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Experimental Investigation of Damping Structural Vibrations Using the Acoustic Black Hole Effect

by

Elizabeth P. Bowyer

DOCTORAL THESIS

Submitted in partial fulfillment of the Requirements for the award of

Doctor of Philosophy

of Loughborough University

August 2012

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Dedicated to my grandparents
John and Moira Bowyer
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This thesis describes the results of the experimental investigations into some new geometrical configurations in plate-like structures materialising one-dimensional (1D) acoustic black holes for flexural waves (wedges of power-law profile) and two-dimensional (2D) acoustic black holes for flexural waves (circular indentations of power-law profile). Such acoustic black holes allow the user to reduce the amplitudes of the vibration responses of plate-like structures to a maximum effect, while not increasing the mass of the structures. This thesis also suggests some new ‘real world’ practical applications for this damping technique.

Initially, the effects of geometrical and material imperfections on damping flexural vibrations in plates with attached wedges of power-law profile (1D black holes) were investigated, demonstrating that this method of damping is robust enough for practical applications. Then, damping of flexural vibrations in turbofan blades with trailing edges tapered according to a power-law profile has been investigated. In addition, experimental investigations into power-law profiled slots within plates have been also conducted.

Another important configuration under investigation was that of circular indentations (pits) of power-law profile within the plate. In the case of quadratic or higher-order profiles, such indentations materialise 2D acoustic black holes for flexural waves. To increase the damping efficiency of power-law profiled indentations, the absorption area has been enlarged by increasing the size of the central hole in the pit, while keeping the edges sharp.

The next step of investigation in this thesis was using multiple indentations of power-law profile (arrays of 2D black holes). It was shown that not only do multiple indentations of power-law profile provide substantial reduction in the damping of flexural vibrations, but also a substantial reduction in radiated sound power. The experimental results have been obtained also for a cylindrical plate incorporating a central hole of quadratic profile. They are compared to the results of numerical predictions, thus validating the results and the experimental technique.

Investigations into the effects of indentations of power-law profile made in composite plates and panels and their subsequent inclusion into composite honeycomb sandwich panels are also reported. These indentations again act as 2D acoustic black holes for flexural waves and they effectively damp flexural vibrations within the panels. It was also demonstrated that these indentations can be enclosed in smooth surfaced panels and that no additional damping layer is required to induce the acoustic black hole effect in composite structures.

In conclusion, it has been confirmed in this thesis that one and two-dimensional acoustic black holes represent an effective method of damping flexural vibrations and reducing the associated structure-borne sound. Furthermore, this thesis has shown that acoustic black holes can be efficiently employed in practical applications, such as trailing edges of jet engine fan blades, composite panels, and composite honeycomb sandwich structures.
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<td>f</td>
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<td>n</td>
<td>Number of microphone positions</td>
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<td>$p_{rms}$</td>
<td>Root mean square pressure</td>
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<td>P</td>
<td>Power</td>
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<td>p</td>
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<td>R</td>
<td>Reflection coefficient</td>
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<td>r</td>
<td>Radius</td>
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<td>$\varepsilon$</td>
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<td>$\pi$</td>
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<td>$\rho$</td>
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<td>$\omega$</td>
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CHAPTER 1

INTRODUCTION

1.1 Introduction

Damping of unwanted structural vibrations remains a major issue in many branches of engineering, particularly in the transportation sector. The need for a reduction of undesirable vibration in structures has led to a significant amount of research into both the prediction of natural frequencies and also different methods which can be employed to manipulate them away from excitation frequencies or eliminate these resonances completely. Excessive vibrations can lead to reduced structural fatigue life resulting in cracks and failure, premature delamination of composite structures and other types of degraded performance. As the amplitude of flexural vibration of a structure is related to the amplitude of radiated structure-borne noise, the attenuation of flexural vibration can also reduce uncomfortable or distracting noise, which contributes to environmental noise pollution, an area that is now strictly regulated. Manufacturers must therefore reduce or manipulate the peak resonant frequencies away from sensitive regions (for example on an A-weighting curve).

There are many existing vibrational problems that have yet to find a solution. For example, in the automotive industry there is a major problem in modern SUV’s where the vibration from the engine is transmitted through the bulk head into the passenger and driver foot wells. The technique employed by car manufactures over the economic crisis has been to apply current damping techniques and then last minutes fixes before production. As the industry is now recovering, the emphasis has reverted to research and development. Permanent effective first time solution are now being sort.

The air transportation industry is not exempt from problems either, with numerous vibration and sound radiation problems emanating from aircraft engines and other inflight vibration sources such as air turbulence. Flexural waves are transmitted from the aircraft skin panels into the main structural components of the airframe. This vibration not only affects the fatigue life of the structure itself, but also the internal equipment that are affected by the structure-borne vibratory energy that can lead to equipment degradation or failure. These flexural vibrations are also transmitted to the aircraft cabin via the vibration of internal skin panels. The level of sound radiation from these panels can also affect the comfort level of the passengers within the aircraft.

Aircraft engines themselves are a sound radiation and vibration challenge for acousticians. Any damping measures cannot interfere with airflow or stringent safety regulations. There is however strict legislation governing the sound and vibration levels that an aircraft engine can produce. New efficient damping methods are thus required to enable these targets to be met.

Another such industry in need of new vibration damping techniques is that of space. For example, the vibro-acoustic levels at launch are such that delicate equipment and payloads such as sensors, optics and satellites can be destroyed.
It is not however, just the transportation industry that suffers from vibration issues, such problems can be found in machinery such as chain saws and road maintenance equipment, where vibrations that are not/cannot be controlled effectively result in severe medical conditions such as Vibration-induced white finger (VWF). This is the most common condition among the operators of hand-held vibrating tools. The vibration can cause damage to the tendons, muscles, bones and joints, and can also affect the nervous system. Collectively, these effects are known as Hand-Arm Vibration Syndrome (HAVS).

From the examples above it is clear that vibration damping has become an important factor in modern day structure and equipment design and is increasingly prominent in government legislation. There are many different methods and theories on how best to achieve the highest damping level which have been applied with varying levels of success. Integration of such damping techniques into existing structures can also prove problematic or impractical, resulting in undesirable side effects such as weight gain.

Despite the many vibration damping techniques available, there is as always room for improvement not only in damping efficiency but in applicable frequency range and reduction of undesirable side effects. There are also many situations where these current methods cannot be applied, leaving room for the development and application of new or recently proposed techniques to ‘real world structures’.

1.2 Review of the key methods of passive damping

In vibration control there are three standard methods used to reduce the vibration of a system; reduce the strength of the source, isolate the source, and damping. Damping can be split into three sub categories; passive, semi-active (also known as adaptive-passive) and active damping. This study is concerned with passive damping techniques.

Material damping is the most obvious and basic form of damping available. Different materials have different attenuation properties (material loss factors), and therefore some are more efficient at dissipating vibration energy. Materials with a higher material loss factor will dissipate transmitted energy more effectively than one with a low loss factor. It is not always possible to choose the material with the highest loss factor for each application as many other design considerations need to be considered.

There are many other methods of damping structural vibrations with increasing degrees of complexity. The most common of which being attaching absorbing layers to the surfaces of vibrating structures resulting in increase of structural wave attenuation (Heckl et al, 1998; Mead et al, 1998; Graff, 1975; and Ross, 1959). Visco-elastic damping attenuates the resonant vibrations excited in a structure by the shear forces induced in the material causing extension and contraction of the molecular bonds within it; this motion requires energy consumption to take place. This energy comes from the propagating wave as it causes the structure to deform. As the molecular bonds within the
damping layer return to their undisturbed state under their own elasticity, the energy which caused the extension in the first place is then dissipated as heat. However, it is well known that applying a layer of damping material directly to a vibrating plate structure does not necessarily lead to a significant reduction in the amplitude of vibration, as the plate bending stiffness and damping will dominate the response (Heckl et al, 1998; Mead et al, 1998). This method of damping also does not perform well at high temperatures.

Increasing the mass of a structure will alter its stiffness and location of its resonant frequencies, shifting the resonances to a higher frequency, which can be more easily damped using other damping methods such as visco-elastic damping mentioned above. A tuned mass damper may also be added to the structure so that a resonance can be effectively damped by acting against the motion of the initial vibration in order to cancel the vibration at the resonance. A tuned mass damper damps the vibration response of a structure at a particular resonant frequency and is commonly used in treating narrow-band noise and vibration problems.

Friction damping, as the name suggests, utilises friction via moving parts sliding over each other to convert from vibrational energy to heat. This type of absorber is frequently used in the automotive and aeronautical industries in brake pads and propulsion systems, as well as in earthquake protection for buildings to name but a few. This type of damper needs moving parts to enable friction to be produced and therefore can’t be applied to stationary systems.

Impact damping is another common damping method. Single particle damper damps the vibration of a system by utilizing a particle free to move on the surface of the component. When the structure is subjected to excitation the particle oscillates on and off the structure surface. This movement dissipates the vibration energy through loss of energy via friction and noise. The main disadvantage of impact dampers is that they are noisy and causing damage to the component when the particle collides with the structure. This method of damping is used to Earthquake-proof buildings, the particle in this case acting like a pendulum to counteract the vibrations in the building.

Particle damping is a derivative of impact damping. Unlike impact damping as such, which refers to only a single (usually quite large) mass in a cavity or building, particle damping uses small sized granular particles inside a cavity, Figure 1.1, or attached externally to the system in a container. The system of damping employed by this method is that of dissipation of vibration energy via a combination of loss mechanisms, including friction and momentum exchange. The vibration energy is dissipated during the impact of granular particles which move freely within the boundaries of a cavity with each other and the cavity walls.

It has been shown theoretically that this method of damping could be utilised within the space industry to reduce vibration in high power turbo-pumps that supply the propellants to the combustion chambers of rocket (Ehrgott, in press) and in turbine blades (Panossian, 1992). This method has even been applied to tennis rackets (Ashley, 1995).
Particle dampers are efficient at damping resonant frequencies through a wider range of temperatures and frequencies than an impact damper, they also due to the lack of large impacts result in a longer system and component life. However they still add mass to the system and require cavities to encase the particles or additional space on the outer surface of the structure to attach containers containing the particles.

Rheological fluids or smart fluids can also be used to damp a vibrating system. These fluids are ones whose viscosity can be controlled via the application of an electric current (Electro-rheological) or a magnetic field (Magneto-rheological). The change in viscosity is used to oppose the motion of the component and therefore attenuate the vibration response. One example of an application for this method of damping is in washing machines (Carlson, 2002) where magneto-rheological sponges are used to damp vibration of the washing machine during the spin cycle. In this type of damping the magnetic field is induced when the drum reaches resonance and is deactivated when the drum passes the resonant frequency. Development of this damping method in other industry applications has been restricted due to a lack of suitable fluids (Stanway, 1996).

An alternative way of damping applicable to finite structures is to reduce reflections of structural waves from their free edges and thus to reduce the amplitudes of structural resonances (Wise, 1984; Heckl et al, 1998; Mironov, 1988). One such method for attenuating structural wave reflections at the edges of plates and bars is by using a graded impedance interface. It has been shown (Vemula et al, 1996) that as much as 60-80% reductions in response can be achieved in a frequency range of 2-10 kHz, with greater damping occurring at higher frequencies for steel with a composite strip around the outer edge. Introducing additional damping in the gradation layers increases the damping efficiency of this method. This method of damping is complicated and manufacturing is not a simple process, the structure is also usually weakened towards it outer edge.

Another method of vibration damping that utilizes this principle is the use of wedges. It has been previously shown that as the thickness of the plate material decreases, the overall composite damping increases (Graff, 1975; Ross, 1959).
1.3 Theoretical and experimental background of the Acoustic black hole

As discussed above, a common means of damping resonant flexural vibration in a structure is to reduce the reflection of flexural elastic waves from the structures free edges. The ‘acoustic black hole effect’ utilizes this method of damping via the use of a wedge of power law profile. As was mentioned in the previous section, using wedges as dampers is not a new concept and has been utilized for many years (Graff, 1975; Ross, 1959), but no previous wedge dampers have shown the same potential as the ‘acoustic black hole’ in performance, ease of manufacture and feasibility. It is this method of damping flexural vibration in structures which this thesis endeavours to develop further into a method that can be utilized to provide substantial damping over a wide frequency range in a wide variety of applications. This section gives a review of the progression of the acoustic black hole effect.

The first theoretical paper describing propagation of a flexural wave in a wedge of power-law profile (quadratic wedge) was published by Mironov in 1988. It has been shown in that paper that a wedge of power law profile with the power exponential $m \geq 2$ acts as an acoustic black hole for flexural waves propagating towards its sharp edge by asymptotically decreasing the phase speed of the wave in such a way that the wave will never reach the end of the wedge and thus never reflect back into the plate (Mironov, 1988), Figure 1.1(a).

The power law profile described above is illustrated in Figure 1.1(b) and is defined by;

$$h(x) = \varepsilon x^m,$$  \hspace{1cm} (1)

where $h(x)$ is the local thickness, $m$ is the above-mentioned power exponential (a non-dimensional positive constant), $\varepsilon$ is a dimensional constant, and $x$ is the coordinate measured along the wedge axis.

It has been shown (Mironov, 1988) that reflections from a real (truncated) wedge are far from zero, and therefore in the majority of practical situations such wedges cannot be used for reduction of wave reflections. To resolve this problem it has been proposed (Krylov, 2001, 2002) to attach a damping
layer at the tip of a power-law wedge. The addition of a damping layer reduces the amplitude of the reflected wave very efficiently due to the decreased thickness of the profile at the wedge tip, this leads to an overall reduction in the plates vibration response. Such reduction of reflected waves achieved by using a combination of a wedge of power-law profile and of an attached damping layer has been termed the ‘Acoustic black hole effect’ (Krylov, 2001). This is due to the fact that when the wedge is viewed from a point outside of the profile a flexural wave enters the profile but ideally never returns, having been attenuated by the damping layer attached to the surface at the wedge tip.

In order to understand the basic principle of the Acoustic black hole effect it is instructive to consider the simplest case of a plane flexural wave propagating in the normal direction towards the edge of a free wedge described by a power-law relationship $h(x)=\varepsilon x^m$. Since flexural waves propagation in such wedges can be described in the geometrical acoustics approximation (Krylov et al, 2004; Krylov, 2004), the integrated wave phase $\Phi$ resulting from the wave propagation from an arbitrary point $x$ on the wedge medium plane to the wedge tip ($x=0$), which defines the wave solution in the geometrical acoustics approximation, can be written in the form:

$$\Phi = \int_0^x k(x)dx. \tag{2}$$

Here $k(x)$ is a local wavenumber of a flexural wave for a wedge in contact with vacuum: $k(x)=12^{1/4}k_p^{1/2}(\varepsilon x^m)^2$, where $k_p=\omega/c_p$ is the wave number of a symmetrical plate wave, $c_p=2c_l(1-c_l^2/c_t^2)^{1/2}$ is its phase velocity, and $c_l$ and $c_t$ are longitudinal and shear wave velocities in a wedge material, and $\omega=2\pi f$ is circular frequency.

It can be seen that the integral in Equation (2) diverges for $m \geq 2$. This means that the integrated phase $\Phi$ becomes infinite under these circumstances and the wave never reaches the edge. Therefore, it never reflects back either, i.e. the corresponding reflection coefficient $R_0$ is zero and the wave becomes trapped, thus indicating that the above mentioned ideal wedges can be considered as ‘acoustic black holes’ for incident flexural waves.

Real fabricated wedges, however, always have truncated edges, as was mentioned above. And this adversely affects their performance as acoustic black holes. If for ideal wedges of power-law profile (with $m \geq 2$) it follows from Equation (2) that even an infinitely small material attenuation, described by an imaginary part of $k(x)$, would be sufficient for the total wave energy to be absorbed, this is not so for truncated wedges. Indeed, for truncated wedges the lower integration limit in Equation (2) must be changed from 0 to a certain value $x_0$ describing the length of truncation. This implies that for typical values of attenuation in wedge materials, even very small truncations $x_0$ result in values of $R_0$ being as large as 50–70% (Krylov et al, 2004; Krylov, 2004), which destroys almost completely the expected acoustic black hole effect.
To radically improve the situation for real manufactured wedges (with truncations), it was proposed to cover the wedge surface by a thin damping layer (film) (Krylov et al, 2004; Krylov, 2004; Krylov et al, 2007) e.g. by polymeric films. Using the approach based on the geometrical acoustics approximation (Krylov et al, 2004;, Krylov, 2004), the corresponding analytical expressions have been derived for the reflection coefficients of flexural waves from the edges of truncated wedges covered by thin absorbing layers. For example, for a truncated wedge of quadratic shape, \( h(x) = \varepsilon x^m \), covered by a thin damping layer on one surface only, the following analytical expression has been derived for the resulting reflection coefficient \( R_0 \) at arbitrary distance \( x \) from the centre of the coordinate system \( x = 0 \) associated with the tip of an ideal (non-truncated) wedge (Krylov et al, 2004;, Krylov, 2004):

\[
R_0 = \exp(-2\mu_1 - 2\mu_2),
\]

where:

\[
\mu_1 = \frac{12^{1/4} k_p^{1/2} \eta}{4 \delta^{1/2}} \ln \left( \frac{\chi}{\chi_0} \right),
\]

\[
\mu_2 = \frac{3 \cdot 12^{1/4} k_p^{1/2} \nu \delta}{8 \delta^{3/2}} \frac{E_2}{E_1} \frac{1}{\chi_0^3} \left( 1 - \frac{x^2}{\chi^2} \right),
\]

Here \( \chi_0 \) is the length of the truncation, \( \eta \) and \( \nu \) are the energy loss factors for the wedge and film materials respectively, \( \delta \) is the film thickness, and \( E_1 \) and \( E_2 \) are Young’s moduli of the wedge and film materials; other notations have been explained earlier.

Note that Equations (3–5) for reflection coefficients of flexural waves are valid if the thickness of absorbing layers (films) \( \delta \) is much smaller than the local thickness of the main wedge. Therefore, although being very useful and simple, these expressions can only be applicable either to wedges covered by very thin absorbing layers (thin films) or to wedges with large values of truncation \( \chi_0 \).

To extend the analysis to smaller values of wedge truncation and/or to thicker damping films one has to consider flexural wave propagation in wedges covered by damping layers of arbitrary thickness. Such an analysis has been undertaken (Krylov, 2004) using a more general approach to the description of the effect of damping layers on complex flexural rigidity of a sandwich plate. Using this approach, a more general expression for the reflection coefficient \( R_0 \) of a quadratic wedge covered by an absorbing layer on one side has been derived, which is not reproduced here for shortness. Although this expression has been derived under a number of simplifying assumptions, it is rather cumbersome and contains the integration that should be carried out numerically.

Numerical calculations according to the above-mentioned simple Equations (3–5) show that for typical parameters of metal wedges and polymeric damping films the values of the reflection coefficient \( R_0 \) at frequencies around 10 kHz can be as low as 3–6%. Thus, in the presence of a damping film the values of the reflection coefficient from the tip of a truncated wedge of power-law profile are usually much smaller than those for a wedge with the same value of truncation \( \chi_0 \), but without a film. Obviously, it is both the specific geometrical properties of a power-law-shaped wedge in respect of flexural wave propagation and the effect of thin damping layers that result in such a
significant reduction in the reflection coefficient. Note that almost all absorption of the incident wave energy takes place in the vicinity of the sharp edge of a wedge. This is why it is sufficient to attach only a narrow strip of damping layer at the tip of a wedge in order to achieve the maximum damping.

As discussed above the length of truncation of a wedge of power-law profile is fundamental in the effectiveness of the wedge as a damper. When a wedge of a set power-law profile is truncated at increasingly shorter lengths the truncation thickness increases along with the reflection coefficient from the truncated end, Figure 1.3 (Krylov, 2004). This result is vital when considering practical application of the Acoustic black hole effect.

![Figure 1.3](image)

Figure 1.3 Effect of truncation of a wedge of power-law profile on the reflection coefficient for an uncovered wedge (solid line) and a wedge of power-law profile with damping layer (dashed line)

The efficiency of wedges of power-law profile with attached damping layers for vibration damping has been confirmed experimentally by measuring point mobilities on an experimental wedge of quadratic profile (Krylov et al, 2007).

The first theoretical paper proposing to establish 2D Acoustic black holes as a solution to the damping of flexural vibrations in plates and considering the behaviour of bending waves in the vicinity of both 1D and 2D Acoustic black holes has been published in 2007 (Krylov, 2007). A 2D Acoustic black hole is described as a nearly protruding cylindrically symmetric pit of power-law profile in a plain plate, Figure 1.4, with a thin absorbing layer attached to the bottom of the plate beneath the centre of the pit, (Krylov, 2007). This investigation has shown promising theoretical results that, with thin absorbing layers on the wedge tips of power-law wedges and bars (1D ABH) or in the centres of pits (2D ABH), very low reflection coefficients of flexural waves result from the
plate/rod edges or pit centres. This paper also suggests possible practical applications for Acoustic black holes in engine turbine blades and in tennis or badminton racket handles.

![2D Acoustic black hole](image)

Figure 1.4 2D Acoustic black hole

The incorporation of the above-mentioned 2D damped indentations of power-law profiles into elliptical plates have been studied experimentally in several papers (Gautier et al, 2008; Cuenca, 2009; Georgiev et al, 2009; Georgiev et al, 2011), where substantial reductions in mobility are seen on the plate when the forcing is in the focus of the profile. In particular, the first experimental results for an elliptical plate with 2D Acoustic black hole located at one of its focuses has demonstrated the possibility to focus the waves towards the black hole (indentation of power-law profile), (Gautier et al, 2008). This paper showed a comparison of mobility results which clearly displayed the damping patterns associated with the Acoustic black hole effect. Considerable research has been conducted also into other practical methods of focusing of flexural waves into 2D Acoustic black holes and into the modeling of the effect. (Cuenca, 2009; Georgiev et al, 2011). This work considered three types of plate: elliptical, rectangular with parabolic edge and an arbitrary polygon plate with a parabolic edge, Figure 1.5. The elliptical plate showed that the parabolic end of the ellipse acted as a wave lense, directing reflected waves towards the Acoustic black hole located at the focus of an elliptical plate. The plates were excited via a shaker placed at the second focus of the elliptical plate, and the mobility response was recorded using a scanning laser vibrometer.

It was found (Cuenca, 2009) that as much as a 10dB reduction in peak amplitude could be achieved using the focusing method described above. Furthermore it was shown that Acoustic black holes with focusing can be effective dampers even on more complicated non-conventional shaped plates.

![Rectangular plate with parabolic edge, elliptical plate, and an arbitrary polygon plate with a parabolic edge](image)

Figure 1.5 Rectangular plate with parabolic edge, elliptical plate, and an arbitrary polygon plate with a parabolic edge

It was also found that similar velocity profiles as found when using 1D Acoustic black holes could be achieved by controlling the Young’s modulus and material density along a strip (Cuenca, 2009).
This was achieved through the use of a shape-memory polymer which uses temperature to control the Young’s modulus along edge of the strip, Figure 1.6. As the wave progresses through the strip the wave length decreases with the increase in temperature towards the free end.

![Figure 1.6](image1.png) Thermal gradient applied to a strip and thermal gradient applied to a wedge of power-law profile

When this effect is combined with a wedge of power-law profile, Figure 1.11, the resultant damping is practically theoretical with the reflection coefficient close to zero. The experimental and numerical predictions agreed with the experimental findings. This combination method is however limited in its applications due to the need and location of a heat source. It would ideally be applied where a heat gradient was already present and allowed for correct orientation of the wedge.

A numerical model of a rectangular plate with a 1D wedge on one end was presented and validated experimentally, the length and thickness of the damping layer was also explored (O’Boy et al, 2010a). Figure 1.7 above shows the composite damping of the wedge when a damping layer is applied to the tip. It clearly shows that the loss factor is initially small at the start of the damping layer 2cm from the wedge tip but rapidly increases towards the wedge tip. From the example discussed above it is clear that a damping layer applied to the tip of the wedge profile provides the most efficient damping of the wedge and to apply a longer damping layer not only increases the mass of the plate but does not increase the damping performance of the wedge itself. It was also proven that the lower the value of the damping layer/wedge thickness ratio the less effective the damping layer is.

![Figure 1.7](image2.png) Composite damping for a strip of damping material applied to the wedge

Figure 1.6 Thermal gradient applied to a strip and thermal gradient applied to a wedge of power-law profile

Figure 1.7 Composite damping for a strip of damping material applied to the wedge
The 1D model was again presented along with a numerical model and experimental measurements of point mobility for a 2D acoustic black hole with a quadratic profile (O’Boy et al, 2010b). In both cases, the numerical predictions are validated by the experimental results. These results also show significant reductions in resonant peaks of the response of the plates.

Following on from the numerical and experimental results presented above a further investigation was carried out to create a numerical model of circular plates containing tapered holes of quadratic power-law profile with attached damping layers (O’Boy et al, 2011a, 2011b). The experimental results, that represent part of this thesis (see Chapter 6), were compared to the developed model as a means of validation. Three styles of plate were investigated a constant thickness circular plate, a constant thickness plate with a damping layer and a plate with a quadratic power-law profile in the centre with a damping layer attached.

The first experimental investigation into application of the Acoustic black hole effect as a means of damping flexural vibrations in badminton racquets was undertaken in 2008 (Kralovic et al, 2008). Before this investigation, the damping of flexural vibrations in tapered rods of power-law profile was considered, Figure 1.8, (Kralovic et al, 2007).

It was found that the addition of a damping layer to the rod itself had a negligible effect on the attenuation of the vibration response of the rod. When a thin damping layer was placed on the Acoustic black hole tip, a significant reduction in the vibration response of the rod was recorded. This was attributed to the reduction in the reflection co-efficient seen at the tip of the power-law section of the rod. A maximum reduction of 10dB was seen in badminton ratchets incorporating rods with a taper of power-law profile with damping layer into the handle. This investigation concluded that further investigation was necessary before this damping method could be implemented.

