Spatial diversity and optimization techniques for cognitive radio networks

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Spatial Diversity and Optimization Techniques for Cognitive Radio Networks

Thesis submitted to Loughborough University in candidature for the degree of Doctor of Philosophy.

Kanapathippillai Cumanan

Advanced Signal Processing Group
Department of Electronic and Electrical Engineering
Loughborough University
2009
Abstract

The explosive growth of wireless applications and high demand for wireless resources have created spectrum crisis. Many spectrum occupancy measurements have shown that most of the allocated spectrum experiences inefficient utilization. Hence radically new approaches are required for better utilization of spectrum.

This has motivated the concept of opportunistic spectrum access in the licensed bands namely cognitive radio technology. This intelligent wireless system has the potential to improve the spectrum utilization by enabling unlicensed users to access the licensed bands without disturbing the licensed users.

In this thesis, spatial multiplexing techniques are studied for underlay cognitive radio networks where transmit beamformers are designed to satisfy quality of service and interference constraints using convex optimization techniques. Robust schemes are also proposed in the presence of imperfect channel state information at the basestation. To overcome the infeasibility issues encountered in the beamformer design, a joint resource allocation and admission control technique is proposed using the branch and bound optimization method. Finally, signal-to-interference and noise ratio (SINR) balancing techniques are developed for different types of interference constraints on the primary users using a max-min fairness approach. These SINR balancing techniques also solve the problem of infeasibility. The performance of all these new schemes has been verified using MATLAB simulation results.
I dedicate this thesis to my parents.
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Statement of Originality

The novelty of Chapter 4 is the design of robust beamforming techniques for single user and multiuser underlay cognitive radio networks. The contribution of Chapter 5 is on the joint resource allocation and admission control techniques for underlay cognitive radio networks based on branch and bound optimization method. The contribution of Chapter 6 is on the SINR balancing technique for underlay cognitive radio networks for different types of interference constraints on the primary users. The novelty of the work is attested by the following publications:


5. K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, 'A Robust Beamforming Based Interference Control Technique and its Per-


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Finally, I am at loss to find suitable words that express my gratefulness for my parents, my sister, Ahalya and my brother, Vaheesan for their continuous support, encouragement throughout my PhD. Special thanks also go to my uncles Kathirgamanathan and Thasam for their support.
### List of Acronyms

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<th>Description</th>
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<tr>
<td>AOA</td>
<td>Angle of Arrival</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BnB</td>
<td>Branch and Bound</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>CMA</td>
<td>Constant Modulus Algorithm</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>FDD</td>
<td>Frequency-Division-Duplex</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
</tr>
<tr>
<td>LCMV</td>
<td>Linearly Constrained Minimum Variance</td>
</tr>
<tr>
<td>LMS</td>
<td>Least Mean Square</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum Ratio Combiner</td>
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<tr>
<td>MSE</td>
<td>Mean Square Error</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>MUSIC</td>
<td>Multiple Signal Classification</td>
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<tr>
<td>MVDR</td>
<td>Minimum Variance Distortionless Response</td>
</tr>
<tr>
<td>NP</td>
<td>Non-deterministic Polynomial-time</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>QCQP</td>
<td>Quadratic Constrained Quadratic Programming</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>QP</td>
<td>Quadratic Programming</td>
</tr>
<tr>
<td>RLS</td>
<td>Recursive Least Squares</td>
</tr>
<tr>
<td>SDP</td>
<td>Semidefinite Programming</td>
</tr>
<tr>
<td>SDR</td>
<td>Semidefinite Relaxation</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Interference and Noise Ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input Single-Output</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SOCP</td>
<td>Second-Order Cone Programming</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TDD</td>
<td>Time-Division-Duplex</td>
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List of Symbols

\begin{itemize}
  \item $E\{\cdot\}$ Statistical expectation
  \item $\text{Var}\{\cdot\}$ Variance
  \item $(\cdot)^T$ Transpose
  \item $(\cdot)^H$ Hermitian transpose
  \item $(\cdot)^*$ Complex conjugate
  \item $\|\cdot\|_F$ Frobenius norm
  \item $\|\cdot\|_2$ Euclidean norm
  \item $(\cdot)^+$ Pseudo-inverse
  \item $\mathbf{1}_M$ $M \times 1$ vector of ones
  \item $\text{Re}\{\cdot\}$ Real part
  \item $\text{Im}\{\cdot\}$ Imaginary part
  \item $\text{Tr}\{\cdot\}$ Trace operator
  \item $\mathcal{L}\{\cdot\}$ Lagrangian function
\end{itemize}
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Chapter 1

INTRODUCTION

1.1 Introduction

Wireless communications have been growing exponentially in the last decade and has influenced a part of every single communication network. Having a mobile phone has become one of the basic needs of human life globally. There are around two billion mobile users worldwide and this figure is growing consistently every year. In addition, many wireless applications such as smart homes, sensor networks, ad hoc networks, and automated surveillance systems have added to the list requiring wireless access. These applications and large numbers of wireless users have created huge demand for the available wireless resources and opened up new challenges. To satisfy this huge demand with limited resources, new technologies and different approaches need to be introduced in future wireless systems.

The exponential growth of wireless applications along with limited availability of spectrum resources has brought the necessity of efficient spectrum utilization in wireless communications [1-3]. Traditionally, frequency bands are divided into sub-bands and each frequency band is licensed by spectrum regulatory bodies. The continuous growth in wireless services have caused spectrum crisis and saturation in the frequency allocation table. Hence, spectrum shortage has become one of the key issues in spectrum allocation. On the other hand, different spectrum occupancy measurements showed that most of the time the licensed frequency bands are unoccupied highlighting
inefficient utilization of radio spectrum rather than a real shortage of spectrum [1, 4].

The idea of sharing the spectrum bands with licensed users and exploiting spectrum holes motivated the concept of opportunistic spectrum access and paved the way for cognitive radio technology, which was first introduced by Joseph Mitola [5]. This intelligent wireless communication system has tremendous potential to improve spectrum utilization and can be developed on software defined platforms [5, 6]. This cognitive radio technology exploits the spectrum by enabling unlicensed users (also known as secondary users) to access frequency bands allocated to licensed users (referred to as primary users) as long as the coexistence of the secondary users and primary users in a specific frequency band does not harm the transmission quality of the latter. The main functions of a cognitive radio are spectrum sensing and exploitation of available spectrum by adjusting the transmission parameters such as modulation scheme, transmit power and beamforming pattern.

Spectrum utilization efficiency, i.e., how efficiently the licensed spectrum is exploited for communications between secondary users, is one of the key factors that significantly influences the performance of cognitive radio networks [7]. The performance of the detection schemes is mainly affected by channel fading and shadowing. It may be very difficult to differentiate the attenuated primary signal from a white noise spectrum. This spectrum sensing problem has been widely studied and different spectrum sensing schemes have been proposed to improve the detection performance [8–14]. Spectrum detection techniques can be classified based on the type of detection techniques employed at the receiver: energy detection [15], coherent detection [16] and cyclostationary feature detection [17]. Energy detection is optimal when the information on the primary signal is limited. Coherent detection can be efficiently employed when the primary pilot signal is known, whereas a cyclostationary detector has the potential to distinguish
the primary signal energy from the local noise energy.

There are three classes of spectrum sharing arrangements, namely, interweave, overlay, and underlay, which have been strongly supported for the development of cognitive radios [18]. The interweave approach is motivated by the idea of opportunistic communication. Recent spectrum occupancy measurements show that major part of the licensed spectrum is not utilized most of the time and yields spectrum holes as shown in Figure 1.1. In addition, these spectrum holes change with time and geographical location and can be exploited by secondary users communication. In this scheme, cognitive transmitters are required to sense availability of spectrum and transmit signals only when frequency holes are available. This is also known as white space filling.

In the overlay paradigm, the secondary users coexist with primary users

![Interweave scheme spectrum](image)

**Figure 1.1.** Interweave scheme spectrum. Green and red represent the spectrum occupied by the primary users and secondary users respectively.

and use part of the transmit power to assist primary users' communications by relaying the primary users' messages to the primary receiver. The power
allocations for secondary users' communication and to relay primary users' messages ensure that the primary users' signal to noise ratios (SNRs) are kept in the same values by assisting primary users' communications. Hence, there is no loss in primary users' SNR by secondary users spectrum access. To mitigate the interference leakage to primary users, different sophisticated coding techniques can be employed according to the availability of sufficient information for example channel state information (CSI) between secondary users and primary users.

In the underlay scheme, the secondary users access the spectrum occupied by primary users without causing harmful interference to primary users' communications. In this approach, the secondary users ensure that interference leakage to the primary users is below an acceptable level as shown in Figure 1.2. In the interweave paradigm, it has been understood that identifying spectrum holes, specifically in the absence of cooperation between primary and secondary networks is a very challenging task. For example, a secondary transmitter could be in the shadow region of the primary transmitter which will falsely indicate (to the secondary transmitter) availability of spectrum. The secondary transmission based on this false indication may harm the primary receivers. This hidden terminal problem is deemed to be very challenging and a limiting factor for the employment of interweave cognitive radio networks. The overlay cognitive radio network is very interesting in terms of its theoretical advantages, however, there are even more challenges in terms of practical implementation as this requires the secondary transmitter to have prior knowledge of the primary user transmitted signal. Hence, the underlay scheme is more realistic and easy to implement compared to the other schemes, and this is the focus of this thesis.
Figure 1.2. Underlay paradigm spectrum. Green and red represent the spectrum occupied by the primary users and secondary users respectively.

1.2 Thesis Outline

In wireless communications, spatial diversity techniques significantly can enhance the spectrum utilization by employing multiple antennas at the transmitter and the receiver. These techniques developed for traditional wireless networks cannot be directly applied to a cognitive radio network due to the additional interference constraints on the primary users. Hence the work in this thesis mainly focuses on the spatial diversity techniques for underlay cognitive radio networks using convex optimization techniques. Resource allocation is one of the key problems in a cognitive radio network, where a number of secondary users request access to the available limited resources. Therefore, power allocation and admission control techniques also have been researched in this thesis.

The next chapter provides a survey of spatial diversity techniques, such as beamforming used in conventional wireless networks. Initially, statistically optimum receiver beamforming techniques are introduced for a narrowband
beamformer, where the multiple sidelobe canceller, minimum mean square error (MMSE) based beamformer design, signal-to-interference plus noise ratio (SINR) maximization approach and linearly constrained minimum variance (LCMV) are briefly discussed. Following on from this, adaptive beamforming techniques based on some statistical sense are described. Transmit beamforming techniques based on power minimization and worst-case SINR maximization are introduced and merits and demerits are also discussed briefly. Finally, recent work related to the beamforming techniques, power allocation and admission control techniques for cognitive radio network is surveyed.

Chapter 3 outlines the basic concepts of convex optimization theory. This chapter describes the most generic classes of convex problems, namely, linear programming (LP), quadratic programming (QP), quadratic constrained quadratic programming (QCQP), second-order cone programming (SOCP) and semidefinite programming (SDP). In addition to these problems, Lagrangian and dual problems are presented with Karush-Kuhn-Tucker (KKT) conditions, which allow feasible convex problems to be solved numerically and analytically.

Chapter 4 focuses on multiuser spatial multiplexing techniques with constraints on interference leakage to the primary users in cognitive radio networks. A beamforming design based on SDP is provided for underlay cognitive radio networks by setting constraints on the interference leakage to the primary users and SINRs of the secondary users. In addition, it is assumed that perfect CSI is available at the secondary network basestation. However, there are practical difficulties to perfectly estimate the channel between the secondary network basestation and primary users and secondary users as well. In the presence of errors in CSI, the beamformer designed based on the assumption of perfect CSI will violate the primary user interference constraint most of the time. Therefore, robust beamforming techniques are
proposed by incorporating the worst-case possible error in the CSI at the secondary network basestation. Simulation results validate the performance of the robust scheme compared to the non-robust scheme.

The problem discussed in Chapter 4 would not be feasible all the time and it is quite difficult to predict whether the problem with certain SINR targets, interference thresholds and total transmit power is feasible. These infeasibility issues can be avoided by dropping some of the secondary users from the cognitive radio network. Therefore, Chapter 5 provides admission control techniques for underlay cognitive radio networks. Optimal and suboptimal admission control techniques are presented based on the branch and bound (BnB) method together with complexity analysis. Simulation results compare the performance of the optimal and suboptimal schemes.

SINR balancing is another approach to overcome the infeasibility issues seen in Chapter 4. Hence, Chapter 6 focuses on the SINR balancing technique for underlay cognitive radio networks, where the ratio between the achieved SINR and the target SINR is balanced for all secondary users while satisfying the primary users' interference and total transmit power constraints. Optimal and suboptimal algorithms are proposed to balance the SINR ratios with individual secondary user interference and total interference constraints respectively. To validate the optimality of beamformer design and power allocations, the results obtained by these proposed algorithms are compared with SDP based results. Merits and demerits are discussed and future directions are identified.

Conclusions are drawn in Chapter 7. A brief summary is also provided and potential areas of future directions are identified.
Chapter 2

SPATIAL DIVERSITY TECHNIQUES

2.1 Introduction

In the past few decades, applications based on wireless communications have experienced an exponential growth and have become part of everyday life in this world. This explosive growth has opened up new challenges in system design. One of the major challenges for system design is the limited availability of frequency spectrum. In wireless communications, various techniques have been employed to improve spectrum utilization. Channel reuse is a common strategy to increase the capacity of a wireless system by reusing the same channel (frequency) in a different cell that is located apart by a certain distance. However, this causes cochannel interference which could degrade the quality of received signals. One of the promising approaches to mitigate cochannel interference is array processing. Antenna array processing techniques such as beamforming techniques can be employed to transmit or receive multiple signals which have the potential to enhance spectrum utilization while mitigating the cochannel interference. This spatial processing technique provides extra degrees of freedom to mitigate interference when compared to time only processing [19, 20]. Interference can also be efficiently controlled by employing appropriate resource allocations techniques.
such as power allocation and admission control techniques. In this chapter, beamforming techniques for general wireless communication systems are introduced, followed by beamforming techniques for an underlay cognitive radio network. Resource allocation techniques for a cognitive radio network, namely power allocation and admission control techniques are also explained.

2.2 Beamforming Techniques

Beamforming is a general signal processing technique used in the physical layer of a communication to control the directionality of transmission or reception of a signal using an array of antennas [21]. In the receiver beamforming design, the objective is to estimate the desired signal in the presence of noise and interference. A narrowband beamformer is depicted in Figure 2.1. The beamformer output can be written as

\[
y(n) = w^H r(n),
\]

(2.2.1)
where \(n\) is the time index, \(\mathbf{r}(n) = [r_1(n)\; r_2(n)\; \cdots\; r_M(n)]^T\) is the \(M \times 1\) vector of array observations and \(\mathbf{w} = [w_1\; w_2\; \cdots\; w_K]^T\) is the complex beamforming weight vector. The array observation vector is given by

\[
\mathbf{r}(n) = \mathbf{d}(n) + \mathbf{i}(n) + \boldsymbol{\eta}(n),
\]

(2.2.2)

where \(\mathbf{d}(n)\), \(\mathbf{i}(n)\) and \(\boldsymbol{\eta}(n)\) are the desired signal, interference and receiver noise respectively. If the desired signal is a far field point source and has a time-invariant wavefront, then it can be written as

\[
\mathbf{d}(n) = s(n)\mathbf{a}_s(\theta),
\]

(2.2.3)

where \(s(n)\) is the source signal and \(\mathbf{a}_s(\theta)\) is an \(M \times 1\) steering vector in the direction of \(\theta\) or the channel response between the transmitter and the array of antennas.

The beamforming design can be classified into two categories, namely, data independent beamformer, and statistically optimum beamformer [21]. In data independent beamformer design, the beamforming weight vector is obtained independent of array observations to present a specified response for all signal and interference scenarios. The beamforming weight vector in statistically optimum design is chosen based on the statistics of the array observations to optimize the array response. In the next subsections, a number of different statistically optimum beamforming techniques are provided in detail.

### 2.2.1 Multiple Sidelobe Canceller

The multiple sidelobe canceller is the earliest statistically optimum beamformer in the context of the beamforming literature. This beamformer consists of a main channel and a number of auxiliary channels as illustrated in Figure 2.2. The main channel is chosen a high directional response in the
desired signal direction. It is assumed that interfering signals enter through the sidelopes of the main channel. The auxiliary channels also receive the interfering signals. The aim is to determine the auxiliary channel weights to remove the main channel interference signals. These weights are usually determined by minimizing the expected value of the total output power as follows:

$$\text{minimize } \mathbf{w}_a \ E \left\{ |y_m(n) - \mathbf{w}_a^H x_a(n)|^2 \right\},$$

where $y_m(n)$ and $x_a(n)$ are the main channel output and auxiliary channels output respectively. The vector $\mathbf{w}_a$ contains the auxiliary channel weights. The optimum weight vector is given by

$$\mathbf{w}_a = \mathbf{R}_a^{-1} \mathbf{r}_{ma},$$

where $\mathbf{R}_a = E \{ x_a(n)x_a(n)^H \}$ and $\mathbf{r}_{ma} = E \{ y_m(n)x_a(n) \}$. In this beamformer, since the total output power includes the desired signal power and is minimized, it might remove the desired signal. Hence this beamformer will be more effective in the scenarios where the desired signal is very weak relative to the interference signals. The weights could be determined when the
desired signal is absent and can be used when it is present. The advantage of this beamformer is simplicity but it requires the absence of the desired signal to obtain the weights.

### 2.2.2 Minimum Mean Square Error Based Beamformer

If the desired signal is known at the receiver, then the beamforming weights could be determined by minimizing the mean square error (MSE) between the beamformer output and the desired signal. The knowledge of the desired signal at the receiver eliminates the need for beamforming. However, in some applications where the statistics of the desired signal may be known or the knowledge of the desired signal would be enough to generate a reference signal. Hence, the beamforming weights are chosen to minimize the mean square error between the beamformer output and the reference signal as follows:

\[
\min_w E \{ |y(n) - y_d(n)|^2 \} = \min_w E \{ |w^H r(n) - y_d(n)|^2 \},
\]

(2.2.6)

where \( y_d(n) \) is the reference signal. The optimum beamforming weights can be determined by

\[
w = R_r^{-1} r_{x_d},
\]

(2.2.7)

where \( R_r = E \{ r(n)r(n)^H \} \) and \( r_{x_d} = E \{ r(n)y_d(n) \} \).

### 2.2.3 Maximization of SINR

The optimal beamforming weights can be obtained by means of maximizing the SINR

\[
\max_w \frac{w^H R_d w}{w^H R_{i+n} w},
\]

(2.2.8)

where \( R_d = E \{ d(n)d(n)^H \} \) and \( R_{i+n} = E \{ [l(n) + \eta(n)][l(n) + \eta(n)]^H \} \) are the signal and interference-plus-noise covariance matrices. The optimum
MSE based beamforming weight is given by

$$\mathbf{R}_{i+n}^{-1} \mathbf{R}_d \mathbf{w} = \lambda_{\text{max}} \mathbf{w}, \quad (2.2.9)$$

where $\lambda_{\text{max}}$ is the maximum eigenvalue of the matrix $\mathbf{R}_{i+n}^{-1} \mathbf{R}_d$.

**Proof:** See the Appendix.

In the case of a far-field point signal source, $\mathbf{R}_d$ can be written as

$$\mathbf{R}_d = \sigma_n^2 \mathbf{a}_s(\theta) \mathbf{a}_s(\theta)^H, \quad (2.2.10)$$

where $\mathbf{R}_d$ is a rank-one matrix. The SINR can be expressed as

$$\text{SINR} = \frac{\sigma_n^2 |\mathbf{w}^H \mathbf{a}_s(\theta)|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}. \quad (2.2.11)$$

In this case the optimum weights can be provided by

$$\mathbf{w} = \alpha \mathbf{R}_{i+n}^{-1} \mathbf{a}_s(\theta), \quad (2.2.12)$$

where $\alpha$ is non-zero complex constant. When there are only noise components available in the system, the beamforming weight vector could be modified as

$$\mathbf{w} = \alpha \mathbf{R}_n^{-1} \mathbf{a}_s(\theta), \quad (2.2.13)$$

where $\mathbf{R}_n = \mathbb{E}\{\eta(n)\eta(n)^H\}$. If the noise components are spatially uncorrelated, then $\mathbf{R}_n^{-1}$ will be a diagonal matrix. In this special case, the beamformer is called the maximum ratio combiner (MRC), where the receiver combines the output of each antenna as follows:

$$y(n) = \sum_{m=1}^{M} \frac{a_s(m)}{\sigma_m^2} x_m(n), \quad (2.2.14)$$
where $\sigma_n^2$ is the noise variance at the $m$th antenna and $a_s(m)$ is the $m$th element of $a_s(\theta)$. Moreover, in the case of equal noise variance with unity power, the beamforming weight vector can be expressed as

$$w = \alpha a_s(\theta).$$

Since the beamforming weight vector matches to the array response vector, it is known as matched filtering in the signal detection context.

### 2.2.4 Linearly Constrained Minimum Variance Beamformer

This beamformer design requires some knowledge of the desired signal. In certain applications, it may not be possible to collect the required information of the desired signal. The desired signal may be of unknown strength and might always be present, resulting in signal cancellation with the multiple sideloop canceller and preventing estimation of signal and noise plus interference covariance matrices in the SINR maximization approach. However, these limitations can be solved by setting a linear constraint on the beamformer design. The basic idea behind the LCMV approach is to limit the response of the beamformer such that the signal from the direction of interest is received with a specified gain and phase $g$.

The beamforming weights are determined to minimize the output power subject to this linear constraint on the beamformer weight vector. This has the effect of preserving the desired signal while minimizing the noise and interference signals that arrive from other directions than the direction of interest. The output power of the beamformer can be written as

$$E\{|y(n)|^2\} = w^H R_w w,$$  

(2.2.16)
where \( R_r = E \{ r(n)r(n)^H \} \). The optimization problem for the LCMV beamformer design can be stated as

\[
\begin{align*}
\text{minimize} & \quad w^H R_r w \\
\text{subject to} & \quad w^H a_s(\theta) = g,
\end{align*}
\tag{2.2.17}
\]

where \( a_s(\theta) \) and \( g \) are the array response vector for an angle of arrival (AOA) \( \theta \) and the required gain and phase in the direction of interest \( \theta \). The optimum beamforming weight vector is given by

\[
w = \frac{gR_r^{-1}a_s(\theta)}{a_s^H(\theta)R_r^{-1}a_s(\theta)}. \tag{2.2.18}
\]

When \( g = 1 \), it is called the minimum variance distortionless response (MVDR) beamformer. It is also known as the Capon receiver [22].

### 2.3 Adaptive Beamforming Techniques

The beamforming techniques discussed so far require knowledge of second order statistics. These statistics are usually unknown or may change over time. To circumvent these problems, adaptive algorithms can be used to determine the beamforming coefficients by optimizing certain criteria such as MSE and constant modulus dispersion (blind techniques) [23–28].

#### 2.3.1 Least Mean Square Adaptive Beamforming

In least mean square (LMS) adaptive beamforming [29], the weights are updated by minimizing the square of the instantaneous error which is the difference between the desired signal and the signal estimated from array of antennas as shown in Figure 2.3. The error signal is written as

\[
e(n) = d(n) - w^H(n)r(n), \tag{2.3.1}
\]
where \(d(n)\) is the desired signal, \(w(n) = [w_1(n) \cdots w_M(n)]^T\) and \(r(n) = [r_1(n) \cdots r_M(n)]^T\) and \(r_k(n)\) is the input signal to the \(k\)th antenna in the array. The cost function is defined as the square of the instantaneous error,

\[
J = |e(n)|^2
\]  

(2.3.2)

which defines a quadratic error surface. The Hessian of the quadratic surface is the covariance matrix of the noisy data given as

\[
R_r = E \{r(n)r(n)^H\}.
\]  

(2.3.3)

This covariance matrix \(R_r\) is a positive definite matrix and defines a convex error surface. The shape of the error surface depends on the eigenvalues of the covariance matrix. The weights are updated by the following adaptive equation

\[
w(n + 1) = w(n) - \mu \frac{\partial J}{\partial w^*} \bigg|_{w^* = w(n)}
\]  

(2.3.4)
where $\mu$ is a positive step size chosen small enough to satisfy the convergence condition $0 \leq \mu \leq 2/\lambda_{\text{max}}(R_r)$, where $\lambda_{\text{max}}(R_r)$ denotes the largest eigenvalue of the covariance matrix $R_r$. The beamforming weight vector is updated by using the following equation:

$$w(n + 1) = w(n) + \mu e(n)^* r(n)$$  \hspace{1cm} (2.3.5)

The main advantage of the LMS algorithm is its simplicity. However, its convergence characteristics depend on the shape of the error surface. When the eigenvalues of the covariance matrix are widely spread, the LMS algorithm shows poor convergence.

