The effect of mass distribution on cricket bat playing properties

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ABSTRACT

Unlike most major sports, the game of cricket has seen little development in its implements based on modern technologies. It is likely that the first bat was an unfashioned branch from a tree and was used to defend against a suitably round stone in the games from which cricket evolved. Since this time, the design of the bat has been an intrinsic part of the games development, but these changes have been empirical and the effectiveness of the bat has relied upon the skill and knowledge of the bat maker and batsman. A significant part of this research project, supported by Dunlop Slazenger International, is to understand the science behind cricket bat performance.

Golf club manufacturers have improved perceived 'feel' and measurable performance of golf club drivers by re-distributing the mass of a solid club head towards the outer surface, in the form of an aluminium or titanium 'shell like' structure. The improvement in performance is such that modern driver heads are predominantly hollow and this development has also occurred in contemporary baseball bat and tennis racket design. However, the benefits of a hollow implement construction have largely eluded cricket. This study has investigated the possible advantages of a hollow wooden cricket bat in comparison to a more conventional, solid design through the manufacture and testing of hollow bat prototypes. Experimental procedures have been developed to accompany this research in the measurement of bat weight distribution and impact performance testing. Prototypes of a multiple wood layer construction were modelled in a computer-aided design environment and realised using computer numerically controlled machining. Further work included the use of finite element analysis to simulate observed impacts between bat and ball and the definition of a bat grading system based upon player perception of bat mass properties.

The research has shown that modification of bat playing properties can be achieved by the design of internal geometry. The hollow bats manufactured in this study demonstrated significant changes in moments of inertia and impact properties in comparison to a solid bat of similar external shape and size. Considering the traditional nature of the game, the ability to generate a range of playing characteristics within the confines of a conventional external shape may be significant for future bat design.
ACKNOWLEDGEMENTS

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Thanks are due to Steve Carr and more recently Andrew Hallam for devoting their time and engineering abilities to making bat prototypes and test machines. Nev Carpenter, Derrick Hurrell, Pete Wileman and Rob Doyle also deserve recognition for their assistance in this study. I would like to thanks Dr. Hugh Mansfield-Williams for his advice, expertise, and the use of the test facilities at the Forest Products Research Centre.

I would also like to thank Dunlop Slazenger for supporting this research and for providing bats and other materials. In particular, I would like to thank Brian Machin, Martin Aldridge and David Pennett for their enthusiasm and support during project meetings.

Finally, I would like to thank my parents Robin and Anick for their support, both emotional and financial, throughout my time in higher education.
Adrian Harms from the BBC interviews Dennis Lillee, the Australian fast bowling legend:

AH: One of the things you're most famous for is that aluminium bat - didn't quite catch on did it?

DL: It did catch on actually! We sold tens of thousands of them to start with, and then all of a sudden the MCC in their wisdom told Mike Brearley to get it stopped. But the way we invented it, my partner came up with the idea of a piece of aluminium railing from a staircase, with a handle welded on, covered in a bit of rubber. So he said "What do you think of this?" and we had a bit of a hit with it and it hit beautifully. So we made a load of them, sold thousands and then Brearley had it stopped. When that happened I got both teams to sign it and when Mike signed it he wrote at the top "Good luck with the sales Dennis"! Of course there weren't any sales after that...
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<td>$\dot{\varepsilon}$</td>
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<tr>
<td>$\alpha$</td>
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<td>$G$</td>
<td>Modulus of rigidity</td>
</tr>
<tr>
<td>$h$</td>
<td>Beam height</td>
</tr>
<tr>
<td>$I$</td>
<td>Inertia</td>
</tr>
<tr>
<td>$I_{sect}$</td>
<td>Second moment of area of a 2-D section</td>
</tr>
<tr>
<td>$K$</td>
<td>Proportional constant</td>
</tr>
<tr>
<td>$k$</td>
<td>Spring constant</td>
</tr>
<tr>
<td>$L$</td>
<td>Longitudinal (wood orientation)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$M$</td>
<td>First moment of force</td>
</tr>
</tbody>
</table>
\( N \)  Maximum stress or effective elastic modulus  
\( P \)  Strength or elastic modulus parallel to wood fibre orientation  
\( Q \)  Strength or elastic modulus perpendicular to wood fibre orientation  
\( q' \)  Loss coefficient  
\( R \)  Radial (wood orientation)  
\( r \)  Radius  
\( R \)  Reference intensity  
\( r_g \)  Radius of gyration  
\( S \)  Kendall-Rank sum  
\( S \)  Specific strength  
\( T \)  Kendall-Rank mean  
\( T \)  Tangential (wood orientation)  
\( t \)  Time, contact time  
\( U \)  Elastic energy  
\( v_{\text{in}}, (u) \)  Inbound (initial) object velocity  
\( v_{\text{out}}, (v) \)  Outbound (final) object velocity  
\( v_{\text{rel}} \)  Relative collision velocity between two objects  
\( x \)  Displacement  
\( 1\text{-D} \)  One-dimensional  
\( 2\text{-D} \)  Two-dimensional  
\( 3\text{-D} \)  Three-dimensional  
ASA  American Softball Association  
ASTM  American Society for Testing and Materials  
BBS  Batted ball speed  
BESR  Ball exit speed ratio  
BPF  Bat performance factor  
CAD  Computer-aided design  
CMM  Co-ordinate measurement machine  
CNC  Computer numerically controlled  
COM  Centre of mass  
COP  Centre of percussion  
DMA  Dynamic mechanical analyser  
DOF  Degrees of freedom  
DSI  Dunlop Slazenger International  
EFFM  Extended form feature modelling  
FE  Finite element  
FEA  Finite element analysis  
FPRS  Forest Products Research Centre  
FPS  Frames per second  

xx
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>GRP</td>
<td>Glass reinforced plastic</td>
</tr>
<tr>
<td>HSV</td>
<td>High speed video</td>
</tr>
<tr>
<td>ICR</td>
<td>Instantaneous centre of rotation</td>
</tr>
<tr>
<td>IPP</td>
<td>Image Pro Plus software</td>
</tr>
<tr>
<td>ISF</td>
<td>International Softball Federation</td>
</tr>
<tr>
<td>JND</td>
<td>Just noticeable difference</td>
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<tr>
<td>KE</td>
<td>Kinetic energy</td>
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<tr>
<td>MCC</td>
<td>Marylebone Cricket Club</td>
</tr>
<tr>
<td>MLB</td>
<td>Major League Baseball</td>
</tr>
<tr>
<td>MOI</td>
<td>Moment of inertia</td>
</tr>
<tr>
<td>MS</td>
<td>Modified silane (polyether adhesive)</td>
</tr>
<tr>
<td>MV</td>
<td>Minimum vibration (region)</td>
</tr>
<tr>
<td>NCAAA</td>
<td>National Collegiate Athletic Association</td>
</tr>
<tr>
<td>NSA</td>
<td>National Softball Association</td>
</tr>
<tr>
<td>PC</td>
<td>Percussion (region)</td>
</tr>
<tr>
<td>PVA</td>
<td>Polyvinyl acetate</td>
</tr>
<tr>
<td>PF</td>
<td>Phenol-formaldehyde</td>
</tr>
<tr>
<td>RMS</td>
<td>Root mean square</td>
</tr>
<tr>
<td>TTC</td>
<td>Tungsten tipped carbide</td>
</tr>
<tr>
<td>UF</td>
<td>Urea-formaldehyde</td>
</tr>
<tr>
<td>USSSA</td>
<td>United States Specialty Sports Association</td>
</tr>
<tr>
<td>WF</td>
<td>Weber fraction</td>
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</table>
CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Cricket is a popular game played mainly in the United Kingdom and Commonwealth countries. It is considered 'traditional' and for many this enhances the game's unique appeal. It may also account for the fact that the game has not been subject to the same level of research and development observed in other sports such as baseball, golf, and tennis. There has been relatively little scientific investigation into current bat design and manufacture and today it still remains a craftsman's art. This thesis aims to understand the nature of cricket bat impact performance and apply this knowledge in developing a novel hollow cavity bat design.

1.2 A BRIEF HISTORY OF CRICKET AND THE CRICKET BAT

Barty-King [1] stated that it is not clear exactly when the so called 'Cat and Dog' games evolved to what we now know as cricket. The Cat was simply an object that was hit, and the Dog, the striking implement. It was the starting point for a variety of modern games such as hockey, lacrosse, croquet, and cricket. As various games developed on this theme, each version demanded changes in the shape of the implements to suit its own requirements. The development of cricket from around 1620 necessitated advances to the design of bat and ball in keeping with changes made to the rules of the game. The bat began with a curve in the blade, but is now predominantly a straight piece of wood. A spliced handle made from cane was added around 1850 to lessen the 'sting' of impact and provide flexibility [1]. Modern handles are now a combination of cane, cork, and rubber, to increase flexibility still further, and use rubber covers to improve a batsman's grip. The blades have become larger and the back faces more sculpted. The Gray-Nicolls Scoop, first produced in 1974, is a good example of this change and was claimed by the company to give a significantly larger 'sweet spot'. It was produced from a larger than normal cleft of willow, with the weight distributed to the edges of the bat by 'scooping' material out of the centre [2]. More recently, Hunts-County machined a cavity into the blade of the bat which was then filled with a composite material with 'honeycomb' geometry - claimed by the manufacturer to reduce weight and enhance performance. Unfortunately, no quantifiable evidence was provided to substantiate improvements in either of these bats. Production has become more automated with labour intensive, craft based jobs supplanted by hand operated machinery [1]. The Slazenger cricket bat manufacturing process, detailed in Appendix 1, used a fully automated, computer numerically controlled (CNC) Wadkins machine to generate the shape of the back face and toe of the bat, which require only finishing by hand. Two bats can be produced every four minutes in this manner, increasing output to keep up with modern production demands.
Policy governing implement technology in cricket is the responsibility of the Marylebone Cricket Club (MCC). The MCC is recognised as the sole authority for drawing up the rules for the game and for all subsequent amendments. The latest revision of the laws [3], include design constraints upon the implements of the game. The bat is governed by a maximum length and width and requires the blade be made solely of wood. A surface covering the face of the bat is permitted for protection, strengthening, and repair of the wood. A constraint upon the thickness of the covering is imposed to limit its ability to enhance other characteristics, such as ball rebound. The rules concerning bat design are shown in Appendix 2. The policy adopted by the MCC towards the arrival of new technology has resulted in a more limited scope for product innovation and resulted is less published research on cricket bat design and development in comparison to other major sports such as tennis, golf, and baseball.

1.3 SUMMARY OF LITERATURE CONCERNING SPORTS BAT DESIGN

Literature concerning the science of cricket is reviewed in this thesis together with relevant studies from other sports. The review has focused upon human factors, bat and ball impact modelling, and material properties.

In relation to other major sports, the volume of published information regarding cricket is small. Research concerned with baseball bats has been more widely published and of significant value. Although a different game, baseball and cricket share a number of significant similarities with respect to the nature of the bat used and its interaction with the ball. Also, the use of hollow baseball bats, has generated several studies examining the difference in performance between the solid and hollow bat constructions [4-7]. Research concerned with golf club head design [8-13] is also relevant since modern golf club driver heads are predominantly hollow.

1.3.1 Biomechanics and Human Factors

Studies of particular interest concern both the kinematic and kinetic measurement of batsmen in cricket and baseball. These two measurements can be used to understand the motion and forces involved in striking a ball. Studies in cricket illustrate the differences in player motion between types of shot [14-16], whilst studies in baseball and golf [17-23] have investigated the effects of bat or club mass properties on achievable swing speed. As bat mass and mass distribution are likely to affect the batsman's ability to manoeuvre and impart speed to the bat during a shot, these studies are particularly relevant.

1.3.2 Bat and Ball Impact Mechanics

Major League Baseball (MLB) bats are currently made from solid wood and are of a size, shape, and weight similar to a cricket bat as defined by the rules of the game [24]. Baseball bats used at other levels, such as collegiate, and youth leagues, can be of hollow aluminium or composite construction [25]. A common area of baseball bat research is the experimental and theoretical investigation of impact performance to determine values such as the coefficient of restitution (COR), the speed of the
ball after impact, termed the 'batted ball speed' (BBS) and the nature of the resulting motion of the bat following impact. The 'sweet spot' is a term generally used by players to describe a region of the bat from which an impact results in enhanced rebound ball speed, complemented by desirable sensory feedback from the shot. Research studies in baseball [7, 26-31] have attempted to characterise the nature of the 'sweet spot' and investigated the physical properties of the bat that influence BBS and the perceived characteristics. Similar studies in cricket also exist which attempt to characterise and optimise the impact performance of the bat [4, 32-36]. Finite element analysis (FEA) packages have become an increasingly effective means to accurately model the mechanics of bat and ball interaction and have been used to investigate the sensitivity of impact performance on bat design parameters in cricket [33, 37] and baseball [38, 39].

1.3.3 Materials and Manufacture
Although several other woods have been used, Salix alba coerula is currently the preferred material for the bat blade. Since the MCC rules of the game [3] currently disallow the use of any material other than wood, it is important to investigate the mechanical properties of wood to fully understand bat performance. The design and manufacture of new bats in this study should also adhere to the MCC rules. However, as the current bat rule is ill-defined, an investigation into wood composites may lead to further possibilities in material selection. Plywood and particle board are examples of wood based composites, possessing different properties from natural wood, potentially useful in bat design. Changes to bulk mechanical properties can be made through selection of wood species, fibre and layer orientation, and the modification or addition of other materials such as adhesives.

Determination of mechanical properties is important to defining valid FEA material models. Mechanical properties are widely available for a number of wood species. However, to measure the bulk mechanical properties of specific bat components within conventional and prototype bats, BS-EN 408 [40] is of particular relevance.

There are a number of patents related to hollow striking implement design. Notable designs include a baseball bat made from several pieces of maple wood, joined to enclose an internal volume [41], a baseball bat made from planar laminates which surround a solid core or a cavity [42], and a hollow composite baseball bat with a multilayer construction of wood and other fibre reinforced materials [43]. An additional patent also exists for a hollow baseball or cricket bat made from a thin deformable front face engineered to produce enhanced ball rebound [44]. However, in review of the patent search, a hollow cricket bat has not yet been successively manufactured by a company which adheres to the rules of the game whilst providing quantifiable evidence for improved performance.

1.4 RESEARCH OBJECTIVE AND PROPOSED APPROACH
The aim of this thesis is to investigate the science of cricket bat impact characteristics and to design, manufacture, and evaluate a novel cricket bat concept.
An investigation of the mass and impact characteristics of existing bats will act as a performance benchmark upon which new designs can be measured. Impact performance of hollow striking implements widely published for baseball, includes work by Nathan [6, 31, 45], Brody [27, 46], and Cross [28, 29]. Studies involving golf have been published by Hocknell [13] and Cochran [10, 12]. These studies are used to assist in the development of numerical models of bat and ball impacts and FEA is used to provide further insight to the performance of conventional and hollow cricket bats. Hollow prototype bats are manufactured using a laminate construction and a standard impact test procedure is developed following a review of current ASTM standards in baseball, in order to measure bat performance and validate FEA models. An evaluation of both conventional and prototype bat performance is made and the validity of a hollow cricket bat design is discussed.

1.5 THESIS OUTLINE

Chapter 2 develops a system to categorise cricket bats with respect to mass properties and describes the test procedure and equipment used to determine player sensitivity to bat mass properties.

Chapter 3 investigates wood and adhesive properties important to the manufacture and performance of cricket bats.

Chapter 4 reviews the current literature on the impact dynamics of striking implements and defines three significant areas of bat impact performance which can be addressed by a hollow bat design.

Chapter 5 describes the generation of a solid cricket bat computer-aided design (CAD) model and the subsequent design of a hollow bat concept based upon improving bat mass properties.

Chapter 6 describes the use of spring damper models to investigate a hollow cricket bat 'trampoline' effect.

Chapter 7 details the FEA of bat and ball impacts and development of ball and bat models. Simulations are conducted using both solid and hollow bats and compared against experimental impact data. The significance of handle stiffness, bat vibration, and local damping effects are also considered.

Chapter 8 describes the processes used in the manufacture of a multilayer hollow cricket bat prototype, involving the use of bat computer models to facilitate the CNC machining of external and internal hollow geometries.

Chapter 9 develops performance metrics for a cricket bat based upon current ASTM standards [47-51] in baseball. The impact performance of conventional and hollow bat designs is measured against established metrics using a laboratory impact test procedure.
Chapter 10 presents the results of the impact tests, discussing the centreline performance of conventional bats, hollow prototype bats, and prototype striking faces. The off-centreline impact performance of the hollow bat design is also discussed.

Chapter 11 develops a standard game condition in which the results of the laboratory impacts tests can be used to predict the playing performance of the bat.

Chapter 12 presents the conclusions of this research study whilst chapter 13 discusses the significant areas of investigation for further work.
CHAPTER 2

CRICKET BAT GRADING SYSTEM

2.1 INTRODUCTION

Many sports share a similar requirement that a player must accelerate the striking implement in a 'swinging' motion to make contact with a ball or similar object. This motion can be fairly infrequent as with golf, where considerable attention to the consistency of the swinging action is made or swinging motions can be more frequent and varying, such as in tennis. Choosing a suitable bat or racket to play with would normally involve practising the swinging motion with a variety of designs and choosing that which 'feels' most appropriate. The feel of the implement throughout the swinging motion is an important characteristic that exists across a broad range of striking sports and the swing generally involves an acceleration of the implement by the player to impact with a ball or similar object. There are countless designs of rackets and clubs that are commercially available to the player and therefore to assist the selection process, a grading system which relates to the feel associated with swinging the implement is considered to be of value.

The bat grading system currently used in cricket is based upon an assessment of the aesthetic qualities of wood - The Dunlop Slazenger bat grading system for 2000 is illustrated in Appendix 3. It is anticipated that an objective grading system based on mass properties would be of significant use to a batsman in selecting an appropriate bat. This study is also relevant to the development of hollow prototype bats with modified mass properties since experimental measurement of those properties can be achieved with the instruments and techniques developed in this study.

This chapter defines relevant implement mass properties and reviews current grading systems that are used in sport. A test procedure is established to determine player sensitivity to bat mass properties and a grading system is proposed based upon the results of the test.

2.2 SPORTS IMPLEMENT MASS PROPERTIES

In describing the mass properties of an sports implement, it is anticipated that the Implement mass, first moment of force, and moments of inertia are of importance. A brief description of each follows:

2.2.1 Mass

Mass is a measure of the inertia of a body, which is its resistance to a change in velocity and is defined by Newton's Second Law of Motion, shown in Equation 2.1. Mass is also a property by which objects experience a mutual attraction. The mass of the earth generates an attractive force with objects on its surface equal to the mass of the object multiplied by the acceleration due to gravity and is commonly
termed ‘weight’. Since sport implements have mass, a force is required of the player in order to impart linear acceleration to the implement and to resist its weight.

\[ F = m \cdot a \]  \hspace{1cm} \text{[2.1]}

### 2.2.2 First Moment of Force

A force which does not act through an object’s centre of mass (COM) tends to change the rate of rotation of the object. The moment of the force is proportional to both the value of force and the perpendicular distance, \( d \) from the axis of rotation to the line of action of the force. The magnitude of the moment is expressed in Equation 2.2, and it is relevant to a sports bat or racket since the implement handle is normally located a distance from the COM. It is therefore likely that the player will exert a torque on the implement in order to resist implement weight under static conditions.

\[ M = F \cdot d \]  \hspace{1cm} \text{[2.2]}

### 2.2.3 Moment of Inertia (MOI)

MOI is measure of the distribution of mass with respect to an axis of rotation, as shown in Equation 2.3.

\[ I = \int r^2 \, dm \]  \hspace{1cm} \text{[2.3]}

where \( I \) is the moment of inertia and \( r \) is the perpendicular distance of mass elements \( dm \) from the axis of rotation. Newton’s Second Law of Motion states that the mass of an object determines the value of linear acceleration attained for an applied force. Similarly, if the object is in rotation then the mass distribution determines the angular acceleration for an applied moment. Therefore, MOI can be described as the resistance to a change in rotational velocity due to the radial distribution of mass around an axis of rotation. A similar Equation to 2.1 can be used to describe the relationship between the applied moment of force, MOI, and an object’s rotational acceleration, as shown in Equation 2.4.

\[ \sum M_{COM} = I_{COM} \cdot \alpha \]  \hspace{1cm} \text{[2.4]}

where \( \sum M_{COM} \) is the sum of moments about an object’s centre of mass, \( I_{COM} \) is the moment of inertia through the centre of mass, and \( \alpha \) is the angular acceleration of the object. Three axis of rotation are commonly used to address the important inertia properties of a sports bat or racket and they are briefly described below. A more detailed description of implement inertia, in chapter 4, provides a justification for these measurements and describes the role of mass distribution in determining the outcome of an impact. The three MOI and corresponding axes of rotation are illustrated for a cricket bat in Figure 2.1.
1. The property $I_{sw}$, termed the 'swing' MOI, is often measured about a point in handle grip of a bat with a rotation axis, $x$, orientated parallel to the striking face and perpendicular to the longitudinal axis of the bat, such that the implement rotation represents that of a swing or playing stroke. The swing MOI affects the value of torque required of a player during a swing and is sometimes referred to as the 'dynamic swing weight'.

2. The property $I_{uu}$, is measured about the principal bat axis, $\bar{x}$, equivalent to the COM. The axis orientation is such that it is significantly parallel to the striking face and perpendicular to the longitudinal axis of the bat. $I_{uu}$ is important to the behaviour of the implement subsequent to an eccentric impact and the value can also be used to determine the swing MOI with the parallel axis theorem, given in Equation 2.5.

$$I_{uu} = \bar{I}_{uu} + md^2$$

where $m$ is the bat mass and $d$ is the distance between the $x$ and $\bar{x}$ axes of rotation.

3. The property $I_{yy}$, termed the 'polar' MOI, is significant to the twisting action generated by the batsman about the longitudinal axis of the handle or from ball impact which deviates from the axis of bat symmetry as shown in Figure 2.1.
2.3 CURRENT GRADING SYSTEMS AND MEASUREMENT OF MASS PROPERTIES

2.3.1 Golf

Golf is played using a set of no more than 14 clubs and each usually has a different design incorporating different shaft lengths, loft angles, or materials, etc. This offers the player a choice in selecting the most appropriate club design for a particular shot application. As a general rule, it is accepted that each club in a set should be chosen such that the characteristic ‘feel’ of the club when swung by the player is appropriate to the set. Whether an attempt is made to have a consistent ‘swing feel’ across the set of clubs, or to have intentional differences in ‘feel’ between certain clubs is a decision for the player. In order to assist club selection in both instances, methods of quantifying and measuring the ‘feel’ of a club during the swing have been established.