Some initial theoretical investigations into the application of a wedge of power-law profile on to model fan blades, (Bayod, 2011) have also been carried out. The area of research concentrates on the theoretical evaluation of the damping effectiveness of elastic wedge theory in blades. Although the author concludes that the results show that theoretically (using elastic wedge theory combined with non-polymeric damping material) an elastic wedge can be applied to turbine blades as an effective passive damper, the ‘blades’ investigated in this paper are in fact plane rectangular plates, the blades true aerofoil shape or twist has not been considered. The results presented are in agreement with previous models of wedges of power-law profile.
In conclusion, the previous work on the acoustic black hole effect is by no means comprehensive and there are many facets of this promising damping technique still to be explored. For example; there is a need to investigate 2D black holes in arbitrary positions inside rectangular plates (previously they were studied experimentally only for focal locations inside elliptical plates). Also, some of the previously obtained theoretical results need to be validated experimentally, for example the effect of damping layer placement on either side of a power-law wedge. More generally, experimental investigations into imperfections due to black hole manufacturing should be carried out. There is also an obvious need for the acoustic black hole effect to be explored in real world applications. This would include the consideration of the effects of common bonding techniques and aspects of safety associated with sharp edges at the wedge tips. The behaviour of the acoustic black hole effect in different non-metal materials deserves a special investigation, and possible practical applications of such materials should be explored. Some specific real world applications should be also considered, for example using the acoustic black hole effect for vibration damping in fan blades and in composite honeycomb sandwich panels.

1.4 Summary of the thesis structure and purpose

The main purpose of this thesis is to develop and investigate experimentally some new geometrical configurations of acoustic black holes that would allow to reduce the amplitude of the vibration response of a plate/strip structures to a maximum effect and to suggest some new possible ‘real world’ practical applications for this damping technique, via experimental investigations.

As described in section 1.2 above, there are many passive damping techniques available on the market. However, acoustic black holes offer the unique possibility of substantial damping of resonant peaks over a broad frequency range, while not increasing the mass of the structure.

This thesis can be split into two Parts; the first consists of three chapters and discusses the 1D Acoustic black holes (wedges of power-law profile) in relation to the reductions of flexural vibrations in plates and strips, and concluding with a suggested application. The second Part forms the major contribution to this thesis, consisting of four chapters, it mirrors that of the first with the 2D Acoustic black holes (circular indentations of power-law profile) becoming the focus of the investigation. Finally conclusions are given.

Chapter 2 looks at the effect of geometrical and material imperfections on the damping of flexural vibrations in plates with attached wedges of power-law profile. This Chapter focuses on four investigations concerning steel strips, it considers strips with wedges of power-law profile both in a homogeneous sample and ones where the wedges have been bonded to the samples. It also considers the effects of tip damage and early truncation of a wedge and the most effective surface on which to place the damping material.
Chapter 3 considers the first experimental investigation into practical application of a 1D Acoustic black hole with the aim of damping flexural vibrations in straight and twisted turbofan blades via the introduction of a wedge of power-law profile to the trailing edge of the blade. The aerodynamic implications of the introduction of such a wedge is also considered, along with the blades vibration response when placed in an airflow.

Chapter 4 introduces tapered slots of power-law profile into steel plates as a mean of damping flexural vibrations and with the aim of moving the exposed wedges on the outer edge of the plate so that they are located in slots within the plates. This not only results in the removal of the exposure of a structurally weak edge with a health and safety risk but also allowing for the integration of Acoustic black holes into panels/plates that need securing at the edges.

Chapter 5 considers the damping of flexural vibrations in rectangular plates containing circular indentations of power-law profile (2D acoustic black holes). This chapter not only looks at optimising the damping performance of such indentation but also at increasing the number of these indentations into arrays of multiple indentations of power-law profile.

Chapter 6 compliments Chapter 5 in that it compares the experimental results for a cylindrical plate incorporating a central hole of quadratic profile and compares the results to the published numerical predictions (O’Boy et al, 2010; 2011), thus validating the results and experimental technique.

Chapter 7 reflects on another vitally important aspect associated with vibrations damping, the sound radiation from rectangular steel plates containing circular indentations of power-law profile. This Chapter also investigates the effect of re-distribution of normal vibration amplitudes over the plate associated with the black holes in relation to the observed sound radiation level.

Chapters 8 When investigating practical applications of ‘Acoustic black holes’ one is drawn to materials of increasing popularity and versatility; composites. This chapter investigates the damping of flexural vibrations in glass fibre composite plates containing one and two dimensional acoustic black holes. It also looks at the application of 2D Acoustic black holes into smooth surfaced composite panels. It also considers the subsequent application of the smooth surfaced composite panels into glass fibre honeycomb sandwich panels.

Chapter 9 details the conclusions to this thesis.
Finally, it is worth to mention that there is some controversy over the term ‘Acoustic black hole’, especially if to compare it with better known ‘cosmological black holes’. To clarify the terms used in this thesis, ‘Acoustic black hole’ and ‘wedge/indentation of power-law profile’ are used interchangeably. The term 1D Acoustic black hole refers to a wedge of power-law profile and a 2D Acoustic black hole refers to an indentation of power-law profile. “What is in a name? a rose by any other name would still spell so sweet”. The Acoustic black hole is an effective damping method and should be judged on this alone not whether its title is deemed accurate.

1.5 Experimental set up and technique

This section aims to clarify and justify the main choices made in regards to the experimental set up and techniques used in this thesis. The first consideration was the experimental set up itself. The main requirements of the experimental set up were to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate edges and introduce minimal damping to the system. Three methods were considered, Figure 1.9. The first method supported the plate on a metal rig cushioned by four pieces of foam placed under non wedge/indentation sections of the plate. This method was not practical for heavy or unsymmetrically shaped samples as the foam would either be unevenly compressed resulting in non-square attachment to the force transducer or compressed to the point of changing the boundary conditions to simply supported.

![Figure 1.9](image)

(a) Plate supported via foam supports, (b) Plate balanced on taught strings, (c) Plate suspended via wires.

The second method balanced the plate on taught strings; this removed the difficulty posed by heavy or unsymmetrically shaped samples. However to allow for correct orientation on to the force transducer this method is very time consuming as to ensure the plate/sample is horizontal a spirit level and many adjustments to the sample are required. The tension in the strings must also be altered in order for the sample to reach the correct height for attachment to the shaker, so the stinger is not
compressed or extended beyond its optimal range. If a sample is light then it could not be suspended in this manner.

The final method suspended the plate via wires from the top of an A frame rig. When the correct length suspension wires are selected the sample hangs at the correct height for shaker attachment. As the shaker can be moved on the shelf the stinger is always in the optimal position. Different sample sizes can be accommodated easily due to the numerous suspension widths that are available. Once hung the sample requires no alterations. It is this final method that was used for the experimental set up for this thesis for the reasons given above, with the exception of Chapter 2 which used the second method.

When measuring the response of a structure it is advantageous to select a response parameter that gives the flattest frequency spectrum in order to best utilize the dynamic range of the instrumentation (Brüel &Kjael, 1982). Accelerance was chosen as the response parameter as this study as it not only best fore fills the criteria stated above but also accounts for the consideration of a high frequency response.

Each sample was tested a minimum of 10 times on separate occasions. Where possible, multiple samples were manufactured and tested for each investigation. Ideally multiple samples would have been made and tested for each type of sample however this is not time or cost effective. The samples created for this investigation were manufactured within tolerance and allowed for the variation in manufacturing applicable to the real world. Where many similar samples were manufactured it could easily be observed from the response of a sample if the sample were faulty or spurious result that required further investigation had occurred. In this case a minimum of one other sample was manufactured to confirm the results of the test.
CHAPTER 2

EFFECT OF GEOMETRICAL AND MATERIAL IMPERFECTIONS ON DAMPING FLEXURAL VIBRATIONS IN PLATES WITH ATTACHED WEDGES OF POWER-LAW PROFILE


Abstract

In this chapter, the results of the experimental studies of the effects of manufacturing intolerances on damping flexural vibrations in wedge-like structures of power-law profile are presented. In particular, the effect of mechanical damage resulting from the use of cutting tools to wedge tips is investigated, including tip curling and early truncation, as well as the placement of absorbing layers on different wedge surfaces. Also, the effects of welded and glued bonding of wedge attachments to basic rectangular plates (strips) are investigated. The results show that, although the above-mentioned geometrical and material imperfections reduce the damping efficiency to various degrees, the method of damping structural vibrations using the acoustic black hole effect is robust enough and can be used widely without the need of high precision manufacturing.

2.1 Introduction

Following on from previous research into the Acoustic black hole effect discussed in Chapter 1, it is obvious that the geometry and material properties of wedges of power-law profile are very important for damping systems utilising the acoustic black hole effect. However, there were no investigations so far in which the influence of geometrical or material imperfections of power-law wedges on damping structural vibrations has been examined. The aim of this chapter is to investigate the effects of geometrical and material imperfections on vibration damping in structures containing elastic wedges of power-law profile.

Initially, the effect of mechanical damage to wedge tips resulting from the use of cutting tools will be examined, including tip curling and early truncation. Also, the placement of absorbing
layers on different wedge surfaces will be studied. Engineering structures are rarely constructed from a single sheet of material and are likely to have joints (bonded sections). Another aspect simulated in this investigation is the bonding of a wedge to an existing structure, thus exploring practical ways of integrating this phenomenon into existing structures. The effects of bonding a wedge to basic rectangular plates (strips), via different welding techniques and gluing will therefore be investigated. Finally, conclusions will be drawn regarding the effects of material and geometrical imperfections on damping structural vibrations.

2.2 Experimental samples and their manufacturing

The experimental samples in this Chapter were manufactured from 5 mm thick, hot-drawn mild sheet steel. This type of steel is more resistant to the mechanical stresses incurred during manufacture than cold drawn steels. This resilience to mechanical stresses results in fewer internal defects that could lead to increased elastic wave scattering. Plate and damping layer material properties are listed in Table 2.1, whereas sample dimensions are given in Table 2.2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness</th>
<th>Young’s modulus</th>
<th>Density</th>
<th>Poissons ratio</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>5.04 mm</td>
<td>190 GPa</td>
<td>7000 kg/m³</td>
<td>0.3</td>
<td>0.6 %</td>
</tr>
<tr>
<td>Electrical tape</td>
<td>0.08 mm</td>
<td>-</td>
<td>300 kg/m³</td>
<td>-</td>
<td>6 %</td>
</tr>
</tbody>
</table>

Table 2.1. Material properties of plates and damping layers.

<table>
<thead>
<tr>
<th>Sample Type</th>
<th>Reference strip</th>
<th>Extended samples</th>
<th>Homogeneous sample</th>
<th>Bonded samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>300mm</td>
<td>349mm</td>
<td>339mm</td>
<td>339mm</td>
</tr>
<tr>
<td>Width</td>
<td>50mm</td>
<td>50mm</td>
<td>50mm</td>
<td>50mm</td>
</tr>
</tbody>
</table>

Table 2.2. Sample dimensions.

Wedges of power law profile, with \( m = 2.2 \), were milled at one end of the plates (strips). The milling machine was accurate to 0.01 mm, and the cutter head was inclined to reduce machining stresses. The cutter rotated at speed of 600 rpm in a manner that resulted in a pushing cut. This safeguarded the leading edge, thus reducing the damage. The main problem incurred by this method of manufacture was in obtaining a thin cut at the end of the wedge. To produce the correct power-law profile the plate was cut extremely thin, and as a result, it was susceptible to curling and damage to the tip of the profile due to heat and machining stresses, Figure 2.1.

For the production of welded and glued samples see Figure 2.2. The following order of manufacture was used; the metal sheet was first cut into strips. The glued and homogeneous
samples were profiled before the glued wedges were cut off. In the case of welded samples, to avoid damage to the profile due to the heat from welding, the un-profiled ends were cut and welded before profiling. A mild steel filler was used for both welds (Figure 2.2), and the excess weld material was removed from the top and bottom of the strip to minimise weight gain and ensure similar strip shape. This was to ensure that the welded samples remained as similar as possible to the reference strip.

Figure 2.1       Machine damage to a wedge tip.

Figure 2.2       Welded and glued samples: (a) -TIG welded wedge, (b) -MIG welded wedge, (c) - glued wedge (standard ‘super glue’), (d) - homogeneous wedge.

Two types of weld have been used, as shown in Figure 2.3: (a) - Tungsten inert gas (TIG) weld (this method applies a separate filler rod and welding torch, and it is used for straight edge
truncation welds); (b) – Metal inert gas (MIG) weld (in this method the filler rod is incorporated into the welding torch and generally requires a slanted cut in the materials to bond them together).

Figure 2.3 Types of weld: (a) - Tungsten inert gas (TIG) weld; (b) – Metal inert gas (MIG) weld; dark grey – weld material/fusion zone, Light grey – heat affected area, White – base material.

2.3 Experimental set-up

The experimental set-up was designed to allow almost free vibration of the sample plates, take the weight of the plate and introduce least damping to the system. A string suspension arrangement, Figure 2.4 was chosen to support the test samples as it provides the least damping effect and nearly free boundary conditions, while still maintaining adequate strength to support the weight of the test sample.

An electromagnetic shaker provided the excitation input to the centre of the plate via a force transducer (Bruei and Kjaer Type 8200) attached to the surface of the plate via wax. The response was recorded by a broadband accelerometer (Bruel & Kjaer Type 4371) that was attached to the upper surface of the test sample, directly above the force transducer, also via wax, Figure
2.5. A broadband white noise signal was generated by the analyser and transferred to the test piece via a shaker. A Bruel & Kjaer Type 2035 analyser and amplifier were used to acquire the results. A schematic view of the experimental set-up is shown in Figure 2.6. Point accelerance was measured in each test case. A measurement range of 0-9 kHz was used. A profile of \( m=2.2 \) was chosen for all samples.

![Figure 2.5](locations-of-shaker-force-and-accelerometer-response-on-an-experimental-sample.png)

**Figure 2.5** Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample.

![Figure 2.6](schematic-of-experimental-set-up.png)

**Figure 2.6** Schematic of the experimental set-up utilising the Bruel & Kjaer Analyser.

### 2.4 Results and discussion

#### 2.4.1 Effect of an attached wedge of power-law profile on damping flexural vibrations

Two sets of initial measurements are described in this section: the effect of profiling a wedge with an added damping layer at the edge of a homogeneous narrow plate (strip), when compared to a reference strip, and the effect of adding a damping layer to a homogeneous wedge, when compared to an uncovered wedge.

Figure 2.7 shows a measured accelerance for a homogeneous profiled sample with an additional damping layer, as compared to the reference strip of constant thickness. It can be seen that there is
little wedge-induced damping in the low frequency range, below 1.5 kHz. Then, there is a noticeable reduction (of about 6 dB) at 1.50 kHz. A smaller reduction of 4.5 and 2.5 dB compared to the reference peak is then recorded at 3.7 and 4.6 kHz respectively. The resonant peaks after this show a growing reduction with increasing frequency, ranging from 5.5 - 8.0 dB between 5.5 and 7.2 kHz. Finally, there is a slightly reduced damping effect shown at 7.2 kHz, where the reduction in peak amplitude is about 6 dB, with the greatest reduction of 8 dB occurring at 5.8 kHz. Due to the structural changes incurred when a wedge is profiled at one edge of a strip, a shift in resonant peaks is expected and can clearly be seen in Figure 2.7. Note that the damping effect of a wedge of power-law profile has been documented earlier [Krylov et al, 2004, Krylov, 2004 and Krylov et al, 2007]. The present measurements extend these earlier results to the case of a wedge added to a strip.

Figure 2.7 Measured accelerance for the homogeneous profiled sample ($m = 2.2$) with additional damping layer (solid curve) compared to the reference plate of constant thickness (dashed curve).

Figure 2.8 shows measurement results for a homogeneous sample with early truncation and no wedge tip damage, with and without an additional damping layer. It can be seen that generally there is a small reduction in peak amplitudes for a wedge with added damping layer, increasing with frequency and ranging between 1.6 and 5 dB. These results confirm that the attachment of a damping layer to the wedge tip results in reduction in resonant peak amplitudes in comparison with
a free wedge. The observed reductions though are much smaller than those observed in [Krylov et al, 2007], which is likely to be caused by the large value of edge truncation in the present sample.

Figure 2.8  Measured accelerance for the homogeneous profiled sample ($m = 2.2$) with early truncation and no wedge tip damage with (solid curve) and without an added damping layer (dashed curve).

2.4.2 Effect of tip damage and early truncation of a wedge

This section describes the effect of tip damage in a wedge of the maximum possible (extended) length of 49 mm on its performance when compared to the same wedge when it has been cut to a reduced length of 39 mm, i.e. with a premature truncation, Figure 2.9.

Figure 2.9  Wedge with tip damage and wedge with premature truncation

Figure 2.10 shows the values of measured accelerance for a TIG welded sample (extended sample) of power-law profile compared to the TIG welded truncated sample. It can be seen that, in spite of the wedge tip damage, the resonant peaks of the extended sample show an increased amplitude reduction with increasing frequency. It ranges from 8.5 to 12.5 dB between 3.8 and 7.8
kHz, with a larger reduction of 15 dB recorded at around 8 kHz. This agrees with the predictions of [Krylov et al, 2004 and Krylov, 2004] the reflections from truncated wedge tips increase with the increased truncation, thus resulting in poor damping characteristics in samples containing truncated power-law wedges.

The same measurements were performed with the glued bond. The results followed a similar trend as in the case of the TIG welded sample, with the extended sample consistently performing better than the truncated sample. The maximum reduction of 7 dB occurred at 7.2 kHz in this case. The MIG welded sample and the homogeneous sample followed the same damping trend again, but with smaller increases in damping performance. The extended MIG welded sample achieved the maximum increase of 5.5 dB compared to the truncated sample and the extended homogeneous sample showed a 4.5 dB reduction over the truncated sample.

![Figure 2.10](image)

**Figure 2.10** Measured accelerance for a TIG welded sample (solid curve) with power-law profile \( m = 2.2 \) compared to the TIG welded truncated sample (dashed curve).

Despite the damage to the extended wedge tip, the increased length and resultant decrease in tip thickness provided the most efficient damping, thus demonstrating that the longer and thinner the wedge tip the greater the contribution of the acoustic black hole effect into the overall vibration damping.
2.4.3 Effect of the damping layer placement

According to the geometrical acoustics theory of the acoustic black hole effect [Krylov et al., 2004, Krylov, 2004], the position of the damping layer in relation to which surface it is attached to should make no difference to the level of damping achieved. However the measured results shown below seem to contradict this statement when real manufactured wedges are considered.

Figure 2.11 Positioning of damping layer on a homogeneous wedge \((m = 2.2)\)

The damping layer has been placed on the flat underside of the wedge tip and also on the profile side, as can be seen in Figure 2.11. The results are shown in Figure 2.12. It can be seen that a greater damping effect occurs when the damping layer is placed on the underside flat surface of the wedge, the difference in the damping effect being quite noticeable. Even at low frequencies there is a difference between the two.

In order to obtain some statistical backing for this observation, the above measurements have been repeated for different pieces of damping tape taken from the same roll. The results showing the difference in the damping associated with placement of different pieces of damping tape on flat and curved surfaces can be seen in Table 2.3 for several peak frequencies. As it can be seen, the difference in the damping effect is preserved.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Peak Frequency (kHz)</th>
<th>Homogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3.5</td>
</tr>
<tr>
<td>1</td>
<td>1.87</td>
<td>4.56</td>
</tr>
<tr>
<td>2</td>
<td>1.78</td>
<td>4.32</td>
</tr>
<tr>
<td>3</td>
<td>2.05</td>
<td>4.95</td>
</tr>
<tr>
<td>4</td>
<td>1.85</td>
<td>4.68</td>
</tr>
<tr>
<td>Average Reduction (dB)</td>
<td>1.89</td>
<td>4.63</td>
</tr>
</tbody>
</table>

Table 2.3. Peak amplitude reduction due to the repeated attachment of pieces of damping tape to the flat side of a homogeneous wedge, in comparison with their attachment to the profiled side.
Figure 2.12 Effect of the positioning of the damping layer in relation to which surface of the wedge it is applied to: damping layer placed on the profiled surface (dashed line), damping layer placed on the flat surface (solid line).

Possible reasons for this difference lie in the differences between the theoretical assumptions and the practicality of producing a wedge to meet the theoretical standards. The most likely reason is that due to the stepped nature of the top of the profile the damping layer does not make full contact with the entire surface beneath the strip, thus resulting in the reduced damping effect. Following from these results, the damping layers in all subsequent experiments of the present work have been placed on the flat underside of the wedge profile to attain the maximum damping possible.

2.4.4 Effect of bonding of a wedge to a strip

The present section describes the effects of bonding wedges to a constant thickness strip, Figure 2.2. The wedges were bonded to a strip using two different types of weld, TIG and MIG (see Figure 2.3), and also by glue. Each of these structures is compared to a reference strip and to a homogeneous structure that contains a wedge. Figure 2.13 shows measured values of accelerance for a sample with the TIG-welded in comparison with the results for the reference plate. It can be seen that there is a significant reduction in resonant peaks with increasing frequency, up to 8.0 - 10 dB between 5.5-9 kHz, with the greatest reduction of about 10 dB occurring at 7.2 kHz.
Figure 2.13  Measured accelerance for a sample with the TIG welded power-law profile \((m = 2.2)\) (solid curve) compared to the reference plate (dashed curve).

Figure 2.14  Measured accelerance for a sample with the MIG welded power-law profile \((m = 2.2)\) (solid curve) compared to the reference plate (dashed curve).
Figure 2.14 presents the measured accelerance for a sample with the MIG-welded compared to the reference plate. Again, there is little damping in the low frequency range, below 1.5 kHz (a slight increase can in fact be seen at 0.29 and 0.81 kHz). A reduction of 6 dB compared to the reference peak is recorded at 3.7 kHz, and there is no change to the peak at 4.60 kHz. The resonant peaks after this show an increasing reduction with increasing frequency ranging from 6.5 to 7.5 dB between 5.5-9 kHz, with the greatest reduction of 7.5 dB occurring at 5.5 kHz.

Figure 2.15 shows the accelerance for a sample with the glued wedge compared to the reference plate. There is little to no damping in the low frequency range, below 1.50 kHz, except for a reduction of about 3 dB in the amplitude of the reference resonant peak located at 0.81 kHz. This reduction increases to 7.5 dB at 1.50 kHz. The resonant peaks then show an increased reduction with increasing frequency ranging from 8.5 to 10 dB between 4.6-9 kHz.

In order to account for statistical deviations in the effect of imperfections in the joining method used, the glued sample bond was broken, cleaned and reattached again. This procedure has been repeated five times. The results for the damping at several peak frequencies (compared to the reference plate) are shown in Table 2.4. As can be seen, in all five cases the amplitudes of the resonant peaks altered by no more than 2dB. Any discrepancies can be put down to the application method of the bonding agent and that it was not always possible to ensure an identical amount was added in each case. The trend however is consistent.
Table 2.4. Peak amplitude reduction for five glued samples containing a wedge of power-law profile, in comparison with the reference plate.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Glued</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Frequency (kHz)</td>
<td>1.8</td>
</tr>
<tr>
<td>1 Peak Reduction (dB)</td>
<td>7.5</td>
</tr>
<tr>
<td>2 Peak Reduction (dB)</td>
<td>7.2</td>
</tr>
<tr>
<td>3 Peak Reduction (dB)</td>
<td>6</td>
</tr>
<tr>
<td>4 Peak Reduction (dB)</td>
<td>6.9</td>
</tr>
<tr>
<td>5 Peak Reduction (dB)</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Finally, Figure 2.16 shows the measured results for a homogeneous sample with wedge compared to the TIG-welded sample. There is little difference in damping performance in the frequency range below 4.5 kHz. After 4.5 kHz, the TIG-welded sample consistently performs better than the homogeneous sample, with the reductions in comparison with the homogeneous sample by 4.5-6.0 dB.

Figure 2.16 Measured results for a homogeneous sample with power-law profile \((m = 2.2)\) (dashed curve) compared to the TIG welded sample (solid curve).

It can be seen that the MIG-welded wedge performed the worst out of the three bonded samples, with a maximum damping of only 7.5 dB occurring at 5.5 kHz. The glued wedge performs better than the TIG-welded wedge at frequencies below 3.7 kHz, while the TIG-welded wedge performs
better than the glued wedge over the remaining frequency range 3.7 – 9 kHz. Out of the three bonds tested, the TIG-welded wedge proved to yield the most significant reductions in comparison with the amplitudes of the reference strip resonance peaks over the largest range. The greatest reduction seen across the three samples was 10 dB occurring at 7.2 kHz, this reduction was seen in both the TIG and glued samples.

As expected, neither of the welds matches the damping performance of the homogeneous sample, however the TIG-welded sample above 4.5 kHz unexpectedly performs better than the homogeneous sample. The glued wedge also has a better damping performance than the homogeneous sample after this point.

The above differences may be due to flexural wave reflections from the material boundaries. For the weld used in this investigation, a mild steel filler was used for mild steel sheet metal, however the properties of the two metals were not identical, and the difference could increase after heating. Obviously, higher frequency waves could be affected more by the break in the plate and subsequent bonding. Note that the thermal cycles endured by the base metal during the welding process tend to alter the structure and properties of the metal considerably (Minnick, 2007). The most prominent defect resulting from heat in the welding process is the formation of a Martensite polycrystalline structure that could result in increased wave scattering and reflections on either side of the weld. Note that this effect can be reduced by preheating the steel before welding.

Although these increased reflections as a result of Martensite formation hinder waves from propagating into the wedge, it is possible that to some extent the waves that enter the wedge and are reflected from the truncation will in turn be reflected back into the wedge. The initial reflection is however the dominant factor with respect to the damping performance of the wedge. Therefore, the greater the Martensite formation the less effective the welded wedge would be.

The TIG-welded wedge still shows significant reductions in the magnitude of the resonance peaks. Theoretically this is expected as the wave should propagate more efficiently and with less reflection through this type of weld as there is very little weld material between the wedge and strip.

