Adaptive strategies for the active control of helicopter vibration

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ADAPTIVE STRATEGIES FOR THE ACTIVE CONTROL OF HELICOPTER VIBRATION

by

John T. Pearson

A Doctoral Thesis
Submitted in partial fulfilment of the requirements for the award of the degree of
Doctor of Philosophy
of the Loughborough University of Technology

November 1994

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Alexia
ABSTRACT

Helicopter fuselage vibrations are of significant levels and produce a unique vibration problem. Reducing the vibration levels increases passenger comfort, reduces crew fatigue, allows higher cruise speeds to be achieved and improves equipment reliability. Vibration Reduction Techniques can be divided into two distinct categories, either passive or active techniques. Active control techniques offer the potential for good vibration reduction performance over significant areas of the fuselage.

This study describes strategies for the Active Control of Structural Response. The technique aims to minimise the structural vibration of a helicopter. Accelerometers measure the vibration at several key points on the fuselage. A multivariable control algorithm processes this information and calculates a set of control forces for a set of hydraulic actuators, located at strategic points in the structure. Vibration reductions result from the superposition of the actuator induced vibrations forces with those induced in the fuselage by the rotor.

This research presents a number of different active control strategies for the reduction of helicopter fuselage vibration. Two distinct active vibration control approaches are a frequency domain controller and a time domain controller, and the Thesis establishes the advantages and disadvantages of each of the control strategies. The time domain option is based upon direct feedback of vibration through constant gain matrices. The subsequent vibration waveform contains information over a wide spectrum of frequencies and consequently control is possible over a range of frequencies. Alternatively the frequency algorithms are specifically concerned with the control of discrete frequencies, the blade-passing frequency being dominant in the case of a helicopter. This thesis describes a third novel approach to the design of an adaptive controller for the reduction of Helicopter vibration. This new technique is a hybrid time/frequency domain solution combining the advantages from both the time domain linear quadratic feedback controller and the frequency domain quasi-static controller. Both fixed gain and adaptive control designs have been implemented, and comparisons of the performance of the various control approaches to the problem of minimising vibration in helicopter structures are made. An estimator provides control system adaptability that permits the periodic update of the fuselage model and produces robustness to changes in the structural dynamics.

A simulation study presents results for the performance and robustness of the control strategies. Experimental investigations considered the effects of linear and nonlinear actuator dynamics, the performance of the strategies during aircraft manoeuvres and the robustness the strategies to changes in the structural response. Experimental validation of the strategies was achieved by testing on a helicopter airframe test rig at the premises of Westland Helicopter Ltd, who collaborated in the SERC funded project within which this work was carried out.
ACKNOWLEDGEMENTS

I am deeply indebted to everyone who has played a part in the completion of this thesis. Namely, Roger Goodall my supervisor for his guidance, advice and invaluable support throughout the project.

Also the representatives of the Advanced Technology Department at Westland Helicopters Limited: Dan Wells for overseeing the later stages of the project, Alan Staple for his help in setting up the project and for his guidance in the initial stages of the research, Ian Lyndon for his technical input and advice at all stages of the research, and Ian Woodrow for his time and patience during long hours of rig testing.

Thanks also to Mathew Stacey from the Aerodynamics Department, and to Ian Morgan from the Flight Test Department, for their advice and expertise regarding modern helicopter flight.

Also Dominic Jenkins of Anthony Best Dynamics for his invaluable assistance in the development of the Active Controller software.

And not forgetting all the members of the Control Group at Loughborough University for their useful discussions and advice, particularly Peter Holme, Dipesh Patel, Jonathan Paddison, Kenneth Nai, Mustafa Abuzeid, Ian Pratt, Malcolm Fraser, and Michael Oliver.

On a more personal note, thanks to my Mum and Dad without whom this thesis would not exist, and to my brother David, and to Daphne for their support. Finally, thanks to my wife Alexia, for her support, help and encouragement throughout my research, and who now knows more about Active Vibration Control than she ever wanted.

I would also like to acknowledge the Science and Research Engineering Council and Westland Helicopters Limited, who jointly funded this research.
PUBLICATIONS

1) Pearson, J.T.  Frequency versus Time Domain Adaptive Control Algorithms for the Active Control of Helicopter Structural Response
   Goodall, R.M.  Control 91 IEE International Conference, Edinburgh, pp 712-717, March 1991

2) Pearson, J.T.  Methods for the Active Control of Helicopter Vibration
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4) Pearson, J.T.  Active Control of Helicopter Vibration
   Goodall, R.M.  IEE Computing and Control Engineering Journal, December 1994
   Lyndon, I.
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<th>Description</th>
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<tr>
<td>A</td>
<td>Actuator Piston Area</td>
</tr>
<tr>
<td>$A(j\omega)$</td>
<td>Frequency shaped weighting Matrices for cost function</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Amplitude of the $b^{th}$ Rotor Harmonic</td>
</tr>
<tr>
<td>$[A_1]$</td>
<td>System matrix of the state space frequency weighting filter</td>
</tr>
<tr>
<td>$[A_2]$</td>
<td>System matrix of the state space Helicopter model</td>
</tr>
<tr>
<td>$[A_3]$</td>
<td>System matrix of the state space actuator model</td>
</tr>
<tr>
<td>$[A_4]$</td>
<td>System matrix of the state space four actuator model</td>
</tr>
<tr>
<td>$[A_{22}]$</td>
<td>Diagonal matrix with elements $[-2\zeta_m\omega_m]$, $m$ is the mode number, $m = 1$ to $d$</td>
</tr>
<tr>
<td>$[A_{21}]$</td>
<td>Diagonal matrix with elements $[-\omega_m^2]$, $m$ is the mode number, $m = 1$ to $d$</td>
</tr>
<tr>
<td>a</td>
<td>Number of ACSR actuators</td>
</tr>
<tr>
<td>$[B_2]$</td>
<td>Input matrix for the state space Helicopter model</td>
</tr>
<tr>
<td>$[B]$</td>
<td>Input matrix for the state space frequency weighting filter</td>
</tr>
<tr>
<td>$[B_{so}]$</td>
<td>Input matrix for the rotor head forces of the state space Helicopter model</td>
</tr>
<tr>
<td>$[B_{su}]$</td>
<td>Input matrix for the actuator control forces of the state space Helicopter model</td>
</tr>
<tr>
<td>$[B_{A2}]$</td>
<td>Input matrix for the structural states of the actuator model</td>
</tr>
<tr>
<td>$[B_{Af}]$</td>
<td>Input matrix for the demanded force of the actuator model</td>
</tr>
<tr>
<td>$[B_{c2}]$</td>
<td>Input matrix for the structural states of the four actuator model</td>
</tr>
<tr>
<td>$[B_{cul}]$</td>
<td>Input matrix for the demanded forces of the four actuator model</td>
</tr>
<tr>
<td>$\mathbf{b}$</td>
<td>Background Frequency Domain vibration vector at $j\omega$</td>
</tr>
<tr>
<td>b, c</td>
<td>Indices</td>
</tr>
<tr>
<td>C</td>
<td>Damping in actuator hydraulic column</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Stochastic/deterministic switch in the optimal control solution</td>
</tr>
<tr>
<td>[C]</td>
<td>Matrix of coupling coefficients for the actuator control forces and fuselage modes</td>
</tr>
<tr>
<td>$[C_A]$</td>
<td>Output matrix of the state space actuator model</td>
</tr>
<tr>
<td>$[C_c]$</td>
<td>Output matrix of the state space four actuator model</td>
</tr>
<tr>
<td>$[C_s]$</td>
<td>Output matrix of the state space frequency weighting filter</td>
</tr>
<tr>
<td>$[C_h]$</td>
<td>Output matrix of the state space helicopter</td>
</tr>
<tr>
<td>$\mathbf{Q}_m$</td>
<td>Vector representing the coupling of the actuator control forces into the $m^{th}$ fuselage mode</td>
</tr>
<tr>
<td>$D_A$</td>
<td>Displacement of Actuator Piston</td>
</tr>
<tr>
<td>$[D]$</td>
<td>Matrix of coupling coefficients for the rotorhead forces and fuselage modes</td>
</tr>
<tr>
<td>$D_m$</td>
<td>Vector representing the coupling of the rotor head forces into the $m^{th}$ fuselage mode</td>
</tr>
</tbody>
</table>
Symbols

[D_s] Feedforward matrix for the state space Helicopter model

[D_r] Feedforward matrix for the state space frequency weighting filter

[D_{sr}] Feedforward matrix for the rotor head forces of the state space Helicopter model

[D_{su}] Feedforward matrix for the actuator control forces of the state space Helicopter model

[D_{ru}] Feedforward matrix for the structural states of the actuator model

[D_{rf}] Feedforward matrix for the demanded force of the actuator model

[D_{cx}] Feedforward matrix for the structural states of the four actuator model

[D_{cu}] Feedforward matrix for the demanded forces of the four actuator model

d Number of modes in the fuselage mathematical model

E Frequency domain estimator error vector

e Observer/estimator error

[F] Generalised forcing matrix

[E_e] Vector of external rotor forces and moments, and ACSR actuator forces

[F_{out}] Force output from the actuator

F_{in} Force demand signal to the actuator

[E(s), \mathbf{E}] Frequency Domain vector of rotor head forces

[f(t), \mathbf{f}] Time Domain vector of rotor head forces

[f_b] Force produced by the b^{th} Rotor Harmonic

[H] Hybrid feedback gain matrix

[H_k] Rotor force distribution matrix

I, i(t) Servo valve current mA

J Frequency domain performance Index

J_p Frequency shaped time domain performance index

J_r Time domain performance index (state weighting)

J_v Time domain performance index (output weighting)

[K] Stiffness matrix

[K'] Principal stiffness matrix

[K_o] Observer/estimator gain matrix

[K_s] Full state feedback gain matrix

K Actuator oil column stiffness

K_c Frequency domain estimator gain matrix

K_s Isolator Spring Stiffness

K_n Servo-valve current/force gain constant N/\text{mA}

K_p Actuator Force/Flow Feedback Gain
Symbols

$K_a$  Servovalve current/flow gain
$K_v$  Actuator Flow Gain
$K_{sv}$  Servovalve electric motor gain (m/mA)
$K_{ae}$  Sensitivity Gain of the Accelerometers
$k$  Number of ACSR accelerometers

$[M_a]$  Matrix of time domain measurements for offline identification
$[M]$  Mass matrix
$[M']$  Principal mass matrix
$[M_c]$  Transfer Matrix, transforms from finite element degrees of freedom to sensors

$m$  Index

$N$  Estimator memory length
$N_r$  Normal Operating Speed of the Main Rotor (100%$N_o$)

$n$  Number of Main Rotor Blades
$P(t)$  Solution of matrix Riccati equation
$P_1, P_2$  Actuator Piston Pressures
$P_s$  Actuator Supply Pressure
$P_r(t)$  Actuator Load Pressure
$P_r(s)$  Frequency shaped penalty function or filter

$[P]$  Estimator covariance matrix
$[P_0]$  Initial estimator covariance matrix
$[Q]$  Matrix of weighting factors for states
$[Q_x]$  Covariance Matrix for Gaussian disturbance $v_x$
$[Q_w]$  Matrix of weighting factors for outputs (accelerations)
$[Q_v]$  Matrix of weighting factors for frequency shaped outputs (accelerations)

$Q, Q_1, Q_2$  Actuator hydraulic Flows
$Q_R$  Actuator rated flow
$q$  Index
$q(t)$  Hydraulic oil flow rate

$[R]$  Matrix of weighting factors for actuators
$[R_{au}]$  Matrix of weighting factors for rate of change of control inputs
$[R_f]$  Matrix of weighting factors for inputs in the frequency shaped cost function
$[R_v]$  Covariance Matrix for Gaussian disturbance $v_v$

$R_c$  Weighting factor for the $c^{th}$ actuator
$R_x$  Spanwise Reaction at the Blade Root
$R_y$  In-plane Reaction at the Blade Root
$R_z$  Vertical Reaction at the Blade Root
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{\text{element}} )</td>
<td>Radius of Blade Element</td>
</tr>
<tr>
<td>( r_{\text{yss}} )</td>
<td>Steady State Component of ( R_y )</td>
</tr>
<tr>
<td>( r_{\text{ycos} m} )</td>
<td>Cosine Component of the ( m^{th} ) Harmonic of ( R_y )</td>
</tr>
<tr>
<td>( r_{\text{ysin} m} )</td>
<td>Sine Component of the ( m^{th} ) Harmonic of ( R_y )</td>
</tr>
<tr>
<td>( r_{\text{yss}} )</td>
<td>Steady State Component of ( R_y )</td>
</tr>
<tr>
<td>( r_{\text{ycos} m} )</td>
<td>Cosine Component of the ( m^{th} ) Harmonic of ( R_y )</td>
</tr>
<tr>
<td>( r_{\text{ysin} m} )</td>
<td>Sine Component of the ( m^{th} ) Harmonic of ( R_y )</td>
</tr>
<tr>
<td>( {T} )</td>
<td>Fuselage Receptance matrix at ( n\omega )</td>
</tr>
<tr>
<td>( \mathbf{U}(s), \mathbf{U} )</td>
<td>Frequency Domain vector of actuator control forces</td>
</tr>
<tr>
<td>( \mathbf{u}_d(t), \mathbf{u}_s )</td>
<td>Time Domain vector of actuator control forces</td>
</tr>
<tr>
<td>( \mathbf{u}_A )</td>
<td>Input vector for actuator model</td>
</tr>
<tr>
<td>( \mathbf{u}_C )</td>
<td>Input vector for the four actuator model</td>
</tr>
<tr>
<td>( V_1, V_2 )</td>
<td>Volume of actuator piston sections</td>
</tr>
<tr>
<td>( V_A )</td>
<td>Velocity of actuator piston</td>
</tr>
<tr>
<td>( V_0 )</td>
<td>Nominal actuator piston area</td>
</tr>
<tr>
<td>( V_{\text{element}} )</td>
<td>Velocity of Blade Element</td>
</tr>
<tr>
<td>( V_h )</td>
<td>Forward Velocity of the Helicopter</td>
</tr>
<tr>
<td>( V_r = \omega R_y )</td>
<td>Velocity of the Blade Tips</td>
</tr>
<tr>
<td>( {W} )</td>
<td>Matrix of weighting factors for sensors (Frequency domain)</td>
</tr>
<tr>
<td>( {W_s} )</td>
<td>Transformation matrix, giving displacements across the actuators</td>
</tr>
<tr>
<td>( W_q )</td>
<td>Weighting factor for the ( q^{th} ) accelerometer</td>
</tr>
<tr>
<td>( x )</td>
<td>Structural displacement across actuator</td>
</tr>
<tr>
<td>( X )</td>
<td>Longitudinal Axis - Force acting on the Main Rotor (Drag Force)</td>
</tr>
<tr>
<td>( \mathbf{x}_A )</td>
<td>State vector of the state space Helicopter model</td>
</tr>
<tr>
<td>( \mathbf{x}_A )</td>
<td>State vector of the state space Actuator model</td>
</tr>
<tr>
<td>( \mathbf{x}_C )</td>
<td>State vector of the state space four Actuator model</td>
</tr>
<tr>
<td>( \mathbf{x}_E )</td>
<td>State vector of the extended state space Helicopter model</td>
</tr>
<tr>
<td>( Y )</td>
<td>Lateral Axis - Force acting on the Main Rotor (Side-slip Force)</td>
</tr>
<tr>
<td>( Y(s), \mathbf{Y} )</td>
<td>Frequency Domain vector of sensor accelerations</td>
</tr>
<tr>
<td>( \mathbf{y}(t), \mathbf{y} )</td>
<td>Time Domain vector of sensor accelerations</td>
</tr>
<tr>
<td>( \mathbf{y}_E )</td>
<td>Output vector of the extended state space Helicopter model</td>
</tr>
<tr>
<td>( Y_n )</td>
<td>Output of frequency weighting filter</td>
</tr>
<tr>
<td>( Z )</td>
<td>Vertical Axis - Force acting on the Main Rotor (Lift Force)</td>
</tr>
</tbody>
</table>
Symbols

\( \beta \)  
Bulk Modulus of Oil

\( \Delta H \)  
Incremental change in the Hybrid gain matrix

\( \Delta P \)  
Change in actuator piston pressure

\( \Delta Q \)  
Change in actuator hydraulic flow

\( \Delta U \)  
Incremental change in frequency domain actuator control vector \( \mathbf{U} \)

\( \Delta V \)  
Change in actuator volume

\( \Delta Y \)  
Incremental change in frequency domain vibration vector \( \mathbf{Y} \)

\( \lambda_i \)  
Eigenvalue of the \( i \)th mode

\( \lambda \)  
Estimator forgetting factor

\( \mathbf{n} \)  
Vector of modal displacements

\( u_{m,b} \)  
Modal amplitude of mode \( m \) at sensor \( b \)

\( u_{s,m} \)  
Cumulative effect of the \( m \)th mode on the \( k \) sensors

\( \omega \)  
Rotational Speed of the Main Rotor

\( \omega_{sv} \)  
Natural frequency of the servovalve (rads/s)

\( \omega_{ae} \)  
Natural frequency of the accelerometers (rads/s)

\( \omega_m \)  
Natural Frequency of the \( m \)th fuselage flexible mode

\( \{ \Omega \} \)  
Transfer Matrix, transforms from modal accelerations to sensor accelerations

\( [\phi] \)  
Matrix of modal eigenvectors

\( \phi \)  
General modal eigenvector

\( \phi_m \)  
Displacement of the \( m \)th mode

\( [\Theta_n] \)  
Matrix of estimated parameters

\( \mathbf{Q}_n \)  
Vector of estimator measurements

\( \sigma_n \)  
Estimator measurement noise variance

\( \psi \)  
Azimuth Angle of Rotor Blade

\( \nu_x \)  
Gaussian noise disturbance vector (states)

\( \nu_y \)  
Gaussian noise disturbance vector (inputs)

\( \nu_R \)  
Estimator noise variance

\( \zeta_m \)  
Damping of the \( m \)th fuselage flexible mode

\( \zeta_{sv} \)  
Damping coefficient of the servovalve

\( \zeta_{ae} \)  
Damping coefficient of the accelerometer

\( \Sigma_o \)  
Frequency domain estimator sensitivity
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## ACKNOWLEDGEMENTS


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CHAPTER 1
INTRODUCTION

INTRODUCTION
The concept of "the Helicopter" was first envisaged as long ago as the mid 15th century. The work of Leonardo DaVinci (1452-1519) contains references to both early helicopter design and to the transmission of vibration, though at this point the two were not related. However it would be another 400 years before the first helicopter, as we know it today, would be successfully built and flown by Igor Sikorsky (1941).

Vibration has been an inherent and unfortunate feature of the helicopter throughout its history, and it continues to be a problem for modern helicopters with their more stringent comfort requirements, lighter airframes, higher cruise speeds and greater manoeuvrability. These vibrations restrict the usefulness of the helicopter in both civil and military roles. Therefore the reduction of these high vibration levels is of primary interest in helicopter design and technology.

This study describes a novel strategy for the active control of helicopter vibration, in which control forces calculated by a microprocessor, are applied to the vibrating fuselage from a set of actuators located at strategic points in the structure.

1.1 CHRONOLOGICAL DEVELOPMENT OF THE HELICOPTER

In the 15th Century Leonardo DaVinci made sketches of a machine which used a screw type propeller for vertical flight (Figure 1.1). This is probably the earliest attempt at a design for a helicopter. It wasn't until three hundred years later that George Cayley (1796, England) made this innovation a reality by constructing a series of successful models. At a similar time in Russia, Mikhail Lomonosov demonstrated a spring powered model to the Russian Academy of Sciences. In the second half of the 19th Century many inventors, such as W.H. Phillips (1842, England), E. Forlanini (1878, Italy) and T. Edison (1880, United States), were involved with the helicopter. Progress was mainly related to steam driven models, though Edison used an electric motor and identified the requirement for a light weight engine that would produce a large amount of power.
Towards the end of the 19th and the beginning of the 20th Century the internal combustion reciprocating engine became a viable technology, making possible aeroplane flight and eventually helicopter flight as well.

The first helicopter to carry a man aloft was constructed by Breguet (France) in 1907. However this only remained airborne for a few seconds, it had no means of directional control and was tethered. In the same year Paul Cornu (France) achieved the world’s first free flight in his helicopter, which achieved an altitude of 12 inches for about 20 seconds. In 1909 Emile and Henry Berliner (United States) built a two-engine aircraft with counterrotating rotors which lifted a pilot untethered. One of the largest helicopters of the time was built by De Bothezat in 1921 (United States). This machine had four rotors, each with six blades, at the ends of intersecting beams. It was powered by a 180 h.p. engine, and it took off at well over 4,000 pounds, carrying three passengers, but at altitudes of only several feet.

The first effective demonstration of cyclic control of the main rotor was achieved by Pescara (1924, Spain), who built a coaxial helicopter that set a distance record of 736m. In 1929, Von Baumhauer (Holland) built the first single rotor helicopter with a vertical tail rotor for torque counteraction. In the following year Dr d’Asconio (1930, Italy) built a coaxial helicopter which had two superimposed, two bladed, counterrotating rotors. For several years this helicopter held records for altitude (59 feet), endurance (8 minutes and 45 seconds) and distance (3500 feet).

In 1935 Dr Heinrich Focke (Germany) built a helicopter with two three-bladed rotors, side by side, rotating in opposite directions. This machine set several new records for endurance (1 hour and 20 minutes), altitude (11,200 feet), distance (143 miles) and speed (75mph). Later in 1937 Flettner (Germany) developed a helicopter with side by side intermeshing rotors, it carried two passengers and was capable of speeds up to 90mph.

Igor Sikorsky (United States) produced the VS300 helicopter in 1941. This machine represents a significant milestone in the invention of the helicopter. It had the well-known configuration of today’s helicopters, i.e. a main rotor and a tail rotor being driven by a shaft from the main rotor. The pilot’s controls also set the standard for today’s helicopter (cyclic stick, pedals, and a collective stick with a twist grip throttle). Sikorsky broke the endurance record set by Focke in 1935, by remaining aloft for 1 hour and 32 minutes.
Sikorsky had experimented with helicopters over a considerable period, and indeed had started as early as 1907. One of his other projects, a helicopter with comparable performance to the VS300 but with two instead of three blades on the main rotor, was abandoned due to extremely high levels of vibration.

The invention of the helicopter was complete by the 1940/1950s. Later work was primarily concerned with research and development, and therefore is not included in this brief list of significant projects in the invention of the helicopter. Gessow and Myers [1967], and Fay [1976] both provide comprehensive information on the history of the helicopter. A modern helicopter, such as the EH101 shown in Figure 1.2, can stay on station for up to 3 hours, and has a range of 580 miles (with 30 passengers) and a maximum cruise speed of 150kts (175mph at 5,000 feet).

1.2 FUNDAMENTAL PRINCIPLE

The fundamental physical process by which a number of superposed waves produce local zones of attenuation was clearly observed by Newton (1642-1727) in his famous optical experiments [Bell 1960], although he did not recognise this phenomenon as interference but as a special case of refraction and reflection. The modern scientific meaning of interference, i.e. when two or more waves combine such that the minima of one wave meet the maxima of another wave, only became apparent to physicists in the late 18th century. The importance of this phenomenon was first recognised by Thomas Young (1773-1829) [Gjertsen,1986], who in 1802 developed the general law of the interference of light, which stated

'when two undulations from different origins, coincide in Direction, their joint effect is a combination of the motions belonging to each.' In particular the joint motion may be the sum or difference of the separate motions accordingly as similar or dissimilar parts of the undulations are coincident.'

The fundamental principle of active vibration control is one of superposition: Simply this implies that an unwanted primary vibration can be attenuated by deliberately generating a secondary anti-phase vibration and superposing it upon the primary. The basic concept of reducing vibration in a system by adding more vibration at first does appear to be rather improbable, since in general any additional sources of disturbances lead to a worsening of the situation.
Introduction

Attempting to attenuate an unwanted disturbance by deliberately generating a secondary source acting in antiphase is not a new concept. Early attempts seemed not to be feasible because of technological problems. There are, however, some very early examples of successful vibration control devices. In 1892 Sir Alfred Yarrow mounted reciprocating masses driven by connecting rods in a motor torpedo boat to successfully reduce a 4Hz component of the hull vibration.

Another early example of vibration control was given in 1896 by A. Mallock, in this application hull vibration was the problem. The problem was solved by linking the twin steam turbines to ensure that the turbines were accurately phase locked. Results clearly showed the reductions in vibrations when the engines were synchronised in antiphase, and the severe beating effects which resulted when the engines were not correctly synchronised.

1.3 ACTIVE CONTROL

Technology has now advanced to the point where many active control systems are possible. The term Active Control implies the use of an actuator to apply forces to a system, which in some way modify the response of that system. The forces generated may be functions of many variables, some of which may be remotely measured. An active control system therefore generally comprises actuators to apply forces, sensors to measure variables, a feedback controller to generate actuator commands from sensor signals and an external power source.

The recent advances in active control, particularly of vibration and noise, can be attributed to a number of breakthroughs. The first was in the early 19th century by Jean Baptiste Joseph Fourier (a French mathematician). His breakthrough, the well-known Fourier theorem, arose from the analysis of heat waves, and is at the heart of many active control systems which require some form of spectral analysis.

In the 1950s and 1960s a breakthrough in control theory was made, now known as Modern Control Theory. Modern control theory embodies state space methods and optimal control, which along with other advanced control techniques are an essential part of many control systems.

The rapid growth of high speed compact computing power, in particular the development of Microprocessors and Digital Signal Processors (DSPs), has provided the hardware platforms
capable of implementing in real time the complex control and spectral analysis algorithms required by active control systems. DSPs in particular have enabled the implementation of one of the more complex control strategies, namely adaptive algorithms. These controllers are now possible because DSPs have the speed to monitor and control the system simultaneously. Other factors which have aided this advancement are improvements in high frequency actuation technology, and the development of reliable and cost effective transducers for vibration measurement.

Active control systems have the potential to provide excellent performance, and offer significant advantages over conventional passive systems. Applications of Active Control Technology cover a broad range of engineering disciplines, transportation being one of the biggest areas. Reviews of Railway active suspensions are given by Goodall and Kortüm [1983,1990], Hedrick [1981], and Hedrick and Wormley [1975]. Road Vehicle Suspensions have been reviewed by Sharp and Crolla [1987]. These applications are often associated with improvements in either comfort and ride or performance, and the following sections identify a number of the areas.

1.3.1 Control of Civil Structures
Active control offers a very great advantage in the design of buildings, especially with respect to seismic disturbance where the major dynamic disturbance is intermittent, but also to stochastic disturbances such as wind loading [Abdel-Rohman, 1982]. Yao [1972] presented one of the earliest technical papers on the use of active control for the suppression of wind and seismic excitation in buildings. Austin et al [1992] investigated the use of optimal control methods for placing of the actuators within the structure.

1.3.2 Control of Space Structures
Large space structures are lightweight flexible structures that have to operate under stringent requirements for pointing accuracy, shape fidelity, vibration suppression and attitude orientation. These structures have many low frequency resonant modes and their natural damping is very low (approximately 0.5 percent critical) [Garibotti, 1984]. A major problem with the control of these structures is that the prediction of their behaviour in space is difficult to achieve by testing 'on-Earth'. They range from central rigid bodies with large flexible appendages to the solar electric propulsion spacecraft and the solar power station satellite which as described by Balas [1982] "would be a structure nearly the size of Manhattan Island". In the same paper Balas provides an excellent review of published papers on the subject of
flexible space structures.

A particular difficulty with designing control systems for large space structures is that extremely large finite element models are generated, which need to be reduced to facilitate control system design. This leads to the well known problem of control spillover, between controlled modes and uncontrolled modes [Canavin, 1978].

A number of different control approaches range from the modal controllers described by Strunce and Henderson [1984], through to the robust H∞ controller proposed by Wie et al. [1992]. Wie [1992] also investigates the use of classical controllers for controlling the flexible modes of large space structures. In this study the controller is designed one loop at a time or mode by mode. Optimal control strategies are studied by Strunce et al [1979] and by Iwens et al [1981]. In contrast to these studies, Sesak, et al. [1979] developed a decentralised control approach which attempted to minimise the problem of control spillover.

1.3.3 Active Noise Control

Active Noise Control (ANC) was first patented in 1936 by Paul Lueg (Patent Number 2,043,416 entitled 'Process of Silencing Sound Oscillations'). The patent describes the concept of constructive and destructive interference when sound waves are mixed, with resulting intensification and attenuation of the sound field respectively. Twenty years later Olsen and May (1953) of RCA Labs suggested null point generators for suppressing sound in a zone of limited size. They also speculated about several future applications such as headsets, helmets, and local control in aeroplane and automobile interiors.

The next major step occurred in the late 1960s, with Jessel and Magiante who performed some of the earliest practical Active Noise Control experiments in ducts. An absorptive attenuator of different configuration was subsequently developed by Swinbanks [1981].

Much of the ANC work is based upon Huygens principle. Christian Huygens (1629-1695) also worked on the description of light. He developed the concept of a wavefront, and the idea of replacing a section of a wave spanning an aperture by a set of imaginary wave sources is known as Huygens' Principle. This can be reinterpreted from a noise control point of view, stating that:

The sound field at any point in a control volume due to a noise source outside the control volume can be reproduced exactly by an infinite array of secondary sources distributed around the control volume.

Thus, by inverting the phase of the secondary sources, perfect noise reduction would be achieved in the control volume. However, Huygens’ principle has a number of practical limitations such as the control sources must be of a finite size and number.
An LMS filter was used by Elliot et al (1987, 1988) to model the acoustic plant in an Active Noise Control system. The algorithm minimises the sum of the squares of a number of microphone (sensor) outputs by adjusting the magnitude and phases of the sinusoidal inputs to a number of loudspeakers. The main advantage claimed is the inherent stability of the LMS algorithm, which allows large gains to be used and this in turn provides rapid adaptation.

The active control of sound in three dimensions is considerably more difficult than the control of sound in ducts, mainly because the three dimensional system has more complex dynamics. One of the first attempts to control a three dimensional sound field was by Bullmore et al (1987). The control system minimised the sound level in a head height plane extending through the length of a cabin. The ANC system consisted of 24 error sensors and 8 speakers. A later study by Elliot et al (1989) evaluated this system during actual flight tests. The ANC system in this study consisted of 16 loudspeakers and 32 microphones (error sensors), and was based on the LMS algorithm developed by Elliot earlier. The results showed a reduction of 11-14dB in the fundamental tone throughout the area containing the microphones.

In contrast to this minimisation of a subregion, the study performed by Lester and Fuller (1986, 1987) into active control of propeller induced noise, attempted to obtain global noise reductions in a simplified model of an aircraft fuselage. Fuller and Jones (1987), Jones et al (1988, 1989, 1990) conducted several studies into the reduction of propeller induced noise in a cylindrical aircraft model by controlling the vibrational response of the fuselage. Microphones were used as error sensors and shakers mounted point wise to the shell (Fuselage) were used as actuators. It should be noted that the interior sound field was being minimised and not the vibrational response of the shell. Good results were reported, with global reductions of approximately 30dB being achieved.

Simpson et al (1989) also investigated active noise control using vibrational inputs. A 50 foot section of a McDonnell Douglas DC-9 fuselage was used as the test structure in this study. Shakers were attached to the engine pylons to simulate structural vibrations caused by the engines. Control forces were applied using shakers connected to the interior of the fuselage and microphones were used as the error signals. The Active Noise Control system reduced interior sound levels by 10dB for a pure tone.

All of these projects contain a common underlying important feature; if good performance is to be achieved, the active sources must be capable of reproducing precisely the same wave distribution as the unwanted disturbance.
1.3.4 Aerospace Applications

While the successful operation of a rigid aircraft is often achieved in the desired flight envelope by suitable aerodynamic design, there will always remain structural dynamic problems such as flutter, vibration and dynamic loading of the aircraft and its substructures. These problems may lead to restrictions in the total useability of the flight envelope. Military aircraft often need to add external stores either beneath the wings or onto the fuselage itself. In many cases these stores will consist of new equipment designed many years after the original aircraft, in some cases the addition of these stores will have detrimental effects on the flight envelope, for example by increasing wing flutter or fuselage vibration. The cost of modifying the structure to accept these new stores is often prohibitive, therefore the use of active control to either suppress flutter or control vibration becomes an attractive solution to the problem.

Sensburg et al [1980] discuss both the use of existing ailerons to control flutter modes and also adding controllable vanes to the stores which act in such a way as to counteract the motion of the stores and hence reduce the flutter modes. The active control of noise and vibration is of particular interest to the aerospace industry, and some of these applications have already been mentioned in the previous section. The active control of helicopter vibration is discussed in some detail in the next chapter.

1.4 STATEMENT OF PURPOSE OF STUDY

The aims of the study research were threefold:

i) to investigate strategies for the active control of helicopter fuselage vibration,

ii) to compare the performance of the strategies with an established frequency domain algorithm, and

iii) to implement the chosen strategies on the airframe test rig.

1.5 OVERVIEW OF STUDY

The research study consisted of three distinct phases. The aim of the first phase was to gain an overall perspective of the problem area, including existing applications. The second phase evaluated control strategies for the reduction of helicopter vibration in non real time simulation,
which allowed performance predictions to be made and identified potential limitations. The final phase was devoted to implementing the chosen strategies in real time using a digital signal processor. The algorithms were tested on the airframe test structure to validate the simulation results from the second phase.

The underlying methodology for the strategies includes measurement of the fuselage vibration at a number of key locations. Vibration is minimised by the introduction of secondary control forces from a number of hydraulic actuators located at strategic points in the structure. This technique is termed the Active Control of Structural Response.

1.6 ORGANISATION OF CHAPTERS

Chapters 2 to 7 are organised as follows:

Chapter 2 - this chapter describes the causes of fuselage vibration in helicopters. Passive techniques for the reduction of helicopter vibration are reviewed and their disadvantages are identified. Next, Active Control techniques are reviewed, and their relative merits and limitations are discussed.

Chapter 3 - this chapter is concerned with the models and modelling techniques used in the simulation studies. Flexible body modelling, finite element modelling and the extension to full helicopter models are detailed. Actuator dynamics are considered and both linear and nonlinear actuator models are developed.

Chapter 4 - this chapter details the control strategies used for the Active Control of Structural Response. Two distinct active vibration control strategies are a frequency domain controller and a time domain controller, and the assumptions on which the strategies are based are discussed. The advantages and disadvantages of the strategies are established. A third novel approach to the design of a controller for the reduction of helicopter vibration is described. This new approach is a hybrid of the time and frequency domain approaches, combining the advantages of the time domain and frequency domain controllers.

Chapter 5 - the purpose of this chapter is to present the results from the simulation study. The simulation study considered the effects of linear and nonlinear actuator dynamics, the performance of the strategies for steady state flight conditions and during aircraft manoeuvres. The robustness of the strategies to changes in the forcing frequency, forcing amplitude and to changes in the structural response are presented and discussed. The control strategies are
implemented in both fixed gain and adaptive frameworks, and results are included for both cases. The effects on the performance of the adaptive controllers to changes in the estimator parameters are also considered.

Chapter 6 - in this chapter results from the experimental validation of the control strategies are presented. The control strategies were implemented using a digital signal processor on the helicopter test rig facility at Westland Helicopters Limited. Controller implementation issues are discussed, and the helicopter test rig and the test signals are described.

Chapter 7 - this chapter summarises the results from the previous chapters. Conclusions from the findings of the research study are made and discussed, and recommendations for further work are made.
Introduction

Figure 1.1 DaVinci's Helicopter 15th Century

Figure 1.2 EH101 Helicopter -1990's (Courtesy of Westland Helicopters Limited)
CHAPTER 2

HELICOPTER VIBRATION

INTRODUCTION

A clear understanding of the causes of helicopter vibration is essential before discussing the techniques that can be employed to reduce fuselage vibration levels. Fuselage vibration is an inherent consequence of driving the rotor blades edgewise through the air. Therefore, a basic outline of the helicopter aerodynamic principles is required. The analysis presented in this chapter is only a simplified approach to the causes of helicopter vibration.

The aerodynamic forces on a helicopter rotor blade vary considerably as it rotates. In turn, the response of the blade to these forces leads to oscillatory moments and shear forces at the blade root, which, when transferred to the fixed system through the rotor hub, excite the airframe response.

Severe vibration is a characteristic of all helicopters, and will potentially continue to remain the main problem for the next generation of helicopters. With the advances in rotary wing technology and associated steadily increasing cruise speeds, the importance of reducing fuselage vibration is increasing. This is particularly important if the helicopter is to be accepted as an alternative form of air transport. The techniques for reducing fuselage vibration can be classified into two broad categories:

- Passive Vibration Control Techniques, and
- Active Vibration Control Techniques.

Both categories will be discussed and the relative merits of the various approaches are detailed.

It is the purpose of this chapter to identify what forces the rotor and helicopter may encounter and the nature of the fuselage response to these forces. Also, the various methods for reducing the fuselage vibration to acceptable levels are identified and discussed. These include structural optimisation of the rotor and fuselage, blade and rotor pendulum absorbers, rotor isolation and active vibration control devices.
2.1 HELICOPTER FLIGHT

The rotor blade of a helicopter can be considered to be a wing, similar to that on an aeroplane, except in this case the wing is moved through the air by rotating it around the aircraft rather than driving the whole aircraft forwards. Like a fixed wing the aerofoil shape changes along the length (span) of the blade so that it is aerodynamically efficient over the helicopter's flight envelope. However, unlike a fixed wing each section of a rotor blade experiences a different velocity than its adjacent sections, and this velocity changes periodically as the blade rotates.

The way a rotor generates lift can be readily understood when the helicopter is in the hover. All the blades have the same velocity and they also have identical pitch settings (angle of attack), this is identical to the way a conventional propeller develops thrust on a fixed wing aircraft [Dommasch, 1953]. Unlike propeller blades which are relatively stiff, helicopter rotor blades are flexible due to the large diameter rotor required for efficient vertical flight and the high aspect ratio of the blades required for good aerodynamic efficiency of the rotating wing.

The lift generated by a rotor can be increased by altering the pitch on all the blades simultaneously, this is known as collective pitch control. The pitch of the blades is changed collectively rather than by increasing the rotor speed, since this would considerably complicate the vibration problem, as the excitation frequency would vary over a range rather than remain fixed. As the pitch angle of the blades is increased to provide more lift their resistance to the airflow also increases. Consequently, in order to keep the rotor speed constant more power will be required from the engines. Generally a control unit on the engines is used to achieve this by regulating the power supplied.

The analysis of the helicopter in forward flight is complicated by the fact that as the helicopter moves forward the rotor now experiences an unsteady aerodynamic field. In the hover the axial airflow through the rotor is symmetric providing a steady airfield, but in forward flight this axial symmetry is lost [Gessow and Myers, 1967, and Liptrot and Woods, 1955].

If the forward speed of the helicopter is \( V_h \), then each blade has a velocity component \( V_r \) due to its speed of rotation and an additional component due to \( V_h \). Therefore, depending on where the blades are (i.e. their azimuth position \( \psi \)) they have a different velocity, see Figure 2.1. Between \( \psi = 0^\circ \) and \( \psi = 180^\circ \), the rotor blades are said to be advancing. In this half of the disk the air speed is increased by the forward speed of the helicopter. At an azimuth position of
\( \psi = 90^\circ \) the two velocities add to produce the maximum velocity a rotor blade will experience around the rotor disk i.e. \( V_h + V_r \). Between \( \psi = 180^\circ \) and \( \psi = 360^\circ \), the rotor blades are said to be retreating, and the minimum velocity experienced by a blade is when the azimuth angle \( \psi = 270^\circ \), where the two velocities subtract to produce a velocity of \( V_r - V_h \). At other rotor azimuth positions, the velocities \( V_h \) and \( V_r \) are not collinear and are therefore treated as vectors, which results in their sum producing a velocity which is not perpendicular to the leading edge of the blade.

Due to these variations in the velocities of the blades a large asymmetry in the lift generated by the rotor would exist because of the higher average speed on the advancing side of the rotor than that on the retreating side. This was the case in early helicopters and resulted in an uncontrollable left rolling tendency in forward flight [Saunders, 1975].

Most modern helicopters compensate for this lift asymmetry by altering the pitch on the blades individually as they rotate. That is, the pitch on the advancing blades is reduced, and the pitch on the retreating blades is increased. This is known as cyclic pitch control, and is generally achieved by allowing the blades to flap up and down as they rotate.

As the blades enter the advancing side of the rotor disk their airspeed increases and consequently so does the lift on the blade. Due to the increased lift the blade flaps upwards, and in doing so the lift on the blade is reduced because the effective angle of attack of the blade will be diminished.

In contrast to this, the airspeed of the blades reduces as they enter the retreating side of the rotor disk. Therefore, the lift is reduced and the blade flaps down, this increases the pitch of the blade which in turn increases the lift on the blade. The consequence of these increases and decreases in lift due to flapping is that they cancel with the variations in lift resulting from the differing airspeeds encountered by advancing and retreating blades. Therefore, equal lift is provided by both sides of the rotor.

An important function of the cyclic pitch control of the main rotor blades is that it provides directional control for the helicopter. To move the helicopter forwards, backwards or sidewardly the rotor disc is tilted to provide a horizontal component to the lift being generated by the main rotor. This is achieved by changing the pitch on the blades cyclically, thereby causing them to flap up or down at other azimuth positions which in turn tilts the rotor and provides a horizontal component to the rotor thrust.
If the vertical lift component of the thrust being generated by the rotor remains constant as the rotor is tilted, then the helicopter will move in the desired direction while remaining at the same altitude. To keep the vertical lift component constant the pitch on the main rotor blades is increased collectively to provide more thrust, some of which is resolved into the horizontal component. This extra thrust also produces an increase in main rotor torque which is counteracted by increasing the tail rotor thrust. Figure 2.2 shows a diagrammatic representation of the pitch control mechanism for a four bladed rotor (only two blades are shown for clarity), also cyclic and collective pitch changes are shown.

2.1.1 Higher Harmonic Motions

Higher Harmonic motions are those motions of the blades around the rotor disk which have a frequency higher than \(1\omega\), and are a result of the blades being flexible. These higher harmonic motions are caused by the asymmetrical loading of the rotor and the periodic changes in blade velocity, as described previously.

The velocity of a blade element at an azimuth angle \(\psi\) and with a radius \(r_{\text{element}}\) is given by:

\[
V_{\text{element}} = \omega r_{\text{element}} + V_h \sin \psi
\]  

(2.1)

Since the forces acting on the blade, which produce the fundamental blade flapping motion, are proportional to the square of the blade velocity, terms with \(\sin^2 \psi\) result and this yields higher harmonic terms as follows

\[
(V_{\text{element}})^2 = \left(\omega r_{\text{element}}\right)^2 + 2\omega r_{\text{element}} V_h \sin \psi + (V_h \sin \psi)^2
\]

(2.2)

\[
(V_{\text{element}})^2 = \left(\omega r_{\text{element}}\right)^2 + 2\omega r_{\text{element}} V_h \sin \psi + \frac{1}{2} V_h^2 - \frac{1}{2} V_h \cos 2\psi
\]

(2.3)

Therefore velocity variations alone produce first and second harmonic force variations which result in higher harmonic blade motions. Higher harmonic motions of the blade are important, not only because of the forces they may transmit to the rotor hub, but also because of the high alternating blade stresses they cause. They also affect rotor performance but to a much lesser degree.

2.1.2 Limiting Forward Speed

A helicopter increases its forward speed by increasing the horizontal component of thrust developed by the main rotor. This is achieved by increasing the collective pitch on the blades.
and by tilting the rotor disk forward using cyclic pitch control. As explained previously the retreating blade is given a large angle of attack to counteract the asymmetry in main rotor lift. Eventually this increasing forward speed leads to two fundamental effects which limit the maximum achievable forward speed of the helicopter [Gessow and Myers, 1967].

The first limiting effect is the stalling of the retreating blade. In forward flight there is a section of the retreating blade near the root which experiences reverse air flow (Figure 2.1). This is where the relative air flow moves from the trailing edge to the leading edge, because it is rotating at a lower speed than the forward speed of the helicopter. This causes a complete loss of lift from the blade section, and as the forward speed increases this section spreads out from the root towards the blade tip. This area causes turbulence, and as the area increases the turbulence increases which induces severe structural vibrations and stresses. The end result is that the blade tip produces all the lift on the retreating blade. Eventually the angle of attack on the retreating blade will be so great that the tip stalls, since this was the only part of the blade producing lift the whole blade has effectively stalled. This results in a dramatic loss of lift and consequently a large lift asymmetry and an associated large increase in fuselage vibration, particularly at $1\omega$. The stall of a helicopter blade limits the maximum forward speed of the helicopter. This can be contrasted with the stall of an aeroplane wing which limits the low speed characteristics of the aeroplane.

Retreating blade stall could be delayed by increasing the rotor speed, thus permitting a higher forward speed. However, this method leads to the second limiting effect. This effect is due to the advancing blade reaching high velocities as the forward speed of the helicopter increases. The blade tip encounters some of the problems experienced by fixed wing aircraft flying at or near the speed of sound, the primary one being severe vibration.

2.2 ROTOR BLADE RESPONSE

The rotor blade operates in a severe aerodynamic environment, which worsens considerably near the boundary of the flight envelope e.g. at maximum cruise speed. The environment includes such effects as:

- stalled and reversed flow on the retreating blade
- transient Mach number effects on the advancing blade (compressibility)
- atmospheric turbulence
At high forward speeds, the tip of the advancing blade may approach the speed of sound. In the hover, typical blade tip velocities can be in the range 300 to 500 knots. Consequently, shock waves can be formed near the tip and this is referred to as a compressibility effect. Prouty [1990] and Saunders [1975] discuss in some detail the theory of rotor aerodynamics, including such effects as compressibility and retreating blade stall.

The variations in blade velocity and pitch angle with blade azimuth angle described in the previous section cause oscillatory rotor blade loads. These can be extreme near the helicopter's maximum cruise speed, where at the blade tip, the local airspeed can vary from near sonic to near gross stall. This can typically happen as often as five times per second. Such rapid variations of local airspeed leads to large oscillatory forces and moments being generated. It should be noted that when in the hover the airflow through the rotor is practically symmetric, and consequently the vibration is lower than when in forward flight.

In forward flight the spanwise distribution of the aerodynamic loads on the blades strongly depends on the shape of the blades. As shown in Figure 2.3, a major difference in the aerodynamic lift loading is that on the retreating blades the loading is concentrated near the blade tip whereas on the advancing blade the load is distributed more evenly. The rotor blades are flexible structures and these variations in load excite the blade flexible modes in both the flapping and in-plane directions. Consequently, there is a substantial motion of the rotor blades in response to the aerodynamic forces in the rotor environment. These blade responses are in addition to the \( 1\omega \) required for control purposes and are called higher harmonic motions (see 2.1.1). Higher harmonic motions can produce high blade stresses in the blades and large moments at the blade root, which are transmitted through the hub to the fuselage. The influence of the trailing vortices from the blades produces significant higher harmonic blade loading and therefore also has a significant effect on vibration [Hooper, 1984]. This leads to a very rough ride for the rotor blades and the aircraft they carry, even in still air. Clearly the problem of blade bending is complicated, and is discussed further by Bramwel [1976].

In summary, the rotor blades of a helicopter not only produce periodic forces which sum to provide the steady response forces of lift and thrust, but they also produce oscillatory forces at the rotor hub. Fuselage vibration is ultimately caused by the combination of these forces, produced by the rotor blades through the rotor hub.
2.3 VIBRATION AT THE ROTOR HUB

Both directional and lift forces are generated by the main rotor, and this creates the need for both collective and cyclic pitch controls. This results in the rotor hub being a very complex and critical area of the helicopter.

The rotor hub connects the blades to the main gearbox and power is transferred to the rotor hub from the engine(s)/gearbox through the rotor shaft. This shaft cannot be moved forwards or backwards or in any other direction to provide pitch changes on the blades. The pitch on each blade is only controlled by mechanical linkages called pitch control levers, these levers transfer both collective and cyclic pitch changes to the blades from either a swash plate (Figure 2.2) or a spiderarm mechanism, the former being the more common method. The type of hub and the number of blades significantly influence which harmonics and the magnitude of the blade vibration transmitted to the fuselage. The rotor hub has a number of possible configurations such as rigid, semi-rigid and articulated.

The articulated rotor has a number of hinges to relieve blade moments. Since the bending moment is zero at the blade hinge, it must be low throughout the root area, and no hub moment is transmitted through the blade root to the helicopter. This configuration makes use of the blade motion to relieve the bending moments that would otherwise be generated at the blade root. A major problem with an articulated rotor is that when there are a large number of blades the hub becomes very bulky and may contribute a large proportion of the total aircraft drag. For the articulated rotor such as used on the Sea King (Figures 2.4 and 2.6), it has been shown that the flap moment transmitted to the airframe is significantly less than that transmitted by a semi-rigid rotor, as used on a Lynx.

The semi rigid rotor has blades which connect to the hub without hinges [Seddon, 1990], but have flexible elements near the root to allow the blade to flap and feather, thereby allowing lift equalisation. The main advantages of the semi-rigid hub are its simpler design and more aerodynamic shape. It also provides greater hub moments leading to improved control (i.e. greater manoeuvrability), but this leads to its primary disadvantage which is a rise in forces transmitted by the hub. The Westland Lynx rotor hub is an example of the simplicity of the semi rigid rotor, and is shown in Figures 2.5 and 2.7.

The effect of the aerodynamic excitation of the blades at the rotor hub is that each blade generates a spectrum of oscillatory forces and moments at frequencies which are harmonic
orders of the rotor rotational frequency. Therefore, if the rotational frequency is $\omega$, the forces and moments at the root of each blade are $1\omega, 2\omega, 3\omega, 4\omega$ etc. Many of these oscillatory forces and moments cancel between the blades across the main rotor hub. However, some reinforce each other and are transmitted down through the main gearbox, and it is these which produce fuselage vibration at the blade passage frequency $n\omega$ on a rotor with $n$ blades. Smaller forcings at $2n\omega, 3n\omega, 4n\omega$ etc are also present. Essentially the rotor hub is therefore acting as a filter, transmitting to the fuselage only harmonics which are multiples of the blade passing frequency. This makes the design of the fuselage a little simpler because only a few frequencies need to be considered. This filtering effect has the added advantage of filtering out the low harmonics, which tend to have the largest magnitudes.

2.3.1 Rotor Filtering

The number of blades determine which harmonics of the blade loads result in fuselage vibration, this is best explained using an example in which a four bladed rotor system with identical blades is considered. In one quarter of a revolution one complete cycle of $4\omega$ will occur, the $4\omega$ flap response of the blades is in phase i.e. they all flap up together. At the rotor hub the $4\omega$ flap forces therefore add together producing a vertical $4\omega$ force on the fuselage, however the $4\omega$ flap moments cancel between blades across the rotor hub, since these are opposite. This will also occur for harmonics of $8\omega, 12\omega$ etc, i.e. only harmonics which are an integer multiple of the number of blades contribute to the vertical hub alternating forces. In one quarter of a rotor revolution three quarters of a cycle of $3\omega$ will have occurred, and the $3\omega$ flap response of the blades will be 90° out of phase i.e. as one blade flaps up the opposite blade flaps down - opposite blades are 180° out of phase. Therefore there is no resultant $3\omega$ vertical load, however there will be a resultant $3\omega$ moment at the rotor hub. This will also occur for $7\omega, 11\omega$ et cetera. A similar line of reasoning can be applied to the $5\omega$ flap inputs, i.e. there is no resultant vertical $5\omega$ load but this harmonic does result in a moment being produced at the rotor hub, again this applies to $9\omega, 13\omega$ et cetera. Figure 2.8 shows various higher harmonic blade motions around the rotor disk.

The $4\omega$ fuselage pitch and roll moments (i.e. in the fixed frame) result from the $3\omega$ and $5\omega$ blade flap moments at the rotor hub (i.e. in the rotating frame). Appendix I describes in more detail how blade forces combine to produce harmonics at the rotor hub. To summarise the results from Appendix I, for a symmetric rotor with $n$ blades the $n\omega$ fuselage vibration is produced in the following manner:
Vertical Shears from $n\omega$ blade flap shears
In-plane Shears from $(n-1)\omega$ and $(n+1)\omega$ blade lag and radial shears
Pitch and Roll moments from $(n-1)\omega$ and $(n+1)\omega$ blade flap and torsion moments
Yaw Moments from $n\omega$ blade lag moments

This result is based on the underlying assumption that all the blades are matched (identical).
If the blades are not matched then other harmonics such as $1\omega$, $2\omega$, $3\omega$ etc may also introduce moments and shears at the rotor hub. In practice there will always be some mismatch, since the blades are tracked within finite tolerances. This tolerance results in some $1\omega$ vibration component being transmitted to the fuselage, which although not significant will not be as large as the $n\omega$ component which dominates the fuselage spectrum generated by real rotor systems.

A number of authors ([King, 1987][Hooper, 1984]) have shown that there is significant reductions in the blade response to harmonics higher than $3\omega$, i.e. with increasing frequency the amplitude decreases. Therefore for low vibration and consequently higher speeds, the rotor should have as many blades as possible. For example, using a five bladed rotor the fuselage $5\omega$ ($n\omega$ where $n = 5$) vibration will be generated by $4\omega$, $5\omega$ and $6\omega$ harmonics from the rotor hub which are smaller than the $1\omega$, $2\omega$ and $3\omega$ harmonics. This leads to helicopters with lower vibration levels than those with fewer blades, as long as the modal frequencies are separated from the blade passing frequency and its harmonics.
However, it should be noted that there is an engineering trade off. Increasing the number of blades introduces extra vibration problems due to the larger number of trailing vortices and their closer proximity to each other. Currently five bladed rotors offer the best compromise for this trade off [Brown, 1987].

2.4 FUSELAGE VIBRATION

The helicopter and rotor represent a highly complex and coupled system, with unique vibration and ride quality problems both of which are self generated. The dynamic characteristics of the rotor and the airframe and the way in which these two systems are coupled at the rotor hub determine the manner in which the helicopter airframe responds to various dynamic loads. The helicopter has many sources of vibration to which the fuselage will respond, including:
- gear meshing
- engines
Helicopter Vibration

main rotor
aerodynamic forces on the rotor blades
tail rotor
aerodynamic forces on the fuselage
interaction of rotor vortices and the fuselage

The primary source of vibration in the helicopter fuselage is caused by the aerodynamic excitation of the main rotor blades. Because the magnitude of the harmonic airloads generally decreases with increasing harmonic number, the lower harmonics of the loads occurring at $n\omega$ (and sometimes $2n\omega$) are usually more important with respect to vibrations than the higher harmonics. Since the engines try to maintain a constant rotor speed, required changes in power being achieved by altering the pitch angle on the blades (section 2.1), vibrations caused by the main rotor occur at a number of distinct and constant frequencies.
Random vibrations do occur, for example vibrations caused by aerodynamic forces on the fuselage, but are in general an order of magnitude smaller than those caused by the main rotor. Ultimately, it is the response of the fuselage to the various sources of excitation, particularly the main rotor, that is responsible for fuselage vibration levels.

Due to the filtering effect of the rotor and for a properly balanced dynamic system, the only forces transmitted by the rotor hub are the blade passing frequency and its harmonics. The forces and moments are transmitted to the fuselage through complicated load paths (predominantly through the main lift frames in the helicopter fuselage), exciting the fuselage flexible modes. Flexible modes can transmit the rotor hub vibrations to various locations in the airframe with significant amplification of magnitude and changes in phase.

Therefore the fuselage vibration is dominated by the blade passing frequency $n\omega$ and its harmonics $2n\omega$, $3n\omega$ etc. Such frequencies characterise the fuselage spectrum typical of modern helicopters (see Figure 2.9). Clearly the vibration is an inherent consequence of driving the rotor blades edgewise through the air i.e. the flight principle itself.

Equally important as the design of the rotor system is the design of the fuselage. Fuselage vibration levels depend on the response of the fuselage to the moments and forces generated by the rotor. The phenomenon of resonance makes it imperative that the natural frequencies of the major fuselage modes should not coincide with the blade passing frequency $n\omega$ or its harmonics. Even though much progress has been made in the helicopter industry with the use of finite element packages, such as NASTRAN, for predicting fuselage modes. One of the biggest causes of high vibration levels in the fuselage is the proximity of fuselage modes to the
blade passing frequency. The fuselage is designed to avoid resonances with \( n\omega \), its harmonics (in particular \((n+1)\omega\), \((n-1)\omega\) and \(1\omega\)), whereas the blades are designed to avoid resonances with \(1\omega\), \(2\omega\), \(3\omega\) etc. Fuselage response can occur at other frequencies due to structural resonances in the blades or airframe, although for a well designed aircraft these responses will be small.

Vibration at \( n\omega \) and its harmonics can never be completely eliminated due to its fundamental nature and in fact even with the use of careful design techniques the resultant fuselage levels are often unacceptably high. In contrast to the blade passing frequency, vibration at the \(1\omega\) frequency can be controlled by careful rotor track and balance. Balancing and tracking procedures are performed in a whirl tower during manufacture, and routinely in service on the aircraft during ground running tests.

Therefore the important difference between fuselage vibration at \( n\omega, 2n\omega, 3n\omega\) etc and other harmonics is that the latter can be eliminated by careful control of manufacturing tolerances, whereas the former are more fundamental and cannot be eliminated by rotor tuning.

Balmford (1977), Gable (1977), Loewy (1984) and Reichert (1981) all provide excellent reviews of the problem of helicopter vibration and how it can be controlled.

### 2.4.1 Structural Optimisation

At the design stage, if there is a possibility of resonant excitation, the structural design is modified. In the past helicopters have been designed to performance requirements, and such changes in structural design would have been made intuitively, relying on the experience of structural and flight engineers to solve excessive vibrations during ground and flight testing phases of development (Done, 1977). More recently structural optimisation techniques have emerged. Done and Rangacharyulu (1979) discuss the application of mathematical optimisation techniques to helicopter vibration control by structural modification. The basic idea behind airframe structural optimisation for vibration reduction is to design the airframe in a way that the vibratory responses in the areas of interest are minimised (Sreekanta and Murthy, 1993).

An obvious pre-requisite for these techniques is an accurate model of the helicopter structure. The main difficulty of structural optimisation lies in the formulation of the cost function for the optimisation process. The passive control of vibration by structural modification and design has been discussed in a survey paper by Friedmann (1990), that categorises and compares the approaches available.
2.4.2 Benefits

One of the driving forces behind vibration reduction research, i.e. the reduction of helicopter direct operating costs, is highlighted by Veca [1973] and is regularly emphasised in helicopter literature. This is the need for reduced vibration because of its impact on the civil market acceptance of scheduled helicopter services.

Any aircraft with excessive levels of fuselage vibration compromises passenger comfort, increases crew fatigue and introduces control difficulties for the crew. Vibrations produce a hostile environment for all types of equipment, it makes instruments difficult to read and it makes sights/weapons hard to aim. Structural vibrations usually mean oscillatory strain and therefore fatigue of the fuselage and of equipment. This increases maintenance costs and reduces the availability and reliability of the aircraft.

From the vibration point of view the helicopter is a much more difficult problem than the fixed wing aeroplane, since it is subjected to disturbances of a much lower frequency. The high amplitude of the vibration in this lower frequency range causes serious problems for the transport of ill and injured people.

The problem vibration poses to the helicopter cannot be over emphasised. In the late 1960s it was estimated that helicopters were frequently operated at speeds reduced by as much as 20% below those they might otherwise achieve, because of the extreme vibration environments (Calcaterra and Schubert, 1968).

Significant reductions in vibration would therefore improve the comfort of passengers and have a large effect on the acceptance of helicopter transportation. Noise levels also play an important role in passenger and crew perceptions of comfort (Stephens and Leatherwood, 1982). Generally noise is caused by fuselage vibrations of higher frequencies than those associated with human comfort, a reduction in vibration would undoubtedly reduce noise levels.

Also, the operational life of most electrical and mechanical components would be increased, reducing maintenance times and associated costs. The extent of the impact both operationally and economically has not been properly assessed. There would also be cost benefits from the reduction of secondary damage due to structural vibration, and this also loads to improvements in flight safety. Thereby increasing the usefulness of the helicopter in both an economical and operational sense. In its military role, other advantages include reduced crew fatigue and consequently improved effectiveness, and an easing of the problems of weapon sighting and delivery.

However, the psychological factor, i.e. the ride perceived by the passengers, is probably more important than all the other factors in gaining acceptance of the helicopter as a routine means of transport.
To summarise, the benefits of reducing helicopter fuselage vibration are mainly self evident and include:

- reduced crew fatigue
- improved passenger comfort
- enhanced systems reliability
- improved visual acuity
- prolonged fatigue lives (airframe and equipment)
- reduced maintenance costs
- reduced operating costs
- increased forward flight speed
- increased manoeuvrability

Although this list is not exhaustive, it can be readily seen that significant reductions in vibration levels must be accomplished if these benefits are to be realised, allowing the next generation of helicopters to reach their full potential.

2.4.3 Trends

Considerable progress has been made in reducing helicopter vibration from levels exceeding 0.5g, typical of many helicopters in the 1950s and 1960s, to levels approaching as low as 0.05g which can be found on some modern helicopters. Helicopters now operate at higher forward speeds, at higher altitudes and with lower vibration levels. The trend for higher cruise speeds has aggravated the problem, since the magnitude of many components of the rotor induced vibration increase rapidly as the forward speed increases. Even though much progress has been made, the level of vibration reduction achieved in newer helicopter designs has been in most cases, only marginally acceptable or insufficient in meeting the increasingly stringent vibration requirements which are being imposed.

The progress to date has been achieved by using techniques such as dynamic optimisation of the fuselage and rotor, and passive dynamic isolation and absorption devices. The improvements that have been made more recently are smaller and it seems that a limit of what can realistically be achieved by passive vibration reduction techniques is being approached. Further reductions are always possible by making the structure stiffer or by the extensive use of passive vibration control devices, but both of these options are unacceptable since they involve significant weight penalties. Active techniques are becoming increasingly attractive for vibration reduction since they potentially offer significant improvements on current vibration levels without significant weight penalties.
Considerable efforts have been expended by the helicopter industry in general, into the research and demonstration of techniques for helicopter vibration reduction, and in particular active methods of control. The overall aim being to reduce the high vibration levels of the helicopter to levels comparable to levels typical of other forms of transport, this has led to the so called 'Jet Smooth Ride' goal. To achieve this goal will require the development of advanced design methodologies.

