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performance of longswings
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CONTRIBUTIONS TO THE PERFORMANCE OF LONGSWINGS ON RINGS

by

Mark Adrian Brewin

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

September 1998

Supervisors: Dr M.R. Yeadon
Dr D.G. Kerwin

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ABSTRACT

Contributions to the performance of longswings on rings
M.A. Brewin, Loughborough University, 1998

Rings is one of six disciplines in Men's Artistic Gymnastics. Judging criteria stipulate that a routine must incorporate two swinging elements completed in a motionless handstand. Performing backward and forward longswings in a routine fulfils this requirement. During both types of longswing, gymnasts typically use large angle changes at their hip and shoulder joints and move their arms laterally. Previous studies have ignored these three-dimensional arm movements, possibly neglecting a crucial aspect of technique. Using a computer simulation model this study investigated the contributions of hip and shoulder elevation angle changes and lateral arm movements to the performance of backward and forward longswings.

A three-dimensional video and cable tension analysis of several backward and forward longswings performed by two elite gymnasts was conducted. The data provided accurate three-dimensional descriptions of backward and forward longswing techniques and the forces experienced by the gymnasts. In addition, data describing deformations of the rings frame and the extension of the gymnast were determined.

A simulation model representing the three-dimensional movements of the rings cables and arms of a gymnast was developed. The model represented the right side of the gymnast and rings apparatus and comprised five segments: rings cable, arm, torso with head, thigh, and shank with foot. Damped linear springs represented the elasticity of the apparatus and gymnast. The model was evaluated against actual backward and forward longswing performances of two elite gymnasts. Actual joint angle time histories describing the gymnasts' techniques, together with subject specific inertia parameters, were used for this procedure. The RMS differences between values estimated by the model and actual values for the orientation of the gymnast and rings cable, the cable tension and the body extension were 4.3°, 2.1°, 161 N and 0.1 m respectively.

The evaluated model was used to determine the contributions of each aspect of technique to the performance of longswings. Hip and shoulder elevation angle changes are important in producing the required rotation of the gymnast in both types of longswing. Without these components of technique the gymnast generated up to 113° less rotation. Lateral arm movements performed during backward longswings resulted in 40% less shoulder elevation torque required to complete the element and a 0.8 bodyweights decrease in peak force experienced at the shoulder joints. When lateral arm movements were omitted during forward longswings the gymnast produced 49° less rotation, and failed to reach the final handstand. This study shows that lateral arm movements make an important contribution to the performance of longswings on rings.
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DEDICATION

Dedicated to the memory of my father, Colin Richard Brewin, who passed away on the 19th September 1996.

(29th February 1936 to 19th September 1996)

The dedication is extended to the rest of the Brewin family; Mum, Melanie, Ian and Luke

and finally, to Denise (Dee) Johnson.
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CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter provides a brief overview outlining the area of interest of this study. After establishing the context of the investigation the purpose of the thesis is formalised into a single statement.

In order to provide a framework for the study several specific questions are posed. These questions are progressive, with the answers from one question often providing some input of knowledge into the subsequent question. It is the intention of the thesis to satisfy the stated purpose by accumulating knowledge from each specific question.

Finally, a summary details the organisation of the entire thesis.

1.2 Overview of the area of study

In the Olympic sport of Men's Artistic Gymnastics the rings apparatus is one of six disciplines in which a gymnast may compete. The remaining five comprise the floor, pommel horse, vault, parallel bars and horizontal bar.

Swinging elements form an essential component of routines on the pommel horse, rings, parallel bars and horizontal bar. However, owing to the structure of the rings frame and cables (Figures 2.1) there is more scope for movement of the rings apparatus, when performing swinging elements, than for the relatively rigid and stable pommel horse, parallel bars and horizontal bar. In turn, such a characteristic of the rings apparatus may increase the difficulty in performing swinging and static elements in combination.

The method by which the rings cables are attached to the frame allows the rings cables to move in three dimensions (Figure 2.2). This capability is utilised by gymnasts when performing both static and swinging elements by moving their arms laterally. The most renowned element which displays this capability is the Iron Cross position, where the gymnast holds a crucifix type configuration, with his arms out to his sides and parallel to the floor. This inherent three-dimensional motion of the gymnast and rings cables increases the complexity of analysing and understanding their motions and interactions.

In order to gain the highest scores in competition gymnasts must comply with the judging criteria specified by the international governing body of gymnastics, the Fédération Internationale de Gymnastique (F.I.G.). For rings the judging criteria stipulate that a routine must comprise both swinging and strength elements in approximately equal...
proportions. Consequently, for a gymnast to attain the highest scores for his performance, he must become proficient in performing elements in both of these categories. In addition special requirements for routines on rings state that it is compulsory for gymnasts to execute two held handstands from swinging elements. One handstand must be completed from a backward swing, whilst a second must be performed from a forward swing. Both the backward and forward longswings fulfil these special requirements. Graphics sequences depicting these two longswing elements are in Section 2.3.1 (Figures 2.4 and 2.5). As a result many elite gymnasts perform both elements in their routines and therefore proficient performances of these two elements may be considered to reflect the skill of the gymnast.

From personal observations of elite gymnasts performing backward and forward longswings it appears that different techniques (changes in the gymnast's body configuration) are often adopted by different gymnasts. Furthermore, these differences in technique are often advocated by different coaches. For example, during a backward longswing the arms of some gymnasts remain nearly parallel throughout the descending phase of the swing. This aspect of technique is encouraged in the coaching article of Kormann (1984). Other gymnasts, however, move their arms laterally during this phase, away from the mid-line of their bodies. Such differences in techniques raise important questions including; what benefits in performance, if any, are gained by adopting one of these techniques in preference to the other? How does each component of technique contribute to the performance? Does a gymnast experience greater forces when adopting one technique rather than the other? What is the mechanics underlying such benefits?

This simple example illustrates the uncertainty which exists in the understanding of how each component of technique contributes to performances of longswings on rings. In a more general sense, what is not clear from the coaching and research literature is how, in mechanical terms, each component of technique contributes to the performance. Without such general knowledge it is extremely difficult to establish how these more subtle differences influence performance.

At present, there is little coaching literature which attempts to address these uncertainties. Since many ideas which coaches have concerning gymnastic techniques are passed on from coach to coach within the gymnastics club, it is conceivable that few coaches know the reasons for encouraging their gymnasts to use a particular technique. Without mechanical tenets endorsing their suggested techniques or explanations, it is likely that they are based simply on trial and error, intuitive guesses and the personal experiences of other coaches (Yeadon, 1984).

From the research literature differences in technique between elite and non-elite gymnasts for backward longswings have been identified (Nissinen, 1983; Brüggemann, 1987). The major distinction in techniques constituted differences in the timing of changes in the gymnast's configuration at the hips and shoulder during the
swing beneath the rings. However, how these differences contribute to the performance was not established. Furthermore the descriptive, experimental and theoretical studies produced to date have all reduced the three-dimensional nature of swinging activities on rings into two dimensions. This means the contribution of lateral arm movements to performance, an aspect of technique used by many elite gymnasts, has been totally neglected. Such a reduction in complexity may not be acceptable and may reduce the validity of the findings.

Much of the descriptive research on rings has focused upon the patterns of force in the rings cables and experienced by gymnasts when swinging on rings. Even with elastic structures incorporated into the apparatus, as directed by the F.I.G., forces as large as 4.5 bodyweights are experienced by gymnasts at each shoulder joint. These excessive peak forces have been linked to an increased potential for injury to the shoulder joints of the gymnasts (Caraffa, Cerulli, Rizzo, Boumpade, Appoggetti & Fortuna, 1996; Nissinen, 1995). Suggestions regarding how these peak forces may be attenuated have been made. These tend to include increasing the elasticity of the rings apparatus (Caraffa et al., 1996). However, without knowing how much the present elastic mechanisms incorporated into the rings apparatus actually decrease peak forces it may be difficult to determine how elastic the apparatus should be. Furthermore, from the literature the implications of significantly altering the elasticity of the rings apparatus on the gymnasts techniques are not well understood.

It would seem that at present relatively little is understood about the mechanics underlying a gymnast swinging on rings. This suggestion may be substantiated by the lack of studies of swinging on rings in the most recent of reviews concerning gymnastics techniques (Brüggemann, 1994).

1.3 Statement of purpose

The purpose of this study is summarised in the following statement:

To increase the scientific knowledge and understanding of the mechanical factors which contribute to the performance of longswings on rings in Men's Artistic Gymnastics.

In order to provide a focus for the study the backward and forward longswings are used as examples of swinging elements. Mechanical factors which influence the performance of longswings on rings may include the technique adopted by the gymnast and the type of rings apparatus on which he is swinging. By addressing the specific questions posed in the following section it is intended that a greater knowledge of what influences performances of longswings on rings is gained.
1.4 Research questions

The following questions are pertinent to increasing the knowledge of the mechanics behind a gymnast swinging on rings. A résumé of the current knowledge relevant to each question is also provided.

Question 1
What inherent problems are faced by a gymnast swinging on rings?

Smith (1982) highlighted that the rings cables place a constraint upon the motion of the gymnast. It was suggested that this constraint effectively forces the mass centre of the gymnast to move in a vertical direction only. However, is this relatively simple proposal of Smith (1982) accurate? In addition, how does the constraint of the rings cables affect the motion and forces experienced by the gymnast during a swing from handstand?

Question 2
Do the elastic structures specifically incorporated into the rings apparatus by manufacturers of gymnastics equipment reduce the peak forces experienced by gymnasts?

The extremely large forces produced during backward and forward longswings on rings are transmitted to the shoulder joints of the gymnast via the rings cables (Brüggemann, 1987). Tentative evidence linking the magnitude of tension and rate of tension development in the rings cables to the injury potential of the shoulder joints have been put forward (Brüggemann, 1987; Gielo-Perczak, 1991; Nissinen, 1995). Such a potential for injury resulted in directives from the F.I.G. stipulating that the apparatus must possess some elasticity (F.I.G., 1989). Since the amount and type of elasticity is not specified, manufacturers of gymnastic equipment have utilised different strategies to introduce this elasticity. However, the effectiveness of these structures in reducing the peak forces experienced by gymnasts when performing longswings on rings has not been determined.

Question 3
Can changes in a gymnast's body configuration while swinging underneath the rings reduce the peak forces experienced by the gymnast? If they do, what is the mechanics behind the reduction in force?

When swinging beneath the rings during a backward longswing rapid reductions in angles at the hip and shoulder joints of a gymnast from an arched configuration have been identified in coaching and research literature as being essential for a proficient
performance (Hesson, 1975; Kormann, 1984; Nissinen, 1983; Brüggemann, 1987). However, Hiley (1998) has alluded to the possibility that during giant circles on the horizontal bar similar joint actions may serve to reduce peak reaction forces. Therefore, can a gymnast performing on rings reduce the peak forces he experiences by changing his body configuration through the bottom of the swing?

**Question 4**

Can a gymnast produce a backward longswing to still handstand on rings? What is the mechanics explaining the production of the longswing?

Using a computer simulation model, which represented a gymnast by three rigid linked segments, Sprigings, Lanovz, Watson & Russell (1998) showed that from an initial handstand with swing the angular motion in the final handstand position could be reduced by an appropriately timed backward longswing. They also highlighted that using their modelling approach, which uses joint torques as input, the gymnast could not produce a motionless final handstand. It would be advantageous to investigate this finding, though using joint angle changes as input to describe the technique of the modelled gymnast.

**Question 5**

What techniques are actually used by elite gymnasts to perform backward and forward longswings?

At present few descriptions of a gymnast's technique when performing backward and forward longswings on rings have been documented. Indeed, descriptions which have been detailed are either qualitative in format (Hesson, 1975; Kormann, 1984) or restricted to planar quantitative descriptions (Nissinen, 1983; Brüggemann, 1987). Hence, detailed quantitative descriptions of the three-dimensional kinematics describing techniques used for both types of longswing are needed if a greater understanding of swinging on rings is to be gained.

**Question 6**

Do any other structures of the rings apparatus possess elastic properties? What influence does the overall elasticity of the rings apparatus have on longswing performance and peak forces experienced by the gymnasts?

Material from which the rings apparatus is manufactured include metal, leather and synthetic webbing. Therefore, besides the elasticity incorporated into the apparatus as a result of the F.I.G. directives (Figure 2.2), it is possible these other structures possess elastic properties. Knowledge of which of these structures of the apparatus exhibit these
elastic properties, and how they combine with the intentional elastic structures, is essential to gain a more complete understanding of what factors affect longswing performance. Furthermore, the elastic components which a gymnast possesses must be considered (Hiley, 1998). Therefore the question arises as to what effect the elasticity of the gymnast has on the peak forces he experiences.

Exact details describing where and how the elasticity should be incorporated into the rings apparatus is not specified by the F.I.G.. This open ended recommendation has therefore led to several different designs of rings apparatus. In a recent venture Gymnova, manufacturers of gymnastics equipment, developed a structure which enables the rings apparatus to possess variable amounts of elasticity. Such variability in the elasticity of the apparatus may be expected to influence the performance of swinging elements and forces experienced by the gymnast. Therefore, if a gymnast maintains his technique when swinging on two sets of rings apparatus which possess different amounts of elasticity will his performance be the same?

**Question 7**

What are the relative contributions of actions at the knee, hip and shoulder joints to the performance of backward and forward longswings?

During both backward and forward longswing performances gymnasts exhibit changes in joint angles at their knee, hip and shoulder joints throughout the performance. In addition, gymnasts use extensive lateral movements of the arms, which is a characteristic unique to swinging on rings. At present it is not known which components of technique are essential to the performance, which are less important, or how each component contributes to the longswing performance.

**Question 8**

Can two different gymnasts adopt exactly the same techniques for backward and forward longswings and produce proficient performances for each longswing?

"Copying the champion" is a method often adopted by gymnastics coaches to determine the technique which they should teach their gymnasts for a particular skill. However, it is quite possible that differences in the inertial characteristics of two gymnasts mean different techniques need to be adopted by the gymnasts. If this is found to be the case, it may reduce the usefulness of such a coaching practice.
Question 9

How critical is the coordination of the joint actions which constitute a gymnast's technique to his overall longswing performance?

By comparing elite and non-elite gymnasts performing backward longswings Nissinen (1983) and Brüggemann (1987) identified the importance of coordinated joint angle changes at the hip and shoulder joints to the performance. However, what remained unresolved was how critical the timing of these coordinated joint angle changes was in producing a proficient performance. Hence, the amount by which a gymnast may mistime these joint actions and still produce a longswing to handstand has not been determined.

Summary

The questions posed will be addressed through a combination of experimental and theoretical approaches. By reducing the complexity of a situation, fundamental concepts are often emphasised and may be more easily observed and explained (Yeadon & Challis, 1994; Alexander, 1990). Hence, simple two-dimensional simulation models of a gymnast swinging on rings will be used to address Questions 1 to 4 and provide an insight into the basic tenets governing swinging on rings.

In order to address Question 5 kinetic and kinematic data from actual performances will be collected and utilised.

Questions 6 to 9 will be addressed using a three-dimensional simulation model which more realistically models the gymnast and rings apparatus. It is expected that a more realistic simulation model will be able to determine more accurately the contributions of selected components of technique to the longswing performances.

1.5 Organisation of the chapters

Chapter 1 provides a general overview of the area to be studied, including a brief introduction to the reasons why backward and forward longswing elements are prolific in the rings routines of elite gymnasts in Men's Artistic Gymnastics. Within this chapter the questions to be addressed within the scope of this thesis are presented.

Chapter 2 reviews critically the coaching and research literature concerned with aspects of swinging on rings, paying particular interest to literature concerned with forward and backward longswings. A more in-depth examination regarding the rings apparatus and techniques used by gymnasts to perform both longswing types is provided. Secondary questions to be addressed, supplementing those already established, are also
posed. Finally this chapter reviews the techniques of investigation which may be considered applicable and proposes those which are best suited to this particular study.

Descriptions of all computer simulation models are presented in Chapter 3 together with the methods used to derive the equations of motion. A brief introduction to Kane's method for formulating the equations of motion and the dynamics software package AUTOLEV™3, which uses this method, are provided. This software package was used to produce the three-dimensional simulation model of a gymnast swinging on rings.

Chapter 4 details the procedure used to acquire kinetic and kinematic data for backward and forward longswings on rings. The techniques for analysing the image and force data are presented together with the methods used to estimate model parameters for the three-dimensional simulation model.

Results and brief conclusions concerning the two-dimensional simulation models are presented in Chapter 5. This chapter also presents the results from the data collection and establishes full three-dimensional kinetic and kinematic descriptions of techniques used by the gymnast to perform the longswings on rings. Results relevant to estimates of model parameters for the three-dimensional model are also provided.

Chapter 6 provides the procedure for the evaluation of the four segment three-dimensional simulation model. The results of this procedure indicate the accuracy of the model when simulating a gymnast swinging on rings.

Within Chapter 7 the three-dimensional simulation model is used to address the questions posed in the first two chapters. Conclusions drawn from the simulations used to address each question are also provided.

Chapter 8 summarises the findings in relation to specific questions posed within the thesis. Possible applications of the four segment three-dimensional simulation model to further research are proposed.
CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

This chapter provides brief descriptions depicting the development of the modern sport of artistic gymnastics and its judging criteria, together with an overview of the modern rings apparatus. Subsequent to this, coaching descriptions and mechanical explanations of both backward and forward longswings on the rings apparatus are given.

A review of research studies investigating gymnasts swinging on rings provides an overview of the current understanding in this area.

Finally, the techniques of investigation applicable for this study are reviewed. The advantages and disadvantages of each technique are discussed and reasons for using the techniques employed in this study are established.

2.2 The development of gymnastics and the rings apparatus

2.2.1 A brief history of Men's Artistic Gymnastics

From the possible origins of gymnastics 2600 years B.C. in China and Egypt, and the subsequent extensive use in Ancient Greece, the development of Artistic Gymnastics to its modern competitive form is well documented (Munrow 1963; Tatlow, 1978; Goodbody, 1982; Aykroyd, 1987). Deriving its name from the greek word gymnos, meaning naked, the role of gymnastics in Ancient Greece was in the training of male athletes for other sports or simply as a form of physical exercise. It is clear that vast changes and developments have occurred since these two periods of history in order for gymnastics to have evolved into the modern competitive sport it is today.

Probably the greatest instigators in the development of the modern Olympic sport of Artistic Gymnastics were the German educationalist Jahann Friedrich Jahn (1778-1852) and the Swedish academic Pehr Henrik Ling (1766-1839) (Goodbody, 1982). Jahn has been credited for the invention of the horizontal bar, parallel bars and pommel horse which now form the basis of the men's apparatus, while Ling was responsible for developing the vaulting horse (Loken, 1959). The contributions of other influential philosophers, educationalists and politicians to gymnastics, have also been discussed (Munrow, 1963; Goodbody, 1982; I.O.C., 1985, Aykroyd, 1987).

The sport of Artistic Gymnastics has been included in every modern Olympic
Games. However, during the first Games in 1896 the sport was quite different from its present form. There was, for example, no women's competition while other disciplines, such as rope climbing and standing long and high jumps, were included in the men's competition. This non-standardised form of competitive gymnastics continued up to and including the 1948 Olympic Games in London.

Only since the 1952 Olympic Games in Helsinki has the present format of Artistic Gymnastics been defined and used. At these Games male and female gymnasts competed for the first time on apparatus similar to those at present day competitions. Apparatus for men, as they are now, were: floor, pommel horse, rings, vault, parallel bars and horizontal bar and for women; vault, asymmetric bars, beam and floor. It was also for these Games that the international governing body of gymnastics, the Fédération Internationale de Gymnastique (F.I.G.), implemented rules regarding the standardisation of apparatus at national and international competitions. Furthermore, judging criteria for each apparatus, where each element was categorised depending on its technical difficulty, were described and used. Together, these changes mean this competition can be described as the first present day Artistic Gymnastics competition (Aykroyd, 1987).

Since the 1952 Games two changes to the format of Men's Artistic Gymnastics are apparent: technological advances improving the structure of each apparatus and the development and refining of the judging criteria. As gymnasts have improved their performances, through more specific training and coaching techniques, gymnastic elements that were once thought of as difficult have become the norm in the routines of elite gymnasts. This increased skill of gymnasts has been dealt with by the appropriate modifications to the judging criteria. In order to differentiate between good gymnasts and those who excel at the sport the judging criteria, documented in the Code of Points, were developed. Revisions to the Code of Points have generally been made every four years since 1964 (F.I.G., 1997). Each new revision has attempted to reflect the increased skill of elite gymnasts and to allow differentiation between standards of gymnasts.

The most recent judging criteria were issued by the F.I.G. in 1997. At the elite standard any routine in Men's Artistic Gymnastics is evaluated from a maximum of 10.00 points. The most recent Code of Points evaluates all recognised gymnastic elements on all apparatus by how technically difficult each element is to execute. The technically simple elements, termed 'A' elements, are given the lowest mark, while the most technically difficult elements, termed 'E', are awarded the highest. The performance of 'E' elements is not mandatory in all competitions.

Evaluation factors within a routine comprise four areas: the summation of the difficulty of elements performed, the technical presentation of the routine, special requirements for each particular apparatus and lastly bonus points (F.I.G., 1997). Of the maximum points a gymnast may be awarded, 50% are concerned with the technical presentation of the routine in terms of the execution (or technique) and body posture (or
aesthetics) of each performed element. A total of 24% of the points that may be awarded are concerned with the overall difficulty of the routine, while 12% of the points are for the execution of three compulsory requirements, which are specific to each apparatus. The final 14% of points that may be awarded are deemed bonus points. These points are only awarded if the gymnast performs the high difficulty 'E' elements with correct technical execution, or combinations of high difficulty elements (F.I.G., 1997).

During the rings routine of an elite gymnast swinging, strength and held elements must be performed in approximately equal portions. While performing held elements the rings must be still, or points are deducted. In addition straight arms must predominate throughout the routine (F.I.G., 1997).

Compulsory requirements of a rings routine include a static strength element of 'B' difficulty and swings to held handstand from backward and forward swings. The backward and forward straight arm longswings fulfil the swinging requirements. As these elements are classed as 'C' difficulty, they also aid the gymnast in scoring highly. Consequently these elements are performed by the majority of elite gymnasts during their rings routines. As these two elements are fundamental to elite performances it may be beneficial for the gymnasts and coaches to know which of the various components of a gymnast's technique are crucial to performance and which are less important. Such knowledge may produce better performances by focusing on those aspects which are of greatest importance. This knowledge may be gained using mechanical principles and scientific method, instead of relying on intuition, experience or trial and error, which are often used by gymnastics coaches.

2.2.2 Development of the rings apparatus

The origin of the rings apparatus lies in the circus, where in the 18th century Francis Amores of Spain developed the trapeze bar. By the mid 19th century the trapeze bar had been developed into two separate triangular stirrups connected to individual ropes, each rope being attached to high frame or ceiling. This description bears some resemblance to the modern apparatus, though this apparatus was still used as a form of trapeze with gymnastic elements performed with swinging rings (Loken, 1959).

By the early 20th century the structure of the rings apparatus had developed to one which was similar to its modern counterpart, though during the first half of the 20th century the rings apparatus was used predominantly for static strength (Hunn, 1978).

**Structure of the modern rings apparatus**

Prior to the 1952 Olympic Games there were no specifications regarding the design,
structure and mechanical properties of any of the gymnastics apparatus. Since these Games, and the introduction of guidelines from the F.I.G., the only major modifications to the rings apparatus has been the materials used for its construction.

The material used for the modern rings frame is a metal alloy. Although the shape of the frame is determined by the manufacturers, each must adhere to several dimensions defined by the F.I.G. (1989) (Figure 2.1).

Each rings cable must be 300 cm in length, which includes a 70 cm strap made of leather or an equivalent synthetic material and a ring. The inner diameter of the rings are 18 cm, with a 2.8 cm diameter of profile. The inner edge of the lowest part of each ring must be 255 cm above the top of a 20 cm thick landing mat placed beneath the rings. The two rings cables are placed at a fixed distance of 50 cm apart when the cables are vertical. Each rings cable is suspended from the frame using a pivoting mechanism. At each suspension point the pivoting mechanism must allow the rings cables to move freely in all directions and to rotate about their long axis. These properties of the modern rings apparatus allow the gymnast to perform the various complex swinging activities without the rings cables restricting the motion of the gymnast.

![Figure 2.1. The modern rings apparatus as specified by the F.I.G. (1989).](image)

During swinging elements large forces are experienced by the gymnast's joints, especially the shoulders (Nissinen, 1995). With the increase in the amplitude of swinging
elements during the 1970s an F.I.G. directive was introduced, stating that the rings apparatus must possess some elasticity. The engineered elasticity of the rings apparatus was introduced in an attempt to decrease the large forces experienced by the gymnasts and hence reduce the risk of injury when performing the swinging elements.

In a modern rings apparatus the elasticity is evident in various structures. Firstly the horizontal beam of the frame is designed to allow some deformation (Figure 2.1). A second more obvious source of introduced elasticity is above the suspension points of the rings cables (Figure 2.2). Although manufacturers of gymnastic equipment engineer this feature in a variety of ways a standard technique is shown in Figure 2.2, providing a general representation of the method.

![Diagram of damped elastic device (DED)](image)

Figure 2.2. The rings cable and damped elastic device (DED).

The pivoting mechanism for each rings cable passes through the horizontal beam of the frame and is secured by a set of lock-nuts. Placed in series between the lock-nuts and the top of the rings frame are a set of curved metal disc springs. These disc springs deform when the rings cable is placed under tension, effectively acting to dampen transient forces.

The rings cables, inclusive of these damped elastic devices, must behave in an elastic manner. This property is tested by statically loading and unloading both rings cables with a weight of 4000 N. For the rings cables to exhibit elastic properties no permanent deformation should be observed after the unloading phase (F.I.G., 1989). However, the maximum deformation produced by the load is not specified by the F.I.G., meaning the mechanical parameter of stiffness is not specified. The lack of a specified
deformation under the load means the test is of limited usefulness if specific values are required for standardising the elasticity of competition apparatus. Furthermore, the damping characteristics of the elastic properties cannot be estimated by static testing, further reducing any standardisation between different rings frames. It may therefore be useful if tests were designed which enabled the mechanical properties of stiffness and damping for the rings frame to be established, aiding in the standardisation of apparatus.

Although two structures possessing elastic properties are named, it is possible that other less obvious structures in the rings apparatus possess similar qualities. For example, the rings cables themselves may stretch, dissipating or storing energy, or the whole rings frame may move, acting to dampen the large forces experienced by the gymnast. These suggestions will be investigated within this study.

The apparatus regulations also stipulate that the elasticity of the rings apparatus should only facilitate a reduction of the risk of injury and not assist the gymnast in performing the swinging elements through the production of 'springy' or 'counter' swings (F.I.G., 1989). However, as there are no values specifying the elasticity of the rings apparatus at present, each rings frame may possess different elastic properties. This raises questions concerning the effect of such differences for a gymnast. For instance, how does the elasticity of the apparatus affect performance? Would a gymnast have to significantly alter his technique depending on the make of the rings apparatus he is performing on?

Technological advances in gymnastics equipment and the response to the problem of large forces experienced by the gymnasts are highlighted by the most recent rings frame design produced by the manufacturers Gymnova (Figure 2.3).

![Diagram of rings apparatus](image)

Figure 2.3. The top of the rings apparatus with variable stiffness properties, made by Gymnova, manufacturers of gymnastics equipment.
This design allows the rings apparatus to display variable elastic properties. Instead of the rings cables passing through the horizontal beam of the frame (Figure 2.1), they are attached to a flexible fibreglass bar positioned beneath the beam. By varying the apparent 'working' length of the flexible bar through altering the position of 'silint-blocks', the elasticity displayed by the bar can be varied from 'stiff' (position a) to 'flexible' (position b).

Such developments in the apparatus are, in part, due to the recognition of the large forces experienced by the gymnast and the increased risk of injury to the gymnast. How much elasticity, however, is required to make the forces experienced by a gymnast swinging on rings safe, or equivalent to those experienced on horizontal bar? In addition such alterations to the mechanical properties of the apparatus may have implications for the techniques adopted by gymnasts for longswing performances.

2.3 Descriptions of longswings on rings

At the turn of the 20th century, the modern rings were used to display the strength of the gymnasts by performing static strength elements such as the crucifix (Kaneko, 1976; Hunn, 1978). This trend remained until the 1952 Olympic Games in Helsinki where swinging elements were performed during the same routines as static strength elements, especially in the routines of the U.S.S.R. gymnasts (Kaneko, 1976). Since this introduction the F.I.G. has made swinging elements compulsory in rings routines. To comply with the most recent Code a gymnast must perform backward and forward swings to a 2 second held handstand. Both the backward and forward longswings fulfil these criteria, whilst exhibiting the largest amplitude of swing on rings by starting and finishing in held handstand positions. These elements have also been named the wheel, giant swing or the felge to handstand from handstand.

2.3.1 Descriptions of the longswings

The backward longswing

Whichever specific technique is employed by a gymnast performing the backward longswing on rings, it will resemble those described by Fukushima & Russell (1980), Hesson (1975) and Kormann (1984). A typical backward longswing technique, where the arms remain straight throughout the element, is shown graphically in Figure 2.4.

The gymnast starts in a stationary handstand position with the rings cables motionless (Figure 2.4a). From this position the gymnast initiates the rotation and
descent, anticlockwise as shown, by moving his mass centre away from the point of support over his hands. Consequently the rings rotate forward of the gymnast (Figure 2.4b).

During the descending phase the gymnast typically arches, displaying angles greater than $180^\circ$ at the hip and shoulder joints (Figure 2.4b-c). Often during the decent the arms are also moved laterally, away from the mid-line of his body (Figure 2.4b). The extent of the lateral arm movements depends on the technique of the individual gymnast.

![Figure 2.4. The backward longswing on rings.](image)

The gymnast swings through the hang position, during which an extended configuration (Figure 2.4d) is formed from the previously arched configuration (Figure 2.4c). As the gymnast passes through the hang position the rings cables are near vertical, with the arms near parallel, directly above the gymnasts head.

The ascending phase of the swing then follows (Figure 2.4d-h). During the first part of this phase the gymnast assumes a piked body configuration (Figure 2.4e) by continuing to reduce the angles at the hips and shoulders joints from the descending phase. Any reduction in the angles at the hip and shoulder joints ceases soon after the hips of the gymnast are at the same vertical level as the rings (Figure 2.4f). The exact timing of changes to the gymnast's configuration depends on the technique of the gymnast, which in turn may depend on his coach. During the swing through hang and early part of the ascending phase the rings cables exhibit a rotational motion which is opposite in direction to the gymnast, in a clockwise direction in Figure 2.4c-f.

During the latter stages of the ascending phase the angles at the hip and shoulder joints start to increase, and generally a slight hyper-extension at the hip joints is shown (Figure 2.4f-g). Furthermore, during this part of the ascending phase a gymnast may move his arms laterally, resulting in lateral motion of the rings cables (Figure 2.4g). Once again the extent of the lateral arm movements depends on the particular gymnast.

The backward longswing is completed when the final handstand position is attained.
(Figure 2.4h). Ultimately, the rings cables should be stationary when this final position is reached.

With changes in the angles at each joint viewed as separate components of technique this simple description raises several issues concerning how a backward longswing is performed. For instance, although changes to the hip and shoulder angles occur during both phases of the swing, how does each component contribute to the overall performance? Are the actions at the hip joints less important than those at the shoulders for a proficient performance? Many coaches advocate a straight body during the ascending phase, yet personal observation indicates that an arching of the body occurs. Thus, what effect does this arching have on the performance or the loading of the gymnast?

In relation to the lateral arm movements Kormann (1984) advocates parallel arms during the descending phase, while gymnast’s often exhibit lateral arm movements. What are the effects of the lateral arm movements during the descending phase on both the performance and the forces experienced at the hands of the gymnast? In reference to the ascent of the longswing, why do gymnasts use lateral arm movements during this phase?

The forward longswing

The following description provides an overview of a typical forward longswing performed on rings, shown graphically in Figure 2.5.

The element starts with the gymnast in a stationary handstand position with the rings still (Figure 2.5a). From this position the gymnast initiates the swing, clockwise in this illustration, by moving his mass centre away from the point of support over his hands. During the first portion of the descending phase the rings cables rotate backwards of the gymnast (Figure 2.5b) in the same angular direction.

Figure 2.5. The forward longswing on rings.
During the descending phase the gymnast typically assumes a dished configuration, displaying angles less than 180° at the hip and shoulder joints (Figure 2.5b-c). Similar to techniques used for backward longswings lateral arm movements occur during this phase (Figure 2.5b), although the extent of these movements is dependent on the individual. During the second half of the descent the rings cables start to rotate in the opposite direction to the gymnast, anticlockwise as shown in Figure 2.5c-d.

While swinging through the hang position the gymnast forms an extended configuration (Figure 2.5d), during which the rings cables rotate to a near vertical orientation, with the arms near parallel above the gymnast's head.

For forward longswings the most significant changes to the gymnast's configuration are made during the ascending phase (Figure 2.5d-h). Firstly, the gymnast adopts an arched configuration (Figure 2.5e), while simultaneously moving the arms laterally. These lateral arm movements are increased, circling the arms to the sides of the body, as the ascent and rotation of the gymnast continues (Figure 2.5e-g). The timing of these components of technique will differ between elite gymnasts, though all seem to use this aspect of technique. In addition, these lateral movements of the arms result in lateral movements of the rings cables, increasing the complexity of any mechanical analysis.

During the final stages of the ascent the arms are brought in front of the body. As this takes place the arch in the body is reduced in preparation for the final handstand position (Figure 2.5h). On attaining the handstand the rings cables and gymnast should be static.

With regards to technique and forward longswings, it seems that all elite gymnasts use extensive lateral movements of the arms. However it is not obvious, in mechanical terms, how this component of technique benefits the gymnast. Besides the lateral movements of the arms being of importance, the relative contribution of the other components of technique to the performance are also unknown.

Presently, answers to questions regarding the contribution of each component forming a gymnast's technique for both backward and forward longswings are few in number. Of those explanations provided, few have included underlying mechanical explanations, being more often than not, simply unsupported speculations.

2.3.2 Mechanical concepts and longswings on rings

Descriptions in coaching texts involving mechanical concepts for backward and forward longswing are sparse. Even more scarce are mechanical explanations describing to what extent and how each component of technique contributes to performances of longswings.

and Readhead (1987) have either identified, or inferred, that a gymnast swinging on rings may be simplified into a double pendulum (Figure 2.6). Although simplistic, knowledge of the interaction between the two objects of a physical double pendulum may give rise to a better mechanical understanding of swinging on rings.

![Diagram of a double pendulum and a gymnast on rings](image)

Figure 2.6. A physical double pendulum and a gymnast on rings (Adapted from Smith & Smith, 1990).

For a double pendulum the motion of each object is dependent on the physical properties of both objects (Smith & Smith, 1990). Therefore, with the gymnast and rings cables effectively acting as a double pendulum their motions are dependent on each other. Physical properties of each object which alter these interactions are the mass, mass centre location and principal moments of inertia. If any of these physical properties of either object varies the interaction between the two objects will also alter.

With the dimensions of the rings apparatus specified by the F.I.G. the physical properties of the rings cables should not differ greatly between apparatus. The physical properties of gymnasts, however, certainly varies between gymnasts. These differences in the physical profiles may increase the difficulty in predicting the overall motion of a gymnast. Although the coaching texts insinuated this theory behind swinging on rings, the influence of the physical properties of the rings cables and gymnasts were not clarified. By determining appropriate values for the physical properties of the gymnasts and cables, such interactions may be established, and with it an increased mechanical understanding of swinging on rings.

These same coaching texts also speculate that the path of a gymnast's centre of mass
during swinging elements on rings will move only vertically. When simplified to a double pendulum, this result may or may not be close to reality. Unfortunately, in reality a gymnast also alters his configuration while swinging on rings, especially at the hip and shoulder joints. This further increases the complexity of the swing. Owing to these changes in body shape it would seem likely that the double pendulum theory can only offer some explanation of the mechanics behind swinging on rings.

The lateral arm movements made by gymnasts, especially prevalent during both types of longswing, further increases the complexity of any mechanical descriptions. Very few coaching texts attempt to describe the mechanical benefits of these lateral arm movements and hence the contributions of these arm movements to performance are unknown. These lateral movements also nullify the assumption that motion is only in a vertical plane, meaning a three-dimensional analysis is required.

At present it is clear that from a mechanical vantage point there is a lack of an understanding regarding what contributes to either a backward or forward longswing performance. The description of the gymnast on rings as a double pendulum probably forms the closest representations in coaching texts of mechanical descriptions of swinging on rings. It may also provide a starting point for a mechanical investigation.

### 2.4 Research concerned with swinging on rings

Historically, two approaches to research in performance-related sports biomechanics are apparent: experimental and theoretical. Both, however, are essential parts of scientific method, as presented by Yeadon & Challis (1994) (Figure 2.7).

#### 2.4.1 Scientific method and approaches to research in performance-related sports biomechanics

Investigations within sports biomechanics typically start with a descriptive study. These studies are often exploratory in nature, aiming to present a detailed and accurate record of the chosen event. An illustration of this first investigation may be a kinematic description of a technique used by a gymnast during a backward longswing on rings. This study forms the initial stage of the theory-experimental cycle of scientific method (Yeadon & Challis, 1994).

From this initial description a theory may be put forward which predicts a performance for a different situation. For instance, one theory might be that if a gymnast does not use lateral arm movements during a longswing the overall performance will be adversely affected. Experiments may then be performed to evaluate this prediction.
Having obtained the experimental result, a comparison between this result and the theoretical prediction can be made. This comparison procedure has a dual role. It allows the accuracy of the theory to be estimated as well as an evaluation of the theory to be made. If the theory is shown to be inadequate, refinements may be made, effectively providing a revised theory. In the same way as the original theory was tested, the revised theory must undergo the same procedure of theoretical prediction and comparison to the experimental result. It is this refinement of a theory, and its subsequent testing, which forms the cycle of scientific method.

![Diagram of the theory-experimental cycle of scientific method](Adapted from Yeadon & Challis, 1994).

Whether a classical experimental or theoretical approach to an investigation is chosen may depend on the purpose of the study. However, both approaches have their own limitations and strengths. Often the limitations of an experimental approach are the strengths of a theoretical study, and vice versa.

To illustrate this, problems associated with the internal and external validity for both approaches are briefly reviewed. Internal validity within an experiment relates to how much of the resulting change in the dependent variable can be credited solely to changes in the independent variable. External validity, also termed ecological validity, is concerned with how applicable the experimental results are to the actual situation.

**Internal validity**

Typically, during experimental studies involving human subjects it is not possible to have complete control over all variables, known or unknown (Yeadon, 1987). For instance, when asked to alter a specific part of technique during a longswing, such as the lateral arm movements, a gymnast may also change other aspects, like the actions at the hips. Effectively, this means that while the independent variable has been altered, other secondary and extraneous variables have also been changed. The dependent variable, in this case the resulting performance, may therefore be ascribed to both changes in technique, while the actual influence of the lateral arm movements on performance...
remains unclear.

Greater control in an experimental study may be achieved by the careful selection of the independent and dependent variables, though secondary and extraneous variables will still intervene. Kerwin, Harwood & Yeadon, (1993), for example, used this approach for data obtained in competition to analyse different hand placement techniques in vaulting and evaluate their effects on post-contact flight variables from the vaulting horse.

In contrast, within a theoretical setting, such as a simulation model, all variables may be defined, providing maximum internal validity. Through the complete control of the independent variable, the outcome of the dependent variable can be credited solely to the independent variable. This allows a direct cause and effect relationship between a variation in technique and an outcome in performance to be established (Yeadon, 1987; Yeadon, Atha & Hales, 1990).

In section 2.4.2 it will become apparent that experimental research of swinging on rings using real gymnasts often halts at the descriptive stage of scientific method. Problems relating to direct intervention and controlling secondary and extraneous variables during experiments may constitute the largest reason for this.

**External validity**

Similar to experimental studies theoretical ones have limitations to overcome. One such limitation is the external validity of the study, or how the study applies to reality.

In all cases within performance-related sports biomechanics the theoretical representation or model of the sporting activity is less complex than reality. This reduced complexity has the advantage of making the activity more manageable to comprehend, but is also based upon a series of assumptions, which may or may not be valid. It is the validity of the assumptions which determines the external validity of the model. If the difference between reality and the theoretical situation is great the external validity of the study is reduced. Hence, any conclusions and generalisations made from the theoretical model of the sporting activity may be inappropriate when transferred to reality.

Attempts must therefore be made to ensure a high external validity in theoretical studies. By comparing a prediction from a theoretical model for a particular situation to a description of the same situation the model is evaluated (Yeadon & Challis, 1994). Similarities between the predicted and actual performances can be determined. From this evaluation modifications to the model can be made until a satisfactory level of difference between the theoretical prediction and the actual event is observed. In essence this stage forms part of scientific method, where the theory here is that the model represents reality. Yeadon & Challis (1994) noted that it is this stage of theoretical studies which is most often omitted and which invariably decreases the applicability of a study's findings to reality.
Within this study a theoretical approach, in the form of a simulation model, is adopted to address the research questions posed in Chapter 1. The model is evaluated and the external validity, assessed, as suggested by Yeadon & Challis (1994). Hence, the results obtained for a gymnast swinging on rings using the evaluated simulation model should be similar to those obtained if an actual gymnast performed the same technique.

2.4.2 Descriptive and experimental research

*Studies of longswings on rings*

Biomechanical analyses have investigated elements performed by gymnasts in static positions (Rozin, 1973), while swinging (Nissinen, 1983, 1995) as well as dismounting from the apparatus (Yeadon, 1994a, b). Although often termed experimental the majority of research concerned with gymnasts swinging on rings has been descriptive in nature, forming the first stage in the scientific method (Figure 2.7).

Biomechanical studies of swinging elements on rings date back to 1968. Minor studies concentrating on longswings include those by Peek (1968), Fetz & Opavski (1968) and Dusenbury (1968) (op. cit. Brüggemann, 1994). These early investigations provided indications of the large forces experienced by a gymnast, the orientation of a gymnast at peak values of these forces and tension in the rings cables throughout the elements. In the study by Dusenbury (1968), for example, two-dimensional inverse dynamics analyses were used to estimate the peak combined tension in the rings cables during both backward and forward longswings. Peak values of between 6.0 and 9.0 bodyweights were calculated for both longswings.

The techniques used by gymnasts to perform longswings during the 1960s and early 1970s are different to those used by modern gymnasts. Gymnasts used bent arms when performing backward longswings, which in present day competitions result in marking deductions and as a consequence this technique has become obsolete in the modern sport. Descriptions of gymnastics techniques and conclusions drawn from these early studies are therefore of limited use to the study of modern longswings.

A common theme of these early studies is the reduction of the analysis into a single vertical plane. Such an analysisneglects the lateral movements of the arms evident in both types of longswings (Figures 2.4 and 2.5), meaning full three-dimensional descriptions of gymnasts' techniques were not produced. More significant, however, is the apparent lack of efforts in more recent studies to overcome this inadequacy. For both backward and forward longswings full three-dimensional descriptions of techniques therefore remains undocumented. Producing meaningful descriptions of these longswings forms one of the aims of this study (Chapter 1).
Synchronisation of measured cable tension and gymnastic techniques was first performed in a study of the shoot-to-handstand (Sale & Judd, 1974). The shoot-to-handstand element starts in an inverted pike position. From this initial position the gymnast performs a kipping action while circling the arms to the front of the body as the body arrives in a near horizontal orientation. From this orientation the swing is similar to a backward longswing, though bent arms are used during the ascending phase.

Tension in the rings cables was indirectly measured by a statically calibrated load cell placed in series within one rings cable. Performances of the shoot-to-handstand were filmed from a lateral view. Synchronisation of these data was achieved by filming an oscilloscope trace of the measured strain onto the film recording the gymnasts' techniques.

Peak values of 5.0 bodyweights, occurring as the gymnast passed through the lowest phase of the swing, were recorded for the combined cable tension during the shoot-to-handstand. Differences in peak cable tension between this study and that of Dusenbury (1968) may be explained by the positions of the gymnasts at the start of the respective elements. In the handstand position a gymnast possesses more potential energy than in the near static position attained after the kipping action of the shoot-to-handstand. With greater potential energy in the handstand position, more energy may be transferred into kinetic energy. Consequently a swing from handstand would be expected to produce a greater angular velocity at the bottom of the swing, which may result in larger peak cable tension values. Comparison of results from these two studies seem to substantiate this theory.

Sale & Judd (1974, 1976) also attempted to identify key components of technique which produced a superior performance. It was concluded that a superior performance required a large downswing, a forceful hip flexion and shoulder extension early in the upswing, followed soon after by a forceful hip extension. Since these aspects of technique were associated with larger cable tension the conclusions suggest that the best shoot-to-handstand would be produced when tension in the rings cables was maximised at these phases of the swing. This conclusion would seem unlikely for several reasons. At its extreme, the best shoot-to-handstand would be produced when peak cable tension was as large as the gymnast could tolerate, which would probably increase the risk of injury to the gymnast.

As straight arm longswings became more prevalent during the 1970s investigations were undertaken to describe the differences between the technically more difficult straight arm technique and the outdated bent arm versions. Using cinematography and dynamometry Valliere (1976) observed that straight arm backward longswings produced a greater combined peak cable tension. The largest peak combined cable tension recorded in this study was 7.0 bodyweights for the straight arm longswings (Brüggemann, 1994).

The study of Nissinen (1983) attempted to observe differences in the techniques
used by gymnasts to perform longswings and use these observations to account for
differences in performance. Using gymnasts from club to international standards,
synchronised kinetic and two-dimensional kinematic data were collected for over 60
longswings using bent and straight arm techniques. Reaction forces exerted on the rings
frame were directly measured by two Kistler force transducers placed inside the metal
frame of the rings just above the attachment of the rings cables (Figure 2.8).

Unlike the study of Sale & Judd (1974), the force measurement system was not
placed directly in the rings cables. From the location of the force transducers, it is unclear
whether the reaction forces measured by this system are equal to cable tension or whether
they are a function of cable tension. It is possible therefore that measurements of reaction
forces are not directly comparable to those of cable tension.

Due to the symmetrical nature of a gymnast's technique about the sagittal plane,
reaction forces for both transducers were similar in magnitude throughout a longswing.
This finding has significant implications for future research. Measurements from one
transducer may be used to obtain accurate estimations of combined reaction forces
experienced by the gymnast, reducing the volume of kinetic data to be collected by half.

As speculated by Sale & Judd (1976) the reaction force time history for a straight
arm backward longswing showed qualitative similarities to the patterns of the
shoot-to-handstand. However, the peak combined reaction force, of 6.5 to
9.2 bodyweights, was greater than recorded for the backward longswings of
Valliere (1976). The greater forward displacement of the rings cables on the descending
phase exhibited by the elite gymnasts was thought to produce these larger forces. It was
suggested that this greater forward displacement of the rings was to simply increase the
kinetic energy of the gymnast, though this supposition was not quantified.

Noticeable differences were observed in the reaction force time histories between
gymnasts of different skill levels when swinging through the hang position and into the ascending phase of the longswing (Brüggemann, 1987). During an elite performance two localised peaks in reaction force (circled in Figure 2.9) were recorded. Non-elite performances did not exhibit these intricacies. It was believed these features in the reaction force time histories were idiosyncrasies of the gymnasts' techniques.

A two-dimensional kinematic analysis comparing elite and non-elite gymnasts indicated that the first of the localised peaks reaction force patterns in elite gymnast could be explained by the rapid reduction in the hip angles followed a decrease in the angles at the shoulder joints. The second localised peak was produced as a result of increasing the shoulder angles, forming an extended configuration in preparation for the final handstand position (Figure 2.10).

![Figure 2.9. Reaction force time history for a backward longswing performed by an elite gymnast (Adapted from Nissinen, 1983).](image)

![Figure 2.10. Time histories of relative vertical velocities for the legs, trunk and arms for an elite and non-elite gymnast (Adapted from Nissinen, 1983).](image)
The estimated net muscle moments at the hip and shoulder joints also demonstrated these coordinated actions at the hip and shoulder joints for the superior performer (Brüggemann, 1987) (Figure 2.11).

![Figure 2.11. Net muscle moments acting at joints for a backward longswing by an elite gymnast (Adapted from Brüggemann, 1987).](image)

The detailed and coordinated joint actions displayed by the elite gymnasts were not found for the non-elite gymnasts (Figure 2.10), leading to the suggestion that the lack of such movements was the reason for the inferior performance. Hence, not only is the initiation, duration and cessation of joint actions essential for proficient performances, but also the coordination of the actions at all joints. The following question may therefore be raised: how much can a gymnast mistime these joint actions and still produce a proficient performance? Such a question is addressed in Chapter 7.

The differences between elite and non-elite gymnasts suggest that gymnasts should not attempt to actively push the rings backward at the end of the ascending swing (Brüggemann, 1987), even though this active push of the rings is advocated in some coaching manuals (Taylor, Bajin & Zivic, 1972; Hunn, 1978; Murdock & Stuart, 1989). Furthermore, the results also imply that the shoulders should remain below the rings until the extension of the hip joints has occurred. To some extent this technique is advocated in the coaching manuals of Readhead (1987) and Turoff (1991).

**Studies of other swinging elements**

Elements on the rings used as coaching progressions for longswings have been studied in an attempt to gain insight into those desirable and undesirable components of technique. The dislocate is one such element for the backward longswing (Chapman & Borchardt 1977; Cheetham, Sreden & Mizoguchi, 1987). Through
exploratory correlations between judging scores and kinetic and two-dimensional kinematic variables Chapman & Borchardt (1977) showed that two aspects of technique were associated with better 'dislocate' performances. These aspects were the peak cable tension from the kipping action at the start of the dislocate as well as peak cable tension produced during the swing. Such findings may be thought to agree with those of Sale & Judd (1974, 1976), though few explanations were given for these conclusions.

In another study Cheetham et al. (1987) related injury risk to aspects of technique during dislocates. Synchronised reaction force measurements and a video camera were used to obtain the necessary data. Peak combined reaction forces of between 5.1 and 7.9 bodyweights were observed. The lower of these values is similar to those obtained by Chapman & Borchardt (1977).

Reaction force time histories were used to indicate the effectiveness of each gymnast's technique for the dislocates. A progressive increase in peak reaction forces over three consecutive dislocates was used to indicate an increase in the amplitude of the dislocate and therefore good technique. This conclusion was supported by simple observations from the video images. The pattern in reaction forces spanning the peak reaction force was also used to indicate the standard of technique. It was proposed that a 'double peak' in the reaction forces indicated a jerky technique and that this would lead to an increased injury risk. No evidence was given to substantiate these claims, although more recent work provides some support (Gielo-Perczak, 1991; Nissinen, 1995).

Much work has focused upon the forces experienced by the gymnast. It would seem that the magnitude and timing of these forces are reasonably well known. Discrepancies in the peak values across studies may to some extent be due to different measurement systems or standards in performance. However, from the literature only a few important components of technique have been identified, such as the joint actions made at the shoulders and hips (Nissinen, 1983). The extent to which these aspects of technique are important has not been determined.

Possible questions which have not been addressed include: how much does each components of technique contribute to the performance of a longswing? Which of these components are crucial for the successful performance of straight arm longswings? How sensitive is the performance to the coordinated timing of the components forming the gymnast's technique?

Furthermore, owing to the two-dimensional analyses undertaken so far, it is possible that other components of technique, such as lateral arm movements have not been determined; are these components significant to performances on rings?

Injuries and swinging on rings

Owing to the large forces experienced by gymnasts performing longswings on
rings, several studies have estimated the loading of the gymnasts' joints. Unpublished results from the study of Nissinen (1983) on joint loading were subsequently presented by Brüggemann (1987). This review paper speculated that increased incidences of pain and injury in the shoulder joints occurs with increased amplitude of swing. Internal joint forces for the hip and shoulders were calculated using inverse dynamics. The largest internal joint forces in the shoulders and hips joints occurred when the gymnast passed through the bottom of the swing in an extended configuration. For an elite gymnast maximum internal joint forces were 2100 N and 1500 N for the shoulders and hips respectively. Surprisingly non-elite gymnasts showed larger maximum values for these joint forces. It was concluded that these extremely large forces experienced at these joints during longswings would increase the risk of ligament or muscular damage, especially for non-elite gymnasts. However no evidence was given to substantiate the claims relating the increased incidences of pain to elements displaying larger amplitude of swings.

Since these speculations, at least three studies have investigated the occurrence of shoulder pain and injuries during swinging on rings. Using finite element analysis, Gielo-Perczak (1991) showed that the pattern of cable tension spanning the peak (for example Cheetham et al., 1987) greatly influenced the forces experienced by the gleno-humeral joint. As patterns in the time history of cable tension are due to a gymnast's technique, appropriate modifications to technique may decrease the forces exerted on the gleno-humeral joint and therefore decrease the risk of injury. This conclusion was also advocated in a study of Nissinen (1995) where further speculation to the injury potential for gymnasts performing these longswing movements was given by X-ray data of gymnasts' shoulder joints.

In two papers utilising electromyography (EMG) on the musculature surrounding the shoulder joints, Caraffa, Cerulli, Rizzo, Buompadre, Appoggetti & Fortuna (1996) and Cerulli, Caraffa, Ragusa & Pannacci (1998) proposed mechanisms by which lesions are produced in the joints. During backward longswings EMG data indicated a dramatic reduction (90%) in the activity of the muscles surrounding the shoulders as the gymnast passed beneath the rings, where the shoulder forces are at their greatest. It was stated that without the recruitment of shoulder muscles in a protective role, such large forces would lead to failure of the various structures surrounding the shoulder joints.

Such problems of large forces may be overcome by an increase in the elasticity of the rings frame. An increase in elasticity would effectively increase the energy stored or dissipated by the rings apparatus and as a result decrease the peak forces experienced by the gymnasts (Caraffa et al., 1996). However, what effect would such changes to the mechanical properties of the apparatus have on performance? Would the gymnast have to significantly alter his technique if the elasticity of the rings frame were doubled? How elastic would the apparatus have to become to make the peak forces 'safe'? Questions on this theme are addressed in Chapter 7.
**Force measurement methods**

The studies reviewed have generally been concerned with the measurement or estimation of tension in the rings cables, though measurement of rings cable tension has not been restricted to swinging activities (Leikin, 1978; Robertson, Paul & Nicol, 1985). Determination of cable tension has been achieved in several ways. Inverse dynamics allows the estimation of cable tension (Dusenbury, 1968), though this procedure relies on estimating acceleration data, which is not always accurate. Strain gauge technology has been utilised (Sale & Judd, 1974) though this requires the accurate calibration of the load cell prior to data collection. More recently piezo-electric quartz force links have been used. Such instrumentation has the advantage over strain gauge technology in that it is pre-calibrated, omitting the need for lengthy calibration procedures.

An indirect method of estimating cable tension has been described by Mills (1997, 1998). This method uses the fact that tension in the rings cables produces vertical movements of the DEDs and horizontal beam of the rings frame (Figure 2.2). During a longswing where peak combined cable tension is 7.5 bodyweights the maximum vertical movement of the top of the DEDs is 30 mm. This method allows the estimation of forces during competition, as instrumentation is only required for calibration purposes of the DEDs vertical movements prior to competition. Initial results indicate the accuracy of this method is within 2% of the cable tension range.

Of all the methods, indirect or direct, piezo-electric force links are advantageous since they do not require additional calibration, and thereby reduce the volume of data to be collected. It may therefore be recommended that they are used whenever possible.

**Summary**

Advances in coaching methods and the construction of the rings apparatus have combined to produce improved longswing performances through changes in the gymnasts' techniques. These changes in technique reduce the usefulness of research studying techniques which have been superseded.

Through comparisons between differing standards of gymnasts, components of technique that produce superior performances for the backward longswing have been determined. However, it is not known which of these components contributes the most to the performance, or how sensitive the performance is to small changes in the timing or magnitude of each component.

A major weakness of previous work has been the two-dimensional analysis of a three-dimensional event (Section 2.2.1). This weakness is addressed in this thesis.

As the majority of research has focused on the backward longswing or its coaching progressions little is known about the techniques used during forward longswings.
Observations indicate a significant difference in technique during the ascending phase of the swing when compared to backward longswings (Section 2.3.1). These differences are determined in this study using three-dimensional kinematic analysis.

2.4.3 Theoretical research

Bauer (1987) highlighted the increasing use over recent years of the theoretical approach in sports biomechanics, including mathematical, physical and computer simulation models.

To the present day, theoretical research into Men's Artistic Gymnastics has focused upon tumbling and vaulting activities and especially swinging on horizontal bar. On vault, differences in the preflight characteristics required to perform different types of vaults have been analysed (King, Yeadon, Kerwin and Sprigings, 1995; Sprigings & Yeadon, 1997), while on floor contributions to the performance of multiple somersaults have been evaluated (King, 1998). On horizontal bar the mechanics of a gymnast performing swinging activities have been investigated on several occasions (Dainis, 1968; Morlock & Yeadon, 1988; Hiley, 1998). All of these studies utilised computer simulation models for their respective investigations. Although this approach has been extended to investigations of a gymnast swinging on rings, there is still a lack of theoretical studies in this particular area.

Due to this lack of theoretical studies this review also incorporates relevant aspects of work on horizontal bar. Results from such studies may be tentatively applied to a gymnast swinging on rings due to the common theme of swing. However, structural differences in the apparatus may mean specific results from horizontal bar are not directly pertinent to swinging on rings. As identified in Section 2.3.2, the gymnast swinging on rings may be thought of as a double pendulum, which is more complex than a single pendulum which a gymnast swinging on horizontal bar may be represented by. Furthermore, the motion of a gymnast on horizontal bar may be considered to be only in the vertical plane (Hiley, 1998), whereas the motion of a gymnast's arms and the rings cables is three-dimensional. Hence, caution must be demonstrated when inferring specific findings from theoretical studies of swinging on horizontal bar.

Theoretical studies of a gymnast swinging on rings

Of the limited number of theoretical studies and simulation models concerned with a gymnast swinging on rings the common feature is the planar analysis in a single vertical plane, of what is essentially a three-dimensional activity.

Provias (1994) used a two-dimensional model to simulate a gymnast performing
straight arm backward longswings on rings. The rings frame and cables were assumed to be rigid. The gymnast comprised three rigid segments, with shoulder and hip joints, though joint angle constraints at the hips ensured that changes only in the shoulder joint angles were investigated. Owing to the use of inappropriate anthropometric values for the gymnast, results obtained from this study are misleading and effectively redundant.

In two complementary papers Sprigings, Watson, Lanovaz & Russell (1996) and Sprigings, Lanovaz, Watson & Russell (1998) developed a two-dimensional simulation model to investigate the removal of unwanted swing in a handstand through the performance of a backward longswing. In the second paper the gymnast was represented firstly as a two segment body with a shoulder joint, and secondly as a three segments body with both shoulder and hip joints. The rings frame and cables were assumed to be rigid, modelling the rings cables as a single segment.

Muscular joint-torque generators placed at each joint were allowed to activate twice during a swing, enabling the gymnast to alter his configuration and hence the mechanical energy of the system. The activation of the muscular joint-torque was exponential in profile, to a maximum value of 200 Nm (Figure 2.12). In addition, protective joint torque activators were implemented at each joint (maximum of 300 Nm) forming a mechanism which eliminated anatomically unrealistic angles at the gymnast's joints.

![Activation profile for the development of maximum joint torques](Adapted from Sprigings et al., 1996).

In general the studies highlighted how a gymnast may alter the mechanical energy within the gymnast-apparatus system as well as the path of his mass centre by changes in joint angles, or technique. More specific results from both studies indicate that the angular momentum of the gymnast could be reduced by 85% during the backward longswing when the initial handstand displayed a $10^\circ$ amplitude of swing. This reduction was only accomplished if two appropriately timed muscular actions at the shoulder and hip joints were used, combined with protective joint torques. The correct initiation of the
swing, $2^\circ$ prior to the vertical handstand while swinging in the same direction as the longswing, was also imperative for the removal of swing. The two studies also showed that if a gymnast performed a backward longswing from an initially still handstand a completely motionless handstand at the end of the longswing could not be achieved. If correct, this result has major implications for a gymnast performing on rings.

Factors influencing the reduction of swing after the completion of a backward longswing include the timing of the muscular actions, the initiation of the swing from handstand and the amplitude of swing in the initial handstand position. Factors shown by the study to be less critical are the muscular actions at the hip joints as the two segment gymnast produced similar longswings to the three segment gymnast. This result however, was obtained using a different muscle strategy (or technique) meaning the actual contribution of the hips to a longswing remains unknown. Their actual contribution to the backward longswing may be better evaluated by removing the actions at the hips during an otherwise identical technique (Chapter 7).

Although the model highlighted factors influencing swinging on rings the model was not evaluated in a quantitative manner. This reduces the external validity of the results. The lack of elasticity in the rings frame and cables means their influence on the loading of the gymnast and performance remain unresolved. The use of protective joint torques meant that the muscles of the gymnast were not always active during the ascending phase, but worked in bursts (Figure 2.13). Such a representation of muscle activation may not be realistic and this is addressed in Chapter 6.

![Graph of joint torques used during a backward longswing with a $10^\circ$ swing in the initial handstand position](Adapted from Sprigings et al., 1998).

In all three of these studies the lateral movement of the arms and rings cables during the swing were omitted. As lateral arm movement may constitute a significant part of a gymnast's swinging technique, a two-dimensional analysis may be too simplified to adopt. Furthermore, the contributions of the lateral arm movements to performance may
only be fully evaluated if the model is able to reproduce the movements. The use of lateral arm movements relate to the questions raised in Chapter 1, and are addressed in Chapter 7.

Theoretical models of a gymnast swinging on horizontal bar

Modelling a gymnast swinging on horizontal bar has basically been accomplished in three ways. Bauer (1983) proposed a mathematical model of swinging, where the gymnast was represented by a point mass swinging at a variable distance r about a stationary point of rotation. Changes to the distance r were assumed to be instantaneous. Using this simple mathematical model it was shown that the most efficient method of increasing the gymnast's mechanical energy was to change the distance r at two key points during a swing. During a giant circle, for example, the distance r should decrease when the point mass is directly below the point of rotation and then increase when the point mass is directly above the point of rotation. Timings of these changes in distance r were determined using equations relating to the mechanical energy and angular momentum of the point mass representing the gymnast. In a more recent study, however, Hiley (1998) showed that because gymnasts comprise more that one segment, and each segment possesses inertia, changes in the gymnast's configuration to maximally increase energy during one backward giant circle should be made prior to the bottom and top of the backward giant circle. Through a more realistic representation of a gymnast, an revision to one theory is superseded by another and forms part of scientific method.

Subsequently, Bauer (1985, 1987) developed a series of physical models, reflecting the increased sophistication with which a gymnast swinging on horizontal bar could be modelled (Figure 2.14).

Figure 2.14. Stages in the development of a physical model representing a gymnast swinging on horizontal bar (Adapted from Bauer, 1985).
The physical models (c) and (d) show that a gymnast can initiate swing from a stationary hanging position as well as alter the mechanical energy of the gymnast by appropriately timed muscular joint actions. From a stationary hang position, changes in the joint angles momentarily change the mass centre location of the gymnast to a position which is not directly beneath the bar. This in turn creates torques about the bar due to the weight of the gymnast. With the correct coordinated changes to the configuration of the gymnast, swing can be developed. The amplitude of swing may be altered by the use of correct joint actions, increasing, maintaining or even decreasing the mechanical energy of the gymnast.

Computer simulation models have been developed which represent the gymnast swinging on a rigid horizontal bar by two segments (Morlock & Yeadon, 1988) and three segments (Dainis, 1968). More recently Arampatzis & Brüggemann (1995) and Hiley (1998) increased the complexity of models representing a gymnast on horizontal bar by representing the elasticity of the bar through super-elements (comprising rotatory and linear springs) or damped linear springs. Hiley (1998) also incorporated the elasticity of the gymnast in his model. As the elasticity of the gymnast and apparatus were found to affect the gymnast's motion these features should also be incorporated into a realistic simulation model of a gymnast swinging on rings.

In the earlier models of Morlock & Yeadon (1988) and Dainis (1968) input was in the form of joint torques. To control the configuration of the gymnast the torques required to maintain or change the joint angles must be calculated. This leads to additional computation difficulties if precise control over joint angle changes is required (Dainis, 1968). If control over technique is paramount a simulation model which uses joint angles as model input may be considered a more appropriate method (Hiley, 1998). A limitation of using joint angles as input is that the joint actions must be physically possible by a gymnast. Hence, estimations of the maximal joint torques should be obtained to ensure the strength limitations of the gymnast are not exceeded (Hiley, 1998).

General results from these two simulation models concerning joint torques may have some application to swinging on rings. Morlock & Yeadon (1988) determined the joint torques required at the hip joint for a horizontal bar swing from near handstand, where the gymnast's body maintained a straight body configuration throughout the swing. To perform the swing it was found that a gymnast must exert muscular torques at the hip joint in order to remain straight (with a maximum of around 30 Nm). During the descending phase of the swing the joint torque acts as though it is reducing the angle at the hips (flexing). Without this flexing torque the hip joint would hyper-extend during the descending phase. Throughout the ascending swing the joint torque serves to increase the angle at the hips (extending). If the gymnasts were to relax while ascending, ceasing any torque about the hips, the hip joints would produce a flexing action. The temporal pattern of the joint torques was shown to be symmetrical about the hang position, and directly
related to the angular acceleration of the gymnast's mass centre about the rigid bar.

This simple model also showed that mechanical energy may be altered due to appropriately timed changes to the gymnast's configuration. Using a three segment model of the gymnast swinging on horizontal bar Hiley (1998) further explained the increase in energy by the type of joint torque used by the gymnast. Concentric joint torques were associated with an input of mechanical energy into the system, while eccentric joint torques were concomitant with a loss in mechanical energy. Using these definitions, the model was used to explain which joint actions were associated with the greatest increases in energy and the mechanical reasons for this.

When considering a gymnast swinging on rings the concepts and explanations for altering the mechanical energy of the gymnast should be the same as those on horizontal bar. However, owing to the additional movement of the rings cables the magnitude and timing of the joint torques required to produce swing may differ to those on horizontal bar. To some extent this has already been shown in the study of Sprigings et al. (1998).

The problems faced by a gymnast performing a longswing on rings are further exacerbated as a gymnast does not simply need to increase his total mechanical energy. Also to be considered is the motion in the final handstand, which must be minimal to ensure no scoring deductions are made. It may therefore be speculated that the way in which joint angle changes influence the path of the gymnast's mass centre is also very important.

Summary

There have been few theoretical studies of a gymnast swinging on rings. This may be due, in part, to the three-dimensional movements of the arms and rings cables which occur during the swinging activities. Studies simplifying the activities to two-dimensions have identified the need for coordinated muscular actions at the shoulders and hips of the gymnast during backward longswings. However, the relative contributions of these joint actions to performances of longswings remains unclear, while the influence of lateral arm movements on longswing performances remains unknown. Furthermore, as previous models have assumed the rings apparatus is rigid the affect of apparatus elasticity on performance and loading of the gymnast has not been investigated.

Explanations of how the mechanical energy of a gymnast swinging on horizontal bar may be altered have also be highlighted. It may be expected that these explanations will also be applicable while swinging on rings apparatus.

Finally, from studies simulating swinging on horizontal bar it would seem that if accurate control of the gymnast's technique is paramount, model input should take the form of joint angle time histories, actual or hypothetical, from which the subsequent motion can be derived (Hiley, 1998).
2.5 Techniques of investigation

The following sections of Chapter 2 examine critically techniques of investigation which may be used to address the questions posed in the study. Reasons for adopting certain techniques, in preference to others, are provided and justified in each sub-section.

2.5.1 Simulation modelling

Modelling, as applied to sports biomechanics, may be divided into two categories: inverse dynamics and forward dynamics (Yeadon & Challis, 1994). In each case the model, which obeys Newton's Laws, typically comprises a set of rigid hinged bodies, effectively reducing the complexity of the actual situation, and providing a more comprehensible version of reality to study.

For inverse dynamics, input to the model consists of the kinematics of the system. Using the equations of motion for the model together with kinematic inputs, the kinetics, such as internal joint forces and joint torques producing the motion, are derived. Models used in inverse dynamics studies include those of Dixon (1996) who modelled the ankle and lower leg region while investigating the loading of the achilles tendon during running and Kerwin (1997) who modelled the whole body during simple jumping activities. Inverse dynamics models have been used widely in the field of injury-related sports biomechanics (Yeadon & Challis, 1994), allowing a greater understanding of forces involved in sporting activities to be gained.

The second category of modelling used in sports biomechanics is forward dynamics, also termed simulation modelling. Inputs for simulation models often comprise the kinetics acting in the system while output is the resulting kinematics, as in the models of Alexander (1990), Greig (1998) and King (1998) which simulate athletic jumping. However, as mentioned in Section 2.4.3 simulation models may also use joint angles and angular velocities as input from which to infer the kinetics of the system. Such procedures may be more effective if control over a certain technique is of paramount importance (Hiley, 1998; Yeadon, 1990). Furthermore, in contrast to inverse dynamics models, simulation models also require an integration procedure which progresses the solution to the model's equations of motion though time.

A simulation model may be used to alter a single aspect of the activity without additional undesirable changes to the activity being made. This ability, to control the direct intervention of chosen variables in the study, is unlike an experimental situation using human subjects. It allows the researcher to establish a direct cause and effect relationship between a variation in technique and an outcome in performance (Yeadon, 1987).
Within the context of scientific method (Figure 2.7) a simulation model may also be used to produce theoretical predictions and explore scenarios which may not be performed by an actual gymnast for fear of injury. Furthermore, although humans may fatigue, a computer does not and this means many more simulated performances may be made than a gymnast can actually perform (Vaughan, 1984). Finally, simulation models may also allow a more complete understanding of the underlying mechanics of an activity than can be gained from the actual performances (Yeadon & Challis, 1994). Simulation models therefore tend to be more applicable to the analysis of sports technique and performance.

As this study is concerned with contributions to performances of longswings on rings, and therefore requires predictions, simulation modelling would seem an appropriate technique to address the questions posed.

Within sports biomechanics simulation models with varying complexity have been used to investigate twisting during aerial movements (Van Gheluwe, 1981; Yeadon, 1990), pre-flight contributions to jumping (Alexander, 1990) and the optimal timing of muscular strategies in jumping activities (Hatze, 1981b; Pandy & Zajac, 1991; Van Soest, Schwab, Bobbert & Van Ingen Schenau, 1993). More specifically, within the sport of gymnastics, simulation models have been used to gain a greater understanding of the motion of the gymnast while performing on the following apparatus:

- rings; Sprigings, Watson, Lanovaz & Russell (1996)
- vault; King (1998); Sprigings, Yeadon, Watson, & King (1994)
- high bar; Arampatzis & Brüggemann (1995); Dainis (1968); Hiley (1998); Morlock & Yeadon (1988).

Several aspects must be considered when developing a simulation model of any sporting activity.

*Model complexity*

The complexity of the simulation model representing a chosen activity depends on the purpose of the model. Essentially the model should represent those components of the activity which make the major contributions to the motion, while omitting those of negligible importance. Components to be modelled may also include any apparatus used by the athlete: the rings frame and cables in the case of a gymnast swinging on rings. By reducing the complexity of the actual situation to one which is more manageable a fundamental understanding may be gained. Descriptive studies may be used to decide which components of the system should be incorporated into the simulation model. Such
studies form the first stage of the theory-experimental cycle.

At one extreme a model may bear little resemblance to the activity it is simulating. The model of Hubbard & Trinkle (1985), which models the high jumper as a single straight rigid rod, is an example of this. As the majority of high jumpers use the 'Fosbury' technique, where the spine arches significantly during the flight phase, this model of the human may be too simplified, and as a consequence the model may possess little external validity.

However, simple models can be effective. Using a simple two segment planar model of jumping; Alexander (1990) showed that different touch-down conditions are required for the take-off phases in optimal high and long jump performances. In a further example, Morlock & Yeadon (1988) modelled a gymnast swinging round a horizontal bar using only two rigid segments linked by a frictionless pin joint with a torque actuator. This simple model increased the understanding of the mechanics involved while swinging on horizontal bar.

At the other end of the spectrum, a model displaying too much complexity may obscure the underlying tenets of the activity. Hatze (1981b) developed a model representing a human with 17 body segments (42 degrees of freedom), and 46 muscle groups. With this many segments and muscles to consider the difficulty in interpreting the importance of each component of the model is increased. Secondly, gaining accurate values for so many model parameters may limit the application of the model. Yeadon & Challis (1994) warned against the use of overly sophisticated models requiring many parameters, observing that an activity may be mimicked without knowing whether each parameter has a realistic value.

Model evaluation

Bauer (1987) stated that a researcher must continually check and validate any model against both static and dynamic features of the actual system. In sports biomechanics this advice has not always been taken. A lack of realism, or external validity of the model, may be a consequence of failing to evaluate the model against the sporting activity it is simulating until satisfactory realism is obtained. This evaluation stage may be thought of as the coupling of experimental and theoretical research (Yeadon & Challis, 1994). By comparing a model's prediction of motion for a particular activity to an actual description of the same activity the model may be evaluated. From such an evaluation, modifications to the simulation model may be made until a satisfactory level of agreement between the simulation model's prediction and the actual activity is observed. By performing this evaluation procedure the realism, or external validity of the simulation model, is increased.
Model parameters

When using a model in which parameters, such as segmental lengths, masses and moments of inertia, are required from the athlete, there is opportunity to personalise the model to specific athletes. In the example of techniques used by gymnasts while swinging on rings this may allow specific advice to be given to a particular gymnast to improve his performance of certain gymnastic elements. Specific advice may not be possible if average values for these parameters of elite gymnasts were used, as they may not represent any actual gymnast. However, in order to gain insight into the mechanics underpinning the movement, representative values for the parameters may be used. By implementing a sensitivity analysis, where parameters are varied, the effect of different parameters on performance may be assessed.

Summary

Remarks made in this section highlight the benefits of a theoretical approach to research. These benefits may be considered to outweigh the limitations of realism and external validity often associated with theoretical studies. Obviously, if the accuracy of the model is poor then the external validity of the study is adversely affected. Attempts are required to overcome this possible problem inherent in theoretical studies.

In the particular case of simulating a gymnast swinging on rings the model should simplify the apparatus and gymnast, while ensuring the model contains the most important factors which go towards producing the activity. In the case of a gymnast swinging on rings, joint angle changes at the hips and shoulders cannot be ignored; neither can the three-dimensional motion of the gymnast and apparatus. These attributes should be incorporated into the model if the results obtained from it are to be valid.

2.5.2 Estimating segmental inertia parameters for human subjects

The inertial properties of any object are characterised by mass, mass centre location and principal moments of inertia. In order to perform quantitative kinetic analyses of humans in motion, the inertial characteristics specific to the subject are required (Hay, 1974; Hatze, 1983). If subject specific values are not estimated accurately it may not be possible to perform accurate inverse dynamics analyses or produce valid simulation models for the investigation of human movement (Reid & Jensen, 1990). The accurate estimation of the inertial properties for a human body is therefore of great importance for biomechanical studies of human movement.

During three-dimensional motion the body configuration, and therefore the inertial
properties of the subject, may vary substantially. For kinetic analyses estimates for the inertial properties of the subject in all body configuration are required throughout the motion. The principal moments of inertia for a human may be determined for each body configuration produced during the motion. This method is impractical as it is generally not possible to recreate all of the required body configurations. The second method uses estimates of the subject's segmental inertial parameters (Hay, 1973). This method is advantageous since the whole body inertial properties of a subject can be calculated in any body configuration, without the need for further measurements. Furthermore, estimates for segmental inertia parameters which are personalised to a gymnast are required. This review therefore concentrates on those techniques which allow estimates of segmental inertia parameters.

**Experimental techniques**

Several experimental techniques have been developed which can estimate some or all of the segmental inertia parameters directly from living subjects. These techniques may be further categorised, in which a sub set of techniques possess common attributes. One such sub-section includes the techniques of Gamma Mass Scanning (GMS), Computer Tomography (CT) and Magnetic Resonance Imaging (MRI). Common features in this sub-section include: the scanning and sectioning of the subject's segments into thin sections (as little as 5 mm in thickness), extremely expensive equipment to perform the scanning process, and the extensive time required to collect personalised data. Each technique determines the volume, mass and density of each thin sliced solid, from which all segmental inertia parameters may be estimated by summation and standard computational procedures.

The technique of GMS uses the principal that γ photons, emitted by Cobalt-60 and Caesium-137, are absorbed by a material at a known rate. The rate of photon absorption is dependent on the density of the object being analysed. This technique has been utilised by Brooks & Jacobs (1975) on legs of lamb and later, on human subjects by Zatsiorsky & Seluyanov (1983, 1985).

Brooks & Jacobs (1975) used the GMS technique to estimate the mass, mass centre location and moments of inertia for legs of lamb. Criterion values for these inertial parameters were calculated using other experimental techniques. Accurate results for the estimated mass of the leg (mean of 1% error), mass centre location (maximum of 2.1% error, expressed as percentage of leg length) and moments of inertia (maximum of 4.5% error) were obtained. Zatsiorsky & Seluyanov (1983, 1985) applied this same technique to 100 human subjects. Adaptations to the amount of radiation received by the subjects were made so that the radiation levels were deemed 'safe'. Although no criteria were given to assess the accuracy of the technique for estimating segmental inertia parameters,
errors similar in magnitude to those observed by Brooks & Jacobs (1975) might be expected.

Computer tomography (CT), which also uses a radiation source, has been applied to living human subjects (Ackland, Blanksby & Bloomfield, 1988a; Wei & Jensen, 1995). Ackland, Henson & Bailey (1988b) used CT to obtain mass and density measurements for a cadaver leg and a leg in vivo. Owing to the different proportions of tissue types throughout a leg the densities of both legs were found to vary by up to 10% throughout the longitudinal axis (Figure 2.15).

![Graph](image)

Figure 2.15. The variation of density throughout a leg (in vivo) (Adapted from Ackland et al., 1988b).

When average and actual density values were used to calculate segmental inertia parameters the differences in predicted masses and moments of inertia were insignificant. Similar conclusions concerning the assumption of constant segmental density on the calculation of segmental parameters were put forward by Chen & Jensen (1995). As the majority of the mathematical inertia modelling techniques assume tissue density is constant throughout each segment (e.g. Jensen, 1978; Yeadon, 1990b) these results provide support that this assumption does not significantly affect the accuracy of these techniques.

The use of radiation sources in both GMS and CT may deter researchers from using these techniques. The technique of Magnetic Resonance Imaging (MRI) however, overcomes the need for relatively large radiation doses during data collection (Mungiole & Martin, 1986). Mungiole & Martin (1990) used MRI to determine the segmental inertia parameters for the lower legs of 12 athletic subjects. Although estimates for segmental inertia parameters determined by MRI were close to those
predicted by regression techniques, the MRI estimates tended to be larger. It was suggested that using the regression equations for the 12 athletic subjects in the study was not appropriate. Athletes could be expected to have a greater proportion of muscle in the lower leg, and a lower proportion of fat, producing a greater segmental density than cadavers.

At present it is doubtful that any of these techniques will be used extensively in sports biomechanics, mainly because of the sophisticated and expensive equipment which is required. Furthermore, the relatively high radiation doses required by the GMS and CT scan procedures remain a contentious issue.

A second branch of experimental techniques uses mechanical principles and relationships to calculate inertial properties of segments. Many of these techniques do not permit the estimation of all segmental inertia parameters.

The quick release technique is one such method which has been used to estimate the moment of inertia of a segment about one of its joint axes (Drillis, Contini & Bluestein, 1964). This technique uses the relationship between a known angular acceleration and applied torque in order to calculate the moment of inertia about a point of rotation. Bouisset & Pertuzon (1968) successfully used this technique to determine the moment of inertia of the forearm and hand about the humero-ulnar joint, obtaining a comparable estimate for this segment to that of Dempster (1955).

Hatze (1975) developed a technique for estimating the mass centre location and moment of inertia of a limb about the joint centre in the transverse axis by forcing the limb to perform damped simple harmonic oscillations. The time period and decay of the oscillations allow the inertia parameters to be estimated. Additionally, estimates for the joint's damping coefficient, due to the skin, muscles and ligaments crossing the joint, may be determined. Values for the moment of inertia of the whole leg about the hip joint were similar to those calculated using a combination of techniques. Taking a more simplified approach Peyton (1986) developed a different oscillatory technique and applied it to the forearm of a subject. The simplifications to the equations for damped oscillations meant the moment of inertia of the forearm about the elbow joint could be determined from the period of oscillation only.

The relationship between the moment of inertia of an object about the suspension point and the oscillation period of a pendulum forms the basis for another experimental technique, the compound pendulum (Hay, 1974). This technique has been used to determine moments of inertia of cadaver segments about their proximal end, (Dempster, 1955) and segments in vivo.

In a similar vein a torsional pendulum has been developed which may be used to determine moments of inertia of segments and of the whole body for all three principal axes. Drillis et al. (1964) briefly describes the theory behind the technique. The usefulness of this technique is restricted on two counts. Firstly the subject is required to
remain in a rigid configuration while being oscillated and secondly knowledge of the physical properties of the pendulum are required. These factors may hinder the accurate determination of segmental moments of inertia using this technique.

The reaction board technique, described by Contini (1972), may be used to determine the mass and mass centre of a segment in vivo. However the calculation of one of these parameters is only possible if one of the inertial parameters is known prior to the calculation of the desired parameter.

The mass of any segment of the human body can be calculated when density is known and volume can be determined (Contini, Drillis & Bluestein, 1963; Contini, 1972). A technique used by Dempster (1955), Contini et al. (1963) and Drillis et al. (1964) for determining segmental volume is that of water immersion, which uses the principle of Archimedes. Although increasingly sophisticated methods have been developed to determine accurately segmental volume using immersion techniques, its straightforward use for living subjects is restricted to the limbs. Great difficulties arise if this method is used to determine central segments, such as the torso. Furthermore, densities derived from cadaver studies are employed to estimate the mass of a segment. It has also been suggested that segmental mass centre locations can be determined by immersion techniques by assuming that the mid-segmental volume is equivalent to the mass centre location (Contini, 1972). This is only true if the density of the segment is constant throughout the segment. As this is known to be an over-simplification (Figure 2.15) the mass centre and mid-volume location will not necessarily coincide for a human segment (Hay, 1973).

Many of these experimental techniques have been used in cadaver studies to determine segmental inertia parameters of the cadaver’s body segments. Techniques widely used in the cadaver studies include those of immersion, reaction board and compound pendulum (Dempster, 1955). In addition, experimental techniques to determine all three principal moments of inertia for each segment of six cadavers were used by Chandler, Clauser, McConville & Reynolds (1975).

With exception of the scanning techniques, the majority of experimental techniques cannot solely provide estimates of all inertial parameters for a single segment. Furthermore "experimental methods .... are difficult to apply to central segments such as the abdomino-thoracic and abdomino-pelvic segments" limiting their applicability (Hatze, 1980). As experimental techniques are generally only appropriate for limb segments, other techniques which are able to produce a full set of segmental inertia parameters for a subject should be selected.

Regression analysis techniques

The thrust behind regression analysis for estimating segmental inertia parameters is
that all inertial properties for an individual subject may be estimated from only a few anthropometric measurements. Depending on the application, it may be advantageous if the samples used to generate the regression equations were as large as possible, encompassing all body types, or alternatively from a suitably sized but appropriate sub-population. On the whole, populations used to generate the regression equations have been from discrete populations, including cadavers (Barter, 1957; Clauser, McConville & Young, 1969), the elderly, and athletes or Physical Education students (Zatsiorsky & Seluyanov, 1985). Furthermore, the sample size used to formulate the equations has generally been minimal.

Several studies have formulated linear regression equations which estimate segmental inertia parameters from cadaver data. Barter (1957) used the work of Dempster (1955), combined with the earlier cadaver studies of Braune & Fischer (1889) to produce equations which estimate segmental mass from total body mass. Differences in dissection procedures between the two cadaver studies may however, reduce the validity of the equations. Clauser et al. (1969) used anthropometric measurements from 14 cadavers of various sizes to produce multiple regression equations for segmental inertia data determined using experimental techniques. From these equations segmental mass, volume and mass centre location can be estimated from three anthropometric measurements. Hinrichs (1985) used cadaver data from Chandler et al. (1975) to derive equations estimating transverse and longitudinal moments of inertia from length and perimeter data only.

The use of cadavers to formulate the regression equations may limit their accuracy when estimating segmental inertia values for elite athletes owing to possible differences in body type and tissue composition. To overcome the problem of subject and sample compatibility a combination of techniques has been implemented. Ackland et al. (1988a) used a mathematical model (Jensen, 1978) to obtain segmental inertia parameters for the lower limbs and trunk regions of male adolescents. Multiple linear regression equations were formulated in an attempt to estimate all segmental inertia parameters from up to five anthropometric measurements taken from each segment. However, no anthropometric variables were found to significantly correlate with segmental mass centre location. This resulted in an incomplete inertia set when using the regression equations, limiting the use of this method.

The extensive use of linear regression for estimating moments of inertia has been shown to be dimensionally incompatible (Morlock & Yeadon, 1986) and therefore non-linear regression equations have been developed for estimating these parameters (Jensen & Wilson, 1988). Such incompatibility was acknowledged in an earlier study where Widule (1976) scaled the cadaver data of Dempster using mass and square of height to personalise estimates of segmental moments of inertia. Later, Yeadon & Morlock (1989) developed linear and non-linear regression equations from the
cadaver data from Chandler et al. (1975). Standard errors for the linear equations were similar to the non-linear equations in a cross-validation procedure. However, extremely large errors were observed when the linear equations were applied outside the sample range used to generate the equations, while little change in error estimates were found for the non-linear equations.

Although regression equations may avoid the need for extended periods of time spent on experimental procedures, if full segmental inertia sets cannot be accurately determined their applications are limited.

