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CONTRIBUTIONS TO PERFORMANCE IN DYNAMIC JUMPS

by

Mark Arthur King

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of
Doctor of Philosophy of Loughborough University

March 1998

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ABSTRACT

CONTRIBUTIONS TO PERFORMANCE IN DYNAMIC JUMPS

by Mark Arthur King

Loughborough University, March 1998

The performance of dynamic jumps is the result of complex interactions between many factors, including preflight characteristics, muscle strength and activation timings, and the elastic properties of external contact surfaces. The aim of this study was to determine the contributions of these factors to the performance of dynamic jumps and to gain a greater understanding of the underlying mechanics.

Theoretical computer simulation models were developed incorporating muscle representations and elastic interfaces between the model and the external contact surfaces for vaulting and tumbling takeoffs in gymnastics. The simulation models were customised to represent the elite male gymnast analysed in this study by calculating subject specific inertia and muscle parameters from experimental testing with the gymnast. The simulation models were evaluated by comparing simulations of each movement with actual vaulting and tumbling performances by the elite male gymnast and then used to quantify the contributions to vaulting and tumbling performance.

The characteristics of the preflight were found to have a major influence on both vaulting and tumbling performance. In addition, for tumbling, the takeoff strategy (activation timings of the muscles) was also crucial, with it being possible to produce a range of postflight performances by just changing the strategy used during the takeoff. Vaulting and tumbling performances were found to be relatively insensitive to changes (within realistic limits) in the elastic nature of the contact surfaces and for vaulting the elasticity of the shoulder joint had a considerable effect on performance. In addition the use of the hand / foot was found to prolong the duration of contact with an external surface.
PUBLICATIONS AND AWARDS

Journals


Conference presentations


EUROPEAN COLLEGE OF SPORT SCIENCE

Young Investigators Award
1997

On behalf of the Jury, Professor Dr. Paavo Komi, President of the ECSS, certifies that the

FIRST PRIZE (US $ 5,000)

is awarded to:

KING, M.

for his / her oral presentation:

Contributions of pre-flight, shoulder torque, and elasticity to
postflight performance in the Hecht vault

Second Annual Congress of ECSS

COPENHAGEN, AUGUST 20 - 23, 1997

Prof. Dr. Paavo Komi
President of the ECSS

Supported by Mars, Incorporated
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- Austin Woods for his assistance with the data collection sessions
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- DeMontfort University for the use of their Penny and Giles goniometers
- BBSRC for making this study financially possible
- My Mother, Father and Norma for their support and encouragement
- The Biomechanics Team at Loughborough (Lesley, Matt, Mike, Mark, Jo, John and Sharon)
DEDICATION

To Norma

“For being a constant river of love and support in everything I do.”
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CHAPTER 1

INTRODUCTION

Evolution has resulted in the development of many specialised animals which can perform dynamic jumps very effectively (for example kangaroos). Humans though have evolved in a different way, being far from specialised for any particular jumping task. Instead humans aspire to jumping higher and further through training and changes in technique.

Many sports at the elite level involve different types of maximal dynamic jumps, which can generically be defined as those activities that include an approach followed by an impact, contact and takeoff from an elastic surface. For example dynamic jumps are involved in the performance of vaulting and tumbling in gymnastics and the jumping events in athletics (high jump, long jump, triple jump and pole vault). The performance achieved in dynamic jumps is the result of a complex interaction between many factors, including preflight characteristics, muscle strength and activations, and properties of external elastic contact surfaces. Although dynamic jumps are vital to many sports, a fundamental understanding of these movements and a mechanical explanation of the techniques used is far from complete. Furthermore there is a lack of understanding of the interaction between the muscle-tendon complex and an external elastic surface during dynamic jumps (Brüggemann, 1994).

A number of sources can help to improve the understanding of the performance of dynamic jumps and give a mechanical explanation of the techniques used:

Cinematographic and video studies have established the techniques used by athletes to perform dynamic jumps. Furthermore these studies identify the aspects of technique that are associated with better performance. This type of study is limited in the understanding it can give to the performance of a movement as it is only based upon a few examples of each movement. Such studies cannot investigate the effect of varying technique outside the range of observed performances of a movement and are limited to speculating on what is the best technique for a particular movement and athlete.

Controlled experimental studies can give a greater understanding of the technique used for a given movement. Direct intervention can be used to vary the approach and technique used by an athlete to investigate the effect of altering technique on
performance. The power of such experiments can be limited as changes in one aspect of
technique may inadvertently introduce other changes to the movement (Yeadon and
Challis, 1994).

Theoretical studies allow complete control and flexibility over the technique used
for the performance of dynamic human movements. In these studies researchers have
developed simulation models for jumping. The models have ranged in complexity from
the simple models for long jumping and high jumping (Alexander, 1990) to complex
models for vertical jumping (van Soest, Schwab, Bobbert and van Ingen Schenau, 1993,
Pandy, Zajac, Sim, and Levine, 1990) and the very complex model of Hatze (1981a) for
the long jump takeoff. Simulation models have improved the general understanding of
the mechanical principles governing jumping activities although none of the models has
been completely evaluated.

A number of common factors which influence the performance of dynamic jumps
have been identified in the literature:

Preflight characteristics

The approach velocity, angular momentum and the orientation of the segments at
contact have been shown to have a substantial influence on the performance of a jump.
The horizontal approach velocity has been found to be a key factor in high jumping, too
fast or too slow a velocity can limit the possible performance (Alexander, 1990).

Muscle strength and activation

By optimising jump height, two research teams have shown that vertical jump
performance is sensitive to muscle strength, and to the sequencing and timing of muscle
activation. For example Bobbert and de Bruin (1994) optimised jump height using a
computer simulation model, by varying the sequencing and timing of muscle activation
for a specified muscle strength. The muscle strength was then increased but the jump
height did not improve unless the timing of the muscles was re-optimised for the
increased muscle strength.

Tendon stiffness

Theoretical models of jumping have included series elastic elements (tendon) in
their representations of muscle. These studies have shown the effect on performance of
varying tendon stiffness. For example Anderson and Pandy (1993) found that decreasing tendon stiffness to the point of tendon rupture (10% strain) resulted in a 3% increase in jump height.

Elasticity of the contact surface

Dynamic jumps are performed from very different contact surfaces ranging from the relatively inflexible athloprene for jumping in athletics to the sprung track for tumbling in gymnastics. McMahon and Greene (1979) have shown how running performance is affected by track stiffness and Toderov and Cooper (1989) have speculated on the effect of tumbling on different surfaces.

The four factors identified as influencing the performance of dynamic jumps have been formulated into a number of general questions which will be answered in this study:

**Which factors contribute to the performance of dynamic jumps?**

The preflight characteristics, muscle strength, sequencing and timing of muscles, tendon stiffness and elasticity of the contact surface have been identified as factors which influence performance. This study will establish the contributions of each of these factors to the performance of dynamic jumps, and investigate the contribution of other factors such as the inertia characteristics and joint elasticity on performance. In addition the sensitivity of the performance to each of these factors will be investigated. The effect of each factor on the performance of dynamic jumps can then be investigated:

**What is the effect of varying the preflight characteristics on the performance of dynamic jumps?**

Alexander (1990) demonstrated the general importance of the approach velocity and the plant angle to performance in long jumping and high jumping. This study will investigate the importance of each of the preflight variables (mass centre velocity, joint configurations, segment angular velocities and whole body angular momentum) on the performance of dynamic jumps.

**How does muscle strength affect the performance of dynamic jumps?**

Muscle strength is often presumed to play an important role in the performance of dynamic jumps, although the high angular velocities reached by the body segments during
dynamic jumps may limit the influence on performance. This study will consider the effect of muscle strength on the performance of dynamic jumps.

How does tendon stiffness affect the performance of dynamic jumps?

Most theoretical studies of jumping include series elastic elements in their muscle representations, although the effect of tendon stiffness on jumping performance is far from complete. This study will investigate how sensitive the performance of dynamic jumps is to changes in tendon stiffness.

What is the effect of the elasticity of the contact surface on dynamic jumps?

Dynamic jumps are performed from very different contact surfaces which may affect the optimum technique for a dynamic jump. This study will investigate how sensitive the performance of dynamic jumps is to changes in the stiffness and damping of the contact surface.

What is the effect of joint elasticity on dynamic jumps?

Dynamic jumps are accompanied by varying degrees of movement at the joints in the body, however, the effect of joint movement on performance has not been considered in the literature. This study will examine the Hecht vault in gymnastics where movement of the shoulder joints occurs and consider the effect of varying the stiffness and damping of the shoulder on vaulting performance.

The aims of this study are summarised below in the statement of purpose:

Statement of purpose

[a] To increase the understanding of the mechanics of dynamic jumps.
[b] To identify those factors which characterise successful performance of dynamic jumps.
[c] To quantify the contributions made to the successful performance of dynamic jumps.

To fulfil these objectives, planar link segment computer simulation models for dynamic jumps will be developed and applied to the takeoff phase of the Hecht vault and layout somersault takeoffs. These jumps were chosen as they are fundamentally planar movements which preserve left-right symmetry. The models will be customised to an elite gymnast through measurements and evaluated by comparing simulated performances
with actual performances by the same gymnast. The models will then be used to quantify the contributions to the performance in the Hecht vault and layout somersaults. The simulation models developed will give a greater understanding of the techniques used for dynamic jumps and will enable the determination of the contributions to the performance of dynamic jumps.

Chapter organisation

Chapter 2

A review of the relevant literature is covered in Chapter 2 with particular attention given to three main areas. Firstly descriptions of dynamic jumps including vaulting, tumbling and jumping takeoffs are presented. Secondly a more detailed examination of these activities is considered with reference to the research literature. Finally the techniques available for pursuing this study are considered.

Chapter 3

Three simulation models of dynamic jumps are developed in Chapter 3. The first model is a simple two segment simulation model for vaulting without muscle or elastic elements during the contact phase with the horse. Two further models are then developed for simulating vaulting and tumbling takeoffs using the Autolev computer software package. These models include representations of muscle and tendon, and elastic interfaces between the gymnast and the contact surfaces.

Chapter 4

This chapter presents the image analysis procedures used to obtain kinematic data for the vaulting and tumbling takeoffs. General descriptions of the vault and tumbling takeoffs are given with the salient features of each activity highlighted. The kinematic data obtained are presented for both the vault and tumbling performances. The kinematic data are used in Chapter 6 to evaluate each simulation model.

Chapter 5

Subject specific inertia, muscle and elastic parameters are calculated in Chapter 5. The inertia parameters are calculated from anthropometric measurements on the subject.
using the inertia model of Yeadon (1990b). The muscle parameters are calculated from maximal effort muscle torque measurements using an isokinetic dynamometer at the ankle, knee, hip and shoulder joints. The elastic parameters for the interface between the model and the vaulting horse and tumbling track are estimated from drop tests.

Chapter 6

The simulation models developed in Chapter 3 are evaluated in this chapter using kinematic data from previous studies (Yeadon, King and Sprigings, in press) and data obtained in Chapter 4, the inertia, muscle and elastic parameters obtained in Chapter 5 and an optimisation procedure using the Simulated Annealing algorithm. The methods used for the evaluation of each model are described and the results obtained are reported.

The simulation models are used to establish the contributions made to the Hecht vault and tumbling performances and in addition the sensitivity of each performance to small changes in the subject specific parameters, preflight characteristics, and the timing and sequencing of the muscle activations is investigated.

Chapter 7

The questions asked in Chapter 1 are discussed in Chapter 7 along with areas for improvement in the techniques used. The findings of this study are summarised and discussed and avenues for further investigation highlighted.
CHAPTER 2

LITERATURE REVIEW

The review of literature is divided into three main areas:

Firstly descriptions of dynamic jumps including vaulting, tumbling and jumping takeoffs are presented. Secondly a more detailed examination of these activities is considered with reference to the research literature. Finally the techniques available for pursuing this study are considered.

DESCRIPTION OF JUMPING TECHNIQUES

Many sports involve maximal dynamic jumps, which are all based upon the same mechanical principles (Jacoby and Fraley, 1995). This section describes the mechanics behind dynamic jumps with particular reference to vaulting, tumbling and jumping which include takeoffs from either the hands or the feet.

Maximal dynamic jumps and their mechanical descriptions

All maximal dynamic jumps consist of three common phases:
1. Approach and preflight prior to the final contact
2. Impact, contact and takeoff
3. Postflight

Description of the approach and preflight

The approach and preflight are used to prepare the athlete for the final contact with an external surface and typically consist of an approach run to gain horizontal velocity followed by an adjustment step to put the body in the optimum position for the final contact (Hay, 1985). For most maximal jumping activities a near maximal / maximal horizontal velocity is required (Jacoby and Fraley, 1995). During the final stride while the athlete is in free-fall, the velocity of the mass centre is governed by constant acceleration equations and the total angular momentum about the mass centre is conserved (Hay, 1985). Therefore, during the final stride the horizontal velocity of the mass centre and the angular momentum about the mass centre are constant, however the athlete can control the rotation by altering the moment of inertia of the body (Hay, 1985).
Description of the final contact and takeoff

Although the contact phase is very short (< 0.2 s), it is during this phase that the athlete alters the path travelled by the mass centre and changes the angular momentum about the mass centre by muscular actions and the reaction from the external surface. This allows the characteristics / potential of the approach and preflight to be converted into the takeoff characteristics at the start of postflight phase (Hay, 1985). Therefore the final contact phase is critical as it determines the angular momentum that the athlete will have during the flight phase and also the time of flight. Typically the contact phase consists of an initial eccentric absorbing phase followed by a concentric push off phase; for vaulting Dainis (1981) labels the two parts as compression and repulsion. The contact phase ends when the vertical reaction force on the body equals zero.

Description of the postflight phase

During the postflight phase the effectiveness of the previous two phases is observed, with the athlete only able to make configuration changes to best utilise the characteristics of the takeoff (Hay, 1985). The mechanics of the postflight phase are the same as for the preflight phase with the velocity of the mass centre governed by constant acceleration equations and the conservation of angular momentum (Hay, 1985). For example in the high jump the athlete uses the vertical and horizontal velocity of the mass centre along with the angular momentum about the mass centre generated during the approach and contact phases to clear the bar with the high jumper changing his body configuration during the postflight phase to best utilise the takeoff characteristics.

Summarising, during the approach and preflight phases the athlete prepares, during the contact phase the athlete converts the kinematics of the approach and preflight into the takeoff characteristics for the postflight and during the postflight phase the effectiveness of the previous phases is realised.

Although all maximal dynamic jumping activities have three common phases the technique used for different jumps varies greatly. The technique used for a given skill or movement depends on the requirement of the movement. For example in high jumping the athlete is trying to jump as high as possible over a bar whereas in long jumping the athlete is trying to jump as far as possible. The purpose of the takeoff is to therefore produce 'the optimum, non-maximal production of takeoff velocities’ (Brüggemann,
Thus for any takeoff there exists an optimum combination of linear and angular velocities that will produce the best performance in postflight. Clearly the optimum technique varies from one movement to another. In addition how does the optimum technique vary between athletes performing the same skill as athletes have different inertial characteristics and strengths. Barker (1976) suggested that gymnastic coaching has been hampered by the misconception that there is a single best way to perform a particular movement. Also there is the question of how the optimum technique varies with different takeoff surfaces. For example how does the optimum technique change when tumbling on a tumbling track rather than a gymnastics floor, when the elastic properties of the track and floor are different?

Examples of maximal dynamic jumps

Approach and prefight

As noted previously the approach and prefight consist of a run-up to generate horizontal velocity followed by an adjustment in the body orientation and configuration to prepare for the penultimate contact and subsequent phases.

For tumbling the adjustment usually consists of a round-off flic flac prior to the final takeoff (Toderov and Cooper, 1989). The round-off flic flac (Figure 2.1) is required to generate a high angular velocity and put the body in the correct position for the final contact (Toderov and Cooper, 1989). Although the round-off flic flac increases the angular momentum there is still the requirement to build up linear momentum (Barker, 1976; Boone, 1976). It is therefore necessary to develop a fast run, round-off flic flac without loss of horizontal speed or possibly an increase in horizontal velocity during the round-off flic flac if possible (Toderov and Cooper, 1989). This leads to the question of what is the optimum combination of linear and angular momentum at the final contact? Should the approach be as fast as possible with a detrimental effect on angular momentum or should rotation be maximised with a reduction in linear momentum. In addition to the production of linear and angular momentum during the approach the gymnast can also vary the body position at the start of the final contact phase. For example in the double straight somersault the final snap down prior to contact should be slightly shorter than for the single somersault takeoff (Toderov and Cooper, 1989). Furthermore Toderov and Cooper (1989) noted that the angle of attack (the angle from the horizontal to the line of
the legs at contact) will vary with the quality of the tumbling track, the individual's characteristics and the level of preparation of the body. With a soft track the attack angle should be small and on a hard track the attack angle should be large and the hip angle must be no smaller than 140-145° (Figure 2.2).

Figure 2.1. Round-off flic flac (Boone, 1976, p. 23).

Figure 2.2. Angle of attack and hip angle (Toderov and Cooper, 1989, p. 4).
For vaulting in gymnastics the approach and the preflight are the most important phases and are complicated by the inclusion of a springboard prior to contact with the horse (Taylor, Bajin, and Zivic, 1972; Hunn, 1978). For men's vaulting the approach consists of a run-up (maximum 25 m: FIG, 1993) followed by an adjustment (hurdle step) to make a two-footed contact with the springboard and a short preflight before contact with the horse (Figure 2.3). Boone (1976) emphasised the importance of the approach noting that a poor approach will lead to a poor vaulting performance. The hurdle step and contact with the springboard is used to convert the linear momentum generated in the approach into the optimum vertical, horizontal and angular momenta for a specific vault with a loss of some horizontal momentum during contact with the springboard (Boone, 1976; Hay, 1985). Readhead (1987) noted that continuous rotation vaults require more rotation during preflight than counter-rotation vaults. Thus the optimum horizontal, vertical and angular momenta at takeoff from the springboard vary depending on the vault being performed. Fukushima and Russell (1980) suggested that a low preflight is required for the Hecht vault (counter-rotation vault) whereas for the handspring somersault vault a quick preflight is preferred. In addition Fukushima and Russell (1980) noted that a more vertical takeoff position is required for handspring vaults compared with the takeoff position for the Hecht vault which should be leaning forward.

![Figure 2.3. The approach and preflight of a vault.](image)

For high jumping, Sohi (1980), Reid (1984), Hackett (1987) and Jacoby (1987) noted that the approach is the most important phase with speed being a major factor which must be high and maintained throughout the curved approach (Figure 2.4). However none of the authors quantify what the best approach velocity is and how the optimum velocity changes between athletes. The curved approach has been preferred in high jumping since
the development of the Fosbury flop technique although there is some debate over the effect of a curved approach on performance (Tan, 1997). Tidow (1993), discussing the question of optimum technique, quotes Viitasalo: ‘...there is presumably no optimal universal style but only an individually optimal style.’ As well as the question of approach velocity Jacoby (1987) and Hackett (1987) discussed the effect on performance of plant angle and position of the hips at contact prior to takeoff. Both authors noted that there is a trade-off between maintaining horizontal velocity and keeping the hips low at contact prior to the final takeoff, with Jacoby acknowledging that there is much debate over the best trade-off.

As with high jumping similar principles are apparent for long jumping even though the object of the jump is different. The general principles for a good long jump approach are: to reach maximum horizontal velocity in the approach, lower the hips in the penultimate stride and then increase the mass centre height to maximum height at takeoff while maintaining horizontal velocity (Hay, 1985; Jacoby and Fraley, 1995).
For all the jumping activities described above the potential for a good performance begins with the approach. With a poor approach it is not possible to produce a good performance. The sort of questions which need to be answered for all the activities are:

What is the optimum approach for a given athlete in a particular activity?
How does the optimum change for different performers and different conditions?
How does the stiffness of the contact surface affect the performance?
What is the optimum body position at initial contact with the ground?
How does the optimum body position change with different athletes?

Alexander (1990) answers some of the above questions concerning the approach and preflight of dynamic jumps. Alexander used a simple two segment simulation model for high and long jumping to show that the approach velocity should be slower for high jumping than for long jumping. The explanation is based upon the effect of horizontal velocity on the time of contact and the subsequent impulse exerted on the ground by the jumper. For high jumping a longer time of contact is required so that a larger vertical impulse can be exerted whereas for the long jump a faster run-up is better as the horizontal component of velocity at takeoff is the most important factor. Although Alexander demonstrates the general relationships and increases the understanding of dynamic athletic movements there is still much work to be done before dynamic jumping activities can be fully understood and explained.

Final contact and takeoff

As noted earlier the final contact phase is very short, with the approach characteristics converted into the optimum distribution of linear and angular momentum for the postflight phase.

For tumbling, the takeoff is where the accumulated potential of the approach is converted into height and rotation (Barker, 1976). To achieve maximum height Barker (1976) noted that it is important that the body and legs are straight at takeoff and for a double backward salto the body should be as vertical as possible with the arms vertical and then backward during the takeoff. Sale (1976) referred to the problem of generating high muscular forces as the joint angular velocities during the takeoff are very high. He suggested that most of the force is generated at the beginning of takeoff when the
velocities are smaller. For example in the standing backward somersault the impulse the
gymnast can exert is limited by the short time of contact and the increasing velocity of the
movement (Sale, 1976). Furthermore Toderov and Cooper (1989), describing the takeoff
for the double straight somersault, noted that the body should be straightened forcefully
with the arms and body swept upwards and backwards. In addition if the gymnast has
insufficient tension in the body at contact, then the gymnast will not receive the impulse
from the track and if the arms do not lift there will be insufficient angular momentum at
takeoff. Finally if the angle of takeoff is too small the gymnast will have good height but
poor rotation and if the angle of takeoff is too large the gymnast will have too much
rotation and insufficient height.

For vaulting, the contact and pushoff must adhere to the technical requirements of
the vault, alter the preflight to give vertical velocity and maintain sufficient momentum
for the postflight (Boone, 1976). The gymnast rotates about the hands while in contact
with the horse either forwards (in a handspring) or backwards (in a Hecht). For the Hecht
vault two different techniques are identified: either a downward push against the horse
(tends to create backward rotation) or a pulling action over the horse (Boone, 1976).

For the high jump the plant angle is small with a near straight leg, allowing the
muscles to stretch slightly during the first part of the plant before extending (Reid, 1984;
1987). Sohi (1980) and Tidow (1993) agreed with Reid noting that the high jumper has
approximately 0.15 s for the takeoff and therefore the takeoff leg may only bend slightly
before extending. In addition Tidow noted that the leg is pre-tensed prior to contact
helping the jumper to resist the flexing of the leg and allowing the muscle to change the
type of contraction in the takeoff leg from eccentric to concentric. Jumpers also use their
arms and free leg to increase the mass centre height at takeoff and increase the pressure on
the takeoff leg (Reid, 1987) through using a range of different techniques (Tidow, 1993).

For a long jump takeoff a large vertical force should be created with the mass centre
moved from a low to a high position while losing as little horizontal speed as possible. In
addition the knee and hip must not flex too much and the jumper needs to be as upright as
possible at takeoff to give a high mass centre position (Jacoby and Fraley, 1995). Jacoby
and Fraley (1995) identified four takeoff techniques which vary the movements made by
the arms and free leg during the takeoff: the kick style, the double arm style, the sprint
takeoff and the power sprint or bounding takeoff.
For dynamic jumps a number of different techniques have been identified for the contact and takeoff phase. The type of questions which should be addressed concerning the contact and takeoff phases include:

How do the muscles aid performance during the contact phase?
How could the optimum performance of an athlete improve with strength training?
What role do the free limbs play in aiding performance?
How does the sequencing and timing of muscles affect performance?
How does optimum technique vary between athletes?

The simulation model of Alexander (1990) can be used to address some of the above questions about the contact phase. Alexander (1990) showed that performance is 'remarkably insensitive' to changes in strength and in addition that the mass centre position at contact should be higher for long jump compared with high jump. Alexander did not consider how technique should change for different athletes with different inertia characteristics and could not consider how the timing and sequencing of different muscle groups would affect performance as his model had only one muscle.

Postflight

For all dynamic jumps the effectiveness of the previous phases is observed during the postflight phase (e.g. Boone, 1976) with the mass centre following a parabolic path and angular momentum being conserved (Frederick, 1979; Toderov and Cooper, 1989; Boone, 1976; Tidow, 1993). The athlete can only alter the rotation of the body by changing the configuration of the body. For example a gymnast can tuck to increase the angular velocity of the body (Frederick, 1979).

Summary of jumping techniques

The above sections have highlighted the main principles for a wide range of athletic activities. The questions raised concerning each phase of the technique will be addressed in the subsequent parts of this chapter and in the remainder of this study.
INVESTIGATIONS OF DYNAMIC ACTIVITIES

Research into dynamic activities can be divided into two types: experimental studies and theoretical studies (Yeadon and Challis, 1994), although the most effective method is one that combines the information obtained from both types of research. This section will highlight the main findings of each type of investigation.

EXPERIMENTAL STUDIES

Experimental studies can record movements during competition where the researcher has no control over the performance (observational study). Alternatively the researcher can intervene to control the technique used by the athlete and observe the effect on performance in a controlled experimental study. Observational studies have been used to show what is possible and also to improve the mechanical understanding of dynamic athletic activities across a wide range of sports. Typically observational studies report the kinematics of the approach, preflight and postflight and use statistical analysis to look for relationships between the data. Controlled experimental studies give additional information on the approach and preflight strategies used by elite athletes and the effect of varying the preflight characteristics on performance.

In addition many studies perform statistical analyses from observational data obtained on a number of different athletes during competition. This technique has more 'unknown factors' since the effects of muscular strength and inertial characteristics differ between athletes as well as between performances. Even so observational studies can identify some relationships and identify criteria which are important for success. However, unless relationships or theories are supported by mechanical explanations caution should be used when interpreting the results obtained.

For the purposes of this review the emphasis will be on research carried out on vaulting, tumbling and jumping activities. For each activity the main findings from the research literature will be presented.

Vaulting

Dainis (1979) analysed handspring vaults from ten female gymnasts and applied a three segment model to analyse the performances. He found that the preflight variables at takeoff from the board were 'very similar' for all the gymnasts although at horse contact
Dainis found that there was considerable variation in the vertical velocity of the mass centre with the better vaults having higher vertical velocities at horse contact. All gymnasts lost horizontal velocity and angular momentum during the contact phase.

Dillman, Cheetham and Smith (1985) filmed eight handspring front somersault and eight Tsukahara type vaults from the Men’s Individual Olympic vaulting final in 1984 using three phase-locked high speed Locam cameras operating at 100 Hz. The mean values for the main approach and preflight variables were given for the handspring front vault (Table 2.1).

Table 2.1. Mean values for preflight variables during vaulting (Dillman et al., 1985)

<table>
<thead>
<tr>
<th>variable</th>
<th>mean value</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal approach velocity</td>
<td>7.79</td>
<td>0.13</td>
</tr>
<tr>
<td>vertical approach velocity</td>
<td>-0.27</td>
<td>0.20</td>
</tr>
<tr>
<td>Takeoff horizontal velocity</td>
<td>5.11</td>
<td>0.26</td>
</tr>
<tr>
<td>Takeoff vertical velocity</td>
<td>4.49</td>
<td>0.16</td>
</tr>
<tr>
<td>body angle at takeoff</td>
<td>74.8°</td>
<td>5.0°</td>
</tr>
<tr>
<td>Touchdown horizontal velocity</td>
<td>5.11</td>
<td>0.26</td>
</tr>
<tr>
<td>Touchdown vertical velocity</td>
<td>2.75</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Takei conducted one of the most comprehensive analyses of vaulting over a period of several years during the late 1980’s and early 1990’s (Takei, 1988, 1989, 1991; Takei and Kim, 1990). Although Takei analysed many vaults (handspring and forward salto vaults) his work has been mainly descriptive using linear regression to look for significant relationships between variables. The main findings were that high horizontal velocity and angular momentum at takeoff from the springboard were important determinants for successful results. Other factors identified by Takei included: a short preflight time, a large gain in vertical velocity while in contact with the horse, and large vertical and horizontal velocities at horse takeoff. During horse contact Takei (1988) noted that it was better to use a sharp, quick blocking action than a slow one for increasing vertical velocity, although he failed to give any explanation to support his findings.
Yeadon et al. (in press) compared the preflight strategies of the Hecht and handspring somersault vault. The vertical and horizontal velocity of the mass centre, body angle and shoulder angle at horse contact were identified as important variables for success in the Hecht vault. Yeadon et al. (in press) explained mechanically the effect of varying the preflight variables on the postflight performance of the Hecht vault by modelling the contact as a passive rebound. Yeadon et al. (in press) showed that to perform the Hecht vault required a longer, lower and faster preflight with slower rotation at horse contact compared with the handspring somersault vault (Table 2.2).

Table 2.2. Approach and preflight velocities for Hecht and handspring somersault vaults [m.s⁻¹]

<table>
<thead>
<tr>
<th></th>
<th>Hecht [Yeadon et al., in press]</th>
<th>handspring [Takei and Kim, 1990]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>horizontal approach velocity</td>
<td>7.14</td>
<td>0.27</td>
</tr>
<tr>
<td>vertical approach velocity</td>
<td>-0.97</td>
<td>0.25</td>
</tr>
<tr>
<td>horizontal velocity at board takeoff</td>
<td>5.56</td>
<td>0.36</td>
</tr>
<tr>
<td>vertical velocity at board takeoff</td>
<td>3.48</td>
<td>0.18</td>
</tr>
<tr>
<td>horizontal velocity at horse contact</td>
<td>5.56</td>
<td>0.36</td>
</tr>
<tr>
<td>vertical velocity at horse contact</td>
<td>1.00</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Brüggemann (1987) reviewed the research literature on gymnastics vaulting. His discussion was based upon his own work and that of a number of other researchers for continuous rotation vaults (handspring and Tsukahara type vaults). Brüggemann divided the vault into six phases: approach, takeoff, preflight, support, postflight and landing and used the experimental data of Brüggemann and Nissinen (1981) on three groups of ten gymnasts. The three experimental groups of gymnasts were a world class group, a national group and a junior class group from Germany. For the approach Brüggemann found that the better gymnasts had higher horizontal velocities (mean 7.98 m.s⁻¹, max 8.4 m.s⁻¹) with correlation coefficients of 0.78 and 0.83 between horizontal approach velocity and postflight height and distance respectively. During the takeoff from the springboard the horizontal velocity of the mass centre decreased while the vertical velocity and
angular momentum increased. He found that the better gymnasts had significantly higher angular momentum values at takeoff, with a correlation of 0.75 between takeoff angular velocity and postflight height. In addition a mean angular velocity of 6.8 rad.s$^{-1}$ was found for the best group at takeoff. A correlation of -0.75 between the time of preflight and the maximum height in postflight was found, indicating that a short preflight time is advantageous (mean 0.20 s for best group). At horse contact a positive (upwards) vertical velocity of the mass centre was found for handspring vaults. Bruggemann noted that to shorten the preflight gymnasts must have higher angular velocities during preflight or else the body angle at horse contact will be too low. Furthermore the angular velocity and horizontal velocity reduced and the vertical velocity increased during the support phase for nearly all the vaults. The height and distance reached by the mass centre, and the angular momentum during postflight were identified as important indicators of success with these three variables being a direct result of the kinetics of the previous phases. Although Bruggemann conducted an extensive review of the vaulting literature, there is little explanation as to why certain variables are important for success and why elite gymnasts use the techniques they do.

Kerwin, Harwood and Yeadon (1993) considered the effect of two different hand placement techniques to the performance of Tsukahara and Kasamatsu vaults. They found that those gymnasts who contacted the end of the horse with one hand and the top of the horse with the other had reduced horizontal velocity in postflight and no enhancement in rotation compared with those gymnasts who contacted the top of the horse with both hands.

**Tumbling**

Hwang, Seo and Liu. (1990) examined the biomechanical characteristics of the takeoff phase for three different types of double backward somersault at the 1988 Seoul Olympic Games. Seven layout double backward somersaults, seven twisting double backward somersaults and seven tucked double backward somersaults were analysed. Hwang et al. (1990) noted that the gymnast must achieve sufficient linear and angular momentum during the takeoff to perform the airborne somersaults, but also that there must be the correct balance between linear and angular momentum at takeoff. The most important mechanical factors at takeoff were identified as the jump height and the angular
momentum about the mass centre. Hwang et al. (1990) observed that the gymnasts performing the double back in the straight position, tried to maximise the angle of touchdown and takeoff (measured to the horizontal) resulting in greater rotation and a reduction in lift at takeoff. Hwang et al. (1990) found the following mean values for the straight double backward somersault (Table 2.3):

Table 2.3. Mean values for straight double backward somersault (Hwang et al., 1990)

<table>
<thead>
<tr>
<th>variable</th>
<th>mean value</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Touchdown angle (cm to toes)</td>
<td>51.4</td>
<td>3.7</td>
</tr>
<tr>
<td>Takeoff angle (cm to toes)</td>
<td>85.2</td>
<td>2.9</td>
</tr>
<tr>
<td>Takeoff duration [s]</td>
<td>0.145</td>
<td>0.006</td>
</tr>
<tr>
<td>Touchdown horizontal velocity [m.s⁻¹]</td>
<td>4.3</td>
<td>0.39</td>
</tr>
<tr>
<td>Touchdown vertical velocity [m.s⁻¹]</td>
<td>0.31</td>
<td>0.48</td>
</tr>
<tr>
<td>Takeoff horizontal velocity [m.s⁻¹]</td>
<td>3.1</td>
<td>0.21</td>
</tr>
<tr>
<td>Takeoff vertical velocity [m.s⁻¹]</td>
<td>3.75</td>
<td>0.25</td>
</tr>
<tr>
<td>Angular momentum (touchdown) [kg.m².s⁻¹]</td>
<td>155</td>
<td>-</td>
</tr>
<tr>
<td>Angular momentum (takeoff) [kg.m².s⁻¹]</td>
<td>112</td>
<td>-</td>
</tr>
</tbody>
</table>

Payne and Barker (1976) compared the techniques used to perform the flic flac and backward somersault. Four gymnasts performed flic flacs and backward somersaults from a standing start on a force plate while being filmed using a 16 mm cine camera. The major difference in technique was attributed to the body angle at takeoff. For the flic flac the body angle was approximately 42° past the vertical whereas for the backward somersault the angle was approximately 20° past the vertical. Small differences in the force traces and angular momentum values for the two movements were reported.

Brüggemann (1983, 1987) analysed the takeoff for single and double backward somersaults by considering the movement of single individual segments with respect to time and used a statistical analysis to find the most important variables for successful performance. For single and double backward somersaults he identified that the height reached by the mass centre and the angular momentum about the mass centre during
postflight were important indicators of successful performance, however for all somersaults analysed, there was little difference in angular momentum during postflight. A comparison was made of skilled and highly skilled performers of backward (20 vs. 20) and double backward (13 vs. 13) somersaults. Table 2.4 shows the results found for the double backward somersault. Brüggemann also found that the horizontal and vertical forces during contact were not typical of other sports due to the effect of the damping mats on the initial peaks in horizontal and vertical directions. Brüggemann demonstrated that the arms do not play an important role in terms of angular momentum production during takeoff with the legs producing the most angular momentum. This is in contrast to Toderov and Cooper (1989) who suggested that the arms help to produce angular momentum during the takeoff phase. Brüggemann also showed that the angular momentum and horizontal velocity at touchdown were closely related to the height achieved in postflight ($r = 0.81$). Brüggemann concluded that for a good backward somersault the conditions at touchdown are most important and therefore great attention to the round-off or flic flac approach is needed. In addition he noted that the importance of joint torques needs to be investigated in the future.

<table>
<thead>
<tr>
<th>variable</th>
<th>mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Takeoff duration [s]</td>
<td>0.123</td>
</tr>
<tr>
<td>Touchdown horizontal velocity [m.s$^{-1}$]</td>
<td>4.12</td>
</tr>
<tr>
<td>Touchdown vertical velocity [m.s$^{-1}$]</td>
<td>-0.42</td>
</tr>
<tr>
<td>Takeoff horizontal velocity [m.s$^{-1}$]</td>
<td>2.85</td>
</tr>
<tr>
<td>Takeoff vertical velocity [m.s$^{-1}$]</td>
<td>4.57</td>
</tr>
<tr>
<td>Angular momentum (touchdown) [kg.m$^2$s$^{-1}$]</td>
<td>128.9</td>
</tr>
<tr>
<td>Angular momentum (takeoff) [kg.m$^2$s$^{-1}$]</td>
<td>64.5</td>
</tr>
</tbody>
</table>
Jumping

Hay (1993) reviewed the ‘current knowledge’ concerning long and triple jumps identifying two requirements for the approach in the long jumper:

1. High horizontal velocity. Jumpers must develop as much horizontal velocity as can be used effectively during the approach. Hay calculated a correlation of 0.95 between horizontal approach velocity and official jump distance for 36 male and 28 female long jumpers.

2. Position at takeoff. Hay described the observed technique used by long jumpers during the final stride prior to takeoff. Jumpers tended to lower their mass centre and increase the touchdown distance of the final stride although no explanation was given as to why this is done.

Hay noted that during takeoff the vertical and horizontal ground reaction forces tend to affect the angular acceleration of the jumper about the transverse axis. The resultant moment of the ground reaction forces acts on the athlete initially tending to rotate the athlete backwards before tending to rotate the athlete forwards for the majority of the takeoff. Overall there is an increase in the forwards angular momentum about the transverse axis during the takeoff. During postflight the long jumper must maximise the potential of the previous two stages by controlling the forwards angular momentum or else over rotation will occur and reduce the distance jumped.

Dapena and Chung (1988) examined the takeoff phase of high jumping and explained the mechanisms involved by comparing the high jump with the vertical jump takeoff. They showed that the mass centre rises throughout the majority of the takeoff, rather than dropping as the legs contract eccentrically in the vertical jump. Thus the large eccentric forces generated during the first part of the contact phase help to increase the vertical velocity of the mass centre rather than just helping to reduce the downwards velocity of the mass centre. Dapena and Chung (1988) suggest that the technique used by high jumpers of running in fast and planting the takeoff foot in front of the mass centre allows maximum use of the forces that can be produced by muscles during eccentric conditions while also allowing the mass centre to rise throughout the majority of the takeoff phase.

Greig, Yeadon and Kerwin (1996) used a ‘single subject design’ to consider the effect of varying approach variables on the height jumped during postflight for high
jumping. The study was conducted in the training environment with the approach velocity of the jumper varied by using different lengths of approach and asking the jumper to run in at fast, medium or slow pace. Greig et al. (1996) found that the maximum height reached by the mass centre was significantly correlated at the 1% level to a quadratic function of approach speed and plant angle at touchdown. In contrast a linear relationship was found between knee angle and jump height with the straighter the knee at touchdown the greater the height jumped during postflight.

THEORETICAL STUDIES

Mathematical modelling of human motion has clear advantages and disadvantages which must be remembered when interpreting results obtained from simulation models. The confidence in results obtained, depends on the accuracy of the model, which is often unknown. To determine the accuracy of a model requires an evaluation which usually entails comparing the model output with a real performance. This requires the simulation model to be specific to a subject and therefore requires subject specific parameters to be determined. Where simulation models have not been evaluated, some confidence can be gained in the results obtained where independently derived models/approaches have given similar results.

To simulate dynamic movements in contact with the external world usually requires the forces produced by muscles to be calculated and modelled (Yeadon and Challis, 1994) as well as the physical characteristics of the human body. Typically a linked segment model is used to model the physical characteristics of the body. Most of the muscle models used in sports biomechanics are based upon the model of Hill (1938) although a number of variations of the Hill model have been used to represent the forces produced by the muscles.

The remainder of this section will highlight the main findings from theoretical studies using simulation models and discuss the suitability of each type of model to simulate dynamic jumps.

Sprigings, Yeadon, Watson and King (1994) used a two segment model with no muscle to show how varying the preflight characteristics could determine the performance that could be produced in postflight for vaulting in gymnastics. They found that varying the preflight characteristics allowed the major features of the postflight performance to be
reproduced. Although the general principles of vaulting were identified the model did not relate very closely to reality with simulations having zero contact times with the vaulting horse.

Hubbard (1980) showed that a planar three segment model with torque generators at the hip, shoulder and wrist joints could reproduce the major features of pole vaulting. The model was not evaluated but showed general relationships and principles in pole vaulting finding that the performance was sensitive to the initial mass centre velocity.

Alexander (1990) used a simple model consisting of two segments with a single torque generator to model high and long jump takeoffs. Alexander found that the performance was sensitive to the approach velocity, plant angle and knee angle at contact. For high jumping a smaller plant angle was beneficial compared with a steeper plant angle for long jumping, in addition a near straight knee angle was found to be beneficial for the high jump takeoff.

Two groups of authors developed simulation models for vertical jumping; one group is based at Amsterdam, The Netherlands (Bobbert, van Soest and van Ingen Schenau, e.g. van Soest et al. (1993)) and the other at Stanford, USA (Pandy and Zajac, e.g. Pandy et al. (1990)). Both simulation models include muscle models of individual muscles in the lower leg. Although both groups have demonstrated that their models 'reproduce the major features of a maximum height squat jump' by comparing the model output with filmed performances of vertical jumping their models are not subject specific and have not been evaluated. This is a draw back of this type of model and can limit the confidence in any findings, for example the two groups of authors found contradicting results over the effect of the biarticular gastrocnemius to jumping performance. The difference in findings has since been attributed to differences in the moment arm length for the gastrocnemius (van Soest et al., 1993). Ultimately it may be that neither model is correct for an individual, or that the results are sensitive to individual differences in anatomy / inertial characteristics. Even so the models have demonstrated that vertical jump performance is very sensitive to the timing of the muscle activations with slight changes in the coordination of the muscles drastically affecting the performance and increasing muscle strength only improved performance when accompanied by the correct timing of the muscle activations.
Overall the work of both the Amsterdam and Stanford groups has greatly improved the understanding of the factors that influence vertical jumping such as strength, coordination and contribution of the various muscles and segments to the final performance.

A model without muscle can help with the basic understanding of the relationships involved between preflight and postflight variables although its use in simulating elite athletic performances is limited. Models using torque generators have shown the general relationships apparent in a number of dynamic activities with the results being more realistic than the models with no muscle. However there has been a lack of evaluation of the models; the models have been too simple to represent elite athletes and the torques used have not been shown to be attainable by elite athletes. Human body models including models of individual muscles have reproduced the general features of dynamic jumping, although the usefulness of any approach which tries to model the forces produced by individual muscles will always be limited until methods are developed to obtain subject specific muscle parameters from the athlete.

Summary of research findings

The great majority of the experimental studies have concentrated on the approach and preflight identifying relationships between the approach / preflight and performance (Hay, 1993). Typical values for approach and preflight variables during dynamic movements have been obtained from observational studies giving an insight into what elite athletes can achieve. The approach / preflight has been identified as an important factor for successful performance in dynamic athletic movements. The contact phase has been given less attention with the approach / preflight characteristics being related to postflight performance regardless of the technique used during the contact phase. In most cases the relationships identified have not been supported by a mechanical explanation. The relationships which have been identified between approach / preflight and postflight suggest that the characteristics of the approach / preflight play a major role in determining the success of a performance and that although important, the role of muscles during the contact phase may not differentiate between the performance produced during postflight by different athletes.
Controlled experimental studies have highlighted the differences in performance between elite and non-elite athletes (e.g. Brüggemann, 1983) by comparing the performances of athletes with different abilities. This adds to the research findings of observational studies, giving more information on the actual range of approach and preflight variables achieved by elite athletes during dynamic movements. However there is a lack of mechanical explanations for the results obtained and little information is available on the techniques used during the contact phase of the athletic movements.

Theoretical studies have found that the approach and preflight characteristics have a large influence on the possible performance of a dynamic jump and given mechanical explanations of the techniques used for dynamic jumps. Studies on vertical jumping have demonstrated that performance is sensitive to the timing and sequencing of the muscles during contact as well as muscle strength.

Relatively simple models appear to be capable of reproducing the important features of dynamic jumps although for all the models reviewed there is a need for a method for calculating subject specific muscle parameters. Of the simulation models developed, those using torque generators to model the net effect of all the muscles around a joint give the most opportunity for calculating muscle parameters from direct measurements on the athlete, since the net performance of all the muscles around a joint can be measured experimentally.

TECHNIQUES OF INVESTIGATION

The contributions to the performance of dynamic jumps can be investigated using a number of different techniques. The following section reviews the techniques involved in the areas of simulation modelling, parameter determination and image analysis.

SIMULATION MODELLING

The techniques used to develop a simulation model can be divided into four parts. There is the development of the model, the evaluation of the model, the optimisation of a performance using a model and a sensitivity analysis.

Development of a simulation model

The complexity of a simulation model and the methods used to formulate the equations of motion for a simulation model will be discussed in this section.
Model complexity

The required complexity of a simulation model depends on the proposed use of the model. For example if the question is; how do muscles generate force, then the model will need to represent the contractile mechanism in fine detail (e.g. Huxley and Simmons, 1971). If the question is; what is the effect of biarticular muscles on vertical jumping performance then the simulation model will require a muscle representation which includes biarticular muscles (e.g. van Soest et al., 1993). For the purposes of this work a subject specific simulation model is required which can be evaluated and used to quantify the contributions to the performance of dynamic jumps. In practice the simplest simulation model which will exhibit the required features should be used so that as many parameters as possible can be determined experimentally (Yeadon and Challis, 1994).

To date the complexity of muscle driven simulation models has ranged from the simple models of Sprigings et al. (1994), Hubbard (1980) and Alexander (1990, 1991) to the complex models of Bobbert / Pandy for vertical jumping and the very complex model of Hatze (1981a) used for the long jump takeoff:

Sprigings et al. (1994) used a simple two segment model with no muscle to show how varying the preflight characteristics can determine the performance that could be produced in postflight for gymnastic vaulting.

Hubbard (1980) used a planar three segment model for pole vault takeoff with torque generators used to model the forces produced at the hips, shoulder and wrist joints. In addition a time delay function was used to represent the motor unit recruitment and stimulation rates in each torque generator. The joint torques used were calculated on a trial and error basis until the model agreed with reality. Since the joint torques were not determined experimentally it would be very difficult to optimise the takeoff as the possible torques at each joint were not known.

Alexander (1990) used a model consisting of two segments representing the thigh and shank along with a single extensor muscle to simulate high and long jump takeoffs. The single extensor muscle was represented as a torque generator which consisted of a contractile element with a optional series elastic element (equations (2.1) and (2.2)).

\[
\text{contractile element: } T = T_{\text{max}} \frac{(\omega_{c, \text{max}} - \omega_c)}{(\omega_{c, \text{max}} + G\omega_c)} \\
\text{series elastic element: } T = C\phi
\]
The contractile element was based on Hill's equation, but was expressed in an angular form. The muscle parameters for equation (2.1) were chosen so that the ground reaction forces obtained during simulations were realistic. The series elastic element was expressed as a function of the joint angle, the parameter C being chosen so that the stored strain energy approximated values reported in the literature by Ker, Bennett, Bibby, Kester, and Alexander (1987). Inputs to the model were the activation of the single torque generator, the velocity of the mass centre at touchdown together with the initial orientation and configuration and the output from the model was the mass centre velocity at takeoff.

Alexander (1991) investigated simple throwing tasks using a model with two torque generators and optimised the speed of release by varying the timing of the two torque generators. Using bang-bang control Alexander was able to explain the optimum sequencing of a two muscle system.

The Amsterdam and Stanford groups have examined vertical jumping in great detail using four segment models and extensor muscles in the lower extremities. The Amsterdam group modelled six muscles in the lower extremity and the Stanford group modelled eight muscles of the lower extremity. Input to the models comprised the activation time histories of the various muscles, together with the initial orientation and configuration. Output from the models comprised the time histories of the segment kinematics and the height jumped. For a number of years there was disagreement over the effect of biarticular muscles on vertical jumping performance. The difference between the models has been attributed to differences in the moment arm length of the gastrocnemius (van Soest et al., 1993). This emphasises the importance of obtaining the correct parameters for a model and evaluating the model before placing too much confidence in any results obtained. For both models muscle parameters were estimated from the literature.

Hatze (1981a) used a very complex 17 segment model to simulate the long jump takeoff. The model incorporated 46 muscle groups with a very complex muscle model used to represent each muscle. The model required the activation of each of the 46 muscle groups along with the segment configurations and orientations. Hatze (1981b) claimed that he had determined parameters for an individual which then allowed an evaluation of the model for a long jump takeoff. Inertia parameters for the 17 segments
were determined from 242 anthropometric measurements (Hatze, 1980) and muscle parameters were estimated from torque and EMG measurements during isometric and isotonic maximum effort contractions (Hatze, 1981b). It is hard to believe that it would be possible to determine all the muscle parameters from direct measurements without using a lot of assumptions when isometric and isotonic measurements in situ can only measure the net torque produced by all the muscles around a joint. Hatze claimed that models which disregard the ‘intricate internal dynamics of muscle’ are ‘doomed to failure’. Although this may be true for models which are trying to unlock the mechanisms used to produce muscular force, it is hard to believe that a simulation model of such complexity is required to simulate dynamic jumps where the total time of the movement is typically less than 0.2 s.

A range of muscle driven simulation models of varying complexity have been developed with there being a reliance on the literature for muscle parameter values. None of the simulation models reviewed derive all the muscle parameters required from experimental testing, and none of the models have been properly evaluated. Relatively simple models using torque generators to represent the forces produced by muscles appear to offer the best opportunity for the development of a subject specific muscle driven simulation model for dynamic jumps.

Formulation of the equations of motion for the simulation model

From the simulation models that have been developed a number of methods have been used to obtain the equations of motion. Some researchers develop their models from first principles (e.g. Yeadon, 1984; Alexander, 1990) and others use automated computer packages to develop the simulation models (e.g. the SPACAR package; van Soest, 1992 and the DADS package; van den Bogert, 1990).

Both methods have their advantages and disadvantages: Writing the equations from first principles allows complete control over the model although there is a much greater chance of making mistakes in the formulation of the equations and the writing of the computer code. The benefit of using a computer package to formulate the equations is that the chance of making mistakes in the code is reduced although one must ensure that the package is working correctly. Additionally it is much quicker to write the code using
a computer package which allows one to focus on the problem to be solved rather than the numerical methods used (van Soest, 1992).

van Soest (1992) describes a number of criteria which must be considered when using a general purpose software package:
1. Flexibility of the package to allow the user to adapt the software.
2. Accuracy of the package. Local numerical accuracy must be high to cope with the numerical integration process.
3. User-friendliness of the package with respect to the definition of the model and control of the output.
4. Calculation speed and numerical efficiency need to be high especially for optimisation of models where many simulations may be required.

One computer package available on the market, is the Autolev package which uses Kane’s method (Kane and Levinson, 1985) to formulate the equations of motion for systems of rigid and elastic bodies and produces ready to compile Fortran code. The benefits of the Autolev package include full access to the Fortran code allowing the user to customise the program to the task required, instead of having to use the code as it stands. In addition the Autolev package incorporates a variable step size Kutta-Merson method so as to reduce integration problems of high accelerations of the segments and vibrations of springs (Schaechter, Levinson and Kane, 1991).

Researchers who have used the Autolev package to simulate dynamic movements include Preston, Subic and Glover (1996) and Hubbard (personal communication).

Evaluation of the simulation model

Alexander (1990) showed that his model of jumping could demonstrate the general relationships between approach velocity, plant angle and performance. The simulation models of vertical jumping (e.g. van Soest et al., 1993; and Pandy and Zajac, 1991) have been shown to reproduce the major features of vertical jumping. The model of Hatze (1981a) for long jump takeoff gives good agreement for the reaction forces however the body configurations do not appear to resemble a long jump takeoff (Yeadon and Challis, 1994).

Overall there is little evaluation of muscle driven models for simulating dynamic jumps. To evaluate a muscle driven simulation model completely requires the model to
be customised to an athlete and the simulated performance of the model compared with the same athlete’s actual performance. For very complex models with a lot of parameters there is a danger that the model has sufficient flexibility to simulate a movement with a different strategy to the one used by the athlete. With a very complex model like Hatze (1981a) a much more thorough evaluation than just comparing the ground reaction forces should be performed (for example compare takeoff orientation and configuration as well).

Optimisation of a performance

The optimisation of a performance using a simulation model can be divided into three parts:

Firstly an objective (cost) function of the given activity must be formulated which can be maximised or minimised by varying the input variables to the model within realistic limits. For jumping simulations the objective function is simply the jump height or distance but for more complex movements where rotation is also of importance a more complex function incorporating rotation is required. The challenge for formulating an objective function is to obtain / calculate weightings for each variable in the function as the weightings will affect the optimum solution.

Secondly realistic limits need to be established for each of the variables which can be varied including the kinematic variables at touchdown, and the activation levels for each muscle. In addition the activation patterns of each muscle need to be constrained in some way to keep calculation time manageable and ensure a well defined optimum solution (van Soest et al., 1993).

Thirdly an algorithm which is capable of minimising or maximising the objective function needs to be used which will find the global optimum rather than a local optimum. If the objective function is linear, then a linear equation solver can be used to find the global solution (e.g. Stewart, 1973). However for a non-linear objective function there is no guarantee of finding the global optimum (Goffe, Ferrier, and Rogers, 1994). Goffe et al. (1994) noted that there is often a poor match between the power of a method (e.g. non-linear least squares method) and the numerical algorithms used to implement them. In addition Press, Flannery, Teukolsky and Vetterling (1988) noted that 'virtually nothing is known about finding global extrema in general' with there being ‘...no perfect optimisation algorithm’. Press et al. (1988) recommended trying more than one method and suggested that there are two options for solving non-linear equations:
1. Find local extrema starting from widely varying starting values of the independent variables.

2. Perturb a local extrema by taking a finite amplitude step away from it, and then see if the routine returns to a better point, or always to the same.

Of the many algorithms available to solve non-linear equations, two routines are:

- The Simplex algorithm (Nelder and Mead, 1965)
- The Simulated Annealing algorithm (Goffe et al., 1994)

The Simplex algorithm (Nelder and Mead, 1965) is a well-known reliable, robust algorithm for minimising functions in multi-dimensions efficiently (Corana, Marchesi, Martini, and Ridella, 1987). The Simplex algorithm requires \( N+1 \) (\( N = \) number of unknowns) initial starting guesses to start the algorithm which uses a straightforward downhill procedure with no special assumptions about the function to find an optimum solution (Press et al., 1988). The algorithm will only accept downhill solutions, therefore there is a risk of getting stuck at a local optimum. Corana et al. (1987) finds that the Simplex algorithm sometimes follows the 'gross behaviour' of a function to find the global optimum but sometimes gets stuck at a local optimum. The algorithm does not require derivatives to be calculated and has a storage requirement of the order \( N^2 \).

The Simulated Annealing algorithm has been shown to be more effective than the Simplex algorithm for finding optimum solutions using test functions although it is far more costly in computer time (Corana et al., 1987; Goffe et al., 1994). The Simulated Annealing algorithm is based on how liquids freeze and crystallise and on how metals cool and anneal. The routine is based upon the analogy that a slow cooling process allows ample time for the redistribution of atoms as they lose energy and therefore allows a lower energy state to be reached than through a fast cooling or quenching (Press et al., 1988). The Simulated Annealing algorithm effectively searches for a low energy state, whereas other routines correspond to a fast cooling or quenching going for the quick nearby solution (Press et al., 1988).

Figure 2.5 shows the minimisation algorithm used by Simulated Annealing (Corana et al., 1987). The algorithm starts at some 'high' temperature and creates a sequence of points choosing which points to accept and reject (both uphill and always downhill). After a specified number of points have been tried the temperature is reduced. The
The Simulated Annealing algorithm is then repeated focusing on the most promising area with the step length (and temperature) reduced each time, until the termination criteria are satisfied and the global optimum is found.

Figure 2.5. The Simulated Annealing minimisation algorithm (Corana et al., 1987).

The Simplex and Simulated Annealing algorithms have been tested against each other on test functions and problems such as the 'Travelling Salesman Problem' (Press et al., 1988; Corana et al., 1987, Goffe et al., 1994). In virtually all tests the Simulated Annealing algorithm found the global optimum apart from a very complex neural network test where the Simulated Annealing algorithm found a very good local optimum (still considerably better than the Simplex algorithm) while the Simplex routine sometimes found a local optimum. The Simulated Annealing algorithm was not dependent on the
starting position as starting the optimisation from 100 different positions resulted in the same solution. However this was not the case for the Simplex algorithm which failed to find a solution from some starting positions (Goffe et al., 1994).

Examples of researchers who have optimised dynamic movements include:

Alexander (1990) optimised long and high jump performance by varying the approach velocity and plant angle as the single torque generator in his model was switched maximally on throughout the contact phase.

For more complex models of dynamic performances the sequencing and activation of the torque generators must also be optimised as well as the kinematics at touchdown.

Pandy et al. (1990); Pandy and Zajac (1991) optimised vertical jump height with a Mayne-Polak algorithm to gain insights into the neuromuscular control of vertical jumping. To optimise performance Pandy allowed the muscles to change from zero activation to maximal activation as often as required using bang-bang control.

van Soest et al. (1993) optimised the vertical jump takeoff to investigate the effect of the biarticular gastrocnemius on vertical jump performance with greater constraints imposed upon the activation of each muscle than Pandy et al. (1990):
1. The initial activation level was set to maintain a static squat position (to prevent counter-movement).
2. The activation level was then allowed to switch to the maximum value once and stay there until takeoff.

These constraints were justified by the performance of the model but also from the optimisation of vertical jumping by Pandy et al. (1990) who found that once a muscle was maximally activated it remained maximally activated until takeoff in most cases. Using constraints meant that the optimisation procedure was simplified to a control space of only six dimensions which comprised the time each muscle was switched to being maximally activated from the initial activation state. To optimise the objective function a sequential quadratic programming algorithm was used which was reported to converge to an ‘optimal’ solution. To confirm that the solution found was optimal, the optimisation was started with different starting values and was found to give identical solutions. To ensure that the constraints imposed were not too severe, greater freedom was given to the
optimisation by allowing the muscles to be switched on twice with a period of
deactivation between. The jump height was improved by only 2 mm.

Sensitivity analysis

A sensitivity analysis is a very powerful tool which can be used with simulation
models to show how sensitive an optimum solution is to the parameters used. For
example Alexander (1990) found that the optimum techniques for high and long jumps
were not sensitive to the muscle parameters used in the single torque generator, and was
thus able to gain some confidence in the results obtained. The technique used to conduct
a sensitivity analysis is to perturb the parameter values used to give the optimum solution
and see how the performance varies. The sensitivity analysis can be used on the
parameters (inertia, muscle) and also on the variables such as the muscle activations and
the kinematic variables. For the other simulation models described in this section there
was a lack of sensitivity analyses performed.

Summary of simulation modelling

The above section has gone through the main sections to the development and use
of a simulation model. A relatively simple model using torque generators would appear
to offer the best opportunity for the successful development of a subject specific muscle
driven simulation model. The next section will go through the types of parameters
required for a subject specific simulation model.

PARAMETER DETERMINATION

Three types of individual parameters need to be determined experimentally for the
development and evaluation of muscle driven simulation models of athletic movements:
1. Inertia parameters
2. Muscle parameters
3. Spring parameters

This section will review the possible techniques of investigation for the
determination of each type of parameter.
Inertia parameters

A number of different methods have been employed to calculate segmental inertia parameters. Each method will be considered together with the advantages, disadvantages and accuracy. For the purposes of this study a quick and simple method for the determination of the inertia parameters was required with minimal inconvenience to the athlete.

1. Cadaver data

There have been a number of studies which have determined the segmental inertia parameters of cadavers (e.g. Dempster, 1955; Clauser, McConville and Young, 1969; Chandler, Clauser, McConville, Reynolds and Young, 1975). To obtain segmental inertia parameters of cadavers consists of dissecting the body into various segments and measuring the mass, mass centre location and moment of inertia of each segment. The mass of each segment is found by weighing, the mass centre location by using a balancing plate and the moment of inertia by using a compound pendulum technique (Reid and Jensen, 1990). The use of cadaver data to obtain the segmental inertia parameters is not appropriate in this study as the data would not be subject specific.

2. Measurement

Researchers have measured segmental volumes using water immersion, mass centre locations using reaction boards and moment of inertia using oscillation techniques (e.g. Drillis, Contini, and Bluestein, 1964; Hatze, 1975). However it is not possible to measure all the parameters required, e.g. the inertia parameters of the central segments (Yeadon and Challis, 1994). Measuring the segmental inertia parameters is not a practical method as a combination of methods would be required to attempt to calculate the inertia parameters.

3. Statistical modelling

Dempster (1955) expressed segment mass and segment mass centre location as percentages of subject mass and segment length and moment of inertia data separately for eight cadavers. More complex types of statistical modelling consist of using regression equations to relate the segmental inertia parameters to body measurements taken on the
subject. Cadaver data have been used to develop regression equations for segmental inertia parameters as functions of anthropometric measurements (Challis and Kerwin, 1992). The use of such a technique is of greatest value when the athlete is not available for measurement or average values are required for the analysis. Yeadon and Morlock (1989) developed regression equations from the cadaver data of Chandler et al. (1975) and concluded that non-linear equations were superior to linear equations for providing estimates of segmental moments of inertia, especially when the anthropometric measurements were outside the sample range used to develop the equations.

4. Geometric Modelling

The technique of using a geometric mathematical model to determine segmental inertia parameters has been used by Jensen (1978), Hatze (1980) and Yeadon (1990b). Although their models vary in complexity they are all based upon the same principles. The body is represented as a series of geometric solids of different shapes, with the size of the solids determined from anthropometric measurements and the density of the segments taken from cadaver data. A weakness of this type of approach is that the density values may not be correct although the models have been shown to estimate the total body mass to within 2%:

Jensen (1978) divided the body into 20 mm thick elliptical zones with density values taken from Dempster (1955). The size of the elliptical solids were determined by digitising photographs of the athlete. Jensen reports a maximum error in total body mass for three subjects of different body shapes as 1.8%.

Hatze (1980) developed an 17 segment mathematical model requiring 242 direct measurements which require approximately 80 minutes to measure. The model used combinations of geometric solids to model the segments and did not require the segments to be symmetrical. Density values were varied along the segments incorporating the density values of Clauser et al. (1969) and Dempster (1955). For the four subjects tested the maximum total body mass error was 0.32%.

Yeadon (1990b) developed an eleven segment model requiring 95 anthropometric measurements which require approximately 20-30 minutes to measure. Total body mass errors of approximately 2% were reported. Yeadon found that the total body mass errors were comparable with those found by Jensen (1978) but greater than those reported by
Hatze (1980). Yeadon explained that a major problem in taking the anthropometric measurements is obtaining accurate torso perimeters due to the effect of breathing. If the lungs contained an extra one litre of air the volume increase will cause the estimate of the total body mass to be increased by 1.5% for a 70 kg subject. Yeadon concluded that an error of 2.0% is quite reasonable. In addition Yeadon noted that the total mass of the subject is the only measurement that can be taken directly on the subject and used for evaluating the inertia model and therefore any systematic errors in the segmental inertia parameters are difficult to identify. However Yeadon also noted that the inertia model can be considered adequate since there is good agreement between his simulation model using calculated inertia parameters (Yeadon, Atha, and Hales, 1990) and filmed performances showing that there are not any major errors in the calculated segmental inertia parameters.

5. Image analyses

Segment lengths have been measured from digitised film data and combined with regression equations to estimate segment perimeters. Effectively the method is the same as for mathematical modelling except the measurements are taken by digitising film data. The great advantage of such an approach is that the presence of the subject is not required. Yeadon, Challis and Ng (1994) proposed a method for estimating segmental inertia parameters from a limited number of segmental length measurements (which could be calculated from recorded images) and a few measured perimeters. Regression equations were then used with the limited number of segmental lengths and perimeter measurements (reduced data set) to calculate a complete set of 95 anthropometric measurements required to calculate segmental inertia parameters using the mathematical model of Yeadon (1990b). Yeadon found that comparable results were obtained using the full data set of 95 anthropometric measurements and the reduced data set.

Baca (1996) developed the technique of obtaining measurements from video by using an automated system to estimate 220 of the 242 measurements required for the geometric model of Hatze (1980) from video images of four different body configurations. The remaining 22 measurements which could not be obtained from video were estimated using regression equations. Baca compared the estimated segmental inertia parameters directly from video with those found from anthropometric
measurements. For the three subjects analysed the average error in the total mass of the subject was less than 5%.

Although it is possible to obtain anthropometric measurements by digitising images of the body it is preferable to obtain direct measurements of the athlete where possible.

6. Computer-aided tomography (CT scan) / magnetic resonance imaging (MRI)

The use of CT scanning (Rodrigue and Gagnon, 1983) and magnetic resonance imaging (Mungiole and Martin, 1990) can provide accurate estimates of segmental inertia parameters although such techniques are not widely available and are expensive. In addition there may be potential medical and ethical problems in using such techniques.

Summary of inertia parameters

The use of a mathematical model to calculate segmental inertia parameters from anthropometric measurements taken on the athlete and density values taken on cadavers has been shown to give acceptable errors of between 1% and 2% for total body mass. This method has the advantage of being relatively simple, requiring only 30 minutes of the subject's time to take the measurements (Yeadon, 1990b). The use of CT scans / MRI may provide better estimates of segmental inertia parameters in the future although it is not a practical alternative at the present time.

Muscle parameters

Subject specific muscle parameters are required for simulation models of dynamic human movement so that under any set of kinematic conditions the maximum torque that an athlete can produce at each joint in the model can be calculated. Many researchers have relied on in vitro muscle data to obtain muscle parameters (e.g. van Soest et al., 1993). Ideally a method is required so that subject specific torque measurements can be obtained directly from measurements taken on a subject with the number of parameters required depending on the method used to model the muscles. A number of different approaches become evident from the muscle driven simulation models that have been developed to simulate dynamic movements with there being a broad division into two groups. Some researchers calculate the forces produced in individual muscles in the body (e.g. Hatze, 1981a; van Soest et al., 1993) whereas others model the net effect of all the muscles around one or more joints (e.g. Alexander, 1990).
The different techniques used to model the muscles during dynamic movements will be considered with the advantages and disadvantages of each method for the experimental determination of subject specific muscle parameters discussed in this section. For the purposes of this study a muscle model is required which will adequately describe the torques produced during dynamic athletic movements and allow the experimental determination of subject specific muscle parameters.