It is difficult to identify with any accuracy the vibration levels typical of helicopters for a given period. The main reasons for this are that sources reporting helicopter vibration levels are rarely at the same speed, or at the same position in the fuselage. Also vibration levels vary considerably depending on the type of helicopter and the role it is intended for. Figure 2.10 shows the trend of helicopter fuselage vibration [Reichert, 1980]. Also shown in the Figure are vibration levels specified by various bodies.

Throughout the years the requirements have been for lower values than could be realised with production helicopters. A special example is the levels specified for the US Army in the AAH/UTTAS specification (UTTAS draft RFQ of 1971) which originally required vibration levels lower than 0.05g. None of the competitors could fulfil this specification, and finally the specification had to be raised to 0.1g to provide a more realistic design goal consistent with the technology available at the time. The vibration levels on current helicopters can typically reach a few tenths of a g at maximum cruise speeds, a level significantly higher than the vibration levels on air liners which are typically only a few hundredths of a g.

The NASA research and Technology Advisory council subpanel on Helicopter Technology in 1976 recommended a desirable level of 0.02g, the "jet smooth ride" helicopter. This level is a long way off. And indeed the trend seems to be converging to an asymptote of approximately 0.1g/0.05g with current techniques. Requirements for the next generation of helicopters will be in the 0.02-0.05g region. It seems that such low vibration levels will only be realised in the next generation of helicopters by active control techniques, such as Higher Harmonic Control, Individual Blade Control, and/or the Active Control of Structural Response.

2.4.4 Comfort and Ride Criteria
The helicopter ride comfort problem is a very complex one involving mechanical, physiological and psychological factors. Many of these factors are hard to quantify. A common denominator in all these factors is vibration, and clearly any vibrations felt by the crew and passengers are detrimental to ride comfort [Butkunas, 1966].
Requirements for helicopter vibration levels in terms of ride, can be divided into three categories. There are requirements for equipment, these concern the reliability and maintainability of equipment in the helicopter vibration environment. There are requirements concerning the fuselage integrity and structural fatigue caused by rotor vibrations. Finally there are human factor requirements, these relate to the sensitivity of the crew and passengers to the vibration environment. The human factor criteria is the most stringent and difficult to meet, and consequently has received the most emphasis.

Helicopter ride quality is largely determined by the vibration level in the fuselage. Blade passing frequency is in the range of 10-30Hz for most helicopters. Unfortunately, this is a range of frequencies to which people are very sensitive. Different parts of the body are susceptible to different frequency ranges: vision is susceptible to the range 10-100Hz, the head, neck and face to the range 4-27Hz and the trunk and abdomen are particularly sensitive to the range 7-20Hz. Therefore it is the blade passing frequency which dominates the passengers perception of the helicopter ride. The International Standards Organisation [ISO, 1974] produced a set of sensitivity curves. These take account of the varying susceptibility of the human body to vibrations of different frequencies.

Vibration manifests itself in a number of ways, predominately it is felt through the floor and seating, it can be seen by crew as motions on instruments panels and by passengers on window and door frames. At higher frequencies the perception of vibration motion diminishes and gradually the perception of vibration as noise takes over. These secondary effects can be evident even when seat and floor vibration levels are good.

Calcaterra [1972] identified a number of different levels corresponding to increasing vibration levels:

- First perceptible sensation
- Decreased comfort
- Distraction
- Impairment of visual functions
- Impairment of psychological functions
- Positive discomfort
- Fatigue
- Extreme loss of control of arms and legs
- Pain and intolerance for task continuation
- Physical injury
Figure 2.11 shows the well known Goldman data [1948]. Goldman plotted equal sensation contours in terms of peak amplitude and frequency for three levels of sensation: just perceived, unpleasant, and intolerable. Helicopters with vibration levels higher than 0.1 g, really are not comfortable. From the figure it can be seen that if the 'jet smooth' ride level of 0.02 g could be achieved, then helicopter vibration would cease to be a major problem and industry's efforts would be focused on other problems.

More recent helicopter vibration criteria are based on the Aeronautical Design Standard (ADS) entitled "Requirements for Rotorcraft Vibration Specifications, Modelling and Testing". This was released as ADS-27 by the United States Aviation System Command (AVSCOM) on the 18th November 1986, and this is described in detail by Crews (1987). The ADS-27 standard uses the intrusion index as a criteria for fuselage vibration. The intrusion index is calculated by normalising triaxial accelerometer data for the four largest spectral peaks for the frequency range where there are significant helicopter inputs in each direction, by the values shown in Figure 2.12. The root mean square value of this data is calculated, this provides a single scalar quantity which can be used to evaluate ride. Since the most troublesome frequencies in the fuselage are at the blade passing frequencies and its harmonics, the logic of this approach is readily ascertained.

From Figure 2.12 it can be seen that the Intrusion Index weighs vertical vibrations most heavily, and that it weighs low frequency components more heavily than higher frequency components. This is because of the human susceptibility to vibrations in the low frequency range (4-30 Hz). Most requirements take this into account, the vibration requirement becoming much less stringent at higher frequencies [Schrage and Peskar, 1977].

The ADS-27 standard requires a pilot seat intrusion index of less than 1.2, and a weapon system operator of less than 1.0 in level flight. Crews (1987) provides an example for a helicopter having 0.05 g vertical vibration at 25 Hz will have an intrusion index of about 0.5.

2.5 PASSIVE VIBRATION CONTROL TECHNIQUES

Control of helicopter vibration remains one of the most serious problems facing the designers and users of rotary wing aircraft. The most obvious way to make a structure more rigid and durable is to build it stronger and stiffer, however this almost always involves an increase in weight and the helicopter designer cannot afford to oversize the structure, every pound of weight costs in terms of vehicle performance.
Very few helicopters built to date have managed to attain satisfactory levels of vibration by simply dealing effectively with fundamentals, i.e. the design of the fuselage and rotor system. As a result many different and ingenious methods for controlling helicopter vibration have been developed. In addition to the optimisation of rotor and fuselage design discussed earlier, the techniques used to reduce helicopter fuselage vibration to acceptable levels are predominately passive.

In the past, the focus of the effort to reduce helicopter vibration has been on the addition of tuned vibration absorbers [Paul, 1969] and isolators [Flannelly, 1966]. These passive devices influence vibrations only after they have been generated, and generally require a large weight penalty. Passive vibration reduction devices have been invented in a wide variety and are reviewed by Loewy (1984). They basically consist of springs and dampers, where the springs support the load and the damper can be used to dissipate energy, although in some devices, such as passive antiresonant isolators, damping is undesirable. The spring can typically be metallic, elastomeric, pneumatic or composite. Examples of some of the many passive devices available include:

- Soft mounts
- DAVI mounts (Kaman Aircraft Company) [Flannelly, 1976].
- IRIS Isolation system (Boeing)
- SARIB system (Aerospatial Helicopters Ltd.)
- Nodal mounts (Bell - Nodamagic) [Gaffey and Balke, 1976]
- Focal Pylon isolation system
- BIFILAR absorbers (Sikorsky) [Paul, 1970]
- Blade mounted absorbers (pendulums) [Taylor and Teare, 1974]
- Rotor head absorbers [King, 1987]
- Spring mass absorbers
- LIVE isolation system [Halwes, 1981]
- Cabin vibration absorbers
- ARIS isolation system [Strehlow et al, 1977]
- UREKA absorber

Despite the numerous passive devices available, they can be divided by their location (on the rotor, at the rotor fuselage to rotor interface, or in the fuselage), or by their principle of operation (isolation or absorption).
2.5.1 Passive Isolation

Isolation devices can be used locally to isolate particular installations such as avionics equipment or crew seats, from the fuselage vibrations. Alternatively, they can be used globally to isolate the entire fuselage from the main rotor and gearbox.

Passive isolation devices generally consist of a parallel arrangement of a spring and a force generator, shown in Figure 2.13. At a certain excitation frequency the dynamic part of the spring force in the force generator (produced by the relative motion across the device, which in the case of gearbox isolation is between the fuselage and the transmission unit) are opposite and equal at the fuselage attachment point of the isolator element.

The use of soft mounts to isolate the main gearbox from the fuselage is one of the simplest solutions [Braun, 1982], but it has been virtually impossible to achieve good vibration reduction at the blade passing frequency without having unacceptably soft springs. The large static forces and moments, coupled with transient flight conditions result in large static deflections across the fuselage/main-gearbox interface. Such large deflections are difficult for the flight control systems, engine drive shafts, tailrotor drive shafts and other functional components to accommodate. The spring stiffness required to obtain a good level of vibration isolation depends on the ratio of the isolated mass to the non-isolated mass. The closer this ratio is to unity the higher the allowable stiffness, typically the ratio between fuselage mass and gearbox mass is around eight.

It is relatively simple to design a passive system to provide good isolation, however, it is quite difficult to meet the displacement requirements simultaneously, and this is the main compromise with all isolation systems.

Simple isolation systems have no problems with tuning and are relatively insensitive to changes in rotor speed, unlike isolation systems based on the more complex antiresonance principle which are tuned to a single frequency. Figure 2.14 also shows the transmissibility plots for a conventional isolator and for an isolator based on antiresonance principles. The conventional isolator has a low resonant frequency, above which the device isolates with increasing efficiency with increasing frequency reaching 100% isolation at infinite frequency. An antiresonant isolator has a higher resonant frequency, but then has a specific anti-resonant frequency at which 100% isolation is achieved with no damping (Figure 2.14). Increasing damping severely reduces the level of isolation that can be achieved.

Gearbox isolation systems are designed to respond to the new blade passing frequency. At the design frequency the forces generated by the springs and inertias of the isolation system cancel...
with the nco forcing generated by the main rotor. Since these systems are tuned to a single frequency, if the rotor speed changes then the performance of the isolation system can be seriously degraded.

Attempts have been made to reduce wear and the number of moving parts in passive devices, these attempts have produced isolators with hydraulic fluids in containers with variable cross sections. The more successful devices are described in more detail in the following sections.

DAVIs (Dynamic Antiresonant Vibration Isolator(s)) are an improvement over the simple soft mounts, since they allow higher static stiffnesses in the gearbox isolation mounts, while isolating the fuselage from the blade passing frequency (Figure 2.15 shows a simple representation of a DAVI mount.). The small relative motion between the fuselage and gearbox is transferred into a large opposite displacement of a small mass. Figure 2.16 shows the principle of operation.

DAVI mounts were developed by the Kaman aircraft company, for isolation of crew seats and also complete rotor systems. The device has been successfully implemented on the Bell UH1, Bolkow BO105 and Boeing UTTAS helicopters [Hooper and Desjardins, 1976].

The main disadvantages of the devices are that they require careful tuning and this increases maintenance requirements, the lever arm must be rigid, the performance of the device is sensitive to the amount of damping, performance can also be seriously affected unless they isolate all load paths between the rotor and fuselage, and performance is also affected by variations in rotor speed. For a gearbox with four mounting points, to completely isolate the fuselage from the blade passing frequency would require twelve DAVI mounts (one acting vertically, one acting laterally and one acting longitudinally at each of the four gearbox mounting points). Such a system would be prohibitively heavy. To overcome this, a common solution is to use just four DAVIs to isolate the fuselage from the gearbox in the vertical direction only.

A number of manufacturers have produced devices also based on the antiresonant isolation principle, and are therefore equivalent to the DAVI in terms of operation. DAVI equivalent devices include:

The ARIS Anti Resonant Isolation System was developed by MBB [Strehlow et al, 1977], this system has been successfully tested on ground and in flight on the MBB BO108 helicopter. Braun (1982) describes a passive nodal rotor isolation system based on ARIS. The system consists of five local uniaxial force isolators which operate using the antiresonance principle and are tuned to the no blade passing frequency.
The Liquid Inertia Vibration Eliminator (LIVE) system for mounting the main gearbox is another DAVI equivalent system. This system is being developed by Bell helicopretors and is described by Halwes [1981]. The aim of the system is to offer similar performance to the Nodal Beam Isolation technique but for considerably less weight.

The principle difference in these designs are the method of operation of the force generation. In the DAVI (Kaman) and IRIS (Boeing) systems, mechanically driven pendulums are used as the force generators. In the LIVE (Bell) and hydraulic DAVI (MBB) systems, hydraulic pendulums are used as force generators.

The Nodamatic system described by Gaffey and Balke [1976], is a gearbox mounting system. This system was developed by Bell Helicopters and has been successfully demonstrated. The gearbox is mounted on a beam arrangement, and the fuselage is connected to the beam at its node points when vibrating in response to the rotor hub forcing. The system is based on the principle that a flexible beam vibrating in its fundamental free-free mode has two node points near the end of the beam. If the beam carries a mass at its mid point and is supported at these nodes, then no vibration should be transmitted from the beam through the supports to the structure. Figure 2.17 shows the principle of nodal isolation.

2.5.2 Passive Absorption

Dynamic absorption is an alternative form of passive vibration control. These devices are based on the principle of a spring mass absorber. They can be applied at various positions within the airframe where the effect is generally localised, or they can be mounted on the rotor itself where of course, their effect is more widespread. If an aircraft has a vibration problem throughout its fuselage, then the most efficient position for an absorber would be to attach it to the rotor head.

In its simplest form a vibration absorber consists of a relatively small mass on a spring, its natural frequency tuned to the frequency of the vibration excitation. An opposing force is thus generated by the sprung mass vibrating in resonance with the excitation. To be effective the damping must be low so that its amplification at resonance is large. The effect of the device when attached to the structure is to produce a node at the point of attachment.

One of the major limitations encountered with many passive techniques is that their effect is often only local in nature. By attaching absorbers as close as possible to the source of vibration a global effect can be achieved, an example of this is to mount the absorber on the main rotor
hub itself, for example the Westland spring head absorber [King, 1987]. The Westland rotor head absorber is a fixed frequency absorber consisting of fibre-composite springs. This absorber operates in the plane of the rotor disk and is tuned to the blade passing frequency. The main disadvantages of such devices are that the weight and drag penalties associated with rotor head absorbers can be high, and the device is sensitive to changes in the rotor speed. The rotor head absorber developed by Sikorsky relies on the centrifugal field of the main rotor to provide the stiffness, and this provides a certain amount of self tuning with regard to changes in rotor speed.

If there are significant levels of vibration in all directions, then the absorber needs to work in all directions, i.e. rotationally as well as translationally. This is difficult, if not impossible, to achieve with a single device. For this reason it has been found that, in some cases, the rotor head absorber does not reduce vibration throughout the entire structure.

Vibration at the rotor head can also be reduced by fitting pendulum absorbers to the rotor blades or to the rotor hub. These devices are tuned to counteract and cancel the vibratory forces at their source before they can be transferred through the rotor hub to the airframe. There are various types of pendulum absorbers

1- blade mounted pendulum dynamic absorbers
2- rotor hub dynamic absorber or bifilar centrifugal pendulum
3- monofilar centrifugal pendulum
4- mercury centrifugal pendulum

Simple pendulums can be attached to the blades of a helicopter to reduce or suppress the reactions at the blade root during forward flight. A pendulum absorber generates forces at its attachment point with the blade, which if correctly tuned, redistribute the loads in the blade such that the shears and moments transmitted by the blade to the hub are attenuated.

Hamouda and Pierce [1981] studied the effect of both flap and lead lag pendulum absorbers. This study reported that the pendulum weight can be typically 10% of the blade weight, and since at least one pendulum is required for each blade this represents a significant contribution to Helicopter weight. Figure 2.18 shows a diagrammatic representation of the operation of a blade mounted pendulum.

The major disadvantage of this type of absorber is that each troublesome mode needs a separate absorber and this would make a complete combined system too complex and expensive. They also carry a weight and drag penalty, particularly because they operate in the aerodynamic field of the main rotor. Despite these disadvantages they have been extensively used by a number of manufacturers including Boeing, Vertol, Hughes, Augusta and MBB.
Bifilar absorbers are a means of producing a pendulum absorber with a very small radius of gyration. The frequency of operation of the device is principally determined by the radius of the effective mass and the radius of the mounting position.

The Bifilar absorber is a centrifugal pendulum absorber developed by Sikorsky Helicopters and has proved to be quite successful [Paul, 1970 and 1969]. Bifilar absorbers have been in use for a long time and many companies have considerable experience of these devices, for example Boeing Vertol, Hughes, and MBB. Strehlow et al [1990] reports the successful use of centrifugal pendulum absorbers on the MBB B0105 helicopter.

In summary the main disadvantages of this type of device are
1- they have bearings on rolling surfaces, and hence they are subject to wear
2- they do detune when oscillation amplitudes become too large
3- they tend to produce aerodynamic drag, which is clearly an undesirable feature in an era of high performance helicopters.

2.5.3 Summary
Passive vibration reduction systems have proved to be successful in the past and have been favoured until recently, mainly because computing and actuator technology were not suitably advanced to make active vibration reduction systems a reality. Modern helicopters have higher cruise speeds and greater manoeuvrability, and associated with these advances are ever increasing stringent comfort requirements. These factors limit the usefulness of passive systems, because the weight penalties associated with devices that would be capable of reducing the fuselage vibration to the required levels may be prohibitively large. For some passive devices the weight penalty can be considerable and may well be in excess of 1% of the gross weight of the helicopter, an example of a passive device with a severe weight penalty is the main rotor head absorber. Rotor head absorbers also have the disadvantage of increasing the aircraft drag, but regardless of these drawbacks this is a popular solution since it is close to the source of vibration. A loss of aircraft performance due to drag penalties is also often attributed to centrifugal pendulum absorbers. Therefore dynamic absorbers in the rotating system are not desirable if the stringent vibration requirements of the next generation of helicopters are to be fulfilled.

Passive systems are relatively simple, they have no direct power supply requirements and have good stability, however, they tend to be characterised by a lack of adaptability with respect
to changes in flight condition (forward speed), rotor rotational frequency (blade passing frequency) and changes in structural dynamics (due to cargo, fuel and passenger changes), and this is a particularly severe limitation.

Passive systems are difficult to tune, and once tuned are sensitive to parameter variations. For example, passive devices are generally tuned to provide maximum vibration reduction at blade passing frequency. If the rotor RPM changes, the exciting frequency may move into an unfavourable range where the performance may be degraded, and may even worsen the situation. If either an isolation system or a dynamic absorber spring rate is too low, the suspended mass will bottom in manoeuvres or under other transient loads. Both Isolation and Absorption systems remain in tune only when amplitudes are small enough to stay within the linear range of the mounting springs. Tuning of passive devices increases the maintenance costs associated with these systems.

Many passive systems only produce reductions in vibration that are local to the device, the exceptions to this being the rotor head absorber which absorbs or reduces the rotor forces, and the main gearbox and rotor isolation devices.

2.6 ACTIVE VIBRATION CONTROL TECHNIQUES

As identified in the previous section passive systems have a number of fundamental disadvantages; the cost of employing passive systems in terms of weight and drag is often very high, they also have a lack of adaptability to changing flight conditions. In recent years many active vibration control techniques have been proposed to overcome these problems, and are under investigation throughout the helicopter industry. Active systems offer the potential to meet more and more stringent comfort requirements over significant areas of the aircraft structure, even with the extended flight envelopes and lighter fuselages of modern helicopters. As a principle these techniques generate forces which oppose the existing forces causing fuselage vibration, they can be applied either at the rotor (Active Rotor Control), at the Rotor to Fuselage interface (Active Isolation), or they can be applied at the structure (Active Control of Structural Response).

As has been explained in sections 2.2 to 2.4 the fuselage vibration is primarily caused by the higher harmonic loading of the main rotor. Consequently, much of the early research into active techniques, and a majority of the research to date, has concentrated upon Higher Harmonic Control (HHC), a technique aimed at reducing the vibratory forcing within the main rotor.
through superposition of a controlled pitch excitation on the main rotor blades onto the primary uncontrolled vibration response of the rotor. HHC is an Active Rotor Control scheme which utilises active swash plate control, the other Active Rotor Control scheme is known as Individual Blade Control (IBC) which uses active pitch link actuators. IBC is a generalisation of HHC, but HHC is more widely researched. Rotor active control techniques have a broad scope, they have the potential to expand the flight envelope of the helicopter as well as reduce structural vibrations. Aside from Higher Harmonic Control, the next most significant area of current research involves the use of some form of rotor isolation scheme.

The advantages of Active Control techniques mean that they have the potential to overcome many of the disadvantages encountered with existing passive devices. The primary advantages of Active techniques are:

1) active systems can supply or absorb power using actuators- passive systems can only dissipate or store energy.
2) active systems can produce local forces as a function of many variables, some of which may be remotely measured. Passive systems generate forces related to local motion only.
3) active systems can be modified as desired by control algorithms to establish certain performance specifications, passive systems do not have this possibility at all.

Various types of actuator are available, for example servohydraulic, electromechanical, electromagnetic, and/or pneumatic. Servohydraulic are the preferred type in the helicopter industry for a number of reasons; they are compact, they are powerful, and practically all helicopters already have a hydraulic power supply.

2.6.1 Higher Harmonic Control

Since the fuselage vibration originates with the airloads causing higher harmonic motions of the rotor blades, intuitively it would be attractive to try and control these higher harmonic motions at the source in order to reduce fuselage vibration. This can be achieved by applying controlled blade pitch changes at harmonics of the pitch control frequency $1\omega$ required for primary flight control.

This technique is known as Higher Harmonic Control (HHC), and aims to reduce the vibratory forces at the rotor head by imposing small magnitude oscillatory pitch changes on the rotor blades. HHC inputs are applied in addition to and quite separate from the pilots collective and cyclic inputs which the pilot uses for flight control, and because they are at frequencies higher
than the $1\omega$ required for collective and cyclic control, they do not interfere with them in any fashion. The term Multicyclic control is often used synonymously with higher harmonic control. HHC also has the potential to improve rotor performance, although the majority of research has been concerned with the use of HHC for vibration reduction.

Controlled rotor blade pitch changes at any harmonics of the blade passing frequency $n\omega$, have the ability to vary the $n\omega$ rotor loads. In practice, the further the blade pitch frequency departs from the blade passing frequency the less effect it has. Generally control is performed at three frequencies $(n-1)\omega$, $n\omega$, and $(n+1)\omega$, since it is these that have been shown to cause fuselage vibration at $n\omega$ (see section 2.3.1 and Appendix I).

The aim of the system is to create blade oscillations which, when properly phased, exactly cancel the rotor head forces and moments created by the flight condition. Cancelling these forces and moments at the source reduces the vibrations transmitted to the airframe. Transducers mounted at key locations in the fuselage measure the vibration. During each control cycle, the algorithm harmonically analyses the measured variables (either mast forces or fuselage accelerations). The result is the $n\omega$ sine and cosine components of the measured force or accelerations. Based upon these measurements a control algorithm determines the phase and the magnitudes of the signals to be sent to the actuators in order to minimise the forces transmitted to the fuselage at the blade passing frequency and consequently minimise the fuselage vibration. The control calculations use a transfer matrix, also known as the T matrix. This is a frequency domain model of the helicopter, which relates the $n\omega$ sine and cosine components of the HHC input to the sine and cosine components of the $n\omega$ response of the helicopter. The actuators vary the blade pitch at harmonics higher than the first ($2\omega$, $3\omega$, $4\omega$ etc) which influence the rotor oscillatory loads, which in turn alters the fuselage vibration.

The two main methods for implementing HHC place servo-actuators in series with either the rotating (known as Individual Blade Control -IBC) or non-rotating sections of the primary flight controls, and these actuation options are discussed later. The actuators act independently to the primary flight controls, and apply high frequency pitch motions of low authority. Figures 2.19 and 2.21 show the concept of HHC.

The potential of HHC was first identified in the early 1950s, and following this many experimental and theoretical studies have been performed. One of the earliest flight tests of a HHC system was reported by Wernicke and Drees [1963]. Even though their implementation
was successful they found that the performance was limited. Wood et al [1983] later identified that since their implementation was on a two bladed rotor, the performance in limitation was probably due to the fact the $2\omega$ couples strongly with $1\omega$ and $3\omega$, and $1\omega$ air loads are also produced by the primary flight controls of the helicopter. In the same study it was demonstrated, using a wind tunnel model, that HHC could successfully suppress vibration with a HHC pitch amplitude of less than 1 degree. Most of the early research used Wind tunnel models to determine the feasibility of HHC [McCloud and Kretz, 1974], [Lehman, 1984].

Theoretical studies carried out by Hughes and Staple [1989] indicate that reductions of up to 90% can be achieved using HHC, providing vibration levels of 0.05g and below over a large part of the helicopter’s flight envelope. The HHC pitch changes required for the rotor blades are relatively quite small, typically only 0.5 degrees, compared to the 5 to 10 degrees required for cyclic control (at $1\omega$) of the rotor blades during high speed forward flight.

A research program launched by Aerospatiale Helicopter Division flight tested one of the first experimental HHC systems [Polychroniadis and Achache, 1986, and Achache and Polychroniadis, 1987]. The aircraft selected for these tests was a SA 349 experimental three bladed helicopter which was derived from the SA 342 Gazelle. The program validated the concept of reducing vibration through a HHC system, a reduction of 80% in the cabin was demonstrated.

Hughes helicopters Inc., the US Army and NASA combined in a program of research directed at flight testing a HHC vibration control system [Wood and Powers, 1980][Ropelewski, 1984]. The helicopter used in this program was an OH-6A, and the initial results are presented by Wood et al [1983].

The HHC controller tested by Wood et al [1983] responded to slow changes in the flight condition, such as accelerations and decelerations, maintaining vibration levels to those achieved under steady flight conditions. However, it was found that the controller was too sluggish to track rapidly changing transient flight conditions. This is of primary importance for any vibration control system.

The transfer matrix approach, linking the HHC inputs to the rotor vibratory response, was used by McCloud and Kretz [1974]. In this work the rotor’s response to the HHC inputs was linearised allowing the use of optimal control theory. The control problem then consists of harmonically analysing the measured vibration, identifying the relationship between the
measured outputs and HHC inputs in matrix form, and determining the optimal HHC inputs to minimise the vibration. [Taylor et al, 1980] also formulated the vibration control problem on the assumption that the helicopter can be represented by a linear, quasi-static transfer matrix (T matrix) relationship between the harmonics of vibration and the harmonics of the control inputs.

It is well known that the blade loads and consequently the fuselage vibration change significantly with flight condition. Also the response of the rotor to higher harmonic control inputs varies with flight condition. Therefore, for a practical HHC system, either some form of adaptive or robust control algorithm is required to determine the best mix of HHC frequencies to apply to the rotor in order to minimise the measured fuselage vibration. This variation in rotor dynamics and consequently transfer function with flight condition has resulted in the use of controllers which often employ kalman filter routines to provide a degree of adaptation.

The first tests using adaptive control were conducted by Hammond [1980], in this study the concepts of HHC and optimal control theory were successfully combined. In this formulation a cost function is used which consists of a weighted sum of the squares of the measured vibration and of the HHC inputs, the solution of this cost function determines the HHC inputs required to minimise the vibration. The controller utilised a Kalman filter and stochastic (cautious) control.

Shaw and Albion [1980] constructed an adaptive controller around a single board microcomputer, and the performance of the system is described in Shaw and Albion [1980,2]. The controller also used Kalman filters to track the Transfer matrix relationship as the flight condition changes. The controller was implemented on a wind tunnel model and good vibration reductions of 88% were reported, however the same degree of vibration reduction could not be achieved during simple transient flight conditions such as a steady climb.

In the following year Molusis et al [1981] extended this research to include fixed gain as well as adaptive controllers for HHC. They concluded that the transfer matrix was non-linear and consequently the fixed gain controllers were unsatisfactory. The effects of nonlinearity on HHC performance was investigated further by Molusis [1983] in contrast to this work, Shaw et al [1985] reported that the transfer matrix was linear, and that a fixed gain control law could provide 90% vibration reduction. Similar results were reported by Nygren and Schrage [1989] using computer simulation studies. Further research by Shaw et al [1989] tested fixed gain, gain scheduled and adaptive regulators on a dynamically scaled model of a three bladed CH-47D Chinook rotor. This study showed that a fixed gain controller provided 90% vibration reduction. In these tests Shaw used sensors mounted in the rotating frame.
From these contradictory reports it can be concluded that there is still considerable uncertainty regarding the higher harmonic control problem. The question of the linearity of the transfer matrix may depend on the placement of the sensors. The study by Molusis used sensors in the fixed frame, while Shaw used sensors in the rotating frame, however further research is required to solve these contradictions.

Real time identification of the Transfer matrix is intended to increase the robustness of the HHC algorithm parameter uncertainty, system non-linearities and changing flight condition. An excellent review of self tuning systems is provided by Johnson [1982]. Johnson discusses various self tuning controllers that have appeared prior to 1982 in both theoretical and experimental studies. Davis [1984] evaluates several alternative controller configurations (deterministic and stochastic). In this study the helicopter simulation used is a non-linear aerelastic helicopter vibration computer analysis that models a generic rotor and fuselage.

Molusis et al [1983] describes the use of stochastic adaptive controllers in order to account for parameter uncertainty and for changing flight condition. A modified Linear Quadratic (LQ) regulator which allows for state and control weightings that are functions of frequency is described by Gupta and DuVal [1982] and by Hall and Wereley [1989]. The cost function places a large weighting on fuselage accelerations at the blade passing frequency, and the optimisation of this frequency shaped cost function leads to a state feedback law. Simulation results are presented for the controller which demonstrate its performance.

2.6.1.1 HHC Actuation Options

A number of different methods have been proposed for HHC. The oscillatory pitch inputs at harmonics higher than the first can be applied to the blades through either the conventional primary flight control system (actuator in the fixed system), or by a system using individual blade actuation (actuator in the rotating system). HHC applied by individual blade pitch actuators has been evaluated by McCloud [1980] and Ham [1980]. The advantage of individual blade actuation is that it allows pitch variations at every harmonic, whereas actuation through the conventional control system can only vary the pitch on blades at \((n-1)\omega, n\omega, \text{ and } (n + 1)\omega\) harmonics.

In the case of rotating frame actuation the response of each actuator must be identical, otherwise a major advantage of the filtering effect of the rotor hub will be lost. Systems which are in the rotating frame have problems of getting power and signals into the rotating frame.
A rotating hydraulic manifold and slipping assembly would be required, obviously actuators in the fixed frame overcome this requirement. Rotating frame actuation systems also have the additional problem of sending reaction forces to the fixed frame of reference.

The primary flight control actuators (in the fixed frame) could be used to apply HHC inputs, though this is undesirable for airworthiness reasons. A more attractive solution is to use a dedicated set of HHC actuators which operate in parallel with the flight control actuators.

In most investigations [Johnson, 1980, McHugh and Shaw, 1976] the control is obtained by oscillating the swashplate. When the actuators are mounted in the rotating frame, the technique is known as Individual Blade Control (IBC). The fundamental difference between HHC and IBC is that IBC can control a range of frequencies and not just discrete frequencies.

As described previously, the rotor acts as a filter. The harmonics \((n-1)\omega, n\omega \) and \((n + 1)\omega\) are combined at the rotor hub and cause \(n\omega\) vibration in the fuselage. By reversing this process it should be possible to excite the rotor hub from the fixed frame at \(n\omega\) to cause harmonics at \((n-1)\omega, n\omega\) and \((n + 1)\omega\) in the rotating frame and thereby minimise the \(n\omega\) fuselage vibration. This is the aim of using actuators to oscillate the swashplate at a frequency of \(n\omega\), in vertical, pitch and roll directions.

### 2.6.1.2 Active Blade Control

Individual Blade Control (IBC) is a generalisation of the Higher Harmonic Control (HHC) concept. The IBC system consists of actuators attached directly to each blade to control the pitch response of the blades. Drive signals for the actuators are derived by a control system and algorithm which uses signals from sensors mounted on the fuselage and on the blades themselves. The actuators, sensors and multiple feedback loops all rotate with the blades. Since the actuators are rotating with the blades this essentially allows broad band control of the blade dynamics, as opposed to the HHC limitation of discrete frequencies namely \((n + 1)\omega, n\omega,\) and \((n-1)\omega\). Therefore, potentially IBC is a more versatile Active Rotor System than HHC, but its implementation has some major technical difficulties.

By applying feedback techniques and controlling the blades individually, it is possible to reduce the vibration caused by higher harmonic motions of the blades, or possibly improve the performance of the rotor. The main rotor provides lift, propulsion and directional control for the helicopter, by improving the blade response to pilot flight control inputs and to flight conditions, and by reducing the effects of aerodynamic turbulence, retreating blade stall and so on, would provide a helicopter with superior manoeuvring abilities.
Ham and McKillip (1980) report that further developments may lead to such features as automatic blade tracking, and eventually to the possible elimination of the swashplate itself, leading to a truly fly by wire helicopter, though this has very serious airworthiness implications. Obviously, the elimination of the swashplate is a major conceptual and functional modification of the helicopter.

An IBC system used to control the first blade elastic flatwise bending mode is reported by Ham (1983). The system uses a servomotor to control the pitch angle of each blade. Three accelerometers mounted on each blade at different spanwise locations are used to derive the flatwise acceleration and displacement of each blade. The controller uses these signals to calculate the actuator demands, the control algorithm increases the effective inertia, damping and stiffness of the first elastic flatwise bending mode of each blade. Other IBC research has been reported by Zwicke (1980) and McKillip (1984).

2.6.1.3 Limitations of HHC

Research work has shown that HHC is an effective method for reducing helicopter vibration in steady flight conditions. However, there are many significant technical risks which must be overcome before it can be claimed that HHC is a technology mature enough for production active control systems. The major technical risks that require further research can be identified as:

1) System transient response, it is important that the system provides an adequate speed of response (of either a non-adaptive or adaptive system) to changes in the flight condition, and ensure system stability. Since the model used by almost all HHC controllers is assumed to be quasi-static, the flight condition must not change significantly over the identification period. This places restrictions on the capability of the HHC controller to respond to changing flight conditions, such as those encountered in fast manoeuvres. The stability of controllers based on linear control theory can be determined, however adaptive algorithms are inherently non-linear making the stability of adaptive controllers difficult to predict [Rohrs et al, 1983].

2) Ensuring system performance given practical engineering constraints such as, available actuator power. The hydraulic power demands of HHC systems are high since moderate vibratory blade pitching requires large control effort and consequently large hydraulic flows, to the highly loaded (steady) primary flight control actuators. Since rotor loads increase dramatically with forward speed, the performance of the HHC system is limited at high forward speeds. Seal wear on these actuators is also of considerable concern since HHC vibratory
actuator motions are continuous and usually of high frequency (e.g. 15-30Hz for a 4 bladed rotor).

3) HHC may degrade rotor performance as well as increase the rotor loads [Reichert, 1980]. This degradation may occur due to the conflicting requirements between retreating blade stall and the demand for vibration reduction. A feature of some concern is the increase in pitch at 270 degrees. Balcerak et al [1969] showed that under certain conditions the main rotor blade stalled for approximately 20 degrees of the azimuth rotation under certain conditions when a HHC system was used to control helicopter vibration. This could, near the flight envelope boundary, have the effect of introducing a premature onset of blade stall. Some experimental studies have predicted an increase in blade loads. Though they were not large, this increase would clearly have to be considered during rotor design. Therefore, this risk may be summarised as reconciling the demands of the HHC system with the degradations in rotor system performance.

4) HHC raises important airworthiness considerations. This applies to all the possible actuation options for HHC, obviously this is even more critical to the implementations that use the primary flight control actuators themselves and to IBC. By its very nature HHC impinges directly on the primary flight controls, consequently HHC control computers/system have to be designed to costly flight critical standards to provide for sufficient redundancy. In the event of a failure with the HHC system, the helicopter must be able to operate and land safely.

2.6.1.4 HHC Conclusions

HHC embodies the largest amount of research with regard to the active control of helicopter vibration to date. The research outlined above has, and in particular the flight testing of HHC control systems, conclusively demonstrated that reduced vibration levels in helicopters can be achieved through higher harmonic blade pitch control.

The use of HHC to reduce fuselage vibration has been demonstrated by Boeing Vertol during wind tunnel testing of model rotor systems [McHugh and Shaw, 1977]. Extensive HHC research has been conducted by Hughes Helicopter Inc, including analytical studies, wind tunnel tests and flight tests [Wood et al, 1983]. Several studies have been performed by Sikorsky including flight tests on an S-76A helicopter [Walsh, 1986] and on a UH-60A Black Hawk [O'Leary et al, 1980]. The study on the S-76A helicopter included an investigation of both open-loop and closed-loop HHC systems.

The UH-60A study concluded that 90% reductions in fuselage vibrations were possible with an associated weight saving of 50% when the HHC system replaced an existing passive
vibration control system of bifilars and absorbers. Analytical studies have been carried out by Westland Helicopters Limited [Hughes and Staple, 1984].

In addition to vibration reduction, other studies have shown that application of HHC has the potential for other benefits:

1- HHC has the ability to reduce blade stresses [McCloud and Kretz, 1974]. Kretz [1975] reported having achieved 40% reduction of stress and 48% reduction of vibration.

2- Also improvement in the performance of the rotor by delaying the onset of retreating blade stall is possible [Wernicke and Drees, 1963], [Kretz and Larche, 1980]. The solution consists of sensing instantaneous rotor behaviour and acting to maintain healthy aerodynamic flow over the whole of the rotor disc area.

3- And, HHC can be used for gust alleviation [Ham and Whitaker, 1978]. It is generally believed that it is not currently possible to achieve more than one of these benefits at a time, and therefore by improving rotor performance fuselage vibrations may be made worse.

While the HHC approach has been shown great promise it has yet to be widely accepted as a practical proposition for production helicopters. The main reasons for the reluctance of the helicopter Industry to accept HHC as a production system were identified in section 2.6.1.3, and require further research. The most important of these are being the constraints in terms of power requirements and high speed performance.

2.6.2 Active Isolation
One of the difficulties with isolation schemes, particularly rotor isolation, is that transient lift loads (manoeuvres, gusts etc) and static lift loads must be carried by the same load paths along which isolation is required. Passive techniques solved this using anti-resonant principles, an alternative solution is to use active control techniques. In this case the fuselage is suspended below the gearbox using hydraulic actuators, and these form the only load paths between the gearbox and fuselage.

In practice an active isolation system consists of an actuator controlled by a servo-valve and an amplifier, in parallel with a spring and damper. The spring and damper act as a reserve suspension in case of actuator failure and also take the main steady state loads.
The main problem with passive isolation systems was the conflicting requirement to maintain the static displacements between the rotor/gearbox and airframe within reasonable limits. With an active system it is possible to modify the stiffness of the actuator such that there is no resistance to motion at the blade passing frequency, i.e. the system appears to be very soft to sinusoidal excitation at the blade passing frequency and possibly to higher frequency deflections typical of rotor vibrations, but appears very stiff to the sustained acceleration during manoeuvres and landing. This can be achieved using local feedback control loops around each actuator. Control laws can also be designed which derive actuator signals from sensors located in the crew and passenger areas of the fuselage.

Figure 2.14 shows the isolation concept, the active system is obtained by replacing the pendulum arrangement of the passive system with a hydraulic actuator. The dynamic characteristics of the actuator being determined by a control system. Two feedback signals are sent to the servo-amplifier. One is the acceleration signal from an accelerometer on the fuselage, and the other is a relative displacement signal from a transducer mounted across the gearbox fuselage interface.

As for the passive devices twelve actuators are required to completely isolate a gearbox with four mounting points.

The safety issues of structurally connecting the fuselage and gearbox via hydraulic actuators must be considered [Allen and Calcaterra, 1982].

The tasks which an active isolation system has to perform can be summarised as follows:

1) Control of the airframe vibration by isolating the fuselage from rotor induced blade passing harmonics in the operating range of the rotor speed (disturbance rejection), which are responsible for passenger discomfort and component fatigue.
2) Control of the displacement across the gearbox fuselage interface due to limitations imposed by interfacing systems such as engines and controls, using a low frequency control loop to limit the static and quasi-static relative displacements in level and manoeuvre flight.

Early active isolation controllers were based on classical control theory and were designed for a particular operating condition (speed) of the helicopter. If the operating condition changed then the performance of the controller degraded [Rruzicka and Schubert, 1969].

One of the earliest concepts for a fully active rotor isolation system was developed by the Barry Wright corporation in the late 1960s and is described by Calcaterra and Schubert, 1968). A simple solution is to use a notch filter feedback of vibration output/transmitted isolator force.
or acceleration at the airframe attachment point. The low frequency displacements can be controlled using integral feedback of the isolator deflection. However, for this system to be practical it would need to incorporate automatic tracking of the rotor speed to ensure that the notches occur at the right frequencies for variations in the rotor speed.

Disturbance rejection controllers for active isolation systems have been designed using modern control theory for multi-variable systems [Strehlow et al, 1977]. Strehlow describes an active system that has been successfully implemented on a MBB BK117 helicopter. The system isolated the fuselage from the main gearbox and good results were reported. The subject of disturbance rejection using feedback control is of fundamental importance for many kinds of vibration control. Disturbance rejection using a feedback controller for the linear time invariant multivariable systems was considered by Davison (1972).

Linear Quadratic feedback techniques have also been investigated. Experimental results of a single axis, full scale laboratory research model using modern control techniques were reported by Mehlhose et al [1979].

As described previously advanced rotor systems, such as the semi-rigid rotor, increase rotor head forces and therefore increase the requirements of the rotor isolation system.

In the mid 1970s an active isolation system was developed by MBB, and was implemented on the MBB BO-105 helicopter [Schulz, 1978]. This active system was used as a force isolator and simply the electrohydraulic actuators replaced existing DAVI mounts.

The main advantages of an active isolation system can be summarised as:

1. The active isolation system can be easily adapted dynamically and optimised
2. Isolation of other frequencies are possible without further physical complications (the complexity of the controller and control laws may be increased).
3. It remains effective for variations in rotor speed, as long as a rotor speed sensor is used.
4. It provides automatic trim of quasi-static displacements across the gearbox fuselage interface.
5. Advanced servo-hydraulic technology means that active isolation systems compare well with passive systems in terms of weight.

Active isolation has many applications such as: reducing aircraft cabin-noise, engine mounts for ships, cars and trains.
The main disadvantages of active isolation systems are:

1. Not all helicopters have appropriate structures that will allow an isolation system to be installed.
2. Other load paths seriously compromise the performance of an isolation system, and the weight and complexity of an isolation system that isolated all the load paths effectively would be prohibitive.

Many of the theoretical studies present very good results, often in excess of what can be achieved practically. The main reason for this is that rigid body dynamics are often used, and clearly a mass weighing 8000Kg will respond with a much smaller amplitude to an excitation force, than will a flexible helicopter airframe weighing 8000Kg, particularly if the frequency of the excitation force coincides with the frequency of one of the dominant fuselage modes.

2.6.3 Active Control of Structural Response

This final section on active control techniques, describes a new approach to the helicopter vibration problem, termed Active Control of Structural Response (ACSR) and has been pioneered by Westland Helicopters Limited.

The principle of operation of ACSR is fundamentally different to that of Higher Harmonic Control (HHC) and Active Isolation, although the goal is the same i.e. to reduce fuselage vibration levels. HHC attempts to reduce the level of blade passing frequency forcing at the rotor hub by applying higher harmonic pitch oscillations to the blades through the rotor hub. Active Isolation attempts to control the vibratory load paths across an isolation interface. While ACSR seeks to reduce the overall vibration response of the helicopter fuselage by applying an additional set of vibratory forces at strategic points within the fuselage. It is desirable for reasons explained later that the ACSR actuators are located in the primary load paths, which is similar to isolation, however, the ACSR concept does not attempt to isolate the fuselage from the excitation.

The basic control philosophy behind ACSR is the principle superposition (refer to section 1.2), A controlled secondary excitation is applied to a structure which is being excited by a primary source, such that the response of the structure to the primary source is minimised. In the case of the helicopter the structural response is modified by secondary vibration sources which are actuators, and they input controlled forces into the helicopter fuselage such that they cancel/reduce or minimise the vibratory response of the fuselage caused by the rotor hub forces, which are the primary sources.
In practice the principle consists of connecting a number of hydraulic actuators between strategic points on the fuselage and applying control forces to the structure through these actuators with the objective of minimising fuselage vibration. The magnitude and phase of the loads generated by the actuators are calculated by an optimum control algorithm to minimise vibration measured at a number of key locations in the fuselage using accelerometers, with the system being controlled by a multivariable adaptive control algorithm. Reductions in the fuselage vibration result through superposition of the vibrations caused by the actuator control forces with the vibrations caused by the rotor head forces, thereby causing cancellation in the dominant vibratory modes. The Active Control of Structural Response (ACSR) strategy is outlined in Figure 2.20.

The ACSR strategy was pioneered by Westland helicopters in the UK, in collaboration with whom the work described in this thesis was undertaken. The ACSR principle has been successfully proven by Westland Helicopters Limited on the Westland 30 [Staple, 1989] helicopter and more recently the EH101 [Lyndon, 1991] helicopter, flight trials have demonstrated that reductions of 80-90% are possible [King and Staple, 1986]. These systems typically consist of ten accelerometers used to measure the fuselage vibration, and an adaptive control unit which uses these measurements to control vibratory forces produced by four actuators.

If only four actuators are used to control the fuselage vibration at four accelerometer positions, then in theory the vibration at these four positions can be reduced to zero. However, the vibration levels at unmonitored fuselage locations may not be reduced, and may even be worsened.

The Westland ACSR system operates in the frequency domain, and as such it controls fuselage vibration at a discrete frequency, usually the blade passing frequency \( n_{\text{co}} \). The controller uses a reference signal from the main rotor to track any changes in the rotor speed. The control law is described in detail by Staple (1989). More than one frequency can be controlled by operating a number of controllers in parallel.

Although the frequency domain control strategy has proven successful in steady flight conditions, it does not respond adequately to transient manoeuvre conditions. One of the main objectives of the research described here was to investigate different control laws which would enable a significant improvement in performance under rapidly changing conditions compared with what can be achieved with the frequency domain technique.

1 with respect to a baseline aircraft with no active vibration control
2.6.3.1 Comparison with HHC

ACSR is a powerful technique for reducing helicopter vibration, and is generally applicable to any helicopter. The competing technologies are HHC and Active Isolation, and HHC is the more versatile and mature of these two technologies. Therefore in this section a comparison is made between ACSR and HHC.

King [1988] reports that the performance of HHC depends upon whether HHC is implemented using swash-plate actuation (fixed frame) or individual blade actuation (rotating frame). Studies carried out at Westlands and reported by King show that HHC with fixed frame actuation can reduce fuselage vibration levels by 90%, and by 95% with rotating frame actuation. These results did not include any aerodynamic constraints on rotor performance. Simulation results for ACSR based on a system consisting of four actuators and ten accelerometers have shown reductions in fuselage vibration levels of 90% [Pearson and Goodall, 1991]. Therefore the performance of ACSR compares very favourably with that of HHC.

HHC research has shown that when it is used to reduce fuselage vibration it can increase blade loads and stresses, and it can reduce the performance of the rotor by increasing retreating blade stall. Since ACSR is implemented using actuators in the fuselage, it has no effect on rotor loads and stresses, or rotor performance.

As identified previously, HHC alters the pitch of the blades directly and therefore by its very nature has an impact on the primary flight control system. The HHC system needs to be designed to flight critical standards. Any failure of the HHC system must allow for the safe operation of the helicopter, and this means that there may be some redesign of the flight control system. Again, since ACSR consists of actuators across strategic points in the fuselage, which in some cases are in the primary flight load paths, minimal airworthiness issues are involved.

The main performance constraints of HHC are the power demands of the system, these are
high since moderate vibratory blade pitching requires large control effort to the highly loaded primary flight control actuators. Since rotor loads increase dramatically with forward speed, the performance of the HHC system is limited at high forward speeds.

ACSR overcomes many of these constraints. As forward speed increases so do the fuselage vibration levels and therefore an increase in actuator power is required to control these larger vibration levels, however the extra power associated with the increase in fuselage vibrations is not as large as the extra power associated with the increase in blade loads at high forward speeds.

### 2.6.3.2 Implementation Issues

The ACSR system consists of actuators, sensors and a controller, and each of these components needs to be carefully considered for a successful implementation.

If an actuator is fixed across two points of the helicopter fuselage and applies a force, the effect will be to excite all the modes of the fuselage which possess relative motion between the attachment points of the actuator. It is not practical or necessary to attempt to control all of the fuselage modes, since the blade passing frequency fuselage vibration will be dominated by a range of modes. Typically blade passing frequency is in the range 10 to 30Hz, and the fuselage response will be dominated by modes in the range from zero to twice the blade passing frequency [King and Staple, 1986]. A fundamental requirement for the successful implementation of ACSR is therefore the strategic placement of the actuators. Intuitively, and also for analytical reasons, the actuators need to be located across points in the fuselage which have relative motion in the dominant fuselage modes at the blade passing frequency.

An actuator connected between node points of a particular mode clearly will not be able to affect the response of that mode, and an actuator located close to the node points of a mode will only be able to exert a small influence, consequently large actuator forces would be required to control the mode in this case. The location of the actuators determines the controllability of the system [Friedland, 1975], and is a key issue for the successful implementation of any active control scheme. Hughes [1990] derives criteria which indicate the degree of contrability of each mode for a given actuator placement. Austin et al [1992] describes the use of Linear Quadratic design techniques for determining the optimum placement of actuators within large civil structures. Finding such locations will not necessarily be easy, unless the airframe already incorporates a significant area of flexibility [King and Staple, 1986].

A similar requirement applies to the sensors, and is related to the observability of the problem. Placing a sensor at a node of a one of the dominant modes will yield no information about that
mode, it is therefore important to have a sufficient number of sensors at suitable locations to measure the overall fuselage vibration environment. In particularly, the sensors should provide information about the modes which contribute to the vibration at the blade passing frequency. The proper selection of sensor and actuator number and locations can lead at best to improved system performance, a poor selection may result in a degradation of performance, or in the worst case may result in an unstable control scheme.

The last implementation issue concerns the controller. The main requirement for the controller, is that it is capable of using the information from the sensors to control the actuators such that it minimises the measured fuselage vibration. The control algorithm options are discussed in detail in Chapter 4.

In summary, the location of the accelerometers used to measure the fuselage vibration and the location of the actuators used to inject the control forces to the fuselage, are critical with respect to obtaining useful vibration information and keeping actuator force and displacement amplitudes at low levels. Given these practical constraints active vibration control by modification of the structural response, can in theory be applied to a wide range of structures.

2.7 CONCLUSIONS

Enormous progress has been made in reducing helicopter vibration levels, but requirements are becoming ever more stringent, demanding levels lower than can be realised by the careful design of the rotor and fuselage.

In the past the aim of the helicopter Industry was just to reduce the vibration level of the helicopter. Human factor research related to the helicopter have now produced criteria which clearly define the environment for a good ride. More importantly, technology has now reached a level where such a ride may be achievable. Consequently, the reduction of helicopter vibration is becoming increasingly important to the helicopter industry, due to increasing higher cruise speeds and improved comfort goals. Consideration must be given at the design stage to the fuselage and rotor for a successful helicopter. The blade passing frequency \( n_{\text{PB}} \), as has been identified is a fundamental form of vibration for the helicopter and cannot be eliminated by careful manufacturing processes. It can, however, be controlled. This chapter has identified three areas which must be considered when producing helicopters with minimal vibration levels:

1) optimisation of rotor dynamics
2) optimisation of fuselage dynamics
3) attenuation of the excitation forces, using either passive or active devices

The magnitude of the rotor loads play an important part in determining the magnitude of the fuselage vibration. Therefore by choosing the number of blades, type of blades, type of rotor hub and so on, the blade loads can be reduced. The design of the rotor is particularly important, since if a helicopter exhibits unacceptable vibration it may be possible to modify the fuselage structure at the development stage, whereas modifications to the structure of the rotor system are virtually impossible. However, fuselage structural modifications at the development stage are expensive and time consuming processes and should therefore be avoided.

The response of the fuselage to the excitation also depends upon the transmissibility of the load paths between the rotor and the fuselage. By designing the fuselage correctly the response to \( n \) and its harmonics can be minimised. Such considerations as these have contributed to the high forward speeds and reduced vibration levels achieved by modern helicopters.

Even with good rotor and airframe design, the vibration levels may still be undesirable. If this is the case, then other means of vibration control need to be considered, and the likely benefits weighed against the weight penalty.

Dynamic optimisation of the rotor and fuselage are necessary but not sufficient for low helicopter vibration, and is not considered to be an alternative to the use of active and passive vibration control techniques. The next generation of helicopters will only achieve the new low vibration requirements by successfully combining structural optimisation techniques with active control technologies.

In the past passive techniques have been used. However, passive systems have a number of limiting drawbacks, namely the increase in weight and the lack of adaptability. The many different passive vibration control devices available indicates the importance and complexity of the problem. Recently, considerable effort has been devoted to research into active methods for vibration reduction.

For the next generation of helicopters these techniques are the most promising methods to meet the more stringent comfort requirements while allowing the helicopter to meet its performance requirements. Active techniques include Isolation, Higher Harmonic Control, Individual Blade Control and Active Control of Structural Response.

Much of the research into active vibration control techniques has concentrated on higher harmonic control and active isolation. Research has shown that there are substantial increments
Helicopter Vibration

to be obtained from the introduction of higher harmonic control to the rotor. Since individual blade control is a generalisation of higher harmonic control, similar benefits can be expected, and preliminary investigations have confirmed this.

Most active control techniques, whether it be HHC, isolation or ACSR comprise actuators, sensors and controller. However, the fundamental principle of operation of ACSR is quite different from HHC and isolation. ACSR offers equal performance to HHC, but with lower power requirements and without the airworthiness issues associated with the implementation of HHC. Control of a range of frequencies is possible with ACSR, including the $1\omega$ frequency, however, this is undesirable since pilots monitor this frequency to indicate any degradation in the rotor tracking.

If an active technique such as ACSR could be used to control the fuselage vibration. Then HHC could be used to increase rotor performance, by delaying the occurrence of retreating blade stall. Such a combination of active techniques would yield a truly superior helicopter in terms of performance and comfort.

Some modern multiengined helicopters have the ability to vary the speed of the main rotor to optimise the rotor performance, and therefore it is important for vibration reduction devices to be able to deal with such changes by being either robust or adaptive. Active techniques are ideally suited to resolving the requirements generated by the various missions possible with advanced helicopters. They offer good vibration reduction over significant areas of the fuselage, and also provide the capability to adapt to changing rotor speed, changes in flight conditions and changes in structural dynamics (such as fuel, cargo).

The are no significant weight penalties, and possibly there may even be weight savings. Karnop [1973] identified that structures are generally over designed (they are stiffer and heavier than they need to be to satisfy vibration requirements), by using an active system the structure can be made lighter and more flexible.

The main disadvantages of active systems are that they require an external power source (including pumps, hoses, connections etc), they increase complexity and cost due to the requirement for special actuators, sensors and computer systems and they decrease reliability. Reliability and maintainability are important issues since they contribute directly to the cost effectiveness of the helicopter. However, as technology advances (microprocessor, actuation and sensor technology), active systems may eventually surpass passive systems in terms of price, weight and even reliability.
Figure 2.1 Helicopter Blade Velocities
Figure 2.2 Altering Blade Pitch Angles (The Swash Plate Mechanism)
Figure 2.3 Typical Main Rotor Blade Loadings During Forward Flight
Figure 2.4 SeaKing Main Rotor Hub - Articulated (Courtesy of Westland Helicopters Limited)

Figure 2.5 Lynx Main Rotor Hub - Semirigid (Courtesy of Westland Helicopters Limited)
Figure 2.6 SeaKing Helicopter (Courtesy of Westland Helicopters Limited)

Figure 2.7 Lynx Helicopter (Courtesy of Westland Helicopters Limited)
Figure 2.8 Higher Harmonic Blade Motions
Figure 2.9 Typical Helicopter Fuselage Spectrum

Figure 2.10 Trends in Helicopter Vibration Levels
Figure 2.11 Human Response to Vibration

Figure 2.12 Intrusion Index Normalisation Curve
Figure 2.13 Passive and Active Isolation
Figure 2.14 Conventional and Anti-resonance Isolation

CONVENTIONAL ISOLATION

Rotor Forces

\[ M_f \]

Gearbox

\[ k \]

Spring

\[ c \]

Damper

\[ M_f \]

Fuselage

ANTI-RESONANCE ISOLATION

Rotor Forces

\[ M_f \]

\[ M \]

Pendulum
Figure 2.15 DAVI Mounts
Figure 2.16 DAVI Mounts - Operation Principle
Figure 2.17 Nodal Isolation
Figure 2.18 Flapping Pendulum Operation
Figure 2.19 Higher Harmonic Control (and IBC) Block Diagram

Figure 2.20 Active Control of Structural Response Block Diagram
CONVENTIONAL ROTOR SYSTEM

LOADS TRANSMITTED TO THE FUSELAGE
AT \((n+1)w\), \(nw\) and \((n+1)w\)

ROTATING SWASH PLATE
NON-ROTATING SWASH PLATE

PILOT CONTROLS

FLIGHT CONTROL ACTUATORS
(FIXED FRAME)

ROTATOR SHAFT

HIGHER HARMONIC CONTROL

\((n+1)w\), \(nw\) and \((n+1)w\)

HIGHER HARMONIC CONTROL ACTUATORS
(FIXED FRAME)

FLIGHT CONTROL ACTUATORS
(FIXED FRAME)

CONTROLLER

NON-ROTATING SENSOR (GEARBOX, FUSELAGE)

INDIVIDUAL BLADE CONTROL

ROTATING SENSOR

1w, 2w, 3w, ...

INDIVIDUAL BLADE CONTROL ACTUATORS
(ROTATING FRAME)

FLIGHT CONTROL ACTUATORS
(FIXED FRAME)

CONTROLLER

NON-ROTATING SENSOR (GEARBOX, FUSELAGE)

Figure 2.21 HHC and IBC Active Rotor Systems
CHAPTER 3

MATHEMATICAL MODELLING

INTRODUCTION
Mathematical models are necessary to simulate or predict the response of the helicopter fuselage to assumed external forces. They are also required to design and evaluate the controller parameters for vibration control systems.

The helicopter fuselage is a complex flexible structure which can only be successfully represented by a large number of flexible modes using finite element methods, and therefore the chapter begins with a brief description of finite element techniques.

Chapter 2 described the vibration problem and introduced the Active Control of Structural Response (ACSR) technique, which is investigated in this thesis. The ACSR system consists of actuators, sensors and a controller, as shown in Figure 3.1. Actuator dynamics greatly influence the performance of any Active Vibration Control technique, consequently actuator and sensor models are also developed and described in this chapter.

3.1 MODELLING

Eykhoff (1974) defined a model as:-

"a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in a usable form."

Most systems can be represented by a mathematical model, which can often be considered as a collection of smaller models representing sub-systems. When developing models it is important that they are complex or comprehensive enough to represent adequately the characteristics of the system being studied. However, model simplicity is also desirable, since simpler models have fewer parameters to be determined or identified and are therefore easier to understand.

The airframe is composed of several main components, namely the fuselage, tail boom, vertical fin, landing gear, main rotor pylon, main rotor shaft and so on. The vibration at any particular point on a helicopter depends on the response of the fuselage to dynamic forces as described in chapter 2.
When considering the fuselage vibration levels it is not satisfactory to consider the fuselage as a lumped mass, because the flexibility of the fuselage must also be modelled. A body with several flexible modes near 20Hz may respond with an amplitude several times that to which a rigid body with the same mass, subject to the same force at 20Hz will respond. As with any structure responding to vibratory forces, it is important to model the natural frequencies and modes. The helicopter fuselage has a significant degree of structural complexity and can be considered as a system with many degrees of freedom. Clearly, the fuselage has many modes, each with its own natural frequency and mode shape. The model needs to represent the flexibility of the fuselage accurately. A reliable mathematical model of the structure is a necessity in order to attempt to solve a vibration problem on any system, and therefore experimental validation of the model is important.

The complete system is represented in Figure 3.1; The helicopter fuselage is a complex dynamic system. Its inputs are both the periodic disturbance forces from the rotor and a set of control forces from the actuators. The outputs are anything that the sensors measure, typically fuselage accelerations, but velocities or displacements are also possible.

Once the problem has been defined the next stage in any dynamic system analysis or control system design is to identify the dynamic properties of the system or structure. The representation of the airframe as a superposition of natural modes has been the classical method used in fuselage analysis and design studies. The control problem, described in the previous chapter, is to derive a control law which drives the actuators to minimise the set of measured vibrations, and this is considered in detail in the next chapter. Two models are required, one to design the control systems described in Chapter 4, and a model for the simulation studies of the complete system described in chapter 5.

An analytical model can be formulated from the fuselage’s material properties, geometry and boundary conditions, which describes the dynamic properties of the fuselage in terms of mass, damping and stiffness distribution. Equations of motion generated from the model are solved to determine the fuselage’s response to any externally applied forces. Generally, the equations are formulated such that the response motions of the fuselage are functions of time, and therefore the description of the fuselage’s mass, damping and stiffness distribution is called a time domain model. Transforming the equations into a system of first order equations results in the well known state space formulation, and in this case the model is known as a state space model.
A time domain model can also be formulated in modal form, which defines each of the fuselage's component modes of vibration. Each mode is represented by a coupled second order differential equation and is defined by four parameters (modal parameters); an undamped natural frequency, a measure of energy dissipation or damping, a characteristic deflection shape (mode shape), and a measure of inertial scaling (modal mass).

Determining the modal model of a fuselage is known as modal analysis, and can be considered as a procedure for identifying the resonant frequencies of the fuselage and the associated deflected shapes when the fuselage is excited at one of these frequencies.

Basically there are two techniques whereby the modal model of a fuselage can be obtained, mathematical (analytical) modelling and modal (dynamic) testing.

The modal model and the time domain model are equivalent descriptions of a fuselage's dynamic properties. An alternative model of the fuselage, which describes the steady state frequency response of the fuselage, can be developed from either the time domain or modal model. These models are described in detail later in this chapter.

The major system components are:
1- the dynamic characteristics of the helicopter fuselage
2- servo actuator dynamics and feedback control
3- active controller
4- filter characteristics
5- sensor dynamics

The following sections develop a mathematical model for a generalised dynamic system, including the effects of rotor hub disturbance and actuator forces, and these are translated into both frequency domain and state-space formulations for subsequent use in developing control laws.

3.2 MODES

A structural mode is a unique deformation that the structure assumes when excited only in that mode of vibration (i.e. at a certain frequency and with certain excitation forces). It is not easily observable and is defined by abstract mathematical parameters (Ewins, 1991). A mode shape is represented mathematically by a vector, known as a modal vector. Each element of the modal vector represents the deflection of one degree of freedom (dof) in the structure (Figure
Mathematical Modelling

3.2), therefore a given mode shape can be constructed by superimposing the deflections of the corresponding modal vector onto the undeformed structure.

