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COLD EXTRUDED RODS - RESIDUAL STRESSES
AND MECHANICAL PROPERTIES

by

PREM SAGAR MIDHA

'A Doctoral Thesis'

Submitted in partial fulfilment of the requirements
for the award of

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of Technology

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© by Prem Sagar Midha, 1976
To my wife, Asha
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SUMMARY

The effect of extrusion ratio and die angle on residual stress distribution in cold extruded rods and the hardness, tensile, impact and fatigue properties of these rods have been studied. 'In-process' control of residual stresses by employing double reduction dies and the effect of low temperature annealing on these stresses has also been investigated.

Four extrusion ratios, namely 2.25:1, 2.78:1, 3.52:1 and 4:1, and three included die angles 60, 90 and 120° at each extrusion ratio have been used. The longitudinal and tangential residual stresses in the extruded rods are found to be compressive along the axis and tensile towards the surface. The residual stress distribution across the cross-section of the rod depends both on the extrusion-ratio and die angle. With an increase in the die angle and a reduction in the extrusion ratio, the stresses tend to become less compressive at the extrusion axis and less tensile at the surface. Residual stresses have also been computed from the stress distribution in the deformation zone predicted by the FLOW FUNCTION method of analysis and a qualitative agreement has been obtained between the observed and analytical results. The analytical results indicate that the friction at the die-metal interface could significantly affect the state of residual stress in the extruded rods. It has also been indicated that after emerging from the deformation zone the material could undergo further yielding under the influence of the die-land and this, in turn, could have an important effect on the residual stress distribution.
By using a double reduction die where a subsequent small reduction (~2%) is imparted at the die exit to the extruded rod after the main reduction at the die entry, the surface tensile stresses become compressive and the general level of the residual stresses is reduced. After a stress relief anneal at 500°C, the surface tensile stress level is reduced considerably and most of the improvement in hardness and strength is retained.

The hardness, tensile strength, and the fatigue limit of the extruded bar show considerable improvement over the unextruded material. For the range of extrusion ratios studied, the gain over the initial improvement, with the increase in extrusion ratio is very small. The effect of die angle also is not very significant.

Except at the lowest extrusion ratio the impact properties of extruded rods show a considerable improvement over those of the unextruded material, in terms of a lowering of the transition temperature from ductile to brittle fracture, although there is some impairment in the energy required for ductile fracture.
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CHAPTER 1

Introduction
1. **INTRODUCTION**

As a means of producing certain steel parts in large numbers, 'Cold Extrusion' offers many economic and other advantages over hot working and machining. These advantages are principally high material utilisation and good dimensional accuracy. Since the feasibility of the process was first demonstrated, much research effort has been expended in improving its economic and technical features. This is evident from an examination of the programmes of the first three international conferences on 'Cold Extrusion' held at 1953, 1960 and 1965 in this country. Many of the papers presented in these conferences were concerned either with various aspects of the process, e.g., tool design, billet cropping and lubrication, prediction of extrusion loads, etc. or with the ways in which this process could be adopted in a variety of industrial applications, the attendant economic and production benefits being examined. The result of this internationally-concerted effort has been that the process is now well developed and an increasing number of steel parts is being produced by cold extrusion.

The increasing use of cold extruded components makes it important to gain a knowledge of their mechanical properties and of the way in which these properties depend on such production variables as extrusion ratio and die angle. Some studies were undertaken in the 1950's and early 1960's [57, 62, 63, 66] to investigate the hardness and tensile properties of extruded steel. However, the major effort in this area has been more recent. The growing interest in the properties of the extruded product is
evident from the programme of the 4th International Cold Forging Conference held in October, 1970, in Germany. Whereas in the 3rd International Conference there had been hardly any discussion on properties, in the 4th International Conference one complete session out of five was devoted to this subject.

Plastic deformation in cold extrusion is not uniform and, as a result of this residual stresses are present in the cold extruded product. These stresses can have a considerable bearing on the success of the working process in the avoidance of difficulties connected with cracking. In addition, residual stresses can produce problems of dimensional changes in subsequent machining operations and can affect mechanical properties, particularly fatigue properties. Surface tensile stresses are generally harmful in the service life of a component and are often eliminated by annealing the component: the benefits of cold working in terms of the increased hardness and strength are thus lost. For these reasons it was considered important to incorporate an assessment of residual stresses in a test programme to evaluate the properties of extruded rods. In fact, it was found that tensile surface residual stresses could be present after extrusion. Methods of modifying the residual stress distribution by alteration of die design were therefore investigated. Investigation of residual stress distribution after stress-relief anneal was also carried out to determine a suitable temperature at which a considerable relief of residual stresses could be obtained whilst retaining most of the hardness and strength gained by cold working.
Attempt was also made to correlate the observed residual stresses with those determined analytically by the use of the Flow Function method of analysis. The analytical study of residual stresses has pointed towards the importance of such variables as friction at the die-metal interface, on residual stresses and has indicated that further yielding of the material could occur in the region of the die-land after its exit from the deformation zone, thus affecting the final distribution of residual stresses in the extruded rod.
CHAPTER 2

Residual Stresses - Causes and Effects
2.1 ORIGIN, CLASSIFICATION AND EFFECTS

2.1.1 INTRODUCTION

The subject of Residual Stresses is of relatively recent origin compared to other branches of science of mechanics of materials. It was Professor Heyn [1] who first gave a comprehensive review of this subject in his May lecture before the Institution of Metals in 1914. His pioneering work gave an impetus to the extensive research which followed in this field. For a better understanding of the effect of residual stresses on various mechanical properties, it was essential to determine more accurately the nature and magnitude of these stresses. Mesnager [2] and Sachs [3] have an important contribution in this direction. During and after World War II, engineers and scientists were confronted by the need for better materials and a better understanding of their failure mechanisms. As residual stresses are known to have a significant effect on the performance of a part in its service life, it becomes important to find out or predict residual stresses in components manufactured by different processes. The need to determine residual stresses coupled with the availability of sophisticated strain measuring devices like electrical resistance strain gauges has encouraged many to undertake such investigations. This has resulted in the development of extensive literature on the residual stress patterns in components processed by different ways. Whereas it is well recognised that the presence of residual stresses affects certain physical and mechanical properties, conflicting views exist about the actual contribution of residual stresses to these effects. A better understanding of these phenomena is certainly more
desirable, but a mere recognition of the fact that beneficial residual stresses improve mechanical properties can help the designer or the engineer to produce components which will have better service life.
Residual stresses are defined as those stresses which would exist in an elastic solid body if all external forces (external loads or thermal gradients) were removed. These are present in a body if some part of it is constrained by its surroundings into a space differing in size or shape from that which it would occupy if it were separated from the body. The constraints on different parts of the body are mutual (and are caused by the fact that the body - or parts of the body - maintain continuity after undergoing different volume or shape changes). If the dimensions of parts affected in different amounts are comparable with the specimen size, stresses arising out of their mismatch are termed macro or body stresses. The second category of stresses are called micro or structural stresses because their domain is of microscopic magnitude. Body stresses can result from mechanical, chemical or thermal operations performed on the body even if its material is perfectly homogeneous. Microstresses, on the other hand, are due to structural inhomogeneities (e.g., the grain structure of a metal) which can give rise to internal stresses even under macroscopically uniform deformation, thermal or chemical changes.

Residual Stresses due to non-uniform Volume Change

(a) VOLUME CHANGE OF CHEMICAL ORIGIN:

In the large majority of cases such stresses are produced
by chemical or physio-chemical changes propagating from the surface to the interior. When a steel part is carburised or nitrided, the iron carbides and nitrides formed in the surface layer produce a volume dilation to a depth dependent on the extent of diffusion, but generally small compared to the thickness of the specimen. A microstress distribution develops in and about each particle of compound together with a body stress distribution, compressive in the hardened skin and correspondingly tensile below the skin. The phenomena that attend the quenching of steel are complex, but it will be sufficient to say here that the surface cracking due to quenching is also a result of residual stresses produced as a result of volume change due to phase change. It is well-known that the resistance of a metal to corrosion depends very often on the size and magnitude of the volume change during the chemical reactions between the metal and surrounding corrosive medium. The products of reaction of the metal with the corrosive medium form a more or less adherent film on the metal surface. The volume of the film being larger than the metal contributing to it, the film is compressed and the metal underneath is stretched. Metal corrosion is inhibited by the surface film and the compressive stresses have a stabilising effect on it (by preventing it from rupturing under tensile stresses). The magnitude of the surface compressive stresses depends upon the size and magnitude of the volume change of the film, and so is the resistance to corrosion of metal.
(b) RESIDUAL STRESSES DUE TO THERMAL VOLUME CHANGES:

Solidification and subsequent cooling seldom proceed uniformly as to the rate and manner. The residual stress distribution attending the non-uniform cooling of a single phase casting is very well illustrated by Baldwin [5]. The outer layers of the cooling and contracting cylindrical ingot, for example, lose heat more rapidly, exerting compressive stresses on the hot and plastic interior in longitudinal, radial and circumferential directions. Under such loading the centre contracts plastically and permanently in the radial direction with a correspondingly longitudinal extrusion. (If liquid remains in the interior at this stage, it may actually be forcibly expelled towards the outside.) The further thermal contraction of the core as it cools to its final temperature is resisted by the already cold sleeve of higher yield strength. Yet, inevitably, the core shrinks and so long as it remains coherent with the sleeve, it must be distended by it, while the sleeve itself is correspondingly strained in compression. The cause of failure found in any particular form of casting may be usefully rationalised in terms of internal stresses set up during inhomogeneous shrinkage.

Residual Stresses due to non-uniform Geometry Changes

The forming operations required to convert metals to finished and semi-finished shapes rarely produce homogeneous deformation of the metal. Inhomogeneous deformation arises because of the constraint
on the flowing metal by geometrical factors associated with the tools and friction between flowing metals and tools. This nonuniform deformation results in residual stresses in the product. In surface rolling, for example, surface fibres of the sheet are cold-worked and tend to elongate while the centre of the sheet is practically unchanged. Since the sheet must remain a continuous whole, the surface and the centre of the sheet must undergo a strain accommodation. The centre fibres, therefore, tend to restrain the surface fibres from elongating, while the surface fibres seek to stretch the central fibres of the sheet. The result is a residual stress pattern in the sheet which consists of high compressive stresses at the surface and tensile residual stresses at the centre.

If an indenter is pressed into a plastic body (Figure 2.1.1) it produces a local plastic compression normal to the surface, accompanied by extension parallel to it. The tangential expansion is resisted by the surrounding material, and so tangential compressive stresses arise. When the indenter is removed, the normal compressive stress largely disappears, but much of the tangential stress remains. Repetition of the process over the surface as in shot peening produces a state of residual compression with its well-known beneficial effects upon the fatigue strength of the body. The compressive stresses in the surface layers are balanced by tensile stresses in the interior. If the body is both sufficiently ductile and large compared with the size of the individual indentation, the tensile stress is spread over a large cross-section and has no significant injurious effect; if it is not, however, it can cause internal fracture.
The effects of residual stresses can be assigned to the following fundamental attributes of residual stress state existing in a body:

(a) Coincident with the residual stress distribution there is a strain distribution related to it by the generalised form of Hook's law.

(b) Residual stresses are in internal static equilibrium. If the distribution is disturbed by outside forces, thermal relief or removal of a part of the body containing residual stresses, static equilibrium is restored on subsequent removal of the external influence.

(c) The residual stress state represents a storage of energy within a body.

Effects on Mechanical Properties

Effects of cold-work on mechanical properties are discussed separately in Chapter 3. The discussion, here, will be confined to the effects on properties when residual stresses are more significant than the structural changes (brought about, for example, by cold-work).

Effects on Properties under Static Loads

The reaction of a part to static loading can be markedly influenced by the presence of residual stresses. In a tension test,
for example, the internal stress resulting from external loading is added to any pre-existing residual stress-distribution. The total internal stress is the sum of the two components, the residual and the reaction. When, at any point in the body, an appropriate yield criterion is satisfied by this sum, plastic flow ensues. Thus, while testing in tension, a specimen having initially some significant residual tensile stress, plastic yielding will be observed at a lower value of applied load, than in their absence. The calculated yield stress may be true for the specimen, but is lower than that inherent in the material. Although the yield point is apparently reduced by residual tensile stresses, the reduction is not necessarily abruptly accomplished. With the residual stresses distributed gradually, rather than sharply, across the specimen section, yielding initiates in relatively few fibres and then extends over greater portions of the section as loading increases. The result is a gradual, rather than sharp sloping-off of the stress-strain curve. The usual determination of the elastic modulus from the slope of the straight line portion of the stress-strain curve, where Hook's law is assumed valid, is thrown in error by falling off from ideal linearity.

In studying the effects of residual stresses on FLOW STRENGTH of a material, MacGregor [6] distinguishes between two types of loads (from any source); one, which would tend to produce (if released) residual stresses of the same kind (sign and distribution) as the residual stresses already present in the body, and the other which do not. He calls the first type of load 'Homoplasic' and the second type 'Non-homoplasic'. A loading system re-applied to a body
after causing residual stresses would be homoplastic. Many cases exist, however, where different types of loads produce essentially similar states of residual stresses. For example, the state of axial, tangential and radial stresses, produced through inhomogeneous plastic flow by internal pressure in a thick-walled cylinder can be developed also by water quenching from inside, or by centrifugal casting.

Whereas the presence of residual stresses increases the flow strength of a material when the material is subjected to homoplastic loads, the effect on flow strength is not very significant when the load is non-homoplastic. In the former case, residual stresses counteract the effect of reaction stresses due to the applied load; and, therefore, the loads required to cause yielding are greater (depending upon the magnitude of residual stresses). The picture is quite different in the case of non-homoplastic loads; reaction stresses due to applied load and the residual stresses are added together and, wherever in the body, their sum exceeds yield stress of the material, yielding occurs. Due to this local yielding, residual stresses are relieved and redistributed. As the load increases, there is more relief of residual stresses (due to local plastic flow), until they are completely wiped out when the load reaches the yield strength of the material. The gross plastic yielding of the material starts only when the loads reach the yield strength. There is, thus, no appreciable change in the yield strength of the material due to the presence of residual stresses. The load carrying capacity of residually stressed I-beam, for example, is not at all affected when a non-homoplastic load is applied [7].
To separate the effects of strain-hardening and residual stress, MacGregor conducted his studies on a non-strain-hardening material. A thick-walled tube (made of the material similar to gun-metal for which the stress-strain curve is shown in Figure 2.1.2) was subjected to an internal pressure $P_0$ so that the tube partially yielded to a depth $R$ (Figure 2.1.3). The tube was unloaded and on re-applying the load, yielding did not begin until pressures had been reached which were greatly in excess of those required for initial yielding. Another example by Nadai [8] quoted here, was of a thin disc of radius '$a$', rotating with an angular velocity '$ω$' about an axis perpendicular to the plane of the disc (Figure 2.1.4). Here also no work hardening was assumed and the peripheral velocity for complete yielding of the disc was found to be about 12% higher than that for initial yielding. The increase in strength is attributed to favourable residual stresses. Similarly, in the plastic bending of a rectangular bar of an ideally plastic material, it is shown that the bending moment required to make the bar yield throughout is 50% greater than the moment for initial yielding.

**Effect on Brittle Fracture**

Most of the work done on the effect of residual stresses has been in connection with the welded structures. This is probably due to the fact that the majority of failures at very low stresses have occurred in ships and other welded structures. Different opinions have been held in the past on the contribution of residual stresses on brittle fracture and one finds authoritative statements as contradictory as the following:
locked-in stresses do not contribute materially to failure (Ship Structure Committee, 1947);

the hypothesis that residual stresses lead to failure in fabrication has been of great utility in that modified practices based on the validity of this hypothesis have been proved successful over many years of development in this field of activity [9].

Whereas the first opinion probably comes very near to the actual contribution of residual stresses, the second opinion is at least valid so far as evolution of the modified fabrication practices is concerned.

Two different concepts have generally been used to account for brittle fracture. One concept developed by Griffiths [10] and others relates cleavage fracture to pre-existing cracks. According to this, brittle cleavage fracture occurs when the normal stress needed to propagate pre-existing cracks is less than the yield stress. The critical size 'C' of existing crack according to this approach is given by Griffith-Orowan [11] equation:

\[ C = \frac{E \gamma}{\sigma_f^2} \]

where \( E \) is the Young's modulus, \( \gamma \) is the surface energy of the crack and \( \sigma_f \) is the brittle fracture stress.

The existence of the microcracks in metals of the size required by the above equation may be possible, for example, during
working operations or in the neighbourhood of microstructural constituents such as carbides in steel [12]. However, the Griffith criterion alone cannot account completely for the brittle fracture of metals and alloys. This is because of the observation made in many studies that even when metals break in a brittle manner they do show some evidence of plastic deformation in the region of the fracture (except, possibly, where the grain boundaries are very weak).

The second concept proposed by Zener [13] takes this fact into consideration, and is, therefore, well accepted. According to this, cracks which are responsible for brittle fracture are not initially present but are produced by the plastic deformation which precedes fracture. Cracks are thought to originate in localised region of high stress generated by slip dislocations or deformation twins. According to this concept, residual stresses should be dissipated at the time of plastic deformation required to nucleate the cracks, and hence should not have any contribution on brittle fracture.

In the actual in-service brittle fractures of structures which occur at very small stress levels, there has generally been no evidence of any plastic deformation. Yet the same structural steels, when loaded statically under the worst conditions of stress concentrations and at temperatures below brittle transition, have been found to have all the necessary ductility to fracture only at net average stress of yield level [14]. The different behaviour in two cases, as suggested by Mylonas [15], is due to
metallurgical changes in the steel which are brought about during fabrication or service. The metallurgical changes embrittle the steel and sufficiently reduce the original ductility for the type of stressing and constraint occurring near the notch, so as to make the large strains needed at high loads impossible and so permit low-net stress fracture.

Mylonas actually achieved brittle fracture in notched plates (pre-strained in compression) at low stress levels, the lowest being at 12% of the virgin yield. In some cases the cracks started at low loads at the root of the notch but were arrested at some distance. They frequently did not restart even at loads producing general yielding of the section. This observation is contrary to the Griffith theory which postulates an average stress at fracture inversely proportional to the square root of the crack length. This behaviour was explained to be due to the change of properties of steel in the region of the notch which was prestrained in compression. The characteristic of this change is the inability of the material to deform sufficiently without breaking: the prestrain had exhausted too much of original ductility of steel at region of notches. After the crack had gone through the embrittled material around the notch root and entered more ductile steel, it would stop unless it had picked up enough velocity to be able to propagate at the existing low stress level.

Summarising the effect of residual stresses on brittle fracture, Mylonas states that initiation of brittle fracture on steel occurs due to the reduction in original ductility which depends on
the strain history of the material at the points of stress concentration. The controlling factor is the relative magnitude of elastic strains produced by residual stresses as compared with the plastic deformation preceding fracture. If the material is ductile enough, the residual stresses will be wiped out. If it is not ductile enough, the residual stresses will contribute to the fracture, but not to any important extent; if they were absent, stresses equal to their magnitude could be produced by an additional local straining of the order of 0.001 (maximum elastic strain produced by residual stresses in the steel considered), and fracture could start with an external load only little higher than when they were present. Thus, whether the material yields sufficiently to wipe out the residual stresses, though not enough to allow the attainment of nett stress of yield level, or the material does not yield enough even to wipe out the residual stresses, the basic cause is insufficient ductility at the root of the notch and is indicated by a low average nett fracture stress. The significant factor is the plastic strain history and aging in the region of notch and the effect it has on the ductility of steel. Consideration should also be given to the size of the field of residual stresses. The same ductility which is usually sufficient to wipe out the usual localised residual stresses may be incapable of preventing low stress fracture in the presence of large fields of reaction stresses.

Fatigue

Under static loads, the effect of residual stresses depends on whether or not the residual stress sign and distribution is identical with that which would be produced by an external load (if
released). Under repeated loading, the sign of surface residual stresses is most significant. This is due to the fact that in fatigue the loads applied are such that large plastic flow does not occur and while the residual stresses may be changed gradually, they can still be effective during the life of a part.

The role of residual stresses in fatigue will be further discussed in Chapter 3.

**Other Effects of Residual Stresses**

**1. DIMENSIONAL STABILITY:**

Relief of residual stresses, thermal or otherwise, constitutes a redistribution tending to upset the static equilibrium, which then must be continuously established. When relaxation occurs under annealing or on removal of a part of the stressed material, flow is more rapid under regions of higher stress, and the stresses are correspondingly redistributed. Inequality of stress relaxation results in distortion, for the required maintenance of equilibrium is accomplished by dimensional changes to neutralise the forces, and bending to neutralise the moments, generated by disturbance of equilibrium. This results in upsetting the dimensional stability, which is one of the most irritating manifestations of residual stresses.

Distortion can also cause secondary effects. A hole drilled into a stressed material may close slightly as a result of release of residual stresses. This may cause the snapping-off of the
drills and reamers as a consequence of binding.

2. BREAKING OF METALS BECAUSE OF STRESS RELAXATION:

Due to removal of a part of a stressed brittle material, the change in the residual stresses may, in some cases, be so great as to raise the strain above the fracture strength of a material at some point, resulting in failure. Figure 2.1.5 shows a split caused by milling a notch in the Charpy bar of a high strength brittle steel [16]. Instead, cracking may be followed by a complete fracture of the material after a time interval. Examples of such failure include the delayed explosion of steel ball bearings upon removal from a tumbling barrel; the spontaneous cracking of quenched and tempered steel helmets weeks after being hit by test bullets, etc.

Effects on Corrosion:

(The process which is responsible for metal attack or corrosion in aqueous solutions is usually electrochemical in nature, i.e., chemical reaction is accompanied by the passage of electric current. For this to occur, a potential difference must exist between one part of the structure and another. In the case of a residually stressed metal, the potential difference is provided by regions of unequal stress [17]. The more stressed parts are usually anodic and corrode more readily. This is evidenced in the etching characteristics of metallographic specimens. Sites of locally higher strain energy, grain and twin boundaries, and deformation markings are preferentially attacked.)
In an environment with which a metal may react, it does so with the formation of products which may be more or less adherent to the metal surface and which necessarily have larger volume than the metal contributing to them. This corresponds to an inhomogeneous expansion of the metal surface with consequent generation of residual stresses in both film and underlying metal. If the film be adherent and strong, it may impede further reaction; this may account for the passivity of such metals as aluminium after developing an oxide film. Such developed stresses are additive to pre-existing residual stresses. Presence of the latter in sufficient magnitude may, therefore, accelerate corrosion by providing the extra force necessary to continuously break down an otherwise protective film.

Often precipitation in grain boundaries renders them more susceptible to corrosion. Deep etching there may have two consequences: generation of cracks to act as stress raisers, the opening of which will admit more corrodant; or the formation of expanding wedges of corrosion products, a source of high internal stress. The existing stresses, applied or residual, may aid the propagation of the cracks or add to the corrosion stresses at the apices of the chemical wedges. Corrosion, accelerated by stress, thus leads to accelerated failure by both stress and corrosion. Such failure, rapid and catastrophic, is STRESS CORROSION CRACKING. As might be expected, in some instances at least, it has been established that stress-corrosion cracking is favoured by residual tensile stress and suppressed by the residual compressive stress [18].
In almost all cases of stress-corrosion cracking, failure is intergranular. (Chromium-nickel stainless steels exposed to chlorides, and magnesium alloys are exceptions, as are some of the failures of brass in ammonical atmospheres.) It occurs only when the grain boundaries are more subject to attack than interiors, and under agents preferentially attacking boundaries, and leaving the rest passive. Transgranular cracking may be produced in almost any material and with almost any corrodant under the fatigue conditions of cyclic loading [19]. Here the most rapid path of attack is presumed along the worked material on operative slip planes. This failure, termed as CORROSION FATIGUE, is probably an instance where corrosion augments the damage of fatigue and fatigue augments the damage of corrosion. Stress relief annealing or introduction of compressive residual surface stresses may significantly improve the resistance of a metal to failure by 'corrosion fatigue' and 'stress corrosion cracking'.
2.2 RESIDUAL STRESSES DUE TO COLD WORKING

2.2.1 RESIDUAL STRESS PATTERNS

With the exception of shapes which are cast to their final useful form, metal requires further fabrication to attain utility. Fabrication usually means plastic deformation. Even in processes like machining, grinding and stamping which are generally not classified as fabrication processes, some plastic deformation is involved. As plastic deformation is seldom applied in an ideally homogeneous manner, the fabricated product is seldom devoid of residual stresses. A vast literature exists on residual stress state arising from various fabrication processes. Residual stresses in cold extruded parts, however, have received relatively little attention, probably because of the more recent origin of this process. However, as the mechanism by which residual stresses are produced is similar in most cold working processes, a brief general review will be made here of the interrelation between residual stresses and coldwork.

**Rolling:**

It is frequently difficult to predict the distribution of residual stresses in a cold worked metal since the flow of metal in cold working operations varies considerably depending upon the conditions of operations. Baker, Ricksecker and Baldwin [20] conducted comprehensive studies of residual stress distributions found in rolled bearing bronze after various schedules in one mill; the salient discussion has been summarised by Baldwin [5]. The distribution of plastic strain through the thickness being rolled is a function
of the ratio of strip thickness to the length of contact in the rolling direction between the roll and the strip, and therefore, of the ratio of strip thickness to roll diameter. If this ratio is large plastic deformation is concentrated in the surface zone. If this ratio is small, deformation extends throughout the thickness of strip. In the former case, which may occur in surface rolling, the surface concentration of deformation tends to elongate the affected layers in the rolling direction by a greater amount than the central zone. The surface layers, therefore, exert a residual tensile stress on the central layers and are themselves constrained in longitudinal compression by the centre.

In the second case, when deformation extends through the strip thickness, tensile stresses are commonly found at the surface. One qualitative explanation that has been offered [21] suggests that in this instance frictional forces between rolls and strip surfaces restrain the elongation of the surface layers while the central zone, in effect, is extruded between them to a greater elongation. Then in the emergent material the centre stretches the surface in tension and is in turn contracted in compression.

*Cold Drawing, Extrusion*

Not dissimilar considerations apply to stresses found in drawn rod and wire. Longitudinal stresses are found to be compressive in surface zones of rods reduced by small amounts in drawing. It has been pointed out [5] that the explanation cannot be as simple as this, in view of the simultaneous existence of compressive tangential stresses at the surface. Radial stresses are correspondingly tensile at the core, necessarily vanishing at the free surface. This stress pattern is, thus, similar to that of a quenched bar.
If a shallow depth of plastic deformation is responsible, this must be presumed to have triaxially distended the case compared to the core, i.e., to have made the deformed outer cylinder not only relatively larger than the core, but also relatively thinner, and of relatively greater circumference. The deformation suggested here is a radial compression of the case, attended by an extension, more or less uniform, in the longitudinal and tangential directions—a mechanism somewhat easier to visualise in cases of swaging and shot peening.

In the limited literature available on the residual stress patterns in extruded bars, there exists conflicting evidence as to the distribution of these stresses across the cross-section of the rod specimen. In a study of residual stresses in 1.5 inch (38mm) diameter aluminium rod extruded from 4.3 inch (109.4mm) diameter billet, FRISCH and THOMSEN [22] found compressive longitudinal stresses at the surface. MEYER [23], based on Frisch and Thomsen's work, has indicated a similar residual stress pattern in extruded rods. A completely different distribution, but similar to that in drawn rods and tubes, has been suggested by FIORENTINO et alia [24] in rods extruded through conventional dies. The existence of surface longitudinal and circumferential cracks in extruded rods, in some cases [25], suggests that the surface residual stresses are tensile and not compressive (in agreement with the distribution postulated by Fiorentino et alia.). This distribution of residual stresses, i.e. tensile on the surface and compressive in the central layers, has also been found in the recent experimental studies of residual stresses in hydrostatically extruded copper, steel and aluminium rods by OSAKADA et alia [4] and MIURA et alia [33].
2.2.2 EFFECT OF THE AMOUNT OF REDUCTION ON RESIDUAL STRESSES:

(a) Drawn Rod and Wire

For very small reductions, when plastic deformation is confined to outer fibres of the material, the residual stresses are generally compressive on the surface and tensile in the central fibres. This type of pattern was observed in steel rods drawn by very light amounts (less than 1.0% reduction in area) by Bühler and Buchholtz [26] as shown in Figure 2.2.1. In the second case when plastic deformation penetrates through the cross-section, residual stresses are tensile at the surface and compressive in the central layers. Stresses at surface increase with increasing amounts of deformations (total deformation being achieved in single reduction) up to a certain limit and then start decreasing if the deformation or percentage reduction increases beyond that. This pattern in drawn wire is indicated in the approximate data of Linicus and Sachs [27] and is shown in Figure 2.2.2.

Drawn and Sunk Tubes

The nature and magnitude of surface residual stresses in drawn and sunk tubes depend on the difference in the percentage reductions of bore and outside diameter (O.D.). This difference (i.e. % reduction bore - % reduction O.D.), is termed by Meadows [28] as STRAIN DISPARITY. In the case of sunk tubes where the strain disparity is generally positive for commonly employed reductions, the residual stresses are tensile at the surface and compressive at the bore. With the increase in the percentage
reduction of O.D. the strain disparity increases and so does the magnitude of surface residual stresses. Meadows' [28] data indicates this trend, Figure 2.2.3. In Figure 2.2.4, the maximum tensile residual stresses corresponding to various amounts of O.D. reductions, are also plotted against strain disparity at those reductions, establishing that, for sunk tubes, strain disparity is a function of percentage reduction of outside diameter. Crampton's [29] approximate data on sunk tubes shows that the magnitude of circumferential stresses at the outer tube surface increases with increasing reductions in O.D., almost duplicating the yield stress of the alloy studied, (Figure 2.2.5). If an analogous behaviour between wire drawing and tube sinking exists, these trends would not be expected to continue at higher reductions. Exploratory tests by Spear [30] confirm this as shown in Figure 2.2.6.

In the case of plug-drawn tubes, the strain disparity can be positive or negative (depending on the diameter of plug used), irrespective of the amount of percentage reduction of outside diameter or of the area. It is, therefore, best to relate the magnitude and nature of residual stresses to the strain disparity for plug-drawn tubes. The surface residual stresses are tensile if the strain disparity is positive and compressive if it is negative. Figure 2.2.7 shows the tangential stress distribution for 70:30 brass tubes drawn with 30% reduction in area to various sizes [28]. These sizes, together with the reduction in the outside and bore diameter, are listed in Table 2.2.1. The relationship between strain disparity and maximum
tensile stresses is shown in Figure 2.2.8. It is evident from this that the minimum in the residual stress curve exists about a point where the strain disparity is zero, i.e. where inside and outside diameters are reduced in the same proportion.

In the data quoted above for sunk- and plug-drawn tubes, residual stresses are maximum at about 0.005" (~1.27mm) below the outer surface. Sachs and Epsey [31] have, similarly, reported a sharp maximum at about a quarter of the way in from the surface for tubes sunk to three different diameters. It is suggested by Baldwin [5] that the maximum is caused by bending action which the tube wall suffers as it quits the die.

*Hydrostatically Extruded Rod*

The effects of the reduction in area on the residual stress patterns in hydrostatically extruded rods have been studied by Osakada et alia [4] and Miuro et alia [33]. The results reported in their work indicate that peak tensile longitudinal stresses near the surface decrease with the increase of the extrusion ratio from 1.25 to 3.7. The compressive residual stresses in the centre of the rods increase first, when the extrusion ratio is increased from 1.25 to about 2.25. Further increase in extrusion ratio to 3.7 makes these stresses less compressive.
2.2.3 EFFECT OF DIE ANGLE

Die angle is an important variable in a metal-working process in that the stress distributions in the deformation zone are influenced to a great extent by the die angle. As the magnitude and nature of the residual stresses in a metal part depend upon the stress distribution existing in the metal at the die opening (just before the metal comes out of the die), die angle influences the distribution of residual stress as well. Not much work has been done on the effect of die angle on residual stresses. Linicu and Sachs [27], however, studied this effect and showed that the longitudinal residual stress at the wire surface was greater for a given reduction, the greater the angle of the die (Figure 2.2.2).

Similar effect of the die angle has been reported on the surface longitudinal residual stresses in hydrostatically extruded steel and aluminium rods [33].
2.3 RELIEF AND REDISTRIBUTION OF RESIDUAL STRESSES

2.3.1 INTRODUCTION

In the general case any change in the residual stresses in a metal may be termed redistribution. The term 'inducement' is commonly used when new patterns of stresses are introduced, whereas "relief" generally implies the reduction in the magnitude of stresses already existing within the part. An example of each case is shown in Figure 2.3.1.

The importance of this subject stems from the fact that the effects of residual stresses are frequently important. Broadly classified, these effects are associated with dimensional stability, static and dynamic mechanical properties, e.g., fatigue, metallurgical, chemical, electrical and magnetic properties. Many investigations in this subject have been prompted by a need for improving dimensional stability, minimising stress corrosion cracking tendencies, and improving fatigue properties.

Residual stresses are due to mismatching of various regions of a part. To relieve these stresses the mismatching regions must be altered in such a way that they become more compatible. Inducement on the other hand involves changing the nature of the mismatch. Both these redistributions are achieved by plastic deformation; relief by thermal action and inducement by mechanical action.
2.3.2 RELIEF OF RESIDUAL STRESSES BY LOW TEMPERATURE ANNEALING

Low temperature annealing or stress relief annealing, as it is sometimes called, is heat-treatment such that the residual stress level is reduced through a suitable combination of heat treatment temperature and holding time. It is a rate process in which the atoms tend to move into more stable locations associated with lower strain energy. Increasing the temperature serves to activate the atoms and increases the probability of a change to a more stable state; hence, the rate at which stresses are relieved is temperature dependent.

Looking at it from continuum approach, stress relieving operations involve the introduction of further inhomogeneous deformation, which counteracts those inhomogeneities that are responsible for the residual stress in the first place. This is achieved by heating the component and thus causing the material to creep under the influence of residual stresses. Regions of high tensile residual stresses will elongate if the temperature is raised sufficiently to lower the yield stress below the level of the internal stress. Since such regions are in a state of tension because their original deformation is less than that in the neighbouring regions, the plastic elongation reverses the initial inhomogeneities (depending upon the temperature), and thus the level of residual stresses is reduced. This reversal in inhomogeneity, if permitted to go to completion by holding for an adequate time at a temperature where the yield stress is very small, enables the residual stresses to be reduced to a very low level.
2.3.3 STRESS RELIEF VERSUS STRENGTH

Whereas it is, sometimes, desirable to relieve residual stresses in a cold worked metal for reasons mentioned earlier, complete relief of residual stresses can only be achieved by sacrificing the improved strength properties acquired as a result of cold working. These two factors are often taken into consideration when selecting a stress relief temperature, i.e. a temperature is selected where a desirable reduction in the residual stresses is achieved at the same time not losing considerable hardness and strength.

When a metal is cold worked, vast numbers of dislocations and point defects are generated which are responsible for the higher strength of cold worked metal (this is further discussed in Chapter 3). The strain field of these dislocations give rise to the elastic energy which constitutes the stored energy in a cold worked metal. Various processes by which annealed state is reached from a deformed state are included under the general terms 'recovery' and 'recrystallisation'. The term recovery has been used to cover processes which do not result in the replacement of deformed grains by new grains, but nevertheless, lead to structural changes on a fine scale within the existing grains. Recrystallisation, on the other hand, results in the absorption of strained and fragmented grains by new equi-axed strain-free grains. After the new grains are formed, they start growing in size if kept at the same temperature. This is termed as grain growth. These stages - recovery, recrystallisation, and grain growth, are shown schematically in Figure 2.3.2.
The driving force for recovery and recrystallisation is the reduction in strain energy achieved by the removal of excess point defects and dislocations. The strain energy is released in the form of heat. Thus, if two specimens, one deformed and the other annealed, are heated in two separate furnaces, the power necessary to maintain the same rise of temperature is less in the case of deformed specimen than that required for annealed specimen. The power difference $\Delta P$ is proportioned to the released energy and is plotted against temperature to provide an overall view of the processes occurring during annealing \[34\]. A typical result for deformed polycrystalline copper of commercial purity quoted by Honeycombe \[34\] from a study of Clarebrough et al \[35\] is shown in Figure 2.3.3. Release of energy starts at about 70°C and occurs steadily until a sharp peak is reached just below 400°C, which can be shown by metallographic and X-ray methods to correspond to the "softening" of a new grain structure, i.e., recrystallisation. The whole of the energy released below 350°C is due to recovery and results from processes taking place in the original deformed grains. The same figure also shows the changes in hardness, density and electrical resistivity which all alter markedly in the recrystallisation range, but also undergo smaller changes at lower temperatures. The changes in resistivity and density are more significant as compared to hardness changes, below the crystallisation temperature. Electron microscope examination of copper recovered in the range 100°C - 350°C indicated that the as-deformed dislocation density had changed very little. Similar results are reported in Figure 2.3.4 \[36\].

In the recovery stage therefore, properties that are most affected are those which are sensitive to point defects (e.g., electrical
conductivity, density, etc). It has also been found that lattice strain, as measured with X-rays, is appreciably reduced during recovery [36]. The strength properties which are mainly controlled by dislocations, are not significantly affected at recovery temperatures. Figures 2.3.5 (a) and (b) show the residual stress distributions in an as-cold drawn bar and after a stress relief treatment at a temperature below the recrystallisation temperature. It was found that there was not any appreciable loss in strength after the stress relief operation [37].

Unless the cold worked part is to be reworked, when it would be desirable to anneal it to regain the ductility, stress relief operations are often carried out at temperatures in the recovery range. At these temperatures the intensity of residual stress is reduced to a level where their harmful effects are appreciably reduced, and at the same time there is not a significant loss in the strength properties achieved through cold working.
It is not always possible to eliminate residual stress by heating. For example, where stress precipitation cracking is a factor, any attempt to stress relieve the metal thermally brings on the very precipitation that causes cracking.

In other instances mere reduction of stress does not give the maximum benefits. Tensile stresses may be detrimental to fatigue life; relief of them may increase fatigue life, but substituting compressive stresses for tensile stresses will double the benefits gained by mere relief. For these reasons it may be desirable to relieve or change residual stresses by mechanical methods. There are basically two different ways in which residual stresses are redistributed by mechanical action - redistribution due to removal of stressed metal; and redistribution due to plastic deformation caused by external forces. Warpage during machining can be explained by the first type of redistribution, and this is also the basis of stress measurement by Sachs boring, and several other mechanical methods. The second type of redistribution, which is also called inducement, will be of interest here. This requires stressing into the plastic range. The sum of the applied stress and the residual stress at some point within the part must exceed the yield stress of the material. The irrecoverable deformation produced by such stressing changes the nature of mismatch in various regions and a redistribution of stress occurs. The stress pattern may be changed in a variety of ways in this case, since the plastic deformation may involve certain portions of the part and it may vary in degrees. The stressing sufficient to cause plastic flow may be localised to areas of stress
concentration such as exist at the edge of a hole or at the root of a notch. In other cases only the surface of the part may be involved and in still other cases, the plastic flow may be rather general — involving perhaps the entire cross-section of the part. In all cases the effect is likely to be more or less non-uniform due to heterogeneity of plastic flow process itself.

It has been found that the final step of the sequence of deformation operations largely determines the ultimate residual stress distribution. Baldwin [5] verified this fact in an experiment on strip rolling and found that the ultimate residual stress distribution was the same in these two cases: whether a 2% final reduction following a single pass of 16% reduction, or a series of 2% passes to the same total reduction (18%). This effect, coupled with the fact that small amounts of plastic deformations introduce compressive residual surface stress, has been utilised in modifying the surface stress patterns to the beneficial ones. Trueting [38] demonstrated this effect by shot peening a strip previously rolled up to 37% reduction; the resultant stress pattern is shown in Figure 2.3.6. The surface stresses after shot peening the rolled strip are of the same nature and magnitude as the stresses produced by shot peening alone. In the interior the stress is more nearly additive.

(It has been shown by Bühler and Schultz [39] that the residual stress pattern in drawn bar can be beneficially modified by a subsequent drawing operation in which a small reduction (generally 1–2% reduction in area) is effected. This principle of small reductions introducing compressive surface stresses in incorporated in the die design by several investigators. Longitudinal cracking of
steel bar and wire has been prevented by using two tandem dies with the second die used for very small second reduction. Bühler [40] obtained patent in this area in 1934-35. Extensive work on tandem dies at Battelle in the late 1940s resulted in a U.S. patent by Sims and Landis [41]. Misra and Polakowski [42] have also applied this method to the in-process control of residual stresses in drawn tubing. Fiorentio et al [24] have successfully used a double reduction die (with second reduction of the order of 2%) to restrict the circumferential cracking in the hydrostatic extrusion of brittle materials. They attribute the avoidance of cracking to the axial compressive stresses which are exerted by the bottom land when the extruded part is in between the top and bottom land and due to modification of residual stresses by the second small reduction, when the extruded bar comes out of the die.
Fig. 2.1.1 Generation of surface compression by plastic indentation.

Fig. 2.1.2 Stress-strain curve for a typical gun-steel
Fig. 2.1.3 Partially yielded tube subjected to internal pressure $P_0$.

Fig. 2.1.4 Thin solid rotating disc

Fig. 2.1.5

(a) Crack Frequently Found at Root of Notch Stilled in Brittle Metals Having Residual Stresses.
(b) Residual stress in the original blank causes cracking when the notch is milled.
(c) Removal of metal at bottom of blank removes the tensile force. Remaining stress pattern changes from dashed lines to solid lines and sends stress over fracture stress at root of notch.
## Table 2.2.1

Die and Plug Sizes for 30% Reduction in Area of 1 × 0.056-in. Tube and the Associated Strain Disparities of Outside and Bore Diameters

<table>
<thead>
<tr>
<th>Die Size, in.</th>
<th>Plug Size, in.</th>
<th>Wall Thickness, in.</th>
<th>% Redn. in Thickness</th>
<th>% Redn. in External Dia.</th>
<th>% Redn. in Bore</th>
<th>Strain Disparity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.712</td>
<td>0.600</td>
<td>0.056</td>
<td>0</td>
<td>28.9</td>
<td>32.4</td>
<td>3.6</td>
</tr>
<tr>
<td>0.715</td>
<td>0.627</td>
<td>0.054</td>
<td>3.6</td>
<td>23.5</td>
<td>29.4</td>
<td>2.9</td>
</tr>
<tr>
<td>0.759</td>
<td>0.655</td>
<td>0.052</td>
<td>7.1</td>
<td>21.1</td>
<td>26.2</td>
<td>2.1</td>
</tr>
<tr>
<td>0.785</td>
<td>0.685</td>
<td>0.050</td>
<td>10.7</td>
<td>21.0</td>
<td>22.9</td>
<td>1.4</td>
</tr>
<tr>
<td>0.814</td>
<td>0.713</td>
<td>0.048</td>
<td>14.3</td>
<td>13.6</td>
<td>19.1</td>
<td>0.5</td>
</tr>
<tr>
<td>0.845</td>
<td>0.753</td>
<td>0.046</td>
<td>17.5</td>
<td>15.2</td>
<td>15.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.879</td>
<td>0.791</td>
<td>0.044</td>
<td>22.4</td>
<td>12.1</td>
<td>10.9</td>
<td>1.2</td>
</tr>
<tr>
<td>0.959</td>
<td>0.879</td>
<td>0.040</td>
<td>28.6</td>
<td>4.1</td>
<td>1.0</td>
<td>3.1</td>
</tr>
</tbody>
</table>
Fig. 2.2.1

Longitudinal and Tangential Residual Stresses in Steel Rod Drawn Eight Amounts

Approximate Longitudinal Residual Stress at the Surface of the Rod in Cold Drawn Brass Wire, as a Function of Reduction in Area (W. Linicus and G. Secia, 1932)

Fig. 2.2.2

Distribution of residual stress in tube sunk from 1 x 0.056 in. to:
(a) 0.850 in.
(b) 0.825 in.
(c) 0.775 in.
(d) 0.883 in.
(e) 0.750 in.
(f) 0.725 in.
(g) 0.807 in.
(h) 0.781 in.

