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<table>
<thead>
<tr>
<th>AUTHOR/FILING TITLE</th>
<th>HALLSWORTH, M</th>
</tr>
</thead>
<tbody>
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</tr>
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SOME ASPECTS OF OBJECTIVE TESTING
IN MATHEMATICS FOR THE PURPOSE OF
DIAGNOSIS AND SELECTION

by

MICHAEL HALLSWORTH, B.Sc.

A Master's Dissertation submitted
in partial fulfilment of the
requirements for the award of the
degree of M.Sc. in Mathematics
Education of the Loughborough
University of Technology,
December 1979.

Supervisor: Dr. R.P. Knott M.Sc. Ph.D.

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ABSTRACT

The project investigates the feasibility of using objective tests for diagnostic purposes and of using certain items from these tests in the construction of selection/attainment tests. The possibility of using the information gained from both types of test to form the basis of a mathematical record for a child is also discussed.

The tests are devised to reveal the algorithmic difficulties encountered by children in the four computational skills of addition, subtraction, multiplication and division. To show the various sub-stages required for mastery of a particular skill, each of the four computational skills was divided into its component sub-stages. Ideally each of these sub-stages would have been tested at the appropriate point in the child's school career but due to the shortage of time for the project all the sub-stages for a particular computational skill were tested in a single test. To ensure that the tests were not too long three items were set to represent each sub-stage and five tests were given; these being Addition, Subtraction, Multiplication (Part 1), Multiplication (Part 2) and Division.

A major problem arose when constructing the distractors for the items of the Objective Diagnostic Tests because these had to be constructed from algorithmic considerations and no books could be found which contained these in enough detail.
Thus it was necessary for the writer to construct his own tests which would be analysed to reveal the necessary information.

The investigation into the feasibility of employing certain items used in the diagnostic tests for selection/attainment purposes arose out of practical considerations. If it were possible that the diagnostic tests could generate certain items for the selection/attainment tests then the teacher would be saved much time and effort by not having to write (at least) two sets of items.

Finally the information gained from both types of tests is briefly discussed as the basis for a Mathematical Record of each child.
ACKNOWLEDGEMENTS

I wish to thank Dr. Knott for the valuable comments and suggestions he has made during the writing of the dissertation.

I am also grateful to Mr. Greenhow, the Headmaster of Berry Hill Middle School, Mansfield and Mr. D.K. Bednall, the Head of Mathematics at that school, for allowing me to administer my tests to a considerable number of children within the school.

Special words of thanks are due to my wife, Kath, for her patience and help during the preparation of this submission.
## CONTENTS

<table>
<thead>
<tr>
<th>Chapter 1</th>
<th>A brief consideration of examinations in general and Diagnostic Tests in particular</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A need for tests</td>
</tr>
<tr>
<td>1.2</td>
<td>Factors which contribute to the suitability of tests</td>
</tr>
<tr>
<td>1.3</td>
<td>The features of Diagnostic Tests</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>An Evaluation of Objective Tests</td>
</tr>
<tr>
<td>2.1</td>
<td>The suitability of the use of Objective Tests for the project</td>
</tr>
<tr>
<td>2.2</td>
<td>Types of objective tests</td>
</tr>
<tr>
<td>2.3</td>
<td>The choice of objective tests</td>
</tr>
<tr>
<td>2.4</td>
<td>A consideration of the strengths and weaknesses of multiple-choice objective tests</td>
</tr>
<tr>
<td>2.5</td>
<td>Some considerations of the presentation of the tests</td>
</tr>
<tr>
<td>2.6</td>
<td>Measures available through the use of multiple-choice objective testing</td>
</tr>
<tr>
<td>Chapter 3</td>
<td>A more detailed consideration of Diagnostic Tests</td>
</tr>
<tr>
<td>3.1</td>
<td>Possible uses of diagnostic tests</td>
</tr>
<tr>
<td>3.2</td>
<td>Facets of mathematical ability to be stressed</td>
</tr>
<tr>
<td>3.3</td>
<td>Analysis of the topic into items</td>
</tr>
<tr>
<td>3.4</td>
<td>The need to consider the child's reading ability</td>
</tr>
<tr>
<td>3.5</td>
<td>The time allocated to diagnostic tests</td>
</tr>
<tr>
<td>3.6</td>
<td>The applicability of diagnostic tests to a variety of schools</td>
</tr>
<tr>
<td>3.7</td>
<td>The reliability and validity of diagnostic tests</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>18</td>
</tr>
<tr>
<td>21</td>
</tr>
<tr>
<td>22</td>
</tr>
<tr>
<td>26</td>
</tr>
<tr>
<td>27</td>
</tr>
<tr>
<td>43</td>
</tr>
<tr>
<td>44</td>
</tr>
<tr>
<td>45</td>
</tr>
<tr>
<td>45</td>
</tr>
<tr>
<td>47</td>
</tr>
<tr>
<td>47</td>
</tr>
<tr>
<td>47</td>
</tr>
<tr>
<td>48</td>
</tr>
<tr>
<td>Chapter 4</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>4.1</td>
</tr>
<tr>
<td>4.2</td>
</tr>
<tr>
<td>4.3</td>
</tr>
<tr>
<td>4.4</td>
</tr>
<tr>
<td>4.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 5</th>
<th>The pre-Objective Diagnostic Tests and their analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>The pre-objective diagnostic tests</td>
</tr>
<tr>
<td>5.2</td>
<td>The analysis of the pre-objective diagnostic tests</td>
</tr>
<tr>
<td>5.3</td>
<td>General comments concerning the analysis</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 6</th>
<th>Designing the Objective Diagnostic Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>6.2</td>
<td>The criteria for choosing the distractors</td>
</tr>
<tr>
<td>6.3</td>
<td>A note on long multiplication and long division</td>
</tr>
<tr>
<td>6.4</td>
<td>The Objective Diagnostic Tests</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 7</th>
<th>A trial of the Objective Diagnostic Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>7.2</td>
<td>Instructions for the Objective Diagnostic Tests</td>
</tr>
<tr>
<td>7.3</td>
<td>The Objective Diagnostic Addition Test</td>
</tr>
<tr>
<td>7.4</td>
<td>The Objective Diagnostic Addition Test answer sheet</td>
</tr>
<tr>
<td>7.5</td>
<td>The Objective Diagnostic Addition Test item response chart</td>
</tr>
<tr>
<td>7.6</td>
<td>The four sample answer sheets to be analysed</td>
</tr>
<tr>
<td>7.7</td>
<td>Analysis of the Objective Diagnostic Addition Test</td>
</tr>
<tr>
<td>7.8</td>
<td>The Objective Diagnostic Subtraction Test</td>
</tr>
<tr>
<td>7.9</td>
<td>The Objective Diagnostic Subtraction Test answer sheet</td>
</tr>
<tr>
<td>7.10</td>
<td>The Objective Diagnostic Subtraction Test item response chart</td>
</tr>
</tbody>
</table>
### Chapter 7

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.11</td>
<td>The four sample answer sheets to be analysed</td>
<td>149</td>
</tr>
<tr>
<td>7.12</td>
<td>Analysis of the Objective Diagnostic Subtraction Test</td>
<td>154</td>
</tr>
<tr>
<td>7.13</td>
<td>The Objective Diagnostic Multiplication Test (Part 1)</td>
<td>158</td>
</tr>
<tr>
<td>7.14</td>
<td>The Objective Diagnostic Multiplication Test (Part 1) answer sheet</td>
<td>161</td>
</tr>
<tr>
<td>7.15</td>
<td>The Objective Diagnostic Multiplication Test (Part 1) item response chart</td>
<td></td>
</tr>
<tr>
<td>7.16</td>
<td>The four sample answer sheets to be analysed</td>
<td>162</td>
</tr>
<tr>
<td>7.17</td>
<td>Analysis of the Objective Diagnostic Multiplication Test (Part 1)</td>
<td>163</td>
</tr>
<tr>
<td>7.18</td>
<td>The Objective Diagnostic Multiplication Test (Part 2)</td>
<td>168</td>
</tr>
<tr>
<td>7.19</td>
<td>The Objective Diagnostic Multiplication Test (Part 2) answer sheet</td>
<td>171</td>
</tr>
<tr>
<td>7.20</td>
<td>The Objective Diagnostic Multiplication Test (Part 2) item response chart</td>
<td>176</td>
</tr>
<tr>
<td>7.21</td>
<td>The four sample answer sheets to be analysed</td>
<td>177</td>
</tr>
<tr>
<td>7.22</td>
<td>Analysis of the Objective Diagnostic Multiplication Test (Part 2)</td>
<td>178</td>
</tr>
<tr>
<td>7.23</td>
<td>The Objective Diagnostic Division Test</td>
<td>183</td>
</tr>
<tr>
<td>7.24</td>
<td>The Objective Diagnostic Division Test answer sheet</td>
<td>185</td>
</tr>
<tr>
<td>7.25</td>
<td>The Objective Diagnostic Division Test item response chart</td>
<td>191</td>
</tr>
<tr>
<td>7.26</td>
<td>The four sample answer sheets to be analysed</td>
<td>192</td>
</tr>
<tr>
<td>7.27</td>
<td>Analysis of the Objective Diagnostic Division Test</td>
<td>193</td>
</tr>
<tr>
<td>7.28</td>
<td>Summary and general comment</td>
<td>198</td>
</tr>
</tbody>
</table>

### Chapter 8

**Designing an Objective Selection Test**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>209</td>
</tr>
<tr>
<td>8.2</td>
<td>General construction of selection/attainment tests</td>
<td>210</td>
</tr>
<tr>
<td>8.3</td>
<td>Instructions for the test</td>
<td>217</td>
</tr>
<tr>
<td>8.4</td>
<td>The Arithmetic Test</td>
<td>218</td>
</tr>
<tr>
<td>8.5</td>
<td>The Arithmetic Test answer sheet</td>
<td>224</td>
</tr>
</tbody>
</table>
# Table of Contents

## Chapter 8
- 8.6 Marking the test 225
- 8.7 Characteristics of the Test 226
- 8.8 Analysis of the Selection Test 227
- 8.9 Summary and conclusions 235

## Chapter 9
- 9.1 Introduction 237
- 9.2 The Mathematical Record 238

## Bibliography
- Appendix A The standard algorithms 254
- Appendix B The computer program and analysis 256
CHAPTER 1

A BRIEF CONSIDERATION OF EXAMINATIONS IN GENERAL AND DIAGNOSTIC TESTS IN PARTICULAR

1.1 A need for tests

It is desirable that, at the earliest possible time in a child's life in a new school, the teachers have an idea of the abilities and aptitudes of the pupil. Information from former schools may be severely limited and it is the purpose of this project to devise a series of tests which will enable the diagnosis of a child's mathematical deficiencies and for these same tests to generate further tests which will allow the teacher to rank the children in his class or school, as the case may be. The diagnosis and ranking of children will be particularly useful when the children are transferred to other schools especially, as in the case of my own school, when children are transferred en bloc from the middle to the upper school. The rankings will be of some relevance, although it should not be the only yardstick, when the children are placed in ability groups at the upper school.

Now that the 11+ examination has disappeared there is more need for primary and secondary school teachers to meet and discuss the problems associated with transfer. The secondary teacher needs to consider what information he wishes to have about the children and the means of obtaining this information. For many purposes assessment by subjective judgement is satisfactory but often a more objective and less
parochial measurement is required. This measurement usually takes the form of a test or tests and if the teacher is to use such a method to obtain this information and wishes to know how the children tested compare with other children in the class, school or nation then he needs to use a norm-referenced test. If he wishes to know the deficiencies of a child then a diagnostic test should be used.

However, as Creswell (1977 p.25) states:

"There has recently been a fashion amongst test designers and test users for 'diagnosis' and a distinction has been drawn between standardised and 'diagnostic' tests but, while it is true that most standardised tests are concerned with generalities and that the more specific type of diagnostic test is usually unstandardised, there seems to be no technical reason why this should be so. Diagnostic information can be obtained from general standardised tests and the addition of normative information would make diagnostic tests more flexible."

This is my point of view also and, had there been sufficient time, I would have investigated the feasibility of standardising the diagnostic tests which I will have produced. However, I will use may tests as a source for selecting suitable items (discussed in Chapter 8) which will enable me to construct further tests which will allow the teacher to rank the children.

It must be stressed now that the children can only be ranked according to the raw score they obtain and that the test is not standardised.
1.2 Factors which contribute to the suitability of tests

Before constructing any tests there are a number of different factors which need to be considered which contribute to the suitability of tests and these factors need to be constantly checked during the construction of the test. These factors are:

(a) Is the age-range of the test right?
(b) Are there any topics that the children have not experienced which are in the test?
(c) Is the approach of the test similar to the approach of the teaching which the children have received?

It is most important that the test contents are closely scrutinised before the results can be properly interpreted. Unless the content of the test has some relevance to the children then no useful information is gained.

Having defined the need for a test or tests and having defined the purpose of the project one now needs to consider the type of test which best fits this purpose.

When applying tests of mathematics it must be remembered that, as Scott (1973 p.97) writes:

"Dealing as it does with facts and not judgments, and with questions which are clearly right or wrong, mathematics is, they maintain, admirably suited to objective marking and there is no difficulty in producing a scheme which will produce a one-to-one correspondence between examination scripts and examination marks .... This complacency should be dispelled for the investigation of Hartog and Rhodes revealed a
considerable difference in marks and in ranking when a set of twenty three final mathematics papers was marked by six different experienced markers."

Thus it seems that the marking of mathematics tests is not as simple and straightforward as would first appear and the factors contributing to the variability of the candidates' performances need to be considered. The following factors (Scott, 1973 p.100) may well contribute to variability of a candidate's performance.

1. **Sample of knowledge**

   If we are to set examination papers in which a year's course has to be covered by nine questions, it is inevitable that in any particular paper only a fraction of the possible topics will be represented. Since few candidates will present themselves equally well prepared over the whole of the syllabus, and in fact many will deliberately adopt a selective strategy of preparation, it is inevitable that random variations will be introduced and that a considerable element of luck is an inherent feature of the examination procedure.

2. **Variable difficulty of questions**

   In spite of the rubric "All questions carry equal marks" it does not mean that all questions are of equal difficulty (Hunt, 1975 pp. 28-29). We think that in giving wider choice we are helping the candidates, but we are adding to our difficulties of assessment, since the outcome is that each student could well be taking a different paper. If a student who attempts the five most difficult questions obtains the same score as a student who attempts the five easiest, does this
represent equality of performance? In effect, one of the skills we are testing and rewarding highly is the candidates ability to select, in the short time available, the questions that he can do best. It appears that this skill is not highly developed and on one occasion a candidate may be successful in picking 'winners' while on another occasion he may be equally unlucky.

3. Tension induced in the examinations

According to Scott (1973 p. 100) a study of some examination scripts revealed that sometimes an uncharacteristically low mark was obtained because a student had become entangled through a mistake early in a question and spent a disproportionate time in trying to sort out the error and complete the solution. This, of course, we all recognise as bad examination technique, but the first question tackled corresponds to the work which the student feels he knows best and the psychological effect of failure is therefore greater. Hence, the desperate attempt to complete the solution, in order to re-establish his confidence, followed by the panic situation after the realisation that the remainder of the paper must be completed in about half the time which should have been allowed.

4. Physical and psychological states

It goes without saying that on different occasions a candidates performance may be adversely affected by the state of his physical health and also by his mental attitudes. This factor is certainly significant and could be minimised
by replacing maybe one three hour test by several short tests
taken on different days. Thus the test or tests set to serve
the purpose of the project need to eliminate or at least
minimise these factors.

Sample of knowledge can be minimised by ensuring that the
test deals with only one topic or maybe even part of a topic
and that there are numerous questions covering all aspects of
that topic. Obviously the problem arises concerning the
number of questions to be set in a test but this will be
discussed in Chapter 2. The problem of variability of
questions can be eliminated by allowing the child no choice,
the child in fact has to answer all the questions. The tension
induced in the examination is difficult to eliminate as is the
physical and psychological state of the child but all can
probably be reduced by making the tests more frequent and
shorter, thus the tests might become a more familiar and relaxed
occurrence in the life of a child.

1.3 The features of Diagnostic Tests

Having discussed some features of tests in general we now
need to consider Diagnostic Tests in particular.

According to Bloom (1971 p.87) the main facets of 'diagnostic
evaluation' are:

1. "...... to discover the underlying causes of
deficiencies in student learning as the instruction
unfolds."

2. its use "...... to place the student properly at the
outset of instruction."
These two facets are exactly aligned with the aim of this project. It was hoped to fulfill the first facet by devising a series of tests to be taken at suitable stages in the child's school career, usually during or after a topic is being taught. As this would be very time consuming, the tests needing to be given throughout the child's four years in the middle school, I propose for the purpose of this project to concentrate the tests on the four computational skills of addition, subtraction, division and multiplication and for these tests to be given to one age group only (the 12-13 years old in my school).

The second facet will hopefully be fulfilled in two ways. The first of these is to use the information, derived from the test, concerning the mathematical deficiencies of the child as part of a mathematical record. The second of these is to use items from the diagnostic tests to construct a further test which will allow the teacher to rank the children according to ability. Hopefully, both the mathematical record and the ranking will enable the child to be placed in a suitable ability group, particularly when the child is transferred from one school to another.

Whereas attainment tests attempt to discover what a child does know, diagnostic tests are designed to find out not only what he does not know, or cannot do, but to point out the exact stage in the 'instruction programme' at which he is failing. Once this stage has been pinpointed it can be broken down into constituent steps and the performance of the child
at each of these can be investigated, and this process of analysis can be repeated until it becomes clear exactly which aspect of the topic the child is failing to grasp. When this aspect has been discovered the teacher must attempt to diagnose the reason for the child failing. Only when the causes have been found and treated can remedial teaching hope to be fully successful.

Bloom (1971 p. 90) when discussing formative tests in their diagnostic aspect states that the overall score on a formative test is "not very meaningful" and because of this he classes formative tests as "criterion referenced measures". Thus the scores arising from the tests, to fulfil the first of Bloom's facets, will only be of concern to the individual child and these test results must be analysed on an individual basis to search out the problems that the child may be experiencing. However it is proposed to use items from those tests to devise later tests in which the overall score is meaningful in allowing the teacher to rank the children. Bloom quotes Glaser (1963 p. 519) who postulates the existence of a "continuum of knowledge acquisition ranging from no proficiency at all to perfect performance". An individual's test results place him at some point along this continuum. The criteria for each such point is defined by means of a pupils' required behaviour for proficiency at that point, so that when a person has been tested with regard to the field of knowledge under consideration his performance can be referred to the criteria for each point and the point on
the continuum line can be located. This gives a 'criterion referenced result'.

The interpretation of criterion referenced scores will be self evident because the scores should be given in a form which specifically states the behaviour, skills, content and so on which a pupil can cope with.

One of the main aims of a diagnostic test is to find the point in the learning sequence at which the child is failing. It would seem to follow that unless the child is seen to fail no diagnostic test will be necessary. However, from experience, it is not always possible to 'see' the child fail due to a variety of reasons such as the child 'bluffing' his way through the problems or copying. Thus rather than take the risk of not recognising that a child has problems it is proposed that the tests be quite short and that all the children will participate in them. Even children who the teacher 'feels' have mastered the skills or concepts involved will take the test as the time taken to complete a test will be short and the teacher will be able to say with a great deal of conviction that these children have mastered the objectives of that test. Obviously it is very necessary for the children who are having difficulties to take the test as a first step to pinpointing their deficiencies.

Nehrens and Lehman (1969 p.130) advise a two stage approach to the diagnostic process. The children are initially given a survey battery (of tests) as a preliminary screening device. It will identify the children who require further
investigation together with the topics in which they are suspect. The children are then given a "diagnostic test in the area of specified weakness". This I feel is a good approach for children entering a new school with no mathematical record to guide the teacher on the child's mathematical ability. As previously stated, had time permitted it was hoped that, in this study, the children would have taken the diagnostic tests frequently throughout their middle school careers when the occasion warrants, such as at the end or during the study of a particular topic. Because this is not possible the tests that would have been devised will be shortened, three items being used instead of ten, and these shortened tests combined to give four larger tests, each of the larger tests separately covering the various stages of addition, subtraction, multiplication and division. Hence the survey test as such is not required in this project but it would prove, as previously stated, invaluable in selecting areas of weakness that a new child to a school may have, particularly if that child had no mathematical record.

To fulfil the first of Bloom's facets the diagnostic test will need to cover a narrower band of children's work than an attainment test would, and the coverage of that band will be more thorough.

Katz (1972 p.177) writes:

"...... a placement test may be wide ranging but a diagnostic test is likely to be intensive rather than extensive".

Meihrens and Lehman (1969 p.139) state:

"...... if we recall that diagnostic tests are primarily
designed to assist the teacher in locating the genesis of some deficiency, it is not unexpected that we find the diagnostic test to have a thorough coverage of a limited area."

Even when the exact stage at which a child is failing has been diagnosed it is still not possible immediately to launch into remedial teaching. Such a plan would only likely succeed if it was the original teaching which was at fault. But the reason for the lack of learning may be with the child and not the teacher. Bloom (1971 p.87) postulates that these may be "physical, emotional, cultural or environmental in nature", all of which are outside the scope of this project but which may be interesting to investigate at a later date.
2.1 The suitability of the use of objective tests for the project

In order for tests to be diagnostic they must be very detailed and the questions numerous so that each concept and skill can be tested. Thus, for the project, the tests used need to cover all the concepts and skills involved in computation and, whilst many types of test can give this, the type which is particularly suitable is the Objective Test.

Objective tests can be structured to test (Vernon 1956) all the education objectives mentioned in the cognitive domain of Bloom's Taxonomy (1956) i.e. knowledge, comprehension, application, analysis, synthesis and evaluation. Objective tests also allow a wide coverage of the syllabus, from both a content and ability point of view, because a relatively large number of questions can be set which can be answered in a short time. This aspect implies a high reliability since it reduces the possibility for a candidate to 'spot' questions. Examiners are thus able to set papers with different items testing different objectives.

Thus objective tests would seem to suit the need of being able to give wide syllabus coverage and to test individual objectives; both considerations making them desirable for use in the project. Now it is necessary to discuss objective tests in general to see if they are suitable in other aspects.
2.2 Types of objective tests

There are four main types of objective tests. These are:-

1. Multiple choice

This is the most commonly used objective test and generally regarded as superior to all others.

Bonney Rust (1973 p.62) thinks it probable that:

"...... the multiple choice test, with a minimum of four choices, is the most widely used and educationally respectable form of objective testing."

Ebel (1972 p.87) states that:

"...... multiple choice items are currently the most highly regarded and widely used form of objective test item."

Unlike Bonney Rust, however, Ebel considers that good items can be written with only two or three alternatives.

Gronlund (1965 p.140) also considers the multiple choice item to be the most widely applicable and useful form of objective item, and points out that:

"the use of a number of plausible alternatives makes the results amenable to diagnosis."

Thus besides giving a wide syllabus coverage multiple choice objective testing can be used for diagnosis; both aspects particularly important to this project. Indeed Dunning (1954 pp.121-211) and Lewis (1965 pp.136-139) support Gronlund's point of view in suggesting that multiple choice items are particularly suitable for pinpointing the non-factual and higher level objective.

The standard method for presenting and analysing the
results of multiple choice tests is in the form of tables. One of the most important of these tables shows for each item the number of candidates choosing each response. Besides indicating the proportion of students choosing the correct response (the key) it also reveals the extent to which they are attracted to the false responses (or distractors). The distractors are chosen by the item writer in the light of known weaknesses in students' knowledge, and the amount of attention which should be given to the correction of these misapprehensions, possible with the present class and certainly with future classes, can be judged from this table. No other type of item can give such precise and compact information about the performance of a class on the test, and the chief reasons for failure to select the correct response. Because of this feature, satisfactory selection, induction and progress tests can in general be composed entirely of multiple choice items.

2. Multiple response

Multiple response items, although superficially similar to multiple choice, have more in common with the true/false type. The true/false item has a stem consisting of a factual statement and the candidate has to decide whether it is true or false. Items of this kind are very vulnerable to guessing. True/false items are no better at identifying students' weaknesses than short-answer questions which are less resistant to guessing.
Multiple response resemble multiple choice items but the number of correct responses to an item is not restricted to one; it can be any number from zero to the number of responses presented to the candidate. Scoring presents a difficulty here. If each item carries one mark, a student who indicates, say, all but one of the correct responses is awarded a zero mark, the same as the student who did not indicate a single one of them. On the other hand, if a mark is provided for each correct response, then a four-choice multiple response item is the same as four true/false items, with the disadvantages described above. Ebel (1972 p.170) considers if:

"the statements were presented and scored as independent true/false statements, they would yield more detailed and reliable information concerning the examinee's knowledge than they can do in multiple response form".

Multiple response items therefore seem to be inferior to multiple choice, and the scoring and the analysis of the results are more complicated.

3. Matching

Matching items provide a way of testing a number of pieces of knowledge with a single item. The item consists of two lists of, say, words, dates, etc., as the subject demands and to avoid the possibility of the last match being determined solely by elimination, the lists usually contain different numbers of elements. Each element in the first list has to be matched with one in the second, sometimes an element can be
used more than once. These items can be completely objective and they offer little chance of successful guessing. However a table showing the possible responses would be impractically large and of negligible use except with the very largest classes; for instance, if the first list contained five elements and the second six, such a table would contain thirty columns.

4. Short answer questions

Short answer questions, and especially the "completion" type in which the candidate supplies a number, symbol, word or short phrase, are sometimes called objective although not in general conforming to the definition of mechanical marking; possibly this terminology arises from the property which such questions have in common with truly objective items - brevity and the consequent possibility of wide syllabus coverage in a short time. It is therefore desirable to give some attention to this method of testing at this point.

Some short answer questions are objective but those requiring verbal answers are unlikely to be; synonyms are common in the English language and not unknown in mathematics, so some skilled judgment is called for in marking. Truly objective completion questions have one advantage over multiple choice items, however; the chance of successful guessing is less in the former. It is the ease with which the extent of the popularity of wrong responses can be measured and displayed with multiple choice tests which give them a telling advantage over the short answer types.

2.3 The choice of objective test

From the foregoing discussion it is concluded that
objective tests in general and the multiple choice type in particular will serve this project well, in that they can be used to test educational objectives and cover a wide syllabus. Thus they are particularly suitable for diagnostic purposes, the multiple choice having the added advantage that the responses can be analysed easily. Having selected the multiple choice objective test we need to consider its strengths and weaknesses.

2.4 A consideration of the strengths and weaknesses of multiple choice objective tests

2.4.1 Strengths of objective tests

1. They are fair to all candidates since in each test candidates answer the same questions and therefore the results are directly comparable. By contrast a traditional choice type paper gives a larger number of possible combinations.

2. Candidates need spend little or no time writing and so may devote much of their time to thinking. Coupled with this advantage there is the reduced emphasis on penmanship. Candidates are not required to express their answers in their own words, their performance is determined by their knowledge and understanding.

3. The tests permit specified sections of the syllabus to be covered more fully, so that candidates relying on "question spotting" are unlikely to do so well and maximum reward is given to those candidates who have studied and understood all the aspects of the syllabus covered by the test.

4. The tests are constructed according to an agreed specification which ensures the consistency of tests from year
to year. In this way, the same abilities are tested in the same pre-determined proportions, and proper discrimination between candidates of varying abilities is assured.

5. The items in a multi-choice paper are pre-tested, thus enabling the examiners to be reasonably certain that they are free from ambiguity, and that they are of an appropriate level of difficulty.

6. The marking is completely objective and totally free from subjective judgments of examiners which are an inevitable part of an essay type of examination.

7. Results can be issued more quickly providing that the examination is wholly objective.

8. A detailed specification can also avoid overlap whereas in essay-type tests, no matter how carefully designed, a candidate can utilize the same limited material to answer more than one question.

2.4.2 Weaknesses and objections to objective testing

1. Objective tests cannot cover every quality. For example they cannot test written expression or the ability to develop an argument. If the ability to write clearly and concisely is an expressed aim, then objective testing can only form part of the examination. The candidate's approach to the solution of a problem cannot be determined. Was it a deliberate logical analysis? An inexplicable leap of mind without much prior thought? A mere guess? If it is necessary to understand the thought process to a given solution then inevitably long written answers and rigorous oral examinations
must be used. However, in mathematics I believe that by careful setting of the questions and careful choice of distractors one can deduce the thought process which may have led to the chosen answer.

2. Good multiple choice questions are difficult to set but examination boards attempt to overcome this problem by training their item writers who attend a 'workshop'.

3. A common criticism of objective testing is that the questions are too easy but research shows that the questions can be sufficiently searching and are answered correctly only by the better candidate.

4. Some credit can be gained through the undesirable process of guessing. This is one aspect of objective testing that is often raised in discussion. On a philosophical level Schofield (1973 pp.170-72) regards the encouragement given by teachers to students to guess, as dishonest and disturbing. He thinks that this attitude reduces the situation to a game, instead of it being a serious attempt to obtain a serious assessment. This view of the situation is contradicted by Ebel (1972) who sees the encouragement of guessing as ordinarily not creating any moral or educational evil. He justifies his point of view by the fact that most decisions, both for examinations and life in general, are made not on a basis of complete understanding, nor of complete ignorance. In practice examination boards have found that if a correction factor for guessing is applied no significant effect on the rank order results.

5. No credit is given unless the answer is completely
correct. I feel that this is a most valid criticism particularly in relation to multiple completion and the assertion/reason types of questions. A candidate can be 75% correct but will be marked in exactly the same way as one who answers the question completely incorrectly.