For the $j^{th}$ mode the eigenvector is given by

$$\Phi_j = [\phi_{1j}, \phi_{2j}, \phi_{3j}, \ldots, \phi_{nj}]^T \quad (3.1)$$

where $\phi_{ij}$ is the deflection of the modal mass of the $i^{th}$ element for the $j^{th}$ mode.

Modal vectors show only the relative motion associated with each of the structure's modes. The actual total deformation of the structure resulting from external excitation is dependent not only on the magnitude and location of the external forces but also on the contribution of each mode in the structure's total response.

Each mode has associated with it an eigenvalue which corresponds to the modal frequency, an eigenvector which corresponds to the mode shape, and a damping factor. Damping in solids and structures is not as well understood as stiffness and mass properties. Damping can be proportional or non-proportional, proportional damping can be further divided into Rayleigh Damping \cite{Rayleigh_1945}, Relative Damping, Absolute Damping and Modal Damping.

3.2.1 Proportional Damping

Proportionally damped systems are those where the energy dissipation mechanisms or damping are distributed proportional to either the mass or stiffness. Structures with proportional damping have real (or normal) mode shapes, and this means that when a mode (real) is excited, all the points on the structure vibrate at the same frequency and reach their maximum displacements and pass through their equilibrium positions at exactly the same time. More precisely, all the points on the structure are moving in phase and only their relative displacements are different. Normal modes can be considered to be standing waves with fixed node points. Typically, when a structure has very light damping it exhibits normal modes.

3.2.2 Non-Proportional Damping

Structures where the damping is not distributed proportional to either the mass or stiffness are called non-proportionally damped systems. Non-proportional damped systems have complex mode shapes. When a structure is oscillating in a complex mode, all the points on the structure vibrate at the same frequency but they do not reach their maximum or pass through their equilibrium positions at the same time. Complex mode shapes can be considered as propagating waves with no stationary node points.
3.3 LUMPED PARAMETER MODELLING

Although all structures are continuous, in some cases their dynamics can be represented adequately by assuming that they behave as an assembly of rigid masses connected together by combinations of spring and damper elements. The equations of motion for these lumped parameter models can be readily derived by applying Newton's Second Law to each rigid mass. This will generate one equation for each degree of freedom of each mass in the system. Combining all the equations for each of the masses yields the equations of motion for the structure, which are often written in matrix form as:

\[ \begin{bmatrix} M \end{bmatrix} \ddot{x} + \begin{bmatrix} C \end{bmatrix} \dot{x} + \begin{bmatrix} K \end{bmatrix} x = \begin{bmatrix} H \end{bmatrix} E \] (3.2)

\([M]\) is called the mass matrix and contains all the inertial properties of the lumped parameter system (masses and dynamics coupling terms).
\([C]\) is called the damping matrix and defines the dissipative forces due to the damping elements.
\([K]\) is the stiffness matrix and defines the restoring forces and static coupling terms due to the stiffness elements in the lumped parameter model.
\(x\) is a vector of displacements, and contains all the degrees of freedom of the model as functions of time.
\(E\) is the force vector containing all the external forces acting at the various degrees of freedom in the model as functions of time.
\([H]\) is a matrix which defines the coupling of the external forces onto the structural degrees of freedom.

3.4 FINITE ELEMENT MODELLING

The helicopter fuselage is a very complicated structure with distributed mass and stiffness. The overall geometry of which cannot be described by a single mathematical expression. The material properties of the fuselage also vary. To calculate the response of the fuselage to forces generated by both the rotor and by an active control system, it is necessary to know the modal frequencies and mode shapes of the fuselage. Unfortunately, due to the complexity of the helicopter fuselage, these frequencies and mode shape are difficult to calculate accurately. The problem cannot be solved using lumped parameter techniques, since they do
not adequately represent the distributed elastic behaviour of the fuselage. The helicopter fuselage is therefore modelled using numerical methods that provide approximate but acceptable solutions. Consequently, the generation of helicopter models is usually achieved using the principle of finite elements, which is one of the most powerful and popular mathematical modelling techniques. The Finite Element models described in this thesis have been generated using the NASTRAN (NASA Structural Analysis) package to estimate the natural frequencies and mode shapes of the helicopter. Finite element analysis of the helicopter fuselage is standard practice for today's major helicopter manufacturers.

The fundamental concepts behind Finite Element techniques have originated independently in the fields of physics, mathematics and engineering. Hrenikoff (1941), in the field of engineering, described the idea that the elastic behaviour of a physically continuous plate would be similar, under certain loading conditions, to a framework of physically separate one dimensional rods and beams, connected together at discrete points. In 1956 Turner et al advanced the framework concept a step further by modelling the odd shaped wing panels of high speed aircraft as an assembly of smaller panels of simple triangular shape. This was a conceptual breakthrough because it made it possible to model structures as an assembly of similar 2 or 3 dimensional pieces, rather than 1 dimensional rods. The term finite elements first appeared in Clough's paper in 1960.

The eventual output of the finite element method is a mathematical model of the structure which can be expressed in the same form as those obtained from a lumped parameter model (equation (3.2)), although the mass, damping and stiffness matrices do not contain values which are readily associated with lumped regions of the structure. Rather, these matrices contain coefficients necessary to satisfy energy balances between the various finite elements.

There are several stages in generating the final FE model. The first step is to enter the geometrical data of the fuselage (domain), this includes mesh generation information, the physical properties of the fuselage, loads, outputs, data that defines the geometric shape of the fuselage and its material properties. This is achieved by interfacing with a CAD program to download the geometric information from aircraft design drawings. Next the fuselage is divided into substructures, based on the CAD drawings. For each of the substructures a mesh is generated, which is refined, based upon engineering experience and expertise.

Finite element models subject to dynamic loads can become extremely difficult to analyze
rigorously in their original physical coordinates. These difficulties can be avoided by using the natural modes of vibration as generalised coordinates. When this path is followed the equations of undamped motion become uncoupled. In these coordinates each equation may be solved as if it pertained to a system with only one degree of freedom.

Superposition of these single degrees of freedom results is accomplished through a transformation back to the original coordinates. By this means it is possible to evaluate time varying nodal displacements, internal stresses, and support reactions for the analytical model. An important advantage is that only the significant modal responses need be included in a dynamic analysis. The other modal responses may often be omitted without much loss of accuracy. This technique is known as modal truncation.

The result of the first stage of the finite element analysis is a set of dynamical equations of the form:

\[
[M] \ddot{x} + [K] x = 0
\]

(3.3)

where \( M \) is a symmetric mass matrix, and \( K \) is a symmetric stiffness matrix. The NASTRAN program compiles the natural frequencies \((\omega_m, s)\), a generalised stiffness matrix \([K']\), a diagonal generalised mass matrix \([M']\), and the generalised eigenvector matrices \([\phi]\), such that:

\[
[\phi^T] [K] [\phi] = [K']
\]

(3.4)

\[
[\phi^T] [M] [\phi] = [M']
\]

(3.5)

\([M']\) and \([K']\) are also known as the principal mass and stiffness matrices.

In order to determine the modal frequencies \(\omega_m\) and the mode shapes \(\Phi_m\) of free vibration of a structure, it is necessary to solve the linear eigenproblem:

\[
[\lambda_m^2 [M] - [K] \Phi_m = 0
\]

(3.6)

where \( m \) is the mode number, the eigenvalue of the \( m^{th} \) mode \(\lambda_m\) (\( = j\omega_m\)), \( \omega_m \) is the frequency of the \( m^{th} \) mode, and \(\Phi_m\) is the mode shape vector of the \( m^{th} \) mode.

The effects of external forces applied at various locations on the fuselage can be included by modifying equation (3.3) to give
where \([H_r]\) is a force distribution matrix, and \(F_g\) is a vector of external forces (in the x, y and z directions in a body fixed coordinate system) and moments (about the x, y and z axes) due to the rotor, and also actuator forces.

Using equations (3.4) and (3.5), equation (3.7) can be written in modal form

\[
[M']\ddot{n} + [K']n = [\Phi^T] [H_r]E_g
\]  

(3.8)

where \(n\) are generalised displacements, known as principal coordinates, and \(x = [\Phi] n\). Since \([M']\) is a positive diagonal matrix, its inverse is well defined and the differential equation can be written as

\[
\ddot{n} + [M']^{-1} [K']n = [M']^{-1} [\Phi^T] [H_r]E_g
\]  

(3.9)

where

\[
[M']^{-1} [K'] = \text{diag}(\omega_1^2, \omega_2^2, \omega_3^2, \ldots, \omega_n^2)
\]  

(3.10)

is a diagonal matrix of the natural frequencies.

The measurement distribution matrix \([M_D]\) specifies the contribution of each degree of freedom to the accelerometer outputs \(y\).

\[
y = [M_D] \ddot{x} = [M_D] [\phi] \ddot{n}
\]  

(3.11)

One weakness of analytical modelling methods is that the damping distribution in a structure (i.e. the damping matrix) is not specifically determined. A commonly used approach, particularly in finite element modelling, is to assume that the structure is proportionally damped. Modal damping is usually estimated or assigned arbitrarily, and a (nominal) 2% of critical is often used. Values between 2% and 5% have been found from many vibration tests, to be typical for helicopter structures at low frequencies, although often there is higher damping in the higher frequency modes. The modal equations were augmented with a damping term as follows:

\[
C'_{i,t} = 2 \zeta_i \omega_i
\]  

(3.12)

and defining
to be a generalised damping matrix. Equation (3.7) then becomes

$$\dot{\phi} \mathcal{C} \phi = \mathcal{K} \phi$$

(3.13)

The NASTRAN finite element modelling program was used to calculate models of the EH101 and W30 helicopters. For the EH101 99 modes were modelled, and for the W30 24 modes were modelled. For the vibrational modes included, natural frequencies, generalised stiffnesses, generalised masses and mode shapes (generalised eigenvectors) at various locations throughout the fuselage (sensor locations and node points) were calculated. The models are included in Appendix II.

3.4.1 Modal Model

The time simulations are based on a decoupled modal model of the fuselage dynamics. The structural dynamics are derived from the truncated finite element models described in the previous section. The modal model is defined by expressing equation (3.14) in modal form giving

$$\ddot{\Phi} + [M']^{-1} \mathcal{C} \Phi + [M']^{-1} \mathcal{K} \phi = [M']^{-1} [\Phi^T] [H_p] E_e$$

(3.15)

defining

$$[A22] = - [M']^{-1} \mathcal{C} = \text{diag} \{-2 \zeta_i \omega_i\}$$
$$[A21] = - [M']^{-1} \mathcal{K} = \text{diag} \{-\omega_i^2\}$$

(3.16)

and partitioning the vector of forces and the distribution matrix $[M']^{-1} [\Phi^T] [H]$ into sub-matrices for the rotor forces and moments, and for the actuator control forces, yields the following equation

$$\ddot{\Phi} - [A22] \dot{\Phi} - [A21] \Phi = [D] f + [C] u$$

(3.17)

$u = [u_1, u_2, \ldots, u_n]^T$ denotes the control inputs to the force actuators, and $[C]$ is the actuator influence matrix. The rotor head forces acting on the fuselage are denoted by $f$ and the corresponding influence matrix is $[D]$. Both $[C]$ and $[D]$ are matrices of modal coefficients generated by NASTRAN (translational and rotational eigenvectors). The model is based on NASTRAN generated parameters, and a modal damping of 2% critical is assumed. This level of damping will give rise to transient decay times in excess of 4 seconds.
Equation (3.11) is modified and gives the structural vibration at the points of interest in the structure by a linear transformation of the modal accelerations:

\[ \mathbf{y} = [\Omega] \mathbf{\ddot{u}} \]  

(3.18)

where \( \mathbf{y} = [y_1, y_2, \ldots, y_s]^T \) is the vector of structural vibrations (accelerations) and \([\Omega]\) defines the coupling from the modes to the measurement points.

For an individual mode equations (3.17) and (3.18) can be expressed as

\[ \ddot{\mathbf{\eta}}_m(t) + 2\zeta_m \omega_m \dot{\mathbf{\eta}}_m(t) + \omega_m^2 \mathbf{\eta}_m(t) = \sum_{q=1}^6 D_{m,q} f_q(t) + \sum_{c=1} d C_{m,c} \mathbf{u}_c(t) \]  

(3.19)

where \( \mathbf{\eta}_m, \zeta_m, \text{ and } \omega_m \) are the respective modal displacement, relative damping and natural frequency of the \( m^{th} \) mode of vibration. The resulting fuselage vibration at sensor \( b \) is given by a linear combination of the modal accelerations:

\[ y_b(t) = \sum_{m=1}^d [\Omega]_{mb} \mathbf{\ddot{\eta}}_m(t) \]  

(3.20)

for \( m = 1 \) to \( d \) (the number of modes), where \( y_b \) is the fuselage vibration measured at sensor \( b \) and \([\Omega]\) is a transfer matrix, comprising NASTRAN generated coefficients.

Using modal analysis techniques results in a model with approximately 40 modes (modes with frequencies up to twice the blade passing frequency - \( 2n\omega \) are included). This gives a good representation of the helicopter fuselage dynamics.

### 3.4.2 State Space Model

For time domain strategies a state space representation of the structural response is used. Since the aim is to minimise the vibrations measured at sensors, it is necessary to transform from the modal vibrations to vibrations measured at sensors via the transformation given in (3.18). By manipulating equations (3.17) and (3.18) the following state space representation is formulated:
where $x_s$ is a vector of displacements and velocities for each vibration mode, and the matrices are given by:

$$[A_s] = \begin{bmatrix} 0 & I \\ \{A21\} & \{A22\} \end{bmatrix}$$

(3.23)

$$[B_{sf}] = \begin{bmatrix} 0 \\ [D] \end{bmatrix} \quad [B_{sv}] = \begin{bmatrix} 0 \\ [C] \end{bmatrix}$$

(3.24)

$$[C_s] = \begin{bmatrix} \Omega \{A21\} \\ \Omega \{A22\} \end{bmatrix}$$

(3.25)

$$[D_{sf}] = [D] \quad [D_{sv}] = [C]$$

(3.26)

3.4.3 Quasi-static Model

In the frequency domain a linear quasi-static model of the structural response can be derived, this being the steady state frequency response of the above equations. The basic assumption used is that a linear relationship exists between the control inputs and the vibration outputs of the form:

$$y = [T] u + B$$

(3.27)

where:

$$[T] = [M] [A]^{-1} [C]$$

(3.28)

$$B = [M] [A]^{-1} [D] E$$

(3.29)
Mathematical Modelling

\[
[A] = \left\{ [I] + \frac{J}{n\omega} [A22] + \frac{1}{(n\omega)^2} [A21] \right\} \tag{3.30}
\]

[T] is the system receptance matrix (or rotor transfer matrix) which characterises the structure, in which element \(T(i,j)\) represents the vibratory output \(Y(i)\) caused by a unit input \(U(j)\) at the forcing frequency, in this case written as the blade-passing frequency \(n\omega\). The residual vibration vector \(B\) is a vector of Fourier sine and cosine coefficients of the vibration component representing the background (uncontrolled) vibration. \(Y\) is the measured vibration vector, comprising the \(n\omega\) sine and cosine Fourier vibration components at the accelerometer positions throughout the helicopter fuselage. The actuator force input vector \(U\) is represented by the sine and cosine Fourier \(n\omega\) force components.

This linear quasi-static model, referred to as the \(T\) matrix, is quite simply a linear static model of the input-output relationship expressing the observed accelerations \(Y\) as a constant linear combination of the inputs \(U\). Clearly, this is not a valid model over the entire frequency range of the helicopter's operation, but it can be used to express accurately the input/output characteristics at a single frequency.

Equation (3.27) is termed the global model representation and needs to be linear over the entire range of control. A local model can be used which is a linearisation of the response about the current operating point can be defined by considering successive controller cycles:

\[
\begin{align*}
Y_{n+1} &= [T]U_{n+1} + B \\
Y_n &= [T]U_n + B
\end{align*}
\tag{3.31}
\]

The local model becomes

\[
\Delta Y = [T] \Delta U \tag{3.32}
\]

where \(\Delta Y\) and \(\Delta U\) are the differential changes of the \(Y\) and \(U\) vectors respectively per control cycle. The local linearisation assumes that the background vibration \(B\) does not change, although if it does, its effect will be picked up in the measurement of \(Y\).

The convention of representing the a helicopter by a constant control response matrix \([T]\), that relates the Fourier coefficients of specific harmonics of the control input to the same harmonics of the output (measured vibration), has been adopted by a number of authors in the field of HHC research (Shaw et al 1985, McCloud 1980, Wood and Powers 1980, and Molusis et al 1981).
3.4.3.1 Relationship with State Space Models

Using equations (3.21) and (3.22), taking Laplace transforms and comparing with the quasi-static model in equation (3.27) can be used to derive equivalent expressions using the state space matrices. The equivalent expression for the $[T]$ matrix, i.e. the transfer function between sensor outputs and actuator input forces, is

$$ [T] = ([C_3] (sI - [A_3])^{-1} [B_{su}] + [D_{su}]) $$  \hspace{1cm} (3.33)

the expression for the $B$ vector is

$$ B(s) = ([C_3] (sI - [A_3])^{-1} [B_{sf}] + [D_{sf}]) E(s) $$  \hspace{1cm} (3.34)

where $B(s)/E(s)$ is the transfer function between sensor outputs and rotor head forces, for $s = jn\omega$

$$ [T_\omega] = ([C_3][jn \omega I - [A_3]]^{-1} [B_{su}] + [D_{su}]) $$  \hspace{1cm} (3.35)

and the background vibration vector $B_\omega$ in terms of the rotor head forces at $n\omega$

$$ B_\omega = ([C_3][jn \omega I - [A_3]]^{-1} [B_{sf}] + [D_{sf}]) E(jn \omega) $$  \hspace{1cm} (3.36)

3.4.4 Mode Selection

The structural models required to represent a helicopter fuselage can be very complex. The original finite element model, which consisted of several thousands or even tens of thousands of physical degrees of freedom, has already undergone several stages of reduction. The technique of component mode synthesis using super-elements (or substructures) was the first stage in model reduction, following which modal analysis was performed resulting in approximately 40 modes being identified. In order to reduce the computational load, only a subset of the modes which significantly contribute to the overall fuselage vibration was chosen for inclusion in the controller implementation, and this subset will probably contain 5 to 10 modes. A slightly larger number of modes were selected for the simulation model, and this approach allows the for investigation of spillover effects as identified by Balas [1976].

Assuming that the original FE model is an accurate representation of the physical structure, the challenge is then to chose a reduced order model which represents the FEM as closely as possible.
Not all of the modes generated contribute substantially to the overall fuselage vibration measured at the accelerometers in the frequency range of interest. The aim is to select modes which contribute strongly to specific input and output locations in the desired frequency range. Clearly the choice and number of these locations has a significant impact on the selection of the modes required in order to calculate the final reduced order model.

To select the modes with the greatest effect on fuselage vibration for frequencies around the blade passing frequency \( n_\omega \), the amplitude of the response of each mode measured at the accelerometers was estimated. The helicopter model in modal form is given by,

\[
\ddot{\mathbf{u}} - [A^{22}] \dot{\mathbf{u}} - [A^{21}] \mathbf{u} = [D] \mathbf{f} + [C] \mathbf{u} \tag{3.37}
\]

Taking the Laplace transform of equation (3.37) and assuming zero initial conditions:

\[
s^2 \mathbf{\ddot{u}}(s) - [A^{22}] s \mathbf{\dot{u}}(s) - [A^{21}] \mathbf{u}(s) = [C] \mathbf{U}(s) + [D] \mathbf{E}(s) \tag{3.38}
\]

\[
s^2 \mathbf{\ddot{u}}(s) \left[ I_{\text{hub}} - \frac{[A^{22}]}{s} - \frac{[A^{21}]}{s^2} \right] = [C] \mathbf{U}(s) + [D] \mathbf{E}(s) \tag{3.39}
\]

If the actuator input is assumed to be zero, the modal responses can be estimated by assuming that the vibrational forces in all six directions (three forces and three moments) are of equal magnitude and phase at \( n_\omega \). For the five bladed EH101 helicopter, \( n_\omega = 110 \) rad/s, \( n \) is the number of blades and \( \omega \) is the rotor angular velocity. Therefore from equation (3.39):

\[
s^2 \mathbf{\ddot{u}}(s) = \mathbf{\ddot{u}}(s) = \frac{[D] \mathbf{E}(s)}{I_{\text{hub}} - \frac{[A^{22}]}{s} - \frac{[A^{21}]}{s^2}} \tag{3.40}
\]

Then the amount of excitation of each mode to the six forces and moments can be estimated by the sum of the modal responses at the rotor hub. Substituting \( s = j(n_\omega) \) in equation (3.40) and solving for mode number \( m \):

\[
\tilde{u}_m(jn_\omega) = \sqrt{\frac{|[D]_m F(jn_\omega)|}{\left(\frac{\omega_m^2}{(n_\omega)^2} - 1\right)^2 + \left(\frac{2\omega_m\omega_n}{n_\omega}\right)^2}} \tag{3.41}
\]

The numerator includes information corresponding to the flight condition for the six degrees of
freedom at the rotor hub. The steady state response can then be easily calculated using the corresponding modal amplitude at the accelerometers. Modal amplitude of mode \( m \) at sensor \( b \) is given by:

\[
v_{m,b} = [M]_{m,b} \ddot{v}_m
\]

(3.42)

The cumulative effect of the \( m \)th mode on the \( k \) sensors is

\[
v_{s,m} = \sum_{q=1}^{k} v_{m,q}
\]

(3.43)

Table 3.1 lists ten fuselage modes used in the helicopter model, and ranks them using the strategies described previously. The table shows that modal amplitudes vary with changing flight condition.

3.4.5 Model Verification

Validation refers to establishing the range and accuracy of an analytical model for predicting the behaviour of a dynamic system in response to external disturbances. The verification of analytical models is important to establish confidence in the responses predicted by the models, since they are only as good as their physically verifiable results.

Most finite element models are fairly large and contain many degrees of freedom. Consequently it is possible that small modelling errors can escape undetected and the accuracy of the results will be degraded.

Discrepancies between the dynamic response of the structure and that of the finite element model can occur for a number of reasons. Errors can occur when entering the model parameters, for example node point coordinates, material properties and so on. More fundamental errors can occur because the element displacement functions do not approximate the physical situation accurately enough. For instance, the model may be much stiffer than the structure due to the use of too coarse a mesh (i.e. an inadequate number of elements) or unrealistic model boundary conditions.

Another limitation is that simplified functions are chosen to represent conventional finite elements. In the vicinity of the corners of a cutout in a structure for example, there is a sharply varying stress intensity. Thus, considerable refinement of the grid is needed to describe this variation when an understanding of it is essential to design.
Modal parameters obtained by dynamic testing (shake tests) of pre-production or prototype structures can be used to verify and/or refine the analytical model (i.e. a finite element model), such methods are described by Loewy [1984], and Stoppel and Degener [1982]. The analytically predicted natural frequencies and damping can be compared directly to those measured. Similarly, the eigenvectors from the analytical model can be compared to the measured mode shapes. There is normally a good correlation between model predictions and test results for fuselage natural frequencies. The NASTRAN finite element models used for this study have been validated against the aircraft, using such a process.

3.5 MODELS

In order to study the algorithms for the active control of helicopter vibration, three models have been used for simulation and control design studies.

3.5.1 Three Mass Model
The first is an idealised three mass model with two actuators and three measurement points, details of which are shown in Figure 3.3. The system parameters are selected such that the damping and principal modes are representative of those found in typical helicopter structures, i.e. modal frequencies in the region of 18Hz and the transient response being approximately 2% critically damped. The model is based on a lumped parameter system. The model neglects nonlinearities and actuator dynamics. The advantage of developing the control strategies on this idealised model is that it allows some of the fundamental characteristics of the control strategies to be identified. It also enabled some baseline performance figures for the different strategies to be calculated and compared. Once the controller has been developed on this model, it is a relatively simple task to extend it to the helicopter model.

3.5.2 Helicopter Models
The other models are more complex helicopter models. These models were derived from full finite element structural analyses, of the EH101 and W30 helicopters, and consequently contained a large number of modal degrees of freedom. Damping parameters are specified to give modal damping factors which are representative of those found in typical helicopter structures. However, not all modes contribute substantially to the fuselage vibration measured at the accelerometers. Therefore, a subset of the modes which significantly contributed to the
overall fuselage vibration was chosen. Modes were selected by ranking them in terms of their effect on fuselage vibration near the blade passing frequency \( n \). Typically the control is based on the minimisation of vibration at ten positions in the fuselage using four actuators, although a reduced order system based on four measurement points and two actuators was also used to compare with the results from the rig tests. A finite element model of a helicopter is shown in Figure 3.4. A list of modes for the two helicopter models is provided in tables 3.2 and 3.3.

3.5.3 Actuator Models

There has been considerable research into various control laws to reduce the vibrations of large, flexible structures. These control laws have generally been based solely on the dynamic properties of the structure to be controlled. More recently, the importance of actuator (and sensor) dynamics has been recognised in the literature. Gaughey and Goh [1983] have shown that actuator dynamics, if not properly treated, may cause an otherwise stable system to become unstable. Other research has shown that neglecting actuator dynamics can lead to an over optimistic prediction of controller performance [Goodall et al, 1993].

Actuators have significant lags and attenuations at required control frequencies and must be considered in any practical control system design. Unmodelled actuator dynamics would therefore result in different magnitude and phase relationships between the inputs and outputs of the simulation model.

Current design practice has investigated the use of actuators at discrete points to control the vibrations of distributed elastic systems. These systems are continuous, and therefore in theory possess an infinite number of degrees of freedom. Most control schemes require the system model to be truncated to a finite number of discrete modes. Balas [1978] identified the difficulty in choosing the number of modes to represent the physical system as well as reconciling the location of the sensors and actuators. In the same paper Balas also describes the phenomena of control and observation spillover.

Actuators can act between two points or at a single point in the structure. Single point actuators are small self contained systems that may be used in a wide range of control applications, an example of which is described by Zimmerman et al [1984]. It is not attached to ground allowing it to be placed nearly anywhere in the structure. The actuator has the disadvantage of requiring a mass to provide an inertia to work against. New actuator technologies, for example piezoelectric film actuators, intuitively seem ideal for
vibration control applications. Burke and Hubbard (1988) describe the application of piezoelectric film actuator to the active control of a simple beam. However, these technologies are still maturing and currently do not provide sufficient force levels for the control of helicopter vibration.

Another important development in actuator technology is the integration of electronics on the actuator. This may lead with time to actuators with more intelligent properties (Isermann and Raub, 1993).

The actuators used for the ACSR system are electro-hydraulic. Hydraulic actuators are currently favoured by the aerospace industry. The main reasons for this are their high power to weight ratio and compactness, when compared with electromechanical and electromagnetic actuators (Clark, 1969). The main disadvantage is that they require a hydraulic supply. These factors ideally suit the helicopter, since compactness and weight are crucially important, and also the helicopter has several hydraulic power supplies used to drive a variety of devices, such as the primary flight control actuators.

The dynamics of the overall control system are complex and non-linear, they include sensor dynamics, actuator dynamics and any signal conditioning filter dynamics. Hydraulic actuators have the a significant effect and they are a major limiting factor when considering system performance. During normal operation, the parameters of the actuating system may vary widely.

In this application the actuators are used to their full capacity and consequently the pressure/flow characteristic is non-linear with a square root profile. If they are to be used successfully then this inherent non-linearity has to be considered and modelled in the control system design.

Time domain methods are based on linear plant models, i.e. a sum of second order structural modes which are typically formulated into a set of first order equations for control system design. Consequently only linear actuator dynamics can be included, often with unmeasurable states. Therefore, a linearised actuator model is also derived in this chapter.

3.5.3.1 Non-linear Actuator Equations

The flow and pressure to the actuator are provided from a constant hydraulic supply pressure $P_s$, controlled by a servo-valve. The servo-valve controls the oil flow into the actuator in proportion to the electrical input current, and the actuator attempts to extend or shorten as required against the load.
The flow/pressure characteristic for an ideal servo-valve is given by

\[ q(t) = K_v i(t) \sqrt{P_s - P_L(t)} \]  \hspace{1cm} (3.44)

Clearly, for a constant load pressure \( P_L \), the actuator flow \( q(t) \) is linearly directly proportional to the servo-valve current \( i(t) \). Also for a constant servo-valve current, the flow is related to the load pressure in a square-root characteristic.

When the load pressure is equal to the supply pressure, the actuator flow will be zero regardless of the servo-valve current (i.e. the load is stalled). For a real actuator this is not the case as there will be some flow due to spool leakage effects. Such effects along with servo-valve null characteristics are important for position control actuators [Neal, 1974], however in this application they can be neglected. The relationship between servovalve flow, current, and load pressure is shown in figure 3.5, for a 40l/min servo-valve operating with a supply pressure of 210 bar, which are the particular valves used for the ACSR system.

Servovalves are highly complex devices that have nonlinear responses [Neal, 1974 and Thayer, 1965]. The dynamic response of a servo-valve varies with operating conditions such as supply pressure, input signal level, valve loading, hydraulic fluid temperature etc. These effects are insignificant for small variations about design points, but need to be considered where wide excursions are present, such as for the ACSR system. Thayer [1965] identified that many operational and environmental variables produce significant differences in the actual dynamic response making it difficult to assume explicit transfer functions for electro-hydraulic servovalves. Most of the sources and effects of non-linearities inherent in electrohydraulic servovalves are well documented in the literature [Arafa and Rizk, 1987].

For small signal levels the system behaviour is especially influenced by the nonlinearities of the servovalve. For larger signals the flow characteristic of the servovalve is the dominant nonlinearity.

A first or second order transfer function is often a good approximation to represent the servovalve dynamics, since the servovalve itself is not the primary dynamic element in the system. The helicopter fuselage dynamics are modelled between approximately 1 and 40Hz, and therefore a second order transfer function for the servovalve dynamics is adequate [Thayer, 1965]. Typically the natural frequency of the servovalve is in the region of 100Hz.
The compressibility of the oil and the corresponding resonance with the mass of the load must be carefully considered. Hydraulic oil compliance is expressed in terms of the fluid bulk modulus which is the change in pressure required to cause a unit change in volume of the oil. The oil compliance results in a loss of stiffness in the actuator caused by compression of oil both within the actuator and between the servovalve and the actuator.

Neal [1974] describes many of the features associated with electrohydraulic actuators, such as oil column resonance, and these are included in the block diagram model shown in Figure 3.6. Other important characteristic which require careful consideration and accurate modelling are the load dynamics and their interaction with the actuator.

The nonlinear actuator model is developed by considering each side of the actuator piston separately, Figure 3.7. For a piston displacement $D_A$, the change in volume on either side of the piston is given by $V_1$ and $V_2$.

\[
V_1 = V_0 + A \cdot D_A \\
V_2 = V_0 - A \cdot D_A
\]  

(3.45)

where $A$ is the actuator piston cross sectional area and $V_0$ is the nominal volume of the actuator on either side of the piston when the piston is central.

The pressure on either side of the piston is given by the integral of the difference between the flow rate into the piston and the flow due to the velocity of the piston.

\[
P_1 = \int \frac{\beta}{V_1} (Q_1 - A \cdot V_A) \, dt
\]

(3.46)

\[
P_2 = \int \frac{\beta}{V_2} (Q_2 + A \cdot V_A) \, dt
\]

(3.47)

where $\beta$ is the bulk modulus of the hydraulic oil, $P_s$ is the hydraulic supply pressure and $V_A$ is the velocity of the piston. Equations (3.46) and (3.47) are correct for $P_1$ and $P_2$ less than the supply pressure $P_s$, which is within the normal operating conditions for the actuators.

The flow into either side of the actuator is given by the square root characteristic defined earlier. For a positive servovalve current the flow is defined as:

\[
Q_1 = I \cdot K_r \cdot \sqrt{(P_2 - P_1)} \quad Q_2 = -I \cdot K_r \cdot \sqrt{P_1}
\]

(3.48)
and for a negative servovalve current the flow is defined as:

\[ Q_1 = I \cdot K_v \cdot \sqrt{P_2}, \quad Q_2 = -I \cdot K_v \cdot \sqrt{(P_2 - P_2)} \quad (3.49) \]

where \( I \) is the servovalve current, and \( K_v \) is the servovalve gain constant. (Note: in some cases the areas either side of the piston are different due to the piston rode, but here the same area is assumed)

The force produced by the actuator is proportional to the pressure difference across the piston, and can be calculated from:

\[ F_{\text{out}} = (P_1 - P_2) \cdot A \quad (3.50) \]

The servovalve current is given by:

\[ I = \frac{(K_v \cdot (F_n - A \cdot (P_1 - P_2)))}{\left( 1 + \frac{0.8}{\omega_v} + \frac{1}{\omega_v^2 s^2} \right)} \quad , \quad 20mA \leq I \leq 20mA \quad (3.51) \]

where \( K_n \) is the servovalve current/pressure gain.

### 3.5.3.2 Linear Actuator Equations

The previous section showed that fundamentally the actuator is non-linear. Since some of the control strategies described in the next chapter are linear, the actuator characteristics need to be linearised.

Figure 3.8 shows a block diagram of the linear actuator model, which models the fundamental flow characteristics of the oil into the piston. The model also includes oil column resonance, since this is in the region of 70Hz for the actuators considered here (short stroke and large area), but can be much lower depending on the physical characteristics of the actuator, it can have a serious effect on the performance of control systems.

The parameters for the linear actuator model were calculated at an operating point of \( \% \) of the supply pressure at the rated servo-valve current giving the rated flow [Goodall and Whitfield, 1985]. The model parameters are given in Table 3.4. The linear parameters for the model are derived as follows and Figure 3.9 shows the design point on the pressure flow characteristic of the servovalve.

The servovalves have a flow rate of 40l/min, and the rated current \( i_R = 20mA \), therefore the rated flow is \( Q_n = 6.7 \times 10^4 \text{ ms}^{-1} \).
The servovalve flow gain is calculated as

\[ k_i = \frac{Q_R}{i_R} = \frac{6.7 \times 10^{-4}}{20 \times 10^{-3}} = 0.0335 \text{m}^3\text{s}^{-1}/\text{A} \]  

(3.52)

The flow equation is given by

\[ Q = i_k v \sqrt{P_s - P_d} \]

\[ Q_R = i_k v \sqrt{P_s - \frac{2}{3} P_s} = i_k v \sqrt{\frac{1}{3} P_s} \]  

(3.53)

\[ k_v = \frac{Q_R}{i_k v \sqrt{\frac{1}{3} P_s}} = \frac{6.7 \times 10^{-4}}{20 \times 10^{-3} \sqrt{0.7 \times 10^7}} = 1.266 \times 10^{-5} \text{m}^3\text{s}^{-1}/\text{AN/m} \]  

(3.54)

The maximum flow rate is given by

\[ Q_{\text{max}} = i_k v \sqrt{P_s} = 20 \times 10^{-3} \times 1.266 \times 10^{-5} \times \sqrt{2.1 \times 10^7} = 11.6 \times 10^{-4} \text{m}^3\text{s}^{-1} \]  

(3.55)

The pressure flow feedback constant is calculated using

\[ K_F = \left. \frac{dQ}{dP} \right|_{\frac{1}{3} P_s} = \left[ \frac{i_k v}{2 \sqrt{P_s} K_F} \right] \frac{1}{\frac{1}{3} P_s} \]

\[ = 4.8 \times 10^{-11} \text{m}^3\text{s}^{-1}/\text{NM}^2 \]  

(3.56)

The following equations define the linear actuator model operating at a design point of ½ of the supply pressure:

the flow equation is defined as

\[ Q = K_Q I_v - K_F K D_A \]  

(3.57)

the actuator spool displacement is given by

\[ D_A = \frac{1}{s A} Q - x \]  

(3.58)
the actuator force equation is

$$F_{\text{out}} = K D_A - C \dot{x}$$  \hspace{1cm} (3.59)

and the servovalve current is defined by

$$I = K_N (F_{\text{in}} - F_{\text{out}})$$  \hspace{1cm} (3.60)

$$I = I_v \left( 1 + \frac{0.8}{\omega_v} s + \frac{1}{\omega_v^2} s^2 \right)$$  \hspace{1cm} (3.61)

These linear actuator equations can be transformed into a state space model of the form

$$\dot{x}_A = [A_A] x_A + [B_{AU}] u_A + [B_{AF}] E_u$$  \hspace{1cm} (3.62)

$$y_A = [C_A] x_A + [D_{AU}] u_A + [D_{AF}] E_u$$  \hspace{1cm} (3.63)

where

$$x_A = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{1}{I_v} & 0 \\ \frac{1}{I_v} & 1 \end{bmatrix} \hspace{1cm} \dot{x}_A = F_{\text{out}} \hspace{1cm} u_A = \begin{bmatrix} \dot{x}_A \\ \ddot{x}_A \end{bmatrix}$$  \hspace{1cm} (3.64)

and the state space matrices are defined as:

$$[A_A] = \begin{bmatrix} -\frac{K_p}{A} & K_p & 0 \\ 0 & 0 & 1 \\ 0 & -\omega_v & -\omega_v \omega_p \end{bmatrix} \hspace{1cm} [C_A] = \begin{bmatrix} K \\ A \\ 0 \end{bmatrix}$$  \hspace{1cm} (3.65)

$$[B_{AU}] = \begin{bmatrix} K_p \omega & K_p \omega \end{bmatrix} \hspace{1cm} [B_{AF}] = \begin{bmatrix} K_p \omega \omega_p \\ 0 \end{bmatrix}$$

$$[D_{AU}] = [-C \quad -K] \hspace{1cm} [D_{AF}] = [0]$$

The linear actuator model can be incorporated into the structural model of the helicopter as follows. The first stage is to expand the actuator model to include dynamic models for each
of the actuators in the system, in this case four:

\[
\begin{bmatrix}
    \dot{x}_{A1} \\
    \dot{x}_{A2} \\
    \dot{x}_{A3} \\
    \dot{x}_{A4}
\end{bmatrix} =
\begin{bmatrix}
    A_A & 0 & 0 & 0 \\
    0 & A_A & 0 & 0 \\
    0 & 0 & A_A & 0 \\
    0 & 0 & 0 & A_A
\end{bmatrix}
\begin{bmatrix}
    x_{A1} \\
    x_{A2} \\
    x_{A3} \\
    x_{A4}
\end{bmatrix}
+
\begin{bmatrix}
    B_{AU} & 0 & 0 & 0 \\
    0 & B_{AU} & 0 & 0 \\
    0 & 0 & B_{AU} & 0 \\
    0 & 0 & 0 & B_{AU}
\end{bmatrix}
\begin{bmatrix}
    u \\
    0 \\
    0 \\
    0
\end{bmatrix}
+
\begin{bmatrix}
    B_{AF} & 0 & 0 & 0 \\
    0 & B_{AF} & 0 & 0 \\
    0 & 0 & B_{AF} & 0 \\
    0 & 0 & 0 & B_{AF}
\end{bmatrix}
\begin{bmatrix}
    x_{A1} \\
    x_{A2} \\
    x_{A3} \\
    x_{A4}
\end{bmatrix}
+
\begin{bmatrix}
    W_1 \\
    0 \\
    0 \\
    0
\end{bmatrix}
\begin{bmatrix}
    x_s \\
    u
\end{bmatrix}
\]

where \([W_1]\) is a transformation matrix which transforms the state vector of the fuselage model \(x_s\), into displacements and velocities across the actuators. Equation (3.66) can be re-expressed as:

\[
\begin{bmatrix}
    \dot{x}_c \\
    \dot{y}_c
\end{bmatrix} =
\begin{bmatrix}
    A_c & [B_{CZ}] [W_1] [B_{CU}] & u_c
\end{bmatrix}
\begin{bmatrix}
    x_s \\
    y_s
\end{bmatrix}
\]

\[
\begin{bmatrix}
    x_c \\
    y_c
\end{bmatrix} =
\begin{bmatrix}
    C_c & [D_{CZ}] [W_1] [D_{CU}] & u_c
\end{bmatrix}
\begin{bmatrix}
    x_s \\
    y_s
\end{bmatrix}
\]

The combined actuator model defined by equations (3.67) and (3.68) can be embedded into the structural model of the fuselage, defined by equations (3.21) and (3.22), to give:

\[
\begin{bmatrix}
    \dot{x}_s \\
    \dot{y}_s
\end{bmatrix} =
\begin{bmatrix}
    A_s + B_{SU} D_{CZ} W_i \\
    D_{SU} W_i
\end{bmatrix}
\begin{bmatrix}
    B_{SU} C_c \\
    A_c
\end{bmatrix}
\begin{bmatrix}
    x_s \\
    y_s
\end{bmatrix}
+
\begin{bmatrix}
    B_{SF} & B_{SU} D_{CZ} W_i & 0 & B_{SU} D_{CU}
\end{bmatrix}
\begin{bmatrix}
    E \\
    u_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
    y_s \\
    y_c
\end{bmatrix} =
\begin{bmatrix}
    C_s + D_{SU} D_{CZ} W_i \\
    D_{SU} C_c
\end{bmatrix}
\begin{bmatrix}
    x_s \\
    y_s
\end{bmatrix}
+
\begin{bmatrix}
    D_{SF} & D_{SU} D_{CZ} W_i & 0 & D_{SU} D_{CU}
\end{bmatrix}
\begin{bmatrix}
    E \\
    u_c
\end{bmatrix}
\]

and this is shown diagrammatically in Figure 3.10.

3.5.4 Sensor Model

The accelerometers used by the ACSR system can also be represented by second order transfer functions which typically have natural frequencies in the range 200 to 700Hz. They have negligible influence on the overall performance and stability of the system, and so a proportional characteristic is used.
This chapter has provided a mathematical description of the flexible helicopter fuselage and ACSR actuators. Experimental validation of the model is important, since analytical models are only as good as their physically verifiable results. Experimental validation allows errors to be identified and the model refined. Once the model is validated it can be used to assess the performance of various vibration control strategies.

Finite element methods can give good results but care is needed in the modelling to ensure the model reflects the actual build standard of the aircraft. Experimental data should always be obtained at the earliest possible time for model validation purposes. Accurate prediction of vibration requires accurate estimation of the applied forces as well as an accurate model of the structural response.

The dynamics of the overall system are complex and non-linear, and include sensor dynamics, fuselage dynamics, actuator dynamics and any signal conditioning filter dynamics. Hydraulic actuators have the greatest effect on the overall control system and they are the major limiting factor when considering system performance. Actuators have significant lags and attenuations at required control frequencies and therefore they must be modelled so that they can be considered in any practical control system design.
<table>
<thead>
<tr>
<th>Modal Frequency (Hz)</th>
<th>Modal Rank using equation (3.43) at 114Kts Forcing</th>
<th>Modal Rank using equation (3.43) at 150Kts Forcing</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.31</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>7.43</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>13.65</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>14.71</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>15.29</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>16.56</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>17.18</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>17.78</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>20.02</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>20.69</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 3.1: Modal Ranking Results, for selection of modes to be included in a control system design.
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Frequency (Hz)</th>
<th>Mode Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>Heavy</td>
<td></td>
</tr>
<tr>
<td>6.53</td>
<td>6.31</td>
<td>First Vertical Bending</td>
</tr>
<tr>
<td>7.55</td>
<td>7.43</td>
<td>First Lateral Bending</td>
</tr>
<tr>
<td>11.48</td>
<td>11.54</td>
<td>No.1 and No.3 Engines F/A Antiphase - Gearbox Yaw</td>
</tr>
<tr>
<td>13.59</td>
<td>13.37</td>
<td>Gearbox Roll Fuselage Lozenging</td>
</tr>
<tr>
<td></td>
<td>13.65</td>
<td>Second Vertical Bending</td>
</tr>
<tr>
<td>15.33</td>
<td>14.29</td>
<td>Gearbox Pitch</td>
</tr>
<tr>
<td></td>
<td>14.71</td>
<td>Gearbox Bounce Nose Vertical Bending</td>
</tr>
<tr>
<td>16.58</td>
<td>15.91</td>
<td>Second Lateral Bending</td>
</tr>
<tr>
<td>17.87</td>
<td>16.99</td>
<td>Gearbox Bounce, Fuselage Vertical Bending</td>
</tr>
<tr>
<td>19.52</td>
<td>18.41</td>
<td>Fuselage Roll Antiphase to Gearbox Roll</td>
</tr>
<tr>
<td>22.02</td>
<td>22.69</td>
<td>Fuselage Vertical Bending No.1 and No.3 Engines F/A Inphase</td>
</tr>
<tr>
<td></td>
<td>22.22</td>
<td>Tailplane Antisymmetric Flap</td>
</tr>
<tr>
<td>23.07</td>
<td></td>
<td>Undercarriage Lateral Antiphase - Gearbox Pitch</td>
</tr>
<tr>
<td>23.31</td>
<td></td>
<td>Intermediate Gearbox Vertical Main Undercarriage Lateral Antiphase</td>
</tr>
<tr>
<td>24.88</td>
<td>25.24</td>
<td>Tailcone Pitch and Yaw</td>
</tr>
<tr>
<td>27.83</td>
<td>26.39</td>
<td>No.2 Engine F/A Inphase with Tailcone Vertical Bending</td>
</tr>
<tr>
<td>32.82</td>
<td>33.11</td>
<td>Accessory Gearbox F/A</td>
</tr>
</tbody>
</table>

Table 3.2 Comparative Table of Measured Lightweight and Heavyweight Aircraft Mode Shapes for Helicopter model 1.
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Frequency (Hz)</th>
<th>Mode Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>Heavy</td>
<td></td>
</tr>
<tr>
<td>8.81</td>
<td>9.52</td>
<td>First Lateral Bending</td>
</tr>
<tr>
<td>9.39</td>
<td>9.91</td>
<td>First Vertical Bending</td>
</tr>
<tr>
<td>15.33</td>
<td>17.29</td>
<td>Raft Pitch/Fuselage Shear</td>
</tr>
<tr>
<td>18.04</td>
<td>19.90</td>
<td>Engines Lateral Antiphase</td>
</tr>
<tr>
<td>18.26</td>
<td>18.26</td>
<td>Raft Rear Deck Torsion</td>
</tr>
<tr>
<td>19.52</td>
<td>19.55</td>
<td>Tailplane Vertical Bending</td>
</tr>
<tr>
<td>22.30</td>
<td>22.40</td>
<td>Fin Torsion</td>
</tr>
<tr>
<td>23.15</td>
<td>24.74</td>
<td>Engine Lateral Antiphase</td>
</tr>
<tr>
<td>24.16</td>
<td>25.06</td>
<td>Engine Lateral Inphase</td>
</tr>
<tr>
<td>26.64</td>
<td>29.68</td>
<td>Fuselage Torsion/Raft Roll</td>
</tr>
<tr>
<td>29.93</td>
<td>30.99</td>
<td>Engine Roll. Lateral Antiphase</td>
</tr>
<tr>
<td>30.22</td>
<td>30.99</td>
<td>Boom Vertical</td>
</tr>
<tr>
<td>32.76</td>
<td>33.44</td>
<td>Boom and Fin</td>
</tr>
<tr>
<td>35.17</td>
<td>35.49</td>
<td>Fuselage Vertical</td>
</tr>
<tr>
<td>36.14</td>
<td>36.56</td>
<td>Engines Axial Antiphase</td>
</tr>
</tbody>
</table>

Table 3.3 Comparative Table of Measured Lightweight and Heayweight Aircraft Mode Shapes for Helicopter model 2.
### Table 3.4 Table of Linear Actuator Model Parameters.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servo-valve Flow Gain</td>
<td>( K_s )</td>
<td>0.033 ( \text{m}^3 \text{s}^{-1} \text{A} )</td>
</tr>
<tr>
<td>Actuator Cross Sectional Area</td>
<td>( A )</td>
<td>1.1 \times 10^{-3} ( \text{m} )</td>
</tr>
<tr>
<td>Flow/Force Constant</td>
<td>( K_p )</td>
<td>4.3 \times 10^4 ( \text{m}^3 \text{s}^{-1} \text{N} )</td>
</tr>
<tr>
<td>Oil Column Stiffness</td>
<td>( K )</td>
<td>4.6 \times 10^8 ( \text{N/m} )</td>
</tr>
<tr>
<td>Oil Column Damping</td>
<td>( C )</td>
<td>20 \times 10^3 ( \text{N/ms}^{-1} )</td>
</tr>
<tr>
<td>Actuator Diameter</td>
<td>( da )</td>
<td>40 ( \text{mm} )</td>
</tr>
<tr>
<td>Supply Pressure</td>
<td>( P_s )</td>
<td>210 ( \text{bar} )</td>
</tr>
<tr>
<td>Servo-valve Flow Rate</td>
<td>( Q )</td>
<td>40 ( \text{l/min} )</td>
</tr>
<tr>
<td>Servo-valve Frequency</td>
<td>( \omega_{sv} )</td>
<td>600 ( \text{rads}^{-1} )</td>
</tr>
</tbody>
</table>

The servo-valve dynamics are modelled as a second order transfer function

\[
H_{sv}(s) = \frac{K_{sv}}{(s^2 + \frac{2\zeta_{sv}\omega_{sv}}{\omega_{sv}^2} + 1)}
\]

where

- \( K_{sv} \) is the servo-valve electric motor gain \( \text{m/mA} \)
- \( \omega_{sv} \) is the natural frequency of the servo-valve \( \text{rads/s} \)
- \( \zeta_{sv} \) is the damping coefficient of the servo-valve

The natural frequency of the servo-valve is typically in the region 100 to 200Hz, with a damping factor of 0.7.
Figure 3.1 Model Sub-systems

Figure 3.2 Mode Shape
PARAMETERS

<table>
<thead>
<tr>
<th>MASS 1</th>
<th>4 Kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>MASS 2</td>
<td>4 Kg</td>
</tr>
<tr>
<td>MASS 3</td>
<td>8 Kg</td>
</tr>
</tbody>
</table>

SPRINGS:

<table>
<thead>
<tr>
<th>K1</th>
<th>10 KN/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>K2</td>
<td>10 KN/m</td>
</tr>
<tr>
<td>K3</td>
<td>30 KN/m</td>
</tr>
</tbody>
</table>

DAMPERS:

<table>
<thead>
<tr>
<th>D1</th>
<th>10 N/m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>10 N/m/s</td>
</tr>
<tr>
<td>D3</td>
<td>30 N/m/s</td>
</tr>
</tbody>
</table>

Figure 3.3 Three mass dynamic model

Figure 3.4a : Helicopter Finite Element Model
Figure 3.4b: Helicopter Finite Element Model
Figure 3.4c: Helicopter Finite Element Model
**Figure 3.5** Hydraulic Actuator Pressure Flow Characteristic

\( P_s \) = supply pressure

**Figure 3.6** Actuator Schematic

Positive Servo-valve current
Q1 positive
Q2 negative
SUPPLY PRESSURE ~ PRESSURE FEEDBACK

\[ \frac{1}{A} \]

\[ \text{FORCE IN SERVO VALVE} \]

\( \frac{1}{1 + 0.8 s + 1\alpha^2} \)

\[ \text{SERVO VALVE CURRENT} \]

\[ K_v \]

\[ \frac{1}{A} \]

\[ \text{FLOW} \]

\[ K \]

\[ \text{LOAD DISPLACEMENT} \]

\[ \text{LOAD VELOCITY} \]

Figure 3.7 Non-linear Actuator Block Diagram

FLOW FEEDBACK

\[ K_p \]

\[ \text{FORCE IN SERVO VALVE} \]

\( \frac{1}{1 + 0.8 s + 1\alpha^2} \)

\[ \text{SERVO VALVE CURRENT} \]

\[ K_q \]

\[ \frac{1}{A} \]

\[ \text{FLOW} \]

\[ K \]

\[ \text{FORCE OUT} \]

\[ \text{LOAD DISPLACEMENT} \]

\[ \text{LOAD VELOCITY} \]

Figure 3.8 Linear Actuator Model
Hydraulic Flow (m³/s)

Pressure/Flow curve for rated servovalve current

Maximum Flow \( Q_{\text{max}} \)

Rated Flow \( Q_R \)

Design Point (Gradient \( K_p \))

Valve Pressure (N/m²)

\[ \frac{2}{3} p_s \]

\[ p_s = \text{supply pressure} \]

\[ 2.1 \times 10^7 \]

Figure 3.9 Pressure Flow Characteristic and Linear Actuator Design Point

---

Figure 3.10 Embedding Actuator Dynamics into the Fuselage Dynamic Model
CHAPTER 4

CONTROL STRATEGIES

INTRODUCTION

In order to implement the active control of helicopter structural response technique, a strategy is required which measures the vibration at a number of fuselage locations and controls the actuator forces in such a way as to reduce the fuselage vibration. If there are as many actuators as measurement positions, in principle the vibration at all the measured points could be reduced to zero. However, since practically there will almost inevitably be fewer actuators than accelerometers, which means that the vibration cannot be reduced to zero at all the points, the control strategies are naturally formulated using optimisation techniques. The optimal control formulation can be applied to problems in both the time and the frequency domains, because the control strategy aims to minimise the measured structural vibration while maintaining the actuator control forces within practical limits. Since an optimal control formulation is applied, the control algorithms are based upon the minimisation of a performance index.

An important consideration is that accurate information of the actual helicopter dynamics is not available. It is therefore necessary to design a control strategy which is either robust to inaccurately modelled dynamics and changes in actual helicopter dynamics (e.g. cargo change or fuel usage), or which adapts to these changes. The control algorithms used to implement an active vibration control strategy can be formulated in either the frequency or time domain, and this chapter summarises the concepts behind the two control approaches and identifies the limitations of each. A new approach is also described which is a hybrid of the time and frequency domain strategies, and which overcomes many of the limitations.

4.1 CONTROL STRATEGIES

The aim of the control strategy is to minimise the fuselage vibration using actuators located at strategic points within the airframe. Accelerometers measure the fuselage vibration, and this is used by a multivariable controller which calculates the drive signals for the actuators. The actuator control forces applied to the fuselage are calculated by the control strategy such that the vibration measured by the accelerometers is minimised.
There are a number of important issues concerned with the control of helicopter vibration. As discussed in Chapter 3, there is uncertainty associated with the modal parameters of the fuselage flexible modes. Initially a theoretical model was developed from a Finite Element model, from which the models required by the various control strategies are developed. The structural modes identified by the Finite Element method have a degree of uncertainty associated with them, in terms of damping level, natural frequency or mode shape. A number of authors have identified errors of 20% in the natural frequencies identified in Finite Element models [Williams 1980 and Balas 1978]. A large number of modes are required to model the flexibility of the fuselage, and extra dynamics are also introduced by the control system itself, for example sensor dynamics and more significantly actuator dynamics. Consequently, controllers are often designed using reduced models that neglect certain fuselage modes, and that either neglect sensor and actuator dynamics or use simple representations. Uncertainties and short comings in the control system model will limit the achievable control system performance. Any controller designed using a reduced or incomplete structural model will interact to some extent with the unmodelled part of the actual structure, Balas (1978) termed this the residual (uncontrolled) modes. Balas also used the phrase control spillover to describe the phenomenon of the control inputs exciting the residual modes, and observation spillover for the phenomenon of the sensors detecting energy in the residual modes. This is shown diagrammatically in Figure 4.1. Spillover can lead to a deterioration in system performance, and in some cases can cause instability [Balas, 1980][Strunce et al, 1979]. Therefore any Active Vibration Control strategy should reduce or eliminate the undesirable effects of control and observation spillover into and from the vibration modes neglected in the system model. This is particularly a problem with state feedback controllers, and these are discussed later in this chapter.

Since the number and location of sensors and actuators relate to system controllability and observability, careful selection can lead to improved system performance and minimise effects such as control and observation spillover. A corollary of this is that a poor selection may degrade system performance or even result in unstable controller performance [Hughes and Skelton, 1979][Juang and Rodriguez, 1981].

The complexity of the control method depends on whether or not the algorithm needs to be adaptive. The blade loads and consequently the fuselage vibration change significantly with flight condition, and also changes in aircraft dynamics can occur due to variations in weight.
Another important consideration is that accurate data for the actual helicopter dynamics are not available. Therefore, for a practical ACSR system, the control algorithm either needs to be robust or needs to adapt to inaccurately modelled dynamics, changes in flight condition and changes in fuselage dynamics (e.g. cargo change or fuel usage).

The difference between non-adaptive robust control and adaptive control is that the former operates on a nominal system with a large robustness margin, whereas an adaptive controller operates on a large number of nominal systems, but at the expense of a reduction in the stability margin associated with each nominal system.

Robust control design methods yield fixed compensators which give satisfactory performance over a specified range of system variations from the nominal. Adaptive controllers utilise estimation techniques to extract information about the system parameters on line and redesign the control laws [Hill et al., 1986].

Two basic categories for vibration control algorithms exist, either time domain or frequency domain strategies, depending upon whether they feedback real time signals or frequency domain information about the signals. Time domain strategies are based upon the direct feedback of vibration through constant gain matrices, and consequently the control signals contain information over a wide range of frequencies. The advantages of such an approach are that its implementation is computationally simple and its stability is readily analysed by standard linear systems analysis techniques.

Frequency domain algorithms control discrete frequencies, the blade passing frequency being dominant in the case of a helicopter, and require a transformation of the feedback data to the frequency domain. Clearly, the dynamic response of a frequency domain controller is limited by its ability to track the time varying vibration components in the frequency domain. The time domain strategies do not need to transform the data to the frequency domain, and the dynamic response is inherently more rapid than that which could be obtained by a frequency domain controller.

The time domain strategy studied in this research is the well known Linear Quadratic control strategy. There are a number of other control strategy design techniques which have been developed for generating control laws for multivariable systems. For example, a popular time domain method developed for controlling vibration in large dimensional space structures is the Independent Modal Space Control (IMSC) strategy [Lindberg and Longman, 1984][Meriovitch and Oz, 1980], although the technique has many similarities with Linear Quadratic control.
Essentially, the derivation of the control gains is based upon the minimisation of a quadratic performance index for each mode to be controlled. Minimisation of this index gives a feedback formula for each controlled structural mode. The primary difference is that the IMSC technique decouples the fuselage modes, and solves a 2x2 matrix Riccati equation for each of the modes to be controlled, whereas the optimal full state feedback technique solves the full state space Riccati equation without decoupling.

The IMSC technique requires one actuator for each mode to be controlled, in order to decouple the control modes. However, for the helicopter fuselage the practical number of actuators is significantly less than the number of dominant modes. Lindberg and Longman also proposed a suboptimal version of IMSC where the restriction of one actuator per mode is relaxed. Both the IMSC and LQ techniques require some form of modal filtering using observers or Kalman filters to estimate modal information, and consequently the identification of modal dynamics is likely to be equally difficult for both techniques. The main drawback with the IMSC technique is that it is very difficult to include actuator dynamics, and consequently this technique was not investigated any further. Spillover of control action into unmodelled and uncontrolled modes can also be a problem. Baz et al (1989) considered a modified form of IMSC, where the control spillover as well as optimum placement and time sharing of actuators is considered.

The $H_{\infty}$ control technique is a robust control method that allows a specified degree of robustness to be built into a system by permitting constraints to be placed on the frequency characteristic of the closed loop system. The technique originated from the influential work of Zames [1981], and good introductions to the $H_{\infty}$ technique are given by Williams [1991], Postlethwaite [1990], and Safonov et al (1987). $H_{\infty}$ is an optimisation based control design technique for multivariable (MIMO) systems which guarantees robustness. Design trade-offs for system performance and robustness are easily incorporated into the optimisation process. Performance specifications are incorporated by shaping the sensitivity function, which is the transfer function matrix between the system disturbances and the system outputs. Robustness specifications are included in the optimisation process by using the reciprocal of the maximum allowable modelling error. Weighting functions for both the sensitivity function and the reciprocal of the modelling error can be introduced, and these reflect design goals. The optimisation process generates a controller which aims to minimise a measure of the gain (maximum singular value) of both the weighted sensitivity function and the reciprocal of the modelling error transfer function matrices.
4.1.1 Performance Indices

Since an optimal formulation is applied the control algorithms are based on the minimisation of a performance index that is a quadratic function of the measured vibration and the actuator forces. The optimal control input is calculated by minimising this performance index. The index is chosen such that the fuselage vibration levels are minimised, but not at the expense of large actuator forces. The performance index can be defined as either deterministic or stochastic, in either the time or frequency domains.

The performance index comprises a weighted sum of the vibration measurements and the weighted sum of the actuator demands. The relative size of the accelerometer and actuator weightings determines the overall reductions achieved and the degree of actuator force limiting. Furthermore, such an approach allows the control algorithm to optimise vibration levels in specific areas of the fuselage by altering the relative sensor and actuator weights. Translating a real control problem into weighting matrices for a cost function, such that the original control objectives are represented, can often be very difficult to achieve and requires experience and engineering judgement. However, this is one of the very few applications where there are no direct engineering trade-offs required in the formulation of the cost function, and the selection of the elements in the weighting matrices is relatively straightforward.

In principle it is only necessary to decide the relative weighting between the accelerations and the actuator forces, although more sophisticated weighting strategies can be adopted by considering the vibration requirements defined by the vibration environment specification. ADS-27 [Crews, 1987] for example specifies that the vertical accelerations should be more heavily weighted than both the longitudinal and lateral directions. In theory, for a system with an equal number of sensors and actuators, the controller will attempt to achieve zero vibration at sensor positions, although the actuator weighting is also an influence. However, this may worsen the vibration levels at other unsensed locations in the fuselage. It is therefore better to attempt to reduce the vibration at a larger number of locations to relatively low levels, rather than to attempt to achieve zero vibration at a few points. Since the aim of the system is to provide reductions throughout the airframe, it is important to have enough suitably located sensors which measure the response of all the dominant fuselage modes.
4.2 ADAPTIVE CONTROL

The main reason for constructing an adaptive controller is due to the wide range of dynamic characteristics the helicopter is expected to assume as the flight conditions and/or structural configurations change. An adaptive scheme can be defined as one which adjusts its controller parameters in response to changes in the system or in the operating conditions.

Adaptive controllers consist of a controller subsystem and a parameter estimator subsystem, the estimator provides the controller subsystem with estimates of the system parameters which can then be used to calculate the controller parameters. In addition, for controllers based on state space concepts, estimation of the states of an internal description of the system is necessary in order to provide sufficient information to the controller subsystem. The key to the problem is then to find a convenient method for changing the controller parameters in response to changes in the system dynamics and disturbances, i.e. the control law. Adaptive control laws are normally derived using the certainty equivalence principle. This means that a system model is determined and a controller is designed as if the model was exact. Clearly, problems may arise when the identified system model is in some way inaccurate.

General surveys of the work on adaptive control are found in Landau [1985], Astrom [1981], Kutz-Isermann and Schumann [1980], and Martin-Sanchez [1976].