### 2.3.2 Recursive Least Squares Adaptive Beamforming

Due to the slow convergence of the LMS algorithm, the exponentially weighted recursive least squares (RLS) algorithm has been proposed in [25]. The cost function is defined as

$$J(n) = \sum_{i=1}^{n} \nu^{(n-i)} |e(i)|^2,$$  \hspace{1cm} (2.3.6)

where $\nu$ is the exponential weighting factor or forgetting factor and is chosen close to one but less than one. The memory of the algorithm is approximately measured by $1/(1 - \nu)$. The weights are updated by the following equations

$$w(n) = w(n - 1) + k(n) \zeta(n),$$  \hspace{1cm} (2.3.7)

where $\zeta(n)$ is a priori estimation error written as

$$\zeta(n) = d(n) - r^H(n)w(n - 1),$$  \hspace{1cm} (2.3.8)

$$k(n) = \frac{\nu^{-1} P(n - 1) r(n)}{1 + \nu^{-1} r^H(n) P(n - 1) r(n)}.$$  \hspace{1cm} (2.3.9)
and
\[
P(n) = \nu^{-1} P(n-1) - \frac{\nu^{-2} P(n-1)r(n)r^H(n)P(n-1)}{1 + \nu^{-1} r^H(n)P(n)r(n)}.
\] (2.3.10)

The weights are initialized with zeros and \( P(0) \) is chosen as \( \delta^{-1}I \) with small \( \delta \) value. \( I \) represents an identity matrix. Since the RLS algorithm is independent of the eigenvalue spread of the covariance matrix \( R_r \), it converges faster than the LMS algorithm.

### 2.3.3 Constant Modulus Algorithm Based Adaptive Beamforming

The constant modulus algorithm (CMA) based beamformer has the ability to recover the signal without requiring a pilot signal or training signal. This algorithm exploits the constant modulus property of the signal of interest to steer the beam in the direction of a signal source while cancelling the interference from other directions. Since it does not require any knowledge of the signal of interest, it is called a blind beamforming technique. This is also known as a constant modulus array [26-28]. The weights of this beamformer are adapted based on the minimization of the following non-convex cost function proposed by Godard [30] as

\[
\tilde{j} = E \left[ \left( |y(n)|^p - R_p \right)^2 \right],
\] (2.3.11)

where \( p \) is a non-zero positive constant, \( y(n) \) is the output of the beamformer and

\[
R_p = \frac{E \left\{ |s(n)|^{2p} \right\}}{E \left\{ |s(n)|^p \right\}},
\] (2.3.12)
where $s(n)$ is the transmitted signal. The weights are modified by the following equations at each time instant,

$$w(n + 1) = w(n) - \mu \frac{\partial J}{\partial w(n)^*}$$  \hfill (2.3.13)

$$= w(n) + \mu x(n)\tilde{e}^*(n),$$  \hfill (2.3.14)

and

$$\tilde{e}(n) = y(n)|y(n)|^{p-2}(R_p - |y(n)|^p)$$  \hfill (2.3.15)

When $p = 2$ in the cost function in (2.3.11), the weights are updated with,

$$\tilde{e}(n) = y(n) \left[ R_2 - |y(n)|^2 \right].$$  \hfill (2.3.16)

The algorithm in the special case with $p = 2$ is referred as CMA in the literature [31].

### 2.4 Transmitter Beamforming Techniques

Beamforming at the transmitter (downlink beamformer) is substantially different in several aspects from using a beamformer at the receiver. In the latter, the design will only determine the performance of a specific user whereas the transmit beamformer will affect not only the desired user but also all the users in the coverage area. Hence the downlink beamforming design should ideally take into consideration the system level performance, i.e. all the users in the reception area rather than a specific user. Another fundamental difference is the channel knowledge. For receiver beamformer design, the receiver could estimate the channel coefficients using the training signal, whereas, for blind beamformers, the receiver determines the beamforming weight vectors without training signal by exploiting certain property of the desired signal as discussed in the previous section. For transmitter beamformer design, the channel knowledge could be made available to the
transmitter by sending the estimates of the CSI from the receiver through a finite rate feedback channel [32–35].

The focus of this section is on multiuser downlink beamformers. The transmitter beamformers can be designed to satisfy quality of service (QoS) requirements for each user. The target SINR for each user is considered as QoS requirement in this section. Consider a wireless network basestation equipped with $N_t$ transmit antennas serving $K$ users. Each user is equipped with a single antenna. The signal transmitted by the basestation is given by

$$x(n) = \mathbf{W}_k s(n), \quad (2.4.1)$$

where $s(n) = [s_1(n) \cdots s_K(n)]^T$, $s_k(n)$ ($k = 1, 2, \ldots, K$) is the symbol intended for the $k$th user, $\mathbf{W}_k = [w_1 \cdots w_K]$ and $w_k \in \mathbb{C}^{N_t \times 1}$ is the transmit beamforming weight vector for the $k$th user. The received signal at the $k$th user can be written as

$$y_k(n) = h_k^H x(n) + \eta_k(n), \quad (2.4.2)$$

where $h_k$ is the channel coefficient vector between the basestation and the $k$th user and $\eta_k(n)$ is receiver noise. The downlink beamforming problem based on SINR requirements can be formulated as minimization of transmitted power at the basestation subject to each user SINR being greater than a target value [36,37].

$$\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \|w_i\|_2^2 \\
\text{subject to} & \quad \frac{\|w_i^H h_i\|_2^2}{\sum_{k=1, k \neq i}^{K} \|w_k^H h_i\|_2^2 + \sigma_i^2} \geq \gamma_i \quad i = 1, \ldots, K, \quad (2.4.3)
\end{align*}$$

where $\sigma_i^2$ is the noise variance at the $i$th receiver. The problem in (2.4.3) is a quadratically constrained non-convex problem. Nevertheless, this prob-
lem can be converted into a SDP with Lagrangian relaxation and can be efficiently solved using convex optimization toolboxes [38-40]. However, it is quite difficult to predict in advance whether the problem in (2.4.3) with a given set of target SINRs and total transmit power at the basestation is feasible.

To overcome this infeasibility issue, this problem can be formulated into a more attractive framework based on a max-min fairness approach where the worst-case user SINR is maximized while using the available total transmit power [41]. This is known as the SINR balancing technique and can be stated as [41-44]

\[
\begin{align*}
\max_{\mathbf{U}, \mathbf{p}} & \quad \min_{1 \leq i \leq K} \frac{\text{SINR}_i(\mathbf{U}, \mathbf{p})}{\gamma_i} \\
\text{subject to} & \quad \mathbf{1}^T \mathbf{p} \leq P_{\text{max}},
\end{align*}
\]

(2.4.4)

where \( \mathbf{U} = [\mathbf{u}_1 \cdots \mathbf{u}_K] \), \( \| \mathbf{u}_k \|_2 = 1 \), and \( \mathbf{p} = [p_1 \cdots p_K]^T \). Here \( \mathbf{u}_k \in \mathbb{C}^{N_r \times 1} \) and \( p_k \) are the transmit beamforming weight vector and the corresponding allocated power for the \( k^{th} \) user respectively. In [41], an iterative algorithm has been proposed using uplink-downlink duality, where the solution balances the ratio between the achieved SINR and the target SINR for all users while using all the transmit power available at the basestation.

### 2.4.1 Zero Forcing Beamforming Technique

The zero forcing technique is a simple method to design the spatial multiplexing weight vector at the transmitter when the number of users to be served is less than or equal to number of transmitter antennas with independent channels between users [45]. This method decouples the multiuser channels into multiple independent sub-channels and reformulates the problem into a simple power allocation problem. This is also known as block
diagonalization [46–51]. The received signal at the $k^{th}$ user is

$$y_k(n) = h_k^H x(n) + \eta_k(n),$$

(2.4.5)

where $h_k = [h_1^{(k)} \cdots h_{N_t}^{(k)}]^H$ is the channel coefficient vector between the basestation and the $k^{th}$ user. The $\eta_k(n)$ term is the receiver noise and

$$x(n) = W s(n),$$

(2.4.6)

where $W = [w_1 \cdots w_K]$, $w_k \in \mathbb{C}^{N_t \times 1}$ is the beamforming weight vector for the $k^{th}$ user, $s(n) = [s_1(n) \cdots s_K(n)]^T$ and $s_k(n)$ is the symbol intended for the $k^{th}$ user. The received signals for all users can be formulated into the following matrix equation:

$$y(n) = H x(n) + \eta(n),$$

(2.4.7)

where $H = [h_1 \cdots h_K]^H$ and it is assumed that $H$ is full row-rank, $y(n) = [y_1(n) \cdots y_K(n)]^T$ and $\eta(n) = [\eta_1(n) \cdots \eta_K(n)]^T$. The SINR of the $k^{th}$ user can be defined as

$$\text{SINR}_k = \frac{[HW]_{k,k}^2}{\sum_{j \neq k} [HW]_{k,j}^2 + \sigma_k^2},$$

(2.4.8)

where $\sigma_k^2$ is the noise variance at the $k^{th}$ user and it is assumed $\mathbb{E}\{s(n) s^H(n)\} = I$. Setting the target SINR values at each user, this problem can be solved using the SDP approach. However, the zero forcing beamforming solution provides a promising trade off between complexity and performance. Here $W$ is designed such that the interference between the users is zero, i.e., $[HW]_{k,j} = 0$ when $k \neq j$. In addition, without loss of generality, it is assumed that $[HW]_{k,k} > 0$ for $k = 1, \cdots, K$. Equivalently, these conditions
can be stated as
\[ [\mathbf{HW}] = \text{diag}\{\sqrt{\gamma}\}, \]  
(2.4.9)

where \( \sqrt{\gamma} = [\sqrt{\gamma_1} \cdots \sqrt{\gamma_K}]^T \) and \( \gamma_k \) is the target SINR of the \( k^{th} \) user. These restrictions yield a simple beamforming design and decouple the multiuser channel into \( K \) independent sub-channels as
\[ y_k(n) = \sqrt{q_k} s_k(n) + \eta_k(n), \]  
(2.4.10)

where \( [\mathbf{HW}]_{k,k} = \sqrt{q_k} \). The optimal solution can be determined by
\[ \mathbf{W} = \mathbf{H}^+ \text{diag}\{\sqrt{\gamma}\}, \]  
(2.4.11)

where \( \mathbf{H}^+ \) is the pseudo inverse of \( \mathbf{H} \). For example, maximizing throughput for a given transmit power can be simply formulated into a concave problem with zero forcing beamforming as follows:
\[
\begin{align*}
\text{maximize} & \quad \sum_k \log(1 + q_k) \\
\text{subject to} & \quad \sum_k q_k \left[ (\mathbf{HH}^H)^{-1} \right]_{k,k} \leq P_{\text{max}},
\end{align*}
\]  
(2.4.12)

where \( P_{\text{max}} \) is the available transmit power.

### 2.5 Joint Transmitter-Receiver Beamforming Design

So far, either receiver beamforming or transmit beamforming has been discussed. When multiple antennas are available at the transmitter and the receiver, the problem of joint transmitter and receiver beamforming arises [52-54]. A communication system with multiple antennas at the transmitter and the receiver is referred to as a multiple-input multiple-output (MIMO) system. The multiple antennas can be used to either increase the data rate and/or the diversity performance. Multiplexing can be obtained by decom-
posing the MIMO channel matrix into various independent sub-channels that are used to transmit different data streams independently. This has the potential to increase the data rate up to a factor that is the same as the rank of the MIMO matrix as compared to the single antenna system [55]. Consider a narrowband point-to-point MIMO channel with $M_t$ transmit antennas and $M_r$ receive antennas as shown in Figure 2.4. The received signal is given by

$$y(n) = Hx(n) + \eta(n),$$

(2.5.1)

where $y = [y_1(n) \cdots y_{M_r}(n)]^T$ and $y_r(n)$ is the received signal at the $r^{th}$ receiver antenna. $H \in \mathbb{C}^{M_r \times M_t}$ and $h_{ij}$ is the channel gain between the $i^{th}$ transmitter antenna and $j^{th}$ receiver antenna. $x(n) \in \mathbb{C}^{M_t \times 1}$ and $\eta(n) \in \mathbb{C}^{M_r \times 1}$ are the transmitted symbol vector and the noise vector at the receiver end respectively. It is assumed that the channel gain matrix $H$ is known to both the transmitter and the receiver. The MIMO channel matrix $H$ can be
decomposed using the singular value decomposition (SVD) as [56]

$$H = \tilde{U} \Sigma \tilde{V}^H,$$

(2.5.2)

where $\tilde{U} \in \mathbb{C}^{M_r \times M_r}$ and $\tilde{V} \in \mathbb{C}^{M_t \times M_t}$ are unitary matrices. $\Sigma \in \mathbb{R}^{M_r \times M_t}$ is a diagonal matrix with the singular values $(\nu_i)$ of $H$. $R_H$ number of singular values are nonzero, so that $R_H$ is the rank of matrix $H$. The singular value satisfies the property $\nu_i = \sqrt{\lambda_i}$, where $\lambda_i$ is the $i^{th}$ eigenvalue of $HH^H$. These MIMO sub-channels are obtained using linear transformation of the input signal and the output signal through transmit precoding and receiver shaping. In transmit precoding, the modulated symbol stream is precoded as

$$x = \tilde{V} \tilde{x},$$

(2.5.3)

where $\tilde{x}$ is the modulated symbol stream. Similarly, the received signal is shaped as

$$\tilde{y} = \tilde{U}^H y$$

(2.5.4)

as shown in Figure 2.5. Such transmit precoding and receiver shaping de-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.5.png}
\caption{Transmit precoding and receiver shaping}
\end{figure}
single-output (SISO) channels as follows:

$$\tilde{y} = \tilde{U}^H (Hx + \eta)$$
$$= \tilde{U}^H \tilde{U} \Sigma \tilde{V}^H \tilde{V} x + \tilde{U}^H \eta$$
$$= \Sigma \tilde{x} + \tilde{\eta} \quad (2.5.5)$$

where $\tilde{\eta} = \tilde{U}^H \eta$. The resulting parallel sub-channels are shown in Figure 2.6. They are independent from each other in the sense that signals through each sub-channel do not interfere with each other. Hence this MIMO channel can support up to $R_H$ times the data rate of a SISO channel. The performance of each channel depends on its gain $v_i$. Here, the transmit precoding and receiver shaping matrices can be considered as transmit and receiver beamformers. The channel capacity of this system, provided that the MIMO channel matrix is known to the transmitter and the receiver, is equal to the sum of capacities of each independent parallel channels

$$C = \max_{\mu: \sum_{i=1}^{R_H} \mu_i \leq P} \sum_{i=1}^{R_H} B \log_2 \left( 1 + \frac{v_i^2 P_i}{\sigma_i^2} \right), \quad (2.5.6)$$
where $P$ and $P_i$ are the total transmit power and power allocated to the $i^{th}$ independent channel respectively. $B$, $v_i$ and $\sigma_n^2$ are the bandwidth, $i^{th}$ independent channel gain and the noise power at the receiver respectively.

The power allocation problem can be formulated into a convex optimization framework as

$$\text{maximize} \quad \sum_{i=1}^{R_H} B \log_2 \left( 1 + \frac{v_i^2 p_i}{\sigma_n^2} \right)$$

subject to

$$\begin{align*}
1^T p &= P, \\
p_i &\geq 0,
\end{align*}$$

(2.5.7)

where $1 \in \mathbb{R}^{R_H \times 1}$ is a vector with all elements equal to one and $p = [p_1 \cdots p_{R_H}]^T$. From KKT conditions, the following are obtained [57].

$$\lambda_i \geq 0 \quad \forall \ i,$$

(2.5.8)

$$\lambda_i p_i = 0 \quad \forall \ i,$$

(2.5.9)

$$\left( \frac{B}{1 + \frac{v_i^2 p_i}{\sigma_n^2}} \right) v_i^2 + \lambda_i = \mu \quad \forall \ i,$$

(2.5.10)

where $\lambda_i$ \footnote{Please note that the same notation has been used for eigenvalues earlier} and $\mu^2$ are Lagrangian variables. From complementary slackness as in (2.5.9) and (2.5.10), the power allocations are obtained as

$$p_i = \begin{cases} 
\frac{B}{\mu} - \frac{\sigma_n^2}{v_i^2}, & \lambda_i = 0; \\
0, & \lambda_i \neq 0.
\end{cases}$$

(2.5.11)

The optimal value of $\mu$ is given by

$$1^T p = \sum_{i=1}^{R_H} \max \left\{ 0, \frac{B}{\mu} - \frac{\sigma_n^2}{v_i^2} \right\} = P.$$  

(2.5.12)
This is called the water-filling solution where the water level is equal to $\frac{B}{\mu^*}$ as shown in Figure 2.7. The parameter $\mu^*$ is the optimal value of $\mu$.

![Figure 2.7. Water filling power allocation](image)

### 2.6 Resource Allocation Techniques

The power allocation and beamforming problems for multiple users have been widely studied to control interference between users in [36, 37, 58, 59]. In [60], an optimal downlink power assignment technique has been proposed for a given set of beamforming weight vectors. This power allocation problem is formulated into an eigenvector matrix equation and the optimal power allocations have been obtained by finding the eigenvector corresponding the largest eigenvalue of the matrix. The property that all elements of the eigenvector corresponding to the largest eigenvalue of a non-negative matrix are always positive [61], has been exploited in [60]. In [58], an iterative algorithm has been proposed to jointly design the beamforming weight vectors and the power allocation vectors in the uplink and the downlink. This design ensures that SINR of each user is above a threshold while minimizing the total transmitted power. The same problem has been formulated into a semidefinite programming in [36, 37] by using Lagrangian relaxation and it
has been solved using interior point methods [62]. This relaxed problem provides a rank-one solution for each user and the optimal beamforming weight vector has been determined by extracting the eigenvector corresponding to the positive eigenvalue of the matrix. In addition it has been proved that the relaxed problem always yields an optimal rank-one solution. In [59], a special scenario has been considered where the transmitter sends the same data to multiple users known as multicasting. In the multicasting setup, the SDP formulation might not always provide a rank-one solution. To overcome this problem, a randomization technique [63] has been recommended to find an optimal solution. The problem of transmit beamforming to multiple cochannel multicast groups is considered in [64,65] with perfect CSI at the transmitter, where QoS and the max-min fairness approach have been presented using convex optimization and randomization techniques. For the same problem, a robust approach has been discussed for a network with imperfect CSI at the transmitter [66]. Robust adaptive beamforming techniques have been presented for imperfect CSI [67–73]. In [67,74], a robust approach based on worst-case performance optimization has been discussed, where the original non-convex problem was formulated into a convex optimization framework known as SOCP and solved efficiently using interior point methods. In [73], robust downlink beamforming using worst-case performance optimization and positive semidefinite covariance constraints has been presented. In [68], outage probability based robust beamforming techniques have been proposed for the downlink. The relationship between probability constrained and worst-case performance based robust beamforming for minimum variance beamformers has been presented in [69], where it is shown that both in the cases of circularly symmetric Gaussian and worst-case distributions of the steering vector mismatch, the resulting problems are mathematically equivalent.

The approaches developed in [58] and [36] use the criterion of minimizing
the total transmit power subject to SINR constraints for each user. However, the resulting problem might become infeasible due to high SINR constraints or insufficient total transmitter power. To avoid the problem of infeasibility, an SINR balancing technique has been proposed in [41]. Here an iterative algorithm has been developed to maximize the worst-case user SINR, where the beamforming weight vectors are designed in the virtual uplink mode have been employed in the downlink using the principle of uplink-downlink duality [42, 43, 75, 76]. The solution to this iterative algorithm provides a balanced ratio of the individual achieved SINR and the target SINR value for all users.

The beamforming and the power allocation problems for cognitive radio networks, however, substantially differ from that of traditional wireless networks due to additional interference constraints imposed by primary users. In [77], spatial diversity has been exploited in the downlink to improve the throughput of the secondary user, while imposing constraints on the secondary user transmit power and the primary user interference power. A beamforming approach has been proposed to maximize the ratio between the received secondary user signal power and the interference power leakage to the primary users in [78]. In [79], joint beamforming and power allocation techniques have been provided for an uplink cognitive radio network. A multi-level water filling algorithm and a recursive decoupled power allocation algorithm have been presented to maximize the sum-rate of the secondary users. A multicast beamforming technique based on convex optimization has been presented for a QoS aware spectrum sharing underlay cognitive radio network in [80]. In [81], two joint power control and beamforming algorithms have been proposed based on a weighted least squares approach and admission control technique for an underlay cognitive radio network. In [82-84], robust cognitive radio network beamformers have been designed that meet specific target secondary user SINRs in the presence of CSI errors. In [85],
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a robust beamforming technique has been developed to limit the probability of the interference leakage to the primary users.

The beamforming problem formulation based on the QoS requirements might become infeasible due to insufficient transmit power and high QoS requirements. To overcome these issues, some of the users can be dropped using appropriate admission control techniques. Admitting optimal number of users is an NP (non-deterministic polynomial-time) hard problem. Therefore suboptimal low complexity algorithms have been proposed [86–90]. In [86], convex approximation techniques have been proposed for joint multiuser downlink beamforming and admission control. This technique maximizes the number of users that can be served with their desired QoSs. The same approximation techniques have been applied for multicast beamforming in [88] and [89]. In [91], a joint beamforming and scheduling algorithm has been proposed for a cognitive radio network with a primary user and multiple secondary users. The proposed algorithm jointly admits a certain number of secondary users and maximizes the throughput while satisfying an interference constraint on the primary user. This algorithm computes the solution in two steps. The first step selects a set of secondary users based on an orthogonality criterion. In the second step, beamforming weight vectors are designed for admitting secondary users using a zero forcing beamforming technique. In [92], an algorithm has been presented to jointly solve admission control and power allocation problem for cognitive radio networks. This NP hard problem is converted into a smooth optimization problem and solved using a gradient descent based algorithm.
2.7 Appendix

2.7.1 Proof: Maximization of SINR

\[
\text{maximize}_w \quad \frac{w^H R_d w}{w^H R_{i+n} w} = \text{maximize}_w \quad \frac{w^H R_d w}{w^H R_{i+n}^{1/2} R_{i+n}^{1/2} w} \quad (2.7.1)
\]

This maximization is equivalent to

\[
\text{maximize}_u \quad u^H R_{i+n}^{-1/2} R_d R_{i+n}^{-1/2} u
\]
subject to \( u^H u = 1 \), \quad (2.7.2)

where \( u = R_{i+n}^{1/2} w \) and the solution of (2.7.2) will be the eigenvector corresponding to the largest eigenvalue of \( R_{i+n}^{-1/2} R_d R_{i+n}^{-1/2} \).

\[
R_{i+n}^{-1/2} R_d R_{i+n}^{-1/2} u = \lambda_{\text{max}} u
\]

\Rightarrow \quad R_{i+n}^{-1} R_d R_{i+n}^{-1/2} u = \lambda_{\text{max}} R_{i+n}^{-1/2} u

\Rightarrow \quad R_{i+n}^{-1} R_d w = \lambda_{\text{max}} w \quad (2.7.3)

This completes the proof. \( \blacksquare \)
The use of optimization methods has become vital in numerous problems in signal processing and communication [57,93,94]. Many problems in communications can be appropriately formulated into a constrained optimization framework. These constrained problems are either naturally convex or can be expressed in convex form after some mathematical manipulation [93–95].

Once it has been formulated into convex form, it can be efficiently solved using interior point methods [62,96]. Convex optimization has influenced most problems of practical interest, since a local optimum is also the global optimum for convex problems and they can be solved with polynomial time complexity. One of the attractive features of convex problems is that they allow verification of the optimality of the solutions using KKT conditions and duality gaps. The widely available software and tool boxes to solve convex problems also make convex optimization more attractive in many engineering applications [38–40]. However, most problems are naturally not in convex form. Therefore, recognizing the problems which can be solved using convex optimization and formulating the problem into a convex form are the major challenges in the application of convex optimization.
3.1 Fundamentals of Convex Optimization

In this section, the fundamentals of convex optimization are introduced briefly.

3.1.1 Convex Sets

A convex set $S \subseteq \mathbb{R}^n$ can be expressed as follows [57]:

$$\theta x + (1 - \theta) y \in S, \quad \forall \theta \in [0, 1] \text{ and } x, y \in S.$$  \hspace{1cm} (3.1.1)

A set can be classified as a convex set if all the points of a line segment, which is formed by connecting two points from the set by a straight line, should be in the same set.

3.1.2 Convex Cones

A set $K$ is said to be a convex cone, if for each $x \in K$ and each $\alpha \geq 0$, $\alpha x \in K$ and $K$ is convex [57]. This can be mathematically expressed as

$$\theta_1 x + \theta_2 y \in K, \quad \forall \theta_1 \geq 0, \theta_2 \geq 0 \text{ and } x, y \in K.$$  \hspace{1cm} (3.1.2)

Convex cones arise in various forms in engineering applications. The most common convex cones are

1. Nonnegative orthant $\mathbb{R}_+^n$

2. Second-order cone (ice cream cone)

$$K = \{(t, x) \mid t \geq \| x \| \}$$

3. Positive semidefinite matrix cone

$$K = S_+^n = \{X \mid X \text{ symmetric and } X \succeq 0\}$$
3.1.3 Convex Functions

A function \( f(x) : \mathbb{R}^n \to \mathbb{R} \) is convex if \( \text{dom } f(x) \) is a convex set and if for all \( x, y \in \text{dom } f(x) \) the following inequality is satisfied [57]:

\[
f(\theta x + (1 - \theta) y) \leq \theta f(x) + (1 - \theta) f(y), \quad \forall \theta \in [0, 1]. \tag{3.1.3}
\]

In other words, along any line segment in \( \text{dom } f(x) \), \( f(x) \) is less than or equal to the value of the linear function agreeing with \( f(x) \) at the end points. The function \( f(x) \) is concave if \(-f(x)\) is convex. If \( f(x) : \mathbb{R}^n \to \mathbb{R} \) is continuously differentiable, the convexity of \( f(x) \) is equivalent to

\[
f(y) \geq f(x) + \nabla f(x)^T(y - x) \quad \forall x, y \in \mathbb{R}^n. \tag{3.1.4}
\]

Moreover, if \( f(x) : \mathbb{R}^n \to \mathbb{R} \) is twice continuously differentiable, then the convexity of \( f(x) \) is equivalent to

\[
\nabla^2 f(x) \succeq 0 \quad \forall x \in \mathbb{R}^n. \tag{3.1.5}
\]

i.e. its Hessian is positive semidefinite on its domain [57]. Thus, for example a linear function is always convex, while a quadratic function \( x^T P x + a^T x + b \) is convex if and only if \( P \succeq 0 \).