Fig. 2.2.3
Relationship between residual stress and strain disparity for tubes sunk from 1 x 0.056 in.

Fig. 2.2.4

Approximated Residual Hoop Stress at Outer Surface of Sunk Tube as a Function of Reduction in Diameter. Reductions were effected in a single pass.

Fig. 2.2.5
Distributior of residual stress in tubes plug-drawn from 1 X 0.056 in. to the sizes shown: 30% reduction in area.

(a) 0.959 X 0.040 in.  
(b) 0.859 X 0.044 in.  
(c) 0.959 X 0.040 in.  
(d) 0.859 X 0.044 in.  
(e) 0.814 X 0.048 in.  
(f) 0.814 X 0.048 in.  
(g) 0.785 X 0.050 in.  
(h) 0.785 X 0.050 in.

Fig. 2.2.7

Relationship between residual stress and strain disparity for tubes plug-drawn from 1 X 0.056 in.; 30% reduction in area.

Fig. 2.2.8
Fig. 2.3.1

Schematic drawing indicating recovery, recrystallization, and grain growth and the chief property changes in each region.

Fig. 2.3.2
Release of stored energy, plotted as power difference $\Delta P$, from commercial copper deformed 33 per cent elongation in tension at room temperature. Changes in hardness (V.H.N.), resistivity ($R$) and density ($D$) are also shown. (After Chrisnook and Hargreaves, 1963, Recovery and Recrystallization of Metals (2.1.1.1.2). Edited by L. Himmel, Interscience, New York and London.)

The property changes in 95 per cent cold-worked iron with heating temperatures (1 hr). The temperature intervals: $A \rightarrow B$, stress relief; $B \rightarrow C$, recrystallization; and $C \rightarrow D$, grain growth, signify the important phenomena occurring.

Residual stress. (a) In a cold-drawn aged bar $1\frac{1}{2}$ in. in diameter 20 per cent cold drawn, 0.45C steel. (b) After stress-relieving bar.
Fig. 2.3.6
CHAPTER 3

Effect of Cold Work on Mechanical Properties
3.1 INTRODUCTION

It is usual to distinguish between physical and mechanical properties of metals. The characteristics that are used to describe the material under conditions that exclude any external force are called physical properties. These properties like density, specific heat, coefficient of thermal expansion, etc. are generally not sensitive to microstructure (or simply structure) of the metal. (There are a few physical properties, however, such as electrical conductivity, magnetic behaviour which are structure-sensitive). The mechanical properties, on the other hand, are characteristics of the materials that describe their behaviour under various and varying external conditions of forces. These properties, such as tensile strength, yield strength, hardness, ductility, fatigue strength, etc., are mainly governed by the structure of the metal.

The structure of a given metal or alloy is generally altered by the various processing operations which are required to transform it to a semi-finished or finished product. With the change in structure, the mechanical properties are also changed. Thus, quoting English and Backofen [43] structure may be taken as the "memory" of the material within which the conditions of processing history are recorded and preserved.

A cold worked metal is characterised by the presence of vast numbers of dislocations (dislocation populations between $10^3 - 10^{12}$/cm$^2$ as compared to $10^6 - 10^8$/cm$^2$ in an annealed metal), generated during cold working; point defects are also generated. Many property changes brought about by cold working can be attributed to the
dislocation interactions and to the point defects. Since in most cold working processes one or two dimensions of the metal are reduced at the expense of an increase in the other dimensions, cold working produces elongation of the grains in the direction of working. Severe deformation produces a reorientation of the crystallographic axes of the grains into preferred orientation and alignment of inclusions in the main direction of working (mechanical fibering). Another effect of cold work is that it may lead to the generation of structural damage in the metal, and thus influence the response of the metal to external forces. In what follows, these various effects of cold work will be discussed with respect to the particular properties affected by them.
3.2.1 STRAIN HARDENING

An important characteristic of the cold working of metals is that it increases the hardness and strength of the metal as a result of strain hardening and decreases its ductility. The relative changes in these properties depend upon the amount of cold work. Several mechanisms for strain hardening have been proposed. A common factor in these mechanisms is the presence of large numbers of dislocations which are produced as a result of cold working. The proposed mechanisms differ in the way the large number of dislocations are thought to cause strain hardening. According to one class of mechanisms these dislocations interact with each other and other structural defects, i.e. inclusions, grain boundaries etc. They would eventually be stopped in their motion unless the applied stress is continually increased to overcome the rising tide of interactions. That is, each additional amount of plastic deformation requires an additional increment of stress.

In the second type of mechanism, plastic deformation (which generates a large number of dislocations) is thought to be accompanied by dynamic recovery \[46\] resulting in the grouping of the dislocations into cells which, in turn, form the cellular substructure. Work done by Gay et. al. \[47\] and Embury and Fisher \[45\] appears to consolidate this hypothesis. Embury et. al. also show in their work on some ferrous materials that the cellular substructure developed during the initial stages of cold working is refined further with the increase of plastic deformation. Fig. 3.2.1 shows a relationship between
drawing strain and substructural cell diameter for ferrovac E (a commercially pure iron), copper and an eutectoid steel. The compositions of these materials and other materials used in the above work is given in Table 3.2.1. Quantitative measurement of the flow stress and the scale of substructure indicated that the strength of heavily worked materials was largely determined by the spacing of the cell walls (Fig. 3.2.2). This suggests an analogy with the Hall-Petch relationship, widely applied to the variation of flow stress with the grain size in polycrystals. Gay et. al. [47] also found a similar relationship between the scale of the structure and flow stress.

Fisher et. al. also observed that the rate of dynamic recovery in the case of b.c.c. metals was much smaller than in f.c.c. metals. This is indicated in Figure 3.2.1 where the firm line for copper shows a positive deviation from the dashed line (the dashed lines represent the case where the initial cell size shown by the first data point is plotted against strain). This difference is suggested by the authors to be due to the presence of interstitials in the b.c.c. materials which provide a stable locking of the barriers. Similar results regarding the substructure size and its relationship to flow stress have been reported in the case of aluminium and aluminium-aluminium oxide drawn products [50].

3.2.2 STRUCTURAL DAMAGE IN COLD WORKING

In the previous section the microstructural changes responsible for strengthening the metal were discussed. Cold work also brings about structural changes which affect the deformation capacity of the materials in terms of its strain to fracture.
Coffin and Rogers [52] define structural damage as the microstructural changes occurring during deformation processes that lead to degradation of mechanical properties. More specifically, it can be described in terms of void formation at inclusions or at grain boundaries, particularly at triple points or wherever the metal locally has great difficulty in conforming to the uniform deformation requirements of the bulk. The non-uniform deformation generates triaxial state of tension which then influences the tendency for damage (void formation). Such voids are generally nucleated at rather small strains, continued deformation leading to void growth, additional void nucleation and coalescence to form crack. Propagation of this crack results in ductile fracture.

Several instances have been reported where the evidence of structural damage was found in cold formed products. Coffin and Rogers [52] studied this effect in drawn sheets of cold rolled copper, an aluminium alloy (1.0% Mg, 0.6% Si, 0.25% Cr, balance Al) and of \( \alpha - \beta \) brass (60% Cu, 40% Zn). They found the structural damage mostly on the midplanes of the sheets (where the hydrostatic stress was highly tensile) and the damage increased with the increasing reduction in thickness (and hence increasing hydrostatic tensile stress). In the sheet drawn with the same reduction but with different die angles, damage increased with increasing die angle.

Evidence of structural damage is also found in the studies of Latham and Cockroft [53], undertaken at the National Engineering Laboratories. Once the microstructural damage starts in a metal during plastic deformation it may progressively lead to macroscopic cracks with increasing deformation [54]. Thus, the damage produced in metal
during a working process controls the workability of the metal, which is defined by Latham and Cockroft to be the maximum strain that can be imposed on a metal during a mechanical working operation without cracks being formed. The damage is a function of the hydrostatic stress, the sign and magnitude of which depends upon the applied stress, the nature of the mechanical working operation and different variables (e.g. die angle, lubrication, etc.) involved in that operation. Workability thus is controlled to a great extent by the stress system.

Whereas in most metal working operations the tendency of the metal to crack was found to be increasing with increasing strain, in cold extrusion, sound products were obtained both at low strains and high strains and there was a limited range of intermediate strains where cracking occurred [54]. This behaviour, which is difficult to explain on the basis of simple maximum stress or maximum strain criteria of fractures, follows easily from a criterion proposed by Latham and Cockroft based on the combination of stress and strain. According to this criterion, fracture occurs in a material when

$$\int_0^{\epsilon_f} \frac{\sigma^*}{\sigma} \left( \frac{\sigma^*}{\sigma} \right) d\epsilon = C^*$$

(3.2.1)

where $\sigma$ and $\epsilon$ are the effective stress and effective strain,

$\sigma^*$ = local maximum principal tensile stress,

$\epsilon_f$ = equivalent strain to fracture, and

$C^*$ is a constant for the given material at a given temperature, strain rate, etc.
At smaller extrusion ratios a tensile stress is set up at the axis in the middle of the deformation zone. This stress decreases and eventually becomes compressive as the extrusion ratio is increased. Small extrusion ratios, therefore, correspond to high stress, but small strains and large extrusion ratios correspond to high strains but small (or compressive) stresses. In both cases the value of the left-hand side of equation (3.2.1) is less than $C^*$ and therefore sound products are obtained. In the intermediate range both the stress and the strain are moderately large, making the left-hand side of equation (3.2.1) greater than $C^*$ and, therefore, cracking occurs.

In addition to the effect of stress system and the local strain, the amount of structural damage is strongly dependent upon the structure of the metal. Coffin and Rogers [52] found that the tendency for void formation was less in the case of (purer) OFHC copper than in (relatively less pure) tough pitch copper, when both were drawn under the same conditions. In steels, pearlite spacing and pearlite orientation have a major effect; coarse pearlite is easily damaged. Finer perlites and, particularly, spheroidal structures are much less susceptible to damage [54].

3.2.3 PREVIOUS WORK ON THE TENSILE PROPERTIES OF EXTRUDED PRODUCTS

Much work has been done in recent years on various aspects of cold extrusion of steel at the National Engineering Laboratory, East Kilbride, Glasgow. One of the areas of study has been the effect of process variables like extrusion ratio and die angle on
the properties of extruded product. Cruden [60] has studied the
strength characteristics of extruded cans and tubes of EN2A steel.
These were subjected to internal pressures and bursting strengths
of extruded cans were compared with those of the cans of similar
dimensions machined from EN2A and some low alloy steels heat-treated
to different hardness values. The bursting strength of the extruded
cans was found to be twice that of the similar cans machined from
the same material and was comparable to the bursting strength of
cans machined from low alloy steels. Extruded tubes also showed
improvement in strength. The gain in strength increased with the
increase in extrusion ratio, but there appeared to be a limiting
value of the reduction in area at which the improvement in strength
was maximum and after which it started to decrease. In a separate
investigation [61] Cruden studied the effect of prior cold work
(by extrusion) and the die angle on the workability of steel in
subsequent extrusion operations. Whereas a combination of higher
extrusion ratios for both prior and subsequent extrusion operation
gave sounder products than when the extrusion ratios for these two
operations were smaller, the die angle also had a significant effect
on the soundness of the products. Lower die angles gave sounder
products than those obtained with higher die angles. Figure 3.2.3
shows this effect. $R_B$ and $R_F$, here are extrusion ratios in the prior
and subsequent extrusion operations (can and tube extrusion).

A study at the Heintz Manufacturing Company, U.S.A.,
provides useful information on the tensile properties and hardness
of some steels (0.08% to 0.38% C) after cold extrusion. The effect
of stress relieving at some temperatures, on the properties of
extruded steels is also given. The detailed results of this study
have been summarised by Feldman [62]. These data indicate an increasing trend in strength properties with increasing reduction in area (up to 80 per cent, the maximum studied in this work).

Work on various aspects of cold extrusion of steel has also been done at Production Engineering Research Association (PERA) [63]. Figure 3.2.4 (a) and (b) depicts, from one of the studies undertaken there, the effects of extrusion ratio and die angle on tensile properties. In Figure 3.2.5 are plotted the hardness values across sections of rods extruded with different extrusion ratios and through various die angles. These data indicate a similar trend in the tensile properties found in the earlier studies. In percentage elongation (in a tensile test) there seems to be a sudden drop after 20% reduction by extrusion, whereas the percentage reduction in area values decreases gradually with the increase in extrusion ratio. At the same extrusion ratio, there is a slight increase in the strength and hardness with the increase in the die angle. The value of hardness increases gradually from the centre to the periphery of the extruded sections. The variation is small in the case of smaller die angles and greater extrusion ratios and the difference increases with the increase of die angle and decrease in extrusion ratio.

The effect of die angle can be rationalised in terms of the redundant work. Dies with larger die angles give rise to more inhomogeneous deformation and hence more redundant work. The redundant strain is assumed to cause an increase in strain hardening equal to that due to the same strain in simple tension. Figure 3.2.6 illustrates this point [64] based on an analogy in wire drawing. The stress-strain curve for a drawn wire is superimposed on the curve for the same material in the annealed condition, the origin of the former being
displaced along the strain axis by an amount equal to the reduction in drawing expressed as natural strain. The curve for the drawn wire does not fit (it rarely fits perfectly) onto the curve for the annealed material. This suggests that drawing has worked the wire more than has tensile stretching, to give the same apparent strain, i.e. in drawing there has been redundant work. To make the curves coincide, that for the drawn wire must be moved to the right (to dotted position) by an amount that may be taken as a measure of the redundant strain. The area swept out by this movement represents the work done per unit volume if it is assumed that the work done per unit volume in effecting this increased strain is the same in wire-drawing as in simple tension [65], an assumption which appears not unreasonable [64].

The effect of die angle was also studied by Hauttmann [66] and the hardness (probably the average hardness over specimen cross-section) was found to increase with increasing die angle, up to $126^\circ$ (included angle). Increasing die angle from $126^\circ$ to $180^\circ$ did not affect the hardness very much.

Work done on the tensile properties of hydrostatically extruded rods [32] indicates that the hardness distribution over the cross-section of the extruded rod (see Table 3.2.2 for the materials and process parameters used) is approximately uniform, and that the mean hardness values of the hydrostatically extruded rods are only marginally different from those for rods produced by lubricated conventional extrusion. While at smaller extrusion ratios the changes in mechanical properties for hydrostatic extrusions showed trends similar to those found in conventional extrusions, at higher
ratios the effects were reversed and the ultimate tensile strength, 0.1% proof stress and mean hardness were reduced and elongation and reduction in area increased as a result of thermal softening.
3.3 DUCTILE-BRITTLE TRANSITION TEMPERATURE

3.3.1 INTRODUCTION

Ductile to brittle transitions -- the type displayed by iron, ferritic steels, and other body centred cubic metals and alloys -- are observed when the ductile, fibrous mode of fracture, absorbing much energy, is replaced by a brittle fracture absorbing little energy. This intervention of brittle fracture at some temperature is not an unalterable feature of the atom or the crystal lattice. The resistance to brittle fracture can be radically altered by changes in composition and processing [67]. The actual range of temperature over which transition occurs depends upon many factors, e.g. composition, structure, strain rate, the presence of notches, etc. In what follows, the criterion for ductile-brittle transition will be outlined and the effect of structure and property changes brought about by cold work, on some of the variables in this criterion will be discussed.

3.3.2 CRITERION FOR DUCTILE-BRITTLE TRANSITION

Recent theories have attempted to determine as to which is the most difficult part of the fracture process to achieve, from the three stages which can be differentiated; (a) generation or unlocking of slip dislocations; (b) formation of crack nuclei by these and (c) propagation of these cracks leading to a brittle fracture [68]. There have been different views on which of these three stages is the most difficult to carry out. The original dislocation crack nucleation theory developed by Stroh [69] assumed
that crack nucleation was the major obstacle and that once a nucleus formed it would propagate readily. On the other hand, Cottrell and Petch [70] decided that the propagation stage of fracture was likely to be the significant one, because microcracks which do not propagate are frequently observed. In support of this view Smallman [71] quotes the example of a notched plate which, when it is thick remains brittle to a higher temperature than a similarly notched thin plate, indicating that the presence of hydrostatic tension increases the brittleness of the material. This observation implies that at the critical stage in the fracture process, a crack must already be present if the hydrostatic stress is to exert an influence. Cottrell and Petch have independently presented similar tentative fracturing laws to explain such behaviour on the basis of dislocation pile-ups. Cottrell's theory leads to an equation which is consistent with much that is known about cleavage fracture [72]. Polakowski's analysis [72] leads to the development of this equation, which is as follows:

\[ \sigma k_y d^{1/2} = \beta G \gamma \]  

(3.3.1)

where \( \sigma \) is the applied normal stress
\( k_y \) is a temperature dependent constant
\( d \) is half the grain diameter
\( \beta \) is twice the ratio of the shear to the normal stress
\( G \) is the shear modulus
\( \gamma \) is the surface energy per unit area of dislocation crack faces

The yield and fracture stress are considered approximately equal in deriving equation (3.3.1), therefore substituting \( \sigma = \sigma_i + k_y d^{-1/2} \).
from Petch relationship, equation (3.3.1) can be alternately written as:

\[ k_y (\sigma_1 d^2 + k_y) = \beta G \gamma \]  

Equations (3.3.1) and (3.3.2) provide a criterion for fracture just to take place at the yield point. The material is ductile when the left-hand side becomes smaller than the right-hand side. These equations bring out the effect of such variables as grain size, temperature (which affects \( \sigma_1 \) and \( k_y \)), rate of loading, composition and microstructure (due to their influence on \( \sigma_1 \)) on the left-hand side; slow strain rates encourage ductile behaviour and thus a lower transition temperature, by utilising thermal energy to help overcome obstacles to dislocation movement [68]. The variables governing the right-hand side of equations (3.3.1) and (3.3.2) are \( \beta \) and \( \gamma \). The value of \( \beta \) changes with the mode of deformation and the presence of any stress concentrators such as notches. The surface energy value \( \gamma \) is thought to increase by the extra work required for the propagation of cracks due to presence of a tougher phase in the matrix, fibred structure or due to the change in orientation at grain boundaries. The presence of these increases the resistance to the propagation of crack by

(i) creating cleavage steps,

(ii) causing localised deformation and

(iii) tearing near the grain boundaries or at weak interfaces [71].

Cold working of metals brings about changes in some of the variables mentioned above, thus altering the ductile-brittle transition temperature of the unworked material. These changes due to cold working
will be discussed in what follows:

3.3.3 GRAIN SIZE

Whereas the substructural grain size in a cold worked metal is related to flow strength in a manner similar to the one in Hall-Petch equation [45], fracture toughness is reduced by the small subgrain size. This effect is contrary to the ordinary grain refinement which lowers the transition temperature. A possible explanation suggested by English and Backofen [43] for this anomaly is the misorientation at the sub-boundaries. These boundaries are obstacles to, and tend to increase, the flow stress but cannot effectively limit the length of the cleavage crack nucleus, nor seriously reduce the ease of propagation.
3.3.4 **FRICTION STRESS (σ_f)**

Small amounts of plastic deformation at room temperature, which overcome the yield point and unlocks some of the dislocations, improve the ductility at low temperature [71]. In general, however, plastic deformation which leads to work hardening embrittles the metal because it raises the σ_i contribution, due to the formation of intersecting dislocations, vacancy aggregates and other lattice defects.

3.3.5 **EFFECT OF MECHANICALLY FIBRED TEXTURE**

The fundamental hypothesis underlying this discussion is that mechanical fibring effects arise from the presence of pre-existing matrix dis-continuities in the form of internal flaws, pores or inclusions. These structural elements are distorted and aligned by the metal flow during deformation. In subsequent mechanical testing or in service, the material behaviour is influenced by the likelihood of stress or strain concentration near inclusions as well as the possibility of inclusion cracking or separation of the matrix-inclusion interface. The relevance of these events to the fracture process is that nucleation may be accomplished by inclusion cracking or interface separation, and crack propagation, particularly with respect to crack path, may take advantage of an array of fibred interfaces or particles [43].

The existence of such interfaces is a natural consequence of the aligning effect of metal flow, combined with the frequently met low adhesion between matrix phase and second-phase particles.
These interfaces contribute to a toughening effect provided they lie in planes perpendicular to the plane of the fracture path.

English [72] has lucidly reviewed the effects of mechanical fibring on the fracture process. The relevant features from his discussion will be presented here.

An inclusion structure has two basic effects on notch toughness in, say, a notch bend test. For fully ductile fibrous separation, the energy absorbed is reduced with increasing inclusion content [74]. This is completely analogous to the reduction in tensile ductility produced by inclusions as such, regardless of the presence of fibring [75].

A second more interesting effect is on the ductile-brittle transition temperature. The critical structural element in this appears to be the weak interface normal to the fracture path. Any anisotropy of the array of interfaces may lead to corresponding differences in transition temperatures and fracture mode among specimens cut out in various ways from the wrought product. Similarly different degrees of fibring intensity confer varying effectiveness in resisting crack propagation. The depression of transition temperature is only felt under conditions of substantial stress triaxiality, accompanied normally by at least some local plasticity. These requirements are fulfilled to various degrees in tests of notched specimens, but the fibring influence on transition temperature is not felt, for instance, in the ductile-brittle transition in mild steel when evaluated in unnotched tension testing.
Mechanical fibring lowers the ductile-brittle transition temperature in the following ways:

1. In a notch bend test hydrostatic stress caused by the notch is a maximum at the elastic-plastic boundary and leads to peak values there. Within the small volume, cleavage of the matrix will be initiated when a critical normal stress is attained. If, however, fissuring occurs at the inclusion-matrix interface (as shown in Figure 3.3.1) before maximum tensile stress exceeds the critical value for cleavage, the maximum stress will be reduced by the amount of the hydrostatic component. For the cleavage to occur the maximum stress has to reach the critical value by some means which raises the yield stress. Since yield stress increases with decreasing temperature, the effect of fissuring is to lower the transition temperature. On this basis the transverse stress required to produce fissures ought to be low compared to that for cleavage, possibly about half the cleavage stress.

Evidence of this effect in ship steel is seen in Figure 3.3.2 [73]. The transition of controlled rolled stock (where fibring intensity is accentuated) is lower than that of the standard rolled stock for the same grain size.

2. Fibred structure increases the resistance to crack propagation by delamination along inclusions. Just as in the previous case, stress triaxiality may open a sufficiently weak interface slightly ahead of the propagating crack. When this occurs, the crack is deflected perpendicular to the original path and
must be reinstated on the opposite side of the fissure. This is illustrated in Figure 3.3.3. In rolled plate, this situation can be produced by notching a bend specimen on the top surface rather than on the side or edge. In a round stock like an extruded bar, the position of the notch is obviously immaterial so long as it is perpendicular to the direction of working.

Delamination effect was observed by Stokes and Li [76] in wrought tungsten bars. McEvily and Bush [77] found the same effect in an ausformed alloy steel which had been quenched and tempered before the Charpy V-notch tests. The laminations appeared to follow the prior austenitic grain boundaries, highly elongated in the ausforming step and it was concluded that the boundaries were embrittled by locally intense carbide precipitation.

3.3.6 EFFECT OF MICROSTRUCTURAL DAMAGE PRODUCED DURING COLD WORKING

Microscopic damage and its effects on ductility has already been discussed in Section 3.2. The presence of voids and micro-cracks brings about concentration of stress and strain and in this way produces a reduction in plastically deformed volume [43]. Fracture energy is reduced by this effect, whatever the inherent ductility of the matrix metal. The structural damage thus, will be expected to raise the ductile-brittle transition temperature.
3.3.7 EXPERIMENTAL EVIDENCE OF THE EFFECT OF COLD WORK ON DUCTILE-BRITTLE TRANSITION TEMPERATURE

It is generally recognised that cold working impairs the ability of ferritic steels to resist the initiation of brittle fracture. In actual practice, the effects of strain ageing are superimposed on the effect of cold work. Lankford [78] carried out a survey of the effect of cold work on transition temperature. In most of the examples he has cited (Tables 3.3.1, 3.3.2, 3.3.3) this effect was studied only after a maximum prestrain of 10% by cold work, and a general increase in the transition temperature was observed. The main effect, which dominates when cold working is carried out up to strains mentioned above, is the increase of friction stress $\sigma_f$ by strain hardening and then by ageing. Accordingly, the value of the left hand side of Equation 3.3.6 is increased resulting in a rise in the ductile-brittle transition temperature. An increase in transition temperature from 30°C to 50°C by 5 to 10% prestrain at room temperature is also indicated in an extensive summary of data on notch bar transition temperature in various annealed or normalised mild steels [79]. For very large prestrain, embrittlement may be anisotropic. The reduction in ductility in iron, for instance, is greatest for tension applied transverse to the direction of prior metal flow [80]. For rolled plate, this means low transverse reduction in area but relatively unimpaired longitudinal reduction in area. Relatedly, tension tests on bars at right angles to the axis of precompression have shown increased ductility, whereas uniformly precompressed
mild steel subsequently tested in unnotched tension in the same direction may fracture in a very brittle manner \[81\].

Not many data are available on the impact properties of steels after severe cold work, common in drawing and extrusion. Ebert's work \[57\] on the mechanical properties of cold drawn bars of three carbon steels (Table 3.3.1) however, includes information on the impact properties as well, figure 3.3.4. It is evident from this that for small amounts of cold work (up to about 20% reduction in area) the transition temperature rises progressively with the increase of cold work. Energy absorbed in the Charpy V-notch test also decreases continuously up to a certain amount of cold work (depending upon the test temperature and carbon content of the steel) and then start rising when the amount of cold work increases after this. For one steel (SAE 1016) this rising trend in the energy absorbed is reversed after about 60% reduction in area. Transition temperature nevertheless, continues to fall for higher reductions after an initial rise for small amounts of reductions. At lower reductions the transition temperature and the energy absorbed are mainly influenced by the strain-hardening, whereas the effect of mechanically fibred texture seems to play a dominant role at higher reductions. The transition temperature increases and the energy absorbed decreases generally with the increase in carbon content.

The impact properties of extruded rods of steels of different carbon contents, some of them Al-killed (Table 3.3.2) were studied by Hautmann \[66\]. The impact strengths were determined by Izod tests. The effect of die angle on impact strength of rods (extruded with 65% reduction in area) is shown in Figure 3.3.5.
which indicates that at higher die angles the impact strength of some steels decreases slightly. This is probably due to the increase in friction stress ($\sigma_1$) and greater structural damage produced at higher die angles. Figure 3.3.6 shows the effect of the amount of reduction on the impact strength. The impact strength of extruded rods generally is extraordinarily high. In the case of steels 10 and 11, the starting impact strengths are even exceeded. Whereas the impact strength is generally low at smaller reductions and high at greater reductions, the Al-killed and non-Al containing steels behave differently. In the Al-free steels, the impact strength is low at small reduction and increases at high reductions. The impact strength of Al-killed steels, on the other hand, drops to a minimum in the range of reductions of about 50%. Beyond 75% reduction, the impact strength almost equals that starting impact strength of the metal. This difference in behaviour can possibly be attributed to the increased value of friction stress ($\sigma_1$) for ageing steels as compared with the non-ageing steels.

Recent work at N.E.L. [44] on the impact properties of a cold worked steel (to British Standard 970: EN3) points towards trends similar to those mentioned above, i.e. impact properties deteriorate after small reductions of area (of the order of 13%) and improve vastly when the amount of plastic deformation is increased. In heavily deformed products (after about 75% reduction in area), a high energy absorption coupled with a transition temperature below -75°C was observed. This work also highlights the importance of the mode of plastic deformation and therefore the nature of stress distribution resulting from this in a particular process. The impact properties
of rods extruded at an extrusion ratio of 2:1 were found to be inferior to those produced by swaging with similar reduction in area. The reason for this was advanced to be the unfavourable stress system occurring in the former case which caused the microstructural damage and hence the degradation of the impact properties.
3.4 COLD WORK AND FATIGUE

3.4.1 INTRODUCTION:

In so far as cold work affects fatigue properties, there are three main aspects to be considered:

1. Residual stress fields produced by inhomogeneous plastic deformation.

2. The enhanced mechanical strength values due to cold work.

3. The effect of cyclic loading.

Whereas the fatigue performance of a cold worked metal is influenced by the net effect of these factors, the contribution of each of these depends upon

(a) composition of the material,

(b) the amount of cold work imparted to it, and

(c) the range of applied stress.

3.4.2 EFFECT OF RESIDUAL STRESSES

Fatigue usually begins at a surface, because stresses are normally higher there, particularly since most parts experience bending loads resulting in substantially higher stresses in the outermost fibres. Apart from this, stress raisers are likely to be present as a result of surface irregularities inherent in the design, produced in service or resulting in processing.
The effect of residual stresses can be considered identical to stresses produced by an external load. Thus, the addition of a compressive residual stress, which exists at a point on the surface, to an externally applied tensile stress on that surface, decreases the likelihood of fatigue failure at that point. Figure 3.4.1 illustrates this effect. Figure 3.4.1 (a) shows the effect of elastic-stress distribution in a beam with no residual stress. A typical residual-stress distribution, such as would be produced by shot peening is shown in Figure 3.4.1 (b). In Figure 3.4.1 (c) the stress distribution due to the algebraic summation of the external bending stresses and residual stresses is shown. The maximum tensile stress at the surface is reduced by an amount equal to the surface compressive residual stress. The peak tensile stress is displaced to a point in the interior of the specimen. The magnitude of this stress depends upon the gradient of the applied stress and the residual stress distribution. Thus, subsurface initiation of fatigue is possible under these conditions. However, as the surface is weaker than the subsurface, the reduction of maximum tensile stress by residual compressive stress results in longer fatigue life. It follows from Figure 3.4.1 that surface compressive stresses are most beneficial under loading conditions which produce high surface tensile stresses, as in bending [33]. However, some improvement in the fatigue performance of axially loaded fatigue specimens also results from surface compressive residual stresses.

There are many studies in the literature on fatigue, where cold work was imparted by processes like shot-peening and
surface-rolling. In these processes plastic deformation is confined to the surface layers and the core of the metal is strained elastically resulting in high compressive surface residual stresses. Coombs et al. \cite{Coombs1984} found an improvement of 20 to 40 per cent in the fatigue strength of the peened polished specimens over the fatigue strength (at $10^6$ cycles) of polished specimens and of the as-heat treated specimens. Mattson \cite{Mattson1985} calls the compressive residual stresses the unharnessed trapped forces and shows their importance by giving examples of some automotive parts where fatigue properties were improved by surface rolling. Figure 3.4.2 \cite{Coombs1984} is a fatigue chart showing a comparison between peened and non-peened spur and helical gears on the basis of average life. The use of surface rolling to improve fatigue strength has been discussed by Horger \cite{Horger1987}. From the experiments on unnotched shafts he obtained an increase in fatigue strength ranging from 20 to 80 per cent, as a result of surface rolling. The fatigue strength of components, such as crankshafts, has been increased by cold rolling the fillets with a steel ball. Love \cite{Love1988} obtained an increase of 60\% under completely reversed bending and 80\% under repeated bending, using a pressure sufficient to produce a small but a detectable deformation of the fillet; similar results were obtained on forged and cast steel shafts. Frost et al. \cite{Frost1990} quote examples from several studies which illustrate the effect of residual stresses on fatigue properties. The data from these examples is compiled in Table 3.4.1. From these and numerous other similar studies, it is well established that surface compressive residual stresses, indeed, improve the fatigue performance of a part.
3.4.3 EFFECT OF WORK-HARDENING

In most cold working operations the nature of the plastic deformation is such that residual stresses and work-hardening are coincident. In order to obtain an indication of the effect of work-hardening alone on the fatigue properties, fatigue tests have to be carried out on the material which is plastically deformed uniformly over its cross-section by a direct stress loading, and is thus free of residual stresses. Materials having high rates of work-hardening, such as mild and austenitic steels, show the greatest change in the fatigue limit. For example [90], fatigue tests on specimens cut longitudinally from mild steel cylindrical blocks, which had been compressed to various nominal compressive plastic strains, showed that the fatigue limit increased from a value of ±250 MNm\(^{-2}\) for as-received bar to a maximum value of ±350 MNm\(^{-2}\) at a nominal compressive pre-strain of about 50%. Pre-strains above this value did not result in any further increase in fatigue limit; in fact, the fatigue limit dropped drastically to a value of about 270 MNm\(^{-2}\) at a compressive pre-strain of about 70%. However, the hardness of the specimen did not show a corresponding decrease and it was suggested that the excessive cold-work caused incipient cracking, so resulting in the drop in the fatigue limit. Similar results are reported by Eckert [91]. He found an optimum hardness of 50-55 Rc. and decreased fatigue life for medium carbon steel shafts of greater hardness (e.g. Rc. 60). In another study quoted by Frost [90] the fatigue strength (10\(^7\) cycles) of a titanium alloy could be increased by about 30% by pre-straining in tension to 10% of the fracture strain. However, only smaller additional increases in fatigue strength occurred for pre-strains of up to 60%. On the other hand the fatigue limits of
alloy steels cut from pre-strained blanks show little difference from that of the unworked material; for example the fatigue limit of Ni-Cr steel (tensile strength 1000 MNm$^{-2}$) specimens cut from blanks compressed to 50% plastic strain was only increased by about 10%.

3.4.4 RELIEF AND REDISTRIBUTION OF RESIDUAL STRESSES

Relief of residual stresses occurs when the resultant of the applied and residual stress at any point in the specimen exceeds the condition of yielding. Plastic flow then ensues, resulting in relief and redistribution of residual stresses. This effect is, therefore, likely to be more pronounced in comparatively softer metals than in hard metals. For example, 1% Mg-Al alloy (softened condition - tensile strength 177 MNm$^{-2}$, and hardened condition - tensile strength 320 MNm$^{-2}$) plate specimens, 76 mm wide and having two opposite edge notches, 18 mm deep and 2.5 mm root radius, were preloaded to a nominal net area stress equal to 80% of the yield stress of the material, prior to fatigue testing in plane-bending. The magnitude of the maximum induced residual stress in the material at the notch root was determined by x-ray diffraction techniques at intervals throughout the test. The fatigue strength ($10^7$ cycles) of the softened plates was unaffected by either a tensile or compressive pre-loading, but that of the hardened plates was increased from ±59 MNm$^{-2}$ to ±76 MNm$^{-2}$ by a tensile pre-load and reduced to ±42 MNm$^{-2}$ by a compressive pre-load. These results were consistent with the residual stress measurements taken because, whereas the values of the initially induced residual stresses in the plates of the hardened material were unaltered by subsequent fatigue stressing during the entire life of the specimen, those in the plates of the softened material were relaxed.
Residual stresses may be relaxed similarly in soft steels, whereas they may not be in harder steels. For example, rotating bending fatigue tests were carried out (as quoted in ref. 90) on notched specimens (20 mm outside diameter, having either a semi-circular notch of 2.5 mm radius or a vee-notch 3.8 mm deep and 0.025 root radius) of three steels (0.2% C, 0.4% C and 4% Ni) which were pre-loaded by applying either a static tensile or compressive loading prior to fatigue testing. The vee-notched specimens were pre-loaded to the 0.1% proof stress of the material multiplied by the area of the minimum cross-section; the semi-circular notches were pre-loaded so that the notch width was either extended or decreased by 0.125 mm. The fatigue limits of the semi-circular notched carbon steel specimens were unaffected by the pre-loading, but those of the corresponding Ni steel specimens were increased by the tension pre-loading and decreased by compression pre-loading. The fatigue limits of the vee-notched Ni steel specimens were also increased by the tension and decreased by the compression pre-loadings. Similar trends in the relief of the residual stresses in carbon and Ni steels are reported in the work of Bühler and Buchholtz [98]. Results by the same authors on carbon steel tubular specimens having residual tensile stresses indicate that these stresses actually changed into compression after various degrees of bending stress and a number of reversals were applied. Fatigue tests on these specimens disclosed that the endurance limit was not decreased by this initial value of the residual tensile stresses. The finding that these tensile stresses go into compression is a possible explanation of this observation.
Fading of residual stresses was also established by x-ray measurements made by Glocker [99]. Decrease of residual stresses after fatigue testing of shot peened parts was reported by Moore [100] as well. Rosenthal [101] found a progressive relief of residual stresses with increasing applied loads (Figure 3.4.3). He established the "maximum shear stress criterion" for the relief of residual stresses as a result of residual and applied loads (Figure 3.4.4). The law of constant sum of maximum shear stresses is indicated by the diagonal. The reduction of residual stresses produced by cyclic loads (marked F) was generally on the lower side as compared to the reduction produced by static loads (marked S). Rosenthal suggested the gradual fading of residual stresses during cyclic loading as a possible explanation for that. Kudryavtsev's [102] results (Figure 3.4.5) also indicate that relief of residual stresses occur during the fatigue life of a part and this relief is proportional to the intensity of cyclic loads.

3.4.5 MICRO AND MACRO-CRACKS

These terms are used in the next section and are therefore defined here as follows:

In a ductile metal, micro-cracks are the slip-band cracks which are created by continual cyclic slip on certain crystallographic planes, under the action of the resolved cyclic shear stresses created by the external loading cycle. A micro-crack opens and closes in a loading cycle, owing to stresses acting across its faces, the amount of opening increasing with the increase of crack size. However, until the micro-crack reaches a depth at which the magnitude of the cyclic opening is sufficient to subject an element of metal immediately ahead of its tip to a tensile strain sufficient to fracture a length of metal which does not rebond completely
during the compression half of the loading cycle, a micro-crack can only continue to develop by the to-and-fro slip movements that created it initially. When the micro-crack has penetrated to this critical depth, it becomes a macro-crack. The growth rate characteristics of the macro-crack depend on the magnitude of the cyclic loading and the imposed mean stress; if any (external or residual), and its direction of growth is normal to the maximum cyclic tensile stress. Its rate of growth is also much faster than when it was growing as a micro-crack.

3.4.6 GENERAL DISCUSSION

The effect of residual stress on the fatigue life of a material is similar to that of a superimposed static mean stress, except that a mean stress is maintained constant throughout a test by means of an external load, whereas a residual stress may be relaxed by any subsequent cyclic plastic deformation induced by the cyclic stressing. Obviously, if the magnitude of the cyclic stressing is sufficient to relax the residual stresses early in the test, they can have little effect on the subsequent fatigue strength. It follows therefore that induced residual stresses have the greatest effect on the fatigue strength of notched components where the applied nominal stresses are less than the plain fatigue limit of the material, the effect increasing with both notch sharpness and the yield stress/fatigue limit ratio of the material. Unrelaxed residual stresses would not be expected to have a pronounced effect on the minimum alternating shear stress required to initiate a micro-crack but may affect profoundly the development of these cracks and the subsequent propagation of macro-cracks. Thus, residual compressive stresses of
sufficient magnitude and depth will prevent a macro-crack opening and so either retard its growth or cause it to remain dormant until the stress level is raised. On the other hand, tensile residual stresses will open a crack, so making its subsequent propagation easier, in situations where this results in an increase in the tensile stress range. Compressive residual stresses will, for a given stress range, increase the changeover length between the micro-crack and macro-crack growth stages. Thus, if the stress level applied to either a plane specimen having high compressive residual stresses induced in the surface layers or a notched specimen having high compressive residual stresses induced around the notch root is such as to cause it to break after a long endurance, it is found that the crack has to grow an appreciable distance as a micro-crack in order to penetrate the volume of material influenced by the residual stresses before finally growing as a macro-crack normal to the loading direction.

If residual compressive stresses affect only a very thin surface layer, which for a given cyclic loading and material is less than the depth at which a micro-crack changes to a macro-crack, a surface micro-crack will be able to grow through it (under the action of resolved cyclic shear stresses) at a stress level either equal to or only slightly greater than that required to develop it to the same depth in a material in which residual stresses are absent. It follows therefore that if the fatigue limit of either plain or notched specimens is increased markedly by the introduction of surface compressive residual stresses, in all probability micro-cracks will have formed in the surface layers but will have been prevented from growing. The slope of the finite-life portion of their S-N curves will be flatter than those of the corresponding untreated specimens because, once a crack has
penetrated the affected layer of material, its growth rate at a
given stress level will be as in an untreated specimen, the difference
in the life being the number of cycles necessary for the crack to
penetrate the affected layer. The difference will become less, the
higher the cyclic stress level - for in addition to it now being,
easier for a micro-crack to reach the macro-crack stage, there will
be a greater tendency for the residual stresses to be relaxed.

Whether or not residual stresses are relaxed depends on the
magnitude of the subsequent cyclic stressing in relation to the yield
stress of the material. It has been suggested (as quoted in ref. 90)
that residual stresses will be relaxed if, when added to the external
stresses acting in the same direction, the resulting value of the
maximum resolved shear stress exceeds the yield stress of the material;
they will be reduced to such a value that the sum of the external and
residual stresses equals the yield stress.
<table>
<thead>
<tr>
<th>Material</th>
<th>Composition, %</th>
<th>Annealing Treatment</th>
<th>Subsequent Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferrovac E</td>
<td>C 0.004, Mn 0.001, S 0.005</td>
<td>½ hr 800°C</td>
<td>Furnace cooled</td>
</tr>
<tr>
<td>Eutectoid carbon steel</td>
<td>C 0.93, Mn 0.37, Si 0.20, P 0.02, S 0.03.</td>
<td>5 min at 980°C</td>
<td>Isothermally transformed at 496°C</td>
</tr>
<tr>
<td>Fe-2% Cu</td>
<td>Cu 2.2, C 0.006, Mn 0.003</td>
<td>1 hr 840°C</td>
<td>Brine quenched and aged 24 hr at 700°C</td>
</tr>
<tr>
<td>Fe-C-Mo</td>
<td>Mo 0.47, C 0.1, Mn 0.49  P 0.018, S 0.019, Si 0.18</td>
<td>2 hr at 720°C, brine quenched</td>
<td>A, aged at 250°C for 140 hr and 60°C for 16 hr. B, aged at 250°C for 140 hr and 100°C for 10 hr</td>
</tr>
<tr>
<td>Low carbon martensite</td>
<td>C 0.11, Mn 0.81, Si 0.28  Ni 1.75, Cr 0.77, Mo 0.22</td>
<td>3 min at 950°C, brine quenched</td>
<td>As quenched</td>
</tr>
<tr>
<td>18/8 stainless steel</td>
<td>C 0.05, Mn 0.48, P 0.01, S 0.013, Ni 9.32, Cr 17.34</td>
<td>1 hr at 1070°C</td>
<td>Brine quenched</td>
</tr>
<tr>
<td>Copper</td>
<td>Cu 99.9, O 0.03, S 0.0016, Fe 0.0016, As 0.002</td>
<td>½ hr 800°C</td>
<td>Furnace cooled</td>
</tr>
</tbody>
</table>
### Table 3.2.2 Details of Materials and Process Parameters used in ref. [32]

<table>
<thead>
<tr>
<th>Material and Heat Treatment</th>
<th>Process Parameters Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrolytic tough-pitch high conductivity copper to B.S. 1306 : 1964, annealed at 600°C for one hour and water quenched</td>
<td>4, 6, 10 and 16</td>
</tr>
<tr>
<td>99.5 grade Aluminium to B.S. 1476 : 1963, annealed at 360°C and air cooled</td>
<td>4, 6, 10, 16, 25, 40, 60, 100 and 160</td>
</tr>
</tbody>
</table>
A graph showing the observed cell diameter as a function of true drawing strain for a variety of materials. The dashed lines represent the predicted relationship of cell diameter with strain in the absence of dynamic recovery.