The ‘swing weight’ of a club is a term used to describe the first moment of mass of a club pivoted about a point on the shaft. In golf, three scales are currently in use that incorporate incremental values of first moment within the range exhibited by commercially available clubs. The values of moment are represented by a single number or an alphanumeric unit depending upon the scale employed. A commonly used scale to represent swing weight is the ‘Lorythmic’ scale. The alphanumeric scale and corresponding values of first moment are shown for a section of the scale in Figure 2.2. Several swing weight scales are in existence but for the Lorythmic scale, the moment is measured from a pivot point 14 inches from the end of the grip of every club.

Attempts have been made to introduce a system of dynamic measurement to match golf clubs. Wishon [20] reports two attempts made by manufacturers to produce sets of clubs matched using values of MOI. First, in the 1970’s, a company named Sounder Golf offered sets of woods and irons which were purported to have the same swing MOI, which could be demonstrated by noting the synchronous time period of two clubs set in pendulum motion about the grip end [20]. The company produced all clubs to one value of swing MOI which Wishon reported as being a contributing factor to it poor success, since different players may prefer clubs with different swing MOI values. In the 1990’s, the Tommy Armour Golf company introduced their ‘EQL’ clubs to the market. The company made all the ‘woods’ the same length and same total mass as the 5-wood, and all the ‘irons’ the same length and total mass as the 6-iron and the club set was reported to have a more consistent swing MOI.

![Figure 2.2 - The Lorythmic scale](image)
2.3.2 Tennis

The term 'swing weight' can also apply to a dynamic measurement rather than the static moment as commonly used in golf. The Babolat Star 4 tennis racket diagnostics machine is an example a commercially available instrument used to measure values of dynamic 'swing weight' based upon racket swing MOI about an axis in the handle. The machine generates an oscillatory motion using two springs fixed to a rotating shaft to which the racket is attached. The time period of oscillation is measured and is a function of the racket swing MOI.

Brody [52] described an experiment to determine the polar MOI of a tennis racket. The test used a metal wire as a torsional pendulum and the racket was suspended from the wire through its COM and was perturbed about its polar axis to initiate motion. The torsional displacement of the wire generated a restoring torque which in turn produced a simple harmonic motion of the implement. The time period $\tau$ for a torsional oscillator is shown in Equation 2.6, and can be re-arranged to solve for MOI as in Equation 2.7.

\[
\tau = 2\pi\sqrt{I/k}, \tag{2.6}
\]

\[
I = k\tau^2 / 4\pi^2 \tag{2.7}
\]

where $k$, is the torsional stiffness of the wire which can be determined by measuring the time period of suspended objects with known MOI values. Typically, these objects are of simple geometric shape such as a rectangular or cylindrical block and their value of MOI can be calculated using simple Equations.

Brody also described an experimental arrangement to determine swing MOI in which the racket is set into motion as a physical pendulum using a pivot point about which the MOI is of particular interest.

2.3.3 Baseball

Heald [18] investigated the usefulness of the swing weight standard for golf clubs and established a similar method of classifying baseball bats. The author described how the concept of swing weight was developed to provide comparison between clubs of differing weight, weight distribution and length, and suggested these were the key properties in determining hit ball distance. The author suggested that MOI was more difficult to measure in comparison to a static swing weight measurement and for the purpose of the study, swing weight was measured in Ounces by resting the barrel end of the bat on weighing scales, whilst supporting the bat at the handle above the scales on an incline. The study established a swing weight measured in ounces from 0-30 oz, equally divided into three categories labelled A (1-10), B (11-20), and C (21-30). The number after the category label denoted further division equal to 1 oz within each category.
2.4 PERCEIVED SENSITIVITY TO IMPLEMENT MASS PROPERTIES

2.4.1 Golf

A study by Harper [53] investigated the worth of the golf swing weight systems currently in place. Manufacturers invest in producing clubs whose mass distributions consistently adhere to specific values on the 'Lorythmic' or 'official' scale. The resolution of the scale determines the accuracy to which the manufacturers must make a club in order that it has the correct mass distribution for its intended swing weight. The study suggested that elite players cannot perceive the difference between single swing weight increments but, players could reliably determine perceived differences as the difference in swing weight between two clubs became equal or greater than three swing weight points.

2.4.2 Tennis

Tennis rackets were used in a study by Davids et al [54] to investigate the sensitivity of both children and adults to haptic information and their interpretation of such information in selecting the most suitable racket. Three groups were tested and included ten children, ten adults who had little experience in wielding a racket, and ten adults who were experienced tennis players. The study demonstrated the ability of each group to determine differences in racket weight distribution and place in order of preference their playing suitability. Separate tests were conducted with and without the opportunity to visually inspect each racket. The swing MOI for each of the six rackets was modified by adding a 50 g mass. The distance of the additional mass increased from the grip end by 10 cm with each racket to provide a range of swing MOI values from 0.0334 kgm² to 0.0512 kgm². Racket mass and first moment about the handle would also increase due to the addition of extra mass, but this was not mentioned by the author. The first test allowed both tactile and visual inspection of the rackets and resulted in experienced adult players preferring rackets with similar values of swing MOI. These values were typically at the high end of the swing MOI values available. Inexperienced adults selected rackets that were in general less contiguous, whilst the children preferred a broad range of swing MOI. The non-visual test illustrated a broader range of preference in both adult groups, whilst the children's group showed a higher degree of clustering in comparison to the visual test, although the selected range remained broader than for both adult groups. The study highlighted the ability of non tennis playing adults to draw upon their experience of interacting with every-day objects and apply it to an order of racket preference. The adults with task specific experience showed the greatest consistency in selecting their preferred rackets, across both sets of tests, and were able to differentiate between smaller incremental steps in swing MOI.

Brody [55] classified three moments used to describe the mass properties of a tennis racket. The zero moment was described as the weight of the implement, the first moment was the static balance of the racket, and the second moment was the 'swing' or dynamic moment (MOI). The study explained the method in which mass can be distributed to affect one or more moments of the racket. For example, an extra mass centred about the axis of rotation or close to the grip position would increase the zero moment without significantly affecting the other two moments. If mass was taken from the handle and placed at the tip of the racket, the zero moment would be unaffected, but the first and second
moments would increase. However, if mass was subtracted from half way between grip and racket tip and half of this mass returned to the tip of the racket then the first moment would be unaffected, whilst the zero moment would be lower and second moment would increase. The study assessed the ability of proficient tennis players to detect changes in racket swing MOI. The mass distribution of two identical rackets was modified by adding lead tape at either the balance point or the tip and butt of the racket whilst maintaining a constant value of mass. The results illustrated that elite players could successfully distinguish between swing MOI of a difference greater than 5%, but this ability became statistically indistinguishable at differences less than 1.2%.

Player sensitivity to changes in polar MOI was tested by adding mass to the rim of the racket in a configuration such that the mass, the first moment and the swing MOI remained consistent, but the polar MOI was modified. The study determined that elite players could correctly distinguish between values whose difference was greater than 10%. At a value of 5%, the players had difficulty in correctly identifying which of the rackets had a greater resistance to twist. Brody suggested that in relative terms, the players had greater ability to detect differences in swing MOI, although in absolute terms they were more sensitive to changes in polar MOI since its value for tennis rackets is typically 20 times less than for swing MOI.

2.4.3 Non Sport Related Studies

A number of psychological studies have been conducted to assess the ability of humans to perceive differences in the intensity of tactile sensation. Work concerning the ability to determine differences in weight or mass have been conducted to ascertain a threshold of sensory discrimination using a range of reference weights under a variety of lifting conditions.

To measure a sensory threshold, tests can be conducted using a reference value of sensation and the change in the value necessary for correct discrimination 50% of the time can be determined. The concept was proposed many years ago by Fechner [56] and is known as ‘just noticeable difference’ (JND) and in this study it was concluded that the value of JND, which is applicable to all sensory perceptions, was linearly proportional to the stimulus intensity, as shown in Equation 2.8.

$$\frac{\Delta R}{R} = WF$$  \[2.8\]

Where $R$ is the reference intensity, $\Delta R$ is the change in stimulus intensity, and $WF$ is the ‘Weber Fraction’. $WF$ values for most stimuli are in the range from 0.01 to 0.25 and linearity remains true across all intensities except at very low levels, where the reference value is of the same order as the threshold value, and at very high intensities where the sensory systems act in a distorted manner [57].

$WF$ for force and weight have been reported in several studies, a value of 0.02 was determined by Teghtsoonian [58] for weight. Karwowski [59] studied the manual lifting of boxes with a range of
masses from 4.5-29.1 kg and observed WF values of 0.07 to 0.02, respectively. Ross and Brodie [60] measured a WF of 0.07 for 400g which increased to 0.11 for a 50 g mass.

The values WF given in [60] were part of a study to understand the effects of weightlessness experienced during manned space flights. The study approached the mass and the weight of an object as two separate properties. On earth, a force generated on a mass is equal to approximately 9.81 Newtons for every 1 kg. The study suggested that the perceived weight of the object could be sensed through the reactive force generated when the object is in contact and rests on the human body or through the effort required to maintain the object in a raised position. Inertia properties may additionally be sensed through the ratio of applied force to the achieved acceleration through pressure receptors in the hand and the rate of change of hand movements through the kinaesthetic system [60]. The quality of sensation is the same for mass and weight but on earth the perception of an objects 'heaviness' can be assessed by taking advantage of both properties. For instance, on earth, an object can be held stationary to determine how 'heavy' it feels and it may also be moved or 'jiggled' to assist discrimination. This fact has been noted in other studies [61-63] which have also investigated the worth of 'jiggling' an object to evaluate 'heaviness'.

In a weightless environment, the static weight of an object is absent and therefore the ability to discriminate masses relies upon their inertia characteristics. A study by Ross [61] observed that thresholds for mass discrimination under zero gravity are 1.8 times higher than those for weight discrimination before flight. This suggests that humans are not as sensitive to inertial mass as they are to the combined effects of inertial mass and weight. This is relevant to investigating the sensitivity of the batsman to the mass properties of a cricket bat as both the mass and inertial characteristics of the implement may affect perceived 'heaviness'. It also suggests that despite the relevance of swing MOI as a measurement of dynamic bat movement, the batsman's ability to assess changes in swing MOI may not be as acute as their perception of the bats static properties. The reason for poorer discrimination from inertia characteristics for all intensities in [60, 62] was due to the intermittent nature of the tactile information for inertia as this is lost when the hand is stationary or moved at a uniform velocity. Ross [62] suggested that imparting high accelerations improves the quality and quantity of sensory information for inertia and if a mass permits rapid movement or a 'jiggle' [63], then it may assist mass discrimination but could equally interfere with static weight perception.

The tests conducted in studies [58-64] were concerned with weight and mass. As a bat is held at the handle located at one end of the bat, its orientation can be such that a component of the bat's weight no longer acts through the handle and a reactive torque must be generated to hold the bat stationary. To move the bat, this value of torque can be altered to impart acceleration. A study by Woodruff [64] examines the human ability to discriminate between values of torque using a metal bar upon which a movable mass was positioned at intervals along its length. The mass could be adjusted to provide a variety of torque values without change in overall mass of the bar. The results showed that sensitivity
to torque measured is similar to the sensitivity to tactile pressure observed through holding static weights. This study is relevant as the values of torque used (1.54 Nm - 2.75 Nm) [64] are similar in magnitude to those observed for cricket bats (3.9 Nm - 4.51 Nm).

2.5 SELECTION AND MEASUREMENT OF BAT MASS PROPERTIES

2.5.1 Mass Property Selection

The swing action generated during a defensive or attacking shot involves the rotation of the bat. The process of generating torque to accelerate an implement towards the target might suggest that swing MOI is the most relevant property to the define the perceived resistance or effort involved. However, psychophysical investigations [60-62] have illustrated that human sensitivity to inertial characteristics are approximately half that of weight.

Within this research, the weight, first moment and swing MOI of a cricket are investigated to evaluate the significance of each, to the intensity of the tactile sensation perceived during the action of holding and swinging a bat. To determine values of first moment and swing MOI, the position of a pivot axis was required. The bat pivot axis was defined at the midpoint between the handle top and the bat 'shoulder', approximately located between the batsman's hands. Although it is likely that the pivot point and therefore the value of swing MOI will change during a shot playing stroke, this definition was considered sufficient for determining relative differences in values of first moment and swing MOI for different bat mass distributions.

2.5.2 Measuring Bat Mass and First Moment of Bat Mass

The total mass of the bat was measured using digital scales with the appropriate weight added. Values of first moment were obtained by measuring the position of bat COM with the addition of the weight using a balance, and multiplying it distance, \(d\) from the rotation axis with the total mass of the bat including the added weight, \(m_{\text{total}}\), as in Equation 2.9. This value is equal to the maximum static value of first moment generated about the handle midpoint due to bat weight.

\[
T = d \cdot m_{\text{total}} \tag{2.9}
\]

2.5.3 Measuring MOI

Computer models of cricket bats, detailed in chapter 5, indicated that values of \(I_{xy}\) would typically be \(~50-60\) times less than \(I_{xx}\) and so two instruments were designed to accommodate the different bat orientations and the relative magnitudes of measured values. Both instruments however, determine MOI using the same principle.

An effort was made within this research to measure MOI about the principal axes of the bat for the following reasons:
1. The bat COM provided a datum for all makes of bat regardless of their geometric differences.

2. The value of MOI about the principal axes would result in minimum MOI values facilitating the design of a more sensitive instrument.

3. Measurement of \( \bar{I}_{xx} \) allowed for subsequent calculation of swing MOI using the parallel axis theorem as shown in Equation 2.5, and illustrated the relative contributions of \( \bar{I}_{xx} \) and \( m d^2 \) to the value of swing MOI.

Values of bat \( \bar{I}_{xx} \) were measured and were typically 1-2% greater than \( \bar{I}_{xx} \), in agreement with values obtained from bat CAD models, in chapter 5. To determine values of swing MOI, the value of \( \bar{I}_{xx} \) was used in preference to \( \bar{I}_{xx} \) since the ease of bat COM alignment and the reduced weight of the clamping mechanism for this orientation, improved the consistency of measurement. Due to the \( m d^2 \) component, values of swing MOI determined using values of \( \bar{I} \) in the \( \bar{z} \) and \( \bar{x} \) axes deviated by less than 1% which was considered an acceptable error, given the improvement in consistency and ease of measurement.

The bat was clamped to the bottom of the instrument shaft and oscillatory motion was generated using to tension springs connected to the ends of an arm located radially through the shaft, as shown in Figure 2.3. A potentiometer was attached to the top of the shaft and connected to a potential divider circuit across a power supply. The change in voltage relating to shaft angular displacement was measured and recorded on an oscilloscope. The time period of oscillation was averaged across five to eight complete oscillations before signal quality was lost due to mechanical damping. Values for average time period and standard deviation were measured and repeated five times. A photographic illustration of the instruments used to measure \( \bar{I}_{xx} \) and \( \bar{I}_{yy} \) are shown in Figures 2.4 and 2.5, respectively.

The time period measured was used to calculate the moments of inertia using the Equation defined for a torsional pendulum, shown in Equation 2.10.

\[
I = C \tau^2 
\]  

[2.10]

where \( \tau \) is the time period of oscillation and \( C \) is the calibration constant. The accuracy of measurement relied on the fact that viscous damping was below a significant level resulting in a frequency of oscillation equivalent to the natural frequency. Measurement of bat MOI is based on the change in the natural frequency of oscillation of the pendulum resulting from the addition of the bat. The calibration constant \( C \) is determined by adding an object with a known value of MOI and recording the time period \( \tau_c \). The object is removed and the 'tare' time period of the instrument and holder \( \tau_e \) is measured. The value of \( C \) can be calculated using Equation 2.11.

\[
C = \frac{I}{\tau_c^2 - \tau_e^2} 
\]  

[2.11]
A dual spring mechanism was used to generate the necessary torque in both instruments and a linear relationship between angular displacement and restoring torque could not be assumed. Therefore, several calibration objects were measured on each machine to characterise the relationship between I and r. The repeated measurements of the calibration weights returned MOI values within ±3%.

Figure 2.3 - Mechanism used to determine bat principal moments of inertia
Figure 2.4 - Photograph of instrument used to measure $J_x$.

Figure 2.5 - Photograph of instrument used to measure $J_y$. 
2.6 TEST METHODOLOGY

2.6.1 Test Bat Specification

A Slazenger V500 bat was modified by inserting ten nylon ferrules in holes drilled at 50 mm intervals along the face of the blade. The ferrules were drilled and tapped to accept four custom built weights of 25 g, 50 g, 75 g, and 100 g which could be bolted to the face at the ferrule locations. Out of the ten positions, four were chosen to act as sites where a weight was added giving rise to 16 configurations of distributed mass, provided a single weight was used for each configuration. Figure 2.6 shows the distance of the four ferrules in relation to the defined swing axis.

Figures 2.7 to 2.9 illustrate the effect of additional weight on bat mass, first moment and swing MOI, respectively. The value of each mass property is shown relative to the value of added mass and its position on the bat. Figure 2.10 illustrates the relative contribution of $I_w$ and $md^2$ to the value of swing MOI for each of the 16 configurations, showing a value of $I_w$ approximately one third of the total swing MOI value.

The polar MOI was not intentionally altered by the addition of the weights but their attachment position was not co-incident with the $y$ axis of rotation resulting in a measurable increase in polar MOI. However, the increase in resistance to twist about the longitudinal axis of the handle did not emerge as a perceived dimension during the testing procedure and was not considered as part of the grading system. Figures 2.11 to 2.13 illustrate the first moment, swing MOI, and polar MOI of the test bat in all weight configurations together with a sample of 17 commercially available bats. Since the general shape of these bats was similar, the values have been plotted against bat mass to illustrate a relationship.

![Figure 2.6 - Relative position of weight attachment points to swing axis](image)
Figure 2.7 - Combined mass of bat and added weight

Figure 2.8 - First moment vs. the addition of weight

Figure 2.9 - Swing MOI vs. addition of weight
Figure 2.10 - The contribution of $\bar{J}_z$ and $md^2$ to swing MOI

Figure 2.11 - Experimental values of first moment about the bat grip vs. bat mass

Figure 2.12 - Experimental values of swing MOI about the bat grip vs. bat mass
2.6.2 Subjective Player Testing

An experiment was developed to assess a player’s ability to perceive differences in the weight and weight distribution of a cricket bat and to examine if there were mass properties to which the batsman was more sensitive. Sixteen elite players were chosen for the study, seven of whom were of an international standard, five of county standard, and three of club standard. The number of players chosen was based upon work done by Roberts [65] and Harper [53] who had concluded that a minimum of 15 players was sufficient to extract valid and statistically significant data regarding player perception.

Each player evaluated all 16 weight configurations, two Latin squares were used to eliminate order effects, the first square determining the order of weight and the second the order of attachment position. The test was non-visual and relied upon the haptic ability of the player to determine bat differences. Players were blindfolded and encouraged to ‘swing’ or ‘pick-up’ the bat in preparation for a shot to help ascertain the nature of the implement. The same four questions were asked for each weight configuration and were designed to explore a player’s ability to distinguish possible differences relating to the bat weight, the weight distribution or ‘balance’, the perceived weight during the ‘pick-up’ motion and the resistance to a swinging motion. Player responses were requested on a one to nine scale with descriptive words used to give each scale an orientation.

To highlight any trends between player ratings and measured mass properties, the average player rating for ‘heaviness’, ‘weight distribution’, ‘pick-up’, and ‘ease of swing’ at each of the 16 mass and positions combinations were plotted with standard deviations. Roberts [66] discusses the lack of a reference level common to all players as an issue concerning scaled responses. The average rating given by each player was therefore subtracted from the value of each individual response, in order to normalise the data across players. Additionally, there is no suggestion that each level on the scale corresponds to an equal change in perceived ‘feel’, but this is of little concern as the function of the scale is to provide ordinate data only.
To simplify the data, added mass and position combinations with similar values of first moment and swing MOI were averaged together as were the associated ratings given for these values. As each question alluded to different properties of mass, the aim was to determine whether significant relationships exist between player ratings for these questions and particular mass properties.

Each added mass was positioned at the four locations consecutively during the testing and this had the effect of dividing the test into four smaller comparative tests. For each of these tests the bat mass was constant but the first moment and swing MOI of the bat changed as the added weight was moved between each of the four positions. To establish a meaningful grading system it is important not only to appreciate which mass properties are most relevant but also the player's sensitivity to changes in the value of the property in question. By maintaining a constant mass and varying the first moment and swing MOI, the players' threshold in discerning differences in these two properties could be investigated.

To determine the player ability to discriminate between differences in mass properties the 'threshold' or 'difference' study is generally accepted as a more suitable method for establishing player sensitivity [58-64]. In this type of test, the player would be asked to compare a two or three implements together in an attempt to rank them in order of perceived 'heaviness' or 'pick-up', for example. It is arguable that this type of test procedure is more sensitive and would establish a lower threshold of perceived difference in mass characteristics. Although the test procedure outlined in this study does not target the player ability in this manner, it is argued that the sensitivity results from this study remain valid with values possibly being slightly greater than threshold values obtained in a study using direct comparisons.

2.7 RESULTS

The mean and standard deviation of scaled responses for each of the 16 mass distributions was calculated across all players. Figure 2.14 illustrates the raw data of player rating and bat first moment. Figure 2.15 shows the same information with a single grouped value for similar first moments against normalised player ratings.
Figures 2.14 and 2.15 demonstrate a positive trend of perceived heaviness against first moment of mass. Each series in Figure 2.14 relates to a different added mass. For similar values of first moment it is observed that a heavier mass is frequently given a higher rating than the value of moment produced using a lighter mass, this is clearly reflected in Figure 2.16 showing the relationship between mass and perceived 'heaviness'. Equally, the high values of standard deviation observed in Figure 2.16 reinforce the argument that mass distribution is significant even when the question relates to a term commonly associated with weight.
The relationship between player ratings 'heaviness' and 'pick-up' are similar. Figures 2.17 and 2.18 illustrate grouped values of first moment and moment of inertia versus rating given for perceived value of 'pick-up'. Standard deviations for pick up ratings show a similar trend to those observed for 'heaviness'. Standard deviations are lowest at the greatest values of swing MOI or first moment, suggesting that as the mass properties increases there is a greater likelihood that the players will be more consistent in rating. As a first estimate of player sensitivity to changes in mass property, significant differences can be determined by properties differing by more than two standard errors. For player ratings of 'pick-up' in Figure 2.17 and 2.18, a difference of two standard errors in player rating exists at approximately 0.4 Nm and 30,000 kgmm², respectively.
Figure 2.19 shows the relationship between the player rating for 'weight distribution' versus the first moment. A good relationship between 'weight distribution' and first moment was anticipated as the question alludes to the position of mass and not simply its weight. However, it is clear from the standard deviations that there is greater disparity between player responses, even at the highest values of first moment. Figure 2.20 shows the relationship between swing MOI and player ratings for 'ease of swing'. It was anticipated that the question would focus the player's assessment on the resistance or effort involved in swinging the bat and might therefore correlate best with swing MOI. A positive relationship exists but the relatively high values of standard deviation reduce its overall significance. It is possible that intermittent nature of the sensation [60-62], together with differences in individual player strength and weight, led to higher standard deviations.
In summary, the question pertaining to ‘heaviness’ against bat first moment provides the clearest and most significant relationship with perception and shows relatively low standard deviations.