3.2 Convex Optimization Problems

A convex optimization problem can be defined in the following standard form:

\[
\begin{align*}
\text{minimize} & \quad f_0(x) \\
\text{subject to} & \quad f_i(x) \leq 0, \quad i = 1, \ldots, m, \\
& \quad h_i(x) = 0, \quad i = 1, \ldots, p,
\end{align*} \tag{3.2.1}
\]
where the vector \( x \in \mathbb{R}^n \) is the optimization variable of the problem, the functions \( f_0, \ldots, f_m \) are convex functions and the functions \( h_1, \ldots, h_p \) are linear functions. The function \( f_0 \) is the objective function or cost function. The inequalities \( f_i(x) \leq 0, \quad i = 1, \ldots, m \) are called the inequality constraints and equalities \( h_i(x) = 0, \quad i = 1, \ldots, p \) are called the equality constraints.

If there are no constraints, then the problem can be classified as an unconstrained problem. The domain of the optimization problem (3.2.1) is the set of points for which the objective and the constraints are defined and is denoted as

\[
D = \bigcap_{i=0}^{m} \text{dom} f_i \cap \bigcap_{i=0}^{p} \text{dom} h_i. \quad (3.2.2)
\]

A point \( x \in D \) is feasible, if it satisfies all the constraints \( f_i(x) \leq 0 \quad i = 1, \ldots, m \) and \( h_i(x) = 0 \quad i = 1, \ldots, p \). Problem (3.2.1) is said to be feasible if there exists a feasible point and is infeasible otherwise. The optimal value or the solution of the optimization problem is achieved at the optimal point \( x^* \) if and only if

\[
f_0(x^*) \leq f_0(x) \quad \forall x \in D. \quad (3.2.3)
\]

### 3.3 Canonical Optimization Problems

In this section, the most general form of the canonical optimization problem formulations are provided.

#### 3.3.1 Linear Programming

A convex optimization problem is called linear programming (LP), when the objective and constraint functions are all affine [57,97]. A general LP can
be defined as follows:

\[
\begin{align*}
\text{minimize} & \quad c^T x + d \\
\text{subject to} & \quad Gx \preceq h, \\
& \quad Ax = b,
\end{align*}
\]

(3.3.1)

where \( G \in \mathbb{R}^{m \times n} \) and \( A \in \mathbb{R}^{p \times n} \).

### 3.3.2 Quadratic Programming

When the objective function is quadratic (convex) and the constraint functions are affine, then the convex optimization problem is called QP. A QP can be expressed as follows:

\[
\begin{align*}
\text{minimize} & \quad x^T P x + q^T x + r \\
\text{subject to} & \quad Gx \preceq h, \\
& \quad Ax = b,
\end{align*}
\]

(3.3.2)

where \( P \in \mathbb{S}^n_+ \), \( G \in \mathbb{R}^{m \times n} \), and \( A \in \mathbb{R}^{p \times n} \). In QP, a convex quadratic function is minimized over a polyhedron. QP includes LP as a special case. This may be obtained by setting \( P = 0 \) in the objective of (3.3.2).

### 3.3.3 Quadratically Constrained Quadratic Programming

The convex optimization problem is called a QCQP, when both objective and constraint functions are quadratic. This has the form

\[
\begin{align*}
\text{minimize} & \quad x^T P_0 x + q_0^T x + r_0 \\
\text{subject to} & \quad x^T P_i x + q_i^T x + r_i \leq 0, \quad i = 1, 2, \ldots, m, \\
& \quad Ax = b,
\end{align*}
\]

(3.3.3)
where $P_i \in S_+^n$, $i = 1, 2, \ldots, m$. In a QCQP, a quadratic convex function is minimized over a feasible region that is the intersection of ellipsoids. In QCQP, by setting $P_i = 0$, $i = 0, 1, 2, \ldots, m$ in the constraints of (3.3.3), an LP can be obtained.

### 3.3.4 Second-Order Cone Programming

An SOCP can be written as \[57, 98\]

$$\begin{align*}
\text{minimize} & \quad f^T x \\
\text{subject to} & \quad \|A_i x + b_i\|_2 \leq c_i^T x + d_i, & i = 1, 2, \ldots, m, \\
& \quad Fx = g,
\end{align*}$$

where $x \in \mathbb{R}^n$ is the optimization variable, $A_i \in \mathbb{R}^{n \times n}$ and $F \in \mathbb{R}^{p \times n}$. The first constraint in (3.3.4) is a second order cone constraint in $\mathbb{R}^{k+1}$. Setting $c_i = 0$, $i = 1, 2, \ldots, m$ and squaring both sides of the constraints, a QCQP will be obtained. Similarly, if $A_i = 0$, $i = 1, 2, \ldots, m$, then the SOCP reduces to a LP. In general, SOCPs are more widely used in convex optimization applications.

### 3.3.5 Semidefinite Programming

Every canonical optimization problem can be considered as a special case of SDP. The most general of all the forms is an SDP. An SDP can be written as \[57, 99\]

$$\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad x_1 F_1 + x_2 F_2 + \ldots + x_n F_n + G \preceq 0, \\
& \quad Ax = b,
\end{align*}$$

(3.3.5)
where $x \in \mathbb{R}^n$ is the optimization variable and $G, F_1, \ldots, F_n \in S^{k \times k}$ are symmetric matrices and $A \in \mathbb{R}^{p \times n}$. The inequality constraints in (3.3.5) are also called linear matrix inequality (LMI). An SDP reduces to an LP if the matrices $G, F_1, \ldots, F_n$ are all diagonal.

### 3.3.6 Geometric Programming

A geometric programming (GP) problem consists of monomial functions and posynomial functions. A monomial function is defined as $f(x) : \mathbb{R}^+ \to \mathbb{R}$

$$f(x) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n},$$

where $c > 0$ and $a_i \in \mathbb{R}$, $i = 1, \ldots, n$. A posynomial is a sum of monomials and can be defined as

$$f(x) = \sum_{k=1}^{K} c_k x_1^{q_{k1}} x_2^{q_{k2}} \cdots x_n^{q_{kn}},$$

where $c_k > 0$. A standard GP can be expressed as follows:

\[
\begin{align*}
\text{minimize} & \quad f_0(x) \\
\text{subject to} & \quad f_i(x) \leq 1, \ i = 1, \ldots, m, \\
& \quad h_i(x) = 1, \ i = 1, \ldots, p,
\end{align*}
\]

where $f_0, \ldots, f_m$ are posynomials and $h_1, \ldots, h_p$ are monomials. The domain of this problem is $D = \mathbb{R}^n_+$. 

### 3.4 Duality and KKT Conditions

The basic idea in Lagrangian duality is to take the constraints in (3.2.1) into account by augmenting the objective function with a weighted sum of the constraint functions. The *Lagrangian* $L : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}$ for the original
problem in (3.2.1) can be defined as [57]

\[ L(x, \lambda, \nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x), \quad (3.4.1) \]

where \( \lambda_i \) and \( \nu_i \) are the Lagrange multipliers associated with the \( i \)th inequality \( f_i(x) \leq 0 \) and equality \( h_i(x) = 0 \) constraints respectively. The objective \( f_0(x) \) in (3.2.1) is called the primal objective and the optimization variable \( x \) is termed the primal variable. Lagrange multipliers \( \lambda \) and \( \nu \) associated with the problem (3.4.1) are called the dual variables. The Lagrange dual objective or the Lagrange dual function \( g(A, \nu) \) is defined as the minimum value of the Lagrangian over \( x \) for \( A \in \mathbb{R}^m \), \( \nu \in \mathbb{R}^p \) [57]

\[ g(A, \nu) = \inf_{x \in D} \left( f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x) \right). \quad (3.4.2) \]

The Lagrange dual function is always concave regardless of whether the original problem is convex or not. This is because the dual function is the pointwise infimum of a family of affine functions of \( (\lambda, \nu) \) [57]. The dual function \( g(\lambda, \nu) \) yields a lower bound on the optimal value \( f_0(x^*) \) of the problem (3.2.1) [57]. For any \( \lambda \geq 0 \) and any \( \nu \),

\[ g(\lambda, \nu) \leq f_0(x^*) \quad (3.4.3) \]

This can be shown for any feasible set \( (x, \lambda, \nu) \) as follows:

\[
\begin{align*}
  f_0(x) &\geq f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x) \\
  &\geq \inf_{x \in D} \left( f_0(z) + \sum_{i=1}^{m} \lambda_i f_i(z) + \sum_{i=1}^{p} \nu_i h_i(z) \right) \\
  &= g(\lambda, \nu) \quad (3.4.4)
\end{align*}
\]

_Duality gap_ is the measure of the difference between the primal objective \( f_0(x) \) and the dual objective \( g(\lambda, \nu) \). When the inequality in (3.4.3) is
satisfied with strict inequality, then it holds a \textit{weak duality}. If the inequality in (3.4.3) is satisfied with equality, it holds \textit{strong duality} between the primal problem and the dual problem. To obtain the best lower bound of the original problem, the following dual problem is solved:

\[
\begin{align*}
\max_{\lambda, \nu} & \quad g(\lambda, \nu) \\
\text{subject to} & \quad \lambda \geq 0.
\end{align*}
\]

(3.4.5)

The \textit{Lagrange dual problem} is always a convex problem, since the objective function (concave function) in (3.4.5) is maximized with convex constraints. This always holds regardless of the nature of the primal problem (3.2.1) [57]. The following conditions are called KKT conditions which provide the facility to validate the optimality of the solutions.

1. Primal constraints: \( f_i(x) \leq 0 \quad i = 1, 2, \ldots, m, \)
   \[ h_i(x) = 0 \quad i = 1, 2, \ldots, p, \]

2. Dual constraints: \( \lambda \succeq 0 \)

3. Complementary slackness \( \lambda _i f_i(x) = 0 \quad i = 1, 2, \ldots, m, \)

4. Gradient of Lagrangian with respect to \( x \) vanishes:

\[
\nabla f_0(x) + \sum_{i=1}^{m} \lambda_i \nabla f_i(x) + \sum_{i=1}^{p} \nu_i \nabla h_i(x) = 0.
\]

(3.4.6)

These KKT conditions are necessary conditions for optimality in general but not sufficient conditions. For convex and non-convex problems, if strong duality holds, then the KKT conditions will be satisfied. But if the KKT conditions are satisfied, it does not mean that strong duality holds. However, for convex optimization problems, if the KKT conditions hold, then the strong duality holds between the primal problem and the dual problem. In addition, primal and dual variables are optimal [57].
3.5 Summary

In this chapter, various convex optimization problems have been discussed briefly. These problems can be effectively solved using interior point methods. The concepts of Lagrange duality and KKT conditions have also been presented. However, in this thesis, the focus will be on SOCP and SDP to solve the optimization problems in cognitive radio networks. More details about these convex optimization problem formulations, applications of convex optimizations, complexity analysis and interior point methods can be found in [57,62,93,100,101] and the references therein.
Employing multiple antennas at the basestation significantly enhances the spectral efficiency in wireless communications. Spatial diversity techniques developed for traditional wireless network can not be used directly in a cognitive radio network due to the additional interference constraints on primary users. In this chapter, initially, robust beamforming techniques are proposed for single user underlay cognitive radio networks by incorporating the errors in the CSI. These robust beamforming techniques are developed using a worst-case optimization approach for different types of error bounds in the CSI. Next, multiuser beamforming techniques are proposed based on an SDP approach. This framework is then extended to robust beamforming techniques using a worst-case performance optimization approach in the presence of uncertainty in CSI.
4.1 Introduction

For a single secondary user underlay cognitive radio network, a transmit beamformer can be designed by maximizing the received power at the secondary user terminal while satisfying the interference constraints on the primary users. On the other hand, for multiuser transmit beamformers, the design should consider minimizing the required total transmit power while achieving a specific set of QoS for multiple secondary users and setting an upper bound on the interference leakage to the primary users. Solutions for downlink beamformer design for underlay cognitive radio networks are proposed based on semidefinite constraints which can be solved using interior point methods [62]. In [77], spatial diversity techniques have been proposed to improve channel capacity between a pair of secondary users while imposing constraints on transmit power and the interference leakage to the primary users. In [79], joint beamforming and power allocation techniques have been presented for an uplink network. Downlink spatial multiplexing techniques require CSI at the basestation. In [77] and [79], the channels between the secondary network basestation and the primary users as well as the secondary users have been assumed to be perfectly known at the secondary network basestation.

However, there are practical difficulties to have perfect CSI at the secondary network basestation. Moreover, in practice it may be hard to estimate the channel between the secondary network basestation and the primary users. Nevertheless, the channel between the secondary network basestation and the secondary users can be obtained using CSI feedback for frequency-division-duplex (FDD) systems. For time-division-duplex (TDD) systems, the estimate of the uplink channel can be used in the downlink using reciprocity. The channels between secondary network basestation and the primary users can be obtained by requesting feedback in the cooperation
between networks. However, in the absence of cooperation, it is important to assume that the primary users use the same frequency bands for the uplink and downlink transmission to their basestation as in TDD. In this case, the channel can be estimated based on the AOA of the signal from the primary users to the secondary network basestation. A detailed discussion on the protocols for obtaining CSI in a cognitive radio environment can be found in [79] and [102]. Any estimation error due to this assumption should be incorporated to satisfy QoS requirements at the secondary users and interference constraints on the primary users.

Therefore, in this chapter, robust beamforming techniques are proposed to satisfy the SINR targets (QoS for secondary users) and interference constraints on the primary users. Here, in addition to randomly generated channels, channels based on AOAs, which can be estimated using, for example, Capon [22] or multiple signal classification (MUSIC) [103] algorithms, are also considered for the simulations. These robust beamforming techniques have been developed based on the worst-case performance optimization approach to satisfy the SINR targets and interference constraints all the time regardless of the errors in the CSI. In the next section, robust beamforming techniques are proposed for single secondary user underlay cognitive radio networks.

### 4.2 Beamforming Design for Single User Cognitive Radio Networks

In this section, robust beamforming techniques are proposed for single user cognitive radio networks using worst-case performance optimization. Different types of error bounds are considered in the CSI, namely, constant error bound and ellipsoid error bound. These problems are also formulated within a convex optimization framework with constraints on worst-case errors. The
performance of the robust beamformer is compared with a non-robust beamformer in terms of bit error rates (BERs) of the primary users and probability density function (PDF) of the interference leakage to the primary users.

### 4.2.1 Problem Statement

A cognitive radio network with \( L \) primary users and a single secondary user is considered in an underlay approach. The secondary network basestation has been assumed to have \( N \) antennas while both the primary and the secondary users have only a single antenna. The received signal at the secondary user can be written as

\[
y_d(n) = w^H h_d s(n) + \eta_d(n),
\]

where \( s(n) \) is the symbol transmitted to the secondary user, \( h_d = [h_1 \ h_2 \ \cdots \ h_N]^T \) represents the complex channel coefficients between the secondary user and the basestation, \( w = [w_1 \ w_2 \ \cdots \ w_N]^T \) is the transmitter beamformer weight vector. It is assumed that \( \eta_k(n) \) is a zero mean circularly symmetric additive white Gaussian noise (AWGN) component with variance \( \sigma^2 \). The received signal power at the secondary user can be written as

\[
P_d = w^H h_d h_d^H w \sigma^2_s,
\]

where \( \sigma^2_s \) is the variance of the transmitted symbols. The transmit power at the secondary network basestation can be written as

\[
P_t = w^H w \sigma^2_s.
\]

The interference leakage to the \( l \)th primary user due to the secondary user transmission can be written as

\[
P_{il} = w^H h_l h_l^H w \sigma^2_s,
\]
where $h_l$ consists of channel coefficients between the secondary network basestation and the $l$th primary user. The beamformer is designed to maximize the received signal power at the secondary user while imposing constraints on the interference leakage to the primary users and available transmit power at the secondary network basestation. The problem can be stated as

$$\max_w \|w^H h_d\|^2$$
subject to $\|w\|^2 \leq \frac{P_m}{\sigma_s^2}$,
$$\|w^H h_l\|^2 \leq \frac{\varepsilon_l}{\sigma_s^2}, \quad l = 1, 2, \ldots, L. \quad (4.2.5)$$

In the above problem, the domain of both constraints is a convex set, and the cost is a non-concave function. Since any arbitrary phase rotation in $w$ does not alter the cost value, this problem can be converted into a SOCP without loss of generality,

$$\max_w \text{Re}\{w^H h_d\}$$
subject to $\text{Im}\{w^H h_d\} = 0$,
$$\|w\|^2 \leq \frac{P_m}{\sigma_s^2},$$
$$\|w^H h_l\|^2 \leq \frac{\varepsilon_l}{\sigma_s^2}, \quad l = 1, 2, \ldots, L. \quad (4.2.6)$$

### 4.2.2 Worst-Case Performance Optimization

Here also, the above beamformer design in (4.2.6) assumes perfect CSI at the secondary network basestation. Therefore robust algorithms are proposed assuming uncertainty in the CSI. The true channel can be written as

$$h_l = \tilde{h} + e,$$  \quad (4.2.7)
where \( \mathbf{h} \) is the CSI available at the secondary network basestation and \( \mathbf{e} \) is a possible error vector. The beamformer should be designed such that the interference leakage to the primary users should be less than the target values all the time regardless of the error in CSI. The original problem can therefore be modified as

\[
\begin{align*}
\text{maximize} & \quad \Re \{ \mathbf{w}^H (\mathbf{h}_d + \mathbf{e}_d) \} \\
\text{subject to} & \quad \Im \{ \mathbf{w}^H (\mathbf{h}_d + \mathbf{e}_d) \} = 0, \\
& \quad \| \mathbf{w} \|^2 \leq \frac{P_m}{\sigma_h^2}, \\
& \quad \| \mathbf{w}^H (\mathbf{h}_l + \mathbf{e}_l) \|^2 \leq \frac{\varepsilon_l}{\sigma_h^2}, \quad l = 1, 2, \ldots, L.
\end{align*}
\]

(4.2.8)

The approach is based on the worst-case error performance optimization [74].

### 4.2.3 Constant Error Bound

Here, it is assumed the norms of error vectors in the CSI are bounded above by known constants as follows:

\[
\| \mathbf{e}_d \| \leq \gamma_d, \quad \| \mathbf{e}_l \| \leq \gamma_l.
\]

(4.2.9)

The true channel coefficients therefore belong to the following convex hull:

\[
\begin{align*}
\mathcal{A}_d(\gamma_d) & \triangleq \{ \mathbf{h}_{td} \mid \mathbf{h}_{td} = \mathbf{h}_d + \mathbf{e}_d, \| \mathbf{e}_d \| \leq \gamma_d \}, \\
\mathcal{A}_l(\gamma_l) & \triangleq \{ \mathbf{h}_{tl} \mid \mathbf{h}_{tl} = \mathbf{h}_l + \mathbf{e}_l, \| \mathbf{e}_l \| \leq \gamma_l \}.
\end{align*}
\]

(4.2.10)

In the worst-case performance optimization, the minimum value of the cost function over all possible errors should be maximized using a set of constraints on the worst possible errors. The worst-case beamformer for single
user underlay cognitive radio is obtained as

\[
\begin{align*}
\text{maximize} \quad & \min_{h_{d} \in A_{d}} \ \text{Re}(w^{H}h_{d}) \\
\text{subject to} \quad & \text{Im}(w^{H}h_{d}) = 0, \\
& \|w\|^{2} \leq \frac{P_{m}}{\sigma_{d}^{2}}, \\
& \max_{h_{l} \in A_{l}} \|w^{H}h_{d}\|^{2} \leq \frac{\varepsilon_{l}}{\sigma_{d}^{2}}, \quad l = 1, 2, \cdots, L. 
\end{align*}
\]  

(4.2.11)

First consider the minimum value of the cost function.

\[
C_{\text{min}} = \min_{h_{d} \in A_{d}} \{ \text{Re}(w^{H}h_{d}) \} \\
= \min \{ \text{Re}(w^{H}\tilde{h}_{d} + w^{H}e_{d}) \} \\
= \text{Re}(w^{H}\tilde{h}_{d}) + \min \{ \text{Re}(w^{H}e_{d}) \}. 
\]  

(4.2.12)

\(\text{Re}(w^{H}e_{d})\) is maximized when \(w\) is colinear with \(e_{d}\) [74]. Hence \(C_{\text{min}}\) is obtained as

\[
C_{\text{min}} = \text{Re}(w^{H}\tilde{h}_{d}) - \gamma_{d}\|w\|_{2}. 
\]  

(4.2.13)

Next the worst case constraint is considered for

\[
C_{l,\text{max}} = \max \|w^{H}h_{l}\|^{2}. 
\]  

(4.2.14)

In order to obtain the worst case error for \(\max \|w^{H}h_{l}\|^{2}\), the cost function is formulated using a Lagrangian multiplier as follows:

\[
J = (w^{H}\tilde{h}_{l} + w^{H}e)^{H}(w^{H}\tilde{h}_{l} + w^{H}e) + \lambda(e^{H}e - \gamma_{l}^{2}), 
\]  

(4.2.15)

where \(\lambda > 0\) is the Lagrangian multiplier. For small errors in the CSI, the second order error term in \((w^{H}\tilde{h}_{l} + w^{H}e)^{H}(w^{H}\tilde{h}_{l} + w^{H}e)\) can be ignored
as compared to the other three terms. Hence (4.2.15) is approximated as

\[ J \approx \tilde{h}_t^H w w^H \tilde{h}_t + \tilde{h}_t^H w w^H e + e^H w w^H \tilde{h}_t + \lambda (e^H e - \gamma_i^2). \] (4.2.16)

By differentiating \( J \) with respect to \( e \) and setting it to zero, the worst-case error is obtained as

\[ w^H e_{l,\text{opt}} = \frac{\gamma_l \|w\|_2}{\|w^H \tilde{h}_t\|_2} w^H \tilde{h}_t. \] (4.2.17)

Hence \( C_{t_{\text{max}}} \) is obtained as follows:

\[
C_{t_{\text{max}}} = \max \|w^H \tilde{h}_t + w^H e\|_2^2
= (\|w^H \tilde{h}_t\|_2 + \gamma_l \|w\|_2)^2. \] (4.2.18)

The worst-case performance based optimization can be stated in the epigraph form using an auxiliary variable \( t \) [57] as

maximize \( t \)
subject to \( \text{Re}\{w^H \tilde{h}_d\} - \gamma_d \|w\|_2 \geq t, \)
\( \text{Im}\{w^H \tilde{h}_d\} = 0, \)
\( \|w\|_2 \leq \sqrt{\frac{P_m}{\sigma^2}}, \)
\( \|w^H \tilde{h}_t\|_2 + \gamma_l \|w\|_2 \leq \sqrt{\frac{\epsilon_l}{\sigma^2}}, \ l = 1, \ldots, L. \) (4.2.19)

### 4.2.4 Ellipsoid Error Bound

Here it is assumed that the error vectors in the CSI are bounded by the following inequalities:

\[ e_d^H P_d^{-1} e_d \leq 1, \]
\[ e_l^H P_l^{-1} e_l \leq 1, \] (4.2.20)
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where \( \mathbf{P}_d \) and \( \mathbf{P}_l \) are positive definite matrices. The true channel coefficients therefore belong to the following ellipsoids:

\[
\mathbf{h}_{td} \in \mathcal{E}_d = \{ \tilde{\mathbf{h}}_d + \mathbf{P}_d \mathbf{u} | \| \mathbf{u} \| \leq 1 \},
\]
\[
\mathbf{h}_d \in \mathcal{E}_l = \{ \tilde{\mathbf{h}}_l + \mathbf{P}_l \mathbf{u} | \| \mathbf{u} \| \leq 1 \}.
\]  (4.2.21)

The worst-case beamformer for single user cognitive radio is obtained as

\[
\begin{align*}
\text{maximize} & \quad \min_{\mathbf{h}_{td} \in \mathcal{E}_d} \text{Re}\{\mathbf{w}^H \mathbf{h}_{td}\} \\
\text{subject to} & \quad \text{Im}\{\mathbf{w}^H \mathbf{h}_{td}\} = 0, \\
& \quad \|\mathbf{w}\|_2^2 \leq P_m/\sigma_s^2, \\
& \quad \max_{\mathbf{h}_d \in \mathcal{E}_l} \|\mathbf{w}^H \mathbf{h}_d\|_2^2 \leq \epsilon_l/\sigma_s^2, \quad l = 1, 2, \cdots, L.
\end{align*}
\]  (4.2.22)

First the minimum value of the cost function is considered.

\[
C_{\text{min}} = \min_{\mathbf{h}_{td} \in \mathcal{E}_d} \{ \text{Re}(\mathbf{w}^H \mathbf{h}_{td}) \}
\]
\[
= \min_{\mathbf{e}_d} \{ \text{Re}(\mathbf{w}^H \tilde{\mathbf{h}}_d + \mathbf{w}^H \mathbf{e}_d) \}
\]
\[
= \text{Re}(\mathbf{w}^H \tilde{\mathbf{h}}_d) + \min_{\mathbf{e}_d} \{ \text{Re}(\mathbf{w}^H \mathbf{e}_d) \} 
\]  (4.2.23)

\( \text{Re}(\mathbf{w}^H \mathbf{e}_d) \) is maximized when \( \mathbf{w} \) is colinear with \( \mathbf{e}_d \) [74]. Hence \( C_{\text{min}} \) is obtained as

\[
C_{\text{min}} = \text{Re}(\mathbf{w}^H \tilde{\mathbf{h}}_d) - \| \mathbf{P}_d^{\frac{1}{2}} \mathbf{w} \|_2.
\]  (4.2.24)

Next the worst-case constraint is considered for

\[
C_{\text{t max}} = \max \| \mathbf{w}^H \mathbf{h}_d \|_2^2.
\]  (4.2.25)
In order to obtain the worst case error for \( \max \|w^H \hat{h}_l\|_2^2 \), the cost function is formulated using a Lagrangian multiplier as follows:

\[
J = (w^H \tilde{h}_l + w^H e)^H (w^H \tilde{h}_l + w^H e) + \lambda (e^H P_t^{-1} e - 1)
\]

\[
= \tilde{h}_l^H w w^H \tilde{h}_l + \tilde{h}_l^H w w^H e + e^H w w^H \tilde{h}_l + e^H w w^H e
\]

\[+ \lambda (e^H P_t^{-1} e - 1) \tag{4.2.26} \]

where \( \lambda > 0 \) is the Lagrangian multiplier. For small errors in the CSI, the third term in \((w^H \tilde{h}_l + w^H e)^H (w^H \tilde{h}_l + w^H e)\) (i.e. the second order error term) can be ignored as compared to the other three terms. Hence (4.2.26) is approximated as

\[
J \approx \tilde{h}_l^H w w^H \tilde{h}_l + \tilde{h}_l^H w w^H e + e^H w w^H \tilde{h}_l + \lambda (e^H P_t^{-1} e - 1). \tag{4.2.27} \]

By differentiating \( J \) with respect to \( e \), the worst-case error is obtained as

\[
w^H e_{l, opt} = \frac{P_t^{\frac{1}{2}} w}{\|w^H \tilde{h}_l\|_2} w^H \tilde{h}_l. \tag{4.2.28} \]

Hence \( C_t_{\max} \) is obtained as follows:

\[
C_t_{\max} = \max_{e^H P_t^{-1} e \leq 1} \{ \|w^H \tilde{h}_l + w^H e\|_2^2 \}
\]

\[= (\|w^H \tilde{h}_l\|_2^2 + \|P_t^{\frac{1}{2}} w\|_2^2). \tag{4.2.29} \]
The worst-case performance based optimization can be stated using an auxiliary variable $\tau$ [57] as

\[
\begin{align*}
\text{maximize} & \quad \tau \\
\text{subject to} & \quad \text{Re}\{w^H \tilde{h}_d\} - \|P_d^{\frac{1}{2}}w\|_2 \geq \tau, \\
& \quad \text{Im}\{w^H \tilde{h}_a\} = 0, \\
& \quad \|w\|_2 \leq \sqrt{\frac{P_m}{\sigma^2}}, \\
& \quad \|w^H \tilde{h}_l\|_2 + \|P_l^{\frac{1}{2}}w\|_2 \leq \sqrt{\frac{\varepsilon_l}{\sigma^2}}, \quad l = 1, \cdots, L.
\end{align*}
\]

(4.2.30)

4.2.5 Simulation Results

In order to assess the performance of the proposed robust beamformer, the BER performance of the primary users is evaluated for various targets on interference leakage to the primary users. A network with a single secondary user and two primary users is considered in an underlay approach. The secondary network basestation consists of four transmitting antennas. The channels between the secondary network basestation and the primary users as well as the secondary user are assumed to be in error. The maximum power available at the transmitter is restricted to unity.

