends on the shape of the barrier used in the model or operating in a real metal [Hull and Bacon 1984].

![Figure 2.4](image-url)  

Figure 2.4 Flow Stress versus Temperature for the Barriers Represented in Figure 2.3b.

### 2.3.1 PROMINENT CONSTITUTIVE MODELS

This section very briefly describes some of the better known constitutive models. The Armstrong-Zerilli model is used extensively in this work and hence is described in some detail.
Armstrong-Zerilli Model

Zerilli and Armstrong [1987] have developed two constitutive equations to describe the behaviour of FCC and BCC metals. The main short-range barrier in BCC metals is the Peierls-Nabarro stress. The main short-range barrier in FCC metals is the intersection of dislocations on the slip plane with dislocation forests. The spacing of the Peierls-Nabarro barriers is equal to the lattice spacing and therefore independent of strain. However, the spacing of the dislocation forest barriers will decrease with increasing dislocation forest density, which in turn increases with strain. The activation volume as defined in equation (2.49) can be defined in terms of an activation area \( A \) (m²):

\[
V = b d \lambda = A b \tag{2.56}
\]

The activation area represents the area swept by a dislocation in overcoming a thermal barrier. The activation area is therefore independent of strain for BCC metals and dependent on strain for FCC metals.

If \( l \) is assumed to be constant in equation (2.30) then:

\[
\varepsilon = k \rho \tag{2.57}
\]

\[
k = \frac{b l}{M}
\]

where:

- \( M \) = orientation factor to convert shear strain to longitudinal strain
- \( l \) = average distance moved by dislocation (m).

The dislocation density is related to the dislocation spacing by:

\[
\rho \approx \frac{1}{d^2} \tag{2.58}
\]
Combining equations (2.56), (2.57) and (2.58) gives:

\[ A = \lambda \left( \frac{b}{M} \right)^{1/2} e^{-1/2} \]  \hspace{1cm} (2.59)

From Armstrong [1967] the thermal flow stress can be expressed as:

\[ \sigma^* = B \exp(-\beta T) \]  \hspace{1cm} (2.60)

\[ B = \frac{M \Delta G_0}{A_0 b} \]
\[ \beta = \frac{\ln\left( \frac{A}{A_0} \right)}{T} - \frac{\ln \left( 1 + \left( \frac{kT}{\Delta G_0} \right) \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right)}{T} \]

where:
\[ A_0 = \text{activation area at } 0 \text{ K} \left( \text{m}^2 \right). \]

\[ \beta \] in equation (2.60) is dependent on strain and strain-rate and is usually expressed in the form [Armstrong and Campbell 1973]:

\[ \beta = -C_3 + C_4 \ln \dot{\varepsilon} \]  \hspace{1cm} (2.61)

where:
\[ C_3, C_4 = \text{material constants} \left( \text{K}^{-1} \right). \]

Since the activation area is independent of strain for BCC metals, equation (2.60) can be expressed as:

\[ \sigma^* = C_1 \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) \]  \hspace{1cm} (for BCC metals)  \hspace{1cm} (2.62)
where:
\( C_1 \) = material constant (Pa).

The activation area for FCC metals from equation (2.59) is proportional to \( \varepsilon^{-1/2} \), therefore equation (2.60) can be expressed as:

\[
\sigma^* = C_2 \varepsilon^{1/2} \exp\left(- C_3 T + C_4 T \ln \dot{\varepsilon}\right) \quad \text{(for FCC metals)} \tag{2.63}
\]

where:
\( C_2 \) = material constant (Pa).

Using the Hall-Petch equation (equation (2.35)) and adding the athermal component of stress, the final Armstrong-Zerilli constitutive equations for FCC and BCC metals can be expressed as:

FCC:
\[
\sigma = \sigma_0 + C_2 \varepsilon^{1/2} \exp\left(- C_3 T + C_4 T \ln \dot{\varepsilon}\right) + k D^{-1/2} \tag{2.64}
\]

BCC:
\[
\sigma = \sigma_0 + C_1 \exp\left(- C_3 T + C_4 T \ln \dot{\varepsilon}\right) + C_5 \varepsilon^n + k D^{-1/2} \tag{2.65}
\]

where:
\( \sigma \) = flow stress (Pa)
\( n \) = work hardening exponent
\( k \) = locking parameter (Pa m\( ^{1/2} \))
\( D \) = grain diameter (m)
\( C_5 \) = material constant (Pa).

Since work hardening is independent of strain-rate and temperature in BCC metals, the work hardening term in equation (2.65) is added to the other terms.
Variants of the Armstrong-Zerilli Model

At very high strain-rates mechanical twinning for iron becomes important. Therefore, Armstrong and Zerilli refined equation (2.65) to include these effects, by assuming that the presence of twins within the grains introduced further barriers to slip and so reduced the effective grain diameter in the Hall-Petch term. This increased the athermal stress component and led to better agreement with data from Taylor tests [Harding 1989].

To model copper more accurately, equation (2.64) was modified to take into account viscous drag [Armstrong and Zerilli 1988]:

\[
\sigma_{th} = 0.5\sigma^* \left( 1 + \left( 1 + 4C_0 \dot{\varepsilon} T / \sigma^* \right)^{1/2} \right)
\]  

(2.66)

\[
\sigma^* = C_2 \dot{\varepsilon}^{1/2} \exp \left( - C_3 T + C_4 T \ln \dot{\varepsilon} \right)
\]

where:

\(\sigma_{th}\) = drag-affected thermal stress (Pa)

\(C_0\) = material constant (Pa s K\(^{-1}\)).

\(C_0\) has a phonon drag term which describes the dislocation running time between barriers. Good agreement was found over a large strain-rate range \((10^{-4} - 10^4 \, \text{s}^{-1})\) between experimental results for copper and the Armstrong-Zerilli model employing the drag-affected thermal stress term.

Goldthorpe [1991] has proposed a modified form of the Armstrong-Zerilli model applicable to annealed and explosively shocked iron:

\[
\sigma = C_1 + C_2 \exp \left( C_3 + C_4 \ln \dot{\varepsilon} \right) T + \left( C_5 \dot{\varepsilon}^n + C_6 \right) \mu_T / \mu_{293}
\]

(2.67)

where:

\(C_i\) = material constants

\(\mu_T\) = shear modulus at prevailing temperature (Pa)
\( \mu_{293} \) = shear modulus at 293 K (Pa).

The work hardening due to shock loading was assumed to increase the athermal stress through \( C_6 \). The last term in equation (2.67) assumes work hardening is independent of temperature; this was proved to be approximately true for small strains but not at higher strains. However, it was assumed that the temperature dependence of the shear modulus also influenced work hardening. Corrections applied for the shear modulus brought better correlation with experimental data. Noble et al. [1999] have found that this model accurately predicts the end loading of Remco iron specimens deformed in a tensile Hopkinson bar system. The accurate prediction of loads during a test as opposed to the deformed specimen shape provides a more sensitive constitutive model validation technique [Noble and Harding 1994b].

A second Armstrong-Zerilli variant has been proposed by Goldthorpe, Butler and Church [1994] for iron and steel:

\[
\sigma = \left( C_1 + C_{\dot{\varepsilon}}^{\infty} \right) \frac{\mu_T}{\mu_{203}} + C_2 \exp \left( (C_3 + C_4 \ln \dot{\varepsilon})T \right)
\]  

(2.68)

The term in this model describing the thermally activated barriers had an almost identical value for the iron and steel tested. The thermal softening of steel acts through the temperature dependence of the shear modulus, i.e. the strain-rate sensitivity is reduced (\( C_1 \) for steel is much greater than \( C_1 \) for iron).

**Johnson-Cook Model**

\[
\sigma = \left( \sigma_0 + B\dot{\varepsilon}^{\infty} \right) \left( 1 + C \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right) \left( 1 - \left( T^* \right)^n \right)
\]  

(2.69)

\[
T^* = \frac{T - T_r}{T_m - T_r}
\]

where:
B, C, n and m are material constants

\[ \sigma_0 = \text{reference stress measured at temperature } T, \text{ (Pa)} \]

\[ \dot{\varepsilon}_0 = \text{reference strain-rate usually made equal to one (s}^{-1}) \]

\[ T^* = \text{thermal softening term} \]

\[ T = \text{absolute temperature (K)} \]

suffix r: reference temperature, suffix m: melting temperature.

At a given strain and temperature equation (2.69) shows the semi-logarithmic dependence of stress on strain-rate often observed experimentally. The actual shape of the stress-strain curve will be independent of temperature and strain-rate. One of the drawbacks of this model is that the plastic strain is coupled with the strain-rate and temperature (all the terms are multiplied together) [Harding 1989]. This model although semi-empirical gives reasonable results and is easy to apply. Therefore it is a very widely used model [Meyers 1994].

**Klepaczko Model**

Klepaczko’s model takes into account the path dependence of the flow stress. (Path dependency in FCC and BCC metals has been investigated by Hartley and Duffy [1984] by the use of step tests). This is achieved by expressing the flow stress in terms of internal state variables which are functions of strain-rate and temperature. The total shear stress can be expressed as the sum of the thermal and athermal components:

\[
\tau(\dot{\gamma}, T, S_j) = \tau_o[S_j(\dot{\gamma}, T)] + \tau^*(\dot{\gamma}, T, S_j(\dot{\gamma}, T)) \quad (2.70)
\]

where:

\[ S_j = \text{internal state variables.} \]

The principal internal state variables are the mobile and immobile dislocation densities [Klepaczko and Chiem 1986, Klepaczko 1989, Meyers 1994].
One problem with this constitutive relationship is that there are a large number of material constants: 13 for the low temperature deformation of FCC aluminium, and rising to 15 for BCC steels.

Surprisingly, better agreement is found between experimental data and predictions from the Klepaczko model if the viscous drag term is left out at higher strain-rates (even though viscous drag is meant to be active at these higher strain-rates) [Harding 1989].

**Mechanical Threshold Stress Model**

The mechanical threshold stress is the stress at 0 K at which a metal would deform in a given microstructural state. The mechanical threshold stress is therefore equal to the height of the barrier in Figure 2.3b. Specimens are first tested dynamically to a given strain and then each one is reloaded quasistatically at different decreasing temperatures. The resulting reload yield stress versus temperature curves can then be extrapolated back to 0 K to give the mechanical threshold stress. The effect of temperature and strain-rate on the mechanical threshold stress can then be deduced assuming a reasonable thermal barrier model has been employed. This technique is essentially semi-empirical in that the mechanical threshold state under many different conditions is established via testing. Follansbee and Kocks [cited in Meyers 1994] state that the increased strain-rate sensitivity seen in copper at higher strain-rates (up to $10^4 \text{ s}^{-1}$) can be explained by the increase in the mechanical threshold stress [Meyers 1994].

Even though the mechanical threshold stress model requires a more complex testing methodology (low-temperature testing) than some other constitutive equations, it is still widely applied. Gourdin and Lassila [1991,1992], Follansbee [1986,1989], and Follansbee and Gray [1991] have applied the model to FCC metals. Follansbee [1989] has applied the model to BCC metals.
CHAPTER 3

3 TESTING TECHNIQUES

The initial purpose of this work was to characterise a number of different metals (copper, pure iron and two armour plate steels) under different conditions of temperature and strain-rate. Three test temperatures (20, -40 and -100 °C) were chosen to allow the metals' temperature sensitivities to be determined. Three strain-rates (one quasi-static and two dynamic) were chosen to allow the metals' strain-rate sensitivities to be determined. As the testing program progressed more emphasis was put upon the problems associated with testing using the SHPB. The problems that cause the most error in the measured flow stress are shown to be friction at the specimen/loading face interfaces and specimen adiabatic softening. Therefore the data analysis presented later in this chapter incorporates flow stress corrections for friction and adiabatic heating.

3.1 TEST MATERIALS

Materials for this study have been kindly provided by Mr. B. D. Goldthorpe of the Defence Research Agency (DRA), Fort Halstead, Kent. The specimens supplied are all cylindrical compression specimens with nominal dimensions of 4.000 mm length by 8.000 mm diameter. The specimens are:

- Pure iron. This has a HV30 = 85 and has been thermomechanically worked to form a fine even grain structure with any carbides distributed throughout, not at the grain boundaries. The axes of the specimens are longitudinal with respect to the rolling direction. Designated PFE.

- Armour plate steel. This has a HV30 = 340±5 corresponding to a tensile strength of 1135 MPa. These were cut from a standard UK specification Rolled Homogeneous plate 12.5 mm thick. The axes of the specimens are longitudinal with respect to the rolling direction. Designated ARP.

- Armour plate steel. This has a HV30 = 305±5 corresponding to a tensile strength of 990 MPa. These were cut from a standard UK specification 1.5% Cr Ni Rolled
Homogeneous Armour plate 100 mm thick. The axes of the specimens are longitudinal with respect to the rolling direction. A typical analysis for this material would be:

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>Cr</th>
<th>Mo</th>
<th>Ni</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.26</td>
<td>0.67</td>
<td>1.47</td>
<td>0.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

This material, although not exactly the same as ARP armour plate steel, was supplied because it was available in much larger quantities. Designated RHA/UK100 or UKP.

- Copper. This material is called XM copper by Fort Halstead meaning that it has been processed to reduce the grain size to 15 microns. The specimen faces have a mirror finish. Designated XM.

- Copper. This material is again XM copper. The specimen faces have what appears to be a turned finish. Designated AXM.

3.2 QUASISTATIC STRAIN-RATE TESTING

Quasistatic strain-rate tests were performed on a Hounsfield H50KM testing machine. For tests at room temperature the specimen was positioned between two hard roller bearings (solid cylinders) which were in turn positioned between the testing machine platens. The roller bearings served a dual purpose. They protected the surfaces of the testing machine platens and provided adequate space for the positioning of a displacement transducer (if required).

Tests at -40°C used an insulated copper cooling jacket filled with liquid nitrogen. Pilkington K glass disks 25.4 mm thick were used between the main machine platens and the smaller roller bearing platens adjacent to the specimen. The glass disks kept heat transfer from the main machine platens to the specimen at a minimum allowing the specimen to be cooled to -40°C. Both the small platens adjacent to the specimen and the specimen had K-type thermocouples soldered on to them to monitor the temperature. An initial comparison of stress-strain curves for specimens with and without
thermocouples attached, proved that the attachment of the thermocouple did not affect the deformation process in any way. The thermocouple output was fed through a 100X gain amplifier and the output of that in turn into a digital voltmeter. The temperature could be measured to an accuracy of approximately ±0.3 °C. To perform a test the specimen was first cooled to approximately -45 °C and then allowed to warm-up (at a rate of approximately 1 °C min⁻¹) to -40 °C. No thermal gradients were observed across any of the specimens tested at -40 °C. Unfortunately, it was not possible to cool the specimen to -100 °C without severe thermal gradients across the specimen. Therefore no quasistatic tests were performed at -100 °C.

The specimen (except where stated) was lubricated on the upper and lower faces with a Dow Corning silicone based high vacuum grease, to help minimise the effects of friction. At the start of a test a very small gap was usually left between the upper surface of the upper roller bearing and the machine platen. This gap enabled the starting zero load point to be established with greater ease from the test data.

The crosshead speed for all quasistatic tests was set at 0.5 mm min⁻¹ (which corresponds to a strain-rate of approximately 10⁻³ s⁻¹ for the materials tested in this study). The strain-rate for a given test was an average value calculated over a given time period dependent on the specimen material.

The load on the specimen was measured by a load cell in the crosshead of the testing machine. An output of 1 V from a connector on the rear of the testing machine corresponds to the maximum load value set on the control board (i.e. 1 V = 2.5, 5, 10, 25, or 50 kN).

The specimen extension was measured using a displacement transducer, a strain gauge attached directly on to the specimen and by using the compliance method. The compliance method used the displacement output of 1 V from a connector on the rear of the testing machine corresponding to the maximum extension set on the control board (i.e. 1 V = 10, 20, 50, 100 or 1000 mm). In the compliance method the load-extension curve for the testing machine (without a specimen) is subtracted from the load-extension curve for a test (i.e. with a specimen) at given load values. The resulting load-extension curve is for the specimen only with any testing machine softness removed. All three measurement techniques gave the same results to within the experimental error in the flow stress of approximately ±2%. The compliance method was used most extensively.
due to positionment and temperature problems at -40 °C for the displacement transducer and the strain gauge. Also, another drawback with the strain gauge method was that the strain gauge only recorded up to approximately 2% strain before it was broken.

The load and extension outputs were fed into a Philips PM3335 analogue/digital storage oscilloscope. The traces could then be immediately dumped from the oscilloscope to a Hewlett-Packard compatible plotter, or transferred to an IBM compatible personal computer (PC) via an RS232 connection lead and stored using Philips "DSOCOM" software. (For some tests the load and extension outputs were fed to a chart-recorder for comparison with the digitally recorded traces, the results were the same).

The axial true strain in the specimen was calculated as follows (taking compressive strain as positive):

\[
\varepsilon_t = - \int_{L_0}^{L} \frac{dL}{L} = - \ln \left( \frac{L}{L_0} \right) = - \ln (1 - \varepsilon_s)
\]  

(3.1)

where:

- \(\varepsilon_t\) = true strain
- \(L\) = instantaneous specimen length (m)
- \(L_0\) = original specimen length (m)
- \(\varepsilon_s\) = engineering strain.

The true radial strain (taken as negative in compression), \(\varepsilon_r\), can be calculated from the true axial strain using Poisson's ratio (= 0.5 for all tests in this study), \(\nu\), hence:

\[
\varepsilon_r = - \ln \left( \frac{D}{D_0} \right) = \nu \ln \left( \frac{L}{L_0} \right) = - \nu \varepsilon_t
\]  

(3.2)

where:

- \(D\) = instantaneous specimen diameter (m)
- \(D_0\) = original specimen diameter (m).
Considering the true stress $\sigma_t = F/A$ and the engineering stress $\sigma_e = F/A_0$:

$$\frac{\sigma_t}{\sigma_e} = \frac{A_0}{A} = \left(\frac{D_0}{D}\right)^2$$

and

$$\ln\left(\frac{\sigma_t}{\sigma_e}\right) = 2 \ln\left(\frac{D_0}{D}\right) = -2 \ln\left(\frac{D}{D_0}\right)$$

Using equations (3.2) and (3.3):

$$\ln\left(\frac{\sigma_t}{\sigma_e}\right) = -2\nu e_i$$

hence the true stress can be calculated using:

$$\sigma_t = \sigma_e e^{-2\nu e_i}$$

(3.4)

All stress-strain calculations for quasistatic tests were performed using Microsoft Excel.

### 3.3 DYNAMIC STRAIN-RATE TESTING - THE SHPB SYSTEM

The Loughborough University SHPB is shown schematically in Figure 3.1. The projectile seated in a PTFE guide is driven along an evacuated gas gun barrel under atmospheric pressure. The velocity of the projectile is controlled by the area of the aperture used to allow air into the gun barrel. When the projectile impacts the end of the smoothing bar, a compressive stress pulse travels down through the smoothing bar. As the stress pulse travels along this low strength (0.2% proof stress of 700 MPa) smoothing bar, the high frequency oscillations produced at the initial projectile impact are severely damped. Hence the stress pulse travelling through the loading bar (or
"incident bar") towards the specimen is virtually oscillation free for all but the lowest projectile velocities. When the stress pulse reaches the specimen, part of it is reflected back along the loading bar and part of it is transmitted into the transmitter bar.

Figure 3.1 The Loughborough University SHPB System.

The sizes of these reflected and transmitted pulses is dependent upon the mechanical properties of the specimen sandwiched between the loading and transmitter bars. There is a gap between the transmitter bar and the momentum bar. This gap allows the compressive (transmitted) pulse to reach the end of the transmitter bar and be reflected as a tensile pulse. This tensile pulse pulls the transmitter bar face away from the specimen so the reflected compressive pulses in the loading bar cannot reload the specimen. The
momentum stored in the momentum bar is absorbed by a momentum trap (not shown in Figure 3.1) at the end of the momentum bar.

The magnitudes of the incident, reflected and transmitted pulses are recorded by strain gauge pairs mounted on the loading and transmitter bars. The strain gauges are connected via the strain gauge bridge to a Philips PM3335 analogue/digital storage oscilloscope. A 2200 pF capacitor is connected in parallel across each of the oscilloscope's inputs to act as a low pass filter cutting off high frequency electrical noise. The strain gauge traces can then be immediately plotted or saved in the same manner as the traces recorded in the quasistatic tests. This quick and efficient data storage technique allows a succession of tests to be completed in a short period of time. Data stored on the PC is then analysed with a program written in ANSI C (please refer to Section 3.3.5).

3.3.1 SHPB STRAIN GAUGES

The strain gauges used on the SHPB have a gauge length of either 1 or 6 mm. Assuming that the wave velocity in the pressure bars is approximately 5 mm µs⁻¹, then the strain gauge temporal resolutions are approximately 0.2 µs and 1.2 µs for the 1 and 6 mm strain gauges respectively. Apart from the better temporal resolution, the 1 mm strain gauges also appear to adhere to the surface of the pressure bars better than the 6 mm strain gauges. This greater adherence may be due to the fact that the maximum pressure differential over the length of the 1 mm strain gauge is much less than that over the 6 mm strain gauge during the initial rise-time of a given measured pulse. A lower pressure differential over the length of the strain gauge also causes a lower pressure differential over the strain gauge adhesive, so lengthening the life of the strain gauge. This fact also means that 1 mm strain gauges are less likely to fail when testing at high strain-rates.