A Kendall-Rank correlation is a non-parametric measure of the strength of dependence between two variables [67]. A Kendall-Rank order analysis was performed to examine the degree of correlation between mass properties and player perception. Although, general trends have shown that bat mass gives the weakest relationship with player rating, it has been argued that its contribution is still evident in player ratings. Therefore to reduce this influence, comparisons of the first moment and swing MOI were conducted for identical bat masses. For each of the four bat masses, six comparisons exist between the four mass distributions.

The analysis involves scoring each comparison a value of +1, 0, or -1 depending upon the ratings given by the batsmen. If the batsman correctly gave the bat with the higher first moment or swing MOI a higher rating then a value score of +1 was awarded, if a bat with a lower value of first moment or swing MOI received a higher rating from the player then a score of -1 was awarded, and a score of 0 was awarded if the two bats were given the same rating, the score given for each comparison is termed the ‘$S$’ value. The sum of the ‘$S$’ values was then divided by the number of players to give a value ranging between $-1$ and $+1$, with $+1$ indicating a perfectly positive correlation and $-1$ a perfectly negative correlation [67], this mean value is termed the ‘$T$’ value.

Table 2.1 is an example of the analysis performed illustrating the awarded $S$ values for each player with respect to a bat comparison of known difference in mass property.
Values of $T$ were plotted against measured differences in both first moment and swing MOI. Figures 2.21 and 2.22 illustrate the relationship between $T$ values derived from the question concerning 'heaviness' and the positive difference in first moment and swing MOI between each bat comparison. Figures 2.21 and 2.22 demonstrate a positive relationship, suggesting that larger differences in mass property are more easily ranked.
Figure 2.22 - T values for differences in swing MOI relating to the question of 'heaviness'

Figure 2.23 - T values for differences in swing MOI relating to the question of 'pick-up'

Figure 2.24 - T values for differences in swing MOI relating to the question of 'ease of swing'
Figures 2.23 and 2.24 illustrate the difference in swing MOI against values of T for the questions relating to 'pick-up' and 'ease of swing', respectively. It was noted during the analysis of general trends that player questions which sought the level of perceived 'heaviness' and 'pick-up' gave the most clear relationships to measured values of mass properties. These two questions also give the most clear results when comparing the T values against differences in mass properties. Figure 2.23 shows a relatively good relationship between swing MOI and T values relating to 'pick-up'. However, 'ease of swing' shown in Figure 2.24 does not show a clear increase in T value and greater variability in the T values for similar values of swing MOI suggests that players are detecting differences in mass when asked about the bat's 'ease of swing'. The clearest relationships between bat mass properties and player perception are the ratings given for 'heaviness' against first moment of bat mass and 'heaviness' against swing MOI.

It is argued that a bat grading system would benefit from incremental steps in value of mass property for which a player's ability to determine difference is statistically significant. Previous psychophysical studies in this area have concentrated on determining values of the WF which define the human ability to successfully discriminate between weights 50% of the time [57]. However, in proposing a grading system which will encompass all makes of bat and all player abilities it is argued that a higher confidence level, for example a 95% level, is important to establishing a more comprehensible and meaningful system.

For all possible T values ranging between -1 and +1 the probability that every possible value occurring by chance can be calculated. The significance of the T value must therefore be scrutinised by determining whether it is above an acceptable level of probability for the given value occurring by chance. As an example, Table 2.2 illustrates all possible outcomes that can occur for two players.

The probability of each T value can be examined by noting the frequency of each value derived from all possible combinations of player response. In this example a T value of 0 occurs three times, T values of 0.5 and -0.5 each occur twice and values of +1 and -1 each occur once. For the results of the bat study a T distribution was computed using a Matlab® routine which calculated T values and probability for all possible combinations of player response. Figure 2.25 is a graphical representation of the distribution of T for 15 players.

It is accepted that a T value which occurs at a probability of less than or equal to 0.05 (5%) can be regarded as significant [67]. The shaded area in Figure 2.25 represents the values of T for which the combined sum of their probabilities 'x', equals 0.038 and so:

\[ p(-x < T < +x) = 0.962 \] [2.12]
This provides a satisfactory T threshold and results that are equal to or above this level are regarded as significant.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>S value</th>
<th>T value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>0.11</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0.11</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-0.5</td>
<td>0.22</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-0.5</td>
<td>0.11</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
<td>0.22</td>
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<td>1</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 2.2 - Probability of T values occurring by chance for two players

Figure 2.25 - T value distribution resulting from 15 player tests

2.8 PROPOSED GRADING SYSTEM

While both mass and swing MOI may have contributed to the overall perceived sensation, the grading system would be most effective if only one mass property was used to determine a grading scale. Figure 2.26 re-illustrates the T values for 'heaviness' against the difference in first moment and the shaded area highlights changes in first moment which correspond to T values above the confidence threshold.
The question relating to 'heaviness' gives the lowest significant change in first moment, equal to 0.16 Nm. It is argued that 'heaviness' is the most appropriate characteristic to describing the quality of sensation and the first moment is most appropriate single measurement of perceived mass characteristics.

The resolution of the grading scale could therefore be in increments of 0.16 Nm. However, if two bats were selected from opposing ends of two consecutive grades then their difference in first moment could be a maximum of ~3.2 Nm. Therefore by halving the incremental step to 0.08 Nm, the likelihood of two bats in contiguous levels having values of first moment greater than 0.16 Nm apart is eliminated. This also increases the possibility that a difference in first moment between two bats in contiguous levels may not be discernable. However, too fine a resolution is considered to be less problematic than a resolution which is too coarse and therefore the difference between levels on the first moment scale should be 0.08 Nm. The proposed scale in Figure 2.27 has 50 discrete steps but, as the majority of modern cricket bats have a first moment between 3.32 Nm - 4.26 Nm it is envisaged that only 13 steps will be commonly used.

It has been shown that human sensitivity to mass and torque behave similarly [64] and that sensitivity to inertial characteristics are less than that for weight [60-62]. Therefore, it is worthwhile calculating the ratio of change in intensity to the reference intensity to determine whether the resolution of the proposed 'swing weight' for cricket is of an appropriate order [58-64]. The scale employs a fixed resolution throughout its range and therefore the average torque value exhibited by commercially available bats can be used to give an estimate of the intensity ratio. The mean first moment for 17 commercial available bats measured in this study was 3.87 Nm and the change in torque value proposed for a point change on the scale is 0.16 Nm. Therefore the intensity ratio denoted by the
The proposed scale is 0.04. This value is within the range of published values of WF (0.02-0.08) [58-64] which suggests the resolution of the scale is of an appropriate order.

Figure 2.27 - Proposed bat 'heaviness' scale relating to first moment, highlighting the range in which commercial bats are anticipated to reside.
CHAPTER 3

CRICKET BAT MATERIAL PROPERTIES

3.1 INTRODUCTION

This chapter describes the anatomy and properties of wood that are relevant to its performance as a cricket bat material. The differences in mechanical properties and expected changes in bat performance due to the construction of a layered system are also investigated as a laminate concept is central to the design aspect of this study.

3.2 THE ANATOMY OF WOOD

All species of wood possess common material characteristics due to generic structural similarities. Wood is cellular in structure and the each cell is generally a hollow tube like structure, 0.6 mm - 2.3 mm in length with a wall that consists of a number of distinct layers. The majority of these cells are arranged side by side, are bonded together longitudinally in the vertical direction of the tree, and they give rise to the predominant mechanical properties of the wood. The radial components at right angles to the primary axis in hardwoods, such as English willow, are large diameter vessels, known as Rays, that transport and retain fluid but they do not make a significant contribution to the mechanical strength. Although annual growth rings are sometimes visible in the tangential direction, no wood fibres have this orientation. All wood exhibits different material properties when tested in these three directions due to this arrangement of cells with respect to the horizontal and vertical axes of the tree.

The principal components and mass quantities within woody cells for all wood species are approximately 45% cellulose, 30% non-cellulose carbohydrates, and 25% lignin, Bodig and Jayne [68]. The last two exist within each cell in a semi-amorphous state. Extractives commonly exist in the heartwood located towards the centre of the tree trunk, typically 2-5% by mass, extractives can increase durability and modify the colour and odour of wood [68].

Cellulose is the predominant component of a cell wall consisting of a long chain of glucose units, they combine in a parallel array of 50-80 cellulose molecules aligned on the same axis to form elementary fibrils. These in turn are bundled together into larger units by means of hydrogen bonding to form micro fibrils. The micro fibrils are arranged in layers that make up a large portion of the secondary wall within a fibre cell. The primary wall is relatively thin and has little effect upon the physical behaviour of the cell. The secondary wall is formed from three distinct layers termed 's1', 's2' and 's3' and the axial direction of micro fibrils in the 's2' layer are aligned closely to the longitudinal cell axis and the structural properties of the 's2' layer tend to dominate the mechanical strength of a wood cell [68].
The growth of a tree occurs through the production of organic cells immediately under the bark and the diameter is increased by the formation of successive layers of cellular tissue. Willow and other hardwoods grown in a temperate climate produce one growth ring each year and this consists of two distinct parts. The wood formed in the early part of the growing season has large cell cavities used primarily for the transportation of fluids through the tree. The latter portion of growth consists of smaller tightly packed cell cavities giving rise to a more dense region of cell wall material, these regions are clearly visible on the face of a cricket bat as dark lines that exist along the length of the blade as shown in Figure 3.1. The presence of growth rings in addition to radial cells account for the variation of mechanical properties between radial and tangential planes [68].

![Figure 3.1 - Annual growth rings on the front face of a cricket bat](image)

3.3 THE MECHANICAL PROPERTIES OF WOOD AND WOOD COMPOSITES

3.3.1 Introduction

Wood is generally comprised from the same chemical constituents and the differences in mechanical properties arise through the structural organisation of these elements. In addition to cellular structure and organisation, there are many other factors that can influence the mechanical properties of wood such as moisture content, temperature and growth features. Wood density, strength and durability, stiffness, and damping properties are relevant materials characteristics to bat performance and are discussed in this section. Moisture content can significantly modify all of these properties and therefore a description of its effects is included.

3.3.2 Wood Density

Bat mass distribution is a function of bat shape, density, and uniformity of material. The density of wood is defined for a wood species at a known value of moisture content. It must be noted that variability exists within species with regard to density and therefore a value should be obtained empirically. If a laminate bat construction is adopted the mass of the adhesive will contribute to the overall mass of the specimen and to the overall volume. Specific gravity $\gamma$, is a measure of the relative weight per unit volume of a substance in comparison to water and the specific gravity of any wood sample, including a cricket bat, can be calculated through measurement of volume and mass. Moisture content affects the specific gravity through the mass of absorbed or desorbed water and the associated change in wood volume due to dimensional swelling or shrinkage as shown in studies by Bodig [68] and Kollman [69].
3.3.3 Wood Stiffness

An orthotropic model is used to represent the mechanical behaviour of wood as it exhibits distinctly different properties in the longitudinal (L), radial (R), and tangential (T) directions. Assumptions are however, necessary to reduce certain natural characteristics into a more simplified structural state in order to apply the model. Growth characteristics such as the taper of the tree trunk, knots and eccentricity in cross section and length are ignored [68], as shown in Figure 3.2. An idealised cylinder of wood exhibits symmetry about its central axis and can be referenced to a cylindrical co-ordinate system for which the primary axis is the longitudinal axis. The orientation of the wood rays is radial from this axis and the curvature of the growth rings lie on a tangential plane. Figure 3.3 shows how the L, R, and T directions can be modelled as mutually perpendicular planes by considering the growth rings as planar growth layers [68]. An orthotropic material model can be used to represent the material behaviour of this simplified arrangement. The error introduced through this simplification is a function of the size of the specimen and its location with respect to the centre of the circular stem as illustrated in Figure 3.4. [68]

An orthotropic material is characterised by 12 elastic moduli, nine of which are independent. There are three ratios of normal stress to normal strain in the principal directions, known as the moduli of elasticity (E), three ratios of shear stress to shear strain in the orthotropic planes, known as the moduli of rigidity (G), and six ratios of lateral strain to axial strain, known as Poisson’s ratios. The elastic moduli are determined from the linear portion of a stress strain curve - a typical example of this stress strain relationship for wood is shown in Figure 3.5 demonstrating the normal stress and strain in the longitudinal direction.
Table 3.1 shows the predicted moduli of elasticity and moduli of rigidity for a selection of woods that are presently or were previously synonymous with sports implements [68] and [35]. Table 3.2 gives examples of their use within the sports goods industry.
Figure 3.6a-c [68] show the effective modulus for Sitka spruce subject to variation of applied loading angle in the LT, LR and RT planes [68]. An angle $\phi = 0^\circ$ corresponds to a loading axis equivalent to the L direction in Figure 3.6a and b, and an angle $\theta = 0^\circ$ corresponds to a loading axis equivalent to the T direction in Figure 3.6c. This example demonstrates that small changes in grain angle can have a substantial effect upon modulus of elasticity. The effect can be calculated from an empirical Equation known as the Hankinson’s formula [68, 69] which is expressed in Equation 3.1.

$$N = \frac{PQ}{P\sin^2\theta + Q\cos^2\theta}$$  \[3.1\]

The Equation applies equally to the strength as well as the stiffness of wood. $P$ and $Q$ are the strengths or elastic moduli in the directions parallel and perpendicular to the grain respectively, $\theta$ is the angle between the direction of applied load and the longitudinal direction of the wood cells, and $N$ is the resulting maximum stress or the effective elastic modulus at this angle.

<table>
<thead>
<tr>
<th>Species</th>
<th>Density</th>
<th>$E_L$</th>
<th>$E_R$</th>
<th>$E_T$</th>
<th>$G_{LR}$</th>
<th>$G_{LT}$</th>
<th>$G_{RT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern White Ash</td>
<td>0.54</td>
<td>13100</td>
<td>1227</td>
<td>620</td>
<td>951</td>
<td>704</td>
<td>236</td>
</tr>
<tr>
<td>European Birch</td>
<td>0.55</td>
<td>15251</td>
<td>1256</td>
<td>640</td>
<td>971</td>
<td>721</td>
<td>241</td>
</tr>
<tr>
<td>Hickory-Mockernut</td>
<td>0.64</td>
<td>16795</td>
<td>1447</td>
<td>814</td>
<td>1159</td>
<td>872</td>
<td>299</td>
</tr>
<tr>
<td>Maple (Black)</td>
<td>0.52</td>
<td>8120</td>
<td>1174</td>
<td>586</td>
<td>910</td>
<td>671</td>
<td>223</td>
</tr>
<tr>
<td>English Willow(^1)</td>
<td>0.44</td>
<td>6600</td>
<td>440</td>
<td>220</td>
<td>330</td>
<td>330</td>
<td>33</td>
</tr>
</tbody>
</table>

Table 3.1 - Elastic moduli of woods which have been in sporting goods [68], [35]\(^1\)

<table>
<thead>
<tr>
<th>Species</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern White Ash</td>
<td>MLB bats (current material)</td>
</tr>
<tr>
<td>Hickory-Mockernut</td>
<td>Shafts for golf clubs &amp; lacrosse sticks, laminated tennis rackets (no longer in use)</td>
</tr>
<tr>
<td>Maple (Black)</td>
<td>Laminated tennis rackets &amp; golf club driver heads (no longer in use)</td>
</tr>
<tr>
<td>English Willow(^1)</td>
<td>Cricket bats (current material)</td>
</tr>
</tbody>
</table>

Table 3.2 - Application of some woods in the sports goods industry
3.3.4 Stiffness of Wood Layered Composites

Since the effective stiffness of wood is dependent upon the direction of loading with respect to the wood fibre orientation. The orientation of each wood layer is an important design consideration in determining the overall mechanical properties of a laminated bat. Figure 3.7a-d demonstrates the effect of layer orientation upon the average stiffness and directionality of the resulting sheet. Figure 3.7a illustrates the variation of stiffness $E_\theta$ with angle $\theta$ for a single wood layer in the LT plane. If a sheet is made with uni-directional layers then the 'in-plane' average stiffness characteristics of the sheet are the same as for a single layer, as shown in Figure 3.7b. For a sheet made with mutually perpendicular layers, as shown in Figure 3.7c, the values of stiffness in the $l$ and $2$ axes of the sheet are equivalent providing the combined cross-section area of each layer orientation is equal. In this case the magnitude of $E_1$ and $E_2$ for the bi-directional sheet is equal to half the value of $E_1$ in the uni-directional sheet. If many layers are used to make a laminate block with layer $L$ directions separated by small angles, as in Figure 3.7d, then the combined orthotropic laminae results in near isotropic bulk properties for in-plane loading. For more complex loading conditions, such as bending and torsion, the distance of each layer with respect to the neutral axis of the specimen must be taken into consideration [68].
3.3.5 Wood Strength and Toughness

An idealised stress strain relationship for wood, shown in Figure 3.5, demonstrates a linear region where the strain induced in a piece of wood is proportional to the applied stress. Under static loading, the elastic behaviour of the wood corresponds to the straight line portion of the curve. Beyond this proportional limit, the deformation is non-linear. Inelastic deformation increases beyond the proportional limit until the specimen undergoes failure and the manner in which the material fails is a reflection upon its toughness. Woods that fail gradually over a significant period of strain are 'tough' and woods that fail abruptly over low values of strain are 'brittle'. As most woods are made from the same constituent elements that combine to make the cell wall material, it is the amount of solid cell wall material within a unit volume that can be used as an index to predict the strength properties. The empirical Equation 3.2 defines a general relationship between strength and specific gravity [69].

\[ S = K(y)^n \]  

[3.2]
where \( S \) equals the specific strength in compression, tension, shear, or in bending, \( K \) is a proportional constant that differs for each specific measurement of strength, \( \gamma \) is the specific gravity of the specimen, and \( n \) is the exponent of the relationship curve. Maximum compressive and tensile strengths are measured through the application of axial loading of the wood cells in both parallel and perpendicular directions to the \( L \) direction. Shear strength can be measured through the \( LR \) and \( LT \) planes as the value measured in the \( RT \) plane (across the fibres) is significantly greater than the values obtained for the all other measurements of strength [69]. For most solid materials, for example mild steel and concrete, the compressive strength is larger than the tensile strength for wood however, the opposite is true, as wood has a lower value of strength in compression for stress parallel to the \( L \) direction that can be attributed to the porous and fibrous structure of wood [69]. Since ball impacts cause deformation local to the region of contact, it is anticipated that compression strength perpendicular to the \( L \) direction is relevant to this study.

The modulus of rupture is a widely used measurement for the bending strength of wood [69]. It is anticipated that modulus of rupture is a useful comparative index for wood selection, since the mode of bat deformation resulting from impact is not unlike the mode of bending observed in this test.

Table 3.3 shows the specific gravity and various strength values for woods synonymous with sports implements [70]. Figure 3.8, taken from Panshin and De Zeeuw [71], describes the general trend between specific gravity and the strength of wood. The specific gravity of wood has a positive effect on strength properties in a similar fashion to its affect on the modulus of elasticity. Therefore, it is anticipated that cricket bat durability would benefit from a wood with high specific gravity providing higher values of strength.

<table>
<thead>
<tr>
<th>Species of wood</th>
<th>Specific gravity</th>
<th>Modulus of rupture</th>
<th>Compression strength parallel to grain</th>
<th>Compression strength perpendicular to grain</th>
<th>Shear strength parallel to grain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern White Ash</td>
<td>0.54</td>
<td>65</td>
<td>28</td>
<td>4.6</td>
<td>9.3</td>
</tr>
<tr>
<td>European Birch</td>
<td>0.55</td>
<td>57</td>
<td>23</td>
<td>3.0</td>
<td>7.6</td>
</tr>
<tr>
<td>Hickory-Mockernut</td>
<td>0.64</td>
<td>76</td>
<td>31</td>
<td>5.6</td>
<td>8.8</td>
</tr>
<tr>
<td>Maple (Black)</td>
<td>0.52</td>
<td>55</td>
<td>23</td>
<td>4.1</td>
<td>7.8</td>
</tr>
<tr>
<td>English Willow(^2)</td>
<td>0.37</td>
<td>31</td>
<td>14</td>
<td>-</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Table 3.3 - Some mechanical properties for wood species used in the sports industry Bodig [68] and Lavers [72]\(^2\)
3.3.6 Wood Dynamic Properties

The long cellulose chains in wood exhibit time dependent strain. An applied stress will initially result in an elastic response followed by subsequent deformation or 'creep' that is time dependant - known as rheological behaviour [68]. The effect of this particular response is that specimens can fail under sustained loads which are less than the ultimate static load determined from experiment. Equally, the use of increased stress levels may be employed in designing wood structures that are loaded for very short periods of time.

Fatigue resistance is the ability of wood to stand repeated, reversed, or short duration cyclic loads without failure. Generally, the fatigue stress limit of wood increases with specific gravity and can be expressed in the form of the following Equation, Kollman [69].

\[ \sigma_f = C \times \gamma_s \]  \[3.3\]

where \( \sigma_f \) is the ultimate fatigue stress; \( C \) is a constant; and \( \gamma_s \) is a specific gravity of oven dry weight of wood.

Shock resistance can be described as the ability of a solid body to absorb energy during impact and to dissipate or return the energy from impact without structural failure and is often reflected in the total
area below a load-deflection curve [69]. Measurement of shock resistance can take the form of single or successive blows from a pendulum or vertical drop hammer that generate a bending moment within a specimen along its span. A calculation of the energy absorbed can be taken from successive maximum deformations or the loss in hammer energy resulting from specimen failure. Kollman [69] shows that the fibre angle has a significant effect upon the shock resistance of wood, Figure 3.9.