The structure for the remainder of this section is to firstly describe the main characteristics of muscle-tendon complexes. A description of the muscle models used by other researchers is then given with the advantages and disadvantages of each method discussed. The possible methods for testing muscle strength in-vivo are then considered before focusing on the use of an isokinetic dynamometer to test muscle strength.

Although two basic groups of models are evident in the research literature, they can both be classed as phenomenological models as they do not try to model the contractile process (Huijing, 1992a). Phenomenological models are usually made up of a number of components including; the contractile element (CE), the series elastic component (SEC) and the parallel elastic element (PEE), (Chapman, 1985).

Contractile element (CE)

The force produced by the contractile element is dependent on many factors including (Chapman, 1985):

1. The velocity of contraction
2. The length of the muscle
3. Muscle activation
4. The prior history of the state of contraction

1. The velocity of contraction

The work of Hill (1938) has been fundamental to the understanding of the force / velocity relationship of whole muscle for concentric contractions. Hill performed experiments on whole sartorius muscle of the frog and developed a comparatively simple hyperbolic relationship (equation (2.3)) for the force produced by a muscle as a function of its velocity at a fixed muscle length (Figure 2.6).
Although Hill’s equation has been the basis of many simulation models, the validity of it has been recently questioned by Edman (1988) who found for experiments on frog fibres that the force / velocity relationship had a double hyperbolic shape (Figure 2.7). The difference between the curves appeared at forces greater than 78% of maximum isometric force. Edman found the forces for single fibres to be smaller than those that Hill’s equation would predict. In addition Edman found as the load (force) increased above isometric the force / velocity curve followed a smooth continuation of the curve recorded between 78% of isometric force and isometric force, with the velocity of contraction between 90% and 120% of isometric force hardly changing (Figure 2.7). Edman found that the force / velocity curve could be fitted by two hyperbolas one to the data points below 78% of isometric force, and one to the data points above 78% of isometric force or by a single exponential function for the whole curve. Edman noted that the double-hyperbolic shape of the force / velocity curve suggested that the kinetics of cross-bridge function change as the load exceeds approximately 78% of isometric force.

Edman (1988) also investigated the effect of the length of fibres on the force velocity relationship, finding at longer lengths a smaller curvature of the force / velocity curve.
Figure 2.7. The force / velocity relationship of muscle fibres (Edman, 1988, p. 315).

Since the work of Edman is based upon single fibre contractions and the work of Hill is from whole muscle it would be expected that the same underlying characteristics would be evident in both cases, as both methods are examining the force / velocity relationship in the contractile part of the muscle when maximally stimulated.

To relate the force / velocity relationship of a fibre or whole muscle in vitro to a torque / angular velocity relationship in vivo is harder to justify, as there are a number of additional factors to include. However it would be expected that the torque / angular velocity relationship would have similar underlying characteristics even if it is masked by the effect of other variables.

2. The length of the muscle

The force / length relationship of single fibres (Figure 2.8) for isometric contraction is well documented as being bell-shaped with small tensions at the extremes of length and maximal in between (Gordon, Huxley, and Julian, 1966; Edman, 1992). Chapman (1985) noted that the bell-shaped force / length relationship of muscle has been reproduced for single fibres, whole isolated muscle and muscle in vivo.
3. **Muscle activation**

The activation of the muscle affects the amount of force that is generated in the muscle on a scale from 0 to 1 with 0 being zero activation and 1 being maximally activated. The recognised method to measure muscle activation is the electromyogram (EMG), (Chapman, 1985). A technique to calculate the force produced from an EMG signal was proposed by Hof and van den Berg (1981a-d). The major problem with trying to calculate muscle force from EMG signals, apart from obtaining reliable EMG recordings, is that muscle force is affected by many other factors such as muscle velocity, length and prior history of events (Chapman, 1985).

4. **The prior history of the state of contraction**

The force produced in a muscle is also affected by the previous state of the muscle, making the relationship much harder to understand. The effect of actively stretching the muscle prior to contraction has been well documented with the stretch resulting in an enhancement of the force produced (Chapman, 1985). The enhancement for concentric force is that for a given velocity the force produced is greater. Chapman, however, noted that there is no evidence to show that the maximum velocity of shortening increases by a prior stretch. The velocity of the stretch and the time elapsed after the stretch before the
contraction affect the amount of enhancement of the force produced during concentric contractions (Edman, Elzinga and Noble, 1978).

Series elastic component (SEC)

The CE of muscle is connected in series with a passive element called the SEC which has the characteristics of a spring (Huijing, 1992a). The SEC represents the elastic properties of muscle which are in series with the CE, the major component of which is located in the tendinous tissue of the muscle (Huijing, 1992b). The series elastic component transfers tension developed in the CE to the limbs, and is usually represented as a conservative system (Huijing, 1992a). Since the SEC almost behaves like a conservative system the force / velocity characteristics of tendon are not very important with tendons having a unique force / length relationship (Huijing, 1992b).

The importance of the SEC is in its ability to store energy when deformed by a force and to recoil after being stretched (Shorten, 1987), the previous history / state of the SEC therefore also affects the force in the SEC. The performance of dynamic jumps is therefore affected by the characteristics of the SEC as the muscles are actively stretched eccentrically before contracting concentrically (stretch-shorten cycle).

Parallel elastic element (PEE)

The PEE is used to represent the passive structures within the muscle including the connecting tissue surrounding and invading whole muscle and the sarcolemma in single fibres (Chapman, 1985). The PEE is thought to be an unimportant factor in the production of force within the normal working ranges of the joints (Chapman, 1985).

Other factors

As well as all the above factors which influence the force produced by muscles, there is also the complication of muscle architecture, which includes how all the muscles are attached to the body and the make-up of each muscle. For example some muscles are bi-pennate, and some muscles cross more than one joint e.g. gastrocnemius. To attempt to incorporate all of the above factors in a muscle model and calculate parameters experimentally for a subject may appear a daunting task. However if the major influences on the muscle force can be reproduced experimentally and the forces can be measured it may be possible to represent the forces produced during dynamic activities and therefore
not have to model the individual components of muscle in detail. Whichever method is used to model the force produced by the muscles, it must be sufficiently complex to incorporate the main factors which influence the force produced by the muscles.

The following section will describe how other researchers have modelled muscle and possible ways to determine muscle parameters.

**Contractile element parameters**

Modelling individual muscles

To model the individual muscles in the body and incorporate the above CE, SEC and PEE requires complex representations including many elements which contribute to the force generated by the muscles. Realistically it is not possible to incorporate every feature of muscle in a model, instead it is necessary to model the muscle in sufficient detail for the purpose that is required.

The structure of the muscle models used to represent individual muscles in the body range in complexity from relatively simple simulation models which include CE, SEC and PEE (e.g. van Soest et al. (1993) and Pandy et al. (1990)) to the very complex model of Hatze (1981a), (Figure 2.9). The major drawback of modelling the individual muscles is that there are many parameters which must be estimated from the literature. Estimating parameters from the literature results in a simulation model that is general but does not represent a particular athlete. Therefore although the simulation models produce and exhibit all the characteristics of a performance, they could not be used to differentiate between the techniques used by two jumpers with different muscular strengths.

![Figure 2.9. Muscle representation used by Hatze (1983, p. 9).](image-url)
Challis (1992) developed a method for estimating individual muscle forces experimentally for three elbow flexor muscles during a dumbbell curl. Although the method calculates muscle parameters for the three elbow flexors there is still a reliance on data from the literature. The method is also quite complex and impractical for the development of a muscle driven simulation model of athletic activities and in addition has only been applied to concentric contractions of the elbow flexors.

The very complexity of simulation models which include models of individual muscles makes it very unlikely that the muscle parameters could be obtained experimentally. A method which does not place a reliance on muscle parameter values from the literature is required so that the simulation model can be customised to an individual’s performance.

Representing muscles as torque generators

Modelling all the muscles around a joint as a single torque generator considerably simplifies the very complex nature of muscle. Whether a single torque generator is too great an assumption can be partly answered by the simulation models which have used torque generators to model muscles and have given sensible results (e.g. Alexander, 1990). The real challenge with representing muscles as torque generators is to be able to produce a function which will realistically represent the net effect of all the muscles around a joint during a given activity:

‘In all cases the model is only as good as the number of facts which are known and the number of assumptions which have to be made’ (Chapman, 1985).

Modelling all the muscles crossing a joint as a single torque generator has the major advantage that it is possible to directly measure the net performance of all the muscles around a joint and does not require assumptions about how the total torque produced is shared between the various muscles crossing a joint.

To date the limits on torque generators used in simulation models have not been based on values obtained experimentally on a subject. Instead values have been approximated from the literature or the torques varied until the performance of the model agrees with reality (e.g. Hubbard, 1980). The torque generator at each joint must be able to represent the characteristics of muscle in sufficient detail so as to be representative of the net torque produced at each joint.
Testing in vivo muscle function

Taylor, Cotter, Stanley, and Marshall (1991a) identified two techniques for testing in vivo muscle function:
1. Controlled loading protocols (isotonic testing)
2. Controlled shortening velocity protocols (isokinetic testing)

The isotonic method involves measuring torque or force exerted against a constant load (e.g. Wilkie, 1950) whereas isokinetic testing consists of measuring torques exerted at a constant angular velocity. The isokinetic method has the advantage that the effect of angular acceleration on the torque produced does not need to be accounted for as the movement is at a fixed velocity. As a consequence isokinetic methods became popular, especially during the late 1960's when they became commercially available (Osternig, 1986; James, Sacco, Hurley, and Jones, 1994), and were used extensively (Herzog, 1988).

Isokinetic machines allow the subject to work maximally against a crank that is moving at a constant angular velocity over a range of angles. There are two main types of isokinetic machine available (Wilk, 1990):
1. Passive machines
2. Active machines

The passive type machines include the popular Cybex machine and the active type machines include the Biodex, Kin-Com and Lido machines. Both types have been shown to give valid and reliable results (Wilk, 1990). The disadvantage of the passive machines is that they only allow concentric muscular contractions whereas the active machines allow both concentric and eccentric contractions. The various manufacturers of isokinetic machine allow different ranges of angular velocities (Table 2.5).

Table 2.5. Maximum angular velocities for isokinetic machine (Wilk, 1990)

<table>
<thead>
<tr>
<th>Isokinetic machine</th>
<th>Maximum concentric angular velocity [°s⁻¹]</th>
<th>Maximum eccentric angular velocity [°s⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cybex II</td>
<td>300</td>
<td>n/a</td>
</tr>
<tr>
<td>Biodex</td>
<td>450</td>
<td>120</td>
</tr>
<tr>
<td>Kin-Com III</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Lido</td>
<td>400</td>
<td>250</td>
</tr>
</tbody>
</table>
The use of isokinetic dynamometers to measure maximum torque values should offer the opportunity to calculate subject specific muscle parameters, although a number of authors question whether isokinetic torque data would be representative of the torques produced during dynamic movements: Wickiewicz, Roy, Powell, Perrine, and Edgerton (1984) extrapolated the moment / angular velocity relationship for isokinetic movements finding maximum angular velocities of 12.5 rad.s\(^{-1}\) and 6.8 rad.s\(^{-1}\) for knee extension and plantar flexion respectively. Bobbert, Huijing and van Ingen Schenau (1986) noted that these values were too low as van Soest, Roebroeck, Bobbert, Huijing and van Ingen Schenau (1985) reported angular velocities as high as 16 rad.s\(^{-1}\) for jumping and Cooper and Glassow (1972) reported peak angular velocities of 22 rad.s\(^{-1}\) for the knee extensors during kicking. Bobbert et al. (1986) concluded that muscles behave differently during isokinetic and 'complex movements' using an explanation based upon tendon length changes and the effect of biarticular muscles: e.g. during ankle plantar flexion, knee angle is constant for isokinetic testing but during jumping the knee angle is changing which affects the plantar flexion moment due to the gastrocnemius which crosses the knee joint as well as the ankle joint. Furthermore Bobbert (1988) noted that due to the very complex behaviour of muscles and the effect of an eccentric contraction prior to concentric contractions in jumping activities the 'bare results' of isokinetic experiments are not sufficient to predict muscular output during complex movements.

Although a number of authors have reservations about using isokinetic data to predict muscular torque produced during jumping movements, no researchers have tried to reproduce the eccentric-concentric conditions in jumping during isokinetic exercise. If it is possible to measure torques under an eccentric-concentric protocol the torques produced may resemble the torques produced during complex jumping activities.

The remainder of this section on the calculation of muscle parameters will review the research literature on isokinetic exercise with a view to using an isokinetic machine for collecting experimental data on an elite athlete. The review of isokinetic research will be divided into a two of sections:

1. Terminology and methodological considerations
2. Results
Terminology and methodological considerations for isokinetic exercise

The term isokinetic exercise has been associated with exercise at a constant angular velocity. However since isokinetic machines were introduced there has been some debate over the definition of the term ‘isokinetics’ (Hinson, Smith, and Funk, 1979). Hinson et al. (1979) noted that there should be a distinction between constant angular velocity of the limb and constant velocity of muscular contraction. He showed through use of a simple mathematical model that a constant angular velocity of the limb does not imply a constant linear velocity of the muscle and concluded that the term ‘isokinetics’ be used for a constant rate of limb movement. In addition it has been shown that isokinetic machines that allow the machine crank to move at constant velocity do not necessarily mean that the human limb is moving at a constant angular velocity (Herzog, 1988).

The ease of use of the various isokinetic machines has resulted in many papers presenting torque data at the knee, ankle and elbow joint. However conflicting results have been found. Taylor et al. (1991a) noted that comparing results between papers should only be done where methodological errors have been taken into account. Therefore the fact that many of the early papers using isokinetic machines did not account for methodological errors could account for the differences in results obtained.

Accuracy of isokinetic data

The accuracy of isokinetic data was first questioned during the 1980’s (Winter, Wells, and Orr., 1981; Sapega, Nicholas, Sokolow, and Saraniti, 1982; Osternig, Hamill, Sawhill and Bates, 1983a; Osternig, Sawhill, Bates, and Hamill, 1983b and Herzog, 1988) and then later by Taylor, Sanders, Howick, and Stanley (1991b). Three main areas of concern were highlighted by these authors:

1. Error due to gravity
2. Acceleration and deceleration of the crank
3. Crank / joint angle differences

Error due to gravity

Winter et al. (1981) and Herzog (1988) noted that large errors result from not correcting torque data for the weight of the machine crank and human limb. For movements in the horizontal plane there is no correction to make (Sapega et al., 1982) but
in the vertical plane the error can be as great as 24% of the total torque produced (Herzog, 1988). The influence of limb and crank weight is greatest when the limb and crank reach the horizontal position. This typically occurs towards the end of the range of motion where the torque produced by the muscles is smallest and therefore the influence of weight is a much larger proportion of the total than at other parts of the range of motion (Herzog, 1988). Torque values are underestimated if the subject is working against gravity and overestimated if the subject is working with gravity (Winter et al., 1981). Both authors recommended that the effect of crank and limb weight must be accounted for in isokinetic measurements taken in the vertical plane, although Winter does recognise that for some isokinetic machines which measure force at the cuff between the limb and the crank, only the weight of the limb needs to be corrected for. To correct for the influence of gravity Winter suggested adjusting the total work for a whole trial based on potential energy. Herzog though showed how to correct for crank and limb weight throughout the time history of the torque data obtained by modelling the crank-limb system as two rigid bodies (Figure 2.10).

\[ T = T + \frac{W_c L_1 L_3 \sin \theta}{L_2} + W_{sf} L_4 \sin \alpha \]  

(2.4)

From equation (2.4) it can be seen that the correction is not a constant but is a function of joint angle \( \alpha \) and the crank angle \( \theta \) which are both measured from the vertical. When \( \alpha \) and \( \theta \) reach 90° the correction for the weight of the limb and crank is greatest.

Figure 2.10. Free body diagram of Cybex arm and shank-foot segment (Herzog, 1988, p. 6).
Acceleration and deceleration of the crank

Sapega et al. (1982) and Herzog (1988) highlight the errors in torque values obtained due to the initial acceleration of the crank to the pre-set angular velocity. For the Cybex II isokinetic dynamometer, it was found that the crank velocity exceeded the pre-set velocity and then decelerated until the velocity was reached. This overspeeding and subsequent deceleration of the crank resulted in spikes in the torque data which were not due to surges of muscular activity but were an artifact of the machine design (Sapega et al., 1982). Similarly for the end of the trial where the crank is decelerated from the pre-set velocity, elevations in the torque values are observed (Osternig et al., 1983a, b). The authors recommend that only the central portion of a trial where the crank and limb are not accelerating and decelerating should be used, so as to minimise the effects of non-constant angular velocity and eliminate the effect of ‘torque overshoot’.

Herzog (1988) quantified the errors due to either the crank or limb accelerating during the central portion of each isokinetic trial where the crank is meant to be moving isokinetically. Herzog found that errors due to the crank accelerating were very small with a maximum error of 0.3% of the total torque produced although for the limb much larger errors were found (up to 5% of the total). The increased error was attributed to the limb moving relative to the crank during the trial. Therefore although the crank was moving isokinetically the limb may not be, even though the limb was attached firmly to the crank and the joint centres are aligned.

To ignore the initial and end portions of a trial has the disadvantage of reducing the range of motion where muscle data can be used. Furthermore as the pre-set angular velocity increases, the range of motion which is isokinetic is reduced (Osternig et al., 1983a, b; Taylor et al., 1991b; James et al., 1994). To overcome this a larger than required range of motion should be used so that there is sufficient data after the initial and end portions have been removed (Osternig et al., 1983a, b).

Crank / joint angle differences

Most authors assume that the crank angle is equivalent to the joint angle for the data collected from an isokinetic machine. Herzog (1988) questioned this assumption for knee extension with a Cybex II dynamometer by calculating the joint and machine angle through digitisation of each trial. He found that by carefully aligning the joint centres
before each trial the error due to movement between the crank and limb could be minimised (1-2%). However with reference to the absolute joint and machine angles Herzog only said that they 'were similar throughout the knee extension exercises' the actual differences between joint and crank angle were not presented. Taylor et al. (1991b) considered the difference between crank and knee angle for knee extension on the Biodex dynamometer for a range of angular velocities from $60^\circ\text{s}^{-1}$ to $450^\circ\text{s}^{-1}$. Two dimensional kinematic data were obtained from digitising high speed video camera recordings of each trial. Taylor found that as the velocity increased the difference between the crank and joint angle decreased. Taylor does not give absolute values for the difference between the crank and joint angle but does quantify the difference between the crank and joint angular velocity. At the slowest angular velocity considered ($60^\circ\text{s}^{-1}$) the maximum difference was 196% of the average crank angular velocity at that velocity and at the fastest angular velocity considered ($450^\circ\text{s}^{-1}$) the maximum difference was 111%. Taylor also noted that at the beginning of each trial the limb moved relative to the crank while the compliance in the system was taken up. This relative movement was due to compression of the lever padding, compression of soft tissues and changes in alignment of the central axis of the knee from the axis of the crank arm. The authors suggested that methods be adopted to minimise the compression of the soft tissues and the lever arm padding during limb movement by securing the limb to the lever arm.

Although a number of authors have identified that differences between the crank and limb angle occur, the effect of such differences on the shape of the torque / angle / angular velocity relationship has not been considered. The error due to differences between the joint and machine angle has only been considered at the knee joint. At other joints where more movement of the joint occurs (e.g. shoulder) and where isolation of the joint during testing is difficult, errors could be much larger.

**Summary of the accuracy of isokinetic data**

When using torque data collected from an isokinetic dynamometer the effect of limb weight must always be taken into account for torque data collected in the vertical plane. The beginning and end of trials where the crank and limb are accelerating and decelerating should not be used and the effect of relative movement between the machine crank and human limb should be considered.
Precision of isokinetic measurements

The precision of isokinetic measurements has been established by taking repeated measurements and randomising the order of the trials.

Mohtadi, Kiefer, Tedford, and Watters (1990) found for knee extension using the Kin-Corn machine that there were only small differences between the original and re-test values for peak torque at 60°s⁻¹. Thorstensson, Grimby, and Karlsson (1976) and Westing, Seger, Karlson, and Ekblom (1988) found from repeated measurements that the overall variation in torque for knee extension was approximately 8-14%. In addition Thorstensson found that randomising the order of the trials at different angular velocities or in sequence made no difference to the variation and Westing found the percentage variation for eccentric measurements to be higher than for concentric movements (10.6% and 8.1% respectively).

Although a variation of around 10% appears quite high, a complex procedure is required to obtain the torque data. A figure of 10% variation indicates the difficulties in obtaining reliable data but it may also suggest that there are other variables which have not been controlled between trials including:

Subject effort and maximal activation during isokinetic testing

The subject is usually encouraged to give maximal effort although this may not always be the same between and during trials. To obtain constant muscle activation both between and during trials requires external electrical stimulation to be used (Thomas, White, Sagar, and Davies, 1987). Thomas found that the largest intrasubject coefficient of variation of torque to be 9.7% for ankle plantar flexion over a range of angular velocities.

The question of whether muscle torques obtained from isokinetic machines are maximal has been raised by a number of authors including James et al. (1994). In particular the force production at low concentric velocities has been examined where measured force values have been found to be lower than predicted from Hill’s equation (Perrine and Edgerton, 1978; Wickiewicz et al., 1984). EMG activity has been measured during collection of muscle torque data to assess whether ‘maximal’ force can be produced by voluntary contraction, (Fuglevand 1987; James et al., 1994). Fuglevand (1987) found that the plateau in the torque / angular velocity relationship was not
complemented by a plateau in the EMG activity of the vastus lateralis muscle for knee extension. By using different exercise protocols authors have assessed whether torques produced by voluntary contraction are maximal:

James et al. (1994) identified three different protocols:

1. Simple voluntary contractions
2. Releases from maximal voluntary contractions
3. Releases from isometric femoral nerve stimulation

James found no evidence for voluntary contractions producing sub-maximal forces; in fact voluntary contractions were found to give higher force values than by stimulation. The author attributed the greater forces at low angular velocities to extension at the hip during contractions at the knee. The stimulated force / velocity curve followed the classic force / velocity relationship of Hill at low velocities although the voluntary contractions protocol produced a flatter higher force curve. Releasing from maximal voluntary isometric contraction was found to give the least reliable results with the subjects finding it difficult to change from isometric to isokinetic contractions, this was shown by a drop in EMG activity as well as torque values. James recommended that if femoral nerve stimulation was not possible then reliable results could be obtained from voluntary contractions if hip extension was prevented.

Aligning joint centres and non rigidity of the limb-crank system

The joint centres of the machine and the limb are usually aligned before each trial. The limb joint centre, however, may move during the trial and there may be movement of the limb relative to the crank (Herzog, 1988).

Clearly when conducting isokinetic experiments, every effort must be made to keep conditions constant between trials, so that extrinsic errors are kept to a minimum.

Calibration and reliability of isokinetic dynamometers

The calibration of isokinetic machines has been achieved by measuring the angle / angular velocity / torque from the dynamometer and comparing them with known values. The reliability of isokinetic machines has been established through repeated measurements:
Angle values

Seger, Westing, Hanson, Karlson, and Ekblom (1988) evaluated the angle output from the SPARK dynamometer by comparing the output angle with a measured known angle using a bubble clinometer. The authors found no significant difference ($p > 0.05$) between the known crank angle and the output from the SPARK system.

Mayhew, Rothstein, Finucane, and Lamb (1994) compared the angle output from the Kin-Com isokinetic dynamometer (model #500-11) with known angles measured using a gravity-referenced protractor in a range of $110^\circ$. Mayhew found that there was a 'nearly perfect linear relationship' ($R^2 = 0.99$) between the actual angle and output angle from the Kin-Com machine.

Angular velocity values

Seger et al. (1988) found during subject testing that the actual angular velocity was up to $6^\circ s^{-1}$ different from the preset angular velocity.

Mayhew et al. (1994) tested the angular velocity output from the Kin-Com machine by comparing the output angular velocity with the angular velocity determined through differentiating the known angle data. He found a 'nearly perfect linear relationship' ($R^2 = 0.99$) between the actual angular velocity and output angular velocity from the Kin-Com machine.

Force / torque values

Seger et al. (1988), Moffroid, Whipple, Hofkosh, Lowman, and Thistle (1969) and Mayhew et al. (1994) evaluated torques from the SPARK, Cybex and Kin-Com dynamometers respectively by hanging weights of known mass from the machine crank while it was horizontal. In all cases no significant difference between the calculated and measured force was found with Mayhew et al. (1994) reporting a nearly linear relationship between the known and measured force.

Summary of the precision of isokinetic measurements

From the testing of the various isokinetic machines available it appears that all the machines give accurate, reliable measurements. Any isokinetic machine should be calibrated before being used by comparing the angle, angular velocity and force output...
from the machine with known angle, angular velocity and force data. (Seger et al., 1988; Moffroid et al., 1969; Mayhew et al., 1994).

Results of isokinetic studies

The main results from isokinetic studies will be covered giving an insight into typical data that can be obtained from isokinetic machines. The results have been split into the following sections:

1. Bilateral Deficit
2. Maximum velocity of shortening
3. Electromechanical delay
4. Torque / angular velocity relationship
5. Torque / angle relationship

The torque / angular velocity and torque / angle relationships are reported with a view to identifying the general shape of each relationships.

Bilateral Deficit

Isokinetic machines typically measure the strength at one joint centre at a time (unilateral contraction). The simulation models developed in this study combine left and right sides of the body and therefore the strength measurements obtained need to be scaled to estimate the torque that can be produced by left and right limbs together (bilateral contraction). A number of researchers have considered how the maximum force exerted by a single limb during a bilateral contraction compares with the force produced for a unilateral contraction during maximal isometric contractions (Howard and Enoka, 1991; Schantz, Moritani, Karlson, Johansson and Lundh, 1989; Secher, Rube and Elers, 1988). Typically the force produced during a unilateral contraction is more than half the force produced during a bilateral contraction with most authors concluding that the deficit is due to neural factors (Howard and Enoka, 1991). Secher et al. (1988) found for leg extension that the two-leg strength was on average 82% of the sum of left and right legs (leg-strength ratio) although after 5 weeks familiarisation with the equipment the leg-strength ratio had improved to 97%. In addition for arm extension Secher et al. (1988) found that the strength ratio was on average 102% for 15 subjects.
Howard and Enoka (1991) found that on average untrained subjects exhibited a bilateral deficit (strength ratio of 92%) while for cyclists and weightlifters there was no evidence of a bilateral deficit for knee extension.

Although it is generally accepted that the force produced during a bilateral contraction is less than double the force produced during a unilateral contraction (strength ratio $< 100$), there is evidence to suggest that through familiarisation with the equipment and for well trained athletes the bilateral deficit is reduced or does not exist at all.

Maximum velocity of shortening

The maximum velocity of shortening for whole muscle in vivo cannot be obtained directly from measurements taken on an isokinetic dynamometer (Houston, Norman, and Froese 1988), as the maximum speed of dynamometers is only approximately 30% of the maximal unloaded velocity (de Koning, Binkhorst, Vos, and van’t Hof, 1985). The maximum speed of shortening has been calculated experimentally for a number of different joints using a goniometer to measure changes in joint angle (Larsson, Grimby and Karlsson, 1979), (Table 2.6). Alternatively a movement has been filmed and the maximum velocity calculated from the digitised locations of the limbs throughout the movement (Table 2.6).

<table>
<thead>
<tr>
<th>action</th>
<th>method</th>
<th>maximum velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pertuzon and Bouisset (1971) elbow flexion</td>
<td>goniometer</td>
<td>1000</td>
</tr>
<tr>
<td>Thorstensson et al. (1976) knee extension</td>
<td>goniometer</td>
<td>687</td>
</tr>
<tr>
<td>Larsson et al. (1979) knee extension</td>
<td>goniometer</td>
<td>630</td>
</tr>
<tr>
<td>Houston et al. (1988) knee extension</td>
<td>goniometer</td>
<td>700</td>
</tr>
<tr>
<td>Yeadon (1984) arm abduction</td>
<td>video</td>
<td>864</td>
</tr>
</tbody>
</table>

Electromechanical delay

This is the time between the onset of muscle electrical activity and the beginning of the limb acceleration. From EMG data taken during isokinetic contractions the following results have been found:
Table 2.7. Electromechanical delay times calculated from isokinetic testing

<table>
<thead>
<tr>
<th></th>
<th>joint</th>
<th>EMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Houston et al. (1988)</td>
<td>unloaded knee extension</td>
<td>39 ms ± 12 ms</td>
</tr>
<tr>
<td>Viitasalo and Komi (1981)</td>
<td>forearm flexion</td>
<td>38 ms</td>
</tr>
<tr>
<td>Norman and Komi (1979)</td>
<td>forearm flexion</td>
<td>41 ms</td>
</tr>
</tbody>
</table>

Torque / angular velocity relationship

The majority of research has concentrated on the torque / angular velocity relationship for concentric muscle actions, with little research on eccentric contractions (Westing et al., 1988). The concentric and eccentric parts of the torque / angular velocity curve will be considered separately:

Concentric muscle action

Torque / angular velocity curves have been presented in two ways; some authors plot the peak torque for a given angular velocity independent of angle (Thorstensson et al., 1976) and others express torque as a function of angular velocity at a given angle (Jarvis et al., 1994). Expressing maximum torque independently of joint angle, immediately increases the error, as torque is also a function of joint angle (Yates and Kamon, 1983; Thomas et al., 1987; James et al., 1994). Thus neglecting the effect of joint angle on the relationship makes it much harder to interpret the resulting curves. The torque / velocity relationship independent of angle has been shown to fit Hill's classic force / velocity equation by some authors (Thorstensson et al., 1976; Fugl-Meyer, 1981; Ivy, Withers, Brose, Maxwell, and Costill 1981). Taylor et al. (1991a) also showed that using peak torque independent of angle produced the classic force / velocity relationship but found that the inclusion of angle dependence resulted in a plateau in the torque values at low angular velocities.

Most researchers who have expressed torque as a function of angular velocity for a given angle have observed a plateau at high force / low angular velocity (Perrine and Edgerton, 1978; Froese and Houston, 1985; Marshall, Mazur, and Taylor, 1990; Taylor et al., 1991a; MacIntosh, Herzog, Suter, Wiley, and Sokolosky, 1993; James et al., 1994). These researchers have attributed the plateau to a number of factors; neural / mechanical
artifacts (Perrine and Edgerton 1978; Wickiewicz et al., 1984; Marshall et al., 1990; James et al., 1994), biphasic force / velocity relationship of Edman (1988) for in-vitro muscle (Marshall et al., 1990 and Taylor et al., 1991a). However others have found less clear cut results:

Fuglevand (1987) found that the shape of the torque / angular velocity curve changed with angle for knee extension with straighter leg positions showing more of a plateau. Fuglevand could not explain why the torque / angular velocity curve changed shape with increased velocity other than the muscle / length relationship was interacting with the torque / angular velocity relationship.

Marshall et al. (1990) found for knee extension that the torque / angular velocity relationship varied between subjects with some subjects following the classic shape of Hill’s equation and others showing a plateau at low angular velocities. From normalising and averaging six subject’s data, Marshall found that there was a plateau at low angular velocities before dropping at higher angular velocities, he suggested that the data may reflect the biphasic force / velocity relationship shown by Edman (1988).

Taylor et al. (1991a) combined the results of five studies for ankle plantar flexion, arm flexion and knee extension, showing that once the torques were normalised they all followed a curve which approximated Edman’s (1988) biphasic relationship. Taylor is cautious to suggest that the plateau in the isokinetic data is due to Edman’s (1988) biphasic relationship since the torque / velocity relationship below 30°s⁻¹ is not well understood he does however suggest that similar relationships are apparent at different joints despite their different structures.

One of the few authors to fit the torque / angular velocity relationship for isokinetic concentric contractions other than using Hill’s equation was Thomas et al. (1987). Thomas found that Hill’s equation gave poor fits for involuntary electrically stimulated plantar flexor data for seven subjects. Instead Thomas used an exponential equation with three constants to fit the torque / angular velocity relationship at a fixed angle:

\[ v = a \left( e^{kb} - e^{kb} \right) \]  

(2.5)

Thomas suggested that the plateaus observed in torque / angular velocity relationships for voluntary contractions may be due to lower activation levels since for involuntary contractions no plateau was found in the torque / angular velocity relationship.
Eccentric muscle action

The eccentric torque / angular velocity relationship has been given much less attention by researchers (Westing et al., 1988) even though athletic movements consist of eccentric as well as concentric types of motion. The majority of studies that have considered the eccentric phase have found higher torques than during either concentric or isometric phases (Komi, 1973; Jorgensen and Bankov, 1971; Rodgers and Berger, 1974; Jorgensen, 1976; Westing et al., 1988). In addition Jorgensen and Bankov (1971), Jorgensen (1976), Hanten and Ramberg (1988) and Westing et al. (1988) found that eccentric force did not increase significantly with faster eccentric velocities. Westing et al. (1988) noted that the plateau in eccentric force in human muscle may be due to an inhibitory feedback loop which prevents the muscle tension reaching higher values than have been found for isolated muscle.

Torque / angle relationship

Marshall et al. (1990); Fuglevand (1987); James et al. (1994) and Taylor et al. (1991a) report a general convex shaped torque / angle relationship which varied with angular velocity for knee extension. At faster velocities maximum torque was found to occur at a straighter leg position with a flatter shape to the whole curve (Marshall et al., 1990; Fuglevand, 1987). Marshall thought the shift and plateau of the torque / angle relationship was due to the interaction of the different muscles as each muscle would have a different optimum length for maximum torque production.

Summary of torque / angle / angular velocity relationships

The consensus of opinion is that the established force / velocity equation of Hill (1938), does not apply to whole muscle in vivo, with Hill’s equation overestimating the force produced at low velocities. Some authors have attempted to explain why force produced does not fit Hill’s equation, suggesting that neural / mechanical factors are the cause while others suggest that the force / velocity relationship for whole muscle reflects the biphasic relationship of Edman (1988). It is clear that more research is needed to establish whether a general torque / angular velocity relationship exists for concentric muscle actions in whole muscle.

The force produced under eccentric conditions for whole muscle in vivo has found to be higher than isometric or concentric force with the eccentric force reaching a plateau.
and not increasing with velocity. Westing et al. (1988) suggested that the plateau in eccentric force with increased velocity is due to an inhibitory feedback loop which prevents the muscle tension becoming too high.

From the literature for both concentric and eccentric muscle contractions there is no clear-cut relationship between torque and angular velocity. Two general relationships have been suggested for the concentric part of the curve but there is disagreement over which (if either) is correct. A number of questions remain unanswered:

What is the effect of a prior eccentric contraction on a concentric contraction?

What is the shape of the torque / velocity curve up to maximum velocity of shortening?

Only the ankle, knee and elbow joints have been examined in any detail. Are similar torque / angular velocity relationships apparent for shoulder flexion and extension?

The force / length relationship for 'in vitro muscle preparations' has been documented by a number of authors (Evans and Hill, 1914; Gordon et al., 1966) and a quadratic convex relationship was found. For whole muscle in vivo, the relationship appears more complex when the interaction of different muscles is incorporated (Marshall et al., 1990). From the research on the torque / angle relationship for knee extension a general convex curve was observed with the curvature varying with angular velocity.

**Series elastic element parameters**

A number of researchers have attempted to measure the in vivo stiffness of the series elastic element, some have adapted methods used for testing isolated muscle (e.g. quick release method) and others have modelled the body / limb as a mass-spring system and calculated the visco-elastic properties of the spring from the natural oscillation of free vibrations (Shorten, 1987). Typically, testing has been for plantar flexor muscles with the stiffness of all muscles combined together. For the simulation models developed in this study an angular stiffness value was required:

Hof and van den Berg (1981a-d) measured the angular stiffness of the plantar flexors with a tilting torque platform using an oscillating method which required the subject to bounce on the torque platform. The elasticity of the contracted calf muscles was then calculated from the resonant frequency of the vibrating system made up of the elasticity and the total body mass. Hof and van den Berg report an average angular
stiffness of 593 Nm.rad\(^{-1}\) with a minimum and maximum of 380 Nm.rad\(^{-1}\) and 900 Nm.rad\(^{-1}\) respectively for seven subjects tested.

**Spring parameters**

For the simulation of the Hecht and tumbling takeoffs, two types of spring parameters are required:

1. Equipment parameters
2. Body parameters

The calculation of equipment parameters requires the characteristics of the vaulting horse/tumbling track to be determined, and for the body parameters the elastic nature of the shoulders needs to be determined. A simple procedure is required which will give accurate results. From the literature little research has been carried out in these two areas.

**Equipment parameters**

To calibrate the vaulting horse and gymnastic floor, Continental (1994) dropped weights vertically onto the equipment. A similar procedure was carried out by Martin Liptal, Yerby and Williams (1994) to test the impact of a mass on a padded surface. Dropping weights onto equipment allows vertical spring parameters to be calculated but does not allow horizontal spring parameters to be determined. The procedures and results of the calibration of the vaulting horse and floor by Continental (1994) are as follows:

**Vaulting horse**

To test the vaulting horse Continental (1994) used the following procedure:

1. A 10 kg steel testing body was dropped from a height of 0.4 m
2. Five measuring points were used, with 10 trials at each point
3. Trials three to ten were used to determine the arithmetic mean
4. The maximum impact force, depression and height of rebound were reported (Table 2.8)

<table>
<thead>
<tr>
<th>parameter</th>
<th>max. impact force [N]</th>
<th>depression [mm]</th>
<th>height of rebound [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>norm</td>
<td>&lt; 2200</td>
<td>34 - 44</td>
<td>130 - 200</td>
</tr>
</tbody>
</table>

Table 2.8. Norms testing results for the vaulting horse (Continental, 1994)
Gymnastic floor

To test the gymnastic floor Continental (1994) used the following procedure:

1. A 20 kg steel testing body was dropped from a height of 0.8 m
2. Nine measuring points were used, with 10 trials at each point
3. Trials three to ten were used to determine the arithmetic mean
4. The maximum impact force, depression and height of rebound were reported (Table 2.9)

Table 2.9. Norms testing results for the gymnastic floor (Continental, 1994)

<table>
<thead>
<tr>
<th>parameter</th>
<th>max. impact force [N]</th>
<th>depression [mm]</th>
<th>height of rebound [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>norm</td>
<td>&lt; 5500</td>
<td>60 - 70</td>
<td>245 - 335</td>
</tr>
</tbody>
</table>

Body parameters

The majority of the major joints in the body are modelled as simple pin joints in simulation models of athletic human motion (e.g. vertical jumping; van Soest et al., 1993). However, for the shoulder joint / complex however there is a potential for considerable movement in the shoulder joint with no obvious single centre of rotation (Engin and Peindl, 1987). A simple pin joint is perhaps therefore insufficient to represent the shoulder. For the simulation of the Hecht vault takeoff in this study parameters are required so that the movement in the shoulder joint can be represented in a simulation model.

Engin and Peindl (1987) determined the 3D maximal voluntary range of motion in the shoulder complex using a sonic digitising technique which plotted the movement of the shoulder joint in each of the three planes of motion. This technique was extended by Engin, Peindl, Berme, and Kaleps (1984a, b) to measure passive resistance of the shoulder complex to external loads. Forces were applied to the upper arm which forced the upper arm to move axially for a given upper arm orientation, displacement of the shoulder joint was plotted as a function of the load applied (Figure 2.11).

Figure 2.11 shows the displacement of the shoulder joint from the neutral position. It can be seen that the relationship is curvilinear with the extension almost reaching a maximum point as the load is increased until it becomes uncomfortable to increase the load any further. A maximum displacement of 20 cm from a neutral shoulder position
was observed when a load of 280 N was applied in the anterior direction with the upper arm at right angles to the trunk. It should be noted that these experiments were carried out to measure the passive nature of the shoulder complex. No studies have been found which try to measure the movement in the shoulders while the muscles of the shoulder are actively being used.

Figure 2.11. Applied force and displacement of the shoulder joint (Engin et al., 1984b, p. 216).
Summary of parameter determination

The above sections have discussed the three types of parameters required for the development and use of a muscle driven simulation model. The next section will describe the procedures needed to record and analyse actual athletic performances.

IMAGE ANALYSIS

An image analysis of actual performances is required so that realistic inputs can be obtained for the simulation models of the Hecht vault and tumbling takeoffs. In addition the time histories for each of the variables which define the motion of the simulation models are required to evaluate the models. Yeadon (1990a) noted that time histories of the angles which specify body configuration and orientation may be used to evaluate the accuracy of a simulation model. However for simulation models which involve a ground contact phase or a moveable joint centre, the time history of the point of contact with the ground or the moveable joint centre is also needed to completely define the movement of the model. The most common way to obtain time histories of various body landmarks during athletic movements is to digitise cine or video recordings of the performance (Yeadon and Challis, 1994). Alternative methods include automated motion analysis which is becoming more popular as technology improves (e.g. MacReflex which uses reflective markers to track the motion of the body parts). An automated system has the advantage of not having to manually digitise body landmarks frame by frame but has the problems of markers falling off, and points getting lost during complex somersault activities. In addition an automated system gives the location of the markers and not the joint centres.

The procedure for obtaining the time histories of various body landmarks from recorded images throughout athletic movements can be divided into the following sections:

1. Video and cine techniques
2. 2D and 3D techniques
3. 2D and 3D reconstruction
4. Lens distortion correction
5. Synchronisation of camera views
6. Smoothing / curve fitting / differentiation
Video and cine techniques

Until recently cine-film was considered to be a better quality recording medium to use than video although with the development of high resolution video digitising systems such as Target (Kerwin, 1995) the accuracy of video has improved and is now comparable with film (Tan, Kerwin and Yeadon, 1995). Video and cine systems have been used successfully to record athletic activities with both systems having advantages and disadvantages. For this study detailed time histories of various body landmarks during the contact phase of vaulting and tumbling takeoffs are required. The duration of both events are short (0.1 s - 0.2 s) and with standard video cameras which operate at 50 Hz would give between four and eight fields during the contact phase. Since the body landmarks are moving very quickly during the contact phase four to eight fields would be insufficient. Therefore a technique of recording the motion at a higher rate is required, e.g. high speed cine techniques which have been used successfully and shown to give accurate data at high speed framing rates (Dapena and Chung, 1988).

2D and 3D techniques

A two-dimensional technique to reconstruct the digitised landmarks assumes that all the digitised points are in a single plane. Points which are actually outside the plane of reconstruction are projected on to the plane of reconstruction (Walton, 1981). Movements outside the plane are ignored and this may result in inaccurate findings (Yeadon and Challis, 1994). A comparison of 2D and 3D reconstruction techniques for a given movement should be performed to justify using a 2D analysis (Yeadon and Challis, 1994). For the chosen activities of the present study (vaulting and tumbling), previous studies have predominantly used 2D reconstruction techniques (e.g. Brüggemann, 1987) although without justification. The advantage of a 2D analysis is that only a single cine/video camera is required to record the activity from which the body landmarks can be digitised and reconstructed. In contrast a 3D analysis requires at least two cameras and is far more complex (Yeadon and Challis, 1994).

2D and 3D reconstruction

The most common method for reconstructing coordinates of digitised body landmarks filmed using cameras with fixed orientation is the Direct Linear
Transformation (DLT) technique (Abdel-Aziz and Karara, 1971). Other methods include those that require the cameras to be placed in known locations and orientations (e.g. Yeadon, 1984). The DLT technique allows arbitrary placement of the cameras but requires that the control points used to calibrate the system are evenly distributed throughout the control space (Abdel-Aziz and Karara, 1971). The DLT reconstruction relates the digitised and spatial coordinates using two equations with 11 parameters. The two equations are formed from assuming an ‘ideal camera-digitiser lens system’ where the point in space, the centre of the lens and the image point lie on a straight line (Yeadon, 1996):

\[
\begin{align*}
    u &= \frac{L_1x + L_2y + L_3z + L_4}{L_9x + L_{10}y + L_{11}z + 1} \\
    v &= \frac{L_5x + L_6y + L_7z + L_8}{L_9x + L_{10}y + L_{11}z + 1}
\end{align*}
\]  

Where \(L_1 - L_{11}\) are the 11 DLT parameters which are functions of 11 geometrical parameters which define the ‘ideal camera-digitiser system’, \((u, v)\) are the digitised coordinates and \((x, y, z)\) is the 3D location of the digitised point. Of the 11 geometrical parameters, 6 define the location and orientation of the camera and the other 5 define the characteristics of the digitising measurement system.

The DLT method requires calibrating each camera view by calculating the values of the 11 parameters for each camera from the digitised coordinates of control points in known locations. A minimum of six control points are required to solve for the 11 parameters using a least squares technique although more points are often used (Wood and Marshall, 1986).

A number of researchers have assessed the accuracy of the DLT method for 3D reconstruction. Wood and Marshall (1986) and Chen, Armstrong and Raftopoulos (1994) found that the reconstruction accuracy was affected by the number and distribution of the control points used in the calibration of each camera. Chen et al. (1994) found that the accuracy improved as the number of control points increased from 8 to 24 and demonstrated that the ‘best accuracy’ was when the control points were evenly distributed throughout the control space. Both Wood and Marshall (1986) and Chen et al. (1994) found significant increases in reconstruction errors outside the calibration volume. In addition Wood and Marshall (1986) recommended a camera set-up where the camera optical axes are perpendicular.
Once the 11 DLT parameters for each camera have been calculated the parameters can be used to reconstruct the locations of the digitised body landmarks, giving the location of the digitised points from each camera view. For a 3D analysis with two cameras, a least squares technique can then be used to calculate the 3D coordinates of each digitised point in space (Karara, 1980).

Hatze (1988) modified the DLT procedure (MDLT) by constraining one of DLT parameters using a constraint equation, this effectively reduced the number of parameters to 10. Hatze found a large increase in reconstruction accuracy from 5 mm to 1 mm.

The DLT technique assumes an ideal camera-digitiser lens system and the different camera views need to be synchronised. A number of techniques are available to correct for lens distortion errors and problems with synchronisation:

**Lens distortion**

A number of researchers have accounted for the effects of non-linear lens distortion in the DLT procedure by adding extra parameters to the two DLT equations (Abdel-Aziz and Karara, 1971; Wood and Marshall, 1986; Hatze, 1988; Challis, 1992).

Wood and Marshall (1986) added one extra parameter to the 11 parameter DLT to 'partially correct for non-linear symmetrical lens distortion'. No increase in the reconstruction accuracy was found, suggesting that this may have been due to the high quality of the lenses or the small number of control points used.

Hatze (1988) added lens correction to his MDLT method to account for both radial and asymmetrical lens distortion (up to 5 extra parameters). However, only a slight improvement in reconstruction accuracy was found.

Challis (1992) examined in detail the corrections for lens distortion using a number of different lens distortion models which corrected for radial and asymmetrical distortions. Challis tested the various models for lens distortion using a calibration volume of 1.0 m by 0.6 m by 1.0 m with a maximum of 51 control points. He found that correcting for lens distortion only produced 'very small' increases (0.1 mm) in reconstruction accuracy which occurred at the precision limit of the system.

Tan et al. (1995) reported RMS digitisation reconstruction errors for 30 calibration points spread over a 9 m field of view of 14.1 mm and 5.9 mm horizontally and vertically for a 2D DLT reconstruction. These reconstruction errors were found to be reduced to
8.7 mm and 4.8 mm when a single extra parameter was added to the DLT equations to correct for radial lens distortion (Tan, 1997).

Corrections for lens distortion have improved the reconstruction accuracy for video systems (Tan, 1997) but for film systems there has been little improvement (Hatze, 1988).

**Synchronisation of camera views**

To synchronise two camera views requires the two cameras to be genlocked / phase locked to give exposures at the same instants in time. In addition a knowledge of the relationship between the time bases of each camera is required by identifying a pair of matching video / film frames (Yeadon, 1996). If it is not possible to synchronise the exposures of the two cameras then synchronous data sets may be obtained by interpolation between fields / frames (Yeadon and Challis, 1994). However this requires a timing device in the field of view of each camera to know how the framing rate of one camera relates to the framing rate of the second camera. For example Walton (1981) used a timing device in the field of view of both cameras to establish corresponding frame times and then linearly interpolated the data of the faster camera over the time base of the slower camera to obtain synchronous data. If a timing device is not available or it is not possible to place one in the field of view of both cameras (e.g. during competitions) then the digitised data can be used to determine the time of a particular field in the time scale of the other camera. For example Yeadon (1989) synchronised two camera views for ski jumping at takeoff and landing, at the takeoff it was assumed that the mass centre of the ski jumper was in a vertical plane which bisected the in-run and at landing the velocity vector of the ski jumper was used.

**Smoothing / curve fitting / differentiation of film / video data**

The process of obtaining three-dimensional coordinates of body landmarks introduces errors inherent in the digitisation and reconstruction process. Thus three-dimensional data consists of the true value plus some error. The error in the data can be divided into systematic and random errors (Challis and Kerwin, 1988). The systematic errors can often be minimised although it is difficult to reduce the random errors. For a mechanical analysis of a movement and the implementation of a simulation model, displacement and velocity time histories of the kinematic variables defining the system are required. By the very nature of numerical differentiation techniques, ‘noisy data’ will
give poor velocity estimates as errors are amplified upon differentiation and so a
technique is required to remove as much of the noise from the data as possible without
over-smoothing.

All of the available numerical techniques involve using a mathematical function to
represent the three-dimensional data and can be separated into three main techniques
when derivatives are required:

1. Spline curve fitting
2. Fourier series truncation
3. Digital low-pass filters

Spline curve fitting

Splines offer the advantage that they do not require data that are equidistant along
the time base (Challis and Kerwin, 1988). Quintic splines are to be preferred to cubic
splines where second or higher order derivatives are required (Phillips and Roberts, 1983)
as a cubic spline has a piecewise linear second derivative with restrictions placed upon the
endpoint conditions (Challis and Kerwin, 1988; McLaughlin, Dillman and Lardner,
1977). There are two types of splines used in the literature. There are those which
require error estimates for each point in the spline and an S value to be adjusted to give
the correct fit (cubic spline: Reinsch, 1967 and quintic spline: Wood and Jennings, 1979)
and there are generalised cross-validated splines (Woltring, 1986) which automatically fit
a spline to the data by performing an internal consistency check (Challis and Kerwin,
1988). Fitting a spline to the data allows derivatives to be obtained without any extra
processing of the data. The advantage of using splines is that they are very flexible,
allowing data that is not equispaced to be fitted, although problems at the endpoints may
occur and a large number of coefficients are required to define a spline (Wood, 1982).

Fourier series truncation

A Fourier series technique involves transferring the data to the frequency domain
using a Fourier transformation (Bartlett, 1992). A cut-off frequency is then chosen with
the data reconstructed up to the cut-off frequency, the problem with this technique is
choosing the value of the cut-off frequency. The filtered data is analytic in form and
therefore derivatives can be obtained directly (Bartlett, 1992). Hatze (1981c) fitted a
Fourier series to angular displacement data obtained for elbow flexion that had been
linearly detrended. He showed that satisfactory approximations of the displacement and acceleration data could be obtained using the Fourier series technique. Hatze went on to show that the Fourier series technique could still give good acceleration estimates when an extra 5% error was introduced to the original data. Hatze though assumes that the second derivatives are zero at the endpoints, this limits the applicability of his method when the derivatives are not zero at the endpoints. If the derivatives at the end points are not zero then extra data can be added to the endpoints (padding) so that within the required range the derivatives are non zero (Vint and Hinrichs, 1996). Overall the Fourier series technique appears to offer a good method for estimating high order derivatives and has the advantage that relatively few coefficients are required to define a series although the original data must be equispaced (Wood, 1982).

Digital low-pass filters

Butterworth filters are often used to remove high frequency noise from digital data with a cut-off frequency of 4 Hz or 8 Hz used for human movements (e.g. Brüggemann, 1987). There are a number of disadvantages in using a digital filter including: the data must be equispaced (Wood, 1982), a cut-off frequency must be selected and there can be problems at the endpoints (Vint and Hinrichs, 1996).

The choice of technique between the three described above has been addressed by a number of authors including:

Yeadon (1984) noted that numerical integration techniques using variable step lengths require variables and their derivatives at arbitrary times, therefore mathematical functions which allow smoothing and interpolation are appropriate. Challis and Kerwin (1988) tested quintic splines, truncated Fourier series and Butterworth filters using a series of test functions and found that quintic splines were superior in the determination of second derivatives from noisy data. Vint and Hinrichs (1996) also tested Butterworth filters, cubic splines, quintic splines and Fourier series on modified raw angular displacement data from Pezzack, Norman and Winter (1977) and like Challis and Kerwin (1988) found quintic splines to give the most accurate acceleration estimates.
Summary of image analysis

The procedures required to record and analyse actual performances have been documented in the above section. To give detailed information of the contact phase will require high speed cine-film recordings as video would not give sufficient information. In addition a 3D analysis will probably be required and it would appear that quintic splines give an accurate method for fitting ‘noisy data’.

CHAPTER 2 SUMMARY

Chapter 2 has covered three main sections of the literature with reference to dynamic jumping activities. The first section described the main features of jumping activities, the second section documented the research literature on dynamic jumps and the third section discussed the techniques required to develop a muscle driven simulation model of dynamic jumps in the areas of simulation modelling, parameter determination and image analysis. In the next chapter simulation models will be developed for the simulation of the Hecht vault and tumbling takeoffs.
CHAPTER 3

DEVELOPMENT OF THE SIMULATION MODELS

This chapter is divided into two main sections:

The first section develops a two segment simulation model of gymnastic vaulting derived from first principles using Newtonian mechanics to:

1. Investigate the extent to which a linked segment model without joint torque and elasticity can simulate a dynamic movement.
2. Gain an understanding of the mechanics of vaulting and experience in developing simulation software.

The second section describes more complex simulation models for gymnastic vaulting and tumbling using the computer software package Autolev.

TWO SEGMENT SIMULATION MODEL OF VAULTING

The ability to perform a gymnastic vault is dependent on many factors including the preflight parameters at horse contact, the elastic properties of the horse and the gymnast, and the joint torques exerted while in contact with the horse. Although there are many factors which will affect the performance, this section will describe a relatively simple inelastic torque-free two segment simulation model. The model will be used to demonstrate the importance of the preflight phase to the performance of the Hecht vault.

Method

Simulation model

A planar two segment computer simulation model comprising an arm segment and a body segment was used to simulate a vault from Reuther board takeoff, through horse contact, until landing. Input to the model comprised the values of five variables at the time of horse contact while output variables described the postflight phase (Table 3.1). To describe the orientation and configuration of the model the shoulder angle $\alpha$ was defined as the angle between the two segments and the body angle $\phi$ as the angle of the body segment (shoulders S to feet F) relative to the horizontal (Figure 3.1). The
mechanical analysis of the vault was divided into preflight, impact, postflight and landing phases.

Table 3.1. Input and output variables of the two segment simulation model

<table>
<thead>
<tr>
<th>Input variables at horse contact</th>
<th>symbol</th>
<th>Output variables from the model</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal velocity of the mass centre</td>
<td>u</td>
<td>maximum height of the mass centre</td>
<td>h</td>
</tr>
<tr>
<td>vertical velocity of the mass centre</td>
<td>v</td>
<td>landing distance of the mass centre</td>
<td>d</td>
</tr>
<tr>
<td>body angle</td>
<td>φ</td>
<td>landing angle</td>
<td>γ</td>
</tr>
<tr>
<td>shoulder angle</td>
<td>α</td>
<td></td>
<td></td>
</tr>
<tr>
<td>angular velocity of the body</td>
<td>ω</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Initial conditions

Input to the model was specified at horse contact and the initial conditions at takeoff from the Reuther board were back-calculated from the horse contact conditions using projectile equations of motion and conservation of angular momentum. This procedure was preferred as it allowed the conditions at horse contact to be varied directly rather than by indirectly changing the initial conditions at takeoff from the Reuther board. The equations used to calculate the initial conditions at takeoff from the Reuther board from the input variables are given in the next section.

Preflight

The preflight phase is from Reuther board takeoff to the initial contact of the hands with the horse. The motion of the model during this phase was determined by the orientation and configuration of the segments, the velocity of the mass centre and the angular velocity of the body at takeoff from the Reuther board. Projectile equations of motion ((3.1) and (3.2)) and conservation of angular momentum (equation (3.3)) were used to calculate the orientation and location of the segments at a given time.
Equations of motion for the preflight phase

horizontally \[ x = ut \]  
vertically \[ z = vt - \frac{1}{2} gt^2 \]

where
- \( x \) = horizontal location of the mass centre at time \( t \)
- \( z \) = vertical location of the mass centre at time \( t \)
- \( u \) = horizontal velocity of the mass centre at takeoff from the Reuther board
- \( v \) = vertical velocity of mass centre at takeoff from the Reuther board
- \( t \) = time
- \( g \) = acceleration due to gravity

\[ h = Io \]

where \( h \) = (constant) angular momentum about the mass centre

\( I \) = moment of inertia about a transverse axis through the mass centre

\( \omega \) = angular velocity about a transverse axis

Since a rigid configuration is maintained by the model during the preflight phase, \( I \) is constant and hence \( \omega \) is also constant. An iterative integration process was used to calculate the motion of the model. The step size (0.005 s) was selected so that further reductions in step size lead to changes in the output variables \( h, d \) and \( \gamma \) that were smaller than 0.001 m, 0.001 m and 0.1° respectively. A rigid configuration was maintained throughout the preflight phase and the orientations of the segments were determined at the time the hands (i.e. the end of the arm segment) reached horse level. Due to the stepwise procedure used the hands were above the horse at the end of one iteration and below the horse surface at the next iteration. To calculate the time of horse contact linear interpolation between iterations was used.

Impact and contact

The model made contact with the horse when the hands reached the level of the horse. At the initial impact of the model with the horse, the velocity of the hands was reduced to zero instantaneously and the angular velocities of the two segments immediately after impact were determined using the conservation of angular momentum (equations (3.4) and (3.5)). Angular momentum was conserved for the whole body.
about the contact point of the hands as the moment arm length of the large impulsive reaction forces produced at the contact point O was zero. Similarly angular momentum was conserved for the body segment about the shoulder point S as the moment arm length of the reaction forces produced at the shoulders S was zero.

**Equations of motion for the impact:**

Angular momentum $h_o$ of the whole system about the hand contact point O:

Before impact:

$$h_o = l_g \omega_g + M_g v_g d_{og}$$

After impact:

$$h_o = l_a \omega_a' + l_b \omega_b' + M_a \omega_a' d_{oa} + M_b v_b' d_{ob}$$  \(3.4\)

Angular momentum $h_s$ of the body segment about the shoulder point S:

Before impact:

$$h_s = l_b \omega_b + M_b u_b' d_{sb}$$

After impact:

$$h_s = l_b \omega_b' + M_b u_b' d_{sb}$$  \(3.5\)

where $\mathbf{I} = $ moment of inertia about the transverse axis

$\mathbf{M} = $ mass

$\mathbf{d_{ij}} = $ distance from point i to j

$\omega$ and $\omega' = $ the angular velocity before and after impact respectively

$v$ and $v' = $ the velocity component perpendicular to the line joining the mass centre to O before and after impact respectively

$u$ and $u' = $ the velocity component perpendicular to the line joining the mass centre to S before and after impact respectively

subscripts g, a and b represent the whole body, the arm segment and the body segment respectively and are represented by i and j above.

Equations (3.4) and (3.5) were solved for two unknowns, the angular velocity of the arms $\omega_a'$ and the angular velocity of the body $\omega_b'$ after impact. The linear velocity of the body after impact was not an unknown as it could be expressed as a combination of the angular velocity of the arms and the body.
During the remainder of the contact phase it was assumed that there was no torque at the shoulders and only the torque due to gravity affected the rotation of the model (Figure 3.1).

Newton's Second Law was used to calculate the angular acceleration of each segment and the vertical reaction force at the hands (equations (3.6) to (3.11)). To solve the six equations in six unknowns ($\dot{\theta}, \ddot{\theta}, N, F, R_h,$ and $R_v$), a linear least squares technique was used (Stewart, 1973). A second order Runge-Kutta method was used to advance the solution of the second order system of differential equations one step. Takeoff from the horse occurred when the vertical reaction force $N$ at the hands reached zero.
Equations of motion for the contact phase

Resolving perpendicular to the arm segment:

\[ R_h \sin \theta - R_v \cos \theta + N \cos \theta - F \sin \theta - M_a g \cos \theta = M_a a \ddot{\theta} \]  

(3.6)

Resolving parallel to the arm segment:

\[ R_h \cos \theta + R_v \sin \theta - N \sin \theta - F \cos \theta + M_a g \sin \theta = M_a \dot{\theta}^2 \]  

(3.7)

Taking moments about the mass centre of the arm segment:

\[ R_h (c-a) \sin \theta - R_v (c-a) \cos \theta - N a \cos \theta + F a \sin \theta = I_a \ddot{\theta} \]  

(3.8)

Resolving perpendicular to the body segment:

\[ R_v \cos \phi - R_h \sin \phi - M_b g \cos \phi = M_b \left[ b \ddot{\phi} + c \dot{\phi} \cos (\theta - \phi) - c \dot{\theta}^2 \sin (\theta - \phi) \right] \]  

(3.9)

Resolving parallel to the body segment:

\[ -R_v \sin \phi - R_h \cos \phi + M_b g \sin \phi = M_b \left[ b \ddot{\phi} + c \dot{\phi}^2 \cos (\theta - \phi) + c \dot{\phi} \sin (\theta - \phi) \right] \]  

(3.10)

Taking moments about the mass centre of the body segment:

\[ R_h b \sin \phi - R_v b \cos \phi = I_b \ddot{\phi} \]  

(3.11)

where

- \( a \) = distance from hands to mass centre of arm segment
- \( b \) = distance from shoulders to mass centre of body segment
- \( c \) = length of arms
- \( \theta \) = angle of arm segment above the horizontal
- \( \phi \) = angle of the body segment above the horizontal
- \( M \) = mass
- \( I \) = moment of inertia about the transverse axis
- \( N, F \) = normal reaction and frictional force at hands
- \( R_v, R_h \) = vertical and horizontal reaction forces at the shoulder joint

The subscripts \( a \) and \( b \) represent the arm and body segments respectively.
Postflight

The motion in the postflight phase was determined by the orientation and configuration of the segments, the velocity of the mass centre and the angular velocity of the two segments at the moment the vertical reaction force became zero. The arms were allowed to continue circling forwards relative to the body at a constant angular velocity determined at takeoff from the horse. Projectile equations (3.1), (3.2) were used to calculate mass centre kinematics, and conservation of angular momentum was used to determine the angular velocity of the body using equation (3.12)). The postflight phase ended when the feet reached the level of the mat. Linear interpolation between iterations was used to calculate the time of landing.

*Equations of motion for the postflight*

Conservation of angular momentum:

The angular momentum about the mass centre for a two segment system can be expressed in the following format (Yeadon, 1990c):

\[ h = h_\phi \dot{\phi} + h_\alpha \dot{\alpha} \]  

(3.12)

where \( \alpha \) = the angle between the arm and body segments. \( h_\phi \) and \( h_\alpha \) are functions of \( \alpha \) calculated algebraically by considering two special cases of equation (3.12), where it was possible to determine \( h_\phi \) and \( h_\alpha \).

**case [1] \( \dot{\alpha} = 0 \)**

If \( \dot{\alpha} = 0 \) the two segment model behaves as a rigid body and therefore equation (3.12) gives:

\[ h = I_\phi \dot{\phi} \]  

(3.13)

where \( h_\phi = I_\phi \) with \( I_\phi = \) moment of inertia of the whole system about the mass centre, which from Figure 3.2 using the parallel axis theorem gives:

\[ I_\phi = I_a + I_b + M_a d_a^2 + M_b d_b^2 \]

Then since \( M_a d_a = M_b d_b \):

\[
\frac{d_b}{M_b} = \frac{d_a}{M_a} = \frac{d_a + d_b}{M_a + M_b}
\]
Therefore

\[ d_a = \frac{M_b}{M_{ab}} \cdot d_{ab}, \quad d_b = \frac{M_a}{M_{ab}} \cdot d_{ab} \]

where

\[ d_{ab} = d_a + d_b, \quad M_{ab} = M_a + M_b \]

Using the Cosine Rule

\[ d_{ab}^2 = a'^2 + b'^2 + 2a'b' \cos \alpha \]

allows equation (3.13) to be determined throughout the postflight phase.

case [2] \( \dot{\phi} = 0 \)

With \( \dot{\phi} = 0 \), equation (3.12) gives:

\[
\begin{align*}
    h &= h_{\alpha} \dot{\alpha} = I_a \ddot{\alpha} + M_a d_a \times (\dot{\alpha} \times \mathbf{S}) \\
    &= I_a \ddot{\alpha} + M_a \frac{M_b}{M_{ab}} \cdot d_{ab} \cos \beta \dot{\alpha}
\end{align*}
\]

Using the Cosine Rule:

\[
\begin{align*}
    b'^2 &= a'^2 + d_{ab}^2 - 2a'd \cos \beta \\
    d_{ab}^2 &= a'^2 + b'^2 + 2a'b \cos \alpha
\end{align*}
\]

which combine to give

\[ d_{ab} \cos \beta = a'^2 + a'b \cos \alpha \]

and therefore:

\[
    h = \left( I_a + \frac{M_a M_b}{M_a + M_b} [a'^2 + a'b \cos \alpha] \right) \ddot{\alpha}
\]

(3.14)

where

\[ h_{\alpha} = I_a + \frac{M_a M_b}{M_a + M_b} [a'^2 + a'b \cos \alpha] \]

Equation (3.12) can therefore be written as:

\[
    h = I_g \ddot{\phi} + \left( I_a + \frac{M_a M_b}{M_a + M_b} [a'^2 + a'b \cos \alpha] \right) \ddot{\alpha}
\]

(3.15)

Equation (3.15) is valid at any time and allows the calculation of the angular velocity of the body given the angular velocity of the arms, as the total angular momentum \( h \) is constant throughout the postflight phase.
Landing

The landing of the gymnast was modelled as an instantaneous impact similar to the contact phase with the horse. After impact the two segment model was treated as a single rigid body which rotated about the contact point with the mat, with gravitational torque used to slow down the rotation of the model. Newton's Second Law was used to calculate the angular acceleration of the single segment using equations (3.16) - (3.18)). To solve the three equations in three unknowns a linear least squares technique was used (Stewart, 1973). A second order Runge-Kutta method was used for the numerical integration to calculate the body angle until the body reached a vertical position or the body stopped rotating forwards.