4.2.6 Simulation Results for Constant Error Bound

The BER performance of the primary users is computed for various values of SNR and for various interference power upper bounds. Here the SNR is defined as the ratio between the received useful signal power (from the primary user basestation) and the power of AWGN present at the primary user terminal. The channels between the primary user basestation and the primary receivers have been generated using zero mean, circularly symmetric AWGN with unity variance. A Monte-Carlo experiment is performed with
Figure 4.1. The BER performance of the primary user for different upper bounds on the interference power.

$10^4$ independent random channels generated using zero mean unity variance, circularly symmetric AWGN noise. The norm of the errors in the channel estimation has been set to $\gamma_d = 0.1$, $\gamma_1 = 0.1$, and $\gamma_2 = 0.1$. Figure 4.1 depicts the BER performance of the robust scheme and the non-robust scheme for various upper bounds on the interference power. From the results, the robust scheme outperforms the non-robust scheme. Next, the distribution of interference leakage to the primary user is computed for the robust scheme and non-robust scheme. Figure 4.2 depicts the PDF of the interference power leakage to the primary user. In the robust scheme, interference power is always below the predefined target on interference power. But in the non-robust scheme, it exceeds the target 50 percent of the time.
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Figure 4.2. The distribution of the interference power leakage to the primary user for robust scheme and non-robust scheme. The upper bound has been set to 0.1.

4.2.7 Simulation Results for Ellipsoid Error Bound

The BER performance of the primary user and secondary user is computed for different upper bounds on the primary user interference power. The same cognitive radio network is considered as in the previous section. The channel coefficients between primary and secondary users and the secondary network basestation are modelled using AOAs as follows:

\[
\mathbf{h} = \alpha_0 \begin{bmatrix} 1 & e^{-j\theta_d} & e^{-j2\theta_d} & e^{-j3\theta_d} \end{bmatrix}^T,
\]  

(4.2.31)

where \(\alpha_0\) is the channel gain (assumed to be unity) and \(\theta_d\) is the AOA generated using uniformly distributed random variables in the range \(-\frac{\pi}{2}\) to \(\frac{\pi}{2}\). Figure 4.3 and Figure 4.4 depict the BER performance of the secondary user and the primary user respectively for different upper bounds on primary user interference power. The BER performance is compared with a null-space method where beamformer weight vectors are obtained by linear
Figure 4.3. The BER performance of the secondary user for different upper bounds on the interference power of the primary user. The upper bounds on the interference power have been shown next to the legends in the graph.

combination of null-space vectors of primary user channel matrix. As it can be seen from Figure 4.3 and Figure 4.4, increasing the upper bound on the possible interference power could slightly degrade the primary user BER performance, however it results into significant improvement on the BER performance of the secondary user. Hence by controlling the interference power, the BER performance of the secondary user can be controlled.

In order to assess the performance of the proposed robust beamformer, the BER performance of the primary users is evaluated for various targets on interference leakage to the primary users. The channels between the secondary network basestation and the primary as well as the secondary users are assumed to be in error. The maximum power available at the transmitter is restricted to unity. The BER performance of the primary users is computed for various values of SNR and for various interference power upper bounds. Here the SNR is defined as the ratio between the received useful signal power (from the primary basestation) and the power of AWGN.
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Figure 4.4. The BER performance of the primary user for different upper bounds on the interference power. The upper bounds on the interference power have been shown next to the legends in the graph.

Present at the primary user terminal. The channels between secondary network basestation and primary receivers have been generated using random AOAs. A Monte-Carlo experiment is performed with $10^4$ independent random channels generated using AOAs uniformly distributed between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$. The errors in the channel estimates have been bounded within ellipsoids generated by the positive definite matrices $P_d, P_1$ and $P_2$:

$$P_d = P_1 - P_2 = \begin{pmatrix}
0.1 & 0.05 & 0.05 & 0.05 \\
0.05 & 0.2 & 0.05 & 0.05 \\
0.05 & 0.05 & 0.3 & 0.05 \\
0.05 & 0.05 & 0.05 & 0.4
\end{pmatrix}.$$  \hspace{1cm} (4.2.32)

Figure 4.5 depicts the BER performance of the robust scheme and the non-robust scheme for various upper bounds on the interference power. The
upper bound on the interference at the primary users has been set to 0.1. The robust scheme outperforms the non-robust scheme. Next, the distribution of interference leakage to the primary user is computed for the robust scheme and the non-robust scheme. Figure 4.6 depicts the PDF of the interference leakage to the primary user for both schemes. In the robust scheme, the interference power is always below the upper bounds (target) whereas in the non-robust scheme, it exceeds the upper bound 50 percent of the time. In the next section, non-robust and robust beamforming techniques are proposed for multiuser underlay cognitive radio networks.

4.3 Beamforming Design for Multiuser Cognitive Radio Networks

In this section, initially a beamforming design for multiuser underlay cognitive radio networks is considered with perfect CSI at the secondary network basestation. Next, robust beamforming techniques have been proposed using
Figure 4.6. The distribution of the interference leakage to the primary user for robust scheme and non-robust scheme. The upper bound on interference is 0.1.

the worst-case performance optimization. The solutions for these beamforming designs are obtained by solving SDPs. Simulation results validate that the robust scheme outperforms the non-robust scheme.

### 4.3.1 Problem Statement

A cognitive radio network with \( L \) primary users and \( K \) secondary users is considered in an underlay approach. The secondary network basestation has been assumed to have \( N \) antennas while both the primary and the secondary user terminals have only single antenna. The signal transmitted by the secondary network basestation is

\[
x(n) = Ws(n),
\]

where \( s(n) = [s_1(n) \ s_2(n) \ \cdots \ s_K(n)]^T \) and \( s_k(n), \ k = 1, 2, \cdots, K \) is symbol intended for the \( k^{th} \) secondary user, \( W = [w_1 \ w_2 \ \cdots \ w_K] \) and \( w_k \in \mathbb{C}^{N \times 1} \)
is the transmit beamforming weight vector for the $k^{th}$ secondary user. The received signal at the $k^{th}$ secondary user can be written as

$$ y_{dk}(n) = h_{dk}^H x(n) + \eta_{dk}(n), \quad (4.3.2) $$

where the signal is distorted by the complex channel coefficients $h_{dk} = [h_{d1}^k h_{d2}^k \ldots h_{dN}^k]^H$, between the $k^{th}$ secondary user and the secondary network basestation. It is assumed that $\eta_{dk}(n)$ is a zero mean circularly symmetric AWGN component with variance $\sigma_k^2$. The transmit power at the secondary network basestation can be written as

$$ P_t = \sigma_k^2 \sum_{i=1}^{K} w_i^H w_i, \quad (4.3.3) $$

where $\sigma_k^2$ is the variance of the transmitted signal $s_k(n)$. Since signals are transmitted to multiple secondary users in the same frequency band, signals transmitted to a particular secondary user could interfere with other secondary users. The SINR for the $k^{th}$ secondary user can be written as

$$ \text{SINR}_k = \frac{w_k^H h_{dk} h_{dk}^H w_k \sigma_k^2}{\sigma_k^2 \sum_{i \neq k} w_i^H h_{dk} h_{dk}^H w_i + \sigma_k^2}. \quad (4.3.4) $$

Since the secondary users and the primary users could share the same frequency band, communication between the secondary users introduces interference to the primary users. The interference leakage to the $l^{th}$ primary
user due to the secondary user transmission can be written as

\[ P_{ll} = E\{|h_l^H(\sum_{i=1}^{K} w_i s_i(n))|^2\} \]

\[ = E\{h_l^H \sum_{i} w_i s_i(n) \sum_{j} w_j^H s_j^*(n) h_l \} \]

\[ = h_l^H \sum_{i} \sum_{j} w_i E\{s_i(n) s_j^*(n)\} w_j^H h_l \]

\[ = h_l^H (\sum_{i=1}^{K} w_i w_i^H) h_l \sigma_s^2, \quad (4.3.5) \]

where \( h_l \) consists of the channel coefficients between the secondary network basestation and the \( l \)th primary user. In (4.3.5), it is assumed that multiple secondary user signals are uncorrelated so that \( E\{s_i(n) s_j^*(n)\} = 0 \) for \( i \neq j \).

### 4.3.2 Semidefinite Programming Based Beamformer Design

The beamformer is designed to minimize the total transmit power (4.3.3) at the secondary network basestation while achieving target SINRs for the secondary user terminals and imposing the constraint on interference leakage to the primary users below a set of target values. This problem can be stated as

\[
\text{minimize} \quad \sum_{i=1}^{K} ||w_i||_2^2 \\
\text{subject to} \quad \frac{w_i^H h_d k h_k^H w_k}{\sum_{i 
eq k} w_i^H h_d k h_k^H w_i + \sigma_s^2} \geq \gamma_k, \quad k = 1, 2, \ldots, K, \\

h_l^H (\sum_{i=1}^{K} w_i w_i^H) h_l \leq \varepsilon_l / \sigma_s^2, \quad l = 1, 2, \ldots, L. \\
\quad (4.3.6)
\]

By introducing a new variable \( W_i = w_i w_i^H \), this problem can be converted to the following optimization problem with some additional constraints [36,37].
Section 4.3. Beamforming Design for Multiuser Cognitive Radio Networks

To formulate this problem into a convex form, (4.3.6) is written as

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \text{Tr}\{W_i\} \\
\text{subject to} & \quad \text{Tr}\{W_k H_{dk}\} - \gamma_k \sum_{i \neq k} \text{Tr}\{W_i H_{dk}\} \geq \gamma_k \frac{\sigma_k^2}{\sigma_n^2}, \quad k = 1, 2, \ldots, K, \\
& \quad \sum_{i=1}^{K} \text{Tr}\{H_i W_i\} \leq \epsilon_l / \sigma_n^2, \quad l = 1, 2, \ldots, L, \\
& \quad W_i = W_i^H, \quad i = 1, 2, \ldots, K, \\
& \quad W_i \succeq 0, \quad i = 1, 2, \ldots, K, \\
& \quad \text{rank}\{W_i\} = 1, \quad i = 1, 2, \ldots, K, \quad (4.3.7)
\end{align*}
\]

where \(H_{dk}\) and \(H_l\) denote \(h_{dk} h_{dk}^H\) and \(h_l h_l^H\) respectively and \(W_i \succeq 0\) indicates that \(W_i\) is a positive semidefinite matrix. For a convex optimization problem, the objective function and the constraints should be convex. The cost \(\sum_{i=1}^{K} \text{Tr}\{W_i\}\) is an affine function in terms of \(W_i\). Affine functions are both convex and concave. Since the cost is minimized, it can be treated as convex. Similarly, each term in the SINR constraints and interference constraints is affine and defines the domain of the convex sets. Obviously, \(W_i \succeq 0\) is a positive semidefinite cone and it is a convex constraint. But, in the above problem, the domain of the constraint \(\text{rank}\{W_i\} = 1\) is not convex. By using semidefinite relaxation (SDR), the non-convex constraint can be relaxed and the problem can be converted to a semidefinite constrained problem [36,37]. This relaxation formulates the optimization problem in (4.3.7) into a convex optimization problem where the cost and constraints
are convex in nature.

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \text{Tr}\{W_i\} \\
\text{subject to} & \quad \text{Tr}\{W_k H_{dk}\} - \gamma_k \sum_{i \neq k} \text{Tr}\{W_i H_{dk}\} \geq \frac{\gamma_k \sigma_k^2}{\sigma_s^2}, \\
& \quad \sum_{i=1}^{K} \text{Tr}\{H_l W_i\} \leq \epsilon_l / \sigma_s^2, \quad l = 1, 2, \cdots, L, \\
& \quad W_i \succeq 0, \quad i = 1, 2, \cdots, K, \\
& \quad W_i = W_i^H, \quad i = 1, 2, \cdots, K. \quad (4.3.8)
\end{align*}
\]

Once \(W_i\) is determined, \(w_i\) could be obtained by extracting the eigenvector corresponding to the non-zero eigenvalue of the rank-one matrix \(W_i\). The above problem can be solved using interior point methods [62] as available in many optimization toolboxes [38–40].

### 4.3.3 Worst-Case Performance Optimization

The transmit beamformer design in (4.3.8) assumes perfect CSI at the base-station. In addition, the transmit beamformer design is very sensitive to the errors in the CSI. Therefore robust algorithms are proposed assuming uncertainty in the CSI. The true channel can be written as

\[
h_l = \hat{h} + e, \quad (4.3.9)
\]

where \(\hat{h}\) is the CSI available at the base-station and \(e\) is a possible error vector. The beamformer should be designed such that the required QoS (in terms of SINRs) at the secondary user should be achieved and the interference leakage to the primary users should be below the specific values all the time regardless of the error in the CSI. The original problem is therefore modified
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as

\[ \begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \text{Tr}\{W_i\} \\
\text{subject to} & \quad \text{Tr}\{W_k(\tilde{H}_{dk} + \Delta_{dk})\} - \gamma_k \sum_{i \neq k} \text{Tr}\{W_i(\tilde{H}_{dk} + \Delta_{dk})\} \geq \gamma_k \frac{\sigma_k^2}{\sigma_2^2}, \\
& \quad \sum_{i=1}^{K} \text{Tr}\{(\tilde{H}_i + \Delta_i)W_i\} \leq \epsilon_i / \sigma_i^2, \quad l = 1, 2, \ldots, L, \\
& \quad W_i \succeq 0, \quad i = 1, 2, \ldots, K, \\
& \quad W_i = W_i^H, \quad i = 1, 2, \ldots, K, 
\end{align*} \]  

(4.3.10)

where \( \tilde{H} \) is the channel matrix formed from the erroneous CSI available at the secondary network base station and \( \Delta \) is the corresponding error introduced in the channel matrix. This approach is based on the worst-case error performance optimization [74]. It is assumed the Frobenious norms of the error matrices are bounded above by known constants as

\[ \|\Delta_d\|_F \leq \alpha, \quad \|\Delta_i\|_F \leq \beta. \]  

(4.3.11)

The true channel matrices, therefore, belong to the following convex hulls [57]:

\[ \mathcal{A}_d(\alpha) \triangleq \{ H_{td} \mid H_{td} = \tilde{H}_{td} + \Delta_{td}, \quad \|\Delta_{td}\|_F \leq \alpha \}, \]

\[ \mathcal{A}_i(\beta) \triangleq \{ H_{ti} \mid H_{ti} = \tilde{H}_{ti} + \Delta_{ti}, \quad \|\Delta_{ti}\|_F \leq \beta \}. \]  

(4.3.12)

In the worst-case performance optimization, the beamformer weight vectors should be designed using a set of constraints on the worst possible errors.
The worst-case beamformer for the cognitive radio network is obtained as

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \text{Tr}\{W_i\} \\
\text{subject to} & \quad \min_{\|\Delta_{dk}\|_F \leq \alpha} \text{Tr}\{W_k(\tilde{H}_{dk} + \Delta_{dk})\} - \gamma_k \sum_{i \neq k} \text{Tr}\{W_i(\tilde{H}_{dk} + \Delta_{dk})\} \geq \gamma_k \frac{\sigma^2_k}{\sigma^2_k}, \\
& \quad k = 1, 2, \ldots, K, \\
& \quad \max_{\|\Delta_l\|_F \leq \beta} \sum_{i=1}^{K} \text{Tr}\{('\tilde{H}_l + \Delta_l\)W_i\} \leq \epsilon_l / \sigma^2_k, \quad l = 1, 2, \ldots, L, \\
& \quad W_i \succeq 0, \quad i = 1, 2, \ldots, K, \\
& \quad W_i = W_i^H, \quad i = 1, 2, \ldots, K. \\
\end{align*}
\] (4.3.13)

To solve the above problem, the worst-case is obtained as follows [74]:

\[
\begin{align*}
\min_{\|\Delta_{dk}\|_F \leq \alpha} & \quad \text{Tr}\{W_k(\tilde{H}_{dk} + \Delta_{dk})\} - \gamma_k \sum_{i \neq k} \text{Tr}\{W_i(\tilde{H}_{dk} + \Delta_{dk})\} \\
& \quad = \text{Tr}\{W_k(\tilde{H}_{dk} - \alpha I)\} - \gamma_k \sum_{i \neq k} \text{Tr}\{W_i(\tilde{H}_{dk} + \alpha I)\}. \\
\end{align*}
\] (4.3.14)

Similarly,

\[
\begin{align*}
\max_{\|\Delta_l\|_F \leq \beta} & \quad \sum_{i=1}^{K} \text{Tr}\{(H_l + \Delta_l)W_i\} \\
& \quad = \sum_{i=1}^{K} \text{Tr}\{(\tilde{H}_l + \beta I)W_i\}. \\
\end{align*}
\] (4.3.15)
Therefore the worst-case performance based optimization can be stated as follows:

\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \text{Tr}\{W_i\} \\
\text{subject to} & \quad \text{Tr}\{W_k(\tilde{H}_{dk} - \alpha I)\} - \gamma_k \sum_{i \neq k} \text{Tr}\{W_i(\tilde{H}_{dk} + \alpha I)\} \geq \frac{\sigma_k^2}{\sigma_s^2}, \\
& \quad \sum_{i=1}^{K} \text{Tr}\{(\tilde{H}_l + \beta I)W_i\} \leq \varepsilon_l/\sigma_s^2, \quad l = 1, 2, \ldots, L, \\
& \quad W_i \succeq 0, \quad i = 1, 2, \ldots, K, \\
& \quad W_i = W_i^H, \quad i = 1, 2, \ldots, K.
\end{align*}

(4.3.16)

Next, the complexity of solving the problem is analyzed using interior point methods. The original non-robust optimization problem is an SDP problem expressed in the primal standard form and it can be efficiently solved by any general purpose SDP solver using interior point methods [62]. The original primal problem consists of \(K\) matrix variables of size \(N \times N\) and \(K + L\) linear constraints. Interior point methods will take \(O(\sqrt{K^3}N^6 + (K + L)KN^2)\) arithmetic operations in the worst-case [62]. But the actual complexity will be far less than this worst-case bound. For example, a scenario with two secondary and two primary users, the algorithm converges typically within ten iterations which is known and generally accepted in the optimization community [71]. For the robust scheme, the structure of the problem in terms of dimension of the matrices involved is the same as that of the non-robust optimization problem because, fortunately, the worst-case errors have been included using closed form solutions. Therefore the robust optimization problem has the same order of complexity as the non-robust optimization problem. Hence, there is no additional computational complexity in solving the robust optimization.
problem in (4.3.16).

4.3.4 Simulation Results

In order to assess the performance of the proposed multiuser transmit beamformer in a cognitive radio network, the average transmit power required to achieve the target SINRs of the secondary users is computed for various interference power values of the primary users. A network with two secondary users and two primary users is considered in an underlay approach. The secondary network basestation consists of four transmitting antennas. In the first phase of the simulation, the channels between the secondary network basestation and the secondary user terminals as well as the primary user terminals are assumed to be perfectly known to the secondary network basestation. The maximum power available at the transmitter is restricted to unity. For a bad set of channels between the secondary network basestation and the secondary users, the transmitter could consume a significant amount of power in an attempt to achieve the target SINRs and to compress interference power to the primary users. Hence, if the required transmit power is higher than unity, the secondary network basestation is expected to remain silent. The outage probability for this will also be determined in the simulation.

The required average transmit power is computed for different SINR targets. The noise power at the primary user and the secondary user receivers has been set to $\sigma^2 = 0.01$ W. A Monte-Carlo experiment is performed with 50000 independent random channels generated using zero mean unity variance, circularly symmetric AWGN noise. Figure 4.7 depicts the required average power against various upper bounds on interference power for different target SINRs. Next to each curve, the outage probabilities are also provided, i.e. the probability that the instantaneous transmit power exceeds unity (the values have been shown next to the legends). However this
outage probability could be reduced further if the maximum instantaneous transmit power at the secondary network basestation is allowed to increase above unity.

Figure 4.8 depicts the PDF of the instantaneous transmit power at the secondary network basestation for four different levels of interference leakage to the primary users. As the requirement on interference power is relaxed, the total transmit power requirement and the outage probability are reduced significantly. When the bound on the interference perceived at the secondary user is decreased, the domain of the optimization problem is shrunk resulting in less degrees of freedom for the secondary network basestation to steer beams towards the secondary users and to place nulls towards the primary users. Hence the outage probability as well as the power dissipation at the secondary network basestation increases. On the other hand when the bound
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Figure 4.8. Probability distribution of the transmitted power at the secondary network basestation for different upper bounds on the interference power at the primary users. The target SINR of the secondary user is 10 dB.

As the interference power is increased, the interference constraints tend to be inactive at the optimum point most of the time. This is equivalent to increasing the domain of the optimization problem as the domain will be influenced only by the SINR constraints. As the domain of the optimization problem is increased, the transmit power (which is the cost function) is decreased further, as otherwise this optimum point will be infeasible with tighter constraints on interference. This results in reduction in transmit power as seen in Figure 4.7 and Figure 4.8.

Next, the effect of interference power on the BER performance of the primary users is studied. Here, the transmission between the primary user basestation and the primary user terminals should be modelled as well. The transmitted signal is uncoded binary phase shift keying (BPSK). The basestation of the primary user has been assumed to have a single antenna. The target SINRs for both the secondary users have been set to 10 dB. The chan-
Figure 4.9. The BER performance of the primary user for different upper bounds on the interference power. The average transmitted powers at the secondary network basestation and the outage probabilities are shown next to the corresponding graphs. The target SINR of the secondary user is 10 dB.

The channel between the secondary network basestation and all the secondary users as well as the channel between the primary user basestation and the primary user terminals have been generated using zero mean, circularly symmetric AWGN with unity variance. The total available transmit power at the secondary network basestation has been set to unity. The BER performance of the primary user is computed for various values of SNRs and various values of interference power as shown in Figure 4.9. Here the SNR is defined as the ratio between the received useful signal power (from the primary user basestation) and the power of AWGN present at the primary user terminal. The secondary network basestation transmit power and the outage probabilities are also shown in the BER curves. It is observed that allowing a small amount of interference leakage to the primary user significantly saves
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![Graph showing required power at the secondary network basestation for different angle of arrivals.](image)

**Figure 4.10.** The variation of required power at the secondary network basestation for different angle of arrivals.

The transmit power while resulting in only a small degradation in the BER performance of the primary user.