Mathematical modelling techniques

Several authors have suggested that the human body may be represented by a number of rigid segments connected by a series of links to estimate segmental inertia parameters. These rigid segments may be described by one or more geometric solids with the links representing articulations of the human body (Hatze, 1980; Yeadon, 1990b). Using anthropometric measurements taken directly from a subject the dimensions of each geometric solid may be estimated and the volume calculated using standard mathematical equations. By combining estimates of segmental densities and calculated volumes for the geometrical solids, all segmental inertia parameters can be calculated using standard mathematical equations for mass, mass centre location and principal moments of inertia. These basic tenets are used in all mathematical inertia models of the human body.

Hay (1974) describes two such mathematical inertia models, those of Whitsett (1963) and Hanavan (1964). The models, comprising 14 and 15 solids respectively, are similar in structure, with only the trunk and feet being modelled differently. In both models the regression equations of Barter (1957), which have the limitations described previously, are used to estimate segmental masses. Errors associated with these equations may be introduced into the modelling process.

In the model of Whitsett, mass centre locations for all segments were determined from the cadaver work of Dempster (1955). Segmental inertial parameters calculated from this model were the three principal moments of inertia for each segment. For the model of Hanavan (1964) 25 anthropometric measurements are taken directly from the subject to estimate the dimensions of each geometric solid. Evaluation of the model took the form of comparing previously experimentally determined whole body mass centre locations and principal moments of inertia in a number of different body configurations (Santschi, Dubois & Omoto, 1963). Although whole body mass centre locations were deemed accurate (error of 0.018 m), errors in estimating the principal moments of inertia were generally large with half of the estimates showing errors larger than 20%.

Both of these early modelling attempts utilised an elliptical cylinder for the trunk region which, it may be argued, does not sufficiently represent this region of the human
body. Furthermore, using only 14 or 15 geometric solids to represent the human body may be too few in number when considering the subtle variations in the shapes of body segments (Jensen, 1978). It is suggested that these models are insufficiently accurate to be employed in this study.

To reduce errors in inertial estimates produced from ignoring segmental shape fluctuations Jensen (1978) developed a mathematical technique for estimating segmental inertial parameters for a 16 segment representation of a human subject. The technique involves a combination of photogrammetry and digitisation, coupled with segmental density estimates of Dempster (1955). The entire body is sectioned into 2 cm elliptical zones by digitising calibrated photographs of two views of the subject (Figure 2.16). Using the dimensions determined from the digitised data the volume of each 2 cm zone is calculated assuming the cross-section is elliptical. From these volumes the mass of each zone is determined, and segmental mass is calculated through their summation. With the assumption that segmental axes are principal axes, the segmental mass centre location and principal moments of inertia are calculated.

To illustrate that the inertia model could accommodate variations in body shapes and sizes it was initially applied to three prepubescent male subjects of varying somatotype. The calculated whole body mass was less than 2% different to the measured body mass for all three subjects. When 12 young subjects were used the mean error in estimates of whole body mass was 0.7%, while using the model of Hanavan (1964) the mean error was 12.4%. Comparisons of actual and estimated whole body mass suggest the model is sufficiently accurate for this parameter, though it is difficult to establish its accuracy for determining segmental moments of inertia as criterion values are unknown.

Figure 2.16. The mathematical inertia model of Jensen (1978) (Adapted from Jensen, 1978).

As this mathematical model takes into account fluctuations in segment shape and measurements are subject specific, it has been used to model diverse human populations,
from the elderly (Jensen & Fletcher, 1994) to the very young (Jensen, 1981). As only two synchronised photographs are required from a subject the "inconvenience to the subject is minimal" (Jensen, 1978). However, the time required to digitise the subject vastly increases the data analysis. In an attempt to reduce the lengthy process of manually digitising the photographs, Sarfaty & Ladin (1993) developed a sophisticated video system which automatically determines the elliptical zones after segmental boundaries are delimited by the operator. The video system was evaluated by estimating segmental inertia parameters for a thigh. Reasonable agreement was found between inertial estimates for the thigh derived from the video based system and those from Zatsiorsky & Seluyanov (1985) which used a similar population of subjects. Such technological advances, if retaining the accuracy of a particular technique, are of benefit to researchers who require estimates of personalised segmental inertia parameters in a short period of time.

Hatze (1980) developed a comparatively complicated mathematical inertia model of the human body to determine all subject specific segmental inertia parameters. Although the human body is modelled using only 17 segments, a total of 319 geometric solids, including elliptical cylinders, cuboids and hemi-spheres, are used to construct these segments. A battery of 242 anthropometric measurements are required to determine the dimensions of the solids. Appropriate segmental density values of Dempster (1955) are used for each solid. In addition a non-linear subcutaneous body fat equation is implemented which allows variations in segmental density to be taken into account.

Using this model errors in calculated and actual whole body mass were less than 0.5% for the four subjects used. Using criterion values for segmental parameters determined by experimental techniques and appropriate cadaver studies the mean errors for all other calculated segmental parameters were reported to be less than 4%.

For the effective application of a mathematical inertia model in sports biomechanics any anthropometric data required may need to be collected quickly because the subject-researcher interaction time is often restricted. Hence, besides model accuracy being a consideration the time required to obtain the anthropometric data must also be taken into account. Hatze (1980) stated that 80 minutes were required to collect all 242 anthropometric measurements. This limits the usability of this model in certain research situations because this length of time for the collection of anthropometric data is not always permissible. To overcome such time factors Baca (1996) developed a video based system for determining the anthropometric measurements required for the model of Hatze (1980). Utilising video images of the subject in four positions 220 of the measurements required for the model of Hatze (1980) were directly measured from the video system. The remaining 22 measurements were calculated from regression equations. Although this video method decreased the time required of the subject a loss in accuracy was observed.
Yeadon (1990b) describes a personalised mathematical model of the human body which is more simplified than the model of Hatze (1980). The whole of the human body is modelled by 40 geometric solids (Figure 2.17a) which are used to produce an 11 segment model (Figure 2.17b). Subject specific values for the dimensions of the 40 solids, including truncated cones, stadium solids for the torso and a semi-ellipsoid of revolution for the cranium of the head, are determined from 95 anthropometric measurements using standard tapes and anthropometric calipers.

![a) 40 geometric solids  b) 11 segments](image)

Figure 2.17. The mathematical inertia model of Yeadon (1990b)
(Adapted from Yeadon, 1990b).

The use of truncated cones for calculating the volume of a leg was advocated by Jones & Pearson (1969) as well as Katch & Katch (1973), while the representation of the torso as a stadium solid has also been proposed previously by Sady, Freedson, Katch & Reynolds (1978). This particular solid appears to more closely represent the cross-section of the thorax than the solids previously used (Yeadon, 1990b). Values for segmental mass, mass centre location and principal mass moments of inertia are calculated using standard equations for such solids in conjunction with the appropriate segmental density values of Dempster (1955).

Using whole body mass estimates to determine the accuracy of the inertia model the maximum error for three subjects was 2.3%. This value of error is slightly larger than that reported by Jensen (1978) for the same parameter. In order to determine the accuracy of the other segmental inertia parameters all values for one subject were utilised in a simulation model performing twisting somersaults. As the simulated and actual performance for a twisting somersault were comparable the segmental inertia parameters representing the human body were deemed to be sufficiently accurate for the analysis of gross human movements.
One advantage of the model of Yeadon (1990b) over mathematical models of a similar accuracy is the time required to collect the anthropometric data. In contrast to the 80 minutes required by the model of Hatze (1980) and the 130 minutes required by the photogrammetry technique of Jensen (1978) the data for the model of Yeadon (1990b) can be easily and accurately collected in 25 minutes.

Summary

Of the three broad techniques which exist for estimating segmental inertia parameters, the method of mathematical modelling would seem the most appropriate if subject specific estimates are required. As the method of Yeadon (1990b) produces acceptably accurate estimates with minimal inconvenience to the subject, this technique may be considered suitable for this study.

2.5.3 Techniques for three-dimensional image analysis

Several techniques for calculating the three-dimensional locations of landmarks on human subjects or inanimate objects have been developed (Abdel-Aziz & Karara, 1971; Dapena, Harman and Miller, 1982; Pigos & Baltzopoulos, 1993; Chow, 1994). These techniques generally involve using at least two static stills, cine or video cameras, which simultaneously record images of the subject (Yeadon & Challis, 1994). Arguably the most widely used three-dimensional reconstruction technique used in sports biomechanics is that developed by Abdel-Aziz & Karara (1971).

Three-dimensional direct linear transformation (DLT)

Abdel-Aziz & Karara (1971) described a method by which non-metric photographic cameras could be used to determine the three-dimensional location of an object in space from at least two images of the object. This technique is termed the Direct Linear Transformation (DLT). Unlike other methods, such as those described by Bergemann (1974) and Dapena et al. (1982), the DLT technique does not require the internal or external parameters of each camera to be calculated. Estimations of internal and external camera parameters are possible sources of error in the non-DLT methods (Shapiro, 1978). The experimental procedure required to perform the DLT technique has been described by several authors, including Van Gheluwe (1978), Wood & Marshall (1986), Challis (1995) and Yeadon (1996).

The DLT procedure assumes that point P in space, the centre of the lens C and the image I on the digitiser, are colinear (Figure 2.18).
This colinearity condition produces a direct linear transformation defined by 11 parameters, termed DLT parameters. The linear transformation relates the digitised coordinates \((u,v)\) of the image \(I\), the coordinates \((u_0,v_0)\) of the centre of the digitiser \(O\), the location of the lens centre \(C\) in space \((x_c,y_c,z_c)\), the focal length \(c\) of the lens and the location \((x,y,z)\) of the point \(P\) in space. The 11 DLT parameters are therefore functions of the six external camera parameters (location and orientation) and five internal camera-digitiser parameters (scaling, shear and image centre).

In order to calibrate the camera-digitiser system a calibration structure, comprising calibration markers, is erected which surrounds the volume of interest. Within the calibration structure the location of at least 6 different calibration markers relative to an origin must be known (Chen, Armstrong and Raftopoulos, 1994). The relationship between the 11 DLT parameters \((L_1 ... L_{11})\), the digitised coordinates of the known calibration points \((u,v)\) and the locations of the known calibration points \((x,y,z)\) can be written as:

\[
\begin{align*}
    u &= \frac{L_1 x + L_2 y + L_3 z + L_4}{L_9 x + L_{10} y + L_{11} z + 1} \\
    v &= \frac{L_5 x + L_6 y + L_7 z + L_8}{L_9 x + L_{10} y + L_{11} z + 1}
\end{align*}
\]  

Twelve equations are produced from the 6 calibration points meaning the calculation of the 11 DLT parameters \((L_1 \text{ to } L_{11})\) for the first camera is performed using a least squares method as the system is over-determined. A set of 11 DLT parameters are
then calculated for the second camera \((M_1 \ldots M_{11})\) using the digitised coordinates \((q,r)\) of the calibration points from the second camera view.

The three-dimensional reconstruction of points of unknown location is performed by combining the calibrated digitiser-camera systems in terms of \((L_1 \ldots L_{11}\) and \(M_1 \ldots M_{11}\)) and determining the digitised coordinates from the two camera views \((u,v)\) and \((q,r)\). Using this procedure four equations are produced which, in combination, contain the unknown three-dimensional locations \((x,y,z)\) of the digitised point. The three-dimensional location of the digitised point is therefore determined using a least squares solution to the following four equations for the over-determined system:

\[
\begin{align*}
(L_1 - L_9 u)x + (L_2 - L_{10} u)y + (L_3 - L_{11} u)z &= (u - L_4) \\
(L_5 - L_9 v)x + (L_6 - L_{10} v)y + (L_7 - L_{11} v)z &= (v - L_8) \\
(M_1 - M_9 q)x + (M_2 - M_{10} q)y + (M_3 - M_{11} q)z &= (q - M_4) \\
(M_5 - M_9 r)x + (M_6 - M_{10} r)y + (M_7 - M_{11} r)z &= (r - M_8)
\end{align*}
\] (2.2)

The 11 parameter DLT technique of reconstructing the three-dimensional locations of points in space has been applied to various sized volumes, from very small \((2.8 \text{ m}^3; \text{Chen et al., 1994})\) to much larger volumes \((13 \text{ m}^3; \text{Wood & Marshall, 1986})\). Considerations concerning the use of DLT include: the simplicity and flexibility of its use, the calibration structure, the accuracy in estimating three-dimensional locations of landmarks and the choice between video or cine cameras and digitising systems. These considerations are critically reviewed within the following sub-sections.

**Accuracy of 11 parameter DLT**

Initially the 11 parameter DLT technique was developed for use with non-metric photographic cameras with high resolution lenses (Marzan & Karara, 1975). In order to study the motion of human subjects, however, a sequence of exposures is required. Therefore 16 mm cine cameras with high framing rates and more recently video cameras have replaced stills cameras, while still adopting the DLT technique (Shapiro, 1978; Wood & Marshall, 1986; Kennedy, Wright & Smith, 1989; Tan, Kerwin & Yeadon, 1995).

Several studies have determined the accuracy of reconstructing three-dimensional locations for a certain horizontal field of view using 11 parameter DLT. Shapiro (1978) filmed a calibration volume containing 48 points of known location using two Locam cine cameras. The horizontal field of view for each camera was 3 m. The accuracy of the 11 parameter DLT technique was assessed during static and dynamic situations. Twenty of the 48 calibrations points were used to determine the 11 DLT parameters for each camera,
with the other known points being used to assess the reconstruction accuracy of the technique for a static situation. Acceleration due to gravity of a free falling golf ball was also estimated from film using 11 parameter DLT. For the static situation the average error associated with the three-dimensional location of the known points was 5 mm, (0.16% of the horizontal field of view). Errors in the estimation of acceleration due to gravity were in the order of 1% and 4%. Similar values for mean errors in accuracy have been determined for other cine based systems including: Wood & Marshall (1986), (5.8 mm, or 0.16% of the horizontal field of view) and Kennedy et al. (1989) (4.8 mm, or 0.14% of the horizontal field of view).

In more recent years the use of video cameras and digitisers for motion analysis, instead of cine based systems, has increased (Angulo & Dapena, 1992). Using 11 parameter DLT the difference in reconstruction accuracy between the two forms of camera and data analysis has been addressed. Kennedy, Wright & Smith (1989) recorded a calibration volume containing 20 points over a 3.5 m horizontal field of view using both cine and video cameras. Using 11 parameter DLT the mean error in reconstruction accuracy for 16 mm cine film was found to be 4.8 mm, while for video the mean error was 5.8 mm (or 0.17% of the horizontal field of view). Although reconstruction errors between video and film were statistically different, the absolute difference of 1.0 mm may be thought of as acceptable for the analysis of gross human motion.

Using an 8 m field of view Angulo & Dapena (1992) also observed larger errors in reconstruction accuracy when video cameras were utilised compared to cine cameras. Within the calibration volume the root mean square errors for video analysis were 10 mm (0.12% of the horizontal field of view), while film analysis errors were 5 mm (0.06% of the horizontal field of view). Suggestions for this difference in accuracy included the resolution of the video image, the difference in quality of the lenses in the cameras, and the resolution of the video digitising system. The suggestion that lens distortion, inherent in the lenses of video cameras, significantly decreases reconstruction accuracy has also been demonstrated by Tan (1997).

Both Kennedy et al. (1989) and Angulo & Dapena (1992) concluded that within the calibration volume both formats of analysis for three-dimensional reconstruction using 11 parameter DLT showed sufficient accuracy for the analysis of gross human motion. Further evidence to support this suggestion can be gained from several other studies. Shapiro, Blow, & Rash (1987) demonstrated the possibility of successfully implementing video based digitisers, providing data of equal accuracy to cine based systems. Challis (1995), combined with a multi-phase calibration volume, observed accuracy values only slightly worse than those using cine based systems (errors of 6.1 mm, or 0.20% of horizontal field of view). Tan, Kerwin & Yeadon (1995) also obtained levels of accuracy for a 10 m field of view which are better than those obtained in studies already mentioned (6.3 mm, or 0.063% of the horizontal field of view). This level of accuracy
was obtained using specific video recording formats (Hi8) combined with a Target high resolution video digitising system (Kerwin, 1995).

Three-dimensional studies in the sport of gymnastics have also implemented 11 parameter DLT (Kerwin, Harwood & Yeadon, 1993; Yeadon, 1994b). Yeadon (1994b) recorded gymnasts dismounting from rings during the 1992 Olympics from two video cameras. The image for each video camera contained a 10 m horizontal field of view. The reconstruction accuracy of the DLT procedure was 17 mm (0.17% of the horizontal field of view), though a comparatively lower resolution video digitiser was used than the one used in the study of Tan et al. (1995).

The current literature would seem to suggest that high resolution video recording formats (sVHS and Hi8 display a horizontal resolution of 400 lines) combined with high resolution video digitising systems (eg Target digitising system; Kerwin, 1995) are able to reproduce levels of accuracy comparable to cine based systems (Kerwin & Challis, 1994). Video based recording and digitising systems may therefore be adopted for accurate three-dimensional image analyses of gross human movements.

*Calibration points and extrapolation*

Motion of a subject in certain sporting situations encompasses a large volume. For accurate three-dimensional reconstruction either the calibration volume must encompass this whole volume whilst maintaining the accuracy of the control points, or extrapolation outside of the calibrated volume is required (Challis, 1995).

Several studies have examined the effect of differing numbers of calibration points within the volume of interest for the calculation of the 11 DLT parameters on the reconstruction accuracy (Wood & Marshall, 1986; Hatze, 1988). Although a minimum of 6 calibration points are required more can be used to calculate the 11 DLT parameters. Both studies highlighted that as more calibration points were used to determine the 11 DLT parameters the reconstruction accuracy increased. However the increase in accuracy was not directly proportional to the number of calibration points used. Shapiro (1978) showed that using more than 12 calibration points did not significantly reduce reconstruction errors. In a similar vein Chen et al. (1994) found similar results indicating 16 calibration points were required, after which limited increases in accuracy were observed. Chen et al. (1994) also suggested that most accurate reconstructions were attained when the calibration points were spread evenly throughout the volume.

It is sometimes difficult to place calibration points throughout the space in which the activity is taking place, meaning that extrapolation outside the calibrated volume is required. However, Wood & Marshall (1986) showed that extrapolation outside of a relatively small calibrated volume was significantly inferior in accuracy to when the whole volume was calibrated. Reconstruction accuracy of points outside the calibration
volume decreased by 100% when compared to accuracy where no extrapolation was required. This finding was maintained even when only 7 control points were used to calibrate the large volume. Angulo & Dapena (1992) also showed that reconstruction accuracy outside of the calibration volume was poor. These findings indicate that every attempt should always be made to ensure the calibration structure encompasses the whole activity when using 11 parameter DLT.

Challis (1995) implemented a multiphase calibration procedure in order to calibrate a large volume (3.6 m³) using a small (0.6 m³) calibration structure moved in a specified manner over the whole space of interest. Using 11 parameter DLT the multiphase procedure showed significantly greater reconstruction accuracy than when known points were reconstructed using extrapolation techniques outside of the initially calibrated volume. Moving the calibration structure is therefore one possible solution to calibrating large volumes. A second approach was adopted by Ball & Pierrynowski (1988), who used points relative only to the calibration points to increase the total calibrated volume.

When extremely large fields of view are required the movement of the cameras in a panning and tilting action may be required. Yeadon (1989) developed a procedure which allowed a subject to be followed during his motion throughout a calibrated field, giving a three-dimensional reconstruction of the subject throughout the motion. Subjects performing a ski jump were analysed using this technique.

For situations where non-panning and tilting cameras are used it would seem the consensus is that at least 12 calibration points positioned throughout the volume are required to accurately calibrate a camera-digitiser system. More calibration points may only slightly increase reconstruction accuracy. Furthermore, greater accuracy may be obtained when the calibration points are spread throughout the volume of interest Chen et al. (1994).

**Lens distortion**

The originally described 11 parameter DLT technique assumes that no distortion of the image occurs owing to the lens of each camera or digitising system. In reality this is not the case, with two sources of lens distortion apparent for cameras. Symmetrical lens distortion causes displacement of an image relative to the optical axis of the camera and results in radial distortion of the image. Asymmetrical lens distortion is caused by the imperfect alignment of lenses resulting in tangential distortion of image (Allard, Blachi & Aissaoui, 1995). Marzan & Karara (1975) identified the possibility of lens distortions and adapted the 11 parameter DLT to include three parameters defining symmetrical lens distortions and a further two parameters for asymmetrical distortions. These adaptations to the DLT procedure resulted in 16 parameters describing the direct linear transformation.
A second method of correcting for lens distortions was demonstrated by Chen, Armstrong & Raftopoulos (1994). In this procedure lens distortions were corrected for by implementing a quadratic function from the centre of a rectangular calibration structure after the digitised points were reconstructed using 11 parameter DLT. Such a method requires a specific calibration structure, encompassing the whole space.

Wood & Marshall (1986) simplified distortions in a photographic camera lens by using one parameter to represent non-linear symmetrical lens distortion (12 parameter DLT). Using up to 30 control points for a calibration volume the accuracy was assessed using 11 and 12 parameter DLT for a field of view slightly greater than 3.5 m. The results indicated that accuracy did not significantly increase when corrections for lens distortion were made. It was suggested that this finding was due to the lenses in stills cameras being of high quality and producing insignificant lens distortion.

Hatze (1988) described a modified DLT technique which also modelled linear and non-linear lens distortions, by implementing 15 parameters to describe the DLT transformation. Cine cameras were used to film a calibration structure in a 2 m horizontal field of view. In contrast to Wood & Marshall (1986) it was found that correcting for lens distortions significantly decreased the reconstruction errors from 4.8 mm to 0.8 mm (0.24% and 0.04% of the horizontal field of view). However, such increases in accuracy were only observed when 30 or more calibration points were used. The increase in reconstruction accuracy was thought to be due to the modelling of lens distortion together with modifications to the original DLT procedure and hence, the effect of solely correcting for lens distortion could not be fully quantified.

Video cameras typically possess lower quality lenses and hence correcting for lens distortions in video-digitiser based systems needs to be addressed (Yeadon, 1996). Such an investigation was undertaken by Tan (1997), using two-dimensional DLT to reconstruct digitised points spread evenly throughout a 10 m horizontal field of view. When points near the extremes of the video image were ignored the overall horizontal reconstruction accuracy decreased from 15 mm to 10 mm in the horizontal plane, highlighting a form of radial lens distortion in the video camera-digitising system. By implementing a further DLT parameter to model radial symmetrical lens distortion in the camera-digitising system, significantly greater horizontal reconstruction accuracy was observed. Using an optimised configuration of video recording format (Hi8) digitising system (Target-Apex high resolution digitising system) and lens correction Tan (1997) showed that horizontal and vertical reconstruction accuracy using video images was comparable to that of 16 mm cine cameras and digitising systems, with errors less than 3 mm horizontally and vertically. For sporting situations where the activity spans the full area of the image, lens distortions may have a large influence on reconstruction accuracy. Corrections for such lens distortions should therefore be considered in order to increase the accuracy of kinematic data.
In agreement with Tan (1997), Chen et al. (1994) observed reconstruction error at the extremes of the video images. However, for activities which remain within the confines of the centre of the image the influence of lens distortions is less apparent. It is suggested therefore that whether lens distortion correction will greatly enhance reconstruction accuracy is dependent on the location of the points to be digitised on the video image.

**Synchronisation of cameras**

When reconstructing the three-dimensional location of a point from video or film data the images recorded by the cameras are required to be taken simultaneously. Simultaneous recording of images from two or more cameras can be achieved by using a physical link, termed gen-locking for video cameras, and phase-locking for cinematographic cameras (Yeadon & Challis, 1994). This physical link ensures the images taken by the cameras are recorded at the same instant in time. By itself this technique does not give a known time base to the images from each camera. In order to provide a time base for the images an easily recognisable and unique signal is required. This unique signal can be achieved by illuminating an LED in the fields of view of all cameras (Hiley, 1998). It may also be possible to use the sporting action being filmed as the unique signal, such as the initial foot contact for a particular stride during a high jump (Greig, Yeadon and Kerwin, 1996).

When it is not practical to physically link cameras, possibly due to their geography in the sporting situation, other methods of producing data at the same instant in time from the recorded images are required. One such method involves making use of the digitised data from each camera and its subsequent interpolation (Yeadon, 1989).

It is suggested, whenever possible, that the physical linking of cameras is utilised together with a unique signal in the fields of view of the cameras. This ensures the simultaneous recording of the chosen activity at the time of data acquisition which reduces the complexity of synchronising image data at a later stage.

**Automatic tracking and motion analysis systems**

Owing to advances in technology, commercially available automatic tracking and motion analysis systems exist which estimate the three-dimensional locations of required landmarks (Yeadon & Challis, 1994). In the case of analysing an athlete such automatic systems require the attachment of specific markers to body landmarks of interest. MacReflex and CODA systems, for example, require the use of reflective markers and LEDs respectively. With the markers secured on the subject, the automatic systems negate the digitisation process required in manual systems to obtain the three-dimensional
location of the body landmarks. These systems may therefore decrease the time needed to obtain the three-dimensional kinematics of the studied motion (Lindsay, 1996).

However, several disadvantages of automatic motion analysis systems may detract from their apparent benefits. In the competitive sporting environment, for example, the attachment of markers may not be permitted. In situations where complex motions occur, such as gymnastics, there is the potential for markers to be dislodged or obscured from the view of all cameras. Furthermore automatic motion analysis systems estimate the three-dimensional location of each marker, whereas it is often the locations of the athletes' joint centres which are of importance. Finally, when the markers require electrical power the source of power must be attached to the athlete. This may hinder the athlete or may even not be possible. Such disadvantages of the automatic tracking systems mean manual systems are often more appropriate for specific studies.

**Synchronisation of force and image data**

The synchronisation of force and video or film images is often required to complete kinetic and kinematic descriptions of motion or for inverse dynamics analyses (Dixon, 1996; Kerwin, 1997). Errors associated with the misalignment of force and image data in inverse dynamics analyses of human gait have been quantified (Cappozzo, Leo, & Peddotti, 1975; Winter, 1990). With the misalignment of force and image derived data by one video field (0.0166 s for NTSC) errors in the maximum joint moments at the knee during walking were estimated to be 59% (Winter, 1990).

Force and video images may be synchronised using a trigger which forms a contact to complete a circuit with known voltage. The completion of the circuit is simultaneously recorded by the ADC sampling the force data and illuminates an LED in the fields of view of the cameras (Dixon, 1996; Kerwin, 1997). The accuracy to which these two sets of data may be synchronised depends largely on the sampling frequencies of cameras, which for standard video cameras is 50 Hz (PAL). O'Conner, Yack & White (1995) stated that using this method of synchronisation a maximum error of almost one video field may be unknowingly obtained. However, using a midpoint alignment procedure (Section 4.6) the maximum potential error in synchronisation of force and image derived data is only ±½ video field.

A method by which greater synchronisation accuracy may be obtained was proposed by O'Conner et al. (1995). This method involves a detailed knowledge of how video cameras produce images. The voltage of the video signal is sampled at a frequency slightly different to that used to sample the force data. Although allowing an accuracy in alignment of 0.00084 s for an NTSC system a dramatic increase in recorded data is required. To accommodate such an increase in data is not always feasible.

The synchronisation of force and video data using using an illuminated LED in the
fields of view of cameras combined with the midpoint method is simple and convenient, proving ±½ video field accuracy. The illumination of the LEDs may also serve a dual purpose, providing a simultaneous 'event' for gen-locked video cameras.

**Summary**

The three-dimensional image analysis technique of DLT is able to produce data of sufficient accuracy for the study of gross human motion. This, however, is dependent on a number of provisions. Firstly, an adequate number of control points must be distributed throughout the space of interest. Secondly a high resolution video format (Hi8 or sVHS) in combination with a high resolution digitising system must be used. The Target system is such a digitising system (Kerwin, 1995), using sVHS tape format and providing a maximum pixel resolution of 12288 x 9216 through zoom facilities and sub-pixel cursor movements. If the digitised points are not located to the extremes of the video image the influence of any lens distortions are limited and 11 parameter DLT may be used without substantially decreasing the accuracy. With these prerequisites satisfied reconstruction accuracy is similar to that of cine based systems.

One possible limitation in using standard video cameras is their fixed sampling frequency of 50 Hz (PAL systems). Although higher sampling frequencies are possible, this is often at great expense and therefore not viable. To avoid the problem of aliasing, the sampling frequency must theoretically be at least twice the frequency of the signal which is of interest. The majority of gross human movements, neglecting high velocity impacts, are at frequencies near or below 6 Hz, (Winter, Sidwall & Hobson, 1974). It may be inferred that the sampling frequency of video cameras is adequate to record gymnasts swinging on rings.

2.5.4 Techniques of curve fitting experimental data

When experimental data are measured the recorded signal invariably contains the true signal with an added 'noise' component (Lees, 1980). The term 'noise' may be used to describe those components of the sampled signal which are not due to the process itself (Winter, 1990). The noise of a signal can be further considered as consisting of systematic and random components. Owing to the introduction of noise in any sampled signal the true signal can never be known, though a best estimate of the true signal should be sort (Yeadon & Challis, 1994). Noise in a signal must be attenuated if derivatives are to be determined from the original data. If the noise components are not reduced prior to the differentiation process the noise in the signal may be amplified to such an extent that derivative values are meaningless.
Various methods are available by which estimates of the true signal may be obtained and derivatives calculated. These methods include: low pass digital filters combined with finite difference techniques, truncated Fourier series, cubic and quintic spline functions and low order polynomials fitted using a least squares difference technique to the data (Wood, 1982). All of these methods have been used with varying success in biomechanical research.

**Digital filters**

Digital filters are frequency selective devices which take equispaced data with noise and produce data of a limited frequency (Wood, 1982). From the filtered data finite difference techniques can be used to determine first and second derivative data. Winter, Sidwall & Hobson (1974) used a low pass second order Butterworth digital filter with a 6 Hz cutoff frequency to attenuate noise in displacement data from film analysis and subsequently determine horizontal acceleration of a toe marker during a stride. The cutoff frequency was determined using a Fourier analysis and identifying those frequencies which could be considered to comprise mostly noise components. The filtering process was shown to attenuate the high frequency content of the raw data by a subjective comparison of time histories for the filtered and raw data.

Challis & Kerwin (1988) more rigorously investigated the use of Butterworth filters in calculating derivative values from noisy data. Data were generated from five mathematical functions which allowed a direct calculation of derivative values to be made. Noise components were then added to the data to reproduce levels incurred during film analysis. Several curve fitting techniques were used to estimate values of the second derivative for each original data set. It was found that a second order Butterworth filter with central difference equations to calculate derivatives was inferior to other curve fitting techniques for all test functions. In contrast Andrews, Cappozzo & Gazzani (1981) used a phase cancelled fourth order Butterworth filter for alternative analytical test functions and found for estimating the second derivative this process was more accurate than other fitting techniques. Similar findings regarding the suitability of using a Butterworth digital filter and first order finite difference methods are provided by Pezzack, Norman, and Winter (1977). These conflicting conclusions concerning digital (Butterworth) filters may in part be attributed to the data used to test the curve fitting techniques and the finite difference techniques used.

Kerwin (1997) identified problems with digitally filtered original data and the subsequent derivative values. End-point errors may be resolved to a degree using several methods, all of which are non-trivial. Methods include padding both ends of the data set by zeros, reflection of the data about the end points from the mid-point outwards, and the collection of more original data than is necessary. Such end-point problems may arise in
all curve fitting techniques, especially in the estimations of derivatives.

**Truncated Fourier series**

A Fourier series provides a method by which a periodic function, of equispaced data, can be expressed by a weighted sum of sine and cosine terms of increasing frequency (Wood, 1982). Signals not showing a full period may still be described by a trigonometric series after correction for the two extreme data points (Lanczos, 1966) or a linear detrending of the data (Cappozzo & Gazzani, 1983; Wood, 1982). Direct differentiation of the sine and cosine terms allows the estimation of first and second order derivatives (Cappozzo, Leo & Pedotti, 1975). The frequency contents of several aspects of human motion have been determined using Fourier analyses and have been used to identify differences in patterns of motion (Winter et al., 1974; Kerwin & Chapman, 1988; Schneider & Chao, 1983).

The capability of the truncated Fourier series for curve fitting a signal and estimating first and second order derivatives has been previously investigated using analytical functions contaminated with noise (Andrews et al., 1981; Wood, 1982; Challis & Kerwin, 1988). In general the frequency of truncation has been calculated by determining the optimal residual difference between the original and curve fitted data (Winter, 1990). Hatze (1981a) however proposed a second method of optimising this truncation process for a Fourier sine series based upon the statistical properties of the signal in the frequency domain. This method negates the often subjective choice of cutoff frequency used in digital filters. Results from Challis & Kerwin (1988) highlighted that the truncated Fourier series performed well when determining the first and second time derivatives for certain functions, such as those displaying a periodic nature. Furthermore, Andrews et al. (1981) observed that the truncated Fourier series produced accurate estimates when used to determine values for the second derivative of a function which was aperiodic.

The Fourier series transformation may also be effectively used to eliminate unwanted specific frequencies within a signal, in a similar manner as a digital notch filter (Press, Flannery, Teukolsky & Vetterling, 1988).

**Spline functions**

A natural spline function comprises a number of piecewise low order (n) polynomials pieced together at a number of knots, providing a continuous function g(t) with 2m-2 continuous derivatives (where m = (n+1)/2). Derivatives of the curve fitted data are determined algebraically from the coefficients of the piecewise splines. Specifications for a spline function, such as the one devised by Reinsch (1967, 1971),
include the closeness of fit of the spline to the experimental data, which is controlled by a parameter, \( S \), and defined by the operator. In effect the value of \( S \) defines the mean error incorporated in the experimental data. Importantly, spline functions do not require the experimental data to be equispaced, unlike digital filter and Fourier series techniques (Wood, 1982). Furthermore, owing to the piecing together of splines at knots the behaviour of the splines in one region of data can be totally independent of the behaviour in another region. This quality allows rapid changes in the original data to be retained (Wood & Jennings, 1979).

Several studies analysing human locomotion have used the cubic spline fitting algorithm, developed by Reinsch (1967), to determine velocity and acceleration data from experimentally determined displacement data. Zernicke, Caldwell & Roberts (1976) and McLaughlin, Dillman & Lardner (1977) both compared the use of cubic splines to least squares polynomial curve fitting for experimentally determined displacement data and the estimation of velocity and acceleration values. A least squares difference polynomial technique determines the function, of order \( n \), which approximates the experimental data in a 'best fit' scenario (Winter, 1990). In both studies simple planar movements were analysed. Displacement data obtained from digitising film sequences were fitted using both techniques and relevant acceleration values determined for the motion. For the kicking movement analysed by Zernicke et al. (1976) the vertical ground reaction force was recorded and also calculated from the acceleration estimates. The mean difference between computed and recorded ground reaction forces were shown to be smaller when a cubic spline was utilised (4.75%) as compared to a fifth order polynomial (10.27%). It was concluded that fitting the displacement data with a cubic spline produced acceleration results which were more accurate than when displacement data were fitted using a low order polynomial. When comparing the use of polynomial and cubic splines McLaughlin et al. (1977) used four different activities: a vertical jump, a kicking action, elbow flexion and a modified squat. In agreement with previous studies McLaughlin et al. (1977) highlighted that cubic splines produced more representative and realistic velocity and acceleration values than curve fitting techniques employing low order polynomial functions.

Both studies noted that the end-points of a cubic spline were zero for the second derivative. Problems associated with this fact were highlighted using an object in free fall (McLaughlin et al., 1977). On curve fitting displacement data for a free falling object acceleration due to gravity was calculated. Owing to the nature of cubic splines the end-points of the second derivative were zero which produced a rapid rise from zero and 'overshoot' for the acceleration values. Both studies suggested that by extending the data series at the beginning and end of a digitised sequence this problem could be negated. As noted previously, the problem of end-points for derivative data is not restricted to spline fitting techniques.
More recently splines of a higher order, termed quintic splines (n=5), have been used in biomechanical studies. Wood & Jennings (1979) further discussed problems with the implementation of cubic splines. For initially curve fitted displacement data the values of the third derivative, or 'jerk', are discontinuous. This characteristic may be thought of as inappropriate for human motion. Visually quintic spline functions were shown to reproduce more accurately acceleration data after differentiation than cubic splines. Furthermore for quintic splines the end-points of the second derivative are not forced to zero, enabling better estimates of acceleration at the extremes of the data sets.

As previously noted Challis and Kerwin (1988) compared the curve fitting technique of quintic splines to other forms of curve fit, including a Butterworth digital filter and truncated Fourier series. It was shown that quintic splines were generally superior to other techniques. Exceptions included the truncated Fourier series for certain data sets, especially those data sets which displayed a periodic nature.

Dowling (1987) suggested the use of different curve fitting techniques for different portions of a data set, utilising the qualities of each curve fitting technique. Using splines, digital filters and truncated Fourier series the second derivative data were calculated, selecting the most appropriate fitting technique to the portions of the signal. Although this method allows the researcher to use the unique characteristics of each curve fitting technique to their full potential, it is both computationally consuming and may still possess problems concerned with end-points and discontinuities between the selected portions.

Polynomials

If a theoretical relationship exists between two variables then that relationship may be used as a basis for the curve fitting technique to be employed. Sprigings, Burko, Watson and Laverty (1987) investigated the accuracy of differing inertia models of humans by using a human in flight. Initially acceleration due to gravity, for a small object was directly determined by fitting a least squares quadratic function to the digitised displacement data. Equations of constant acceleration were employed and acceleration due to gravity was calculated as \(-9.84 \text{ m.s}^{-2}\). With the curve fitting techniques of splines and digital filters adopted to calculate acceleration due to gravity problems of zero end-points and 'overshoot' were again observed. Fitting a least squares quadratic function to the vertical mass centre location time history of a body during free flight provides a theoretical basis for the data fitting technique.

Summary

Various techniques for curve fitting data exist which may be successfully applied to
noisy experimental data and from which accurate derivatives can be determined. Essentially any procedure that can be shown to produce valid results within the context of motion being analysed may be acceptable (Wood, 1982). It is suggested that the most applicable technique depends on the data being analysed (Andrews et al., 1981) and certain curve fitting techniques may be more theoretically applicable than others (Sprigings et al., 1987). For more general data sets quintic splines and truncated Fourier series seem able to produce valid results. Furthermore, the first and second derivatives of the experimentally derived data are easily determined using these methods, negating finite difference techniques of differentiation and their inherent inaccuracies. Errors inherent in end-points and their derivative estimations may be reduced by collecting more original data than necessary. As a result these two curve fitting techniques are used extensively within this study.

2.5.5 Global optimisation techniques

It is often possible for a performance or outcome from a theoretical setting to be specified by a single objective function comprising one or more independent variables. The objective function may be continuous or discontinuous, depending on the type of independent variables and any limits to the values they may take. In the case of an objective function relating to human motion such independent variables may be running speed or muscular strength at a joint. Hence the limits may relate to the fastest running speed possible or maximal strength of a particular joint for the human under study. Typically, the values of the independent variables which produce either a global maximum or minimum value for the objective function are required as these values provide the optimum performance as defined by the objective function.

Besides a single global maximum or minimum (truly the highest or lowest function value) local maxima or minima (the highest or lowest function value in the restricted finite neighbourhood) may be apparent in the function (Press et al., 1988). Care must be taken when attempting to find the global maximum or minimum of a function, else local maxima or minima may be unknowingly observed and mistaken for the global optimum. Such results may lead to incorrect conclusions concerning the objective function. Using computerised optimisation algorithms the optimum value of a function, be it a minimum or maximum, may be determined together with the values of each independent variable required to obtain this solution. From this point onwards the term minimisation is adopted as the specific case for both forms of optimisation.
Computerised global optimisation algorithms

Several methods of determining the minimum value of a linear function, that is a function with only one independent variable, exist. These methods include the **Golden Section Search** which brackets the function using a triplet of points and **Brent's Method** which fits a parabola to evaluated points of the function (Press et al., 1988). However, if the function to be minimised comprises more than one independent variable, linear optimisation methods may not find the global minimum, habitually finding only the first local minima (Corina, Marchesi, Martini & Ridella, 1987).

Owing to the limitations of linear minimisation methods several other techniques exist which minimise a given function consisting of $N$ independent variables. Of these minimisation techniques some require only evaluations of the function from a set of values for the independent variables. Other techniques however, require the calculation of the function's first derivative in order to determine the global minimum. If the objective function is unknown or discontinuous the performance of those techniques which require information regarding the derivative of the function may deteriorate. It is therefore proposed that techniques which make no assumptions concerning the function to be minimised and do not require the calculation of the function's gradient may be best suited to finding the global minimum of a function with unknown behaviour. Techniques which allow minimisation of an $N$ dimensional function and make no assumptions concerning the function to be minimised include the **Simulated Annealing** described by Corana et al. (1987) and the **Downhill Simplex Method** developed by Nelder & Mead (1965).

A simplex is a geometrical object consisting of $N+1$ vertices in $N$ dimensional space. In order to initialise the downhill simplex algorithm $N+1$ evaluations, or starting points, are required to form the simplex. To obtain the global minimum of the function the simplex performs 'steps' by reflecting the highest evaluated point through the opposite face and producing another vertex at a lower function value. By taking many successive steps the simplex 'walks downhill'. On reaching a valley, possibly a local minimum, the simplex may flow along the valley floor. However, as the simplex technique only evaluates the function using a reflection process, when located in a valley (or local minimum) the reflected vertices of the simplex may remain within the constraints of the valley's sides. This minimisation technique may therefore find difficulties in obtaining the global minimum for a function which contains many valleys and hills. Multiple starts with the vertices of the simplex at different evaluation values of the function may partially overcome this imperfection. Restarts from different 'starting points' may also be required to determine whether a located minima is a local or the global minima (Press et al., 1988). These imperfections increase the computational costs when using this algorithm and reduce the confidence in its findings.
The origins of the simulated annealing technique lie in thermodynamics and the process of cooling a liquid metal so that the energy state of the metal obtained after cooling is at a global minimum. Unlike other algorithms the simulated annealing is based on a random evaluation of the objective function throughout \( N \) dimensional space, in such a way that transitions out of a local minimum are possible. The likelihood of transitions out of local minimum is dependent on a temperature parameter controlling the cooling of the function and a Metropolis criterion which rejects or accepts function evaluations (Corana et al., 1987). For a minimisation problem uphill movements are accepted with exponentially decreasing probability, dependent on the increase in the function value from a previously determined point and the value of the temperature parameter (Corana et al., 1987). The possibility of uphill movements in the objective function provides a means by which the algorithm can escape valleys and gives confidence that local minima are avoided and the global minimum found.

Several comparisons have been made between the two described algorithms and other multi-dimensional optimisation algorithms. Corona et al. (1987) used the Rosenbrock test function in 2 and 4 dimensions to assess the usefulness of the simplex, simulated annealing and an Adaptive Random Search (ARS) methods. The discontinuous Rosenbrock test function has \( 10^{10} \) and \( 10^{20} \) local minima in two and four dimensions respectively, with a global minimum of zero. In ten tests carried out for each function the simulated annealing found the global minimum every time. The simplex consistently found the global minimum for the two dimensional function, though for the four dimensional function the algorithm twice settled in local minima. The ARS showed the least reliability, failing to find the global minimum four times throughout all of the tests. Although the simulated annealing found the global minimum more reliably than the simplex method the number of function evaluations required to do so were greater by a factor of 1000. Corana et al. (1987) stated this factor had serious connotations for computational costs in determining the global optimum.

Goffe, Ferrier & Rogers (1994) applied the simulated annealing algorithm to econometric problems. In addition the simplex algorithm and two other algorithms, a conjugate gradient and quasi-Newtonian algorithm which required numerical derivatives of the test function, were used in a comparison study. For a test function provided by Judge, Griffiths, Hill, Lütkepohl & Lee (1985), which has two variables and possesses one local and one global minimum, the simulated annealing always obtained the global minimum. The other algorithms faired no better then a 60% occurrence in finding the global minimum. Further test functions from economics and neural networks were used. These functions used up to 62 independent variables and contained limitations on the values for the independent variables. Although the simulated annealing did not always terminate at the minimum solution for the three test functions the minimum solution found was always less than the minimum values found from multiple restarts using the
other algorithms. As the function to be optimised became more difficult the difference in computer time required between multiple starts of the simplex algorithm and a single start from the simulated annealing became comparable (Goffe et al., 1994).

Summary

Current literature suggests that simulated annealing is the most successful algorithm in finding the global minimum of an $N$ dimensional function. As this quality is essential to this study, and computational costs are not the limiting factor, simulated annealing is the only algorithm to be considered.

2.6 Summary

This chapter reviewed the current understanding of swinging on rings from both coaching and research viewpoints. The questions raised concerning swinging on rings are pertinent to those posed in Chapter 1 and directly addressed as part of this study. Points raised concerning the modelling of a gymnast swinging on rings are used in Chapter 3.

The appropriate techniques of investigation for this study were also highlighted, making use of past studies. Use of these chosen techniques are highlighted in Chapter 4.
CHAPTER 3

DEVELOPMENT OF SIMULATION MODELS REPRESENTING A GYMNAST SWINGING ON RINGS

3.1 Introduction

This chapter describes four simulation models, each representing a gymnast swinging on rings. The order in which the models are presented indicates the increased sophistication with which the models simulate the gymnast and rings apparatus, culminating with the most complex simulation model. The organisation of the chapter represents an enhanced awareness of the complexity of the activity under scrutiny combined with an advancement in modelling techniques.

For each model a diagram and nomenclature are given. Where possible, consistency in nomenclature between models is maintained to allow easier comparison of results between the four models and video analyses. The equations of motion for each model are detailed, together with the methods used to derive them. Subsequent to the equations of motion for each model the numerical integration procedure used to form the simulation is described.

Methods of calculating the various output variables for each model are also highlighted. For example, for each model the time histories of various energy forms are calculated and presented in an output file. Such output, besides being of intrinsic value to the understanding of the motion, may be used to indicate that each model is free from theoretical and programming errors. Furthermore, such output may be used to select suitably small values of time for the numerical integration procedure.

The simulations to be performed using each model are described together with the reasons for their performance. The initial conditions, model parameters and constants used to perform the simulations are also given within the appropriate sub-sections.

3.2 A simulation model of a single segment gymnast swinging on a rigid rings frame: rigid model

3.2.1 Introduction

In order to produce a simulation model which represents a gymnast swinging on rings two components of the model are essential: a gymnast and rings cables. With these two components of the actual system represented, a simulation model may be used to
investigate the interaction between gymnast and cables when attempting to perform a longswing on rings. This interaction, unique to this gymnastics apparatus, would be expected to greatly affect the way in which movements are performed. In addition, values of tension in the rings cables may be estimated.

Although motions of the rings cables and arms are three-dimensional in nature (Chapter 2) these characteristics are not essential to form a simple representation of a gymnast swinging on rings. Hence, to reduce the complexity of such a model it may be assumed that all motions of the rings cables and gymnast are symmetrical and take place in the vertical plane formed by the inertial axes y and z.

The simplest representation of a gymnast swinging on rings, therefore, is a single rigid segment representing the gymnast and a further rigid segment representing both rings cables. As the combined mass of the rings cables (1 kg per rings cable) is relatively small in comparison to the mass of a gymnast (62.88 kg; Takei & Kim, 1990) the modelled rings cable may be regarded as massless, further reducing the complexity of the model.

Using Newtonian mechanics a two-dimensional simulation model representing this scenario was developed. The model comprises a single segment gymnast and massless rigid rings cable linked by a frictionless joint at the hands of the gymnast (Figure 3.1). Air resistance is assumed to be negligible. The rings frame is modelled as a rigid structure, providing the model with the name 'rigid'. The computer language of Fortran is used to perform the calculations required for the simulation model with the program being executed using a SUN SPARC station 1+. A copy of the Fortran program, RIGID, is provided in Appendix A. For a single simulation 30 seconds of CPU time is required.

![Figure 3.1. The rigid model of a gymnast swinging on rings.](image)
3.2.2 Methods

Newtonian mechanics was used to determine the equations of motion for the system forming the rigid model. Figure 3.2, together with the Nomenclature, provides definitions of the lengths and angles which define the orientation and position of the gymnast and cable segments in the Newtonian reference frame. Figure 3.2 also shows the forces acting in the system which were used to determine the equations of motion for the model.

*Free body diagram of the rigid model system*

![Free body diagram of the rigid model](image)

*Figure 3.2. Free body diagram of the rigid model.*

*Nomenclature for the rigid model*

The following Nomenclature not only indicates the meaning of all symbols used in deriving the equations of motion for the rigid model but also shows, in square brackets, the equivalent symbols used in the Fortran code held in Appendix A.

- \(a_1\) \([a1]\) : distance from hands to mass centre of gymnast segment
- \(p\) \([p]\) : length of cable segment
Derivation of the equations of motion for the rigid model

The forces acting on the gymnast segment were resolved horizontally and vertically (Figure 3.2). Applying Newton’s Second Law of motion to the horizontal components produces equation (3.1):

\[ f \sin \phi_c = m_a \ddot{y}_a \]  \hspace{1cm} (3.1)

while applying the Law vertically forms equation (3.2):

\[ f \cos \phi_c - m_a g = m_a \ddot{z}_a \]  \hspace{1cm} (3.2)

A combination of these two equations yields a third equation defining the system. Equation (3.3) is formed by the division of equation (3.1) by (3.2):

\[ \tan \phi_c (\ddot{z}_a + g) = \ddot{y}_a \]  \hspace{1cm} (3.3)

A fourth equation defining the system is obtained by using the rotational version of Newton’s Second Law, which states:
torque = the rate of change of angular momentum for the whole system
about a point O

or \( t = \frac{\delta h_0}{\delta t} \)

about a point O

In general terms angular momentum for a body moving about a point O is defined as:

\[ h_0 = I_g \omega + mvr \]

where:

\( h_0 \) = angular momentum about point O

\( I_g \) = moment of inertia of the object about its mass centre

\( \omega \) = angular velocity of the object

\( m \) = mass of the object

\( v \) = linear velocity of the object

\( r \) = the perpendicular distance of the mass centre from the point O

When applied to the rigid model, taking moments in an anticlockwise direction about (0,0) as positive, it can be shown that:

\[ -m_{a_g}y_a = \frac{\delta (I_{g_a}y_{a_tr} - m_{a_g}y_{a_tr} + m_{a_g}z_{a_tr})}{\delta t} = \frac{d(I_{g_a}y_{a_tr} - m_{a_g}y_{a_tr} + m_{a_g}z_{a_tr})}{dt} \]

and, when expanded and fully simplified, produces equation (3.4):

\[ m_{a_g}y_{a_tr} = m_{a_g}y_{a_tr} - m_{a_g}z_{a_tr} - I_{g_a}\ddot{\phi}_{a_tr} \quad (3.4) \]

The rigid system is fully defined by equations (3.3) and (3.4), each equation possessing terms which define the horizontal and vertical locations and accelerations of the mass centre of the gymnast segment:

\( y_a, z_a, \ddot{y}_a \) and \( \ddot{z}_a \).

The horizontal and vertical components of the mass centre location for the gymnast segment, in respect to the fixed point (0,0), are defined as:

\[ y_a = psin\phi_c - q\sin\phi_{tr} \]

\[ z_a = -pcos\phi_c + q\cos\phi_{tr} \]

The horizontal and vertical components of velocity and acceleration for the mass centre of the gymnast segment are determined by forming the first and second order derivatives of the location of the mass centre of the gymnast segment with respect to time. The horizontal and vertical components of velocity for the mass centre of the gymnast
segment are:

\[
\dot{y}_a = \dot{\phi}_c \cos \phi_c - \dot{\phi}_r a_r \cos \phi_r \\
\dot{z}_a = \dot{\phi}_c \sin \phi_c - \dot{\phi}_r a_r \sin \phi_r
\]

with the horizontal and vertical components of acceleration of the mass centre for the gymnast segment being defined as:

\[
\ddot{y}_a = \ddot{\phi}_c \cos \phi_c - \ddot{\phi}_r a_r \cos \phi_r + \dot{\phi}_r^2 a_r \cos \phi_r \\
\ddot{z}_a = \ddot{\phi}_c \sin \phi_c + \ddot{\phi}_r a_r \sin \phi_r - \dot{\phi}_r^2 a_r \cos \phi_r
\]

Through the substitution of these expressions for the horizontal and vertical components of location and acceleration of the mass centre for the gymnast into equations (3.3) and (3.4), two simultaneous equations are produced in the form:

\[
\ddot{\phi}_r (\text{set of known values}) + \ddot{\phi}_c (\text{set of known values}) = (\text{set of known values})
\]

Substitutions into equation (3.3) produces equation (3.5):

\[
\ddot{\phi}_r (m_a a_r^2 - m_a a_i \cos \phi_i \phi_r) + \ddot{\phi}_c (m_a \rho (a_i \cos \phi_i - \phi_r)) \\
= m_a (g \sin \phi_r - g \sin \phi_c - \dot{\phi}_r^2 a_r \sin \phi_r - \dot{\phi}_r a_i \sin \phi_r)
\]

and substitutions into equation (3.4) forms equation (3.6):

\[
\ddot{\phi}_r (a_i (\cos \phi_i + \sin \phi_i \tan \phi_c)) + \ddot{\phi}_c (-g \tan \phi_c - \dot{\phi}_r a_i \cos \phi_r - \sin \phi_r)
\]

Equations (3.5) and (3.6) simultaneously define the system in terms of two unknown angular accelerations, \(\ddot{\phi}_r\) and \(\ddot{\phi}_c\).

Equations (3.5) and (3.6) are used to determine the values for these unknown angular accelerations for any given instant in time.

**Numerical integration and the simulation model RIGID**

The simulation procedure in the program RIGID is formed by the numerical integration of the angular accelerations determined by equations (3.5) and (3.6). To progress the simulation over a small time increment, \(\Delta t\), a modified Euler procedure is adopted. With known original cable and gymnast angles and angular velocities the two unknown angular accelerations, termed 'original' angular accelerations, are determined
extended handstand configuration.

Initial conditions

The model requires four initial input values: the initial angles of the cable and the gymnast segments, $\phi_c$ and $\phi_r$ (Figure 3.1) and their respective angular velocities.

Output from the model RIGID

Several variables were calculated within the simulation model and presented as time histories throughout a single simulation including the time histories of the orientation angles $\phi_r$ and $\phi_c$ of the gymnast and cable segments. The horizontal and vertical locations of the mass centre of the gymnast segment are also calculated using their respective definitions.

Tension in the rings cable, which is equivalent to combined rings cable tension values, is calculated by rearranging equation (3.2), to give:

$$\text{combined cable tension} = \frac{m_a(z_a + g)}{\cos \phi_c}$$

In addition the maximum combined cable tension value during a single simulation is also reported on completion of a simulation.

With the rings cable being modelled as massless and rigid the positional potential energy for the system, PE, is calculated using:

$$\text{PE} = m_a g z_a$$

with the point of suspension (0,0) defining zero potential energy. The total kinetic energy for the system, KE, is calculated as:

$$\text{KE} = \frac{1}{2} I_g \dot{\phi}_r^2 + \frac{1}{2} m_a (\dot{y}_a^2 + \dot{z}_a^2).$$

The model was constructed in such a way that the total energy in the system should be conserved during a simulation, with the gymnast segment possessing all of the energy. This factor was used to ensure the model was correct, with no theoretical or programming errors and that the numerical integration procedure was satisfactory.

3.2.3 Simulations performed using the simulation model RIGID

In total fourteen simulations were performed. All simulations used the realistic value of 3.014 m for the cable length parameter p (F.I.G., 1989). Realistic values for the model parameters representing the gymnast were determined from anthropometric measurements taken from eleven elite male Canadian gymnasts with an average mass of
58.28 ± 6.0 kg and height of 1.63 ± 0.06 m, and the two subjects A and K (Chapter 4). By normalising the anthropometric measurements for each of the eleven Canadian gymnasts to those values of an average elite male gymnast at the 1988 Olympics (mass 62.88 kg and height 1.67 m respectively, Takei & Kim, 1990) values for one further gymnast were determined. These values may be expected to represent the anthropometry of an average elite gymnast. The three model parameters for these fourteen gymnasts were determined using the mathematical inertia model of Yeadon (1990), the principle of moments and the parallel axis theorem. The mean and standard deviation for each model parameter calculated for the eleven Canadian gymnasts are given in Table 3.1 (n = 11), together with values for the normalised gymnast and subjects A and K.

Table 3.1. Means and standard deviations for the model parameters representing the modelled gymnast

<table>
<thead>
<tr>
<th>gymnast</th>
<th>mass (kg)</th>
<th>transverse moment of inertia about mass centre (kg.m²)</th>
<th>distance of mass centre from the hands (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (n = 11)</td>
<td>58.28</td>
<td>11.02</td>
<td>0.885</td>
</tr>
<tr>
<td>s.d. (n = 11)</td>
<td>5.98</td>
<td>1.60</td>
<td>0.040</td>
</tr>
<tr>
<td>normalised</td>
<td>62.88</td>
<td>12.57</td>
<td>0.910</td>
</tr>
<tr>
<td>A</td>
<td>67.30</td>
<td>13.58</td>
<td>0.947</td>
</tr>
<tr>
<td>K</td>
<td>61.63</td>
<td>11.94</td>
<td>0.921</td>
</tr>
</tbody>
</table>

The initial cable and gymnast angles were set at 2° and 4° respectively, with their respective angular velocities set to 0 rad.s⁻¹ for all simulations. This combination of initial conditions placed the gymnast in a near handstand position with the mass centre of the gymnast below the point of suspension (0,0) prior to performing a swing.

3.3 A simulation model of a single segment gymnast swinging on an elastic rings frame: elastic model

3.3.1 Introduction

When simulating any sporting performance it is imperative that the model takes into account any equipment used by the athlete which affects their performance (Yeadon & Challis, 1994). It is therefore essential that the rings apparatus is represented with sufficient complexity when producing a model of a gymnast swinging on rings.

The RIGID model assumes the rings apparatus is a completely rigid structure,
However, directives presented by the international governing body of gymnastics (F.I.G.) state the rings cables and frame must possess some elasticity. This directive was made in an attempt to reduce forces and loading rates on the joints of the gymnasts (F.I.G., 1989). Such structures, described in Chapter 2, may form energy sinks within the rings apparatus. It may be suggested the role of such elastic structures on performance would be of major importance when performing swinging activities.

As a first approximation damped linear springs may be used to represent the elastic structures of the rings frame. A vertical spring may be used to represent the DEDs and bend of the horizontal beam of the frame, while a horizontal spring may represent any anterior-posterior motion of the rings frame. By retaining the simplified gymnast and cable segments the effect of rings frame elasticity on cable tension and performance may be investigated.

A two-dimensional simulation model representing the elasticity of the rings frame was developed (Figure 3.3). This 'elastic' model comprises three components: a single segment gymnast, a single massless rigid rings cable and two damped linear springs at the top of the cable representing the horizontal and vertical elasticity of the rings frame. To further simplify the modelling of the elastic nature of the rings frame the physical characteristics of the two springs are assumed to be identical. This combined elastic structure may therefore be assumed to act as a single spring. Air resistance is assumed to be negligible and the joint between the hands of the gymnast and the cable is modelled as frictionless.

The computer language Fortran is used to perform the calculations for the simulation model ELASTIC. This program is provided in Appendix B. The ELASTIC program was executed on a SUN SPARC station 1+ with a single simulation taking 1 minute of CPU time.
3.3.2 Methods

Newtonian mechanics was used to determine the equations of motion for the two-dimensional system forming the elastic model. Figure 3.4 and the following Nomenclature, provide definitions for the lengths and angles which define the orientation and location of the gymnast, cable and springs in the Newtonian reference frame. Figure 3.4 also highlights the forces acting in the system which were used to determine the equations of motion for the model.

Free body diagram of the elastic model

![Free body diagram of the elastic model](image)

Figure 3.4. Free body diagram of the elastic model.

Nomenclature for the elastic model

The following Nomenclature indicates the meaning of all symbols used in deriving the equations of motion and, in square brackets, the equivalent symbols used in the Fortran code held in Appendix B.
Derivation of the equations of motion for the elastic model

Equations of motion for the elastic system were derived by applying Newton's Second Law to both the horizontal and vertical components of the forces acting on the
gymnast and cable segments. For the gymnast segment in the horizontal direction equation (3.7) is produced:

\[ f \sin \phi_c = m_a \dot{y}_a \quad (3.7) \]

while in the vertical direction equation (3.8) is formed:

\[ f \cos \phi_c - m_a g = m_a \dot{y}_a \quad (3.8) \]

Through the application of Newton's Second Law to the massless cable segment it can be shown that horizontally:

\[ R_y = f \sin \phi_c \quad (3.9) \]

and vertically:

\[ R_z = f \cos \phi_c \quad (3.10) \]

where the forces \( R_y \) and \( R_z \) are those produced by the horizontal and vertical damped linear springs. In general terms the force produced by, or tension in, a damped linear spring may be expressed as:

\[ \text{tension} = - kx - cx \]

where:
- \( k \) = spring stiffness coefficient
- \( c \) = spring damping coefficient
- \( x \) = extension of the spring
- \( \dot{x} \) = velocity of the spring

Using horizontal and vertical damped linear springs to model the rings frame leads to the following equations for \( R_y \) and \( R_z \):

\[ R_y = c_h y_b + d_h \dot{y}_b \]
\[ R_z = - c_v z_b - d_v \dot{z}_b \]

Hence, equations (3.7) and (3.8) may be rewritten as:

\[ c_h y_b + d_h \dot{y}_b = m_a \ddot{y}_a \quad (3.11) \]
\[ - c_v z_b - d_v \dot{z}_b - m_a g = m_a \ddot{z}_a \quad (3.12) \]

forming equations (3.11) and (3.12).

Using the relationships for the forces \( R_y \) and \( R_z \) and tension in the two damped linear springs a third equation for the elastic system is produced. Equation (3.13) is formed by dividing equation (3.9) by (3.10):

\[ - c_v z_b \sin \phi_c - d_v \dot{z}_b \sin \phi_c = c_h y_b \cos \phi_c + d_h \dot{y}_b \cos \phi_c \quad (3.13) \]

A further equation defining the system was obtained using the rotational version of Newton's Second Law, which requires the determination of the system's angular momentum about the fixed point (0,0). When applied to the elastic model system, taking
moments in an anticlockwise direction about (0,0) as positive, it can be shown that:

\[-m_ay_a = -m_ay_a + m_ay_a \dot{z}_a + m_ay_a \ddot{z}_a\]

and, when fully simplified, produces equation (3.14):

\[m_ay_a = m_ay_a + m_ay_a \dot{z}_a - l_{ga} \dot{z}_a\]

Equations (3.11), (3.12), (3.13) and (3.14) fully define the modelled system, each equation possessing terms which specify the horizontal and vertical location and acceleration of the mass centre of the gymnast segment:

\[ya, za, \dot{y}_a and \dot{z}_a\]

The horizontal and vertical location of the mass centre of the gymnast segment, in respect to the fixed point (0,0), are defined as:

\[ya = y_b + pc \sin \phi_c - a_1 \sin \phi_{tr}\]
\[za = z_b - pc \cos \phi_c + a_1 \cos \phi_{tr}\]

The horizontal and vertical components of velocity and acceleration for the mass centre of the gymnast segment are:

\[\dot{y}_a = \dot{y}_b + \dot{\phi}_c pc \cos \phi_c - \dot{\phi}_{tr} a_1 \cos \phi_{tr}\]
\[\dot{z}_a = \dot{z}_b + \dot{\phi}_c pc \sin \phi_c - \dot{\phi}_{tr} a_1 \sin \phi_{tr}\]

These definitions for the horizontal and vertical locations and accelerations of the mass centre of the gymnast are substituted into equations (3.11) to (3.14) to form equations (3.15) to (3.18). Equations (3.15) to (3.18) are expressed in the form:

\[A \ddot{x} = B\]

where A is a 4 x 4 matrix containing the coefficients for \(\ddot{x}\), which represents the four unknown accelerations of:

\[\ddot{\phi}_{tr}, \ddot{\phi}_c, \ddot{y}_b and \ddot{z}_b\]

The 4 x 1 matrix B represents the right hand sides of all four equations (3.15) to (3.18).

A least squares calculation is performed in the Fortran subroutine 'Solve' (Stewart, 1973) within the simulation model ELASTIC to calculate the components of \(\ddot{x}\).
Subsequent to these substitutions, equation (3.11) forms (3.15):

\[ \ddot{\phi}_{tr} (a_{1} \cos \phi_{tr}) + \ddot{\phi}_{c} (-p \cos \phi_{c}) + \ddot{y}_{b} (-1) \]

\[ = \frac{c_{h} y_{b} + d_{h} \dot{y}_{b} + \dot{\phi}_{tr}^{2} a_{1} \sin \phi_{tr} - \dot{\phi}_{c}^{2} p \sin \phi_{c}}{m_{a}} \]  \hspace{1cm} (3.15)

With substitutions, equation (3.12) produces equation (3.16):

\[ \ddot{\phi}_{tr} (a_{1} \sin \phi_{tr}) + \ddot{\phi}_{c} (-p \sin \phi_{c}) + \ddot{z}_{b} (-1) \]

\[ = \frac{c_{v} z_{b} + d_{v} \dot{z}_{b} - \dot{\phi}_{tr}^{2} a_{1} \cos \phi_{tr} + \dot{\phi}_{c}^{2} p \cos \phi_{c} + g}{m_{a}} \]  \hspace{1cm} (3.16)

and equation (3.13) forms equation (3.17):

\[ \ddot{y}_{b} (-d_{h} \cos \phi_{c}) + \ddot{z}_{b} (-d_{v} \sin \phi_{c}) = \dot{\phi}_{c} c_{v} z_{b} \cos \phi_{c} + c_{v} \dot{z}_{b} \sin \phi_{c} \]

\[ + \ddot{\phi}_{c} d_{v} \dot{z}_{b} \cos \phi_{c} - \dot{\phi}_{c} c_{h} y_{b} \sin \phi_{c} + c_{h} \dot{y}_{b} \cos \phi_{c} - \dot{\phi}_{tr} c_{h} \dot{y}_{b} \sin \phi_{c} \]  \hspace{1cm} (3.17)

Finally, equation (3.18) is produced from substitutions into equation (3.14):

\[ \ddot{\phi}_{tr} (-a_{2} z_{b} \cos \phi_{c} - a_{1}^{2} + a_{1} \cos(\phi_{c} - \phi_{tr}) + a_{1} y_{b} \sin \phi_{tr} - g a_{2}) \]

\[ + \ddot{\phi}_{c} (p z_{b} \cos \phi_{c} - p^{2} + p a_{1} \cos(\phi_{c} - \phi_{tr}) - y_{b} \sin \phi_{c}) \]

\[ + \ddot{y}_{b} (z_{b} - p \cos \phi_{c} + a_{1} \cos \phi_{tr}) \]

\[ + \ddot{z}_{b} (-y_{b} - p \sin \phi_{c} + a_{1} \sin \phi_{tr}) \]

\[ = g y_{b} + g p \sin \phi_{c} - g a_{1} \sin \phi_{tr} \]

\[ - \dot{\phi}_{c}^{2} p(-z_{b} \sin \phi_{c} + a_{1} \sin(\phi_{tr} - \phi_{c}) - y_{b} \cos \phi_{c}) \]

\[ - \dot{\phi}_{tr}^{2} a_{1} (z_{b} \sin \phi_{tr} + p \sin(\phi_{c} - \phi_{tr}) + y_{b} \cos \phi_{tr}) \]  \hspace{1cm} (3.18)

Hence, for any instant in time the angular and linear accelerations \( \ddot{\phi}_{tr}, \ddot{\phi}_{c}, \ddot{y}_{b} \) and \( \ddot{z}_{b} \) can be determined.

**Numerical integration and the simulation model ELASTIC**

The numerical integration of the four accelerations, two angular and two linear, determined simultaneously by equations (3.15) to (3.18), forms the simulation procedure in the program ELASTIC. A modified Euler procedure, equivalent to that used by the RIGID model, was adopted to progress the simulation over the small time increment \( dt \).
With known angles for the gymnast and cable segments and extensions for the linear springs, together with their appropriate velocities, the calculation of the four unknown accelerations at the beginning of each time increment is possible using equations (3.15) to (3.18). As utilised in the RIGID model, equations of constant acceleration are used to determine the best estimate for each of the four unknown accelerations over the time increment \( dt \) by calculating an average value for each acceleration throughout this small time increment.

The modified Euler procedure is repeated for each time increment, with the time increment set to 0.0001 s. Each simulation was made to carry out 25000 iterations, producing a simulation time history of 2.50 s.

**Input to the model ELASTIC**

Input to the ELASTIC model comprises model parameters relating to the gymnast, rings cable and the two damped linear springs, together with initial conditions for these components of the model.

**Model parameters**

The model parameter of cable length was set equal to the length of an actual rings cable (F.I.G., 1989). Three parameters, also necessary in the RIGID model, are required to describe the gymnast segment: mass of the gymnast, the distance of the mass centre from the hands and the moment of inertia about the transverse axis through the mass centre while in an extended handstand configuration.

Additional parameters required by the ELASTIC model relate to the physical properties of the horizontal and vertical springs. These properties are determined by the springs' stiffness and damping coefficients, which were identical for each spring. By ensuring the physical properties of the two springs were identical a single damped linear spring system is formed at the top of the rings cable to reproduce the elastic nature of the rings frame (Figure 3.3).

**Initial conditions**

Eight initial input values are required. These values are the initial orientation angles of the cable \( \phi_c \) and gymnast \( \phi_g \), together with their angular velocities, and the initial spring extensions, \( y_b \) and \( z_b \), and their respective linear velocities.

**Output from the model ELASTIC**

When appropriate, variables calculated within the simulation model are presented as time histories. For example, the horizontal and vertical locations of the mass centre of the
gymnast segment are calculated using their respective definitions. The extensions of the horizontal and vertical springs are also presented as time histories throughout a simulation. Furthermore, time histories of the orientation angles \( \phi_r \) and \( \phi_c \) of the gymnast and rings cable are also reported.

Tension in the cable segment, equivalent to combined cable tension values, is calculated by rearranging equation (3.8) to give:

\[
\text{combined cable tension} = \frac{m_a(z_a + g)}{\cos \phi_c}
\]

In addition the maximum and minimum combined cable tension values during a single simulation are also reported.

With the cable segment being modelled as a massless rigid segment the positional potential energy, \( PE \), for the system is calculated using:

\[
PE = m_agz_a
\]

with the point of suspension \((0,0)\) defining zero positional potential energy. The elastic potential energy, \( EPE \), for each spring is calculated as:

\[
EPE_h = \frac{1}{2} (c_h y_b^2) \text{ for the horizontal spring}
\]

\[
EPE_v = \frac{1}{2} (c_v z_b^2) \text{ for the vertical spring}
\]

As only the gymnast possesses mass the total kinetic energy, \( KE \), for the system is calculated using:

\[
KE = \frac{1}{2} l_a \dot{\phi}_{lr}^2 + \frac{1}{2} m_a (\dot{y}_a^2 + \dot{z}_a^2)
\]

The total energy, \( TE \), in the system at any instant in time is calculated by the summation of all energy forms described above, and hence:

\[
TE = PE + EPE_h + EPE_v + KE
\]

3.3.3 Simulations to be performed using the simulation model ELASTIC

All simulations used realistic values for the parameters representing the gymnast and rings cable. Cable length, \( p \), was set to 3.014 m. For the model parameters representing the gymnast, \( m_a \), \( l_a \) and \( a_1 \), values determined for the normalised gymnast were used in all simulations (Table 3.1). By using parameters which represent the normalised gymnast it may be expected that results obtained from the model are representative of performances by an average elite male gymnast.

In order to investigate the effect of the elasticity of the rings frame on the gymnast's
performance spring parameters were selected to represent the rings frame as three different types: inelastic, elastic and highly elastic. Values for the stiffness of the springs which represent these three types of rings frame are presented in Table 3.2. Parameter values relating to the elastic rings frame represent those determined from video analyses of longswings (Chapter 5). As the model is planar in nature the stiffness and damping values represent the elastic structures at the top of both rings cables. Spring damping coefficients, \(d_h\) and \(d_v\), were systematically varied over a full range for each stiffness type to determine the influence of apparatus damping on patterns in cable tension and performance.

<table>
<thead>
<tr>
<th>rings frame type</th>
<th>(c_v)</th>
<th>(c_h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inelastic</td>
<td>500000</td>
<td>500000</td>
</tr>
<tr>
<td>elastic</td>
<td>140000</td>
<td>140000</td>
</tr>
<tr>
<td>highly elastic</td>
<td>14000</td>
<td>14000</td>
</tr>
</tbody>
</table>

The initial cable and gymnast angles were set to 2\(^{\circ}\) and 4\(^{\circ}\), with their respective angular velocities set to 0 rad.s\(^{-1}\) for all simulations. Initial velocities of both springs were set equal to 0 m.s\(^{-1}\), providing an initially motionless rings frame. The vertical spring extension was calculated so that the initial tension in the vertical spring was equal to 1 bodyweight. The extension of the horizontal spring was calculated to ensure the spring structure was initially in line with the cable segment. These initial conditions placed the gymnast in a near handstand position prior to performing a backward longswing element.

All output described previously were produced for each simulation. In total 20 simulations were performed for each type of modelled rings frame. Several further simulations were performed when investigating and speculating on additional components of elasticity for a gymnast swinging on rings.
3.4 A simulation model of a two segment gymnast swinging on a rigid rings frame:

two segment model

3.4.1 Introduction

Several studies, outlined in Section 2.2.2, observed coordinated changes in angles at the hip and shoulder joints of a gymnast when performing the backward longswing on rings (Nissinen, 1983; Brüggemann, 1987, 1994). The theoretical studies of Sprigings et al. (1996; 1998) modelled a gymnast with a single joint when swinging on rings and similar conclusions to those of Nissinen (1983) were reported. These studies suggest the exact initiation, duration and cessation of joint angle changes are crucial in producing a proficient performance of a backward longswing.

Owing to the importance of these joint angle changes a two-dimensional simulation model of a two segment gymnast swinging on rings was developed using Newtonian mechanics (Figure 3.5). A single rigid massless segment represents the two rings cables of the apparatus. The modelled gymnast is connected to the cable segment at his hands via a frictionless hinge joint. The gymnast comprises two rigid segments joined by a single movable hinge joint, the angle of which is controlled by the operator. For simplicity a gymnast modelled with a shoulder joint will be used to describe the system.

![Figure 3.5. The two segment simulation model of a gymnast (with shoulder joint) swinging on rings.](image)

Four angles define the orientation and configuration of the system (Figure 3.5). The cable elevation angle $\phi_c$, arm angle $\phi_a$ and trunk angle $\phi_tr$ are defined relative to the vertical in the inertial reference frame. The joint angle $\phi_{cl}$ is defined as the angle between...
the body segment and the arm segment. When modelled with a shoulder joint the 'arm' segment represents both arms of the gymnast while the 'body' segment represents the head, torso and both legs. The rings frame is assumed to be rigid and air resistance negligible.

The two segment model may also be used to represent a gymnast with a hip joint. When a hip joint is modelled the 'upper body' segment represents both arms and the torso with head, while the 'leg' segment represents both legs of the gymnast. When the gymnast is modelled with a hip joint $\phi_{e}$ defines the angle between the upper body segment and leg segment and is termed the hip angle. Modelling the gymnast with a different joint is accomplished by altering the values of the inertia parameters for each segment to those which accurately represent the inertial properties of the two new segments. The mechanical and mathematical derivations for the two segment model are identical whether a shoulder or hip joint is used.

The programming language of Fortran is used to perform the calculations for the simulation model TWOSEG. The program was executed on a Hewlett-Packard 9000/700 series UNIX based multi-user system. A copy of the computer program is provided in Appendix C.

3.4.2 Methods

Newtonian mechanics was used to determine the equations of motion for the two segment model. Two figures defining the two segment model are presented. The first figure defines all lengths and angles for the two segment model together with the location of the mass centre for each segment (Figure 3.6). The second figure shows the internal and external forces acting in the system (Figure 3.7). Following these figures a complete Nomenclature is provided. The diagrams and Nomenclature both relate to a gymnast modelled with a shoulder joint. Hence, the segments are termed the 'arm' segment and the 'body' segment, while the angle $\phi_{e}$ is termed the 'shoulder elevation' angle.
Definitions and free body diagrams of the two segment model

Figure 3.6. Definitions of lengths, locations and angles for the two segment model.

Figure 3.7. Free body diagram of the two segment model.
Nomenclature for the two segment model

\( a_1 \) [a1] : distance from the hands to the mass centre of the arm segment
\( a_2 \) [a2] : length of the arm segment
\( e_1 \) [e1] : distance from the shoulders to the mass centre of the body segment
\( e_2 \) [e2] : length of the body segment
\( p \) [p] : length of the cable segment
\( y_a \) [ya] : horizontal location of the mass centre of the arm segment
\( z_a \) [za] : vertical location of the mass centre of the arm segment
\( \dot{y}_a \) [yad] : horizontal velocity of the mass centre of the arm segment
\( \dot{z}_a \) [zad] : vertical velocity of the mass centre of the arm segment
\( \ddot{y}_a \) [yadd] : horizontal acceleration of the mass centre of the arm segment
\( \ddot{z}_a \) [zadd] : vertical acceleration of the mass centre of the arm segment
\( y_c \) [ye] : horizontal location of the mass centre of the body segment
\( z_c \) [ze] : vertical location of the mass centre of the body segment
\( \dot{y}_c \) [yed] : horizontal velocity of the mass centre of the body segment
\( \dot{z}_c \) [zed] : vertical velocity of the mass centre of the body segment
\( \ddot{y}_c \) [yedd] : horizontal acceleration of the mass centre of the body segment
\( \ddot{z}_c \) [zedd] : vertical acceleration of the mass centre of the body segment
\( m_a \) [ma] : mass of the arm segment
\( m_c \) [me] : mass of the body segment
\( I_{ga} \) [ia] : moment of inertia of the arm segment about the transverse axis through the mass centre
\( I_{ge} \) [ie] : moment of inertia of the body segment about the transverse axis through the mass centre
\( \phi_c \) [phc] : orientation of the cable segment
\( \dot{\phi}_c \) [phcd] : angular velocity of the cable segment
\( \ddot{\phi}_c \) [phcdd] : angular acceleration of the cable segment
\( \phi_a \) [pha] : orientation of the arm segment
\( \dot{\phi}_a \) [phad] : angular velocity of the arm segment
\( \ddot{\phi}_a \) [phadd] : angular acceleration of the arm segment
\( \phi_{tr} \) [phtr] : orientation of the body segment
\( \dot{\phi}_{tr} \) [phtrd] : angular velocity of the body segment
\( \ddot{\phi}_{tr} \) [phtrdd] : angular acceleration of the body segment
\( \phi_{el} \) [el] : shoulder angle
\( \dot{\phi}_{el} \) [eld] : shoulder angular velocity
\( \ddot{\phi}_{el} \) [eldd] : shoulder angular acceleration
\( T_y \) [Ty] : horizontal reaction force at the hands of the gymnast
Derivation of the equations of motion for the two segment model

Newton's Second Law of motion was applied to the vertically and horizontally resolved forces for the two segments representing the gymnast (Figure 3.7). Furthermore, for both of these segments, the rotational equivalent of Newton's Second Law was applied, taking moments about the segmental mass centres.

For the arm segment three equations are formed. Utilising Newton's Second Law for the horizontal components of force produces equation (3.19):

\[ R_y - T_y = m_a \ddot{y}_a \quad (3.19) \]

while for the vertical components of force equation (3.20) is generated:

\[- R_z + T_z - m_a g = m_a \ddot{z}_a \quad (3.20) \]

Applying the rotational equivalent of Newton's Second Law to the arm segment about its mass centre produces equation (3.21):

\[ - R_y (a_2 - a_1) \cos \phi_a - T_y (a_2 - a_1) \cos \phi_a + R_z (a_2 - a_1) \sin \phi_a + T_z (a_2 - a_1) \sin \phi_a + T_1 = I_{ga} \ddot{\phi}_a \quad (3.21) \]

Similarly for the body segment three equations are formed using the linear and angular forms of Newton's Second Law:

\[ R_y = m_e \ddot{y}_e \quad (3.22) \]

\[ R_z - m_e g = m_e \ddot{z}_e \quad (3.23) \]

and

\[ R_y e \cos \phi_{tr} + R_z e \sin \phi_{tr} + T_1 = I_{ge} \ddot{\phi}_{tr} \quad (3.24) \]

Finally, the rotational equivalent of Newton's Second Law was applied to the system as a whole. This requires the determination of the system's angular momentum about the fixed point (0,0). Angular momentum about the fixed point (0,0) for the two segment system can be shown to be:

\[ h_0 = I_{ga} \ddot{\phi}_a - m_a \dot{y}_a z_a + m_a \dot{z}_a y_a + I_{ge} \ddot{\phi}_{tr} - m_e \dot{y}_e z_e + m_e \dot{z}_e y_e \]

while the torque about fixed point (0,0) for the whole system is:
\[ t_0 = -m_a g y_a - m_e g y_e \]

When these equations are used in the rotational equivalent of Newton's Second Law, and then fully simplified, equation (3.25) is generated:

\[
m_a g y_a + m_e g y_e = m_a \ddot{y}_a z_a - m_a \ddot{z}_a y_a - I_g a \ddot{\phi}_a + m_e \ddot{y}_e z_e - m_e \ddot{z}_e y_e - I_e \ddot{\phi}_e \tag{3.25}
\]

Equations (3.19) to (3.25) possess the following terms for the mass centre locations and accelerations of the segments representing the gymnast with respect to the fixed point (0,0):

\[
y_a, z_a, y_e, z_e, \dot{y}_a, \dot{z}_a, \dot{y}_e, \dot{z}_e.
\]

Definitions for these terms may be generated using the defined lengths and angles (Figure 3.6). Hence, the horizontal and vertical locations of the mass centre of the arm segment, in respect to (0,0) are defined as:

\[
y_a = psin\phi_c - a_1sin\phi_a,
\]

\[
z_a = -pcos\phi_c + a_1cos\phi_a
\]

meaning the horizontal and vertical velocity and acceleration for the mass centre of the arm segment are:

\[
\dot{y}_a = \dot{\phi}_c pcos\phi_c - \dot{\phi}_a a_1cos\phi_a
\]

\[
\dot{z}_a = \dot{\phi}_c psin\phi_c - \dot{\phi}_a a_1sin\phi_a
\]

\[
\ddot{y}_a = \ddot{\phi}_c pcos\phi_c - \ddot{\phi}_a a_1cos\phi_a + \ddot{\phi}_a a_1sin\phi_a
\]

\[
\ddot{z}_a = \ddot{\phi}_c psin\phi_c - \ddot{\phi}_a a_1sin\phi_a - \ddot{\phi}_a a_1cos\phi_a
\]

Similarly the horizontal and vertical location, velocity and acceleration of the body segment mass centre are defined as:

\[
y_e = psin\phi_c - a_2sin\phi_a - e_1sin\phi_tr
\]

\[
z_e = -pcos\phi_c + a_2cos\phi_a + e_1cos\phi_tr
\]

\[
\dot{y}_e = \dot{\phi}_c pcos\phi_c - \dot{\phi}_a a_2cos\phi_a - \dot{\phi}_tr e cos\phi_tr
\]

\[
\dot{z}_e = \dot{\phi}_c psin\phi_c - \dot{\phi}_a a_2sin\phi_a - \dot{\phi}_tr e sin\phi_tr
\]

\[
\ddot{y}_e = \ddot{\phi}_c pcos\phi_c - \ddot{\phi}_a a_2cos\phi_a + \ddot{\phi}_a a_2sin\phi_a + \ddot{\phi}_tr e cos\phi_tr + \ddot{\phi}_tr e sin\phi_tr
\]

\[
\ddot{z}_e = \ddot{\phi}_c psin\phi_c + \ddot{\phi}_a a_2sin\phi_a - \ddot{\phi}_a a_2cos\phi_a - \ddot{\phi}_tr e sin\phi_tr - \ddot{\phi}_tr e cos\phi_tr
\]

Definitions of horizontal and vertical components of location, velocity and acceleration for the body segment possess the term \( \phi_tr \) and its time derivatives. Using geometry the angle \( \phi_tr \) and its derivatives may also be defined in terms of the joint angle, \( \phi_el \), controlled by the operator through the joint angle time history, and the arm angle \( \phi_a \)
(Figure 3.6). This relationship is:

\[ \phi_{el} = \pi + \phi_a - \phi_{tr} \]

Relationships between the angular velocities and accelerations of these angles are determined by forming the first two time derivatives of the above equation:

\[ \dot{\phi}_{el} = \dot{\phi}_a - \dot{\phi}_{tr} \]

\[ \ddot{\phi}_{el} = \ddot{\phi}_a - \ddot{\phi}_{tr} \]

These three equations allow the calculation of body segment angle, angular velocity and acceleration when the arm and joint angles, angular velocities and accelerations are known. Consequently these relationships are used to reduce the number of unknowns for the two segment system.

Two types of substitutions are required in equations (3.19) to (3.25) in order to form seven equations, (3.26) to (3.32), with seven unknowns. Firstly the definitions for the horizontal and vertical components for the mass centre locations and accelerations of the two gymnast segments are substituted into the seven equations (3.19) to (3.25). Secondly, for equations (3.22), (3.23) (3.24) and (3.25), the relationships between arm, joint and body angles are utilised to obtain seven final equations in terms of the following seven unknowns:

\[ \phi_a, \phi_c, R_y, R_z, T_y, T_z \text{ and } T_1. \]

In similarity to the spring model, equations (3.26) to (3.32) are expressed in the form \( Ax = B \): where \( A \) is a \( 7 \times 7 \) matrix containing the coefficients for \( x \), representing the unknown angular accelerations, forces and joint torques, and matrix \( B \), representing the \( 7 \times 1 \) matrix containing the right hand sides of equations (3.26) to (3.32).