![Figure 3.9 - Decrease in toughness with increase of angle between specimen axis and fibre direction](image)

The removal of energy from the bat and ball collision occurs partially through material damping and several sources of damping in both the bat and the ball are likely to exist. Damping mechanisms will be present within each material used in their construction and also at the interfaces or boundaries between the different structural components. The total dissipation of energy is the product of material damping within each element and the interaction between neighbouring elements within the structure. In addition, damping in solid wood can also be viewed as a product of both microscopic damping mechanisms within the molecules and macroscopic interaction of individual cells. This is particularly relevant to understanding the damping behaviour of a conventional willow bat, a laminated bat and a leather covered ball since are they can be considered structures made from several or many discrete components.

The physical mechanisms which dissipate energy, mostly in the form of heat and deformation are numerous and complex. However, the mathematical models commonly used to describe the behaviour of damping in materials and structures do not rely upon intricate models the physical mechanisms. A phenomenological approach is taken since the overall behaviour of the material can be represented without modelling the physical mechanisms involved [68, 73].
A property similar to all damped materials or structures is that the cyclic load deformation curve takes the form of a hysteretic loop and the area prescribed between the loading and unloading branches of the curve is proportional to the energy dissipated. Hysteretic loops are synonymous with inelastic behaviour which can be classified as: [73]

1. Rate dependent and recoverable.
2. Rate dependent and non-recoverable.
3. Rate independent and recoverable.
4. Rate independent and non-recoverable.

Materials that exhibit the behaviour in cases 1 and 2 are commonly described as having 'viscoelasticity'. As the term implies, the overall behaviour material is a product of both elastic and viscous (or flow) behaviour. As a result, the strain resulting from a cyclic load maybe partially (viscoelastic) or completely recoverable over time (anelastic). In case three the inelastic strain is a function of the stress only and is termed 'elastic hysteresis'. Case 4 is generally identified as plastic behaviour in which the strain or deformation resulting from material stress is non-recoverable. In these two latter cases the width of the hysteresis strain curve is a function of the stress value only and is not dependent upon time [73].

Since the magnitude of damping is proportional to the bounded area of a hysteresis curve, damping can also be regarded as rate-dependent or rate independent. The value of rate dependent damping may also be affected by the amplitude of the stress (or load), whereas the value of rate independent damping will only be affected by amplitude [73].

Damping of a bat and ball collision may involve all four inelastic processes. The mechanisms involved will depend upon the properties of the wood and the impact conditions. For example, Ono and Norimoto [74] investigated Young's modulus and loss coefficient for different species of softwood and hardwood. The loss coefficient \( q' \) is proportional to the ratio of damped strain energy to total strain energy in a material as shown in Equation 3.4.

\[
q' = \frac{D}{2\pi U} \tag{3.4}
\]

where \( D \) and \( U \) are the damping energy and elastic energy, respectively. The investigation showed that the value of Young's modulus was much greater in the \( L \) direction than in the \( R \) and \( T \) directions in the order \( E_L >> E_R > E_T \). The loss coefficient \( q' \) showed exactly the opposite tendency whereby \( q_L^{-1} < q_R^{-1} < q_T^{-1} \). The relationship between elastic modulus and \( q' \) was expressed as follows:
Where $b$ and $k$ are constants, $E$ is the elastic modulus, and $\gamma$ is the specific gravity. Additionally, the values of $E$ and $q^1$ for different woods fitted Equation 3.5. Consequently, it was stated that higher elastic modulus resulted in lower internal friction regardless of direction or wood type.

The viscoelastic behaviour of wood was studied by Ouis [75] in which the theoretical relationships for dynamic modulus and loss coefficient against a frequency up to $10^{15}$ Hz were established. Greatest attenuation due to longitudinal excitation in wood occurred at approximately 1.5 MHz. However, for bending vibrations Ouis states that the greatest loss factor could occur at a frequency several orders of magnitude lower than observed under axial excitation due to differences in mechanical energy dissipation. Since a ball impact predominately excites the transverse bending modes of the bat, damping due to bending vibrations are of more interest than axial vibrations in this study. The vibration frequencies that are of particular interest range from $\sim 0$ Hz, associated with quasi-static bending tests, discussed in chapter 7, to 500-600 Hz, approximately equivalent to the third mode of transverse bat oscillation and also the frequency of ball impact. From the values obtained in [75], it is assumed that 500 Hz is significantly lower than the transition frequency associated with bending vibrations of wood and therefore the degree of viscoelastic behaviour observed in the defined frequency domain of this study is relatively small.

This agrees with work by Skudrzyk [76] which illustrated the relationship of loss coefficient against frequency of cyclic flexural bending for wood of different moisture contents, governed by their drying times. Figure 3.10, taken from the study, demonstrates the relationship; the flat region for each curve is governed by elastic hysteresis which is independent of frequency but as the frequency increases the viscoelastic behaviour of the wood becomes more apparent. Kollman and Côte [69] makes reference to James in a study which investigated the internal friction of Douglas fir. It was suggested that the internal friction was a product of the elastic hysteretic behaviour of completely dry wood and the viscoelastic behaviour due to hydroscopically bound water. It was reported that internal friction due to the first mechanism was reduced by adding water, whereas damping due the viscoelastic behaviour increased with moisture content and temperature. The study reported that a moisture content of around 7% produced a minimum combined internal friction at room temperature.

The elastic modulus and loss coefficient will both increase slightly over the frequency domain defined in this study due to the viscoelastic behaviour of wood. From the results of studies [69, 75, 76], it is reasonable to suggest that the energy dissipated in a bat and ball impact will be predominately due to elastic hysteresis and plastic deformation (or structural slip), the latter of which is visibly evident in both the bat and the ball local to the region of contact. Both models are dependent upon the amplitude of stress but are independent of loading frequency. Therefore, it is anticipated that contact force is
more significant than contact time or strain rate in determining the magnitude of energy loss resulting from impact.

![Graph showing loss factor vs. bending frequency in wood dried for different periods of time.](image)

**Figure 3.10 - Loss factor vs. bending frequency in wood dried for different periods of time, taken from Skudrzyk [76]**

### 3.3.7 Wood Moisture Content

Moisture content affects the specific gravity, volume, toughness, modulus of elasticity, damping, and measurements of strength in wood. The hygroscopic nature of most chemicals in wood results in water retention within the cell wall material, the amount of which varies depending upon the vapour pressure of the surrounding atmosphere. The fibre saturation point is described as the amount of water required for saturation of the cell wall, with no free water present within the cell matrix. Free water has no significant effect upon the material properties of wood, apart from mass and mass distribution within a specimen [69]. However, moisture levels below the fibre saturation influence the mechanical properties. The fibre saturation point for most woods grown in a temperate climate range from 20-30%. Kollman [69] illustrates the effect of moisture content on the modulus of elasticity as shown in Figure 3.11a and b. The tensile strength along the grain also increases if wood dries below the fibre saturation point with a maximum tensile strength at approximately 8-10% moisture content.

In static loading conditions it is observed that wood increases in strength and stiffness as it dries below the fibre saturation point. However, this relationship does not hold for values of shock resistance or toughness under dynamic conditions. Kollman [69] illustrated that the impact bending properties of wood were independent of wood moisture content. However, below a moisture content of 5% there was a decline in properties associated with toughness as illustrated in Figure 3.11a.

Laminated bat design must take into account the influence of moisture content on the bulk properties of the laminate construction. In addition, fluctuations in moisture content can lead to delamination of a layered construction due to constant dimensional changes that disrupt the layer adhesion. The
moisture content of willow is conditioned prior to machining in current cricket bat manufacture to a value of 12% (private communication, Dunlop Slazenger). Unless the bat is sealed from its environment, it will gain or lose moisture over a period of time accompanied by dimensional change and alteration of mechanical properties.

Figure 3.11 - The effect of moisture content on the modulus of elasticity parallel to the grain of (a) Spruce, and (b) Oak [69]

3.3.8 Literature Concerning Wood Properties in Cricket and Baseball Bats

The effect of microstructure on the impact dynamics of a cricket bat was investigated by Grant & Nixon [33] to improve the accuracy a cricket bat computer model. The face of a cricket bat blade is pressed during manufacture producing a surface better able to withstand impacts encountered during play. The authors believed this process significantly affected the dynamic behaviour of the bat through modifying the flexural vibration characteristics of the implement. In a previous study, Grant and Nixon [77] concluded that flexural vibration characteristics were significant in determining the elasticity of ball
impact. Samples cut from the surface face of the bat were weighed and subjected to a three point bend test with the pressed face uppermost in accordance with BS 373 [78] and values for flexural stiffness against mean density were determined. A computer model of the test was developed and the values of density and elastic modulus were modified for layers of variable thickness to represent the pressed layer in the specimen. The model showed that if a thin layer of the beam were to have its density and elastic modulus increased, then the mass and flexural stiffness of the beam would also increase. Mass, stiffness and frequency values were plotted against layer thickness and a value of ~1.4 mm was determined to accurately represent the pressed layer. The study did not investigate the relationship between flexural stiffness and bat impact characteristics.

Axtell, Smith and Shenoy [79] investigated the durability of wooden baseball bats used in MLB and developed a bat with improved durability. A laminated bat, constructed from three separate wood layers was developed. Static bending tests indicated that the laminated construction decreased the variability inherent in wood and facilitated an accurate prediction of bat properties. However, the laminated bat did not demonstrate an increase in durability. The authors defined the handle section as the weakest part of the bat and developed a procedure for applying a braided sleeve to the outside of the handle to provide reinforcement. The study used FEA to model the increase in bat stiffness from the addition of reinforcement and compared the results of the model against experimental values. The results were in good agreement and showed an increase in bat stiffness of ~20% due to the additional of the braided sleeve. To assess the durability of braided bats, the authors experimentally determined a measurement of durability by progressively increasing the velocity of impact between bat and ball until failure of the bat occurred. Ball impacts were located either side of the centre of percussion (COP) to simulate what were described as ‘inside’ (nearer the handle) and ‘outside’ (further from the handle) impacts. The durability arising from both types of impacts increased substantially from 50% to 90% for ‘outside’ and ‘inside’ impacts, respectively with the addition of the braided sleeve. The authors suggested that the greater increase observed for ‘inside’ impacts was due to the location of maximum stress, which occurred further away from the grip than for an ‘outside’ impact.

3.4 INTRODUCTION TO WOOD ADHESIVES

The hollow cricket bat prototypes in this study are of a multilayer laminate construction in which individual layers are bonded together and it is important to understand the adhesive properties which may affect bat manufacture and bat impact performance.

Bonding with adhesive is a good method of securing contiguous layers of wood together in forming a unified object as it negates the use of wood fasteners such as screws which may impinge upon the functionality of the bat and the rules of the game. The main considerations in determining suitable wood adhesives for the purpose of this study are:
1. The adhesive bond must have the appropriate mechanical properties to satisfy the performance criteria of the bat.
2. The correct method of application and curing must be readily accommodated within bat prototype manufacture.

The overall performance of the bond is not only dependent upon the adhesive properties but also the nature of the substrate and the conditions during application and curing. In this study, all of the gluing surfaces are wood and therefore wood properties that affect bond quality are investigated. Adhesive properties that determine suitability in terms of manufacturing application are investigated and also the adhesive properties that affect bond performance.

3.5 WOOD PROPERTIES THAT AFFECT BOND QUALITY

The quality of an adhesive bond affects its initial strength and its long term durability. Wood type and condition are significant factors in determining bond quality as well as the surface preparation prior to application.

3.5.1 Wood Extractives

In addition to the cellulose compounds that make up wood cells, there are various combinations of extractives which can migrate towards the surface as the wood is dried after cutting and these can present both physical and chemical barriers to bonding. Physically they present an obstacle for the adhesive in reaching fibre material and chemically they can act as plasticisers which create excessive flow whilst others can react to accelerate or retard the cure rate. Different wood species contain different combinations and amounts of extractives and these levels also depend upon the region of the trunk from which the wood is taken. Likewise, it is possible that components of the adhesive may diffuse into the wood and promote chemical alteration and degradation of the wood structure [80].

3.5.2 Wood Density

The density of the wood surface physically affects the adhesive's absorption and migration into wood. A denser wood has more cellulose material per unit area of the contacting surface and will therefore have fewer voids into which the adhesive can penetrate. This can confine adhesives to the immediate glue line and as the wood is pressed the liquid adhesive migrates towards areas of lower contact pressure which can result in a non-uniform adhesive thickness.

3.5.3 Wood Porosity

The angle at which the fibres are presented at the gluing face affects the porosity of the surface. An end grain surface exhibits fibre orientation perpendicular to the exposed face. Fibres that are cut or plucked from this surface expose a greater percentage of voids than if exposed across tangential or radial directions. This leads to a highly porous surface which may absorb an excessive quantity of adhesive thus weakening the glue line.
3.5.4 Wood Anisotropy
Bonding two different types of wood or bonding wood at different fibre orientation may lead to dissimilar dimensional changes across the glue line. A change in dimension occurs through a change in moisture content which may result from the wood structure absorbing adhesive moisture on application or through change in ambient moisture content. In both cases, this can lead to glue line stress, which may diminish the quality and strength of the bond [80].

3.5.5 Wood Surface Flatness and Smoothness
Surface flatness is particularly important when adhering high density woods. If the cell structure is significantly stiff so that it does not deform under the platen pressure then liquid adhesive will migrate to areas of low contact pressure during the pressing cycle, potentially resulting in regions with adhesive starvation. A more consistent gap between the bonding surfaces lowers the pressure gradients within the gluing area resulting in less adhesive migration and a more uniform glue line thickness. Solid particles are often added to some adhesive formulations which act to maintain a minimum gap between bonding surfaces during the press cycle.

3.5.6 Wood Surface Cleanliness and Activation
The removal of surface extractives, impurities and excessively damaged wood fibres that result from machining and handling can be achieved through sanding. The process of sanding removes the top layer of material exposing a 'fresh' surface. A newly exposed surface exhibits a higher activation energy, as over a period of time, wood compounds exposed at the surface may react with atmospheric oxygen lessening their ability to act as sites for chemical adhesion. Surface 'freshness' therefore significantly affects the bond quality of adhesives that rely upon a chemical bond. Ultra violet light increases the surface reaction rate and so exposure of the surfaces to direct sunlight prior to gluing is undesirable. The correct grade of sand paper also promotes surface roughness suitable for optimum mechanical adhesion.

3.6 ADHESIVE PROPERTIES THAT AFFECT APPLICATION
Adhesives increase in viscosity until fully cured and the properties of the adhesive prior to complete hardening are important to its application.

Viscosity is a measure of the resistance to flow of a liquid. The viscosity of liquid adhesives prior to hardening determines the method of application, and influences the 'wetting' and glue line thickness which can effect bond quality. The hardening process results in complete solidification in most thermoset and thermoplastic adhesives whilst some elastomeric adhesives are designed to remain deformable after curing. The hardening process occurs through one or a combination of chemical reaction, loss of solvent/water from the adhesive or cooling from a melted state. The conditions required to initiate curing are specific to the type of adhesive, as are the rates of hardening, although rate is predominantly governed by the glue line temperature. Adhesive specifications include a number
of terms that are associated with the practicalities of the hardening process. The following definitions are useful in selecting an appropriate adhesive in terms of bat manufacturing capability.

'Pot life' ('working life' or 'gel time') refers to the period of time after curing has been initiated, during which the viscosity remains sufficiently low to allow correct application and the desired level of substrate 'wetting'. Ease of application and bond quality suffer as a result of continued application beyond this time. Certain adhesives, such as PVA, require a period after their application to develop a greater viscosity before the two surfaces are joined, this increase in viscosity prior to assembly is termed 'tack'.

The thickness of adhesive applied to a surface depends upon the level of viscosity and the method and rate of application and these will determine the rate of adhesive consumption. A recommended rate of consumption is given as a mass per unit of area of substrate to generate the correct glue line thickness for greatest bond quality. In general, the glue line thickness of adhesives is between 0.001 mm to 0.1 mm. If the surface texture of the adherent results in mating surfaces which exceed the recommended glue line thickness, then the quality of the bond relies upon the gap filling qualities of the adhesive. The gap filling properties depend upon the percentage volume of adhesive that transforms to a solid upon hardening. For instance, polyvinyl acetate (PVA) glues solidify with 35-50% reduction in volume due to water evaporation and absorption, and therefore its gap filling potential is less than thermoset adhesives which retains 100% volume in the hardened state. Once the adhesive has been applied to the mating surfaces, pressure can be required to bring the surfaces together to aid penetration and wetting of the adhesive and to compensate for the volume loss. Some thermosetting adhesives release gas or water during heat curing and so pressure is also required to prevent the surfaces separating under gas or vapour pressure.

The 'open time' refers to an approximate length of time prior to pressing in which the adhesive viscosity is sufficiently low to allow for the glue line pressure to be successfully applied. Pressure is maintained whilst the hardening process continues and is released after the adhesive has become sufficiently strong to maintain adhesion at the join without disruption to the glue line or mating surfaces, this is known as the 'closed assembly time'. The glue line usually requires a further period after pressing to allow the adhesive to reach a state of complete hardening, resulting in full mechanical strength.

### 3.7 ADHESIVE PROPERTIES RELEVANT TO BOND PERFORMANCE

Bond performance concerns the adhesive properties that exist after the hardening process and considers the immediate mechanical properties of the bond as well as the change in those properties whilst in service. The mechanical properties can be derived from tensile tests on adhesive films to measure values such as stiffness, strength, and elongation to break. These can be compared to the properties of the wood in determining suitability. An adhesive is chosen to provide equal or greater
strength than the substrate whilst having similar values for stiffness, in order to minimise stress concentrations in the bond layer [80]. The service conditions of the bat will also affect the properties of the adhesive and therefore their appropriate selection. The varying susceptibility of adhesives types to service conditions such as moisture or ultra-violet light is dependent upon the substrate as well as the adhesive, and so specific tests are required to accurately quantify bond performance for a particular service condition. Studies by Gillespie and River [81] and Vick and Okkonen [82] are examples of such work in which techniques to accelerate the test procedure are employed.

Substrate and adhesive tests can provide mechanical data for the bond and provide a visual assessment of bond quality following insertion of a knife or chisel into the glue line to produce failure in a 'cleavage' or 'peel' mode. Tests can be quantified in terms of mean load or stress required to rupture the specimen. Measurement of creep under sustained loading is important for some adhesive bonds as is fatigue resistance due to the swell and shrinkage of substrates. Exposure and accelerated exposure tests are used to quantify the change in mechanical properties due to anticipated environmental conditions over a period of time.

The immediate mechanical properties of the bond are of particular interest to this study, the most significant test for the bond being the loading condition that results from an impact of the bat with a cricket ball. In this instance, the durability of the bond defines its ability to withstand multiple ball impacts without failure. It is also anticipated that the adhesive properties will also have an effect upon the COR of impact. Yeh and Brown [83] devised experiment to measure the adhesive damping properties in wood joints. The results illustrated that shear motion was dominant in generating energy loss and the value of damping increased with glue line thickness.

3.8 ADHESIVE SELECTION

The selection of appropriate adhesives for this study considered the quality of adhesion to the substrate, the ease of application in prototype manufacture, the resulting properties of the laminate, and the expected service conditions. Numerous formulations exist within each family type of adhesive which satisfy these criteria to various extents. The functional performance of the bat is of significant concern in this study and successful application must be achieved within reasonable steps. Therefore, the choice of adhesives illustrated below was made with equal consideration of performance and ease of application. The following section briefly describes the adhesives used in this study. The reasons for selecting these adhesives in terms of bat manufacture is discussed in chapter 8 and performance considerations are discussed in chapter 10.

3.8.1 Prototype 1 Bat

A urea-formaldehyde (UF) thermoset resin was used in the manufacture of the prototype 1 bat. The resin is characterised by a rigid, cross-linked molecular structure upon curing and cannot be re-melted or restored to a low viscous state once the reaction has taken place. It is a two part adhesive
consisting of a liquid resin and powder hardener. The adhesive undergoes a chemical reaction forming a rigid, cross-linked, three-dimensional network of macromolecules. Thermoset resins show little elastic deformation under load and as result most structural adhesives requiring sustained load bearing qualities tend to be formulated with thermosetting components [80].

3.8.2 Prototype 2 Bat

Polyvinyl acetate (PVA), used in the prototype 2 bat manufacture. It is a thermoplastic adhesive and does not exhibit a cross-linked, rigid structure. The molecules are either linear or branched and therefore they are susceptible to softening and melting upon application of heat and under sustained loading the adhesives are susceptible to plastic deformation. PVA emulsion forms the basis for the white wood glue commonly used in furniture building and household applications. Once applied, the water is removed from the adhesive by evaporation and diffusion resulting in a solidified mass. Bonding wood requires a set time of 10–180 minutes at room temperature and full strength is achieved after one to seven days [84]. The bond is relatively flexible in comparison to thermoset adhesive bonds.

A Polyurethane and a silane modified polyether adhesive (MS Polymer) were used in the construction of the striking face component of hollow prototype bats. These adhesives are based upon synthetic elastomeric polymers and are particularly suitable for applications that require flexibility and toughness. Polyurethane is a synthetic rubber but adhesive formulations can react to form covalent bonds with the hydroxyl group of the cellulose of the wood. The wood is therefore chemically bonded to the adhesive resulting in a strong bond which is resistant to moisture. MS polymers are classified as high performance sealants, the formulation consists of elastomeric resins to provide flexibility and elongation rather than high tensile strength [85].
CHAPTER 4
CRICKET BAT AND BALL IMPACT DYNAMICS

4.1 INTRODUCTION
In defending a wicket and scoring runs, the batsman combines a number of limb and body movements that transfer energy from the batsman to the bat. A percentage of the kinetic energy transferred to the bat by the player is then imparted to the ball upon contact. The contact time is approximately 1 ms and forces in excess of 14 kN can be generated between the contacting surfaces over this duration. Non-rigid phenomena such as deformation and vibration become significant in understanding the transfer of kinetic energy. The following section examines current knowledge of cricket and baseball bat impacts including both rigid and deformable models.

4.2 RIGID BODY ANALYSIS
Rigid body analysis treats objects as non-deformable entities and the method can be used to determine the post impact linear and angular momentum of objects from knowing the pre-impact conditions. The analysis does not include local impact deformation or vibration of the objects. The accuracy of the model is dependent upon the significance of object deformation in the system. For example, the post impact momentum of two colliding spheres in a game of snooker can be reasonably modelled by neglecting deformation and post impact vibration in this manner. However, in the case of a thin beam colliding with stationary sphere, the vibration characteristics of the beam are more likely to effect the resulting rigid body motion. Advanced modelling techniques have shown that cricket and baseball bats with shapes that essentially resemble beams demonstrate post impact bending modes that significantly affect bat rigid body motion. [35, 86]. Rigid body analysis can however, be used to investigate the rigid body modes following object collision. This is relevant to the cricket bat and ball contact situation as the majority of desirable impacts which occur during play are eccentric.