6. Candidates may remember one of the distractors and think that it is the correct answer. This may well be true to a limited extent, but the same criticism can be levelled with greater justification at traditional questions. In a classroom situation, much can be learned from the discussion of wrong answers particularly as a means to dispelling misconceptions.

2.4.3 A summary of the strengths and weaknesses of objective tests

Thus summarising the points mentioned it is without doubt that objective tests are subject to less sources of errors than the traditional test but it must be appreciated that the former test type is incapable of adequately testing certain factors such as originality, the ability to organise and integrate. So while accepting some of the criticism directed at the objective test system I am in agreement with Hunt (1975 p.49) who writes that:

"..... this form of assessment (objective testing) is more likely to give results of greater validity and reliability in mathematics."

It is sometimes suggested that a weighted or differential scoring system be adopted on objective tests; that is the
more important items should be given more than one mark and, or, a score of minus one allotted for an incorrect response. However, many researchers such as Guilford (1942 pp.15-21), Phillips (1943 pp.151-5), Sabers and White (1969 pp.93-6) conclude that there is little to be gained from weighted scores. Any advantages could be gained more easily by adding more items to the test. Also neither test validity nor reliability are improved by weighted scores. Thus in this project weighted scores will not be used and any important aspect of a topic will be emphasised by increasing the number of items concerning that aspect.

2.5 Some considerations of the presentation of the tests

Having discussed the strengths and weaknesses of objective tests and concluded that they are particularly suitable for the purpose of this project the manner in which the tests are to be presented needs to be considered. Recalling the points raised in Chapter 1, concerning the factors affecting the child taking tests, the problem of sample of knowledge and variable difficulty of questions will be eliminated by objective testing because the child will attempt all questions. The problem of tension induced in tests and the child's physical and psychological states will, hopefully, be reduced by replacing, say, a single three hour test by four tests of unlimited time but which will, on average, last approximately forty minutes. Spreading the tests over four sessions should lessen the affect of adverse physical and psychological states of any one child and the children should
become more familiar and relaxed with the notion of tests.

General consideration in constructing the tests, such as
the correct age range and topics covered will be met by
looking at the scheme of work for the school and ascertaining
the stages in the four computational skills to be tested
that are expected to have been experienced by the 12-13 years
old in my school. Very few of the children, if any, will
have experienced objective tests before and thus it is both
desirable and necessary for them to become familiar with
this type of test and this will be done by the use of a few
sample items.

2.6 Measures available through the use of multiple choice
objective testing

Having decided that multiple choice objective tests
best fit the needs of the project it is necessary to discuss
how they may be used.

Objective tests, and especially those comprising
multiple choice items, are usually scored dichotomously; that
is each item is given either one mark or none. Dichotomous
scoring facilitates the calculation of three important
measures which not only allow the test scores to be interpreted
more precisely, but can also be useful when modifying tests
for future use. These measures are:

(1) Facility Value (F.V.)
(2) Index of Discrimination (I.D.)
(3) Reliability Factor (R.F.)
2.6.1 Facility Value (F.V.)

The F.V. of an item is the proportion of candidates who gave the correct response to that item. For instance for F.V. = 40% then 40% of the candidates must have given the correct response whilst 60% failed to do so.

The F.V. is simply defined as

\[ F.V. = \frac{\text{Number of students answering correctly}}{\text{Total Number of students}} \]

This value can either be expressed as a decimal or a percentage and can range from zero (0%) to +1.0 (100%). The easier the items, the higher the F.V., although the parameter does not provide a uniform scale of difficulty.

An approximate F.V. can be found using the top 27% of the sample and the bottom 27% i.e. using 54% of the total sample. Thus if \( N_h \) represents the number in the top 27% and \( N_L \) represents the number in the bottom 27% and there are n candidates in each group, the facility value is given by:

\[ F.V. = \frac{N_h + N_L}{2n} \]

Since most tests are intended to provide valid discrimination amongst students of a wide range of abilities, test items should provide a challenge for the most able and yet permit weaker students to demonstrate their ability. Thus it is not necessary or even desirable to have each item with a facility value of 0.5. Dawson (1972) suggests that items with F.V. between 0.25 and 0.75 are useful. Macintosh and Morrison (1972) state that items which have a F.V. \( \geq 0.60 \)
err on the easy side and items with $F.V. < 0.40$ err on the difficult side. This is more closely in line with Ebel (1972 pp.396-9) who gives data which shows items of middle difficulty should be favoured in the construction of achievement tests. The reason for this conclusion was based on evidence which showed that the wider the dispersion of facility values the more concentrated were the distribution of test scores and also that the reliability of the test seemed to suffer if the items were too easy or too hard.

2.6.2 Index of Discrimination (I.D.)

The I.D. of an item is a measure of how efficient it has been at distinguishing between the stronger and weaker candidates as indicated by the results of the other items. Since the items in the Diagnostic Tests are set to find areas of deficiencies that a child may have and not to distinguish between the candidates the I.D. of an item is of no particular importance in these tests. Therefore the following discussion on the I.D. refers to the items of the selection test which will be developed from the diagnostic test.

The strict definition of Index of Discrimination is the correlation coefficient over the class taking the test between the scores on the item under consideration and those on all other items in the test. Because of the large amount of calculation required in obtaining this for every item, various simplifications have been devised. In 1939 an article by Truman Kelly (1939 pp.17-24) included a proposal for basing the calculation on only a few scores obtained by the strongest
and weakest candidates. Kelly showed that under certain conditions there was an optimum size for the upper and lower groups as a proportion of the number of candidates. Small numbers would produce a large difference in ability between the two groups but would allow large sampling errors within the groups; large numbers would reduce these sampling errors at the expense of the between-group difference. The most reliable value of discrimination was shown to result from taking upper and lower groups of 27% of the total group, and Kelly further claimed that even when the conditions he had stated did not apply, the choice of 27% was "ordinarily the most serviceable".

Of the simplified forms of discrimination index which have been based on Kelly's proposal, it is likely that the one now in the widest use is that due to A.P. Johnson (1951 pp. 499-504). The I.D. is defined as

\[
\frac{N_U(C) - N_L(C)}{N_T}
\]

where U and L are equal-sized groups taken respectively from the upper and lower ends of the rank-order list of candidates. N(C) is the number of correct responses to the item given by the group specified and N_T is the total number of candidates in that group.

It will be seen that dichotomous scoring is implied in this definition, as otherwise the phrase "correct response" would lack precision. It is universal practice to use the rank order obtained from the results of the entire test; it is impracticable to obtain a different rank order for use with
each item; as would be necessary if that item were to be
excluded from the scores being ranked. This amount of
approximation is seen to be acceptable when it is recognised
that the index is in any case internal to the test, in that
rank order is not determined by any external assessment.

In selection tests, and others where the main aim is to
establish a rank order, the discriminating power is at least
as important as the F.V. in determining the suitability of an
item. Items with I.D. values of less than about 0.3 are
usually regarded as unsuitable for such tests. For instance,
items with I.D. = 0 are allowing the weaker candidates to score
as many marks as the stronger ones, and so are not helping to
rank the candidates correctly. Most authors agree on the
critical I.D. value being around 0.3, but many pay little
attention to a point which is equally important, namely that the
I.D. is of no relevance if the object of the test is to find
whether the class as a whole has an adequate grasp of the
subject matter. In this situation, items with facility value of
0% or 100% may give valuable information about the subject matter
they are testing, but will have I.D. = 0.

Dawson (1971) suggests the following values of I.D. as
a guide:

<table>
<thead>
<tr>
<th>I.D. Value</th>
<th>Discrimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40 and above</td>
<td>item discriminates well</td>
</tr>
<tr>
<td>0.30 - 0.39</td>
<td>item discriminates reasonably well</td>
</tr>
<tr>
<td>0.20 - 0.29</td>
<td>marginal discrimination only</td>
</tr>
</tbody>
</table>
| below 0.20 | inadequate discrimination and item
             | would normally be rejected except
             | for very difficult or very easy |
items that may be desirable to include.

The guide suggested by Macintosh and Morrison (1972) is:

- $0.30 \leq I.D. \leq 0.39$ - item is reasonable but may be improved
- $0.20 \leq I.D. \leq 0.29$ - item is marginal and requires revision
- $I.D. < 0.20$ - item is unsuitable.

Since most tests are designed to distinguish different levels of ability, as opposed to testing "content mastery", items should have an I.D. as high as possible. Tests of content mastery are usually unreliable because of the inclusion of items of low discrimination. However Ebel (1972) suggests that preference for items of high I.D. should not be pushed to the extent of excluding items for which a high I.D. cannot be obtained, provided such items are relevant. However, if the I.D. is negative, that is to say that the correct option has been chosen more frequently by 'poor' candidates, the item should be rejected.

The I.D. is related to the F.V., the former having a larger value for those items of middle order difficulty. For example, if 100% of a sample give correct responses, the discrimination will be zero. If, however, 50% of the sample respond correctly, the maximum discrimination will be +1.0. The relationship between F.V. and I.D. can be elucidated further by considering the following specific cases.

(a) If the F.V. is less then 0.27 it will be possible for all students in the top 27% merit to answer
incorrectly or vice-versa. The maximum and minimum F.V. will be given by:

\[ I.D. = \pm 1.0 \quad \text{i.e.} \quad \left( \pm \frac{F.V.}{0.27} \right) \]

(b) If the F.V. is between 0.27 and 0.73 the I.D. can again have values ranging between +1.0 and -1.0.

(c) If the F.V. is greater than 0.73 then even if all the students in the top 27% group answer correctly, there will be some students in the bottom 27% group who also answer correctly. The maximum I.D. will thus be given by:

\[ I.D. = \frac{0.27 - (F.V. + 0.27 - 1)}{0.27} \]

Therefore \( I.D. = \left( 1 - \frac{F.V.}{0.27} \right) \)

The minimum value of I.D. will be given by:

\[ I.D. = -(1 - F.V.) \]

Therefore for items with F.V. below 0.27 or above 0.73 the maximum and minimum values of I.D. will be less than one.

All values of the F.V. and I.D. parameters will lie within the above hexagon. As the size of the sample becomes larger, the effect of the students within the middle 46%
group becomes less and less significant and the chance of having an I.D. of ±1 will only occur when the F.V. is 0.50 i.e. the horizontal sides of the above hexagon will shrink as the sample size increases.

As stated earlier, this does not mean that the ideal test will have a F.V. of 0.50 although such items are more likely to have a good discrimination.

The reason for choosing a 27% sample in the calculation of F.V. and I.D. is that this size provides the best compromise between two conflicting requirements.

(a) To make the extreme groups as large as possible.

(b) To make the extreme groups as different as possible.

However, although 27% is the optimum size to ensure the above two requirements, the results obtained will not be that much better compared to group sizes of 25% or 33%, so that if these latter size groups are better to work with, they can be used with confidence.

Finally, the degree of uncertainty associated with having relatively small samples can be allowed for by considering the standard error (S.E.). This involves using a statistical method to ascertain the uncertainty in any measured quantity, like the F.V. or I.D.

As an example, the table (Dawson 1971) below shows how standard error of pre-test facility values vary with the size of the population.
Pre test F.V. | Size of population
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>±0.09</td>
<td>±0.06</td>
<td>±0.04</td>
</tr>
<tr>
<td>0.25</td>
<td>±0.13</td>
<td>±0.09</td>
<td>±0.06</td>
</tr>
<tr>
<td>0.50</td>
<td>±0.16</td>
<td>±0.10</td>
<td>±0.07</td>
</tr>
<tr>
<td>0.75</td>
<td>±0.13</td>
<td>±0.09</td>
<td>±0.06</td>
</tr>
<tr>
<td>0.90</td>
<td>±0.09</td>
<td>±0.06</td>
<td>±0.04</td>
</tr>
<tr>
<td>0.95</td>
<td>±0.02</td>
<td>Less than ±0.01</td>
<td></td>
</tr>
</tbody>
</table>

Thus for a pre test F.V. of 0.25, for a student population of 38, the standard error will be ±0.075. The probability of this error can be estimated by means of confidence curves (Dawson 1971 pp.13-14). From these for a student population of 40, there is a one in five chance that a F.V. of 0.20 means a true F.V. of 0.25. Similarly for a sample of 40, there is a one in four chance that a F.V. of 0.30 is less than 0.25.

At the other end of the F.V. spectrum, for a F.V. of 0.90, the table shows that for a student population of 38, the standard error will be approximately ±0.05. From the confidence curves for a population of 40 there is a one in six chance that a pre test F.V. of 0.90 is less than 0.85 and a one in ten chance that a pre test F.V. of 0.76 is greater than 0.85.
2.6.3 Reliability factor

We are not concerned with the reliability factor for the diagnostic tests because the aim of the teacher giving the test is to discover the learning failure of the child and then to remedy that failure. If the teacher does this successfully then any subsequent use of that test on the same children must give a different result because the child no longer has his previous failure.

Therefore the following discussion on the reliability factor refers only to the selection test which will be derived from the diagnostic tests.

Since the measure of facility and discrimination apply to the individual items, measures which evaluate a complete test must now be considered. The most important of these measures is "test reliability". The reliability of a test is the extent to which the results are reproducible, a perfect reproducibility being impossible because of the variability present in all measurements, including test scores. A score must be regarded as an estimate of a notional "true score".

Some measures of reliability use the analysis of variance technique to estimate how much of the variability between scores arises from the difference between candidates as distinct from that which arises from that which occurs as a result of random errors in the responses made (such as those due to careless reading by the candidate or the imperfect writing of an item). This approach leads to the following as the test reliability factor:
\[
1 - \frac{6_c^2}{6^2}
\]

where \(6_c^2\) and \(6^2\) denote respectively the population error variance and total variance. In a perfectly reliable test \(6_c^2\) will be zero as there will be no random errors, that is the reliability factor will be unity; in an unreliable test the error variance will account for nearly all the variability, making the variance ratio so near to unity that the reliability factor approaches zero.

Reliability factors derived from variance considerations are discussed fully by a number of authors notably Ebel (1972) and Thorndike (1951). One of the most widely used form is that given by the Kuder-Richardson (1937 pp. 151-160) formula 20.

That is:

\[
K.R. = k \left( \frac{s^2 - \sum pq}{s^2} \right)
\]

where \(k = \) the number of items in the test
\(s = \) the sample standard deviation of students scores
\(p = \) the proportion of candidates that answers an item correctly
\(q = 1 - p
\)

\(\sum pq = \) the summation of the product \(pq\) over \(k\) items.

In the project the reliability coefficient, \(r\), will be calculated using the approximate Kuder-Richardson formula 20 suggested by Macintosh and Morrison (1972 p. 71) based on the upper and lower 27%, that is 54% of the total sample.

The reliability coefficient, \(r\), is given by:

\[
r = \frac{k}{(k-1)} \left( 1 - \frac{2n\sum(N_U + N_L) - \sum(N_U + N_L)^2}{0.667\sum(N_U - N_L)^2} \right)
\]

where \(k = \) No. of test items
\(n = \) No. of candidates in 27% of sample
where \( k \) = No. of test items

\( n \) = No. of candidates in 27% of sample

\( N_u \) = No. of correct responses per item for those in the top 27%

\( N_l \) = No. of correct responses per item for those in the lowest 27%.

Dichotomous scoring is implied in this formula.

Thorndike describes the formula as "the most generally useful of the formulas for estimating reliability from the relationship of total variance to item variance". Ebel states that this and the related Kuder-Richardson Formula 21 "have become widely accepted as a basis for estimating test reliability".

The derivation of the K-R formula involves a number of assumptions which in practice are only partly justified. The user should not therefore be surprised at obtaining K-R values which are negative; this can happen quite easily with small classes.

Another estimate of reliability can be obtained by the method of Split-Halves. The difficulty of constructing and administering parallel tests led to the concept of splitting a single test into two halves. One method of doing this split was to consider one test to consist of the odd numbered items and the second test to be the even numbered items. The reliability of the whole test could then be determined by using the Spearman-Brown formula:

\[
    r_n = \frac{nr_s}{(n-1)r_s + 1}
\]

where \( r_n \) = reliability of a test \( n \) times as long
as a test of known reliability.

Therefore, to predict the reliability of a test twice as long as a given test, as when using the split-halves method, the formula becomes:

\[ r = \frac{2rs}{(r_s+1)} \]

The estimate of test reliability obtained by this method is a function of the split chosen. Lord (1956 pp. 245-249) has shown how wide this variation can be. However, provided the test is split as nearly as possible into two equivalent halves and the test is reasonably long and homogeneous (the test should not cover too wide a spectrum), the reliability errors are not likely to be serious.

The reliability of a test is related to many features of the test, some of which are discussed below.

(a) Length of test

The Spearman-Brown formula indicates the theoretical statistical relationship between test length and reliability. The relationship is displayed on the graph below, the data for this being obtained by considering a fair item test having a reliability of 0.1. It can be seen that statistically, as the number of items tends to infinity the reliability tends to 1.00. The statistical assumption with this conclusion is that the extra items added to the test have the same statistical properties as the existing items. The psychological assumption is that the addition of extra items will not affect the examinees response to the test. This may not be a valid assumption if the test is made very long, when fatigue and
boredom may make the Spearman-Brown formula erroneous. Thus when setting the test we will have to compromise between having a very reliable test comprising of a considerable number of questions and having a very short test which is very unreliable. Therefore it is proposed to set a test having approximately 25-35 items.

Relation of Test length to test reliability

<table>
<thead>
<tr>
<th>Item</th>
<th>5</th>
<th>20</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>160</th>
<th>320</th>
<th>640</th>
<th>00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability</td>
<td>0.10</td>
<td>0.18</td>
<td>0.31</td>
<td>0.47</td>
<td>0.64</td>
<td>0.78</td>
<td>0.88</td>
<td>0.93</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Spearman-Brown formula $r = \frac{2rs}{1+rs}$ where $r_s = 0.1$
(b) Homogeneity of test

Greater reliability is obtained when test items are homogeneous. Thus the reliability obtained when testing a large syllabus in one test is not as high as that obtained when testing a smaller content of subject matter. The test written therefore, from this point of view, will be quite reliable because we are only considering the four computational skills of addition, subtraction, multiplication and division.

(c) Index of discrimination (I.D.)

If the standard deviation of a test is made as large as possible, the reliability will be increased. The spread of marks will depend, amongst other things, on the technical quality of the test. This means that items with a high discrimination index and by implication, a middle order facility value, are likely to be the most reliable. Ebel (1972 p.429) writes that the best way of increasing test reliability is to use items with a high index of discrimination. The range of I.D.'s and F.V.'s suggested by Macintosh and Morrison (172 p.67) for inclusion in a test are shown below.

<table>
<thead>
<tr>
<th>Facility Value</th>
<th>Index of Discrimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below</td>
<td>0.4</td>
</tr>
<tr>
<td>0.4-0.6</td>
<td>Above</td>
</tr>
<tr>
<td>0.40</td>
<td>Difficult*</td>
</tr>
<tr>
<td>0.30-0.39</td>
<td>Acceptable*</td>
</tr>
<tr>
<td>0.20</td>
<td>Difficult</td>
</tr>
<tr>
<td>0.29</td>
<td>Improvable</td>
</tr>
<tr>
<td>Below 0.20</td>
<td>Rejected</td>
</tr>
<tr>
<td>0.20</td>
<td>Rejected</td>
</tr>
<tr>
<td>Items marked * would be considered suitable.</td>
<td></td>
</tr>
</tbody>
</table>
(d) The effectiveness of the Distractors

The perfect statistical item would be one in which all the lower 27% group would answer wrongly, at the same time distributing their answers equally amongst the distractors, and the top 27% all answer correctly. The response display for this 'perfect' item would be:

<table>
<thead>
<tr>
<th>Percentage responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>Upper group</td>
</tr>
<tr>
<td>Lower group</td>
</tr>
</tbody>
</table>

If it is found that some of the distractors are not operating satisfactorily then it should be replaced.
CHAPTER 3

A MORE DETAILED CONSIDERATION OF DIAGNOSTIC TESTS

3.1 Possible uses of diagnostic tests

The teacher may use a diagnostic test in one of three ways:

(1) As an assessment of entry behaviour. The teacher may wish to know, particularly with a new class, the strengths and weaknesses his class may have concerning the topic and so he may use the test to decide his starting point and to plan the notions which the test reveals as needing emphasis.

(2) As an assessment of progress. The teacher may wish to know if the notions he has taught have been understood before progressing further with the topic. In this case the teacher will give the test during the teaching of the topic. If no weakness is revealed the teacher may safely assume that the children are ready to progress with the topic. Any weaknesses need solving before progression within the topic is made.

(3) To assess mastery of a topic. Once a teacher has completed a topic he may wish to assess who has mastered what. Any parts not mastered may be dealt with there and then or may be used as a starting point when the topic is dealt with again in the child's life. The diagnostic tests in this project will primarily be used in this context and the notions mastered or otherwise will be used as a basis for a mathematical
record of the child which will be used when the child is transferred to another situation, be it a new class, or most likely in the case of the children participating in the tests for the projects, a new school (the Upper School).

3.2 Facets of Mathematical ability to be stressed

The project will consider the diagnostic testing of the computational skills involved in addition, subtraction, multiplication and division. Having chosen the topic or sub-topic to test, it is necessary to decide whether any particular facet of mathematics is to be stressed. Schonell (1957 p.75) says that 'mathematical attainment' has elements of "accuracy, speed of working, methods of work, and extent of processes mastered". The last element is covered by the fact that the teacher has decided at which 'process' the test is to be aimed. 'Speed of working' will only play a very minor part in the test if it occurs at all. This leaves accuracy and methods of work. Both of these fall into the algorithmic aspect of the test and so can be catered for by the same test.

Although the broad splitting of young children's attainment into two main facets of 'accuracy and algorithms' and 'understanding' must not be taken to imply that these two facets are necessarily disjoint, it does lead to the broad splitting of diagnostic tests into tests of skills and tests of concepts. It may well be that the tests turn out not to be disjoint either. However, the tests written for this project will be mainly concerned with the testing of the child's knowledge of the algorithms involved in four chosen topics rather than his
understanding of the concepts involved; although the writer feels that the understanding of concepts and the use of the algorithm are very much entwined.

3.3 Analysis of the topic into items

Having decided upon the skill(s) which we hope to test we need to consider the construction of the test. Mehrens and Lehman (1969 p. 141) write:

"In general a diagnostic test of a topic will be concerned with those factors which experts in teaching that topic think are important".

The basis of the test is that, within the topic to be tested, there exists a progression of sub-topics, competence at each of which is necessary for topic competence to exist. It is not unlikely that the sub-topics themselves can be further analysed into even more basic steps for as Schonell (1957 p. 76) writes:

"A diagnostic test takes into consideration all the vital skills involved in each important aspect of the subject".

Carefully graded examples must then be written so that each example tests one and only one of the above steps. The means of achieving these graded examples will be discussed in Chapter 4.

3.4 The need to consider the child's reading ability

A major consideration with written tests is the reading and comprehension ability of the children. Obviously there could be those children who, while perfectly capable of coping with the mathematics in the test, would appear not to be so
because, having failed to read or comprehend the instructions adequately, they did not know what was required of them.

The term 'reading ability' needs further discussion. Schonell (1957 p.25) says that two aspects of verbal ability enter into arithmetic. The first is the nature and extent of the mathematical vocabulary used, as an example shall the word 'times' be allowed when 'multiplication' is meant? The second is the power to reason with ideas which are expressed in sentences. Note that this is *verbal* ability. If the test is to rely on the child himself reading important parts in it, then it is necessary to expand the above to require that reading ability shall include:

(1) the ability mechanically to read words i.e. to be able to decode the written symbols

(2) the ability to recognise the words so read as being in his vocabulary. This is likely to be an intuitive act rather than a deliberate one

(3) the ability to recall the concepts which are represented by the words

(4) the ability to organise these component concepts, in his mind, so that the meaning of the sentence becomes clear, and

(5) the ability to reason with respect to the ideas expressed in the sentences.

It would seem to be obvious that the restrictions which the necessity to allow for the above abilities would place on the design and use of a test would necessarily impair its value as a *mathematical* diagnostic tool. It follows that diagnostic
tests of mathematics which require the child to read and comprehend the reading in order to be able to follow instructions may be suspect.

For younger or less able children the test constructor should aim to have as little explanatory reading in it as he reasonably can. This will be duly noted in the tests to be constructed by omitting any written instructions. Any instructions required to be given to the children will be read and explained to the class as a whole by the writer, who will administer the tests.

3.5 The time allocated to diagnostic tests

Since in: diagnostic tests, in general, we are concerned with accuracy, algorithms and understanding and not with speed of computation, there is no point in laying down time limits. Indeed imposing a time limit on the children may well put them under extra pressure so that there could be a possibility that a child might fail with a problem worked under a time limit but with which he was normally competent.

3.6 The applicability of diagnostic tests to a variety of schools

If the diagnostic test were to be applicable in a variety of schools, infant, first school, junior or middle schools, the chances are that the curriculum and syllabus followed in that school is unique to that school. However, since mathematics, even in its early stages, is a structured subject, that is for example that in any school children need to be taught addition before they are taught long multiplication, because competence in the latter requires competence in the former, the mathematics
to be tested is likely to be met in much the same order by all children irrespective of their school. It may be that they will each reach a given stage in one school at an earlier (or later) age than they would have met it in another school but this will not matter. Differences in school or syllabus should not therefore, make much difference to the test.

3.7 The reliability and validity of diagnostic tests

Any test constructor whose tests are to stand scrutiny must consider reliability and validity. If a test is valid it measures what it sets out to measure, and if it is reliable then the correlation between the results obtained from successive administrations of the test will be high.

The references to validity, including formulae for testing it, seem to have been written with attainment tests in mind. For instance Copland's (1962 p.15) formula $V=U-L$ where $V$ is the validity, $U$ and $L$ are the percentage of the upper ($U$) and lower ($L$) thirds of the class who got a given item in the test correct requires that the children have been put in an ordered list as a result of their performance in the test. But the mere fact that the list is ordered means that they have been judged against each other.

The Secondary Schools Examinations Council Bulletin No.3 (1964 pp.19-20) gives eight factors which "contribute to the validity of an examination". The first seven of these are mainly concerned with careful selection and identification of objectives to be tested, and these factors will have been covered by the fact that the essence of a diagnostic test is
the extremely careful breakdown of a topic into its constituent sub-topics, so that the test will in fact have validity built into it. The eighth factor given is "comparison with previous examination of known validity". Unfortunately there may not be any previous examinations. However, Schonell (1957 p.81) suggests that there may be "a need to occasionally supplement a diagnostic test by

(1) scrutiny of pupils' work and/or
(2) oral analysis of written work".

But he warns (p.32) that a friendly, work orientated classroom atmosphere is of vital importance, "the teacher must see that the oral analysis does not become oral inquisition".

For small scale teacher constructed classroom tests, the sign that the test is valid will be that, in the majority of cases, the test results bear a fair resemblance to the teacher's expectations.

A brief but clear exposition of the reliability of an examination is given in the Secondary Schools Examinations Council Bulletin No.3 (1964 pp.16-17). Reliability of a measuring instrument is given as the ability of the instrument "to give the same measurement on different occasions".

However, it seems that the above view of reliability cannot be used for diagnostic tests because the aim of the teacher giving the test is to discover the learning failure of the child and then to remedy that failure. If the teacher does this successfully then any subsequent use of that test on the same child must give a different result because the child no longer has his previous failure.
It would seem then, that in the case of diagnostic tests, validity is of far more importance than reliability. However, reliability will have some significance in the selection tests derived from the diagnostic tests.
4.1 The need for pre-objective diagnostic tests

In order for the objective diagnostic test to be both useful and meaningful the distractors need to be chosen with great care. The objective diagnostic test will be set in order to diagnose the children's algorithmic weaknesses and therefore the distractors need to be chosen through the consideration of the algorithmic reasoning of the children. Thus the first step to be taken in constructing the objective diagnostic test was to discover the types of algorithmic errors made by children in general. Few books (Schonell, 1951 and National Council of Teachers of Mathematics, 1978) discuss the algorithmic errors made by children and even these lack the detail required by the project. To overcome this problem the writer needed to construct diagnostic tests which would be analysed and, hopefully, the algorithmic difficulties of children revealed. From this analysis the writer will then be able to construct distractors for the objective diagnostic test.

The pre-objective diagnostic test will be presented to approximately eighty-five 11-12 years old children (note that these are not the children who will be taking the objective diagnostic test for obvious reasons) and it will be of the standard format (see Chapter 5) i.e. the questions will be
presented which the children will answer showing all their working out.

The discussion which follows is, generally, applicable to both the pre-objective and the objective diagnostic tests which will be written; particular reference will be made if this is not the case.

4.2 Essential components of diagnostic tests

Schonell (1957 p.76) writes:

"A diagnostic test ..... takes into consideration all the vital skills involved in each aspect (of arithmetic) and these are tested by a series of carefully graded examples which cover all important steps in the acquirement of that skill".

The three essential basic requirements of a good diagnostic test are pinpointed in the above quotation. They are:

(1) The selection of the topic to be chosen.

(2) The accurate identification of a progression of component sub-stages in the topic which need to be mastered for mastery of the topic itself to be achieved; and

(3) The writing of test items which accurately test the child's mastery, or lack of it, at each sub-stage.