Hence, with appropriate substitutions and rearrangement, equation (3.19) forms (3.26):

\[ \ddot{\phi}_a (-ma_a \cos \phi_a) + \dot{\phi}_c (ma_c \cos \phi_c) + R_y (-1) + T_y (1) = ma_c (\phi_c^2 \sin \phi_c - \phi_a a_c \sin \phi_c) \]  (3.26)

equation (3.20) forms (3.27):

\[ \ddot{\phi}_a (ma_a \sin \phi_a) + \dot{\phi}_c (-ma_c \sin \phi_c) + R_z (-1) + T_z (1) = ma_c (\phi_c^2 \cos \phi_c - \phi_a a_c \cos \phi_c + g) \]  (3.27)

and equation (3.21) forms (3.28):

\[ \ddot{\phi}_a (-I_a) + R_y (-a_z \cos \phi_a) + R_z ((a_z -a_i) \sin \phi_a) + T_y (-a_i \cos \phi_a) + T_z (a_i \sin \phi_a) + T_1 (-1) = 0 \]  (3.28)

Similarly, the equations formed using the body segment, after substitution, take the
following forms. Equation (3.22) produces (3.29):
\[
\ddot{\phi}_a(-m_c(a_2\cos\phi_a + e_1\cos\phi_{tr}) + \ddot{\phi}_c(m_c\cos\phi_c) + R_y(1) = m_c(\ddot{\phi}_c + \dot{\phi}_c\dot{e}_1\cos\phi_{tr} + \dot{\phi}_c\dot{e}_1\sin\phi_{tr} - \frac{\dot{\phi}_a^2}{2}\frac{e_1}{\sin\phi_{tr}}) + m_c\dot{e}_1^2\cos\phi_{tr} - \frac{\dot{\phi}_a^2}{2}\cos\phi_a - \frac{\dot{\phi}_a^2}{2}\sin\phi_a - \frac{\dot{\phi}_a^2}{2}\sin\phi_{tr}) (3.29)
\]
equation (3.23) produces (3.30):
\[
\ddot{\phi}_a(m_c(a_2\sin\phi_a + e_1\cos\phi_{tr})) + \ddot{\phi}_c(-m_c\sin\phi_c) + R_z(1) = m_c(\ddot{\phi}_c + \dot{\phi}_c\dot{e}_1\sin\phi_{tr} + \dot{\phi}_c\dot{e}_1\cos\phi_{tr} + g) (3.30)
\]
and equation (3.24) forms (3.31):
\[
\ddot{\phi}_a(I_{ge}) + R_y(e_1\cos\phi_c) + R_z(-e_1\sin\phi_c) + T_i(-1) = I_{ge}\ddot{\phi}_c (3.31)
\]
Finally, with appropriate substitutions into equation (3.25), equation (3.32) is formed:
\[
\ddot{\phi}_a(I_{ga} + m_a^2 - m_a\dot{\phi}_a + I_{ge} - m_c\dot{e}_1\cos\phi_{tr}) = m_c(\ddot{\phi}_c + \dot{\phi}_c\dot{e}_1\sin\phi_{tr} + \dot{\phi}_c\dot{e}_1\cos\phi_{tr} - \frac{\dot{\phi}_a^2}{2}\frac{e_1}{\sin\phi_{tr}}) + m_c\dot{e}_1^2\sin\phi_{tr} - \frac{\dot{\phi}_a^2}{2}\sin\phi_a - \frac{\dot{\phi}_a^2}{2}\sin\phi_{tr}) + m_a^2 + m_c\dot{e}_1^2 - \frac{\dot{\phi}_a^2}{2}\cos\phi_a - \frac{\dot{\phi}_a^2}{2}\sin\phi_{tr}) = m_c\dot{e}_1\cos\phi_{tr} + \dot{\phi}_c\dot{e}_1\sin\phi_{tr} + \dot{\phi}_c\dot{e}_1\cos\phi_{tr} - \frac{\dot{\phi}_a^2}{2}\frac{e_1}{\sin\phi_{tr}}) + m_c\dot{e}_1^2\cos\phi_{tr} + \dot{\phi}_c\dot{e}_1\sin\phi_{tr} + \dot{\phi}_c\dot{e}_1\cos\phi_{tr} - \frac{\dot{\phi}_a^2}{2}\frac{e_1}{\sin\phi_{tr}}) (3.32)
\]
These seven equations (3.26 to 3.32) are in terms of the seven unknown values and hence, for any instant in time values for the unknowns:
\[
\ddot{\phi}_a, \ddot{\phi}_c, R_y, R_z, T_y, T_z and T_i
\]
can be determined.

Numerical integration and the simulation model TWOSEG

The numerical integration of the two unknown angular accelerations:
\[
\ddot{\phi}_a and \ddot{\phi}_c
forms the simulation procedure in the TWOSEG program. A modified Euler method, described comprehensively in Section 3.2.2, is used to numerically integrate these angular accelerations and progress the simulation over the small time increment \( dt \). This procedure allows average estimates for the angular accelerations over the small time increment to be determined.

The value of \( dt \) was set to 0.0005 s. 5000 iterations were completed for each simulation producing a simulation time history of 2.50 s.

**Input to the model TWOSEG**

TWOSEG requires three types of model input: parameters relating to the gymnast and rings cable segments, initial conditions for these segments, and the joint angle time history parameters defining changes in the angle at the joint.

**Model parameters**

Eight model parameters are required to describe the two segment gymnast. Parameters for each segment include the length, mass and moment of inertia about the transverse axis through the segmental mass centre. In addition, the distance from the hands to the mass centre of the arm segment \( (a_1) \) and the distance from the joint to the mass centre of the body segment \( (e_1) \) are also required (Figure 3.6). Cable length, \( p \), was set equal to the length of an actual rings cable.

**Initial conditions**

Excluding joint angle time history parameters, four initial conditions are required. These values are the initial orientation angles of the cable \( \phi_c \) and the gymnast segment connected to the cables (eg arm or upper body) \( \phi_a \) together with their respective angular velocities.

**Joint angle time history**

One further model input is the joint angle time history, which describes the manner in which the joint angle varies throughout the simulation. The time history is controlled by the operator by defining five joint angle time history parameters. For all simulations the joint angle and its derivatives are expressed as a non-overlapping piecewise quintic function of time (Yeadon, 1984). During the time interval \([0,1]\) the joint angle is described by the quintic function:
\( q(x) = x^7(6x^2 - 15x + 10) \)

with first and second derivatives:

\[
\dot{q}(x) = 30x^2(x - 1)^2 \\
\ddot{q}(x) = 120x(x - \frac{1}{2})(x - 1).
\]

The quintic function ensures joint angle changes occur smoothly, with zero angular velocity and acceleration at the onset and cessation of a joint angle change (Figure 3.8). These attributes make joint angle changes described by this function appropriate for human movement (Yeadon, 1984).

![Graphs showing angle, angular velocity, and angular acceleration over time](image)

Figure 3.8. The quintic function \( q(x) \) used to describe the joint angle in the two segment model (Adapted from Yeadon, 1984).

When changing the joint angle in TWOSEG from \( A_0 \) to \( A_1 \) over the time period \([t_0, t_1]\) angle \( A \) and its derivatives at time \( t \) are given by:
\[ A(t) = A_0 + (A_1 - A_0)q(x) \]
\[ \dot{A}(t) = \frac{(A_1 - A_0)\dot{q}(x)}{(t_1 - t_0)} \]
\[ \ddot{A}(t) = \frac{(A_1 - A_0)\ddot{q}(x)}{(t_1 - t_0)^2} \]

where: \[ x = \frac{(t - t_p)}{(t_1 - t_p)}. \]

For both aspects examined using TWOSEG two changes in joint angle were produced during each simulation. The initial and final joint angles for the simulations were restricted to 180°. Hence, changes in joint angle comprised five parameter values, four times: \( t_0, t_{dur1}, t_{int1} \) and \( t_{dur2} \) and one angle: \( A_1 \) (Figure 3.9).

![Figure 3.9. Example of a hypothetical shoulder joint angle time history for a backward longswing.](image)

The first parameter of the joint angle time history is the instant in time \( t_0 \) at which the joint angle was initially altered from 180° (Figure 3.9b). The joint angle is altered to a chosen value, the angle parameter \( A_1 \) (Figure 3.9c), over the time duration \( t_{dur1} \), which forms the third parameter. The joint angle remains at the angle \( A_1 \) for a specified time interval \( t_{int1} \) the fourth parameter. After completing the time interval \( t_{int1} \) the joint angle returns to a value of 180° over the time duration \( t_{dur2} \) (Figure 3.9d-e).

**Output from the model TWOSEG**

General output presented as time histories include the horizontal and vertical locations of the whole body mass centre of the gymnast relative to (0,0). These values are calculated using mass centre locations for each segment derived from their definitions and the principle of moments for segmental masses taken about inertial axes y and z. A
further angle, termed the body angle $\varepsilon$, is calculated as the angle between the vertical and the vector passing from the hands of the gymnast through his whole body mass centre, denoted by an asterisk in Figure 3.10. This angle is used to represent the average orientation of the gymnast throughout his swing, and together with the cable elevation angle $\phi_e$ is provided as a time history throughout the simulation.

![Figure 3.10. The body angle $\varepsilon$ describing the average orientation of the gymnast.](image)

Tension in the cable segment, equivalent to a combined cable tension value, is calculated using:

$$\text{combined cable tension} = \frac{T_z}{\cos \phi_e}$$

When appropriate, the peak cable tension is reported together with the time and body angle at which it occurred. The joint torque $T_1$ is determined directly from the matrix $x$, $T_1$ being the seventh element of this array. This joint torque is equivalent to that produced by both shoulders owing to the planar nature of the model. The joint angle time history producing the joint torque time history for each attempted longswing is also provided on output.

With the point of suspension $(0,0)$ being zero the positional potential energy $PE$ for the system is calculated using:

$$PE = m_agz_a + m_egz_e$$

Only the gymnast possesses mass and hence, the total kinetic energy $KE$ for the system is calculated using:

$$KE = \frac{1}{2}I_{g_a}\dot{\theta}_a^2 + \frac{1}{2}m_a(\dot{y}_a^2 + \dot{z}_a^2) + \frac{1}{2}I_{ge}\dot{\theta}_{ir}^2 + \frac{1}{2}m_e(\dot{y}_e^2 + \dot{z}_e^2)$$

The total mechanical energy $TE$ in the system is calculated as the summation of these two energy forms, $PE$ and $KE$. 
When investigating the optimum performance of a backward longswing using changes in one joint angle a performance score was also calculated.

Performance score for longswings

The performance score utilises model output which reflects those characteristics essential to a proficient performance of the backward longswing. On completion of the backward longswing a gymnast should attain a motionless handstand with no angles present at his joints. These characteristics are required by the judging criteria (F.I.G., 1993). For this scenario to occur the body angle of the gymnast must be virtually motionless at 360° (handstand, after a full rotation) while the cable angle and angular velocity must be near zero. The performance score reflects the judging criteria by utilising the cable and arm angles $\phi_c$ and $\phi_a$ and angular velocities in a cost function. To further collaborate with judging criteria the performance score is only calculated on the ascending phase of the longswing after completion of all joint angle changes, which ensures a straight final handstand position. Hence, a series of scores is produced for a simulation, with the minimum score from the series representing the orientation and motion of the gymnast closest to the judging requirements during the attempted longswing. The score is defined as:

$$\text{performance score} = \min(k_1\phi_c^2 + k_2\phi_a^2 + k_3(360° - \phi_a)^2 + k_4\phi_a^2).$$

Values for each output in the function are squared in order that a positive score is calculated for any instant in time. The perfect longswing would record a score of zero as the gymnast would be in a stationary handstand position after rotating through 360°, with the cables hanging vertically down. For a less than perfect longswing a positive score is recorded, with poorer performances recording larger positive scores. Hence, for any given attempted longswing, with specific joint angle time history, the performance can be objectively scored.

Constants, $k_1$ to $k_4$, were used so that each of the judging criteria, described by model output, were given equal emphasis in the performance score. Values for the constants were determined using conservation of energy equations for a gymnast near a handstand position. A hypothetical situation of a gymnast swinging with small oscillations near a handstand was used to determine the relationship between the angle and angular velocity of the cable, assuming the gymnast and rings cables formed a simple pendulum. The relationship between cable and body angles was determined using trigonometry for a gymnast near the handstand, assuming the mass centre of the gymnast remained directly below the suspension point of the cables. These relationships led to the following constant values: 15, 3, 5 and 1 for $k_1$, $k_2$, $k_3$, and $k_4$ respectively, and provided equal emphasis to the variables used in the performance score.
3.4.3 Simulations to be performed using the simulation model TWOSEG

TWOSEG was used to investigate two aspects of swinging on rings: the reduction in peak cable tension due to joint actions during the descending phase of the longswing and the production of the optimum backward longswing. Both sets of investigations used a gymnast firstly modelled with a shoulder and subsequently with a hip joint.

For all aspects of investigation the cable length was set equal to 3.014 m. Model parameters describing the inertial characteristics of the appropriate segments were calculated from a set of anthropometric data for the elite gymnast K using the mathematical inertia model of Yeadon (1990b), the principle of moments and the parallel axis theorem. Model parameter values for a gymnast modelled with either joint are presented in Table 3.3.

<table>
<thead>
<tr>
<th>modelled joint</th>
<th>$m_a$</th>
<th>$m_e$</th>
<th>$I_{ga}$</th>
<th>$I_{ge}$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$e_1$</th>
<th>$e_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>shoulder</td>
<td>7.328</td>
<td>54.302</td>
<td>0.241</td>
<td>8.989</td>
<td>0.349</td>
<td>0.601</td>
<td>0.481</td>
<td>1.537</td>
</tr>
<tr>
<td>hip</td>
<td>39.324</td>
<td>22.306</td>
<td>3.014</td>
<td>1.444</td>
<td>0.719</td>
<td>1.146</td>
<td>0.334</td>
<td>0.992</td>
</tr>
</tbody>
</table>

Subject K was similar in height (1.69 m) and body mass (61.3 kg) to the average elite gymnast as presented by Takei & Kim (1990). It may therefore be assumed that values used in this investigation are similar to those of an average elite gymnast.

The first aspect investigated was the reduction of peak cable tension through joint actions during the descending phase of a longswing. The initial conditions for this investigation placed the gymnast in a near handstand position, with the initial cable, arm and joint angles at $2^\circ$, $4^\circ$ and $180^\circ$ respectively, and their respective angular velocities set to 0 rad.s$^{-1}$ for all simulations.

For the production of a backward longswing two scenarios were investigated, requiring different values for the initial conditions. The first scenario involved determining the joint angle time history which produced the optimum backward longswing from an initial handstand position. The initial cable, arm and joint angles and their respective angular velocities were the same as used in the reduction of peak cable tension, placing the gymnast in a stationary and straight handstand position.

The second scenario attempted to represent a dissipation of energy from the system equivalent to the energy dissipated from the rings frame during a swing from handstand. The simulation model SPRING was utilised to determine the total mechanical energy dissipated due to the elasticity of the rings frame during a swing from handstand. Values
for the spring parameters were equivalent to the average values determined for the DEDs from video analyses (Chapter 5). In total 409 J were dissipated from the system during a swing from handstand for subject K. This is equivalent to 37% of the possible change in positional potential energy. To simulate this dissipation of mechanical energy due to the rings frame in TWOSEG, the gymnast was positioned away from handstand in order to reduce his initial positional potential energy. To accomplish this the initial cable, arm and joint angles were 15°, 81° and 180° respectively and their angular velocities equal to 0 rad.s⁻¹. These initial conditions placed the mass centre of the gymnast below the point of suspension (0,0) in a motionless and straight configuration, similar to what occurs during a backward longswing.

The optimisation procedures using TWOSEG

Two different objective functions were used for the two aspects of inquiry. When investigating the reduction in peak cable tension the value of peak cable tension was the objective function to be minimised. When determining the optimum backward longswing the performance score was the objective function to be minimised. In both cases the Simulated Annealing global optimisation algorithm (Corana et al., 1987) was employed to determine the optimum solution through automatic modifications to the joint angle time history.

For both aspects of inquiry several restrictions to the simulations were imposed in an attempt to retain some realism. For both aspects simulations were rejected if cable tension became negative. In addition further restrictions were placed on the timings of joint angle changes and on the magnitude of angle parameter $A_1$.

When minimising peak cable tension, joint angle changes were allowed to occur throughout the swing. The value of joint angle $A_1$ was restricted to two values, 210° and 230°, with optimisations performed for both magnitudes of angle change and joint.

When optimising the backward longswing performance, the angle parameter $A_1$ was allowed to vary between 90° and 178°. Further restrictions were made on the timings of these joint angle changes. When the gymnast was initially in the handstand position, changes in the joint angle time history were restricted to the ascending phase of the backward longswing, corresponding to a body angle $\epsilon$ greater than 180°. This phase of swing has been identified as possessing the majority of joint angle changes (Nissinen, 1983). In each case the minimum performance score and hence the optimum backward longswing was identified. When investigating the production of an optimum backward longswing while mimicking energy dissipation from the system, changes in the gymnast's joint angle time history were allowed to take place throughout the swing.
3.5 A four segment, three-dimensional simulation model of a gymnast swinging on rings

3.5.1 Introduction

This section is concerned with the four segment, three-dimensional simulation model of a gymnast swinging on rings. Firstly, a general description of the four segment model is provided. The assumptions and simplifications of the model are described and explained using relevant literature and results from the video analyses of backward and forward longswings (Chapter 5).

Kane's method of formulating the equations of motion for a rigid system is described briefly together with a summary of the software package AUTOLEV™3 which was used to produce the four segment model. A comprehensive explanation of the development and details of the four segment model are provided in Appendix D.

Several modifications to the 'raw' simulation model, produced directly from AUTOLEV™3, were required to produce the 'customised' model FOURSEG which satisfies the specific needs of this study. These modifications are detailed later. Descriptions of the input required and output produced by the four segment simulation model FOURSEG are also given.

Finally, the investigations to be carried out using the customised simulation model FOURSEG are described.

General description of the four segment model

When modelling any sporting activity a compromise must be made between the realism and simplicity of the model. Where possible the model should simplify the activity under scrutiny while still modelling its main features (Morlock & Yeadon, 1988). With appropriate constraints imposed on a complex model, however, a more simple model may be formed. In adopting this ethos it is necessary to ascertain the important structural properties of the rings apparatus and crucial aspects of a gymnast's technique during swinging skills on rings.

During a rings routine lateral motion of the gymnast's whole body generally leads to deductions. Such motions are therefore generally undesirable. Hence, the vast majority of gymnastic elements on rings require symmetrical changes to the left and right sides of the gymnast's body. These symmetrical alterations ensure the mass centre of a gymnast remains an equal distance from the left and right uprights of the rings frame throughout a swinging element, minimising lateral motion of the gymnast. Although slight asymmetry of the gymnast's technique may occur during any element, it is proposed that the net effect of any asymmetry is negligible as an elite gymnast invariably completes gymnastic
elements with minimal lateral motion. Consequently, if both sides of the gymnast move symmetrically, only one side of the gymnast-rings apparatus system needs to be represented if appropriate constraints replicating this symmetry are incorporated into the model. Such a method was adopted for the four segment model, meaning the four segment model comprises only the right rings cable, right arm and body segments representing only the right hand side of the gymnast.

The following pages detail the reality of a gymnast swinging on rings and relate these features to the four segment simulation model, considering the rings cables and gymnast first. When deciding on the manner in which the gymnast and cables are represented experimental data and judging criteria were considered.

The structure of the rings apparatus makes it possible for the gymnast to move his arms, and hence the rings cables, to the front and side of his body during elements on rings (Figure 3.11). These motions may be described as three-dimensional in nature.

![Diagram](image)

Adapted from Readhead (1987)

Figure 3.11. Cable and gymnast orientations during swinging on rings.

The motion of the rings and cables are not only a feature of the apparatus but also an attribute of the gymnast’s technique. They are, however, of a constrained nature: not all motions of the rings are possible when a gymnast is swinging since his hands must remain in contact with both rings throughout each swinging element.

These three-dimensional motions for the rings cables are simulated in the four segment model for the right rings cable. The orientation of the cable segment is defined by three ordered successive orientation angles $\phi_{cr}$, $\theta_{cr}$ and $\psi_{cr}$ (Appendix D). Angle $\phi_{cr}$ is termed the cable elevation angle. Angle $\theta_{cr}$ is termed the cable abduction angle and reflects lateral motion of the cable (Figure 3.12). Although modelled, the twist angle of the cables, $\psi_{cr}$, is disregarded as it supplies no further useful information concerning cable orientation. Figure 3.12 shows the right rings cable at an orientation of $15^\circ$ elevation $\phi_{cr}$ and $10^\circ$ abduction $\theta_{cr}$.
In order for the modelled gymnast to perform specific techniques during swinging elements the configuration angle at each joint of the gymnast is required to be specified by the operator throughout a simulation. Using a joint angle driven model provides the operator with extensive control over hypothetical situations concerning technique.

Taking into consideration appropriate research (Brüggemann, 1987, 1994; Nissinen, 1983), coaching literature (Readhead, 1987), and results from the video analyses of longswings (Chapter 5), important changes in angles at the shoulder, hip and knee joints have been identified during longswings on rings. To replicate these joint angle changes in the model four rigid segments are required to model the gymnast: a right arm, a torso with head, a thigh and a shank with foot, connected by a 'universal' joint (2 degrees of freedom) for the shoulder and hinge joints (1 degree of freedom) for the hip and knee (Figure 3.13). By using four segments to represent a gymnast the majority of body configurations produced by a gymnast may be simulated.

Judging criteria for rings penalises bent arms during swinging skills (F.I.G., 1993) and little bend at the elbow was observed in the video analyses (Chapter 5). For these two reasons a single rigid segment represents the right arm of the gymnast. The configuration of the arm is described relative to the torso by two ordered successive configuration angles \( \phi_{\text{clr}} \) and \( \theta_{\text{abr}} \) about the shoulder joint centre (Figure 3.14).
configuration of the right arm in Figure 3.14 is described by angles of $180^\circ$ elevation $\phi_{\text{clr}}$ and $20^\circ$ abduction $\theta_{\text{abr}}$. Formal definitions of the configuration angles for the right arm are given in Appendix D.

right side of the gymnast

right arm

shoulder spring

shoulder joint (universal)

torso with head

hip joint (hinge)

thigh

knee joint (hinge)

shank with foot

Figure 3.13. Segments and joints representing the gymnast in the four segment simulation model.

right side of the gymnast

right shoulder abduction angle $\theta_{\text{abr}}$

arm configuration from neutral location

right shoulder elevation angle $\phi_{\text{clr}}$

neutral right arm location

Figure 3.14. Angles defining the configuration of the right arm segment relative to the torso of the gymnast in the four segment model.
The anatomical structure of the shoulder girdle allows extension of the mass centre of the arm away from the whole body mass centre (Engin, Peindl, Berme, & Kaleps 1984a, 1984b; Peindl & Engin, 1987). This extension may be as large as 12 cm when forces as large as 2.5 BW are exerted through each shoulder (Hiley, 1998). Such extensions at this joint, which also exhibits large angle changes during a swinging skill, may influence the overall motion of the gymnast. This extension is simulated in the four segment model by a damped linear spring placed between the torso and the right arm of the gymnast acting in the direction of the arm (Figure 3.13). Having a substantial amount of musculature around this joint (the deltoideus, subscapularis, pectoralis major muscle groups) forced extensions may be expected to be elastic in nature (Engin et al., 1984b).

The second segment of the model represents the right side of the torso and head of the gymnast. Although a gymnast may move the head relative to the torso during swinging elements, this relative motion is restricted by the joints of the cervical vertebrae. The mass of a torso for a 65 kg gymnast is approximately 30 kg while the mass of the head is near 5 kg. If a gymnast were to place his chin on his chest the mass centre location of the modelled torso segment (torso and head) would move closer to his hip joint by around 1.0 cm. On this basis it is proposed that any relative motion of the head during swinging skills on rings would produce negligible effects to the overall motion of the gymnast. Hence, for simplicity, the head and torso of the gymnast are modelled as one segment, with the head fixed in a neutral position. The torso segment is defined relative to the Newtonian reference frame by one orientation angle $\phi_{tr}$. For purposes of input and output in the four segment model and consistency with the two-dimensional models the torso angle $\phi_{tr}$ on input is defined to the vertical (Figure 3.15).

![Figure 3.15. The orientation angle $\phi_{tr}$ of the torso in the Newtonian reference frame in the four segment model.](image)

The third and fourth segments, the 'thigh' and 'shank', represent the right thigh and shank with foot of the gymnast. For the majority of swinging elements on rings the knees
and ankles should be together, so deductions are not made (F.I.G., 1993). Hence, two segments are used to represent the legs, using the assumption that the knees and ankles remain together throughout a swing. In the four segment model the configuration of the thigh is defined by a single angle $\phi_{ulcg}$ relative to the torso. The shank with foot segment is defined relative to the thigh by one angle $\phi_{lleg}$ (Figure 3.16).

![Figure 3.16. Configuration of the thigh relative to the torso and shank relative to the thigh in the four segment model.](image)

Results from the experimental data (Chapter 5) were used to determine which components of the rings apparatus were to be represented in the four segment model. Specifications produced by the F.I.G. on apparatus design state the rings frame should possess some elasticity to reduce the potential of injury to the gymnast's joints (F.I.G., 1989). Owing to these directives several structures incorporated into the frame and cables possess elastic properties which facilitate the reduction of forces in the rings cables and subsequently the joints of the gymnasts (Figure 3.17).

These structures include the damped elastic devices DEDs, constructed from a series of concave washers (Figure 2.2), connected to the cables at the top of the rings frame (Figure 3.17). Additionally, the horizontal beam at the top of the frame, to which the DEDs are attached, has the potential to move vertically owing to its elastic properties and the 'elbow' structures of the frame. The combined maximum vertical movement of the DEDs and horizontal beam during a longswing is approximately 30 mm (Chapter 5; Mills, 1997). The combined effect of these two elastic elements are represented in the four segment model by a vertical damped linear spring placed at the top of the cable segment (Figure 3.18).

The method by which the apparatus is erected, using guide wires connected midway up and at the top of the rings frame, introduces elastic properties to the whole apparatus (Figure 3.17). When a gymnast swings the frame moves in an anterior-posterior direction. This anterior-posterior motion of the frame is modelled using
a horizontal damped linear spring placed at the top of the cable, acting in conjunction with the vertical damped linear spring (Figure 3.18).

The mass of the rings frame is also incorporated into the four segment model. This is accomplished by a massive particle, equivalent to half the mass of the frame, being concentrated into a single point at the top of the rings frame (Figure 3.18). The motion of this effective mass was constrained to the anterior-posterior direction, ensuring that it only affected the horizontal motion of the top of the rings cable.

The rings cables are constructed from 2.13 m of 6 mm diameter steel wire in series with 0.70 m of nylon webbing or leather and stretch as tension within them increases. As the majority of the materials from which they are constructed is metal it may be assumed the cables also possess elastic properties. Hence, the cable is modelled with a damped linear spring placed in series at the bottom of an inextensible rod (Figure 3.19). The wooden rings, with 18 cm inner diameter, are assumed to remain in line with the cable. The construction of the combined rings and rings cables ensures each unit acts as one continuous structure when under tension, a simplification used in the previous two-dimensional models. The hand of the gymnast is connected to the rings cable using a frictionless pin joint. Unlike a gymnast swinging on high bar, there is relatively little motion of the hands around the wooden rings and therefore any frictional forces which may occur may be assumed to be negligible. Air resistance in the model is also assumed
to be negligible.

The final structure of the four segment model is shown in two dimensions in Figure 3.18 and Figure 3.19. Full details of the model's structure are presented in Appendix D.

![Diagram of a gymnast swinging on rings](image)

Figure 3.18. Four segment model of a gymnast swinging on rings (side view of the model).
3.5.2 Methods

*Introduction to Kane's method of formulating equations of motion for a rigid system*

A variety of methods exist, such as Lagrangian and Newton-Euler, to formulate the equations of motion for a linked system of rigid bodies dictated by Newtonian mechanics. Equations of motion may be formulated from first principles using Newton's First and Second Laws combined with their rotational equivalents. This method is demonstrated in the formulation of the equations of motion for the planar models of a gymnast swinging on rings, in Sections 3.2, 3.3 and 3.4. However, for more complex systems, such as three-dimensional models comprising several segments, these methods tend to be
cumbersome, leading to extremely long and complex equations (Kane, Likins & Levinson, 1983). One method of formulating equations of motion for such a system, while making the equations more concise and convenient, is Kane's method.

Formally, Kane's method reads:

"There exists reference frames \( N \) such that, if \( S \) is a system possessing \( p \) degrees of freedom in \( N \), and \( F_r \) and \( F_r^\ast \) \((r = 1, \ldots, p)\) are, respectively, the constrained generalised active forces and the constrained generalised inertia forces for \( S \) in \( N \), then the equations

\[
F_r + F_r^\ast = 0 \quad (r = 1, \ldots, p)
\]
govern all motions of \( S \) in any reference frame. The reference frames \( N \) are called Newtonian or inertial reference frames and the equations are known as Kane's dynamical equations" (Kane & Levinson, 1996).

Constrained generalised active forces are forces which occur due to contact between two bodies in the system (e.g. internal joint forces between the gymnast's body segments) as well as those forces which result from distance (e.g. forces due to springs and the weight of a segment). Constrained generalised inertia forces are those forces which are dependent on the motion and mass distribution of the bodies of system \( S \) (e.g. the body configuration of the gymnast and relative motions of segments).

Using Newton's Second Law, if \( \mathbf{R}_i \) (a vector) is the resultant of all contact and distance forces acting on a particle \( \mathbf{P}_i \) of system \( S \), and \( \mathbf{a}_i \) (a vector) is the acceleration of \( \mathbf{P}_i \) in a Newtonian reference frame \( N \), then:

\[
\mathbf{R}_i - m_i \mathbf{a}_i = 0 \quad (i = 1, \ldots, v)
\]
where: \( m_i \) is the mass of \( \mathbf{P}_i \) and \( v \) is the number of particles comprising \( S \). Multiplying both side of this equation by the partial velocities of \( \mathbf{P}_i \) in \( N \), \( N\mathbf{v}_i \), together with summation gives:

\[
\sum_{i = 1}^{v} N\mathbf{v}_i \mathbf{P}_i \cdot \mathbf{R}_i + \sum_{i = 1}^{v} N\mathbf{v}_i \mathbf{P}_i \cdot (-m_i \mathbf{a}_i) = 0.
\]

The first summation in the equation above relates to constrained generalised active forces while the second summation relates to the the constrained generalised inertia forces for the system \( S \).

The differential equations which form the partial velocities of \( \mathbf{P}_i \) in \( N \) \((N\mathbf{v}_i)\) may be formulated in terms of generalised speeds, \( \mathbf{U}_i \) \((i = 1, \ldots, v)\), which characterise the velocity vectors of the system. Hence, Kane's method of formulating the equations of motion for system \( S \) in frame \( N \) leads to the dynamical equations of motion for the system being defined by a set of first order differential equations in \( \mathbf{U}_i \), where \( \dot{\mathbf{U}}_i \) is the first order time derivative of the generalised speed \( \mathbf{U}_i \). These first order differential equations may then be solved simultaneously.

As a consequence of using Kane's method for formulating equations of motion for a system, forces and torques acting between segments which do not contribute to the vector
component of force $\mathbf{R}_i$ are not explicitly in evidence in the equations of motion. Such forces and torques are termed non-contributing. This characteristic of Kane's method is in contrast to other methods. Using TWOSEG as an example, the internal joint forces $R_y$ and $R_z$ and joint torque $T_1$ are determined directly in the matrix $\mathbf{x}$ relating to the equations of motion for that system without further manipulation. In order to obtain values for non-contributing torques and forces using Kane's method auxiliary generalised speeds are used to represent them in such a manner as to elicit partial velocities in $N$, and hence non-zero values for their contribution to $\mathbf{R}_i$. By representing these non-contributing forces and torques in such a way it is possible to obtain their numerical values.

For a more comprehensive review of Kane's method of formulating equations of motion for a multi-link system the reader is directed to the following references: Kane, Likins & Levinson (1983) and Kane & Levinson (1996).

Kane's method of formulating equations of motion for a system is used by the software package AUTOLEV™3, and was used to produce the 'raw' four segment simulation model of a gymnast swinging on rings.

Introduction to the software package AUTOLEV™3

AUTOLEV™3 is an advanced symbols manipulation software package which is executable on several operating systems (e.g. DOS, UNIX). It may be used to automatically formulate the equations of motion for a chosen system, relieving the operator of performing this task by hand. It requires the operator to use specific commands which describe the structure, motion, external forces and torques of the system to be simulated. Commands used to produce the four segment model are given in Appendix D.

With the structure of a model correctly described using AUTOLEV™3 commands the potential for typographic or algebraic errors when formulating equations of motion for a large multi-link system is vastly reduced. In addition, the process of formulating the equations of motion for a given system is less time consuming. Consequently operators may focus their attention to choosing the best structure of the model to represent the activity being studied. Furthermore, alterations to the structure of the model at a later stage are more easily implemented than if the equations are derived from first principles.

Besides formulating the equations of motion for a defined system AUTOLEV™3 has the capability of producing a 'ready-to-run' simulation model of the system, in the form of 'ready-to-compile' Fortran code. The flow diagram below highlights the procedure used to create a simulation model using AUTOLEV™3 (Figure 3.20).
Decide on the structure of the simulation model.

Produce an input file of AUTOLEV™3 commands which defines the structure, motion, external forces and external torques of modelled system.

Read this AUTOLEV™3 command input file, defining the system, into the AUTOLEV™3 software.

Three files are produced by the AUTOLEV™3 software:
- Simulation model file in Fortran code,
- Input file for the simulation model,
- Directory of output files which are produced when the simulation model is executed.

Compile the Fortran code for the simulation model and set values for model constants, parameters and initial conditions in the input file for the model.

Execute the Fortran code for the simulation model and view the results in the appropriate output files using the graphics package ALPLOT.

Figure 3.20. Flow diagram highlighting the procedure used to produce and execute a simulation model using AUTOLEV™3.

In order to produce a simulation model of any mechanical system in AUTOLEV™3 the structure and motion of the system is required, together with any external forces and torques acting on in the system. Knowledge of a system, relating directly to Kane's method of formulating equations of motion, include:
• the angular velocity and acceleration of each body of the system in the inertial reference frame
• the linear velocity and acceleration of the mass centre of each body of the system and of the points at which forces and torques act in the inertial reference frame which relate to the constrained generalised inertia forces,
• expressions for the forces and torques which act on the system which relate to the constrained generalised active forces.

Formulating the equations of motion for the four segment model

The equations of motion for the four segment model were formulated using Kane's method and AUTOLEV™3: Professional. The AUTOLEV™3 command file, fourseg.al, describes the structure of the model, as shown in Appendix D, (Figures D.8, D.9, D.10 and D.11) and is arranged in such a manner as to follow the requirements of Kane's method. Expressions relating to constrained generalised inertia forces, \( F_{c*} \), and constrained generalised active forces, \( F_c \), were determined and hence the equations of motion formulated.

Throughout this study the AUTOLEV™3: Professional software was executed on a Research Machines, Personal Computer (PC-S166) with 166 MHz Pentium Processor and 32 MB of EDO RAM with 1.6 GB of IDE hard disc. To produce the 'raw' ready-to-compile four segment simulation model from the AUTOLEV™3 command file required two minutes.

Numerical integration and the four segment simulation model

The 'raw' simulation model produced by AUTOLEV™3 utilises a Kutta-Merson numerical integration algorithm, which uses a Runge-Kutta integration method to advance the solution of the equations of motion. The Kutta-Merson algorithm possesses a variable step length routine which reduces the time increment between iterations if too large an error is produced during the numerical integration procedure. The time increment may also be increased when errors due to numerical integration are below an appointed tolerance level, thus decreasing the total time to complete a simulation. The magnitude of the error due to numerical integration is controlled using input values to the simulation. Using this sophisticated integration procedure should ensure that errors arising from numerical integration are insignificant.

The Fortran code representing the simulation model was compiled and executed on a Hewlett-Packard 9000/700 series UNIX based multi-user system, taking around two minutes to simulate three seconds of motion for a gymnast swinging on rings.
Modifications to the 'raw' model to 'customise' the simulation model FOURSEG

Customisation of the Fortran code representing the 'raw' four segment model was required in order to address the questions posed in Chapter 1. Although AUTOLEV™3 performs within the operating system DOS the final model was executed on a UNIX based system. This change in operating system required several alterations to the format of the 'raw' model. Further modifications were made to certain aspects of the model to meet the specific needs of the study. None of these modifications, which lead to the production of the 'customised' four segment simulation model FOURSEG, affected the equations of motion for the system.

Redefining appropriate joint angle time histories

Within the 'raw' model joint angle changes describing alterations in the gymnast's configuration were defined as a linear function in time, producing zero acceleration throughout the joint angle change. Such a definition of joint angle changes may be thought of as unrealistic for human movement. This method was used in the 'raw' model to reduce the number of parameters required to describe the model while formulating the equations of motion in AUTOLEV™3. As parameters defining joint angle changes over time do not directly affect the equations of motion for the system it is possible to alter the parameters so that changes in joint angles are described in a more realistic manner for human movement.

In this respect two modifications were made to the 'raw' model to develop the 'customised' simulation model FOURSEG. The first was to produce an operator option whereby joint angle data described by continuous functions for a gymnast are used as input into the model. For the video analyses of longswings (Chapter 5) joint angle time histories and their first two derivatives for the knee, hip and two shoulder angles were obtained by fitting the original joint angle data with quintic splines (Wood & Jennings, 1979). The procedure of fitting quintic splines to the angle data is dealt within Section 4.2.2. This option allows the modelled gymnast in FOURSEG to perform the same techniques as an elite gymnast.

The second modification to the 'raw' model was to produce an operator option for defining all joint angle changes and their derivatives at the modelled joints of the gymnast by non-overlapping quintic functions of time (Yeadon, 1984). The equations and suitability of this quintic function for describing human movement are provided in Section 3.4.2. In order to replicate all joint angle changes made during a longswing on rings the following joint actions are represented during a simulation:

- a reduction followed by an enlargement of the knee angle
- an enlargement, then reduction followed by an enlargement for the hip angle
• an enlargement, reduction then enlargement for the shoulder elevation angle
• an enlargement, reduction, enlargement and subsequent reduction for the shoulder abduction angle.

Data determined from the video analyses of longswings were used to decide on the number and type of angle changes to replicate.

Calculation of the initial cable abduction angle $\theta_{cr}$

Motion constraints for the four segment model arise for two reasons. Firstly there is the need for the right hand of the gymnast to remain in contact with the right rings cable at all times. Secondly, in order to ensure the model represents a complete gymnast the resultant velocity vector of the midpoint of the torso (MPTR) defined from the right side of the gymnast must be in a vertical (sagittal) plane only. These constraints were built directly into the simulation model using AUTOLEV™ (Appendix D).

To utilise these motion constraints correctly and ensure the modelled gymnast swings as though both the left and right hand sides of the gymnast and apparatus are represented, a further condition must be satisfied. This condition is that the point MPTR must start in the plane described by the y and z axes of the Newtonian reference frame. By ensuring this is fulfilled the assumption of symmetrical motion between the left and right sides of the gymnast and rings apparatus is satisfied. In order to satisfy this condition the initial cable abstraction angle is calculated so the initial location of the point MPTR is on the plane described by the Newtonian reference axes y and z. The constraint equations for the system then ensures the point MPTR remains on this plane throughout a simulation. Equation (3.33) calculates the required initial cable abstraction angle $\theta_{cr}$ to satisfy the condition and to produce symmetry to the model:

$$\theta_{cr} = \sin^{-1}\left(\frac{(BUNG D - (0.5L T_4 + \sin\theta_{abr}(L A_2 + M_r)))}{(L C_2 + CABS_r)}\right)$$

(3.33)

Definitions for the symbols may be found in the Nomenclature for the four segment model, Appendix D.

Input to the customised model FOURSEG

The customised four segment model requires values for various constants and parameters relating to the gymnast and modelled apparatus. Initial values for several model inputs are also required. Depending on the use of the simulation model the initial conditions required and the methods of calculating other conditions vary slightly. The difference in use stems from the manner in which the gymnast's technique (changes in the configuration angles) is defined: from experimentally determined data or hypothetically
using the quintic function.

Model constants

The distance from the origin to the horizontal spring at the top of the right cable, forms one constant BUNGĐ. This distance is set to 0.25 m, which is consistent with regulations governing the structure of the apparatus. Acceleration due to gravity is assigned a constant value of \(-9.81 \text{ m.s}^{-2}\).

Model parameters

Model parameters for the customised four segment model may be divided into five categories: segmental inertia parameters, parameters representing the dimensions of the apparatus, spring parameters, joint angle time history parameters and those relating to numerical integration.

Segmental inertia parameters include the mass, principal moments of inertia and segmental dimensions for all segments representing the gymnast (Figures D.10 and D.11). Values for these parameters were estimated using the mathematical inertia model of Yeadon (1990b) for a human body. These anthropometric measurements, the parallel axes theorem and the principle of moments were used to calculate all segmental inertia parameters for the gymnast in the four segment model. Mass centre locations of all solids comprising each segment in the four segment model are assumed to be situated on a common axis.

Parameters describing the dimensions of the rings apparatus include the length of a rings cable \(L_C\), the mass \(m_c\) and mass centre distance from the top of the cables \((L_C_1)\) and the principal moments of inertia \(I_C_L\), \(I_C_F\) and \(I_C_T\). The mass centre location and principal moments of inertia for the rings cable were calculated by modelling the cable as a solid circular cylinder. The dimensions of the rings cable related to the F.I.G. specifications, taking into consideration results from the data collection described in Chapter 4. Furthermore a value for the effective mass, which represents the mass of the rings frame is also required. This value was determined by using an analogy between linear and angular displacements for low amplitude oscillations for the rings frame (Chapter 4).

Parameters relating to the springs include the stiffness and damping coefficients for each spring in the model (Figures 3.18 and 3.19). Methods of estimating values for these parameters are presented within Chapter 5.

Two options are available in FOURSEG to alter the gymnast's joint angles, and hence technique. The first option is to alter joint angles in the model using quintic spline fitted joint angle changes determined from video analyses of longswings on rings. These data are in the form of continuous and smooth functions and require the quintic spline coefficients for each spline fitted set of angle data.
The second option available in FOURSEG is to alter joint angles in the manner described by the quintic function described in Section 3.4.3. Four model parameters are required to describe a single joint angle change: two angles, the initial angle $\alpha_0$ and final angle $\alpha_1$ and two times, the time $t_0$ to initiate angle change and the duration $t_1-t_0$ over which angle change occurs. An additional time parameter is required to connect single joint angle changes together. This parameter is the time interval $t_{\text{int}}$ over which the angle remains constant at $\alpha_1$ and therefore defines the start of the subsequent angle change. A complete joint angle time history for one joint consists of several joint angle changes.

To fully describe the joint angle time history for the knee joint 7 parameters are required: 3 angles and 4 time values, allowing a reduction followed by an enlargement of the knee angle. The hip and shoulder elevation angles require 10 parameters each: 4 angles and 6 times to represent an enlargement, then reduction followed by an enlargement of these joint angles during a longswing. For the shoulder abduction angle, which undergoes an enlargement, reduction, enlargement and subsequent reduction during a longswing, 13 parameters are required to define the joint angle time history.

The final model parameters for FOURSEG are those relating to the numerical integration routine. AUTOLEV™3 uses a Kutta-Merson algorithm for numerical integration with variable step length. This algorithm requires two error tolerance levels to be defined by the operator. For all simulations these tolerance levels were set to those advised by the software manufacturers.

Initial input conditions

Initial input values include the orientation $\phi_\text{tr}$ of the gymnast's torso in the Newtonian reference frame and the cable elevation $\phi_\text{cr}$ and twist $\psi_\text{cr}$ angles. Depending on the use of the simulation model these values are either taken directly from video data or appropriate hypothetical values are chosen.

Additionally initial values for the generalised speeds $U_1$, $U_3$ and $U_4$ are required ($U_2$ is the dependent generalised speed and as such values are calculated in the constraint equations). The method of defining the gymnast's technique throughout the longswing, actual or hypothetical, dictates the initial values for these angular velocities. Generalised speed $U_4$ represents the angular velocity of the torso about $\text{TR}_3>$ in the Newtonian frame. $U_1$ however is the generalised speed for the cable segment about the internal axis $\text{CR}_3>$, and therefore relates to the derivative of the cable elevation angle $\phi_\text{cr}$ by:

$$\dot{\phi}_\text{cr} = \frac{\sin(\psi_\text{cr})}{\cos(\theta_\text{cr})} U_2 + \frac{\cos(\psi_\text{cr})}{\cos(\theta_\text{cr})} U_1$$

As the initial value of the cable twist angle is always set to zero and the initial angle of $\theta_\text{cr}$ is always close to zero the angular velocity of the cable elevation angle is extremely close to the value for $U_1$. Hence, in the model FOURSEG the initial value of $U_1$ is
assumed to be equal to the angular velocity of $\phi_c$.

The initial lengths of all model springs are calculated within FOURSEG. The methods of calculating the cable and vertical spring lengths, $CABS_r$ and $VB_r$, respectively, are dependent on the option used to define the configuration of the gymnast throughout a simulation. When the gymnast's configuration is defined by the quintic spline data obtained from video analyses the appropriate synchronised cable tension value is used to determine the initial spring lengths $CABS_r$ and $VB_r$. When using hypothetical descriptions of the gymnast's technique calculations for the spring lengths $CABS_r$ and $VB_r$ are made assuming the gymnast to be in a stationary handstand position. This ensures the initial combined cable tension value is bodyweight.

The method of calculating $M_r$ within FOURSEG remains the same irrespective of the application. The initial value for the spring length $M_r$ is calculated on the assumption that the shoulder spring acts to accelerate the mass centre of the torso away from the hands while under compression as the gymnast is in handstand. Hence the gymnast possesses a negative shoulder spring length when in handstand. Furthermore the angle of the torso segment from the vertical is taken into account:

$$M_r = \frac{\cos \phi_{tr} \cdot (m_{tr} + m_{uleg} + m_{ileg}) \cdot g}{ASTIF}$$

(3.34)

The initial spring length for the horizontal spring, $HF_r$, is always set to zero. Furthermore, the initial spring velocities, $U_5$ to $U_8$, are set equal to zero for all springs in all simulations. From these angles and initial spring extensions the cable abduction angle $\theta_{cr}$ is calculated to ensure the midpoint MPTR of the gymnast is initially on the plane defined by the $y$ and $z$ Newtonian axes (Equation 3.33).

Finally, the model requires the step length to be specified for the numerical integration algorithm, together with the initial and final times of the simulation.

**Output from the customised model FOURSEG**

AUTOLEV™3 commands were utilised to produce the majority of the output for the four segment model FOURSEG. In total 29 output files are produced from one simulation, the contents of which are documented inside the file `fourseg.dir`. In each file a series of time histories is recorded.

Various forms of mechanical energy are calculated for the system and provided on output as a time history. Energy forms include the total kinetic energy of the system (KEALL), the potential energy of the system (PEALL) as well as the elastic potential energy of the springs (EPE) (Appendix D). Calculating total energy for the whole system may be used to check that the equations of motion and structure of the model are correct. With no energy sinks, such as spring damping or forced changes to the gymnast, the
system is one in which energy is conserved. This was used to show the numerical integration algorithm was working effectively and the integration error tolerance levels were acceptable.

The model produces the combined cable tension time history as output. Cable tension is calculated using the general equation for a damped linear spring. Using the cable spring, cable tension is calculated as:

\[ \text{cable tension} = (\text{CSTIF} \times \text{CABS}_r + \text{CDAMP} \times \dot{\text{CABS}}_r) \]  

(3.35)

To form the combined cable tension time history, CTEN, this value is doubled and given in units of the gymnast's bodyweight. The time history of the tension STEN in the shoulder joints is obtained in a similar fashion:

\[ \text{shoulder tension} = - (\text{ASTIF} \times M_r + \text{ADAMP} \times \dot{M}_r) \]  

(3.36)

with the combined shoulder tension being equal to doubling this value and altered to units of the gymnast's bodyweight. Also provided on output are the time histories for the extensions of each spring.

The time history of the joint angle and joint torque acting about the joint are produced as output from the model. Further model output comprising angles are time histories of cable elevation and abduction angles $\phi_e$ and $\theta_e$ (Figure 3.12) and their respective angular velocities.

The time history of the mass centre location for the gymnast is calculated in the inertial reference frame. The body angle $\varepsilon$ is also calculated and is defined as the angle between the vertical and a line joining the mass centre of the gymnast to the mid-point of the hands. This angle provides an estimate of the overall orientation of the gymnast at any instant in time. In handstand the value for this angle is approximately 0°. After a backward longswing has been completed $\varepsilon$ is equal to 360° while after a forward longswing $\varepsilon$ is equal to -360°.

Combined lengths calculated throughout a simulation include the wrist to ankle length for the gymnast (WALEN) and the total cable length (TCABLEN). Both of these values include the lengths of the relevant springs, $M_r$ and $\text{CABS}_r$.

To calculate the angular momentum of the system and gymnast about the point O for all three Newtonian axes the appropriate AUTOLEV™3 commands were used. Time histories for these values are found in two different output files.

3.5.3 Simulations to be performed using the simulation model FOURSEG

The customised four segment model will be employed to investigate the techniques of the two gymnasts performing backward and forward longswings from the data
collection. Both methods of altering body configuration of the modelled gymnast will be utilised for these investigations.

Firstly the contribution of each major aspect of the gymnasts' techniques to the performance of a longswing will be examined. Using the evaluated simulation model, driven by quintic spline joint angle data from the video analyses, each major aspect of the gymnasts' techniques will be selectively removed and the resulting performance determined by the model. From these experiments the contribution to performance for each aspect of technique may be determined and an understanding of the mechanical reasons for their contribution obtained.

For the backward longswing the evaluated four segment simulation model will be used to investigate the contributions to performance of the following aspects of the gymnasts' techniques:

- the lateral movements of the arms
- the changes in shoulder elevation
- the changes in hip angle
- the timings of joint actions at both the hip and shoulder joints.

Similarly the contributions of the same aspects of technique to the performance of a forward longswing will be investigated. Furthermore, the effect of the various aspects of technique on the maximum joint torques produced, combined peak cable tension and maximum forces at the shoulder joint will be explored.

The influence of the elasticity of the rings frame and apparatus on performance and patterns of rings cable tension and forces in the shoulder joint will also be examined.

3.6 Summary

The simulation model RIGID forms the most simple representation of a gymnast swinging on rings. The 14 simulations to be performed will investigate the interaction between the gymnast and cables when attempting to perform a longswing on rings. Owing to the rigid nature of the model it may be expected that forces and angular accelerations produced by the model are far greater than those experienced by gymnasts. However, inferences concerning the performance of a longswing on rings may be made.

The simulation model ELASTIC builds on the previous model by attempting to replicate the elastic nature of the rings frame. The simulations to be performed should provide an insight into the influence of rings frame elasticity on cable tension patterns and performance. Although overly simplified, further deductions may be made concerning the possible role of other energy sinks within the apparatus and gymnast system.
The simulation model TWOSEG is to be used to investigate two different aspects of swinging on rings. Firstly, the model will be used to investigate the influence of joint actions during the descending phase of the swing on peak cable tension values. Secondly, the model will be used to investigate the production of a backward longswing using changes in body configuration at a single joint, for two different scenarios.

The evaluated four segment three-dimensional simulation model, FOURSEG, will be utilized to investigate the contribution of various aspects of technique to the performance of backward and forward longswings. Each major aspect of the techniques used by the two gymnasts in the data collection to perform backward and forward longswings will be evaluated.

Secondly, the influence of the elasticity of the rings frame on the loading of the cables and shoulders of the gymnast will be determined.
CHAPTER 4

DATA COLLECTION AND ANALYSES OF LONGSWINGS ON RINGS

4.1 Introduction

To date, experimental research concerning a gymnast swinging on rings has concentrated on measuring and relating tension in the rings cables during swinging elements to the orientation and body configuration of the gymnast (Nissinen, 1983; Brüggemann, 1987). One shortfall of previous studies is the simplified planar analyses of the three-dimensional kinematics of the gymnasts' arms and rings cables. Owing to this simplification previous descriptions of techniques used by gymnasts to perform backward and forward longswings are incomplete. Consequently changes in body configuration which are three-dimensional in nature remain largely undocumented for swinging elements on rings.

In order to ensure that results obtained from experiments performed using a simulation model are realistic, there is a need for the model to be evaluated against appropriate athletic performances. Yeadon & Challis (1994) highlighted that the evaluation procedure has been neglected in many previous studies which have utilised simulation models of athletic performance. The confidence in results determined from such simulation models is in part determined by their realism. Hence, actual kinetic and kinematic data for gymnasts performing backward and forward longswings are required for the evaluation procedure of the four-segment, three-dimensional simulation model described in Chapter 3.

Data obtained during the data collection session described in the following sections therefore has two applications. The first application is to produce a full description of the three-dimensional kinematics and associated cable tension time histories for gymnasts performing backward and forward longswings. The second application of these data is to evaluate the four-segment simulation model, as described in Chapter 6. These two applications of experimental data are concerned with the first and last steps in the description of scientific method respectively.

The next section of this chapter, Section 4.2, describes the data collection session where anthropometric, force and video data for gymnasts performing backward and forward longswings were measured and recorded. Details of the equipment and computer hardware and software used during the data collection are given. Appropriate details concerning the gymnasts and the data collection protocol are also supplied.

Subsequent sections of this chapter are concerned with the analyses of the various forms of data recorded during this session, namely: anthropometric, force and video.
Analyses performed on each type of data are dealt with independently in each section.

A method of synchronising force and video data is illustrated, together with the limitations of the chosen method. When considering left and right side symmetry of the gymnasts and their movements, a method of establishing whether cable tension is equal in each rings cable during longswing elements is also presented.

Finally, the procedures used to estimate subject and apparatus specific model parameters for the four segment model are described.

4.2 Data collection session

The location for the data collection session was Lilleshall, National Sports Centre, the National Centre of British Gymnastics. The data collection was performed in the training environment during a single training session. The subjects were two elite male gymnasts, both members of the British National squad. Informed consent was gained from the subjects, referred to as A and K. The attire of each subject comprised a leotard and contrasting coloured shorts only. This enabled landmarks on their bodies, such as joint centres, to remain visible throughout a longswing. No other markers were placed on the subjects.

4.2.1 Collecting anthropometric data

Anthropometric data were collected on each subject using the method outlined in Yeadon (1990b). A researcher experienced in this method performed the 95 anthropometric measurements on each subject: 34 lengths, 41 perimeters, 17 widths and 3 depths. The measurements for each subject are shown in Appendix E. Length measurements were taken using a 5 m steel tape, perimeter measurements were taken using a 1.5 m fibreglass tailor's tape. Width and depth measurements were determined using a set of clinical anthropometric calipers. These data were subsequently supplied to the mathematical inertia model of Yeadon (1990b) in order to determine personalised segmental inertia parameters for each subject. Furthermore, the mass and height of each subject were determined using a set of electronic scales (Seca alpha, model 770) and 5 m steel tape measure respectively.

4.2.2 Measuring and recording cable tension data

All longswings during the data collection were performed on a competition rings
frame (Continental Sports Manufacturers). Prior to data collection a rings cable was constructed which allowed a calibrated quartz force link (Kistler 9331A) to be placed in series within the rings cable. The working range of the force link is ±20 kN, which more than encompassed the magnitudes of forces experienced in the cables during swinging elements performed on rings. The force link was secured in the rings cable by two eye-bolts and D-shackles (Figure 4.1).

![Diagram of quartz force link in rings cable](image)

**Figure 4.1.** The quartz force link (Kistler 9331A) in the adapted rings cable.

The analogue signal from the force link was transmitted directly into a charge amplifier (Kistler 5007) via an M4 to BNC connecting cable (C1500). The charge amplifier converts the charge produced by the quartz force link into a proportional electrical voltage. The sensitivity of the amplifier was set to produce a full scale deflection of ±10 kN over the electrical output scale of ±10 V. The accuracy of the amplifier at this setting is greater than ±1% of the full scale deflection. The time constant switch was set to *Medium* to avoid drift of the signal, as directed in the accompanying manual. At data collection the charge amplifier was securely fitted to the top horizontal beam of the competition rings frame. Electrical power for the charge amplifier was provided through an extended electrical lead attached to the upright of the frame. In order to reset the charge amplifier a remote trigger was used, the lead for which was firmly attached to the upright of the frame.

**Figure 4.2** schematically shows the rings frame used for the data collection with the force link in series within the adapted rings cable. This arrangement of the equipment ensured the force link and accompanying electrical cables did not hinder the natural motion of the rings cable during swinging elements.
Figure 4.2. Instrumentation of the rings frame.

The analogue signal from the charge amplifier was passed into an analogue to digital converter (CED1401 ADC) (Figure 4.3). The CED1401 utilised an unsigned 12 bit ADC, providing 4096 ADC counts for the full range of ±10 V, with count 2048 equal to 0 V. With the sensitivity of the charge amplifier set to ±10 kN each ADC count represented 4.88 N of force. The accuracy of the CED1401 is ±½ the least significant bit, in this case equal to 2.44 N. The analogue voltage signal from the charge amplifier was sampled at 1000 Hz. This high sampling frequency was used to reduce the likelihood of errors due to aliasing as such errors cannot be rectified at a later stage (Winter, 1990). As the majority of non-impact whole body human motion and locomotion is associated with frequencies of 6 Hz and lower (Winter et al., 1974) it may be proposed that errors due to aliasing would be minimal during this analogue to digital conversion.

The digital output from the ADC was recorded in binary format onto the hard disc of an Acorn computer (A5000) using a CED1401/BBC micro interface. The data
acquisition software for this purpose was written by Dr D.G. Kerwin (© D.G. Kerwin, 1995). This software also converted the digital voltage signal into equivalent measurements of force using the chosen sensitivity of the charge amplifier.

Each trial was named by subject, trial number and type of longswing. At the end of each trial a graph of cable tension against time was displayed on the monitor of the Acorn computer. This graph enabled an assessment of whether the cable tension was recorded correctly and acceptable. As a secondary check on the cable tension data a digital storage oscilloscope was used to display the trace. On deeming the trial 'acceptable', a comma separated variable (CSV) ASCII file of the cable tension time history for that particular trial was produced at a sampling frequency of 1000 Hz. The software used to convert binary data files to CSV ASCII format was written by Dr D.G. Kerwin (1997). Output values of cable tension in the CSV ASCII files were given to the nearest integer. For example: an ADC count of 2038 count represented a force of -48.8 N ± 2.44 N, while the equivalent force value in the ASCII file read -49.0 N. Negative values for force data highlighted the force link was experiencing forces under tension. To initiate the recording of tension data a two way trigger was pressed. This trigger also created an event signal for the ADC (Figure 4.3) and initiated the storage of cable tension data in a binary format.

![Diagram](image)

Figure 4.3. Equipment used to measure and record cable tension data during data collection.
The data acquisition software coupled with the ADC measured cable tension in a continuous loop using PERI-TRIGGERING. This enabled cable tension data to be recorded 1.5 s prior to the event produced by pressing the trigger. In total 4.999 s of cable tension data were recorded per trial.

Pressing the two way trigger also completed a further electrical circuit, simultaneously closing circuitry associated with an array of light emitting diodes (LED), and resulting in their illumination (Figure 4.4). A schematic diagram of all equipment used for the processing of cable tension data is shown in Figure 4.3.

The LED array comprised two-blocks of 9 ultra bright LEDs, each individual LED being 10 mm in diameter. Each block of 9 LEDs were arranged in a square with the centres of each adjacent horizontal and vertical LEDs 25 mm apart. The two blocks of LEDs were hinged in the middle (Figure 4.4: view from above) which allowed both cameras to see at least one block of LEDs head on. The LED array was powered by a 12 V battery (Figure 4.3).

4.2.3 Video recordings

The calibration volume and all longswings were recorded at 50 Hz by two Hi8 format video cameras, a Sony Hi8 Hyper HAD (EVW-300P) and a Sony video Hi8 HandyPRO (CCD-VX1E). The cameras were positioned on balconies approximately 4.5 m above the ground at a distance of 35 m and 25 m from the rings frame respectively (Figure 4.5).

The two cameras were gen-locked in order to synchronise their shutters, which were
set to 1/250 s and 1/300 s respectively. The electronic facilities on each camera were adjusted to obtain the best video images possible. The horizontal field of view of each camera was approximately 8 m, ensuring the whole rings frame and gymnast were visible throughout each longswing. Additionally, the LED array and trial numbers were visible in the view of each camera during the subjects' performances.

For a three-dimensional analysis of the longswing using a three-dimensional direct linear transformation technique (3D DLT) a calibration volume is required. The calibration structure comprised 12 markers, each marker being either a sphere or forming a cube-like solid. The calibration volume, of approximately 18 m³, was designed to incorporate the space in which the gymnasts performed.

Markers 1 to 6, made of 50 mm wide white tape, were fixed directly onto the rings frame, forming cube-like solids. Markers 7 to 12, white polystyrene spheres of 80 mm diameter, were positioned on two 5.2 m vertical poles at previously measured locations up the pole. The centre of each solid or sphere signified the calibration point of known location (Table 4.1). One 5.2 m vertical pole was placed 1.0 m in front of the rings frame, with the other behind the rings frame (Figure 4.6). The origin of the calibration volume was approximately the centre of the rings frame, in the middle of the two frame uprights. The inertial reference frame for the volume is shown in Figure 4.6. The locations of the centres for all 12 markers were measured, using steel tapes, from the origin of the calibration structure.
Figure 4.6. Calibration volume comprising 12 markers (view from HandyPRO camera).

Table 4.1. Three-dimensional locations of the calibration points

<table>
<thead>
<tr>
<th>Calibration point</th>
<th>x location (m)</th>
<th>y location (m)</th>
<th>z location (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.423</td>
<td>0.000</td>
<td>0.500</td>
</tr>
<tr>
<td>2</td>
<td>-1.465</td>
<td>0.000</td>
<td>2.722</td>
</tr>
<tr>
<td>3</td>
<td>-0.762</td>
<td>0.000</td>
<td>5.700</td>
</tr>
<tr>
<td>4</td>
<td>1.423</td>
<td>0.000</td>
<td>0.500</td>
</tr>
<tr>
<td>5</td>
<td>1.465</td>
<td>0.000</td>
<td>2.724</td>
</tr>
<tr>
<td>6</td>
<td>0.762</td>
<td>0.000</td>
<td>5.700</td>
</tr>
<tr>
<td>7</td>
<td>0.009</td>
<td>1.000</td>
<td>0.597</td>
</tr>
<tr>
<td>8</td>
<td>0.009</td>
<td>1.000</td>
<td>3.176</td>
</tr>
<tr>
<td>9</td>
<td>0.009</td>
<td>1.000</td>
<td>5.160</td>
</tr>
<tr>
<td>10</td>
<td>-0.004</td>
<td>-1.000</td>
<td>0.597</td>
</tr>
<tr>
<td>11</td>
<td>-0.004</td>
<td>-1.000</td>
<td>3.176</td>
</tr>
<tr>
<td>12</td>
<td>-0.004</td>
<td>-1.000</td>
<td>5.160</td>
</tr>
</tbody>
</table>
4.2.4 Protocol

Prior to any performances by the subjects the calibration volume was erected and videoed by both cameras. The calibration poles were then removed and appropriate mats placed under the rings frame.

Each subject was then asked to perform several proficient forward and backward longswings from a near motionless initial handstand. Trial numbers were placed near the rings frame in view of each camera to indicate the trial number on the video images. The proficiency of each trial was assessed and recorded by a National gymnastics coach. Each longswing took between two and three seconds to complete. The two way trigger was pressed as the gymnast started on the descending phase of the longswing. This simultaneously initiated cable tension data capture and illuminated the LED array, which formed the synchronisation signal between the cable tension and video data. The accuracy of this method is discussed in later sections.

The subjects were asked to perform forward longswings until several proficient performances with acceptable cable tension data were recorded. Similarly, the gymnasts then performed several backward longswings until an adequate number of trials were accepted. Both gymnasts were allowed to rest between trials in order to reduce the affects of fatigue and maintain the standard of their performances.

For each type of longswing the gymnasts performed trials facing one wall of the gymnasium and then performed trials facing the opposite wall. Proficient trials with acceptable cable tension data were required by both subjects facing both directions for each type of longswing. This procedure was carried out in order to assess whether the assumption of symmetrical tension in the two rings cables could be justified.

In total 22 trials were performed: 5 forward and 6 backward longswings by subject K, 8 forward and 3 backward longswings by subject A. Within these trials, both types of longswings were performed in both directions. From these 22 trials three forward and three backward longswings were deemed proficient performances by a Grade 1 National Gymnasts Coach and satisfactory for analysis (Table 4.2).

<table>
<thead>
<tr>
<th>trial</th>
<th>subject</th>
<th>longswing type</th>
<th>direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>K</td>
<td>forward</td>
<td>normal</td>
</tr>
<tr>
<td>K05</td>
<td>K</td>
<td>forward</td>
<td>opposite</td>
</tr>
<tr>
<td>A09</td>
<td>A</td>
<td>forward</td>
<td>normal</td>
</tr>
<tr>
<td>K15</td>
<td>K</td>
<td>backward</td>
<td>normal</td>
</tr>
<tr>
<td>A20</td>
<td>A</td>
<td>backward</td>
<td>normal</td>
</tr>
<tr>
<td>A21</td>
<td>A</td>
<td>backward</td>
<td>opposite</td>
</tr>
</tbody>
</table>
4.3 Analysis of anthropometric data for video analysis

The sets of anthropometric data for subjects A and K are provided in Appendix E. These data relate to the measurements required for the mathematical inertia model of Yeadon (1990b). Segmental density values determined by Dempster (1955), Clauser et al. (1969) and Chandler et al. (1975) were used separately for the segments in the inertia model. The three different estimates of segmental densities were used to assess the influence of such properties on the calculated whole body mass. The inertia model was used to calculate the lengths, masses and mass centre locations for fourteen segments: head, trunk, the left and right upper arms, lower arms, hands, thighs, shanks, and feet for both subjects using the three density sets. In addition total body mass and whole body density values were calculated. The total body mass for each subject was compared to that determined at data collection by electronic scales. These comparisons were used to estimate the accuracy of the personalised segmental inertia parameters for the two subjects using the three different segmental density sets. The density set which provided the most accurate total body mass estimates was used to calculate all subject specific inertia parameters.

The inertia model also calculated the mass centre location of each segment from the proximal segment end as a ratio of the segment length. Furthermore, the mass of each segment was calculated as a ratio of the calculated whole body mass. These values of mass centre location ratios of each segment from the proximal end and segmental mass ratios were used in the video analysis program VIDEO.

4.4 Analysis of cable tension data

On obtaining cable tension data in CSV ASCII format for the six chosen longswings, each trial was analysed to ascertain the frequency content of the data using a battery of Fortran programs. For each trial attenuation of the higher frequency components of the signal was carried out in order to reduce random errors in the cable tension data. Systematic errors were also corrected using the frequency content of the signals.

4.4.1 Pre-data collection testing and calibration of force link, charge amplifier, ADC and software system

Prior to the data collection performed at Lilleshall N.S.C. the Kistler force link, charge amplifier, CED1401 and computer software were subjected to a series of static
calibration tests. Several objects of known mass were suspended from a horizontal beam to ensure force values determined by the force link - charge amplifier - ADC - software system were correct. Each object of known mass was suspended using a metal chain of known mass. The force link was placed in series at the top of the chain so both the chain and object were supported by the force link. The charge amplifier was reset to zero with no chain or object suspended. Prior to the measurement and recording of tension in the force link the known mass was placed on the metal chain and lowered to a freely suspended static position. For this simple system, tension in the force link is equal to the weight of the known object and metal chain. Further tests were performed with no chain or object suspended from the force link.

Two observations were made which were critical to the analysis of cable tension data for the longswing:

- when the charge amplifier was reset and no load added the recorded force in the link was on average +15 N. When objects of known mass (the load) were suspended the recorded tension data (tension being expressed by negative values) were, on average, incorrect by -15 N. The force link could therefore be considered to be under permanent compression. This systematic error is easily rectified by the addition of a -15 N offset to all data recorded.
- when determining the combined weight of the load and chain, random errors in the tension data were observed. Random errors in the tension data were as large as 90 N in the pre-data collection testing.

Immediately prior to the recording of cable tension at the data collection a similar testing of the force link - charge amplifier - ADC - software system was performed. This testing reinforced the earlier observations of the 15 N systematic offset and random errors as large as 90 N. Observations at all testings were considered in the analyses of the cable tension time histories for the six chosen longswings.

4.4.2 Fourier analysis of sampled data

For any analogue signal which has been measured and sampled the final recorded signal may be thought of as the true signal with an added error or 'noise' component (Lees, 1980). The term 'noise' may be used to describe those components of the sampled signal which are not due to the process itself (Winter, 1990). The noise element of the sampled signal may be further considered as consisting of systematic and random noise. Systematic noise may be described as an additional signal to the true signal, which occurs in the same manner throughout the sampled signal, and as such may be detected and
removed. Random noise, often termed white noise, is generally small in amplitude and may occur throughout the whole frequency spectrum, though its actual distribution may not be precisely known (Lees, 1980). This characteristic makes the detection and reduction of random noise in a sampled signal very difficult. Owing to the introduction of noise in any sampled signal the true signal can never be known, though a best estimate of the true signal should be sort (Yeadon & Challis, 1994).

It may be assumed that the original cable tension data for each trial contains all three components of a general sampled signal. Both systematic and random errors observed in pre-data collection testings may be expected to be present in the cable tension data. For several trials recorded in the data collection rapid oscillations of large amplitude, or 'buzz', in the cable tension data were noted throughout the time history. At the time of data collection these errors seemed to be of a systematic nature, and therefore their effects were believed to be rectifiable.

As outlined in Section 2.4.5 a number of techniques exist which may be used to fit experimentally obtained data and reduce random errors (Challis & Kerwin, 1988). Truncated Fourier series have been shown to appropriately attenuate errors in data presented at equal intervals in time and displaying oscillations over a whole period throughout the time history. Furthermore, the use of Fourier series allows the reduction of a particular frequency content of the signal. Owing to the oscillating data noted during the data collection it is proposed that truncated Fourier series is the most appropriate method of error reduction in the cable tension data.

The Fourier series

Any function may be described in both the time domain and frequency domain. In the frequency domain the function is described by a series of frequencies, each exhibiting a specific amplitude. In order to determine the frequencies and amplitudes of a function it is represented as a series of weighted cosine and sine functions. This representation is termed a trigonometric or a Fourier series. The weightings for each of the cosine and sine functions are termed the Fourier coefficients, $a_k$ and $b_k$. To complete the Fourier transformation $2n+1$ data samples are required giving $2n$ intervals. The general trigonometric series given to fit a function with $2n+1$ points is:

$$f_n(x) = \frac{1}{2}a_0 + a_1\cos(x) + \ldots + \frac{1}{2}a_n\cos(nx)$$
$$+ b_1\sin(x) + \ldots + b_n\sin((n-1)x) \quad (4.1)$$

(Lanczos, 1966)

Using this trigonometric series the Fourier coefficients, $a_k$ and $b_k$, of the function $f_n(x)$ are determined through summation, owing to the orthogonal nature of cosine and sine functions over a full oscillatory period. The Fourier coefficients, $\frac{1}{2}a_0$, $a_k$ and $b_k$, for
a data set of \(2n+1\) samples are calculated as:

\[
\frac{1}{2}a_0 = \frac{1}{n} \sum_{\alpha=0}^{2n} f(x_\alpha)
\]

(4.2)

\[
a_k = \frac{1}{n} \sum_{\alpha=0}^{2n} f(x_\alpha) \cos(kx_\alpha)
\]

(4.3)

\[
b_k = \frac{1}{n} \sum_{\alpha=0}^{2n} f(x_\alpha) \sin(kx_\alpha)
\]

(4.4)

where:

\[x_\alpha = \frac{\pi}{n} \alpha, \quad (\alpha = 0 \ldots 2n)\]

\[f(x_\alpha) = \text{signal value at time } x_\alpha\]

\[k = 1 \ldots n\]

If the signal does not demonstrate a complete oscillation over the sampled time period both the first and last data points may be replaced by the arithmetic mean of these two values to ensure a complete oscillation (Lanczos, 1966). Each Fourier coefficient, \(a_k\) and \(b_k\), relate to a single harmonic within the function, which exhibits \(n\) harmonics in total. The coefficient \(\frac{1}{2}a_0\) is equal to twice the arithmetic mean of the signal in the time domain, corresponding to the offset of the data from zero. The frequency \(f_k\) and amplitude \(c_k\) of each harmonic, defined by \(a_k\) and \(b_k\), are calculated using the formulae 4.5 and 4.6:

\[
f_k = \frac{k}{T}
\]

(4.5)

\[
c_k = \sqrt{a_k^2 + b_k^2}
\]

(4.6)

where:

\(a_k\) and \(b_k\) = the \(k^{th}\) Fourier coefficients

\(c_k\) = the amplitude of the \(k^{th}\) frequency

\(f_k\) = the \(k^{th}\) frequency

\(T\) = the total time of the signal

A Fortran code *trigfilter* representing these calculations was used to perform the frequency analysis of the cable tension data for each trial.

An inverse Fourier transform reverses the procedure above by using the Fourier coefficients \((a_k\) and \(b_k)\) to determine the values of the function in the time domain. Thus, if a sampled data set in the time domain, showing a full period, were firstly Fourier transformed and subsequently inverse Fourier transformed the two sets of data in the time domain would be identical.
4.4.3 Methods of rectifying systematic errors and reducing random errors in cable tension data

Cable tension data for all trials were measured and recorded in the same manner. Hence, any procedures used to rectify systematic errors and attenuate random errors should be applicable to all trials. Cable tension data for each trial, as measured and recorded at the data collection and comprising systematic and random errors, were termed 'raw' data. For each longswing trial 4.999 seconds of raw cable tension data were recorded at 1000 Hz, corresponding to 5000 samples. The Fourier analysis described previously however requires 2n+1 data samples. To ensure the conditions of the transformation were met the 5001st value for cable tension (at t = 5.000 s) was set equal to the 5000th value (t = 4.999 s). For the selected trials the gymnasts were in a handstand position at this instant in time. With the sampling frequency at 1000 Hz it may be assumed that cable tension would not alter significantly over 0.001 s at this point of the longswing and therefore this method of estimating the value for cable tension at 5.000 s would not significantly affect the subsequent analyses.

For each of the selected trials (Table 4.2, excluding trial A20 which required additional treatment prior to frequency analyses) a Fourier analysis was performed on the raw cable tension time history using the Fortran program trigfilter. The Fortran program trigfilter calculated the 5001 Fourier coefficients: 2500 a_k coefficients and 2500 b_k coefficients describing the combined cosine and sine fit, together with the constant \( \frac{1}{2}a_0 \). The frequency and amplitude of each harmonic were calculated (Equations 4.4 and 4.5). Figure 4.7 shows raw cable tension data for two longswings in the frequency domain.

![Figure 4.7](image_url)

**Figure 4.7.** Frequency spectra of raw cable tension data for trials A09 and A21.

These initial analyses showed that trial A21 possessed a strong component of signal at a frequency of 50 Hz, the amplitude of which was 55 N. This systematic error in trial A21 (or "buzz") was observed at the time of data collection. Owing to the frequency of
this systematic error its appearance was attributed to electrical mains interference with the change amplifier, although this speculation was never fully verified. Furthermore frequencies between 49.6Hz and 50.4Hz inclusive displayed amplitudes of around 5N, this phenomena being spectral smearing from the 50 Hz signal. Owing to the appearance of this systematic error in the raw cable tension data for trial A21 signals at a frequency of 50 Hz (49.6 Hz to 50.4 Hz inclusive to reduce the effect of spectral smearing) were removed in all trials by the reduction of the Fourier coefficients for these frequencies to zero and the subsequent inverse Fourier transformation of the data to the time domain using the Fortran program triginverse. This procedure has been termed notch filtering (Press et al., 1988). The data in the time domain representing raw cable tension data with all systematic errors removed at a frequency of 50 Hz is termed raw50 data.

Procedures to reduce random errors in each of the recorded cable tension data were made using the newly formed raw50 cable tension data. The nature of random errors identified at pre-testing were generally of a high frequency. Attenuation of higher frequencies may therefore eradicate the majority of random errors in the cable tension data. For each trial raw50 cable tension data were compared to raw50-fc cable tension data, which had been further modified by removing all components of the signal greater than a specified cutoff frequency fc. A range of cutoff frequencies was used, encompassing the majority of the frequency spectrum: 300 Hz, 250 Hz, 200 Hz, 150 Hz, 100 Hz, 75 Hz, 50 Hz, 45 Hz, 40 Hz, 30 Hz, 20 Hz and 10 Hz.

Comparisons were made between the raw50 and the raw50-fc data by calculating the root mean squared difference (RMS) between the two estimates of the same cable tension time history for each trial. The 12 different values for fc were compared for all longswing trials.

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \hat{x}_i)^2}$$

where:

N = the number of data samples
x_i = the value for the i_th sample from the first estimate
\hat{x}_i = the values for the i_th sample from the second estimate

From the pre-testing results it was known that random errors as large as 90 N may be observed in the cable tension data due to the force-link-amplifier-ADC-software system. Hence, the maximum difference between the data sets at any instant in time was also determined for each of the five trials. Using the results for the RMS and maximum differences between raw50 cable tension and raw50-fc cable tension for all longswings the appropriate cutoff frequency was determined to be 45 Hz (Section 5.5). This value of fc ensured random and systematic errors in the cable tension data for each trial were greatly
reduced and a more accurate representation of the true signal obtained.

Finally the systematic error of -15 N was rectified for each trial and produced the final filtered cable tension data for each trial at 1000 Hz. In order to synchronise the filtered cable tension data with the 50 Hz kinematic data determined from video the procedure outlined in Section 4.6 was adopted. The synchronised filtered cable tension data at 50 Hz was used in all subsequent analyses, including estimating the spring parameters of the four segment simulation model and the subsequent evaluation.

The reduction of errors in cable tension data for trial A20

Cable tension data for trial A20 posed an additional set of error problems. At certain times in the cable tension time history the measured tension was recorded as being equal to the full scale deflection of the amplifier - ADC system over several consecutive samples. These errors were of extremely high frequency (500 Hz) and large magnitude (20 kN). The cause of these errors was never confirmed. These large full scale deflection errors rendered the 'raw' data unrealistic and therefore trial A20 was omitted from the procedure used to determine the method of reducing systematic and random errors in the cable tension data. However, owing to the nature of other longswing performances, trial A20 was required in the evaluation of the four segment model. The eradication of these full scale deflection errors in the cable tension data of A20 was therefore required.

The following procedure was used to reduce the full scale deflection errors in the cable tension data for trial A20. Firstly, the full cable tension time history, comprising all errors, was sampled at 100 Hz to form a new cable tension data set. This was performed a further nine times, each new set starting at a time increment of 0.001 s after the previous set. In effect this produced 10 estimates for the cable tension time history for A20, each at 100 Hz, with a time delay of 0.01 s between each new data set.

Of these 10 data sets, created from the raw cable tension data, the set which started at time equal to 0.008 s displayed no full scale deflection errors, though several large random errors were still present. These data were then assumed to start at 0.01 s, a 0.002 s error. This assumption may lead to an error one fifth in size of that associated with the synchronisation procedure used to synchronise force and video derived data. Hence, any errors due to this assumption may be deemed negligible (Section 4.6).

The large random errors were attenuated by calculating a new value from the average of adjacent data values. This modified 100 Hz data set was then subjected to a Fourier transformation and truncated inverse Fourier transformation with all frequencies above 45 Hz reduced to zero, and then rectified for the -15 N systematic error. By adopting this procedure the cable tension time history for trial A20 was subjected to the same process in random and systematic error reduction as the other five trials. This modified data set was used in all further analyses concerning trial A20.
The battery of five Fortran programs, written to reduce the random and systematic errors in the cable tension data, were implemented in the following order:

**trigfilter**: calculated the Fourier coefficients for the cable tension time data with 2n+1 samples.

**reduce**: reduced to zero the Fourier coefficients relating to frequencies greater than the cutoff frequency $f_c$.

**triginverse**: performed an inverse Fourier transform using modified Fourier coefficients, producing raw50-$f_c$ cable tension data.

**res**: calculated the residual and maximum difference between the two sets of raw50 and raw50-$f_c$ data.

**sample**: rectified the systematic offset of -15 N and produced a 50Hz sampled time history and synchronised the cable tension data and kinematic data derived from the video analysis using the procedure outlined in Section 4.5.

### 4.5 Analysis of video data

Analysis of the video data comprised several steps. Firstly, synchronised video images of the calibration volume, gymnast and rings frame were digitised. Using a direct linear transformation technique both camera-digitiser systems were calibrated by 11 parameters for each system. The three-dimensional locations of the digitised points on the gymnast and rings frame were reconstructed using data from both camera views. Orientation and configuration angles for the gymnast and cables were determined from these three-dimensional locations, as well as deformations and movements of the rings apparatus. Original data were fitted using quintic splines to obtain fitted estimates of angles, displacements and their derivatives.

#### 4.5.1 Preparation of the Hi8 video tapes for digitising

To digitise the calibration volume and longswing performances copies of the original Hi8 video images were made in an sVHS format. The horizontal resolution of Hi8 and sVHS formats are identical (400 vertical lines) which ensures the quality of the copies were similar to the original recordings. Video tapes in the sVHS format are used in the Target-Apex high resolution video digitising system (Kerwin, 1995). During this copying procedure, a vertically interlaced timecode (VITC) was incorporated onto the sVHS recordings using timecoding equipment (IMP Electronics V-9000). The VITC provides a method of identifying each field of the recording in the Target-Apex high resolution digitising system.
4.5.2 Digitisation protocols

The Target-Apex high resolution video digitising system was used for all video digitising. Two digitising protocols were used, one for the calibration structure and a second for the digitisation of the gymnast and rings cables during performances of the longswings. For both camera views every field was digitised giving a sampling rate of 50 Hz. Digitised data derived from the camera views were labelled 'F' for the HyperHAD camera and 'S' for the HandyPRO camera (Figure 4.5).

Digitisation of the calibration volume

The digitisation protocol for the calibration volume involved digitising the centres of the 12 calibration markers in a video field. To obtain better estimates for the digitised locations of the calibration markers the calibration structure was digitised over five consecutive fields, providing five estimates of the same digitised points. The mean value for each marker was used to represent its best digitised estimate. This procedure was repeated for both camera views. Digitisations were performed in numerical order 1 to 12 shown in Figure 4.6. The actual three-dimensional coordinates of the markers are provided in Table 4.1.

Digitisation of the gymnast and rings apparatus

Prior to digitising images of the gymnast and apparatus, the start and completion of each longswing was determined. These instants in time indicated the timecode readings at which digitisation was to commence and finish for each trial. The start of each longswing was determined by estimating when the gymnast was in a handstand position, half a pendulum swing prior to the descent. Completion of the longswing was determined as the instant at which one pendulum swing from handstand to handstand had been completed by the gymnast. For each trial the VITC timecodes were used in conjunction with the illuminated LED array to ensure digitisations from the two gen-locked cameras views were synchronised. For each trial the same number of fields prior to and proceeding the initial illumination of the LED array from each camera view were digitised. This ensured digitised data from each camera view for each trial were synchronised. The field in which the LED array was first illuminated was also used for the synchronisation of force and kinematic data obtained from the video analysis.

The digitisation protocol for the longswings involved digitising the gymnast, cables and selected landmarks on the apparatus. In each video field 25 points were digitised. Fifteen of these points related to the gymnast: the joint centres of the left and right wrists, elbows, shoulders, hips, knees, ankles, metatarsal-phalangeal joints and middle of the
head. This produced an 11 segment model of the gymnast which incorporated a torso, head, left and right arms, thighs, shanks and feet. Furthermore the points at which the left and right hands of the gymnast gripped the rings were digitised.

Eight other landmarks, relating to the rings apparatus, were also digitised in each field: the top of both damped elastic devices (DED), the bottoms of the attachments for both cables, two markers placed on the strap of each cable and markers 3 and 6 of the calibration volume. Markers 3 and 6 were used later in the video analysis to indicate whether any movement or change in focal length of the cameras had occurred during the data collection session.

All six longswing trials were digitised from both camera views. The number of video fields digitised from each view for each trial is provided in Table 4.3.

<table>
<thead>
<tr>
<th>trial</th>
<th>subject</th>
<th>longswing type</th>
<th>video fields / camera view</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>K</td>
<td>forward</td>
<td>187</td>
</tr>
<tr>
<td>K05</td>
<td>K</td>
<td>forward</td>
<td>190</td>
</tr>
<tr>
<td>A09</td>
<td>A</td>
<td>forward</td>
<td>166</td>
</tr>
<tr>
<td>K15</td>
<td>K</td>
<td>backward</td>
<td>146</td>
</tr>
<tr>
<td>A20</td>
<td>A</td>
<td>backward</td>
<td>173</td>
</tr>
<tr>
<td>A21</td>
<td>A</td>
<td>backward</td>
<td>178</td>
</tr>
</tbody>
</table>

The Target software produced an ASCII format file comprising the (X,Z) locations for each digitised point in each field from each camera view for the calibration volume and for the longswings. Coordinates for points digitised from the view of the Hyper HAD camera were labelled (FX,FZ), coordinates for the images from the HandyPRO camera view were labelled (SX,SZ).

4.5.3 Video analysis protocol

In order to transform the X-Z digitised coordinates of the 25 landmarks from each camera view to their three-dimensional locations in space a direct linear transformation technique (DLT) was used. The theory behind this procedure is described in Section 2.4.4. A series of Fortran programs, developed by Dr M.R. Yeadon, were modified and subsequently used to firstly calibrate the camera-digitiser systems and then reconstruct the three-dimensional locations of unknown points digitised in the longswing trials.
Calibration of the camera-digitiser systems

Prior to the calibration of both camera-digitiser systems all digitised coordinates of the markers comprising the calibration structure were scaled to pseudo-scaled values by assuming a 10 m horizontal field of view. Using the five video fields of the calibration volume the centre of the digitised image was determined for both camera views, a value required during the direct linear transformation. Furthermore two diagonals were formed by markers 1 with 4 and 3 with 6. These diagonals were used to detect and, if required, adjust for camera movements or focal length changes which may have occurred during the data collection.