The MIG-welded sample appears to have been more adversely affected by the welding process than the TIG-welded sample, and as a result, its reflection coefficient is greater, reflecting a greater proportion of the waves passing through the weld. It is quite possible that, due to the shape of the MIG weld, the materials towards the points at the centre of the weld have been affected to a greater extent than the straight edges of the TIG weld. This could result in higher levels of reflection from the Martensite area. To reduce the reflection coefficient at the two boundaries, materials with similar velocities and mass densities should be used for the filler and wedge.

In the case of the glued sample, the glue was pasted thinly and evenly between the wedge and the strip, ensuring a uniformed bond with little to no air gaps. Although the glue creates a greater impedance change in the bond than that of the welds, the glue layer itself is very thin. Moreover,
the metal surrounding the bond is not exposed to the thermal cycles induced by weld formation, and therefore not affected by the heat defects that are associated with welded bonds. There are however limited practical applications for a glued bond as it is weak and can be easily broken.

### 2.5 Conclusions

In the present work, the effects of deviations of real manufactured wedge-like structures from ideal elastic wedges of power-law profile on damping flexural vibrations have been investigated experimentally. In particular, the effect of mechanical damage to wedge tips has been investigated, including tip curling and early truncation, as well as the placement of absorbing layers on different wedge surfaces. Also, the effects of welded and glued bonding of wedge attachments to basic rectangular plates (strips) have been studied.

It has been demonstrated that the effect of tip damage (curling) in a wedge of the maximum possible (extended) length allowed by manufacturing is not detrimental for its performance when compared to the same wedge that has been cut to a reduced length (truncated) in order to avoid curling. Despite the damage to the extended wedge tip, the increased length and resultant decrease in tip thickness provided the most efficient damping of flexural vibrations. The longer and thinner the wedge tip the greater the contribution of the acoustic black hole effect into the overall vibration damping, in spite of the resulting technological damage to the tip. Despite this increased damping performance, the practical applications of an extended tip are limited though due to an increased possibility of the tip breaking off because of its increased fragility, although this can in part be countered by the addition of a damping layer.

It has been shown that the position of the damping layer in relation to which surface it is attached does make difference to the level of damping achieved, contrary to the predictions following from the geometrical-acoustics theory applied to idealised wedges. The measured results show that a greater damping effect occurs when the damping layer is placed on the underside flat surface of the wedge. The most likely reason for that is that due to the stepped nature of the top of the profile the damping layer does not make full contact with the entire surface beneath the strip, thus resulting in the reduced damping effect.

It has been demonstrated that attaching power-low profiled wedges to a rectangular plate (strip) by welding or via glue results in damping performances that generally isn’t any worse than the performance of a homogeneous sample containing the same wedge.

In particular, the experiments show that the glued wedge performs better than the TIG-welded wedge at frequencies below 3.7 kHz, while the TIG-welded wedge is performing better than the glued wedge over the remaining frequency range 3.7 –9 kHz. Out of the three bonds tested, the TIG-welded wedge proved to yield the most significant reductions in the amplitude of the reference strip resonance peaks over the largest frequency range.
In all four samples, the TIG-welded, MIG-welded, glued and homogeneous sample, the increased extended length results in an overall increased damping effect despite significant machining damage to the extended tip. The tears and small holes also result in enhanced damping performance due to increased scattering.

The main conclusion that can be drawn from the present experimental work is as follows. Although the above-mentioned geometrical and material imperfections generally reduce the damping efficiency to various degrees, the method of damping structural vibrations using the acoustic black hole effect is robust enough and can be used widely without the need of high precision manufacturing.
CHAPTER 3
DAMPING FLEXURAL VIBRATIONS IN FAN BLADES
CONTAINING WEDGES OF POWER-LAW PROFILE

This chapter in part was submitted to the Journal of Sound and Vibration in December 2012. A preliminary version of the results described herein was presented at the Anglo-French Conference ‘Acoustics 2012”, Nantes, France, 23-27 April 2012.

Abstract
In this chapter, the results of the experimental investigations into damping of flexural vibrations in turbofan blades with trailing edges tapered according to a power-law profile are reported. Edges of power-law profile (wedges), with small pieces of attached absorbing layers, materialise as one-dimensional acoustic black holes for flexural waves that can absorb a large proportion of the incident flexural wave energy. The NACA 1307 aerofoil was used as a base model for experimental samples. This model was modified to form four samples of non-engine-specific model fan blades. Two of them were then twisted, so that a more realistic fan blade could be considered. All model blades, the ones with tapered trailing edges and the ones of traditional form, were excited by an electromagnetic shaker, and the corresponding frequency response functions have been measured. The results show that the fan blades with power-law tapered edges have the same pattern of damping that can be seen for plates with attached wedges of power-law profile, when compared to their respective reference samples. The resonant peaks are reduced substantially once a power-law tapering is introduced to the sample. The obtained results demonstrate that power-law tapering of trailing edges of turbofan blades can be a viable method of reduction of blade vibrations.

3.1 Introduction
The fan is located at the front of a jet engine and usually forms the first stage of the low pressure (LP) compressor, Figure 3.1. The fan in modern Bypass Turbofan engines usually has one stage consisting of a rotor and stator row, however the fan can have as many as three stages as is often seen in a Turbo jet engine. The purpose of the stationary stator row is to direct and smooth the airflow on to the next stage of rotors with the correct angle of incidence. The fan operates at a relatively low temperature compared to the turbine section of the engine. The fan blades are therefore made of a alloys with a lower temperature capability, and are un-cooled.
Aircraft engines require that engine fan blades operate at maximum efficiency. Designs for aerodynamically efficient, high-aspect-ratio compressor blades that can comply with all the current legislation both environmental and safety orientated necessitate the damping of blade vibration. One of the major causes of fan blade failure in jet engines is from the undamped vibration of the blades resulting in a high cyclic fatigue (Rao, 1991, Tsai, 2004 and Poursaeidi et al, 2008). To reduce the vibration in the blades would result in lower stress levels on the blade and ultimately a longer fatigue life. Vibration in fan blades arises as a result of the combination of many vibration sources. These sources include aircraft/ground vortices, distortion of inlet airflow resulting from flight manoeuvres or cross wind, speed and altitude effects, and thrust reverser operation. One of the main contributing sources of this harmonic force on the blade is that caused by a fluctuating lift force that acts on the blade aerofoil as it rotates about the central annulus in front of the stationary stator row. Other sources include atmospheric turbulence and viscous wake interaction along with turbulence from mechanical components such as pitot tubes and struts. All these factors result in a variance of fan blade resonance dependent on engine design, so a frequency specific damping system could not be widely implemented.

Due to the variance of fan blade resonance for the reasons mentioned above a broad frequency damper would be ideal solution as it would allow for variation in resonant peak frequencies brought about by varying engine designs. Two traditional methods of damping structural vibrations are the addition of layers of highly absorbing materials to the structure in order to increase energy dissipation of propagating (mostly flexural) waves (Heckl et al, 1988, Mead, 1998 and Ross et al, 1959) and the second is the suppression of resonant vibrations of different structures is to reduce reflections of structural waves from their free edges (Vemula et al, 1996). There are existing damping methods used on jet engine fan blades, each presenting individual difficulties (Logan et al, 2003 and Beards, 1996). These methods include slip damping, gas damping and damping wires.
This Chapter looks at the integration of wedges of power-law profile initially on to the trailing edge of a straight then twisted fan blade and its effect on the damping of flexural vibrations in the blade. An initial investigation into the aerodynamic implications of this method was then considered utilizing a flow visualisation technique and final the response of the fan blades was considered when excited via airflow over the blade.

3.2 Current damping techniques

This section will give a brief review of the vibration problems suffered by fan blades and some of the damping solution currently in use. The aim of research into fan blade vibration has four main drivers; increase safety, reduce sound radiation, increase the fatigue life of the blades and finally increase engine efficiency resulting in reduced cost and environmental impact. There have been many case studies and reports produced on the causes of fan blade failure within turbofan fan blades. Many have concluded that failure occurs due to high cycle fatigue within the blades resulting from undamped vibrations in the blades (Rao 1991, Hou, 2002, and Rolls Royce, 2001).

One such case as reported on by Rolls Royce, 2001 and the Australian Transport Safety Bureau, 2002 occurred at Melbourne International Airport, where a Boeing 777-300 A6-EMM aborted its take-off run at low speed as a result of a catastrophic engine failure, Figure 3.2. The RB211 Trent 892 engine failed due to the release of a fan blade, the blade caused extensive damage to the remainder of the fan and the intake shroud. When examined the blade was found to have suffered from progressive fatigue cracking which resulted in two major sections from the blade dovetail root. The remainder of the blade could no longer support the centrifugal loads associated with the accelerating engine and failed in ductile shear, allowing for the release of the blade.

![Figure 3.2. Royce RB-211 Trent 892 Turbofan Engine Boeing 777-300 fan blade failure (Rolls Royce, 2001)](image)
From major incidents such as the one mentioned above, it can be concluded that in order to minimise the stress levels in the blade it is necessary to minimise the vibration levels within the blade. This will have the effect of maximising the fatigue life of the blades; increasing safety, reducing sound radiation, reducing maintenance and increasing engine efficiency resulting in reduce cost and environmental impact.

As mentioned previously the vibration of a fan blade is the resultant of excitation from a number of different sources. Fluctuation of lift forces acting on the fan blades as they rotate is the main contributing factor in the harmonic forcing of the blade. As described in the introduction a fan blade stage consists of a rotor and stationary stator row, it is this stator row that is responsible for this changing lift force. As the airflow hits this stator row some of the air is blocked recirculating resulting in a change in the air flow over the lifting surface of the blade and causing a periodic force on the blade which induces vibration. Other objects in the airflow such as struts, pitot tubes, subsequent blade stages, along with atmospheric turbulence and viscous wake interaction also contribute to this effect.

This fluctuation in lift force is known as ‘blade flutter’ and most commonly occurs at engine start up and run down, it is harmonic and occurs at the nozzle bypass frequency and the blade resonant frequencies. If these frequencies coincide then large fatigue stresses occur in the blade, ultimately leading to blade failure (Rao, 1991) as seen above. This problem rarely occurs at standard engine operating conditions, however problems can occur when there is an inherent blade defect or a blade is damaged resulting in a change in its resonant frequencies.

There are many other considerations when investigating blade behaviour and damping solutions. For example, the many different sources of excitation (as described above) and their interaction resulting in the coupling of the modes. Rotation of the blade and the increase in blade stiffness, due to centrifugal forces also has the effect of altering the natural frequencies of the blade (Logan, 2003).

Five of the most widely used traditional damping methods for blades in within a jet engine are categorised as follows; material damping, blade construction, gas damping, slip damping and the use of damping wires. The main techniques employed by these methods are to alter the natural frequencies of the blades by adding mass or stiffness to the structure.

Material damping is the most obvious and basic form of damping available. Different materials have different attenuation properties (material loss factors) and therefore some are more efficient at dissipating vibration energy. Materials with a higher material loss factor will dissipate transmitted energy more effectively than one with a low loss factor. When a structure is loaded (excited) and undergoes a level of deformation, the basic loss mechanism can be observed through analysis of a hysteresis loop for the structure under test (Blevins 1977). The area inside the loop is equivalent to the kinetic and strain energy converted to and dissipated as heat (Goodman, 1976), this process occurs through molecule interaction during displacement for each loading cycle. The damping
coefficient can be defined as the area enclosed within the loop divided by the energy stored in the system.

In engine blades there are several considerations when choosing a material in addition to damping; weight, strength and temperature tolerance to name a few so a balance must be found between the materials damping properties and other factors mentioned above. Considerable work has gone into the research of damping properties of materials and their response characteristics under different types of excitation (Lazan, 1968). From work such as this and the other considerations mentioned; titanium is the most commonly used material in fan blade construction. However as with most metal its material loss factor is low in comparison to other materials such as composites and additional damping is required to damp the blades sufficiently. It is the strength and density of the material which determines its suitability for use in fan blades as the blades have to pass stringent safety tests such as the bird strikes simulation. Increasing the mass of the fan blade by adding additional material or selecting another heavier material is an effective damping method; however its application contradicts the greater need in modern jet engines for reduced weight and increased efficiency to meet environmental targets and customer demands, and of course reduced cost.

Another advance on material damping is the use of hard (primarily ceramic) coatings sprayed on to the surface of the blade (Patsias et al, 2002). Such layers reduce the stress levels on the blade during operation and therefore reduce the fatigue stress on the blade and prolong its life. It has been shown in this research that such sprayed ceramic coating can provide damping over a broad temperature range.

There is also current research at The University of Sheffield in to moving away from traditional titanium used in fan blade construction and instead using multi-scale reinforced thermoplastic composite materials. There are still no published results from this research.

![Figure 3.3](image)

Figure 3.3. (1) Solid blade, (2) Blade with a hollowed out structure, (3) Blade with a hollowed out structure filled with damping foam.

When considering fan blade construction as a damping method, a clear progression can be seen, Figure 3.3 from a solid blade to blade with a hollowed out structure (primarily for reduced mass) to filling these cavities with a damping foam to reduce vibration of the blade. This method enables a
reduction in blade weight by 27-30% with some damping results. Work on improving this method by combining it with material damping to produce a hybrid composite foam-filled engine fan blades has been under investigation by Seng C. Tan of the American Department of Defence since 2006.

![Damping wire in a fan blade row](image1)

![Schematic of fan blades with damping wire](image2)

Figure 3.4. (a) Damping wire in a fan blade row, (b) Schematic of fan blades with damping wire (Swikert et al, 1968).

Damping wires, Figure 3.4 are another method of damping compressor and fan blade vibration. This method requires a wire to be threaded through each blade in the stage (Chubb, 1967). During engine operation the fan blades rotate resulting in the centripetal force, forcing the wire against the blade. The resulting frictional force between the wire and blade results in damping in the blade due to increased stiffness and a frequency shift due to the blades appearing shorter in length. By making the blades appear shorter there is an increase in the frequencies at which resonance occurs. When the blade is simply modeled as a cantilever beam this phenomena can be explained. The natural frequencies for a cantilever beam are given by:

\[
\omega_i = \left(\frac{nL}{2}\right)^2 \sqrt{\frac{EI}{mL^4}}
\]  

(1)

Where \(\omega_i\) is the natural frequency, \(EI\) is the bending stiffness of the beam, \(m\) is the beam mass, \(L\) is its length and \((nL)^2\) is a solution to a transcendental equation depending upon the mode excited. From the above relationship it can be seen that the natural frequencies are inversely proportional to the square of the length of the beam, therefore the shorter the beam or in our case blade the higher the frequencies at which resonance occurs. When using damping wires material selection is vital for non destructive/longer life of blade to wire interactions.
The main disadvantages of damping wires are that heavy fretting can eventually cause the holes to widen to an extent that the rotor has to be re-bladed and they can be undesirable from an aerodynamic point of view as they interfere with the airflow through the engine causing turbulent flow which can lead to aerodynamic losses in the engine cycle.

As the air passes through the engine it is compressed and its temperature increases resulting in a denser fluid medium through which the blades have to pass. This denser air damps the motion of the blades and engine structure and also allows for energy to be radiated from the structure in the form of sound. This phenomenon is known as Gas damping and is efficient at damping vibration above the fifth harmonic (Logan et al, 2003). As fan blades are located at the front of the engine where the airflow is relatively cool and un-compressed the effect of Gas damping will be minimal.

Finally Slip damping is the most commonly used form of damping in jet engines making use of friction between the two surfaces at the root as the blade rotates and attempts to vibrate, Figure 3.5. Slip damping dissipates energy as a result of cyclic shear strain at the blade root and disc interface, (Goodman, 1976). This may occur as a result of Coulomb friction; dry sliding of the blade root (Rao 1991), viscous forces; lubricated sliding of the blade root, or if present damping in a visco-elastic layer between the blade root and disc surfaces; cyclic strain in a separating adhesive.

![Figure 3.5. Fan blade root located in disc (AERMEC Laboratory)](image)

The main problems encountered with this method of damping include stress concentration in the blade root, additional damping layers add weight and the rotational inertia of the blade will eventually ‘lock’ the blade in place so much so that it is not able to slip and remove energy in this way, so can only be used up to a set engine rotational speed.

Whilst all the damping methods above are utilized to some extent in the damping of vibrations in fan blades, they are neither easily integrated in to the engine nor provide an optimum damping performance. Therefore there is still room for investigation into new methods of achieving more effective damping levels within the fan blades of a jet engine.

When considering the aforementioned vibration and subsequent fatigue issue found in fan blades the acoustic black hole seems to present a unique damping solution. A one dimensional acoustic black hole does not incorporate the addition of large quantities of damping material, nor is
it influenced by the type of material, it does not require a certain rotational velocity to be reached in order for it to work, nor is it limited to a single frequency band, finally it could be machined or cast in to the blade at manufacture. The acoustic black hole therefore presents itself as an attractive alternative to traditional fan blade damping techniques.

Some initial theoretical investigations have been performed in this area, (Boyod, 2011). The area of research concentrates on the evaluation of the damping effectiveness of elastic wedge theory in blades. Although the author concludes that the results show that theoretically (using elastic wedge theory combined with non-polymeric damping material) an elastic wedge can be applied to turbine blades as an effective passive damper, the ‘blades’ investigated in this paper are in fact plane rectangular plates, the blades true aerofoil shape or twist has not been considered. The results presented are in agreement with previous models of wedges of power-law profile.

### 3.3 Manufacturing of experimental samples

Four fan blade samples were machined out of aluminium block, Figure 3.6(a) using a CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm. The NACA 1307 aerofoil was used as a base model and then manipulated to form a non-engine specific model fan blades, two of the sample were then twisted so the effect of adding an Acoustic black hole on to a more realistic fan blade could be considered, Figure 3.6(b). The dimensions of the fan blade are given in Table 3.1. When a twist was added to the blade it was done post manufacture of the blade and wedge.

<table>
<thead>
<tr>
<th>Length</th>
<th>Root chord</th>
<th>Tip chord</th>
<th>Twist Angle</th>
<th>Wedge length</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>300mm</td>
<td>100mm</td>
<td>120mm</td>
<td>11 degrees</td>
<td>43.5 mm</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 3.1: Blade dimensions

(a) Manufacturing of a fan blade, (b) Fan blade profile with (top) and without (lower picture) wedge of power-law profile.
The main problem encountered when utilizing this method of manufacturing was the complexity of the profiles combined with the wedge of power-law profile. Recreating identical twists in the blade was also difficult. The four samples consist of a straight reference fan blade (Figure 3.7(a)), a straight fan blade with a wedge of power-law profile (Figure 3.7(b)), a twisted reference fan blade (Figure 3.7(c)) and a twisted fan blade with a wedge of power-law profile (Figure 3.7(d)).

![Referencing images](a) (b) (c) (d)

Figure 3.7. (a) Reference fan blade - straight, (b) Fan blade with power-law wedge - straight, (c) Reference fan blade - twisted, (d) Fan blade with power-law wedge - twisted

### 3.4 Experimental set up

Three experimental set ups were utilised in the acquisition of results for this chapter. The first experimental set up was used to acquire a vibration response. This experimental set-up has been designed to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate edges and introduce minimal damping to the system, see Figure 3.8(a).

The excitation force was applied centrally on the blade via an electromagnetic shaker and attached using ‘glue’ and fed via a broadband signal amplifier. The response was recorded by an accelerometer (B&K Type 4371) attached to the one surface, directly in line with the force transducer (B&K Type 8200), also attached using ‘glue’, Figure 3.8(b). The acquisition of the point accelerance was utilised using a Bruel & Kjaer 2035 analyser and amplifier. A frequency range of 0-9 kHz was investigated.
The second experimental set up utilises a closed circuit wind tunnel to produce flow visualisation diagrams of the fan blades when placed in an air flow, Figure 3.9. The wind tunnel was run at maximum speed; 30.4 m/s, although this speed is not a true representation of normal engine running speed it is sufficient to get a basic indication of the effect of a power-law wedge on the trailing edge of the fan blade especially at engine start/wind up.

In order for the white flow visualization patterns to be clearly visible on the final photographs the blade were spray painted black. The samples where secured in the working section of the wind tunnel and the flow visualization fluid painted on to the blade. The wind tunnel was then ran up to speed and the flow allowed to stabilize. At this point with the tunnel still running a still was taken of the blade. This process was performed on the top and underside of the blade and at 0 and 10 degrees to the airflow.
The final set up again utilized the wind tunnel. This time the response of the blade was measured in acceleration. The excitation force was provided by the airflow from the wind tunnel (30.4 m/s). The response was recorded using an accelerometer (B&K Type 4371) in the same location as for the first experimental set up. A RT Pro Phonon analyser was used to process the results. As with the flow visualisation the blade was screwed to the dummy balance, representing a clamped beam structure. Again this experiment was carried out at 0 and 10 degrees to the airflow.

For all three experiments the damping layer consisted of a single 40mm x 300mm piece of ducting tape attached to the profiled side of the wedge. This damping layer has a loss factor of 0.06. 10 degrees incidence was chosen as the upper angle of incidence due to the airspeeds under consideration. The usual angle of incidence ranges from 0 to 60 degrees depending on the aerofoil and application it is used for.

### 3.5 Results and Discussion

#### 3.5.1 Introduction of a wedge of power-law profile to a straight fan blade

This section looks at the introduction of a wedge of power-law profile to a fan blade and whether this could produce an ‘Acoustic black hole effect’ as seen in previously tested steel samples, (Krylov et al, 2007). In this section two types of sample were tested a straight reference blade and a straight blade with a machined wedge of power-law profile (1D Acoustic black hole). As discussed in the introduction it has already been ascertained that an additional damping layer is required to produce an ‘Acoustic black hole effect’, therefore all samples with a wedge also have a damping layer attached to the wedge tip.

A comparison of a straight blade with and without a power-law profile wedge is shown in Figure 3.10. As seen in previous work the addition of a wedge of power-law profile to the end of an aluminum fan blade, shows the same trends seen in steel plates (O’Boy et al, 2010a). There is no difference between the two samples below 1.4 kHz. After this point an increase in the reduction of the resonant peaks is seen up until a maximum reduction of 12 dB from the reference sample seen at 4.2 kHz. Above this frequency the response is smoothed with resonant peaks heavily damped if not completely removed.
3.5.2 Introduction of a wedge of power-law profile on to a twisted fan blade

After a promising result on a straight fan blade the next step was to apply the wedge of power-law profile on to a twisted (11 degrees) blade and compare it to a twisted reference blade. The straight and twisted reference blades were also compared. A twisted blade more accurately represents the real world engine fan blades these samples are emulating. This section looks at the effect of the addition of such a wedge and also at the effect of twisting the blade has on the damping performance of the samples.

Figure 3.11 shows the results for a twisted reference compared to a straight reference blade. Below 1 kHz there is correlation in the resonances, however after this there is little to no duplication of resonant frequencies. The twist in the blade creates different interaction between the modes resulting not only in peak shifts but entirely different resonances. This result confirms the need for the experiments on the twisted fan blades in order that this damping method can be proven on a more realistic structure.
Figure 3.11. Twisted Reference blade (solid line) compared to Straight Reference blade – straight (dashed line)

Figure 3.12. Reference fan blade – twisted (dashed line) compared to Fan blade with wedge of power-law profile and damping layer – twisted (solid line)
From Figure 3.12 it can be seen that when the twisted reference blade is compared to the twisted blade with a wedge of power-law profile, a damped response as with the straight blades is clearly viable. Below 1.4 kHz there is little to no damping, although an obvious peak shift is already visible. Between 1.4 – 6.8 kHz there are reductions in the resonant peaks of 3-10 dB with some resonances damped completely. A maximum reduction from the reference plate by the profiled sample of 10.5 dB can be seen at 4.1 kHz. After 6.8 kHz the response is smoothed with resonant peaks heavily damped if not completely removed.

In order to ascertain that all the damping seen in the blades was due to the combined wedge of power-law profile and damping layer and not just due to the visco-elastic damping layer, the twisted reference blade was tested with and without a damping layer and the results compared.

![Figure 3.13. Reference fan blade – twisted with (solid line) and without a damping layer (dashed line)](image)

Figure 3.13 shows the results for the comparison of the twisted reference blade with and without a damping layer. Below 2 kHz little to no damping is seen. The next resonant peak at 2.4 kHz shows the maximum reduction of 3 dB by the reference blade with the damping layer. After this frequency there is a reduction of the peak amplitudes by the reference plate with damping layer by 1 dB over the remainder of the frequency range. This result confirms unequivocally that the damping seen in these samples is due to the presence of the 1D ‘Acoustic black hole’.
3.5.3 Flow visualisation of a fan blade with a wedge of power-law profile

This section looks at the results of the flow visualisation for the straight fan blade. The fan blade is at an incline of 10 degrees incidence to the airflow. The aim of this investigation was to prove that with adaption of the damping layer attached to the wedge of power-law profile, the airflow over the underside of the blade could be returned to a similar state as that seen over the reference blade.

![Flow visualization diagram](image)

Figure 3.14. Flow visualization diagram for (a) Reference fan blade, (b) Fan blade with power-law wedge, (c) Fan blade with power-law wedge with single damping layer and (d) Fan blade with power-law wedge and shaped damping layer

Figure 3.14 shows the progression of the flow visualization tests from a reference blade to a fan blade with a wedge of power-law profile and a shaped damping layer. Looking at the reference fan blade (Figure 3.14(a)) the flow visualisation shows lamina flow across the blade surface with no separation. The effects on the airflow as a result of the presence of the wedge are immediately obvious (Figure 3.14(b)) when looking at the flow visualisation it shows a clear transition line and lamina separation bubble and the flow then reattaches towards the trailing edge of the blade.

The same type and position of damping layer as used in the vibration test was then attached and the test carried out. From Figure 3.14(c) a clear line of transition can be seen between the upstream lamina flow and the turbulent flow after the start of the damping layer. This flow is too turbulent to reattach to the blade. It is worth noting that with the damping layer attached in this way there is a step between the blade surface and the damping layer, this is responsible for the increased turbulence of the airflow in the wedge area.
Any deviation from the original design specification of the blade will not only have the increased drag results seen above resulting in lower efficiency, it will also affect the designed airflow angle into the next stage of the engine, Figure 3.15.