The first stage is usually chosen by the teacher to coincide with the teaching of that particular topic. However, the four topics chosen in the project were chosen as vehicles for investigating the feasibility of the use of objective tests for the purpose of diagnosing a child's algorithmic deficiencies.
4.2.1 The analysis of the computational skills into their component stages

The second stage involves the analysis of the skills into their component stages. One method of analysing the components of a skill is to consider the simplest stage and then to consider variations from that starting point. With the aid of Yardsticks (Milward and Fraser, 1975) the writer now proposes the component stages for the four basic computational skills of addition, subtraction, multiplication and division. It must be stressed that the sequencing of the stages is not unique and others may choose different sequential stages for them. For example, one may choose to consider the addition of two 1-digit numbers with no carrying to be followed by the addition of a 1-digit number to a 2-digit number with no carrying and so on; that is, be concerned with the addition of digits with no carrying before considering the use of carrying figures.

4.2.2 The component stages for addition

<table>
<thead>
<tr>
<th>Sub-stage A</th>
<th>Sub-stage B</th>
<th>Sub-stage C</th>
<th>Sub-stage D</th>
<th>Sub-stage E</th>
<th>Sub-stage F</th>
<th>Sub-stage G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Number bonds to 10</td>
<td>Addition number bonds to 18</td>
<td>To add a 2-digit number and a 1-digit number: no carrying</td>
<td>To add a 2-digit number and a 1-digit number: carrying units as tens</td>
<td>To add two 2-digit numbers: no carrying</td>
<td>To add two 2-digit numbers: carrying units</td>
<td>To add two 3-digit numbers: no carrying</td>
</tr>
</tbody>
</table>
Sub-stage H: To add two 3-digit numbers: carrying units
Sub-stage I: To add two 3-digit numbers: carrying where necessary (one place only)
Sub-stage J: To add two 3-digit numbers: with repeated carrying
Sub-stage K: To add numbers with up to 4 digits: carrying where necessary
Sub-stage L: To add numbers with up to 5 digits: carrying where necessary.

4.2.3 The component stages for subtraction

Sub-stage A: Subtraction number bonds to 10
Sub-stage B: Subtraction number bonds to 18
Sub-stage C: To subtract a 1-digit number from a 2-digit number: no borrowing
Sub-stage D: To subtract a 1-digit number from a 2-digit number: with borrowing
Sub-stage E: To subtract using two 2-digit numbers: no borrowing
Sub-stage F: To subtract using two 2-digit numbers: with borrowing
Sub-stage G: To subtract using two 3-digit numbers: no borrowing
Sub-stage H: To subtract using two 3-digit numbers: borrowing units
Sub-stage I: To subtract using two 3-digit numbers: borrowing tens
Sub-stage J: To subtract using two 3-digit numbers: borrowing where necessary (one place only)
Sub-stage K To subtract using two 3-digit numbers: with repeated borrowing

Sub-stage L To subtract using numbers with up to 4 digits: with repeated borrowing

Sub-stage M To subtract using numbers with up to 5 digits: borrowing where necessary

4.2.4 The component stages for multiplication

Sub-stage A To multiply by 1 and 0

Sub-stage B To do simple multiplication by 2
To do simple multiplication by 3
To do simple multiplication by 4
To do simple multiplication by 5
To do simple multiplication by 6
To do simple multiplication by 7
To do simple multiplication by 8
To do simple multiplication by 9
To do simple multiplication by 10

Sub-stage C To multiply a 2-digit number by a 1-digit number: no carrying

Sub-stage D To multiply a 3-digit number by a 1-digit number: no carrying

Sub-stage E To multiply a 2-digit number by a 1-digit number: carrying units

Sub-stage F To multiply a 3-digit number by a 1-digit number: carrying units

Sub-stage G To multiply a 3-digit number by a 1-digit number: carrying where necessary (one place only)
Sub-stage H  To multiply a 3-digit number by a 1-digit number: with repeated carrying
Sub-stage I  To multiply a whole number by 10 or 100
Sub-stage J  To multiply a 2-digit number by 20, 30, 40 or 50
Sub-stage K  To multiply two 2-digit numbers: carrying units only
Sub-stage L  To multiply two 2-digit numbers: carrying where necessary
Sub-stage M  To multiply a 3-digit number by a 2-digit number: carrying where necessary (one place only)
Sub-stage N  To multiply a 3-digit number by a 2-digit number: with repeated carrying
Sub-stage O  To multiply two 3-digit numbers: carrying where necessary

4.2.5 The component stages for division
Sub-stage A  To divide by 1 and 0
Sub-stage B  To do simple divisions by 2
To do simple divisions by 3
To do simple divisions by 4
To do simple divisions by 5
To do simple divisions by 6
To do simple divisions by 7
To do simple divisions by 8
To do simple divisions by 9
To do simple divisions by 10
Sub-stage C  To do simple division by a 1-digit number: no remainder
Sub-stage D  To do simple division by a 1-digit number: with remainder
Sub-stage E  To divide a 3-digit number by a 1-digit number: no remainder
Sub-stage F  To divide a 3-digit number by a 1-digit number: with remainder
Sub-stage G  To divide a 4-digit number by a 1-digit number: with and without remainder
Sub-stage H  To divide a 3-digit number by 20, 30, 40 or 50: no remainder
Sub-stage I  To divide a 3-digit number by a 2-digit number: no remainder
Sub-stage J  To divide a 3-digit number by a 2-digit number: with remainder
Sub-stage K  To divide a 4-digit number by a 2-digit number: with and without remainder.

4.3 Writing the test questions

   The third essential for the test is the writing of questions which will test mastery of individual sub-stages.

4.3.1 The number of questions to be written

   The first consideration is to decide how many questions should be set to test each sub-stage. Mehrens and Lehman (1969 p.149) write that "content validity is increased by numerous items of the same kind". For addition there are twelve sub-stages, for subtraction thirteen sub-stages, for
multiplication fifteen sub-stages and for division there are eleven sub-stages. Ideally each sub-stage would be tested individually at the appropriate point in the teaching process; in this situation ten items would be used to test each sub-stage. However, for reasons already stated (Chapter 1) the sub-stages of each topic will be tested together in a single test with three items being used to represent each sub-stage. As it is considered that the multiplication test will consist of too many items the writer will divide this test into two parts; Part 1 will deal with the sub-stages up to and including I and Part 2 will deal with the sub-stages from J to O. The tests will also assume that, as an entry behaviour, the children have mastery of each of the first two sub-stages of the computational skills to be tested. Those that do not have this mastery will soon become evident and will need special remedial help. Thus the addition test will consist of thirty items, the subtraction thirty-three items, the multiplication (Part 1) twenty-one items, the multiplication (Part 2) eighteen items and the division twenty-seven items.

4.3.2 Consideration of format

There were several considerations of format. The first was whether the test questions should be written in vertical form, horizontal form; or whether there should be examples of each. Mixing the formats might present the children with an added difficulty and if it is required that both formats be tested then separate sections of a single test should be used. For this project all the problems will be written in vertical form although
it is realised that there is a very real need to test the knowledge of solving problems in horizontal form.

Similarly the use of such as 't.u.' as headings over columns needs to be considered. In this study no column headings will be used although, where the teacher thought it necessary, they could be inserted.

Another format consideration was the position of the operations sign. For example the position of the + sign may vary. Alternatives are:

(a) \[29 \quad + \quad 32\]
(b) \[29 + 32\]
(c) \[29 + 32\]

For all these positions there are logical explanations:
(a) and (c) are justified by a lateral reading of the question, (b) requires a vertical reading. The format which will be used in the test will be exemplified by the following:

For addition -
27 + 3
\[\underline{229}\]
+ 146

For subtraction -
19 - 8
\[\underline{147}\]
- 132

For multiplication -
27 x 3
\[\underline{146}\]
x 26

For division -
\[4 \div 25\]
\[\underline{22 \div 746}\]

4.4 Test instructions for the pre-objective diagnostic tests

One very important consideration is exactly what instructions shall be contained in the test for the children.
The children will be given no written instructions for either the pre-objective or the objective diagnostic tests for the reasons explained in the last chapter. The pre-objective test is of the standard form and needs no special instructions apart from emphasising to the children the need to show all the working out used to solve a particular problem.

4.5 The use of aids

Quite a difficult problem connected with the administration of the test was to decide how much the use of aids should be allowed. That is, should the children be allowed the use of counters, some form of structured apparatus, or some similar aid to calculation. A discussion with practicing teachers revealed the general feeling that the use of those aids which the children currently used in their classroom should be allowed. This is eminently reasonable and not to allow such aids could penalise some of the children. The test instructions should clearly state whether the use of aids is permissible, and if so, which ones may be used.
CHAPTER 5

THE PRE-OBJECTIVE DIAGNOSTIC TESTS
AND THEIR ANALYSIS

5.1 The pre-diagnostic tests

The following five pre-objective diagnostic tests were administered to approximately eighty-five 11-12 years old children and the responses analysed to reveal the various algorithmic approaches used. The code letters correspond to the sub-stages listed in Chapter 4. Note that enough space was left to allow each child to show all his working out for each problem.

5.1.1 The Addition Test

(C) 1. 41 2. 6 3. 82
+ 6 +13 + 7

(D) 1. 29 2. 6 3. 55
+ 4 +47 + 7

(E) 1. 12 2. 23 3. 72
+13 +34 +22

(F) 1. 38 2. 46 3. 18
+52 +39 +17

(G) 1. 621 2. 246 3. 157
+258 +530 +121
(H) 1. 275 2. 339 3. 353  
     +116          +354          +219  
     —            —            —

(I) 1. 768 2. 671 3. 276  
     +217          +266          +803  
     —            —            —

(J) 1. 119 2. 738 3. 456  
     +283          +184          +654  
     —            —            —

(K) 1. 893 2. 2988 3. 1375  
     +19           +2988         +527  
     —            —            —

(L) 1. 14577 2. 54321 3. 31673  
     +5828         +89          +48717  
     —            —            —

5.1.2 The Subtraction Test

(C) 1. 48 2. 19 3. 67  
     -6 -4 -3  
     — — —

(D) 1. 34 2. 40 3. 87  
     -9 -8 -9  
     — — —

(E) 1. 76 2. 68 3. 98  
     -50 -23 -38  
     — — —

(F) 1. 72 2. 95 3. 74  
     -45 -49 -38  
     — — —
5.1.3 The Multiplication Test (Part 1)

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>516</td>
<td>964</td>
<td>654</td>
</tr>
<tr>
<td></td>
<td>-305</td>
<td>-904</td>
<td>-123</td>
</tr>
<tr>
<td>H</td>
<td>412</td>
<td>674</td>
<td>762</td>
</tr>
<tr>
<td></td>
<td>-305</td>
<td>-518</td>
<td>-435</td>
</tr>
<tr>
<td>I</td>
<td>619</td>
<td>504</td>
<td>347</td>
</tr>
<tr>
<td></td>
<td>-246</td>
<td>-120</td>
<td>-184</td>
</tr>
<tr>
<td>J</td>
<td>556</td>
<td>508</td>
<td>208</td>
</tr>
<tr>
<td></td>
<td>-127</td>
<td>-337</td>
<td>-172</td>
</tr>
<tr>
<td>K</td>
<td>721</td>
<td>700</td>
<td>687</td>
</tr>
<tr>
<td></td>
<td>-468</td>
<td>-369</td>
<td>-199</td>
</tr>
<tr>
<td>L</td>
<td>5840</td>
<td>1700</td>
<td>1034</td>
</tr>
<tr>
<td></td>
<td>-1761</td>
<td>-913</td>
<td>-955</td>
</tr>
<tr>
<td>M</td>
<td>29567</td>
<td>1800</td>
<td>3721</td>
</tr>
<tr>
<td></td>
<td>-4563</td>
<td>-1762</td>
<td>-98</td>
</tr>
</tbody>
</table>

(C) 1. 43 x 2 2. 40 x 2 3. 32 x 3

(D) 1. 213 x 3 2. 304 x 2 3. 120 x 4
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(E)</td>
<td>1.</td>
<td>29</td>
<td>x 3</td>
<td>1.</td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td>17</td>
<td>x 5</td>
<td>3.</td>
</tr>
<tr>
<td></td>
<td>3.</td>
<td>19</td>
<td>x 4</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>1.</td>
<td>116</td>
<td>x 5</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>208</td>
<td>x 4</td>
<td>3.</td>
<td>217</td>
</tr>
<tr>
<td></td>
<td>x 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(G)</td>
<td>1.</td>
<td>329</td>
<td>x 3</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>181</td>
<td>x 5</td>
<td>3.</td>
<td>242</td>
</tr>
<tr>
<td></td>
<td>x 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(H)</td>
<td>1.</td>
<td>253</td>
<td>x 4</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>127</td>
<td>x 6</td>
<td>3.</td>
<td>153</td>
</tr>
<tr>
<td></td>
<td>x 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(I)</td>
<td>1.</td>
<td>5</td>
<td>x100</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>x100</td>
<td>3.</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>x100</td>
<td></td>
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</tr>
</tbody>
</table>

5.1.4 The Multiplication Test (Part 2)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>(J)</td>
<td>1.</td>
<td>12</td>
<td>x 20</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>x 50</td>
<td>3.</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>x 40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(K)</td>
<td>1.</td>
<td>12</td>
<td>x 37</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>56</td>
<td>x 18</td>
<td>3.</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>x 29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L)</td>
<td>1.</td>
<td>26</td>
<td>x 57</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>x 24</td>
<td>3.</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>x 87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M)</td>
<td>1.</td>
<td>121</td>
<td>x 32</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>113</td>
<td>x 26</td>
<td>3.</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>x 32</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.1.5 The Division Test

(C) 1. \[ 5 \overline{130} \] 2. \[ 8 \overline{56} \] 3. \[ 7 \overline{777} \]

(D) 1. \[ 6 \overline{56} \] 2. \[ 7 \overline{68} \] 3. \[ 9 \overline{83} \]

(E) 1. \[ 5 \overline{125} \] 2. \[ 3 \overline{42} \] 3. \[ 8 \overline{44} \]

(F) 1. \[ 4 \overline{826} \] 2. \[ 9 \overline{709} \] 3. \[ 8 \overline{795} \]

(G) 1. \[ 3 \overline{2638} \] 2. \[ 9 \overline{7777} \] 3. \[ 4 \overline{9368} \]

(H) 1. \[ 30 \overline{270} \] 2. \[ 40 \overline{560} \] 3. \[ 50 \overline{950} \]

(I) 1. \[ 14 \overline{518} \] 2. \[ 19 \overline{798} \] 3. \[ 28 \overline{672} \]

(J) 1. \[ 24 \overline{853} \] 2. \[ 57 \overline{946} \] 3. \[ 13 \overline{811} \]

(K) 1. \[ 84 \overline{8507} \] 2. \[ 34 \overline{3181} \] 3. \[ 13 \overline{1001} \]
5.2 The analysis of the pre-objective diagnostic tests

The responses to the tests were analysed according to the sub-stages for which they were set. The analysis involved investigating each item of a sub-stage for each of the eighty-five children participating in the tests and then categorising the algorithm. Each algorithm is labelled (the choice of label being the writer's) and exemplified by a typical response from a child; if the algorithm re-occurs in a later sub-stage then only the label is used. The number enclosed in the circle alongside each algorithm label is the number of responses made to that particular algorithm in that sub-stage.

5.2.1 Analysis of the Addition Test

Sub-stage C  

i) Diagonal addition  

\[ \begin{array}{c}
41 \\
+ 6 \\
\hline 
107 
\end{array} \]  

\[ \text{e.g.} \]

ii) Addition of all the digits i.e. the pupils ignored place value  

\[ \begin{array}{c}
41 \\
+ 6 \\
\hline 
17 
\end{array} \]  

\[ \text{e.g.} \]

iii) Vertical and horizontal addition  

\[ \begin{array}{c}
41 \\
+ 6 \\
\hline 
57 
\end{array} \]

\[ \text{e.g.} \]
Sub-stage D  
i) Diagonal addition (including addition of the carrying figure)  
e.g.  
\[
\begin{array}{c}
29 \\
+ 4 \\
\hline
73
\end{array}
\]

ii) Addition of all the digits including the carrying figure  

iii) Failure to add carrying figure  
e.g.  
\[
\begin{array}{c}
29 \\
+ 4 \\
\hline
23
\end{array}
\]

Sub-stage E  
No difficulties were encountered by any child on this sub-stage.

Sub-stage F  
i) Failure to add the carrying figure i.e. the pupil 'dropped' the ten that should have been grouped with the digits in the tens place  
e.g.  
\[
\begin{array}{c}
38 \\
+ 52 \\
\hline
80
\end{array}
\]

Sub-stage G  
No algorithmic difficulties encountered by any child on this sub-stage although two gave incorrect responses because of incorrect addition bonds.

Sub-stage H  
i) Failure to add carrying figure  

Sub-stage I  
i) Failure to add carrying figure
Sub-stage J  
i) Failure to add carrying figure  

ii) Reversal of the carrying digit when the column sum is 10

\[ \begin{align*}
\text{e.g.} & \quad 456 \\
& + 654 \\
& \hline
& 1111
\end{align*} \]

Sub-stage K  
i) Failure to add carrying figure

ii) Failure to add final digit

\[ \begin{align*}
\text{e.g.} & \quad 1375 \\
& + 527 \\
& \hline
& 902
\end{align*} \]

iii) Reversal of the carrying digit when the column sum is 10

Sub-stage L  
i) Failure to add at least one of the carrying figures

\[ \begin{align*}
\text{e.g.} & \quad 14577 \\
& + 5823 \\
& \hline
& 20305
\end{align*} \]

ii) Reversal of the carrying digit when the column sum is 10

5.2.2 Analysis of the Subtraction Test

Sub-stage C  
i) Diagonal subtraction

\[ \begin{align*}
\text{e.g.} & \quad 67 \\
& - 3 \\
& \hline
& 34
\end{align*} \]
Sub-stage D  
i) Lack of borrowing concept  
\[ \begin{align*} 
\text{e.g.} & \quad 40 \quad 87 \\
\text{\quad} & \quad -8 \quad -9 \\
\text{\quad} & \quad \frac{40}{38} \quad \frac{80}{79} 
\end{align*} \]

ii) Failure to add on the residue after borrowing  
\[ \begin{align*} 
\text{e.g.} & \quad 78 \\
\text{\quad} & \quad -9 \\
\text{\quad} & \quad \frac{71}{69} 
\end{align*} \]

iii) Inverted subtraction  
\[ \begin{align*} 
\text{e.g.} & \quad 34 \quad \text{i.e.} \quad 9-4=5 \\
\text{\quad} & \quad -9 \\
\text{\quad} & \quad \frac{05}{05} 
\end{align*} \]

iv) Inverted subtraction for convenience:  
\[ \text{i.e.} \quad \text{the pupil subtracted the lesser digit from the greater digit regardless of whether it is in the subtrahend or in the minuend} \]
\[ \begin{align*} 
\text{e.g.} & \quad 34 \\
\text{\quad} & \quad -9 \\
\text{\quad} & \quad \frac{35}{25} 
\end{align*} \]

Sub-stage E  
i) Borrowing when not necessary  
\[ \begin{align*} 
\text{e.g.} & \quad 16 \\
\text{\quad} & \quad \frac{50}{16} 
\end{align*} \]

ii) Incorrect borrowing procedure  
\[ \begin{align*} 
\text{e.g.} & \quad 76 \\
\text{\quad} & \quad -58 \\
\text{\quad} & \quad \frac{34}{32} 
\end{align*} \]

iii) Mixed subtraction procedure  
\[ \begin{align*} 
\text{e.g.} & \quad 76 \quad \text{i.e.} \quad 0-6=0 \\
\text{\quad} & \quad \frac{50}{20} \quad 7-5=2 
\end{align*} \]
Sub-stage F
i) Failure to add on the residue after borrowing
ii) Failure to modify the total after borrowing
   e.g. $\frac{95}{-49} = \frac{56}{9}$
iii) Inverted subtraction for convenience
iv) Incorrect borrowing procedure

Sub-stage G
No algorithmic difficulties encountered by any child on this sub-stage

Sub-stage H
i) Inverted subtraction for convenience
ii) Failure to add on the residue after borrowing
iii) Borrowing when not necessary
iv) Failure to modify the total after borrowing

Sub-stage I
i) Failure to add on the residue after borrowing
ii) Inverted subtraction for convenience
iii) Failure to modify the total after borrowing

Sub-stage J
i) Failure to modify the total after borrowing
ii) Inverted subtraction for convenience
iii) Failure to add on the residue after borrowing
Sub-stage K
i) Failure to modify the total after borrowing
ii) Inverted subtraction for convenience
iii) Failure to add on the residue after borrowing
iv) Lack of borrowing concept when dealing with zero

\[ \begin{array}{c}
\text{e.g.:} \\
700 \\
-369 \\
\hline
400
\end{array} \]

Sub-stage L
i) Borrowing when not necessary
ii) Borrowing from the extreme left

\[ \begin{array}{c}
\text{e.g.:} \\
\hline
700 \\
-913 \\
\hline
\end{array} \]

iii) Lack of borrowing concept when dealing with zero
iv) Inverted subtraction for convenience
v) Failure to add on the residue after borrowing

Sub-stage M
i) Ignoring digits

\[ \begin{array}{c}
\text{e.g.:} \\
28557 \\
-4563 \\
\hline
4994
\end{array} \]

ii) Lack of borrowing concept when dealing with zero
iii) Inverted subtraction for convenience
iv) Failure to add on the residue after borrowing
v) Failure to modify the total after borrowing
5.2.3 Analysis of the Multiplication Test (Part 1)

Sub-stage C  i) Incorrect multiplication by zero  

\[ \begin{array}{c}
i.e. 40 \\
x 2 \\
\hline 82
\end{array} \]

Sub-stage D  i) Incorrect multiplication by zero  

Sub-stage E  i) Carrying digit placed in the answer  

i.e. the pupil wrote both digits of the basic facts products in the final product  

\[ \begin{array}{c}
e.g. 29 \\
x 3 \\
\hline 627
\end{array} \]

ii) Failure to add on the carrying figure  

Sub-stage F  i) Incorrect multiplication by zero  

ii) Carrying digit placed in the answer  

Sub-stage G  i) Failure to add on the carrying figure  

ii) Carrying the wrong digit  

\[ \begin{array}{c}
e.g. 329 \\
x 3 \\
\hline 2112
\end{array} \]

iii) Carrying digit placed in the answer  

Sub-stage H  i) Failure to add on the carrying figure  

ii) Carrying digit placed in the answer  

Sub-stage I

i) Incorrect multiplication by 100
   (adding too many zeros)
   e.g. 37
       \times 100
       37000

ii) Incorrect multiplication by zero
   e.g. 37 \times 0 = 37
       
       37
       \times 0
       37
       370
       3700
       4107

For this sub-stage very few children showed any working out. They were obviously aware that 'it had something to do with adding noughts' but from the number of incorrect responses it was quite clear that they didn't know what this 'something' was.

5.2.4 Analysis of the Multiplication Test (Part 2)

Sub-stage J

i) Multiplication only by the tens digit
   e.g. 97
       \times 40
       280
       360
       640

ii) Vertical multiplication
    e.g. 20
        \times 50
        100

iii) Reversing the place values of the digits of the multiplier
    e.g. 12
         \times 20
         120
         24
         144

(Note 0 \times 2 = 2)
(0 \times 1 = 1)
Sub-stage K

i) Ignoring the place value shift of tens digit i.e. pupils misaligned addition portion because the place value shifts were ignored when multiplying.

- Example 1 -
  12
  \[ \times 37 \]
  \[ \equiv 36 \]
  \[ \equiv 120 \]

ii) Reversing the place value of the digits of the multiplier.

- Example 2 -
  12
  \[ \times 37 \]
  \[ \equiv 40 \]
  \[ \equiv 876 \]

iii) Vertical and diagonal multiplication.

- Example 3 -
  56
  \[ \times 18 \]
  \[ \equiv 608 \]

iv) Vertical multiplication.

v) Placing the answers to the basic multiplication adjacent to each other.

- Example 4 -
  12
  \[ \times 37 \]
  \[ \equiv 37 \]
  \[ \equiv 3604 \]

vi) Wrong place value shift.

- Example 5 -
  12
  \[ \times 37 \]
  \[ \equiv 36 \]
  \[ \equiv 4440 \]

vii) Multiplying only the units digit of the multiplicand by the multiplier.

- Example 6 -
  56
  \[ \times 18 \]
  \[ \equiv 108 \]
Sub-stage L

i) Multiplication by the units digit only
   e.g. \[26 \times 57 = 152\]

ii) Ignoring the place value shift of the tens digit
   e.g. \[26 \times 57 = 1442\]

iii) Basic product placed on one line with no carrying of digits
   e.g. \[26 \times 57 = 10300\]

iv) Wrong place value shift

v) Placing the answers to the basic multiplication adjacent to each other

vi) Vertical multiplication
   e.g. \[26 \times 57 = 142\]

vii) Vertical and diagonal multiplication

Sub-stage M

i) Ignoring the place value shift of the tens digit
   e.g. \[26 \times 57 = 142\]

ii) Basic product placed on one line with no carrying of digits

iii) Placing the answers to the basic multiplication adjacent to each other

iv) Vertical multiplication

v) Wrong place value shift

vi) Multiplying only the units digit of the multiplicand by the multiplier
vii) Vertical and diagonal multiplication

Sub-stage N
i) Ignoring the place value shift of the tens digit

ii) Basic product placed on one line with no carrying of digits

\[ \begin{array}{l}
\text{e.g.} & 414 \\
\times & 89 \\
\hline
36936 \\
32320 \\
\hline
365256 \\
\end{array} \]

iii), Wrong place value shift

iv) Multiplying only the units digit of the multiplicand by the multiplier

v) Placing the answers of the above basic multiplication adjacent to each other

vi) Vertical multiplication

Sub-stage O
i) Ignoring the place value shift of the tens and hundreds digit

ii) Placing the answers to the basic multiplication next to each other

iii) Vertical multiplication

iv) Wrong place value shift

5.2.5 Analysis of the Division Test

Sub-stage C  No algorithmic difficulties were encountered by the children on this sub-stage

Sub-stage D
i) Remainder omitted

\[ \frac{9}{6156} \]

ii) Incorrect remainder

\[ \frac{9}{6183} \]

iii) Lack of carrying concept

\[ \frac{01}{6156} \]
Sub-stage E

i) Division of digits only exactly divisible by divisor
   e.g. \[ \frac{001}{5} \equiv 25 \]

ii) Division of each digit individually
    - no carrying
   e.g. \[ \frac{21}{3} \equiv 642 \]

Sub-stage F

i) Internal remainder ignored
   e.g. \[ \frac{201}{4} \equiv 828 \]

ii) Omitting zero in quotient
    e.g. \[ \frac{26}{4} \equiv 626 \]

iii) Incorrect remainder
     e.g. \[ \frac{78}{9} \equiv 709 \]

iv) Division of each digit individually
    i.e. lack of carrying concept
     e.g. \[ \frac{001}{9} \equiv 709 \]

Sub-stage G

i) Internal remainder ignored
   e.g. \[ \frac{864}{9} \equiv 7777 \]

ii) Remainder omitted
    e.g. \[ \frac{864}{9} \equiv 7777 \]

Sub-stage H

The children encountered many problems on this sub-stage but they seemed to lack any algorithmic approach to the problems; most simply guessed. Obviously this is a sub-stage which needs greater emphasis and the teaching methods probably require revising. No algorithmic errors were categorised.
Sub-stage I
i) Failure to 'bring down' the necessary digit
   e.g.
   \[
   \begin{array}{c}
   40 \\ 19798 \\
   \hline
   \end{array}
   \]
ii) Incorrect subtraction
   e.g.
   \[
   \begin{array}{c}
   38 \\ 14518 \\
   \hline
   -42 \\
   -118 \\
   \hline
   -118000
   \end{array}
   \]

Sub-stage J
i) Failure to 'bring down' the necessary digit
ii) Remainder omitted
iii) Incorrect subtraction

Sub-stage K
i) Omitting zero in quotient
   e.g.
   \[
   \begin{array}{c}
   23 \\ 84 \overline{8507} \\
   \hline
   \end{array}
   \]
ii) Incorrect subtraction
iii) Failure to 'bring down' the necessary digit

5.3 General comments concerning the analysis

Although these tests were only used as a tool to enable the writer to select suitable distractors for the objective diagnostic tests it proved very enlightening and a useful course to pursue might be to research the ways the children arrive at the various incorrect algorithmic notions. There certainly appears to be a need for it for, from the writer's experience, there appears to be few books written concerning
the subject and even these lack detail. However, it must be stated now that the analysis was an extremely time consuming process.

The analysis of the tests has been recorded very briefly for the purpose of it was to categorise algorithms in order that suitable distractors for the objective diagnostic test could be constructed (i.e. the distractor will be arrived at by following the various algorithmic processes categorised for a particular sub-stage). No attempt will be made to 'explain' these algorithmic misnomers although, as stated, much research could and should be spent on this facet; maybe by understanding their origins we could eliminate many of them. No attempt will be made here to diagnose the algorithmic deficiencies of individual children for this was not the purpose of the tests. Diagnosis of algorithmic deficiencies of individual children will be discussed in Chapter 7 after analysing the responses to the objective diagnostic tests.

It should be noted in the analysis of the Subtraction test that all the children applied, or attempted to apply, the method of decomposition. This is exactly as the writer expected as this is the method of subtraction taught throughout the school. The point to be stressed is that if 'equal addition' was the algorithm taught then a different set of distractors would be required.
CHAPTER 6

DESIGNING THE OBJECTIVE DIAGNOSTIC TESTS

6.1 Introduction

The objective diagnostic tests need to take into consideration all the facets discussed in Chapter 4 concerning the pre-objective diagnostic tests; the philosophy and construction of the tests being identical. A major difference between the two is that the pre-objective diagnostic tests allowed an 'open' response from the child whilst in the objective diagnostic tests the child is presented with a number of responses from which he has to choose one. The need now is to select suitable distractors for the objective diagnostic test.