Both camera-digitiser systems were calibrated using the known three-dimensional locations of the 12 calibration markers and their respective pseudo-scaled digitised \((X,Z)\) locations. The mean digitised values for each calibration marker over five video fields were used in the calculation of the 11 DLT parameters (Section 2.4.4). With 12 markers in the calibration procedure 24 equations were formed from which the 11 parameters were calculated in a least squares solution using a simultaneous linear equation solver (Stewart, 1973). The parameters \(L_1\ldots L_{11}\) describe the camera-digitiser system comprising the HyperHAD camera while the parameters \(M_1\ldots M_{11}\) describe the system using the HandyPRO camera (Equation 2.1).

Three-dimensional reconstruction accuracy of the calibration markers

The accuracy of the three-dimensional DLT method for this particular calibration volume was calculated using the 11 DLT parameters for both camera-digitiser systems and the digitised values of the 12 calibration markers to reconstruct their three-dimensional locations. This procedure was performed using the reconstruction equation for DLT (Equation 2.2). The overall root mean squared difference between the known three-dimensional locations of the 12 calibration markers and the reconstructed three-dimensional locations was calculated. The largest and smallest differences between known and reconstructed three-dimensional locations were also calculated.

Three-dimensional reconstruction of landmarks digitised in the longswing trials

Prior to the three-dimensional reconstruction of landmarks in the video images of longswings two procedures were performed. Firstly, the coordinates of all digitised landmarks on the gymnast and rings frame from both camera views in all six trials were scaled to pseudo-scaled digitised points assuming a 10 m horizontal field of view. Secondly, the ratio calculated from the diagonal lengths in the calibration video fields were also calculated for the video fields of all longswing trials. The ratios of the
diagonals calculated during the calibration and longswings were used to establish whether changes in the locations of the cameras or focal lengths were made during the data collection and, if observed, corrected for such changes. The results from these calculations indicated that neither camera had moved or had any changes made to their focal lengths during the data collection session.

The video analysis program VIDEO was used to reconstruct the three-dimensional locations of all 25 digitised landmarks in each field of each longswing. Three-dimensional reconstruction using DLT requires the 11 DLT parameters for each camera together with the synchronised digitised coordinates for each landmark from both camera views throughout each longswing (Equation 2.2). Digitised coordinates of the HyperHAD camera (FX,FZ) are equivalent to the coordinates (u,v) in Equation 2.2 while digitised coordinates of the HandyPRO camera (SX,SZ) relate to (q,r) in the same set of equations. The least squares simultaneous solution for each equation was determined using an equation solver (Stewart, 1973) and the three-dimensional locations of each digitised point were calculated.

The three-dimensional coordinates of all digitised landmarks produced directly from the DLT procedure were termed original estimates. Within the Fortran program VIDEO further output values were calculated using these original estimates. Where applicable the left and right sides of the apparatus were used to establish changes in its structure. Output calculated as an average of the three-dimensional locations of landmarks on the left and right sides of the apparatus include the vertical and horizontal movement of the DEDs and cable lengths (Sections 4.7.2 to 4.7.4).

Further output calculated in VIDEO, which used the original estimates of the three-dimensional locations of landmarks include the mass centre location of the gymnast and the orientation and configuration of the gymnast and cables. The definitions and methods of calculating these output are described in Sections 4.5.4 though to 4.5.6.

4.5.4 Calculation of the mass centre location of the gymnast

In order to calculate the three-dimensional location of the gymnast's whole body mass centre the program VIDEO requires the subject specific anthropometric data for each segment calculated by Yeadon's mathematical inertia model. These ratios of segmental masses and centre of mass locations from proximal end-points allows the calculation of the whole body mass centre location of the gymnast in the inertial reference frame. The calculation of the gymnast's mass centre throughout each longswing is based upon the original location data for segmental end-points.
4.5.5 Definitions and calculations of gymnast orientation and configuration angles

Results from the kinetic and kinematic data collection have two applications. Firstly, a full three-dimensional angular kinematic description of the techniques used by these elite gymnast will be obtained. Descriptions of this nature have not been previously established in the literature. Previous research has reduced the complexity of the gymnast in kinematic studies to a two-dimensional three link system of arms, body and legs (Nissinen, 1983; Brüggemann, 1987, 1994).

Secondly the kinematic data will be used to drive the simulation model FOURSEG in the evaluation procedure and subsequently to determine the influence of the different aspects of technique on performance. Hence, orientation and configuration angles are required which describe the techniques used by the gymnasts to perform the longswings and which also relate to the angles defined in FOURSEG.

For the angular kinematic description of the gymnast's technique a gymnast was modelled as a 5 segment system comprising two arms, a torso with head, a thigh and a shank with foot. Modifications to Yeadon's VIDEO program enabled the calculation of angles which were the average of the left and right sides of the gymnast. Hence, one orientation angle and four configuration angles describe the technique of a gymnast, assuming the left and right sides of the gymnast are mirror images. Evidence suggesting that this assumption is close to reality is provided by cable tension and mass centre location data. In addition one further angle, the body angle $\varepsilon$, was calculated. This angle describes the overall orientation of the gymnast, as described in Chapter 3. Orientation and configuration angles were calculated using the method of Yeadon (1984) and modifications thereof. This method utilises reference frames with mutually perpendicular axes defined within body segments.

The orientation of a right-handed triad $f$ comprising units vectors $(f_1, f_2, f_3)$ from another triad $i$ comprising unit vectors $(i_1, i_2, i_3)$ may be described generally by three successive ordered rotations of $\phi, \theta, \psi$ about $i_1, i_2, i_3$ respectively. Similarly reference frame $f$ may be brought into alignment with frame $i$ through the reverse ordered rotations of $-\psi$ about $f_3$, $-\theta$ about $f_2$ and $-\phi$ about $f_1$. Letting $R_1(-\phi)$, $R_2(-\theta)$, and $R_3(-\psi)$ be the rotation matrices describing the alignment of frame $f$ with frame $i$ then:

$$R(-\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$
\[
R(-\theta) = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]

\[
R(-\psi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The general matrix \( S_{fi} \) transforms the \( f \) coordinates of a vector into \( i \) coordinates:

\[
S_{fi} = \begin{bmatrix}
F1(1) & F2(1) & F3(1) \\
F1(2) & F2(2) & F3(2) \\
F1(3) & F2(3) & F3(3)
\end{bmatrix}
\]

where \( F_k(j) \) denotes the \( j \)th component of \( f_k \) in frame \( i \).

As \( S_{fi} = R_1(-\phi_g) \cdot R_2(-\theta_g) \cdot R_3(-\psi_g) \) then:

\[
S_{fi} = \begin{bmatrix}
\cos \theta \cos \psi & -\cos \theta \sin \psi & \sin \theta \\
\cos \phi \sin \psi + \sin \phi \sin \theta \cos \psi & \cos \phi \cos \psi - \sin \phi \sin \theta \sin \psi & -\sin \phi \cos \theta \\
\sin \phi \sin \psi - \cos \phi \sin \theta \cos \psi & \sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi & \cos \phi \cos \theta
\end{bmatrix}
\]

Comparison of these two definitions for the matrix \( S_{fi} \) allows the cosine and sine for all three angles to be calculated using the three-dimensional locations of landmarks on the gymnast as:

\[
\sin \theta = F3(1)
\]
\[
\cos \theta = (1 - \sin^2 \theta) \text{ with } \theta \text{ lying within the range } \pm \pi/2
\]
\[
\sin \phi = -F3(2)/\cos \theta
\]
\[
\cos \phi = F3(3)/\cos \theta
\]
\[
\sin \psi = -F2(1)/\cos \theta
\]
\[
\cos \psi = F1(1)/\cos \theta
\]

This method of calculating all three ordered rotation angles forms the basis of all angle calculations in VIDEO for the gymnast and cable. In most cases only two of the angles were required (those relating to \( \phi \) and \( \theta \)) though estimates of \( \psi \) values were also calculated.

The three-dimensional locations of the 15 joint centres are used to calculate the mutually perpendicular reference axes in each segment. For configuration angles the frames \( f \) and \( i \) are represented by adjacent body segments. Segments were defined by three-dimensional vectors between appropriate joint centres or other calculated points. The following configuration angles, which define the technique of the gymnast, were
calculated using the methods described above: knee, hip, shoulder elevation and shoulder abduction.

Knee angle

A single angle describes the magnitude of bend at both knees of the gymnast with the assumption that the knees are hinge joints. The knee angle is defined as the first ordered rotation angle between the reference frames defining the thighs and shanks of the gymnast. Shank segments were defined by three-dimensional vectors from the knee to ankle joint centres. Thigh segments were defined by three-dimensional vectors from hip to knee joint centres. The average cosine of this angle for the left and right sides of the gymnast is used to represent the single knee angle. For a straight leg the knee angle equates to $180^\circ$. A reduction in knee angle indicates the average bend at the knees of the gymnast (Figure 4.8a).

![Figure 4.8](image)

Figure 4.8. Definitions of (a) the knee angle and (b) the hip angle in the video analysis.

Hip angle

The hip angle represents the average angle between the thigh segments and torso of the gymnast. This angle is equivalent to the first ordered angle between these two reference frames. The torso segment was defined by a three-dimensional vector joining the mid-points of the shoulder and hip joint centres. The average value for the angle between the torso and the left and right thigh segments is used to represent the angle at the hip joint. The hip angle is defined as being less than $180^\circ$ when the legs of the gymnast are in front of the body, such as in a piked position (Figure 4.8b), and greater
than 180° when the legs are behind the gymnast, in an arched position.

**Shoulder elevation and abduction angles**

Two ordered rotation angles, $\phi$ and $\theta$, define the configuration of each arm segment from the torso of the gymnast. These two angles at the shoulder joint are termed shoulder elevation and abduction respectively. Figure 4.9 highlights the rotations corresponding to these angles for the right arm of the gymnast.

![Diagram showing right shoulder abduction angle and elevation angle](image)

**Figure 4.9. Definitions of the shoulder elevation and abduction angles in the video analysis.**

When calculating the shoulder elevation and abduction angles the arm segments are defined by the three-dimensional vector from the shoulder to the hand on the same side of the gymnast. In order to align the arm segment from its known configuration to a neutral position at the side of the gymnast's torso two successive rotations, of angles $\phi$ and $\theta$ are required about the mutually perpendicular reference axes within the arm.

The shoulder elevation angle is positive when an arm is lifted forwards of the gymnast from an initial position at the side of the torso. An elevation angle of 180° indicates the arm is directly above the head of the gymnast. The shoulder abduction angle describes the rotation of the arm away from the sagittal plane of the gymnast in the plane defined by the shoulder elevation angle. Positive values for this abduction angle denotes a displacement of the gymnast's hands away from the mid-line of his body. The shoulder abduction angle, therefore, is a measure of how wide the gymnast's arms are during the
longswings. The average values for elevation and abduction angles for the left and right sides of the gymnast were calculated, taking into account the direction of rotation. These average values represent the shoulder elevation and abduction angles of the gymnast throughout a performance.

Orientation angles of the gymnast

The orientation of the torso segment was calculated with respect to the inertial frame. For these calculations frame i represented the inertial reference frame, while f represented the torso segment of the gymnast. As the motion of the longswing elements on rings are essentially in the plane described by the y and z inertial axes only the first rotation angle $\phi$ was calculated (Figure 4.10). Angles describing tilt $\theta$ and twist $\psi$ of the torso in space (Yeadon, 1984) were not required. With a calculated orientation angle of zero degrees the torso segment was in an inverted position. Hence, when in handstand on rings, a value close to $0^\circ$ was calculated which retained consistency between video determined values and values in FOURSEG.

![Diagram](image)

Figure 4.10. Definitions of the torso angle and 'body angle' orientations in the video analysis.

A second angle was used to describe the average orientation of the gymnast. This angle, termed the body angle $\varepsilon$, was defined as the angle between the vector joining the mid-point of the hands to the mass centre and the vertical (Figure 4.10). Zero degrees indicated a gymnast in the handstand position. After the completion of a backward longswing the value of this angle is near $360^\circ$. For a completed forward longswing the body angle is close to $-360^\circ$. 
The gymnast configuration angles of knee, hip, shoulder elevation and abduction defined in the video analysis are consistent with their definitions used in the model FOURSEG. The two gymnast orientation angles (torso and body) are also consistent with the orientation angles defined in the simulation model. The configuration angles are used to drive the simulation model in the evaluation process, and hence derivatives for each angle are required. The method of estimating derivatives for all angles is provided in Section 4.5.7.

4.5.6 Determination and definition of orientation angles for the rings cables

The orientation of the rings cables in the inertial frame is described by the two rotation angles $\phi_c$ and $\theta_c$, termed cable elevation and abduction respectively (Figure 4.11).

![Diagram of cable orientation angles](image)

Figure 4.11. Definitions of the right cable orientation angles of elevation and abduction in the video analysis.

The right rings cable was defined by a three-dimensional vector from the right hand of the gymnast to the right pivot point. The left rings cable was defined in the same manner for the left side of the system, these vectors forming the general frame $f$. The inertial reference frame was equivalent to frame $i$ in the general method of calculating angles by Yeadon (1990a). For both cable orientation angles the average of the left and
right values were used to represent the cable elevation and abduction, taking into consideration the sign convention of the rotations.

The cable elevation angle describes how far the rings cables have rotated in the anterior-posterior direction. This angle is equivalent to the cable angle described in the two-dimensional simulation models as well as the model FOURSEG. Where there is an increasing positive body angle $\theta$ when performing a backward longswing, the cable elevation angle also increased during the descent of the longswing.

The rotation describing the cable abduction angle occurs in the plane defined by the cable elevation angle. This second angle is therefore a measure of how far apart laterally the rings cables become during the longswings. Positive values indicate the cables are spread to the sides of the gymnast (ie when shoulder abduction angles are positive). With the rings cables hanging vertically both cable orientation angles equal zero degrees. The cable orientation angles are used in the evaluation of the simulation model FOURSEG.

All angles described were derived from the original estimates of the reconstructed three-dimensional locations of the digitised points. These first estimates of angles are reported in Chapter 5. In addition the initial angle and velocity of the cable elevation are required in the evaluation process to ensure the simulation model starts with the correct motion. In order to obtain derivatives for all angles (cable and gymnast orientation and configuration) the angle data were fitted using the quintic spline routine of Jennings.

4.5.7 Curve fitting angle data to estimate angles and their derivatives

Values for all angles calculated from the original estimates will contain some errors, both systematic and random. For any signal containing random errors the influence of these errors are magnified during the differentiation process (proportional to the frequency of the data) when calculating derivative data (Winter, 1990). In order to obtain accurate estimates of angular velocities and accelerations for all configuration angles, errors within the angle data were reduced. The reduction in random errors was accomplished by fitting the original angle data with a quintic spline (Jennings' quintic spline algorithm), with estimates of error in the data based upon the original data.

*Estimating error in data: local and global variance*

The following method forms the basis of calculating errors, local and global, for all video derived data. Where necessary variations to this general method are highlighted within each section. The original data and calculated error estimates derived in this way were supplied to the quintic spline routine of Jennings to calculate spline fitted data and corresponding derivatives. Spline fitted data are reported when appropriate (Chapter 5).
Let \( x_n \) be a set of values for a transient variable recorded over equal time intervals, where \( x_{g-1}, x_g \) and \( x_{g+1} \) are three consecutive estimates of the variable in time. These data may also be expected to possess random errors, and the variance for each datum may be expected to equal \( \sigma^2 \). In order to obtain an estimate of error in a datum a second estimate of the variable is required for each time interval. This second estimate may be established using data from adjacent fields. In general terms the second estimate of the value of variable at \( x_g \) is calculated as the average of \( x_{g+1} \) and \( x_{g-1} \) and termed \( x'_g \):

\[
x'_g = \frac{(x_{g+1} + x_{g-1})}{2} \quad (\text{where } g = 2 \ldots n-1)
\]

This technique produces a pseudo data set \((x')\) for the variable based upon data from adjacent fields. The values of the first and last data of the pseudo data, \( x'_1 \) and \( x'_n \), are set equal to the original data \( x_1 \) and \( x_n \) as these data do not have two adjacent values.

If the variance for \( x_{g+1} \) and \( x_{g-1} \) are equal to \( \sigma^2 \) then the variance \( (V'_g) \) for the calculated \( x'_g \) is:

\[
V'_g = \frac{1}{2}\sigma^2 \quad (\text{Yeadon, 1984; op. cit. Shchigolev, 1965}).
\]

The deviation \( d_g \) between the pseudo value \( x'_g \) and \( x_g \) is:

\[
d_g = (x_g - x'_g).
\]

Hence, the total variance \( V_g \) for the estimate of the variable at \( x_g \) is:

\[
V_g = \text{variance } x_g + \text{variance } x'_g = \sigma^2 + \frac{1}{2}\sigma^2
\]

The local variance \( V_{local} \) for the data at \( x_g \) is \( 2/3 \) of the total variance owing to the method of producing the pseudo data and the calculation of error estimates from the pseudo data. The global variance \( V_{global} \) for the original estimates of the transient variable is the mean of all local variances.

The quintic spline routine of Jennings requires error estimates for each data point. A combination of local and global variances was used as a measure of error in the original data. For all angle data, error estimates were formed by the square root of the sum of 50\% global variance \( V_{global} \) and 50\% local variance \( V_{local} \).

For other video derived data different ratios of local and global error estimates were used. The actual proportions used are provided in each case.

The quintic spline routine of Jennings also requires a value for the 'S' or 'smoothness' parameter of Reinsch (1971). In all cases this value was set to \( N \), the total number of data points used in the analysis.
4.6 Synchronisation of cable tension and video data

In order to synchronise filtered cable tension data and data derived from video analyses simultaneous events were delivered to both data sets during each trial. For video data this event comprised the illumination of the LED array. For cable tension data a switch contact was used to PERI-TRIGGER the ADC data collection 1.5 s prior to the contact, and collect for 3.5 s after contact (Section 4.2.2). Figure 4.12 highlights the method of synchronising cable tension and video data. Video images at 50 Hz are represented by white rectangles. The effect of a shutter speed less than 1/50 s is shown by each rectangle spanning a time period less than 0.02 s. The time scale for tension data, at a frequency of 1000 Hz, is indicated on the lower horizontal axis.

For all trials it was assumed the simultaneous events occurred in the middle of two successive video fields, over which the LED array was illuminated and recorded as such in the video images (fields A and B). This means the first field in which the LED became illuminated is equal to 1.51 s in the time base for cable tension. Establishing the video field which first displays the illuminated LED (eg B) for each trial forms the synchronisation process.

![Diagram of synchronisation process](image)

Figure 4.12. Method of synchronising cable tension and video derived data.

This procedure enabled the synchronisation of the filtered cable tension data and kinematic data derived from the video analysis to an accuracy of ±0.01 s. For all regression analyses described in Section 4.7 only synchronised filtered cable tension data for a single cable sampled at 50 Hz were used.

**Investigating the assumption of equal tension in the rings cables**

During the data collection session, tension in only one rings cable was measured
and recorded under the assumption that tension in the other rings cable was identical. Several previous studies have either indicated or implied that this assumption is valid (Nissinen, 1983; Cheetham et al., 1987). During the present study an attempt to verify this assumption was made by asking the gymnasts to perform both types of longswings in each direction and measuring cable tension.

Suppose two separate longswings are performed by the same gymnast. Although the performances are unlikely to be identical, if both are initiated from similar conditions and completed with similar technique and competency then it may be expected that the cable tension time histories would be similar. This expectation may be addressed by reversing the direction in which the gymnast initially faces over two separate trials, effectively recording cable tension on each side of the gymnast's body. A comparison of these two independent recordings of cable tension patterns may then be used to investigate the assumption.

The following analyses were performed in conjunction with this assumption. The cable tension data sampled at 50 Hz for trials K03 and K05 for forward longswings and A20 and A21 for backward longswings were used. Table 4.2 highlights that for both types of longswings cable tension was effectively measured in both rings cables. As these longswings were performed over different time bases, that is the collection of tension data was initiated at slightly different parts of the swing for each trial, synchronising the data for the two trials was required. This synchronisation was accomplished by matching peak cable tensions for the two trials being used to analyse the assumption, under the conjecture that peak tension occurs at the same gymnast orientation during a longswing. All overlapping data from the two cable tension time histories were used to calculate the root mean square difference between the trials. For the backward longswing trials this gave 4.80 s of data while for the forward longswings 4.10 s were available. The root mean squared difference was expressed in Newtons and also as a percentage of the average maximum peak cable tension observed in both trials.

4.7 Estimating parameters for the three-dimensional simulation model

The four segment simulation model requires values for several parameters, including those relating to the modelled gymnast and rings apparatus. Results derived from the anthropometric data were used to establish segmental inertia parameters specific to each subject. By simplifying the structure and movement of the rings frame, an estimate of the effective mass of the rings frame was determined. Cable tension and video derived data were used to obtain estimates for model parameters specific to the elasticity of the gymnast and rings apparatus.
4.7.1 Personalised segmental inertia parameters representing the gymnast in the four segment model

Estimates of length, mass, mass centre location and principal moments of inertia for each segment in the four segment simulation model were required. These parameters are collectively known as segmental inertia parameters. In order to obtain subject specific estimates of these parameters the anthropometric data recorded for each gymnast at the data collection session were used in conjunction with the mathematical inertia model of Yeadon (1990b). Besides calculating mass and mass centre locations for the 14 segments in the 'extended' inertia model, their principal moments of inertia were also calculated. By combining appropriate segments from the inertia model of Yeadon, values corresponding to the segments comprising the four segment simulation model were also calculated (Table 4.4).

Table 4.4. Segments from the inertia model of Yeadon which represent the segments in the FOURSEG simulation model

<table>
<thead>
<tr>
<th>FOURSEG model</th>
<th>inertia model of Yeadon (1990b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>upper arm (1A, 1B), forearm (2A, 2B), hand (3A, 3B)</td>
</tr>
<tr>
<td>torso with head</td>
<td>head (H), trunk (PTC),</td>
</tr>
<tr>
<td>thigh</td>
<td>thigh (1J, 1K)</td>
</tr>
<tr>
<td>shank with foot</td>
<td>calf (2J, 2K), foot (3J, 3K)</td>
</tr>
</tbody>
</table>

Segments of the inertia model which were combined to represent those segments of the four segment simulation model were assumed to lie on a common axis. The parallel axis theorem and principle of moments were used to calculate the subject specific segmental inertia parameters for both subjects, A and K, for the four segment simulation model. For the arm, thigh and shank segments of the four segment model the average results from the inertia model for the left and right sides of the gymnast were used. When calculating inertia parameters for the arm segment, the wrist to knuckle lengths of the gymnast was used owing to the grip of the gymnasts' hands on the rings. As the simulation model only represents the right side of the gymnast the masses and principal moments of inertia for the torso, thigh and shank segments were adjusted to provide the appropriate values for the segmental inertia parameters for these segments.

Video data were also used to determine the average value for LT₄, the distance between the right and left shoulders of the gymnast, for each trial. The magnitude of the vector from the right shoulder to the left shoulder was determined for each field of each trial. For each trial the mean value for LT₄ was calculated. Subject specific values for
LT₄ were based on the average of the mean values over the three trials for each subject. These values defining the subject specific length of LT₄ were used in all subsequent simulations and analyses.

4.7.2 Estimating the effective mass representing the rings frame in the four segment model

The effective mass in the four segment model represents half the mass of the rings frame through a single massive point at the top of the rings cable. The mass of this massive point was estimated using an analogy between linear and angular displacements for small oscillations about a fixed point, a simplification concerning the motion of the rings frame and a relationship between the point of force application and the moment of inertia through the base of the rings frame about the Newtonian x axis. Figure 4.13 highlights these relationships and assumptions.

![Figure 4.13. Hypothetical motion of the rings frame when a gymnast is swinging.](image)

When a gymnast is swinging on rings it may be assumed the rings frame rotates about the floor O in the plane defined by the y-z axes of the Newtonian frame with a small amplitude of oscillation (Figure 3.21). The force F represents the horizontal component of cable tension exerted at the top of the rings frame. Using this simplified approach:

\[ s = h\gamma \]

and for small values of \( \gamma \):

\[ y = s \]

and therefore:

\[ y = h\gamma. \]
By substitution and differentiation with respect to time the following relationship is established:
\[ \ddot{y} = \frac{\dot{y}}{h} \]

If \( I_o \) is the moment of inertia of the rings frame about point O in the x axis then the torque \( T \) about point O forms the following relationship:
\[ T = I_o \ddot{y}. \]

As torque \( T \) is also equal to the product of force \( F \) and moment arm \( h \) with rearrangement of the above equations into the form of Newton's Second Law the following equation is produced:
\[ F = \frac{I_o \dot{y}}{h^2}. \]

Hence, for small amplitudes of motion \( I_o/h^2 \) forms the effective mass of the complete rings frame.

The manufacturer's designs for the rings frame were used to calculate the total mass of the rings frame and the value of \( I_o \) using the principle of moments and parallel axis theorem. The total mass of the rings frame was calculated to be 72 kg. Using \( I_o/h^2 \), the effective mass representing the whole frame is 33.5 kg. Hence, the effective mass \( em \), representing only the right side of the rings apparatus, is equal to 16.75 kg. This value for the effective mass of the rings frame was kept constant throughout the study.

4.7.3 Estimating spring parameters representing the rings frame in the four segment model

In the four segment, three-dimensional simulation model the horizontal movement of the rings frame is represented by a damped linear spring with the effective mass \( em \), attached to one end (Figures 3.18 and 3.19). The effective mass represents the right side of the rings frame and is restricted to movements parallel to the inertial y axis.

For each analysed longswing the three-dimensional locations of the DEDs were calculated using DLT. The average locations of the left and right DEDs were determined for each video field throughout each trial. The average locations of the DEDs in the z direction were used to identify the video field in which the DEDs were at their lowest vertical location during each trial. For each trial this video field was termed the 'PEAK' field. This field denoted a key stage in the subsequent analyses.

Pseudo data sets were calculated for the average three-dimensional locations of the DEDs for each trial using the average of adjacent fields. These pseudo data were used to estimate the global variance and the local variance for the average x, y and z locations for each longswing trial using the methods outlined in Section 4.4.7. Customised versions of
all local variances were utilised in Jennings' quintic spline routine to reduce errors in the average (of left and right) y and z locations of the DEDs and subsequently estimates of velocity and acceleration.

Location data of the DEDs in the y direction were used to determine the physical properties of the horizontal spring in the four segment model. Owing to the 50 Hz sampling rate of video and the oscillation of the DEDs, a single datum often identified the peaks in the oscillations of the DEDs location in the y direction. Using data from adjacent video fields to calculate local error estimates in such data may overestimate errors for data identifying peaks. Figure 4.14 shows a hypothetical situation where a single point defines the peak of an oscillation. When using data from adjacent video fields to determine local variance, the value for point A is substantially larger than for data either side of the peak. This discrepancy is due to the process of calculating local variance and not the process of obtaining the data. If, in the cases of the DEDs, the digitised landmarks remain in view throughout a trial, it may be expected that local error estimates would not increase so dramatically. Any approach to fitting the data which depends on local estimates of error would therefore oversmooth for that particular datum, and an incorrect amplitude for the oscillation would be obtained. There is, it seems, an inherent inaccuracy in using the procedure for calculating local variance for oscillating data. Furthermore, estimates of global variance are also affected by oscillations in the data, rendering its sole use in such specific cases as suspect. This inaccuracy may be vastly reduced by observing original data and ensuring inaccurate error estimates are not formed.

![Graph showing data and error](image)

Figure 4.14. Hypothetical example highlighting the possible inaccuracy of calculating error estimates using data from adjacent video fields for oscillating data.

Taking these possible inaccuracies into consideration, Jennings' quintic spline
routine was implemented to fit the smoothest spline (parameter $S' = 1.0*\text{N}$) to the average locations of the DEDs in the $y$ direction for each trial using customised local variances for error estimates. Error estimates were set to 100% local error estimates from the first video field to two prior to the identified PEAK for each trial, where the PEAK field coincided with the first amplitude of the oscillations. From two fields prior to the PEAK field through to the final video field, error estimates were reduced by a factor of 4 to ensure the original average data were fit closely to the data points identifying the peaks of the oscillations. Fitting quintic splines to the data allowed the data to be interpolated and the velocity of the DEDs in the $y$ direction to be determined.

The time history of the DEDs $y$ location resembled that of a mass undergoing simple harmonic motion, with the motion produced by an underdamped linear spring. As this motion is representative of the horizontal spring in the four segment simulation model these data were used to experimentally determine the stiffness and damping parameters of this spring.

The general equation, in terms of time, describing the displacement $x$ of an oscillating mass from its neutral position due to an underdamped linear spring is:

$$x = C e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)$$  \hspace{1cm} (4.7)

where:

- $C$ = constant, representing maximum amplitude of oscillation
- $\zeta$ = the spring damping ratio
- $\omega_n$ = the natural angular frequency
- $\omega_d$ = the damped natural frequency
- $t$ = time
- $\phi$ = phase angle
- $x$ = displacement of spring from neutral position.

Equation 4.7 provides a time history for the underdamped linear spring which is an exponentially decreasing harmonic function of time. This time history is shown in Figure 4.15.

The damped natural frequency $\omega_d$ and damped period $\tau_d$ of the oscillations (Figure 4.15) are given by:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$\tau_d = \frac{2\pi}{\omega_d}$$

so that:

$$\omega_n = \frac{2\pi}{\tau_d \sqrt{1 - \zeta^2}}$$

These properties of the oscillations are related to the stiffness $k$ and damping $c$ of the underdamped linear spring in the following manner:
\[ k = m\omega_n^2 \]
\[ c = 2\zeta m\omega_n \]

where \( m \) is the mass of the particle which is undergoing damped simple harmonic motion.

The ratio of the displacements \( x_1 \) and \( x_2 \) of the oscillating mass over two consecutive peaks (times \( t_1 \) and \( t_1 + \tau_d \) (Figure 4.15) is equal to:

\[
\frac{x_1}{x_2} = \frac{Ce^{-\zeta\omega_n t_1}}{Ce^{-\zeta\omega_n (t_1 + \tau_d)}} = e^{\zeta\omega_n \tau_d}.
\]

The logarithmic decrement \( \delta \) for this ratio is defined as:

\[
\delta = \ln\left(\frac{x_1}{x_2}\right) = \zeta\omega_n \tau_d
\]

Substituting \( \tau_d \) in terms of \( \omega_n \) and \( \zeta \) and then solving for \( \zeta \) produces:

\[
\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}
\]

This provides a method of determining the damping ratio \( \zeta \) for the spring. With a known ratio of spring damping and a known mass being oscillated, the stiffness and damping values of the spring allowing such oscillations can be determined (Meriam and Kraige, 1987). These values represent the underdamped linear spring in the four segment simulation model.

Figure 4.15. The exponentially decreasing harmonic function describing the motion of an underdamped linear spring.

For each trial the following procedure was performed to determine the stiffness and damping of the spring constraining the oscillations of the DEDs in the actual
performances. The quintic spline fitted data for the average y location of the DEDs were used, producing data at a frequency of 100 Hz. The value for the damped period \( \tau_d \) was calculated as the average value of the first two full periods of oscillation. Taking into account the symmetrical nature of the response about the neutral location a value representing \( x_1 (x_1^*) \) was determined from the first full oscillation, using the value from peak to peak. Similarly a value representing \( x_2 (x_2^*) \) was estimated from the second full oscillation using the peak to peak value. These methods of estimating values for \( \tau_d, x_1 \) and \( x_2 \), were adopted to decrease errors associated with digitising and the reconstruction process. The damping ratio \( \zeta \) was determined and the value of \( \omega_n \) calculated. The stiffness and damping values for the spring in the four segment simulation model were calculated with the mass \( m \) equal to the effective mass \( m_e \) (16.75 kg) of the right side of the rings frame. The mean and standard deviations of stiffness and damping for the 6 trials were calculated providing a range of values for the horizontal spring in the simulation model. In addition the coefficient of variation was calculated for the two spring parameters.

4.7.4 Estimating spring parameters representing the DEDs and the horizontal beam of the rings frame in the four segment model

The four segment simulation model reproduces the combined elasticity of the DEDs and horizontal beam of the rings frame by a damped linear spring restricted to movements parallel to the inertial z axis (Figures 3.18 and 3.19). The physical characteristics of this spring should represent the combined elasticity of the DEDs and horizontal beam of the rings frame. A combination of cable tension data and vertical location data of the DEDs were used to estimate the stiffness and damping values of this spring in the simulation model.

Tension produced by a damped linear spring is proportional to the extension of the spring from its natural length under equilibrium conditions as well as the rate of extension. The general equation for a damped linear spring is:

\[
tension = -kx - cx
\]

(4.8)

where:
- \( k \) = stiffness coefficient
- \( c \) = damping coefficient
- \( x \) = extension of spring
- \( \dot{x} \) = rate of spring extension

Figure 4.16 highlights the tension exerted by a damped linear spring on an object where compression and extension of the spring is produced for two hypothetical
situations. The first situation shows the force exerted by a spring on the object when extended and compressed under static conditions. The second situation shows the force exerted by the spring when being actively extended and compressed, and therefore possessing a rate of spring extension.

![Graph showing force vs. extension for static and dynamic situations](image)

**Figure 4.16.** Two situations for a damped linear spring highlighting the force exerted by a spring undergoing both extension and compression: static (open circles) and dynamic (closed circles).

For a linear spring under the first condition (open circles) the gradient of the slope is equal to the stiffness of the spring. In the second situation (closed circles) the magnitude of force exerted by the spring on the object is different owing to the damping component of the spring. It therefore follows that if the extension, rate of spring extension, and force are known then values for the spring stiffness and damping can be calculated. If the spring is known to be, or assumed to act, as a damped linear spring then the a multiple linear regression may be used to estimate the stiffness and damping of the spring. By estimating the force in the spring as a function of extension and rate of extension, the coefficients of the two predictor variables represent the stiffness and damping of the spring respectively.

The following procedure was performed for each analysed longswing to provide estimates for the vertical spring in the four segment model. The average vertical component of the three-dimensional locations for the DEDs was determined for each video field. Digitising the tops of the DEDs ensured that the combined vertical movements of the DEDs and horizontal beam of the rings frame were reflected in the three-dimensional data. Pseudo data for the average locations of the DEDs were formed using adjacent fields for each trial. Global and local error estimates were determined
using the pseudo data and the procedure described in Section 4.5.7. Jennings' quintic spline routine was used to fit the smoothest spline (parameter 'S' = 1.0 N) to the average vertical DEDs location, using the calculated local variance to obtain the error estimates with the exception of data for five video fields. For five video fields, PEAK and two video fields prior and subsequent to this field, local error estimates were reduced by a factor of 20. This reduction in error estimates ensured a closer fit to the rapidly oscillating data, reducing the inaccuracies demonstrated in Figure 4.14.

Fitting splines to the data enabled the rate of change in vertical location of the DEDs to be determined giving the velocity of the damped linear spring.

As the rings cables are in tension throughout all longswings the three-dimensional orientations of the cables define the direction of the tension vector. With the magnitude of cable tension measured and recorded, the vertical component of the tension vector may be calculated. If \( \mathbf{r} \) is a vector in inertial space with components \( (x, y, z) \) the cosine of the angle \( \gamma \) between the vertical axis and vector \( \mathbf{r} \) is given by:

\[
\cos \gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}} \tag{4.9}
\]

The three-dimensional vector from the right hand of the gymnast to the right pivot was used to determine the cosine of the angle between the vertical and the right rings cable (Equation 4.9). In the same manner the left hand of the gymnast and left pivot were used to determine the cosine of the angle between the vertical and the left rings cable. The average of these two values describes the mean cosine of the angle between the vertical and the rings cables. This calculation was performed throughout each trial. Although cable tension in only one cable was measured, the average of the left and right rings cables was used to obtain a more accurate estimate of the cosine of the angle. The product of the mean cosine of the angle and the synchronised and filtered force data forms the vertical component of force acting on the DEDs and horizontal beam of the rings frame during each longswing.

The spline fitted vertical location and velocity of the DEDs were synchronised to the vertical component of the filtered cable tension at a sampling frequency of 50 Hz. The DEDs and horizontal beam of the rings frame were assumed to act as a damped linear spring. A multiple linear regression analysis was performed, regressing the vertical component of cable tension (\( \text{vten} \)) against the vertical location (\( \text{sded} \)) and velocity (\( \text{vded} \)) of the DEDs. From this initial analysis the neutral location of the theoretical damped linear spring was calculated, corresponding to the spring length and velocity values which produce zero force. The vertical location data were then modified by subtracting the neutral location of the spring from the actual vertical location to form the extension of the theoretical spring from its natural length (\( \text{esded} \)) throughout the trial. A further multiple linear regression was performed regressing the vertical component of cable tension against the displacement and velocity of the DEDs while forcing the regression through
the origin. The coefficients from this regression for 'esded' and 'vded' corresponded to the stiffness and damping values respectively.

This procedure was repeated for all six trials providing six estimates for each parameter of stiffness and damping for the vertical spring in the four segment model. The mean and standard deviation of these estimates were calculated, together with the coefficient of variation.

4.7.5 Estimating spring parameters representing the elasticity of the rings cables in the four segment model

The elastic nature of the rings cables is represented by a damped linear spring acting in line with the cable segment in the four segment model (Figures 3.18 and 3.19). A combination of cable tension and cable length data throughout the longswings was used to estimate the stiffness and damping of the spring in the four segment model.

Similar to the procedure used to estimate stiffness and damping for the vertical spring the rings cables were assumed to behave as a damped linear spring. Hence, tension in the cables was assumed to be proportional to the extension of the rings cables and rate of change of the extension (Figure 4.16).

For each analysed longswing the following procedure was completed to estimate the stiffness and damping of the damped cable spring in the four segment model. For each video field the three-dimensional vector from the right hand of the gymnast to the right pivot was used to provide an estimate of cable length. The magnitude of the three-dimensional vector, \( l \), with components \( (x,y,z) \), is calculated as:

\[
|l| = \sqrt{x^2 + y^2 + z^2}
\]

(4.10)

Similarly, the length of the left rings cable was calculated as the magnitude of the vector from the left hand of the gymnast to the left pivot, providing a second estimate of cable length. For each digitised video field the average of the right and left cable lengths was calculated. Pseudo data for the average cable length data were produced using data from adjacent fields. The original and pseudo data were used to determine global and local error estimates for the original average cable length data. The average cable length data were fit by the smoothest quintic spline (\( S = 1.0*N \)) using Jennings' quintic spline algorithm with the square root of local variance values as error estimates, for all but five data values. For five data, those corresponding to the PEAK field and two either side of the PEAK field, local error estimates were reduced by a factor of 20. This was performed to counteract the overestimations in local variance using the average values of data from adjacent video fields for oscillating data (Figure 4.14).

Data for the average cable length and rate of change in cable length determined
from the quintic splines were sampled at intervals of 0.02 s and synchronised with the filtered and sampled cable tension data. Multiple linear regressions were then performed on these data.

For each trial a succession of multiple linear regression analyses were performed. The first regression for each trial expressed cable tension (ten) as a function of the average cable length (cl) and rate of change in cable length (vel). This first regression allowed the natural length of the theoretical damped linear spring to be calculated; this length corresponding to a static spring producing zero force. The average cable length data were then modified by subtracting the calculated natural length of the cables from the average cable length to form the extension of the theoretical spring from its natural length (eel). A further multiple linear regression was performed expressing cable tension as a function of the extension and velocity of the cable while forcing the regression through the origin. The coefficients from this regression for 'eel' and 'vel' were the stiffness and damping values respectively, representing the theoretical spring in the four segment model.

This procedure formed six sets of estimates for the stiffness and damping of the damped linear spring representing the elasticity of the cable in the four segment model. The mean, standard deviation and coefficient of variation were calculated for each spring parameters.

4.7.6 Estimating spring parameters representing the elasticity of the gymnast in the four segment model

Within the human body several structures possess elastic properties of differing nature which allow the gymnast to stretch under the action of large external forces. Such structures include the muscles, tendons and ligaments surrounding the joints of the upper and lower limbs as well as the vertebral column, though only some of these structures are passive. Of all the joints in the human body the shoulders probably exemplify the greatest magnitude of extension owing to their muscular surroundings and relatively shallow joint setting, though this is not to say that the shoulder is a passive structure. However, taking the passive extension of a gymnast's shoulders into account, the four segment model represents the elasticity for the whole of the gymnast by a damped linear spring placed at the shoulder joint in series with the arm segment. In order to determine the stretch of the gymnast due to external forces during a swinging element on rings, a measure which is representative of the whole length of the gymnast is required. The distance from the wrist to the ankle joint of the gymnast may suffice for such a measurement.

When the gymnast is in handstand it may be expected that he is under compression
owing to the actions of the external forces in this position. In contrast, when swinging beneath the rings the gymnast may be expected to experience extension, owing to the external forces acting at this time. Hence, the modelled linear spring at the shoulder joint must demonstrate both compression and extension.

For each longswing trial the following procedures were used to estimate values for the stiffness and damping parameters of the shoulder spring in the four segment model.

The three-dimensional locations of the wrist, elbow, shoulder, hip, knee and ankle joints for both sides of the gymnast were used to calculate the wrist to ankle length of the gymnast for each field throughout a longswing. The magnitudes of the vectors from successive joint centres were calculated using equation 4.10. The combined lengths of wrist to elbow and elbow to shoulder joints for the left and right sides of the gymnast were determined and the average calculated. This provided the average length of the gymnast's arms. The combined length of the hip to knee and knee to ankle joints for the left and right sides of the gymnast were also determined and the average calculated. This provided an estimate of the average length of the legs. The average torso length was calculated as the magnitude of the vector from the midpoint of the shoulder joints to the midpoint of the hip joints. The sum of these average values provides an estimate of the average wrist to ankle length of the gymnast for each field.

Pseudo data for the average wrist to ankle length were calculated using data from adjacent fields. Global and local error estimates were calculated using the original and pseudo wrist to ankle length data. Jennings' quintic spline algorithm was used to fit the smoothest splines \( S = 1.0*N \) to the original wrist to ankle length data. Error estimates were calculated from a mixture of global (25%) and local (75%) variances. The rates of change in the average wrist to ankle length were also calculated throughout the longswing using the spline fitted data.

The four segment model represents the elasticity of the whole gymnast at the shoulder joint by a damped linear spring placed in series within the arm segment. Consequently it is the resultant force in the direction of the arm segment which the spring experiences. From the experimental data an analogous resultant force, acting in the line of the arm segment, is required to determine the stiffness and damping of the shoulder spring.

The magnitude of one three-dimensional vector in the direction of another vector may be calculated using the scalar product. The scalar product of two vectors, \( \mathbf{a} \) and \( \mathbf{b} \), is defined as:

\[
\mathbf{a} \cdot \mathbf{b} = \| \mathbf{a} \| \| \mathbf{b} \| \cos \theta
\]

\[ (4.11) \]

where \( \| \mathbf{a} \| \) and \( \| \mathbf{b} \| \) represent the magnitudes of vectors \( \mathbf{a} \) and \( \mathbf{b} \) and \( \theta \) is the angle between the vectors \( \mathbf{a}, \mathbf{b} \). Using the scalar product the resultant force acting in the direction of each arm may be determined. Owing to the location of the spring in the model, contributions
to the resultant force in the direction of each arm include the external forces of cable tension and the weight of the gymnast's arms.

The vector describing the right rings cable defines the direction of the cable tension vector, the magnitude of which is known. The vector describing the right arm was defined from the right hand to shoulder of the gymnast. The cosine of the angle between these two vectors was determined using the scalar product. The cosine of the angle between the left rings cable and the left arm was calculated in the same manner. The average of these values was then used as the estimate of the cosine of the angle between the arms and the rings cables. The product of the filtered cable tension value (single cable) and the average cosine of the angle, forms the average magnitude of cable tension acting in the direction of one arm, with the direction from hand to shoulder defined as positive.

The second force contributing to the resultant force in the direction of the arm is the weight of the arm. As the arm's weight acts vertically the cosine of the angle between the vertical and the vector describing the arm was calculated using equation 4.9. The cosine of the angle between the vertical and the arm was calculated for each arm, with the average of the right and left values being used. The product of the average weight of the arms and the average cosine of the angle between the arms and the vertical defines the magnitude of the force in the direction of the arm. The positive direction is from the hand to shoulder. The resultant force acting in the average direction of the arms is the vector sum of these two contributions. These calculations were performed for each field of each trial, forming time histories of the resultant force acting in the average direction of one arm of the gymnast. Positive resultant forces in the arms indicate a force acting to accelerate the mass centre of the torso away from the arms. Negative resultant forces indicate a force acting to accelerate the torso towards the arms.

The values of resultant force in the direction of the arms were used in the multiple linear regression analyses, similar to those performed to determine spring parameters for the cable spring. The simulation model assumes the spring at the shoulders is damped and linear and therefore the force experienced or exerted by the spring is directly proportional to its extension and rate of extension.

The first regression expressed resultant force in the direction of the arm (tenarm) as a function of the average wrist to ankle length (walen) and velocity (wavel). This regression allowed the natural length of the theoretical damped linear spring at the shoulders to be calculated. The average wrist to ankle length data were then modified by subtracting the natural length of the spring from the average length. This formed the extension (or compression) of the theoretical spring from its natural length (eswalen). A further multiple linear regression was performed expressing the force in the direction of the arm as a function of the wrist to ankle extension and velocity while forcing the regression through the origin. The coefficients from this regression for 'eswalen' and
'wavel' were the stiffness and damping values which represent the theoretical shoulder spring in the four segment model.

This procedure was repeated for all six of the trials. Taking into account the subject specific nature of these parameters, three estimates for the stiffness and damping of the damped linear shoulder spring representing the elasticity of the gymnast in the four segment model were produced for each gymnast. The mean, standard deviation and coefficient of variation were calculated for these spring parameters, taking into account the estimates were subject specific.

4.8 Summary

This chapter describes the data collection session used to obtain appropriate kinetic and video data for a full three-dimensional description of gymnasts' techniques when performing backward and forward longswings on rings. Such descriptions of gymnasts' techniques are currently unavailable in research literature on rings.

The procedures used to calculate the kinetic and kinematic data are also presented, including techniques for synchronising the data and methods to reduce errors in the cable tension and video derived data. Approaches used to obtain values for the subject and apparatus specific model parameters in the four segment simulation model are also presented.

Results from all of the procedures are presented in Chapter 5, providing a complete three-dimensional description of a gymnast's technique while performing longswings on rings. Estimates for the four segment simulation model parameters are also given in Chapter 5, using the approaches outlined in the current chapter.

Data describing the gymnasts' techniques, in terms of orientation and configuration angles, and cable tension time histories are also used in the evaluation of the four segment simulation model of a gymnast swinging on rings. Details of the model's evaluation procedure are dealt within Chapter 6. The influences of several aspects of the gymnasts' techniques on performance are considered in Chapter 7, together with the influence of the elasticity of the apparatus.
CHAPTER 5

RESULTS FROM THE TWO-DIMENSIONAL SIMULATION MODELS
AND THE ANALYSES OF LONGSWINGS

5.1 Introduction

The following chapter is divided into three sections, which comprise:

- results and conclusions drawn from the simulations performed using the two-dimensional computer simulation models
- results from the kinetic and kinematic analyses of backward and forward longswings, providing a description of the techniques used by the two subjects for these elements
- estimates for the personalised segmental inertia and spring parameters for the two subjects together with spring parameters for the rings apparatus in the four segment, three-dimensional computer simulation model.

RESULTS FROM THE TWO-DIMENSIONAL SIMULATION MODELS

5.2 Results from simulations using the rigid model

As described in Section 3.2, the rigid model comprises a single segment gymnast swinging on a rigid rings frame. Using the simulation model RIGID a total of 14 simulations was performed, each reproducing a swing from handstand. Each simulation used a different set of inertia parameters to represent different gymnasts accurately, as outlined in Section 3.2.3. A résumé of the inertia parameters of the gymnasts is provided in Table 3.1. In the cases of the RIGID and ELASTIC models the body angle \( \varphi_r \) is equal to the gymnast orientation angle \( \phi_r \). To maintain consistency with subsequent results the term body angle is adopted from this point onwards. Each simulation was termed complete when the body angle of the gymnast no longer increased, indicating the gymnast was at the apex of his swing.

The simulation for the normalised gymnast is used in all following Figures since performance and kinetic indicators (Table 5.1) for this gymnast fell within the standard deviation of the mean in all cases. Results obtained for the normalised gymnast may therefore be considered representative of all the modelled gymnasts.
Figure 5.1 highlights the changes to all mechanical energy forms in the whole system throughout the simulated swing. As the modelled rings cable is massless only the gymnast possesses energy. Furthermore, as the rigid model has no energy sinks, the total energy within the system should remain constant throughout a simulated swing. This was found to be the case, providing confidence in the mechanical derivation of the model and in the accuracy of the numerical integration procedure.

During the descending phase of the swing the total kinetic energy of the gymnast increased while a corresponding decrease in potential energy occurred. Throughout this phase the relative contributions of linear and rotational kinetic energy, which constitute total kinetic energy, varied. During the first 1.0 s of the descent linear kinetic energy provided the greater contribution to the gymnast's total kinetic energy. By 1.06 s the vertical velocity of the gymnast's mass centre had increased to a maximum of -4.67 m.s\(^{-1}\), while the gymnast possessed a relatively low angular velocity (357°.s\(^{-1}\)).

During the final 0.14 s of the descent the rotational component of kinetic energy increased rapidly, while a rapid reduction in the linear kinetic energy occurred. As the gymnast passed through the bottom of the swing the total kinetic energy of the gymnast was maximal. When at a body angle of 177° rotational kinetic energy formed 99% of the total kinetic energy, while linear kinetic energy formed only 1%. The linear velocity of the gymnast's mass centre at this instant was close to zero. The constraint formed by the rings cable on the motion of the gymnast resulted in the gymnast at this instant rotating rapidly about his near stationary mass centre at an angular velocity of 760°.s\(^{-1}\).

During the ascending phase of the swing (a body angle greater than 180°) the kinetic energy was converted back to potential energy as the mass centre of the gymnast rose. As in the descending phase, the relative contributions of linear and rotational kinetic energy forms varied throughout the ascent. During the early stages of the ascending phase the majority of the kinetic energy was of the rotational form. However, within
0.08 s of the peak rotational kinetic energy (body angle 177°), the linear kinetic energy was maximal (690 J) and formed the majority (74%) of the gymnast's total kinetic energy.

Figure 5.1 also provides the combined cable tension time history for the same simulation. This time history exhibits some temporal similarities to experimentally recorded cable tension data for backward longswings (Figure 2.9). However, detailed variations in cable tension contained within measured data were not observed in the simulated data. Peak cable tension occurred at a body angle of 177° and coincided with the maximum kinetic energy of the gymnast. Furthermore, the peak combined cable tension during the swing was 22.7 bodyweights. This value is substantially larger than values recorded experimentally in the current study (Section 5.5) and in previous studies (Nissinen, 1983, 1995).

Figure 5.2 shows the body angle and cable elevation angle time histories for this simulated swing. The maximum body angle reached by the gymnast was 346° after which the gymnast rotated in the opposite direction, decreasing the body angle. Hence, a full backward longswing to handstand was not accomplished. This result is consistent for all simulations (Table 5.1). The total angular range of motion for the rings cable was 36.5°. This value is similar to that observed in the current study and, not unexpectedly, larger than those observed for dislocates which exhibit a smaller amplitude of swing (Chapman and Borchardt, 1977). The resulting angular motion of the gymnast and rings cable suggest the general structure of the model defining the gymnast and apparatus are appropriate.

![Figure 5.2. Body angle and cable elevation angle time histories for the normalised gymnast during a swing from handstand: rigid model.](image)

The path of the gymnast's mass centre (Figure 5.3) was produced by the combined motions of the gymnast and rings cables. For consistency with kinematic data determined from the data collection the coordinates in the vertical (z) direction are translated by 5.84 m. The path of the modelled gymnast's mass centre is in some agreement with the theoretical analysis of swinging on rings proposed by Smith (1982), which suggested
motion would be restricted to vertical movements only. Since an elongated elliptical shape was formed by the mass centre path these results demonstrate that the proposal of Smith (1982) was somewhat simplified. It should also be noted that at the start of the swing the mass centre was fractionally higher (0.025 m) than at the completion of the swing.

The gymnast's mass centre initially moved in the negative horizontal direction. The only external forces acting on the gymnast were the gymnast's bodyweight and cable tension. Of these external forces only cable tension may possess a horizontal component during the swing. With the orientation of the rings cable at 2° the horizontal component of cable tension is directed in the negative y direction. This resulted in the initial horizontal movement of the gymnast's mass centre in the negative y direction (Figure 5.3).

![Figure 5.3. Mass centre path for the normalised gymnast during a swing from handstand: rigid model.](image)

Figure 5.4 presents a graphics representation of the swing from handstand for the normalised gymnast. The orientations of the gymnast and rings cables at the initiation of the ascending phase, at the instant of peak combined cable tension and at the greatest amplitude of swing are labelled A, C, and E respectively (Figure 5.4). If the amplitude of swing is defined by the excursion of the body angle, this result signifies the amplitude of swing is reduced during this swing from handstand (Figure 5.4). The body angle of the gymnast away from the final handstand (14°) is larger than that from the initial handstand position (4°). This result occurs even though the total energy within the system remains constant. It is also in agreement with the observation made regarding the location of the gymnast's mass centre at the start and end of the swing.
Figure 5.4. Graphics sequence of the swing from handstand for the normalised gymnast: rigid model.

Table 5.1. Performance and kinetic indicators for the swing from handstand: rigid model

<table>
<thead>
<tr>
<th></th>
<th>peak combined cable tension (BW)</th>
<th>body angle at peak cable tension (°)</th>
<th>maximum body angle θ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (n = 14)</td>
<td>22.9</td>
<td>176.4</td>
<td>346.5</td>
</tr>
<tr>
<td>range</td>
<td>20.3 - 24.5</td>
<td>176 - 177</td>
<td>346 - 348</td>
</tr>
<tr>
<td>s.d.</td>
<td>1.11</td>
<td>0.50</td>
<td>0.80</td>
</tr>
</tbody>
</table>

5.2.1 Conclusions from simulations using the rigid model

Several conclusions concerned with swinging on rings may be drawn from the results of the rigid model. Such conclusions are concerned with the underlying relationship between the motions of the gymnast and rings cable and factors which may reduce peak forces experienced by the gymnast during longswings on rings.

In the rigid model the gymnast and rings cable form an energy conservative system. It might therefore be expected that no decrease in the amplitude of the swing would occur since the mechanical energy within the system remains constant throughout a swing. Taking the example of a simple model representing gymnast in a rigid configuration swinging on a rigid and frictionless horizontal bar, no loss in the amplitude of swing is observed (Morlock & Yeadon, 1988). Thus, if the body angle of the gymnast prior to the swing on horizontal bar was 4° from the handstand position, the gymnast would attain a body angle 4° short of the final handstand position. Results from the rigid model indicate this is not the case for a gymnast swinging on rings.
From an initial body angle of $4^\circ$ the largest body angle was $346.5^\circ \pm 0.8^\circ$, or $13.5^\circ$ short of the vertical. This amounts to a $9.5^\circ$ reduction in the amplitude of swing. The vertical location of the mass centre at the beginning and end of the swing also supports this finding. As no energy loss occurred, this decrease in the amplitude of swing can be attributed to the interaction between the gymnast and rings cables, which effectively form a double pendulum. This interaction means the gymnast possesses only linear kinetic energy and positional potential energy at the apex of the swing, but no energy due to angular motion. The rings cables therefore place an inherent constraint on the motion of the gymnast during swinging activities. This motion constraint is different to those gymnasts experience during swinging activities on horizontal bar.

This interaction between the rings cables and gymnast has implications for performances of both backward and forward longswings. In order for a gymnast to complete a longswing he must alter the path of his mass centre to compensate for this interaction and allow the final handstand to be obtained. The path of his mass centre may be altered by changing his body configuration during the swing. Such a speculation is dealt within Section 5.4.3, where changes in angles at the hip and shoulder joints are determined to produce optimal backward longswings.

The modelling of the gymnast and rings frame as rigid entities may be expected to over-estimate maximum accelerations and forces since no mechanical energy is dissipated from the system. In reality both the apparatus and gymnast have elastic properties (Section 2.2). Values for peak combined cable tension from the simulation model are more than twice as large as those observed experimentally. The results from the rigid model concerning peak forces are therefore consistent with the rigid nature of the model.

It has been shown that large forces experienced by gymnasts while swinging beneath the rings lead to an increased potential for injury to the shoulder joints (Gielo-Perczak, 1991; Nissinen, 1995; Caraffa et al., 1996). It may be proposed that gymnasts would not survive unscathed if they experienced the peak combined cable tension predicted by the rigid model. This, in turn, implies that within the combined rings/gymnast system mechanisms must exist which are responsible for reducing the peak forces experienced by gymnasts while performing longswing elements. Indeed, these factors must be of vital importance in ensuring the joints of the gymnast are not injured.

It is surmised that these mechanisms take the form of inanimate structures in the apparatus, biological structures of the gymnast and the technique used by the gymnast. In Chapter 2 it was stated that elastic structures are present in the rings apparatus in the form of the damped elastic devices (Section 2.2). Using the elastic model the influence of the elasticity of the apparatus on peak combined cable tension is investigated. The results of these investigations are presented in Section 5.3.

It has been speculated that changes to the body configuration of a gymnast as he passes through the bottom of the swing may also serve to reduce peak cable tension
(Hiley, 1998; Sprigings et al., 1998). The effect of a gymnast using joint angle changes during the descending phase of a swing from handstand on peak combined cable tension is investigated using the two segment model (Section 5.4).

The potential for any elasticity within the rings cables, frame and gymnast are also incorporated in the four segment model. Investigations determining the effect of these elastic mechanisms during backward and forward longswings are reported in Chapter 7.

The extent to which each of these factors decrease peak forces will, to some degree, be established by the other two-dimensional simulation models. However, it is the results from the rigid model which indicate such mechanisms must exist within the gymnast and rings apparatus.

5.3 Results from simulations using the elastic model

The elastic simulation model, described in Section 3.3, furthers the modelling process by incorporating elasticity into the rings frame while retaining the simplified rigid representations of the gymnast and rings cables. The springs in the elastic model may be considered to represent the elasticity of the DEDs and horizontal beam of the rings frame. These two structures, or ones of a similar nature, are specifically incorporated into the rings apparatus by manufacturers of gymnastics equipment under directives from the F.I.G.

Over 60 simulations were performed to investigate the influence of the stiffness and damping characteristics of the rings frame on peak combined cable tension during a swing from an initial handstand position. The initial orientations of the gymnast and cables in the handstand position were the same as those used in the rigid model. Inertia parameters relating to the normalised gymnast were also used to maintain consistency with the result from the rigid model. Each simulation was termed complete when the body angle of the gymnast no longer increased, indicating the gymnast was at the apex of his swing. Direct comparisons may therefore be made between the motion derived from the ELASTIC simulation model and that predicted by the RIGID simulation model. Table 5.2 provides the ranges of damping used for each specified type of rings frame.

Figure 5.5 shows the effect of spring damping on peak combined cable tension produced during a swing from an initial handstand for the three different modelled rings frames. As expected the inelastic rings frame, which effectively forms a rigid structure, produced results which were in close agreement with those of the rigid model. During a swing from handstand for the highly stiff and damped rings frame (500000 N.m·s and 400000 N.s.m·t) the peak combined cable tension was 22.7 bodyweights produced at a body angle of 176.4°.
Table 5.2. The ranges of damping values used for each modelled rings frame

<table>
<thead>
<tr>
<th>rings frame type</th>
<th>spring stiffness (N.m⁻¹)</th>
<th>range of spring damping (N.s.m⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inelastic</td>
<td>500000</td>
<td>4000 to 400000</td>
</tr>
<tr>
<td>elastic</td>
<td>140000</td>
<td>500 to 400000</td>
</tr>
<tr>
<td>highly elastic</td>
<td>14000</td>
<td>50 to 400000</td>
</tr>
</tbody>
</table>

However, the maximum body angle attained during this simulation was 339.4°, while in the rigid model the maximum body angle was 345.9°. This discrepancy in body angle achieved by the same modelled gymnast may be accounted for by the 14.2 J of mechanical energy dissipated during the simulation. This dissipation of mechanical energy was due to the damping components of the springs in the elastic model. In addition a relatively minor amount of energy (0.4 J) was stored as elastic energy within the springs at the end of the swing.

For the inelastic rings frame, variations in spring damping had little influence on the peak combined cable tension produced during a swing from handstand (Figure 5.5). The range of combined peak cable tension observed over the entire range of spring damping was only 1.24 bodyweights (Table 5.3). The equivalent results for the elastic and highly elastic rings frames were larger, being 3.83 and 13.66 bodyweights respectively. Hence, when more realistically modelling the elasticity of the rings frame, the damping component has a larger influence on the peak combined cable tension. This has implications when attempting to model a gymnast swinging on rings more accurately using a four segment model (Chapter 6).

Figure 5.5. The influence of spring damping on combined peak cable tension during a swing from handstand for the three rings frame types: elastic model.
The value of stiffness chosen for the elastic rings frame (140000 N.m⁻¹) is representative of the mean stiffness for the vertical spring in the four segment model, which represents only the right hand side of the DEDs and horizontal beam of the rings frame (68045 N.m⁻¹ for a single vertical spring, Section 5.6). The elastic properties of the springs in the elastic rings frame may therefore be considered to reflect realistically the elasticity incorporated into the rings frame by the manufacturers of the apparatus.

Throughout the selected range in damping for the elastic rings frame, peak combined cable tension was much greater than that experimentally measured (Nissinen, 1983; Section 5.5). The minimum peak combined cable tension was 18.8 bodyweights (Figure 5.5). This was produced when the damping component of each spring was 3500 N.s.m⁻¹. This value of damping for the vertical spring in the elastic model is equivalent to that estimated for the single vertical spring representing the DEDs and horizontal beam of the rings frame in the four segment model (1052 N.s.m⁻¹ for a single vertical spring). Results for the elastic rings frame, when realistic stiffness and damping values are implemented, are therefore in conflict with the peak combined cable tension experienced by actual gymnasts swinging from handstand.

Realistic peak combined cable tension values of around 9 bodyweights were only observed when the rings frame was modelled as highly elastic and damping was at an appropriate level (Figure 5.5; Table 5.3). The springs in the elastic simulation model are meant to represent the combined elasticity of the DEDs and horizontal beam of the frame only. However, in order to produce the realistic peak combined cable tension value of 8.9 bodyweights the vertical spring in the highly elastic rings frame underwent a maximal extension of 0.36 m. In reality the maximum combined vertical deflection of the DEDs and horizontal beam of the frame is near 0.03 m (Section 5.6). This inconsistency suggests that other mechanisms exist, besides the DEDs and horizontal beam of the rings frame, which reduce peak combined cable tension. This conjecture is addressed in the following sections and in more detail within Chapter 7.

Table 5.3. Range of peak combined cable tension for each modelled rings frame

<table>
<thead>
<tr>
<th>rings frame type</th>
<th>from</th>
<th>to</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>inelastic</td>
<td>23.06</td>
<td>21.82</td>
<td>1.24</td>
</tr>
<tr>
<td>elastic</td>
<td>22.62</td>
<td>18.79</td>
<td>3.83</td>
</tr>
<tr>
<td>highly elastic</td>
<td>22.55</td>
<td>8.89</td>
<td>13.66</td>
</tr>
</tbody>
</table>

Besides affecting peak combined cable tension during a swing from handstand, the stiffness and damping of the rings frame also influence the maximum body angle attained.
by the gymnast and the amount of mechanical energy dissipated from the system.

Figure 5.6 highlights the influence of damping for each type of rings frame on the maximum body angle attained by the gymnast when swinging from the initial handstand position. As the damping of the modelled rings frame decreased, the body angle attained by the gymnast also decreased, effectively reducing the amplitude of the gymnast's swing.

In conjunction with these results, Figure 5.7 provides the relationship between the damping of the modelled rings frame and the mechanical energy dissipated from the system. With a decreased damping component for the rings frame the mechanical energy dissipated during a swing increased for all three types of rings frame. Since the amplitude of swing is the result of the mechanical energy possessed by the gymnast coupled with the interaction of the gymnast and rings cables (Section 5.2), this result is consistent with the reduction in the maximum body angle attained, shown in Figure 5.6.

Figure 5.6. The influence of spring damping on maximum body angle attained during a swing from handstand for the three rings frame types: elastic model.

Figure 5.7. The influence of spring damping on energy dissipation during a swing from handstand for the three rings frame types: elastic model.
5.3.1 Conclusions from simulations using the elastic model

From the results obtained using the elastic model it is clear that both the stiffness and damping of the rings apparatus influence the motion of the gymnast and rings cables. With the rings frame possessing the appropriate amount of stiffness and damping, peak combined cable tension may be reduced. Concurrently, the amplitude of the gymnast's swing from an initial handstand position is also reduced. To illustrate the mechanical reasons for the decrease in these two performance indicators, a single simulation of a gymnast swinging from handstand on an elastic rings frame is described in detail. This simulation is also used to highlight the discrepancy identified between actual peak combined cable tension and that predicted by the elastic model (Figure 5.5).

The following simulation may be considered realistic since the stiffness and damping components of the springs representing the DEDs and horizontal beam of the rings frame are similar to values determined experimentally within this study (stiffness of 140000 N.m⁻¹ and damping of 3500 N.s.m⁻¹, representing both the DEDs and horizontal beam of the rings frame, Section 5.6). This simulation therefore predicts what would occur if the gymnast maintained an extended body configuration and only the DEDs and horizontal beam of the rings frame possessed elastic qualities.

Figure 5.8 provides a pictorial sequence of the simulated swing. The orientations of the gymnast and rings cables are shown at the start of the ascending phase (A), at the instant of peak combined cable tension (C) and at the greatest amplitude of swing (E).

![Figure 5.8. Graphics sequence of the swing from handstand using realistic values for the DEDs and horizontal beam of the rings frame: elastic model.](image)

The maximum body angle attained by the gymnast was 276.7°. This represents a 69° reduction in the amplitude of swing compared to the motion of the gymnast when the rings frame was modelled as a rigid structure (Figure 5.4). The interaction between the gymnast and rings cable also resulted in a different motion of the rings cable when
compared to the motion predicted by the rigid model.

The combined cable tension time history for this simulation is provided in Figure 5.9. The peak combined cable tension was 18.8 body weights, which occurred at a body angle of 184°. In comparison with the simulation on a rigid rings frame the elasticity of the rings apparatus produced a temporal shift of the peak combined cable tension and a 3.9 body weight reduction in its value. However, the peak combined cable tension produced during this swing is still much greater than any experimentally recorded (Section 5.6).

![Figure 5.9. Combined cable tension and path of the gymnast's mass centre using realistic values for the DEDs and horizontal beam of the rings frame: elastic model.](image)

Figure 5.9 also presents the path of the gymnast's mass centre throughout the simulation. The reduction in the amplitude of swing is underlined by the location of the gymnast's mass centre being 0.675 m below the initial location on the completion of the swing. Since the gymnast possessed a small linear resultant velocity (0.7 m.s\(^{-1}\)) at the apex of the swing (maximum body angle reached) the lower vertical location of the mass centre may be equated with an overall loss in total mechanical energy.

Figure 5.10 exhibits the time histories of the various mechanical energy forms present in the elastic model. The extensions of both springs representing the elastic structures of the rings frame are also provided. The transfer of mechanical energy during the descending phase of the swing from potential to kinetic (rotational and linear) is similar to that which occurred when the rings frame was rigid. This similarity is due to the relatively low forces acting on the damped springs as the gymnast descends. However, extensive differences exist in the transfer of mechanical energy between the gymnast and the elastic and rigid rings frames as the gymnast swings underneath the rings.

Owing to the large forces involved as the gymnast swings beneath the rings the springs are forced to extend rapidly, particularly the vertical spring (Figure 5.10). Concurrently, there is a dissipation of energy from the system since damping forces are
proportional to the rate of spring extension. This dissipation of mechanical energy from the system can be identified in Figure 5.10 by the decline in the total energy.

![Figure 5.10. Time histories for the energies and extensions of the springs using realistic values for the DEDs and horizontal beam of the rings frame: elastic model.](image)

At the instant of peak cable tension 174 J of energy had been dissipated from the system. This resulted in the system possessing less total mechanical energy than at the beginning of the simulation. Furthermore, at the instant of peak cable tension the extensions of the vertical and horizontal springs were 0.054 m and 0.003 m respectively. These spring extensions resulted in 202 J being stored as elastic energy in the system.

The storage of energy within the springs combined with the dissipation of energy resulted in significantly less mechanical energy being possessed by the gymnast than at the start of the swing. Hence, less energy was available for kinetic energy as the gymnast completed the descent. Consequently, at the bottom of the swing where the linear velocity of the gymnast's mass centre is effectively zero, the gymnast possessed a maximum angular velocity of 625°.s⁻¹. This maximum angular velocity is somewhat smaller than the 760°.s⁻¹ produced when the rings frame was rigid. The reduced peak angular velocity of the gymnast resulted in the lower peak combined cable tension of 18.8 bodyweights. Thus, the dissipation of mechanical energy combined with the redistribution and transfer of energy resulted in a reduced peak combined cable tension.

During the initial phase of the ascending swing the extension of the springs reduced. Since the springs were in motion mechanical energy was still dissipated from the system. In addition, as the springs recoiled the stored elastic energy was converted into kinetic and potential energy forms. However, due to the dissipation of energy, the total energy which is converted back to these forms of energy was less than in the case of a rigid rings frame (Figure 5.1). This resulted in the lower amplitude of swing for the gymnast.

In the elastic model the vertical spring represents the combined elasticity of the DEDs and horizontal beam of the rings frame. When realistically reflecting their elastic properties, the peak combined cable tension was 18.8 bodyweights. Since actual peak
combined cable tension values are closer to 9 bodyweights, the result suggests the elasticity formed by the DEDs and horizontal beam of the rings frame is not the sole mechanism for reducing peak combined cable tension during longswings on rings. It may therefore be concluded that other structures are also responsible for reducing peak forces experienced by gymnasts while swinging on rings.

To conclude, the damping components of the elastic structures act as consumers of mechanical energy (Meriam & Kraige, 1987) and produce a dissipation of mechanical energy from the system (Figure 5.10). The elastic energy stored in the springs also reduces the energy available to be converted to kinetic (rotational and linear) energy forms. This dissipation and storage of mechanical energy produced by elastic components of the rings apparatus acts to reduce the peak forces experienced by gymnasts performing longswings.

The elasticity of the DEDs and horizontal beam of the rings frame incorporated into the rings apparatus by the manufacturers of gymnastics equipment is therefore beneficial to gymnasts in reducing the risk of injury by excessive forces. Together they produced a 17% reduction in the peak combined cable tension when compared to a rigid rings frame. From this result it may be speculated that other mechanisms must exist which reduce the peak combined cable tension during longswings on rings. These mechanisms may include the elasticity of the rings cables as well as the gymnast. The roles of these elastic structures in reducing peak forces produced during longswings are investigated using the four segment simulation model. In addition, the two segment model is used to examine whether peak forces experienced by the gymnast may be reduced by appropriately timed changes in a gymnast's body configuration during a swing from handstand.

Dissipation of mechanical energy by the damping components of the elastic structures within the rings apparatus raises further implications for the production of a longswing on rings. As mechanical energy is dissipated from the system the gymnast must also seek to input mechanical energy into the system to compensate for this loss and ensure the final handstand position is attained. Mechanisms by which the gymnast may increase the total mechanical energy of the system may include appropriately timed joint actions (Section 2.4.3). Methods by which a gymnast may increase the mechanical energy while swinging on rings is examined in the following section using the two segment model.

5.4 Results from simulations using the two segment model

5.4.1 Comparison of results between the two segment and rigid models

In order to gain confidence that the two segment model was theoretically correct
two preliminary simulations were carried out. During these preliminary simulations the gymnast performed a swing from handstand and maintained a rigid configuration in the extended handstand position. In all simulations the segmental inertia parameters estimated for subject K were used. Subject K possesses a similar total body mass (modelled as 61.63 kg) and height (1.69 m) to the normalised gymnast used in investigations performed with the rigid and elastic models. The first simulation modelled the gymnast with a hip joint while the second modelled the gymnast with a shoulder joint.

Both simulations are the same as those performed with the rigid model, Section 5.2. Hence, the same resulting motion of the gymnast and rings cables should be produced. By comparing the predicted motion of the gymnast and cables from both models confidence in the correct mechanical derivation of the two segment model may be gained.

During both preliminary simulations the modelled gymnast attained a maximum body angle of 346°. Peak combined cable tension of 23.7 bodyweights was produced at a body angle of 178°. The path of the gymnast’s mass centre for both simulations was the same as when subject K was utilised in the rigid model. With no relative motion of the gymnast's segments the total mechanical energy of the system was found to be conserved (Figure 5.11). All of these results are the same as the motion obtained for subject K using the rigid model. These two simulations therefore provide confidence in the correct derivation of the two segment model, and the modified Euler procedure utilised for numerical integration.

![Graph showing joint torques and mechanical energy](image)

Figure 5.11. Mechanical energy and joint torques for a swing from handstand in an extended and rigid configuration: two segment model.

The two preliminary simulations also provided further insight into the mechanics of swinging on rings. Figure 5.11 shows the joint torque time histories required at the hip and shoulder joints to maintain the extended body configuration throughout the swing from handstand. Throughout the descending phase a positive joint torque, that is joint torque $T_1$ acting in an anticlockwise direction in Figure 3.7, was required to maintain the rigid configuration. Maximum positive hip and shoulder torques were 561 Nm and
1007 Nm respectively and occurred at a body angle of 164°. Throughout the ascending phase a negative joint torque, acting in an clockwise direction in Figure 3.7, was required at both joints to maintain the extended configuration. The largest negative hip and shoulder joint torques were -568 Nm and -1017 Nm at a body angle of 189°. A larger magnitude of joint torque was required at the shoulders than at the hips throughout the swing. This result is consistent with the moment of inertia about the mass centre of the distal segment of the gymnast. When a shoulder joint is modelled, the body segment (body and legs) possesses a greater moment of inertia about the segmental mass centre than when a hip joint is modelled and the distal segment is the legs. A greater torque is therefore required to maintain the extended shape with a shoulder joint since the moment of inertia is greater while the angular acceleration is identical for both segments.

In the two segment model positive joint torques act to reduce the angle at the joint. Throughout the descending phase both the hip and shoulder joint torques were positive, yet no reduction in joint angle occurred. Hence, in order for the gymnast to maintain a straight configuration during a swing from handstand he must exert positive torques at his joints. If the modelled gymnast ceased the torques at either joint the joint angle would increase and he would assume an arched configuration. During the ascending phase of the swing the torques at both joints were negative. In the two segment model negative joint torques act to increase the joint angle. If, during the ascending phase, the gymnast ceased the torque at either joint the joint angle would decrease. This would result in the gymnast assuming a piked configuration.

These findings are generally consistent with those of Morlock & Yeadon (1988) and Hiley (1998) for a gymnast performing giant circles on horizontal bar. However, both the magnitudes and timings of joint torques while swinging on rings differ greatly to those on horizontal bar. For a swing from handstand on a rigid horizontal bar in a fixed configuration the maximum shoulder joint torque is typically near 30 Nm. When the mass centre is directly below the hands the joint torque is zero (Morlock & Yeadon, 1988). However, owing to the different type of constraint on the gymnast’s motion when swinging on rings the maximum joint torques for both joints are much greater on rings than on horizontal bar (Figure 5.11). Furthermore, the timing of maximal torques differed from those on horizontal bar, since the change from positive to negative joint torques occurred at a body angle of 177°.

When comparing these results with joint torques used during actual longswings on rings the maximal joint torques predicted by the simulation model are twice as large as those reported by Brüggemann (1987). This result is not unexpected. Since the model possesses no elasticity it also possesses the inadequacies highlighted for the rigid model, of producing unrealistic angular accelerations and kinetics for the gymnast.

From a knowledge of the joint torques and the rate of change in joint angle (joint angular velocity) the type of action produced at a joint may be determined. Consider only
the descending phase of the swing from handstand produced during the preliminary simulations. If no change in joint angle occurs the muscle contraction may be termed isometric. Hence, all joint actions were isometric throughout the preliminary simulations. If the gymnast were to increase the positive joint torque a reduction in the joint angle would occur. This would result in a piked configuration. For a joint torque which acts to reduce the joint angle and results in a joint angle reduction, the joint action itself may be termed concentric. In contrast, if a positive joint torque less than that required to maintain the extended configuration were produced, the joint angle would increase. This would result in the gymnast assuming an arched shape. Such a joint action, where the joint torque acts in opposition to the change in joint angle, may be termed eccentric. Similar labelling of joint actions has been implemented by Hiley (1998) to determine whether changes in joint angles produce a net increase or decrease in the total mechanical energy of a gymnast swinging on horizontal bar. Concentric joint actions are associated with net increases in mechanical energy, while eccentric joint actions produce net reductions. As illustrated in the preliminary simulations, isometric joint actions produce no change in the total mechanical energy of a system.

The following sub-sections present the results obtained for the investigations of reducing peak combined cable tension and the production of proficient longswings. Since these investigations relate to alterations of the gymnast’s configuration and total mechanical energy of the system, a knowledge of the joint torques used to alter the gymnast’s configuration is required.

5.4.2 Optimum reduction in peak cable tension during a swing from handstand

Results from the spring model (Section 5.3) indicated that, during a swing from handstand, peak combined cable tension may be significantly reduced by the elasticity of the rings apparatus. However, since peak combined cable tension remained larger than observed in reality it was inferred that other mechanisms must exist which serve to reduce peak cable tension. Such mechanisms reduce the peak forces experienced by the gymnast to tolerable levels and reduce the risk of injury. Video derived data (Section 5.5.3) show that gymnasts alter their body configuration rapidly through the bottom of the swing. Such changes in body configuration may serve to reduce the peak external forces experienced by the gymnast.

Using the optimisation procedure described in Section 3.4.3, the two segment model was used to investigate whether joint angle changes made during a swing from handstand were able to decrease peak combined cable tension. Changes in the gymnast’s body configuration (limited to an increase and decrease in the joint angle) which produced the minimum peak combined cable tension were determined. Attempts to incorporate reality
into simulations during this particular investigation are also described in Section 3.4.3.

Table 5.4 presents the minimum peak combined cable tension produced during a swing from handstand. The modelled joint and maximum joint angle is also provided.

In all cases appropriate changes in the gymnast's body configuration reduced the peak combined cable tension produced during the swing from handstand. The smallest peak combined cable tension values were associated with the greatest arching of the gymnast (joint angles of 230°). In terms of the modelled joint, arching through the shoulder at the appropriate times produced a smaller peak combined cable tension than changes in the hip joint.

Table 5.4. Minimum combined peak cable tension produced during a swing from handstand for subject K: two segment model

<table>
<thead>
<tr>
<th>joint</th>
<th>maximum joint angle 210°</th>
<th>maximum joint angle 230°</th>
</tr>
</thead>
<tbody>
<tr>
<td>hip</td>
<td>18.89</td>
<td>15.95</td>
</tr>
<tr>
<td>shoulder</td>
<td>17.22</td>
<td>13.50</td>
</tr>
</tbody>
</table>

Figure 5.12 shows the time histories for the optimal reductions in peak combined cable tension produced using the two segment gymnast.

Figure 5.12. Combined cable tension time histories for the optimal reductions in peak combined cable tension (shoulder and hip joints): two segment model.

During the descending phase of a backward longswing gymnasts actually produce, though not concurrently, hip and shoulder elevation angles near 220° (Section 5.5). The hypothetical joint angle change from 180° to 230° is therefore similar to actual changes in body configuration made by gymnasts. With the gymnast modelled with a shoulder joint and the shoulder elevation angle altering from 180° to 230°, the peak combined cable
tension was 13.50 bodyweights. This represents a 47% reduction in the peak combined cable tension during the swing from handstand due solely to optimal changes in the shoulder joint during the descent. This combination of shoulder joint, maximum joint angle of $230^\circ$ and appropriately timed joint angle changes produced the greatest reduction in peak combined cable tension from the 23.7 bodyweights produced in a rigid and extended configuration. This simulation will now be considered in greater detail.

Figure 5.13 provides the shoulder elevation time history for the optimal simulation where the maximum shoulder elevation angle was $230^\circ$. Joint angle changes are also presented as a function of body angle. The resulting motion of the gymnast and rings cable are presented pictorially in Figure 5.14. Figure 5.15 shows the combined cable tension time history for this simulation, providing indications of the orientations of the gymnast and rings cables and corresponding combined cable tension.

Figure 5.13. Joint angle changes during the optimal reduction in peak combined cable tension when modelled with a shoulder joint ($230^\circ$): two segment model.

Figure 5.14. Graphics sequence of the optimal reduction in peak combined cable tension (shoulder joint, $230^\circ$): two segment model.
Figure 5.15. Combined cable tension time history for the optimal reduction in peak combined cable tension (shoulder joint, 230°): two segment model.

The shoulder torques required to produce these joint angle changes and reduce peak combined cable tension are shown in Figure 5.16. The types of joint actions being performed by the gymnast are also presented together with the time histories of the changes in the various forms of mechanical energy present in the system.

During the swing from handstand two phases of eccentric joint actions were produced. The first of these, during which the gymnast increased the shoulder elevation angle to 230°, resulted in a slight net decrease in total mechanical energy (6.2 J). Although the second eccentric joint action was performed over a shorter time period the effect on the total mechanical energy of the gymnast was more dramatic. As the gymnast passed through the bottom of the swing (body angle from 163° to 205°) the net decrease in the total mechanical energy of the system was 783 J. Hence, eccentric joint actions were associated with a net dissipation of energy from the system.

Figure 5.16. Torque and energy time histories for the optimal reduction in peak combined cable tension (shoulder joint, 230°): two segment model.

Net reductions in total mechanical energy occurred for all the optimal simulations, during which eccentric joint actions were performed. As well as producing a reduced
peak combined cable tension the dissipation of mechanical energy from the gymnast manifests itself in a reduction in the maximum body angle attained (Table 5.5).

Table 5.5. Maximum body angle attained during the optimal reduction in peak combined cable tension for subject K: two segment model

<table>
<thead>
<tr>
<th>joint</th>
<th>maximum joint angle 210°</th>
<th>maximum joint angle 230°</th>
</tr>
</thead>
<tbody>
<tr>
<td>hip</td>
<td>289.9</td>
<td>255.0</td>
</tr>
<tr>
<td>shoulder</td>
<td>264.8</td>
<td>232.4</td>
</tr>
</tbody>
</table>

To assess whether the results for reducing peak combined cable tension during a swing from handstand were particular to the subject, the same optimisation procedure was performed using inertia parameters relating to subject A. The total body mass of subject A was 67.3 kg, 5.67 kg greater than subject K. The two subjects may be regarded as suitably different to determine whether the results discussed so far are subject specific or more generally applicable. Table 5.6 presents the peak combined cable tension values associated with the different joints and maximum joint angles used in the joint angle time histories.

Table 5.6. Minimum combined peak cable tension produced during a swing from handstand for subject A: two segment model

<table>
<thead>
<tr>
<th>joint</th>
<th>peak combined cable tension (BW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>hip</td>
<td>maximum joint angle 210°</td>
</tr>
<tr>
<td>shoulder</td>
<td>maximum joint angle 230°</td>
</tr>
<tr>
<td>hip</td>
<td>19.19</td>
</tr>
<tr>
<td>shoulder</td>
<td>16.57</td>
</tr>
<tr>
<td></td>
<td>17.45</td>
</tr>
<tr>
<td></td>
<td>13.78</td>
</tr>
</tbody>
</table>

A comparison of the results for both subjects indicate that with each joint and maximum joint angle similar minimum peak combined cable tension values were produced. Figure 5.17 presents the joint angle changes for the simulations producing minimum peak cable tension. Since the body angle determines the phase of swing, joint angles are expressed in terms of the body angle. When utilising a maximum hip angle of 210° or shoulder angle of 230° both subjects performed the body configuration change at similar phases of the swing. However, for the other two optimal performances the subjects employed different gymnastic techniques. The majority of these differences
occurred as the joint angle increased to its maximum angle.

![Graphs showing hip and shoulder elevation angles for subjects K and A.](image)

Figure 5.17. Joint angle changes for both subjects for all four simulations producing minimum peak combined cable tension: two segment model.

This sensitivity analysis suggests that a similar possibility of significantly reducing peak combined cable tension exists for both gymnasts. In order to produce the reduction the joint angle was increased as the gymnast passed through the bottom of the swing, from body angles of 160° to 200°. However, it seems that the exact changes in body configuration required to achieve the reduction in peak forces is specific to the inertial characteristics of the gymnast (Figure 5.17).

Since only one joint is represented in this simulation model, the contributions of actions at each joint to reducing peak combined cable tension cannot be established. Such contributions will be determined using the evaluated four segment model (Chapter 7). However, the comparative simplicity of the two segment model enables the mechanical reasons for the reduction in peak forces to be determined. In addition, the reduction in peak combined cable tension is comparable to that produced when realistic elastic properties representing the DEDs and horizontal beam of the rings frame were used in the ELASTIC model.

In more general terms, this investigation provides evidence that arching actions performed by the gymnast during the descending phase of backward longswings
(Section 5.6) may reduce the peak forces experienced by the gymnast. Similarly, while performing forward longswings, the dished body shape adopted during the descending phase has a comparable effect.

5.4.3 Optimum performance for a backward longswing using the two segment model

This section provides the results for the production of the optimum backward longswing from an initial handstand position. For a given modelled joint the optimum backward longswing was defined by the longswing which obtained the minimum performance score (Section 3.4.2). Since the gymnast's body was in a straight configuration when the performance score for each longswing was calculated, the body angular velocity is equal to the angular velocity of the arm segment utilised in this score. Hence, the term body angular velocity is adopted in the following results to maintain consistency with body angle.