4.2.1 Bat Rotation
The rotational characteristics of the bat are dependant upon the location and orientation of the axes of rotation. This study uses values of inertia to investigate the properties of cricket bats and therefore it is important that appropriate axes are chosen about which the inertia measurements are made. The resistance of a 3-D rigid body to forces tending to cause rotation about three mutually perpendicular axes, $x$, $y$, $z$, can be written in the form of an inertia matrix, as shown in Equation 4.1, Ginsberg [87].

\[
[I] = \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{yx} & I_{yy} & -I_{yz} \\
-I_{zx} & -I_{zy} & I_{zz}
\end{bmatrix}
\] [4.1]
where $I_{xx}$, $I_{yy}$ and $I_{zz}$ are the moments of inertia and $I_{xy}$, $I_{yx}$ and $I_{yz}$ and are the products of inertia. The moments of inertia describe the object's resistance to rotation about each axis of rotation and the products of inertia, which involve two axes simultaneously, describe the symmetry of mass distribution relative to the co-ordinate planes [87].

Figure 4.1 illustrates the location of a differential element of mass $dm$ from the origin of arbitrary axes $x, y, z$. Since elements $dm$ fill the region occupied by the bat, the inertia about the $x, y, z$ axes can be determined from the integral of all elements in the bat.

The moments of inertia about each axis are a function of the element mass and the square of the shortest distance from the axis to the element, as shown in Equations 4.2 i, ii and iii. The products of inertia are a function of the element mass and the product of the shortest distance between the respective axes and the element as shown in Equations 4.3 i, ii and iii.

\begin{align}
(\text{i}) \quad I_{xx} &= \int_{\text{bat}} (y^2 + z^2) \, dm \\
(\text{ii}) \quad I_{yy} &= \int_{\text{bat}} (x^2 + z^2) \, dm \\
(\text{iii}) \quad I_{zz} &= \int_{\text{bat}} (x^2 + y^2) \, dm 
\end{align} \tag{4.2}

\begin{align}
(\text{i}) \quad I_{xy} &= \int_{\text{bat}} xy \, dm \\
(\text{ii}) \quad I_{yx} &= \int_{\text{bat}} xz \, dm \\
(\text{iii}) \quad I_{yz} &= \int_{\text{bat}} yz \, dm 
\end{align} \tag{4.3}

The products of inertia are non-zero values for an arbitrary co-ordinate system $x, y, z$. However, there exists a particular set of orthogonal axes for which the products of inertia are zero, resulting in a principal inertia matrix in the form of Equation 4.4.

\[
[\hat{I}] = \begin{bmatrix}
\hat{I}_{xx} & 0 & 0 \\
0 & \hat{I}_{yy} & 0 \\
0 & 0 & \hat{I}_{zz}
\end{bmatrix} \tag{4.4}
\]
where $I_x$, $I_y$, and $I_z$ are the principal MOI. Characterising the rotation of an object about the principal axes involves only the values of principal MOI. The principal axes are aligned with respect to the bat such that two of the principal MOI demonstrate maximum and minimum values of MOI.

The principal axes were determined for idealised CAD models of a solid and hollow cricket bat, discussed in chapter 5. Figure 4.2 illustrates the location and orientation of these principal axes, which are approximately equivalent for both designs. It can be seen that the $y$ axis is approximately parallel to the geometric axis of handle symmetry and is off-set in the $z$ direction by a small distance. Since the principal axes are orthogonal, the $z$ axis is approximately perpendicular to the handle axis and normal to the surface of the striking face (if the face is considered as planar). Table 4.1 illustrates the relationship between the principal axes and the axis of handle symmetry for the solid and hollow bat CAD models.

<table>
<thead>
<tr>
<th>OFFSET (mm)</th>
<th>DEVIATION ANGLE (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$</td>
</tr>
<tr>
<td>Solid bat</td>
<td>0</td>
</tr>
<tr>
<td>Hollow bat</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1 – Location and orientation of principal axes in relation to axis of handle symmetry

Figure 4.2 - Bat principal axes and the handle axis of symmetry
Bat Motion due to the Actions of the Batsman

In chapter 2, three MOI were defined to investigate the perceived effort in welding a bat. It was anticipated that a bat swinging motion during a shot would be in an axis parallel to the principal axis, $x$ and therefore measured values of $I_{xx}$ were used to determine swing MOI about a point in the handle using the parallel axis theorem, shown in Equation 2.4. This assumption is reasonable since the plane of the bat face is perpendicular to the direction of motion for a rotation about this axis and would present the largest striking area to the direction of motion, as shown in Figure 4.3. Although the player may alter the face orientation with respect to its direction of travel, it is argued that the value of swing MOI in an axis parallel to $x$ will be the predominant rotation inertia value in affecting the perceived effort during a typical batting shot.

The polar MOI was defined in chapter 3 as a bat property which may be significant to the batsman and was measured as the MOI about the principal axis, $y$. Since the batsman grips the bat about the handle, it is anticipated that the value of $I_{yy}$ will be the predominant inertia value in affecting the angular momentum applied or resisted through rotation about the axis of handle symmetry as this axis is approximately equivalent to the $y$ axis, as shown in Table 4.1.

![Figure 4.3 - Bat principal axis and parallel axis representing the rotation of the bat during a swing](image)

Bat Motion due to Ball Impact

The action of the bat during and for a short period after ball impact can be modelled as if the bat were without constraint, since the gripping condition provided by the hands is not sufficient to affect the
initial behaviour of the bat resulting from the collision impulse, (Brody [46], Nathan [45], and Cross [28]). Therefore, the free linear and rotational motion of a bat following an eccentric impact is relevant to understanding the impact performance of the bat. Figure 4.4a and b illustrate eccentric ball impacts with ball position and velocity components coincident with the \( yz \) and \( zx \) principal planes, respectively.

It is reasonable to assume that for both normal ball impacts (1) and oblique ball impacts, such as (2) and (3), there will be a tendency for the bat to rotate predominantly about the \( x \) axis for impacts in Figure 4.34a and in the \( y \) axis for impacts in Figure 4.34b. Therefore, it is anticipated that values of \( I_{xx} \) and \( I_{yy} \) are the predominant inertia values in affecting the bat angular momentum for impacts equivalent to the \( yz \) and \( zx \) principal planes, respectively.

![Figure 4.4 - Eccentric ball impacts equivalent to the \( yz \) and \( zx \) principal planes](image)

Figure 4.5 illustrates examples of impacts which deviate from both the \( yx \) and \( zx \) planes. An analytical solution to the rotation of the bat following such impacts involves the determination of the axes of rotation and values of MOI and products of inertia. However for such impacts, it is anticipated that there will be tendencies for the bat to rotate (at least initially) about the \( x \) and \( y \) axes. Values of \( I_{xx} \) and \( I_{yy} \) are used in this study as a measure of the change in inertia properties between solid and hollow bat designs. These values are predominant in affecting bat rotation for impacts equivalent to the \( yz \) and \( zx \) planes and it is anticipated that the general difference observed between designs will remain true for impacts which deviate from these planes. Therefore, values of \( I_{xx} \) and \( I_{yy} \) are used to indicate general differences in the rotational characteristics of different bat designs for impacts on the front face of the bat.
4.2.2 'Effective' Bat Mass Concept ($m_e$)

The 'effective' bat mass $m_e$ represents the mass of the bat which results in the observed post impact velocity of the bat and ball at point of contact [31, 45]. Figures 4.6 and 4.7 illustrate the effective mass concept showing how a 2-D model of the bat and ball impact can be represented more simply by a 1-D model using point masses with single degrees of freedom.
The value of effective mass for a rigid bat is a function of the distance of impact from the COM, the MOI about the COM with an axis perpendicular to the plane of motion, and the total mass, as shown in the following Equations:

The impulse from impact results in a change in both linear and angular bat momentum:

Linear momentum:  \[ \int_0^1 F \, dt = M \cdot v_{cm} \]  \[ \text{[4.5]} \]

Angular momentum  \[ d \cdot \int_0^1 F \, dt = I_{cm} \cdot \omega \]  \[ \text{[4.6]} \]

Where limits 0 and 1 define the contact duration. An identical impulse occurring in the point mass model results in a linear change of momentum in mass \( m_e \):

\[ \int_0^1 F \, dt = m_e \cdot v_e \]  \[ \text{[4.7]} \]

For the point mass model to be representative of the bat and ball model, \( v_e \) must equal the velocity of the bat at point of contact \( v_{com} \):

\[ v_e = v_{com} + \omega \cdot d \]  \[ \text{[4.8]} \]
Substituting Equations 4.5 to 4.7 into Equation 4.8 gives:

\[
\frac{1}{m_t} \int F \, dt = \frac{1}{M} \int F \, dt + \frac{d^2}{I_{cm}} \int F \, dt \tag{4.9}
\]

The impulse function can be eliminated from Equation 4.9 resulting in a solution for effective mass in the form of Equation 4.10.

\[
\frac{1}{m_e} = \frac{1}{M} + \frac{d^2}{I_{cm}} \tag{4.10}
\]

The radius of gyration \( r_g \) is the root of the MOI divided by the total mass of the object, as shown in Equation 4.11. The value has units of distance and can be described as distance from the axis of rotation about which the entire mass could be concentrated to exhibit the same MOI as the original volume, for a given axis of rotation [88].

\[
r_g = \sqrt{\frac{I}{m}} \tag{4.11}
\]

\( r_g \) can be substituted into Equation 4.10 resulting in an expression for the ratio of effective mass to total mass in the form:

\[
\frac{m_t}{m_{bat}} = \frac{r_g^2}{(r_g^2 + d^2)} \tag{4.12}
\]

Ideally, the value \( \frac{m_t}{m_{bat}} \) would equal 1, where by the total bat mass contributes to the change in ball momentum. However, the effective mass reduces as the distance \( d \) of eccentric impact increases. This effect is reduced if the bat radius of gyration is increased. Figures 4.8 and 4.9 illustrate values of bat \( m_t \), calculated for a conventional bat of \( m = 1.15 \text{ kg} \) and \( I = 5.1 \times 10^4 \text{ kgmm}^2 \), as function of bat \( y \) and \( x \) distance from the COM, respectively. Values 'P1-P9' along the \( x \) axis represent impact locations at discrete distances from the top of the bat handle used during experimental testing and computer simulation. In referring bat impacts, the 'P' value is used in this study to identify the impact location on the striking region of the bat.
4.2.3 Bat Recoil Factor ($r$)

The bat recoil factor $r$, [31] is the ratio of ball mass to effective bat mass and describes the inertia relationship between both objects at point of contact, as in Equation 4.13.

$$ r = \frac{m_{\text{ball}}}{m_{b}} $$

[4.13]

4.2.4 Centre of Percussion (COP)

A normal bat and ball impact results in both translation and rotation of the bat. For a ball struck below the bat COM, there will be a point located in the bat handle defined as the instantaneous centre of rotation (ICR). At this point the relative translation of the bat away from the ball after impact is effectively cancelled by the forward rotation of the handle about the bat COM. Equation 4.17 can be used to obtain the distance between the points, known as conjugate points, and the bat COM. Figure 4.10 illustrates conjugate points $S$ and $P$ - for an impact at $P$ there is no net movement at point $S$ and point $P$ is known as the centre of percussion (COP), relative to point $S$ and visa-versa. Figure 4.10 shows a cricket bat with mass $m$, moment of inertia $I$ and COM located at point $C$. The ball impacts the
bat at point $P$ with an average force $F$ which generates bat transactional acceleration $a$, to the left and an angular acceleration $\alpha$, about $C$. From Newton's Second law of motion:

$$a = \frac{F}{m} \quad \text{and} \quad \alpha = \frac{F.l_p}{l}$$

[4.14]

after time $t$:

$$v = u + a.t = \frac{F}{m} l \quad \text{and} \quad \omega = \frac{F.l_p}{l} t$$

[4.15]

Therefore net velocity at $S$.

$$v_s = \frac{F.l_p.L_s}{l} - \frac{F}{m} l = \frac{F.t}{M} \left( \frac{L_p.L_s}{l} - \frac{1}{M} \right)$$

[4.16]

If $L_p = \frac{l}{m} \frac{1}{L_s}$ then $v_s = 0$

[4.17]

where $L_p$ and $L_s$ are the distances of ball impact and the conjugate percussion point in the handle form the bat COM, respectively. From Equation 4.17, an impact at a distance $L_p$ from the bat COM results in no net velocity at point $S$. The result of this analysis is significant as it is likely that the reaction force generated between the batsman’s hands and the handle will be minimised for a hand position coincident with point $S$.

![Diagram](image)

**Figure 4.10 - The centres of percussion following impact**

### 4.2.5 Coefficient of Restitution (COR, $e$)

Figure 4.11 shows the momentum of the bat and ball modelled as a direct central collision between two deformable masses at three stages during the impact. Figure 4.12 illustrates the change in contact force over the duration of contact. Upon contact at time $t_0$ the objects undergo increasing deformation
until \( t_1 \) at which time the contact area between the two objects is at a maximum and both objects are travelling at the same velocity \( v_{\text{def}} \). During the remainder of the contact period, the objects undergo restoration and separate at time \( t_2 \), with outbound velocities \( v_{\text{ball}} \) and \( v_{\text{bat}} \) [89].

The conservation of linear momentum for the impact can be shown in the form of Equation 4.18 which is true regardless of the elasticity of collision.

\[
m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2
\]  

[4.18]

To determine the values of \( v_1 \) and \( v_2 \), an additional Equation is required that reflects the capacity of the colliding bodies to recover elastically from impact. The relationship is expressed by the ratio of the magnitude of restoration impulse to the magnitude of deformation impulse and is known as the coefficient of restitution (COR) [89].

The contact force can be considered as two parts, the deformation force \( F_d \), resulting in the initial deformation of both objects and the restoration force \( F_r \), resulting in the restoration of shape following greatest deformation. If the time taken for maximum deformation to occur is \( t_1 \), and the total time of contact is \( t_2 \), then the ratio of restoration impulse to deformation impulse is equal to the change in momentum. Equation 4.19 illustrates this relationship in the direction of force observed by the ball.

\[
e = \frac{\int_{t_0}^{t_1} F_d \, dt}{\int_{t_0}^{t_2} F_d \, dt} = \frac{m_{\text{ball}}(v_{\text{ball}} - v_{\text{def}})}{m_{\text{ball}}(u_{\text{ball}} + v_{\text{def}})}
\]  

[4.19]

Equally, the ratio of restoration to deformation impulse in the direction of the force observed by the bat is given in Equation 4.20.

\[
e = \frac{\int_{t_0}^{t_1} F_d \, dt}{\int_{t_0}^{t_2} F_d \, dt} = \frac{m_{\text{bat}}(v_{\text{def}} + v_{\text{bat}})}{m_{\text{bat}}(u_{\text{bat}} - v_{\text{def}})}
\]  

[4.20]

Eliminating \( v_{\text{def}} \) between Equations 4.19 and 4.20 gives the COR as the ratio of the relative outbound velocity to the relative inbound velocity [89].

\[
e = \frac{v_{\text{ball}} - v_{\text{bat}}}{u_{\text{ball}} + u_{\text{bat}}}
\]  

[4.21]

The value of COR is dependent upon the properties of both colliding bodies. In the case where one of the colliding bodies is significantly more stiff than the other, such as a cricket ball, with a bulk elastic
modulus of 90 MPa impacting a thick steel plate of elastic modulus = 200 GPa. The stiff body can be considered perfectly rigid as its deformation is negligible and does not significantly influence the behaviour of the ball, as shown by the spring damper models in chapter 6. In this instance the elasticity of the impact is a predominantly a function of the ball.

4.2.6 Collision Efficiency ($e_A$)
Collision efficiency $e_A$ is the ratio of final to initial ball velocity, relative to the inbound velocity of the bat. This ratio, has greater application in baseball and softball than COR since the response of the bat after impact is not required in order to calculate values of $e_A$ [31].

$$e_A = \frac{v_{\text{final}} - v_{\text{initial}}}{v_{\text{initial}} + v_{\text{initial}}}$$  \[4.22\]

The relationship between $e_A$ and COR can be shown in the form of Equation 4.23 [31].

$$e_A = \frac{e - r}{1 + r}$$  \[4.23\]

Figure 4.11 - Change in momentum between bat and ball
Deformation per body time $t_d \rightarrow t_d$

$F_d, F_0$

effective mass $m_{e}$

Restoration period time $t_r \rightarrow t_r$

$F_r, F_t$

effective mass $m_{e}$

Figure 4.12 - Change in impulse between bat and ball

4.2.7 Kinetic Energy Transfer From Impact

Figure 4.13 shows a simple 1-D impact scenario involving an effective bat mass, $m_e$, moving at initial velocity, $u_e$ and colliding with a stationary ball of mass, $m_b$. After impact, both masses have a final velocity in the same direction, $v_e$ and $v_b$, respectively.

The kinetic energy (KE) of the ball is defined as:

$$K.E_{\text{ball}} = \frac{1}{2} m_b v_e^2 = \frac{1}{2} \frac{m_e m_b}{m_e + m_b} u_e^2 (1 + \epsilon)^2$$  \[4.24\]

The efficiency in transferring bat KE to ball KE can be described as the ratio of final kinetic energy of the ball relative to the initial kinetic energy of the effective mass:

$$\eta = \frac{\text{Final } K.E_{\text{ball}}}{\text{Initial } K.E_{\text{bat}}} = \frac{m_e m_b (1 + \epsilon)^2}{(m_e + m_b)^2} = \frac{\lambda(1 + \epsilon)^2}{(1 + \lambda)^2}$$  \[4.25\]

where, $\lambda = \frac{m_e}{m_b}$

$\eta_{\text{max}}$ occurs when:

$$\frac{\partial \eta}{\partial \lambda} = 0$$

$$\frac{\partial \eta}{\partial \lambda} = \frac{(1 + \epsilon)^2 (1 + \lambda)^2 - 2 \lambda (1 + \lambda) (1 + \epsilon)^2}{(1 + \lambda)^2} = 0$$  \[4.26\]

Equation 4.26 is satisfied when $\lambda = 1$, therefore maximum efficiency occurs when $m_e = m_b$. The following Equations describe the effect of the change in $m_e$ along the $y$ axis of the bat, upon the final kinetic energy of the ball.
If \( m_e = \lambda m_b \), substituting into Equation 4.24 becomes:

\[
K.E_{\text{ball}} = \frac{\lambda^2}{(1+\lambda)^2} \frac{m_b}{2} \mu_e^2 (1+e)^2
\]  

[4.27]

For an impact location equivalent to the bat COM, the effective bat mass is equivalent to the total bat mass \( \sim 1.2 \) kg. A ball mass \( \sim 0.16 \) kg gives a value of \( \lambda = 7.5 \) and using \( e = 0.6 \), then:

\[
K.E_{\text{ball}} = 0.16u_e^2
\]  

[4.28]

and from Equation 4.25, \( \eta = 0.27 \)

For an impact near the toe region of the bat, at a distance of \( \sim 270 \) mm further from the handle than the bat COM, the value of effective mass \( \sim 0.44 \) kg. Therefore, \( \lambda = 2.75 \) and the ball KE and the efficiency of energy transfer are equal to:

\[
K.E_{\text{ball}} = 0.11u_e^2 \quad \text{and} \quad \eta = 0.50
\]  

[4.29]

Therefore, for the same impact velocity, an impact at the bat COM will result in a ball KE \( \sim 45\% \) greater than an impact at the toe. However, the impact is \( \sim 45\% \) less efficient in transferring the effective mass KE. It is unlikely that the efficiency of kinetic energy transfer between bat and ball is the most significant factor in determining optimum bat mass properties, since ball KE (and therefore velocity) is a more meaningful measure of performance during the game. Therefore, increasing the value of \( m_e \) above that of the ball is desirable, despite the reduction in efficiency. However, Equation 4.27 highlights a diminishing return in ball KE for increases in \( m_e \). For example, increasing \( m_e \) to 20 kg only results in 1.98 times the ball velocity relative to the \( m_e = m_b \) condition. It is also likely that a limitation exists from the batsman to increasing bat mass, unremittingly. It is likely that the empirical design evolution of the bat has resulted in commercially available designs for which the mass is sufficient to be effective in changing ball momentum, whilst allowing the player to manoeuvre the bat effectively during the game.

Figure 4.13 - 1-Dimensional impact scenario involving two objects
4.2.8 Literature Review Concerning Rigid Body Analysis of Sports Implements

This section reviews studies concerning rigid body models of tennis rackets and cricket/baseball bats. The studies highlight the usefulness of momentum based solutions and also the limitations these models can exhibit when used to predict post impact characteristics.

Brody [52] illustrated the orientation of the principal axes and 'swing' axes of a tennis racket, shown in Figure 4.14. The tennis racket demonstrates a degree of geometric symmetry through which the principal planes are aligned. Brody described how these principal axes also coincide with the directions of a simplified model of racket motion, as the axis of rotational 'swing' of the racket is approximately parallel to the x axis with the string bed face held perpendicular to the direction of movement. The value of \( I_{xx} \) is referred to as the 'swing weight' of the racket as the majority of swinging actions that take place during a game of tennis rotate about an axis parallel to this axis. However, the position of a pivot point during the rotation of the racket at any instance relies upon the relative movement of the player's limbs. The MOI through the \( y \) axis, described as the polar MOI, was stated as being significant to the resistance to twist through the centreline of the handle. Brody [55] suggested the ball COR is sensitive to the distance of an impact that deviates from the polar axis of the racket more so than impacts that deviate towards the handle or the tip of the racket. The author suggested that this was due to the relatively low value for polar MOI, such that small deviations would result in the racket twisting upon impact and energy being absorbed in racket rotation. However, increasing the polar MOI by adding mass to the head of the racket at its widest part had the additional effect of increasing the swing MOI of the racket. Brody suggested there was a 'trade off' between racket manoeuvrability and size of the 'sweet spot' across the face of the racket, with better players opting for a smaller racket and benefiting from a reduced swing weight as they are able to hit the ball more accurately. The author mentioned that some players add lead tape to the racket to increase both swing weight and polar moments of the racket resulting in lower swing speed but greater perceived ball 'control'. Brody suggested that the moment of inertia \( I_{zz} \) is only of concern when the motion of the racket is adjusted to provide spin on the ball, in this case the direction of motion of the racket is no longer perpendicular to the \( x \) axis and therefore the value of \( I_{zz} \) becomes significant. Measured values of \( I_{zz} \) were \( \sim 5\% \) greater than \( I_{xx} \) for the rackets used in the study and 30-60\% greater than the values of \( I_{yy} \). The percentage difference between \( I_{zz} \) and \( I_{xx} \) was only 2\%, since the value of \( m d^2 \) remains consistent between the two quantities.
Heald [18] investigated the relationship between baseball bat swing weight and the distance a ball will travel upon being struck by hitters of varying physical stature. The study showed that as the swing weight of the bat increased from very low values, the hit distance also increased. At an intermediate value of swing weight, the hit distance reached a maximum and further increases in swing weight resulted in a reduction in hit distance. If the swing weight was reduced then the relationship showed a more rapid loss of hit ball distance than if swing weight were increased beyond the optimum value. The study modelled the swing weight vs. hit distance characteristics for six sizes of player by modifying the value of torque used to accelerate the bat within the mathematical model. The study showed that there is an optimum and unique value of swing weight for each particular ‘strength’ of hitter.