*Fig. 3.2.1*

The flow stress of drawn materials expressed as a function of cell diameter.

*Fig. 3.2.2*
SECOND STAGE PRODUCTS SHOWING EFFECT OF DIE ANGLE ON BORE

Fig. 3.2.3
Effect of Reduction on the Mechanical Properties of Extruded Components.
(Die angle—90°)

Fig. 3.2.4 (b)—Effect of Die Angle on the Tensile Properties of Extruded Tubes.
(Reduction in area — 66 per cent; Extrusion ratio—2.9x)

Fig. 3.2.5—Effect of Die Angle on Hardness of Extruded Rods.

Fig. 3.2.6

Stress—natural-strain curves for annealed and drawn wires
### Table 3.3.1
Chemical Composition of Steels, %

<table>
<thead>
<tr>
<th>Steel (SAE No)</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016</td>
<td>0.13</td>
<td>0.18</td>
<td>0.60</td>
<td>0.90</td>
<td>0.040</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1040</td>
<td>0.37</td>
<td>0.64</td>
<td>0.60</td>
<td>0.90</td>
<td>0.040</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1060</td>
<td>0.55</td>
<td>0.65</td>
<td>0.60</td>
<td>0.90</td>
<td>0.040</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8630</td>
<td>0.28</td>
<td>0.33</td>
<td>0.70</td>
<td>0.90</td>
<td>0.040</td>
<td>0.040</td>
<td>0.20</td>
<td>0.35</td>
</tr>
</tbody>
</table>

---

### Table 3.3.2
STEEL ANALYSES AND MECHANICAL PROPERTIES

<table>
<thead>
<tr>
<th>Running Number</th>
<th>Designation</th>
<th>Dia. in.</th>
<th>Thick., inches</th>
<th>Heat</th>
<th>Treatment</th>
<th>Chemical Composition</th>
<th>Yield Strength, 1000 P.S.I.</th>
<th>Tensile Strength, 1000 P.S.I.</th>
<th>Elongation, L-5 in. %</th>
<th>Reduction in Area %</th>
<th>Impact Strength, ft-lb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ms B</td>
<td>1.6</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.09</td>
<td>0.40</td>
<td>50.2</td>
<td>49.0</td>
<td>78</td>
</tr>
<tr>
<td>2</td>
<td>AR 20</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.16</td>
<td>0.04</td>
<td>31.8</td>
<td>31.2</td>
<td>52.8</td>
</tr>
<tr>
<td>3</td>
<td>Non-aging</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.09</td>
<td>0.32</td>
<td>34.2</td>
<td>34.2</td>
<td>22.9</td>
</tr>
<tr>
<td>4</td>
<td>Killed</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>25.8</td>
<td>25.8</td>
<td>22.9</td>
</tr>
<tr>
<td>5</td>
<td>Boiler Plate I</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>6</td>
<td>BS 20</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>7</td>
<td>Non-aging</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>8</td>
<td>CK 15</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>9</td>
<td>Non-aging I</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>10</td>
<td>Manganese</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>11</td>
<td>Pre-hardening</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
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</tr>
<tr>
<td>12</td>
<td>Pre-hardening</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>13</td>
<td>C 45</td>
<td>2.92</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>14</td>
<td>T 45 Thomas</td>
<td>2.92</td>
<td></td>
<td></td>
<td></td>
<td>Normalized</td>
<td>0.12</td>
<td>0.08</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Notes:
1. These data were obtained with a DVM-Test specimen and reported in pounds/square inch. The reported values have been multiplied by 6 which converts DVM response to Charpy values, respectively.
2. Compressed 10 pct., annealed 1; heat at 225°C. Notch extremely to surface.
Probable stages in the growth of a crack approaching a plane of weakness in a material.

Fig. 3.3.1

Transiton temperature vs. ASTM grain size number for ship plate in notched slow-bend test. Controlled rolling produces more intense fibering and lower transition temperature.

Fig. 3.3.2

Crack propagation in presence of transverse weak interface. Delamination of this type can produce increased notch toughness.

Fig. 3.3.3
Effect of cold drawing and of stress relieving on the mechanical properties of three plain carbon steels and one low-alloy steel. Solid lines represent as-drawn material; dashed lines, material stress relieved after cold drawing. Test specimens were taken from half-radius of bars aged for 4 hr at 212 F after cold drawing, to simulate the condition of steel after several months of storage at room temperature. Tests were performed on one heat of each grade that had been hot worked down to a 2-in. bar by conventional practices, then normalized and cold drawn to the reductions indicated. Note that the larger reductions are well beyond commercial ranges. Bars stress relieved after cold drawing were treated as follows: carbon steels, 2 hr at 900 F; 8630, 2 hr at 1000 F. (L. J. Ebert. Report WAL 310/60-85 to Watertown Arsenal, 1965)

Fig. 3.3.4
Fig. 3.3.5 Effect of die angle on the impact strength of 4.6 in cylinders extruded with 65% reduction in area from the different steels in Table 3.3.2

Fig. 3.3.6 Effect of the type of steel (Table 3.3.2) and reduction in area on the impact strength. Final diameter of rods was 0.393 in; die angle was 126°.
<table>
<thead>
<tr>
<th>Srl No.</th>
<th>Details of Material, Test Piece etc.</th>
<th>Method by which residual stresses were induced</th>
<th>Magnitude and nature of induced residual stresses (when quoted)</th>
<th>Method of fatigue testing (when given)</th>
<th>Effect on the fatigue properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.17% C steel 12.5 mm square</td>
<td>Specimens bent to produce a surface plastic strain equal to six times the elastic yield point strain, annealed at 650° and re-straightened</td>
<td>Surface residual stress (tensile on one face and compressive on the other) 100 MNm⁻²</td>
<td>Reversed plane bending</td>
<td>Fatigue limit reduced from 230 MNm⁻² to 205 MNm⁻² because of the presence of tensile residual stresses. The reduction in the fatigue limit was not very significant because most residual stresses relaxed during fatigue testing</td>
</tr>
<tr>
<td>2</td>
<td>4½% Cu-Al 12.5 mm square</td>
<td>Bent to give a surface plastic strain of twice the elastic strain at 0.2% proof stress, then re-straightened without any heat treatment</td>
<td>Surface, ±115 MNm⁻²</td>
<td>Reversed plane bending</td>
<td>Fatigue strength (10⁷ cycles) reduced from 206 MNm⁻² to 150 MNm⁻². Residual stresses did not relax during fatigue testing</td>
</tr>
<tr>
<td>3</td>
<td>5½% Zn-Al alloy, Notch - 12 mm deep x 0.05 root radius</td>
<td>Strained in tension to 570 MNm⁻²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5½% Zn-Al alloy, Notch - 12 mm deep x 0.05 root radius</td>
<td>Strained in compression to 570 MNm⁻²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4% Cu-Al alloy, Vee Notch - 1 mm deep x 0.1 mm root radius</td>
<td>Strained in tension to 390 MNm⁻²</td>
<td></td>
<td>Rotating bending</td>
<td>Fatigue strength (5 x 10⁶ cycles) increased from 58 MNm⁻² to 123 MNm⁻²</td>
</tr>
<tr>
<td>6</td>
<td>4½% Cu-Al alloy, Specimens 51 mm wide x 5.1 mm thick containing a central hole 6.4 mm diameter</td>
<td>Strained in tension on the net area to 80% of the material's 0.1% proof stress</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean Load MNm⁻²</td>
<td>Change in Fatigue Strength (2 x 10⁷ cycles) MNm⁻²</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>73 → 89</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+93</td>
<td>50 → 73</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+185</td>
<td>46 → 46</td>
</tr>
</tbody>
</table>
Superposition of applied and residual stresses.

Fig. 3.4.1

Fig. 3.4.2

Fatigue chart of carburized automotive-type spur and helical gears, shot-peened, and non-peened.
Fig. 3.4.3

Relief of residual stress by a longitudinal tensile load applied to the plate which had been heated locally; I, II, III, IV, V, residual stresses left after increasing loading.
The validity of the maximum-shear law for the relief of the residual stresses.

* see Reference [101]

Fig. 3.4.4

Fig. 3.4.5 Change in size of residual stresses in the surface layers of rolled samples depending on the working conditions of the test.

Number of cycles in testing - 2,000,000 except in specified cases.
CHAPTER 4

Determination of Residual Stresses
4.1 INTRODUCTION

After Heyn's [1] pioneering work in 1912, on the importance and measurement of Residual Stresses, a number of methods have been proposed by various researchers for determination of these stresses. The basic principle underlying most of these methods is the same; that residual stresses are the result of mismatch of different regions of a body. Due to this mismatch, they are under elastic strain and when a stressed portion of the body is removed, the remaining portion undergoes partial stress relaxation resulting in change in dimensions. From this change of dimensions or strain, the stresses existing in the removed portion can be calculated from elastic theory formulae. All mechanical methods of residual stress determination are based on this principle. X-ray diffraction technique, however, is based on the measurement of the change of interatomic distance of surface lattice, caused by the residual micro or macro stresses. Vast literature exists on the various experimental techniques on residual stress determination and these have been reviewed by several workers, perhaps the latest being by Denton [111] in 1966.

For a complete determination of the magnitude and nature of residual stresses in solid bars, Sach's boring method gives the most dependable results and, therefore, it was decided to use this method in the present investigation.

The mechanical methods of residual stress determination are not direct methods in the sense that in these methods the residual stresses in a particular region of the body are determined
from the strains which are induced in the remainder of the body by
the removal of the former part. A more direct approach would be
to find out the state of stress in the body just before the
unloading takes place after the deformation that introduces the
residual stresses, and to determine the stresses left over in the
body (residual stress) as a result of the stress relaxation after
the forming load is removed. In this approach the accuracy of the
residual stress determination depends on the accuracy with which the
stress distribution in the deformation zone of the body is arrived
at. Several methods are available for the determination of the
state of stress in the deformation zone of the body but in all these
certain simplifying assumptions are made about the material and the
way in which it is deformed. In spite of these assumptions some of
the methods are claimed to give good agreement with the experimental
results. By making use of such methods it should be possible to
determine the state of residual stress in a body which, although it
may not be very accurate, can still reveal the nature of the stress
distribution along the cross-section of the body and can provide a
useful check on the experimentally determined residual stresses.
4.2 SACH'S BORING METHOD

4.2.1 GENERAL

Bauer and Heyn [112] worked out equations relating longitudinal residual stresses with change in length of specimen. These were subsequently replaced by the more rigorous equations due to Mesnager [2] and, then, later simplified by Sachs [3]. The general technique of boring out a cylinder or tube in stages and measuring the resulting longitudinal and circumferential strains at the outer surface caused by the release of residual stress is now known as "Sachs boring method".

The following assumptions are implicit in this method:

1. The metal is effectively isotropic and has a constant value of Young's modulus and Poisson's ratio.

2. The residual stresses are distributed with rotational symmetry about the axes of the bar and the principal axes coincide with the axial, radial and circumferential directions.

3. After the boring operation, the tube left is circular in section and inner and outer wall surfaces are concentric.

4. The specimen is sufficiently long to prevent lateral bending.

The stress equations derived by Sachs are:

\[ \sigma_\ell = E' \left[ (Fa - F) \frac{d\ell}{dP} - \lambda \right] \]  

(4.2.1)
\[ \sigma_t = E' \left[ (F_a - F) \frac{d\theta}{dF} - \frac{F_a + F}{2F} \theta \right] \]

(4.2.2)

\[ \sigma_r = E' \left[ \frac{F_a - F}{2F} \theta \right] \]

(4.2.3)

where, \( E' = \frac{E}{1 - \nu^2} \), \( \nu \) being the Poisson's ratio and \( E \) Young's modulus.

\( F_a = \pi R^2 \), \( R \) being the external radius of cylinder

\( F = \pi r^2 \), \( r \) being the current radius of cylinder

\( \lambda = \lambda + \nu \theta \) and \( \theta = \theta + \nu \lambda \),

\( \lambda \) and \( \theta \) being the longitudinal and circumferential strains.

\( \sigma_x, \sigma_t \) & \( \sigma_r \) are longitudinal, tangential and radial stresses respectively.

A derivation of these equations is given in Appendix II.

4.2.2 LIMITATIONS OF THIS METHOD

1. It is limited to cylindrical parts in which stresses vary in one cartesian direction (radial) and are constant in the other two (longitudinal and circumferential).

2. In order to obtain the value of stresses from the above equations, it is necessary to plot \( \lambda \) and \( \theta \) as functions of \( F \). Between the final cut in boring, and the outside diameter, there are no experimental points on such curves and surface stresses must be obtained by extrapolation. This introduces large errors in some cases, particularly where the difference in \( F_a \) and \( F \) is
large. This can be quite significant with surface hardened specimens where it is not easy to machine the hardened layer. Many attempts have been made in the past to overcome this difficulty. Barker and Hardy [113] suggest that Sachs equations can be rewritten thus:

$$\sigma_\theta = E' \frac{d}{dF} \left[(F_a - F) \lambda\right]$$

$$\sigma_t = \frac{E'}{2} \frac{d}{dr} \left[\frac{F_a - F}{r} \Theta\right]$$

(4.2.4)

(4.2.5)

the expression for radial stress remains unchanged.

The values of the functions \((F_a - F)\lambda\) and \(\frac{F_a - F}{r} \Theta\) are zero at both inner and outer surfaces, so that extrapolation is to zero instead of an unknown value. Hence values of \(\sigma_\theta\) and \(\sigma_t\) extrapolated by this method between the last bore diameter and the outside diameter are more accurate than the same extrapolated by using Sachs original equations.

This difficulty has been overcome by some workers by removing material from the cylindrical specimen by etching out the material till a very small thickness of the order of 0.01" (0.254 mm) of the tube is left. The remaining tube is then slit longitudinally on the sides of the axial strain gauges and tongues are cut out to release the remaining stresses. Also the method of metal removal by etching is better than machining because it eliminates errors caused by the possible introduction of stresses by conventional machining. The fact that acid etching does not introduce any additional stresses is verified by Loxley [114].
Another approach [115] to eliminate inaccuracies due to extrapolation in Sachs method, especially in cases where the stresses change sign in the outermost wall thickness or where the surface stresses are very high compared to the stresses in the remaining section is to start material removal from outside to inside. The Sachs' equations are consequently modified for outer layer removal. This method presupposes the existence of a bore where strains are measured while the tube is turned from outside. A survey of the literature showed that a number of different stress equations are quoted for this method [116 - 121]. These equations were, therefore, derived and the derivation is given in Appendix III. The modified stress equations are:

\[
\sigma_L = -E' \left[ (F - Fa) \frac{d\Delta}{dF} + \Delta \right] \quad (4.2.6)
\]

\[
\sigma_t = -E' \left[ (F - Fa) \frac{d\Theta}{dF} + \frac{F + Fa}{2F} \Theta \right] \quad (4.2.7)
\]

\[
\sigma_r = -E' \frac{F - Fa}{2F} \Theta \quad (4.2.8)
\]

where \( F = \pi r^2 \) and \( Fa = \pi r_o^2 \), \( r \) and \( r_o \) being the current and the bore radius.

In case of solid cylinders boring is carried out to a diameter where it is possible to cement strain gauges on the bore. Remaining material is then removed by turning from outside till a thin shell is left. Extrapolation over the remaining thickness is now between the known values and hence more accurate. Alternatively, where possible, the specimen length is divided into two equal parts. One half is analysed completely by Sachs boring method (boring from
inside) and the second half is first bored out to a diameter where it is convenient to take measurements by strain guages or by some other means, and then turned down from outside. Stresses calculated in the second part by the above formulae are corrected for the stresses which were already present in the bored out material. The corrections to be applied are derived in Appendix III. By this method it is possible to measure surface stresses accurately as well as to apply checks on the accuracy of the method by comparing the stresses in the overlapping region which should be the same.

Another source of error in Sachs boring method is from the tangents to \( \lambda / F \) and \( \theta / F \) curves, as it is difficult to construct these tangents accurately. Bühlér and Schreiber [122] overcame this difficulty by using "step by step" method. In this, the total area \( F_a \) of the specimen is divided into \( N \) equal parts, each part being bored out in one step. The Sachs' equations then become:

\[
\sigma_{kn} = E' \left[ \frac{N - n}{2} (\lambda_{n+1} - \lambda_{n-1}) - \lambda_n \right] 
\]

(4.2.9)

\[
\sigma_{kn} = E' \left[ \frac{N - n}{2} (\theta_{n+1} - \theta_{n-1}) - \frac{N + n}{2n} \theta_n \right] 
\]

(4.2.10)

\[
\sigma_{rn} = E' \frac{N - n}{2n} \theta_n 
\]

(4.2.11)

The equally spaced individual points of cross-sectional area are, here, designated by the integers \( n = 1, 2, 3, \ldots, N \). These integers have likewise been used as subscripts of \( \lambda \), \( \theta \) and \( \sigma_k \), \( \sigma_t \) and \( \sigma_r \). The same authors [123] also fitted cubic equations between the points \( F, \lambda \) and \( F, \theta \) and differentiated these equations.
to get the values of $\frac{d\Lambda}{dF}$ and $\frac{d\varnothing}{dF}$. Stress calculations by this method were fairly accurate.

4.2.3 CHECKS ON SACHS BORING METHOD:

Static checks should be made of the resulting longitudinal, tangential and radial stress distribution curves obtained by the boring out method, to establish that equilibrium stress conditions have been expressed. The force equilibrium requires that:

1. The sum of the longitudinal stresses over the cross-sectional area must be equal to zero, i.e.,

$$\int_0^L \sigma_L \, dF = 0 \quad (4.2.12)$$

2. The sum of the tangential stresses over the diametrical section of the cylinder must be zero, i.e.

$$\int_0^L \sigma_t \, dr = 0 \quad (4.2.13)$$

Apart from this, a check can be made at any point at radius $r$ where,

$$\sigma_t = \sigma_r + r \frac{d\varnothing}{dr} \quad (4.2.14)$$

Also the radial and tangential stresses should be equal at the centre of the solid cylinder. These checks do not throw any light on the correctness of the experimental data; whatever may be the values of strain readings and therefore of $\Lambda$ and $\varnothing$, (from which $\sigma_L$ and $\sigma_t$ are calculated), equations (4.2.12) and (4.2.13)
are satisfied. This can be seen by substituting the expressions for $\sigma_\theta$ and $\sigma_t$ (from Sachs equations) in Equations (4.2.12) and (4.2.13) and partially integrating these equations. The sum of the partial integrals is zero, as shown by Buhler [124].

The checks are, however, useful in ascertaining the accuracy of the calculations carried out, e.g., whether the tangents

$$\left( \frac{dA}{dF} \text{ and } \frac{d\sigma}{dF} \right)$$

are drawn correctly etc. Equations (4.2.12) and (4.2.13) are also used in finding the value of residual stresses between the final cut and the outside diameter in the case of the boring out method or between the final cut and bore diameter in the case of the turning down method.

4.2.4 EFFECT OF PARTING-OFF SAMPLES

Normally a small length is parted off from a cylindrical part for residual stress analysis, on the assumption that the residual stress pattern found in the specimen is representative of the stress distribution existing in the cylindrical part. It has been verified by Buhler [124] that if the length-diameter ratio of detached specimen exceeds 2 (or slightly less in the case of a thin wall tube), the parting off does not change the residual stress pattern in the specimen from that of the parent part. This fact has also been confirmed by Loxley [125].
4.2.5 PLASTIC YIELDING ON LAYER REMOVAL

There exist a number of examples of residual stress determination by Sachs method, where the calculated maximum stresses are considerably higher than the yield stress of the material in simple tension. In cases where the longitudinal and circumferential stresses are high and radial stress is low, this, as explained by Barker and Hardy [113], is caused by plastic yielding of the remaining material when a layer is removed from a region where residual stress level is at or very near to the yield value. They illustrate this point by taking a hypothetical case of a cylinder where longitudinal stress distribution is as shown in Figure 4.2.1 (a), radial and tangential stresses being zero.

A yield criterion is assumed, such that the residual stresses can nowhere be greater than \( x \), the original maximum residual stress. This is roughly the same as assuming the yield point of the material to be \( x \). After 30% area is bored out from the centre, stresses are redistributed, (Figure 4.2.1 b), to maintain equality of areas above and below the axis. This increases the tensile stress near the new bore to \( 1.4x \) which will, therefore, yield plastically to \( x \) which in turn further reduces the compressive stress near the outside diameter. In the Sachs method, the change in surface strain is the quantity determined experimentally. In this example the correct value would be \( \Delta S \) at the stage considered but the measured value would be greater, \( \Delta S' \). If now the values of \( \Delta S' \) so obtained are used to calculate the residual stresses by means of the Sachs equations, the resulting calculated stresses will be as shown in Figure 4.2.1 (c). It can be seen that the bore stress is
too high by a factor of 2 and the shape of the distribution curve is quite different from the actual one.

If \( b \) was measured in the bore, the cylinder being turned down, the outer surface stress would be high by a factor of 2.

The half of the stress distribution away from the surface removed by machining remains unaltered in both cases, no plastic deformation occurring in this region. Thus, the occurrence of plastic yielding in one part of the cylinder does not affect the calculated residual stress distribution in the part which has not yielded.

This fact can be used in finding the correct values of residual stresses in hollow cylinders where the stresses at the bore and the outside surface are very high, i.e., yielding is likely to occur when the cylinder is bored or turned down. The length of the cylinder is divided into two halves. One half is used to determine stresses by boring from inside and in the second half stresses are determined by turning from outside. In the former, yielding is likely to occur or it does occur near the bore but the stresses calculated at some distance from the bore up to the outside diameter are the true stresses. The correct stresses near the bore are given in the latter case when the stresses are calculated by turning down from outside. Thus by carrying out the stress determinations on two parts of the cylinder by boring out and turning down methods, the true residual stresses across the cylinder cross-section can be calculated.
Alternately an approximate estimate of the effect of plastic yielding can be made and the calculated stresses can subsequently be corrected \([113]\). In a hollow cylinder, for example, the residual stresses at the bore are such that \(\sigma_b\) and \(\sigma_t\) are very nearly of the same order and \(\sigma_r\) should be zero. Assuming the maximum shear stress criterion for yielding, yielding will occur when,

\[
(\sigma_b - \sigma_r) = \sigma_o
\]

where \(\sigma_o\) = yield stress in uniaxial tension.

In other words when \(\sigma_b\) exceeds \(\sigma_o\), yielding will occur.

Now if for a hollow cylinder the calculated residual longitudinal stress state is as indicated in Fig. 4.2.2, it is evident that yielding has occurred after material removal near the bore. The calculated residual longitudinal stress is, therefore, not the true value of the residual longitudinal stress. The calculated stress can be approximately corrected as follows:

Suppose the first boring step is of radius \(r\) (Fig. 4.2.3). Let \(S_r\) be the average value of calculated residual longitudinal stress in this layer. In fact \(S_r\) should never have been more than \(\sigma_o\) and the excessive value of \(S_r\) is caused from the excessive value of the observed surface strain due to plastic yielding. The additional observed surface strain, \(\Lambda a\), can be found thus,

\[
\frac{\pi(r^2 - r_0^2)(S_r - \sigma_o)}{\pi(R^2 - r^2)} = \frac{E}{1 - \nu^2} \Lambda a
\]
The observed surface strain $\Lambda$ can now be corrected, and the correct value $\Lambda c$ is given by

$$\Lambda c = \Lambda - \Lambda a$$

The correct value of longitudinal stress can be found by substituting $\Lambda c$ for $\Lambda$ in the appropriate Sachs equation.

Similar corrections can be made at other points where the calculated stress is more than $\sigma_o$ and approximately true longitudinal residual stress distribution can thus be found.

In the case of a solid bar, the residual stress state at the centre is such that $\sigma_r = \sigma_t$. If the Von Mises yield criterion is assumed, yielding will occur when

$$12 \left[ (\sigma_L - \sigma_T)^2 + (\sigma_T - \sigma_r)^2 + (\sigma_r - \sigma_L)^2 \right]^{1/2} = \sigma_o$$

Substituting $\sigma_t$ for $\sigma_r$, equation becomes

$$\sigma_L - \sigma_T = \sigma_o$$

Generally the value $\sigma_L - \sigma_T$ is not very high. It is, therefore, possible in this case that the values of $\sigma_L$ and $\sigma_T$ may be higher than $\sigma_o$ and still yielding would not occur.

4.2.6 CONCLUSION

The improved experimental techniques of material removal and strain measurements, and the use of computers to minimise calculation errors, have made Sachs boring method fairly reliable
for residual stress analysis for cases where axisymmetric stress distribution exists. Stresses determined by this method can sometimes be higher than the yield stress of the material in uniaxial tension. This can be either due to triaxial state of stress or due to plastic yielding of material caused by stress redistribution after material removal. From the calculated values of stress it is possible to find out whether the higher values of stresses are due to the state of stress or plastic yielding. Plastic yielding invalidates this method and the stresses calculated are not the true stresses existing in the material.
Residual stresses are produced in a body when it is unloaded from a state where it is undergoing non-uniform plastic deformation. If the state of stress in the body is known just before it is unloaded then, the residual stresses in the body can be determined. Suppose $\sigma_{ij}$ denotes the state of stress at a particular point at the instant when the loading is interrupted in the body. Let $\sigma'_{ij}$ be the system of stresses that would act if the body were elastic under identical external conditions. When the two stress systems are combined, then assuming no plastic deformation during unloading, the difference in stresses, $\sigma_{ij} - \sigma'_{ij} = \sigma''_{ij}$, defines the system of residual stresses after complete unloading.

The state of stress $\sigma_{ij}$ in a deforming body can be found by the application of the mechanics of plastic deformation. Much effort has been devoted to this subject resulting in several methods which give detailed mechanics when certain simplifying assumptions are made about the material and the way in which it is deformed. The mechanics of plastic deformation is, therefore, not an exact science and the solutions obtained e.g. state of stress $\sigma_{ij}$ in the deforming body, are only approximate. This approximation is also reflected in the residual stresses which are computed from the stress state $\sigma''_{ij}$. However, in spite of this lack of accuracy, the information obtained on the nature and distribution of residual stresses in a body as a function of certain process variables, is quite useful. In addition residual stress thus determined provide a useful comparison with those measured experimentally.
One method of determining the state of stress in plastic deformation zone of a deforming body is by the application of plain-strain slip-line field theory. There are, however, some limitations in applying this theory to the determination of residual stresses. As mentioned earlier, residual stresses arise as a result of elastic recovery of the material on emerging from the deformation zone. As the material is assumed rigid-plastic in the formulation of 'slip-line' theory, no elastic recovery can take place and hence there would not be any residual stresses. Even if this assumption is taken only as far as the determination of the state of stress in the deformation zone and ignored while calculating residual stresses, most slip line fields for plain-strain extrusion (of the type shown in fig. 4.3.1a) yield constant axial stress at exit boundary (line AC) which can only result in zero residual stress. In the case of curved slip line at the die opening (fig. 4.3.1b), however, the stresses vary along this line giving rise to the residual stresses in the deformed material. Some workers have estimated residual stresses using slip line fields of latter type (fig. 4.3.1b) but their conclusions about the nature of surface residual stresses are contradictory. In the slip-line field of fig. 4.3.1(b) axial stress is compressive at the surface at the die opening. According to Osakada et. al. [4] this can only result in compressive residual stress at the surface whereas surface residual stress is tensile according to their and most others' experimental findings. Using a similar type of slip line field (fig. 4.3.2) the analysis of Miura et. al [33], yields tensile residual stress at the surface. There are, however, two drawbacks in their analysis. First is their assumption of constant axial stress at the die opening (presumably the exit boundary of deformation zone) which itself rules out the existence of any residual stresses, and second, is the neglect of tangential and radial residual.
stresses in their calculations which if not carried out could lead to different results. From the work of the above authors it appears that in order to make use of slip-line field solutions for residual stress determinations, further work needs to be done.

4.3.2 VISIOPLASTICITY

Visioplasticity is an experimental-analytical technique for obtaining information on the mechanics of the deforming body, based on the observations of the rate of distortion of grid patterns applied to the billet prior to the deformation being studied. This technique was used by Frisch & Thomsen [22] to determine the state of stress in the deformation zone and then to derive residual stresses from this state of stress, in an extruded aluminium rod. Their analysis yielded compressive residual stresses on the surface and tensile residual stress in the central regions of the rod. This residual stress distribution had good qualitative agreement with their experimental results.

The actual measurement of the deformed grid patterns to obtain complete solutions is tedious and time consuming and there is no way of checking the accuracy of these measurements. These difficulties are overcome to some extent, by the flow function method of analysis which enables a computer to be programmed for carrying out the detailed calculations. The only experimental data used in this method are the position of the axial grid lines (flow lines). Velocity fields are computed from the analysis of the axial grid lines data. Once the velocity fields are known, strain rate fields and stress fields can be computed by the application of equilibrium equations and stress-strain rate relations. Certain checks can be applied on the accuracy of the data obtained by this
method by, for example, comparing the positions of the actual transverse grid lines with those computed from the velocity fields. After applying this and other similar checks Medrano & Gillis [51] concluded that this method yields fairly accurate information on the mechanics of the deforming body.

4.3.3 DETERMINATION OF THE MECHANICS OF EXTRUSION FROM A PROPOSED THEORETICAL MODEL

The accuracy of information furnished by the application of flow function analysis to viscoplasticity depends on the accuracy in the observation of flow lines and in determining the deformation boundaries. It is not always easy to determine accurately the position of flow lines and the deformation boundaries from a deformed billet. In addition, the requirement that an experiment has to be performed in order to observe the flow pattern sets certain limitation on the application of the above method.

Lambert & Kobayashi [55] have proposed a theoretical model of the extrusion process from which information on the detailed mechanics can be obtained. This is based on choosing a certain flow pattern for the material shown, for example, in fig. 4.3.3, with arbitrary chosen velocity discontinuity curves AQ & BP. A large number of such flow patterns e.g. BP, AQ, BP, AQ etc. (fig. 4.3.4) are then superimposed using flow function analysis. The superposition of a large number of flow patterns results in nearly complete elimination of velocity discontinuities in the admissible velocity fields. Velocities and strain rates are calculated for the resultant deformation pattern BP, AQ and these are used to determine the upper bound load. This procedure is repeated with different sets of values of 'd, de'. The set of values of 'd, de' which gives the least
upper bound load is taken to be the optimum for the deformation pattern. Using the optimum values of \( d_1 \), \( d_2 \) the state of stress in the deformation zone is computed and residual stresses derived therefrom.

In the present investigation the theoretical determination of residual stresses has been carried out by the above method. This method is, therefore, described here in some detail.

For axisymmetric deformation, a function \( \psi \) of \( r \) and \( z \) can be defined such that:

\[
\frac{u}{r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \text{and} \quad \vartheta = -\frac{1}{r} \frac{\partial \psi}{\partial r} \quad 4.3.1
\]

Where \( u \) and \( v \) are the radial and axial velocity components in the cylindrical co-ordinate system \((r, \theta, z)\), and the circumferential velocity is, of course, zero.

Then \( \psi \) always satisfies the incompressibility condition:

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial z} = 0
\]

It can be shown that by defining velocity components by \( \psi \) as in Eq. 4.3.1, the condition of continuity across the velocity discontinuity is also satisfied.

Then a velocity field which is defined by \( \psi \) has only to satisfy the boundary conditions to be admissible. Thus the selection of \( \psi \) for an admissible velocity field is arbitrary so long as it satisfies the boundary conditions. Since the flow lines can be expressed by \( \psi = C_k \), where \( C_k \) is a constant for each flow line, the arbitrary selection of \( \psi \) implies that the flow lines can be drawn arbitrarily with specified boundary conditions. As for example, the fields characterized by flow lines in Fig. 4.3.5 a, b, c, for a
particular extrusion condition are all admissible velocity fields.

From Eq. 4.3.1 the flow functions \( \psi_1 \) in region 1 and \( \psi_3 \) in region 3 (fig. 4.3.3) are given by

\[
\psi_1 = \frac{r^1}{2} \quad \text{and} \quad \psi_3 = \frac{r^1}{2}\alpha.
\]

(4.3.2)

In region 2 the flow lines are

\[
z = or + d
\]

(4.3.3)

Where coefficients \( \alpha \) & \( d \) are determined for each flow line from given velocity discontinuity curves.

Velocity fields derived from the flow pattern of fig. 4.3.5 (c) are reported to have best agreement with those determined experimentally [55]. This type of flow pattern has therefore been chosen in the present investigation. The deformation boundaries of the flow pattern are curved. Their equations, chosen to be of 2nd order, are derived as follows:

Referring to fig. 4.3.3 let the equations of boundaries AQ and BP be

\[
z = g_2(r) = Ar^2 + Br + c \quad \text{and} \quad z = g_1(r) = Kr^2 + L
\]

respectively

Then in region 2, the flow line is given by

\[
z = \frac{g_2(R_k) - g_1(R_k)}{(1 - b) R_k} r + \frac{g_1(b R_k) - b g_1(R_k)}{1 - b}
\]

(4.3.4)

or

\[
z = \frac{AR_k^2 + BR_k + C - (K^2 R_k^2 + L)}{(1 - b) R_k} r + \frac{(K^2 R_k^2 + L) - b(AR_k^2 + BR_k + c)}{1 - b}
\]

(4.3.5)

The condition that the order of the equation in \( R_k \) should not exceed 2 (to facilitate an explicit formulation for \( R_k \) in region 2), permits the elimination of one unknown coefficient, while the four that
remain can be found from the points $A, B, P, Q$ in Fig. 4.3.3. In this way the boundaries $ADQ$ and $BCP$ have their respective equations

$$z = g_2(r) = \left(\text{Cot} \alpha - \frac{d_1}{b}\right) r^2 + d_1 \left(\frac{1}{b} - m\right) r + md_1$$

and

$$z = g_4(r) = \left(b \text{Cot} \alpha - d_1\right) \left(\frac{r^2}{b}\right) + d_1$$

Substituting Equation 4.3.6 in Eq. 4.3.4, $R_K$ can be found in region 2. The flow function in this region is given by

$$\psi_2 = \frac{1}{2} (R_K)^2$$

### 4.3.4 SUPERPOSITION OF FLOW PATTERNS

The superposition of the basic flow patterns of Fig. 4.3.3 is shown in Fig. 4.3.4. The points $P_1$ and $P_n$ are selected arbitrarily and the number of intermediate points, depending on the number of patterns $n$, is equally distributed as

$$d_i = d_1 + \frac{1}{n-1} (d_n - d_1)$$

The corresponding points $Q_i$ are determined by $m$.

A mesh point system is selected as shown in Fig. 4.3.6 with the spacing of the mesh points being minimum allowed by the capacity of the computer. Table 4.3.1 gives the spacings of the mesh points in $r$ & $z$ directions for various extrusion conditions.

At each mesh point $R_K$ is calculated for each of the $n$ flow patterns. The superposition of $R_K$ is made according to

$$\psi = \frac{1}{2} (R_K)^2 = \frac{1}{2} \left[\frac{1}{n} \left(R_{K_1} + R_{K_2} + \ldots + R_{K_n}\right)\right]^2$$

Having computed $R_K$ at each mesh point, the velocities are calculated according to Eq. 4.3.1 analytically or where required by numerical differentiation. Strain rates are then calculated by

$$\dot{\varepsilon}_r = \frac{\partial u}{\partial r}, \quad \dot{\varepsilon}_\theta = \frac{u}{r}, \quad \dot{\varepsilon}_z = \frac{\partial u}{\partial z}$$

and

$$\dot{\gamma}_{r_2} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}$$

(4.3.7)
Assuming constant $\overline{\sigma}$, the external work rate $\dot{E}_a$, according to upper bound theorem is expressed by

$$\dot{E}_a \leq \overline{\sigma} \int \dot{\varepsilon}(\gamma, \beta) \, d\gamma + \frac{\kappa'}{\beta} \int V_T \, dS$$

(4.3.8)

Where $\kappa'$ is a constant friction factor (between 0 & 1), and $V_T$ is the absolute velocity along the die surface.

The first term in equation 4.3.8 expresses the power for internal deformation over the volume of the billet and the second term gives the shear power over the die surface.

Expressing $\dot{E}_a = p_{av} A_0 |v_o|

Where $p_{av}$ is the average extrusion pressure, $A_0$ the initial x-sectional area of the billet and $v_o$ the ram velocity, and introducing the values of $A_0 = \pi$ and $v_o = -1$, the upper bound for the extrusion pressure is given by

$$p_{av} = \frac{2}{\overline{\sigma}} \left[ \int \dot{\varepsilon}(\gamma, \beta) \, d\gamma \, d\beta \right] + \frac{2k'}{\beta} \int V_T \, d\gamma$$

$$= \frac{p_{av}}{\overline{\sigma}} + \frac{p_{av}}{\overline{\sigma}}$$

4.3.5 SELECTION OF PARAMETERS $d_1$ AND $m$ OF THE BOUNDARIES OF DEFORMATION ZONE

If the boundaries of the deformation zone $BP_1$ and $AQ_n$ (see Fig. 4.3.8) are assumed to be perpendicular to the axis of symmetry and not of the form given by equations 4.3.6 then,

$$d_1 = b \cot \omega$$

and

$$d_c = \cot \omega$$

since $d_c = m^2 d_1$ (see Fig. 4.3.4)

$$m = \sqrt{\frac{1}{d_1}}$$

(4.3.8 a)

The experimental evidence in the literature suggests that the points $P_1$ and $Q_n$ of the boundaries of the deformation zone generally lie away from the positions indicated in Fig. 4.3.8, in the direction opposite to the extrusion direction. This deviation increases with
the increase in friction along the die surface.

In the present investigation the rods were extruded under normal industrial conditions and no attempt was made to make the experimental conditions frictionless. The minimum value of \( d_1 \) was taken to be \( 1.05 \times b \cot \alpha \) which is very close to that indicated in Fig. 4.3.8. The value of \( d_1 \) was optimised between this value and the arbitrarily chosen maximum of \( 1.5 \times b \cot \alpha \).

The values of \( m \) were varied equally on both sides of the value given by the expression 4.3.8A.

4.3.6 **OPTIMISATION OF \( d_1, de \) VALUES**

The minimum and maximum values of \( d_1 \) are set out to be

\[
d_1 \text{ min} = 1.05 \times b \cot \alpha \\
d_1 \text{ max} = 1.5 \times b \cot \alpha
\]

Total of 5 values of \( d_1 \) are used in the optimisation procedure starting with \( d_1 \text{ min} \) and then increasing successively by the interval \( (d_1 \text{ max} - d_1 \text{ min})/4 \). With each value of \( d_1 \), 5 values of \( m \) are used so that there are five values of \( de \) \( (=m \times d_1) \) for each value of \( d_1 \). The value of \( 'm' \) is set out to be

\[
m \text{ min} = L - 0.2 \\
m \text{ max} = L + 0.2
\]

where \( L = \sqrt{\frac{1}{b}} \)

The five values of \( 'm' \) used with each value of \( d_1 \) are, starting with \( m \text{ min} \) and then increasing successively by the interval \( m \text{ max} - m \text{ min} / 4 \).

So for each extrusion condition i.e. extrusion ratio and die angle, 25 sets of \( (d_1, de) \) values are used and \( \rho_0 \nu / \sigma \) calculated for
each \((d_1, \delta_e)\) value, taking \(k' = 0\). The values \(d_1, \delta_e\) which give the minimum \(\rho_0\) are taken to be the optimum at that particular extrusion condition for the frictionless case.

In the present investigation rods were not extruded under frictionless conditions but also no attempt was made to determine the actual friction conditions. In the absence of any friction data, it was not possible to determine the \(d_1, \delta_e\) values and hence compute residual stresses for the actual extrusion conditions. To overcome this difficulty residual stresses at a certain extrusion ratio and die angle were computed for a number of \(d_1, \delta_e\) values, with \(d_1\) deviating on either side of the optimum \(d_1\) for frictionless case. The number of \(d_1, \delta_e\) values at which residual stresses were computed varied between six and twelve depending on the nature of the agreement in the first place, between the experimentally determined residual stresses and those calculated theoretically for the frictionless case. The \(d_1, \delta_e\) values for a certain extrusion condition which gave the best agreement with the experimentally determined residual stresses were taken to be the optimum for that particular extrusion condition.

### 4.3.7 Determination of the Stress Distributions in Deformation Zone

Since the strain rates are known at every mesh point according to Eq. 4.3.7, one can solve for the four stresses from the two equilibrium equations \((\tau_2 \theta = \gamma_0 \beta_2 = 0\) being zero by symmetry),

\[
\begin{align*}
\frac{\partial \tau_1}{\partial \gamma} + \frac{\partial \gamma_2}{\partial \beta_1} + \sigma_\gamma - \frac{\sigma_\beta}{\gamma} &= 0 \\
\frac{\partial \gamma_2}{\partial \gamma} + \frac{\partial \gamma_2}{\partial \beta_2} + \frac{\tau_1 \beta_2}{\gamma} &= 0
\end{align*}
\]

(4.3.9)
and the three stress-strain rate relations:

\[ \sigma_\gamma = \sigma_{\theta} + \frac{2}{3} \bar{\sigma} \left( \frac{\dot{\varepsilon}_\gamma - \dot{\varepsilon}_\theta}{\bar{\varepsilon}} \right) \]
\[ \sigma_{\theta} = \sigma_\gamma + \frac{1}{3} \bar{\sigma} \left( \frac{\dot{\varepsilon}_\theta - \dot{\varepsilon}_\gamma}{\bar{\varepsilon}} \right) \]
\[ \tau_{\gamma \theta} = \frac{1}{3} \left( \frac{\gamma_{\gamma \theta}}{\bar{\varepsilon}} \right) \]  
(4.3.10)

The shear stress \( \tau_{\gamma \theta} \) at each mesh point is found by the last equation of 4.3.10. The normal stress components are determined as follows:

Referring to fig. 4.3.7 let \( R (o,a) \) be a reference point inside the deformation zone and let \( P(r,z) \) be any point at which the normal stress components are to be found. These can be expressed in terms of the known strain-rate components and stresses at the reference point \( R \) by integrating the equilibrium equations along the path \( RQP \). The axial stress at the point \( R \) is found from the condition that the load in the axial direction over the surface \( AC \) is zero. The other two stresses \( \sigma_\gamma \) & \( \sigma_\theta \) at point \( R \) are then, found from equations 4.3.10.

At point \( Q, \sigma_\gamma \) is determined by integrating the first of the equilibrium equations along \( HQ \) and using the stress-strain rate relations of 4.3.10. The other two stress components at \( Q \) are then found by equations 4.3.10. At point \( P, \sigma_\gamma \) is computed by integrating the second equilibrium equation along \( QP \), the other two stresses being found by equations 4.3.10.