Table 5.7 gives a summary of the performance indicators for the optimum backward longswings when using either a hip or shoulder joint. The two segment gymnast was not able to complete a backward longswing from an initially still handstand to a final motionless handstand using only two joint angle changes during the ascending phase of the swing. In both cases the gymnast displayed both angular displacement ($\phi_c$) and motion of the rings cables and did not attain the handstand position ($e = 360^\circ$). This result is in general agreement with the findings of Sprigings et al. (1998), as discussed in Section 2.4.3.

Table 5.7. Performance score indicators for the optimum backward longswings from an initial handstand position: two segment model

<table>
<thead>
<tr>
<th>joint</th>
<th>$\phi_c$ (°)</th>
<th>$\dot{\phi}_c$ (rad.s$^{-1}$)</th>
<th>$\varepsilon_a$ (°)</th>
<th>$\dot{\varepsilon}_a$ (rad.s$^{-1}$)</th>
<th>performance score</th>
</tr>
</thead>
<tbody>
<tr>
<td>hip</td>
<td>3.57</td>
<td>0.16</td>
<td>356.4</td>
<td>-0.14</td>
<td>587.5</td>
</tr>
<tr>
<td>shoulder</td>
<td>3.55</td>
<td>0.17</td>
<td>356.2</td>
<td>-0.12</td>
<td>596.0</td>
</tr>
</tbody>
</table>

In both optimal performances the orientations of the gymnast and rings cables near the handstand position suggest that the gymnast would be able to maintain the final handstand through additional minor alterations to his body configuration. A backward longswing would therefore be completed. Such additional changes to the joint angles were not considered in this particular investigation. The orientations of the modelled rings cables and gymnast close to the final handstand, together with their respective
angular velocities, are lower than those measured for actual gymnasts from the data collection. For example, the body angular velocity for the real gymnasts on completion of the backward longswings ranged from 0.50 rad.s\(^{-1}\) to 0.83 rad.s\(^{-1}\), while for the simulated gymnast the mean was 0.13 rad.s\(^{-1}\) (Table 5.7). Since, during these longswings, the handstands were maintained using additional movements of the body segments, the speculation for the modelled gymnast may be considered realistic.

The joint angle changes used to perform the optimum backward longswings using either a hip or shoulder joint are given in Figure 5.18.

The resultant performances for both optimum longswings using changes in either the hip or shoulder joints were very similar (Table 5.7). Figure 5.19 therefore presents only the longswing performed using changes in the hip joint.

Since joint angle changes were restricted to the ascending phase of the swing, joint
actions were made after a body angle of 180°. The maximum angle attained at the hip joint was greater than at the shoulder during these optimum longswings. However, both joint angle changes were comparatively small in comparison to those observed during actual performances.

Figure 5.20. Path of the gymnast's centre of mass for the optimum backward longswings (hip and shoulder joint): two segment model.

Figure 5.20 shows the path of the gymnast's mass centre during the optimum backward longswings utilising different joints. It should be noted that vertically the gymnast's mass centre at the beginning and end of the simulation were within 0.0036 m of each other. In both cases it was the horizontal (y) displacement of the mass centre which was inappropriate for the final handstand position, being 0.23 m from the suspension point of the rings cable.

The two segment model does not possess any damped elastic components. Mechanical energy is therefore not automatically dissipated from the system. Since the total mechanical energy is not necessarily reduced in this scenario the modelled gymnast possessed sufficient energy at the beginning of the simulation to attain the final handstand. However, the rigid model identified the inherent constraint imposed by the rings cables on the gymnast's motion. This constraint resulted in a reduced amplitude of swing with no changes to the gymnast's body configuration. For these optimum longswings, therefore, changes to the gymnast's body configuration were concerned with altering the interaction of the gymnast and rings cables in order to increase the amplitude of swing sufficiently and place the hands of the gymnast vertically beneath his mass centre. It would therefore seem likely that an insignificant alteration in the total mechanical energy of the system occurred.

Figure 5.21 provides the time histories of the joint torques and types of joint actions which produced the modifications to the gymnast's body configuration during the optimum longswing where the gymnast possessed a hip joint. Also shown are the time
histories for all forms of energy within the system. When the gymnast comprised a hip joint the overall change in the total mechanical energy for the optimum longswing was 14.6 J. For the optimum longswing where a shoulder joint was implemented the total change in energy was 14.5 J. These results are in agreement with the suggestions made previously.

Figure 5.21. Torque and energy time histories for the optimum backward longswing implementing a hip joint: two segment model.

The concentric and eccentric joint actions produced during the swing are similar in duration, since the total energy of the system only increased slightly by the end of the simulation. Due to the rigid nature of the model, the joint torques required to perform these optimum backward longswings are unrealistic. However, the mechanical reasons behind the motion are more understandable with this simple model.

In order to assess the sensitivity of the optimum strategies for producing the backward longswings the same procedure was carried out using subject A. Figure 5.22 presents the strategies implemented by both gymnasts which suggests that similar techniques were employed by both subjects when performing optimum backward longswings from an initial handstand.

Figure 5.22. Hip and shoulder joint angle changes for the optimum backward longswings from a handstand for both subjects: two segment model.
Optimum backward longswings when representing an energy loss from the system

A second scenario regarding the backward longswing was also investigated since in reality the rings apparatus possesses damped elastic components which dissipate energy from the system. Since the two segment model does not incorporate such components the energy loss was represented by placing the gymnast in a motionless, nearly horizontal orientation. The orientation of the rings cables was such that the mass centre of the gymnast was below the suspension points of the rings cable. This meant the gymnast initially possessed $407 \text{ J}$ less energy than in the handstand position.

Table 5.8 consists of the performance indicators for the optimum backward longswings. Both optimum performances produced better scores than for the longswings where there was no artificial energy loss. In both cases the gymnast and rings cable were near vertical and motionless. From these orientations it could be assumed the gymnast would maintain the held handstand position with additional minor joint actions.

Table 5.8. Performance score indicators for the optimum backward longswings when representing energy loss from the system: two segment model

<table>
<thead>
<tr>
<th>joint</th>
<th>$\phi_c , (^\circ)$</th>
<th>$\dot{\phi}_c , (\text{rad.s}^{-1})$</th>
<th>$\varepsilon_a , (^\circ)$</th>
<th>$\dot{\varepsilon}_a , (\text{rad.s}^{-1})$</th>
<th>performance score</th>
</tr>
</thead>
<tbody>
<tr>
<td>hip</td>
<td>1.93</td>
<td>0.08</td>
<td>358.5</td>
<td>-0.07</td>
<td>150.4</td>
</tr>
<tr>
<td>shoulder</td>
<td>1.24</td>
<td>0.05</td>
<td>359.1</td>
<td>-0.05</td>
<td>59.1</td>
</tr>
</tbody>
</table>

Figure 5.23 presents the joint angle changes for the two optimum backward longswings. In both cases a smaller minimum joint angle was used than during the previous scenario.

Figure 5.23. Hip and shoulder joint angle changes for the optimum backward longswings when representing a dissipation of energy: two segment model.
These joint angle changes produced the following paths for the gymnast's mass centre (Figure 5.24). In both cases the vertical location of the gymnast's mass centre was higher than at the start of the swing. Since at the beginning and end of the swing the gymnast displayed either zero or insignificant motion (Table 5.8) this overall increase in mass centre location may be attributed to an overall increase in mechanical energy.

![Figure 5.24](image)

**Figure 5.24.** Path of the gymnast's centre of mass for the optimum backward longswings when representing a dissipation of energy: two segment model.

![Figure 5.25](image)

**Figure 5.25.** Graphics sequence of the optimal backward longswing when representing a dissipation of energy (shoulder joint): two segment model.

The shoulder torques required at the shoulder joint in order to produce the optimum longswing shown in Figure 5.25, are presented in Figure 5.26, together with the associated changes in the mechanical energy of the system.
Two significant concentric joint actions were made throughout the longswing, both of which were associated with an increase in the total energy of the system (Figure 5.26). The greater increase in total energy (373 J) was created by the reduction in the shoulder elevation angle during the latter portion of the descending phase and the swing beneath the rings ($\varepsilon$ from 98° to 193°). Such a large increase in energy is possible through a decrease in joint angle since typically during this stage of the swing concentric joint actions are required to decrease the joint angle (Figure 5.11). In addition a slight decrease in total energy was produced at the end of this reduction in shoulder elevation angle, produced by a negligible eccentric joint action from body angles (197° to 228°).

The combined effect of both concentric joint actions was an increase of 411 J in the total mechanical energy. This increase in energy enabled the gymnast's mass centre to be raised to an appropriate level for the final handstand. These changes in joint angles also overcame the interaction between the gymnast and rings cables to ensure the correct orientation of the cables in time for the final handstand (Figure 5.25).

The inertia parameters for gymnast A were then used to determine whether a different gymnast would adopt the same changes in body configuration for the optimum longswings when replicating a loss of energy from the system. The performance scores obtained for the simulated optimum backward longswings for subject A were similar to those of subject K. When the gymnasts possessed a shoulder joint the technique adopted by subject A was similar to subject K (Figure 5.27). However, when the gymnasts possessed a hip joint they used different techniques, requiring different minimum joint angles, as shown in Figure 5.27.

However, a general trend in these joint actions is apparent. Typically, since the gymnasts required an increase in their total mechanical energy, a decrease in the joint angle occurred during the descending phase of the swing. Such a decrease in joint angle was produced by concentric torques which resulted in the required net increase in mechanical energy.
Figure 5.27. Hip and shoulder joint angle changes for the optimum backward longswings representing a dissipation of energy for both subjects: two segment model.

Both investigations regarding the production of a backward longswing provide an insight into the mechanics involved in producing a longswing on rings. The two segment model showed that appropriately timed changes in a gymnast's body configuration may sufficiently overcome the constraint to the gymnast's motion provided by the rings cable and adequately compensate for any mechanical energy dissipated from the system.

5.4.4 Conclusions from simulations using the two segment model

The rigid nature of the two segment model decreases its external validity. For instance, the joint torques required to maintain or produce joint angle changes are unrealistic when compared to those estimated for gymnasts performing longswings on rings (Brüggemann, 1987).

However, the relative simplicity of the model enabled the mechanical reasons underlying the movements of the gymnast to be explained. For example, appropriately timed joint actions were shown to be able to reduce the peak combined cable tension significantly during a swing from handstand. Since actual gymnasts perform similar changes in body configuration during longswings on rings, such joint actions probably form a significant mechanism by which the peak external forces experienced by the gymnast are decreased. This point will be addressed in more detail within this thesis by implementing the evaluated four segment simulation model.

Similarly the two segment model showed that by specifically timed joint angle changes the interaction between the gymnast and rings cables can be modified to produce a handstand position at the end of a longswing. Furthermore, concentric joint actions were shown to be able to provide a net increase in the mechanical energy while eccentric actions produced a net loss. This finding is consistent with that of Hiley (1998). In the
cases regarding the performance of a backward longswing the gymnasts did not attain a motionless final handstand position. This finding is in agreement with that of Sprigings et al. (1998). However, when energy dissipation from the system was represented the gymnast and rings cables were extremely close to being motionless on completion of the longswing and were considered adequate to maintain the final handstand position.

On this point it is important to note the particular constraints placed on the longswing investigations. Firstly, only one decrease and subsequent increase in joint angle was allowed. Secondly, only one joint was modelled for any particular situation. Both of these restrictions limit the possibility for altering the path of the gymnast's mass centre and the interaction between the gymnast and rings cable. No elastic components were modelled which, as shown by the elastic model, also alter the relationship between the gymnast and rings cables. Finally, the model was restricted solely to planar movements of the arms and hence rings cables. Three-dimensional movements of the arms are used by elite gymnasts during longswings (Section 5.5). Such movements, absent in the two segment model, will increase the possibility for further modifications to the gymnast/rings cable interaction which were not explored in these investigations. Similar limitations concerning three-dimensional movements, elastic components and number of joint actions were also imposed by Sprigings et al. (1998). Such limitations must be considered when interpreting their finding concerning swinging on rings.

5.4.5 Summary

The accuracy of each two-dimensional simulation model was only addressed for particular cases. Extensive evaluations of each model's accuracy were not performed. To illustrate, differences between peak combined cable tension values and peak joint torques were highlighted together with the total excursions of the rings cable. However, the purpose of these models was to gain a greater insight into the mechanics involved in swinging on rings and this was accomplished. For example, several different concepts concerning the interaction between the gymnast and rings cables were identified.

In order to make more accurate judgements concerning specific influences on the performances of longswings a more accurate and possibly more sophisticated model is required. For this reason the four segment three-dimensional simulation model was developed. As described in Chapter 3 this model incorporates all components considered essential to simulate a gymnast swinging on rings accurately. However, before the model is used to estimate specific influences, the accuracy should be determined against actual longswing performances. This evaluation procedure is undertaken in Chapter 6 using the longswing performances described in the following section.
5.5 Results from the kinetic and kinematic analyses of longswings

The following section establishes the effects of filtering the cable tension data and provides indicators for the validity of the procedure. Also presented are measures of reconstruction accuracy and precision for the digitising and DLT procedure. Estimates of precision when calculating orientation and configuration angle data are also established.

Subsequently, descriptions of the techniques used by the gymnasts to perform backward and forward longswings are given. The three-dimensional angle definitions provided in Chapter 4 are used to describe the gymnasts' techniques, using one example from each type of longswing. Quantitative comparisons between different trials are used to estimate intra-subject variations in technique for each type of longswing, effectively establishing the consistency of the gymnasts' techniques.

5.5.1 Results of filtering cable tension data

The following results were obtained using the procedures outlined in Section 4.4.3, for filtering cable tension data with systematic variations in the cutoff frequency \( f_e \). This procedure was performed on five trials; K03, K05, A09, K15 and A21.

Theoretically the root mean square (RMS) difference between an original and filtered signal increases as the cutoff frequency \( f_e \) decreases. When the filtering process reduces mostly random noise within the signal the RMS difference between the original and filtered signal increases gradually. When the frequency of cutoff is smaller than frequencies in the true signal the filtering process significantly attenuates the true signal. Consequently the RMS difference between the original and filtered signals rapidly increases. Such a procedure, often termed residual analysis (Winter, 1990), has been used to define the resolution of a recording system (Cappozzo et al., 1975). The resolution of a system is estimated from the RMS differences between the original and filtered signals over the cutoff frequencies which produce only small changes in the RMS deviations.

Figure 5.28 highlights the effect of the cutoff frequency \( f_e \) on the RMS difference between raw\(^{50}\) and raw\(^{50} \cdot f_e \) for trial K15. The range in \( f_e \) was 300 Hz to 10 Hz inclusive. For this trial the results indicate a rapid increase in the RMS difference at a cutoff frequency of 30 Hz. This result suggests that only random noise is attenuated when the cutoff frequency is greater than 30 Hz. Similar findings were observed for the other four trials.

The findings of the pre-data collection testing of the force link-amplifier-ADC-
computer system (Section 4.4.1) indicated that the amplitude of random noise in the system was approximately 90 N. With a decrease in cutoff frequency a greater maximum difference between the original and filtered data would be expected. When cutoff frequencies less than 45 Hz were used in trial K15 the maximum difference between the data sets increased significantly, above the expected 90 N (Figure 5.28). Similar results were observed for trials K03, A09 and A21. The same analysis of trial K05, however, suggests that random noise of a slightly larger amplitude was present in the cable tension data (127 N).

![Figure 5.28. The RMS and maximum differences between raw and filtered data for trial K15.](image)

Taking these results into consideration all cable tension data (raw) were filtered using a cutoff frequency of 45 Hz, producing raw-45 Hz data. This cutoff frequency ensured a reduction in the random noise contained in all cable tension data without significantly affecting the true signal. For all trials the systematic offset of -15 N was then added to the raw-45 Hz cable tension data.

To provide further evidence that a cutoff frequency of 45 Hz reduced random noise in the signal without significantly affecting the true signal, each cable tension time history was subjected to a signal energy analysis. The power of a signal at a discrete frequency is equal to the square of the amplitude at that frequency (Press et al., 1988). If the amplitude at each discrete frequency is equal to $c_k$, (Equation 4.6.) the power $p$ at each discrete frequency $k$ may be calculated as:

$$p_k = c_k^2$$  \hspace{1cm} (5.1)

where:

- $c_k$ = the amplitude of the $k^{th}$ frequency
- $p_k$ = the power of the $k^{th}$ frequency
- $f_k$ = the $k^{th}$ frequency

The total signal energy $E_s$ may be calculated as the sum of all powers at each
discrete frequency including 0 Hz (Press et al., 1988):

$$E_s = \sum_{k=0}^{k=k} p_k$$

where:

$$E_s = \text{total signal energy}$$

In the signal energy analysis, the frequencies below which 99% and 99.9% of the energy was contained in the signal were calculated. These two frequencies indicate the frequencies above which the signal is expected to have an insignificant amount of true signal and therefore comprise mostly random error or white noise (Winter et al., 1974). A Hanning window was implemented on the raw 50 data prior to the signal analysis. The calculations for the complete energy analysis were performed in the Fortran program psd.

For all raw 50 cable tension data (after the 50 Hz notch filter was implemented) the cutoff frequency of 45 Hz ensured that more than 99.9% of the energy contained in the signal was retained (Table 5.9). This result provides further evidence that, by selecting this cutoff frequency, random errors in the cable tension data were attenuated without significantly affecting the true signal.

Table 5.9. Frequencies below which 99% and 99.9% of the energy in the signal is contained for the five analysed trials

<table>
<thead>
<tr>
<th>trial</th>
<th>frequency $f_{99%}$ (Hz)</th>
<th>frequency $f_{99.9%}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>7.60</td>
<td>10.60</td>
</tr>
<tr>
<td>K05</td>
<td>7.40</td>
<td>20.00</td>
</tr>
<tr>
<td>A09</td>
<td>7.60</td>
<td>10.80</td>
</tr>
<tr>
<td>K15</td>
<td>6.60</td>
<td>11.40</td>
</tr>
<tr>
<td>A21</td>
<td>7.40</td>
<td>14.80</td>
</tr>
</tbody>
</table>

Filtering method for trial A20

The modified 100 Hz data set for trial A20 was subjected to a cutoff frequency of 45 Hz and rectified for the systematic -15 N offset. Since the Nyquist frequency for this data is 50 Hz a cutoff frequency of 45 Hz was able to be used. This procedure ensured the cable tension time history for trial A20 was subjected to the same process of random and systematic error reduction as the other five trials.
Cable tension time histories

For all trials the raw^50-45 Hz synchronised cable tension data adjusted for the systematic offset were used in all subsequent analyses and the evaluation of the four segment simulation model. Figures 5.29 and 5.30 show original and filtered (raw^50-45 Hz) cable tension time histories for trial A21. The reduction in random and systematic errors, together with the offset in the original cable tension data, are clearly visible.

![Figure 5.29. Original and filtered (raw^50-45 Hz) cable tension time histories for trial A21 at 0.02 s intervals.](image)

![Figure 5.30. Original and filtered (raw^50-45 Hz) cable tension time histories for trial A21 at 0.001 s intervals for times t = 1.7 s to 2.0 s (boxed section in Figure 5.29).](image)
The separation of the original and filtered signals towards the end of trial A21 (Figure 5.29) is a result of sampling the original data with 50 Hz systematic error (buzz) at 0.02 s intervals. In this case, the troughs of the oscillating data were sampled, producing an alias of the original data, and resulting in an artificial reduction in the magnitude of cable tension in Figure 5.29. Figure 5.30 shows the 50 Hz oscillations in the original cable tension more clearly. The filtered cable tension does not have this 50 Hz systematic error since a notch filter was used. With the systematic offset also incorporated into the filtered (raw 50-45 Hz) tension data the filtered signal passes through the original signal at a value 15 N larger than the mean of the oscillating data.

Assumption of equal tension in the rings cables during longswings

The following results vindicate the assumption that tension in the rings cables during swinging elements are of equal magnitudes. Table 5.10 presents differences between two similar trials for backward and forward longswings which were performed facing opposite directions. In effect this meant tension in both rings cables was measured, as detailed in Section 4.6. Results suggesting the gymnasts' techniques were similar over the two different performances for each type of longswing are presented in Section 5.5.3. The difference in cable tension for the left and right cables was calculated as the RMS difference. This difference was also expressed as a percentage of the average maximum peak cable tension.

Figure 5.31 shows the raw 50-45 Hz cable tension time histories for forward longswings K03 and K05.

![Synchronised cable tension time histories for forward longswings K03 and K05.](image)
In both cases the RMS difference in cable tension between the left and right rings cables was less than 2.5% of the average peak cable tension. This is a somewhat diminutive difference and may be accounted for by the slightly different techniques used in the trials. This finding, implying the magnitudes of tension in the rings cables are equal during swinging elements on rings, is in agreement with the work of Nissinen (1983). This conclusion also provides evidence to justify the assumption used in the four segment simulation model, which assumes the left and right sides of the gymnast, rings cables and frame act in symmetry. The procedure used to synchronise the cable tension time histories for these analyses may be justified since peak cable tension occurred at similar gymnast orientation angles (182.6° and 183.8° for backward longswings A20 and A21; -184.2° and -181.1° for forward longswings K03 and K05).

Table 5.10. Differences between cable tension in the left and right rings cables throughout similar longswings

<table>
<thead>
<tr>
<th></th>
<th>RMS difference (N)</th>
<th>RMS difference expressed as % of peak cable tension (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>backward: A20 vs A21</td>
<td>53.2</td>
<td>2.4</td>
</tr>
<tr>
<td>forward: K03 vs K05</td>
<td>47.6</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Using the procedure outlined in Section 4.3 the filtered cable tension data for each trial were synchronised to the appropriate video derived data. These synchronised data were used in the estimation of all parameters for the four segment model. Slightly reduced ranges were used for the evaluation of the four segment simulation model, using approximately 2.0 seconds of the available data. These reduced ranges encapsulate the complete swinging phase of the longswings, which is the phase of interest in this study.

Peak combined cable tension

Table 5.11 details the combined peak cable tension produced during each longswing. Combined cable tension values are reported for comparison with previous studies on rings. The average peak combined cable tension for backward longswings was 6.8 bodyweights. The mean for forward longswings was 7.5 bodyweights. These values are similar to those reported in the study of straight arm longswings by Valliere (1976).

When compared to the studies of Nissinen (1983, 1995) these values are at the lower end of the range. In these two studies combined peak reaction forces ranged from 6.5 to 11.0 bodyweights. At present it is not possible to say whether this apparent difference between this study and those of Nissinen are due to the different measurement
systems, as discussed in Section 2.3.2, or the types of longswings performed in the studies. For instance, backward longswings often used prior to a dismount from rings are initiated from a swing which passes through the handstand position. These longswings, where the initial position is not a held handstand position, may produce greater forces. Such longswings may be responsible for the largest forces reported by Nissinen (1995).

Table 5.11. Peak combined cable tension for backward and forward longswings

<table>
<thead>
<tr>
<th></th>
<th>backward longswings</th>
<th>forward longswings</th>
</tr>
</thead>
<tbody>
<tr>
<td>cable tension</td>
<td>K15</td>
<td>A20</td>
</tr>
<tr>
<td>combined peak (N)</td>
<td>4012</td>
<td>4198</td>
</tr>
<tr>
<td>combined peak (BW)</td>
<td>6.56</td>
<td>6.45</td>
</tr>
<tr>
<td>mean (BW) (n= 3)</td>
<td>6.80</td>
<td></td>
</tr>
</tbody>
</table>

From data presented in Table 5.11 it may be inferred that for these two subjects peak cable tension produced during forward longswings was greater than during backward longswings. This inevitably leads to the gymnasts experiencing greater forces during forward longswings. Current literature does not reveal information or evidence regarding differences in the peak combined cable tension during backward and forward longswings. However, if a difference does exist the forward longswings may increase the risk of injury to gymnasts. With only three samples of each type of longswing the difference between peak cable tension values were not tested statistically. However, this finding warrants further investigation.

5.5.2 Accuracy and precision of the DLT technique

Reconstruction accuracy

Table 5.12 highlights the three-dimensional reconstruction accuracy of the camera-digitiser system. Errors were estimated by calculating the RMS difference between the reconstructed and known locations of the 12 calibration markers. When estimating errors in the reconstruction procedure the inter-dependency of the 11 DLT parameters was accounted for by constraining the degrees of freedom of the system to 6.5 (n-5.5). The overall error in the three-dimensional reconstruction was 0.010 m. This error is equivalent to 0.125% of the horizontal field of view.
Table 5.12. Estimates of reconstruction accuracy for the camera-digitiser system

<table>
<thead>
<tr>
<th>accuracy estimates (m)</th>
<th>x axis</th>
<th>y axis</th>
<th>z axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.010</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>maximum</td>
<td>0.015</td>
<td>0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>minimum</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
</tr>
</tbody>
</table>

These levels of reconstruction accuracy for 11 parameter DLT are in conformity with previous studies over similar horizontal fields of view (Angulo & Dapena, 1992; Tan, Kerwin & Yeadon, 1995; Section 2.5.3).

**Digitising precision for gymnast and rings apparatus landmarks**

Table 5.13 presents the digitising precision for images from both camera views. The horizontal fields of view were both 8 m. The overall digitising precision, estimated as the RMS difference between real and pseudo data, was 0.007 m.

Table 5.13. Estimates of digitising precision for gymnast and rings apparatus landmarks from all six longswings

<table>
<thead>
<tr>
<th>precision estimates (m)</th>
<th>HyperHAD camera</th>
<th>HandyPRO camera</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FX axis</td>
<td>FZ axis</td>
</tr>
<tr>
<td>mean</td>
<td>0.007</td>
<td>0.009</td>
</tr>
<tr>
<td>maximum</td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td>minimum</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Landmarks with low precision were those on the body of the gymnast, for example the knees (0.010 m) and toes (0.010 m). These body landmarks define the locations of the gymnasts' major articulations and as such require subjective judgements. It might be expected that for these landmarks larger errors would be introduced into the digitising process, producing a lower precision. The precision of body landmarks, compared with apparatus landmarks, is in agreement with this rationale.

Landmarks which had greater digitising precision, being generally less than the
overall estimates, were those on the rings apparatus, such as the DEDs (0.003 m). These landmarks were identified by sharp and often contrasting edges in the video images. Less subjectivity was therefore involved in their estimation. Consequently the greater precision observed when digitising these landmarks is not unexpected.

Estimates of reconstruction precision for gymnast and rings apparatus landmarks

The reconstruction precision of digitised landmarks on the gymnast and rings apparatus is also a factor in accurate three-dimensional analyses. Table 5.14 presents estimates of precision for the reconstructed three-dimensional locations of all digitised landmarks from all longswings. These estimates were calculated using the method described in Section 4.5.7. The overall precision in estimating the three-dimensional location of any digitised landmark was 0.006 m. This is similar to the reconstruction accuracy of the camera-calibration-digitiser system.

Table 5.14. Estimates of reconstruction precision for all landmarks using all six longswings

<table>
<thead>
<tr>
<th>precision estimates (m)</th>
<th>x axis</th>
<th>y axis</th>
<th>z axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.006</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>maximum</td>
<td>0.010</td>
<td>0.010</td>
<td>0.011</td>
</tr>
<tr>
<td>minimum</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Landmarks displaying the least precision include those obscured from view for some part of the digitised sequence, such as the elbows during the circling motion of the arms (overall 0.007 m). Also having low precision were those landmarks which exhibited a relatively large velocity, such as the toes of the gymnast (overall 0.009 m).

In accordance with the results for digitising precision, landmarks on the rings apparatus possessed the greatest precision in three-dimensional reconstruction. By way of illustration, the overall reconstruction precision of the DEDs was 0.003 m.

Estimates of precision for orientation and configuration angles

Estimates of precision in determining each orientation and configuration angle for the gymnast and orientation angles for the rings cables were calculated using the procedure described in Section 4.5.7. Table 5.15 presents estimates of precision for each
orientation and configuration angle of each longswing trial.

Except for the shoulder elevation angle, the precision estimates for the configuration angles of the gymnast during backward and forward longswings are similar. When expressed as a percentage of their respective ranges the precision of the hip and two shoulder angles are within 2% (knee angle within 8%). Such precision may be considered adequate for the analysis of the gymnasts' techniques undertaken in the following section.

For trial K03, a forward longswing, lower precision was observed for the shoulder elevation angle (3.1°). This may, in part, be due to the circling and lateral movements of the arms which occur concurrently during the ascending phase of a forward longswing. Such combined arm movements allow the arms to pass from behind the gymnast's body to the front. Owing to the definition of these two shoulder angles these arm configurations may be represented by shoulder elevation angles anywhere between 90° or 270°. Consequently, discrepancies between video fields for the estimated locations of the shoulders and hands of the gymnast may in one video field place the arms initially behind the gymnast and in the next field to the front. This may lead to rapid changes in the shoulder elevation angle, producing less precise estimates for this particular angle.

Table 5.15. Precision estimates for orientation, body configuration and cable angles during backward and forward longswings

<table>
<thead>
<tr>
<th>angle</th>
<th>precision estimate (°)</th>
<th>backward longswings</th>
<th>forward longswings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K15</td>
<td>A20</td>
</tr>
<tr>
<td>body angle</td>
<td></td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>knee</td>
<td></td>
<td>2.3</td>
<td>1.2</td>
</tr>
<tr>
<td>hip</td>
<td></td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>shoulder elevation</td>
<td></td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>shoulder abduction</td>
<td></td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>cable elevation</td>
<td></td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>cable abduction</td>
<td></td>
<td>0.09</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The high precision observed in estimating the body angle for the gymnast and two orientation angles for the rings cables is not unexpected. The body angle of the gymnast uses the mass centre location of the gymnast, which itself may be seen as a weighted mean of the 15 body landmarks of the gymnast. As a number of estimates are used to
produce the mass centre location the precision in the body angle would be expected to be high.

The high precision (0.08°) in estimating the orientation angles of the rings cables may be explained by the length of the cables. As the rings cables are 3.0 m in length and each end of the cable may be digitised with a precision of 0.005 m (Table 5.15) the effect of imprecise digitisations on the calculation of these two orientation angles is small.

Results concerning reconstruction accuracy, digitising precision and reconstruction precision indicate that this particular camera-calibration-digitiser system is acceptable for the analysis of gross human motion during swinging activities on rings. Considering that movements of the gymnast and cables encompass a volume of approximately 20 m³, which includes a maximum vertical range of 5.00 m (top of rings frame to toes of the gymnast in hang), the overall errors in accuracy and precision in locating landmarks and the calculation of orientation and configuration angles may be considered to be very small.

5.5.3 Descriptions of techniques used by gymnasts when performing backward and forward longswings on rings

This section presents amalgamated results from the kinematic and kinetic analyses of the backward and forward longswings described in Chapter 4. Hence, the actual techniques used by elite gymnasts to perform backward and forward longswings are evaluated and quantified. A gymnast's technique for a particular element may be described by the body configuration throughout the performance. For gymnastic elements the orientation of the gymnast relative to the inertial reference frame may be used to define the phase of the element which the gymnast is at. For a gymnast swinging on rings, the orientation of the cables is also of importance since the motion of the gymnast and rings cables are dependent. This dependency was highlighted by the two-dimensional simulation models.

One orientation angle and four body configuration angles are used to describe the gymnasts' techniques when executing both types of longswings. Two angles are used to describe the orientation of the rings cables throughout the longswings. The definitions of these angles, which are consistent with those of the four segment simulation model, are presented in Chapter 4. The filtered cable tension data (raw 50-45 Hz) are synchronised to the kinematic data to establish key moments during the longswings in terms of cable tension and its relationship to the gymnasts' techniques. Finally, the consistency of the gymnasts' techniques is considered for both types of longswings.
Three performances of the backward longswing were analysed, one by subject K (trial K15) and two by subject A (trials A20 and A21). Results from trial K15 are presented initially, providing an overview of the technique used by subject K to perform a backward longswing. Figure 5.32 shows the body angle time history for trial K15, indicating the element is a backward longswing, with a final body angle of around 360°. The data are the first estimates of the body angles, which are denoted in this and all subsequent figures in this section by closed symbols. Curve fitted angle data, using quintic splines, are denoted by open symbols. Angular velocity and acceleration estimates were calculated by determining the first and second derivatives of the quintic splines. Hence, these kinematic variables are also denoted in subsequent figures by open symbols.

The gymnast took approximately 2.2 seconds to complete the full 360° rotation of the backward longswing (Figure 5.32). During the final 0.7 s of the trial the gymnast maintained the handstand position, indicating a proficient performance. A body angle of 180° indicates the mass centre of the gymnast is directly below the mid-point of his hands and defines the interface between the descending and ascending phase of the backward longswing.

The majority of the descending phase of the longswing, defined by a body angle less than 180°, was characterised by an increasing rate of change in body angle. However, this characteristic was reversed for the last 38° of the descending phase, where the angular velocity of the body angle rapidly decreased from 358° s⁻¹ to 138° s⁻¹ (Figure 5.33). This rapid decrease in body angle angular velocity suggests that extensive alterations in body configuration of the gymnast occurred during the final portion of the descent from handstand.
Figure 5.33. Body angular velocity time history for the backward longswing K15.

The ascending phase of the swing, with a body angle greater than 180°, was initially characterised by the reduced body angular velocity initiated during the descending phase. This observation suggests the change in body configuration initiated during the descent continued into the ascending phase of the longswing. The body angle then rapidly increased as the ascending phase continued. By a body angle of 238° the angular velocity had increased to 345°.s⁻¹, a similar value to the peak angular velocity produced during the descending phase.

A competent performance of the backward longswing requires the final handstand to be attained with little or no motion of the gymnast or rings cables. The latter stage of the ascending phase (body angle 238° to 360°) was characterised by a reduction in the angular velocity of the body to a sufficiently low value. This decrease in angular velocity may aid the gymnast in controlling and maintaining the final handstand. As trial K15 possesses some body angular velocity at 360°, approximately 20°.s⁻¹, some angular motion occurs in the handstand from 2.2 s onwards (Figure 5.33).

The technique used by gymnast K when performing the backward longswing may be described in terms of changes in his body configuration throughout the longswing. Figure 5.34 shows the knee angle time history for trial K15. In terms of judging criteria knee bend is a negative aspect of technique which is deductible. Hence little knee bend would be expected. Figure 5.34 shows only slight fluctuations occurred in the knee angle, with the exception of one estimate at 1.24 s (knee angle of 168°). This spurious estimate may be due to digitising errors as such a change in knee angle is not evident in the original video images.
With the exception of this datum the knee angle displays a small range of variation throughout the longswing (10°). From this finding it may be proposed that changes in the angle at the knee joint are not an essential part of the gymnast's technique. These angle data also suggest a systematic error in the estimation of the joint centres for the ankle, knee and hip, which together define the knee angle. From observations of video images with the gymnast in the handstand position the legs may be regarded as straight. However, knee angle data corresponding to this body orientation indicate an angle of around 190°. Hence a straight leg may be defined by a knee angle of 190°.

Several studies have identified important changes in angles at the hip and shoulder joints during backward and forward longswings (Brüggemann, 1987; Nissinen, 1983). Figure 5.35 provides the joint angle time histories for the hip and shoulder elevation angles during trial K15.

Three large changes in hip angle were observed during the longswing, covering a range of 105°. From an initial angle of 189°, the hip angle increased to 217° by 1.22 s. This was followed by a rapid decrease to an angle of 112° over a period of 0.22 s. From
this minimum the hip angle rapidly increased to 200° when the final handstand was attained. These extensive changes in hip angle may be regarded as a significant part of the gymnast's technique when performing the backward longswing. This proposal is in agreement with findings from previous research (Nissinen, 1983) and with coaching material (Hesson, 1975; Kormann, 1984).

Changes in the shoulder joint angle have also been documented as an essential part of a gymnast's technique for a backward longswing (Nissinen, 1983; Fukushima & Russell, 1980). Figure 5.35 provides the joint angle time history for the shoulder elevation angle during trial K15. The time history is qualitatively similar to the hip angle. Firstly an increase in the shoulder elevation angle occurs, from 180° to 224°, followed by a rapid reduction to a minimum of 74°. Similarly to the hip angle, the shoulder elevation angle increased from this minimum angle to an angle near 180°, which is appropriate for the final handstand. It is suggested that these extensive changes in the shoulder angle also constitute a major part of the gymnast's technique. Again, this suggestion is in accordance with findings of previous studies (Nissinen, 1983) and present coaching opinions.

A second angle at the shoulder, that of abduction, was also calculated in this study. Owing to the two-dimensional analyses performed in previous studies of longswings, such as Nissinen (1983) and Brüggemann (1987), this angle has been neglected. Figure 5.36 presents the shoulder abduction time history for trial K15. Two major changes in the shoulder abduction angle were made during the backward longswing, indicating the arms of the gymnast moved laterally during both the descending and ascending phase of the longswing. Owing to the magnitude of the shoulder abduction angle (with two local maxima of 30° and 51°) it would seem plausible that it also forms a major component of the gymnast's technique.

![Figure 5.36. Shoulder abduction angle time history for trial K15.](image-url)
segment simulation model (Chapter 7).

The extensive changes in the hip and two shoulder configuration angles occur at slightly different times to form the gymnast's technique. It is also probable that these changes occur in a coordinated manner, as suggested by Nissinen (1983). In order to show in which phases of the swing these joint angle changes occur, the hip and shoulder elevation and abduction angles are expressed with respect to the body angle.

Figure 5.37 displays the hip, shoulder elevation and shoulder abduction angles throughout the longswing as a function of body angle. These graphs illustrate the coordinated changes in these angles throughout the swing.

The majority of changes in hip angle occurred over a smaller portion of the swing than the shoulder elevation angle, especially during the ascending phase. This may suggest that of these two components of technique, changes in the shoulder elevation angle are of greater importance in the performance of a backward longswing. The two distinctive alterations in the shoulder abduction angles may also have a large influence on the resulting longswing performance. Such suggestions are covered by the questions posed at the beginning of the thesis and will be investigated using the four segment simulation model. Furthermore, the observations of Nissinen (1983) and Brüggemann (1987) regarding the timing and coordinated actions at the hip and shoulder joints for elite gymnasts, described in Section 2.4.2, were observed for this gymnast.

Figure 5.38 shows a graphics sequence of the backward longswing K15, and the synchronised combined cable tension.
In the initial handstand the gymnast displayed a slightly arched upper body and slightly rounded lower body configuration, with his arms only slightly spread laterally (Figure 5.38: A). Since the gymnast was in a nearly motionless handstand position the combined cable tension was around 1 bodyweight. As the gymnast initiated the descending phase of the swing his arms moved laterally, away from the mid-line of his body, and his hip and shoulder elevation angles increased to assume an arched body configuration (Figure 5.38: B). With the gymnast descending and rings cables rotating forward of the gymnast the combined cable tension was 0.1 bodyweights.

Prior to a body angle of 135° the arms started to move close together above the head of the gymnast (abduction angle of 18.8°) as the shoulder elevation angle reached its maximum value of 224°. At this body angle the hip angle was still increasing. The initiation of a reduction in the shoulder elevation angle at a body angle of 135° was accompanied by a further, more extensive, decrease in the shoulder abduction angle. This
indicated that the hands of the gymnast moved closer together as the body angle increased. During this phase the hip angle increased to its maximum value of 217° at a body angle of 169°.

Prior to the completion of the descending phase, the gymnast rapidly reduced both the hip and shoulder elevation angles. This produced an extended body configuration as the gymnast passed through the bottom of the swing (Figure 5.38: D). The reductions in the angles at the shoulders and hips continued into the ascending phase of the swing. At the bottom of the swing the hands were close together, indicated by a shoulder abduction angle of 4° and negative cable abduction angle (Figure 5.39). By forming a nearly straight body configuration at the end of the descending phase the moment of inertia in the transverse axis of the gymnast about his mass centre was nearly maximal. This might be expected to produce a decrease in the body angular velocity during the swing through the bottom of the swing. Such an expected decrease is consistent with the recorded body angular velocity (Figure 5.33). At this phase of the swing the combined cable tension was at a maximum of 6.6 bodyweights.

During the initial portion of the ascending phase the hip and shoulder elevation angles decreased while the arms remained close together. These joint angle changes produce a piked configuration soon after the bottom of the swing, at a body angle of 199° (Figure 5.38: E), at which point the hip angle was at a minimum (112°). At this instant the shoulder elevation angle was still decreasing towards its minimum value (74°) and the arms were moving laterally, indicated by the increasing abduction angle. The body configuration in Figure 5.38: F was attained at a body angle of 259°. From this minimum shoulder elevation angle the gymnast continued to move his arms laterally, while starting to increase the shoulder elevation angle. The changes to the gymnast's body configuration resulted in the sudden increase in the combined cable tension between labels E and F.

By a body angle of 330° the arm abduction angle was a maximum of 51°, indicating the arms of the gymnast were spread wide. Concurrently the shoulder elevation angle was still increasing and the hip angle had reached 200°, an angle appropriate for the final handstand (Figure 5.38: G). At this instant combined cable tension was equal to 0.5 bodyweights.

The final phase of the ascent is characterised by a rapid increase in shoulder elevation angle to around 175°, coupled with lateral movements of the hands bringing the hands closer together. This led to a more extended body configuration at the end of the swing, reducing the body angle angular velocity at this stage. This reduction in body angular velocity may be associated with the increase in the gymnast's moment of inertia about his mass centre in the transverse axis as the extended body configuration of a handstand was assumed. During this final stage little change in the hip angle was evident, remaining near an angle appropriate for the final handstand. Since the gymnast possessed
some motion on the completion of the swing the combined cable tension oscillated around 1.0 bodyweight.

While the gymnast made these changes in body configuration the orientation of the rings cables also altered as the cables moved. Figure 5.38 shows the orientation of the cables pictorially, while Figure 5.39 details the two cable angles with respect to body angle.

![Figure 5.39. Cable elevation and abduction angles for K15.](image)

In the initial handstand position the cable elevation angle is near 0° while the cable abduction angle is -2°, indicating the cables were initially near vertical. The initial cable abduction angle also indicates the hand separation of the gymnast is slightly smaller than the separation of the suspension points of the rings cables. During the descending phase of the swing the cable elevation angle increased to a maximum value of 18° at a body angle of 82°. During the same phase the cable abduction angle increased to 2.4° due to the lateral movements of the gymnast's arms. During the second half of the descent both cable angles reduced. When the gymnast attained a body angle of 180° the cables were once again in a near vertical orientation. This is in agreement with the motion predicted by the two-dimensional simulation models.

During the ascending phase the cable elevation angle became negative, reaching a value of -12° at a body angle of 265°. A negative cable elevation angle indicates the rings were orientated backwards, or behind the gymnast. Throughout the majority of the ascending phase the gymnast's arms moved laterally. These lateral arm movements manifested themselves in the rings cable abduction angle, which reached a maximum of 7° abduction after 322° of body angle rotation. With the longswing completed, the cable angles reverted to values appropriate for a near vertical cable orientation and the final handstand.

Owing to the dependent motion of the rings cables and gymnast, the path described by the gymnast's mass centre in the y-z inertial plane resembles a thin stretched ellipse (Figure 5.40). At the start of the descending phase the mass centre of the gymnast moved in the positive y direction. This initial movement is consistent with the movement
described for the simple rigid model. Owing to the rings cables ability to rotate about their pivot point, together with their small masses and moments of inertia, the gymnast does not swing about the rings. Essentially the combined rotations of the gymnast and cables represent a double pendulum and result in the path of the gymnast's mass centre representing a thin vertical ellipse (Figure 5.40). This mass centre path is in close agreement with the simulated backward longswing produced using the two segment model.

![Diagram of mass centre path](image)

Figure 5.40. Path of the gymnast's mass centre for K15 in the y-z plane and time history of the mass centre location in the x direction.

Figure 5.40 also highlights the location of the gymnast's mass centre in the inertial x direction throughout the backward longswing. These data show that although the mass centre deviated from the y-z plane in the inertial frame of reference, the movement was only slight (0.14 m range of movement). When compared to the range in movement in the z direction (2.0 m) such lateral movements may be considered to be negligible. This result, in conjunction with those stemming from the magnitudes of cable tension in both rings cables (Section 5.5.1), provides evidence to support the notion that the majority of changes made by the gymnast to his body configuration are symmetrical about the gymnast's sagittal plane.

**Consistency of the backward longswing technique**

Two further backward longswings from the data collection were analysed, both by gymnast A (A20 and A21). These trials were used to estimate the consistency in technique for a gymnast performing a backward longswing. Differences between the two trials of gymnast A, in terms of the angles which define the gymnast's technique (hip, shoulder elevation, shoulder abduction) throughout the longswing, were calculated at each body angle. Since technique is dependent on which phase of longswing the gymnast is in, the two longswings were synchronised by aligning the fields in which the gymnast
was closest to a body angle of 180°. This ensured that consistency in technique in each phase of the longswing was determined.

Figure 5.41 shows the hip and shoulder elevation angles for both longswings as a function of body angle. These graphs provide a qualitative insight into the consistency of technique used by gymnast A when performing a backward longswing.

Figure 5.41. Hip, shoulder elevation and shoulder abduction angles for backward longswings A20 and A21.

Angle data in Figure 5.41 suggest that the gymnast perfomed similar techniques for both trials. The RMS differences between these angles are provided in Table 5.16.

Table 5.16. Consistency in technique of gymnast A during backward longswings A20 and A21

<table>
<thead>
<tr>
<th>configuration angle</th>
<th>RMS difference (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee</td>
<td>3.5</td>
</tr>
<tr>
<td>hip</td>
<td>3.6</td>
</tr>
<tr>
<td>shoulder elevation</td>
<td>5.5</td>
</tr>
<tr>
<td>shoulder abduction</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Since the precision in estimating each configuration angle (Table 5.15) is greater than the differences observed, the data suggest that slightly different techniques were used during the two longswings. This result, however, is not unexpected as it is unlikely that a gymnast is able to reproduce exactly the same technique time after time.

Slight differences in technique may also have occurred due to different body configurations and cable angles in the initial handstand. Owing to the interaction of the gymnast and rings cables slight differences in their motion at the onset of a longswing may require the gymnast to adjust his technique in order to attain the final handstand. Such changes may include using slightly different body configurations throughout the longswing. Furthermore, as these initial alterations in joint angles are made, further adaptations to technique may be required due to the continual interaction of the rings cables and gymnast (Figure 5.42), increasing the average difference in techniques used for two successive performances by the same gymnast. Hence, such small overall differences in configuration angles provide support for a high consistency in backward longswing technique for this gymnast.

![Figure 5.42. Cable elevation and abduction angles for backward longswings A20 and A21.](image)

The slight adjustments made by the gymnast to his technique produced similar motions of the rings cables and ensured a proficient performance in both longswings.

It would seem that the techniques of gymnasts remain quite consistent between trials. Furthermore, it seems that a gymnast is able to respond to slight differences in his motion during a swing and adjust his technique to ensure a proficient performance. However, the extent to which gymnasts are able to adapt their technique cannot be deduced from this data alone.
Forward longswings

Three forward longswing were analysed, two by subject K (trials K03 and K05) and one by subject A (trial A09). Results from trial A09 are presented first, providing an overview of the technique used by subject A to perform the forward longswing. Figure 5.43 shows the body angle time history for trial A09, indicating the element is a forward longswing, with a final body angle of around -360°. To complete the full 360° of rotation for the forward longswing took approximately 2.7 s. The final 0.7 s of data from this trial indicates the gymnast maintained the handstand position on completion of the element.

In similarity to the backward longswing the majority of the descending phase of the forward longswing was characterised by an increased rate of change in the body angle (Figure 5.43).

This increase in body angle angular velocity occurred until a body angle of -135° was reached, after which it rapidly decreased from -351°.s⁻¹ to -175°.s⁻¹ as the gymnast passed underneath the rings. This provided a relatively slow body angle angular velocity during the change from the descending to the ascending phases of swing. During the early stages of the ascending phase the body angle angular velocity rapidly increased to a maximum of -496°.s⁻¹ by a body angle of -289°. As with the backward longswing the time histories of the body angle and associated angular velocity reveal that major changes in body configuration were initiated prior to the completion of the descending phase, and continued through to the ascending phase of the longswing.

Greater differences in the general trend of body angle time history between backward and forward longswings were observed during the ascending phase. While the initial portion of the ascending phase of a forward longswing is characterised by an increased body angular velocity, the later stages (a body angle greater than 290°) saw the
the body angular velocity decrease at a much greater rate than during a backward longswing. In 0.26 s during the ascending phase, corresponding to a change in body angle from $-289^\circ$ to $-349^\circ$, the body angular velocity decreased from the peak value of $-496^\circ \cdot s^{-1}$ to near zero. The body angle and angular velocity data indicate the gymnast was almost in an inverted orientation, with his mass centre almost vertically above his hands, around 0.5 s before the longswing was completed. This is in contrast to the findings for the backward longswing, where a more gradual reduction in the body angular velocity was observed.

Figure 5.44 highlights the body configuration time histories for this forward longswing.

![Graphs showing body configuration angles](image)

Figure 5.44. Time histories for all gymnast configuration angles for trial A09.

In the final stages of the swing, changes in the configuration of the arms can be identified, while only small changes occurred at the hips and knees. During this phase of the longswing the orientation of the gymnast is correct for the handstand, though the gymnast is not in the handstand position. Effectively, only the arms of the gymnast are still moving and therefore these lateral arm movements form the majority of the gymnast's technique throughout the final stages of the forward longswing.

In the cases of the knee, shoulder elevation and shoulder abduction configuration
angles, the overall ranges of angles in the forward longswings were found to be larger than those observed during any backward longswing (Table 5.17). However in all cases the largest changes in the hip angle occurred during the backward longswings (Table 5.17).

The maximum and minimum values during each longswing type differ between the two types of longswings. The hip angle exemplifies this observation. Thus, both the maximum range of angle changes and the actual angles which each joint reaches are specific to the longswing type.

<table>
<thead>
<tr>
<th>angle</th>
<th>trial</th>
<th>knee (°)</th>
<th>hip (°)</th>
<th>shoulder elevation (°)</th>
<th>shoulder abduction (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>min</td>
<td>max</td>
<td>range</td>
<td>min</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>K03</td>
<td>K05</td>
<td>A09</td>
<td>K15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>147.3</td>
<td>149.4</td>
<td>151.7</td>
<td>167.6</td>
</tr>
<tr>
<td>knee (°)</td>
<td></td>
<td>195.8</td>
<td>201.7</td>
<td>199.3</td>
<td>196.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>48.5</td>
<td>52.3</td>
<td>47.6</td>
<td>28.4</td>
</tr>
<tr>
<td>max</td>
<td></td>
<td>195.8</td>
<td>201.7</td>
<td>199.3</td>
<td>196.0</td>
</tr>
<tr>
<td>range</td>
<td></td>
<td>48.5</td>
<td>52.3</td>
<td>47.6</td>
<td>28.4</td>
</tr>
<tr>
<td>min</td>
<td></td>
<td>137.0</td>
<td>134.9</td>
<td>148.5</td>
<td>111.7</td>
</tr>
<tr>
<td>shoulder elevation (°)</td>
<td></td>
<td>230.3</td>
<td>226.9</td>
<td>235.7</td>
<td>216.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>93.3</td>
<td>92.0</td>
<td>87.2</td>
<td>104.8</td>
</tr>
<tr>
<td>max</td>
<td></td>
<td>294.1</td>
<td>270.1</td>
<td>267.4</td>
<td>224.1</td>
</tr>
<tr>
<td>range</td>
<td></td>
<td>214.9</td>
<td>179.2</td>
<td>163.0</td>
<td>150.4</td>
</tr>
<tr>
<td>min</td>
<td></td>
<td>1.4</td>
<td>2.4</td>
<td>1.9</td>
<td>1.7</td>
</tr>
<tr>
<td>shoulder abduction (°)</td>
<td></td>
<td>86.7</td>
<td>78.5</td>
<td>87.4</td>
<td>50.7</td>
</tr>
<tr>
<td>max</td>
<td></td>
<td>85.3</td>
<td>76.1</td>
<td>85.5</td>
<td>49.0</td>
</tr>
<tr>
<td>range</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.45 relates the configuration angles which define the gymnast's technique to the body orientation angle $\varepsilon$ for longswing A09.

From these graphs it is clear that a large variation in knee angle occurred during the longswing. For certain gymnastic elements, such as longswings on rings, variations in the knees angles are not desirable, leading to deductions in competition. These changes in the knee angle were initiated during the descending phase, and span the swing underneath the rings into the ascending phase of the swing. During this part of the swing the knee angle varies from $192^\circ$ to $152^\circ$ and returns to $196^\circ$. This variation is larger than observed during any backward longswing. As the variation in knee angle is over a short period of time and relatively small in comparison to variations in the hip and shoulder angles, it is
proposed that changes in the knee angle do not form an essential part of the gymnast's technique. It may be further speculated that if this aspect of technique was removed it would not significantly affect the gymnast's overall performance. This proposition is investigated using the evaluated four segment simulation model (Chapter 7).

Gymnast A performs three extensive changes in the hip angle and four in the shoulder elevation angle during the forward longswing. This indicates that a large part of the gymnast's technique is formed by changes in the angles at these joints. As with the backward longswing, the most extensive changes in hip angle occurred over a smaller body angle range than changes in shoulder elevation angle (Figure 5.45). This provides support to the suggestion, addressed in Chapter 7, that changes in the shoulder angle are of greater importance than those at the hip.

Figure 5.45 also highlights the shoulder abduction angle used by gymnast A during this longswing. Two local maxima were displayed, 26° and 83°. Hence, extensive lateral arm movements occurred during both phases of the swing. The magnitude of change in this angle also suggests that this aspect of technique constitutes a major part of the
gymnast's overall technique. What contribution this component of technique provides to the gymnast's performance will be determined in Chapter 7.

The combination of changes in body configuration is shown in the graphics sequence (Figure 5.46). The associated synchronised combined cable tension time history is also given. In the initial handstand, (Figure 5.46: A), the gymnast displayed a reasonably straight body position, with a slightly reduced shoulder elevation angle. By adopting this initial configuration, which possessed little motion, the combined cable tension was equal to 1.0 bodyweight. From this initial handstand the gymnast increased his hip and shoulder elevation angles, producing an arched-body configuration at the initiation of the descending phase of the swing. From this initially arched shape the majority of the descending phase was characterised by the gymnast adopting a piked or dished body configuration.

By a body angle of -62° both the hip and shoulder elevation angles had decreased, while the arms moved laterally to a shoulder abduction angle of 24°. Near a horizontal orientation (a body angle of -83°) the shoulder elevation angle reached its minimum value of 120° while the hip angle continued to decrease (Figure 5.46: B). These joint actions produced a dished body configuration. At this point of the longswing the arms were slightly apart displaying a shoulder abduction of 18°. The orientations and motion of the gymnast and rings cables produced a combined cable tension of 0.2 bodyweights at this phase of the swing.

The hip angle displayed its minimum value of 149° when the body angle was -162°. At the same time the arms of the gymnast moved closer together becoming almost parallel as the shoulder elevation angle increased. The body configuration at this point of the longswing may be described as piked (Figure 5.46: C).

Prior to the completion of the descending phase, both the hip and shoulder elevation angles rapidly increased, producing a slightly arched body configuration through the bottom of the swing. At the bottom of the swing the shoulder elevation and abduction angles indicate that the arms of the gymnast were almost directly above his head (177° and 3° respectively). This change in body shape during the swing beneath the rings resulted in a decrease in the body angular velocity to 179°.s⁻¹, from its peak value of 351°.s⁻¹ at a body angle of -135°. This decrease in body angular velocity as the gymnast swings through the bottom of the longswing may have been caused by the increase in the gymnast's moment of inertia in the transverse axis about his mass centre. At this point in the longswing the gymnast experienced the peak combined cable tension of 7.5 bodyweights.
At the start of the ascending phase the hip and shoulder elevation angular velocities were 684°.s⁻¹ and 453°.s⁻¹. This indicates the increases in hip and shoulder elevation angles initiated during the final stage of the descending phase were continued into the early part of the ascending phase. At a body angle of -198° the body was in the configuration shown in Figure 5.46: E, which may be described as extremely arched. The knee, hip and shoulder elevation angles were 154°, 236° and 204° respectively, with the hip angle at the maximum value attained during the longswing. Furthermore, the shoulder abduction angle was 7° with a positive angular velocity. This indicates that lateral movements of the arms were taking place.

From this extremely arched body configuration the hip angle decreased while the shoulder elevation and abduction angles continued to increase. At a body angle of -279° (near the horizontal) the shoulder elevation angle attained a maximum of 267° while the
abduction angle was 74°. These angles relate to a configuration where the arms of the gymnast were held wide and behind the torso (Figure 5.46: F). During this stage of the longswing the body angular velocity increased rapidly to its peak of -496°.s⁻¹ (Figure 5.43). In addition the cable tension experienced by the gymnast was near 0.4 bodyweights.

Over the final 90° of body angle rotation the movement of the arms formed the major characteristic of the gymnast's technique. From an configuration where the arms were spread very wide and behind his body, the gymnast circled his arms in front of his torso with an accompanied increase in the shoulder abduction angle to 87°. These angles define the arms as being almost perpendicular to the torso, spread as wide as possible. As the arms passed further in front of the gymnast the shoulder elevation angle rapidly decreased to a minimum of 104° at which point the arms were still extremely wide (77° shoulder abduction angle). This placed the gymnast in a position where the mid-point of his hands was beneath his mass centre. During this extensive change in both shoulder angles, the body angular velocity peaked at -496°.s⁻¹ and rapidly decreased to -164°.s⁻¹. During this rotation the hip angle decreased from 221° to 198° in preparation for the final handstand. This was a relatively minor change in comparison to that made in the shoulder angles.

The phase of the longswing from a body angle of -339° to -360°, was characterised by a rapid decrease in the body angular velocity to near zero. The gymnast performed a rapid increase in the shoulder elevation angle combined with a rapid reduction in the shoulder abduction angle (Figure 5.45). During this final stage of the longswing the mass centre of the gymnast was almost below the gymnast's hands, indicated by the body angle being near to -360° for the majority of this phase. The actions of the arms formed the appropriate configuration for the final handstand while the hip angle remained reasonably steady at 198° (Figure 5.45).

It would seem that the emphasis of the ascending phase of the forward longswing was to obtain an orientation whereby the mass centre of the gymnast is directly above the mid-point of his hands. This is in contrast to the backward longswing where less extensive changes to the body configuration are produced once the mass centre of the gymnast is above his hands. By moving his arms laterally and then circling them to the front and above his head the mass centre does not need to be raised as high as it would if the arms remained parallel to each other in order to position the mass centre above the mid-point of his hands. The importance of such circling movements of the arms on the performance will be investigated using the evaluated four segment model.

The following Figure presents the two cable orientation angles during the forward longswing A09, which correspond to the cable orientations shown graphically in Figure 5.46.
The initial cable orientation indicates the cables were close to vertical during the initial handstand position. During the descending phase of the swing the cable elevation angle increased to a maximum magnitude of \(-17^\circ\) when the orientation of the gymnast was near horizontal. The negative sign of the cable orientation indicates the rings cables rotated in a manner which allowed the rings to move behind the gymnast. Prior to this the cable abduction angle increased to \(2^\circ\) in response to the increased shoulder abduction angle which occurred during this stage. During the rest of the descending phase both cable angles decreased. At the bottom of the swing (body angle \(180^\circ\)) the angles corresponded to a near vertical orientation \((1.4^\circ\) and \(-1.6^\circ\) respectively).

During the initial stage of the ascending phase the cable angles increased rapidly, though at differing rates. At a body angle of \(230^\circ\) the cable elevation angle was \(11^\circ\), while the cable abduction angle was \(5^\circ\). Owing to the changes in the two shoulder angles during the final stages of the forward longswing, the cable abduction angle reached \(10^\circ\) at a body angle of \(326^\circ\). On completion of the longswing the cables reached a vertical orientation while possessing some cable elevation angular velocity. This indicates that on arriving in the handstand the gymnast and rings possessed some linear velocity. In terms of judging criteria the longswing may be considered imperfect.

The combined effects of the gymnast's body orientation and configuration and the cable movements produces the path for the mass centre of the gymnast in the \(y-z\) plane (Figure 5.48). As in the case of the backward longswing the general shape of the path described by the gymnast's mass centre may be termed a thin ellipse. Owing to the initial orientation and motion of the rings cables at the start of descending phase the horizontal component of tension in the rings cables results in the mass centre of the gymnast moving in the negative \(y\) direction.
The motion of the gymnast's mass centre in the inertial x direction is shown in Figure 5.48. These data suggest that changes in body configuration angles may be considered symmetrical for the left and right hand sides of the gymnast's body. If changes in body configuration angles were far from symmetrical, greater deviations than 0.08 m in the x direction may have occurred due to unequal external forces acting at the hands of the gymnast. These data, and those from the backward longswing, support the symmetry assumption (between the left and right sides on the system) used in the four segment simulation model.

**Consistency of the forward longswing technique**

The other two forward longswings analysed from the data collection were used to estimate the consistency in technique for a gymnast performing a forward longswing. Differences between the two trials of gymnast K (K03 and K05) in terms of the angles defining the gymnast's technique (hip, shoulder elevation, shoulder abduction) were calculated at each body angle throughout the longswings. The two longswings were synchronised by aligning the fields in which the gymnast was closest to a body angle of 180°. This meant that the consistency of technique during each specific phase of the longswing was determined.

Figure 5.49 presents the knee, hip, shoulder elevation and abduction angles for both longswings as a function of body angle and provides an insight into the consistency of technique used by gymnast K when performing a forward longswing. The RMS deviations between the knee, hip, shoulder elevation shoulder abduction angles for the longswings are given in Table 5.18.
Figure 5.49. Knee, hip, shoulder elevation and shoulder abduction angles for forward longswings K03 and K05.

These data, together with the precision data in Table 5.15 suggest the gymnast used slightly different techniques during both trials.

Table 5.18. Consistency in technique of gymnast K during forward longswings K03 and K05

<table>
<thead>
<tr>
<th>configuration angle</th>
<th>RMS difference (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee</td>
<td>4.1</td>
</tr>
<tr>
<td>hip</td>
<td>7.0</td>
</tr>
<tr>
<td>shoulder elevation</td>
<td>12.1</td>
</tr>
<tr>
<td>shoulder abduction</td>
<td>2.4</td>
</tr>
</tbody>
</table>

The most pronounced difference in technique was in the shoulder elevation angle, which on average was 12.1° different (RMS). The precision in estimating this angle is 2.1° (Table 5.15). A difference in technique is therefore apparent, especially considering...
that during the ascending phase the difference was as large as 35° (body angle -278°). Large differences in the shoulder elevation angle may to some extent be due to the method and definition of the shoulder elevation angle. As described previously, the calculation of this angle is dependent on the location of the hands and shoulders of the gymnast. However, the overall RMS difference suggests that a different technique was adopted by the gymnast. The consistency in technique for this gymnast performing forward longswings may therefore be deemed to be lower than used by subject A for backward longswings.

Differences in hip and shoulder elevation angles at a body angle of 0° indicate the initial conditions of the two trials differed slightly. Taking this into consideration the consistency of the technique might not be expected to be high. Indeed, owing to the interaction of the cables and gymnast, slightly different techniques would be expected. Figure 5.50 also provides evidence that the longswings were slightly different from the outset. However, slight adjustments to the gymnast's technique produced similar motions of the rings cables which were required to ensure a proficient performance. On completion of the longswings the rings cables possessed the similar orientations and angular velocities required for a stable handstand.

![Graph 5.50 Cable elevation and abduction angles for forward longswings K03 and K05](image)

Figure 5.50. Cable elevation and abduction angles for forward longswings K03 and K05.

It would seem that even if the technique used by the gymnast is not too consistent between trials, the gymnast is able to respond and adjust his technique during the performance to ensure the final performance is proficient. Such adjustments may be required due to different initial conditions. During a rings routine different initial conditions are inevitable owing to the nature of a routine and the requirements of the judging criteria. It is of importance therefore that gymnasts are able to make alterations to their techniques whilst swinging. However, the extent to which gymnasts are able to modify their techniques cannot be deduced from this data alone.
5.6 Results of model parameter estimates for the three-dimensional simulation model

Estimates of the gymnast and apparatus specific parameters used in the four segment three-dimensional simulation model are presented in this section. Subject specific segmental inertia parameters are presented first. Estimates of the accuracy for these data are also provided. Secondly, results estimating the values of model parameters representing the elasticity of the gymnasts and rings apparatus are provided, together with estimates of precision for these values.

5.6.1 Estimates of personalised segmental inertia parameters for subjects A and K

The anthropometric measurements for subjects A and K were used in the calculation of personalised segmental lengths, masses, and principal moments of inertia for the four segments of the modelled gymnast using the mathematical inertia model of Yeadon (1990b). To calculate segmental inertia parameters the inertia model requires estimates for the density of each segment of the human body. Initially three different sets of segmental density values: Dempster (1955), Clauser et al. (1969) and Chandler et al. (1975) were used for the appropriate segments of the inertia model. The accuracy of the inertia model for each set of density values was determined through the comparison of calculated and measured total body mass. Total body mass was used to evaluate the inertia model and determine which density set to use as this measurement is easily and accurately determined, unlike segmental masses or principal moments of inertia.

Table 5.19 highlights the percentage difference in calculated total body mass from measured total body mass using the different segmental density sets. On average this difference was smallest when the density values of Dempster (1955) were used (1.5% and -1.1%). These percentage errors values are within those which might be expected using this mathematical technique (Yeadon, 1990b). Density values of Chandler et al. (1975) led to under-estimations of the total body mass for both subjects (-5.6% and -8.2%), while the density values of Clauser et al. (1969) led to over-estimations of total body mass for both subjects (3.2% and 0.4%). The segmental density estimates of Dempster (1955) were therefore used in the production of all segmental inertia parameters for both subjects.
Table 5.19. Estimations of total body mass and errors using the method of Yeadon (1990b) with different segmental density estimates

<table>
<thead>
<tr>
<th>density sets</th>
<th>subject</th>
<th>A</th>
<th>K</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mass (kg)</td>
<td>% error</td>
<td>mass (kg)</td>
<td>% error</td>
</tr>
<tr>
<td>Dempster (1955)</td>
<td>A</td>
<td>67.30</td>
<td>1.5</td>
<td>61.63</td>
<td>-1.1</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>68.39</td>
<td>3.2</td>
<td>62.51</td>
<td>0.4</td>
</tr>
<tr>
<td>Clauser et al. (1969)</td>
<td>A</td>
<td>62.57</td>
<td>-5.6</td>
<td>57.22</td>
<td>-8.2</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>measured</td>
<td>66.30</td>
<td>62.30</td>
<td></td>
</tr>
</tbody>
</table>

Estimates for the segmental parameter defining the distance LT₄ from the right to the left shoulder centre are presented in Table 5.20. These results indicate a difference of 0.06 m between anthropometric and digitised estimates for both subjects. As digitised estimates were obtained during the longswing performances they may be considered more representative of this distance than a gymnast stood erect and measured using the technique of Yeadon (1990). Hence, the mean value of the digitised derived estimates of LT₄ for each subject is used in the four segment three-dimensional simulation model.

Table 5.20. Estimates of the subject specific parameter LT₄

<table>
<thead>
<tr>
<th>trial</th>
<th>LT₄</th>
<th>trial</th>
<th>LT₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>0.2576</td>
<td>A09</td>
<td>0.2742</td>
</tr>
<tr>
<td>K05</td>
<td>0.2711</td>
<td>A20</td>
<td>0.2752</td>
</tr>
<tr>
<td>K15</td>
<td>0.2423</td>
<td>A21</td>
<td>0.2790</td>
</tr>
<tr>
<td>mean (n = 3)</td>
<td>0.2570</td>
<td>mean (n = 3)</td>
<td>0.2761</td>
</tr>
<tr>
<td>anthropometric</td>
<td>0.314</td>
<td>anthropometric</td>
<td>0.338</td>
</tr>
</tbody>
</table>

Table 5.21 provides values for all segmental inertia parameters required in the four segment model for subjects A and K. These values were used for personalised representations of the gymnasts in the three-dimensional simulation model.
Table 5.21. Segmental inertia parameters representing both subjects for the modelled segments in the four segment model

**Arm segment**

<table>
<thead>
<tr>
<th>Subject</th>
<th>(m_a) (kg)</th>
<th>(L_{A_2}) (m)</th>
<th>(L_{A_1}) (m)</th>
<th>(I_{ARMT}) (kg.m^2)</th>
<th>(I_{ARMF}) (kg.m^2)</th>
<th>(I_{ARML}) (kg.m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3.888</td>
<td>0.607</td>
<td>0.355</td>
<td>0.127</td>
<td>0.127</td>
<td>0.004</td>
</tr>
<tr>
<td>K</td>
<td>3.664</td>
<td>0.601</td>
<td>0.350</td>
<td>0.119</td>
<td>0.119</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**Torso and head segment**

<table>
<thead>
<tr>
<th>Subject</th>
<th>(m_{tr}) (kg)</th>
<th>(L_{T_1}) (m)</th>
<th>(L_{T_4}) (m)</th>
<th>(L_{T_5}) (m)</th>
<th>(I_{TRT}) (kg.m^2)</th>
<th>(I_{TRF}) (kg.m^2)</th>
<th>(I_{TRL}) (kg.m^2)</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>17.199</td>
<td>0.127</td>
<td>0.276</td>
<td>0.475</td>
<td>0.814</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>K</td>
<td>15.998</td>
<td>0.119</td>
<td>0.257</td>
<td>0.461</td>
<td>0.770</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

**Thigh segment**

<table>
<thead>
<tr>
<th>Subject</th>
<th>(m_{uleg}) (kg)</th>
<th>(L_{U_2}) (m)</th>
<th>(L_{U_1}) (m)</th>
<th>(I_{ULEGT}) (kg.m^2)</th>
<th>(I_{ULEGF}) (kg.m^2)</th>
<th>(I_{ULEGL}) (kg.m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8.145</td>
<td>0.410</td>
<td>0.175</td>
<td>0.118</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>K</td>
<td>6.965</td>
<td>0.382</td>
<td>0.165</td>
<td>0.088</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

**Shank and foot segment**

<table>
<thead>
<tr>
<th>Subject</th>
<th>(m_{ileg}) (kg)</th>
<th>(L_{L_2}) (m)</th>
<th>(L_{L_1}) (m)</th>
<th>(I_{ILLEGT}) (kg.m^2)</th>
<th>(I_{ILLEGF}) (kg.m^2)</th>
<th>(I_{ILLEGL}) (kg.m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4.420</td>
<td>0.615</td>
<td>0.235</td>
<td>0.120</td>
<td>0.120</td>
<td>0.017</td>
</tr>
<tr>
<td>K</td>
<td>4.189</td>
<td>0.610</td>
<td>0.233</td>
<td>0.102</td>
<td>0.102</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Definitions of the symbols can be found in Appendix D.

5.6.2 Estimates for model parameters representing the elasticity of the rings apparatus and gymnast in the three-dimensional simulation model

Using the procedures outlined in Sections 4.7.2 to 4.7.5 estimates of the physical characteristics for the four damped linear springs in the three-dimensional simulation model are presented. Physical characteristics of damped linear springs are their stiffness and damping coefficients. These are generally termed \(k\) and \(c\) respectively. Estimates of errors for each characteristic are presented as standard errors about the mean (s.e.). Certain estimates of stiffness and damping were regarded as unsuitable on theoretical
grounds and were not used in subsequent calculations of means and error estimates. Reasons for the omissions are described and highlighted in the appropriate sub-sections.

**Spring parameters representing the rings frame**

The horizontal spring in the three-dimensional simulation model, together with the effective mass of the rings frame, allows the actual anterior-posterior movement of the rings frame to be simulated. Figure 5.51 provides an example of the anterior-posterior motion of the rings frame during the forward longswing A09.

![Graph showing DEDs location in inertial y direction](image)

Figure 5.51. Time history of the anterior-posterior location of the DEDs during the forward longswing A09.

**Table 5.22. Estimates of stiffness and damping values for the horizontal spring in the three-dimensional simulation model**

<table>
<thead>
<tr>
<th>trial</th>
<th>$x_1^*$ (m)</th>
<th>$x_2^*$ (m)</th>
<th>$\zeta$</th>
<th>$\tau_d$ (s)</th>
<th>$\omega_n$ (rad.s$^{-1}$)</th>
<th>$k$ (N.m$^{-1}$)</th>
<th>$c$ (N.s.m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>0.0354</td>
<td>0.0133</td>
<td>0.1539</td>
<td>0.190</td>
<td>33.47</td>
<td>18762</td>
<td>173</td>
</tr>
<tr>
<td>K05</td>
<td>0.0292</td>
<td>0.0182</td>
<td>0.0750</td>
<td>0.170</td>
<td>37.06</td>
<td>23011</td>
<td>93</td>
</tr>
<tr>
<td>A09</td>
<td>0.0293</td>
<td>0.0235</td>
<td>0.0351</td>
<td>0.175</td>
<td>35.93</td>
<td>21619</td>
<td>42</td>
</tr>
<tr>
<td>K15</td>
<td>0.0281</td>
<td>0.0159</td>
<td>0.0903</td>
<td>0.180</td>
<td>35.05</td>
<td>20577</td>
<td>106</td>
</tr>
<tr>
<td>A20</td>
<td>0.0262</td>
<td>0.0173</td>
<td>0.0659</td>
<td>0.195</td>
<td>32.29</td>
<td>17466</td>
<td>71</td>
</tr>
<tr>
<td>A21</td>
<td>0.0338</td>
<td>0.0245</td>
<td>0.0511</td>
<td>0.185</td>
<td>34.01</td>
<td>19372</td>
<td>58</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20134</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>s.e.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>822</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.22 presents the estimates of the stiffness and damping for the horizontal spring in the simulation model.
Values for the stiffness and damping of the horizontal spring in the simulation model are 20134 N.m⁻¹ and 91 N.s.m⁻¹ respectively. Coefficients of variation associated with these physical characteristics are 10% and 52% respectively. The coefficients of variation indicate the low accuracy in the estimates of the spring parameters, particularly the damping, for the horizontal spring representing the anterior-posterior motion of the rings frame. In order to obtain more accurate estimates an optimisation process involving the simulation model was employed. This process is described in Chapter 6.

Spring parameters representing the DEDs and horizontal beam of the rings frame

The vertical spring in the three-dimensional simulation model represents the combined elasticity of the damped elastic devices (DEDs) and horizontal beam of the rings frame in the vertical direction. An example of the time history for the vertical location and velocity of the DEDs is presented in Figure 5.52. As the top of the DEDs were digitised changes in their vertical location also incorporated the vertical movement of the horizontal beam of the rings frame.

![Graph of DEDs location and velocity](image)

Figure 5.52. Time history of the vertical location and velocity of the DEDs during trial A09.

Figures 5.53 and 5.54 are graphs of the regression analyses from trial A09 used to estimate the stiffness and damping values for this vertical spring in the simulation model. The method and theoretical basis for these analyses are presented in Section 4.7.3. This trial displayed the lowest error estimate (s.e.) to the final fit (Table 5.22). The data may therefore be considered to be the most representative of a damped linear spring. Figure 5.53 depicts the vertical component of cable tension (vten) plotted against the synchronised spline fitted vertical location (sded). The vertical component of cable tension (vten) estimated as a function of the spline fitted vertical location (sded) and velocity (vded) of the DEDs is highlighted by the regression line and equation. The linear regression equation estimating 'vten' as a function of 'sded' and 'vded', is:
\[ v_{ten} = 348412 - 59651 \, \text{sded} - 711 \, \text{vded}. \]

![Graph showing the relationship between vertical component of cable tension and vertical location of DEDs](image)

**Figure 5.53.** Vertical component of cable tension against splined vertical location and estimated as a function of splined vertical location and velocity of the DEDs for trial A09.

The neutral location of the spring was calculated by using the specific condition of zero velocity of the DEDs and zero vertical component of cable tension in the spring. The neutral spring location for trial A09 is 5.8408 m. The neutral spring locations for all trials are shown in Table 5.23.

The vertical location data were detrended from the neutral location, providing the vertical displacement data of the DEDs (\( \text{esded} \)). The estimation of the vertical component of cable tension using displacement of the DEDs from the neutral location and velocity produced the following equation for A09:

\[ v_{ten} = -59794 \, \text{esded} - 711 \, \text{vded}. \]

Theoretically a damped linear spring with zero displacement from the neutral location and velocity has zero tension. Hence, this multiple regression is forced to pass through the origin. The magnitude of the coefficient for 'esded' is equal to the stiffness of the spring, while the magnitude of the coefficient for 'vded' is equal to the damping of the spring.
Table 5.23. The neutral spring lengths for the DED vertical location, cable length and subject specific wrist to ankle lengths

<table>
<thead>
<tr>
<th>trial</th>
<th>DED vertical location (m)</th>
<th>cable length (m)</th>
<th>wrist to ankle length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>5.8405</td>
<td>2.9141</td>
<td>1.8479</td>
</tr>
<tr>
<td>K05</td>
<td>5.8429</td>
<td>2.9242</td>
<td>1.8626</td>
</tr>
<tr>
<td>A09</td>
<td>5.8404</td>
<td>2.9330</td>
<td>1.9135</td>
</tr>
<tr>
<td>K15</td>
<td>5.8469</td>
<td>2.9191</td>
<td>1.8631</td>
</tr>
<tr>
<td>A20</td>
<td>5.8330</td>
<td>2.9186</td>
<td>1.8944</td>
</tr>
<tr>
<td>A21</td>
<td>5.8439</td>
<td>2.9271</td>
<td>1.8917</td>
</tr>
<tr>
<td>mean (n = 6)</td>
<td>5.8412</td>
<td>2.9227</td>
<td>A = 1.8579, K = 1.8998†</td>
</tr>
<tr>
<td>s.e. (n = 6)</td>
<td>0.0019</td>
<td>0.0028</td>
<td>A = 0.0049, K = 0.0069†</td>
</tr>
</tbody>
</table>

† as subject specific. for the wrist to ankle lengths n = 3

Figure 5.54. Vertical component of cable tension against splined vertical displacement of the DEDs and estimated as a function of splined vertical displacement and velocity of the DEDs for trial A09.

Table 5.24 presents the estimates of the stiffness and damping for the vertical spring from all 6 longswing trials. Since a damped linear spring must possess a damping
component, both variables were incorporated in all regression analyses. In all trials the coefficients for stiffness and damping were significant \((p < 0.01)\). The adjusted coefficient of determination (adjusted \(r^2\)) and s.e. of each fit of the multiple linear regression are provided. The means and standard deviations of the estimated stiffness and damping coefficients are presented.

Table 5.24. Estimates for the stiffness and damping of the vertical spring in the three-dimensional simulation model

<table>
<thead>
<tr>
<th>trial</th>
<th>(k) (N.m(^{-1}))</th>
<th>(c) (N.s.m(^{-1}))</th>
<th>(r^2)</th>
<th>s.e. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>58567</td>
<td>1170</td>
<td>0.46</td>
<td>233</td>
</tr>
<tr>
<td>K05</td>
<td>78870</td>
<td>1905</td>
<td>0.61</td>
<td>200</td>
</tr>
<tr>
<td>A09</td>
<td>59794</td>
<td>711</td>
<td>0.88</td>
<td>125</td>
</tr>
<tr>
<td>K15</td>
<td>59592</td>
<td>820</td>
<td>0.67</td>
<td>218</td>
</tr>
<tr>
<td>A20</td>
<td>83937</td>
<td>1076</td>
<td>0.76</td>
<td>171</td>
</tr>
<tr>
<td>A21</td>
<td>67511</td>
<td>628</td>
<td>0.71</td>
<td>198</td>
</tr>
<tr>
<td>mean (n = 6)</td>
<td>68045</td>
<td>1052</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s.d. (n = 6)</td>
<td>10949</td>
<td>467</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s.e. (n = 6)</td>
<td>4470</td>
<td>191</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using the same procedure further regression analyses were performed to produce secondary estimates for the stiffness and damping coefficients of the vertical spring. Data for which the vertical component of cable tension was greater than 0.5 bodyweights were selected for these analyses. This produced a time window of approximately 0.25 s spanning only the peak cable tension (approximately 13 data points) for each trial. This time window incorporated the largest variations in cable tension and resultant vertical displacement of the DEDs. It was therefore expected that the effect of errors in the displacement data would be reduced and this selected data would provide more accurate estimates of the stiffness and damping for the vertical spring. The secondary estimates are shown in Table 5.25. For all of these regression analyses both the damping and stiffness coefficients were significant in all trials \((p < 0.01\) level).

The mean values in Tables 5.24 and 5.25 represent two estimates of the stiffness and damping for the vertical spring in the simulation model, 68045 N.m\(^{-1}\) (or 75428 N.m\(^{-1}\)) and 1052 N.s.m\(^{-1}\) (or 1514 N.s.m\(^{-1}\)) respectively. Coefficients of variation in these estimates are around 17% for the stiffness of the spring and 40% for the damping. It may be speculated from these analyses that the exact values for these two spring parameters
are not known. Using an optimisation process described in Chapter 6, these estimates are used to evaluate the three-dimensional simulation model through limited alterations to the spring parameters within the standard deviation of the means.

Table 5.25. Secondary estimates for stiffness and damping of the vertical spring in the three-dimensional simulation model

<table>
<thead>
<tr>
<th>trial</th>
<th>k (N.m⁻¹)</th>
<th>c (N.s.m⁻¹)</th>
<th>r²</th>
<th>s.e. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>62032</td>
<td>1435</td>
<td>0.93</td>
<td>156</td>
</tr>
<tr>
<td>K05</td>
<td>79782</td>
<td>2469</td>
<td>0.81</td>
<td>289</td>
</tr>
<tr>
<td>A09</td>
<td>62579</td>
<td>984</td>
<td>0.88</td>
<td>232</td>
</tr>
<tr>
<td>K15</td>
<td>67244</td>
<td>1237</td>
<td>0.83</td>
<td>214</td>
</tr>
<tr>
<td>A20</td>
<td>85291</td>
<td>1696</td>
<td>0.77</td>
<td>279</td>
</tr>
<tr>
<td>A21</td>
<td>95639</td>
<td>1261</td>
<td>0.96</td>
<td>130</td>
</tr>
<tr>
<td>mean (n = 6)</td>
<td>75428</td>
<td>1514</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s.d. (n = 6)</td>
<td>13684</td>
<td>524</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s.e. (n = 6)</td>
<td>5586</td>
<td>214</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Spring parameters representing the elasticity of the rings cables*

The cable spring in the three-dimensional simulation model represents the elasticity of the rings cables. The time history of rings cable length and rate of change in length is provided in Figure 5.55. Figures 5.56 and 5.57 detail the regression analyses for trial K03 to estimate the stiffness and damping values for the cable spring in the simulation model.

This trial showed the lowest standard error of fit in the final linear regression and therefore represents the best approximation to a damped linear spring (Table 5.26). The method and the theoretical basis for all of these analyses are presented in Section 4.7.4. Figure 5.56 is a graph of cable tension (ten) against the spline fitted cable length (scl) for trial K03. The straight line indicates the multiple linear regression equation predicting cable tension (ten) as a function of cable length (scl) and velocity (vcl). The regression equation estimating 'ten' in terms of 'scl' and 'vcl' is:

\[
\text{ten} = -86294 + 29613 \text{scl} + 547 \text{vcl}.
\]
From this first regression equation the natural length of the rings cables was calculated. For trial K03 the natural length was 2.9141 m. The calculated natural length of the rings cables for each trial is presented in Table 5.23. The rings cable length data were detrended from the natural length providing extension data for the rings cable (escl). The estimation of cable tension using the extension of the rings cable from the natural length and rate of extension produced the following equation:

\[ \text{ten} = 29624 \text{ escl} + 547 \text{ vel.} \]

Theoretically a damped linear spring with zero displacement or velocity has zero tension. Hence, this regression is forced to pass through the origin.
Figure 5.57. Cable tension against splined extension of the cable and estimated as a function of extension and velocity of the cable for K03.

The magnitudes of the coefficients for 'escl' and 'vel' are equal to the stiffness and damping of the spring respectively. Table 5.26 presents estimates from all six trials for the stiffness and damping values of the cable spring in the model.

Table 5.26. Estimates of stiffness and damping values for the cable spring in the three-dimensional simulation model

<table>
<thead>
<tr>
<th>trial</th>
<th>k (N.m(^{-1}))</th>
<th>c (N.s.m(^{-1}))</th>
<th>r(^2)</th>
<th>s.e. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>29624</td>
<td>547</td>
<td>0.66</td>
<td>186</td>
</tr>
<tr>
<td>K05</td>
<td>28278</td>
<td>470</td>
<td>0.41</td>
<td>247</td>
</tr>
<tr>
<td>A09</td>
<td>35802</td>
<td>631</td>
<td>0.55</td>
<td>244</td>
</tr>
<tr>
<td>K15</td>
<td>13576(^\dagger)</td>
<td>-362(^\dagger)</td>
<td>0.16</td>
<td>348</td>
</tr>
<tr>
<td>A20</td>
<td>35544</td>
<td>299</td>
<td>0.61</td>
<td>216</td>
</tr>
<tr>
<td>A21</td>
<td>25405</td>
<td>854</td>
<td>0.31</td>
<td>307</td>
</tr>
</tbody>
</table>

| mean (n = 5) | 30931 | 560 |
| s.d. (n = 5) | 4590  | 205 |
| s.e. (n = 5) | 2053  | 92  |

\(^\dagger\) not used in the calculation of mean, s.d. and s.e.

By definition, a damped linear spring must display some level of damping. Hence,
both variables were used in all analyses. For five trials the coefficients in the regression analyses representing the stiffness and damping of the springs were significant \((p < 0.05)\). For trial K15 the \(p\) value for the damping coefficient was equal to 0.108 and a negative value obtained for the damping coefficient. As a damped linear spring cannot demonstrate negative damping, this trial was disregarded from all subsequent calculations. Of the data sets used to determine the physical characteristics of the cable spring for trial K15 it is suggested that errors in the cable length and velocity estimates produced this spurious result.

For all trials the adjusted coefficient of determination (adjusted \(r^2\)) of the multiple linear regressions are given, together with the s.e. of each fit. The means and standard deviations of the estimated stiffness and damping coefficients are presented excluding trial K15 \((n = 5)\).