The loading and transmitter bar strain gauge pairs are placed equidistant from the specimen. A distance of 0.45 m was used between each strain gauge pair and the specimen. This distance is long enough to ensure that the loading pulse is not overlapped by the reflected pulse on the oscilloscope trace.
SHPB Strain Gauge Circuitry and Formulae

Strain gauge pairs (two diametrically opposed strain gauges connected in series) are used to eliminate bending wave effects and to double the output from longitudinal waves. The "strain gauge bridge" is a simple potential divider circuit. The strain gauge pair (resistance = $R_s$) is connected in series with a ballast resistor ($R_b = 2.2 \, k\Omega$) and a Farnell E350 stabilised voltage power supply ($E = 90.00 \, V$).

The strain ($\varepsilon$) recorded by a strain gauge is related to the change in its electrical resistance ($dR_s$) by:

$$\varepsilon = \frac{1}{F} \frac{dR_s}{R_s}$$  \hspace{1cm} (3.5)

where:

- $F$ = strain gauge factor (= 2.14)
- $R_s$ = strain gauge resistance ($\Omega$).

From simple analysis of the strain gauge bridge circuit:

$$V_s = \frac{R_s}{R_b + R_s} E = \frac{E}{n + 1}$$  \hspace{1cm} (3.6)

where:

- $V_s$ = voltage across the strain gauge pair (V) and $n$ is given by:

$$n = \frac{R_b}{R_s} = \frac{IR_b}{IR_s} = \frac{E - V_s}{V_s}$$  \hspace{1cm} (3.7)

where:

- $I$ = current through the circuit (A).
Differentiating equation (3.6) with respect to n gives:

\[ \frac{dV_s}{dn} = -\frac{E}{(n+1)^2} \tag{3.8} \]

Differentiating the first two terms in equation (3.7) with respect to \( R_s \) gives:

\[ \frac{dn}{dR_s} = -\frac{R_b}{R_s^2} \tag{3.9} \]

Combining equations (3.8) and (3.9) gives:

\[ \frac{dV_s}{dR_s} = -\frac{R_b}{R_s^2} \left( \frac{E}{(n+1)^2} \right) \tag{3.10} \]

Hence:

\[ dR_s = R_s \frac{(n+1)^2}{R_s E} dV_s \tag{3.11} \]

Substituting for \( dR_s \) in (3.5):

\[ \epsilon = \frac{(n+1)^2}{nFE} dV_s \tag{3.12} \]

Equation (3.12) gives a direct relationship between the strain in the pressure bars and the voltage across the strain gauge. It is assumed that \((n+1)^2/nFE\) is a constant. Although the resistance of the strain gauge (and hence the value of n) varies throughout a test, because the strains measured in the pressure bar are very small the error in this assumption is small (of the order of ±0.5%).
3.3.2 SHPB THEORY
(Please refer to Section 3.3 for a general description of the SHPB).

Elastic wave theory enables the calculation of the velocities of the pressure bar faces in contact with the specimen from the pressure bar strain gauge records. From this data true stress-true strain curves can be calculated.

Using simple elastic theory:

\[ F = \sigma A = E_b A \varepsilon \]  \hspace{1cm} (3.13)

where:

- \( F \) = axial force in the pressure bar (N)
- \( \sigma \) = axial stress in the pressure bar (Pa)
- \( A \) = cross-sectional area of the pressure bar (m\(^2\))
- \( E_b \) = elastic modulus of the pressure bar (Pa)
- \( \varepsilon \) = axial strain in the pressure bar.

Using one dimensional elastic wave propagation theory:

\[ \sigma = \rho c_0 \dot{u} \]  \hspace{1cm} (3.14)

where:

- \( \rho \) = density of the pressure bar (kg m\(^{-3}\))
- \( \dot{u} \) = particle velocity (m s\(^{-1}\))
- \( c_0 \) = elastic wave velocity in pressure bar (m s\(^{-1}\))

and given that the elastic wave velocity in the bar is:

\[ c_0 = \frac{E_b}{\sqrt{\rho}} \]  \hspace{1cm} (3.15)
then:

\[ \dot{u} = \frac{\sigma}{\rho c_0} = \frac{\varepsilon E_v}{\rho \sqrt{E_o / \rho}} = \varepsilon c_0 \]  

(3.16)

Let subscripts I, R, and T refer to the incident, reflected and transmitted pulses respectively, and subscripts 1 and 2 refer to the end faces of the incident and transmitter bars adjacent to the specimen. Then the forces on the pressure bar faces adjacent to the specimen are:

\[ F_I = E_v A (\varepsilon_1 + \varepsilon_R) \]  

(3.17)

\[ F_T = E_v A \varepsilon_T \]  

(3.18)

From equation (3.16) the velocities of the pressure bar faces adjacent to the specimen are:

\[ \dot{u}_1 = c_o (\varepsilon_1 - \varepsilon_R) \]  

(3.19)

\[ \dot{u}_2 = c_o \varepsilon_T \]  

(3.20)

Displacements of the pressure bar faces adjacent to the specimen from equations (3.19) and (3.20) are:

\[ u_1 = c_o \int_0^t (\varepsilon_1 - \varepsilon_R) \, dt \]  

(3.21)

\[ u_2 = c_o \int_0^t \varepsilon_T \, dt \]  

(3.22)

where:

\[ u \] = particle displacement (m).
Therefore for the specimen from equations (3.17) and (3.18):

\[
\sigma_s = \frac{F_1 + F_2}{2A} = 0.5E_b (\varepsilon_1 + \varepsilon_R + \varepsilon_T)
\]  
(3.23)

and from equations (3.21) and (3.22):

\[
\varepsilon_s = \frac{u_1 - u_2}{L_0} = \frac{c_0}{L_0} \int_0^t (\varepsilon_1 - \varepsilon_R - \varepsilon_T) dt
\]  
(3.24)

where:
\[
\sigma_s \quad \text{= engineering stress in the specimen (assuming the area of the specimen is the same as the area of the pressure bars) (Pa)}
\]
\[
\varepsilon_s \quad \text{= engineering strain in the specimen}
\]
\[
L_0 \quad \text{= original specimen length (m)}.
\]

Neglecting effects due to wave propagation in the specimen:

\[
F_1 = F_2
\]  
(3.25)

Hence:

\[
\varepsilon_T = \varepsilon_1 + \varepsilon_R
\]  
(3.26)

and:

\[
\sigma_s = E_b \varepsilon_T
\]  
(3.27)

\[
\varepsilon_s = \frac{-2c_0}{L_0} \int_0^t \varepsilon_R dt
\]  
(3.28)
\[ \dot{\varepsilon}_s = \frac{-2c_0 \varepsilon_R}{L_0} \quad (3.29) \]

where:

\[ \dot{\varepsilon}_s = \text{engineering strain-rate of the specimen} \quad (s^{-1}). \]

Before analysing the reflected and transmitted pulses, the starting points of these reflected and transmitted are altered so that they both start simultaneously. For ease of calculation the engineering strain equation (3.28) is expressed in the following form:

\[ \varepsilon_s = \frac{-2c_0}{L_0} \sum_{t=0}^{t} \varepsilon_R \Delta t \quad (3.30) \]

The diameter of the specimen is always smaller than that of the pressure bars to allow for transverse expansion during the test. A modified form of equation (3.27) takes into account the difference in diameters:

\[ \sigma_s = E_t (A/A_s) \varepsilon_T \quad (3.31) \]

where:

\[ A_s = \text{the instantaneous cross-sectional area of the specimen} \quad (m^2). \]

Equation (3.29) shows that the specimen engineering strain-rate is directly proportional to the reflected pulse. Equation (3.31) shows that the specimen engineering stress is directly proportional to the transmitted pulse. The engineering stress and strain calculated using equations (3.31) and (3.30) can be converted into true stress and strain using equations (3.1) and (3.4). All the data analysis for the SHPB tests is performed using an ANSI C MS-DOS program called SHPB1.C (see Section 3.3.5).
3.3.3 SHPB EXPERIMENTAL PROCEDURE

The initial setting-up of the SHPB apparatus requires that a checklist of procedures is followed. This step by step approach ensures the best possible results in terms of repeatability, maximum data resolution and minimisation of background electrical noise in the strain gauge records.

The following steps represent the basic test procedure for a test at room temperature:

1. Clean the ends of the pressure bars with a suitable solvent.
2. Align the end faces between the smoothing bar, loading bar, and transmitter bar. Check for gaps between the end faces of the bars by shining a small handheld torch behind the touching faces of the bars and looking for any chinks of light on the other side. Shining a torch between the platens of a digital micrometer has shown that it is easy for the naked eye to detect chinks of light corresponding to gaps of less than 5 μm. This suggests that the axial misalignment between the pressure bars is less than ±0.02°.
3. If there are any gaps work along the bars from the incident bar to the transmitter bar adjusting the optical mounts supporting the pressure bars.
4. Smear a thin layer of Dow Corning high vacuum silicone based grease between the smoothing and loading bars and allow them to gently come together under the force of the elastic strap restraints.
5. Mark the specimen circumference with a dot using an overhead projector marker pen.
6. Now the pressure bars are initially aligned, insert the specimen in between the incident and transmitter bars with the alignment dot uppermost. Check again using a torch for any gaps between the specimen and the pressure bars. If there are any gaps, adjust only the optical mounts supporting the transmitter bar.
7. Now the system is aligned with the specimen in place, use an overhead marker pen to draw alignment lines across the ends of the pressure bars.
8. Carefully remove the specimen from the SHPB. Smear a thin layer of Dow Corning high vacuum silicone based grease between the incident and transmitter bars. Do not
bring together the incident and transmitter bar end faces. Smear vacuum grease over the end faces of the specimen. Reposition the specimen in between the incident and transmitter bars using the alignment dot. Slide the plastic bag over the specimen and the adjacent pressure bar ends (to catch the specimen if it falls from the pressure bars after the test).

9. Check the tension in the elastic straps pulling the transmitter bar towards the incident bar. Replace elastic if slack and repeat the setting-up procedure from stage 1.

10. Align momentum bar with transmitter bar. Leave a gap (≈1 mm) between the transmitter and momentum bar (to prevent reloading of specimen by reflected pulses in incident bar). Position the free end of the momentum bar in the momentum trap.

11. Turn on all data recording equipment. Set Philips scope for maximum resolution possible. Check for electrical background noise. If noise is greater than ±0.5 mV try grounding pressure bars in different positions.

12. Remove the projectile from the freezer. The projectile is kept in a freezer to ensure that it always has the same diameter when placed in the gun barrel. This ensures repeatable projectile velocities. Position the projectile in the end of the gas gun barrel. Set the required aperture in the aperture plate (this controls the rate of air ingress into the barrel and hence the projectile velocity). Seat the aperture plate in the end of the gas gun and slide up the release plate.

13. Begin evacuation of the gas gun from the aperture plate end. When the pressure inside the gas gun is of the order of 1 mbar, begin evacuation at both ends of the gas gun. When the pressure has again reached the order of 1 mbar, seal off the pressure gauge.

14. Record the strain gauge power supply voltage, and the voltages across the strain gauge pairs. These voltages will be used in the analysis if the strain gauge(s) fail during the test, otherwise these voltages recorded immediately after the test are usually used.

15. Check data recording equipment is armed. Quickly slide release plate allowing air to rush into gas gun and propel the projectile.

16. Immediately after the test record the strain gauge power supply voltage, times used for calculating the projectile velocity from the laser diode timing system, and voltages across the strain gauge pairs. Lock data on Philips scope.
17. To return the projectile to the aperture plate end of the gas gun, evacuate the gas gun and then slowly let air in from the impact end of the gas gun.

18. Slowly allow air into the pressure gauge. Return projectile to freezer ready for the next test.

For tests at below room temperature, an insulated copper cooling jacket (in place of the plastic bag in Step 8) is placed around the specimen and the ends of the adjacent pressure bars. The cooling jacket is filled with liquid nitrogen down to approximately 10 K below the required temperature. A K-type thermocouple soldered to the specimen is connected to a Philips PM3335 analogue/digital storage oscilloscope via a 100X gain amplifier to monitor the specimen’s initial temperature and the temperature rise during the test. Tests with and without a thermocouple attached show that attachment of the thermocouple does not affect the measured flow stress. This thermocouple arrangement is also used for room temperature tests when the temperature rise during the test needs to be recorded. When the specimen has warmed up to the required test temperature (at a rate of about 2 K min\(^{-1}\)) the test is performed.

3.3.4 PROBLEMS ASSOCIATED WITH THE SHPB

Problems associated with the SHPB have been covered extensively in the literature survey in Section 2.1.3. This section will examine these problems to assess the magnitude of the error that each one introduces into the final flow stress curve determined by the SHPB. Corrections for these problems are only applied if they affect the portion of the flow stress used to calculate the Armstrong-Zerilli models.

**Inertia**

The effects of inertia are greatest during the highest strain-rates experienced by the specimen in the early stages of a test. These inertial stresses can be calculated by combining equations (2.20) and (2.22) derived by Gorham [1989]. Ignoring terms in \(v\) which are very small gives:
\[ P_2 - P_1 = \frac{\rho h^2 (\dot{e} + \ddot{e})^2}{2} \]  

(3.32)

\( P_2 - P_1 \) represents the difference in stress measured at the two faces of the specimen due to inertia. If inertia plays a significant roll then the stress measured at the loading bar/specimen interface should be greater than that measured at the specimen/transmitter bar face.

The highest strain-rate test with the densest material is X5M23B4, a copper specimen tested at 20 °C and 5300 s\(^{-1}\). This test suffers from the highest level of inertial stress in all the tests included in this study. Using equation (3.32), \( P_2 - P_1 \) for X5M23B4 is at a maximum of 42 MPa after 19 \( \mu \text{s} \). However, after 25 \( \mu \text{s} \) \( P_2 - P_1 \) drops to 1.6 MPa (approximately 0.3% of the flow stress) and continues to decrease throughout the remainder of the test. Because no data from the first 25 \( \mu \text{s} \) of any dynamic strain-rate test is used in any of the analyses no correction for inertia has been applied to the flow stress measured using the SHPB.

Attempts at accurately predicting the inertial stress from experimental values of the stresses on either specimen face are confounded by the effects of multiple wave reflections within the specimen. This together with inaccuracies due to pulse start times (they can only be resolved using the present system to a resolution of 1 \( \mu \text{s} \)) makes the use of equation (3.32) even more convenient.

**Multiple Wave Reflection within the Specimen**

The time required for the internal stress equilibrium to be reached within the specimen depends upon the elastic (and perhaps plastic) wave propagation times within the specimen, and the impedance match/mismatch between the specimen and the pressure bars. It is difficult from experimental results to separate the effects of inertia from those of multiple wave reflections within the specimen. An appreciation of the time required for the specimen to reach equilibrium can be gleamed from the stress equilibrium factor, \( \sigma_{EQ} \), defined by Parry et al. [1994] as:
\[ \sigma_{eq} = \frac{\sigma_T}{\sigma_I + \sigma_R} \]  

(3.33)

where:

\( \sigma_T \) = transmitted stress in the transmitter bar (Pa)

\( \sigma_I \) = incident stress in the loading bar (Pa)

\( \sigma_R \) = reflected stress in the loading bar (Pa).

The stress equilibrium factor will equal unity when equilibrium has been reached.

Figure 3.2 shows the stress equilibrium factor for AR6P28B4, an armour plate steel specimen tested at 20 °C and 930 s\(^{-1}\). The exact shape of the stress equilibrium factor versus time curve in the first 20 μs of a test is highly dependent upon the choice of the starting points for the incident, reflected and transmitted pulses. The stress equilibrium factor reaches unity for all tests after approximately 20 μs. If the stress equilibrium factor was governed only by the elastic wave velocity in the specimen and the impedance match
between the specimen and the pressure bars, then stress equilibrium should be reached earlier in the iron and steel tests than in the copper tests. It appears that the maximum inertial stresses that coincide with the maximum strain-rate during a test control the time required for stress equilibrium to be reached. In all tests the maximum strain-rate is reached after approximately 19 μs.

Because no data from the first 25 μs of any dynamic strain-rate test is used in any of the analyses no corrections for wave propagation (and/or inertia) have been applied to the flow stress measured using the SHPB.

**Wave Dispersion**

As mentioned in Sections 2.1.3 and 3.3 the Loughborough University SHPB system uses a 431 smoothing bar to remove high frequency oscillations from the loading pulse. Extensive studies by Parry, Walker and Dixon [1995] have shown that dispersion effects have a negligible effect on the flow stress curves measured using the Loughborough University SHPB system. Measurements of the loading pulse in the loading and transmitter bars with no specimen present in between show no discernible difference in the shape or size of the loading pulse. Therefore no corrections are applied to the flow stress curves for wave dispersion effects.

**Friction: Avitzur Ring Tests**

It was decided that ring specimens would be used to determine the levels of friction that exist in the quasistatic and dynamic tests. When a ring is compressed the inner radius might increase or decrease depending on the friction conditions prevalent. The ring tests presented in Chapter 4 clearly indicate that corrections for friction are necessary for all the metals tested in this study. The Avitzur [1964] correction for friction detailed in Section 2.1.3 has been used to estimate levels of friction from ring tests in terms of a constant shear factor between the specimen and the loading platen interfaces. The constant shear factor derived from a ring specimen is used to correct a solid specimen tested under the same conditions of strain-rate and temperature.
All specimens in this study were initially solid disc specimens of 8.000±0.005 mm diameter by 4.000±0.005 mm in length. Tight tolerances on the specimen length ensured that the specimens had a high degree of parallelism. These specimen dimensions were chosen as a compromise to try and minimise the effects of friction against inertia and wave propagation effects in the specimen. Solid specimens were drilled to produce ring specimens of 8.000 mm outer diameter, 4.00±0.05 mm inner diameter, by 4.000 mm in length. The hole was drilled using a lathe to ensure concentricity; a series of four drills (1, 2, 3 and 4 mm diameter) was used to slowly make the hole expand to the desired diameter without distortion of the specimen. A cooling/cutting fluid was employed to assist the drilling. Pure iron specimens always distorted when transformed into ring specimens. Therefore only copper and armour plate steel specimens were used for ring tests. Constant shear factors for pure iron were determined by interpolation from the copper and armour plate steel ring tests. Some solid specimens had to be reduced to a diameter of 6.90±0.01 mm to allow the required stress to be reached. Flow stress curves of all 6.90 mm and 8.000 mm solid specimens tested under the same temperature and strain-rate conditions overlapped within experimental error (±2%). Therefore constant shear factors derived from 8.000 mm outer diameter ring tests were applied to both 6.90 mm and 8.000 mm diameter solid specimens.

The length and outer diameter of a specimen were measured using a digital micrometer with a resolution of 1 μm. The diameter of the hole in the ring specimen was measured as close as possible to each specimen end face using a hole gauge. This was to ensure that the hole diameter did not vary by more than 20 μm through from one side of the specimen to the other. The hole gauge was then measured using the digital micrometer. An average of ten readings was taken for each dimension. The diameter of the hole in the ring specimen and the specimen length after the test were measured and used in the Avitzur analysis. The diameter of the hole in the ring specimen was used as opposed to the outer diameter as it is a more accurate indicator of friction and did not seem to be affected by barrelling.

The Avitzur analysis predicts final specimen dimensions dependent upon a given set of data. This data was fed into a QBASIC program AVITZUR1.BAS (see Appendix 1) to perform the analysis for the ring specimens. Different values of the constant shear factor were used until a value for the diameter of the hole in the ring specimen was
calculated that matched the experimentally determined value. The data required by AVITZUR1.BAS are the constant shear factor \( m \), the equivalent platen velocity \( v \), the time interval for the iteration process \( t \), the original specimen inner and outer radii and the original and final specimen lengths. The equivalent platen velocity is the average rate at which the loading bar face approaches the transmitter bar face during the test. This equivalent platen velocity was calculated as follows:

The average strain-rate during a dynamic test was calculated as:

\[
\dot{\epsilon}_{\text{AVE}} = \frac{\dot{\epsilon}_{170} - \dot{\epsilon}_{130}}{40 \times 10^{-6}} \tag{3.34}
\]

where:

\( \dot{\epsilon}_{170} \) = true strain after 70 \( \mu \)s

\( \dot{\epsilon}_{130} \) = true strain after 30 \( \mu \)s.

The period of time in which the specimen was deforming plastically was calculated by dividing the total true plastic strain measured from the specimen by the average strain-rate:

\[
t_p = \frac{-\ln\left(\frac{L}{L_0}\right)}{\dot{\epsilon}_{\text{AVE}}} \tag{3.35}
\]

Hence the equivalent platen velocity (defined as being negative):

\[
v = \frac{L - L_0}{t_p} \tag{3.36}
\]

The equivalent platen velocity calculated in this manner agrees to within \( \pm 2\% \) of that calculated by considering the difference in velocities of the pressure bar faces either side of the specimen. The equivalent platen velocity was calculated in all cases using the
procedure detailed above. This was because many of the Avitzur ring tests had been performed before the new ANSI C SHPB program SHPB1.C had been written which includes the pressure bar face velocity calculations.

The iteration time interval was determined by continuously reducing its value until using a smaller value had no effect on the AVITZUR1.BAS results. A time interval of 0.5 $\mu$s was used for all dynamic tests and a value of 0.1 s for all quasistatic tests.

Now the constant shear factor is known for a given set of test conditions it was applied to solid specimen tests. For a solid specimen either of equations (2.5) or (2.6) yield the same equation when $R_i = R_o = 0$, namely:

\[
\frac{P_{\text{ave}}}{\sigma_0} = 1 + \frac{2mR_o}{3\sqrt{3}L_i} \tag{3.37}
\]

where:
- $P_{\text{ave}}$ = uncorrected flow stress (Pa)
- $\sigma_0$ = flow stress corrected for friction (Pa)
- $m$ = constant shear factor
- $R_o$ = instantaneous specimen outer radius (m)
- $L_i$ = instantaneous specimen length (m).