6.2 The criteria for choosing the distractors

For each item there will be one key (the correct response) and three distractors; the distractors being constructed through the use of the incorrect algorithms revealed in the analysis of the pre-objective tests (Chapter 5). All the sub-stages and nearly all the items will correspond to those given in the pre-objective diagnostic test (some items were altered because a distractor for that particular item could have been constructed through the use of more than one algorithm; as no 'working out' is shown the writer would have no means of deciding which algorithm had been employed

\[ \begin{align*}
24 & \quad +42 \\
\hline
\end{align*} \]
The response 66 would be arrived at by employing the correct algorithm, or by horizontal addition. This difficulty was overcome by replacing such items.

As the sub-stages and items of the two tests correspond the distractors were generally constructed using the algorithms analysed in that particular sub-stage. If for any particular sub-stage more than three 'algorithmic errors' occur then the most frequently occurring algorithmic error will be used as a distractor for each of the three items of that sub-stage. If one of these algorithmic errors only occurs once then it might be only used to construct one distractor. In other words, all the algorithmic errors of a sub-stage will usually be tested at least once with the most frequently occurring errors being tested three times for that sub-stage (the same algorithmic error, of course, may occur in another sub-stage and be tested there).

The above discussion is exemplified by the following. Consider the analysis of the Subtraction Test, sub-stage K (Chapter 5 p.71). This revealed the following algorithmic errors:

i) A failure to modify the total when borrowing

ii) Inverted subtraction for convenience

iii) A failure to add on the residue after borrowing

iv) A lack of borrowing concept when dealing with zero

The following three items and their distractors will be set for the Subtraction Test, Sub-stage K. The choice of algorithm to construct the distractor is written under the
corresponding distractor.

1.  721
   -468
   ___

(A)  347
     Inverted subtraction for convenience

(C)  342
     Failure to add on the residue

(B)  253
     Correct

(D)  363
     Failure to modify the total

2.  700
   -359
   ___

(A)  400
     Lack of borrowing concept when dealing with zero

(C)  441
     Failure to modify the total

(B)  469
     Inverted subtraction for convenience

(D)  331
     Correct

3.  687
   -199
   ___

(A)  488
     Correct

(C)  598
     Failure to modify the total

(B)  411
     Failure to add on the residue

(D)  512
     Inverted subtraction for convenience
If, for a particular sub-stage, three algorithmic errors are revealed then the errors are used for constructing the distractors for each of the three items. For example, consider the analysis of the Multiplication Test, Sub-stage G (Chapter 5 p.72). This revealed the following algorithmic errors:

i) A failure to add on the carrying figure

ii) Carrying the wrong digit

iii) Carrying digit placed in the answer

The following three items and their distractors will be set for the Multiplication Test, Sub-stage G. The choice of algorithm to construct the distractor is written under the corresponding distractors.
1. 329
   x 3
   
   (A) 1212
      Carrying the wrong digit
   (B) 987
      Correct
   (C) 9627
      Carrying both digits in the answer
   (D) 967
      Failure to add carrying figure

2. 181
   x 5
   
   (A) 545
      Carrying the wrong digit
   (B) 5405
      Carrying both digits in the answer
   (C) 905
      Correct
   (D) 505
      Failure to add the carrying figure

3. 242
   x 4
   
   (A) 8168
      Carrying both digits in the answer
   (B) 868
      Failure to add carrying figure
   (C) 1418
      Carrying the wrong digit
   (D) 968
      Correct

If the analysis of a sub-stage revealed less than three algorithmic errors then two alternative courses of action were used to construct the three distractors of an item. Wherever possible an algorithmic error from a previous sub-stage was used to construct the distractor but if, as occurs for Subtraction, Sub-stage C, this is not possible then the application of incorrect number bonds is used to construct the distractor. It must be stressed now that the writer did not wish to apply this option because such a
distractor is wasteful in that it does not test an algorithm, which is the aim of the test. Luckily this did not occur too often and generally only when the simplest algorithm was being tested. Nevertheless, it is hoped that the 'incorrect number bond' will be replaced in future tests when the appropriate algorithmic errors are revealed (which might be during the teaching of the topic).

Occasionally the writer has included distractors constructed through using algorithms that have not been revealed in the analysis but which have occurred during his teaching experience. This is quite legitimate for the pre-objective diagnostic test was written merely to reveal such algorithms.

6.3 A note on Long Multiplication and Long Division

In order to master the process of long multiplication a child must:

(a) know his multiplication tables
(b) be au fait with the multiplication algorithm and
(c) be able to add the two multiplicands.

Since the objective of the test is to concentrate on the multiplication algorithm the children, if they desire will be allowed to use a multiplication card from which they can look up or check the multiplication fact which they require. If these are not allowed then each incorrect answer must be scrutinised to decide the reason for it being wrong and this will take the teacher considerably longer to mark.

In some examples of the pre-diagnostic test there were so
many misconceptions displayed that Schonell's (1957 p. 81) advice that "oral analysis in doubtful cases is helpful" is itself helpful. Consider the example:

\[
\begin{array}{c}
27 \\
\times 36 \\
\hline
2160 \\
1236 \\
\hline
3396
\end{array}
\]

The multiplication of the first multiplicand i.e. \(2 \times 3 = 6\) and \(3 \times 7 = 21\) to give 2160 indicates either a lack of the multiplication algorithm, or lack of an understanding of the decimal structure of the number system, or both. Then in the second line \(6 \times 7 = 36\) indicates table deficiencies. This child needs to be led orally through some similar examples and his method of working discussed with him.

As was to be expected, a test of multiplication is a more difficult test to write and analyse than a test of addition because the former process requires more knowledge or processes than the latter. Because of the difficulties discussed above it is proposed to include in the test the various alternative algorithmic 'workings out' for each distractor of the Long Multiplication items; the child will then indicate his choice of 'working out' i.e. the algorithm he would use.

By similar consideration the Long Division items will also include the various workings out used to arrive at the distractors.
6.4 The Objective Diagnostic Tests

The items of the tests will now be presented along with the algorithm used to arrive at the various distractors. This section will be of immense importance when the analysis of the child's responses to the various items of the tests is performed.

6.4.1 The Objective Diagnostic Addition Test

Sub-stage C  To add a 2-digit number and a 1-digit number (no carrying)

1. 41
   + 6
   (A) 107  Diagonal addition
        (B) 11  Addition of all the digits
        (C) 47  Correct
        (D) 57  Vertical and horizontal addition

2. 6
    + 13
    (A) 19  Correct
    (B) 79  Diagonal addition
    (C) 10  Addition of all the digits
    (D) 49  Vertical and horizontal addition

3. 82
    + 7
    (A) 109  Horizontal and vertical addition
    (B) 159  Diagonal addition
    (C) 89  Correct
    (D) 17  Addition of all the digits
Sub-stage D  To add a 2-digit number and a 1-digit number
(carrying units as tens)

1.  29
    + 4
    ___

(A)  16
     Addition of all the digits including the carrying figure

(B)  33
     Correct

(C)  73
     Diagonal addition including the carrying figure

(D)  23
     Failure to add carrying figure

2.  6
    + 47
    ___

(A)  43
     Failure to add carrying figure

(B) 113
     Diagonal addition including the carrying figure

(C)  18
     Addition of all the digits including the carrying figure

(D)  53
     Correct

3.  55
    + 7
    ___

(A)  52
     Failure to add carrying figure

(B) 132
     Diagonal addition including carrying figure

(C)  62
     Correct

(D)  18
     Addition of all the digits including carrying figure
Sub-stage E  To add two 2-digit numbers (no carrying)

1.  \[22 + 13\]
   - (A) 44
   - (B) 8
   - (C) 35
   - (D) 53

   Horizontal addition
   Addition of all the digits
   Correct
   Diagonal addition

2.  \[23 + 24\]
   - (A) 47
   - (B) 11
   - (C) 65
   - (D) 56

   Correct
   Addition of all the digits
   Diagonal addition
   Horizontal addition

3.  \[73 + 12\]
   - (A) 85
   - (B) 103
   - (C) 13
   - (D) 94

   Correct
   Horizontal addition
   Addition of all the digits
   Diagonal addition

Sub-stage F  To add two 2-digit numbers (carrying units)

1.  \[38 + 52\]
   - (A) 80
   - (B) 90
   - (C) 135
   - (D) 19

   Failure to add the carrying figure
   Correct
   Diagonal addition
   Addition of all the digits including the carrying figure
2. \[ \begin{array}{c}
90 \\
\hline
46 \\
+39 \\
\hline
\end{array} \]

\[ (A) \quad 23 \]
Addition of all the digits including the carrying figure

\[ (B) \quad 139 \]
Diagonal addition

\[ (C) \quad 75 \]
Failure to add the carrying figure

\[ (D) \quad 85 \]
Correct

3. \[ \begin{array}{c}
236 \\
\hline
18 \\
+17 \\
\hline
\end{array} \]

\[ (A) \quad 18 \]
Addition of all the digits including the carrying figure

\[ (B) \quad 89 \]
Diagonal addition

\[ (C) \quad 25 \]
Failure to add the carrying figure

\[ (D) \quad 35 \]
Correct

Sub-stage G  To add two 3-digit numbers (no carrying)

1. \[ \begin{array}{c}
621 \\
\hline
+258 \\
\hline
\end{array} \]

\[ (A) \quad 915 \]
Horizontal addition

\[ (B) \quad 26 \]
Addition of all the digits including the carrying figure

\[ (C) \quad 879 \]
Correct

\[ (D) \quad 24 \]
Addition of all the digits not including the carrying figure

2. \[ \begin{array}{c}
246 \\
\hline
+530 \\
\hline
\end{array} \]

\[ (A) \quad 776 \]
Correct

\[ (B) \quad 118 \]
Horizontal addition

\[ (C) \quad 22 \]
Addition of all the digits including the carrying figure

\[ (D) \quad 20 \]
Addition of all the digits not including the carrying figure
3. 157
+121

(A) 135
Horizontal addition

(B) 17
Addition of all the digits not including the carrying figure

(C) 18
Addition of all the digits including the carrying figure

Sub-stage H To add two 3-digit numbers (carrying units)

1. 275
+116

(A) 381
Failure to add the carrying figure

(B) 148
Horizontal addition

(C) 22
Addition of all the digits

(D) 391
Correct

2. 339
+354

(A) 683
Failure to add the carrying figure

(B) 27
Addition of all the digits

(C) 693
Correct

(D) 1512
Horizontal addition

3. 353
+219

(A) 672
Addition of the carrying digit to wrong column

(B) 572
Correct

(C) 562
Failure to add the carrying figure

(D) 1112
Horizontal addition
Sub-stage I  To add two 3-digit numbers (carrying where necessary - one place only)

1. 768
   +217
   ________

   (A) 2110
   Horizontal addition

   (B) 985
   Correct

   (C) 1075
   Addition of the carrying figure to the wrong column

   (D) 975
   Failure to add the carrying figure

2. 339
   +354
   ________

   (A) 783
   Addition of the carrying figure to the wrong column

   (B) 693
   Correct

   (C) 683
   Failure to add the carrying figure

   (D) 27
   Addition of all the digits

3. 276
   +803
   ________

   (A) 79
   Failure to add the carrying figure

   (B) 179
   Zero omitted

   (C) 1511
   Horizontal addition

   (D) 1079
   Correct

Sub-stage J  To add two 3-digit numbers (with repeated carrying)

1. 119
   +283
   ________

   (A) 492
   Addition of the carrying figure to the wrong column

   (B) 402
   Correct

   (C) 392
   Failure to add the carrying figure

   (D) 411
   Reversal of the carrying digit i.e. for 12 carry the 2
2.  
738  
+184  
- 93 -

(A)  
812  
(B)  
922  
Correct  
Failure to add the carrying figure  
Reversal of the carrying digit  
Correct  
Reversal of the carrying digit  

(C)  
1111  
(D)  
1112  
Carrying to the extreme left each time  
Carrying to the extreme left each time  

3.  
456  
+654

(A)  
1110  
Correct  
Failure to add the carrying figure  
Reversal of carrying digit  
Correct  
Carrying to the extreme left each time  

(C)  
1011  
(D)  
1200  
Carrying to the extreme left each time  
Carrying to the extreme left each time  

Sub-stage K  
To add numbers with up to 4 digits (with repeated carrying where necessary)

1.  
893  
+ 19

(A)  
802  
(B)  
1011  
Reversal of the carrying digit  
Failure to add the carrying figure  
Correct  
Failure to add the final digit  

(C)  
912  
(D)  
12  
Correct  
Failure to add the final digit  

2.  
2988  
+2988

(A)  
7866  
(B)  
12121  
Reversal of carrying figure  
Carrying to the extreme left each time  

(C)  
4866  
(D)  
5976  
Failure to add the carrying digit  
Correct
3. \[
\begin{array}{c}
1375 \\
+527 \\
\hline
1892
\end{array}
\]

(A) 1892  
Failure to add the carrying figure  
(B) 892  
Failure to add the final digit  
(C) 1902  
Correct  
(D) 1911  
Reversal of the carrying digit

Sub-stage L  
To add numbers with up to 5 digits (carrying where necessary)

1. \[
\begin{array}{c}
14577 \\
+5828 \\
\hline
20405
\end{array}
\]

(A) 71111  
Reversal of the carrying digit  
(B) 10405  
Failure to add the final digit  
(C) 19395  
Failure to add the carrying figure  
(D) 20405  
Correct

2. \[
\begin{array}{c}
54321 \\
+89 \\
\hline
54410
\end{array}
\]

(A) 54410  
Correct  
(B) 110  
Failure to add the final digit  
(C) 54300  
Failure to add the carrying figure  
(D) 54311  
Reversal of the carrying figure

3. \[
\begin{array}{c}
31673 \\
+48717 \\
\hline
79380
\end{array}
\]

(A) 79380  
Failure to add the carrying figure  
(B) 80390  
Correct  
(C) 91181  
Reversal of the carrying figure  
(D) 99380  
Carrying to the extreme left each time
6.4.2 The Objective Diagnostic Subtraction Test

Sub-stage C  To subtract a 1-digit number from a 2-digit number (no borrowing)

1. 48  - 4

(A)  8  Diagonal subtraction  (B)  43  Incorrect number bond
(C)  45  Incorrect number bond  (D)  44  Correct

2. 39  - 2

(A)  37  Correct  (B)  19  Diagonal subtraction
(C)  36  Incorrect number bond  (D)  38  Incorrect number bond

Sub-stage D  To subtract a 1-digit number from a 2-digit number (with borrowing)

1. 34  - 9

(A)  30  Lack of borrowing concept  (B)  25  Correct
(C)  35  Inverted subtraction for convenience  (D)  21  Failure to add on the residue after borrowing
2.  
\[
\begin{array}{c}
\text{(A) 47} \\
\text{Inverted subtraction for convenience}
\end{array}
\quad
\begin{array}{c}
\text{(B) 33} \\
\text{Correct}
\end{array}
\]  
\[
\begin{array}{c}
\text{(C) 40} \\
\text{Lack of borrowing concept}
\end{array}
\quad
\begin{array}{c}
\text{(D) 32} \\
\text{Failure to add on the residue after borrowing}
\end{array}
\]

3.  
\[
\begin{array}{c}
\text{(A) 71} \\
\text{Failure to add on the residue after borrowing}
\end{array}
\quad
\begin{array}{c}
\text{(B) 82} \\
\text{Inverted subtraction}
\end{array}
\]  
\[
\begin{array}{c}
\text{(C) 80} \\
\text{Lack of borrowing concept}
\end{array}
\quad
\begin{array}{c}
\text{(D) 78} \\
\text{Correct}
\end{array}
\]

Sub-stage E  
To subtract using two 2-digit numbers
(no borrowing)

1.  
\[
\begin{array}{c}
\text{(A) 16} \\
\text{Borrowing when not necessary}
\end{array}
\quad
\begin{array}{c}
\text{(B) 34} \\
\text{Incorrect borrowing procedure}
\end{array}
\]  
\[
\begin{array}{c}
\text{(C) 26} \\
\text{Correct}
\end{array}
\quad
\begin{array}{c}
\text{(D) 71} \\
\text{Diagonal subtraction}
\end{array}
\]

2.  
\[
\begin{array}{c}
\text{(A) 45} \\
\text{Correct}
\end{array}
\quad
\begin{array}{c}
\text{(B) 36} \\
\text{Diagonal subtraction}
\end{array}
\]  
\[
\begin{array}{c}
\text{(C) 35} \\
\text{Borrowing when not necessary}
\end{array}
\quad
\begin{array}{c}
\text{(D) 55} \\
\text{Incorrect borrowing procedure}
\end{array}
\]
### Sub-stage F

To subtract using two 2-digit numbers (with borrowing)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>72</td>
<td>-45</td>
<td></td>
</tr>
<tr>
<td>(A)</td>
<td>33</td>
<td>Inverted subtraction for convenience</td>
<td>(B)</td>
</tr>
<tr>
<td>(C)</td>
<td>27</td>
<td>Correct</td>
<td>(D)</td>
</tr>
<tr>
<td>2.</td>
<td>95</td>
<td>-49</td>
<td></td>
</tr>
<tr>
<td>(A)</td>
<td>41</td>
<td>Failure to add on the residue after borrowing</td>
<td>(B)</td>
</tr>
<tr>
<td>(C)</td>
<td>46</td>
<td>Correct</td>
<td>(D)</td>
</tr>
<tr>
<td>3.</td>
<td>74</td>
<td>-38</td>
<td></td>
</tr>
<tr>
<td>(A)</td>
<td>44</td>
<td>Inverted subtraction for convenience</td>
<td>(B)</td>
</tr>
<tr>
<td>(C)</td>
<td>46</td>
<td>Failure to modify the total after borrowing</td>
<td>(D)</td>
</tr>
</tbody>
</table>
Sub-stage G  To subtract using two 3-digit numbers
(no borrowing)

1.  516
    -305

(A)  211  \hspace{2cm}  (B)  201
Correct  Incorrect number bonds

(C)  210  \hspace{2cm}  (D)  212
Incorrect number bonds  Incorrect number bonds

2.  964
    -904

(A)  160  \hspace{2cm}  (B)  900
Incorrect number bonds  Incorrect number bonds

(C)  60  \hspace{2cm}  (D)  65
Correct  Incorrect number bonds

3.  654
    -123

(A)  631  \hspace{2cm}  (B)  530
Incorrect number bonds  Incorrect number bonds

(C)  531  \hspace{2cm}  (D)  532
Correct  Incorrect number bonds

Sub-stage H  To subtract using two 3-digit numbers
(borrowing units)

1.  412
    -305

(A)  113  \hspace{2cm}  (B)  107
Inverted subtraction  Correct
for convenience

(C)  105  \hspace{2cm}  (D)  117
Failure to add on the  Failure to modify the
residue after borrowing  total after borrowing
<p>| | | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>674</td>
<td></td>
<td>164</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-518</td>
<td></td>
<td>Inverted subtraction for convenience</td>
<td></td>
</tr>
<tr>
<td>(A) 156</td>
<td>Correct</td>
<td>(B) 166</td>
<td>Failure to modify the total after borrowing</td>
<td></td>
</tr>
<tr>
<td>(C) 152</td>
<td>Failure to add on the residue after borrowing</td>
<td>(D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>762</td>
<td></td>
<td>337</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-435</td>
<td></td>
<td>Failure to modify the total after borrowing</td>
<td></td>
</tr>
<tr>
<td>(A) 327</td>
<td>Correct</td>
<td>(B) 333</td>
<td>Inverted subtraction for convenience</td>
<td></td>
</tr>
<tr>
<td>(C) 325</td>
<td>Failure to add on the residue after borrowing</td>
<td>(D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td>Sub-stage I</td>
<td>To subtract using two 3-digit numbers (borrowing tens)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A) 473</td>
<td>Failure to modify the total after borrowing</td>
<td>(B) 363</td>
<td>Failure to add on the residue after borrowing</td>
<td></td>
</tr>
<tr>
<td>(C) 433</td>
<td>Inverted subtraction for convenience</td>
<td>(D) 373</td>
<td>Correct</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A) 384</td>
<td>Correct</td>
<td>(B) 484</td>
<td>Failure to modify the total after borrowing</td>
<td></td>
</tr>
<tr>
<td>(C) 404</td>
<td>Lack of borrowing concept</td>
<td>(D) 424</td>
<td>Inverted subtraction for convenience</td>
<td></td>
</tr>
</tbody>
</table>
Sub-stage J  To subtract using two 3-digit numbers
(borrowing where necessary - one place only)

1.  556
    - 127

   (A)  443
        Failure to add on the residue after borrowing

   (B)  439
        Failure to modify the total after borrowing

   (C)  431
        Inverted subtraction for convenience

   (D)  429
        Correct

2.  508
    - 337

   (A)  201
        Lack of borrowing concept when dealing with zero

   (B)  231
        Inverted subtraction for convenience

   (C)  171
        Correct

   (D)  271
        Failure to modify the total after borrowing

3.  208
    - 172

   (A)  106
        Lack of borrowing concept when dealing with zero

   (B)  36
        Correct

   (C)  176
        Inverted subtraction for convenience

   (D)  136
        Failure to modify the total after borrowing
Sub-stage K  To subtract using two 3-digit numbers
(with repeated borrowing)

1.  

(A) 347
Inverted subtraction for convenience

(B) 253
Correct

(C) 343
Failure to add on the residue after borrowing

(D) 363
Failure to modify the total after borrowing

2.  

(A) 400
Lack of borrowing concept when dealing with zero

(B) 469
Inverted subtraction for convenience

(C) 441
Failure to modify the total after borrowing

(D) 331
Correct

3.  

(A) 488
Correct

(B) 411
Failure to add on the residue after borrowing

(C) 598
Failure to modify the total after borrowing

(D) 512
Inverted subtraction for convenience

Sub-stage L  To subtract using numbers with up to 4 digits
(with repeated borrowing)

1.  

(A) 3349
Failure to add on the residue after borrowing

(B) 4189
Failure to modify the total after borrowing

(C) 4121
Inverted subtraction for convenience

(D) 4079
Correct
2. 1700
   - 913
   ———

   (A) 813
    Lack of borrowing concept when dealing with zero
   (B) 787
    Correct

   (C) 887
    Failure to modify the total after borrowing
   (D) 1213
    Inverted subtraction for convenience

3. 1034
   - 55
   ———

   (A) 1021
    Inverted subtraction for convenience
   (B) 979
    Correct

   (C) 955
    Failure to add on the residue after borrowing
   (D) 989
    Failure to modify the total after borrowing

Sub-stage M To subtract using numbers with up to 5 digits (borrowing where necessary)

1. 29557
   - 4563
   ———

   (A) 25041
    Inverted subtraction for convenience
   (B) 26547
    Failure to add on the residue after borrowing

   (C) 25094
    Failure to modify the total after borrowing
   (D) 24994
    Correct

2. 1800
   - 1762
   ———

   (A) 38
    Correct
   (B) 138
    Failure to modify the total after borrowing

   (C) 162
    Inverted subtraction for convenience
   (D) 100
    Lack of borrowing concept when dealing with zero
3. \[ \begin{array}{c}
3721 \\
- 98
\end{array} \]

- (A) 3777
  Inverted subtraction for convenience

- (C) 3733
  Failure to modify the total after borrowing

- (B) 3623
  Correct

- (D) 3612
  Failure to add on the residue after borrowing

6.4.3 The Objective Diagnostic Multiplication Test (Part 1)

Sub-stage C To multiply a 2-digit number by a 1-digit number (no carrying)

1. \[ \begin{array}{c}
43 \\
x 2
\end{array} \]

- (A) 86
  Correct

- (C) 106
  Incorrect number bond

- (B) 64
  Incorrect number bond

- (D) 85
  Incorrect number bond

2. \[ \begin{array}{c}
40 \\
x 2
\end{array} \]

- (A) 82
  Incorrect multiplication by zero

- (C) 80
  Correct

- (B) 100
  Incorrect number bond

- (D) 40
  Incorrect number bond

3. \[ \begin{array}{c}
32 \\
x 3
\end{array} \]

- (A) 66
  Incorrect number bond

- (C) 64
  Incorrect number bond

- (B) 94
  Incorrect number bond

- (D) 96
  Correct
Sub-stage D  To multiply a 3-digit number by a 1-digit number (no carrying)

1.  213  
    x 3

(A)  609  (B)  642
Incorrect number bonds  Incorrect number bonds

(C)  939  (D)  639
Incorrect number bonds  Correct

2.  304  
    x 2

(A)  628  (B)  608
Incorrect multiplication by zero  Correct

(C)  610  (D)  632
Incorrect number bond  Incorrect number bond

3.  120  
    x 4

(A)  480  (B)  484
Correct  Incorrect multiplication by zero

(C)  490  (D)  500
Incorrect number bond  Incorrect number bond

Sub-stage E  To multiply a 2-digit number by a 1-digit number (carrying units)

1.  29  
    x 3

(A)  87  (B)  627
Correct  Carrying digit placed in answer

(C)  67  (D)  132
Failure to add the carrying figure  Carrying the wrong digit
2. 17
x 5

(A) 103
Carrying the wrong
digit

(B) 55
Failure to add the
carrying figure

(C) 535
Carrying digit placed
in answer

(D) 85
Correct

3. 19
x 4

(A) 76
Correct

(B) 103
Carrying the wrong
digit

(C) 436
Carrying digit placed
in the answer

(D) 46
Failure to add the
carrying figure

Sub-stage F To multiply a 3-digit number by a 1-digit
number (carrying units)

1. 116
x 5

(A) 550
Failure to add the
carrying figure

(B) 580
Correct

(C) 5530
Carrying digit placed
in the answer

(D) 553
Carrying the wrong digit

2. 208
x 4

(A) 823
Carrying the wrong
digit

(B) 802
Failure to add the
carrying figure

(C) 872
Incorrect
multiplication by
zero

(D) 832
Correct
3. \[ \begin{array}{c} 217 \\ \times 4 \end{array} \]

(A) 868  
Correct

(B) 8428  
Carrying digit placed in the answer

(C) 348  
Failure to add the carrying figure

(D) 1012  
Carrying the wrong digit

Sub-stage G  
To multiply a 3-digit number by a 1-digit number  
(carrying where necessary - one place only)

1. \[ \begin{array}{c} 329 \\ \times 3 \end{array} \]

(A) 1212  
Carrying the wrong digit

(B) 987  
Correct

(C) 9627  
Carrying digit placed in the answer

(D) 967  
Failure to add the carrying figure

2. \[ \begin{array}{c} 181 \\ \times 5 \end{array} \]

(A) 545  
Carrying the wrong digit

(B) 5405  
Carrying digit placed in the answer

(C) 905  
Correct

(D) 505  
Failure to add the carrying figure

3. \[ \begin{array}{c} 242 \\ \times 4 \end{array} \]

(A) 8168  
Carrying digit placed in the answer

(B) 868  
Failure to add the carrying figure

(C) 1418  
Carrying the wrong digit

(D) 968  
Correct
Sub-stage H  

To multiply a 3-digit number by a 1-digit number (with repeated carrying)

1. 253
   x 4
   ______
   (A) 1012  Correct
   (B) 802   Failure to add the carrying figure
   (C) 82012 Carrying digits placed in the answer
   (D) 1021  Carrying the wrong digit

2. 127
   x 6
   ______
   (A) 762   Correct
   (B) 1014  Carrying the wrong digit
   (C) 622   Failure to add the carrying figure
   (D) 61242 Carrying digits placed in the answer

3. 153
   x 5
   ______
   (A) 765   Correct
   (B) 52515 Carrying digits placed in the answer
   (C) 531   Carrying the wrong digit
   (D) 555   Failure to add the carrying figure

Sub-stage I  

To multiply a whole number by 10 or 100

1. 5
   x 100
   ______
   (A) 555 Incorrect multiplication by zero
   (B) 500 Correct
   (C) 5000 Incorrect number bond
   (D) 50 Incorrect number bond
2. \[
\begin{array}{c}
\text{(A)} \quad 6000 \\
\text{Correct}
\end{array}
\quad \begin{array}{c}
\text{(B)} \quad 6660 \\
\text{Incorrect multiplication by zero}
\end{array}
\begin{array}{c}
\text{(C)} \quad 600 \\
\text{Incorrect number bond}
\end{array}
\quad \begin{array}{c}
\text{(D)} \quad 60000 \\
\text{Incorrect number bond}
\end{array}
\]

3. \[
\begin{array}{c}
\text{(A)} \quad 4107 \\
\text{Incorrect multiplication by zero}
\end{array}
\quad \begin{array}{c}
\text{(B)} \quad 3700 \\
\text{Correct}
\end{array}
\begin{array}{c}
\text{(C)} \quad 370 \\
\text{Incorrect number bond}
\end{array}
\quad \begin{array}{c}
\text{(D)} \quad 37000 \\
\text{Incorrect number bond}
\end{array}
\]

6.4.4 The Objective Diagnostic Multiplication Test (Part 2)

Sub-stage J To multiply a 2-digit number by 20, 30, 40 or 50

1. \[
\begin{array}{c}
\text{(A)} \quad 12 \\
x20
\hline
40 \\
20 \\
60
\end{array}
\quad \begin{array}{c}
\text{(B)} \quad 12 \\
x20
\hline
20
\end{array}
\]

Multiplication by the tens digit only

\[\text{Vertical multiplication}\]

\[
\begin{array}{c}
\text{(C)} \quad 12 \\
x20
\hline
000 \\
240 \\
240
\end{array}
\quad \begin{array}{c}
\text{(D)} \quad 12 \\
x20
\hline
12 \\
240 \\
252
\end{array}
\]

Correct Incorrect multiplication by zero
2. \[ \begin{align*} 
(A) \quad 20 & \times 50 \\
 & \underline{100} \\