Adaptive controllers can be classified into three groups: Gain Scheduling, Model Reference and Self Tuning. The basic concepts behind this classification are described by Warwick [1990]. Gain scheduling adaptive controllers design fixed point control strategies off line, i.e. control strategies are designed for a specific operating condition or point. The task of the controller is then to choose the appropriate strategy to meet the current conditions. Gain scheduling strategies have the advantage of quick response of the controller parameters to changes in the system, compared with the convergence time associated with self tuning schemes. Also, the adaptation mechanism does not cause instability. However, gain scheduling strategies have a number of disadvantages:

- it is an openloop controller (that is the adaptation is openloop), therefore an incorrect schedule is not compensated for until the next controller update.
- if there are many operating points, then the controller design process may be time consuming.

Gain scheduled controllers are probably the simplest form of adaptive control, since all the control design calculations are performed off-line. The computational requirements for practical
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implementation are also less. However, they do require the measurement of some variable to schedule the gains, for example flight condition (airspeed, rotor angle of attack, etc). In addition, preflight testing is required to determine the necessary matrices for gain calculation, the number of design points may also be large resulting in the storage of a large number of controller gains.

A model reference adaptive controller aims to make a real system with some unknown parameters behave like a prescribed known mathematical model. The model is selected such that its performance is that required for the overall system. The inputs to the system are applied to the model, then the error between the system and model outputs is used to vary the controller parameters in a predefined manner. The adaptation mechanism basically adjusts feedback gains or injects additional feedback signals into the system to achieve the desired behaviour. A comprehensive review of model reference adaptive control is given by Landau [1974].

The final category of adaptive controllers is Self tuning control, and these have received much attention since the early ideas reported by Astrom and Wittenmark [1973], including practical implementations [Isermann, 1980]. Good introductions to self tuning control are given by Harris and Billings [1985], and Warwick [1988]. The basic idea is to perform on-line system identification and control system design simultaneously. As the flight condition or structural dynamics change, so does the system being identified and appropriate changes are made to the control law. Hence, control system adaptability is provided. A self tuning control scheme essentially has two loops. The outer loop consists of a recursive parameter estimator and a controller design routine; the parameter estimates are used in the calculation of the controller parameters for the inner loop. Many different estimation schemes have been used, e.g. stochastic approximation, least squares, extended least squares, instrument variables, kalman filtering and the maximum likelihood method. A self tuning controller utilising a recursive least squares estimation scheme and a state feedback controller is described by Warwick [1981]. The advantages of such a control scheme are that it is capable of tuning itself to optimal settings, and further retuning itself if the system dynamics change.

Self tuning controllers can be one of two types:

1- explicit self tuning algorithms: The control law is calculated (indirectly) via an analytical design mechanism, using the parameter estimates for a model of the system.

2- implicit self tuning algorithms: It is sometimes possible to reparameterize the
problem so that it can be re-expressed in terms of the controller, hence the controller parameters are directly estimated. This gives a significant simplification of the controller because the control design calculations are eliminated.

The explicit self tuning algorithms are used in this study, due to the difficulty in reparameterising the problem in terms of the controller parameters. For the explicit self tuning algorithms the design principle is to consider the parameter estimates to be the true parameters for the purpose of using them in the controller design stage. This methodology was first used by Astrom in 1973 and is usually referred to as the certainty equivalence principle.

It is important to further understand the closed loop adaptive system properties. However, understanding the intrinsic properties of an adaptive controller is difficult because the controller is itself nonlinear and therefore its properties cannot be analyzed using classical linear methods.

4.3 FREQUENCY DOMAIN APPROACH

Since the disturbance forces from the rotor head are predominantly periodic, it is natural to try and formulate the problem in the frequency domain by assuming a quasi-static linear relationship between the measured vibrations resulting both from the disturbance forces and the actuator forces at the blade passing vibration frequency $\omega_0$. This relationship is mathematically described by equation (4.1), and has been discussed in detail in chapter 3.

$$Y = [T]U + B$$ (4.1)

This model of the aircraft's structural dynamics is only valid at the blade passing frequency, and is necessarily quasi-static in nature, since both the fuselage and rotor transients must be allowed to decay before vibration measurements are taken. Since it is a quasi-static relationship Discrete Fourier Transform (DFT) techniques can be used over an appropriate time interval to extract the $\omega_0$ vibration component from the measured fuselage response. Additional sets of quasi-static equations are needed if control is required at other frequencies for which there are significant vibration effects.

4.3.1 Frequency Domain Control Strategy

The control strategy hinges upon the description of the rotor and fuselage response to inputs as a linear quasi-static frequency domain representation. The parameters for this linear model are known to vary with helicopter flight condition. It is known that the background vibration
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$\mathbf{B}$ varies with flight condition and that the transfer matrix $[T]$ changes with variations in payload and fuel. Therefore, a non-adaptive or fixed gain algorithm will be in error, since both $[T]$ and $\mathbf{B}$ are assumed to be constant for those algorithms. Changes in the fuselage dynamics and in the flight condition can be accounted for by identifying the transfer matrix and the uncontrolled vibration during each control cycle using parameter estimation techniques. The actual inputs are derived through the minimisation of a quadratic performance index using the substituted background parameter estimates. The control process is iterative, where the parameter estimates and consequently, the control inputs are updated at discrete intervals.

The whole measurement period consists of a delay time to permit the transients to decay following the last change in actuator forces, a measurement time to carry out the discrete Fourier Transform by which the nco components of the vibration are determined, and finally the computation time to perform the estimation and control calculations.

The fuselage transient response will typically be between 2% and 5% critically damped, and this level of damping will give rise to transient decay times in region of 3 seconds. The use of DFTs requires a signal processing time of 2 or 3 rotor revolutions to identify the blade passing frequency, which is typically in the region of 20Hz for most helicopters (with 4 or 5 blades) and this gives a signal processing time of between 400 and 600ms [King and Staple, 1986].

The implementation of the Frequency domain algorithms in an adaptive framework has shown update rates to be of the order of 1 second, this comprises 600ms for signal processing and 400ms for algorithm calculations. Although faster processing capability can be used to reduce the computation time, the delay and measurement times are not affected because they determine the accuracy of the measurement. Recent progress has reduced this controller update time to 0.5 seconds. However, even with this reduced update time the performance of the frequency domain system is limited, since during manoeuvres the rotor head forces and consequently the fuselage vibration can change rapidly when compared to the control update time.

The frequency domain algorithm represents the measured vibration and actuator control forces by in-phase and quadrature components (relative to a controller reference signal) at the controller operating frequency nco. The controller can be implemented either in a fixed frequency configuration or with a feedforward reference signal. In the latter case the reference is derived from a rotor sensor such that the reference signal frequency is the blade passing frequency. In the fixed frequency configuration the frequency of the actual signals is that which is deduced during the measurement period, for which the magnitudes and phases are strictly applicable. If a feedforward reference signal is used, the measured vibration and actuator force

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components are then specified relative to this reference signal, so that if the forcing frequency changes the actuator frequency changes accordingly, thereby allowing any variations in the blade passing frequency to be tracked, although strictly they no longer have the correct phases and magnitudes.

Frequency domain techniques operate at single frequencies and control inputs are updated on a cyclic basis. The frequency domain controller is basically open loop for the duration of each control cycle, i.e. the sinusoidal actuator forces are maintained at constant magnitudes. In parallel with this operation the algorithm performs a number of calculations during each control cycle. The control algorithm calculates the optimum actuator control forces to minimise the measured fuselage vibration during the next control cycle. The algorithm calculations can be separated into three distinct phases. The first stage extracts the blade passing frequency content of the fuselage vibration using Discrete Fourier Transform (DFT) techniques. The vibration measurements from the accelerometers are decomposed into sine and cosine components at the operating frequency $\omega$ using a correlation method, described in Appendix III. The reference signal is used to generate sine and cosine signals of the operating frequency, the required sine and cosine coefficients are obtained by summing the products of these signals with the measured vibration at each discrete sampling interval. Since the rotor reference signal is continually fed forward in the time domain, the frequency domain ACSR algorithm can be considered to be a feed forward algorithm [Lyndon, 1991]. The reference signal continually corrects for changes in phase and frequency of the rotor, the changes in amplitude involve a lag due to the quasi-static assumptions in the signal processing technique.

The next stage uses the frequency domain representation of the measured vibration in a parameter estimation algorithm to update the estimates of the transfer relationship between the measured vibration from the sensors and the actuator demands, at the blade passing frequency. The transfer relationship is a quasi-static linear model of the aircraft structural dynamics at the blade passing frequency. The estimation algorithm is based upon recursively adjusting the estimate of the structural dynamics according to the error between the internal controller prediction of the vibration and the actual measurement. The estimator provides the controller with robustness, since it is able to adapt to changes in the fuselage dynamics caused by variations in the main rotor speed and changes in the flight condition which causes variations in the airframe loading and distribution. The final stage uses the updated parameter estimates and the measured vibration in an optimal control algorithm to generate the actuator demands required to minimise the structural vibration for the next control cycle. The optimal
control algorithm uses a quadratic performance index, which comprises the weighted sum of the squares of the \( n_\omega \) fuselage vibration and the \( n_\omega \) actuator force inputs. The sine and cosine components for the control actuators are then transformed to a time domain signal at the end of the controller calculations during the scheduling phase, such that the frequency of the sinusoidal control forces is synchronised to the controller reference signal.

Control system adaptability is therefore provided through the periodic updating of the parameter estimates, which results in robustness to changes in structural dynamics, variations in rotor speed, changes in rotor head loadings and to errors in the initial parameter estimates of the transfer function relationship.

Making the control system adaptive allows the algorithm to be implemented without a priori knowledge of the system being controlled. It also has the advantage that it is readily transportable from one helicopter to another, since it does not require flight testing to generate controller parameters that are functions of flight condition and aircraft configuration.

4.3.2 Frequency Domain Cost Function

In the frequency domain the basic performance index has the form:

\[
J = \sum_{q=1}^{k} W_q Y_q^2 + \sum_{s=1}^{\delta} R_s U_s^2 = Y^* [W] Y + U^* [R] U
\] (4.2)

The matrices \([W]\) and \([R]\) are matrices of weighting factors for sensors and actuators respectively, and \(Y^*\) represents the complex conjugate of \(Y\). Complex conjugates are used since the \(Y\) and \(U\) vectors are phasors representing the \(n_\omega\) frequency components of the measured vibration and control inputs respectively. With zero off diagonal elements the second representation of the index can be used, where the weighting elements \(W_q\) and \(R_s\) are the diagonal terms. The weighting matrix \([W]\) allows for the possibility of certain fuselage locations being more important with respect to vibration than others, although usually equal weighting has been used.

An advantage of the frequency domain performance index is that it is relatively easy to include extra design goals or constraints, for example the performance index (or cost function) defined by equation (4.2) can be extended to include a term which penalises the rate of change of the actuator control force magnitude. This allows the controller to exercise 'caution' through
indirect rate limiting of the control inputs. Rate limiting has been found to be very important for enhanced controller stability [Davis, 1984] and performance for nonlinear systems, or for systems where initial parameter estimates are poor. This is achieved using the matrix \([R_{u}]\), which weights the incremental control vector \(\Delta U\) (i.e. penalises large control changes).

\[
J = J' [W] Y + U' [R] U + \Delta U' [R_{u}] \Delta U
\]  \hspace{1cm} (4.3)

Since the control input is specified in the frequency domain this weighting has the effect of limiting the change in magnitude of the control signal in the time domain between controller cycles. The weighting matrix \([R]\) constrains the amplitude of the control, and \([R_{u}]\) constrains the rate of change of control. The above performance index is for a deterministic controller and is based upon the assumption that all parameters are known explicitly and ignores the fact that only estimates are available.

Alternatively, terms representing the uncertainty of the estimated values of the model may be included in the cost function definition and the controller is now termed stochastic. Caution is introduced by these terms since large changes of input will not be made if the uncertainty of identification is high.

Many physical systems have inherent uncertainties either in the system itself or in the measurements made, and these uncertainties can be appropriately modelled as stochastic processes. In some of these cases the use of deterministic control theory may not be possible or may be inappropriate, and then stochastic control theory can be used [Athans and Mitter, 1977]. The stochastic nature of the vibration problem arises primarily due to three effects:

1) the instrumentation used to measure the vibration amplitude is contaminated with random noise
2) the amplitude of vibration varies with flight condition and is not precisely known, and
3) the parameters of the control system model relating ACSR inputs to vibration outputs vary with flight condition and are only approximately known.

Since the vibration amplitude and fuselage model vary with flight condition and require a statistical description, a stochastic control formulation can be used.

The stochastic controllers make allowance for the fact that the \([T]\) and \(B\) matrices are estimated by taking the expected value of the performance function [Davis, 1983][Johnson, 1982]. Thus, the stochastic cost function involves uncertainty and has the form
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\[ J_s = E \left[ \sum_{j=1}^{k} W_j Y_j^2 + \sum_{c=1}^{a} R_c U_c^2 + \sum_{c=1}^{a} R_{\Delta U_c} \Delta U_c^2 \right] \]  

(4.4)

where \( E[.] \) denotes the expected value. The elements \( W \) and \( R \) are basically chosen to satisfy the primary design criteria, but in this case also reflect the uncertainty in the measurements of \( Y \) and \( U \) respectively. The use of this performance index gives rise to the stochastic controller. The control algorithm is therefore based upon the minimisation of a performance index that is the expected value of a weighted sum of squares of the measured vibration and the actuator control inputs.

A further extension is the cautious controller which recognises that some parameters are only available as estimates. The cautious controller attempts to account for these uncertainties by including a term involving the covariance of the estimated parameters (covariance is a statistical matrix indicating the relative uncertainty in an associated set of variables). This term is clearly dependent upon the uncertainty in the estimated T-matrix. As the covariance of the T matrix increases, the rate of change in the control allowed is reduced. This results in reduced amplitude, or cautious, control for uncertainty in the parameter estimates. The larger the uncertainty in the transfer matrix \([T]\), the larger the covariance \([P]\), and thus the controller exhibits cautious control. The cautious control reduces to the deterministic control as the parameter uncertainties approach zero.

Clearly the effect of the term involving the covariance matrix is similar to the effect of the incremental control weighting matrix \([R_{\Delta U}]\), however the rate limiting effect due to this term will depend upon the uncertainty in the identified T matrix.

Hammond [1980] investigates both deterministic and stochastic HHC controllers using a linearised global model. To limit control input magnitudes the deterministic controller used external rate limiting, whereas the stochastic controller used a stochastic caution term which reflected the level of uncertainty present in the estimation of the identified system parameters. The results from this investigation showed that the performance of the cautious controller was slightly better than the deterministic controller, in that it was less erratic.

Molusis et al [1980] extended the work reported by Hammond, and confirmed Hammonds finding that the cautious controller was superior to the cautious controller with external rate limiting, this work was also in the field of HHC.
An adaptive vibration controller has two tasks to perform:

1) it must estimate the system parameters on line, and
2) it must exercise control in order to minimise fuselage vibration.

The estimation algorithm identifies the parameters most accurately when the vibration levels and the control signals are large, since the signal to noise ratio is maximised. The control algorithm aims to reduce the fuselage vibration with minimal control effort. Clearly, to some extent these two tasks conflict. This dual property is widely known in stochastic control theory, and was pointed out by Feldbaum [1973]. As discussed in Wittenmark [1975], the two tasks of trying to improve system identification and of trying to provide control are, in general, counter productive. Good identification may require large control inputs, while good control may require small control inputs.

In an attempt to deal with this conflict a class of adaptive controllers known as Dual Controllers has emerged. Mookerjee et al [1986] investigate the use of dual control concepts applied to adaptive HHC vibration controllers for helicopters. A dual or active-adaptive controller (Goodwin and Payne, 1977), actively probes the system to reduce the parameter errors. The control is used for learning, to improve the parameter estimates, but at the expense of short term deterioration of the closed loop performance. Optimal dual controllers have so far had limited success due to their complexity, and have not been considered further in this thesis.

The general cost function used in the implementation of the Local Linear control algorithms includes extra terms for rate limiting and stochastic caution, and is defined as:

\[ J = Y'WY + U'R^1U + \Delta U'(R_{\Delta U}) \Delta U + \Delta U'(C_\rho P) \Sigma \{W\}_{ij} \Delta U \quad (4.5) \]

and for the Global control algorithms is:

\[ J = Y'WY + U'R^1U + \Delta U'(R_{\Delta U}) \Delta U + U'(C_\rho P) \Sigma \{W\}_{ij} U \quad (4.6) \]

where \([R_{\Delta U}]\) is a matrix of weighting factors for the change in control inputs and \([P]\) is the covariance matrix, and is calculated by the parameter identification routine. The parameter \(C_\rho\) is a switch which is set to 1 for a cautious controller and 0 for a deterministic controller formulation since in this formulation the system parameters are assumed to be known explicitly. By making \(C_\rho\) a value other than 0 allows added flexibility to be included in the cost function, this effectively weights the influence of the caution term. The deterministic formulation ignores the fact that only estimates for the \(T\) matrix (and also the background vibration \(B\)) are available from the parameter estimation.
As system identification becomes worse the elements in the covariance matrix increase representing the increased uncertainty in the T matrix, thus the controller becomes more cautious. As system identification improves and the covariance Matrix [P] tends to zero, the term vanishes and the performance index reduces to that for the deterministic controller.

The optimal control algorithm is obtained by solving the partial derivative of the cost function J defined earlier, with respect to the control input U, i.e.

\[
\frac{\partial J}{\partial U} = 0
\]  \hspace{1cm} (4.7)

and using the equation for the quasi-static frequency domain representation of the helicopter model, equation (4.1). Obviously, for the local control algorithms the partial derivative is taken with respect to \( \Delta U \). Minimisation of the global performance index using the global formulation equation (4.1) produces an optimal control law as follows:-

\[
\Delta U = -[G] \left[ \begin{bmatrix} T & W \end{bmatrix} \begin{bmatrix} T & W \end{bmatrix} + \begin{bmatrix} R \end{bmatrix} + C_r P \sum_j W_j \right] U + \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} B \end{bmatrix} + C_r P \sum_j W_j \right] \]  \hspace{1cm} (4.8)

where

\[
[G] = \left[ \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} T \end{bmatrix} + \begin{bmatrix} R \end{bmatrix} + [R_{\Delta U}] + C_r P \sum_j W_j \right]^{-1}
\]  \hspace{1cm} (4.9)

The resulting expression for the global optimal control input given by equation (4.8) depends upon an estimate of the uncontrolled vibration level B.

The system model represented by equation (4.10) is termed the local model to indicate linearisation of the system T matrix about the current operating point. In contrast the global model linearises the system T matrix about a zero input.

\[
\Delta Y = [T] \Delta U \]  \hspace{1cm} (4.10)

The equivalent algorithm for the local linear model can be obtained by substituting an incremental control vector \( \Delta U \) for U.

\[
\Delta U = -[G] \left[ \begin{bmatrix} R \end{bmatrix} U + [T]'W \right] \]  \hspace{1cm} (4.11)

As can be seen from equation (4.10) the local linear optimal control law, which is a linearisation of the response about the current operating point, depends upon the measured vibration from the previous controller cycle.
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Note that the update in control for the local model is dependent on an estimate of the $T$ matrix and the computed vibration response from the last update $Y$. For the local model the covariance matrix $[P]$ is the covariance of the $T$ matrix, since only the $T$ matrix is identified. For the global model, the control update is based on an estimate of both the $T$ matrix and the uncontrolled vibration $B$. For the global model the covariance matrix now includes the covariance of the $T$ matrix and $B$, since both are identified.

For practical implementation, where during each control cycle (denoted by the subscript $N$) the controller measures the fuselage vibration, and calculates the optimal actuator inputs for the next controller cycle, equations (4.8) and (4.10) can be re-expressed as:

$$U_{N+1} = U_N - [G][([T]W)[T] + [R] + C \Sigma \Sigma_j [W]_y] U_N + [T]W B + C \Sigma \Sigma_j [W]_y$$

and

$$U_{N+1} = U_N - [G][[R]U_N + [T]W Y_N]$$

respectively.

Deterministic controllers for Higher Harmonic Controllers have been investigated by Shaw and Albion [1980], and Johnson [1980]. Johnson also investigated stochastic HHC strategies. Molusis et al [1981] extended Johnson's work to include cautious controllers.

Figure 4.2 shows the frequency domain adaptive control scheme. The actuator inputs and accelerometer outputs are transformed to the frequency domain, and then a Recursive Least Squares (RLS) estimator is used to provide an estimate of the receptance matrix $[T]$, which provides adaptation to changes in structural dynamics, aircraft dynamics and flight condition. The information from the estimator is then used by the control law designer, which determines the control vector for the actuator forces to minimise the cost function equation. The actuators are then driven by sinusoidal signals at the vibration frequency with amplitudes and phases defined using this vector until the next measurement period updates its values.

4.3.3 Summary

The frequency domain controller provides feedback control but it is fundamentally designed to minimise the steady state fuselage response. One of the main problems with the use of the frequency domain techniques relates to the assumption of a static linear background model.
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This means that the magnitudes of actuator control forces are fixed for the duration of each controller cycle.

For the first 2/3 of each cycle, the algorithm uses signal processing to extract the blade passing frequency content from the fuselage vibrations, with the consequence that the response to changes can be slow. Performance of the frequency domain control system to fast manoeuvres and disturbances is limited by the requirement to extract accurately the steady forcing component from the transient response. In reality this involves a delay in the controller update time, while the transients from the previous actuator input are allowed to decay. The active control system can take several controller iterations (several seconds) to adapt, and this may result in a rise in fuselage vibration until the controller catches up.

The performance of the controller is therefore limited by its update rate, since this determines the time taken for the controller to respond to changes. Since the controller is based upon quasi-static principles (i.e. the need to accurately measure the steady state response of the airframe), a fast update rate may make the assumption invalid and lead to poor or even unstable controller performance. Therefore a reduction in the controller update rate should improve the response of the controller during manoeuvres. Reducing the update rate too far may lead to instabilities [Staple 1989].

In summary, the main advantages of the frequency domain algorithm are:

- It is inherently adaptive, but only at discrete instants in time;
- Actuator dynamics are implicitly incorporated into the structural model by the algorithm;
- The frequency domain transfer function is independent of pre-assumed models - this is important for the initial assessment of natural system order, dominant mode locations, control spillover and stability characteristics;
- The estimated frequency domain model, though only valid at a single frequency, can be measured quite accurately.
- Extra terms to limit the rate of change of control magnitudes, or to make the controller cautious for example, can be added to the cost function with relative ease.

The main disadvantages of the algorithm are:

- It only controls at a single frequency (\( n_0 \), the blade passing frequency);
- Its operation is cyclic, i.e. unchanging during a measurement cycle;
- The quasi-static nature limits its ability to track rapidly varying effects.
4.4 LINEAR QUADRATIC CONTROL APPROACH

Using the time domain formulation of the helicopter mathematical model, linear quadratic multivariable control techniques may also be applied to the vibration reduction problem. This is no longer quasi-static, giving effective control through the transient changes, and vibration reductions over all frequencies are possible.

The time domain strategies are based on the direct feedback of the vibrations, through an optimal gain matrix to give the control forces for the actuators, in order to minimise the structural vibration. Therefore, the frequency component does not need to be extracted from the measured vibration, and consequently vibration reductions over all frequencies are possible.

Since there is no need to identify the data in the frequency domain, or to allow structural transients to decay, the dynamic response of a time domain controller should be more rapid than that which could be obtained by a frequency domain controller. In theory time domain methods are only limited by the operational bandwidth of the actuators used, although the main benefit is the potential for an enhanced transient response. Time domain strategies generate the control inputs from the continuous feedback of the vibration time response. In principle the controller is much more simple (consisting only of a gain matrix) and provides effective control over the range of frequencies for which the model is representative of the real system.

Much literature exists on optimal control, this include Athans [1966], Bryson and Ho [1969], Kwakernaak and Sivan [1972], and more recently Grimble [1986]. The theory has crystallised from research in the 1960's, which is reported in text books and published literature, for example Bellman [1957], Athans and Falb [1966], Noton [1972], and others. The bibliography by Mendel and Gieseking [1971] on Linear quadratic control contains over 900 references. The main advantage of Linear Quadratic techniques is the ease with which multivariable systems can be handled. Disturbances can also be dealt with as long as they can be appropriately modelled as Gaussian white noise processes.

The robustness result of Safonov and Athans [1977] showed that multivariable linear quadratic optimal controllers have very good robustness properties, including guaranteed classical gain margins of -6dB to 60dB and phase margins of 60° in all channels. However, if the full state vector is not available and a state estimator is used (Observer or Kalman Filter) then there is no guarantee that these robustness properties will hold. In fact, stability can be compromised [Doyle, 1978]. Although there are many applications of optimal control, few have been in the area of helicopter vibration control.
DuVal and Saberi (1988) describe the use of a feedback control law using Linear Quadratic Theory (LQR), for a HHC system. This solution required the feedback of all states, which included a number of rotor states which are unmeasurable and would need to be estimated.

The state space formulation of the helicopter model was described in Chapter 3. The states are modal displacements and velocities, and the outputs are the fuselage accelerations measured by the sensors. The models used by the LQ controller are generated from the Finite Element models of the fuselage. The large system order precludes the active control of the entire state vector and hence the controller is designed using a reduced order model. Fuselage models are truncated with respect to modes of vibration, and so only the control of a relatively small number of modes is described. The selection of the modes was also discussed in Chapter 3. The reduced order model must be chosen and controlled in such a manner that the entire system remains stable and meets performance specifications. It is well established that the uncontrolled modes affect system performance and can cause instability due to control and observation spillover.

The subset of modes is selected and a modern controller, consisting of a state estimator and a linear feedback control law, is designed as though no other modes were present in the structure.

The familiar form of a time domain quadratic performance index can be used:

\[
J_T = \int_0^T \left( x_s^T(t) [Q] x_s(t) + u^T(t) [R] u(t) \right) dt
\]  

(4.14)

If all the parameters in the model are known, a deterministic controller is obtained. With unknown, estimated parameters, the certainty-equivalence principle may be applied, then the deterministic control solution is used with the estimated parameter values. Alternatively, a cautious controller (Wittenmark, 1975) can be obtained by minimising the expected value of the performance index.

\[
\dot{x}_s = [A_s] x_s + [B_s] u_s + v_s
\]  

(4.15)

\[
x_s = [C_s] x_s + [D_s] u_s + v_y
\]  

(4.16)

\(v_s\) and \(v_y\) are disturbance vectors containing white, Gaussian noise. The other parameters in the above equations were defined in Chapter 3.
With the structural model in state space form, equations (4.15) and (4.16), it is possible to find a linear time domain feedback control law, and the optimal control vector using full state feedback is given by

\[ u(t) = -[K_s] \dot{x}_s(t) \]  \hspace{1cm} (4.17)

where, \( x_s \) represents a vector of modal displacements and velocities, \( [K_s] \) is the feedback gain matrix and is given by:

\[ [K_s] = [R]^{-1} [B_s]^T [P] \]  \hspace{1cm} (4.18)

and the matrix \( P \) is a positive definite solution of the Matrix Riccati equation. The generation and solution of the Matrix Riccati equation is described in Appendix IV.

As with the Frequency domain control strategy, the stochastic nature of the control problem can be included in the controller formulation by taking the expected value of the cost function and using information about the uncertainty in the measurements.

\[ J_s = E \left[ \int_0^T \begin{pmatrix} x_s^T(t) & u_s^T(t) \end{pmatrix} \begin{bmatrix} Q_s & R_s \end{bmatrix} \begin{bmatrix} x_s(t) & u(t) \end{bmatrix} dt \right] \]  \hspace{1cm} (4.19)

where \( [Q_s] \) and \( [R_s] \) are the covariance matrices of the disturbances \( v_s \) and \( v_u \) respectively. This is known as the LQG feedback design problem [Stein and Athans, 1987].

### 4.4.1 Optimal Control with Output weighting

The aim of the strategy is to control the fuselage accelerations. Since accelerations are the outputs from the state space model, the cost function needs to be modified. For the case where direct control of the output vector is required, a performance index, such as equation (4.20) is minimised

\[ J_y = \int_0^T \begin{pmatrix} y_s^T(t) & u_s^T(t) \end{pmatrix} \begin{bmatrix} Q_y & R_y \end{bmatrix} \begin{bmatrix} y_s(t) & u(t) \end{bmatrix} dt \]  \hspace{1cm} (4.20)

with

\[ \dot{x}_s = [A_s] x_s + [B_s] u_s \]  \hspace{1cm} (4.21)

\[ y_s = [C_s] x_s + [D_s] u_s \]  \hspace{1cm} (4.22)
The matrix Riccati equation for this formulation is:

\[ \dot{P}(t) = -P(t)A_s - A_s^*P(t) - Q_o + P(t)R_o^{-1}B_s^*P(t) \]  \hspace{1cm} (4.23)

where

\[ A_o = A_s - B_s \left( R + D_s^*Q_wD_s \right)^{-1}D_s^*Q_wC_s \]  \hspace{1cm} (4.24)

\[ R_o = R + D_s^*Q_wD_s \]  \hspace{1cm} (4.25)

\[ Q_o = C_s^* \left[ Q_w - Q_wD_sR^{-1}D_s^*Q_w \right] C_s \]  \hspace{1cm} (4.26)

The optimal control vector is defined by equation (4.27)

\[ u_s(t) = - \left( R + D_s^*Q_wD_s \right)^{-1} \left[ D_s^*Q_wC_s + B_s^*P(t) \right] x_s(t) \]  \hspace{1cm} (4.27)

and

\[ K_s(t) = \left( R + D_s^*Q_wD_s \right)^{-1} \left[ D_s^*Q_wC_s + B_s^*P(t) \right] \]  \hspace{1cm} (4.28)

P(t) is a positive definite solution of the matrix Riccati equation (4.23). The full derivation of this controller is given in Appendix IV.

4.4.2 Optimal Control with Frequency Weighted Outputs

As highlighted in previous sections a draw back in the straight forward application of state feedback techniques is that the states contain all frequencies. Therefore, a state feedback controller designed to minimise the no frequency component may aggravate unmodelled dynamics.

Work by Gupta and DuVal [Gupta 1980, Gupta and DuVal 1981, DuVal et al 1981] has shown that it is possible for frequency dependent penalties to be placed on states and controls. The frequency shaping approach, in essence, augments the plant with frequency shaped filters so as to penalise their outputs in addition to other cost terms in the performance index.

This frequency weighting is also desirable in this application since the fuselage vibration is dominated by the blade passing frequency, and therefore it is sensible to place more control effort on reducing vibration at this frequency, as opposed to weighting all the frequencies equally.
The method adopted here is to pass the outputs (fuselage vibration measured by accelerometers) in which vibration is to be suppressed, through a second order filter tuned to resonate at the blade passing frequency $n_0$. Standard Linear Quadratic design techniques can then be applied to obtain the feedback gains [Bryson and Ho, 1969]. The control law obtained from this procedure is simple to implement because it is, in effect, a constant gain regulator with filters in the feedback loops.

The frequency shaped controller is based upon a state space model of the plant and optimizes a cost function which places a large weighting on the $n_0$ component of the fuselage vibration. The frequency shaping filters, appended to the plant outputs, effectively penalise the bandwidth of interest in the LQ problem formulation. These filters can be thought of as providing dynamically compensated outputs for feedback control. Only that frequency which is dominant in the vibration spectrum is fed back through the controller so as not to excite any other vibration modes.

In order to place the outputs of the filters in the cost function a modification to the standard LQ cost functional is required. The continuous time state space model has the form

$$\dot{x}_s = [A_s] x_s + [B_s] u_s \tag{4.29}$$

$$y_s = [C_s] x_s + [D_s] u_s \tag{4.30}$$

Subject to these dynamical constraints, minimisation of the following frequency shaped cost functional, proposed by Gupta and DuVal [1981], is desired

$$J_F = \int_{-\infty}^{\infty} \left[ Y^*(j\omega) A(j\omega) Y(j\omega) + U^*(j\omega) B(j\omega) U(j\omega) \right] d\omega \tag{4.31}$$

where

$$A(j\omega) = P^*_m(j\omega) P_m(j\omega) \tag{4.32}$$

$$P_m(j\omega) = P_r(j\omega) \text{diag}(\alpha_1, \alpha_2, \ldots) \tag{4.33}$$

$$B = I \tag{4.34}$$

The $\alpha_i$'s are scalar weighting factors. $P_r(s)$ is the frequency shaped penalty function or filter.

In order to reduce vibrations at the $n_0$ frequency, the function $A(j\omega)$ must have a large gain at the $n_0$ frequency, therefore a simple second order transfer function is used to define a filter tuned to $n_0$.  

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The poles and zeros of $P_p(s)$ are determined by $h_0$, $h_1$, $h_2$, $\zeta_1$, and $\zeta_2$. Assuming a measurement of the rotor angular velocity is available, the filters can be continuously tuned to resonate at the blade passing frequency $n\omega$, thus tracking any variations in the rotor frequency.

The frequency weighting provided by filter $P(s)$ is shown in Figure 4.3, for the following filter parameters $h_0 = h_2 = 0$, $h_1 = 1$, $\zeta_1 = \zeta_2 = 5\times 10^{-4}$, this level of damping may prove difficult to implement practically, but has been used here for comparison purposes. The filter structure defined by these parameters provides a gain of unity at the blade passing frequency, which allows the same weighting matrices used in the Frequency domain controller to be used for this controller. The lower the damping in the filter the larger the weighting given to the centre frequency of the filter, however this also makes the filter sensitive to variations in the blade passing frequency. Therefore using a small amount of damping makes the system more robust but at the expense of vibration reduction at the centre frequency.

A state space model for the frequency shaping filters can be obtained from the transfer function $P_p(s)$.

\[
P_p(s) = \frac{h_0 s^2 + h_1 2\zeta_1 n\omega s + h_2 (n\omega)^2}{s^2 + 2\zeta_2 n\omega s + (n\omega)^2}
\]

\[\dot{x}_n = [A_n]x_n + [B_n]Y_n \]

\[Y_n = [C_n]x_n + h_0 Y_1\]

where

\[[A_n] = \begin{bmatrix} 0 & 1 \\ -(n\omega)^2 & -2\zeta n\omega \end{bmatrix}\]

\[[B_n] = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}\]

\[[C_n] = \begin{bmatrix} 1 & 0 \end{bmatrix}\]
Control Strategies

\[ \beta_1 = 2 \zeta n \omega (h_1 - h_0) \]  
(4.41)

\[ \beta_2 = (n \omega)^2 [h_2 - h_0 + 4 \zeta^2 (h_0 - h_1)] \]  
(4.42)

\( x_n \) is now a controlled variable. That is, the vibration measurement at the \( n \omega \) frequency is now heavily weighted. Finally, substitution of (4.33) into equation (4.31), and using Parsevals theorem, equation (4.31) can be rewritten as

\[ J_f = \int_0^1 \left[ Y_1^T(t) [Q_f] Y_1(t) + u^T(t) [R_f] u(t) \right] dt \]  
(4.43)

where

\[ Q_f = \text{diag}[\alpha_1^2, \alpha_2^2, \ldots, \alpha_N^2] \]  
(4.44)

Thus the frequency weighted optimisation problem is transformed into a Linear Quadratic design problem, by augmenting the state space model with additional states corresponding to those of a state space time domain realisation of the frequency shaping filters.

Augmenting the helicopter state space model with the filter states produces the state space model given by equations (4.45) and (4.46). This state space formulation can then be used to design the control law.

Augmented state space model for frequency weighted output control

\[
\begin{bmatrix}
X_S \\
X_{11} \\
X_{12} \\
\vdots \\
X_{1N}
\end{bmatrix} = 
\begin{bmatrix}
A_S & 0 & 0 & 0 & \ldots & 0 \\
B_i C_S(1,:) & A_f & 0 & 0 & \ldots & 0 \\
B_i C_S(2,:) & 0 & A_f & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
B_i C_S(N,:) & 0 & 0 & 0 & \ldots & A_f
\end{bmatrix}
\begin{bmatrix}
X_S \\
X_{11} \\
X_{12} \\
\vdots \\
X_{1N}
\end{bmatrix} + 
\begin{bmatrix}
B_{S} \\
B_{S}(1,:) \\
B_{S}(2,:) \\
\vdots \\
B_{S}(N,:)
\end{bmatrix}
\begin{bmatrix}
\mu_S \\
\mu_{S}(1,:) \\
\mu_{S}(2,:) \\
\vdots \\
\mu_{S}(N,:)
\end{bmatrix}
\]

(4.45)
The quadratic cost function defined by equation (4.43), can be re-expressed in terms of this extended model

\[
J_E = \int_0^t \left[ x_E^T(t) [Q_E] x_E(t) + u^T(t) [R_E] u(t) \right] dt \tag{4.47}
\]

where \( x_E = [x_0 \ Y_{11} \ Y_{12} \ Y_{13} \ldots \ Y_{1N}]^T \). Thus the frequency shaped cost function can be transformed to a time domain Linear Quadratic Regulator problem. The solution of the linear quadratic problem with output weighting was described in the previous section, and provides a matrix of optimal feedback gains, \( G \), and the control law is of the form

\[
u_s(t) = -[G] x_e(t) \tag{4.48}
\]

where \( x_e = [x_0 \ x_{11} \ x_{12} \ x_{13} \ldots \ x_{1N}]^T \), i.e. the state vector includes the original states of the helicopter model (which may need to be estimated) and the states of the each of the filters on the outputs. Obviously these do not need to be estimated since they are generated internally by the controller, but they do add considerably to the overall complexity of the controller.

4.4.3 Output Feedback

In many practical instances of optimal controller design, the full state feedback information required to implement the steady state control described in the previous sections is not available. Such controller implementations fall into the category of output feedback, this category can be further divided into two sections.

The first section consists of controllers utilising direct output feedback, i.e. an output feedback system applies control gains directly to sensor outputs, and these signals are fed back to the control actuators [Porter, 1977; Levine and Athans, 1970]. A number of output feedback implementations can be found in the control of flexible spacecraft, for example Canavin [1978], Balas [1979], Strunce and Carman [1984]. Strunce et al [1978] describe a number of output feedback strategies, including pole assignment and suboptimal output feedback. A major
Control Strategies

\[
\begin{bmatrix}
Y_s \\
Y_{f1} \\
Y_{f2} \\
\vdots \\
Y_{fN}
\end{bmatrix} =
\begin{bmatrix}
C_s & 0 & 0 & 0 & \ldots & 0 \\
0 & C_f & 0 & 0 & \ldots & 0 \\
0 & 0 & C_f & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \ldots & C_f
\end{bmatrix}
\begin{bmatrix}
x_s \\
x_{f1} \\
x_{f2} \\
\vdots \\
x_{fN}
\end{bmatrix} +
\begin{bmatrix}
D_s \\
h_0 D_s(1,:) \\
h_0 D_s(2,:) \\
\vdots \\
h_0 D_s(N,:)
\end{bmatrix} u_s
\]

extended model

\[
J_{ex} = \int_0^1 [y_{ext}^T(t) [Q_e] y_{ext}(t) + u^T(t) [R_e] u(t)] dt
\]

where \( y_{ext} = [y_s \ Y_{f1} \ Y_{f2} \ \ldots \ldots \ Y_{fN} ]^T \). Thus the frequency shaped cost function can be transformed to a time domain Linear Quadratic Regulator problem. The solution of the linear quadratic problem with output weighting was described in the previous section, and provides a matrix of optimal feedback gains, \( G \), and the control law is of the form

\[
u_{ext}(t) = -[G] x_{ext}(t)
\]

where \( x_{ext} = [x_s \ x_{f1} \ x_{f2} \ \ldots \ldots \ x_{fN} ]^T \), i.e. the state vector includes the original states of the helicopter model (which may need to be estimated) and the states of the each of the filters on the outputs. Obviously these do not need to be estimated since they are generated internally by the controller, but they do add considerably to the overall complexity of the controller.

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limitation of many of these techniques is that they assume that the feedforward matrix is zero (i.e. $D = 0$), which in this application it is not.

This approach can be compared to the linear optimal control approach, which uses a state estimator to reconstruct the state vector and applies control gains to this estimated state as though it were the actual state vector. With the direct output feedback approach no state estimator is involved and consequently the internal complexity of the controller is reduced.

With state estimation the control law is designed assuming that the full state vector is available. Then the state estimator is designed that recreates the states which are not available. In a deterministic framework the state estimator is called an observer or Luenberger observer [Luenberger, 1966 and 1971]; in a stochastic framework it is known as a Kalman Filter [Kalman and Bucy, 1961]. The observer or Kalman filter uses both the control inputs and available outputs of the system whose state is to be approximated.

The observer states $\hat{x}_e(t)$ are driven towards the plant states $x_s(t)$ by comparing the measured output of the plant $y_o(t)$ with the recreated variables $y_e(t)$, and using the resulting error to drive the observer model. An observer gain matrix $[K_o]$ has to be chosen, such that it ensures stability and that it will follow the plant closely.

The observer system is formulated for the continuous time case as follows:

$$\dot{\hat{x}}_e = [A_e]x_e + [B_e]u_s + [K_o](y_s - y_e) \quad (4.49)$$

Defining the estimation error as the difference between the estimated state vector and the actual state vector i.e. $e = x_s - \hat{x}_e$, then the error is governed by

$$\dot{e} = ([A_s] - [K_o][C_s])x_s - ([A_e] - [K_o][C_e])x_e - ([B_s] - [B_e]) + [K_o][D_s] - [K_o][D_e]u_s \quad (4.50)$$

which simplifies to the following equation if the estimator dynamics match the system dynamics

$$\dot{e} = ([A_s] - [K_o][C_s])e \quad (4.51)$$

The observer gain matrix $[K_o]$ determines the speed with which these errors are reduced. Increasing the gain of the feedback matrix will cause the observer to follow the plant more closely, it also makes it more susceptible to noise so that observer design is a compromise
between state reconstruction and noise rejection.
In practice the observer dynamics will be a subset of the actual system dynamics, giving rise
to the effects observation and control spillover discussed earlier (refer to Figure 4.1).
The observer gain matrix can be determined using any of several methods. For simple systems
a trial and error method can be used, but for more complex systems a more formalised system
is needed, for example the Kalman filter [Kalman and Bucy, 1961].

By defining $[Q_v]$ and $[R_v]$ as the covariance matrices of the disturbances $v_x$ and $v_y$ respectively,
an optimal estimator gain matrix can be calculated by solving an algebraic Riccati equation
which is the dual of the one in the optimal control problem [Kwakernaak and Sivan, 1972]. The
estimator (or Kalman gain) is then defined as:

$$ K_o = \hat{P} C_s^T \hat{R}^{-1} $$  \hspace{1cm} (4.52)

The estimator gain is optimal in the sense of providing a state vector that is the best weighted
least squares estimate of the actual state vector in the presence of disturbances.

4.4.4 Adaptive Control for Time Domain Strategies
A fuselage model derived for one flight condition or structural configuration may prove
unsuitable for other conditions. Therefore the controller may need to be adaptive. The complete
controller consists of a number of subsystems. The controller subsystem is implemented using
a Linear Quadratic control law. The state estimation subsystem utilises a Kalman filter whose
system parameters are the outputs of a multi-input multi-output parameter identification
algorithm.

The controller subsystem uses the outputs from the estimation and identification subsystem
to calculate feedback control parameters. In a full-state feedback controller, an estimator is
then used to obtain optimal estimates of the states in the dynamical model of the system based
on past and present inputs and outputs of the system.

4.4.5 Summary
The time domain strategies are based on the direct feedback of the vibrations, through an
optimal gain matrix, to give the control forces for the actuators, in order to minimise the
structural vibration. Since there is no need to identify the data in the frequency domain, or to
allow structural transients to decay, the dynamic response of a time domain controller should be more rapid than that which could be obtained by a frequency domain controller. In theory time domain methods are only limited by the operational bandwidth of the actuators used, although the main benefit is the potential for an enhanced transient response. Time domain strategies provide effective control over the range of frequencies for which the model is representative of the real system.

It was observed previously that with the frequency domain controller any actuator dynamics are implicitly incorporated into the model. However, with the time domain controller, actuator dynamics need to be modelled and incorporated explicitly into the structural model. Combined with the fact that the structural dynamics are not accurately known and may also change with time, the essential simplicity of the controller itself is therefore overthrown by the need for adaptation, for example on the basis of parameter estimation.

Since an optimal approach gives a natural solution to minimising the measured vibration, a self-tuning adaptive controller is an obvious choice. Figure 4.4 shows the scheme, in which a recursive least squares parameter estimator provides model parameters for an optimal control law designer [Fortescue et al, 1981], which in turn sets the coefficients for the gain matrix.

The state vector consists of modal velocities and displacements and therefore some form of modal filtering using observers or Kalman filters will be required. Since an Observer consists of a model of the plant, the combination of an optimal controller and an observer results in a controller with a similar order to that of the system being controlled. With regard to the frequency weighted LQ control, the states of the frequency shaped filters are artificial and can be calculated based on the frequency shaping filter parameters, which are known and do not therefore need to be identified and consequently do not add to the observer complexity.

In summary, the main features of the time domain approach are:

- It is computationally simpler to implement than the frequency domain strategy, as long as an observer is not needed;
- Vibration reduction over a range of frequencies is possible;
- The time domain algorithm has a superior transient response, giving the potential to respond to fast manoeuvres, changes in dynamics caused by stores release or rapid changes in forcing caused by turbulent disturbances;
- The method is naturally suited to MIMO identification since the model can be of arbitrary order and structure.
- It is readily designed and analyzed using well known linear systems theory.
The main disadvantages are:

Actuator dynamics need to be modelled and incorporated explicitly into the structural model, hence results are dependent on presumed model structure and order;
Since the states contain all frequencies, a vibration minimising controller utilising state feedback may excite unmodelled fuselage dynamics, though recent advances in LOG methods allow for frequency dependent weightings to be placed on states and controls.
The optimal controller requires the feedback of all states. In practice it may not always be possible to have access to all of the states and therefore an observer or filter is required to reconstruct the unavailable states, adding further to the controller's complexity.
The calculation of an optimal gain matrix in a real time implementation may prove difficult due to the large dimension of the system.
Sensitive to unmodelled dynamics.

4.5 HYBRID CONTROL APPROACH

In this section the concept of a hybrid time/frequency domain control algorithm is introduced, combining some of the desirable characteristics from the frequency and time domain control algorithms discussed previously. Before discussing the Hybrid control approach, it is informative to discuss the motivation for its derivation in terms of the limitations and requirements of the time and frequency domain approaches.

4.5.1 Limitations of Frequency and Time Domain Approaches
The frequency domain controller is inherently adaptive, and the dynamics of both the structure and the actuators are implicitly incorporated into the frequency domain model, which can be accurately measured. The effects of structured and unstructured uncertainties in the system are included in the measurements, and even non-linearities are accommodated as long as a local linearisation about the operating point is appropriate. The main limitation is that it is fundamentally an approach for dealing with steady-state vibrations: the rotor head forces and corresponding fuselage vibration may change rapidly during manoeuvres, but it can only react to such changes via the process of adaptation, a process which has a relatively slow update rate to ensure the accuracy of the frequency domain model.
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From previous research studies (Pearson and Goodall, 1991), it has been shown that the frequency domain controller, though only strictly correct at a single frequency (n0), is robust to changes in the forcing frequency, maintaining good vibration reduction for such changes. The time domain controller offers the possibility of vibration reduction over a range of frequencies with a superior transient performance allowing fast changes in loading or forcing to be controlled. The main difficulty is that structural and actuator dynamics must be incorporated explicitly into the model, and since adaptation is essential the results are dependent upon the presumed model structure and order. This has a number of implications, particularly with respect to the "spillover" effect of unmodelled fuselage dynamics. Also an observer will be needed to reconstruct the unavailable states, adding further to the controller's complexity.

4.5.2 Computation Requirements

The frequency and time domain control laws clearly have quite different computational requirements, and an assessment of these will help to set in perspective the advantages and disadvantages which were identified in the previous sub-section.

The computation will be dominated by the identification process and the subsequent control law calculation. Both methods use an RLS estimator, but the number of parameters to be calculated is quite different. Consider a system with m structural modes, k sensors (outputs), a control actuators and p forcing inputs (generally six, consisting of three forces - vertical, lateral and longitudinal, and three moments - pitch, roll and yaw). The time domain algorithm requires the estimation of an A matrix of dimension 2d by 2d, and a B matrix of size 2d by a. The C and D matrices are also required for output weighting strategies, or for the implementation of an observer, the C matrix has dimensions k by 2d and the D matrix has dimensions k by a. The inclusion of actuator dynamics in the state space model add considerably to the overall system complexity. The frequency domain algorithm (local linear formulation) requires a complex T matrix of size k by a to be estimated, equivalent to a real matrix of size 2k by 2a.

For the typical helicopter system described in Chapter 3, in which k = d = 10, a = 4 and p = 6, this gives the following estimator requirements (see also Table 4.1):

- Time domain parameters to be identified = 720
- Frequency domain parameters to be identified = 160
If second order actuator dynamics are included for the four actuators, then the estimator requirements become:

- Time domain parameters to be identified = 1216
- Frequency domain parameters to be identified = 160

Also the update of the estimator parameters is required much more often for the time domain controllers than for the frequency domain controllers.

Both control laws require the minimisation of a performance index to be calculated in real time to achieve the self-tuning effect. This is much simpler for the frequency domain approach which requires the solution of the quadratic equation (4.2), whereas the time domain method requires the solution of a matrix Riccati equation. It should be noted that a state estimator will also be needed for the time domain control law.

4.5.3 Basic Principles of the Hybrid Control Strategy

This section of the thesis describes a new approach for self-tuning regulators which is particularly appropriate for the active vibration control problem, but which may also have application to a more general class of adaptive control problem. The concept is quite simple: since the vibration measurements contain components at the frequency to be controlled, we can construct a gain matrix which will produce the desired control signals for the actuators by feedback of the measurements, rather than specifically generating the control signals from the magnitude and phase information. In this way we keep the effectiveness of the frequency domain formulation, but also provide continuous feedback of the measurements to give a much improved transient response.

The vast majority of adaptive control research to date has been based on parameteric estimation combined with modern control law design. By using frequency domain estimation technique that is non-parametric, the model order assumption can be relaxed leading to reduced computation requirements when compared to time domain techniques. There has been some work on frequency domain estimation [Lilja, 1990][Middleton, 1988], adaptive frequency domain filters [Bitmead and Anderson, 1981][Parker and Bitmead, 1987], and adaptive controllers using frequency domain information [LaMaire et al, 1987], [Hagglund and Astrom, 1990], [Balchen and Lie, 1986] and [Tang and Ortega, 1990], but none of this provides what is needed for the active vibration control problem.

It has been explained that the steady state output of a system in response to vibration at a particular frequency can be reduced from $B$ to $Y$ by applying optimal control forces at the same
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frequency, whose magnitudes are given by the vector $U$ in equation (4.8). Since the outputs contain components at the desired frequency it is possible to create the required sinusoidal control inputs at frequency $n_0$, with magnitude and phase specified by $U$, by feeding the outputs directly through a gain matrix $[H]$, chosen such that

$$U = [H] Y$$  \hspace{0.5cm} (4.53)

This will satisfy the same minimisation process, and will achieve the same vibration reduction performance as the frequency domain approach. Note that the actual control law for implementation will of course be in the time domain:

$$u(t) = [H] [M_p] \ddot{z}(t) = [H] x(t)$$  \hspace{0.5cm} (4.54)

The time domain control input is generated by feeding back the measured time domain accelerations (outputs) through the hybrid gain matrix. Using frequency domain information it is possible to formulate the feedback gain matrix $[H]$, hence the concept of a hybrid time/frequency domain algorithm is introduced. The frequency domain information is in the form of a set of $n_0$ coefficients for uncontrolled vibration levels, actuator control forces and the calculated reduced vibration levels. A gain matrix is needed which will generate the required control action from the time domain system outputs, to minimise the fuselage vibration. From equations referred to above, the following quantities are known at the blade passing frequency $n_0$: the background vibration vector $\dot{B}$, the controlled vibration vector $\dot{Y}$, the actuator control vector $\dot{U}$, in which all three vectors are complex. The frequency domain estimator provides the following measurements at the blade passing frequency $n_0$: the background vibration vector $\dot{B}$, the controlled vibration vector $\dot{Y}$, and the actuator control vector $\ddot{U}$, in which all three vectors are complex.

These quantities can be redefined as real vectors:

$$\ddot{U} = [Re (\dot{U}) \ Im (\dot{U})]$$  \hspace{0.5cm} (4.55)

$$\ddot{Y} = [Re (\dot{Y}) \ Im (\dot{Y})]$$  \hspace{0.5cm} (4.56)

$$\ddot{B} = [Re (\dot{B}) \ Im (\dot{B})]$$  \hspace{0.5cm} (4.57)

The hybrid gain matrix needs to satisfy the following equations:

$$\ddot{U} = [H] \ddot{Y}$$  \hspace{0.5cm} (4.58)

i.e. generate the required control input from the calculated reduced vibration levels.
\[ \bar{U} = [H] \bar{B} \]  

(4.59)

i.e. generate the required control input from the current vibration levels.

Since the values of \( \bar{B}, \bar{Y} \) and \( \bar{U} \) are known in equations (4.58) and (4.59), the hybrid gain matrix \([H]\) can be calculated from the following equation:

\[ [H] = [\bar{U} \bar{U}] [\bar{Y} \bar{B}]^{-1} \]  

(4.60)

The aim of the Hybrid strategy [Pearson and Goodall, 1992 and 1994] is to construct a gain matrix from frequency domain measurements. This strategy is based upon the same assumptions as the frequency domain strategy outlined in the previous section.

Using the frequency domain performance index

\[ J = Y^T [W] Y + U^T [R] U + \Delta U^T [R \Delta U] \Delta U + \Delta U^T (C \sum_j [W]_j) \Delta U \]  

(4.61)

and the local linear frequency domain relationship between control inputs and vibration outputs

\[ \Delta Y = [T] \Delta U \]  

(4.62)

the following optimal control law is derived

\[ \Delta U_{n-1} = -[G] \left( [R] U_N + [T]^T [W] Y_N \right) \]  

(4.63)

Alternatively, minimisation of the global performance index using the global formulation equation produces an optimal control law as follows:

\[ \Delta U_{n-1} = -[G] \left( [T]^T [W] [T] + [R] + [R \Delta U] + C \sum_j [W]_j \right) U_N + [T]^T [W] B + C \sum_j [W]_j \]  

(4.64)

where in both cases

\[ [G] = \left( [T]^T [W] [T] + [R] + [R \Delta U] + C \sum_j [W]_j \right)^{-1} \]  

(4.65)

and

\[ \Delta U_{n-1} = U_{n-1} - U_N \]  

(4.66)
4.5.4 Adaptive Hybrid Control

Experience with operating the frequency domain controller has shown that for an adaptive controller it is more robust to linearise around the current operating point and use a local linear controller.

This local linear approach can also be used for calculating the change in the hybrid gain matrix as follows, in which the subscripts denote the controller calculation cycle:

\[
[\Delta H] = [H]_{n+1} - [H]_n
\] (4.67)

The incremental change in the coefficients of the controlled vibration is given by

\[
\Delta Y = \bar{Y}_{n+1} - \bar{Y}_n
\] (4.68)

and the incremental change in the control coefficients is

\[
\Delta U = \bar{U}_{n+1} - \bar{U}_n
\] (4.69)

For the \((n+1)\)th controller cycle

\[
\begin{align*}
\bar{U}_{n+1} & = [H]_{n+1} \bar{Y}_{n+1} \\
& = ([\Delta H] \bar{Y}_{n+1} + [H]_n \bar{Y}_{n+1} \\
& = [\Delta H] \bar{Y}_{n+1} + [H]_n \Delta Y + [H]_n \bar{Y}_n \\
& = [\Delta H] \bar{Y}_{n+1} + [H]_n \Delta Y + \bar{U}_n
\end{align*}
\] (4.70)

Rearranging gives

\[
\Delta U = [\Delta H] \bar{Y}_{n+1} + [H]_n \Delta Y
\] (4.71)

Second set of equations can be derived for the previous measurement cycle:

\[
\begin{align*}
\bar{U}_{n+1} & = [H]_{n+1} \bar{Y}_n \\
& = ([\Delta H] + [H]_n) \bar{Y}_n \\
& = [\Delta H] \bar{Y}_n + [H]_n \bar{Y}_n \\
& = [\Delta H] \bar{Y}_n + \bar{U}_n
\end{align*}
\] (4.72)

\[
\Delta U = [\Delta H] \bar{Y}_n
\] (4.73)

Solving equations (4.71) and (4.73) simultaneously provides a solution to the incremental
hybrid gain matrix and rearranging the equations gives:

\[
[\Delta H] = (\Delta U - [H]_a \Delta Y)([\bar{Y}_{a+1}])^{-1}
\]

\[
[\Delta H] = (\Delta U)[\bar{Y}_a]^{-1}
\]  

(4.74)

Solving for \(\Delta H\)

\[
[\Delta H] = [(\Delta U - [H]_a \Delta Y) (\Delta U)] [\bar{Y}_{a+1} \bar{Y}_a]^{-1}
\]

(4.75)

Thus giving a solution for

\[
[H]_{a+1} = \Delta H + [H]_a
\]  

(4.76)

from which a new value of \([H]\) for the next computation cycle can be calculated. With this local linear formulation there is no longer a requirement to estimate the background vibration vector \(\bar{Y}\) in an adaptive controller framework.

For the frequency domain approach control is provided at more than one frequency of vibration by having a number of parallel controllers, the outputs of each being added to form a composite actuator drive signal. The hybrid control law can cope with more than one frequency by including measurements corresponding to the extra frequencies in the calculation of \([H]\). The next sub-section will explain that there will normally be sufficient degrees of freedom in the calculation for this to be possible.

Figure 4.5 shows the self tuning adaptive hybrid control scheme. A parameter estimator provides a recursive estimate of the transfer function between the actuator inputs and the accelerometer outputs to give the \(T\) matrix in exactly the same way as the frequency domain controller. The control law designer then uses this frequency domain information to determine the feedback gain matrix coefficients in accordance with the method described.

By comparing Figure 4.5 with Figure 4.4 and with Figure 4.2, it can be seen that the hybrid scheme combines the simplicity of the frequency domain controller with the feedback structure of the time domain controller.

4.5.5 Solution Methods for the Hybrid Gain Matrix

In many cases equation (4.60) will be underdetermined and this means that other factors can be incorporated to determine the particular solution matrix. The mathematics reveal that a
unique solution is achieved when there are twice as many outputs as frequencies of
optimisation, irrespective of the number of control inputs.

Adding dimensions to equation (4.53) and including the possibility of incorporating
measurements at more than one frequency, we obtain.

\[ U_{(a \times 2b)} = [H]_{(a \times k)} [Y]_{(k \times 2b)} \]  (4.77)

where \( a \) is the number of control inputs, \( k \) is the number of outputs and \( f \) is the number of
frequencies used in the minimisation. It is clear that there are \( 2fa \) equations to determine the
\( ks \) unknown coefficients in \( [H] \), meaning that if the number of outputs is greater than twice the
number of frequencies used \( (k > 2f) \), as will generally be the case in more complex systems,
the gain matrix \( [H] \) cannot be fully determined from the measurements i.e. there are a large
number of gain matrices which will satisfy the minimisation. This is evident when considering
the helicopter model (four actuators and ten sensors) for which it can be seen that with a single
measurement frequency there are only 8 equations for 40 unknowns.

There are a number of possible approaches for constraining the solution of equation (4.60):

1) optimise performance at multiple frequencies rather than at a single frequency
2) minimise a suitable matrix norm, e.g. singular value decomposition.
3) limit the variations of \( [H] \) from some nominal value, e.g. seeding the hybrid solution with
another gain matrix solution such as an optimal output feedback gain matrix,
4) make some assumptions about the likely form of the system, enabling the closed loop
eigenvalues to be suitably placed.

In the work reported in this thesis we have used the first two approaches, but the other
possibilities remain to be explored.

1) multiple frequencies

For measurements at the frequency \( \omega 1 \), let

\[ \bar{U}_{\omega 1} = [\text{Re}(U) \text{ Im}(U)] \]  (4.78)

\[ \bar{Y}_{\omega 1} = [\text{Re}(Y) \text{ Im}(Y)] \]  (4.79)

\[ \bar{B}_{\omega 1} = [\text{Re}(B) \text{ Im}(B)] \]  (4.80)
and for the frequency \( \omega_2 \)

\[
\bar{U}_{\omega_2} = [\text{Re}(U) \ \text{Im}(U)]
\]

(4.81)

\[
\bar{Y}_{\omega_2} = [\text{Re}(Y) \ \text{Im}(Y)]
\]

(4.82)

\[
\bar{B}_{\omega_2} = [\text{Re}(B) \ \text{Im}(B)]
\]

(4.83)

where \( U, B \) and \( Y \) have been calculated separately at \( \omega_1 \) and \( \omega_2 \). Then a Hybrid gain matrix can be calculated as before, but using the information from two frequencies:

\[
[H] = \begin{bmatrix}
\bar{U}_{\omega_1} & \bar{U}_{\omega_2} & \bar{U}_{\omega_2} \\
\bar{Y}_{\omega_1} & \bar{Y}_{\omega_2} & \bar{Y}_{\omega_2} \\
\bar{B}_{\omega_1} & \bar{B}_{\omega_2} & \bar{B}_{\omega_2}
\end{bmatrix}^{-1}
\]

(4.84)

2) minimalising a suitable matrix norm, e.g. singular value decomposition

3) seeding the hybrid solution with another gain matrix solution such as an optimal output feedback gain matrix. The solution to (4.53) is constrained to be as close as possible, in a least squares sense, to a specified gain matrix obtained using another method such as Linear Quadratic output control methods, or pole placement methods.

From equation (4.58) we have a set of linear nonhomogeneous equations

\[
\bar{U} = [H] \bar{Y}
\]

(4.85)

if \( H \) is any given solution of (4.85) and \( H_0 \) is an arbitrarily chosen solution of (4.85). Then

\[
\bar{U} = [H] \bar{Y} \quad \bar{U} = [H_0] \bar{Y}
\]

(4.86)

and, therefore

\[
([H] - [H_0]) \bar{Y} = 0
\]

(4.87)

This shows that the difference \([H] - [H_0]\) between any solution \([H]\) of (4.85) and any fixed solution of (4.85) is a solution, say \([H_n]\), of the corresponding homogeneous system of linear equations

\[
0 = [H] \bar{Y}
\]

(4.88)

Hence all the solutions of (4.85) are obtained by letting \([H_n]\) run through all the solutions of the homogeneous system (4.88). It can be shown for the homogeneous system equations that if \([H_n]\) is a solution of (4.88) then so is \(c[H_n]\), where \(c\) is an arbitrary constant.
Therefore, the solutions to (4.85) are of the form

\[ [H] = [H_0] + c[H_a] \]  \hspace{1cm} (4.89)

41 pole placement approach. This idea is similar to (3), the solution to (4.53) is chosen such that the poles of the system are moved to, or moved as close as is possible to, some desired locations.