![Air flow velocity diagram from rotor to stator](image)

Figure 3.15. Air flow velocity diagram from rotor to stator

There is an obvious solution to the flow effects seen in Figure 3.14(b) and (c) and to recreate the flow pattern seen in Figure 3.14(a); the original profile of the blade has to be restored. One method of partly achieving this is to shape the damping layer in order to recreate the profile. This was achieved by building up layers of the damping material which when covered by a layer of damping material the same width as the wedge would reproduce the original profile. The final diagram (Figure 3.14(d)) shows the resultant flow over the blade with this shaped damping layer. There is still a clear line of transition but the flow quickly reattaches to blade. This line of transition will always be seen with the ridge at the edge of the damping layer. This result shows that if the damping layer could be more effectively blended in to the blade the line of transition would disappear and a lamina flow would cover the blade.

Another obvious observation is that although on the samples tape could be applied as viscoelastic damping layer, it would not be practical on a real world engine to have this type of damping layer as it could become detached and ingested by the engine and therefore it would be more realistic to incorporate a different style of damping layer. One such example to solve the transition problem and the one just mentioned would be an alloy with a greater loss factor that could be cast on to the blade at manufacture ensuring a strong bond and continuous surface with no transition visible between the blade and damping layer.
3.5.4  *Effect of a built up damping layer on damping performance*

To ensure that the solution found to the aerodynamic problems associated with the addition of a wedge of power-law profile to a fan blade does not affect its damping performance a vibration test with a built up damping layer was carried out. The response of a fan blade with a built up damping layer will be compared to a fan blade with a single damping layer. The experimental set up is the same used in sections 3.5.4 and 3.5.5. Figure 3.16 shows there is little to no difference in the response of a sample with a standard single damping layer and one with a built up damping layer.

![Graph showing comparison of vibration response](image)

**Figure 3.16.** Response for a fan blade with a wedge of power-law profile with a single standard damping layer (dashed line) compared to a fan blade with a wedge of power-law profile with a built up damping layer (solid line)

3.5.5  *Investigation into an airflow excited response in a fan blade*

In order to gain an insight into the effect of airflow on the vibration response of the fan blades the fan blades were excited by the airflow and their response recorded in acceleration. The acceleration was used as an accurate force value could not be calculated. The blades were tested at $0^\circ$ and $10^\circ$ incidence to the airflow. Where a damping layer was attached it was of the ‘built up’ type, to reform the profile as described previously. The more realist twisted fan blades were used for this investigation.
Again the wind tunnel was run at maximum to give the fastest airflow possible, although this air speed is not representative of actual engine speed it can allow for initial conclusions to be drawn and to give insight in to the periods of start-up and run-down on an engine. As mentioned in the literature the period of start-up and run-down cause significant vibration and fatigue on the fan blades.

The first thing to note is that as expected the greater the angle of incidence to the airflow, the greater the amplitude of the response. This is obvious from Figure 3.17, where a comparison of the reference blade is made at 0 and 10 degrees incidence to the airflow. This result is also compared to the response of the bench to the airflow, thus allowing for confidence in the responses shown emanating from the samples and not the bench.

There are two resonances visible in the fan blade samples one at 60 Hz and one at 350 Hz, the bench has only one at 60 Hz. the blade inclined at 10 degrees shows an increase in response of 0.8 m/s² and 0.7 m/s² respectively at each resonance when compared to the blade at normal incidence. Although this bench resonance corresponds to the first resonance seen in the other two samples in frequency, the amplitude of the response is negligible in comparison to the responses from the fan blades. It can therefore be concluded that the resonances seen are those of the blade and not of the bench.
The following results in this section show the responses of the blades inclined at 10 degrees to the airflow. The blades at normal incidence follow the same trends but with a lower amplitude response.

Figure 3.18 shows a comparison of the twisted fan blade inclined at 10 degrees to the airflow with wedge of power-law profile, with and without a built up damping layer. The blade with a built up damping layer shows a reduction in peak amplitude of 0.7 m/s$^2$ and 0.5 m/s$^2$ at 60 Hz and 360 Hz respectively when compared to the same blade without a damping layer.

![Figure 3.18](image)

Figure 3.18. Response in acceleration for a twisted blade with a wedge of power-law profile with (solid line) and without built up damping layer (dashed line)

Finally Figure 3.19 shows the response of the twisted fan blade with a wedge of power-law profile and built up damping layer compared to the twisted reference blade. The twisted fan blade with a wedge of power-law profile and built up damping layer shows a 50% reduction in response amplitude when compared to the twisted reference blade. A reduction in peak amplitude of 1.4 m/s$^2$ and 1.25 m/s$^2$ at 60 Hz and 360 Hz respectively is observed.
The final consideration is the effect the accelerometer has on the airflow and the effect the subsequent turbulence has on the response of the blade. The maximum disruption to the airflow occurs at the largest angle of incidence, it can be seen from flow visualization of a fan blade at 10° to the airflow that with an accelerometer attached that there is a substantial effect on the airflow.

Figure 3.20 shows the flow visualization of the effect of an accelerometer attached to the fan blade at 10° incidence while in the airflow. It is clear that the accelerometer has a strong influence on the airflow over the blade. Starting in front of the accelerometer the air is deflected around the accelerometer to form a horse shoe vortex has it spreads around the accelerometer till it reaches the trailing edge. Directly behind the accelerometer flow is an area of low pressure the air disturbed by the accelerometer is deflected up before returning to form an area of reverse flow becoming entrained along the lower surface before separating of the blade, the bright white line represents this separation line.

These results show that an accelerometer produces a large area of turbulent flow resulting in high levels of drag. As the blades were all tested under the same drag conditions the damping effects seen can be put down to the wedge of power-law profile. Testing with a smaller accelerometer would alleviate some but remove the disturbances to the flow seen below.
Figure 3.20. Flow visualization of the effect of an accelerometer attached to the fan blade at $10^\circ$ incidence while in the airflow

### 3.6 Conclusions

Fan blades with a wedge of power-law profile ‘1D Acoustic Black Hole’ and attached damping layer is an effective method of damping flexural vibrations in the blade. The maximum damping achieved in a straight fan blade was 12 dB at 4.2 kHz and 10.5 dB at 4.1 kHz in the twisted fan blade. Vibration tests in the wind tunnel confirm these results with a 50% reduction in peak amplitude from the reference sample seen by a blade with a wedge of power-law profile and built up damping layer.

Using flow visualisation it can be seen that the wedge can be incorporated in to the fan blades trailing edge and when an appropriate damping layer is applied the aerofoil can be restored to its original profile with limited to no interruption in flow over the blade surface. This built up damping layer has little to no effect on the damping of the blade.

These initial results show that the use of ‘1D Acoustic black holes’ on engine fan blade could be a viable way of reducing flexural vibration in the blade, therefore reducing internal stresses on the blade and increasing its fatigue life.
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CHAPTER 4

DAMPING OF FLEXURAL VIBRATIONS IN PLATES CONTAINING SLOTS OF POWER-LAW PROFILE

This chapter in part was submitted to the journal ‘Applied Acoustics’ in December 2012. Preliminary versions of the results described herein were presented at the Anglo-French Conference ‘Acoustics 2012”, Nantes, France, 23-27 April 2012.

Abstract

It has been shown in Chapter 1 that the addition of power-law profiled wedges to edges of rectangular plates or strips results in substantial increase in damping of resonant flexural vibrations in plates or strips due to the acoustic black hole effect associated with power-law wedges. One of the problems faced by this method of damping is having the wedge tip exposed on the outer edge of the plate or strip. One of the solutions to the problems listed above is to move the wedges inside a plate, so that they form edges of power-law slots within the plate. This chapter reports the results of the experimental investigations into the effects of such slots on damping flexural vibrations. Four experimental investigations are described: the effect of power-law tapered slots on vibration damping in steel and composite plates, the effect of positioning of slots in a plate, and finally the effect of a combined slotted plate. The obtained experimental results show that introducing power-law profiled slots within plates is an effective method of damping flexural vibrations, which is comparable with the method using power-law wedges at plate edges.

4.1 Introduction

The beneficial damping effects seen when a wedge of power-law profile is added to a plate has been documented (D.J. O’Boy et al, 2010) and further explored in Chapters 2 and 3 of this thesis. One of the main problems faced by this method of damping is having the wedge tip exposed on the outer edge of the plate or strip. The tip of the wedge is not only delicate but sharp, thus presenting an exposed structurally weak edge with a health and safety risk. A wedge on the edge of a strip or plate also presents a difficulty in integrating this damping technology into panels/plates that need securing at the edges.

One possible solution to the problems listed above is to move the wedges so that they are located in slots within the plates. This solution will be the focus of this chapter. Four investigations are explored; the effect of the introduction of a tapered slot into steel and composite plates on the
damping of flexural vibrations, the directionality of a slotted plate and finally, the effect of a combined slotted plate.

### 4.2 Samples and Manufacturing

Eleven samples were created for this investigation; six of which were manufactured from 5 mm thick hot-drawn mild steel sheets; which are more resistant to mechanical stresses incurred in the manufacturing process than cold-drawn steel sheets, resulting in fewer internal defects. The dimensions of the steel rectangular plates are 320 x 240 mm, with a slot size of 100 x 75 mm. The other five samples were made from 5 mm thick carbon composite. These samples were made using pre-preg carbon composites sheets that were layed up, cured, and then machined. The dimensions of the carbon composite plates were 280 x 175 mm, with a slot size of 140 x 80 mm. Estimated material properties of plates and viscoelastic damping layers are listed in Table 4.1.

A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm was used to produce the slots in both the steel and composite plates. There are two main problems encountered when utilizing this method of manufacturing. The first being that at the centre of the indentation, where the machining stresses and resulting heat are high and the material thickness is less than 0.2 mm, blistering in the steel can occur, see Figure 4.1(a). This can lead to inaccurate results during test due to varying thickness. The second is wedge tip tearing. This is

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Steel Plate</th>
<th>Carbon composite plate</th>
<th>Damping layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>190 GPa</td>
<td>85 GPa</td>
<td>-</td>
</tr>
<tr>
<td>Density</td>
<td>7000 kg/m³</td>
<td>1600 kg/m³</td>
<td>300 kg/m³</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Loss factor</td>
<td>0.6</td>
<td>0.1–0.2</td>
<td>6 %</td>
</tr>
</tbody>
</table>

Table 4.1: Geometrical and material properties of plates and damping layers.

A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm was used to produce the slots in both the steel and composite plates. There are two main problems encountered when utilizing this method of manufacturing. The first being that at the centre of the indentation, where the machining stresses and resulting heat are high and the material thickness is less than 0.2 mm, blistering in the steel can occur, see Figure 4.1(a). This can lead to inaccurate results during test due to varying thickness. The second is wedge tip tearing. This is
particularly prominent in the composite samples where the tip thickness is less than 0.2mm, see Figure 4.2(b). This effect can occur during the profiling of the wedge itself or during the insertion of the central gully.

The six steel samples consisted of a plain reference plate, a punched slot reference plate and Samples A-D, Figure 4.1-2, the longitudinal cross-sections of which are shown in Figure 4.3. The wedges in the slots are of power-law profile with m=2.2. The five carbon composite samples consisted of a plain reference plate, a combined reference plate and Samples E-G, the longitudinal cross-sections of which are shown in Figure 4.3. The wedges in this case are of power-law profile with m=4.
4.3 Experimental set up

The experimental set-up has been designed to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate edges and introduce minimal damping to the system, see Figure 4.4(a).

Figure 4.4 (a) Experimental Set up, (b) Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample

The excitation force was applied to three locations on the plate via an electromagnetic shaker attached to the plate using ‘glue’ and fed via a broadband signal amplifier. Position 1 – top dead centre, position 2 - to the side centrally located and position 3 - bottom dead centre, Figure 4.4 (b).

The response was recorded by an accelerometer (B&K Type 4371) that was attached to the one surface, directly in line with the force transducer (B&K Type 8200), also attached using ‘glue’. The acquisition of the point accelerance was utilised using a Bruel & Kjaer 2035 analyser and amplifier, a schematic is shown in Figure 4.5. A frequency range of 0-9 kHz for the steel samples and 4.5 kHz for the carbon composite samples were used.

Figure 4.5 Schematic of the experimental setup utilising a Bruel & Kjaer Analyser
4.4 Results and Discussion

4.4.1 Tapered slots in steel plates

Two different styles of steel reference plate were considered in order to determine if there is any significant difference between them. A plain plate, Figure 4.6(a), and a plate with a punched rectangular hole, Figure 4.6(b), were considered as both had merit as a reference. Both reference plates were of the same dimensions as the profiled plates, the punched rectangular hole plate contained a through hole of the same dimensions and position as the machined power-law profile slots.

![Figure 4.6](image)

(a) Plain reference plate – steel, (b) Punched slot reference plate – steel

The plain plate more accurately represents a practical situation, with the profile being cut into an existing structure, it also has reduced internal defects resulting from machining stress, when compared to the punched hole plate. However, it does not account for the reduction in mass and equivalent stiffness of the plate. A plain reference plate was used for comparison in this section. All tests in this section were performed at position 1, as shown in Figure 4.4(b).

![Figure 4.7](image)

Photograph and longitudinal cross-section of Sample C – steel
Initially, a long narrow wedge, of dimensions 95mm x 70mm similar in style to that seen on the external edge of a strip or plate (Krylov, 2004) was machined into the slot in the steel plate. The tip of the wedge was free in order to allow for a more accurate physical comparison to an external wedge. This wedge is almost double the length of the wedges machined on the external edge of the plates that have previously been tested. The sides of the wedge in the slot however are fixed from wedge root to tip, Figure 4.7.

![Figure 4.8 Accelerance for Sample C (solid line) compared to a Reference Plate (dashed line)](image)

Figure 4.8 shows the results for the Sample C when compared to a plain reference plate. The effect of adding an internal wedge into the slot is immediately obvious, with considerable damping of resonant peaks easily observed. Below 900 Hz little to no damping is seen. The peak at 1.4 kHz appears to have been split into two smaller resonances, one either side of the original peak. The reduction in resonant peaks and therefore damping increases throughout the frequency range. A matching of peak amplitudes between the two samples occurs at 6.5 kHz after which the response of Sample C flattens out the distinct resonant peaks seen in the reference sample. A maximum reduction from the reference plate of 11 dB can be observed at 4 frequencies 1.4, 2.2, 5, and 8.3 kHz.

The next step was to investigate the effect of doubling the wedges within the slot, Figure 4.9. Although this would half the length of the existing wedge, it was expected that by doubling the number of internal wedges the damping capabilities of the slot would increase. The internal flat
reflective surface present 5mm from the wedge tip would be removed and the waves instead of being reflected would instead propagate into the wedge. There are now two wedges within the slot, both 47mm x 70mm.

Figure 4.9 Photograph and longitudinal cross-section of Sample D – steel

Figure 4.10 Accelerance for Sample D (solid line) compared to Reference Plate (dashed line)

The results for Sample D, when compared to a plain reference plate are shown in Figure 4.10. Again, below 900 Hz little to no damping is seen and the damping of resonant peaks increases throughout the frequency range. In this case the second matching of peak amplitudes between the two samples has shifted to 7.3 kHz, after which the response of Sample D flattens out the distinct resonant peaks seen in the reference sample. The peak seen at 1.4 kHz in the reference sample is
not split as with Sample C, but simply the amplitude is reduced. A maximum reduction from the reference plate of 10 dB occurs at 8.2 kHz, with further reductions of 8 and 5 dB seen at two frequencies 3 and 5.3 kHz.

![Figure 4.11 Accelerance for Sample C (solid line) compared to Sample D (dashed line)](image)

A comparison of Samples C and D is shown in Figure 4.11. As in previous work (Bowyer et al, 2012), a peak shift to the left from reference is seen to varying degrees in the slot samples. In this case Sample D has a more pronounced peak shift compared to Sample C. There is no difference between the two samples below 1 kHz. Sample C oscillates in most efficient damping performance with Sample D, with the later outperforming Sample C in the ranges 4.5-5.7 kHz and 6-7 kHz. A maximum reduction of 7 dB is achieved by Sample C at 2.2 kHz.

When the plates are excited from position 1 directly above the entrance to the wedge root, the principle discovered in Chapter 2; that a longer wedge is move effective at damping flexural vibrations in lower frequencies than a shorter one of the same profile, can also be applied to multiple wedges in certain frequency ranges. However the benefit of the additional wedge is visible in the increased damping performance seen in the two ranges where Sample D acts as a more effective damper.

This result provides motivation for the consideration of a greater tip width and if it performed more effectively at damping flexural vibration than a wedge of shorter width. In order to accommodate such an increase in width the wedges would have to be rotated in the slot, aligning
the wedge tip line in parallel to the excitation to produce Sample B with two short wide wedges (see Figure 4.12). Wedge dimensions are 95mm x 37mm. A two-wedge configuration was chosen due to the increased damping attained in certain frequency ranges and also the flexibility in excitation position gained with the addition of a second wedge.

![Figure 4.12](image)

**Figure 4.12** Photograph and longitudinal cross-section of Sample B – steel

![Figure 4.13](image)

**Figure 4.13** Accelerance for Sample B (solid line) compared to Reference Plate (dashed line)

From Figure 4.13 it can be seen that when Sample B is compared to a plain reference plate and excited from position 1, the damping achieved is less than that achieved when Sample C or Sample D configurations are used. As expected, below 900 Hz there is little to no damping and at 1 kHz an increase in 4 dB can be seen. Another variation from the previous samples is that there are two
frequencies where the amplitudes match, this occurs at 5.2 and 6.4 kHz, the latter matches that of Sample C. A maximum reduction from the reference plate of 8.5 dB can be seen at 7.6 kHz.

Sample B was then compared to the Sample C and Sample D. When comparing these samples it must be remembered that they are excited at position 1. Figure 4.14 shows the results for Sample B and C. Sample C outperforms Sample B by 2-10 dB. As expected, there is no difference between the two samples below 900 Hz. There is a good correlation between peaks, however Sample B does not split the peak at 1.4 kHz. A maximum reduction from Sample B by Samples C of 10 dB can be seen at 1.4 kHz.

The results for the comparison of Samples B and Samples D are shown in Figure 4.15. Below 900 Hz no difference is seen. Between 2.5-3.8 kHz and 4-7 kHz it is clearly visible that Samples D outperforms Samples B. This is more than likely due to the configuration of the wedges and the increased length and orientation of Sample D. A maximum reduction from Sample B by Samples D sample of 9 dB can be seen at 5.2 kHz.
One of the drivers towards creating the slot plates was the protection of the wedge tip, however they are still fragile and prone to tearing during and post manufacture another solution was to remove the central gully creating a ‘slot pit’, Sample A, Figure 4.16.

A comparison of Sample A to the plain reference plate is shown in Figure 4.17. Below 900 Hz little to no damping is seen. A maximum reduction from the reference plate of 9 dB can be seen at 3 kHz, with a 6dB reduction at 1.4 and 8.3 kHz, despite this large reduction in peak amplitude the majority of peaks are reduced by only 0-3dB.
Figure 4.17: Accelerance for Sample A (solid line) compared to Reference Plate (dashed line)

Figure 4.18: Accelerance for Sample C (solid black line) compared to Sample D (solid grey line) and Sample A (dashed line)
From Figure 4.18 it can be seen that when Sample C, Sample D and Sample A are compared, Sample A is clearly the least effective at damping flexural vibration in the steel plate. As expected, there is no difference between the three samples below 900 Hz. A maximum reduction from Sample A of 9 dB to Sample D can be seen at 3 kHz, as seen in the previous result. The gap at the end of the wedge tip is essential for increased damping performance; this allows a free edge with no wave transmission through the gap and a free vibration at the tip.

4.4.2. Tapered slots in carbon composite plates

Composites have always been revered as the material of the future and with their increasing popularity in industry it would be a miss not to consider the effects of new damping methods on such materials. In this section the effects of a tapered slot on a composite plate is investigated.

![Figure 4.19](image)

Plain reference plate—carbon fibre composite

In this section the effects of a tapered slot on a composite plate is investigated. A plain reference plate, Figure 4.19, was used for comparison in this section. No additional visco-elastic damping layer was added to any of the composite samples. All tests in this section were performed at position 1, as shown in Figure 4.4(b).

![Figure 4.20](image)

Carbon fibre composites Sample E (a) and Sample F (b)
When used in a practical application, it is unlikely that the excitation of the plate will be from a single point source, and it will more than likely be excited from multiple sources. The two slot configurations therefore chosen to be tested in carbon composite were Sample E and Sample F, Figure 4.20 (a-b), as they offer the greatest flexibility in excitation position.

Figure 4.21 Accelerance for Sample F (solid line) compared to a reference plate (dashed line)

Figure 4.21 shows the results for Sample F compared to a plain reference plate. Below 250 Hz little to no damping is seen, the second resonance after this area at 600 kHz does however show an increase in peak amplitude by 6 dB compared to the reference plate. After this point the response is smoothed, with resonant peaks heavily damped if not completely removed. A maximum reduction from the reference plate of 11.5 dB can be seen at 800 kHz.

The results for Sample E compared to a plain reference plate are shown in Figure 4.22. Again, below 250 Hz there is little to no damping, there is also an increase in the second peak amplitude of Sample E, increasing the reference value at 650 kHz by 5 dB. A maximum reduction from the reference plate of 12 dB can be seen at 800 kHz despite this slightly higher maximum.
Figure 4.22  Accelerance for Sample E (solid line) compared to a reference plate (dashed line)

Figure 4.23  Accelerance for Sample F (solid line) compared to Sample E (dashed line)
When compared to each other, the two samples are expected to show the same trends as their steel plate counterparts. The only differences in the observed trends are down to the changes in material properties. There is an initial area of no reduction, then an increasing reduction in resonant peaks as the frequency increases. The sample with the wedge root directly adjacent to the excitation position should perform the most effectively.

A comparison of Sample E and F frequency responses are shown in Figure 4.23. There is no difference between the two samples below 250 Hz. After 1.5 kHz Sample F is consistently the more effective damper of the two samples when excited from position 1. A maximum reduction is achieved by Sample F of 3 dB, it can be seen at 1.6 and 2.4 kHz.

One of the most interesting results obtained from this section is that due to the large material loss factors of composites, no additional damping layer is needed to obtain substantial reductions in the amplitude of the resonance peaks.

4.4.3. The effect of Vibration Source Position on Achievable Levels of Damping in slotted plates

This section investigates the levels of damping that can be obtained when comparing the position of the excitation point of the sample to the position of the start of the wedge profile (wedge root) within the slot. When different excitation positions are considered, different modes are excited within the plates. Three steel samples were used; Sample C, Sample D and Sample B along with the two composite samples. First, Sample C was excited in three different positions in relation to the slot, Figure 4.4(b).

From Figure 4.24 it can be seen that when Sample C was excited from positions 1, 2 and 3, the position of the wedge root relative to the excitation position determined the level of damping achieved. Considering the general trend of these three excitation positions on Sample C; excitation position 1 has closest proximity to the wedge root and has the greatest damping capability. When excited at position 2, the damping capability of the sample is less than that at position 1. The increase in the damping performance over the frequency range is small; approximately only 1-2 dB per rotation of excitation position.

Figure 4.25 shows the results for Samples B when excited from positions 1 and 2. This result shows more clearly that when excited from position 2, a greater general reduction takes place than for the same sample excited from position 1.
Figure 4.24 Accelerance comparison for Sample C excitation positions 1 (solid black line), 2 (grey line) and 3 (dashed line)

Figure 4.25 Accelerance for Sample B excitation positions 1 (dashed line) and 2 (solid line)
Figure 4.26 Accelerance for Sample D excitation positions 1 (dashed line) and 2 (solid line)

The results for Sample D for excitation positions 1 and 2 are shown in Figure 4.26. The results in this case are less clear than in Sample C. Below 4.5 kHz the sample is more effective at reducing peak amplitudes that when excited from position 2.

Excitation at any of the three excitation positions will cause flexural waves to propagate out in all directions from the excitation point. The excitation position directly adjacent to the wedge root as expected to result in the greatest reduction in resonant peaks due to the waves propagating directly into the wedge. For Sample C this situation occurs at Excitation position 1, Samples B and E preform more effectively when excited from position 2. When exited from position 1 Sample C and D outperformed the Double sample. The samples configurations that had the wedge root positioned adjacent to the excitation point yielded the greatest reduction in resonant peaks. The composite sample results concurred with the conclusions obtained above. Sample D performed more efficiently when excited from position 1 and Sample B was a more efficient damper when excited from position 2.
4.4.4. Combined Slot Composite Plate

Considering the effect of the source position in relation to the wedge root on the damping performance found in all samples, the possibility of combining two styles of slotted plates was considered in an attempt to obtain a more efficient damping performance. The composite samples that would best suit amalgamation were Samples E and F, as they could be easily bonded together using a long cure epoxy resin. These configurations when combined were expected to provide the greatest damping performance over the greatest number of excitation positions. Sample G, Figure 4.27, was excited in both position 1 and 2 and was compared to the sample that produced the most effective damping for each respective excitation point. Both excitation positions for the combination plate were then compared.

![Figure 4.27 Photograph and longitudinal cross-section for Sample G](image)

From Figure 4.28 it can be seen that when Sample G was compared to Sample F for excitation position 1 that after 1 kHz the samples respond in much the same way, with little to no defined resonant peaks visible. The peak at 200 Hz from Sample F has been completely damped and the peaks at 400 and 600 Hz have been reduced by 4 and 5 dB respectively. An additional peak in Sample G has emerged however at 850 Hz and is likely due to the bonding of the two plates. A maximum reduction for Sample F of 18 dB can be seen at 200 Hz.

Figure 4.29 shows the results for Sample G when compared to Sample E for excitation position 2. For this comparison after 1.5 kHz the samples respond in much the same way, with little to no defined resonant peaks visible. As with the peak in Sample F at 200 Hz the equivalent peak in Sample E has been completely damped. An additional resonance in Sample G is again seen around 800 Hz. A maximum reduction by Sample G of 18 dB can be seen at 200 kHz.