Equations of motion for the landing

Resolving perpendicular to the line joining the mass centre to the contact point O:

\[ N \cos \gamma - F \sin \gamma - M_g \cos \gamma = M_g \frac{d\gamma}{dt} \]  (3.16)
Resolving parallel to the line joining the mass centre to the contact point O:

\[-N \sin \gamma - F \cos \gamma + M_g g \sin \gamma = M_g d \ddot{\gamma}^2\]  \hspace{1cm} (3.17)

Taking moments about contact point O:

\[-M_g g d \cos \gamma = \left(I_g + M_g d^2\right) \ddot{\gamma}\]  \hspace{1cm} (3.18)

where \(\gamma\) = angle between the line joining the mass centre to O and the horizontal
\(d\) = fixed distance between the mass centre and O

Numerical integration

A second order Runge-Kutta method was used to progress the solution to the differential equations throughout the contact phases with the horse and at landing. [During the flight phases it was not necessary to use the second order Runge-Kutta method as the mass centre followed a parabolic flight path which could be calculated using constant acceleration equations (3.1) and (3.2)]. The second order Runge-Kutta method progressed the solution to the differential equations by calculating an average acceleration for a time interval \(\delta t\) using the following method:

1. Calculate the acceleration at the start of the interval \(a_t\).
2. Calculate the acceleration at the end of the interval \(a_{t+\delta t}\) assuming constant acceleration \(a_t\) during the interval.
3. Average the accelerations at the start and end of the interval to give an average acceleration value for the whole interval \(\bar{a}_t\).
4. Use the average acceleration value over the whole interval to calculate the displacement and velocity at the end of the interval.
5. Re-calculate the acceleration at the start of the next interval \(a_t\).

Inertia parameters

Anthropometric measurements available on 11 elite gymnasts from previous studies and the mathematical model of Yeadon (1990b) were used to calculate segmental masses, mass centre locations, link lengths and moments of inertia. The inertia data were then normalised to a total body mass of 62.88 kg and a standing height of 1.67 m.
corresponding to the average gymnast from the 1988 Olympic Games (Takei and Kim, 1990). Normalisation was carried out using the procedure of Dapena (1978) in which segment mass is assumed to be proportional to body mass, segment length proportional to standing height and segmental moment of inertia proportional to mass times length squared. In addition the normalised inertia data were averaged to give a single inertia data set from which inertia parameters were calculated for the two segment simulation model.

Table 3.2. Means and standard deviations of average normalised segmental inertia parameters

<table>
<thead>
<tr>
<th></th>
<th>arm</th>
<th>body</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass [kg]</td>
<td>$M_a = 7.14 \pm 0.59$</td>
<td>$M_b = 55.74 \pm 0.59$</td>
</tr>
<tr>
<td>length [m]</td>
<td>$c = 0.524 \pm 0.03$</td>
<td>$d = 1.508 \pm 0.04$</td>
</tr>
<tr>
<td>CM location [m]</td>
<td>$a = 0.276 \pm 0.03$</td>
<td>$b = 0.467 \pm 0.02$</td>
</tr>
<tr>
<td>moment of inertia [kg.m$^2$]</td>
<td>$I_a = 0.214 \pm 0.05$</td>
<td>$I_b = 9.115 \pm 0.42$</td>
</tr>
</tbody>
</table>

Note: See Figure 3.1. for nomenclature

**SIMULATION MODELS USING AUTOLEV**

The software package Autolev is an advanced symbol manipulation program which is based upon Kane's method for formulating the equations of motion for mechanical systems. The Autolev package reduces the amount of time required to produce simulation programs, with there being less chance for mistakes and allows direct access to the Fortran code produced so that simulation models can be customised for specific tasks.

The remainder of this chapter will describe Kane's method for determining the equations of motion and discuss the information required by Autolev to produce simulation models. Two simulation models of vaulting and tumbling will then be described.

Full details of the two simulation models developed, including the nomenclature for each model and listings of the commands used to develop the models using Autolev are documented in Appendix A.
Kane's method for obtaining the equations of motion

Kane’s method for obtaining the equations of motion for a system S is based upon re-writing the variables used in Newton’s Second Law (F = ma) in a more convenient way.

Kane re-writes Newton’s Second Law as:

\[ \mathbf{F}_r + \mathbf{F}_r^* = 0 \quad (r = 1, \ldots, n) \] (3.19)

where:
- \( r \) = the number of degrees of freedom in reference frame N
- \( \mathbf{F}_r^* \) = the generalised inertia forces
- \( \mathbf{F}_r \) = the generalised active forces

Generalised coordinates and speeds

To calculate the generalised inertia forces, generalised coordinates and speeds must first be assigned. Kane’s method allows generalised speeds to be chosen so that the formulation of the equations of motion is simplified (Schaechter et al., 1991, pg. 43):

If there is a system S characterised by n generalised coordinates \( q_1, \ldots, q_n \) then \( n \) generalised speeds \( u_1, \ldots, u_n \) must be chosen such that:

\[ u_r = \sum_{s=1}^{n} G_{rs} \dot{q}_s + H_r \quad (r = 1, \ldots, n) \] (3.20)

where \( G_n \) and \( H_r \) are functions of \( q_1, \ldots, q_n \) and the time \( t \).

In addition the functions \( G_n \) and \( H_r \) must be chosen such that the equations can be solved uniquely for \( \dot{q}_1, \dot{q}_2, \dot{q}_3 \). That is functions \( W_r \) and \( X_s \) of \( q_1, \ldots, q_n \) and \( t \) must exist such that:

\[ \dot{q}_s = \sum_{r=1}^{n} W_{sr} u_r + X_s \quad (s = 1, \ldots, n) \]

Typically the generalised coordinates are lengths or angles and are denoted by the letters \( q_1, \ldots, q_n \). The generalised coordinates can then be differentiated with respect to time to give the generalised speeds \( u_1, \ldots, u_n \).

In addition to the generalised speeds, velocities of all points / mass centres in the system must be calculated, this is achieved by differentiating with respect to time the position vectors of each point / mass centre.
Accelerations

The final set of variables to be calculated is the set of accelerations of each point / mass centre in the system. These can be calculated by differentiating the velocities of each point / mass centre.

Generalised inertia forces

Once the generalised coordinates, generalised velocities, velocities and accelerations have been defined / calculated the generalised inertia forces can be calculated using equation (3.21).

Kane defines the generalised inertia forces as:

\[ \mathbf{F}_r^* = \sum_{i=1}^{u} \mathbf{v}_{rP_i} \cdot (-m_i \mathbf{a}_{P_i}) \quad (r = 1, \ldots, n) \] (3.21)

where:  
\( u \) = number of particles in the system S  
\( P_i \) = the \( i^{th} \) particle of the system S  
\( m_i \) = the mass of the \( i^{th} \) particle of the system S  
\( \mathbf{v}_{rP_i} \) = the \( r^{th} \) partial velocity of \( P_i \) in N  
\( \mathbf{a}_{P_i} \) = the acceleration of \( P_i \) in N

Generalised active forces

To calculate the generalised active forces requires all the forces / torques acting on the system to be defined in terms of magnitude direction and point of application. Kane defines the generalised active forces as:

\[ \mathbf{F}_r = \sum_{i=1}^{u} N \mathbf{v}_{rP_i} \cdot \mathbf{F}_{P_i} \quad (r = 1, \ldots, n) \] (3.22)

where:  
\( u \) = number of particles in the system S  
\( P_i \) = the \( i^{th} \) particle of the system S  
\( N \mathbf{v}_{rP_i} \) = the \( r^{th} \) partial velocity of \( P_i \) in N  
\( \mathbf{F}_{P_i} \) = the resultant of all forces acting on \( P_i \)
Equations of motions

To formulate the equations of motion each generalised inertia and active force are added together and the sum set equal to zero (equation (3.19)).

Autolev requirements and procedures

The Autolev package requires the following information in order to generate the equations of motion for a mechanical system (Schaechter et al., 1991):

1. An expression for the inertial angular velocity of each rigid body in the system.
2. An expression for the inertial velocity of each particle, each rigid body mass centre, and each point at which a force that contributes to generalised active forces is applied.
3. An expression for the inertial angular acceleration of each rigid body in the system.
4. An expression for the inertial acceleration of each particle and each rigid body mass centre.
5. Expressions for the forces and / or torques that contribute to generalised active forces.

The five requirements by Autolev to generate the equations of motion would perhaps suggest that it is very difficult to build a mechanical system using Autolev. However in practice the Autolev package is very simple to use. To formulate the equations of motion requires expressions for the positions and orientations of each body in the system to be defined since Autolev can then calculate the velocities and accelerations of each body by differentiation. In addition expressions for the forces / torques acting on the system are required.

Simulation models

Two simulation models were developed using Autolev for vaulting and tumbling. The vaulting model simulated the vault from wrist contact with the horse through to finger takeoff from the horse, and the tumbling model simulated the whole of the final takeoff from a tumbling track (toe down to toe off). Segmental inertia, spring stiffness / damping and muscle torque parameters were calculated from measurements taken on the subject / equipment used, and through comparing simulations with real performances (Chapters 4, 5). The Fortran code generated by Autolev was customised to meet the specific requirements of simulating each activity.

The remainder of this chapter will describe the two simulation models developed, full details of the two models are given in Appendix A.
Five segment model for tumbling

A planar five segment model consisting of a foot, leg, thigh, trunk + head and arm + hand segments with left and right sides of the body averaged was developed for simulating the final contact phase in tumbling (Figure 3.3). The model has torque generators at the ankle, knee, hip and shoulder joints (TA, TK, TH, TS respectively) which can extend each joint. An elastic interface is used between the model and the tumbling track which allows the track to depress horizontally and vertically.

Inputs to the model correspond to the initial conditions at the start of the final contact with the tumbling track and the time each torque generator is activated:

- Mass centre velocity
- Orientation of each segment
- Angular velocity of each segment
- Time each muscle in the model is activated

The outputs from the model corresponded to the time of takeoff from the track:

- Whole body angular momentum about the mass centre
- Mass centre velocity
- Orientation of each segment
- Angular velocity of each segment

![Diagram of the five segment model for tumbling](image)

**Figure 3.3.** Five segment model for tumbling.
Five segment model for vaulting

A planar five segment model with left and right sides of the body averaged consisting of four rigid segments (foot + leg, thigh, trunk + head and arm) and an elastic hand segment was developed for simulating the horse contact phase of vaulting (Figure 3.4). The model has a torque generator at the shoulder (TS) which can close the shoulder angle and the hip / knee angles are constrained to specified angles throughout each simulation. An elastic shoulder joint and elastic interface between the model and the vaulting horse allow the shoulder to depress in line with the arms and the vaulting horse to depress horizontally and vertically. The elastic hand segment is included in the interface between the model and the vaulting horse which allows the model to remain in contact with the horse until the vertical reaction at the fingers reaches zero.

Inputs to the model correspond to the initial conditions as the wrists make contact with the horse and the time the torque generator at the shoulders is activated:

- Mass centre velocity
- Orientation of each segment
- Angular velocity of each segment
- Time each muscle in the model is activated

The outputs from the model corresponded to the time of takeoff from the horse:

- Whole body angular momentum about the mass centre
- Mass centre velocity
- Orientation of each segment
- Angular velocity of each segment

![Diagram of the five segment model for vaulting](image-url)

**Figure 3.4.** Five segment model for vaulting.
Elastic vaulting horse / tumbling track

The contact between the model and the vaulting horse / tumbling track was modelled using horizontal and vertical linear massless springs, with the force in the springs dependent on the depression and velocity of depression of the springs (Figure 3.5; equation (3.23)).

\[ F = -K_1 d - K_2 v \]  

where  
\( F \) = force in spring  
\( K_1 \) = stiffness coefficient  
\( K_2 \) = damping coefficient  
\( d, v \) = depression and velocity of depression

For the vaulting model it was assumed that there was one point of contact between the model and the vaulting horse at the wrists. Two springs were used to model the contact of the gymnast with the horse, one horizontal and one vertical spring at the wrists.

For the tumbling model it was assumed that there were up to two points of contact between the model and the tumbling track at the ball of the feet and the heels. Three springs were used to model the contact of the gymnast with the tumbling track, one horizontal and one vertical spring at the ball of the feet, and one vertical spring at the ankle to model the effect of the heel in contact with the tumbling track.

Each vertical spring was restricted to only act on the model when in contact with the vaulting horse / tumbling track. The horizontal spring in each model acted until takeoff from the vaulting horse / tumbling track had occurred. The spring parameters were
determined from experiments on the equipment and by comparing the depression of the
equipment in actual performances with simulated performances (Chapters 4, 5, 6).

Hand segment for the vaulting model

The hand segment was included in the simulation model to consider the effect of the
hand keeping the gymnast in contact with the horse until the fingers came away from the
horse. The hand segment was included in the vertical linear spring between the wrists and
the vaulting horse.

During the first part of the horse contact phase the hands were not represented with
the vertical spring having a natural length of zero and only representing the compression
of the elastic interface between the gymnast and the vaulting horse (Figure 3.5, equation
(3.24)). After the wrists had started to spring back from maximum depression the natural
length of the vertical spring was changed to equal the height of the wrists above the horse
top at takeoff from the horse as calculated from filmed performances of the vault (Chapter
4). To ensure the model remained in contact with the horse until the wrists reached the
height above the horse determined from filmed performances, the formula for the vertical
force in the spring was changed from equation (3.24) to (3.25). As equation (3.25) does
not include a damping component, the force in the spring only reaches zero when the
spring reaches its natural length and therefore the model only takes off when the wrists
are the required height above the horse top. The spring stiffness parameter in equation
(3.25) was determined so as to ensure that the vertical reaction force remained continuous
throughout the change from equation (3.24) to (3.25); (equation (3.26)).

The vertical reaction force $R_Z$ before the change in the natural length of the vertical
spring between the wrists and the horse top is:

$$ R_Z = -K_3 z - K_4 v $$  \hspace{1cm} (3.24)

where $z, v$ = vertical depression and velocity of depression of the horse
$K_3, K_4$ = spring stiffness and damping parameters

The vertical reaction force $R_Z$ after the change in the natural length of the vertical spring
between the wrists and the horse top from zero to $w_h$ above the horse top is:
\[ RZ = -K_{3n}(z - w_h) \] (3.25)

where \( K_{3n} \) = new spring stiffness parameter

The new stiffness parameter \( K_{3n} \) at time \( t \) when the natural length of the vertical spring is changed to \( w_h \) above the horse top is:

\[ K_{3n} = \frac{-RZ_t}{z_t - w_h} \] (3.26)

where \( K_{3n} \) = spring stiffness

\( RZ_t \) = vertical reaction force at time \( t \)

\( z_t \) = vertical depression at time \( t \)

Shoulder elasticity

For the vaulting model one linear massless spring was used at the shoulders joining the arm segment to the trunk segment in line with the arms to model the elastic nature of the shoulder joint (Figure 3.4; Figure 3.6). The force in the spring was dependent on the depression and velocity of depression of the spring (equation (3.23)). The stiffness and damping parameters were determined from comparing actual performances of the vault (Chapter 4) with simulated performances (Chapter 6) as it was not possible to obtain the parameters from experimental testing (Chapter 5).

\[ \text{torque} \]

\[ \begin{array}{c}
\text{trunk} \\
\downarrow \\
\text{shoulder spring} \\
\downarrow \\
\text{arms}
\end{array} \]

Figure 3.6. Shoulder spring for the vaulting model.

Torque generators

Torque generators were used at the ankle, knee, hip and shoulder for the tumbling model and at the shoulder for the vaulting model to represent the forces produced by
muscles across each joint (Figure 3.3, Figure 3.4). For the tumbling model each torque generator extended a joint and for the vaulting model the torque generator 'closed' the shoulder angle (see Figure 3.4). Two types of torque generator were used which were both activated using a bang-bang approach, which allowed each torque generator to be switched to maximum activation once and remain at maximum activation until takeoff from the vaulting horse / tumbling track.

Case 1. Contractile element

The knee, hip and shoulder joints in the tumbling model (TK, TH and TS in Figure 3.3) used torque generators which calculated the torque that could be produced using an 18 parameter function of the joint angle and angular velocity (equation (3.27)). The 18 parameters for the contractile element were determined from experimental data on the subject (Chapter 5).

\[ T = A(t)F(\theta, \dot{\theta}) \]  

where

- \( T \) = the torque produced by the contractile element at time \( t \)
- \( A(t) \) = activation function which switches full on after time \( t \)
- \( F \) = 18 parameter function of the joint angle \( \theta \) and joint angular velocity \( \dot{\theta} \) (Chapter 5)

Case 2. Elastic and contractile elements in series

The ankle joint in the tumbling model (TA in Figure 3.3) and the shoulder joint in the vaulting model (TS in Figure 3.4) have torque generators consisting of an elastic and a contractile element in series. This set-up was used to include the effect of a series elastic element as well as a contractile element on the torque produced by the muscles during a performance. To model the contractile and elastic elements in series an intermediary segment was included in the simulation models with negligible mass and moment of inertia (Figure 3.7). The elastic element acted between the arm / foot and the intermediary segment, and the contractile element acted between the intermediary segment and the trunk / shank. The torque produced by the contractile element was modelled using an 18 parameter function (equation (3.28)). The torque produced by the series elastic element was modelled as a linear function of the angle \( \theta_{ee} \) (equation (3.29)).
Muscle parameters for the elastic and contractile elements were determined from experimental testing with the subject and from the literature (Chapter 5).

\[ T_{ce} = A(t)F(\theta_{ce}, \dot{\theta}_{ce}) \]  

where \( T_{ce} \) = the torque produced by the contractile element at time \( t \)
\( \theta_{ce} \) = the angle between the intermediary segment and the trunk / shank
\( A(t) \) = activation function which switches full on after time \( t \)
\( F \) = 18 parameter function in terms of \( \theta_{ce} \) and \( \dot{\theta}_{ce} \) (Chapter 5)

\[ T_{ee} = P_e \theta_{ee} \]  

where \( T_{ee} \) = the torque produced by the elastic element at time \( t \)
\( P_e \) = stiffness parameter
\( \theta_{ee} \) = angle between the arm / foot and the intermediary segment

Figure 3.7. Set-up of the elastic and contractile elements in series at the shoulder and the ankle.

Joint angle constraint

The hip and knee angles in the vaulting model (AH and AK in Figure 3.4) were set to fixed values throughout each simulation.

Customisation of the Autolev Fortran code

The Autolev package produced Fortran code for each model, although the code needed to be customised to meet the specific requirements of each model. There were three main requirements for each simulation model:
1. Allow single simulations to be run.
2. Optimise the spring parameter values by comparing the model’s performances with actual recorded performances.
3. Allow the evaluation of the models by comparison with recorded performances.

The Fortran code generated by Autolev for both simulation models was developed in the same three stage format (Figure 3.8) which allowed similar changes to be made to the Fortran code produced for each model.

1. Declare parameters, variables and integers

2. Read: initial conditions / parameter values
   - maximum time of simulation
   - time interval for printing
   - starting step length for simulation

3. main loop
   - call subroutine to solve equations of motion
   - print out variables at a chosen frequency

Figure 3.8. Basic format for Autolev Fortran code.

The Autolev code was customised by converting the main segment of the code into a subroutine and adding extra lines to allow the different options for each program to be run. The Autolev code was then called from a template Fortran program whenever a simulation needed to be run. To fulfil the different options for the simulation model, the template was written with different options which were chosen depending on the value of variables which were read in from an input file.
CHAPTER 3 SUMMARY

This chapter has described the development and structure of three simulation models. Parameters for the five segment models will be determined in Chapter 5 and all three models will be evaluated in Chapter 6.

The next chapter in this study documents the recording and analysis of vaulting and tumbling performances, which will be used to evaluate the five segment simulation models developed in this chapter.
CHAPTER 4

FILM / VIDEO ANALYSIS

This chapter describes the recording and analysis of vaulting and tumbling performances which are used to obtain kinematic data for the evaluation of the 5 segment tumbling and vaulting models developed in Chapter 3 and to identify key aspects of each performance. The kinematic data required for the evaluation of the two segment model of vaulting was taken from a previous study (Yeadon et al., in press).

The new kinematic data from the image analysis were required for the following:

- To obtain realistic values for the initial input to the two simulation models. Input was required as the wrists made contact with the horse (vaulting model) and as the toes made contact with the tumbling track (tumbling model).
- To measure the movement of the contact surfaces for both performances.
- To measure the movement at the shoulder during contact with the vaulting horse so that the spring parameters for the shoulder in the vaulting model could be determined by comparing the actual movement at the shoulders with the simulated movement for different spring parameter values.
- To obtain kinematic data on actual performances by the subject so that the two simulation models could be evaluated by comparing the output of the model with actual performances.

An overall description of the vaulting and tumbling performances is given in this chapter with the salient features of each performance highlighted. Full details of the kinematic data obtained on the vaulting and tumbling performances are documented in Appendix B. The descriptions and highlighted features of each performance are investigated and discussed in Chapters 6 and 7.

Method

This section describes the procedures used to record and analyse the Hecht vault and a series of tumbling movements. The recordings were made on day 1 of a two day data collection, where on day 2 the subject’s strength was tested using an isokinetic dynamometer (Chapter 5).
Data collection

Three video cameras and one high speed 16 mm film camera were used to record Hecht vaults and tumbling movements at Lilleshall National Sports Centre on 26/7/95. The subject was ranked in the top 10 for Great Britain at artistic gymnastics and trained full-time at Lilleshall. Additional lighting was provided by two Lanebeam 800W quartz colour spotlights.

Camera set up

Three video cameras [Sony Hi8 Handycam PRO CCD-VXIE (Small Sony), Sony Hi8 Camcorder EVW-300P (Large Sony) and Panasonic F15 (F15)] and a 16 mm film camera [Locam II 16 mm high speed (Locam)] were used to record all performances. The cameras were set up with fields of view which could be used for recording the vault and the tumbling performances (Figure 4.1). Details of the camera set up for each camera are given in Table 4.1.

<table>
<thead>
<tr>
<th>Camera</th>
<th>Field of View [m]</th>
<th>Frame Rate [Hz]</th>
<th>Exposure Time [s]</th>
<th>Recording Medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Sony</td>
<td>5</td>
<td>50</td>
<td>1/300</td>
<td>Hi8 tape</td>
</tr>
<tr>
<td>Large Sony</td>
<td>6</td>
<td>50</td>
<td>1/250</td>
<td>Hi8 tape</td>
</tr>
<tr>
<td>F15</td>
<td>6</td>
<td>50</td>
<td>1/250</td>
<td>SVHS tape</td>
</tr>
<tr>
<td>Locam</td>
<td>5</td>
<td>200</td>
<td>1/600</td>
<td>16 mm VNF film</td>
</tr>
</tbody>
</table>

The Small Sony was positioned in the balcony perpendicular to the vault runway covering the whole of the vault and tumbling performances (Figure 4.1). The Large Sony, F15 and Locam cameras were positioned in the balcony parallel to the vault runway and between them covered the hurdle step of the vault / flic-flac of the tumbling movement through to landing (Figure 4.1). The Small and Large Sony cameras recorded directly onto Hi8 tape, the F15 recorded onto SVHS video tape with a portable recorder (Panasonic AG7450) powered by an AC-Adapter (AG B640) and the Locam recorded onto 16 mm colour film (Kodak Ektachrome VNF 7250, ASA 400).
Locam recording frequency

The nominal recording frequency for the Locam was 200 Hz. To calculate the actual recording frequency of the Locam a timing light box (Darlington decade counter, DDC) was filmed after the data collection consisting of 4 columns of light emitting diode (LED) lights displaying the time to 1 ms. The recording of the DDC showed that the average recording frequency over a four second period after the collection of the movement data was 200.6 Hz.

Genlock

The Small Sony was genlocked to the Large Sony and the F15 by splitting the genlock signal from the Small Sony to the other two video cameras. To check that the three cameras would genlock together the DDC was used before the data collection at Lilleshall in the laboratory at Loughborough. When all three cameras were set to the same exposure time (frame rate constant at 50 Hz) the view from each camera had the same LED's lit on the DDC. When the cameras were switched on at Lilleshall for the data collection session with the same set-up the genlock light did not come on for the F15 camera; the genlock cables were left connected and the genlock system was subsequently re-tested at Loughborough after the data collection session. Repeating the experiment with the DDC the two Sony Hi8 cameras genlocked but the F15 did not. It was therefore assumed that the Small Sony and Large Sony were genlocked during the data collection but the F15 was not genlocked with the Small Sony.

Figure 4.1. Locations and field of views of the cameras.
Calibration set up

13 calibration poles were used to calibrate the movement space for the vault and tumbling. Each pole was 2.20 m high with three markers on each pole located at 0.10 m, 1.10 m and 2.10 m measured from the bottom of the pole with a three point base used to erect each pole vertically. The height of each base was set to 0.03 m above floor level before the calibration.

The calibration poles were placed where possible around the vault and tumbling areas so that the same calibration set up could be used for the vault and the tumbling performances (Figure 4.2). The 3D locations of the calibration poles relative to an origin (0,0) on the floor (Figure 4.2) were measured twice independently using steel tapes (Table 4.2). The calibration poles would only stand on a firm surface therefore it was not possible to stand the calibration poles evenly throughout the movement space due to the location of the runways and mats. Each camera view had a different number of calibration poles visible (Table 4.3) and pole 13 (Figure 4.2) was only recorded by the Locam camera. Only the calibration points visible from both cameras were used in the calibration of each camera view. The exception was for the pairing of the Small Sony and Locam cameras, where it was necessary to include pole 3 for the Small Sony and pole 13 for the Locam (which were not visible from the other camera view) to ensure that the calibration volume for the tumbling encompassed the whole movement space.

Figure 4.2. Locations of the calibration poles.
Table 4.2. 3D locations of calibration poles

<table>
<thead>
<tr>
<th>calibration pole: b - bottom, m - middle, t - top</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (b)</td>
<td>0.000</td>
<td>3.000</td>
<td>0.155</td>
</tr>
<tr>
<td>2 (b, m, t)</td>
<td>0.625</td>
<td>2.600</td>
<td>0.130, 1.130, 2.130</td>
</tr>
<tr>
<td>3 (m, t)</td>
<td>3.500</td>
<td>0.870</td>
<td>1.130, 2.130</td>
</tr>
<tr>
<td>4 (b, m, t)</td>
<td>0.000</td>
<td>0.174</td>
<td>0.130, 1.130, 2.130</td>
</tr>
<tr>
<td>5 (b, m, t)</td>
<td>0.400</td>
<td>-0.100</td>
<td>0.130, 1.130, 2.130</td>
</tr>
<tr>
<td>6 (m, t)</td>
<td>-0.400</td>
<td>-0.100</td>
<td>1.130, 2.130</td>
</tr>
<tr>
<td>7 (b, t)</td>
<td>0.400</td>
<td>-1.565</td>
<td>0.130, 2.130</td>
</tr>
<tr>
<td>8 (b, m, t)</td>
<td>-0.400</td>
<td>-1.565</td>
<td>0.130, 1.130, 2.130</td>
</tr>
<tr>
<td>9 (m, t)</td>
<td>3.500</td>
<td>-1.630</td>
<td>1.130, 2.130</td>
</tr>
<tr>
<td>10 (b, t)</td>
<td>0.000</td>
<td>-1.865</td>
<td>0.180, 2.180</td>
</tr>
<tr>
<td>11 (b, t)</td>
<td>-1.535</td>
<td>-3.975</td>
<td>0.180, 2.180</td>
</tr>
<tr>
<td>12 (b, t)</td>
<td>5.700</td>
<td>-8.330</td>
<td>0.130, 2.130</td>
</tr>
<tr>
<td>13 (m, t)</td>
<td>3.500</td>
<td>-0.130</td>
<td>1.130, 2.130</td>
</tr>
</tbody>
</table>

Table 4.3. Calibration poles visible from each camera

<table>
<thead>
<tr>
<th>camera</th>
<th>calibration poles</th>
<th>number of calibration points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Sony [SS]</td>
<td>1 - 11</td>
<td>22</td>
</tr>
<tr>
<td>Large Sony [LS]</td>
<td>1 - 10</td>
<td>20</td>
</tr>
<tr>
<td>F15</td>
<td>4 - 12</td>
<td>19</td>
</tr>
<tr>
<td>Locam</td>
<td>4 - 10, 13</td>
<td>16</td>
</tr>
</tbody>
</table>

Data recording

The subject performed 12 handspring vaults prior to performing 3 Hecht vaults, 2 layout back somersaults and 3 double layout back somersaults. The 12 handspring vaults were required for a different study and are not analysed here.
Data processing

All the video recordings were copied onto SVHS tape with time code added (VITC, IMP Electronics 9000). The 16 mm film was developed and pushed ½ a stop during the processing.

Video / 16 mm film digitisation

The video was digitised using the (Target software) Apex high resolution digitising system (Kerwin, 1995) and the 16 mm film was projected onto a high resolution tablet digitiser (TDS HR48). Every video field and cine frame were digitised throughout each movement sequence. The digitisation process was split into two parts, giving a calibration file and a movement file.

Calibration file

The calibration file from each camera view consisted of the digitised locations of the calibration poles and two control points in five consecutive frames. These digitised locations were used to calibrate each camera view and to correct for camera movement and focal length changes between the recording of the calibration poles and the movement sequences.

Movement files

Each movement file consisted of two digitised reference frames followed by the digitised movement sequence. The two reference frames contained the digitised locations of key points in the field of view and the two control points. The movement sequence contained the digitised locations of 15 points on the body and the two control points. The 15 points on the body were the wrist, elbow, shoulder, hip, knee, ankle and toe on the left and right sides of the body plus the centre of the head. For each camera view the movement sequence consisted of every frame where the whole body was visible. The digitised locations of the two control points in the reference frames were used to correct for focal length changes. The digitised locations of the two control points in the movement sequence were used to correct for camera movement during the recording of each movement sequence.
Corrections for camera movement and focal length changes

Corrections were made for camera movement and focal length changes between the recordings of the calibration and the movement. These corrections were based on the digitised locations of the two control points in the calibration and movement files:

**Calibration file**

Two control points were digitised in the five calibration frames for each calibration file. The average horizontal and vertical digitiser coordinates of the two control points were calculated (equation (4.1)) along with the average resultant distance $r_0$ between them (equation (4.2)). The average horizontal and vertical digitiser coordinates of the two control points were then averaged to give the location of the average control point $(x_0, z_0)$ for each camera view (equation (4.3)).

$$x_i = \frac{\sum_{j=1}^{5} x_{ji}}{5} ; \quad z_i = \frac{\sum_{j=1}^{5} z_{ji}}{5} \quad (i = 1, 2)$$  

$$r_0 = \frac{\sum_{j=1}^{5} \sqrt{(x_{j1} - x_{j2})^2 + (z_{j1} - z_{j2})^2}}{5}$$  

$$x_0 = \frac{x_1 + x_2}{2} \quad z_0 = \frac{z_1 + z_2}{2}$$  

Each digitised point $(x, z)$ in the calibration file was translated so that the origin was at the centre of the image and was scaled so that the horizontal field of view was 10 units (equation (4.4)). For the Target video digitiser with an overall resolution of 12288 x 9216 (768x16, 576x16) the image centre $(x_{or}, z_{or})$ was known (6144, 4608). However for the 16 mm film digitiser an approximate centre of the image (800, 400) was calculated from the range of data obtained for the digitised coordinates of the calibration poles. The scale factors $(sf)$ for the video and 16 mm film were 1228.8 (1536x8/10) and 100 respectively based upon a 10 unit horizontal field of view (pseudo metres).

$$X = \frac{(x - x_{or})}{sf} \quad Z = \frac{(z - z_{or})}{sf}$$  

$$
**Movement files**

For the video recordings an average control point location \((x_1, z_1)\) and resultant distance \(r_l\) were calculated from the two reference fields in each movement file. For the Locam recordings the average control point location \((x_1, z_1)\) was calculated in each frame, and an average resultant distance \(r_l\) over all the digitised movement frames.

Camera movement corrections were made based upon the difference between the average control point location from the calibration and movement files. Focal length corrections were made based upon the ratio of \(r_0\) to \(r_l\) (equation (4.5)). If the ratio of \(r_0\) to \(r_l\) was less than 1.005 no scaling correction was made.

The overall corrected coordinates \(x_{ji}\) were obtained using the transformation:

\[
x_{ji} = \frac{(x_{ji} - x_1)r_0}{r_l} + x_0 \quad z_{ji} = \frac{(z_{ji} - z_1)r_0}{r_l} + z_0
\]

\((j = 1, \text{nf}; i = 1, \text{np})\) (4.5)

\(\text{nf} = \text{number of fields digitised} \quad \text{np} = \text{number of points digitised per field}\)

For the video recordings small corrections for camera movement and no corrections for focal length changes were made. For the Locam recordings both camera movement and focal length changes were corrected for (Table 4.4). The focal length correction for the Locam increased from the first trial recorded to the last, which suggests that the camera focal length was changing slowly due to vibrations. To prevent the focal length changing the camera could have been taped at a fixed focal length prior to recording. After the corrections for camera movement and focal length changes had been made the digitised movement files were transformed in the same way as the digitised calibration files were transformed using equation (4.4).

<table>
<thead>
<tr>
<th>Camera</th>
<th>Camera movement [pseudo metres]</th>
<th>focal length corrections [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>horizontal</td>
<td>vertical</td>
</tr>
<tr>
<td>Small Sony</td>
<td>0.010</td>
<td>0.003</td>
</tr>
<tr>
<td>Large Sony</td>
<td>0.012</td>
<td>0.022</td>
</tr>
<tr>
<td>F15</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td>Locam</td>
<td>0.017</td>
<td>0.136</td>
</tr>
</tbody>
</table>

Table 4.4. Average camera movement and focal length corrections
Corrections for camera movement during recordings

To check for image movement (wobble) of the digitised movement sequences the vertical locations of the two control points were inspected to see if there was any systematic movement of the control points. For all the digitised movement sequences no systematic movement of the control points was found.

Fitting splines to data

It was necessary to fit splines to allow the digitised data from different cameras to be synchronised, reduce the propagation of errors, and calculate velocities. Quintic splines (Wood and Jennings, 1979) were fitted to the digitised movement data, and the 3D reconstructed coordinates. Standard error estimates for each point fitted were required. The error estimates ($e_{ui}$ and $e_{vi}$) were calculated from the difference between the real coordinates and a pseudo data set generated by averaging coordinate values from adjacent frames (equation (4.6)).

$$u_i^* = \frac{u_{i-1} + u_{i+1}}{2}, \quad v_i^* = \frac{v_{i-1} + v_{i+1}}{2}$$

(4.6)

where $(u_i, v_i), (u_i', v_i') =$ real and pseudo coordinates at time $i$

$i =$ frame number

From the real and pseudo data time histories the error estimate in each real coordinate was calculated as a combination of the local error variance (equation (4.7)) and the global error variance (equation (4.8)).

$$\sigma_{ui}^2 = \frac{2}{3} (u_i - u_i')^2, \quad \sigma_{vi}^2 = \frac{2}{3} (v_i - v_i')^2 \quad i = 1, n$$

(4.7)

$$\sigma_{ug}^2 = \frac{1}{n} \sum_{i=1}^{n} \sigma_{ui}^2, \quad \sigma_{vg}^2 = \frac{1}{n} \sum_{i=1}^{n} \sigma_{vi}^2 \quad n =$ no. of frames

(4.8)

The constant $2/3$ in the calculation of the local error variance (equation (4.7)) was required to take into account the reduction in the variance between a real and pseudo data sets compared with two independent data sets. The statistics underlying the calculation of the error variance in the real data set from the difference between the real and pseudo data sets was calculated as follows:
Let $x_1$ and $x_2$ be real and pseudo data sets where $x_2$ is calculated from averaging the data sets $y_1$ and $y_2$ in adjacent frames. Assume the data sets $x_1$, $y_1$ and $y_2$ each have equal variance $\sigma^2$ then the variance in the pseudo data set $x_2$ is calculated as follows:

$$\text{var}(x_1) = \sigma^2, \quad \text{var}(y_1) = \sigma^2, \quad \text{var}(y_2) = \sigma^2, \quad x_2 = \frac{(y_1 + y_2)}{2}$$

$$\text{var}(x_2) = \frac{\text{var}(y_1) + \text{var}(y_2)}{4} = \frac{\sigma^2 + \sigma^2}{4} = \frac{\sigma^2}{2}$$

The variance in the difference $d$ between the real and pseudo data sets $x_1$ and $x_2$ can then be expressed as:

$$\text{var}(d) = \text{var}(x_1 - x_2) = \text{var}(x_1) + \text{var}(x_2) = \sigma^2 + \frac{\sigma^2}{2} = \frac{3\sigma^2}{2}$$

Therefore the error estimate calculated from the difference between the real and pseudo data sets has standard deviation:

$$\sigma = \sqrt{\frac{2}{3} \text{var}(d)} \quad \text{where} \quad \text{var}(d) = (u_i - u_i')^2; \quad \text{var}(d) = (v_i - v_i')^2$$

There was a danger of over-fitting the data at the moment of contact and takeoff, due to the rapid accelerations of the points on the body contacting the horse / tumbling track. The error variances were adjusted to prevent over smoothing by changing the calculation of the pseudo data sets for the wrists, toes and ankles at the moment of contact and takeoff with the horse / tumbling track. The pseudo data sets were usually calculated as the average of points in adjacent frames (equation (4.6)). At the moments of contact and takeoff, if the difference between the real and pseudo data values was too large, the pseudo data value was replaced with the real value in the frame after contact and the real value in the frame before takeoff respectively. Reducing the difference between the real and pseudo data values at the moment of contact and takeoff reduced the error variance and the danger of over-smoothing the data values.

The calculation of the local and global error variance allowed the calculation of the standard error estimates ($e_{ui}$ and $e_{vi}$) for each point. The standard error estimates for the digitised movement data were calculated as 75% local error and 25% global error
For the 3D reconstructed coordinates error estimates were calculated as 50% local error and 50% global error ($k = 0.50$; equation (4.9)).

$$e_{ui}^2 = k\sigma_{ui}^2 + (1-k)\sigma_{ug}^2 \quad e_{vi}^2 = k\sigma_{vi}^2 + (1-k)\sigma_{vg}^2 \quad i=1, n; 0 < k < 1 \quad (4.9)$$

For the digitised movement data a $k$ value of 0.75 was chosen as it was expected that there would be some large errors in the data to be smoothed and the 25% global error prevented the error estimate at any point from being too small. In addition the digitised movement data were fitted 10 times tighter than the calculated error estimates would predict by changing the $s$ value from $s = 1.00n$ to $s = 0.01n$ in the quintic spline routine. This effectively meant that a interpolating spline was fitted with 90% of the error in the original digitised data preserved, which was necessary to prevent over smoothing of the reconstructed 3D coordinates. Setting $s = 0.01n$ was preferred to fitting an interpolating spline ($s = 0.00n$), as the output was very similar and more stable.

For the 3D reconstructed coordinates a $k$ value of 0.50 was chosen with an $s$ value of 1.00n. The 50% global error prevented the error estimates from being smaller than 25% of the global error, and the 50% local error gave sufficient flexibility for the splines to smooth the data where the local error estimates were large.

Calibration of each camera view

Each camera view was calibrated separately by solving for the DLT parameters in the method of Abdel-Aziz and Karara (1971). For the video cameras a 12 parameter DLT reconstruction was used which included a correction for symmetrical lens distortion (equations (4.11) and (4.12)). For the Locam camera an 11 parameter DLT reconstruction was used (equation (4.10)). The 12 parameter reconstruction was preferred for the video recordings due to the radial distortion in video systems (Tan, 1997).

$$
\begin{align*}
\mathbf{u} &= \frac{L_1x + L_2y + L_3z + L_4}{L_9x + L_{10}y + L_{11}z + 1} \\
\mathbf{v} &= \frac{L_5x + L_6y + L_7z + L_8}{L_9x + L_{10}y + L_{11}z + 1} \\
\end{align*}
\quad (4.10)
$$

where \((u, v)\) were the digitised locations

\((x, y, z)\) were the 3D locations of the digitised points

$L_1 - L_{11}$ were the DLT parameters

$$
\begin{align*}
\mathbf{u}' &= \frac{L_1x + L_2y + L_3z + L_4}{L_9x + L_{10}y + L_{11}z + 1} \\
\mathbf{v}' &= \frac{L_5x + L_6y + L_7z + L_8}{L_9x + L_{10}y + L_{11}z + 1} \\
\end{align*}
\quad (4.11)
$$
where $u'$ and $v'$ are the undistorted digitised coordinates and:

\[
\begin{align*}
    u' &= u + \Delta u \\
    &= u + (u - u_0) r^2 L_{12} \\
    v' &= v + \Delta v \\
    &= v + (v - v_0) r^2 L_{12}
\end{align*}
\]

(4.12)

where $u, v$ are the distorted digitiser coordinates

$u_0, v_0$ is the centre of the image which is at $(0, 0)$ as the digitiser values have been translated to the centre of the image

$r$ = distance between digitised point and the centre of the lens

$L_{12}$ = lens distortion parameter

To calculate the DLT parameters the digitised locations of the calibration points, and their corresponding known $(x, y, z)$ locations were substituted into equations ((4.10) / (4.11) and (4.12)), and solved using a linear least squares equation solver (Stewart, 1973). For the 11 parameter equations the least squares solution was found directly, however the 12 parameter equations were non-linear and had to be solved with an iterative approach using the linear equation solver. The first iteration solved for the 12 DLT parameters starting with a zero guess (no lens correction) for the 12th DLT parameter, the values for $u'$ and $v'$ were then adjusted using equation (4.12) and the 12 parameters solved for again. Five iterations were required for the solution to converge for the 12 DLT parameters.

3D reconstruction of digitised points

The four cameras used allowed the 3D locations of digitised points throughout the vaulting and tumbling performances to be calculated. The Small Sony camera was positioned at approximately $90^\circ$ to the three side cameras in the balcony perpendicular to the vault runway (Figure 4.1). The Small Sony covered the whole of the vault / tumbling performances and each side camera covered part of the performances. The Small Sony was paired with each of the side cameras to obtain 3D locations of digitised points throughout the vault / tumbling performances. The reconstructed 3D locations of both the digitised calibration points and the body points were determined. For the digitised calibration points there was no problem of synchronising the camera views but for the digitised movement data the three side cameras needed to be synchronised with the Small Sony camera.
Reconstruction of the calibration points

To reconstruct the 3D location of each digitised calibration point the DLT equations (4.11) were rearranged to give two equations for each camera view relating the 3D location of each digitised point and the digitised coordinates (equation (4.13)). For the 16 mm film camera the 12\textsuperscript{th} parameter which corrects for radial distortion was set to zero.

\[
\begin{bmatrix}
L_1 - L_9 u'_1 & L_2 - L_{10} u'_1 & L_3 - L_{11} u'_1 \\
L_5 - L_9 v'_1 & L_6 - L_{10} v'_1 & L_7 - L_{11} v'_1 \\
L_1 - L_9 u'_2 & L_2 - L_{10} u'_2 & L_3 - L_{11} u'_2 \\
L_5 - L_9 v'_2 & L_6 - L_{10} v'_2 & L_7 - L_{11} v'_2
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
u'_1 - L_4 \\
v'_1 - L_8 \\
u'_2 - L_4 \\
v'_2 - L_8
\end{bmatrix}
\] (4.13)

where \( u'_1 = u_1 + u_1 r_1^2 L_{12} \)
\( v'_1 = v_1 + v_1 r_1^2 L_{12} \)
\( r_1^2 = u_1^2 + v_1^2 \)
\( u'_2 = u_2 + u_2 r_2^2 L_{12} \)
\( v'_2 = v_2 + v_2 r_2^2 L_{12} \)
\( r_2^2 = u_2^2 + v_2^2 \)

where \( u_1, v_1; (i=1, 2) \) are the digitised coordinates from the first and second cameras.

Equation (4.13) gives four equations for three unknowns \((x, y, z)\). The equations were normalised so that the right hand side of equation (4.13) corresponded to the distance of each of the four planes from \((0,0,0)\) and multiplied by a weighting factor to allow each equation to have a specified different weighting (equation (4.14)). The 3D location of each digitised point was then calculated from the normalised version of equation (4.13) using a least squares technique (Stewart, 1973). Comparing the known location of each point with its reconstructed location gave an estimate of the reconstruction accuracy.

\[
a_{i1} = \frac{a_{i1} w_i}{m_i}; \quad a_{i2} = \frac{a_{i2} w_i}{m_i}; \quad a_{i3} = \frac{a_{i3} w_i}{m_i}; \quad b_i = \frac{b_i w_i}{m_i}
\] (4.14)

where \( a_{ij} = x, y, z \) coefficients \((j=1, 3)\) for each equation in (4.13)
\( b_i = \) right hand side of each equation in (4.13)
\( i = 1, 4 \)
\( w_i = \) weighting for equation \( i \)
Synchronisation of digitised movement data

The times of contact and takeoff from each camera view were identified. This allowed the Small Sony and Large Sony to be synchronised as these two cameras were genlocked. For the F15 and Locam cameras identifying common events in each camera view only synchronised these cameras with the Small Sony to the nearest field / frame as these cameras were not genlocked.

To synchronise the F15 and Locam cameras with the Small Sony, the normalised version of equation (4.13) with equal weightings for the four equations in the matrix was used to calculate the time offset between digitised sequences of the same performance from different camera views.

The least squares solution to the normalised version of equation (4.13) resulted in the 3D location which was closest to the four planes by minimising the sum of squares of the residual distances (equation (4.15)).

\[
\text{minimise } [b_1^2 + b_2^2 + b_3^2 + b_4^2] \quad (4.15)
\]

where \(b_1, b_2, b_3, b_4\) are the residual distances to the four equations in equation (4.13).

The RMS distance \(d\) of each 3D location from the four planes was calculated (equation (4.16)):

\[
d = \sqrt{\frac{b_1^2 + b_2^2 + b_3^2 + b_4^2}{4}} \quad [b_i = \text{distance from each plane}] \quad (4.16)
\]

The RMS distance was calculated for each point digitised in each field throughout the whole movement. To obtain an overall RMS error estimate \(D\) for each trial over all the points and fields the global RMS distance was calculated (equation (4.17)).

\[
D = \sqrt{\frac{\sum_{j=1}^{n_f} \sum_{i=1}^{15} d_{ij}^2}{15n_f}} \quad (4.17)
\]

where \(n_f = \text{number of fields / frames}\)
\(15 = \text{number of points digitised on the body in each field}\)

The global RMS distance represented an overall error estimate of the reconstruction accuracy of the digitised body landmarks and was affected by a number of factors including digitisation errors, lens distortion and synchronising errors.
The global RMS distance was expected to be smallest when the digitised data sets were correctly synchronised since the other errors would be the same for different time offsets. The different digitised data sets were therefore synchronised by varying the time offset between the two data sets until the global RMS distance was minimised.

To minimise the global RMS distance 11 estimates were obtained for the global RMS distance using different time offsets. The 11 time offsets were firstly selected around the estimated time offset at a time interval equivalent to the sampling frequency (200 Hz for Locam; 50 Hz for video). The time interval was then reduced around the minimum point of the global RMS distance by a factor of five so that the new 11 time offsets spanned one frame either side of the estimated minimum. A least squares quadratic was fitted to the 11 estimates of the global RMS errors and the minimum point of the quadratic function calculated to give the offset between the time scales of the two digitised data sets.

To interpolate the digitised data at the same frequency and vary the time offset between the digitised data from each camera view a quintic spline as previously described \((s = 0.01; k = 0.75)\) was fitted to the digitised coordinates and interpolated at the required frequency.

The method presented for synchronising digitised movement sequences was evaluated on the digitised data from the Small Sony and Large Sony video cameras. These two video cameras were genlocked together which allowed the actual time offset to be calculated from identifying the times of contact and takeoff from both camera views.

**Movement data**

For the reconstruction of the digitised data from the Small Sony with the Large Sony and F15 the normalised version of equation (4.13) with equal weightings for all equations in the matrix was used. For the reconstruction of the digitised data from the Small Sony with the Locam, it was necessary to interpolate the quintic spline fitted to the video digitised data to give four times as much information as was digitised. To take into account the reduced amount of digitised data from the video recording the weightings for the two equations involving the video data in equation (4.13) were reduced to 0.25 the weighting of the film data. In addition the vertical component of the video camera data had greater movement and variability and therefore the weighting was reduced to 0.09 for that equation to give the best estimates of the 3D locations. The reduced amount of
information available from the Small Sony camera at 50 Hz should not be a great limitation as the gymnast is travelling towards the Small Sony. The digitised body landmarks are therefore not changing very rapidly in the Small Sony view so the interpolation to give four times as much information as was digitised may not be too inaccurate.

**Data required**

The mass centre location and velocity, the movement at the shoulders during horse contact, the depression of the horse / tumbling track, and the orientation and configuration angles were required. The mass centre location and velocity were calculated from the 3D locations of all 15 points digitised on the body. The movement at the shoulders during horse contact was calculated from the average wrist, shoulder, and hip joint centre locations along with the centre of the head location. The depression of the horse / tumbling track, and the orientation and configuration angles were calculated from the digitised locations of one side of the body as it was only possible to see one side of the body clearly throughout each performance from the side cameras. For the vault the right hand side of the body was used and for the tumbling the left hand side of the body was used.

The following section will describe the procedures used to calculate the required kinematic data for the vault and tumbling performances from the reconstructed 3D locations of the digitised body landmarks.

**Time base for each trial**

Once each trial had been synchronised it was possible to use the same time base for all the recordings of each trial. The calculated time offset translated the time base for the Small Sony onto the time base of each of the second cameras which resulted in a different time base for each pair of cameras used for a given performance. To obtain the same time base, all the recordings of each trial were translated onto the time base used for the 200 Hz film data (equation (4.18)). In addition all time bases were translated so that the contact of the fingers with the horse / final contact of the toes with the track corresponded to time zero for each trial (equation (4.19)).
\[ t_{n^*} = t_0 + \frac{(\text{sync}_{c} - \text{sync}_s)}{50} \]  

\[ t_{n^*} = t_{n^*} - t_{c0} \]  

where \( * \) = the \( s \) second (Large Sony), \( t \) third (F15) and \( c \) cine (Locam) cameras

\( t_0 \) = old time base

\( t_{n^*} \) = new time base

\( \text{sync}_s \) = time offset for each camera synchronised with the Small Sony

\( t_{c0} \) = time offset used to translate the time base for the cine camera so that the contact of the fingers with the horse / final contact of the toes with the track corresponded to time zero

Times of contact and takeoff

Seven times of contact and takeoff were identified for each trial (Table 4.5). To identify these times throughout each digitised sequence, the displacement of the wrists, toes and ankles were used.

The times of contact with the horse and foot contact with the tumbling track were identified to the nearest frame from the 200 Hz film with video data. The times of contact with the Reuther board / hand contact with the tumbling track and landing were identified to the nearest field from the video recordings of the Small Sony, Large Sony and F15.

The values of the required kinematic variables for each trial were then determined at the seven times identified.

Location and velocity of the mass centre

Table 4.6 shows the camera views used to determine the mass centre velocity during the vault and the tumbling performances. In each field / frame the mass centre location was determined from the digitised locations of the joint centres and the inertia data obtained on the subject from anthropometric measurements and the inertia model of Yeadon (1990b); (see Chapter 5). For the vault the mass centre locations were translated so that the origin was a point on the floor below the midpoint of the wrists at the start of contact with the horse, such that the midpoint had the location \((0.00 \, \text{m}, 1.35 \, \text{m})\) for each
vault. For the tumbling performances the origin was set at the midpoint of the toes at the start of the foot contact phase with the tumbling track.

The velocity of the mass centre was calculated from the known locations of the mass centre using projectile equations of motion. In all tumbling sequences the flight phase prior to the hands contacting the track was not in the field of view of the side cameras and so it was not possible to calculate the mass centre velocity as the hands contacted the tumbling track. The horizontal velocities of the mass centre during preflight and postflight were assumed to be constant and were calculated from the digitised data from the Small Sony and 16 mm Locam.

Table 4.5. Times identified throughout each performance

<table>
<thead>
<tr>
<th>vault</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td>contact of the toes with the Reuther board</td>
<td>contact of the fingers with the track</td>
</tr>
<tr>
<td>takeoff of the toes from the Reuther board</td>
<td>takeoff of the fingers from the track</td>
</tr>
<tr>
<td>contact of the fingers with the vaulting horse</td>
<td>contact of the toes with the track</td>
</tr>
<tr>
<td>contact of the wrists with the vaulting horse</td>
<td>contact of the heel with the track</td>
</tr>
<tr>
<td>takeoff of the wrists from the vaulting horse</td>
<td>takeoff of the heel from the track</td>
</tr>
<tr>
<td>takeoff of the fingers from the vaulting horse</td>
<td>takeoff of the toes from the track</td>
</tr>
<tr>
<td>landing of the toes on the mats</td>
<td>landing of the toes on the mats</td>
</tr>
</tbody>
</table>

Table 4.6. Camera views used to determine the mass centre location at each time

<table>
<thead>
<tr>
<th>vault</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td>approach</td>
<td>Small Sony with Large Sony</td>
</tr>
<tr>
<td>board / track takeoff</td>
<td>Small Sony with Large Sony</td>
</tr>
<tr>
<td>horse / track contact</td>
<td>Small Sony with Locam</td>
</tr>
<tr>
<td>horse / track takeoff</td>
<td>Small Sony with Locam</td>
</tr>
<tr>
<td>landing</td>
<td>Small Sony with F15</td>
</tr>
</tbody>
</table>
Shoulder movement during horse contact

The movement at the shoulders in line with the arms $dSS'$ and the body $dS'B$ were calculated along with the distance from the wrists to the shoulders $dWS'$ (Figure 4.3). The distance $dWS'$ was calculated as the distance between the digitised wrist $W$ and shoulder $S$ locations $dWS$ plus the distance $dSS'$. The distance $dSS'$ was calculated as the distance from the digitised shoulder location $S$ to a point $S'$ where the point $S'$ was calculated as the intersection of the wrist to shoulder line with the head to hip line. The distance $dS'B$ was calculated as the distance from the point $S'$ to a point $B$ where the point $B$ was calculated as the mid point of the head $H$ to hip $O$ line. Quintic splines were fitted to the three distances in order to reduce the propagated digitising errors as previously described.

Figure 4.3. Calculation of the shoulder movement during horse contact.

Depression of tumbling track / vaulting horse

The depression of the tumbling track and vaulting horse during the Hecht and tumbling performances were determined from the time histories of the wrists, ankles and toes during the contact phase with the tumbling track / vaulting horse. Quintic splines were fitted to the time histories of the wrists, ankles and toes with standard error estimates adjusted to take into account the contact with the horse / tumbling track as previously described.
Orientation and configuration angles / angular velocities

For the vault and tumbling performances the angles of the arms, trunk, thigh, shank and foot relative to the horizontal were required. In addition the hip and knee angles were required for the Hecht performance, these two angles were calculated from the trunk, thigh and shank angles. The time histories of various angles and angular velocities were calculated during the last part of preflight, to midway through postflight from the 3D locations at 200 Hz obtained from the Small Sony and Locam data. In addition angles and angular velocities during contact with the Reuther board / the hand contact with the tumbling track and landing were calculated from the 3D locations obtained from the Small Sony with the two video side cameras. All angles were determined from the 2D coordinates of the joint centres ignoring the effect of movement of the joint centres away from a vertical plane running parallel to the runways. The angles were calculated from the sine and cosine of each angle using the method described in Yeadon (1990a) which ensured that the angle time histories were continuous.

\[ \text{Figure 4.4. Angle definitions.} \]

To smooth the time histories of the calculated angles and to calculate angular velocities for each segment, quintic splines were fitted to the time histories of each angle as previously described. Fitting a spline to the complete time history of each angle resulted in the angular velocity estimates around contact with the horse / track being over
smoothed due the rapid changes during the impact phase. Therefore for each angle time history two additional splines were fitted, one to the preflight until contact with the horse / track and one to the postflight from takeoff through to landing.

Angular momentum

The model of Yeadon (1990c) was used to calculate the time history of the angular momentum from quintic spline coefficients fitted to the orientation and configuration angles throughout each performance. The angular momentum estimates for each digitised frame of preflight were averaged to give a better estimate of the angular momentum at contact with the horse or tumbling track. A similar procedure was used for postflight to estimate the angular momentum at takeoff from the horse or tumbling track.

Results and Discussion

This section gives the results obtained from the video and film recordings of the Hecht vault and tumbling performances, and a discussion of the salient features of each performance. The results are split into 3 sections; Error analysis and reconstruction accuracy; Description of the Hecht vault performances and Description of the tumbling performances.

Error analysis and reconstruction accuracy

_**Camera calibration and reconstruction of calibration points**_

Table 4.7 shows the 12 DLT parameters calculated for each camera. There are three sets of parameters for the Small Sony camera which was paired with each of the other cameras. The three sets of parameters for the Small Sony camera varied slightly as different combinations of calibration points were used to calibrate the Small Sony for each camera pairing (Table 4.7).

To estimate the reconstruction accuracy the locations of the digitised calibration markers were reconstructed. Average unbiased estimates of the coordinate errors of the reconstructed locations of the calibration markers were found to be 0.013 m, 0.011 m and 0.009 m in the x, y and z directions respectively (Table 4.8). The 3D reconstruction errors for the calibration points were larger than the digitising precision of landmarks on the body (Table 4.11) due to errors in the measured locations of the calibration points.
Table 4.7. 12 DLT parameters for each camera pairing

<table>
<thead>
<tr>
<th>Preflight pairing</th>
<th>Postflight pairing</th>
<th>Contact pairing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small Sony</td>
<td>Large Sony</td>
</tr>
<tr>
<td>1.002</td>
<td>0.989</td>
<td>-0.570</td>
</tr>
<tr>
<td>-0.410</td>
<td>-0.404</td>
<td>-1.285</td>
</tr>
<tr>
<td>0.118</td>
<td>0.119</td>
<td>0.098</td>
</tr>
<tr>
<td>-2.191</td>
<td>1.260</td>
<td>-2.176</td>
</tr>
<tr>
<td>0.065</td>
<td>0.065</td>
<td>0.212</td>
</tr>
<tr>
<td>0.441</td>
<td>-0.162</td>
<td>0.445</td>
</tr>
<tr>
<td>1.007</td>
<td>1.749</td>
<td>0.999</td>
</tr>
<tr>
<td>-0.556</td>
<td>-2.631</td>
<td>-0.542</td>
</tr>
<tr>
<td>0.019</td>
<td>0.042</td>
<td>0.018</td>
</tr>
<tr>
<td>0.050</td>
<td>-0.017</td>
<td>0.053</td>
</tr>
<tr>
<td>-0.020</td>
<td>-0.006</td>
<td>-0.021</td>
</tr>
<tr>
<td>-0.000</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

Table 4.8. Average 3D reconstruction errors for the calibration markers

<table>
<thead>
<tr>
<th>camera combination</th>
<th>no. of markers</th>
<th>x error [m]</th>
<th>y error [m]</th>
<th>z error [m]</th>
<th>mean error [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Sony and Large Sony</td>
<td>19</td>
<td>0.0133</td>
<td>0.0108</td>
<td>0.0080</td>
<td>0.0109</td>
</tr>
<tr>
<td>Small Sony and F15</td>
<td>16</td>
<td>0.0119</td>
<td>0.0097</td>
<td>0.0082</td>
<td>0.0100</td>
</tr>
<tr>
<td>Small Sony and Locam</td>
<td>12</td>
<td>0.0150</td>
<td>0.0131</td>
<td>0.0106</td>
<td>0.0130</td>
</tr>
<tr>
<td>mean</td>
<td>16</td>
<td>0.0134</td>
<td>0.0112</td>
<td>0.0089</td>
<td>0.0113</td>
</tr>
</tbody>
</table>

Evaluation of the synchronisation method

The error in the synchronisation method was evaluated for the digitised movement data from the Small and Large Sony cameras which were genlocked together (Table 4.9).
The maximum error in the synchronisation method was 0.0036 s for the third Hecht trial (h3). This corresponds to a synchronisation to within 1/5 of the sampling interval. The average synchronisation error over the eight trials was 0.00083 s which corresponds to a synchronisation to within 1/24 of the sampling interval.

Table 4.9. Errors in the synchronisation method for the Small and Large Sony cameras

<table>
<thead>
<tr>
<th>trial</th>
<th>actual offset [fields]</th>
<th>calculated offset [fields]</th>
<th>sync error [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>h1</td>
<td>-2.00</td>
<td>-2.034</td>
<td>0.00068</td>
</tr>
<tr>
<td>h2</td>
<td>-2.00</td>
<td>-2.035</td>
<td>0.00070</td>
</tr>
<tr>
<td>h3</td>
<td>-3.00</td>
<td>-3.180</td>
<td>0.00360</td>
</tr>
<tr>
<td>t1</td>
<td>0.00</td>
<td>0.009</td>
<td>0.00018</td>
</tr>
<tr>
<td>t2</td>
<td>0.00</td>
<td>-0.040</td>
<td>0.00080</td>
</tr>
<tr>
<td>t3</td>
<td>-1.00</td>
<td>-1.002</td>
<td>0.00004</td>
</tr>
<tr>
<td>t4</td>
<td>0.00</td>
<td>0.018</td>
<td>0.00036</td>
</tr>
<tr>
<td>t5</td>
<td>0.00</td>
<td>0.014</td>
<td>0.00028</td>
</tr>
<tr>
<td>average error</td>
<td></td>
<td></td>
<td>0.00083</td>
</tr>
</tbody>
</table>

where h1-3: three Hecht performances; t1-5: five tumbling performances

3D RMS movement error estimates

Each 3D body location was calculated as a least squares solution. The residuals from the least squares solution gave an indication of the accuracy with which each point was located (Table 4.10). The smaller RMS error estimates for the Small Sony with Locam cameras were due to the better precision of the Locam camera (Table 4.11).

Table 4.10. 3D RMS movement error estimates [m]

<table>
<thead>
<tr>
<th>3D RMS error estimate</th>
<th>Small Sony with:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Large Sony</td>
</tr>
<tr>
<td>3D RMS error estimate</td>
<td>0.012</td>
</tr>
</tbody>
</table>
Precision estimates for digitised body landmarks

Average RMS digitising precision estimates were 0.007 m and 0.008 m in the x and z directions respectively for the 15 digitised body landmarks throughout each performance (Table 4.11). The poorest precision estimates were for the side video cameras where one side of the body was obscured during each performance, and the best precision estimates were for the Locam which had the largest image size. On the body the largest precision estimates were for the elbows and knees, and smallest for the centre of the head. The difficulty in locating the elbow and knee centres was probably due to the arms and legs being straight for the majority of the Hecht and tumbling performances.

Table 4.11. Average digitising precision for the digitised body landmarks [m]

<table>
<thead>
<tr>
<th>body landmark</th>
<th>Small Sony x</th>
<th>Small Sony z</th>
<th>side video camera x</th>
<th>side video camera z</th>
<th>Locam x</th>
<th>Locam z</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>l. wrist</td>
<td>0.005</td>
<td>0.007</td>
<td>0.010</td>
<td>0.010</td>
<td>0.004</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td>l. elbow</td>
<td>0.007</td>
<td>0.009</td>
<td>0.013</td>
<td>0.013</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>l. shoulder</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.008</td>
<td>0.007</td>
<td>0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>l. hip</td>
<td>0.005</td>
<td>0.006</td>
<td>0.008</td>
<td>0.008</td>
<td>0.004</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>l. knee</td>
<td>0.008</td>
<td>0.011</td>
<td>0.011</td>
<td>0.010</td>
<td>0.008</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>l. ankle</td>
<td>0.006</td>
<td>0.011</td>
<td>0.011</td>
<td>0.012</td>
<td>0.003</td>
<td>0.003</td>
<td>0.008</td>
</tr>
<tr>
<td>l. toe</td>
<td>0.007</td>
<td>0.012</td>
<td>0.014</td>
<td>0.014</td>
<td>0.003</td>
<td>0.003</td>
<td>0.009</td>
</tr>
<tr>
<td>r. wrist</td>
<td>0.006</td>
<td>0.009</td>
<td>0.010</td>
<td>0.010</td>
<td>0.005</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>r. elbow</td>
<td>0.009</td>
<td>0.012</td>
<td>0.012</td>
<td>0.014</td>
<td>0.007</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>r. shoulder</td>
<td>0.007</td>
<td>0.008</td>
<td>0.010</td>
<td>0.010</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>r. hip</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
<td>0.008</td>
<td>0.005</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td>r. knee</td>
<td>0.008</td>
<td>0.012</td>
<td>0.013</td>
<td>0.011</td>
<td>0.008</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>r. ankle</td>
<td>0.006</td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
<td>0.003</td>
<td>0.003</td>
<td>0.007</td>
</tr>
<tr>
<td>r. toe</td>
<td>0.007</td>
<td>0.012</td>
<td>0.013</td>
<td>0.015</td>
<td>0.004</td>
<td>0.004</td>
<td>0.009</td>
</tr>
<tr>
<td>head</td>
<td>0.004</td>
<td>0.007</td>
<td>0.006</td>
<td>0.007</td>
<td>0.004</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>average</td>
<td>0.006</td>
<td>0.009</td>
<td>0.011</td>
<td>0.011</td>
<td>0.005</td>
<td>0.005</td>
<td>0.008</td>
</tr>
</tbody>
</table>
Description of the Hecht vault performances

Figure 4.5 shows the three performances of the Hecht vault which are all similar. The sequence of graphics in Figure 4.5 progress from left to right starting with the hurdle step and ending with the landing.