Next, in order to validate the performance of the proposed scheme for location based channel models, a cognitive radio network with two primary users and two secondary users is considered. The secondary network basestation is equipped with five transmit antennas. The channel between the secondary network basestation and the secondary users as well as the primary users are modelled using AOAs as follows:

\[
\mathbf{h} = \alpha_0 \begin{bmatrix} 1 & e^{-j\theta_d} & e^{-j2\theta_d} & e^{-j3\theta_d} & e^{-j4\theta_d} \end{bmatrix}^T,
\]

where \(\alpha_0\) is the channel gain and \(\theta_d\) is the AOA of the signal. In the simulation, \(\alpha_0\) is assumed to be one. First the power consumption at the secondary network basestation is evaluated for different AOAs. The AOAs of one of
Figure 4.11. The required average transmit power at the secondary network basestation for different target SINRs of the secondary users and various interference power of the primary users. Noise power at the primary and the secondary receivers has been set to 0.01. The maximum instantaneous transmit power has been limited to 5.

the secondary users and the two primary users are fixed at -2.1, 2.6 and 1.5 respectively (in radians). The target SINRs for the secondary users and the upper bound on the interference leakage to the primary users have been set to 15dB and 0.01 respectively. The noise power at the primary and the secondary receivers has been set to 0.01. By changing the AOAs of the remaining secondary user, the required transmit power at the secondary network basestation is computed. From the results depicted in Figure 4.10, it can be seen that the secondary network basestation requires excessive transmit power, when the second secondary user is located close to the primary users or the first secondary user.

Next, the required average transmit power is evaluated for different
4.3. Beamforming Design for Multiuser Cognitive Radio Networks

SINR targets. The AOAs between the secondary network basestation and the secondary users as well as the primary users are randomly (uniformly distributed) generated between $-\pi$ and $\pi$. The maximum transmit power available at the secondary network basestation is restricted to 5. For the channels where the users locate close to each other, the secondary network basestation tends to consume more power or the corresponding optimization problem becomes infeasible. Hence, the secondary network basestation has been set to be silent, if the required transmit power increases above 5, resulting in an outage scenario. Figure 4.11 depicts the required average power against various upper bounds on interference leakage to the primary users for different target SINRs. The outage probability for some selected points is also included in Figure 4.11. It can be seen that when the upper bound on interference power leakage to the primary users is increased, the required transmit power for the secondary network basestation is reduced as similar to the results in Figure 4.7 and Figure 4.8. Also the required transmit power becomes constant beyond certain upper bounds on interference constraints. This is because when the upper bound is increased the interference constraint becomes inactive and does not influence the power optimization problem.

4.3.5 Simulation Results for Robust Scheme

In order to verify the effectiveness of the proposed robust beamformer for cognitive radios, the distributions of SINRs achieved at the secondary user terminals are evaluated for the robust and non-robust schemes. A cognitive radio network with two secondary users and two primary users has been considered in the simulation. The secondary network basestation is equipped with four transmit antennas and each of the primary and the secondary users consists of a single antenna. The channels between the secondary network basestation and the primary as well as the secondary users are assumed to
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Figure 4.12. The distribution of the achieved SINRs at the secondary users for robust scheme and the non-robust scheme. The target SINR has been set to 7 dB.

be in error. The channels are generated using zero mean, circularly symmetric complex AWGN with unit variance. The noise power and the target SINRs at the secondary user terminals have been set to $\sigma^2 = 0.01$ and 7 dB respectively.

In order to validate the performance of the proposed robust scheme, the errors generated using random noise are added to the CSI available at the secondary network base station and multiuser beamformers are designed using the robust and non-robust schemes. The upper bound on the error (i.e. $\alpha$ and $\beta$ in (4.3.16)) of the channel matrix has been set to 0.1. The distribution of SINRs achieved at the secondary user terminals is computed for the robust and the non-robust schemes as depicted in Figure 4.12. In the robust scheme, the SINR is always above the target SINR regardless of the errors introduced in the CSI whereas in the non-robust scheme, the target SINR is attained only 50 percent of the time. The PDF of the interference power...
Figure 4.13. The distribution of the interference power at the primary user for robust scheme and the non-robust scheme. The interference power upper bound has been set to 0.05.

leakage to the primary users as depicted in Figure 4.13. In the optimization problem, the upper bound on the interference leakage to the primary users was set to 0.05. In the robust scheme, the interference power is always below this upper bound whereas in the non-robust scheme, it occasionally exceeds the required upper bound.

4.4 Conclusions

In this chapter, firstly robust beamforming techniques have been proposed for single user underlay cognitive radio networks using worst-case performance optimization. These robust beamforming techniques have been developed for different types of error bounds, namely, constant and ellipsoid error bounds in the available CSI at the base station. Then, non-robust and robust beamformer schemes have been proposed for multiuser underlay cogni-
Section 4.4. Conclusions

tive radio networks using SDP and worst-case performance optimization approaches. In addition, it was demonstrated that by allowing a small amount of interference to the primary users, a significant reduction in the total transmit power can be achieved, while attaining target SINRs for the secondary users and maintaining satisfactory BER performance for the primary users. Simulation results have been provided to validate the performance of the proposed robust schemes over the non-robust schemes. The proposed robust schemes always satisfy the constraints regardless of the error in the CSI.
Chapter 5

JOINT RESOURCE ALLOCATION AND ADMISSION CONTROL TECHNIQUES

In this chapter, a network of secondary users coexisting and sharing the spectrum with primary users is considered for an underlay cognitive radio system. Specifically, a cognitive radio network is considered wherein the number of secondary users requesting channel access exceeds the number of available frequency bands and spatial modes. In such a setting, a joint fast optimal radio resource allocation and beamforming algorithm is proposed to accommodate the maximum possible number of secondary users while satisfying QoS requirement for each admitted secondary user, transmit power limitation at the secondary network basestation and interference constraints imposed by the primary users. Recognizing that the original admission control problem is an NP hard, a mixed-integer programming framework is used to formulate the joint admission control and beamforming problem. Subsequently, an optimal algorithm based on the BnB method has been derived. In addition, a suboptimal algorithm is proposed based on BnB method to reduce the complexity of the proposed optimal algorithm. Specifically, the
suboptimal algorithm has been developed based on the first feasible solution it achieves in the fast optimal BnB algorithm. Simulation results have been provided to verify the theoretical results and to compare the performance of optimal and suboptimal algorithms.

In order to enhance the spectrum utilization, the proposed solution should admit as many secondary users as possible in the optimal frequency bands while satisfying the transmit power constraint and QoS requirement (here considered as SINR targets) for each admitted secondary user. In addition, power allocation and beamforming design for secondary users should ensure that the interference power leakage to the primary users is less than a predefined threshold. These different constraints in the problem lead to a multi-objective optimization problem with conflicting goals - admitting as many secondary users as possible while satisfying all constraints and minimizing transmit power at the secondary network basestation.

In this chapter, the above mentioned multi-objective optimization problem is formulated with a single cost function using a mixed-integer programming and SDP framework to accommodate as many secondary users as possible while jointly satisfying QoS requirement of admitted secondary users, interference constraints for primary users and transmit power constraint at the secondary network basestation. Subsequently, a joint fast optimal radio resource allocation and beamforming algorithm is proposed for cognitive radio networks based on BnB method. BnB is a method used to find a global optimal solution in a non-convex problem with integer variables and it is known for solving the class of integer linear programming and mixed-integer programming [104]. The proposed algorithm simultaneously allocates the optimum frequency bands and determines the corresponding beamformer for each secondary user while having a guaranteed QoS for each admitted secondary users and maintaining a priority to accommodate as many secondary users as possible. A mixed-integer programming [104] framework is
used to formulate the joint admission control and beamforming problem, for which a fast optimal algorithm is developed based on the BnB method [105].

5.1 System Model and Problem Statement

A cognitive radio network is considered with $K$ secondary users and $L$ primary users. The number of available licensed frequency bands for underlay approach for secondary users is denoted by $M$. It is assumed that the secondary network basestation is equipped with $N$ antennas while each secondary user consists of a single antenna. The signal transmitted by the secondary network basestation in the $m^{th}$ frequency band can be written as

$$x_m(n) = W_m s_m(n), \quad m = 1, \ldots, M,$$  \hspace{1cm} (5.1.1)

where $s_m(n) \in \mathbb{C}^{N_m \times 1}$ indicates the signal vector consists of $N_m$ users data in the $m^{th}$ frequency band and $W_m \in \mathbb{C}^{N \times N_m}$ is a matrix with $i^{th}$ column (denoted as $w_{i,m}$) as the beamforming vector for the $i^{th}$ user in the $m^{th}$ frequency band. The received signal at the $k^{th}$ secondary user terminal in the $m^{th}$ frequency band can be written as

$$y_{k,m}(n) = h_{k,m}^H x_m(n) + \eta_{k,m}(n),$$  \hspace{1cm} (5.1.2)

where the signal in the $m^{th}$ frequency band is distorted by the complex channel coefficients $h_{k,m} = [h_{k,m}^1 \ h_{k,m}^2 \ldots h_{k,m}^N]^H$, between the $k^{th}$ secondary user and the secondary network basestation in the $m^{th}$ frequency band. It is assumed that $\eta_{k,m}(n)$ is a zero-mean circularly symmetric AWGN component with variance $\sigma_{k,m}^2$. Assuming that the transmitted symbols are uncorrelated, the transmitted power at the secondary network basestation can be written as

$$P_s = \sigma_s^2 \sum_{m=1}^{M} \sum_{k=1}^{N_m} w_{k,m}^H w_{k,m},$$  \hspace{1cm} (5.1.3)
where $\sigma_s^2$ is the variance of the transmitted symbols. Since more than one secondary user could be allocated in each frequency band, signals transmitted to a particular secondary user could interfere with other secondary users in the frequency band. The SINR for the $k^{th}$ secondary user in the $m^{th}$ frequency band can be written as

$$\text{SINR}_k = \frac{w_{k,m}^H b_{k,m} h_{k,m}^H w_{k,m} \sigma_s^2}{\sigma_s^2 \sum_{i=1, i \neq k}^N w_{i,m}^H b_{i,m} h_{i,m}^H w_{i,m} + \sigma_{i,m}^2}. \quad (5.1.4)$$

The interference power at the $l^{th}$ primary user due to the secondary user transmission in the $m^{th}$ frequency band can be formulated as

$$P_{l,m} = g_{l,m}^H \left( \sum_{k=1}^N w_{k,m} w_{k,m}^H \right) g_{l,m} \sigma_s^2, \quad (5.1.5)$$

where $g_{i,m}$ consists of the channel coefficients between the secondary network basestation and the $i^{th}$ primary user in the $m^{th}$ frequency band. In this chapter, a scheme is proposed for allocating secondary users in optimal frequency bands and designing the corresponding beamformers to minimize the total transmitted power (5.1.3), at the secondary network basestation while jointly satisfying QoS constraints for each secondary user and interference power constraints imposed by the primary users.

The optimization problem is now formulated according to

\[
\begin{align*}
\text{maximize} & \quad \sum_{m=1}^M N_m \\
\text{minimize} & \quad \sum_{m=1}^M \sum_{k=1}^N \| w_{k,m} \|^2 \\
\text{subject to} & \quad \frac{w_{k,m}^H b_{k,m} h_{k,m}^H w_{k,m} \sigma_s^2}{\sigma_s^2 \sum_{i=1, i \neq k}^N w_{i,m}^H b_{i,m} h_{i,m}^H w_{i,m} + \sigma_{i,m}^2} \geq \gamma_{k,m}, \\
& \quad k = 1, \ldots, N_m, \ m = 1, \ldots, M, \\
& \quad g_{l,m}^H \left( \sum_{k=1}^N w_{k,m} w_{k,m}^H \right) g_{l,m} \sigma_s^2 \leq \xi_{l,m}, \\
& \quad l = 1, \ldots, L_m, \ m = 1, \ldots, M,
\end{align*}
\]
where $L_m$ denotes the number of primary users in the $m^{th}$ frequency band and $\gamma_{k,m}$ represents the SINR target for the $k^{th}$ secondary user in the $m^{th}$ frequency band. Note that the problem has three goals; the first goal is to admit as many secondary users as possible in the system, the second goal is to determine optimal frequency bands for admitted secondary users while the third goal is to minimize the total transmit power consumption at the secondary network basestation while satisfying QoS and interference constraints. These goals are chased by optimally allocating the secondary users to the frequency bands and jointly designing the spatial beamformers when more than one secondary user is allocated in the same frequency band.

Finding the optimum frequency band for each feasible secondary user in the above mentioned optimization problem is an NP hard problem. Once, the optimum frequency is found, the beamformer design problem can be formulated into an SDP [57]. By introducing a new variable $W_{k,m} = w_{k,m} \mathbf{w}_{k,m}^H$, the beamformer design in the $m^{th}$ frequency band can be stated as [37,82]

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{N_m} \text{Tr}\{W_{k,m}\} \\
\text{subject to} & \quad \text{Tr}\{W_{k,m}H_{k,m}\} - \gamma_{k,m} \sum_{i=1, i \neq k}^{N_m} \text{Tr}\{W_{i,m}H_{k,m}\} \geq \gamma_{k,m}\sigma^2_{k,m}, \\
& \quad k = 1, \ldots, N_m, \\
& \quad \sum_{k=1}^{N_m} \text{Tr}\{W_{k,m}G_{l,m}\} \leq \varepsilon_{l,m}, \quad l = 1, \ldots, L_m, \\
& \quad W_{k,m} = W_{k,m}^H \succeq 0, \quad k = 1, \ldots, N_m, \\
& \quad \text{rank}\{W_{k,m}\} = 1, \quad k = k = 1, \ldots, N_m,
\end{align*}
\] (5.1.6)

where $H = h^H h$, $G = g^H g$ and $W_{k,m} \succeq 0$ indicates $W_{k,m}$ is a positive semidefinite matrix. For a convex optimization problem, the objective function and the constraints should be convex. The cost $\sum_{i=1}^{N_m} \text{Tr}\{W_i\}$ is an affine function in terms of $W_i$. Affine functions are both convex and con-
cave. Since the cost is minimized, it can be treated as convex. Similarly, each term in the SINR constraints and interference constraints is affine and defines the domain of the convex sets. Obviously, $W_i \succeq 0$ is a positive semidefinite cone and it is a convex constraint. The last constraint $\text{rank}(W_{k,m}) = 1$ is non-convex. However, the last constraint can be relaxed using SDR [36, 37] to write

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{N_m} \text{Tr}(W_{k,m}) \\
\text{subject to} \quad & \text{Tr}(W_{k,m}H_{k,m}) - \gamma_{k,m} \sum_{i=1, i \neq k}^{N_m} \text{Tr}(W_{i,m}H_{i,m}) \geq \gamma_{k,m}\sigma_{k,m}^2, \\
& \sum_{k=1}^{N_m} \text{Tr}(W_{k,m}G_{l,m}) \leq \epsilon_{l,m}, \quad l = 1, \cdots, L_m, \\
& W_{k,m} = W_{k,m}^H \succeq 0, \quad k = 1, \cdots, N_m.
\end{aligned}$$

(5.1.7)

The cost and constraints are now convex and they formulate a convex optimization problem.

**Lemma 1:** Provided the problem in (5.1.7) is feasible, the SDR provides rank-one matrices $W_i, \quad i = 1, \cdots, N_m$, establishing equivalence between solutions of (5.1.6) and (5.1.7).

**Proof:** See Appendix (Section 5.7).

Once matrix $W_{k,m}$ has been determined, $w_{k,m}$ could be obtained by extracting the eigenvector corresponding to the non-zero eigenvalue of the rank-one matrix $W_{k,m}$. The above problem can be solved using interior point methods [62] as available in many optimization tools [38–40].
5.2 Mixed-Integer Programming Problem Formulation

In this section, the overall problem is formulated within a mixed-integer programming framework. First, a vector \( s_j \in \mathbb{R}^{M \times 1} \), \( j = 1, 2, \ldots, K \) is defined, for the \( j \)th secondary user such that at most one element of this vector is non-zero and equal to one. Hence, it can take only one of the following \( M + 1 \) possible combinations,

\[
\mathbf{s}_j \in \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix}
\]  

(5.2.1)

If the \( m \)th element of \( s_j \) is one, then it represents that the \( j \)th secondary user is admitted to the \( m \)th frequency band. On the other hand, when all of the elements of this vector are zeros, then the corresponding secondary user will not be accommodated in the network. By introducing this vector, the original optimization problem in (5.1.6) can be formulated within a mixed-integer programming framework as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{m=1}^{M} \sum_{k=1}^{K} \|w_{k,m}\|^2 - \lambda \sum_{j=1}^{K} s_j^T 1_M \\
\text{subject to} & \quad \sum_{m=1}^{M} \sum_{k=1}^{K} \|w_{k,m}\|^2 \leq P_{\text{max}}, \\
& \quad \frac{w_{k,m}^H h_{k,m} h_{k,m}^H w_{k,m} - \frac{1}{\rho_{k,m}} (s_j^T 1_M - 1)}{\sum_{i=1, i \neq k}^{K} w_{i,m}^H h_{k,m} h_{k,m}^H w_{i,m} + \sigma^2_{k,m}} \geq \gamma_{k,m}, \\
& \quad \mathbf{g}_l^H \left( \sum_{k=1}^{K} w_{k,m}^H w_{k,m} \right) \mathbf{g}_l \sigma^2_{l,m} \leq \varepsilon_{l,m}, \\
& \quad s_j \in \{0, 1\}, \quad j = 1, 2, \ldots, K.
\end{align*}
\]  

(5.2.2)
The second part of the cost function $\lambda \sum_{j=1}^{K} s_j^T 1_M$ ensures that accommodating more number of secondary users significantly reduces the cost of the above problem, i.e. the more secondary users admitted the less the cost achieved. Moreover a positive $\lambda$ is chosen such that accommodating a secondary user reduces the cost of the problem more than dropping a secondary user from the cognitive radio network.

In order to find a solution for (5.2.2), the problem should be formulated such that $w_{k,m}$ should be designed and should satisfy the SINR constraint of the secondary user and interference constraint of the primary users, if the $k^{th}$ secondary user is allocated to the $m^{th}$ frequency band. Otherwise the corresponding beamforming weight vectors should be zero. Furthermore $\rho_{k,m}$ is designed to automatically satisfy the SINR constraint when the $k^{th}$ secondary user is not allocated in the $m^{th}$ frequency band. A solution for $\rho_{k,m}$ can be found according to [86]

$$\rho_{k,m} \leq \min_{1 \leq k \leq N_m} \frac{\gamma_{k,m}^{-1}}{P_{\max} \max_{1 \leq k \leq N_m} \| h_{k,m} \|_2^2 + \sigma_{k,m}^2},$$ (5.2.3)

where one can show that $\rho_{k,m}$ satisfies the SINR constraint when $s_j^T 1_M = 0$. By introducing a new variable $W_{k,m} = w_{k,m}w_{k,m}^H$ and using semidefinite relaxation, the optimization problem (5.2.2) is converted into a mixed-integer
programming framework as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{m=1}^{M} \sum_{k=1}^{K} \text{Tr}(W_{k,m}) - \lambda \sum_{j=1}^{K} s_j^T 1_M \\
\text{subject to} & \quad \sum_{m=1}^{M} \sum_{k=1}^{K} \text{Tr}(W_{k,m}) \leq P_{\text{max}}, \\
& \quad \frac{\text{Tr}(W_{k,m} H_{k,m}) - \frac{1}{\rho_{k,m}} (s_j^T 1_M - 1)}{\sum_{i=1, i \neq k}^{K} \text{Tr}(H_{k,m} W_{i,m}) + \sigma_{k,m}^2} \geq \gamma_{k,m}, \\
& \quad \sum_{k=1}^{K} \text{Tr}(W_{k,m} G_{l,m}) \leq \epsilon_{l,m}, \\
& \quad s_j \in \{0, 1\}, \quad j = 1, 2, \ldots, K, \\
& \quad W_{k,m} = W_{k,m}^H \geq 0, \quad k = 1, 2, \ldots, K, \quad m = 1, 2, \ldots, M, \\
& \quad (5.2.4)
\end{align*}
\]

where \(H_{k,m} = h_{k,m} h_{k,m}^H\). Having obtained the mixed-integer programming framework, an algorithm is proposed to solve the above problem based on BnB method in the next section.

5.3 Fast Optimal Algorithm Based on Branch and Bound Method

In this section, the basic idea of a BnB method is introduced and then an algorithm is proposed based on a BnB method to jointly allocate the frequency bands and to design the beamforming weight vectors in a cognitive radio network for the problem in (5.2.4).

A typical BnB algorithm performs two main steps namely branching and bounding. The branching step divides the feasible set of a problem into subsets and formulates the corresponding subproblems with those subsets. The bounding step finds the upper and lower bounds for those subproblems within the corresponding subset. Subsets are further divided into smaller
subsets which generate a tree structure. In the BnB algorithm, some of the branches of the tree can be efficiently removed or pruned. For example, when the lower bound of a branch $P$ is greater than the upper bound of the branch $Q$ at the last level, then branch $P$ can be removed without any effect on the tree. At each level, the global lower bound is updated, if the minimum of the lower bounds of all subsets is less than the global lower bound. Similarly the global upper bound is updated. This procedure continues until it satisfies the accuracy $\epsilon$ which is the difference between the global lower bound and global upper bound.

The fast optimal radio resource allocation algorithm is developed based on BnB method from the original mixed-integer programming formulation (5.2.4). First, the integer constraint in equation (5.2.4) can be relaxed as follows:

$$0_M \leq s_j \leq 1_M, \quad j = 1, 2, \ldots, K. \quad (5.3.1)$$

By relaxing the integer constraints as in (5.3.1), mixed-integer programming is converted to SDP with positive semidefinite matrix variables and continuous vector variables between zero and one. After relaxing the integer constraints, the cost function and the constraints in (5.2.4) are convex. The first and second parts of the cost function are affine. Since the cost is minimized, the first and second part of the cost function can be treated as convex and concave respectively. This SDP relaxation provides a lower bound for the original problem. If all the components of every vectors are integer solutions, then the problem will be deemed to have been solved with optimal solution and the algorithm will be terminated at this stage. Furthermore, if the proposed relaxed problem is infeasible, then the original problem is also infeasible.

The branching step will be performed in the algorithm, when the relaxed problem yields a solution with non-integer values. The computational com-
plexity of the BnB algorithm mainly depends on the choices of branching and bounding steps. Inappropriate choices of those steps might significantly increase the computational complexity of the algorithm. In this algorithm, the branching step is introduced by trying all the possibilities of vector variables. This step creates the subproblems and it is repeated until an optimal solution of the problem is found.

The proposed algorithm based on the BnB method is summarized in Table 5.1. At each level, different nodes are generated by branching steps. For example, a subproblem created at the third level is

\[
\text{minimize } \sum_{m=1}^{M} \sum_{k=1}^{K} \text{Tr}\{W_{k,m}\} - \lambda \sum_{j=1}^{K} s_j^T 1_M
\]

subject to

\[
\sum_{m=1}^{M} \sum_{k=1}^{K} \text{Tr}\{W_{k,m}\} \leq P_{\text{max}},
\]

\[
\frac{\text{Tr}\{W_{k,m} H_{k,m}\} - \frac{1}{\eta_{k,m}} (s_j^T 1_M - 1)}{\sum_{j \neq k} \text{Tr}\{W_{j,m} W_{k,m}\} + \sigma_{k,m}^2} \geq \gamma_{k,m},
\]

\[
N_m \sum_{k=1}^{K} \text{Tr}\{W_{k,m} G_{i,m}\} \leq \varepsilon_{l,m}
\]

\[
0 \leq s_j \leq 1, \quad j = 1, 2, \cdots, K,
\]

\[
s_1(1) = 1, \quad s_1^T 1_M = 1, \quad s_2(1) = 1, \quad s_2^T 1_M = 1,
\]

\[
W_{k,m} = W_{k,m}^H \succeq 0, \quad k = 1, 2, \cdots, K, m = 1, 2, \cdots, M.
\]

(5.3.2)

This subproblem is created when the first and second secondary users are to be allocated in the first frequency band. Moreover, a node is pruned or removed from the tree when the subproblem associated with that particular node is infeasible. It is obvious that any subproblems created from any nodes which are children nodes of an infeasible node are also infeasible. A global variable $\text{GlobalCost}$, which is initialized with infinity, is assigned to
Section 5.3. Fast Optimal Algorithm Based on Branch and Bound Method

1. Initialize $\text{GlobalCost} = \infty$, $\text{Cost} = 0$, $\text{Solution} = \text{infeasible}$, $\text{Level} = 0$, $U = \{1, 2, \ldots, K\}$, $\text{Temp} = \emptyset$, $\text{Node} = 0$.

2. Solve SDP relaxation problem in (5.2.4) with relaxed integer constraints and find the objective value.
   
   $\text{if}$ all solution vectors are with integer components $\text{then}$
   
   $\text{Solution} \leftarrow [s_1, s_2, \ldots, s_K]$ and go to 10.
   
   $\text{elseif}$ Objective value $< \text{GlobalCost}$ $\text{then}$
   
   $\text{Cost} \leftarrow$ objective value and go to 3.
   