Sample G, the combined composite plate allows for a more uniform damping response independent of excitation position. Combining of the two styles of slot plate; Sample E and Sample F mean that modes damped by one sample and not the other can be damped at the same time with only one sample.
Figure 4.28 Accelerance for Sample G (solid line) compared to Sample F (dashed line) excitation position 1

Figure 4.29 Accelerance for Sample G (solid line) compared to Sample E (dashed line) excitation position 2
Figure 4.30 Accelerance for Sample G; Combined composite for excitation positions 1 (solid line) and 2 (dashed line)

The results for Sample G for excitation positions 1 and 2 are shown in Figure 4.30. Above 1.7 kHz the samples respond in much the same way. The second peak is most likely due to bonding and is present in both samples at the same frequency. The third peak from both samples, although shifted due to the different excitation point, is within a 3 dB amplitude range. Sample G is only fractionally more effective at damping resonant frequencies when excited in position 1.

Sample G, the combined composite plate allows for a more uniform damping response independent of excitation position.

4.5 Conclusions

Plates containing slots of power-law profile materialise an effective method of damping flexural vibrations. The maximum damping achieved using a steel slot plate was by utilizing the configuration Sample C, where a reduction of 11 dB was observed at 4 frequencies: 1.4, 2.2, 5 and 8.3 kHz, when excited at position 1. Sample A, although providing some damping, is relatively ineffective when compared to other slot wedge combinations. Carbon composite slot plates follow the same trends as the steel slot plates No damping layer is required on the composite plates to achieve comparable damping performance to the steel plates. This is due to the large values of the material loss factor for composites. The maximum damping achieved using a composite slot plate
was by utilizing the configuration of Sample E, where a reduction of 12 dB at 800 Hz was achieved.

The shaker position does influence the damping performance of the slot plates. Damping performance of the slot layout is dependent upon the excitation point in relation to the wedge root. The combined carbon composite plate, Sample G, damps all the modes above 1.5 kHz that are damped by the individual slot plates before amalgamation, irrespective of excitation point. Sample G completely damps a resonant peak at 200 Hz produced by the reference plate, with a maximum reduction of 18 dB.
CHAPTER 5

DAMPING OF FLEXURAL VIBRATIONS IN PLATES CONTAINING TAPERED INDENTATIONS OF POWER-LAW PROFILE

This chapter in part was accepted for publication in the journal ‘Applied Acoustics’ in October 2012. A preliminary version of the results described herein was presented and published at the International Conference ISMA 2010 (Leuven, Belgium) in September 2010 and presented at the Meeting of the Acoustical Society of America in Kansas City, USA 2012 and was published in the proceedings; the Journal of the Acoustical Society of America 132(3), p2041.

Abstract

In this chapter, the results of the experimental investigation of damping flexural vibrations in rectangular plates containing tapered indentations (pits) of power-law profile with the addition of a small amount of absorbing material are described. In the case of quadratic or higher-order profiles, such indentations materialise two-dimensional ‘black holes’ for flexural waves. In the present investigation, pits have been made in different locations of rectangular plates. It has been found that basic power-law indentations with no central hole produce rather small reduction in resonant peak amplitudes, which may be due to the relatively small absorption area covered by a damping material. To increase damping in the present investigation, this absorption area has been enlarged by increasing the size of the central hole in the pit, while keeping the edges sharp. As expected, such pits, being in fact curved power-law wedges, result in substantially increased damping. When multiple indentations are used the resultant damping is comparable if not greater than that achieved by one-dimensional wedges of power-law profile.

5.1 Introduction

Using circular indentations inside plates offers a range of benefits in comparison with wedges of power-law profile attached to plate edges. First of all, the potentially dangerous sharp edges of power-law wedges are eliminated, so that all exposed sides of the plate are of the same nominal thickness, which brings a safety benefit. Secondly, two-dimensional pits can be applied to suppress just some selected resonant peaks, when placed in certain positions. Finally the delicate wedge tips are protected to a greater extent.
In the following sections of this chapter, the manufacturing method used to produce the experimental plates with two-dimensional power-law indentations is described, followed by the description of the experimental set-up. The results of the measurements are then discussed, followed by the conclusions.

It is demonstrated in this Chapter that basic power-law pits that are just protruding over the opposite plate surface (no central hole) result in rather small reduction in resonant peak amplitudes, which may be due to their relatively small absorption cross-section capturing a relatively small number of incident flexural wave rays. Note that, as described in Chapter 1, for elliptical plates this disadvantage has been overcome by focusing of flexural waves in the pit (O’Boy et al, 2010, O’Boy et al, 2010 and Georgiev et al, 2011).

To increase the damping efficiency of power-law profiled indentations, the absorption cross-section has been enlarged by increasing the size of the central hole in the plate while keeping the edges sharp. As expected, such large pits, being in fact curved power-law wedges, result in substantially increased damping. However, to achieve or exceed the damping provided by one-dimensional wedges of power-law profile, multiple circular indentations must be made within a plate. The effect of indentation position is also considered.

5.2 Manufacturing of experimental samples

All experimental samples in the present work were manufactured from 5 mm thick hot drawn mild steel sheets; which are more resistant to mechanical stresses incurred in the manufacturing process than cold drawn steel sheets, thus resulting in fewer internal defects. Dimensions of the produced rectangular plates were 400 x 300mm and profile m=4. Estimated material properties of plates and visco-elastic damping layer are listed in Table 5.1.
Table 5.1 Geometrical and material properties of plates and damping layers.

<table>
<thead>
<tr>
<th></th>
<th>Thickness</th>
<th>Young’s modulus</th>
<th>Mass density</th>
<th>Poisson’s ratio</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>5.04 mm</td>
<td>190 GPa</td>
<td>7000 kg/m$^3$</td>
<td>0.3</td>
<td>0.6 %</td>
</tr>
<tr>
<td>Damping layer</td>
<td>0.08 mm</td>
<td>-</td>
<td>300 kg/m$^3$</td>
<td>-</td>
<td>6 %</td>
</tr>
</tbody>
</table>

A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm was used to produce circular fourth-power indentations into the plates. Three types of experimental samples were produced for this investigation, a plate with a singular circular indentation (Figure 5.2), a plate with a singular circular indentation with a drilled central hole, and plates containing multiple profiled circular indentations with central holes; these samples are shown in Figures 5.3 and 5.4.

Figure 5.2 Plate containing a circular indentation with no central hole.

Figure 5.3 Profiled circular indentations with central holes: (a) - two indentations, (b) - three indentations, (c) - four indentations.
There are three main problems encountered when utilizing this method of manufacturing. The first being that at the centre of the indentation, where the material thickness is less than 0.4 mm and the machining stress and resulting heat are high, there may be blistering (see Figure 5.5) leading to inaccurate results during experimental testing. Secondly, it is the formation of a machine line, as the cutters movement through the indentation is computer controlled. It merely moves from one programmed height to another, gouging the material and creating a raised line in the indentation, which, as with blistering, could lead to increased elastic wave scattering at higher excitation frequencies. Finally, additional damage can occur when a hole is drilled into the centre of the circular indentation. Due to the reduced thickness of the material at this point, it is more susceptible to tearing.
5.3 Experimental setup

The experimental set-up has been designed to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate edges and introduce minimal damping to the system, see Figure 5.6(a).

The excitation force was applied to the centre of the plate via an electromagnetic shaker attached to the plate using ‘glue’ and fed via a broadband signal amplifier. The response was recorded by an accelerometer (B&K Type 4371) that was attached to the upper surface, directly above the force transducer (B&K Type 8200), also via ‘glue’, see Figure 5.6(b). The acquisition of the point accelerance was utilised using a Brue & Kjaer 2035 analyser and amplifier over a frequency range of 0-9 kHz, a schematic is shown in Figure 5.7. The results presented in this chapter have been averaged over five measurements in order to ensure a statistically accurate representation of the point accelerance.

Two different styles of reference plate can be considered for comparison: a plain plate and a plate with a punched through hole. Both reference plates are of the same dimensions as the profiled plates, the punched hole plate contains a through cylindrical hole of the same diameter and position as the machined power-law profiles. The plain plate more accurately represents a practical situation, with the profile being cut into an existing structure. It also has reduced internal defects resulting from machining stress, when compared to the punched hole plate, although the latter accounts for the reduction in mass and equivalent stiffness due to the profiled indentation. Keeping the above in mind, a plain reference plate was used for comparison in this Chapter.
5.4 Experimental results and discussion

5.4.1. A profiled circular indentation with and without a small central hole

Two sets of experimental results are described in this section: the effect of adding a circular indentation of power-law profile into a plate, when compared to a reference plate, and the effect of drilling a 2mm hole in the centre of the indentation.

The initial measurements with a single circular indentation without a central hole demonstrated that in this case there were no noticeable damping effects when compared to a reference plate, apparently because of a rather thick central area due to limitations of the manufacturing method used. There was little to no improvement in damping by the addition of a damping layer. To the contrary, in some frequency ranges the presence of the indentation actually increased the level of vibrations in the plate. From these initial observations it has been decided that a central hole has to be drilled into the centre of the indentation.

Note that due to manufacturing limitations at the centre of the circular pit, there is an area of almost equal thickness of about 0.4 mm that extends from the central point out to a diameter of approximately 3 mm. A hole of 2mm diameter can therefore be drilled without affecting the minimum tip thickness.

Figure 5.8 shows the measured accelerance for an indentation of power-law profile with a 2 mm central hole with and without a damping layer compared to a reference plate. When comparing an indentation with a 2mm central hole and damping layer to a reference plate it can be seen that below 4.7 kHz there is little to no damping. A slight increase is seen in some of the peaks between 1.5 – 3 kHz and 5.5 -6 kHz. A maximum damping of 6 dB occurs at 6.5 kHz. These results show that the effect of drilling a hole in the centre of the circular indentation of power-law profile...
increases the overall losses at higher frequencies due to increased absorption of the incident flexural wave by the indentation in this case.

![Graph](image)

Figure 5.8 Measured accelerance for a profiled circular indentation with a 2 mm central hole with (solid line) and without (dashed line) an additional damping layer compared to a reference plate (solid grey line)

These measurements, however, also show some specific behaviour as a result of adding a damping layer to the tip of the circular inclusion. Earlier, it has been shown for power-law wedges (Krylov et al, 2007) that in order to achieve significant damping there is a requirement to add an additional damping layer to the tip of the wedge. However, this is not the clear case for a circular indentation with a 2 mm central hole, with only a 1-2 dB additional reduction occurring. Damping obtained by the addition of a damping layer is dependent on the stiffness of the damping layer itself.

5.4.2. A profiled circular indentation with a large central hole

In an attempt to improve the damping efficiency of the profiled indentations, the central hole size was increased progressively by 2 mm until a central hole size of 14 mm and an indentation diameter of 100 mm was produced. As the central hole size increased so did the damping performance of the circular indentation. This is most likely due to the increased boundary area of
the tapered surface and the indentation more closely resembling a wrapped around wedge. As the central hole increases so does the diameter of the indentation.

Figure 5.9 shows a direct comparison between the profiled circular indentations with 2 mm, 10 mm and 14 mm central holes. Below 3.5 kHz, the responses of all three samples are almost identical, varying by no more than approximately 1-2 dB. In all other regions the 14 mm central hole has an increased damping performance compared to the 2 mm and 10 mm central holes. In the region of 6-8 kHz, the maximum difference can be seen with the response of the 14 mm central hole. It shows a reduction by a maximum of 2.5 dB compared to the sample containing the 10 mm central hole and by 6 dB compared to the sample containing a 2 mm central hole. Increasing the central hole diameter thus increases damping performance of the circular indentation as it increasingly resembles a curved wedge as the diameter of the central hole is enlarged and the curvature of the free edge diminishes.

Figure 5.9  
Measured accelerance for a damped circular indentation with a 14 mm central hole (solid black line) compared to a damped circular indentation with a 2 mm central hole (dashed line) and a 10 mm central hole (solid grey line).

Unlike in the case of the indentation containing a 2 mm central hole, the addition of a damping layer increases the damping performance of the inclusion up until 8 kHz, after which it has a reduced effect. This pattern of varying damping is consistent with the theory of power-law profiled wedge damping (Krylov et al., 2004, Krylov, 2004 and Krylov et al., 2007). The greatest increase in
damping performance was achieved in the frequency range between 3.5-8 kHz, when an additional damping layer was attached.

A comparison of the results for a profiled circular indentation with a 14 mm central hole and an additional damping layer to the results for a reference plate are shown in Figure 5.10. Again, Figure 5.10 shows that below 2 kHz the circular pit provides little to no damping. In the region of 3.8 – 9 kHz, damping varies between 1 – 10 dB, and maximum damping occurs at 6.5 kHz.

5.4.3. Damping effect of increasing the diameter of the circular inclusion

This section describes the effect of increasing the diameter of the circular indentation from 100 mm to 114 mm. By increasing the size of the indentation, the length of the profiled area on the plate increases. The two samples tested had a hole of 14 mm in the centre of the indentation. The results are shown in Figure 5.11. From Figure 5.11, it can be seen that increasing the diameter of the indentation increases the damping performance. Below 2.5 kHz there is little to no discernable difference between the two samples, as the flexural wavelength is longer than the diameter of the inner aperture of the smaller diameter hole. Above 2.5 kHz the damping performance of the larger diameter hole increases,
compared to the smaller diameter hole. A maximum of a 5 dB increase in damping occurs with the 114 mm diameter indentation at 4 kHz.

![Graph](image)

Figure 5.11 Measured accelerance for a damped circular indentation of diameter 114 mm (solid line) compared to a circular indentation of diameter 100 mm (dashed line), both with a 14 mm central hole.

5.4.4. Multiple circular indentations

In the previous section, the comparison of a reference plate was made with a similar rectangular plate with a damped circular indentation of 14 mm diameter, leading to a reduction in amplitude of up to 10 dB at 6.5 kHz. In this section, the results of increasing the number of damped circular indentations on a centrally excited plate for two, three, four, five and six profiles are presented. They are compared to each other and to a reference plate. Naturally, the multiple-indentations samples were expected to show higher damping than the plates with one circular indentation, and the damping performance was expected to increase with the number of circular indentations due to a cumulative effect. The diameters of the circular indentations were 114 mm and the central holes were 14 mm in diameter. Unless otherwise stated all indentation had an additional damping layer.
Figure 5.12  Measured accelerance for a plate containing two profiled circular indentations with 14 mm central holes and additional damping layers (solid line), as compared to a reference plate (dashed line).

The results for a plate containing two profiled circular indentations with 14 mm central holes and additional damping layers, as compared to a reference plate, are displayed in Figure 5.12. Below 3 kHz little to no damping is seen. A substantial reduction in all peak amplitude can be seen above 4 kHz. Maximum damping occurs at 6.5 kHz with a reduction of 11 dB. When referred back to Figure 5.10, the addition of an extra indentation can clearly be seen. The sample containing two indentations shows a far greater reduction in peak amplitude above 4 kHz, for example the peaks at 4.5, 5 and 7 kHz are reduced by a further 3-5 dB, when compared to a single indentation sample.

Figure 5.13 shows the results for a plate containing three profiled circular indentations with 14 mm central holes and additional damping layers, when compared to a reference plate. Maximum damping occurs at 6.5 kHz with a reduction of 11.8 dB. A substantial reduction in peak amplitude can again be seen above 4 kHz, however at 6 kHz there is a matching peak, where the amplitude of the response for the reference plate and three indentation sample coincide. This is the only multiple indentation sample to display this trait and is most likely due to the layout of the indentations and therefore the modes they affect.
Figure 5.13  Measured accelerance for a plate containing three profiled circular indentations (solid line), as compared to a reference plate (dashed line).

Figure 5.14  Measured accelerance for a plate containing four profiled circular indentations (solid line), as compared to a reference plate (dashed line).
The results for a plate containing four profiled circular indentations with 14 mm central holes and additional damping layers, as compared to a reference plate are shown in Figure 5.14. Below 2 kHz little to no damping is seen. A substantial reduction in all peak amplitude can be seen above 3 kHz, there again no matching peaks in this region. Maximum damping has shifted slightly and now occurs at 6 kHz with a reduction of 12.5 dB. Again there is an increase in the maximum reduction of peak amplitude and the frequency at which substantial reductions in peak amplitude can be seen to start at a lower frequency, compared to plates with fewer indentations.

Figure 5.15 Measured accelerance for a plate containing five profiled circular indentations (solid line), as compared to a reference plate (dashed line).

A comparison of the results for a plate containing five profiled circular indentations with 14 mm central holes and additional damping layers, as compared to a reference plate, are shown in Figure 5.15. Below 1.5 kHz little to no damping is seen. A substantial reduction in all peak amplitude can be seen above 2.5 kHz, there is a matching peak at this frequency, the two resonances below this frequency show considerable damping, 9dB. Maximum damping has shifted slightly and now occurs at 6.5 kHz with a reduction of 13.5 dB. Again there is an increase in the maximum reduction of peak amplitude and a reduction in the frequency at which substantial reductions in peak amplitude can be seen to start, compared to plates with fewer indentations.
The results for a plate containing six profiled circular indentations with 14 mm central holes and additional damping layers, compared to a reference plate, are shown in Figure 5.16. This configuration by far has the greatest damping at higher frequencies out of the samples tested, and it even outperforms a wedge of the same power-law profile. Below 1 kHz there is little to no damping, as expected, and between 1 – 2 kHz a reduction of peak amplitude by up to 4 dB is observed, after which damping increases until almost all peak responses are flattened. A maximum damping of 14 dB occurs at 6.5 kHz.

![Figure 5.16](image_url)

Figure 5.16  Measured accelerance for a plate containing six profiled circular indentations with 14 mm central holes and additional damping layers (solid line), as compared to a reference plate (dashed line).

Figure 5.17 shows the results for the plates containing two, four and six profiled circular indentations with 14 mm central holes and additional damping layers. It is seen that the greater the number of indentations in the plate, the greater the damping obtained. The frequency at which a substantial reduction in peak amplitude is first seen decreases as the number of indentations increases. For the two, four and six indentation plates this occurs at approximately 4, 3 and 2 kHz respectively, as the lower frequency mode shapes start to correspond more closely with parts of the indentations patterns. After 4 kHz, it can be seen that the response clearly flattens increasingly with indentation number, thus increasing the level of broad band damping achieved compared to the high level of damping perceived at specific frequencies as seen in the one indentation plate.
Figure 5.17  Measured accelerance for a plate containing two (dashed line), four (solid grey line) and six (solid black line) profiled circular indentations.

Figure 5.18  Plate containing six circular indentations (solid black line), as compared to a reference plate (dashed line) and to a reference plate covered by a damping layer with the total surface area equal to that of a plate containing six circular indentations (solid grey line).
To ensure that the damping effect shown by the plate containing six profiled circular indentations was in fact due to the presence of the indentations and not just merely due to the increase in surface area of the plate covered by the visco-elastic damping layer, a comparison was made between a plain reference plate, a reference plate with the equivalent area of attached visco-elastic damping that is present on a plate containing six indentations and a plate containing six indentations. The results are shown in Figure 5.18. It is clear to see that some degree of damping is obtained through the addition of the visco-elastic damping layer alone, however it does not account for the full extent of the damping performance seen when it is combined with the circular indentations of power-law profile. For example, at approximately 5 kHz the visco-elastic damping layer sees a reduction in amplitude in the range of 3 dB compared to the reference plate, and at the same instance the plate containing six circular indentations produces a reduction in the range of 12 dB.

As expected, the damping performance of the plates containing a singular indentation is not greater than the performance of 1D wedges of the same profile. However, the multiple indentation plates were able to compete with the damping performance provided by the tapered wedges. The position of the holes is linked to performance, and different combinations may result in greater levels of damping for particular resonance peaks.

5.4.5. Effect of indentation position on a five indentation plate

Finally, the effect of positioning the indentations was considered. Using the five indentation sample as an example the fifth indentation was positioned along the vertical axis of the plate and then compared to the previously tested non-symmetrical alternative, Figure 5.19.

Figure 5.19 (a) Plate containing five indentations non-symmetrical about the vertical axis, (b) Plate containing five indentations symmetrical about the vertical axis
A comparison of the results for a plate containing five profiled circular indentations five indentations symmetrical about the vertical axis, compared to a plate containing five indentations non-symmetrical about the vertical axis, are shown in Figure 5.20. Below 1.3 kHz and above 3 kHz there is little to no difference in the responses of the plates. Between these ranges there are two resonances of interest, one for each plate in which the amplitude of the response is significantly different from the other. For the symmetrical sample the resonant peak located at 1.6 kHz has been reduced by 6dB when the indentation is moved to its non-symmetrical position. For the non-symmetrical sample the resonant peak located at 2.3 kHz has been reduced by 6dB when the indentation is moved to its symmetrical position.

By altering the position of the indentations the stiffness and modes that are excited change within the plates. Different indentation positions cause different modes to be excited and different modes to be damped. Therefore depending on the frequency range of interest the indentation can be moved to ensure maximum damping in that range/specific frequency as required.
5.5 Conclusions

It has been demonstrated in this chapter that basic power-law indentations with no central hole cause very small reduction in resonant peak amplitudes of plate vibrations, which may be due to their relatively small absorption cross-section capturing a relatively small number of flexural wave rays. Introduction of a 2 mm central hole increasing the absorption cross-section improves the situation and increases damping.

To increase damping even more, the absorption cross-section has been enlarged by increasing the size of the central hole in the indentation up to 14 mm, while keeping the edges sharp. As expected, such indentations, being in fact curved power-law wedges, resulted in substantially increased damping performance that was comparable with that achieved by one-dimensional wedges of power-law profile. Also, it has been shown that the larger the diameter of the indentation the greater the damping effect due to increased length of the profiled area on the plate.

The indentation position will of course have an effect on the modes that are damped; therefore indentation position can be used to target different resonances within the lower frequency range. For example below 3kHz for a 5 indentation plate an extra 6dB reduction in peak amplitude can be achieved at 2.2 kHz by changing the position of one of the indentations.

As expected, the introduction of multiple-hole plates in the current layout clearly increases the damping performance of the two-dimensional indentations of power-law profile. As the number of indentations increases so does the damping performance. The six indentation sample showed a maximum increase in resonant peak damping over a large frequency range. The obtained results show that using combinations or arrays of several circular indentations of power-law profile with large central holes covered by small pieces of absorbing materials can be a very efficient method of damping flexural vibrations in different plate-like structures.
CHAPTER 6

VIBRATION DAMPING IN A CYLINDRICAL PLATE
INCORPORATING A CENTRAL INDENTATION OF
POWER-LAW PROFILE

This chapter in part has been published in the Journal of the Acoustical Society of America, Volume 329, Issue 22, 25 October, 4672-4688 (2011). A preliminary version of the results described herein were also presented at the RASD 2010 conference.

Abstract

This chapter describes the results of a series of experimental measurements of point accelerance carried out on circular plates containing tapered central holes of quadratic profile with attached damping layers. These experimental results are then compared to the earlier developed numerical model (O’Boy, 2010), as a means of validation.

The driving point accelerance measurements and predictions are shown for a plain reference disc, a plain reference disc with a layer of damping film and a disc with an indentation of quadratic power-law profile machined into the center, again with a thin layer of damping material attached. The experimental results show a substantial suppression of the amplitude of resonant peaks. This is in agreement with a numerical model, which is based on the analytical solution available for the vibration of a plate with a central quadratic power-law profile.

6.1 Introduction

When considering purely experimental work, the results of which constantly display a substantial suppression of the amplitude of resonant peaks over a wide frequency range and in a variety of different media, as shown throughout this thesis, there are obvious questions that should be considered. In order to ensure the experimental procedure is sound, that the post processing and analysis is correct, confirmation of the results can be sort by comparison to a numerical model.

A validation case where experimental measurements can be directly compared to numerical predictions would not only allow a physical insight into the wave propagation in the tapered section of the plate, but also will ensure that the experimental procedure is sound. As stated in Chapter 1, every sample was tested for repeatability a minimum of 10 times. Where possible, multiple samples of the same type were made. However, due to financial and manufacturing time constraints, this
was not always logistically possible. Validation of these results via a numerical model ensures the upmost confidence in the experimental procedures utilized throughout this thesis. This numerical model is a simple case study for validation, due to the exact solution of the equation of motion. No such exact solution is available for the 2D rectangular case; the closest is an asymptotic approximation.

![Image of a circular disc with a quadratic power-law profile indentation](image)

Figure 6.1 Circular disc incorporating a circular indentation of quadratic power-law profile at the center (not to scale), (a) illustration (b) Numerical model notation.

The cylindrical plate with a circular indentation of quadratic power-law profile at the center consists of a constant thickness annular disc and a smaller profiled disc as shown in Figure 6.1(a), the notation used for the numerical model is shown in the cross section of half of the disc shown Figure 6.1(b).

As the experimental measurements are compared directly to the results of the numerical predictions, a short background to the numerical model method (O’Boy et al, 2010) is given in Section 6.2. In the results section, the experimental method is shown and discussed. These experimental results are then compared to the theoretical results of a numerical model in order to validate the experimental procedure by demonstrating a match in the actual and theoretical amplitude reductions and increased overall loss factor which can be achieved through the use of cylindrical power-law profiles in plate structures. Finally, the accuracy of the total loss factor for the three samples mentioned above will be calculated.
6.2 Background to numerical method

There are many analytical solutions to the bending plate equation of motion where the thickness is a constant; however the addition of a variable thickness section increases the complexity to level that results in a reduced availability of complete analytical solutions. The numerical model for point accelerance utilized in this chapter was provided by the O’Boy numerical model for point mobility of a disc containing an indentation of quadratic power-law profile (O’Boy, 2010) and then converted to accelerance, for comparison. The following section reviews this method.

The process required to calculate the numerical model utilizes three stages; first, solve the bending plate equation of motion for a constant thickness disc, then solve the bending plate equation of motion for the profiled central disc using wave propagation techniques, and finally, combine the two solutions using continuity of displacement and force at the join between the two discs (common boundary conditions). Both the sections of the disc are assumed to have the same material properties.