(B) \quad 20 & \times 50 \\
 & \underline{400} \\
 & \underline{1000} \\
 & \underline{1400} \\

Vertical multiplication & Multiplication by the tens digit only
\end{align*} \]

Reversing the place values of the digits of the multiplier and incorrect multiplication by zero

3. \[ \begin{align*} 
(A) \quad 97 & \times 40 \\
 & \underline{3880} \\
 & \underline{3880} \\

3880 & \underline{3880} \\

Correct & Vertical multiplication
\end{align*} \]

Reversing the place values of the digits of the multiplier

(B) \[ \begin{align*} 
97 & \times 40 \\
 & \underline{360} \\

3880 & \underline{360} \\
 & \underline{640} \\

Multiplication by the tens digit only
\end{align*} \]
Sub-stage K  To multiply two 2-digit numbers
( carry with units)

1.  \[ 12 \times 37 \]

(A) \[
\begin{array}{c}
12 \\
x37 \\
\hline 84 \\
36 \\
120
\end{array}
\]

(B) \[
\begin{array}{c}
12 \\
x37 \\
\hline 840 \\
36 \\
876
\end{array}
\]

Place value shift
ignored

Reversing the place
values of the digits
of the multiplier

(C) \[
\begin{array}{c}
12 \\
x37 \\
\hline 3604 \\
\end{array}
\]

(D) \[
\begin{array}{c}
12 \\
x37 \\
\hline 84 \\
360 \\
444
\end{array}
\]

Answers to basic
products placed in
one line

Correct

2.  \[ 56 \times 18 \]

(A) \[
\begin{array}{c}
56 \\
x18 \\
\hline 108 \\
\end{array}
\]

(B) \[
\begin{array}{c}
56 \\
x18 \\
\hline 448 \\
560 \\
1008
\end{array}
\]

Multiplication only of
units digit of the
multiplicand

Correct

(C) \[
\begin{array}{c}
56 \\
x18 \\
\hline 448 \\
56 \\
504
\end{array}
\]

(D) \[
\begin{array}{c}
56 \\
x18 \\
\hline 608
\end{array}
\]

Place value shift
ignored

Vertical and diagonal
multiplication
3. 63
   x29

(A)  63  
    x29  
    5670 
    126  
    5796 

Reversing the place 
values of the digits 
of the multiplier

(B)  63  
    x29  
    567  
    126  
    693 

Place value shift ignored

(C)  63  
    x29  
    567  
    1260  
    1827 

Correct

Vertical and diagonal multiplication

Sub-stage L

To multiply two 2-digit numbers
(carrying where necessary)

1. 26  
    x57  

(A)  26  
    x57  
    132  
    1300  
    1482 

Correct

(B)  26  
    x57  
    182  
    130  
    312 

Place value shift ignored

(C)  26  
    x57  
    142  

Vertical multiplication

(D)  26  
    x57  
    1342  

Vertical and diagonal multiplication
## Exercise 2

<table>
<thead>
<tr>
<th>Column</th>
<th>Expression</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>93 x 24</td>
<td>192</td>
</tr>
<tr>
<td>B</td>
<td>93 x 24</td>
<td>1872</td>
</tr>
<tr>
<td>C</td>
<td>93 x 24</td>
<td>372</td>
</tr>
<tr>
<td></td>
<td>1860</td>
<td>2232</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>Vertical multiplication</td>
</tr>
<tr>
<td>D</td>
<td>93 x 24</td>
<td>372</td>
</tr>
<tr>
<td></td>
<td>186</td>
<td>558</td>
</tr>
</tbody>
</table>

**Correct Place value shift ignored**

## Exercise 3

<table>
<thead>
<tr>
<th>Column</th>
<th>Expression</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14 x 87</td>
<td>108</td>
</tr>
<tr>
<td>B</td>
<td>14 x 87</td>
<td>108</td>
</tr>
<tr>
<td>C</td>
<td>14 x 87</td>
<td>1148</td>
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<td></td>
<td>98</td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td>112</td>
<td>210</td>
</tr>
</tbody>
</table>

**Correct Vertical multiplication**

**Place value shift ignored**

**Vertical and diagonal multiplication**
Sub-stage M  

To multiply a 3-digit number by a 2-digit number (carrying where necessary - one place only)

1. \[ 121 \times 32 \]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>121</td>
<td>x 32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>362</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Vertical multiplication

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(C)</td>
<td>121</td>
<td>x 32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>242</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>362</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3872</td>
</tr>
</tbody>
</table>

Correct Place value shift ignored

2. \[ 113 \times 26 \]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>113</td>
<td>x 26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6678</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2260</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8878</td>
</tr>
</tbody>
</table>

Basic product placed on one line with no carrying of digits

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(C)</td>
<td>113</td>
<td>x 26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>678</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2260</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2938</td>
</tr>
</tbody>
</table>

Correct Wrong place value shift

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(D)</td>
<td>113</td>
<td>x 26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>678</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2260</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2938</td>
</tr>
</tbody>
</table>

Correct Place value shift ignored
3. \[ \begin{array}{c}
\text{(A)} & \text{(B)} \\
231 \times 32 & 231 \times 32 \\
693462 & \underline{462} \\
\end{array} \]

Answers to basic products placed on one line

Correct

(C) \[ \begin{array}{c}
\text{(C)} \\
231 \times 32 \\
\underline{462} \\
693 \\
7155 \\
\end{array} \]

Place value shift ignored

Wrong place value shift

Sub-stage N To multiply a 3-digit number by a 2-digit number (with repeated carrying)

1. \[ \begin{array}{c}
\text{(A)} & \text{(B)} & \text{(C)} & \text{(D)} \\
414 \times 89 & 414 \times 89 & 414 \times 89 & 414 \times 89 \\
\underline{36936} & \underline{3726} & \underline{33120} & \underline{3726} \\
328320 & 33120 & 364256 & 368460 \\
\end{array} \]

Answer to basic product placed on one line with no carrying of digits

Correct

Place value shift ignored

Wrong place value shift
2. \[ \begin{array}{c}
(A) \ 267 \\
\times \ 65 \\
\hline
1335 \\
1602 \\
2937 \\
\hline
(B) \ 267 \\
\times \ 65 \\
\hline
13350 \\
160200 \\
173550 \\
\end{array} \]

Place value shift ignored
Wrong place value shift

(C) \[ \begin{array}{c}
267 \\
\times \ 65 \\
\hline
1335 \\
16020 \\
17355 \\
\end{array} \]
Correct

Vertical multiplication

3. \[ \begin{array}{c}
(A) \ 987 \\
\times \ 78 \\
\hline
78960 \\
690900 \\
769860 \\
\hline
(B) \ 987 \\
\times \ 78 \\
\hline
7896 \\
69090 \\
76986 \\
\end{array} \]
Wrong place value shift
Correct

(C) \[ \begin{array}{c}
987 \\
\times \ 78 \\
\hline
7896 \\
6909 \\
14805 \\
\end{array} \]
Place value shift ignored
Vertical multiplication
Sub-stage 0  To multiply two 3-digit numbers (carrying
where necessary)

1.  \[ 137 \times 246 \]

- **A)** 137
- **B)** 137
- **C)** 137
- **D)** 137

```
(A) 137
x246
822
548
274
1644
---
337020
```

Ignoring place value shift

Wrong place value shift

2.  \[ 638 \times 208 \]

- **A)** 638
- **B)** 638
- **C)** 638
- **D)** 638

```
(A) 638
x208
3104
0000
127600
132704
---
133004
```

Correct

Wrong place value shift

```
(C) 638
x208
3104
0000
127600
6380
---
133004
```

Ignoring place value shift

Incorrect number bond

Correct product placed on one line with no carrying
3. \[ \begin{array}{c c c}
(A) & 805 & \times 354 \\
& & \\
\hline
& 4020 & \\
& 40250 & \\
& 241500 & \\
& 285770 & \\
\end{array} \]

Incorrect number bond

\[ \begin{array}{c c c}
(C) & 805 & \times 354 \\
& & \\
\hline
& 3220 & \\
& 40250 & \\
& 241500 & \\
& 284970 & \\
\end{array} \]

Wrong place value shift

6.4.5 The Objective Diagnostic Division Test

Sub-stage C To do simple division by a 1-digit number (no remainder)

1. \[ 5 \overline{30} \]

(A) 0 Lack of carrying concept

(B) 6 Correct

(C) 0.3 Lack of internal carrying concept

(D) 7 Incorrect number bond

2. \[ 8 \overline{56} \]

(A) 7 Correct

(B) 1.3 Tens digit not exchanged for units

(C) 0 Lack of carrying concept

(D) 8 Incorrect number bond
Sub-stage D  

To do simple division by a 1-digit number
(with remainder)

1. \( \frac{6156}{6} \)

(A) 1  
Lack of carrying concept

(B) 9  
Remainder omitted

(C) 9 \%  
Correct

(D) 1\%  
Placing internal remainder as external remainder; lack of place value notion

2. \( \frac{7364}{7} \)

(A) 0  
Lack of carrying concept

(B) 1\%  
Tens digit not exchanged for units

(C) 9 \%  
Correct

(D) 9  
Remainder omitted

3. \( \frac{933}{9} \)

(A) 9 \%  
Correct

(B) 1\%  
Tens digit not exchanged for units

(C) 0  
Lack of carrying concept

(D) 9  
Remainder omitted
Sub-stage E To divide a 3-digit number by a 1-digit number (no remainder)

1. \[ \frac{5}{\overline{525}} \]
   (A) 15
   Zero omitted
   (B) 11
   Lack of carrying concept including omission of zero
   (C) 101
   Lack of carrying concept
   (D) 105
   Correct

2. \[ \frac{3}{\overline{554}} \]
   (A) 211
   Lack of carrying concept
   (B) 211\(\frac{1}{2}\)
   No internal carrying of the remainder
   (C) 218
   Correct
   (D) 219
   Incorrect number bond

3. \[ \frac{8}{\overline{944}} \]
   (A) 118
   Correct
   (B) 100
   Lack of carrying concept
   (C) 106\(\frac{1}{2}\)
   Placing the remainder externally each time
   (D) 116
   Incorrect number bond

Sub-stage F To divide a 3-digit number by a 1-digit number (with remainder)

1. \[ \frac{4}{\overline{328}} \]
   (A) 201\(\frac{1}{2}\)
   Lack of carrying concept
   (B) 26\(\frac{3}{4}\)
   Zero omitted
   (C) 206\(\frac{3}{4}\)
   Correct
   (D) 206
   Remainder omitted
Sub-stage G To divide a 4-digit number by a 1-digit number
(with and without remainder)

1. 312638

(A) 212 \(\frac{2}{3}\)  
Lack of carrying concept

(B) 879  
Remainder omitted

(C) 879 \(\frac{2}{3}\)  
Correct

(D) 879 \(\frac{2}{3}\)  
Remainder inverted

2. 977777

(A) 0  
Lack of carrying concept

(B) 864 \(\frac{2}{3}\)  
Correct

(C) 864  
Remainder omitted

(D) 864 \(\frac{2}{3}\)  
Remainder inverted

3. 43568

(A) 2342  
Correct

(B) 2012  
Lack of carrying concept

(C) 2112 \(\frac{2}{3}\)  
Internal remainder carried as units each time

(D) 2112 \(\frac{2}{3}\)  
Internal remainder carried as units each time and external remainder inverted
Sub-stage H  To divide a 3-digit number by a multiple of ten (no remainder)

1. \( \frac{30}{270} \)
   - (A) 3 Incorrect number bond
   - (B) 10 Incorrect number bond
   - (C) 7 Incorrect number bond
   - (D) 9 Correct

2. \( \frac{40}{560} \)
   - (A) 14 Correct
   - (B) 140 Incorrect number bond
   - (C) 13 Incorrect number bond
   - (D) 1400 Incorrect number bond

3. \( \frac{50}{2500} \)
   - (A) 500 Incorrect number bond
   - (B) 50 Correct
   - (C) 5 Incorrect number bond
   - (D) 5000 Incorrect number bond

Sub-stage I  To divide a 3-digit number by a 2-digit number (no remainder)

1. \( \frac{14}{518} \)
   - (A) 30 \( \frac{30}{9} \)
   - (B) 38 \( \frac{38}{6} \)

 Failure to bring down the next digit
   - (C) 217 \( \frac{217}{9} \)
   - (D) 37 \( \frac{37}{6} \)

 Leaving internal remainder greater than divisor
   - (C) 217 \( \frac{217}{9} \)
   - (D) 37 \( \frac{37}{6} \)

 Incorrect subtraction
2. \(19 \overline{798}\)

(A) \[
\begin{array}{c}
42 \\
19 \overline{798} \\
-76 \\
38 \\
-38 \\
00 \\
\end{array}
\]
Correct

(B) \[
\begin{array}{c}
40 \frac{2}{9} \\
19 \overline{798} \\
-76 \\
3 \\
\end{array}
\]
Failure to bring down the next digit

(C) \[
\begin{array}{c}
47 \frac{5}{9} \\
19 \overline{798} \\
-76 \\
138 \\
-133 \\
5 \\
\end{array}
\]
Incorrect subtraction

(D) \[
\begin{array}{c}
312 \\
19 \overline{798} \\
-57 \\
228 \\
-228 \\
000 \\
\end{array}
\]
Leaving the internal remainder greater than the divisor

3. \(28 \overline{672}\)

(A) \[
\begin{array}{c}
24 \\
28 \overline{672} \\
-56 \\
112 \\
-112 \\
000 \\
\end{array}
\]
Correct

(B) \[
\begin{array}{c}
20 \frac{28}{2} \\
28 \overline{672} \\
-56 \\
12 \\
\end{array}
\]

(C) \[
\begin{array}{c}
20 \frac{28}{2} \\
28 \overline{672} \\
-56 \\
11 \\
\end{array}
\]
Failure to bring down the next digit

(D) \[
\begin{array}{c}
114 \\
28 \overline{672} \\
-28 \\
392 \\
-392 \\
000 \\
\end{array}
\]
Leaving the internal remainder greater than the divisor
Sub-stage J To divide a 3-digit number by a 2-digit number (with remainder)

1. \[24 \div 35\]

(A) \[
\begin{array}{c}
24 \overline{)35} \\
-72
\end{array}
\begin{array}{c}
53
-120
\end{array}
\begin{array}{c}
13
\end{array}
\]

Remainder omitted

(B) \[
\begin{array}{c}
24 \overline{)35}\ 35 \div 35
\end{array}
\begin{array}{c}
-72
\end{array}
\begin{array}{c}
13
\end{array}
\]

Remainder inverted

(C) \[
\begin{array}{c}
24 \overline{)35}\ 35 \div 35
\end{array}
\begin{array}{c}
53
-120
\end{array}
\begin{array}{c}
13
\end{array}
\]

Correct

(D) \[
\begin{array}{c}
24 \overline{)35}\ 35 \div 35
\end{array}
\begin{array}{c}
-72
\end{array}
\begin{array}{c}
13
\end{array}
\]

Remainder inverted

2. \[57 \div 37\]

(A) \[
\begin{array}{c}
57 \overline{)16}
\end{array}
\begin{array}{c}
-57
\end{array}
\begin{array}{c}
342
\end{array}
\begin{array}{c}
34
\end{array}
\]

Remainder omitted

(B) \[
\begin{array}{c}
57 \overline{)16}\ 16 \div 37
\end{array}
\begin{array}{c}
-57
\end{array}
\begin{array}{c}
376
\end{array}
\begin{array}{c}
34
\end{array}
\]

Remainder inverted

(C) \[
\begin{array}{c}
57 \overline{)16}\ 16 \div 37
\end{array}
\begin{array}{c}
-57
\end{array}
\begin{array}{c}
376
\end{array}
\begin{array}{c}
342
\end{array}
\]

Remainder inverted

(D) \[
\begin{array}{c}
57 \overline{)16}\ 16 \div 37
\end{array}
\begin{array}{c}
-57
\end{array}
\begin{array}{c}
376
\end{array}
\begin{array}{c}
342
\end{array}
\]

Correct
Sub-stage K To divide a 4-digit number by a 2-digit number (with and without remainder)

1. 84 \overline{507}

(A) \[
\begin{array}{c}
84 \overline{507} \\
-84 \\
\hline
1
\end{array}
\]

Correct

(B) \[
\begin{array}{c}
84 \overline{507} \\
-84 \\
-107 \\
\hline
-23
\end{array}
\]

Failure to bring down the necessary digits

(C) \[
\begin{array}{c}
84 \overline{507} \\
-84 \\
-107 \\
\hline
-23
\end{array}
\]

Remainder omitted

(D) \[
\begin{array}{c}
84 \overline{507} \\
-84 \\
-107 \\
\hline
-23
\end{array}
\]

Zero omitted

3. 13 \overline{8711}

(A) \[
\begin{array}{c}
13 \overline{8711} \\
-78 \\
-26 \\
\hline
5
\end{array}
\]

Remainder omitted

(B) \[
\begin{array}{c}
60 \frac{3}{3} \\
-78 \\
\hline
3
\end{array}
\]

Failure to bring down the next digit

(C) \[
\begin{array}{c}
62 \frac{3}{3} \\
13 \overline{8711} \\
-78 \\
-26 \\
\hline
5
\end{array}
\]

Correct

(D) \[
\begin{array}{c}
62 \frac{3}{3} \\
13 \overline{8711} \\
-78 \\
-26 \\
\hline
5
\end{array}
\]

Remainder inverted
2. \( \frac{93}{34} \) \( \frac{3181}{34} \)

(A) \[
\begin{array}{c}
34)3181 \\
-306 \\
\hline
121 \\
-102 \\
\hline
19
\end{array}
\]

Correct

(B) \[
\begin{array}{c}
34)3181 \\
-306 \\
\hline
121 \\
-102 \\
\hline
19
\end{array}
\]

Remainder inverted

(C) \[
\begin{array}{c}
34)3181 \\
-306 \\
\hline
121 \\
-102 \\
\hline
19
\end{array}
\]

(D) \[
\begin{array}{c}
34)3181 \\
-306 \\
\hline
121 \\
-102 \\
\hline
19
\end{array}
\]

Failure to bring down the next digit

3. \( \frac{209}{13} \) \( \frac{2718}{13} \)

(A) \[
\begin{array}{c}
13)2718 \\
-26 \\
\hline
118 \\
-117 \\
\hline
001
\end{array}
\]

Zero omitted

(B) \[
\begin{array}{c}
13)2718 \\
-26 \\
\hline
118 \\
-117 \\
\hline
001
\end{array}
\]

Correct

(C) \[
\begin{array}{c}
13)2718 \\
-26 \\
\hline
118 \\
-117 \\
\hline
001
\end{array}
\]

(D) \[
\begin{array}{c}
13)2718 \\
-26 \\
\hline
118 \\
-117 \\
\hline
001
\end{array}
\]

Remainder omitted

Remainder omitted

Zero and remainder omitted
CHAPTER 7

A TRIAL OF THE OBJECTIVE DIAGNOSTIC TESTS

7.1 Introduction

The five tests compiled by the writer were administered to approximately ninety children in the fourth year of a middle school; the children being 12-13 years old. The tests were administered over a period of a school term (Autumn Term, 1979), hence the numbers of children who completed each test varied due to absenteeism.

The marking of the test was simple; a mark of 1 being given for a correct response and 0 for an incorrect response. Thus the actual marking was very quick but the analysis was somewhat slower. Although the final score was of little or no importance for the diagnostic test, this marking system did pinpoint the erroneous responses very quickly. The writer noted the incorrect response on the answer sheet and compared it to that of the relevant test in Chapter 6 which was used as the key to reveal the algorithm employed to record that response.

The diagnostic tests were set to reveal the deficiencies of individual children but because of the numbers participating in the tests, it was not practical in this project to analyse the responses of each child. Therefore the writer proposes to analyse the responses offered by a fairly random sample of four children for each of the tests. The reason the sample was not totally random was that it generally consisted of two boys.
and two girls who had made a number of incorrect responses. The sample of four children so chosen was different for each of the tests and so, in all, the responses of twenty different children were analysed. It is hoped that the analysis of twenty different children's responses will show that the tests are suitable for revealing the deficiencies of a variety of children rather than a select band of just a few children (usually the least able).

The aim of the analysis was to suggest various ways in which the tests may be used to identify deficiencies and as such there was, deliberately, no set pattern of analysis for the five tests. For each of the tests the analysis was varied in order to indicate how the tests could be used to reveal deficiencies arising in particular sub-stages and also how, when occurring in a number of sub-stages, these deficiencies can be categorised.

Besides being used to reveal the deficiencies of the individuals the tests can also be used to reveal the deficiencies, or otherwise, of the whole population participating in the tests. Some methods of doing this are mentioned in the analysis of the tests. Again these methods are by no means exhaustive and are merely presented to indicate the versatility of the tests in analysing the algorithmic deficiencies of not only the individual but of the whole population.

In the analysis reference is made to various 'standard' algorithms. The standard algorithm is taken to be the method
that one usually applies to various computation skills and will be exemplified in Appendix A.

Often in the analysis individual distractors are referred to. For ease of writing these have been written in code form but are easily understood as exemplified by the following two examples:

- \text{F.1.B} \text{ refers to sub-stage F, item 1, distractor B}
- \text{K.3.D} \text{ refers to sub-stage K, item 3, distractor D.}

The first letter refers to the sub-stage, the digit refers to the item and the second letter refers to the distractor. The lettered sub-stage refers to the sub-stages discussed in Chapter 4.

7.2 Instructions for the Objective Diagnostic Tests

None of the children had experienced objective tests and so there was a need for them to be introduced to such tests. The following examples were used to exemplify the tests; the children were instructed to circle the letter on the answer key corresponding to their response.

\begin{align*}
\text{Example 1} & \quad 8 \\
& +2 \\
& \quad \text{(A) 9} \quad \text{(B) 10} \quad \text{(C) 12} \quad \text{(D) 6} \\
\text{Example 2} & \quad 9 \\
& -3 \\
& \quad \text{(A) 5} \quad \text{(B) 7} \quad \text{(C) 12} \quad \text{(D) 6} \\
\text{Example 3} & \quad 5 \\
& \times 5 \\
& \quad \text{(A) 10} \quad \text{(B) 25} \quad \text{(C) 20} \quad \text{(D) 30}
\end{align*}
The corresponding answer key for a child might be:

1. A B C D
2. A B C D
3. A B C D

No children had any difficulty in grasping the method of indicating their responses to a particular item.

A particular instruction that was emphasised was that if a child could not attempt an item then he was not to guess but should draw a line through all the responses on the answer key for that particular item. To help reduce the problem of guessing it was emphasised that the final score was of no importance and that the aim of the tests was to discover areas of deficiencies in their knowledge in order that a teacher might help them.

Because, as explained in Chapters 5 and 6, we cannot predict all the responses made by each and every child, the writer asked the children to write down their responses, complete with working out, if it differed from those of the distractor. As the correct response was always available among the distractors a differing response was treated as a non-response in the analysis. The writer felt that this would help reduce guessing and also help in modifying the distractors for later tests; any distractor having a facility value of zero could be replaced by one of these differing responses if it were appropriate.

There was enough space left on the question sheets for the child to perform any working out that may have been required.
Any working out that was required was written on the question sheet and then the appropriate response selected; this response was indicated on the answer key by circling the appropriate letter. The children were informed that there was no time limited and that they could use aids (except calculators) as required. (The actual aids used were multiplication squares.)

7.3 The Objective Diagnostic Addition Test

C.

1. 41 + 6
   (A) 107 (B) 11 (C) 47 (D) 57

2. 6 + 13
   (A) 19 (B) 79 (C) 10 (D) 49

3. 82 + 7
   (A) 109 (B) 159 (C) 89 (D) 17

D.

1. 29 + 4
   (A) 16 (B) 33 (C) 73 (D) 23

2. 6 + 47
   (A) 43 (B) 113 (C) 18 (D) 53

3. 55 + 7
   (A) 52 (B) 132 (C) 62 (D) 18
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|    | 1. | 119 | +283 |   |
| (A) 492 | (B) 402 | (C) 392 | (D) 411 |
| 2. |   | 738 | +184 |   |
| (A) 812 | (B) 922 | (C) 1111 | (D) 1112 |
| 3. |   | 456 | +654 |   |
| (A) 1110 | (B) 1000 | (C) 1011 | (D) 1200 |</p>
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7.4 The Objective Diagnostic Addition Test answer sheet

Name: __________________________

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When a child has finished the test he transfers his answers to this sheet. Any questions he cannot answer he must put a line through all the letters A, B, C, D for that question.
7.5 Objective Diagnostic Addition Test item response chart

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7.6 The four sample answer keys to be analysed

A correct response on the answer sheet is indicated by a 1 whilst an incorrect response is indicated by a 0. Occasionally a child may have considered that all the alternative answers were incorrect and he may have offered his own solution which is indicated by his having ruled a line through the alternative offered and then writing in his own answer.
## Objective Diagnostic Addition Test answer sheet

Name: Vanda

| C. | 1. A | B | C | D | 0 |
| 2. A | B | C | D | 1 |
| 3. A | B | C | D | 1 |

| D. | 1. A | B | C | D | 0 |
| 2. A | B | C | D | 0 |
| 3. A | B | C | D | 0 |

| E. | 1. A | B | C | D | 1 |
| 2. A | B | C | D | 1 |
| 3. A | B | C | D | 1 |

| F. | 1. A | B | C | D | 1 |
| 2. A | B | C | D | 1 |
| 3. A | B | C | D | 0 |

| G. | 1. A | B | C | D | 1 |
| 2. A | B | C | D | 1 |
| 3. A | B | C | D | 1 |

| H. | 1. A | B | C | D | 1 |
| 2. A | B | C | D | 1 |
| 3. A | B | C | D | 1 |

| I. | 1. A | B | C | D | 1 |
| 2. A | B | C | D | 0 |
| 3. A | B | C | D | 1 |

| J. | 1. A | B | C | D | 1 |
| 2. A | B | C | D | 1 |
| 3. A | B | C | D | 1 |

| K. | 1. A | B | C | D | 1 |
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| 3. A | B | C | D | 1 |

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Objective Diagnostic Addition Test answer sheet

Name: Angela

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Objective Diagnostic Addition Test answer sheet

Name:   Paul R.

C. 1. A B C D 1  
    2. A B C D 1  
    3. A B C D 1

D. 1. A B C D 1  
    2. A B C D 1  
    3. A B C D 1

E. 1. A B C D 1  
    2. A B C D 1  
    3. A B C D 0

F. 1. A B C D 1  
    2. A B C D 0  
    3. A B C D 1

G. 1. A B C D 1  
    2. A B C D 1  
    3. A B C D 1

H. 1. A B C D 1  
    2. A B C D 1  
    3. A B C D 0

I. 1. A B C D 1  
    2. A B C D 1  
    3. A B C D 1

J. 1. A B C D 1  
    2. A B C D 1  
    3. A B C D 0

K. 1. A B C D 0  
    2. A B C D 0  
    3. A B C D 0

L. 1. A B C D 0  
    2. A B C D 0  
    3. A B C D 0
7.7 Analysis of the Objective Diagnostic Addition Test

Ninety-six 12-13 years old children participated in the test and the item response chart (Section 7.5) indicates, as was to be expected, that few children experienced difficulty with the items. The 'differing answer' option was little used and in each case arose through erroneous number bonds rather than through the use of algorithms differing from those catered for in the construction of the distractors. Thus from the analysis it can be fairly confidently stated that, overall, these particular ninety-six 12-13 years old children had a sound grasp of the addition algorithm which is, of course, what one would hope for and expect. However the test did reveal algorithmic difficulties that certain individuals experienced. Three items were set to represent each sub-stage and, as a general rule, it seems reasonable to assume that if a child gave at least two incorrect responses per sub-stage then that child was experiencing some difficulty. Consider the responses proposed by Vanda, Paul C., Angela and Paul R. (Section 7.6).

Vanda's responses indicate an inconsistency when applying the addition algorithm for she answered what one feels to be the items of the easy sub-stage D (the addition of a 2-digit number and a 1-digit number: carrying units as tens) completely incorrectly and yet proceeded to answer the other sub-stages fairly successfully. Obviously Vanda needs to experience more of the activities of the type involved in sub-stage D and her difficulties arising from this type of
problem discussed. Analysis of Vanda's choice of distractors reveals that, in all but one of the cases, an incorrect response was due to her having ignored the carrying figure. Hopefully, a few sessions spent discussing and experiencing similar situations will help her overcome this problem.

Paul C.'s responses, like Vanda's, again show a lack of consistency in the application of the addition algorithm and again it was the items of the earlier sub-stages he found difficulty with. For each of the sub-stages C and D Paul only gave one correct response. In both these sub-stages he applied the 'diagonal' addition algorithm and, indeed, indicated his awareness of the 'carrying' concept by including the carrying figure in his answers for sub-stage D. It seems that Paul's difficulties arise when dealing with the addition of smaller numbers when the option is open for him to apply the diagonal algorithm (he is probably trying to apply a multiplication algorithm to addition). It appears that when dealing with large numbers Paul cannot apply this algorithm and he has to resort to the standard algorithm with which he is not too confident; in both the sub-stages F and G he prefers to add all the digits together, ignoring their place value, whilst in sub-stage K he applied the standard algorithm but is unsure of the placement of the carrying figure. Clearly Paul has problems applying the standard addition algorithm. He is aware of this algorithm but he is not at all confident in its application and, indeed, he prefers to use others where possible. A great deal of time needs to be devoted to Paul
in explaining the errors of his own algorithms and in giving him the experience in the consistent application of the standard algorithm.