Using equations (3.2), (3.3) and (3.4):

\[
\frac{R_o}{L_i} = \frac{D_0e^{\varepsilon_{(1+v)}}}{2L_0} \tag{3.38}
\]

Hence equation (3.37) becomes:

\[
\sigma_0 = \frac{P_{\text{ave}}}{1 + \frac{mD_0e^{\varepsilon_{(1+v)}}}{\sqrt{27L_0}}} \tag{3.39}
\]
Two programs were written that employ equation (3.39) to correct solid specimen flow stress curves for friction. AVITZUR2.C (please see Appendix 1) asks for the original specimen length, the original specimen diameter, Poisson's ratio and the Avitzur constant shear factor. It outputs the ratio $P_{av}/\sigma_0$ for true strains of 0 to 10% in 1% increments, for true strains of 12% and 14% and for true strains of 15 to 50% in 5% increments. These values are very useful for correcting quasistatic or dynamic tests. SHPB1.C also uses equation (3.39) to correct all dynamic flow stress curves for friction (please refer to Section 3.3.5).

Friction: Deformed Specimen Observations

As the armour plate steel specimens appeared to suffer the greatest amounts of interfacial friction, measurements of deformed armour plate steel specimen characteristics were made in order to try and relate these characteristics to the level of friction predicted by the Avitzur ring tests.

Figure 3.3 A Typical Barrelled Specimen.
During testing some specimens barrelled indicating that friction was playing a significant role. Figure 3.3 shows a typical barrelled armour plate steel specimen tested at 20 °C, 2790 s\(^{-1}\) to a strain of 25%. The barrelling of specimens tested at various temperatures and strain-rates was measured by first photographing the specimens using fine grain Ilford Pan F Plus film with lighting to enhance the edge contrast, and then enlarging (total magnification X60) and tracing the negative image on a microfiche reader. The barrelling radius of curvature was measured by overlapping a set of pre-drawn arcs over the traced images. Pre-drawn arcs with radii greater than 200 cm were difficult to produce with accuracy and hence radii of curvature greater than 3.3 cm could not be measured.

During testing a ring (the "major ring") forms around the edges of the specimen faces. This major ring has "minor rings" within it. The face of the specimen illustrating barrelling in Figure 3.3 is given in Figure 3.4.

![Figure 3.4 The Face of a Typical Dynamic Test Specimen.](image-url)
The width of the major ring and the number of minor rings were measured with a travelling microscope with a resolution of 0.01 mm. Each of these measurements was an average of eight readings per specimen.

**Friction: Specimen Surface Finish**

Specimen faces of ARP and RHA/UK100 have been examined using a Burleigh Personal SPM atomic force microscope in contact mode. Dynamic tests have been performed on all materials using different surface finishes. Details are given in Chapter 4.

**Adiabatic Softening**

The results presented in Chapter 4 clearly indicate that corrections have to be applied to the armour plate steels and pure iron for adiabatic softening in dynamic tests. Adiabatic softening in copper is shown to be so small as to be negligible and so no correction is applied for copper. The adiabatic softening correction technique devised by Dixon and Parry [1991] detailed in Section 2.1.3 of the literature survey has been used in this study. Temperature sensitivities have also been determined from re-tests (please refer to Chapter 4).

Temperature rises calculated by considering the area under the flow stress curve were calculated using the specific heat capacities for pure iron and copper shown in Figure 3.5. The exact variation of the specific heat capacity with temperature for the armour plate steels was not known and so an estimate of 510 J kg\(^{-1}\) K\(^{-1}\) was used for all temperatures.

The temperature rise calculated from an uncorrected flow stress curve consists of two components: the temperature rise due to the work done in loading the specimen and the work done against the forces of friction. If the temperature rise measured during a test using the K-type thermocouple is equal to the temperature rise calculated from a flow stress curve not corrected for friction, then the work done to heat conversion factor can be considered equal to 1 (100% conversion). Tests for all the metals indicated that the mechanical work done to heat conversion factor was in the range 0.95 to 1. Therefore in all temperature calculations the mechanical work done to heat conversion
factor was taken as 1. A typical K-type thermocouple trace for an armour plate steel specimen tested at -40 °C and 640 s\(^{-1}\) (AR1730A4) is shown in Figure 3.6.

![Figure 3.5 The Variation of the Specific Heat Capacity with Temperature for Pure Iron and Copper.](image)

The baseline for the thermocouple trace was taken from a digital voltmeter connected in parallel with the K-type thermocouple. The maximum temperature rise from the thermocouple was 17.9 K compared with 18.3 K from the uncorrected flow stress curve. The response time of the K-type thermocouple and solder blob is not fast enough to record the instantaneous temperature rise during the dynamic test. However, the specimen does remain at the final temperature reached at the end of the test for at least one or two tenths of a second. This gives the thermocouple and solder blob time to reach the final specimen temperature. Therefore an accurate indication of the specimen total temperature rise has been determined.
The new SHPB program also analyses the test in more detail and gives far more information to enable the understanding of a given test. In particular the program corrects the measured flow stress for temperature and friction.

The new program SHPB1.C (please see Appendix 2) has been written in a highly structured format in ANSI C. It has been compiled using Borland Turbo C++ V3 for MSDOS.

The Philips oscilloscope traces are downloaded to the computer via an RS232 link using the Philips DSOCOM software. The traces are saved as two separate files e.g. test1.asc and test2.asc. test1.asc refers to the loading bar strain gauge record and test2.asc refers to the transmitter bar strain gauge record. The SHPB1.C program can then be run by typing SHPB1.EXE at the MSDOS prompt. The main() function in the program calls five other functions in the order shown below:

- **open_pulse_file()**: this function is called twice by the main() function to load in the two pressure bar strain gauge records.

- **expt_params()**: this function asks the user to input the experimental parameters or accept suggested defaults.

- **stpoints_bslines()**: this function calculates the strain gauge record baselines and the pulse starting points automatically. A sloping baseline for the reflected pulse is calculated (if required) to ensure that the strains calculated from the start and end points of the reflected pulse are zero. This is necessary because sometimes the "tail" of the incident pulse interferes with the starting point of the reflected pulse. The user is then prompted to use the default values or enter their own.

- **analysis()**: this function calculates all the results for a given set of tests.

- **write_results()**: this function writes the results to the path and filename specified by the user. It is suggested that all filenames are in the format ?????????.csv. The ".csv" will tell any spreadsheet package that the results file is in comma separated variable format.
The parameters used (or calculated) by the program are:

- params[0]: strain gauge factor of strain gauges on the loading bar
- params[1]: strain gauge factor of strain gauges on the transmitter bar
- params[2]: original specimen length (m)
- params[3]: original specimen diameter (m)
- params[4]: power supply voltage of strain gauge bridge circuitry (V)
- params[5]: voltage across strain gauges on the loading bar (V)
- params[6]: voltage across strain gauges on the transmitter bar (V)
- params[7]: Poisson's ratio
- params[8]: start of incident pulse (μs)
- params[9]: start of reflected pulse (μs)
- params[10]: start of transmitted pulse (μs)
- params[11]: baseline of loading bar strain gauge record (V)
- params[12]: baseline of transmitter bar strain gauge record (V)
- params[13]: number of points to analyse
- params[14]: pressure bar Young's modulus (N m⁻²)
- params[15]: pressure bar's density (kg m⁻³)
- params[16]: pressure bar's diameter (m)
- params[17]: projectile Young's modulus (N m⁻²)
- params[18]: projectile's density (kg m⁻³)
- params[19]: projectile length (m)
- params[20]: specimen density (kg m⁻³)
- params[21]: specimen specific heat capacity (J kg⁻¹ K⁻¹) (or enter 1 for iron, 2 for armour plate steel or 3 for copper)
- params[22]: temporal digitising step (s)
- params[23]: projectile velocity calculated by program from incident pulse height (m s⁻¹)
- params[24]: gain of loading bar strain gauge amplifier
- params[25]: gain of transmitter bar strain gauge amplifier
- params[26]: Avitzur constant shear factor (for correcting solid specimens only)
- params[27]: test temperature (K)
- params[28]: thermal sensitivity factor 1 (MPa K⁻¹)
params[29]: temperature below which params[28] is valid (K)
params[30]: thermal sensitivity factor 2, valid above params[29] (MPa K⁻¹)
params[31]: mechanical work done to heat conversion factor (0 to 1)

The columns of results produced by the program are:

A: time (µs)
B: true stress (calculated from transmitted pulse) (MPa)
C: true strain (%)
D: engineering stress (calculated from transmitted pulse) (MPa)
E: engineering strain (%)
F: true strain-rate (s⁻¹)
G: incident pulse in terms of stress (MPa)
H: reflected pulse (with corrected baseline) in terms of stress (MPa)
I: transmitted pulse in terms of stress (MPa)
J: incident-reflected pulse in terms of stress (MPa)
K: true stress on the specimen face adjacent to the loading bar (MPa)
L: average specimen true stress (B+K)/2 (MPa)
M: stress equilibrium factor
N: velocity of loading bar face (adjacent to specimen) (m s⁻¹)
O: velocity of transmitter bar face (adjacent to specimen) (m s⁻¹)
P: velocity of loading bar face-velocity of transmitter bar face (N-O) (m s⁻¹)
Q: temperature rise in the specimen calculated from the uncorrected flow stress B (K)
R: true stress (calculated from transmitter pulse) corrected for friction (MPa)
S: temperature rise in the specimen calculated from the flow stress corrected for friction R (K)
T: true stress (calculated from transmitter pulse) corrected for friction and temperature (MPa)
U: incident pulse in terms of force (kN)
V: reflected pulse (with corrected baseline) in terms of force (kN)
W: transmitted pulse in terms of force (kN)
X: oscilloscope record of the reflected pulse (with corrected baseline) (V)
Therefore for a typical analysis there are a total of three files: test1.asc, test2.asc and test.csv. All the experimental parameters and results are stored in just one file: test.csv.

### 3.3.6 FITTING THE ARMSTRONG-ZERILLI MODELS

The Armstrong-Zerilli models are fitted to the flow stresses that have been corrected for friction and temperature. The Armstrong-Zerilli model for FCC metals from Section 2.3.1 is:

\[
\sigma = \sigma_0 + C_2 \varepsilon^\frac{1}{2} \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) + kD^{-1/2}
\]  

(3.40)

and for BCC metals is:

\[
\sigma = \sigma_0 + C_3 \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) + C_5 \dot{\varepsilon}^n + kD^{-1/2}
\]  

(3.41)

For the purposes of fitting these equations to this study's results equations (3.40) and (3.41) were modified slightly. For FCC metals equation (3.40) becomes:

\[
\sigma = C_0 + C_2 \varepsilon^\frac{1}{2} \exp(-C_3 T + C_4 T \ln \dot{\varepsilon})
\]  

(3.42)

and for BCC metals equation (3.41) becomes:

\[
\sigma = C_0 + C_3 \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) + C_5 \dot{\varepsilon}^n
\]  

(3.43)

where:

- \( C_0 \) = material constant (Pa).
**FCC Metals**

Manipulating equation (3.42) and taking logs gives:

\[
\ln(\sigma - C_0) = \ln(C_2 \varepsilon^{1/2}) + T(-C_3 + C_4 \ln \dot{\varepsilon})
\]  

(3.44)

\(C_0\) was determined by incrementing its value in a plot of \(\ln(\sigma - C_0)\) versus \(T\) (at constant \(\varepsilon\) and \(\dot{\varepsilon}\)) until a straight line was obtained. Then \(\ln(\sigma - C_0)\) was plotted versus \(T\) (at a constant \(\dot{\varepsilon}\)) for true strains of 0.10, 0.15 and 0.25. The intercepts of these lines at 0 K were determined and plotted against \(\varepsilon^{1/2}\) to determine a value of \(C_2\).

Manipulating equation (3.42) and taking logs gives:

\[
\ln(\sigma - C_0) = \ln(C_2 \varepsilon^{1/2}) - C_3 T + C_4 T \ln \dot{\varepsilon}
\]  

(3.45)

\(C_4\) was determined from a plot of \(\ln(\sigma - C_0)\) versus \(\ln \dot{\varepsilon}\) at constant \(T\) and \(\varepsilon\). Finally \(C_3\) was determined at a strain of 0.20 directly from equation (3.42) as an average of various values determined at different \(T, \sigma\) and \(\dot{\varepsilon}\).

**BCC Metals**

At constant \(T\) and \(\dot{\varepsilon}\) the terms

\[C_0 + C_1 \exp(-C_3 T + C_4 T \ln \dot{\varepsilon})\]

from equation (3.43) are constant, say \(K(T, \dot{\varepsilon})\) and equation (3.43) can be written as:

\[
\ln(\sigma - K(T, \dot{\varepsilon})) = \ln C_3 + n \ln \varepsilon
\]  

(3.46)

A plot of \(\ln(\sigma - K(T, \dot{\varepsilon}))\) versus \(\ln \varepsilon\) (at constant \(T\) and \(\dot{\varepsilon}\)) will yield a straight line provided strain hardening is independent of \(T\) and \(\dot{\varepsilon}\). (This is shown to be true in
Chapter 4 for pure iron and for both the armour plate steels. \( C_3 \) and \( n \) were determined by incrementing the value of \( K(T, \dot{\varepsilon}) \) in a plot of \( \ln(\sigma - K(T, \dot{\varepsilon})) \) versus \( \ln \varepsilon \) (at constant \( T \) and \( \dot{\varepsilon} \)) until a straight line was obtained.

Manipulating equation (3.43) and taking logs gives:

\[
\ln(\sigma - C_0 - C_3\varepsilon^n) = \ln C_1 - C_3T + C_4T \ln \dot{\varepsilon} \quad (3.47)
\]

Values of \( C_0 \) and \( C_4 \) were determined by incrementing the value of \( C_0 \) in a plot of \( \ln(\sigma - C_0 - C_3\varepsilon^n) \) versus \( \ln \varepsilon \) at constant \( T \) and \( \dot{\varepsilon} \) until a straight line was obtained.

Finally \( C_1 \) and \( C_3 \) were determined from a straight line plot of \( \ln(\sigma - C_0 - C_3\varepsilon^n) \) versus \( T \) at constant \( \varepsilon \) and \( \dot{\varepsilon} \).
CHAPTER 4

4 RESULTS

The experimental results presented in this chapter are discussed in Chapter 6.

4.1 TEST MATERIALS

Nomenclature

For each material a simple coding system is used to identify an individual specimen. The name used to identify a given specimen is the same as the filename used to store the specimen analysis results but excluding the filename extension. For example, if the results file produced using SHPB1.C was X8M22A4.CSV, then the specimen is referred to as X8M22A4. The length of the specimen name is limited to the MSDOS standard of a maximum of eight characters. For convenience X8M22A4 may also be referred to as just X8M.

The first three or four letters of any filename indicate the material tested, if the test was the first test or a re-test, and the actual sample number, for example:

AR_P Material tested was Armour Plate Steel.
First test.

RR_P Material tested was Armour Plate Steel.
Re-test.

UK_P Material tested was RHA/UK100 Armour Plate Steel.
First test.

RK_P Material tested was RHA/UK100 Armour Plate Steel.
Re-test.

X_M Material tested was XM Copper.
First test.
<table>
<thead>
<tr>
<th>M</th>
<th>Test Details</th>
</tr>
</thead>
</table>
| RM | Material tested was XM Copper.  
    | Re-test. |
| AX | Material tested was AXM Copper.  
    | First test. |
| RX | Material tested was AXM Copper.  
    | Re-test. |
| PFE| Material tested was Pure Iron.  
    | First test. |
| RFE| Material tested was Pure Iron.  
    | Re-test. |

The number (represented by "_" above) inserted between the first three or four letters of the filename is the sample number. When the sample number has two digits the third letter from the left will be removed from the filename (except for XM Copper), for example:

- P10F.... Material tested was Pure Iron.  
  First test.  
  Sample number 10.
- X10M.... Material tested was XM Copper.  
  First test.  
  Sample number 10.

The next number in the filename is the projectile velocity for a given test, for example:

- P10F27... Projectile impact velocity 27 m s\(^{-1}\).
X5M4... Projectile impact velocity 4 m s\(^{-1}\).

The final letter in the filename represents a given set of analysis conditions (i.e. shifting of reflected and transmitted pulses relative to each other and baseline levels for pulses), for example:

P10F27A. First analysis A.

P10F27B. Second analysis B, e.t.c.

The final number in the filename indicates if the data is direct test data, smoothed data, or averaged data:

P10F27A4 "4" indicates this data is directly from a test and has not been manipulated in any way.

P10F27A5 "5" indicates this data has been smoothed.

P10F27A6 "6" indicates this data has been averaged with other data.

The legends used in the graphs in this chapter also contain other test information, for example:

UK3324A4, 1020 s\(^{-1}\), -40 °C, 34.5 K, δ6.9 mm
Material tested RHA/UK100 armour plate steel.
Specimen number 33.
Projectile velocity 24 m s\(^{-1}\).
Strain-rate 1020 s\(^{-1}\).
Test temperature -40 °C.
Maximum temperature rise calculated from area under curve 34.5 K.
Specimen diameter 6.9 mm.
UK3324A4, 1020 s⁻¹, -40 °C, φ6.9 mm, m=0.30, ts=−1.55 MPa K⁻¹

Material tested RHA/UK100 armour plate steel.

Specimen number 33.

Projectile velocity 24 m s⁻¹.

Strain-rate 1020 s⁻¹.

Test temperature -40 °C.

Specimen diameter 6.9 mm.

Flow stress corrected for friction using an Avitzur constant shear factor of 0.30.

Flow stress corrected for adiabatic softening using a temperature sensitivity (determined at a true strain of 5 %) of -1.55 MPa K⁻¹ using the Dixon and Parry [1991] technique (see Section 3.3.4).

**General Information**

All the test specimens are cylindrical with nominal dimensions of 8.000 mm diameter by 4.000 mm length. Specimens with smaller diameters are tested to allow higher stresses/strains to be reached. Specimens with diameters smaller than 8.000 mm are labelled in the graph legends to this effect. All the Avitzur ring specimens have nominal dimensions of 8.000 mm outer diameter by 4.000 mm inner diameter by 4.000 mm length.

The experimental error in the uncorrected flow stress is estimated from the self-consistency of the results to be ±2%. The main source of error introduced by the Avitzur friction correction technique is due to the accuracy to which the radius of the hole in a ring specimen can be measured. The accuracy of this measurement affects the accuracy of the constant shear factor calculated from this measurement. The experimental error in the flow stress corrected for friction is estimated to be ±3%. The main source of error introduced by the adiabatic softening correction depends upon the method used to derive the temperature sensitivity values. The experimental error in the flow stress corrected for friction and temperature is estimated to be ±6%.

The lubricant used for all the tests (unless stated otherwise in the graph legend) is a Dow Corning silicone based vacuum grease.
4.2 RESULTS: COPPER

XM and AXM copper are the same material, the reason why they have different designations is that they were received as two separate batches with different surface finishes on their faces. XM copper has a mirror finish on its faces and AXM copper has what appears to be a turned finish.

A typical strain gauge record for a copper specimen is given in Figure 4.1a and the stress-strain curve calculated from this record is given in Figure 4.1b.

Figure 4.1a Typical Copper Specimen (X5M23B4) Strain Gauge Record.
Figure 4.1b Typical Copper Stress-Strain Curves: Uncorrected and Corrected for Friction.

Figure 4.2 Typical XM and AXM Copper Stress-Strain Curves (AX3314A4 and X3M14B4) Together with a Stress-Strain Curve (AX7M15A4) Measured Using Strain Gauges Attached with a Faulty Adhesive.
When the first tests were performed on AXM copper it was found to be much weaker than XM copper. AX7M15A4 has a much lower flow stress than X3M14B4 in Figure 4.2. It was originally thought that if XM and AXM copper are the same materials then the difference between the flow stress curves could be due to the different specimen surface finishes. However a small dip in the baseline after the transmitted pulse was observed for all the "low strength" AXM copper tests. This dip and the low values of flow stress obtained for some AXM copper tests were found to be due to a faulty batch of cyanoacrylate strain gauge adhesive (and nothing at all to do with the specimen surface finish). Tests using a new batch of cyanoacrylate strain gauge adhesive showed that flow stresses measured for XM and AXM copper specimens agreed within experimental error (AX3314A4 and X3M14B4 in Figure 4.2).

![Figure 4.3 The Effect of Different Specimen Surface Finishes on the Stress-Strain Curve for AXM Copper.](image)

To double check that the surface finish was indeed not creating the difference in the measured flow stress between XM and AXM copper, both XM and AXM copper specimens were given the same surface finish. As shown in Figures 4.3 and 4.4 the surface finish (whether it is a mirror finish, a turned finish or one produced using 320
grade silicon carbide sandpaper) appears to make no difference to the measured flow stress within experimental error.

Some of the specimens used for re-tests were ground to ensure good parallelism. This ground surface finish was also proven to make no difference to the measured flow stress.