   $\text{else}$
   
   go to 10.
   $\text{end if}$

3. Update $\text{Level} = \text{Level} + 1$, $\text{LevelTemp} = \emptyset$ and go to 4.

4. if $\text{Cost} \leq \text{GlobalCost}$ $\text{then}$
   
   $m \leftarrow 1$, $s_{\text{Level}} \leftarrow 0$ and go to 5.
   
   $\text{else}$
   
   go to 9.
   $\text{end if}$

5. Generate the node.
   
   $\text{if}$ $m = M + 1$ $\text{then}$
   
   $s_{\text{Level}} \leftarrow 0$.
   
   $\text{else}$ $s_{\text{Level}}(m) \leftarrow 1$.
   $\text{endif}$

   $\text{Node} \leftarrow \text{Node} + 1$, $\Omega(\text{Node}) \leftarrow [\Omega(\text{parentNode}) s_{\text{Level}}]$, store $\Omega(\text{Node})$ to this node and go to 6.

6. Solve SDP relaxation problem with the values stored to this node.
   
   $\text{Cost} \leftarrow$ objective value.

   $\text{if}$ the subproblem is feasible $\text{then}$
   
   $\text{if}$ $\text{Level} \neq K$ $\text{then}$
   
   Store the objective value to this node and append this node $\text{LevelTemp}$ and go to 7.
   
   $\text{else}$
   
   if $\text{Cost} \leq \text{GlobalCost}$ $\text{then}$
   
   $\text{GlobalCost} \leftarrow \text{Cost}$, $\text{Solution} \leftarrow [s_1, s_2, \ldots, s_K]$.
   
   $\text{endif}$
   
   $\text{endif}$

   $\text{else}$
   
   prune this node and go to 7.
   $\text{endif}$

7. Update $m = m + 1$

   $\text{if}$ $m \leq M + 1$ $\text{then}$
   
   go to 5.
   
   $\text{else}$
   
   go to 8.
   $\text{endif}$

8. Sort $\text{LevelTemp}$ in descending order according to the corresponding objective values and append to $\text{Temp}$. Empty $\text{LevelTemp}$.

9. if ($\text{Temp} \neq \emptyset$)

   Pick the last node from $\text{Temp}$ and get the stored values with this node and go to 4.

   $\text{else}$

   Go to 10.
   $\text{endif}$

10. Stop and display the $\text{Solution}$.

**Table 5.1.** Joint fast optimal radio resource allocation and beamforming algorithm based on BnB method
the objective values at the last level. By comparing all objective values at last level, the algorithm provides an optimal frequency band allocation which consumes the minimum transmit power while admitting as many as secondary users as possible.

5.4 Fast Suboptimal Algorithm Based on the BnB Method

The fast optimal algorithm based on the BnB method, proposed in the previous section, has relatively less complexity when compared to a full search algorithm. However, the complexity of the proposed algorithm can still be too high to find the optimal solution. As such, in this section, a suboptimal algorithm is proposed for the optimization problem in (5.1.6). The algorithm is developed based on the first feasible solution that the fast optimal algorithm achieves in the BnB method, i.e. the first branch it completes in the last level of the tree. Initially, this algorithm will solve the SDR problem with relaxed integer constraints (continuous vector variables with each element between zero and one). If all elements of every vectors are integer solutions, then this sub-optimal algorithm will yield an optimal solution and the algorithm will be terminated as in the optimal algorithm. In addition, if the relaxed problem is infeasible, then the original problem is also infeasible. Next, the branching step will be introduced to create subproblems by trying all possibilities of vector variables. Hence, the branches in the first level will be generated by allocating the first secondary user in all available frequency bands and the next level branch will be chosen with the minimum cost from the first level branches. Once, the algorithm has selected the branch to proceed to the next level, it will prune the rest of the branches in the previous level. Hence, it does not need to keep the cost of all branches and this significantly reduces the memory that is in use in the progress of the algorithm. But in the optimal algorithm, it is necessary to keep the
cost and frequency band allocation at each node until it is compared with
the node that generated in the last level and pruned. The selection of the
branch with minimum cost will lead to the next level and this will be con-
tinued until the last level. Since this suboptimal algorithm does not solve
most of the branches at each level in the progression, it potentially reduces
the complexity and solution time compared to the optimal algorithm. The
proposed suboptimal algorithm is summarized in Table 5.2.

The complexity of the optimal and the suboptimal algorithms is ana-
lyzed using interior point methods. In the worst-case, the optimal algorithm
will generate \( \sum_{i=1}^{K} (M + 1)^i + 1 \) number of subproblems for \( K \) number of
secondary users and \( M \) number of available frequency bands. Each sub-
problem is an SDP problem expressed in the primal standard form and it
can be efficiently solved by any general purpose SDP solver using interior
point methods [62]. In addition, each subproblem consists of \( KM \) matrix
variables of size \( N \times N \) and \( KM + ML_m \) linear constraints. Interior point
method will take \( O \left[ \sqrt{KM \log(\frac{1}{\epsilon})} \right] \) iterations to converge with \( \epsilon \) solution ac-
curacy at the termination of the algorithm. Each iteration requires at most
\( O \left[ K^3 M^3 N^6 + (KM + ML_m)KMN^2 \right] \) arithmetic operations in the worst-
case [62]. But actual complexity will be far less than this worst-case bound.
In the suboptimal algorithm, it will only solve \( K(M + 1) + 1 \) subproblems.
This number of subproblems significantly reduces the computational com-
plexity of the suboptimal algorithm as compared to the optimal algorithm.

5.5 Simulation Results

In order to assess the performance of the proposed admission control and
beamforming algorithms in a cognitive radio network, the number of admitt-
ted secondary users is computed and the corresponding frequency bands and
beamformers are determined. A cognitive radio network is considered with
Section 5.5. Simulation Results

1. Initialize $Cost = 0$, $Solution = infeasible$, $Level = 0$, $\Omega = \emptyset$, $CostTemp = \infty$.

2. Solve SDP relaxation problem in (5.2.4) with relaxed integer constraints and find the objective value.
   - if all solution vectors are with integer components then
     $Solution \leftarrow [s_1 \ s_2 \ \cdots \ s_K]$ and go to 7.
   - elseif problem is feasible then
     go to 3.
   - else
     go to 7.
   endif

3. Update $Level = Level + 1$, $m = 1$, $s_{Level} = 0$, $CostTemp = \infty$

4. Generate the node.
   - if $m = M + 1$ then
     $s_{Level} \leftarrow 0$ and go to 5.
   - else
     $s_{Level}(m) \leftarrow 1$ and go to 5.
   endif

5. Solve SDP relaxation problem with current value of $s_{Level}$ and $\Omega$.
   - if $CostTemp >$ objective value then
     $CostTemp \leftarrow$ objective value
   - if $Level = K$ then
     $Solution \leftarrow [s_1 \ s_2 \ \cdots \ s_K]$ and go to 7.
   - else
     $\Omega \leftarrow s_{Level}$.
   endif
   - else
     Prune this node.
   endif

6. Update $m = m + 1$
   - if $m \leq M + 1$ then
     go to 4.
   - else
     go to 3.
   endif

7. Stop and display the $Solution$.

Table 5.2. Joint fast suboptimal radio resource allocation and beamforming algorithm based on BnB method
Section 5.5. Simulation Results

six secondary users requesting channel access and two primary users who operate in two different frequency bands. The basestation of the secondary network consists of six transmitting antennas. Moreover, each secondary user and primary user terminals is equipped with a single receiver antenna. The channel gains between the secondary network basestation and secondary users as well as the primary users, are modelled using AOAs as follows:

\[ h = \alpha_0 \begin{bmatrix} 1 & e^{-j\theta_d} & e^{-j2\theta_d} & \ldots & e^{-j5\theta_d} \end{bmatrix}^T, \]  

(5.5.1)

where \( \alpha_0 \) is the channel power gain and \( \theta_d \) is the AOA of the signal. The AOA model is only considered to demonstrate the performance using beam patterns, however the proposed algorithm will also handle other random channels as described in the subsequent simulations. In the simulation, the channel power gain is set to \( \alpha_0 = 1 \). Two frequency bands are assumed to be available, one for each primary user. Frequency allocation and the corresponding beamformer design are evaluated for a particular set of AOAs of secondary users and primary users. The AOAs of six secondary users are fixed at \(-45^\circ, 45^\circ, 50^\circ, 55^\circ, -55^\circ\) and \(-50^\circ\) in both frequencies. Moreover, the AOAs of two primary users are fixed at \(180^\circ \) and \(180^\circ \) respectively. The AOAs in both frequency bands have been set identical only to demonstrate the principle but they can be set to different values without affecting the scope of the algorithm. The target SINRs for the secondary users and the upper bound on the interference power constraint of the primary users have been set to 10 dB and 0.1, respectively. The noise power at the primary and secondary receivers has been set to 0.01. Figure 5.1 illustrates the corresponding beam patterns for the secondary users admitted in the both frequency bands. For this particular channel scenario, only five out six secondary users have been allocated, three in the first frequency band and two in the second frequency band. The proposed algorithm does not allocate
those secondary users that are close to each other in the same frequency band.

The performance of the proposed suboptimal algorithm in a cognitive radio network is assessed by evaluating the required average transmit power and average number of secondary users admitted to the cognitive radio network for optimal and suboptimal algorithms as shown in Figure 5.2 and Figure 5.3 respectively. In particular, a cognitive radio network is considered with four secondary users and two primary users. The basestation of the secondary network consists of four transmitting antennas and each primary and secondary user is equipped with a single receiver antenna. The channels between the secondary network basestation and all the secondary users as well as the primary users have been generated using zero-mean, circularly symmetric AWGN with unity variance. The total transmit power available at the secondary network basestation has been set to 5. Moreover, the noise power at each secondary user receiver and the upper bound

**Figure 5.1.** The frequency allocation and corresponding beamforming pattern in first frequency (left) and the second frequency (right).
Section 5.5. Simulation Results

Figure 5.2. The required average transmit power at the secondary network basestation for different target SINRs of the secondary users obtained using fast optimal and suboptimal algorithms. The maximum available power at the secondary network basestation is restricted to 5.

on the interference power constraint for each primary user have been set to 0.01 and 0.1, respectively. Figure 5.2 depicts the required average transmit power at the secondary network basestation for different target SINRs for optimal and suboptimal algorithms. Figure 5.3 depicts the average number of secondary users served while satisfying the QoS constraints (SINR in the x-axis) for optimal and suboptimal algorithms. The average values are computed using 10000 Monte carlo experiments. As can be seen in Figure 5.2 and Figure 5.3, the suboptimal algorithm requires more power while admitting less number of users compared to the optimal algorithm. However, suboptimal algorithm requires less computational complexity as compared to the optimal algorithm. In this specific problem, the optimal algorithm is four times more complex than the suboptimal algorithm.

Next, in order to validate the performance of the optimal algorithm, a
Section 5.5. Simulation Results

4.5

Figure 5.3. The average number of secondary users served with different QoSs by using optimal and suboptimal algorithms. The maximum available power at the secondary network basestation is restricted to 5.

The outage probability of not being able to admit all four users with the required QoS is also shown next to the graph. The optimal algorithm accommodates all four users with a higher probability while consuming less
transmit power as compared to the non-optimal scheme (fixing users in the frequency bands).

5.6 Conclusions

A cognitive radio network was considered where the number of secondary users requesting channel access exceeds the number of available frequency bands and spatial modes. This led to a multi-objective optimization problem with admission control and resource allocation problems under interference power constraints imposed by the primary users. In addition, SINR constraints are imposed by the secondary users as a QoS requirement. This multi-objective optimization problem has been formulated using a single cost function based on a mixed-integer programming framework to accommodate
as many secondary users as possible while satisfying all constraints including transmit power at the secondary network basestation. Subsequently, a fast optimal and suboptimal radio resource allocation and admission control algorithm have been proposed based on BnB to allocate optimally the primary frequency bands to multiple secondary users in a cognitive radio network. The optimal algorithm simultaneously and optimally assigns the maximum possible number of secondary users to the available frequency bands and designs the corresponding optimal beamformers to achieve the target SINRs for each secondary user while satisfying the interference constraints of the primary users. While the suboptimal algorithm performs inferior to the optimal algorithms in terms of transmit power and number of secondary users admitted, it has the advantage of reduced computational complexity.

5.7 Appendix

Proof: First it is shown that the original and the relaxed problems have the same dual problems. The original problem is defined as follows:

minimize $\mathbf{w}_{k,m}$

$$\sum_{k=1}^{K} \mathbf{w}_{k,m}^H \mathbf{w}_{k,m}$$

subject to

$$\mathbf{w}_{k,m}^H \mathbf{H}_{k,m} \mathbf{w}_{k,m} - \gamma_{k,m} \sum_{i=1,i \neq k}^{N_m} \mathbf{w}_{i,m}^H \mathbf{H}_{k,m} \mathbf{w}_{i,m} \geq \gamma_{k,m} \sigma^2_{k,m},$$

$$k = 1, \cdots, N_m,$$

$$\sum_{k=1}^{N_m} \mathbf{w}_{k,m}^H \mathbf{G}_{l,m} \mathbf{w}_{k,m} \leq \varepsilon_{l,m}, \quad l = 1, \cdots, L_m.$$  

(5.7.1)
The Lagrangian of the original problem can be written

\[
\mathcal{L}(w, \lambda, \mu) = \sum_{k=1}^{K} w_{k,m}^H w_{k,m} - \sum_{k=1}^{K} \lambda_k \left( w_{k,m}^H H_{k,m} w_{k,m} - \gamma_{k,m} \sum_{i=1, i \neq k}^{N_m} w_{i,m}^H H_{i,m} w_{i,m} - \gamma_{k,m} \sigma_{k,m}^2 \right) \\
+ \sum_{l=1}^{L} \mu_l \left( \sum_{k=1}^{N_m} w_{k,m}^H G_{l,m} w_{k,m} - \epsilon_{l,m} \right)
\]

\[
= \sum_{k=1}^{K} \lambda_k \gamma_{k,m} \sigma_{k,m}^2 - \sum_{l=1}^{L} \mu_l \epsilon_{l,m}
+ \sum_{k=1}^{K} w_{k,m}^H \left( I - \lambda_k H_{k,m} + \sum_{i=1, i \neq k}^{N_m} \lambda_i \gamma_{k,m} H_{i,m} + \sum_{l=1}^{L} \mu_l G_{l,m} \right) w_{k,m}
\]

(5.7.2)

where \( \lambda \) and \( \mu \) are non-negative Lagrangian multipliers and the dual problem is defined as

\[
\text{maximize } \lambda, \mu \quad \sum_{k=1}^{K} \lambda_k \gamma_{k,m} \sigma_{k,m}^2 - \sum_{l=1}^{L} \mu_l \epsilon_{l,m}

\text{subject to } I - \lambda_k H_{k,m} + \sum_{i=1, i \neq k}^{N_m} \lambda_i \gamma_{k,m} H_{i,m} + \sum_{l=1}^{L} \mu_l G_{l,m} \geq 0,
\]

\( k = 1, \ldots, K. \)

(5.7.3)
Similarly, the dual problem of the relaxed problem in (5.1.6) is derived from Lagrangian as follows:

\[ \mathcal{L}(W, \lambda, \mu) = \sum_{k=1}^{K} \text{Tr}\{W_{k,m}\} \]

\[ -\sum_{k=1}^{K} \lambda_k \left( \text{Tr}\{W_{k,m}H_{k,m}\} - \gamma_{k,m} \sum_{i=1, i\neq k}^{N_m} \text{Tr}\{W_{i,m}H_{k,m}\} - \gamma_{k,m} \sigma_{k,m}^2 \right) \]

\[ + \sum_{l=1}^{L} \mu_l \left( \sum_{k=1}^{N_m} \text{Tr}\{W_{k,m}G_{l,m}\} - \varepsilon_{l,m} \right) - \sum_{k=1}^{N_m} \text{Tr}\{Z_k W_k\} \]

\[ = \sum_{k=1}^{K} \lambda_k \gamma_{k,m} \sigma_{k,m}^2 - \sum_{l=1}^{L} \mu_l \varepsilon_{l,m} \]

\[ + \sum_{k=1}^{K} \text{Tr}\left\{W_{k,m} \left( I - \lambda_k H_{k,m} + \sum_{i=1, i\neq k}^{N_m} \lambda_i \gamma_{k,m} H_{k,m} + \sum_{l=1}^{L} \mu_l G_{l,m} - Z_k \right) \right\} \]

(5.7.4)

where \( \lambda \) and \( \mu \) are non-negative Lagrangian multipliers and \( Z \) is a positive semidefinite matrix. Since \( Z \) is a positive semidefinite matrix, the dual problem can be stated as follows:

**maximize** \( \lambda, \mu \)

\[ \sum_{k=1}^{K} \lambda_k \gamma_{k,m} \sigma_{k,m}^2 - \sum_{l=1}^{L} \mu_l \varepsilon_{l,m} \]

**subject to** \( I - \lambda_k H_{k,m} + \sum_{i=1, i\neq k}^{N_m} \lambda_i \gamma_{k,m} H_{k,m} + \sum_{l=1}^{L} \mu_l G_{l,m} \succeq 0, \)

\[ k = 1, \ldots, K. \]

(5.7.5)

From both the dual problems, it is shown that both original and relaxed problems have the same dual problem. However since the relaxed problem is convex and satisfies Slater’s condition, it holds a strong duality between relaxed problem and its dual problem [57]. Next the Hessian matrix of the
Lagrangian of the original problem is derived as

$$\nabla_{ww'} = \sum_{k=1}^{K} \left( I - \lambda_k H_{k,m} + \sum_{i \neq k} \lambda_i \gamma_{k,m} H_{k,m} + \sum_{l=1}^{L} \mu_i G_{l,m} \right).$$

(5.7.6)

According to the work in [106], provided the Hessian is positive semidefinite, there is a strong duality between the original problem and its dual problem. It is apparent, provided that the relaxed problem as in (5.1.6) is feasible, that the constraint of its dual problem in (5.7.5) is satisfied. This implies that it holds strong duality between the original problem and its dual problem. Hence both the original and the relaxed problem have same the dual problem. Therefore the minimum achieved in the relaxed problem is the global minimum of the original problem which completes the proof.
SINR BALANCING
TECHNIQUES FOR
UNDERLAY COGNITIVE
RADIO NETWORKS

SINR balancing technique is a typical approach used in a multiuser communication system to allocate transmit power between users fairly when there is only an inadequate power to satisfy the SINR requirements of all users [41]. This approach jointly designs the transmit beamformers and the power allocations such that the ratios between the achieved SINR and the target SINR are balanced for all users. This is performed by maximizing the worst SINR while satisfying the transmit power constraint. In this chapter, SINR balancing technique for an underlay cognitive radio network is explored with different types of interference constraints on primary users.

6.1 Introduction

The power allocation and beamforming problems for multiuser systems have been widely studied for traditional wireless network over the past decade [36,37,58–60,107]. In [60], an optimal downlink power assignment technique
has been proposed for a given set of beamforming weight vectors. This power allocation problem is solved by finding the eigenvector corresponding the largest eigenvalue of the matrix. In [58], an iterative algorithm has been proposed to jointly design the transmit beamformers and the power allocations to satisfy the SINR constraints where the uplink-downlink duality was first observed. Later, it was shown that both uplink and downlink have the same SINR achievable regions, when the total available transmit power approaches infinity with absence of noise in [44, 108]. In [41, 42, 75], it has been proved that these achievable regions are the same with equal noise power at the receiver and the transmitter. SINR balancing technique has been presented in [41], where the beamformers designed in the uplink, have been employed in the downlink using the concept of uplink-downlink duality [43].

The beamformer design for cognitive radio networks, however, substantially differs from that for traditional wireless networks due to additional interference constraints on primary users. In [82, 83], robust cognitive radio network beamformers have been designed that meet specific target secondary user SINRs in the presence of CSI errors. The latter approach obtains the beamformer weights and corresponding power allocations through achieving the target SINR for each secondary user within the given power budget while ensuring that the interference leakage to primary users is below specific thresholds. However, it is quite difficult to predict in advance whether the problem with certain given SINR targets, interference thresholds and total power budget is feasible. In this chapter, to avoid infeasibility issues, two SINR balancing techniques have been proposed for an underlay cognitive radio network. The first approach is based on direct extension of the SINR balancing technique proposed in [41] for conventional multiuser network. According to this method, the transmit power is indirectly controlled by the primary user interference limit. A drawback associated with this
scheme is that the total available transmit power may not be fully utilized by controlling the beam pattern towards the primary user direction. To overcome this, a novel SINR balancing approach is proposed for cognitive radio network. These SINR balancing techniques have been developed using max-min fairness approach with additional primary user interference constraints. An SNR balancing technique has been proposed in the context of broadcast cognitive radio network beamforming [80]. However, the broadcast approach of [80] assumes that the same data stream is transmitted to all users and, therefore, it determines a single beamforming weight vector without taking into account cross-talk interference. Hence, cognitive radio network downlink beamforming problem considered in this chapter (which assumes that different data streams are sent to different users and employs the SINR rather than SNR criterion) significantly differs from that of [80].

6.2 System Model

In this chapter, a cognitive radio network is considered with \( L \) primary users and \( K \) secondary users. Each of the primary user and the secondary users consists of single antenna. The secondary network base station is equipped with \( N_t \) transmit antennas. The signal transmitted by the secondary network base station is

\[
x(n) = \hat{U} s(n),
\]

where \( s(n) = [s_1(n) \ s_2(n) \ \cdots \ s_K(n)]^T \) and \( s_k(n), \ k = 1, 2, \ldots, K \) is symbol intended for the \( k^{th} \) secondary user, \( \hat{U} = [\sqrt{p_1}u_1 \ \sqrt{p_2}u_2 \ \cdots \ \sqrt{p_K}u_K] \), \( \|u_k\|_2 = 1 \) and \( u_k \in \mathbb{C}^{N_t \times 1} \) is the transmit beamformer weight vector for the \( k^{th} \) secondary user. The variance of the symbol \( s(n) \) is assumed to be unity, i.e. \( \sigma_{s(n)}^2 = 1 \). The received signal at the \( k^{th} \) secondary user can be written as

\[
y_k(n) = h_{k}^H x(n) + \eta_k(n),
\]
where $h_k = [h_1^{(k)} h_2^{(k)} \cdots h_{N_t}^{(k)}]^{H}$ is the channel responses from the secondary network basestation to the $k^{th}$ secondary user and $\eta_k(n)$ is assumed to be zero mean circularly symmetric AWGN component with variance $\sigma_k^2$. Further, $p_k$ is the power allocated to the $k^{th}$ secondary user and the total transmit power at the secondary network basestation is denoted as $P_{\text{max}}$. The SINR of the $k^{th}$ secondary user can be written as

$$\text{SINR}_k(U, p) = \frac{p_k u_k^H R_k u_k}{\sum_{i \neq k} p_i u_i^H R_k u_i + \sigma_k^2}$$

(6.2.3)

where $R_k = h_k h_k^{H}$.

### 6.3 SINR Balancing Technique with Total Interference Constraint

In this section, SINR balancing technique proposed in [41] is extended to serve multiple secondary users in the downlink while imposing constraints on interference temperature of primary users. In this approach, the total interference leakage to primary users is considered in an underlay cognitive radio network. It is shown that when the set interference temperatures is fixed, the proposed SINR balancing technique will always have a unique solution that is identical to SDP based optimal solution. The advantages and disadvantages of the SINR balancing technique and SDP based techniques are also discussed. The interference leakage to the $l^{th}$ primary user can be written as

$$\varepsilon_l = E\{h_l^H x(n)x(n)^H h_l\}$$

$$= h_l^H E\{[\sum_{k=1}^K \sqrt{p_k} u_k s_k(n)] [\sum_{m=1}^K \sqrt{p_m} u_m s_m(n)]^H\} h_l$$

$$= h_l^H (\sum_{k=1}^K p_k u_k u_k^H) h_l$$

$$= \sum_{k=1}^K p_k \|h_l^H u_k\|^2$$

(6.3.1)
where $h = [h_1^{(l)} \ h_2^{(l)} \ \ldots \ h_N^{(l)}]^H$ is the channel responses from the secondary network basestation to the $i^{th}$ primary user and $p_k$ is the power allocated to the $k^{th}$ secondary user. Let

$$g_l = [||h_1^H u_1||_2^2 \ ||h_2^H u_2||_2^2 \ \ldots \ ||h_N^H u_N||_2^2]^T.$$  \hspace{1cm} (6.3.2)

The interference leakage to the $i^{th}$ primary user is

$$\epsilon_i = g_l^T p,$$  \hspace{1cm} (6.3.3)

where $p = [p_1 \ p_2 \ \ldots \ p_K]^T$.

The SINR balancing for the downlink of a cognitive radio network is defined as

$$\max_{\mathbf{U}, \mathbf{p}} \ \min_{1 \leq k \leq K} \ \frac{\text{SINR}_k(\mathbf{U}, \mathbf{p})}{\gamma_k} \hspace{1cm} \text{subject to} \hspace{1cm} 1^T p \leq P_{\text{max}},$$

$$g_l^T p \leq P_{\text{int}}, \quad l = 1, \ldots, L,$$  \hspace{1cm} (6.3.4)

where $\mathbf{U} = [u_1 \ u_2 \ \ldots \ u_K]$, $\gamma_k$ is the target SINR of the $k^{th}$ secondary user, $P_{\text{int}}^{(l)}$ is the interference limit on the $l^{th}$ primary user and $1 = [1 \ 1 \ \ldots \ 1]$.