Figure 4.5. Computer graphics of the three Hecht vaults analysed.

The six different variables highlighted in the methods section of this chapter for the Hecht vault will be presented and discussed. Average results for the three vaults are presented in the text with full details of each trial documented in Appendix B. The key for
the graphs within this section is: a thick line represents trial 1, a dashed line represents trial 2 and a thin line represents trial 3.

**Time**

The mean durations of the preflight, horse contact and postflight phases of the Hecht vault were 0.23 s, 0.18 s, and 0.69 s respectively, giving a total duration of the vault from board takeoff to landing of 1.1 s. The horse contact phase was only 16% of the total duration of the vault and can be subdivided into the phases where the fingers/wrists are in contact with the horse. During the initial 0.01 s of the contact phase only the fingers were in contact with the horse. The wrists contacted the horse after 0.01 s and remained in contact for 0.09 s before coming away from the horse, but the fingers remained in contact for a further 0.07 s. The effect of the fingers prolonging the contact time with the horse has not been considered in the literature.

For the subsequent figures on vaulting in this section, time $= 0.00$ s corresponds to the fingers making contact with the horse, time $= 0.01$ s is the time the wrists made contact with the horse, time $\approx 0.10$ s is the time the wrists came away from the horse and time $\approx 0.17$ s is the time the fingers came away from the horse.

**Location and velocity of the mass centre**

The mass centre travelled up to 5 m horizontally from Reuther board contact to landing and reached a maximum height of 2.23 m above floor level during postflight. Figure 4.6 and Figure 4.7 show the mass centre location and velocity during the last part of preflight through to the start of postflight for the three vaults analysed. At horse contact the mass centre was 0.80 m behind the location of the hands and at horse takeoff the mass centre had travelled forwards to be approximately vertically above the contact point of the hands with the horse (Figure 4.6).

The average horizontal approach velocity of the mass centre for the Hecht vaults was $7.1 \text{ m.s}^{-1}$. During contact with the Reuther board the horizontal velocity was reduced to $6.0 \text{ m.s}^{-1}$ and the vertical velocity increased from $-1.3 \text{ m.s}^{-1}$ to $3.2 \text{ m.s}^{-1}$. During preflight the mass centre followed a parabolic flight path, the horizontal velocity was constant and the vertical velocity had decreased to $0.9 \text{ m.s}^{-1}$ as the fingers contacted the horse. Between the fingers and the wrists contacting the horse, the velocity of the mass centre changed by a maximum of $0.1 \text{ m.s}^{-1}$ (Figure 4.7). During the first half of horse
contact the horizontal velocity decreased rapidly, and the vertical velocity increased rapidly. After the wrists had lost contact the horizontal velocity remained almost constant and the vertical velocity started to decrease (Figure 4.7). At takeoff from the horse the average horizontal and vertical velocity of the mass centre were 4.2 m.s\(^{-1}\) and 2.5 m.s\(^{-1}\) when the wrists came away from the horse and 4.3 m.s\(^{-1}\) and 1.9 m.s\(^{-1}\) when the fingers came away from the horse (Figure 4.7). During postflight the horizontal velocity of the mass centre was constant and the vertical velocity reduced to -4.9 m.s\(^{-1}\) at landing.

\[\text{Figure 4.6. Horizontal [A] and vertical [B] location of the mass centre during each Hecht vault, [origin such that wrist location at horse contact is (0.00 m, 1.35 m).}\]

\[\text{Figure 4.7. Horizontal [A] and vertical velocity [B] of the mass centre during each Hecht vault.}\]

\textit{Shoulder movement during horse contact}

Figure 4.8 shows the time history of the movement at the shoulder during contact with the horse (see Figure 4.3), in line with the arms [A and B] and the body [C]. Comparing Figure 4.8 [A] and [C] shows that the majority of the movement is in line with the arms and not the body. In line with the arms the shoulders depressed up to 0.11 m and
sprang back 0.05 m before the wrists came away from the horse and 0.13 m by the end of the contact phase with the horse. In line with the body the shoulders depressed up to 0.04 m and sprang back less than 0.01 m before the wrists came away from the horse.

![Graph A](image1.png)

![Graph B](image2.png)

![Graph C](image3.png)

**Figure 4.8.** Shoulder movement in line with the arms dSS' [A], dWS' [B] and in the line of the body dS'B [C] during horse contact.

**Depression of vaulting horse**

The movement of the hand-horse interface was measured from the movement of the wrists during horse contact (Figure 4.9). The maximum depression of the wrists during horse contact was 0.021 m with an average of 0.013 m. Horizontally the wrists moved forward slightly during the initial contact of the fingers and wrist with the horse, but then remained in a fixed location until the wrists started to come off the horse top. At finger takeoff the wrists were on average 0.017 m above the horse top.
Figure 4.9. Wrist movement during horse contact: [A] horizontal, [B] vertical.

Orientation and configuration angles / angular velocities

trial 1

trial 2

trial 3

Figure 4.10. Computer graphics of the horse contact phase with a 0.5 m spacing between figures.

The configuration of the body remained in a relatively fixed orientation during preflight. At horse contact the configuration of the body started to change (Figure 4.10). The arm angle increased steadily during horse contact (see Figure 4.4 for definitions of
angles), which had the effect of closing the shoulder angle as the body remained horizontal throughout most of the horse contact phase (Figure 4.11). The knee and hip angles remained almost constant throughout the horse contact phase although the hip angle closed rapidly after horse takeoff (Figure 4.11).

During postflight the arms rotated forwards relative to the body and the body piked near landing. The total rotation of the arms during postflight was up to $494^\circ$ and the average hip angle at landing was $118^\circ$.

![Graphs of arm, trunk, hip, and knee angles](image)

Figure 4.11. Arm, trunk, hip and knee angles during each vault.

**Angular momentum**

Figure 4.12 shows the time history of the angular momentum about a transverse axis through the mass centre for each vault. The variability in the angular momentum
during preflight and postflight gives an indication of the errors in the data as during the flight phases the angular momentum should be constant.

During the time interval from 0.10 s to 0.17 s where only the fingers were in contact with the horse, the angular momentum drops on average from -2 to -15 kg.m².rad.s⁻¹, this drop in angular momentum plays an important role in the performance of the vault. Without the fingers remaining in contact with the horse, the gymnast may not be able to reduce the angular momentum about the mass centre sufficiently to produce the required backwards rotation during postflight.

![Figure 4.12. Time history of the angular momentum [kg.m².rad.s⁻¹] about a transverse axis through the mass centre for each vault.](image)

**Summary of vaulting performances**

The three Hecht vaults analysed were all similar in terms of the approach, preflight and postflight performance. The key features to take from the performances are:

- The horizontal approach velocity is high giving a high horizontal preflight velocity.
- At horse contact the mass centre has a low vertical velocity, high horizontal velocity and the body has a low angular momentum about the mass centre.
- The majority of the movement at the shoulders is along the line of the arms.
- The hand-horse interface depresses horizontally and vertically during horse contact.
- The body angle at horse contact is approximately horizontal.
- The use of the fingers extends the time of contact with the horse.
- The angular momentum changes substantially during the final part of the horse contact phase when only the fingers are in contact with the horse.
• Towards the end of the horse contact phase and during the first part of postflight the gymnast pikes the body before extending.
• To help produce backwards rotation in postflight the arms rotate forwards relative to the body and the body pikes just prior to landing.
• The gymnast lands behind the vertical to control the forwards rotation at landing.

Description of the tumbling performances

Figure 4.13 and Figure 4.14 show computer graphics sequences of the five tumbling performances. The computer graphics sequences progress from left to right starting with the hand contact with the tumbling track and ending with the landing. Trials 1 and 2 are similar single layout somersault performances, trials 3 and 4 are similar double layout somersault performances and trial 5 is a double layout somersault performance with a slightly different technique to trials 3 and 4.

The five different variables highlighted in the method section of this chapter for the tumbling performances will be presented and discussed. Graphs for trials 1, 3 and 5 are presented in the text with full details of all trials documented in Appendix B. The key for the graphs within this section is: a thick line represents trial 1, a dashed line represents trial 3 and a thin line represents trial 5.

Time

The mean durations of the preflight, contact of the feet with the tumbling track and postflight phases of the tumbling performances were found to be 0.07 s, 0.12 s, and 1.1 s respectively, giving a total duration of the performance from hand takeoff to landing of 1.3 s. During the foot contact phase with the tumbling track, the toes contacted the tumbling track first followed by the heel, the heel came away from the tumbling track first followed by the toes.

For the tumbling performances in this section, time = 0.00 s corresponds to the toes making contact with the tumbling track and time ≈ 0.12 s is the time the toes came away from the track. There were only slight differences between the times of contact and takeoff for the five trials (Appendix B).
Figure 4.13. Computer graphics sequences of two layout performances.
Figure 4.14. Computer graphics sequences of the three double layout performances.
Location and velocity of the mass centre

The movement of the mass centre during postflight varied between the performance of the single and double layout somersaults with the single layout somersaults travelling higher and further (Table 4.12). Both single layout somersaults reached similar maximum heights, as did the first two double layout somersaults, however the last double somersault performed (trial 5) was 0.2 m lower than the other two double layout somersaults. Horizontally the mass centre travelled up to 3.0 m during postflight with the single layout somersaults travelling up to 0.6 m further than the double layout somersaults (Table 4.12).

Table 4.12. Maximum height and distance travelled by the mass centre during postflight for each tumbling performance [m]

<table>
<thead>
<tr>
<th></th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>2.31</td>
<td>2.35</td>
<td>2.15</td>
<td>2.13</td>
<td>1.93</td>
</tr>
<tr>
<td>distance</td>
<td>2.99</td>
<td>3.02</td>
<td>2.61</td>
<td>2.88</td>
<td>2.39</td>
</tr>
</tbody>
</table>

Figure 4.15 shows the mass centre location during the last part of preflight through to the start of postflight. At the beginning of the foot contact phase with the tumbling track the mass centre was 0.50 m in front of the location of the toes and at the end of the takeoff the mass centre had travelled backwards to be approximately vertically above the contact point of the toes with the tumbling track (Figure 4.15).

Figure 4.15. Horizontal [A] and vertical [B] location of the mass centre [origin is the toe location at contact with the tumbling track].
The velocity of the mass centre at the start of the foot contact phase with the tumbling track varied slightly for each trial (Table 4.13; Figure 4.16). The single layout somersaults tended to have lower horizontal and greater downwards vertical velocities at the end of the preflight phase compared with the double layout somersaults at contact. During contact the horizontal velocity dropped steadily and the vertical velocity increased before dropping slightly at the end of the contact phase. At takeoff from the tumbling track the single layout somersaults had higher vertical and similar horizontal velocities compared with the double layout somersaults (Table 4.13; Figure 4.16).

Table 4.13. Velocity of the mass centre at the start and end of the foot contact phase with the tumbling track [m.s⁻¹]

<table>
<thead>
<tr>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>vx</td>
<td>vz</td>
<td>vx</td>
<td>vz</td>
<td>vx</td>
</tr>
<tr>
<td>contact</td>
<td>4.50</td>
<td>4.66</td>
<td>4.80</td>
<td>4.85</td>
</tr>
<tr>
<td></td>
<td>-0.70</td>
<td>-0.59</td>
<td>-0.33</td>
<td>-0.52</td>
</tr>
<tr>
<td>takeoff</td>
<td>2.93</td>
<td>4.98</td>
<td>2.79</td>
<td>2.66</td>
</tr>
<tr>
<td></td>
<td>4.50</td>
<td>5.09</td>
<td>4.53</td>
<td>4.50</td>
</tr>
</tbody>
</table>

Figure 4.16. Horizontal [A] and vertical velocity [B] of the mass centre.

Depression of tumbling track

The tumbling track was depressed horizontally and vertically during the contact phase of the feet with the track (Figure 4.17). Horizontally the toes moved on the track surface up to 0.06 m and vertically the toes depressed the track up to 0.07 m. There were
no systematic differences between the movement of the toes on the tumbling track for the single and double layout somersaults.

The heels of the feet also contacted the tumbling track during four of the five trials, with a maximum depression of the track at the heel of 0.01 m, there was no heel contact with the tumbling track during trial 5.

Figure 4.17. Movement of the toes horizontally [A] and vertically [B] during contact with the tumbling track.

Orientation and configuration angles / angular velocities

Figure 4.18 and Figure 4.19 show computer graphics sequences for the contact phase of the five tumbling performances. At contact with the tumbling track the body configuration and orientation were similar for all five trials, with the legs almost straight at contact. During the contact there was very little bending of the knees, and the hips extend continuously. At takeoff there were clear differences between the body configuration and orientation in the single and double somersault performances. The single layout somersaults had straighter body positions compared with the double layout somersaults which had an arched body position.
Figure 4.18. Computer graphics sequences of the contact phase with 0.5 m spacing between graphics for the two layout performances.
Figure 4.19. Computer graphics sequences of the contact phase with 0.5 m spacing between graphics for the three double layout performances.
Figure 4.20 shows the time histories of the foot, shank, thigh and trunk angles (see Figure 4.4 for angle definitions). The foot and shank angles were similar for all performances and the thigh and trunk angles for the single layout somersault were smaller at takeoff compared with the double layout performances.

![Time histories of the foot, shank, thigh and trunk angles.](image)

Figure 4.20. Time histories of the foot, shank, thigh and trunk angles.

**Angular momentum**

Figure 4.21 shows how the angular momentum time history varied between the single and double layout somersault performances. For all five performances the angular momentum at contact with the tumbling track were similar. The angular momentum was reduced during contact to 49% and 83% of the angular momentum at contact for the single and double layout somersaults respectively. The differences in the angular momentum of the single and double layout somersaults at takeoff appears to be due to
different techniques used during the foot contact phase and not due to different angular momenta at contact with the tumbling track.

Figure 4.21. Time history of the angular momentum [kg.m².rad.s⁻¹] about a transverse axis through the mass centre for each tumbling performance.

Summary of tumbling performances

The key features to take from the performances of the single and double layout back somersaults are:

- The single and double layout somersaults had slightly different mass centre velocities at contact with the tumbling track.
- The vertical velocity at takeoff was greater for the single layout somersaults.
- At contact the body configuration was similar for all five performances.
- During contact the knees flexed slightly before extending whereas the hips extended continuously.
- At takeoff the body configuration for the single layout somersaults was straight compared with an arched body shape for the double layout somersaults.
- Similar depressions of the tumbling track were observed for the five tumbling performances.
- The angular momenta at the start of the foot contact phase were similar but at takeoff the double layout somersaults had approximately 58% more angular momentum than the single layout somersaults.
CHAPTER 4 SUMMARY

This chapter has described the recording and analysis of three Hecht vaults and five tumbling performances by an elite male gymnast. The data obtained from these performances has highlighted the salient features of each performance and will be used in Chapter 6 to evaluate the simulation models developed in Chapter 3.

The next chapter documents the determination of parameters for the simulation models from measurements taken on the same elite male gymnast who was analysed in this chapter.
For the development and evaluation of the five segment simulation models of dynamic movements (Chapter 3); inertia, muscle and spring parameters were required.

The inertia and muscle parameters were determined from measurements taken on the same subject who performed the Hecht vault and series of tumbling movements analysed in Chapter 4. The inertia parameters were calculated from anthropometric measurements and the muscle parameters were calculated from experimentally determined maximum muscle torque measurements over a range of joint angles and angular velocities. The spring parameters were calculated through experimentation / computer simulation for the equipment used (vaulting horse and tumbling track) and image analysis / computer simulation for the shoulder.

This chapter will describe the methods used and the results from the determination of inertia, muscle and spring parameters.

INERTIA PARAMETERS

The mass, mass centre location and moment of inertia of each segment of the body are required for the simulation models developed (Chapter 3). These segmental inertia parameters are also needed for the isokinetic muscle data collected so that the torque values can be corrected for segment weight (Chapter 5) and for the video / film analysis so that the location of the mass centre can be calculated (Chapter 4).

There are a number of methods available for the calculation of segmental inertia parameters. The only method which is practical and will allow the experimental determination of the required segmental inertia parameters quickly and easily with as little inconvenience to the subject as possible, is to use a geometric model. Geometric models require lengths, widths, depths and heights from various parts of the body along with the density of the various geometric solids. The most accurate way to obtain the measurements is to take anthropometric measurements on the subject, and the density values are usually taken from cadaver studies. A number of researchers have developed mathematical models based on anthropometric measurements reporting similar and acceptable percentage errors for the prediction of the total body mass.
Method

The inertia model of Yeadon (1990b) with 95 anthropometric measurements taken on the subject and density values from Chandler et al. (1975) was used in this study to determine the required segmental inertia parameters. The geometric model of Yeadon (1990b) split the body into 11 segments, calculating the inertia parameters (mass, mass centre location and principal moment of inertia) for each of the 11 segments. Where the segmental inertia parameters were required for two or more segments combined together the mass of the combined segments was calculated by adding the masses of the individual segments. The location of the combined mass centre was calculated as a weighted average of the individual segments mass centre locations and the parallel axis theorem (Smith and Smith, 1983) was used to calculate the moment of inertia of the combined segment about the combined mass centre location.

Results and Discussion

The density values from Chandler et al., (1975) were preferred to the values of Dempster (1955) used by Yeadon (1990b) because they had resulted in closer estimates for the total body mass of various subjects from previous unpublished studies.

Segmental inertia parameters are presented for input to the simulation models (Table 5.2) and to correct the experimental torque data for the limb weight (Table 5.1).

<table>
<thead>
<tr>
<th>segment</th>
<th>mass [kg]</th>
<th>mass centre location [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>foot (left)</td>
<td>0.79</td>
<td>0.08</td>
</tr>
<tr>
<td>shank and foot (left)</td>
<td>3.98</td>
<td>0.23</td>
</tr>
<tr>
<td>straight leg (left)</td>
<td>11.46</td>
<td>0.33</td>
</tr>
<tr>
<td>straight arm (right)</td>
<td>3.95</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Table 5.2. Segmental inertia parameters for the simulation model

<table>
<thead>
<tr>
<th>segment</th>
<th>mass [kg]</th>
<th>mass centre location [m]</th>
<th>length of segment [m]</th>
<th>moment of inertia [kg.m^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>foot</td>
<td>1.59</td>
<td>0.08</td>
<td>0.20</td>
<td>0.01</td>
</tr>
<tr>
<td>shank</td>
<td>6.53</td>
<td>0.19</td>
<td>0.40</td>
<td>0.08</td>
</tr>
<tr>
<td>thigh</td>
<td>15.41</td>
<td>0.17</td>
<td>0.40</td>
<td>0.21</td>
</tr>
<tr>
<td>trunk + head</td>
<td>34.45</td>
<td>0.35</td>
<td>0.47</td>
<td>1.62</td>
</tr>
<tr>
<td>arm + hand</td>
<td>7.79</td>
<td>0.25</td>
<td>0.55</td>
<td>0.25</td>
</tr>
<tr>
<td>upper arm</td>
<td>4.22</td>
<td>0.11</td>
<td>0.25</td>
<td>0.03</td>
</tr>
<tr>
<td>lower arm + hand</td>
<td>3.58</td>
<td>0.16</td>
<td>0.30</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The lengths of the body segment (trunk + head) and the lower arm + hand segment are adjusted:

The length of the trunk represents the body segment and therefore is the length from the hips to the shoulders. The length of the lower arm segment was adjusted to be equal to the length of the lower arms plus 0.2 times the hand length, this was chosen because for the vault the distance from the point of contact with the horse to the elbow joint was considered greater than the length of the lower arms. Likewise the length of the arm was the shoulder to wrist length plus 0.2 times the hand length.

The measured total body mass of the subject was 65.0 kg with a standing height of 1.675 m. The predicted total body mass from the geometric model of Yeadon (1990b) was 65.78 kg; therefore the error in the prediction of total body mass was 1.19% which is within the 2% reported error by Yeadon (1990b). It was not possible to evaluate the accuracy of the segmental inertia parameters as it was not practical or feasible to measure the segmental inertia parameters individually. However if the simulation model gives realistic results with the calculated inertia parameters then the segmental inertia parameters may be thought to be realistic.
MUSCLE MODEL PARAMETERS

A lumped parameter muscle model was used in this study. The muscle model at the ankle (tumbling model) and shoulder (vaulting model) comprised contractile and series elastic elements and the muscle model at the knee, hip and shoulder (tumbling model) comprised a contractile element (see Chapter 3 for details). The determination of muscle parameters for use in the simulation models of dynamic movements consisted of two main parts. The first part considers how muscle and tendon interact and move during an isokinetic movement and the second describes the methods used to obtain parameters for the series elastic and contractile elements in the muscle model. Series elastic parameters were required for the ankle (tumbling model) and shoulder (vaulting model), while parameters for the contractile element were required for all the muscle models.

Introduction

The simulation models developed in Chapter 3 require the torques produced during vaulting and tumbling to be known for ankle plantar flexion, knee extension, hip extension, shoulder flexion and shoulder extension. The torques produced at each joint are represented by torque generators in the simulation models which allow the maximum torque that can be produced by all the muscles around a joint to be calculated from the joint angle and angular velocity. The torque generator at each joint defines a surface of the maximum torque that can be produced at that joint as a function of joint angle and joint angular velocity. The curvature of the surface is defined by a number of muscle parameters which are calculated from experimental data collected on the subject using an isokinetic dynamometer.

SIMULATION OF ISOKINETIC MOVEMENTS

Isokinetic dynamometers have been used by many researchers to test muscular strength at various joints over a wide range of angles and angular velocities. However the data obtained from dynamometers has been questioned by a number of researchers with one area of concern being how the isokinetic movement of the joint / crank relates to the movement of the muscle-tendon complex. To examine the movement of the muscle-tendon complex during isokinetic movements a simple simulation model was used.
Method

The simulation model consisted of a rigid, light segment free to move in a plane about a frictionless point at one end O. Two torque generators $T_m$ and $T_c$ were used to represent the torque produced by the muscle-tendon complex and isokinetic dynamometer respectively. The torque produced by the muscle-tendon tended to close the joint angle and the torque produced by the dynamometer opened the joint angle (Figure 5.1). The muscle-tendon model consisted of contractile and elastic elements in series.

The position of the segment OP (defined by the angle $\phi$ to the left horizontal) was specified as a function of time using a cubic spline fitted to the joint angle time history of an arbitrary trial on a dynamometer. The isokinetic trial used consisted of two repetitions of concentric-eccentric movement at an angular velocity of $150^\circ\cdot s^{-1}$ over a $75^\circ$ range of motion (Figure 5.2). This was a typical trial over a realistic range of motion and an intermediate angular velocity.

The motion of the segment OP was defined by equation (5.1).

$$ T = I \alpha $$

where $T =$ net torque

$\alpha =$ angular acceleration of the segment OP

However since $\phi$ was defined as a function of time, the movement of the segment OP was always known and therefore the equation of motion above was not required to determine the position of the segment OP during a simulation.

![Figure 5.1. Free body diagram for simulation of isokinetic movements.](image-url)
Figure 5.2. Arbitrary isokinetic dynamometer trial used to determine the joint angle during a simulation of an isokinetic movement.

Muscle-tendon complex

The angle $\phi$ of the joint was split into two parts to represent the length of the muscle and extension of the tendon (equation (5.2)).

$$\phi = \phi_m + \phi_t$$

(5.2)

where $\phi_m =$ angular length of muscle

$\phi_t =$ angular extension of tendon

The torque produced by the contractile component of the muscle was determined using a Hill type muscle model with three parameters which was independent of the muscle length:

$$T_m = \frac{a}{1 + ce^{-p\omega_m}}$$

(5.3)

where a, c and p are positive constants

$T_m =$ torque produced by contractile elements

$\omega_m =$ angular velocity of contractile muscle (lengthening)

The parameters a, c and p were chosen arbitrarily to give an angular equivalent of Hill's classic force / velocity relationship ($a = 150; c = 1$ and $p = 1$); (Figure 5.3). Since $\omega_m$ was the velocity of lengthening the torque / angular velocity relationship in Figure 5.3 looks the opposite way round to Hill's force / velocity relationship with the eccentric plateau on the right side of the graph.
\( \phi_t \) represented the extension of the series elastic element of the muscle-tendon complex with the tendon having a resting length of zero. The torque produced by the series elastic element in the muscle-tendon complex was determined using a massless linear spring:

\[
T_t = r \phi_t
\]  

(5.4)

where \( r \) = positive angular stiffness constant

\( T_t \) = torque produced by the tendon

\( \phi_t \) = angular extension of the tendon

200 Nm.rad\(^{-1}\) and 500 Nm.rad\(^{-1}\) were used for the stiffness \( r \) of the tendon (Hof et al. 1981b). A stiffness of 200 Nm.rad\(^{-1}\) allowed a maximum extension of 0.75 rad when a torque of 150 Nm was applied compared with an extension of 0.3 rad for a stiffness of 500 Nm.rad\(^{-1}\).

An equation of motion was required for the movement of the muscle-tendon complex during the isokinetic trial as the muscle and tendon were free to change length throughout the movement. Since the contractile and elastic elements were in series and massless, the torque produced by the contractile element was equal to the torque produced by the elastic element:

\[
T_t = T_m
\]  

(5.5)

Substitute for \( T_t \) and \( T_m \):

\[
r \phi_t = \frac{a}{1 + ce^{-r \omega m}}
\]
Differentiate with respect to time: 

\[ r\omega_t = \frac{acpe^{-p_0\alpha_m}}{(1 + ce^{-p_0\alpha_m})^2} \]

rearrange to give:

\[ \alpha_m = \frac{(1 + ce^{-p_0\omega_t})^2 r(\omega - \omega_m)}{acpe^{-p_0\omega_t}} \]  \( (5.6) \)

where \( \alpha_m \) = angular acceleration of the muscle

\[ \omega_t = \omega - \omega_m \]

Equation (5.6) defines the angular acceleration of the muscle throughout a movement. Since the movement of the joint and the movement of the muscle are defined, the movement of the tendon is defined from equation (5.2) and its derivatives.

To advance the simulation one step forward the simple Euler method was used which assumed that over a small interval the angular acceleration of the muscle was constant. With a small time interval the simple Euler method proved sufficient as the torque produced by the tendon and contractile elements agreed. If the step length was too large the accelerations became inaccurate and the torque produced by the tendon and contractile elements did not agree.

**Results and Discussion**

Figure 5.4 shows the simulation of an isokinetic trial using typical parameter values for tendon stiffness of 500 and 200 Nm.rad\(^{-1}\). With a tendon stiffness of 500 Nm.rad\(^{-1}\) the tendon remains at a fairly fixed length during the isokinetic parts of the trial and the angular velocity of the joint is close to the angular velocity of the muscle throughout the majority of the trial. The tendon remains at a fixed joint angle as the torque produced by the contractile element is assumed to be independent of joint angle and the effect of gravity has been neglected. In reality both the muscle length and segment weight would affect the torque produced in the muscle and therefore cause slight changes in tendon length. With a tendon stiffness of 200 Nm.rad\(^{-1}\) unrealistically high tendon extensions of up to 0.75 rad are observed which result in the motion of the muscle not matching the movement of the joint.

Figure 5.4 shows that there is a systematic difference between the joint angle and muscle angle throughout the simulations which is proportional to the torque produced by
the contractile element with the greater the stiffness of the tendon the less the difference between the joint and muscle angle.

![Graph showing the effect of tendon stiffness on muscle movement](image)

**Figure 5.4.** The effect of tendon stiffness on the movement of muscle during an isokinetic simulation.

**Summary of the simulation of isokinetic movements**

Simulation of an isokinetic movement with realistic muscle and tendon parameter values shows that the angular velocity of the muscle is equal to the angular velocity of the joint throughout the majority of the range of motion. Therefore during isokinetic movements the angular velocity of the joint can be assumed to be equal to the angular velocity of the muscle. Simulation also shows that there is a systematic difference between the joint angle and muscle angle throughout an isokinetic trial.

**SERIES ELASTIC PARAMETERS**

The series elastic parameters required for the ankle (tumbling model) and for the shoulder (vaulting model) were estimated from the literature.

The series elastic element was modelled as a linear spring with natural length zero. In addition the spring was defined in angular units and so the torque produced by the series elastic element was dependent on the angle of the tendon (extension) around the joint:

\[ T = r \phi_t \]  

(5.7)

One parameter was therefore needed to define the torque in the series elastic element at the ankle and the shoulder for any extension \( \phi_t \).

The length of the muscles, tendons and moment arms were estimated from the literature for the ankle (Bobbert, 1988) and through personal communication with Dr. van
der Helm for the shoulder (Table 5.3). Maximum joint torques were estimated from the experimental testing using an isokinetic dynamometer with the subject in this study.

Table 5.3. Estimated lengths of the muscle, tendon, moment arm and maximum joint torque

<table>
<thead>
<tr>
<th></th>
<th>ankle</th>
<th>shoulder</th>
</tr>
</thead>
<tbody>
<tr>
<td>muscle length</td>
<td>0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>tendon length, $t_L$</td>
<td>0.40</td>
<td>0.10</td>
</tr>
<tr>
<td>moment arm length, $ma_L$</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>maximum joint torque</td>
<td>150</td>
<td>250</td>
</tr>
</tbody>
</table>

The series elastic element stretches about 4% during normal activities with the tendon rupturing at around 8% stretch (Dixon, 1996).

A stretch of 4% of tendon length would be equivalent to the following angle change:

$$4\% \text{ stretch} = \frac{0.04 \times t_L}{ma_L}$$

For the ankle this gives an extension $= 0.32 \text{ rad}$
For the shoulder this gives an extension $= 0.13 \text{ rad}$

From equation (5.7) the stiffness of tendon $r$ required to give a maximum stretch equivalent to 4% of tendon length would be:

For the ankle $r = 469 \text{ Nm.rad}^{-1}$
For the shoulder $r = 1923 \text{ Nm.rad}^{-1}$

The calculated stiffness value for the ankle series elastic element is similar to the angular stiffness values determined by Hof et al. (1981c) who found an average stiffness of 593 Nm.rad$^{-1}$. An ankle stiffness of 500 Nm.rad$^{-1}$ was chosen as a initial estimate for the tendon stiffness at the ankle which was a round figure between the average value found by Hof and the value calculated above.

For the shoulder no values in the literature could be found for the stiffness of the series elastic element. Van der Helm (personal communication) suggested that the stretching of the muscle should be included in the total stretch of the series elastic
element. To incorporate the stretching of the muscle the stiffness at the shoulder was reduced to 1500 Nm.rad\(^1\).

Both stiffness values are estimates of the stiffness of the series elastic element, from which a sensitivity analysis will be used to establish the effect of varying the stiffness value on the performance of the models.

**CONTRACTILE ELEMENT PARAMETERS**

Contractile element parameters were required for ankle plantar flexion, knee extension, hip extension, shoulder flexion and shoulder extension. The process to calculate the parameters was divided into two parts; the first part concerns the collection of maximal torque data from a isokinetic machine and the second considers fitting a function to the data collected.

**JOINT TORQUE MEASUREMENT**

Isokinetic dynamometers

Isokinetic machines allow the experimental determination of the maximum joint torque possible over a range of angles and angular velocities. Reservations have been made regarding the use of isokinetic data to predict the torques produced during dynamic activities. This section will describe a method for measuring the torques produced during eccentric-concentric isokinetic conditions in order to reproduce the eccentric-concentric conditions experienced during dynamic jumps and to ensure that the muscles are maximally activated throughout each trial.

In addition isokinetic machines output the torque produced at the crank as a function of crank angle and crank angular velocity; however the simulation models require the maximum joint torque at each joint as a function of joint angle and joint angular velocity. A correction therefore is needed to relate the machine torque, angle and angular velocity to joint torque, angle and angular velocity. To correct isokinetic data for differences between the movement of the crank and the movement of the joint requires the time histories of the crank and joint angles and angular velocities to be known along with the time history of the crank torque.

The isokinetic dynamometer used in this study recorded the time histories of the crank angle, angular velocity and force.
Joint angle measurement

Two methods were available to obtain the time histories of the joint angle and angular velocity:

- Attach a goniometer across the joint being tested and measure the joint angle directly.
- Video the movement and through digitisation calculate the time history of the joint angle for each trial.

A goniometer was chosen to measure the joint angle time history in preference to obtaining the joint angle data from video for a number of reasons:

- In a preliminary investigation it was found very difficult to position a camera so that the subject's limb could be clearly visible throughout each trial as either the dynamometer or the subject's other limb would obscure the view depending on where the camera was placed.
- It would take a considerable amount of time to digitise the video data from each trial compared with directly measuring the joint angle using a goniometer.
- Previous studies using goniometers to measure joint angles have found acceptable error values between the measured and known joint angle (2° Kettlekamp Johnson, Smidt, Chao and Walker, 1970; and more recently less than 0.5° Liu and Panjabi, 1996).

Method

The following section will describe the procedure used for obtaining experimental torque data using an isokinetic dynamometer and a goniometer.

Introduction: Isokinetic dynamometer (Kin-Com 125E, Operators Manual, 1992)

The active isokinetic dynamometer used in this study is very versatile allowing the subject and the crank of the machine to be positioned so as to minimise movements of other limbs during testing of different joints (Figure 5.5). The dynamometer allows the angular velocity of the crank to be controlled during maximal effort joint flexions and extensions in the range of angular velocities from -250°s⁻¹ (eccentric) to +250°s⁻¹ (concentric). The dynamometer is controlled by an IBM compatible computer (Figure 5.5) using a 'user friendly' touch screen control interface which allows the exercise protocols to be customised. The computer records the angle and angular velocity of the crank along with the force produced and has additional software to allow ASCII data files
to be produced of each trial at a frequency of 100 Hz which could be accessed and copied to floppy disc for subsequent analysis.

Figure 5.5. The dynamometer used in this study (Kin-Com 125E).

The force produced by the subject tangential to the crank axis of rotation is measured by a load cell attached to the crank of the machine. The load cell consists of a full bridge strain gauge whose signal is amplified by an instrumentation amplifier and then fed to the computer for analysis and recording. The angle of the crank is measured by a potentiometer which is connected via an analogue to digital converter to the computer, where it is converted into an angle in degrees. To measure the angular velocity of the crank an internal tachometer is attached directly to the motor which measures the rotational speed of the motor and sends a digital signal to the computer for analysis. A further check on the angular velocity of the crank is performed by the computer by comparing the angular velocity from the tachometer with the mathematically calculated angular velocity from the potentiometer. If a discrepancy exists the machine shuts down.

The angular velocity of the crank is controlled to specified angular velocities by a servo amplifier consisting of four quadrature amplifiers which controls the current that the motor receives. In addition a real time feedback loop via the internal tachometer allows for 'minor' corrections to the programmed angular velocity. The dynamometer uses a high performance DC servo system motor with a 100 to 1 gearbox which multiplies the motor torque allowing a small motor to produce the required torque output. The
dynamometer-computer system measures and makes corrections at a frequency of 100 Hz. The dynamometer measures forces up to a maximum of 2000 N with a resolution of 1 N and calculates torque by multiplying the measured force by the moment arm length. The angle and angular velocity of the crank are measured by the dynamometer to a resolution of 0.01° and 0.25°s⁻¹ respectively.

Introduction: Multi-axis goniometer (Penny and Giles)

A multi-axis goniometer (Penny and Giles 'M' series twin axis goniometer, size M180) was used to measure the joint angle throughout each trial on the isokinetic dynamometer. This goniometer was chosen as it allowed flexibility in its positioning around the joint and did not require the goniometer and joint axes to be aligned. The output from the goniometer was ± 5 volts DC which was connected via a 12 bit analogue to digital converter (CED1401) interfaced to an Archimedes A5000 computer (Figure 5.6). The output from the analogue to digital converter (ADC) was recorded in a 16 bit unassigned binary format. An ASCII data file of each trial containing time and ADC count at a frequency of 500 Hz were written to a file. The data capture was triggered using a control switch (PERI triggered) with two seconds before and eight seconds after the trigger event.
Data collection

Written consent was obtained from the subject prior to testing and the dynamometer and goniometer were calibrated.

Written consent

The subject and his coach signed a written consent form (Appendix C) agreeing to the testing, with the option to withdraw from the experiment at any time.

Calibration of the dynamometer

Internal computer software was used to calibrate the Kin-Com dynamometer (Operators Manual, 1992) which involved a number of tests to check the angle, angular velocity and force measurements of the machine. The adjustments to the dynamometer were carried out as directed by the manufacturers.

Independent tests were also carried out to check the angle and force measurements of the machine. A spirit level was placed on the crank when in a horizontal and vertical position to confirm the angle measurements for the crank. To check the force measurements of the machine a load (231 N) was hung vertically while the crank was in a horizontal position and the force recorded.

Calibration of the goniometer

The goniometer was calibrated before and after testing with the subject. The calibration procedure consisted of positioning the goniometer at a known angle and recording the goniometer's output. Five angles (0°, 90°, 180°, -90°, -180°) were chosen for the calibration. These angles were chosen as they spanned the whole range of motion of the goniometer from -180° to +180° and were simple to produce accurately using a flat surface and a set square. At each angle 10 s of data at a frequency of 500 Hz were recorded.

Subject and equipment preparation

The same experimental procedure was used at each joint with the joints tested in the sequence: knee (extension), shoulder (extension), shoulder (flexion), hip (extension) and ankle (plantar flexion). This sequence of testing was chosen starting with the easiest joint to become familiar with and finishing with the hardest (based upon previous experience).
The subject and equipment were prepared for testing at each joint using the following procedure:

1. Attach goniometer across the joint

The goniometer was attached across the joint being tested (knee, hip and shoulder) using sticky tape (Figure 5.7, Figure 5.8, and Figure 5.9) with the goniometer in a straight position when the joint was straight and fully open (As recommended by the manufacturers). It was not possible to measure the ankle angle with the goniometer as the dynamometer attachments prevented the goniometer from being placed around the joint.

Figure 5.7. Positioning and securing of the goniometer around the joint (knee).
Figure 5.8. Positioning and securing of the goniometer around the joint (hip).

Figure 5.9. Positioning and securing of the goniometer around the joint (shoulder).
2. Position subject on machine so that crank and joint axes are aligned

The subject was positioned and secured on the machine as directed by the manufacturers for the testing of each joint, with the joint axis and crank axis aligned:

For the ankle joint data collection the subject was seated on the machine (Figure 5.10) with his legs straight and horizontal. The dynamometer used a complex attachment which isolated the ankle joint (Figure 5.11) and allowed the ankle joint to plantar flex.

For the knee joint data collection the subject was seated upright (Figure 5.12) with the upper legs and body secured using Velcro straps to prevent excessive movement.

For the hip joint data collection the subject was positioned on his back (Figure 5.13) to isolate the hip extension movement with Velcro straps used to prevent excessive movement.

For the shoulder joint data collection the subject was positioned on his back for both shoulder flexion and shoulder extension (Figure 5.14). Velcro straps were used to isolate the shoulder joint, without restricting the torque produced at the shoulders.

When the subject was positioned and secured for testing the subject’s limb being used was strapped to the dynamometer via a cuff attached to the machine crank.

![Figure 5.10. Positioning for testing the ankle joint.](image-url)
Figure 5.11. Attachment for the ankle joint testing.

Figure 5.12. Positioning for testing the knee joint.
Figure 5.13. Positioning for testing the hip joint.

Figure 5.14. Positioning for testing the shoulder joint.
3. Extra safety precautions

To ensure the complete safety of the subject ‘mechanical stops’ were used to prevent the dynamometer exceeding the specified range of motion at each joint. In addition the subject held a ‘patient interrupt switch’ throughout all testing which allowed him to stop a trial at any time.

4. Zero crank angle using spirit level

A spirit level was used to set a common reference angle of zero degrees for each joint tested when the crank was horizontal.

5. Range of motion

The range of motion used at each joint was programmed into the computer based upon the ranges of motion observed from video data of Hecht vaulting and straight double back somersault takeoffs (Table 5.4). Where possible the ranges of motion were extended outside the ranges observed from video although the subject had final control over the ranges of motion used.

<table>
<thead>
<tr>
<th>joint</th>
<th>crank angle ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee</td>
<td>94° - 167°</td>
</tr>
<tr>
<td>shoulder (extension)</td>
<td>22° - 110°</td>
</tr>
<tr>
<td>shoulder (flexion)</td>
<td>61° - 166°</td>
</tr>
<tr>
<td>hip</td>
<td>90° - 185°</td>
</tr>
<tr>
<td>ankle</td>
<td>92° - 136°</td>
</tr>
</tbody>
</table>

6. Perform handle jog

Before testing at each joint a ‘handle jog’ was performed with the dynamometer, this notified the computer which direction the tangential force was going to be applied to the crank of the machine.

7. Warm up and familiarisation

At each joint tested the subject was given a few trials to become familiar with the exercise protocol and prepare for testing.
Exercise protocol

Each trial consisted of two repetitions of concentric-eccentric exercise at a pre-set angular velocity apart from the trials at 20°s⁻¹ where one repetition was performed. The range of angular velocities used varied from 20°s⁻¹ to 250°s⁻¹ and the sequence of angular velocities used was 20, 20, 50, 100, 150, 200, 250, 250, 250, 20 and 20°s⁻¹. For the first trial at 20°s⁻¹ the subject worked maximally for the concentric part of the first repetition and then on the second trial at 20°s⁻¹ worked maximally on the eccentric part of the first repetition. For the trials at 50, 100, 150, 200 and 250°s⁻¹ the subject worked maximally for both repetitions. The testing at each joint was completed with two repeated trials at the slowest angular velocity where the subject worked maximally for the concentric part of the first repetition at 20°s⁻¹ and then worked maximally on the eccentric part of the first repetition at 20°s⁻¹.

Data recording

At each joint tested the subject was given a few trials to become familiar with the exercise protocol prior to recording. When the subject was ready to start a trial the Kin-Com and goniometer computers were manually contact triggered independently; the subject was then instructed to start. The dynamometer started a trial when the minimum threshold force of 50 N was exceeded (as recommended by the manufacturers), this prevented the crank from moving until the subject had actively exerted a force on the crank. Throughout each trial the subject was given verbal encouragement. The Kin-Com computer captured the crank angle, angular velocity and torque data at a frequency of 100 Hz until a key was pressed to stop the data capture. For the goniometer the Archimedes computer captured the joint angle for a 10 s period at frequency of 500 Hz from 2 s prior to being manually triggered for the start of each trial.

Between trials the subject was allowed to rest while secured to the dynamometer until he was ready to perform the next trial. Between testing each joint the subject was free to get up and relax before preparing for the next joint to be tested.

The ASCII data files of each trial were copied from the Kin-Com and goniometer computers onto floppy disc after testing had finished.
Data Analysis

The analysis of the data was divided into three sections to obtain a data set of joint torque as a function of joint angle and angular velocity. The first section involved the calibration of the dynamometer and goniometer and editing the complete data time histories to the sections of each trial that were required for analysis. The second section involved synchronising the dynamometer data time histories with the joint angle time histories from the goniometer. The third section used the synchronised dynamometer and goniometer data to correct for differences between the joint and crank: angle, angular velocity and torque, and also to correct the torque values for segment weight. In addition the torque / angle data were extrapolated to give the required range of motion for each joint.

Calibration of the equipment and editing the movement data files

*Calibration and resolution of the dynamometer*

Additional tests to those recommended by the manufacturers of the dynamometer were performed to check the accuracy of the internal calibration of the machine. For the angle measurements the measured angle using the spirit level was within $1^\circ$ for the crank in a horizontal and vertical position, and for the force measurements the average force recorded by the dynamometer over a 5 s period with the crank in a horizontal position was within 2 N of the load hung from the machine (Table 5.5). These errors were within the tolerance quoted by the manufacturers and of the same order of magnitude as the resolution of the machine and were therefore thought to be acceptable.

<table>
<thead>
<tr>
<th>trial</th>
<th>load [N]</th>
<th>measured force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>231</td>
<td>229 (2)</td>
</tr>
<tr>
<td>2</td>
<td>231</td>
<td>231 (2)</td>
</tr>
</tbody>
</table>
Calibration of the goniometer

From the static calibration of the goniometer the average ADC count for each trial was calculated (Table 5.6). At first a linear relationship was tried between the known angle and the measured angle. Linear regression using the statistical package 'MINITAB' (Sevin, 1992) found the best fit straight line between the known angle and the measured angle. The equation of the straight line fit was:

\[ \text{angle} = 0.451779 \times \text{adc} - 246.650 \]  \hspace{1cm} (5.8)

Although the correlation coefficient between the known and measured angle was 0.9996 the root mean square difference was 3.7°; this was quite high and suggested that there existed a non-linear relationship between the joint angle and the goniometer output. To obtain a better relationship a cubic spline (Reinsch, 1967) was fitted to the known angle using the average ADC count as the time base. Error estimates for each angle (Table 5.7) were calculated by fitting a cubic spline to the average of two estimates for each known angle (one taken before testing and one after) using the known angle as the time base. The RMS error for each point was calculated as 50% local error / 50% global error and was converted from ADC count units to degrees by multiplying each RMS value by the conversion factor 0.451779 calculated through linear regression. Evaluating the cubic spline reduced the root mean square difference between the known angle and the calculated angle to 1.4° (Table 5.8).

The calculated cubic spline coefficients from the calibration data were used to evaluate the goniometer data files for each movement trial giving time histories of the joint angle throughout each trial in degrees. The goniometer data files were then transformed so that 180° referred to each joint being fully closed.
Table 5.6. Summary of output from goniometer for calibration

<table>
<thead>
<tr>
<th>trial</th>
<th>angle</th>
<th>mean [ADC count]</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°</td>
<td>550.8</td>
<td>1.4</td>
</tr>
<tr>
<td>2</td>
<td>-90°</td>
<td>353.5</td>
<td>1.4</td>
</tr>
<tr>
<td>3</td>
<td>-180°</td>
<td>140.8</td>
<td>1.7</td>
</tr>
<tr>
<td>4</td>
<td>90°</td>
<td>738.8</td>
<td>1.4</td>
</tr>
<tr>
<td>5</td>
<td>180°</td>
<td>947.1</td>
<td>1.4</td>
</tr>
<tr>
<td>6</td>
<td>0°</td>
<td>555.3</td>
<td>1.3</td>
</tr>
<tr>
<td>7</td>
<td>-90°</td>
<td>357.9</td>
<td>1.4</td>
</tr>
<tr>
<td>8</td>
<td>-180°</td>
<td>139.6</td>
<td>1.8</td>
</tr>
<tr>
<td>9</td>
<td>180°</td>
<td>945.2</td>
<td>1.5</td>
</tr>
<tr>
<td>10</td>
<td>90°</td>
<td>729.9</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 5.7. Error estimates for each angle in calibration

<table>
<thead>
<tr>
<th>angle</th>
<th>error estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>-180°</td>
<td>0.8°</td>
</tr>
<tr>
<td>-90°</td>
<td>1.1°</td>
</tr>
<tr>
<td>0°</td>
<td>1.1°</td>
</tr>
<tr>
<td>90°</td>
<td>1.6°</td>
</tr>
<tr>
<td>180°</td>
<td>0.9°</td>
</tr>
</tbody>
</table>
Table 5.8. Relationship between known and measured angle

<table>
<thead>
<tr>
<th>trial</th>
<th>known angle</th>
<th>calculated angle</th>
<th>difference°</th>
<th>calculated angle</th>
<th>difference²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°</td>
<td>2.2°</td>
<td>5.0</td>
<td>-0.6°</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>-90°</td>
<td>-86.9°</td>
<td>9.4</td>
<td>-90.8°</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>-180°</td>
<td>-183.0°</td>
<td>9.0</td>
<td>-179.8°</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>90°</td>
<td>87.2°</td>
<td>8.1</td>
<td>90.3°</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>180°</td>
<td>181.2°</td>
<td>1.6</td>
<td>180.6°</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>0°</td>
<td>4.3°</td>
<td>18.1</td>
<td>1.6°</td>
<td>2.6</td>
</tr>
<tr>
<td>7</td>
<td>-90°</td>
<td>-84.9°</td>
<td>25.6</td>
<td>-88.9°</td>
<td>1.2</td>
</tr>
<tr>
<td>8</td>
<td>-180°</td>
<td>-183.5°</td>
<td>12.4</td>
<td>-180.3°</td>
<td>0.1</td>
</tr>
<tr>
<td>9</td>
<td>180°</td>
<td>180.4°</td>
<td>0.1</td>
<td>179.8°</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>90°</td>
<td>83.1°</td>
<td>47.2</td>
<td>86.1°</td>
<td>14.9</td>
</tr>
<tr>
<td></td>
<td>r.m.s. 3.7</td>
<td></td>
<td></td>
<td>r.m.s. 1.4</td>
<td></td>
</tr>
</tbody>
</table>

*Editing movement data files from the dynamometer and goniometer*

The ASCII data files from the dynamomometer for each trial consisted of the time histories of the crank angle, crank angular velocity and measured force at a frequency of 100 Hz (Figure 5.15). Each file was edited leaving the central eccentric-concentric part of the first two repetitions (Figure 5.16). This procedure was used in an attempt to ensure that the subject had reached maximum voluntary activation, and that the effect of contracting the muscles eccentrically before contracting concentrically would be recorded in the second concentric repetition. In addition by completing the second eccentric repetition attempted to ensure that the subject remained at maximum voluntary activation throughout the central eccentric-concentric part of the first two repetitions.
Figure 5.15. Example of an ASCII date file from the Kin-Com machine.
Figure 5.16. Example of an edited ASCII data file from the Kin-Corn machine.

The isokinetic sections of the eccentric and concentric phases were then identified and defined by the values p1, p2, p3 and p4 for each trial (Figure 5.17), (p1 = first isokinetic eccentric value, p2 = last isokinetic eccentric value, p3 = first isokinetic concentric value and p4 = last isokinetic concentric value).
Figure 5.17. Identifying the isokinetic parts of each trial.

The ASCII data files from the goniometer for each trial after being converted from ADC counts to degrees consisted of the joint angle time history at 500 Hz (Figure 5.18). The ASCII data files were edited to match the central eccentric-concentric part of the motion (Figure 5.19).
The trials at 20°s⁻¹ at the beginning and end of the testing at each joint were treated differently with the concentric part of the first trial combined with the eccentric part of the second trial to form a complete data set.

Synchronisation of dynamometer and goniometer data files

Synchronising the dynamometer and goniometer output directly would have required alterations to the dynamometer control software. Therefore to synchronise the edited crank angle and joint angle time histories for each trial an optimisation procedure was used which assumed that the turning point in the joint angle time history occurred at the same time as the turning point in crank angle time history. The turning point in the crank angle occurred when the crank changed from moving concentrically to eccentrically and vice versa. This resulted in three turning points for each trial apart from the trials at
the slowest angular velocity (20°s⁻¹) where only two turning points occurred (Figure 5.20).

![Graph showing 20 deg/s and 50 deg/s](image)

**Figure 5.20.** Examples of crank and joint angle time histories prior to synchronisation.

To synchronise the crank and joint angle time histories data sets of equal length and frequency were required. Cubic splines (Reinsch, 1967) were fitted to the crank and joint angle time histories and evaluated at a frequency of 250 Hz over the greater interval between the crank and joint for each trial to give time histories of equal length and frequency (Figure 5.21).

Error estimates for the splines fitted to the crank angle time histories were calculated from the difference between the crank and pseudo crank angle time histories where the pseudo crank angle time history was calculated by averaging the crank angle
values at adjacent times. Standard error estimates for each point in the crank angle time history of 75% local error and 25% global error were used, so that if the local error was very small the global error would prevent the spline from being fitted too tightly.

Global error estimates were calculated for the splines fitted to the joint angle time histories by averaging standard deviation values obtained from the trials at 20°s⁻¹ at each joint. The standard deviation values were obtained using linear regression on parts of the joint angle time history where the crank angular velocity was constant (Table 5.9). This method was used as, on close examination of the joint angle time histories, it was found that there was 50 Hz oscillation throughout the time histories which was not showing with the small differences between the raw and pseudo joint angle data sets at a frequency of 500 Hz. A filtering technique could have been used to remove the 50 Hz oscillation in the data, however it would still have been necessary to smooth the data further. Fitting a cubic spline with error estimates that incorporated the 50 Hz oscillation, and any other errors in the data, meant that the data was only fitted once.

![Crank and joint angle time histories](image)

Figure 5.21. Example of extrapolated time histories for the crank and joint angle (prior to synchronisation).
Table 5.9. Error estimates (standard deviations) for the joint angle time histories [°]

<table>
<thead>
<tr>
<th>joint</th>
<th>first con.</th>
<th>first ecc.</th>
<th>second con.</th>
<th>second ecc.</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee</td>
<td>0.80</td>
<td>0.82</td>
<td>0.78</td>
<td>0.60</td>
<td>0.745</td>
</tr>
<tr>
<td>hip</td>
<td>0.57</td>
<td>0.44</td>
<td>0.73</td>
<td>0.45</td>
<td>0.55</td>
</tr>
<tr>
<td>sh. vaulting</td>
<td>0.93</td>
<td>1.03</td>
<td>0.95</td>
<td>1.25</td>
<td>1.04</td>
</tr>
<tr>
<td>sh. tumbling</td>
<td>1.25</td>
<td>0.68</td>
<td>1.83</td>
<td>3.44</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Optimisation procedure

To synchronise the crank and joint angle time histories a non-linear optimisation procedure was used (minimisation). Two packages (Simplex (Nelder and Mead, 1965) and Simulated Annealing (Goffe et al., 1994)) were used to perform the optimisation. The Simplex algorithm always found a solution quickly, although the optimum was not always as good as the one found by the Simulated Annealing algorithm. The optimisation procedure used was to find a solution using the Simplex algorithm and then to use the Simulated Annealing package to confirm / improve the solution:

The crank and joint angle time histories for each trial were synchronised by minimising the sum of squares difference between the crank and joint angles. The crank and joint angle time histories were related by a transformation which translated the joint angle time history onto the crank angle time history:

\[ \phi_1(t) = x_2 \phi(t + x_1) + x_3 \]  \hspace{1cm} (5.9)

where

- \( \phi_1(t) \) transformed joint angle at time t
- \( \phi(t) \) joint angle at time t
- \( x_1 \) constant which allows the joint angle to be translated horizontally
- \( x_2 \) constant which stretches the joint angle vertically
- \( x_3 \) constant which translates the joint angle vertically

The objective function \( f \) was minimised by optimising the values of \( x_1, x_2 \) and \( x_3 \) for each trial:
\[ f = \sum_{t=1}^{n} (\theta(t) - \phi_t(t))^2 \]  

or substituting equation (5.9) in equation (5.10):

\[ f = \sum_{t=1}^{n} (\theta(t) - x_2 \phi_1(t + x_1) - x_3)^2 \]  

where \( \theta(t) \) = crank angle at time \( t \)

The optimisation procedure failed for some of the slowest trials at 20°s\(^{-1}\), where the data capture period of 10 s missed part of the movement. As a consequence all of the 20°s\(^{-1}\) trials were synchronised by plotting the crank and goniometer time histories and estimating the horizontal translation factor \( x_1 \) (Figure 5.21). The coefficients \( x_2 \) and \( x_3 \) were then obtained by minimising the objective function \( f \) with \( x_1 \) fixed.

*Synchronised joint angle and joint angular velocity time histories*

Three methods were available for obtaining the synchronised joint angle and joint angular velocity time histories from the optimised values of \( x_1, x_2 \) and \( x_3 \):

1. Calculate the synchronised joint angle from the known crank angle by rearranging equation (5.9) and replacing the transformed joint angle with the crank angle (equation (5.12)).

2. Evaluate the cubic spline fitted to the joint angle time history with the time base adjusted by the synchronisation factor \( x_1 \) for each trial.

3. From the synchronised time \( x_1 \) obtained through optimisation with the whole time history of each trial, re-optimise the values of \( x_2 \) and \( x_3 \) using only the isokinetic parts of each time history.

Method 3 was preferred for the following reasons:

For some trials the original joint angle time histories were incomplete and required extrapolation of the cubic spline outside the data range which could potentially cause inaccuracies (see Figure 5.22, where the extrapolation of joint angle time history (thin line) is not very good).

Using the transformation of the crank angle gives a new data set that is the best fit to the splined joint angle data but keeps the general shape of the crank angle and the turning points apparent in the crank angle data.
Keeping the general shape of the crank angle data allows a new joint angular velocity to be calculated from the rate of change of the joint angle. Whereas if the original joint data were extrapolated and interpolated there would not be a constant angular velocity and there may be large errors where extrapolation would be necessary.

Method 3 of re-optimising the values of the $x_2$ and $x_3$ coefficients was preferred to the method 1 of using the original coefficients as for some trials the joint angle data around the second turning point were missing and skewed the optimised values of the coefficients $x_2$ and $x_3$ (Figure 5.22). Re-optimising the values of $x_2$ and $x_3$ using only the isokinetic portions of each time history (method 3) minimised any error in the transformation giving the best fit to the original joint angle time history (Figure 5.23).

![Figure 5.22. Synchronised crank and joint time histories using the full data sets.](image)
20 deg/s

50 deg/s

Figure 5.23. Synchronised crank and joint time histories using the isokinetic data sets.

Calculation of joint angle time history for each trial

The joint angle was calculated from the crank angle by re-arranging equation (5.9):

\[ \phi(t) = \frac{(\theta(t) - x_3)}{x_2} \]  

(5.12)

The joint angular velocity time history for each trial was calculated by differentiating the joint angle time history:

\[ \omega(t) = \frac{\theta(t)}{x_2} \]  

(5.13)
Results

The output from the synchronisation of the crank and joint angle time histories for each joint was a series of coefficients for each trial $x_1$, $x_2$ and $x_3$ (Table 5.10 - Table 5.13). In addition an ASCII data file containing the time histories of the synchronised joint angle, the crank angle, the crank angular velocity and measured force was produced.

Table 5.10. Optimised coefficients for the knee

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>rmsd</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°s⁻¹ concentric</td>
<td>0.000</td>
<td>1.208</td>
<td>-59.06</td>
<td>3.77</td>
</tr>
<tr>
<td>50°s⁻¹</td>
<td>-0.043</td>
<td>1.260</td>
<td>-68.01</td>
<td>1.70</td>
</tr>
<tr>
<td>100°s⁻¹</td>
<td>-0.013</td>
<td>1.312</td>
<td>-78.75</td>
<td>1.32</td>
</tr>
<tr>
<td>150°s⁻¹</td>
<td>0.084</td>
<td>1.342</td>
<td>-78.08</td>
<td>1.32</td>
</tr>
<tr>
<td>200°s⁻¹</td>
<td>-0.038</td>
<td>1.359</td>
<td>-81.63</td>
<td>1.89</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>-0.029</td>
<td>1.394</td>
<td>-91.70</td>
<td>1.14</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.024</td>
<td>1.393</td>
<td>-90.46</td>
<td>1.99</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>-0.020</td>
<td>1.426</td>
<td>-97.64</td>
<td>1.52</td>
</tr>
<tr>
<td>20°s⁻¹ concentric</td>
<td>-0.160</td>
<td>1.095</td>
<td>-38.37</td>
<td>3.58</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>0.500</td>
<td>1.368</td>
<td>-82.40</td>
<td>1.77</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>1.100</td>
<td>1.458</td>
<td>-103.9</td>
<td>2.00</td>
</tr>
</tbody>
</table>
Table 5.11. Optimised coefficients for the hip

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th></th>
<th>x3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20°s⁻¹ concentric</td>
<td>-0.100</td>
<td>1.840</td>
<td>-174.12</td>
<td>3.77</td>
</tr>
<tr>
<td>50°s⁻¹</td>
<td>0.457</td>
<td>1.826</td>
<td>-171.07</td>
<td>2.58</td>
</tr>
<tr>
<td>100°s⁻¹</td>
<td>0.208</td>
<td>2.034</td>
<td>-206.91</td>
<td>2.68</td>
</tr>
<tr>
<td>150°s⁻¹</td>
<td>0.336</td>
<td>1.874</td>
<td>-175.58</td>
<td>2.64</td>
</tr>
<tr>
<td>200°s⁻¹</td>
<td>0.249</td>
<td>1.652</td>
<td>-132.04</td>
<td>2.28</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>-0.156</td>
<td>1.817</td>
<td>-170.27</td>
<td>1.13</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.118</td>
<td>1.605</td>
<td>-130.33</td>
<td>2.30</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>-0.211</td>
<td>1.811</td>
<td>-165.92</td>
<td>1.77</td>
</tr>
<tr>
<td>20°s⁻¹ concentric</td>
<td>0.000</td>
<td>1.614</td>
<td>-129.80</td>
<td>1.43</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>0.040</td>
<td>1.774</td>
<td>-159.12</td>
<td>2.51</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>0.020</td>
<td>1.739</td>
<td>-146.37</td>
<td>0.42</td>
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</tbody>
</table>

Table 5.12. Optimised coefficients for the shoulder (vaulting)

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th></th>
<th>x3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20°s⁻¹ concentric</td>
<td>0.000</td>
<td>1.112</td>
<td>-6.26</td>
<td>2.79</td>
</tr>
<tr>
<td>50°s⁻¹</td>
<td>0.155</td>
<td>1.345</td>
<td>-38.65</td>
<td>1.67</td>
</tr>
<tr>
<td>100°s⁻¹</td>
<td>-0.034</td>
<td>1.277</td>
<td>-30.48</td>
<td>1.34</td>
</tr>
<tr>
<td>150°s⁻¹</td>
<td>0.002</td>
<td>1.420</td>
<td>-44.59</td>
<td>1.88</td>
</tr>
<tr>
<td>200°s⁻¹</td>
<td>0.021</td>
<td>1.279</td>
<td>-28.17</td>
<td>1.29</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.082</td>
<td>1.344</td>
<td>-34.97</td>
<td>1.79</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.000</td>
<td>1.360</td>
<td>-31.58</td>
<td>2.33</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.043</td>
<td>1.380</td>
<td>-28.63</td>
<td>1.45</td>
</tr>
<tr>
<td>20°s⁻¹ concentric</td>
<td>0.000</td>
<td>1.338</td>
<td>-25.69</td>
<td>5.41</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>0.290</td>
<td>1.499</td>
<td>-35.18</td>
<td>0.45</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>0.030</td>
<td>1.319</td>
<td>-32.79</td>
<td>3.21</td>
</tr>
</tbody>
</table>
Table 5.13. Optimised coefficients for the shoulder (tumbling)

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>rmsd</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°s⁻¹ concentric</td>
<td>0.000</td>
<td>1.160</td>
<td>-63.03</td>
<td>16.94</td>
</tr>
<tr>
<td>50°s⁻¹</td>
<td>0.346</td>
<td>2.093</td>
<td>-214.96</td>
<td>11.27</td>
</tr>
<tr>
<td>100°s⁻¹</td>
<td>0.342</td>
<td>2.345</td>
<td>-224.71</td>
<td>8.12</td>
</tr>
<tr>
<td>150°s⁻¹</td>
<td>0.152</td>
<td>2.334</td>
<td>-219.10</td>
<td>4.37</td>
</tr>
<tr>
<td>200°s⁻¹</td>
<td>0.019</td>
<td>1.840</td>
<td>-156.25</td>
<td>4.53</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.011</td>
<td>1.676</td>
<td>-154.11</td>
<td>3.42</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.116</td>
<td>1.516</td>
<td>-126.28</td>
<td>2.21</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>0.037</td>
<td>1.614</td>
<td>-135.47</td>
<td>2.54</td>
</tr>
<tr>
<td>20°s⁻¹ concentric</td>
<td>0.000</td>
<td>1.761</td>
<td>-141.98</td>
<td>17.12</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>0.000</td>
<td>5.026</td>
<td>-649.82</td>
<td>11.72</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>0.000</td>
<td>3.076</td>
<td>-361.56</td>
<td>9.48</td>
</tr>
</tbody>
</table>
Corrections to the synchronised muscle data

The measured force values from the dynamometer were converted to an equivalent joint torque so that the torque values could be used in the simulation models. For the knee, hip and shoulder joints the measured force time histories for each trial were converted to torque values, corrected for the limb weight and converted from a crank torque to an equivalent joint torque. For the ankle joint where the joint movement was not measured the recorded force data were converted to torque values and corrected for the limb weight.

Conversion of crank force to crank torque

The dynamometer measured and recorded the force produced at the cuff on the crank. The force time history for each trial was converted to a torque time history by multiplying each force reading by the measured moment arm:

\[ T_c = F d_c \] (5.14)

where \( T_c \) = crank torque
\( F \) = measured force at cuff of the dynamometer
\( d_c \) = measured moment arm distance

The moment arm distance \( d_c \) was constant for each trial and measured using a ruler as the perpendicular distance from the point where the force was measured \( C \) to the centre of rotation of the crank \( O \) (Figure 5.24).

Segment weight correction

The dynamometer used in this study measured the force using a force transducer located at the cuff on the crank which attached the limb to the crank during each trial. To give the torque produced during each trial by the subject required the weight of the
moving limb to be corrected for (Winter et al., 1981). To correct for limb weight required
the mass and mass centre location of each limb and the time history of the limb angle
relative to the vertical throughout each trial (equation (5.15)). The mass and mass centre
location for each limb were calculated from anthropometric measurements taken on the
subject using the mathematical model of Yeadon (1990b). The angle of the limb relative
to the vertical throughout each trial was unknown as the goniometer measured the internal
joint angle. The angle of the limb relative to the horizontal was assumed to be equal to
the crank angle \( \theta \) for the purposes of limb weight correction allowing the torque due to
the segment weight to be corrected.

\[
T_c = T_c \pm Mg d_j \cos \theta
\]

where (Figure 5.25):

- \( T_c \) = crank torque
- \( M \) = mass of limb
- \( g \) = acceleration due to gravity
- \( d_j \) = perpendicular distance from mass centre location to joint centre
- \( \theta \) = crank angle relative to the horizontal

![Figure 5.25. Limb weight correction.](image)

From equation (5.15) it can be seen that the closer the limb is to the horizontal
position the greater the correction and the \( \pm \) sign shows that the correction can increase or
decrease the measured torque values. When the subject is working against gravity the
weight correction increases the measured torque, but when the subject is working with
gravity the weight correction decreases the measured torque.