The mean values of 30931 N.m\(^{-1}\) for stiffness and 560 N.s.m\(^{-1}\) for damping serve as the best estimates for the physical characteristics representing the elasticity of the cables. However, the coefficients of variation (stiffness being 15% and damping equal to 37%) indicate that the actual values of the spring parameters are not well estimated. Using the same optimisation process alluded to for the estimates of the vertical spring, more accurate representations for the elasticity of the rings cables may be obtained.

Further information regarding the dimensions of the rings apparatus was also determined from these regression analyses. The natural lengths of the rings cables determined from the multiple regressions more closely represent their actual length than standardised values. Hence, this value for cable length may be considered more realistic than simply using dimensions presented by the F.I.G.. The mean natural length from all six trials was used as the cable length parameter \(L_{C2}\) in the simulation model. Table 5.27 presents all of the inertia parameters representing the rings cable in the simulation model, through modelling this segment as a right circular cylinder. These calculated parameter values for the rings cable segment were used in the three-dimensional model for all simulations.

<table>
<thead>
<tr>
<th>(m_c)</th>
<th>(L_{C2})</th>
<th>(L_{C1})</th>
<th>(I_{CT})</th>
<th>(I_{CF})</th>
<th>(I_{CL})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(kg)</td>
<td>(m)</td>
<td>(m)</td>
<td>(kg.m(^2))</td>
<td>(kg.m(^2))</td>
<td>(kg.m(^2))</td>
</tr>
<tr>
<td>1.0</td>
<td>2.9227</td>
<td>1.46135</td>
<td>0.712</td>
<td>0.712</td>
<td>0.001</td>
</tr>
</tbody>
</table>

*Spring parameters representing the elasticity of the gymnasts*

The shoulder spring in the three-dimensional simulation model represents the
elasticity of the gymnast throughout his body. The spring is placed at the shoulder since it may be considered to possess the greatest potential for extension. If the extension of the gymnast is assumed to be passive, that is no muscular actions are used to alter the total length of the gymnast's body, then extension of the gymnast may be expected to be linearly related to external forces acting on the body.

Figures 5.58 and 5.59 show the regression analyses for trials A09 used to estimate the stiffness and damping values for the shoulder spring in the simulation model. This trial demonstrated one of the lowest standard errors in the multiple linear regressions and therefore represents one of the best approximations of the gymnast as a passive and damped linear spring (Table 5.28). The procedure for these regression analyses are presented in Section 4.7.5.

Figure 5.58 shows the resultant forces in the direction of the arms (tenarm) plotted against the spline fitted wrist to ankle length (swalen).

\[
\text{tenarm} = 18557 - 9698 \text{ swalen} - 173 \text{ vwalen}.
\]

Figure 5.58. Resultant forces in line with the arms against splined wrist to ankle length and estimated as a function of splined wrist to ankle length and velocity for trial A09.

The equation estimating the resultant forces in the direction of the arms (tenarm) is a function of the spline fitted wrist to ankle length (swalen) and velocity (vwalen). For this trial the regression equation estimating 'tenarm' in terms of 'swalen' and 'vwalen', is:

\[
\text{tenarm} = 18557 - 9698 \text{ swalen} - 173 \text{ vwalen}.
\]

From this relationship the natural length of the shoulder spring was calculated. For trial A09 the natural wrist to ankle length was 1.9135 m. Natural wrist to ankle lengths for all trials are shown in Table 5.23.
The wrist to ankle length data were detrended from the natural length providing extension data for the shoulder spring (eswalen). Estimates of forces in line with the arm as a function of the extension of the shoulder spring from the natural length and rate of extension produced the following equation:

\[
tenarm = -9699\text{ eswalen} - 173\text{ vwalen}.
\]

Theoretically a damped linear spring with zero displacement and velocity has zero tension. Hence this regression was forced through the origin. The magnitudes of the coefficients for 'eswalen' and 'vwalen' are equal to the stiffness and damping of the shoulder spring respectively. Table 5.28 provides estimates for the stiffness and damping of the shoulder spring for all six trials.

![Graph](image)

**Figure 5.59.** Resultant forces in line with the arms against splined extension of the wrist to ankle length and estimated as a function of splined extension of the wrist to ankle length and velocity for trial A09.

As a damped linear spring must possess some damping both variables were forced into all regression analyses. In all trials the coefficients for stiffness were significant (p < 0.01 level). The damping coefficient, however, was significant for only 4 trials (p < 0.05). The p values for 2 trials indicated a lack of fit for the velocity term and calculated a negative damping value (Table 5.28). As a damped spring cannot demonstrate negative damping these trials were disregarded from all subsequent calculations. For all trials the adjusted coefficient of determination (adjusted r²) of the multiple linear regressions are given, together with the s.e. of each fit.
Table 5.28. Estimates of stiffness and damping values for the shoulder spring in the three-dimensional simulation model

<table>
<thead>
<tr>
<th>trial</th>
<th>k (N.m(^{-1}))</th>
<th>c (N.s.m(^{-1}))</th>
<th>(r^2)</th>
<th>s.e. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>11124</td>
<td>-22(^\dagger)</td>
<td>0.78</td>
<td>191</td>
</tr>
<tr>
<td>K05</td>
<td>7841</td>
<td>-18(^\dagger)</td>
<td>0.49</td>
<td>294</td>
</tr>
<tr>
<td>A09</td>
<td>9699</td>
<td>173</td>
<td>0.80</td>
<td>204</td>
</tr>
<tr>
<td>K15</td>
<td>-7078</td>
<td>-251</td>
<td>0.46</td>
<td>336</td>
</tr>
<tr>
<td>A20</td>
<td>6428</td>
<td>198</td>
<td>0.40</td>
<td>340</td>
</tr>
<tr>
<td>A21</td>
<td>10059</td>
<td>222</td>
<td>0.58</td>
<td>298</td>
</tr>
<tr>
<td>mean (n = 6)</td>
<td>8705</td>
<td>211(^**)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s.d. (n = 6)</td>
<td>1857</td>
<td>33(^**)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^\dagger\) not used in the calculation of mean and s.d.

\(^**\) (n = 4)

From Figure 5.59 it may be inferred that the theoretical shoulder spring does not behave linearly. This non-linearity may be due to muscular actions performed by the gymnast (such as shoulder girdle elevation). Such muscular actions produce an increase in the wrist to ankle length without the appropriate increase in the external forces acting in the line of the arm. Such muscular actions may be identified in Figure 5.60 which presents time histories for the wrist to ankle length for trials A09 and K15, forward and backward longswings respectively.

Figure 5.60. Time histories of the wrist to ankle length for trials A09 and K15.

During the descending phases of both longswings (0.5 s to 1.2 s approximately) extensive increases in the wrist to ankle length were observed. At these instants in time
external forces in line with the arms are small (cable tension being small and arms near 90° to the cable tension vector). These changes must therefore be due to active muscular actions. Such muscular activity at the lower extremes of forces will have a large influence on calculations estimating the elasticity of the shoulder spring. During the ascending phase of the swings, however, changes in the wrist to ankle lengths may be thought to more closely resemble a passive structure.

During extreme forces in line with each arm, for instance those greater than 0.5 bodyweights, active muscular actions may be considered to have less of an effect on changes in the wrist to ankle length. For this reason two more regression analyses were performed on all trials to ascertain further estimates representing the elasticity of the gymnast.

Firstly, external forces resolved in the direction of the arms spanning only the peak force values (approximately 0.25 s) were analysed, together with the corresponding splined wrist to ankle length and velocity data. The same procedure of multiple linear regression analysis was again used to estimate the elastic characteristics of the shoulder spring. While these large external forces are acting, the increase in wrist to ankle length may be expected to exhibit damped linear spring characteristics. This procedure formed secondary estimates of the physical properties of the shoulder spring.

The same procedure was performed on second sets of data for all trials, which comprised full time histories of data excluding data which exhibited external forces in the direction of the arms greater than 0.5 bodyweights. This formed tertiary estimates for the stiffness and damping for the shoulder spring. With external forces in the direction of the arms less than 0.5 bodyweights the gymnasts may more easily produce muscular actions. Thus, relatively large changes in the wrist to ankle data may be expected, without the corresponding increase in external forces. During these phases of the swing the gymnasts might be expected to exhibit the least spring-like behaviour of the three conditions. Tables 5.29 and 5.30 provide the secondary and tertiary estimates of the stiffness and damping of the shoulder springs.

The secondary and tertiary estimates of stiffness and damping for the shoulder spring are very different to the first estimates where all data were used (Table 5.28). This suggests the gymnast did not behave like a damped linear spring during the longswings. Such a result is not completely unexpected. From the wrist to ankle length time histories it becomes obvious that muscular actions are performed during the descending phase of the swing. It may be speculated that the extension in the wrist to ankle lengths during the descending phase is part of the gymnasts' techniques. Effectively this alters the motion of the gymnast and his interaction with the rings cables. Such muscular actions also invalidate the assumption that the elasticity of the gymnast may be represented by a damped linear spring.
Table 5.29. Secondary estimates of stiffness and damping values for the shoulder spring

<table>
<thead>
<tr>
<th>trial</th>
<th>( k ) (N.m(^{-1} ))</th>
<th>( c ) (N.s.m(^{-1} ))</th>
<th>( r^2 )</th>
<th>s.e. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>40040</td>
<td>-302( ^{\dagger} )</td>
<td>0.90</td>
<td>177</td>
</tr>
<tr>
<td>K05</td>
<td>27042</td>
<td>-39( ^{\dagger} )</td>
<td>0.70</td>
<td>364</td>
</tr>
<tr>
<td>A09</td>
<td>31189</td>
<td>774</td>
<td>0.86</td>
<td>246</td>
</tr>
<tr>
<td>K15</td>
<td>5280( ^{\dagger} )</td>
<td>530</td>
<td>0.61</td>
<td>286</td>
</tr>
<tr>
<td>A20</td>
<td>9116( ^{\dagger} )</td>
<td>1052</td>
<td>0.67</td>
<td>314</td>
</tr>
<tr>
<td>A21</td>
<td>20835</td>
<td>732</td>
<td>0.89</td>
<td>199</td>
</tr>
</tbody>
</table>

*mean (n = 4)*: 29777, 772
*s.d. (n = 4)*: 8057, 215

\( ^{\dagger} \) not used in the calculation of mean and s.d.

Table 5.30. Tertiary estimates of stiffness and damping values for the shoulder spring

<table>
<thead>
<tr>
<th>trial</th>
<th>( k ) (N.m(^{-1} ))</th>
<th>( c ) (N.s.m(^{-1} ))</th>
<th>( r^2 )</th>
<th>s.e. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>4976</td>
<td>55</td>
<td>0.29</td>
<td>123</td>
</tr>
<tr>
<td>K05</td>
<td>-1601( ^{\dagger} )</td>
<td>-56( ^{\dagger} )</td>
<td>0.06</td>
<td>130</td>
</tr>
<tr>
<td>A09</td>
<td>4480</td>
<td>11</td>
<td>0.51</td>
<td>97</td>
</tr>
<tr>
<td>K15</td>
<td>1568</td>
<td>-142( ^{\dagger} )</td>
<td>0.24</td>
<td>125</td>
</tr>
<tr>
<td>A20</td>
<td>616</td>
<td>-193( ^{\dagger} )</td>
<td>0.15</td>
<td>138</td>
</tr>
<tr>
<td>A21</td>
<td>2357</td>
<td>-125( ^{\dagger} )</td>
<td>0.19</td>
<td>136</td>
</tr>
</tbody>
</table>

*mean (n = 5)*: 2799, 33\( ^{**} \)
*s.d. (n = 5)*: 1874, 31\( ^{**} \)

\( ^{\dagger} \) not used in the calculation of mean and s.d.

\( ^{**} \) (n = 2)

The results obtained from these two new conditions, secondary and tertiary, suggest that the elasticity within the gymnast may be non-linear. This result, highlighted in Figure 5.61, indicates that the stiffness of the gymnast increases as the amount of stretch increases. The majority of stretch may be assumed to be at the shoulder joint. This finding is consistent with those of Engin et al. (1984b) for the shoulder joint. Engin et al. (1984b) also showed the dependency of the shoulder stiffness on the joint...
orientation. As the shoulder joint is such a complex joint (Peindl & Engin, 1987) a full and accurate representation may be difficult to obtain. As a first approximation, however, a damped linear spring, with the appropriate stiffness and damping may allow some reflection of the elasticity of this joint and the gymnast in the simulation of swinging activities on rings.

![Graph](image)

Figure 5.61. Resultant forces in line with the arms against splined wrist to ankle length and estimated as a function of splined wrist to ankle length and velocity for trial A09 using secondary and tertiary estimates.

Although the shoulder spring does not strongly exhibit damped linear spring characteristics, estimates for the stiffness and damping are implemented in the four segment simulation model. Accurate values representing the elasticity of the gymnasts at their shoulder joints are not known. Therefore an optimisation process, described in Chapter 6, is employed to determine the stiffness and damping for the shoulder spring which best represents the actual stretch of the gymnast. Within the optimisation process alterations to both stiffness and damping values were limited to the full range of estimates obtained for both spring parameters.

5.6.3 Implementation of quintic spline fitted data and derivatives

The time histories of each orientation and configuration angle were fitted with quintic splines using the methods outlined in Chapter 4. Figure 5.62 shows that the first two derivatives of a quintic spline are smooth and continuous (represented by a continuous line). Such properties may be thought of as representative of human motion (Yeadon, 1984).
The process of curve fitting the angle data reduces the random noise due to digitisation and allows more accurate estimates of the angular velocity and acceleration for each angle. By retaining the six coefficients for each quintic spline the angle, angular velocity and acceleration may be evaluated at any point in time.

Table 5.31 presents the largest angular velocities at each joint observed for each trial. With the exception of shoulder elevation angular velocities during forward longswings, the estimates of joint angular velocities are comparable with those observed in other gymnastic events, such as accelerated longswings on horizontal bar (Hiley, 1998) and vaulting and tumbling activities (King, 1998). This comparison provides some confidence that the appropriate fit to the raw data has been attained.

On inspection the shoulder elevation angular velocities for forward longswings K03 and A09 seem excessive. Owing to the method of defining this joint angle rapid changes can occur when the shoulder abduction angle is near 90°. When the hands of the gymnast are located to the front of the gymnast, and then behind the gymnast, or vice-versa, the shoulder elevation angle rapidly changes from an angle greater than 180° to one less than 180°. During forward longswings the shoulder elevation angle prior to the above scenario is often near 270° and rapidly changes to near 90°. Thus, the calculated shoulder
elevation angular velocity is excessive, when little alteration to the hand location relative to the gymnast occurs. It can be shown these excessive shoulder elevation angular velocities coincide with instants when the shoulder abduction angle is near 90°.

Table 5.31. Maximum and minimum joint angular velocities during longswings on rings

<table>
<thead>
<tr>
<th>Angular velocity</th>
<th>Trial</th>
<th>K03</th>
<th>K05</th>
<th>A09</th>
<th>K15</th>
<th>A20</th>
<th>A21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee (°.s⁻¹)</td>
<td>Min</td>
<td>-298</td>
<td>-345</td>
<td>-400</td>
<td>-4.4</td>
<td>-453</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>231</td>
<td>232</td>
<td>342</td>
<td>4.8</td>
<td>364</td>
<td>3</td>
</tr>
<tr>
<td>Hip (°.s⁻¹)</td>
<td>Min</td>
<td>175</td>
<td>220</td>
<td>169</td>
<td>-768</td>
<td>-817</td>
<td>-844</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>628</td>
<td>653</td>
<td>684</td>
<td>382</td>
<td>482</td>
<td>449</td>
</tr>
<tr>
<td>Shoulder elevation (°.s⁻¹)</td>
<td>Min</td>
<td>-1472</td>
<td>-841</td>
<td>-2474</td>
<td>-473</td>
<td>-517</td>
<td>-564</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>493</td>
<td>474</td>
<td>566</td>
<td>289</td>
<td>348</td>
<td>304</td>
</tr>
<tr>
<td>Shoulder abduction (°.s⁻¹)</td>
<td>Min</td>
<td>-254</td>
<td>-216</td>
<td>-214</td>
<td>-151</td>
<td>-185</td>
<td>-163</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>326</td>
<td>316</td>
<td>293</td>
<td>158</td>
<td>186</td>
<td>182</td>
</tr>
</tbody>
</table>

As the quintic spline fitted configuration angles define the gymnast's technique, they may be used to determine the validity of the angle driven four segment simulation model. By providing the correct initial conditions for the modelled gymnast and rings cable and by driving the modelled gymnast using the techniques used by the actual gymnasts, the model may be evaluated. Subsequently, if the model is sufficiently accurate, the contributions of various components of the gymnasts' techniques and the physical properties of the rings apparatus can be investigated thoroughly.

In the following chapter the four segment three-dimensional simulation model is evaluated, an essential part of the theory-experimental cycle of scientific method (Figure 2.7). Investigations pertinent to the questions posed in the Introduction are then addressed in Chapter 7.
CHAPTER 6

EVALUATION OF THE FOUR SEGMENT THREE-DIMENSIONAL SIMULATION MODEL

6.1 Introduction

The evaluation of a computer simulation model may be considered an illustration of the theory-experimental cycle of scientific method (Figure 2.7). In performance-related sports biomechanics the evaluation of a simulation model typically compares kinetic and kinematic data from an actual athletic performance with that predicted by the model for a simulation of the performance (Hiley, 1998; King, 1998; Van Gheluwe, 1981; Yeadon et al., 1990). This procedure provides criteria from which the accuracy of the simulation model can be objectively determined. Failure in adequately evaluating simulation models has been noted as a major limitation of many previous theoretical studies of sporting performance (Bauer, 1983; Vaughan, 1984; Yeadon & Challis, 1994).

If the accuracy of the simulation model is either unknown or poor then confidence in results predicted in hypothetical situations is reduced. This concept is pertinent to the external validity of a study, addressed in Section 2.4.1. Furthermore, incorrect conclusions may be drawn from simulation models predicting the motion of humans without appropriate accuracy and without constraints on the ranges of motion and strength of an athlete. This is especially significant when investigating optimal techniques for sporting performances (Hiley, 1998).

Longswings on rings constitute both swinging and hand balancing phases. Both of these aspects are essential for a successful performance. However, this study is concerned with the mechanical aspects of swinging. Since only swinging is of interest the simulation model was evaluated against only those phases which constitute swing for both the backward and forward longswings. Such phases are initiated when a gymnast adopts an orientation, or angular velocity, from which the handstand may not be maintained.

The following sections describe the procedure used to evaluate the four segment three-dimensional simulation model (Section 3.5). The evaluation procedure provides information indicating the accuracy of the four segment model in predicting the motion of the gymnast and rings cables during a longswing. A sensitivity analysis was also conducted to investigate further the sensitivity in the accuracy of the model to the different parameters. Finally, additional output calculated within the four segment model is presented. This output provides further insight into changes which occur in mechanical variables throughout longswings on rings.
6.2 Evaluation procedure for the four segment three-dimensional simulation model

In order to determine the accuracy of the four segment model for simulating the motion of the gymnast and rings cables, changes to the configuration of the modelled gymnast (or technique) must be identical to that performed during an actual performance. Since the four segment simulation model is joint angle driven, this may be achieved by using the spline fitted angle data derived from the kinematic analyses of actual performances (Section 5.5.3). Comparisons between actual and simulated performances may then be made in terms of variables produced as a result of the gymnast's configurational modifications. Such variables include the orientations of the gymnast and rings cables, cable tension and internal forces experienced by the gymnast.

The model requires estimates for several parameters to reflect accurately the elastic and physical properties of the gymnast and rings apparatus. Errors associated with the elastic parameters are provided in Tables 5.22 to 5.30. These errors arise from the assumption that the elastic properties of these structures are linear and from errors associated with digitising, three-dimensional reconstruction and cable tension data. As no criteria values are known, it is feasible that the mean values for these spring parameters, presented in Tables 5.22 through to 5.30, are themselves not representative of the elastic components for the rings apparatus and gymnasts. The actual stiffness and damping of the modelled springs therefore remain unknown. Since the damped linear springs in the simulation model represent the elasticity of the rings apparatus and gymnast, they should accurately reflect these structures when the model is evaluated.

In an attempt to determine more representative estimates for the stiffness and damping parameters of each spring in the model an optimisation process was implemented. Firstly, an objective function was created to evaluate the accuracy of the simulation model. Secondly, the objective function was used in the optimisation process to obtain more representative values for each spring parameter. These new estimates for each spring parameter may be considered to represent more closely the elasticity of the rings apparatus and gymnast. Finally, using the new estimates for each spring parameter the model was evaluated. The evaluation procedure was conducted using all six analysed longswings.

6.2.1 The objective function for comparing actual and simulated performances

For an objective comparison between two data sets, quantitative descriptions of the differences must be sought. Differences between kinetic and kinematic data from actual and simulated performances, which form the evaluation of the model, may be described by a single objective function. Depending on the application, this function may comprise
one or several different components.

In the case of the four segment model components forming the objective function should include those which indicate a backward or forward longswing has been completed. By including these indicators a comparison of important features which define a longswing performance is made. A further consideration is that the chosen indicators must be calculated within the four segment model.

Variables which indicate a longswing has been completed include the orientations of the gymnast and rings cables, together with tension in the rings cables and forces experienced by the gymnast. Such indicators are calculated in the four segment model and are known for actual performances (Section 5.5). Owing to the interaction between the gymnast and rings cables, the cables undergo certain angular motions as the gymnast completes the longswing elements. Concurrently, tension in the rings cables, and therefore forces experienced by the gymnast, also change. Since all of these indicators change simultaneously they may all be used throughout a longswing to indicate the completion of either a backward or forward longswing.

A method which suitably combines comparisons of dimensionally different quantities was therefore formulated. In order to provide equal emphasis for each of the components, which possess different ranges as well as units of measurement, weighting factors were implemented. These weighting factors reflected the range and units of measurement of each indicator in the objective function. The rationale for including each indicator of performance into the objective function and associated weightings is discussed next. In all cases the components of the objective function and their weightings were derived from the kinetic and kinematic data obtained from actual performances in Chapter 5.

Body angle $\varepsilon$

The kinematic analysis of longswings (Section 5.5.3) described the orientation of the gymnast by the body angle $\varepsilon$. During either a backward or forward longswing the total angular excursion of the body angle is close to $360^\circ$. This requirement may be used to indicate the completion of either longswing element. The body angle $\varepsilon$ of the gymnast therefore formed one component of the objective function for the evaluation procedure and optimisation process.

Cable elevation angle $\phi_c$

The interaction between the rings cables and gymnast, identified using the two-dimensional simulation models, ensures the rings cables also undergo angular motion during a longswing. Of the two cable orientation angles, changes in the cable elevation
angle $\phi_c$ are required to complete a longswing. Angular motion in the cable abduction angle $\theta_c$, is not directly related to the completion of a longswing. This was demonstrated by the planar two segment simulation model (Section 5.4.3).

Results from the kinematic data indicate the total angular excursion of the elevation angle for the rings cables during a longswing is approximately 60°. Hence, the angular excursion of the rings cables throughout a longswing may be used to form another component of the objective function.

A weighting factor was employed to ensure that the body angle and cable elevation angle had equal emphasis in the objective function. During a longswing the ratio for the total excursions of these two components is 6:1. The inverse of this ratio provided a suitable weighting to ensure these components had an equal emphasis in the final objective function.

**Cable tension**

Cable tension data from actual performances indicate that for each subject the cable tension time history remains similar over successive performances. Hence, for a longswing to be completed a certain cable tension time history must be produced. A cable tension time history may therefore be used to identify that a longswing has been completed and consequently was incorporated into the objective function.

Over the six longswing trials the average range of tension in a single rings cable was close to 2200 N (Table 5.11). During a longswing this full range is produced as the gymnast swings underneath the rings (Figures 5.38 and 5.46), which for backward longswings relates to body angles from 150° to 210°. Relating this range in cable tension to the 60° angular displacement of the body angle over which it occurs allows a weighting factor to be established and ensure equal emphasis in the objective function. Using this analogy the ratio of cable tension to body angle is 220:6. The inverse of this ratio provided these two components with equal emphasis in the objective function.

**Wrist to ankle length**

Owing to the large forces experienced by gymnasts as they swing beneath the rings the wrist to ankle length increases by approximately 0.15 m (Figure 5.60). In the initial and final handstands the wrist to ankle length is relatively constant, and approximately 0.15 m less than the largest length observed. The increase in wrist to ankle length, represented in the model by the damped spring at the shoulder joint, may be considered to represent the forces experienced by the gymnast during a longswing. Such an indicator of these forces may be used to identify longswing performances and therefore was incorporated into the objective function. Furthermore, because values for the shoulder
spring parameters are unknown, the inclusion of an indicator representing this phenomenon may help to obtain appropriate estimates for this elastic component.

The fluctuation in external forces experienced by the gymnast may be considered to occur over the same period as cable tension and therefore body angular displacement. The total excursion in wrist to ankle length during this body angle change is nearly 0.3 m. This gives a ratio of body angle excursion to wrist to ankle length excursion of 60:0.03. Implementing the inverse of this ratio as a weighting factor ensured these two components have equal emphasis with all other components in the objective function.

To calculate the simulated wrist to ankle lengths, hand to wrist and ankle to toe lengths were required for the modelled gymnasts. Subject specific estimates for these two lengths were calculated from video derived data in the analysis program VIDEO using the method for calculating segmental lengths. The average values of these lengths for each subject throughout the longswings is provided in Table 6.1.

Table 6.1. Estimates of subject specific hand to wrist and ankle to toe lengths

<table>
<thead>
<tr>
<th>trial</th>
<th>hand to wrist (m)</th>
<th>ankle to toe (m)</th>
<th>trial</th>
<th>hand to wrist (m)</th>
<th>ankle to toe (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K03</td>
<td>0.051</td>
<td>0.136</td>
<td>A09</td>
<td>0.044</td>
<td>0.131</td>
</tr>
<tr>
<td>K05</td>
<td>0.042</td>
<td>0.136</td>
<td>A20</td>
<td>0.045</td>
<td>0.137</td>
</tr>
<tr>
<td>K15</td>
<td>0.053</td>
<td>0.126</td>
<td>A21</td>
<td>0.046</td>
<td>0.133</td>
</tr>
<tr>
<td>mean (n = 3)</td>
<td>0.049</td>
<td>0.133</td>
<td>mean (n = 3)</td>
<td>0.045</td>
<td>0.134</td>
</tr>
</tbody>
</table>

Also shown are the mean subject specific values for both lengths over all trials. These values were used in the evaluation procedure. The hand to wrist and ankle to toe lengths were subtracted from the simulated hand to toe lengths to give simulated wrist to ankle length data for comparison with the corresponding value from video.

The objective function

Comparisons between actual and simulated data were made at 0.02 s time intervals throughout each simulation. This is equivalent to each video field of the actual synchronised kinetic and kinematic data. In order to provide each component with equal emphasis in the objective function, weightings for each component were determined by combining the ratios relating components to each other. The objective function \( f \) which describes the difference between actual and simulated performances is defined by the following equation:
\[ f = \sqrt{\frac{\sum_{i=1}^{n_0} (\varepsilon_i^v - \varepsilon_i^s)^2}{n_f}} + \sqrt{\frac{\sum_{i=1}^{n_0} (\phi_c_i^v - \phi_c_i^s)^2}{n_f}} + \sqrt{\frac{\sum_{i=1}^{n_0} (\text{cten}_i^v - \text{cten}_i^s)^2}{n_f}} + \sqrt{\frac{\sum_{i=1}^{n_0} (wa_i^v - wa_i^s)^2}{n_f}} \]

where:
\( \varepsilon \) denotes the body angle in degrees
\( \phi_c \) denotes the cable elevation angle in degrees
'cten' denotes tension in a single rings cable in Newtons
'wa' denotes the wrist to ankle length in metres
superscript v denotes video derived data
superscript s denotes simulation derived data
subscript i denotes the video field
\( n_f \) is the total number of video fields simulated

Since only the swinging phase of each longswing was simulated, each simulation commenced at a time greater than 0.0 s in the video time base of each trial (Section 5.5). The swinging phase of each longswing was determined by a combination of torso angle, cable elevation angle and the respective angular velocities. All of these factors affect the gymnast's body angle. The initial time for the simulation was therefore dependent on the gymnast being at an appropriate body angle. Table 6.2 provides the actual initial and final body angles over which the simulation model was evaluated for each trial. These ranges envelop the vast majority of the swinging phases for all longswings. Both prior and subsequent to these selected body angles the gymnast is concerned with the maintenance and balance of the handstand position, an area of research beyond the scope of this study.

In order to perform the optimisation process automatically the simulated annealing algorithm, described in Section 2.4.7, was implemented. Parameters varied by the algorithm during the optimisation process were the stiffness and damping parameters for each spring in the model. The fact that actual longswings were performed facing in different directions during the data collection session (Table 4.2) was taken into consideration, ensuring that all actual data were comparable with the simulation model.
Table 6.2. Actual initial and final body angles and times over which the evaluation was performed for each longswing

<table>
<thead>
<tr>
<th>longswing</th>
<th>initial</th>
<th>final</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>body angle (°)</td>
<td>time (s)</td>
</tr>
<tr>
<td>backward</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K15</td>
<td>2.0</td>
<td>0.10</td>
</tr>
<tr>
<td>A20</td>
<td>1.8</td>
<td>0.70</td>
</tr>
<tr>
<td>A21</td>
<td>5.1</td>
<td>0.42</td>
</tr>
<tr>
<td>forward</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K03</td>
<td>-6.5</td>
<td>1.26</td>
</tr>
<tr>
<td>K05</td>
<td>-9.5</td>
<td>1.14</td>
</tr>
<tr>
<td>A09</td>
<td>-7.4</td>
<td>0.54</td>
</tr>
</tbody>
</table>

6.2.2 The optimisation process

The objective function defines the difference between the actual and simulated performances. Supposing that appropriate values for all spring parameters were utilised in the four segment model, it might be expected that the overall difference between actual and simulated longswing performances would be relatively small. Hence, a global minimum was sought in order to obtain the greatest agreement between simulated and actual performances. Implementing a root mean square difference (RMS), rather than selecting just one or several instants in time, was thought to be more representative of the overall difference between two data sets. The RMS difference between actual and estimated data from the model throughout a longswing was therefore adopted for each component of the objective function.

Minimisation of the objective function was achieved through the selection of the appropriate spring parameters, since these parameters must closely reproduce the elastic properties of the rings apparatus and gymnast in order for the model to simulate reality. As the model comprised four damped linear springs up to 8 spring parameters could be varied.

The simulated annealing algorithm allows values for each parameter to be varied only within specified upper and lower limits. To ensure that values for the spring parameters determined using the optimisation process were realistic, appropriate values for the upper and lower limits were required. In general, estimates for the parameters obtained from multiple regression analyses performed on actual data (Section 5.6.2) were used to determine these limits.
Upper and lower limits for the spring parameters

Prior to the optimisation process and evaluation procedure a battery of test simulations was performed. These initial simulations highlighted that mean estimates for the stiffness (20134 N.m⁻¹) and damping (91 N.s.m⁻¹) of the horizontal spring (Section 5.6.2) produced similar movements to those observed during actual longswing performances. As a result, these values were used throughout the evaluation procedure to represent the anterior-posterior movement of the rings frame. By reducing the number of parameters to be varied during the optimisation process to 6 the computational efficiency of the task increased. Such an increase in efficiency allowed a more complete search of the other 6 spring parameters within their boundaries and increased the likelihood of obtaining a global minimum.

For the 6 spring parameters which were varied, namely the stiffness and damping values for the vertical, cable and shoulder springs, values for the upper and lower limits were required. Limits representing stiffness properties of the springs incorporated the full ranges of estimates obtained in Section 5.6.2 (Tables 5.22 to 5.30). This ensured the values obtained through the optimisation process were similar to those determined experimentally. Utilising the full range of estimated values allowed the largest flexibility in the stiffness parameters during the optimisation process.

Table 6.3. Upper and lower limits of the spring parameters for the optimisation process

<table>
<thead>
<tr>
<th></th>
<th>stiffness *</th>
<th>damping †</th>
<th>stiffness *</th>
<th>damping †</th>
<th>stiffness *</th>
<th>damping †</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertical</td>
<td>90000</td>
<td>5000</td>
<td>36000</td>
<td>5000</td>
<td>40000</td>
<td>10000</td>
</tr>
<tr>
<td>cable</td>
<td>56000</td>
<td>300</td>
<td>25000</td>
<td>300</td>
<td>1000</td>
<td>300</td>
</tr>
<tr>
<td>shoulder</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* units of stiffness, N.m⁻¹
† units of damping, N.s.m⁻¹

Estimates from the regression analyses for the damping properties of each spring typically possessed errors three times larger than those associated with stiffness estimates. Hence, for all springs the upper limits for damping were extended beyond the ranges observed in the regression analyses. In contrast, the lower limits for spring damping were set to 300 N.s.m⁻¹. This was done in an attempt to reduce the possibility of rapid vibrations at the start of a simulation which may have influenced the motion of the gymnast and rings cables significantly. Table 6.3 presents the upper and lower limits for the damping characteristics of each spring. Less stringent upper limits were imposed for
the damping component of each spring. However, by constraining all spring parameters to vary within seemingly realistic values the new estimates for the elastic properties of all springs should also be realistic.

Implementation of the simulated annealing algorithm

Familiarization with the simulated annealing algorithm was gained through the work with the two segment model (Section 5.4). This familiarization enabled the appropriate selection of values for the parameters which control the automatic optimisation within the algorithm for this application.

The starting temperature \( T \) and temperature reducing factor \( RT \) of the annealing algorithm both affect the search for the optimum solution of the objective function. An appropriately high starting temperature was selected to ensure that the full six-dimensional terrain of the objective function was examined. Additionally, the parameter defining the rate of temperature reduction of the algorithm was set to produce a slow cooling effect. A slow cooling of the annealing reduces the chance of a quenching effect and therefore aids in the search of the global minimum for the objective function.

For the initial optimisation of each longswing the maximum number of simulations to search for the global minimum of the objective function was 20000. Initial values for the six spring parameters were, in most cases, set to the middle of their respective upper and lower limits. If, after 20000 simulations uncertainty existed as to whether the final solution was the global minimum or simply a local one, a further optimisation was performed until a satisfactory optimum (which was considered to be global) was found. Between 4000 and 20000 simulations were performed for these secondary optimisations.

The six spring parameters which defined the global minimum of the objective function were provided on termination of the algorithm. Values for these spring parameters, the objective function and the unweighted RMS differences for all components of the objective function were also provided. These spring parameter values may be considered the most appropriate representations of the elasticity of the rings apparatus and gymnast according to the objective function. Furthermore, they also provide the model with its greatest accuracy for simulating each longswing.

The four segment simulation model and the optimisation process

The four segment model required the initial angles and angular velocities of the rings cables (elevation \( \phi_c \) only) and torso of the gymnast. These initial values were set equal to the conditions at the appropriate time of the performance being simulated. The model also required the initial extensions of the springs. These values were calculated, as described in Section 3.5.2, in order for the model to start the simulation with the same
conditions as the actual performance. For simplicity, all spring velocities were set to zero on the assumption that near handstand no significant movements of the elastic components occurred.

The inertia parameters specific to each subject were required for the appropriate trials (Table 5.21). Changes to the gymnasts' body configurations (or technique) throughout each longswing were also required. These were provided by the spline fitted angle data which ensured that the modelled gymnast performed the same changes in body configuration as the actual gymnast for each particular trial. As the configuration angles and first two derivatives described by the quintic splines are continuous functions, the simulation model was able to integrate numerically over appropriately small time intervals. This allowed the Kutta-Merson numerical integration algorithm, which allows variable time intervals for the numerical integration procedure, to be fully utilised. A suitable error estimate for the numerical integration algorithm forms the basis of the variable time intervals over which numerical integration occurs. The error estimate for numerical integration was set to $1.0 \times 10^{-8}$ on advice from the AUTOLEV™ manual.

### 6.3 Results of the evaluation procedure for the four segment model

The following tables and figures provide results indicating the accuracy of the four segment simulation model for simulating the motion of the gymnast and rings cables during longswings on rings. Table 6.4 presents the RMS differences between components of the objective function for actual and simulated longswings. The objective function score for each evaluation is also given.

When expressed as a percentage of their total ranges the model estimated the body angle, cable elevation angle and rings cable tension to within 1.2%, 3.5% and 7.3% of the actual values respectively. The simulation model, however, was less accurate in predicting changes in the wrist to ankle length of the gymnast. When expressed as a percentage of the total excursion of the wrist to ankle length (0.30 m) the accuracy of the model for predicting this length was 33%.

The relatively poor accuracy in predicting changes to this length is not surprising. Firstly, the regression analyses performed in Section 5.6.2 suggest the elasticity of the each gymnast was non-linear. Moreover, data presented in Section 5.6.2 indicated that each gymnast actively increased his wrist to ankle length through muscular actions. These factors reduce the accuracy of the model for predicting changes in the wrist to ankle length of the gymnast during a longswing on rings.
Table 6.4. The RMS differences for components of the objective function in each evaluated longswing

<table>
<thead>
<tr>
<th>indicators</th>
<th>backward longswings</th>
<th>forward longswings</th>
</tr>
</thead>
<tbody>
<tr>
<td>body angle (°)</td>
<td>K15 A20 A21</td>
<td>K03 K05 A09</td>
</tr>
<tr>
<td>cable elevation angle (°)</td>
<td>5.32 5.12 2.92</td>
<td>3.73 4.41 3.94</td>
</tr>
<tr>
<td>cable tension (N)</td>
<td>1.12 1.32 0.76</td>
<td>0.66 2.35 4.09</td>
</tr>
<tr>
<td>wrist-ankle length (m)</td>
<td>142 115 133</td>
<td>168 193 194</td>
</tr>
<tr>
<td>function value</td>
<td>0.09 0.09 0.06</td>
<td>0.10 0.18 0.03</td>
</tr>
</tbody>
</table>

The precision estimates for the actual orientations of the gymnast and rings cables are shown in Table 5.15. Therefore, the accuracy of the model is lower than the precision in estimating these indicators during actual performances. This suggests the overall performances were different. However, since the simulation model is a simplification of reality this difference was expected.

Figures 6.1 to 6.4 highlight the accuracy of the model in predicting the gymnast and rings cable motion during both backward and forward longswings. The time histories of each component forming the objective function $f$ are presented. In terms of the model's accuracy these two trials may be considered typical since both give the median objective function scores for their respective types of longswing (Table 6.4).

Table 6.5 provides values for the spring parameters which produced the minimum objective function value for each longswing trial (Table 6.4). The combination of these values may be thought to represent the elasticity of both the gymnast and rings apparatus more closely than estimates derived from video and cable tension data. Since generally the upper and lower limits for each parameter were set within those obtained from regression analyses, their values may be termed realistic.

Trends in the values of the spring parameters can be identified. The optimum stiffness for the vertical and cable springs is consistent for each type of longswing but not between types of longswing. To illustrate, the mean stiffness for the vertical spring when evaluating the model using backward longswings was 57414 N.m$^{-1}$, whilst using forward longswings the mean stiffness was 89302 N.m$^{-1}$. The same trend is present in the stiffness of the cable spring. Since these springs represent inanimate structures, it would be expected that their elastic properties would remain constant over a short period of time. However, this was not observed. From Table 6.5 it also appears that no trend is present in the damping properties of these springs, which for inanimate objects would be expected.
Figure 6.1. Time histories of actual and predicted values for each component of the objective function for backward longswing A20: four segment model.

Figure 6.2. Graphics sequence indicating the typical accuracy of the simulation model when simulating backward longswings (A20): four segment model.
Figure 6.3. Time histories of actual and predicted values for each component of the objective function for forward longswing A09: four segment model.

Figure 6.4. Graphics sequence indicating the typical accuracy of the simulation model when simulating forward longswings (A09): four segment model.
Table 6.5. The physical properties of the springs in the model for each evaluated longswing using the optimisation process.

<table>
<thead>
<tr>
<th>longswing</th>
<th>vertical stiffness *</th>
<th>damping †</th>
<th>cable stiffness *</th>
<th>damping †</th>
<th>shoulder stiffness *</th>
<th>damping †</th>
</tr>
</thead>
<tbody>
<tr>
<td>backward</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K15</td>
<td>56002</td>
<td>4996</td>
<td>25004</td>
<td>4148</td>
<td>2757</td>
<td>2919</td>
</tr>
<tr>
<td>A20</td>
<td>56503</td>
<td>4987</td>
<td>25184</td>
<td>4165</td>
<td>2418</td>
<td>8367</td>
</tr>
<tr>
<td>A21</td>
<td>59737</td>
<td>2099</td>
<td>25507</td>
<td>806</td>
<td>5374</td>
<td>9959</td>
</tr>
<tr>
<td>forward</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K03</td>
<td>89803</td>
<td>911</td>
<td>35957</td>
<td>665</td>
<td>1708</td>
<td>7428</td>
</tr>
<tr>
<td>K05</td>
<td>88382</td>
<td>518</td>
<td>35969</td>
<td>501</td>
<td>1022</td>
<td>4107</td>
</tr>
<tr>
<td>A09</td>
<td>89723</td>
<td>2555</td>
<td>35870</td>
<td>1165</td>
<td>34695</td>
<td>338</td>
</tr>
</tbody>
</table>

* units of stiffness, N.m⁻¹
† units of damping, N.s.m⁻¹

Little consistency is exhibited in the elastic characteristics of the gymnasts. This is not unexpected, since the wrist to ankle length data suggest the elasticity of each gymnast is non-linear. In addition the elasticity of the shoulder spring must attempt to represent the gymnast's active extension of his body during the swing (Figure 5.60) since this was incorporated into the objective function. The timing and extent of this muscle activity may be specific to the longswing being performed. In turn this may result in the shoulder spring attempting to replicate different features of the gymnasts' techniques for each type of longswing, and therefore requires different elastic properties.

It is therefore speculated that inconsistencies in the spring parameters were partly due to the type of longswing being performed. The stiffness and damping properties of a spring interact to influence the overall response of the spring (Section 5.2). This interaction may be reflected in the spring parameters selected by the optimisation procedure. Since the elastic properties of the gymnasts are not fully represented and muscular activations are not included in the simulation model, the springs present in the model may compensate for these inadequacies. This compensation may take the form of adopting different stiffness or damping properties for the other springs, and yet still provide similar motions of the gymnast and rings cables.

However, for any particular longswing the combination of spring parameters highlighted in Table 6.5 represents the elastic nature of the whole system accurately. This
is indicated by the accuracy of the model in replicating the motion of gymnast and rings cables. Hence, for each longswing the model accurately simulates the gymnast swinging on rings. It also highlights, however, that the model is sensitive to the combination of spring parameters adopted to simulate the motion.

Using optimal spring parameters for a simulation the combined motion of the rings cables and gymnast produce an elongated elliptical shape for the path of the gymnast's mass centre (Figure 6.5). The accuracy in simulating the motion of the gymnast and rings cables means the path of the gymnast's mass centre produced by the model is also similar to reality (Figure 6.5).

![Diagram](attachment:image.png)

**Figure 6.5.** Actual and simulated path of the mass centre of gymnast A during backward A20 and forward longswing A09: four segment model.

The simulated forward longswing A09 obtained a greater score for the objective function than A20 (Table 6.4). This indicates a lower accuracy, which is reflected in the path of the mass centre being less close to what actually occurred than for the backward longswing (Figure 6.5). However, generally the path of the gymnast's mass centre provides further evidence to suggest that agreement between the actual and simulated performances is sufficient for the model to be used to investigate the influence of selected aspects of technique and elastic properties of the rings apparatus on longswing performances.

Owing to the planar nature of previous simulation models described in the literature the cable abduction angle $\theta_a$ has been dismissed. Figure 6.6 presents the actual and simulated time histories for this angle during backward longswing A20 and forward longswing A09. These indicate the accuracy of the model when simulating the lateral movements of the gymnasts' arms, since lateral arm movements result in an increased cable abduction angle. The high accuracy of the model in estimating this cable angle is due to the rings cables, arms and chest of the gymnasts forming a closed link system. By
accurately representing the lengths of the rings cables, arms and chests of the gymnasts, high accuracy in estimating the motion of the rings cables using this angle would be expected.

The four segment model therefore accurately simulates the three-dimensional motion of the gymnast and rings cables inherent in swinging on rings. This has not been accomplished in previous simulation models of a gymnast swinging on rings.

6.4 Kinetic variables calculated within the four segment simulation model

Several additional variables, described in Section 3.5.2, were calculated within the four segment model. These variables include the internal joint forces experienced at the shoulders of the gymnasts and the joint torques at the knees, hips and shoulders required to produce their techniques. Since the four segment three-dimensional model accurately simulates a gymnast swinging on rings these additional variables may be considered to represent what actually occurred during the longswings. By comparing these variables to those determined from descriptive studies of gymnasts performing longswings on rings (Section 2.4.2), the accuracy of the simulation model may be further scrutinised.

6.4.1 Joint torques

Torques required at the knee, hip and shoulder joints to perform the techniques used by the gymnasts were calculated in the simulation model. Owing to the structure of the model, these values represent joints only on the right hand side of the gymnast. Owing to the two-dimensional nature of former studies values for joint torques from the literature tend to present combined values representing the left and right sides of the gymnast (Figure 2.11). Hence, for comparative reasons, combined values are presented where
applicable. Combined values were calculated by assuming symmetry through the sagittal plane of the gymnast and doubling the values for the torques calculated by the model.

By adopting the same definition used for the two segment model, the type of joint actions being performed by the gymnast (isometric, concentric or eccentric) were determined. For example, positive joint torques which act to close the joint angle and also produce a reduction in joint angle are termed concentric joint actions. The joint actions being performed by the gymnasts throughout the longswings are presented in graphs displaying joint torque time histories by a broken line. Horizontal broken lines at the upper limits of the graphs indicate times during which concentric joint actions were utilised, while broken lines at the lower limits of the graphs indicate times during which the gymnasts were using eccentric joint actions. Times during which isometric joint actions were used by the gymnasts are indicated by a broken line on the horizontal axis.

Figure 6.7 displays time histories for the combined hip joint torques for both backward and forward longswings. Peak positive combined torques for the hips were 183 Nm during backward longswing A20 and 209 Nm during forward longswing A09. Peak combined negative values were -116 Nm and -184 Nm during the backward and forward longswings respectively. Surrounding these peak hip torques two stylised changes in the combined hip joint torques may be identified. For the backward longswing a large positive hip torque was required from 1.6 s to 1.84 s, which relates to a change in body angle from $117^\circ$ to $187^\circ$. A large negative torque was subsequently required, from 1.86 s to 2.1 s. The equates to a change in body angle from $192^\circ$ to $261^\circ$.

During the forward longswing a reverse of this stylised hip torque pattern was produced. From 1.26 s to 1.5 s a large negative torque was required at the hips, over a body angle of $-140^\circ$ to $-198^\circ$. Subsequently from 1.5 s to 1.6 s (body angle from $-198^\circ$ to $-220^\circ$) a large positive hip torque was required. The pattern of hip torque spanning the swing of the gymnast beneath the rings is therefore coincidental with the direction of longswing. This finding was also observed by Brüggemann (1987).

![Figure 6.7](image)

Figure 6.7. Time histories of the combined hip joint torques during backward longswing A20 and forward longswing A09: four segment model.
Owing to the three-dimensional nature of the simulation model three joint torques were calculated at the shoulder of the gymnast. Since the modelled arm of the gymnast is not allowed to rotate about its longitudinal axis only two of these torques are considered. The first to be considered is the torque which acts about the third internal axis of the modelled arm. This torque is analogous to that calculated in two-dimensional studies, and is associated with maintaining and altering the shoulder elevation angle $\phi_{elr}$. This torque is therefore termed the shoulder elevation torque ($\text{torque}^3$). Figure 6.8 presents the time histories for this combined shoulder elevation torque for both the backward and forward longswings.

![Figure 6.8: Time histories of the combined shoulder elevation torque ($\text{torque}^3$) during backward longswing A20 and forward longswing A09: four segment model.](image)

The temporal pattern in the shoulder elevation torque for the backward longswing is similar to the torque required at the hip joint to complete the element. The pattern of shoulder elevation torque required throughout the forward longswing also resembles that required at the hip joints for the same longswing. However, severe oscillations occurred in the simulated shoulder elevation torque for the forward longswing which may be considered unrealistic. These oscillations may be attributed to oscillations which occurred in the springs throughout this simulation. The oscillations can be identified in the simulated cable tension time history (Figure 6.4). Such oscillations may have occurred due to the large spring stiffness selected for the shoulder spring and relatively low damping value. This shoulder elevation torque data highlights a possible limitation of the simulation model for predicting the required joint torques for hypothetical simulated performances.

It should be noted that the maximum combined shoulder elevation torque required to produce the backward longswing is 10 fold smaller than that required by the rigid simulation model to produce a backward longswing to handstand. This reduction may be
attributed to the elasticity incorporated into this model, together with the real gymnast having three joints which may be utilised during the longswing.

The second shoulder torque calculated is that associated with producing lateral arm movements. This torque acts about the second internal axis of the modelled arm of the gymnast and is termed the shoulder abduction torque (torque$^2$). Owing to the planar analyses adopted by previous studies this joint torque has not previously been estimated. The temporal pattern for shoulder abduction torque is shown for both longswings A20 and A09 (Figure 6.9).

![Time histories of the single shoulder abduction torque (torque$^2$) during backward longswing A20 and forward longswing A09: four segment model.](image)

Figure 6.9. Time histories of the single shoulder abduction torque (torque$^2$) during backward longswing A20 and forward longswing A09: four segment model.

The greatest shoulder abduction torque required during backward longswing A20 occurred as the gymnast brought his arms closer together from a wide lateral configuration ($\theta_{\text{abu}}$, 49.7°) during the final stages of the ascending phase (2.34 s to 2.5 s). Since the arms were brought closer together, resulting in a reduction in the shoulder abduction angle, and the shoulder abduction torque was positive, the joint action was concentric.

During the forward longswing A09 much larger shoulder abduction torques were required to complete the element. The largest torques were evident over a change in body angle from -317° to -348° (1.88 s to 2.04 s). Between these body angles the shoulder abduction angle $\theta_{\text{abu}}$ decreased from 84.7° to 60.0°. The shoulder abduction angle continued to decrease until the arms were in a configuration appropriate for the final handstand. During this period the shoulder abduction torque was positive and, by definition, formed a concentric joint action. Caution must be shown to the magnitudes of the shoulder abduction torque required to perform this joint angle change. The magnitudes of the joint torques may be partly attributed to the oscillations of the springs during this particular trial. However, the large differences apparent in the magnitudes of
shoulder abduction torque required during each longswing may be partly attributed to the extent of lateral arm movements made by the gymnast. These movements are much more extensive during a forward longswing (Section 5.4), and result in larger shoulder abduction torques being required to perform the element.

*Comparison with the research literature*

Joint torques estimated by the simulation model for backward longswings compare favourably with estimates derived from the descriptive studies of Brüggemann (1987). Taking into account different definitions of joint torques, similar temporal patterns are evident (Figures 2.11, 6.7 and 6.8). The magnitudes of joint torques estimated by the simulation model however are lower than those calculated by Brüggemann (1987). In the study of Brüggemann (1987) peak combined shoulder elevation torque values were equivalent to +700 Nm and -450 Nm. Similarly the combined hip torques were calculated to be in excess of ±400 Nm. Joint torque values calculated by Brüggemann (1987) were typically twice as large as those estimated by the four segment model. This difference may be partly due to different inertial characteristics between the gymnasts used in the studies. However, it is unlikely that differences in inertial characteristics account for a doubling of peak torque values. Hence, it is speculated that a more likely reason for the differences is in the accuracy of angular acceleration data utilised in both studies.

Since no other studies have attempted to estimate values for shoulder abduction torque (torque²) it is not possible to corroborate these findings. However, the mechanical explanations given for the joint torques and actions required to complete the longswings are consistent with what might be expected to occur during the final stages of both types of longswing.

6.4.2 Shoulder tension

Internal joint forces acting at the shoulder joint were also calculated throughout each longswing. Positive internal joint forces, which indicate the gymnast is under compression, occurred when the gymnast was near the handstand position, for example from 0.7 s to 0.9 s for the backward longswing. Negative forces denote extension forces, effectively stretching the gymnast. Peak combined extension forces in the shoulders of the gymnast were equivalent to 7.0 bodyweights in the backward longswing A20 and 7.5 bodyweights in the forward longswing A09 (Figure 6.10). These peak extension forces occurred as the gymnast passed beneath the rings and cable tension was at or near its peak value.
Comparison with the research literature

Brüggemann (1987) estimated the peak internal joint forces in each shoulder joint of an elite gymnast to be in excess of 2100 N during a backward longswing on rings. The four segment model estimated the peak internal joint force in each shoulder at 2116 N for the backward longswing A20. It is acknowledged that different gymnasts were used in the studies and without data on the mass of the gymnast used in the study of Brüggemann (1987) a direct comparison cannot be made. However, the similarity in peak values provides further independent evidence that the simulation model accurately represents a gymnast swinging on rings.

6.5 Sensitivity analysis

During the optimisation process forming the evaluation procedure, the spring parameters representing the elasticity of the gymnast and rings apparatus were determined for a particular set of initial conditions. These initial conditions manifest themselves in the initial body angle of the gymnast which, for all evaluated longswings, was less than $10^\circ$ from a vertical orientation (Table 6.2). In order to determine the sensitivity of the model's accuracy to the initial conditions and optimal spring parameters a sensitivity analysis was conducted.

6.5.1 Sensitivity of the simulation model and optimal spring parameters to initial conditions

Without re-optimising the spring parameters each evaluated longswing was
simulated using an initial time 0.2 s later than that used during the optimisation process. By conducting this analysis the accuracy of the simulation model was re-assessed. In addition, the sensitivity of the model to the initial conditions used to obtain the optimal spring parameters was determined.

Table 6.6 presents the body angles of the gymnasts from which the simulations were initiated for the sensitivity analysis. For backward longswings this equated to a 7.1° mean difference in the initial body angle from that used in the evaluation procedure. For forward longswings the mean difference in initial body angles between the simulations carried out in the evaluation procedure and those of the sensitivity analysis was 19.6°.

Table 6.6. Initial body angles (actual) of the gymnasts for each longswing implemented in the sensitivity analysis

<table>
<thead>
<tr>
<th>Initial body angle (°)</th>
<th>K15</th>
<th>A20</th>
<th>A21</th>
<th>K03</th>
<th>K05</th>
<th>A09</th>
</tr>
</thead>
<tbody>
<tr>
<td>backward longswings</td>
<td>6.0</td>
<td>5.8</td>
<td>18.3</td>
<td>-24.6</td>
<td>-31.3</td>
<td>-26.2</td>
</tr>
<tr>
<td>forward longswings</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The sensitivity analysis revealed that the optimal spring parameters obtained from the optimisation / evaluation procedure were sensitive to the initial conditions from which they were determined. For backward longswings different initial orientations of the gymnast and rings cables resulted in an average RMS difference between actual and simulated body angles of 59.2°. For forward longswings the equivalent value was 49.2° (Table 6.7). This corresponds to the simulation model being considerably less accurate at simulating the motion of the rings cables and gymnast.

Table 6.7. The RMS differences between actual and simulated body angle ε for each longswing in the sensitivity analysis

<table>
<thead>
<tr>
<th>RMS difference between actual and simulated</th>
<th>K15</th>
<th>A20</th>
<th>A21</th>
<th>K03</th>
<th>K05</th>
<th>A09</th>
</tr>
</thead>
<tbody>
<tr>
<td>backward longswings</td>
<td>59.6</td>
<td>77.0</td>
<td>41.2</td>
<td>26.8</td>
<td>69.9</td>
<td>50.8</td>
</tr>
<tr>
<td>forward longswings</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Some of the decreased accuracy may be accounted for by assumptions concerned with the initial velocity of the springs. Since in the evaluation procedure the gymnast was in handstand the springs were assumed to have zero velocity. However, for the sensitivity
analysis the gymnasts were further from the initial handstand position. Hence, the elastic components may be expected to display a non-zero velocity. This reduces the validity of the assumption used for the initial velocity of the springs. By incorporating an accurate estimate for the initial spring velocity this effect may be reduced.

This analysis highlighted one limitation in the accuracy of the four segment model for simulating a gymnast swinging on rings. The model is sensitive to the initial conditions from which a simulation is initiated. In order to maintain the accuracy of the model in further investigations only the evaluated longswings are implemented with the same initial conditions used to determine the optimal spring parameters.

6.6 Summary

Results from the evaluation procedure suggest the four segment three-dimensional simulation model is sufficiently accurate for simulating a gymnast swinging on rings. In addition, independent comparisons with previous descriptive studies of longswings on rings indicate similar estimates for internal joint forces at the shoulder joints, and torques required at the shoulder and hip joints to produce the longswings. These comparisons lend further support to the accuracy of the model.

However, it was found that the accuracy of the model was sensitive to the initial conditions required by the simulation model. Hence, only the evaluated longswings, with the optimal spring parameters combined with the initial conditions, were used in further investigations.

To conclude, with evidence from the evaluation procedure, it is proposed that the evaluated simulation model is sufficiently accurate to investigate a gymnast swinging on rings in more detail and address the questions posed in Chapter 1. These questions are directly addressed in the following sections of Chapter 7 while the findings are summarised in Chapter 8.
CHAPTER 7

INVESTIGATIONS USING THE FOUR SEGMENT THREE-DIMENSIONAL SIMULATION MODEL

7.1 Introduction

This chapter presents the investigations carried out using the evaluated four segment three-dimensional simulation model. Four separate investigations were conducted, each relevant to a particular question posed in Chapter 1. By adopting the theoretical approach of a simulation model total control is maintained over the intervention applied to each investigation. The resulting motion of the gymnast and rings cable can therefore be entirely ascribed to the intervention applied to the gymnast or apparatus (Section 2.4.1).

The four investigations examined the following aspects of swinging on rings:

- the influence of a gymnast's inertial characteristics on longswing performance
- the influence of the elasticity of the rings apparatus on the performance of longswings and peak forces experienced by a gymnast
- the contribution of each component of technique to the performance of longswings and forces experienced by a gymnast
- the sensitivity of the coordination of a gymnast's joint actions on longswing performance.

For each investigation the simulations produced by the evaluation procedure (Chapter 6) were used as the criteria performances against which all hypothetical performances were compared. Throughout this chapter the six simulations which formed the evaluation of the model (3 backward and 3 forward longswings) are termed actual performances. This term is appropriate since the accuracy of the model provides confidence in these simulations closely representing the actual performances. Furthermore, by comparing actual simulated performances against hypothetical simulations the effect of systematic differences between the model and reality are overcome.

The appropriate subject specific inertia parameters, initial times for each simulation, elastic properties of each damped spring (Tables 5.21, 6.2 and 6.5 respectively) and joint angle time histories were used for the hypothetical simulations. Intervention to create the hypothetical simulations was solely dependent on the aspect of investigation. For instance, when the influence of rings apparatus elasticity was examined, only values for
the spring parameters representing the elasticity of the rings apparatus were altered from the actual performances. All other model parameters and initial conditions remained the same as those used for the actual performance. The intervention used in each investigation is provided within the sections.

Performance indicators used to describe differences between hypothetical simulations and actual performances comprise the body angle $\theta$ of the gymnast, the peak combined shoulder joint forces and the joint torques required to perform the joint actions. Differences in other mechanical variables between actual and hypothetical situations appropriate to the investigations are also described.

7.2 The influence of a gymnast's inertia characteristics on longswing performance

7.2.1 Introduction

A gymnast's inertial properties may, to some extent, determine the exact technique he adopts when performing backward and forward longswings on rings. To assess the influence of a gymnast's inertial characteristics on performance for a certain technique two gymnasts possessing different inertial properties must perform the single technique. Using a simulation model this may be achieved by taking the inertial characteristics of one gymnast and making this gymnast perform the technique adopted by a different gymnast. The effect of a gymnast's inertial characteristics on performance is determined by comparing both performances, which differ only in the gymnast who performed the element.

This particular investigation focused upon the influence of a gymnast's inertial characteristics on performance for both backward and forward longswings. This is directly relevant to Question 8 posed in Chapter 1.

7.2.2 Methods

In terms of total body mass (estimated) subject A was 5.67 kg heavier than subject K (Table 5.19). When expressed as a percentage of total body mass the difference in the inertial characteristics of the gymnasts was 8.5%. This difference in body mass and differences in other segmental inertial parameters (Table 5.21) make these two subjects suitable to assess the influence of inertial characteristics on performance for a specific gymnastic technique.

The actual simulated longswing performances of subject A, 2 backward longswings, (A20 and A21) and 1 forward longswing (A09), were used in this analysis. While
retaining the same elasticity of the rings apparatus, initial conditions and joint angle time histories of subject A, the inertia parameters for gymnast K were implemented in each longswing. Effectively this ensured that subject K performed the same technique for each longswing as subject A.

Differences between longswing performances by both subjects were evaluated as the RMS difference in body angle since this variable indicates whether a gymnast has completed each element. In addition, the peak forces experienced by the gymnasts at their shoulder joints were evaluated and compared.

7.2.3 Results

Figure 7.1 shows the backward longswing A20, performed by subject A as well as the predicted performance of subject K while using the technique of subject A. Figure 7.2 depicts the performance of subject A for forward longswing A09 and the predicted performance of subject K whilst adopting the technique of subject A.

![Figure 7.1](image1)

Figure 7.1. Graphics sequences showing backward longswing A20 performed by subject A and the predicted performance of subject K when adopting the technique of subject A.

During the descending phase of the backward longswing (the first four figures from the left) the performances of the two gymnasts were very similar (Figure 7.1). A greater
difference was apparent during the ascending phase of the longswing. While the performance of subject A was proficient the performance of subject K became inadequate. Finally, while subject A completed the backward longswing, subject K possessed insufficient rotation to attain the final handstand position, and started to swing back on himself (anticlockwise as shown in Figure 7.1).

Figure 7.2. Graphics sequences showing forward longswing A09 performed by subject A and the predicted performance of subject K when adopting the technique of subject A.

A result similar to the backward longswings was observed for the forward longswing performances. Figure 7.2 reveals that the descending phases of the two forward longswings were very similar. Large deviations in performance occurred only after the gymnasts passed through the bottom of the swing (fourth figure from the left). Using the technique of subject A gymnast K once again did not possess enough rotation to attain the final handstand position. Thus, when subject K adopted the same technique as subject A, his performance was poor. This result is in agreement with the findings regarding the backward longswing performances.

Figure 7.3 provides the time histories of the body angle for the actual performances by subject A and the predicted performances for subject K for both types of longswings. The particular moments where the performances of gymnast K started to diverge from that of subject A are close to 240° of body angle rotation (-240° for the forward longswing).

Differences between final body angles attained by gymnast A and K for all
longswings were: 142.4° for forward longswing A09, 72.8° for backward longswing A20 and 53.1° for backward longswing A21. For backward longswing A20 the RMS difference in body angle between the actual performance by gymnast A and that predicted for gymnast K was 23.5°. For backward longswing A21 the RMS difference in body angle between the two performances was 16.7°, while for forward longswing A09 this difference was 44.4°.

Figure 7.3. Time histories of body angle $\epsilon$ for the actual simulated performances for gymnast A and the predicted performances for gymnast K.

Figure 7.4 presents the combined forces at the shoulder joints experienced by the gymnasts throughout the longswings.

Figure 7.4. Time histories of combined shoulder joint forces for the actual simulated performances of gymnast A and the predicted performances for gymnast K.

During the longswing performances peak shoulder joint forces experienced by gymnast K were less than those of subject A (Table 7.1). The smaller forces experienced by subject K, produced as a result of differing inertial characteristics, may have contributed to the less proficient performances.
Table 7.1. Peak combined shoulder joint forces during longswings for subjects A and K when adopting longswing techniques of subject A

<table>
<thead>
<tr>
<th>Longswing technique</th>
<th>Subject A</th>
<th>Subject K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward longswing A09</td>
<td>-7.45</td>
<td>-6.40</td>
</tr>
<tr>
<td>Backward longswing A20</td>
<td>-6.98</td>
<td>-6.25</td>
</tr>
<tr>
<td>Backward longswing A21</td>
<td>-7.77</td>
<td>-7.10</td>
</tr>
</tbody>
</table>

7.2.4 Conclusions

Results from the simulation model indicate that for a particular technique the inertia characteristics of a gymnast greatly influence longswing performance. When gymnast K adopted the techniques of gymnast A for all three longswings subject K possessed insufficient rotation to achieve the final handstand (Figure 7.3).

Differences in body angle caused as a direct result of differences in inertia, suggest that for a particular technique forward longswing performances are more sensitive to the inertial characteristics of the gymnast than backward longswings. On average the 8.5% change in the inertial characteristics of the gymnast resulted in a 6% overall change in performance, when expressed as a percentage of the total body angular excursion of the actual performance. The equivalent result for the forward longswing was 12%. When the effect of the 8.5% change in inertia is expressed in terms of the inadequate rotation produced by gymnast K for the backward longswings the inertia accounted for a mean difference of 18%. For the forward longswing the influence of inertia accounted for a 38% reduction. In both cases these results indicate that forward longswing performances are twice as sensitive as backward longswings to changes in the gymnast's inertia.

The inertial properties of a gymnast affect the timing of his swing and hence his interaction with the rings cables. This feature was highlighted by the range of maximum body angles attained by the gymnast as predicted by the planar rigid model (Table 5.1). By affecting the combined movements of the gymnast and rings cables the effect of subsequent joint angle changes on performance are also altered. For these longswings this compound effect resulted in gymnast K losing more mechanical energy as he passed beneath the rings and consequently experiencing lower peak forces. Since the orientation of the gymnast at which subsequent joint angle changes are made is different from that required for a good performance, the changes in body configuration produced during the
ascending phase of the swing do not have the desired effects and therefore result in a degradation of performance. This finding was also demonstrated using the two segment model (Section 5.4) and can be identified in Figure 7.3 by the significant parting of the body angles after 240° of body angle rotation.

The greater sensitivity of forward longswing techniques to the inertial characteristics of the gymnast may be due partly to the more prominent lateral movements of the arms during the ascending phase. In addition, since the body angular velocity of the gymnasts as they swing beneath the rings is slightly larger for forward longswings, differences in the timing of joint angles will be more exaggerated than for backward longswings. Any inappropriate timings of joint actions for forward longswings may result in a gymnast not attaining the near vertical orientation with his arms to the sides of his body (Section 5.5) which is a major feature of forward longswings. Any subsequent circling of the arms may then result in the rings cables remaining in front of the gymnast, reducing his body angle rotation when the arms are brought closer together. Such an effect is highlighted in Figure 7.2.

On average the peak combined shoulder forces experienced by gymnast K were smaller than gymnast A by 0.8 bodyweights (Table 7.1). This decrease in peak forces is also a result of altering the interaction between the gymnast and rings cables, which in turn is a result of changing the inertial characteristics of the gymnast.

This investigation indicates that different gymnasts possessing different inertial characteristic must adopt slightly different techniques in order to produce proficient longswing performances. This does not mean that similar techniques cannot be adopted by two different gymnasts. Indeed, results concerning the techniques employed by the two subjects in this study suggest their techniques are qualitatively similar. However, for a particular longswing performance the exact timings of the joint angles changes are specific to the inertial properties of the gymnast.

7.3 The influence of the elasticity of the rings apparatus on longswing performance and peak forces experienced by the gymnast

7.3.1 Introduction

The development of the rings apparatus to its modern structure was detailed in Section 2.2.2, where a particular make of rings apparatus was described. The absence of specific directives from the F.I.G. concerning the elasticity of the modern rings apparatus means there is no criterion against which this property can be assessed. Since manufacturers of gymnastics equipment introduce the obligatory elasticity in the apparatus using a variety of methods it is likely that rings apparatus manufactured by
different companies possess different elastic qualities. What is not clear, however, is the
effect different elastic properties of the rings apparatus have on a gymnast's longswing
performance. For example, can a gymnast use the same technique on two different rings
apparatus which possess different elastic qualities and expect to produce the same
performance?

Recently a revolutionary advancement in the structure of the rings apparatus was
made by the manufacturers Gymnova. The newly designed rings frame, which is
intended for use within the training environment, possesses the ability to vary the
effective elasticity of the rings frame (Figure 2.3). According to the manufacturer's
instructions, when set to 'very supple', the increased elasticity of the apparatus
considerably reduces the forces experienced by the gymnast. Increasing the elasticity of
the rings apparatus to reduce peak forces experienced by a gymnast, and therefore the risk
of injury to a gymnast's shoulders, was advocated by Caraffa et al. (1996) and
Cerulli et al. (1998). However, by how much does a significant increase in the elasticity
of the rings frame alter the peak forces experienced by a gymnast when performing
longswing elements?

The following investigation examined the influence of the stiffness of the rings
apparatus in relation to the performance of gymnasts using specific techniques for
backward and forward longswings. Effectively, this investigation addresses Question 6
posed in Chapter 1. Differences in peak forces experienced by the gymnast when
swinging on different apparatus possessing varied elasticity were also determined for each
longswing performance. The evaluated four segment simulation model was implemented
in this investigation.

7.3.2 Methods

The six evaluated longswings were used as criteria against which the influence of
rings apparatus' elasticity was evaluated. Values for spring parameters representing the
elasticity of the rings apparatus in these simulations were termed standard (Table 6.4).
The term 'rings apparatus' was considered to comprise the elastic structures representing
the apparatus in the four segment simulation model: rings cables, horizontal motion of the
frame and the DEDs combined with the horizontal beam of the rings frame. These
structures are represented by the cable, horizontal and vertical springs respectively.

Two variants of the standard rings apparatus, in terms of elasticity, were modelled:
inflexible and flexible. The inflexible rings apparatus was modelled by increasing the
stiffness values of the springs representing the apparatus by a factor of 1.33, while the
flexible apparatus was represented by decreasing the stiffness values of the springs by a
factor of 1.33. Hence, values for the stiffness coefficients of the springs for the flexible
rings apparatus were effectively 75% of their original values.

Realistic values for each stiffness parameter were specific to the values determined from the evaluation procedure (Table 6.5). Hence, stiffness values specific to each performance were adjusted by the appropriate amounts to model the hypothetical apparatus. No changes to the damping properties of the springs were made.

Simulations predicting the performance of each gymnast using their respective techniques on the two hypothetical rings apparatus were conducted using the four segment simulation model. Each simulation utilised the initial conditions, shoulder spring parameters, joint angle time histories and subject specific inertia parameters associated with the evaluated performance. Thus, the only intervention made was the rings apparatus on which the gymnast was swinging.

The final body angle attained for each longswing on all three types of apparatus (standard, inflexible and flexible) were recorded together with the peak combined shoulder joint forces. The RMS difference in body angle between the actual longswing performance of each gymnast on the standard rings apparatus and the hypothetical performances on the inflexible and flexible apparatus were also calculated.

7.3.3 Results

Tables 7.2 and 7.3 present the final body angles attained during the performances on each modelled rings apparatus (100% standard, 133% inflexible, 75% flexible). In all cases the inflexible rings apparatus produced a longswing which rotated through the handstand position, illustrated by the body angle being of a greater magnitude than the final body angle on the standard apparatus. In contrast, swinging on the flexible rings apparatus with the same technique used on the standard rings resulted in the gymnasts possessing insufficient rotation to attain the final handstand.

The peak combined shoulder joint forces experienced by the gymnasts swinging on the flexible rings apparatus were consistently smaller than those produced on the standard and inflexible apparatus. This finding is in agreement with the proposal of Caraffa et al. (1996) and Cerulli et al. (1998). However, the maximum differences in peak shoulder joint forces between the inflexible and flexible rings apparatus for backward longswings was only 0.32 bodyweights. The corresponding value for the forward longswings was 0.34 bodyweights. These reductions in peak forces at the shoulders of the gymnasts may be regarded as small, when considering the influence on the gymnasts' performances. Thus, peak forces experienced by the gymnast during both types of longswings were not vastly altered by an extensive change (58%) in the elasticity of the apparatus.
Table 7.2. Final body angles attained and peak combined shoulder forces experienced by the gymnasts during backward longswings

<table>
<thead>
<tr>
<th></th>
<th>K15</th>
<th>A20</th>
<th>A21</th>
</tr>
</thead>
<tbody>
<tr>
<td>apparatus elasticity</td>
<td>100% 133% 75%</td>
<td>100% 133% 75%</td>
<td>100% 133% 75%</td>
</tr>
<tr>
<td>final body angle (°)</td>
<td>368.3 385.4 349.1</td>
<td>356.3 376.4 334.5</td>
<td>359.2 363.6 346.6</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-7.78 -7.93 -7.61</td>
<td>-6.98 -7.08 -6.84</td>
<td>-7.77 -7.83 -7.60</td>
</tr>
</tbody>
</table>

Table 7.3. Final body angles attained and peak combined shoulder forces experienced by the gymnasts during forward longswings

<table>
<thead>
<tr>
<th></th>
<th>K03</th>
<th>K05</th>
<th>A09</th>
</tr>
</thead>
<tbody>
<tr>
<td>apparatus elasticity</td>
<td>100% 133% 75%</td>
<td>100% 133% 75%</td>
<td>100% 133% 75%</td>
</tr>
<tr>
<td>final body angle (°)</td>
<td>-365.9 -415.1 -296.0</td>
<td>-370.8 -393.3 -311.7</td>
<td>-376.8 -378.6 -337.7</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-6.69 -6.81 -6.50</td>
<td>-5.91 -6.01 -5.74</td>
<td>-7.44 -7.64 -7.30</td>
</tr>
</tbody>
</table>

Table 7.4. The RMS differences in body angle between simulated performances of backward longswings on the standard, inflexible and flexible apparatus

<table>
<thead>
<tr>
<th></th>
<th>K15</th>
<th>A20</th>
<th>A21</th>
</tr>
</thead>
<tbody>
<tr>
<td>apparatus elasticity</td>
<td>100% 133% 75%</td>
<td>100% 133% 75%</td>
<td>100% 133% 75%</td>
</tr>
<tr>
<td>RMS difference in body angle from actual performance (°)</td>
<td>0.0 4.53 4.85</td>
<td>0.0 4.68 5.50</td>
<td>0.0 1.62 3.85</td>
</tr>
</tbody>
</table>

Mean differences in performance, measured as the RMS difference in body angle between actual and predicted performances (Tables 7.4 and 7.5), were 4.4° for backward longswings and 12.2° for forward longswings. This result indicates that the forward...
longswing techniques were more sensitive to the elasticity of the rings apparatus than
techniques adopted for the backward longswings.

Table 7.5. The RMS differences in body angle between simulated performances of
forward longswings on the standard, inflexible and flexible apparatus

<table>
<thead>
<tr>
<th>forward longswings</th>
<th>K03</th>
<th>K05</th>
<th>A09</th>
</tr>
</thead>
<tbody>
<tr>
<td>apparatus elasticity</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>133%</td>
<td>133%</td>
<td>133%</td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td>75%</td>
<td>75%</td>
<td></td>
</tr>
<tr>
<td>RMS difference in body angle from actual performance (°)</td>
<td>0.0</td>
<td>12.71</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>19.64</td>
<td>6.03</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>14.9</td>
<td>0.65</td>
<td>9.02</td>
</tr>
</tbody>
</table>

These results are illustrated using longswings K15 and K05 in graphs of the body angle time history for each performance (Figure 7.5). Both simulations represent the median alterations in performance as a direct result of elasticity alterations (Tables 7.2 to 7.5). These performances are therefore considered typical examples demonstrating the influence of changing the elasticity of the rings apparatus on longswing performance.

Figure 7.5. Body angle ε time histories for backward longswing K15 and forward longswing K05 on the standard, inflexible and flexible rings apparatus.
Figure 7.6. Graphics sequences of the simulated backward longswing performances (longswing K15) on the standard, inflexible and flexible rings apparatus.

The graphics sequences in Figures 7.6 and 7.7 highlight the influence of rings apparatus' elasticity on the performance of backward and forward longswings using specific techniques. The increased rotation caused by the inflexible rings apparatus can be identified to the right of the middle sequence in Figures 7.6 and 7.7. The lack of sufficient rotation when the gymnast performed on the flexible apparatus can be observed to the right of the lowest sequence in the same two figures.
7.3.4 Conclusions

From these results it is clear that the elasticity of the rings apparatus influences the longswing performance of a gymnast. If a gymnast performs exactly the same technique on three different rings apparatus possessing different stiffness properties, the resulting performances are noticeably different.

The manner in which the performance differs with a change in the stiffness of the apparatus was found to be predictable (Tables 7.2 and 7.3). When both gymnasts performed on the inflexible rings apparatus they completed all backward and forward longswings. It may be argued that the gymnast possessed too much rotation to be able to
retain the handstand position, but in all cases the required 360° of rotation was produced. However, the consequences of performing on the flexible apparatus may be considered more detrimental since the gymnast did not complete any longswing (Table 7.2 and 7.3).

The mechanical reasons underpinning these results are two fold. Firstly, the increased stiffness of the apparatus effectively alters the interaction between the gymnast and rings cables. Since the gymnasts in the simulations cannot adapt their technique all subsequent joint actions made by the gymnasts compound any alterations to this interaction.

Besides affecting the timing of the interaction between gymnast and rings cables a different apparatus stiffness leads to different changes in the mechanical energy of the gymnast and deformation of the apparatus. These differences manifest themselves in the total mechanical energy of the gymnast. With an inflexible rings apparatus the elastic structures deform less than when the apparatus was either standard or flexible. The change in the loading of the apparatus effectively acts in the same manner as described for the elastic model (Section 5.3). The planar elastic model indicated than for a specific damping of the apparatus, a greater spring stiffness produced a greater maximum body angle when the gymnast performed a swing from handstand. In the elastic model this result was accounted for by the mechanical energy dissipated from the system through the damping forces and the energy stored in the apparatus which could be returned to the gymnast.

Both of these factors play roles in affecting the longswing performances for a particular technique when swinging on all three types of rings apparatus. Time histories for the total mechanical energy of the gymnast and orientation of the rings cables (φe) suggest that for backward longswings changes to the interaction between the gymnast and cables contributed more to the alterations in performance than changes in energy.

![Figure 7.8. Total mechanical energy of the gymnast during backward K15 and forward longswing K05 on the standard, inflexible and flexible rings apparatus.](image-url)
This conclusion is reached since the total mechanical energy of the gymnast on completion of the simulations is similar for performances on all three rings apparatus (Figure 7.8). However, there is still a build-up in the deviation of the orientation of the rings cables ($\phi_c$) from that produced when swinging on the standard apparatus (Figure 7.9). For forward longswings it seems that both factors play a substantial role in altering the gymnasts' performances with a change in the elasticity of the apparatus. Differences in mechanical energy of the gymnast at the end of the simulation and a build up in the deviation of the rings cable elevation angle $\phi_c$ from that produced on the standard apparatus are evident in Figures 7.8 and 7.9.

In similarity to the results obtained when investigating the influence of inertia it was found that forward longswings were more sensitive to changes in the stiffness of the apparatus than backward longswings. Combining results from these two investigations it may be inferred that timing in a gymnast's technique is more crucial in forward longswings than for backward longswings. Essentially there is a smaller time window for errors in technique. This factor is examined in greater detail in Section 7.5.

It may also be inferred from this investigation that when a gymnast swings on a different apparatus he must alter his technique in some way to adjust for changes in the interaction between the gymnast and rings cables. From the body configuration data for the real longswing performances (Section 5.5.3) it was speculated that gymnasts may make slight adjustments throughout the swing in order to alter the interaction between the gymnast and rings cables. Thus, if a gymnast learns a particular technique on one type of rings apparatus, he will have to adjust his technique substantially if the subsequent apparatus differs greatly in its elastic properties.

The peak forces experienced by the gymnasts during the longswings varied depending on the simulated stiffness of the apparatus (Tables 7.2 and 7.3). Although
decreases in peak shoulder forces were produced as a direct result of increasing the elasticity of the apparatus. The decrease was only in the order of 0.17 bodyweights or 2.5% of the peak value for forward longswings and a 0.16 bodyweight, or 2.1%, reduction for backward longswings. These are small differences considering the stiffness of the apparatus decreased substantially from the standard apparatus (25%). Several studies have suggested that a decrease in the stiffness would reduce peak forces experienced by the gymnasts and hence reduce the risk of injury (Caraffa et al., 1996; Cerulli et al., 1998). These results, however, suggest that enormous changes to the stiffness of the apparatus are required to produce a reasonable decrease in peak shoulder joint forces. To illustrate, peak reaction forces experienced by the gymnast when performing giant circles on horizontal bar range from 3.6 bodyweights for regular circles (Kopp & Reid, 1980) to 5.0 bodyweights during accelerated circles (Hiley, 1998). From these investigations it is concluded that dramatic changes to the stiffness of the rings apparatus (greater than 25%) are required in order to obtain forces which are comparable to those that a gymnast experiences when swinging on the horizontal bar. In addition such extensive changes will also have a wide reaching effect on the techniques adopted by the gymnasts to perform the longswings, as demonstrated in this particular investigation.

The joint torques required by the gymnasts to perform, or attempt to perform, the longswings did not vary greatly throughout each longswing, except where simulated performances greatly deviated from actual performances. A difference in joint torques would be expected at such instants since effectively the gymnastic elements being performed become different and therefore cannot be compared.

When expressed as a percentage of the total body angular excursion the backward longswing performances on different sets of rings apparatus differed by only 1.2% (4.4°). The forward longswing performances on average deviated by only 3.4% (12.2°). Taking into account the final body angles attained by the gymnasts, differences in performance for backward longswings were only 4.4% (15.8°) and 11.1% (40.25°) for forward longswings. When comparing the influence of a change in the stiffness of the rings apparatus by a factor of 1.33 (reduction or increase) to an 8.5% change in a gymnast’s inertia (Section 7.2) these results strongly suggested that the technique adopted for a performance is more sensitive to the inertial characteristics of the gymnast than the stiffness properties of the apparatus on which the gymnast is swinging.
7.4 The contribution of components of technique to performances of longswings on rings

7.4.1 Introduction

Coordinated angle changes at the hip and shoulder joints of a gymnast have been identified as essential components of technique for proficient performances of backward and forward longswings on rings (Nissinen, 1983; Brüggemann, 1987). Coaching literature also regards these two components of technique as crucial for elite performances of longswings (Hesson, 1976; Kormann, 1984). However, the relative contribution of these two components of technique to longswing performances is unresolved in both research literature and coaching articles. Furthermore, owing to the two-dimensional nature of previous studies, the contribution of lateral arm movements to backward and forward longswing performances, although mentioned in texts, has been neglected and is therefore unknown.

From the present knowledge of a gymnast swinging on rings the manner in which components of technique enable the gymnast to perform the longswing elements is unclear. Factors which may limit a gymnast's ability to perform an element may include the peak joint forces which he is able to withstand and the maximum strength at each joint. How, therefore, does each component of technique affect the peak forces experienced by the gymnast or the joint torques he requires to exert in order to complete the longswing elements? Examining these factors may also indicate the contribution of each component of technique to performance.

Using the evaluated four segment simulation model this investigation examined the contribution of each component of technique to the performance of backward and forward longswings on rings. In addition, the effect of each component on peak forces and joint torques required to perform the elements were analysed. By evaluating the contribution of each component of technique to performance, insight into why gymnasts adopt certain techniques may be established: in addition how each component benefits the gymnasts' performances may be determined.

The components of technique were partitioned into four angle changes at the three joints in the four segment simulation model: knee angle, hip angle, shoulder elevation angle and shoulder abduction angle. The following sections are divided into sub-sections to reflect this partitioning of a gymnast's technique. This investigation is pertinent to Question 7 posed in Chapter 1 and directly relevant to the purpose of the thesis.
7.4.2 Methods

Changes in the knee, hip, shoulder elevation and shoulder abduction angles throughout each longswing were each considered to represent the individual components of a gymnast's technique. This resulted in four components of technique being investigated for each evaluated longswing.

The four segment model was used in conjunction with each evaluated longswing. Using the initial conditions (Table 6.2), spring parameters (Table 6.5) and appropriate inertial parameters of the gymnasts (Table 5.21) for the evaluated longswings each component of technique was systematically removed from the gymnasts' techniques and simulations performed. The simulation model predicted the resulting motion of the gymnast and cables without the specified component of technique. Thus, for each evaluated longswing the contribution to performance from changes in the knees, hips, shoulder elevation and shoulder abduction angles was evaluated.

The removal of only one component of technique was achieved by forcing the angle describing the chosen component to retain its initial value throughout the whole simulation. In effect, the eliminated joint angle remained at an angle representative of a handstand throughout the attempted longswing. Since no angle changes occurred at the chosen joint this removed any contribution to performance which the angle changes produced. Only one component of a gymnast's technique was removed for each attempted longswing, thus isolating the contribution of each component to performance. The contribution of each component of technique to the performance of all six evaluated longswings was determined, which resulted in 24 simulations being conducted for this investigation.

The hypothetical performances with particular components of technique removed were compared to the evaluated longswings in order to determine their contributions to performance. The maximum body angle of the gymnast during each hypothetical performance was used to indicate the contribution of each component to the longswing performance. A large difference in maximum body angle indicated a large contribution. In addition, comparisons of peak joint forces and torques required at each joint for hypothetical and actual simulated performances were made to gain insight into how each component helps the gymnast to complete the longswing.

7.4.3 Results

The following results are divided into sub-sections consisting of the four components of technique examined in this investigation. The results highlight the contribution of each component of technique to longswing performance. In addition, the
influence of each component on peak forces experienced by the gymnasts and joint torques required to perform the longswings are presented.

Contribution of joint actions at the knees

Angle changes at the knees of a gymnast are deductible during competition and as such are undesirable. However, since changes at the knees were observed from the video analyses of longswings (Section 5.5.3) their contribution to performance was evaluated.

Table 7.6 highlights the contribution of changes in angles at the knee joints to the gymnasts' longswing performances. The term 'with' indicates the component of technique was used by the gymnast, this performance being identical to the evaluated longswing. The term 'without' indicates the omission of the named component, and therefore indicates the contribution of the component to performance.

The removal of joint angle changes at the knee joint for the backward longswings resulted in only small differences in the performance. The mean magnitude of difference in maximum body angle between utilising the component of technique and omitting it was 18.7°. For backward longswings the mean RMS difference in body angle with and without this component was only 7.9°.