4.3.8 RESIDUAL STRESS DETERMINATIONS

The normal stress components at the exit boundary deformation zone are calculated by interpolation from known at the mesh points on either side of the exit boundary. constitute the state of stress \( \sigma_{\epsilon_{ij}} \) at the instant when is interrupted i.e., the material is about to leave the
zone and become the extruded rod. On emerging from the deformation zone the material undergoes elastic stress relaxation. Because of the non-uniform distribution of $\sigma_{r'y}$, the material remains residually stressed after the elastic stress relaxation has taken place. Let $\sigma_{r'y}$ be the state of residual stress in the extruded bar. Then the static equilibrium requires that

$$
2\pi \int_0^L \sigma_{\theta'} (r,z) r \, dr = 0
$$

$$
\int_0^L \sigma_{\theta} (r,z) \, dr = 0, \text{ and}
$$

$$
\sigma_{\theta} (r,z) = \sigma_{r'} (r,z) + r \frac{d\sigma_{r'} (r,z)}{dr}
$$

Assuming elastic stress relaxation to be uniform, the residual stress components are given by

1. $\sigma_{v'} (r,z) = \sigma_{3'} (r,z) - \sigma_{3'} (r,z)$

$$
\text{where } \sigma_{3'} = \frac{2\pi}{\pi \xi^2} \int_0^L \sigma_{v'} r \, dr
$$

2. $\sigma_{\theta} (r,z) = \sigma_{\theta} (r,z) - \sigma_{\theta} (r,z)$

$$
\text{where } \sigma_{\theta} = \int_0^L \sigma_{\theta} \, dr
$$

and

$\sigma_{r'}$ is calculated as follows:

Solving the last equation of 4.3.11 for $\sigma_{r'}$ we have:

$$
\frac{d}{dr} (r \sigma_{r'}) = \sigma_{\theta}
$$

or

$$
\left. r \sigma_{r'} \right|_0^R = \int_0^L \sigma_{\theta} \, dr
$$

Where $R$ is any radius at which $\sigma_{r'}$ is to be determined.
Then, \( (\sigma_T'')_{r=0} = \frac{4}{\pi} \int_0^R \sigma_0'' \, dr \), \hspace{1cm} (4.3.14)

\[ \text{because} \quad (\sigma_T'')_{r=L} = 0 \]

4.3.9 MAGNITUDE OF RESIDUAL STRESS

The theoretical analysis gives residual stresses in terms of the stress ratios \( \sigma''/\sigma \) where \( \sigma \) is the effective stress. A constant effective stress for each extrusion condition (i.e. extrusion ratio and die angle) is assumed and is taken to be the average of the yield stress of the billet and the extruded bar. By multiplying the stress ratios with this value of effective stress, the magnitude of residual stresses is calculated along the cross section of the bar.

4.3.10 COMPUTER PROGRAMMES

Three main computer programmes were used to carry out the residual stress analysis theoretically. Two out of these were adaptations of the Lambert & Kobayashi's [49] programmes, and the third programme for computing the residual stresses from the known stress distribution along the exit boundary of the deformation zone was written and is given in Appendix IV. The programmes used were as follows:

1. MAIN
2. MAIN 4  \hspace{1cm} \text{Adapted from Ref. [49]}
3. RESIDUAL

The flow chart incorporating the main points of these programmes is given in Fig. 4.3.9.
Fig. 4.2.1 Hypothetical stress distribution demonstrating the effect of plastic yielding.

Fig. 4.2.2

Fig. 4.2.3
### Table 4.3.1  Mesh Point Spacings used in the Computer Programmes

<table>
<thead>
<tr>
<th>Extrusion Ratio</th>
<th>Die Angle</th>
<th>Mesh Point Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r-DIRECTION</td>
<td>z-DIRECTION</td>
</tr>
<tr>
<td>2.25</td>
<td>60</td>
<td>0.019</td>
</tr>
<tr>
<td>&quot;</td>
<td>90</td>
<td>0.026</td>
</tr>
<tr>
<td>&quot;</td>
<td>120</td>
<td>0.026</td>
</tr>
<tr>
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<td>60</td>
<td>0.019</td>
</tr>
<tr>
<td>&quot;</td>
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</tr>
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<td>0.02</td>
</tr>
<tr>
<td>&quot;</td>
<td>90</td>
<td>0.025</td>
</tr>
<tr>
<td>&quot;</td>
<td>120</td>
<td>0.027</td>
</tr>
<tr>
<td>4.00</td>
<td>60</td>
<td>0.02</td>
</tr>
<tr>
<td>&quot;</td>
<td>90</td>
<td>0.025</td>
</tr>
<tr>
<td>&quot;</td>
<td>120</td>
<td>0.025</td>
</tr>
</tbody>
</table>
FIG. 4.3.1 SLIP LINE FIELD FOR 
PLANE STRAIN EXTRUSION

FIG. 4.3.2 DISTRIBUTION OF HYDROSTATIC 
PRESSURE IN A SLIP LINE FIELD FOR SHEET DRAWING

FIG. 4.3.3 A FLOW PATTERN FOR 
ASYMMETRIC EXTRUSION
**Fig. 4.3.4** Superposition of Flow Patterns

**Fig. 4.3.5** Various Admissible Velocity Fields at a Certain Extraction Ratio and Dr Angle

**Fig. 4.3.6** Meshpoint System

**Fig. 4.3.7** Stress Integration Path

**Fig. 4.3.8**
FLOW CHART

Input data: EXTRUSION RATIO, DIE ANGLE, NUMBER OF PATTERNS TO BE SUPERIMPOSED AND NUMBER OF MESH POINTS AT THE EXIT SECTION

GENERATION OF DATA (d₁, de etc.) FOR 25 DEFORMATION ZONES at a particular extrusion ratio and die angle. SUPERPOSITION OF PATTERNS for each set of d₁, de values, CALCULATION OF FLOW FUNCTION, velocities and strain rates at mesh points

Calculation of upper bound extrusion pressure

SELECTION OF OPTIMUM SET (d₁, de)

INPUT DATA - SAME AS FOR 'MAIN' + STRAIN RATES COMPUTED IN 'MAIN' and ε

Calculation of longitudinal, tangential and radial stress components at all mesh points, calculation of these stress components along the exit boundary of the deformation zone by interpolation

INPUT DATA - longitudinal, tangential and radial stress components along the exit boundary of the deformation zone computed in 'MAIN 4'

Calculation of longitudinal, tangential and radial residual stress distributions along the cross section of the extruded rod

Fig. 4.3.9
CHAPTER 5

Strain Measurements - Instrumentation

and Experimental Technique
5.1 STRAIN MEASUREMENTS - INSTRUMENTATION

5.1.1 ELECTRICAL RESISTANCE STRAIN GAUGES AND STRAIN MEASURING BRIDGE

The electrical resistance strain gauges have been the most popular strain sensing devices to be used in the residual stress-analysis by Sachs boring-method since they became commercially available. They owe their popularity to their small size, high sensitivity and accuracy, and the ease of handling. There are several types of resistance gauges which are commercially available but in most static stress analyses, where ultra-precise measurement is not a major problem, the most rugged and economical metal-foil strain gauges are generally used.

An electrical resistance strain gauge acts only as a strain sensing device or a pick-up, i.e. it converts the mechanical quantity (strain) into an electrical quantity (resistance), which are related to each other by the gauge factor as shown in the following relationship:

\[
S_g = \frac{\Delta R}{R} e_{xx} \\
\text{when } e_{yy} = -\mu e_{xx}
\]

Here \(S_g\) = gauge factor

\(R\) = electrical resistance of the strain gauge

\(e_{xx}\) = strain in the direction of x-axis along which the strain gauge is mounted

\(e_{yy}\) = strain in the direction of the y-axis

\(\mu\) = Poisson's ratio of the material in which the strain measurements are being carried out.
In order to apply the electrical resistance strain-gauge in any experimental stress-analysis, the quantity \( \frac{\Delta R}{R} \) must be measured and converted to the strain which produced the resistance change. Two electrical circuits, the potentiometer and the Wheatstone bridge, are commonly employed to convert the value \( \frac{\Delta R}{R} \) to a voltage signal (\( \Delta E \)) which can be measured with a recording instrument.

A null-balance type bridge with a commercial circuit based on the Wheatstone-bridge principle was used in the present investigation. The condition of balance of the bridge circuit could be brought about by switching the measuring switches to different positions, and the balanced condition was detected visually on a 'cathode ray tube'. The difference between the initial and final readings of the switches (before and after the application of strain) gave the value of strain in \( \mu \) in/in. The bridge was designed for a gauge factor of 2 and if the gauges used had a different gauge factor, say \( S_g \), the true values of strain readings could be obtained by applying a correction as follows:

\[
\text{True strain} = \text{Measured strain} \times \frac{2}{S_g}
\]

5.1.2 SELECTION OF STRAIN GAUGES

The main consideration which influenced the choice of gauges and, in fact, the cement and the whole installation procedure, was the duration of the read-out period. It was necessary to record strains over periods of up to two months, while the specimens were constantly under strain and there was no opportunity to check the zero reading of the gauges during this period. All factors which influence the behaviour of strain gauges, such as temperature, humidity, and gauge creep, can have a significant effect on strain
measurements when the read-out periods are long. All possible precautions were, therefore, taken to minimise the influence of these factors.

120 ohm "Advance" foil gauges with epoxy backing were used throughout the investigation. "Advance" is a copper-nickel alloy (55% Cu, 44% Ni and 1% Mn) which has the following advantages:

1. The value of the strain sensitivity \( (S_a) \) is linear over a wide range of strain and it does not change as the material goes plastic. This implies that the gauge calibration will not vary with the strain level - therefore, a single calibration constant is adequate. A typical curve showing the percentage change in resistance \( \frac{\Delta R}{R} \) as a function of strain for this alloy is reproduced from [128] in Figure 5.1.1.

2. The alloy has a high specific resistance \( \rho = 49 \times 10^{-6} \text{ ohm/cm} \). This is beneficial in constructing a small gauge with a relatively high resistance as is evident from the relationship,

\[
R = \rho \frac{L}{A},
\]

where \( L \) is the length of a uniform conductor with a cross-sectional area \( A \).

3. The alloy itself has an excellent thermal stability and is not influenced appreciably by the temperature changes, when mounted on commonly employed structural materials.

Foil gauges were used instead of wire gauges since foil gauges place less stress on the cement and hence limit the creep. For long-term use it is better to use gauges with maximum possible length because longer gauges have better heat-dissipation properties.
and they also tend to minimise the stress placed on the adhesive. Gauges with 15 mm x 4 mm overall dimensions were used. This was the maximum length which could be accommodated on the specimen along with the terminal strips, as shown in Figure 5.1.2. After the gauge installation, the room left on the ends was just sufficient for gripping the specimens in the fixture for machining.
5.2 STRAIN MEASUREMENTS - EXPERIMENTAL TECHNIQUE

5.2.1 PREPARATION OF SPECIMENS

Specimens in the form of rods approximately 2\(\frac{1}{4}\)" (63.5 mm) long were taken from the region in the extruded rods where the hardness distribution had reached a steady state. These specimens were parted-off from the extruded rods with a mechanical saw using a copious supply of coolant. Ends were then faced, avoiding any heating of the specimens. The curved surfaces were then hand polished using 120 and 220 grade silicon-carbide paper to remove the phosphate coating and to get an appropriate rough surface for proper adherence of the strain gauges on them.

The greater part of the investigation for stress-analysis was carried out by metal removal from inside in steps until a very thin shell of approximately 0.015" (0.38 mm) thickness was left. The shell was then slit and tongues were cut out from it to relieve the stress in the outer layers. However, to determine the surface stresses more accurately experiments were also carried out in which specimens were first bored to a diameter (without any strain gauges on them) such that it was convenient to mount the strain gauges in the bore and the strain measurements were taken while the specimens were turned from outside. Changes in strains, due to metal removal were measured in the former case by two pairs of strain gauges mounted in longitudinal and circumferential directions while only two gauges were used in the outer layer removal method.
5.2.2 MOUNTING OF STRAIN GAUGES:

Specimens were carefully polished and the surfaces intended for mounting the strain gauges and the strain gauges themselves were cleaned in trichloroethylene and dried with a gentle blast of warm air. For external mounting, surfaces were marked in the axial and circumferential directions by scratching lines which were later coincided with the centre line of gauges for their accurate positioning. An epoxy cement was used and the recommended proportions of monomer and hardening agent were carefully weighed and mixed thoroughly. A thin layer of adhesive was applied on the back surface of the strain gauges and these were then placed in the identified positions. The gauges were then cured at room temperature under the recommended pressure applied through a cushion of foamed polyurethane.

The procedure was slightly different for inside mounting. Slits were cut in a piece of paper which simulated the inside surface of the specimen. These slits were the corresponding intended positions of the gauges. Gauges were then positioned in these slits by using cellulosic tape. A thin layer of cement was applied on the back of the gauges and the whole assembly (paper and gauges) was carefully placed in the bore of the specimen. Pressure in this case was applied by inflating a rubber tube in the bore, following the method used by Bühler [124]. The inflated tube was left in position for twenty-four hours for the cement to be cured under pressure.

Terminal strips were used to provide intermediate connection between the lead-out wires from the strain gauge and the connecting-cables. Lead-out wires and connecting-cables were soldered to the
terminal strips prior to the mounting of gauges. This was done to avoid the possibility of damage to the adhesive properties of the cement by overheating while soldering was carried out.

5.2.3 GAUGE PROTECTION:

Gauges and terminal strips were covered (after curing) with a thin layer of adhesive used for cementing the gauges. This adhesive was recommended by the manufacturer for use as a protective coating as well and was claimed to have excellent protection against humidity and mechanical damage. The terminal strips provided additional protection of gauges against disturbance caused by the handling of connecting cables during measurements. After the complete installation procedure described above, specimens were left for at least a week before any boring was started. This was done to allow complete polymerisation of the cement and thus, to prevent any zero shift in the gauges once the boring was started.

5.2.4 CALIBRATION OF GAUGES:

Following calibration procedure was followed for strain gauges.

The strain gauge was cemented on a 0.5 in (12.7 mm) nominal diameter tensile specimen machined from an annealed EN-2E rod. The specimen was gripped in a 50 ton (500 KN) tensile testing machine*

* This machine is calibrated to British Standard: 1610, 1964.
and loaded gradually up to 2.5 tons (25 KN) corresponding to the stress below the yield stress of the material. Strain readings were taken from the strain-bridge corresponding to 0.25 ton (2.5 KN) increment in lead. These were then plotted against load as shown in Figure 5.2.1. A linear relationship between the recorded loads and strains was obtained which was how it should have been, as the load applied was below the yield point of the material.

As the strain measured by a strain gauge is a function of its resistance change, the relationship indicated in Figure 5.2.1 is, in fact, the relation between the resistance change of the gauge and the load applied. In other words, the linear relationship of Figure 5.2.1 also established that the resistance change of the strain gauge was proportional to the applied load (or strain).

The gauge factor ($S_g$) was specified by the manufacturer. For comparison it was actually calculated from the data obtained in the above test. This is illustrated in the following analysis which is based on the load and strain readings corresponding to 1.5 ton load.

Actual diameter of specimen $\rho = 0.50126$ inch
Cross-section area $A = 0.19734$ sq.in.
Strain-bridge reading corresponding to 1.5 tons load $e_B = 0.000607$ in. per in.

Actual strain corresponding to 1.5 ton load (taking modulus of elasticity, $E = 30 \times 10^6$ psi)

$$e_A = \frac{1.5 \times 2240}{0.19734 \times 30 \times 10^6} = 0.000567$$
The difference between $e_B$ and $e_A$ can be due to two factors,

(a) the strain-bridge gives erroneous readings, or

(b) the gauge factor $S_g$, of the strain gauge is not the same for which the strain-bridge is calibrated.

Assuming that the bridge is not faulty, the gauge factor $S_g$, is given by

$$e_A = \frac{e_B \times 2}{S_g}$$

where 2 is the gauge factor for which the strain-bridge is calibrated.

Using actual data in the above equation

$$0.000567 = \frac{0.000607 \times 2}{S_g}$$

or

$$S_g = \frac{0.000607 \times 2}{0.000567} = 2.14$$

which is the same as specified by the manufacturer (2.14 ± 0.01).

Gauges were supplied in packets of ten. Calibration on a gauge from another batch yielded similar results. The gauges from other batches were, therefore, not calibrated and the gauge factor specified by the manufacturer was used for strain measurements.

5.2.5 CALIBRATION OF STRAIN-BRIDGE:

The bridge was designed and fixed for a gauge factor ($S_g$) of 2.0. This meant that the bridge used in conjunction with a strain gauge of gauge factor 2.0 was supposed to satisfy the relationship
(5.1.2) i.e.,

\[ e_B = \frac{\Delta R/R}{S_g} = \frac{\Delta R/R}{2} \quad \text{as} \quad S_g = 2 \quad (a) \]

where \( e_B \) is the strain indicated by the bridge.

The purpose of the calibration was to check whether the above relation was satisfied and if \( e_B \) was not related to \( \Delta R/R \) by a factor of 2.0, then to find the actual value of this factor.

This was done by introducing known resistance change in the arm of the Wheatstone-bridge containing the active strain gauge. A resistance of 60,000 ohms was connected in parallel with the 120 ohm strain gauge. The difference between the bridge readings taken before and after connecting the additional resistance in parallel with the strain gauge was 1,000 micro-inch/inch, i.e.

\[ e_B = 0.001 \text{ inch/inch} \quad (b) \]

The value \( \Delta R/R \) was calculated as shown below:

\[ R = 120 \text{ ohm} \]
\[ \Delta R = 120 - \left( \frac{1}{60,000} + \frac{1}{120} \right) \]
\[ = 120 - \frac{60,000}{501} = \frac{120}{501} \text{ ohm}. \]
\[ \Delta R/R = \frac{1}{501} \quad (c) \]

Substituting (b) and (c) into (a)
\[ -0.001 = \frac{1}{501} \times \frac{1}{2} \]

\[ \approx 0.001 \]  
(exact value = 0.000998)

Performing a similar analysis with 30,000 ohm resistance in parallel with 120 ohm strain gauge yielded similar results. It was therefore established that the strain-bridge was accurately calibrated by the manufacturer.

5.2.6 MATERIAL REMOVAL AND STRAIN MEASUREMENTS:

As conventional machining is thought to introduce stresses, thus altering the stress pattern existing in the material, it was first decided to bore out material by electro-chemical machining (E.C.M.). A rig was designed for this purpose but had to be abandoned because, with the facilities available, the material could not be machined with a constant diameter along the length of the specimen; also the holes machined were not concentric.

Two important items of equipment required for E.C.M. are a corrosion resistant pump capable of pumping electrolyte at more than 150 lbf/in² (1.0342 MN·m⁻²) and a rectifier to give constant values of high current of the order of 150 amperes and low voltage 10 - 20V. These items are very expensive and it was, therefore, thought to be a better economic proposition to get the E.C.M. done from some external source. The project of material removal was then given to PERA (Production Engineering Research Association, Melton Mowbray) since they had proper facilities for E.C.M. The accuracy of holes machined by them was much better than those attempted at
the departmental laboratories, but it was not good enough for the purpose. The accuracy desired was ±0.001 inch (0.025 mm) on diameter and the same on the concentricity of the hole, but the accuracy achieved by PERA was of the order of ±0.005 inch (0.125 mm) both on diameter and concentricity. After these unsuccessful attempts, the attempt to use E.C.M. for material removal was abandoned.

The possibility of using acid etching for material removal was then explored and after a few successful preliminary tests, it was decided to use the method throughout the investigation. A rig was designed for the purpose (Figures 5.2.2. a, b), similar to the one used by Botros [125]. In this equipment the 15% aqueous solution of nitric acid is pumped through a rotating perspex chamber containing the specimen. The advantage of pumping acid solution under pressure is that it prevents deposition of hydrogen bubbles on the specimen and to some extent removes the oxide layer as well, thus attacking the exposed surface more or less uniformly.

Strain gauges and the unexposed surfaces of the specimen were protected from the acid attack as shown in Figure 5.2.3.

The procedure followed for metal removal was as follows:

Rods were initially bored out to approximately 0.25" (6.35 mm) by conventional machining. The starting hole in each case was 1/16" (1.59 mm) diameter which was enlarged to 0.25" (6.35 mm) diameter in steps of 0.02" (0.5 mm) using end mills. Extreme care was taken for the drills and end mills to be sharp and for the
specimen to be kept cool by copious use of lubricants. Boring was carried out on a lathe using a fixture in which the specimens were clamped lightly on the outside diameter at both ends. The fixture was designed for the biggest diameter of specimen and the smaller diameter specimens were held in the fixture using brass split rings on the outside diameter.

Attempts made to enlarge smaller diameter than 0.25 inch (6.35 mm) hole by acid etching were not satisfactory because the smaller diameter holes were not uniformly etched. However, most of the material from specimens for bore diameters greater than 0.25 inch was removed by etching. Though the bore diameter after each etching operation was fairly uniform along the axis, yet small inaccuracies were taken care of by measuring the diameter at three places along the specimen length and taking the average of the three readings. To avoid accumulation of inaccuracies in diameter, the bores were corrected by conventional machining after two or three consecutive etching operations. This sequence was continued until about 0.05 inch (1.25 mm) wall thickness was left which was thinned down to about 0.015 inch (0.38 mm) by etching only. Tongues were subsequently cut out from these thin shells and saw cuts were made on two sides of the longitudinal strain gauges to release the stresses from the final shell. A typical sequence of metal removal is shown in Table 7.2.1 in Chapter 7.

The specimens for stress determination by material removal from outside, were first polished by silicon carbide paper to remove the phosphate coating. Etching was then carried out from outside in the same rig used for internal etching. The strain gauges, connecting cables, the bore of the specimen and the sides were
protected from-acid attack as shown in Figure 5.2.4. External
diameters were corrected after 2 - 3 consecutive etching operations
by turning the specimen on a lathe. A typical sequence of metal
removal operations is shown in Table 7.2.1 in Chapter 7.

Following precautions were taken to minimise errors in
strain measurements due to temperature changes and change in contact
resistance between gauge and strain-bridge connecting wires.

1. After each metal removal operation, specimens were kept in a
temperature-controlled room for eighteen hours, before the strain
readings were taken. This minimised the temperature differences
between the specimens under investigation, and the dummy specimen.
The strain measuring bridge and the dummy specimen were stored
permanently in the temperature controlled room.

2. The contact resistance was maintained constant by the use of mercury
pools through which the leads were connected at the time of each
strain measurement.

5.2.7 CALCULATION OF STRESSES FROM EXPERIMENTAL DATA:

Where two strain gauges were used in one direction, the
average of the two readings was used for subsequent calculations.
Computation of the stresses was done by computer (ICL 1905).
Two main programmes were written in Fortran IV - one for computation
of stresses in specimens which were bored out only, and the second
for specimens which were partly bored out and partly turned from the
outside. The only experimental data which were supplied to the
computer were the bored and turned diameters and the corresponding
strain readings. The programmes do the following computations:
1. calculate the areas corresponding to the given diameter of bores (F);

2. calculate the values of \( A \) and \( \theta \);

3. fit polynomials up to 8th order by least square method, between areas calculated above and the corresponding values of \( A \) and \( \theta \);

4. differentiate the equations of curves at all values of areas (F) i.e., find \( \frac{dA}{dF} \) and \( \frac{d\theta}{dF} \) and compute stresses by using Sachs equations;

5. for externally turned specimens, compute stresses as above and apply corrections at all points to give the actual values of stresses;

6. calculate the areas under the curves \( F \) vs \( \sigma_\theta \) and \( r \) vs \( \sigma_t \);

7. calculate the values of \( \sigma_r + r \frac{d\sigma_r}{dr} \) at all values of \( r \);

Programs were also written to plot the following curves

1. \( F \) vs \( A \) and \( F \) vs \( \theta \) curves;

2. \( F \) vs \( \sigma_\theta \), \( r \) vs \( \sigma_\theta \), \( r \) vs \( \sigma_t \), and \( r \) vs \( \sigma_r \) curves.

These programs not only eliminated errors inherent in drawing tangents to \( F \) vs \( A \) and \( F \) vs \( \theta \) curves but also eliminated the drudgery of complicated and time-consuming mathematics.

A listing of the programs for Sachs boring method and combined inner and outer layer removal method are given in Appendix IV.
Fig. 5.1.1. Percent change in resistance as a function of percent strain for an advance alloy.

Fig. 5.1.2 Layout of strain gauges on extruded specimen

1. Specimen
2. Strain gauges
3. Lead-out wires
4. Connecting wires
5. Terminal strip
Fig. 5.2.2 (b) Rig for material removal by acid etching.
CHAPTER 6

Experimental Investigation Planned

and

Description of Apparatus
6.1 INVESTIGATION PLANNED

The experimental work can be divided into three main parts:

1. production of extruded specimens;
2. determination of residual stresses;
3. testing of extruded specimens for mechanical properties.

The objective of the investigation was to study the residual stress patterns and mechanical property changes brought about in steel rods by the cold extrusion process; and also to find out the effect of such production variables as extrusion ratio and die angle on the residual stresses and the properties. It was decided to use four extrusion ratios fairly representative of those commonly employed. At each extrusion ratio three die angles, considered to be common for an extrusion process were used.

The four extrusion ratios were 2.25:1, 2.78:1, 3.52:1, and 4:1, corresponding to 55.5%, 64%, 71.5% and 75% reduction in area respectively. Specimens were extruded at each extrusion ratio with three die angles, namely 60°, 90° and 120° (included). Other considerations in the design of the extrusion tools were to obtain the specimens of a size and shape such that the required mechanical properties could be conveniently tested with the existing equipment and satisfactory specimens could be obtained from the extruded bars for residual stress determination.

At a later stage in the project when the residual stress determination in most of the extruded specimens was completed and
surface stresses in all cases were found to be tensile, it was
decided to find out the possibility of modifying the surface stresses
to compressive in the actual extrusion process. This was done by
designing two dies corresponding to two extrusion ratios (2.78:1 and
4:1) and one die angle (120°), with the exit diameter smaller than
the entry diameter, so that after the major reduction in the die
orifice the rods underwent another 2% reduction at the exit diameter.
For reasons of comparison, the total reduction including the final 2%
reduction was the same as in the case of rods extruded through
conventional dies.

In practice, products containing residual stresses,
particularly tensile surface stresses, are often stress-relieved
to eliminate the harmful stresses. Some of the extruded rods were
therefore, stress-relieved at three temperatures, namely, 500°, 550°
and 600°C and subsequently tested for hardness and tensile properties
and residual stresses. The aim was to find a temperature at which
the level of residual stresses should be appreciably reduced but at
the same time most of the improvement in the strength acquired through
cold working should be retained.

The mechanical properties determined were tensile, hardness,
fatigue and impact properties, most commonly used in the design of a
product.
6.2 DESCRIPTION OF APPARATUS

6.2.1 WORK MATERIAL

The material used was a low carbon aluminium killed mild steel to British Standard 970: EN2E. Two batches of material were used, the major portion of the investigation being carried out with the batch having the chemical composition given in Table 6.2.1 (a). The second batch had a slightly different chemical composition (Table 6.2.1 (b)) and was used only to extrude some specimens for tensile testing.

The material was received as 1½" (38·1 mm) nominal diameter (slightly oversize) hot rolled bar. Before machining billets out of the bars, these were heat treated as follows, to give a microstructure of ferrite + pearlite at 940°C for half an hour and furnace cooled. Unless otherwise stated the material was used in this condition. One batch was, however, subjected to an additional heat treatment at 700°C for eighteen hours, which caused spheroidisation of the pearlite.

6.2.2 PREPARATION AND LUBRICATION OF BILLETS

The annealed bars were turned down to 1·5 - 0·005 inch (- 38·1mm) diameter and billets of three different lengths, 2 inch, 1½ inch and 1¼ inch (50·8, 44·8 and 38·1 mm respectively), were parted off, faced and chamfered. The billets were degreased in a boiling solution of Pyroclean 9 (25 gms/litre) - a commercial alkali cleaner. These were then pickled with 10% sulphuric acid solution, followed
by a hot and cold water rinse.

By far the most common lubrication practice for cold extrusion of steel is phosphating of the billets followed by a coating of soap lubricant. Zinc phosphate is invariably used because of the zinc soap produced as a result of the following chemical reaction with soap (generally sodium stearate),

\[
Zn_3(PO_4)_2 + 6C_{17}H_{35}COO Na \rightarrow 3(C_{17}H_{35}COO)_2Zn + 2Na PO_4
\]

Zinc stearate is known to be a highly efficient extreme pressure lubricant chemically bonded to the phosphate coating. After a complete phosphating and soap treatment, the billet has a coating of consecutively Zinc phosphate, Zinc stearate and the unreacted soap.

For the billet to get an effective coating of phosphate and soap, it is important that the temperature of the phosphate and soap solutions should be maintained at a specified temperature. This was achieved by surrounding the stainless-steel tanks containing these solutions, with hot water in a constant temperature bath (Figure 6.2.2), whose temperature was maintained at 80°C ± 1°C.

After the degreasing and pickling treatment the billets were immersed in the phosphate tank for 8 to 10 minutes. These were then given a hot and cold water rinse and transferred to the soap tank. An immersion for 5 - 7 minutes gave a good soap coating to the billets.
6.2.3 DESIGN OF EXTRUSION TOOLS

Fig 6.2.3 shows the assembled tool. This tool, which will be referred to as Tool 'A', was used for extruding rods for the major part of the investigation. A similar tool on a smaller scale was designed to extrude small diameter rods for the investigation of the fatigue properties of the as-extruded specimens. This tool will be referred to as tool 'B' and is shown with the press, in Figure 6.2.5. The schematic diagram of tool "A" is shown in Figure 6.2.6.

Extrusion Chamber

The extrusion chamber is of the pre-stressed three ring type construction designed for the simultaneous yielding of the three rings [129 - 131]. The support rings were designed to have an interference of 0.004 inch per inch diameter, between the mating diameters of the outer and inner support ring and the chamber liner. The relationship between the liner diameter and the support ring diameters was based on the work of Wilson [129].

A 1° taper was provided on the mating diameters of liner and support rings so that the three could be forced into one another by a hydraulic press. The bottom face of the liner was bevelled to have a narrow annular flat adjacent to the bore. The bevel localises the clamping force between the die and the liner and so prevents the sideways flashing of the work material and escape of the lubricant during extrusion. The liner is of high speed steel (18-4-1) and the support rings are made from high tensile alloy steel, EN 30B : BS 970.
Twelve conventional dies and two double reduction dies were used with the tool 'A' and four dies corresponding to four extrusion ratios were used with the tool 'B'. The details of the die are shown in Figure 6.2.7 and the dimensions of the entry and exit diameters are given in Table 6.2.2. The die insert was made from high speed steel (18-4-1) and was surrounded by a support ring of EN 30B. The interference and taper between the mating diameters of the die insert and the support ring is the same as for the extrusion chamber. The maximum diameter of the die throat (Dt, Figure 6.2.7) was made 0.003 to 0.005 inch (0.076 - 0.127 mm) smaller than the bore of the liner so that the extrusion chamber did not overhang. Only two support rings were made and dies were pressed into these in turn. To facilitate changing of the dies in support rings, the tapered mating surfaces were lightly lubricated with a suspension of molybdenum disulphate in alcohol, before each assembly operation.

A solid punch of 1.5 inch (~38.1 mm) nominal diameter with 4½ inch (124 mm) parallel length was used with tool 'A'. It is made from high speed steel (18-4-1), hardened to RC 62-65. The punch for tool 'B' was of ½ inch (12.7 mm) nominal diameter and is of similar design as the punch for tool 'A'.

6.2.4 PRESS AND RECORDING EQUIPMENT

300 ton (~3 MN) "Dennison" compression testing machine with
a built-in load cell was used for extruding rods with the tool 'A'.

Extrusion in this press was carried out at a ram speed of approximately \( \frac{1}{2} \) - 1 inch/min (0.21 - 0.42 mm/sec). The tool 'B' was used in conjunction with a 100 ton (~1 MN) hydraulic press. Ram speed in this case was 5 in/min (2.1 mm/sec). The Hewlett Packard X - Y recorder was used with both presses to record load displacement curves. A layout of the tool 'A' with the press and recorder is shown in figure 6.2.8.

In most cases extruded rods were pushed through the die consecutively without ejection.

6.2.5 HARDNESS TESTING

Extruded rods were slit longitudinally so that one complete half of the extruded rod was available for hardness testing. Hardness surveys were carried out along longitudinal as well as radial directions of the specimen by making a number of impressions at regular intervals. All hardness measurements were made on a Vickers hardness testing machine using a 30 kgf load.

6.2.6 TENSILE TESTING

Tensile tests were carried out on a 50 ton (500 KN) 'Dennison' tensile testing machine. In most cases Hounsfield extensometer with 2 inch (50.8 mm) gauge length was used for strain measurements.

Specimens were machined from the extruded rods in accordance with British Standard : 18. Except for specimens machined from rods extruded at 2.25:1 extrusion ratio, the test diameter of
the machined specimens was 0.505 inch (~12.8 mm). Rods extruded at 2.25:1 extrusion ratio were not long enough to conform to the recommended diameter/gauge length ratio for 0.505 inch test diameter. They were therefore machined to a smaller test diameter of 0.357 inch (~9.07 mm). As the gauge length in this case was only 1.8 inch (~45.72 mm), the Hounsfield extensometer could not be used and strain measurements were made with strain gauges. The test specimens werethreaded on collars and sleeves with corresponding internal threads were used for gripping the specimens in the testing machine.

Attempts made to test the as-extruded specimens were not very successful because the specimen failed at the threaded portions even before giving enough points on the stress-strain curve for 0.1% proof-stress determination. In some cases, however, it was possible to determine the 0.02% proof-stress from the readings obtained.

6.2.7 FATIGUE TESTING

Fatigue testing was carried out on a four-point loading rotating-beam fatigue testing machine (giving a uniform distribution of bending moment along the test section) at 1500 r.p.m. Tests were carried out on both machined and as-extruded specimens. The machined specimens with test section of 0.375 inch (9.52 mm) diameter were prepared in accordance with the British Standard 3518, pt 2. Specimens were machined from rods extruded with three extrusion ratios, 2.78:1, 3.52:1 and 4:1 and 120° die angle. Extruded rods with the smallest extrusion ratio, i.e., 2.25:1 were not long enough to give
the standard test specimens. The as-extruded specimens for fatigue testing were extruded with tool 'B' at two higher extrusion ratios, 3.52:1 and 4:1, and die angle 120° to give specimens of diameter 0.265 and 0.25 inch (~6.73 and 6.35 mm) respectively. Collars of epoxide resin filled with iron powder were cast around both ends of the as-extruded specimens. These collars were then turned down concentric with the rods to 0.5 inch (12.7 mm) diameter. There was a likelihood of these small diameter specimens being bent if they were loaded with the gripping arrangement provided on the machines. Special grips were therefore designed (figure 6.2.9) which were first held in the machine with the existing gripping arrangement. Specimens were then gripped in these collets by allen screws with only a light pressure. In spite of the precautions to avoid stress concentration at the ends by providing epoxide collars, most of the specimens fractured under these collars. Nonetheless, the results did, at least, provide a lower limit for the fatigue properties of the unmachined specimens.

6.2.8 IMPACT TESTING

Impact properties were determined on standard V-notch Charpy specimens, with 'Avery' impact testing machine. The specimens were machined from the extruded rods according to British Standard 131, pt. 2, the notches being perpendicular to the rod axis. Specimens for testing at different temperatures below room temperature were cooled to these temperatures by immersion in acetone chilled with liquid nitrogen. For a particular test temperature, the temperature of the acetone was maintained constant.
by gradual additions of liquid nitrogen, the acetone being stirred continuously. Sufficient immersion time was given to ensure that the specimens attained the test temperature uniformly. Tests were carried out on specimens containing pearlite microstructure and also on specimens containing spheroidised pearlite.

Residual Stresses

Experimental procedure for the determination of residual stresses has already been described in Chapter 5.
Fig 6.2.5 Extrusion tool 'B' (set up in the press).
Heat treatment of Parts 1, 6, 9

Preheat to 400°C. Then heat to 900°C in a salt bath, transfer to salt bath at 1200°C for 3 minutes, oil quench, temper at 560°C for one hour, air cool, retemper at 560°C.
Table 6.2-5

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<th>Serial Number</th>
<th>Extrusion Ratio</th>
<th>Entry Diameter $D_i$</th>
<th>Exit Diameter $D_e$</th>
<th>Die Angle (included)</th>
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Fig 6.2.8 Layout of the extrusion tool with the press and the recorder.

Fig 6.2.9 Special grips used for the fatigue testing of as-extruded specimens.
### Chemical Composition, per cent

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<th>S</th>
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<tr>
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<td>0.06</td>
<td>0.032</td>
<td>0.033</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 6.2.1*
CHAPTER 7

Results
7.1 EXTRUSION PRESSURE

A typical example of the load-displacement curve is shown in Figure 7.1.1. All curves showed a maximum value at the beginning of the extrusion operation and it is the load calculated from this maximum which is used to derive the extrusion pressure.

7.1.1 EFFECT OF EXTRUSION RATIO AND DIE ANGLE

The relationship between the maximum extrusion pressure and the included die angle for each extrusion ratio is shown in Figure 7.1.2. It is evident from this that, at any particular extrusion ratio, the extrusion pressure decreased as the die angle was decreased. The fact that extrusion pressure increases with the increase of extrusion ratio can also be seen from Figure 7.1.2.

The extrusion pressures in all cases were within 10% (mostly on the lower side) of those obtained at PERA [13] and by McKenzie [13]. There was no any significant difference between the extrusion pressures required to extrude material with ferrite + pearlite structure and that with spheroidised pearlite.
7.2 RESIDUAL STRESSES

7.2.1 STRAIN MEASUREMENTS

The technique of strain measurements is described in Chapter 5. For both methods, i.e., material removal from inside to the outside diameter and vice versa, the material was removed by acid etching with intermediate trueing up of the bore or outside diameter by conventional boring and turning operation. A typical sequence of layer removals with strain readings at each step is shown in Table 7.2.1.

7.2.2 ESTIMATION OF THE ACCURACY OF STRESSES DETERMINED

The accuracy of strain measurements could be affected by the following sources of error,

(i) strain gauge tolerances;

(ii) temperature variations during the period over which a particular specimen was analysed;

(iii) possibility of some change in the contact resistance between the strain gauge and instrument leads;

(iv) possibility of zero shift of the instrument;

(v) possibility of gauge creep due to incomplete curing of the cement, during the initial read-out period.

The tolerance on the gauge factor was ±1% and the temperature sensitivity of the gauge was 0.0015%/°C. As the average of the readings of two strain gauges used in one direction was taken to be the strain
reading in that direction, the error due to gauge factor could either be averaged out or duplicated. Temperature variation, however, was not thought to introduce any error because of

(i) the variation of temperature of the constant temperature room, within $\pm 2^\circ$C. (Strain readings were taken only after the specimens had been in the constant temperature room for 18 hours.)

(ii) the use of a dummy specimen which was maintained at the specimen temperature.

The zero shift of the instrument was checked from time to time and necessary correction was incorporated in the strain readings.

Other errors could be due to items (iii) and (v) and due to the possibility of any stresses introduced during material removal by conventional machining.

It is very difficult to make an overall estimate of accuracy of strain measurements due to all these variables. Some estimate can, however, be made by carrying out stress determination on a residual stress-free specimen.

Strain measurements taken in an annealed specimen during the sequence of material removal as followed in the extruded specimens yielded strain values of the order of $\pm 10 \mu \text{in/in}$. The $\lambda$ and $\theta$ (defined in Chapter 4) values calculated from the strain readings were insignificant.

Complete stress distribution carried out in an extruded specimen (extrusion ratio 4, die angle 60$^\circ$) after the stress relief anneal at 600$^\circ$C where stresses were thought to be almost completely relieved
indicated a stress level in the range of ±30 MN/m² over the cross-section. It can, therefore, be concluded that the residual stress values calculated from the strain measurements were fairly accurate and could be within ±30 MN/m² of the actual stresses.

7.2.3 STRESS PATTERNS

In the rods extruded through conventional dies, the residual longitudinal and tangential stresses are found to be compressive at the interior and tensile near the free surface. Radial stresses are compressive in the interior and, as would be expected, they vanish at the free surface. The stress distribution is modified in the case of rods extruded through double-reduction dies. Stress level, in general, is lower than in the conventionally extruded rods and the surface stresses change sign from tension to compression.

7.2.4 EFFECT OF EXTRUSION RATIO

Residual longitudinal and tangential stress distributions at each die angle corresponding to the four extrusion ratios are shown in Figures 7.2.1 - 3. A similar relationship for radial residual stresses, corresponding to the die angle 120° is shown in Figure 7.2.3 (c). The maximum longitudinal and tangential tensile stresses exist, in all cases, at a small distance (0.5 - 1.0 mm) below the surface and they drop off to lower values at the surface. The difference between the peak values of tensile stresses for the four extrusion ratios is not large, probably, because of the small difference (~20%) between the maximum and minimum reductions of areas employed. The difference in the surface tensile stresses corresponding
to different extrusion ratios is even smaller and in some cases this
difference is within the estimated experimental error. In the rods
extruded through dies with the two smaller die angles, namely 60° and
90°, longitudinal and tangential residual stresses at the axis
become more negative as the extrusion ratio increases. This is
indicated in figures 7.2.1 (a,b) and 7.2.2 (a,b). The trend is,
however, not consistent in the case of rods extruded through dies
with 120° die angle (figure 7.2.3 (a,b)).

7.2.5 EFFECT OF DIE ANGLE

To show the effect of die angle the longitudinal and
tangential stress distributions are plotted at the three die angles
corresponding to each extrusion ratio in figures 7.2.4 - 7. The
surface tensile stresses and the peak values of tensile stresses
near the surface do not follow any definite trend dependent on the
die angle. In fact, the difference in the magnitude of surface
tensile stresses at various die angles is very small and in most
cases is within the experimental error. The effect of die angle on
surface residual stresses is likely to be concealed in such a case.
Residual stresses at the axis of the extruded rod do, however, show
a definite trend dependent on the die angle at the three higher
extrusion ratios, i.e. 2.78:1, 3.52:1 and 4:1. Residual stresses
become more compressive as the die angle decreases, as seen in figures
7.2.5 - 7.2.7. At the smallest extrusion ratio, namely 2.25:1, this
trend is reversed as indicated in figures 7.2.4 (a,b).
7.2.6 EFFECT OF THE FINAL 2% REDUCTION

The longitudinal and tangential stress distributions in the bars extruded through conventional dies and through double reduction dies incorporating a small (~2%) reduction at the die exit (total reduction remaining the same in two cases), are compared in Figures 7.2.8 - 9. A similar comparison is made for radial stress distributions for one extrusion ratio, namely, 4:1, in Figure 7.2.9 (c). The effect of the final small reduction on the residual stress distributions is similar for the two extrusion ratios studied. The level of the compressive residual stresses in the interior is lowered and the stresses at and near the surface become compressive. The peak tensile stresses are shifted back towards the centre.

7.2.7 SURFACE STRESSES DETERMINED BY OUTER LAYER REMOVAL METHOD

Residual stresses were determined in one specimen (Extrusion ratio 3.52:1 and die angle 120°) by combined inner and outer layer removal method. The stress distribution was found to be the same as in other specimens, namely, stresses were compressive in the interior and tensile at and near the surface. In other cases, surface stresses were determined in specimens similar to those for which the complete stress distributions had been previously carried out by material removal from centre to outside. The surface stresses determined independently by the two method are compared in Figure 7.2.10. It can be seen from this that the surface stresses remain tensile, in both cases, although the magnitude of the stresses determined by the outer layer removal method generally is lower, particularly in rods extruded at the largest die angle, namely, 120°.
7.2.8 EFFECT OF STRESS RELIEF ANNEALING

The complete stress distributions carried out in the extruded bars after stress relief at 500°C and 550°C for one hour per inch (25.4 mm) diameter, are shown in Figures 7.2.11-12. Rods extruded at two extrusion ratios, namely, 2.78:1 and 4:1, both with dies having 60° included angle, were used for this study. It can be seen from Figure 7.2.11 a,b,c that the residual stress level is appreciably reduced by the stress relief operation at 500°C and the residual stresses almost vanish after treatment at 550°C. In the rods extruded at 4:1 extrusion ratio the reduction in stress level brought about by the stress relief operation at 500°C is even more marked than in the case of lower extrusion ratio, 2.78:1. At 550°C stresses are almost completely relieved (Figures 7.2.12 a,b,c). In one case the stresses near the extrusion axis, after the stress relief operation at 550°C, show a sudden rise (figure 7.2.12, a). This is probably due to the experimental error in strain measurements.