![Stress-Strain Curve for XM Copper](image)

**Figure 4.4** The Effect of Different Specimen Surface Finishes on the Stress-Strain Curve for XM Copper.

Figures 4.5 to 4.12a describe the behaviour of copper over a range of temperatures and strain-rates. Only general trends in terms of strain-rate and temperature sensitivities can be gained from the uncorrected results presented in these figures. In general copper appears to be very temperature and strain-rate insensitive at both -40 °C and -100 °C. The strain-rate and temperature insensitivity at these temperatures is clearly demonstrated by the overlapping of dynamic flow stress curves at different strain-rates. The temperature insensitivity is also highlighted by the tests and re-tests at a given temperature. There is no difference in the flow stress measured at the end of a test and the start of its re-test for tests and re-tests at -40 °C and -100 °C. At 20 °C copper appears to be slightly more temperature and strain-rate sensitive.
The Avitzur ring test flow stresses are in excellent agreement with the flow stresses measured from solid specimens tested under the same experimental conditions up to a true strain of approximately 20%. Above approximately 20% true strain the flow stress measured from an Avitzur ring test is fractionally lower than that measured from the equivalent solid specimen test.
Figure 4.5 Copper: 20 °C Tests, 20 °C Re-tests.
Figure 4.6 Copper: 20 °C Avitzur Ring Tests.
1 mm diameter holes were drilled into the sides of some specimens and a K-type thermocouple inserted and held in place using solder (see Figure 4.7). A volume correction was applied to enable a reasonable approximation of the flow stress to be
This technique was used on only a few specimens before the more convenient technique of soldering the K-type thermocouple to the side of the specimen was employed. Due to the approximate nature of correcting the flow stress of specimens drilled to accept thermocouples, none of these specimens' results have been used in any of the subsequent analyses in this study.

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**Figure 4.8** Copper: -40 °C Test, -40 °C Re-test.
Figure 4.9 Copper: -40 °C Avitzur Ring Tests.
Figure 4.10 Copper: -100 °C Tests, 20 °C Re-tests.
Figure 4.11 Copper: -100 °C Tests, -100 °C Re-tests.
Figure 4.12a Copper: -100 °C Avitzur Ring Tests.
Many of the highest strain-rate -100 °C Avitzur ring tests for both RHA/UK100 armour plate steel and copper showed the skewed barrelling illustrated in Figure 4.12b. In all cases of skewed barrelling the specimen tapered towards the loading bar, i.e. the specimen's face labelled "high friction surface" in Figure 4.12b was adjacent to the loading bar face. In such cases the Avitzur constant shear factor was calculated from an
average inner hole radius determined from values measured at each end of the hole. Ridges of metal approximately 10 µm in length extended out in the same plane as the high friction face. These ridges of metal were only observed on very high friction (m≥0.3) dynamically tested specimens.

![Figure 4.13 Copper: Stress/Temperature Variation for Two Strain-Rates.](image)

The temperature sensitivity of copper at the three test temperatures is very low. At a true strain of 5% the temperature sensitivity for both strain-rates from Figure 4.13 is approximately -0.1 MPa K⁻¹. Therefore no correction has been applied to copper for adiabatic softening effects. Figure 4.13 suggests that the temperature sensitivity of copper may be slightly higher at 20 °C than at -40 °C or -100 °C. The thermal sensitivity of tests at 20 °C and the higher dynamic rate seems to increase with increasing strain.
The friction corrections calculated from the Avitzur ring tests for copper are given in Table 4.1. $P_{AV/E}/\sigma_0$ values in all the friction correction tables presented in this chapter are calculated for solid specimens unless otherwise indicated and represent an average value ($P_{AV/E}/\sigma_0$ increases throughout a test).

The constant shear factor for copper decreases with increasing strain-rate and temperature.

**Armstrong-Zerilli Model for Copper**

The Armstrong-Zerilli model applied to copper has the form:

$$\sigma = C_0 + C_4 \varepsilon^{1/2} \exp(-C_3 T + C_4 T \ln \dot{\varepsilon})$$

where:

$C_0 = 60 \text{ MPa}$
The material constants in the Armstrong-Zerilli model were determined from flow stress curves that have been corrected for friction. In general, as shown in Figures 4.14 to 4.16, the Armstrong-Zerilli model accurately describes the copper flow stress curves except for tests exceeding a true strain of approximately 25%.

Figure 4.14 Copper: 20 °C Armstrong-Zerilli Model.
Figure 4.15 Copper: -40 °C Armstrong-Zerilli Model.

Figure 4.16 Copper: -100 °C Armstrong-Zerilli Model.
4.3 RESULTS: RHA/UK100 ARMOUR PLATE STEEL

A typical strain gauge record for an RHA/UK100 specimen is given in Figure 4.17a and the stress-strain curve calculated from this record is given in Figure 4.17b.

![Strain Gauge Record](image)

**Figure 4.17a** Typical RHA/UK100 Specimen (UK3324A4) Strain Gauge Record.

Figures 4.18a and 4.18b show a typical surface topography for an RHA/UK100 specimen. RHA/UK100 specimens have a fine ground surface finish with an average roughness of about 120 nm. The surface finish on some RHA/UK100 specimen faces was altered by abrasion with 320 grade silicon carbide sandpaper and by re-grinding. As with the copper specimens, the three specimen surface finishes used with the RHA/UK100 specimens made no difference to the measured flow stress.

Some of the specimens used for re-tests were re-ground to ensure good parallelism.
Figure 4.17b Typical RHA/UK100 Stress-Strain Curves: Uncorrected, Corrected for Friction and Corrected for Friction and Adiabatic Softening.

Figure 4.18a Atomic Force Micrograph of the Surface of a Typical Untested RHA/UK100 Specimen.
Figures 4.19 to 4.30 describe the behaviour of RHA/UK100 over a range of temperatures and strain-rates. Only general trends in terms of strain-rate and temperature sensitivities can be gained from the uncorrected results presented in these figures. In general RHA/UK100 appears to be quite temperature and strain-rate sensitive over the range of test temperatures employed. It is difficult to appreciate the temperature sensitivity from tests and re-tests at a given temperature for two reasons. Firstly the first test specimens are only strained to a few percent and hence the difference between a test and a re-test at a given temperature would be small. Also because the adiabatic softening correction is only a small percentage of the total flow stress at low strains it cannot easily be resolved on the graph scale or separated from the graph digitising steps. However the temperature sensitivity can easily be observed by comparing tests at different temperatures or by noticing the drop in flow stress between re-tests tested at a higher temperature than their first tests.

The Avitzur ring test flow stresses are in excellent agreement with the flow stresses measured from solid specimens tested under the same experimental conditions up to a true strain of approximately 15%. Above approximately 15% true strain the flow stress measured from an Avitzur ring test is fractionally lower than that measured from the equivalent solid specimen test.
Figure 4.19 RHA/UK100: 20 °C Tests.
Figure 4.20 RHA/UK100: 20 °C Tests, 20 °C Re-tests.
Figure 4.21 RHA/UK100: 20 °C Avitzur Ring Tests.
Figure 4.22 RHA/UK100: -40 °C Tests.
Figure 4.23 RHA/UK100: -40 °C Test, 20 °C Re-test.
Figure 4.24 RHA/UK100: -40 °C Tests, -40 °C Re-tests.
Figure 4.25 RHA/UK100: -40 °C Avitzur Ring Tests.
Figure 4.26 RHA/UK100: -100 °C Tests.
Figure 4.27 RHA/UK100: -100 °C Tests, 20 °C Re-tests.
Figure 4.28 RHA/UK100: -100 °C Tests, -100 °C Re-tests.
Figure 4.29 RHA/UK100: -100 °C Avitzur Ring Lubricant Tests.

A set of lubricants was tested at -100 °C and about 700 s⁻¹ (Figure 4.29). The flow stress curves obtained with these lubricants overlap within experimental error.
indicating little difference in their lubricating properties. The Avitzur constant shear factor for all of these tests was approximately 0.30.

A similar set of lubricants was tested at 20 °C and about 1000 s⁻¹ using ARP armour plate steel solid specimens (please see Figure 4.39). The lubricant giving the lowest value for the specimen flow stress was adjudged to have the lowest friction. The lubricant order from highest to lowest friction is given in the Figure 4.39 legend. Although PTFE spray produced the lowest friction, the Dow Corning vacuum grease was chosen as best overall because it was easier to apply consistently and gave a measured yield stress only 30 MPa higher than the PTFE spray.
Figure 4.30 RHA/UK100: -100 °C Avitzur Ring Tests.
Figure 4.31 RHA/UK100: Stress/Temperature Variation at \( \approx 1000 \, \text{s}^{-1} \).

Figure 4.32 RHA/UK100: Stress/Temperature Variation at \( \approx 2300 \, \text{s}^{-1} \).
Table 4.2 RHA/UK100 Temperature Sensitivities From Figures 4.31 and 4.32.

The temperature sensitivities presented in Table 4.2 were obtained using the Dixon and Parry [1991] technique. Temperature sensitivities obtained from tests and re-tests lie in the range -1.0 to -2.5 MPa K⁻¹.

Table 4.3 RHA/UK100: Deformed Specimen Observations and Friction Corrections Calculated from Avitzur Ring Tests.

<table>
<thead>
<tr>
<th>RHA/UK100</th>
<th>TRUE STRAIN (%)</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≈ 1000 s⁻¹</td>
<td>ts (MPa K⁻¹)</td>
<td>-1.55</td>
<td>-1.43</td>
<td>-1.33</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>≈ 2300 s⁻¹</td>
<td>ts (MPa K⁻¹)</td>
<td>-1.76</td>
<td>-</td>
<td>-</td>
<td>-1.34</td>
<td>-1.27</td>
</tr>
</tbody>
</table>

Table 4.3 RHA/UK100: Deformed Specimen Observations and Friction Corrections Calculated from Avitzur Ring Tests.
Table 4.3 shows friction information for 6.900 mm and 8.000 mm outer diameter (d) by 4.000 mm length solid specimens, and 8.000 mm outer diameter by 4.000 mm inner diameter by 4.000 mm length Avitzur ring specimens. Specimens tested without a lubricant are labelled NL. Radii of curvature (p) greater than 3.3 cm were difficult to measure and are quoted as >3.3 cm. Radii of curvature quoted as <3.3 cm are less than 3.3 cm but not perfect arcs. Two radii of curvature were measured per specimen.

In similar trends to those observed for copper, the level of friction for RHA/UK100 specimens appears to decrease with increasing strain-rate and temperature. These trends are not as clearly defined for the RHA/UK100 armour plate steel as they were for copper.

During testing a ring (the “major ring”) forms around the edges of the specimen faces. This major ring has “minor rings” within it. The width of the major ring (MRW_{AVE}), and the number of minor rings (No. R_{MIN}) are presented in Table 4.3.

The predicted major ring width (MRW_{PRED}) was calculated as half the difference between the specimen’s initial and final outer diameters. MRW_{PRED} is therefore the major ring width assuming the edges of the specimen interlock with the loading faces at the beginning of the test; this occurs when friction is very high. MRW_{AVE}/MRW_{PRED} is a rough guide to the level of interfacial friction taking into account the difference in the specimen diameter when necessary. In general, this ratio decreases with increasing strain-rate, an indication of friction decreasing with increasing strain-rate. More minor rings appear in dynamic tests with lower rather than higher friction at a given strain-rate. Whereas the major ring width is a good indicator of friction, the barrelling radius of curvature appears to be more a function of strain than friction.

**Armstrong-Zerilli Model for RHA/UK100**

The Armstrong-Zerilli model applied to RHA/UK100 has the form:

\[
\sigma = C_0 + C_1 \exp(-C_2 T + C_3 T \ln \varepsilon) + C_5 \varepsilon^n
\]

where:

\[
C_0 = 760 \text{ MPa}
\]
\[ C_1 = 895 \text{ MPa} \]
\[ C_3 = 0.00660 \text{ K}^{-1} \]
\[ C_4 = 0.000440 \text{ K}^{-1} \]
\[ C_5 = 541 \text{ MPa} \]
\[ n = 0.39. \]

The material constants in the Armstrong-Zerilli model were determined from flow stress curves that have been corrected for friction and adiabatic softening. In general, as shown in Figures 4.33 to 4.35, the Armstrong-Zerilli model accurately describes the RHA/UK100 flow stress curves except for approximately the first 5% true strain of tests at 20 °C and -40 °C.

![Graph](image)

**Figure 4.33** RHA/UK100: 20 °C Armstrong-Zerilli Model.
Figure 4.34 RHA/UK100: -40 °C Armstrong-Zerilli Model.

Figure 4.35 RHA/UK100: -100 °C Armstrong-Zerilli Model.
4.4 RESULTS: ARP ARMOUR PLATE STEEL

A typical strain gauge record for an ARP specimen is given in Figure 4.36a and the stress-strain curve calculated from this record is given in Figure 4.36b.

![Figure 4.36a Typical ARP Specimen (AR6P28B4) Strain Gauge Record.]

Figures 4.37a and 4.37b show a typical surface topography for an ARP specimen. ARP specimens have a turned and then honed surface finish which at this magnification appears quite random with an average roughness of about 70 nm. The surface finish on some ARP specimen faces was altered by abrasion with 320 grade silicon carbide sandpaper and by re-grinding. As with the copper and RHA/UK100 specimens, the three specimen surface finishes used with the ARP specimens made no difference to the measured flow stress.

Some of the specimens used for re-tests were re-ground to ensure good parallelism.
Figure 4.36b Typical ARP Stress-Strain Curves: Uncorrected, Corrected for Friction and Corrected for Friction and Adiabatic Softening.

Figure 4.37a Atomic Force Micrograph of the Surface of a Typical Untested ARP Specimen.
Figures 4.38 to 4.42 describe the behaviour of ARP armour plate steel over a range of temperatures and strain-rates. Only general trends in terms of strain-rate and temperature sensitivities can be gained from the uncorrected results presented in these figures. As expected from this material's processing conditions ARP armour plate steel is slightly stronger than RHA/UK100. ARP is very similar to RHA/UK100 in terms of temperature and strain-rate sensitivity.
Figure 4.38 ARP: 20 °C Tests, 20 °C Re-tests.
Figure 4.39 ARP: 20 °C Solid Specimen Lubricant Tests.
(Please refer to the comments under Figure 4.29).
Figure 4.40 ARP: 20 °C Avitzur Ring Tests.
Figure 4.41 ARP: -40 °C Tests, 20 °C Re-tests.
Figure 4.42 ARP: -100 °C Test, 20 °C Re-test.
The temperature sensitivity calculated from Figure 4.43 for ARP armour plate steel is -1.60 MPa K$^{-1}$. This is in very close agreement with the corresponding RHA/UK100 temperature sensitivity of -1.55 MPa K$^{-1}$.

Because of the small numbers of ARP specimens supplied there is not a large enough set of ARP data available to allow the Armstrong-Zerilli model for this material to be accurately determined.

Friction corrections determined by the ARP 20 °C ring tests agreed with those determined for RHA/UK100. Assuming that the ARP friction trends follow those of RHA/UK100 at -40 °C and -100 °C, typical ARP flow stresses have been corrected for friction and adiabatic softening (Figure 4.44).
Figure 4.44 ARP: Stress-Strain Curves at 20 °C, -40 °C and -100 °C Corrected for Friction and Adiabatic Softening.
4.5 RESULTS: PURE IRON

A typical strain gauge record for an iron specimen is given in Figure 4.45a and the stress-strain curve calculated from this record is given in Figure 4.45b.

![Figure 4.45a Typical Iron Specimen (P5FE24A4) Strain Gauge Record.](image)
Figures 4.46 to 4.53 describe the behaviour of iron over a range of temperatures and strain-rates. Only general trends in terms of strain-rate and temperature sensitivities can be gained from the uncorrected results presented in these figures. Iron appears to have similar strain-rate and temperature sensitivities to those of both the ARP and RHA/UK100 armour plate steels.

The level of strain reached by the iron first tests is in most cases much larger than that reached by either the ARP or RHA/UK100 specimens. Hence the temperature rise in the iron first tests is quite large and adiabatic softening can clearly be seen from tests and re-tests at a given temperature.

Some of the specimens used for re-tests were ground to ensure good parallelism. This ground surface finish was proven to make no difference to the measured flow stress.
Figure 4.46 Iron: 20 °C Tests, 20 °C Re-tests.
A set of step tests was performed at 20 °C to illustrate the adiabatic softening that is present in the iron dynamic tests. The strain increase for each step test is small enough to ensure that the step test remains reasonably isothermal. Figure 4.47 shows
how adiabatic softening can reduce the flow stress of a dynamic test to below that of a lower strain-rate test.

Figure 4.48 Iron: -40 °C Tests, 20 °C Re-tests.
Figure 4.49 Iron: -40 °C Tests, -40 °C Re-tests.
Figure 4.50 Iron: -100 °C Tests, 20 °C Re-tests.
Figure 4.51 Iron: -100 °C Tests, -100 °C Re-tests.
Figure 4.52 Iron: Stress/Temperature Variation at $\approx 2100 \text{ s}^{-1}$.

Figure 4.53 Iron: Stress/Temperature Variation at $\approx 3500 \text{ s}^{-1}$.
Table 4.4 Iron Temperature Sensitivities From Figures 4.52 and 4.53.

The iron temperature sensitivities calculated from the tests and re-tests (taking an average of values at low and high dynamic strain-rates) are approximately -1.6 MPa K\(^{-1}\) at 20 °C, -2.2 MPa K\(^{-1}\) at -40 °C and -2.3 MPa K\(^{-1}\) at -100 °C. The temperature sensitivity appears to increase very slightly (i.e. becomes more negative by approximately -0.05 MPa K\(^{-1}\)) from the low to the high dynamic strain-rates.

Table 4.5 Constant Shear Factors for Iron Estimated from Copper and RHA/UK100 Avitzur Ring Tests and Calculated Friction Corrections.

The constant shear factors for iron were estimated from those determined for copper and RHA/UK100. It was assumed that the iron specimen surface finish (turned) would not play a role in determining the constant shear factors (which appear to be
independent of surface finish for copper and RHA/UK100). The level of friction was adjudged to be roughly proportional to the specimen flow stress level. Therefore the constant shear factors for iron given in Table 4.5 fall within those established for copper and RHA/UK100.

**Armstrong-Zerilli Model for Iron**

The Armstrong-Zerilli model applied to iron has the form:

\[
\sigma = C_0 + C_1 \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) + C_5 \dot{\varepsilon}^n
\]

where:

- \( C_0 = 40 \text{ MPa} \)
- \( C_1 = 1256 \text{ MPa} \)
- \( C_3 = 0.00544 \text{ K}^{-1} \)
- \( C_4 = 0.000247 \text{ K}^{-1} \)
- \( C_5 = 352 \text{ MPa} \)
- \( n = 0.52 \).

The material constants in the Armstrong-Zerilli model were determined from flow stress curves that have been corrected for friction and adiabatic softening. In general, as shown in Figures 4.54 to 4.56, the Armstrong-Zerilli model reasonably accurately describes the iron flow stress curves except for approximately the first 8% true strain of each test.
Figure 4.54 Iron: 20 °C Armstrong-Zerilli Model.

Figure 4.55 Iron: -40 °C Armstrong-Zerilli Model.
Figure 4.56 Iron: -100 °C Armstrong-Zerilli Model.
CHAPTER 5

5 HYDROCODE MODELLING

The results presented in this chapter are based upon a series of three visits to Fort Halstead hosted by Mr. P. Church. The purpose of these visits was to familiarise the author of this work with some of the numerical modelling packages used at Fort Halstead and to complete a small set of Hopkinson Bar simulations. These simulations were run to gain a further insight into the effects of friction on specimen deformation. In particular, a comparison between friction corrections predicted by the code and the Avitzur ring tests was required. Good agreement between friction corrections calculated by these two separate techniques will allow greater confidence to be placed in the Avitzur corrections used within this work.

5.1 HYDROCODE SOFTWARE

Three packages were used to develop, run and analyse the Hopkinson Bar simulations: INGRID, DYNA2D and ORION. These three packages were developed by the Lawrence Livermore National Laboratories, CA, USA, and the work presented in this report used Fort Halstead’s development versions of these codes.

INGRID

INGRID is a general purpose, three dimensional mesh generator developed for use with finite element, non-linear, structural dynamics codes. INGRID generates the input data file required for DYNA2D. An annotated example INGRID file for a two bar and projectile simulation is given in Appendix 3.

DYNA2D

DYNA2D is a vectorised, explicit, two dimensional, axisymmetric and plane strain finite element program for analysing the large deformation dynamic and hydrodynamic response of inelastic solids. A contact-impact algorithm permits gaps and sliding with friction along material interfaces.
ORION

ORION is an interactive postprocessor used for the analysis of the binary plot data files generated by DYNA2D. ORION can plot contour and fringe plots of strain, force and pressure as well as displaying meshes. Plots can be viewed on a monitor or output to a Postscript graphics file.

5.2 CONSTITUTIVE EQUATIONS

This hydrocode modelling study was performed before all the mechanical testing on the UKP armour plate steel had been completed. Therefore the constitutive equation used in the code to simulate this material was based upon that derived by Goldthorpe, Butler and Church [1994] for a similar armour plate steel:

\[ \sigma = \left( C_1 + C_5 \varepsilon^* \right) \frac{\mu_T}{\mu_{293}} + C_2 \exp\left[ (C_3 + C_4 \ln \dot{\varepsilon})T \right] \]  

where \( C_1 \) to \( C_5 \) and \( n \) are constants and \( \sigma, \varepsilon, \dot{\varepsilon} \) and \( T \) are respectively stress, strain, strain-rate and temperature in Kelvin. \( \mu_{293} \) is the shear modulus at 293 K and \( \mu_T \) is the shear modulus at the current temperature where:

\[ \mu_T = \mu_{293} \left( 1.13 - 0.000445T \right) \]  

The values used for the constants were:

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>710</td>
<td>575</td>
<td>-0.0048</td>
<td>0.00032</td>
<td>567</td>
<td>0.41</td>
</tr>
</tbody>
</table>

where the stress units are in MPa.