Bahill [23] studied the possibility of maximising the hit velocity of a baseball, for an individual player by selecting and distributing the bat mass to suit particular swing speeds and swing geometries generated by individual players. The author noted that the advent of hollow baseball bats has allowed greater flexibility in the positioning of bat mass without affecting the external size or shape of the bat. The study agreed with the fundamental criteria for batted ball speed (BBS), described by Nathan [45] and considered the mass, the mass distribution, the COR and characteristics of the players’ swing. Bahill described the practical tests and mathematical calculations involved in selecting the ideal mass characteristics for an individual. Values of MOI were used as an accurate measurement of the weight distribution characteristics that affect a players’ swing. The values were determined from the time period of a bat under simple harmonic motion, pivoted about the end of the handle. The values of COR were derived for different impacts speeds and player swing speed was determined using light gates measuring the linear speed of the ‘sweet spot’ just before impact. Players were asked to use a number of bats that had varying MOI and the relationship between swing speed and MOI was produced for individual players. The relationship in the form of Equation 4.30 was defined and relates bat MOI and bat velocity for a limited range of MOI.
where $u_{bat}$ is the velocity of the bat before impact and $I_{knob}$ is the bat moment of inertia about the end of the handle. As COR is the ratio of inbound to rebound velocity. Bahill combined relationship between COR and impact velocity with the relationship of swing speed and MOI to understand the relationship between the velocity of the ball after impact and the bat MOI, for an individual player. The results of this study indicated that the rebound speed of the ball can be increased by distributing the weight of the bat towards the end of the barrel resulting in an 'end loaded bat' whereby the MOI is increased without increasing the mass. Bahill comments that some manufacturers have included end loading in commercially available bats, but these designs do not provide a MOI equal to the optimum values determined in the study.

Kirkpatrick [90] illustrated the diminishing return for increasing the mass of a baseball bat beyond approximately three times that of the baseball. The study also determined that a bat mass 3.4 times that of the ball, resulted in a rebound ball velocity equivalent to the inbound velocity for the least initial bat kinetic energy. However, the author noted that commercially available baseball bats are approximately seven times the ball mass, suggesting that players are more interested in effectiveness than efficiency. The study highlighted that the empirically determined value of bat mass for optimum effectiveness from the player's perspective is likely to be a compromise between two desirable but incompatible extremes. To maximise bat velocity, the ideal bat mass would approach zero at which point the limiting speed would be a function of the players ability to overcome their own inertia. Whereas, to maximise the change in ball momentum, a bat with the largest possible mass is required.

A study by Elliot and Ackland [91] investigated the physical and impact characteristics of aluminium and wood cricket bats. The study was prompted by the use of an aluminium cricket bat during the first test between Australia and England in 1979. The MCC subsequently banned use of all bats other than those made from wood. However, its appearance led to speculation regarding its performance relative to more conventional designs. The authors tested a total of ten cricket bats and the range included junior and senior versions of both aluminium and wooden designs. Mass and geometric properties for each bat were measured including values for MOI and locations of COM. The authors described both the aluminium and wooden bats to have similar mass properties and suggested that in preparation of a shot, the batsman would have little to distinguish between the two. The experimental set-up was designed to simulate an impact encountered during playing conditions. Each bat was accelerated through the handle secured at a position corresponding to an axis through which the grip of the index finger of the lower hand would normally reside, as suggested by Plagenhoef [92]. A pitching machine delivered a leather cricket ball ~ 80 kph to impact the bat and the resulting motion was measured in the handle using force transducers. The impact was recorded using stroboscopic photography in order to evaluate the correct position of bat and ball upon impact and to measure bat and ball velocities. The balls were fired randomly at four discrete zones on and either side of the theoretical COP the position
of which was determined for each bat. The largest differences between theoretical and experimental COP positions were found with the wood bats. This was assumed to be due to an error in the theoretical model which did not account for the flexible nature of the wooden design. The rigidity of the aluminium design resulted in better correlation between theoretical and experimental COP locations. The author defined an ‘area of percussion’ as the area in which the recoil impulse was within 1 Ns of the minimum recorded value. In the study, both junior and senior versions of the aluminium bat exhibited smaller areas of percussion than the wooden bats. The recoil impulses experienced in the handle were three to four times larger than the values found for the wood bats. The senior aluminium bats did however, produce greater rebound coefficients from three of the four zones. The authors suggested that there could be trade off between ball velocity and recoil impulse for the senior batsman using an aluminium bat where greater rebound velocities could be imparted at the expense of larger impulses experienced in the handle. The authors concluded by stating that the aluminium bats were more durable than wood bat but they did not appear to present impact characteristics conducive to a viable commercial product.

An area of interest in baseball concerns the performance of aluminium bats against those made from wood. In 1974 the National Collegiate Athletic Association (NCAA) legalised the use of the aluminium baseball bat. Since then, the majority of colleges have endorsed their use as opposed to the MLB who use wood bats exclusively. Bryant [93] investigated the opinions of leading college coaches of the time who held the belief that aluminium bats offered a larger ‘sweet spot’ and imparted greater ball velocity than their wood counterparts. This study employed six collegiate baseball players, using both bats, throughout a number of batting sessions against a pitching machine. Struck balls along a flight path consistent with a ‘clean’ shot - landing within a narrow section of the playing field and beyond a certain length, were designated for post impact ball speed measurement. The results demonstrated that each subject obtained a higher mean velocity with the aluminium bat and suggested a possible difference in bat mass properties. Subsequently, the COP was measured and a surrounding zone was defined in which similar hitting characteristics to the COP were attributed, known as the ‘effective’ COP. Both types of bat were supported at the handle and struck at different positions along the length of the bat, with a ball of constant linear momentum. The data from this experiment described a section along the length of the aluminium bat for which there was little resultant force at the handle. However, the wood baseball bats demonstrated a very limited ‘effective’ COP.

Noble [94] investigated the COP through empirical analysis of 17 collegiate baseball hitters, the study focused upon the position of the impact reaction axis. The COP was defined as the position along the handle where the summation of linear and angular translation due to impact reaction were zero. Noble suggested that a ball struck at the COP would result in maximum transfer of energy from bat to ball, maximising post impact velocity of the ball for a given collision. A camera mounted above the hitters was used to capture the impacts with an aluminium baseball bat. A number of impacts were selected upon the hitters’ subjective interpretation as to whether the ball hit the ‘sweet spot’. From the impact
images, key positions along the length of the bat were digitised, including those that defined the position of the hands on the bat. The position of the COP in the handle was calculated and compared to these key positions and it was found to coincide with the top hand rather than between the two hands as was originally expected. The author referred to Bryant [93] and suggested that the hitters were attempting to use the furthest point of the effective COP corresponding to an impact further towards the barrel end of the bat.

A study by Wood and Dawson [36] highlighted the limitations of modelling the cricket bat as a rigid body. The COP for a number of conventional bat designs were theoretically calculated from measurement of bat inertia properties. An experimental rig was used to impact bats at different locations along the striking region of the blade, whilst pivoting about the handle. The authors experimentally determined the COP by measuring the motion of the handle with a force transducer from discrete impact locations along the length of the blade. The authors used the root mean square (RMS) of the waveform over a period of 25 ms following impact to determine the percussion point on the handle. However, it is likely that these waveforms measured the modal excitation of the bat in addition to rigid body modes. It is reasonable to speculate that this contributed to the error which the authors observed between theoretical and experimental COP locations.

4.3 QUASI-STATIC ANALYSIS

In addition to the bulk oscillations excited upon impact, local deformation can be experimentally observed within both bat and ball. Gradual deterioration in the shape and integrity of both objects indicates that a degree of plasticity is involved. The Hertz contact law, originally used to describe the deforming nature of elastic spheres under static loading, is widely accepted to accurately predict impact parameters that can be experimentally verified during a sports ball and implement impact. It is however, limited to the immediate area of contact and cannot predict the stress history of areas removed from direct object interaction.

Hertz's theory is widely accepted to accurately predict the behaviour of sports ball impacts [95-98] and so a brief outline of the theory and its significance in this study is discussed. The theory concerns the mutual static compression of two elastic, homogeneous spherical objects. The model can be simplified by replacing one of the balls with an object of infinite mass and radius, to represent a flat immovable surface, such as the fixed steel plate used in testing. The theory states that the relationship between contact force $F$ and ball deformation $d$ is as follows:

$$d^{3/2} = \frac{3F(E_i + E_s)}{4r^{3/2}}$$

where $r$ is the radius of the ball and $E_i$ and $E_s$ are the elastic coefficients of the ball and the steel plate/second ball, respectively. Integrating Equation 4.31 gives an elastic potential energy function
which can be used to model the dynamic impact, based on the assumption that the sum of kinetic energy and elastic stored energy during the collision remains constant [97]. From this, maximum ball deformation can be expressed as function of the elastic coefficients, ball mass and radius, and the inbound ball velocity. The theory assumes that the elastic coefficients remain constant throughout impact. The relationship between deformation and velocity is as follows [97]:

\[ d \propto v_m^{4/5} \]  

[4.32]

Similarly, contact time can be related to inbound velocity [97]:

\[ t \propto v_m^{-1/5} \]  

[4.33]

Equations 4.32 and 4.33 are shown graphically in Figures 4.15 and 4.16 to illustrate the these relationships.

![Figure 4.15 - Hertz model of ball deformation vs. \( v_m \)](image)

![Figure 4.16 - Hertz model of contact time vs. \( v_m \)](image)

4.4 DYNAMIC ANALYSIS

The fact that long and thin striking implements vibrate following impact suggests that a portion of the impact energy available to the ball is translated into bat excitation modes. Higher inbound velocities lead to greater excitation of a flexible bat and make the limitation of rigid body analysis more apparent. In addition, the position of impact with respect to the nodes of these natural frequencies as well as the contact duration determines which frequencies are excited and the level to which they exist. Modelling the vibration characteristics of such striking implements therefore requires investigation into the detail of both contacting objects and not simply their pre-and post impact velocities.

4.4.1 Literature Review Concerning Non-FEA Dynamic Analysis of Sports Bats

Models of elastic bulk oscillations and local contact deformation can be used in conjunction to provide a more realistic predictive simulation of the hitting performance of rackets and bats. The following section reviews non-FEA based studies which have incorporated the propagation of internal forces and in some cases considered the energy absorbed through localised plastic deformation.
Brody [46] used both wooden and aluminium baseball bats in his study which aimed to verify that a hand held baseball bat behaves at impact as if it were a free body. This assumption facilitates the investigation of dynamic performance of baseball bats by negating the inclusion of grip variation in the analysis of results. The principle adopted to absolve the free body model, concerned the different vibration characteristics that can be observed between a bat whose handle is clamped and another whose ends are not constrained. Brody described unique modes of vibration dependent upon clamping condition, which he observed experimentally. The second phase of the experiment was conducted using a hand gripped bat. Modal analysis of consequent impacts illustrated the vibration similarities between a gripped bat and one that is unconstrained. The tightness of the grip appeared to have no effect upon the modal characteristics. A tight grip did however, increase the rate at which subsequent vibrations are damped out. Brody concluded therefore, that grip firmness should not influence the post impact velocity of the ball but speculated that grip is an important factor in terms of controlling the swing. As a result of the findings, Brody disagreed with work presented by Plagenhoef [92] and Griffing [99] who argued the concept of adding some fraction of the hand and arm mass to an effective striking mass to determine bat impact characteristics under playing conditions.

Studies in baseball by Van Zandt [100] and Nathan [45] and in cricket by Gutaj [37] have used beam deflection theory to model the transverse vibration of the bats. The bats were discretised into a number of parallel ‘slices’ of uniform thickness along the length of the bat. The relative displacement of contiguous slices due to the action of shear forces and bending moments were resolved to characterise the bat transverse mode shapes and frequencies. Van Zandt suggested that the lower modes of bat vibration were sensitive to changes in bat cross section and the shape of handle was predominant in modifying the node position and frequencies of these modes, since it is the least rigid component. Van Zandt suggested it may be possible to ‘tune’ the lower frequencies in order to return energy coupled with the lower modes of vibration to the ball during contact. The study by Nathan [45] characterised the modes of bat vibration and also the ball damping characteristics in a bat and ball impact model. A notable result was the delayed reaction of the handle such that the momentum transfer between bat and ball had been completed before the deformation impulse had reached the handle, agreeing with the study by Brody [46]. Nathan determined that the energy coupled with the first mode of bat vibration was predominant, determining the overall energy loss due to bat vibration, and that the outbound ball velocity is not significantly affected by modes higher than the second mode of vibration.

Brody [27] defined a baseball bat ‘sweet spot’ as the point of maximum COR and suggested this value and its location was related to bat flexural deformation. If the impact time is less than half the period of a bat vibration mode, then the energy coupled with this mode of vibration is not transferred to the ball during contact. Greater levels of deformation were associated with greater energy loss, resulting in lower values of COR. Brody modelled a baseball bat swing through ball impact and based on the principles of conservation of linear and angular momentum and the definition of COR, the bat impact location resulting in maximum ball velocity was determined. The resulting expression was independent
of the location of COP and was a function of the bat to ball mass ratio, the ratio of angular bat speed to linear ball speed and bat MOI.

A study by Knowles, Mather and Brooks [34] investigated cricket bat design through impact vibration modelling. Theoretical and experimental modal analysis was used to investigate the vibration characteristics of a conventional bat made from willow and a prototype bat made from glass reinforced plastic (GRP). A number of discrete locations on each bat face were impacted and an accelerometer placed at the tip of the bat measured impact excitation. A computer model of impact, representing the ball as a sphere, the bat blade as a beam, and the handle as a torsion spring was used to determine the bat deflection caused by impacts at nine locations along the length of the blade. The authors evaluated changes in flexural rigidity, handle stiffness and mass per unit length of the GRP bat model with respect to the output velocity of the ball. The authors suggested that a stiff and heavy bat would be capable of hitting the ball the greatest distance but accepted that in practice, the hit distance would depend upon the ability of the batsman to swing the bat effectively.

Fallon and Sherwood [101] investigated the effects of baseball bat barrel construction on performance and durability. The authors suggested that the baseball bat should be engineered to match a player’s strength and provide just enough durability to optimise bat performance. The study identified the handle as the more flexible bat component but stated that its design has a smaller effect on performance than the barrel. The study investigated the performance of both solid (wood) and hollow (corked wood, aluminium, titanium) baseball bats. The authors suggested that one of the key reasons for variations in bat performance was the amount of energy stored in the hollow barrel designs which was subsequently transferred to the ball. The study determined that for a solid bat only 2% of the deformation energy was stored in the bat and 98% of deformation energy was stored in the ball in contrast to a hollow aluminium bat which stored 10% of the deformation energy. In addition, the energy efficiency of the aluminium barrel was stated as being ~99% in comparison to wood which was approximated to be 71%.

Naruo [102] investigated the vibration characteristics of baseball bats in relation to the affect on ball COR. The study used hollow pipes made from fibre re-enforced plastic, each having different fibre alignment with respect to the longitudinal axis of the pipe. The fibre alignment modified the values of pipe flexural and circumferential stiffness which were measured prior to ball impact testing. The impact test results showed COR increased with higher flexural stiffness. This agreed with studies on baseball bats [38, 45] and cricket bats [35, 77] which suggested high flexural stiffness reduced the energy loss through flexural vibration. Lower circumferential pipe stiffness also resulted in higher COR and the author suggested that a lower circumferential stiffness resulted in greater pipe deformation and a more efficient transfer of energy. The experiment was repeated using four baseball bats, the circumferential stiffness of the bats was measured and a similar relationship was observed with the lowest circumferential stiffness corresponding to the highest values of COR.
4.4.2 Literature Review Concerning FEA of Sports Bats

The following section reviews studies that made use of FEA to simulate and predict the impact performance of cricket and baseball bats. Models have been developed to investigate the effects of impact velocity, bat material stiffness and bat geometry on bat vibration and COR.

Fallon and Sherwood [101] investigated the performance of solid and 'corked' baseball bats. The 'corked' bats were of laminate construction, with a wall thickness of approximately 20 mm and a central, circular cavity of 20 mm diameter, filled with cork. The cork insert is lighter than the hardwood it replaces and is used by some players to reduce bat mass and swing MOI. From experimental impact testing, the authors reported that the corked bats were damaged more easily from ball impact, but demonstrated a small increase in performance. Two-dimensional FEA models defining the cross section of a solid and a corked bat were developed. A static load was applied to the top surface of each cross section and the resulting deformation was simulated. The models predicted a greater level of hoop strain in the 'corked bats' resulting an a small increase in ball outbound velocity of ~0.9 mph in comparison to a solid wood bat.

Grant and Nixon [77] investigated the dynamic performance of cricket bats using an FEA model and suggested that improved performance may be possible if the frequency of one or more of the significant modes of bat vibration could be raised above the excitation spectrum of impact. The parametric model allowed for the manipulation of geometric design features currently employed in some commercial bat designs. Two vertical 'scoops' of variable length and thickness and the provision for up to three springs within the handle of variable thickness were included in the bat model. The bat design parameters were modified and the effect on the third mode of bat vibration was measured. The results indicated that bat design features such as scoops and perimeter weighting were ineffective at increasing the natural frequencies. However, removal of the handle spring components, an increase in handle thickness, and an increase in the ridge height of the blade increased the frequency of bat vibration. The authors were aware that by removing the spring components, the batsman might be exposed to painful hand sensation, but argued that if the bat were stiff enough, the amplitude of vibrations would be significantly reduced and the batsman would perceive less handle vibration as a result.

John and Li [103] measured and simulated the vibration characteristics of a cricket bat. Modal analysis was used to experimentally determine the first four flexure modes of bat vibration. The bat was rigidly clamped at the handle and the response of each mode was measured from seven impacts along the central, longitudinal axis of the blade. The results showed that the first three modes account for ~95% of the total energy coupled with bat vibration. The model evaluated the addition of fibre reinforced rubber layers into the cane handle and it was determined that handle stiffness increases significantly as do the first three modes of bat vibration. Therefore the energy coupled with bat vibration was reduced with the addition of handle reinforcement. The authors concluded that rubber strips were
present in the handle to increase damping, but they also reduced the stiffness of the handle resulting in a reduction in impact performance. The introduction of handle fibre re-enforcement reduced the energy coupled with bat vibration without the need to remove the rubber components.

4.5 CONCEPTS FOR IMPROVED PERFORMANCE

Three significant areas governing the performance of a cricket bat are apparent from the review of cricket and baseball bat research. They relate to the rigid body behaviour of the bat, the local deformation of the bat and ball, and the transverse vibration of the bat. This section illustrates how a hollow bat concept can address these areas and potentially improve bat impact performance.

4.5.1 Bat Rigid Body Behaviour and Distribution of Mass

The location of the blade conjugate percussions points for a grip at the handle is dependent upon bat mass, \( \bar{I}_m \), and the location of bat COM. These values can be modified for a hollow bat design without change to its external bat shape. The COP region or point is significant since it can potentially minimise the impulsive rigid body modes of the bat as perceived by the batsman, although it has been suggested that ball rebound is independent of COP. It is anticipated that its location relative to the nodes of transverse vibration and bat ICR will be significant in minimising handle rigid body motion for impacts locations which enhance ball rebound.

For a cricket bat swing, the linear velocity of the bat varies along the length of the implement, increasing with distance from the handle [14, 15]. Bat mass properties can be modified to reduce the swing MOI and increase the swing speed of the bat and the relative velocity between bat and ball at point of contact or be modified to increase the effective mass at point of contact to increase the outbound momentum of the ball. As suggested in studies [18, 23, 90] a compromise is normally sought between these two properties in order to enhance a batsman's effectiveness. A hollow design of equivalent mass to a solid design has an increased value of \( \bar{I} \) in all three principal axes, as the integral of the mass multiplied by its distance from the COM is increased. It is anticipated that this will increase in effective bat mass for eccentric bat impacts, as shown in Equation 4.10. Therefore for the same initial impact conditions and value of COR, there will be less angular momentum transferred to the bat during impact resulting in a higher ball rebound velocity for the hollow bat design. Provided the mass and location of COM remain constant between hollow and solid designs, it is anticipated that the value of \( md^2 \) (Equation 2.5) will remain constant and therefore the change in swing MOI will be minimal as demonstrated for tennis rackets [55]. A hollow design may therefore demonstrate an increase in effective mass without significant increase in swing MOI.

4.5.2 Bat Vibration and Flexural Rigidity

Baseball and cricket bat studies [27, 33, 34, 37, 45, 46, 100-104] have investigated bat vibration characteristics. The first three flexure modes were predominantly investigated as they account for the majority of energy coupled with bat vibration. Modification to bat shape and material, particularly in the handle, affected mode frequency and node positions. Studies [45, 101] also concluded that only a
fraction of the striking implement mass has a direct effect upon the rebound characteristics of the ball
and it is therefore likely that an extreme component such as the handle would have little effect upon
the ball velocity. Since this study is primarily concerned with bat impact performance relating to the
ball rebound velocity, the design of the blade is the main focus of the study. However, it is
acknowledged that the handle may have significant effect upon the sensations felt in the hands.

The flexural stiffness of a uniform cross section beam can be calculated as the product of material
stiffness and the section’s second moment of area $I_{sec\cdot y}$ as shown in Equation 4.34.

$$\text{Flexural Rigidity} = EI_{sec\cdot y} \quad [4.34]$$

The elastic modulus of wood was discussed in chapter 3 and a general rule was described, whereby the
mechanical stiffness and strength properties in a particular direction could be attributed to the density
of the wood species. To ensure a similar blade mass, a more dense material must be used in a hollow
blade and it is anticipated that a wood can be selected for which the value of $E_L$ is greater than for
willow.

Since the hollow blade has a reduced cross section area for the same outside shape as a solid bat, the
value of $I_{sec\cdot y}$ for a hollow blade will be lower than for a solid blade. In order to minimise the reduction
in $I_{sec\cdot y}$ the volume of material in the hollow design must be positioned so as to maximise the blade’s
section moment of area $I_{sec\cdot y}$. This is achieved by positioning material at the furthest distance from the
neutral axis of the blade’s section.