Paul R. is fairly successful in applying the addition algorithm to all the items of the sub-stages except for those of K and L which involve the addition of large numbers (the numbers not necessarily having the same number of digits). Even in the earlier sub-stages Paul shows a lack of consistency in the application of the addition algorithm and in sub-stage E he gave an incorrect response due to him adding the digits horizontally whilst in sub-stage F he added all the digits (ignoring place value) including the carrying figure. In sub-stages K and L all the responses were incorrect showing Paul's inability to apply the addition algorithm to numbers with more than 3 digits, especially if the amount of digits in each number is unequal. In sub-stage K two items were set in which the digits in the two numbers being added were unequal and in both cases Paul's responses indicate a failure to add the final digit. For the item involving the addition of two 4-digit numbers he placed all his carrying figures to the extreme left each time and thus the value of the carrying figure was ignored and they were all included in the addition of the thousands column. In sub-stage L Paul offered his own answers to the first two items, both of which were incorrect, whilst for the third example he reversed the place value of the carrying digit. Obviously Paul R., like the others discussed, has many
difficulties in applying the standard addition algorithm consistently and, because of the variety of the algorithms he employs when dealing with larger numbers, it would seem easier to discuss with him and for him to experience the activities involved in the early sub-stages with which he is experiencing success; the algorithm he applies to these could be developed and applied to the later sub-stages.

Angela's problems arise in sub-stages I, J, K and L which involve items requiring her to use a carrying figure. Ten of her eleven incorrect responses were due to the failure to add on the carrying figure. The other incorrect responses (I.3.B) is due to the omission of a zero in the answer. This test clearly highlights Angela's difficulties and she obviously needs to experience more situations involving the 'carrying concept'.

Using the test the teacher could analyse the responses of each individual and diagnose any algorithmic difficulties they might have. In the cases of Vanda and Angela their main difficulty arises through their failure to include the carrying figure whilst in the cases of Paul R. and Paul C. they are seen to be very inconsistent in their use of the standard addition algorithm and their many problems are revealed.

Besides being used to serve the need in diagnosing the deficiencies of the individual child (the main purpose) the test could also be used for global purposes, as indicated at the beginning of the section, in that any difficulties common to the class or year in general would be highlighted (Section 7.5).
For addition there are no such difficulties but any which arose in future administerings of the test would help the teacher in indicating that either his approach or emphasis to the teaching of a particular algorithm needed to be either modified or completely changed.

7.8 The Objective Diagnostic Subtraction Test

C.  
1.  
(A) 8  (B) 43  (C) 45  (D) 44  
2.  
(A) 37  (B) 19  (C) 36  (D) 38  
3.  
(A) 66  (B) 65  (C) 64  (D) 37

D.  
1.  
(A) 30  (B) 25  (C) 35  (D) 21  
2.  
(A) 47  (B) 33  (B) 40  (D) 32  
3.  
(A) 71  (B) 82  (C) 80  (D) 78
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7.11 The four sample answer sheets to be analysed

A correct response on the answer sheet is indicated by a 1 whilst an incorrect response is indicated by a 0. Occasionally a child may have considered that all the alternative answers to a particular item were incorrect and he may have offered his own solution which is indicated by his having ruled a line through the alternatives offered and then writing in his own answer.
Objective Diagnostic Subtraction Test answer sheet

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Objective Diagnostic Subtraction Test answer sheet

Name:  Dawn

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Objective Diagnostic Subtraction Test answer sheet

Name: Darren

C. 1. A B C  
    2. A B C  
    3. A B D  

D. 1. A B D  
    2. A B C  
    3. A B C  

E. 1. A B C  
    2. A B C  
    3. A B C  

F. 1. A B C  
    2. A B C  
    3. A B C  

G. 1. A B C  
    2. A B C  
    3. A B C  

H. 1. A B C  
    2. A B C  
    3. A B C  

I. 1. A B C  
    2. A B C  
    3. A B C  

J. 1. A B C  
    2. A B C  
    3. A B C  

K. 1. A B C  
    2. A B C  
    3. A B C  

L. 1. A B C  
    2. A B C  
    3. A B C  

M. 1. A B C  
    2. A B C  
    3. A B C  

Number of corrected subtraction test.
7.12 Analysis of the Objective Diagnostic Subtraction Test

Ninety-six 12-13 years old children participated in the test. The four children chosen whose responses were analysed were Nigel, Colin, Dawn and Darren (Section 7.11).

Nigel's difficulties arose with the items posed in sub-stage I and onwards. His responses indicate that his main difficulties arise from a failure to modify the total after borrowing (6 out of 10 incorrect responses were due to this), a failure to include the residue on borrowing (2 incorrect responses due to this) and the use of inverted subtraction (2 incorrect responses due to this). It would appear that Nigel is very much aware of the subtraction algorithm but needs to experience many situations involving the borrowing concept which will enable him to put together the two facets of modifying the total on borrowing and adding on the residue to that which is borrowed. It seems that Nigel, being unsure of the subtraction algorithm, reverts to the use of inverted subtraction for his convenience. The writer considers that once Nigel has grasped the notions of the modification of the total after borrowing and including the residue on borrowing the need to use the inverted subtraction algorithm will be overcome.

Colin shows an inconsistency in the use of the subtraction algorithm in that he gave completely correct responses to the items of sub-stages C, D, E, F and K, L and M and yet appeared to have a number of difficulties with those of sub-stages in between, namely G, H, I and J. The
difficulties in Sub-stage G arise from the incorrect knowledge of the subtraction number bonds. The main difficulties in the other sub-stages are a failure to modify the total after borrowing and a lack of the borrowing concept when subtracting from zero. Clearly Colin needs more experience of the subtraction number bonds and to have explained to him the need to modify the total after borrowing. The lack of the borrowing concept when subtracting from zero is one which the writer has met many times in his teaching career and these examples serve to highlight the problems children have when dealing with zero. Clearly Colin needs to experience more subtraction activities involving zero.

Darren is able to apply the subtraction algorithm very successfully until he reached sub-stage I. For sub-stage I he gave an incorrect response for each item whilst for sub-stage K and L he gave two incorrect responses for each sub-stage. His responses indicate that his two main difficulties arise from a failure to modify the total after borrowing and a failure to add on the residue to that which is borrowed. Darren is clearly aware of the subtraction algorithm but is inconsistent in applying the two above facets together. Darren needs to experience more subtraction activities involving a need to borrow along with an explanation of the algorithm.

Dawn, like Colin, is able to apply the subtraction algorithm to the items for the early and late sub-stages but has difficulties with the middle order ones; namely sub-stages
G, H, I and J. Like Colin, Dawn's difficulties with sub-stage G arise from an incorrect knowledge of the subtraction number bonds. Dawn is also inconsistent in the application of the subtraction algorithm for she fails to modify the total after borrowing (five times), uses inverted subtraction (once), fails to add on the residue after borrowing (once) and shows a lack of borrowing concept when subtracting from zero (once). Dawn is clearly aware of the correct subtraction algorithm and is able to apply it successfully and so she must be made aware of her tendency to fail to modify the total after borrowing and of her inconsistency in the use of the algorithm.

Analysing the responses for the whole year it is clear, as might be expected, that more children have difficulties with subtraction than with addition. The item response chart (Section 7.10) indicates that the choice of distractors has been fairly well selected by the number of responses to the individual distractors. However certain distractors have not been selected at all and these could well need replacing, when the test is next administered, if more suitable algorithmic errors arise and are noted in class work. From the number of correct responses indicated in the item response chart it appears that, in the main, the children find difficulty from sub-stage H onwards. This test clearly indicates to the teacher areas of common difficulties amongst the children.

Consider the responses of the items of sub-stage J.

<table>
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<tr>
<th>Sub-stage J</th>
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<th>D</th>
<th>No response</th>
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<td>1.</td>
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<td>5</td>
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<td>(76)</td>
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Those numbers which are circled indicate the number of correct responses.
Reponses 1.B, 2.D and 3.D indicate the response when a child has failed to modify the total after borrowing. This would seem to be a difficulty that a fair proportion of the children have and it serves to indicate to the teacher the need to emphasise this aspect of the subtraction algorithm. Similarly the none-use of distractor 1.A indicates that, for this particular sub-stage, the children have grasped the aspect of the algorithm which requires them to include the residue on borrowing.

Similarly if we consider the responses to the items of sub-stage H.

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<th>Sub-stage H</th>
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Those numbers which are circled indicate the number of correct responses.

The responses 1.A, 2.B and 3.D indicate that a fair proportion of the children apply the inverted subtraction algorithm. Responses 1.C, 2.C and 3.C indicate that the failure to add on the residue after borrowing is only a minor problem for this particular sub-stage.

These two examples show how a teacher might use the test to indicate areas of common difficulties that children may have.
7.13 The Objective Diagnostic Multiplication Test (Part 1)

C.
1. \[ \frac{43}{2} \times 2 \]
   - (A) 86
   - (B) 64
   - (C) 106
   - (D) 85

2. \[ \frac{40}{2} \times 2 \]
   - (A) 82
   - (B) 100
   - (C) 80
   - (D) 40

3. \[ \frac{32}{3} \times 3 \]
   - (A) 66
   - (B) 94
   - (C) 64
   - (D) 96

D.
1. \[ \frac{213}{3} \times 3 \]
   - (A) 609
   - (B) 642
   - (C) 939
   - (D) 639

2. \[ \frac{304}{2} \times 2 \]
   - (A) 628
   - (B) 608
   - (C) 610
   - (D) 632

3. \[ \frac{120}{4} \times 4 \]
   - (A) 480
   - (B) 484
   - (C) 490
   - (D) 500

E.
1. \[ \frac{29}{3} \times 3 \]
   - (A) 87
   - (B) 627
   - (C) 67
   - (D) 132

2. \[ \frac{17}{5} \times 5 \]
   - (A) 103
   - (B) 55
   - (C) 535
   - (D) 85

3. \[ \frac{19}{4} \times 4 \]
   - (A) 76
   - (B) 103
   - (C) 436
   - (D) 46
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1.  \( \frac{5}{x100} \)

\[ \begin{array}{cccc}
(A) & 555 & (B) & 500 \\
(C) & 5000 & (D) & 50 \\
\end{array} \]

2.  \( \frac{100}{x60} \)

\[ \begin{array}{cccc}
(A) & 6000 & (B) & 6660 \\
(C) & 600 & (D) & 60000 \\
\end{array} \]

3.  \( \frac{37}{x100} \)

\[ \begin{array}{cccc}
(A) & 4107 & (B) & 3700 \\
(C) & 370 & (D) & 37000 \\
\end{array} \]
7.14 The Objective Diagnostic Multiplication Test (Part 1)

answer sheet

Name: ___________________________

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7.16 The four sample answer sheets to be analysed

A correct response on the answer sheet is indicated by a 1 whilst an incorrect response is indicated by a 0. Occasionally a child may have considered that all the alternative answers to a particular item were incorrect and he may have offered his own solution which is indicated by his having ruled a line through the alternatives offered and then writing in his own answer.
**Objective Diagnostic Multiplication Test (Part 1) answer sheet**

**Name:** Nigel H.

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Objective Diagnostic Multiplication Test (Part 1) answer sheet

Name: Marie

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| D | 1. | A | B | C | D | 1 |
|   | 2. | A | B | C | D | 0 |
|   | 3. | A | B | C | D | 0 |

| E | 1. | A | B | C | D | 0 |
|   | 2. | A | B | C | D | 0 |
|   | 3. | A | B | C | D | 1 |

| F | 1. | A | B | C | D | 0 |
|   | 2. | A | B | C | D | 0 |
|   | 3. | A | B | C | D | 1 |

| G | 1. | A | B | C | D | 1 |
|   | 2. | A | B | C | D | 0 |
|   | 3. | A | B | C | D | 1 |

| H | 1. | A | B | C | D | 0 |
|   | 2. | A | B | C | D | 1 |
|   | 3. | A | B | C | D | 0 |

| I | 1. | A | B | C | D | 1 |
|   | 2. | A | B | C | D | 0 |
|   | 3. | A | B | C | D | 0 |
Objective Diagnostic Multiplication Test (Part 1) answer sheet

Name: Jane

C. 1. A B C D 1
   2. A B C D 1
   3. A B C D 1

D. 1. A B C D 1
   2. A B C D 0
   3. A B C D 0

E. 1. A B C D 1
   2. A B C D 1
   3. A B C D 1

F. 1. A B C D 1
   2. A B C D 0
   3. A B C D 1

G. 1. A B C D 1
   2. A B C D 1
   3. A B C D 1

H. 1. A B C D 1
   2. A B C D 1
   3. A B C D 1

I. 1. A B C D 1
   2. A B C D 0
   3. A B C D 0
Objective Diagnostic Multiplication Test (Part 1) answer sheet

Name: Andrew

C.
1. A B C D 1
2. A B C D 1
3. A B C D 1

D.
1. A B C D 1
2. A B C D 1
3. A B C D 1

E.
1. A B C D 0
2. A B C D 76 0
3. A B C D 1

F.
1. A B C D 1
2. A B C D 1
3. A B C D 0

G.
1. A B C D 0
2. A B C D 1
3. A B C D 1

H.
1. A B C D 1
2. A B C D 1
3. A B C D 1

I.
1. A B C D 1
2. A B C D 1
3. A B C D 0
7.17 Analysis of the Objective Diagnostic Multiplication Test

(Part 1)

Ninety-two 12-13 years old children participated in the tests which involved multiplication by a 1-digit number with the exception of sub-stage I which required the knowledge of multiplication by multiples of 10. The four children chosen whose responses were analysed were Nigel H., Marie, Jane and Andrew (Section 7.16).

Andrew made five incorrect responses and all were due to algorithmic inconsistencies. Although his responses indicate that he has some degree of mastery of the standard algorithm there is a need for the teacher to discuss with him the inconsistent use of the algorithm. Andrew's difficulties include a tendency to place both digits in the answer e.g. E.1.B which was constructed in the following manner was chosen.

\[
\begin{array}{c}
29 \\
x 3 \\
\hline
627
\end{array}
\]

He also, on occasions, carries the wrong digit as exemplified by the choice of F.3.D which was constructed as follows:

\[
\begin{array}{c}
217 \\
x 4 \\
\hline
1012
\end{array}
\]

His responses to the items of sub-stage G shows the occasional failure to include the carrying figure whilst the responses to sub-stage I indicates a lack of knowledge when
dealing with multiples of ten. It can be deduced from the above that most of Andrew's difficulties arise when having to deal with carrying figures and he obviously needs to experience activities which will help him overcome this.

Jane made six incorrect responses to the test and these responses indicate that her difficulties arise through an incorrect multiplication by zero. She has a sound grasp of the standard algorithm and once this problem has been overcome Jane should experience a great deal more success.

Marie, with only nine correct responses out of a possible twenty-one, clearly has a great many difficulties with the multiplication algorithm. Her responses involve incorrect multiplication by zero (D.2.A and D.3.B), carrying the wrong digit (E.1.D, E.2.A, F.1.D, F.2.A, G.2.A, H.3.C), a failure to include the carrying figure (H.1.B) and a lack of knowledge when dealing with multiples of ten. Although Marie obviously has a great many difficulties with the multiplication algorithm they can at least be categorised as above. She needs to experience activities indicated by the early sub-stages which involve carrying in order to correct her difficulties with the carrying figure. Obviously her understanding of place value is weak and this needs to improve in order to help her overcome her difficulties.

Remedial activities for Marie would probably involve work enabling her to understand place value before she experiences the early sub-stages, particularly those involving the carrying concept.
Nigel H. only made four incorrect responses but these indicate areas of weakness which need to be discussed with him. One of these responses (C.3.B) was due to incorrect number bonds, two incorrect responses (G.2.D and H.2.C) were due to a failure to include the carrying figure whilst the other (H.1.D) was due to carrying the wrong digit. Thus Nigel would seem to have a fair understanding of the multiplication algorithm but he is inclined to make careless mistakes which cause him to offer incorrect solutions.

The year group, in general, did not experience too many algorithmic difficulties with the items of the sub-stages up to and including sub-stage H although seven children chose F.2.C which indicated that a fair number of children were uncertain about multiplication by zero. Sub-stage I involving multiplication by multiples of ten showed that many children experienced difficulty in this skill and that more time needs to be devoted to overcome this difficulty, particularly as mastery of this skill is vital when one deals with long multiplication.
7.18 The Objective Diagnostic Multiplication Test (Part 2)

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|   | 328320 | 331200 | 33120 | 33120 |   |   |   |   |
|   | 364256 | 368460 | 36846 | 36846 |   |   |   |   |
| 2. |   | 267 | x 65 |   |   |   |   |   |
|   | (A) | 267 | (B) | 267 | (C) | 267 | (D) | 267 |
|   | x 65 | x 65 | x 65 | x 65 |   |   |   |   |
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987 & \times 78 \\
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805 & \times 354 \\
4020 & \\
40250 & \\
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284970 & \\
\end{align*}
\]
7.19 The Objective Diagnostic Multiplication Test (Part 2)

answer sheet

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### 7.20 Objective Diagnostic Multiplication Test (Part 2)

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7.21 The four sample answer sheets to be analysed

A correct response on the answer sheet is indicated by a 1 whilst an incorrect response is indicated by a 0.

Occasionally a child may have considered that all the alternative answers to a particular item were incorrect and he may have offered his own solution which is indicated by his having ruled a line through the alternatives offered and then writing in his own answer.
Objective Diagnostic Multiplication Test (Part 2) answer sheet

Name: Emma P.

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   3. A B C D 1

K. 1. A B C D 0
   2. A B C D 1
   3. A B C D 1

L. 1. A B C D 1
   2. A B C D 1
   3. A B C D 1

M. 1. A B C D 1
   2. A B C D 1
   3. A B C D 1

N. 1. A B C D 1
   2. A B C D 1
   3. A B C D 1

O. 1. A B C D 0 - 137
   2. A B C D 1
   3. A B C D 1
Objective Diagnostic Multiplication Test (Part 2) answer sheet

Name: Jayne

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Objective Diagnostic Multiplication Test (Part 2) answer sheet

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Objective Diagnostic Multiplication Test (Part 2) answer sheet

Name: Christopher

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7.22 Analysis of the Objective Diagnostic Multiplication Test (Part 2)

Ninety-one 12-13 years old children participated in the second part of the multiplication test which was concerned with long multiplication. The four children's responses to be analysed were Emma P., Jayne, Anna and Christopher (Section 7.21).

Emma gave three incorrect responses to the test, these being J.1.A, K.1.A and 0.1. For the latter she considered that all the alternatives were incorrect and accordingly wrote down her own solution. The choice of distractor J.1.A indicates that Emma was multiplying by the tens digit only but as she gave the correct response to problems J.2 and J.3 it would appear that this was due to carelessness; nevertheless the point does need clarifying and discussing with Emma. Analysis of Emma's own solution to 0.1 indicate that she omitted to add a carrying figure. Thus it appears that Emma has a fairly sound grasp of the long multiplication algorithm but she is prone to careless errors; she needs to be encouraged to check her work.

Jayne made four incorrect responses, three of which occurred in sub-stage 0 which involved multiplication by a 3-digit number. Obviously Jayne needs more experience of similar situations as those indicated by the items of sub-stage 0, particularly as her errors were due to a wrong place value shift. Analysis of the rest of her responses indicate that Jayne has a sound grasp of the multiplication algorithm when multiplying by a 2-digit number.
Of Anna's five incorrect responses three were made in sub-stage J. These responses indicate that she applies a vertical multiplication algorithm when multiplying by multiples of ten. Her other two incorrect responses were due to incorrect multiplication number bonds and not to any incorrect use of the standard multiplication algorithm. The responses to the test show that Anna has a sound understanding of the multiplication algorithm but needs to experience more activities involving multiplication by multiples of ten.

Christopher's five incorrect responses would indicate that he is unsure of the place value shift aspect of the algorithm. He is inconsistent in its use, sometimes putting a zero for the place value when it is not necessary whilst at other times he ignores it altogether. Thus although Christopher is aware of the standard long multiplication algorithm the teacher needs to discuss with him, and allow him to experience various situations which will ensure that he understands and is consistent in the use of the place value shift.

As was to be expected more children experienced difficulty with this test than with the others analysed to date. By inspecting the item response chart (Section 7.17) the year group, in general, did not perform too well in sub-stage 0 which required the multiplication of two 3-digit numbers (carrying where necessary). One hundred and fifty of the incorrect responses were due to children ignoring the place value shift when multiplying by the tens and hundreds.
digit. Twenty-six incorrect responses occurred through children choosing the distractor M.1.B which meant that the children were using a place value shift when multiplying by the units digit, and two place value shifts when multiplying by the tens digit. Where the option was given twenty-nine children chose to use the vertical and diagonal multiplication algorithm. Clearly these particular children were little aware of the standard long multiplication algorithm and were trying to apply their knowledge of ordinary multiplication to multiply by two 2-digit numbers. Thus this very brief analysis of the responses would indicate to the teacher a need to emphasise the place value shift effect of multiplying by the tens and hundreds digit.

7.23 The Objective Diagnostic Division Test

C.

1. \[ \frac{5}{30} \]
   (A) 0  (B) 6  (C) 0-3  (D) 7

2. \[ \frac{8}{56} \]
   (A) 7  (B) 1-3  (C) 0  (D) 8

3. \[ \frac{7}{77} \]
   (A) 11  (B) 10  (C) 9  (D) 12
D. 1. 6156
   (A) 1 (B) 9 (C) 9\% (D) 1\%
   2. 7164
   (A) 0 (B) 1\% (C) 9\% (D) 9
   3. 9\sqrt{33}
   (A) 9\% (B) 1\% (C) 0 (D) 9

E. 1. 5\sqrt{25}
   (A) 15 (B) 11 (C) 101 (D) 105
   2. 3\sqrt{54}
   (A) 211 (B) 211\% (C) 218 (D) 219
   3. 8\sqrt{944}
   (A) 118 (B) 100 (C) 106\% (D) 116

F. 1. 4\sqrt{25}
   (A) 201\% (B) 26\% (C) 206\% (D) 206
   2. 9\sqrt{709}
   (A) 78 (B) 1 (C) 78\% (D) 78\% 
   3. 8\sqrt{795}
   (A) 10 (B) 99\% (C) 99 (D) 1
G. 1. 3\sqrt{2638}
   (A) 212 3/4 (B) 879 (C) 879 3/4 (D) 879 3/4
   2. 9\sqrt{7777}
   (A) 0 (B) 864 3/4 (C) 864 (D) 864 3/4
   3. 4\sqrt{9368}
   (A) 2342 (B) 2012 (C) 2112 3/4 (D) 2112 3/4

H. 1. 30\sqrt{270}
    (A) 3 (B) 10 (C) 7 (D) 9
    2. 40\sqrt{560}
    (A) 14 (B) 140 (C) 13 (D) 1400
    3. 50\sqrt{2500}
    (A) 500 (B) 50 (C) 5 (D) 5000

I. 1. 14\sqrt{518}
    (A) 39 3/4
        14\sqrt{518}
    (B) 38 3/4
        14\sqrt{518}
    -112
        -112
    6

    (C) 217
        14\sqrt{518}
    (D) 37
        14\sqrt{518}
    -42
        -42
    88
        -98
    00
        00
2. \[19 \sqrt{798}\]

(A) \[
\begin{array}{c}
42 \\
19 \sqrt{798} \\
-76 \\
38 \\
-38 \\
0
\end{array}
\]

(B) \[
\begin{array}{c}
40 \frac{3}{4} \\
19 \sqrt{798} \\
-76 \\
3 
\end{array}
\]

(C) \[
\begin{array}{c}
47 \frac{5}{9} \\
19 \sqrt{798} \\
-76 \\
138 \\
-133 \\
5
\end{array}
\]

(D) \[
\begin{array}{c}
312 \\
19 \sqrt{798} \\
-57 \\
228 \\
-228 \\
000
\end{array}
\]

3. \[28 \sqrt{672}\]

(A) \[
\begin{array}{c}
24 \\
28 \sqrt{672} \\
-56 \\
112 \\
-112 \\
000
\end{array}
\]

(B) \[
\begin{array}{c}
20 \frac{13}{18} \\
28 \sqrt{672} \\
-56 \\
12
\end{array}
\]

(C) \[
\begin{array}{c}
20 \frac{13}{28} \\
28 \sqrt{672} \\
-56 \\
11
\end{array}
\]

(D) \[
\begin{array}{c}
114 \\
28 \sqrt{672} \\
-28 \\
392 \\
-392 \\
000
\end{array}
\]

J. 1. \[24 \sqrt{853}\]

(A) \[
\begin{array}{c}
35 \\
24 \sqrt{853} \\
-72 \\
133 \\
-120 \\
13
\end{array}
\]

(B) \[
\begin{array}{c}
35 \frac{20}{3} \\
24 \sqrt{853} \\
-72 \\
133 \\
-120 \\
13
\end{array}
\]

(C) \[
\begin{array}{c}
35 \frac{20}{24} \\
24 \sqrt{853} \\
-72 \\
133 \\
-120 \\
13
\end{array}
\]

(D) \[
\begin{array}{c}
30 \frac{17}{24} \\
24 \sqrt{853} \\
-72 \\
13
\end{array}
\]
2. \[ \begin{array}{c}
(A) \quad \frac{16}{57} \frac{946}{376} \frac{342}{34} \\
(B) \quad \frac{16}{57} \frac{946}{376} \frac{342}{34}
\end{array} \]

3. \[ \begin{array}{c}
(A) \quad \frac{62}{13} \frac{811}{78} \frac{26}{5} \\
(B) \quad \frac{62}{13} \frac{811}{78} \frac{26}{5}
\end{array} \]

K.

1. \[ \begin{array}{c}
(A) \quad \frac{100}{84} \frac{8507}{84} \frac{1}{1} \\
(B) \quad \frac{101}{84} \frac{8507}{84} \frac{107}{23}
\end{array} \]

(C) \[ \begin{array}{c}
\frac{101}{84} \frac{8507}{84} \frac{107}{23}
\end{array} \]

(D) \[ \begin{array}{c}
\frac{1}{84} \frac{107}{23}
\end{array} \]
2. \[ \begin{array}{c|c} 
34)3181 & 34)3181 \\
(A) & (A) \\
\underline{34)3181} & \underline{34)3181} \\
-306 & -306 \\
\underline{121} & \underline{121} \\
-102 & -102 \\
\underline{19} & \underline{19} \\
34)3181 & 34)3181 \\
(D) & (D) \\
\underline{34)3181} & \underline{34)3181} \\
-306 & -306 \\
\underline{121} & \underline{121} \\
-102 & -102 \\
\underline{19} & \underline{19} \\
\end{array} \]

3. \[ \begin{array}{c|c} 
13)2718 & 13)2718 \\
(A) & (A) \\
\underline{13)2718} & \underline{13)2718} \\
-26 & -26 \\
\underline{118} & \underline{118} \\
-117 & -117 \\
\underline{001} & \underline{001} \\
13)2718 & 13)2718 \\
(C) & (C) \\
\underline{13)2718} & \underline{13)2718} \\
-26 & -26 \\
\underline{118} & \underline{118} \\
-117 & -117 \\
\underline{001} & \underline{001} \\
\end{array} \]
7.24 The Objective Diagnostic Division Test answer sheet

Name: ____________________________

C.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

D.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

E.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

F.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

G.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

H.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

I.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

J.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D

K.  1. A  B  C  D
    2. A  B  C  D
    3. A  B  C  D
7.25 Objective Diagnostic Division Test item response Chart

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<th>Item</th>
<th>A</th>
<th>B</th>
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<th>D</th>
<th>Response</th>
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7.26 The four sample answer sheets to be analysed

A correct response on the answer sheet is indicated by a 1 whilst an incorrect response is indicated by a 0. Occasionally a child may have considered that all the alternative answers to a particular item were incorrect and he may have offered his own solution which is indicated by his having ruled a line through the alternatives offered and then writing in his own answer.
Objective Diagnostic Division Test answer sheet

Name: Phillip

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Objective Diagnostic Division Test answer sheet

Name: Gillian

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Objective Diagnostic Division Test answer sheet

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7.27 Analysis of the Objective Diagnostic Division Test

Eighty-four 12-13 years old children participated in the test. Item 3 of sub-stage F was declared void because it became apparent, on marking the responses, that the divisor had become unclear on some of the question papers and that some children were treating it as an eight whilst others treated it as a three. Thus in this analysis the responses to this particular item were ignored. The four children whose responses were analysed were Phillip, Gillian, Neil and Emma S. (Section 7.26).

Phillip gave four incorrect responses, two (E.1.C and E.2.B) of which were very surprising. Both these responses would indicate that he lacked knowledge of the carrying concept and yet he gave correct responses to all the items of the other sub-stages with the exception of K. As all these sub-stages required the internal remainder to be carried to the next digit (which he did correctly) then it would seem that these two incorrect responses were due to carelessness. Nevertheless the teacher needs to discuss with Phillip the reasons for these two responses in order to make sure that the above assumption is correct. The other two incorrect responses occurred in sub-stage K which involved division by a 2-digit number. Response K.1.C indicates a failure to include the remainder whilst response K.3.A indicates an omission of a zero in the quotient. Phillip has a sound grasp of the division algorithm but, as his incorrect responses indicate, he is prone to carelessness.
Gillian gave seven incorrect responses, five of which were responsible for her getting the items in sub-stages F and K completely wrong. One of the incorrect responses was because she couldn't attempt the problem whilst four of the remaining six were due to Gillian omitting the zero in the quotient (E.1.A, F.1.B, K.1.D, K.3.A). The other two incorrect responses were due to her writing the remainder incorrectly. Gillian's difficulties should be quickly overcome for she has a sound grasp of the division algorithm; she needs to have explained to her the importance of zero in being a place value holder and the correct method of arriving at the fractional remainder.