Downlink beamforming is more complicated compared to the uplink beamforming because for the downlink, SINR of each user is a function of beamforming weight vectors of all other users. In this problem, beamforming weight vectors and power allocation must be jointly optimized while satisfying the power constraints and interference constraints of all primary users. Moreover, in the uplink, power gain between the secondary user and the primary user is not a function of beamforming weight vectors, however in the downlink, the power gain between the secondary network basestation and the primary user is a function of beamforming weight vectors. Hence, in the downlink any changes in the beamforming weight vectors will change
the power gain, but this difficulty does not arise in the uplink problem.

6.3.1 Solution to the Single Primary User

The SINR balancing problem in a cognitive radio network with single primary user case is similar to the problem under the sum power constraint in [41]. Here, an iterative algorithm is proposed to obtain the joint optimal power allocation and beamforming matrix for SINR balancing in a cognitive radio network. The optimum beamforming matrix and power allocation must satisfy the following conditions to achieve a balanced SINR for all the secondary users.

\[ \frac{\text{SINR}_k(U, p)}{\gamma_k} = C_1(U, P^{(1)}_{\text{int}}), \quad \text{for} \quad k = 1, \ldots, K, \]

\[ p^T p \leq P_{\max}, \]

\[ g_k^T p \leq P^{(1)}_{\text{int}}, \]

where \( C_1(U, P^{(1)}_{\text{int}}) \) is the balanced SINR value and either the interference constraint or the total transmit power constraint must be satisfied with the equality. The first step of the iterative algorithm is to determine the optimum power allocation for a given beamforming matrix \( U \). The following equation is formulated using the power gain between the secondary network basestation and the primary user. This equation is identical to the one presented in [41], except for the explicit control on interference \( P^{(1)}_{\text{int}} \).

\[ \frac{1}{C_1(U, P^{(1)}_{\text{int}})} \mathbf{p} = \mathbf{D}\Psi(U)\mathbf{p} + \mathbf{D}\sigma, \quad (6.3.5) \]

where \( \sigma = [\sigma_1^2, \ldots, \sigma_K^2] \) and

\[ \mathbf{D} = \text{diag}\{\gamma_1/(u_1^H R_1 u_1), \ldots, \gamma_K/(u_K^H R_K u_K)\}. \quad (6.3.6) \]
Moreover $\Psi(U)$ is defined as follows:

$$\left[\Psi(U)\right]_{ik} = \begin{cases} u^T_k R_i u_k, & k \neq i; \\ 0, & k = i, \end{cases} \quad (6.3.7)$$

The following equation is obtained by multiplying both sides of (6.3.5) by $g_i = [g_1, \ldots, g_K]^T$. Note that power gain between the secondary network base station and the primary user is a function of the beamforming weight vectors.

$$\frac{1}{C_1(U, P_{\text{int}}^{(l)})} = \frac{1}{P_{\text{int}}^{(l)}} g_i^T D \Psi(U) p + \frac{1}{P_{\text{int}}^{(l)}} g_i^T D \sigma. \quad (6.3.8)$$

From (6.3.5) and (6.3.8), extended coupling matrix is formed as

$$\phi(U, P_{\text{int}}^{(l)}) = \begin{bmatrix} D \Psi(U) & D \sigma \\ \frac{1}{P_{\text{int}}^{(l)}} g_i^T D \Psi(U) & \frac{1}{P_{\text{int}}^{(l)}} g_i^T D \sigma \end{bmatrix}. \quad (6.3.9)$$

Note (6.3.9) is similar to (12) in [41], but instead of $P_{\text{max}}$, $P_{\text{int}}$ is directly used together with the primary user power gain vector $g$. By defining an extended power vector

$$p_{\text{ext}} = \begin{bmatrix} p \\ 1 \end{bmatrix}, \quad (6.3.10)$$

(6.3.5) and (6.3.8) can be formulated into an eigenvector problem as

$$\frac{1}{C_1(U, P_{\text{int}}^{(l)})} p_{\text{ext}} = \begin{bmatrix} D \Psi(U) & D \sigma \\ \frac{1}{P_{\text{int}}^{(l)}} g_i^T D \Psi(U) & \frac{1}{P_{\text{int}}^{(l)}} g_i^T D \sigma \end{bmatrix} p_{\text{ext}}. \quad (6.3.11)$$

From (6.3.11), it can be seen that $\frac{1}{C_1(U, P_{\text{int}}^{(l)})} (-\lambda_{DL})$ and $p_{\text{ext}}$ represent the eigenvalue and the corresponding eigenvector, respectively, of the extended coupling matrix. As stated in [41], from Perron-Frobenius theory [61], it is always possible to find a positive eigenvalue and the corresponding positive eigenvector for a non-negative matrix. Moreover, in [60], it has been proven
that only maximal eigenvalue and the corresponding eigenvector satisfy the positivity requirement. Next step of the iterative algorithm is to determine the optimum beamforming matrix for a given interference level for the primary user. From the uplink-downlink duality [43, 75, 76], it is well known that the beamforming vectors used in the uplink can be used to achieve the same SINR values in the downlink. In a cognitive radio network, the interference leakage to the primary user should be incorporated in finding optimum beamforming vectors in the uplink. To incorporate the primary user interference in the design of optimum beamforming vectors in the uplink, the following Lemma is required:

**Lemma 1:** For the downlink SINR balancing, for a given set of channels and beamforming matrix, each interference level for the primary user has a corresponding unique total power at the secondary network basestation. i.e. each \( P_{\text{tot}} \) has a corresponding unique \( P_{\text{tot}} \).

See Appendix (Section 6.6) for the proof.

Next determining the optimum beamforming weight vectors is considered in the virtual uplink. In [41], an iterative algorithm has been proposed to obtain the optimum beamforming weight vectors for sum power constraint. The uplink coupling matrix is defined as follows:

\[
\phi_2(U, P_{\text{tot}}) = \begin{bmatrix}
D\Psi(U)^T & D\sigma \\
\frac{1}{P_{\text{tot}}} 1^T D\Psi(U)^T & \frac{1}{P_{\text{tot}}} 1^T D\sigma
\end{bmatrix},
\]  

(6.3.12)

where the last row of the matrix controls the available total power. The power allocation of each user in the uplink is determined by finding the
eigenvector corresponding the maximum eigenvalue as follows [41]:

\[ \lambda_{UL} \mathbf{q}_{ext} = \begin{bmatrix} D\Psi(U)^T & D\sigma \\ \frac{1}{P_{tot}}1^T D\Psi(U)^T & \frac{1}{P_{tot}}1^T D\sigma \end{bmatrix} \mathbf{q}_{ext}, \quad (6.3.13) \]

where

\[ \mathbf{q}_{ext} = \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix}, \quad (6.3.14) \]

and \( \mathbf{q} \) is the uplink power allocation for each user in the virtual uplink mode. Similarly, in a cognitive radio network, power allocation can be controlled by primary user interference. In order to find the beamformers, the uplink coupling matrix is modified by incorporating the interference to the primary user as follows:

\[ \lambda(U, P_{int}^{(1)}) = \begin{bmatrix} D\Psi(U)^T & D\sigma \\ \frac{1}{P_{int}^{(1)}} \mathbf{g}_i^T D\Psi(U)^T & \frac{1}{P_{int}^{(1)}} \mathbf{g}_i^T D\sigma \end{bmatrix}, \quad (6.3.15) \]

where last row of the modified uplink coupling matrix ensures that the interference leakage to the primary user does not exceed the threshold. Moreover, it defines the total power that can be allocated in the downlink through \( P_{int} \) and power gain between the secondary network basestation and the primary user. The beamformer will be formulated by maximizing each virtual uplink SINR separately as presented in [41]

\[ \mathbf{u}_k = \arg \max \mathbf{u}_k \frac{\mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{\hat{Q}}_k \mathbf{u}_k}, \quad \text{subject to } \| \mathbf{u}_k \|_2 = 1, \quad (6.3.16) \]

where \( \mathbf{\hat{R}}_k = \mathbf{R}_k / \sigma_k^2 \) and \( \mathbf{\hat{Q}}_k = \sum_{i=1, i \neq k}^K q_i \mathbf{R}_i / \sigma_i^2 + 1 \). In [41], it has been proven that for a given amount of total power, the set of beamforming weight vectors obtained using the iterative algorithm is unique. Further, the last row of both the coupling matrices defines the same total power. Hence the
Section 6.3. SINR Balancing Technique with Total Interference Constraint

beamforming weight vectors obtained using both the coupling matrices will be the same. Hence every $P_{tot}$ has a unique $P_{int}$ and interestingly $P_{tot}$ and $P_{int}$ forms a one to one mapping. In [41], it was shown that the beamforming matrix and power allocation obtained using iterative algorithm are optimal. Since, the proposed algorithm generates the same beamforming matrix and power allocation, they are optimal.

The steps used to find optimum beamforming weight vectors are given

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>initialize: $q^{(0)} = [0, \cdots, 0]^T$</td>
</tr>
<tr>
<td>2.</td>
<td>$\hat{R}_k = R_k/\sigma_k^2$, $k = 1, \cdots, K$</td>
</tr>
<tr>
<td>3.</td>
<td>$\sigma_k^2 = 1$, $k = 1, \cdots, K$.</td>
</tr>
<tr>
<td>4.</td>
<td>repeat</td>
</tr>
<tr>
<td>5.</td>
<td>$n = n+1$</td>
</tr>
<tr>
<td>6.</td>
<td>$Q_k^{(n)} = \sum_{m=1, m \neq k}^{K} q_m^{(n-1)} \hat{R}_k + I$, $k = 1, \cdots, K$</td>
</tr>
<tr>
<td>7.</td>
<td>$u_k^{(n)} = \text{generalized eigenvector of } \hat{R}_k \text{ and } Q_k^{(n)}$</td>
</tr>
<tr>
<td>8.</td>
<td>$u_k^{(n)} = u_k^{(n)}/|u_k^{(n)}|_2$, $k = 1, \cdots, K$</td>
</tr>
<tr>
<td>9.</td>
<td>solve $\lambda(U, P_{int}^{(1)}) \begin{bmatrix} q^{(n)} \ 1 \end{bmatrix} = \lambda_{\max}(n) \begin{bmatrix} q^{(n)} \ 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>10.</td>
<td>until $\lambda_{\max}(n) - \lambda_{\max}(n-1) &lt; \varepsilon$</td>
</tr>
<tr>
<td>11.</td>
<td>solve $\phi(U, P_{int}^{(1)}) \begin{bmatrix} p^{(opt)} \ 1 \end{bmatrix} = \lambda_{\max}(n) \begin{bmatrix} p^{(opt)} \ 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>12.</td>
<td>if $1^T p^{(opt)} &lt; P_{max}$</td>
</tr>
<tr>
<td>13.</td>
<td>$P_{opt} = p^{(opt)}$</td>
</tr>
<tr>
<td>14.</td>
<td>else</td>
</tr>
<tr>
<td>15.</td>
<td>Replace step 9 by</td>
</tr>
<tr>
<td>16.</td>
<td>solve $\phi_2(U, P_{max}) \begin{bmatrix} q^{(n)} \ 1 \end{bmatrix} = \lambda_{\max}(n) \begin{bmatrix} q^{(n)} \ 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>17.</td>
<td>repeat steps 1 to 10.</td>
</tr>
<tr>
<td>18.</td>
<td>end</td>
</tr>
</tbody>
</table>

Table 6.1. Algorithm for SINR balancing for single primary user.

in Table 6.1. In the original problem in (6.3.4), two constraints are defined: primary user interference and available total power. In the design of beamforming weight vectors, the modified uplink coupling matrix incorporates only the primary user interference. After finding the power allocation in the downlink according to the beamforming weight vectors obtained in the
uplink, the required power could turn out to be greater than the maximum available power $P_{\text{max}}$. Hence, the available total power should be considered as a constraint in the algorithm. In order to introduce total power constraint, the following Lemma is required.

**Lemma 2:** Suppose $P_{\text{tot}}^{(1)}$ is the total power allocated using the primary user interference $P_{\text{int}}^{(1)}$. Any power allocation $P_{\text{tot}}^{(2)}$ less than $P_{\text{tot}}^{(1)}$ will cause an interference $P_{\text{int}}^{(2)}$ less than $P_{\text{int}}^{(1)}$. i.e. if $P_{\text{tot}}^{(1)} > P_{\text{tot}}^{(2)}$, then $P_{\text{int}}^{(1)} > P_{\text{int}}^{(2)}$.

See Appendix (Section 6.6) for the proof.

Hence, if the power utilization obtained using primary user interference control is more than the total power constraint, then the beamforming weight vectors can be formulated using uplink coupling matrix instead of modified coupling matrix.

### 6.3.2 Solution to Multiple Primary Users

For simplicity, however without loss of generality, first two primary users are only considered. The problem can be expressed as

$$\begin{align*}
\text{maximize} & \quad U, \mathbf{p} \\
\text{min} & \quad \min_{1 \leq k \leq K} \frac{\text{SINR}_k(U, \mathbf{p})}{\gamma_k} \\
\text{subject to} & \quad \mathbf{1}^T \mathbf{p} \leq P_{\text{max}}, \\
& \quad \mathbf{g}_1^T \mathbf{p} \leq P_{\text{int}}^{(1)}, \\
& \quad \mathbf{g}_2^T \mathbf{p} \leq P_{\text{int}}^{(2)}. \quad (6.3.17)
\end{align*}$$

The above problem based on two interference constraints can be decoupled into the following two sub-problems with single interference constraint:
Sub-problem 1:

\[
\begin{align*}
\text{maximize} \quad & \sum_{1 \leq k \leq K} \frac{\text{SINR}_k(U,p)}{\gamma_k} \\
\text{subject to} \quad & \mathbf{1}^T p \leq P_{\text{max}}, \\
& g^T_k p \leq p^{(1)}_\text{int}.
\end{align*}
\]  

(6.3.18)

Sub-problem 2:

\[
\begin{align*}
\text{maximize} \quad & \sum_{1 \leq k \leq K} \frac{\text{SINR}_k(U,p)}{\gamma_k} \\
\text{subject to} \quad & \mathbf{1}^T p \leq P_{\text{max}}, \\
& g^T_2 p \leq p^{(2)}_\text{int}.
\end{align*}
\]  

(6.3.19)

Next, it will be shown that only one of the solutions of the sub-problems in (6.3.18) or (6.3.19) will be optimal and will satisfy the constraints in both the sub-problems. Suppose \((U_1, p_1)\) and \((U_2, p_2)\) are the optimal solutions for the sub-problem 1 and sub-problem 2 respectively. In order to prove the decoupling property, the following two Lemmas are required.

**Lemma 3:** The solutions of both the sub-problems can not satisfy simultaneously, i.e. \(g^T_1 p_2 \leq p^{(1)}_\text{int} \) and \(g^T_2 p_1 \leq p^{(2)}_\text{int} \).

**Proof:** Let the optimal solution of sub-problem 1 satisfies the sub-problem 2. At the optimal solution of sub-problem 1, the interference constraint holds equality. i.e. \(g^T_1 p_1 = p^{(1)}_\text{int} \). Since it satisfies the sub-problem 2, the interference leakage to the second primary user should be less than the threshold. i.e. \(P^{(2,\text{new})}_\text{int} = g^T_2 p_1 \leq p^{(2)}_\text{int} \). For the single primary user case, it has been already proven that primary user interference and total power allocation at the secondary network base station form a one to one mapping. The amount of power that can be allocated using interference \(P^{(2)}_\text{int} \) is higher than the power
allocated using interference \( P_{int}^{(1)} \). Then, it is obvious that the inequality 
\[
\mathbf{g}_1^T \mathbf{p}_2 \leq P_{int}^{(1)}
\]
can not be satisfied. Hence, it is infeasible that both solutions of the sub-problems satisfy simultaneously.

Lemma 4: The inequalities \( \mathbf{g}_1^T \mathbf{p}_2 \geq P_{int}^{(1)} \) and \( \mathbf{g}_2^T \mathbf{p}_1 \geq P_{int}^{(2)} \) can not hold simultaneously.

Proof: Similar to proof of Lemma 3.

Next, it has been proven that the solution to one of the sub-problems will be the global optimum. For the single primary user case, it has been proven that the primary user interference and the total power at the secondary network basestation form a one to one mapping. In finding beamforming matrix, interference power can be replaced by the corresponding total power. Hence, any amount of power which is more than the corresponding total power would cause more interference to the primary user. In [41], it has been proven that the beamforming matrix and power allocation vector are unique and globally optimal for a given total power. From Lemma 3 and Lemma 4, both solutions of the sub-problems can not satisfy simultaneously. Hence, one of the solutions of the sub-problems which satisfies the constraints of both sub-problems is globally optimum.

From the above Lemmas and global optimality, it has been proven that two primary users SINR balancing problem can be decoupled into two sub-problems. Moreover, using mathematical induction method, it can be shown that the two primary user SINR balancing problem can be extended to multiple primary users. The SINR balancing algorithm for multiple primary users is given in Table 6.2.
Section 6.3. SINR Balancing Technique with Total Interference Constraint

| 1. initialize: n = 0  
| 2. repeat  
| 3. n = n + 1  
| 4. Find optimal beamforming matrix $U_o^{(n)}$ and power allocation $p_o^{(n)}$ for sub-problem n.  
| 5. Check whether $U_o^{(n)}$ and $p_o^{(n)}$ satisfy other sub-problems.  
| 6. if yes  
| 7. exit  
| 8. else  
| 9. continue  
| 10. until n = L |

Table 6.2. Algorithm for SINR balancing for multiple primary users.

6.3.3 Simulation Results

In order to validate the performance of SINR balancing in a cognitive radio network, the balanced SINR and the allocated total transmit power are computed for a given interference level of the primary user. A network with three secondary users and a primary user is considered in the simulation. The basestation of the secondary user consists of five transmitting antennas while both the primary user and the secondary users have only single antenna. The channels between the secondary network basestation and the secondary users as well as the primary user are assumed to be known to the secondary network basestation, and they have been generated using zero mean, unity variance, circularly symmetric AWGN. The primary user interference level and the noise power at the secondary user receiver have been set to $P_{\text{int}}^{(1)} = 1$ and $\sigma_k^2 = 0.05$ for $k = 1, 2, 3$ respectively. Target SINR of each user has been set to 20 dB and the available total transmit power at the secondary network basestation is restricted to 2. Table 6.3 provides the power allocation of each secondary user and balanced SINR for five different set of random channels. Then the achieved balanced SINR values have been set as target SINRs for the semidefinite optimization based problem [36,37,82,83].
Section 6.3. SINR Balancing Technique with Total Interference Constraint

<table>
<thead>
<tr>
<th>Channels</th>
<th>User 1 Power</th>
<th>User 2 Power</th>
<th>User 3 Power</th>
<th>Total Power</th>
<th>Balanced SINR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel 1</td>
<td>0.69</td>
<td>0.55</td>
<td>0.41</td>
<td>1.64</td>
<td>15.77</td>
</tr>
<tr>
<td>Channel 2</td>
<td>0.22</td>
<td>0.32</td>
<td>0.28</td>
<td>0.82</td>
<td>11.27</td>
</tr>
<tr>
<td>Channel 3</td>
<td>0.25</td>
<td>0.16</td>
<td>0.10</td>
<td>0.51</td>
<td>8.48</td>
</tr>
<tr>
<td>Channel 4</td>
<td>0.43</td>
<td>0.39</td>
<td>0.17</td>
<td>0.98</td>
<td>12.02</td>
</tr>
<tr>
<td>Channel 5</td>
<td>0.38</td>
<td>0.40</td>
<td>0.40</td>
<td>1.18</td>
<td>12.31</td>
</tr>
</tbody>
</table>

Table 6.3. Power allocation and balanced SINR by using the proposed SINR balancing technique for a cognitive radio network with three secondary users and one primary user.

<table>
<thead>
<tr>
<th>Channels</th>
<th>Target SINR (dB)</th>
<th>Required Total Power</th>
<th>User 1 Power</th>
<th>User 2 Power</th>
<th>User 3 Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel 1</td>
<td>15.77</td>
<td>1.64</td>
<td>0.69</td>
<td>0.55</td>
<td>0.41</td>
</tr>
<tr>
<td>Channel 2</td>
<td>11.27</td>
<td>0.82</td>
<td>0.22</td>
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<tr>
<td>Channel 3</td>
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<td>0.51</td>
<td>0.25</td>
<td>0.16</td>
<td>0.10</td>
</tr>
<tr>
<td>Channel 4</td>
<td>12.02</td>
<td>0.98</td>
<td>0.43</td>
<td>0.39</td>
<td>0.17</td>
</tr>
<tr>
<td>Channel 5</td>
<td>12.31</td>
<td>1.18</td>
<td>0.38</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 6.4. Target SINR and required total power using SDP.

When the balanced SINR values obtained in Table 6.3 have been set as the targets and the interference temperature has been set to 1, the SDP based solution provided the same power allocation and beamforming weight vector for each user. This confirms that proposed SINR balancing techniques performs as good as the SDP based solutions. However, there are advantages and disadvantages over both methods. First, if SINR balancing could not attain a specific SINR, it will always provide a solution with a balanced SINR less than the specific SINR value. However, for the SDP based method, if the target can not be achieved with the given maximum available transmit power, the problem will be classified as infeasible, and it should be tried a different set of target SINRs less than the original target SINRs until a feasible solution is found. This is the disadvantage of SDP based solution for this cognitive radio problem.

However, the disadvantage associated with the proposed SINR balancing
method is as follows. In the SINR balancing method, a maximum interference temperature has been set, and allowed the SINR balancing method to obtain its natural solution in terms of the transmit power and balanced SINR values. For example, for the first simulation result in Table 6.3, an SINR value of 15.77 dB has been achieved for all three users with a power allocation of 1.64. Suppose, if the allocated power is increased above 1.64, the achieved SINR values will also increase, as well as the interference leakage to the primary users. Hence, an SINR value above 1.64 can not be obtained without increasing the interference temperature for the SINR balancing method.

For the SDP based problem, however, it is possible to set a higher SINR target (for example 18 dB) with the same interference temperature, and if the solution is feasible, the SDP will find this solution with a power consumption above 1.64. In this case, a different set of beamformer weight vectors will be obtained, possibly by steering nulls along the directions of the primary users. However, the proposed SINR balancing technique does not have this capability, i.e. steering appropriate nulls along the direction of primary users and increase the transmit power to achieve a higher target SINR values. This requires further study and development of new SINR balancing algorithms for cognitive radio. Hence, in the next section, a novel SINR balancing technique is proposed for a cognitive radio network by maximizing the worst case secondary user SINR and fully utilizing the available total transmit power. However, this problem is solved by considering interference leakage by each secondary user rather than total interference leakage from all secondary users.
In this section, a novel SINR balancing technique is proposed for a downlink cognitive radio network wherein multiple secondary users coexist and share the licensed spectrum with the primary users using the underlay approach. The proposed beamforming technique maximizes the worst secondary user SINR while ensuring that the interference leakage to primary users is below specific thresholds. Here also, due to the additional interference constraints imposed by primary users, the principle of uplink-downlink duality used in the conventional downlink beamformer design cannot be directly applied anymore. To circumvent this problem and the disadvantages discussed in the previous method, using an algebraic manipulation on the interference constraints, a novel SINR balancing technique is proposed for cognitive radio networks based on uplink-downlink iterative design techniques. Simulation results illustrate the convergence and the optimality of the proposed beamformer design.

The technique proposed in the previous section is a direct extension of the SINR balancing technique of [41], that is, the uplink and downlink power allocation algorithms have been modified in to explicitly include the primary user interference constraint. However, in the approach proposed in this section, the uplink-downlink duality is used iteratively to obtain the optimum beamformer weights in the virtual uplink mode and power allocations in the downlink mode. This way, the worst secondary user SINR is maximized and, at the same time, it is ensured that the interference leakage to primary users is below given thresholds. The underlying idea is to use the downlink power allocation values obtained in the previous iterations to optimally determine the attenuation required along the direction of the primary users when beamformers are designed in the virtual uplink mode. In each itera-
tion, the beamformers designed in the virtual uplink mode will be used in the downlink mode to optimally determine the downlink power allocations.

The interference power leakage to the $l$th primary user due to the $k$th secondary user transmission is given by

$$
\varepsilon_{l,k} = p_k \| \tilde{h}_l^H \mathbf{u}_k \|_2^2,
$$

(6.4.1)

where $\tilde{h}_l = [\tilde{h}_l^{(1)} \cdots \tilde{h}_l^{(L)}]^H$ is the channel vector from the secondary network basestation to the $l$th primary user. The SINR balancing technique with individual interference constraints on the primary users can be stated as

$$
\text{maximize } U, p \quad \min_{1 \leq k \leq K} \frac{\text{SINR}_k(U, p)}{\gamma_k}
$$

subject to

$$
1^T p \leq P_{\text{max}},
$$

$$
p_k \| \tilde{h}_l^H \mathbf{u}_k \|_2^2 \leq P_{\text{int}}^{(l)},
$$

$$
k = 1, \ldots, K, \quad l = 1, \ldots, L,
$$

(6.4.2)

where $U = [\mathbf{u}_1 \cdots \mathbf{u}_K]$, $\gamma_k$ is the target SINR for the $k$th secondary user, $P_{\text{int}}^{(l)}$ is the interference limit on the $l$th primary user due to the transmission of each secondary user, $p = [p_1 \cdots p_K]^T$ and $1 \triangleq [1 \ 1 \ \cdots \ 1]$. Note that another formulation has been proposed in [109] where the SINR has been maximized by combining multiple linear constraints into a single constraint via auxiliary variables. These auxiliary variables are optimized in [109] using a subgradient algorithm which normally requires a large number of iterations.