For the first stage, the bending plate equation of motion for a constant thickness cylindrical disc is given by,

\[-D\nabla^4 w(r, \theta, t) = \rho h \frac{\partial^2 w(r, \theta, t)}{\partial t^2},\]  

Where:

- $D$ : Bending stiffness
- $E$ : Young’s modulus
- $\rho$ : Density
- $\nu$ : Poisson’s ratio
- $h$ : Plate thickness
- $\nabla$ : Differential operator
- $\omega$ : Frequency
- $w(r, \theta, t)$ : Plate displacement normal to surface

It can be shown that the Bessel’s functions can provide the solution to the displacement on any part of the constant thickness disc (Graff, 1975):

\[w(r, \theta, \omega) = c_1 J_n(\beta r) + c_2 Y_n(\beta r) + c_3 I_n(\beta r) + c_4 K_n(\beta r)e^{in\theta}e^{-i\omega t}.\]  

The constants $c_1$, $c_2$, $c_3$, $c_4$ can be found via the applications, of boundary conditions to the outer edge of the disc, where the radius $r = r_o$ (the disc radius) and $\beta^4 = \rho h \omega^2 / D$.

For the second stage, the bending plate equation of motion for a profiled indentation with a varying thickness in the radial direction only is required. This profiled indentation starts at a radius $r = r_i$ and the bending stiffness will vary with radial position. The profiled center section is described by the equation of motion given by: (Harris, 1967):

\[\rho h \frac{\partial^2 w(r, \theta, t)}{\partial t^2} = (1 - \nu) \nabla^4 \{D, w(r, \theta, t)\} - \nabla^2 [D, \nabla^2 w(r, \theta, t)].\]  


There is an exact solution available to an indentation with a thickness profile defined by a quadratic power-law (Conway, 1958):

\[ w(r, n = 0, \omega) = c_5 r^{k_1} + c_6 r^{k_2} + c_7 r^{k_3} + c_8 r^{k_4}. \] (4)

This solution is valid for the first angular order, \( n=0 \), although solutions for subsequent mode numbers can also be calculated numerically.

Finally the common boundary conditions are applied, where the constant thickness disc to the inner profiled section, and a combined numerical solution can be found. The parameter of interest in this Chapter is the driving point accelerance, the frequency transfer function where the ratio of normal surface acceleration \( a = \dot{w} \) is co-located with the excitation force \( p \) applied as an impulse, \( \dot{w}(r) / p(r) \) (the dots represent a double differentiation with respect to time).

The final set of results to be mentioned in this brief review concerns the total loss factor of the samples, this was achieved both for the experimental and predicted results by utilizing the half-power bandwidth method, as the total loss factor can be difficult to measure directly.

This method calculates an estimate for the total loss factor via the analysis of the half-power bandwidth of various resonant peaks is given by \( \eta_{\text{Total}} = \Delta \omega_{3\text{dB}} / \omega \), it is applicable for small values of the composite damping (Mandal et al 2004).

### 6.3 Samples and their manufacturing

Three cylindrical plates were manufactured for this Chapter, Figure 6.3. The discs were manufactured from 5mm thick mild steel and an outer diameter of 500 mm. The diameter of the punched hole and profiled area was 200mm. Where a central hole was present the diameter was 10mm.

A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm was used to produce the circular quadratic power-law indentations into the plates. The four samples consisted of; a plate with a plain reference disc (Figure 6.2a), a punched hole reference disc (Figure 6.2b), and a disc with a circular indentation of quadratic power-law profile and central hole (Figure 6.2c).

<table>
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<th>( \rho )</th>
<th>( \nu )</th>
<th>( r_o )</th>
<th>( r_i )</th>
<th>( r_i )</th>
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<td>0.25m</td>
<td>0.1m</td>
<td>0.005m</td>
</tr>
</tbody>
</table>

Table 6.1 Estimates of the material parameters for the experimental steel plates
Figure 6.2  (a) plate with a plain reference disc, (b) a punched hole reference disc, and (c) a disc with a circular indentation of quadratic power-law profile and central hole

The same three main problems encountered when utilizing this method of manufacturing as described in Chapter 5 are encountered again here; the formation of a machine line, blistering and tearing at the profile tip. The visco-elastic damping layer used in this investigation is electrical tape. The numerical predictions in this chapter utilize estimates of the material parameters for the experimental steel plates given in Table 6.1.

6.4 Experimental set up

The experimental set-up has been designed to simulate free boundary conditions on the edges of the plate, by suspending the sample the weight is taken off the plate edges and therefore minimal additional damping is introduced into the system, see Figure 6.3(a).

Figure 6.3  (a) Experimental set up, (b) Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample.
The excitation force was applied at a distance of 300mm from the center of the plate via an electromagnetic shaker attached to the plate using ‘glue’ and fed via a broadband signal amplifier. The response was recorded by an accelerometer (B&K Type 4371) that was attached to the upper surface, directly above the force transducer (B&K Type 8200), also via ‘glue’, see Figure 6.3(b).

The acquisition of the point accelerance was utilised using a Brüel & Kjaer 2035 analyser and amplifier over a frequency range of 0-6 kHz, with a frequency resolution of 4Hz, a schematic is shown in Figure 6.4. The results presented in this paper have been averaged over ten measurements in order to ensure a statistically accurate representation of the point accelerance.

![Schematic of the experimental setup utilising the Brüel & Kjaer analyser.](image)

**Figure 6.4** Schematic of the experimental setup utilising the Brüel & Kjaer analyser.

### 6.5 Results and discussion

#### 6.5.1 Cylindrical plates containing circular indentations of power-law profile

This section considers the experimental results for the introduction of an indentation of power-law profile into cylindrical discs. Two different styles of reference disc can be considered for comparison: a plain disc and a disc with a punched through hole. Both reference discs are of the same dimensions as the profiled plates, the punched hole plate contains a through cylindrical hole of the same diameter and position as the machined power-law profiles. Other than a peak shift to the left by the punched hole sample due to its reduced mass there is little to no difference in the two reference samples. The plain disc more accurately represents a practical situation, with the profile being cut into an existing structure. It also has reduced internal defects resulting from machining stress, when compared to the punched hole plate, although the latter accounts for the reduction in mass and equivalent stiffness due to the profiled indentation. Keeping the above in mind, a plain reference plate was used for comparison in this Chapter.
A comparison of the results for a disc with a profiled circular indentation and damping layer compared to a plain reference plate are shown in Figure 6.5. As expected, the introduction of an indentation of power-law profile is immediately obvious. Below 2.3 kHz there is little or no difference between the two samples. Above this frequency an increase in the reduction of peak amplitude is seen until a smaller reduction of 3 dB is seen at the matched peak at 5.4 kHz. A maximum reduction of peak amplitude of 13 dB occurs at 4.4 kHz.

6.5.2 Numerical validation of the experimental results

For the numerical validation of the results a comparison will be made between a plain reference disc, plain reference disc covered with a layer of viscoelastic damping material and a disc containing an indentation of power-law profile with a layer of visco-elastic damping material. The comparison will be made over a frequency range of 0-6 kHz.

The results for the experimental response of a plain reference plate compared to its numerical prediction are shown in Figure 6.6. There is a good agreement between these two sets of results, with the predicted locations and the experimental measurements of the resonant frequencies matching up. There is less than a five percent error in prediction for the majority of modes; this
error does increase slightly at higher frequencies. This is thought to be because in the prediction calculations shear deformation and rotary inertia terms in the equation of motion are neglected.

Figure 6.6  
Driving point accelerance for the cylindrical reference plate: numerical predictions (dashed line) compared to the experimental measurements (solid line).

When a damping layer is placed on the surface of the plain reference plate, the higher frequency amplitudes of resonant peaks are reduced by an average of only 1–4dB. This is due to the inherent damping in the plate dominating the response; the plates stiffness is much greater than that of the damping layer. When compared to the damping achieved with a damping layer and profiled indentation, the results shown in Chapter 5 are again confirmed; the high levels of damping achieved, are due to the indentation of power-law profile and damping layer and not just the damping layer itself. This result shows that again there is a close correlation between the predicted and experimental response both in resonant amplitude and frequency.

Figure 6.7 compares the numerical prediction to the experimental results for a disc containing a circular indentation of power-law profile with an attached damping layer. When the damped indentation is incorporated into the plate, the numerical model indicates that the peak accelerance amplitude will decrease, as expected.
For all three cases the numerical predictions for the frequency location of the resonant peaks are accurate to within five percent. This provides confidence in the experimental results, confirming that the boundary conditions are met and also that the response is as expected. The amplitude is slightly more variable however, but is within 1-3 dB, this slight variation occurs in the profiled plate where the effect of machining quality can produce slight variation in the response.

As with the rectangular plate containing a circular indentation of power-law profile, a significant reduction in the resonant peak amplitudes is seen experimentally and confirmed via the numerical predictions.

6.5.3 Predicted and experimental loss factors for cylindrical discs

Finally an investigation of the predicted and experimental total loss factor was carried out. The same three test samples used above were utilized; the plain reference plate, the reference plate with damping layer, and the disc incorporating an indentation of power-law profile and damping layer.

The material loss factor of steel at a temperature of approximately 18°C (64°F) is estimated, from literature as $\eta = 0.004$, this value is within the range 0.002 to 0.01, 0.0005 to 0.01, and
0.001 to 0.008 given in published literature (DeSilva, 1999, Mead, 1998 and Rosset et al, 1959 respectively). The combined reference plate with damping layer as an estimated loss factor of $\eta_{\text{com}} = 0.00593$. The equivalent loss factor for the disc incorporating an indentation of power-law profile and damping layer is $\eta_{\text{equiv}} = 0.148$.

Again good correlation between these results above 2 kHz provides further validation of the experimental results.

Estimates of the experimental and predicted total loss factor $\eta_{\text{Total}}$ for the three different styles of discs used in this investigation by utilizing the half-power bandwidth method is shown in Figure 6.8. When considering the predicted and experimental total loss factors it can be seen that there is little difference in the relative magnitudes of the loss factors. Both the experimental and predicted results show similar trends for each respective sample. The damped reference plate experimental and predicted total loss factors are almost identical over the entire frequency range.

The main difference between the experimental and predicted loss factors for the plain reference plate and the disc with an indentation of power-law profile and damping layer occur below 2 kHz. The numerical predictions for the disc with a circular indentation of quadratic power-law profile with a damping layer tapers to an asymptotic faster than the experimental measurement. This may be due to the loss factor of the plate and damping film being initial approximations. From the difference in the loss factor of the plain reference disc below 2 kHz it can be seen that the total loss factor is less than that determined experimentally. It also appears that the estimated magnitude of the inherent damping estimated for the steel is larger than experimentally determined.

At very high frequencies, the total loss factor becomes independent of the damping material loss factor (Ungar et al, 1964) and it can be seen that all graphs are asymptotically converging toward the higher frequencies (convergence would occur outside the frequency range of this investigation). The loss factor rises at lower frequencies as expected, where the material damping is proportional to the displacement and loading (Mandal et al 2004).

From this comparison of results, the half-power bandwidth method can be used to determine the inherent loss factor for the plain reference disc and the damping material loss factor of the reference disc with damping layer. However, when this method is applied to the frequency response curve for a disc with an indentation of power-law profile and damping layer, it is not possible to separate out the losses from the constant thickness section of the disc, the tapering section, and the edge reflection (where the reflection coefficient is likely less than 100 percent) (O’Boy et al, 2010). Another drawback to this method is that at lower frequencies where the resonant peaks are clear enough to apply the method, there are only a small number of data points available. This can limit the accuracy of the results.
Figure 6.8  Estimates of the total loss factor $\eta_{\text{Total}}$ for the three different styles of discs used in this investigation by utilizing the half-power bandwidth method

(O’Boy et al 2010)
However the trends shown in the loss factor and accelerance response in both predictions and experimental results show the same conclusions; the greater damping efficiency of a disc with an indentation of power-law profile and damping layer when compared just to a visco-elastic damping layer and the reference disc.

### 6.6 Conclusions

The experimental findings have illustrated that the incorporation of damped circular indentation of power-law profile into a steel disc, as with steel plates, leads to a reduction in the peak amplitude of flexural vibration, especially at higher frequencies above the fundamental mode shapes. With the attached damping layer, a maximum reduction in peak amplitude of 13 dB is seen at 4.4 kHz.

For all three cases investigated the numerical predictions for the frequency location of the resonant peaks are accurate to within five percent, above 1.5 kHz, and are accurate below this value. The predicted amplitudes are within 1-3 dB, thus providing confidence in the experimental results and confirming that the response is as expected.

It has been shown by utilizing the half power bandwidth measurements that the total equivalent loss factor for the disc with a circular indentation of power-law profile and damping layer is greater than the total loss factor when reference disc is just covered by a visco-elastic damping layer.
This chapter in part was submitted to the Journal of the Sound and Vibration in August 2012. A preliminary version of the results presented herein was presented in the meeting of the Acoustical Society of America, Kansas 201 and published in the proceeding in the Journal of the Acoustical Society of America 132(3), p2041

Abstract

In this Chapter, the results of the experimental investigations into the sound radiation of rectangular plates containing tapered indentations of power-law profile are reported. Such tapered indentations materialise two-dimensional acoustic black holes for flexural waves that result in absorption of a large proportion of the incident wave energy. A multi-indentation plate was compared to a plain reference plate of the same dimensions, and the radiated sound power was determined (ISO 3744). It was demonstrated that not only do such multiple indentations provide substantial reduction in the damping of flexural vibrations within the plates, but also cause a substantial reduction in the radiated sound power. As the amplitudes of the flexural vibrations of a plate are directly linked to the amplitude of radiated sound from the same plate, this paper also considers the effect of redistribution of the amplitude of the plate’s response due to the presence of acoustic black holes on the amplitudes of the radiated sound. This investigation shows that, despite an increase in the amplitudes of the displacements at the centres of black holes, the overall reduction of vibration response over the plate is large enough to result in substantial reduction in the resulting sound radiation of plates containing indentations of power-law profile.

7.1 Introduction

Government targets for NVH, especially within the transport industry are continuously changing, requiring greater and greater reductions in noise and vibration levels. Noise can be generated by a variety of different mechanisms. This Chapter will consider one of these mechanisms and will investigate sound radiation as a result of structural vibration, i.e. ‘structure borne sound’. 
The obvious reason for reducing the levels of sound heard by an individual is to reduce the potential damage that can be caused to the ear drum above 85 dB (sound pressure level – SPL). However this limit has for a long while been standard and the field of NVH now looks at improving the perceived quality of an environment by the reduction of sound radiating from a structure. This is either achieved by direct noise reduction or by altering the frequency in order to move a resonance into a more ‘acceptable frequency range’. For example lower frequency noise has been reported to have a greater effect on perceived environment quality (Fahr et al, 1988) and so it is favourable to shift resonances to higher frequencies.

As one of the main aims of this thesis is to explore practical applications for acoustic black holes it is natural to consider the sound radiation from structures containing these indentations. The amplitudes of the flexural vibration of a plate are directly linked to the amplitude of radiated noise from the plate structure. It has been shown in the previous chapters that the acoustic black hole damps flexural vibrations in steel plates over a broad frequency range, and therefore one would expect that sound radiation of the structure should also be reduced. However, it has also been shown that the amplitude of displacement at the wedge or indentation tip of power-law profile is greatly increased. Therefore, this investigation will consider the implications of this increase in the amplitude of the displacement at the indentation tip, in particular if the overall reduction in the displacement amplitude over the constant thickness section of the plate is large enough to result in substantial reductions in the overall vibration response and therefore in the resulting sound radiation of plates.

In this chapter the experimental samples are presented and the experimental set up and procedures are then described and a brief background to the sound power calculation is given. The results section of this Chapter will describe the sound radiation, in terms of sound power, from a rectangular steel plate containing multiple indentations of power-law profile and compare it to a reference sample. Finally, these sound radiation results will be compared to visual representations of the displacement over the samples at given frequencies, obtained using a scanning laser vibrometer, in order to compare the sound reduction with the displacement amplitude variation of the plates.

### 7.2 Samples and manufacturing

Two of the samples used in Chapter 5 were used for this investigation: the plain reference plate and the plate containing six indentations of power-law profile (m=4), Figure 7.1. The dimensions of these samples remain the same with plate dimensions of 300 x 400 mm and indentation diameter of 110mm and a central hole size of 14mm. Both samples were made from 5mm thick steel. The same visco-elastic damping layer used in Chapter 5 was applied to the samples where stated.
7.3 Experimental set up and procedure

Three experiments were carried out in this investigation. The first tests followed ISO 3744 in order to determine and compare the sound power levels of the two styles of plates. These plates were then tested using a Scanning laser vibrometer in order to compare displacement amplitudes with sound radiation. Finally the amplitudes of response were considered at the indentation tip again utilizing the scanning laser vibrometer. This section will present the experimental set up and outline the method used to calculate sound power from the measured sound pressure. The experimental set up for the measurement of vibration using a scanning laser vibrometer is also described.

7.3.1 Experimental set up for sound power measurements

The sound radiation experiments were conducted in an anechoic chamber in order to ensure; no reflected sound interference and that the free field conditions were met. The plates were suspended vertically from the test rig used in the testing in previous Chapters.

Figure 7.2 Microphone positions in relation to the sample (ISO 3744)
The tests were conducted in accordance with ISO 3744, the British engineering standard for the calculation of sound power, the microphone positions given in ISO 3744 are shown in Figure 7.2. The geometrical distances of the microphone from the sample are given in Table 7.1.

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</table>

Table 7.1 Microphone geometrical distance from the centre point of the plate (m)

The excitation force was applied centrally on the plate via an electromagnetic shaker with force transducer (B&K Type 8200) attached to the plate using ‘glue’ and fed via a broadband signal amplifier. A microphone and pre amplifier was connected to the RT Pro Phonon analyser and the sound pressure obtained. This was then converted to sound power using the method detailed below. A schematic of the experimental set up is shown in Figure 7.3.

Figure 7.3 Schematic of sound radiation set up

The main source for error when considering results obtained using the set up above is the effect of acoustic short-circuiting due to the free boundary condition and lack of a baffle. It has been documented (Putra et al, 2010 and Oppenheimer et al, 1997) that these conditions can have a significant effect of the measured sound power and radiation efficiency of the panel. They concluded that this effect is greatest at low frequency (dipole-type radiation) and reduces as the frequency increases, they also conclude that the results can be corrected using a uniform scaling factor across the frequency range. As the plates tested in this section were subjected to the same experimental conditions, the results are accurate with respect to each other and as the general trend
is under consideration, this correction has not been applied. This effect is prominent at low frequencies. The main area of consideration for this investigation is not within the affected range.

### 7.3.2 Sound power calculation using sound pressure

As this thesis has so far been primarily concerned with vibration, it would seem useful to first show the derivation of sound power from the time average acoustic intensity associated with harmonic fluctuations in sound pressure and particle velocity. For simplicity, we first consider the case of constant acoustic intensity \( I \) over the surface area of the hemisphere \( S \) surrounding the source of radiation. In this case the expression for sound power \( P \) can be written as:

\[
P = \int I \, dS = IS
\]  

(1)

The acoustic intensity is the time averaged rate at which energy is transferred through the unit surface area and is given by:

\[
i = \frac{1}{2} \text{Re}[p'v^*]
\]  

(2)

where: \( p \) is acoustic pressure, and \( v \) is acoustic particle velocity.

Expanding equation (2), using real and imaginary parts of the complex pressure and particle velocity, and taking the complex conjugate of the velocity gives:

\[
i = \frac{1}{2} \text{Re}[p'v^*] = \text{Re}\{[p_a + ip_b](v_a - iv_b)\} = \text{Re}\{p_a v_a - ip_a v_b + ip_b v_a - i^2 p_b v_b\}
\]  

(3)

Taking the real part of equation (3) allows the acoustic intensity to be defined as:

\[
i = \frac{1}{2}\{p_a v_a + p_b v_b\}
\]  

(4)

For sufficiently large distances between the source and the surface \( S \), larger than 1m for the frequencies used in this investigation, the incident wave on the boundary of the hemisphere can be approximately considered plain, and therefore acoustic particle velocity can be defined as:
\[ \rho = \frac{P}{\rho_0 c_0} \]  

where: \( \rho_0 \) is mass density of air and \( c_0 \) is speed of sound

Substitution of equation (5) into equation (4) gives:

\[ I = \frac{1}{2} \left( \frac{\rho_0^2}{\rho_0 c_0} + \frac{P^2}{\rho_0^2 c_0^2} \right) = \frac{\rho_0^2}{2\rho_0 c_0^2} \]

Substituting equation (6) into equation (1) results in the definition for power becoming:

\[ P = \frac{\rho_0^2}{2\rho_0 c_0^2} S \]

Sound power level, \( L_w \) in dB is calculated by:

\[ L_w = 20 \log \frac{P}{P_0} = 10 \log \left( \frac{1}{2} \frac{\rho_0^2}{\rho_0 c_0^2} \right) S = 10 \log \left( \frac{\rho_0^2}{\rho_0 c_0^2} \right) + 10 \log \left( \frac{S}{S_0} \right) \]

Substituting in sound pressure level \( L_p \):

\[ L_p = 20 \log \left( \frac{P}{P_0} \right) \]

Into equation (8) results in the expression for sound power in dB:

\[ L_w = L_p + 10 \log \left( \frac{S}{S_0} \right) \]

The following method outlines the procedure of calculating sound power from experimentally recorded sound pressure. Note that in this case we have to consider a more general and more realistic case where sound pressure varies over the surface of the hemisphere. It can be shown that in such a case the simple expression (10) still can be applicable if instead of \( L_p \) one uses the spatially averaged value of measured sound pressure.

Sound power in this Chapter has been calculated in accordance with the International Standard ISO 3744. The calculation of sound power should ideally be conducted in a free field, as a minimum a semi-anechoic chamber is required for accurate measurements (ISO 3744, Beranek,
This standard requires a minimum of 20 microphone positions to obtain the spatially averaged sound pressure, and it warns of possible sound reflection from the floor at lower microphone positions. This experimental procedure including microphone positions is detailed in Section 7.3.1 of this Chapter.

The above-mentioned sound pressure root mean square value \( \mu \text{rms} \) required for the calculation of sound power in accordance with ISO 3744 can be calculated from the recorded sound pressure, not necessarily time-harmonic, according to the following expression:

\[
\mu \text{rms} = \sqrt[2]{\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} p^2(t)dt} \quad \text{or} \quad \overline{p^2} = \frac{1}{T} \int_{0}^{T} p^2(t)dt \tag{11}
\]

where \( \overline{p^2} \) = The mean square pressure, \( T \) = The integration interval

This step can be performed directly with most analyser software, thus removing the need for this post processing step. The Sound pressure level, \( L_p \) (in dB) for each microphone position needs to be calculated from the measured sound pressure using the formula:

\[
L_p = 10 \log_{10} \left( \frac{\mu \text{rms}^2}{\mu \text{ref}^2} \right) = 20 \log_{10} \left( \frac{\mu \text{rms}}{\mu \text{ref}} \right) \tag{12}
\]

The reference value \( \mu \text{ref} \) as stated by ISO3744 is 20 µPa. This reference value is used as it is considered the threshold of average human hearing at 1 kHz (Fahy, 1998) and corresponds to a sound pressure level of 0 dB.

Calculation of the average Sound pressure level, \( \overline{L_p} \) over the 20 microphone positions is then required. By averaging the SPL over the 20 microphone positions the energy average of the time averaged SPLs is found:

\[
\overline{L_p} = 10 \log_{10} \left[ \frac{1}{N} \sum_{i=1}^{N} 10^{0.1L_{pi}} \right] \tag{13}
\]

Here \( L_{pi} \) is the SPL corrected for background noise, \( N \) is the number of microphone positions

Finally the sound power level, \( L_w \) is calculated from the averaged Sound pressure level:

\[
L_w = \overline{L_p} + 10 \log \left( \frac{S}{S_0} \right) \quad \text{or} \quad L_w = 10 \log_{10} \left( \frac{W}{W_{ref}} \right) \tag{14}
\]
where; $S$ is the area of measurement and $S_0 = 1\text{m}^2$ assuming that the acoustic impedance of the medium equals 400 Pa·s/m.

Sound power in Watts can be calculated from the Sound power level in dB, by simply rearranging (14) as given below:

$$W = W_{\text{ref}} \times 10^{(L_w \times 0.1)}$$

(15)

### 7.3.3 Experimental set up for vibration response measurements using a scanning laser vibrometer

The use of the scanning laser vibrometer OFV 056, allowed for accurate visual representation of the mode shapes, displacement amplitudes at any given frequency in the test range. The set up for this experiment is shown in Figure 7.4(a). The response was recorded by a combined accelerometer and force transducer (PCB Type 208 B02) that was attached to the opposite surface to that being scanned, using ‘glue’, Figure 7.4(b). The laser vibrometer recorded the point accelerance of the plate along with the amplitude of deflection over the entire plate surface. A frequency range of 0-9 kHz was initially investigated however above approximately 6 kHz the coherence became unacceptable, due to the angle of reflection of the laser beam when aimed inside the circular indentations.

![Figure 7.4](image_url)

(a) Experimental Set up, (b) Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample

The procedure above was repeated for the final investigation which concentrated the measurements on two of the indentations. The tip deflection was then considered, one indentation had a thinner damaged tip compared to a respectively thicker undamaged tip on the other indentation. The input level for the sound radiation tests was kept at the same level as the vibration tests to allow for comparison.
7.4 Results and Discussion

7.4.1 Sound Radiation of a steel plate containing six circular indentations of power-law profile

This section considers the sound radiation of the sample containing six indentations of power-law profile with and without a damping layer and a plain reference plate. The Sound power level in dB and Sound power in Watts were calculated as described in Section 7.3.2 above.

It has been well documented that the addition of a thin visco-elastic damping layer to the indentation tip when a central hole is present considerably increases the damping performance of the indentation of power-law profile, Chapter 5. It is therefore expected that the addition of such a damping layer to the indentation tip will also reduce the sound radiated from the plate.

Figure 7.5 shows a comparison of the Sound power level in dB for a plate containing six indentations of power-law profile with and without a damping layer. As with the reduction in the vibration response, there is a reduction in the sound power levels of the plate when a damping layer is attached to the indentations, showing that the vibration energy is not released as sound but more likely converted to heat as expected through the damping layer. Below 1.2 kHz the damping layer provide no increased level of sound reduction. A maximum reduction in sound power level of 8 dB occurs at 1.7 kHz.