Neil's nine incorrect responses were somewhat 'scattered' among the sub-stages. However, analysis of his responses reveals one of the virtues of the test in that his difficulties can be readily categorised. One of the incorrect responses was due to Neil not being able to attempt to answer the item whilst half the remaining incorrect responses were due to him ignoring the remainder in the quotient and a further incorrect response was due to him inverting the remainder. Of the other three incorrect responses one was due to him omitting the zero in the quotient (E.1.A) whilst the other two occurred in substage I where he failed to 'bring down' the remaining digits in the long division algorithm. Thus a first priority for the teacher in helping Neil overcome his difficulties is to discuss the use and importance of fractions with him and to allow him to
experience activities involving fractional remainders. He also seems unsure of the long division algorithm and needs more experience with these types of problems.

Emma S., like Neil, gave nine incorrect responses but of these seven were in the last three sub-stages. These sub-stages involve the long division algorithm and it would appear on first inspection that Emma is very unsure of the application of the algorithm. However, four of these incorrect responses were due to her omitting the remainder whilst two others were due to inverting the remainder. Thus, like Neil, she needs to experience activities which will enable her to understand the correct writing of the fraction and the need to include it as a remainder. This comment is reinforced by Emma's responses to item 2 of sub-stage G (g.2.D) which was incorrect because the fractional remainder had been inverted.

The year group, in general, found this test more difficult than any of the others. It is quite clear from looking at the number of none-responses (Section 7.25) that a small number of children have little or no understanding of the 'short' division algorithm and that quite a few more have no understanding of the long division algorithm. The writer is not surprised at this for in his experience division tends to be a neglected topic. However, in the school in which the tests were administered division is certainly experienced by the children. However the responses do indicate that the children find more difficulties with division than with the
other computational skills (maybe this is why it's a neglected topic!)

The distractors, especially for the last four sub-stages, were well chosen with some being chosen so often as to raise some concern. For example, considering sub-stage K only, K.1.D was chosen by twenty children and K.3.A was chosen by fourteen children; analysis of these revealed that a zero was omitted from the quotient. Twenty-one incorrect responses in the sub-stage were due to the remainder being omitted from the quotient. It seems that the children, generally, have knowledge of the standard long division algorithm but some are so unsure of it that they are making what would appear to be elementary errors. The importance of zero as a place value holder needs to be stressed, as does the need to include the fractional remainder in the quotient. The problem of neglecting the zero in the quotient is further highlighted by twenty children having chosen F.1.B and eighteen having chosen E.1.A.

The distractors J.1.A, J.2.A and J.3.A were responsible for thirty incorrect responses and all were constructed so as to omit the remainder. Further analysis of the choice of distractors indicate that the main difficulties arising when using the division algorithm are the omission of both zero and the remainder in the quotient. Obviously, these are points which the teacher, when teaching the topic again, needs to emphasise and explain.
7.28 Summary and General comments

The preparation required before the objective diagnostic tests could be written was very time consuming because the tests required the distractors to be constructed according to various algorithms which the children might use. To select the distractors through an algorithmic criteria the writer had to have knowledge of the various erroneous algorithms a child might apply to the computational skills which were to be tested. In order to gain this knowledge the writer compiled the five tests discussed in Chapter 5 which were administered to approximately eighty-five 11-12 years old children. The responses to each question were analysed and categorised as discussed in Chapter 5, Section 5.2. This was very time consuming but it gave the writer the necessary insight into the difficulties that children experience with the various algorithms and in particular highlighted those common to a number of children. Having categorised the erroneous algorithms each distractor was then constructed meaningfully according to one of the algorithms (Chapter 6 Section 6.4). Wherever possible the distractors were constructed by using one of the erroneous algorithms but occasionally, because there were not enough erroneous algorithms to apply to three distractors, the writer had to resort to use of incorrect number bond in constructing a distractor. This was wasteful because the aim of the tests was to reveal algorithmic deficiencies that children might have and not to reveal their knowledge of number bonds.
Nevertheless the writer included the distractors so constructed so that four alternatives could be presented for each item and each sub-stage represented by three items.

This summary of the construction of the objective diagnostic tests indicates the considerable time spent in constructing the tests. However once this was done and the tests administered the analysis of the tests revealed a number of advantages, details of which were presented in the early sections of this chapter. The choice of the actual numbers used in the various items are not really significant as long as they conform to the criteria of that particular sub-stage in which they are placed. In the analysis the writer has tried to indicate various ways in which the tests may be used to aid both the pupil and teacher. The pupil is aided because his algorithmic deficiencies can be analysed through his choice of response to the test items whilst the teacher is aided because the tests can be analysed for the whole population participating in them to reveal algorithmic deficiencies common to many children as well as to the individual child. The writer feels that this two-fold feature of the tests makes them very worthwhile.

One way this two-fold feature of the tests could be put to good effect would be to give the relevant test to the class before the topic is taught in order to diagnose the algorithmic deficiencies of each child and those common to a number of children. The teacher, on being aware of these deficiencies, could then construct his teaching program so that the children
could overcome these deficiencies. When the children had completed the course the teacher could set the same test again to reveal the effectiveness of his planned program.

One of the main advantages of the tests are that they can be very quickly marked and therefore can be presented to a large number of children if the situation required. Once marked the incorrect responses can be analysed quickly and this should prove useful to teachers in middle and junior schools, particularly those teaching large classes, who probably have to teach many subjects. Allied to this latter comment is the fact that many teachers in those schools are not mathematics specialists; at the time of writing, in the writer's own middle school thirteen teachers out of a staff of seventeen teach mathematics at some time during the week and of those only three might be termed mathematics specialists. Many of those 'non-specialists' would have difficulties in diagnosing a child's deficiencies but by employing these objective diagnostic tests the child's particular response categorises his deficiencies immediately. Of course the tests only indicate the deficiencies a child may have and the non-specialist mathematics teacher would probably still have to consult the mathematics advisor within the school to discuss the situations the child must experience in order to overcome his difficulties. It is not the purpose of this project to suggest remedial action but the writer does feel it important that the teacher discusses with the child the reasons for his particular algorithm failing rather than
merely just ignoring it and presenting him with 'another' algorithm.

Two possible disadvantages of the objective diagnostic tests are that they are 'open' to guesswork on the part of the child (an opinion already discussed in Chapter 2) and that the teacher pre-judges child's responses by limiting him to four alternatives. The writer feels that the problem of guesswork is not too serious because on administering the tests it was stressed to the children that the final mark gained was of no importance and that the test was to help them in that it would reveal areas of weakness in their knowledge which we, as teachers, could help them overcome. There is also a built-in factor in the tests which will enable the teacher to decide whether a child is guessing. This factor is that the test is highly structured and if the child were merely guessing his choice of responses would be very inconsistent indicating immediately to the teacher that the child had difficulties with many aspects of the topic. However, if the child were consistently using an erroneous algorithm this would be indicated by his responses and the teacher would become aware of his difficulties.

The second possible disadvantage of the tests, that the choice of distractors pre-judges the child's response, is very valid and the writer tried to compensate for this by allowing the child the choice of writing down his own solution if he disagreed with all of the alternatives offered. The writer realises that the test is then no longer
strictly objective but as the key was always presented to
the child, it was felt that the objectivity of the test was
not impaired. It was hoped that the option could reveal
'new' algorithms which could be used later, if the tests
needed modifying, by using these new algorithms to construct
distractors to replace those that may not have been used in
that particular sub-stage. In the tests the children very
rarely resorted to this option which seemed to indicate that
the choice of distractors was reasonable. However, where a
distractor is little or not used then the tests could be
modified by replacing it with a more suitable one. The
pre-objective diagnostic tests revealed many erroneous
algorithms employed by children but, obviously, many more
were probably not highlighted. Thus the teacher, when
teaching the topics, needs to note any 'new' erroneous
algorithm as it occurs so that he may use it to replace any
distractor not used in the test and in particular to replace
those distractors constructed through the use of number bonds.

It must be emphasised that the writer realises that
the tests are crude and that there is much need for
modification but they do serve the purpose of indicating how
useful they could be for diagnostic purposes. A first
modification would be to replace the distractors constructed
through the use of incorrect number bonds. It must be
emphasised that even though a particular distractor was not
chosen by any child for these tests it does not necessarily
mean the distractor should be replaced; it may happen that
if the test were to be administered again to different children then that distractor may then be chosen. Obviously if, after several administerings of the test, the distractor was still no longer chosen then the distractor was probably serving no purpose and should be replaced.

Another modification would be to test each sub-stage individually and at the appropriate point in the teaching of a topic rather than to test them altogether on completion of the topic (as in the trial). Each sub-stage would then be probably represented by ten items and the tests should prove far more useful by indicating which children had not mastered that particular sub-stage (a criteria for this could be that mastery of the sub-stage is shown if the child proposes at least eight correct responses). Those who didn't show mastery would need to experience more and possibly new activities to try and encourage mastery. These tests could even prove useful in providing a mathematics profile on the individual child by noting any difficulties arising at a particular sub-stage and using this to form part of the record; the record would comprise of the results and any written comments arising from a series of such tests covering the whole mathematics spectrum. This is an idea that needs to be developed but could prove useful.

Even in their present form the tests have proved very useful in diagnosing the deficiencies of groups of children. Because objective tests have been used than a future
development could be to use computers to mark and analyse the tests thus allowing the teacher more time to actually correct any deficiencies a child might have (after all it's no good spending all one's time diagnosing the deficiencies of children and never having time to correct them). Indeed a computer could be used to store the results of such diagnostic tests and update the information if future tests were completed. The trial tests are only concerned with the four computational skills of addition, subtraction, multiplication and division but the writer anticipates no problems which would prevent objective diagnostic tests being applicable to other aspects of mathematics.
8.1 Introduction

One of the aims of the project is to investigate the feasibility of using items arising from the objective diagnostic test for the purpose of compiling an objective selection test. It has been called a selection test rather than an attainment test because its main purpose in this project is to rank the children according to their raw score. However similar arguments and comments are applicable to both. The diagnostic tests would be used to supply suitable items for the selection test. There are possible problems arising from this because diagnostic and selection tests are set for different reasons and yet it is proposed that the same items be used for two types of test. The basic difference is that items in diagnostic tests are set to test an individual's mastery of a particular topic or skill and discrimination between the abilities of different children is not important: those items in selection tests will need to discriminate well between the abilities of different children particularly as the ranking of the children is of upmost importance.

Before considering the particular objective selection test to be compiled it is useful to consider the general construction of such a test in order to clarify the reasons for proposing to use the items generated by the objective diagnostic tests.
8.2 General construction of selection/attainment tests

A flow chart for test construction might be the following:

- Specification Grid
- Allocation of topics to item writer
- Submitted items
  - Categorising
  - Editing
  - Item modification into abilities
- Assembling items for pre-testing
- Pre-testing
- Re-tested
  - Item Analysis
  - Extensively
  - Re-appraised
  - Revised Items
  - Rejection
- Accepted Item
- Minor modification
- Final examination
The first question to consider is concerned with the purpose of the test. Most usually this will be to ascertain the level of achievement, with perhaps a rank order of students being obtained. Obviously in the case of selection tests which may be used for placement of children into ability groups this ranking is of major importance although it needs to be stressed that this placement should **never** be done on the evidence of one test. The placement should be done on the evidence of several such tests and with full consultation with the child's mathematics teacher. The test compiled in this project will be used only to investigate the feasibility of using certain items employed in the diagnostic tests for selection tests.

8.2.1 Specification Grid

Having decided upon the purpose of the test the next step is to construct a specification grid which may be a syllabus or some part of it. The test to be compiled will be concerned with the computational skills and therefore the specification grid will be based on the sub-stages discussed in Chapter 4, section 2. The skills to be tested were chosen so that the child might show his overall ability in the computational skills but, obviously, the grid would be altered to suit the needs of the particular teacher.
<table>
<thead>
<tr>
<th>Sub-stage</th>
<th>Ability</th>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
<th>Number of Question</th>
<th>Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>To add two 3-digit numbers: with repeated carrying</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>4%</td>
</tr>
<tr>
<td>L</td>
<td>To add numbers with up to 5 digits: carrying where necessary</td>
<td>8, 10</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>J</td>
<td>To subtract using two 3-digit numbers: borrowing where necessary (one place only)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>4%</td>
</tr>
<tr>
<td>K</td>
<td>To subtract using two 3-digit numbers: with repeated borrowing</td>
<td>2, 7</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>M</td>
<td>To subtract using numbers with up to 5 digits: borrowing where necessary</td>
<td>6, 11</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>H</td>
<td>To multiply a 3-digit number by a 1-digit number: with repeated carrying</td>
<td>1, 3</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>I</td>
<td>To multiply a whole number by 10 and 100</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>4%</td>
</tr>
<tr>
<td>J</td>
<td>To multiply a 2-digit number by 20, 30, 40 or 50</td>
<td>5, 9</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>L</td>
<td>To multiply two 2-digit numbers: carrying where necessary</td>
<td>13, 15</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>G</td>
<td>To multiply two 3-digit numbers: carrying where necessary</td>
<td>19, 25</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>G</td>
<td>To divide a 3-digit number by a 1-digit number: with or without remainder</td>
<td>17, 18, 22</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>12%</td>
</tr>
<tr>
<td>H</td>
<td>To divide a 3-digit number by 20, 30, 40 or 50: no remainder</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>4%</td>
</tr>
<tr>
<td>J</td>
<td>To divide a 3-digit number by a 2-digit number: with remainder</td>
<td>20, 21</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>K</td>
<td>To divide a 4-digit number by a 2-digit number: with or without remainder</td>
<td>23, 24</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>8%</td>
</tr>
</tbody>
</table>

| No. of question | 3 | 5 | 9 | 8 |
| Weightings      | 12% | 20% | 36% | 32% |
The specification grid indicates that the skills and concepts involved in multiplication will be tested most whilst those of addition will be tested least. The specification grid is a concise method of stating the skills and concepts to be tested along with the relative importance the test places on those skills and concepts. The importance of a skill or concept that is to be tested can be emphasised by including more items to test that particular skill or concept. This is shown by the weighting. The specification grid of the test to be compiled is somewhat limited due to the nature of the project but there is no reason why the specification grid cannot be enlarged to include many other aspects of mathematics when other selection/attainment tests are written.

8.2.2 Item writing

Obviously the more quality items there are the easier the test will be to construct. Macintosh and Morrison (1972) suggest that in any case there should be approximately three times the number of questions to choose from than set in the final examination. To expect a single teacher to write this number of items seems a little unrealistic unless the teacher has little else to do. It becomes more realistic, however, if within a school or college, item writing sessions were organised so that groups of teachers could devise quality questions. Dawson (1971) suggests that if objective tests were used within a department, all the academic staff should be requested to produce items from each lesson given.
The items bank could be enlarged further by interchanging items with neighbouring schools. The writer agrees with these suggestions but in practice they are difficult to implement because, from my own experience, many teachers are unaware of objective testing and certainly none of my neighbouring schools employ this type of test. The suggestion that all members of a department should participate in devising suitable questions is admirable but in many junior and middle schools it is difficult to define 'the department' because so many staff are involved in the teaching of mathematics. The majority of these staff are not mathematics specialists and would not welcome the extra burden of devising mathematics questions. In a junior or middle school there are usually, at most, only two mathematics 'specialists' who would be likely to have an interest in writing items. However, diagnostic testing is a tool which is frequently employed in these types of schools and therefore, to relieve the burden of writing items for both the diagnostic tests and the selection/attainment tests, the writer chose to investigate the feasibility of employing the same items for both types of test. Hopefully the diagnostic tests would serve to provide items for the selection/attainment tests.

8.2.3 Pre-testing the items

Having compiled sufficient items, ideally they should be pre-tested. The difficulty of doing this is to find a sample of students who are as similar as possible to the group for whom the test is intended. For the classroom
teacher the normal pre-testing procedure could involve the following:

1. Short tests could be administered on completion of each major topic. The two major disadvantages of this are:
   (a) the students will have seen the questions when they eventually appear on the major examination. However, provided that the time elapse is sufficiently great this is not too serious a problem
   (b) students will not be in the same state of preparedness for the pre-test as for the major examination. This can be overcome to some extent if the pre-tests are used as part of the course work requirement. Such a continuous assessment method in itself may have desirable educational features and would also help to produce more reliable items. Also, since each topic should be fresh in the students' minds, they should score higher than on the complete final examination, so that the Facility Value may vary in the two situations.

2. Parallel examinations can be given immediately before or after a 'final' examination. If this method is used the students, provided they believe that the test will be important, will be in the right state of mind and will have completed the entire course syllabus. The disadvantage is that the results of the pre-test will be obtained too late for the students on the course at that particular time.
However, provided the course is to be repeated with similar student populations, the pre-test sample could be regarded as representative.

Untested items could also be included in the actual major examination provided, of course, it was objective in form. These items would be scattered amongst scoring items of the test and the students would not be told which were the non-scoring items; his final score being calculated from the established items. The major disadvantage of this method is that only a few items can be included in each paper (approximately 20% of the total) so that this method is only really suitable for adding to an already established bank.

Having discussed the general ideas concerning pre-testing it should be realised that we have already pre-tested the items for the population we are considering when the objective diagnostic tests were administered. The data from these diagnostic tests will yield, amongst other things, the facility value of the items (Chapter 7, sections 7.5, 7.10, 7.15, 7.20).

The next step in the chain of the test construction is to re-appraise the items before finally making up the balance, tested examination paper. As the flow chart indicates, there can be a certain amount of recycling of pre-test items and obviously any items not used in the final examination, but which are acceptable, can be stored for
future use in an item bank. The criterion used for selecting suitable items from those used in the diagnostic test will be that discussed in Chapter 2, section 2.6.1 i.e. the items should have $0.4 < \text{Facility Value} < 0.6$. Note that the Discrimination Index of each item is not used in this criterion because we do not know this value as it is not applicable to calculate the I.D. value for items of diagnostic tests (Chapter 2, section 2.6.2). Part of this study is to investigate whether items used in diagnostic tests can be suitably selected for use in attainment tests through the above criterion. Comparing the criterion above with the facility values of the items of the diagnostic tests it can be seen that certain items have F.V.'s greater than 0.6. However, as the children were allowed aids in the diagnostic test which will not be allowed in the selection test it can be expected that these F.V.'s are somewhat high. The items chosen for the selection test will be those best fitting the criterion mentioned above but other items not conforming to this will be included in order that the constraints of the specification grid can be met.

8.3 Instructions for the test

The children, after experiencing the objective diagnostic tests, were now very familiar with the procedure for objective testing. As in the diagnostic tests, they were told to select one of the four alternatives offered for each item and to indicate their choice by circling the appropriate letter on the answer sheet. Ample time (35 minutes) was
allowed to complete the test because the Kuder-Richardson
Formula 21 was being used to obtain the reliability
coefficient. Johansen and Johnson (1969 p.35) write:

"With the use of the formula (the K-R 21), reliability
coefficients can be obtained from a single administration of
a complete test. However, for the K-R formula to be
applicable, it is essential that the test be homogeneous
and that the test not be a speeded test".

The test was certainly homogeneous and the 35 minutes
allowed everyone to complete the test with time to spare.
No aids were allowed for this test.

8.4 The Arithmetic Test

The selection test was given the title of Arithmetic
Test to indicate the aspect of mathematics to be tested. The
actual test was easy to write because the necessary decisions
had been made and the items available. It was simply a matter
of looking at the skill to be tested, finding the appropriate
sub-stage of the respective diagnostic test and then selecting
the items which best fitted the F.V. criterion laid down in the
last sub-section. The test was constructed to consist of 25
items and these were ordered so that the item with the highest
Facility Value was first and that with the lowest Facility Value
was last. The reason for this was so that the children should
experience the easier items first, thus enabling them to relax
and gain confidence in order that they might perform their
best in the test.
The Arithmetic Test

1. \[ \begin{array}{c}
127 \\
\times 6 \\
\end{array} \]

(A) 762  
(B) 1014  
(C) 622  
(D) 61242

2. \[ \begin{array}{c}
721 \\
-468 \\
\end{array} \]

(A) 347  
(B) 253  
(C) 343  
(D) 365

3. \[ \begin{array}{c}
253 \\
\times 4 \\
\end{array} \]

(A) 1012  
(B) 802  
(C) 32012  
(D) 1021

4. \[ \begin{array}{c}
456 \ 
+654 \\
\end{array} \]

(A) 1110  
(B) 1000  
(C) 1011  
(D) 1200

5. \[ \begin{array}{c}
97 \\
\times 40 \\
\end{array} \]

(A) 97  
(B) 97  
(C) 97  
(D) 97

\[ \begin{array}{c}
00 \\
360 \\
3880 \\
3880 \\
\end{array} \]

(A) 3777  
(B) 3623  
(C) 3733  
(D) 3612
7. \[ \begin{array}{c}
400 \\
+ 469 \\
- 369 \\
\hline
79380 \\
\end{array} \]

(A) 400  (B) 469  (C) 441  (D) 331

8. \[ \begin{array}{c}
31673 \\
+ 48717 \\
\hline
80390 \\
\end{array} \]

(A) 79380  (B) 80390  (C) 91181  (D) 99380

9. \[ \begin{array}{c}
12 \\
x 20 \\
\hline
240 \\
\end{array} \]

(A) 12  (B) 12  (C) 12  (D) 12

\[ \begin{array}{c}
\times 20 \\
\times 20 \\
\times 20 \\
\hline
40 \\
20 \\
12 \\
\end{array} \]

\[ \begin{array}{c}
+20 \\
+20 \\
\hline
60 \\
240 \\
240 \\
\end{array} \]

\[ \begin{array}{c}
240 \\
240 \\
252 \\
\end{array} \]

10. \[ \begin{array}{c}
14577 \\
+ 5828 \\
\hline
20405 \\
\end{array} \]

(A) 71111  (B) 10405  (C) 19395  (D) 20405

11. \[ \begin{array}{c}
1800 \\
- 1762 \\
\hline
38 \\
\end{array} \]

(A) 38  (B) 138  (C) 162  (D) 100

12. \[ \begin{array}{c}
50 \\
\underline{2500} \\
\hline
5000 \\
\end{array} \]

(A) 500  (B) 50  (C) 5  (D) 5000
13. \[ \begin{array}{l}
(A) \times 14 \\
(B) \times 14 \\
(C) \times 14 \\
(D) \times 14
\end{array} \]
\[ \begin{array}{r}
98 \\
+1120 \\
1218
\end{array} \]
\[ \begin{array}{l}
+108 \\
\underline{1148}
\end{array} \]

14. \[ \begin{array}{l}
(A) 106 \\
(B) 36 \\
(C) 176 \\
(D) 136
\end{array} \]
\[ \begin{array}{r}
208 \\
-172
\end{array} \]
\[ \begin{array}{l}
13.
\end{array} \]

15. \[ \begin{array}{l}
(A) 26 \\
(B) 26 \\
(C) 26 \\
(D) 26
\end{array} \]
\[ \begin{array}{r}
182 \\
+1300 \\
1482
\end{array} \]
\[ \begin{array}{l}
182 \\
+130 \\
312
\end{array} \]
\[ \begin{array}{l}
\underline{1342}
\end{array} \]

16. \[ \begin{array}{l}
(A) 6000 \\
(B) 6660 \\
(C) 600 \\
(D) 60000
\end{array} \]

17. \[ \begin{array}{l}
7 \div \overline{709}
\end{array} \]
\[ \begin{array}{l}
(A) 78 \\
(B) 1 \\
(C) 78\% \\
(D) 78\frac{3}{4}
\end{array} \]

18. \[ \begin{array}{l}
5 \div \overline{525}
\end{array} \]
\[ \begin{array}{l}
(A) 15 \\
(B) 11 \\
(C) 101 \\
(D) 105
\end{array} \]
19. \[ \begin{array}{c}
\text{(A)} \quad 137 \\
\times 246 \\
\hline
822 \\
548 \\
274 \\
\hline
1644
\end{array} \quad \begin{array}{c}
\text{(B)} \quad 137 \\
\times 246 \\
\hline
822 \\
548 \\
274 \\
\hline
1644
\end{array} \quad \begin{array}{c}
\text{(C)} \quad 137 \\
\times 246 \\
\hline
822 \\
548 \\
274 \\
\hline
1644
\end{array} \quad \begin{array}{c}
\text{(D)} \quad 137 \\
\times 246 \\
\hline
822 \\
548 \\
274 \\
\hline
1644
\end{array} \]

\[ \quad = 274543822 \]

20. \[ \begin{array}{c}
28 \sqrt{672} \\
\hline
28 \text{ goes into } 672 \text{ at } 24 \\
\hline
228 \\
228 \\
\hline
000
\end{array} \]

21. \[ \begin{array}{c}
19 \sqrt{798} \\
\hline
19 \text{ goes into } 798 \text{ at } 42 \\
\hline
38 \\
38 \\
\hline
00
\end{array} \]

22. \[ \begin{array}{c}
4 \sqrt{826} \\
\hline
3 \text{ goes into } 826 \text{ at } 207 \frac{1}{2} \\
\hline
19 \\
19 \\
\hline
0
\end{array} \]

23. \[ \begin{array}{c}
34 \sqrt{3181} \\
\hline
34 \text{ goes into } 3181 \text{ at } 93 \frac{1}{2} \\
\hline
306 \\
306 \\
\hline
121 \\
121 \\
\hline
102 \\
102 \\
\hline
19
\end{array} \]
24. \[ \frac{2718}{13} \]

<table>
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<th>(29\frac{3}{13})</th>
<th>(209\frac{9}{13})</th>
<th>(209)</th>
<th>(29)</th>
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<tbody>
<tr>
<td>A</td>
<td>13 (\overline{2718})</td>
<td>13 (\overline{2718})</td>
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<td>(-001)</td>
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25. \[ 805 \times 354 \]

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<td></td>
<td>285770</td>
<td>9660</td>
<td>2820720</td>
<td>284970</td>
</tr>
</tbody>
</table>
8.5 The Arithmetic Test answer sheet

<p>| | | | | |</p>
<table>
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<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>B</td>
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<td>24</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>25</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>
8.6 Marking the test

The test was very quickly marked: a 1 being given for a correct response and a 0 for an incorrect response. These results were analysed using the computer (see program in Appendix B) to give the following:

1. the Facility Value of each item
2. the Index of Discrimination of each test item
3. the Reliability coefficient for the test
4. the rank ordering of the children.

The first three values are given in the following table.
### 8.7 Characteristics of the Test

<table>
<thead>
<tr>
<th>Item</th>
<th>Facility Value from the Diagnostic Test</th>
<th>Facility Value from the Selection Test</th>
<th>Difference in Facility Value</th>
<th>Index of Discrimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.93</td>
<td>0.80</td>
<td>-0.13</td>
<td>0.32</td>
</tr>
<tr>
<td>2</td>
<td>0.92</td>
<td>0.86</td>
<td>-0.06</td>
<td>0.28</td>
</tr>
<tr>
<td>3</td>
<td>0.90</td>
<td>0.90</td>
<td>0.0</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>0.88</td>
<td>0.94</td>
<td>+0.06</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>0.88</td>
<td>0.72</td>
<td>-0.16</td>
<td>0.56</td>
</tr>
<tr>
<td>6</td>
<td>0.87</td>
<td>0.90</td>
<td>+0.03</td>
<td>0.20</td>
</tr>
<tr>
<td>7</td>
<td>0.86</td>
<td>0.90</td>
<td>+0.04</td>
<td>0.20</td>
</tr>
<tr>
<td>8</td>
<td>0.85</td>
<td>0.88</td>
<td>+0.03</td>
<td>0.24</td>
</tr>
<tr>
<td>9</td>
<td>0.85</td>
<td>0.86</td>
<td>+0.01</td>
<td>0.28</td>
</tr>
<tr>
<td>10</td>
<td>0.84</td>
<td>0.86</td>
<td>+0.02</td>
<td>0.28</td>
</tr>
<tr>
<td>11</td>
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<td>0.80</td>
<td>-0.03</td>
<td>0.40</td>
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<td>0.70</td>
<td>-0.10</td>
<td>0.44</td>
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<tr>
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<td>0.80</td>
<td>0.68</td>
<td>-0.12</td>
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<td>14</td>
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<td>15</td>
<td>0.78</td>
<td>0.66</td>
<td>-0.12</td>
<td>0.60</td>
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<tr>
<td>16</td>
<td>0.77</td>
<td>0.70</td>
<td>-0.07</td>
<td>0.52</td>
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<td>19</td>
<td>0.71</td>
<td>0.56</td>
<td>-0.15</td>
<td>0.64</td>
</tr>
<tr>
<td>20</td>
<td>0.70</td>
<td>0.64</td>
<td>-0.06</td>
<td>0.72</td>
</tr>
<tr>
<td>21</td>
<td>0.68</td>
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<td>0.68</td>
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<td>22</td>
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<td>0.60</td>
<td>-0.02</td>
<td>0.56</td>
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<tr>
<td>23</td>
<td>0.55</td>
<td>0.58</td>
<td>+0.03</td>
<td>0.84</td>
</tr>
<tr>
<td>24</td>
<td>0.54</td>
<td>0.56</td>
<td>+0.02</td>
<td>0.80</td>
</tr>
<tr>
<td>25</td>
<td>0.46</td>
<td>0.58</td>
<td>+0.12</td>
<td>0.76</td>
</tr>
</tbody>
</table>

**Key**

- * item discriminates well
- o item discriminates reasonably well
- \ item has marginal discrimination
- x item should be rejected
- - means F.V. was lower in selection test
- + means F.V. was higher in selection test

Reliability coefficient = 0.84
8.8 Analysis of the Selection Test

Davis (1964 p. 24) writes:

"Experience has shown that for measuring characteristics of individuals, scores with a reliability coefficient below 0.75 are rather inefficient. For measuring the average characteristics of groups the size of many classes, say 25 to 50, scores with a reliability coefficient as low as 0.50 may often be serviceable. With average scores in larger groups, even lower reliability coefficients are frequently useful".