4.5.6 Summary

Essentially the hybrid approach constructs an output feedback gain matrix corresponding to the optimisation of a frequency domain performance index, since the vibration measurements contain components of the frequency to be controlled. This gain matrix replaces a much more complex state feedback matrix which would involve state estimation and the solution of a matrix Riccati equation. Since the structural dynamics are not accurately known and may vary with time a self tuning adaptive controller is required. A key element of the hybrid control strategy is that the estimation and optimisation are performed in the frequency domain.

The main features of the Hybrid time/frequency domain approach are:

- It is computationally simpler than the time domain method to implement.
- Vibration reduction at a number of frequencies is possible, the transfer function may be tracked at a number of frequencies using parallel estimators.
- It is adaptive, but, like the frequency domain controller, only at discrete instants in time.
- Actuator dynamics are implicitly incorporated into the structural model by the algorithm.
- A rotor reference signal is no longer strictly required, since the actuator forces are generated directly from the sensor measurements.

The main disadvantages are:

- Its stability and robustness need further investigation.

4.6 PARAMETER ESTIMATION

To calculate the optimal control input to minimise the fuselage vibration, the controller requires a model of the fuselage. The helicopter fuselage is a very complex system, and although analytical models are good and can be improved by tuning them using empirical data, they may
still not be accurate enough to design a control system that will achieve the high levels of performance required. The main areas of deficiency of these models are:

- **Damping** is poorly understood, and the values of damping derived empirically from shake tests may produce incorrect responses.

- **Actuator Dynamics** are complex and nonlinear, and for a successful control system design they must be included in the system model.

- **Model Complexity**: The fuselage is modelled as a set of modes represented by linear second order equations. As well as the fuselage it is necessary to model the complete system, this includes actuators, sensors, signal conditioning, delays and so on.

Since accurate information of the actual helicopter dynamics is not available it is necessary for the control strategies to adapt to any changes in structural dynamics, changes in flight condition and system nonlinearities. Accurate identification of the system is important for good vibration reduction since Frequency domain control algorithms depend on the system model which varies with flight condition. Similarly, for the time domain strategies, accurate identification of the state space matrices \([A_s], [B_s], [C_s], \text{ and } [D_s]\) is needed. System identification therefore provides a formal mathematical procedure for fitting a mathematical model to a series of measurements of the system responses [Ljung, 1988].

System identification can be performed either on-line or off-line:

With on-line identification the controller sampling period is critical in determining the amount of processor time available for estimation and control calculations. In adaptive control, on-line identification is also known as real-time identification.

Conversely with off-line identification it is assumed that all data regarding the input and output data is available prior to the analysis.

A robust controller could use the parameters identified off-line whereas an adaptive controller requires the parameters to be identified on-line. A gain scheduled controller would use a library of parameters stored in the controller, but calculated off-line. On-line identification has a significant advantage over off-line identification, since it can track any changes in helicopter dynamics and hence allows an adaptive controller to alter the control law accordingly. The disadvantages of adaptive control are the added controller complexity and the increased concerns for stability.

The identification method selected must compliment the control law used, therefore these methods also fall into time domain and frequency domain categories [Tischler and Kalatka,
Frequency domain identification methods use spectral techniques to transform the data to the frequency domain, an estimation technique is then used to identify the transfer function matrix from this frequency domain data. Since no model structure has been assumed these results can be considered to be non-parametric (i.e. a series of gains and phases are calculated). Time domain methods first require the selection of a state space model structure. Model parameters are identified from the time response histories using an estimation technique.

The identification of a mathematical model for a system can be divided into two parts: identification of the model structure, and identification of the parameters. The frequency domain algorithms use the input/output data in the frequency domain to identify a classical transfer function representation of the system. The time domain algorithms use the input/output data in the time domain to identify a state space representation of the system.

The disadvantages of the frequency domain identification are:
1- more preprocessing is required to transform the data to the frequency domain
2- long data records are required to obtain sufficient frequency resolution
The main advantages are:
1- the model to be identified is much simpler
2- an accurate representation of the system at a particular frequency can be identified

Time domain identification methods tend to dominate the literature on control engineering on system identification. The survey paper by Astrom and Eykhoff [1970] provides a good guide to this literature. There have been a number of practical implementations of parameter estimation techniques to identify frequency domain parameters: Bergman and Ljung [1988] describe the use of the RLS algorithm to identify fourier coefficients, Ljung [1985] uses system identification techniques to estimate transfer functions, and Bitmead et al [1986] describe the use of a kalman filtering approach to identify harmonic components. Ljung and Glover [1981] discuss basic time and frequency domain identification techniques, the main conclusion from the paper is that the techniques are complementary rather than competing techniques.

The recursive form of the least squares was first arrived at in 1950 by Plackett. The idea was enhanced even further by Kalman in 1960 due to the advances in digital computers and digital filters. The Recursive Least Squares (RLS) is simple to calculate and is widely used due to its general applicability to different areas of engineering [Astrom and Wittenmark, 1984][Iserman,
The derivation of the basic least squares estimator equations is relatively straightforward, and can be found in any good textbook on system identification [Soderstrom and Stoica, 1989] [Ljung, 1988]. By defining an estimation criterion that is an a function of the error squared gives rise to a least squares identification method, other criterions give rise to other methods.

RLS is an attractive and popular choice in adaptive controllers due to its reliable convergence and takes little computational effort [Soderstrom et al., 1974]. Its main disadvantage is that it suffers from bias in the estimates unless the noise in the system is white noise and the signal to noise ratio is large. Other methods have been developed to overcome this disadvantage by estimating the noise parameters themselves. Most of these methods are variations of the RLS method, for example Generalised RLS, Recursive Extended Least Squares (RELS), Recursive Instrument Variables (RIV), and Recursive Maximum Likelihood (RML) method. Some of these methods have superior convergence and robustness properties, but at the expense of extra computational effort [Astrom and Eykhoff, 1970]. A number of papers contrast these methods, for example Graupe et al [1980], Saridis [1974] and Isermann et al. [1974].

The method used to identify these structural parameters on-line and therefore used to provide control system adaptability is a recursive least squares parameter estimation algorithm [Fortescue, 1981].

The response of the aircraft, either in the time domain or the frequency domain, is compared with the output from the identified model. The differences are minimised by the identification methods which iteratively adjusts the model parameters.

A major limitation of standard RLS algorithms is that as the parameter estimates converge to correct values, the estimator gain and uncertainty approach zero and this makes the estimator unresponsive to changes in the system parameters, i.e. the ability of the controller to adapt is reduced.

Clearly the estimator needs be able to respond to periodic changes in the system parameters. One technique to achieve this is to use a forgetting factor which adjusts the weighting applied to past data depending upon the current estimator conditions. The forgetting factor is a gain which has a value between 0 and 1.

One strategy is to vary the forgetting factor according to the information content of the system variables. Astrom [1980] and Fortescue et al. [1981] suggested a time varying forgetting factor which is automatically set to appropriate values by using the prediction error as the measure of information in the data.
Constant forgetting factors can be used but these need to be carefully chosen, otherwise estimator problems such as covariance blow up can occur. This can be particularly difficult in systems where the rate of change of system parameters may vary. In such systems it is necessary to vary the forgetting factor accordingly [Zarrop, 1981], [Hagglund, 1983]. When the system parameters are constant or if the system has not been excited the estimator error will be small, the best strategy is then to retain as much information as possible and therefore the forgetting factor should be 1. If the system parameters change the estimator error will increase. The forgetting factor value should then lower which reduces the estimator’s memory length (shorter data window) meaning that only more recent and accurate data is used by the estimator, until the parameters are readjusted and the estimator errors reduce.

The equation used to vary the forgetting factor tries to maintain a constant amount of dynamic information in the estimator. Reconsidering the situation when the system parameters are constant the forgetting factor is set to 1 and this corresponds to no dynamic information. As the system parameters change the dynamic information increases and therefore the forgetting factor is reduced. The amount of dynamic information in the estimator is determined by a constant N called the asymptotic memory length. This also determines the speed of adaptation of the estimator [Gawthrop, 1986]

\[ N = \frac{1}{(1 - \lambda)} \]  

(4.90)

the above equation implies that the information dies away with a time constant of N sample intervals. As can be deduced from equation (4.90) forgetting factors of 0,0.99 and 1 correspond to memory lengths of 1,100 and infinity.

The asymptotic memory length N controls the sensitivity of the estimator. If the estimator is too sensitive it will react to measurement noise and the covariance matrix \( [P] \) will converge even if the system parameters are constant. In order to compensate for measurement noise, which appears as dynamic information to the estimator, an estimate of the measurement noise variance \( \sigma^2_n \) is used in the calculation of the estimator’s sensitivity:

\[ \epsilon_s = N \sigma_n^2 \]  

(4.91)

The value of the forgetting factor is determined primarily by the estimator error and the estimator sensitivity, upper and lower limits are often placed on the forgetting factor to provide additional control.

The upper limit for the forgetting factor \( \lambda_{\text{max}} \) defines a threshold for the estimator, above which old data will not be discarded. Once the forgetting factor drops below this threshold data will
start to be discarded or forgotten by the estimator. This threshold also determines the accuracy of the estimated parameters \((T\text{ matrix})\). By considering the extreme case of setting this limit low it can be deduced that during parameter changes the estimator's effective memory length will be too long causing it to use information which is essentially out of date. If measurement noise and consequently estimator error are significant the covariance matrix may diverge but this can be avoided by setting this limit at a suitable level. The lower limit for the forgetting factor \(\lambda_{\min}\) is used to restrict the maximum rate of change of parameter estimates by the estimator.

Adopting such an estimation strategy enables the parameter estimates to follow both slow and sudden changes in the system dynamics.

4.6.1 Estimator Equations

This section contains the equations specific to the implementation of the estimator. The estimator algorithm has the following stages:

1- prediction: the output of the system is predicted using current parameter estimates and a set of system measurements.

2- error: the error between the measured system output and the predicted system output is calculated.

3- forgetting: a forgetting factor is calculated which is used after the memory length of the estimator by weighting estimator information.

4- gain: an estimator gain is calculated which along with the estimator error, is used to update the parameter estimates.

5- estimates: the parameter estimates are updated using the estimator error and gain.

6- covariance: the covariance matrix is updated. This matrix reflects the uncertainty in the parameter estimates.

The Recursive Least Squares estimator minimises the sum of the squares of the error between the estimated vibration levels and the measured vibration levels. The local linear controller formulation requires an estimate for the \(T\) matrix only, whereas the Global controller formulation requires estimates for both the \(T\) matrix and the uncontrolled fuselage vibration \(B\).

The time domain strategies require an estimate of the \([A_s]\) and \([B_s]\) matrices for full state feedback with state weighting, and for state estimation and/or output weighting they require the estimation of the \([C_s]\) and \([D_s]\) matrices as well [Moonen et al, 1989]. The estimator formulation for each of the controller types is outlined next.
It is convenient to write the estimation problem as follows:

$$\mathbf{y} = \mathbf{[\Phi]} \mathbf{\Theta}$$

(4.92)

where $[\Phi]$ is a matrix of the unknown parameters, $\mathbf{\Theta}$ is a vector of measurements and $\mathbf{y}$ is a vector of predicted outputs. The subscript $N$ denotes the estimation cycle. The task is to estimate the values for $[\Phi]$ that will best fit equation (4.92), where best is judged on the basis of the minimum squared error criterion.

The least squares cost criterion on the parameter vector estimates is

$$J_y = \sum_{k=0}^{N} \left[ \mathbf{y}(k) - \mathbf{[\Phi]} \mathbf{\Theta}(k) \right]^2$$

Minimisation of this cost function is accomplished by differentiating with respect to the parameter vector $\mathbf{\Theta}$ and setting this gradient to zero. Preserving the recursive nature of the equations, the recursive least squares identification algorithm is obtained.

Frequency Domain Local Linear Controller Formulation

The $[\mathbf{T}]$ matrix estimate is obtained by using a number of estimators in parallel, one for each row of the $[\mathbf{T}]$ matrix (2m rows where m is the number of sensors). A vector for each estimator is constructed which contains a corresponding row of the $[\mathbf{T}]$ matrix, the index $i$ refers to the $i^{th}$ RLS estimator.

$$\mathbf{[\Phi]}^i_N = \begin{bmatrix} [\mathbf{T}]_{i,1} & [\mathbf{T}]_{i,2} & \cdots & [\mathbf{T}]_{i,2m} \end{bmatrix} \quad \text{for } i = 1 \text{ to } 2m$$

(4.94)

For the Local linear controller the measurement is defined as the change in the control input:

$$\mathbf{\Theta}_N = \begin{bmatrix} \Delta U_1, \Delta U_2, \ldots, \Delta U_N \end{bmatrix}$$

(4.95)

For the Hybrid controller the change in control is given by

$$\Delta U_N = [H]_N \mathbf{x}_N - [H]_{N-1} \mathbf{x}_{N-1}$$

(4.96)
Control Strategies

Prediction The predicted change in vibration levels is the measurement vector (change in control) multiplied by the estimated T matrix

\[ \Delta \hat{Y}_N = \Theta_N [\Phi]_N \]  

(4.97)

Error The error vector is defined as the difference between the change in the actual vibration levels and the change in the predicted vibration levels. The actual vibration level is that measured by the signal processing (DFT) routine.

\[ E_N = \Delta Y_N - \Delta \hat{Y}_N \]  

(4.98)

Frequency Domain Global Controller Formulation

The estimator formulation for the Global controller is very similar to that for the Local linear controller. An extra element in each of the 2m estimation vectors is required to include an estimate for the uncontrolled vibration level \( \beta \). The state vector is therefore defined as:

\[ \Phi_i^1 = [ \tau_{i,1} \tau_{i,2} \ldots \tau_{i,N} \beta_i ] \quad \text{for } i = 1 \text{ to } 2m \]  

(4.99)

The measurement vector also contains an extra element which corresponds to the estimate for the \( \beta \) vector

\[ \Omega_N = [ U_1, U_2, \ldots, U_N, 1 ] \]  

(4.100)

For the Hybrid controller the control action is defined as:

\[ U_N = [ H ] Y_N \]  

(4.101)

Prediction The predicted vibration levels are given by the measurement vector multiplied by the estimated T matrix

\[ \hat{Y}_N = \Theta_N [\Phi]_N \]  

(4.102)

i.e.

\[ \hat{Y}_N = [\hat{\Phi}]_N U_N + \hat{\beta} \]  

(4.103)

where \( \cdot \) represents an estimate.
Error The error vector is defined as the difference between the actual (measured) vibration levels and the predicted vibration levels:

$$E_N = Y_N - \hat{Y}_N$$  (4.104)

Time Domain Controller Formulation
For the time domain algorithms the matrix of state vectors is constructed as follows

$$[\phi] = \begin{bmatrix} [A_s] & [B_s] \\ [C_s] & [D_s] \end{bmatrix}$$  (4.105)

The measurement matrix is defined as

$$\Theta_N = \begin{bmatrix} x_{s,N} \\ u_N \end{bmatrix}$$  (4.106)

where the state vector $x_{s,N}$ is an estimate obtained from the state estimation sub system.

Prediction The predicted acceleration is given by

$$\hat{y}_{s,N} = \Theta_N[\phi]$$  (4.107)

Error The error vector is defined as the difference between the actual vibration levels and the predicted vibration levels.

$$E_N = \begin{bmatrix} \hat{x}_{s,N} \\ \hat{y}_{s,N} \end{bmatrix} - \hat{y}_{s,N}$$  (4.108)

Equations Common to all Controller Formulations
The following equations are common to all of the controller configurations.

Forgetting The forgetting factor equation is:

$$\lambda_{N+1} = 1 - (1 - \Omega_N K'_{N+1}) \left( \frac{2n \int E^2}{\sum_{l=1}^{\infty} \Theta'_N} \right) \left( \frac{1}{\Sigma_o} \right)$$  (4.109)

where $K'_{N+1}$ is the estimator gain and $\Sigma_o$ is the estimator sensitivity.
Control Strategies

The forgetting factor limits are (as explained in the main section)

\[ \text{if } \lambda > \lambda_{\text{max}} \text{ then } \lambda = \lambda_{\text{max}} \]  

(4.110)

\[ \text{if } \lambda < \lambda_{\text{min}} \text{ then } \lambda = \lambda_{\text{min}} \]  

(4.111)

**Gain** The gain $K'$ is essentially the ratio of uncertainty between the uncertainty in the estimated model parameters and the uncertainty in the measured vibration parameters. When the uncertainty in the parameters is small (i.e. $|\mathbf{P}|$ is small), then the parameter estimates will change less, even though there may be a difference between the measured and predicted vibration. Estimator gain

\[ K'_{N+1} = \left[ \mathbf{P}_N \mathbf{Q}_N \left( \mathbf{Q}_N \mathbf{P}_N \mathbf{Q}_N^T + \mathbf{v}_N \right)^{-1} \right] \]  

(4.112)

where $\mathbf{v}_N$ is the estimated noise variance. Alternatively the estimator gain can be defined by

\[ K'_{N+1} = \left[ \mathbf{P}_N \mathbf{Q}_N \left( \mathbf{Q}_N \mathbf{P}_N \mathbf{Q}_N^T + \mathbf{v}_N \right)^{-1} \right] \]  

(4.113)

**Estimates** The updated estimates are obtained by combining the vectors from the parallel estimators

\[ [\mathbf{\Phi}]_{N+1} = [\mathbf{\Phi}]_N + K'_{N+1} \mathbf{E}_N \]  

(4.114)

**Covariance** The covariance matrix is updated as follows

\[ [\mathbf{P}]_{N+1} = \frac{([\mathbf{P}]_N - K'_{N+1} \mathbf{Q}_N [\mathbf{P}]_N)}{\lambda_{N+1}} \]  

(4.115)

In practice the covariance matrix is factorised into a $[U][D]$ formulation, since it has been shown by Bierman [1977] that there are numerical advantages in using this technique. Bierman showed that numeric errors can be dramatically reduced by using covariance factorization algorithms, and that the U-D covariance factorization is computationally efficient and compact. The technique relies on the fact that a positive definite matrix (all covariance matrices are positive definite) can be factored into a product of three matrices, an upper triangular matrix with unit diagonal elements, a diagonal matrix with positive elements, and the transpose of the upper triangular matrix, the so called U-D algorithms. The method is described in Appendix V.
The local linear controller formulation may give better control for very nonlinear systems since it linearises locally around the current operating point. However, since the identification algorithm for the local linear configuration depends upon $\Delta U$ and not $U$ (as for the global configuration) for small changes in the control signal the error signal used by the estimator becomes very small.

The global controller configuration requires the estimator to identify another parameter $\mathcal{B}$. Since this is an estimate and not a measurement the identification algorithm may be less accurate for the global configuration.

Regardless of which system model is used, the identification algorithm requires only the current vibration measurements and error covariance to identify the required system parameters.

4.6.3 Off-Line Estimation

For the non-adaptive algorithms it is necessary to have a good estimate of the system dynamics before the controller is initialised. This is also desirable for the adaptive algorithms, so that they provide good initial control.

For the non-adaptive frequency domain controller it is necessary to measure $[T]$ and $\mathcal{B}$ off-line by exciting the helicopter fuselage dynamics using the control inputs. By measuring the fuselage response to several different input combinations $U_j$, it is possible to calculate the corresponding values for $[T]$ and $\mathcal{B}$. With no control the fuselage response $Y$ is then simply $\mathcal{B}$. The calculation of $[T]$ is slightly more difficult; the change in the fuselage response $Y$ caused by a unit value of a single control vector component $U_j$ gives the $j^{th}$ column of the $T$ matrix. A larger control input can be used to improve identification, in which case the resulting vector needs to be scaled according to the control signal magnitude. The $j^{th}$ column of the $T$ matrix is then

$$
I_j = \frac{(Y_j - \mathcal{B})}{|U_j|} \tag{4.116}
$$

where $j = 1$ to $b$ (number of actuators), $Y_j$ is the vibration vector measurement corresponding to the $j^{th}$ control input. In practice the fuselage is excited with a sine waveform followed by a cosine waveform (with respect to the rotor reference signal) using each of the control actuators in turn.
Due to system nonlinearities and noise, a more satisfactory approach may be to use a least squares approach (Goodwin and Payne, 1977) to fit the model parameters \( T \) and \( B \) to a set of results from several identification trials. Since these identified parameters are based on more data, they are more likely to be acceptable. This approach is also required for the off-line identification of the state space models.

For the time domain algorithms the matrix of parameter estimates can be partitioned as follows

\[
[\Phi] = \begin{bmatrix}
[A_2] & [B_2] \\
[C_2] & [D_2]
\end{bmatrix}
\]  

(4.117)

A set of \( n \) measurements are made and stored in a measurement matrix \([M] \), using a predetermined set of control inputs, which are stored in an input matrix \( [\theta] \). The number of measurements made must be greater than the number of parameters to be estimated. For the global frequency domain algorithms the input matrix consists of

\[
[\theta] = \begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_n
\end{bmatrix}
\]  

(4.118)

and for the local frequency domain algorithms the input matrix is defined as

\[
[\theta] = \begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_n
\end{bmatrix}
\]  

For all the frequency domain algorithms the measurement matrix is defined as \([M] = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_n \end{bmatrix} \). For the time domain algorithms the input and measurement matrices is defined as

\[
[\theta] = \begin{bmatrix}
\dot{x}_{s,1} & \dot{x}_{s,2} & \cdots & \dot{x}_{s,n} \\
\dot{x}_{s,1} & \dot{x}_{s,2} & \cdots & \dot{x}_{s,n} \\
\dot{x}_{s,1} & \dot{x}_{s,2} & \cdots & \dot{x}_{s,n}
\end{bmatrix}
\]  

(4.119)

Considering the sum of the squares of the errors

\[
S = \sum_{n=1}^{N} ([M_n] - [\Phi][\theta])^2
\]  

(4.120)

The solution that minimises \( S \) is the least squares estimate

\[
[\Phi] = ([\theta]^T[\theta])^{-1}[\theta]^T[M_n]
\]  

(4.121)
4.7 CONCLUSIONS

This chapter has examined a variety of control strategies by which sensor measurements of vibration can be used to generate actuator control forces to minimise the vibration. Practically sensors are light, cheap and easy to install when compared to actuators, so it is advantageous to use more sensors than actuators and minimise a cost function based on the sum of weighted sensor levels. Sensors are placed at fuselage positions where vibration reduction is most critical, and in sufficient numbers to sense motions in all the dominant modes.

The fundamental difference between the time and frequency domain algorithms is in the way the control is exercised. Frequency domain algorithms provide better vibration minimisation than the time domain algorithms, since they only control the blade passing frequency \( n_0 \) (and by using parallel controllers at appropriate harmonics if required), which dominates the fuselage vibration. Also, the frequency domain methods are simpler to implement practically. The time domain methods feedback vibration information through a gain matrix in real time, therefore they can potentially control over a range of frequencies rather than just the blade passing frequency component.

With the frequency domain techniques any actuator dynamics and even nonlinearities are implicitly incorporated into the model. However, with the time domain strategies actuator dynamics need to be modelled and incorporated explicitly into the structural model.

The main disadvantage of the Frequency domain controller is that it is based upon quasi-static principles, and this limits its ability to deal with rapid changes in the fuselage vibration. The main disadvantages of the Time domain controller are that it requires actuator dynamics to be modelled and explicitly incorporated into the controller model. The controller model is based on a subset of fuselage modes and consequently control and observation spillover may be a problem. Also, the controller requires the complete state vector for feedback and since this consists of modal velocities and displacements and actuator states a state estimator will be required.

An important consideration in the practical implementation of any estimation algorithm is the number of parameters to be identified, since this, along with factors such as the type of estimation algorithm used and the speed of the microprocessor, determines the computation time. It has been clearly shown that the time domain algorithms require a significantly larger number of parameters to be identified.
The improved control approach combines the advantages of the frequency domain algorithm (simplicity, steady state performance, incorporation of unmodelled dynamics) with the advantages of time domain algorithms (transient response). It may prove to be even simpler to implement than the frequency domain algorithms since it strictly no longer requires a reference signal from the main rotor, and also since the actuator forces are generated directly by multiplying the sensor signals by a feedback gain matrix there is no scheduling phase where the frequency domain actuator coefficients are transformed into time domain signal.

Essentially the hybrid approach constructs an output feedback gain matrix corresponding to the optimisation of a frequency domain performance index, replacing a much more complex state feedback matrix which would involve state estimation and the solution of a matrix Riccati equation. Since the structural dynamics are not accurately known and may vary with time a self-tuning adaptive controller is required. A key element of the hybrid control strategy is that the estimation and optimisation are performed in the frequency domain.

A number of approaches for constraining the solution of the hybrid feedback gain matrix have been identified, i.e. multiple frequency minimisation, matrix norm minimisation, gain matrix seeding and pole placement, and these need to be explored further.
### Control Strategies

<table>
<thead>
<tr>
<th>Parameters to be Identified</th>
<th>sensors k</th>
<th>modes d</th>
<th>actuators a</th>
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<td>Strategy</td>
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| Time Domain                | 720       | 2200    | 660         |
| Time Domain + Actuators    | 1226      | 3016    | 1150        |
| Frequency Domain           | 160       | 160     | 80          |
| Frequency Domain + Actuators| 160      | 160    | 80          |

Table 4.1 Variation of the number of parameters to be identified with changes in system dimensions
Figure 4.1 Observation and Control Spillover
Figure 4.2: Frequency domain adaptive control system

Figure 4.3: Frequency Shaping Filter magnitude response
Figure 4.4 Time domain adaptive control system

Figure 4.5 Hybrid adaptive control system
CHAPTER 5

SIMULATION STUDIES

INTRODUCTION

The previous chapters have outlined helicopter models, described time and frequency domain vibration control strategies, and introduced a novel hybrid control strategy for the active control of helicopter vibration. The inherent non-linearity of all adaptive algorithms makes them unsuitable for analysis by classical control techniques and thus their development relies heavily upon simulation studies. This chapter investigates the performance of the control strategies using simulations of the helicopter fuselage. The purpose of the simulation study was to compare and evaluate the alternative control strategies, and also to refine and fully understand the effects of tuning parameters within the algorithms.

5.1 ACTIVE VIBRATION CONTROL

The ACSR system is implemented using the control strategies described in the Chapter 4, and a mathematical model of the helicopter fuselage in a computer based simulation. The simulation consists of a modal model of the fuselage, accelerometers to measure the broad band vibratory response, force actuators to input control forces and a control strategy.

The mathematical models used for the simulation study are derived from NASTRAN finite element models which have been validated against the aircraft, and have been described in detail in Chapter 3.

In the course of the simulation study several important issues were addressed. These include:

i) steady state response (straight and level forward flight at speeds of 114, 125, 136 and 150 knots),

ii) transient response during basic manoeuvres,

iii) robustness (changes in forcing frequency, forcing amplitude, structural dynamics),

iv) adaptation to the same types of problem as in section iii), and

v) the effects of real actuators on control system performance and stability.

Simulations have been carried out using Matlab in order to make a comparison between the various strategies in terms of their ability to reduce fuselage vibration, and to identify their
relative merits. Accelerations are used to characterise the vibration since it is these, rather than
displacements or velocities, to which people and equipment are sensitive, and of course they
can readily be measured in practice with accelerometers.

The important questions in a comparison between strategies can be summarised as follows:-
1) How effective are they in reducing vibrations under quasi-static (steady state)
   conditions?
2) How quickly do the strategies respond during changing flight conditions?
3) How well do they adapt to changes in the system parameters?

Three stages of simulation have been used to answer these questions.
Stage 1 : Quasi-static Performance This is a straightforward comparison of the basic vibration
   reduction capabilities of the strategies for an accurate system model, but with constant
   vibration forces.
Stage 2 : Non-adaptive The second stage involved a robustness assessment of the strategies
   without any adaptation, but with changes to the vibratory inputs and to other system
   parameters. Three different types of robustness have been considered: response to changing
   frequency of the disturbing vibratory forces, to changing amplitude of the vibratory forces, and
   to changing parameters of the structure.
Stage 3 : Adaptive This stage was a performance assessment of the strategies with adaptation,
   and with the same kind of changes used for stage 2. It included recursive least squares
   estimators with variable forgetting factors.

The assessment was carried out using two models, a simple 3 mass model and a much more
complex model to represent the fuselage. The next section describes a preliminary assessment
using the simpler model which enabled the fundamental characteristics of the time and
frequency domain control strategies to be identified prior to a more comprehensive assessment
using the fuselage model.

5.2 PRELIMINARY ASSESSMENT USING 3 MASS MODEL

The dynamic characteristics and parameters of the 3 mass model were described in Chapter
3. Throughout the assessment, the performance of the strategies is calculated using the
average of the r.m.s vibration levels.
5.2.1 Quasi-static Performance Assessment: Stage 1
The testing, carried out at 20Hz which is a typical blade passing frequency, showed that both time and frequency domain strategies give nearly 90% reductions in vibration.

- Frequency Domain Controller: 89.74%
- Time Domain Controller: 87.70%

Since both strategies used the same weighting factors in the quadratic minimisation process the similarity in performance was to be expected, although the frequency domain approach gives slightly better results because it is anti-resonant (controls at a single frequency), compared with the non-resonant properties of the time domain approach (controls across a range of frequencies). A value of 1/(3^2) was used to weight each of the acceleration outputs, and a value of 1/(30000^2) was used to weight each of the actuator control inputs. These values correspond to the reciprocal of the maximum value squared of the fuselage accelerations and actuator forces respectively. The results confirmed that the algorithms were working properly, as well as providing a baseline for the overall performance likely to be achieved.

5.2.2 Non-Adaptive Assessment: Stage 2
Three different types of robustness have been considered: response to changing frequency of the disturbing vibratory forces, to changing amplitude of the vibratory forces, and to changing parameters of the structure.

- Frequency Robustness: The ability of the algorithms to control vibrations over a range of frequencies was analysed by varying the frequency of the disturbance force, and the percentage vibration reductions as a function of frequency are shown in Figure 5.1. The frequency domain algorithm was implemented with both fixed frequency and feedforward configurations (described in Chapter 4). The basic fixed frequency controller is extremely sensitive to variations in the forcing frequency, as would be expected. The time domain algorithm, and the frequency domain algorithm with a feedforward reference signal, were not unduly sensitive to forcing frequency variations. It is perhaps surprising that the two results are relatively similar, given that the time domain strategy is still strictly optimal, whereas the frequency domain strategy is (strictly) wrong at frequencies other than 20Hz.

- Amplitude Robustness: The performance of the time domain strategy is independent of the forcing amplitude, as Figure 5.2 demonstrates. However, whilst the estimation of the $[T]$ matrix for the frequency domain algorithms is amplitude independent, the amplitude of the actuator forces will be incorrect if the excitation amplitude alters, and the figure also shows this effect. The results indicate that it is better to under-compensate for the vibrations rather than over-compensate (negative changes in forcing amplitude correspond to overcompensating).
Parameter Robustness For this study the time and frequency domain controllers were calculated for a known accurate system model. The system model was then degraded by varying a number of the parameters, and the controller applied to this degraded system model. Figure 5.3 gives results for variations in mass 3. The effects of other parameter variations were assessed but have not been included because the results for variations in mass 3 are typical. Figure 5.3 shows that the time and frequency domain algorithms have similar parameter robustness characteristics, with the frequency domain controller being marginally less effective for larger parameter changes.

5.2.3 Adaptive Assessment: Stage 3
For this stage the controllers were augmented with a recursive least squares parameter estimator, allowing the control strategies to adapt to the changes described in stage 2. The previous stage identified that frequency robustness is not a problem, as long as the feedforward approach is used, and so the results are presented for changes in the amplitude of the vibratory forces, and in the parameters of the model. The results are in the form of time simulations which enable the speeds of response to be demonstrated as well as the levels of vibration; in each case about 0.5 seconds is allowed at the start without the controller operating, and this shows the uncontrolled vibration levels.

Amplitude Robustness Figures 5.4 to 5.7 contrasts the results for the two approaches with a 20% increase in amplitude (the change in forcing occurs at a point 2 seconds into the simulation). The time domain controller of course does not need adaptation, and gives the same percentage reduction in vibration after the change as shown in Figures 5.5 and 5.6. The frequency domain approach does not initially respond particularly well, which is a consequence of the use of the local linear strategy which does not include an estimate of the background vibration. Initially therefore it starts to make changes to the model estimate, but after a while "catches" the changes and eventually results in a slightly lower vibration than the time domain strategy (consistent with the quasi-static results) and this is shown in Figures 5.4 and 5.5.

Parameter Robustness Figures 5.8 to 5.11 show typical results for a 50% reduction in mass 3. This is of course a large change, to which the time domain strategy adapts very quickly. The frequency domain strategy is slower by virtue of its update time, although it still gives an overall reduction in vibration once it has settled out. The frequency domain adaptation initially makes the vibration level worse than it would be without adaptation. A similar effect is seen if the global formulation is used, even though it estimates the background vibration as well as the T matrix. Note that despite the size of the change, there is still an overall reduction in the vibration level even while the adaptation is taking place.
5.3 **STEADY STATE PERFORMANCE**

This section corresponds to stage 1 from the preliminary assessment described previously, and contrasts the quasi-static or steady state performance of the various control strategies when implemented on the helicopter models. From the results of the preliminary assessment, the novel control strategy which is a hybrid of the time and frequency domain strategies, was developed, and has already been described in Chapter 4. The control strategies assessed are the Frequency domain and Hybrid approaches, and a number of variations on the optimal time domain scheme. These variations included state weighting, output weighting and frequency weighted outputs, all of which have also been described in Chapter 4.

When implemented on the helicopter models the ACSR system consists of ten accelerometers, four actuators and one of the control strategies just mentioned. Typical accelerometer time histories for the frequency domain controller are shown in Figure 5.12, and the corresponding actuator time histories are shown in Figure 5.13. The ACSR system is initialised after 0.5 seconds, and the overall reduction in average vibration corresponds to an improvement of 88%.

The performance of the various strategies was investigated at a number of baseline speeds (steady state flight conditions). Figure 5.14 shows the average r.m.s fuselage vibration levels for the different control strategies on the EH101 model, at different forward speeds. The corresponding performances in terms of percentage vibration reduction are shown in Table 5.1. The Frequency domain, Hybrid and Time domain (Frequency weighted outputs) strategies offer exactly the same steady performance, which was to be expected since they use the same weighting matrices in the cost function, which is used to weight fuselage accelerations at the blade passing frequency. The time domain controller with output weighting performs almost as well (typically a drop of 10% in performance when compared with the frequency weighted case). This reduced performance is because it is optimised for a range of frequencies as opposed to a single frequency. The strategy which offered the least reduction was the time domain scheme with state weighting (typically a drop of 20% in performance compared to the frequency weighted case), again this is optimised for a range of frequencies, and weights the states which are modal velocities and displacements. At a frequency of 1 rad/s the weighting on displacement, velocity or acceleration is equivalent, at other frequencies there are factors proportional to the frequency or the square of the frequency in the weighting. Therefore, weighting the states (modal displacements and velocities) as opposed to outputs (fuselage accelerations) effectively penalises the lower frequencies below 1 rad/s more, and those above less.
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Figure 5.15 shows the corresponding results for the strategies implemented on the W30 helicopter model. The results show that all the strategies also perform very well under a variety of steady state conditions with a different helicopter model.

These results are summarised for each of the strategies in Table 5.1. The results for the time domain optimal control strategy with frequency weighted outputs were included to demonstrate the equivalence between the strategies, but the approach is not considered any further due to its complexity. The strategies considered further are the Frequency domain, Hybrid and Time domain with output weighting.

In addition to the basic ACSR system performance, the effects of reducing the system configuration were investigated. Figure 5.16 shows the baseline and controlled fuselage vibration levels for the various control strategies at each of the ten accelerometer positions, for an ACSR system aiming to minimise the fuselage vibration levels at the ten accelerometers using four actuators on the EH101 model. Figure 5.17 shows the corresponding results for the W30 model. The system configuration was altered by varying the number of accelerometers used in the optimisation from 10 to 1, and by varying the number of actuators used from 4 to 1. Tables 5.2 and 5.3 show the overall performance for the Frequency domain/Hybrid and Time domain control strategies respectively, for different system configurations. The numbers in the tables represent the overall system performance, i.e. a positive number represents a percentage reduction in baseline vibration. The tables show that large numbers of accelerometers provide better system performance, even if only one or two actuators are used. It should be noted that it may be possible to improve on these performances by careful selection of accelerometers and actuators (positions), which relates to the problem of observability and controllability.

A fundamental difference between the time domain and Frequency domain/Hybrid control approaches is that since the Frequency domain/Hybrid approaches minimise at a single frequency, compared to the broad minimisation of the time domain approach. It is therefore possible, for example, to reduce the vibration to zero at three accelerometers using three actuators. This is not possible with the time domain approach since it controls across a range of frequencies. Figures 5.18 and 5.19 show this property for a 3 accelerometer and 3 actuator minimisation, for a 4 accelerometer and 4 actuator minimisation, and for a 10 accelerometer and 4 actuator minimisation. The results show that, even though the acceleration is reduced at the accelerometers being controlled, it can be made considerably worse at other locations. For the 3 by 3 minimisation using the Frequency domain strategy, the results show that despite the acceleration at 3 of the accelerometers being reduced to zero, the overall performance of
the system is only slightly better than the baseline case (an improvement of 4%). This indicates that it is important to have enough accelerometer measurements to represent the whole fuselage vibration, even if their individual levels of vibration cannot be reduced to zero. The results are summarised in a tabular form in Tables 5.4 and 5.5 for the Frequency domain/Hybrid and Time domain strategies respectively.

5.4 ROBUSTNESS

This section performs a robustness analysis of all the control strategies on the helicopter models. As for the three mass model, three different types of robustness have been considered: response to changing frequency of the disturbing vibratory forces, to changing amplitude of the vibratory forces, and to changing parameters of the structure. As well as these three types of robustness, two further robustness issues have been considered. The first is the ability of the strategies to perform during aggressive manoeuvres. The second issue applies to the time domain strategies only, and concerns the related issues of state estimation and unmodelled dynamics.

5.4.1 Frequency Robustness

The percentage vibration reductions as a function of frequency for the various control strategies are shown in Figure 5.20. For this assessment a blade passing frequency of 17.5Hz is assumed, which is a typical value for the EH101 helicopter. At 17.5Hz the values correspond to those given in Table 5.1. Away from this frequency the performance generally deteriorates, with the degree of deterioration depending upon the control strategy. The results agree with those from the 3 mass model and confirm that the frequency domain method is more sensitive to changes in the forcing frequency than the other methods. The time domain method offers the best robustness to such changes. For the hybrid strategy results are included both with one frequency in the optimisation process and with four frequencies. Its performance is significantly less sensitive than the frequency domain controller, and when four frequencies are used it approaches the robustness of the time domain method.

5.4.2 Amplitude Robustness

The amplitude robustness of the strategies predicted by the helicopter model is very similar to the results produced from the 3 mass model. The performance of the time domain and of the hybrid strategies is independent of the amplitude of the disturbance force, as Figure 5.21
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shows, since they are both based on a feedback structure. The lack of robustness to amplitude changes of the frequency domain controller is clearly shown in Figure 5.21 and the reasons for this were discussed in 5.2.2.

5.4.3 Parameter Robustness

As for the robustness assessment on the 3 mass model, the controllers were calculated for a known accurate system model. The system model was then degraded by varying the modal frequencies of the helicopter structure by a small percentage, and the controller applied to this degraded system model. Figure 5.22 gives the percentage vibration reduction for variations in the modal frequencies, and shows that the time domain and hybrid algorithms have similar robustness to shifts in the modal frequencies, with the frequency domain controller being somewhat less effective. Again, by including more than one frequency in the optimisation process for the hybrid controller, its robustness performance comes very close to that of the time domain approach.

Figure 5.23 demonstrates the robustness of the strategies to different forcing conditions corresponding to different flying speeds. In order to model manoeuvre response, linear interpolation is used between steady rotor force components for the dynamic simulation. By varying the flight condition the magnitudes and phases of the elements of the rotor forces input vector are varied, and consequently the amplitude and distribution of the vibrations across the structure are altered. The figure shows time simulations for the frequency domain, hybrid and time domain controllers respectively under the same sequence of forcing conditions, with the first 2 seconds of each response indicating the uncontrolled background vibration levels. The controllers are optimised for the initial forcing condition corresponding to 150kts, and after five seconds the forcing condition changes gradually over five seconds to that for 114kts. This represents a deceleration from 150kts to 114kts over five seconds, during which period significant changes occur in the amplitudes, phases and distribution of the rotor forces.

The time domain controller provides approximately 78 percent vibration reduction initially, which remains constant as forcing is changed. Both the hybrid and the frequency domain controllers provide 88 percent reduction initially. The performance of the frequency domain controller drops to -27 percent when the forcing is changed, actually making the fuselage vibration worse, whereas the performance of the hybrid controller only drops to 22 percent. The poor performance of the Frequency domain controller can be attributed to the fact that for this test it is openloop, and therefore as the background vibration changes it does not
reschedule the actuator demands. The hybrid controller performs better due to its feedback structure, although it is far from optimal. Figure 5.24 displays a similar set of results, but in this case the hybrid and frequency domain controllers measure the fuselage vibration and calculate a new set of control gains every second. Both controllers provide the same reduction of 86 percent once a steady flight condition has been reached and following several controller iterations. The time domain controller remains strictly optimal throughout this test. These performance levels are consistent with the steady-state performance indicated in Table 5.1.

5.4.4 Manoeuvre Robustness

There is a high level of vibration during transitions because of the rotor wake influence on the blade loading. The highest vibration levels will be reached during the flare manoeuvre, i.e. the transition from forward flight to hover, with a change of the rotor angle of attack from the normal negative value of forward flight to a positive value. Part of the reason for this is that the blade vortex interactions are strongly pronounced. The duration of the extreme flare vibrations is only a few seconds causing a rapid increase in the baseline fuselage vibration. However, because of their severity they are a real problem for many helicopters.

Figure 5.25 illustrates the performance of the frequency domain controller during this manoeuvre. Since the controller update rate is 1.0 second, during which time it remains openloop, the rapid increases in vibration are not fully compensated for until a new measurement is taken, by which time the vibration may have changed again. Therefore the controller may take several control cycles before satisfactory performance is once again achieved, during which time the controller may have only reduced vibration levels slightly or may even have made them worse. This periodic openloop nature of the frequency domain controller can be clearly seen by the sawtooth appearance of the reduced vibration.

Figures 5.26 and 5.27 show the responses of the time domain and hybrid controllers respectively. The periodic openloop nature of the frequency domain controller can also be seen in the actuator forces, which remain constant for the duration of each control cycle. Figure 5.28 shows the time history of forces for actuator number 3. This can be compared with the feedback nature of the hybrid controller which is shown again by the time history of the actuators. Figure 5.29 shows the time history for actuator number 3 and the envelope of the actuator force clearly follows the envelope of the fuselage vibration.

Figure 5.30 compares the performance of all the strategies. Both the time domain and hybrid strategies offer similar performance with the hybrid providing marginally better performance.
It is clear from the above results that the important factor for manoeuvre response is the rate of change of rotor hub forces and the corresponding rate of change of background vibration. It is found that using a one second control update rate, the vibration reduction capability of the frequency domain algorithms during quick manoeuvres is severely limited. An update time of 0.5 seconds enables the changes in rotor forcing and fuselage vibration to be tracked more accurately. This results in a marginal improvement in performance during aggressive manoeuvres and is shown in Figure 5.31. Reducing the update time further may result in contamination of the vibration measurements by the fuselage transients, resulting in erratic controller performance. A further possibility is that the controller may become unstable.

5.4.5 State Estimation

The time domain strategies require that the states of the system are fed back through a gain matrix to generate the required optimal control inputs. In this formulation the state vector consists of modal displacements and modal velocities, which are extremely difficult to measure, and therefore need to be estimated.

An important factor in the design of the estimator is how many fuselage modes need to be modelled in order to provide an adequate level of performance. Neglecting fuselage modes in the controller design gives rise to the phenomena of control and observation spillover, and this was discussed in Chapter 4.

For the first stage of analysis it was assumed that the modal states can be perfectly identified, i.e. with no observation spillover. Modes to be included in the controller design and in the estimator model were selected using the technique described in Chapter 3, i.e. the modes were selected based upon their contribution to the overall vibration at the blade passing frequency.

The simulation of the fuselage used all 24 modes. Based upon these assumptions and a steady forcing corresponding to 125 knots forward flight, the following performances were predicted for different numbers of modes included in the controller:

- 24 modes: 70% vibration reduction
- 10 modes: 67% vibration reduction
- 5 modes: 46% vibration reduction

These results show that even a small level of reduction can be achieved with the inclusion of only a few modes. Table 5.6 summarises the performance of the controller for different numbers of modes included in the controller. The reduction in the controlled modes and the uncontrolled modes is also given in each case. Generally the vibration of the controlled modes was reduced more effectively than the uncontrolled modes. This is what would be expected, although the performance does depend on the modes selected.
In practice a Kalman filter would be used to reconstruct the state vector and this will be subject to both control and observation spillover. The estimator contains a subset of the structural modes and has as its inputs the actuator control forces, therefore the estimated outputs will be in error due to two factors: the first is due to the unmodelled dynamics, the second is that the estimator only has the actuator forces as inputs whereas the actual structure is also excited by rotor forces and moments, which are of course unmeasurable.

Ten modes were selected to be included in the estimator, which was a reasonable compromise between performance and complexity. The predicted performance of the complete controller including a Kalman Filter becomes

10 modes 55% vibration reduction

Clearly, the performance of the time domain controller is significantly affected when practical implementation issues are considered. This section has identified the need for a Kalman filter and at least identified some of the likely degradations in performance, it is however acknowledged that with further investigation it may be possible to improve upon the predicted performance given here for the controller when a Kalman filter is included.

5.5 ADAPTIVE STRATEGIES

The previous stages have clearly shown that the fixed gain controllers prove to be robust, providing effective control under a wide range of flight conditions and parameter uncertainties. The adaptive controllers are designed to reduce the uncertainties through updating of the system parameters. The previous stages also identified that frequency robustness for the frequency domain controller is not a problem, as long as the feedforward approach is used. This stage gives a performance assessment of the strategies with adaptation. The controllers include recursive least squares estimators with variable forgetting factors [Fortescue, Kershenbaum and Ydstie, 1981].

5.5.1 Parameter Robustness

Results are presented for a 5 percent shift in the frequencies of the structural modes. The results given in Figures 5.32-5.34 are time simulations which enable the speed of response to be demonstrated as well as the levels of vibration. In each case about 0.5 sec is allowed at the start without the controller operating, and this shows the uncontrolled vibration levels. It can be seen that the time domain controller (Figure 5.34) offers the best robustness, providing an initial vibration reduction of 83 percent, and 77 percent vibration reduction when the structural
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modes are shifted. No noticeable degradation in performance is actually seen because the percentage reduction is quoted with respect to the background vibration level, which changes for each forcing condition. The hybrid controller (Figure 5.33) provides 85 percent vibration reduction initially, falling to 82 percent following the modal shift. It reduces further to 76 percent as the controller first tracks the changes in the structure, but finally provides 82 percent following the next controller calculation. The frequency domain controller (Figure 5.32) initially provides 85 percent reduction, falling to 55 percent following the structural change and then improving to 78 percent vibration reduction as the controller tracks the changes in the structure.

Generally, the estimates for the time domain controller take more iterations to converge due to the larger number of parameters to be identified, and this was discussed in Chapter 4. However, the controller update rate is considerably quicker for the time domain strategy when compared to the other strategies. For these simulations the controller update rate for the time domain controller is 100ms, whereas that for the hybrid and frequency domain controllers is 1 second. The results emphasise the poor transient performance of the frequency domain controller, even with adaptation, and indicate clearly that the hybrid controller offers a much improved transient performance, approaching that of the time domain controller.

5.5.2 Estimator Parameters

The next set of results demonstrate the effect of varying the estimator parameters. The initial parameter estimators are poor and after 30 seconds the flight condition is changed from a steady state flight condition of 114knots to 150knots. The results are for the adaptive hybrid control strategy, and time histories for the estimator's forgetting factor, error and covariance, and for the performance of the strategy in terms of percentage vibration reduction are shown.

At the first iteration of the recursive least squares estimator, an initial covariance matrix $P_o$ is required. In the same way that the size of $P_o$ determines the degree of knowledge about the system (confidence in the chosen initial parameters), it also fixes the way in which the searching (or tuning) for the correct controller is conducted. A large value of $P_o$ gives a rapid search with rapidly varying control. A small $P_o$, on the other hand, gives a cautious search, which may in fact not be able to move to the correct control configuration. The usual trend is to choose these values to be high (of the order of $10^5$-$10^6$). Therefore, when accurate initial estimates are used smaller initial covariance elements should be used.

The other parameter which significantly alters the behaviour of the estimator is the estimator
sensitivity $\Sigma$. This parameter determines how the estimator responds to errors between the predicted and estimated outputs. As the sensitivity is decreased (i.e. the magnitude of $\Sigma$ is increased) the estimator becomes progressively less responsive.

Figures 5.35 to 5.38 are for an estimator with a sensitivity ($\Sigma$) of 0.1, and an initial covariance matrix with its diagonal elements set to 100. The forgetting factor is changed rapidly in response to the change in flight condition. During the change in the flight condition the actuator inputs for the adaptive controller fluctuate as the parameter estimates are adjusted to reduce the residual error between measured and predicted vibration. The covariance matrix is set to a large number initially and decreases as the parameters are more accurately identified.

The effect of a lower covariance is shown in Figures 5.39 to 5.42. The main difference between these results and the previous results is the slower convergence of the parameter estimates. This is shown in the time taken for controller performance to reach the level predicted for ideal estimates (shown by a dotted line, Figure 5.42), and by the slower reduction in the estimator error (Figure 5.40). This is emphasised even more in the results for a covariance of 0.001 (Figures 5.43 to 5.46), which also includes measurement noise giving a signal to noise ratio of 25dB.

5.5.3 Effects of Measurement Noise

Increasing the measurement noise to significant levels causes the covariance matrix to quickly diverge resulting in divergent parameter estimates and a degradation in fuselage vibration. Figures 5.47 to 5.50 show the results for an estimator with sensitivity $\Sigma = 0.1$, lower sensitivities than which result in the estimator reacting to the measurement noise. Measurement noise therefore effectively places a lower level on the estimator sensitivity. Again increasing $\Sigma$ results in insensitive estimators that do not react to changes. Increased noise levels reduce the convergence rate of the parameter estimates and this can be seen by comparing the results in Figures 5.35 to 5.38 with Figures 5.47 to 5.50. Figures 5.43 onwards include the effects of measurement noise.

5.5.4 Limiting

Practically, limiting is required to satisfy hardware requirements of the actuators used to implement the ACSR controller. When limiting is achieved by weighting the control action $U$ in the performance index to reduce control amplitudes as much as possible without increasing fuselage vibration levels, it is known as Internal limiting. The performance index and weighting matrices were described in Chapter 4. Control can be limited externally, after the control forces have been calculated and hence without regard to optimality.
Simulation Studies

For small values in the control weighting matrix \( [R_v] \), the controller is still able to achieve about the same overall vibration reduction with smaller, but properly phased control inputs. However, for large weighting values, the controller can no longer command sufficient amplitudes to reduce fuselage vibration effectively. Large transients occur when there are large changes in control inputs, and these transients take longer to decay. Thus the controller is working with larger errors in system measurements which in turn leads to changes being made to the estimated \([T]\) matrix that may not be appropriate. Thus, even if the system is linear, large control inputs can lead to oscillatory behaviour and even instability.

Internal Rate Limiting The use of internal limiting improves the stability and performance of the controller, because internal rate limiting takes into account the desire to implement relatively small changes in control when calculating a new solution.

Very high values in the control rate limiting matrix \( [R_{av}] \) cause very slow, smooth relations in vibration, which may prove too slow for manoeuvres (Figures 5.51 to 5.54). Careful choice of \( [R_{av}] \) can result in a controller with very desirable performance properties, shown in Figures 5.55 to 5.58. While \( [R_{av}] \) has a significant impact on the rate of convergence, it does not inhibit the magnitude of control inputs that can be commanded and therefore does not limit the final steady state performance of the controller.

External Limiting/External Rate Limiting External limiting is not taken into account when calculating the solution needed to minimise vibration. Rather the solution is calculated using an unconstrained optimisation algorithm under the assumption that any set of control inputs is acceptable. Then after the fact, limits, or rate limits are applied to the magnitude of the calculated optimal solution to keep it within the capabilities of the hardware.

The solutions obtained using internal and external limiting are different. Figures 5.59 and 5.60 show the variation in magnitudes and phases of each of the actuators for different control input weighting using internal limiting (i.e. the limiting is performed within the cost function). These graphs show that the phases and relative magnitudes vary considerably for different weighting magnitudes. Presumably, these phases and relative magnitudes provide the optimum distribution of control forces for minimising vibration with inputs of this order of magnitude. External limiting would yield different phases and relative magnitudes. Davis [1984] identified that internal rate limiting provided better performance and stability than external rate limiting for HHC applications.
5.5.5 Stochastic Control

The stochastic control constant $\lambda_c$ also has a significant effect on the controller performance in much the same way that $|R_u|$ and $|R_u|$ effect the controllers performance, since the stochastic caution term increases the effective weighting on $\Delta U$ and $U$. Large values of $\lambda_c$ cause very slow, smooth reductions in vibration, which may be too slow for manoeuvres.

Employing a stochastic control strategy restricts the changes in the actuator inputs for poor parameter estimates, and effectively makes the controller cautious thereby improving the baseline transient response. In this investigation it was found that the internal rate limiting strategy proved to be more effective.

5.6 EFFECTS OF REAL ACTUATORS

The dynamics of the overall system are complex and non-linear, and include sensor dynamics, fuselage dynamics, actuator dynamics and any signal conditioning filter dynamics. Hydraulic actuators have the greatest effect on the overall system and they are the major limiting factor when considering system performance. In this application the actuators are used to their full capacity and consequently the nonlinear pressure/flow characteristic has to be considered and modelled in the control system design if they are to be used successfully.

Time domain methods are based on linear plant models, i.e. a sum of second order structural modes which are typically formulated into a set of first order equations for control system design. Consequently only linear actuator dynamics can be included, often with unmeasurable states. However, it was noted previously that with the frequency domain controller any actuator dynamics are implicitly incorporated into the model.

A simulation study was performed to assess the identification issues associated with linear and non-linear actuators. The first stage involved the development and validation of both linear and non-linear actuator models. These models were presented in Chapter 3, and Figures 5.61 and 5.62 show openloop step responses for flow and load velocity respectively for both actuator models. The oil column resonance (which is approximately 70Hz for these actuators) is visible. The nonlinear model provides a higher flow rate at full demand, which is because the linear model is based on an operating point at $\%$ of the supply pressure.

The main controller options with regard to the actuators are:

i) to ignore any actuator dynamics

ii) to include linear actuator dynamics in the controller model

iii) to provide an adaptive controller
The possible simulation options are summarised in Table 5.7. The initial stage considered an actuator driving a simple mass and spring arrangement, and a Recursive Least Squares (RLS) estimator was used to identify model parameters. The identification of the linear actuator was straightforward. The next stage involved identifying a set of parameters for a linear model using a non-linear actuator in the simulation. A set of parameters could be successfully identified which gave dynamics broadly similar to the linear model as long as the actuator was not being driven hard into the nonlinear regions of the pressure/flow characteristic. Figure 5.63 shows the load position when the non-linear actuator model is driven under different conditions. The first uses a 10Hz small amplitude sinusoidal signal allowing the actuator to operate in the linear region of its dynamics, the second is a larger amplitude 10Hz signal causing the actuator to operate over its full non-linear range. Figure 5.64 shows the estimator errors resulting from the two operating conditions. Figure 5.65 shows the estimated eigenvalues (real parts), clearly when the actuator is operating in its linear region linear parameters can be successfully identified. However when operating over its full range the estimator continuously adjusts the model.

The actuators were then incorporated into the helicopter models. Simulations have shown the excellent performance of the algorithms with perfect system models, but when actuator dynamics were included in the simulation the controller's performance was degraded. When adaptation was included the performance of the hybrid and frequency domain controllers was improved, but the time domain strategy became unstable. The results are summarised in Table 5.8.

5.7 CONCLUSIONS

The simulation results show significant vibration reduction at all steady state flight conditions tested. The steady vibration levels achieved show the optimal performance of the frequency domain and hybrid controllers, with reductions in average vibration from 0.6g to under 0.08g (representing an 88% reduction). Significant vibration reduction was also achieved during changing flight conditions.

Comparing the controller's performance, the frequency domain and hybrid algorithms are better in terms of overall vibration reductions, although the time domain controller's performance is only a little inferior. The frequency domain controller suffers from a poor transient response as
the damping in the modes is unaffected by the control. The time domain controller offers the best transient performance but is very complex once adaptation and state estimation have been included. The improvement reflects the fact that the time domain strategy increases the damping in the dominant modes of vibration. The hybrid controller gives a transient performance which approaches that of the time domain controller, but is computationally simpler and inherently includes effects such as actuator dynamics which are problematic with the time domain approach.

The steady state performance of the time domain controller is slightly less than the hybrid and frequency domain algorithms at the blade passing frequency. This is because the optimal controller weights all states (and hence frequencies) equally, but its performance can be improved to match the performance of the frequency domain and hybrid controllers by using frequency shaped cost functionals in the performance index. These weight the blade passing frequency content of the fuselage vibration. This results in a reduction in the transient performance of the controller and greatly increases the complexity of the controller, since a second order tuned filter is required for each output.

From the robustness results it can be seen that the frequency domain algorithm, though only strictly correct at a single frequency, still provides vibration reduction at other frequencies. The further away the forcing frequency is from the design point, the worse the degradation in the controller's performance. The hybrid algorithm has the same vibration reduction performance at the blade passing frequency and provides slightly better performance away from the design point, which can be further improved by optimising at a number of frequencies. The hybrid controller also has slightly better robustness to different forcing conditions.

The better robustness of the hybrid approach can be partly explained by the fact that since the hybrid controller feeds the outputs back through a gain matrix, it compensates for any changes in amplitude directly, leaving only the phase uncompensated. The frequency domain controller is an open loop system and therefore it compensates for neither changes in magnitude or phase directly. The time domain LQ controller exhibits very good robustness properties, and good performance across a range of frequencies.

The dynamics of the overall system are greatly influenced by the hydraulic actuators and they will always limit the performance regardless of the controllers processing capability. If they are to be used to their full capacity then the inherent non-linearity has to be considered in the control system design.
With the Frequency domain strategies difficulties can occur if the period of the FFT is different from the period of the signal. In this case the values at the beginning and the end of the FFT period are different, and the transform into the frequency domain is strictly incorrect. One preventative measure is the application of a window function, such as the Hanning or Hamming Window.

When considering steady state flight conditions the more time allowed for transient decay, the better the accuracy of the identification of the blade passing frequency components and hence improved controller performance. However, when considering the capability of the controller to reduce vibration due to transient flight conditions this may not be true. As the time allowed for transient decay decreases, there is a degradation in controller performance due to inaccurate system identification, but an improvement in the ability of the controller to respond to transient flight conditions.

There are a number of difficulties which must be studied before the benefits of the time domain strategy can be fully realised. Firstly, as already mentioned, the system non-linearities must be studied more fully, particularly those introduced by the hydraulic actuators. Secondly, unmodelled dynamics are inevitable in a structure such as a helicopter airframe, and the spillover effects will need to be carefully analysed. Practical implementation of LQ time domain controllers also require that all states are available for feedback. With any but the simplest system however this is clearly an impossibility and it is either necessary to compromise the design with the instrumentation that is available, or recreate unmeasured states with the aid of an observer. While it is possible to achieve the theoretical regulator performance with an observer, the observer is only as good as the system model used to design it. The implementation of an adaptive time domain optimal controller adds even further to the overall complexity of this strategy. All of these factors make the time domain approach computationally more extensive than the other methods.

The restrictions imposed by assuming linear actuator models, and the requirements of state estimation make state feedback control an unlikely proposition for practical implementation. Therefore the conclusion of this study was that time domain state feedback designs are unsuitable for this application at the moment but have been included in the linear simulations for comparison purposes. Only the frequency domain and hybrid control strategies have been carried through into the final testing phase, implemented practically and tested. The results are presented in the next chapter.
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<td></td>
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<td>91.1</td>
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<td>Hybrid</td>
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<td>91.1</td>
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|             |           |                  | 136   | 150  |
|             |           | Time Domain - Optimal State weighting | 71.3  | 67.5 |
|             |           | Time Domain - Optimal Output Weighting | 77.7  | 78.6 |
|             |           | Time Domain - Optimal Frequency Weighting | 90.5  | 88.2 |
|             |           | Frequency Domain | 90.5  | 88.2 |
|             |           | Hybrid           | 90.5  | 88.2 |

|             |           |                  | 114   | 125  |
|             |           | Time Domain - Optimal State weighting | 54.6  | 57.2 |
|             |           | Time Domain - Optimal Output Weighting | 64.9  | 70.0 |
|             |           | Time Domain - Optimal Frequency Weighting | 89.1  | 87.9 |
|             |           | Frequency Domain | 89.1  | 87.9 |
|             |           | Hybrid           | 89.1  | 87.9 |

Table 5.1 Steady State Performance of the Control Strategies, assessed in terms of Percentage Vibration Reduction

178
## Table 5.2 Frequency Domain and hybrid Controller Performance for various System Configurations

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Table 5.3 Time Domain Controller Performance for various System Configurations
Table 5.4 Vibration Reduction at Individual Accelerometer Positions for the Frequency Domain and Hybrid Controllers using different Configurations

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<th>Controller Performance</th>
<th>Minimisation (Actuators by Sensors)</th>
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<td>Average 87.3 4.4 91.1</td>
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### Table 5.5 Vibration Reduction at Individual Accelerometer Positions for the Time Domain Controller using different Configurations

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<th>Controller Performance</th>
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### Table 5.6 Time Domain Controller Performance with number of controlled modes

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<th>Percentage Reduction (Uncontrolled Modes)</th>
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* alternative selection of modes
### Simulation Studies

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<th>Simulation Options</th>
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Table 5.7 Simulation options for actuator models.