7.2.9 RESIDUAL STRESSES - ANALYTICAL RESULTS

Figures 7.2.13 to 7.2.24 show the longitudinal and tangential residual stresses in rods extruded with 12 different combinations of extrusion ratio and die angle. The curves 'a' correspond to the experimental results. Out of the several residual stress distributions derived from alternative velocity fields for a particular combination of extrusion ratio and die angle, the distributions represented by the curves 'd' gave the best agreement with the experimental results, on the consideration of the magnitude and sign of the residual stresses along the cross-section of the extruded rod. In the majority of cases
the velocity fields of curves 'd' were displaced from the optimum velocity fields as indicated in table 7.2.2: the displacement (except for some large die angles and extrusion ratios) being in the direction opposite to the extrusion direction. Residual stresses calculated from the optimum velocity fields (curves 'c') generally gave poorer agreement with the experimental results. The agreement with the experimental results remained poorer (as indicated by curves 'b') when the residual stresses were calculated from the velocity fields which were displaced relative to the optimum velocity fields or the velocity fields of curves 'd' in the direction of extrusion. Table 7.2.2 gives the boundaries of the regions of deformation corresponding to curves b, c and d of Figures 7.2.13 to 7.2.24. The ratio of the extrusion pressure (assuming zero friction) to the mean yield stress derived from the above velocity fields is also given in table 7.2.2.

At a particular extrusion ratio and die angle residual stresses computed from several alternative velocity fields were found to be very sensitive, particularly at and near the axis of the extruded rod, to the location of the regions of deformation. A typical example of the dependence of the residual stress distribution on the location of the regions of deformation is shown in figure 7.2.25.
7.3 MECHANICAL PROPERTIES:

7.3.1 HARDNESS TESTS

(i) AS EXTRUDED SPECIMENS

Figure 7.3.1 is typical of the hardness surveys carried out on the longitudinal sections of the extruded bars. These surveys indicate that the hardness rises from the leading end and that a steady state distribution of hardness across the specimen is reached at a distance of approximately 4R from the leading end, 2R being the rod diameter. In the steady state portion, the radial distribution of hardness is dependent upon the extrusion ratio and die angle as shown in Figure 7.3.2. The increase in hardness on the extrusion axis with the increase in extrusion ratio from 2.25:1 to 4:1 is not very significant, the figures being approximately 215 and 230 HV30 respectively. At a particular extrusion ratio the hardness increases with the increase in the die angle, the difference being more at the periphery than at the extrusion axis.

(ii) EFFECT OF STRESS RELIEF

Hardness surveys carried out on the radial sections of stress relieved specimens are shown in Figure 7.3.3 a,b. The decrease in hardness on the extrusion axis, after stress relieving at 500°C is 20 and 30 HV30 respectively for the two extrusion ratios studied, i.e., 2.78 and 4. The hardness after the stress relief operation at 550°C drops significantly (by ~ 90 HV30) for the extrusion ratio 4:1 but the difference is only 40 HV30 for
the lower extrusion ratio. Stress relief at 600°C brings the hardness level to nearly that of the unworked material, in both cases.

7.3.2 TENSILE PROPERTIES

(i) As-EXTRUDED SPECIMENS

The results of the tensile tests on machined specimens from the unworked material and from the extruded bars are collected in Table 7.3.1. A comparison of the tensile properties of the unworked and extruded material shows that there is a considerable increase in the yield strength (in terms of 0.1% proof-stress) and the U.T.S. of the extruded material over those of the unworked material. The increase in the yield strength is proportionately more than the increase in the U.T.S. The yield strength for the lowest extrusion ratio is nearly twice that of the unworked material and the increase is even more at the higher extrusion ratios. Ductility is terms of % elongation and % reduction in area of the extruded material decreases appreciably.

Table 7.3.1 indicates comparatively higher values of proof stress and U.T.S. at 60° die angle than at other two angles, corresponding to extrusion ratio 2.78:1 and 3.52:1. The reason for this is that these two specimens were extruded from the material with higher carbon content (0.15%); the rest of the specimens contained 0.09% carbon. Taking this fact into consideration it can be seen that the tensile properties of machined specimens at a particular extrusion ratio are not much
different for different die angles. The effect of increasing extrusion ratio is to slightly increase the values of proof-stress and U.T.S., whereas the drop in the % elongation and % reduction in area values from those of annealed material remain practically the same at all extrusion ratios.

From some of the results obtained from the tensile tests on unmachined specimens (Table 7.3.2), it appears that at a particular extrusion ratio the 0·02% proof stress increases with the increase in die angle.

(ii) STRESS RELIEVED SPECIMENS

The tensile test results on the machined specimens from the stress-relieved extruded bars are given in Table 7.3.3 along with those on the machined specimens from the as-extruded bars. The figures obtained indicate that after the stress-relief treatment at 500°C most of the gain in tensile strength (proof stress and U.T.S.) is retained. The 0·1% proof stress, for example, remains almost double that of the unworked material in the case of 2·78:1 extrusion ratio and even more at extrusion ratio 4:1. The ductility in terms of % elongation is considerably improved after stress relief at this temperature. The stress relief operation at 550°C produces a greater change in properties in rods extruded at 4:1 extrusion ratio: ductility is nearly completely restored and the proof stress and U.T.S. are only slightly higher than those of the unworked material. For the extrusion ratio 2·78:1, the proof stress and U.T.S. after stress relief at 550°C are still considerably higher than those of the
7.3.3 FATIGUE TESTS

The results of the fatigue tests on the machined specimens are given in Figure 7.3.4 (a) and on the as-extruded specimens in Figure 7.3.4 (b). Whereas the fatigue limit of the specimens machined from the extruded rods shows a general considerable improvement over the fatigue limit of the unworked material, this improvement is not appreciably affected as the extrusion ratio is increased from 2.78 to 4.

Tests on the as-extruded specimens were unsatisfactory in that fracture frequently occurred in the grips. Nevertheless, it can be seen from Figure 7.3.4 (b) that the fatigue limit must be, in excess of ±300 MN/m², the fatigue limit of the unworked material being 235 MN/m².

7.3.4 IMPACT PROPERTIES

The results of the impact tests on the machined specimens with a ferrite + pearlite microstructure and those containing spheroidised pearlite are collected in Figures 7.3.5 (a) and (b). In the pearlite condition the transition temperature rises from the value for the unextruded material for the lowest extrusion ratio, namely, 2.25:1; thereafter the transition temperature is shifted to lower temperatures with the increasing extrusion ratios. The level of the energy absorbed in the ductile fracture of the extruded specimen is less than that required for the ductile
fracture of the unextruded steel: in the "brittle" condition the reverse is true. These changes were accompanied by the changes in the appearance of the fractured test pieces. It is seen from figure 7.3.6 that in the case of extruded specimens, the fracture tends to branch down planes roughly parallel to the specimen axis giving rise to a very torn and ragged appearance. This effect is more marked as the extrusion ratio increases.

In the case of specimens containing spheroidised pearlite, the changes in fracture appearance and fracture energy are much the same as those described for pearlite specimens, but the transition temperature decreases monotonically with the increasing extrusion ratio.

7.3.5 MICROSTRUCTURAL STUDIES

A study of the microstructure at the mid-planes of rods extruded at various extrusion ratios and die angles revealed that in most cases there was not any indication of structural damage in the worked material. In the bar extruded at 2.25:1 extrusion ratio and 120° die angle, however, micro-cracks were found in the pearlite colonies. Photomicrographs indicating this evidence are shown in figures 7.3.7 (a,b). Microstructure of the stress-relieved specimens was also studied. It is interesting to note that in the bars extruded at 2:78 extrusion ratio where no evidence of structural damage was found before stress relief annealing, microcracks were observed in the pearlite colonies and in the inclusions.
Table 7.2.1  Sequence of Material Removal from centre to outside

Extrusion ratio: 4

die angle: 120°

Diameter of extruded bar = 0·75 inch (19·05 mm)

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<th>Sr. No.</th>
<th>Bore Diameter mm</th>
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<td>* 20</td>
<td>13·03</td>
<td>-73·5</td>
<td>30</td>
</tr>
<tr>
<td>21</td>
<td>13·59</td>
<td>-78</td>
<td>11</td>
</tr>
<tr>
<td>22</td>
<td>14·02</td>
<td>-76</td>
<td>0</td>
</tr>
<tr>
<td>* 23</td>
<td>14·63</td>
<td>-95</td>
<td>-16·5</td>
</tr>
<tr>
<td>24</td>
<td>14·97</td>
<td>-100</td>
<td>-8·5</td>
</tr>
<tr>
<td>* 25</td>
<td>15·49</td>
<td>-117·5</td>
<td>-24·5</td>
</tr>
<tr>
<td>26</td>
<td>16·33</td>
<td>-131</td>
<td>13</td>
</tr>
<tr>
<td>27</td>
<td>16·64</td>
<td>109</td>
<td>15</td>
</tr>
<tr>
<td>28</td>
<td>17·14</td>
<td>176</td>
<td>34</td>
</tr>
<tr>
<td>29</td>
<td>17·37</td>
<td>84</td>
<td>83</td>
</tr>
<tr>
<td>30</td>
<td>17·7</td>
<td>95</td>
<td>15</td>
</tr>
<tr>
<td>31</td>
<td>17·88</td>
<td>115</td>
<td>83</td>
</tr>
<tr>
<td>32</td>
<td>18·03</td>
<td>108</td>
<td>94</td>
</tr>
<tr>
<td>33</td>
<td>19·05</td>
<td>144</td>
<td>971</td>
</tr>
</tbody>
</table>

* Bored out by conventional machining

† Remaining tube split and a tongue cut out at this diameter
Table 7.2.2 Parameters of Deformation Zone (refer to Figs. 7.2.13 to 7.2.24)

<table>
<thead>
<tr>
<th>Extrusion Ratio</th>
<th>Die Angle 2α deg</th>
<th>Parameters of Deformation Zone which (in most cases) is displaced relative to the Deformation Zone 3 on the opposite side of Deformation Zone 2</th>
<th>Parameters of optimum deformation zone for minimum extrusion pressure, frictionless conditions</th>
<th>Parameters of deformation zone assessed to give best agreement with observed residual stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Parameters: d1, de, m, Pav/(\bar{\sigma})</td>
<td>Parameters: d1#, de, m, Pav/(\bar{\sigma})</td>
<td>Parameters: d1, de, m, Pav/(\bar{\sigma})</td>
</tr>
<tr>
<td>2.25</td>
<td>60</td>
<td>d1: 0.699, de: 1.047, m: 1.224, Pav/(\bar{\sigma}): 1.189</td>
<td>d1#: **1.170, de: 1.770, m: 1.230, Pav/(\bar{\sigma}): 0.9744</td>
<td>1.341, de: 2.009, m: 1.229, Pav/(\bar{\sigma}): 1.093</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>d1: 0.403, de: 0.604, m: 1.224, Pav/(\bar{\sigma}): 1.686</td>
<td>d1#: **0.774, de: 1.160, m: 1.224, Pav/(\bar{\sigma}): 1.137</td>
<td>0.924, de: 1.620, m: 1.324, Pav/(\bar{\sigma}): 1.295</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>d1: 0.628, de: 1.047, m: 1.291, Pav/(\bar{\sigma}): 1.387</td>
<td>d1#: **0.490, de: 0.993, m: 1.424, Pav/(\bar{\sigma}): 1.457</td>
<td>0.949, de: 0.993, m: 1.424, Pav/(\bar{\sigma}): 1.295</td>
</tr>
<tr>
<td>2.78</td>
<td>60</td>
<td>d1: 1.050, de: 1.770, m: 1.298, Pav/(\bar{\sigma}): 1.176</td>
<td>d1#: **1.089, de: 1.815, m: 1.291, Pav/(\bar{\sigma}): 1.173</td>
<td>1.089, de: 2.107, m: 1.391, Pav/(\bar{\sigma}): 1.222</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>d1: 0.636, de: 0.605, m: 1.291, Pav/(\bar{\sigma}): 1.843</td>
<td>d1#: **0.480, de: 0.929, m: 1.391, Pav/(\bar{\sigma}): 1.612</td>
<td>0.696, de: 1.346, m: 1.391, Pav/(\bar{\sigma}): 1.365</td>
</tr>
<tr>
<td>3.52</td>
<td>60</td>
<td>d1: 0.559, de: 1.048, m: 1.369, Pav/(\bar{\sigma}): 1.578</td>
<td>d1#: **0.935, de: 1.770, m: 1.376, Pav/(\bar{\sigma}): 1.390</td>
<td>1.072, de: 2.020, m: 1.373, Pav/(\bar{\sigma}): 1.491</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>d1: 0.322, de: 0.603, m: 1.368, Pav/(\bar{\sigma}): 2.099</td>
<td>d1#: **0.875, de: 1.770, m: 1.422, Pav/(\bar{\sigma}): 1.514</td>
<td>0.679, de: 1.093, m: 1.269, Pav/(\bar{\sigma}): 1.556</td>
</tr>
<tr>
<td>4.00</td>
<td>60</td>
<td>d1: 0.525, de: 1.050, m: 1.414, Pav/(\bar{\sigma}): 1.705</td>
<td>d1#: **0.650, de: 1.050, m: 1.314, Pav/(\bar{\sigma}): 1.674</td>
<td>0.357, de: 0.878, m: 1.568, Pav/(\bar{\sigma}): 1.880</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>d1: 0.302, de: 0.604, m: 1.414, Pav/(\bar{\sigma}): 2.256</td>
<td>d1#: **0.432, de: 0.990, m: 1.514, Pav/(\bar{\sigma}): 1.915</td>
<td>0.367, de: 0.841, m: 1.514, Pav/(\bar{\sigma}): 1.963</td>
</tr>
</tbody>
</table>

* Pav, average extrusion pressure; \(\bar{\sigma}\), mean yield stress taken as the average of the yield stress before and after deformation.

** Analysis of deformation zone with parameter d1 less than d1\# was outside the bounds of the computer programme.

\# Best agreement between observed and assessed residual stress distributions is given by the deformation zone of parameters 2 i.e. parameters 2 and 3 are identically the same.

\#\# Residual longitudinal stresses assessed from the deformation zone of parameters 2 have best agreement with the observed longitudinal residual stresses. Deformation zone of parameters of 3 give the best agreement between observed and assessed residual tangential stresses.

Residual Stress distributions in Figs. 7.2.13 to 7.2.24 are represented as follows:

(a) experimental
(b) assessed from deformation zone 1
(c) assessed from deformation zone 2
(d) assessed from deformation zone 3
### TENSILE TEST RESULTS

#### Table 7.3.1 Machined Specimens

<table>
<thead>
<tr>
<th>Extrusion ratio</th>
<th>Unextruded</th>
<th>2.25</th>
<th>2.78</th>
<th>3.52</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die angle (included) degrees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 90 120</td>
<td>60 90 120</td>
<td>60 90 120</td>
<td>60 90 120</td>
<td>60 90 120</td>
<td></td>
</tr>
<tr>
<td>0.1% Proof Stress, MN/m²</td>
<td>279 557 540 534</td>
<td>683 600 580</td>
<td>722 637 655</td>
<td>668 674 687</td>
<td></td>
</tr>
<tr>
<td>U.T.S., MN/m²</td>
<td>389 632 631 646</td>
<td>722 673 683</td>
<td>763 710 739</td>
<td>718 740 764</td>
<td></td>
</tr>
<tr>
<td>Reduction in area, %</td>
<td>66.91 45.21 45.35 47.11</td>
<td>34.55 42.44 42.28</td>
<td>34.36 41.36 40.76</td>
<td>41.57 40.58 40.35</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 7.3.2 Unmachined Specimens

<table>
<thead>
<tr>
<th>Extrusion ratio</th>
<th>Unextruded</th>
<th>2.25</th>
<th>2.78</th>
<th>3.52</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die angle (included) degrees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 90 120</td>
<td>60 90 120</td>
<td>60 90 120</td>
<td>60 90 120</td>
<td>60 90 120</td>
<td></td>
</tr>
<tr>
<td>0.02% Proof Stress, MN/m²</td>
<td>269 405 437 451</td>
<td>474 448 579</td>
<td>506 575 595</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Table 7.3.3**  Tensile Test Results of Specimens machined from Stress-Relieved Rods  

Die angle 60°

<table>
<thead>
<tr>
<th>Extrusion Ratio</th>
<th>0.1% Proof Stress MN/m²</th>
<th>U.T.S. MN/m²</th>
<th>Elongation, %</th>
<th>Reduction in area %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unextruded</td>
<td>As-extruded</td>
<td>2.78</td>
<td>Annealed</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>500°C, 1&quot;/hour</td>
<td>550°C, 1&quot;/hour</td>
</tr>
<tr>
<td>0.1% Proof Stress MN/m²</td>
<td>279</td>
<td>683</td>
<td>571.1</td>
<td>502</td>
</tr>
<tr>
<td>U.T.S. MN/m²</td>
<td>389</td>
<td>722</td>
<td>646.7</td>
<td>549.9</td>
</tr>
<tr>
<td>Elongation, %</td>
<td>39.96</td>
<td>9.41</td>
<td>16.95</td>
<td>22.25</td>
</tr>
<tr>
<td>Reduction in area %</td>
<td>66.91</td>
<td>34.55</td>
<td>39.8</td>
<td>61.14</td>
</tr>
</tbody>
</table>
Extrusion Ratio: 2.25:1
Die Angle: 90°
Extrusion Ratio

- □ 4:1
- △ 3.52:1
- ○ 2.78:1
- ◇ 2.25:1

DIE ANGLE, DEGREES

Fig. 7.1.2
Die angle: 60°

A-A  Extrusion ratio: 2.25
B-B  "        "  2.78
C-C  "        "  3.52
D-D  "        "  4

Fig. 7.2.1 (a)
Die angle: 60°
Extrusion ratio:

A-A  Extrusion ratio: 2.25
B-B  "    " 2.78
C-C  "    " 3.52
D-D  "    " 4

Fig. 7.2.1 (b)
Die angle: $90^\circ$

Extrusion ratio:

- A-A: 2.25
- B-B: 2.78
- C-C: 3.52
- D-D: 4

**Fig. 7.2.2 (a)**
Die angle: 90°

Extrusion ratio:
- A-A: 2.25
- B-B: 2.78
- C-C: 3.52
- D-D: 4

Fig. 7.2.2. (b)
Die angle 120°

A-A extrusion ratio: 2.25
B-B " 2.73
C-C " 3.52
D-D " 4

Fig. 7.2.3 (a)
Die angle $120^\circ$

- A-A: extrusion ratio: 2.25
- B-B: 2.78
- C-C: 3.52
- D-D: 4

Fig. 7.2.3 (b)
Die angle $120^\circ$

A-E \quad \text{extrusion ratio: 2.25}
B-E \quad 2.78
C-E \quad 3.52
D-E \quad 4

Fig. 7.2.3 (c)
Extrusion ratio: 2.25

Die angle:
- 60°
- 90°
- 120°

Fig. 7.2.4 (a)
Extrusion ratio: 2.25

Die angle: 60°

" " 90°

" " 120°

Fig. 7.2.4 (b)
Extusion ratio: 2.78

Die angle: 60°
90°
120°

Fig. 7.2.5 (a)
Extrusion ratio: 2.78

--- Die angle: 60°
--- " " 90°
--- " " 120°

Fig. 7.2.5 (b)
Extrusion ratio: 3.52

- Die angle: 60°
- " " 90°
- " " 120°

Fig. 7.2.6 (a)
Extrusion ratio: 3.52

- - - - - Die angle: 60°
- - - - - " " 90°
- - - - - " " 120°

Fig. 7.2.6 (b)
Extrusion ratio: 4:1
Die angle: 60°
90°
120°

Longitudinal stress, psi

Fig. 7.2.7 (a)
Extrusion ratio: 4:1

Die angle:

- - - - 60°
- - - - 90°
- - - - 120°

Fig. 7.2.7 (b)
Extrusion ratio: 2.78, Die angle: 120°

- Conventional die
- Double reduction die

Fig. 7.2.8 (a)
Extrusion ratio: 2.78, Die angle: 120°

- Conventional die
- Double reduction die

Fig. 7.2.8 (b)
Extrusion ratio: 4:1, Die angle: 120°

- conventional die
- double reduction die

**Fig. 7.2.9 (a)**
extrusion ratio: 4, Die angle: 120°

---

conventional die

double reduction die

---

TANGENTIAL STRESS

RADIUS

centre...outside

Fig. 7.2.9 (b)
Extrusion ratio: 4:1, Die angle: 120°

- conventional die

- double reduction die

Fig. 7.2.9 (c)
Extrusion Ratio: 3.52:1
Die Angle: 90°

Extrusion Ratio: 4:1
Die Angle: 60°

Extrusion Ratio: 4:1
Die Angle: 90°

Extrusion Ratio: 4:1
Die Angle: 120°

Centre — Surface
Centres — Surface

RADIUS

By material removal from inside

By material removal from outside

Fig. 7.2.10 (a)
Fig. 7.2.10 (a) continued
Extrusion Ratio: 2.25:1
Die Angle: 90°

Extrusion Ratio: 2.25:1
Die Angle: 120°

Extrusion Ratio: 2.78:1
Die Angle: 60°

Extrusion Ratio: 2.78:1
Die Angle: 90°

---

By material removal from inside

---

By material removal from outside

RADIUS

Fig. 7.2.10 (b)
Extrusion Ratio: 2.78:1
Die Angle: 120°

Extrusion Ratio: 3.52:1
Die Angle: 60°

Extrusion Ratio: 3.52:1
Die Angle: 90°

Extrusion Ratio: 4:1
Die Angle: 60°

Extrusion Ratio: 4:1
Die Angle: 90°

Extrusion Ratio: 4:1
Die Angle: 120°

**Fig. 7.2.10 (b) continued**
Extrusion ratio: 2.78, Die angle: 60°

--- As extruded
--- Stress relieved at 500°C
--- " " " 550°C

Fig. 7.2.11 (a)
Extrusion ratio: 2.78, Die angle: 60°

- As extruded
- Stress relieved at 500°C
- " " " 550°C

Fig. 7.2.11 (b)
Extrusion ratio: 2.78, Die angle: 60°

--- As extruded
--- Stress relieved at 500°C
--- " " " 550°C

Fig. 7.2.11 (c)
Extrusion ratio: 4, Die angle: 60°

- As extruded
- Stress relieved at 500°C
- " " " 550°C

Fig. 7.2.12 (a)
Extrusion ratio: 4, Die angle: 60°

- As extruded
- Stress relieved at 500°C
- Stress relieved at 550°C

Fig. 7.2.12 (b)
Extrusion ratio: 4, Die angle: 60°C

- As extruded
- Stress relieved at 500°C
- " " " 550°C

Fig. 7.2.12 (c)
**Fig. 7.2.13**

*FOR EXPLANATION OF DIFFERENT CURVES IN Figs 7.2.13 TO 7.2.24, REFER TO TABLE 7.2.2*
Extrusion Ratio: 2.25
Die Angle: 90°
EXTRUSION RATIO: 2.25
DIE ANGLE: 120°

FIG. 7.2.15
Extrusion Ratio: 2.78

Die Angle: 60°

Figure 7.2.16
EXTRUSION RATIO : 2.78
DIE ANGLE : 120°

FIG. 7.2.18
Extrusion Ratio: 3.52
Die Angle: 60°

Res. Long. Stress, MN/m²

Res. Tan. Stress, MN/m²

Figure 7.2.19
Extrusion Ratio: 3.52
DIE ANGLE: 90°

FIG. 7.2.20
Extrusion Ratio: 3.52
Die Angle: 120°

FIG. 7.2.21
EX. RATIO: 4, DIE ANGLE: 60°

RES. TAN. STRESS, MN/m²

RES. LONG. STRESS, MN/m²

FIG 7.2.22
EXTRUSION RATIO: 4
DIE ANGLE: 90°

FIG. 7.2.23
EXTRUSION RATIO: 4
DIE ANGLE: 120°

RES. LONG. STRESS, MN/m²

RES. TAN. STRESS, MN/m²

CENTER RADIUS OUTSIDE

FIG. 7.2.24
<table>
<thead>
<tr>
<th>Ex. Ratio: 2.32</th>
<th>a</th>
<th>d</th>
<th>φ</th>
<th>( \phi' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.446</td>
<td>0.782</td>
<td>1.324</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.490</td>
<td>0.838</td>
<td>1.313</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.533</td>
<td>0.796</td>
<td>1.324</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.583</td>
<td>0.924</td>
<td>1.320</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.568</td>
<td>0.913</td>
<td>1.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.566</td>
<td>0.948</td>
<td>1.23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Refer to Fig. 4-34.
Fig. 7.3.1 Hardness contours. Numbers indicate hardness HV30.
(The leading and tail end of the extruded rod have been machined square to enable it to be gripped during the slitting and grinding operations.)
Extrusion Ratio: 2.78:1, Die Angle: 120°

Fig. 7.3.1 Diagram is twice the natural size.
Fig. 7.3.2

Die angle

- - - 120°
- - 90°
--- 60°

Extrusion Ratio

a  4 : 1
b  3.52 : 1
c  2.78 : 1
d  2.25 : 1

Distance from centre
0  5  10 mm
Extrusion ratio: 2.78, Die angle: 60°

- As extruded
- Stress relieved at 500°C
- " " 550°C
- " " 600°C

Fig. 7.3.3 (a)
Extrusion ratio: 4, Die angle: 60°

--- As extruded

--- Stress relieved at 500°C

--- " " " 550°C

--- " " " 600°C

Fig. 7.3.3 (b)
Extrusion ratio

- □ 4
- △ 3.52
- ○ 2.78
- ▽ unextruded

Fig. 7.3.4 (a)
Fig. 7.3.4 (b)

Number of cycles

Stress, MN/m²
Fig. 7.3.5 (b)
Micrograph showing the fracture path in an extruded specimen.
Fracture through plane away from main fracture × 120.
Extrusion ratio 2:25:1;
Die angle 120°; test temperature -80°C.
Fig 7.3.7. Micrograph showing micro-cracks in the pearlite colonies of the extruded structure into 570. Extrusion Ratio 2:28:1; die angle 120°.
CHAPTER 8

Discussion of Results
8.1 RESIDUAL STRESSES

8.1.1 RESIDUAL STRESS PATTERNS

Except in cases where the extruded bars were given a final 2% reduction, the longitudinal and tangential stresses were found to be tensile on and near the surface, and compressive in the interior. These stress patterns are very similar to those reported for cold drawn [27] and hydrostatically extruded rods [4,33]. The work of Fiorentino et. al. [24] who imparted a final small reduction in order to extrude brittle material free from the surface cracks is based on the premise that residual tensile stresses occur and that these tensile stresses could be beneficially modified by a subsequent small reduction.

While machining the tail-end of an extruded bar (extrusion ratio 3.52:1, die angle 120°) during the present investigation, four radial cracks were formed as shown in Fig. 8.1.1. (the fourth crack is not easily visible). This implies high tensile residual stresses in that portion of the rod removed by machining. Although the residual stresses in the tail end of the extruded bar may be of higher magnitude than those in the steady state region, the sign of the residual stresses is expected to be the same in both regions (because of a similar normal stress distribution in the billet in both cases).

One of the earlier studies on residual stresses in extruded rods by Frisch and Thomsen [22] indicates an apparently anomalous stress pattern in the aluminium rods, i.e. compressive stresses at the surface and tensile stresses at and near the centre. However, the extrusion ratio and included die angle used by Frisch and Thomsen
(8:1 and 180° respectively) are outside the range of these variables studied in the present investigation and in the work with hydrostatically extruded rods [4,33]. When these differences in experimental conditions are taken into account, the various patterns of residual stress reported may be regarded as not being contradictory.

8.1.2 THEORETICAL vs EXPERIMENTAL RESULTS

Figures 7.2.13 to 7.2.24 indicate that in most cases the residual stresses derived from the velocity fields which are located relative to the optimum velocity field for frictionless conditions, in a direction opposite to the extrusion direction, give a better qualitative agreement with the experimental results than those which are derived from the rest of the velocity fields investigated. The latter include the optimum velocity fields for frictionless conditions and those which are displaced relative to it towards the extrusion direction, opposite to the extrusion direction, but further removed from the velocity fields which give the best qualitative agreement with the experimental results.

The above observation is consistent with the experimental conditions which were such as to minimise the friction between the material and tool boundaries by appropriate lubrication but in no way to eliminate friction completely. That the increasing friction causes the boundaries of the deformation zone to move progressively in a direction opposite to the direction of extrusion is observed experimentally and established analytically in studies of Lambert and Kobayashi [55]. Similar effects of friction are also reported elsewhere [39].

The results of Fig. 7.2.25 show the effect of the movement of the boundaries of deformation zone on the residual stress patterns.
From these results and the observation that increasing friction moves the deformation zone in the direction opposite to the extrusion direction and also enlarges the same, it may be concluded that friction has a significant influence on the magnitude and nature of the residual stresses. With a displacement of the deformation zone which would be caused by increasing friction, the residual stresses at the axis become more compressive. Similar effect of the contribution of friction may be seen in the results of the present investigation. As will be seen later, a reduction in die angle which gives rise to increased friction at the die, results in increased compressive stresses at the centre of the rods.

8.1.3 EFFECT OF DIE ANGLE AND EXTRUSION RATIO ON RESIDUAL STRESS PATTERNS

The effects of these variables will be discussed on the residual longitudinal stresses only. The tangential residual stresses, as can be seen from the experimental and analytical results follow similar patterns and are generally lower in magnitudes than the longitudinal stresses. The comments on the effect of the process variables on residual longitudinal stresses, therefore, are applicable to residual tangential stresses as well. Where experimental results are compared with the theoretical results in the subsequent discussion, the theoretical results are those taken from the stress distributions 'd' in Figs. 7.2.13 to 7.2.24.

8.1.4 EFFECT OF DIE ANGLE

Residual longitudinal stresses at the centre of the rods are plotted in terms of the stress ratios (residual stress/mean yield stress) as a function of the die angle, in Figs. 8.1.2 (a) and (b) respectively for experimental and theoretical results. Fig. 8.1.2 (b)
also includes the results of the residual stresses which were computed from the axial stress distribution (for frictionless conditions) in the deformation zone given in the work of Lambert and Kobayashi [133].

It can be seen from these figures that except at the smallest extrusion ratio, residual stresses become more compressive with the smaller die angles. This effect is more pronounced in the theoretically computed residual stresses (Fig. 8.1.2 (b) ) than in the experimental results (Fig. 8.1.2 (a) ) It is interesting to note that the curve 'e' for which the residual stress data were computed from a completely independent source [133] fits in well with the trends found in the results of the present investigation.

At the smallest extrusion ratio investigated (i.e. 2.25:1), the effect of the die angle on the residual stresses at the centre of the rods is, however, opposite to that at the higher extrusion ratios, i.e. residual stresses become more compressive as the die angle is increased.

The residual stresses on or near the surface of the extruded rods do not indicate any consistent trend with the change of the die angle. This is true both in the case of experimental and analytical results. The reason for this could be that it is difficult to determine accurately the stress distribution at the die-metal interface in the deformation zone. Also the surface residual stresses, especially for the extrusion conditions investigated, are likely to undergo further modification at the die-land. This effect is discussed further separately.
8.1.5 EFFECT OF EXTRUSION RATIO

The effect of extrusion ratio on the residual stresses at the centre of the rods is summarised in Fig. 8.1.3 (a) and (b). The trends in the experimental and analytical results are similar for 30° and 60° die angles, although the variation in the magnitude of the residual stresses with the change in extrusion ratio is more gradual in experimental than in the analytical results. The effect of the extrusion ratio on the experimental results for the 60° die angle is similar to that reported in the hydrostatically extruded rods [4].

In Fig. 8.1.3 are also included the results for residual stresses derived from the axial stress distribution in the deformation zone reported by Lambert and Kobayashi [133]. Again the agreement with the results of the present investigation is quite good.

The effect of extrusion ratio on the experimentally determined peak tensile residual stresses near the surface is demonstrated in Fig. 8.1.4 for the three die angles. It can be seen from this that the peak tensile stresses increase first with the increase in extrusion ratio, reach a maximum and then start to decrease with further increase in extrusion ratio. This trend is consistent with the observation by Pugh as quoted by Miura et. al. [33], that surface cracks appear only in rods extruded with intermediate extrusion ratios. The cracks are absent when low or high extrusion ratios are employed.

The reduction in the peak tensile surface stresses with the increase in the extrusion ratio is also reported in hydrostatically
extruded rods by Miura et. al. [33]. Their work, however, indicates
that these stresses decrease monotonically when the extrusion ratio
is increased from 1.25 to 3.7.

8.1.6 MAGNITUDE OF RESIDUAL STRESSES

The magnitude of residual stresses at the extrusion axis
in some instances, is higher than the yield stress of the material
in uniaxial tension. The existence of such high stresses may, at
first, appear to be anomalous but if the triaxiality of the stresses
is taken into consideration, it can be seen that residual stresses
of higher magnitude than the yield stress of the material in
uniaxial tension can exist. Taking for example the stress distri­
butions in Fig. 8.1.5, the longitudinal, tangential and radial
compressive stresses at the extrusion axis are 1000, 530 and
530 MN/m² respectively. The effective stress from Huber-von-Mises
criterion (as discussed in Chapter 4) will be

\[1000 - 530 = 470 \text{ MN/m}^2\]

The 0.1% proof stress of the specimen machined from the
extruded bar, the residual stress distribution for which is cited
above, is 687 MN/m². The proof stress of the material at the
extrusion axis may be less than 687 MN/m² but the difference is
not likely to be appreciable. The proof stress determination was
made on the machined specimen where the outer severely deformed
material is already removed.

8.1.7 EFFECT OF THE FINAL 2% REDUCTION

The final 2% reduction was achieved by making the die­
exit diameter smaller than the die-entry diameter. The exit of the
die, therefore, acted as a square edged die for the bar coming out of the entry end. Had this die been used for an annealed material, the residual stress pattern would have been high compressive stresses on the surface and comparatively low tensile stress in the interior. In the present case the bar which undergoes a small reduction through this die has already tensile residual stresses on the surface and comparatively high compressive stresses in the interior. The residual stress pattern due to the final small reduction will be approximately superimposed on the residual stress pattern already existing in a manner indicated in Fig. 8.1.6. The resultant stress pattern will, therefore, be compressive surface stresses. The stresses in the interior will remain compressive but their magnitude will be reduced.

8.1.8 STRESS RELIEF ANNEALING

At both extrusion ratios, namely 2.78 and 4, stress relief annealing at 500°C reduces the residual stress level considerably. The surface tensile stress left is too low to have any appreciable harmful effect. On the other hand, hardness, yield strength and tensile strength are not greatly reduced after 500°C stress relief annealing. Restoration processes are known to occur at lower temperatures as the degree of prior cold work is increased. Therefore, in rods extruded at 4:1, extrusion ratio stress relieving at 550°C causes greater loss of strength and hardness than in rods extruded at 2.78:1 extrusion ratio. The residual stress level is reduced to very low values, in both cases, after the stress relief annealing at 550°C. Stress relief annealing at 600°C brings the
hardness and strength values of the extruded rods, in both cases, nearly down to the hardness, yield strength and U.T.S. values of the annealed material. Residual stresses, of course, vanish at this temperature.

8.1.9 GENERAL COMMENTS

Although a large number of velocity fields had been examined at a particular extrusion condition (extrusion ratio and die angle), it has not been possible to obtain more than a qualitative agreement between the theoretically derived and experimentally determined residual stress patterns. It is possible that a better agreement with the experimental results could have been obtained had the theoretical analysis been more accurate, i.e. (i) instead of assuming a constant yield stress, a yield stress as a function of the strain at each point in the deformation zone had been used; (ii) a larger number of velocity fields had been examined; and (iii) the stress solution had been more accurate (in the stress solution used the equilibrium is not completely satisfied) and the stresses at the die-metal interface could be determined more accurately. However, a complete agreement with the experimental results may not have been possible. This is because of the fact that a modification of the residual stress patterns is likely to take place when the material passes over the 'die-land' as indicated below.

There is both experimental and theoretical evidence to suggest that the exit of the deformation zone is a curved surface which lies ahead of the start of the die land. After exit from the deformation zone there is supposed to be no relative motion (apart from that due to elastic stress relaxation) between the elements of the extrusion and
it would seem that the effects of the elastic stress relaxation would start. If as a result of elastic stress relaxation there is a tendency for the extrudate to dilate, this would be prevented by the die land which, in turn, would impose compressive radial stress on the extrudate. The radial stress would act in combination with (a) the residual stresses being developed in the extrudate immediately on exit from the deformation zone and (b) the frictional stresses at the die land etc. Wherever this combination results in the effective yield stress in the surface layers being exceeded, the surface material would yield. This indeed happens as is implied from the observations made by Fiorentino et. al. [24]. While extruding beryllium with an extrusion ratio of 4:1 through a 90° (included) conical die, they observed that circumferential cracks in the material developed only after leaving the deformation zone and entering the die land; no evidence of cracking was found within the deformation zone. This, as proposed by the authors, was due to the fact that the combination of the radial compression (imposed by the die land) and the developing tensile residual stresses exceeded the fracture stress of the brittle material (beryllium) and caused its cracking. That the surface cracks initiate in the extrusion only after its exit from the deformation zone is also reported by Yamada et. al. [56]. In their work on the hydrostatic extrusion of an Al-Cu-Si alloy (see Table 8.1.1 for chemical composition) through conical dies of varying length (L) of the die land (see Fig. 8.1.7), they found that the back pressure required to prevent the occurrence of surface cracking of the rods increased with the increase in length, L, of the die land. For instance, when extruding with an extrusion ratio of 1.4 through a conical die of included angle 30°, the surface cracks were prevented by a back pressure
of 1800 kg/cm² (177 MN/m²) when the die had no land (i.e. L = 0). This pressure had to be increased as the length L increased and was maximum at 3600 kg/cm² when L = 2.5 mm. Any further increase in L did not significantly affect the back pressure.

The authors suggest that the surface cracks were caused by the presence of axial tensile stresses on the surface and these stresses developed after the exit of the extrusion from the deformation zone. They propose that the factors which contributed to these surface stresses were (i) tensile residual stresses; (ii) reaction stresses from the die surface; (iii) frictional stress between the billet and die surface, and extruded rod and the die land; and (iv) elastic stress caused by the pressure from the die bearing. From this work again, the contribution of the die land in effectively increasing the surface tensile stress, and hence in causing either the cracking or the yielding of the surface material, appears quite significant.

The dilation of the extrudate on its exit from the deformation zone is possibly caused by the developing residual surface hoop tensile stresses. This could be so as evidenced in the work reported by Pugh et al. [137]. Rods extruded by them with extrusion ratios and die angles similar to those used in the present investigation (and therefore having surface tensile residual stresses) were found to be larger than the diameter of the die. The increase in diameter was of the order of 0.001 to 0.0015 in. (0.025 to 0.038 mm) for a die diameter of 0.5 in (12.7 mm). In another study [58] the diameter of the bars which were cold drawn with a reduction in area of 10 to 20 per cent, was also found to be larger than the die diameter. The difference between the rod and die diameters was, however, reduced considerably.
when the drawn rods were given a second small reduction in area of the order of 2 per cent. For example, rods drawn with a 20 per cent reduction in area through a 43.7 mm diameter die were found to be 0.162 mm larger (in diameter) than the die diameter. A second reduction in area of about 2 per cent narrowed the difference between die and rod diameter down to 0.02 mm. It is thought the second small reduction imparted to the drawn rods brought down considerably the level of residual surface tensile stresses and possibly even changed these into compressive stresses. This change in the nature of the residual stresses then reduced the diameter of the drawn rods and narrowed down the difference between the rod and die diameters.

When the dilation of the extrudate is large enough for the die land to cause yielding of the extrusion surface material, then the action of the die land may be considered analogous to the second reduction in a double reduction die. The difference between the two is that whereas the second reduction in a double reduction die, being of the order of 2 per cent reduction in area, causes a penetration of the plastic deformation to some depth below the surface, the die land affects the surface material to a considerably smaller depth. Nevertheless, any plastic deformation of the surface material would cause a modification of the residual stresses as shown schematically in Fig. 8.1.8. It can be seen from this that the die land would cause the residual stresses at the central region of the rod to become less compressive and at the surface to become less tensile. Away from the surface, however, the level of the tensile residual stresses is raised. Thus, after a modification of the residual stress by the die land, the peak tensile stresses occur near the surface rather than on the surface.
an observation which is consistent with the experimental results of the present and other \([4,33]\) investigations.

In the present investigation the theoretical analysis of the residual stresses has been carried out assuming a constraint-free elastic stress relaxation of the material on exit from the deformation zone. The residual stresses thus determined in the central region of the rod are compared with the experimental results in Figs. 8.1.2-3(a) and (b). It can be seen from these figures that the experimental residual stresses are less compressive than the theoretical stresses. This trend is consistent with the results of the analysis presented above.
8.2 MECHANICAL PROPERTIES

The hardness and tensile properties are in agreement with the work of other authors [62,63,66,135]. In general, an increase in the die angle at any given extrusion ratio causes an increase in the redundant shearing during extrusion, and this is reflected in a greater variation in hardness over the specimen cross-section. In the region near the extrusion axis, the increase in hardness with the increase in die angle is not significant. Similar effect is seen in the proof stress values of machined specimens. (The higher proof stress values in two cases are due to higher carbon content of the specimens as discussed in Chapter 7.) The die angle, however, affects the proof stress of the unmachined specimens in a manner similar to its effect on hardness. The proof stress of the unmachined specimens is the average of the proof stresses of the more or less uniformly deformed material in the region near the extrusion axis and of the outer severely deformed material. As redundant shearing of the outer material increases with the increase in die angle, the outer material will be work-hardened more at higher die angles. This will be reflected in the higher average values of proof stress over the cross-section of the extruded bar at higher die angles.

The change in the percentage elongation and % reduction in area with the increasing extrusion ratio from 2.25 to 4 is negligible. This is because these values drop sharply at low reductions and after a certain reduction the further change is almost negligible [135].
The fatigue properties of machined extruded rods show a considerable improvement over the properties of the unworked material. No attempt was made to calculate surface residual stresses after machining; however, from the discussion in previous sections, the residual surface stresses resulting from the redistribution after machining would be expected to be tensile. Moreover, the results of fatigue tests on as-extruded specimens having surface tensile stresses also show an improvement in the fatigue limit over the unextruded material. It can therefore be concluded that the improvement in the fatigue properties of the extruded rods was due to increased strength and hardness of material. However, the effect of the surface tensile stresses cannot be entirely ruled out. In fact, the tensile residual stresses on the surface were very small (this is further confirmed from the stresses measured by outer layer removal techniques) and even if they had some detrimental effect, the effect of strain-hardening seems to have dominated, resulting in considerable improvement in the fatigue properties. It is quite possible that the fatigue properties of the rods extruded through double reduction dies would have been better than those of the conventionally extruded rods. However, due to lack of time, the former could not be tested for fatigue.

Fatigue tests on the as-extruded specimens were not satisfactory because of

(i) the problems of gripping them in the machine without introducing stress concentration in the ends;

(ii) the surface finish of the extruded rods was not very good
and the surface finish changed from one specimen to another because of the different surface finish of the billets;

(iii) the as-extruded rods were slightly bent. The rods could not, of course, be straightened because of the change in the residual stress pattern which this would have introduced.

In spite of all these drawbacks, it can be seen that the true fatigue limit is considerably higher than that of the unworked material.