Table 7.6. Maximum body angles and peak combined shoulder forces experienced by the gymnasts during longswings with and without knee joint actions

<table>
<thead>
<tr>
<th>Backward longswings</th>
<th>K15</th>
<th>A20</th>
<th>A21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee joint actions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum body angle (°)</td>
<td>368.3</td>
<td>403.5</td>
<td>359.6</td>
</tr>
<tr>
<td>Peak combined shoulder force (BW)</td>
<td>-7.79</td>
<td>-8.06</td>
<td>-6.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Forward longswings</th>
<th>K03</th>
<th>K05</th>
<th>A09</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee joint actions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum body angle (°)</td>
<td>-365.9</td>
<td>-329.2</td>
<td>-370.8</td>
</tr>
<tr>
<td>Peak combined shoulder force (BW)</td>
<td>-6.69</td>
<td>-6.86</td>
<td>-5.91</td>
</tr>
</tbody>
</table>
Figure 7.10. Graphics sequences showing the contribution of joint angle changes at the knees to the performance of backward longswing A21.

Figure 7.11. Graphics sequences showing the contribution of joint angle changes at the knees to the performance of forward longswing K05.
The graphics sequence in Figure 7.10 shows the small difference in performance when angle changes at the knee joint were omitted from the gymnast's technique during a backward longswing. The contribution of joint angle changes at the knees to the performance of backward longswings may therefore be regarded as negligible.

The removal of this component of technique during forward longswings resulted in a larger difference in performance than for the backward longswings. This is highlighted in the mean magnitude of difference in maximum body angle (51.2°), the mean RMS differences in body angle between performances with and without the component (39.7°), and the graphic form presented in Figure 7.12.

In the video analysis of the longswings knee angle changes were shown to be more extensive in forward longswings than backward longswings (Table 5.17). The result regarding its contribution to longswing performance supports the findings of the video analysis. The contribution of angle changes at the knee joints may be regarded as more important to the performance of forward longswings than backward longswings.

![Figure 7.12. Time histories of body angle ε for backward longswing A21 and forward longswing K05 with and without changes in the knee angle.](image)

The removal of changes in the knee joint angle also had a small effect on the peak joint forces experienced by the gymnasts in their shoulders. For backward longswings the average increase in peak combined shoulder joint forces was 0.13 bodyweights, while the equivalent value for forward longswings was a decrease of 0.05 bodyweights.

**Contribution of joint actions at the hips**

Table 7.7 provides a résumé indicating the contribution of hip joint actions to the performances of longswings on rings. For backward longswings the removal of this component of technique resulted in a substantial reduction in the maximum body angle attained during the performance. The mean difference in maximum body angle between
performances with and without this component was 99.0°, while the mean RMS difference was 47.5°. In all cases, as a result of omitting this component of technique, the gymnast produced insufficient rotation to complete the longswings to handstand.

Similar findings were observed for the forward longswing performances. The performances without hip joint actions produced a mean difference in the maximum body angle of the gymnast of 100.6°. The mean RMS difference in body angle between the with and without performances was 77.3°.

These influences, highlighted in the graphics sequence in Figures 7.13 and 7.14, indicate that changes in the hip angles contribute significantly to the performance of backward and forward longswings on rings. If the maximum body angle attained by the gymnast is used to represent the magnitude of a component's contribution to performance, these results show that hip angle changes contribute considerably more to performance than changes in the knee angle. Similarly, the extensive changes in the hip angle made during forward longswings contribute more to performance than changes in the knee angle for the techniques used during these particular longswings.

Table 7.7. Maximum body angles and peak combined shoulder forces experienced by the gymnasts during longswings with and without hip joint actions

<table>
<thead>
<tr>
<th></th>
<th>backward longswings</th>
<th></th>
<th>forward longswings</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K15</td>
<td>A20</td>
<td>A21</td>
<td>K03</td>
</tr>
<tr>
<td>hip joint actions</td>
<td>with</td>
<td>without</td>
<td>with</td>
<td>without</td>
</tr>
<tr>
<td>maximum body angle (°)</td>
<td>368.3</td>
<td>243.4</td>
<td>359.6</td>
<td>290.8</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-7.79</td>
<td>-7.94</td>
<td>-6.98</td>
<td>-10.16</td>
</tr>
<tr>
<td></td>
<td>K03</td>
<td>K05</td>
<td>A09</td>
<td></td>
</tr>
<tr>
<td>hip joint actions</td>
<td>with</td>
<td>without</td>
<td>with</td>
<td>without</td>
</tr>
<tr>
<td>maximum body angle (°)</td>
<td>-365.9</td>
<td>-267.4</td>
<td>-370.8</td>
<td>-283.2</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-6.69</td>
<td>-7.81</td>
<td>-5.91</td>
<td>-7.76</td>
</tr>
</tbody>
</table>

Figure 7.15 presents the body angle time histories for backward longswing A21 and forward longswing K03 both with and without joints actions at the hips. The contribution
of this component to the performance of these two longswings is representative of those determined for all of the evaluated longswings (Table 7.7).

Figure 7.13. Graphics sequences showing the contribution of joint angle changes at the hips to the performance of backward longswing A21.

Figure 7.14. Graphics sequences showing the contribution of joint angle changes at the hips to the performance of forward longswing K03.
The omission of angle changes at the hips during longswing performances also had a substantial effect on peak forces experienced by the gymnasts (Table 7.7). For backward longswings the peak combined shoulder joint forces increased by 1.78 bodyweights when hip angle changes were removed from the performance. Similarly for forward longswings an increase of 1.29 bodyweights was observed when joint actions at the hips were removed.

Angle changes at the hips, especially prevalent through the bottom of the swing, therefore facilitate a reduction in the peak forces which gymnasts experience when swinging from handstand. This reduction is highlighted in Figure 7.16 which presents the time histories of combined shoulder joint forces throughout the two longswings both with and without the component of hip angle changes. The mechanics underlying this reduction in peak forces were demonstrated using the planar two segment simulation model (Section 5.4.2).

Figure 7.15. Time histories of body angle ϵ for backward longswing A21 and forward longswing K03 with and without changes in the hip angle.

Figure 7.16. Time histories of combined shoulder tension for backward longswing A21 and forward longswing K03 both with and without changes in hip angle.
Besides altering the performances and peak forces experienced by the gymnasts the removal of joint angle changes at the hips also serves to alter the hip torques required by the gymnasts to produce the performance. Figure 7.17 presents time histories for combined hip joint torques during backward and forward longswings with and without this component of technique. The hip joint torques for these longswings are used to highlight the alteration in the magnitude of joint torques required to produced the performance as a direct result of omitting actions at the hip joints.

Since the performances deviate substantially in the ascending phase, only the descending phase and the initial section of the ascent are considered when highlighting the major differences in performance produced as a result of the hip component of technique. Essentially, without actions at the hip joints, the peak torques required at the hips were increased in both backward and forward longswings (Figure 7.17). The hip joint actions were isometric, since no angle changes occurred. However, similar increases in the magnitudes of torques at the shoulder joints were also produced as a result of removing actions at the hip joints.

These results indicate that when the gymnast utilises appropriately timed joint angle changes at the hips during longswings he reduces the joint torques required to produce the performance. In effect using joint angle changes at the hips enables a gymnast to perform the element with less effort, or reduces the strength required for the performance.

**Contribution of shoulder elevation joint actions**

The contribution of changes in the shoulder elevation angles to longswing performance is shown in Table 7.8. For forward longswing K05 the gymnast swung in a direction opposite to that of the actual performance. This simulation was therefore ignored in subsequent analyses.
In similarity to the results regarding the hip component of technique the removal of changes in the shoulder elevation angle from the gymnasts' techniques substantially altered their longswing performances.

Table 7.8. Maximum body angles and peak combined shoulder forces experienced by the gymnasts during longswings with and without shoulder elevation angle changes

<table>
<thead>
<tr>
<th>shoulder elevation actions</th>
<th>backward longswings</th>
<th>forward longswings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K15</td>
<td>A20</td>
</tr>
<tr>
<td>maximum body angle (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with</td>
<td>368.3</td>
<td>224.8</td>
</tr>
<tr>
<td>without</td>
<td>235.8</td>
<td>235.8</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with</td>
<td>-7.79</td>
<td>-9.01</td>
</tr>
</tbody>
</table>

In terms of maximum body angle attained by the gymnast, performances with and without this component of technique differed by 113.8° (mean) for backward longswings and 109.7° (mean) for forward longswings. The mean RMS differences in body angle between performances with and without this component were 74.9° and 88.1° for backward and forward longswings respectively. The large contribution to longswing performances from changes in the shoulder elevation angle is highlighted in the graphics sequences shown in Figures 7.18 and 7.19 and in the body angle time histories provided in Figure 7.20.

The RMS differences in body angles indicate that this component of technique has a slightly greater contribution to performance than angle changes at the hip joint. The results also highlight that both components of technique are more significant than angle changes at the knees for both types of longswings.
Figure 7.18. Graphics sequences showing the contribution of shoulder elevation angle changes at the shoulders to the performance of backward longswing A20.

Figure 7.19. Graphics sequences showing the contribution of shoulder elevation angle changes at the shoulders to the performance of forward longswing K05.
When changes in the shoulder elevation angle were removed, peak combined shoulder joint forces increased by 2.17 bodyweights for backward longswings (Figure 7.21). This effect is larger than that observed when the hip joint actions were omitted from backward longswings.

A slight decrease in peak shoulder joint forces was produced as a result of removing shoulder elevation angle changes for longswing K05. However, the removal of this component for longswing A09 resulted in a substantial increase (6.86 bodyweights) in peak forces at the shoulders. This result indicates that the influence of changes in technique are specific to the gymnast's initial technique, making it difficult to generalise on occasions.

The massive change in performance which resulted from removing shoulder elevation angle changes manifests itself in the joint torques required by the gymnast to
perform the predicted performances. Often, in these hypothetical situations, the gymnast is not physically able to produce the large joint torques required to perform the simulated performances. Since the simulation model calculates the joint torques it is possible to determine whether a gymnast would be able to complete the joint angle changes. In the case of omitting shoulder elevation angle changes, the peak combined elevation shoulder torque (torque³) required to produce the resulting performance is excessive (Figure 7.22). Data derived using isokinetic dynamometers concerned with the maximum shoulder elevation torques of an elite male gymnast show that in reality the gymnast would not be able to produce shoulder torques of this magnitude (Hiley, 1998; King, 1998).

![Figure 7.22. Time histories of combined shoulder elevation torque (torque³) for backward longswing A20 and forward longswing K05 both with and without shoulder elevation angle changes.](image)

However, this finding still has great importance to understanding the role of joint angle changes during longswings. Since joint torques and peak joint forces are typically lowered when a gymnast produces changes in the hip and shoulder elevation angles as he passes beneath the rings the strength limitations of the gymnast are not superseded. Hence, a possible reason for performing extensive joint angle changes as the gymnast swings beneath the rings is to make the performance humanly possible.

**Contribution of shoulder abduction joint actions**

Owing to the three-dimensional nature of the four segment simulation model the contribution of shoulder abduction angle changes to performance was evaluated. Table 7.9 presents the changes in maximum body angle and peak shoulder joint forces as a direct result of omitting shoulder abduction angle changes during longswing performances.

For backward longswings the mean magnitude of change in maximum body angle attained by the gymnast was 24.1°. The mean RMS difference in body angle time
histories between performances with and without the shoulder abduction component of technique was 10.3°. On an initial inspection, the data suggest the contribution of the gymnasts' lateral arm movements to performance was only slightly greater than changes in the gymnasts' knee angles. This apparently small contribution to performance is highlighted in the graphics sequences in Figure 7.23.

Table 7.9. Maximum body angles and peak combined shoulder forces experienced by the gymnasts during longswings with and without shoulder abduction angle changes

<table>
<thead>
<tr>
<th></th>
<th>K15</th>
<th>A20</th>
<th>A21</th>
</tr>
</thead>
<tbody>
<tr>
<td>shoulder abduction actions</td>
<td>with</td>
<td>without</td>
<td>with</td>
</tr>
<tr>
<td>maximum body angle (°)</td>
<td>368.3</td>
<td>375.7</td>
<td>359.6</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-7.79</td>
<td>-8.54</td>
<td>-6.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>K03</th>
<th>K05</th>
<th>A09</th>
</tr>
</thead>
<tbody>
<tr>
<td>shoulder abduction actions</td>
<td>with</td>
<td>without</td>
<td>with</td>
</tr>
<tr>
<td>maximum body angle (°)</td>
<td>-365.9</td>
<td>-328.4</td>
<td>-370.8</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-6.69</td>
<td>-6.68</td>
<td>-5.91</td>
</tr>
</tbody>
</table>

In contrast, the mean change in maximum body angle attained by the gymnast for forward longswings was 48.8°, while the mean RMS difference in body angle time histories between performances with and without this component was 36.1°. These results indicate that the contribution of lateral arm movements to the performance of the forward longswings was substantial. Figures 7.24 and 7.25 highlight the contribution to performance of lateral arm movements made during forward longswings.

By removing the lateral arm movements made by the gymnasts, peak forces increased in both backward and forward longswings (Table 7.9). For backward longswings the mean increase was 0.74 bodyweights. For forward longswings the equivalent increase was only 0.11 bodyweights. Hence, increases in forces experienced by the gymnast performing backward longswings are greater than those for the forward
longswings. However, these increases are not of the same magnitude as those observed when changes in the hip and shoulder elevation angles were removed.

![Graphics sequences showing the contribution of shoulder abduction angle changes to the performance of backward longswing K15.](Image)

![Graphics sequences showing the contribution of shoulder abduction angle changes to the performance of forward longswing K05.](Image)
Figure 7.25. Time histories of body angle $\varepsilon$ for backward longswing K15 and forward longswing K05 with and without shoulder abduction angle changes.

Changes to the magnitudes of combined shoulder elevation torque (torque$^3$) and shoulder abduction torque (torque$^2$) throughout the longswing performances were produced as a result of retaining the arms in a near parallel configuration. The time histories of combined shoulder elevation torque and single shoulder abduction torque for backward longswing K15 performed with and without lateral arm movements are shown in Figure 7.26.

For backward longswing K15 peak values for shoulder elevation torque increased by 37% (278.4 Nm to 392.8 Nm and from -254.4 Nm to -337.0 Nm) when the lateral arm movements were removed from the gymnast's technique. In addition, in order to maintain the arms in their near parallel configuration as the gymnast passed beneath the rings, a single shoulder abduction torque of -157.3 Nm was required. This represents an increase of over 73% when compared to using lateral arm movements (-90.7 Nm to -157.3 Nm).
These results show that when gymnast K used lateral arm movements he needed to produce smaller shoulder elevation torques throughout the longswing. In effect, this meant the gymnast did not have to be as strong or put in as much effort to complete the element. Hence, during backward longswings lateral arm movements reduce the magnitudes of shoulder elevation and abduction torques required to produce the element. This factor may be one reason why gymnasts adopt lateral arm movements when performing backward longswings on rings.

Coupled with this effect lateral arm movements during backward longswings reduce peak shoulder joint forces. This may lower the injury potential for the gymnast and may also reduce the discomfort often felt by gymnasts performing longswings on rings (Caraffa et al., 1996).

When lateral arm movements were not utilised during forward longswings the peak shoulder elevation torque was significantly increased to a value which cannot be reproduced by an elite gymnast (Figure 7.27) (Hiley, 1998; King, 1998). In addition, a similar increase in the peak shoulder abduction torque to that observed during the backward longswing was produced. It is clear that such large increases in the shoulder joint torques cannot be produced by elite gymnasts.

![Figure 7.27. Time histories of combined shoulder elevation torque (torque) and single shoulder abduction torque (torque) for forward longswing K05 with and without shoulder abduction angle changes.](image)

These results indicate that lateral arm movements are required to complete forward longswings on rings. The anatomical and strength limitations of a gymnast's shoulders may be overcome by moving his arms laterally and circling them to the front. It is concluded that these are the reasons for gymnasts using lateral arm movements to produce forward longswings on rings.
7.4.4 Conclusions

All of the components of technique which were examined were found to contribute to some extent for either one or both types of longswings.

Changes in the gymnasts' knee angles, although deductible under the judging criteria of the F.I.G. (1997), were shown to contribute extensively to forward longswing performances. A similar contribution for knee angle changes was not observed for the backward longswings.

When reflecting on the knee angle changes observed from the actual performances (Section 5.5.3) these findings are not surprising. Little knee bend was observed for the backward longswings, which in itself indicates the contribution of knee angle changes to performance is negligible. However, the magnitude of changes in the gymnasts' knee angles during forward longswings was much greater. The average range of changes in the knee angles during forward longswings was 49.5° (Table 5.17). In addition, these changes occur exclusively as the gymnast swings beneath the rings and during the early phase of the ascent (body angle $\epsilon$ from $-160^\circ$ to $-240^\circ$).

Essentially knee angle changes contribute to the gymnasts' forward longswing performance. Without them the interaction between the gymnasts and rings cables is altered, meaning the gymnasts do not attain the near vertical orientation before they circle their arms (Figure 7.12). Furthermore, the hypothetical simulations imply that if the gymnasts were to remove this deductible component of technique they would also have to alter other components of technique to make up for the loss of one which helps the performance.

For both types of longswings the contribution of angle changes at the hip joints to performance was extremely large. Without this component of technique the gymnasts were not able to complete the full $360^\circ$ of rotation required for the longswings (Table 7.7). For forward longswings this component of technique may be considered twice as important as that of the knee.

From the hypothetical simulations it can also be inferred that this component of technique also contributes to performance by reducing the peak forces experienced by the gymnasts. Without using angle changes at the hips the forces at their shoulder joints increased by around 1.5 bodyweights. Such an increase in force would lead to a greater chance of injury (Nissinen, 1995). Furthermore, by using actions at the hip joints the joint torques required to perform the longswing are reduced through the bottom of the swing. In effect this means the gymnasts are able to perform the element with less effort, since the actions at the hip joint reduce the joint torques required to produce the technique.

The contribution of shoulder elevation angle changes to both types of longswings was considered slightly greater than that of the hip component of technique. However,
from the results it is speculated that if a gymnast does not fully utilise both of these components of technique, for either longswing, he is unable to complete the longswings. The requirement for the combination of these two components of technique for elite performance was stated by Nissinen (1983) and Brüggemann (1987) and is emphasised in some coaching material (Hesson, 1976). However this investigation provided evidence of the magnitude of the contribution to performance of these two components of technique.

Lateral arm movements contributed to the performance of both backward and forward longswings. For forward longswings the contribution of these movements to performance was considered slightly greater than those at the knee-joint. However, when no lateral arm movements were used during forward longswings extremely large shoulder elevation torques were required to perform the shoulder elevation angle changes. Furthermore, the gymnasts still did not attain the final handstand.

In effect, lateral arm movements must therefore be used during forward longswings for a gymnast to complete the element. Without them a gymnast is not strong enough to produce the joint torques necessary from a configuration which may be described as an extremely arched 'C' shape (Figure 7.24, fifth figure in the lower sequence). Thus, although the contribution to performance was deemed to be only as large as that of the knee, the results from the hypothetical simulations suggest that lateral arm movements are essential for the performance of forward longswings on rings unless a completely new technique is developed.

The contribution of lateral arm movements to the performance of backward longswings was more subtle than for forward longswings. On first inspection the results seem to indicate a relatively small contribution to performance. However, although changes to the performance were limited, the magnitudes of both shoulder elevation torques and shoulder abduction torques differed greatly. By adopting the lateral arm movements the peak joint torques required to complete the longswing were reduced by nearly 40%. Hence, in reality lateral arm movements may contribute to a gymnast's performance by reducing the effort required to complete the element, or by actually enabling a less strong gymnast to produce the backward longswing.

It is speculated that this is one reason why gymnasts often adopt lateral arm movements during the ascending phase of the backward longswing. In addition, the hypothetical simulations indicate that lateral arm movements performed during the descending phase of the swing may reduce the risk of shoulder injuries since peak forces are reduced when these joint actions are adopted.

On a more general level, the results concerning lateral arm movements highlight that if these components of technique are neglected or misrepresented in certain circumstances, conclusions concerned with swinging activities on rings may be incorrect.
7.5 The influence of joint action timings on longswing performance

7.5.1 Introduction

The requirement of joint actions at a gymnast's hip and shoulder joints for proficient longswing performances was identified by both Nissinen (1983) and Brüggemann (1987). This observation was confirmed in the previous investigation (Section 7.4) where the proficiency of the gymnasts' performances were significantly affected when joint actions at either the hips or shoulders were removed from their techniques.

In the cited papers the authors also remark that the elite gymnasts in the studies performed changes at the hip and shoulder joints in a precisely timed and coordinated manner. However, the non-elite gymnasts did not display the same coordinated actions at their joints, leading to a less proficient performance. Therefore, not only is the timing of the gymnast's changes in joint angles important to performance but also the coordination of angle changes at all joints.

This investigation examined the influence of mistiming the coordination of joint actions which, if correctly timed, would produce a proficient longswing. Since changes in the hip and shoulder elevation angles contribute to longswing performance (Tables 7.7 and 7.8), the influence of mistiming these joint actions on performance was examined. The effect of altering the timing of these joint actions on peak forces produced during the longswing was also evaluated.

The evaluated four segment simulation model was used to perform the hypothetical simulations which directly address Question 9 of the research questions.

7.5.2. Methods

Two longswings, one backward and one forward, were used in this investigation. These two longswings, A21 and K03 respectively, are the most accurately simulated performances by the four segment model (Table 6.4).

Two hypothetical situations using altered timings of joint angle changes were used. In the first situation the gymnasts retained the techniques they adopted for the actual longswing (evaluated) performances except at the hip joint. The time history of angle changes at the hips was forced out of coordination with the other angle changes by -0.10 s. Hence, this situation looked at the effects on performance when angle changes at the hip joint were not correctly coordinated with angle changes at the other joints. Two simulations were executed for this situation, one each for the backward and forward longswings, A21 and K03 respectively.

In the second situation the gymnasts retained their techniques used for the actual
longswings (evaluated) except for changes in the shoulder elevation angles. Like the changes in hip angle in the first situation, the shoulder elevation time history was forced out of coordination with the other angle changes by -0.10 s. Simulations performed in this situation therefore examined the effect on performance when shoulder elevation angle changes were not appropriately coordinated with angle changes at other joints. Again two simulations, one for each type of longswing, were conducted using the four segment model.

In order to determine the effect of uncoordinated joint angle changes on performance the hypothetical performances were compared to those of the evaluated longswings. The maximum body angle attained by the gymnast for each hypothetical performance was used to indicate the influence of mistimed joint angle changes during longswings. Comparisons of peak combined shoulder forces provided an insight into how mistimed joint actions alter the forces experienced by gymnasts.

7.5.3 Results

The graphics sequences provided in Figures 7.28 and 7.29 show the effect of mistiming angle changes at the hip joints for backward and forward longswings. The lack of coordination in actions at the hip joints can be easily recognised as the gymnast passes beneath the rings (fourth figure from the left). In both types of longswing the mistiming of actions at the hip joints, although by only -0.10 s, substantially reduced the proficiency of the overall performance. In both cases the gymnast did not possess sufficient rotation to attain the final handstand position.

Table 7.10 provides the maximum body angles attained by the gymnast when using 'coordinated' and 'uncoordinated' joint angle changes at the hips. For the backward longswing the mistimed hip actions resulted in a reduction of 89.2° of rotation, while for the forward longswing a 125.7° reduction in rotation was produced.

Table 7.10. Maximum body angles and peak combined shoulder forces experienced by the gymnasts using coordinated and uncoordinated actions at the hip joint

<table>
<thead>
<tr>
<th>hip actions</th>
<th>backward longswing A21</th>
<th>forward longswing K03</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coordinated</td>
<td>uncoordinated</td>
</tr>
<tr>
<td>maximum body angle (°)</td>
<td>359.2</td>
<td>270.0</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-7.77</td>
<td>-6.81</td>
</tr>
</tbody>
</table>
Changes in the gymnasts' performances as a result of mistimed actions at the hips are of the same magnitude as those observed when hip joint actions were totally removed from their technique (Table 7.7).

![Coordinated hip actions](image1)

![Uncoordinated hip actions](image2)

**Figure 7.28.** Graphics sequences showing the effect of uncoordinated actions at the hips on the performance of backward longswing A21.

![Coordinated hip actions](image3)

![Uncoordinated hip actions](image4)

**Figure 7.29.** Graphics sequences showing the effect of uncoordinated actions at the hips on the performance of forward longswing K03.
During both hypothetical longswings the peak combined shoulder joint forces decreased as a result of the mistimed hip joint actions. The mean reduction was 0.86 bodyweights. This result provides further evidence that joint actions performed at certain times during a longswing can substantially reduce peak forces experienced by the gymnast. The mechanics underlying such a reduction were determined using the two segment model (Section 5.4).

Table 7.11 summarizes the effects of mistiming changes in the shoulder elevation angle on performances and forces produced during the longswings.

Table 7.11. Maximum body angles and peak combined shoulder forces experienced by the gymnasts using coordinated and uncoordinated changes in shoulder elevation angles

<table>
<thead>
<tr>
<th>shoulder elevation actions</th>
<th>backward longswing A21</th>
<th>forward longswing K03</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum body angle (°)</td>
<td>359.2</td>
<td>-365.9</td>
</tr>
<tr>
<td>-</td>
<td>284.9</td>
<td>-347.9</td>
</tr>
<tr>
<td>peak combined shoulder force (BW)</td>
<td>-7.77</td>
<td>-6.69</td>
</tr>
<tr>
<td>-</td>
<td>-6.23</td>
<td>-10.72</td>
</tr>
</tbody>
</table>

As with the mistimed hip joint actions, a reduced coordination in shoulder elevation angle changes for a specific technique resulted in a less competent performance. This result is indicated by the 74.3° reduction in maximum body angle attained during the backward longswing and the 18° reduction for the forward longswing. The results also suggest that the performance of forward longswings was less sensitive to mistiming shoulder elevation angle changes than backward longswings. This result is contrary to the observations for mistimed angle changes in the hip joint and other influences on performance examined in these four investigations.

Peak forces experienced by the gymnast also altered as a result of mistimed shoulder elevation angle changes. However, peak forces altered in a different manner, depending on the longswing. During the backward longswing a reduction of 1.54 bodyweights was produced as a result of altering the timing of shoulder elevation actions. In contrast a greater peak combined shoulder force was produced during the forward longswing (-10.72 bodyweight or 3239 N per shoulder joint). This force represents a 55% increase in the peak forces experienced by the gymnast during the longswing as calculated by Brüggemann (1987). It is suggested that a gymnast would probably be injured as a result of such large forces. Hence, although the simulation model performs the performance, if a real gymnast mistimed his joint actions in such a manner it is highly probable that he would sustain a shoulder injury.
coordinated shoulder elevation actions

uncoordinated shoulder elevation actions

Figure 7.30. Graphics sequences showing the effect of uncoordinated shoulder elevation angle changes on the performance of backward longswing A21.

coordinated shoulder elevation actions

uncoordinated shoulder elevation actions

Figure 7.31. Graphics sequences showing the effect of uncoordinated shoulder elevation angle changes on the performance of forward longswing K03.
7.5.4 Conclusions

This investigation examined the effects of mistiming hip and shoulder elevation joint actions for a particular technique on the performance of both backward and forward longswings. Although the mistiming of these joint actions, -0.10 s in this case, may be considered small, the effect on the gymnasts' performances was substantial.

For the backward longswing the effect of mistimed hip actions on performance was comparable (90% of the difference) to that observed when actions at the hips were completely removed from the gymnast's technique. A similar finding was observed when shoulder elevation angle changes were incorrectly timed (65% of the difference). Hence, incorrectly timed joint actions may degrade a gymnast's backward longswing performance almost as much as totally neglecting the components of technique.

The results also imply that for a backward longswing the correct timing of hip joint actions is more crucial to performance than shoulder elevation angle changes. This is in contrast to findings regarding the contribution of each component of technique to performance. One possible explanation for this may be the relative angular velocities of the hip and shoulder elevation angles as the gymnast swings beneath the rings. Since the hip angle decreases more vigorously than the shoulder elevation angle during this phase of the swing, changes to the timing of hip actions may lead to a greater alteration in the motion of the gymnast than changes in the timing of the shoulder angle.

For the forward longswing different effects of mistimed joint actions were observed, though both resulted in less competent longswing performances by the gymnasts. The importance of correctly timed hip angle changes for a technique was shown to be almost as great as the component itself. This finding is similar to that for the backward longswing.

A contrasting result was observed for uncoordinated shoulder elevation angle changes (Table 7.11 and Figure 7.31), which produced only a 18° difference in maximum body angle. A major effect of altering the timing of this component of technique, however, was the large increase in peak forces acting at the shoulders of the gymnast. It is proposed that a gymnast could not withstand such large forces without sustaining an injury to the shoulder region. Hence, the correct timings of joint actions are critical to maintain the forces experienced by the gymnast at a tolerable level without increasing the injury potential.

Although the investigation did not determine the time interval over which joint angle timings may be mistimed for a particular technique, the results highlight that without major adjustments to technique, the interval must be less than 0.10 s. During the initial stages of a longswing this time interval equates to only 5° or 10° of body angle rotation. However, at the end of the descending phase and during the start of the ascending phase this time interval is equivalent to nearly 40° of body angle rotation. This
provides an indication of the temporal window inside which gymnasts must be able to coordinate their technique.

In a more general sense, this investigation confirms the findings of Nissinen (1983) and Brüggemann (1987) who highlighted that not only were hip and shoulder joint actions important to elite gymnasts' performances but also the coordination of these joint actions. This investigation provided an indication of the level of importance of coordinated actions for performance and reducing peak forces experienced by the gymnasts.
CHAPTER 8

SUMMARY AND DISCUSSION

8.1 Introduction

The purpose of this study was to increase knowledge and understanding of mechanical factors which contribute to the performance of longswings on rings. This chapter consolidates the findings of this study by re-addressing each question posed in Chapter 1. In doing so, the findings of the study are placed into the wider context of the present understanding of contributions to performance and enhance this understanding.

Limitations of the simulation models and other aspects of the study are identified and appraised. Possible solutions to these limitations are discussed with a view to future investigations.

Finally, possible investigations and applications of the four segment three-dimensional simulation are contemplated. Difficulties which may arise for these potential investigations are also considered, given the limitations associated with this study.

8.2 Addressing the questions

The nine questions posed at the beginning of this thesis are specifically addressed in this section.

Question 1

What inherent problems are faced by a gymnast swinging on rings?

Fundamental problems confronting a gymnast swinging on rings were determined using the rigid computer simulation model (Section 5.2).

One problem is the interaction between the gymnast and rings cables, which has been alluded to in coaching articles (Fukushima & Russell, 1980) and in the text of Smith (1982). Effectively the gymnast and rings cables combine to form a double pendulum. Creating this double pendulum means the exact interaction between a gymnast and rings cables alters with the inertial characteristics of the gymnast. This increases the difficulty in predicting the motion of a gymnast on rings. Evidence of the varied interaction between gymnast and rings cables was provided by an average 13° reduction in the amplitude of swing from an initial handstand position for 14 different
modelled gymnasts. The reduction in swing occurred without a decrease in the mechanical energy of the gymnasts, which demonstrates the interaction alone creates problems for a gymnast attempting to swing to handstand.

Investigations conducted using the evaluated four segment, three-dimensional simulation model also demonstrated the inherent interaction between the gymnast and rings cables. If changes to a gymnast's body configuration are altered, the interaction between gymnast and rings cables is also altered, and typically results in a less proficient performance. Furthermore, the exact extent of this interaction was shown to alter with the elasticity of the rings apparatus.

To summarize, many factors alter the inherent interaction between the rings cables and gymnast. These factors include the inertial characteristics, technique and elasticity of the gymnast, together with the elasticity of the rings apparatus.

A second fundamental problem faced by gymnasts swinging on rings is the magnitude of forces which he could potentially experience. During the descending phase of the swing, tension in the rings cables becomes close to zero. When this occurs a gymnast is almost falling freely under gravity. However, the constraint placed on a gymnast's motion by the rings cables compels a gymnast to rotate extremely quickly as he swings beneath the rings. With no form of elasticity or changes to his configuration a gymnast could be expected to experience forces as large as 23 bodyweights. Since a gymnast cannot tolerate such large forces without being injured, this indicates that mechanisms within the gymnast and rings apparatus must reduce the peak forces produced during longswings.

Question 2

Do the elastic structures specifically incorporated into the rings apparatus by manufacturers of gymnastics equipment reduce the peak forces experienced by gymnasts?

Primarily this question was addressed using the two-dimensional simulation model representing the elastic properties of the DEDs and horizontal beam of the rings frame (Section 5.3). Using realistic stiffness and damping values in the springs representing the elasticity of these structures, the peak force experienced by a gymnast performing a swing from handstand was close to 19.0 bodyweights. This represents a reduction of nearly 4.0 bodyweights due to the obligatory elasticity incorporated into the rings apparatus, which is equivalent to an 18% reduction.

However, peak combined forces experienced by gymnasts swinging from handstand typically range from 6.5 bodyweights (recorded in this study) to 11.0 bodyweights (Nissinen, 1995). Mechanisms which reduce the peak forces experienced by gymnasts performing longswings, other than those specifically incorporated into the rings apparatus under F.I.G. directives, must therefore be present. These mechanisms, examined in more
detail for subsequent questions, include additional elasticity within the rings apparatus and gymnast and changes in the gymnast's body configuration during the swing.

**Question 3**

Can changes in a gymnast's body configuration while swinging underneath the rings reduce the peak forces experienced by the gymnast? If they can; what is the mechanics behind the reduction in force?

Using the two segment simulation model, appropriately-timed joint actions were shown to reduce the peak force experienced by a gymnast swinging from handstand. This finding is consistent with those expressed by Sprigings et al. (1998), and the speculation of Hiley (1998).

The results from these simulations suggest there is a large potential for reducing peak forces by this mechanism. For example, using an arching technique at the shoulder joint during the descending phase of a backward longswing, followed by a vigorous reduction in shoulder angle, reduced the peak force experienced by the gymnast by 9.5 bodyweights (Table 5.4). This reduction is equivalent to a 41% decrease, which is potentially a more efficient mechanism for decreasing peak forces than the elasticity of the DEDs.

The mechanics underlying the reduction in peak forces experienced by the gymnast consisted of producing large eccentric joint actions, which dissipate energy from the system. Eccentric joint actions occur when joint torques exerted by the gymnast result in a joint angular velocity in the opposite direction to that which the torque acts. For a swing from handstand into a backward longswing the potential for large eccentric joint actions was produced by the gymnast adopting an arched configuration during the descent. During actual performances gymnasts also perform this arching, suggesting this mechanism contributes to the reduction in peak forces. The simplicity of the two segment model enabled the mechanics to be more easily determined and understandable. This was one purpose for developing this simple model of a gymnast swinging on rings.

However, there are several limitations associated with this planar model, which reduce the external validity of these results. The most prominent of these is the rigid nature of the model. Since the model possesses no elastic components the angular velocities reached by the gymnast are 50% greater than in actual performances. In turn, this increases the potential for the gymnast to lose mechanical energy using eccentric joint actions, and therefore overestimates the potential of this mechanism for decreasing peak forces.

A more accurate evaluation of the potential of joint actions for reducing peak forces may be determined using the results concerning the contributions to performance from the four segment model. When changes in either the shoulder elevation angle, or hip angle,
were omitted during backward longswings, the peak combined shoulder forces increased by 1.5 bodyweights and 2.0 bodyweights respectively. Combining these effects suggest that joint angle changes performed as the gymnast swings beneath the rings contribute to a reduction in peak forces of at least 3.5 bodyweights. Hence, this mechanism for reducing the peak forces experienced by gymnasts during longswings on rings can be considered to be as effective as the obligatory elasticity of the rings apparatus.

**Question 4**

Can a gymnast produce a backward longswing to still-handstand on rings? What is the mechanics explaining the production of the longswing?

Elite gymnasts are able to complete longswings to handstand. However, invariably the gymnasts and cables possess some angular motion, which is deemed a technical fault by the F.I.G. judging criteria. The motion in handstand is illustrated in the kinematic data of backward and forward longswings (Section 5.5.3).

Using a torque driven two-dimensional simulation model of a three segment gymnast swinging on rings, Sprigings et al. (1998) showed that a longswing to a motionless handstand could not be performed using a two pulse muscular control strategy. These findings suggest it is difficult for gymnasts to complete a longswing to a still handstand.

A similar investigation was conducted to determine whether a longswing to still handstand could be produced. Using a different approach to the modelling procedure, the model, a two segment representation of a gymnast swinging on rings, was joint angle driven. It was found that using two joint angle changes the longswing could be completed but the gymnast still possessed some motion. When energy loss from the system was included to represent energy dissipation through the damped elastic components of the rings apparatus, the gymnast nevertheless completed the longswing. However, the modelled gymnast still possessed some motion. These results are consistent with those of Sprigings et al. (1998), though they were obtained using a different approach.

The mechanics underlying the production of a backward longswing was described in terms of concentric and eccentric joint actions acting to alter the mechanical energy of the system and interaction between the gymnast and rings cables. In the case of the gymnast swinging from handstand, little change in the total mechanical energy of the system occurred. Joint angle changes were concerned with altering the path of the gymnast's mass centre to reach the handstand.

When the energy loss was incorporated in the investigation large concentric joint actions were used extensively. These joint actions are associated with increases in mechanical energy. Hence, for this situation changes in the gymnast's configuration were
involved with increasing energy whilst overcoming the interaction of the rings cables to reach the final handstand. The simplicity of the two segment model allowed these features to be observed.

In reality, it is likely that for the production of longswings on rings, changes in a gymnast's body configuration are involved with both of these aspects: altering the interaction with the rings cables and compensating for energy loss through the damped elastic components of the rings apparatus.

Limitations of the approach used to address Question 4 include the rigid nature of the two segment model, the use of a single modelled joint; limiting the gymnast to two joint angle changes (a reduction followed by an enlargement), and the unrestricted strength of the modelled gymnast. With more joints, the modelled gymnast would be allowed to alter his interaction with the rings cables in a greater variety of ways. Similarly more joint angle changes, such as an enlargement, reduction, and enlargement of a joint angle, would allow further changes to this interaction. By increasing the complexity in these ways the model more closely represents a gymnast. Finally, a significant omission from the two segment model and that of Sprigings et al. (1998) is the lateral arm movements. Since movements of the arms laterally play a significant part in a gymnast's technique (Question 7) they should not be neglected from future investigations concerning technique.

**Question 5**

What techniques are actually used by elite gymnasts to perform backward and forward longswings?

Using a three-dimensional video analysis, descriptions of techniques used by two elite gymnasts were determined. Such descriptions are currently not documented in the literature.

The techniques used for backward longswings started with arching during the descending phase, with shoulder elevation and hip angles typically reaching values of $220^\circ$ and $210^\circ$ respectively. The gymnasts' arms also moved laterally during this phase. As the gymnasts swung beneath the rings the hip angle and shoulder elevation angle vigorously decreased and the gymnasts adopted piked configurations. To illustrate, the hip and shoulder elevation angles were typically $120^\circ$ and $100^\circ$ respectively at a body angle of $210^\circ$. During the ascending phase of the swing the gymnasts increased their hip and shoulder elevation angles, while also moving their arms laterally to a shoulder abduction angle of around $50^\circ$. From this wide arm configuration the gymnasts brought their arms in while simultaneously changing the hip and shoulder elevation angles for the final handstand.

During the descending phase of the forward longswings the gymnasts typically
produced a dished shape, with the shoulder elevation and hip angles at 110° and 165° after a quarter of the swing (body angle -90°). Typically the lateral movements of the gymnasts' arms during this phase were small (25° shoulder abduction). As the gymnasts passed through the bottom of the swing the hip angle and shoulder elevation angle rapidly decreased. The gymnasts quickly produced arched configurations, where maximum hip and shoulder elevation angles were generally 230°. During the early stages of the ascending phase the gymnasts also started to move their arms laterally. By a body angle of -300° the majority of the arched configuration was removed and replaced by an extremely wide arm configuration (nearly 90° shoulder abduction). The gymnasts continued to rotate in this configuration until almost all of the rotation required to complete the swing was produced. During the last part of the element the major aspect of technique was the lateral movements of the arms being brought closer together.

These descriptions indicate the extensive use of lateral arm movements to the performance of backward and forward longswings on rings. However, the magnitude of the contributions and the mechanical reasons for their use cannot be gauged from these descriptions alone.

Question 6
Do any other structures of the rings apparatus possess elastic properties? What influence does the overall elasticity of the rings apparatus have on longswing performance and peak forces experienced by the gymnasts?

From the three-dimensional video and cable tension analysis (Chapter 4) two structures of the rings apparatus, other than the DEDs, were identified as possessing elastic qualities. These structures are the rings cables and the rings frame oscillating in an anterior-posterior direction (Figure 3.17).

Results from the video analysis indicate the rings cables extend by approximately 0.04 m when a gymnast performs a longswing. Since the combined vertical deformation of the DEDs and horizontal beam of the rings frame was almost 0.03 m, extension of rings cables was of a similar magnitude. This may suggest the influence of the rings cables' elasticity on reducing peak forces is similar to the obligatory elasticity of the rings frame. For this reason the elasticity of the rings cables was incorporated into the four segment model.

The anterior-posterior motion of the top of the rings frame displayed an oscillatory amplitude of slightly more than 0.02 m. This motion is initially created by the large forces in the rings cables acting to jerk the rings frame in either a forward or backward direction, depending on the type of longswing and the direction of swing. The subsequent oscillations, or forced vibrations, may be caused as a result of the guide wires which hold the rings frame in suspension (Figure 3.17).
The influence of the overall elasticity of the rings apparatus on longswing performance was shown to be substantial. For a particular technique, a reduction of 25% in the stiffness of the rings apparatus eventually resulted in a 16° difference in body angle rotation for backward longswings and a 40° difference for forward longswings. Thus, changes in the elasticity of the rings apparatus do not result in uniform changes to performance. The effects are dependent on the magnitude of change in the apparatus' elasticity and the element being performed. This result has implications for gymnasts performing on different rings apparatus, for example going to a competition venue away from their clubs. In order to produce competent longswings they must be able to alter their technique appropriately for each element.

The change in performance was partly accounted for by alterations in the interaction between the rings cables and gymnast. Modifications in the transfer and dissipation of mechanical energy between the gymnast and the elastic components of the rings apparatus were also noted to contribute to changed performances.

Modifications to the peak forces experienced at the gymnasts' shoulder joints as a result of substantially altering the elasticity of the rings frame were not large. With a 25% reduction in the stiffness of the rings apparatus peak combined shoulder forces were found to reduce by less than 0.20 bodyweights. The method proposed by Caraffa et al. (1996) to reduce the potential of injury to the shoulders may therefore not be successful unless dramatic changes (>25%) to the elasticity of the apparatus are introduced. Larger changes to the elasticity of the apparatus will also have greater effects on performance for a particular technique of longswing.

A final mechanism noted to decrease peak forces during longswings is the elasticity of the gymnast. Gymnasts extend by 0.15 m during a longswing, part of which is due to the large forces they experience as they swing beneath the rings. This extension will decrease the peak forces they experience using a similar mechanism described for the DEDs. From the results concerning other mechanisms which facilitate the reduction in peak forces, it is speculated that extension of the gymnast has the same potential for decreasing the peak forces as the DEDs. However, further investigations are needed to clarify this speculation.

Question 7

What are the relative contributions of actions at the knee, hip and shoulder joints to the performance of backward and forward longswings?

The relative contributions of angle changes at the knees, hips and shoulders have not been previously investigated. In addition, the two-dimensional nature of previous experimental and theoretical studies has neglected any contribution made by lateral arms movements to longswing performance (Brüggemann, 1987; Nissinen, 1983;
Sale & Judd, 1974; Sprigings et al., 1996; Sprigings et al., 1998). Using the four segment three-dimensional simulation model the contributions to performance of angle changes at each joint were evaluated.

For backward longswings the hip and shoulder elevation angle components of technique are essential to performance. It was shown that the relative contributions to performance were similar, though hips contributed slightly less. With either of these components removed the gymnast produced at least $99^\circ$ less rotation than required to complete the longswing.

The contribution of knee angle changes in backward longswings was shown to be insignificant. This finding is in accordance with judging criteria on rings which regards bending at the knees as a technical fault and deducts accordingly.

The contribution of lateral arm movements to performance, in terms of the rotation of the gymnast to handstand, was less substantial than contributions made by the hips and shoulder elevation angles. Omitting this component resulted in $24^\circ$ difference in the maximum body angle reached by the gymnasts, which may be regarded as important. However, in all cases the gymnasts still produced the $360^\circ$ of body angle rotation to complete the swing. Thus, in contrast to hip and shoulder elevation components, the contribution to the production of rotation from lateral arm movements is insignificant.

By comparing shoulder joint torques for longswings with and without lateral arm movements a possible reason for their use was established. Evidence strongly suggests that their contribution to performance is concerned with the magnitude of joint torques required to complete the longswing. By using lateral arm movements the peak shoulder torques a gymnast has to exert to complete a longswing are reduced by nearly 40%. Effectively this means lateral arm movements enable a gymnast to put less effort into a performance, or alternatively he can be less strong and still be able to complete the element.

For forward longswings all components of technique were shown to contribute to performance. Hip and shoulder elevation angle changes are vital for the performance. Both components contribute by producing some of the required $360^\circ$ body angle rotation to complete the longswing. With either of these components removed between $99^\circ$ and $133^\circ$ less body angle rotation is produced.

The lateral arm movements contributed to performance by increasing the body angle rotation of the gymnast by $49^\circ$. It was also found that if lateral arm movements were not used by the gymnasts, much greater shoulder elevation torques ($>1000$ Nm) were required to perform the longswing. Since such large torques cannot be produced by elite male gymnasts (Hiley, 1988; King, 1998), this provides evidence to suggest that without lateral arm movements it is probable that forward longswings could not be performed.

Although considered a technical fault, angle changes at the knee were also shown to
contribute to performance in forward longswings by adding to the gymnasts' total body angle rotation. Thus, if these particular gymnasts wish to remove this component of technique they would have to modify other aspects of technique in order to compensate for the removal of knee actions.

**Question 8**

Can two different gymnasts adopt exactly the same techniques for backward and forward longswings and produce proficient performances for each longswing?

Using the four segment model, different standards of performance were produced by two different gymnasts who adopted the same techniques for backward and forward longswings. With two gymnasts possessing different inertial properties the differences (RMS) in the body angles $\epsilon$ throughout the performances were found to be $20^\circ$ for backward longswings and $44^\circ$ for forward longswings (Figure 7.1 and 7.2).

The inertial characteristics of the gymnasts were found to be more critical to performance than the elasticity of the apparatus on which the performance was completed. In answer to Question 8, therefore, it is highly unlikely that two gymnasts who possess different inertial characteristics and adopt the same technique will both produce proficient longswings. The difference in performance occurs because different inertial characteristics of the gymnast alter the interaction between gymnast and rings cables. This effectively changes the body orientation at which certain joint angle changes occur, and subsequently alters their effects on the motion of the gymnast.

However, qualitative similarities in technique were observed from the video analysis. This suggests that a general technique may be adopted by different gymnasts but the exact timings and intricacies of the techniques are particular to an individual gymnast. These slight variations enable both gymnasts to produce competent performances.

**Question 9**

How critical is the coordination of the joint actions which constitute a gymnast's technique to his overall longswing performance?

The need for coordinated changes in the hip and shoulder elevation angles for elite longswing performances was identified by Nissinen (1983) and Brüggemann (1987) by examining differences in techniques used by elite and non-elite gymnasts. Neither of these papers detailed the effect of performing "uncoordinated" changes in the gymnasts' body configuration on performance, or the manner in which performance would alter.

Using the four segment model the coordination of hip and shoulder elevation angle changes was shown to be critical to longswing performances. This result is consistent
with the two previous studies. A 0.1 s delay in the timing of hip angle changes with respect to the other components of technique resulted in the gymnasts producing $90^\circ$ less rotation for backward longswings and $125^\circ$ for forward longswings. The same time delay for shoulder elevation angles resulted in $74^\circ$ and $18^\circ$ less rotation in the performances of backward and forward longswings respectively. These results highlight the significance of coordinated joint actions for longswing performances.

This investigation also showed that for a given technique which typically produces an excellent performance, a gymnast may not mistime the major components by more than 0.1 s and still produce a competent longswing, unless he subsequent alters other components to compensate for the mistiming. This time interval corresponds to $35^\circ$ of body angle rotation as the gymnast swings beneath the rings but only $10^\circ$ when near the handstand position. This tolerance in the timing and coordination of technique may be considered to be small.

**Proposed developments of the four segment simulation model**

Results from the four segment simulation model were used to provide assessments of contributions to the performance of backward and forward longswings. In the evaluation procedure the general accuracy of the model was considered to be sufficient to perform these more detailed investigations. However, of the indicators used to assess the accuracy of the model the wrist to ankle length of the gymnast was the weakest simulated aspect.

The reduced accuracy in simulating this aspect of swinging on rings forms one limitation of the four segment model. Greater accuracy may be obtained with modifications to the structure of the present four segment model.

Wrist to ankle length time histories suggest that gymnasts actively extend their bodies during the descending phases of backward and forward longswings. Since the four segment model presently represents the elasticity of a gymnast by a damped linear spring, the model is only able to represent passive changes in the wrist to ankle length. By introducing an active spring into the model, where forces can be generated in the spring at a given time, direction and over a specified duration, the muscular activity of the gymnast may be better represented. Such a modification may improve the accuracy of the model in this aspect, though more subject specific parameters to define this active spring would be required.

A second possible area for improvement includes the modelling of all elastic components in the system. At present the elasticity in the real system is represented by damped linear springs in the model. The analyses of the DEDs and anterior-posterior movement of the rings frame indicate this assumption is sufficient for these structures. However the wrist to ankle length data strongly suggest the elasticity of the gymnasts is
non-linear, with the stiffness of the gymnasts increasing as the extension increases. By incorporating this into the simulation model further improvements to its accuracy may be made.

By considering these developments, the findings of the four segment model in its present state are not discredited. The evaluation procedure highlighted the model estimated tension in the rings cables, the rotation of gymnast and the cables to within 194 N, 5.32° and 4.09° (RMS) of the actual longswing performances, respectively. These proposed developments therefore serve only to highlight the potential for improvement to the model.

8.3 Future investigations

Since the four segment model is regarded as sufficiently accurate to simulate a gymnast swinging on rings, future investigations using this model may be considered. The model, for example, may be used to assess the contribution of known components of technique to the performance of other swinging elements. For those elements where no common technique is employed by elite gymnasts, the optimal technique may be determined using the model.

Optimisation of technique

Where more than one general technique is used by elite gymnasts it is possible that one technique has benefits which the others do not. Using the four segment model the optimum technique for a swinging element may be determined using hypothetical joint angle changes, such as those implemented in the two segment model.

The backward longswing may be used to illustrate this point. From personal observations it is clear that different gymnasts use lateral arm movement to different extents during the ascending phase of the longswing. Although this study highlighted a mechanical reason for adopting such movements of the arms, what remains unknown is the extent to which the arms should move laterally during this phase to make the performance 'optimal'. Hence, what is the optimum amount of lateral arm movements a gymnast should use during the ascending phase of a backward longswing? A response to this question may be determined using the simulation model. However, what are the considerations for this task?

In order to allow the modelled gymnast to use a similar number of joint angle changes as a real gymnast, 33 joint angle time history parameters are required: 10 for the hip angle (three major angle changes), 10 for the shoulder elevation angle (three major angle changes) and 13 for the shoulder abduction angle (four major angle changes). As
changes in the knee angle result in score deductions during a performance; the knee angle may be assumed to remain at 180°.

Since the investigation would be totally theoretical, constraints regarding the gymnast must also be incorporated into the study to ensure the results are applicable. For example, the maximum tolerance of the gymnast must be included, so that only realistic performances are considered in the optimisation. Maximum tolerance may be in terms of peak forces a gymnast is able to tolerate and maximum joint torques he is able to exert. The elasticity of the rings apparatus must also be accurately reflected by the model to ensure the results are not detached from reality.

This example highlights some of the many factors which have to be considered for future investigations concerning the optimisation of technique. By no means have all the factors to be considered been identified; only a selection of what is required has been provided.

Investigations of other swinging skills

Little is understood about the contributions to performance within other swinging elements performed on rings. The lack of understanding is illustrated by the deficiency of scientific papers on other swinging elements on rings; for example the swinging skills of Honma and Johnasson (forward somersaults inside the rings to support and to swing respectively). Although the accuracy of the four segment model for simulating longswings is known, it is not known for these elements. An evaluation procedure for these elements would be required before detailed investigations could be performed. However, the potential for studying many more swinging elements on rings exists using the model in its present state.

A final note

Simulation modelling is an extremely powerful tool which can facilitate the sports biomechanist in understanding the techniques used in sporting situations, and with it the potential to improve athletic performance (Yeadon & Challis, 1994). However, without incorporating realistic constraints, such as the strength of a gymnast, or representing the key features of the sporting activity, results obtained from models may be detached from reality or even incorrect. Unlike previous studies, which have neglected three-dimensional aspects of technique, this study has shown that lateral arm movements contribute to the performance of longswings on rings and are an essential feature of a gymnast's technique.
REFERENCES


APPENDIX A

Listings of the Fortran 77 code for the RIGID model, described in Chapter 3.

PROGRAM RIGID

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This program represents a one segment gymnast connected to
a massless cable, connected to a stationary point (O).

The program uses rigid.in as its input file. An example input
file for a simulation used in the thesis is given.

Model parameters defining the inertial characteristics of the
gymnast, such as mass, are changed within the program.

The equations of motion are derived using the nomenclature and
system described in Chapter 3 for the RIGID model.

Nomenclature:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>al</td>
<td>distance from hands to mass centre of gymnast</td>
</tr>
<tr>
<td>p</td>
<td>length of cable segment</td>
</tr>
<tr>
<td>ya</td>
<td>horizontal position of mass centre of gymnast</td>
</tr>
<tr>
<td>yad</td>
<td>horizontal velocity of mass centre of gymnast</td>
</tr>
<tr>
<td>yadd</td>
<td>horizontal acceleration of mass centre of gymnast</td>
</tr>
<tr>
<td>za</td>
<td>vertical position of mass centre of gymnast</td>
</tr>
<tr>
<td>zad</td>
<td>vertical velocity of mass centre of gymnast</td>
</tr>
<tr>
<td>zadd</td>
<td>vertical acceleration of mass centre of gymnast</td>
</tr>
<tr>
<td>ma</td>
<td>mass of the gymnast segment</td>
</tr>
<tr>
<td>ia</td>
<td>moment of inertia of gymnast segment about mass centre</td>
</tr>
<tr>
<td>phc</td>
<td>cable elevation angle</td>
</tr>
</tbody>
</table>
c  phcd  : cable elevation angular velocity
 c  phcd0 : cable elevation angular acceleration
 c  phtr  : orientation of gymnast segment
 c  phtr0 : orientation angular velocity of gymnast segment
 c  phtrdd : orientation angular acceleration of gymnast segment
 c  all-dl2 : hold the equations of motion for the system
 c  dt  : time interval for integration
 c  t  : time
 c  g  : acceleration due to gravity
 c  f  : tension in the cable
 c  fmax : maximum tension in the cable
 c  pi  : pi
 c  rtd : radians to degrees conversion
 c  PE  : potential energy of the system (measured for 0)
 c  RE  : rotational kinetic energy of the system
 c  LE  : linear kinetic energy of the system
 c  TTE : total translatational energy of the system
 c  TE  : total energy of the system

c

double precision
* al, p, g,
* ia, ma, t, dt,
* ya, yad, yadd, za, zad, zadd,
* f, fmax,
* phc, phcd, phcd0, phcf, phcddf, phcddf0,
* phtr, phtrd, phtrdd, phtrf, phtrdf, phtrddf,
* phtrdd0, phcd0,
* pi, rtd,
* PE, RE, LE, TTE, TE,
* all, bl1, cll, dll, al2, bl2, cl2, dl2,
* allf, bl1f, cllf, dllf, al2f, bl2f, cl2f, dl2f

integer i, n

character output1*20
character output2*20
character output3*20
character output4*20

read the input file for output file names, initial conditions
for the simulation, and the variables for the integration
procedure

read(*,*), output1
read(*,*), output2
read(*,*), output3
read(*,*), output4
read*, phc
read*, phtr
read*, phcd
read*, phtrd
read*, dt
read* , n

open (10, file = output1)
open (20, file = output2)
open (30, file = output3)
open (40, file = output4)

set up the headings for the output files

write(10,*), t(S) phc(o) phcd phtr(o) phtrd phtrdd Ten( *N) PE TTE TE(J)

write(20,*), ya(m) za(m)

write(30,*), yad(m/s) zad(m/s) yadd(m/s2) zadd(m/s2)

write(40,*), t(S) PE RE, LE TTE TE (J)

initialise all variables

three sets of inertia parameters to represent different gymnasts

ma 61.63 ia 11.94 al 0.9206 for subject K
ma 67.30 ia 13.58 al 0.9471 for subject A
ma 62.88 ia 12.57 al 0.9103 for normalised subject

ma 62.88
ia 12.57
p 3.014
al 0.9103
pi 3.14159265358
rtd = 180.0 / 3.14159265358

initialise fmax and change all angles from degrees to radians

fmax = 0.0

phtr = phtr / rtd
phc = phc / rtd
n = n + 1

do 100, i = 1, n

t = (i-1)*dt

equations of motion for the system

equations of motion for the system

\[
al_1 = ma*al^2 + ma*al*p*cos(phc - phtr) + ia
\]
\[
b11 = ma*p*(p - al*cos(phtr - phc))
\]
\[
c11 = 0
\]
\[
d11 = ma*(g*al*sin(phtr) - g*p*sin(phc) - al*p*phc^2 * *sin(phc - phtr) - al*p*phtr^2*sin(phtr - phc))
\]
\[
al_2 = al*(cos(phtr) + sin(phtr)*tan(phc))
\]
\[
b2 = -p*(tan(phc)*sin(phc) + cos(phc))
\]
\[
c2 = 0
\]
\[
d2 = g*tan(phc) - phtr^2*al*(tan(phc)*cos(phtr) * *sin(phtr))
\]

use simultaneous equations all to d12 to calculate initial angular accelerations for the system

\[
phtrdd0 = (d12*b11 - b12*d11) / (b11*a12 - b12*a1)
\]
\[
phcdd0 = (d11 - a1*phtrdd0) / b1
\]

calculate final angles and angular velocities using the initial angles, angular velocities and accelerations

\[
phtrf = phtr + phtr^2*dt + 0.5*phtrdd0*dt^2
\]
\[
phtrdf = phtrd + phtrdd0*dt
\]
\[
phcf = phc + phc^2*dt + 0.5*phcdd0*dt^2
\]
\[
phcdf = phcd + phcdd0*dt
\]

find the average angular accelerations by calculating the final angular accelerations using the equations of motion for the system

\[
al_1f = ma*al^2 - ma*al*p*cos(phcf - phtrf) + ia
\]
\[
b11f = ma*p*(p - al*cos(phtrf - phcf))
\]
\[
c11f = 0
\]
\[
d11f = ma*(g*al*sin(phtrf) - g*p*sin(phcf) - al*p*phcf^2 * *sin(phcf - phtrf) - al*p*phtrdf^2*sin(phtrf - phcf))
\]
\( a_{12f} = a_l \cdot (\cos(\phi_{trf}) + \sin(\phi_{trf}) \cdot \tan(\phi_{cf})) \)
\( b_{12f} = -p \cdot (\tan(\phi_{cf}) \cdot \sin(\phi_{trf}) + \cos(\phi_{cf})) \)
\( c_{12f} = 0 \)
\( d_{12f} = g \cdot \tan(\phi_{cf}) \cdot \phi_{trf}^2 \cdot a_l \cdot (\tan(\phi_{cf}) \cdot \cos(\phi_{trf}) \cdot \sin(\phi_{trf})) \)

Using simultaneous equations of allf to dl2f calculate the final angular accelerations:

\[
\begin{align*}
\phi_{trddf} &= (d_{12f} \cdot b_{11f} - b_{12f} \cdot d_{11f}) / (b_{11f} \cdot a_{12f} - b_{12f} \cdot a_{11f}) \\
\phi_{cddf} &= (d_{11f} - a_{11f} \cdot \phi_{trddf}) / b_{11f}
\end{align*}
\]

Calculate the average angular accelerations using the average angular accelerations between the final and initial angular accelerations:

\[
\begin{align*}
\phi_{trdd} &= (\phi_{trdd0} + \phi_{trddf}) / 2 \\
\phi_{cdd} &= (\phi_{cdd0} + \phi_{cddf}) / 2
\end{align*}
\]

Calculate the required output:

\[
\begin{align*}
ya &= p \cdot \sin(\phi_{cf}) \cdot a_l \cdot \sin(\phi_{trf}) \\
za &= a_l \cdot \cos(\phi_{trf}) \cdot p \cdot \cos(\phi_{cf})
\end{align*}
\]

\[
\begin{align*}
yadd &= \phi_{trdd} \cdot a_l \cdot \cos(\phi_{trf}) + \phi_{trdd}^2 \cdot a_l \cdot \sin(\phi_{trf}) \\
zadd &= \phi_{cdd} \cdot p \cdot \sin(\phi_{cf}) + \phi_{cdd}^2 \cdot p \cdot \cos(\phi_{cf}) \cdot \phi_{trdd} \cdot a_l \cdot \sin(\phi_{trf}) \\
f &= (m_a \cdot (zadd + g)) / \cos(\phi_{cf}) \\
PE &= m_a \cdot g \cdot za \\
RE &= 0.5 \cdot i_a \cdot \phi_{trdd}^2 \\
LE &= 0.5 \cdot m_a \cdot (yadd^2 + zadd^2) \\
TTE &= RE + LE \\
TE &= PE + TTE
\end{align*}
\]

Write out values for the output variables to the output files in CSV format.
if (mod(i,100).eq.1) then

105 format (f6.4,' ',f10.3,' ',f8.3,' ',f10.3,' ',f7.1,' ',f7.2*
	*, ',f7.2,' ',f9.0,' ',f8.0,' ',f7.0,' ',f8.0)
115 format (f6.4,' ',f9.4,' ',f9.4)
125 format (f6.4,' ',3(f10.4,' '),f10.4)
135 format (f6.4,' ',5(f10.3,' '),f10.3)

write(10,105) t, phc*rtd, phcd, phcdd0, phtr*rtd, phtrd,
* phtrdd0, f, PE, TTE, TE
write(20,115) t, ya, za
write(30,125) t, yad, zad, yadd, zadd
write(40,135) t, PE, RE, LE, TTE, TE
endif

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
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cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
c c recalculate phc, phcd, phtr, phtrd using equations for constant
acceleration
c
phc = phc + phc*dt + 0.5*phc*dt**2
phcd = phcd + phcdd*dt
phtr = phtr + phtr*dt + 0.5*phtr*dt**2
phtrd = phtrd + phtrrd*dt

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
c c determine the value of fmax using an if statement
c
if (f.gt.fmax) then
fmax = f
end if

100 continue

write(10,*)
write(10,*)'THE MAX CABLE TENSION IS',fmax/(ma*g),' BW'

c c END END END END END END END END END END END END

APPENDIX B

Listings of the Fortran 77 code for the ELASTIC model, described in Chapter 3.

```
PROGRAM ELASTIC

Copyright (c) Mark A Brewin, 1998

This program represents a one segment gymnast swinging on a massless rigid rings cable with vertical and horizontal damped linear springs at the top of the cable representing the elasticity of the damped elastic devices (DEDs) and horizontal beam of the rings frame only.

The springs at the end of the massless rings cable are connected to the stationary point (O). The stiffness and damping coefficients are ch and cv and dh and dv respectively. For these simulations the vertical and horizontal springs possessed equal stiffness and damping characteristics.

This program is used to determine the peak combined cable tension and corresponding spring extension for differing spring stiffness damping coefficients.

The program uses elastic.in as the input file. An example input file for a simulation used in the study is provided.

Example input file elastic.in

elastic << eof | compiled version of program
angT | angle output file
posT | position output file
linT | linear output file
eneT | energy output file
sprT | spring output file
fmaxT | maximum force and spring extension output file
140000 3500 | stiffness and damping values for springs
2 | initial cable orientation angle, phc (deg)
4 | initial gymnast orientation angle, phtr (deg)
0 | initial cable angular velocity, phcd (rad/s)
0 | initial gymnast angular velocity, phtrd (rad/s)
0.00005 | time interval for integration
45000 | number of iterations
eof | end of file
```

Model parameters defining the inertial characteristics of the modelled gymnast, such as mass, are changed within the program.
The equations of motion are derived using the nomenclature described in Chapter 3 for the ELASTIC model.

Nomenclature:

- \( a_l \): distance from hands to mass centre of gymnast
- \( p \): length of cable segment
- \( y_a \): horizontal position of mass centre of gymnast
- \( y_{ad} \): horizontal velocity of mass centre of gymnast
- \( y_{add} \): horizontal acceleration of mass centre of gymnast
- \( z_a \): vertical position of mass centre of gymnast
- \( z_{ad} \): vertical velocity of mass centre of gymnast
- \( z_{add} \): vertical acceleration of mass centre of gymnast
- \( y_b \): extension of the horizontal spring
- \( y_{bd} \): velocity of the horizontal spring
- \( y_{bdd} \): acceleration of the horizontal spring
- \( z_b \): extension of the vertical spring
- \( z_{bd} \): velocity of the vertical spring
- \( z_{bdd} \): acceleration of the vertical spring
- \( c h \): horizontal spring stiffness
- \( c v \): vertical spring stiffness
- \( d h \): horizontal spring damping
- \( d v \): vertical spring damping
- \( m a \): mass of the gymnast segment
- \( i a \): moment of inertia of gymnast segment about mass centre
- \( p h c \): cable elevation angle
- \( p h c p c t \): cable elevation angle at peak cable tension
- \( p h c d \): cable elevation angular velocity
- \( p h c d d \): cable elevation angular acceleration
- \( p h t r \): orientation of gymnast segment
- \( p h t r p c t \): orientation of gymnast segment at peak cable tension
- \( p h t r m a x \): maximum orientation of gymnast segment
- \( p h t r d \): orientation angular velocity of gymnast segment
- \( p h t r d d \): orientation angular acceleration of gymnast segment
- \( a(6,5) \): array holding the equations of motion for solve
- \( b(4) \): array holding the equations of motion for solve
- \( d t \): time interval for integration
- \( t \): time
- \( g \): acceleration due to gravity
- \( f \): tension in the cable
- \( f s p r v \): tension in vertical spring
- \( f s p r h \): tension in horizontal spring
- \( f m a x \): maximum tension in the cable
- \( t f m a x \): time at fmax
- \( f m i n \): minimum tension in cable
- \( p i \): pi
- \( r t d \): radians to degrees conversion
- \( P E \): potential energy of the system (measured for 0)
- \( R E \): rotational kinetic energy
- \( L E \): linear kinetic energy
- \( T T E \): total translational energy of the system
- \( E E H \): total energy in the horizontal spring
- \( E E V \): total energy in the vertical spring
- \( T E \): total energy of the system
- \( i n r g \): initial total energy of the system
- \( f n r g \): final total energy of the system
double precision
* aI, p, g,
* a, b,
* ia, ma, t, dt,
* ya, yad, yadd, za, zad, zadd,
* f, fsprv, fsprh, fmax, tfmax, fmin,
* phc, phcd, phcdd, phcf, phcdf, phcddf, phcpc1,
* phtr, phtrd, phtrdd, phtrf, phtrddf,
* phtrd0, phcdd0, phtrmax, phtrpc1,
* yf, ybd, ybdf, zb, zbf, ybdf, zbf, zbd, zbd0,
* ybf, ybdf, ybddf, zbf, zbd, zbddf,
* ybdd0, zbdd0,
* ch, cv, dh, dv,
* pi, rtd,
* PE, RE, LE, TTE, TE, EEH, EEV, irng, fnng,
* a(6), b(4), x(4), y(4)

integer i, mm, m, nnn, n

character output1*20
character output2*20
character output3*20
character output4*20
character output5*20
character output6*20

read the input file for output file names, stiffness and damping
of the springs, the initial conditions and the variables for the
integration procedure

read(*,*) output1
read(*,*) output2
read(*,*) output3
read(*,*) output4
read(*,*) output5
read(*,*) output6
read*, cv, dv
read*, phc
read*, phtr
read*, phcd
read*, phtrd
read*, dt
read*, nnn

open (10, file = output1)
open (20, file = output2)
open (30, file = output3)
open (40, file = output4)
open (50, file = output5)
open (60, file = output6)
set up the headings for the output files

write(10,*), t(s) phc(o) phcd phcdd pht(o) phtrd p
*htrdd Force(BK)
*
write(20,*), t(s) ya(m) za(m)
*
write(30,*), t(s) yad(m/s) zad(m/s) yadd(m/s2) zadd(m/s2)
*
write(40,*), t(s) PE RE LE TTE
* EEH EEV TE(J)
write(50,*), t(s) yb ybd ybdd zb
* zbd zbdd
*
initialise all variables and set parameters

make the vertical and horizontal springs identical

three sets of inertia parameters for different gymnasts

ma = 61.63 ia = 11.94 al = 0.9206 for subject K
ma = 67.30 ia = 13.58 al = 0.9471 for subject A
ma = 62.88 ia = 12.57 al = 0.9103 for normalised subject

ma = 62.88
ia = 12.57
al = 0.9103
p = 3.014
pi = 3.14159265358
rtd = 180.0 / 3.14159265358
t = 0
g = 9.81

define fmax, tfmax, zbfmax and zbmin for spring stretch

and phtrmax for amplitude of swing

fmax = 0.0
fmin = 0.1
tfmax = 0.0
zbmax = 0.0
zbmin = 0.0
tzmin = 0.0
phtrmax = 0.0
phtrmax = 0.0
phcmax = 0.0
phcmax = 0.0
phcmax = 0.0
phcmax = 0.0
phtrpct = 0.0
phcmax = 0.0
phcmax = 0.0
phcmax = 0.0
phcmax = 0.0

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

c change all angles from degrees to radians
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

phtr = phtr / rtd
phc = phc / rtd

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

c calculate the initial yb and zb according to the initial cable angle from the input file to ensure the spring is initially in line with the rings cable
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

zb = -(ma*g)/cv
zbd = 0
yb = zb*tan(phc)
ybd = 0

nnn = nnn + 1
do 100, i = 1, nnn
t = (i-1)*dt

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

c call the equations to calculate phcdd, phtrdd, ybdd and zbdd
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

call equation(al, p, g, ia, ma, ch, cv, dh, dv, phtr, phtrd, *phc, phcdd, yb, ybd, zb, zbd, a, b)

mm = 6
m = 4
n = 4
call solve(x,a,b,mm,m,n)

phtrdd0 = x(1)
phcdd0 = x(2)
ybdd0 = x(3)
zbdd0 = x(4)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
calculate the final angles, angular velocities using initial angles, angular velocities and accelerations

calculate the final spring extensions and spring velocities using initial spring extensions, velocities and accelerations

cphtrf = phtr + phtrd*dt + 0.5*phtrdd*dt**2
phtrdf = phtr + phtrdd0*dt
phcf = phc + phcd*dt + 0.5*phcdd*dt**2
phcdf = phc + phcdd0*dt

ybf = yb + ybd*dt + 0.5*ybdd0*dt**2
ybdf = yb + ybdd0*dt
zbdf = zb + zbd*dt + 0.5*zbdd0*dt**2
zbdf = zb + zbdd0*dt

calculate the final angular and linear accelerations using equation2

call equation2(al, P, g, ia, ma, ch, cv, dh, dv, phtrf,* phtrdf, phcf, phcdf, ybf, ybdf, zbf, zbdf, a, b)
call solve(y,a,b,mm,m,n)

phtrddf = y(l)
phcddf = y(2)
ybddf = y(3)
zbddf = y(4)

calculate the average angular and linear accelerations of the system using the average between the final and initial accelerations

cphtrdd = (phtrdd0 + phtrddf) / 2.0
phcdd = (phcdd0 + phcddf) / 2.0
ybdd = (ybddd + ybddf) / 2.0
zbdd = (zbddd + zbddf) / 2.0

calculate the required output

ya = yb * al*sin(phtr) + p*sin(phc)
za = zb + al*cos(phtr) * p*cos(phc)
\[ y_{ad} = y_{bd} - \text{phtrd}^*a_{1}^*\cos(\text{phtr}) + \text{phcd}^*p^*\cos(\text{phc}) \\
\]

\[ z_{ad} = z_{bd} + \text{phcd}^*p^*\sin(\text{phc}) - \text{phtrd}^*a_{1}^*\sin(\text{phtr}) \]

\[ y_{add} = y_{bd} - \text{phtrdd}^*a_{1}^*\cos(\text{phtr}) + \text{phtrd}^*a_{1}^*\sin(\text{phtr}) + \phcd^*p^*\cos(\text{phc}) - \phcd^*p^*\sin(\text{phc}) \]

\[ z_{add} = z_{bd} + \text{phcd}^*p^*\sin(\text{phc}) + \text{phcd}^*p^*\cos(\text{phc}) \]

\[ f = \frac{(\text{m}a^*)(z_{add} + g)}{\cos(\text{phc})} \]

\[ \text{fsprv} = -c_{v}^*z_{b} - d_{v}^*z_{bd} \]

\[ \text{fsprh} = c_{h}^*y_{b} + d_{h}^*y_{bd} \]

\[ \text{PE} = \text{m}^*g^*z_{a} \]

\[ \text{RE} = 0.5^*i_{a}^*\text{phtrd}^*^2 \]

\[ \text{LE} = 0.5^*\text{m}a^*(y_{ad}^*^2 + z_{ad}^*^2) \]

\[ \text{TTE} = \text{RE} + \text{LE} \]

\[ \text{EEH} = (c_{h}^*y_{b}^*^2) / 2.0 \]

\[ \text{EEV} = (c_{v}^*z_{b}^*^2) / 2.0 \]

\[ \text{TE} = \text{PE} + \text{TTE} + \text{EEH} + \text{EEV} \]

\[ \text{if}(i.\text{eq.1}) \text{ then} \]

\[ \text{ing} = \text{TE} \]

\[ \text{endif} \]

\[ \text{if}(f.\gt.f_{\text{max}}) \text{ then} \]

\[ f_{\text{max}} = f \]

\[ t_{\text{bfmax}} = t \]

\[ z_{\text{bfmax}} = z_b \]

\[ \text{phtrpct} = \text{phtr} \]

\[ \text{phcpct} = \text{phc} \]

\[ \text{end if} \]

\[ \text{if}(f.\lt.f_{\text{min}}) \text{ then} \]

\[ f_{\text{min}} = f \]

\[ \text{end if} \]
use an if statement to obtain the maximum vertical downward extension of the vertical spring--zb

if (zb.lt.zbmin) then
  zbmin = zb
  tzbmin = t
end if

use an if statement to determine the maximum body angle attained by the gymnast

if (phtr.gt.phtrmax) then
  phtrmax = phtr
end if

write out values for the output variables to the output files in CSV format

if (mod(i,100).eq.1) then
  105 format (f6.3,' ','2(f7.3,' ','f7.3,' ','f9.4,' ','f8.3))
  115 format (f6.4,' ','f9.4)
  125 format (f6.4,' ','3(f10.4,' ','f10.4))
  135 format (f6.3,' ','f10.2,' ','f10.2,' ','2(f9.2,' ','f9.2)
  145 format (f6.4,' ','5(f10.4,' ','f10.4))
  write(10,105) t, phc*rtd, phcd, phcdd0, phc*rtd, phtrd,
* phtrd0,f/(ma*g)
  write(20,115) t, ya, za
  write(30,125) t, yad, zad, yadd, zadd
  write(40,135) t, PE, RE, LE, TTE, EEH, EEV, TE
  write(50,145) t, yb, ybd, ybdd0, zb, zbd, zbdd0
end if

use an if statement to stop the simulation near, though just after the gymnast reaches the apex of his swing

if ((phtr+0.0175).lt.phtrmax) then
  fnrg = TE
goto 9898
endif

c recalculate phtr, phtrd, phc, phcd, yb, ybd, zb and zbd using
equations for constant acceleration

phtr  = phtr  + phtrd*dt + 0.5*phtrdd*dt**2
phtrd = phtrd + phtrdd*dt
phc  = phc  + phcd*dt + 0.5*phcdd*dt**2
phcd = phcd + phcdd*dt

yb    = yb    + ybd*dt + 0.5*ybdd*dt**2
ybd = ybd + ybdd*dt
zb    = zb    + zbd*dt + 0.5*zbdd*dt**2
zbd = zbd + zbdd*dt

100 continue

c use the 9898 label to stop the swing at the apex

9898 continue

c write out the max cable tension, the vertical spring extension,
and the gymnast and cable angles and the instant in time at
which they occur

write (60,*), ch/cv, dh/dv, t(s), fmax (N), fmax(BW)
write (60,155) cv, dv, tfmax, fmax, fmax/(ma*g)
write (60,*)
write (60,*), 'zb at fmax(m), phtr(o), phc(o), phtrmax'
write (60,175) zbmax, phtrpct*rtd, phc pct*rtd, phtrmax*rtd
write (60,*)
write (60,*), 'time (s) min zb(m), phtrmax'
write (60,185) tzbmin, zbmin, phtrmax*rtd
write (60,*)
write (60,*), 'energy lost (J), inrg, fnrg'
write (60,195) (fnrg·inrg), inrg, fnrg
if (fmin.lt.0.01) then
write(60,*) 'cable tension became negative'
write(60,165) fmin, fmin/(ma*g)
endif

close(10,60)
stop
end

the following subroutines, equation and equation2, relate to
the equations of motion for the elastic model

subroutine equation(al, P, g, ia, ma, ch, cv, dh, dv, phtr,
* phtrd, phc, phcd, yb, ybd, zb, zbd, a, b)
double precision
* al, P, g,
* ia, ma, ch, cv, dh, dv,
* phtr, phtrd, phc, phcd, yb, ybd, zb, zbd,
* a(6,5), b(4)

a(1,1) = al*cos(phtr)
a(1,2) = -p*cos(phc)
a(1,3) = -l
a(1,4) = 0

b(1) = ((ch*yb)/ma)+((dh*ybd)/ma)+phtrd**2*al*sin(phtr)
* - phcd**2*p*sin(phc)

a(2,1) = al*sin(phtr)
a(2,2) = - p*sin(phc)
a(2,3) = 0
a(2,4) = -l

b(2) = ((cv*zb)/ma)+((dv*zbd)/ma)+g - phtrd**2*al*cos(phtr)
* + phcd**2*p*cos(phc)

a(3,1) = 0
a(3,2) = 0
a(3,3) = - dh*cos(phc)
a(3,4) = -dv*sin(phc)
b(3) = ch*ybd*cos(phc) - ch*yb*phcd*sin(phc) - 
* dh*ybd*phcd*sin(phc) + cv*zb*phcd*cos(phc) + 
* cv*zbd*sin(phc) + dv*zbd*phcd*cos(phc)

c

a(4,1) = p*a1*cos(phtrf) - al**2 - zb*a1*cos(phtrf)
* + a1*yb*sin(phtrf) - ia/ma
a(4,2) = p*a1*cos(phtrf) - phtrf**2 + p*zb*cos(phtrf)
* - p*yb*sin(phtrf)

b(4) = g*yb + g*p*sin(phtrf) * g*al*sin(phtrf)
* - phtrf**2*p*(-yb*cos(phtrf) - zb*sin(phtrf)) + ...
* al*sin(phtrf-phec) - phtrf**2*a1*(yb*cos(phtrf) +
* zb*sin(phtrf)-p*sin(phtrf))

c

c

a(1,1) = al*cos(phtrf)

a(1,2) = -p*cos(phtrf)

a(1,3) = -1

a(1,4) = 0

b(1) = ((ch*ybf)/ma)+((dh*ybdf)/ma)+phtrdf**2*al*sin(phtrf)
* - phcdf**2*p*sin(phcf)

c

a(2,1) = al*sin(phtrf)

a(2,2) = -p*sin(phtrf)

a(2,3) = 0

a(2,4) = -1

b(2) = ((cv*zbf)/ma)+(dv*zbdf)/ma)*g-phtrf**2*al*cos(phtrf)
* + phcdf**2*p*cos(phcf)

c

a(3,1) = 0

a(3,2) = 0

a(3,3) = -dh*cos(phcf)

a(3,4) = -dv*sin(phcf)

c

b(3) = ch*ybdf*cos(phcf) - ch*ybf*phcdf*sin(phcf) - 
* dh*ybdf*phcdf*sin(phcf) + cv*zbdf*phcdf*cos(phcf) + 
* cv*zbf*phcdf*cos(phcf) + dv*zbdf*phcdf*cos(phcf)
\[
a(4,1) = p^*a1^*\cos(phcf-phtrf) - a1^*2 - zbf^*a1^*\cos(phtrf)
\]
\[
+ a1^*ybf^*\sin(phtrf) - ia/ma
\]
\[
a(4,2) = p^*a1^*\cos(phcf-phtrf) - p^*2 + p*zbf^*\cos(phcf)
\]
\[
- p*ybf^*\sin(phcf)
\]
\[
a(4,3) = zbf^* - p^*\cos(phcf) + a1^*\cos(phtrf)
\]
\[
a(4,4) = -ybf^* - p^*\sin(phcf) + a1^*\sin(phtrf)
\]

\[
b(4) = g*ybf + g*p^*\sin(phcf) - g*a1^*\sin(phtrf)
\]
\[
- phcdf^*2*p^*{-}\cdot ybf^*\cos(phcf) - zbf^*\sin(phcf) +
\]
\[
a1^*\sin(phtrf-phcf)) - phtrdf^*2*a1^*{(ybf^*\cos(phtrf) +
\]
\[
- zbf^*\sin(phtrf)+p^*\sin(phcf-phtrf))
\]

This subroutine solves the matrix equation \(ax=b\).

The least squares calculation. The algorithm used is based
on Algorithms 3.8 and 3.10 in: G. W. Stewart's "Introduction to

This algorithm, unlike the conventional normal equations
approach, is virtually bomb proof, allowing models with a large
number of degrees of freedom to be used without worry.

This section essentially factors the matrix \(A\) (algorithm 3.8)

subroutine solve(x, a, b, mm, m, n)

this standard subroutine is not included though the reference for
the subroutine solve is given above

return
end
APPENDIX C

Listings of the Fortran 77 code for the TWOSEG model, described in Chapter 3.

PROGRAM TWOSEG

Copyright (C) Mark A Brewin, 1998

This program represents a two segment gymnast swinging on a massless rigid rings' cable about a suspension point (0).

The joint connecting the two segments may either a shoulder or hip joint. The inertial characteristics of the two segments are altered to choose which joint is modelled.

This particular program is designed for producing time histories of the mechanical variables for the optimum swings. The equations of motion for the system do not change, only the application of the model.

Variations of this program for optimising aspects of swing, either performance or peak cable tension, incorporate the simulated annealing algorithm.

The program uses twoseg.in as its input file. An example input file for a simulation used in the thesis is given.

c example input file, twoseg.in

twoseg << eof

angAhip | angle output file
posAhip | position output file
linAhip | linear velocity output file
nrgAhip | energy output file
torAhip | joint torque output file
optAhip | longswing score
2 | initial cable orientation angle, phc (deg)
4 | initial arm orientation angle, phtr (deg)
180 | initial joint angle, el (deg)
0 | initial cable angular velocity (rad/s)
0 | initial arm angular velocity (rad/s)
180 160.9 | 1st change in joint angle (from: to)
1.4673 1.7673 | times for 1st joint angle change
160.9 180 | 2nd change in joint angle (from: to)
1.7673185 1.96 | times for 2nd joint angle change
0.0005 | time interval for integration
4500 | number of iterations
eof | end of file

c cccecccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc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The type of muscle contraction is also determined using the angular velocity of the joint and the direction of the torque. The type of muscle contraction is presented within the output.

Model parameters defining inertial characteristics of the modelled gymnast, such as mass, are changed within the program.

The equations of motion are derived using the nomenclature and system described in Chapter 3 for the TWOSEG model.