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Table 5.8 Controller performance with actuator dynamics
Simulation Studies

Figure 5.1 Change in Forcing Frequency for non-adaptive strategies

Figure 5.2 Change in Forcing Amplitude for non-adaptive strategies

Figure 5.3 Change in Parameter (Mass 3) for non-adaptive strategies
Simulation Studies

Figure 5.4 Adaptive Frequency Domain Controller
Acceleration Time Histories

Figure 5.5 Performance of Adaptive Frequency Domain Controller

Figure 5.6 Adaptive Time Domain Controller
Acceleration Time Histories

Figure 5.7 Performance of Adaptive Time Domain Controller
Figure 5.8 Adaptive Frequency Domain Controller Acceleration Time Histories

Figure 5.9 Performance of Adaptive Frequency Domain Controller

Figure 5.10 Adaptive Time Domain Controller Acceleration Time Histories

Figure 5.11 Performance of Adaptive Time Domain Controller
Simulation Studies

Figure 5.12 Simulated Accelerometer Time Histories. ACSR system initialised after 0.5 seconds.
Figure 5.13 Actuator Time Histories for 4 Actuators. ACSR system initialised after 0.5 seconds.
Figure 5.14 Fuselage Vibration Levels for EH101 Helicopter

Figure 5.15 Fuselage Vibration Levels for W30 Helicopter
Figure 5.16 Fuselage Vibration Levels for EH101 Helicopter at 125knots (control minimisation based upon 4 actuators and 10 sensors)

Figure 5.17 Fuselage Vibration Levels for W30 Helicopter at 125knots (control minimisation based upon 4 actuators and 10 sensors)
Figure 5.18 Fuselage Vibration Levels for EH101 Helicopter at 125knots (control minimisation based upon 4 actuators and 4 sensors)

Figure 5.19 Fuselage Vibration Levels for EH101 Helicopter at 125knots (control minimisation based upon 3 actuators and 3 sensors)
**Simulation Studies**

![Graph](image)

**Figure 5.20** Change in Forcing Frequency for non-adaptive strategies (Helicopter Model)

**LEGEND**

- Hybrid Control (Multiple Frequencies)
- Time Control (Output Weighting)
- Hybrid Control
- Frequency Control

![Graph](image)

**Figure 5.21** Change in Forcing Amplitude for non-adaptive strategies (Helicopter Model)

![Graph](image)

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Figure 5.24 Average RMS Vibration levels during a change in Forward Speed from 150 knots to 114 knots
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CHAPTER 6

EXPERIMENTAL VALIDATION

INTRODUCTION

This chapter evaluates the strategies on a ground based full size helicopter, extending the work of Chapter 5, which assessed the control strategies in non-real time simulation. The effectiveness of the system is proved by a helicopter shake test. For this, the helicopter, which is suspended by the rotor hub, is loaded by realistic rotor forces and moments by the use of electromagnetic shakers. The work concentrates on a comparison between the established Frequency domain controller and the new Hybrid control algorithm.

The controller hardware is described, followed by a description of the software implementation of the Hybrid ACSR control algorithm. The test rig arrangement is outlined, along with the proposed rig tests. Finally the results from the rig tests are presented and discussed. The main aim of this chapter is to validate the simulation results, and to assess the practicability and effectiveness of the newly developed control approaches.

Figure 6.1 shows a block diagram of the overall system set up. For a combination of reasons the system to be implemented was restricted to using four out of the possible ten accelerometers, and two out of the four actuators, both located towards the front of aircraft.

The active vibration controller hardware consisted of the DSP card and the analogue interface card. A second computer contained and provided a user interface for the active vibration controller hardware. This computer also stored the controller software, and was linked to the accelerometers via the accelerometer drive rack and to the actuators via the actuator drive rack.

6.1 CONTROLLER HARDWARE DESCRIPTION

The Active Vibration Control system is shown in Figure 6.2 and can be divided into a number of sub-systems:

1) Vibration sensors (accelerometers) fixed at various locations on the helicopter fuselage. These accelerometers measure the structural vibration and pass the signals to the active vibration controller via an accelerometer driver, amplifier and signal
conditioning unit, so that it can calculate the actuator control inputs. The actual locations and the accelerometers used by the controller are detailed in section 6.3 of this chapter.

2) an Analogue Interface for sampling the vibration measured by the sensors (using 4 ADCs), and for generating the control inputs for the actuators calculated by the controller (using 2 DACs),
3) a Digital Signal Processor (DSP) controller which computes the control algorithm and generates the optimum control inputs using the algorithms described previously,
4) Electro-hydraulic servo-actuators which apply the control inputs to the fuselage in order to minimise fuselage vibration. Each actuator is mounted in parallel with a load bearing compliant structural element so that the actuator can provide the controlling vibratory forces while the load bearing element can accommodate the quasi-static primary flight loads. A local force feedback loop around each device ensures that the actuator does not react to these quasi-static loads,
5) a PC which provides an interface between the control system and the user.

6.1.1 Controller
A floating point digital signal processor was chosen to implement the algorithms, mainly due to the computationally intensive nature of the algorithm calculations, which must be accomplished within a very short time interval. A floating point processor was selected instead of a fixed point processor to avoid problems associated with variable scaling, etc. The development board used to implement the controller was the Vector 32C/256 produced by Surrey Medical Imaging systems Ltd, which is provided with 64 kwords (32 bit) zero wait state memory. The board contains an AT&T DSP32C digital signal processor, a high performance floating point Digital Signal Processor capable of executing up to 25 million arithmetic operations per second.

The DSP32C can be programmed using a standard C compiler. The development board is connected to a PC via the PC bus, which allows files, code etc to be transferred between the controller and the PC host system.

6.1.2 Analogue Interface Board
The board is available from Loughborough Sound Images Ltd and interfaces to the main DSP board using the DSPLINK (data link). The board contains 4 analogue input channels and 2 analog output channels. The 4 analogue input channels each have an input buffer, a low pass
filter and a sample and hold device. The four channels are multiplexed to a fast 12 bit analogue to digital converter (ADC). Channel selection is performed in software using the Control Register. The conversion time is 5\mu s which allows a single channel sampling frequency of 200K samples/second, or 50K samples/second when all 4 channels are being sampled simultaneously. A +/− 2.5 volt analog input range provides full scale operation of the ADC. The 2 analogue output channels each have a low pass filter and buffer amplifier. Each converter is digitally double buffered to allow simultaneous updating of both analogue output channels.

Filtering for all channels is via 3rd order low pass filters which are selectable via resistor changes. A typical settling time of 3\mu s allows sample output rates up to 300K samples/second. Clocking may be from an on board timer provided by a 16 bit counter. Alternatively both the ADC and DAC sections can be triggered using an external trigger signal or by software. The board is memory mapped so that variables can be assigned to addresses from assembler or cross compiled code.

6.2 CONTROLLER SOFTWARE DESCRIPTION

This section describes the software used to implement the control algorithms discussed in Chapter 4. The code for the DSP was developed in C using a cross-compiler resident in the PC. The software was developed in a number of stages:

Stage 1: developed the optimisation and estimation routines using MATLAB generated files as inputs, and validated the outputs against those obtained by the MATLAB routines in the simulation studies. In this stage the procedures were implemented using the PC's processor.

Stage 2: developed C and assembler routines to handle the input and output of data. This included ADC and DAC initialisation, reading from the ADCs and writing to the DACs. The code was cross compiled to run on the DSP. As before input files were generated by MATLAB, and output files were imported back into MATLAB so as to validate the operation of the routines for reading from the ADCs and writing to the DACs.

Stage 3: combined the routines developed in stages 1 and 2 and developed the linking code required to interface these routines. To validate and refine the integrated software an electronic simulation was designed and built. This consisted of electronic circuits to simulate a number of the flexible fuselage modes of a helicopter, and is described in
Appendix VII. It has four outputs corresponding to four accelerometers on the fuselage, and three inputs, corresponding to the two ACSR actuators and one for the rotor forcing input. This stage allowed the controller software to be validated in real time before implementing the controller on the full size airframe.

The controller has two sets of tasks to perform. One set is to update the actuator control inputs and to measure the vibration, and this is executed during each sample period (many times per second). The other is to calculate new control gains and to perform parameter estimation, and is executed during each controller update cycle (once or twice per second).

Controller tasks performed during each sample period are:

1. Sample 4 acceleration signals,
2. Calculate the actuator control signals:
   i) for the frequency domain algorithm this involves updating pointers for sine and cosine tables and multiplying by the optimal frequency domain control vector.
   ii) for the hybrid algorithm the sampled vibration vector (consisting of ADC values) is multiplied by the hybrid gain matrix.
3. Update the DAC output signals (actuator drive signals).

The controller tasks performed during each control cycle consist of computing three primary interrelated algorithms:

1. A real time optimal control algorithm for vibration minimisation,
2. A real time identification algorithm to identify and track the fuselage T matrix in real time,
3. A harmonic analyser for obtaining real-time harmonic components of the measured fuselage vibration. The harmonic analyser includes a delay to allow structural transients to decay prior to any signal processing, and signal processing (correlation DFT) to measure the fuselage vibration.

6.2.1 Hybrid Controller

This section describes an overview of the software used to implement the Hybrid time/frequency domain controller algorithms. Both the frequency domain and hybrid control algorithms have been implemented in software, in adaptive and non-adaptive forms. In this chapter the adaptive Hybrid controller is taken as an example for description since many of the routines are common or very similar. Appendix VIII contains a complete listing and detailed decryption of the software for the adaptive hybrid controller.
Experimental Validation

The control algorithm essentially consists of two loops, as shown by the software chart in Figure 6.3. One loop is interrupt driven and the primary task is to feed back the measured vibration through a gain matrix to provide drive signals for the actuators. The interrupt service software contains functions: which initialise and read from the 4 ADCs, initialise and write to the 2 DACs, correlate the measured vibration, digitally filter and update the control signals for the actuators. Figure 6.4 shows the sequence of events for the interrupt routine. The other loop acts in parallel with this operation. For a given controller configuration, the initialisation consists of zeroing the control inputs, defining the initial parameters for the estimation algorithm, and initialising the T matrix. Since the controller can identify and track the T matrix, the only criteria for defining the initial T matrix is that it will maintain stability and not generate vibration when the controller is activated. This is best accomplished using an open-loop identification scheme. The initial value of the covariance matrix (covariance of T matrix estimate - \( \mathbf{P}_0 \)) can be set to a large value to reflect the uncertainty in the estimate of the T matrix. The control algorithm then calculates an update for the feedback gain matrix in a number of stages, which are shown in Figure 6.5:

Transience Decay Since the algorithm is based on a quasi-static assumption structural transients must be allowed to decay before any measurements are taken. The controller allows for a time delay before signal processing begins.

Digital Signal Processing All control optimisation calculations are performed in the frequency domain. Therefore the noise content of the vibration signals, measured by the accelerometers is calculated using a Discrete Fourier Transform (DFT).

Parameter Estimation The estimator uses a Recursive Least Squares (RLS) technique incorporating a variable forgetting factor. Actuator control and vibration measurement data is used by the estimator to predict fuselage vibration levels. The estimator recursively updates the estimate of the fuselage transfer function based upon the error between the predicted and actual vibration levels.

Control Optimisation The measurement of the fuselage vibration and the estimate of the fuselage transfer function are used to calculate the optimal control input for the next controller cycle. Based on this information a feedback gain matrix is calculated which produces the required control action by feeding back the accelerometer signals.

6.2.2 Initial System Identification

Both the Hybrid and the Frequency Domain controllers require an initial estimate of the T matrix, because although the adaptive controllers include an on line parameter estimator an accurate initial estimate of the T matrix will help convergence. There are two schemes which
could be used to accomplish this initialisation. The first is to load a predetermined T matrix, either from calculations or from a previous test. The other scheme is to perform an openloop identification of the system. This is accomplished by applying drive signals to each actuator individually and measuring the resulting vibration levels (in practice a steady state \( n \omega \) sine signal followed by a \( n \omega \) cosine signal is applied sequentially by each actuator). If the baseline vibrations are measured prior to the application of the first input, then the respective columns of the T matrix may be determined. A period to allow the fuselage transients to decay is included between the successive application of control inputs.

Once the controller has been successfully initialised, it functions as described previously. A new estimate of the T matrix (and the background vibration vector \( \mathbf{B} \)) are obtained at each controller iteration for the adaptive controllers, and the optimal control inputs are calculated based on these updated estimates. The structure of the openloop identification software is very similar to the structure of the Hybrid and Frequency Domain controller software.

Prior to the closed loop ACSR tests, a series of openloop tests were conducted to establish the baseline fuselage response to rotor head and ACSR actuator input excitations. In this case the background vibration was zero and consequently the measured fuselage responses gave the columns of the T matrix directly.

### 6.3 TEST RIG DESCRIPTION

Testing was carried out on a Westland 30 series 160 aircraft. This aircraft is used as a research and development test rig at Westland Helicopters Ltd. The aircraft is configured as flight standard, but is suspended by soft springs through a dummy rotor head in order to excite the free-free fuselage vibration response. Power for the Active Vibration Control system was provided by the normal aircraft systems fed from external hydraulic and electrical ground supplies. The test rig is equipped with ten airframe vibration sensors (accelerometers) and four force producing electrohydraulic actuators. Figure 6.6 shows an overview of the complete configuration and Figure 6.7 shows a photograph of the test rig. A dedicated computer controls an electromagnetic shaker, which acts at the dummy rotor head providing excitation forces to the airframe. This computer also stores flight data recorded from aircraft flight trials, enabling rotor head loads representative of the flight environment to be applied to the aircraft. The active vibration control software was stored on a second computer, this was linked to the
accelerometers via the accelerometer drive rack and to the actuators via the actuator drive rack (shown in Figure 6.8). A detailed diagram of the instrumentation arrangement is shown in Figure 6.9.

6.3.1 Accelerometers
Accelerometers measure fuselage vibration at various positions on the airframe. The test rig is fitted with ten Endevco 7251-100 piezoelectric accelerometers. These have a number of advantages including good linearity, low weight, broad dynamic range, strong construction and simple design. The accelerometer locations and their use are listed below:

<table>
<thead>
<tr>
<th>Accelerometer</th>
<th>Location</th>
<th>Direction</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Mid Cabin Floor</td>
<td>Vertical</td>
<td>Controlled</td>
</tr>
<tr>
<td>9</td>
<td>Top Door Port</td>
<td>Lateral</td>
<td>Controlled</td>
</tr>
<tr>
<td>8</td>
<td>Co-pilot's feet</td>
<td>Vertical</td>
<td>Monitored</td>
</tr>
<tr>
<td>7</td>
<td>Top Door Starboard</td>
<td>Lateral</td>
<td>Controlled</td>
</tr>
<tr>
<td>6</td>
<td>Floor Starboard</td>
<td>Vertical</td>
<td>Controlled</td>
</tr>
<tr>
<td>5</td>
<td>Top Door Port</td>
<td>Lateral</td>
<td>Unused</td>
</tr>
<tr>
<td>4</td>
<td>Floor Port</td>
<td>Vertical</td>
<td>Unused</td>
</tr>
<tr>
<td>3</td>
<td>Co-pilot's Seat</td>
<td>Vertical</td>
<td>Unused</td>
</tr>
<tr>
<td>2</td>
<td>Top Door Starboard</td>
<td>Lateral</td>
<td>Unused</td>
</tr>
<tr>
<td>1</td>
<td>Pilot's Seat</td>
<td>Vertical</td>
<td>Monitored</td>
</tr>
</tbody>
</table>

The actuators used by the controller to generate control input drive signals for the actuators are labelled "controlled". This selection covered the cabin and cockpit areas in both vertical and lateral directions. The accelerometers labelled "monitored", are used by a data acquisition unit. The controller is interfaced to the accelerometers via an accelerometer driver, amplifier and signal conditioning unit. The system included anti-aliasing filters at 100Hz. These filters formed part of the signal conditioning cards that interface the accelerometers to the controller.

6.3.2 Actuators
The electro-hydraulic servo-actuators are located at the gearbox/fuselage interface and operate in parallel with structural elements (elastomer springs). The aircraft’s 2000psi hydraulic supply (generated by a ground rig) drives the ACSR actuators. A Moog Series 30 servovalve controls each actuator and the actuator pistons have an effective area of one square inch. The actuators provide approximately a maximum force of +/− 9 kN at blade passing frequency, with a maximum vibratory displacement of 0.25cm.

The controller calculates the drive signals for the servovalves. Pressure transducers measure the differential pressure across the actuator and produce a force feedback signal. Moog
industrial cards interface the actuators to the controller; provide a pressure feedback signal (using a PID local feedback arrangement) and generate a servovalve drive current from the pressure feedback signal and control voltages.

6.3.3 Data Acquisition

The data acquisition unit consisted of a commercially available integrated hardware and software toolbox installed in a third PC. It makes use of a National Instruments Lab PC card, and is capable of acquiring up to eight channels of data, at a maximum sampling frequency of 8kHz. The software is a toolbox attached to the MATLAB application. The sampled data can be acquired from the system through the toolbox directly into the MATLAB environment for analysis. The unit also has the capability of playing two channels of data, allowing signals created within the MATLAB environment to be played through the output channels.

The system sampled the accelerometer and actuator signals at 1000Hz independently of the active vibration controller. A frequency response analyser and oscilloscope were also connected to provide immediate indication of the controller's performance (typical displays are shown in Figures 6.10 and 6.11). Figure 6.10 shows a spectrum of the fuselage vibration. Peaks at the blade passing frequency \( n_o \) (20Hz) and its harmonics are clearly visible. Figure 6.11 contains a dual display, the top display shows a spectrum of the time domain signal shown in the bottom trace. This trace shows the fuselage vibration measured at an accelerometer, after approximately one second the ACSR controller is turned off and an increase in vibration can be seen.

The data acquisition unit recorded the two actuator drive signals, the four accelerometer signals which are to be controlled (fed back by the controller). An important consideration in the evaluation of an active vibration control system is the degree to which the vibration at locations not directly controlled are reduced, and therefore two other accelerometer signals are recorded to monitor the vibration at uncontrolled locations in the fuselage. The additional accelerometers accessible to the data acquisition unit were listed earlier (labelled "monitored").

6.4 RIG TESTS

The main objectives of testing were to confirm the performance of the Hybrid algorithm predicted by the simulation studies, to verify the controller software developed using the bench test rig, to assess the controller's steady state performance, to assess the robustness of the
Experimental Validation

system to variations in rotor speed, and to analyze the response of non-adaptive and adaptive controllers to transient flight conditions. The tests themselves also include openloop tests to characterise the fuselage dynamics, represented by the frequency response relationship at the blade passing frequency between actuator inputs and the resulting measured structural response.

The testing methodology adopted can be summarised as follows:

1) the test signal is played to the electromagnetic shaker using the computer connected to the rig shaker,
2) the dedicated data acquisition computer records the accelerometer and actuator signals,
3) the Active Vibration Controller is initialised and step (2) is repeated.

The output data is in a form which can be loaded into the Matlab environment and analysed using the supplied system identification functions.

6.4.1 Closed Loop Tests

With the aircraft suspended by the bungee system, excitation was applied to the main rotor hub and the baseline fuselage vibration was measured. The ACSR system was then activated to suppress the fuselage vibration levels which were measured again. Considering the average r.m.s value of the measured controlled and uncontrolled vibration signals provides an indication of the controller's performance. A number of different test signals were used to drive the rig shaker and these are detailed in the following sections. The performance of the Hybrid and Frequency Domain Control strategies was assessed by exciting the test rig with these signals.

Steady State Tests Two types of steady state test signals were used. The first was simply a sinusoidal signal with constant amplitude and frequency constructed using Matlab. The other type were generated from recorded flight data for steady state forward flight conditions at both 40 and 140 knots.

Gust Tests The signals used for these tests were generated in Matlab. Each signal consisted of a 20Hz sine wave with shaped magnitude envelope. For the first few seconds the amplitude of the signal is constant, then the magnitude of the signal increases and then decreases, following a sine² profile, back to the original value which is held constant for several more seconds. A number of signals were generated with different sine² profile durations. Durations of 1 second, 2 seconds, 4 seconds and 8 seconds were constructed.
Experimental Validation

Transition to Hover (EH101 Aircraft)
The transition from steady state forward flight to the hover is called the Transition to Hover manoeuvre. The transition to hover is characterised by a rapid increase in the baseline fuselage vibration amplitude and phase during 1 to 2 seconds and therefore provides a good indication of the performance of a controller. This is one of the most severe manoeuvres and it was thought the most likely to affect the stability of the controller. Flight data was used, recorded during a Transition to Hover Manoeuvre performed by EH101 and Lynx helicopters. The Lynx helicopter has an unusual rotor system which gives the aircraft outstanding manoeuvrability, but also means that the rotor generates relatively large vibratory loads at the blade passing frequency. This feature, coupled with its high cruising speeds, results in an aircraft with potentially high vibration at high forward speeds. Consequently the data for the transition to hover manoeuvre contains considerably more rapid changes in baseline vibration magnitude and phase than does the data for the EH101 helicopter. Four test signal variants were generated from this data:

Transition to Hover- Noisy (EH101 Aircraft) The raw unfiltered data was used as one test signal.

Transition to Hover- Filtered (EH101 Aircraft) The data was low pass filtered to remove the high frequency components.

Transition to Hover- Smoothed (EH101 Aircraft) The data was again low pass filtered, this time using a filter with a lower break frequency, to provide data with a smooth envelope.

Transition to Hover Filtered (Lynx Aircraft) Low pass filtered.

6.5 TEST RIG RESULTS

The results presented in this section are a summary of the shake tests using the signals described above. The vibration response for the baseline dynamics is compared with that for the ACSR system. The important questions in the overall assessment of the strategies were summarised in Chapter 5, and are: -

1) How effective are they in reducing vibrations under quasi-static conditions?
2) How quickly do the strategies respond during changing flight conditions?
3) How well do they adapt to, or cope with changes in the system parameters?

Section 6.5.1 presents steady state results to answer the first question. Sections 6.5.2 to
6.5.5 assess the performance under changing conditions, both with and without adaptation, in order to address the second question. In the final sub-section 6.5.6, robustness performance is measured so as to answer the last question.

6.5.1 Steady State Results
These tests assess the steady state performance of the controllers. For the first series of tests a simple 20Hz sinusoidal signal was used to excite the fuselage. Both the Frequency Domain and Hybrid Controllers provided the same levels of performance. These tests investigated the effect of the actuator weighting term in the controller cost function. As would be expected the trend shows that for lower control weightings (allowing more control effort) larger improvements in the reduction of fuselage vibration can be achieved. Table 6.1 contains figures for the performance of the controllers in terms of percentage vibration reduction, for different actuator weightings. Figure 6.12 shows the r.m.s baseline vibration and the r.m.s controlled vibration levels measured at each sensor diagrammatically.

Figures 6.13 and 6.14 show the Power Spectral Densities of the fuselage vibration for different controller conditions. If the control weighting was set to very low values the Hybrid controller sometimes excited other fuselage frequencies, a consequence of the feedback structure of the Hybrid controller and the fact that these frequencies are not modelled. The results for a control weighting of 0.2 clearly show that fuselage vibration at 40Hz is made considerably worse (refer to Figure 6.14), although the 20Hz component is being controlled very well. This problem was solved by including a digital filter into the controller structure. The filter was second order and was tuned to 20Hz with 3% damping, thereby allowing only the 20Hz blade passing frequency to be fed back.

The final steady state rig tests used recorded flight data for steady state flight conditions of 40knots and 140knots. Figure 6.15 show the power spectrum density at one of the sensor positions for a steady state flight condition of 40knots. The reduction at the blade passing frequency can be seen, and the uncontrolled vibration peaks at harmonics of this frequency. Figure 6.16 displays the average r.m.s accelerations measured at each sensor position for baseline and controlled vibration levels for a steady state flight condition of 140 knots. Table 6.2 contains the results for the performance of the Frequency Domain and Hybrid Controllers under these different flight conditions.
6.5.2 Gust Results - Non Adaptive

The Gust tests enable the response of the controllers to changing flight conditions to be analysed. The gust tests used represent a fairly simple change in flight condition and the controller was implemented in a non-adaptive framework, thus enabling the fundamental characteristics of the controllers to be observed. Various gust signals were applied to the fuselage ranging from a short duration harsh gust of one second to a longer duration eight second gust. A selection of results are presented to summarise the findings.

Figure 6.17 shows the average r.m.s vibration levels for the baseline and for the frequency domain and Hybrid controllers. As can be seen the Hybrid strategy controls the fuselage response to the gust slightly better than does the Frequency domain controller. This is again shown in Figure 6.18 which converts the r.m.s figures into percentage vibration reduction. Both controllers provide approximately 70% vibration reduction (this includes the responses at controlled and uncontrolled accelerometers), but the performance of the Hybrid controller is not degraded as much by the gust.

Figures 6.19 and 6.20 show the actuator demand signals for the Hybrid controller. As can be clearly seen the frequency domain controller does not respond to the gust at all and this can be partially attributed to the control update period. Due to the feedback structure of the Hybrid controller it manages to respond to the gust.

A corresponding set of results are shown for the four second gust. The periodic openloop nature of the controller is shown by the sawtooth form of the controlled vibration response, which is caused by the controller reoptimising every second. Figure 6.21 shows the vibration response in terms of r.m.s values, and Figure 6.22 shows the performance in terms of percentage vibration reduction.

Figures 6.23 and 6.24 show the actuator demand signals for actuator 2, for the Hybrid and Frequency domain controllers respectively. The Hybrid controller’s demand follows the fuselage vibration response, whereas the periodic openloop nature of the Frequency domain controller is very evident in the stepped appearance of the actuator demand.

The results for the two and eight second gust responses displayed the same trends and therefore are not included. High actuator weights combined with low filter damping in some instances lead to an instability for the Hybrid controller, and an example is shown in Figure 6.25. For this test the damping in the digital filters used to filter the output from the Hybrid controller was increased from 3% to 6%, and the oscillatory variation in the vibration level is clearly seen.
From these results the periodic openloop nature of the Frequency domain controller is clearly visible. As the length of the gust duration increases the performance of the two controllers converges, but even then the switching effect of the Frequency domain controller is observable as the controller attempts to compensate for the changes in baseline vibration since the last control update. The Hybrid controller, although having a consistent control update rate (estimation and optimisation), does not suffer from these problems due to its feedback structure. These differences are clearly seen in the demand signals for the actuators.

6.5.3 Transition to Hover Results - Non Adaptive

The Transition to Hover test signals can be used to assess the response of the strategies to changing flight conditions. From an airframe vibration perspective, the transition to hover manoeuvre remains one of the most severe cases. The results show that ACSR maintains a reduced level of no vibration throughout the manoeuvre, which is most pronounced when the baseline response is at a maximum. The results from these tests show the basic performance of the Frequency domain and Hybrid controllers, implemented in a non-adaptive framework, for changing flight conditions.

Figure 6.26 shows the average r.m.s vibration levels for baseline, Frequency domain and Hybrid controllers using the smoothed transition to hover signal. The results clearly reveal the periodic update of the frequency domain controller, seen as a saw tooth waveform Figure 6.27 shows the performance of the controllers in terms of percentage vibration reduction. The Hybrid controller provides a fairly constant 70% vibration reduction throughout the manoeuvre, whereas the performance of the Frequency domain controller degrades significantly during rapid changes of the baseline vibration.

Figure 6.28 shows a comparison between baseline, Frequency domain and Hybrid controller using the filtered transition to hover signal. The performance of the two controllers can be seen, and the ability of the hybrid controller to control the rapidly changing vibration is visible. Comparing the actuator signals in Figure 6.29 and Figure 6.30 the difference in the fundamental controller structures can be seen, the frequency domain’s periodic openloop nature can be compared with the hybrid’s feedback structure.

It was possible to improve the performance of the Frequency domain controller by reducing the control update period from 1.0 second to 0.55 seconds. Figure 6.31 shows results for a control update period of 0.55 seconds. The performance of the Frequency domain controller was improved, although the general observations made earlier still apply. The improvement in
performance of the frequency domain controller (0.55 secs) can also be seen in the actuator signal. Comparing with Figure 6.30, Figure 6.33 shows that the transition to hover envelope is followed more closely.

As can be seen from the figures both controllers reduce the baseline vibration significantly, again the Hybrid controller providing better control even when the control update period of the Frequency domain controller is reduced.

Figure 6.34 shows results for a noisy version of the transition to hover signal, which provides a test condition worse than would be experienced in actual flight. Even with this signal, both controllers reduce the fuselage vibration throughout the transition. The controller update time used by the Frequency domain controller for this test was 0.55 seconds.

Figure 6.35 displays results using a transition to hover signal recorded from a Lynx helicopter, a signal which contains some very rapid changes in baseline vibration. The results clearly show that the Hybrid controller reduces baseline vibration compared with the baseline, whereas the Frequency domain controller actually worsened it for certain periods. Looking at the actuator demands (Figures 6.36 and 6.37) it can be seen that the Frequency domain controller does not respond to the changes in the vibration. In Figure 6.38 the controller update period is reduced to 0.55 seconds, but this only provided very marginal improvements in the performance of the Frequency domain controller, and again for some periods the Frequency domain controller actually made the vibration worse. The actuator demands for this test are shown in Figure 6.39 and 6.40.

6.5.4 Gust Results with Adaptation

The previous sections analysed the control strategies in a non-adaptive framework, in this section the controllers are implemented in an adaptive framework. The Eight second gust is used to analyse the Hybrid and Frequency Domain control strategies. To provide an indication of the adaptive controller’s performance various parameters are recorded by the controller: the sum of the diagonal elements of the covariance matrix, the sum of the estimator errors; and the estimator forgetting factor.

Various estimator configurations were implemented, with estimator sensitivities ($E_0$) of 10, 1 and 0.1, and initial covariances of 1000, 10 and 0.1. These combinations provide for a range of estimators with high and low levels of uncertainty in the initial parameter estimates. With high values for $E_0$ (i.e. an insensitive estimator), both controllers provide similar results since the estimator does not react to the errors. Regardless of the value used for the initial...
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covariance the estimator errors are the same and the forgetting factor does not change. In each case the covariance value is reduced on each control cycle because the initial estimates are good.

The next two sets of results show the internal operation of the controller's estimation algorithm. Firstly, Figures 6.41 to 5.46 show results for a sensitive controller ($\Sigma = 0.1$) with a low covariance value (0.1). Both controllers respond similarly, in each case the covariance gradually increases to reflect the uncertainty in the estimates (Figures 6.41 and 6.42). The estimator error and forgetting factor are similar for both controllers (Figures 6.43 and 6.44, and Figures 45 and 46 respectively). As the magnitude of the fuselage vibration rapidly increases at the beginning of the gust, an error is generated in the estimator due to the difference in the predicted and measured fuselage vibration levels. The combination of these estimation errors and the estimator sensitivity causes the forgetting factor to drop as the estimator attempts to reduce its memory length due to the changing measurements.

Figures 6.47 to 6.52 show the results with a sensitivity of 1 and a covariance of 1000, for which it can be seen that the estimator errors are similar for both controllers, but the estimator performance is different. The Hybrid controller's estimator error is smaller than that for the Frequency Domain controller (Figures 6.49 and 6.50 respectively), which is due to the feedback nature of the controller compensating for the increase in fuselage vibration. Both controllers reduce the covariance at each control cycle, from the initial very high value to a lower value reflecting the better parameter estimates. The Frequency Domain controller reduces the covariance at a slower rate then the Hybrid controller (Figures 6.47 and 6.48). Changes in the forgetting factor for the Frequency Domain controller can be seen (Figure 6.52) and these again correspond to the estimator errors at the beginning and the end of the Gust. With even higher values of covariance the Frequency Domain estimator error increases, and is larger than the estimator error for the Hybrid controller.

The final group of results show the average r.m.s vibration levels for each controller configuration. Results for different sensitivities and covariances are plotted on the same graphs, so that the sensitivity of the controller to a particular parameter can be seen. The Frequency Domain controller is much more sensitive and its performance is degraded significantly during transient changes (Figures 6.53 to 6.55). The results show that the performance of the Hybrid controller only deteriorates slightly with high values of covariance and very sensitive estimators (Figures 6.56 to 6.58).
6.5.5 Transition to hover Results with Adaptation

This section uses the filtered transition to hover signal in order to repeat the tests described in section 6.5.3, but this time with the Adaptive controllers. The Hybrid controller is implemented with a 1.0 second control update, and the Frequency Domain controller is implemented with a 0.55 second control update.

Figure 6.59 shows the r.m.s vibration levels for baseline, hybrid and frequency domain controllers set up with a sensitivity of 1 and a covariance of 10. Figures 6.60, 6.62, and 6.64 shows the sum of the covariance matrix elements, sum of estimator errors, and the estimator forgetting factor for the Hybrid controller. Figures 6.61, 6.63, and 6.65 show the corresponding results for the Frequency Domain controller. Both results are similar - the estimator's response to the changing fuselage vibration can be seen in the changes in the covariance matrix, estimator error and forgetting factor.

Figures 6.66 to 6.72 show results for the controllers with an estimator sensitivity of 1 and a covariance of 1000. Figure 6.66 shows the Average r.m.s vibration levels for Baseline, Hybrid and Frequency Domain Hybrid controllers, when compared with Figure 6.59 it can be seen that the Frequency Domain controller's performance has been degraded by this higher covariance value. Figures 6.67, 6.69, and 6.71 shows the sum of the covariance matrix elements, sum of estimator errors, and the estimator forgetting factor for the Hybrid controller. Figures 6.68, 6.70, and 6.72 show the corresponding results for the Frequency Domain controller. The results for this estimator set up are similar to those presented in the previous paragraph, except that the covariance has been set to very high value which the estimator reduces at each control cycle to a more appropriate level.

Figures 6.73 show the Average r.m.s vibration levels for the Hybrid controller for three estimator configurations. Figure 6.74 show the corresponding results for the Frequency Domain controller. In both cases the best controller performance is achieved with a covariance of 10 and a sensitivity of 1.

Figure 6.75 compares the performance of the Hybrid controller with controller update periods of 0.55 and 1.0 seconds. In both cases the estimator had a covariance of 10 and an estimator sensitivity of 1. Reducing the controller update time from 1.0 seconds to 0.55 seconds for the Hybrid controller actually causes a slight degradation in performance. This is probably a consequence of the larger errors in the signal processing routine due to the transients not dying out.
6.5.6 Robustness Results

Frequency Sweep Test A Test signal which had a constant amplitude, but varied in frequency from 17 to 23 Hz over 12 seconds, was used to excite the fuselage response. This represents a change of +/- 15% in the blade passing frequency, and for this test the controllers were implemented in a non-adaptive framework.

Figure 6.76 shows the Average r.m.s acceleration levels for baseline vibration, for the Frequency Domain controller, and for the Hybrid controller. Figure 6.77 redisplay this information in terms of percentage vibration reduction. Figures 6.78 and 6.79 show the corresponding actuator demand signals.

From the results the Hybrid controller reduces the baseline vibration significantly over the frequency range tested. The performance of both controllers would be improved if a reference signal was used to synchronise the signal processing routine. The performance of the Frequency Domain controller would be improved even further since the reference signal would also synchronise the actuator frequency to the blade passing frequency vibration.

Dual Frequency Test The fuselage was excited with a dual frequency signal consisting of a 20Hz and a 25Hz sinusoidal signal. Only results for the Hybrid controller are presented, since this has the potential to control a number of frequencies simultaneously. It is possible to control a number of discrete frequencies with the Frequency Domain strategy, but in this case a parallel controller has to be used for each frequency to be controlled.

Figure 6.80 shows the Power Spectral Density (PSD) for accelerometer 1, both baseline and controlled vibration levels are shown. Twin digital filters were also implemented one tuned to 20Hz and the other to 25Hz. These allow the control frequencies to be feedback and filter out unwanted frequencies. The results for this case are not shown, but the percentage vibration reductions of the various options, including a range of actuator weightings, are listed in tables 6.3 and 6.4 for 20 and 25Hz respectively.

Figure 6.81 shows the percentage vibration reduction achieved at each accelerometer by the Hybrid controller with an actuator weighting of 0.5, at 20Hz and 25Hz respectively when compared with the baseline vibration levels. (Figures 6.82 and 6.83 show the r.m.s vibration levels at each accelerometer for baseline and for the Hybrid controller, at 20Hz and 25Hz respectively). The results clearly show that the Hybrid controller successfully controlled the 20Hz and 25Hz fuselage vibration simultaneously, using only one feedback gain matrix. The
only discrepancy is that the 25Hz vibration level at accelerometer 2 is actually increased a little, although the baseline level starts off lower than all the others.

Parameter Robustness Tests For these tests the aircraft was lowered until the undercarriage just came into contact with the hangar floor, and the electromagnetic shaker was moved to the other side of the dummy rotor head. This effectively changed the rotor/fuselage receptance matrix (T-matrix). The tests were performed with non-adaptive controllers using the T-matrix identified when the aircraft was suspended in the air and with the shaker in its original position. The first test also replayed the transition to hover signal at 18.5 Hz instead of 20Hz, with both the controllers set up to control 20Hz vibration. Figure 6.84 shows the average r.m.s vibration levels for baseline, hybrid and frequency domain controllers. The Hybrid controller manages to reduce the baseline vibration even with the incorrect T-matrix and with the error of 7.5% in the blade passing frequency. The Frequency domain controller fails to control the vibration, which is mainly due to the fixed clock signal used to drive the actuator demands at 20Hz. The signal processing algorithms used by both controllers use the fixed clock signal and are therefore set to measure 20Hz signals only. If a signal from the rotor was used as a reference for the clock signal then the performance of both controllers would undoubtedly be improved, particularly the frequency domain controller which quite simply is generating actuator demands at the wrong frequency.

The second test used the correct blade passing frequency of 20Hz, and both controllers were initialised with the incorrect T matrix as before. The results for this test are shown in Figure 6.85. Both controllers reduce the fuselage vibration, but the performance of the frequency domain controller is degraded slightly more than the Hybrid controller when compared with previous test results (Figure 6.28).

6.6 CONCLUSIONS

This chapter has summarised the experimental assessment of the frequency domain and hybrid controllers on a full size airframe rig at the premises of Westland Helicopters. The experimental work was more limited than the simulation studies, using only 4 acceleration measurements and 2 actuators, and also the time domain controller was not included because it is practically quite difficult to achieve for the reasons identified in Chapter 4 of this thesis.

The rig tests have assessed the performance of the Hybrid strategy in comparison with the Frequency Domain control strategy for active control of helicopter vibration. A variety of tests
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were conducted including: steady state tests, steady state tests using flight data, transient tests, transient tests using a variety of flight data, adaptive controller tests, robustness tests and dual frequency tests. The results from the rig tests presented in this thesis confirm the performance of the Hybrid control strategy predicted from theory and computer simulations. Throughout the shake tests, an impressive indication of the performance of the ACSR system was given by the virtual elimination of structure borne acoustic noise.

The main conclusions that can be drawn from the rig test study are:

The shake test results for the steady state conditions confirmed the significant reductions in vibration indicated by theoretical predictions. Under quasi-static conditions both strategies provided similar vibration reduction figures. For the configuration tested (4 accelerometers and two actuators), both controllers achieved approximately 75% vibration reduction.

The Hybrid strategy feeds back all frequencies, even though it is only optimised for the blade passing frequency. For very low actuator weightings or noisy signals, it was found that the controller excited other fuselage frequencies and became unstable. This problem was solved by including digital filters on the controller output signals. These filters were tuned to the blade passing frequency and proved to be robust to significant changes in the blade passing frequency.

The performance of the Hybrid strategy during transient flight conditions is most impressive, demonstrating its ability to provide large reductions in vibration throughout severe manoeuvres. Its performance in this respect is significantly better than the Frequency Domain strategy, which is mainly due to the fundamental structural differences of the two controllers: the Hybrid strategy has a time domain feedback structure, whereas the Frequency Domain strategy is periodically openloop.

The performance of the Frequency Domain strategy can be improved by reducing the control update time, but reducing this update time to far will invalidate the basic quasi-static assumption upon which this strategy is based.

The frequency robustness of the strategies was tested by varying the blade passing frequency. The Hybrid controller maintained significant reductions in the fuselage vibration for variations of as much as ±15% in the blade passing frequency. As expected the Frequency Domain controller had a poor robustness, since both controllers were implemented using a fixed reference frequency. The performance of the Frequency Domain controller would be greatly
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Improved if a rotor reference signal were used, allowing the correlation process and actuator forces to be correctly synchronised to the rotor frequency.

The ability of the strategies to adapt to changing flight conditions was analyzed by implementing both strategies in an adaptive controller framework. Both strategies provided similar results. The compromise for the adaptive controllers is between being sensitive enough to respond to any structural changes, but not responding unduly to changes in the flight condition. To tune the estimator algorithms successfully further information regarding the likely changes in structural parameters is required.

In theory the Hybrid strategy has the ability to control a number of frequencies simultaneously, and this was confirmed by the rig test. The Hybrid strategy successfully reduced vibration at 20Hz and 25Hz by 60% and 40% respectively.

The main limitation of the Hybrid strategy is the issue of stability. From the rig tests it was found that in certain cases the Hybrid controller excited other (non-controlled) frequencies, although this problem was solved by using of digital tuned filters on the outputs.
Figure 6.1 Controller Hardware Block Diagram
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Figure 6.2 Active Vibration Controller System Diagram
Experimental Validation

Figure 6.3 Hybrid Controller Software Flow Diagram
Figure 6.4 Timing diagram for Hybrid Algorithm Sample Period

Figure 6.5 Timing Diagram for Hybrid Algorithm Controller Cycle
Figure 6.6 Test Rig Overall Scheme
Figure 6.7 Test Rig Photograph
Figure 6.8 Test Rig Equipment Photograph
Figure 6.9 Test Rig Control and Instrumentation Equipment
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Figure 6.10 Frequency Response Analyser Display

Figure 6.11 Time History from Frequency Response Analyser Display
Figure 6.12 Steady State Fuselage Vibration Levels for the Helicopter Test Rig

Table 6.1 Controller Performance for different Control Weighting factors

<table>
<thead>
<tr>
<th>Actuator Weighting</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
<th>Average</th>
</tr>
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<tbody>
<tr>
<td>Hybrid (R=5.0)</td>
<td>42.87</td>
<td>35.51</td>
<td>46.84</td>
<td>47.09</td>
<td>43.08</td>
</tr>
<tr>
<td>Hybrid (R=2.0)</td>
<td>51.42</td>
<td>45.20</td>
<td>53.60</td>
<td>52.59</td>
<td>50.70</td>
</tr>
<tr>
<td>Hybrid (R=1.0)</td>
<td>60.04</td>
<td>56.03</td>
<td>61.13</td>
<td>62.03</td>
<td>60.05</td>
</tr>
<tr>
<td>Hybrid (R=0.5)</td>
<td>68.64</td>
<td>66.79</td>
<td>70.73</td>
<td>70.29</td>
<td>69.11</td>
</tr>
<tr>
<td>Hybrid (R=0.2)</td>
<td>74.24</td>
<td>70.34</td>
<td>78.68</td>
<td>77.07</td>
<td>75.09</td>
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Table 6.2 Controller Performance for Recorded Flight Data

<table>
<thead>
<tr>
<th>Flight Condition</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
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<tr>
<td>140 knots</td>
<td>73.10</td>
<td>77.46</td>
<td>77.54</td>
<td>77.14</td>
<td>75.25</td>
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<tr>
<td>40 knots</td>
<td>72.45</td>
<td>79.03</td>
<td>80.51</td>
<td>81.98</td>
<td>79.91</td>
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</table>
Figure 6.13 PSD of Baseline and Controlled Vibrations at Sensor 1 - Control weighting = 0.5

Figure 6.14 PSD of Baseline and Controlled Vibrations at Sensor 1 - Control weighting = 0.2
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Figure 6.15 Frequency Response at Sensor 1 for 40 knots Steady State Forward Flight

Figure 6.16 RMS Baseline and Controlled Vibration Levels at 140 knots
Figure 6.17 Average RMS Vibration Levels for Baseline, Frequency Domain and Hybrid Controllers

Figure 6.18 Average Percentage Vibration Reduction at Controlled Sensors for the Frequency Domain Controller
Figure 6.19 Time History of Actuator 1 for the Hybrid Controller

Figure 6.20 Time History of Actuator 1 for the Frequency Domain Controller
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Figure 6.21 Average RMS Vibration Levels at Controlled Sensors for Baseline, Actuators Active and Frequency Domain Controller

Figure 6.22 Average Percentage Vibration Reduction at Controlled Sensors for the Frequency Domain Controller
Figure 6.23 Time History of Actuator 1 for the Hybrid Controller

Figure 6.24 Time History of Actuator 1 for the Frequency Domain Controller
Figure 6.25 Average RMS Vibration Levels for Baseline and Hybrid Controller
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Figure 6.26 Vibration Levels for Transition to Hover

Figure 6.27 Controller Performance for Transition to Hover
Figure 6.28 Average RMS Baseline Vibration Levels for the Frequency Domain and Hybrid Controllers during a Transition to Hover
Figure 6.29 Time History of Actuator 2 for the Hybrid Controller

Figure 6.30 Time History of Actuator 2 for the Frequency Domain Controller (1 second Update)
Figure 6.31 Average RMS Baseline Vibration Levels for the Frequency Domain and Hybrid Controllers
Figure 6.32 Time History of Actuator 2 for the Hybrid Controller

Figure 6.33 Time History of Actuator 2 for the Frequency Domain Controller (0.55 second Update)
Figure 6.34 Average RMS Baseline Vibration Levels for the Frequency Domain and Hybrid Controllers - Noisy Transition to Hover.

RMS Vibration Magnitude (g)

Time (secs)

Baseline
Hybrid
Frequency Domain
Figure 6.35 Average RMS Baseline vibration levels for the Frequency Domain and Hybrid

RMS Vibration Magnitude (g)

Time (secs)

Baseline

Frequency Domain

Hybrid

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Figure 6.36 Time History of Actuator 2 for the Hybrid Controller

Figure 6.37 Time History of Actuator 2 for the Frequency Domain Controller (1 Second Update)
Figure 6.38 Average RMS Baseline Vibration Levels for the Frequency Domain and Hybrid Controllers - Lynx Transition to Hover
Figure 6.39 Time History of Actuator 2 for the Hybrid Controller

Figure 6.40 Time History of Actuator 2 for the Frequency Domain Controller (0.55 Second Update)
Figure 6.41 Sum of Covariance Matrix Elements for the Adaptive Hybrid Controller (Covariance = 0.1, Sensitivity = 0.1)

Figure 6.42 Sum of Covariance Matrix Elements for the Adaptive Frequency Domain Controller (Covariance = 0.1, Sensitivity = 0.1)

Figure 6.43 Sum of Estimator Errors for the Adaptive Hybrid Controller (Covariance = 0.1, Sensitivity = 0.1)

Figure 6.44 Sum of Estimator Errors for the Adaptive Frequency Domain Controller (Covariance = 0.1, Sensitivity = 0.1)

Figure 6.45 Estimator Forgetting Factor for the Adaptive Hybrid Controller (Covariance = 0.1, Sensitivity = 0.1)

Figure 6.46 Estimator Forgetting Factor for the Adaptive Frequency Domain Controller (Covariance = 0.1, Sensitivity = 0.1)
Figure 6.47 Sum of Covariance Matrix Elements for the Adaptive Hybrid Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.48 Sum of Covariance Matrix Elements for the Adaptive Frequency Domain Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.49 Sum of Estimator Errors for the Adaptive Hybrid Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.50 Sum of Estimator Errors for the Adaptive Frequency Domain Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.51 Estimator Forgetting Factor for the Adaptive Hybrid Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.52 Estimator Forgetting Factor for the Adaptive Frequency Domain Controller (Covariance = 1000, Sensitivity = 0.1)
Figure 6.53 Average RMS Vibration Levels at Controlled Accelerometers for the Adaptive Frequency Domain Controller - Sensitivity = 0.1, and Covariance = 1000, 10 and 0.1.

Figure 6.54 Average RMS Vibration Levels at Controlled Accelerometers for the Adaptive Frequency Domain Controller - Sensitivity = 1, and Covariance = 1000, 10 and 0.1.

Figure 6.55 Average RMS Vibration Levels at Controlled Accelerometers for the Adaptive Frequency Domain Controller - Sensitivity = 10, and Covariance = 1000, 10 and 0.1.
Figure 6.56 Average RMS Vibration Levels at Controlled Accelerometers for the Adaptive Hybrid Controller with Sensitivity = 0.1, and Covariance = 1000, 10 and 0.1.

Figure 6.57 Average RMS Vibration Levels at Controlled Accelerometers for the Adaptive Hybrid Controller with Sensitivity = 1, and Covariance = 1000, 10 and 0.1.

Figure 6.58 Average RMS Vibration Levels at Controlled Accelerometers for the Adaptive Hybrid Controller with Sensitivity = 10, and Covariance = 1000, 10 and 0.1.
Figure 6.59: Average RMS Vibration Levels for the Frequency Domain and Hybrid Controllers (Covariance = 10, Sensitivity = 1).

RMS Vibration Magnitude (g)

Time (secs)
Figure 6.60 Sum of Covariance Matrix Elements for the Adaptive Hybrid Controller (Covariance = 10, Sensitivity = 1)

Figure 6.61 Sum of Covariance Matrix Elements for the Adaptive Frequency Domain Controller (Covariance = 10, Sensitivity = 1)

Figure 6.62 Sum of Estimator Errors for the Adaptive Hybrid Controller (Covariance = 10, Sensitivity = 1)

Figure 6.63 Sum of Estimator Errors for the Adaptive Frequency Domain Controller (Covariance = 10, Sensitivity = 1)

Figure 6.64 Estimator Forgetting Factor for the Adaptive Hybrid Controller (Covariance = 10, Sensitivity = 1)

Figure 6.65 Estimator Forgetting Factor for the Adaptive Frequency Domain Controller (Covariance = 10, Sensitivity = 1)
Figure 6.66 Average RMS Vibration Levels for the Frequency Domain and Hybrid Controllers (Covariance = 1000, Sensitivity = 1)
Figure 6.67 Sum of Covariance Matrix Elements for the Adaptive Hybrid Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.68 Sum of Covariance Matrix Elements for the Adaptive Frequency Domain Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.69 Sum of Estimator Errors for the Adaptive Hybrid Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.70 Sum of Estimator Errors for the Adaptive Frequency Domain Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.71 Estimator Forgetting Factor for the Adaptive Hybrid Controller (Covariance = 1000, Sensitivity = 1)

Figure 6.72 Estimator Forgetting Factor for the Adaptive Frequency Domain Controller (Covariance = 1000, Sensitivity = 1)
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Figure 6.73 Average RMS Vibration Levels for different estimator parameters for the Adaptive Hybrid Controller

Figure 6.74 Average RMS Vibration Levels for different estimator parameters for the Adaptive Frequency Domain Controller
Figure 6.75 Average RMS Vibration Levels for the Hybrid Controller with update times of 1.0 and 0.55 seconds (Covariance = 10, Sensitivity = 1)
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Figure 6.77 Percentage Vibration Reduction for Swept Frequency Test
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Figure 6.79 Frequency Domain Controller - Actuator 2 Demand for Swept Frequency Test
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Figure 6.81 Percentage Vibration Reduction for Hybrid Controller with Dual Frequencies, Actuator Weighting = 0.5
Figure 6.82 RMS Vibration Levels at 20Hz for the Hybrid Controller with Dual Frequencies, Actuator Weighting = 0.5

Figure 6.83 RMS Vibration Levels at 25Hz for the Hybrid Controller with Dual Frequencies, Actuator Weighting = 0.5
## Experimental Validation

<table>
<thead>
<tr>
<th>Actuator Weighting</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>R = 2.0</td>
<td>8.0</td>
<td>15.0</td>
<td>17.4</td>
<td>15.8</td>
<td>13.8</td>
</tr>
<tr>
<td>R = 1.0</td>
<td>57.1</td>
<td>53.9</td>
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<td>57.5</td>
<td>57.0</td>
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<td>63.6</td>
<td>62.5</td>
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<td>38.5</td>
<td>44.9</td>
<td>43.6</td>
<td>42.3</td>
</tr>
</tbody>
</table>

Table 6.3 Dual Frequency Tests with the Hybrid Controller - Percentage Vibration Reduction Achieved at 20 Hz

<table>
<thead>
<tr>
<th>Actuator Weighting</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>R = 2.0</td>
<td>9.3</td>
<td>61.4</td>
<td>18.8</td>
<td>23.9</td>
<td>19.1</td>
</tr>
<tr>
<td>R = 1.0</td>
<td>50.5</td>
<td>44.1</td>
<td>40.5</td>
<td>49.7</td>
<td>42.3</td>
</tr>
<tr>
<td>R = 0.5</td>
<td>58.7</td>
<td>-94.0</td>
<td>45.1</td>
<td>56.2</td>
<td>43.5</td>
</tr>
<tr>
<td>Filters</td>
<td>49.0</td>
<td>18.0</td>
<td>43.6</td>
<td>48.1</td>
<td>45.4</td>
</tr>
</tbody>
</table>

Table 6.4 Dual Frequency Tests with the Hybrid Controller - Percentage Vibration Reduction Achieved at 25 Hz
Figure 6.84: Average RMS Baseline Vibration Levels for the Frequency Domain and Hybrid Controllers (Blade Passing Frequency = 18.5 Hz)

- Baseline
- Hybrid
- Frequency Domain
Figure 6.85: Average RMS Baseline Vibration Levels for the Frequency Domain and Hybrid Controllers (Incorrect T matrix)

Baseline
Hybrid
Frequency Domain

RMS Vibration Magnitude (g)

Time (secs)
CHAPTER 7

CONCLUSIONS

Research into high performance adaptive strategies for the Active Control of Helicopter Vibration has been described. A new hybrid control approach has been developed for the reduction of fuselage vibration. The potential of this method has been demonstrated both analytically using computer simulations, and through experimental studies on a full size helicopter fuselage, achieving 90 percent vibration reduction at the blade passing frequency.

7.1 CONTRIBUTION OF THE RESEARCH

The study has assessed the performance of the hybrid controller in comparison with time and frequency domain strategies for active vibration control, and has identified advantages and disadvantages for each. In summary, the frequency domain and hybrid controllers offer the best steady-state vibration reduction at frequencies of optimisation, although the time domain controller’s performance is only a little inferior, due to the broader minimisation. The time domain controller offers the best transient performance but is very complex once adaptation and state estimation have been included. The performance of the hybrid strategy during transient conditions is most impressive, providing large reductions in vibration throughout severe manoeuvres. Its performance approaches that of the time domain controller, but is computationally simpler and inherently includes effects such as actuator dynamics which are problematic with the time domain approach. For changing flight conditions the Hybrid strategy performs better than the Frequency Domain strategy. The difference in the transient performance between the hybrid and the frequency domain controllers is mainly due to the fundamental structures of the two controllers. The hybrid strategy has a time domain feedback structure, whereas the Frequency Domain strategy is periodically openloop.

The disadvantages of the time-domain scheme relate to the imprecise modelling of the actuator dynamics and the structural modes; also, for a typical helicopter with 4 actuators, 10 sensors and a 20th order (at least) system, the computational load would be considerable.

A further problem with the implementation of the time domain strategies is due to a pure time delay that has been identified on the helicopter test rig. This delay is on average between 5 and
10ms which corresponds to a phase change of 36° to 72° at 20Hz. This is easily incorporated in the system transfer function by the frequency domain algorithms, but cannot be easily included into time domain control designs. If the controller is designed ignoring this time delay effect, i.e. assuming a linear system, then the delay can cause the controller to become unstable.

Essentially the hybrid technique may be summarised as an approach for creating a self-tuning type adaptive controller in which frequency domain measurements are made to characterize the open loop system, an optimisation is carried out in the frequency domain, and the results are then used to generate an output feedback gain matrix, replacing a much more complex state feedback matrix which involves state estimation and the solution of a matrix Riccati equation. The advantages are that structured and unstructured uncertainties in the system are implicitly included in the measurements, and even non-linearities are accommodated as long as a local linearisation around an operating condition as appropriate. It is also probable that the controllers will overall be simpler, particularly for more complex control systems. A possible disadvantage is that the need to make frequency domain measurements may restrict the technique to systems having relatively slowly-varying parameters, but this has yet to be resolved.

Although the Hybrid control algorithm is based on linear control principles, it is also adaptive enabling the controller to alter its feedback gains to compensate for structural non-linearities and variations in helicopter structural dynamics experienced in flight, yielding more efficient control solutions. A key element of the hybrid control strategy is that the estimation and optimisation are performed in the frequency domain.

The new method potentially gives the best of both the time and frequency domain approaches, and in the context of the Active Vibration Control project can be considered a hybrid frequency/time domain strategy. Since there is the danger of the term "hybrid" causing confusion with mixed continuous/discrete solutions, it may be more appropriate to use the phrase "Frequency Optimised Self Tuning Regulator" (FOSTR).

A number of research groups have been investigating frequency response methods in adaptive control. The relay self-tuner of Åström and Hägglund [1990] uses a frequency domain approach; it is appropriate for simple regulators, and has been commercially exploited by SattControl Instruments in Sweden. Åström and Hägglund's frequency domain approaches have been extended to more complex strategies including the use of feedforward control [4],
but they are primarily concerned with classical design methods. Other work has concerned itself with achieving a given frequency response specification for the closed loop system \([5,6]\) and in some senses may be considered a frequency-domain Model Reference Adaptive scheme. The hybrid or FOSTR approach is distinct from these because the design is guided by the choice of weighting factors used for the optimisation. It clearly is appropriate when inputs or disturbances are predominantly periodic, but it also appears to be applicable to more complicated schemes such as MIMO systems, for which the computational advantages are likely to be most significant. There are well established and efficient frequency domain identification techniques which can be employed.

The results from the rig tests presented in this thesis confirm the performance of the hybrid control strategy predicted from theory and computer simulations. Significant vibration reductions were obtained at more than one rotor speed and during manoeuvres demonstrating the robust nature of the hybrid ACSR strategy for active vibration control. Throughout the shake tests, an impressive indication of the performance of the ACSR system was given by the virtual elimination of structure borne noise. In theory the hybrid strategy has the ability to control a number of frequencies simultaneously, and this was confirmed by the rig tests.

Although the hybrid controller has been developed in the context of active vibration control, it is believed that it offers another approach for self-tuning regulator design in more general applications, in which an optimised controller is designed in the frequency domain using frequency domain measurements.

An important characteristic of the active controller is that it operates recursively and, after being initialised and activated, is completely independent of theoretical predictions of helicopter response or flight test measured helicopter response.

The ACSR technique has significant potential for helicopter reduction, is far superior to the performance of passive systems and compares favourably to other Active Control techniques such as HHC. Furthermore ACSR is both generally applicable to any structure and has minimal associated airworthiness issues.

The way forward for future advanced helicopters may well be a combination of an active control of structural response system for improving ride, and an active rotor control system such as HHC for improving the performance of the helicopter.

Even with active vibration control systems, the design of the rotor and of the airframe need careful consideration, otherwise some of the potential benefits may not be realised.
The main tasks carried out during this research include:
- clear understanding of the source of the problem
- identified the existing technologies for reducing helicopter vibration
- mathematical model of the helicopter fuselage for simulation studies
- investigation of different control strategies
- simulation of control strategies in adaptive and non-adaptive frameworks
- implementation of the control strategy on a DSP platform
- rig tests using a helicopter airframe at WHL.

7.2 RECOMMENDATIONS FOR FURTHER WORK

The results of the hybrid controller tests are very promising, and based upon these results several areas requiring further attention have been identified and the following recommendations for further work are made.

- Implementation of the control system using more actuators and sensors, for example 4 actuators and 10 accelerometers. This would allow the simulation predictions to be verified fully, since a reduced controller was implemented for the rig tests (2 actuators and 4 accelerometers). It would also provide an opportunity to investigate the effects of degraded system operation, i.e. the loss of actuator(s) and/or accelerometer(s).
  With regard to both actuator and accelerometer failures there are two conditions that can be considered, either a detected or an undetected failure. With an undetected failure the controller continues to provide demand to an actuator or use the signal from an accelerometer.
  The system would either need to be robust to such failures or use some form of algorithm management to compensate for component/system failure by reconfiguring the system (rescheduling controller parameters). If the controller can detect the failure then it may be possible to adjust the elements in the weighting matrices such that the failed accelerometer or actuator is weighted out of the control calculations.

- Optimisation of the placement of actuators and sensors within the fuselage, since successful vibration reduction is critically dependent on both the observability and controllability of structural response.
Conclusions

• A more extensive process and measurement noise sensitivity analysis should be conducted in order to ascertain the degradation in controller performance with increasing noise levels.

• The theoretical basis for stability with the hybrid controller needs clarification, since stability is a basic requirement for any control system.

• Unmodelled dynamics are inevitable in a structure such as a helicopter fuselage. The effects of these unmodelled dynamics (which cause spillover for time domain strategies) as well as nonlinear fuselage dynamics will need to be analysed. Further studies should be performed on a simulation that includes nonlinear fuselage effects.

• Methods for constraining the solution to the hybrid gain matrix in the undetermined case need further study. Some preliminary ideas have been discussed in this Thesis.

• Investigation of other application areas for the hybrid/FOSTR strategy.

• At the expense of increases controller complexity, it may be possible to implement a variable weighting matrix as a function of time, or of controller performance, or of flight condition. Davis (1984) proposed a scheme where a new vibration weighting matrix is calculated based upon the current vibration level with a gain factor that is indicative of the uncontrolled vibration level.
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APPENDIX I

FORCES TRANSMITTED BY THE ROTOR HUB

INTRODUCTION
This appendix mathematically describes how the forces generated at the blade roots, due to blade flexibility and aerodynamic forces, are combined and transmitted through the rotor hub to the fuselage. Some harmonics cancel across the rotor hub, while others are reinforced and it is these which then excite the fuselage flexible modes causing structural vibration.

1.1 MULTIBLADE SUMMATION
Assuming a rotor with $n$ blades, then the force contribution from the first blade can be expressed in general as

$$f_b = A_b \cos b \psi$$  \hspace{1cm} (1.1)

where $b$ is the harmonic number. The contribution of the remaining blades will have a phase difference corresponding to the angular separation from the first blade, the total effect on the hub is therefore

$$A_b \cos b \psi + A_b \cos b \left( \psi + \frac{2\pi}{n} \right) + A_b \cos b \left( \psi + \frac{2\pi}{n} \right) + \ldots + A_b \cos b \left( \psi + \frac{2\pi (n-1)}{n} \right)$$  \hspace{1cm} (1.2)

This can be reexpressed as a summation

$$C = A_b \sum_{p=1}^{n} \cos b \left( \psi + \frac{2\pi (p-1)}{n} \right)$$  \hspace{1cm} (1.3)

Since in general the forces can be considered to consist of both sine and cosine components, a similar expression can be obtained consisting of sine terms only

$$S = A_b \sum_{p=1}^{n} \sin b \left( \psi + \frac{2\pi (p-1)}{n} \right)$$  \hspace{1cm} (1.4)
Appendix I

And the force can therefore be expressed as a complex combination of sine and cosine terms

\[ C + jS = A_b \sum_{n=1}^{\infty} e^{j\frac{2\pi(n-1)}{n}} = A_b e^{j\theta} \sum_{n=1}^{\infty} e^{j\frac{2\pi(n-1)b}{n}} \]  

1.5

It is readily deduced that the terms in a summation area geometric series

\[ C + jS = A_b e^{j\theta} \left( \frac{e^{(2\pi b)}}{e^{\frac{2\pi b}{n}} - 1} \right) \]  

1.6

This expression needs to be evaluated for three different cases: 1) \( b \) is not an integer, 2) \( b \) is an integer, but not a multiple of the number of blades \( n \), and 3) \( b \) is an integer multiple of the number of blades.