The maraging steel Hopkinson pressure bars and projectile were both treated as linear elastic-plastic materials. Deformation was considered to be elastic up to a yield
stress of 1400 MPa with a Young's modulus of approximately 187 GPa. To accurately model the elastic wave speed a bar density of 8050 kg m\(^{-3}\) was used.

### 5.3 SIMULATION RESULTS

Many of the initial runs were used to familiarise the author with the software. In this initial period the effects of mesh density and the advantages and disadvantages in using a direct impact (top bar and bottom bar) or three bar (projectile, incident bar and transmitter bar) simulation were studied. In general, the finer the mesh in the top (incident) bar, the better the loading pulse resolution. The finer mesh in this top bar also allowed the generation of a plastic zone in the top bar to be observed. The advantage of the three bar simulation was that it allowed the loading pulse to be observed and easily compared with actual experiments. This comparison showed that the finer the mesh, the more accurately the simulation modelled the actual experimental pulses observed. The disadvantage of the three bar simulation compared to the direct impact simulation was the extra amount of processor time required to model the passage of the loading pulse through the top bar.

![Graph](image)

**Figure 5.1** UK46P Corrected for Friction Using the Avitzur Technique.
Three simulations are presented in the following sections to illustrate the results achieved in modelling the Loughborough University SHPB system. All three simulations are modelling one experiment: a test at 20°C and approximately 2800 s\(^{-1}\) using a solid specimen 4 mm in length by 6.8 mm in diameter (UK46P). The experimental flow stress and flow stress corrected for friction are given in Figure 5.1. The friction correction derived from Avitzur ring tests ($\mu = 0.09$, $m = 0.17$) predicts an average decrease in the flow stress of approximately 75 MPa (approximately 5%).

The test specimen (UK46P) modelled in all the simulations was actually reduced to 6.808 mm in diameter. A 6.8 mm diameter was used in all the simulations. Most specimens tested are nominally 8 mm in diameter. This reduced diameter specimen was modelled to try and emphasise two mechanisms connected with friction. A smaller diameter specimen can be tested to greater strains. Lubricant breakdown may occur more readily in real tests at greater strains. If lubricant breakdown does occur at higher strains in real tests an increase in flow stress might be observed. This flow stress increase would not be seen in the simulations which assume a constant coefficient of friction. A smaller diameter specimen might also increase the elastic punching or indentation of the pressure bar faces. As the pressure bar faces curve around the ends of the specimen the effective contact area between the specimen and pressure bar faces might be reduced. This pressure bar face curvature might also restrict the radial flow of the outer edges of the specimen's faces.

**Simulation A**

This simulation is a three bar simulation with the incident and transmitter bars both 1 m in length and the projectile 25 cm in length. Both bars and projectile are 12.7 mm in diameter. The projectile velocity is 31 m s\(^{-1}\). A friction coefficient of $\mu = 0.1$ is assumed.

Part of the mesh in the pressure bars used for all the simulations is shown in Figure 5.2. Axial symmetry is used to reduce the processor time required to run the simulations. The incident bar has 10 radial elements and 116 axial elements. The transmitter bar has 10 radial elements and 29 axial elements. The greater mesh density in the incident bar close to the bar/specimen interface allows better resolution of the loading.
pulse. The specimen is comprised of 35 equi-spaced radial elements and 56 equi-spaced axial elements. The high specimen mesh density is required to resolve the non-uniform stress and strain distributions within the specimen. The projectile is comprised of 10 equi-spaced radial elements and 10 equi-spaced axial elements.

Figure 5.2 Mesh Adjacent to the Specimen Used for All the Simulations.

Figure 5.3 shows the simulated and experimental loading pulses in terms of effective stress. The simulated loading pulse has Pochhammer-Chree oscillations that are removed in the real test by the 431 smoothing bar. The rise-times of the simulated and
experimental pulses agree well. The experimental incident pulse stress level is fractionally higher than the simulation. This difference in pulse heights is because the projectile velocity used in the simulation was determined from the fibre optic projectile velocity measuring system. This measurement system has inaccuracies due to circuit response times greater than first assumed. The projectile velocity calculated from the incident pulse stress level is 33.02 m s\(^{-1}\). If a projectile velocity of 33 m s\(^{-1}\) had been used in the simulation, the simulation loading pulse would increase by approximately 6%. Hence the simulation and experimental loading pulse would be in excellent agreement. However the use of a fractionally lower projectile velocity is fortunate. The flow stress and ductility predicted by the specimen constitutive model are such that the correct final axial specimen strain is calculated in these simulations (please see later in this section). The simulation loading pulse along the surface of the incident bar (effectively at a radius of 6.35 mm) is identical to the simulation loading pulse measured at the centre of the bar. Very little pulse distortion occurs along the outer faces of the top bar. Therefore the values of strain measured by the strain gauges can be taken as very accurately describing the level of strain across the complete bar cross-section.

![Graph](image)

**Figure 5.3** Experimental Loading Pulse Measured Using Strain Gauges and Simulated Loading Pulses.
The experimental loading pulse is fractionally wider than the simulated loading pulse. This is due to the dispersion effects experienced by the experimental loading pulse whilst travelling through the 431 smoothing bar.

![Average Axial Stress in the Specimen](image)

**Figure 5.4** Average Axial Stress in the Specimen.

The average axial stress in the specimen (Figure 5.4) was calculated by averaging the forces acting on each element across the plane parallel to the specimen faces passing through the specimen centre. The stress in the specimen increases approximately 190 \( \mu s \) after projectile impact: the time required for the loading pulse to travel the length of the incident bar. For convenience all test times will be relative to the time at which the specimen loading begins taken as 190 \( \mu s \). The average axial pressure is approximately 100 MPa lower than the experimental flow stress. This difference is due to the constitutive model used to simulate the specimen response. The simulated specimen material is effectively weaker than the real UK46P armour plate steel specimen. In spite of this difference in material response, the simulated specimen material is still a high strength armour plate steel. Hence it is still valid to identify trends from these simulations and relate them to the deformation mechanisms of the UKP (and ARP) specimens.
A profile of effective stress in the specimen after 70 µs is shown in Figure 5.5. Effective stress \( \bar{\sigma} \) is defined as:

\[
\bar{\sigma} = \frac{\sqrt{2}}{2} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)^{1/2}
\]

\[ (5.3) \]

where:
\( \sigma_1, \sigma_2, \sigma_3 \) = principal stresses (Pa).

The effective stress is calculated along a plane parallel to the specimen faces passing through the specimen centre. The average effective stress agrees very well with the average axial stress in Figure 5.4. This is very encouraging because the SHPB system and the usual one dimensional analysis of the results calculates the average axial stress.

Profiles of axial stress after 30 µs and 70 µs are shown in Figure 5.6. The axial stress is calculated along a plane parallel to the specimen faces passing through the specimen centre. After 30 µs the profile of axial stress is greatest at the specimen centre and lowest at the specimen edge. This variation in axial stress with radial distance is
commonly described as the "friction hill". The frictional shear stresses at the specimen/bar interfaces lead to a radial pressure within the specimen. This radial pressure due to friction is zero at the specimen edges and builds up to a maximum at the specimen centre. Hence the axial stress follows the same trend. The axial stress measured at the edge of the specimen (approximately 1020 MPa) should provide a reasonable estimate of the axial stress under frictionless conditions. In fact this stress underestimates the frictionless flow stress by more than 150 MPa (please refer to Simulation C). This difference is probably due to the non-uniform stress distribution caused by specimen barrelling due to friction in this simulation. The profile of axial stress develops an unexpected maximum after approximately 40 µs. The radial position of this axial stress maximum does not appear to vary significantly with increasing strain.

Figure 5.6 Profiles of Axial Stress in the Specimen after 30 µs and 70 µs.

Figure 5.7 shows the deformed mesh plot. The final axial true strain for the specimen from the simulation was 23.1% across the specimen centre and 23.6% across the specimen edge. These values agree reasonable well with the experimental true strain measured as 24.0%. The greater strain measured across the specimen's edge implies that the pressure bar faces might be curving around the edge of the specimen's faces. This
elastic punching of the pressure bar faces might be contributing to the locking of the specimen edges with the pressure bar loading faces. Only the 35 radial elements of the specimen are in contact with each pressure bar face. However the process of metal folding over from the edge of the specimen to the face of the specimen is just beginning. The simulation radius of curvature was 1.1 cm. The experimental radius of curvature was 2.2 cm. These values imply that the value of 0.1 used for the coefficient of friction in this simulation was too high.

Figure 5.7 Final Deformed Specimen Mesh.

The non-uniformity of specimen deformation with friction is graphically illustrated by the fringes of effective plastic strain shown in Figure 5.8.
Effective plastic strain $\bar{e}$ is defined as:

$$
\bar{e} = \left( \frac{2}{3} \left( \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 \right) \right)^{1/2} \tag{5.4}
$$

where:

$\varepsilon_1, \varepsilon_2, \varepsilon_3 = $ principal strains.
The highest levels of plastic strain occur where the specimen edges interlock with the pressure bar loading faces. A plastic zone has formed within the incident bar near to the loading faces after approximately 50 µs. This is very surprising considering that the maximum specimen axial stress is approximately 1280 MPa and the pressure bar quasistatic yield strength is 1400 MPa. The level of plastic strain in the incident bar is known only to fall within the range 0 to 0.0544. If the plastic strain in the incident bar predicted by the simulation is very low (e.g. 0.0001), then it may have been caused by numerical inaccuracies within the DYNA2D code. However, if a plastic zone is forming in the incident bar then the usual one dimensional Hopkinson bar analysis is invalidated as it assumes the pressure bars always remain elastic. A plastic zone was not detected in the transmitter bar probably because of the lower mesh resolution.

**Simulation B**

This simulation is a direct impact version of Simulation A using a velocity of 31 m s\(^{-1}\) for the top bar. All other test parameters are the same as those of Simulation A.
The main reason for running a direct impact simulation is to greatly reduce the computer run time (and hence expense) by dispensing with the time required for the loading pulse to travel down the incident/top bar. This simulation should produce almost identical results to that of Simulation A. The main difference between the two simulations is the higher strain-rate in the early stages of Simulation B due to the much shorter loading stress rise-time. If Simulation A and Simulation B are in very good agreement then the direct impact version will be used for the frictionless Simulation C.

Figure 5.9 shows the stress pulses generated in the top and bottom bars. Both stress pulses agree well together in both size and shape. The stress pulse in the top bar is obviously much better resolved because of the finer mesh. Indeed the main criterion that was originally used to determine the mesh density in the top bar was that the incident pulse rise-time in Simulation A should be accurately simulated.

The average axial stresses in the specimen for Simulations A and B are given in Figure 5.10. The plots have been shifted so that the plastic portions overlap. There is excellent agreement between these two simulations in terms of average axial stresses. This indicates that the direct impact simulation would be suitable for modelling the Loughborough University SHPB system.

![Figure 5.10 Average Axial Stress in the Specimen.](image-url)
As in Simulation A the profile of effective stress in the specimen shown in Figure 5.11 agrees very well with the average axial stress given in Figure 5.10.

The profile of axial stress in the specimen after 100 µs is shown in Figure 5.12. Once again the unexpected maximum that develops after approximately 40 µs is present. Also as in Simulation A there is a lower than expected value of the axial stress at the specimen edge.

The deformed mesh plot for Simulation B looks exactly the same as that of Simulation A. The axial true strain for the specimen from the simulation after 100 µs was 26.3% across the specimen centre and 26.7% across the specimen edge. This implies the possibility of elastic punching of the pressure bar loading faces. Again the radius of curvature was 1.1 cm. The experimental radius of curvature was 2.2 cm. These values imply that the value of 0.1 used for the coefficient of friction in this simulation was too high.

A plastic zone formed in the top bar after approximately 20 µs. This plastic zone has developed approximately 30 µs earlier than the one observed in Simulation A. This implies that the formation of this plastic zone might be associated with inertial stresses that peak at the highest transient strain-rates present in the early stages of the test.
As stated earlier there is very close agreement between the axial flow stresses predicted by Simulations A and B. Because a comparison of flow stresses at different levels of friction is required, it was decided that a direct impact zero friction simulation would be suitable.

**Simulation C**

This simulation is exactly the same as Simulation B except that the coefficient of friction is now zero.

The average axial stress in the specimen shown in Figure 5.13 was much lower for this simulation (without friction) compared to Simulation B with friction. A change in the coefficient of friction from 0.1 (Simulation B) to zero (Simulation C) lowers the average axial stress level by approximately 60 MPa. This is a stress reduction of approximately 5%. This is very encouraging because the Avitzur correction assuming the coefficient of friction was 0.09 also reduces the true stress levels by approximately 5%. It is interesting to note that the friction correction reduces very slightly with strain from the simulation results, but increases very slightly with strain from the Avitzur correction. The
correction should increase very slightly with strain due to the increase in specimen diameter with strain.

**Figure 5.13** Average Axial Stress in the Specimen for Different Levels of Friction.

**Figure 5.14** Profiles of Effective Stress and Axial Stress in the Specimen after 100 µs.
The axial stress in the specimen shown in Figure 5.14 does not show the characteristic friction hill seen in Simulations A and B. Indeed the axial and effective stresses in the specimen are almost equal implying an almost one dimensional stress state. The axial stress is fractionally higher than the effective stress probably due to the effects of axial and radial inertia.

The deformed mesh plot is shown in Figure 5.15 after 100 μs. Under frictionless conditions no barrelling, material fold over or locking of the specimen edges with the pressure bar loading faces is observed. The axial true strain for the specimen from the simulation was 29.4% across the specimen centre and 30.4% across the specimen edge. This implies elastic punching of the pressure bar loading faces may have occurred. This may be responsible, together with inertial stresses, for the very slight decreases in axial and effective stresses with radial distance shown in Figure 5.14. The curvature of the pressure bar loading faces around the specimen faces might impose a small radial stress in a similar manner to that imposed by friction.

No plastic strain was observed in the pressure bars. This is significant because the point at which plastic strain appears in the loading bars is governed by the value of the friction coefficient over a very narrow range (0 to 0.1). Therefore, it still cannot be stated for certain from these simulations whether or not plastic deformation has occurred in the loading bars of the Loughborough University SHPB. From tests on the Loughborough University SHPB without a specimen to test the alignment of the pressure bars, the incident pulse agrees perfectly in size and shape with the transmitted pulse. If there was a small zone of plastically deformed metal in one of the pressure bars, the shapes of the incident and transmitted pulses should be different. Therefore it is highly unlikely that any plastic deformation has occurred within the body of the Loughborough University SHPBars.
5.4 CONCLUSIONS

1. Interfacial friction can be modelled with a reasonable degree of accuracy. However, the variations of friction with specimen radial distance cannot be easily modelled. Therefore predictions of major ring widths are impossible from these simulations.

2. The friction corrections predicted by the Avitzur technique and by the simulations agree very well. Therefore a high degree of confidence can be placed in the Avitzur corrections used in this study.

3. The specimen radius of curvature predicted by the simulations when compared with the experimental value suggested that the coefficient of friction used in the...
simulations was too large. Barrelling predicted from simulations is not useful as a guide to friction in real tests.

4. Elastic punching of the pressure bars' loading faces probably does occur in real tests. The effects of this on the measured or simulated flow stresses are difficult to predict from these simulations.

5. The formation of plastic zones in the pressure bars is unlikely. Simulating a test with a smaller than usual diameter probably increases the likelihood of plastic zone formation. However tests without a specimen should be performed regularly to check pressure bar alignment and to detect for possible plastic zone formation.

6. The average axial stress in the specimen agrees very well with the effective stress in the specimen. Therefore the flow stress calculated from the SHPB tests, effectively the average axial stress, is a very good measure of the effective stress in the specimen.

7. The average experimental value of the coefficient of friction taken over the face of the specimen does not increase (or decrease) significantly with strain. The experimental flow stress with friction remains parallel with the simulated flow stress with friction.

8. The strain gauges on the surface of the pressure bars accurately describe the level of strain across the complete pressure bar cross-section.
6 ANALYSIS AND DISCUSSION

The following sections discuss the testing techniques used to obtain the original flow stress data and the accuracy of the procedures employed to correct for problems inherent in these techniques. The Armstrong-Zerilli models are shown to reasonably describe the behaviour of copper, iron and RHA/UK100 armour plate steel. Therefore the trends in material characteristics derived from the corrected flow stress data are augmented throughout this chapter using the Armstrong-Zerilli models.

6.1 TESTING TECHNIQUE

This section describes the alterations made to the existing quasistatic and dynamic testing techniques (the Hounsfield H50KM testing machine and the SHPB). Problems inherent in these techniques are discussed in the following sections.

The temperature of specimens tested both quasistatically and dynamically had been measured by drilling a small hole into the specimen and affixing a K-type thermocouple with molten solder. This technique has the obvious drawback in that the stress distribution within the specimen must be greatly affected, making an accurate flow stress determination difficult. Also this technique is very time consuming. Attachment of the K-type thermocouple to the outer surface of the specimen with a blob of solder takes very little time and has been shown to yield accurate readings. Attachment of a K-type thermocouple in this manner has been shown not to produce any measurable effect on the flow stress. The only drawback with this procedure is that for the occasional very high strain-rate dynamic test the thermocouple/solder blob combination does become detached during the test.

The strain gauges used on the SHPBars originally had a gauge length of 6 mm. These gauges failed after approximately 30 tests. Using smaller 1 mm gauge length strain gauges in excess of 50 tests can be performed with the added benefit of providing better temporal resolution of the pressure bar pulses if required. As mentioned in Section 3.3.1 the greater longevity of the smaller strain gauge is perhaps due to the lower pressure differential experienced over the length of the strain gauge during the initial rise-time of a given measured pulse.
The projectile used to generate the SHPB loading pulse is a maraging steel bar supported in a PTFE sabot. The diameter of the PTFE sabot is approximately the same as that of the gas gun barrel. Therefore the projectile was kept in a refrigerator to always ensure a consistent fit in the gas gun barrel. During the evacuation of the gas gun barrel the projectile's PTFE sabot warms-up. If the PTFE sabot is not cold enough then its diameter can expand and friction between the sabot and the gas gun barrel walls can increase leading to a reduction in projectile velocity. It was found that more consistent projectile velocities could be obtained if the projectile was kept at a lower temperature (in the freezer).

The processing of the experimental data has been simplified by the new SHPB analysis program. This program produces a more detailed analysis of the dynamic tests than the previous QBASIC program including corrections for friction and adiabatic softening.

6.2 FRICTION

Friction at the specimen faces has been shown to vary greatly depending on the lubricant chosen. Dow Corning vacuum grease was chosen as the lubricant for all the tests in this study because of its performance against other lubricants (Figures 4.29 and 4.39) and ease of application.

Avitzur ring tests have clearly shown for copper and RHA/UK100 tested with vacuum grease as a lubricant, that interfacial friction decreases with increasing strain-rate and temperature. Lichtenberger, Lach and Bohmann [1994] also found the same variation of friction with strain-rate for tests on a steel and copper. The reduction in friction with increasing strain-rate is thought to be due to the higher velocity (at higher strain-rates) at which the lubricant is squeezed out or jetted from in between the specimen and loading bar faces. At high dynamic strain-rates (compared to low dynamic strain-rates) the specimen is tested to higher stress levels (and higher strains). The higher axial interfacial stresses increase the amount of jetting and pressure within the lubricant layer. As long as lubricant breakdown does not occur, then the increased jetting appears to improve lubrication. A small increase in friction due to an increase in pressure within the lubricant layer is only observed for large increases in the flow stress levels (for
example between copper and RHA/UK100). Hydrodynamic lubrication is assumed to occur over the whole surface of a specimen tested at dynamic strain-rates apart from a small ring around the outer edge of the face (the major ring). Jetting of the lubricant or loss of lubricant by any mechanism might be expected to increase friction. However, when the lubricant is squeezed from between the surfaces of the loading bars and the specimen it has to pass across the major ring on both the specimen's faces. The major ring is the area where hydrodynamic lubrication breaks down and interfacial friction is at its highest. Therefore any lubricant being forced across the interface between the specimen face's major ring and the loading bar's face improves lubrication exactly where it is most required. At high strain-rates the local heating in the specimen together with that caused by friction might lower the lubricant's viscosity and improve jetting and lubrication.

Interfacial friction decreasing with increasing temperature can also be explained in terms of lubricant jetting. At lower temperatures the vacuum grease lubricant becomes more viscous and this increase in viscosity reduces the lubricant jetting velocity. A reduction in lubricant jetting velocity increases friction under hydrodynamic conditions. Increased levels of friction at lower temperatures might also be due to the "frost" created on the bars. The water might mix with the vacuum grease lubricant at the edges of the specimen's faces decreasing the vacuum grease lubricant's effectiveness.

The lubrication for all quasistatic tests is very poor. This is because most of the lubricant is squeezed out from the specimen/loading platen interfaces within the first few seconds of the test. There is no appreciable difference between the levels of friction for quasistatic RHA/UK100 tests performed at 20 °C and -40 °C. Friction is lower for the quasistatic copper test at 20 °C than for the equivalent RHA/UK100 test. This may be due to the major ring forming on the copper specimen at a far lower stress level than for the RHA/UK100 specimen. The major ring forming on the copper specimen's faces could form before the major ring on the RHA/UK100 specimen's faces. Therefore it may trap more of the little remaining lubricant between the platen loading faces and the copper specimen's faces, thereby improving lubrication.