Thus the reliability coefficient for the Selection Test of 0.84 is encouragingly high and we can discuss with some conviction the values calculated for the test.

The means of selecting the items from the Diagnostic Test was fully discussed in Section 8.2. The writer had thought it likely that the Facility Value (F.V.) of the items in the Diagnostic Tests would be a little higher than those of the Selection Test because in the former aids were allowed whilst in the latter test they were not. On comparing the values for both tests (Section 8.7) it can be seen that the F.V.'s in most cases were very similar. The number of tests have not been sufficient to analyse these results statistically but nevertheless I feel that some comments can be made. For one item the F.V.'s were identical and for 15 items the F.V.'s were slightly lower for the Selection Test than the Diagnostic Test. Indeed, for 18 of the 25 items there was a difference in F.V.'s for the items
in the two tests of 7% or less. The similarity of the Facility Values for the two tests should have been expected because the distractors were constructed through algorithmic consideration and the use of aids such as multiplication cards in the Diagnostic Tests should not have aided the child if he were applying the wrong algorithm.

It is necessary in selection tests to consider the Index of Discrimination (I.D.) of the items for, as discussed in Chapter 2, in tests where the main aim is to establish a rank order of the discrimination power is at least as important as the Facility Value in determining the suitability of an item. Dawson (1971) suggests the following criteria:

<table>
<thead>
<tr>
<th>I.D. values</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40 and above</td>
<td>item discriminates well</td>
</tr>
<tr>
<td>0.30 to 0.39</td>
<td>item discriminates reasonably well</td>
</tr>
<tr>
<td>0.20 to 0.29</td>
<td>item shows marginal discrimination</td>
</tr>
<tr>
<td>below 0.20</td>
<td>item shows inadequate discrimination; it is normally rejected.</td>
</tr>
</tbody>
</table>

Applying this criteria 15 items of the test discriminate well, 1 item discriminates reasonably well, 8 items show marginal discrimination whilst item 4 should be rejected. Since the I.D. of an item is a measure of how efficient it has been at distinguishing between the stronger and the weaker candidates the writer considers that it is a more important value than the Facility Value in selection tests where ranking is important.
The reliability coefficient of the test has already been shown to be encouragingly high. The high reliability coefficient for the test was not unexpected because the features related to test reliability (Chapter 2, section 2.6.3) were noted in order that the test be reliable. An unknown factor which might have affected the test reliability was the Index of Discrimination. Ebel (1972 p.428) writes that the best way of increasing test reliability is to use items with a high Index of Discrimination. In order to satisfy reliability needs Macintosh and Morrison (1972 p.67) suggest the following range of I.D.'s and F.V.'s for inclusion in a test.

<table>
<thead>
<tr>
<th>Index of Discrimination</th>
<th>Facility Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above 0.4</td>
<td>Difficult*</td>
</tr>
<tr>
<td>0.3 to 0.39</td>
<td>Difficult</td>
</tr>
<tr>
<td>0.20 to 0.29</td>
<td>Difficult</td>
</tr>
<tr>
<td>below 0.20</td>
<td>Reject</td>
</tr>
</tbody>
</table>

Items marked * would be considered suitable

Using the above criteria only 5 items in the test would be considered suitable, from the test reliability point of view. However, as has been stated, where the aim of the test is to establish rank order the Index of Discrimination is at least as important as the Facility Value and the writer considers that items marked + would also be suitable. This
criteria then becomes very similar to Dawson's (1971) and again 15 items for the test would be considered suitable. The high reliability coefficient for the test would seem to confirm that this criteria is reasonable.

The Selection Test was constructed primarily to investigate the feasibility of choosing items from diagnostic test, by virtue of their Facility Value, for use in Selection/Attainment tests. The ranking of the children was a very minor part of the project but it can be shown to have a practical use without undue importance being given to the ranking of a single test.

As was to be expected for a test consisting of 25 items given to 92 children it was not very successful in ranking the children. There was a great deal of clustering around each score as shown in Table 8.8.1 and it would be difficult to decide the dividing line if the test were used, say, to divide the year group into four ability groups as at present happens in the school were the test was administered. However it could serve a purpose in selecting candidates for the lowest ability group. The writer considers that the test should have been easily answered by children of 12-13 years of age and yet there were children who scored less than half marks. Thus when considering the computational skills the test could serve a useful purpose in selecting the very poor children. The teacher would have to decide where the dividing line was and this would probably be dictated by the size of the group he wanted. The test would serve to
bring to the teacher's attention possible candidates for the low ability group; obviously the teacher would take into account other considerations and not solely rely on the result of a single test. The test would not be very useful in selecting children for other ability groups because of the considerable amount of clustering. Obviously more testing would have to be done and it must be remembered that this test only concerned itself with the computational skills.
8.8.1 A Comparison of rank ordering and ability grouping

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Ability Group</th>
<th>Score</th>
<th>Ranking</th>
<th>Ability Group</th>
<th>Score</th>
</tr>
</thead>
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8.8.1 A Comparison of rank ordering and ability grouping (continued)

<table>
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<th>Rating</th>
<th>Ability Group</th>
<th>Score</th>
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<td>4</td>
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<td>15</td>
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</tr>
</tbody>
</table>
When the answer sheets were marked they were placed in order with the candidates having the highest scores placed first and so on. The candidates were then labelled by numbers according to this ranking. When the results were analysed by the computer it, among other things, ranked the children. This was used as a checking device and it can be seen that candidates 31 and 32 had been wrongly placed and that is the reason they are out of sequence.

An interesting exercise using the selection test was to compare the ranking of the candidates with the ability groups in which the school had placed them (Table 8.3.1). Obviously no firm conclusions can be drawn from a single test, particularly one concerned with such a narrow aspect of mathematics, but a few points were raised.

As expected there is a fair amount of mingling when comparing the candidates ability groups with their ranking. In general the ability groups seem to have been reasonably well chosen but there are a few notable exceptions. Candidate 65, who is in Ability Group 1, is very low in the ranking and is somewhat isolated from the rest of her group. Similarly with candidate 74 and candidate 82 who are again isolated from the rest of the candidates in their ability groups. These children may have been placed in a higher Ability Group then they ought to have been and as a consequence the level of teaching and mathematics experienced may be beyond them. Of course they might just have performed
poorly in the test but at least they have been highlighted and close observation over a period of time should reveal whether they are coping with the work presented to them.

Contrary to the above argument we have candidate 50 who is high in the ranking and yet has been placed in ability group 4. This child could be underachieving in lessons and he needs to be observed. Obviously the test can only be used to indicate the misplacement of children in ability groups but it does highlight those children who need to be closely observed.

8.9 Summary and conclusions

Writing suitable items for a selection/attainment test is very time consuming. In the preceding discussion it has been shown that the teacher can save himself much time and effort by choosing suitable items from the appropriate diagnostic test. Items are usually chosen to meet both Facility Value and Index of Discrimination requirements (section 8.8) but, as discussed in Chapter 2, the Index of Discrimination plays no part in Diagnostic Tests. Therefore there was a need to investigate the feasibility of choosing items from the Diagnostic Tests using the Facility Value only.

The Facility Value was chosen because it has some meaning in Diagnostic Tests and is easily calculated. Although the number of tests have not been sufficient to analyse the results statistically the comparison of Facility Value in section 8.7 lends one to tentatively suggest that
items used in Diagnostic Tests can be chosen for use in Selection/Attainment tests through the use of these values. If this assertion is correct (much more testing needs to be done to verify this) then it could prove invaluable in saving time and effort. Instead of writing items for two sets of tests the same items could be used for both and the many steps required in constructing a selection/attainment test would be greatly reduced. The diagnostic test would serve the selection/attainment test in providing items which had already been pre-tested (Section 8.2.2). Once these items are used in a Selection Test both their Facility Values and Index of Discrimination can be calculated along with the Reliability Coefficient of the test to reveal valuable information (all these characteristics of the tests are fully discussed in Chapter 2).

From the brief analysis of the test it can be concluded that 15 items would be suitable for use in future tests. These items could be used to start an item bank by typing them onto card along with the appropriate I.D., F.V. and Reliability Coefficient of the test in which they were used. The analysis shows that 9 items need modifying whilst 1 needs rejecting.
CHAPTER 9

A MATHEMATICAL RECORD

9.1 Introduction

It was suggested in Chapter 1 and Chapter 8 that the diagnostic and selection tests might be used as the basis of a mathematical record for individual children. This record would provide valuable information concerning the child's mathematical ability and would prove invaluable when the child was transferred to another school or, indeed, to another class within the same school. The mathematical record which is suggested will be very detailed, being based on the component sub-stages of the computational skills (Chapter 4), and will be easy and quick to fill in. This latter point is most important because it is of little use having a very detailed record which is time-consuming to upkeep, for the teacher simply hasn't much time and eventually the records will be neglected.

The mathematical record proposed will only be concerned with the computational skills but there are no reasons for this not being extended to include other aspects of mathematics. The record will list all the skills/concepts that will be required to be mastered; the criterion that a child has shown mastery will be that he gave at least two correct responses to the items of the corresponding sub-stage in the Diagnostic Tests. Mastery will be
indicated on the record by inserting, in blue, the date on which the test was taken at the appropriate point on the matrix. Failure to show mastery will be indicated by inserting, in red, the date on which the test was taken. If a child fails to show mastery then he would be tested at a later date and mastery or not would be indicated accordingly.

There will be a section on the record for the results of the selection/attainment tests. Although only one has been compiled in this project the writer envisages that many will be given throughout a child's school career covering all aspects of mathematics. Each of these tests will have their own specification grid which means that each test should be consistent over a number of years in the objectives it is testing even though the items themselves may change. The important information required for the record are the name of the test, its reliability and the ranking of the child.

The results from the Diagnostic and Selection tests for three children will be used to exemplify the mathematical record.

9.2 The Mathematical Record

Three typical mathematical records which are based on the results from the tests will follow.
Mathematical Record

Name: Vanda

Skill/Concept

Addition

<table>
<thead>
<tr>
<th>Task</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>To add a 2-digit number and a 1-digit number: no carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td>one wrong - vertical and horizontal addition algorithm employed</td>
</tr>
<tr>
<td>To add a 2-digit number and a 1-digit number: carrying units as tens</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td>all wrong - failure to add carrying figure</td>
</tr>
<tr>
<td>To add two 2-digit numbers: no carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td>one wrong - failure to add carrying figure</td>
</tr>
<tr>
<td>To add two 2-digit numbers: carrying units</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
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<td>To add two 3-digit numbers: no carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td>To add two 3-digit numbers: carrying units</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td>one wrong - failure to add carrying figure</td>
</tr>
<tr>
<td>To add two 3-digit numbers: carrying where necessary (one place only)</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td>one wrong - adding all digits place value ignored</td>
</tr>
<tr>
<td>To add two 3-digit numbers: with repeated carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>To add numbers with up to 4 digits: carrying where necessary</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td>one wrong - failure to add carrying figure</td>
</tr>
<tr>
<td>To add numbers with up to 5 digits: carrying where necessary</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Skill/Concept

Subtraction

<table>
<thead>
<tr>
<th>Task</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>To subtract a 1-digit number from a 2-digit number: no borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td>all wrong - inverted subtraction algorithm employed</td>
</tr>
<tr>
<td>To subtract a 1-digit number from a 2-digit number: with borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Comments</td>
<td></td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>To subtract using two 2-digit numbers: no borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using two 2-digit numbers: with borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: no borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: borrowing units</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: borrowing tens</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: borrowing where necessary (one place only)</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: with repeated borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using numbers with up to 4 digits with repeated borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract using numbers with up to 5 digits: borrowing where necessary</td>
<td>Sept '79</td>
</tr>
</tbody>
</table>

**Skill/Concept**

**Multiplication**

| To multiply a 2-digit number by a 1-digit number: no carrying             | Oct '79 |
| To multiply a 3-digit number by a 1-digit number: no carrying             | Oct '79 |
| To multiply a 2-digit number by a 1-digit number: carrying units          | Oct '79 |
| To multiply a 3-digit number by a 1-digit number: carrying units          | Oct '79 |

*All wrong - inverted subtraction algorithm employed.*

*All wrong - inverted subtraction algorithm employed.*

*All wrong - inverted subtraction algorithm employed.*

*All wrong - inverted subtraction algorithm employed.*

*All wrong - inverted subtraction algorithm employed.*

*All wrong - inverted subtraction algorithm employed.*

*One wrong - incorrect multiplication by zero.*

*Two wrong - incorrect multiplication by zero.*

*One wrong - carrying digit placed in answer.*

*All wrong - both inverted subtraction algorithm and a failure to modify total after borrowing employed.*
To multiply a 3-digit number by a 1-digit number: carrying where necessary (one place only)  

| Oct | 79 |

To multiply a 3-digit number by a 1-digit number: with repeated carrying  

| Oct | 79 |

To multiply a whole number by 10 or 100  

| Oct | 79 |

To multiply a 2-digit number by 20, 30, 40 or 50  

| Oct | 79 |

To multiply two 2-digit numbers: carrying with units only  

| Oct | 79 |

To multiply two 2-digit numbers: carrying where necessary  

| Oct | 79 |

To multiply a 3-digit number by a 2-digit number: carrying where necessary (one place only)  

| Oct | 79 |

To multiply a 3-digit number by a 2-digit number: with repeated carrying  

| Oct | 79 |

To multiply two 3-digit numbers: carrying where necessary  

| Oct | 79 |

Skill/Concept

Division

To do simple division by a 1-digit number: no remainder  

| Nov | 79 |

To do simple division by a 1-digit number: with remainder  

| Nov | 79 |

To divide a 3-digit number by a 1-digit number: no remainder  

| Nov | 79 |
To divide a 3-digit number by a 1-digit number: with remainder

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov '79</td>
</tr>
<tr>
<td>one wrong - zeros omitted in quotient</td>
</tr>
</tbody>
</table>

To divide a 4-digit number by a 1-digit number: with and without remainder

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov '79</td>
</tr>
</tbody>
</table>

To divide a 3-digit number by 20, 30, 40 or 50: no remainder

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov '79</td>
</tr>
</tbody>
</table>

To divide a 3-digit number by a 2-digit number: with remainder

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov '79</td>
</tr>
</tbody>
</table>

To divide a 4-digit number by a 2-digit number: with and without remainder

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov '79</td>
</tr>
</tbody>
</table>

could not attempt

Selection/Attainment Tests

<table>
<thead>
<tr>
<th>Name of Test</th>
<th>Reliability of Test</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic Test</td>
<td>0.84</td>
<td>78 out of 92</td>
</tr>
</tbody>
</table>
Mathematical Record

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td></td>
</tr>
<tr>
<td>To add a 2-digit number and a 1-digit number: no carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add a 2-digit number and a 1-digit number: carrying units as tens</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add two 2-digit numbers: no carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add two 2-digit numbers: carrying units</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add two 3-digit numbers: no carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add two 3-digit numbers: carrying units</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add two 3-digit numbers: carrying where necessary (one place only)</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add two 3-digit numbers: with repeated carrying</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add numbers with up to 4 digits: carrying where necessary</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To add numbers with up to 5 digits: carrying where necessary</td>
<td>Sept '79</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td></td>
</tr>
<tr>
<td>To subtract a 1-digit number from a 2-digit number: no borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract a 1-digit number from a 2-digit number: with borrowing</td>
<td>Sept '79</td>
</tr>
</tbody>
</table>
### Comments

<table>
<thead>
<tr>
<th>Task</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>To subtract using two 2-digit numbers: no borrowing</td>
<td>Sept 79</td>
</tr>
<tr>
<td>To subtract using two 2-digit numbers: with borrowing</td>
<td>Sept 79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: no borrowing</td>
<td>Sept 79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: borrowing units</td>
<td>Sept 79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: borrowing tens</td>
<td>Sept 79</td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: borrowing where necessary (one place only)</td>
<td>Sept 79</td>
</tr>
<tr>
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<td>Sept 79</td>
</tr>
<tr>
<td>To subtract using numbers with up to 4-digits with repeated borrowing</td>
<td>Sept 79</td>
</tr>
<tr>
<td>To subtract using numbers with up to 5-digits: borrowing where necessary</td>
<td>Sept 79</td>
</tr>
</tbody>
</table>

*Notes: The items not attempted—probably omitted accidentally but this needs checking.*

### Skill/Concept

#### Multiplication

<table>
<thead>
<tr>
<th>Task</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>To multiply a 2-digit number by a 1-digit number: no carrying</td>
<td>Oct 79</td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 1-digit number: no carrying</td>
<td>Oct 79</td>
</tr>
<tr>
<td>To multiply a 2-digit number by a 1-digit number: carrying units</td>
<td>Oct 79</td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 1-digit number: carrying units</td>
<td>Oct 79</td>
</tr>
<tr>
<td>Comments</td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td></td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 1-digit number: carrying where necessary (one place only)</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 1-digit number: with repeated carrying</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply a whole number by 10 or 100</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply a 2-digit number by 20, 30, 40 or 50</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply two 2-digit numbers: carrying with units only</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply two 2-digit numbers: carrying where necessary</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 2-digit number: carrying where necessary (one place only)</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 2-digit number: with repeated carrying</td>
<td>Oct '79</td>
</tr>
<tr>
<td>To multiply two 3-digit numbers: carrying where necessary</td>
<td>Oct '79</td>
</tr>
</tbody>
</table>

**Skill/Concept**

**Division**

| To do simple division by a 1-digit number: no remainder | Nov '79 |
| To do simple division by a 1-digit number: with remainder | Nov '79 |
| To divide a 3-digit number by a 1-digit number: no remainder | Nov '79 |

*One wrong - 2 zeroes omitted in quotient.*
To divide a 3-digit number by a 1-digit number: with remainder

To divide a 4-digit number by a 1-digit number: with and without remainder

To divide a 3-digit number by 20, 30, 40 or 50: no remainder

To divide a 3-digit number by a 2-digit number: no remainder

To divide a 3-digit number by a 2-digit number: with remainder

To divide a 4-digit number by a 2-digit number: with and without remainder

Selection/Attainment Tests

<table>
<thead>
<tr>
<th>Name of Test</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic Test</td>
<td>0.84</td>
<td>48 out of 92</td>
</tr>
</tbody>
</table>
### Mathematical Record

**Skill/Concept**

**Addition**

<table>
<thead>
<tr>
<th>Description</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>To add a 2-digit number and a 1-digit number: no carrying</td>
<td>Sept '79</td>
</tr>
<tr>
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<tr>
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</tr>
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<td>To add two 2-digit numbers: carrying units</td>
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</tr>
<tr>
<td>To add numbers with up to 5 digits: carrying where necessary</td>
<td>Sept '79</td>
</tr>
</tbody>
</table>

**Skill/Concept**

**Subtraction**

<table>
<thead>
<tr>
<th>Description</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>To subtract a 1-digit number from a 2-digit number: no borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>To subtract a 1-digit number from a 2-digit number: with borrowing</td>
<td>Sept '79</td>
</tr>
<tr>
<td>Comments</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>To subtract using two 2-digit numbers: no borrowing</td>
<td></td>
</tr>
<tr>
<td>To subtract using two 2-digit numbers: with borrowing</td>
<td></td>
</tr>
<tr>
<td>To subtract using two 3-digit numbers: no borrowing</td>
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</tr>
<tr>
<td>To subtract using two 3-digit numbers: borrowing units</td>
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<tr>
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<tr>
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<td>To subtract using numbers with up to 4 digits with repeated borrowing</td>
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</tr>
<tr>
<td>To subtract using numbers with up to 5 digits: borrowing where necessary</td>
<td></td>
</tr>
</tbody>
</table>

**Skill/Concept**

**Multiplication**

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>To multiply a 2-digit number by a 1-digit number: no carrying</td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 1-digit number: no carrying</td>
</tr>
<tr>
<td>To multiply a 2-digit number by a 1-digit number: carrying units</td>
</tr>
<tr>
<td>To multiply a 3-digit number by a 1-digit number: carrying units</td>
</tr>
</tbody>
</table>
### Comments

<table>
<thead>
<tr>
<th>Task</th>
<th>Skill/Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>To multiply a 3-digit number by a 1-digit number: carrying where necessary (one place only)</td>
<td><strong>Oct '79</strong></td>
</tr>
<tr>
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<td><strong>Oct '79</strong></td>
</tr>
<tr>
<td><strong>Skill/Concept</strong></td>
<td></td>
</tr>
<tr>
<td>Division</td>
<td></td>
</tr>
<tr>
<td>To do simple division by a 1-digit number: no remainder</td>
<td><strong>Nov '79</strong></td>
</tr>
<tr>
<td>To do simple division by a 1-digit number: with remainder</td>
<td><strong>Nov '79</strong></td>
</tr>
<tr>
<td>To divide a 3-digit number by a 1-digit number: no remainder</td>
<td><strong>Nov '79</strong></td>
</tr>
</tbody>
</table>

**Two wrong - incorrect multiplication by 20, 1979.**

**Couldn't attempt.**

**One wrong - lack of internal carrying concept.**
To divide a 3-digit number by a 1-digit number; with remainder

To divide a 4-digit number by a 1-digit number; with and without remainder

To divide a 3-digit number by 20, 30, 40 or 50; no remainder

To divide a 3-digit number by a 2-digit number; no remainder

To divide a 3-digit number by a 2-digit number; with remainder

To divide a 4-digit number by a 2-digit number; with and without remainder

Selection/Attainment Tests

<table>
<thead>
<tr>
<th>Name of Test</th>
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</tr>
</thead>
<tbody>
<tr>
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THE STANDARD ALGORITHMS

A.1 Introduction

In Chapter 7 reference was made to the 'standard algorithms' for Addition, Subtraction, Multiplication and Division. These algorithms are given in great detail in Chapter 6, Section 6.4, for they are the ones used to construct the keys for the various items. However the various standard algorithms will be exemplified below.

Addition

\[
\begin{array}{c}
4217 \\
+ 396 \\
\hline
4613
\end{array}
\]

The digits in the columns are added vertically starting with the units column. When exchanging occurs the carrying figure is placed as shown above.

Subtraction

Only the method of decomposition will be considered because this is the method taught in the school where the tests were administered.

\[
\begin{array}{c}
80175 \\
- 3926 \\
\hline
16249
\end{array}
\]

The digits are subtracted vertically starting with the units column. Exchanging is performed as indicated by the example.
**Multiplication**

(a) 'Short' multiplication

\[
\begin{array}{c}
247 \\
\times \ 8 \\
\hline
35 \\
1976
\end{array}
\]

When carrying is necessary the figure is placed as shown.

(b) Long multiplication

\[
\begin{array}{c}
362 \\
\times \ 98 \\
\hline
2896 \\
+32580 \\
\hline
35476
\end{array}
\]

Note that multiplication by units digit is performed first, followed by the tens digit and so on. This is the algorithm considered as standard for the project because it is the one taught in the school where the tests were administered.

**Division**

(a) 'Short' division

\[
\begin{array}{c}
619 \div 6 \\
\hline
103 \\
\hline
49
\end{array}
\]

For short division the internal remainder was carried as indicated in the above example.

(b) Long division

\[
\begin{array}{c}
330 \\
39 \overline{12899} \\
-117 \\
\hline
119 \\
-117 \\
\hline
29
\end{array}
\]

For long division, unlike short division, complete analysis of the solution is shown.
B.1 The program

The Basic program used for the analysis of the results of the Selection Test was written by Dr. R.P. Knott of Loughborough University.

The following print outs list the statements of the program, show the ranking of the students, list the items of the test with their respective Facility Value and Discrimination Index and give the Reliability Coefficient for the test.
1. REM THIS PROGRAM CREATES A DATA FILE FOR "RPK2". THE DATA IS ENTERED
2. REM IN GROUPS OF FIVE ALONG THE ROWS OF THE DATA MATRIX.
3. REM THE DECIMAL EQUIVALENT OF THE BINARY EXPRESSION OF EACH GROUP OF
4. REM FIVE IS ENTERED. THERE ARE L SUCH GROUPS, ANY EXTRA ITEMS ARE ENTERED
5. REM AS O.1'S AT THE END OF THE DATA INPUT
10 DIM B$(22)
17 DIM C(200).
20 DEFINE READ FILE #1="RPKBINDT"
30 DEFINE FILE #2="RPKDF1"
40 FOR I=0 TO 31
50 READ LINE #1,B$(I)
60 NEXT I
62 PRINT 'HOW MANY STUDENTS'
63 INPUT N
64 PRINT 'INPUT DATA IN GROUPS OF FIVE'
65 PRINT 'HOW MANY INPUTS'
68 INPUT L
85 MAT C=ZER(L)
90 PRINT 'WHAT DO YOU WANT THE FILE CALLED'
100 INPUT A$
110 DEFINE FILE #3=A$, ASC SEP
115 FOR J=1 TO M
120 PRINT 'COMMENCE INPUT OF SCORES FOR STUDENT J'
130 MAT INPUT C:
162 PRINT 'ARE THESE ENTRIES CORRECT'
163 INPUT A$
164 IF A$="NO" GOTO 120
165 FOR I=1 TO L
166 WRITE #2,B$(C(I))
167 NEXT I
170 PRINT 'IS THAT ALL'
180 INPUT A$
190 IF A$="YES" GOTO 220
200 PRINT 'INPUT LAST LINE'
210 INPUT A$
220 WRITE #2,A$
230 PRINT 'FILE END'
240 NEXT J
250 REMIND #2
260 PRINT 'WHAT IS THE NUMBER OF QUESTIONS'
270 INPUT N
275 MAT C=ZER(N)
280 FOR I=1 TO M
285 WRITE #3, I
290 MAT READ #2., C
310 MAT WRITE #3., C
315 NEXT I
316 CLOSE #2
317 CLOSE #2
320 END
THE PROGRAM (CONTINUED)

OK, SLIST RPKM5C1

60 10 DIM S(200), P(200), Z(200), D(200), R(200) 40 PRINT 'WHAT IS THE NUMBER OF STUDENTS'

70 INPUT N

80 PRINT 'WHAT IS THE NUMBER OF QUESTIONS'

90 INPUT M

100 PRINT 'WHERE IS THE DATA, GIVE FILENAME'

110 INPUT AT

120 MAT S=ZER(N)

130 MAT P=ZER(M)

140 MAT Z=A

150 MAT Q=A

160 MAT C=ZER(M)

170 DEFINE READ FILE #1=A$

180 FOR I=1 TO M

190 READ #1, X

200 MAT READ #1, S

210 FOR J=1 TO N

220 A(I)=A(I)+S(J)

230 C(J)=C(J)+S(J)

240 NEXT I

250 NEXT J

260 PRINT 'MATRIX OF STUDENT SCORES'

270 MAT PRINT A

280 PRINT 'MATRIX OF QUESTION SCORES'

290 MAT PRINT C

300 MAT Z=A

310 C=0

320 FOR I=1 TO M-1

330 IF Z(I+1)<=Z(I) THEN 410

340 X=Z(I)

350 Y=Q(I)

360 Z(I)=Z(I+1)

370 Q(I)=Q(I+1)

380 Z(I+1)=X

390 Q(I+1)=Y

400 C=C+1

410 NEXT I

420 IF C>0 THEN 310

430 PRINT 'HERE IS THE ORDERED SCORE'

440 PRINT 'STUDENT NO., 'SCORE'

450 FOR I=1 TO M

460 PRINT Q(I), Z(I)

470 NEXT I

480 PRINT
THE PROGRAM (CONTINUED)

480 X=INT(.27*N)
490 MAT P=ZER(N)
495 MAT D=ZER(N)
500 PRINT "NO OF OBJECTS IN GROUP IS = X"
502 REWIND #1
504 FOR I=1 TO N
505 READ #1, H
506 MAT READ #1, S
508 FOR J=1 TO X
510 IF I=Q(J) THEN 520
512 IF I=Q(M+1-J) THEN 520
514 NEXT J
516 NEXT I
518 GOTO 560
520 FOR K=1 TO N
522 D(K)=D(K)+S(K)
524 NEXT K
526 GOTO 514
530 FOR K=1 TO N
532 P(K)=P(K)+S(K)
534 NEXT K
536 GOTO 514
560 PRINT 'MATRIX UPPER SCORES'
570 MAT PRINT D
575 PRINT 'MATRIX OF LOWER SCORES'
580 MAT PRINT P
600 T=0
610 U=0
620 S=0
630 PRINT 'QUESTION NO.', 'FACILITY VALUE', 'DISCRIMINATION INDEX'
640 FOR I=1 TO N
650 G=D(I)+P(I)
660 F=G/(2+X)
670 T=T+G*F
680 S=S+G
690 G=G-2*P(I)
700 D=D/N
710 U=U+F
720 PRINT I, F, D
725 NEXT I
730 R=(N*(N-1))*(1-(2*X*S-T)/(.667*U+U))
740 PRINT 'RELIABILITY COEFFICIENT IS = R'
900 END
THE RANKING OF THE CHILDREN

Here is the ordered score:

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<tr>
<th>Student No.</th>
<th>Score</th>
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</table>
LIST OF VALUES FOR THE TEST

<table>
<thead>
<tr>
<th>QUESTION NO.</th>
<th>FACILITY VALUE</th>
<th>DISCRIMINATION INDEX</th>
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RELIABILITY COEFFICIENT: 0.2325