**Conversion of crank torque to joint torque**

The torque time histories obtained, after converting the measured force to torque
and correcting for the segment weight, were the torques produced at the crank \( T_c \) and not
the required torques produced at the joint \( T_j \). The relationship between the joint and
crank torques was calculated as follows:
\[ T_j = F d_j \]  
\[ T_j = F d_c \frac{d_j}{d_c} \]

from equation (5.14):
\[ T_j = T_c \frac{d_j}{d_c} \]

where \( T = \) torque
\( F = \) measured force at cuff
\( d = \) moment arm length

and the subscripts \( c \) and \( j \) represent the crank and joint respectively.

The crank moment arm \( d_c \) was constant throughout each trial and was measured prior to testing each joint; the moment arm for the joint \( d_j \) measured from the joint centre to the point of force application on the crank was not constant. \( d_j \) varied throughout each trial as there was some relative movement between the limb and crank. To estimate the relationship between the joint moment arm and the crank moment arm a simple geometric model (Figure 5.25) was used (equation (5.19)).

\[ \frac{d_j}{d_c} = \frac{\sin \theta}{\sin \phi} \approx \frac{\theta}{\phi} \]  
(for small \( \theta \) and \( \phi \))

From equation (5.19) the relationship between the moment arms of the crank and joint can be represented by the ratio of crank and joint angles. The synchronisation of the crank and joint angle time histories resulted in a relationship between the crank and joint angles. By rearranging equation (5.12) the coefficient \( x_2 \) relates the crank angle \( \theta \) and joint angle \( \phi \) giving:
\[ \theta(t) = x_2 (\phi(t)) + x_3 \]  

The best fit factor \( x_2 \) between the crank and joint angle time histories was used to relate the crank and joint angle time histories as this gave the average relationship between the crank and joint throughout each trial, rather than using a variable moment arm length dependent on individual estimates of the crank and joint angles. The crank torque time history for each trial was therefore corrected using equation (5.21) with a different value of \( x_2 \) for each trial (Table 5.14).

\[ T_j = x_2 T_c \]  

### Table 5.14. Moment arm correction factor \( x_2 \) used for each trial

<table>
<thead>
<tr>
<th>trial</th>
<th>knee</th>
<th>hip</th>
<th>shoulder (vaulting)</th>
<th>shoulder (tumbling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°s⁻¹</td>
<td>1.208</td>
<td>1.840</td>
<td>1.112</td>
<td>1.160</td>
</tr>
<tr>
<td>50°s⁻¹</td>
<td>1.260</td>
<td>1.826</td>
<td>1.345</td>
<td>2.093</td>
</tr>
<tr>
<td>100°s⁻¹</td>
<td>1.312</td>
<td>2.034</td>
<td>1.277</td>
<td>2.345</td>
</tr>
<tr>
<td>150°s⁻¹</td>
<td>1.342</td>
<td>1.874</td>
<td>1.420</td>
<td>2.334</td>
</tr>
<tr>
<td>200°s⁻¹</td>
<td>1.359</td>
<td>1.652</td>
<td>1.279</td>
<td>1.840</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>1.394</td>
<td>1.817</td>
<td>1.344</td>
<td>1.676</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>1.393</td>
<td>1.605</td>
<td>1.360</td>
<td>1.516</td>
</tr>
<tr>
<td>250°s⁻¹</td>
<td>1.426</td>
<td>1.811</td>
<td>1.380</td>
<td>1.614</td>
</tr>
<tr>
<td>20°s⁻¹</td>
<td>1.095</td>
<td>1.614</td>
<td>1.338</td>
<td>1.761</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>1.368</td>
<td>1.774</td>
<td>1.499</td>
<td>5.026</td>
</tr>
<tr>
<td>20°s⁻¹ eccentric</td>
<td>1.458</td>
<td>1.739</td>
<td>1.319</td>
<td>3.076</td>
</tr>
</tbody>
</table>

In most cases the moment arm correction factor \( x_2 \) was between 1.0 and 2.0 (Table 5.14). However for the shoulder (tumbling) the two eccentric trials gave very high moment arm correction factors. These were clearly incorrect and were replaced with the average of the other trials for that movement. The moment arm correction factors for the
two eccentric trials were very high due to having only one turning point in the joint angle data which was therefore not very well defined.

Ranges of motion for each joint

The ranges of motion over which the muscle data were collected at each joint had been chosen so as to exceed the approximate observed ranges required for the performance of the Hecht vault (shoulder only), and for tumbling (ankle, knee, hip and shoulder) from video (Table 5.15). The ranges of motion chosen were greater than required so that, after the data had been processed and the isokinetic parts selected for analysis, there would still be sufficient ranges of motion for the simulation models. The simulation models required as input joint torque as a function of joint angle and joint angular velocity. Therefore the torque data were required for any possible angle or angular velocity. Table 5.15 shows the chosen range of motion for the crank, the actual isokinetic range of motion for the crank and the joint, and the observed ranges of motion from video (Chapter 4).

Table 5.15. Summary of ranges of motion at each joint [°]

<table>
<thead>
<tr>
<th>range of motion</th>
<th>ankle</th>
<th>knee</th>
<th>hip</th>
<th>shoulder (vaulting)</th>
<th>shoulder (tumbling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>chosen</td>
<td>92-136</td>
<td>94-167</td>
<td>90-185</td>
<td>22-110</td>
<td>61-166</td>
</tr>
<tr>
<td>crank (min)</td>
<td>106-117</td>
<td>113-145</td>
<td>112-163</td>
<td>43-92</td>
<td>.78-131</td>
</tr>
<tr>
<td>crank (max)</td>
<td>92-136</td>
<td>94-167</td>
<td>90-185</td>
<td>22-110</td>
<td>61-166</td>
</tr>
<tr>
<td>crank (mean)</td>
<td>99-129</td>
<td>102-159</td>
<td>98-177</td>
<td>29-103</td>
<td>68-156</td>
</tr>
<tr>
<td>joint (min)</td>
<td>-</td>
<td>148-169</td>
<td>155-182</td>
<td>55-87</td>
<td>141-162</td>
</tr>
<tr>
<td>joint (max)</td>
<td>-</td>
<td>123-188</td>
<td>136-195</td>
<td>26-110</td>
<td>107-197</td>
</tr>
<tr>
<td>joint (mean)</td>
<td>-</td>
<td>136-179</td>
<td>145-190</td>
<td>46-101</td>
<td>128-173</td>
</tr>
<tr>
<td>video ranges</td>
<td>65-144</td>
<td>123-170</td>
<td>110-178</td>
<td>45-100</td>
<td>135-169</td>
</tr>
</tbody>
</table>

The isokinetic range of motion of each trial was smaller than the chosen range of motion as it took time for the dynamometer to reach the chosen angular velocity. Figure 5.27 shows the effect of the angular velocity on the isokinetic range of motion. It can be seen that the faster the angular velocity the smaller the isokinetic range of motion. In
addition there is a difference between the range of motion for the crank and joint. These differences are due to relative movement between the crank and limb during each trial.

Figure 5.27. Example of change in range of motion with angular velocity for the knee.

*Extrapolation of torque / angle relationship*

Although the dynamometer could achieve any range of motion specified, it was found from the analysis of the data obtained that the isokinetic ranges of motion obtained were insufficient for the simulation models (Table 5.16). The reasons for the lack of data were:

1. The range of motion for the crank was selected from video of vaulting and tumbling and then checked with the subject to ensure that it was okay. If the subject was unhappy with the selected ranges of motion, the subject was given the opportunity to reduce the range of motion used. The subject had the final choice in the range of motion used, although where the subject did want to go through the whole range of motion required the maximum range of motion the subject was happy with was used. Therefore in some cases (e.g. ankle) the required range of motion was not achieved.

2. The chosen range of motion for each joint was for the crank and not the joint. The analysis of the data found that the crank range of motion was not always equivalent to joint range of motion. Therefore for some trials the range of motion was not sufficient as although the crank moved through the required range the joint did not. This effect also proved advantageous for some trials as the crank did not move through a sufficiently large range but the joint did.
3. For all the muscle data collected only the isokinetic parts of each trial were used for analysis. This resulted in smaller ranges of isokinetic motion for the trials at faster angular velocities as the time taken to reach the pre-set angular velocity was longer (Figure 5.27), and therefore the angle moved through was greater before the crank reached the pre-set angular velocity.

Table 5.16. Ranges of motion from isokinetic data and actual performances [°]

<table>
<thead>
<tr>
<th>joint</th>
<th>ankle</th>
<th>knee</th>
<th>hip</th>
<th>shoulder (vaulting)</th>
<th>shoulder (tumbling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>joint (min)</td>
<td>148-169</td>
<td>155-182</td>
<td>55-87</td>
<td>141-162</td>
<td></td>
</tr>
<tr>
<td>(isokinetic data)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>observed range</td>
<td>65 - 144</td>
<td>123 - 170</td>
<td>110 - 178</td>
<td>45 - 100</td>
<td></td>
</tr>
<tr>
<td>(video)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>angle range for</td>
<td>65 - 145</td>
<td>120 - 190</td>
<td>110 - 190</td>
<td>20 - 100</td>
<td></td>
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<tr>
<td>surface fits</td>
<td></td>
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<td></td>
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</tbody>
</table>

For all the joints tested some extrapolation of the torque / angle relationship was required. A number of methods were considered to extend the torque / angle data:

1. Extrapolate with a constant torque value equal to last known torque value
2. Fit a spline to the torque angle data and extrapolate outside the range
3. Fit a least squares quadratic function through the torque / angle data

The first method although simple did not try to approximate the torque / angle relationship but was stable and robust. The second method of fitting a cubic was found to be very dependent on the endpoint conditions and resulted in unacceptable extrapolations. The last method of fitting a least squares quadratic to the torque angle data had the flexibility to give a quadratic shape to the torque / angle plot while being constrained from giving spurious results. To prevent the quadratic function from giving large discontinuities at the endpoints, the fitted quadratic was forced to pass through the first and last data points of each torque / angle time history. Constraining the quadratic function to go through the first and last points reduced the number of unknown
coefficients from three to one. The procedure for fitting quadratic functions through the
torque / angle data for each trial is described below:

Fit a straight line through the first and last data points \((a, y(a))\) and \((b, y(b))\) in the
torque / angle data series:

\[
y_s = mx + c \tag{5.22}
\]

where

\[
m = \frac{y(b) - y(a)}{b - a}
\]

and

\[
c = y(b) - mb
\]

Subtract equation (5.22) from the measured torque values \(y_t\) to give a series of data points
starting at zero when the angle equals \(a\) and finishing at zero when the angle equals \(b\):

\[
y = y_t - y_s
\]

Translating the torque data series to start and finish at zero torque resulted in a
simpler equation for the quadratic function passing through the first and last points of the
data series (equation (5.23)).

\[
f_k(x) = k(x - a)(x - b) = k\phi(x) \tag{5.23}
\]

where \(k\) = unknown coefficient to be solved for.

The objective function \(f\) was defined as the sum of squares difference between the
translated torque values and the quadratic function \(f_k(x)\):

\[
f = \min \sum [y - f_k(x)]^2 = \min \sum [y - k\phi(x)]^2
\]

The objective function \(f\) had a unique minimum when:

\[
\frac{\partial f}{\partial k} = 0 \tag{5.24}
\]

\[
2 \sum [y - k\phi(x)]\phi(x) = 0
\]

which rearranges to give

\[
k = \frac{\sum y\phi(x)}{\sum \phi(x)\phi(x)} \tag{5.25}
\]
Once the value of $k$ had been obtained the quadratic equation (5.23) was defined. The quadratic equation (5.23) and the straight line equation (5.22) could then be used to obtain the value of the torque $y_q$ at any angle:

$$y_q = f_k(x) + y_s$$ (5.26)

Within the range of motion for each trial a cubic spline was fitted to the data and interpolated to obtain torque / angle time histories for each trial.

Outside the range of motion the quadratic fit might be expected to be stable, and constrained from giving spurious extrapolations. However it was found that where extrapolation was required over a large range of angles, the quadratic fit did not do very well giving too high or low torque values. This resulted in distorted torque / angle relationships compared to the torque / angle relationship within the range of motion. To prevent the extrapolation from distorting the torque / angle relationship, the safest method of extrapolating the torque angle data with a constant value for each trial equal to the last known value in that trial was used. This method was found to be the most stable and robust and did not distort the torque / angle relationship.

For each trial the torque / angle relationship was determined at 100 increments over the required range.

**Special cases**

Due to the nature of the experimentally derived data there were some unexpected rogue data points. For the muscle torque data collected the majority of the data looked sensible. However for the knee the trials at -150 and -200°s⁻¹ looked spurious and incorrect as the torque values were much lower than the other eccentric trials. The two spurious trials were replaced with the average of the other six eccentric trials so as to not skew the eccentric data.

**Results and Discussion**

An ASCII data file consisting of the torque time history at 100 increments over the required angle range for each trial was produced for input to the muscle model. Within the range of real data for each trial a cubic spline was interpolated to give the torque value and outside the angle range the last known torque value was used. The number of 100 angles was chosen as an arbitrary figure which did not result in a huge data file but was
considered sufficiently large so that information would not be lost. The angular velocity for each trial was calculated as the average angular velocity for the concentric and eccentric isokinetic sections of each trial.

MUSCLE MODEL

The muscle model developed used a ‘black box’ approach fitting the experimental angle, angular velocity and torque data to give a smooth surface of maximum torque as a function of angle and angular velocity.

Introduction

The muscle torque data obtained from the dynamometer were in the range of angular velocities for the crank from $-250$ to $+250\text{s}^{-1}$ for each joint tested. However for sports movements angular velocities as high as $800\text{s}^{-1}$ have been recorded. Therefore if torque data obtained on an isokinetic dynamometer is to be used to predict torques produced during sports movements a method for extrapolating the muscle torque data to a much higher range of angular velocities is required. The problem is aggravated when the angular velocity of the crank is converted to an angular velocity of the joint as the range of angular velocities for the joint is smaller than the crank.

The following section will set out the methods used to fit the torque data collected on the isokinetic dynamometer so that the torque produced at the joints could be predicted for the simulation models.

Method

A number of methods were attempted to fit the torque / angle / angular velocity data for each joint tested:

1. Cubic spline
2. Hill exponential fit
3. Edman double exponential fit

Cubic spline

The torque / angular velocity data was fitted using a cubic spline with error estimates for each point calculated from the repeated measurements taken at the fastest
and slowest angular velocities (Table 5.17). The error estimates were calculated as the root mean square difference between the repeated trials at $20^\circ s^{-1}$ and $250^\circ s^{-1}$:

$$
\sigma = \sqrt{\frac{\sum_{i=1}^{n_a} \sum_{j=1}^{n_r} \left( T_{ij} - \overline{T_i} \right)^2}{n_a (n_r - 1)}}
$$

(5.27)

where $n_a = 100$ (the number of angles in the range of motion)

$n_r = $ the number of repeated trials at an angular velocity

$T_{ij} = $ actual torque for known angle and trial at pre-set angular velocity

$\overline{T_i} = $ average torque over $n_r$ repeated trials at a known angle

$\sigma = $ error estimate

Where one trial was collected at a given angular velocity the error estimate was calculated as a percentage of the error estimates at $20$ and $250^\circ s^{-1}$ (Table 5.17).

The problem with fitting splines was that there was no control over the direction the spline would extrapolate to outside the known interval of angular velocities. Therefore using splines to extrapolate the torque / angular velocity data resulted in unrealistic torque values outside the range of known angular velocities (Figure 5.28).

![Figure 5.28. Extrapolation of the torque / angular velocity relationship (cubic spline).](image)

Fitting a cubic spline to the torque / angular velocity data in order to extrapolate to faster eccentric and concentric angular velocities was not suitable. A method was required which constrained the shape of the torque / angular velocity relationship outside the range of angular velocities tested as well as smoothing the torque data obtained.
Table 5.17. Example of error estimates for each angular velocity based upon repeated trials

<table>
<thead>
<tr>
<th>pre-set angular velocity [°s⁻¹]</th>
<th>weighted formula to calculate error estimates</th>
<th>measured angular velocity [°s⁻¹]</th>
<th>error estimate [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-250</td>
<td>1.0σ₋₂₅₀</td>
<td>-164.7</td>
<td>35.4</td>
</tr>
<tr>
<td>-200</td>
<td>0.8σ₋₂₅₀ + 0.2σ₋₂₀</td>
<td>-142.7</td>
<td>35.4</td>
</tr>
<tr>
<td>-150</td>
<td>0.6σ₋₂₅₀ + 0.4σ₋₂₀</td>
<td>-110.2</td>
<td>35.4</td>
</tr>
<tr>
<td>-100</td>
<td>0.4σ₋₂₅₀ + 0.6σ₋₂₀</td>
<td>-75.4</td>
<td>35.4</td>
</tr>
<tr>
<td>-50</td>
<td>0.2σ₋₂₅₀ + 0.8σ₋₂₀</td>
<td>-39.4</td>
<td>35.4</td>
</tr>
<tr>
<td>-20</td>
<td>1.0σ₋₂₀</td>
<td>-14.6</td>
<td>35.4</td>
</tr>
<tr>
<td>20</td>
<td>1.0σ₂₀</td>
<td>17.2</td>
<td>21.5</td>
</tr>
<tr>
<td>50</td>
<td>0.2σ₂₅₀ + 0.8σ₂₀</td>
<td>39.5</td>
<td>19.5</td>
</tr>
<tr>
<td>100</td>
<td>0.4σ₂₅₀ + 0.6σ₂₀</td>
<td>75.6</td>
<td>17.5</td>
</tr>
<tr>
<td>150</td>
<td>0.6σ₂₅₀ + 0.4σ₂₀</td>
<td>110.8</td>
<td>15.5</td>
</tr>
<tr>
<td>200</td>
<td>0.8σ₂₅₀ + 0.2σ₂₀</td>
<td>144.1</td>
<td>13.4</td>
</tr>
<tr>
<td>250</td>
<td>1.0σ₂₅₀</td>
<td>175.7</td>
<td>11.4</td>
</tr>
</tbody>
</table>

Shape of the torque / angular velocity relationship

Many authors have studied the force / velocity relationship in muscle. The two main researchers are Hill and Edman. Hill (1938) found a hyperbolic relationship for the force / velocity relationship in whole muscle (Figure 5.29) while Edman found a double hyperbolic relationship (Figure 5.30) for the force / velocity relationship in single fibres.

Hill fitted a hyperbolic function to the force / velocity data obtained (equation (5.28)) which was only valid for concentric muscle actions, and Edman used a modified version of Hill’s equation to fit his experimental force / velocity data.

Hill’s Equation: 

\[(P + a)(V + b) = (P₀ + a)b\]  \hspace{1cm} (5.28)

Where: \(P\) = force, \(V\) = velocity of shortening and \(a, b, P₀\) = constants
The muscle data collected in this study showed a closer resemblance to the results of Edman than Hill with a plateau in the torque values at low concentric angular velocities. The general characteristics of the double hyperbolic relationship of Edman were:

1. Plateau in torque values at high eccentric velocities
2. Steep drop around zero velocity
3. Plateau in torque values at low concentric velocities
4. Second drop (less steep) at high concentric velocities

_Fitting a function to torque / angular velocity data_

A function was required which would adequately fit the experimental data collected. An exponential function with the general characteristics of the double hyperbolic...
relationship of Edman (1988) was used as this resembled the experimental data better than a Hill type hyperbola.

The exponential function used to fit the experimental data was developed from a simpler Hill type relationship (Figure 5.29). However Hill’s equation (5.28) was only defined for the concentric phase so an exponential function was used which was valid for the whole range of velocities:

Three parameter exponential relationship

An exponential function was used to extend the force/velocity relationship of Hill to the complete range of eccentric and concentric velocities which tended to a plateau \( a \) at high eccentric velocities, and an asymptote to zero torque at high concentric velocities (Figure 5.31):

\[
T = \frac{a}{1 + ce^{p0}}
\]  

(5.29)

where \( a, c \) and \( p \) are positive constants, \( T \) = torque and \( \omega \) = angular velocity

The three parameter function differed from Hill’s equation in that the three parameter function tended to zero torque as the concentric angular velocity increased, whereas the Hill relationship reached zero force at the maximum velocity of shortening.

Six/seven parameter exponential relationship

The three parameter exponential function was extended to a six parameter exponential function with the flexibility to represent the double hyperbolic relationship of Edman (1988). The six parameter function consisted of two exponential functions:

exponential with four parameters:

\[
T = \frac{a + be^{p0}}{1 + ce^{p0}}
\]  

(5.30)

exponential with two parameters:

\[
T = \frac{1}{1 + d e^{q0}}
\]  

(5.31)

where: \( a, b, c, d, p, \) and \( q \) are positive constants, \( T \) = torque and \( \omega \) = angular velocity

Equation (5.30) with four parameters was similar to the three parameter equation used to approximate Hill’s equation, however the four parameter exponential tended to
b/c as $\omega$ increased (Figure 5.32). Equation (5.31) with two parameters tended to unity as $\omega$ decreased and to zero as $\omega$ increased (Figure 5.33).

Figure 5.31. Three parameter torque / angular velocity relationship.

Figure 5.32. General shape of the four parameter exponential.

Figure 5.33. General shape of the two parameter exponential.
The four and two parameter exponential functions were then combined to give a double exponential relationship (Figure 5.34) with six parameters:

\[
T = \frac{a + be^{\omega_0}}{(1 + c \cdot e^{\omega_0})(1 + d \cdot e^{\omega_0})} \tag{5.32}
\]

where \(a, b, c, d, p,\) and \(q\) are constants, \(T = \) torque and \(\omega = \) angular velocity

Figure 5.34. General shape of the six parameter double exponential function.

The six parameter function had the general shape of Edman’s double hyperbolic data with the following characteristics:

1. Plateau at high eccentric velocities
2. Steep drop through isometric
3. Plateau at low concentric velocities
4. Second drop at high concentric
5. Asymptote to zero torque at high concentric velocities

An alternative seven parameter function was also formulated which allowed the function to reach zero torque by subtracting a constant \(f\) from the six parameter equation:

\[
T = \frac{a + be^{\omega_0}}{(1 + c \cdot e^{\omega_0})(1 + d \cdot e^{\omega_0})} - f \tag{5.33}
\]

where \(a, b, c, d, f, p,\) and \(q\) are constants, \(T = \) torque and \(\omega = \) angular velocity
The difference between the six and seven parameter fits was with the shape of the curve at high concentric angular velocities. The seven parameter fit had the potential to cross the x axis, where as the six parameter fit had an asymptote to the x axis at high concentric angular velocities and therefore never reached zero torque. To test the six and seven parameter functions, Edman’s (1988) data was used.

**Testing the six and seven parameter functions on Edman’s (1988) data**

To obtain a data set from Edman (1988) a photocopy of the force / velocity graph on page 315 in Edman (1988) was fixed to the digitising tablet (HR48 TDS) with the axes of the graph approximately horizontal and vertical. The data points were then digitised and an ASCII data file of the digitised points obtained. Examining the digitised points showed that the graph had been placed slightly off vertical so a rotational matrix was used to rotate all the digitised points:

\[
\begin{bmatrix}
X \\
Z
\end{bmatrix} = \begin{bmatrix}
\cos\alpha & \sin\alpha \\
-\sin\alpha & \cos\alpha
\end{bmatrix} \begin{bmatrix}
x \\
z
\end{bmatrix}
\]

(5.34)

where \( \alpha \) = angle rotated digitised points through
\( x, z \) = digitised points
\( X, Z \) = rotated points

The rotation of the digitised data \( \alpha \) was less than 0.5°. The digitised data were then scaled to give pseudo muscle force values (Figure 5.35).

The six and seven parameter functions were then fitted to Edman’s data by minimising the difference between each function and Edman’s data. The least squares solution was found by varying the coefficients of the function using the Simplex optimisation routine (Nelder and Mead, 1965) until the optimum solution was found. Figure 5.36 and Figure 5.37 show the best fits to the Edman data with six and seven parameter functions.

The six and seven parameter fits to Edman’s data gave very similar results, demonstrating the flexibility of the function to fit a double hyperbolic relationship. The main difference between the curves was that the six parameter function tended to zero force at high concentric velocities, where as the seven parameter function did not reach zero force.
Although both functions fitted the complete data set from Edman satisfactorily, the isokinetic torque / angular velocity data obtained was not over the complete isokinetic range. Therefore to test the six and seven parameter fits on a similar range of velocities Edman’s data was truncated with the force values at the eight fastest velocities (Figure 5.38) not used in the fitting of the two functions.
Figure 5.38. Truncated force / velocity data set from Edman (1988).

Figure 5.39. Optimum six parameter fit to Edman’s truncated data.

Figure 5.40. Optimum seven parameter fit to Edman’s truncated data.

From Figure 5.39 and Figure 5.40 the six parameter function offers greater stability since it tends to zero at high concentric velocities, where as the seven parameter function tends to the value of $-f$. If $f$ is positive the function crosses the x axis and reaches zero torque, however if the value of $f$ is negative the function can remain at an unrealistically
high force at high concentric velocities. The six parameter equation was therefore preferred for fitting the data as it was less likely to give spurious results.

**Extension of the torque / angular velocity relationship to include angle dependence**

Two options were considered to extend the 6 parameter function to incorporate angle changes as well as angular velocity changes:

- Fit a 6 parameter function to each torque / angular velocity plot and link the independent functions at each angle.
- Fit a global surface function incorporating angle and angular velocity changes.

Fitting independent six parameter functions to each torque / angular velocity relationship throughout the range of motion, fitted each torque / angular velocity relationship very well. However the fits at each angle were independent of the other fits at different angles, therefore there was the potential for large discrepancies between one fit and an adjacent one. By fitting a surface to all the data at each joint at once resulted in a global optimum which used all the data to give the best global fit. A global fit was therefore preferred to the separate independent fits.

To determine the type of function required for a global fit, the six parameters in the torque / angular velocity relationship were each plotted as a function of angle. It was found that a quadratic relationship would be sufficient to fit each parameter as a function of joint angle. Expressing each of the six parameters as a quadratic function of the joint angle increased the number of parameters to 18 parameters:

\[
T = \frac{a + be^{p\omega}}{(1 + ce^{q\omega})(1 + de^{q\omega})}
\]

with:

\[
\begin{align*}
a &= x_1\theta^2 + x_2\theta + x_3 \\
b &= x_4\theta^2 + x_5\theta + x_6 \\
c &= x_7\theta^2 + x_8\theta + x_9 \\
d &= x_{10}\theta^2 + x_{11}\theta + x_{12} \\
p &= x_{13}\theta^2 + x_{14}\theta + x_{15} \\
q &= x_{16}\theta^2 + x_{17}\theta + x_{18}
\end{align*}
\]

where: \(x_1 - x_{18}\) = parameters, \(\omega\) = joint angular velocity, \(\theta\) = joint angle and \(T\) = torque
**Optimisation procedure**

For each of the functions used to fit the torque/ angular velocity relationship, a least squares optimisation method was used to obtain the optimum set of coefficients. To minimise the objective function (equation (5.36)) two non-linear optimisation routines were used (Simplex and Simulated Annealing).

\[
S = \sum_{i=1}^{N} \left[ T_i - f(\theta_i, \omega_i) \right]^2 
\]  
(5.36)

where \( T_i = \) The measured torque values and \( f = \) The function used to fit the data

**Constraints to the optimisation procedure**

After optimising the 18 parameters for the surface fits it was found that the surface function did not always keep its general specified shape. Examining the results showed that two constraints were required. Firstly \( a, b, c, d, p \) and \( q \) were constrained to be positive by giving the objective function a value of 500 if any of the 6 parameters were negative. Secondly it was found that in some cases the function did not keep decreasing as the angular velocity went from high eccentric to isometric to high concentric. The reason for this was associated with the values of the parameters \( a, b \) and \( c \). In equation (5.30) for a four parameter exponential function, the function tended to \( b/c \) as the angular velocity increased. However if \( a < b/c \), the function increased to the plateau at \( b/c \) rather than decreasing (Figure 5.41). To prevent an optimisation from allowing \( a < b/c \), the objective function was forced equal to 500 if \( a < b/c \), forcing the optimisation to find a solution where the torque decreased as the angular velocity increased.

![Figure 5.41. Four parameter equation with \( a < b/c \) and \( a > b/c \).](image-url)
Although constraining the function by setting the objective function to 500 resulted in discontinuities in the value of the objective function for some combinations of parameters the Simulated Annealing package always found a sensible solution. It was not possible to know whether the solution found in each case was optimal but viewing a graph of the surface confirmed whether the solution looked sensible.

**Alternative forms of 18 parameter equation**

The 18 parameters were found for each joint. In some cases the equation became undefined due to the exponential function becoming too large. To prevent the function from being undefined two alternatives of the same equation were used depending on the values of $p_\omega$ and $q_\omega$:

$$\begin{align*}
  p_\omega < 0 \text{ and } q_\omega < 0 & \quad T = \frac{a + be^{p_\omega}}{(1 + ce^{p_\omega})(1 + de^{q_\omega})} \tag{5.37} \\
  p_\omega > 0 \text{ and } q_\omega > 0 & \quad T = \frac{ae^{-(p+q)_\omega} + be^{-q_\omega}}{(e^{-p_\omega} + c)(e^{-q_\omega} + d)} \tag{5.38}
\end{align*}$$

**Range of data for surface fit**

The surface fits for each joint were optimised over different ranges of motion: firstly over a range where there was no extrapolation of the torque / angle data for any trial (minimum range), secondly over the required range for the simulation model (maximum range) and thirdly over a range of motion which used half real and half extrapolated data at the end points (average range). At all the joints the surface fits over the minimum range resulted in surface fits which looked sensible and gave small RMS errors between the surface fit and raw data. Over the maximum range the surface fits at most of the joints were distorted with larger RMS errors. The average range of motion was chosen, for the surface fits, as it used as much real data as possible without distorting the shape of the surface fit and increasing the RMS difference between the surface fit and the raw data (Figure 5.42 - Figure 5.46).
Table 5.18. RMS differences between the surface function and the raw data

<table>
<thead>
<tr>
<th></th>
<th>minimum range</th>
<th>maximum range</th>
<th>average range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>range</td>
<td>RMS</td>
<td>range</td>
</tr>
<tr>
<td>ankle</td>
<td>107° - 118°</td>
<td>12.7°</td>
<td>65° - 145°</td>
</tr>
<tr>
<td>knee</td>
<td>148° - 169°</td>
<td>24.5°</td>
<td>120° - 190°</td>
</tr>
<tr>
<td>hip</td>
<td>155° - 182°</td>
<td>39.5°</td>
<td>110° - 200°</td>
</tr>
<tr>
<td>shoulder (vaulting)</td>
<td>55° - 87°</td>
<td>16.6°</td>
<td>20° - 110°</td>
</tr>
<tr>
<td>shoulder (tumbling)</td>
<td>142° - 162°</td>
<td>22.7°</td>
<td>100° - 200°</td>
</tr>
</tbody>
</table>

For the shoulder (tumbling) the average range did not give a good fit so the range was reduced to 142° - 173°. This range gave a much better surface fit with an RMS of 21.2° which was actually lower than the minimum range fit.

For the ankle the average range did not give a good fit as the torque at high concentric angular velocities did not drop to zero for some joint angles. The minimum range was therefore used which did drop to zero for all angles within the range (Figure 5.42).

Extending the surface function for use in the simulation model

The surface function was well defined within the range of motion of the data fitted. However the function was not well defined outside the range of motion used, as the function was not constrained in the torque / angle plane at a fixed angular velocity. Although the surface fit had been extended by extrapolation it would be possible for a simulation to require the torque outside the extended range of the surface function. To prevent the surface function from ever giving spurious results, the surface function was constrained to the endpoint value at that joint angle outside the range. This resulted in the function being well defined for any combination of angle and angular velocities. Therefore an unrealistically high or low torque value would not be used if the simulation model required a torque value for a joint angle outside the range used for the surface fit.
Results

The output from fitting the experimental data was a data file containing the 18 parameters defining the torque / angle / angular velocity surface for each joint (Table 5.19). In the output files each parameter was written to 15 decimal places which resulted in the torque values being accurate to 7 decimal places when the surface was reconstructed from the parameter values.

Table 5.19. Optimised 18 parameters for each joint

<table>
<thead>
<tr>
<th>parameter</th>
<th>ankle</th>
<th>knee</th>
<th>hip</th>
<th>shoulder (vaulting)</th>
<th>shoulder (tumbling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>-41.0210</td>
<td>-26.3796</td>
<td>-51.7811</td>
<td>-35.2491</td>
<td>-35.9371</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-29.0466</td>
<td>42.2015</td>
<td>52.4603</td>
<td>81.9888</td>
<td>-2.2331</td>
</tr>
<tr>
<td>$x_3$</td>
<td>344.7666</td>
<td>294.5829</td>
<td>548.2718</td>
<td>157.0884</td>
<td>414.3696</td>
</tr>
<tr>
<td>$x_4$</td>
<td>-41.6529</td>
<td>-65.3208</td>
<td>-20.8836</td>
<td>5.8163</td>
<td>-100.4339</td>
</tr>
<tr>
<td>$x_5$</td>
<td>51.1625</td>
<td>91.0452</td>
<td>-11.9846</td>
<td>10.4583</td>
<td>1000.0000</td>
</tr>
<tr>
<td>$x_6$</td>
<td>164.1148</td>
<td>547.8304</td>
<td>269.3935</td>
<td>15.8065</td>
<td>1000.0000</td>
</tr>
<tr>
<td>$x_7$</td>
<td>-0.1286</td>
<td>-0.5501</td>
<td>0.0641</td>
<td>0.4062</td>
<td>22.4606</td>
</tr>
<tr>
<td>$x_8$</td>
<td>0.8824</td>
<td>3.5272</td>
<td>-0.4342</td>
<td>-0.6455</td>
<td>-19.6464</td>
</tr>
<tr>
<td>$x_9$</td>
<td>-0.1354</td>
<td>-3.6646</td>
<td>1.0251</td>
<td>0.4411</td>
<td>-60.7471</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>0.0000</td>
<td>0.0076</td>
<td>-0.0027</td>
<td>-0.0158</td>
<td>0.1594</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>0.0000</td>
<td>-0.0409</td>
<td>0.0161</td>
<td>0.0407</td>
<td>0.1143</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>0.0000</td>
<td>0.0553</td>
<td>-0.0207</td>
<td>-0.0177</td>
<td>-1.2625</td>
</tr>
<tr>
<td>$x_{13}$</td>
<td>-178.1169</td>
<td>-42.1302</td>
<td>-196.5371</td>
<td>-246.1986</td>
<td>11.1390</td>
</tr>
<tr>
<td>$x_{14}$</td>
<td>9.8581</td>
<td>-68.0807</td>
<td>688.0178</td>
<td>27.1609</td>
<td>-260.1231</td>
</tr>
<tr>
<td>$x_{15}$</td>
<td>882.6583</td>
<td>624.7731</td>
<td>-109.0162</td>
<td>723.3179</td>
<td>700.3085</td>
</tr>
<tr>
<td>$x_{16}$</td>
<td>55.2415</td>
<td>-0.8728</td>
<td>0.3058</td>
<td>0.4612</td>
<td>4.9355</td>
</tr>
<tr>
<td>$x_{17}$</td>
<td>-218.1757</td>
<td>5.5386</td>
<td>0.3500</td>
<td>-0.9882</td>
<td>-24.4664</td>
</tr>
<tr>
<td>$x_{18}$</td>
<td>216.1268</td>
<td>-6.2977</td>
<td>-0.6456</td>
<td>1.9229</td>
<td>30.3214</td>
</tr>
</tbody>
</table>
Figure 5.42. 3D surfaces of the torque / angle / angular velocity relationship for the ankle.
Figure 5.43. 3D surfaces of the torque / angle / angular velocity relationship for the knee.
Figure 5.44. 3D surfaces of the torque / angle / angular velocity relationship for the hip.
Figure 5.45. 3D surfaces of the torque / angle / angular velocity relationship for the shoulder (vaulting).
raw data

Figure 5.46. 3D surfaces of the torque / angle / angular velocity relationship for the shoulder (tumbling).
Using surfaces in the simulation models

The surface fits obtained for each joint predicted the maximum torque produced at one joint. The simulation model required a prediction of the maximum torque $T$ produced by left and right sides of the body together. To do this the activation function $A(t)$ for the surface fits was allowed a maximum value of 2.0 which would correspond to double the torque produced around a single joint:

$$T = A(t)T(\theta, \omega)$$

where $T(\theta, \omega) =$ torque produced by 18 parameter surface

The surface fits for the knee, hip and shoulder (tumbling) were used directly in the simulation model of tumbling as the simulation model could calculate the joint angle directly. For the ankle and shoulder (vaulting) surfaces, however, corrections needed to be made as the torque generators at these joints included a series elastic element as well as a contractile element. In addition a further correction needed to be made for the ankle as the surface was in terms of the crank angle and not the joint angle.

Muscle-tendon correction

The muscle model for the ankle (tumbling) and the shoulder (vaulting) consisted of series elastic and contractile elements:

![Figure 5.47. Set-up for the muscle-tendon complex at the ankle and shoulder (vaulting).](image)

For each iteration of the simulation model the contractile element angle $\theta_{ce}$ and the series elastic element angle $\theta_{ee}$ were known and the maximum torque that could be produced by the contractile element needed to be calculated. The 18 parameter surfaces fitted to the isokinetic data defined the torque that could be produced by the contractile element as a function of the joint angle $\theta_{iso}$ and angular velocity $\omega_{iso}$ during isokinetic
conditions. The joint angle $\theta_{iso}$ from the surface fit, however, was not equal to the joint
angle $\theta$ in Figure 5.47 as in the simulation model the contractile and series elastic
elements were both accelerating.

The simulation of the muscle-tendon complex earlier in this chapter demonstrated
that the joint angular velocity was approximately equal to the contractile element angular
velocity throughout the isokinetic parts of each trial used to fit the surfaces. However, the
joint angle was not equal to the angle of the contractile element and therefore the joint
angle under isokinetic conditions needed to be back-calculated from the contractile
element angle.

The known contractile element angle was used as an initial estimate of the joint
angle. This allowed an initial estimate of the torque produced by the contractile element
to be calculated as:

$$T_i = f(\theta_{ce}, \omega_{ce})$$

where $f = 18$ parameter surface, $\theta_{ce}$, and $\omega_{ce} =$ contractile angle and angular velocity

The torque produced by the contractile element equals the torque in the series elastic
element during an isokinetic trial as the contractile and series elastic elements are not
changing velocity. Thus an initial estimate of the extension of the series elastic element
can be calculated from the initial estimate of the torque $T_i$ and the known stiffness $k$ of
the tendon:

$$\theta_{eei} = \frac{T_i}{k}$$

The joint angle $\theta_{iso}$ can then be estimated from the contractile element angle and
the series elastic element angle:

- ankle: $\theta_{iso} = 2\pi - (\theta_{ce} + \theta_{eei})$
- shoulder: $\theta_{iso} = \pi - (\theta_{ce} + \theta_{eei})$

A new estimate for the contractile element torque can then be estimated for the new
estimate of the joint angle $\theta_{iso}$ and contractile element angular velocity $\omega_{ce}$:

$$T_i = f(\theta_{iso}, \omega_{ce})$$

Repeating the above procedure improved the estimate for the series elastic element
angle at each iteration allowing the joint angle to be calculated for a known contractile
element angle and angular velocity. The method was found to converge within five
iterations for the prediction of the torque produced by the contractile element.
Ankle joint / crank correction

For the ankle joint an additional correction was needed to correct for the difference between the joint and crank angle, as the surface was fitted to the crank angle. A simple translation of the joint angle in the simulation model was made to give the equivalent crank angle. The magnitude of the translation was estimated from a photograph of the ankle secured to the Kin-Corn machine during testing (Figure 5.10). In the photograph the crank angle was $90^\circ$ and the joint angle was estimated as $119^\circ$. Therefore the relationship between the crank and joint angle was simply:

$$\text{crank angle} = \text{joint angle} - 29^\circ$$

As the correction to the joint / crank angle is a constant translation, the joint angular velocity is equivalent to the crank angular velocity. The joint / crank correction is an approximation, however, the shape of the surface for the ankle has very little angle dependence and therefore any errors in the correction should not affect the predicted torque greatly.

**SPRING PARAMETERS**

Spring parameters were required for the simulation models of Hecht vaulting and tumbling takeoffs. The required spring parameters were divided into two parts; spring parameters for the equipment and spring parameters for the shoulder joint. The spring parameters for the equipment were required to model the elastic behaviour of the vaulting horse / tumbling track, and for the shoulder spring parameters were required to model the movement of the shoulder joint centre during the vault takeoff.

Ideally to calculate the spring parameters an independent experimental test is required which will allow the spring parameters to be calculated. For the equipment it was possible to drop weights onto the surface of the horse / tumbling track to calculate spring stiffness and damping parameters. However for the shoulder joint attempts to experimentally determine the spring parameters proved difficult, so spring parameters for the shoulder joint were calculated from the film data of the gymnast performing the Hecht vault obtained for the evaluation of the simulation models using simulation and inverse dynamics (Chapter 6).

Horizontal and vertical stiffness and damping parameters were required for the vaulting horse and the tumbling track where the gymnast had contacted the horse / track during the data collected for evaluating the simulation models. The point where the
gymnast made contact with the horse / tumbling track was determined from the video / film recordings (Chapter 4) and used for the spring tests.

**Method**

**Data collection**

A shot of mass 7.05 kg was dropped / thrown on to the point where the gymnast had contacted the vaulting horse / tumbling track. All trials were video recorded using one camcorder (Small Sony Hi8) operating at 50 fields per second with a 3 ms shutter and a portable video recorder (Panasonic AG 7450) via a time coded box (IMP Electronics, V9000) on to SVHS. The camera was placed perpendicular to the plane in which the shot was thrown pointing approximately horizontal. 12 calibration markers on six calibration poles were placed around both the vaulting horse and tumbling track in known locations spanning approximately 0.5 m either side of the intended contact point of the shot with the vaulting horse / tumbling track. The locations of these calibration markers were measured, video recorded and then removed.

Two types of test were conducted to measure the stiffness and damping of the horse / tumbling track. Firstly the shot was dropped vertically from three different heights (0.4, 0.6 and 0.8 m) onto the vault / tumbling track. The shot was then thrown onto the vault / tumbling track to give the shot a horizontal and vertical velocity at impact which approximated the velocity of the gymnasts wrists / ankles as they contacted the vaulting horse / tumbling track.

**Data Analysis**

The video was digitised using the (Target software) Apex high resolution digitising system (Kerwin, 1995). Each of the 12 calibration points were digitised in five consecutive fields and the coordinate data used together with the known locations of the points to calculate the minimum least squares solution for the 11 DLT parameters (Abdel-Aziz and Karara, 1971):

\[
\begin{align*}
    u &= \frac{L_1x + L_2y + L_3z + L_4}{L_9x + L_{10}y + L_{11}z + 1} \\
    v &= \frac{L_5x + L_6y + L_7z + L_8}{L_9x + L_{10}y + L_{11}z + 1}
\end{align*}
\]
To reconstruct digitised locations \((u, v)\) required solving the two DLT equations from the single camera view for the three unknowns \((x, y, z)\). The shot was thrown in the \(x\)-\(z\) plane perpendicular to the optical axis of the camera. The \(y\) value was assumed to be constant throughout each trial and measured prior to testing giving two equations for two unknowns which were solved using a linear equation solver (Stewart, 1973).

For each trial the velocity at contact and takeoff from the horse / track and the duration of contact were required to calculate the horizontal and vertical stiffness and damping parameters:

The vertical velocity of the shot at contact and takeoff was calculated by differentiating a quadratic function fitted to the vertical displacement of the shot during the free flight phases before contact / after takeoff. The quadratic functions were then extrapolated to the calculated vertical height of the shot at contact / takeoff. The parameters of the best fit quadratic functions for both flight phases were calculated by partially differentiating the sum of squares difference between the shot location and each quadratic curve with respect to each of the parameters of the quadratic function giving three equations which would all equal zero at the minimum point. This gave three equations in three unknowns which were solved algebraically for the three parameters for a quadratic fit.

The horizontal velocity of the shot at contact and takeoff was calculated as the gradient to a straight line fitted to the horizontal displacement of the shot during the free flight phases before contact / after takeoff. The parameters of the best fit straight line were calculated for both flight phases by partially differentiating the sum of squares difference between the shot location and each straight line with respect to both parameters of the linear function giving two equations which would both equal zero at the minimum point. This gave two equations in two unknowns which were algebraically solved for the two parameters for a straight line fit.

The duration of contact was calculated as the difference between the times of contact and takeoff calculated from the extrapolation to points of contact and takeoff using a quadratic fit to the vertical displacement of the shot.

*Calculation of stiffness and damping parameters*

The stiffness and damping parameters were calculated by simulating the shot contacting a sprung surface and minimising the difference between the velocity of the
shot at takeoff and the duration of contact, given the velocity at touchdown. The shot was simulated as a two-dimensional mass - spring system (Figure 5.48).

\[ RX = -k_1x - k_2\dot{x} \]
\[ RZ = -k_3z - k_4\dot{z} \]

Figure 5.48. Two-dimensional mass - spring system.

The equations of motion for the mass - spring system were calculated using Newton's Second Law:

Resolving horizontally $\rightarrow$ 
\[ -k_1x - k_2\dot{x} = m\ddot{x} \quad (5.41) \]

Resolving vertically $\uparrow$ 
\[ -k_3z - k_4\dot{z} - mg = m\ddot{z} \quad (5.42) \]

rearranging gives:
\[ \ddot{x} = \frac{- (k_1x + k_2\dot{x})}{m} \quad (5.43) \]
\[ \ddot{z} = \frac{- (k_3z + k_4\dot{z} + mg)}{m} \quad (5.44) \]

where:

- $m$ = mass of shot
- $x, z$ = horizontal and vertical displacement of the sprung surface
- $\dot{x}, \dot{z}$ = horizontal and vertical velocity of the sprung surface
- $\ddot{x}, \ddot{z}$ = horizontal and vertical acceleration of the sprung surface
- $k_1, k_3$ = horizontal and vertical spring stiffness parameter
- $k_2, k_4$ = horizontal and vertical spring damper parameter
The objective function used weightings calculated so that a 10% change in spring stiffness and damping values resulted in an equal change in the three contributions to the value of the objective function:

\[ f = w_u (u_a - u_s)^2 + w_v (v_a - v_s)^2 + w_t (t_a - t_s)^2 \]  \hspace{1cm} (5.45)

where

- \( w \) = weighting factor
- \( u \) = horizontal velocity at takeoff
- \( v \) = vertical velocity at takeoff
- \( t \) = duration of contact

The subscripts \( a \) and \( s \) denoting the actual and simulated values respectively.

To minimise the objective function the Simulated Annealing algorithm (Goffe et al., 1994) was used which varied the four spring parameters until the optimum set of spring parameters were found for each trial.

**Results**

Table 5.20 - Table 5.25 show initial velocities at contact for the testing of the vaulting horse, the contact time, horizontal / vertical velocity at contact and takeoff for the vault and tumbling track respectively along with the calculated spring parameters.
Table 5.20. Initial velocities at contact for the testing of the vaulting horse

<table>
<thead>
<tr>
<th>trial</th>
<th>initial velocities [m.s(^{-1})]</th>
<th>vertical</th>
<th>horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.72</td>
<td>3.11</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-4.04</td>
<td>3.10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-4.04</td>
<td>2.84</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-3.94</td>
<td>2.77</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-4.00</td>
<td>2.97</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-4.02</td>
<td>3.25</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-3.48</td>
<td>3.34</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-4.00</td>
<td>3.13</td>
<td></td>
</tr>
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</table>

Table 5.21. Results from testing of the vaulting horse

<table>
<thead>
<tr>
<th>trial</th>
<th>velocities at takeoff [m.s(^{-1})]</th>
<th>vertical</th>
<th>horizontal</th>
<th>contact time [s]</th>
<th>score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>actual</td>
<td>simulated</td>
<td>actual</td>
<td>simulated</td>
<td>actual</td>
</tr>
<tr>
<td>1</td>
<td>2.77</td>
<td>2.77</td>
<td>1.76</td>
<td>1.76</td>
<td>0.046</td>
</tr>
<tr>
<td>2</td>
<td>3.05</td>
<td>3.05</td>
<td>1.74</td>
<td>1.74</td>
<td>0.043</td>
</tr>
<tr>
<td>3</td>
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<td>3.00</td>
<td>1.59</td>
<td>1.59</td>
<td>0.044</td>
</tr>
<tr>
<td>4</td>
<td>2.96</td>
<td>2.96</td>
<td>1.51</td>
<td>1.51</td>
<td>0.043</td>
</tr>
<tr>
<td>5</td>
<td>2.90</td>
<td>2.90</td>
<td>1.70</td>
<td>1.70</td>
<td>0.044</td>
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<tr>
<td>6</td>
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<td>3.03</td>
<td>1.84</td>
<td>1.84</td>
<td>0.044</td>
</tr>
<tr>
<td>7</td>
<td>2.68</td>
<td>2.68</td>
<td>1.92</td>
<td>1.92</td>
<td>0.046</td>
</tr>
<tr>
<td>8</td>
<td>2.94</td>
<td>2.94</td>
<td>1.74</td>
<td>1.75</td>
<td>0.043</td>
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</table>
Table 5.22. Calculated spring parameters for the vaulting horse

<table>
<thead>
<tr>
<th>trial</th>
<th>vertical stiffness [N.m⁻¹]</th>
<th>horizontal stiffness [N.m⁻¹]</th>
<th>vertical damping [N.s.m⁻¹]</th>
<th>horizontal damping [N.s.m⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>494078</td>
<td>92</td>
<td>139</td>
</tr>
<tr>
<td>2</td>
<td>24729</td>
<td>471075</td>
<td>77</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>33140</td>
<td>538127</td>
<td>95</td>
<td>171</td>
</tr>
<tr>
<td>4</td>
<td>27383</td>
<td>499332</td>
<td>83</td>
<td>131</td>
</tr>
<tr>
<td>5</td>
<td>31452</td>
<td>507906</td>
<td>97</td>
<td>157</td>
</tr>
<tr>
<td>6</td>
<td>24535</td>
<td>470591</td>
<td>77</td>
<td>35</td>
</tr>
<tr>
<td>7</td>
<td>31124</td>
<td>503768</td>
<td>80</td>
<td>165</td>
</tr>
<tr>
<td>8</td>
<td>26674</td>
<td>452240</td>
<td>88</td>
<td>167</td>
</tr>
<tr>
<td>mean</td>
<td>28842</td>
<td>492140</td>
<td>86</td>
<td>127</td>
</tr>
<tr>
<td>s.d.</td>
<td>3401</td>
<td>26871</td>
<td>8</td>
<td>54</td>
</tr>
</tbody>
</table>

Table 5.23. Initial velocities at contact for the testing of the tumbling track

<table>
<thead>
<tr>
<th>trial</th>
<th>initial velocities [m.s⁻¹]</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>2</td>
<td>-4.96</td>
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<tr>
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<td>-4.95</td>
</tr>
<tr>
<td>4</td>
<td>-5.13</td>
</tr>
<tr>
<td>5</td>
<td>-5.25</td>
</tr>
</tbody>
</table>
Table 5.24. Results from testing of the tumbling track

<table>
<thead>
<tr>
<th>trial</th>
<th>velocities at takeoff [m.s⁻¹]</th>
<th>contact time [s]</th>
<th>score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>vertical</td>
<td>horizontal</td>
<td></td>
</tr>
<tr>
<td>actual</td>
<td>simulated</td>
<td>actual</td>
<td>simulated</td>
</tr>
<tr>
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<td>1.87</td>
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<td>0.063</td>
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<tr>
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<td>1.74</td>
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<td>0.058</td>
</tr>
<tr>
<td>3</td>
<td>1.75</td>
<td>1.28</td>
<td>0.060</td>
</tr>
<tr>
<td>4</td>
<td>1.74</td>
<td>1.49</td>
<td>0.062</td>
</tr>
<tr>
<td>5</td>
<td>1.62</td>
<td>1.86</td>
<td>0.063</td>
</tr>
</tbody>
</table>

Table 5.25. Calculated spring parameters for the tumbling track

<table>
<thead>
<tr>
<th>trial</th>
<th>vertical stiffness [N.m⁻¹]</th>
<th>horizontal stiffness [N.m⁻¹]</th>
<th>vertical damping [N.s.m⁻¹]</th>
<th>horizontal damping [N.s.m⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
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<td>13487</td>
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<td>229</td>
<td>98</td>
</tr>
<tr>
<td>2</td>
<td>15108</td>
<td>375340</td>
<td>259</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>14544</td>
<td>338415</td>
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<td>119</td>
</tr>
<tr>
<td>4</td>
<td>13482</td>
<td>339714</td>
<td>254</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>12689</td>
<td>285628</td>
<td>272</td>
<td>116</td>
</tr>
<tr>
<td>mean</td>
<td>13862</td>
<td>321572</td>
<td>253</td>
<td>73</td>
</tr>
<tr>
<td>s.d.</td>
<td>959</td>
<td>43545</td>
<td>16</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 5.26 / Table 5.27 show a comparison between the spring parameters calculated for the testing of the vaulting horse and tumbling track used with standard data obtained from Continental for the vaulting horse and gymnastic floor and also from a preliminary study testing a new vaulting horse from Gymnova.
Table 5.26. Comparison of spring parameters for the vaulting horse

<table>
<thead>
<tr>
<th></th>
<th>own testing</th>
<th>Gymnova vault</th>
<th>Continental testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertical stiffness</td>
<td>28842</td>
<td>20561</td>
<td>37594</td>
</tr>
<tr>
<td>[N.m⁻¹]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertical damping</td>
<td>86</td>
<td>121</td>
<td>180</td>
</tr>
<tr>
<td>[N.s.m⁻¹]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>horizontal stiffness</td>
<td>492140</td>
<td>363515</td>
<td>-</td>
</tr>
<tr>
<td>[N.m⁻¹]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>horizontal damping</td>
<td>127</td>
<td>207</td>
<td>-</td>
</tr>
<tr>
<td>[N.s.m⁻¹]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.27. Comparison of spring parameters for tumbling track

<table>
<thead>
<tr>
<th></th>
<th>own testing</th>
<th>Continental testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertical stiffness</td>
<td>13862</td>
<td>50100</td>
</tr>
<tr>
<td>[N.m⁻¹]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertical damping</td>
<td>253</td>
<td>344</td>
</tr>
<tr>
<td>[N.s.m⁻¹]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>horizontal stiffness</td>
<td>321572</td>
<td>-</td>
</tr>
<tr>
<td>[N.m⁻¹]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>horizontal damping</td>
<td>73</td>
<td>-</td>
</tr>
<tr>
<td>[N.s.m⁻¹]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion / Discussion

The vertical spring parameters obtained for the vaulting horse and tumbling track compare well to those reported by Continental. For the horizontal spring parameters no data were available for comparison. The average values obtained from testing were initially used for the simulation models of vaulting and tumbling (Table 5.28).

Table 5.28. Average spring parameters for the vaulting horse and the tumbling track

<table>
<thead>
<tr>
<th></th>
<th>vault</th>
<th>tumbling track</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertical stiffness</td>
<td>28842</td>
<td>13862</td>
</tr>
<tr>
<td>[N.m⁻¹]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertical damping</td>
<td>86</td>
<td>253</td>
</tr>
<tr>
<td>[N.s.m⁻¹]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>horizontal stiffness</td>
<td>492140</td>
<td>321572</td>
</tr>
<tr>
<td>[N.m⁻¹]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>horizontal damping</td>
<td>127</td>
<td>73</td>
</tr>
<tr>
<td>[N.s.m⁻¹]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 5 SUMMARY

This chapter has described the procedures used to determine inertia, muscle and spring parameters for the five segment simulation models developed in Chapter 3. The inertia parameters were determined from anthropometric measurements on the subject using the inertia model of Yeadon (1990b), the muscle parameters were determined from isokinetic measurements on the subject and from the literature, and the spring parameters were determined through experimental testing. In addition the values of the spring parameters will also be calculated through simulation in Chapter 6.

The next chapter combines the work done in Chapters 3, 4, and 5 for the evaluation of each simulation model. The simulation models are then used to quantify the contributions made to vaulting and tumbling.
CHAPTER 6

EVALUATION OF THE SIMULATION MODELS AND CONTRIBUTIONS TO PERFORMANCE

The three simulation models developed in Chapter 3 are each evaluated in this chapter. The evaluation procedure varied slightly for each model, although the basic principle was to compare the simulated performance of each model with actual performances.

Video data of an actual performance of the Hecht from Yeadon et al. (in press) is used to evaluate the simulated performance of the Hecht vault with the two segment simulation model.

Hecht vault and tumbling performances analysed in Chapter 4 are used to evaluate the simulated performance of the Hecht vault and tumbling takeoffs with the five segment simulation models.

The five segment simulation models are then used to determine the contributions to vaulting and tumbling performances.

EVALUATION OF THE TWO SEGMENT MODEL OF VAULTING

The horse contact phase of the vault was modelled as an inelastic impact with no torque exerted at the shoulder joint. These assumptions were made since it was thought that the preflight characteristics are the major determinants of postflight performance. In order to test whether these assumptions are reasonable, the simulated performance of the model was compared with an actual performance.

Method

A simulation was performed with the same initial conditions just prior to contact with the horse (Table 6.1) as the highest scoring Hecht vault from a national competition (Yeadon et al., in press). These values were used as input to the simulation of the contact phase. The postflight characteristics of the simulation after impact were compared with the video analysis of the actual performance in order to assess whether there was reasonable agreement between them.
Results and Discussion

Good agreement was found between the simulated and actual performance at the start of the postflight phase with the exception of the shoulder angle which remained unchanged by the instantaneous simulated impact (Table 6.1). The postflight performance of the model, however, will depend primarily on the mass centre velocity and body angular velocity which are in reasonable agreement with actual performance values. This suggests that the assumptions of inelastic impact and zero shoulder torque do not produce major errors.

Table 6.1. Comparison of simulation and video analysis

<table>
<thead>
<tr>
<th></th>
<th>end of preflight</th>
<th>start of postflight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>simulation</td>
<td>video</td>
</tr>
<tr>
<td>horizontal velocity of the mass centre [m.s⁻¹]</td>
<td>6.01</td>
<td>3.51</td>
</tr>
<tr>
<td>vertical velocity of the mass centre [m.s⁻¹]</td>
<td>0.96</td>
<td>3.07</td>
</tr>
<tr>
<td>body angle at contact [°]</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>shoulder angle at contact [°]</td>
<td>141.7</td>
<td>141.7</td>
</tr>
<tr>
<td>body angular velocity [rad.s⁻¹]</td>
<td>2.92</td>
<td>-2.44</td>
</tr>
</tbody>
</table>

The evaluation of the two segment simulation model has shown that even though some characteristics of the human body have been ignored, the model can still reproduce the major features of vaulting performance. The two segment model demonstrates the importance of the preflight characteristics for vaulting performance and indicates that the model, although simplified, incorporates the main elements of vaulting.
The evaluation of the five segment simulation models was more complicated than the evaluation of the two segment model previously described, as the five segment models have several ‘unknown’ parameters. For an actual performance the muscle activation patterns are not known, the spring parameters for the shoulder (vaulting model) are not known and the spring parameters for the contact surfaces have only been estimated through experimental testing (Chapter 5).

The aim of the evaluation was to show that some combination of muscle activations and spring stiffness parameters resulted in simulations that closely matched the actual performances analysed in Chapter 4. Simulations were compared with each performance analysed in Chapter 4; there were three trials for comparison for the vaulting model and five trials for comparison for the tumbling model.

Both simulation models were customised to represent the gymnast analysed in Chapter 4 using subject specific inertia and muscle parameters calculated in Chapter 5.

Method

To evaluate the simulation models required information from each of the performances analysed in Chapter 4 so that the simulated performance could be compared with each actual performance:

- The initial conditions for the start of each simulation
- The time histories of how the contact surfaces move
- The time history of the shoulder movement (for vaulting performances)
- The takeoff characteristics
- The body orientation and configuration

The initial conditions were required so that the simulation models would start with the same initial characteristics as each actual performance. The movement of the shoulders and contact surfaces, the takeoff conditions and the body orientation and configuration were required to compare simulations with the actual performances.
Initial conditions

Table 6.2 and Table 6.3 show the kinematic data obtained from the analysis of the Hecht vault and tumbling takeoffs at the time the wrists made contact with the vaulting horse and the time the toes made contact with the tumbling track respectively (Chapter 4).

Table 6.2. Kinematic data for each vault at the time the wrists made contact with the vaulting horse

<table>
<thead>
<tr>
<th>trial</th>
<th>$u_g$</th>
<th>$v_g$</th>
<th>$a_a$</th>
<th>$a_{\omega}$</th>
<th>$tr_a$</th>
<th>$tr_{\omega}$</th>
<th>$h_a$</th>
<th>$h_{\omega}$</th>
<th>$k_a$</th>
<th>$k_{\omega}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.71</td>
<td>0.62</td>
<td>49</td>
<td>358</td>
<td>6</td>
<td>304</td>
<td>163</td>
<td>-319</td>
<td>178</td>
<td>-243</td>
</tr>
<tr>
<td>2</td>
<td>5.94</td>
<td>0.90</td>
<td>49</td>
<td>440</td>
<td>6</td>
<td>232</td>
<td>167</td>
<td>-57</td>
<td>181</td>
<td>-21</td>
</tr>
<tr>
<td>3</td>
<td>6.20</td>
<td>0.86</td>
<td>46</td>
<td>272</td>
<td>8</td>
<td>103</td>
<td>169</td>
<td>-2</td>
<td>180</td>
<td>-80</td>
</tr>
</tbody>
</table>

where $u_g =$ horizontal velocity of the mass centre [m.s$^{-1}$]

$v_g =$ vertical velocity of the mass centre [m.s$^{-1}$]

$a, tr, h$ and $k$ represent: arm, trunk, hip and knee respectively with the subscripts $a$ and $\omega$ denoting angle [$^\circ$] and angular velocity [$^\circ$s$^{-1}$]

Table 6.3. Kinematic data for the tumbling performances at the time the feet made contact with the track

<table>
<thead>
<tr>
<th>trial</th>
<th>$u_g$</th>
<th>$v_g$</th>
<th>$f_f$</th>
<th>$f_{\omega}$</th>
<th>$s_s$</th>
<th>$s_{\omega}$</th>
<th>$t_t$</th>
<th>$t_{\omega}$</th>
<th>$tr_a$</th>
<th>$tr_{\omega}$</th>
<th>$a_a$</th>
<th>$a_{\omega}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.50</td>
<td>-0.70</td>
<td>119</td>
<td>42</td>
<td>53</td>
<td>325</td>
<td>75</td>
<td>672</td>
<td>3</td>
<td>772</td>
<td>-22</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>4.66</td>
<td>-0.59</td>
<td>115</td>
<td>-221</td>
<td>48</td>
<td>218</td>
<td>82</td>
<td>837</td>
<td>6</td>
<td>841</td>
<td>-20</td>
<td>633</td>
</tr>
<tr>
<td>3</td>
<td>4.80</td>
<td>-0.33</td>
<td>123</td>
<td>539</td>
<td>48</td>
<td>121</td>
<td>83</td>
<td>499</td>
<td>16</td>
<td>1034</td>
<td>-15</td>
<td>725</td>
</tr>
<tr>
<td>4</td>
<td>4.85</td>
<td>-0.52</td>
<td>121</td>
<td>10</td>
<td>49</td>
<td>185</td>
<td>81</td>
<td>596</td>
<td>6</td>
<td>793</td>
<td>-17</td>
<td>762</td>
</tr>
<tr>
<td>5</td>
<td>4.66</td>
<td>-0.37</td>
<td>123</td>
<td>73</td>
<td>50</td>
<td>277</td>
<td>80</td>
<td>884</td>
<td>1</td>
<td>707</td>
<td>-22</td>
<td>827</td>
</tr>
</tbody>
</table>

$f, s, t, tr$ and $a$ represent: foot, shank, thigh, trunk and arm respectively with the subscripts $a$ and $\omega$ denoting angle [$^\circ$] and angular velocity [$^\circ$s$^{-1}$]

Inputting the values presented in Table 6.2 and Table 6.3 to the simulation models, allowed the mass centre location to be calculated at the start of each simulation. The
calculated mass centre location for each trial using (a) the simulation model and (b) the image analysis in Chapter 4 (Table 6.4 and Table 6.5) did not agree very closely. To establish the reason for the differences the measured segment lengths used in simulation model were compared with the average segment lengths calculated from the digitised body landmarks (Chapter 4).

Table 6.4. Mass centre locations from the image analysis and simulation model at the start of wrist contact for each vault

<table>
<thead>
<tr>
<th>trial</th>
<th>simulation model</th>
<th>image analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_s$</td>
<td>$z_s$</td>
</tr>
<tr>
<td>1</td>
<td>-0.66</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>-0.66</td>
<td>0.37</td>
</tr>
<tr>
<td>3</td>
<td>-0.68</td>
<td>0.37</td>
</tr>
</tbody>
</table>

where $x_s$ = horizontal location of the mass centre relative to the wrists at contact [m]  
$z_s$ = vertical location of the mass centre relative to the wrists at contact [m]

Table 6.5. Mass centre location from the image analysis and simulation model at the start of the contact phase for each tumbling performance

<table>
<thead>
<tr>
<th>trial</th>
<th>simulation model</th>
<th>image analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_s$</td>
<td>$z_s$</td>
</tr>
<tr>
<td>1</td>
<td>-0.47</td>
<td>0.77</td>
</tr>
<tr>
<td>2</td>
<td>-0.47</td>
<td>0.78</td>
</tr>
<tr>
<td>3</td>
<td>-0.44</td>
<td>0.81</td>
</tr>
<tr>
<td>4</td>
<td>-0.46</td>
<td>0.78</td>
</tr>
<tr>
<td>5</td>
<td>-0.45</td>
<td>0.75</td>
</tr>
</tbody>
</table>

where $x_s$ = horizontal location of the mass centre relative to the contact point of the toes [m]  
$z_s$ = vertical location of the mass centre relative to the contact point of the toes [m]
Table 6.6. Segment lengths from anthropometric measurements and image analysis [m]

<table>
<thead>
<tr>
<th></th>
<th>measured</th>
<th>vaulting</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>0.51</td>
<td>0.55</td>
<td>0.52</td>
</tr>
<tr>
<td>trunk</td>
<td>0.47</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td>thigh</td>
<td>0.40</td>
<td>0.43</td>
<td>0.42</td>
</tr>
<tr>
<td>shank</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>foot</td>
<td>0.20</td>
<td>0.21</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The measured lengths of the arm and trunk were different from the image analysis values from vaulting and tumbling. The average hip to wrist distance from the image analysis was 0.095 m longer than the anthropometric measurement due to shoulder movement in the vaulting and tumbling performances. The anthropometric measurements did not take into account the more distal shoulder joint centre in the lengths of the trunk and arm. In addition the arm and trunk lengths from the image analysis of the vaulting and tumbling performances were not the same. For vaulting the arm and trunk were both 0.04 m longer than the anthropometric measurements whereas for tumbling the arm was 0.01 m longer and the trunk 0.09 m longer. Whether the increase in segment lengths was in the arm or trunk was dependent on where the shoulder joint was digitised.