It has also been noted that there is little difference in hardness on the specimen axis for the different extrusion ratios used; also the difference in the proof stresses at the three higher extrusion ratios is not very large. (The lower proof stresses at the lowest extrusion ratio may be due to the smaller test diameter of the tensile specimens used in this case.) This is because the strains involved in the extrusion process correspond to the region of work hardening at high strains where hardness rises only slowly with increasing strain. Hence little difference is to be expected as a result of an increase in the extrusion ratio (Figure 7.3.2).

The impact behaviour of extruded specimens exhibits a number of unusual features (Figure 7.3.5a,b). Except for the lowest extrusion ratio, namely 2.25:1, the transition temperature for the three higher extrusion ratios is lower than that of the unextruded material. This is followed by the higher energies required to
fracture extruded specimens in the brittle condition. In fact the energy required to fracture is so high for the extrusion ratio 4:1 that the term "brittle fracture" becomes a misnomer.

The beneficial effect of high extrusion ratios on the impact behaviour is probably caused by delamination along inclusions elongated during the working process. This effect can be seen from the appearance of the fractured test pieces; the fracture tends to branch down planes roughly parallel to the specimen axis (Figure 7.3.6) giving rise to a very torn and ragged appearance. The harmful effect of small amounts of deformation on the impact properties of mild steel is well-known [78], and is caused by the rise in the yield strength of the deformed pearlite. At small deformations a mechanically fibred structure is not produced and the dominant effect is therefore due to the increased yield strength. Elongated inclusions in the fracture path cause the relief of the fracture energy - and at the same time due to branching off of the fracture path the area, and therefore the surface energy of the fracture faces, is increased. The observed transition curves are thus explicable in terms of the competition between the increased yield strength and effects of the mechanically fibred structure. Structural damage may play a small part in determining the detailed behaviour of the steel and is a possible explanation of the difference observed between the pearlite and the spheroidised microstructures.

The above discussion refers only to the impact behaviour of specimens having notches perpendicular to the extrusion axis; it is probable that the resistance to cracks propagated parallel to the extrusion axis would be poor.
From the limited work carried out on the determination of structural damage in the extruded bars it became evident that structural damage was a factor of the tensile stresses developed in the zone near the extrusion axis, during extrusion. As discussed earlier the tensile stresses in the region near the extrusion axis are likely to be developed at small extrusion ratios and large die angles. Evidence of structural damage was found only in the specimen extruded at the lowest extrusion ratio, i.e. 2.25:1 employed in this investigation and at the largest die angle used, namely 120°. The revelation of structural damage in the bars extruded at 2.78:1 extrusion ratio and 60° die angle after stress relief anneal at 500°C could be due to the fact that in the as-extruded condition the microcracks were closed by the high residual compressive stresses present in the interior and were opened up when these stresses were relieved to a large extent.
The residual longitudinal and tangential stresses are found to be compressive along the axis and tensile towards the surface for the conical dies used in this investigation. The distribution of the residual stress across the cross-section depends on the particular extrusion ratio and die angle employed. It is difficult to predict the influence of either the extrusion ratio or the die angle without considering them together. At a particular extrusion ratio residual stresses at the centre become more compressive and surface residual stresses become more tensile with the decreasing die angle. The effect of decreasing extrusion ratio at a particular die angle is opposite, i.e., the residual stresses at the axis become less compressive and surface stresses become less tensile as the extrusion ratio decreases.

By the use of a subsequent small reduction in area the residual stress pattern can be modified to yield compressive stresses at the surface. The residual stress level can be reduced considerably and at the same time most of the gain in strength and hardness can be retained by employing a suitable temperature for stress relief annealing. This temperature for EN-2E was found to be 500°C.

An attempt to correlate the observed residual stresses with those computed from the stress distribution in the deformation zone predicted by the use of flow function method of analysis, indicates that the residual stresses depend upon the details of the velocity field selected and that friction at the die surface, as one of the factors affecting the velocity field, thus affects the pattern of residual stresses.
Under certain experimental conditions (extrusion ratio, die angle and friction at the die-metal interface etc.), plastic deformation of the extruding material possibly occurs even after its exit from the deformation zone. This, amongst other factors, is caused when a high radial pressure is imposed on the extruding material by the die land. It is suggested that the difference in the experimental and analytical residual stress results are partly due to the above phenomenon. The occurrence of the peak residual tensile stresses near the surface rather than on the surface is also thought to be caused because of the yielding of the extruding material after its exit from the deformation zone.

The hardness, yield strength and tensile strength of steel show a general marked increase after extrusion and ductility in terms of % elongation indicate a sharp decline. There is not much difference between the hardness and yield strength of rods extruded in the range of extrusion ratios employed. These properties show only slight improvement with the increasing extrusion ratios and depend to some extent on the die angle as well.

The fatigue properties, as measured by the use of machined specimens, improve markedly and this improvement can be attributed to the enhanced strength. Surface residual stresses, being small, do not seem to contribute materially to the fatigue properties and the effect of work hardening remains dominant. This is confirmed from the fatigue behaviour of unmachined specimens. The fatigue properties are not markedly inferior to those of the machined specimens.

The notch impact transition temperature is lower than that of the unworked material, at high extrusion ratios; the energy for
ductile fracture is reduced whilst that for brittle fracture is raised. The fracture showed marked tearing roughly parallel to the extrusion direction.
SUGGESTIONS FOR FUTURE WORK

The results of the present investigation have indicated that friction at the die surface can significantly affect the residual stress state in the extruded rod. It is suggested that this effect of friction be verified by studying residual stress patterns in rods which have been extruded through dies with different lubrication conditions.

Further work also needs to be done in order to investigate the possibility of improving the accuracy of the theoretical analysis of residual stresses. For this, it is suggested that residual stresses in the above rods be determined analytically from the actual velocity fields obtained experimentally. The theoretical analysis should incorporate (i) the varying properties of the material throughout the deformation zone and (ii) the effect of the die land. The residual stresses thus determined should then be compared with the experimental results in order to establish the accuracy of the theoretical analysis.

The fatigue property investigation of the extruded bars has been carried out only for those cases in which residual tensile stresses were present at the surface. In spite of the presence of surface tensile stresses, fatigue properties of the extruded material have been found to be markedly superior to those of the unextruded material, and this improvement is thought to be due to the improved strength of the extruded material. It will be interesting to compare these results with those obtained by the fatigue testing of rods which are extruded through double reduction dies and which have thus compressive residual surface stresses. This, it is thought, may lead to more definite conclusion about the contribution of surface residual stresses on the fatigue properties of extruded rods.
<table>
<thead>
<tr>
<th>Cu</th>
<th>Zn</th>
<th>Fe</th>
<th>Mn</th>
<th>Ni</th>
<th>Si</th>
<th>Mg</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥3.0</td>
<td>&lt;1.0</td>
<td>&lt;1.2</td>
<td>&lt;0.8</td>
<td>&lt;0.5</td>
<td>≥6.0</td>
<td>&lt;0.5</td>
<td>Remainder</td>
</tr>
</tbody>
</table>

Table 8.1.1 Chemical Composition of Al-Cu-Si Alloy used in Ref. 56 (Wt %)
Fig 8.1.1 (Radial cracking occurring as a result of machining of extruded rod) Extrusion ratio
3.52:1; die angle 120°.
Fig. 8.1.3 Res. Stresses in the Centre of Rods

Fig. 8.1.4 Surface Res. Stresses Experimental Results
extrusion ratio: 4, die angle: 120

A-A -------- longitudinal stress
B-B -------- tangential stress
B-C -------- radial stress

Fig. 8.1.5
Fig. 8.1.6

Schematic residual stress patterns (longitudinal)
**Fig. 8.1.7 Conical Die**

**Residual Long Stress**

![Graph showing residual long stress distribution](image)

- **Q-Q** Residual stress distribution immediately on exit from deformation zone.
- **C-C** Resultant of Q-Q and L-L.
- **L-L** Residual stress distribution due to small plastic deformation of the surface layers caused by the die land.
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APPENDIX II

DERIVATION OF SACHS EQUATIONS

There are two approaches which have been followed to find out the residual stresses by layer removal technique. One approach is to calculate the stresses in the removed layer from the observed strains and then correct these stresses by calculating the amount by which these stresses have changed due to the previous material removals. The second approach is to calculate the stresses in the material, which has not yet been bored out, from the total observed strains due to all the layer removals and then relate these stresses to the apparent stresses which are introduced in the remaining material due to the sum of the stresses in the removed layers. This second approach which is adopted by Lambert [126] is used here to derive the Sachs equations. It is also assumed that the "apparent" stresses which cause the strains that are measured can be superimposed on the residual stresses originally present without disturbing the residual stress distribution. This assumption is valid if the effective stress due to the resultants of the apparent and the residual stresses does not exceed the critical yield stress. If it does, then plastic flow will occur, resulting in the partial relief and redistribution of residual stresses. The Sachs boring method in that case will not give the true values of residual stresses.

Use is made of the following relationships from the theory of elasticity. Tangential stress at the outside surface of a thick walled cylinder of inside radius \( r \) and outside radius \( R \)
(Figure 11.1), subjected to a uniform internal pressure $p_i$ is given by [127]:

$$\sigma'_{t} = \frac{2r^2 p_i}{R^2 - r^2} \quad (1)$$

and from the generalised Hook's Law,

$$\sigma'_{z} = \frac{E}{1 - \nu^2} (\lambda + \nu\theta) = E'\lambda \quad (2)$$

$$\sigma'_{t} = \frac{E}{1 - \nu^2} (\theta + \nu\lambda) = E'\theta \quad (3)$$

Now consider that the specimen is bored out to an arbitrary radius $r$. A uniform pressure $p_i$ has been removed from the inside boundary due to the removal of material containing polar symmetric residual stresses. At radius $r$, the $p_i$ removed is equal to the original radial stress at that point before boring started, therefore:

$$\sigma'_{r} = p_i \quad (4)$$

Substituting from Equations (1) and (3),

$$\sigma'_{r} = \frac{R^2 - r^2}{2r^2} \sigma'_{t}$$

$$\sigma'_{r} = E' \frac{R^2 - r^2}{2r^2} \theta \quad (5)$$

* Notations are defined at the end of the Appendix.
Equation (5) can be rewritten in the form used by Sachs by multiplying the numerator and denominator by \( n \) such that

\[
\sigma_r = \frac{F_a - F}{2F} \theta
\]

where \( F_a = \pi R^2 \) and \( F = \pi r^2 \). Equation (6), therefore, gives the radial residual stress as a function of \( r \) and the strains measured at various stages of boring.

For the tangential stress, consider the slug of material of radius \( r \) which has been bored out. This, too, was subject to the uniform pressure \( p_i \). The relationship between the tangential stress and the external pressure \( p_i \) is obvious from the free body diagram of half the slug. If the slug is split by a plane through its axis, the equilibrium of the forces acting gives:

\[
2r p_i = \int_{0}^{r} \sigma_t(\rho) \, d\rho
\]

or

\[
r p_i = \int_{0}^{r} \sigma_t(\rho) \, d\rho
\]

where \( \rho \) is the variable of integration \( \leq r \).

Substituting \( p_i \) and \( \sigma_t \) from Equations (1) and (3),

\[
\int_{0}^{r} \sigma_t(\rho) \, \rho = \frac{R^2 - r^2}{2r} \theta
\]

Differentiating both sides of equations with respect to \( r \)
To put into the form used by Sachs, multiply the first term in the brackets by $\pi/\pi$ and for the second term use:

$$\frac{d\Theta}{dr} = \frac{d\Theta}{dF} \cdot \frac{dF}{dr} = \frac{d\Theta}{dF} \cdot 2\pi r$$

(10)

The value of $\sigma_t$ at any radius $r$ is, therefore, given by

$$\sigma_t = E' \left[ (F_a - F) \frac{d\Theta}{dF} - \frac{F_a + F}{2F} \Theta \right]$$

(11)

For the longitudinal stress, it is assumed that the specimen is long enough so that plane strain conditions are valid. When the specimen is bored out to a radius $r$, a longitudinal boundary force $P_a$ is removed from the inner surface of the remaining hollow cylinder.

Considering the slug which was removed:

$$P_a = \int_0^r \sigma_L(\rho) \, dF \rho$$

$$= 2\pi \int_0^r \sigma_L(\rho) \, \rho \, d\rho, \quad (F \rho = \pi \rho^2)$$

(12)

$P_a$ sets up the apparent stress $\sigma'_L$ in the remaining cylinder, such that

$$\sigma'_L = \frac{P_a}{\pi (R^2 - r^2)}$$

(13)
Substituting the value of $P a$ from Equation (12) and the value of $\sigma'_x$ from Equation (2), Equation (13) becomes

$$2 \int_{0}^{r} \sigma_x(\rho) \: \rho \: d\rho \: = \: E' (R^2 - r^2) \Lambda$$

(14)

Differentiating both sides with respect to $r$:

$$2 \sigma_x (r) \: = \: E' \left[ -2 \pi \Lambda + (R^2 - r^2) \frac{d \Lambda}{dr} \right]$$

or

$$\sigma_x = E' \left[ -\Lambda + \frac{R^2 - r^2}{2 \pi} \frac{d \Lambda}{dr} \right]$$

(15)

To get the equations in the form used by Sachs, proceed as in Equations (9) to (11) with

$$\frac{d \Lambda}{dr} = 2 \pi r \frac{d \Lambda}{d \phi}$$

$$\sigma_x = E' \left[ (P a - F) \frac{d \Lambda}{d \phi} - \Lambda \right]$$

(16)
NOTATIONS USED IN APPENDIX II

\[ E = \text{modulus of elasticity} \]

\[ E' = \frac{E}{1 - \nu^2} \]

\[ \nu = \text{Poisson's ratio} \]

\[ \lambda = \text{measured axial strain due to layer removal} \]

\[ \theta = \text{measured tangential strain due to layer removal} \]

\[ \Lambda = \lambda + \nu \theta \]

\[ \Theta = \theta + \nu \lambda \]

\[ \sigma_A, \sigma_T, \sigma_R = \text{polar symmetric axial, tangential and radial stresses respectively} \]

\[ \sigma'_A, \sigma'_T, \sigma'_R = \text{apparent longitudinal, tangential and radial stresses respectively which cause the measured surface strains on layer removal} \]
APPENDIX III

DERIVATION OF EQUATIONS FOR RESIDUAL STRESS DETERMINATION BY OUTER-LAYER REMOVAL

In Figure III.1, $2r_o$ is the internal diameter of the tube. Strains are measured on the tube bore as the external diameter is reduced from the original value $2R$. The radial, tangential and longitudinal residual stresses ($\sigma_r$, $\sigma_t$ and $\sigma_z$ respectively) are again assumed to depend only on $r$, and are assumed to be principal stresses.

Let $\lambda$ and $\theta$ be the longitudinal and circumferential strains found when the external diameter has been reduced to $2r$: let $\sigma'_z$ and $\sigma'_t$ be the corresponding changes in longitudinal and tangential stress at the bore surface. Then

$$\frac{\sigma'_z}{E} - \frac{\nu \sigma'_t}{E} = \lambda$$

and

$$\frac{\sigma'_t}{E} - \frac{\nu \sigma'_z}{E} = \theta$$

Hence

$$\sigma'_z = \frac{E}{1 - \nu^2} A \quad \text{where } A = \lambda + \nu \theta \quad \text{III.1}$$

and

$$\sigma'_t = \frac{E}{1 - \nu^2} \theta \quad \text{where } \theta = \theta + \nu \lambda \quad \text{III.2}$$

Consider the hollow tube, diameter $2r$: originally the
reaction of the surrounding material provided an external negative pressure of magnitude $\sigma_r$. When released from the surrounding material, the external pressure is zero. Hence $\sigma'_0$ may be considered as caused by the application of pressure $\sigma_r$ to the outside surface of the hollow cylinder of external diameter $2r$. Using the appropriate equations for a thick cylinder, we obtain

$$\sigma'_0 = -\sigma_r \left( \frac{2r^2}{r^2 - r_0^2} \right)$$

and hence, from equation III.2

$$\sigma_r = -\frac{E}{1 - \nu^2} \left[ \left( \frac{r^2 - r_0^2}{2r^2} \right) \theta \right]$$

III.3

or

$$\sigma_r = -\frac{E}{1 - \nu^2} \left[ \frac{F - F_a}{2F} \theta \right]$$

III.4

where $F = \pi r^2$, $F_a = \pi r_0^2$.

Since the residual stresses $\sigma_r$ and $\sigma_t$ must obey the equilibrium equation

$$\sigma_t = \sigma_r + r \frac{d\sigma_r}{dr}$$

it may readily be shown that

$$\sigma_t = -\frac{E}{1 - \nu^2} \left( \frac{r_0^2 - r^2}{2R} \frac{d\theta}{dr} + \frac{r^2 + r_0^2}{2r^2} \theta \right)$$

$$= -\frac{E}{1 - \nu^2} \left[ (F - F_a) \frac{d\theta}{dF} + \frac{F + F_a}{2F} \theta \right]$$
The axial tensile load in the material ground away as the external diameter is reduced from \( R \) to \( r \) is

\[
P = \int_{r}^{R} \sigma_{L} 2\pi \rho \, d\rho
\]

Since \( P \) acted to compress the remaining hollow cylinder, its release causes an apparent tensile stress \( \sigma'_{L} = \sigma'_{L0} \) uniform across the section such that

\[
P = \pi \sigma'_{L} (r^2 - r_0^2)
\]

From equation III.1 we therefore obtain

\[
\int_{r}^{R} 2\sigma_{L} \rho \, d\rho = \frac{E}{1 - v^2} (r^2 - r_0^2) \Lambda
\]

Differentiating both sides with respect to \( r \)

\[
-2\sigma_{L} r = \frac{E}{1 - v^2} \left[ \left( r^2 - r_0^2 \right) \frac{d\Lambda}{dr} + 2r \Lambda \right]
\]

Hence the two equivalent equations for \( \sigma_{L} \) may easily be obtained, namely

\[
\sigma_{L} = \frac{E}{1 - v^2} \left[ \left( \frac{r^2 - r_0^2}{2r} \right) \frac{d\Lambda}{dr} + \Lambda \right]
\]

and

\[
\sigma_{L} = \frac{E}{1 - v^2} \left[ (F - F_a) \frac{d\Lambda}{dF} + \Lambda \right] \quad \text{III.6}
\]

* \( \rho \) is the variable of integration \( \geq r \)
This method of derivation follows that given by Lambert [126] and equations III.4 - 6 are those found by Bühler [115] and quoted by Botros [116].

**Correction for removal of bore**

The residual stresses \( \sigma_z, \sigma_t \) and \( \sigma_r \) derived above are the stresses which were left in the tube when the boring out was terminated at the internal radius of \( r_o \). For determining the original residual stresses in the solid bar, \( \sigma_z, \sigma_t \) and \( \sigma_r \) must be corrected for the changes caused by the removal of the bore from the solid bar. Let \( \sigma_z'', \sigma_t'' \) and \( \sigma_r'' \) be corrected stresses and \( \sigma_z', \sigma_t' \) and \( \sigma_r' \) be corrections applied to \( \sigma_z, \sigma_t \) and \( \sigma_r \) respectively. These corrections are obtained as follows:

\[
\sigma_z' = \sigma_z - \frac{E}{1 - v^2} \Lambda_o,
\]

This is, in fact, the stress relieved (+ve or -ve) due to the removal of axial force, caused by the removal of the core (of outside radius \( r_o \)). Let \( \Lambda_o \) be value of \( \Lambda \) found at the surface of the solid bar when boring-out was terminated at radius, \( r_o \). Then the uniform stress \( \sigma_z' \), relieved from the tube cross-section due to \( \Lambda_o \) is given by

\[
\sigma_z' = \frac{E}{1 - v^2} \Lambda_o,
\]

and hence

\[
\sigma_z'' = \sigma_z - \frac{E}{1 - v^2} \Lambda_o.
\]
This release of the radial stress at the surface of the bore corresponds to the imposition of an internal pressure into a thick-walled cylinder of the dimensions of the remaining tube. The value of this internal pressure is given by equation 5 in Appendix II. The tangential and radial stresses induced due to this internal pressure at a radius \( r \) in the wall of the tube are the values of \( \sigma_t' \) and \( \sigma_r' \) at that radius.

Using appropriate equations for a thick-walled cylinder, we obtain

\[
\sigma_t' = \frac{1}{2} \left( \frac{E}{1 - \nu^2} \right) \left( 1 + \frac{R^2}{r^2} \right) \theta_0
\]

\[
\sigma_r' = \frac{1}{2} \left( \frac{E}{1 - \nu^2} \right) \left( 1 - \frac{R^2}{r^2} \right) \theta_0
\]

hence

\[
\sigma_t'' = \sigma_t - \frac{1}{2} \left( \frac{E}{1 - \nu^2} \right) \left( 1 + \frac{R^2}{r^2} \right) \theta_0
\]

and

\[
\sigma_r'' = \sigma_r - \frac{1}{2} \left( \frac{E}{1 - \nu^2} \right) \left( 1 - \frac{R^2}{r^2} \right) \theta_0.
\]
Fig II.1

Fig III.1
APPENDIX IV

COMPUTER PROGRAMS FOR THE CALCULATION OF RESIDUAL STRESSES

Program I

This program calculates the residual stresses by using Sachs equations when material removal is carried out from centre to the outside diameter.

The program does the following computation:

(i) Calculates areas *F* corresponding to diameters at material removal steps.
(ii) Calculates *θ* and *A* from *θ* and *λ*.
(iii) Fits up to eighth order polynomial in *F* vs *θ* and *F* vs *λ*.
(iv) Calculates \( \frac{d\theta}{dF} \) and \( \frac{dA}{dF} \) at all values of *F*.
(v) Calculates *σ*₁, *σ*₉ and *σ*₉ (in lbs f/sq.in) using Sachs equations.
(vi) Calculates positive and negative areas under *σ*₁ vs *F* and *σ*₉ vs *r* curves.
(vii) Converts DS, *F*, *σ*₁, *σ*₉ and *σ*₉ in SI units.
(viii) If required, plots curves *σ*₁ vs *F*, *σ*₉ vs *r*, *σ*₉ vs *r* and *σ*₉ vs *r*.
(ix) Makes the equilibrium check

\[
\sigma_r = \sigma_t + r \frac{d\sigma_r}{dr}
\]

The following parameters are used to denote data and the results:

* These notations are defined in Chapter 4.
Data

Card 1:

MA = 2 \times \text{number of specimens for which the stresses are to be calculated.}

Card 2:

ISW2 = 1 \text{ for coefficients for each order fitted}\n= 0 \text{ for coefficients of highest order only.}

ISW3 = 1 \text{ table observed vs calculated for each order}\n= 0 \text{ for highest order only.}

NCASES = \text{Number of cases to be considered}\n= 2 \text{ in this program (one each for } \Theta \text{ vs } F \text{ and } A \text{ vs } F).}

Card 3:

ISPEC = \text{Specimen number.}

Card 4:

N = \text{The number of pairs of points (number of steps of material removal).}

LAST = \text{Highest order of polynomial to be fitted.}

NGR = 0 \text{ if graphs are not required}\n= 1 \text{ for a graph to be plotted of observed } Y \text{ vs } X \text{ and calculated } Y \text{ vs } X, \text{ for each order.}\n= 2 \text{ if the above graphs are plotted only for the highest order.}
XAXIS = size of the x-axis in inches.

YAXIS = size of the y-axis in inches.

NTITLE = 0 if no title is required.

> 0, a further card is to be supplied containing a title for the output (and graph, if applicable) consisting of NTITLE x 8 characters.

Card 5:

K = number of ordinates (odd) required for calculation of areas under the curve \( \sigma_1 \) vs \( F \) and \( \sigma_2 \) vs \( r \).

Card 6:

DB = outside diameter of the specimen.

Card 7: (one or more cards to accommodate all values)

DS = diameters of the bore after each material removal step.

Card 8: (one or more cards to accommodate all values of \( A_1 \) and \( B_1 \))

\( A_1 \) = longitudinal strain readings

\( B_1 \) = tangential strain readings.

In statement No. 61 the factor

\[ 0.95238 = \text{Gauge factor for which the strain measuring bridge is calibrated} \] and \[ \text{gauge factor of the gauge used for strain measurements} \]
0.28 = Poisson's ratio of the material.

Results

\[ x = \text{areas of bored out diameter} \]
\[ y = 0 \text{ or } 1 \]

STLONG = longitudinal residual stress
STTAN = tangential residual stress
STRAD = radial residual stress.

Library Routines used in the Program

UTPOP = to open the graph plotter
UTPCL = to close the graph plotter
UTS2 = chooses the maximum element in an array or column of an array.
UTS3 = As for UTS2 but chooses the minimum element.
UTS8 = Calculates the area under the curve by Simpson's rule.
UTP4A, UTP4B = Graph plotter routines
Program 2

This program calculates stresses in a specimen from which material removal is carried out firstly by boring from centre to an intermediate bore diameter and then from outside diameter to this intermediate bore diameter. In addition to the computation done by program 1, this program calculates stresses at diameters corresponding to material removal steps from outside diameter to the intermediate bore diameter and then applies corrections (given in Appendix III) to these stresses due to the stresses which had been removed when material removal was carried out from the centre to the intermediate diameter.

The parameters used in this program are the same as used in Program 1, except that MA in this program is three times the number of specimens to be analysed.

Additional data cards 4 to 8, containing data for material removal from outside diameter to bore, are required in this program.
Program 3

This program calculates the residual stresses from the stress distribution on the exit boundary of the deformation zone by making use of the static equilibrium equations as outlined in Chapter 4. The stress distribution in the deformation zone is already determined in Program 'MAIN 4' (see Fig 4.3.9) and is read in Program 3 as the data.

The variable names used in this program are as follows:

- $\text{SIGMA}$ = $\sigma$ (average yield stress)
- $\text{ER}$ = Extrusion ratio
- $\text{IANGLE}$ = Die angle (included)
- $\text{ILPHAD}$ = Half die angle ($a$)
- $B$ = $b$
- $D1$ = $d_1$
- $DN$ = $d_n$
- $DEND$ = $d_e$
- $FM$ = $m$
- $RR$ = $r$
- $F$ = $\pi r^2$
- $\text{TTRZ, TTRR, TTHR}$ = $\sigma_z, \sigma_r, \sigma_\theta$ (components of the normal stress distribution in the deformation zone - see Chapter 4)
- $\text{RESZ, RESR, RESTH}$ = $\sigma_z, \sigma_r, \sigma_\theta$ (components of residual stress distribution - see Chapter 4)

For other variable names see ref [49].
PROGRAM 1

DOCUMENT SOURCE:

MASTER STRESS

DIMENSION X(100),A(10,10),SUMX(31),SUMY(10),Y(100)
1,VG(100),XB(100),TITLE(10),WIDTH(20),AREA(20),X1(100),STTONG(10,20)

DIMENSION XBG(100),YG(100)

DIMENSION A1(60),A1(60),TA(60),TB(60),DS(60),A2(60),B2(60)

DIMENSION STNG(100),STN(100),STAD(100)

DIMENSION STN(100),STNG(100),STLAN(100),RSA(100),DSA(100)

DIMENSION STTONG(100),STTON(100),RS(60)

DIMENSION RSA(100),STADG(100),STTFN(100)

DIMENSION A4(60),A4(60)

DIMENSION X1TITLE(1),Y1TITLE(2),X2TITLE(1),Y2TITLE(2),X3TITLE(1),Y3TITLE(2),X4TITLE(1),Y4TITLE(2)

DATA X1TITLE/SHAPEA/,Y1TITLE(1)/16HRES LONG STRESS/,X2TITLE/16HRADIUS/,Y2TITLE(1)/16HRES LONG STRESS/,
X3TITLE/6HRADIUS/,Y3TITLE(1)/6HRADIUS/,
X4TITLE(1)/6HRADIUS/6HRADIUS/,Y4TITLE(1)/16HRES
3S RAD STRESS/

CALL UTPPP

PFAD(1,7)MA

FORMAT(1X)

DO 9905 J=1,MA

71=140/2;

J40=140/2.

IF(X1,FO,J40)GO TO 401

PFAD(1,700)ISW2,ISW3,NCASES

DO 175 ICASE=1,NCASES

IF(ICASF,FO,2)GO TO 55

READ(1,67)ISPFC

62 FORMAT(1X)
READ (1,1000) N, LAST, NGR, XAXIS, YAXIS, NTITLE
READ (1,42) V
42 FORMAT(12)
READ (1,16) DR
16 FORMAT(6X,4)
READ (1,12) (DS(I), I=1, N)
12 FORMAT(100F0,0)
READ (1,19) (A1(I), I=1, N), (B1(I), I=1, N)
19 FORMAT(6*100F0,0)
F = 0, 7854*DB**2
DO 22 I=1, N
X(I) = 0, 7854*DS(I)**2
P1(I) = DS(I)**2
22 CONTINUE
A2(I) = A1(I)
23 CONTINUE
A2(I) = A2(I-1) + A1(I)
DO 23 I=1, N
24 CONTINUE
A2(I) = A2(I-1) + A1(I)
DO 24 I=1, N
27 CONTINUE
B2(I) = B2(I-1) + B1(I)
27 CONTINUE
B2(I) = B2(I-1) + B1(I)
DO 27 I=1, N
61 V(I) = 0, 95238*(A2(I) + R2(I) + 0, 28)
25 CONTINUE
GO TO 70
55 DO 31 I=1, N
55 CONTINUE
31 CONTINUE
9999 IF(NITLE.GT.0) READ(1,1) TITLE
1 FORMAT(10A8)
2 IF(NTITLE.GT.0) WRITE(2,3) TITLE
3 FORMAT(1HO/1HO10A8/)
53 IF(NGP.EQ.0) GO TO 70
   DO 2 I=1,N
   XB(I)=X(I)
   VMAX=V(1) /_ANY=0
   YMAX=UTS0(N,V(1))
    YMIN=UTS3(N,V(1))
   CALL UTP4A(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,TITLE,NTITLE,Y,0)
   XSCALE=XAXIS/(XMAX-XMIN)
   YSCALE=YAXIS/(YMAX-YMIN)
   DO 900 IJK=1,N
    XB(IJK)=(XB(IJK)-XMIN)*XSCALE
    YG(IJK)=(YG(IJK)-YMIN)*YSCALE
   CALL UTP7H*(XBC(IJK),VG(IJK),2)
   XB(IJK)=XB(IJK)/XSCALE+XMIN
   YG(IJK)=YG(IJK)/YSCALE+YMIN
   GO TO 70
401 N=N+1
   DO 402 I=1,N
    X(I)=PS(I+1)
    Y(I)=STAND(I+1)
402 CONTINUE
   NCASES=1
70 SUMX(1)=0,
   SUMX(2)=0,
   SUMX(3)=0.
SUMY(1) = 0;
SUMY(2) = 0;
DO 90 I = 1, N
    CX = X(I)
    CY = Y(I)
    SUMX(1) = SUMX(1) + CX
    SUMX(2) = SUMX(2) + CX
    SUMX(3) = SUMX(3) + CX * CX
    SUMY(1) = SUMY(1) + CY
90 SUMY(2) = SUMY(2) + CX * CY
NINE = 1
100 A(1, J) = SUMX(1)
101 A(I, K) = SUMY(I)
105 I = 1, L
A(I, J) = -1
K = I + 1
GO TO 110
110 A(I, J) = sum
I = 1, L
A(I, J) = A(I, J) + A(I, J-1)
A(I, J) = A(I, J)
120 A(I, J) = A(I, J) + A(I, J) + A(I+1, J)
140 A(I+1, J) = A(I+1, J)

S2=0.
DO 160 J=1,N
YJ=Y(J)
XJ=X(J)
S1=0.
S1=S1+A(I,KK)
DO 150 I=1,NORD
150 S1=S1+A(I+1,KK)*YJ*K
XR(J)=X(J)
YG(J)=S1
160 S2=S2+(S1-YJ)*K
IF (IRW2) 161,161,163
161 IF (NORD-LAST) 171,163,171
163 XS2=S2/REAL(N)
PMSS2=SQR(T(XS2))
WRITE(2,1061)ISPFC
1061 FORMAT(40X14HSPECIMEN NO.=I2,/)!
WRITE(2,1020)NORD,S2,XMS2,PMSS2
1020 FORMAT(1X,15HORDER = 13,1X28HSUM OF SQUARED ERRORS = E1
14.X,/13X28HMEAN SQUARE ERROR = E14.8,/13X28HROOT MEAN SQUA
2RE OF ERROR = E14.8,/)!
WRITE(2,1090)
NO 164 I=1,L
J=I-1
164 WRITE(2,1030)J,A(I,KK)
IF (ISW3) 165,165,167
165 IF (NORD-LAST) 171,167,171
167 IF(Z1.EQ.J40)GO TO 299
WRITE(2,1100)
299 IF(Z1.EQ.J40)GO TO 588
IF(X(1),NE,F)GO TO 589
GO TO 589
589 N=N+1
NS(N)=DB
PS(N)=DB/2
X(N)=X
588 DO 169 I=1,N
S1=0,
S1=A(1,KK)
NYDX=0,0
DO 168 J=1,NNRD
NYDX=NYDX+J*(A(J+1,KK)*X(I)**(J-1))
168 S1=S1*A(J+1,KK)*X(I)**J
IF(71,EQ,0)1GO TO 403
IF(ICASE,0,2)1GO TO 56
STLNG(I)=32.5521*EPS*((F*X(I))*DYDX=S1)
STTAN(I)=0
STRAD(I)=0
GO TO 9998
56 STTAN(I)=32.5521*EPS*((F*X(I))*DYDX=((F+X(I))/(2.0*X(I)))*S1)
STRAD(I)=32.5521*EPS*((F*X(I))/(2.0*X(I)))*S1
9998 S3=Y(I)-F1
WRITE(2,1040)X(I),Y(I),S1,S3,STLNG(I),STTAN(I),STRAD(I)
GO TO 160
403 S3=Y(I)-F1
T=Y(I)+X(I)*DYDX
WRITE(2,604)Y(I),Y(I),S1, S3,T,STTAN(I+1)
404 FORMAT(0F10.5,5G15.6)
1 CONTINUE
IF(HP=1)174,4,8
IF (NORD .NE. LAST) GO TO 5
4  L MN=0
   GO TO 902
174 IF (NORD=LAST) 5, 173, 173
171 IF (NGP.NE.1) GO TO 6
   L MN=1
   GOTO 502
5  NORD=NORD+1
   WRITE (2,6)
6   FORMAT (1HO/1HO/1HO)
     J=2+NORD
     SUMX(J)=O.
     SUMX(J+1)=0,
     SUMY(NORD+1)=O.
   DO 177 I=1,N
     RX=X(I)
     CV=Y(I)
     SUMX(J)=SUMX(J)+RX**J
     SUMX(J+1)=SUMX(J+1)+CX**J
172 SUMY(NORD+1)=SUMY(NORD+1)+CY*CX**NORD
   GO TO 91
902 NG=1
   XBG(1)=X(1)
   YBG(1)=Y(C)
   DO 901 L=2,N
6 IF (L.EQ.'H') GOTO 901
   IF (X(L+1).EQ.XH(L).AND.YG(L+1).NE.YG(L)) GOTO 901
   NG=NG+1
   XBG(NG)=X(L+1)
   YBG(NG)=YG(L+1)
900 CONTINUE
CALL 1TP6B(XPG,YGG+NG,0)
IF(LMN.=.0) GOTO 174
GOTO 5
200 FORMAT(211,13)
1000 FORMAT(11,12,11,?,F4,0,11)
1030 FORMAT(1V,13,5X,F10,12)
1090 FORMAT(4P NO.10X11bCOEFFICIENT//)1100 FORMAT(10X1H64X1H3X50CALC7X5ERROR6X6HSTLONG6X5HSTTAN7X5HSTRAD)
1040 FORMAT(1Y9P12,N,3F12,8,3G12,4)
173 CONTINUE
IF(Z1:FO,J40)GO TO 9993
STLONG=STLONG(2)+X(2)*(STLONG(1)-STLONG(2))/(X(2)-X(1))
STTAN=STTAN(2)+PS(2)*(STTAN(1)-STTAN(2))/(RS(2)-RS(1))
STRAD=STRAD(2)+PS(2)*(STRAD(1)-STRAD(2))/(RS(2)-RS(1))
STTAN=STTAN(2)+C(X)*((STTAN(1)-STTAN(2))/(X(2)-X(1))
STTAN=X=STTAN(N-1)+(STTAN(N)-STTAN(N-1))*(F=x(N-1))/(x(N)-
1X(N-1))
STLONG=STLONG(N-1)+(STLONG(N)-STLONG(N-1))*(F=x(N-1))/(x(N)-
1X(N-1))
STTAN=STTAN(N-1)+(STTAN(N)-STTAN(N-1))*(DB=DS(N-1))/
1(DS(N)-DS(N-1))
STRAD=STRAD(N-1)+(STRAD(N)-STRAD(N-1))*(DB=DS(N-1))/
1(DS(N)-DS(N-1))
WRITE(2,333)
333 FORMAT(1W//)
DASA(1)=0.000
PSA(1)=0.000
PSE(1)=0.00
X(1)=0.000
\[ x_1(1) = 0.00 \]
\[ A_4(1) = 0.00 \]
\[ B_4(1) = 0.00 \]
\[ STLANGE(1) = STLONG1 \]
\[ STTANGE(1) = STTAN1 \]
\[ STKADC(1) = STPADI \]
\[ STTONE(1) = STTANB \]
\[ DS(N) = DB \]
\[ PS(N) = NA/2 \]
\[ X(N) = F \]
\[ A1(N) = 0.0 \]
\[ R1(N) = 0.0 \]

1001 
DO 30 1 = 1, N 
IF (STLONG(1)) 321, 9494, 302 
321 
XA(I+1) = X(I) 
A4(I+1) = A1(I) 
R4(I+1) = R1(I) 
STLANE(1+1) = STLONG(I) 
GO TO 301 
307 
\[
7 = (x(I) - x(I-1)) / (ABS(STLONG(I-1)) + ABS(STLONG(I))) 
\]
\[
XZERO = x(I-1) + 7 
\]
\[
YA(I+1) = XZERO 
\]
\[
A4(I+1) = 0.0 
\]
\[
R4(I+1) = 0.0 
\]
\[
STLANE(1+1) = 0.0 
\]
GO TO 303 
303 CONTINUE 
303 WIDTH(1) = (XZERO - 0, W0001) / (X-1) 
WRITE(2, 341) XZERO, WIDTH(1) 
341 FORMAT(2F10.8)
$\text{STLON}(1) = \text{STLANG}(1)$
$\text{XNEW} = 0.0$
DO 304 $J = 2, K$
$\text{XNEW} = \text{XNEW} + \text{WIDTH}(1)$
CALL INTPPL($\text{Ya}, \text{STLANG}, \text{N}, \text{XNEW}, \text{YNEW}$)
$\text{STLON}(J) = \text{YNEW}$

304 CONTINUE
$\text{AREA}(1) = \text{UTSR}(K, \text{STLANG}(1), \text{WIDTH}(1))$
DO 305 $J = 1, N$
$\text{STLANG}(J+2) = \text{STLANG}(J)$
$\text{XA}(J+2) = \text{X}(J)$
$\text{A}(J+2) = \text{A}(J)$
$\text{R}(J+2) = \text{R}(J)$

305 CONTINUE
$\text{M} = \text{M} + 2$
$\text{WIDTH}(2) = (\text{Y}(N) - \text{Y}(0)) / (K - 1)$
$L = 2 * K - 1$
$\text{XNEW} = \text{Y}(0)$
DO 306 $J = K + 1, L$
$\text{XNEW} = \text{XNEW} + \text{WIDTH}(2)$
CALL INTPPL($\text{Ya}, \text{STLANG}, \text{M}, \text{XNEW}, \text{YNEW}$)
$\text{STLON}(J) = \text{YNEW}$

306 CONTINUE
$\text{AREA}(2) = \text{UTSR}(K, \text{STLANG}(K), \text{WIDTH}(2))$
DO 307 $I = 1, N$
IF ($\text{STTAN}(I) < 22, 494, 308$
322 $\text{RSA}(I+1) = \text{S}(I)$
$\text{DSA}(I+1) = \text{DS}(I)$
$\text{STTAN}(I+1) = \text{STTAN}(I)$
GO TO 307
308  \[ S = \frac{R_S(1) - RS(1-1) + \text{ABS}(\text{STTAN}(1-1))}{(\text{ABS}(\text{STTAN}(1-1)) + \text{ABS}(\text{STTAN}(I)))} \]

309  \[ \text{RZERO} = R_S(1-1) + S \]

310  \[ \text{PSA}(I+1) = \text{RZEROF} \]

311  \[ DSA(I+1) = 2 * \text{RZERO} \]

312  \[ \text{STTAN}(I+1) = 0.0 \]

313  \[ \text{CONTINUE} \]

309  \[ \text{WIDTH}(3) = \frac{(\text{RZERO} - 0) + \text{V0001}}{(K-1)} \]

310  \[ \text{WRITE}(2, 347) \text{ RZERO, WIDTH}(3) \]

311  \[ \text{FORMAT}(2F10,8) \]

309  \[ \text{STTAN}(1) = \text{STTANG}(1) \]

310  \[ \text{XNEW} = 0.0 \]

311  \[ \text{DO 311 } J = 2, K \]

312  \[ \text{XNEW} = \text{XNEW} + \text{WIDTH}(3) \]

313  \[ \text{CONTINUE} \]

311  \[ \text{CALL INTRPL(PSA, STTANG, N, XNEW, YNEW)} \]

312  \[ \text{STTOK}(J) = \text{YNEW} \]

313  \[ \text{CONTINUE} \]

311  \[ \text{AREA}(3) = \text{I:TS8}(K, \text{STTUN}(1), \text{WIDTH}(3)) \]

312  \[ \text{DO 312 } J = 1, N \]

313  \[ \text{CONTINUE} \]

311  \[ \text{PSA}(J+2) = \text{RS}(J) \]

312  \[ \text{DSA}(J+7) = \text{DS}(J) \]

313  \[ \text{CONTINUE} \]

311  \[ \text{STTANG}(J+2) = \text{STTAN}(J) \]

312  \[ \text{WIDTH}(4) = \frac{\text{PS}(N) - \text{RZEROF}}{(K-1)} \]

313  \[ \text{CONTINUE} \]

311  \[ \text{XNEW} = \text{RZEROF} \]

312  \[ \text{DO 312 } J = K+1, L \]

313  \[ \text{CONTINUE} \]

311  \[ \text{CALL INTRPL(PSA, STTANG, M, XNEW, YNEW)} \]

312  \[ \text{STTOK}(J) = \text{YNEW} \]