Table 4.2 illustrates an example of the change in flexural rigidity by considering an identical section
through a solid blade made from willow and a hollow blade made from birch.

<table>
<thead>
<tr>
<th>Wood species: English willow</th>
<th>Wood species: European Birch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section area: 46 cm$^2$</td>
<td>Section area: 19 cm$^2$</td>
</tr>
<tr>
<td>$I_{sec\cdot y}$: 96 cm$^4$</td>
<td>$I_{sec\cdot y}$: 76 cm$^4$</td>
</tr>
<tr>
<td>Stiffness $E_L$: 6600 MPa</td>
<td>Stiffness $E_L$: 15300 MPa</td>
</tr>
<tr>
<td>Flexural rigidity $EI_y$: 6.35e+3 Nm$^2$</td>
<td>Flexural rigidity $EI_y$: 11.6e+3 Nm$^2$</td>
</tr>
</tbody>
</table>

Table 4.2 - Comparison of sectional properties between a solid and hollow bat
A comparison illustrates that if the material chosen for the hollow design is sufficiently stiffer than willow, the flexural rigidity can also be higher despite a decrease in the value of $I_{secy}$. In this example, birch with $E_y = 15300$ MPa compensates for the loss of internal material and provides a section 80% more rigid in accordance with the design intent of lowering vibration amplitude.

4.5.3 Bat and Ball Local Impact Deformation and Damping

Damping due to bat and ball deformation in the region local to impact also has a significant effect on ball rebound as determined in studies [39, 45, 105]. The studies suggest that in contrast to increasing the flexural rigidity of the blade, the stiffness of the impacting face local to the point of contact can be decreased to store and return energy. The compressive stiffness of a cricket ball ~90 MPa is approximately 2.5 times less than the quoted stiffness of willow in the direction parallel to impact [35]. It is anticipated that a lower bat face stiffness would enhance ball rebound provided that the level of face damping is less than that for the ball. This concept was presented in a patent [44] which described how a 'trampoline' effect on the striking face of a sports bat improved the efficiency of the change in ball momentum. It was mentioned that the transverse flexural rigidity of the bat could be tuned to return bat vibration energy to the ball. However, for different impact locations along the length of the bat, the vibration and deflection characteristics would differ resulting in a 'trampoline' effect significantly dependent upon the location of impact. It was argued that reducing the flexural rigidity of the front face was a more effective method for improving the elasticity of impact since consistent face flexural properties could be maintained along the length of the blade.

A 'trampoline' effect can be realised by having a hollow cross section that allows for the front face of the bat to deflect inwards. A cross section through a hollow bat would illustrate a striking face that is only supported at its edges allowing for this mode of deformation. As described by the patent [44] the face can be 'tuned' so that the strain energy stored is returned to the ball whilst it remains in contact with the bat. However, it is anticipated that the enhancement in ball rebound due to the 'trampoline' effect will be reduced if the transverse flexural rigidity of the bat is compromised by lowering the stiffness of the striking face.

4.5.4 Rules and Product Awareness

A notable feature of the MCC rules governing bat design, in Appendix 2, is its use of design constraints and not limits on bat performance. The maximum length and width of the blade are defined and additionally the rule states that 'the blade of the bat shall solely be made of wood'. Adhesive is often used in the repair of commercially available blades, particularly in the 'toe' and 'edge' regions and is used to bond the handle to the blade during manufacture. The use of adhesive is therefore present in the manufacture and repair of commercially available bats. It is argued that the adhesive between laminate layers of a multilayer hollow bat only provides a bond between contiguous layers of wood and does not directly increase the post impact velocity of the ball.
The design approach was to accommodate a hollow technology within a bat whose external shape and form did not deviate radically from current, accepted designs. A traditional 'mindset' prevails in the game and it was perceived that a familiar bat shape would be accepted more readily by players, in contrast to a bat which bore little resemblance to a conventional bat design.

If the use of hollow technology was permitted within the game, then the external blade design could be reviewed as a further development. However, it was anticipated that a hollow bat with a familiar external size and shape would demonstrate modified impact behaviour.
CHAPTER 5

BAT CAD MODELS AND BAT MASS PROPERTIES

5.1 INTRODUCTION

CAD modelling was initially used to assess a hollow bat design by investigating the possible change in bat mass properties. The bat models developed were also useful in many areas of this study including the CNC machining of prototype cricket bats and FEA modelling of bat impacts. This chapter discusses the process used to generate an idealised CAD representation of a solid bat and the development of a hollow cavity bat model. The mass properties of both CAD models and manufactured prototype bats are investigated and the potential improvement in performance is discussed.

5.2 CAD MODELLING TECHNIQUES

Engineering components that are produced with standard machining processes such as milling and turning often incorporate geometric primitives (cylinders, cubes) to facilitate ease of manufacture. Smooth blends between features can exist but are secondary in defining the overall form of the component. In contrast, a sculptured product can be defined as that in which there are no obvious geometric primitives that form the predominant features of the product, Mitchell [106]. The golf club iron is an example of a sculptured product, there are no predominant prismatic geometries, but instead, the smooth surface curvatures define the overall form of the product and there are few obvious surface discontinuities that define distinct features. Figure 5.1 illustrates the golf club and an example of a machined component with primitive features. The body of the iron is cast or forged during manufacture and therefore little consideration needs to be given to facilitate standard machining practices in its design. Consideration is given to process limitations such as draft angles and wall thickness but greater freedom to pursue aesthetic and performance considerations remain as a result of this method of manufacture. This culminates in the use of larger blend radii and less surface discontinuities.

The design intent of the bat craftsmen and the hand and machine tools used to realise the design give rise to a distinctive and recognisable cricket bat shape. The use of hand tools in fashioning the majority of the surface results in a sculptured form. The handle top, whose shape was turned to form an cylindrical feature, is the only surface which satisfies a simple geometric definition. Therefore the techniques used in generating a computer representation of the bat are those used to model sculptured products.
5.3 CAD MODEL OF CONVENTIONAL BAT

5.3.1 Modelling Strategy

The Unigraphics NX™ computer aided design application was used to generate the cricket bat model. Since no predominant geometric features were present from which to develop the bat shape, the modelling strategy relied upon generating an accurate surface representation of the bat defining an enclosed volume.

Mitchell [106] used a modelling technique known as extended form feature modelling (EFFM) in order to accurately represent the sculptured form of a golf club iron and a shoe last, using spatial co-ordinate data from a coordinate measurement machine (CMM). This technique requires that the entire surface of the product be divided into discrete regions known as features, which are then categorised into a hierarchy concerned with the feature's influence on the overall shape of the product. Visible surface discontinuities may define the boundary of a feature, for example, a golf club iron face is defined by small radius edges. Intended design features of the product may also provide convenient boundaries. with the golf club iron example the 'face' feature defined by the radius edge boundary coincides with the boundary that defines the 'face' as a recognised design feature. Specific manufacturing processes limited to a region of the product may also provide suitable feature boundaries such as a particular surface finish over a region not defined by surface blends.

The cricket bat exhibits a number of discrete regions that are generally accepted as specific design features by both player and manufacturer alike. The boundaries to the majority of these designs features also coincide with a number of surface discontinuities and some are also defined by separate machining or craft processes. Figure 5.2 illustrates these predominant surface regions, these are regarded as the 'primary' features as they govern the overall shape and size of the bat. The secondary features encompass smaller surface regions that often exist at the boundaries of primary features or where two or more primary features intersect and often define the edges and corners of an object in the form of a blend (radius or fillet). The regions do not significantly influence the overall size and shape of the object but can impact upon the object's perceived 'smoothness'. Figure 5.3 illustrates the regions on the bat that have been defined as secondary features. Tertiary features are defined as ornamental features with little or no structural form and may include product graphics or a surface finish which satisfies the aesthetic objectives of the product without influencing the shape of the object.
For example, cricket bats will generally include a number of graphics to identify the bat manufacturer and bat model.

Blend devolution [106] can be used to establish the order in which the blends should be generated. The first generation of secondary features are formed from the intersection between two primary features only. Second generation blends are formed at the intersection between a primary feature and a first generation blend and so on. The sanding process during manufacture is responsible for creating the majority of secondary features, and a close inspection of the procedure reveals the order of each blend which can assist in structuring their creation within the model.

![Diagram showing the primary features of a cricket bat model](image)

**Figure 5.2 - Primary features of the cricket bat model**
5.3.2 Modelling Limitations

In defining the primary and secondary features, consideration was given to the number of surface features that would result. Efficiently representing the entire surface requires a compromise between the number of features and their complexity. For instance, an object can be described by a few complex surfaces or by many surfaces that are less complex. In addition, the real cricket bat is an assembly of components, although the conventional blade is made from a single piece of willow, the front face often includes a protective plastic film and graphics films that are adhered to the surface. The handle is an assembly of components adhered together which is then wrapped with string and covered with a rubber grip. The CMM only provides surface position and although each component could be digitised separately it was deemed sufficient to model the handle components as one part without string and rubber grip as these could be added separately if required at a further stage. The blade was modelled without protective covering film or graphics as tertiary features do not significantly
affect bat mass properties. Likewise, surface topology of the blade and handle were considered unnecessary for this study.

The ability to accurately reproduce a model of the real bat surface is first limited by the method used to measure its geometry. The only absolute certainty when creating an accurate surface from a point array is that the surface should intersect those points. The CAD surface generated from this information may therefore only be exact in terms of the location of its surface at these discrete positions. Since there is no real information to adhere to in the spaces between points, the software uses polynomial expressions to define smooth transitions between points within the array. The user may specify the degree of the polynomial expressions used in generating surfaces which may influence the accuracy of the surface.

The cricket bat is hand crafted or finished and although symmetry appears to exist through the centreline of the blade and handle, subtle differences in shape and size of features intended to be identical can occur due to human error or preference. Differences in features also exist between bats intended to be identical for the same reason. These differences may be captured as spatial coordinates and translated into the model. Discrepancies can be minimised by modelling an idealised representation of the bat. In contrast to an accurate representation of the bat which inherits all features that are digitised, the idealised representation takes into account the craftsmen’s design intent for each feature. A study of the Dunlop Slazenger International (DSI) manufacturing process, in Appendix 1, was used to filter intentional features from those that should be overlooked in generating the model. The general principle in creating the predominate features was to ensure that surfaces were free from discontinuities except at feature boundaries. In addition, a fundamental manufacturing intention was to create symmetry through the \( yz \) plane of the bat. Therefore, only one side of the bat was modelled and the features were copied and mirrored on completion. Figure 5.4 illustrates the cricket bat plane of symmetry and the orientation of each axis and plane described in this section.
5.3.3 Measurement of Bat Geometry

A commercially available cricket bat with a conventional shape was chosen to provide a set of geometric data used to develop a benchmark CAD model. A CMM was used to digitise the bat surface using a small probe which halts and records spatial co-ordinate data upon sensing contact with the bat surface. As a surface can be described with an infinitesimal number of points, a strategy was required for generating a comprehensive, but efficient set of data.

The length of the bat exceeded the $x$ and $y$ axes travel of the CMM but it could be accommodated diagonally across the machine bed. Mounting the bat diagonally on the machine bed facilitated the complete digitising of all primary features without the need to re-position the bat during the procedure or to manipulate the data. Operating the machine diagonally however, eliminated the ability to use an automatic procedure to measure discrete points along a fixed machine axis. Therefore the probe was manually positioned for each designated point on the surface of the bat. This procedure required that the bat be marked with grid lines whose intersections formed the array of points required. An attempt was made to align one axis of the grid array with an edge or symmetry line of a predominant feature. Since the exact spatial location of each point was recorded by the machine, the accuracy of the bat markings only governed the relative spacing of each point. Therefore, the accuracy of the array
markings and the manual alignment of the probe did not diminish the quality of surface measurement. Figure 5.5 shows an image of the bat mounted on the CMM.

The number of points necessary to define each surface feature was dependent upon the complexity of the surface and took into consideration the number of surface inflexions that existed through the yz and xz planes. Figure 5.6 illustrates how a minimum of three points could be used to efficiently describe a curve of uniform radius. Increasing the number of points would not be improve the representation of the curve and using a large number of points to describe the curve could lead to the appearance of inflexions or ripples that do not appear to exist on the original curve. This is due to inaccuracies in the spatial measurement of the points as shown in Figure 5.7. In contrast, too few points can lead to the simplification of the order of curvature or complete non-detection of surface inflexions or features as shown in Figure 5.8.

In addition to the CMM, a radius gauge allowed for simple measurement of the secondary blend features. Most blend dimensions were one to two orders of magnitude less than the primary feature dimensions and so the accuracy afforded by a radius gauge was considered sufficient for these features.
5.3.4 Surface Generation

The initial stage in modelling the primary surface features was to import the spatial coordinate data as a point cloud into the CAD environment. Each set of points defining a primary surface were considered in turn. The points were used to create an initial surface using the Unigraphics 'point cloud' function purposely designed to generate a smooth surface representation of spatial co-ordinates. The tolerance of the surface position with respect to the coordinate data was dependent upon the polynomial degree and number of 'patches'. In accordance with EFFM procedure, the boundary of this surface was increased to ensure adequate overlap of mating primary features. Using this method of surface generation an overall impression of the surface quality and feature intersections could be assessed using curvature analysis tools.

In order to achieve greater control over the shape of the back face surface and avoid unwanted ripples, the point cloud surface was used as a construction tool to create a swept surface feature. Nine datum planes provided planar intersections at regular intervals along the surface. Four parallel lines in the $y$ direction were generated and projected onto the point cloud surface. The intersection points between the projected lines and datum planes where used to generate third order, two-dimensional splines. The characteristics of each spline could be controlled by the positioning of its two endpoints and two midpoints. The nine section splines characterised the changes in surface section along the length of the bat and the endpoints of each spline were joined and used as guides to assist surface generation. In
summary, a total of 36 points controlled the characteristics of the swept surface feature in contrast to 60 point cloud data points. This allowed for greater ease of surface manipulation and generated a smoother surface feature with less rippling and discontinuity as illustrated in Figure 5.9.

The front face of the bat was idealised with a constant curvature along the length of the blade. Section splines were created in similar fashion to the back face and were averaged to produce a single spline which was linearly extruded along the length of the blade as shown in Figure 5.10. The toe surface was extruded using a similar procedure to that of the back face as shown in Figure 5.11.
Figure 5.10 - Creation of front face feature using one averaged spline, controlled by three points

Figure 5.11 - Creation of toe face feature using three sections splines, each spline controlled by three points
The shoulder and handle features were individually generated in similar fashion, using a point cloud surface as a construction tool to develop a swept surface geometry as shown in Figures 5.12 and 5.13, respectively. The central back face guide spline was extended to incorporate both the shoulder and handle features to facilitate a continuous and smooth edge along the centreline of the bat.

Overlapping surfaces were trimmed along their intersections to form unblended sharp edges and corners between mating primary features. A Boolean operation united the sheet bodies to form a continuous and enclosed surface and a solid body was automatically identified within the enclosed volume. The solid ‘half’ bat was then mirrored along the central yz plane and united to form a complete solid bat model.

Single radius fillets were applied to the face-edge, face-toe and handle side-top, surface intersections in accordance with the values measured using the radius gauge. A variable radius was generated between the two back faces to accommodate the variable surface intersection angle and was adjusted along the length of the bat in order to maintain a realistic blend width.

Finally, the handle was separated from the model using the correct ‘splice’ geometry and the blade and handle were defined as discrete parts.

![Diagram of cricket bat with labeled sections](image-url)

**Figure 5.12 - Creation of swept shoulder feature using two sections splines and three guide splines**
5.3.5 Accuracy of Model

**Dimensional Accuracy**

The co-ordinate data had been used as a construction tool to derive geometry with an emphasis towards surfaces which were smooth and free from undesirable changes in curvature. This resulted in a spatial discrepancy between original co-ordinate data and the final surfaces.

The tolerance of each surface to the CMM data points was measured using a deviation analysis tool provided by Unigraphics. Figure 5.14 illustrates the analysis plot of the back face surface. The analysis lines pass through the data points and are normal to the surface, their length provides a visual indication of the relative deviation between each point and the surface. The maximum deviation resulting from this analysis over the entire surface of the bat from the original CMM data was observed on the back face at 0.726 mm, as illustrated in Figure 5.14.

The volume of the real cricket bat was physically measured using apparatus which captured the volume of water displaced by the complete submersion of the bat into a water bath and was measured to be 2533 cm$^3$. The volume of the CAD model was calculated to within 1 mm$^3$ using the analysis tools provided by the software to be 2523 cm$^3$ resulting in an error 0.4% volume.
Mass Properties

The COM and MOI for the real bat were measured as part of the cricket bat grading study detailed in Chapter 4. Table 5.1 compares the mass characteristics of the real bat and the computer model.

<table>
<thead>
<tr>
<th>Centre of Mass From Handle Top</th>
<th>Principal Moments of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y$ (mm)</td>
</tr>
<tr>
<td>Real</td>
<td>497</td>
</tr>
<tr>
<td>Model</td>
<td>503</td>
</tr>
<tr>
<td>Error</td>
<td>6 mm</td>
</tr>
</tbody>
</table>

Table 5.1 - A comparison of mass properties between the willow bat and the solid computer model.

The mass properties of the CAD model were determined using homogeneous definitions for the blade and handle. Therefore, error in the $z$ location of bat COM and the values of polar MOI could be attributed to the densification of the bat striking face during manufacture. A non-uniform density through the cross section of the real bat would have the effect of moving the COM towards the front face and could increase the value of polar MOI.
5.4 HOLLOW CAVITY BAT CAD MODEL

5.4.1 Design Hypothesis

The prototype 1 bat was developed with the aim of improving upon the mass properties of a conventional, solid bat and CAD software was the predominant tool used. The findings of numerous studies concerning cricket bat, baseball bat, and tennis racket design, discussed in chapter 4, suggest that the design intent should be to maximise polar inertia whilst a more considered approach should be taken to modifying other properties that directly affect the perceived effort in swinging the bat. Therefore, the intent was to modify the values of swing inertia about the COM without significantly affecting the total mass and position of COM of the bat. This approach could then establish improvements to playing properties without fundamental change to the batsman’s perception of mass characteristics in wielding the bat. It was intended that a hollow bat would ‘feel’ like a commercially available bat, with similar mass, first moment and swing inertia about the handle grip. No alteration of playing technique would therefore be required in order for the player to benefit from mass related improvements.

5.4.2 Geometry

A parametric hollow geometry was created within the blade of the original bat model. Wall thicknesses were altered within various regions of the blade and the effect on mass properties assessed using analysis tools within the software. Birch ply was the chosen material for the prototype 1 bat and the density of this material (~820 kg/m³) is approximately twice that of willow (~420 kg/m³). A number of objectives were therefore established as follows:

1. To remove approximately half the volume to retain similar total mass.
2. To minimising the disturbance to the position of COM.
3. To increase the principal MOI about the COM in order to improve performance.
4. To accommodate the handle by including a wall thickness within the blade around the handle interface.

The hollow design describes a uniform wall thickness within each primary region such as the ‘toe’ and front face feature but values across primary regions vary in order to fulfil the objectives. Secondary blend features were not added to the internal wall surfaces since the laser cut section profiles would not represent such features. Figure 5.15 illustrates the final hollow blade geometry with significant dimensions labelled.
5.5 COMPARISON OF SOLID AND HOLLOW MASS DISTRIBUTION

5.5.1 Section and Mass Properties Along the Bat $y$ Axis

To gain a detailed understanding of the difference in mass properties between solid and hollow bat designs, both solid and hollow models were identically 'sliced' into multiple sections along their entire length as illustrated in Figure 5.16. Mass characteristics for each 'slice' were calculated and plotted with respect to their distance from the end of the handle in order to illustrate their contribution to the total value of mass property in question. A slice thickness of 10 mm was arbitrarily chosen to provide a satisfactory resolution in observing the change in property distribution along the length of the bat. The absolute values on each $y$ axis are therefore not as significant as the relative difference in values between the two models.
Figure 5.17 shows the distribution of mass along the length of the bat, the areas underneath the curves are equal both models have the same mass. The prototype bat illustrates two regions of relatively high mass. Firstly, at a distance of about 300 mm from the end of the handle where only a small volume of wood was removed from the blade section. The second peak at about 800 mm from the handle end occurs where the hollow section ends towards the toe of the bat. Since the laminate birch wood is denser than willow, the mass is greater in these regions. The solid bat shows an increase in slice mass from 400 mm - 700 mm as the blade section increases in depth, but the hollow bat, however, does not demonstrate this increase. In the hollow design, the wall thickness remains constant along each primary feature of the blade. Therefore an increase in blade depth along the bat does not result in an increase in section area and mass, as observed for the solid bat. It is important to note therefore, that the mass distribution of a hollow bat with constant wall thickness is not as sensitive to changes in blade depth. Figure 5.18 demonstrates this concept using simple solid and hollow prismatic shape sections. The polar MOI of each bat slice, in Figure 5.19, highlights the improvement of the hollow design. The increase in the value of polar inertia as the profile increases in thickness is reflected in both solid and hollow designs, the two peaks apparent on the hollow bat curve are associated with greater cross section area that exists at both ends of the blade. The MOI distribution, shown in Figure 5.20, is calculated about the bat COM resulting in a minimum slice MOI for both bats at about 500 mm from the handle end, equivalent to the position of COM. The two regions with a higher sectional area in the hollow design account for the relative increase in observed values. Figure 5.21 shows the change in second moment of area along the length of the bat, the hollow section exhibits lower values within the blade region in comparison to the solid design with a largest difference of 33% at a position equivalent to the thickest part of the blade.