Nomenclature:

- $a_1$: distance from hands to mass centre of first segment
- $a_2$: distance from hands to joint
- $el_1$: distance from joint to mass centre of second segment
- $el_2$: distance from joint to end of second segment
- $p$: length of cable segment
- $ya$: horizontal position of mass centre of first segment
- $yad$: horizontal velocity of mass centre of first segment
- $yadd$: horizontal acceleration of mass centre of first segment
- $za$: vertical position of mass centre of first segment
- $zad$: vertical velocity of mass centre of first segment
- $zadd$: vertical acceleration of mass centre of first segment
- $ye$: horizontal position of mass centre of second segment
- $yed$: horizontal velocity of mass centre of second segment
- $yedd$: horizontal acceleration of mass centre of second segment
- $ze$: vertical position of mass centre of second segment
- $zed$: vertical velocity of mass centre of second segment
- $zedd$: vertical acceleration of mass centre of second segment
- $ma$: mass of the first segment
- $me$: mass of the second segment
- $ia$: moment of inertia of first segment about mass centre
- $ie$: moment of inertia of second segment about mass centre
- $phc$: cable elevation angle
- $phcd$: cable elevation angular velocity
- $phcdd$: cable elevation angular acceleration
- $pha$: orientation of first segment
- $phad$: orientation angular velocity of first segment
- $phadd$: orientation angular acceleration of first segment
- $el$: joint elevation angle
- $e1d$: joint elevation angular velocity
- $e1dd$: joint elevation angular acceleration
- $phtr$: orientation of second segment
- $phtrd$: orientation angular velocity of second segment
- $phtrdd$: orientation angular acceleration of second segment
- $a(9,7)$: holds the equations of motion for the system
- $b(7)$: holds the equations of motion for the system
- $asphell$: angle parameters for the joint angle time history
- $aephc14$: ...
- $tsphe14$: time parameters for the joint angle time history
- $Ry$, $Rz$: reaction forces at the joint of the gymnast
- $Ty$, $Tz$: reaction forces at the hand of the gymnast
- $Tl$: torque at the joint of the gymnast
- $ycm$, $zcm$: location of whole body mass centre
eps : "body angle" (hands to mass centre)
epsmax : maximum "body angle" (hands to mass centre)
tepsmax : time at maximum "body angle"
dt : time interval for integration
t : time
g : acceleration due to gravity
ten : tension in the cable
pi : pi
rtd : radians to degrees conversion
PE : potential energy of the system (measured from O)
TTE : total translational energy of the system
RE : total rotational kinetic energy of the system
LE : total linear kinetic energy of the system
TE : total energy of the system
kl - k4 : score weighting coefficients
scor : score for longswing
scorrnin : minimum score for longswing
tscormin : time at minimum score for longswing
muscle : type of contraction, "iso", "con" or "ecc"

c MODEL PARAMETERS

double precision
* p, g,
* a, b, pi, rtd,
* ia, ma, ie, me, t, dt,
* al, a2, el, e2,
* asell, aecell, tsell, teell,
* asell2, aecell2, tsell2, teell2,
* aselli, aecelli, tselli, teelli,
* ase12i, aee12i, tse12i, tee12i

c MODEL VARIABLES

double precision
* phci, phcdi, phai, phadi, eli,
* phc, phcd, phcdd, phcf, phcdf, phcddf, phco, phcko, phcddo,
* pha, phad, phadd, phaf, phaddf, phao, phado, phaddo,
* phtr, phtrd, phtrdd, phtrf, phtrdf, phtrddf, phtro, phtrdo,
* phtrddo,
* el, eld, eldd, elf, eldf, elddf, elo, eldo, elddo,
* yh, zh,
* Ry, Rz, Ty, Tz, Tl,
* Ryf, Rzf, Tyf, Tzf, Tlf,
* Ryo, Rzo, Tyo, Tzo, Tlo,
* a(9,7), b(7), x(7), y(7)

c MODEL OUTPUT

double precision
* ya, yad, yadd, za, zad, zadd,
* ye, yed, yedd, ze, zed, zedd,
  * ten, PE, TTE, TE, RE, LE, ycm, zcm

do double precision
  * epshyp, epsmax, tepsmax, eps, epso, seps, ceps, bb, sb, cb

do double precision
  * cc(4), f, f0, f1, t0, t1, z

do double precision
  * k1, k2, k3, k4, scor, scormin, tscormin

integer i, mm, m, nnn, nnni, n, count, bl

character output1*20
character output2*20
character output3*20
character output4*20
character output5*20
character output6*20

character rnuscle*3

read(*,*) output1
read(*,*) output2
read(*,*) output3
read(*,*) output4
read(*,*) output5
read(*,*) output6
read*, phci
read*, phai
read*, eli
read*, phcdi
read*, phadi
read*, aselli, aeelli
read*, tselli, tseelli
read*, aselll1, ael12i
read *, tsel2i, teel2i
read *, dt
read *, nnni
open (10, file = 'output1')
open (20, file = 'output2')
open (30, file = 'output3')
open (40, file = 'output4')
open (50, file = 'output5')
open (60, file = 'output6')

set up the headings for the output files

write(10,*) ' t(s) phc(o) phcd phcdd pha(o) phad phadd ten(N*) PE TTE TE(J)
*
write(20,*) ' t(s) ycm(m) zcm(m) el(o) eldd eld(rad/s) eldd
*(rad/s2)
write(30,*) ' t yad yadd zad zadd yed yedd
* zed zedd
write(40,*) ' t(s) PE RE LE TTE TE(J)
*
write(50,*) ' t(s) ten(RW) phc(o) pha(o) el(o) TE
* torque(Nm) type eps(o)
*
write(60,*) ' t(s) epsmax(o) t sco(s) score stal(s) endl(*s), sta2(s) end2(s) el(o)

initialise all variables and set inertial parameters

g = 9.81
p = 3.014
pi = 3.14159265358
rtd = 180.0 / pi

these are inertial values for the normalised gymnast with a
shoulder joint

ma = 7.137
these are inertial values for subject K with a shoulder joint
these are inertial values for subject K with a hip joint
these are inertial values for subject A with a shoulder joint
these are inertial values for subject A with a hip joint
\[ \begin{align*}
\text{ma} &= 42.174 \\
\text{me} &= 25.126 \\
\text{ia} &= 2.7943 \\
\text{ie} &= 1.7276 \\
\text{a1} &= 0.6640 \\
\text{a2} &= 1.082 \\
\text{e1} &= 0.3405 \\
\text{e2} &= 1.0245
\end{align*} \]

Define \( \text{tepsmin} \) and \( \text{epsmin} \) for the score for the longswing, this score is used in the optimisation of the longswing.

\[ \begin{align*}
\text{k1} &= \text{constant for cable angle, } \text{phc} \\
\text{k2} &= \text{constant for cable angular velocity, } \text{phcd} \\
\text{k3} &= \text{constant for body angle, } \text{pha} \\
\text{k4} &= \text{constant for body angular velocity, } \text{phad}
\end{align*} \]

Initial times for joint angle changes

\[ \begin{align*}
\text{tsell1} &= \text{tselli} \\
\text{teell1} &= \text{teelli} \\
\text{tsell2} &= \text{tsel2i} \\
\text{teell2} &= \text{teel2i}
\end{align*} \]

Define all variables

\[ \begin{align*}
\text{t} &= 0.0 \\
\text{tscormin} &= 0.0 \\
\text{scormin} &= 9999999 \\
\text{eps} &= 0.0 \\
\text{tepsmax} &= 0.0 \\
\text{epsmax} &= 0.0
\end{align*} \]
nnn = nnni + 1  

do 200, i = 1, nnn  

     t = (i-1)*dt  

cccccc change the input angles to (o)riginal angles 

cccc determine elo, eldo, elddo using varang, vdang and d2ang 

cccc which are equations outlined by M.R. Yeaton (1984)  

cc(1) = asell  
cc(2) = aeell  
cc(3) = tsell  
cc(4) = teell  
f = elo  
call varang(f,cc,t)  
elo = f  

c  
f = 0.0  
call vdang(f,cc,t)  
eldo = f  

c  
f = 0.0  
call d2ang(f,cc,t)  
elddo = f  

c  
for re-changing the shoulder angle use if statements  

cccc if (t.gt.teell.and.t.lt.teel2) then  
cc(1) = asell2  
cc(2) = aeell2  
cc(3) = tsell2  
cc(4) = teell2  
f = elo  
call varang(f,cc,t)  
elo = f  

c  
f = 0.0  
call vdang(f,cc,t)  
eldo = f
c
f = 0.0
call d2ang(f, cc, t)
elddo = f
c
calculate phtr, phtrd, phtrdd
c
call the equations of motion for the system to calculate
the initial estimates for phdd, phcdd, Ry, Rz, Ty, Tz, T1
c
call arrayl(p, g, a, b, ia, ma, ie, me, a1, a2, *el, e2, phao, phado, phaddo, phco, phcdo, phcddo, phtro, *phtrdo, elo, eldo, elddo, Ryo, Rzo, Tyo, Tzo, Tlo)
mm = 9
m = 7
n = 7
call solve(x, a, b, mm, m, n)
phaddo = x(1)
phcddo = x(2)
Ryo = x(3)
Rzo = x(4)
Tyo = x(5)
Tzo = x(6)
Tlo = x(7)
calculate (f)inal angles, angular velocities using the
initial angular accelerations
c
phaf = phao + phado*dt + 0.5*phaddo*dt**2
phadf = phado + phaddo*dt
phcf = phco + phcdo*dt + 0.5*phcddo*dt**2
phcdf = phcdo + phcddo*dt
calculate the final elevation angles

```
elf = elo + eldo*dt + 0.5*elddo*dt**2
eldf = eldo + elddo*dt
elddf = eldo + elddo*dt
```

```
phtrf = pi + phaf - elf
phtrdf = phadf - eldf
```

determine the (f)inal angular accelerations

```
call array2(p, g, a, b, ia, ma, ie, me, a1, a2, *el, e2, phaf, phadf, phaddf, phcddf, phcf, phcdf, phcddf, phtrf, *phtrdf, elf, eldf, elddf, Ryf, Rzf, Tyf, Tzf, Tlf)
call solve(y, a, b, mm, m, n)
```

```
phaddf = y(1)
phcddf = y(2)
Ryf = y(3)
Rzf = y(4)
Tyf = y(5)
Tzf = y(6)
Tlf = y(7)
```

calculate the average angular accelerations using the average between the final and initial acceleration values

```
phadd = (phaddo + phaddf) / 2.0
phcddf = (phcddo + phcddf) / 2.0
Ry = (Ryo + Ryf) / 2.0
Rz = (Rzo + Rzf) / 2.0
Ty = (Tyo + Tyf) / 2.0
Tz = (Tzo + Tzf) / 2.0
Tl = (Tlo + Tlf) / 2.0
phtrdd = phadd - elddo
```

calculate the required output


```
ccc SEGMENT A (ARMS ONLY OR ARMS AND TORSO)

\[
\begin{align*}
\text{ya} &= p \cdot \sin(\phi_{co}) \cdot a_1 \cdot \sin(\phi_{ao}) \\
\text{za} &= a_1 \cdot \cos(\phi_{ao}) \cdot p \cdot \cos(\phi_{co})
\end{align*}
\]

yad = \( p \cdot \phi_{cd0} \cdot \cos(\phi_{co}) + a_1 \cdot \phi_{ado} \cdot \cos(\phi_{ao}) \)

\[
\begin{align*}
\text{zad} &= a_1 \cdot \phi_{ado} \cdot \sin(\phi_{ao}) + p \cdot \phi_{cd0} \cdot \sin(\phi_{co})
\end{align*}
\]

\[
\begin{align*}
\text{yadd} &= -p \cdot \phi_{cd0} \cdot \sin(\phi_{co}) + a_1 \cdot \phi_{ado} \cdot \sin(\phi_{ao}) \\
\text{zadd} &= a_1 \cdot \phi_{ado} \cdot \cos(\phi_{ao}) + \phi_{cd0} \cdot \sin(\phi_{co})
\end{align*}
\]

ccc SEGMENT E (BODY AND LEGS OR LEGS ONLY)

\[
\begin{align*}
\text{ye} &= p \cdot \sin(\phi_{co}) \cdot a_2 \cdot \sin(\phi_{ao}) \cdot \sin(\phi_{tro}) \\
\text{ze} &= a_2 \cdot \cos(\phi_{ao}) \cdot p \cdot \cos(\phi_{co}) + \sin(\phi_{tro})
\end{align*}
\]

\[
\begin{align*}
\text{yed} &= p \cdot \phi_{cd0} \cdot \cos(\phi_{co}) - a_2 \cdot \phi_{ado} \cdot \cos(\phi_{ao}) \\
\text{zed} &= a_2 \cdot \phi_{ado} \cdot \sin(\phi_{ao}) + p \cdot \phi_{cd0} \cdot \sin(\phi_{co})
\end{align*}
\]

\[
\begin{align*}
\text{yedd} &= -p \cdot \phi_{cd0} \cdot \sin(\phi_{co}) + a_2 \cdot \phi_{ado} \cdot \sin(\phi_{ao}) \\
\text{zedd} &= a_2 \cdot \phi_{ado} \cdot \cos(\phi_{ao}) + \phi_{cd0} \cdot \sin(\phi_{co})
\end{align*}
\]

ccc CALCULATE THE WHOLE BODY CM

\[
\begin{align*}
call \text{cmlocation}(m_a, m_e, y_a, z_a, y_e, z_e, y_{cm}, z_{cm})
\text{ycm} &= y_{cm} \\
z_{cm} &= z_{cm}
\end{align*}
\]

ccc CALCULATE THE CM ANGLE EPS (FROM HANDS TO CM)

\[
\begin{align*}
call \text{cmangle}(\text{eps}, \text{epshyp}, y_{h}, z_{h}, y_{cm}, z_{cm}, p, \phi_{co}) \\
\text{epshyp} &= \text{epshyp}
\end{align*}
\]

\[
\begin{align*}
\text{ceps} &= (z_{cm} \cdot z_{h}) / \text{epshyp} \\
\text{seps} &= (y_{h} \cdot y_{cm}) / \text{epshyp}
\end{align*}
\]

\[
\begin{align*}
\text{if (i.eq.1) then} &
\text{call ang0(eps,eps0,seps,ceps)}
\text{endif}
\end{align*}
\]

\[
\begin{align*}
\text{if (i.gt.1) then} &
\text{call csang(eps,eps0,seps,ceps)}
\end{align*}
\]
```
```plaintext
endif

eps = eps0

CABLE TENSION AND ENERGY

\[
\text{ten} = \frac{T_z}{\cos(\phi_c)}
\]

\[
\text{PE} = m_a g z_a + m_e g z_e
\]

\[
\text{RE} = 0.5 m_a (y_{ad}^2 + z_{ad}^2) + 0.5 m_e (y_{ed}^2 + z_{ed}^2)
\]

\[
\text{TTE} = \text{PE} + \text{LE}
\]

T = PE + TTE

SCORE FOR THE BACKWARD LONGSWING

if \( t > t_{ee12i} \) then

\[
\text{scor} = (k_1 (\phi_{c0} \omega_{t0})^2 + k_2 (\phi_{c0} \omega_{t0})^2 + k_3 ((360 - \phi_{ao} \omega_{t0}))^2 + k_4 (\phi_{ao} \omega_{t0})^2)
\]

if (scor < scormin) then

\[
\text{scormin} = \text{scor}
\]

\[
t_{scormin} = t
\]

end if

end if

MAXIMUM CM ANGLE OBTAINED (EPSMAX)

if \( \epsilon > \epsilon_{max} \) then

\[
\epsilon_{max} = \epsilon
\]

endif

DETERMINE THE MUSCLE CONTRACTION TYPE

the general array to predict the muscle contraction type:

\[
T^+ | W^- | C^+ | E^-
\]

this means \( C \) indicates a concentric torque, while \( E \) indicates a eccentric torque

however, owing to the definition of torque being +ve when the joint angular velocity is -ve the array is modified to the specific of the definitions in the Nomenclature

\[
T^+ | E^- | C^+
\]
```
if (eldo.eq.0.0d0) muscle = 'iso'
if (eldo.lt.0.0d0.and.Tl.gt.0.0d0) muscle = 'con'
if (eldo.gt.0.0d0.and.Tl.gt.0.0d0) muscle = 'ecc'
if (eldo.gt.0.0d0.and.Tl.lt.0.0d0) muscle = 'con'
if (eldo.lt.0.0d0.and.Tl.lt.0.0d0) muscle = 'ecc'

write out values for the output variables to the output files in CSV format

if (mod(i,lO).eq.1) then
105 format (f6.4,,f7.3,,f7.3,,f6.1,,f7.3,,f7.3
*,,f6.1,,f8.0,,f8.0,,f7.0,,f8.0)
115 format (f6.4,,4(f9.4,,),f9.4)
125 format (f6.4,,8(f9.4,,),f9.4)
135 format (f6.4,,4(f10.2,,),f10.2)
145 format (f6.4,,f8.3,,f8.3,,f9.3,,f9.2,,f10.2
*,,f9.3,,A4,,f9.3)

write(10,105) t, phcrtd, phcd, phddo, phartd, phad,
* phaddo, ten, PE, TTE, TE
write(20,115) t, ycm, zcm, elo*rtd, eldo, elddo
write(30,125) t, yad, yadd, zad, zadd, yed, yedd, zed, zedd
write(40,135) t, PE, RE, LE, TTE, TE
write(50,145) t, ten/(g*ma+g*me), phcrtd, phartd, elo*rtd,
* TE, Tlo, muscle, eps*rtd
endif

recalculate pha, phad, phc, phcd using the average angular acceleration for each angle and equations of constant acceleration

pha = phao + phado*dt + 0.5*phadd*dt**2
phad = phado + phadd*dt
phc = phco + phcdo*dt + 0.5*phcdd*dt**2
phcd = phcdo + phcdd*dt

write out the maximum body angle and the time at which it occurred and the performance score for the backward longswing
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```
155 format (f6.4,',',f10.4,',',f7.4,',',f12.4,',',4(f7.3,','),f7.1)

write(60,155) tepsmax,epsmax*rtd,tscormin,scormin,
* tsel1,teel1,tsel2,teel2,aeell*rtd

100 continue
   close(10)
   close(20)
   close(30)
   close(40)
   close(50)
   close(60)
   stop
   end

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc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cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
subroutine array1(p, g, a, b, ia, ma, ie, me, al, a2, *el; e2, phao, phado, phaddo, phcdo, phcddo, phtro, *ph trov, elo, eldo, elldo, Ryo, Rzo, Tyo, Tzo, Tlo)

double precision
* p, g, a, b, ia, ma, ie, me, al, a2, el, e2,
* phao, phado, phaddo, phcdo, phcddo, phtro, phtrdo,
* elo, eldo, elldo, Ryo, Rzo, Tyo, Tzo, Tlo,
* a(9,7), b(7)

da(1,1) = -ma*al*cos(phao)
da(1,2) = ma*p*cos(phco)
da(1,3) = -1
da(1,4) = 0
nda(1,5) = 1
nda(1,6) = 0
nda(1,7) = 0

db(1) = ma*(p*phcdo**2*sin(phco) - al*phado**2*sin(phao))

da(2,1) = ma*al*sin(phao)
da(2,2) = -ma*p*sin(phco)
da(2,3) = 0
da(2,4) = -1
nda(2,5) = 0
nda(2,6) = 1
nda(2,7) = 0

db(2) = ma*(g - al*phado**2*cos(phao) - p*phcdo**2*cos(phco))

nda(3,1) = -ia
nda(3,2) = 0
nda(3,3) = - (a2 - al)*cos(phao)
da(3,4) = (a2 - al)*sin(phao)
da(3,5) = - al*cos(phao)
da(3,6) = al*sin(phao)
da(3,7) = -1

db(3) = 0

nda(4,1) = - me*(a2*cos(phao) + el*cos(phtro))
da(4,2) = me*p*cos(phco)
da(4,3) = 1
nda(4,4) = 0
nda(4,5) = 0
nda(4,6) = 0
nda(4,7) = 0

db(4) = me*(phcdo**2*p*sin(phco) - a2*phado**2*sin(phao) - el*
phtro**2*sin(phtro) - elldo*el*cos(phtro))
\begin{verbatim}

a(5,1) = me*(a2*sin(phao) + el*sin(phtro))
a(5,2) = - me*p*sin(phco)
a(5,3) = 0
a(5,4) = 1
a(5,5) = 0
a(5,6) = 0
a(5,7) = 0

b(5) = me*(g + p*phcdo**2*cos(phco) - a2*phado**2*cos(phao)
  * - el*phtrdoo**2*cos(phtro) + elddo*el*sin(phtro))

a(6,1) = ie
a(6,2) = 0
a(6,3) = el*cos(phtro)
a(6,4) = - el*sin(phtro)
a(6,5) = 0
a(6,6) = 0
a(6,7) = -1

b(6) = ie*elddo

a(7,1) = ia + ma*a1**2 - ma*a1*p*cos(phao - phco) + ie
  * - me*a2*p*cos(phao - phco) + me*a2**2 + me*a2*el*
  * cos(phao - phtro) - me*el*p*cos(phco - phco) +
  * me*el*a2*cos(phtro - phao) + me*el**2
a(7,2) = -ma*p*a1*cos(phco - phao) + ma*p**2 + me*p**2
  * - me*p*a2*cos(phco - phao) - me*p*el*cos(phco - phtro)

a(7,3) = 0
a(7,4) = 0
a(7,5) = 0
a(7,6) = 0
a(7,7) = 0

b(7) = elddo*(ie - me*el*p*cos(phtro - phco) + me*el*a2*
  * cos(phtro - phao) + me*el**2) + phado**2*(-ma*a1*p*
  * sin(phao - phco) + me*a2*p*sin(phco - phao) - me*a2*el*
  * sin(phao - phtro) + phtrdoo**2*(me*el*p*sin(phco - phtro)
  * - me*
  * el*a2*sin(phao - phtro)) + phcdo**2*(-ma*p*a1*
  * sin(phco - phao)
  * + me*p*a2*sin(phao - phco) + me*p*el*sin(phco - phco))
  * - ma*g*(p*sin(phco) - al*sin(phao)) - me*g*(p*sin(phco)
  * + a2*sin(phao) - el*sin(phtro))

return
end

subroutine array2(p, g, a, b, ia, ma, ie, me, al, a2,
  *el, el, phaf, phadf, phadff, phcf, phcdf, phcddf, phtrf,
  *phtrdf, elf, eldf, elddf, Ryf, Rzf, Tyf, Tzf, Tlf)

double precision

\end{verbatim}
\[ a(1,1) = -ma*a_{1}\cos(\phi_{af}) \]
\[ a(1,2) = ma*p*\cos(\phi_{cf}) \]
\[ a(1,3) = -1 \]
\[ a(1,4) = 0 \]
\[ a(1,5) = 1 \]
\[ a(1,6) = 0 \]
\[ a(1,7) = 0 \]

\[ b(1) = ma*(p*\phi_{cdf}*2*\sin(\phi_{cf}) - a_{1}\phi_{adf}*2*\sin(\phi_{af})) \]

\[ a(2,1) = ma*a_{1}\sin(\phi_{af}) \]
\[ a(2,2) = -ma*p*\sin(\phi_{cf}) \]
\[ a(2,3) = 0 \]
\[ a(2,4) = -1 \]
\[ a(2,5) = 0 \]
\[ a(2,6) = 1 \]
\[ a(2,7) = 0 \]

\[ b(2) = ma*(g - a_{1}\phi_{adf}*2*\cos(\phi_{af}) + p*\phi_{cdf}*2*\cos(\phi_{cf})) \]

\[ a(3,1) = -ia \]
\[ a(3,2) = 0 \]
\[ a(3,3) = - (a_{2} - a_{1})\cos(\phi_{af}) \]
\[ a(3,4) = (a_{2} - a_{1})\sin(\phi_{af}) \]
\[ a(3,5) = a_{1}\cos(\phi_{af}) \]
\[ a(3,6) = a_{1}\sin(\phi_{af}) \]
\[ a(3,7) = -1 \]

\[ b(3) = 0 \]

\[ a(4,1) = -me*(a_{2}\cos(\phi_{af}) + e_{1}\cos(\phi_{trf})) \]
\[ a(4,2) = me*p*\cos(\phi_{cf}) \]
\[ a(4,3) = 1 \]
\[ a(4,4) = 0 \]
\[ a(4,5) = 0 \]
\[ a(4,6) = 0 \]
\[ a(4,7) = 0 \]

\[ b(4) = me*(\phi_{cdf}*2*p*\sin(\phi_{cf}) - a_{2}\phi_{adf}*2*\sin(\phi_{af}) - e_{1} * \phi_{trdf}*2*\sin(\phi_{trf}) - e_{ddf}*e_{1}\cos(\phi_{trf})) \]

\[ a(5,1) = me*(a_{2}\sin(\phi_{af}) + e_{1}\sin(\phi_{trf})) \]
\[ a(5,2) = -me*p*\sin(\phi_{cf}) \]
\[ a(5,3) = 0 \]
\[ a(5,4) = 1 \]
\[ a(5,5) = 0 \]
\[ a(5,6) = 0 \]
\[ a(5,7) = 0 \]

\[ b(5) = me*(g + p*\phi_{cdf}*2*\cos(\phi_{cf}) - a_{2}\phi_{adf}*2*\cos(\phi_{af})) \]
* \( \text{el} \times \text{phtrdf} \times 2 \times \cos(\text{phtrf}) + \text{elddf} \times \text{el} \times \sin(\text{phtrf}) \)

\[ a(6,1) = \text{ie} \]
\[ a(6,2) = 0 \]
\[ a(6,3) = \text{el} \times \cos(\text{phtrf}) \]
\[ a(6,4) = -\text{el} \times \sin(\text{phtrf}) \]
\[ a(6,5) = 0 \]
\[ a(6,6) = 0 \]
\[ a(6,7) = -1 \]

\[ b(6) = \text{ie} \times \text{elddf} \]

\[ a(7,1) = \text{ia} + \text{ma} \times \text{al} \times 2 \times \text{ma} \times \text{al} \times \text{p} \times \cos(\text{phaf} - \text{phcf}) + \text{ie} \]
\[ \times \text{me} \times \text{a2} \times \text{p} \times \cos(\text{phaf} - \text{phcf}) + \text{me} \times \text{a2} \times 2 + \text{me} \times \text{a2} \times \text{el} \]
\[ \times \cos(\text{phaf} - \text{phtrf}) \]
\[ \times \text{me} \times \text{el} \times \text{p} \times \cos(\text{phcf} - \text{phtrf}) + \text{me} \times \text{el} \times \text{a2} \times \cos(\text{phtrf} - \text{phaf}) \]
\[ + \text{me} \times \text{el} \times 2 \]
\[ a(7,2) = -\text{ma} \times \text{p} \times \text{al} \times \cos(\text{phcf} - \text{phaf}) + \text{ma} \times \text{p} \times 2 + \text{me} \times \text{p} \times 2 \]
\[ \times \text{me} \times \text{p} \times \text{a2} \times \cos(\text{phcf} - \text{phaf}) - \text{me} \times \text{p} \times \text{el} \times \cos(\text{phcf} - \text{phtrf}) \]
\[ a(7,3) = 0 \]
\[ a(7,4) = 0 \]
\[ a(7,5) = 0 \]
\[ a(7,6) = 0 \]
\[ a(7,7) = 0 \]

\[ b(7) = \text{elddf} \times (\text{ie} - \text{me} \times \text{el} \times \text{p} \times \cos(\text{phtrf} - \text{phcf}) + \text{me} \times \text{el} \times \text{a2} \]
\[ \times \cos(\text{phtrf} - \text{phaf}) + \text{me} \times \text{el} \times 2) + \text{phtrdf} \times 2 \times (-\text{ma} \times \text{al} \times \text{p} \]
\[ \times \sin(\text{phaf} - \text{phcf}) + \text{me} \times \text{a2} \times \text{p} \times \sin(\text{phcf} - \text{phaf}) - \text{me} \times \text{a2} \times \text{el} \]
\[ \times \sin(\text{phtrf} - \text{phaf}) + \text{phtrdf} \times 2 \times (\text{me} \times \text{el} \times \text{p} \times \sin(\text{phcf} - \text{phtrf}) \]
\[ - \text{me} \times \text{el} \times \text{a2} \times \sin(\text{phaf} - \text{phtrf})) + \text{phtrdf} \times 2 \times (-\text{ma} \times \text{p} \times \text{al} \]
\[ \times \sin(\text{phcf} - \text{phaf}) + \text{me} \times \text{p} \times \text{a2} \times \sin(\text{phcf} - \text{phaf}) + \text{me} \times \text{p} \times \text{el} \times \sin(\text{phcf} - \text{phtrf}) \]
\[ + \text{me} \times \text{p} \times \text{el} \times \sin(\text{phtrf} - \text{phcf}) \]
\[ + \text{me} \times \text{p} \times \text{al} \times \sin(\text{phaf}) - \text{al} \times \sin(\text{phaf}) - \text{me} \times \text{p} \times \text{sin}(\text{phcf}) \]
\[ - \text{a2} \times \sin(\text{phaf}) - \text{el} \times \sin(\text{phtrf}) \]

return
end

subroutine cmlocation (ma, me, ya, za, ye, ze, ycm, zcm)

double precision
* ma, me, ya, za, ye, ze, ycm, zcm

ycm = (ya * ma + ye * me) / (me + ma)
zcm = (za * ma + ze * me) / (ma + me)

return
end

subroutine cmangle (eps, epshyp, yh, zh, ycm, zcm, p, phco)
double precision
* eps, epshyp, yh, zh, ycm, zcm, p, phco

yh = p*sin(phco)
yh = -p*cos(phco)
epshyp = sqrt((ycm - yh)**2 + (zh - zcm)**2)
c
return end
c
subroutine ango(eps, epso, seps, ceps)
double precision eps, epso, seps, ceps, pi
intrinsic acos
pi = 3.14159265358
if (ceps.gt.1.0d0.or. ceps.lt.-1.0d0) ceps = sign(1.0d0, ceps)
eps = acos(ceps)
if (seps.lt.0.0d0) eps = -eps
epso = eps
c
return end
c
subroutine csang(eps, epso, seps, ceps)
double precision eps, seps, ceps, epso, sepso, cepso, bb, sb, cb
intrinsic acos, cos, sin, sign, sqrt
if (ceps.gt.1.0d0.or. ceps.lt.-1.0d0) ceps = sign(1.0d0, ceps)
if (1.0-ceps**2.ge.0.0d0) seps = sign(sqrt(1.0-ceps**2), seps)
cepso = cos(epso)
sepso = sin(epso)
if (cb.gt.1.0d0.or. cb.lt.-1.0d0) cb = sign(1.0d0, cb)
bb = acos(cb)
bb = sign(bb, sb)
eps = epso + bb
epso = eps
SUBROUTINE VARANG(F,CC,T)
DOUBLE PRECISION F,CC(4),T,F0,F1,T0,T1,ANGQ

F0 = CC(1)
F1 = CC(2)
T0 = CC(3)
T1 = CC(4)
IF (T.LT.T0) GO TO 500
IF (T.LT.T1) GO TO 200
GO TO 500

200 F = ANGQ(F0,F1,T0,T1,T)
500 CONTINUE
RETURN
END

ANGQ IS THE MONOTONIC QUINTIC ON THE INTERVAL T0, T1, WHICH TAKES END-POINT VALUES F0,F1 AND WHICH HAS ZERO FIRST AND SECOND DERIVATIVES AT THE END POINTS (M.R. YEADON, 1984)

DOUBLE PRECISION FUNCTION ANGQ(F0,F1,T0,T1,T)
DOUBLE PRECISION F0,F1,T0,T1,T,Z

Z = (T-T0)/(T1-T0)
ANGQ = F0 + (F1-F0)*(Z*Z*Z)*((6*Z*Z) - (15*Z) + 10)
RETURN
END

VDANG DEFINES DERIVATIVES IN THE SAME WAY THAT VARANG DEFINES FUNCTIONS (M.R. YEADON, 1984)

SUBROUTINE VDANG(F,CC,T)
DOUBLE PRECISION F,CC(4),T,F0,F1,T0,T1,DANGQ

F0 = CC(1)
F1 = CC(2)
T0 = CC(3)
T1 = CC(4)


```plaintext
f = 0.0
if (t.lt.t0) go to 500
if (t.lt.t1) go to 200
go to 500
200 f = dangq(f0,f1,t0,t1,t)
500 continue
return
end
c

dangq is the derivative of the quintic angq (M.R. Yeadon, 1984)
c
double precision function dangq(f0,f1,t0,t1,t)
double precision f0,f1,t0,t1,t,z

z = (t·t0)/(t1·t0)
dangq = (((f1-f0)*((z*z*30)*(z-1)*(z-1)))/(t1·t0))
return
end
c
d2angq is the second derivative of the quintic angq
(c (M.R. Yeadon, 1984)
c
subroutine d2ang(f,cc,t)
double precision f,cc(4),t,f0,f1,t0,t1,ddangq
c
f0 = cc(1)
f1 = cc(2)
t0 = cc(3)
t1 = cc(4)
f = 0.0
if (t.lt.t0) go to 500
if (t.lt.t1) go to 200
go to 500
200 f = ddangq(f0,f1,t0,t1,t)
500 continue
return
end
c
ddangq is the second derivative of the quintic angq
(c (M.R. Yeadon, 1984)
c
double precision function ddangq(f0,f1,t0,t1,t)
double precision f0,f1,t0,t1,t,z

z = (t·t0)/(t1·t0)

ddangq = (((f1-f0)*((z*z*120)*(z-0.5)*(z-1)))/(t1·t0)*(t1·t0))
```

subroutine solve( x, a, b, mm, m, n ) obtains the least squares solution to the matrix equation ax=b.

parameters:

mm : first dimension of array a as declared in the calling program
m : number of equations
n : number of unknowns
'a': 'dp' array of dimension (mm,nn) where:
    mm > m+1 and nn > n
    a is overwritten on exit
b : 'dp' array of dimension at least n
    on entry b contains the rhs of the equation
    on exit b contains the residuals
x : 'dp' array of dimension at least n
    on exit x contains the solution vector


This algorithm, unlike the conventional normal equations approach, is virtually bomb proof, allowing models with a large number of degrees of freedom to be used without worry.

This section essentially factors the matrix A (algorithm 3.8)

subroutine solve(x, a, b, mm, m, n)

this standard subroutine is not included though the reference for the subroutine solve is given above

return
end
APPENDIX D

The following Appendix is divided into two sections. Firstly, the methods by which orientation angles, configuration angles and kinematical differential equations are formulated using AUTOLEV™3 are presented, using body A as an example. The term orientation angle implies the angle is used to define an internal reference frame relative to the Newtonian reference frame, for instance the orientation of the right rings cable segment in the three-dimensional simulation model. The term configuration angle is applied when an angle is used to describe the relationship between one internal reference frame and another, such as the configuration of the shank from the thigh segment for the gymnast. Subsequent to examples relating to body A, further illustrations highlighting the application of these methods to the four segment model are provided.

Secondly, an annotated version of the AUTOLEV™3 command file describing the four segment model, together with relevant diagrams and Nomenclature, is presented. Prior to this, syntax and commands used by AUTOLEV™3 in formulating the equations of motion are described and explained with reference to the four segment three-dimensional simulation model, FOURSEG.

Orientation angles and kinematical equations in AUTOLEV™3

When a massive body (or massless frame) is defined in AUTOLEV™3 the body is assigned a set of three internal, mutually perpendicular unit vectors fixed in the body. These unit vectors represent the internal reference frame of the body. Body A for example, representing a brick, would be assigned unit vectors $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ as shown in Figure D.1.

![Mutually perpendicular unit vectors forming the internal reference frame for body A.](image)

These unit vectors fixed in the body have the following relationships, where $\times$ denotes the cross product:
For three-dimensional analyses it is necessary to describe the orientation of a body with respect to the inertial, or Newtonian reference frame N. The relationship between the unit vectors of any two reference frames employed in AUTOLEV™3 is described by nine dot products. For body A in the Newtonian frame N, they are:

\[ A_i \cdot N_j \quad (i, j = 1, 2, 3) \]

These nine dot products, known as direction cosines, are equal to the cosine of the angle between \( A_i \) and \( N_j \) denoted by the matrix \( C_{ij} \) \( (i, j = 1, 2, 3) \), such that:

\[ C_{ij} = A_i \cdot N_j \quad (i, j = 1, 2, 3) \]

Furthermore, the general relationship between two sets of unit vectors can be described in terms of three angles \( \phi \), \( \theta \) and \( \psi \). These angles denote three ordered successive positive rotations about the internal unit vectors of the body, \( A_3 \), \( A_2 \), and \( A_1 \), termed Body-three: 3-2-1 in AUTOLEV™3, from an initial orientation identical to the frame from which the rotations occur. To illustrate these successive rotations the example of body A, and therefore reference frame A, rotating in a Newtonian frame N will be adopted. As these rotations take place relative to the Newtonian frame the angles used to describe the rotations are termed orientation angles. This method is used in the four segment three-dimensional model, FOURSEG, to define the orientation of the right rings cable segment relative to the Newtonian reference frame.

In the following three diagrams the dashed unit vectors with grey arrow heads indicate the orientation of the frame prior to the described rotation. The Newtonian reference frame is placed along side, with mutually perpendicular unit vectors \( N_1 \), \( N_2 \), and \( N_3 \). Initially the two sets of unit vectors \( A_i \) and \( N_j \) \( (i, j = 1, 2, 3) \) are aligned such that direction cosines \( C_{11} \), \( C_{22} \) and \( C_{33} \) are equal to 1, and all other direction cosines are equal to zero. Body A then rotates by the orientation angle \( \phi \) in a positive direction about unit vector \( A_3 \) (Figure D.2) to an intermediate orientation.

![Figure D.2. Orientation angle \( \phi \) (rotation 1).](image_url)
From this intermediate orientation determined after the first rotation, body A rotates by a positive angle $\theta$ about the unit vector $A_2$ (Figure D.3).

![Figure D.3. Orientation angle $\theta$ (rotation 2).]

The direction cosine matrix $C(\theta)$ which defines the second rotation $\theta$, a rotation about the internal unit vector $A_2$ from the intermediate orientation, is given by:

$$
C(\theta) = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}.
$$

The final positive rotation occurs about the internal axis $A_1$ from the intermediate orientation obtained after the second rotation (Figure D.4).

![Figure D.4. Orientation angle $\psi$ (rotation 3).]

The direction cosine which defines this third rotation is $C(\psi)$:

$$
C(\psi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \psi & -\sin \psi \\
0 & \sin \psi & \cos \psi
\end{bmatrix}.
$$
In this example the combination of all three orientation angles defines the final orientation of the internal reference frame for body A in the Newtonian reference frame N. To establish the direction cosine matrix C3 which describes this relationship the product of the three matrices is determined in the following order 
\[ C_3 = C(\phi) \cdot C(\theta) \cdot C(\psi), \]

where the matrices are ordered in the following order:

\[ C_3 = \begin{bmatrix}
\cos \phi \cos \theta & \sin \phi \cos \psi - \sin \phi \sin \theta \cos \psi & \cos \psi \sin \theta \cos \phi + \sin \phi \sin \psi \\
\cos \phi \sin \theta & \sin \phi \sin \psi + \cos \phi \cos \psi & \cos \psi \sin \phi \sin \theta - \cos \phi \sin \psi \\
-sin \theta & \sin \phi \cos \phi & \cos \phi \cos \theta
\end{bmatrix}. \]

Direction cosine matrix C3 is the template matrix defining the orientation of an internal reference frame in the Newtonian reference frame. For example, in the simulation model FOURSEG, this matrix describes the orientation of the right rings cable segment in the Newtonian reference frame. The actual direction cosine matrix describing the orientation of the rings cable is formed by applying the appropriately named angles \( \phi_{cr}, \theta_{cr}, \psi_{cr} \) to the ordered rotations (Table D.1).

Where only two angles are used to describe the relationship between two reference frames the first two rotations are used, described in general terms by angles \( \phi \) and \( \theta \). The following general direction cosine matrix C2 is determined by the product of 
\[ C_2 = C(\phi) \cdot C(\theta): \]

\[ C_2 = \begin{bmatrix}
\cos \phi \cos \theta & \sin \phi & \cos \phi \sin \theta \\
\cos \phi \sin \theta & \cos \phi & -\sin \phi \sin \theta \\
-sin \theta & 0 & \cos \phi
\end{bmatrix}. \]

The matrix C2 is the template matrix which defines the configuration of two reference frames by two successive rotation angles. In the model FOURSEG matrix C2 is used to describe the configuration of the right arm of the gymnast relative to the torso segment by the two configuration angles \( \phi_{elr} \) and \( \theta_{elr} \) (Table D.1).

Where only one angle is used to describe the relationship between two reference frames the angle \( \phi \) is used. The general direction cosine matrix for this rotation is C1:

\[ C_1 = \begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}. \]

Matrix C1 is the template for all segments in FOURSEG which use only one angle to define the relationship between the two frames. The configuration of the thigh, for example, is defined relative to the torso segment by a single angle \( \phi_{uleg} \), while the shank is defined relative to the thigh by the angle \( \phi_{ileg} \). In addition matrix C1 is used to describe the orientation of the torso segment relative to the Newtonian frame using the orientation angle \( \phi_{tr} \) (Table D.1).

In Kane's method for formulating equations of motion for a system the angular
velocity of a body is defined using generalised speeds. The generalised speeds, \( U_i \) \((i = 1, 2, 3)\) may be used to define all angular kinematics of a body about the internal reference frame relative to another frame, either internal or Newtonian. The angular velocities of the angles may then be related to these generalised speeds. Using body A as the example, Figure D.5 shows how generalised speeds are used to define kinematical differential equations. The kinematical equation is formed by the summation of all products of generalised speeds and the axis about which the speed describes the generalised motion.

\[
\omega_A^N = U_3 * A_1 + U_2 * A_2 + U_1 * A_3
\]

Figure D.5. Defining kinematical differential equations for body A in Newtonian frame N.

The angular velocities of the angles \( \phi \), \( \theta \) and \( \psi \) are then determined in terms of these generalised speeds. In the model FOURSEG the generalised speeds \( U_1 \) to \( U_4 \) are used to characterise the angular velocities of the right rings cable and the torso segments. To determine the angular acceleration of body A in the Newtonian reference frame the kinematical equation above is differentiated with respect to time. This procedure, performed automatically within AUTOLEV™3, gives the following result for body A:

\[
\alpha_A^N = \frac{dU}{dt} = \dot{U}_3 * A_1 + \dot{U}_2 * A_2 + \dot{U}_1 * A_3
\]

Figure D.6. Defining the angular acceleration of body A in Newtonian frame N.
Determining constrained generalised inertial forces

In the four segment model generalised speeds $U_1$ to $U_4$ are used to determine the angular acceleration of the right rings cable and torso segments in the Newtonian frame. These angular motions contribute to the constrained generalised inertial forces. However, to describe all motions which contribute to constrained generalised inertial forces the linear velocity and acceleration of each point and segmental mass centre are also required.

In AUTOLEV™3 the velocity and acceleration of a point or mass centre in the Newtonian frame is determined by the differentiation of the vector describing the point or mass centre in the Newtonian frame. The vector describing the position of a point or mass centre may use the internal and the Newtonian reference frames. Differentiation of a position vector twice with respect to time leads to the velocity and acceleration of a point or mass centre in the Newtonian frame. Using a specific command in AUTOLEV™3 the differentiation process is performed automatically. This process was conducted for all points and mass centres of segments defined in the four segment model (eg MPTR, ULEGo) which, combined with the angular velocities and accelerations, determines the constrained generalised inertial forces for the four segment model.

Determining constrained generalised active forces

To determine all constrained generalised active forces in the four segment model the linear velocity and acceleration in the Newtonian frame of all points at which external forces act on the system are required. These points include the right offset ($Q_r$), the effective mass ($EM_i$), the top and bottom of the rings cable ($TC_i$ and $BC_i$), the shoulder joint ($Q_i$) and the mass centre location of each segment where segmental weight acts (eg ARMRo). The generalised active forces are incorporated into the system by specific AUTOLEV™3 commands relating to the type of force produced at each point. Types of force include the weight of each segment and those due to the damped linear springs of the model. Segmental weights are determined by the product of the segmental mass and acceleration due to gravity. Forces due to damped linear springs require the extension of the spring from its natural length and the rate of change of extension, or spring velocity (Section 3.3.2). In the four segment model the natural length of each spring is zero while the generalised speeds $U_5$ to $U_8$ are used to characterise spring velocities.

Syntax and commands used by AUTOLEV™3 in the command file. fourseg.al

The following syntax and commands were used in producing the four segment three-dimensional simulation model. Definitions of the commands are taken from the tutorial book, which is issued in conjunction with AUTOLEV™3.
Examples for each command relate directly to the four segment model. The first set of commands are required to formulate the equations of motion for the four segment model. The second set are used to furnish the output from the model. For the four segment model the inertial reference frame represented by \( N_1, N_2 \) and \( N_3 \) in AUTOLEV\textsuperscript{TM}3 is synonymous with the frame denoted by \( X, Y \) and \( Z \) (Figure D.7). In the main body of the thesis the results are presented using the \( X, Y \) and \( Z \) axes for consistency with the video analysis and the two-dimensional simulation models.

![Figure D.7. The Newtonian reference frame in terms of \( N_1, N_2 \) and \( N_3 \) and \( X, Y \) and \( Z \).](image)

Syntax used by AUTOLEV\textsuperscript{TM}3

In AUTOLEV\textsuperscript{TM}3 vectors are denoted by a '>' following their declared name. Scalars have nothing following their declared name.

- **P\_O\_OR\>**: denotes a position vector from point \( O \) to point \( OR \).
- **V\_TCR\_N\>**: denotes the velocity of point \( TCR \) in Newtonian frame \( N \).
- **A\_CRo\_N\>**: denotes the acceleration of the point \( CRo \) in Newtonian frame \( N \).
- **W\_CR\_N\>**: denotes the angular velocity of body \( CR \) in Newtonian frame \( N \).
- **ALF\_CR\_N\>**: denotes the angular acceleration of body \( CR \) in Newtonian frame \( N \).

Commands used to formulate the equations of motion for the four segment model

AUTOZ ON: uses a matrix of \( Zs \) for intermediate calculations to form the most efficient method for formulating the equations of motion using Kane's method. This command also increases the efficiency of the Fortran code representing the simulation model.

NEWTONIAN N: declares \( N \) to be a Newtonian (inertial) reference frame with orthogonal unit vectors \( N_1\>, N_2\> \) and \( N_3\> \).

BODIES CR: declares the massive body \( CR \), with the point \( CRo \) as the mass centre of body \( CR \), and the orthogonal unit vectors \( CR1\>, CR2\> \) and \( CR3\> \) fixed in \( CR \).
PARTICLES EMR: declares the massive point EMR.

POINTS O: declares the massless point O.

FRAMES INTR: declares the massless frame INTR comprising orthogonal unit vectors \( \text{INTR}_1, \text{INTR}_2 \) and \( \text{INTR}_3 \).

MASS CR=MSC: assigns MSC as the mass of body CR.

INERTIA CR, ICL, ICF, ICT, 0, 0, 0: assigns the principal moments of inertia for body CR as ICL (longitudinal), ICF (frontal) and ICT (transverse), while assigning zero values to the other central inertia scalars of body CR.

CONSTANTS LC{2}: declares the constants LC1 and LC2.

VARIABLES PHCR', \( U_{17} \)': declares the variables PHCR and its first time derivative PHCR'. Generalised speeds \( U_1, U_2 \) through to \( U_{17} \) are also declared as variables, together with their first time derivatives.

ZEE_NOT [KCTORQ1, . . , STORQR3]: excludes scalar quantities from the AUTOZ function.

SPECIFIED ELR": declares ELR, and its first and second time derivatives, as specified functions of time and other constants.

DIRCOS(N, CR, BODY321, PHCR, THCR, SICR): forms the direction cosine matrix for frames N and CR using a BODY321 rotation, with PHCR (\( \phi_{cr} \)), THCR (\( \theta_{cr} \)) and SICR (\( \psi_{cr} \)) as successive ordered orientation angles from Newtonian frame N to internal reference frame CR.

SIMPROT(TR, INTR, 3, ELR-PI): forms the direction cosine matrix for frames TR and INTR associated with a simple rotation about the axis \( \text{INTR}_3 \) by the radian measure ELR-\( \pi \).

\( \text{P}_O\text{OR} = \text{BUNG}D*N3 \): declares the position vector O to OR equal to the scalar value BUNGD in the direction of the unit vector N3.

\( \text{W}_{CR\text{ N}} = U_3*CR1 + U_2*CR2 + U_1*CR3 \): declares the angular velocity of reference frame CR in Newtonian frame N in terms of generalised speeds \( U_1, U_2 \) and \( U_3 \).
KINDIFFS(N, CR, BODY321, PHCR, THCR, SICR): forms the kinematical differential equations for body CR in frame N using a BODY321 rotation with PHCR (φ_cr), THCR (θ_cr) and SICR (ψ_cr) as successive ordered orientation angles from frame N to frame CR.

\[ V_{TCR_N} = DT(P_{O_{TCR}}, N) \]: determines the time derivative of the position vector \( P_{O_{TCR}} \) in the Newtonian frame N and returns it as \( V_{TCR_N} \).

\[ CCMPTX = DOT(LOOP>, N3)> \]: forms the dot product of the vectors \( LOOP \) and \( N3 \) and assigns the name CCMPTX to this scalar value.

\[ DEPENDENT[1] = DOT(V_MPTR_N>, N3)> \]: forms a constraint equation where the dot product of the two vectors is set equal to DEPENDENT[1], a zero value.

\[ AUXILIARY[1] = U9 \]: sets \( U9 \) to be an auxiliary generalised speed used to determine force and torque measure numbers and is equal to zero.

\[ CONSTRAIN (AUXILIARY[U9, . . ,U17], DEPENDENT [U2]) \]: forms the constraint equations for the system by regarding the dependent generalised speeds to be equal to zero. Also forms force and torque measure numbers by setting the auxiliary generalised speeds equal to zero.

\[ ALF_{CR_N} = DT(W_{CR_N}, N) \]: determines the time derivative of the angular velocity vector \( W_{CR_N} \) in the Newtonian frame N and returns it as \( ALF_{CR_N} \).

\[ A_{TCR_N} = DT(V_{TCR_N}, N) \]: determines the time derivative of the velocity vector \( V_{TCR_N} \) in the Newtonian frame N and returns it as \( A_{TCR_N} \).

\[ GRAVITY(G*N2)> \]: forms a local gravitation force for each body acting in the direction of the unit vector \( N2 \).

\[ FORCE(OR/EMR, -(CH*HFR+DH*HFR')*N1)> \]: forms a force acting between two points OR and EMR, the force acting in the direction of the unit vector \( N1 \). The force shown is equivalent to that produced by a damped linear spring.

\[ ZERO = FR() + FRSTAR() \]: forms the generalised active forces \( FR() \) and generalised inertia forces \( FRSTAR() \) for the system and sets the sum of these equal to zero. This is part of the requirements of Kane's method.
KANE(STORQR1, . . . , KTORQ3): forms Kane's dynamical equations and brings these equations into a form suitable for numerical integration, as well as determining the force and torque measure numbers formed by the auxiliary speeds.

CODE DYNAMICS () //pathname/FOURSEG.FOR, SUBS: creates the Fortran code for the four segment three-dimensional simulation model on rings together with all the necessary subroutines.

Commands used to furnish output of the model

TORQUE(ARMR/TR, STORQR1*TR1> + STORQR2*TR2> + STORQR3*TR3>): forms three torques between the two bodies ARMR and TR, the torques acting about the three internal axes of body TR and, in conjunction with auxiliary speeds, determines values for all joint torques.

CM(O): determines the centre of mass location for the whole system from the point O.

CM(O, ARMR, TR, ULEG, LLEG): determines the vector from O to the centre of mass location for the bodies ARMR, TR, ULEG and LLEG.

XGYCM = DOT(P_O_GYCM>, N3>): determines the value of the x coordinate for the mass centre of the gymnast.

KE(): calculates the kinetic energy for the whole system.

ARRANGE(KEALL, 2, U(1:17)): rearranges the expression for KEALL in terms of the generalised speeds U1 to U17 which reduces the length of the expression for kinetic energy in the Fortran code.

MOMENTUM(ANGULAR,O): forms an expression for the angular momentum vector for the whole system about the point O.

XAMOMALL = DOT(ANGMOM_ALL_N>, N3>): forms the dot product of the angular momentum vector for the whole system and the Newtonian frame unit vector N3>, which is synonymous with the Newtonian x axis.

RHS(HTALL): uses or returns the right hand side of the variable HTALL.
Diagrams of the four segment simulation model

Figure D.8. Bodies and particle defined in the four segment model.

Figure D.9. Points defined in the four segment model.
Figure D.10. Lengths defined in the four segment model.

Figure D.11. Forces defined in the four segment model.
Nomenclature for the four segment model

Bodies:
CR = right rings cable segment
TR = torso segment
ARMR = right arm segment
ULEG = thigh segment
LLEG = shank segment

Particles:
EMR = effective mass of the right side of the rings frame

Masses:
MSC = mass of the right rings cable
MSARM = mass of the right arm
MSTR = mass of the right side of the torso
MSULEG = mass of the right thigh
MSLLEG = mass of the right shank
HMSEM = mass of the effective mass

Inertia:
ICL = moment of inertia about the longitudinal axis of the rings cable
ICF = moment of inertia about the frontal axis of the rings cable
ICT = moment of inertia about the transverse axis of the rings cable
IARML = moment of inertia about the longitudinal axis of the arm
IARMF = moment of inertia about the frontal axis of the arm
IARMT = moment of inertia about the transverse axis of the arm
ITRT = moment of inertia about the transverse axis of the torso
IULEGT = moment of inertia about the transverse axis of the thigh
ILLEGL = moment of inertia about the longitudinal axis of the shank
ILLEGF = moment of inertia about the frontal axis of the shank
ILLEG = moment of inertia about the transverse axis of the shank

Constants:
G = acceleration due to gravity

Frames:
N = Newtonian reference frame
INTR = intermediate internal reference frame for right arm
Points:

O = origin, (0,0,0)
OR = fixed point for the right rings cable on N3 axis
TCR = top of the right rings cable segment
CRo = mass centre of right rings cable
BCR = bottom of the right rings cable segment
HR = right hand of the gymnast
ARMRo = mass centre of right arm
QR = connects right arm to the right shoulder spring
JR = right shoulder joint centre
MPTR = mid-point of the torso defined from the right side of the model
TRo = mass centre of torso
J1 = middle of the hip joint hinge
ULEGo = mass centre of the thigh
J2 = middle of the knee joint
LLEGo = mass centre of the shank

Additional points:

ALLCM = mass centre point for the whole system
GYCM = mass centre point for the gymnast
CABCM = mass centre for the rings cable
MIDHAND = point defined on the x axis by the location of the right hand

Lengths:

BUNGD = between origin and OR
LC1 = between top of the rings cable and the rings cable mass centre
LC2 = rings cable length
LA1 = between hand and arm segment mass centre
LA2 = arm length
LT1 = between MPTR and the torso mass centre
LT4 = between right shoulder joint and theoretical left shoulder joint
LT5 = between MPTR and the hip joint
LU1 = between hip joint and the thigh mass centre
LU2 = between hip joint and the knee joint
LL1 = between knee joint and the shank mass centre

Variable angles:

PHCR = first orientation angle for the right rings cable
THCR = second orientation angle for the right rings cable
SICR = third orientation angle for the right rings cable
PHTR = orientation angle for torso

Generalised speeds:
U1 = represents motion of right rings cable about internal axis CR3
U2 = represents motion of right rings cable about internal axis CR2
U3 = represents motion of right rings cable about internal axis CR1
U4 = represents motion of torso about internal axis TR3
U5 = represents motion of right vertical spring along axis N2
U6 = represents motion of right horizontal spring along axis N1
U7 = represents motion of right cable spring along axis CR1
U8 = represents motion of right shoulder spring along axis ARMR1

Auxiliary generalised speeds:
U9 = represents the torque for the right arm about axis ARMR3
U10 = represents the torque for the right arm about axis ARMR2
U11 = represents the torque for the right arm about axis ARMR1
U12 = represents the torque for the thigh about axis ULEG3
U13 = represents the torque for the thigh about axis ULEG2
U14 = represents the torque for the thigh about axis ULEG1
U15 = represents the torque for the shank about axis LLEG3
U16 = represents the torque for the shank about axis LLEG2
U17 = represents the torque for the shank about axis LLEG1

Specified angles, angular velocities and angular accelerations:
ELR = first configuration angle for the right arm segment
ELR' = angular velocity of ELR
ELR" = angular acceleration of ELR
ABR = second configuration angle for the right arm segment
ABR' = angular velocity of ABR
ABR" = angular acceleration of ABR
PHULEG = configuration angle for the thigh segment
PHULEG' = angular velocity of PHULEG
PHULEG" = angular acceleration of PHULEG
PHLLEG = configuration angle for the shank segment
PHLLEG' = angular velocity of PHLLEG
PHLLEG" = angular acceleration of PHLLEG
Spring parameters:

- CV = stiffness of the vertical spring
- CH = stiffness of the horizontal spring
- CSTIF = stiffness of the cable spring
- ASTIF = stiffness of the arm spring
- DV = damping of the vertical spring
- DH = damping of the horizontal spring
- CDAMP = damping of the cable spring
- ADAMP = damping of the arm spring

Torque measure numbers:

- STORQR1 = representing the torque at the right shoulder joint about axis TR1
- STORQR2 = representing the torque at the right shoulder joint about axis TR2
- STORQR3 = representing the torque at the right shoulder joint about axis TR3
- HTORQ1 = representing the torque at the hip joint about axis ULEG1
- HTORQ2 = representing the torque at the hip joint about axis ULEG2
- HTORQ3 = representing the torque at the hip joint about axis ULEG3
- KTORQ1 = representing the torque at the knee joint about axis LLEG1
- KTORQ2 = representing the torque at the knee joint about axis LLEG2
- KTORQ3 = representing the torque at the knee joint about axis LLEG3

Definitions of orientation and configuration angles for FOURSEG

Table D.1 indicates the reference frame from which each segment is described, the rotation angles which define the orientation or configuration, the names of the angles and the appropriate direction cosine matrix used to describe the orientation or configuration. For example, rotation 1 for the right rings cable segment is defined from the Newtonian frame by the angle \( \phi_{cr} \), rotation 2 by \( \theta_{cr} \) and rotation 3 by \( \psi_{cr} \).

When the orientation angles for the right rings cable segment are all zero the internal reference frame is in line with the Newtonian frame. That is, direction cosines \( C_{311}, C_{322} \) and \( C_{333} \) are equal to 1, with all other direction cosines of \( C_3 \) equal to zero. The first rotation angle for the rings cable (\( \phi_{cr} \)) and the only rotation of the torso (\( \phi_{tr} \)) must place the rings cable and torso segments in a suitable orientation for a gymnast ready to perform a longswing. In order to place the rings cable in an orientation hanging vertical downwards an angle of \( 270^\circ \) is required for \( \phi_{cr} \). To place the torso segment of the gymnast in a vertical handstand position an angle of \( 90^\circ \) for \( \phi_{tr} \) is required. To retain consistency with definitions used in the two-dimensional simulation models and video analyses the Fortran code was modified so that an input value of 0\(^\circ\) for \( \phi_{cr} \) and \( \phi_{tr} \) placed the rings cable in an inverted orientation and the torso of the gymnast in a handstand.
position respectively. Additionally, output from the simulation model FOURSEG was modified from the 'raw' model to produce values for the rings cable and torso segments consistent with the two-dimensional models.

In FOURSEG positive values for the right rings cable abduction angle \( \theta_{cr} \) indicate the right rings cable is closer to the inertial plane y-z. This is equivalent to a narrowing of the distance between the rings in the video analysis, denoted by negative values. In order to retain consistency with video data for comparison purposes, the sign of the right rings cable abduction angle and angular velocity from the model was altered to be consistent with video analyses.

Table D.1. Orientation and configuration angles in the four segment model

<table>
<thead>
<tr>
<th>segment</th>
<th>frame from which segment orientation or configuration is defined</th>
<th>rotation angles used</th>
<th>direction cosine matrix used</th>
</tr>
</thead>
<tbody>
<tr>
<td>right rings cable</td>
<td>Newtonian</td>
<td>( \phi_{cr} ) ( \theta_{cr} ) ( \psi_{cr} )</td>
<td>C3</td>
</tr>
<tr>
<td>torso</td>
<td>Newtonian</td>
<td>( \phi_{tr} )</td>
<td>C1</td>
</tr>
<tr>
<td>right arm</td>
<td>torso</td>
<td>( \phi_{elr} ) ( \theta_{abr} )</td>
<td>C2</td>
</tr>
<tr>
<td>thigh</td>
<td>torso</td>
<td>( \phi_{uleg} )</td>
<td>C1</td>
</tr>
<tr>
<td>shank</td>
<td>thigh</td>
<td>( \phi_{lleg} )</td>
<td>C1</td>
</tr>
</tbody>
</table>

The hands of the gymnast must remain in contact with the rings in the four segment model, meaning not all motions of the system are independent. Motion constraint equations simulating these inter-dependencies were formed by creating an imaginary point MPTR on the mid-line of the torso and defining a vector (LOOP) from the origin O to this theoretical point. The designated constrain equation states that the velocity of the point MPTR in the Newtonian frame only possesses components in the vertical plane. This constraint reduces the degrees of freedom of the four segment three-dimensional model from 8 to 7 and ensures that all segments of the model remain in contact with each other during a simulation.

**AUTOLEV™3 command input file, fourseg.al, for the four segment model**

The following pages are annotated command lines from the input file *fourseg.al*, which produced the 'raw' simulation model of a gymnast swinging on rings.
The methods outlined in Chapter 4.9 in DYNAMICS ONLINE are used to obtain force/torque measure numbers for the model. All angles and angular velocities are defined as positive.

The model forces the right side of a gymnast to slide up and down the plane formed by the inertial y-z axes. The velocity vector for the midpoint of the torso defined from the right side of the gymnast (MPTR) is constrained to remain in the vertical plane.

This file details the structure, motion and external forces which represent a gymnast swinging on rings, modelled as detailed in Figures D.8 - D.11 of Appendix D.

The model comprises five segments:
1 cable, 1 arm, 1 torso with head, 1 thigh, 1 shank with foot
one massive particle:
1 mass simulating the effective mass of the rings frame
four damped linear springs:
1 vertical spring at the top of the rings cable representing the damped elastic devices
1 horizontal spring at the top of the rings cable representing the elasticity of the rings frame
1 spring in line with the rings cable representing the elasticity of the rings cable
1 spring in line with the arm representing the elasticity of the shoulder or gymnast as a whole.

Additional output produced by the model includes:
CTENBW = combined cable tension in BW
STENBW = combined shoulder tension in BW
MIDHAND = the location of the hand on the plane defined by y-z axes
BANG = variable to calculate body angle of the gymnast as defined in the video analyses
KITORQ(3) = knee torques
HTORQ(3) = hip torques
STORQR(3) = shoulder torques
ANG_MOM = angular momentum of system (ALL) and gymnast (GY) about 0 for all Newtonian axes
WALEN = wrist to ankle length
TCABLEN = total rings cable length

Total energy of the system is calculated and used to check the model for errors.

DEFINE THE SYSTEM IN PHYSICAL TERMS

NEWTONIAN N % Newtonian reference frame
BODIES CR, ARM, TR, ULEG, LLEG % cable, arm, torso, thigh, shank
PARTICLES EMR % effective mass of rings frame
POINTS O, OR % fixed points
POINTS TCR, MR, JR, J1, J2 % top of cable, hand, joints
POINTS MPTR % midpoint of torso
POINTS ALLCM, GYCM, CABCM % CM: all, gymnast, cable
POINTS MIDHANO % midhand location
POINTS QR, BCR % point defining spring in arm, and rings cable
FRAMES INTR % intermediate frame for the arm

MASS AND INERTIA OF SEGMENTS
MASS CR=MSC, ARMR=MSARM, TR=MSTR, &
ULEG=MSULEG, ILEGL=MSLLEG, EMR=HMSEM
INERTIA CR, ICL, ICF, ICT, 0, 0, 0
INERTIA ARMR, IARM, IARMF, IARMF, 0, 0, 0
INERTIA TR, ITRL, ITRF, ITRT, 0, 0, 0
INERTIA ULEG, IULEGL, IULEG, IULEGT, 0, 0, 0
INERTIA LLEG, ILLEG, ILEG, ILLEG, 0, 0, 0

CONSTANTS FOR THE SYSTEM
CONSTANTS G % gravity
CONSTANTS LC(2), LA(2) % lengths for cable and arm
CONSTANTS LT(5), LU(2), LL(2) % lengths for torso, thigh and shank
CONSTANTS CV, DV % stiffness and damping for vert. spring
CONSTANTS CH, DH % stiffness and damping for hor. spring
CONSTANTS CSTIF, CDAMP % stiffness and damping for cable spring
CONSTANTS ASTIF, ADAMP % stiffness and damping for arm spring
CONSTANTS BUNGD % distance from origin (0) to hor. spring
CONSTANTS TREE(4), PEE(4) % used for angle changes in AUTOLEVBTM3
CONSTANTS STPHULEG, STPHLLEG % initial angles for knee and hip joints
CONSTANTS STELR, STABR % initial elevation and abduction angles for shoulder joint

VARIABLES FOR THE SYSTEM
VARIABLES U(17)' % generalised speeds: 4 angular, 4 linear, 9 auxiliary
VARIABLES PHCR', THCR', SICR' % angles defining right cable orientation
VARIABLES PHTR' % angle defining torso orientation
VARIABLES HPR', VBR' % lengths and velocities for hor. and vert. springs
VARIABLES CABSR' % length and velocity for cable spring
VARIABLES MR' % length and velocity for arm spring
VARIABLES KTORQ(1:3), &
HTORQ(1:3), STORQR(1:3)
ZEE_NOT = [KTORQ1, KTORQ2, KTORQ3, &
HTORQ1, HTORQ2, HTORQ3, STORQR1, &
STORQR2, STORQR3]

SPECIFIED VALUES FOR THE SYSTEM
SPECIFIED ELR'' % arm elevation angle, vel., accn.
SPECIFIED ABR'' % arm abduction angle, vel., accn.
SPECIFIED PHULEG'' % hip joint angle, vel., accn.
SPECIFIED PHLLEG'' % knee joint angle, vel., accn.
ELR = STELR * (PEE1*T)
ABR = STABR + (PEE3*T)  % angle changes used in AUTOLEVTM3
PHULEG = STPHULEG*TREE5*SIN(PEE5*T) % but modified in the customised
PHLEG = STPHLEG*TREE6*SIN(PEE6*T) % model FOURSEG

GEOMETRY RELATING UNIT VECTORS AND FRAMES

DIRCOS(N,CR,BODY321,PHCR,THCR,SICR)
SIMPROT(N,TR,3,PHTR)
SIMPROT(TR,INTR,3,ELR-PI)
SIMPROT(INTR,ARMR,2,ABR)
SIMPROT(TR,ULEG,3,PI-PHULEG)
SIMPROT(ULEG,LLEG,3,PHLEG-PI).

DEFINE POSITION VECTORS OF ALL POINTS RELATIVE TO THE ORIGIN O

P_O_OR> = BUNGD*N3>
P_O_EMR> = P_O_OR> + HFR>*N1>
P_O_TCR> = P_O_EMR> + VBR*N2>
P_O_CRO> = P_O_TCR> + LC1*CR1>
P_O_BCR> = P_O_TCR> + LC2*CR1>
P_O_HR> = P_O_BCR> + CABS*R*CR1>
P_O_ARMRO> = P_O_HR> + LA1*ARMR1>
P_O_QR> = P_O_HR> + LA2*ARMR1>
P_O_JR> = P_O_QR> + MR*ARMR1>
P_O_MPRTR> = P_O_JR> + 0.5*LT4*TR3>
P_O_TRO> = P_O_MPRTR> + LT1*TR1>
P_O_J1> = P_O_MPRTR> + LT5*TR1>
P_O_ULEGO> = P_O_J1> + LU1*ULEG1>
P_O_J2> = P_O_J1> + LU2*ULEG1>
P_O_LLEG> = P_O_J2> + LL1*LLEG1>

DEFINE ANGULAR VELOCITIES OF ALL SEGMENTS USING GENERALISED SPEEDS
AND DETERMINE THE KINEMATICAL DIFFERENTIAL EQUATIONS FOR ANGULAR
MOTION

W_CR N> = U3*CR1> + U2*CR2> + U1*CR3>
KINDIFFS(N,CR,BODY321,PHCR,THCR,SICR)
PHTR' = U4
W_TR N> = PHTR'*TR3>
VBR' = U5
HFR' = U6
CABS'R' = U7
MR' = U8
W_INTR TR> = ELR'*TR3>
W_ARMR_INTR> = ABR'*INTR2>
W_ULEG_TR> = PHULEG'*TR3>
W_LLEG_ULEG> = PHLEG'*ULEG3>

Use auxiliary generalised speeds, u9 to ul7, to produce measure
numbers for the joint torques.

W_ARMR N> = W_ARMR N> + ul1*ARMR1> + ul0*ARMR2> + u9*ARMR3>
W_ULEG N> = W_ULEG N> + ul4*ULEG1> + ul3*ULEG2> + ul2*ULEG3>
W_LLEG N> = W_LLEG N> + ul7*LLEG1> + ul6*LLEG2> + ul5*LLEG3>
DEFINE LINEAR VELOCITIES FOR ALL POINTS AND DETERMINE THE KINEMATICAL DIFFERENTIAL EQUATIONS FOR LINEAR MOTION

\[ V_{O_N} = 0 \]
\[ V_{OR_N} = 0 \]
\[ V_{EMR_N} = DT(P_{O_EMR}, N) \]
\[ V_{TCR_N} = DT(P_{O_TCR}, N) \]
\[ V_{CRO_N} = DT(P_{O_CRO}, N) \]
\[ V_{BCR_N} = DT(P_{O_BCR}, N) \]
\[ V_{HR_N} = DT(P_{O_HR}, N) \]
\[ V_{ARMRO_N} = DT(P_{O_ARMRO}, N) \]
\[ V_{QR_N} = DT(P_{O_QR}, N) \]
\[ V_{JR_N} = DT(P_{O_JR}, N) \]
\[ V_{MPTR_N} = DT(P_{O_MPTR}, N) \]
\[ V_{TRO_N} = DT(P_{O_TRO}, N) \]
\[ V_{J1_N} = DT(P_{O_J1}, N) \]
\[ V_{ULEGO_N} = DT(P_{O_ULEGO}, N) \]
\[ V_{J2_N} = DT(P_{O_J2}, N) \]
\[ V_{LLEGO_N} = DT(P_{O_LLEGO}, N) \]

DEFINE MOTION CONSTRAINTS AND FORM CONSTRAINT EQUATIONS FOR THE SYSTEM

Constraint equations are produced by forcing the velocity of MPTR (midpoint of torso defined from the right side of the system) to be zero in the x or N3 direction of the Newtonian reference frame.

\[ \text{LOOP} = P_{O_MPTR} \]
\[ CCMPTX = \text{DOT} (\text{LOOP}, N3) \]
\[ MCMPTX = \text{DOT} (CCMPTX) \]
\[ \text{DEPENDENT}[1] = \text{DOT} (V_{MPTR}, N3) \]

Determine the auxiliary equations using:
\[ w_{bodies_n} = w_{bodies_n} + w_{bodies\_relative\_frame} \] in terms of auxiliary speeds.

auxiliary[1] = u9
auxiliary[2] = u10
auxiliary[3] = u11
auxiliary[4] = u12
auxiliary[5] = u13
auxiliary[6] = u14
auxiliary[7] = u15
auxiliary[8] = u16
auxiliary[9] = u17

Determine equations in terms of generalised speeds U1 to U8 (excluding the independent speed U2) for constraints and torque measure numbers.

CONSTRAN (auxiliary[u9,u10,u11,u12,u13,u14,u15,u16,u17], DEPENDENT[U2])

DEFINE ANGULAR ACCELERATIONS FOR ALL BODIES
DEFINE LINEAR ACCELERATIONS FOR POINTS OF THE SYSTEM

\[
\begin{align*}
\mathbf{v}_0 &= 0 \\
\mathbf{v}_1 &= 0 \\
\mathbf{v}_2 &= \mathbf{a}(N) \\
\mathbf{v}_3 &= \mathbf{a}(N) \\
\mathbf{v}_4 &= \mathbf{a}(N) \\
\mathbf{v}_5 &= \mathbf{a}(N) \\
\mathbf{v}_6 &= \mathbf{a}(N) \\
\mathbf{v}_7 &= \mathbf{a}(N) \\
\mathbf{v}_8 &= \mathbf{a}(N)
\end{align*}
\]

STATE EXTERNAL FORCES

Forces due to the weight of the segments.

GRAVITY(G*N2>)

Forces due to the damped linear springs.

\[
\begin{align*}
\text{FORCE} &\left(\mathbf{F}_{\text{fr}}/\mathbf{F}_{\text{fr}}, \mathbf{F}_{\text{br}}/\mathbf{F}_{\text{br}}, \mathbf{F}_{\text{cbr}}/\mathbf{F}_{\text{cbr}}, \mathbf{F}_{\text{shr}}/\mathbf{F}_{\text{shr}}\right) \\
&\left(\mathbf{F}_{\text{fr}}/\mathbf{F}_{\text{fr}}, \mathbf{F}_{\text{br}}/\mathbf{F}_{\text{br}}, \mathbf{F}_{\text{cbr}}/\mathbf{F}_{\text{cbr}}, \mathbf{F}_{\text{shr}}/\mathbf{F}_{\text{shr}}\right)
\end{align*}
\]

Define joint torques acting at each joint in order to obtain joint torque measure numbers.

\[
\begin{align*}
\text{TORQUE} &\left(\mathbf{F}_{\text{fr}}/\mathbf{F}_{\text{fr}}, \mathbf{F}_{\text{br}}/\mathbf{F}_{\text{br}}, \mathbf{F}_{\text{cbr}}/\mathbf{F}_{\text{cbr}}, \mathbf{F}_{\text{shr}}/\mathbf{F}_{\text{shr}}\right) \\
&\left(\mathbf{F}_{\text{fr}}/\mathbf{F}_{\text{fr}}, \mathbf{F}_{\text{br}}/\mathbf{F}_{\text{br}}, \mathbf{F}_{\text{cbr}}/\mathbf{F}_{\text{cbr}}, \mathbf{F}_{\text{shr}}/\mathbf{F}_{\text{shr}}\right)
\end{align*}
\]

DETERMINE CM LOCATION, KE, EPE AND PE FOR THE WHOLE SYSTEM,

GYMNAST AND CABLE

\[
\begin{align*}
P_{\text{all}} &= \mathbf{c}(0) \\
P_{\text{gy}} &= \mathbf{c}(0, \mathbf{ARM}, \mathbf{TR}, \mathbf{ULEG}, \mathbf{LLEG}) \\
P_{\text{cab}} &= \mathbf{c}(0, \mathbf{CR})
\end{align*}
\]

In the model \(N_1\) is equivalent to the y direction, \(N_2\) is equivalent to the z direction.
and $N \rangle$ is equivalent to the $x$ direction, hence form locations in terms of $x,y,z$ coordinates.

\[
\begin{align*}
X_{ALLCM} &= \text{DOT} (P_0_{ALLCM}, N) \\
Y_{ALLCM} &= \text{DOT} (P_0_{ALLCM}, N) \\
Z_{ALLCM} &= \text{DOT} (P_0_{ALLCM}, N) \\
X_{GYCM} &= \text{DOT} (P_0_{GYCM}, N) \\
Y_{GYCM} &= \text{DOT} (P_0_{GYCM}, N) \\
Z_{GYCM} &= \text{DOT} (P_0_{GYCM}, N) \\
X_{CABCM} &= \text{DOT} (P_0_{CABCM}, N) \\
Y_{CABCM} &= \text{DOT} (P_0_{CABCM}, N) \\
Z_{CABCM} &= \text{DOT} (P_0_{CABCM}, N) \\
X_{POSEM} &= \text{DOT} (P_0_{POSEM}, N) \\
Y_{POSEM} &= \text{DOT} (P_0_{POSEM}, N) \\
Z_{POSEM} &= \text{DOT} (P_0_{POSEM}, N) \\
HT_{ALL} &= \text{DOT} (P_0_{ALLCM}, N)
\end{align*}
\]

Determine the KE for the whole system, the gymnast only and the cable only.

\[
\begin{align*}
KE_{ALL} &= KE() \\
\text{ARRANGE} (KE_{ALL}, 2, U[1:17]) \\
KE_{GY} &= KE(ARMR, TR, ULEGG, LLEGG) \\
\text{ARRANGE} (KE_{GY}, 2, U[1:17]) \\
KE_{CABLE} &= KE(CR)
\end{align*}
\]

Determine the elastic potential energy in each set of springs.

\[
\begin{align*}
\text{EPERF} &= 0.5*CH*HFR*HFR \\
\text{EPERV} &= 0.5*CV*VBR*VBR \\
\text{EPERC} &= 0.5*CSTIF*CABSR*CABSR \\
\text{EPERM} &= 0.5*ASTIF*MR*MR
\end{align*}
\]

Form the potential energy (PE) for the whole system. \(-1\) is used in PE equations as on input $G = -9.81$.

\[
\begin{align*}
PE_{ALL} &= -1*RHS(HT_{ALL})*(MSC+MSARM+MSTR+MSULEG+MSLLEGG+HMSEM)*G \\
PE_{GY} &= -1*RHS(Z_{GYCM})* (MSARM+MSTR+MSULEG+MSLLEGG)*G \\
PE_{CABLE} &= -1*RHS(Z_{CABCM})*(MSC)*G
\end{align*}
\]

Determine the total energy for the whole system. Use this as a checking function for model structure. When spring damping for all springs is zero and no forced relative motion occurs between the gymnast's body segments the total energy of the system remains constant.

\[
\begin{align*}
TE_{ALL} &= RHS(KE_{ALL}) + RHS(PE_{ALL}) + RHS(EPERF) + RHS(EPERV) + RHS(EPERC) + RHS(EPERM)
\end{align*}
\]

Calculate more output:

\[
\begin{align*}
\text{CTEN} &= \text{cable tension in N.} \\
\text{CTENBW} &= \text{combined cable tension in bodyweights} \\
\text{STEN} &= \text{shoulder tension in N.} \\
\text{STENBW} &= \text{combined shoulder tension in bodyweights} \\
\text{CTEN} &= (CSTIF*CABSR + CDAMP*CABSR') \% \text{cable tension} \\
\text{CTENBW} &= \text{CTEN}/(MSARM + MSTR + MSULEG + MSLLEGG)*G \% \text{combined cable tension BW}
\end{align*}
\]
Calculate the position vector of the hand and determine the \((y, z)\) location of this point on the \(y-z\) vertical plane. The \((y, z)\) location of the gymnast's centre of mass (\(GYCM\)) is also required to determine the body angle (\(BANG\)) using trigonometry, as defined in the video analysis. The initial handstand position equals zero degrees, which is consistent with planar models and video analyses.

\[
\text{BANG} = \frac{(ZGYCM - ZMIDHAND)}{(YGYCM - YMIDHAND)} \% \text{ altered in Fortran code}
\]

Calculate angular momentum of the system and gymnast about 0 for all three Newtonian axes.

**System:**

\[
\text{ANGMOM\_ALL\_N} = \text{MOMENTUM(ANGULAR, O)}
\]

\[
\text{XAMOMALL} = \text{DOT (ANGMOM\_ALL\_N>, N3>)}
\]

\[
\text{YAMOMALL} = \text{DOT (ANGMOM\_ALL\_N>, N1>)}
\]

\[
\text{ZAMOMALL} = \text{DOT (ANGMOM\_ALL\_N>, N2>)}
\]

**Gymnast:**

\[
\text{ANGMOM\_GY\_N} = \text{MOMENTUM(ANGULAR, O, ARMR, TR, ULEG, LLEG)}
\]

\[
\text{XAMOMGY} = \text{DOT (ANGMOM\_GY\_N>, N3>)}
\]

\[
\text{YAMOMGY} = \text{DOT (ANGMOM\_GY\_N>, N1>)}
\]

\[
\text{ZAMOMGY} = \text{DOT (ANGMOM\_GY\_N>, N2>)}
\]

Calculate the total wrist to ankle length and total cable length

\[
\text{WALEN} = \text{LA2} + \text{MR} + \text{LT5} + \text{LU2} + \text{LL2}
\]

\[
\text{TCABLEN} = \text{LC2} + \text{CABSR}
\]

**USE KANE'S METHOD OF FORMULATING THE EQUATIONS OF MOTION FOR THE SYSTEM**

Retain the torque measure numbers for the joint torque values in the Kane command.

\[
\text{ZERO} = \text{FR()} + \text{FRSTAR()}
\]

\[
\text{KANE (STORQR1, STORQR2, STORQR3, HTORQ1, HTORQ2, HTORQ3, & KTORQ1, KTORQ2, KTORQ3)}
\]

**FORMAT THE UNITS USED FOR INPUT AND OUTPUT**

**UNITS**

\[
\text{T} = \text{SEC}
\]

\[
\text{[UNITS [BUNGD,LC1,LC2,LA1,LA2,LT1,LT2,LT3,LT4,LT5,LU1,LU2,LL1,LL2] = M}
\]

\[
\text{[VBR,HFR,CABSR,MR, WALEN,TCABLEN,XALLCM,YALLCM,ZALLCM] = M}
\]

\[
\text{[CCMPTX,XGYCM, YGYCM,ZGYCM,XMIDHAND,YMIDHAND,ZMIDHAND] = M}
\]

\[
\text{[VBR',HFR',CABSR',MR'] = M/S}
\]

\[
\text{[PHCR,THCR,SICR,PHTR,PHULEG,PHLLEG,ELR,ABR,BANG] = DEG}
\]
UNITS [STABR, STPHULEG, STPHLLEG] = DEG
UNITS [PHCR', THCR', SICR', PHTR', PHULEG', PHLLEG',ELR', ABR'] = RAD/S
UNITS [ELR', ABR', PHULEG', PHLLEG'] = RAD/S^2
UNITS [MSC, MSARM, MST, MSLEG, MSLLEG, HSEM] = KG
UNITS [ICL, ICP, ICT, ITRL, TRF; ITRT, IARML, IARMF, IARM] = KG.M^2
UNITS [IULEGL, IULEGF, IULECT, ILEGL, ILEGT] = KG.M^2
UNITS [PEALL, TEALL, KEALL, EPF, EPER, EPERC, EPERM] = J
UNITS [KEG, PEGY, KECABLE, PECABLE] = J
UNITS [CV, CH, CSTIF, ASTIF] = N/M
UNITS [DV, DH, CDAMP, ADAMP] = N/M/S
UNITS [CTEN, STEN] = N
UNITS [CTENBW, STENBW] = BW
UNITS [STORQR1:3, HTORQ1:3, KTORQ1:3] = N.M
UNITS [XAMOMALL, YAMOMALL, ZAMOMALL] = KG.M^2/S
UNITS [XAMOMGY, YAMOMGY, ZAMOMGY] = KG.M^2/S

% State the order of variables to be written to output files

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OUTPUT T, PHCR', PHTR'
OUTPUT T, THCR, THCR'
OUTPUT T, SICR, SICR'
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OUTPUT T, 2*PEALL, 2*PELV, 2*EPERC, 2*EPERM
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OUTPUT T, CTENBW, STENBW
OUTPUT T, VBR, HFR, CABS, MR
OUTPUT T, YGYCM, ZGYCM, YMIDHAND, ZMIDHAND, BANG
OUTPUT T, PHULEG, PHLLEG
OUTPUT T, PHULEG', PHLLEG'
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OUTPUT T, XAMOMGY, YAMOMGY, ZAMOMGY
OUTPUT T, XPOSEMR, YPOSEMR, ZPOSEMR
OUTPUT T, CCMPFX
OUTPUT T, WALEN, TCABLEN

% SAVE OUTPUT AND PRODUCE FORTRAN CODE FOR THE SIMULATION MODEL

SAVE FOURSEG.ALL
CODE DYNAMICS () FOURSEG.FOR, SUBS
APPENDIX E

This appendix presents the anthropometric measurements taken from subjects A and K using the procedure of Yeadon (1990b). Output from the 11 segment mathematical inertia model of Yeadon (1990b) for both subjects, A and K, are also provided.

Signed written consent forms from both subjects for the collection of anthropometric data and the collection of video and cable tension data during longswing performances are also included.
# Anthropometric Measurements for Segmental Inertia Parameters

**Name:** subject A  
**Age:** 20  
**Height:** 1.712 m  
**Date:** 26/07/95  
**Measurer:** M.H  
**Weight:** 66.30 kg

All measurements in millimetres

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ANTHROPOMETRIC MEASUREMENTS FOR SEGMENTAL INERTIA PARAMETERS

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- All measurements in millimetres

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The inertia data for subject A were calculated using the segmental density estimates of Dempster (1955).

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FORMAT AND SEQUENCE OF DATA PRESENTATION
SEGMENT NAME
MASS, DISTANCE OF MASS CENTRE FROM PROXIMAL JOINT, SEGMENT LENGTH
PRINCIPAL MOMENTS OF INERTIA

SUBJECT: A

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**WHOLE BODY**

MASS = 67.30  DENSITY = 1.026  
MEASUR ED MASS = 66.30

The inertia data for subject K were calculated using the segmental density estimates of Dempster (1955).

STATE OPTION NUMBER:
(1) ANTHROPOMETRIC DATA
(2) DIGITISED DATA FILE

STATE NAME OF SUBJECT
SEGMENTAL INERTIA PARAMETER VALUES

UNITS: MASS IN KG  
DISTANCE IN METRES  
MOMENT OF INERTIA IN KG*M**2

FORMAT AND SEQUENCE OF DATA PRESENTATION
SEGMENT NAME  
MASS, DISTANCE OF MASS CENTRE FROM PROXIMAL JOINT, SEGMENT LENGTH  
PRINCIPAL MOMENTS OF INERTIA

SUBJECT: K
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0.034  0.034  0.016
TRUNK PTC
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<td>0.042</td>
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<tr>
<td>Foot 3J</td>
<td>0.778</td>
<td>0.077</td>
<td>0.210</td>
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<tr>
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<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
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<tr>
<td>Calf 2K</td>
<td>3.409</td>
<td>0.175</td>
<td>0.400</td>
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<td>0.041</td>
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<tr>
<td>Foot 3K</td>
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<td>0.072</td>
<td>0.205</td>
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<tr>
<td></td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Whole Body</td>
<td></td>
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</tr>
<tr>
<td>Mass = 61.63</td>
<td></td>
<td></td>
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<tr>
<td>Density = 1.028</td>
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</tr>
<tr>
<td>Measured Mass = 62.30</td>
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Loughborough University

INFORMED CONSENT FORM

PURPOSE
To obtain anthropometric and kinematic data of a gymnast during longswings on rings in order to develop and evaluate a mathematical model of this activity.

PROCEDURES
Video cameras will be used to collect information during the performance of longswings on rings, in addition cable tension will be measured by a force transducer in a rings cable. A number of trials will be requested, with suitable breaks to minimise fatigue and boredom. Anthropometric data will be collected using tape measures and specialist anthropometers.

QUESTIONS
The researcher will be pleased to answer any questions which you may have at any time.

WITHDRAWAL
You are free to withdraw from the study at any time whatever reason without prejudice.

CONFIDENTIALITY
Your identity will remain confidential in any material resulting from this work.

I have read and understood the information on this form and agree to participate in this study. As far as I am aware I do not have any injury nor infirmity which would be affected by the procedures outlined.

Name...........................................
Signed...........................................
(gymnast) Signed...........................................(parent)

In the presence of:

Name...........................................
Signed...........................................(coach) Date...........................................
Loughborough University

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Name .......................................................... Name ......................................................

Signed ............................................. (gymnast) Signed ........................................... (parent)

In the presence of:

Name ............................................

Signed ............................................ (coach) Date .........