313  \[ \text{CONTINUE} \]
AREA(4) = UTS8(K), STTUN(K), WIDTH(4)
WRITE(2, 772)
772 FORMAT(1X3HAA, 1XRHDIAMETER5X6HRADIUS4X4HAREA3X11HLONG STRAIN1X10HT
1AN STRAIN1X11HLONG STRESS1X10HTAN STRESS/6X5H(DSA)6X5H(RSA) 5X4H(XA
2)6X8HRADINGS3X3HRADINGS3X9H(REVISED)2X9H(REVISED)
M=M-1
DO 1002 J=1,N
1002 STRADG(J+1) = STRAD(J)
I=1
1003 IF(STLAN(I) .EQ. 0, U) GO TO 1004
I=I+1
GO TO 1003
1004 DO 1005 I=I, M
A4(J) = A4(J+1)
XA(J) = XA(J+1)
1005 STLAN(J) = STLAN(J+1)
I=1
1006 IF(STTANG(I) .EQ. 0, U) GO TO 1007
I=I+1
GO TO 1006
1007 DO 1008 J=1, M
DSA(J) = DSA(J+1)
PSA(J) = RSA(J+1)
R4(J) = B4(J+1)
1008 STTANG(J) = STTANG(J+1)
NC 325 J=1,M
WRITE(2, 326) J, DSA(J), RSA(J), XA(J), A4(J), B4(J), STLAN(J), STTANG(J)
326 FORMAT(14, 0P3F10.0, 2F11.7, 3G12.4)
325 CONTINUE
WRITE(2, 331)
531    FORMAT(1HX//)
      WRITE(2,734)
734    FORMAT(1Y3HNO.3X6HSTLONG13X5HSTTAN/6X12HINTERPOLATED5X12HINTERPOLATION)!
      DO 327 J=1,L
      WRITE(2,728),STLONG(J),STTAN(J)
327    CONTINUE
      WRITE(2,732)
322    FORMAT(1HX//)
      WRITE(2,729) AREA(1),AREA(2),AREA(3),AREA(4)
329    FORMAT(1FX,0PF12.4,3X0PF12.4,3X0PF12.4,3X0PF12.4)
      DO 351 I=1,M
351    STLONG(I)=68944E-07*STLANG(I)
5    STTAN(I)=STTANG(I)*68944E-07
5    STRAD(I)=STRADG(I)*68944E-07
5    RS(I)=RSA(I)*25.4
5    DS(I)=DSA(I)*25.4
5    X(I)=XA(I)*25.4*25.4
      WRITE(2,741)
741    FORMAT(1HX//)!
      DO 743 I=1,M
743    CONTINUE
      WRITE(2,742) I, DS(I), RS(I), X(I), STLONG(I), STTAN(I), STRAD(I)
5    FORMAT(13,0PF10.3,3612.4)
      CALL UTPGA(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,X1TITLE,Y1TITLE)
CALL UTP4R(X,STLONu,~,O)
X~AX=RS(~)*25,4*25,4*6/140
CALL UTP4A(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,X2TITLE,1,Y2TITLE,2)
CALL UTP4B(RS,STLONG,M,0)
CALL UTP4A(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,X3TITLE,1,Y3TITLE,2)
CALL UTP4B(RS,STTAH,M,0)
CALL UTP4A(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,X4TITLE,1,Y4TITLE,2)
CALL UTP4B(RS,STRAD,M,0)
903 IF((R40-RA),NE.0) 40 TO 904
904 CONTINUE
905 CALL UTPCL
STOP
END
SUBROUTINE INTRPL(X,Y,N,XNEW,YNEW)
DIMENSION X(100),Y(100)
DO 777 J=1,N
IF(XNEW.EQ.X(J)) GO TO 778
IF(XNEW.GT.X(J)) GO TO 777
IF(J.EQ.1) GO TO 780
YNEW=Y(J-1)*(XNEW-X(J-1))*(Y(J)-Y(J-1))/((X(J)-X(J-1))
GO TO 781
778 YNEW=Y(J)
GO TO 781
780 YNEW=Y(J)*XNEW/X(J)
GO TO 781
781 CONTINUE
RETURN
END
FINISH
PROGRAM 2

LIBRARY (ED, SURF, GROUP, GRAF)
BEGIN
INPUT 1 = CRN
OUTPUT 2 = IP0
END

DIMENSION STRESS, X(100), A(20, 20), SUMX(31), SUMY(30), Y(100)
1, YG(100), XB(100) TITLE(10), WIDHT(20), AREA(20), X1(100), STTONG(10
20)
DIMENSION XRG(100), YGG(100)
DIMENSION A1(60), B1(60), TA(60), TB(60), DS(60), A2(60), B2(60)
DIMENSION STI151N1100), STTAN(100), STRAD(100)
DIMENSION STTANG(100), STLANG(100), XA(100), RSA(100), DSA(100)
DIMENSION STI100), STTON(100), PS(60)
DIMENSION RSA(100), STRADG(100), STTFN(100)
DIMENSION A4(60), B4(60), STRON(100)
DIMENSION STAI(100), SRAD(100)
DIMENSION X1TITLE(1), Y1TITLE(2), X2TITLE(1), Y2TITLE(2),
IX3TITLE(1), Y3TITLE(2), X4TITLE(1), Y4TITLE(2)
DATA X1TITLE/4HAREA/, Y1TITLE(1)/16HRES LONG STRESS/, X2TITLE/
16HRADIUS/, Y2TITLE(1)/16HRES TAN STRESS/, X3TITLE/6HRADIUS/,
2X3TITLE(1)/16HRES RAD STRESS/, X4TITLE/6HRADIUS/, Y4TITLE(1)/16HRE
S LONG STRESS/
CALL UTP0P
READ (1, 7) MA
FORMAT (1X)
DO 9995 140 = 1, MA
Z1 = 140/2,
J140 = 140/2,
IF(I40.EQ.3.OR.I40.EQ.6.OR.I40.EQ.9)GO TO 401
IF(I40.EQ.2.OR.I40.EQ.5.OR.I40.EQ.8)GO TO 631
READ(1,62)ISPEC
62 FORMAT(I??)
   IF(ISPEC.EQ.3.OR.ISPEC.EQ.7)GO TO 639
639 READ (1,200)ISW2,ISW3,NCASES
GO TO 2001
631 N1=N1+1
2001 DO 173 ICASE=1,NCASES
   IF(ICASE.EQ.2) GO TO 55
637 READ (1,1000)N,LAST,MCR,XAXIS,YAXIS,NTITLE
READ (1,42)K
42 FORMAT(1?)
   READ(1,16)DR
16 FORMAT(F7.5)
   READ (1,12)NS(I),I=1,N
12 FORMAT(100F0.0)
   PFAD (1,19) (A1(I),I=1,N),(B1(I),I=1,N)
19 FORMAT(6X100F0.0)
   F=0.7854*DB**2
   DO 22 I=1,N
      X(I)=0.7854*DS(I)**2
      RS(I)=DS(I)/2
22 CONTINUE
A2(I)=A1(I)
   DO 23 I=2,N
      A2(I)=A2(I-1)+A1(I)
23 CONTINUE
   R2(I)=R1(I)
   DO 27 I=2,N
R2(I) = R2(I-1) + S1(I)

27  CONTINUE
   DO 25 I = 1, N
   IF(ISPEC.EQ.9) GO TO 61
   IF(ISPEC.EQ.0) GO TO 61
   IF(ISPEC.EQ.4) GO TO 61
   IF(ISPEC.EQ.10) GO TO 61
   V(I) = 0.9592 * (A2(I) + B2(I) * 0.28)
   GO TO 25

61  V(1) = 0.9592 * (A2(1) + B2(1) * 0.28)

25  CONTINUE
   GO TO 70

55  DO 31 I = 1, N
   IF(ISPEC.EQ.9) GO TO 69
   IF(ISPEC.EQ.4) GO TO 69
   IF(ISPEC.EQ.10) GO TO 69
   IF(ISPEC.EQ.10) GO TO 69
   V(I) = 0.9592 * (A2(I) + A2(I) * 0.28)
   GO TO 31

69  V(1) = 0.9592 * (A2(1) + A2(1) * 0.28)

31  CONTINUE
   GO TO 70

401 N = N + 1
   DO 402 I = 1, N
   X(I) = PI(1+1)
   Y(I) = STRAD(1+1)

402 CONTINUE
   NCASES = 1

70  SUMX(1) = N
   SUMX(2) = N
SUMX(3) = 0
SUMY(1) = 0
SUMY(2) = 0
DO 90 I = 1, K
  CX = X(I)
  CY = Y(I)
  SUMX(1) = SUMX(1) + CX
  SUMX(2) = SUMX(2) + CX
  SUMX(3) = SUMX(3) + CX*CX
  SUMY(1) = SUMY(1) + CY
  SUMY(2) = SUMY(2) + CX*CY
  90 SUMY(2) = SUMY(2) + CX*CY
NORD = 1
L = NORD + 1
KK = L + 1
DO 101 I = 1, L
  DO 100 J = 1, L
    IK = J - 1 + 1
    100 A(I, J) = SUMX(IK)
  101 A(I, KK) = SUMY(I)
  DO 140 I = 1, L
    A(KK, I) = -1,
    KK = I + 1
    DO 170 J = KK, KK
    170 A(KK, J) = .+
    C = 1 / A(I, I)
    DO 120 I = 2, K
      DO 170 J = KK, KK
      120 A(I, J) = A(I, J) + A(I, I) + C
    DO 140 I = 1, L
    DO 140 J = KK, KK
40 A(I,J)=\((I+1,J)
S2=0,
DO 160 J=1,N
YJ=Y(J)
XJ=X(J)
S1=0,
S1=S1+A(I,KK)
DO 150 I=1,NORD
150 S1=S1+A(I+1,KK)*XJ*XJ
X8(J)=X(J)
Y8(J)=S1
160 S2=S2+(S1-YJ)**2
IF (ISW2) 161,161,163
161 IF (NORD-LAST) 171,163,171
163 XMS2=S2/FLOAT(N)
RMSS2=SQRT(XMS2)
WRITE(2,1061)ISPFC
1061 FORMAT(4N14HSPECIMEN NO.\(=\) 12,\\)
WRITE(2,1020)NORD,NORD,S2,RMSS2
1020 FORMAT(1X \(\#\)NORDEP = 13,1X28HSUM OF SQUARED ERRORS \(\(=\) E1
14,8,13X28HMEAN SQUARE ERROR \(\(=\) E14,8,13X28HROOT MEAN SQUA
2RE OF ERROR \(\(=\) E14,8,13X28H)
WRITE(2,1090)
DO 164 I=1,L
J=I-1
164 WRITE(2,1030)J,A(I,KK)
IF (ISW3) 165,165,167
165 IF (NORD-LAST) 171,167,171
167 IF (I40,Eq.3,Mr.I40,Ep.6,Mr.I40,Eq.9)GO TO 299
WRITE(2,1100)
DO 169 I=1,N
S1=0.
S1=4(I+1, KK)
DYDX=0.0
DJ 16A J=1,NORD
DYDX=DYDX+J*4(J+1, KK)*X(I)*J(J+1))
168 S1=A(I+1, KK)*X(I)*J
IF(I4N, EN.3, OR.140, EN.6, OR.140, EN.9) GO TO 403
IF( CASE.EQ.7) GO TO 56
IF (I4N, EN.2, OR.140, EN.5, OR.140, EN.8) GO TO 651
STLONG(I)=32.5521E06*((F-X(I))*DYDX-S1)
STTAN(I)=0
STRAD(I)=0
IF (I, NE, 4) GO TO 99Y8
ALEMDA=81
GO TO 990A
651 STLONG(I)=32.5521E06*(X(I)-X(N))*DYDX+S1
STTAN(I)=0.0
STRAD(I)=0.0
IF (I, NE, 1) GO TO 99Y8
STLONG=32.5521E04+S1
GO TO 990A
56 IF (I4N, EN.2, OR.140, EN.5, OR.140, EN.8) GO TO 652
STTAN(I)=32.5521E06*((F-X(I))*DYDX-(F-X(I))/(2.0*X(I)))S1
STRAD(I)=32.5521E06*(F-X(I))/(2.0*X(I))S1
GO TO 990B
652 STTAN(I)=32.5521E06*(X(I)-X(N))*DYDX+(X(I)*X(N))*S1/(2.0*X(I)))
STRAD(I)=32.5521E06*(X(I)-X(N))*S1/(2.0*X(I)))
IF (I, NE, 4) GO TO 99Y8
STTAN=32.5521E06*(X(I)-X(N))*S1/(2.0*X(I)))
STRAD = 0, 0
S3 = Y(I) = 1
WRITE(2, 1040) X(I), Y(I), S1, S3, STLON(I), SSTAN(I), STRAD(I)
GO TO 160

S3 = Y(I) = 1
T = Y(I) + X(I)*NYDX
WRITE(2, 404) X(I), Y(I), S1, S3, T, SSTAN(I + 1)

FORMAT(0DF10.5, 5G15.6)
CONTINUE

IF(NORD = LAST) 5, 173, 173
IF(NORD, NE, 1) GO TO >
5 NORD = NORD + 1
WRITE(2, 6)

FORMAT(1H0/1H0/1H0)
J = 2*NORD
SUMX(J) = 0.
SUMX(J + 11) = 0.,
SUNY(NORD + 1) = 0.
NO 172 I = 1, N
CX = X(I)
CY = Y(I)
SUMX(J) = SUMX(J) + CX**(J - 1)
SUMX(J + 1) = SUMX(J + 1) + CX**J

172 SUMY(NORD + 1) = SUMY(NORD + 1) + CY*CX**NORD
GO TO 9

2(0) FORMAT(2I11, 13)
1060 FORMAT(1X, 12, 11, 2F4.0, 11)
1030 FORMAT(1X, 13, 5X, E10, 12)
1090 FORMAT(4H ND, 10X11HCOEFFICIENT//)
1100 FORMAT(10X14Y8X14Y6X5HCALC7X5HEREOR6X6HSTLONG6X5HSTTAN7X5HSTRAD)
1040 FORMAT(1X,DPF12.8,3F12.8,3G12.4)
173 CONTINUE
IF(I40.EQ.3,OR.I40.EQ.6,OR.I40.EQ.9) GO TO 9993
IF(I40.EQ.2,OR.I40.EQ.5,OR.I40.EQ.8) GO TO 603
341 STLONI=STLONG(2)+X(2)*(STLONG(1)=STLONG(2))/(X(2)-X(1))
SSTANI=STTAN(2)+RS(2)*(STTAN(1)=STTAN(2))/(RS(2)-RS(1))
STRAD1=STRAD(2)+RS(2)*(STRAD(1)=STRAD(2))/(RS(2)-RS(1))
STLANG(1)=STLONI
STTAN(1)=STTANI
STRAD(1)=STRAD1
634 WRITE(2,333)
333 FORMAT(14X//)
DSA(1)=0.000
RSA(1)=0.000
PSA(1)=0.000
A4(1)=0.000
R4(1)=0.000
DO 301 I=1,N
321 XA(I+1)=X(I)
A4(I+1)=A4(I)
R4(I+1)=R4(I)
STLANG(I+1)=STLANG(I)
322 PSA(I+1)=PSA(I)
DSA(I+1)=DSA(I)
STTAN(I+1)=STTAN(I)
STRAD(I+1)=STRAD(I)
301 CONTINUE
GO TO 9993
603 WIDTH(7)=(X(N)-,00001)/(X-1)
STLON(1)=STLANG(1)
XNEW=0.0
DO 304 J=2, K
XNEW=XNF1+WIDTH(7)
CALL INTRPL(XA,STLANG,N1,XNEW,YNEW)
STLON(J)=YNEW
304 CONTINUE
ARFA(7)=ATS8(K,STLUN(1),WIDTH(7))
STRESS=ARFA(7)/(F-X(N))
STRESS LEMDA=32.541E06*ALEMDA
WRITE(2,671)
671 WRITE(2,672) STRESS,STRESS LEMDA
672 FORMAT(F7.4)
WIDTH(8)=(RS(N)-QV0001)/(K-1)
STRON(1)=STADG(1)
XNEW=0.0
DO 305 J1=2, K
XNEW=XNF1+WIDTH(R)
CALL INTRPL(XA,STADG,N1,XNEW,YNEW)
STRON(J1)=YNEW
305 CONTINUE
C=f4*RS(N)*RS(N)*STRON(K)/(DB*DB=2*RS(N)*RS(N))
DO 605 I=1,N
STAN(I)=C*(1-(DB*DB)/(RS(1)*DS(I))))
605 C=AD(I)-C*(1-(DB*DB)/(DS(I)*DS(I))))
J1=1
681 IF(STLAN(J1).GE.STLON(K))GO TO 682
J1=J1+1
GO TO 681
641  \( n \) = \( 1 + \), \( N \)

\[
\begin{align*}
\text{SLANG}(J1-1+1) &= \text{SLUNG}(N+1-1) - \text{STRESS} \\
\text{STTANG}(J1-1+1) &= \text{STTAN}(N+1-1) - \text{STAN}(N+1-1) \\
\text{STRANG}(J1-1+1) &= \text{STRAD}(N+1-1) - \text{SRAD}(N+1-1) \\
X1(J1+1) &= X(N+1-1) \\
D1(J1+1) &= \text{NS}(N+1-1) \\
-1X(J1+1) &= \text{DS}(N+1-1) \\
A1(J1+1) &= A(N+1-1) \\
4(J1-1+1) &= B(N+1-1) \\
M1 = J1 + N \\
\text{SLANG}(M1) &= \text{SLUNF}-\text{STRESS} \\
\text{STTANG}(M1) &= \text{STTANF}-\text{STRESS} \\
\text{STRANG}(M1) &= \text{STRADF} \\
X1(M1) &= X \\
D1(M1) &= \text{NS}/2,0 \\
\text{WRITE}(2,772)
\end{align*}
\]

777  \text{FORMAT(1X3H10.1,1XRHFILMGR5X6HRADIUS4X4HAREA3X11HLONG STRAIN1X10HT}
1AN STRAIN1X10HTA1N STRESS/6X5H(DSA)6X5H(RSA)5X4H(XA
2)6X8HRHEIGHT53X6HRREADINGS3X9H(REVISED)2X9H(REVISED)1X10HRAD STRESS
3)
NO 325 J = 1, M1
\text{WRITE}(2,734) J, DSA(J), RSA(J), XA(J), A(J), B(J), SLANG(J), STTANG(J)
1, STRAD(J)
326  \text{FORMAT(14,0P5F10.6,2F11.7,3G12.4)}
325  \text{CONTINUE}
331  \text{FORMAT(1X//)}
L1 = M1/2
\text{WIDTH(1) = (XARL1) = .00001) / (K+1)}
WRITE(2,341) XA(L1), WIDTH(1)
FORMAT(2=10,A)
STLON(1)=STLANG(1)
XNEW=0.0
DO 687 J=2,K
XNEW=XNEW+WIDTH(1)
CALL INTRPL(XA,STLANG,MD,XNEW,YNEW)
STLON(J)=YNEW
687 CONTINUE
AREA(1)=UTSB(K,STLUN(1),WIDTH(1))
WIDTH(2)=(XA(MD)-XA(L1))/(K-1)
L=2*K-1
XNEW=XA(L1)
DO 306 J=K+1,L
XNEW=XNEW+WIDTH(2)
CALL INTRPL(XA,STLANG,MD,XNEW,YNEW)
STLON(J)=YNEW
306 CONTINUE
AREA(2)=UTSB(K,STLUN(K),WIDTH(2))
WIDTH(3)=(RSA(L1)-0.0001)/(K-1)
WRITE(2,342) RSA(L1), WIDTH(3)
342 FORMAT(2=10,A)
STTUN(1)=STLANG(1)
XNEW=0.0
DO 311 J=2,K
XNEW=XNEW+WIDTH(3)
CALL INTRPL(RSA,STTANG,MD,XNEW,YNEW)
STTUN(J)=YNEW
311 CONTINUE
AREA(3)=UTSB(K,STTUN(1),WIDTH(3))
\begin{verbatim}
WIDTH(4) = (RSA(MD) - RSA(L1))/(K-1)
XNEW = RSA(L1)
DO 313 J = K+1, L
XNEW = XNEW + WIDTH(4)
CALL INTREL(RSA, STTANG, MD, XNEW, YNEW)
313 CONTINUE
STTON(J) = YNEW
AREA(J) = (STTANG(J) - STTANG(K)) * WIDTH(4)
WRITE(2, 724)
724 FORMAT(1X, 3HNG, 3X, 6HST, 3X, 5X, 6HTAN/6X12HINTERPOLATED5X12HINTERPOLA
1TFO)
DO 327 J = 1, L
WRITE(2, 328), J, STTANG(J), STTON(J)
328 FORMAT(14, 12.4, 6X, G12.4)
327 CONTINUE
WRITE(2, 332)
332 FORMAT(1HX///)
WRITE(2, 329) AREA(1), AREA(2), AREA(3), AREA(4)
329 FORMAT(1X, 0P12.4, 3X, 0P12.4, 3X, 0P12.4, 3X, 0P12.4)
DO 351 I = 1, M
STLANG(I) = 68044E-06 * STLANG(I)
STTANG(I) = 68044E-06 * STTANG(I)
STRADG(I) = 68044E-06 * STRADG(I)
RSA(I) = 25.4 * RSA(I)
DSA(I) = 75.4 * DSA(I)
351 XA(I) = XA(I) * 25.4 * 4
WRITE(2, 741)
741 FORMAT(1HX///)
DO 743 I = 1, M
WRITE(2, 742) I, DSA(I), RSA(I), XA(I), STLANG(I), STTANG(I), STRADG(I)
\end{verbatim}
742 FORMAT(17,AP3F10.3,3G12.4)
743 CONTINUE
XMAX=UTS2(MD,RSA(1))
XMIN=0.0
YMAX=850*254/200
YMIN=1350*254/200
CALL UTP&A(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,X4TITLE,1,Y4TITLE,2)
CALL UTP&R(RSA,STLANG,MD,0)
CALL UTP&A(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,X2TITLE,1,Y2TITLE,2)
CALL UTP&R(RSA,STTANG,MD,0)
CALL UTP&A(XMIN,XMAX,YMIN,YMAX,XAXIS,YAXIS,X3TITLE,1,Y3TITLE,2)
CALL UTP&R(RSA,STRADG,MD,0)
9903 IF((140-MA).GE.0) GO TO 9994
9995 CONTINUE
STOP
9994 CALL UTP&CL
END
SUBROUTINE INTRPL(X,Y,N,XNEW,YNEW)
DIMENSION X(100),Y(100)
DO 777 J=1,N
IF(XNEW.EQ.X(J)) GO TO 778
IF(XNEW.GT.X(J)) GO TO 777
IF(J.EQ.1) GO TO 780
YNEW=Y(J-1)+XNEW-X(J-1)*(Y(J)-Y(J-1))/(X(J)-X(J-1))
GO TO 780
772 YNEW=Y(J)
GO TO 780
780 YNEW=Y(J)*XNEW/X(J)
GO TO 781
777 CONTINUE
RETURN
END
FINISH
PROGRAM 3

MASTER RESIDUAL
DIMENSION XTITLE(1),XTITLE(1),Y5TITLE(3),Y6TITLE(3),Y7TITLE(3)
DIMENSION RR(40),F(40),TTRZ(40),TTRR(40),TTRTH(40),SUM(40),RESZ(40)
RESR(40),RESTH(40),Y1MAX(3),Y1MIN(3),XR(40),YR(40),IMAX(40)
DIMENSION XGRAP(40),ZSUM(40),THSUM(40)
DATA XTITLE(1)/BHRADIUS//XTITLE(1)/SHAREA//Y5TITLE(1)/24HRES
ISIDUAL LONG STRESS//Y6TITLE(1)/24HRESIDUAL RADIAL STRESS//Y7
2TITLE(1)/24HRESIDUAL TAN STRESS/
READ(5) MA
DO 1000 KKH=1,MA
READ(5) SIGMA1,SIGMA2
READ(5) ER,IANGLE
WRITE(2,1511) ER,IANGLE
1511 FORMAT(40X,11HEX RATIO=,F4.2/40X,12HDIE ANGLE=,13//)
READ(5) LPHA,B,D1,DN,DEND,FM,DLREAL,VO,IMAX(1),N,RMAX,ZMAX
WRITE(2,1512) D1,DEND
1512 FORMAT(20X,4HD1=.,F5.3/18X,6HDEND=.,F5.3/)
IMAX=IMAX(1)
SUMZ=B/(FIMAX+1)
NALY=IMAX(1)
DO 107 I=1,NALY
107 READ(5) RR(I),F(I),TTRZ(I),TTRR(I),TTRTH(I)
SUH(I)=0.0
DO 102 I=2,NALY
SUH(I)=SUM(I-1)+(TTRZ(I)+TTRR(I-1))*ZSUM/2.0
102 CONTINUE
FACTOR=SUM(NALY)/F(NALY)
WRITE(2,201) FACTOR
201 FORMAT(30X,F15.5)
DO 103 K=1,NALY
103 RESZ(K)=TTRZ(K)-FACTOR
SUM(1)=0.0
DO 104 I=2,NALY
SUM(I)=SUM(I-1)+(TTRZ(I)+TTRR(I-1))*ZSUM/2.0
104 CONTINUE
FACTOR1=SUM(NALY)/RR(NALY)
WRITE(2,202) FACTOR1
202 FORMAT(30X,F15.5)
DO 105 K=1,NALY
105 RESTH(K)=TTRTH(K)-FACTOR1
RESR(NALY)=0.0
SUM(1)=0.0
DO 106 I=2,NALY
IF(I.EQ.NALY) GO TO 117
L=NALY-1+2
SUM(I)=SUM(I-1)+(RESTH(L)*RESTH(L-1))*ZSUM/2.0
RESR(L-1)=SUM(I)/RR(L-1)
106 CONTINUE
117 RESTH(1)=RESTH(1)
106 CONTINUE
COUNT=143, /R*ZUMS
XGRAF(1)=0.
DO 122 I=2,NALY
122 XGRAF(I)=XGRAF(I-1)*CONST
WRITE(2,108)
108 FORMAT(1X,12,15HRES LONG STRESS,2X,14HRES TAN STRESS,2X,14HRES
1 RAD STRESS,4X,5HGRAF)
DO 109 I=1,NALY
109 WRITE(2,110) I,RESZ(I),RESTH(I),RESR(I),XGRAF(I)
110 FORMAT(1X,12,E14.4,2X,E14.4,2X,E14.4,2X,E9.3)
WRITE(2,119) SIGMA1
119 FORMAT(36X,8HSIGMA1=,E15.5)
WRITE(2,108)
DO 123 I=1,NALY
RESZ(I)=RESZ(I)*SIGMA1
RESTH(I)=RESTH(I)*SIGMA1
RESR(I)=RESR(I)*SIGMA1
WRITE(2,110) I,RESZ(I),RESTH(I),RESR(I),XGRAF(I)
123 CONTINUE
V1MAX(1)=UTS2(NALY,RESZ(I))
V1MIN(1)=UTS3(NALY,RESZ(I))
V1MAX(2)=UTS2(NALY,RESTH(I))
V1MIN(2)=UTS3(NALY,RESTH(I))
V1MAX(3)=UTS2(NALY,RESR(I))
V1MIN(3)=UTS3(NALY,RESR(I))
WRITE(2,1523) V1MAX(1),V1MAX(2),V1MAX(3),V1MIN(1),V1MIN(2),V1MIN(3)
1523 FORMAT(1X,5HMAX= ,E11.4,2X,5HMAX= ,E11.4,1X,5HMAX= ,E11.4/1X,5HM
1TN= ,E11.4,2X,5HMIN= ,E11.4,1X,5HMIN= ,E11.4/)
ZSUM(1)=0.
THSUM(1)=0.
DO 126 I=2,NALY
ZSUM(I)=ZSUM(I-1)+(RESZ(I)+RESZ(I-1))/2.*(F(I)-F(I-1))
THSUM(I)=THSUM(I-1)+(RESTH(I)+RESTH(I-1))/2.*ZSUMS
126 CONTINUE
WRITE(2,227) ZSUM(NALY),THSUM(NALY)
227 FORMAT(15X,6HZSUM= ,E15.4,15X,7HTHSUM= ,E15.4)
GO TO 1521
XMIN=0.
X1MIN=0.
XMAX=F(NALY)
X1MAX=RR(NALY)
XINS=6.
VINS=8.
V1MAX=UTS2(3,V1MAX(1))
V1MIN=UTS3(3,V1MIN(1))
XSCALE=XINS/(XMAX-XMIN)
YSCALE=YINS/(YMAX-YMIN)
X1SCALE=XINS/(X1MAX-X1MIN)
CALL UTP4A(X1MIN,X1MAX,YMIN,YMAX,XINS,YINS,XTITLE,1,Y5TITLE,3)
DO 111 I=1, NALY
YB(I) = (RR(I) - XMIN) * XSCALE
YB(I) = (RESZ(I) - YMIN) * YSCALE
CALL UTP3(XH0, XR(I), YB(I), 2)
111 CONTINUE
CALL UTP4B(RR, RESZ, NALY, 0)
CALL UTP4A(XMIN, XMAX, YMIN, YMAX, XINS, YINS, XTITLE, 1, YTITLE, 3)
DO 112 I=1, NALY
YB(I) = (RESTH(I) - YMIN) * YSCALE
CALL UTP3(XH0, XR(I), YB(I), 2)
112 CONTINUE
CALL UTP4B(RR, RESTH, NALY, 0)
CALL UTP4A(XMIN, XMAX, YMIN, YMAX, XINS, YINS, XTITLE, 1, YTITLE, 3)
DO 113 I=1, NALY
YB(I) = (RESR(I) - YMIN) * YSCALE
CALL UTP3(XH0, XR(I), YB(I), 2)
113 CONTINUE
CALL UTP4B(RR, RESR, NALY, 0)
WRITE(2,132) SIGMA2
132 FORMAT(36X,R36SIGMA2=,E15.4)
WRITE(2,108)
DO 133 I=1, NALY
RESZ(I) = RESZ(I) * SIGMA2/SIGMA1
RESTH(I) = RESTH(I) * SIGMA2/SIGMA1
RESR(I) = RESR(I) * SIGMA2/SIGMA1
WRITE(2,110) I, RESZ(I), RESTH(I), RESR(I), XGRAF(I)
133 CONTINUE
1000 CONTINUE
STOP
END
FINISH
APPENDIX I

PUBLICATIONS

The last of the four papers - "Residual stress relief in cold-extruded rod", has been accepted for publication in METALS TECHNOLOGY.
that low arc energies and high travel speeds will be beneficial to toughness. A reduction in bainite in the grain-coarsened 
HAZ microstructure can be achieved by increasing the cooling 
rate to lower the transformation temperature. Since a mini-
imum preheat temperature is necessary to avoid HAZ hydro-
gen cracking, reductions in cooling rate must be accomplished 
by lowering arc energy which, as just noted, is also beneficial 
in reducing the prior austenite grain size. This has the implica-
tion that by using low hydrogen processes with associated 
lower preheat temperatures, higher arc energies could be used. 
Reduced arc energy values, however, will lower production 
rates.

As regards initiation by microvoid coalescence, the results 
suggest that variations in austenite grain size and martensite 
colony width will have only a small effect on the toughness 
of grain-coarsened HAZ regions. Nor would changes be ex-
pected by altering the proportion of bainite in the HAZ up to 
60% of the applied stress. This evidence is quite consistent with earlier 
tests on weldments and it appears that the probable controlling 
microstructural features are the inclusion and coarse 
carbide population and the dislocation density and arrange-
ment through their effect on work hardening. Fine carbides 
would also contribute through their effect on work hardening. 
Improvements in toughness can be expected if the inclusion 
and coarse carbide content were reduced or the microstructure 
is considerably softened below the levels investigated to give 
a lower yield strength and higher strain-hardening exponent. A 
reduction in sulphur and carbon content would therefore be 
beneficial to toughness. However, it is relevant to note that 
McCintock's model for microvoid growth and coalescence 
which relates fracture strain to inclusion diameter and spacing 
and transverse components of the applied stress as shown below:

\[
\epsilon_f = \frac{h}{2b} \left( n - 1 \right) \frac{\ln \left( \frac{2b}{h} \right)}{\sin \left( n - 1 \right) \sigma_a + \sigma_b}
\]

predicts that a given increase in the fracture strain and there-
fore critical COD becomes progressively more difficult to 
achieve as the volume fraction of inclusions in the steel 
falls. For example, since

\[
\frac{h}{2b} \approx \frac{1}{V_t}
\]

if \( V_t = 10^{-2} \) to double the fracture strain requires \( V_t \) to be 
reduced by a factor of about 100 whereas if \( V_t = 10^{-4} \) the 
volume fraction must be reduced by a factor of about 10000 for 
the same increase in fracture strain.

The fracture strain could also be raised by increasing the arc 
energy sufficiently to give transformation products of lower 
yield strength and higher strain-hardening exponent. This could 
result in microstructures having a much greater susceptibility 
to quasi-cleavage fracture as seen from the trends observed 
in the testing of martensite-bainitic microstructures. The toughness could therefore fall instead of being increased.

It is therefore considered that in the as-welded condition, a 
significant increase in the toughness of untempered grain-
coarsened HAZ regions with respect to ductile fracture will be 
difficult to achieve. For a given steel, the only positive way of 
improving the toughness would be to ensure that no untempered 
regions existed in the structure, by using efficient tempering 
bead techniques, or by post-weld heat treatment. Steels lower 
in inclusion content, of course, would show improved tough-
ness.

**Conclusions**

1. The fracture toughness of martensite structures develop-
ed in a low-alloy quenched and tempered steel, HY80, was 
dependent on prior austenite grain size. An increase in grain 
size lowered the toughness when fracture was initiated by quasi-
cleavage, and the principal controlling microstructural factor 
was concluded to be the martensite colony width. Where 
fracture was initiated by microvoid coalescence, the toughness 
was much less dependent on austenite grain size and the im-
portant factors were the inclusion and carbide population 
and the strain-hardening exponent of the matrix.

2. The introduction of upper bainite into martensite of 
constant prior austenite grain size progressively lowered the 
toughness when initiation was by quasi-cleavage. The evidence 
suggests that bainite contents up to 60% did not influence the 
toughness of martensite significantly, when initiation was by 
microvoid coalescence.

3. An improved resistance to fracture initiation by quasi-
cleavage in untempered weld HAZs in as-welded structures 
of HY80 should be obtained by reducing arc energy or pre-
heat temperature. This is consistent with welding experience.

Changes in the same parameters will have little or no effect on 
HAZ toughness where initiation is by microvoid coalescence.

Improvements in toughness for fracture by this mode will be 
obtained only by reducing the inclusion and carbide contents or 
by softening the microstructure by temper bead techniques.

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followed by 18 h at 700°C and air cooling. (a) resulted in a microstructure of ferrite+pearlite (Fig. 2a), the pearlitic becoming coalesced as a result of (b) (Fig. 2b); the average linear intercepts made by the ferrite were respectively 25 and 33 μm. The cooling rate through the temperature range 720°-700°C was about 0.03 Ks⁻¹ in (a). No artificial aging was applied to the extruded rods, but the period between extrusion and impact testing was not less than three months. The impact energy-test temperature curves are shown in Fig. 1.

In Fig. 1a it may be seen that, at the lowest extrusion ratio (2.25:1) the transition from a high- to a low-energy fracture occurs at a higher temperature than in the unworked material, and that the level of energy absorbed in the high-energy fracture is reduced. At the higher extrusion ratios, the transition is shifted to temperatures below the transition of the unworked steel, and the energy required to cause fracture below the transition increases. At the highest extrusion ratio, the energy required to cause fracture below the transition is so high that the term 'brittle' is no longer appropriate.

In the specimens containing coalesced pearlite, the same trends were found (Fig. 1b) apart from the fact that, for all the extrusion ratios investigated, the transition occurred at a temperature below the corresponding figure of the unworked steel. The experiments making use of a coalesced microstructure had been performed because it had been thought that structural damage, involving the formation of micro-cracks through pearlite colonies, might occur as a result of the extrusion operation, and might subsequently play a part in facilitating crack initiation and propagation during the impact test. In fact, although some evidence of such structural damage in the pearlitic specimens was obtained, its extent was very limited and was not possible to associate the crack formed in the impact test with pre-existent structural damage. No structural damage was found in specimens which had been given heat treatment (b).

At the higher extrusion ratios, the fractures became more ragged in appearance as the fracture path tended to be deflected parallel to the extrusion direction. This effect is, of course, similar to that found in ausformed steels and in steels worked by other methods.  It is probable that the resistance to cracks propagating parallel to the extrusion direction would be very poor.

REFERENCES

Tube and pipe production

Following the Autumn General Meeting of The Iron and Steel Institute, held in London in November 1970, a one and a half day meeting entitled 'Tube and pipe production' took place. After an introductory lecture by Herr Direktor K. Neuhoff, of Mannesmannröhren-Werke, sessions were held on structural and mechanical applications, conveyance, and pressure and temperature tubing. Twelve papers were presented (13 papers including Neuhoff).

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TECHNICAL NOTE

The impact properties of cold-extruded rod

The impact properties of mild-steel rods (BS En2E: 0.09C, 0.39Mn, 0.25Si, 0.045S, and 0.025%P) have been determined after extrusion at four extrusion ratios (2.25:1, 2.78:1, 3.52:1 and 4:1; billet diameter, 1.5in; included die angle, 120°). Standard Charpy V-notch specimens were used, the notches being at right angles to the direction of extrusion. Two different heat treatments were given to the steel before it was machined into billet form: (a) 940°C, \( \frac{1}{2} \) h, furnace cool, (b) as (a).

---

1 Impact energy – test temperature curves

---

a heat treatment (a); b heat treatment (b)
Role of oxygen in weld solidification cracking in low-alloy high-strength steels

In the welding of high-strength steels, a major problem is the occurrence of solidification cracking, often termed 'hot cracking'. In the weld metal and a great deal of effort has been devoted to its solution, as Kammer, Masubuchi and Monroe show in their review. It is generally agreed that cracking is due to the fracture of liquid films at grain boundaries during the last stages of solidification. The effect has been attributed to the impurities sulphur and phosphorus which tend to form low-melting-point segregates, the distribution of which is influenced by the alloying elements. However, recent work concerned with the effects of composition variables on weld solidification cracking in thin-sheet low-alloy steels has shown that the oxygen content of the steels is also a major factor.

Several investigators have shown that quantitative expressions can be developed to relate crack susceptibility to chemical composition. In particular the present authors demonstrated that the following simple linear formula adequately represented a large number of available crack susceptibility-chemical composition data:

\[ \text{CSF} = 36\text{[C]} + 12\text{[Mn]} + 8\text{[Si]} + 540\text{[S]} + 812\text{[P]} + 5\text{[Ni]} + 3.5\text{[Co]} - 20\text{[V]} - 13 \]  

(1)

In this expression CSF is the crack susceptibility factor determined in a weld cracking test developed by Huxley and the concentrations of chemical constituents are expressed as weight percentages. This formula was obtained by regression analysis of CSF on composition for tests on some 80 steels in thin sheet form, welded by the autogenous TIG process, and it includes elements found to be significant at the 5% level.

In the present work this expression has been used to predict the crack susceptibility of a number of additional low-alloy steel samples which were subsequently tested experimentally. While agreement between theoretical predictions and the test results was in general most acceptable, certain individual steels gave CSF values well below those predicted by equation (1). The discrepancies were too great to be explained by errors in chemical analysis or by variation in testing conditions. Since oxygen plays an important role in the chemical reactions occurring during solidification and in particular on the distribution of sulphur, it was considered that the oxygen content of the steels should be investigated.

Vacuum fusion analysis of the steels exhibiting anomalously low crack susceptibility showed that their oxygen content was in general higher than the others. A total of 25 steels were then analysed for oxygen and a further regression analysis of the composition-cracking data was carried out, leading to the following revised expression:

\[ \text{CSF} = 42\text{[C]} + 847\text{[S]} + 265\text{[P]} - 10\text{[Mo]} - 3042\text{[O]} + 19 \]  

(2)

This expression predicts a very powerful crack-inhibiting effect of oxygen, which is quite consistent with the known tendency of oxygen to flux sulphur out of liquid steel to form harmless spherical inclusions.

The most spectacular example of crack inhibition by oxygen observed in these experiments emerges from a comparison of the behaviour of two heats of ASTM A387B steel, see Table I. Both heats were very similar in analysis except for their oxygen contents and give rise to very similar high CSF predictions using equation (1). In practice Heat B showed very little cracking and significantly contained three times the oxygen content of Heat A. Use of equation (2) for crack susceptibility prediction eliminated this anomaly and gave good agreement with the experimental result.

In the light of these results it was apparent that it should be possible to reduce the level of cracking in a susceptible steel by increasing the oxygen content of the weld metal. Experiments were therefore conducted in which oxygen was added to the argon shielding gas in the TIG welding of a crack-susceptible steel (Heat A, ASTM A387B). Significantly there was a marked reduction in the cracking obtained in the Huxley cracking test. The results of these welding tests therefore confirmed those of the regression analysis on the composition-cracking data.

A great deal more work is required to elucidate the effect on cracking of oxygen, both individually and in combination with other elements. It is, however, apparent that oxygen is a powerful inhibitor of solidification cracking and, if present in a suitable concentration may minimize or eradicate the harmful effect of sulphur. Thus control of oxygen content would seem to be as important as the control of sulphur content in specifying the composition of weldable steels. Furthermore, since the weld metal oxygen content is a function of the welding process employed and particularly the oxidizing potential of the shielding gas and/or slag, there would appear to be scope for control of cracking susceptibility by appropriate choice of welding process and consumables.

ACKNOWLEDGMENTS

The authors thank Professor W. O. Alexander of the Department of Metallurgy, University of Aston, for provision of laboratory facilities. They also thank Bristol Aerojet Ltd and the Redheugh Iron and Steel Co. Ltd for the supply of steels. The investigation is being carried out under contract to the Ministry of Defence (Materials Division).

REFERENCES


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868 Journal of The Iron and Steel Institute November 1972
SLIPLINE FIELDS AND THE PREDICTION OF RESIDUAL STRESSES

In a note on the use of slipline fields to calculate deformation stresses, G. F. Modlen of Loughborough University of Technology and P. S. Midha of PERA question some recent assumptions.

In previous articles in *Metals and Materials*, a number of workers have attempted to make use of known slipline fields to account for the axial residual stress pattern found in hydrostatically-extruded rod (Refs. 1, 2). Slipline fields apply to plane strain deformation, so that only qualitative agreement is to be expected when they are applied to axi-symmetric deformation. In addition, a rigid-plastic material is postulated: since there can be no residual strains, the existence of corresponding residual stresses becomes questionable. Nevertheless, extremely good agreement with experimental work was obtained: it is the purpose of this note to show that the agreement was fortuitous.

In the slipline field used by Miura and his co-workers, the hydrostatic stress was given at a network of points in the region of deformation. The field was actually for sheet-drawing, but a comparison of the residual stress patterns in drawn wire and in extruded rod made it appear reasonable to expect that sheet drawing and sheet extrusion would give rise to the same pattern of residual stresses. It was assumed that the axial stress at the die opening (presumably the boundary of the plastic zone) was constant.

The axial residual stress arises from elastic springback when the rod is unloaded on leaving the deformation zone. Since (1) this springback is taken to be uniform across the extrudate; (2) such deformation corresponds to the imposition of a uniform axial stress; and (3) the residual stress is given by the difference between the axial stress imposed on the rod as it leaves the region of deformation and this uniform springback stress, the requirement that there should be no net axial force on the unloaded extrudate means that the axial residual stresses should be zero. The fact that Miura did go on to calculate axial residual stresses, after this initial assumption that ruled out their existence, is due to the neglect of residual stresses in directions other than axial. In fact, as Osakada and others have noted (Ref. 3), calculations of this type based on similar slipline fields yield axial residual stresses that are tensile on the mid-plane.

The method of estimating axial residual stresses by assuming a uniform springback from the stress conditions at (or very near) exit can make use of deformation stresses calculated by methods other than the slipline field method. The illustration, for example, shows such a comparison between experimental and predicted curves, the latter being calculated from the stress distribution in the region of deformation given in Refs. 4 and 5. The agreement is poor, but at least the trend from compression on axis to tension near the surface is reproduced in the two sets of curves.

A detailed approach based on elastic-plastic behaviour, rather than on rigid-plastic behaviour, shows more promise of accounting for observed residual stress patterns (Ref. 6). It is worthy of note that the residual stress patterns quoted for hydrostatically-extruded rod are similar to those found in conventionally-extruded rod (7).

References
5. E. R. Lambert and S. Kobayashi, ibid, p. 253
7. P. S. Midha and G. F. Modlen, 4th International Meeting on Cold Forging, Dusseldorf, 1970, p. 231
Also, Fig. 10 shows that the steels containing titanium additions in excess of the amount required to precipitate TiN gave yield strengths which could not be accounted for by grain size alone, but which are clearly related to precipitation hardening of the ferrite.