Case 1 is not considered here, since \( b \) represents the harmonic number and consequently will always be an integer.

Case 2. If \( b \) is an integer, but not a multiple of the number of blades \( n \), then

\[ C + jS = A_b e^{j\theta} \left( \frac{0}{e^{\frac{2\pi b}{n}} - 1} \right) = 0 \]  

1.7

If \( b \) is an integer multiple of the number of blades \( n \), then

\[ C + jS = A_b e^{j\theta} \left( \frac{0}{0} \right) \]  

1.8

This expression can be evaluated using L'Hopital's Theorem

\[ C + jS = nA_b e^{j\theta} \left( \frac{\frac{d}{db} \left( e^{(2\pi bj)} - 1 \right)}{\frac{d}{db} \left( e^{\frac{2\pi b}{n}} - 1 \right)} \right) \]  

1.9

\[ C + jS = nA_b e^{j\theta} \left( \frac{2\pi j e^{2\pi b j}}{(2\pi j e^{2\pi b j})} \right) = n e^{j\theta} \]  

1.10
Appendix I

This yields

\[ C = n \cos b \psi \quad and \quad S = n \sin b \psi \quad (1.11) \]

In summary, when \( n \) is an integer:
but not a multiple of the number of blades \( n \) and is an integer of the number of blades

\[ C = 0 \quad C = n \cos b \psi \quad (1.12) \]
\[ S = 0 \quad S = n \sin b \psi \quad (1.13) \]

### 1.2 HUB FORCES AND MOMENTS

The forces and moments generated by each blade at the rotor hub can be resolved into components relative to the fixed reference axes of the helicopter, refer to Figure 1.1. The components at the blade root, refer to Figure 1.2, can be defined as follows

\[ X_c = -R_c \cos \psi_c + R_x \sin \psi_c \quad (1.14) \]
\[ Y_c = R_c \sin \psi_c + R_x \cos \psi_c \quad (1.15) \]
\[ Z_c = -R_z \quad (1.16) \]

Where \( \psi \) is the azimuth angle of the reference blade, \( c \) denotes the blade number (\( c = 1 \) to \( n \)), and therefore the azimuth angle of the \( c^{th} \) blade is

\[ \psi_c = \psi + \frac{2\pi (c - 1)}{n} \quad (1.17) \]

The reactions to the blade root are periodic in steady flight and can therefore be expressed as a Fourier series. For the \( c^{th} \) blade, these reactions are given by

\[ R_{xc} = r_{xc} + \sum_{m=-\infty}^{\infty} r_{xc} \cos m \psi_c + \sum_{m=-\infty}^{\infty} r_{xc} \sin m \psi_c \quad (1.18) \]
\[ R_{zc} = r_{zc} + \sum_{m=-\infty}^{\infty} r_{zc} \cos m \psi_c + \sum_{m=-\infty}^{\infty} r_{zc} \sin m \psi_c \quad (1.19) \]
\[ R_{zc} = R_{z0} + \sum_{m=-\infty}^{\infty} r_{z\cos m} \cos m \psi_c + \sum_{m=-\infty}^{\infty} r_{z\sin m} \sin m \psi_c \quad 1.20 \]

1.2.1 Vertical Forces

Considering the total force in the vertical (Z) direction first, the sum of the root reactions of all the blades is given by

\[ Z = -\sum_{c=1}^{n} R_{zc} = -n R_{z0} - \sum_{m=-\infty}^{\infty} \sum_{c=1}^{n} r_{z\cos m} \cos m \psi_c - \sum_{m=-\infty}^{\infty} \sum_{c=1}^{n} r_{z\sin m} \sin m \psi_c \quad 1.21 \]

using the results from section 1.1, the above expression can be simplified to give

\[ Z = -n \left( R_{z0} + \sum_{m=-\infty}^{\infty} r_{z\cos m} \cos m \psi + \sum_{m=-\infty}^{\infty} r_{z\sin m} \sin m \psi \right) \quad 1.22 \]

Equation 1.22 clearly shows that the only harmonics which contribute to the total vertical force are harmonics which are multiples of the number of blades, the other component of the vertical force is of course the steady lift load \( nr_{z0} \).

1.2.2 In-Plane Forces

The forces acting in the plane of the rotor hub, i.e. the rotor drag force \( X \) and rotor side force \( Y \), are considered in this section. The force along the X axis can be expressed as follows:

\[ X = \sum_{c=1}^{n} (-R_{xc} \sin \psi_c + R_{xc} \cos \psi_c) \quad 1.23 \]

Substituting for \( R_{xc} \) and \( R_{xc} \) in the above equation yields

\[ X = + \sum_{c=1}^{n} r_{x0} \sin \psi_c - \sum_{c=1}^{n} r_{x0} \cos \psi_c + \sum_{m=-\infty}^{\infty} \sum_{c=1}^{n} r_{x\cos m} \cos m \psi_c \sin \psi_c \]
\[ + \sum_{m=-\infty}^{\infty} \sum_{c=1}^{n} r_{x\sin m} \sin m \psi_c \sin \psi_c - \sum_{m=-\infty}^{\infty} \sum_{c=1}^{n} r_{x\cos m} \cos m \psi_c \cos \psi_c \quad 1.24 \]

This expression can be simplified using trigonometric relationships, and also the first two expressions in 1.24 cancel if the blades are matched.
When $m + 1$ is an integer number of multiples of the blade number $n$, i.e. when $m + 1 = qn$ or $m = qn - 1$ ($q = 1, 2, 3 \ldots$), the first and third terms reduce to $n\cos q\psi$ and $n\sin q\psi$ respectively. And similarly, the second and fourth terms reduce to $n\sin q\psi$ and $n\cos q\psi$, when $m - 1 = qn$ i.e. $m = qn - 1$.

Equation I.25 therefore becomes

\[
X = -\frac{1}{2} \sum_{m=-n}^{n} \sum_{q=-1}^{\infty} \left( r_{x\cos(qm-1)} + r_{y\sin(qm-1)} + r_{x\cos(qm+1)} - r_{y\sin(qm+1)} \right) \cos qn\psi
\]

\[
- \frac{1}{2} \sum_{m=-n}^{n} \sum_{q=-1}^{\infty} \left( r_{x\sin(qm-1)} - r_{y\cos(qm-1)} + r_{x\sin(qm+1)} + r_{y\cos(qm+1)} \right) \sin qn\psi
\]

A similar expression can be derived for the rotor side force along the $Y$ axis

\[
Y = \frac{1}{2} n \sum_{q=-n}^{n} \left( r_{x\cos(qm-1)} - r_{y\sin(qm-1)} + r_{x\cos(qm+1)} + r_{y\sin(qm+1)} \right) \cos qn\psi
\]

\[
+ \frac{1}{2} n \sum_{q=-n}^{n} \left( r_{x\sin(qm-1)} + r_{y\cos(qm-1)} + r_{x\sin(qm+1)} - r_{y\cos(qm+1)} \right) \sin qn\psi
\]

1.3 TABLES

Tables 11, 12 and 13 use equations I.22, I.26 and I.27 respectively to show which harmonics are transmitted by rotors having 2, 3, 4 and 5 blades.
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<th>Vertical Force (or torque) due to 1 Blade</th>
<th>Number of Blades</th>
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<td>$f_b = A_0$</td>
<td>$2A_0$</td>
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<tr>
<td>$f_b = A_1 \sin \psi$</td>
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</tr>
<tr>
<td>$f_b = A_1 \cos \psi$</td>
<td>$0$</td>
</tr>
<tr>
<td>$f_b = A_2 \sin 2\psi$</td>
<td>$2A_2 \sin 2\psi$</td>
</tr>
<tr>
<td>$f_b = A_2 \cos 2\psi$</td>
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<tr>
<td>$f_b = A_3 \sin 3\psi$</td>
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<td>$f_b = A_4 \sin 4\psi$</td>
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<td>$f_b = A_5 \sin 5\psi$</td>
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<tr>
<td>$f_b = A_8 \cos 8\psi$</td>
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Table 11 Vertical Forces or Torques Transmitted by the Blades of an $n$ Bladed Rotor to the Hub
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<th>In Plane Force due to 1 Blade</th>
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</tr>
<tr>
<td>$f_b = A_8 \cos 8\psi$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 12 Lateral Forces or Torques Transmitted by the Blades of an $n$ Bladed Rotor to the Hub
<table>
<thead>
<tr>
<th>In Plane Force due to 1 Blade</th>
<th>Number of Blades</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>( f_b = A_0 )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_1 \sin \psi )</td>
<td>( A_1 - A_1 \cos 2\psi )</td>
</tr>
<tr>
<td>( f_b = A_1 \cos \psi )</td>
<td>( A_1 \sin 2\psi )</td>
</tr>
<tr>
<td>( f_b = A_2 \sin 2\psi )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_2 \cos 2\psi )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_3 \sin 3\psi )</td>
<td>(A_3 \cos 2\psi)</td>
</tr>
<tr>
<td>( f_b = A_3 \cos 3\psi )</td>
<td>(A_3 \sin 2\psi)</td>
</tr>
<tr>
<td>( f_b = A_4 \sin 4\psi )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_4 \cos 4\psi )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_5 \sin 5\psi )</td>
<td>(A_5 \cos 4\psi)</td>
</tr>
<tr>
<td>( f_b = A_5 \cos 5\psi )</td>
<td>(A_5 \sin 4\psi)</td>
</tr>
<tr>
<td>( f_b = A_6 \sin 6\psi )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_6 \cos 6\psi )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_7 \sin 7\psi )</td>
<td>(A_7 \cos 6\psi)</td>
</tr>
<tr>
<td>( f_b = A_7 \cos 7\psi )</td>
<td>(A_7 \sin 6\psi)</td>
</tr>
<tr>
<td>( f_b = A_8 \sin 8\psi )</td>
<td>0</td>
</tr>
<tr>
<td>( f_b = A_8 \cos 8\psi )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table I3 Longitudinal Forces or Torques Transmitted by a Blade to the Hub
Figure I.1 Helicopter Fixed Axes System

Figure I.2 Reactions at the Blade Root
APPENDIX II

HELICOPTER FUSELAGE MODAL MODEL

INTRODUCTION
This appendix describes one of the models used to describe the helicopter fuselage modes. The parameters are derived from a full structural model generated using the NASTRAN finite element package.

II.1 MATHEMATICAL MODEL FORMULATION

The following equation was derived in Chapter 3 and describes the helicopter fuselage as a set of flexible modes.

\[ \ddot{\mathbf{x}} - [A22] \ddot{\mathbf{u}} - [A21] \dot{\mathbf{u}} = [D] \mathbf{f} + [C] \mathbf{u} \]

\( \mathbf{u} = [u_1, u_2, ..., u_n]^T \) denotes the control inputs to the force actuators, and \([C]\) is the actuator influence matrix. The rotor head forces acting on the fuselage are denoted by \([\mathbf{f}]\) and the corresponding influence matrix is \([D]\). Both \([C]\) and \([D]\) are matrices of modal coefficients generated by NASTRAN (translational and rotational eigenvectors). The model is based on NASTRAN generated parameters, and a modal damping of 2% critical is assumed. The matrices \([A22]\) and \([A21]\) are defined as

\[ [A22] = \text{diag}[-2\zeta_i \omega_i] \]

\[ [A21] = \text{diag}[-\omega_i^2] \]

where \(\eta_i, \zeta_i, \omega_i\) are the respective modal displacement, relative damping and natural frequency of the \(i\)th mode of vibration.

Equation 1 is modified and gives the structural vibration at the points of interest in the structure by a linear transformation of the modal accelerations:

\[ \mathbf{y} = [\Omega] \ddot{\mathbf{x}} \]

where \(\mathbf{y} = [y_1, y_2, ..., y_i]^T\) is the vector of structural vibrations (accelerations) and \([\Omega]\) defines the coupling from the modes to the measurement points.
II.2 MODEL PARAMETERS

The $[C]$ matrix has 24 rows, one for each fuselage mode, and 4 columns, one for each actuator, and is defined as follows:

\[
\begin{array}{cccc}
-2.7037e-004 & 3.1396e-004 & -4.7700e-004 & 1.3890e-004 \\
-2.3770e-004 & -1.8390e-004 & 1.1680e-003 & 1.2427e-003 \\
-4.6190e-003 & -4.5460e-003 & 4.3333e-003 & 4.3654e-003 \\
3.3710e-003 & 3.5074e-003 & 1.1734e-003 & 1.2210e-003 \\
2.1072e-003 & -2.0146e-003 & -9.4471e-004 & 7.1562e-004 \\
5.1780e-004 & -3.7100e-005 & 4.1608e-003 & 4.6771e-003 \\
-3.3800e-004 & 1.0253e-003 & 3.2376e-003 & 2.8884e-003 \\
-2.3110e-004 & -1.3523e-003 & 3.7457e-003 & 3.3206e-003 \\
8.0110e-003 & 6.2910e-003 & 2.0750e-003 & 2.4500e-003 \\
5.9200e-005 & -1.7012e-003 & -1.3857e-003 & -2.1641e-003 \\
1.8870e-004 & 2.3910e-004 & 2.5020e-004 & -3.1860e-004 \\
-3.4886e-003 & 3.0988e-003 & 2.5732e-004 & 3.1490e-003 \\
-1.6815e-003 & -1.5005e-003 & -8.3390e-004 & -1.8370e-004 \\
3.6600e-004 & 3.6382e-004 & 4.1540e-004 & 6.8000e-005 \\
-1.3902e-003 & 4.1390e-004 & -1.5927e-003 & 1.2952e-003 \\
-4.7201e-003 & 5.2498e-003 & -3.7721e-003 & 3.9408e-003 \\
1.6558e-004 & -4.9499e-004 & -1.5320e-003 & -1.8816e-003 \\
-7.2912e-004 & -3.4894e-004 & -3.3123e-003 & -3.0858e-003 \\
1.4487e-003 & -1.4568e-003 & 4.1555e-003 & -4.0680e-003 \\
5.9618e-003 & -8.7699e-003 & -1.5970e-003 & 1.6360e-003 \\
9.3010e-003 & -1.1702e-003 & -1.9370e-003 & 8.9210e-004 \\
2.5421e-003 & -7.3843e-003 & -1.2021e-003 & 4.5383e-003 \\
-3.6570e-003 & 3.7410e-004 & 2.8505e-003 & 1.1910e-004 \\
\end{array}
\]

where each element (6,3) (i.e. row 6 column 3) represents the coupling of actuator number 3 into mode number 6.
The $[D]$ matrix has 24 rows, one for each fuselage mode, and 5 columns, one for each input from the rotor (i.e. vertical, lateral and longitudinal forces, and pitch and roll and moments at the main rotor hub), and is defined as follows:

\[
\begin{array}{cccccc}
-1.4142e-003 & 5.3615e-003 & 7.7514e-004 & -7.5909e-004 & -1.1299e-003 \\
9.7019e-003 & 3.5062e-004 & -4.5469e-003 & 2.2044e-004 & 7.2912e-003 \\
1.9651e-002 & 2.7839e-004 & 1.0974e-002 & -2.3936e-004 & 1.8416e-002 \\
-1.0590e-002 & 4.7709e-004 & -9.3734e-003 & -3.3364e-004 & -4.4152e-003 \\
-3.7694e-004 & -1.3749e-002 & 1.1529e-004 & 6.8293e-003 & -2.9673e-004 \\
3.5722e-003 & -1.4360e-003 & -4.2543e-003 & -6.2491e-004 & 6.4736e-003 \\
1.4193e-003 & 2.8583e-003 & -1.7037e-003 & 6.4956e-004 & 3.7639e-003 \\
-1.1529e-003 & -1.6094e-003 & 3.1347e-003 & 1.4802e-003 & 3.2146e-003 \\
3.9307e-005 & -2.0398e-002 & -1.1526e-003 & 2.8243e-002 & -3.0398e-004 \\
-1.2334e-003 & 9.5779e-004 & -1.7762e-002 & -1.2071e-003 & -4.9743e-003 \\
7.5343e-004 & -2.3398e-003 & 2.1830e-003 & 6.2691e-003 & 9.9752e-004 \\
1.0064e-005 & 1.5116e-003 & -3.5481e-004 & -2.9101e-003 & 8.5418e-007 \\
1.3635e-003 & 5.5049e-004 & 5.4128e-003 & 1.7767e-003 & 1.5812e-003 \\
1.0019e-003 & 2.5296e-004 & 2.3907e-003 & -5.2325e-003 & 1.3621e-003 \\
-2.5976e-004 & -5.3276e-005 & -3.8895e-004 & 1.8990e-003 & -1.0393e-003 \\
3.3183e-004 & 2.1050e-003 & 6.7958e-004 & -1.2300e-002 & 2.5102e-004 \\
-1.2320e-002 & -5.9821e-004 & 3.0999e-003 & 9.6852e-004 & -8.3289e-003 \\
-2.1560e-002 & 3.2210e-004 & 5.8944e-003 & -6.3161e-004 & -1.5869e-002 \\
1.4450e-004 & -3.4348e-003 & -3.8579e-005 & 1.4206e-002 & 8.5155e-005 \\
2.3631e-003 & -1.1255e-002 & 9.1844e-004 & 1.1443e-002 & 3.6714e-003 \\
-7.2866e-003 & -7.8977e-003 & -2.3113e-003 & 1.1568e-002 & -1.2774e-002 \\
6.1506e-003 & -7.3777e-003 & 7.7792e-004 & 1.8864e-002 & 1.3520e-002 \\
5.3571e-003 & 2.6860e-003 & 9.3356e-004 & -9.6189e-003 & 1.8153e-002 \\
\end{array}
\]
The modal frequency ($\omega$, for the $i^{th}$ mode) for each of the 24 modes are listed below, these are listed in Hz. For the model they are required in radians per second and therefore should be multiplied by $2\pi$.

8.4521
8.8815
15.4388
16.7001
18.0752
19.5172
20.1156
22.8515
23.8478
24.4398
25.2913
25.7115
26.5598
27.5959
28.9328
29.1683
30.1376
31.5970
31.7689
32.9960
34.1101
35.2369
37.6895
40.1739
The coupling of each of the fuselage modes to each of the accelerometers is defined by the \( \Omega \) matrix, since there are 24 modes and 10 sensors this matrix has dimensions 10 by 24. For clarity the matrix is transposed and partitioned into two, coupling of the modes for the first 5 sensors and coupling of the modes into the second set of five sensors (sensors 6 to 10).

\[
\begin{bmatrix}
-1.4140e-003 & -1.7406e-002 & -3.2594e-003 & -1.7398e-002 & -1.2219e-003 \\
-1.7560e-002 & 2.3506e-005 & -1.7691e-002 & 2.1272e-004 & -8.4579e-003 \\
1.0720e-002 & 2.1818e-005 & 1.0083e-002 & -1.2810e-004 & 7.3751e-003 \\
-4.4625e-003 & 5.0494e-003 & 4.4151e-003 & 5.0460e-003 & -2.9205e-003 \\
6.2483e-004 & 3.4324e-003 & -1.0983e-003 & 3.4304e-003 & 3.6633e-003 \\
-2.3093e-003 & -6.5178e-003 & 1.2044e-003 & -6.5094e-003 & -5.2710e-004 \\
-1.1395e-003 & 8.8894e-004 & -3.5660e-003 & 8.6974e-004 & 6.3519e-004 \\
2.2771e-002 & -3.8215e-003 & -1.9992e-002 & -3.8536e-003 & 1.5528e-002 \\
4.5964e-003 & 2.5348e-005 & 5.2415e-003 & -4.7665e-005 & 9.2133e-004 \\
-5.4871e-003 & -5.0190e-003 & -1.6168e-002 & -4.6471e-003 & 6.0686e-003 \\
-7.2312e-003 & 2.9837e-003 & -4.0912e-004 & 3.1032e-003 & -4.5125e-004 \\
3.8654e-003 & 4.0315e-003 & 1.1170e-002 & 3.7735e-003 & -3.1639e-003 \\
-5.8978e-004 & 5.9114e-004 & 1.1957e-002 & 4.8510e-004 & 2.0681e-003 \\
-3.7588e-003 & -1.5546e-002 & 3.1056e-004 & -1.5471e-002 & 6.2553e-003 \\
2.2449e-003 & -1.1039e-003 & 2.5626e-003 & -1.2048e-003 & 4.9276e-004 \\
6.0420e-003 & 8.1584e-003 & 2.2305e-003 & 7.9898e-003 & -3.6310e-004 \\
-1.1272e-002 & 9.6354e-004 & -6.9967e-003 & 1.4105e-003 & -8.2991e-004 \\
-5.6542e-003 & 1.0560e-002 & -1.2559e-002 & 1.0166e-002 & -4.5076e-003 
\end{bmatrix}
\]
The second partition of matrix $[\Omega]$, for sensors 6 to 10, is defined below:

\[
\begin{align*}
-1.9564e-003 & \quad 5.6848e-004 & \quad -9.0620e-003 & \quad 1.7058e-004 & \quad -5.5057e-003 \\
9.5948e-003 & \quad -2.3402e-003 & \quad -1.3722e-003 & \quad 6.1230e-003 & \quad -7.6522e-004 \\
-8.6302e-003 & \quad -6.5569e-003 & \quad -9.7721e-007 & \quad -4.5327e-003 & \quad 6.9218e-005 \\
6.9671e-003 & \quad 5.4421e-003 & \quad -4.9991e-005 & \quad 7.9229e-003 & \quad -3.6627e-004 \\
2.8417e-003 & \quad 6.7396e-005 & \quad 1.6367e-003 & \quad -5.5417e-003 & \quad 1.8507e-003 \\
1.6946e-003 & \quad -1.1669e-003 & \quad 9.2557e-004 & \quad 5.8670e-003 & \quad -8.0007e-004 \\
3.5460e-003 & \quad -3.2041e-004 & \quad -1.8470e-003 & \quad 9.3506e-004 & \quad 1.0121e-003 \\
-1.5493e-003 & \quad 5.0502e-003 & \quad 1.3143e-004 & \quad 3.8525e-003 & \quad -5.9263e-004 \\
-1.6370e-002 & \quad 3.4643e-004 & \quad -3.4532e-003 & \quad 2.1439e-002 & \quad -1.1001e-002 \\
1.5235e-003 & \quad -4.2310e-003 & \quad 1.5928e-005 & \quad -1.8305e-003 & \quad 1.0069e-004 \\
-2.1120e-005 & \quad -6.1453e-003 & \quad -1.9093e-003 & \quad 1.0879e-002 & \quad -2.5034e-003 \\
2.9213e-003 & \quad -1.6035e-003 & \quad 1.2141e-003 & \quad -9.0884e-004 & \quad 1.2074e-003 \\
-9.4989e-003 & \quad 5.4620e-003 & \quad -8.0579e-004 & \quad -1.7571e-002 & \quad 1.4265e-004 \\
-1.9065e-003 & \quad -1.2356e-004 & \quad 1.1593e-003 & \quad -7.8748e-003 & \quad 4.8601e-004 \\
6.2369e-003 & \quad 4.6592e-003 & \quad -5.0646e-005 & \quad -2.1053e-003 & \quad 3.8889e-004 \\
2.0966e-004 & \quad 5.5932e-004 & \quad -4.6205e-006 & \quad -5.2418e-003 & \quad -5.0630e-004 \\
-6.2181e-003 & \quad 2.8024e-004 & \quad -3.7007e-003 & \quad 6.3549e-003 & \quad -2.0609e-003 \\
6.1113e-004 & \quad 3.3354e-003 & \quad 2.6168e-004 & \quad 2.7353e-003 & \quad 6.0677e-005 \\
1.0235e-004 & \quad 4.8763e-004 & \quad -1.9922e-004 & \quad 4.4287e-004 & \quad -6.1924e-005 \\
9.5540e-003 & \quad -5.1589e-005 & \quad -3.0733e-003 & \quad -1.9124e-002 & \quad -1.0123e-003 \\
3.5637e-004 & \quad -1.3722e-003 & \quad 1.7989e-003 & \quad -2.9160e-003 & \quad 2.5219e-005 \\
1.3981e-003 & \quad 1.0967e-003 & \quad 2.3541e-004 & \quad -9.7726e-004 & \quad 5.6573e-004 \\
8.0773e-004 & \quad 4.0404e-004 & \quad -5.4944e-006 & \quad 2.6925e-003 & \quad 2.0053e-003 \\
-4.5113e-003 & \quad 1.6896e-002 & \quad -2.3920e-003 & \quad 1.2063e-002 & \quad -2.9156e-003 \\
\end{align*}
\]
INTRODUCTION
This appendix describes the correlation process used by the frequency domain controller to extract the blade passing frequency component of the fuselage vibration from the accelerometer measurements.

III.1 FREQUENCY CORRELATION

The vibration measurements from the accelerometers are decomposed into sine and cosine components at the operating frequency $n\omega$ using a correlation method. For the algorithms implemented a look up table is used and the required sine and cosine coefficients are obtained by summing the products of these tables with the measured vibration at each discrete sampling interval, i.e. essentially an on-line Fourier analysis.

The vector of cosine coefficients can be defined as:

$$Y_{\cos} = \frac{2}{h} \sum_{k=0}^{h} y(k) \cdot \cos(n\omega k \tau)$$  \hspace{1cm} (III.1)

and the corresponding vector of sine coefficients is:

$$Y_{\sin} = \frac{2}{h} \sum_{k=0}^{h} y(k) \cdot \sin(n\omega k \tau)$$  \hspace{1cm} (III.2)

where $y$ is the vector of measured vibration, $n\omega$ is the operating frequency, $\tau$ is the sample period and $ht$ the measurement period, which must of course be over an integer number of periods of the operating frequency.

Equations (III.1) and (III.2) are of course equivalent to a Discrete Fourier Transform (DFT) at a single frequency. Depending upon the variation in the blade passing frequency or if multiple frequencies are required then a Fast Fourier Transform algorithm may be used as an alternative.
III.2 CORRELATION ROBUSTNESS

A key feature of the frequency correlation method used to decompose the measured accelerations into sine and cosine components at the operating frequency \( n_\omega \), is that the influence of extraneous noise on these measurements can be reduced to very low levels by increasing the averaging time \( T \).

Figure III.1 shows a system where the true system output is corrupted by additive noise \( n(t) \). Because the noise is additive, the sine and cosine channels at the analysis frequency \( n_\omega \), will each be in error an amount \( \Delta R(t) \) and \( \Delta I(t) \) given by:

\[
\Delta R(T) = \frac{1}{T} \int_0^T n(t) \sin(n_\omega t) \, dt \quad (\text{III.3})
\]

\[
\Delta I(T) = \frac{1}{T} \int_0^T n(t) \cos(n_\omega t) \, dt \quad (\text{III.4})
\]

Considering the sine channel error only (the cosine error behaves in a similar manner) and interpreting the action of the averaging process as a filtering operation, equation (III.3) can be restated as:

\[
\Delta R(T) = \frac{1}{T} \int_0^T n(t) \sin[n_\omega (T-t)] \, dt \quad (\text{III.5})
\]

because \( \sin(n_\omega t) = -\sin(n_\omega (T-t)) \), when \( T = N2\pi/n_\omega \). Equation (III.5) is a time domain convolution between the disturbance \( n(t) \) and a filter impulse response of the form:

\[
h(\tau) = \frac{1}{T} \sin(n_\omega \tau) \text{ for } 0 < \tau < T
\]

\[
= 0 \quad \text{elsewhere}
\]

(III.6)

The frequency response \( H(\omega) \) of this filter is obtained by noting that the impulse response consists of the product of a ‘boxcar’ function \( W(\tau) \) defined by:

\[
W(\tau) = \frac{1}{T} \text{ for } 0 < \tau < T
\]

\[
= 0 \quad \text{elsewhere}
\]

(III.7)

and the sine wave, \( \sin n_\omega \tau \).
The sine wave can be expressed as
\[ \sin \pi \omega t = \frac{1}{2j} (e^{j \omega t} - e^{-j \omega t}) \]  
(III.8)

Hence, by the shift theorem in the frequency domain
\[ H(j\omega) = \frac{1}{2j} [L(j\omega + jn\omega) - L(j\omega - jn\omega)] \]  
(III.9)

where
\[ L(j\omega) = \int l(t) e^{-j\omega t} dt = \frac{1}{T} \int e^{-j\omega t} dt \]  
(III.10)

Hence
\[ L(j\omega) = \frac{1}{j\omega T} (1 - e^{-j\omega T}) \]  
(III.11)

\[ e^{-j\omega T} \frac{T}{2} \sin \omega T \]  
\[ = \frac{\omega \frac{T}{2}}{2} \]  
(III.12)

Combining equations (III.9) and (III.12), the filter transfer function for \( T = 2\pi/n\omega \) is
\[ H(j\omega) = \frac{n\omega (1 - e^{-j\omega T})}{T(\omega^2 - (n\omega)^2)} \]  
(III.13)

It can be seen from the gain function of \( H(j\omega) \), Figure III.2, that the averaging associated with the correlation acts as a band pass filter with centre frequency \( n\omega \). As the averaging time increases, the bandwidth of the filter becomes narrower. Thus, the corrupting influence of wide band noise \( n(t) \) is increasingly filtered out as the correlation time is increased.
Figure III.1 Frequency Correlation

Figure III.2 Rejection curves for N cycles of integration
APPENDIX IV

LINEAR QUADRATIC CONTROL

INTRODUCTION
This appendix describes the solution of the time domain cost function, yielding a linear quadratic controller. The cost function is then modified to include output terms and the derivation is repeated.

IV.1 LINEAR QUADRATIC CONTROL

The Linear Quadratic control strategy is described in Chapter 4. This section describes the mathematical solution of the cost function:

\[ J_T = \int_0^T (x^*_s(t) Q x_s(t) + u^*_s(t) R u_s(t)) \, dt \]  

(IV.1)

The performance index, equation (IV.1), which is subject to the constraint equation specified by

\[ \dot{x}_s = [A_s] x_s + [B_s] u_s \]  

(IV.2)

can be solved by adjoining the constraint equation to the performance index using Lagrange multipliers \( \lambda_1, \lambda_2, \ldots, \lambda_n \). An augmented functional \( L \) can be defined as

\[ L = \int_0^T \left[ x^*_s(t) Q x_s(t) + u^*_s(t) R u_s(t) + \lambda^*_s(t) (A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t)) \right] \, dt \]  

(IV.3)

The terms in equation (IV.3) involving Lagrange multipliers are in the form shown to ensure \( L = L^* \). Lagrange multipliers are also known as adjoint vectors or covectors. The minimisation of this augmented functional \( L \) is equivalent to the minimisation of the cost functional \( J \), equation (IV.1), when it is subject to the equality constraint defined by equation (IV.2).
In order to minimise the integrand

\[ F = x_s^*(t) Q x_s(t) + u_s^*(t) R u_s(t) + \lambda^*(t) \left( A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) \right) + \left( A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) \right) \lambda(t) \]  

equation (IV.4) needs to be differentiated with respect to each of its component vectors \( x_s(t) \), \( u_s(t) \) and \( \lambda(t) \), and set the results equal to zero. From the computational view point, however, it is convenient to differentiate \( F \) with respect to the complex conjugates of \( x_s(t) \), \( u_s(t) \) and \( \lambda(t) \). Note that a signal and its complex conjugate contain the same mathematical information.

The integrand \( F \) is a function of three variables \( x_s(t) \), \( u_s(t) \) and \( \lambda(t) \), and so there are three Euler equations

\[ \frac{\partial F}{\partial x_s^*} - \frac{d}{dt} \left( \frac{\partial F}{\partial \dot{x}_s^*} \right) = 0 \quad \text{gives} \quad \frac{\partial f_0}{\partial x_s^*} + \lambda \frac{\partial f^*}{\partial \dot{x}_s^*} + \dot{\lambda} = 0 \]  

\[ \frac{\partial F}{\partial u_s^*} - \frac{d}{dt} \left( \frac{\partial F}{\partial \dot{u}_s^*} \right) = 0 \quad \text{gives} \quad \frac{\partial f_0}{\partial u_s^*} + \lambda \frac{\partial f^*}{\partial \dot{u}_s^*} = 0 \]  

\[ \frac{\partial F}{\partial \lambda^*} - \frac{d}{dt} \left( \frac{\partial F}{\partial \dot{\lambda}^*} \right) = 0 \quad \text{gives} \quad f = 0 \]  

Defining

\[ f_0 = x_s^*(t) Q x_s(t) + u_s^*(t) R u_s \]  

\[ f = A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) \]  

\[ H = f_0 + \lambda^* f + \dot{\lambda} f^* \]

Equation (IV.10) is defined as the Hamiltonian.
Equations (IV.5), (IV.6) and (IV.7) can be rewritten as

\[ \dot{\lambda} = -\frac{\partial H}{\partial x^*_s} \]  

and \[ 0 = \frac{\partial H}{\partial u^*_s} \]  

and \[ 0 = \frac{\partial H}{\partial \lambda^*} \]  

Equation (IV.11) is known as the adjoint equation, and can be expressed as

\[ -\frac{\partial H}{\partial x^*_s} = \dot{\lambda}(t) = -Qx_s(t) - A_s\lambda(t) \]  

Similarly, equations (IV.12) and (IV.13) can be expressed as

\[ \frac{\partial H}{\partial u^*_s} = Ru_s(t) + B^*\lambda(t) = 0 \]  

\[ \frac{\partial H}{\partial \lambda^*} = A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) = 0 \]  

Equation (IV.16) is simply the state equation. Solving equation (IV.15) for \( u_s(t) \) gives

\[ u_s(t) = -R^{-1}B_s^*\lambda(t) \]  

substituting equation (IV.17) into (IV.16)

\[ \dot{x}_s(t) = A_s x_s(t) - B_s R^{-1}B_s^*\lambda(t) \]  

In order to obtain the solution to the minimisation problem, equations (IV.18) and (IV.14) need to be solved simultaneously. The optimal control vector \( u_s(t) \) can be obtained in the following closed loop form
where $K_s(t)$ is the feedback matrix.

To obtain the optimal control vector $u_s(t)$ in the closed loop form, it is first necessary to derive the Riccati equation. Assuming that the solution to equation (IV.11), $\Delta(t)$, can be written in the following form

$$\Delta(t) = P(t)x_s(t) \quad (IV.20)$$

Differentiating equation (IV.20) with respect to $t$ gives

$$\dot{\Delta}(t) = \dot{P}(t)x_s(t) + P(t)\dot{x}_s(t) \quad (IV.21)$$

Eliminating $\Delta(t)$ from equations (IV.18) and (IV.14)

$$\dot{P}(t)x_s(t) + P(t)\dot{x}_s(t) = -Qx_s(t) - A_s^*P(t)x_s(t) \quad (IV.22)$$

$$\dot{x}_s(t) = A_s x_s(t) - B_s R^{-1}B_s^*P(t)x_s(t) \quad (IV.23)$$

Substituting equation (IV.23) into (IV.22)

$$\dot{P}(t)x_s(t) = -P(t)A_s x_s(t) + P(t)B_s R^{-1}B_s^*P(t)x_s(t) - Qx_s(t) - A_s^*P(t)x_s(t) \quad (IV.24)$$

This last equation must hold for all $x_s(t)$, hence

$$\dot{P}(t) = -P(t)A_s - A_s^*P(t) - Q + P(t)B_s R^{-1}B_s^*P(t) \quad (IV.25)$$

Equation (IV.25) is called the Riccati equation. Referring to equation (IV.17), the optimal control vector now becomes

$$u_s(t) = -R^{-1}B_s^*P(t)x_s(t) \quad (IV.26)$$

where

$$K_s(t) = R^{-1}B_s^*P(t) \quad (IV.27)$$

The matrix $P(t)$ is a positive definite solution of the Riccati equation (IV.25). For the inverse of matrix $R$ to exist, it is necessary that $R$ be positive definite.
For equations (IV.26) and hence (IV.27) to be true, the following Jacobian matrix must be positive definite, i.e.

\[
\begin{bmatrix}
\frac{\partial H}{\partial x_s(t)} & \frac{\partial H}{\partial x_s(t)\partial u_s(t)} \\
\frac{\partial H}{\partial u_s(t)\partial x_s(t)} & \frac{\partial H}{\partial u_s(t)}
\end{bmatrix} > 0
\]  
\text{(IV.28)}

which reduces to

\[
\begin{bmatrix}
Q & 0 \\
0 & R
\end{bmatrix} > 0
\]  
\text{(IV.29)}

Since \( R \) is positive definite, then \( Q \) must be at least positive semi-definite for equation (85) to be true.

### IV.2 OPTIMAL CONTROL WITH OUTPUT WEIGHTING

For the case where direct control of the output vector is required, a performance index, such as equation (IV.30) is minimised

\[
J_y = \int_0^T \left( y_s^*(t) Q y_s^*(t) + u_s^*(t) R u_s(t) \right) dt
\]  
\text{(IV.30)}

with

\[
\dot{x}_s = [A_s]x_s + [B_s]u_s
\]  
\text{(IV.31)}

\[
y_s = [C_s]x_s + [D_s]u_s
\]  
\text{(IV.32)}

Substituting for \( y_s(t) \) in equation (IV.30), and adjoining the constraint equation (IV.2), (IV.31) using Lagrange multipliers, to form an augmented cost functional
\[ J_{\gamma} = \int_{0}^{\gamma} \left[ (C_s x_s(t) + D_s u_s(t))^\top Q (C_s x_s(t) + D_s u_s(t)) + u_s^*(t) R u_s(t) ight. \\
\left. + \lambda^*(t) \left( A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) \right) + \left( A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) \right)^\top \lambda(t) \right] \, dt \]  

Proceeding as before, the associated Hamiltonian, \( H \), can be expressed as

\[ H = (C_s x_s(t) + D_s u_s(t))^\top Q (C_s x_s(t) + D_s u_s(t)) + u_s^*(t) R u_s(t) \]

\[ + \lambda^*(t) \left( A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) \right) + \left( A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) \right)^\top \lambda(t) \]  

For \( H \) to be minimised with respect to \( u_s(t) \), then

\[ \frac{\partial H}{\partial u_s^*} = D_s^\top Q C_s x_s(t) + D_s^\top Q D_s u_s(t) + R u_s(t) + B^* \lambda(t) = 0 \]  

or

\[ u_s(t) = -\left( R + D_s^\top Q D_s \right)^{-1} \left[ D_s^\top Q C_s x_s(t) + B_s^\top \lambda(t) \right] \]  

The adjoint equation

\[ \dot{\lambda} = -\frac{\partial H}{\partial x_s^*} \]  

is then

\[ -\frac{\partial H}{\partial x_s^*} = \dot{\lambda}(t) = -C_s^\top Q C_s x_s(t) - C_s^\top Q D_s u_s(t) - A_s^\top \lambda(t) \]  

The solution to equation (IV.37) is

\[ \lambda(t) = P(t) x_s(t) \]  

\[ \frac{\partial H}{\partial \lambda^*} = A_s x_s(t) + B_s u_s(t) - \dot{x}_s(t) = 0 \]  

which is simply the system state equation.

Substituting (IV.39) into (IV.36) and (IV.38)
Substituting equation (IV.40) into equation (IV.42) and then eliminating \( u_s(t) \) using equation (IV.41) gives

\[
\dot{x}_s(t) + P(t)A_s x_s(t) - P(t)B_s (R + D_s^T Q D_s)^{-1} \left[ D_s^T Q C_s x_s(t) + B_s^T P(t) x_s(t) \right] = (IV.43)
\]

\[
-C_s A_s x_s(t) + C_s B_s (R + D_s^T Q D_s)^{-1} \left[ D_s^T Q C_s x_s(t) + B_s^T P(t) x_s(t) \right] - A_s^T P(t) x_s(t)
\]

The last equation must hold for all \( x_s(t) \), hence

\[
\dot{\theta}(t) = -P(t)A_s + P(t)B_s (R + D_s^T Q D_s)^{-1} \left[ D_s^T Q C_s + B_s^T P(t) \right] - A_s^T P(t) (IV.44)
\]

Equation (IV.44) can be re-expressed in the standard matrix Riccati format

\[
\dot{\theta}(t) = -P(t)A_s - A_s^T P(t) - Q_s + P(t)B_s R_s^{-1} B_s^T P(t) (IV.45)
\]

where

\[
A_s = A_s - B_s (R + D_s^T Q D_s)^{-1} D_s^T Q C_s (IV.46)
\]

\[
R_s = R + D_s^T Q D_s (IV.47)
\]

\[
Q_s = C_s [Q - Q D_s R_s^{-1} D_s^T Q] C_s (IV.48)
\]

The optimal control vector is defined by equation (IV.49)

\[
u_s(t) = -\left( R + D_s^T Q D_s \right)^{-1} \left[ D_s^T Q C_s + B_s^T P(t) \right] x_s(t) (IV.49)
\]

and

\[
K_s(t) = \left( R + D_s^T Q D_s \right)^{-1} \left[ D_s^T Q C_s + B_s^T P(t) \right] (IV.50)
\]

\( P(t) \) is a positive definite solution of the matrix Riccati equation (IV.45).
APPENDIX V

UDU COVARIANCE FACTORISATION

INTRODUCTION
This appendix describes the $[U][D]$ formulation used to factorise the covariance matrix. This factorisation relies upon the fact that a positive definite matrix (all covariance matrices are positive definite) can be factored into a product of three matrices, an upper triangular matrix with unit diagonal elements, a diagonal matrix with positive elements, and the transpose of the upper triangular matrix.

V.1 UDU FACTORISATION
Bierman [1977] showed that numeric errors can be dramatically reduced by using covariance factorization algorithms, and that the U-D covariance factorization is computationally efficient and compact. The technique relies on the fact that the covariance matrix $[P]$ is symmetric and positive definite and can therefore be decomposed as follows:

$$[P] = [U][D][U]^T$$

where, $[D]$ is a diagonal matrix and $[U]$ is an upper triangular matrix with unit values on the leading diagonal.

To calculate the $[D]$ and $[U]$ matrices, the following equations are cycled through for $s = m, (m-1), \ldots, 2$ where $m$ is the number of sensors

$$[D]_{ss} = [P]_{ss}$$

$$[U]_{js} = \frac{[P]_{js}}{[D]_{ss}}$$

$$[U]_{ss} = 1$$

where the subscript denotes the matrix element, and for $j, l = 1$ to $(s-1)$
\[ [P]^l = [P]^l - [U]^l [D]_{ss} [U]^s \] \hspace{1cm} \text{V.5}

set

\[ [D]_{1,1} = [P]_{1,1} \] \hspace{1cm} \text{V.6}
\[ [U]_{1,1} = 1 \] \hspace{1cm} \text{V.7}
\[ [U]_{1} = 0 \quad \text{for } j/l = 1 \text{ to } m \] \hspace{1cm} \text{V.8}

Next the factorised matrices \([U]\) and \([D]\) are calculated along with the estimator gain vector \([K]\). First temporary work vectors \([f, v, h]\) are set up, each \(m\) elements long

\[ f = [U] \Theta^T \] \hspace{1cm} \text{V.9}
\[ v = [D] f \] \hspace{1cm} \text{V.10}
\[ K = [g_1, 0, 0, 0, \ldots, 0]^T \] \hspace{1cm} \text{V.11}

The following scalar terms are also defined

\[ a_1 = R + g_1 f_1 \] \hspace{1cm} \text{V.12}
\[ [\hat{D}]_{1,1} = \frac{R[D]_{1,1}}{(R + g_1 f_1)} \] \hspace{1cm} \text{V.13}
\[ [\hat{U}]_{1,1} = 1 \] \hspace{1cm} \text{V.14}

the following equations are cycled through for \(s = 2\) to \(m\),

\[ a_s = a_{s-1} + v_s f_s \] \hspace{1cm} \text{V.15}
\[ [\hat{D}]_{ss} = \frac{[D]_{ss} a_{s-1}}{a_s} \] \hspace{1cm} \text{V.16}
and for $j = 1$ to $(s-1)$

$$[\hat{U}]_j = [U]_j - \frac{f_s \tilde{K}_j}{\alpha_{s-1}} \quad \text{V.17}$$

$$[\hat{U}]_{ss} = 1 \quad \text{V.18}$$

all other elements $[U']$ are zero, then for $j = 1$ to $s$

$$\tilde{K}_j = \tilde{K}_j + g_s [U]_j \quad \text{V.19}$$

The estimator gain is then given by

$$K_{W1} = \frac{\tilde{K}}{\alpha_N} \quad \text{V.20}$$

the final step is to calculate reconstructed covariance matrix using $[U]$ and $[D]$. 
APPENDIX VI

MATLAB SIMULATION FILES

INTRODUCTION
This appendix contains two files which simulate the Hybrid ACSR controller operating on an helicopter fuselage. The first file set up the helicopter model and the simulation parameters, it also contains the signal processing, control optimisation and parameter estimation routines. The second file defines the equations that need to be integrated with respect to time, these include the flexible structure dynamics and the nonlinear actuator dynamics.

VI.1 MAIN SIMULATION ROUTINE

% Read in EH101 helicopter data files
o = ehom; % natural frequencies for vibration modes
c = ehc; % actuator influence matrix
d = ehd; % rotor head forces-modal influence matrix
phi = ehphi; % sensor eigenvector matrix - transformation
iph = inv(phi); % modal eigenvector matrix - transformation
f1 = ehf(114); % flight data for forcing at 114 Kts
f2 = ehf(125); % flight data for forcing at 125 Kts

% Global Variables Declaration

global f_in f_out ah bh ch dh new_gain wt uf control x_dis x_vel qa qb ps
global yy a
%-----------------------------------------------
% Signal Processing Initial Conditions and Simulation Definitions
%-----------------------------------------------
% parameters for phase and magnitude calculations each cycle
sums = zeros(10,1); % nw correlator sine summation over one period
sumc = zeros(10,1); % nw correlator cosine summation over one period
yw0 = zeros(10,1); % vectors of nw cosine and sine coefficients
yw = zeros(10,1); % extracted using correlator
zeta = 0.05; % Damping
w = 110; % Forcing frequency in rad/s = 17.5 Hz
samplef = 1000; % Sampling frequency for simulation
sample = 1/samplef; % stepsize
n = 7; % number of cycles for correlation period
period = round(n*samplef*2*pi/w); % correlation period
ywflag = 0; % nw vibration vector available (1 = true)
end_time = 3; % end time for simulation in secs
end_sim = end_time*samplef; % number of steps for simulation
control_on = period + samplef/50; % controller switch on point
ind = 1; % index for estimated percentage reduction
old_gain = zeros(4,10); % initial value of hybrid gain matrices
new_gain = zeros(4,10); % set to zeros
f_in = zeros(4,1);
vy = zeros(10,1); % initial vibration vector

%--------------------------------------------------,-~-------------------
% Initialisation parameters for the four actuators
%--------------------------------------------------,-~-------------------
pra = zeros(4,1); % pressure for piston side A
prb = zeros(4,1); % pressure for piston side B
qa = zeros(4,1); % flow
qb = zeros(4,1); % flow

% construct state transformation matrix to give velocities and displacements
% across each of the four actuators
wwv = [ 0 0 0 0 1 0 0 0
        1 0 0 0 0 0 0 0
        0 0 0 0 1 0 0 0
        0 1 0 0 0 0 0 0
        0 0 0 0 0 1 0 0
        0 0 1 0 0 0 0 0
        0 0 0 0 0 0 1 0
        0 0 0 1 0 0 0 0];
c = c';
w2 = [c zeros(c); zeros(c) c];
w = -ww'w2;

%-------------------------------------------------------------
% Formulation of EH101 model
%-------------------------------------------------------------
a21 = diag(0.2); %
a22 = -2*zeta*diag(0);
aa = ((levy(10)) + (i/w)*(a22)) + ((1/(w·2))*(a21));
inv = inv(aa); % construct Transfer function matrix (T matrix)
t0 = phi*invac; % for frequency domain calculations
bwhl = phi*invad*f1; % background vibration vector B for 114Kts
bwhl2 = phi*invad*f2; % background vibration vector B for 125Kts

% state space formulation of helicopter model A,B,C,D matrices
ba = c;
bf = d;
ah = [zeros(10) eye(10); a21 a22];
ch = [phi*a21 phi*a22];
fh = zeros(10,4) zeros(10,6); ba bf];
% x0=[zeros(20,1)]; % initial condition vector for time simulation
load ehx0114; % initial condition vector for time simulation

% Parameter Estimator Initial Conditions
sigma = 1;
cov = 10000;
rr = 1;
decay = round(samplef/3);
% decay = 900;
trans = decay;
% flag = 0;
ysp = zeros(10,1);
% z = rand(4,10);
% ttz = z';
load ttz;
pc = zeros(4);
for diagonal = 1 :4,
    pc(diagonal,diagonal) = cov;
end
meas_vec(1:4,1) = [0 0 0 0];
clc
for scr = 1:11,
    disp(' ');
end
disp('Discrete Time Simulation ');
disp('Start - start of simulation ');

for t = 1:end_sim,
    time = (t-1)/samplef;
    trans = trans + 1;
    if t = 1,
        xcl(:,1) = [0 0 0 0]';
    end
end
\[
\text{cosw} = \cos(w \cdot \text{time}); \quad \% \text{reference signal for rotor forcing and}
\sinw = \sin(w \cdot \text{time}); \quad \% \text{correlator}
\uf = \text{real}(f1) \cdot \sinw + \text{imag}(f1) \cdot \cosw; \quad \% \text{calculate rotor forcing}
\% \text{frequency domain control action - time history}
\uxc(:,t) = \text{real}(\xc(:,t)) \cdot \sinw + \text{imag}(\xc(:,t)) \cdot \cosw;
\]

\% set up vector for the integration/simulation of actuator and helicopter
\% dynamics
\x = \x0;
\s = [\pra; \prb; \x0];
[\tout, \sdot] = \text{ode}23(’ehwhdyn’, \text{time}, \text{time} + \text{sample}, \s);
[\rowq, \colq] = \text{size}(\sdot);
\pra = \sdot(\rowq, 1:4)'; \quad \% \text{extract actuator pressure}
\prb = \sdot(\rowq, 5:8)'; \quad \% \text{extract actuator pressure}
\x0 = \sdot(\rowq, 9:28)'; \quad \% \text{extract state vector}
\uf = \text{real}(f1) \cdot \sinw + \text{imag}(f1) \cdot \cosw; \quad \% \text{actuator outputs}
\uf(:,t) = \uf;
\ufo(:,t) = \ufo;
\uff(:,t) = \uff;
\]

\% calculate vibration magnitudes and phases of nw components in measured
\% vibration using a reference signal - correlation method
\wc(:,t) = \yt(:,t) \cdot \cosw;
\ws(:,t) = \yt(:,t) \cdot \sinw;
\]

\% time history of rotor forces
\% time history of actuator pressures
\% time history of actuator flows
\% recombined input vector - control | forcing
\% calculate vibration outputs
\% time history of vibrations
\% time history of state vector
\% time history of actuator output forces
\% time history of actuator input forces

\text{if (trans = decay + period) & (flag = 1)),}
\text{for wi = 1:period,}
\quad \text{sumc = sumc + wc(:,t-(wi-1));}
\quad \text{sums = sums + ws(:,t-(wi-1));}
\text{end}
\text{ywflag = 1;}
\text{ycos(:,1) = (2/(period))}^* \text{sumc;}
\text{ysin(:,1) = (2/(period))}^* \text{sums;}
\text{sums = \text{zeros}(10,1);}
\text{sumc = \text{zeros}(10,1);}
\text{yw = [ysin(1) + j*ycos(1); ysin(2) + j*ycos(2); ysin(3) + j*ycos(3)}
\text{ysin(4) + j*ycos(4); ysin(5) + j*ycos(5); ysin(6) + j*ycos(6)}
\text{ysin(7) + j*ycos(7); ysin(8) + j*ycos(8); ysin(9) + j*ycos(9)}
\text{ysin(10) + j*ycos(10));}
\text{else,}
\quad \text{ywflag = 0;}
\text{end
}

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% Controller calculations - transients allowed to decay - local linear
% formulation, i.e. dY = T dX - controller update cycle 1 second
% actuator/sensor weighting defined by rcon
if t > control_on,
    tc = ((1 * samplef) - (control_on + 1) + t);
    if rem(tc, (1 * samplef)) == 0,
        rcon = 10^(-8.5);
        x_old = old_gain * yw;
        % calculate optimal frequency domain control inputs
        concalc = ttz * eye(10) * yw + rcon * eye(4) * x_old;
        xc(:, t + 1) = x_old - (inv(ttz * eye(10) * ttz + rcon * eye(4))) * concalc;
        dx = xc(:, t + 1) * x_old;
        dy = ttz * dx;
        ynew = dy + yw;
        % calculate hybrid gain matrix
        % partition control and vibration vectors into real and
        % imaginary parts
        hdx = [real(dx - old_gain * dy), imag(dx - old_gain * dy), real(dx), imag(dx)];
        hdy = [real(ynew), imag(ynew), real(yw), imag(yw)];
        delta_gain = hdx * pinv(hdy);
        oldgain = old_gain;
        new_gain = delta_gain + old_gain;
        old_gain = new_gain;
        trans = 0;
    else
        xc(:, t + 1) = xc(:, t);
    end
else
    xc(1:4, t + 1) = [0 0 0 0]';
end

% transient delay checks - set flags accordingly
if (trans < decay),
    flag = 0;
else,
    flag = 1;
end
flt(t) = flag;
if trans = = 0,
    yw0 = yw;
    xu0 = oldgain * yw0;
    % measured vibration from previous cycle
    % control input from previous cycle
else,
    est_sw = 0;
end
if (ywflag = = 1) & (flag = = 1),
    est_sw = 1;
    ywn = yw;
% set estimator switch
else,
    est_sw = 0;
end
estsw(t) = est_sw;
% if t > 1599,
% f1 = f2;
% bwhl2 = bwhl;
% end
Parameter Estimation

% check estimator switch
meas_vec(1:4,1) = (new_gain*yn)-xuO;
p = pc;

% update covariance matrix
for diagonal = 1:4,
    p(diagonal,diagonal) = pc(diagonal,diagonal);
end

% calculate estimator gain
kal = p*meas_vec*(1/(meas_vec'*p*meas_vec + rr));

% estimate state vector (prediction)
yp = meas_vec'*z;
yps = yp + ywO';

% actual change in Y
ysp = ywn-ywO;

% estimator error - dY_estimated - dY_actual
error = yp-ysp';

% calculate residual
res = (1/(meas_vec'*p*meas_vec + rr)*error;

% update parameter estimates
ztz = z';

% variable forgetting factor calculations
er = (abs(error)).^2;
esq = sum(er)/10;
resd(t) = esq;
lam(t) = 1-(1-meas_vec'*kal)*esq/sigma);

% limits on forgetting factor
if lam(t) < 0.2,
lam(t) = 0.2;
elseif lam(t) > 0.98,
lam(t) = 1;
else
    lam(t) = lam(t);
end

% correct covariance matrix
pc = (1/lam(t))*(p-(kal*meas_vec'*p));

% initial values for time histories
if t = = 1,
    resd(t) = 0;
lam(t) = 0;
yp = zeros(1,10);
else
    yps = yp + ywO';
    resd(t) = resd(t-1);
lam(t) = lam(t-1);
end

% values if estimator is switched off
end
end

%---------------END---------------------------------------
VI.2 DYNAMICS INTEGRATION

function sqddot = ehwhdyn(time,sqd)
%-------------------------------------------------------------------------
% % 1 January 1992 Matlab M_File
% % Actuator model supplied by Ian Lyndon Westland Helicopters Ltd
% function xdot=ehwhdyn(time,x)
% % Helicopter dynamics
% % non-linear actuator model first order dynamics
% %-------------------------------------------------------------------------
%-------------------------------------------------------------------------
% % system definition
% %-------------------------------------------------------------------------

ksa = 0.1e-2;
% gain V/A
% gain - force feedback around actuator
kc = 4.8e-3;
% kc = kf*kh/a
kc = 1.573e-4;
% feedback gain
kf = 2.6e-7;
kh = 1;
% initial actuator hydraulic volume
vo = 11.5e-5;
% actuator piston area
a = 1.653e-3;
% bulk modulus
bm = 1.4e9;
ps = 20.86;
% hydraulic supply pressure
kv = 4.36e-6;
% gain flow/pressure

%-------------------------------------------------------------------------
% % actuator equations
% %-------------------------------------------------------------------------

pa = sqd(1:4,1);
pb = sqd(5:8,1);
x = sqd(9:28,1);
relx = wt*x;
x_vel(1,1) = relx(1);
x_dis(1,1) = relx(2);
x_vel(2,1) = relx(3);
x_dis(2,1) = relx(4);
x_vel(3,1) = relx(5);
x_dis(3,1) = relx(6);
x_vel(4,1) = relx(7);
x_dis(4,1) = relx(8);

% calculate velocities and displacements
% across actuators - from state space vector
f_in = new_gain*flvyl;

% actuator demand derived from vibrations
% fed back through hybrid gain matrix

for act = 1:4,
p1 = pa(act);
p2 = pb(act);
q1 = qa(act);
q2 = qb(act);
end
% servo-valve current - including limits
si(act, 1) = ksa*(kc*f_in(act)-kf*kh*(p1-p2)*1e6);
if si(act) > 10e-3, si(act) = 10e-3; end
if si(act) < -10e-3, si(act) = -10e-3; end

va = vo + a*x_dis(act);
vb = vo-a*x_dis(act);
% servo-valve actuator dynamics

% integrate pressures
pad(act, 1) = 1e-6*bm*(q1-a*x_vel(act))/va;
if p1 > ps, p1 = ps; end % check pressure does not exceed supply
if p1 < 0, p1 = 0; end

pbd(act, 1) = 1e-6*bm+(q2+a*x_vel(act))/vb;
if p2 > ps, p2 = ps; end
if p2 < 0, p2 = 0; end
if si(act) > 0, % calculate flow in actuators
    qa(act, 1) = si(act)*kv*sqrt(abs(ps-p1)*1e6);
    qb(act, 1) = -si(act)*kv*sqrt(abs(p2)*1e6);
else
    qa(act, 1) = si(act)*kv*sqrt(abs(p1)*1e6);
    qb(act, 1) = -si(act)*kv*sqrt(abs(ps-p2)*1e6);
end

if control = 'off', % output force
    f_out(act) = zeros(1,1);
    end
f_out(act, 1) = (p1-p2)*a*1e6; % calculate actuator output force
end

u = [f_out; uf]; % construct input vector
x_dot = ah*x + bh*u; % structural dynamics simulation
yy = ch*x + dh*u; % output calculations
sqddot = [pad;pbd;x_dot]; %

%--------------------------------------------------------END--------------------------------------------------------
APPENDIX VII

BENCH TEST RIG DESCRIPTION

INTRODUCTION

This appendix describes the analogue bench test rig constructed to develop the controller software. The rig is based upon the mathematical models described in Chapter 3.

VII.1 FUSELAGE MODEL

The bench rig comprises a number of electronic circuits, which represent 6 structural modes. Each mode has an equation of the form:

\[ \ddot{\eta}_i + 2\zeta_i \dot{\eta}_i + \omega_i^2 \eta_i = C_{ii} U_1 + C_{2i} U_2 + D_i F \]

where \([U_1, U_2]\) is the input vector and \([C]\) is the input (actuator) influence matrix. \(F\) is the forcing input, representing the rotor head forces, \([D]\) is the rotor forcing influence matrix.

where \(\eta_i\) and \(\zeta_i\) and \(\omega_i\) are the modal displacement, relative damping and natural frequency of the \(i\)th mode of vibration.

\(\mathbf{u} = [U_1, U_2]\) denotes the control inputs the force actuators, and \([C]\) is the actuator influence matrix. The disturbance forces acting on the structure are denoted by \(E\) and the corresponding influence matrix is \([D]\). Both \([D]\) and \([C]\) are matrices of modal coefficients. The resulting structural vibration is given by a linear transformation of the modal accelerations:

\[ \mathbf{y} = [M_D] \ddot{\mathbf{u}} \]

where \(\mathbf{y}\) is the vector of structural vibrations and \([M_D]\) is a transformation matrix.

The coefficients for the equations are given in Tables VII.1 and the transformation matrix \([M_D]\) is given in Table VII.2. The overall configuration of the rig consists of 10 boards (one for each of the 6 modes and one for each of the 4 sensor outputs), and this is shown in Figure VII.1. The circuit schematics for a general mode and a sensor are shown in Figures VII.2 and VII.3 respectively. The PCB layouts for one of the sensor boards and for one of the mode boards from the rig are shown in Figures VII.4 and VII.5.
## Appendix VII

### Table VII.1 Coefficients for the Modes included in the bench rig

<table>
<thead>
<tr>
<th>Mode Number i</th>
<th>Modal Damping</th>
<th>Modal Frequency</th>
<th>[C] Matrix</th>
<th>[D] Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>104.93</td>
<td>0.3507</td>
<td>0.1173</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>122.63</td>
<td>-0.0037</td>
<td>0.4161</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>143.58</td>
<td>-0.1352</td>
<td>0.3746</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>149.84</td>
<td>-0.7284</td>
<td>0.3331</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>158.91</td>
<td>-0.1701</td>
<td>-0.1386</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>189.36</td>
<td>0.5250</td>
<td>-0.3772</td>
</tr>
</tbody>
</table>

### Table VII.2 Coefficients for the Transformation matrix \([M_c]\) (sensor eigenvectors are elements*100)

<table>
<thead>
<tr>
<th>Sensor</th>
<th>mode 1</th>
<th>mode 2</th>
<th>mode 3</th>
<th>mode 4</th>
<th>mode 5</th>
<th>mode 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>sensor 1</td>
<td>1.072</td>
<td>0.063</td>
<td>-0.114</td>
<td>2.227</td>
<td>-0.549</td>
<td>-0.376</td>
</tr>
<tr>
<td>sensor 2</td>
<td>1.001</td>
<td>-0.110</td>
<td>-0.357</td>
<td>-2.000</td>
<td>-1.617</td>
<td>0.031</td>
</tr>
<tr>
<td>sensor 3</td>
<td>0.697</td>
<td>0.170</td>
<td>-0.155</td>
<td>-1.637</td>
<td>-0.000</td>
<td>-0.622</td>
</tr>
<tr>
<td>sensor 4</td>
<td>0.792</td>
<td>0.587</td>
<td>0.385</td>
<td>2.144</td>
<td>1.100</td>
<td>0.636</td>
</tr>
</tbody>
</table>
Forcing Inputs
Input 1
Input 2

Sensor Outputs
Sensor 1
Sensor 2
Sensor 3
Sensor 4

Modal Accelerations

Appendix VII

John Pearson
March 1992

Figure VII.1 Rig Configuration
Figure VII.2 Circuit schematic for general second order mode
Figure VIII.3. Circuit schematic for a general sensor output.
Figure VII.4 PCB Layout for Sensor 2

Figure VII.5 PCB Layout for Mode board 1
INTRODUCTION
This appendix lists the software for the implementation of the adaptive hybrid control strategy on the DSP32C. The control algorithm essentially consists of two loops. One loop is interrupt driven and the primary task is to feed back the measured vibration through a gain matrix to provide drive signals for the actuators. Other tasks include correlating the sampled accelerometer signals with corresponding values from tables for sine and cosine signals at \( \omega \), and digitally filtering the outputs. The other loop acts in parallel with this operation, the control algorithm calculates an update for the feedback gain matrix in a number of stages:
- Transient Decay
- Digital Signal Processing
- Parameter Estimation
- Control Optimisation

The following sections describe the detail operation of the software and list the source files themselves.