In general the Avitzur ring dynamic tests predict slightly lower levels of friction for the copper specimens than for the RHA/UK100 specimens at all three test temperatures when comparing tests at a given strain-rate. This is expected as friction,
especially if it is hydrodynamic in nature over the majority of the specimen's faces, should be a function of the lubricant's properties. This is also supported by the fact that different surface finishes for all specimens tested in this study made no measurable difference on the flow stress determined under a given set of conditions. The flow stress of iron lies between that of copper and RHA/UK100 and friction is independent of the specimen surface finishes employed in this study. Therefore it seems reasonable to predict the lubricant's properties in relation to the level of the iron's flow stress (which controls the axial specimen/loading bar interface stresses). Therefore the constant shear factors used to correct iron for friction were chosen to lie in between those determined for copper and RHA/UK100.

Deformed RHA/UK100 specimen observations were made in an attempt to link specimen barrelling and other macroscopic features to the Avitzur ring predictions of friction. No relationship could be found between the levels of friction in a given test and the specimen's radius of curvature (except that some form of barrelling occurs when friction is present). \( \frac{MRW_{AV}}{MRW_{PRED}} \) in Table 4.3 gives a rough guide to the level of interfacial friction. This ratio decreases with increasing strain-rate indicating friction decreases with increasing strain-rate. The minor rings inside the major ring are probably due to the edge of the specimen sticking and sliding as the material folds around from the sides to the faces of the specimen. More minor rings appear in dynamic tests with lower rather than higher friction at a given strain-rate. This implies that at low levels of friction the edge of the specimen is sticking more times with the loading faces but for shorter time intervals than at higher levels of friction.

Many of the highest strain-rate -100 °C Avitzur ring specimens suffered from skewed barrelling (please see Figure 4.12b). Some of the highest strain-rate copper solid specimens also suffered from skewed barrelling. The high friction face was always adjacent to the loading bar. During approximately the first 20 μs of a test the load on the specimen's face adjacent to the loading bar is higher than the load on the specimen's face adjacent to the transmitter bar due to inertia effects. The slightly greater stresses at the specimen/loading bar interface might increase the lubricant jetting at this interface for the first 20 μs of the test. It is postulated that if too much lubricant is lost by jetting then hydrodynamic lubricant breakdown might occur at the specimen/loading bar interface before the end of the test due to a lack of lubricant. This lubricant breakdown would
explain the difference in friction observed at the two specimen's faces. Gorham [1991a] has also discussed the additional radial stresses present in the specimen due to wave propagation effects in the early stages of a test. The precise nature of the three dimensional stress state within the specimen and its relationship to wave propagation effects within the early stages of a test is poorly understood. However this could be responsible for the stress distribution causing the skewed barrelling within the specimen.

The hydrocode modelling predicts flow stress corrections similar to those from Avitzur ring theory for a given level of friction. This agreement allows even more confidence to be placed in the friction corrections predicted by Avitzur ring theory. The hydrocode modelling also produced evidence that elastic platen punching might be happening in the dynamic tests. If the loading bar faces do curve around the specimen's faces then this will increase the localised high levels of stress around the edges of the specimen's faces (the major ring). Friction was found to generate a non-uniform stress distribution within the specimen. Walley et al. [1997] performed similar tests and hydrocode simulations on drop-weight tested copper and steel ring specimens deformed between glass platens. The major and minor rings were observed in these tests on the faces of the specimens around both the outer edge and the hole, and barrelling was observed at the inner radius. In this study no rings were observed around the Avitzur ring specimen's hole and no barrelling was observed at the inner radius. These differences might be due to the greater elastic punching and curvature of the glass platens around the specimen in Walley et al.'s [1997] tests.

6.3 TEMPERATURE SENSITIVITY AND ADIABATIC SOFTENING

Due to the difficulty of calculating accurate temperature sensitivity values from RHA/UK100 tests and re-tests, the Dixon and Parry [1991] technique was used to correct the armour plate steels for adiabatic softening. Iron was also corrected using this technique so a fair comparison can be drawn between the iron and armour plate steel results.

The temperature sensitivity of copper is illustrated in Figure 6.1. From experimental results the temperature sensitivity of copper was calculated as approximately -0.1 MPa K⁻¹. This is in close agreement with the values predicted from
the Armstrong-Zerilli model of \(-0.16\) MPa K\(^{-1}\) at 2300 s\(^{-1}\) and \(-0.15\) MPa K\(^{-1}\) at 4750 s\(^{-1}\). Due to this low temperature sensitivity no correction has been applied to copper for adiabatic softening.

![Graph showing temperature sensitivity for copper](image)

**Figure 6.1 Copper: Temperature Sensitivity.**

The slight increase in temperature sensitivity observed in the experimental results at the higher end of the test temperature range must be attributed to experimental error. The thermal component of the flow stress decreases with increasing temperature and therefore the temperature sensitivity would also be expected to decrease with increasing temperature.

The temperature sensitivity for copper increases slightly with strain. At higher strains the activation volume decreases resulting in the increased temperature sensitivity.

The temperature sensitivities for RHA/UK100 are illustrated in Figures 6.2 and 6.3. Unfortunately although the variation of stress was known not to be a linear function of temperature, using three test temperatures meant that that only a linear function of stress with temperature could be derived from the experimental data. This introduces a degree of error into the temperature sensitivities determined for RHA/UK100 (and iron).
However, because the levels of strain for all the RHA/UK100 tests are relatively small, the overall error in the flow stress due to the error in the temperature sensitivity is small.

Figure 6.2 RHA/UK100: Temperature Sensitivity at a Strain-Rate of 1000 s\(^{-1}\).

Figure 6.3 RHA/UK100: Temperature Sensitivity at a Strain-Rate of 2300 s\(^{-1}\).
Values of the temperature sensitivity determined using the Dixon and Parry [1991] technique are shown in Table 6.1 together with those predicted by the Armstrong-Zerilli model.

<table>
<thead>
<tr>
<th>RHA/UK100</th>
<th>TEMPERATURE SENSITIVITY AT 5% AND 1000 s(^{-1}) (MPa K(^{-1}))</th>
<th>TEMPERATURE SENSITIVITY AT 5% AND 2300 s(^{-1}) (MPa K(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEMPERATURE (°C)</td>
<td>ARMSTRONG-ZERILLI MODEL</td>
<td>EXPERIMENTAL</td>
</tr>
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<td>20</td>
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<td>-1.55</td>
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<td>-40</td>
<td>-1.39</td>
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<td>-1.72</td>
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The experimentally determined temperature sensitivity increases with strain-rate. Temperature sensitivities determined from RHA/UK100 tests and re-tests lie within the region -1.0 to -2.5 MPa K\(^{-1}\), the lower temperature sensitivity corresponding to the lower dynamic strain-rates.

The variation of the specific heat capacity with temperature for RHA/UK100 is not known and so a constant value was used to correct for the effects of adiabatic softening. Because the temperature correction for specimens tested to low strains is small, then the overall error in the corrected flow stress introduced by using a constant value for the specific heat capacity is small.

ARP armour plate steel has a very similar experimentally determined temperature sensitivity to that of RHA/UK100. At 5% true strain and a strain-rate of approximately 600 s\(^{-1}\) ARP has a temperature sensitivity of -1.60 MPa K\(^{-1}\).

The temperature sensitivities for iron are illustrated in Figures 6.4 and 6.5. Experimentally determined temperature sensitivities increase with increasing strain-rate. Temperature sensitivities calculated for iron using the Dixon and Parry [1991] technique are -1.63 MPa K\(^{-1}\) at 2100 s\(^{-1}\) and -1.73 MPa K\(^{-1}\) at 3500 s\(^{-1}\). These temperature sensitivities fall within the range predicted by the Armstrong-Zerilli models for a given strain-rate (Table 6.2).
The tests and re-tests for iron display the same trend in the temperature sensitivity variation with temperature as derived from the Armstrong-Zerilli model (Table 6.2). The temperature sensitivity (-5.30 MPa K⁻¹) for the low dynamic strain-rate 20 °C is very high. This may be artificially inflated by the depression in stress caused by the lower yield point.

The temperature sensitivity of iron is greater than that of both the two armour plate steels because the flow stress of iron has a much larger thermal component than the flow stress of either armour plate steel. The athermal flow stress of the armour plate steels is much larger than that of iron due to dislocation solute interactions (dispersion strengthening by iron carbide particles). A low temperature sensitivity in an armour plate steel is desirable. When in use as armour a low temperature sensitivity will increase the ability of the armour to resist attack (the degree of thermal softening will be less).

It is interesting to note that for both iron and RHA/UK100 the temperature sensitivity increases quite significantly with increasing strain-rate. The Armstrong-Zerilli models for both iron and RHA/UK100 predict a slight decrease in temperature sensitivity with increasing strain-rate. The experimental trends represent a large departure from the theoretical deformation model.

<table>
<thead>
<tr>
<th>TEMPERATURE (°C)</th>
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<th>EXPERIMENTAL ARMSTRONG-ZERILLI MODEL</th>
<th>EXPERIMENTAL</th>
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<td>-100</td>
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<td>-2.38</td>
</tr>
</tbody>
</table>

Table 6.2 IRON: Temperature Sensitivities (Experimental Temperature Sensitivities Determined using Tests and Re-tests).
Figure 6.4 IRON: Temperature Sensitivity at a Strain-Rate of 2100 s\(^{-1}\).

Figure 6.5 IRON: Temperature Sensitivity at a Strain-Rate of 3500 s\(^{-1}\).
6.4 STRAIN-RATE SENSITIVITY

Strain-rate sensitivity $\lambda$ (Pa) at a given level of strain can be defined as:

$$\lambda = \frac{\partial \sigma}{\partial \log \dot{\varepsilon}} = \frac{\sigma_1 - \sigma_2}{\log \left( \frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2} \right)} \quad (6.1)$$

Therefore if there is a sudden increase in the gradient of a plot of true stress against strain-rate (log scale) this is an indication of a sharp increase in strain-rate sensitivity.

Figure 6.6 illustrates the strain-rate sensitivity variation of the flow stress with temperature for copper. The strain-rate sensitivity increases with temperature. At higher temperatures the thermal component of the flow stress is lower. A lower thermal component of the flow stress corresponds to a larger activation volume. A larger activation volume results in a reduced strain-rate sensitivity. It is difficult to establish if the strain-rate sensitivity increases rapidly at higher strain-rates due to the experimental error and resulting scatter in the experimental data points. However it does appear that there may be some increased strain-rate sensitivity at the highest strain-rates indicating a possible change in the deformation mechanism. The variation in strain-rate sensitivity with strain for copper at 20 °C is shown in Figure 6.7. At higher levels of strain the strain-rate sensitivity increases. This is because at higher levels of strain the thermal component of the flow stress increases, corresponding to a reduction in the activation volume and hence an increase in strain-rate sensitivity.
Figure 6.6 COPPER: Strain-Rate Sensitivity Variation with Temperature.

Figure 6.7 COPPER: Strain-Rate Sensitivity Variation with Strain at 20 °C.
The strain-rate variation with temperature for RHA/UK100 is shown in Figure 6.8. The increase in strain-rate sensitivity at the higher flow stresses is well described by the Armstrong-Zerilli model and therefore is probably not indicative of a change to a viscous flow stress mechanism. However if there is a gradual change to a viscous deformation mechanism then the Armstrong-Zerilli model might be averaging the low and high strain-rate sensitivities characteristics of the two deformation mechanisms. The experimentally determined strain-rate sensitivity of RHA/UK100 is less than that predicted by the Armstrong-Zerilli model. This is discussed in Section 6.5 in relation to the temperature dependence of the shear modulus.

The strain-rate variation with strain for RHA/UK100 is illustrated in Figure 6.9. Because the thermal flow stress and the activation volume are independent of strain for BCC metals, then there is no strain-rate variation with strain. This is in stark contrast to copper in which the thermal flow stress and activation volume are strain dependent.

Due to the similarities between the flow stress curves for ARP armour plate steel and RHA/UK100, it is reasonable to assume that ARP armour plate steel has similar strain-rate sensitivities to that of RHA/UK100.

![Figure 6.8 RHA/UK100: Strain-Rate Sensitivity Variation with Temperature.](image-url)
The variation of strain-rate sensitivity with temperature for iron follows the same trend as RHA/UK100 (Figure 6.10). There is no variation of strain-rate sensitivity with strain at a given temperature (Figure 6.11). The strain-rate sensitivity predicted by the Armstrong-Zerilli model for iron is lower than that of RHA/UK100. However the experimental results indicate that iron is more strain-rate (and temperature) sensitive than RHA/UK100. The experimental results are logical in that a much larger proportion of the flow stress for iron is thermally activated in comparison to RHA/UK100. This discrepancy between the Armstrong-Zerilli model and the experimental results is discussed in Section 6.5.
Figure 6.10 IRON: Strain-Rate Sensitivity Variation with Temperature.

Figure 6.11 IRON: Strain-Rate Sensitivity Variation with Strain.
6.5 ARMSTRONG-ZERILLI MODELS

The BCC Armstrong-Zerilli model applied to iron and RHA/UK100 has the form:

\[ \sigma = C_0 + C_1 \exp(-C_3 T + C_4 T \ln \varepsilon) + C_5 \varepsilon^n \]  
(6.2)

and the FCC Armstrong-Zerilli model applied to copper has the form:

\[ \sigma = C_0 + C_2 \varepsilon^{1/2} \exp(-C_3 T + C_4 T \ln \varepsilon) \]  
(6.3)

where the material constants are given in Table 6.3.

<table>
<thead>
<tr>
<th></th>
<th>( C_0 ) (MPa)</th>
<th>( C_1 ) (MPa)</th>
<th>( C_2 ) (MPa)</th>
<th>( C_3 ) (K(^{-1}))</th>
<th>( C_4 ) (K(^{-1}))</th>
<th>( C_5 ) (MPa)</th>
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Table 6.3 Armstrong-Zerilli Model Material Constants.

The greatest difficulty in accurately determining the material constants for all the BCC materials was due to the variation of the strain hardening coefficient with strain. Table 6.4 shows the variation of the strain hardening with temperature and strain-rate. The strain hardening coefficients listed in Table 6.4 were calculated by applying the following equation to the plastic true strain portion of the flow stress:

\[ \sigma = K \varepsilon^n \]  
(6.4)
where \( K \) (Pa) is referred to as the strength coefficient. It should be noted that the strain hardening material constant in the BCC Armstrong-Zerilli models is calculated from an equation of the following form:

\[
\sigma = K(T, \varepsilon) + C_\varepsilon \varepsilon^n
\]  

(6.5)

where \( K(T, \varepsilon) \) is a constant for a given temperature and strain-rate. Therefore the strain hardening coefficients from equations (6.4) and (6.5) are not equal for a given material.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>TEMPERATURE (°C)</th>
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<th>n</th>
<th>K (MPa)</th>
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<td>1.65x10(^3)</td>
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<td>0.12</td>
<td>740</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3700</td>
<td>0.13</td>
<td>794</td>
</tr>
<tr>
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<td>-40</td>
<td>2830</td>
<td>0.10</td>
<td>911</td>
</tr>
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<td></td>
<td>3960</td>
<td>0.10</td>
<td>911</td>
</tr>
<tr>
<td></td>
<td>-100</td>
<td>2140</td>
<td>0.10</td>
<td>1005</td>
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<tr>
<td></td>
<td></td>
<td>2850</td>
<td>0.10</td>
<td>1050</td>
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<tr>
<td>COPPER</td>
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<td>1.83x10(^3)</td>
<td>0.27</td>
<td>579</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2400</td>
<td>0.41</td>
<td>650</td>
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<td></td>
<td>5300</td>
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<td>2350</td>
<td>0.40</td>
<td>691</td>
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<td></td>
<td>4600</td>
<td>0.38</td>
<td>674</td>
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<tr>
<td></td>
<td>-100</td>
<td>2180</td>
<td>0.40</td>
<td>672</td>
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<td></td>
<td></td>
<td>4950</td>
<td>0.36</td>
<td>664</td>
</tr>
<tr>
<td>RHA/UK100</td>
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<td>0.11</td>
<td>1378</td>
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<td>0.10</td>
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<td>2790</td>
<td>0.09</td>
<td>1642</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td>0.51x10(^3)</td>
<td>0.13</td>
<td>1522</td>
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<td></td>
<td></td>
<td>1020</td>
<td>0.10</td>
<td>1724</td>
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<td></td>
<td></td>
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<td>0.08</td>
<td>1695</td>
</tr>
<tr>
<td></td>
<td>-100</td>
<td>890</td>
<td>0.07</td>
<td>1691</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2080</td>
<td>0.06</td>
<td>1787</td>
</tr>
<tr>
<td>ARP</td>
<td>20</td>
<td>930</td>
<td>0.09</td>
<td>1613</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td>640</td>
<td>0.07</td>
<td>1582</td>
</tr>
<tr>
<td></td>
<td>-100</td>
<td>240</td>
<td>0.04</td>
<td>1594</td>
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</tbody>
</table>

Table 6.4 Variation of Strain Hardening Coefficient and Strength Coefficient with Temperature and Strain-Rate for All Metals Studied.

The work hardening coefficient for copper appears to be insensitive to variations in temperature over the temperature range tested. The work hardening coefficient for all
the BCC metals decreases very slightly with temperature. The temperature sensitivity of the work hardening coefficient for iron is greatest between 20 °C and -40 °C. The temperature sensitivities of the work hardening coefficients for the armour plate steels are greatest between -40 °C and -100 °C. In general, the work hardening coefficient decreases with increasing strain-rate for all the metals. This is because the yield stress is more strain-rate sensitive than the ultimate strength of the metal. Therefore at higher strain-rates the slope of the line connecting the yield stress to the ultimate strength decreases.

The strain hardening coefficients in Table 6.4 were calculated by fitting equation (6.4) over the entire plastic strain region for a given test, therefore they represent average values. The work hardening coefficient is much higher in the early stages of a test due to the high number of mobile dislocations that become restricted in their movement. Therefore if equation (6.5) is applied over the early stages of a test it may not be expected to provide a reasonable fit to the latter stages of the test and vice versa. Because many of the specimens in this study are tested to reasonably low plastic strains then it is difficult to accurately fit equation (6.5) to cover the wide variation in work hardening. As a compromise the work hardening coefficient in equation (6.5) was calculated from the highest strain-rate BCC flow stress curves at approximately half the total plastic strain. This compromise eliminates errors due to the high initial work hardening and yield point effects (yield point effects are not included in the Armstrong-Zerilli model) and reduces errors later on in the test due to possible inaccuracies in the adiabatic softening correction.

In all the plots for iron and RHA/UK100 (Figures 4.33, 4.34, 4.35, 4.54, 4.55, 4.56) the Armstrong-Zerilli model does not accurately represent the first 5 to 10% of the experimental flow stress. This is due to the inadequacy of the model to represent the higher work hardening rate at the start of a test and the yield point phenomena (as explained above).

The flow stress predicted by the Armstrong-Zerilli model for the highest strain-rate copper tests overestimates the experimentally determined flow stress at high strains. This difference is only partly explained by the slightly higher temperature sensitivity for copper predicted by the Armstrong-Zerilli model compared with that derived experimentally. However, the Armstrong-Zerilli model in overestimating the strain-rate
sensitivity of copper (see Section 6.6) might have balanced this error by underestimating the temperature sensitivity. All of the constants in the Armstrong-Zerilli model for copper were derived at true strains up to 25% and the Armstrong-Zerilli model is a very good representation of the experimental flow stress curves up to this strain level. At higher strains the work hardening coefficient decreases. This variation in the work hardening coefficient does not appear to be well catered for by the Armstrong-Zerilli model. The Armstrong-Zerilli model is often applied over a much larger plastic strain range taken at higher plastic strains. The variation in the work hardening coefficient with strain at higher strains is lower. Therefore when the Armstrong-Zerilli model is then applied over these higher strain regions, less variation between the experimentally flow stress and the Armstrong-Zerilli model predicted flow stress is observed.

The difference in the experimental flow stress and the Armstrong-Zerilli model predicted flow stress at high strains for copper might also be due to the assumption that the activation area for FCC metals is proportional to $E^{-0.5}$. This assumption is an approximation and even a small change in the strain exponent would greatly affect the overall shape of the copper flow stress curve.

The material constants for iron and copper were easily established from the experimental data and a high level of confidence can be placed in these values (except for the work hardening coefficient as discussed above). The large value of $C_6$ (representative of the high athermal barrier to dislocation motion) for RHA/UK100 makes the Armstrong-Zerilli model insensitive to the precise values of $C_3$ and $C_4$. Therefore it is difficult to accurately calculate the precise values of $C_3$ and $C_4$. Goldthorpe, Butler and Church [1994] also found this model insensitivity to the precise values of $C_3$ and $C_4$ for an armour plate steel similar to RHA/UK100. This model insensitivity was shown to be due to the fact that the thermal softening in the armour plate steel mainly acts through the temperature dependence of the shear modulus. This would also reduce RHA/UK100's strain-rate sensitivity. Indeed the strain-rate sensitivity predicted by the Armstrong-Zerilli model is greater for RHA/UK100 than for iron. However experimentally the strain-rate sensitivity of RHA/UK100 ($\approx 34$ MPa) is less than that of iron ($\approx 43$ MPa). Therefore a correction for the temperature dependence of the shear modulus in the Armstrong-Zerilli model for BCC metals (as used by Goldthorpe, Butler and Church [1994]) would improve its accuracy.
It is interesting to note that even with all the uncertainty in the precise values of \( C_3 \) and \( C_4 \) for iron and steel, the values of \( C_3 \) for iron are similar to the values of \( C_3 \) for steel and the values of \( C_4 \) for iron are similar to the values of \( C_4 \) for steel (Table 6.3). This would imply that the thermal activation mechanism in iron is similar to that in steel.