Figure 7.5 Sound power level comparison of a plate containing six indentations of power-law profile with (black line) and without a damping layer (grey line)
Figure 7.6 Sound power level comparison of a plate containing six indentations of power-law profile with a damping layer (black line) compared to a reference plate (grey line)

The results for a plate containing six profiled circular indentations with 14 mm central holes and additional damping layers, compared to a reference plate, are shown in Figure 7.6. This configuration by far has the greatest reduction in resonant peak amplitude over the widest frequency range out of the samples tested in Chapter 5. Below 1 kHz there is little to no reduction in the sound pressure level, as was the case with the reduction in vibration response. Between 1 – 3 kHz the sound power level response is reduced from the reference plate response by 10 – 18 dB, with the maximum reduction in the sound radiated occurring at 1.6 kHz. Above 3 kHz almost all peak responses in sound radiation are flattened.

Figure 7.7 shows a comparison of the sound power in Watts of a plate containing six indentations of power-law profile with and without a damping layer compared to a plain reference plate. After 1.2 kHz the sound power in Watts of the six indentation plate with damping layers has been reduced to a level where almost all peaks seen in the reference plate have been removed. The effect of the addition of a damping layer can be seen but is less obvious than that seen in the sound power level plots. A maximum reduction of 5.1x10^-8 Watts is seen at 1.4 kHz.
From the above results it can therefore be concluded that making six indentations of power-law profile (with a damping layer) in a plate is an effective method of reducing the sound radiation of a steel plate in the medium frequency range (~1-4 kHz). Sound at higher frequencies radiates less readily so this method although efficient at damping higher frequency vibration is not expected to reduce higher frequency sound radiation. This is clearly shown in Figure 7.7 above 5 kHz.

### 7.4.2 Comparison of the Sound power response compared to the vibrational response of the plate

The amplitude of the flexural vibration of a plate is directly linked to the amplitude of radiated noise from the same plate. This section considers the amplitude of the plates response in comparison to the amplitude of the sound radiation. A frequency range of 0-5 kHz was implemented for this investigation. There are two reasons for this choice; the first is that above this value the accuracy of the scanning laser vibrometer is reduced due to the high angles of reflection around the centre of the indentations. The second is that the difference in response in sound power is little to non above the upper frequency range value. This is due to the well known fact that plates radiate inefficiently at higher order vibration modes, i.e. at higher frequencies. Three resonances were selected for comparison from this frequency range; a low frequency resonance where there is
little to no difference in the sound power of the two samples, one in the centre, with a substantial
difference in response, and a resonance towards the upper limit of the frequency range. The two
samples considered were the reference plate and the plate containing six indentations of power-law
profile with damping layers.

The first resonance considered was that at 900 Hz, although there is a peak shift in the response,
there is a minimal difference between the amplitude of the resonance in sound power and
accelerance (see result Chapter 5). Figure 7.8 shows the results for the sound power in Watts for
the reference plate compared to the plate containing six indentations with damping layers and the
modal response of the reference plate and the plate containing six indentations with damping
layers. A defined mode shape can be seen with little to no difference in the amplitude of the
response. This corresponds to the limited reduction in sound power that can be seen in Figure
7.8(a). The effect of the indentations has served to slightly alter the mode shape seen at this
frequency.

The second resonance considered was that seen at 2.2 kHz, where a reduction in sound power of
3x10⁻⁸ Watts and a reduction in accelerance of 9 dB from the reference plate can be seen. Figure
7.9 shows the results for the sound power in Watts for reference plate compared to the plate
containing six indentations with damping layers and the modal response of the reference plate and
the plate containing six indentations with damping layers. At this frequency the mode shape seen in
Figure 7.9(b) has been illuminated by the plate containing six indentations of power-law profile
with a damping layer, Figure 7.9(c). This corresponds with the reduction in sound radiation seen in

![Figure 7.8 Results for resonant peak at 900 Hz; (a) Sound power in Watts for reference plate (dashed line) compared to the plate containing six indentations with damping layer, (b) Modal response of the reference plate, (c) Model response of the plate containing six indentations with damping layers, (d) Amplitude of response; key.](image)
Figure 7.9 (a). There is however some sound radiation from the panel. This can be seen not only on the plot, but also in Figure 7.9(c), where some displacement over the constant thickness section can still be seen, this displacement increases in the indentations and then a large amplitude increase is seen at the final 2 cm at the tip of the indentations. Despite this tip deflection being equal in amplitude to that seen on the reference plate the average reduction over the plate with six circular indentations is great enough to result in a considerable reduction in sound radiation when compared to the reference plate.

![Figure 7.9](image)

Figure 7.9 Results for resonant peak at 2.2 kHz; (a) Sound power in Watts for reference plate (dashed line) compared to the plate containing six indentations with damping layers, (b) Modal response of the reference plate, (c) Model response of the plate containing six indentations with damping layers, (d) Amplitude of response; key.

The final resonance considered was that seen at 2.2 kHz where a reduction in sound power of $3 \times 10^{-8}$ Watts and a reduction in accelerance of 9 dB from the reference plate can be seen. Figure 7.11 shows the results for the sound power in Watts for reference plate compared to the plate containing six indentations with damping layers and the modal response of the reference plate and the plate containing six indentations with damping layers. It can clearly be seen, Figure 7.10(c) that other than a radius of 3 cm at the centre of the indentations the amplitude of the response over the entire plate is zero. The mode shape seen in Figure 7.10(b) had been illuminated in Figure 7.10(c). This ‘active’ area on the indentation plate where a response is seen corresponds to the area over which the damping layer was determined to be effective, Chapter 1. The amplitude of the response in the ‘active’ area is approximately 1 m/s$^2$ greater than that seen on the reference plate, however this does not affect the sound radiation of the plate as seen in Figure 7.10(a) where as previously mentioned there is a reduction of 9 dB by the plate containing six indentations of power-law profile.
with damping layers when compared to the plain reference plate. The trends described above are the same for all resonances observed during testing.

![Image](image_url)

**Figure 7.10** Results for resonant peak at 4.75 kHz; (a) Sound power in Watts for reference plate (dashed line) compared to the plate containing six indentations with damping layers, (b) Modal response of the reference plate, (c) Model response of the plate containing six indentations with damping layers, (d) Amplitude of response; key.

### 7.4.3 Deflection of the tip of a circular indentation of power-law profile

It became obvious when considering the Figures 7.9(c) and 7.10(c) and the other modes not shown that one of the indentations was consistently underperforming with less than half the displacement amplitude seen in the other indentations of power-law profile. This prompted a closer physical examination of the indentation tips where it was found that one of the indentations was slightly thicker (0.14 mm) and had no damage (tearring or blistering) (Figure 7.11(d)), whereas the others were all similar (Figure 7.11(c)) with thinner (0.10 mm) slightly damaged tips.

As described in Section 1.3 of Chapter 1, when considering a wedge of power-law profile it was discovered that a thinner tip even if damaged was more effective at damping flexural vibrations than a thicker undamaged wedge tip. This conclusion was derived from the testing of multiple wedges and analysing the response from each wedge, chapter 2.
In this case with the aid of the scanning laser vibrometer the difference in the amplitude of the displacement of the indentation wedge tip of these two cases for ‘2D Acoustic black holes’ can be seen, Figure 7.12 (a/b). As with a wedge of power-law profile, a circular indentation of power-law profile performs more effectively as an ‘Acoustic black hole’ with a thinner damaged indentation tip as it can reach higher deflection amplitudes when compared to a slightly thicker undamaged indentation tip. As a flexural wave enters a circular indentation of power-law profile by definition its wave speed and wave length decrease and its amplitude increases as it passes along the wedge. This result confirms the sensitivity of the thickness of the wedge tip on the performance of the ‘Acoustic black hole’ and the increase in amplitude that can be achieved.

**7.5 Conclusions**

A plate containing six indentations of power-law profile covered by a damping layer has shown a significant reduction in the level of sound radiation. Between 1 – 3 kHz the sound power level response is reduced from the reference plate response by 10 – 18 dB.

As the frequency increases the amplitude of deflection over the constant thickness section of a plate containing circular indentations of power-law profile tends to zero. At lower frequencies, where no reduction in sound radiation or vibration response is seen, the plate behaves as a constant thickness plate, with a little difference from the plate without indentations. In the frequency range where reductions in vibration response and sound radiation are seen the plate vibration pattern changes substantially, with a noticeable amplitude reduction outside the indentations. In the higher frequency range the only displacement on the plate is seen in the last 2cm of the wedge tip. This corresponds to the area of maximum effectiveness of the damping layer.

Despite this tip deflection being equal in amplitude to that seen on the reference plate the average reduction over the plate with six circular indentations is great enough to result in a considerable reduction in sound radiation when compared to the reference plate.
As with a wedge of power-law profile, a circular indentation of power-law profile performs more effectively as an ‘Acoustic black hole’ with a thinner damaged indentation tip, when compared to a slightly thicker undamaged indentation tip.
This chapter in part was submitted to the journal of Composite Materials in November 2012. A preliminary version of the results described herein were presented and published in the Proceedings of the Anglo-French Conference ‘Acoustics 2012’, Nantes, France, April 2012 and was presented in the meeting of the Acoustical Society of America, Kansas 201 and published in the proceeding in the Journal of the Acoustical Society of America 132(3), p2041

Abstract

In this chapter, the results of the experimental investigations into the addition of one-dimensional and two-dimensional acoustic black holes into composite plates and their subsequent inclusion into composite panels, and composite honeycomb sandwich panels are reported. The composite plates in question are sheets of composite with visible one-dimensional or two-dimensional indentations of power-law profile materialising acoustic black holes for flexural waves. A panel is a sheet of composite with the indentations encased within the sample. This makes a panel similar in surface texture to an un-machined composite sheet (reference plate) or conventional honeycomb sandwich panel. In the case of quadratic or higher-order profiles, the above-mentioned indentations act as one- or two-dimensional acoustic black holes for flexural waves that can absorb a large proportion of the incident wave energy. For all the composite samples tested in this investigation, the addition of one- or two-dimensional acoustic black holes resulted in further increase in damping of resonant vibrations, in addition to the already substantial inherent damping due to large values of the loss factor for composites (0.1 - 0.2). Note that due to large values of the loss factor for composite materials used, no increase in damping was seen with the addition of a small amount of absorbing material to the indentations, as expected.

8.1 Introduction

When exploring practical applications of ‘Acoustic black holes’ one is drawn to a material of increasing popularity and versatility; composites. This Chapter looks at glass fibre composite and its integration with both 1D and 2D ‘Acoustic black holes. One of the main problems faced by this
method of damping is having the wedge tip at the centre of a circular indentation exposed, presenting an exposed structurally weak edge with a health and safety risk. A composite panel with smooth outer edges is one of the most commonly found composite structures. The development of such a composite panel that can incorporate the damping abilities of the ‘Acoustic black hole’ forms the initial aim of this chapter.

Having discussed a solution to the problems posed by exposed indentations, further applications incorporating the use of smooth surfaced panels into other structures can now be explored. The natural progression of this investigation led clearly to the incorporation of enclosed indentations of power-law profile into composite honeycomb sandwich panels.

Manmade honeycomb structures are designed for the minimisation of material used, structural weight and by consequence material cost. They consist of two thin solid panels enclosing a honeycomb core, Figure 8.1. There is great flexibility in the choice of solid panel and core materials depending on the design criteria. Even the geometry of the honeycomb itself can vary greatly, with the most common design being that of a column or hexagonal hollow cell. This configuration provides the structure with a high strength to weight ratio. The thin solid plates provide the panel with strength in tension. The most commonly used materials for such structures are glass fibre plates with aluminium honeycomb cores. These panels are used to form simple flat or slightly curved structures; however they can be combined to form more complicated structures, such as those seen in aircraft wings.

![Composite honeycomb sandwich structure](image)

**Figure 8.1** Composite honeycomb sandwich structure

It is because of the high strength to weight ratios that can be achieved by such panels that they are used extensively in the aerospace, aviation, nautical and automotive industries. Excessive vibration is known to cause fatigue problems and lead to delamination of these types of panels, there is therefore a market for research in the damping of flexural vibrations in panels such as these.

One example is that found in satellite dishes deployed in space, in this case multiple indentations of power-law profile embedded throughout the honeycomb sandwich structure of the dish would be an ideal solution to vibration problems during take-off and once deployed. However
in this case the indentations would need to be enclosed in order to obtain a smooth surface finish capable of functioning as if the indentations were not present. This is just one example of where enclosed ‘2D Acoustic black hole’ technology could be utilised.

In this Chapter a brief review of alternative methods of damping incorporated into composite honeycomb sandwich panels is given followed by the manufacturing of the experimental samples. The experimental set up is then detailed. The results of two 1D investigations are presented in this chapter; the introduction of a wedge of power-law profile to a composite strip (1D Acoustic black hole), and the bonding of a composite wedge onto a steel strip.

The results of three 2D investigations into composite panels are also presented: the introduction of a circular indentation of power-law profile to a composite plate (2D ‘Acoustic black hole’), the combining of composite plates containing circular indentations of power-law profile, and a comparison of smooth surface composite panel configurations with enclosed circular indentations of power-law profile.

The replacement of the outer composite panels on a composite honeycomb structure with the smooth surfaced composite panels containing circular indentations of power-law profile designed in the 2D section is then investigated. Finally this chapter investigates the two different methods of manufacturing the Acoustic black holes that are available when utilizing composites.

8.2 Current damping methods

The main methods of passive damping in relation to composite sandwich panels are that of visco-elastic damping, energy absorbing foam and material damping. Material damping has been explained in detail in Chapters 1 and 3. It relies upon the designer to select materials with appropriate strength and weight while fore filling the requirement of a high material loss factor. As stated before, this is not always possible to achieve as a trade-off between properties is almost always required.

Visco-elastic damping manifests itself in two different ways in relation to sandwich panels. The traditional method, as covered in Chapter 1 were a visco-elastic damping layer is applied to the outer surface of the structure, and secondly via the introduction of a visco-elastic core layer between the two composite sheets. The second method relies on the core layer to have a high level of inherent damping and is not applicable to the honeycomb sandwich structures investigated in this Chapter. However, this method could be combined with an outer panel containing enclosed indentations of power-law profile. Composite sandwich panels with a visco-elastic core are however very effective at reducing the vibration response of lightweight and flexible structures (Li, 2005)
This leads on to the final method of damping in honeycomb sandwich panels; the use of energy absorbing foams placed inside the honeycomb pockets. A study into the damping of thin walled honeycomb structures with energy absorbing foam has shown this method to be effective in a small frequency range 1-2.5 kHz (Woody, 2006). This method of damping is again not mutually exclusive from the use of enclosed ‘2D Acoustic black holes’ and there is a possibility of combining the two methods, this may result in increased damping in the lower frequency range if the foam damping exceeds that of the indentations of power-law profile.

There is therefore still a requirement for a new damping technique that can provide damping over a larger frequency range and produces no resulting increase in mass through its application. Enclosed indentations of power-law profile provide high levels of damping over a wider frequency range and result in a favourable decrease in structural mass. Enclosed ‘2D Acoustic black holes’ appear to provide a solution to the damping of flexural vibration in honeycomb and other types of sandwich panels, where indentations of power-law profile can be integrated into the panels enclosing the core.

### 8.3 Manufacturing of experimental samples

Fifteen glass fibre composite samples were created for this investigation; three strips of dimensions 250 x 50mm and a thickness of 6mm; the additional wedge being 50mm long and of power-law profile m=2.2. A wedge of power-law profile m=2.2 was also produced in order to be attached to a steel strip, dimensions being the same as for the composite strip. The eleven glass fibre composite plates were of dimensions 310 x 185 mm and consisted of two 3mm thick plates and nine 6mm thick plates. The circular indentations of power-law profile m=4 had a diameter of 110mm with a central hole of 10mm, leaving a profile length of 50mm.

The glass fibre composite used for these sample was SE84LV- Low Temperature Cure Epoxy Prepreg System. It has a 0/90 woven yarn with a weight of 295g/m². This composite has a high compressive strength and it is widely used in large heavily loaded components, such as yacht hulls, spars and in aviation panels and also in non-structural applications. SE 84LV is also widely used in sandwich structures with honeycomb. Each sheet had a thickness of 0.2mm. The composite plates and strips where laid-up to the required thickness and then cured using the Vacuum bagging process. This involved using the process shown in Figure 8.2, then curing the samples at 120° for 45 minutes.
Fourteen of the profiles were created in the traditional manor A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm with a carbide cutter was used to produce the wedges and circular indentations. The main problem encountered when utilizing this method of manufacturing was that it is not possible to construct directly a panel with internal cavities while utilizing a ‘vacuum only processing’ method. The outer layer needs to be cured separately and attached using epoxy resin. This can lead to a more lengthy manufacturing time.

These thirteen samples consisted of a reference strip and strip with an additional wedge (Figure 8.3(a)), Examples of the other 3 types of plate can be seen in Figure 8.3(b-d). Figure 8.4 displays the cross-sectional view of the plate samples when viewed from the narrow end. The average profile tip thickness for each of the samples is given in the Table 8.1.
Table 8.1 Average profile tip thickness for each sample in mm

<table>
<thead>
<tr>
<th>Sample</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip thickness (mm)</td>
<td>0.10</td>
<td>0.11</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Two types of glass fibre composite honeycomb sandwich panels were created for this investigation, Figure 8.5. A Reference Composite honeycomb sandwich panel and a Composite honeycomb sandwich panel containing two Acoustic black holes in each of the composite plates.

There are several methods that could have been employed in the manufacturing of these honeycomb sandwich structures, for this investigation a method that was specific to composites and that would allow the integration of Acoustic black holes into the structure was required. The standard method has four stages, the first being the curing of the outer composite plates, this was done using the same vacuum curing technique described above. The composite plates, aluminium
honeycomb and the adhesive sheets are then cut to size; the final structure can be seen in Figure 8.6. Each composite plate is then cured in turn on to the aluminium honeycomb, Figure 8.7(a). This has to be done in two stages to prevent the adhesive running into the honeycomb. The excess adhesive is removed to form a honeycomb sandwich panel.

![Figure 8.6](image)

**Figure 8.6** Layers required to form a composite honeycomb sandwich panel (Ayres, 2012)

In addition to the method used above, two further stages were added for the construction of the sandwich panel incorporating the ABH’s. The first being the manufacture of the ABH’s, a CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm with a carbide cutter was used to produce the circular indentations in the composite. This stage was introduced prior to the bonding of the composite sheets to the honeycomb, resulting in a honeycomb sandwich panel with exposed indentations; Figure 8.7(b). The second being the fixing of the single sheet of composite as seen in Chapter 8 to the outer surfaces of the panel to enclose the ABH’s leaving a smooth surfaced panel. This stage was added to the end of the manufacturing procedure. A cross-section view displaying the composition of the sandwich panel as seen from the narrow end of the samples can be seen in Figure 8.8.

![Figure 8.7](image)

**Figure 8.7** (a) Composite plate cured on to the aluminium honeycomb, (b) Honeycomb sandwich panel with exposed indentations.
The dimensions of the final samples were 310 x 185mm and a thickness of 20mm; the circular indentations of power-law profile $m=4$ had a diameter of 110mm with a central hole of 10mm, leaving a profile length of 50mm. The composite plates have a thickness of 1.2mm each. The 5052 aluminium honeycomb centre has a thickness of 12.7mm, a cell size of 3 1/16 in, a foil thickness of 15 microns and a mass of 70kg per m2.

A single profile was chosen for this investigation instead of the double profile, as they effects of the heating during the bonding of the composite plate to the honeycomb on the epoxy used to bond the composite sheet used to enclose the sample is not known. There was only a small difference in the damping performance of the two profiles so the above profile was selected to ensure accurate results.

The final samples were manufactured via directly curing the profile in to the sample, without any post curing manufacturing processes. The composite sheets were first cut to size and then laid-up on an aluminium mould, Figure 8.9, the inverse of the power-law profile was machined into the aluminium. The sample was then cured as above.
8.4 Experimental set up

The experimental set-up has been designed to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate edges and introduce minimal damping to the system, see Figure 8.10(a).

![Experimental Set up](image)

Figure 8.10 (a) Experimental Set up, (b) Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample

The excitation force was applied centrally on the plate via an electromagnetic shaker attached to the plate using ‘glue’ and fed via a broadband signal amplifier. The response was recorded by an accelerometer (B&K Type 4371) that was attached to the one surface, directly in line with the force transducer (B&K Type 8200), also attached using ‘glue’, Figure 8.5(b). The acquisition of the point accelerance was utilised using a Bruel & Kjaer 2035 analyser and amplifier. A frequency range of 0-6 kHz was investigated; above this range no discernable response could be detected.

8.5 Results and Discussion

8.4.1 Introduction of a wedge of power-law profile to a composite strip

In the first instance it seemed prudent to first ascertain whether the introduction of a wedge of power-law profile to a composite strip could produce an ‘Acoustic black hole effect’ as seen in previously tested steel samples, Chapter 2. In this section two types of sample were tested: a reference strip and a strip with a machined wedge of power-law profile (1D Acoustic black hole).

It was found during initial testing that a composite sample unlike the steel samples required no additional damping layer to be attached to the wedge tip to produce the ‘Acoustic black hole effect’. There are two main reasons for this effect; the first being the increased loss factor of the material itself (~0.1-0.2). The second is the extra reduced tip thickness that can be achieved when
manufacturing the composites. This meant that the tip thickness was less than that of the electrical tape damping layer. Adding this damping layer increases the tip thickness and therefore its stiffness, reducing the damping effect that can be achieved by the wedge. For this reason all of the following results show the samples without any additional damping layers.

This effect is dependent upon the thickness of the wedge tip. As previously shown in Chapter 2; the thinner the wedge tip the greater the damping performance of the wedge. When the tip thickness was greater than that of the damping layer the addition of such a layer again increased the damping performance of the wedge. This increased performance can then match that of a thinner wedge tip without a damping layer. The upper thickness limit of this effect has not been explored in this Chapter as a thicker tipped wedge with a damping layer is not required for the applications explored in the following sections.

![Figure 8.11](image)

**Figure 8.11** Accelerance for a Strip with wedge of power-law profile (solid line) compared to Reference Strip (dashed line)

A comparison of a strip with and without a power-law profile wedge is shown in Figure 8.11, as seen in previous work (Chapters 2 and 4) the addition of a wedge of power-law profile to the end of a glass fiber composite strip, shows the same trends seen in steel strips. There is no difference between the two samples below 250 Hz. After this point an increase in the reduction of the resonant peaks is seen up until a maximum reduction of 3.5 dB from the reference sample which is seen at 2.2 kHz. At 2.6 kHz the two peaks match and no reductions are seen. Beyond this point the sample with the wedge has damped all remaining peaks.
8.4.2 Bonding of a composite wedge to a steel strip

Another method of damping flexural vibration, discussed in Chapter 1 is to attach a material of different density/damping properties on to the end of a plate or strip. This method has been adapted to incorporate a profiled wedge, Figure 8.12. Combining the damping effect of a different wedge material with the acoustic black hole effect should maximise the damping capability of the wedge. Considerations at this point include the impedance change from the strip to the wedge, the loss factor of the wedge material and the bonding method. This adaption again integrates well with the development of welded/bonded wedges (Chapter 2), opening up the possibility of different applications and increased damping capability. This initial investigation will look at incorporating a composite wedge and applying it to steel strips. As previously demonstrated in the earlier sections in this chapter, composite wedges are efficient dampers of flexural vibrations, the aim in this section is to see if an attached composite wedge can further improve the reduction of peak amplitude of the resonant peaks when compared to an attached steel wedge on a steel strip. Both the steel and composite wedge have a tip profile of m=4, the steel wedge has an additional damping layer and both wedges were attached using glue. Each sample was broken and reattached 10 times to near perfect repeatability.

![Existing method and adaptation of existing method](image)

Figure 8.12 Adaption of existing damping method

Figure 8.13 shows the results of a comparison between a steel strip with an attached steel wedge and the same strip with an attached composite wedge. It can be seen that below 2 kHz, there is little to no difference in the response of the two samples. The usual peak shift is seen due to the reduced weight of the composite wedge sample. Above 2 kHz there is a reduction of almost all the steel wedge resonant peaks. A maximum increase in damping is seen on the resonant peak at 3.8 kHz of 9 dB. For this combination the attachment of a different material wedge has proven successful with an increase in damping performance visible of the same material wedge.
As with the fan blades in Chapter 3, the use of a ‘Super alloy’ with a high loss factor would reduce the impedance change at the boundary and is expected to increase the damping performance of the wedge. It is also expected that this method could be extended to the insertion of different material circular indentations of power-law profile in plates, Figure 8.14. This method would also allow for the introduction of an indentation of power-law profile, in to existing structures with an existing void in structure, be it in circular or slot form. An example of such a structure can be found in aircraft wings. This would not alter the structural performance of the structure just reduce the flexural vibrations within it.
8.4.3 Tapered circular indentation of power-law profile in glass fibre composites

The next step, as with the previously tested steel samples (O’Boy et al, 2011a) was to introduce circular indentations of power-law profile (2D ABH’s) into glass fibre composite plates. This section looks at the effect of the addition of two 2D ABH’s into both a 3mm (Sample 2) and 6mm thick plate (Sample 5) when compared to a respective thickness reference plate (Samples 1 and 4). This section also looks at the effect of a double profiled indentation of the same power-law (Sample 7) compared to the single profiled 6mm thick sample (Sample 5) and the reference plate. The cross-sections of these profiles can be seen in Figure 8.15.

![Figure 8.15: 3mm reference plate (sample 1), 6mm reference plate (sample 4), Cross-sections of samples containing tapered circular inclusions (samples 2, 5 and 7)](image)

![Figure 8.16 Accelerance for Sample 1; 3mm Reference plate (dashed line) compared to Sample 2; 3mm, 2x2D ABH (solid line)](image)
Figure 8.16 shows the results for the 3mm thick samples; Sample 2, when compared to a plain reference plate; Sample 1. The effect of adding two circular indentations of power-law profile is immediately obvious with considerable damping of resonant peaks easily observed. Below 500 Hz little to no damping is seen. An increase in the reduction of the peak responses of the reference plate is seen in the profiled sample until a maximum reduction from the reference plate of 7.5 dB can be observed at 1.2 kHz. After 2.7 kHz the response is smoothed with all resonant peaks seen in the reference sample heavily damped if not completely removed.