To make a sensible correction, the arm and trunk segments were both lengthened by 0.045 m in each simulation model; this correction assumed that the increase in hip to wrist length was evenly distributed between the arm and trunk. The mass centre locations for the arm and trunk were left at a fixed distance from the elbow and hip respectively and the moments of inertia for the arm and trunk were not changed.

The calculated average thigh lengths from the image analysis of the vaulting and tumbling performances were slightly longer than the anthropometric measurements. The differences were probably due to errors in locating the hip and knee joint centres which were sometimes difficult to locate as the body was straight for parts of both performances. The thigh length, mass centre location and moment of inertia used in both simulation models were not changed from the values calculated using the anthropometric measurements.
Similar shank lengths were found from the anthropometric measurements and image analysis. The shank length, mass centre location and moment of inertia were not changed from the values calculated using the anthropometric measurements.

The foot length from the tumbling image analysis was 0.05 m shorter than from the anthropometric measurements. This difference was due to the toes being obscured during contact with the tumbling track. For the tumbling model, the foot represented the ankle to ball of foot distance, which from the anthropometric measurements was 0.15 m. For the tumbling model a foot length of 0.15 m was used with the mass centre location at a fixed distance from the ankle and the moment of inertia of the foot was not changed.

Changing the segment lengths in the simulation models resulted in a different calculated initial mass centre location for each performance (Table 6.7) which still did not agree with the mass centre locations from the image analysis (Table 6.4 and Table 6.5) although the agreement was a lot better. The reason for the differences was that the mass centre location of each segment in the image analysis was calculated as a ratio of the length of each segment. Therefore for the trunk and arm segments the ratio predicted a different mass centre location due to the longer arm and trunk lengths. The agreement for the tumbling model was not as good as the vaulting model as the increase in hip to wrist length was mainly in the trunk from the image analysis which produced a greater change in the mass centre location of the trunk.

Table 6.7. Initial mass centre locations with adjusted segment lengths for the start of each simulation [m]

<table>
<thead>
<tr>
<th>trial</th>
<th>vaulting</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_g$</td>
<td>$z_g$</td>
</tr>
<tr>
<td>1</td>
<td>-0.72</td>
<td>0.40</td>
</tr>
<tr>
<td>2</td>
<td>-0.73</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>-0.75</td>
<td>0.41</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The mass centre locations given by the simulation model with adjusted segment lengths, fixed mass centre locations of the segments and segmental moment of inertias unchanged (as described previously) were considered to give a better estimate of the whole body mass centre location than the locations given by the image analysis. These segmental inertia parameters were therefore used in both simulation models.

Inputting the values for the angle and angular velocity of each segment in the model resulted in an acceptable estimate of the mass centre location and the whole body angular momentum (Table 6.9). However the vaulting model required the hip and knee angles to be constrained to fixed values as the model did not allow for changes in hip and knee angles during a simulation. Average hip and knee angles during horse contact were calculated from the actual performances and used as the fixed hip and knee angles throughout each simulation (Table 6.8). Fixing the hip and knee angles to the average values had the effect of altering the initial mass centre location and whole body angular momentum. To take these differences into account, the initial trunk angle and trunk angular velocity were adjusted (Table 6.8) until the mass centre location and whole body angular momentum agreed with the values calculated by the simulation model when each segment's angle and angular velocity were used.

Table 6.8. Initial conditions for the start of each vault simulation

<table>
<thead>
<tr>
<th>trial</th>
<th>uₗ</th>
<th>vₗ</th>
<th>hₗ</th>
<th>aₗ</th>
<th>a₀₀</th>
<th>tr₁</th>
<th>tr₀₀</th>
<th>h₁</th>
<th>k₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.71</td>
<td>0.62</td>
<td>48</td>
<td>49</td>
<td>358</td>
<td>6</td>
<td>195</td>
<td>168</td>
<td>178</td>
</tr>
<tr>
<td>2</td>
<td>5.94</td>
<td>0.90</td>
<td>53</td>
<td>49</td>
<td>440</td>
<td>7</td>
<td>210</td>
<td>168</td>
<td>178</td>
</tr>
<tr>
<td>3</td>
<td>6.20</td>
<td>0.86</td>
<td>30</td>
<td>46</td>
<td>272</td>
<td>9</td>
<td>115</td>
<td>168</td>
<td>178</td>
</tr>
</tbody>
</table>

hₗ = angular momentum about a transverse axis through the mass centre [kg.m².rad.s⁻¹]
Depression of the shoulders and contact surfaces

The depression of the shoulders and contact surfaces were calculated from the image analysis so that the simulated and actual depressions could be compared. For each actual performance the depression of the shoulders was measured as the change in wrist to shoulder distance $D_{WS'}$ (Chapter 4, Figure 4.3) and the movement of the contact surfaces were estimated from the movement of the toes, ankles and wrists during contact. Similar depressions were observed from each actual performance (Chapter 4); the average values were therefore used for a comparison between the actual and simulated performance.

From the vaulting performances the average maximum depression of the shoulders was $0.11$ m with an average final depression at takeoff from the horse of $-0.02$ m. Therefore from the three performances analysed the wrist to shoulder distance decreased by $0.11$ m before increasing by $0.13$ m during contact with the horse. The average maximum vertical movement of the wrists during horse contact was $0.01$ m and the
average horizontal displacement of the wrists was 0.02 m during the period 0.03 s to 0.07 s of horse contact (0.00 s corresponds to the time the fingers contacted the horse). The average horizontal displacement of the wrists during the period 0.03 s to 0.07 s of horse contact was used as an indicator of the horizontal movement of the horse top rather than the maximum horizontal displacement of the wrists which would have been distorted by the wrist movement after the wrists had come away from the horse top. In addition before the wrists contacted the horse the rest of the hand was in contact with the horse for a brief period (approximately 0.01 s); during this time and the initial 0.02 s of wrist contact the hands appeared to slip along the horse top, although it proved difficult to identify exactly when the hands were slipping.

From the tumbling performances the average maximum movement of the toes horizontally and vertically were estimated to be 0.06 m and 0.06 m respectively although it was difficult to identify the toes at maximum depression as they were obscured by the tumbling track. At takeoff from the tumbling track the toes were in the same position relative to the track as they were at touch down. In addition the average maximum vertical movement of the ankles while in contact with the tumbling track was 0.01 m.

Table 6.10. Summary of shoulder and contact surfaces movement during actual performances [m]

<table>
<thead>
<tr>
<th></th>
<th>vaulting</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum horizontal movement of contact surface ([q_{1\text{max}}])</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>horizontal movement of contact surface at takeoff ([q_{1\text{end}}])</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>maximum vertical movement of contact surface ([q_{2\text{max}}])</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>vertical movement of contact surface at takeoff ([q_{2\text{end}}])</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>maximum depression of shoulders ([q_{4\text{max}}])</td>
<td>0.11</td>
<td>-</td>
</tr>
<tr>
<td>final depression of shoulders ([q_{4\text{end}}])</td>
<td>-0.02</td>
<td>-</td>
</tr>
</tbody>
</table>

Takeoff characteristics

The whole body mass centre velocity and angular momentum at takeoff were required to define the takeoff characteristics for each trial analysed in Chapter 4. Table 6.11 shows a summary of the takeoff characteristics for each trial analysed in Chapter 4.
Table 6.11. Mass centre velocity [m.s\textsuperscript{-1}] and angular momentum [kg.m\textsuperscript{2}.rad.s\textsuperscript{-1}] at takeoff for each vaulting and tumbling performance

<table>
<thead>
<tr>
<th>trial</th>
<th>$u_s$</th>
<th>$v_s$</th>
<th>$h_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.05</td>
<td>1.79</td>
<td>-17</td>
</tr>
<tr>
<td>2</td>
<td>4.32</td>
<td>2.00</td>
<td>-21</td>
</tr>
<tr>
<td>3</td>
<td>4.42</td>
<td>1.91</td>
<td>-20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>trial</th>
<th>$u_s$</th>
<th>$v_s$</th>
<th>$h_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.93</td>
<td>4.98</td>
<td>56</td>
</tr>
<tr>
<td>2</td>
<td>2.79</td>
<td>5.09</td>
<td>59</td>
</tr>
<tr>
<td>3</td>
<td>2.66</td>
<td>4.53</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>2.99</td>
<td>4.50</td>
<td>95</td>
</tr>
<tr>
<td>5</td>
<td>2.63</td>
<td>3.93</td>
<td>107</td>
</tr>
</tbody>
</table>

Body orientation and configuration

For both the Hecht vault and tumbling takeoffs the trunk angle and joint angles were used to define the strategy used during each actual performance. For the Hecht vault the trunk and shoulder angles were used to define the strategy. During contact with the horse the trunk angle remained almost constant and the shoulder angle decreased for all three trials analysed. It was therefore sufficient to just use the final trunk and shoulder angles for comparing the strategy used in a simulation with each actual performance. However the vaulting model required the hip and knee angles to be constrained to fixed values as the model did not allow for changes in hip and knee angle during a simulation. Thus the calculated final trunk and shoulder angles from the actual performances needed to be adjusted slightly to take into account the fixed knee and hip angles. Fixing the hip and knee angles had the effect of altering the mass centre location at takeoff. To take into account the differences, the final trunk angle and corresponding shoulder angle from the image analysis were adjusted until the mass centre location agreed with the values calculated by the simulation model when each segment’s angle was used.

For the tumbling performances the ankle, knee, hip, shoulder and trunk angles were used to define the strategy. During contact with the tumbling track, the ankle and knee angles both decreased before increasing while the trunk, hip and shoulder angles all increased throughout the contact phase for the five actual performances (Chapter 4). The minimum ankle and knee angles as well as the final trunk, hip, knee, ankle and shoulder angles were therefore used to define the strategy used for each tumbling performance.
Table 6.12. Body orientation and configuration at finger / toe takeoff, and minimum ankle and knee angles for each performance analysed [°]

<table>
<thead>
<tr>
<th>trial</th>
<th>$s_a$</th>
<th>$t_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>76</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>82</td>
<td>16</td>
</tr>
</tbody>
</table>

$a_a = \text{ankle angle}$
$a_{amin} = \text{minimum ankle angle}$
$k_a = \text{knee angle}$
$k_{amin} = \text{minimum knee angle}$
$h_a = \text{hip angle}$
$s_a = \text{shoulder angle}$
$t_r = \text{trunk angle}$

Evaluation using an optimisation procedure

The two simulation models were evaluated using the initial conditions, depression of the shoulders and contact surfaces, takeoff characteristics, and body orientation and configuration previously described for each of the trials analysed in Chapter 4. The models were evaluated by comparing simulations of actual performances with kinematic data calculated on each performance (Chapter 4). All simulations of actual performances started with the calculated initial conditions. The simulated and actual performances were compared using several criteria (Table 6.13) with the Simulated Annealing optimisation algorithm used to find the values for a number of ‘unknown parameters’ in each simulation model that resulted in the best agreement between the simulated and actual performance. The criteria used allowed each simulated and actual performance to be compared objectively. The following section will describe the criteria for comparing simulated and actual performances, the formulation of the objective functions for the evaluation of each model using an optimisation procedure and the unknown parameters for each of the models. The results of the evaluation of the tumbling and vaulting models will then be documented.
Criteria for comparing simulated and actual performances

Initially three criteria were formulated for comparing a simulation with an actual performance: performance, elastic, and strategy criteria (Table 6.13). Each criterion consisted of a number of different variables which defined each performance/simulation.

Table 6.13. Criteria for each model

<table>
<thead>
<tr>
<th></th>
<th>elastic</th>
<th>performance</th>
<th>strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>vaulting</td>
<td>( t, q_1_{\text{max}}, q_2_{\text{max}}, q_4_{\text{max}}, q_4_{\text{end}} )</td>
<td>( u_g, v_g, h_g )</td>
<td>( \tau_a, s_a )</td>
</tr>
<tr>
<td>tumbling</td>
<td>( t, q_1_{\text{end}}, q_2_{\text{max}}, q_2_{\text{end}} )</td>
<td>( u_g, v_g, h_g )</td>
<td>( a_{\text{min}}, a_a, k_{\text{min}}, k_a, h_a, \tau_a, s_a )</td>
</tr>
</tbody>
</table>

The three criteria were used either on their own or with each other to compare a simulated performance with an actual performance. The criteria were also used to formulate a number of different objective functions which were minimised using the Simulated Annealing algorithm.

Objective function and weightings

A score was calculated for each simulation as a measure of the difference between a simulation and actual performance using the variables in each criterion (Table 6.13). The scores \( \text{val}_p \) and \( \text{val}_e \) for the performance and elastic criteria were both calculated so that the score represented the average percentage error between the simulated and actual performance. The weightings for the performance and elastic variables were equal to the inverse of the average value of each variable from the actual performances (Table 6.14, equation (6.1)). The exceptions were the weightings for \( q_1_{\text{end}}, q_2_{\text{end}} \) and \( q_4_{\text{end}} \) which were set equal to the weightings for \( q_1_{\text{max}}, q_2_{\text{max}} \) and \( q_4_{\text{max}} \) respectively so that the weightings for maximum and final displacements of each spring were the same. The score \( \text{val}_s \) for the strategy criterion represented the difference between the simulated and actual performance in degrees (equation (6.2)). Table 6.15 shows the weightings used for each strategy variable. In both models each joint was given an equal weighting apart from the trunk angle which was given double the weighting of the other strategy variables because the trunk angle represented the whole body orientation whereas the joint angles defined the configuration.
Table 6.14. Average values from the actual vaulting and tumbling performances for the performance and elastic criteria

<table>
<thead>
<tr>
<th>mean variable</th>
<th>vaulting</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_g$ [m.s$^{-1}$]</td>
<td>4.3</td>
<td>2.8</td>
</tr>
<tr>
<td>$v_g$ [m.s$^{-1}$]</td>
<td>1.9</td>
<td>4.8</td>
</tr>
<tr>
<td>$h_g$ [kg.m$^2$.rad.s$^{-1}$]</td>
<td>-20.0</td>
<td>75.0</td>
</tr>
<tr>
<td>$\bar{t}$ [s]</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td>$q_{1\text{max}}$ [m]</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>$q_{1\text{end}}$ [m]</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>$q_{2\text{max}}$ [m]</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>$q_{2\text{end}}$ [m]</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>$q_{4\text{max}}$ [m]</td>
<td>0.11</td>
<td>-</td>
</tr>
<tr>
<td>$q_{4\text{end}}$ [m]</td>
<td>-0.02</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.15. Weightings for strategy in the vaulting and tumbling models

<table>
<thead>
<tr>
<th>angle</th>
<th>vaulting</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{\text{amin}}$</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>$a_a$</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_{\text{amin}}$</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_a$</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>$h_a$</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$t_r$</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$s_a$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Performance and elastic objective function:
\[ S = 100 \sqrt{\frac{1}{n} \sum_{i=1}^{n} \frac{(s_i - a_i)^2}{\bar{a}_i}} \]  
(6.1)

Strategy objective function:
\[ S = \sqrt{\frac{1}{n} \sum_{i=1}^{n} w_i (s_i - a_i)^2} \]  
(6.2)

where \( S \) = score [units: % for performance and elastic, ° for strategy]

\( i \) = denotes different variables

\( n \) = number of variables used in objective function

\( s_i \) = value of variable \( i \) from simulation

\( a_i \) = value of variable \( i \) from actual performance

\( \bar{a}_i \) = average value of variable \( i \) from actual performances

\( w_i \) = weighting for variable \( i \)

Five objective functions \( \text{val}_{ps}, \text{val}_p, \text{val}_s, \text{val}_{ps}, \) and \( \text{val}_{pes} \) (Table 6.16) were formulated from equations (6.1) and (6.2) for the evaluation of the vaulting and tumbling models. These gave objective measures of how good the comparison was between simulated and actual performance in terms of each of the three criteria. For tumbling it was assumed that degree and percentage errors in \( \text{val}_p, \text{val}_{ps}, \) and \( \text{val}_s \) were of similar magnitudes and since there were similar numbers of variables in \( \text{val}_p, \text{val}_{ps}, \) and \( \text{val}_s \) each criteria was given an equal weighting in the calculation of \( \text{val}_{ps} \) and \( \text{val}_{pes} \). For vaulting the calculation of \( \text{val}_{pes} \) was weighted 80% \( \text{val}_p \) and 20% \( \text{val}_s \) as there were only two variables in \( \text{val}_s \) compared with eight variables in the \( \text{val}_p \).

The procedure for using the five objective functions for the vaulting and tumbling models will be described in a subsequent section where the results of the evaluation of the two models are given.
Table 6.16. Objective functions for evaluation of vaulting and tumbling models

<table>
<thead>
<tr>
<th>criteria</th>
<th>vaulting</th>
<th>tumbling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance and elastic [%]</td>
<td>$\text{val}_{pe} = 100 \sqrt{\frac{8}{i=1} \frac{(s_i - a_i)^2}{\bar{a}_i}}$</td>
<td>$\text{val}_{pe} = 100 \sqrt{\frac{8}{i=1} \frac{(s_i - a_i)^2}{\bar{a}_i}}$</td>
</tr>
<tr>
<td>Performance [%]</td>
<td>$\text{val}_p = 100 \sqrt{\frac{3}{i=1} \frac{(s_i - a_i)^2}{\bar{a}_i}}$</td>
<td>$\text{val}_p = 100 \sqrt{\frac{3}{i=1} \frac{(s_i - a_i)^2}{\bar{a}_i}}$</td>
</tr>
<tr>
<td>Strategy [°]</td>
<td>$\text{val}_s = \sqrt{\frac{2}{i=1} \frac{\sum w_i (s_i - a_i)^2}{\sum w_i}}$</td>
<td>$\text{val}_s = \sqrt{\frac{7}{i=1} \frac{\sum w_i (s_i - a_i)^2}{\sum w_i}}$</td>
</tr>
<tr>
<td>Performance and strategy [% + 0]</td>
<td>$\text{val}<em>{ps} = \sqrt{\frac{(\text{val}</em>{p}^2 + \text{val}_{s}^2)}{2}}$</td>
<td>$\text{val}<em>{ps} = \sqrt{\frac{(\text{val}</em>{p}^2 + \text{val}_{s}^2)}{2}}$</td>
</tr>
<tr>
<td>Performance, elastic and strategy [% + 0]</td>
<td>$\text{val}<em>{pes} = \sqrt{\frac{4\text{val}</em>{pe}^2 + \text{val}_{s}^2}{5}}$</td>
<td>$\text{val}<em>{pes} = \sqrt{\frac{(\text{val}</em>{pe}^2 + \text{val}_{s}^2)}{2}}$</td>
</tr>
</tbody>
</table>

**Unknown parameters**

Initially each model started with nine unknown parameters, which needed to be calculated for each performance (Table 6.17). Although the stiffness and damping of the horse top and tumbling track had been estimated through experimental testing, the simulation models required stiffness and damping parameters which represented not only the elastic properties of the equipment but also the hand / foot of the gymnast. The activation time histories of the torque generators were also unknown, with one time parameter required for each torque generator to switch the activation of the knee, hip and shoulder torque generators in the tumbling model to maximum. However two parameters were required for two of the torque generators which consisted of contractile and elastic elements in series (ankle in the tumbling model and shoulder in the vaulting model). For these torque generators it was necessary to have a gradual increase in the activation to the
contractile element before a rapid increase to maximum activation (Figure 6.1), as there was the potential for the torque generator to vibrate if the torques produced by the contractile and elastic elements were very different.

In addition for the vaulting model the stiffness and damping parameters for the shoulder were unknown and the time at which the length of the vertical spring at the wrist was changed was also allowed to vary.

Table 6.17. Original unknown parameters for each model

<table>
<thead>
<tr>
<th>vaulting model</th>
<th>tumbling model</th>
</tr>
</thead>
<tbody>
<tr>
<td>shoulder activation (2 parameters)</td>
<td>ankle activation (2 parameters)</td>
</tr>
<tr>
<td>time the hand spring length was changed</td>
<td>knee activation</td>
</tr>
<tr>
<td>stiffness of the shoulder spring</td>
<td>hip activation</td>
</tr>
<tr>
<td>damping of the shoulder spring</td>
<td>shoulder activation</td>
</tr>
<tr>
<td>horizontal stiffness of the contact surface</td>
<td>horizontal stiffness of the contact surface</td>
</tr>
<tr>
<td>horizontal damping of the contact surface</td>
<td>horizontal damping of the contact surface</td>
</tr>
<tr>
<td>vertical stiffness of the contact surface</td>
<td>vertical stiffness of the contact surface</td>
</tr>
<tr>
<td>vertical damping of the contact surface</td>
<td>vertical damping of the contact surface</td>
</tr>
</tbody>
</table>

activation

\[
iact = \text{activation at time } ton
\]

\[
\text{ton} = \text{time activation switched to maximum}
\]

Figure 6.1. Activation time history for contractile element in series with elastic element.
Results and Discussion

The simulation model of tumbling was evaluated first and the experience gained used in the evaluation of the vaulting model.

Tumbling model

A seven phase procedure was used to evaluate the tumbling model which involved minimising different objective functions to obtain the best agreement between simulated and actual performance for each of the five actual tumbling performances. This section will describe the optimisations used for the evaluation of the tumbling model and the results obtained.

Evaluation using performance and elastic criteria

The first optimisation (opt1) used val_p as the objective function (Table 6.16) which included the performance and elastic criteria. The optimisation minimised val_p by varying the nine unknown parameters identified for the tumbling model (Table 6.17).

For all five trials the simulation model was able to give good agreement for the performance and elastic criteria with an average error of 7% for val_p. However the comparison between strategies (val_p) was not as good with an average error of 33°. It would appear therefore that the model is able to give good agreement for the performance and elastic criteria without using the correct strategy. This suggests that it is possible to produce the same performance using different strategies.

Average stiffness and damping values were calculated from the values obtained for the five trials with opt1 to give one set of stiffness and damping parameters to be used with each trial.

Evaluation using performance criteria with average elastic parameters

opt2 used val_p as the objective function (Table 6.16) with the average stiffness and damping parameters calculated for the track from the five trials for opt1. The optimisation minimised val_p by varying the five muscle parameters identified for the tumbling model (Table 6.17).

val_p increased from 10% (opt1) to 11% (opt2) for the five trials, demonstrating that it was possible to almost do as well in terms of performance when using the average stiffness and damping parameters. The comparison between strategies (val_p) was again
not as good with an average error of 29° which was a slight improvement on the 33° found for opt1. Interestingly \( \text{val}_{opt} \) which was optimised in opt1 doubled from 7% to 14% for opt2, even though the stiffness and damping of the track were fixed to the average values. This indicates that the strategy used affects the movement of the track.

**Evaluation using strategy criterion with average elastic parameters**

The third optimisation (opt3) used \( \text{val}_{opt} \) as the objective function (Table 6.16) with the average stiffness and damping parameters to try and encourage a solution for each trial that used the correct strategy. The optimisation minimised \( \text{val}_{opt} \) by varying the five muscle parameters identified for the tumbling model (Table 6.17).

For all five trials \( \text{val}_{opt} \) improved with the average value of \( \text{val}_{opt} \) decreasing from 29° for opt2 to 20° for opt3. However there was a detrimental effect on performance with \( \text{val}_{p} \) increasing substantially for all five trials; the average value for \( \text{val}_{p} \) increased from 11% (opt2) to 27% (opt3).

To try and find the best overall solution for each trial that had good performance and strategy, \( \text{val}_{p} \) and \( \text{val}_{opt} \) were combined to produce \( \text{val}_{popt} \).

**Evaluation using performance and strategy criteria with average elastic parameters**

opt4 used \( \text{val}_{popt} \) as the objective function (Table 6.16) with the average stiffness and damping parameters calculated from the five trials for opt1. The optimisation minimised \( \text{val}_{popt} \) by varying the five muscle parameters identified for the tumbling model (Table 6.17).

For all five trials, \( \text{val}_{popt} \) improved with the average value for \( \text{val}_{popt} \) decreasing between opt3 and opt4 from 24 to 20 (\( \text{val}_{popt} \) is a combination of degrees and percent, so units will be expressed as ° (%)). The improvement in \( \text{val}_{popt} \) was due to a large improvement in \( \text{val}_{p} \) between opt3 and opt4, and only a small decrease in \( \text{val}_{opt} \) between opt3 and opt4. In fact comparing the solution for opt4 with opt2 and opt3 showed that \( \text{val}_{p} \) was 12% better than opt3 and only 4% worse than opt2, while \( \text{val}_{opt} \) was 6° better than opt2 and only 2° worse than opt3. Including both the performance and strategy criterion in the objective function resulted in the best overall solution which was slightly worse than the best match to each individual criterion but much better than the match when the alternative criterion was minimised. In addition the strategy adopted was found to affect the way that the tumbling track moved.
This indicates that it is necessary to include performance, elastic and strategy criteria in the objective function, if the overall best match between simulation and performance is to be found.

**Evaluation using performance, elastic and strategy criteria**

opt5 used $\text{val}_{\text{pes}}$ as the objective function (Table 6.16) which was minimised by varying the nine unknown parameters identified for the tumbling model (Table 6.17). For all five trials; a reasonable overall agreement was found with an average value for $\text{val}_{\text{pes}}$ of $14.0^\circ$ (%).

Including the elastic criterion in the objective function resulted in a closer agreement for the movement of the tumbling track during the contact phase compared with opt4. Overall opt5 resulted in the better match between the simulation model and the actual performance for each of the five trials than opt4. However there were still some aspects of the simulation model that did not compare very well with the actual performances:

- For most trials the match with strategy was not as good as the match with performance, in fact it appeared that the simulation model was not able to produce sufficient torque at the joints. Examining the optimum simulations for each trial, showed that the model was not able to exert very much torque during the latter part of the contact phase, when the joints in the body were extending rapidly. This resulted in the joints in the body not being able to extend sufficiently before takeoff (Table 6.24).

- The movement at the ankle joint had the worst match with the actual performances. In the simulation model the ankle joint flexed too far past anatomical limits before extending for all five trials. Making the ankle 50% stronger made little difference to the movement of the foot with the ankle joint still flexing too much. This suggests that it is not the magnitude of the active torque produced at the ankle that results in the ankle flexing too much.

To try and improve the match between the simulation model and each actual performance, an adjustment was made to the torque / angular velocity profile in each torque generator.
Adjustment to the torque / angular velocity profile

The 3D surface fits obtained in Chapter 5 (Figures 5.42 - 5.46) for each joint, were calculated from fitting a function through the known data points, where the maximum range of the data was ± 250°s⁻¹. Outside this range the surfaces tended to a plateau at high eccentric angular velocities and to zero torque at high concentric angular velocities; however there was no information to guide the surfaces at high concentric angular velocities. There was therefore the potential for the function to tend to zero torque at too low an angular velocity. Some studies have determined the maximum angular velocity that can be produced at a joint (Table 2.6) where angular velocities of over 1000°s⁻¹ have been recorded. The 3D surface fits for each joint tended to zero torque at around 400°s⁻¹, where zero torque was taken as being able to produce less than 1% of isometric torque at a given joint. An adjustment was made to the surface fits by stretching them for angular velocities higher than the maximum from the experimental testing to a more realistic maximum concentric angular velocity (equation (6.3)). This equation calculated a new angular velocity from the actual angular velocity so that the each torque generator would be able to produce 1% of the isometric torque at an actual angular velocity of 800°s⁻¹.

\[
\omega_n = \frac{(\omega_a - \omega_{\text{max}})}{(800 - \omega_{\text{max}})} \times (\omega_{1\%} - \omega_{\text{max}}) + \omega_{\text{max}} \quad (\text{for } \omega_a > \omega_{\text{max}}) \tag{6.3}
\]

where:  
\(\omega_n\) = new angular velocity used to calculate torque possible  
\(\omega_a\) = actual angular velocity from simulation model  
\(\omega_{\text{max}}\) = maximum angular velocity for isokinetic data  
\(\omega_{1\%}\) = angular velocity at 1% of mean isometric torque
Table 6.18. Summary of data required to stretch torque / angular velocity profiles at each joint

<table>
<thead>
<tr>
<th>joint</th>
<th>(\omega_{\text{max}}) [°s(^{-1})]</th>
<th>(\omega_{1%}) [°s(^{-1})]</th>
<th>1% of mean isometric torque [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ankle</td>
<td>250</td>
<td>399</td>
<td>1.2</td>
</tr>
<tr>
<td>knee</td>
<td>175</td>
<td>314</td>
<td>1.8</td>
</tr>
<tr>
<td>hip</td>
<td>143</td>
<td>179</td>
<td>2.3</td>
</tr>
<tr>
<td>shoulder (tumbling)</td>
<td>155</td>
<td>380</td>
<td>0.5</td>
</tr>
<tr>
<td>shoulder (vaulting)</td>
<td>181</td>
<td>363</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Figure 6.2. Example of the original [A] and stretched [B] torque / angular velocity profile.

*Evaluation using performance, elastic and strategy criteria with stretched torque / angular velocity profile*

opt6 used \(\text{val}_{\text{pes}}\) as the objective function (Table 6.16) which was minimised by varying the nine unknown parameters identified for the tumbling model (Table 6.17). In addition the torque / angular velocity profile was stretched for high concentric angular velocities as described above.
The average value of $v_{\text{pes}}$ over the five trials was 8.8 which indicated that there was approximately 9 ° (%) error in each simulation model for the performance, elastic and strategy criteria (Table 6.19). This was an improvement of 5 ° (%) on the average value of $v_{\text{pes}}$ from opt5 where the original torque / angular velocity profile was used. This indicates that the stretched torque / angular velocity is an improvement to the model and indicates a weakness in the original function used to produce the 3D surfaces. The stretched torque / angular velocity profiles will therefore be used for all subsequent simulations.

The average value of $v_{\text{a}}$ was 9.9° for opt6 which was a large improvement compared with opt5 where the average value of $v_{\text{a}}$ was 14.7°. Thus the match between the strategies used by the simulations and actual performances was markedly improved with the stretched angular velocity torque profiles (Table 6.24).

Average stiffness and damping values were calculated for the tumbling track from the values obtained for the five trials with opt6 to give one set of stiffness and damping parameters to be used with each trial (Table 6.20).

Evaluation using performance, elastic and strategy criteria with stretched torque / angular velocity profile and average elastic parameters

opt7 used $v_{\text{pes}}$ as the objective function (Table 6.16) with the average stiffness and damping values calculated from the five trials for opt6 and the stretched torque / angular velocity profile. The optimisation minimised $v_{\text{pes}}$ by varying the five muscle parameters identified for the tumbling model (Table 6.17).

There was only a small change in the value of $v_{\text{pes}}$ from 8.8 ° (%) (opt6) to 9.9 ° (%) (opt7) demonstrating that the model was not too sensitive to the stiffness and damping parameters used for the tumbling track. The other criteria also did not change very much, with less than 2 units of difference between opt6 and opt7 in any of the criteria. The ankle was the only joint where the match between simulated and actual performance was not very good.

In the future the movement of the tumbling track and ankle should be examined in more detail, but for the purposes of this study the agreement was acceptable and indicates that a relatively simple model is able to reproduce the major features of tumbling performance.
Table 6.19. Scores for the evaluation of the tumbling model (opt5, opt6 and opt7) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>val_{pe} [%]</th>
<th>val_{p} [%]</th>
<th>val_{s} [°]</th>
<th>val_{ps} [° (%)]</th>
<th>val_{pes} [° (%)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, opt5</td>
<td>8.5</td>
<td>7.8</td>
<td>18.0</td>
<td>13.9</td>
<td>14.1</td>
</tr>
<tr>
<td>1, opt6</td>
<td>5.1</td>
<td>5.8</td>
<td>9.9</td>
<td>8.1</td>
<td>7.9</td>
</tr>
<tr>
<td>1, opt7</td>
<td>7.7</td>
<td>3.3</td>
<td>9.3</td>
<td>7.0</td>
<td>8.6</td>
</tr>
<tr>
<td>2, opt5</td>
<td>11.7</td>
<td>13.4</td>
<td>12.3</td>
<td>12.8</td>
<td>12.0</td>
</tr>
<tr>
<td>2, opt6</td>
<td>3.4</td>
<td>3.2</td>
<td>8.8</td>
<td>6.6</td>
<td>6.7</td>
</tr>
<tr>
<td>2, opt7</td>
<td>5.0</td>
<td>3.9</td>
<td>8.6</td>
<td>7.0</td>
<td>7.0</td>
</tr>
<tr>
<td>3, opt5</td>
<td>9.8</td>
<td>8.7</td>
<td>8.0</td>
<td>8.4</td>
<td>9.0</td>
</tr>
<tr>
<td>3, opt6</td>
<td>7.3</td>
<td>8.3</td>
<td>10.1</td>
<td>9.2</td>
<td>8.8</td>
</tr>
<tr>
<td>3, opt7</td>
<td>7.2</td>
<td>10.6</td>
<td>14.6</td>
<td>12.7</td>
<td>11.5</td>
</tr>
<tr>
<td>4, opt5</td>
<td>11.9</td>
<td>12.1</td>
<td>17.9</td>
<td>15.3</td>
<td>15.2</td>
</tr>
<tr>
<td>4, opt6</td>
<td>8.7</td>
<td>8.7</td>
<td>10.1</td>
<td>9.4</td>
<td>9.4</td>
</tr>
<tr>
<td>4, opt7</td>
<td>9.4</td>
<td>9.4</td>
<td>10.0</td>
<td>9.7</td>
<td>9.7</td>
</tr>
<tr>
<td>5, opt5</td>
<td>21.9</td>
<td>19.9</td>
<td>17.5</td>
<td>18.7</td>
<td>19.8</td>
</tr>
<tr>
<td>5, opt6</td>
<td>12.2</td>
<td>7.0</td>
<td>10.6</td>
<td>9.0</td>
<td>11.4</td>
</tr>
<tr>
<td>5, opt7</td>
<td>14.4</td>
<td>6.9</td>
<td>11.0</td>
<td>9.2</td>
<td>12.8</td>
</tr>
<tr>
<td>average, opt5</td>
<td>12.8</td>
<td>12.4</td>
<td>14.7</td>
<td>13.8</td>
<td>14.0</td>
</tr>
<tr>
<td>average, opt6</td>
<td>7.3</td>
<td>6.6</td>
<td>9.9</td>
<td>8.5</td>
<td>8.8</td>
</tr>
<tr>
<td>average, opt7</td>
<td>8.7</td>
<td>6.8</td>
<td>10.7</td>
<td>9.1</td>
<td>9.9</td>
</tr>
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</table>
Table 6.20. Stiffness [N.m\(^{-1}\)] and damping [N.s.m\(^{-1}\)] parameters for the tumbling track from the evaluation of the tumbling model (opt5, opt6 and opt7) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
<th>k4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, opt5</td>
<td>40319</td>
<td>88</td>
<td>44149</td>
<td>105</td>
</tr>
<tr>
<td>1, opt6</td>
<td>60220</td>
<td>10</td>
<td>48504</td>
<td>56</td>
</tr>
<tr>
<td>1, opt7</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
<tr>
<td>2, opt5</td>
<td>39276</td>
<td>186</td>
<td>56187</td>
<td>96</td>
</tr>
<tr>
<td>2, opt6</td>
<td>54373</td>
<td>271</td>
<td>69199</td>
<td>33</td>
</tr>
<tr>
<td>2, opt7</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
<tr>
<td>3, opt5</td>
<td>63313</td>
<td>95</td>
<td>89883</td>
<td>10</td>
</tr>
<tr>
<td>3, opt6</td>
<td>65218</td>
<td>179</td>
<td>89928</td>
<td>15</td>
</tr>
<tr>
<td>3, opt7</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
<tr>
<td>4, opt5</td>
<td>57657</td>
<td>10</td>
<td>39135</td>
<td>327</td>
</tr>
<tr>
<td>4, opt6</td>
<td>41895</td>
<td>378</td>
<td>65027</td>
<td>54</td>
</tr>
<tr>
<td>4, opt7</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
<tr>
<td>5, opt5</td>
<td>50980</td>
<td>59</td>
<td>48807</td>
<td>76</td>
</tr>
<tr>
<td>5, opt6</td>
<td>33631</td>
<td>331</td>
<td>54502</td>
<td>45</td>
</tr>
<tr>
<td>5, opt7</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
<tr>
<td>average, opt5</td>
<td>50309</td>
<td>88</td>
<td>55632</td>
<td>123</td>
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<td>234</td>
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</tr>
<tr>
<td>average, opt7</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
</tbody>
</table>

k1 = horizontal stiffness  
k2 = horizontal damping  
k3 = vertical stiffness  
k4 = vertical damping
Table 6.21. Activation times [s] for each torque generator and the initial activation of the ankle torque generator from the evaluation of the tumbling model (opt5, opt6 and opt7) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>ankle</th>
<th>knee</th>
<th>hip</th>
<th>shoulder</th>
<th>iact</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, opt5</td>
<td>0.030</td>
<td>0.015</td>
<td>0.000</td>
<td>0.130</td>
<td>0.172</td>
</tr>
<tr>
<td>1, opt6</td>
<td>0.030</td>
<td>0.016</td>
<td>0.000</td>
<td>0.130</td>
<td>1.319</td>
</tr>
<tr>
<td>1, opt7</td>
<td>0.005</td>
<td>0.020</td>
<td>0.000</td>
<td>0.129</td>
<td>0.050</td>
</tr>
<tr>
<td>2, opt5</td>
<td>0.036</td>
<td>0.005</td>
<td>0.001</td>
<td>0.115</td>
<td>1.992</td>
</tr>
<tr>
<td>2, opt6</td>
<td>0.014</td>
<td>0.005</td>
<td>0.003</td>
<td>0.067</td>
<td>1.050</td>
</tr>
<tr>
<td>2, opt7</td>
<td>0.016</td>
<td>0.005</td>
<td>0.005</td>
<td>0.066</td>
<td>1.318</td>
</tr>
<tr>
<td>3, opt5</td>
<td>0.000</td>
<td>0.000</td>
<td>0.052</td>
<td>0.032</td>
<td>0.965</td>
</tr>
<tr>
<td>3, opt6</td>
<td>0.000</td>
<td>0.001</td>
<td>0.113</td>
<td>0.031</td>
<td>1.711</td>
</tr>
<tr>
<td>3, opt7</td>
<td>0.009</td>
<td>0.001</td>
<td>0.120</td>
<td>0.021</td>
<td>1.407</td>
</tr>
<tr>
<td>4, opt5</td>
<td>0.041</td>
<td>0.019</td>
<td>0.000</td>
<td>0.130</td>
<td>0.160</td>
</tr>
<tr>
<td>4, opt6</td>
<td>0.000</td>
<td>0.027</td>
<td>0.000</td>
<td>0.109</td>
<td>0.896</td>
</tr>
<tr>
<td>4, opt7</td>
<td>0.002</td>
<td>0.027</td>
<td>0.000</td>
<td>0.107</td>
<td>0.373</td>
</tr>
<tr>
<td>5, opt5</td>
<td>0.050</td>
<td>0.021</td>
<td>0.003</td>
<td>0.130</td>
<td>0.667</td>
</tr>
<tr>
<td>5, opt6</td>
<td>0.006</td>
<td>0.041</td>
<td>0.000</td>
<td>0.130</td>
<td>0.151</td>
</tr>
<tr>
<td>5, opt7</td>
<td>0.005</td>
<td>0.035</td>
<td>0.000</td>
<td>0.129</td>
<td>0.008</td>
</tr>
<tr>
<td>average, opt5</td>
<td>0.031</td>
<td>0.012</td>
<td>0.011</td>
<td>0.107</td>
<td>0.791</td>
</tr>
<tr>
<td>average, opt6</td>
<td>0.017</td>
<td>0.017</td>
<td>0.025</td>
<td>0.095</td>
<td>1.215</td>
</tr>
<tr>
<td>average, opt7</td>
<td>0.007</td>
<td>0.018</td>
<td>0.025</td>
<td>0.090</td>
<td>0.631</td>
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</table>
Table 6.22. Values of the variables for the performance criterion from the evaluation of the tumbling model (opt5, opt6 and opt7) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>$u_e$ [m.s$^{-1}$]</th>
<th>$v_e$ [m.s$^{-1}$]</th>
<th>$h_e$ [kg.m$^2$.rad.s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, actual</td>
<td>2.93</td>
<td>4.98</td>
<td>56</td>
</tr>
<tr>
<td>1, opt5</td>
<td>3.22</td>
<td>4.57</td>
<td>56</td>
</tr>
<tr>
<td>1, opt6</td>
<td>2.93</td>
<td>5.46</td>
<td>56</td>
</tr>
<tr>
<td>1, opt7</td>
<td>3.01</td>
<td>5.22</td>
<td>57</td>
</tr>
<tr>
<td>2, actual</td>
<td>2.79</td>
<td>5.09</td>
<td>59</td>
</tr>
<tr>
<td>2, opt5</td>
<td>3.26</td>
<td>4.31</td>
<td>60</td>
</tr>
<tr>
<td>2, opt6</td>
<td>2.89</td>
<td>5.27</td>
<td>61</td>
</tr>
<tr>
<td>2, opt7</td>
<td>2.92</td>
<td>5.25</td>
<td>62</td>
</tr>
<tr>
<td>3, actual</td>
<td>2.66</td>
<td>4.53</td>
<td>96</td>
</tr>
<tr>
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<td>3.02</td>
<td>4.29</td>
<td>100</td>
</tr>
<tr>
<td>3, opt6</td>
<td>3.02</td>
<td>4.82</td>
<td>98</td>
</tr>
<tr>
<td>3, opt7</td>
<td>3.15</td>
<td>4.54</td>
<td>100</td>
</tr>
<tr>
<td>4, actual</td>
<td>2.99</td>
<td>4.50</td>
<td>95</td>
</tr>
<tr>
<td>4, opt5</td>
<td>3.23</td>
<td>3.74</td>
<td>87</td>
</tr>
<tr>
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<td>2.81</td>
<td>4.84</td>
<td>86</td>
</tr>
<tr>
<td>4, opt7</td>
<td>2.78</td>
<td>4.94</td>
<td>87</td>
</tr>
<tr>
<td>5, actual</td>
<td>2.63</td>
<td>3.93</td>
<td>107</td>
</tr>
<tr>
<td>5, opt5</td>
<td>3.28</td>
<td>3.51</td>
<td>89</td>
</tr>
<tr>
<td>5, opt6</td>
<td>2.81</td>
<td>4.42</td>
<td>108</td>
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<tr>
<td>5, opt7</td>
<td>2.79</td>
<td>4.43</td>
<td>107</td>
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Table 6.23. Values of the variables for the elastic criterion from the evaluation of the tumbling model (opt5, opt6 and opt7) for each trial

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<th>trial, optimisation</th>
<th>t [s]</th>
<th>$q_{1 \max}$ [m]</th>
<th>$q_{1 \text{end}}$ [m]</th>
<th>$q_{2 \max}$ [m]</th>
<th>$q_{2 \text{end}}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, actual</td>
<td>0.120</td>
<td>-0.060</td>
<td>0.000</td>
<td>-0.060</td>
<td>0.000</td>
</tr>
<tr>
<td>1, opt5</td>
<td>0.135</td>
<td>-0.069</td>
<td>-0.002</td>
<td>-0.059</td>
<td>-0.001</td>
</tr>
<tr>
<td>1, opt6</td>
<td>0.128</td>
<td>-0.060</td>
<td>-0.003</td>
<td>-0.060</td>
<td>-0.004</td>
</tr>
<tr>
<td>1, opt7</td>
<td>0.133</td>
<td>-0.050</td>
<td>0.001</td>
<td>-0.056</td>
<td>-0.002</td>
</tr>
<tr>
<td>2, actual</td>
<td>0.120</td>
<td>-0.060</td>
<td>0.000</td>
<td>-0.060</td>
<td>0.000</td>
</tr>
<tr>
<td>2, opt5</td>
<td>0.132</td>
<td>-0.073</td>
<td>0.002</td>
<td>-0.059</td>
<td>0.000</td>
</tr>
<tr>
<td>2, opt6</td>
<td>0.130</td>
<td>-0.060</td>
<td>-0.001</td>
<td>-0.059</td>
<td>-0.002</td>
</tr>
<tr>
<td>2, opt7</td>
<td>0.131</td>
<td>-0.064</td>
<td>-0.001</td>
<td>-0.062</td>
<td>-0.002</td>
</tr>
<tr>
<td>3, actual</td>
<td>0.110</td>
<td>-0.060</td>
<td>0.000</td>
<td>-0.060</td>
<td>0.000</td>
</tr>
<tr>
<td>3, opt5</td>
<td>0.108</td>
<td>-0.060</td>
<td>-0.004</td>
<td>-0.047</td>
<td>0.000</td>
</tr>
<tr>
<td>3, opt6</td>
<td>0.114</td>
<td>-0.057</td>
<td>-0.008</td>
<td>-0.058</td>
<td>0.000</td>
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<tr>
<td>3, opt7</td>
<td>0.117</td>
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<td>-0.003</td>
<td>-0.059</td>
<td>-0.002</td>
</tr>
<tr>
<td>4, actual</td>
<td>0.120</td>
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<td>0.000</td>
<td>-0.060</td>
<td>0.000</td>
</tr>
<tr>
<td>4, opt5</td>
<td>0.146</td>
<td>-0.056</td>
<td>0.000</td>
<td>-0.054</td>
<td>-0.004</td>
</tr>
<tr>
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<td>-0.003</td>
<td>-0.060</td>
<td>-0.003</td>
</tr>
<tr>
<td>4, opt7</td>
<td>0.141</td>
<td>-0.056</td>
<td>-0.006</td>
<td>-0.058</td>
<td>-0.002</td>
</tr>
<tr>
<td>5, actual</td>
<td>0.120</td>
<td>-0.060</td>
<td>0.000</td>
<td>-0.060</td>
<td>0.000</td>
</tr>
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<td>-0.047</td>
<td>-0.003</td>
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<td>-0.005</td>
<td>-0.059</td>
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<td>0.154</td>
<td>-0.047</td>
<td>-0.006</td>
<td>-0.054</td>
<td>-0.002</td>
</tr>
</tbody>
</table>

$q_{1 \max}$ = maximum horizontal depression  
$q_{1 \text{end}}$ = final horizontal depression  
$q_{2 \max}$ = maximum vertical depression  
$q_{2 \text{end}}$ = final vertical depression
Table 6.24. Values of the variables for the strategy criterion from the evaluation of the tumbling model (opt5, opt6 and opt7) for each trial [°]

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>$a_{\text{amin}}$</th>
<th>$a$</th>
<th>$k_{\text{amin}}$</th>
<th>$k_a$</th>
<th>$h_a$</th>
<th>$t_r$</th>
<th>$s_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, actual</td>
<td>73</td>
<td>125</td>
<td>142</td>
<td>174</td>
<td>176</td>
<td>78</td>
<td>154</td>
</tr>
<tr>
<td>1, opt5</td>
<td>44</td>
<td>108</td>
<td>133</td>
<td>155</td>
<td>148</td>
<td>73</td>
<td>184</td>
</tr>
<tr>
<td>1, opt6</td>
<td>47</td>
<td>143</td>
<td>137</td>
<td>196</td>
<td>176</td>
<td>76</td>
<td>159</td>
</tr>
<tr>
<td>1, opt7</td>
<td>48</td>
<td>145</td>
<td>132</td>
<td>188</td>
<td>172</td>
<td>78</td>
<td>157</td>
</tr>
<tr>
<td>2, actual</td>
<td>74</td>
<td>128</td>
<td>139</td>
<td>177</td>
<td>183</td>
<td>82</td>
<td>158</td>
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<tr>
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<td>110</td>
<td>137</td>
<td>161</td>
<td>161</td>
<td>79</td>
<td>153</td>
</tr>
<tr>
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<td>47</td>
<td>145</td>
<td>137</td>
<td>188</td>
<td>179</td>
<td>80</td>
<td>154</td>
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<td>47</td>
<td>145</td>
<td>136</td>
<td>188</td>
<td>179</td>
<td>81</td>
<td>155</td>
</tr>
<tr>
<td>3, actual</td>
<td>73</td>
<td>125</td>
<td>141</td>
<td>169</td>
<td>203</td>
<td>100</td>
<td>153</td>
</tr>
<tr>
<td>3, opt5</td>
<td>49</td>
<td>107</td>
<td>145</td>
<td>171</td>
<td>205</td>
<td>101</td>
<td>160</td>
</tr>
<tr>
<td>3, opt6</td>
<td>49</td>
<td>141</td>
<td>144</td>
<td>191</td>
<td>211</td>
<td>102</td>
<td>162</td>
</tr>
<tr>
<td>3, opt7</td>
<td>47</td>
<td>143</td>
<td>144</td>
<td>200</td>
<td>220</td>
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<td>172</td>
</tr>
<tr>
<td>4, actual</td>
<td>76</td>
<td>130</td>
<td>141</td>
<td>169</td>
<td>200</td>
<td>100</td>
<td>155</td>
</tr>
<tr>
<td>4, opt5</td>
<td>29</td>
<td>98</td>
<td>127</td>
<td>148</td>
<td>181</td>
<td>101</td>
<td>172</td>
</tr>
<tr>
<td>4, opt6</td>
<td>41</td>
<td>145</td>
<td>132</td>
<td>175</td>
<td>196</td>
<td>102</td>
<td>153</td>
</tr>
<tr>
<td>4, opt7</td>
<td>41</td>
<td>144</td>
<td>132</td>
<td>172</td>
<td>195</td>
<td>102</td>
<td>152</td>
</tr>
<tr>
<td>5, actual</td>
<td>78</td>
<td>141</td>
<td>125</td>
<td>153</td>
<td>209</td>
<td>116</td>
<td>149</td>
</tr>
<tr>
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<td>34</td>
<td>154</td>
<td>112</td>
<td>163</td>
<td>195</td>
<td>114</td>
<td>182</td>
</tr>
<tr>
<td>5, opt6</td>
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<td>147</td>
<td>123</td>
<td>158</td>
<td>201</td>
<td>116</td>
<td>153</td>
</tr>
<tr>
<td>5, opt7</td>
<td>38</td>
<td>146</td>
<td>121</td>
<td>159</td>
<td>199</td>
<td>113</td>
<td>151</td>
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</table>
Computer graphics for tumbling evaluation

A visual comparison between the simulated and actual performance from the start of the final contact through to landing was made using computer graphics software developed at Loughborough University. The simulation model of Yeadon (1990c) was used to simulate the postflight phase for each tumbling performance using the takeoff characteristics calculated for each trial. The simulation model calculated the whole body orientation during postflight using the same configuration changes as each actual performance and the angular momentum at takeoff as determined in the simulation of the contact phase. The position of each graphic for the postflight was calculated from the mass centre velocity at takeoff using projectile equations of motion.

Computer graphics of the best overall comparison (opt7) between the simulated and actual performance with fixed stiffness and damping parameters for the tumbling track are presented below for trials 2 and 4 which gave the best match between the simulated and actual performance (valpes smallest for opt7) for a layout and double layout performance; although all five trials had acceptable agreement with the actual performances (Table 6.19). The graphics sequences show the effect of the difference in takeoff conditions between simulated and actual performance on the postflight of each tumbling trial.

Figure 6.3 shows the actual and simulated performance of a layout somersault (trial 2). The first two pictures in each sequence are the same, as the simulation started with the same initial conditions at the start of the contact phase with the tumbling track. At takeoff the simulation model has very similar characteristics to the actual performance, although the knee has extended too far. In the future it may be desirable to constrain the model to discourage/prevent solutions with unrealistic joint angles being achieved.

For the double layout somersault, the agreement with the actual performance is not quite so good (Figure 6.4). At takeoff the simulated performance has greater vertical velocity but less angular momentum which results in a slightly longer flight time but a slower rate of rotation.
Figure 6.3. Computer graphics sequences of the actual and simulated performance (opt 7) of trial 2 a layout performance.
Figure 6.4. Computer graphics sequences of the actual and simulated performance (opt 7) of trial 4 a double layout performance.
Vaulting model

A two phase optimisation procedure was used to evaluate the vaulting model using $v_{\text{pes}}$ as the objective function. This function was used as it proved to be the most appropriate for the evaluation of the tumbling model. To simplify the optimisation the single torque generator in the vaulting model was assumed to be maximally activated throughout the contact phase with the stretched angular velocity profile used to calculate the torque produced.

**Evaluation using performance, elastic and strategy criteria with stretched angular velocity profile**

opt1 used $v_{\text{pes}}$ as the objective function (Table 6.16) with a stretched angular velocity profile. The optimisation minimised $v_{\text{pes}}$ by varying seven unknown parameters identified for the vaulting model (Table 6.17).

The average value of $v_{\text{pes}}$ for the three trials was 15.7 ° (%), which is quite high. However looking at the individual results shows that most of the variables are in good agreement with the actual performances (Table 6.25 - Table 6.29) apart from the time of contact, the horizontal velocity of the mass centre and the shoulder angle. The time of contact is a bit short, the horizontal velocity of the mass centre is rather high and the shoulder angle is too small for the simulated performances compared with the actual performances. There are a number of possible reasons for the differences:

- From the image analysis of the actual performances the hands initially appear to slide along the horse top and this is not permitted in the model. This sliding would have the effect of gradually slowing down the horizontal velocity during the initial part of horse contact. One possible way to represent the hands sliding along the horse top would be to use a velocity damper instead of a horizontal linear spring. This would have the effect of slowing down the velocity of the hands but not allow them to spring back.

- The present model uses linear springs at the hands and the shoulders. In reality it is more likely that the movement at the hands and the shoulders would be better represented by non-linear springs, although preliminary findings on using a non-linear spring at the shoulders did not greatly improve the match between the simulated and
actual performance. Using non-linear springs at the hand and shoulder would probably allow increased time of contact with the horse.

These ideas for improving the performance of the simulation model were not investigated in this study, although it is hoped to address these areas in the future (see Chapter 7 for further details).

To consider the effect of using the same stiffness and damping parameters for the springs at the hands and shoulders the average stiffness and damping values were calculated for the three trials (opt1) to give one set of stiffness and damping parameters to be used with each trial.

*Evaluation using performance, elastic and strategy criteria with stretched angular velocity profile and average elastic parameters*

opt2 used \( \text{val}_{\text{pes}} \) as the objective function (Table 6.16) with the average stiffness and damping values calculated from the three trials for opt1 and a stretched angular velocity profile. The optimisation minimised \( \text{val}_{\text{pes}} \) by varying the time the hand spring length was changed (Table 6.17).

There was only a small decrease in the value of \( \text{val}_{\text{pes}} \) between opt1 and opt2 of 1 ° (%) demonstrating that the model was not too sensitive to the stiffness and damping parameters used for the vaulting horse and shoulder.

The other criteria also did not change very much with less than 2 units of difference in any of the criteria between opt1 and opt2.

In the future the movement of the hands and surface of the vaulting horse should be examined in more detail but for the purposes of this study the agreement was acceptable and indicates that a relatively simple model is able to reproduce the major features of vaulting performance.
Table 6.25. Scores for the evaluation of the vaulting model (opt1 and opt2) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>$val_p$ [%]</th>
<th>$val_p$ [%]</th>
<th>$val$ [°]</th>
<th>$val_{ps}$ [° (%)]</th>
<th>$val_{pes}$ [° (%)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, opt1</td>
<td>14.1</td>
<td>15.9</td>
<td>6.6</td>
<td>12.2</td>
<td>13.0</td>
</tr>
<tr>
<td>1, opt2</td>
<td>16.0</td>
<td>15.5</td>
<td>6.4</td>
<td>11.9</td>
<td>14.6</td>
</tr>
<tr>
<td>2, opt1</td>
<td>21.0</td>
<td>24.0</td>
<td>5.2</td>
<td>17.4</td>
<td>18.9</td>
</tr>
<tr>
<td>2, opt2</td>
<td>21.8</td>
<td>26.6</td>
<td>5.5</td>
<td>19.2</td>
<td>19.7</td>
</tr>
<tr>
<td>3, opt1</td>
<td>15.2</td>
<td>16.0</td>
<td>15.8</td>
<td>15.9</td>
<td>15.3</td>
</tr>
<tr>
<td>3, opt2</td>
<td>15.9</td>
<td>15.8</td>
<td>15.9</td>
<td>15.9</td>
<td>15.9</td>
</tr>
<tr>
<td>average, opt1</td>
<td>16.8</td>
<td>18.6</td>
<td>9.2</td>
<td>15.2</td>
<td>15.7</td>
</tr>
<tr>
<td>average, opt2</td>
<td>17.9</td>
<td>19.3</td>
<td>9.3</td>
<td>15.7</td>
<td>16.7</td>
</tr>
</tbody>
</table>

Table 6.26. Stiffness $[N.m^{-1}]$ and damping $[N.s.m^{-1}]$ parameters for the vaulting horse and shoulder, and the time the hand spring length was changed $[s]$ from the evaluation of the vaulting model (opt1 and opt2) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$k_5$</th>
<th>$k_6$</th>
<th>thand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, opt1</td>
<td>492</td>
<td>1983</td>
<td>46494</td>
<td>7491</td>
<td>20282</td>
<td>10</td>
<td>0.072</td>
</tr>
<tr>
<td>1, opt2</td>
<td>2814</td>
<td>1943</td>
<td>23097</td>
<td>7245</td>
<td>20961</td>
<td>10</td>
<td>0.070</td>
</tr>
<tr>
<td>2, opt1</td>
<td>1230</td>
<td>1889</td>
<td>14169</td>
<td>7152</td>
<td>22377</td>
<td>10</td>
<td>0.042</td>
</tr>
<tr>
<td>2, opt2</td>
<td>2814</td>
<td>1943</td>
<td>23097</td>
<td>7245</td>
<td>20961</td>
<td>10</td>
<td>0.039</td>
</tr>
<tr>
<td>3, opt1</td>
<td>6719</td>
<td>1956</td>
<td>8629</td>
<td>7091</td>
<td>20224</td>
<td>10</td>
<td>0.062</td>
</tr>
<tr>
<td>3, opt2</td>
<td>2814</td>
<td>1943</td>
<td>23097</td>
<td>7245</td>
<td>20961</td>
<td>10</td>
<td>0.065</td>
</tr>
<tr>
<td>average, opt1</td>
<td>2814</td>
<td>1943</td>
<td>23097</td>
<td>7245</td>
<td>20961</td>
<td>10</td>
<td>0.059</td>
</tr>
<tr>
<td>average, opt2</td>
<td>2814</td>
<td>1943</td>
<td>23097</td>
<td>7245</td>
<td>20961</td>
<td>10</td>
<td>0.058</td>
</tr>
</tbody>
</table>

$k_1 = \text{horizontal stiffness}$ \hspace{1cm} $k_2 = \text{horizontal damping}$

$k_3 = \text{vertical stiffness}$ \hspace{1cm} $k_4 = \text{vertical damping}$

$k_5 = \text{shoulder stiffness}$ \hspace{1cm} $k_6 = \text{shoulder damping}$
Table 6.27. Values of the variables for the performance criterion from the evaluation of the vaulting model (opt1 and opt2) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>( u_g ) [m.s(^{-1})]</th>
<th>( v_g ) [m.s(^{-1})]</th>
<th>( h_g ) [kg.m(^2).rad.s(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, actual</td>
<td>4.05</td>
<td>1.79</td>
<td>-17.0</td>
</tr>
<tr>
<td>1, opt1</td>
<td>5.16</td>
<td>1.85</td>
<td>-15.2</td>
</tr>
<tr>
<td>1, opt2</td>
<td>5.15</td>
<td>1.88</td>
<td>-15.6</td>
</tr>
<tr>
<td>2, actual</td>
<td>4.32</td>
<td>2.00</td>
<td>-21.0</td>
</tr>
<tr>
<td>2, opt1</td>
<td>5.48</td>
<td>2.46</td>
<td>-16.8</td>
</tr>
<tr>
<td>2, opt2</td>
<td>5.50</td>
<td>2.37</td>
<td>-14.7</td>
</tr>
<tr>
<td>3, actual</td>
<td>4.42</td>
<td>1.91</td>
<td>-20.0</td>
</tr>
<tr>
<td>3, opt1</td>
<td>5.56</td>
<td>1.81</td>
<td>-18.9</td>
</tr>
<tr>
<td>3, opt2</td>
<td>5.53</td>
<td>1.80</td>
<td>-18.7</td>
</tr>
</tbody>
</table>

Table 6.28. Values of the variables for the elastic criterion from the evaluation of the vaulting model (opt1 and opt2) for each trial

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>( t ) [s]</th>
<th>( q_{1\text{max}} ) [m]</th>
<th>( q_{2\text{max}} ) [m]</th>
<th>( q_{4\text{max}} ) [m]</th>
<th>( q_{4\text{end}} ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, actual</td>
<td>0.169</td>
<td>-0.019</td>
<td>-0.013</td>
<td>-0.113</td>
<td>0.024</td>
</tr>
<tr>
<td>1, opt1</td>
<td>0.127</td>
<td>-0.019</td>
<td>-0.013</td>
<td>-0.102</td>
<td>0.011</td>
</tr>
<tr>
<td>1, opt2</td>
<td>0.126</td>
<td>-0.019</td>
<td>-0.015</td>
<td>-0.101</td>
<td>0.008</td>
</tr>
<tr>
<td>2, actual</td>
<td>0.159</td>
<td>-0.019</td>
<td>-0.013</td>
<td>-0.113</td>
<td>0.024</td>
</tr>
<tr>
<td>2, opt1</td>
<td>0.115</td>
<td>-0.019</td>
<td>-0.013</td>
<td>-0.095</td>
<td>-0.005</td>
</tr>
<tr>
<td>2, opt2</td>
<td>0.117</td>
<td>-0.017</td>
<td>-0.012</td>
<td>-0.097</td>
<td>-0.002</td>
</tr>
<tr>
<td>3, actual</td>
<td>0.164</td>
<td>-0.019</td>
<td>-0.013</td>
<td>-0.113</td>
<td>0.024</td>
</tr>
<tr>
<td>3, opt1</td>
<td>0.121</td>
<td>-0.019</td>
<td>-0.013</td>
<td>-0.094</td>
<td>0.012</td>
</tr>
<tr>
<td>3, opt2</td>
<td>0.121</td>
<td>-0.021</td>
<td>-0.012</td>
<td>-0.093</td>
<td>0.013</td>
</tr>
</tbody>
</table>

\( q_{1\text{max}} \) = maximum horizontal depression  
\( q_{2\text{max}} \) = maximum vertical depression  
\( q_{4\text{max}} \) = maximum shoulder depression  
\( q_{4\text{end}} \) = final shoulder depression
Table 6.29. Values of the variables for the strategy criterion from the evaluation of the vaulting model (opt1 and opt2) for each trial [°]

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>tr₁</th>
<th>s₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, actual</td>
<td>6</td>
<td>73</td>
</tr>
<tr>
<td>1, opt1</td>
<td>4</td>
<td>58</td>
</tr>
<tr>
<td>1, opt2</td>
<td>4</td>
<td>59</td>
</tr>
<tr>
<td>2, actual</td>
<td>10</td>
<td>76</td>
</tr>
<tr>
<td>2, opt1</td>
<td>7</td>
<td>65</td>
</tr>
<tr>
<td>2, opt2</td>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>3, actual</td>
<td>16</td>
<td>82</td>
</tr>
<tr>
<td>3, opt1</td>
<td>3</td>
<td>58</td>
</tr>
<tr>
<td>3, opt2</td>
<td>2</td>
<td>58</td>
</tr>
</tbody>
</table>

Computer graphics for vaulting evaluation

A visual comparison between the simulated and actual performance from the start of the final contact through to landing was made using computer graphics software developed at Loughborough University. The simulation model of Yeadon (1990c) was used to simulate the postflight phase for each vaulting performance using the takeoff characteristics calculated for each trial. The simulation model calculated the whole body orientation during postflight using the same configuration changes as each actual performance and the angular momentum at takeoff as determined in the simulation of the contact phase. The position of each graphic for the postflight was calculated from the mass centre velocity at takeoff using projectile equations of motion.

Computer graphics of the best overall comparison (opt2) between the simulated and actual performance with fixed stiffness and damping parameters for the vaulting horse are presented below for trial 1 (Figure 6.5). The graphics sequences show the effect of the difference in takeoff conditions between simulated and actual performance on the postflight performance of each vaulting performance.
At takeoff from the horse the simulation model has too much horizontal velocity which results in the model travelling further in postflight than the actual performance.

Figure 6.5. Computer graphics sequences of the actual and simulated performance (opt 2) of trial 1 a Hecht vault.
CONTRIBUTIONS TO VAULTING PERFORMANCE USING THE TWO SEGMENT MODEL OF VAULTING

The two segment simulation model was used to examine the contributions of the preflight and inertia characteristics to the performance of the vault.

Method

The two segment simulation model was used to simulate two completely different types of vault (Hecht and handspring somersault) by adjusting the preflight characteristics of the simulation model at horse contact. In addition for the handspring somersault vault the arms were brought to the sides of the body at takeoff from the horse and remained there throughout the postflight phase.