VIII.1 INTERRUPT SERVICE ROUTINE DESCRIPTION
The interrupt timer is set using the `init_adc` function, this function also initialises the ADCs and is discussed later. Each time an interrupt is generated on INTREQ1 the set-up routine for the interrupt service function is called (`set_int.s`). The first task of this function is to disable the interrupts using the program control word `pcw`, so that the interrupt service routine can not interrupted by itself. Next the accumulators and registers are saved to temporary locations. A temporary stack (1kb in size) is initialised for use by the interrupt service routine. Once these tasks have been completed the interrupt service routine can be called, since all accumulators and registers were saved and a local stack is used this function was written in C. When the interrupt service routine has finished the accumulators and registers are restored, then the interrupts are re-enabled.

`init_adc` This routine is used to initialise the ADCs and the interrupt timer. The analogue interface board is memory mapped and occupies 4 locations in the I/O addressing range of
Appendix VIII

DSPLINK master processor board. Addresses for the Control and Status Registers are set up. Bits within the Status Registers reflect the current operational state of the board and provide pollable flags indicating end of conversion and timeout events. The least significant two bits of the Control Register determine the channel to be converted. Other bits control the HOLD flip-flop, end-of-conversion and timeout interrupts, operation of the timer, initiation of the SAMPLE/HOLD calibration cycle, and an output flag for controlling external circuitry. Figure VIII.1 shows the registers used on the Analogue Interface board. The Sample Rate Timer Register address is also set up, the timer consists of a 16 bit up-counter running with an 8 MHz clock.

The counter has a programmable input register from which the counter is loaded every time the maximum count value (FFFF) is reached. Writing to the timer automatically generates a trigger signal and also loads the timer from its input register. The following formula can be used to calculate the value needed to be loaded into the timer

\[
\text{COUNT} = \left\lfloor \frac{F_{\text{clock}}}{F_{\text{sampling}}} - 1 \right\rfloor \quad \text{VIII.1}
\]

where \(F_{\text{clock}}\) is the counter clock (8000000), and \(F_{\text{sampling}}\) is the required sampling frequency in Hz, for example:

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Period</th>
<th>Count in Hex</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 Hz</td>
<td>5 msecs</td>
<td>63C1</td>
</tr>
<tr>
<td>1 kHz</td>
<td>1 msecs</td>
<td>EOC1</td>
</tr>
<tr>
<td>5 kHz</td>
<td>200 usecs</td>
<td>F9C1</td>
</tr>
<tr>
<td>10 kHz</td>
<td>100 usecs</td>
<td>FCE1</td>
</tr>
</tbody>
</table>

Before the Sample Rate Timer is set with a sample rate the 4 channel Sample and Hold device is calibrated. This has the effect of nulling any DC offsets in the Sample/Hold and multiplexer components. Calibration is achieved by writing a '1' to bit 5 of the Control Register (Figure VIII.1). Then once calibration has been initiated bit 5 of the Control Register is returned to a '0' state. During the calibration cycle (which lasts for approximately 10msec) bit 5 of the Status Register remains low and returns high at the end of calibration. Next the ADC register is read, this has the effect of clearing the status flags associated with the end of conversion and overflow of the sample rate timer. The ADC is then set up to enable a sample to be taken from channel 0, this is achieved by writing 0x73 to the Control Register. This is an 'AND' combination of the channel number, TRACK which sets bit 6 of the Control Register to a '1' setting the inputs to track, and TIMER_ON which sets bit 4 of the Control Register to a '1'
enabling the timer.

Finally the Sample Rate Timer Register is loaded with the sample rate, for this application a sample rate of 1 kHz was selected (i.e. EOC1 was loaded). This process is shown in Figure VIII.2.

init_dac Each DAC has two 12 bit registers, an input register which is loaded by writing to the appropriate board addresses defined by \texttt{DAC\_O\_ADDRESS} and \texttt{DAC\_I\_ADDRESS}, and a DAC register which controls the current output level. The contents of the input register are transferred to the DAC register on receipt of a DAC update signal supplied by the sample rate timer. This double buffering has the advantage that new values can be loaded to the DAC input registers and then all DAC outputs updated simultaneously. This function sets up an address for each of the DAC input registers.

write_dac A pointer \texttt{*p\_dac\_buffer} is set to point to the buffer used to store the values which are to be sent to the DACs. The value pointed to by the pointer is then written to the first DAC input register, the address of which is pointed to by the pointer \texttt{*DAC\_O}, the pointer used by the DAC buffer is incremented and the new value is then written to the second DAC input register (address pointed to by \texttt{*DAC\_I}). The DAC registers are then updated simultaneously by the sample rate timer.

sample_adc The Sample/Hold Control bit (Bit 6 of the Control Registers) must be set to ‘0’ ("Hold") before any conversion other than the last one of a set channels takes place. Setting this to 0 before the conversion therefore holds all the inputs to the Sample and Hold device constant. The Sample/Hold Control bit is then set to 1 ("Sample") before the fourth conversion (last in the series). This is because the operation of the Sample/Hold control signal takes effect at each rising edge of the End-Of-Convert (EOC) Flag signal. Setting the Sample/Hold Control bit to 1 causes the input channels of the Sample/Hold device to track their respective inputs. The conversion process is initiated by a rising edge on the trigger signal, which causes the Sample/hold device to enter the hold mode if the Sample/Hold Control bit is set to 1 ("hold"). There is a 1 usec delay for the multiplexer to settle and a delay of 3 usec for the conversion process.

The process of conversion is indicated by Bit 4 (ADC Busy) of the Status Register, which is zero during conversion and 1 otherwise. The end of conversion (EOC) is signalled to the processor by a transition from 0 to 1 on the EOC Flag bit of the Status Register (Bit 7).
EOC Flag Bit is reset low by the software reading the ADC value. Another channel is then selected by writing the appropriate control word to the Control Register. This will again be followed by a 1usec delay for the multiplexer to settle and a 3usec delay for the conversion process. Figure VIII.3 shows the timing of the software for sampling the four analogue input channels.

The sample_adc routine is shown in Figure VIII.4 and functions as follows. First pointers are set up for the buffer to which ADC samples are to be written (adc_buffer) and for the control words used to control the Sample/Hold device (one for each channel). For the first channel the EOC Flag Bit of the Status Register (Bit 7) is checked, when the flag is set (i.e. conversion complete) the value of the ADC is read into the ADC buffer. The pointer to the ADC buffer is then incremented. The next channel number is then written to the Control Register using the control word pointed to by the pointer *p_flags, this pointer is then incremented ready for the next channel. Then as before, once the conversion is complete the ADC is read. This is repeated for each input channel, once the last channel has been read the Sample/Hold Control Bit is set to 1 so that the inputs to the Sample/Hold device are set to track mode.

VIII.2 HYBRID CONTROLLER SOFTWARE DESCRIPTION

The main section will be discussed first and then the interrupt service routine will be detailed.

Algorithm Initialisation The first stage includes the declaration of variables, pointers and tables required by the software. Two look up tables are initialised sint and cost, one contains a single cycle of a 20Hz sine wave and the other a single cycle of a 20Hz cosine wave, both sampled at 1000Hz thus giving 50 points. These tables are used in the correlation process and for the control signals for the DACs. Pointers for each ADC are set up which point to the ADC buffer, and similarly pointers are set up for the DACs.

When the Hybrid gain matrix is updated it is important that all elements of the matrix are switched immediately, otherwise an indeterminate control action can occur which may significantly degrade performance. Therefore two gain matrices are set up, each consisting of two vectors: a_gain_1 and a_gain_1, and b_gain_1 and b_gain_2. One matrix to contain the current control gain matrix and the other to contain the next gain matrix. A pointer for each of these matrices is set up *c_gain and *e_gain, and then pointers to these pointers are initialised - instantaneous gain matrix switching is then achieved by switching these pointers, this is shown in Figure VIII.5.
Vectors to contain the coefficients used to implement the filters on the DAC outputs are initialised. These filters are second order tuned digital filters based on the delta operator and are tuned to the blade passing frequency. The Menu software can be used to automatically calculate these coefficients by selecting the level of damping required.

Controller Initialisation Two functions are used to load the controllers initial parameters. `load_set` loads the transient decay time `decay`, the controller update rate `update` and the correlation period `corr_p` from the ASCII file `set_up.dat`, and the name for the file to which results are written `result_file` from the ASCII file `file_res.dat`. Both `set_up.dat` and `file_res.dat` are initialised by the menu programme, alternatively they can be edited using a text editor. The `set_up.dat` file format is: the first line contains the length of the transient decay period, the next line contains the controller update period and third line contains the correlation period, all of these parameters are specified in terms of the sample period. Since the controller is implemented using a sampling frequency of 1000Hz (1ms sample period), a `set_up` file for a controller with a transient decay time of 0.5 seconds, an update rate of 1 second and a correlation period of 0.4 seconds is therefore specified as:

```
500
1000
400
```

`loaddat2` loads the sensor weights `s_wt`, actuator weights `a_wt`, actuator rate limit weights `da_wt`, initial covariance matrix `p_cov`, estimator sensitivity `sigma`, the measured background vibration `b_vector`, a T matrix `t_mat` which includes the output filter dynamics and the filter coefficients `ft_vec`, from the following ASCII files `s_wt.dat`, `a_wt.dat`, `da_wt.dat`, `p_cov.dat`, `sigma.dat`, `b_vec.dat`, `t_math.dat` and `ft_vec.dat`. These files are initialised by the menu software or again they can be edited using a text editor. The sensor weight file `s_wt.dat` contains a weighting factor for each sensor (in this implementation 4 sensors are used), similarly the control weighting matrices `a_wt.dat` and `da_wt.dat` contain weighting elements for each actuator (2 in this case). The `sigma.dat` file contains a single number which represents the sensitivity for the estimator. The covariance matrix file `p_cov.dat` contains a 4 by 4 matrix representing the uncertainty associated with each channel of the estimator, for the global controller this is a 5 by 5 matrix. The measured background vibration vector is stored in the `b_vec.dat` file. The initial sensor/actuator transfer matrix (T matrix) is stored in the `t_mat.dat` file, since there are 4 sensors and two actuators this file contains an 8 by 4 matrix.

The filter coefficients for the second order tuned delta filters used to filter the DAC outputs are
stored in the \textit{ft_vec.dat} file, 5 coefficients are stored in this file - 3 for the numerator and 2 for the denominator. The file for results is initialised. One array is used to contain the samples from the four ADCs over the signal processing period \textit{sample_buffer}, pointers are set for each of the ADCs which point to a corresponding block within this array. Similarly an array and two pointers are set up for the two DACs. Pointers are also set for the sine and cosine look up tables.

\textbf{ADCs, DACs and Interrupt Initialisation} These have already been discussed. The interrupt service routine is called 1000 times per second, it increments a flag used as a timer for the controller cycle, writes control signals to the DACS and writes the sampled ADC signals to a buffer. The operation of the interrupt service routine is discussed later.

\textbf{Main Controller Loop}

The main controller loop for the Hybrid Algorithm has the same stages as the Frequency Domain controller. The main stages are:

- **Transient Decay** When the flag incremented by the interrupt service routine \texttt{(update\_flag)} is greater than the decay period, which has already been defined, the \texttt{transient\_flag} is set. The next stage of the routine is then performed.

- **Signal Processing** When the array (buffer) used to store the sampled ADC signals is full the interrupt service routine sets the \texttt{buffer\_full\_flag}. If the correlation has not already been performed then the \texttt{data\_valid} flag is clear. If the \texttt{buffer\_full\_flag} is set and the \texttt{data\_valid} flag is clear then the correlation vector \texttt{(y\_new)} is transferred to the array \texttt{Yv\_new}, the actual correlation process is performed by the interrupt service routine. If the control cycle is the first one, then the measured vibration (in the frequency domain) for the previous controller cycle \texttt{(y\_old)} is set to be equal to the current vibration vector \texttt{Yv\_new}.

- **Parameter Estimation** A Recursive Least Squares algorithm with a variable forgetting factor is used to perform the parameter estimation. The equations used to implement the estimator are detailed in Chapter 4. The estimator function for the Hybrid controller \texttt{estim} requires the following parameters to be passed to it; the measured vibration vector from the previous controller cycle \texttt{y\_old}, the current measured vibration vector \texttt{Yv\_new}, the gain matrix from the previous controller cycle \texttt{old\_gain}, the current gain matrix \texttt{new\_gain}, the estimated \texttt{T} matrix from the previous controller cycle \texttt{t\_mat}, the covariance matrix \texttt{p\_cov}, the estimator sensitivity \texttt{sigma}, and the forgetting factor \texttt{lam}.

The estimator function returns an updated estimate of the \texttt{T} matrix, an updated covariance matrix and an updated forgetting factor.
Control Calculations

Before any control optimisation is performed the current gain matrix \( new_{gain} \) and the current vibration vector \( Yv_{new} \) are copied to the corresponding vectors for the previous controller cycle \( old_{gain} \) and \( y_{old} \). The control optimisation function for the Hybrid controller \( conh_{opt} \) is then called. This function requires the following parameters to be passed to it; the current vibration vector \( Yv_{new} \), the gain matrix from the previous controller cycle \( old_{gain} \), the estimated T matrix \( t_{mat} \), the sensor weights \( s_{wt} \), the actuator weights \( a_{wt} \), the actuator rate limit weights \( da_{wt} \), the covariance matrix \( p_{cov} \) and a switch which determines whether a deterministic or a cautious control law is used \( deterministic \).

The control function returns a new control vector for the next controller cycle \( new_{gain} \).

Control Cycle

A new controller cycle is started when the flag incremented by the interrupt service routine \( update_{flag} \) is greater than the controller update period \( update \). The new calculated gain matrix is copied to the gain matrix which is not currently being used by the interrupt service routine, either \( a_{gain\_1/2} \) or \( b_{gain\_1/2} \).

For the frequency domain controller, the new calculated control vector is copied to the control vector which is not currently being used by the interrupt service routine, either \( con_{vec\_1} \) or \( con_{vec\_2} \). The pointers to the gain matrices are then swapped so that the new gain matrix is used by the interrupt routine. The flag \( control_{flag} \) used to indicate which gain matrix is being used by the interrupt service routine is then toggled.

The pointers for the ADC sample buffer and for the DAC output buffer are then reset to the beginning of their corresponding blocks. The flag \( buffer_{full_{flag}} \) is reset which enables the interrupt service routine to restart sampling the ADCs and to perform signal processing on the ADC samples. The transient decay flag \( transient_{flag} \) is reset, the controller update flag is reset, the signal processing flag for the main controller loop is reset \( data_{valid} \) and the control cycle number \( control_{cycle} \) is incremented.

Results

At the end of the controller cycle the following are calculated and written to the result file specified earlier: the average RMS vibration, the sum of diagonal elements of the covariance matrix, the forgetting factor and the estimator error \( err \).

Interrupt Service Routine

This is the routine called when an interrupt from the timer occurs (filename \( intbyf9t.c \), function name \( service1() \)). An input buffer to contain ADC samples and an output buffer to contain values to be written to the DACs are created, \( adc_{buffer} \) and \( dac_{buffer} \) respectively. Pointers for the DAC buffer and the ADC buffer are defined, as well as a pointers for the Hybrid gain matrix and for the filter coefficients. Next the four analogue
input channels are sampled using the sample_adc function, the results are stored in the ADC buffer. 

The required output signal is generated by multiplying the sampled inputs by the gain matrix. The output for the first DAC pointed to by *p_dac_buffer is given by the sum of the products of the elements of the first row of the gain matrix (pointed to by *p_gain_1 · incremented after each multiplication) and the ADC samples for the four analogue input channels (pointed to by *p_adc_buffer · incremented after each multiplication). Similarly the output value for the second DAC (pointed to by *p_dac_buffer · incremented once) is given by the sum of the product of the elements of the second row of the gain matrix (pointed to by *p_gain_2) and the ADC samples for the four analogue input channels.

For the frequency domain algorithm the optimal control vector is essentially specified in terms of magnitudes and phases at the blade passing frequency. The frequency is generated by using two tables one containing a single cycle of a sine wave and the other containing a single cycle of a cosine wave at the blade passing frequency. The number of points in the tables is obviously determined by dividing the sampling frequency by the blade passing frequency. By incrementing pointers for these tables each time the interrupt routine is called, and by combining the appropriate sine magnitude with the appropriate cosine magnitude, a control input of the required magnitude, phase and frequency can be generated.

This is achieved by multiplying the first element of the control vector pointed to by *p_control by the value from the sine table pointed to by *p_sin, the control vector pointer is then incremented so that the second element of the control vector can be multiplied by the value from the cosine table pointed to by *p_cos and adding the result together. This value corresponds to the required control output for DAC 1 and is written to the first location in the DAC buffer. This is repeated for the third and fourth values of the control vector incrementing the control vector pointer each time, and the result is written to the second location in the DAC buffer.

The next stage involves a frequency correlation. First the ADC buffer is checked to see if it is full, the length of the buffer is defined by the correlation period corr_p which is set by the user. The buffer stores samples from each of the four analogue input channels over a complete correlation period, pointers associated with each channel point to blocks within this buffer. To check the buffer the address of the pointer associated with the fourth channel p_adc_buffer_3 is compared to the address of the end of the ADC buffer, if the buffer is full the buffer_full_flag is set. If the ADC buffer is not full (i.e. buffer_full_flag = 0) and the transients have decayed (i.e. transient_flag = 1) then the correlation process begins. A vector is set up to contain
recursive sums of the products of the sampled vibration with sine and cosine waveforms
(vector has dimensions 8 by 1). When the correlation period is complete this vector is similarly
divided by the correlation period to give the sine and cosine magnitudes of the measured
vibration. The sample from the first analogue channel pointed to by \*p_adc_buffer is multiplied
by the value in the sine table pointed to by \*p_sin and the result is added to the first value in
the correlation vector pointed to by the correlation vector pointer \*p_y_new. The correlation
vector pointer is then incremented and the process is repeated for the cosine table pointer
\*p_cos. Then the sample is written to the ADC buffer at the location given by the pointer for
the first channel \*p_adc_buffer_0. The correlation vector pointer and the ADC buffer pointer
are both incremented ready for the next channel. This is repeated for the ADC samples from
the other three channels.

The DAC output samples are written to the DAC buffer. Next the pointers for the sine and
cosine tables are incremented ready for the next time the interrupt routine is called. If the ends
of these tables have been reached then the pointers associated with these tables are reset. The
flag used to time the operations of the main control loop (update_flag) is incremented.
Next the two DAC output signals are filtered using second order delta filters, this is a general
second order filter and by choosing the coefficients different filter functions/structures can be
implemented. For this particular implementation a tuned filter structure is used, tuned to the
blade passing frequency with 3% damping. The filter equations and diagram are contained in
Appendix IX. Finally the filtered control signal stored in the DAC buffer is outputted to the
DACs using the write_dac function.
VIII.3 INPUT/OUTPUT ROUTINES

/*-------------------------------*/
/*
* File name inthybft.c
*/
/*
* v1.0 John Pearson (Loughborough University) 28 July 1992
*/
/*
*/
/*-------------------------------*/

/* NOTES:
* Used by the Hybrid controller algorithms contains a second order delta filter tuned to
* 20Hz, 2% damping filters DAC signals
* This is the service routine called by the interrupt
*/

extern int buffer_full_flag; /* flag is incremented by service routine */
extern int update_flag; /* set when sample buffer is full */
extern int transient_flag; /* set when transients have decayed */
extern float **p_c_gain; /* pointer to control gain matrix pointer */
extern int corr_p;
extern int BUFFER_LENGTH;
extern float sample_buffer[3000]; /* pointer for control vector */
extern float output_buffer[1500]; /* set up sample buffer */
extern float *p_adc_buffer_0; /* set up output buffer */
extern float *p_adc_buffer_1; /* pointers to buffers associated */
extern float *p_adc_buffer_2; /* with each of the 4 ADC channels */
extern float *p_adc_buffer_3;
extern float *p_dac_buffer_0; /* extern float p_dac_buffer_1; */
extern float *p_dac_buffer_2;
extern float *p_dac_buffer_3;
extern float *p_sin, *p_cos; /* pointers to correlation tables */
extern float sint[50]; /* sine table 20Hz sampled at 1000Hz */
extern float cost[50]; /* cosine table 20Hz sampled at 1000Hz */
extern float *p_y_new; /* pointer to correlation vector */
extern float y_new[8]; /* correlation vector */
extern float filter_wl[2]; /* delta filter variables and */
extern float filter_xl[2]; /* coefficients, filter tuned to 20Hz */
extern float c0, c1, c2, r1, r2;

void service()
{
    float adc_buffer[NUM_ADC]; /* input buffer for ADCs */
    float dac_buffer[NUM_DAC]; /* output buffer for DACs */
    register float *p_dac_buffer = dac_buffer;
    register float *p_adc_buffer = adc_buffer;
    register float *p_gain_1 = *p_c_gain; /* set pointer to controller gain */
    register float *p_gain_2 = *(p_c_gain + 1);
    register float *p_filter_v1 = filter_v1;
    register float *p_filter_w1 = filter_w1;
    register float *p_filter_x1 = filter_x1;
    int status,i;

    /* sample all ADC's */
    status = sample_adc(adc_buffer);
}
/* multiply ADC's by gain matrix */
*p_dac_buffer = *p_gain_1 + + *p_adc_buffer + +;
*p_dac_buffer + = *p_gain_1 + + *p_adc_buffer + +;
*p_dac_buffer + = *p_gain_1 + + *p_adc_buffer + +;
*p_dac_buffer + + = *p_gain_1 * p_adc_buffer;
p_adc_buffer = adc_buffer;
*p_dac_buffer = *p_gain_2 + + *p_adc_buffer + +;
*p_dac_buffer + = *p_gain_2 + + *p_adc_buffer + +;
*p_dac_buffer + = *p_gain_2 + + *p_adc_buffer + +;
*p_dac_buffer + = *p_gain_2 * p_adc_buffer;

/* check for buffer full */
if (p_adc_buffer_3 > (float *)(sample_buffer + (int) 4 * corr_p) -1)) {
  buffer_full_flag = 1;
}

/* read sample into buffer if buffer flag is not set and transients have decayed */
if ((buffer_full_flag = 0) && (transient_flag = 1)) {
  /* read ADC samples into sample buffer */
  /* and correlate with sine and cosine signals */
  p_y_new = y_new;
  p_adc_buffer = adc_buffer;
  *p_y_new + + = *p_sin * p_adc_buffer;
  *p_y_new + + = *p_cos * p_adc_buffer;
  *p_adc_buffer_0 + = *p_adc_buffer + +;
  *p_y_new + + = *p_sin * p_adc_buffer;
  *p_y_new + + = *p_cos * p_adc_buffer;
  *p_adc_buffer_1 + = *p_adc_buffer + +;
  *p_y_new + + = *p_sin * p_adc_buffer;
  *p_y_new + + = *p_cos * p_adc_buffer;
  *p_adc_buffer_2 + = *p_adc_buffer + +;
  *p_y_new + + = *p_sin * p_adc_buffer;
  *p_y_new + + = *p_cos * p_adc_buffer;
  *p_adc_buffer_3 + = *p_adc_buffer;

  /* read DAC samples into output buffer */
p_dac_buffer = dac_buffer;
  *p_dac_buffer_0 + = *p_dac_buffer + +;
  *p_dac_buffer_1 + = *p_dac_buffer;
}

/* increment table pointers */
*p_sin + +;
*p_cos + +;

/* check for end of correlation tables */
if (p_sin > (float *)(sint + TABLE -1)) {
  p_sin = sint;
  p_cos = cost;
}
update_flag ++;

352
/ * filter DAC signal using a second order delta filter */
 p_dac_buffer = dac_buffer;
p_filter_v1 = filter_v1;
p_filter_w1 = filter_w1;
p_filter_x1 = filter_x1;

*p_filter_v1 = *p_dac_buffer - r1 * *p_filter_w1 - r2 * *p_filter_x1;
*p_dac_buffer += c0 * *p_filter_v1 + c1 * *p_filter_w1 + c2 * *p_filter_x1;

/* delta calculations */
*p_filter_x1 += *p_filter_w1;
*p_filter_w1 += *p_filter_v1 + +;
*p_filter_v1 = *p_dac_buffer - r1 * *p_filter_w1 - r2 * *p_filter_x1;
*p_dac_buffer = c0 * *p_filter_v1 + c1 * *p_filter_w1 + c2 * *p_filter_x1;

/* delta calculations */
*p_filter_x1 += *p_filter_w1;
*p_filter_w1 += *p_filter_v1;

/* write out DAC's */
write_dac(dac_buffer);
}
typedef short unsigned int boolean;

char *CONTROL, *STATUS; /* 8 bit io registers */
short *TIMER, *ADC; /* 16 bit io registers */
short *DAC_0, *DAC_1; /* 16 bit io reg */

/* flags for ADC control */
char flags[4] = {1 | TIMER_ON, 2 | TIMER_ON, 3 | TRACK | TIMER_ON, 0 | TIMER_ON | TIMEOUT_INT};

/* Function to init the ADC addresses */
int init_adc(sample_rate)
int sample_rate;
{
    short dummy;
    /* setup addresses */
    /* PC/CH4 control register (write) */
CONTROL = (char *) (CONTROL_ADDRESS);
    /* PC/CH4 status register (read) */
STATUS = (char *) (STATUS_ADDRESS);
    /* PC/CH4 sample rate timer */
TIMER = (short *) (TIMER_ADDRESS);
    /* PC/CH4 ADC register (read) */
ADC = (short *) (ADC_ADDRESS);

    /* calibrate ADC */
    *CONTROL = 0x00; /* clear the calibration bit */
    *CONTROL = CALIBRATE; /* set the calibration bit on */
    *CONTROL = 0x00; /* clear the calibration bit */
    printf("Calibrate..");
    while (!(STATUS & CALIBRATE_FLAG)); /* wait for end of calibration */
    printf("Done!");
    dummy = *ADC; /* read the ADC to clear status flags */
}
```c
*CONTROL = TRACK | flags[3];
*TIMER = sample_rate;                /* set sample rate  */
    return(0);
}

/*
   Function to init the DAC addresses
*/
int init_dac()
{
    /* set DAC addresses
        /* PC/CH4 DAC channel 0 (write) */
       DAC_0 = (short *)(DAC_0_ADDRESS);
        /* PC/CH4 DAC channel 1 (write) */
       DAC_1 = (short *)(DAC_1_ADDRESS);
    return(0);
}

/*
   Function to write from dac_buffer to all DAC's
*/
void write_dac(dac_buffer)
float dac_buffer[NUM_DAC];
{
    register float *p_dac_buffer = dac_buffer;
    *DAC_0 = (short)*p_dac_buffer + +;
    *DAC_1 = (short)*p_dac_buffer;
}

/*
   Function to read all ADC's into a buffer
*/
int sample_adc(adc_buffer)
float adc_buffer[NUM_ADC];
{
    register int chan;
    register boolean first;
    register int status = 0;
    register float *adcp;
    register char *p_flags;
    adcp = adc_buffer;               /* set pointer to ADC buffer */
    p_flags = flags;                 /* set pointer to control chars */
    for (chan = 0; chan < NUM_ADC; chan + +) {
        while (!(STATUS & EOC_FLAG)) ;
            *adcp + + = (float)*ADC;                /* input new sample from chan */
            *CONTROL = *p_flags + +;                /* output control for next ADC */
    }
    return(status);
}
```
/* File name: 'PC_CH4.H'. */

/* Include file with definitions for the PC/CH4 analogue card. */

/* NOTE addresses are for use with the VECTOR 32C cards. */

/* Control register bits. */
#define EOC_INT_ON 0x80 /* enable EOC interrupt */
#define TRACK 0x40 /* track after conversion */
#define CALIBRATE 0x20 /* initiate calibration */
#define TIMER_ON 0x10 /* enable the on board TIMER */
#define BIT_OUT 0x08 /* output bit on connector */
#define TIMEOUT_INT 0x04 /* enable TIMER interrupt */
#define CHANNEL_0 0x00 /* channel 0 */
#define CHANNEL_1 0x01 /* channel 1 */
#define CHANNEL_2 0x02 /* channel 2 */
#define CHANNEL_3 0x03 /* channel 3 */

/* Status register bits. */
#define EOC_FLAG 0x80 /* '1' at end of conversion */
#define HOLD_FLAG 0x40 /* '0' while inputs are held */
#define CALIBRATE_FLAG 0x20 /* '0' during calibration */
#define BUSY_FLAG 0x10 /* '0' during conversion */
#define BIT_IN 0x08 /* connector input bit */
#define TIMEOUT_FLAG 0x04 /* set '1' by sample trigger */
#define CHANNEL_NUMBER 0x03 /* to read channel number */

/* Timer values for some sample rates. The value loaded to the timer for a given sample */
/* is: TIMER = - [ ( Fclock / Fsample ) - 1 ] */
/* Where the standard clock frequency (Fclock) is 8,000,000. NOTE that the timer values */
/* below refer to the quad sample and hold; since four channels are held simultaneously, */
/* up to four channels may be converted for each sample trigger */
#define SR_200_Hz 0x063c1 /* 200 Hz */
#define SR_1_kHz 0x0e0c1 /* 1 kHz */
#define SR_5_kHz 0x091c1 /* 5 kHz */
#define SR_8_kHz 0x0fc1 /* 8 kHz */
#define SR_10_kHz 0x0fe1 /* 10 kHz */
#define SR_15_kHz 0x0fdec /* 15 kHz (approximate) */
#define SR_20_kHz 0x0fe71 /* 20 kHz */
#define SR_32_kHz 0x0ff07 /* 32 kHz */
#define SR_44_kHz 0x0ff4c /* 44.1 kHz (approximate) */
#define SR_48_kHz 0x0ff5b /* 48 kHz (approximate) */
#define SR_50_kHz 0x0ff61 /* 50 kHz */
#define SR_100_kHz 0x0ffb1 /* 100 kHz */

/* Register addresses */
#define DSPlink 0x820000 /* DSPlink base address */
#define CONTROL_ADDRESS DSPlink /* control register */
#define STATUS_ADDRESS DSPlink /* Status register */
#define TIMER_ADDRESS DSPlink + 4 /* timer/counter */
#define ADC_ADDRESS DSPlink + 8 /* ADC sample value */
#define DAC_0_ADDRESS DSPlink + 8 /* DAC0 output value */
#define DAC_1_ADDRESS DSPlink + 12 /* DAC1 output value */
VIII.4 MAIN ROUTINE

/* File name hybadapf.c */
/* v1.0 John Pearson (Loughborough University) 4 July 1992 */

/* NOTES: */
/* Adaptive frequency/time domain controller */
/* 1) an interrupt service routine reads from the 4 ADCs and writes to the 2 DACs */
/* 2) interrupt service routine is initialised this routine is called every (1/samplerate) secs */
/* 3) the estimates for T and B are loaded from files on disk test.dat - T, and best.dat - B */
/* 4) the interrupt routine multiplies the ADCs samples by a gain matrix and outputs the */
/* results to the DACs */
/* 5) the steady state vibration is measured by correlating the ADCs samples with a */
/* 20 Hz signal and the result is stored as a vector */
/* 6) a control law is used to determine a feedback gain matrix which will generate */
/* the optimal control required to minimise the steady state vibrationation Y */
/* 7) a RLS estimator is used to update the estimate for the transfer function T */
/* 8) the resulting RMS vibration levels, covariance matrix trace, forgetting factor */
/* and estimator residue are written to files on disk result.dat */

#include < xio.c.asm > /* assembler macros */
#include < xstdio.h > /* std io lib header */
#include <pc_ch4.h > /* definitions for card */
/*#include <libap.h> */
/*#include <math.h> */
#include <xgraph.h > /* standard AT&T library */
/*#include < xgraph.h > */
/*#include < xgraph.h > */
#include "sampley.h" /* graphics library */
#include "sampleh.c" /* function definitions */
#include "conh_opt.c" /* ADC,DAC routines */
#include "inthybft.c" /* optimal control calcs. */
#include "psd_inv.c" /* interrupt service routine */
#include "svd2.c" /* Pseudo inverse routine */
#include "load_set.c" /* Singular Value Decomp. */
#include "loaddat2.c" /* Load controller set up */
#include "loaddat2.c" /* Load initial data routine */
#include "estim.c" /* RLS estimator routine */

int update_flag; /* flag is incremented by service routine */
int buffer_full_flag; /* set when sample buffer is full */
int transient_flag; /* set when transients have decayed */
int data_valid; /* flag set when correlation data is valid */
int corr_p; /* value for correlation period */
int decay; /* value for transient delay period */
int update; /* value for controller update period */
int BUFFER_LENGTH = 3000; /* max buffer size */

float a_gain_1[4] = {0.0,0.0,0.0,0.0}; /* A gain matrix */
float a_gain_2[4] = {0.0,0.0,0.0,0.0};
float b_gain_1[4] = {0.0,0.0,0.0,0.0}; /* B gain matrix */
float b_gain_2[4] = {0.0,0.0,0.0,0.0};
float *c_gain2 = {a_gain_1,a_gain_2}; /* set controller to A gain matrix */
float *e_gain2 = {b_gain_1,b_gain_2}; /* set estimator to B gain matrix */


```c
float **p_c_gain = c_gain;
float **p_e_gain = e_gain;
float sample_buffer[3000]; /* set up sample buffer */
float output_buffer[1500]; /* output buffer for DACs */
float y_new[8] = {0., 0., 0., 0., 0., 0., 0., 0.}; /* pointers to buffers associated */
float p_adc_buffer_0;
float p_adc_buffer_1;
float p_adc_buffer_2;
float p_adc_buffer_3;
float p_dac_buffer_0;
float p_dac_buffer_1;
float p_dac_buffer_2;
float p_dac_buffer_3;
float p_sin, p_cos; /* pointers to tables used for correlation */
float **p_v_new; /* pointer to vibration vector */
float filter_w1[2]; /* pointers to buffers associated */
float filter_x1[2]; /* pointers to buffers associated */
float filter_v1[2]; /* pointers to buffers associated */
```

---

```c
float sint[50] = {0.000000000000000E+00, 1.2533233564304E-01,
2.4689887164855E-01, 3.68124552684678E-01, 4.81753674101715E-01,
5.97785225292473E-01, 6.84547105928689E-01, 7.70513242775789E-01,
8.4432795502015E-01, 9.0482705246602E-01, 9.51056516295145E-01,
9.82287257028689E-01, 9.98026728428272E-01, 9.98026728428272E-01,
9.82287257028689E-01, 9.51056516295145E-01, 9.0482705246602E-01,
8.4432795502015E-01, 7.70513242775789E-01, 6.84547105928689E-01,
5.97785225292473E-01, 4.81753674101715E-01, 3.68124552684678E-01,
2.4689887164855E-01, 1.2533233564304E-01, 1.2246035382238E-01,
-1.2533233564304E-01, -2.4689887164855E-01, -3.68124552684678E-01,
-4.81753674101715E-01, -5.97785225292473E-01, -6.84547105928689E-01,
-7.70513242775789E-01, -8.4432795502015E-01, -9.0482705246602E-01,
-9.51056516295145E-01, -9.82287257028689E-01, -9.98026728428272E-01,
-9.98026728428272E-01, -9.82287257028689E-01, -9.51056516295145E-01,
-9.0482705246602E-01, -8.4432795502015E-01, -7.70513242775789E-01,
-6.84547105928689E-01, -5.97785225292473E-01, -4.81753674101715E-01,
-3.68124552684678E-01, -2.4689887164855E-01, -1.2533233564304E-01};

float cost[50] = {1.000000000000000E+00, 9.92114701314478E-01,
9.68583161128631E-01, 9.29776485888251E-01, 8.76306680043864E-01,
8.09016994374948E-01, 7.28968627421412E-01, 6.37423987486890E-01,
5.35826794978997E-01, 4.25779291565073E-01, 3.09016994374948E-01,
1.87381314585725E-01, 1.62790519529313E-02, -6.72905195293132E-02,
-1.87381314585725E-01, -3.09016994374948E-01, -4.25779291565073E-01,
-5.35826794978997E-01, -6.37423987486890E-01, -7.28968627421411E-01,
-8.09016994374948E-01, -8.76306680043864E-01, -9.29776485888251E-01,
-9.68583161128631E-01, -9.29776485888251E-01, -1.000000000000000E+00,
-9.92114701314478E-01, -9.68583161128631E-01, -9.29776485888251E-01,
-8.76306680043864E-01, -8.09016994374948E-01, -7.28968627421412E-01,
-6.37423987486890E-01, -5.35826794978997E-01, -4.25779291565072E-01,
-3.09016994374948E-01, -1.87381314585725E-01, -6.27905195293141E-02,
6.27905195293128E-02, -1.87381314585725E-01, 3.09016994374947E-01,
4.25779291565073E-01, 5.35826794978997E-01, 6.37423987486890E-01,
7.28968627421411E-01, 8.09016994374947E-01, 8.76306680043864E-01,
9.29776485888252E-01, 9.68583161128631E-01, 9.92114701314478E-01};

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```
```c
float c0, c1, c2, r1, r2;

main()
{
    int status, loadstatus, sample_rate;
    int dummy, block, determ;
    int i, j, k, y;
    char result_file[25];
    int control_cycle;
    int gain_flag = 0;
    float c_period;
    float Yv_new[8];
    float rate;
    float s_wt[8], a_wt[4], da_wt[4];
    float p_cov[4][4];
    float sumy = 0.;
    float sump = 0.;
    float lam = 0.;
    float err = 0.;
    float sigma;
    register float **temp_gain;
    float b_vector[8], t_mat[8][4];
    float old_gain[2][4], new_gain[2][4];
    float y_old[8];
    FILE *result_stream = (FILE *)NULL;
    float ft_vec[5];
    status = load_set(&decay, &update, &corr_p, result_file);

    /* initialise working arrays/matrices */
    for (i = 0; i < 2; i + +)
    {
        for (j = 0; j < 4; j + +)
        {
            old_gain[i][j] = 0.;
            new_gain[i][j] = 0.;
        }
        filter_w1[i] = 0.;
        filter_x1[i] = 0.;
        filter_v1[i] = 0.;
    }
    for (i = 0; i < 8; i + +)
    {
        y_old[i] = 0.;
    }

    update_flag = 0; /* sample count flag */
    control_cycle = 1; /* controller cycle flag */
    buffer_full_flag = 0; /* sample buffer full flag */
    transient_flag = 0; /* transient decay flag */
    data_valid = 0; /* data ready flag */
    determ = 0;
    c_period = (2.0/((float) corr_pl)); /* correlation period */
    rate = SAMPLE_RATE; /* define sample rate */
    sample_rate = (int)(0xffff - (int)(( 8000000 / rate ) - 1));
}
```
/* file to read in controller initial parameters for */
/* T matrix, B vector, Sensor weightings, Actuator weightings */
/* covariance matrix and estimator sensitivity */
loadstatus = loaddat2(lb_vector, p_cov, &sigma, s_wt, a_wt, t_mat, da_wt, ft_vec);
printf("incov %.3e sigma %.3e", p_cov[0][0], sigma);

/* define filter coefficients */
c0 = ft_vec[0];
c1 = ft_vec[1];
c2 = ft_vec[2];
r1 = ft_vec[3];
r2 = ft_vec[4];

/* initialise files for results */
if ((result_streamhf = fopen(result_file, "w")) == NULL) return(-1);

/* set pointers for sample buffer, 4 blocks one for each ADC */
block = (int) corr_p;
p_adc_buffer_0 = sample_buffer;
p_adc_buffer_1 = p_adc_buffer_0 + block;
p_adc_buffer_2 = p_adc_buffer_1 + block;
p_adc_buffer_3 = p_adc_buffer_2 + block;

/* set pointers for output buffer, 2 blocks, one for each DAC */
p_dac_buffer_0 = output_buffer;
p_dac_buffer_1 = p_dac_buffer_0 + block;

/* set pointers for sine and cosine lookup tables */
p_sin = sint;
p_cos = cost;
p_v_new = v_new;

status = init_adc(sample_rate); /* set up ADCs */
init_dac(); /* set up DACs */
set_int(); /* enable interrupts */
do{
    /* if transients have decayed set flag */
    if (update_flag >= decavl{
        transient_flag = 1;
    }

    /* if buffer is full, correlate and calculate vibration vector */
    /* vector is scaled - ADC size 12 bits - 32767 */
    if ((buffer_full_flag) && (data_valid == 0)) {
        p_v_new = v_new;
        for (i = 0; i < 8; i++) {
            Yv_new[i] = v_new[i] * c_period / SCALE_IN;
            v_new[i] = 0;
        }
        data_valid = 1;
    }
} while (!transient_flag);
if (control_cycle == 1) {
    for (i = 0; i < 8; i++) {
        y_old[i] = Yv_new[i];
    }
}

/* estimator calculations using a RLS estimator with a variable forgetting factor */
estim(y_old, Yv_new, old_gain, new_gain, t_mat, p_cov, &sigma, &lam, &err);

/* set old gain = new gain and y_old = y_new */
for (i = 0; i < 2; i++) {
    for (j = 0; j < 4; j++) {
        old_gain[i][j] = new_gain[i][j];
    }
}
for (j = 0; j < 8; j++) {
    y_old[j] = Yv_new[j];
}

/* controller calculations - update feedback gain matrix */
conh_opt(Yv_new, old_gain, t_mat, new_gain, s_wt, a_wt, da_wt, determ, p_cov);

if (update_flag >= update) {
    /* copy new gain matrix to old gain matrix */
    if (gain_flag == 1) {
        for (i = 0; i < 4; i++) {
            a_gain_1[i] = new_gain[0][i];
            a_gain_2[i] = new_gain[1][i];
        }
    } else {
        for (i = 0; i < 4; i++) {
            b_gain_1[i] = new_gain[0][i];
            b_gain_2[i] = new_gain[1][i];
        }
    }
}

/* swap matrix pointers, use new feedback gain matrix */
temp_gain = p_c_gain;
p_c_gain = p_e_gain;
p_e_gain = temp_gain;
gain_flag = !gain_flag; /* toggle gain matrix flag */

/* reset pointers for sample buffer, 4 blocks one for each ADC */
p_adc_buffer_0 = sample_buffer;
p_adc_buffer_1 = p_adc_buffer_0 + block;
p_adc_buffer_2 = p_adc_buffer_1 + block;
p_adc_buffer_3 = p_adc_buffer_2 + block;

/* reset pointers for output buffer, two DACs */
p_dac_buffer_0 = output_buffer;
p_dac_buffer_1 = p_dac_buffer_0 + block;
/ * reset buffer full flag and get more data */ buffer_full_flag = 0; /* * reset controller update flag, initialises transient decay */ update_flag = 0; control_cycle++; transient_flag = 0; data_valid = 0; /* * calculate and store results - write to file */ sumy = 0.; for (j = 0; j < 8; j++) { sumy += sqrt(Yv_new[j] * Yv_new[j]); } sumy = sumy/8; sump = 0.; for (i = 0; i < 4; i++) { sump += p_cov[i][i]; } /* write results - RMS vibration - to file */ fprintf(result_streamhf, "%f %f %f %f\n", sumy, sump, lam, err); } while (1); fclose(result_streamhf);
VIII.5 CONTROL GAIN CALCULATION ROUTINE

/*-----------------------------------------------*/
/*                                             */
/*     File name conh_opt.c                     */
/*                                             */
/* v1.0  John Pearson  (Loughborough University) 10 June 1992 */
/*                                             */
/*                                             */
/*-----------------------------------------------*/
/* Function to calculate the hybrid gain matrix */
/* based on the optimisation at one frequency of the performance index */
/* J = Y'S Y + U'AU */
/* with a frequency domain representation of the structure: */
/* Y = T U + B or dY = T dU */
/* where Y is the vector of Fourier coefficients of new vibration and */
/* dY is the incremental change in Y from one control cycle to the next */
/* U is the vector of Fourier coefficients for the actuator control */
/* forces. */
/* dU is the incremental change in U from one control cycle to the next */
/* T is the transfer function between the sensors and the actuators */
/* B is the background or uncontrolled vibration vector */
/* S is the matrix of sensor weightings */
/* A is the matrix of actuator weightings */
/* Giving the following optimal control law: */
/* Un + 1 = Un - (inv(T'S T + A)) * (T'S Yn + A * Un) */
/* A hybrid gain matrix is constructed such that */
/* u(t) = H y(t), */
/* a local linear formulation is used since this is an adaptive controller, */
/* and H is calculated using a singular value decomposition routine to perform */
/* the pseudo inverse: */
/* H = [Un + 1 Un + 1] (inv([Yn + 1 Yn])) */
/* File requires vibration level, gain matrix and an estimate of the transfer */
/* function between sensors and actuators from the previous controller cycle, */
/* and also the sensor and actuator weighting matrices. */
/*-----------------------------------------------*/
/* Program format is as follows: */
/* 1) Working matrices and arrays are initialised to zeros */
/* 2) Calculate control action from previous cycle */
/* 3) Calculate T'S T + A diagonal elements */
/* 4) Calculate T'S T + A off diagonal elements making use of symmetry */
/* 5) Calculate inverse of (T'S T + A) - uses pseudo inverse (SVD) */
/* 6) Calculate T'S Yn + A * Un */
/* 7) Calculate new optimal control vector Un + 1 */
/* 8) Check that the actuator limit is not exceeded, if it is reset control */
/* 9) Estimate reduced vibration levels Yn + 1 = T dU + Yn */
/* 10) Calculate incremental change in gain matrix */
/* 11) Calculate new gain matrix */

conh_opt(y_new,old_gain,t_est,new_gain,s_wt,a_wt,da_wt,index,p_cov)

float y_new[8], t_est[8][4];
float old_gain[2][4], new_gain[2][4];
float s_wt[8], a_wt[4], da_wt[4];
float p_cov[4][4];
int index;


```c
{ double itwt[16], twt[16], invy_mat[16], y_mat[16];
  float twy[4], con_old[4], con_new[4];
  float del_v[8];
  float del_con[4];
  float change_h[8], del_gain[8], y_est[8];
  float con_mag[2], con_mat[8];
  float stoch[4];
  float sum_s_wt = 0.; /* sum of diagonal sensor weights */
  float con_lim = 5.0; /* software control limit for */
                     /* actuators - 2.5v (12 bits) */
                     /* 32767, scaled internally */

  int i,j,k,colnt,rowd,cold;

  colnt = 4;
  rowd = 4;
  cold = 4;
  /* initialise working arrays to zeros */
  for (i = 0;i < 16;i + +) {
    twt[i] = 0.;
    itwt[i] = 0.;
    invy_mat[i] = 0.;
  }
  for (i = 0;i < 8;i + +) {
    del_v[i] = 0.;
    change_h[i] = 0.;
    del_gain[i] = 0.;
    sum_s_wt += s_wt[i];
  }
  for (i = 0;i < 4;i + +) {
    twy[i] = 0.;
    con_old[i] = 0.;
    con_new[i] = 0.;
    del_con[i] = 0.;
  }

  /* calculate control action from last cycle Un = Hn * Yn, control vector */
  /* is organised as real,imaginary,real etc */
  for (i = 0;i < 2;i + +) {
    for (j = 0;j < 4;j + +) {
      con_old[2*i] += old_gain[i][j] * y_new[2*j];
      con_old[2*i+1] += old_gain[i][j] * y_new[2*j+1];
    }
  }

  /* calculate stochastic term I * p_cov * sum diag S */
  for (i = 0;i < 4;i + +) {
    stoch[i] = ((float) index) * p_cov[i][i] * sum_s_wt;
  }

  /* calculate T'W T + A, making use of symmetry in calculations */
  /* diagonal elements first */
  for (i = 0;i < 4;i + +) {
```

for (j = 0; j < 8; j++) {
    twt[i + i * colnt] += ((double) (s_wt[j] * t_est[j][i] * t_est[j][i]));
}
    twt[i + i * colnt] += ((double) (a_wt[i] + da_wt[i] + stoch[i]));
*
/* off diagonal elements - note T'W T + A is symmetric */
for (i = 0; i < 4; i++) {
    for (j = i + 1; j < 4; j++) {
        for (k = 0; k < 8; k++) {
            twt[j + i * colnt] += ((double) (t_est[k][i] * s_wt[k] * t_est[k][j]));
            twt[i + j * colnt] = twt[j + i * colnt];
        }
    }
}
/* calculate inverse of (T'W T + A) */
psd_inv(twt, itwt, rowd, cold);
/* calculate T'W Y + A Un */
for (i = 0; i < 4; i++) {
    for (j = 0; j < 8; j++) {
        twy[i] += t_est[j][i] * s_wt[j] * y_new[j];
    }
    for (i = 0; i < 4; i++) {
        twy[i] = a_wt[i] * con_old[i];
    }
/* calculate new frequency domain optimal control vector */
for (i = 0; i < 4; i++) {
    for (j = 0; j < 4; j++) {
        con_new[i] += -((float) (itwt[j + i * colnt]) * twy[j]);
    }
    for (i = 0; i < 4; i++) {
        con_new[i] = con_old[i];
        del_con[i] = con_new[i] - con_old[i];
    }
/* check that calculated control is not greater than control limit */
for (i = 0; i < 2; i++) {
    con_mag[i] = sqrt(((con_new[2*i]*con_new[2*i]) + (con_new[2*i+1] * con_new[2*i+1])));
    if (con_mag[i] > con_lim) {
        con_new[2*i] = ((con_lim/con_mag[i])*con_new[2*i]);
        con_new[2*i+1] = ((con_lim/con_mag[i])*con_new[2*i+1]);
    }
}
/* estimate reduced vibration levels due to new control action */
for (j = 0; j < 8; j++) {
    for (i = 0; i < 4; i++) {


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\[ \text{del}_Y[i] = t_{est}[i] \cdot \text{del}_C[i]; \]
\[ y_{est}[i] = \text{del}_Y[i] + y_{new}[i]; \]

/* calculate hybrid gain matrix based on local linear formulation */

\[
\begin{align*}
\text{y_mat}[i + \text{colnt}] &= (\text{double}) (y_{est}[2 \cdot i]); \\
\text{y_mat}[i + \text{colnt} + 1] &= (\text{double}) (y_{est}[2 \cdot i + 1]); \\
\text{y_mat}[i + \text{colnt} + 2] &= (\text{double}) (y_{new}[2 \cdot i]); \\
\text{y_mat}[i + \text{colnt} + 3] &= (\text{double}) (y_{new}[2 \cdot i + 1]); \\
\end{align*}
\]

for (i = 0; i < 2; i + +) {
for (j = 0; j < 4; j + +) {
    change_h[i] + = old_gain[i][j] * del_y[2 \cdot j];
    change_h[i + 2] + = old_gain[i][j] * del_y[2 \cdot j + 1];
}
}

for (i = 0; i < 2; i + +) {
    con_mat[0 + i \cdot \text{colnt}] = \text{del_con}[2 \cdot i] - change_h[i];
    con_mat[1 + i \cdot \text{colnt}] = \text{del_con}[2 \cdot i + 1] - change_h[i + 2];
    con_mat[2 + i \cdot \text{colnt}] = \text{del_con}[2 \cdot i];
    con_mat[3 + i \cdot \text{colnt}] = \text{del_con}[2 \cdot i + 1];
}

psd_inv(y_mat, invy_mat, rowd, cold);

/* incremental change in hybrid gain matrix */

for (i = 0; i < 2; i + +) {
    for (j = 0; j < 4; j + +) {
        for (k = 0; k < 4; k + +) {
            \[ \text{del_gain}[j + i \cdot \text{colnt}] + = \text{con_mat}[k + i \cdot \text{colnt}] \cdot ((\text{float}) \text{invy_mat}[j + k \cdot \text{colnt}]); \]
        }
    }
}

/* new gain matrix equals old gain matrix plus incremental change */

for (i = 0; i < 2; i + +) {
    for (j = 0; j < 4; j + +) {
        new_gain[i][j] = del_gain[i + \text{colnt}] + old_gain[i][j];
    }
}
VIII.6 ESTIMATION ROUTINE

/* File name estim.c */
/* v1.0 John Pearson (Loughborough University) 4 July 1992 */
/* NOTES: */
/* Routine to implement a Recursive Least Squares estimator with a */
/* variable forgetting factor. Past data is weighted with a variable */
/* factor, chosen to maintain constant a scalar measure of the */
/* information content of the estimator. */
/* A U-D covariance factorisation is used, this has been shown to be */
/* computationally efficient and compact. */
/* Local linear formulation for estimator/measurement matrix, i.e. */
/* Program format: */
/* 1) working arrays are initialised */
/* 2) measurement vector is determined, using data from the previous controller cycle */
/* 3) covariance matrix is decomposed into U'D U format */
/* 4) the U D factors are updated and temporary work vectors are set up */
/* 5) covariance matrix is reconstructed */
/* 6) new transfer function estimate is calculated (updated) */
/* 7) estimator error is calculated */
/* 8) variable forgetting factor calculations */
/* 9) final covariance matrix calculations using forgetting factor */

estim(y_old, y_new, gain_old, new_gain, t_est, p_cov, p_sigma, p_lambda, p_est_residue)

float y_new[8], y_old[8], t_est[8][4];
float gain_old[2][4], new_gain[2][4];
float p_cov[4][4];
float *p_sigma, *p_lambda, *p_est_residue;

{ float con_old[4], con_new[4], fvec[4], hvec[4];
  float up[16], uc[16];
  float est_vector[8];
  float dp[4], dc[4];
  float vvec[4], alpha[4];
  float measurement[4], est_gain[4];
  float est_error[8];
  float lam_min, lam_max, sensitivity;
  int i, j, k, l, ind, col, row;
  float lambda, est_residue;
  sensitivity = *p_sigma;

  /* initialise working arrays */
  for (i = 0; i < 16; i++){
    up[i] = 0.0;
    uc[i] = 0.0;
  }
for (i = 0; i < 8; i++) {
    est_vector[i] = 0.0;
}
for (i = 0; i < 4; i++) {
    con_old[i] = 0.0;
    con_new[i] = 0.0;
    fvec[i] = 0.0;
    hvec[i] = 0.0;
}

lam_min = 0.2;
lam_max = 0.98;
row = 4;

/* set up measurement matrix */
for (i = 0; i < 2; i++) {
    for (j = 0; j < 4; j++) {
        con_old[2*i] += gain_old[i][j] * y_old[2*j];
        con_old[2*i+1] += gain_old[i][j] * y_old[2*j+1];
        con_new[2*i] += new_gain[i][j] * y_new[2*i];
        con_new[2*i+1] += new_gain[i][j] * y_new[2*i+1];
    }
}
for (i = 0; i < 4; i++) {
    measurement[i] = con_new[i] - con_old[i];
}

/* decompose covariance matrix into U'D'U format */
/* making use of positive symmetric properties in calculations */
for (i = 3; i > 0; i--) {
    dp[i] = p_cov[i][i];
    for (j = 0; j < i; j++) {
        up[i + j] = p_cov[i][j] / dp[i];
    }
    up[i + i] = 1.0;
    for (k = 0; k < i; k++) {
        for (l = 0; l < i; l++) {
            p_cov[k][l] += p_cov[k][l] - (up[i + k] * dp[i] * up[i + l]);
        }
    }
}
up[0] = 1.0;
dp[0] = p_cov[0][0];

/* calculate the update factorised estimates U'D'U and K*/
/* temporary work vectors are set up */
/* calculate gain and covariance */
for (i = 0; i < 4; i++) {
    for (j = 0; j <= i; j++) {
        fvec[i] += (up[i + j] * measurement[i]);
    }
    vvec[i] = dp[i] * fvec[i];
}
hvec[i] = 0.;
}
hvec[0] = vvec[0]; alpha[0] = sensitivity + (fvec[0] * vvec[0]);
dc[0] = sensitivity * dp[0]/alpha[0]; uc[0] = 1.;

for (i = 1; i < 4; i++) {
    ind = i - 1;
    alpha[i] = alpha[ind] + (fvec[i] * vvec[i]);
dc[i] = dp[i] * alpha[ind]/alpha[i];
    for (j = 0; j < ind; j++) {
        uc[i + j * row] = up[i + j * row] • (fvec[i] • hvec[i]/alpha[ind]);
    }
    uc[i + i * row] = 1.;
    for (j = 0; j < ind; j++) {
        hvec[k] = hvec[k] + (vvec[i] • up[i + k * row]);
    }
}
uc[0] = 1.;
for (i = 0; i < 4; i++) {
    est_gain[i] = hvec[i]/alpha[3];
}

/* reconstruct covariance matrix */
for (i = 0; i < 4; i++) {
    for (j = 0; j < 4; j++) {
        p_cov[i][j] = 0.;
        for (k = 0; k < 4; k++) {
            p_cov[i][j] += (uc[k + i * row] * dc[k] * uc[k + j * row]);
        }
    }
}

/* calculate new transfer function estimate - T */
for (j = 0; j < 8; j++) {
    for (i = 0; i < 4; i++) {
        est_vector[j] = (measurement[i] • t_est[j][i]);
    }
    est_error[j] = est_vector[j] • y_new[j] + y_old[j];
    for (i = 0; i < 4; i++) {
        t_est[j][i] = t_est[j][i] - (est_gain[i] • est_error[j]);
    }
}
est_residue = 0.;
for (i = 0; i < 8; i++) {
    est_residue += (est_error[i] • est_error[i]);
}
est_residue = est_residue/8;

/* variable forgetting factor calculations */
lambda = 0.;
for (i = 0; i < 4; i++) {

\[ \text{lambda} += \text{measurement}[i] \times \text{est_gain}[i]; \]
\[ \text{lambda} = 1.0 - ((1.0 - \text{lambda}) \times \text{est_residue}/\text{sensitivity}); \]

if (\text{lambda} < \text{lam_min}) \text{lambda} = \text{lam_min};
if (\text{lambda} > \text{lam_max}) \text{lambda} = 1.0;

/* covariance matrix */
for (i = 0; i < 4; i++) {
    for (j = 0; j < 4; j++) {
        \text{p_cov}[i][j] = \text{p_cov}[i][j]/\text{lambda};
    }
}
\text{p_lambda} = \text{lambda};
\text{p_est_residue} = \text{est_residue};
### Appendix VIII

#### CONTROL REGISTER

<table>
<thead>
<tr>
<th>Bit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ROC INT ENAB</td>
</tr>
<tr>
<td>1</td>
<td>SN HOLD CONTR</td>
</tr>
<tr>
<td>2</td>
<td>SIN CAL CONTR</td>
</tr>
<tr>
<td>3</td>
<td>COUNTER ENAB</td>
</tr>
<tr>
<td>4</td>
<td>BIT OUT PLA</td>
</tr>
<tr>
<td>5</td>
<td>TIMEOUT ENAB</td>
</tr>
<tr>
<td>6</td>
<td>CHANNEL SELECT</td>
</tr>
<tr>
<td>7</td>
<td>CHANNEL SELECT</td>
</tr>
</tbody>
</table>

#### STATUS REGISTER

<table>
<thead>
<tr>
<th>Bit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ROC PLA</td>
</tr>
<tr>
<td>1</td>
<td>STATUS HOL EN</td>
</tr>
<tr>
<td>2</td>
<td>CAL BUSY</td>
</tr>
<tr>
<td>3</td>
<td>ADC BUSY</td>
</tr>
<tr>
<td>4</td>
<td>BIT IN STATUS</td>
</tr>
<tr>
<td>5</td>
<td>TIMEOUT PLA</td>
</tr>
<tr>
<td>6</td>
<td>SELECTED CHANNEL</td>
</tr>
<tr>
<td>7</td>
<td>SELECTED CHANNEL</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Register</th>
<th>Bits</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Register</td>
<td>7 0</td>
<td>NOT USED</td>
</tr>
<tr>
<td>Status Register</td>
<td>7 0</td>
<td>NOT USED</td>
</tr>
<tr>
<td>Timer Register</td>
<td>8 7 0</td>
<td></td>
</tr>
<tr>
<td>ADC Result</td>
<td>4 5 5 5 5</td>
<td></td>
</tr>
<tr>
<td>DAC Registers</td>
<td>4 NOT USED</td>
<td></td>
</tr>
</tbody>
</table>

S = Sign Bit  
MS BYTE  
LS BYTE

**Figure VIII.1** Analogue Interface Board Registers
Figure VIII.2 ADC Initialisation Routine

START

Set up addresses
Create and Initialise buffers

Calibrate address
Toggle Calibrate bit 5

Wait for end of calibration

Set

Read ADC to clear status flags

Load timer register with sample rate

Clear status bit

END

Control Register (8 bit) (write)
Status Register (8 bit) (read)
Timer Register (16 bit)
ADC Register (16 bit)

Write a 1 to bit 5 of the Control Register

Cleared
Figure VIII.3 Timing for Analog Interface Board - 4 Channel Read

1,2,3) Software: Read ADC
Write Control Register with new channel number and SAMPLE/HOLD CONTROL = 1

4) Software: Read ADC
Write Control Register with new channel number and SAMPLE/HOLD CONTROL = 0
START

Set pointer to ADC buffer

Set pointer to Control chars.

Wait for end of conversion flag

Set

Read ADC - channel data

Output Control for next channel

Increment channel No.

Not last channel

Last channel

END

Figure VIII.4 ADC Sample Flow Diagram
Pointers to Pointers (Switchable)  Pointers  Address Labels (Pointers)  Data Blocks (Gain Matrices)

Figure VIII.5 Software Control Pointers
CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a higher degree.

(Signed)

30/11/94

(Date)