![Graph showing acceleration](image)

Figure 8.17 Accelerance for the 6mm thick Sample 5 (solid line) compared to the 6mm Sample 7 (grey line) and a 6mm Reference sample; Sample 4 (dashed line)

The same test as above was repeated for 6mm thick plate with the addition of a third sample for comparison; Sample 7, a double profiled sample. From Figure 8.17, it can be seen that when the double profiled Sample 7, is compared to a plain reference plate Sample 4 and Sample 5 a single profile plate, the damping achieved by Samples 5 and 7 is very similar. Sample 7 performs slightly (1-1.5 dB) more effectively than Sample 5 below 1 kHz. Above this frequency this trend is reversed with Sample 5 slightly (again 1-1.5 dB) more effectively than Sample 7. As expected below 450 Hz there is little to no damping. A maximum reduction from the reference plate by both Sample 5 and 7 of 8.5 dB can be seen at 2.4 kHz. After 2.7 kHz both Sample 5 and 7 show a smoothing of the reference plate resonant peaks. Samples 5 and 7 can therefore be classified as interchangeable, whether a sample has a double profile as with Sample 7 or a single profile as with Sample 5 the damping performance obtained is the same.
Figure 8.18 shows the results for Sample 7 when compared to a plain reference plate; Sample 4. Below 500Hz little to no damping is seen. A peak shift to left by the lighter Sample 7 is seen from as low as 150 Hz. At 1 kHz the resonant peak is completely damped. Above 800Hz all reference plate resonant peaks are substantially or completely damped by Sample 7. A maximum reduction of 9 dB occurs at 2.4 kHz.

![Figure 8.18 Accelerance for Sample 4 Reference plate (dashed line) compared to Sample 7 (solid line)](image)

The results of a comparison between Sample 5 and Sample 7 are shown in Figure 8.19. As expected from results obtained in Chapter 2, where a single sided wedge was compared to a double sided wedge; the results for a single and double sided circular indentation follow the same trend. There is fundamentally very little difference between the responses of the two samples. A maximum reduction of 3 dB is obtained by Sample 5 at 2.7 kHz and a maximum reduction of 2 dB occurs at 1.6 kHz for Sample 7. Sample 7 (below 2.7 kHz) has an increased peak reduction by 1-2 dB over that of Sample 5.
8.4.4 Combined plates containing circular indentations of power-law profile

In this section different ways of combining 3mm plates were considered in order to maximise the damping performance of the composite plate. The initial motivation for this combining of plates; sample 6, was due to two main drivers: the first was a profile combination that could easily be converted in to a panel with a continuous outer surface and also to increase the ease of manufacture of the double profiled indentations. The cross-sections of these profiles can be seen in Figure 8.20. The plates were combined using a long cure epoxy resin.

Sample 7 from the previous section was also used for comparison in this section, as it allowed for a direct comparison between a sample manufactured from two separate composite sheets and then combined to a sample manufactured from a single sheet of composite.

Figure 8.19 Accelerance for Sample 7 (solid line) compared to Sample 5 (dashed line)

Figure 8.20 Cross-section of combined composite samples, Sample 3; reference plate, Sample 6 and Sample 7
Two different styles of reference plate were considered in order to gauge the effect of combining the plates and to determine which should be used for comparison. A plain reference plate; Sample 4 (Figure 8.15), and a combined reference plate; Sample 3 (Figure 8.20), were considered as both had merit as a control sample. Both reference plates were of the same dimensions as the profiled plates, the combined plate consisted of two 3mm plates combined with epoxy resin. As seen from Figure 8.21, there is little to no difference between the two styles of reference plate. Sample 4 will be used as a reference sample for the following sections.

![Figure 8.21 Accelerance for Sample 4 (solid line) compared to Sample 3 (dashed line)](image)

A comparison of the two combined double profile Samples 6 and 8 and Sample 7 is shown in Figure 8.22. Below 500 Hz little to no difference in the response of the three samples is seen. In general tend it can be said that the amplitudes of the resonant peaks of Sample 8 are generally lower than the other two samples and therefor it performs more efficiently at damping resonant peaks. Sample 8 shows a maximum reduction from Sample 6 of 3 dB at 640 Hz and the same reduction from both the other two samples at 1.65 and 2.7 kHz. Above 2.9 kHz the response of all three samples is smoothed with resonant peaks heavily damped if not completely removed.

When comparing the manufacturing of Sample 6 and Sample 7, the most time, skill and cost effective method is that used to create Sample 6. Sample 7 requires an extra mould to be created, a greater skill level is needed to accurately machine both sides of the sample, it therefore also takes a lot longer to manufacture. Sample 6 on the other hand can be manufactured in the usual way then simply gluing the two samples together to form the same final profile and thickness plate. The
manufacturing method utilized to create Sample 6 is the most effective and efficient method of manufacture.

![Graph showing acceleration](image)

Figure 8.22 Accelerance for Sample 8 (solid black line) to Sample 7 (solid grey line) to Sample 6 (dashed line)

From Figure 8.23 it can be seen that when comparing Sample 6 to the plain reference plate; Sample 4, below 500 Hz little to no damping is seen, there is however a peak shift from 200 Hz. Again the resonant peak at 1 kHz has been completely damped. Above 800 Hz all resonant peaks are substantially or completely damped with peak reductions in the range of 1.5 to 9.5 dB. Maximum damping of 9.5 dB occurs at 2.4 kHz.

Figure 8.24 shows the results for Sample 8 when compared to a plain reference plate. Below 500 Hz little to no damping is seen. The reduction in the amplitude again increases until a maximum reduction from the reference plate of 10 dB can be seen at 2.4 kHz. After this point the response is smoothed with resonant peaks heavily damped if not completely removed. It should be noted that the resonant peak seen at 1 kHz in the reference sample has been completely damped in Sample 8. The results for Sample 7 compared to the reference plate; Sample 4 where shown earlier in section 8.4.3
Figure 8.23 Accelerance for Sample 4, Reference plate (dashed line), compared to Sample 6 (solid line)

Figure 8.24 Accelerance for Sample 8 (solid line) compared to Reference Plate Sample 4 (dashed line)
8.4.5 **Comparison of composite panel construction**

Finally this section looks at encasing Samples 5-7 with a single sheet of pre-preg glass fibre composite in order to obtain a profile combination that could easily be converted into a panel with a continuous outer surface as shown in Figure 8.25. Despite the Damping performance of Sample 8, the two 10mm diameter holes on the either side of the outer surface of the panel meant that this configuration would be impractical to convert to a smooth surfaced panel without interfering with the circular indentations at the tip. It was therefore decided that this configuration would not be taken forward for experimentation in this section.

![Sample Diagram](image)

Figure 8.25 Cross-section of Samples 9,10 and 11

This section will investigate the responses of the three enclosed panels when compared to each other, a reference plate and finally when compared to the equivalent sample without the additional casing. This final comparison will show if there are any adverse effects to encasing the sample and whether such an effect, if present, can be offset against the advances gained by encasing the panels.

From Figure 8.26 it can be seen that when the composite panels Samples 9, 10 and 11 are compared there is fundamentally very little difference between the three samples. Between Samples 9 and 10 there is no quantifiable difference between the resonant peaks. However as Sample 9 has two 10mm diameter holes on one side of the plate it does not have two smooth surfaces, therefore Sample 10 would best fit the specification. Sample 11 however shows a reduction of 0.5-1 dB at 460 and 700Hz and 2 kHz from the other two samples. Below 460 Hz there is no difference in the response of the three samples and above 3kHz the response is smoothed with resonant peaks heavily damped if not completely removed, thus producing a similar response.
Sample 5 cannot be fully enclosed as the profile shape terminates on the one of the outside edges of the sample, so only one side has been enclosed; Sample 9. If an extra layer of composite were added to this surface the tip thickness would be increased and the central hole removed. The amplitude of the flexural wave could achieve at the profile tip would be restricted and therefore as shown in Chapter 5 with steel; when no central hole is present the damping performance of the acoustic black hole is destroyed. This sample could therefore only be utilized in practical applications where only one side of the panel is vulnerable to damage, poses a safety risk or has the aesthetic requirement of a smooth surface finish.

A comparison of the enclosed panel; Sample 9 compared to a reference plate; Sample 4 is shown in Figure 8.27. Below 500 Hz there is little to no difference in the amplitude of response between the two samples. A peak shift to left is seen from 200Hz by the slightly lighter enclosed sample. The resonant peak at 1 kHz on the reference plate has been completely damped. In fact, above 800 Hz all resonant peaks are substantially or completely damped. However, a matching amplitude peak at 3 kHz can be seen, this is due to model interaction between wave types. A maximum reduction in peak amplitude of 10dB at 2.4 kHz is obtained by Sample 9.
Figure 8.27 Accelerance for Sample 9 (solid line) compared to the Reference plate (dashed line)

Figure 8.28 Accelerance for Sample 10 (solid line) compared to the Reference plate (dashed line)
Figure 8.28 shows the results for Sample 10 when compared to a plain reference plate; Sample 4. Below 500 Hz little to no damping can be seen, a peak shift to the left can be seen in the slightly lighter Sample 10 from 200 Hz. Above 800 Hz all resonant peaks are substantially or completely damped. There is again however, a matching amplitude peak at 3 kHz due to model interaction, and the resonant peak at 1 kHz has been completely damped. A maximum reduction is achieved by Sample 10 of 10 dB at 2.4 kHz.

The most effective damping panel; Sample 11 when compared to a reference plate is shown in Figure 8.29. A peak shift to the left from reference is observed, however in the case of the combined composite plates this effect occurs at a much lower frequency than previously observed, with the peak shift occurring as low as 500 Hz. There is no difference between the two samples below 450 Hz. After this frequency the reduction of the peak amplitudes increases until a maximum reduction of 10 dB from the reference plate is achieved at 2.4 kHz. It can be seen that the reference plate resonant peak at 1 kHz has been damped completely. Again there is a matching amplitude peak at 3 kHz due to model interaction. The combination of composite plates and sheets results in an effective method of damping flexural waves in smooth surfaced composite panels.

Figure 8.29 Sample 11 (solid line) compared to the Sample 4 (dashed line)

The results of a comparison between Sample 5 and its part enclosed counterpart Sample 9 are shown in Figure 8.30. Below 1.3 kHz and above 2.3 kHz there is little to no difference in the response of the samples. A peak shift to the left is seen from 200 Hz by the slightly lighter
unenclosed sample. Between these ranges is where the greatest variation in the samples response is seen, Sample 9 shows a maximum reduction in peak amplitude from Sample 5 of 4 dB at 1.6 kHz.

Figure 8.30 Accelerance for Sample 9 (solid line) compared to Sample 5 (dashed line)

Figure 8.31 Sample 10 (solid line) compared to Sample 7 (dashed line)
From Figure 8.31 it can be seen that when Sample 7 and Sample 10 are compared, again there is little difference between the samples. Below 500 Hz the peak amplitudes are the same with a peak shift to the left by Sample 7; this is due to the slightly reduced mass of the plate. The most noticeable difference in the responses of the two samples occurs at 1.6 and 2 kHz, where Sample 10 shows a reduction in peak amplitude by 2 and 1.5 dB respectively. Again the most likely curse of this reduction is due to the addition of the epoxy resin to secure the cover over the indentations.

![Graph](image)

**Figure 8.32 : Accelerance for Sample 11 (solid line) compared to Sample 6 (dashed line)**

Finally, from Figure 8.32 it can be seen that when Sample 11 is compared to its exposed indentation equivalent; Sample 6, the damping achieved by Sample 11 is in fact greater (1-2 dB) or equal to that of Sample 6. As expected below 450 Hz there is little to no difference in the samples. A maximum reduction of 3 dB can be seen at 650 Hz and 1.6 kHz. The most likely explanation for this increase is the thin layer of epoxy resin between the sample and the composite sheets it is enclosed within providing a slight increase in the loss factor of the sample. This result shows that for glass fiber composites the circular indentations of power-law profile can be successfully enclosed in a smooth surface panel with a positive effect on the damping performance of the plate. There also appears little to no significant ‘drum skin effect’ over the area of the enclosed indentations.

When considering all three smooth surfaced panels compared to their un-enclosed counterparts, other than the damping performance another similarity can be deduced. The resonant peak at 1.6 kHz in all three enclosed samples shows a reduction in peak amplitude. As discussed earlier, a
slight reduction in dB is seen when the accelerometer and force transducer are attached using glue, as the glue itself has a damping effect in this frequency range. When the profiled plates are enclosed to form smooth surfaced panels a layer of epoxy resin is used to attach the outer layer of composite thus introducing a further damping effect as seen with high material loss factor, caused by the resin itself. This result again supports the conclusion that a plate containing indentations of power-law profile can be successfully enclosed to form a smooth surfaced panel with little alteration to damping performance.

8.4.6 Introduction of a indentations of power-law profile to a honeycomb sandwich panel

The main aim of this Chapter is the investigation of a composite honeycomb structure incorporating the smooth surfaced composite panels containing circular indentations of power-law profile designed above.

![Graph](image_url)

Figure 8.33 Measured accelerance for a composite honeycomb sandwich reference panel (dashed line) compared to a composite honeycomb sandwich panel with enclosed indentations of power-law profile (solid line)

Figure 8.33 shows the measured accelerance for a composite honeycomb sandwich reference panel compared to a composite honeycomb sandwich panel with enclosed indentations of power-law profile. As expected, there is a peak shift to the left as a result of reduced mass and stiffness.
Above 1 kHz the resonance peaks of the acoustic black hole plate show increasing reductions in amplitude compared to the reference sample. Above 2.4 kHz the response is smoothed with all resonant peaks seen in the reference sample heavily damped if not completely removed. A maximum reduction of 6 dB is seen at 2.5 and 3.4 kHz.

The investigations in this chapter discussed a composite honeycomb sandwich panel with indentations on both sides of the panel. There may be applications where this is not suitable, for example on boat hulls. In cases such as these indentations can be placed on only one side of the panel, Figure 8.34. A reduction in damping performance will occur but a substantial amount of damping can still be achieved while increasing the strength of the outer surface of the panel.

![Composite honeycomb sandwich panel with one ‘normal side’ and one containing enclosed indentations of power-law profile.](image1)

Theoretically the double profile used in Section 8.4.5, Figure 8.35 would be the most effective profile to use not only due to the damping performance but also a greater contact area would be attached to the honeycomb to reduce the implications of the indentation on the structural performance of the panel.

![Composite honeycomb sandwich panel with a different configuration of enclosed indentations of power-law profile](image2)

### 8.4.7 Effect of manufacturing techniques on damping performance

This final section in this chapter compares the traditional method of manufacture, detailed above in Section 8.3, to a new method unique to composites which involves cutting the composite sheets to the required lengths before the curing process. These sheets are then laid-up on an aluminium mould containing the inverse of the power-law profile required. The power-law profile is then formed under a vacuum in an oven, again detailed in Section 8.3 above.
The results of a comparison between a strip with a machined wedge of power-law profile compared to a strip with a cured wedge of power-law profile is shown in Figure 8.36. Below 500 Hz there is little to no difference in the response of the two samples. The greatest variation in the responses is seen between 1-2.6 kHz with a pronounced peak shift to the left. The greatest reduction in peak amplitude is achieved by the machined sample at 2.5 dB at 2 kHz although between 0.5-1 kHz and 2.5 kHz the cured sample has a reduced response of 0.5-1.5 dB. Above 2.6 kHz the resonance peaks of the reference sample are damped by both samples.

The main reason for the differences in the responses is down to the profile. The machined sample has 50 steps of 0.1 mm where the cured sample had 20 steps of pre-preg composite uncured 0.2 mm. The surface of the cured sample is smooth with the composite resin forming the profile against the mould. The difference in profile formation is responsible for the shift in response. The fact that fibers in the cured sample do not extend to form the profile is the most likely explanation for the distinct sharp responses seen at 1.3 and 2 kHz. The equivalent resonances in the machined sample show a rounded, damped response.

It can therefore be shown that within a 2 dB range and accounting for peak shift the responses are very similar and curing the profile is an acceptable method of manufacture to produce a wedge of power-law profile and the acoustic black hole effect.
The 2D indentation can also be easily manufactured via the curing method of manufacture, in much the same way as the strip circles of increasing diameter are cut out of the composite and laid-up on an inverse profile mould, then cured, Figure 8.37.

![Figure 8.37](a) Cured indentation of power-law profile and (b) inverse mould

### 8.6 Conclusions

Glass fibre composite strips containing wedges of power-law profile and plates containing circular indentations of power-law profile, materialising 1D and 2D ‘Acoustic Black Holes’ respectively, are effective configurations to be used for damping flexural vibrations. A glass fibre composite strip behaves in much the same way as steel when a wedge of power-law profile is added to the end of the strip. A 1D ABH in a strip produces a maximum reduction of 3.5 dB at 2.2 kHz.

A glass fibre wedge can be successfully attached to a steel strip in place of a steel wedge to produce increased reduction of resonant peaks. In theory this method can be used to create 2D acoustic black hole inserts that can be integrated into existing structures with existing holes/slots.

There is little to no difference in the amplitude of the responses when compared a machined wedge to a cured wedge. There is however, a difference in the frequency at which the resonances occur. This observation could be useful when a shift and reduction in resonance is required. The 2D indentation can also be easily manufactured via the curing method of manufacture.

As with wedges of power-law profile, 2D ABH perform well in much the same way as in steel plates, with a maximum reduction of 7.5 dB at 1.2 kHz in a 3mm thick plate and a maximum reduction of 8.5 dB at 2.4 kHz in a 6mm plate, when compared to a reference sample.

The composite plates can be combined to attain more effective damping combination and achieve a more time/cost and skill effective method of production. A combined plate provides an identical if not better reduction of the reference plate resonant peaks as a sample machined out of a
single plate of composite. A maximum reduction of 10 dB at 2.4 kHz was achieved by Sample 8 when compared to a reference sample.

When a composite plate is enclosed by a composite sheet to form a smooth surfaced panel there is relatively little reduction in damping performance when compared to an unenclosed plate. A maximum reduction of 10 dB at 2.4 kHz was achieved by Sample 11 when compared to a reference sample. Little to no drum skin effect is seen over the enclosed ABH’s. Therefore, an enclosed smooth surfaced composite panel can be manufactured to give the same level of damping of flexural waves that can be achieved by a plate with exposed indentations.

It can be shown that curing is an acceptable method of manufacture to produce a wedge of power-law profile and the acoustic black hole effect. This method can be used to mass produce panels incorporating acoustic black holes with relative ease.

A smooth surfaced composite panel is an effective method of damping flexural vibrations in composite panels. These panels can be substituted into applications where a non-structural composite panel is in use and a flexural vibration problem is present as an effective damping method. Such applications would include aircraft internal panels, internal panels in boats and formula 1 vehicles.

As with the composite panels, a composite honeycomb panel with enclosed indentations of power-law profile is an effective method of damping flexural vibrations within the structure. A maximum reduction of 6 dB is seen at 2.5 and 3.4 kHz, with heavy damping or elimination of resonant peaks above 2.4 kHz. The circular indentations as seen before provide substantial reductions in resonant peaks over a large frequency range.

Enclosing the circular indentations of power-law profile within the composite panel results in a surface texture similar to that of an un-machined conventional honeycomb sandwich panel, it also increases the damping performance of the panels. Honeycomb sandwich panels can be made with either both sides or a single side machined depending on the application and damping requirement.

It is important to note that further testing is required into the structural effects of the indentations on the composite and honeycomb panels.
CHAPTER 9

CONCLUSIONS

9.1 Conclusions

As stated in the Introduction, the main purpose of this thesis was to investigate experimentally new geometrical configurations of acoustic black holes to achieve maximum reduction in the structural vibration response and to explore possible ‘real world’ practical applications for this damping technique. The conclusions that can be drawn from the results of this thesis are as follows:

Damping of flexural vibrations using the acoustic black hole effect has proven to be effective despite geometrical and material imperfections due to the manufacturing process. Although these defects generally reduce the damping efficiency to various degrees; the method of damping structural vibrations using the acoustic black hole effect is robust enough and can be used widely without the need of high precision manufacturing.

In particular, it has been demonstrated that the effect of tip damage (curling and tearing) in a wedge of the maximum possible (extended) length allowed by manufacturing is not detrimental for its performance when compared to the same wedge that has been cut to a reduced length (truncated) in order to avoid curling. Despite the damage to the extended wedge tip, the increased length and resultant decrease in tip thickness provided the most efficient damping of flexural vibrations. The longer and thinner the wedge tip the greater the contribution of the acoustic black hole effect into the overall vibration damping, in spite of the resulting technological damage to the tip.

It has been shown that the position of the damping layer in relation to which surface it is attached does make difference to the level of damping achieved, contrary to the predictions following from the geometrical-acoustics theory applied to idealised wedges.

It has been demonstrated that attaching power-low profiled wedges to a rectangular plate (strip) by welding or via glue results in damping performance that generally isn’t any worse than the performance of a homogeneous sample containing the same wedge.

The use of one-dimensional (1D) acoustic black holes on trailing edges of turbofan engine fan blades could be a viable and efficient way of reducing flexural vibration in the blade, therefore reducing internal stresses on the blade and increasing its fatigue life. Vibration tests in the wind tunnel, with self-excited vibrations, confirm these results.

Using flow visualisation, it has been shown that the wedge can be incorporated into the fan blades trailing edge and, when an appropriate damping layer is applied, the aerofoil can be restored to its original profile with limited to no interruption in air flow over the blade surface. This built up
damping layer does not interfere with the acoustic black hole effect and therefore the damping of
the blade.

Use of slots of power-law profile inside plate-like structures allows for the removal of sharp
edges and facilitates attachments at the plate edges. At the same time, slots of power-law profile
materialise an effective method of damping flexural vibrations in steel plates. Slots in carbon
composite plates follow the same trends as the steel slot plates. An additional benefit in this case is
that, due to large values of material loss factor for composite materials, no damping layer is
required to achieve comparable damping performance to the steel plates.

Initial investigations into the damping performance of basic power-law indentations (with no
central hole) resulted in rather limited damping of flexural vibrations. However, it was found
experimentally that the insertion of a central hole into the centre of the indentation increased the
damping effectiveness of the indentation. Further to this result, it was found that, as the diameter
of the central hole was increased (while maintaining the central edges of the indentation sharp ), the
greater reduction in amplitude of resonant peaks was observed, comparable with the damping
obtained by a 1D acoustic black hole (a wedge). It was also shown that the larger the diameter of
the indentation the greater the damping effect due to increased length of the profiled area on the
plate.

The introduction of multiple two-dimensional acoustic black holes into plates clearly increases
the damping performance of the black holes. The individual indentation position, of course, has an
effect on the modes that are damped. Therefore, indentation position can be used to target specific
vibration resonances within the operational frequency range. The obtained results show that using
combinations (or arrays) of several circular indentations of power-law profile with large central
holes covered by small pieces of absorbing materials can be a very efficient method of damping
flexural vibrations in different plate-like structures.

The incorporation of damped circular indentation of power-law profile into a steel disc, as with
steel plates, leads to a reduction in the peak amplitude of flexural vibration, especially at higher
frequencies. The obtained experimental results agree with the published numerical predictions for
the frequency locations of the resonant peak. The predicted amplitudes were within 1-3 dB, thus
providing confidence in both the theoretical and the experimental results.

Measurements of sound radiation have shown that a plate containing six indentations of power-
law profile and damping layers had a much reduced level of sound radiation. Measurements using
scanning laser vibrometer have demonstrated that in the higher frequency range, the only
displacements on the plate are seen in the last 2cm around the indentation edges. Despite this tip
deflections being comparable in amplitude to those seen on the reference plate, the average
reduction of vibration amplitudes over the plate with six circular indentations due to the acoustic
black hole effect is great enough to result in a considerable reduction in sound radiation when
compared to the reference plate. It was also shown that, as with a wedge of power-law profile, a
circular indentation of power-law profile performs more effectively as an acoustic black hole with a thinner damaged indentation tip when compared to a slightly thicker undamaged indentation tip.

Glass fibre composite strips containing wedges of power-law profile and composite plates containing circular inclusions of power-law profile show significant reductions of vibration amplitudes in comparison with the reference plates. A glass fibre composite strip behaves in much the same way as steel when a wedge of power-law profile is added to the end of the strip. A glass fibre wedge can be successfully attached to a steel strip in place of a steel wedge to produce increased reduction of resonant peaks.

The composite plates containing acoustic black holes can be combined to attain more effective damping combination and achieve a more time/cost and skill effective production. These combined panels can then be enclosed by a layer of composite to form a smooth surfaced panel. Little to no drum skin effect is seen over the enclosed black holes. Thus, an enclosed smooth surfaced composite panel can be manufactured to give the same level of damping of flexural waves that can be achieved by a plate with exposed indentations.

There is little to no difference in the amplitude of the responses when compared a machined wedge to a cured wedge. There is however, a difference in the frequency at which the resonances occur. This observation could be useful when a shift and reduction in resonance is required. The 2D indentation can also be easily manufactured via the curing method of manufacture.

A smooth surfaced composite panel with enclosed acoustic black holes is an effective configuration for damping flexural vibrations in composite panels. It was also shown that curing is an acceptable method of manufacture to produce a wedge of power-law profile materializing as the acoustic black hole effect. This method can be used to mass produce panels incorporating acoustic black holes with relative ease.

A composite honeycomb panel with enclosed indentations of power-law profile demonstrates a substantial damping of flexural vibrations within the structure. Enclosing the circular indentations of power-law profile within the composite panel results in a surface texture similar to that of an un-machined conventional honeycomb sandwich panel, whilst not adversely affecting its vibration damping performance.

Reviewing the above, one can come to the general conclusion that 1D and 2D acoustic black holes represent an effective method of damping flexural vibrations. They can be applied to plate-like structures made of different materials, and they can have numerous practical applications.
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