The sensitivity of the performance of the Hecht vault and handspring somersault vault (Figure 6.6 and Figure 6.7) to changes in inertial characteristics were examined for eleven normalised inertia data sets. For the Hecht vault single simulations were run with each of the eleven normalised inertia data sets and fixed preflight conditions at horse contact. For the handspring somersault vault the preflight characteristics at horse contact were allowed to vary for each of the eleven normalised inertia data sets in order to optimise the postflight performance. Upper limits were required for the horizontal preflight velocity and the preflight angular velocity as it was found that faster preflights produced greater height, distance and rotation for the handspring somersault vault. The horizontal preflight velocity was fixed at 6.05 m.s⁻¹ and the preflight angular velocity was fixed at 466°s⁻¹ based upon data for Olympic gymnasts (Takei and Kim, 1990). The shoulder angle \( \alpha \) and the vertical velocity \( v \) of the mass centre at horse contact were allowed to vary.

For a successful handspring somersault simulation the path of the mass centre should be high with the distance travelled by the mass centre at least 2.5 m past the end of the horse as required by the International Gymnastics Federation in order for there to be no deductions from the score. In addition the landing angle needs to be such that the gymnast does not need to take a step forwards or backwards at landing. Simulation was used to estimate the required landing angle \( \gamma_0 \) behind the vertical. An objective function \( f \) defined by equation (6.4) was maximised where there was a range of possible solutions that satisfied all the criteria for a successful simulation. The objective function was based
upon the maximum height $h$ of the mass centre, the landing angle $\gamma$ and the landing distance $d$. The coefficients for each variable were based upon the variation in each variable (Table 6.30) obtained from analysis of competitive handspring somersault vaults (Takei and Kim, 1990) in order to give appropriate weighting to each variable.

$$f = \frac{h}{\sigma_h} + \frac{d}{\sigma_d} + \frac{|\gamma - \gamma_0|}{\sigma_\gamma}$$  \hspace{1cm} (6.4)

where $h$ = the maximum height of the mass centre during postflight

$d$ = the landing distance of the mass centre past the end of the horse

$\gamma$ = the landing angle

$\sigma$ = the standard deviation in each variable

Table 6.30. Standard deviations for the three variables used in the objective function

<table>
<thead>
<tr>
<th>variable</th>
<th>standard deviation (Takei and Kim, 1990)</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum height of the mass centre</td>
<td>0.08 m</td>
</tr>
<tr>
<td>landing distance</td>
<td>0.34 m</td>
</tr>
<tr>
<td>landing angle</td>
<td>$4.2^\circ$</td>
</tr>
</tbody>
</table>

Results and Discussion

By just altering the preflight characteristics a simulation model comprising two rigid segments linked by a pin joint was able to simulate the Hecht vault and handspring somersault vault (Figure 6.6 and Figure 6.7). This suggests that the majority of the backwards rotation during the postflight of a Hecht vault is a result of a good preflight (a low trajectory of the mass centre during preflight with a low vertical velocity of the mass centre and a low angular velocity of the body at horse contact). Similarly the majority of the postflight rotation of a handspring somersault is a result of a high preflight trajectory with a high angular velocity of the body and a high vertical velocity at horse contact. Despite the assumption of inelasticity and zero shoulder torque during horse contact, the two segment model was able to give reasonable agreement with observed values from a competitive vault. This demonstrates the importance of the preflight characteristics for vaulting performance and indicates that the model, although simplified, incorporates some of the main elements of vaulting.
The postflight performance of the Hecht vault was found to be quite sensitive to the inertia characteristics of the gymnast when the preflight was fixed with differences in the foot clearance, maximum height, landing distance and landing angle found for each of the eleven inertia sets (Table 6.31). This result is also true for the handspring somersault simulations.

Table 6.31. Variation in performance of the Hecht vault for 11 sets of inertia parameters

<table>
<thead>
<tr>
<th>inertia set</th>
<th>foot clearance [m]</th>
<th>maximum height [m]</th>
<th>landing angle [°]</th>
<th>landing distance [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>average</td>
<td>0.00</td>
<td>2.28</td>
<td>-25.3</td>
<td>2.32</td>
</tr>
<tr>
<td>1</td>
<td>0.08</td>
<td>2.37</td>
<td>10.6</td>
<td>2.13</td>
</tr>
<tr>
<td>2</td>
<td>-0.03</td>
<td>2.22</td>
<td>-35.2</td>
<td>2.45</td>
</tr>
<tr>
<td>3</td>
<td>0.02</td>
<td>2.28</td>
<td>-27.1</td>
<td>2.34</td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
<td>2.33</td>
<td>-9.9</td>
<td>2.19</td>
</tr>
<tr>
<td>5</td>
<td>-0.02</td>
<td>2.28</td>
<td>-33.5</td>
<td>2.41</td>
</tr>
<tr>
<td>6</td>
<td>0.01</td>
<td>2.31</td>
<td>-27.4</td>
<td>2.34</td>
</tr>
<tr>
<td>7</td>
<td>-0.01</td>
<td>2.27</td>
<td>-21.3</td>
<td>2.28</td>
</tr>
<tr>
<td>8</td>
<td>-0.03</td>
<td>2.25</td>
<td>-44.0</td>
<td>2.54</td>
</tr>
<tr>
<td>9</td>
<td>0.09</td>
<td>2.31</td>
<td>-8.3</td>
<td>2.24</td>
</tr>
<tr>
<td>10</td>
<td>-0.04</td>
<td>2.26</td>
<td>-39.2</td>
<td>2.48</td>
</tr>
<tr>
<td>11</td>
<td>-0.04</td>
<td>2.24</td>
<td>-50.4</td>
<td>2.63</td>
</tr>
<tr>
<td>mean</td>
<td>0.01</td>
<td>2.28</td>
<td>-26.1</td>
<td>2.36</td>
</tr>
<tr>
<td>σ</td>
<td>0.05</td>
<td>0.05</td>
<td>17.6</td>
<td>0.16</td>
</tr>
</tbody>
</table>

On the other hand allowing the preflight variables to vary for each of the eleven inertia data sets in order to optimise the postflight performance of the handspring somersault vault (Table 6.32) shows that the optimum postflight performance is not sensitive to the inertia parameters used. This result is also true for the Hecht simulations. Therefore it may be speculated that a gymnast's inertia parameters do not affect the level
of performance that is attainable but do affect the technique used to achieve optimum performance.

Table 6.32. Variation in optimum preflight variables for the handspring somersault vault for 11 sets of inertia parameters

<table>
<thead>
<tr>
<th>inertia set</th>
<th>optimised preflight variables at horse contact</th>
<th>postflight variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi$ [°]</td>
<td>$\alpha$ [°]</td>
</tr>
<tr>
<td>average</td>
<td>50.0</td>
<td>178.0</td>
</tr>
<tr>
<td>1</td>
<td>50.8</td>
<td>177.2</td>
</tr>
<tr>
<td>2</td>
<td>50.4</td>
<td>179.2</td>
</tr>
<tr>
<td>3</td>
<td>48.8</td>
<td>175.2</td>
</tr>
<tr>
<td>4</td>
<td>48.2</td>
<td>174.0</td>
</tr>
<tr>
<td>5</td>
<td>50.0</td>
<td>177.8</td>
</tr>
<tr>
<td>6</td>
<td>51.4</td>
<td>180.2</td>
</tr>
<tr>
<td>7</td>
<td>50.2</td>
<td>176.2</td>
</tr>
<tr>
<td>8</td>
<td>50.0</td>
<td>177.0</td>
</tr>
<tr>
<td>9</td>
<td>52.8</td>
<td>184.4</td>
</tr>
<tr>
<td>10</td>
<td>51.8</td>
<td>181.8</td>
</tr>
<tr>
<td>11</td>
<td>48.2</td>
<td>175.6</td>
</tr>
<tr>
<td>mean</td>
<td>50.2</td>
<td>178.1</td>
</tr>
<tr>
<td>s.d.</td>
<td>1.5</td>
<td>3.1</td>
</tr>
</tbody>
</table>
Figure 6.6. Hecht vault simulation using the two segment model.

Figure 6.7. Straight handspring somersault simulation using the two segment model.
CONTRIBUTIONS TO TUMBLING PERFORMANCE USING THE FIVE SEGMENT MODEL OF TUMBLING

The five segment tumbling model was used to examine the contributions of the following factors to tumbling performance:

- Preflight characteristics
- Muscle strength and activation
- Series elastic component
- Elastic characteristics of the tumbling track

Method

The following single simulations were run with the best layout (trial 2) and best double layout (trial 4) to investigate the contributions made by each factor to tumbling performance:

- The preflight of the layout with the strategy (muscle timings) of the double layout [sim1]
- The preflight of the double layout with the strategy (muscle timings) of the layout [sim1]
- The torque generators switched off [sim3]
- A 10 fold increase in the stiffness of the SEC [sim4]
- A 10 fold increase in the stiffness of the tumbling track [sim5]

The technique used to perform the layout / double layout was split into two parts; preflight and strategy. The preflight referred to the mass centre velocity, joint angles and angular velocities at touchdown prior to the takeoff and strategy referred to the timings of the torque generators used during the contact phase. Two additional procedures were used to investigate the contributions of the preflight and strategy:

Strategy

Two optimisations (sim2) were run to see if it was possible to do any better than sim1 by optimising the strategy used (activation of the torque generators) to perform a double layout with the preflight of a layout and to perform a layout with the preflight of a double layout. The Simulated Annealing algorithm was used to minimise $v_{1p}$ (Table 6.16) by varying the activation of the torque generators (strategy).
Preflight

To demonstrate the importance of the preflight for the layout and double layout takeoffs the preflight of the double layout was altered (with as few changes as possible) to perform a layout with the strategy (activation of the torque generators) of the double layout (sim6).

Results and Discussion

The contributions to tumbling performance are given in tabular form (Table 6.33 - Table 6.38) and computer graphics form (Figure 6.8 - Figure 6.11).

Preflight characteristics

sim1 showed that the performance was sensitive to changes in preflight (swapping the preflight of the layout and double layout) when the strategies were not changed from opt7 with valp increasing to approximately 24% (Table 6.33, Table 6.35, Figure 6.8, Figure 6.10). Using the preflight of the layout with the strategy of the double layout resulted in takeoff characteristics which were more like the double layout than the layout (2, sim1; Table 6.36). Likewise using the preflight of the double layout with the strategy of the layout resulted in takeoff characteristics which were more like the layout than the double layout (4, sim1; Table 6.36). If the strategies were allowed to vary it was found to be possible to produce the required takeoff characteristics for the layout with the preflight of the double layout and the required takeoff characteristics for the double layout with the preflight of the layout, with a value of valp of less than 1% (Table 6.33, Table 6.35, Figure 6.8, Figure 6.10). This is an important result as it shows that for an actual performance it is possible to produce very similar performances for a range of preflights by just altering the strategy used during the takeoff. However if the preflight is very different then it would not be possible to change the strategy used to produce a similar performance.

sim6 demonstrated the importance of the preflight (Figure 6.12) by changing the preflight of the double layout to produce a layout performance (Table 6.33, Figure 6.12). In fact it was only necessary to change the angular velocity of the trunk and arms at contact with the tumbling track (reduce trunk angular velocity from 793°s⁻¹ to 700°s⁻¹; reduce arm angular velocity from 762°s⁻¹ to 700°s⁻¹) to produce a layout performance with a value of valp of less than 5% (Table 6.33).
Muscle strength and activation

\[ \text{sim1 \ and \ sim2 \ showed \ the \ importance \ of \ muscle \ timings \ to \ the \ performance \ of \ the} \]
\[ \text{layout \ and \ double \ layout, \ with \ there \ being \ an \ optimum \ strategy \ for \ a \ particular \ preflight.} \]
\[ \text{The \ preflight \ of \ the \ layout \ and \ double \ layout \ were \ similar \ (Table \ 6.9) \ but \ the \ muscle} \]
\[ \text{timings \ used \ during \ the \ takeoff \ changed \ the \ performance \ into \ the \ layout \ or \ double \ layout.} \]
\[ \text{Switching \ all \ the \ torque \ generators \ in \ the \ simulation \ model \ off \ during \ the \ takeoff \ (sim3) \}
\[ \text{resulted \ in \ very \ poor \ performances \ of \ the \ layout \ and \ double \ layout \ (Table \ 6.36, \ Figure} \]
\[ \text{6.9, \ Figure \ 6.11). \ For \ the \ double \ layout \ there \ was \ insufficient \ vertical \ velocity, \ too \ much} \]
\[ \text{horizontal \ velocity \ and \ too \ much \ angular \ momentum. \ In \ fact \ the \ horizontal \ velocity,} \]
\[ \text{angular \ momentum \ and \ vertical \ velocity \ did \ not \ change \ very \ much \ from \ their \ initial} \]
\[ \text{values \ at \ contact \ (Table \ 6.9, \ Table \ 6.36).} \]

SEC

Performance of the layout and double layout were found to be insensitive to changes in the stiffness of the series elastic component for a fixed strategy (sim4), with a 10 fold increase in stiffness resulting in approximately 1% change in the layout and double layout performances (Table 6.33, Figure 6.9, Figure 6.11). If the strategy used had been allowed to vary for the increased stiffness of the SEC it may have been possible to slightly improve the performance.

Although increasing the stiffness of the SEC did not improve performance very much, the decrease in the stretch of the SEC due to the higher stiffness resulted in 15° less stretch in the SEC. This had an effect on the movement of the contractile part of the torque generator causing it to stretch further for the stiffer SEC and therefore reach higher angular velocities both eccentrically and concentrically. A stiffer SEC may, therefore, not be of benefit as it would allow less force to be produced during the concentric phase of the movement.

Elasticity of the tumbling track

By increasing the stiffness of the tumbling track 10 fold and keeping a fixed strategy resulted in very poor layout and double layout performances (Table 6.33, Figure 6.9, Figure 6.11). However from the evaluation of the tumbling model (opt6 and opt7) it can be seen that performance is not very sensitive to changes in the stiffness of the
tumbling track when the strategy is allowed to vary (Table 6.19). Therefore it would appear that within reasonable limits the performance is not very sensitive to changes in the stiffness of the tumbling track as long as the strategy used can be changed.

Actual layout [trial 2]

Simulated layout [opt7]

Preflight for layout with double layout activation [sim1]

Preflight for layout with optimised activation for double layout [sim2]

Figure 6.8. Computer graphics demonstrating the contribution of muscle activation (sim1, sim2) to tumbling performance with layout (trial 2) preflight.
Figure 6.9. Computer graphics demonstrating the contributions from muscle strength (sim3), SEC (sim4) and stiffness of the tumbling track (sim5) to layout performance (trial 2).
Actual double layout [trial 4]

Simulated double layout [opt7]

Preflight for double layout with layout activation [sim1]

Preflight for double layout with optimised activation for layout [sim2]

Figure 6.10. Computer graphics demonstrating the contribution of muscle activation (sim1, sim2) to tumbling performance with double layout (trial 4) preflight.
Simulated double layout [opt7]

Simulated double layout with no muscle [sim3]

Simulated double layout with stiff SEC [sim4]

Simulated double layout with stiff tumbling track [sim5]

Figure 6.11. Computer graphics demonstrating the contributions from muscle strength (sim3), SEC (sim4) and stiffness of the tumbling track (sim5) to layout performance (trial 4).
Simulated double layout [opt7]

Simulation with strategy of double layout and preflight adjusted to give layout [sim6]

Figure 6.12. Computer graphics demonstrating the importance of the preflight (sim6) to tumbling performance.
Table 6.33. Scores for the contributions to tumbling performance (sim1 - 6) for trials 2 and 4.

<table>
<thead>
<tr>
<th>trial, simulation</th>
<th>val$_{ps}$ [%]</th>
<th>val$_{p}$ [%]</th>
<th>val$_{v}$ [°]</th>
<th>val$_{ps}$ [° (%)]</th>
<th>val$_{pec}$ [° (%)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, opt7</td>
<td>5.0</td>
<td>3.9</td>
<td>8.6</td>
<td>7.0</td>
<td>7.0</td>
</tr>
<tr>
<td>2, sim1</td>
<td>24.7</td>
<td>28.4</td>
<td>36.6</td>
<td>32.7</td>
<td>31.2</td>
</tr>
<tr>
<td>2, sim2</td>
<td>11.8</td>
<td>0.0</td>
<td>13.7</td>
<td>9.7</td>
<td>12.8</td>
</tr>
<tr>
<td>2, sim3</td>
<td>168.9</td>
<td>159.5</td>
<td>99.1</td>
<td>132.8</td>
<td>138.5</td>
</tr>
<tr>
<td>2, sim4</td>
<td>6.2</td>
<td>4.1</td>
<td>8.0</td>
<td>6.3</td>
<td>7.2</td>
</tr>
<tr>
<td>2, sim5</td>
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<td>34.5</td>
<td>25.7</td>
<td>30.4</td>
<td>35.2</td>
</tr>
<tr>
<td>4, opt7</td>
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<td>9.4</td>
<td>10.0</td>
<td>9.7</td>
<td>9.7</td>
</tr>
<tr>
<td>4, sim1</td>
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<td>19.4</td>
<td>19.4</td>
<td>19.4</td>
<td>17.7</td>
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<td>18.7</td>
<td>21.1</td>
</tr>
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<td>55.7</td>
<td>60.4</td>
<td>56.8</td>
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<td>10.4</td>
<td>10.0</td>
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<td>10.6</td>
</tr>
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<td>4, sim5</td>
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<td>14.1</td>
<td>19.5</td>
<td>17.0</td>
<td>31.5</td>
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<tr>
<td>4, sim6</td>
<td>8.2</td>
<td>4.5</td>
<td>9.7</td>
<td>7.6</td>
<td>9.0</td>
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</tbody>
</table>

Note:

2, sim1 and sim2 are calculated using actual values from trial 4.
2, sim3, sim4 and sim5 are calculated using actual values from trial 2.
4, sim1, sim2 and sim6 are calculated using actual values from trial 2.
4, sim3, sim4 and sim5 are calculated using actual values from trial 4.
Table 6.34. Stiffness [N.m⁻¹] and damping [N.s.m⁻¹] parameters for the tumbling track for the contributions to tumbling performance (sim1-6) for trials 2 and 4

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
<th>k4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, opt7</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
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<tr>
<td>2, sim1</td>
<td>51067</td>
<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
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<td>2, sim2</td>
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<td>234</td>
<td>65432</td>
<td>41</td>
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</tr>
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<td>234</td>
<td>65432</td>
<td>41</td>
</tr>
<tr>
<td>2, sim5</td>
<td>510670</td>
<td>234</td>
<td>654320</td>
<td>41</td>
</tr>
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<td>234</td>
<td>65432</td>
<td>41</td>
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<td>41</td>
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<td>41</td>
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<td>41</td>
</tr>
<tr>
<td>4, sim5</td>
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<td>654320</td>
<td>41</td>
</tr>
<tr>
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<td>234</td>
<td>65432</td>
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</table>
Table 6.35. Activation times [s] for each torque generator and the initial activation of the ankle torque generator for the contributions to tumbling performance (sim1-6) for trials 2 and 4

<table>
<thead>
<tr>
<th>trial, optimisation</th>
<th>ankle</th>
<th>knee</th>
<th>hip</th>
<th>shoulder</th>
<th>iact</th>
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<tbody>
<tr>
<td>2, opt7</td>
<td>0.016</td>
<td>0.005</td>
<td>0.005</td>
<td>0.066</td>
<td>1.318</td>
</tr>
<tr>
<td>2, sim1</td>
<td>0.002</td>
<td>0.027</td>
<td>0.000</td>
<td>0.107</td>
<td>0.373</td>
</tr>
<tr>
<td>2, sim2</td>
<td>0.013</td>
<td>0.013</td>
<td>0.014</td>
<td>0.084</td>
<td>1.148</td>
</tr>
<tr>
<td>2, sim3</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>2, sim4</td>
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<td>0.005</td>
<td>0.005</td>
<td>0.066</td>
<td>1.318</td>
</tr>
<tr>
<td>2, sim5</td>
<td>0.016</td>
<td>0.005</td>
<td>0.005</td>
<td>0.066</td>
<td>1.318</td>
</tr>
<tr>
<td>4, opt7</td>
<td>0.002</td>
<td>0.027</td>
<td>0.000</td>
<td>0.107</td>
<td>0.373</td>
</tr>
<tr>
<td>4, sim1</td>
<td>0.016</td>
<td>0.005</td>
<td>0.005</td>
<td>0.066</td>
<td>1.318</td>
</tr>
<tr>
<td>4, sim2</td>
<td>0.117</td>
<td>0.003</td>
<td>0.042</td>
<td>0.025</td>
<td>0.627</td>
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<td>4, sim3</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<tr>
<td>4, sim4</td>
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<td>0.027</td>
<td>0.000</td>
<td>0.107</td>
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<td>0.027</td>
<td>0.000</td>
<td>0.107</td>
<td>0.373</td>
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</table>
Table 6.36. Values of the variables for the performance criterion for the contributions to tumbling performance (sim1-6) for trials 2 and 4

<table>
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<tr>
<th>trial, optimisation</th>
<th>( u_g ) [m.s(^{-1})]</th>
<th>( v_g ) [m.s(^{-1})]</th>
<th>( h_g ) [kg.m(^2).rad.s(^{-1})]</th>
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</thead>
<tbody>
<tr>
<td>2, actual</td>
<td>2.79</td>
<td>5.09</td>
<td>59</td>
</tr>
<tr>
<td>2, opt7</td>
<td>2.92</td>
<td>5.25</td>
<td>62</td>
</tr>
<tr>
<td>2, sim1</td>
<td>2.97</td>
<td>3.10</td>
<td>125</td>
</tr>
<tr>
<td>2, sim2</td>
<td>2.99</td>
<td>4.50</td>
<td>95</td>
</tr>
<tr>
<td>2, sim3</td>
<td>-2.77</td>
<td>-2.59</td>
<td>-21</td>
</tr>
<tr>
<td>2, sim4</td>
<td>2.97</td>
<td>5.11</td>
<td>61</td>
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<tr>
<td>2, sim5</td>
<td>2.79</td>
<td>3.77</td>
<td>99</td>
</tr>
<tr>
<td>4, actual</td>
<td>2.99</td>
<td>4.50</td>
<td>95</td>
</tr>
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<tr>
<td>4, sim6</td>
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Table 6.37. Values of the variables for the elastic criterion for the contributions to tumbling performance (sim1-6) for trials 2 and 4

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<th>$q_{1 \text{end}}$ [m]</th>
<th>$q_{2 \text{max}}$ [m]</th>
<th>$q_{2 \text{end}}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, actual</td>
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<td>-0.060</td>
<td>0.000</td>
</tr>
<tr>
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<td>-0.001</td>
<td>-0.062</td>
<td>-0.002</td>
</tr>
<tr>
<td>2, sim1</td>
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<td>-0.011</td>
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</tr>
<tr>
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<td>-0.006</td>
<td>-0.052</td>
<td>-0.002</td>
</tr>
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<td>-0.068</td>
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<td>-0.001</td>
<td>-0.067</td>
<td>-0.001</td>
</tr>
<tr>
<td>2, sim5</td>
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<td>-0.001</td>
<td>-0.012</td>
<td>0.000</td>
</tr>
<tr>
<td>4, actual</td>
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<td>0.000</td>
<td>-0.060</td>
<td>0.000</td>
</tr>
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<td>-0.006</td>
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<td>-0.001</td>
<td>-0.057</td>
<td>-0.002</td>
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Table 6.38. Values of the variables for the strategy criterion for the contributions to tumbling performance (sim1-6) for trials 2 and 4 [°]

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<th>$a_{\text{amin}}$</th>
<th>$a_*$</th>
<th>$k_{\text{amin}}$</th>
<th>$k_*$</th>
<th>$h_*$</th>
<th>$tr_*$</th>
<th>$s_*$</th>
</tr>
</thead>
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<td>2, actual</td>
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<td>139</td>
<td>177</td>
<td>183</td>
<td>82</td>
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<td>44</td>
<td>136</td>
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<td>182</td>
<td>174</td>
<td>82</td>
<td>149</td>
</tr>
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</table>
Summary of the contributions to tumbling performance

The activation timings of the torque generators were found to be very important to tumbling performance with there being an optimum strategy for a given preflight (sim2). Switching all the torque generators in the simulation model off during the takeoff resulted in a poor performance with both the layout and double layout having insufficient vertical velocity at takeoff.

The preflight was found to be important for tumbling performance with a poor preflight resulting in a poor performance. sim1 showed that the performance of the layout and double layout were sensitive to changes in the preflight for a fixed strategy but sim2 showed that the possible performance of the layout and double layout was not sensitive to changes in preflight when the strategy used was allowed to change. In addition sim6 showed that it was possible to convert a double layout into a layout performance by just changing the angular velocity of the trunk and arms at initial contact with the tumbling track.

The stiffness of the SEC did not affect the layout or double layout performance very much and the stiffness of the tumbling track did not affect performance as long as the strategy was allowed to vary.

In general it may be speculated that performance is sensitive to changes in all four factors identified when the strategy used is not changed; however if strategy is allowed to vary then similar performances are possible. This is a very important result as it indicates that a gymnast can cope with small differences in preflight / equipment by making slight adjustments to the strategy and still be able to produce a good performance.
CONTRIBUTIONS TO VAULTING PERFORMANCE USING THE FIVE SEGMENT MODEL OF VAULTING

The five segment vaulting model was used to examine the contributions of the following factors to vaulting performance:

- Preflight characteristics
- Effect of the hand
- Muscle strength
- Series elastic component
- Elastic properties of the shoulder
- Elastic properties of the vaulting horse

Method

The best Hecht vault (trial 1) was chosen to consider the effect of each of the above factors on performance by running single simulations with one factor varied and all the other factors unchanged. This allowed the effect of changing the one variable on performance to be isolated.

The following single simulations were run to investigate the contributions made by each factor to vaulting performance:

- A modified preflight to produce a handspring vault [sim1]
- No hand segment [sim2]
- The torque generator at the shoulder switched off [sim3]
- A 10 fold increase in the stiffness of the SEC [sim4]
- A 10 fold increase in the stiffness of the vaulting horse and shoulder [sim5]

Results and Discussion

The contributions to vaulting performance are given in tabular form (Table 6.39-Table 6.43) and computer graphics form (Figure 6.13 - Figure 6.14).

Preflight characteristics

sim1 showed that it was possible to alter the preflight of the Hecht vault to produce a handspring vault (Figure 6.13). Without a good preflight it would not be possible to
perform the desired vault and this shows the importance of the preflight for the successful performance of a vault. This result is in agreement with the findings from the two segment model for vaulting where it was shown that the preflight determined the type of vault that was possible.

Hand

sim2 showed the effect of using the hand on the performance of the Hecht vault. Without the hand the time of contact was reduced by 0.04 s which resulted in only 38% of the required angular momentum at takeoff being produced (Table 6.41, Table 6.42). Therefore an accurate simulation model of vaulting must include a representation of the hand or else a vital element of the vault will be missing.

Muscle strength and SEC

sim3 and sim4 show the effects of not using shoulder torque and increasing the stiffness of the SEC (Figure 6.14). Without using shoulder torque there was a small increase of 7° (%) in the value of $\text{val}_{\text{pers}}$. However, it may be expected that performance would not worsen as much if the strategy was optimised for the condition where no shoulder torque was used. Increasing the stiffness of the SEC had virtually no effect on performance with the value of $\text{val}_{\text{pers}}$ increasing by less than 1° (%).

Elastic properties of the vaulting horse and shoulder

Increasing the stiffness of the horse and shoulder reduced the contact time with the horse to 0.04 s and reduced the movement at the shoulder to less than 0.04 m (Table 6.42). Although the contact time was reduced the model was able to produce sufficient backwards angular momentum and vertical velocity at takeoff from the horse (Table 6.41). In fact by increasing the stiffness of the horse and shoulder resulted in a simulation which was similar to the two segment simulation model where there was a zero contact time with the horse.
Figure 6.13. Computer graphics demonstrating the contributions of the preflight (sim1) and hand (sim2) to vaulting performance (trial 1).
Simulated Hecht vault [opt2]

Simulated Hecht vault with no muscle [sim3]

Simulated Hecht vault with stiff SEC [sim4]

Simulated Hecht vault with stiff vaulting horse and shoulder [sim5]

Figure 6.14. Computer graphics demonstrating the contribution of muscle strength, SEC and elastic properties of the vaulting horse / shoulders to vaulting performance (trial 1).
Table 6.39. Scores for the contributions to vaulting performance (sim1-5) for trial 1

<table>
<thead>
<tr>
<th>trial, simulation</th>
<th>$v_{\text{pe}}$ [%]</th>
<th>$v_{\text{p}}$ [%]</th>
<th>$v_{\text{r}}$ [°]</th>
<th>$v_{\text{pe}}$ [° (%)]</th>
<th>$v_{\text{r}}$ [° (%)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>l, opt2</td>
<td>16.0</td>
<td>15.5</td>
<td>6.4</td>
<td>11.9</td>
<td>14.6</td>
</tr>
<tr>
<td>l, sim1</td>
<td>112.6</td>
<td>162.9</td>
<td>92.8</td>
<td>132.5</td>
<td>108.9</td>
</tr>
<tr>
<td>l, sim2</td>
<td>29.9</td>
<td>35.0</td>
<td>7.1</td>
<td>25.2</td>
<td>27.0</td>
</tr>
<tr>
<td>l, sim3</td>
<td>23.9</td>
<td>19.6</td>
<td>4.4</td>
<td>14.2</td>
<td>21.5</td>
</tr>
<tr>
<td>l, sim4</td>
<td>16.4</td>
<td>15.3</td>
<td>6.0</td>
<td>11.6</td>
<td>14.9</td>
</tr>
<tr>
<td>l, sim5</td>
<td>58.4</td>
<td>37.0</td>
<td>19.7</td>
<td>29.6</td>
<td>53.0</td>
</tr>
</tbody>
</table>

Table 6.40. Stiffness [N.m⁻¹] and damping [N.s.m⁻¹] parameters for the vaulting horse and shoulder, and the time the hand spring length was changed [s] for the contributions to the vaulting performance (sim1-5) for trial 1

<table>
<thead>
<tr>
<th>trial, simulation</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
<th>k4</th>
<th>k5</th>
<th>k6</th>
<th>thand</th>
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<tr>
<td>l, opt2</td>
<td>2814</td>
<td>1943</td>
<td>23097</td>
<td>7245</td>
<td>20961</td>
<td>10</td>
<td>0.070</td>
</tr>
<tr>
<td>l, sim1</td>
<td>2814</td>
<td>1943</td>
<td>23097</td>
<td>7245</td>
<td>20961</td>
<td>10</td>
<td>0.070</td>
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<td>230970</td>
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<td>20961</td>
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</table>
Table 6.41. Values of the variables for the performance criterion for the contributions to vaulting performance (sim1-5) for trial 1

<table>
<thead>
<tr>
<th>trial, simulation</th>
<th>$u_g$ [m.s$^{-1}$]</th>
<th>$v_g$ [m.s$^{-1}$]</th>
<th>$h_g$ [kg.m$^2$.rad.s$^{-1}$]</th>
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<td>2.21</td>
<td>38.5</td>
</tr>
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<td>5.12</td>
<td>1.73</td>
<td>-6.0</td>
</tr>
<tr>
<td>1, sim3</td>
<td>4.96</td>
<td>2.25</td>
<td>-19.2</td>
</tr>
<tr>
<td>1, sim4</td>
<td>5.12</td>
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Table 6.42. Values of the variables for the elastic criterion for the contributions to vaulting performance (sim1-5) for trial 1

<table>
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<tr>
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<th>$q_1_{max}$ [m]</th>
<th>$q_2_{max}$ [m]</th>
<th>$q_4_{max}$ [m]</th>
<th>$q_4_{end}$ [m]</th>
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<td>-0.014</td>
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Table 6.43. Values of the variables for the strategy criterion for the contributions to vaulting performance (sim1-5) for trial 1 [°]

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</tr>
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<td>1, sim1</td>
<td>93</td>
<td>184</td>
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<tr>
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</tr>
<tr>
<td>1, sim5</td>
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Summary of the contributions to vaulting performance

The preflight was found to be very important for vaulting performance with a poor preflight resulting in a poor performance. It is necessary to include a representation of the hand in the model or else the contact time will be too short and an important aspect of the vault will be missed. The performance of the Hecht vault decreased only slightly when no shoulder torque was used and increasing the stiffness of the SEC had very little effect on performance. Increasing the stiffness of the horse and shoulder resulted in a shorter time of contact and a simulation which was similar to those produced by the two segment simulation model.

In general it may be noted that vaulting performance is sensitive to changes in the preflight and the use of different strategies during the contact with the horse does not have a large effect on performance.

CHAPTER 6 SUMMARY

The three simulation models have been evaluated in this chapter and the contributions to the performance of vaulting and tumbling have been examined. The two segment simulation model was able to reproduce the major features of the vaulting and distinguish between the preflights of two completely different types of vault. Stretching the torque / angular velocity profile in the tumbling model resulted in much better
agreement between the simulated and actual performances. In the future the accuracy of the representations of the foot in the tumbling model and the hand-horse interface in the vaulting model need to be improved.

The three simulation models for vaulting and tumbling give a number of common results but also some contrasting results for the contributions to performance:

• For all three models the preflight was important with a poor preflight resulting in a poor performance.
• For all three models it was possible to alter the preflight to produce a different type of performance.
• For tumbling the strategy used could determine the possible performance with it possible to produce a layout performance with the preflight of a double layout and produce a double layout with the preflight of a layout. In contrast for vaulting the strategy used had little effect on performance.
• For all three models performance was insensitive to changes in the stiffness and damping parameters for the tumbling track / vaulting horse / shoulder, as long as the strategy used could be re-optimised. Although if the stiffness and damping parameters were markedly changed it was not possible to get good agreement between simulated and actual performance.
• For vaulting the hand segment makes a large contribution to performance by prolonging the time of contact with the horse.
CHAPTER 7

DISCUSSION

The limitations and improvements to the techniques used in this study will be discussed and then the factors which contribute to the performance of dynamic jumps will be considered before looking to possible future applications of the study.

LIMITATIONS AND IMPROVEMENTS TO THE TECHNIQUES USED

Four areas are considered in this section; simulation modelling, image analysis, parameter determination and evaluation procedures.

Simulation modelling

There were three simulation models developed in this study which have all helped in understanding the contributions to the performance of dynamic jumps. The two segment model for vaulting was the simplest model developed, had a number of limitations and was too simple to be completely evaluated (see later section on evaluation procedures for more details). The limitations of the two segment model will not be discussed in detail as they have been accounted for in the development of the five segment model of vaulting. Both five segment models have a number of common limitations which will be discussed before considering the limitations particular to each model:

- Activation of the torque generators

  Both models assumed that the torque generators at each joint were switched to maximum activation once (bang-bang activation) during a simulation and stayed at maximum activation for the rest of the simulation of the contact phase. The exception was for the two torque generators where a SEC was included: For vaulting it was assumed that the torque generator was maximally activated throughout the simulation of the whole contact phase, and in the tumbling model a two phase activation procedure was used to prevent the CE and SEC from vibrating.

  A bang-bang activation procedure was used as it simplified the optimisation procedure by using only one parameter for the activation of each torque generator. It may be speculated that better agreement with the actual performances would have been found
if the torque generators had been able to have variable activation levels during the simulation of the contact phase, although the improvement would probably have been small. For vaulting, switching the torque generator off had a small effect on the performance of the model and therefore increasing the complexity of the activation procedure would probably not improve the agreement with actual performances. On the other hand, for tumbling, the performance was found to be very sensitive to the activation timings of the torque generators and therefore it may be speculated that some improvement in the agreement between the simulated and actual performances may be obtained by increasing the complexity of the activation procedure. However, the improvement in the agreement would probably not be very large as the agreement between actual and simulated performance was already good, and van Soest et al. (1993) showed that by increasing the complexity of the activation procedure only improved vertical jumping performance by 2 mm.

Furthermore, it is not completely realistic to allow the muscles to be switched to maximum activation instantaneously (bang-bang activation), as muscles take time to reach maximum activation. To incorporate delay functions, activation before contact and ramped activation time histories in the simulation models would have increased the complexity of the models considerably and would have required more parameters to be determined. It could be possible to investigate the effect of ramped activation time histories in the future but in this study the simulation models were kept as simple as possible to limit the number of parameters that needed to be determined. Since other researchers (van Soest et al., 1993) have only slightly improved performance by increasing the complexity of the activation procedure it is not thought to be a major limitation of the simulation models developed in this study.

In the future it may be possible to investigate what the actual activation patterns used during dynamic jumps are using EMG, as at the moment the activations are unknown and therefore the agreement between the simulated and actual performance may be artificially good.

- Passive structures at the joints

In the tumbling model the ankle joint flexed beyond anatomical limits before extending. Increasing the strength of the torque generator by 50% did not improve the movement at the ankle. In the future a passive structure could be included at the ankle to
prevent the joint from flexing beyond anatomical limits and make the movement at the ankle more realistic. Although the movement at the ankle joint was not entirely realistic the overall performance of the tumbling model gave good agreement with the actual performances. Thus it may be that the torque produced around the ankle joint does not make a large contribution to tumbling performance, but the existence of a foot segment allows the model to stay in contact with the tumbling track for longer (like the hand in the vaulting model).

- Both models represented the trunk as a single segment

The two types of dynamic jump (vaulting and tumbling) analysed in this study were chosen as they did not require much flexion or extension of the trunk. This was done so as to minimise any errors due to not having a flexible trunk in each simulation model. A single segment was used to represent the trunk as it simplified the simulation models and it meant that strength parameters were not required for the trunk as these would have been difficult to calculate. In the future it may be desirable to include a flexible trunk when modelling other dynamic jumps although it is not thought to be a major limitation in the simulation models developed in this study.

- Both models used linear springs to represent the interaction with the external surfaces

Realistic movement of the contact surfaces in both simulation models was produced when appropriate spring parameters were used. In actual vaulting the hands appeared to slide along the horse top which would have helped to slow the horizontal velocity of the hands. Modelling the contact between the hands and the horse top in more detail would have probably improved the overall agreement of the model. Improving the modelling of the hands interacting with the horse top could perhaps be achieved by using a horizontal velocity damper to slow down the hands or a rigid segment could be used to represent the hands instead of using a spring. In the future a detailed study of how the body interacts with an external surface is required if an appropriate representation of the interaction is to be produced (see image analysis section for details).

- The vaulting model did not allow for movement at the hip or knee joints

Although the model included hip and knee joints these were fixed to average joint angle values during each simulation in order to simplify the movement. The evaluation of
the vaulting model may have been improved if hip and knee flexion and extension had been incorporated.

**Image analysis**

The techniques used to record and analyse the performances were reasonably successful although one limitation was that the high speed film camera did not give sufficient information on the contact with the vaulting horse or tumbling track:

The Locam 16 mm cine camera with a nominal framing rate of 200 Hz recorded the last part of the preflight the whole of the contact and the first part of the postflight resulting in approximately 30 frames of preflight, 30 frames of contact and 30 frames of postflight. On the whole the frame rate and field of view used allowed good estimates of the displacement of body landmarks to be calculated although there was some variation in velocity estimates. In addition it proved very difficult to identify exactly when contact was made with the horse for the vaulting and the toes were not visible during parts of the contact phase with the tumbling track. In the future a camera zoomed in on the contact point between the external surface and hands or feet of the gymnast would give a clearer picture of how the body interacts with an external surface. In addition a higher framing rate would give more information although this would require additional lighting.

**Parameter determination**

This study aimed to obtain subject specific parameters for the subject used with no reliance on data from the literature. The limitations of the techniques used are as follows:

- **Inertia parameters**

  The mathematical model of Yeadon (1990b) estimated the whole body mass of the subject to within 2% of the actual mass. The effect of different inertia parameters on performance was investigated using the two segment simulation model (Chapter 6). It was found that postflight performance was quite sensitive to the inertia characteristics of the gymnast for a given set of preflight variables. However, similar postflight performances were found when the performance was optimised by varying the preflight within realistic limits. Thus any small errors in the inertia parameters should not greatly affect the results from the simulation models and should not be a limitation of the study.
• Muscle parameters

A complex procedure was used to calculate the contractile element parameters from torque measurements on an isokinetic dynamometer (Chapter 5). In Chapter 6 the torque / angular velocity profile was stretched so that at $800\text{°s}^{-1}$ the model was able to exert 1% of isometric torque. This modification improved the performance of the tumbling model considerably and therefore highlighted a weakness in the original procedure used to fit the raw torque / angle / angular velocity data (Chapter 5). In the future, although the protocol used to collect the torque data appears to have been successful it would be desirable to develop a method which could give a realistic torque / angular velocity profile at high concentric angular velocities without the need for arbitrarily stretching the relationship. This could be achieved by collecting torque data at higher angular velocities (e.g. kicking into a padded surface to prevent injuries at high angular velocities). In addition only one set of muscle parameters have been used in this study; in the future it would be desirable to use other subjects to establish if similar relationships are true.

The series elastic parameters were estimated from the literature. In the future it may be possible to estimate series elastic parameters from experimental testing (e.g. Hof, 1997) although both the vault and tumbling takeoffs were found to be insensitive to changes in the stiffness of the SEC.

• Spring parameters

The spring parameters for the contact surface (vaulting horse and tumbling track) were first estimated using drop tests (Chapter 5). However, when the parameters were used in the simulation models of vaulting and tumbling the agreement with the actual performances was not very good. The values of the spring parameters were then determined using an optimisation procedure whereby the horizontal and vertical stiffness and damping parameters for the vaulting horse / tumbling track were varied until an objective function was minimised using the Simulated Annealing algorithm (Chapter 6). This resulted in values for the spring parameters which gave a good match between the simulated and actual performances. The average stiffness and damping parameters were then determined for the three vaults and the five tumbling performances. Using the average stiffness and damping parameters with each trial did not greatly affect the performance (but did affect the required strategy) and therefore it can be suggested that
the possible performance is not very sensitive to the elasticity of the track or horse as long as reasonable stiffness and damping values are used.

The spring parameters for the shoulder (vaulting model) were also determined using an optimisation procedure, and again using the average stiffness and damping parameters did not greatly affect performance.

Evaluation of the simulation models

The evaluation of the simulation models is a very important part of the development process which has often been overlooked in other studies. This study has tried to evaluate each of the simulation models used:

The two segment simulation model was evaluated by comparing the postflight characteristics of a simulation after impact with the postflight characteristics of the highest scoring Hecht vault from a national competition. Reasonable agreement was found even though the model did not account for a number of factors which affect performance. Although the model had a number of limitations, the model was still able to demonstrate the major relationships between the preflight and postflight and give a good ‘feel’ of the mechanics underlying the vaulting performance.

In fact the two segment vaulting model has many similarities to the two segment jumping model of Alexander (1990) which was able to identify the major differences in preflight for long jumping and high jumping. Both Alexander’s model and my two segment model demonstrated the same fundamental point that the preflight phase (the preparation) prior to the dynamic jump will determine the possible performance in postflight. Therefore other factors (e.g. muscular strength and activation) are not as important since it is not possible to correct for a poor preflight.

A complex procedure was developed for obtaining the overall best match between the simulated and actual performance for each trial with the five segment models.

The five segment simulation model of tumbling was evaluated for two layout and three double layout somersaults with acceptable agreement found between simulated and actual performance for all five trials. The major weakness of the model was the representation of the foot movement, with the simulation model predicting an unrealistically small ankle angle (beyond anatomical limits). In the future the model should include a representation of the passive structures around the ankle joint which
would prevent the ankle from flexing too much and improve the agreement with actual performance.

The five segment simulation model of vaulting was evaluated for three Hecht vaults, with the agreement between simulated and actual performance considered acceptable for this study. The differences between the simulated and actual performance are thought to be due to difficulties in modelling the hand-horse interface. In reality the hands appear to slide along the horse top although it is difficult to know exactly when contact is made with the horse. In the future more information is required on exactly how the hands and horse interact so that the model can be adjusted to give a more realistic representation of the interface between the hands and the horse, and therefore improve the agreement between the simulated and actual performance.

FACTORS THAT CONTRIBUTE TO THE PERFORMANCE OF DYNAMIC JUMPS

This study set out to establish the contributions to the performance of dynamic jumps. In particular there were three main purposes:

- To increase the understanding of the mechanics of dynamic jumps.
- To identify those factors which characterize successful performance of dynamic jumps.
- To quantify the contributions made to the successful performance of dynamic jumps.

Four factors have been identified in the literature as contributing to the performance of dynamic jumps; preflight characteristics, muscle strength and activation, tendon stiffness, and elasticity of the contact surface. To establish the contributions of these four factors and investigate whether other factors contribute to the performance of dynamic jumps; three computer simulation models were developed and applied to vaulting and tumbling takeoffs.

Preflight characteristics

It is well recognised by athletes, coaches and researchers alike that the preflight is of prime importance in the performance of dynamic jumps. However it is not so clear as to exactly how the preflight for one type of dynamic jump should differ from the preflight of another type of dynamic jump, or how the preflight should change for different athletes. Alexander (1990) is one of the few researchers to consider how the preflight should change for different dynamic jumps showing that the approach velocity for long jumping
should be faster than for high jumping. Greig (1998) accounted for 80% of the variation in height jumped through changes in approach velocity and knee angle for high jumping. While Yeadon et al. (in press) identified the horizontal and vertical velocity of the mass centre, body angle and shoulder angle at horse contact as important variables for success in the Hecht vault and showed that performance of the Hecht vault required a lower, faster and slower rotating preflight compared with the handspring somersault vault.

The three models developed in the present study showed to varying degrees that the preflight characteristics were an important factor to vaulting and tumbling performance, with a good preflight necessary to produce a good performance. For all three models making small changes to the preflight resulted in completely different postflight performances. For vaulting, increasing the body angle and angular velocity, and changing the mass centre velocity at horse contact resulted in a Hecht vault performance being converted into a handspring performance. For tumbling, reducing the trunk and arm angular velocities at touchdown for a double layout by approximately 10% resulted in a single layout somersault performance being produced. Thus the importance of the preflight cannot be over-emphasised for vaulting and tumbling.

For tumbling an additional contrasting result was found when the preflight and takeoff strategy used for the performance of the layout and double layout somersaults were examined. The subject analysed in the present study performed two layout and three double layout somersaults (Chapter 4) but there were only small differences in the preflight characteristics at the start of the contact phase prior to the takeoff. When the preflights of a layout and double layout somersault were interchanged and the muscle activations of the opposite performance used, the resulting performance changed to be closer to the performance linked to the muscle activations than the performance linked to the preflight. Furthermore when the muscle activations were re-optimised for the alternative preflight, the simulation model was able to produce a layout with the preflight from a double layout somersault and to produce a double layout with the preflight from a layout somersault. This is a very interesting result and suggests that the preflight is perhaps not as important for tumbling as it is for vaulting. In fact it would appear that for tumbling it is possible to produce a range of postflight performances by changing the strategy used during the takeoff phase as long as the preflight characteristics are within certain limits.
The sensitivity of the performance to changes in preflight and activation timings would suggest that a performer must be able to recognise slight changes in the preflight and be able to adjust the strategy used during the takeoff to produce the required postflight performance. However, the performer would not have time to recognise differences in the preflight during the short contact phase and must therefore decide on the strategy required prior to contact (feedforward).

With respect to dynamic jumps in general, it should be expected that the preflight will always be important. In fact it could be suggested that for all dynamic jumps the preflight characteristics must be within certain limits to enable the performance of a given dynamic jump, although the limits may vary for each jump. For some jumps the range may be quite wide and for other jumps the range might be quite narrow; from the athlete’s point of view a wide range of possible preflight characteristics would be advantageous for the performance of dynamic jumps as it would give the athlete some margin for error in the approach and preflight.

In addition, for those dynamic jumps where the athlete is trying to achieve a maximal performance (e.g. high jumping or long jumping), it might be expected that there is a small range of preflight characteristics that could result in a near optimal performance.

Muscular torque

The effect of strength and activation time histories of muscles on the performance of dynamic jumps has not been examined directly in the literature, although the effect of activation time histories of muscles on vertical jumping performance has been considered in detail by two groups of researchers (Amsterdam group: Bobbert, van Soest and van Ingen Schenau; and the Stanford group: Pandy and Zajac). Both groups have shown that vertical jumping performance is sensitive to the strength and activation time histories in the lower limb.

In the present study the two segment model of vaulting did not employ any muscular torque during the contact phase, and was still able to distinguish between the performance of the Hecht and handspring somersault vaults. This suggests that muscular torque does not provide a major contribution to vaulting performance or else it would not be possible to produce both types of vault by just changing the preflight. The five
segment vaulting model confirmed this by showing that the postflight performance was not greatly affected by the use of shoulder torque. The reason for shoulder torque not providing a major contribution to vaulting performance is due to the nature of the vault which results in the muscles used to produce shoulder torque contracting at high concentric velocities throughout the horse contact phase; therefore, it is not possible to exert very much shoulder torque due to the shape of the force/velocity profile of muscle.

For tumbling a very different result was found; tumbling performance was found to be sensitive to changes in the activation of the torque generators with it being possible to change a layout performance into a double layout performance by just altering the activation of the torque generators in the tumbling model. In addition switching off all the torque generators in the model during the takeoff resulted in very poor performances of the layout and double layout somersault with insufficient vertical velocity, too much horizontal velocity and too much angular momentum. It would therefore appear that muscular torque can provide a large contribution to tumbling performance with the correct activation timings allowing a good performance to be produced from a limited range of preflight characteristics. The reason for muscle providing a large contribution to tumbling performance is due to the muscles across the ankle and knee joints operating eccentrically before operating concentrically during takeoff, thereby allowing high torques to be exerted which change the joint angle configurations during the takeoff and alter the mass centre velocity and angular momentum.

Furthermore stretching the torque/angular velocity profile only improved the agreement between the simulated and actual performance for the tumbling model when the strategy used was re-optimised. This is similar to the findings of Bobbert and de Bruin (1994) who showed that jump height only improved for increased muscle strength when the timings of the muscles were re-optimised for the increased muscle strength.

In general the contribution of muscular torque to the performance of dynamic jumps varies from one dynamic jump to the next. It may be speculated that muscular torque can provide a large contribution for those dynamic jumps where muscles are operating eccentrically prior to concentrically (e.g. tumbling, long jump, high jump) but a smaller contribution for those dynamic jumps where the muscles only contract concentrically (e.g. Hecht vault). In addition muscular torque can only provide a large contribution to the performance of dynamic jumps when the correct coordination of the torque generators
(muscles) is used, with performance being sensitive to changes in the activation time histories of the torque generators. However, for all dynamic jumps the preflight must be within certain limits or else no amount of muscular torque can produce a good performance.

**Series elastic component**

The contribution of the SEC to the performance of dynamic jumps has not been given very much consideration in the literature. Alexander (1990) showed that long jumping and high jumping performances were insensitive to changes in the stiffness of the SEC while Anderson and Pandy (1993) showed that decreasing tendon stiffness to permit 10% strain (point of tendon rupture) resulted in a 3% increase in vertical jump performance.

Both five segment simulation models were used to examine the contribution of the SEC to performance of dynamic jumps. Increasing the stiffness of the SEC ten fold in each model resulted in only slight changes to performance. Therefore the SEC does not make a major contribution to the performance of dynamic jumps, and using values from the literature for the stiffness of the SEC should not have a major affect on the accuracy of each model.

**Elasticity of the contact surfaces**

The interaction of the human body with an external elastic surface has been highlighted as an area which is not very well understood (Brüggemann, 1994). Toderov and Cooper (1989) speculated that for tumbling the attack angle (the angle from the horizontal to the line of the legs at contact) should be smaller for a soft track.

Using the stiffness and damping parameters calculated from the drop tests (Chapter 5) in the simulation models did not result in very good matches between the simulated and actual performances. This was thought to be due to the drop tests giving parameters which represented the elastic nature of the external surface but not the hands or feet.

Calculating the elastic parameters using an optimisation procedure resulted in realistic movements of the equipment and good agreement between the performance of the simulation models and the actual performances for vaulting and tumbling. When average stiffness and damping parameters were used and the strategy re-optimised the agreement between the simulated and actual performance was almost as good. Therefore
the performance was insensitive to the elastic parameters within a realistic range of each of the parameters so long as the strategy used (timings of the torque generators) could be re-optimised to give the best agreement between the simulated and actual performance.

In general it may be speculated that within realistic limits the elastic nature of equipment does not greatly effect the level of performance that can be produced but does affect the strategy used to produce a dynamic jump.

**Joint elasticity**

The effect of movement within joints in the body to the performance of dynamic jumps has not been considered in the literature with most models of dynamic jumps using simple pin joints as representations.

In the present study the five segment vaulting model was used to consider the contribution of shoulder elasticity to performance in the Hecht vault. A linear spring was found to be sufficient to represent the major compression and extension at the shoulder. The movement at the shoulder in the simulation model was reduced by increasing the stiffness of the spring at the shoulder ten fold, resulting in a completely different postflight performance for the same preflight and strategy. Therefore it is vital that a simulation model includes a realistic representation of the elastic nature of the shoulder or else an important feature of vaulting performance will be overlooked. This may have important implications for modelling in the future where it may be necessary to represent the elastic movement at other joints in the body.

**Inertia parameters**

The effect of different inertia parameters to the performance of dynamic jumps has not been considered in the literature. In the present study the two segment model of vaulting was used to consider the effect of 11 different inertia sets on the performance of the Hecht and handspring somersault vaults. Performance was found to be sensitive to the inertia parameters for a fixed preflight. However, when the performance was optimised for each set of inertia parameters by altering the preflight characteristics, similar levels of performance were produced.

It may be speculated that within certain limits, inertia parameters do not limit the level of performance that can be achieved, but do affect the technique used.
Hand segment

The effect of the hand segment to vaulting performance has not been considered in the literature. In the present study it was found that the hand segment increased the time of contact with the horse and had a large effect on the performance of the Hecht vault. Failing to model the hand would have resulted in an important feature of vaulting being omitted. This has important implications for the modelling of dynamic jumps where it is important that small segments such as the foot or the hand that extend the duration of contact, should be represented or else an important feature of performance could be overlooked.

FUTURE APPLICATIONS

The simulation models developed in this study have been evaluated and used to examine the contributions to vaulting and tumbling performances. In the future the simulation models could be used to optimise vaulting and tumbling performances, however, this would require the preflight and the takeoff strategy to be varied and require an objective function to be formulated which could be maximised to define an optimum performance. This would not be an easy task as it would require a lot of computer time to optimise all the possible parameters, and arbitrary weightings would be needed for the objective function. In addition the sensitivity of the performance to small changes in the preflight, inertia, muscle and elastic parameters around the optimum could be examined. The models could also be used to examine the effect of increased strength on performance, and therefore be able to identify aspects of strength or technique that need to be improved for a particular athlete.

Finally the techniques developed in the present study could be used to produce subject specific simulation models in order to clarify the contributions to the performance of other dynamic jumps.
REFERENCES


Section of the British Association of Sport and Exercise Sciences No. 21, 25-28.
Loughborough: BASES.


APPENDIX A

SIMULATION MODELS DEVELOPED USING
THE AUTOLEV SOFTWARE PACKAGE

Appendix A is split into two main sections:
The first section gives the commands used with Autolev and is split into two parts:

- **Commands used by Autolev**
  Gives a description of the commands / functions used to build models with Autolev

- **Additional functions**
  Gives a description of extra functions used with Autolev which are used once the simulation model has been programmed into the Autolev package.

The second section gives details of the two models developed using the Autolev software package and is split into two parts for each model:

- **Nomenclature**
  Definition of each of the labels assigned to variables, constants, etc. used in each of the models.

- **Listings of the commands used**
  The commands used to define each model in Autolev are listed.

**Commands used by Autolev**

The Autolev package uses a number of commands to build a simulation model which are input to the Autolev package at the command line. The Autolev package generates the equations of motion and the Fortran code for the simulation model after the model has been defined.
Degrees of freedom

The number of degrees of freedom $n$ of the system are defined using the command:

$$\text{DOF}(n)$$

= system with $n$ degrees of freedom

Frames

Each frame is used to represent a segment / spring in the system. The frames command informs Autolev which symbols stand for reference frames and assigns a dextral set of mutually perpendicular unit vectors (e.g. A1, A2 and A3 fixed in A). In addition to the reference frames declared Autolev automatically assigns a Newtonian reference frame N. The reference frames used in the system are defined using the command:

$$\text{FRAMES}(A, B)$$

= A and B are frames

Points

A point is a position on a frame where a force is applied / the end of a segment. The names of the various points used in the system are defined using the command:

$$\text{POINTS}(P1, P2)$$

= P1 and P2 are points

Mass / Massless

All points and frames are assumed to have a mass unless they are declared massless. To define a frame / point as massless Autolev uses the command:

$$\text{MASSLESS}(O, A, P1, P2)$$

= O, A, P1 and P2 are massless

Inertia

For each frame that is not massless the moment of inertia about each of the axes must be defined. The following format is used for the inertia values of each frame:

$$\text{INERTIA}(A, I_{11}, I_{22}, I_{33}, I_{12}, I_{13}, I_{23})$$

= moment of inertia of frame A

where $I_{11}, I_{22}, I_{33}$ are the principle moments of inertia

$I_{12}, I_{23}, I_{31}$ are the cross - product terms

Variables

Variables (e.g. $Q_1, Q_2$) are defined by the command:

$$\text{VAR}(Q1, Q2)$$

= Q1 and Q2 are variables
**Constants**

Constants (e.g. L1, L2) are defined by the command:

\[ \text{CONST}(L1,L2) \]

\( \equiv \) L1 and L2 are constants

**Simple rotations and Direction Cosines**

The simple rotation and direction cosine commands (SIMPROT and DIRCOS) are used by Autolev to define the orientation of one frame relative to another. \texttt{SIMPROT} is a special case which is used to define a direction cosine for a rotation about one axis, e.g.:

\[ \text{SIMPROT}(A,B,3,Q1) \]

\( \equiv \) rotation from frame A to B is of magnitude Q1 about the third axis

**Position vectors**

To define the position vector of a point relative to another point Autolev uses the letter \( P \) followed by the labels of the two points:

\[ P cient{1}Pcient{2} \]

\( \equiv \) position vector from point \( P_{1} \) to \( P_{2} \)

**Generalised speeds**

Generalised speeds are defined by the letter \( U \).

**Velocities**

The velocity of every point and mass centre in the system must be defined using the letter \( V \). Velocities are usually calculated from position vectors using the differentiation command \texttt{DERIV} within Autolev e.g.:

\[ \text{VP}_{1}\text{N} = \text{DERIV}(P\text{OP}_{1},T,N) \]

\( \equiv \) velocity of point \( P_{1} \) in N

**Accelerations**

The acceleration of every point and mass centre in the system must be defined using the letter \( A \). Accelerations are usually calculated from velocities using the differentiation command \texttt{DERIV} within Autolev e.g.:

\[ \text{AP}_{1}\text{N} = \text{DERIV}(\text{VP}_{1}\text{N},T,N) \]

\( \equiv \) acceleration of point \( P_{1} \) in N
**Generalised inertia forces**

Autolev uses the command **FRSTAR** to calculate the generalised inertia forces (one for each degree of freedom) after the generalised velocities and accelerations of each particle in the system have been defined.

**Generalised active forces**

The command **FR** is used by Autolev to calculate the generalised active forces once all the forces and torques have been defined. Forces and torques acting on the system are declared using the commands **FORCE** and **TORQUE**, e.g.:

- **FORCE(P1/P2)** = force exerted by point P1 on P2
- **TORQUE(A/B)** = torque exerted by frame A on frame B

**Kane's equations of motion**

The command **KANE** generates Kane's dynamical equations of motion once the generalised inertia and active forces have been defined in Autolev.

**Additional functions**

There are a number of additional functions available with the Autolev software package which can be used once a system has been defined:

**Mass centre location**

Autolev can calculate the mass centre location using the command **CM**, e.g.:

- **CM(O,N)** = calculates the mass centre location in the inertial reference frame relative to O.

**Angular momentum**

The angular momentum of a system can be found by using the command **ANGMOM**. This command can be used to find the angular momentum about any point in any frame, usually the angular momentum is calculated about the mass centre **CM** in the inertial reference frame **N**:

- **ANGMOM(CM,N)** = angular momentum about CM in N
**Kinetic energy**

The kinetic energy of a system is found using the command KE.

**Potential energy**

The potential energy of a system must be defined by the user relative to a point, with the variable name PE used.

**Controls**

The controls function has two purposes, firstly to allow all force / torque expressions to be defined, and secondly to allow the time histories of independent variables to be printed out in the output files of the Fortran simulation code generated by Autolev. The format for the controls function is:

CONTROLS(C1,C2,...,Cn) = defines C1, C2, ..., Cn as controls

**Specified**

The specified function allows variables to be specified as functions of time. The variables that are specified must be formulated as functions of time in the work space or added to the Fortran code in the SPEC subroutine produced by Autolev. The format for the specified function is:

SPEC(S1,S2,...,Sn) = defines S1,S2,...,Sn as specified functions

**Fortran code generation**

Once the simulation model has been programmed into Autolev the command CODE generates a complete Fortran simulation program (filename.FOR) ready to be compiled and run. The format for the code function is:

CODE(filename, SUBS) = generates Fortran code with subroutines

**Record**

The RECORD function is used to output a file (filename.ALL) containing all the input commands plus the Autolev responses. The format for the record function is:

RECORD(filename, ALL) = generates a complete record of the input commands and responses from Autolev
5 segment tumbling model

Figure A.1. Five segment model for tumbling.
Nomenclature

Frames

N = Newtonian reference frame N
A = reference frame A: segment A: horizontal and vertical springs
B = reference frame B: segment B: feet
C = reference frame C: segment C: lower legs
D = reference frame D: segment D: thighs
E = reference frame E: segment E: trunk + head
J = reference frame J: segment J: arms + hands
BC = reference frame BC: segment BC: intermediary segment

Points

O = origin, contact point of the toes with the tumble track
P1 = distal end of segment B
P2 = proximal end of segment B / Distal end of segment C
P3 = proximal end of segment C / Distal end of segment D
P4 = proximal end of segment C / Distal end of segment E
P5 = proximal end of segment D / Distal end of segment J
P6 = distal end of segment J

BSTAR = mass centre location of the feet (segment B)
CSTAR = mass centre location of the lower legs (segment C)
DSTAR = mass centre location of the thighs (segment D)
ESTAR = mass centre location of the trunk + head (segment E)
JSTAR = mass centre location of the arms + hands (segment J)
BCSTAR = mass centre location of the intermediary segment (segment BC)

Generalised Coordinates (variables)

Q1 = length of the horizontal spring joining O to P1 (segment A)
Q2 = length of the vertical spring joining O to P1 (segment A)
Q3 = angle of feet to the horizontal (segment B)
Q4 = angle of shank to the horizontal (segment C)
Q5 = angle of the thighs to the horizontal (segment D)
Q6 = angle of the trunk + head to the horizontal (segment E)
Q7 = angle of the arms + hands to the horizontal (segment J)
Q8 = angle of intermediary segment to the horizontal (segment BC)

Lengths
L1 = distance from P1 to BSTAR
L2 = distance from P1 to P2
L3 = distance from P2 to CSTAR
L4 = distance from P2 to P3
L5 = distance from P3 to DSTAR
L6 = distance from P3 to P4
L7 = distance from P4 to ESTAR
L8 = distance from P4 to P5
L9 = distance from P5 to JSTAR
L10 = distance from P5 to P6
L11 = distance from P2 to BCSTAR

Other angles
AA = ankle angle (180 - Q3 + Q4)
AAT = ankle tendon angle (Q3 - 180 - Q8)
AAM = ankle muscle angle (180 - Q4 + Q8)
AK = knee angle (180 - Q5 + Q4)
AH = hip angle (180 - Q5 + Q6)
AS = shoulder angle (180 - Q6 + Q7)

Generalised velocities
U1 = generalised velocity (horizontal spring) (Q1')
U2 = generalised velocity (vertical spring) (Q2')
U3 = generalised angular velocity (feet) (Q3')
U4 = generalised angular velocity (shank) (Q4')
U5 = generalised angular velocity (thighs) (Q5')
U6 = generalised angular velocity (trunk + head) (Q6')
U7 = generalised angular velocity (arms + hands) (Q7')
U8 = generalised angular velocity (intermediary) (Q8')
### Other velocities

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>WA</td>
<td>ankle angular velocity</td>
<td>( U_4 - U_3 )</td>
</tr>
<tr>
<td>WAT</td>
<td>ankle tendon angular velocity</td>
<td>( U_3 - U_8 )</td>
</tr>
<tr>
<td>WAM</td>
<td>ankle muscle angular velocity</td>
<td>( U_4 - U_8 )</td>
</tr>
<tr>
<td>WK</td>
<td>knee angular velocity</td>
<td>( U_4 - U_5 )</td>
</tr>
<tr>
<td>WH</td>
<td>hip angular velocity</td>
<td>( U_6 - U_5 )</td>
</tr>
<tr>
<td>WS</td>
<td>shoulder angular velocity</td>
<td>( U_7 - U_6 )</td>
</tr>
<tr>
<td>UG</td>
<td>horizontal velocity of the mass centre</td>
<td></td>
</tr>
<tr>
<td>VG</td>
<td>vertical velocity of the mass centre</td>
<td></td>
</tr>
</tbody>
</table>

### Reaction forces

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>RX</td>
<td>horizontal reaction force at P1</td>
<td>(-K_1<em>Q_1 - K_2</em>U_1)</td>
</tr>
<tr>
<td>RZ</td>
<td>vertical reaction force at P1</td>
<td>(-K_3<em>Q_2 - K_4</em>U_2)</td>
</tr>
<tr>
<td>RZ1</td>
<td>vertical reaction force at P2</td>
<td>(-K_3<em>P_2Z - K_4</em>V_2Z)</td>
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</table>

### Muscle torques

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TAT</td>
<td>torque produced by the series elastic element at the ankle joint</td>
<td></td>
</tr>
<tr>
<td>TAM</td>
<td>torque produced by the contractile element at the ankle joint</td>
<td></td>
</tr>
<tr>
<td>TK</td>
<td>torque produced by the contractile element at the knee joint</td>
<td></td>
</tr>
<tr>
<td>TH</td>
<td>torque produced by the contractile element at the hip joint</td>
<td></td>
</tr>
<tr>
<td>TS</td>
<td>torque produced by the contractile element at the shoulder joint</td>
<td></td>
</tr>
</tbody>
</table>

### Spring parameters:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>horizontal spring stiffness of the floor</td>
<td></td>
</tr>
<tr>
<td>K2</td>
<td>horizontal spring damping of the floor</td>
<td></td>
</tr>
<tr>
<td>K3</td>
<td>vertical spring stiffness of the floor</td>
<td></td>
</tr>
<tr>
<td>K4</td>
<td>vertical spring damping of the floor</td>
<td></td>
</tr>
</tbody>
</table>
Listings of commands used for the 5 segment tumbling model

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

FIVE SEGMENT MODEL FOR TUMBLING WITH SPRUNG FLOOR
AND TORQUE GENERATORS AT THE ANKLE, KNEE, HIP AND
SHOULDER.