The general agreement between the Armstrong-Zerilli material constants for the metals in this study and those derived by other authors for similar metals is very good (Table 6.3). The strength of the iron and copper tested by the other authors is very similar to that of the iron and copper examined in this study. There is excellent agreement between both the iron models and both the copper models in Table 6.3. The steel tested by Goldthorpe, Butler and Church [1994] is slightly weaker than RHNUKI00 and this is reflected in the higher values of \( C_0 \) and \( C_1 \) for RHNUK100.

**6.6 ACTIVATION VOLUME**

In general the Armstrong-Zerilli models provide a reasonable description of the flow stress characteristics of the metals in this study. Therefore the Armstrong-Zerilli models will now be used to calculate the activation volume. The activation volume is very important because it is directly related to the deformation mechanism and has a value dependent upon the thermal component of the flow stress. The activation volume can be defined as (Dieter [1986]):

\[
V = \left( \frac{\partial \Delta G}{\partial \sigma^*} \right) \quad (6.6)
\]

where \( \Delta G \) is given by equation (2.46):

\[
\Delta G = kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}} \right) \quad (6.7)
\]
The significance of the activation volume can be clearly demonstrated by considering the constitutive model for a rectangular barrier (equation (2.51)):

\[ \sigma = \sigma_0 + \frac{\Delta G_0}{V} + \frac{kT}{V} \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \]  

(6.8)

The temperature sensitivity (ts Pa K\(^{-1}\)) can be expressed from equation (6.8) as:

\[ ts = \frac{k}{V} \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \]  

(6.9)

and the strain-rate sensitivity defined using natural logarithms (\(\lambda\), Pa) can be expressed from equation (6.8) as:

\[ \lambda = \frac{kT}{V} \]  

(6.10)

Therefore the temperature and strain-rate sensivities are both inversely proportional to the activation volume.

**Activation Volume for BCC Metals**

The thermal component of the flow stress for BCC metals is given by equation (2.62):

\[ \sigma^* = C_i \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) \]  

(6.11)

Manipulation of equation (6.11) gives:

\[ \ln \dot{\varepsilon} = \frac{1}{C_4 T} \ln \left( \frac{\sigma^*}{C_i} \right) + \frac{C_3}{C_4} \]  

(6.12)
The strain-rate is given by equation (2.45):

\[ \dot{e} = \dot{e}_0 \exp \left( -\frac{\Delta G}{kT} \right) \]  

(6.13)

Manipulating equation (6.13) gives:

\[ \ln \dot{e} = \ln \dot{e}_0 - \frac{\Delta G}{kT} \]  

(6.14)

Therefore a plot of \( \ln \dot{e} \) against \( T^{-1} \) gives a straight line. Comparing equations (6.12) and (6.14) gives:

\[ \dot{e}_0 = \exp \left( \frac{C_3}{C_4} \right) \]  

(6.15)

and:

\[ \Delta G = -\frac{k}{C_4} \ln \left( \frac{\sigma^*}{C_1} \right) \]  

(6.16)

A plot of \( \Delta G \) versus \( \sigma^* \) for RHA/UK100 and iron is given in Figure 6.12 (\( 1\text{eV} = 1.6022 \times 10^{-19} \text{J} \)). The values of \( \dot{e}_0 \) for iron and RHA/UK100 are \( 3.7 \times 10^9 \text{ s}^{-1} \) and \( 3.3 \times 10^6 \text{ s}^{-1} \) respectively. These values represent the theoretical maximum strain-rate obtainable at infinite temperature. The activation volume can be calculated directly from the gradient of the plots in Figure 6.12 using equation (6.6). The variation of the activation volume with thermal flow stress for iron and RHA/UK100 is given in Figure 6.13. For example for a test at 20 °C and 1000 s\(^{-1}\) the thermal components of the flow stress for iron and RHA/UK100 are 420 MPa and 315 MPa respectively. These values equate to activation volumes of \( 1.3 \times 10^{-28} \text{ m}^3 \) for iron and \( 1.0 \times 10^{-28} \text{ m}^3 \) for RHA/UK100. Similar activation volumes for iron and RHA/UK100 indicate that the thermal deformation mechanism in both metals is similar.
Figure 6.12 Variation of $\Delta G$ with the Thermal Flow Stress for RHA/UK100 and Iron (Predicted by the Armstrong-Zerilli Models).

Figure 6.13 Variation of the Activation Volume with the Thermal Flow Stress for Iron and RHA/UK100 (Predicted by the Armstrong-Zerilli Models)
Activation Volume for FCC Metals

The thermal component of the flow stress for FCC metals is given by equation (2.63):

\[ \sigma^* = C_2 e^{3/2} \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) \]  

(6.17)

Manipulation of equation (6.17) gives:

\[ \ln \dot{\varepsilon} = \frac{1}{C_4 T} \ln \left( \frac{\sigma^*}{C_2 e^{3/2}} \right) + \frac{C_3}{C_4} \]  

(6.18)

Comparing equations (6.14) and (6.18) gives:

\[ \dot{\varepsilon}_0 = \exp \left( \frac{C_3}{C_4} \right) \]  

(6.19)

and:

\[ \Delta G = -\frac{k}{C_4} \ln \left( \frac{\sigma^*}{C_2 e^{3/2}} \right) \]  

(6.20)

A plot of \( \Delta G \) versus \( \sigma^* \) for copper is given in Figure 6.14 for various strain levels (1eV = 1.6022x10^{-19} J). The gradient of the curves in Figure 6.14 is independent of strain and is the same for a given thermal flow stress. Therefore the variation of the activation volume with thermal flow stress shown in Figure 6.15 is independent of strain. The value of \( \dot{\varepsilon}_0 \) for copper is 2.1x10^8 s^{-1}. 

207
The variation of the activation volume predicted by the Armstrong-Zerilli model for a typical dynamic strain-rate copper test is shown in Figure 6.16. At 10% the activation volume is $0.5 \times 10^{-27} \text{ m}^3$. This compares reasonably well with $2.7 \times 10^{-27} \text{ m}^3$ at 10% strain from Parry and Walker [1989]. One reason for the difference between these two values is that Parry and Walker [1989] assumed that the activation energy was a linear function of stress. Also the Armstrong-Zerilli equation overestimates the strain-rate sensitivity of copper and this would result in a lower activation volume. Assuming that the activation energy is a linear function of stress and ignoring the data at the highest strain-rates (which may be affected by a viscous deformation mechanism) then the activation volume for copper at 10% strain is $2.6 \times 10^{-27} \text{ m}^3$. This value is in excellent agreement with that calculated by Parry and Walker [1989].
The overestimation of the strain-rate sensitivity of copper by the Armstrong-Zerilli equation might be due to two reasons. At the highest strain-rates there might be a slight transition from thermally activated deformation to viscous flow. Therefore the Armstrong-Zerilli model is effectively averaging out the low and high strain-rate sensitivities corresponding to these two deformation mechanisms. Secondly, in a similar manner to that observed for RHA/UK100, copper might suffer thermal softening through the temperature dependence of its shear modulus which would reduce its strain-rate sensitivity. This type of thermal softening is not incorporated in the standard Armstrong-Zerilli model.

The higher activation volume at the start of a copper test explains the lower strain-rate sensitivity at low strains.
Figure 6.16 Variation of the Activation Volume with Strain for Copper Tested at 20 °C and 5000 s^{-1} (Predicted by the Armstrong-Zerilli Model).
CHAPTER 7

7 CONCLUSIONS AND RECOMMENDATIONS

Detailed in the following two sections are the main conclusions that can be drawn from this study and recommendations for future work that will develop both the experimental technique and the theoretical understanding of the deformation mechanisms.

7.1 CONCLUSIONS

• Adiabatic softening can greatly reduce the experimentally determined dynamic flow stresses of the BCC metals in this study. Copper (FCC) has a very low temperature sensitivity and a correction for adiabatic softening for copper is unnecessary. The determination of the temperature sensitivity by tests and re-tests is a convenient method for calculating the temperature sensitivity over a limited temperature range. The determination of the temperature sensitivity using the Dixon and Parry [1991] technique requires a number of tests at different temperatures. The temperature sensitivities predicted by both methods are in reasonable agreement with one another.

• Friction greatly increases the measured flow stress in both quasistatic and dynamic tests for all the metals in this study. In general friction decreases with increasing temperature and strain-rate. Friction was found to be independent of the specimens' surface finishes and more a function of the lubricant and the level of flow stress. In general the level of friction for an RHA/UK100 specimen is fractional higher than that of a copper specimen tested under the same conditions. This difference in friction is thought to be directly related to the magnitude of the flow stress. The friction corrections predicted from Avitzur ring theory agree well with the hydrocode modelling allowing a reasonable level of confidence to be placed upon the calculated friction coefficients.

• Observations of the deformed specimens and the hydrocode modelling clearly demonstrated that the barrelling radius of curvature is not a good indicator of the precise levels of friction present at a specimen's faces. The thickness of the major ring
that forms around the outer edge of a specimen during a test can be used as an indicator of the level of friction at a given temperature. Friction is higher around the edges of the specimen's faces. This is probably due to a combination of lubricant breakdown in this area and the effects of elastic platen punching predicted in the hydrocode modelling.

- The first 20 µs of a SHPB dynamic test are greatly affected by inertia and wave propagation effects. Therefore no use was made of this portion of the flow stress curve for any subsequent analysis.

- RHA/UK100 and iron are both strain-rate and temperature sensitive over the test conditions used in this study. Iron is more temperature and strain-rate sensitive than RHA/UK100. ARP armour plate steel is slightly stronger than RHA/UK100 and has similar strain-rate and temperature sensitivities. Copper is both temperature and strain-rate insensitive in comparison to RHA/UK100.

- The Armstrong-Zerilli models reasonably predict the flow stress properties of RHA/UK100, iron and copper. The Armstrong-Zerilli model overestimates the strain-rate sensitivity for RHA/UK100 because no account is made for the temperature dependence of the shear modulus. The Armstrong-Zerilli models do not accurately describe the variation of the work hardening coefficient over the total plastic strain range for any of the metals in this study.

- The similarity between the material constants C_3 and C_4 in the Armstrong-Zerilli models for iron and RHA/UK100 indicate that the thermally activated deformation mechanisms in both these metals are similar.

### 7.2 RECOMMENDATIONS FOR FUTURE WORK

- Tests should be performed over a greater range of temperatures for the armour plate steels and iron. This would allow the variation of temperature sensitivity with temperature to be more accurately described. Also a better comparison between the
temperature sensitivities predicted by the tests and re-tests and the Dixon and Parry [1991] technique could be made. Tests at much higher temperatures than room temperature would also allow a direct estimation of the athermal stress to be made from the experimental results.

- Tests using smaller specimens to achieve higher strain-rates for all metals in this study would provide firm evidence as to whether there is a transition to a viscous flow mechanism at very high strain-rates. The ends of the SHPBars would probably need protective end caps to eliminate plastic deformation of the pressure bar faces. A re-assessment of the levels of friction and inertia would have to be made. Alternatively expanding ring tests could be performed to achieve very high strain-rates. Tests up to higher plastic strains (than achieved in this study) could be performed using smaller specimens. This would allow the fitting of the Armstrong-Zerilli equations to be performed over a strain range much further away from the yield point in which the variation of the work hardening coefficient with strain is lower.

- The temperature dependence of the shear modulus should be measured for RHA/UK100 and incorporated into the Armstrong-Zerilli equation in a similar manner to that performed by Goldthorpe, Butler and Church [1994].

- Testing all the metals at an intermediate strain-rate (perhaps using a drop-weight system) would allow the characterisation of these metals over a large strain-rate range when taking into account the very high strain-rate tests proposed above.

- The effects of friction in most tests introduces the largest error into the experimentally determined flow stress and also introduces stresses that cause non-uniform deformation. Therefore an extensive search for the best lubricant for a given set of test conditions should be performed to minimise the effects of friction.
REFERENCES


GORHAM, D. A., 1991a. An Effect of Specimen Size in the High Strain Rate Compression Test. In: DYMAT 91 International Conference on Mechanical and


PHYSICS DEPT. Drop Weight Testing. Loughborough University, Internal Documentation.


APPENDIX 1

AVITZUR FRICTION ANALYSIS PROGRAMS

AVITZUR1.BAS: ANALYSES RING SPECIMENS

AVITZUR2.C: FRICTION CORRECTION FOR SOLID SPECIMENS
REM AVITZUR1.BAS
REM FORGING OF HOLLOW DISCS, B. AVITZUR
INPUT "OUTPUT FILENAME"; OPFNS$
INPUT "SHEAR FACTOR m"; M
INPUT "PLATEN VELOCITY U (mm/s)"; U
INPUT "TIME INTERVAL t (s)"; T
INPUT "INNER RADIUS Ri (mm) (Ri = 0 = SOLID DISC)"; RI
INPUT "OUTER RADIUS Ro (mm)"; RO
INPUT "SPECIMEN THICKNESS T (mm)"; H
INPUT "COMPRESSED THICKNESS CT (mm)"; CT
OPEN "O", #1, OPFNS$
NUM = 0
A = 0
C = 0
DO
  NUM = NUM + 1
  IF RI <> 0 THEN A = RI / RO
  B = M * RO / H
  IF RI <> 0 THEN C = RO / RI
  IF RI <> 0 THEN D = LOG(ABS(3 * (C * 2) / (1 + (1 + 3 * (C * 4)) .5))) / (2 - 2 * A)
  IF B <= D AND RI <> 0 THEN
    X = (C * EXP(A * B - B)) .2
    REM E = (Rn/Ro)^2
    E = .5 * (3 .5) * (1 - (A * 4) * (X * 2)) / ((X * (X - 1) * (1 - (A * 4) * X)) .5)
    G = (ABS(E)) .5
    RN = G * RO
    REM F = Pav / YIELD LIMIT.
    F = (1 - A * 2) .1) * (((1 + (E * 2) / 3) .5) - (A * 4 + (E * 2) / 3) .5 + 2 * B * (1 - A * 3) / (3 * (3 .5)))
  ELSEIF RI <> 0 THEN
    REM G = Rn / Ro
    G = (2 * (3 .5) * B / ((C * 2) - 1)) * (((1 + (1 + A) * ((C * 2) - 1) / (2 * (3 .5) * B)) .5) - 1)
    RN = G * RO
    F = ((1 - (A * 2)) .1) * (((1 + (G * 4) / 3) .5) - ((A * 4 + (G * 4) / 3) .5) + 2 * B * (1 + (A * 3) - 2 * G * 3) / (3 * 3 .5))
  ELSE
    F = 1 + (2 * M * RO) / (3 * (3 .5) * H)
REM DRO = CHANGE IN OUTER RADIUS
REM DRI = CHANGE IN INNER RADIUS
DRO = -.5 * (U / H) * RO * (1 - (G^2)) * T
IF RI <> 0 THEN DRI = -.5 * (U / H) * RI * (1 - (RN / RI)^2) * T
FRIC = M / ((3^0.5) * F)
PRINT #1, "", NUM; " "; RI; " "; RO; " "; A; " "; H; " "; B; " "; G; " "; RN; " "; DRO; " "; DRI; " "; F; " "; FRIC
RO = RO + DRO
RI = RI + DRI
H = H + (U * T)
LOOP UNTIL H <= CT
END
AVITZUR.C
*ANSI C AVITZUR SOLID SPECIMEN FRICTION CORRECTION CALCULATOR *
*MARK ASHTON 1999
***************************************************************************/

#include <stdlib.h>
#include <stdio.h>
#include <math.h>

void main()
{
    double sl/*specimen length*/
    double sd /*specimen diameter*/
    double pr/*Poissons ratio*/
    double m/*Avitzur constant shear factor*/
    double ascf /*Avitzur stress correction factor*/
    double ts /*true strain %*/
    char str[15];

    printf("\nA VITZUR SOLID SPECIMEN 
FRICTION 
CORRECTION CALCULATOR\n");

    printf("\n\nInput original specimen length (default=4.00 mm): ");
    gets(str);
    if (str[0] == '0') sl=0.004;
    else sl=atof(str)/1E3;

    printf("\nInput original specimen diameter (default=8.00 mm): ");
    gets(str);
    if (str[0] == '0') sd=0.008;
    else sd=atof(str)/1E3;

    printf("\nInput Poissons ratio (default=0.5): ");
    gets(str);
    if (str[0] == '0') pr=0.5;
    else pr=atof(str);

    printf("\nInput Avitzur constant shear factor (default=0.15): ");
    gets(str);
    if (str[0] == '0') m=0.15;
    else m=atof(str);

    /*Avitzur correction factor for a solid specimen calculated at various 
levels of true strain*/
    for(ts=0.0;ts<=9.0;ts+=1.0)
    {
        ascf=1+(m*sd*exp(ts*(1+pr)/1E2))/(sl*sqrt(27.0));
        printf("\nAvitzur correction factor at a true strain of %2f %%% = %.4f",ts,ascf);
    }
    for(ts=10.0;ts<=14.0;ts+=2.0)
    {
        ascf=1+(m*sd*exp(ts*(1+pr)/1E2))/(sl*sqrt(27.0));
        printf("\nAvitzur correction factor at a true strain of %2f %%% = %.4f",ts,ascf);
    }
    for(ts=15.0;ts<=50.0;ts+=5.0)
    {
        ascf=1+(m*sd*exp(ts*(1+pr)/1E2))/(sl*sqrt(27.0));
        printf("\nAvitzur correction factor at a true strain of %2f %%% = %.4f",ts,ascf);
    }
}
APPENDIX 2

SHPB ANALYSIS PROGRAM: SHPB1.C
/***************************************************************
*SHPB1.C                                               *
*ANSI C VERSION OF HOPKINSON BAR ANALYSIS PROGRAM       *
*MARK ASHTON 1999                                      *
***************************************************************

#include <stdlib.h>
#include <stdio.h>
#include <ermo.h>
#include <math.h>

void open_pulse_file(char *filename, double array[]);
void expt_params(double params[]);
void start_points_lines(double inc[], double trans[], double params[]);
void analyse(double inc[], double trans[], double params[], double results[][20]);
void write_results(char *filename, double results[][20], double params[], double inc[]);

void main()
{
    double inc[1000];
    double trans[1000];
    double params[32];
    double results[180][20];
    char *filename;
    filename = malloc(50);
    printf("An HOPKINSON BAR ANALYSIS PROGRAM\n");
    printf("An MARK ASHTON 1999\n");
    printf("This program analyses the first 1000 data points of\n");
    printf("the pulse records. If the oscilloscope has triggered\n");
    printf("early remove unwanted points at start of pulse records\n");
    printf("using the DOS EDIT program.\n");
    printf("To terminate program at any point press <CTRL-C>\n");
    printf("To accept default values press <RETURN> at prompt.\n");
    printf("Input path and name of incident bar file: \n");
    gets(filename);
    open_pulse_file(filename,inc);
    printf("Input path and name of transmitter bar file: \n");
    gets(filename);
    open_pulse_file(filename,trans);
    printf("Input path and name of incident bar file: \n");
    gets(filename);
    open_pulse_file(filename,inc);
    printf("Input path and name of transmitter bar file: \n");
    gets(filename);
    open_pulse_file(filename,trans);
    printf("Input path and name of incident bar file: \n");
    gets(filename);
    open_pulse_file(filename,inc);
    printf("Input path and name of transmitter bar file: \n");
    gets(filename);
    open_pulse_file(filename,trans);
    printf("Input path and name of results file: \n");
    gets(filename);
    write_results(filename,results,params,inc);
    free(filename);
    printf("Analysis complete, program terminated.\n");
}

void open_pulse_file(char *filename, double array[])
{
    FILE *file;
    int c;
    file = fopen(filename,"r");
    if (file == NULL)
    {
        printf("File cannot be opened, terminating program.\n");
        exit(errno);
    }
}
for(c=0; c<=999; c++)
load first 1000 data points
scanf(file, "%lf,%lf", &array[c]);
}
close(file);