### 6.4.1 Algorithmic Solution to SINR Balancing

Here, an iterative algorithm is proposed to balance the SINR of secondary users while satisfying the constraints in (6.4.2). Using the principle of uplink-downlink duality, the beamformer designed in the virtual uplink mode can be used in the downlink mode to achieve the same SINR values by choosing
appropriate downlink power allocations \([41]\). However, the uplink beamformer design should consider the amount of interference that will leak to the primary users if the same beamformers are to be used in the downlink mode with the corresponding power allocation \(p_k\). Hence, the beamformer design in the virtual uplink mode for the \(k\)th secondary user for a given uplink power allocations \(q_k\) and downlink power allocations \(p_k\) (obtained from the previous iteration) can be stated as follows:

\[
\begin{align*}
\text{maximize} & \quad \frac{u_k^H \tilde{R}_k u_k}{u_k^H \tilde{Q}_k u_k} \\
\text{subject to} & \quad p_k \| \tilde{h}_l^H u_k \|_2^2 \leq P_{\text{int}}^{(l)}, \quad l = 1, \ldots, L, \\
& \quad \| u_k \|_2 = 1,
\end{align*}
\]

(6.4.3)

where \(\tilde{R}_k = R_k / \sigma_k^2\) and \(\tilde{Q}_k = \sum_{i=1, i \neq k}^K q_i R_i / \sigma_i^2 + I\). The problem in (6.4.3), as it stands, is not convex. By introducing a new variable \(w_k = \tilde{Q}_k^{1/2} u_k\) and setting \(w_k^H w_k = 1\), the virtual uplink beamformer design in (6.4.3) can be reformulated as

\[
\begin{align*}
\text{maximize} & \quad w_k^H \tilde{Q}_k^{-1/2} \tilde{R}_k \tilde{Q}_k^{-1/2} w_k \\
\text{subject to} & \quad w_k^H w_k = 1, \\
& \quad p_k \| \tilde{h}_l^H u_k \|_2^2 \leq P_{\text{int}}^{(l)}, \quad l = 1, \ldots, L.
\end{align*}
\]

(6.4.4)

The interference constraints in (6.4.4) have been set with the assumption that \(\| u_k \|_2 = 1\). Therefore, a difficulty arises since one cannot simultaneously set unity norm constraints for both \(w\) and \(u\). Hence, the dependency of \(u_k\)
Section 6.4. SINR Balancing Technique with Individual Interference Constraint

in (6.4.4) is eliminated while ensuring that $\|u_k\|_2 = 1$ as follows:

$$p_k \left\| \tilde{h}_l^H u_k \right\|_2^2 \leq P_{\text{int}}^{(l)}$$

$$\Rightarrow p_k \left\| \tilde{h}_l^H \frac{\tilde{Q}_k^{-1/2} w_k}{\bar{Q}_k^{-1/2} w_k} \right\|_2^2 \leq P_{\text{int}}^{(l)}$$

$$\Rightarrow p_k \frac{w_k^H \tilde{Q}_k^{-1/2} \tilde{H}_l \tilde{Q}_k^{-1/2} w_k}{w_k^H \bar{Q}_k^{-1} w_k} \leq P_{\text{int}}^{(l)}$$

$$\Rightarrow w_k^H \left[ p_k \tilde{Q}_k^{-1/2} \tilde{H}_l \tilde{Q}_k^{-1/2} - P_{\text{int}}^{(l)} \bar{Q}_k^{-1} \right] w_k \leq 0, \quad (6.4.5)$$

where $\tilde{H}_l = \tilde{h}_l \tilde{h}_l^H$. The problem in (6.4.4) is not convex yet. By defining a rank-one matrix $D_k \triangleq w_k w_k^H$, the problem in (6.4.4) can be reformulated as

$$\max_{D_k} \quad \text{Tr} \left\{ \tilde{Q}_k^{-1/2} \tilde{H}_l \tilde{Q}_k^{-1/2} D_k \right\}$$

subject to

$$\text{Tr} \{ D_k \} = 1,$$

$$D_k = D_k^H,$$

$$D_k \succeq 0,$$

$$\text{rank} \{ D_k \} = 1,$$

$$\text{Tr} \left\{ \left[ p_k \tilde{Q}_k^{-1/2} \tilde{H}_l \tilde{Q}_k^{-1/2} - P_{\text{int}}^{(l)} \bar{Q}_k^{-1} \right] D_k \right\} \leq 0, \quad \text{for } l = 1, \ldots, L, \quad (6.4.6)$$

where $D_k \succeq 0$ means that $D_k$ is a positive semidefinite matrix. By relaxing the rank constraint in (6.4.6), the latter problem becomes a SDP, and can be efficiently solved using interior point methods [62]. Using a similar approach as in [36, 37], it can be proved that the solution with the Lagrangian rank relaxation always provides a rank-one matrix $D_k$. The proof is based on deriving the dual functions of both the original and the relaxed problems and demonstrating that the duality gaps for both problems are zero. Therefore, no randomization techniques [63] are required for this problem. Once $D_k$ is
determined, \( w_k \) could be obtained from the principal eigenvector of \( D_k \) [36].

The power allocation method for virtual uplink and downlink modes is similar to that proposed in [41]. The uplink power allocation for a given set of beamformers is determined by using (6.3.13) and (6.3.14) [41]. These equations are presented in Section 6.3 where \( q \) is the uplink power allocation. Similarly, the optimum power allocation in the downlink mode is determined by using (6.3.10) and (6.3.11) [41]. These equations also provided in Section 6.3 where \( p \) is the downlink power allocation. Initially, the optimal virtual uplink beamformers and the optimal power allocation in the downlink mode are determined without any interference constraints, as in traditional case. In the second and subsequent iterations, the beamformers are designed in the virtual uplink mode using (6.4.6). In addition to \( \mathbf{R}_k \) and \( \mathbf{Q}_k \), the design in (6.4.6) also requires values of power \( p_k \) allocated for each secondary user in the downlink mode. For this, the values of transmitted power obtained in the downlink mode of the previous iterations are used. The beamformers obtained in the virtual uplink mode using (6.4.6) are then used in the downlink in (6.3.11) to determine optimum power allocation to maximize the worst downlink SINR in the subsequent iterations. This procedure should be repeated until convergence, as summarized in Table 6.5.

### 6.4.2 Simulation Results

To assess the performance of the proposed algorithm, the balanced SINR values and the power allocations are computed for each secondary user for given total transmit power (5) and primary user interference threshold (0.1). A cognitive radio network with three secondary users and two primary users has been assumed. The secondary network basestation consists of six antennas. The channel coefficients between the secondary network basestation and the secondary users as well as those between the secondary network basestation and the primary users are assumed to be known at the sec-
1. Balance SINR in the downlink mode without any interference constraints.
2. Determine uplink and downlink power allocations, \( p^{(1)} \) and \( q^{(1)} \), respectively.
3. \textbf{repeat}
4. \hspace{1em} \( n \leftarrow n + 1 \)
5. \hspace{1em} \textbf{repeat}
6. \hspace{2em} \( m \leftarrow m + 1 \)
7. \hspace{2em} Solve (6.4.6) with rank relaxation, \( 1 \leq k \leq K \).
8. \hspace{2em} Extract \( w_k \) from \( D_k \), \( 1 \leq k \leq K \).
9. \hspace{2em} \( u_k \leftarrow Q_k^{-1/2} w_k \), \( 1 \leq k \leq K \).
10. \hspace{2em} \( u_k \leftarrow u_k / \|u_k\|_2 \), \( 1 \leq k \leq K \).
11. \hspace{2em} Solve (6.3.13) to obtain \( \lambda_{UL}(m) \) and \( q_{\text{ext}}^{(m)} \).
12. \hspace{2em} until \( \lambda_{UL}(m) - \lambda_{UL}(m-1) < \epsilon \)
13. \hspace{2em} Solve (6.3.11) to obtain \( \lambda_{DL}(n) \) and \( p_{\text{ext}}^{(n)} \).
14. \hspace{2em} until \( \lambda_{DL}(n) - \lambda_{DL}(n-1) < \epsilon \)

\textbf{Table 6.5.} SINR balancing algorithm with individual interference power constraints.

Secondary network basestation, and they have been generated using zero mean circularly symmetric i.i.d. Gaussian random variables. The noise power at each secondary user receiver is set to 0.05. The stopping criterion \( \epsilon \) has been set to 0.001. The power allocations for each secondary user and the balanced SINR values obtained using the proposed algorithm are shown for five different random channels in Table 6.6. As the table reveals, the proposed technique uses all the possible transmit power and maximizes the worst secondary user SINR. To validate the optimality of the proposed beamformer design and power allocation, the results obtained from proposed method is compared with the SDP approach of [82] which is assumed to use the same SINR targets as obtained via the proposed method. For example, according to Table 6.6 the SINR target for Channel 1 and all three users is chosen in the approach of [82] to be 14.19 dB, while the primary user interference threshold has been set to 0.1 (to make it consistent to the proposed method). The results for the approach of [82] are shown in Table 6.7. Comparing Tables
6.4 and 6.7, it can be observed that the beamformer of [82] is equivalent to
the proposed one, as the values of power allocation obtained using the SDP
approach of [82] are the same as that obtained using the proposed method.
It has been also observed that the interference leakage values for all primary
users are equal 0.1 for both schemes. Therefore, both the proposed method
and the design of [82] provide similar solutions in the case considered. Note
that the SDP based method of [82] has been used just to demonstrate the
optimality of the proposed scheme. However, it should be stressed that the
approach of [82] cannot be directly applied to the considered scenario as
the maximum achievable SINR values are not known \textit{a priori}, while setting
other SINR targets can lead to infeasible beamformer designs.

It is also evaluated the number of iterations required for convergence of

<table>
<thead>
<tr>
<th>Channels</th>
<th>User 1 Power</th>
<th>User 2 Power</th>
<th>User 3 Power</th>
<th>Total Power</th>
<th>Balanced SINR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel 1</td>
<td>0.55</td>
<td>2.37</td>
<td>2.08</td>
<td>5</td>
<td>14.19</td>
</tr>
<tr>
<td>Channel 2</td>
<td>1.73</td>
<td>1.45</td>
<td>1.83</td>
<td>5</td>
<td>20.02</td>
</tr>
<tr>
<td>Channel 3</td>
<td>1.96</td>
<td>1.71</td>
<td>1.34</td>
<td>5</td>
<td>18.48</td>
</tr>
<tr>
<td>Channel 4</td>
<td>1.08</td>
<td>0.69</td>
<td>3.23</td>
<td>5</td>
<td>17.54</td>
</tr>
<tr>
<td>Channel 5</td>
<td>1.86</td>
<td>1.66</td>
<td>1.48</td>
<td>5</td>
<td>17.86</td>
</tr>
</tbody>
</table>

Table 6.6. Power allocations and achieved SINRs of the proposed method.

<table>
<thead>
<tr>
<th>Channels</th>
<th>Target SINR (dB)</th>
<th>Maximum available power</th>
<th>User 1 Power</th>
<th>User 2 Power</th>
<th>User 3 Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel 1</td>
<td>14.19</td>
<td>5</td>
<td>0.55</td>
<td>2.37</td>
<td>2.08</td>
</tr>
<tr>
<td>Channel 2</td>
<td>20.02</td>
<td>5</td>
<td>1.73</td>
<td>1.45</td>
<td>1.83</td>
</tr>
<tr>
<td>Channel 3</td>
<td>18.48</td>
<td>5</td>
<td>1.96</td>
<td>1.71</td>
<td>1.34</td>
</tr>
<tr>
<td>Channel 4</td>
<td>17.54</td>
<td>5</td>
<td>1.08</td>
<td>0.69</td>
<td>3.23</td>
</tr>
<tr>
<td>Channel 5</td>
<td>17.86</td>
<td>5</td>
<td>1.86</td>
<td>1.66</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Table 6.7. Target SINRs and achieved user powers of the SDP-based
method.

the proposed algorithm under the same settings as considered in Tables 6.6
and 6.7. Figure 6.1 depicts the balanced SINR values versus the iteration number. It is noticed that, most of the time, the algorithm converges within fewer than seven iterations. To further validate this fact, the probabilities of the number of iterations required are displayed for convergence in Figure 6.2. The results in Figure 6.2 have been averaged over 2000 channel realizations and it has been observed that the algorithm always converges.

![Figure 6.1](image.png)

**Figure 6.1.** The convergence of the proposed SINR balancing technique versus the iteration number.

### 6.5 Conclusions

In this chapter, two SINR balancing techniques have been proposed for underlay cognitive radio networks. In the first approach, an iterative algorithm has been proposed by extending the technique proposed for traditional multiuser network. This algorithm does not have the potential to reduce the interference leakage to the primary users. In addition, total transmit power is also indirectly controlled by the primary user interference limit and hence the available total transmit power is not fully utilized to maximize SINRs of
For a given beamforming matrix, assume that

\[ p_1 = \begin{bmatrix} p_0 \\ 1 \end{bmatrix}, \]  

(6.6.1)
satisfies the matrix equation
\[
\frac{1}{C_1(U, P_{\text{int}}^{(1)})} \mathbf{p}_1 = \phi(U, P_{\text{int}}) \mathbf{p}_1,
\]
where
\[
\phi(U, P_{\text{int}}) = \begin{bmatrix}
\mathbf{D} \Psi(U) & \mathbf{D} \sigma \\
\frac{1}{P_{\text{int}}^{(1)}} \mathbf{g}_{\text{int}}^T \mathbf{D} \Psi(U) & \frac{1}{P_{\text{int}}^{(1)}} \mathbf{g}_{\text{int}}^T \mathbf{D} \sigma
\end{bmatrix}.
\] (6.6.2)

Hence \( \mathbf{p}_0 \) satisfies the following equation:
\[
\frac{1}{C_1(U, P_{\text{int}}^{(1)})} \mathbf{p}_0 = \mathbf{D} \Psi(U) \mathbf{p}_0 + \mathbf{D} \sigma.
\] (6.6.3)

By multiplying both sides of (6.6.3) by \( \mathbf{1}^T \), the following equation is obtained.
\[
\frac{1}{C_1(U, P_{\text{int}}^{(1)})} \mathbf{1}^T = \frac{1}{P_{\text{tot}}} \mathbf{1}^T \mathbf{D} \Psi(U) \mathbf{p}_0 + \frac{1}{P_{\text{tot}}} \mathbf{1}^T \mathbf{D} \sigma,
\] (6.6.4)

where \( P_{\text{tot}} = \mathbf{1}^T \mathbf{p}_0 \).

From the equations (6.6.3) and (6.6.4), the downlink coupling matrix for sum power constraint can be defined as follows:
\[
\phi_1(U, P_{\text{tot}}) = \begin{bmatrix}
\mathbf{D} \Psi(U) & \mathbf{D} \sigma \\
\frac{1}{P_{\text{tot}}} \mathbf{1}^T \mathbf{D} \Psi(U) & \frac{1}{P_{\text{tot}}} \mathbf{1}^T \mathbf{D} \sigma
\end{bmatrix}.
\] (6.6.5)

The same power allocation \( \mathbf{p}_1 \) satisfies the following matrix equation as well.
\[
\frac{1}{C_1(U, P_{\text{int}}^{(1)})} \mathbf{p}_1 = \phi_1(U, P_{\text{tot}}) \mathbf{p}_1
\] (6.6.6)

In [60], it has been already proven that for a non-negative matrix, only maximal eigenvalue and corresponding eigenvector satisfy positivity. Hence \( \mathbf{p}_1 \) satisfies the (6.6.3) and (6.6.6), power allocations obtained using both \( \phi_1(U, P_{\text{tot}}) \) and \( \phi(U, P_{\text{int}}) \) will be the same.
6.6.2 Proof of Lemma 2

It has been already proven that $P_{\text{tot}}$ and $P_{\text{int}}$ are one to one mapping. Then $P_{\text{tot}} vs P_{\text{int}}$ should be either increasing function or decreasing function. It is obvious that when $P_{\text{tot}}$ increases, $P_{\text{int}}$ also will increase.

\[\blacksquare\]
Chapter 7

SUMMARY, CONCLUSION
AND FUTURE WORK

Exponential growth in wireless applications and mobile phone users has created a huge demand for wireless resources and opened up new challenges for researchers in the last decade. Specifically, spectrum has become overcrowded and has created spectrum crisis for the current and future wireless applications. However, different spectrum occupancy measurements revealed that most of the time, the licensed frequency bands are not in use by license holders and resulted in inefficient spectrum utilization. In addition to these results, the idea of opportunistic spectrum access has brought up cognitive radio technology which can be developed on software defined radios. This intelligent wireless communication system enables the secondary users to access the spectrum without disturbing the primary users' communication. Spectrum sensing and exploiting the available spectrum are the main functions of a cognitive radio network and these functions are performed by monitoring the environment and adjusting the transmission parameters such as frequency bands, spatial radiation pattern, operating power and modulation schemes in the system.
7.1 Summary and Conclusions

This thesis has studied various spatial diversity techniques and resource allocation problems for an underlay cognitive radio networks using convex optimization techniques. Spatial diversity techniques have been developed based on SDP and robust schemes have also been proposed for scenarios where there is some uncertainty on the CSI.

The first chapter provided the motivation of the thesis, the problems that need to be addressed for the current and future wireless networks. In addition, an introduction to cognitive radio, different types of cognitive radio networks and the outline of the thesis have also been discussed briefly.

In Chapter 2 various types of beamforming techniques used in conventional wireless networks have been provided. This included multiple sidelobe canceller, MMSE, SINR maximization approach and LCMV for a narrow band beamformer design. Adaptive beamforming, transmit beamforming and joint transmitter-receiver beamforming techniques have also been presented briefly. Recent work related to beamforming techniques, power allocation and admission control techniques have been discussed for traditional wireless networks and cognitive radio networks.

Chapter 4 focused on spatial diversity techniques for an underlay cognitive radio networks. A downlink spatial multiplexing technique has been proposed to enable multiple secondary users to efficiently share the spectrum with primary users. The multiuser transmit beamformers have been designed by setting constraints on the interference leakage to primary users and SINR targets for secondary users. The proposed beamformers have minimized the total transmit power at the secondary network base station while achieving the required QoS for all secondary users. This problem has been formulated into a SDP and solved using convex optimization tool boxes. Initially, it has been assumed that perfect CSI is available at the secondary network.
basestation for both primary users and secondary users. However, there are some practical difficulties to perfectly estimate the channel between secondary network basestation and the primary users as well as the secondary users. Therefore, robust beamforming techniques have been proposed based on the worst-case performance optimization. The results demonstrated advantages of using interference control techniques in cognitive radio networks. Also, robust scheme satisfies interference constraints all the time while the non-robust scheme violates the interference bound 50 percent of the time.

Chapter 5 has provided admission control techniques for an underlay cognitive radio network. The solution to the problem discussed in Chapter 4, may become infeasible and it is very difficult to predict in advance whether the problem could be feasible. To avoid this feasibility issue, some of the secondary users should be dropped from the network and an admission control problem has arisen to optimally admit as many as secondary users as possible. An optimal algorithm has been proposed to admit the maximum possible number of secondary users over a number of available frequency bands, while satisfying the required QoS for each admitted user and interference constraint on the primary users. This algorithm has been developed based on the BnB method and complexity analysis has been provided. In addition, a sub-optimal algorithm has been proposed with less complexity based on BnB method. The performance of both algorithms has been compared in the simulation.

To overcome the infeasibility issue discussed in Chapter 4, an SINR balancing technique has been proposed for an underlay cognitive radio network in Chapter 6. An SINR balancing technique for cognitive radio network is significantly different from that for conventional wireless networks due to the additional interference constraints on primary users. An optimal algorithm has been proposed to balance the SINR values between secondary users with individual secondary user interference constraint. Here, the interference leak-
age to the primary users due to transmission from each secondary user is kept under a predefined threshold. To validate the optimality of the beamformer design and power allocations, the results have been compared with SDP based results. Both schemes have provided the same beamformer designs and power allocations. Another sub-optimal SINR balancing algorithm has been also provided with total primary user interference constraints. Here also, the optimality has been verified by comparing the results with SDP based results in the simulation.

7.2 Future Work

The potential areas of future research have been recognized. The techniques discussed in Chapter 4 using an SDP framework considered spatial multiplexing to serve multiple secondary users simultaneously with target SINR while imposing constraints on primary user interference with a cost function on total transmit power. It would be very interesting to extend this technique with per antenna power constraints in addition to the average transmit power constraint. This problem will probably require an SDP relaxation to exclude the non-convex rank-one constraint. If the solution of the relaxed problem does not yield a rank-one solution, randomization techniques [63] can be developed to compensate the Lagrangian relaxation on rank of the matrix.

So far only a single antenna was considered for the secondary receiver. MIMO systems have tremendous potential to increase the data throughput compared to a MISO system. Hence, in cognitive radio networks, employing multiple antennas at the receiver will significantly improve the data throughput and it will be very interesting to extend the work in Chapter 4 to a MIMO system. Moreover, in underlay cognitive radio network secondary users and primary users can transmit their data simultaneously. There-
fore, employing multiple antennas at the secondary user receivers will also be very beneficial in order to mitigate interference from the primary users. However, this problem will probably turn out to be a non-convex problem and a joint transmitter-receiver beamformer design is generally required. To overcome the non-convex issue, iterative methods can be exploited to design transmitter-receiver beamformers. The robust optimization techniques proposed in this thesis could also be applied to this MIMO based cognitive radio network.

In Chapter 6, an SINR balancing technique has been proposed by considering the interference leakage to each primary user form each secondary user communication. Another possible extension would be SINR balancing technique with total interference constraint, where the SINR ratios should be balanced while ensuring that the total interference leakage to each primary user due to all secondary user transmission is less than a certain value. This problem can be stated as follows:

$$\begin{align*}
\text{maximize} & \quad \min_{1 \leq i \leq K} \frac{\text{SINR}_i(U, p)}{\gamma_i} \\
\text{subject to} & \quad 1^T p \leq P_{\text{max}}, \\
& \quad g_l^T p \leq \epsilon_l, \quad l = 1, \ldots, L.
\end{align*}$$

(7.2.1)

where the matrix $U$ consists of all transmitter beamformers with unity gain and the power vector $p$ denotes the power allocation for all secondary users. The vector $g_l$ is the channel gain between secondary network basestation and the $l^{th}$ primary user. The parameters $P_{\text{max}}$ and $\epsilon_l$ are the total transmit power available at the secondary network basestation and interference threshold for the $l^{th}$ primary user. With total interference constraint on primary users, it is not straight forward to use the uplink-downlink duality to find the optimal power allocation and the beamformers. To solve this problem, iterative method will be required with a modification to the uplink-
downlink duality. Moreover it will be interesting to derive the condition for the convergence of the iterative method.

Transmit beamforming techniques rely on the CSI fed back from the receivers. The CSI available at the transmitter will normally be in error due to quantization of CSI and outdated channel information due to fading and feedback delays. Hence, developing robust SINR balancing technique based on worst-case performance optimization and probabilistic approaches would be another possible extension.

In Chapter 5, a joint resource allocation and admission control algorithms have been proposed based on BnB method. When the number of secondary users and number of available frequency bands for secondary user access increase, the complexity of these algorithm also increases exponentially. This radio resource allocation problem is originally a non-convex problem and NP hard in computational complexity. Therefore, different non-convex optimization methods could be investigated and the original problem can be formulated into one of the non-convex programming techniques. In addition, sub-optimal algorithms could be developed based on subgradient methods [110] and convex approximation techniques. The condition for the convergence and the performance degradation from the optimal algorithm could be investigated and derived analytically. Moreover, the number of iterations required for the convergence and order of computational complexity of the worst-case scenario can be analyzed based on the size of the problem. These algorithms could be extended to take into account possible errors in the CSI and robust beamforming and resource allocation algorithms could be developed using worst-case performance optimization. In [111], sensor selection via convex optimization has been already developed in this direction.

Resource allocation techniques in the absence of a centralized controller (for example basestation) is challenging, but an important problem and specifically relevant problem for cognitive radios. Resource allocation for
Section 7.2 Future Work

Spectrum sharing channels within the context of multiple users competing for available resources has been pursued as an active research recently [112-117]. When multiple users (also called players) compete for available resources such as spectrum and power, a game theory based solution is provided so that all users settle at an equilibrium, called Nash equilibrium and there is no incentive for any users deviating from this equilibrium [118]. The joint beamforming and power allocation can be extended to the decentralized set up where users compete for resources without a centralized controller. Developing the algorithms using game theory based distributed algorithms would be an interesting possible extension. In addition, the analytical challenge will be on the determination of conditions for existence and uniqueness of Nash equilibrium.

In addition to the evaluation of the physical layer performance for the secondary networks and primary networks for various channel fading environment, the performance evaluation of the networks based on the information theoretical methods will reveal more useful information about the networks. Specifically, channel capacity under a spectrum sharing set up with interference constraints to primary users can be derived analytically. There are two different capacity metrics, namely, ergodic capacity, which is the capacity metric suitable in systems with no delay constraint wherein the transmission time is long enough to reveal the long-term ergodic properties of the channel, and outage capacity which defines the rate that can be maintained in all fading states and is a more appropriate capacity notion in wireless systems that carry out real-time delay-sensitive applications. These capacities could be derived analytically in the future work for the techniques discussed in this thesis when only imperfect channel knowledge of the interference link is available to the secondary transmitters.
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