5segtend

The model allows each of the joints to extend by
producing torques at the joints between the segments.
For the ankle joint an elastic element has been added.

ANKLE ANGLE / ANGULAR VELOCITY = AA / WA
KNEE ANGLE / ANGULAR VELOCITY = AK / WK
HIP ANGLE / ANGULAR VELOCITY = AH / WH
SHOULDER ANGLE / ANGULAR VEL. = AS / WS

ANKLE TENDON ANGLE / ANG. VEL. = AAT / WAT
ANKLE MUSCLE ANGLE / ANG. VEL. = AAM / WAM

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

AUTOZ(OFF)

FRAMES(A,B,C,D,E,J,BC)
DOF(8)
POINTS(O,P1,P2,P3,P4,P5,P6,P7)
MASSLESS(O,A,P1,P2,P3,P4,P5,P6,P7)
INERTIA(B,0,IB,IB,0,0,0)
INERTIA(BC,0,IBC,IBC,0,0,0)
INERTIA(C,0,IC,IC,0,0,0)
INERTIA(D,0,ID,ID,0,0,0)
INERTIA(E,0,IE,IE,0,0,0)
INERTIA(J,0,IJ,IJ,0,0,0)
VAR(Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8)
CONST(L1,L2,L3,L4,L5,L6,L7,L8,L9,L10,L11,G)
CONST(PAA,PAB,PAC,PA,PAP,PAQ)
CONST(PKA,PKB,PKC,PKD,PKP,PKQ)
CONST(PHA,PHB,PHC,PHD,PHP,PHQ)
CONST(PAA,PSB,PSD,PS,PSQ)
CONST(PAANG,PKANG,PHANG,PSANG)
CONST(PTEND)
CONST(K1,K2,K3,K4)
CONST(AACT,KACT,HACT,SACT)
CONST(TAON,TKON,THON,TSON)

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

DIRECTION COSINES

SIMPROT(N,B,3,Q3)
SIMPROT(N,B,C,3,Q8)
SIMPROT(N,C,3,Q4)
SIMPROT(N,D,3,Q5)
SIMPRT(N,E,3,Q6)
SIMPRT(N,J,3,Q7)
!

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!

POSITIONS
!
POP1 = Q1*N1 + Q2*N2
PP1BSTAR = L1*B1
PP1P2 = L2*B1
PP2CSTAR = L3*C1
PP2P3 = L4*C1
PP3DSTAR = L5*D1
PP3P4 = L6*D1
PP4ESTAR = L7*E1
PP4P5 = L8*E1
PP5JSTAR = L9*J1
PP5P6 = L10*J1
!
PP2BCSTAR = L11*BC1
!
POP1 = Q1*N1 + Q2*N2
POBSTAR = ADD(POP1,PP1BSTAR)
POP2 = ADD(POP1,PP1P2)
POCSTAR = ADD(POP1,PP2CSTAR)
POP3 = ADD(POP2,PP2P3)
PODSTAR = ADD(POP3,PP3DSTAR)
POP4 = ADD(POP3,PP3P4)
POESTAR = ADD(POP4,PP4ESTAR)
POP5 = ADD(POP4,PP4P5)
POJSTAR = ADD(POP5,PP5JSTAR)
POP6 = ADD(POP5,PP5P6)
!
POBCSTAR = ADD(POP2,PP2BCSTAR)
!

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
GENERALISED SPEEDS (KINEMATIC EQUATIONS)
!
Q1' = U1
Q2' = U2
Q3' = U3
Q4' = U4
Q5' = U5
Q6' = U6
Q7' = U7
Q8' = U8
!

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
VELOCITIES
!
WBN = Q3*N3
WCN = Q4*N3
WDN = Q5*N3
WEN = Q6*N3
WJN = Q7*N3
WBCN = Q8*N3
! VON = 0
VP1N = DERIV(POP1,T,N)
VBSTARN = DERIV(POBSTARN,T,N)
VP2N = DERIV(POP2,T,N)
VCSTARN = DERIV(POCSTARN,T,N)
VP3N = DERIV(POP3,T,N)
VDSTARN = DERIV(PODSTARN,T,N)
VP4N = DERIV(POP4,T,N)
VESTARN = DERIV(POESTARN,T,N)
VP5N = DERIV(POP5,T,N)
VJSTARN = DERIV(POJSTARN,T,N)
VP6N = DERIV(PO6N,T,N)
!
VBCSTARN = DERIV(POB6N,T,N)
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
ACCELERATIONS
!
ALFBN = DERIV(WBN,T,N)
ALFCN = DERIV(WCN,T,N)
ALFDN = DERIV(WDN,T,N)
ALFEN = DERIV(WEN,T,N)
ALFIN = DERIV(WJN,T,N)
!
ALFBCN = DERIV(WBCN,T,N)
!
AON = 0
AP1N = DERIV(VP1N,T,N)
ABSTARN = DERIV(VBSTARN,T,N)
AP2N = DERIV(VP2N,T,N)
ACSTARN = DERIV(VCSTARN,T,N)
AP3N = DERIV(VP3N,T,N)
ADSTARN = DERIV(VDSTARN,T,N)
AP4N = DERIV(VP4N,T,N)
AESTARN = DERIV(VESTARN,T,N)
AP5N = DERIV(VP5N,T,N)
AJSTARN = DERIV(VJSTARN,T,N)
AP6N = DERIV(VP6N,T,N)
!
ABCSTARN = DERIV(VBCSTARN,T,N)
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
GENERALISED INERTIA FORCES
!
FRSTARN
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
GENERALISED ACTIVE FORCES
!
FORCE(BSTAR) = -MASSB*G*N2
FORCE(CSTAR) = -MASSC*G*N2
FORCE(DSTAR) = -MASSD*G*N2
FORCE(ESTAR) = -MASSE*G*N2
FORCE(JSTARN) = -MASSJ*G*N2
REACTION FORCES ARE ONLY USED WHEN P1 / P2 ARE IN CONTACT WITH THE TUMBLE TRACK

FORCE(O/P1) = RX*N1 + RZ*N2
FORCE(P2) = RZ1*N2

TORQUE(B/BC) = TAT*N3
TORQUE(BC/C) = TAM*N3

TORQUE(D/C) = TK*N3
TORQUE(D/E) = TH*N3
TORQUE(E/J) = TS*N3

FR

KANE

MASS CENTRE LOCATION

CM(O,N)
VCMN = DERIV(POCM,T,N)

ANGULAR MOMENTUM CALCULATION ABOUT MASS CENTRE

ANGMOM(CM,N)

ENERGY

KE

PEL1 = DOT(POCM,N2)
PEL2 = DOT(POP2,N2)
PEL3 = 0.5*K1*Q1*Q1 + 0.5*K3*Q2*Q2
PEL4 = 0.5*K3*RIGHT(PEL2)*RIGHT(PEL2)
PEL5 = 0.5*PTEND*(Q3-PAANG-Q8)*(Q3-PAANG-Q8)

TOTMASS = MASSB+MASSC+MASSD+MASE+MASSJ

PE = RIGHT(TOTMASS)*RIGHT(PEL1)*G + RIGHT(PEL3) + RIGHT(PEL4) + RIGHT(PEL5)

EXTRA PRINTOUT VARIABLES / FORCE & TORQUE EQUATIONS

REACTION FORCES AND MASS CENTRE VELOCITY
CONTROLS(RX,RZ,RZ1,XG,ZG,UG,VG)

RX = -K1*Q1 - K2*U1
RZ = -K3*Q2 - K4*U2

TEMP = DOT(POP2,N2)
TEM1 = DOT(VP2N,N2)
RZ1 = -K3*RIGHT(TEMP) - K4*RIGHT(TEM1)

XG = DOT(POCM,N1)
ZG = DOT(POCM,N2)

UG = DOT(VCMN,N1)
VG = DOT(VCMN,N2)


P2X = DOT(POP2,N1)
P2Z = DOT(POP2,N2)
P3X = DOT(POCM,N1)
P3Z = DOT(POCM,N2)
P4X = DOT(POCM,N1)
P4Z = DOT(POCM,N2)
P5X = DOT(POCM,N1)
P5Z = DOT(POCM,N2)
P6X = DOT(POCM,N1)
P6Z = DOT(POCM,N2)


V2X = DERIV(P2X,T)
V2Z = DERIV(P2Z,T)
V3X = DERIV(P3X,T)
V3Z = DERIV(P3Z,T)
V4X = DERIV(P4X,T)
V4Z = DERIV(P4Z,T)
V5X = DERIV(P5X,T)
V5Z = DERIV(P5Z,T)
V6X = DERIV(P6X,T)
V6Z = DERIV(P6Z,T)


P2X = DOT(POP2,N1)
P2Z = DOT(POP2,N2)
P3X = DOT(POCM,N1)
P3Z = DOT(POCM,N2)
P4X = DOT(POCM,N1)
P4Z = DOT(POCM,N2)
P5X = DOT(POCM,N1)
P5Z = DOT(POCM,N2)
P6X = DOT(POCM,N1)
P6Z = DOT(POCM,N2)


V2X = DERIV(P2X,T)
V2Z = DERIV(P2Z,T)
V3X = DERIV(P3X,T)
V3Z = DERIV(P3Z,T)
V4X = DERIV(P4X,T)
V4Z = DERIV(P4Z,T)
V5X = DERIV(P5X,T)
V5Z = DERIV(P5Z,T)
V6X = DERIV(P6X,T)
V6Z = DERIV(P6Z,T)


V2X = DERIV(P2X,T)
V2Z = DERIV(P2Z,T)
V3X = DERIV(P3X,T)
V3Z = DERIV(P3Z,T)
V4X = DERIV(P4X,T)
V4Z = DERIV(P4Z,T)
V5X = DERIV(P5X,T)
V5Z = DERIV(P5Z,T)
V6X = DERIV(P6X,T)
V6Z = DERIV(P6Z,T)


V2X = DERIV(P2X,T)
V2Z = DERIV(P2Z,T)
V3X = DERIV(P3X,T)
V3Z = DERIV(P3Z,T)
V4X = DERIV(P4X,T)
V4Z = DERIV(P4Z,T)
V5X = DERIV(P5X,T)
V5Z = DERIV(P5Z,T)
V6X = DERIV(P6X,T)
V6Z = DERIV(P6Z,T)
CONTROLS(AA,AAT,AAM,AK,AH,AS)

AAT = Q3 - PAANG - Q8
AAM = PAANG - Q4 + Q8

AA = PAANG + Q4 - Q3
AK = PKANG + Q4 - Q5
AH = PHANG + Q6 - Q5
AS = PSANG + Q7 - Q6

CONTROLS(WA,WAT,WAM,WK,WH,WS)

WAT = DERIV(AAT,T)
WAM = DERIV(-AAM,T)

WA = DERIV(AA,T)
WK = DERIV(AK,T)
WH = DERIV(AH,T)
WS = DERIV(AS,T)

ANKLE TORQUE

TOP = PAA + PAB*EXP(PAP*WAM)
BOTL = 1 + PAC*EXP(PAP*WAM)
BOTR = 1 + PAD*EXP(PAQ*WAM)
TAM = (AACT*RIGHT(TOP))/(RIGHT(BOTL)*RIGHT(BOTR))

TOP = PAA*EXP(-(PAP+PAQ)*WAM) + PAB*EXP(-PAQ*WAM)
BOTL = PAC + EXP(-PAP*WAM)
BOTR = PAD + EXP(-PAQ*WAM)
TANM = (AACT*RIGHT(TOP))/(RIGHT(BOTL)*RIGHT(BOTR))

TENDON

TAT = PTEND*AAT

KNEE TORQUE

TOP = PKA + PKB*EXP(PKP*WK)
BOTL = 1 + PKC*EXP(PKP*WK)
BOTR = 1 + PKD*EXP(PKQ*WK)
TK = (KACT*RIGHT(TOP))/(RIGHT(BOTL)*RIGHT(BOTR))
TOP = PKA*EXP(-(PKP+PKQ)*WK) + PKB*EXP(-PKQ*WK)
BOTL = PKC + EXP(-PKP*WK)
BOTR = PKD + EXP(-PKQ*WK)
TKN = (KACT*RIGHT(TOP))/(RIGHT(BOTL)*RIGHT(BOTR))

HIP TORQUE:

TOP = PHA + PHB*EXP(PHP*WH)
BOTL = 1 + PHC*EXP(PHP*WH)
BOTR = 1 + PHD*EXP(PHQ*WH)
TH = (HAC + RIGHT(TOP))/(RIGHT(BOTL)*RIGHT(BOTR))

SHOULDER TORQUE:

TOP = PSA + PSB*EXP(PSP*WS)
BOTL = 1 + PSC*EXP(PSP*WS)
BOTR = 1 + PSD*EXP(PSQ*WS)
TS = (SACT*RIGHT(TOP))/(RIGHT(BOTL)*RIGHT(BOTR))

UNITS FOR OUTPUT

UNITS(U1,M/S,U2,M/S,U3,DEG,U4,DEG,U5,DEG,U6,DEG,U7,DEG,U8,DEG)
UNITS(L1,M,L2,M,L3,M,L4,M,L5,M,L6,M,L7,M,L8,M,L9,M,L10,M)
UNITS(L11,M)
UNITS(Q1,M,Q2,M,Q3,DEG,Q4,DEG,Q5,DEG,Q6,DEG,Q7,DEG,Q8,DEG)
UNITS(T,S,MASS,KG,INERTIA,KGM^2,G,M/S^2)
UNITS(XG,M,ZG,M,UG,MlS,VG,MlS,RZ,N,RX,N,RZI,N)
UNITS(ENERGY,J,ANGMOM,KG,M^2.1S)
UNITS(KI,NIM,K2,NSIM,K3,NIM,K4,NSIM)
UNITS(AA,DEG,WA,DEG,PAANG,DEG,TAM,NM,TANM,NM,TAT,NM,TAON,S,AACT,ACT.)
UNITS(AAT,DEG,AAM,DEG,WAT,DEG,WAM,DEG)
UNITS(AK,DEG,WK,DEG,PKANG,DEG,TK,NM,TKN,NM,TKN,S,KACT,ACT.)
UNITS(AH,DEG,WH,DEG,PHANG,DEG,TH,NM,THN,NM,THON,S,HACT,ACT.)
UNITS(AS,DEG,WS,DEG,PSANG,DEG,TS,NM,TSN,NM,TSON,S,SACT,ACT.)
UNITS(ACT,P2X,M,P2Z,M,P3X,M,P3Z,M)
UNITS(V2X,M/S,V2Z,M/S,V3X,M/S,V3Z,M/S)

GENERATION OF FORTRAN CODE AND FULL OUTPUT
CODE(5SEGTEND, SUBS)
RECORD(5SEGTEND, ALL)
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
UNITS
5 segment vaulting model

Figure A.2. Five segment model for vaulting.
Nomenclature

Frames

N = Newtonian reference frame N
A = reference frame A: segment A: horizontal and vertical springs
B = reference frame B: segment B: arms
C = reference frame C: segment C: shoulder spring
D = reference frame D: segment D: trunk and head
E = reference frame E: segment E: thighs
J = reference frame J: segment J: shank + feet
BD = reference frame BD: segment BD: intermediary segment

Points

O = origin, contact point of the wrists with the vaulting horse
P1 = distal end of segment B
P2 = proximal end of segment B
P3 = distal end of segment D
P4 = proximal end of segment D / Proximal end of segment E
P5 = distal end of segment E / Proximal end of segment J
P6 = distal end of segment J

BSTAR = mass centre location of the arms (segment B)
DSTAR = mass centre location of the trunk + head (segment D)
ESTAR = mass centre location of the thighs (segment E)
JSTAR = mass centre location of the shank + feet (segment J)
BDSTAR = mass centre location of the intermediary segment (segment BD)

Generalised Coordinates (variables)

Q1 = length of the horizontal spring joining O to P1 (segment A)
Q2 = length of the vertical spring joining O to P1 (segment A)
Q3 = angle of arms to the horizontal (segment B)
Q4 = length of the shoulder spring joining P2 to P3 (segment C)
Q5 = angle of the intermediary segment to the horizontal (segment BD)
Q6 = angle of the trunk to the horizontal (segment D)
Specified angles

AH = hip angle
AK = knee angle

Lengths

L1 = distance from P1 to BSTAR
L2 = distance from P1 to P2
L3 = distance from P3 to DSTAR
L4 = distance from P3 to P4
L5 = distance from P4 to ESTAR
L6 = distance from P4 to P5
L7 = distance from P5 to JSTAR
L8 = distance from P5 to P6
L9 = distance from P3 to BDSTAR

Other angles

AS = shoulder angle (Q3 - Q6)
AST = shoulder tendon angle (180 - Q3 + Q5)
ASM = shoulder muscle angle (Q6 - Q5)

Generalised velocities

U1 = generalised velocity (horizontal spring) (Q1’)
U2 = generalised velocity (vertical spring) (Q2’)
U3 = generalised angular velocity (arms) (Q3’)
U4 = generalised velocity (shoulder spring) (Q4’)
U5 = generalised angular velocity (intermediary) (Q5’)
U6 = generalised angular velocity (trunk) (Q6’)

Specified velocities

HDOT = hip angular velocity
KDOT = knee angular velocity

Other velocities

WS = shoulder angular velocity (U3 - U6)
WST = tendon angular velocity (U3 + U5)
WSM = muscle angular velocity (U5 - U6)
UG = horizontal velocity of the mass centre
VG = vertical velocity of the mass centre

Reaction forces
RX = horizontal reaction force at P1 (-K1*Q1 - K2*U1)
RZ = vertical reaction force at P1 (-K3*Q2 - K4*U2)
RS = shoulder reaction force between P2 and P3 (-K5*Q4 - K6*U4)

Muscle torques
TST = torque produced by the series elastic element at the shoulder
TSM = torque produced by the contractile element at the shoulder

Spring parameters
K1 = horizontal spring stiffness of the floor
K2 = horizontal spring damping of the floor
K3 = vertical spring stiffness of the floor
K4 = vertical spring damping of the floor
K5 = spring stiffness at the shoulder
K6 = spring damping at the shoulder
Listings of commands used for the 5 segment vaulting model

```
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
FIVE SEGMENT MODEL FOR VAULTING WITH A SPRUNG HORSE
A SINGLE SPRING AT THE SHOULDERS TO REPRESENT THE
MOVEMENT AT THE SHOULDERS AND SHOULDER TORQUE

5SEGTEND
!

Comments:
!
The torque produced at the shoulders is determined
using a contractile and series elastic element in series.
!
The model controls the hip and knee angles using the
specified command to be functions of time.
The model takes off when the ground reaction force
(RZ) at the hands is less than or equal to zero.
!
KNEE ANGLE / ANGULAR VELOCITY = AK / AKDOT
HIP ANGLE / ANGULAR VELOCITY = AH / AHDOT
SHOULDER ANGLE / ANGULAR VEL. = AS / WS
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
AUTOZ(OFF)
!
FRAMES(A,B,C,D,E,I,BD)
DOF(6)
POINTS(O,P1,P2,P3,P4,P5,P6)
MASSLESS(O,A,C,P1,P2,P3,P4,P5,P6)
INERTIA(B,0,IB,IB,0,0,0)
INERTIA(D,0,ID,ID,0,0,0)
INERTIA(E,0,IE,IE,0,0,0)
INERTIA(J,0,II,II,0,0,0)
INERTIA(BD,0,IBD,IBD,0,0,0)
VAR(Q1,Q2,Q3,Q4,Q5,Q6)
CONST(L1,L2,L3,L4,L5,L6,L7,L8,L9,G)
CONST(PSA,PSB,PSC,PSD,PSP,PSQ)
CONST(PKANG,PHANG,PSANG)
CONST(PEND)
CONST(K1,K2,K3,K4,K5,K6)
CONST(SACT)
CONST(TSON)
SPECIFIED(AK,AH)
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
DIRECTION COSINES
!
SIMPROT(N,B,3,Q3)
```
SIMPROT(N,C,3,Q3)
SIMPROT(N,BD,3,QS)
SIMPROT(N,D,3,Q6)
SIMPROT(D,E,3,AH-PHANG)
SIMPROT(E,J,3,PKANG-AK)
DIRCOS(B,C)
DIRCOS(B,D)
DIRCOS(N,E)
DIRCOS(N,J)

! ! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! POSITIONS

POPI = Q1*N1 + Q2*N2
PP1BSTAR = L1*B1
PP1P2 = L2*B1
PP2P3 = Q4*C1
PP3DSTAR = L3*D1
PP3P4 = L4*D1
PP4ESTAR = L5*E1
PP4P5 = L6*E1
PP5JSTAR = L7*J1
PP5P6 = L8*J1

PP3BDSTAR = L9*BD1

POPI = Q1*N1 + Q2*N2
POBSTAR = ADD(POPI,PP1BSTAR)
POP2 = ADD(POPI,PP1P2)
POP3 = ADD(POP2,PP2P3)
PODSTAR = ADD(POP3,PP3DSTAR)
POP4 = ADD(POP3,PP3P4)
POESTAR = ADD(POP4,PP4ESTAR)
POP5 = ADD(POP4,PP4P5)
POJSTAR = ADD(POP5,PP5JSTAR)
POP6 = ADD(POP5,PP5P6)

POBDSTAR = ADD(POP3,PP3BDSTAR)

! ! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! GENERALISED SPEEDS (KINEMATIC EQUATIONS)

Q1' = U1
Q2' = U2
Q3' = U3
Q4' = U4
Q5' = U5
Q6' = U6

! ! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! VELOCITIES

WBN = Q3*N3
WCN = Q3*N3
WBDN = Q5*N3
WDN = Q6*N3
WED = AH*N3
WEN = ADD(WDN, WED)
WJE = -AK*N3
WJN = ADD(WEN, WJE)
!
VON = 0
VP1N = DERIV(P01, T, N)
VBSTARN = DERIV(P0BSTAR, T, N)
VP2N = DERIV(P02, T, N)
VP3N = DERIV(P03, T, N)
VDSTARN = DERIV(P0DSTAR, T, N)
VP4N = DERIV(P04, T, N)
VESTARN = DERIV(P0ESTAR, T, N)
VP5N = DERIV(P05, T, N)
VJSTARN = DERIV(P0JSTAR, T, N)
VP6N = DERIV(P06, T, N)
!
VBDSTARN = DERIV(P0BDSTAR, T, N)
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
! ACCELERATIONS
!
ALFBN = DERIV(WBN, T, N)
ALFCN = DERIV(WCN, T, N)
ALFBDN = DERIV(WBDN, T, N)
ALFDN = DERIV(WDN, T, N)
ALFED = DERIV(WED, T, N)
ALFJE = DERIV(WJE, T, N)
ALFEN = ADD(ALFDN, ALFED)
ALFJN = ADD(ALFEN, ALFJE)
!
AON = 0
AP1N = DERIV(VP1N, T, N)
ABSTARN = DERIV(VBSTARN, T, N)
AP2N = DERIV(VP2N, T, N)
AP3N = DERIV(VP3N, T, N)
ADSTARN = DERIV(VDSTARN, T, N)
AP4N = DERIV(VP4N, T, N)
AESTARN = DERIV(VESTARN, T, N)
AP5N = DERIV(VP5N, T, N)
AJSTARN = DERIV(VJSTARN, T, N)
AP6N = DERIV(VP6N, T, N)
!
ABDSTARN = DERIV(VBDSTARN, T, N)
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
! GENERALISED INERTIA FORCES
!
FRSTAR
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!
! GENERALISED ACTIVE FORCES
!
FORCE(BSTAR) = -MASSB*G*N2
FORCE(DSTAR) = -MASSD*G*N2
FORCE(ESTAR) = -MASSE*G*N2
FORCE(JSTAR) = -MASSJ*G*N2

FORCE(O/P1) = RX*N1 + RZ*N2
FORCE(P2/P3) = RS*B1

TORQUE(BD/B) = TST*N3
TORQUE(D/BD) = TSM*N3

FR

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! KANES EQUATIONS

KANE

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! MASS CENTRE LOCATION

CM(O,N)
VCMN = DERIV(POCM,T,N)

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! ANGULAR MOMENTUM CALCULATION ABOUT MASS CENTRE

ANGMOM(CM,N)

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! ENERGY

KE

PE1 = DOT(POCM,N2)
PE2 = 0.5*K1*Q1*Q1 + 0.5*K3*Q2*Q2
PE3 = 0.5*K5*Q4*Q4
PE4 = 0.5*PTEND*(PSANG-Q3+Q5)*(PSANG-Q3+Q5)

TOTMASS = MASSB+MASSD+MASSE+MASSJ
PE = RIGHT(TOTMASS)*RIGHT(PEL1)*G+RIGHT(PEL2)+RIGHT(PEL3)+RIGHT(PEL4)

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! EXTRA PRINTOUT VARIABLES / FORCE & TORQUE EQUATIONS

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! REACTION FORCES AND MASS CENTRE VELOCITY

CONTROLS(RX,RZ,RS,XG,ZG,UG,VG)

RX = -K1*Q1 - K2*U1
RZ = -K3*Q2 - K4*U2
RS = -K5*Q4 - K6*U4

XG = DOT(POCM,N1)
ZG = DOT(POCM,N2)

UG = DOT(VCMN,N1)
VG = DOT(VCMN,N2)

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! JOINT CENTRE LOCATIONS
!

P1X = DOT(POPI,N1)
P1Z = DOT(POPI,N2)

P2X = DOT(POP2,N1)
P2Z = DOT(POP2,N2)

P3X = DOT(POP3,N1)
P3Z = DOT(POP3,N2)

P4X = DOT(POP4,N1)
P4Z = DOT(POP4,N2)

P5X = DOT(POP5,N1)
P5Z = DOT(POP5,N2)

P6X = DOT(POP6,N1)
P6Z = DOT(POP6,N2)


V1X = DERIV(P1X,T)
V1Z = DERIV(P1Z,T)

V2X = DERIV(P2X,T)
V2Z = DERIV(P2Z,T)

V3X = DERIV(P3X,T)
V3Z = DERIV(P3Z,T)

V4X = DERIV(P4X,T)
V4Z = DERIV(P4Z,T)

V5X = DERIV(P5X,T)
V5Z = DERIV(P5Z,T)

V6X = DERIV(P6X,T)
V6Z = DERIV(P6Z,T)

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! JOINT ANGLES / ANGULAR VELOCITIES:
!
CONTROLS(AS,AST,ASM)
AS = (Q3 - Q6)

AST = PSANG - Q3 + Q5

ASM = Q6 - Q5

CONTROLS(WS, WST, WSM)

WS = DERIV(AS, T)

WST = DERIV(AS, T)

WSM = DERIV(-ASM, T)

CONTROLS(WS, WST, WSM)

WS = DERIV(AS, T)

WST = DERIV(AST, T)

WSM = DERIV(-ASM, T)

JOINT TORQUES

USE TSM IF WSM < 0.0

USE TSMN IF WSM > 0.0

CONTROLS(TSM, TSMN, TST)

SHOULDER TORQUE:

TOP = PSA + PSB * EXP(PSP * WSM)

BOTL = PSC + EXP(PSP * WSM)

BOTR = PSD + EXP(-PSQ * WSM)

TSM = (SACT * RIGHT(TOP)) / (RIGHT(BOTL) * RIGHT(BOTR))

TOP = PSA * EXP(-PSP + PSQ) * WSM + PSB * EXP(-PSQ * WSM)

BOTL = PSC + EXP(-PSP * WSM)

BOTR = PSD + EXP(-PSQ * WSM)

TSMN = (SACT * RIGHT(TOP)) / (RIGHT(BOTL) * RIGHT(BOTR))

TST = PTEND * AST

UNITS FOR OUTPUT

UNITS(U1, M/S, U2, M/S, U3, DEG, U4, M/S, U5, DEG, U6, DEG)

UNITS(L1, M, L2, M, L3, M, L4, M, L5, M, L6, M, L7, M, L8, M, L9, M)

UNITS(Q1, M, Q2, M, Q3, DEG, Q4, M, Q5, DEG, Q6, DEG)

UNITS(T, S, MASS, KG, INERTIA, KGM^2, G, M/S^2)

UNITS(XG, M, ZG, M, VG, M/S, RX, N, RZ, N, RS, N)

UNITS(ENERGY, J, ANGMOM, KG, M^2/S)

UNITS(K1, N/M, K2, N/M, K3, N/M, K4, N/S, K5, N/M, K6, N/S)

UNITS(AK, DEG, PKANG, DEG)

UNITS(AH, DEG, PHANG, DEG)

UNITS(AS, DEG, WS, DEG, PSANG, DEG, TSM, NM, TSMN, NM, TSON, S, SACT, ACT.)

UNITS(ASM, DEG, AST, DEG, WSM, DEG, WST, DEG, TST, NM)

UNITS(P1X, M, P1Z, M, P2X, M, P2Z, M, P3X, M, P3Z, M)


UNITS(V1X, M/S, V1Z, M/S, V2X, M/S, V2Z, M/S, V3X, M/S, V3Z, M/S)


UNITS(AHDOT, DEG/S, AHDOTDOT, DEG/S^2, AKDOT, DEG/S, AKDOTDOT, DEG/S^2)
!!
!! GENERATION OF FORTRAN CODE AND FULL OUTPUT
!!
CODE(5SEGEND, SUBS)
RECORD(5SEGEND, ALL)
!!
!! UNITS
Full details of the results from the film analysis of each Hecht and tumbling performance are given in this section:

Table B.1 shows the times corresponding to the seven instants during each vault.
Table B.2 shows the calculated segment lengths at horse contact and the overall average segment lengths.
Table B.3 shows the location of the mass centre at the seven instants during each vault.
Table B.4 shows the velocity of the mass centre at the seven instants during each vault.
Table B.5 shows the movement at the shoulders during horse contact.
Table B.6 shows the maximum vertical depression of the vaulting horse.
Table B.7 shows the height of the wrist above the top of the horse at horse takeoff.
Table B.8, Table B.9, Table B.10 and Table B.11 show the segment / joint angles and angular velocities at finger contact, wrist contact, wrist takeoff and finger takeoff from the horse respectively for each trial.
Table B.12 shows the average hip and knee angles during horse contact for each vault.
Table B.13 shows the angular momentum about a transverse axis through the mass centre at the seven instants during each vault.
Table B.14 shows the times corresponding to the six instants during each tumbling performance.
Table B.15 shows the calculated segment lengths during each tumbling performance.
Table B.16 shows the mass centre location relative to the contact point with the track during each tumbling performance.
Table B.17 shows the velocity of the mass centre at the six instants during each tumbling performance.
Table B.18 shows the maximum vertical depression of the vaulting horse calculated from the vertical location of the right wrists during horse contact.
Table B.19, Table B.20, Table B.21 and Table B.22 show the segment / joint angles and angular velocities during the final contact with the tumble track for each trial.
Table B.23 shows the angular momentum about a transverse axis through the mass centre at the seven instants during each trial.
Table B.1. Times of contact and takeoff for each vault [s]

<table>
<thead>
<tr>
<th>instant during vault</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>board contact [toe]</td>
<td>-0.350</td>
<td>-0.330</td>
<td>-0.352</td>
<td>-0.344</td>
</tr>
<tr>
<td>board takeoff [toe]</td>
<td>-0.230</td>
<td>-0.230</td>
<td>-0.232</td>
<td>-0.231</td>
</tr>
<tr>
<td>horse contact [finger]</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>horse contact [wrist]</td>
<td>0.010</td>
<td>0.010</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>horse takeoff [wrist]</td>
<td>0.105</td>
<td>0.100</td>
<td>0.100</td>
<td>0.102</td>
</tr>
<tr>
<td>horse takeoff [finger]</td>
<td>0.179</td>
<td>0.169</td>
<td>0.174</td>
<td>0.174</td>
</tr>
<tr>
<td>landing [toe]</td>
<td>0.848</td>
<td>0.866</td>
<td>0.864</td>
<td>0.859</td>
</tr>
</tbody>
</table>

Table B.2. Calculated segment lengths during each vault [m]

<table>
<thead>
<tr>
<th>instant</th>
<th>segment</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>horse contact [wrists]</td>
<td>arm</td>
<td>0.52</td>
<td>0.52</td>
<td>0.53</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>trunk</td>
<td>0.56</td>
<td>0.56</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>thigh</td>
<td>0.44</td>
<td>0.43</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>shank</td>
<td>0.41</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>foot</td>
<td>0.20</td>
<td>0.21</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>average</td>
<td>arm</td>
<td>0.55</td>
<td>0.54</td>
<td>0.53</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>trunk</td>
<td>0.52</td>
<td>0.50</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>thigh</td>
<td>0.42</td>
<td>0.43</td>
<td>0.43</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>shank</td>
<td>0.42</td>
<td>0.42</td>
<td>0.41</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>foot</td>
<td>0.19</td>
<td>0.20</td>
<td>0.19</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Table B.3. Mass centre location during each vault [m]

<table>
<thead>
<tr>
<th>instant</th>
<th>location</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>board contact</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[toe]</td>
<td>x</td>
<td>-2.78</td>
<td>-2.63</td>
<td>-2.90</td>
<td>-2.77</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>1.07</td>
<td>1.07</td>
<td>1.08</td>
<td>1.07</td>
</tr>
<tr>
<td>board takeoff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[toe]</td>
<td>x</td>
<td>-2.09</td>
<td>-2.06</td>
<td>-2.16</td>
<td>-2.10</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>1.30</td>
<td>1.30</td>
<td>1.24</td>
<td>1.28</td>
</tr>
<tr>
<td>horse contact</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[finger]</td>
<td>x</td>
<td>-0.78</td>
<td>-0.79</td>
<td>-0.81</td>
<td>-0.79</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>1.73</td>
<td>1.71</td>
<td>1.72</td>
<td>1.72</td>
</tr>
<tr>
<td>horse contact</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[wrist]</td>
<td>x</td>
<td>-0.73</td>
<td>-0.74</td>
<td>-0.75</td>
<td>-0.74</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>1.74</td>
<td>1.72</td>
<td>1.73</td>
<td>1.73</td>
</tr>
<tr>
<td>horse takeoff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[wrist]</td>
<td>x</td>
<td>-0.33</td>
<td>-0.34</td>
<td>-0.35</td>
<td>-0.34</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>1.90</td>
<td>1.90</td>
<td>1.88</td>
<td>1.89</td>
</tr>
<tr>
<td>horse takeoff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[finger]</td>
<td>x</td>
<td>-0.03</td>
<td>-0.04</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>2.04</td>
<td>2.03</td>
<td>2.03</td>
<td>2.03</td>
</tr>
<tr>
<td>landing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[toe]</td>
<td>x</td>
<td>2.66</td>
<td>2.96</td>
<td>3.02</td>
<td>2.88</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>0.97</td>
<td>0.96</td>
<td>0.95</td>
<td>0.96</td>
</tr>
</tbody>
</table>

The origin for the mass centre location for each vault was a point on the ground below the midpoint of the wrists at the start of horse contact, such that the midpoint had the location (0.00m, 1.35m).
Table B.4. Mass centre velocity during each vault [m.s⁻¹]

<table>
<thead>
<tr>
<th>instant</th>
<th>velocity</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>board contact</td>
<td>vx</td>
<td>6.79</td>
<td>7.17</td>
<td>7.23</td>
<td>7.06</td>
</tr>
<tr>
<td>[toe]</td>
<td>vz</td>
<td>-1.36</td>
<td>-1.24</td>
<td>-1.26</td>
<td>-1.29</td>
</tr>
<tr>
<td>board takeoff</td>
<td>vx</td>
<td>5.73</td>
<td>5.98</td>
<td>6.28</td>
<td>6.00</td>
</tr>
<tr>
<td>[toe]</td>
<td>vz</td>
<td>3.08</td>
<td>3.26</td>
<td>3.19</td>
<td>3.18</td>
</tr>
<tr>
<td>horse contact</td>
<td>vx</td>
<td>5.73</td>
<td>5.98</td>
<td>6.28</td>
<td>6.00</td>
</tr>
<tr>
<td>[finger]</td>
<td>vz</td>
<td>0.71</td>
<td>0.99</td>
<td>0.94</td>
<td>0.88</td>
</tr>
<tr>
<td>horse contact</td>
<td>vx</td>
<td>5.71</td>
<td>5.94</td>
<td>6.20</td>
<td>5.95</td>
</tr>
<tr>
<td>[wrist]</td>
<td>vz</td>
<td>0.62</td>
<td>0.90</td>
<td>0.86</td>
<td>0.79</td>
</tr>
<tr>
<td>horse takeoff</td>
<td>vx</td>
<td>4.02</td>
<td>4.28</td>
<td>4.38</td>
<td>4.23</td>
</tr>
<tr>
<td>[wrist]</td>
<td>vz</td>
<td>2.41</td>
<td>2.51</td>
<td>2.52</td>
<td>2.48</td>
</tr>
<tr>
<td>horse takeoff</td>
<td>vx</td>
<td>4.05</td>
<td>4.32</td>
<td>4.42</td>
<td>4.26</td>
</tr>
<tr>
<td>[finger]</td>
<td>vz</td>
<td>1.79</td>
<td>2.00</td>
<td>1.91</td>
<td>1.90</td>
</tr>
<tr>
<td>landing</td>
<td>vx</td>
<td>4.05</td>
<td>4.32</td>
<td>4.42</td>
<td>4.26</td>
</tr>
<tr>
<td>[toe]</td>
<td>vz</td>
<td>-4.91</td>
<td>-4.96</td>
<td>-4.96</td>
<td>-4.94</td>
</tr>
</tbody>
</table>
Table B.5. Movement at the shoulders during the contact phase with the horse for each vault [m]

<table>
<thead>
<tr>
<th>instant</th>
<th>value</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>finger contact</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>finger contact dSS'</td>
<td>0.128</td>
<td>0.095</td>
<td>0.117</td>
<td>0.113</td>
<td></td>
</tr>
<tr>
<td>finger contact dWS'</td>
<td>0.648</td>
<td>0.618</td>
<td>0.613</td>
<td>0.626</td>
<td></td>
</tr>
<tr>
<td>finger contact dS'B</td>
<td>0.155</td>
<td>0.174</td>
<td>0.165</td>
<td>0.165</td>
<td></td>
</tr>
<tr>
<td>wrist contact</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wrist contact dSS'</td>
<td>0.122</td>
<td>0.084</td>
<td>0.105</td>
<td>0.104</td>
<td></td>
</tr>
<tr>
<td>wrist contact dWS'</td>
<td>0.635</td>
<td>0.598</td>
<td>0.610</td>
<td>0.614</td>
<td></td>
</tr>
<tr>
<td>wrist contact dS'B</td>
<td>0.159</td>
<td>0.175</td>
<td>0.160</td>
<td>0.165</td>
<td></td>
</tr>
<tr>
<td>maximum depression</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>maximum depression dSS'</td>
<td>0.032</td>
<td>0.009</td>
<td>0.009</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>maximum depression dWS'</td>
<td>0.524</td>
<td>0.502</td>
<td>0.477</td>
<td>0.501</td>
<td></td>
</tr>
<tr>
<td>maximum depression dS'B</td>
<td>0.121</td>
<td>0.129</td>
<td>0.124</td>
<td>0.125</td>
<td></td>
</tr>
<tr>
<td>wrist takeoff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wrist takeoff dSS'</td>
<td>0.039</td>
<td>0.016</td>
<td>0.033</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>wrist takeoff dWS'</td>
<td>0.578</td>
<td>0.554</td>
<td>0.530</td>
<td>0.554</td>
<td></td>
</tr>
<tr>
<td>wrist takeoff dS'B</td>
<td>0.135</td>
<td>0.132</td>
<td>0.124</td>
<td>0.130</td>
<td></td>
</tr>
<tr>
<td>finger takeoff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>finger takeoff dSS'</td>
<td>0.095</td>
<td>0.067</td>
<td>0.077</td>
<td>0.080</td>
<td></td>
</tr>
<tr>
<td>finger takeoff dWS'</td>
<td>0.646</td>
<td>0.629</td>
<td>0.638</td>
<td>0.638</td>
<td></td>
</tr>
<tr>
<td>finger takeoff dS'B</td>
<td>0.175</td>
<td>0.156</td>
<td>0.165</td>
<td>0.165</td>
<td></td>
</tr>
</tbody>
</table>
Table B.6. Maximum vertical depression of the vaulting horse [m](wrist movement relative to contact point of wrist)

<table>
<thead>
<tr>
<th>trial</th>
<th>depression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.012</td>
</tr>
<tr>
<td>2</td>
<td>0.021</td>
</tr>
<tr>
<td>3</td>
<td>0.007</td>
</tr>
<tr>
<td>average</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Table B.7. Wrist height above the initial wrist height at finger takeoff [m]

<table>
<thead>
<tr>
<th>trial</th>
<th>height</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>0.18</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td>average</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Table B.8. Segment and joint angles / angular velocities at finger contact with the horse

<table>
<thead>
<tr>
<th>angle [°]</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>45</td>
<td>44</td>
<td>43</td>
<td>44</td>
</tr>
<tr>
<td>trunk</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>hip</td>
<td>166</td>
<td>167</td>
<td>169</td>
<td>167</td>
</tr>
<tr>
<td>knee</td>
<td>180</td>
<td>181</td>
<td>181</td>
<td>181</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ang. vel. [°s⁻¹]</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>372</td>
<td>422</td>
<td>332</td>
<td>375</td>
</tr>
<tr>
<td>trunk</td>
<td>284</td>
<td>234</td>
<td>177</td>
<td>232</td>
</tr>
<tr>
<td>hip</td>
<td>-232</td>
<td>-50</td>
<td>-38</td>
<td>-107</td>
</tr>
<tr>
<td>knee</td>
<td>-217</td>
<td>-16</td>
<td>-69</td>
<td>-101</td>
</tr>
</tbody>
</table>

Table B.9. Segment and joint angles / angular velocities at horse contact (wrist contact)

<table>
<thead>
<tr>
<th>angle [°]</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>49</td>
<td>49</td>
<td>46</td>
<td>48</td>
</tr>
<tr>
<td>trunk</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>hip</td>
<td>163</td>
<td>167</td>
<td>169</td>
<td>166</td>
</tr>
<tr>
<td>knee</td>
<td>178</td>
<td>181</td>
<td>180</td>
<td>180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ang. vel. [°s⁻¹]</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>358</td>
<td>440</td>
<td>272</td>
<td>357</td>
</tr>
<tr>
<td>trunk</td>
<td>304</td>
<td>232</td>
<td>103</td>
<td>213</td>
</tr>
<tr>
<td>hip</td>
<td>-319</td>
<td>-57</td>
<td>-2</td>
<td>-126</td>
</tr>
<tr>
<td>knee</td>
<td>-243</td>
<td>-21</td>
<td>-80</td>
<td>-115</td>
</tr>
</tbody>
</table>
Table B.10. Segment and joint angles / angular velocities at horse takeoff (wrist takeoff)

<table>
<thead>
<tr>
<th></th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm [°]</td>
<td>89</td>
<td>91</td>
<td>87</td>
<td>89</td>
</tr>
<tr>
<td>trunk [°]</td>
<td>7</td>
<td>8</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>hip [°]</td>
<td>162</td>
<td>162</td>
<td>173</td>
<td>166</td>
</tr>
<tr>
<td>knee [°]</td>
<td>176</td>
<td>169</td>
<td>177</td>
<td>174</td>
</tr>
<tr>
<td>arm [°s⁻¹]</td>
<td>388</td>
<td>403</td>
<td>493</td>
<td>428</td>
</tr>
<tr>
<td>trunk [°s⁻¹]</td>
<td>237</td>
<td>196</td>
<td>70</td>
<td>168</td>
</tr>
<tr>
<td>hip [°s⁻¹]</td>
<td>403</td>
<td>-142</td>
<td>-704</td>
<td>-148</td>
</tr>
<tr>
<td>knee [°s⁻¹]</td>
<td>-79</td>
<td>527</td>
<td>-317</td>
<td>44</td>
</tr>
</tbody>
</table>

Table B.11. Segment and joint angles / angular velocities at horse takeoff (finger takeoff)

<table>
<thead>
<tr>
<th></th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm [°]</td>
<td>113</td>
<td>114</td>
<td>114</td>
<td>114</td>
</tr>
<tr>
<td>trunk [°]</td>
<td>9</td>
<td>16</td>
<td>23</td>
<td>16</td>
</tr>
<tr>
<td>hip [°]</td>
<td>152</td>
<td>144</td>
<td>141</td>
<td>146</td>
</tr>
<tr>
<td>knee [°]</td>
<td>181</td>
<td>184</td>
<td>185</td>
<td>183</td>
</tr>
<tr>
<td>arm [°s⁻¹]</td>
<td>261</td>
<td>324</td>
<td>171</td>
<td>252</td>
</tr>
<tr>
<td>trunk [°s⁻¹]</td>
<td>-170</td>
<td>1</td>
<td>60</td>
<td>-36</td>
</tr>
<tr>
<td>hip [°s⁻¹]</td>
<td>155</td>
<td>-162</td>
<td>-272</td>
<td>-93</td>
</tr>
<tr>
<td>knee [°s⁻¹]</td>
<td>51</td>
<td>-116</td>
<td>148</td>
<td>28</td>
</tr>
</tbody>
</table>
Table B.12. Average hip and knee angles during horse contact for each vault

<table>
<thead>
<tr>
<th>angle</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hip</td>
<td>161</td>
<td>160</td>
<td>166</td>
<td>163</td>
</tr>
<tr>
<td>knee</td>
<td>178</td>
<td>177</td>
<td>180</td>
<td>178</td>
</tr>
<tr>
<td>contact</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wrists</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hip</td>
<td>167</td>
<td>166</td>
<td>175</td>
<td>169</td>
</tr>
<tr>
<td>knee</td>
<td>178</td>
<td>174</td>
<td>182</td>
<td>178</td>
</tr>
<tr>
<td>down</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table B.13. Angular momentum about a transverse axis during the hurdle step, preflight, contact and postflight [kg.m².rad.s⁻¹]

<table>
<thead>
<tr>
<th>time</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>approach</td>
<td>2.52</td>
<td>2.52</td>
<td>1.26</td>
<td>1.89</td>
</tr>
<tr>
<td>mean preflight</td>
<td>50.35</td>
<td>56.02</td>
<td>54.76</td>
<td>54.13</td>
</tr>
<tr>
<td>finger contact</td>
<td>43.43</td>
<td>44.06</td>
<td>41.54</td>
<td>42.80</td>
</tr>
<tr>
<td>wrist contact</td>
<td>37.13</td>
<td>36.50</td>
<td>34.62</td>
<td>35.88</td>
</tr>
<tr>
<td>wrist takeoff</td>
<td>-5.04</td>
<td>-0.63</td>
<td>-0.63</td>
<td>-1.89</td>
</tr>
<tr>
<td>finger takeoff</td>
<td>-14.48</td>
<td>-17.62</td>
<td>-11.96</td>
<td>-14.48</td>
</tr>
</tbody>
</table>

The calculated angular momentum values at finger contact and the mean preflight at finger takeoff and the mean postflight were expected to be the same. The mean values were used as the angular momentum estimates at finger contact and finger takeoff.
Table B.14. Times of contact and takeoff for each tumbling performance [s]

<table>
<thead>
<tr>
<th>instant</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>penultimate contact [toe]</td>
<td>-0.273</td>
<td>-0.285</td>
<td>-0.274</td>
<td>-0.277</td>
<td>-0.271</td>
</tr>
<tr>
<td>penultimate takeoff [toe]</td>
<td>-0.080</td>
<td>-0.070</td>
<td>-0.075</td>
<td>-0.080</td>
<td>-0.055</td>
</tr>
<tr>
<td>final contact [toe]</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>final contact [heel]</td>
<td>0.045</td>
<td>0.040</td>
<td>0.030</td>
<td>0.040</td>
<td>-</td>
</tr>
<tr>
<td>final takeoff [heel]</td>
<td>0.080</td>
<td>0.075</td>
<td>0.070</td>
<td>0.080</td>
<td>-</td>
</tr>
<tr>
<td>final takeoff [toe]</td>
<td>0.120</td>
<td>0.120</td>
<td>0.110</td>
<td>0.120</td>
<td>0.130</td>
</tr>
<tr>
<td>landing [toe]</td>
<td>1.208</td>
<td>1.235</td>
<td>1.162</td>
<td>1.176</td>
<td>1.099</td>
</tr>
</tbody>
</table>

Table B.15. Calculated segment lengths during each tumbling performance [m]

<table>
<thead>
<tr>
<th>instant</th>
<th>segment</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>final track</td>
<td>arm</td>
<td>0.56</td>
<td>0.58</td>
<td>0.58</td>
<td>0.54</td>
<td>0.54</td>
<td>0.56</td>
</tr>
<tr>
<td>contact [toe]</td>
<td>trunk</td>
<td>0.56</td>
<td>0.58</td>
<td>0.54</td>
<td>0.57</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>thigh</td>
<td>0.46</td>
<td>0.45</td>
<td>0.44</td>
<td>0.44</td>
<td>0.46</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>shank</td>
<td>0.41</td>
<td>0.40</td>
<td>0.42</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>foot</td>
<td>0.12</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>average</td>
<td>arm</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>trunk</td>
<td>0.56</td>
<td>0.56</td>
<td>0.54</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>thigh</td>
<td>0.42</td>
<td>0.41</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>shank</td>
<td>0.41</td>
<td>0.41</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>foot</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.16</td>
<td>0.16</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Table B.16. Mass centre location during each tumbling performance [m]

<table>
<thead>
<tr>
<th>instant</th>
<th>coordinate</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>penultimate contact [toe]</td>
<td>x</td>
<td>-1.78</td>
<td>-1.79</td>
<td>-1.79</td>
<td>-1.82</td>
<td>-1.80</td>
<td>-1.80</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>0.81</td>
<td>0.82</td>
<td>0.78</td>
<td>0.77</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
<td>penultimate takeoff [toe]</td>
<td>x</td>
<td>-0.89</td>
<td>-0.84</td>
<td>-0.85</td>
<td>-0.89</td>
<td>-0.77</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>0.77</td>
<td>0.79</td>
<td>0.77</td>
<td>0.77</td>
<td>0.76</td>
<td>0.77</td>
</tr>
<tr>
<td>final contact [toe]</td>
<td>x</td>
<td>-0.53</td>
<td>-0.52</td>
<td>-0.49</td>
<td>-0.50</td>
<td>-0.51</td>
<td>-0.51</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>0.75</td>
<td>0.77</td>
<td>0.78</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>final contact [heel]</td>
<td>x</td>
<td>-0.33</td>
<td>-0.34</td>
<td>-0.36</td>
<td>-0.31</td>
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<td>-0.31</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>0.76</td>
<td>0.78</td>
<td>0.78</td>
<td>0.77</td>
<td>-</td>
<td>0.77</td>
</tr>
<tr>
<td>final takeoff [heel]</td>
<td>x</td>
<td>-0.21</td>
<td>-0.22</td>
<td>-0.21</td>
<td>-0.16</td>
<td>-</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>0.86</td>
<td>0.86</td>
<td>0.87</td>
<td>0.87</td>
<td>-</td>
<td>0.87</td>
</tr>
<tr>
<td>final takeoff [toe]</td>
<td>x</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.10</td>
<td>-0.03</td>
<td>-0.04</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>z</td>
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<td>1.05</td>
<td>1.04</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>landing [toe]</td>
<td>x</td>
<td>2.90</td>
<td>2.93</td>
<td>2.51</td>
<td>2.85</td>
<td>2.35</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>0.74</td>
<td>0.68</td>
<td>0.57</td>
<td>0.50</td>
<td>0.52</td>
<td>0.60</td>
</tr>
</tbody>
</table>

The origin for the mass centre locations was set at the midpoint of the toes at the start of the final contact with the tumbling track.
Table B.17. Mass centre velocity during each tumbling performances [m.s⁻¹]

<table>
<thead>
<tr>
<th>instant</th>
<th>coordinate</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>penultimate contact</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>penultimate</td>
<td>x</td>
<td>4.50</td>
<td>4.66</td>
<td>4.80</td>
<td>4.85</td>
<td>4.66</td>
<td>4.69</td>
</tr>
<tr>
<td>takeoff [toe]</td>
<td>z</td>
<td>0.08</td>
<td>0.09</td>
<td>0.40</td>
<td>0.26</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>final contact</td>
<td>x</td>
<td>4.50</td>
<td>4.66</td>
<td>4.80</td>
<td>4.85</td>
<td>4.66</td>
<td>4.69</td>
</tr>
<tr>
<td>[toe]</td>
<td>z</td>
<td>-0.70</td>
<td>-0.59</td>
<td>-0.33</td>
<td>-0.52</td>
<td>-0.37</td>
<td>-0.50</td>
</tr>
<tr>
<td>final contact</td>
<td>x</td>
<td>4.48</td>
<td>4.52</td>
<td>4.70</td>
<td>4.83</td>
<td>-</td>
<td>4.63</td>
</tr>
<tr>
<td>[heel]</td>
<td>z</td>
<td>-0.69</td>
<td>-0.61</td>
<td>-0.46</td>
<td>-0.63</td>
<td>-</td>
<td>-0.60</td>
</tr>
<tr>
<td>final takeoff</td>
<td>x</td>
<td>2.99</td>
<td>2.86</td>
<td>2.70</td>
<td>3.04</td>
<td>-</td>
<td>2.90</td>
</tr>
<tr>
<td>[heel]</td>
<td>z</td>
<td>5.25</td>
<td>5.34</td>
<td>4.85</td>
<td>4.83</td>
<td>-</td>
<td>5.07</td>
</tr>
<tr>
<td>final takeoff</td>
<td>x</td>
<td>2.93</td>
<td>2.79</td>
<td>2.66</td>
<td>2.99</td>
<td>2.63</td>
<td>2.80</td>
</tr>
<tr>
<td>[toe]</td>
<td>z</td>
<td>4.98</td>
<td>5.09</td>
<td>4.53</td>
<td>4.50</td>
<td>3.93</td>
<td>4.61</td>
</tr>
<tr>
<td>landing [toe]</td>
<td>x</td>
<td>2.93</td>
<td>2.79</td>
<td>2.66</td>
<td>2.99</td>
<td>2.63</td>
<td>2.80</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>-5.61</td>
<td>-5.80</td>
<td>-5.60</td>
<td>-5.68</td>
<td>-5.28</td>
<td>-5.59</td>
</tr>
</tbody>
</table>
Table B.18. Maximum vertical depression of the track during each tumbling performance
[relative to the average height of the toe / ankle at contact with the track]

<table>
<thead>
<tr>
<th>trial</th>
<th>toe height at toe contact</th>
<th>toe height at toe takeoff</th>
<th>ankle height at heel contact</th>
<th>ankle height at heel takeoff</th>
<th>depression [ankle]</th>
<th>depression [toe]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.227</td>
<td>0.221</td>
<td>0.212</td>
<td>0.224</td>
<td>0.008</td>
<td>0.056</td>
</tr>
<tr>
<td>2</td>
<td>0.210</td>
<td>0.213</td>
<td>0.219</td>
<td>0.219</td>
<td>0.008</td>
<td>0.063</td>
</tr>
<tr>
<td>3</td>
<td>0.209</td>
<td>0.215</td>
<td>0.223</td>
<td>0.222</td>
<td>0.013</td>
<td>0.066</td>
</tr>
<tr>
<td>4</td>
<td>0.227</td>
<td>0.216</td>
<td>0.214</td>
<td>0.221</td>
<td>0.012</td>
<td>0.063</td>
</tr>
<tr>
<td>5</td>
<td>0.224</td>
<td>0.213</td>
<td><em>0.229</em></td>
<td><em>0.229</em></td>
<td>-</td>
<td>0.065</td>
</tr>
<tr>
<td>average</td>
<td>0.219</td>
<td>0.216</td>
<td>0.217</td>
<td>0.222</td>
<td>0.010</td>
<td>0.063</td>
</tr>
</tbody>
</table>

* minimum ankle height as heel did not contact the track during trial 5

Table B.19. Segment angles / angular velocities at the final contact of the toes with the tumble track [spline fitted to preflight]

<table>
<thead>
<tr>
<th>angle [°]</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>-22</td>
<td>-20</td>
<td>-15</td>
<td>-17</td>
<td>-22</td>
<td>-19</td>
</tr>
<tr>
<td>trunk</td>
<td>3</td>
<td>6</td>
<td>16</td>
<td>6</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>thigh</td>
<td>75</td>
<td>82</td>
<td>83</td>
<td>81</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>shank</td>
<td>53</td>
<td>48</td>
<td>48</td>
<td>49</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>foot</td>
<td>119</td>
<td>115</td>
<td>123</td>
<td>121</td>
<td>123</td>
<td>120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ang. vel. [°s⁻¹]</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>800</td>
<td>633</td>
<td>725</td>
<td>762</td>
<td>827</td>
<td>749</td>
</tr>
<tr>
<td>trunk</td>
<td>772</td>
<td>841</td>
<td>1034</td>
<td>793</td>
<td>707</td>
<td>829</td>
</tr>
<tr>
<td>thigh</td>
<td>672</td>
<td>837</td>
<td>499</td>
<td>596</td>
<td>884</td>
<td>698</td>
</tr>
<tr>
<td>shank</td>
<td>325</td>
<td>218</td>
<td>121</td>
<td>185</td>
<td>277</td>
<td>225</td>
</tr>
<tr>
<td>foot</td>
<td>42</td>
<td>-221</td>
<td>539</td>
<td>10</td>
<td>73</td>
<td>89</td>
</tr>
</tbody>
</table>
Table B.20. Segment angles / angular velocities at the final contact of the toes with the tumble track [spline fitted to whole time history]

<table>
<thead>
<tr>
<th></th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm [°]</td>
<td>-23</td>
<td>-21</td>
<td>-15</td>
<td>-16</td>
<td>-22</td>
<td>-19</td>
</tr>
<tr>
<td>trunk</td>
<td>5</td>
<td>7</td>
<td>17</td>
<td>7</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>thigh</td>
<td>75</td>
<td>80</td>
<td>81</td>
<td>77</td>
<td>77</td>
<td>78</td>
</tr>
<tr>
<td>shank</td>
<td>51</td>
<td>47</td>
<td>46</td>
<td>48</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>foot</td>
<td>123</td>
<td>117</td>
<td>126</td>
<td>123</td>
<td>125</td>
<td>123</td>
</tr>
<tr>
<td>arm [°/s]</td>
<td>662</td>
<td>676</td>
<td>782</td>
<td>834</td>
<td>774</td>
<td>746</td>
</tr>
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<td>954</td>
<td>859</td>
<td>932</td>
<td>910</td>
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<tr>
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<td>553</td>
<td>295</td>
<td>477</td>
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<td>shank</td>
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<td>26</td>
<td>-5</td>
<td>71</td>
<td>-13</td>
<td>15</td>
</tr>
<tr>
<td>foot</td>
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<td>302</td>
<td>775</td>
<td>434</td>
<td>425</td>
<td>502</td>
</tr>
</tbody>
</table>
Table B.21. Segment angles / angular velocities at the final takeoff of the ankle from the track [spline fitted to time histories from takeoff of the ankle to end of time history]

<table>
<thead>
<tr>
<th>angle [°]</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm</td>
<td>39</td>
<td>41</td>
<td>50</td>
<td>53</td>
<td>19</td>
<td>40</td>
</tr>
<tr>
<td>trunk</td>
<td>61</td>
<td>61</td>
<td>72</td>
<td>71</td>
<td>49</td>
<td>63</td>
</tr>
<tr>
<td>thigh</td>
<td>87</td>
<td>88</td>
<td>83</td>
<td>86</td>
<td>89</td>
<td>87</td>
</tr>
<tr>
<td>shank</td>
<td>56</td>
<td>52</td>
<td>51</td>
<td>54</td>
<td>42</td>
<td>51</td>
</tr>
<tr>
<td>foot</td>
<td>161</td>
<td>156</td>
<td>153</td>
<td>155</td>
<td>143</td>
<td>154</td>
</tr>
<tr>
<td>ang. vel. [°s⁻¹]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>arm</td>
<td>456</td>
<td>504</td>
<td>609</td>
<td>605</td>
<td>1219</td>
<td>679</td>
</tr>
<tr>
<td>trunk</td>
<td>436</td>
<td>448</td>
<td>693</td>
<td>723</td>
<td>631</td>
<td>586</td>
</tr>
<tr>
<td>thigh</td>
<td>-236</td>
<td>85</td>
<td>-323</td>
<td>-116</td>
<td>325</td>
<td>-53</td>
</tr>
<tr>
<td>shank</td>
<td>469</td>
<td>496</td>
<td>395</td>
<td>318</td>
<td>-302</td>
<td>275</td>
</tr>
<tr>
<td>foot</td>
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<td>-238</td>
<td>-554</td>
<td>-813</td>
<td>-54</td>
<td>-542</td>
</tr>
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</table>
Table B.22. Segment angles / angular velocities at the final takeoff of the toes from the track [spline fitted to time histories from takeoff of the ankle to end of time history]

<table>
<thead>
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<th>angle [°]</th>
<th>arm</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>arm</td>
<td>52</td>
<td>60</td>
<td>73</td>
<td>75</td>
<td>85</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>trunk</td>
<td>78</td>
<td>82</td>
<td>100</td>
<td>100</td>
<td>116</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>thigh</td>
<td>82</td>
<td>79</td>
<td>77</td>
<td>80</td>
<td>87</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>shank</td>
<td>76</td>
<td>76</td>
<td>66</td>
<td>69</td>
<td>60</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>foot</td>
<td>131</td>
<td>128</td>
<td>121</td>
<td>119</td>
<td>99</td>
<td>120</td>
</tr>
<tr>
<td>ang. vel. [°s⁻¹]</td>
<td>arm</td>
<td>287</td>
<td>376</td>
<td>538</td>
<td>498</td>
<td>644</td>
<td>469</td>
</tr>
<tr>
<td></td>
<td>trunk</td>
<td>402</td>
<td>408</td>
<td>674</td>
<td>726</td>
<td>811</td>
<td>605</td>
</tr>
<tr>
<td></td>
<td>thigh</td>
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<td>-171</td>
<td>37</td>
<td>-100</td>
<td>-90</td>
<td>-77</td>
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<tr>
<td></td>
<td>shank</td>
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<td>480</td>
<td>307</td>
<td>347</td>
<td>600</td>
<td>440</td>
</tr>
<tr>
<td></td>
<td>foot</td>
<td>-448</td>
<td>-678</td>
<td>-784</td>
<td>-796</td>
<td>-753</td>
<td>-692</td>
</tr>
</tbody>
</table>

Table B.23. Angular momentum about a transverse axis during each tumbling performance [kg.m².rad.s⁻¹]

<table>
<thead>
<tr>
<th>time</th>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>approach</td>
<td>82.45</td>
<td>80.56</td>
<td>88.12</td>
<td>96.93</td>
<td>90.00</td>
<td>87.61</td>
</tr>
<tr>
<td>mean preflight</td>
<td>118.53</td>
<td>115.65</td>
<td>120.04</td>
<td>119.02</td>
<td>121.79</td>
<td>119.01</td>
</tr>
<tr>
<td>contact [toe]</td>
<td>118.96</td>
<td>110.77</td>
<td>117.70</td>
<td>113.29</td>
<td>122.10</td>
<td>116.56</td>
</tr>
<tr>
<td>contact [heel]</td>
<td>75.53</td>
<td>92.52</td>
<td>100.70</td>
<td>108.89</td>
<td>-</td>
<td>94.41</td>
</tr>
<tr>
<td>takeoff [heel]</td>
<td>73.01</td>
<td>76.79</td>
<td>92.52</td>
<td>91.26</td>
<td>-</td>
<td>83.39</td>
</tr>
<tr>
<td>takeoff [toe]</td>
<td>59.79</td>
<td>49.72</td>
<td>90.00</td>
<td>101.96</td>
<td>115.18</td>
<td>83.33</td>
</tr>
<tr>
<td>mean postflight</td>
<td>55.72</td>
<td>59.36</td>
<td>96.44</td>
<td>95.30</td>
<td>106.95</td>
<td>82.75</td>
</tr>
</tbody>
</table>
APPENDIX C.

WRITTEN CONSENT
INFORMED CONSENT FORM

PURPOSE
To obtain anthropometric and kinematic data of a gymnast during long horse vaults and tumbling. To obtain maximum muscular force values at the ankle, knee, hip and shoulder. Which will be used to evaluate a mathematical model of vaulting and tumbling.

PROCEDURES
Video cameras will be used to collect information during the performance of Hecht vaults and tumbling. A number of trials will be requested, with suitable breaks to minimise fatigue and boredom. An isokinetic dynamometer will be used to collect maximum muscle force values at the ankles, knee, hip and shoulder for one side of the body. Anthropometric data will be collected using tape measures and specialist anthropometers.

QUESTIONS
The researcher will be pleased to answer any questions which you may have at any time.

WITHDRAWAL
You are free to withdraw from the study at any time whatever reason without prejudice.

CONFIDENTIALITY
Your identity will remain confidential in any material resulting from this work.

I have read and understood the information on this form and agree to participate in this study. As far as I am aware I do not have any injury nor infirmity which would be affected by the procedures outlined.

Name........................................................... Name............................................................

Signed........................................ (gymnast) Signed........................................... (parent)

In the presence of:

Name..................................................

Signed.............................................(coach) Date.........................................................