Conclusions

In the design of C–Mn steels for reinforcing bar for use in earthquake-resistant structures, strain aging is a major problem and an investigation designed to stabilize steels against strain aging has provided the following information:

1. Titanium added to killed 0·2C–Mn steels effectively stabilizes the nitrogen and eliminates strain aging.
2. The stoichiometric composition of about 3·4:1 (Ti/N) appears to give the optimum properties. Thus, additions of about 0·02–0·03% Ti to a 0·005–0·006% N steel prevented strain aging, increased the ultimate strain, and reduced the impact transition temperature.
3. The impact transition temperature was decreased by about 1·33%Mn steel than in the 0·46%Mn steel when the stoichiometric addition of titanium was made.
4. Increasing the titanium addition in excess of 0·03% resulted in a marked increase in the impact transition temperature. This is thought to be due to the precipitation of TiC which has been shown to occur partly by interphase nucleation on the moving austenite/ferrite boundary during transformation on cooling after hot rolling.
5. The mechanism of toughening is thought to be due to the removal of nitrogen with a small contribution due to grain refinement. A small reduction in grain size was actually produced by adding titanium but the grains were not sufficiently small to account completely for the observed low transition temperature. There was, however, a correlation in that the higher-manganese steels which had the lowest impact transition temperatures also had the finer grain size.
6. The regression equation for impact transition temperature, due to Erasmus, has been modified to allow for the effect of titanium. The resulting equation is:

\[ T_\text{tr}=264 \left(\%C\right)-11.8 \left(d^2\right)+28\%\text{Mn}=140000 \left(F'\right) +3850 \left(\%\text{Na}\right)+18 \left(\%\text{Cr}\right)+50 \left(\%\text{Al}\right)+700 \left(\%\text{Ti}\right)+17.5 \]

Acknowledgments

Thanks are due to T. P. Mak, C. M. Yee, C. J. Chong, and K. H. Lim for their assistance with the experimental work. Dr Keown thanks the University of Canterbury for the provision of a Visiting Lectureship during which this work was carried out.

References

Typical residual stress distributions at various extrusion ratios, included die angle 90°

2 Comparison of residual longitudinal stresses found near the surface by boring-out (full lines) and by materials removal from the surface (broken lines) displaced from the optimum frictionless field in the manner expected as a result of friction at the die surface, namely, displacement of the boundaries of the plastic zone in a direction opposite to the direction of extrusion and an increase in the width of the region of deformation.\textsuperscript{10,11}

Residual longitudinal stresses: a for conventional extrusion, 4:1, included die angle 120°; b as a, but in a double-reduction die incorporating a reduction in area of about 2% subsequent to the main reduction
Residual stresses in cold-extruded rods

Twelve different combinations of extrusion ratio and die angle have been used to produce mild steel rods, the residual stresses in which have been determined by the Sachs bore-out method. The longitudinal residual stresses are found to be tensile near the surface, and it is shown that these can be modified to become compressive by an additional small reduction of about 2% following the main reduction in area in extrusion. The experimental residual stress patterns have been compared with theoretical patterns derived from the stress distribution at the exit boundary of the region of plastic deformation by the flow-function method. In general, selection of the velocity fields to give the minimum work of deformation under frictionless conditions does not give the best agreement with experiment. It is, however, possible to select other velocity fields giving more satisfactory agreement, and these are found to be displaced, in general, from the minimum-energy frictionless fields in a manner expected as a result of friction at the die surface. It therefore appears that residual stresses may be expected to be affected by the detailed mechanism of metal flow in extrusion.

P. S. Midha
G. F. Moden

©1976 The Metals Society. Manuscript received 28 April 1975. Mr Midha is with the Production Engineering Research Association, and Dr Moden is in the Department of Engineering Production, Loughborough University of Technology.

Experimental

The material used was mild steel to British Standard En 2E, nominally 38·1 mm dia. hot-rolled bar, but in fact slightly oversized, and heat treated before extrusion at 940°C for 0·5 h, followed by furnace cooling. After heat treatment, the bar was machined to produce billets of about 38 mm dia. and 51 mm long. Standard phosphate-soap lubrication was used. The billets were extruded at slow speeds on a 3 MN hydraulic compression testing machine. Twelve different extrusion dies were used, made up of three included die angles (60°, 90°, and 120°) at each of four extrusion ratios (2·25:1, 2·78:1, 3·52:1, and 4 to 1). Extrusion loads were within 10% of those given by McKenzie.1 In addition, the effect of a subsequent small reduction of about 2% immediately after extrusion through a 4:1, 120° die was investigated. The distribution of residual stresses was studied in the central region of the extruded rods to minimize effects of non-steady-state conditions connected with extrusion of the leading and tail ends. Resistance strain gauges were fixed to the outer surface of the rod to measure longitudinal and circumferential strains. The rod was bored out by a combination of drilling and chemical etching; in a number of cases, in order to check the residual stress distribution near the surface, strain gauges were cemented to the interior surface after a tube had been produced by boring, and the experiment was continued by surface removal. The methods used for calculation of the residual stresses are described in Appendix 1.

Results and discussion

EXPERIMENTAL RESULTS

Figure 1 shows some typical curves obtained for the distribution of longitudinal and circumferential residual stresses, and Table 1 gives values of the maximum tensile residual longitudinal stress, which was found in all cases to be just below the specimen surface. Figure 2 shows that very similar results were obtained by both the bore-out and surface-layer removal techniques.

These residual stress patterns are similar to those reported for hydrostatically extruded rods2-4 and for cold-drawn bar,5 but differ from those reported for the extrusion of aluminium through a 180° included angle die.6 In this case, the longitudinal residual stresses were found to be compressive near the surface and tensile on the rod axis.

Tensile residual stresses near the surface have been thought to be the cause of cracking in brittle extruded products, and it has been suggested that a small reduction immediately following the main reduction would cause a change in the sign of these surface residual stresses,7 since it is known that very small reductions give rise to compressive surface residual stresses. The effect has been demonstrated for bar drawing.8 Figure 3 shows that the expected modification does indeed occur.

THEORETICAL CONSIDERATIONS

If the stress distribution required for the deformation of the metal is known at the boundary where plastic deformation ceases, then the residual stresses may be found from this stress distribution by making use of the condition that

<table>
<thead>
<tr>
<th>Included die angle, deg.</th>
<th>Extrusion ratio 2·25:1</th>
<th>2·78:1</th>
<th>3·52:1</th>
<th>4:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>520</td>
<td>470</td>
<td>570</td>
<td>390</td>
</tr>
<tr>
<td>90</td>
<td>520</td>
<td>460*</td>
<td>550*</td>
<td>410*</td>
</tr>
<tr>
<td>120</td>
<td>650</td>
<td>410*</td>
<td>540*</td>
<td>450*</td>
</tr>
<tr>
<td></td>
<td>420</td>
<td>710</td>
<td>510</td>
<td>440</td>
</tr>
<tr>
<td></td>
<td>460*</td>
<td>700*</td>
<td>510</td>
<td>380*</td>
</tr>
</tbody>
</table>

*By surface removal method
Examples are shown in Figs. 4–7. Figures 4 and 5 refer to 2:78:1 extrusion through a die of included angle 90°. Figure 4 shows longitudinal stresses as a function of radial position from the centre to the surface, and Fig. 5 the circumferential stresses. Similarly, Figs. 6 and 7 are for 4:1 extrusion, included die angle 120°. In each case, curve a is given by the experimentally determined residual stresses, and curve b by the residual stresses corresponding to the optimum velocity field for frictionless conditions; the qualitative agreement between curves a and b is poor, but the agreement is improved by selecting a velocity field displaced in a direction corresponding to the effect of friction, giving curve c. Displacement in the opposite direction (curve d) causes no improvement in the agreement between the calculated and experimental curves.

Since the residual stress patterns determined from arbitrarily postulated velocity fields appear to be very sensitive to the features of the deformation zone, it would be unrealistic to look for more than some general form of agreement between the experimental and the calculated residual stresses. In addition, a constant yield stress is assumed in the calculations rather than one that is taken as a function of strain. Only a number of fields of a particular type were examined, and hence complete coincidence with that actually occurring in practice is not to be expected; and the possibility of inelastic effects after exit from the region of deformation is not examined. Nevertheless, it appears that the residual stress pattern should be sensitive to the pattern of metal flow in extrusion and that friction, could thus be a factor affecting residual stresses.

Although most of the longitudinal residual patterns that have been reported are such that the stresses are compressive on an axis, there are exceptions, which may probably be attributed to friction effects rather than to any special features of the metal used. Another example of this unusual pattern has been found in extruded aluminium rod (4:1 extrusion ratio, 120° included angle) by the longitudinal slitting technique (the slit closed up, indicative of longitudinal residual compressive stresses near the surface). The slitting technique has also been used to examine the effect of variations in friction brought about by alteration of the die surface finish on the longitudinal residual stresses in 6.35 mm dia. extruded mild steel rods (4:1 extrusion ratio, 120° included angle). The opening of the slit, and

### Table 2 Parameters of deformation zone (see Fig. 8)

<table>
<thead>
<tr>
<th>Extrusion ratio</th>
<th>2x, deg.</th>
<th>Parameters of deformation zone for minimum extrusion pressure, frictionless conditions</th>
<th>Parameters of deformation zone assessed to give best agreement with observed residual stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:25</td>
<td>60</td>
<td>$d_1$, $d_2$, $d_3$</td>
<td>$p_{av}$, $d_1$, $d_2$, $d_3$, $p_{av}/d_1$</td>
</tr>
<tr>
<td>2:25</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2:25</td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2:78</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2:78</td>
<td>90</td>
<td></td>
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<tr>
<td>2:78</td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2:78</td>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4:1</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4:1</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4:1</td>
<td>120</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$p_{av}$ = average extrusion pressure; $d$ = mean yield stress

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Consequently the magnitude of the longitudinal residual stresses, were found to depend on surface finish in the following way:

- Length of slit from leading end of extrudate, mm: 34.92
- Thickness of saw, mm: 0.30
- Surface finish of die, µm (centre-line average): 0.2, 0.64, 1.55, 4
- Approximate coefficient of friction (ring test): 0.024, 0.028, 0.035, 0.055
- Opening of slit, mm: 1.14, 1.29, 1.35, 1.37

On this basis, it is therefore considered possible that modification to the pattern of metal flow in extrusion will give rise to corresponding changes in the pattern of residual stresses and that one cause of such modification may be the existence of friction at the die surface. It is suggested that the estimation of residual stresses by examination of experimentally determined flow functions is worthy of further study. The possibility also arises of determining residual stresses theoretically or semi-empirically if data relating to friction conditions at the metal/die interface are available.

Conclusions

1. The longitudinal residual stresses in cold-extruded mild steel rods are similar to those found in hydrostatically extruded rod and in drawn bar, namely, compressive on the axis with a maximum tensile value near the surface. As in drawn bar, the residual stresses can be modified by a small reduction of approximately 2% immediately following the main die. This causes the surface longitudinal residual stresses to become compressive.

2. An attempt to correlate the observed residual stresses with those predicted by the use of various velocity fields to describe the metal flow indicates that the residual stresses depend upon the details of the velocity field selected and that friction at the die surface, as one of the factors affecting the velocity field, may consequently also affect the pattern of residual stresses.

Acknowledgment

The authors acknowledge with thanks the receipt of a Science Research Council Grant that enabled this work to be carried out.

References

Appendix 1

The equations used to derive the residual stresses are (a) for strain measurements made on the outside, metal being successively removed from the bore:

\[ \sigma_r = E' \left( \frac{F_1 - F}{A} \right) \]

\[ \sigma_\theta = E' \left[ \left( F_1 - F \right) \frac{\theta}{2F} \right] \]

\[ \sigma_z = E' \left[ \frac{F_1 - F}{2F} \right] \] .......................... (1)

and (b) for strain measurements made on the bore surface, material being removed from the outside:

\[ \sigma_r = E' \left( F - F_0 \right) \]

\[ \sigma_\theta = -E' \left( F - F_0 \right) \frac{\theta}{2F} + \frac{F + F_0}{2} \theta \]

\[ \sigma_z = -E' \left[ \frac{F - F_0}{2F} \right] \] .......................... (2)

where \( \sigma_r \), \( \sigma_\theta \), and \( \sigma_z \) are the residual longitudinal, tangential (circumferential), and radial stresses, respectively; \( E' = E/(1 - \nu^2) \), \( E \) being Young's modulus and \( \nu \) Poisson's ratio; \( F_1 \) is the original cross-sectional area of the solid cylinder; in equations (1), \( F \) is the cross-sectional area of the bore corresponding to the radial position of the calculated residual stresses, and in equations (2) \( F \) is the overall cross-sectional area of the tube; \( F_0 \) is the constant bore cross-sectional area; \( A = \lambda + \nu \theta \), where \( \lambda \) and \( \theta \) are the longitudinal and circumferential strains, and \( \Theta = \nu \lambda + \theta \). The residual stresses found by equations (2) have to be corrected for the effect of producing the internal bore.

Sixth-degree polynomials were fitted to \( A = A(F) \) and \( \Theta = \Theta(F) \), and the residual stress patterns were produced by the computer in the form of curves of \( \sigma_r \), \( \sigma_\theta \), and \( \sigma_z \) as functions of the parameter \( r \). A blank test carried out on the annealed specimen gave residual stresses that were zero to within \( \pm 30 \) MN/m².

The derivation of the residual stresses from the experimental data identically ensures the fulfilment of the various equilibrium conditions, for example, the condition that there should be no net longitudinal force transmitted across any section perpendicular to the rod axis:

\[ \int_0^{\gamma_r} \sigma_r dF = 0 \]

Checks were made that the equilibrium conditions were satisfied, but these do not, unfortunately, provide any confirmation of the validity of the original data. In the figures showing longitudinal stresses, these have been plotted against radial position rather than against \( F \) or \( r^2 \), and the total area under the curves is not, therefore, zero.

More complete details, and a derivation of equations (2), which have been quoted incorrectly in a number of publications, have been given elsewhere.\(^{16}\)

Reference


Appendix 2

The basic flow pattern used in the theoretical analysis is of the type shown in Fig. 8a, and the calculation of stresses within the region of deformation follows Lambert and Kobayashi.\(^{9,11}\) Patterns of this type are known to give good agreement with experimentally observed velocity fields and deformed grid patterns, and the optimum upper-bound loads derived from them for frictionless conditions are lower than those derived from alternative fields such as those shown in Fig. 9.\(^{9}\) The flow lines are straight in the region \( ABCD \), and the boundary curves \( CQD \) and \( APB \) (Fig. 8a) are given by the equations:

\[ z = (h \cot \alpha - d_1) \left( \frac{r}{b} \right)^2 + d_1 \] .......................... (3)

and

\[ z = \left( \cot \alpha - d_1 \right) \frac{r^2}{b} + d_1 \left( \frac{1}{b} - m \right) r + md_1 \] .......................... (4)

respectively: that is, for each exit boundary \( CQD \), the corresponding entry boundary is determined by the parameter \( m \).

The deformation boundaries \( CQD \) and \( APB \) correspond, in three dimensions, to surfaces on which the velocity of a particle changes discontinuously. In practice such discontinuities cannot occur, but a more or less smooth transition may be realized by the superposition of a large number (in practice, 50 is adequate\(^{9,11}\)) of basic flow patterns. In order to derive the stress and strain distribution associated with any given flow pattern, the flow-function method was used. Since the flow is steady-state and axisymmetric, the metal may be imagined to flow...
Consequently the magnitude of the longitudinal residual stresses were found to depend on surface finish in the following way:

Length of slit from leading end of extrudate, mm 34-92
Thickness of saw, mm 0-30
Surface finish of die, µm (centre-line average) 0-2 0-64 1-55 4
Approximate coefficient of friction (ring test) 0-024 0-028 0-035 0-055
Opening of slit, mm 1-14 1-29 1-35 1-37

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1. The longitudinal residual stresses in cold-extruded mild steel rods are similar to those found in hydrostatically extruded rod and in drawn bar, namely, compressive on the axis with a maximum tensile value near the surface. As in drawn bar, the residual stresses can be modified by a small reduction of approximately 2% immediately following the main die. This causes the surface longitudinal residual stresses to become compressive.

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Acknowledgment

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References

through pipes of circular cross-section, the flowrate down any pipe being constant and equal to \( \phi \). The intersection of a plane containing the axis of symmetry with any given pipe gives a flow line associated with a particular constant value of \( \phi \). Thus each basic flow pattern corresponds to a family of flow lines, each of constant \( \phi \). The superposition of the basic flow patterns to smooth out the velocity discontinuities may be performed in any way that satisfies the boundary conditions; in fact, the method selected was simply:

\[
\Psi = \sum_{i=1}^{n} \psi_i
\]

where \( \psi_i \) represents the flow functions corresponding to the basic patterns, and \( \Psi \) is the smoothed flow function.

Radial and axial velocities are then given by

\[
u = \frac{1}{2\pi r} \frac{\partial \Psi}{\partial \theta}
\]

and

\[v = \frac{1}{2\pi r} \frac{\partial \Psi}{\partial \theta}
\]

respectively. The circumferential velocity is, of course, zero.

For each extrusion condition (die angle and extrusion ratio), 25 smoothed flow functions, \( \Psi \), were generated by taking 5 values of \( d \), in equal steps from \( d_{\text{min}} = 1.05 \) b cota to \( d_{\text{max}} = 1.5 \) b cota, each with 5 values of \( m \), again in equal steps, from \( m_{\text{min}} = \sqrt{1/b} - 0.2 \) to \( m_{\text{max}} = \sqrt{1/b} + 0.2 \). The 50 basic flow patterns making up each \( \Psi \) had entry boundaries equally spaced between \( d \) (giving \( \psi_1 \)) and \( d_0 = md_1 \) (giving \( \psi_n \), where \( n = 50 \)) (Fig.8b). The values of \( \psi_i \) were calculated on a mesh of points, the mesh size being the minimum possible within the limits set by the computer's capacity, and the \( \psi_i \) combined, according to equation (5), to give \( \Psi \). Velocities, strain rates, axial, radial, and circumferential stresses and extrusion pressure may then be calculated for each \( \Psi \), a mean yield stress being taken for each extrusion ratio.\(^*\) The optimum upper bound for frictionless conditions is given by the \( \Psi \) corresponding to the lowest extrusion pressure.

For each \( \Psi \), therefore, the radial, axial, and circumferential stresses are calculated on each of the mesh points.

### RESIDUAL STRESS CALCULATION

The normal axial, radial, and circumferential stress components \( (\sigma_x', \sigma_y', \text{ and } \sigma_z') \) at the exit boundary of the deformation zone are calculated by interpolation from known stresses at the mesh points on either side of the exit boundary. These then contribute the state of stress at the instant when the loading is interrupted, that is, when the material is about to leave the deformation zone and become the extruded rod. On emerging from the deformation zone the material undergoes elastic stress relaxation. Because of the non-uniform distribution of stress, the material remains residually stressed after the elastic stress relaxation has taken place.

Let \( \sigma_x, \sigma_y, \text{ and } \sigma_z \) be the residual longitudinal, circumferential, and radial stresses in the extruded bar. Then static equilibrium requires that

\[
2\pi \int_0^b \sigma_x r dr = 0
\]

\[
\int_0^b \sigma_y dr = 0
\]

\[
\sigma_z = \sigma_x + \int_0^b \frac{d\sigma_x}{dr}
\]

Assuming elastic stress relaxation to be uniform and denoting the changes in stress caused by elastic relaxation by \( \sigma_x', \sigma_y', \text{ and } \sigma_z' \), the residual stress components are given by

\[
\sigma_x = \sigma_x - \sigma_x'
\]

where

\[
\sigma_x' = \frac{2}{b^2} \int_0^b \sigma_x r dr
\]

\[
\sigma_y' = \sigma_y - \sigma_y'
\]

where

\[
\sigma_y' = \frac{1}{b} \int_0^b \sigma_y dr
\]

and \( \sigma_z \) is calculated as follows. Solving the last equation of (8) for \( \sigma_z \), we have

\[
\frac{d}{dr} (\sigma_z) = \sigma_z
\]

or

\[
\sigma_z = \int_b^R \sigma_z dr
\]

where \( R \) is any radius at which \( \sigma_z \) is to be determined. Then

\[
\sigma_z = \frac{1}{R} \int_b^R \sigma_z dr
\]

because

\[
(\sigma_z)_{r=R} = 0
\]

In general, residual stresses calculated from the optimum upper bound for frictionless extrusion did not give good agreement with those found experimentally, and flow patterns with exit and entry boundaries on either side of the optimum upper-bound pattern were then examined, and the corresponding residual stresses were determined.

---

\* The mean yield stress values for the calculated curves in Figs.4-7 were taken from McKenzie.\(^{1}\)
Recent studies\textsuperscript{1}-\textsuperscript{3} have shown that the fracture toughness of martensitic steels can be improved by austenitizing at higher temperatures than are conventionally used in the heat treatment of such steels. Austenitizing AISI 4340 at 1200°C instead of the usual temperature of 870°C, increased the plane-strain fracture toughness for the as-quenched state by about 100%.

Conventional heat-treatment practice is to heat into the lower austenite range to keep the grain size small and the 'toughness', as measured with a Charpy-type test, high. Dulieu\textsuperscript{4} has reported a rise in plane-strain fracture toughness with increasing austenitizing temperature without an increase in the Charpy impact absorbed energy. In the present investigation the 'toughness' of a tempered martensitic steel austenitized at 850° and 1200°C is determined using Charpy V-notched specimens and Charpy fatigue-cracked specimens and compared with plane-strain fracture-toughness data obtained previously by Clark and Ferguson.\textsuperscript{5}

The energy measured in a Charpy V-notch impact test arises from three main sources: (a) energy for crack initiation, (b) energy for crack propagation, and (c) losses. Under the heading of 'losses' are included mechanical losses in the test machine (vibrational and frictional losses) and the kinetic energy of the broken testpieces. Chipperfield\textsuperscript{6} has found that the crack opening displacement for crack initiation \( \delta_i \) in mild steel is a linear function of the notch root radius down to a radius approximately equal to the inclusion spacing, below which it remains constant. The value of \( \delta_i \) for fatigued specimens was determined by this constant value. Hence \( \delta_i \) for Charpy V-notched specimens, which contain relatively blunt notches, will be much greater than that for Charpy fatigue-cracked specimens and consequently the energy absorbed will be very much greater in initiating a crack in the former type of specimen than in the latter type. Hence Charpy fatigue-cracked specimens will basically give the energy for propagation and in this respect the results from this type of specimen should, apart from strain-rate effects, approximate to the behaviour found using standard fracture-toughness specimens.

**Experimental procedure**

The specimen material was Consteel En25, supplied in the form of centreless-ground bars, 25 mm dia., of the following composition (wt-%): 0.27C-0.30Si-0.63Mn-2.63Ni-0.70Cr-0.55Mo-0.021S-0.023P.

The Charpy V-notched specimens were standard but the fatigue-cracked specimens which had the same dimensions had, instead of a V-notch, a slot 0.25 mm wide and 1.67 mm deep from which a fatigue crack was grown after heat treatment.

Samples of both types of specimen were austenitized at either 850° or 1200°C. Austenitizing at 850°C was carried out in a salt bath for 1 h before oil quenching, whereas at 1200°C austenitizing was carried out in an inert atmosphere furnace for 1 h before oil quenching. For the latter heat treatment the quench was interrupted at 850°C for 1 min to reduce quenching stresses. Two samples of each specimen type were tempered at the following temperatures (with subsequent oil quenching): 200°, 300°, 400°, and 600°C.

The fatigue-cracked specimens were fatigued in three-point bending until the crack was 3-33 mm long, the final stages of crack growth being carried out in accordance with ASTM E399-72.

All impact testing was done at room temperature in a standard Charpy impact tester, and the fractured area of the fatigue-cracked specimens was measured. The hardness of each specimen was measured and a selection of heat-treated specimens etched in a 3% Nital solution in order to determine the prior austenite grain size.

**Results and discussion**

The average grain size for specimens austenitized at 850°C was 17 \( \mu \text{m} \), and for those austenitized at 1200°C was 184 \( \mu \text{m} \). With the etch used it was relatively easy to detect the prior austenite grain boundaries for the 850°C heat treatment, but very difficult for the 1200°C heat treatment. Clark et al.\textsuperscript{7} attributed this effect to grain-boundary segregation for they found that grain boundaries readily grooved when segregated and that the degree of segregation decreased as the austenitizing temperature increased, provided that the specimens were quenched from the austenitizing temperature.

The hardness (HRC) as a function of tempering temperature is shown in Fig.1a. The hardness data indicate that the two austenitizing heat treatments give approximately the same tensile strength. Lai et al.\textsuperscript{8} reported a similar result for as-quenched 4340 steel.

The Charpy V-notched impact energy and the Charpy fatigue-cracked impact energy for the 850° and 1200°C heat treatments are given in Fig.1b. Each point represents the mean of two tests. From now on the Charpy V-notched impact energy will be referred to as \( C_V \) and the fatigue-cracked impact energy as \( C_t \). For purposes of comparison, the fatigue-cracked data have been corrected by multiplying the measured values by:

\[
\frac{\text{fracture area for } C_v}{\text{fracture area for } C_t}
\]

This allows comparison on the basis of the Charpy V-notch test and was done because the two areas are different, being 0.8 cm\(^2\) for the V-notched specimens and about 0.68 cm\(^2\) for the fatigue-cracked specimens. The results show that for both heat treatments \( C_V \) is greater than \( C_t \) for all tempers. For the 850°C heat treatment the minimum
RESIDUAL STRESS RELIEF IN COLD-EXTRUDED ROD

Synopsis.

The residual stresses in cold-extruded mild steel rods have been estimated by the longitudinal slitting method, and the effect of low-temperature annealing on the residual stress level has been investigated. For the extrusion ratios used (2.78:1 and 4:1), residual stress relief appears first at about 250°C, and is substantially complete by approximately 500°C, the tensile strength having decreased over this temperature range by less than 10 percent.

A correction factor is proposed that allows the determination of residual stresses by the longitudinal slitting method to become independent of slit length.

Introduction.

Although there is considerable evidence\(^1\) that residual stresses introduced by cold work may be substantially reduced in certain cases by heat-treatment at temperatures that do not cause extensive loss of the increase in strength imparted by the cold work, there appears to be little information regarding the temperatures required for the relief of residual stresses in mild steel subjected to large strains, or regarding the resultant mechanical properties. The present work was therefore undertaken to investigate residual-stress relief in heavily cold-worked (extruded) mild steel, and also to investigate whether it would be feasible to aim for a more-or-less complete removal of residual stresses without incurring a drastic loss in strength.

Experimental.

The material used was a mild steel to British Standard En 2E (0.09C, 0.39Mn, 0.25Si, 0.045S and 0.025 percent P). The steel, in the form of nominally 1.5 inch (38 mm) diameter black bar, slightly oversize, was heat-treated as follows: 940°C, 1/2 h, furnace cool. Billets, 2 inches long x 1.5 inches diameter (51 x 38.1 mm) were machined from the black bar, shot-blasted and subjected to a proprietary phosphate + soap lubrication treatment. Extrusion was through conical dies having included angles of 60°. The billets were extruded straight through without ejection by use of a following dummy billet.
The specimens for residual stress determination and tensile testing were heat-treated together, (apart from those heat-treated at 500°C and above) with the thermocouple in the centre of the bundle of rods. When the required temperature had been reached, the specimens were held for 45 minutes at temperature and then cooled in air.

The level of residual stress in the extruded rods was determined by longitudinal slitting into two semicircular sections from the leading end of the extrusion, the slit width being 1/32 inch (~0.8 mm) and the length approximately 2.875 inches (~73 mm) or 3.875 inches (~98 mm) for the 2.78:1 and the 4:1 extrusion respectively. Some aspects of the method are discussed in the Appendix, and a method of correction to make the residual stress determination independent of slit length is proposed. Tensile specimens were 0.564 inches (14.45 mm) in diameter, with a gauge length of 2 inches (50.80 mm).

Results and Discussion.

Table I gives the slit openings, \( t \), found for the as-extruded and for the heat-treated rods; Fig. I shows the same results plotted against heat-treatment temperature, but as a percentage of the slit opening for the as-extruded rods. It can be seen that the residual stresses first begin to relax at about 250°C, and that any given percentage relaxation occurs at a lower temperature for the 2.78:1 extrusion than for the 4:1 extrusion. This result may appear anomalous, in that relaxation processes, like recrystallization processes, might be expected to occur more readily in the more highly worked material. However, the peak residual stresses are higher in the case of the 2.78:1 extrusion (see Table 2), and it is this factor that is here determining the relative positions of the two curves.

Fig. 2 shows that there is comparatively little change in tensile strength or 0.1% proof stress up to a heat-treatment temperature of approximately 450 - 500°C. The tensile specimens were machined from the extruded rods, and hence the residual longitudinal stresses in them were displaced from the values in the initial bar by some constant stress \( \sigma' \), to maintain the condition of zero net axial force \( \int_0^R \sigma'_x \rho \, d\rho = 0 \).

Denoting the diameter of the test-piece by \( 2R' \), and the diameter of the initial bar by \( 2R \), it may readily be shown that

\[
\sigma'_x = \frac{2}{\left(R'^2 - R^2\right)} \int_{R'}^R \sigma'_x \rho \, d\rho
\]
In this way, by making use of the residual stress patterns previously determined (5) (Fig. 8), the peak tensile longitudinal residual stresses in the 2.78:1 and 4:1 test-pieces were found to be approximately 140 and 150 MN/m² respectively, the location of these peak stresses being at the specimen surface.

When a tensile stress is applied to a specimen containing residual stress, yielding will first occur where the effect of the applied stress plus that of the internal stress is such that the yield criterion is fulfilled: in the present case, since the peak residual stress occurs at the specimen surface, this merely reduces to the requirement that sum of the residual and applied stresses should exceed some critical value. The initial deviation from linearity on the stress strain curve is thus affected by the presence of residual stresses, but as further plastic flow relieves the residual stresses, the higher proof stresses are little affected. The form of the curves in Fig. 2 thus show little change in 0.1% proof stress and UTS over a range of heat-treatment temperature that causes a marked increase in the 0.01% proof stress and a marked decrease in the residual stress level (200–400°C). It should be noted that there was little evidence of any strain ageing effects, the steel having been aluminium-killed. In addition, it may be seen that a combination of time and temperature (45 mins at 200°C) that would be expected to cause appreciable strain-ageing (if such effects were relevant), in fact does not cause any appreciable change in the 0.01% proof stress. It therefore appears reasonable to ascribe the increase in the 0.01% proof stress to the relaxation of those residual stresses which, when the external stress is applied, give rise to early local yielding in the tensile test. As to the mechanism by which the stress relaxation itself occurs, this must clearly involve the thermally activated movement of dislocations, non-conservative motion possibly being assisted by the vacancies produced in the initial deformation.

Metallographic examination showed that the specimens heat-treated at 600 and 615°C had undergone recrystallization, the rather high lower yield stress (380 MN/m²) corresponding to the small recrystallized grain size (approximately 6.5 μm). Initial recrystallization was observed in the specimens heat-treated at 550°C (Fig. 3, b), whereas there were no signs of recrystallization in those heat-treated at 500°C (Fig. 3, a). The form of the tensile strength-temperature curves, in the region 500–550°C, does, however, indicate that recrystallization is more advanced in the specimens extruded at the 4:1 ratio, in agreement with their greater degree of working.
The results of the present work are compared with existing data in Fig. 4 and Table 2. Some results concerning residual stresses introduced into Armco iron cylinders by quenching from 850°C are included\(^1\): these are remarkable in that they show appreciable stress relief at comparatively low temperatures, for example, 150°C. It is possible that processes connected with diffusion of interstitial atoms are here contributing to the rapid stress relaxation. In a rather similar situation, in steels undergoing tempering, there is some evidence that residual stress relaxation is accelerated.\(^7\) There are two sets of data from previous investigations of residual stress relief in drawn bar: whereas the results of the present investigation and those of Peiter's\(^3\) appear to be mutually consistent, Bühler's\(^1, 6\) results point to much greater stress relief at low temperatures. A possible reason for this discrepancy is the rather high value of peak residual stress reported for the as-drawn bar: 600 MN/m\(^2\) for natural strains of 0.082 and 0.085 respectively, details of die angle, etc., not being available. If this initial value were too high, this would, of course, cause an over-estimate of subsequent stress relief. Peiter's\(^3\) results were obtained with a leaded free-cutting mild steel (Table 2) (C, .13 max; Mn, .9 - 1.3; P, .035 - .1; S, .2 - .27; Pb, .15 - 30 %). It may be seen that the temperature required for a given percentage stress relief is lower for the higher initial peak residual stress, even though the higher stress was the result of a smaller initial deformation. A comparison of the two sets of data indicates that an increase in the initial strain over the range considered (true strains from 0.085 to 1.386) also has the effect of lowering the temperature required for stress relief.
Conclusions.

1. In cold-extruded mild-steel rods (extrusion ratios, 2.78:1 and 4:1, included die angle 60°), residual stresses begin to be removed by heat-treatment at temperatures of approximately 250 - 300°C. At any given temperature, relief is more extensive in the 2.78:1 extrusion, corresponding to the higher initial residual stress.

2. Heat-treatment at 500°C for 45 minutes results in ~90 percent relief of residual stresses with an approximately 10 percent fall in tensile strength. It therefore appears feasible to obtain effective stress relief without appreciable loss of the strength acquired by cold work.

3. The UTS and 0.1 percent proof stress of the extruded material change little as a result of heat treatment at temperatures up to approximately 400°C: over the same range, the 0.01 percent proof stress rises as the residual stresses are relieved, to become approximately equal to the 0.1 percent proof stress.

4. Comparison with other data indicates that two factors determine the temperature required for a given percentage of stress relief. These are the initial peak residual stress and prior strain: an increase in either factor tends to reduce the temperature required for stress relief.

5. A correction factor is proposed to make the determination of residual stresses by the longitudinal slitting method independent of slit length.
APPENDIX.

The longitudinal residual stresses in the as-extruded rods had previously been determined by the Sachs bore-out method and it was therefore of interest to compare the slit openings found in the present work with calculated values based on the previous work.

If the width of the slit is neglected, the second moment of area of each half of semicircular cross-section about its neutral axis, \( I \), is given by \( I = \frac{R^4}{3} \left( \frac{\pi}{6} - \frac{2}{3\pi} \right) \); the bending moment caused by slitting is \( M = 2 \int_0^R \rho^3 \sigma_z d\rho \), where \( 2R = D \) is the rod diameter, and \( \sigma_z \) is the longitudinal residual stress at radial position \( \rho \) before slitting. Hence the neutral axis of each half is bent into a circular arc of radius \( r = \frac{M}{E' I} \) where \( E' = \frac{E}{1 - \nu^2} \), \( E \) being Young's modulus and \( \nu \) Poisson's ratio (following Schepers and Peiter, the semicircular section is regarded as undergoing plane strain during bending).

In addition to the circular curvature giving rise to the opening \( f \), there is also a region of elastic stress relief in advance of the slit, so that the slope \( \frac{dy}{dx} \neq 0 \) at \( x = 0 \) (Fig. 5,a). In order to evaluate this slope, the slit was clamped shut and a second slit made at right angles to the original slit, but some distance back (Fig. 5, b). This causes a region of local stress relief, so that the portion AB of the rod (Fig. 5, b) undergoes a rotation equal to twice the rotation of each half section in Fig. 5, a (the width of the slit at A, 6 mm, being ignored). Hence the slit opening at any position \( x \) may be corrected to \( f' \) corresponding to \( \frac{dy}{dx} = 0 \) at \( x = 0 \). If the half section is deformed into a circular arc, then \( f' = \frac{3r}{2} \), and a straight line should be obtained by plotting \( f' \) against \( x^2 \).

Fig. 6 shows such a diagram for the as-extruded 2.78:1 rod. It can be seen that the initial section of the graph becomes linear as a result of the correction, indicating a constant radius of curvature, \( r_0 \), but for large \( x \) the values of \( f' \) are less than those given by extrapolation of the initial linear section. This is attributed to the fact that the residual stresses build up from the leading end of the extrusion at B (Fig. 5,b), so that \( r \) is greater at large \( x \).
An equivalent increment of slit length, \( \Delta \), may be derived from the value of \( r_0 \) found from Fig. 6, such that the rotation \( \theta = \left( \frac{d\gamma}{dx} \right)_{x=0} \) would occur over a slit length \( \Delta \) (Fig. 7): that is, \( \theta = \left( \frac{d\gamma}{dx} \right)_{x=0} \approx \frac{\Delta}{r_0} \).

With the assumption that \( \Delta \) is proportional to \( D \), \( \Delta \approx 0.29D \).

But \( r' = x^2 \) and hence \( \theta' = \frac{f}{1 + 2\Delta \gamma} = f/(1 + 0.5 \Delta \gamma x) \). The rotation \( \theta \) appears to have been ignored in previous publications dealing with longitudinal slitting (6,7), although the effect has been appreciated in studies involving deflections caused by transverse slitting (9, 10). From Ruiz Fernandez's data (9), a value \( \Delta \approx 0.33D \) may be derived: Buhler and Kreher's results (10) do not appear to permit estimation of \( \Delta \). By postulating a particular form for the distribution of longitudinal residual stress, a maximum longitudinal stress \( \sigma_L \) can be calculated from the uncorrected values of \( f \) and \( x \); although \( \sigma_L \) is known to be dependent on \( x \), this variation had been previously ascribed incorrectly to the existence of ranges of \( x \) in which different approximate relationships between \( x \), \( r \), and \( f \) were valid.

In fact, by correcting \( f \) as shown above, this anomaly disappears. This is shown in Table 3 for data for cold-drawn bar: in this case, it may be assumed that the original residual stresses, and hence the radius of curvature are constant, apart from the material very near original cut end of the bar.

The method of correction outlined above was used to calculate the slit openings expected from the residual patterns shown in Fig. 8 (5). The experimentally determined short-fall in slit opening, ascribed to non-uniformity of the bending moment, was then subtracted from each figure, to give calculated openings of 1.30 and 2.44 mm for the 2.73:1 and the 4:1 extrusion respectively, compared with the experimental figures of 1.43 and 1.92 mm. The agreement in the first case is probably fortuitously good, whereas the discrepancy is of the order of 30 percent in the second case. It must be remembered, however, that the Sachs bore-out method is based on the measurement of strains due to forces arising during the boring out procedure, whereas the slitting method is based on the measurement of deformations due to bending moments.
The calculation of the bending moments from the stress distribution involves the determination of the difference between integrals corresponding to the contributions of the compression and tensile stresses to the bending moment on the semicircular cross-section, and the original errors in the stresses and radial positions are therefore considerably magnified. Removal of the slit causes a shift in the longitudinal stresses required to calculate the bending moment imposed on each semicircular section and also, since the sections are not exactly semicircular, causes a deviation from the second moment of area assumed here. Both these causes of error are greater in the smaller section (4:1 extrusion), since the slit width was constant.

Tests by the Sachs' bore-out method indicated that heat-treatment at 500°C caused a reduction in peak tensile residual stress to 17 and 9% of the values in the as-extruded rod for 2.78:1 and 4:1 extrusion ratios, as compared with 9 and 13% found by the slitting technique. Considering the low levels of residual stress after heat-treatment, one may regard the agreement as satisfactory.
Table 1. Slit opening as a function of heat-treatment temperature.

<table>
<thead>
<tr>
<th>Extrusion ratio</th>
<th>Slit opening, mm, after heat-treatment (0.75h) at:</th>
<th>200°C</th>
<th>300°C</th>
<th>400°C</th>
<th>500°C</th>
<th>550°C</th>
<th>615°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.78:1</td>
<td>as extruded</td>
<td>1.43</td>
<td>1.40</td>
<td>1.06</td>
<td>0.52</td>
<td>0.16</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>0.52</td>
<td>0.16</td>
<td>0.05</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4:1</td>
<td>as extruded</td>
<td>1.92</td>
<td>1.92</td>
<td>1.51</td>
<td>0.94</td>
<td>0.24</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>2.78:1</td>
<td>1.06</td>
<td>0.52</td>
<td>0.16</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 2. Comparison of temperatures for 50% stress relief in cold-drawn and in cold-extruded mild steel.

<table>
<thead>
<tr>
<th>Method of deformation</th>
<th>Peak longitudinal residual stress, MN/m²</th>
<th>True strain</th>
<th>Temperature for approx. 50% relief of residual stress 2 h at:</th>
<th>Temperature for approx. 50% relief of residual stress 0.75 h at:</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drawing</td>
<td>430*</td>
<td>0.085</td>
<td>520°C</td>
<td></td>
<td>Peiter(3)</td>
</tr>
<tr>
<td></td>
<td>430*</td>
<td>0.174</td>
<td>520°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>360*</td>
<td>0.227</td>
<td>540°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extrusion</td>
<td>480**</td>
<td>1.022</td>
<td>360°C</td>
<td>390°C</td>
<td>Present work</td>
</tr>
<tr>
<td></td>
<td>390**</td>
<td>1.386</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Determined by the Heyn and Bauer(3) method
** Determined by the Sachs Bore-out(3,5) method

Table 3. Data for the maximum tensile residual stresses in cold-drawn bar(2)

<table>
<thead>
<tr>
<th>Bar Diameter, 19 mm</th>
<th>Slit length, mm</th>
<th>Uncorrected peak stress, MN/m²</th>
<th>Corrected peak stress MN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>850</td>
<td>650</td>
</tr>
<tr>
<td></td>
<td></td>
<td>550</td>
<td>520</td>
</tr>
<tr>
<td>Bar diameter, 21 mm</td>
<td>Slit length, mm</td>
<td>Uncorrected peak stress, MN/m²</td>
<td>Corrected peak stress MN/m²</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>750</td>
<td>620</td>
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<tr>
<td></td>
<td></td>
<td>470</td>
<td>480</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

Fig. 1 Residual stress remaining after heat treatment, as a percentage of the residual stress in the as-extruded rod.

Fig. 2 Mechanical properties of specimens, machined from extended bars after heat-treatment. Extrusion ratio: (a) 2.78:1 (b) 4:1. The full line corresponds to the UTS, broken line to the 0.1% proof stress and the chain line to the 0.01% proof stress. The lowest curves show the reduction in area.

Fig. 3 (a) Longitudinal section of specimen after heat-treatment at 500°C extrusion ratio 4:1

(b) as (a), but heat-treatment temperature 550°C

Fig. 4 Comparison of the present results with those of previous work. 1, Armco iron, residual stresses produced by quenching(6); 2, cold drawn mild steel(1,6); 3,4,5 cold drawn mild steel, 24 mm diameter bar drawn to 23, 22 and 21 mm respectively(3); 6,7 present work, 2.78:1 and 4:1 extrusion ratio respectively.

Fig. 5 (a) Slit specimen showing rotation of half-sections due to stress relief in shaded areas ahead of the slit.

(b) Transverse slit at A producing only rotation; C marks the end of the prior longitudinal slit.

Fig. 6 Corrected and uncorrected slit openings, f and f' respectively, plotted against $x^2$, the square of the distance from the unopen slit end.

Fig. 7 Correction of slit opening.

Fig. 8 Distribution of longitudinal residual stresses in extruded rods.
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8. A.A. Denton, Metallurgical Reviews, No. 101, 1966 (Vol. 11, p1)
Extrusion ratio

- 2.78:1
- 4:1

Fig. 1.
Figure 4

Key (see caption)

- 1
- 2
- 3, 4
- 5
- 6
- 7

Temperature, °C

Percentage residual stress
Fig. 5