void expt_params(double params[18])
{
    char str[15];
    printf("Input strain gauge factor of strain gauges 1 (incident bar)");
    printf("(default=2.14)");
    gets(str);
    if(str[0]==0) params[0]=2.14; else params[0]=atof(str);
    printf("Input strain gauge factor of strain gauges 2 (transmitter bar)");
    printf("(default=2.14)");
    gets(str);
    if(str[0]==0) params[1]=2.14; else params[1]=atof(str);
    printf("Input gain of strain gauges 1 (incident bar) amplifier");
    printf("(default=1)");
    gets(str);
    if(str[0]==0) params[2]=1.0; else params[2]=atof(str);
    printf("Input gain of strain gauges 2 (transmitter bar) amplifier");
    printf("(default=1)");
    gets(str);
    if(str[0]==0) params[3]=1.0; else params[3]=atof(str);
    printf("Input original specimen length (default=4.00 mm)");
    printf("(default=0.008)");
    gets(str);
    if(str[0]==0) params[4]=0.008; else params[4]=atof(str);
    printf("Input original specimen diameter (default=8.00 mm)");
    printf("(default=0.008)");
    gets(str);
    if(str[0]==0) params[5]=0.008; else params[5]=atof(str);
    printf("Input power supply voltage (default=90.00 V)");
    printf("(default=90.0)");
    gets(str);
    if(str[0]==0) params[6]=90.0; else params[6]=atof(str);
    printf("Input voltage across strain gauges 1 (default=8.8 V)");
    printf("(default=8.8)");
    gets(str);
    if(str[0]==0) params[7]=8.8; else params[7]=atof(str);
    printf("Input voltage across strain gauges 2 (default=8.8 V)");
    printf("(default=8.8)");
    gets(str);
    if(str[0]==0) params[8]=8.8; else params[8]=atof(str);
    printf("Input Poisson's ratio (default=0.5)");
    printf("(default=0.5)");
    gets(str);
    if(str[0]==0) params[9]=0.5; else params[9]=atof(str);
    printf("Input number of points to analyse (maximum 180 microseconds)");
    printf("(default=140 microseconds)");
    gets(str);
    if(str[0]==0) params[10]=140.0; else params[10]=atof(str);
    printf("Input Hopkinson bar's Young's modulus (default=187E9 N/m2)");
    printf("(default=187E9)");
    gets(str);
    printf("Input Hopkinson bar's density (default=8056 kg/m3)");
    printf("(default=8056)");
    gets(str);
    if(str[0]==0) params[12]=8056.0; else params[12]=atof(str);
    printf("Input Hopkinson bar's diameter (default=12.7 mm)");
    printf("(default=12.7)");
    gets(str);
    if(str[0]==0) params[13]=0.0127; else params[13]=atof(str);
    printf("Input projectile bar's Young's modulus (default=187E9 N/m2)");
    printf("(default=187E9)");
    gets(str);
}
if(str[0]=='0') params[17]=18759; else params[17]=atof(str);
printf("Input projectile bar's density (default=8056 kg/m3): ");
gets(str);
if(str[0]=='0') params[18]=8056.0; else params[18]=atof(str);
printf("Input projectile bar's length (default=25 cm): ");
gets(str);
if(str[0]=='0') params[19]=0.25; else params[19]=atof(str)/1E2;
printf("Input specimen's density (default Cu=8900 kg/m3): ");
gets(str);
if(str[0]=='0') params[20]=8900.0; else params[20]=atof(str);
printf("Input temporal digitising step (default=1 microsecond): ");
gets(str);
if(str[0]=='0') params[22]=1E-6; else params[22]=atof(str)/1E6;
printf("Input Avitzur constant shear factor (default=0.15): ");
gets(str);
if(str[0]=='0') params[26]=0.15; else params[26]=atof(str);
printf("Input specimen's specific heat (default Cu, enter 1 for Fe,");}
gets(str);
if(str[0]=='0') params[21]=3.0; else params[21]=atof(str);
printf("Input deformation work to heat conversion factor");
printf("(default=1): ");
gets(str);
if(str[0]=='0') params[31]=1.0; else params[31]=atof(str);
printf("Input Avitzur constant shear factor 2 (default=-0.5 MPa/K): ");
gets(str);
if(str[0]=='0') params[30]=-0.5; else params[30]=atof(str);
void stpoints_blines(double inc[],double trans[],double params[])
{
    int sip; /*start of incident pulse (microseconds)*/
    int sp; /*start of reflected pulse (microseconds)*/
    int spt; /*start of transmitted pulse (microseconds)*/
    int nmp = fabs(params[13]+0.5)-1; /*number of points to analyse*/
    int c; /*counter*/
    int incmax; /*point on incident bar trace with maximum voltage*/
    int incmin; /*point on incident bar trace with minimum voltage*/
    int transmax; /*point on transmitter bar trace with maximum voltage*/
    double b1; /*baseline of incident bar trace (mV)*/
    double b2; /*baseline of transmitter bar trace (mV)*/
    double rpsc; /*reflected pulse start correction*/
    double rpec; /*reflected pulse end correction*/
    double temp1=0.0; /*storage variable*/
    double temp2=0.0; /*storage variable*/
    double temp3=0.0; /*storage variable*/
    char str[15];
    /*the baselines for the incident and transmitter traces are calculated
    by averaging the first 100 points in the incident and transmitter
    bar traces respectively*/
    for(c=0;c<=99;c++)
\{  temp1=templ+inc[c];  
  temp2=temp2+trans[c];  
\}  

bl1=templ/1E2;  
bl2=temp2/1E2;  

printf("Input baseline of incident bar trace (default=%.2f mV): ").1E3*bl1);  
get(str);  
if(str[0]=='0') params[11]=bl1; else bl1=atof(str)/1E3;  
params[11]-=bl1;  

printf("Input baseline of transmitter bar trace (default=%.2f mV): ").1E3*bl2);  
get(str);  
if(str[0]=='0') params[12]=bl2; else bl2=atof(str)/1E3;  
params[12]=bl2;  

/*the starting points of the incident and reflected pulses are found by finding the maximum and minimum voltage points in the incident bar trace and then moving back along the trace until within 0.8 of a voltage digitising step of the baseline. The start point of the transmitter pulse is found using the same method*/  

/*find maximum and minimum voltage points from incident bar trace and maximum voltage point on transmitter bar trace*/  

temp1=inc[0];  
temp2=inc[0];  
temp3=trans[0];  
for(c=0;c<=999;c++)  
\{  
  if(inc[c]>temp1)  
  \{  
    incmax=c;  
    temp1=inc[c];  
  \}  
  if(inc[c]<temp2)  
  \{  
    incmin=c;  
    temp2=inc[c];  
  \}  
  if(trans[c]>temp3)  
  \{  
    transmax=c;  
    temp3=trans[c];  
  \}  
\}  

/*find size of voltage digitising step on incident bar trace and store it in temp2*/  
temp2=1E3;  
for(c=0;c<=998;c++)  
\{  
  temp1=inc[c+1]-inc[c];  
  if(temp1>=1E-6 && temp1<temp2) temp2=temp1;  
\}  

/*find start point of incident pulse, first point is point number 1*/  
c=0;  
while(inc[c+incmax]>(bl1+0.8*temp2) && c<1000) c--;  
sip=incmax+c+1;  

/*find start of reflected pulse, first point is point number 1*/  
c=0;  
while(inc[c+incmin]<(bl1-0.8*temp2) && c<1000) c--;  
srp=incmin+c+1;  

/*find size of voltage digitising step on transmitter bar trace and store it in temp2*/  
temp2=1E3;  
for(c=0;c<=998;c++)  
\{  
  temp1=trans[c+1]-trans[c];  
  if(temp1>=1E-6 && temp1<temp2) temp2=temp1;  
\}
/* find start of transmitted pulse, first point is point number 1 */
c = 0;
while(trans[c]+transmax) >=(bl2+0.8*temp2) && c <=1000) c=;
stp = transmax+c+1;

printf("Input start of incident pulse (default=%i microseconds):

gets(str);
if(str[0]=="\0")
params[8]=sip;
else params[8]=atof(str);
sip = fabs(params[8]+0.5)-1;

printf("Input start of reflected pulse (default=%i microseconds):

gets(str);
if(str[0]=="\0")
params[9]=srp;
else params[9]=atof(str);
srp = fabs(params[9]+0.5)-1;

printf("Input start of transmitted pulse (default=%i microseconds):

gets(str);
if(str[0]=="\0")
params[10]=stp;
else params[10]=atof(str);

/* start correction for reflected pulse baseline */
rpsc = inc[srp]-bl1;
printf("Input reflected pulse start correction (default=%i mV):

gets(str);
if(str[0]=="\0")
rpsc = atof(str)*1000;

/* end correction for reflected pulse baseline */
rpec = inc[srp+nop+1]-bl1;
printf("Input reflected pulse end correction (default=%i mV):

gets(str);
if(str[0]=="\0")
rpec = atof(str)*1000;

/* correcting reflected pulse baseline */
for(c=0;c<nop;c++)

inc[srp+c+1]=inc[srp+c+rpsc-c*(rpec-rpsc)]((nop+1));

void analysis(double inc[], double trans[], double params[], double results[][20])
{

double sgfl = params[0]; /* strain gauge factor of strain gauges 1 (incident bar) */
double sgf2 = params[1]; /* strain gauge factor of strain gauges 2 (transmitter bar) */
double sl = params[2]; /* original specimen length (m) */
double sd = params[3]; /* original specimen diameter (m) */
double psv = params[4]; /* power supply voltage (V) */
double vsg1 = params[5]; /* voltage across strain gauges 1 (V) */
double vsg2 = params[6]; /* voltage across strain gauges 2 (V) */
double pr = params[7]; /* Poisson's ratio */

int sip = fabs(params[8]+0.5)-1; /* start of incident pulse (microseconds) */
int srp = fabs(params[9]+0.5)-1; /* start of reflected pulse (microseconds) */
int stp = fabs(params[10]+0.5)-1; /* start of transmitted pulse (microseconds) */
double bl1 = params[11]; /* baseline of incident bar trace (V) */
double bl2 = params[12]; /* baseline of transmitter bar trace (V) */

double temp = params[13]; /* number of points to analyse */
double brym = params[14]; /* Hopkinson bar's Young's modulus */
double bde = params[15]; /* Hopkinson bar's density */
double bdi = params[16]; /* Hopkinson bar's diameter */
double bmy = params[17]; /* projectile's Young's modulus */
double pde = params[18]; /* projectile's density */
double plc = params[19]; /* projectile length */
double sde = params[20]; /* specimen's density */

/* params[21] contains information describing which material specific heat equation to use, 1 for Fe, 2 for armour plate steel, 3 for Cu, or a constant specific heat value */

double tds = params[22]; /* temporal digitising step */

/*params[22] is the projectile velocity calculated below*/
double gain1 = params[24]; /* gain of strain gauges 1 amplifier */
double gain2 = params[25]; /* gain of strain gauges 2 amplifier */
double m = params[26]; /* Avitzur constant shear factor */
double tt = params[27]; /* test temperature in Kelvin */

/*params[28] thermal softening correction factor */
double vmt = params[29]; /*params[28] OK below valid maximum temperature */
/*params[30] thermal softening correction factor */

double dwhcf = params[31]; /* deformation work to heat conversion factor */

int c = 0; /* counter */
int iplds; /* length of incident pulse in digitising steps */
double n; /* ratio of ballast resistance/strain gauge resistance */
double ewv; /* elastic wave velocity in bar */
double ewvp; /* elastic wave velocity in projectile */
double area; /* area increment under true stress-true strain curve */
double sum=0; /* total of a given set of values */
double ave; /* average of a given set of values */
double ssh; /* specimen specific heat J/kg/K */
double tscf; /* thermal softening correction factor */

/* time column (microseconds) */
for(c=0;c<=nop;c++)
    results[c][0]=c*tts*1E6;

/* calculating size of pulses relative to baselines and scaling size of pulses according to strain gauge amplifier gains */
for(c=0;c<=nop;c++)
{
    inc[sip+c]=(inc[sip+c]-bll)/gain1;
    inc[srp+c]=(inc[srp+c]-hll)/gain1;
    transl[c]=(trans[stp+c]-bll)/gain2;
}

/* incident pulse in terms of stress (MPa) */
n=(psv-vsg1)/vsg1;
for(c=0;c<=nop;c++)
    results[c][6]=bym*pow((n+1,2)*inc[sip+c]/(1E6*n*sfl*psv));

/* reflected pulse in terms of stress (MPa) uses value of n calculated above */
for(c=0;c<=nop;c++)
    results[c][7]=bym*pow((n+1,2)*inc[srp+c]/(1E6*n*sfl*psv));

/* transmitted pulse in terms of stress (MPa) */
n=(psv-vsg2)/vsg2;
for(c=0;c<=nop;c++)
    results[c][8]=bym*pow((n+1,2)*trans[sip+c]/(1E6*n*sfl2*psv));

/* incident pulse stress - reflected pulse stress (MPa) */
for(c=0;c<=nop;c++)
    results[c][9]=results[c][6]-results[c][7];

/* stress equilibrium factor = transmitted/(incident+reflected), 1E-6 prevents division of zero by zero error */
for(c=0;c<=nop;c++)
    results[c][12]=results[c][8]/(results[c][6]+results[c][7]+1E-6);

/* specimen engineering stress (MPa) calculated from transmitted pulse */
for(c=0;c<=nop;c++)
    results[c][3]=(pow(bdi.2)/pow(sd,2))*results[c][8];

/* specimen engineering strain (%) calculated from reflected pulse, area under the curve is calculated using the mid-ordinate rule */
ewv=sqrt(bym/bds);
results[0][4]=0.0;
for(c=1;c<=nop;c++)
    results[c][4]=1E2*ewv*(results[c][7]+results[c-1][7])*1E6*tts/(bym*sl)+results[c-1][4];

/* specimen true strain-rate */
results[0][5]=0.0;
for(c=1;c<=nop;c++)
    results[c][5]=(results[c][4]-results[c-1][4])/(1E2*tts*(1.0-results[c][4]/1E2));

/* specimen true strain (%) calculated from engineering strain */
for(c=0;c<=nop;c++)
    results[c][2]=1E2*log(1.0-results[c][4]/1E2);

/* specimen true stress calculated from engineering stress calculated from transmitted pulse */
for(c=0;c<=nop;c++)
    results[c][1]=results[c][3]*exp(-2*pr*results[c][2]/1E2);

/* true stress on incident face of specimen */
for(c=0;c<=nop;c++)
    results[c][10]=(pow(bdi.2)/pow(sd,2))*results[c][9]*exp(-2*pr*results[c][2]/1E2);
 average specimen true stress (true stress on incident face of specimen + true stress on transmitter face of specimen) / 2

for (c=0; c<nop; c++)
result[c][11] = (result[c][1] + result[c][10]) / 2.0;

velocity of incident bar face (m/s)

for (c=0; c<nop; c++)
result[c][13] = evv * (result[c][6] - result[c][7]) * 1E6 / bym;

velocity of transmitter bar face (m/s)

for (c=0; c<nop; c++)
result[c][14] = evv * result[c][8] * 1E6 / bym;

velocity of incident bar face - velocity of transmitter bar face (m/s)

for (c=0; c<nop; c++)
result[c][15] = result[c][13] - result[c][14];

temperature rise in specimen (K) calculated from area under original true stress-true strain curve (without any corrections applied) using the mid-ordinate rule; specific heats: case 1 for Fe, case 2 for armour plate steel, case 3 for Cu, case 4 user defined constant

for (c=0; c<nop; c++)
result[c][16] = 0.0;
for (c=1; c<nop; c++)
{
switch((int)(fabs(params[21]) + 0.5))
{
case 1: ssh = 74.9691 * (3.37 + 7.1E-3 * (tt + result[c][11][16]) + 0.43E5 * pow((tt + result[c][11][16]), 2)); break;
case 2: ssh = 510.0; break;
case 3: ssh = 65.8861 * (5.41 + 1.5E-3 * (tt + result[c][1][2])) / sqrt(27.0)); break;
default: ssh = params[21];
}
area = result[c][11] + result[c][11][2];
result[c][16] = dwheat(area / sde / ssh) + result[c][17][12];
}

true stress-true strain curve corrected for friction using the Avitzur correction for a solid specimen, corrected stress in MPa

for (c=0; c<nop; c++)
result[c][17] = result[c][11][1] / (1 + (m * sd * exp(result[c][2] * (1 + pr + 1E2)) / (sqrt(27.0))));

temperature rise in specimen (K), calculated from area under true stress-true strain curve corrected for friction using the mid-ordinate rule

for (c=0; c<nop; c++)
result[c][18] = 0.0;
for (c=1; c<nop; c++)
{
switch((int)(fabs(params[21]) + 0.5))
{
case 1: ssh = 74.9691 * (3.37 + 7.1E-3 * (tt + result[c][1][18]) + 0.43E5 * pow((tt + result[c][1][18]), 2)); break;
case 2: ssh = 510.0; break;
case 3: ssh = 65.8861 * (5.41 + 1.5E-3 * (tt + result[c][1][18])) / sqrt(27.0)); break;
default: ssh = params[21];
}
area = result[c][17] + result[c][11][11];
result[c][18] = dwheat(area / sde / ssh) + result[c][17][12];
}

true stress-true strain curve corrected for friction is now corrected for thermal softening using the technique developed by Dixon and Parry, stress corrected for friction and thermal softening in MPa

for (c=0; c<nop; c++)
{
if (result[c][18][tt] < vmt) tscf = params[28]; else tscf = params[30];
result[c][15] = result[c][17] * tscf * result[c][18];
}

projectile velocity (params[23]) from incident pulse height (m/s), value printed to screen so user can use it in results file name if desired. The average pulse height is calculated over the second and third quarters of the pulse

evwp = sqrt(pumple); iplds = fabs(2.0 * ple / evwp) / ds + 0.5;
for (c = iplds / 4; c < iplds / 3 / 4; c++)
sum = sum + results[cl][6];
ave = 1E6 * sum / (iplds / 2.0);
params[23] = ave * (1 / (pde * ewwp) + 1 / (bde * eww));
printf("auProjectile velocity calculated from incident pulse height: ");
printf("%s 21 m/s", params[23]);
}

void write_results(char *filename, double results[][20], double params[], double inc[])
{
    FILE *file;
    int rows;
    int cols;
    int nop = fabs(params[13] + 0.5) - 1; /* number of points to write */
    int srp = fabs(params[9] + 0.5) - 1; /* start of reflected pulse */

double aohb = 3.141592654 * pow(params[16], 2) / 4.0; /* area of SHP bars */

    file = fopen(filename, "w");
    if (file == NULL)
    {
        printf("inFile cannot be opened, terminating program.");
        exit(errno);
    }
    for (rows = 0; rows <= nop; rows++)
    {
        for (cols = 0; cols <= 19; cols++)
        {
            fprintf(file, ".", results[rows][cols]);
            fprintf(file, ".", results[rows][6] * aohb * 1E3);
            fprintf(file, ".", results[rows][7] * aohb * 1E3);
            fprintf(file, ".", results[rows][8] * aohb * 1E3);
            fprintf(file, ".", inc[srp + rows] * params[24]);
            if (rows <= 31)
            {
                fprintf(file, ".", params[rows]);
            }
            else
            {
                fprintf(file, "\n");
            }
        }
    }
    fclose(file);
}
APPENDIX 3

HYDROCODE MODELLING: TYPICAL INGRID FILE
L.U. HOPK BAR RHA/UK.100 (Us=0.4) S4x8 293K SOLID pv=25m/s

DN2D
AXIS
SI 1 SVFP DEFPEN 2.0 FRIC 0.40;
SI 2 SVFP DEFPEN 2.0 FRIC 0.40;
SI 3 SV;
TERM 350.0E-06
PLTI 10.0E-06
PRTI 1.0E-06
START
C *************************************
C PRESSURE BAR (BOTTOM BAR)
  1  11;
  1  20  25  30;
 -1;
  0.0  0.00635
  0.0  0.978  0.993  1.000
C *************************************
MATERIAL 1
SI 1 412 411 M
END
START
C *************************************
C SPECIMEN
  1  36;
  1  29  56;
 -1;
  0.0  0.0034
  1.000  1.002  1.004
  0.0
C SLIDELINE
MATERIAL 2
EPB 1 2 1 ro 0 0 34 0 0 ND 1374
SI 1 1 1 2 1 1 1 S
SI 2 1 1 2 3 1 1 S
SI 2 1 1 2 3 1 2 S
SI+ 1 3 1 2 3 1 2 S 0.0 10.0 0.0
C *************************************
'10' is a vector to specify slideline direction

C Hopkinson Bar (Top Bar)

1 11;
1 5 10 30;
-1;
0.0 0.00635
1.004 1.011 1.026 2.004
0.0

Material I

SI+ 1 1 1 2 1 1 2 M 0.0 -10.0 0.0
SI 1 1 2 4 1 3 M

C Projectile

1 11;
1 11;
-1;
0.0 0.00635
2.004 2.254
0.0

C Slideline

Material I

SI 1 1 1 2 1 1 3 S

Velocity 0.0 -31.0 0.0

End

Material 1

Stainless Steel Type 10 elastic-plastic-hydrodynamic

RO 8.05E+3
G 7.2430E10
PC -1.0E15

NPTS 3

ES 14.0E8 14.0E8 14.0E8
EPS 0.0 0.3 200.0  \[\rightarrow\text{effective plastic strains}\]
ENDMAT
EOS 1 4
HEAD
Mie-Gruniesen equation of state for steel
SP 4.818E03
S1 1.73
GAMMA 1.67
V0 1.0
ENDEOS
C****************************
MAT 2 40  \[\rightarrow\text{material 2}\]
HEAD
Armstrong-Zerilli for unshocked RHA/steel
R0 7.86E3
G 9.545E10
DSG 7.10E8
C1 5.75E8
C2 0.0048
C3 0.0048
C4 0.00032
C5 5.67E8
XN 0.41
TROOM 293.0
EPS0 1.0  \[\rightarrow\text{test start temperature in Kelvin}\]
SPH 450.0
PC -1.0E15
SPT 0.0  \[\rightarrow\text{spall type}\]
PSIF 0.0  \[\rightarrow\text{plastic strain iteration flag}\]
EL1 0.0  \[\rightarrow\text{element 1 data}\]
EL2 0.0  \[\rightarrow\text{element 2 data}\]
EL3 0.0  \[\rightarrow\text{element 3 data}\]
D1 0.0  \[\rightarrow\text{failure parameter}\]
D2 0.0  \[\rightarrow\text{failure parameter}\]
D3 0.0  \[\rightarrow\text{failure parameter}\]
D4 0.0  \[\rightarrow\text{failure parameter}\]
D5 0.0  \[\rightarrow\text{failure parameter}\]
IMF 2.0  ←material flag
SF 0.0  ←shock flag
GS 0.02  ←grain size
XK 0.0  ←user defined constant for material 2
SM1 1.13  ←user defined constant for material 2
SM2 0.000445  ←user defined constant for material 2
SHC 0.0  ←shock constant
SPHF 1.0  ←specific heat flag
ELPT 1.0E-06  ←element print time
ENDMAT
EOS 2 4
HEAD
Mie-Gruniesen equation of state for steel
SP 4.61E03  ←wave velocity
S1 1.73  ←empirical parameter
GAMMA 1.67  ←Gruniesen constant
V0 1.0  ←specific volume at zero pressure
ENDEOS
END  ←end of file