Figure 5.16 - Hollow and solid bats sliced into multiple sections to characterise mass property distribution
Figure 5.17 - Mass of 10 mm thick slices along length of bat

Figure 5.18 - For a similar increase in depth, the hollow section demonstrates a lower relative increase in section area

Figure 5.19 - Values of $f$ for 10 mm thick sections along length of the bat
5.5.2 Mass Property Values

Table 5.2 presents the mass properties of the solid and hollow bat CAD models and those of the real willow bat and hollow prototype bats. Prototype bat mass properties were measured subsequent to their manufacture using the MOI instruments detailed in chapter 3. MOI is dependent upon the total mass and its distribution, since the mass of the willow and prototype bats vary, the radius of gyration, $r_g$ shown in Equation 4.11, provides an alternative measure for comparing bat mass distribution. The percussion (PC) region refers to the range of conjugate points on the striking face corresponding to points from the top to the bottom of the handle. Values of Swing MOI, first moment, and the location of the COM were determined relative to the midpoint of the handle. Table 5.3, illustrates the percentage difference between the solid and hollow CAD models and the percentage difference between the real willow bat and the hollow prototypes.
<table>
<thead>
<tr>
<th>Mass Property</th>
<th>CAD Solid Bat</th>
<th>CAD Hollow Bat</th>
<th>Real Solid Bat</th>
<th>Prototype 1 Bat</th>
<th>Prototype 2-1 Bat</th>
<th>Prototype 2-2 Bat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade Density (kg/m³)</td>
<td>420</td>
<td>820</td>
<td>420</td>
<td>820</td>
<td>814</td>
<td>814</td>
</tr>
<tr>
<td>Bat Material Volume (mm³)</td>
<td>252 x 10⁴</td>
<td>147 x 10⁴</td>
<td>253 x 10⁴</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bat Mass (g)</td>
<td>1142</td>
<td>1152</td>
<td>1147</td>
<td>1373</td>
<td>1333</td>
<td>1545</td>
</tr>
<tr>
<td>COM x (mm)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>COM y (mm)</td>
<td>356</td>
<td>-353</td>
<td>350</td>
<td>352</td>
<td>336</td>
<td>347</td>
</tr>
<tr>
<td>COM z (mm)</td>
<td>-6</td>
<td>-3</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>First Moment xx (Nm)</td>
<td>4.0 x 10³</td>
<td>4.0 x 10³</td>
<td>3.9 x 10³</td>
<td>4.7 x 10³</td>
<td>4.4 x 10³</td>
<td>5.3 x 10³</td>
</tr>
<tr>
<td>Swing tₓ (kgmm²)</td>
<td>1.9 x 10⁶</td>
<td>2.0 x 10⁶</td>
<td>1.9 x 10⁶</td>
<td>2.4 x 10⁵</td>
<td>2.1 x 10⁵</td>
<td>2.6 x 10⁵</td>
</tr>
<tr>
<td>( \bar{t}_{xx} ) (kgmm²)</td>
<td>5.13 x 10⁴</td>
<td>5.7 x 10⁴</td>
<td>5.1 x 10⁴</td>
<td>6.9 x 10⁴</td>
<td>6.3 x 10⁴</td>
<td>7.4 x 10⁴</td>
</tr>
<tr>
<td>( \bar{t}_{yy} ) (kgmm²)</td>
<td>8.5 x 10²</td>
<td>11.1 x 10²</td>
<td>9.6 x 10²</td>
<td>13 x 10²</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \bar{t}_{zz} ) (kgmm²)</td>
<td>5.18 x 10⁴</td>
<td>5.80 x 10⁴</td>
<td>5.0 x 10⁴</td>
<td>7 x 10⁴</td>
<td>6.3 x 10⁴</td>
<td>7.4 x 10⁴</td>
</tr>
<tr>
<td>( \bar{t}_{xy} ) (mm)</td>
<td>211</td>
<td>222</td>
<td>211</td>
<td>224</td>
<td>217</td>
<td>219</td>
</tr>
<tr>
<td>( \bar{t}_{yx} ) (mm)</td>
<td>27</td>
<td>31</td>
<td>29</td>
<td>31</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Swing ( \bar{t}_{xy} ) (mm)</td>
<td>414</td>
<td>417</td>
<td>409</td>
<td>418</td>
<td>397</td>
<td>410</td>
</tr>
<tr>
<td>Face PC region Corresponding to All Points in the Handle (mm)</td>
<td>445 - 553</td>
<td>452 - 573</td>
<td>441 - 553</td>
<td>449-575</td>
<td>431-563</td>
<td>442-565</td>
</tr>
</tbody>
</table>

Table 5.2 - A Comparison of CAD solid and hollow bat mass properties
<table>
<thead>
<tr>
<th>Mass Property</th>
<th>CAD Solid Bat → CAD Hollow Bat % Difference</th>
<th>Real Solid bat → Prototype 1 % Difference</th>
<th>Real Solid Bat → Prototype 2-1 % Difference</th>
<th>Real Solid Bat → Prototype 2-2 % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade Density (kg/m³)</td>
<td>+95%</td>
<td>+95%</td>
<td>+94%</td>
<td>+94%</td>
</tr>
<tr>
<td>Bat Material Volume (mm³)</td>
<td>-41%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bat Mass (g)</td>
<td>+1%</td>
<td>+20%</td>
<td>+16%</td>
<td>+35%</td>
</tr>
<tr>
<td>COM x (mm)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>COM y (mm)</td>
<td>3 mm</td>
<td>2 mm</td>
<td>-14 mm</td>
<td>-3 mm</td>
</tr>
<tr>
<td>COM z (mm)</td>
<td>1 mm</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>First Momentₓₓ (Nmm)</td>
<td>0 mm</td>
<td>+21%</td>
<td>+13%</td>
<td>+36%</td>
</tr>
<tr>
<td>Swing 𝐼ₓₓ (kgmm²)</td>
<td>+5%</td>
<td>+26%</td>
<td>+11%</td>
<td>+37%</td>
</tr>
<tr>
<td>𝐼ᵧᵧ (kgmm²)</td>
<td>+11%</td>
<td>+35%</td>
<td>+24%</td>
<td>45%</td>
</tr>
<tr>
<td>𝐼ₓᵧ (kgmm²)</td>
<td>+31%</td>
<td>+35%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>𝐼ᵧₓ (kgmm²)</td>
<td>+12%</td>
<td>+40%</td>
<td>26%</td>
<td>+48%</td>
</tr>
<tr>
<td>𝐿ᵧᵧ (mm)</td>
<td>+5%</td>
<td>+6%</td>
<td>+3%</td>
<td>+4%</td>
</tr>
<tr>
<td>𝐿ₓᵧ (mm)</td>
<td>+15%</td>
<td>+7%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Swing 𝐿ₓᵧ (mm)</td>
<td>+1%</td>
<td>+2%</td>
<td>-3%</td>
<td>0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PC region Corresponding to All Points in the Handle (mm)</th>
<th>PC region +12% larger and midpoint 14 mm further from handle</th>
<th>PC region +13% larger and midpoint 15 mm further from handle</th>
<th>PC region +18% larger and midpoint 0 mm further from handle</th>
<th>PC region +10% larger and midpoint 6.5 mm further from handle</th>
</tr>
</thead>
</table>

Table 5.3 - A comparison of real solid and hollow bat mass properties

5.6 PREDICTED CHANGES IN BAT PERFORMANCE

The mass and section properties of solid and hollow bats designs are used to provide a first indication of the effect of mass re-distribution on bat performance.

5.6.1 Player Perception

Bat mass, first moment, swing MOI are equivalent in both solid and hollow CAD designs. This suggests that both the static and dynamic mass characteristics perceived by the batsman in wielding bat and playing a shot will be similar for both models. The manufactured prototype bats are 20-35% heavier and therefore the values of first moment and swing MOI increase approximately by the same amount. However, the position of bat COM y and the values of swing radius of gyration 𝐿ₓᵧ are approximately equivalent for solid and prototype bats suggesting that a prototype bat of similar mass would exhibit similar values of first moment and swing MOI, as observed for the CAD models.

5.6.2 Centres of Percussion

Table 5.3 illustrates that the PC regions for the CAD hollow bat and prototype bats are 12-18 % larger than for the solid bats. It is anticipated that the increase will be beneficial to the batsman since this translates to a larger striking zone for which there exists conjugate points in the handle. The midpoints
of the hollow PC regions are a maximum of 15 mm further from the handle in comparison to the solid CAD model and the real bat. It is anticipated that this will be beneficial to the batsman since the rotational component observed in cricket shots, such as the 'on drive' and 'off drive' [15, 107] result in a variation of bat linear velocity along the striking region, increasing with distance from the handle. Therefore a desirable region of impact in terms of minimising rigid body handle motion will also demonstrate higher relative impact velocities.

5.6.3 Effective Mass

A comparison between solid and hollow bat CAD models illustrates that for approximately the same bat mass, the values of \( I_{xw} \) and \( I_{yw} \), defined as being significant for eccentric bat face impacts, are greater by 11% and 31%, respectively, for the hollow CAD design. Since MOI is a function of total mass, the same comparison cannot be made between the real solid bat and the prototype bats, since their values of total mass are different. The radius of gyration provides a more meaningful comparison as its units are independent of mass. Equation 4.12 illustrates that \( r_x \) has a greater effect on the value \( m \), as the distance of impact increases from the COM. Values of \( r_y \) are 5% (CAD) and 3-6% (real) greater for the hollow bats about the \( x \) axis - concerning impacts along the length of the bat, and 15 % (CAD) and 7 % (real) greater in the \( y \) axis - concerning impacts across the width of the face. Figure 5.22 illustrates the change in \( m/m_{bat} \) for impacts along the bat face, equivalent to the bat \( x \) axis, and for an impact at the toe region of the bat, values of \( m/m_{bat} \) are 7% (CAD) and 9% (real) greater for the hollow bats. Figure 5.23 illustrates values of \( m/m_{bat} \) for impacts across the width of the bat face, equivalent to the bat \( y \) axis, and for an impact at the edge of the bat, the values of \( m/m_{bat} \) are 20% (CAD) and 9% (real) greater for the hollow bats. The increase in \( m/m_{bat} \) for the hollow bat design suggests that more of the rigid bat mass will contribute to the transfer of ball momentum for impacts away from the COM, resulting in a greater change in ball velocity for the same initial conditions and value of \( e \).

It is likely that differences between CAD model and real bat comparisons are due to changes in material density for the real bats which would affect bat mass distribution, particularly for the solid bat, in which the top layer of the striking face is compressed during manufacture.

5.6.4 Flexural Rigidity

Values of \( I_{sec} \) were used to calculate flexural rigidity as a function of distance along the bat to indicate the propensity of the bat to undergo flexural deformation from impact. In order to maintain a flexural rigidity that is equivalent or higher than the solid willow design, a wood with an \( E_L \sim 30\% \) greater than willow is required to compensate for the reduction in the hollow design second moment of area. The initial hollow prototypes (prototype 1) were manufactured from birch with a published value of \( E_L \sim 15 \) GPa [68] and this compares favourably to the value for cricket willow measured in this study at about 9 GPa. However, the birch wood was constructed as a ply laminate in which the fibre angle of contiguous layers was mutually perpendicular. This construction reduced the maximum stiffness of the material which was measured to be \( \sim 6.5 \) GPa. This had the effect of further reducing the flexural
rigidity of the blade and offers a reason for the reduction in measured performance of these initial prototypes, discussed in chapter 10. Figure 5.24 illustrates the low flexural rigidity of the prototype 1 blade section, due to the combination of reduced second moment of area and material stiffness. The dashed curve is an estimation of the flexural rigidity of the next generation of hollow bat (prototype 2), built from ash wood with a published longitudinal stiffness of 13 GPa [68]. The majority of the blade was constructed with the fibre orientation of the wood aligned with the y axis of the bat and as a consequence, the anticipated flexural rigidity of the prototype 2 bats are comparable with the conventional solid design made from willow.

![Figure 5.22 - Values of m_y/m_bat for an impacts equivalent to the bat y axis](image)

![Figure 5.23 - Values of m_x/m_bat for impacts equivalent to the bat x axis](image)
Figure 5.24 - The second moment of area of bat cross section along the length of the bat.
CHAPTER 6

SPRING DAMPER IMPACT MODELS

6.1 INTRODUCTION

In this chapter a mathematical modelling procedure widely used to characterise the vibration of engineering designs has been employed to investigate how properties of the striking face of a hollow cricket bat affect the rebound velocity of the ball.

A vibrating system undergoes a transformation of kinetic energy to stored potential energy in a cyclic pattern describing an oscillatory motion. Similarly, a percentage of the kinetic energy of the bat and ball prior to impact will undergo a transformation into stored potential strain energy within the material of both objects during impact and a portion of this energy is then converted back into the kinetic energy of both objects after impact.

Figure 6.1 illustrates the displacement time characteristic of an underdamped vibrating system. The attenuation in amplitude of the signal indicates that energy is being lost within the system, due to the presence of a damping component. Despite this energy loss, a number of oscillations are present as the damping component is not sufficiently large to reduce the displacement time characteristic to aperiodic motion, as observed in an overdamped system.

Figure 6.2 illustrates a representation of the time displacement characteristic observed during a ball impact. This type of impact can be successfully described by one half period of an underdamped vibration response and so the mathematical models used to describe the kinematical quantities of an underdamped vibration can be used to describe the kinematical quantities of a bat and ball impact.

This modelling procedure was used to investigate the effects of different bat mass, damping and stiffness characteristics on the impact behaviour and rebound velocity of the ball. These parameters are determined physically by the construction of the bat and particularly the striking face of the blade. Understanding the how these values affect ball rebound within a lumped parameter model influenced how the blade was constructed to optimise its striking performance.
6.2 LUMPED PARAMETER MODELS IN SPORTING IMPACTS

A spring damper mass model has been used to describe the motion of a vibrating system. The model consists of discrete components that 'lump' together properties of the system being modelled in the form of point masses \( m \), zero mass springs with stiffness \( k \), and zero mass dashpots with damping constant \( c \). A schematic of a single degree of freedom system with viscous damping indicated by the dashpot is shown in Figure 6.3.

![Spring damper model](image)

Figure 6.3 - Spring damper model

The Equation that describes a force applied by the spring, denoted by \( F_s \), to the mass is:

\[
F_s = kx
\]  \[6.1\]

Where \( x \) is the displacement and \( k \) is the spring constant. The force applied by a viscous damper is proportional to velocity rather than displacement:
\[ F_i = c \dot{x}(t) \]  \[ (6.2) \]

Where \( \dot{x} \) is the velocity and \( c \) is damping coefficient. Applying Newton's second law of motion to the system results in the following:

\[ m \ddot{x} = -F_i - F_k \]  \[ (6.3) \]

Which can be substituted and re-arranged to give:

\[ m \ddot{x}(t) + c \dot{x}(t) + kx(t) = 0 \]  \[ (6.4) \]

This differential Equation can be solved by numerical or analytical means. In more complex systems such as in a bat and ball impact which involves the interaction of two or more spring damper masses, a solution may be achieved more easily by numerical means. The analytical solution to this general equation of an underdamped free response is given by Inman [108] as:

\[ x(t) = Ae^{-\zeta\omega_n t} \sin(\omega_d t + \phi) \]  \[ (6.5) \]

\[ A = \sqrt{\left(\omega_d \omega_0 \xi_x \right)^2 \left(\omega_d \omega_0 \xi_x \right)^2 \omega_n^2} \]  \[ (6.6) \]

\[ \phi = \arctan \left( \frac{\omega_d \xi_x}{\omega_0 + \omega_d \xi_x} \right) \]  \[ (6.7) \]

where \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \), \( \omega_n = \sqrt{k/m} \), \( \zeta = \frac{c}{2\sqrt{km}} \)

\( A \) is the maximum value of the displacement response \( x \), \( \phi \) is the phase angle in radians, \( \omega_n \) is the undamped natural frequency, \( \omega_d \) is the damped natural frequency in radians per second, and \( \zeta \) is the damping ratio. Using initial conditions, \( t = 0, \ y(0) = 0 \) and substituting for \( \omega_n \) and \( \zeta \), Equation 6.5 becomes:

\[ x(t) = Ae^{-\zeta t} \sin \omega_d t \]  \[ (6.8) \]

and can be differentiated to give the velocity of mass \( m \), at any time \( t \):

\[ \dot{x}(t) = Ae^{-\zeta t} \left( \omega_d \cos \omega_d t - \frac{c}{2m} \sin \omega_d t \right) \]  \[ (6.9) \]

Both these Equations can be solved providing the parameters, \( c \), \( \omega_d \), and \( A \) are known.
Cottey [109] illustrates the process of determining these values and demonstrates that the relationship:

\[ x_{\text{damp}}(t) = \pm x e^{-\frac{c}{2m} t} \]  

[6.10]

can yield the Equation for damping coefficient \( c \) in terms of inbound and rebound velocity, mass and contact time based upon the premise that the impact is represented by one half of a full oscillation:

\[ c = -\ln \left( \frac{v_{\text{out}}}{v_{\text{in}}} \right) \frac{2m}{t_f} \]  

[6.11]

Similarly, the solution for the angular frequency of a damped oscillation is expressed as:

\[ \omega_d = \sqrt{\frac{k - c^2}{m - 4m^2}} \]  

[6.12]

results in an Equation for ball stiffness \( k \) in terms of ball mass \( m \), inbound and rebound ball velocity and contact time \( t \):

\[ k = m \left( \frac{\pi^2}{t_f^2} - \left[ \ln \left( \frac{v_{\text{out}}}{v_{\text{in}}} \right) \right]^2 \frac{1}{t_f^2} \right) \]  

[6.13]

Cottey [109] used high speed video data of tennis ball impacts with a rigid flat plate to establish values of \( c_{\text{ball}} \) and \( k_{\text{ball}} \) and contact time \( t \) as a function of ball inbound velocity. A state-matrix method was used to solve the general Equation 6.9 and the Runge-Kutta integration method to obtain a numerical solution within Matlab\textsuperscript{®}. The study was able to accurately model the tennis ball impact as half the oscillation of a spring damper vibration and the expressions of \( c_{\text{ball}} \) and \( k_{\text{ball}} \) as a function of \( v_{\text{in}} \) accommodated the effects of strain rate on the observed stiffness and damping properties of the ball.

The model was developed to accommodate the interaction of the ball against a racket string bed. The general Equation 6.9 was modified to include the mass, stiffness and damping properties of the string bed, by incorporating a second mass and spring damper components as shown in Figure 6.4. In calculating the force balance around both masses, a state matrix combining the two Equations was generated. The model was then solved by numerical means in similar fashion to the ball and plate model and gave good agreement with results obtained experimentally.
Nathan [6] modelled the impact between bat and ball in baseball and softball and investigated the 'trampoline' effect observed using hollow metal bats. Initially, a simple model, illustrated in Figure 6.5, was used to describe the effect of altering face mass and stiffness values on the rebound velocity of the ball. The mass of the ball was represented by $m_{\text{ball}}$ and its stiffness by $k_{\text{ball}}$. The effective mass of the bat was represented by $m_{\text{bat}}$ and Nathan notionally used a value four times the mass of the ball. Figure 6.6 illustrates the relationship observed between the ratio of bat to ball spring stiffness and the resulting COR, the shape of the plot is characteristic of result obtained with lumped parameter impact models.

In Figure 6.6, the region where bat stiffness $\to \infty$ describes the behaviour of the bat as being similar to that of a rigid plate. The bat mass is in effect, rigidly coupled to the wall and therefore has little influence over the behaviour of the impact. In this instance the COR of impact is solely determined by the ball damping characteristics. Where bat stiffness tends to zero the mass appears to be completely decoupled from the wall over the short time duration of impact. Therefore the momentum transferred to the bat is not returned to the ball as the bat spring has no time to recoil during the collision. The
The greatest value of COR is achieved at a bat stiffness that is sufficiently high so that its natural period of oscillation is shorter than the impact duration; whilst remaining low enough to maximise the percentage of stored energy in the bat spring. If the natural period of oscillation is greater than the collision time, then the energy that is transferred to the bat cannot be returned to the ball during the period of contact. Nathan described the optimum face as one that maximises the energy transferred to the bat, by having a low stiffness whilst maintaining a sufficiently high natural frequency in order to return the stored energy during the collision. This could be accomplished by having a thin, deformable membrane of low mass; features which are apparent in modern hollow baseball bats.

A spring damper model was also used by Cochran [12] to model the 'trampoline' effect in golf club driver heads. In this study, a non-linear Hertzian spring and non-linear viscous damper were included to more accurately represent the impact behaviour of the ball. The golf club face was modelled as a mass on a linear spring damper attached to a rigid, immovable body. The general Equations of motion for each mass were solved numerically and the relationship between face stiffness and COR were plotted for different values of face damping. The characteristic response demonstrated by Nathan [6] was similarly observed in this study. The effect of an increase in club face damping was to reduce the maximum value of COR obtainable and to shift the response so that the maximum value occurred at a greater face stiffness. Cochran concluded that when half the period of the 'loaded' face oscillation (with both the mass of the face and ball together) is equal to the duration of contact, the optimum value of COR is obtained in the undamped scenario.

A further study by Cochran [10] developed the lumped parameter golf ball and club head model by ascribing further model components to physical parts of the ball and club head. For example, a second mass was added to the ball to represent the hard plastic outer cover, whilst a portion of the club head mass was designated as the deforming region of the striking face, largely responsible for the 'trampoline' effect. The detailed model provided more accurate representation at very low ball inbound velocities. In contrast, higher velocity impacts proved to be less representative of real behaviour and additionally, the extra model parameters that were required added to the complexity in tuning the model.

Johnson [11] also investigated the 'trampoline' effect in golf club drivers using lumped parameter models and high speed experimental impact testing. Golf balls were launched from an air cannon into a stationary but freely suspended metallic disk. The inbound and rebound of the ball was measured in order to assess the effect of disc thickness on ball rebound performance. A spring damper model representing the experiment was additionally used to assess the effects of different plate stiffness. This model differs from studies [6, 12], as the striking face was modelled with two masses; one representing the sprung mass of the striking portion of the plate and the other representing the remaining unsprung club mass. Since the plate mass was not coupled to a fixed rigid component, it allowed the post impact translation of the plate to be modelled. The ball was modelled using two non
linear springs and a linear damper. The general Equations of motion were solved numerically and the resulting motion of the ball and plate compared with those values obtained experimentally. The simulation showed that there was very little enhancement in rebound velocity until the face thickness was reduced to about 4 mm and stated that the improvement continues as the face thickness is reduced but is limited by the onset of plastic deformation which occurred experimentally at 2.5 mm.

Russell [110] investigated the differences in performance of a variety of commercially available aluminium, titanium and composite baseball bats by relating the barrel hoop frequency with hitting performance. Figures 6.7 and 6.8 represent the shapes of the first three bending modes and first three hoop modes along the length of a baseball bat. A lumped parameter model was used to investigate the relationship between hoop frequency and ball rebound by modelling the radial behaviour of the barrel with spring damper components and Figure 6.9 shows the general relationship determined from the model. The shape of the plot is similar to Figure 6.6 except hoop frequency is used in preference to bat to ball stiffness ratio on the x axis. The author refers to Cross [28] in describing how greater performance results if the longitudinal stiffness of the barrel is increased in order to minimise the energy detracting effects of the first three bending modes of the bat. Russell states that enhanced performance can be achieved by maximising these bending frequencies whilst optimising the hoop frequency as demonstrated by the model. This concept is in agreement with work by Nathan [45] discussed earlier in the Chapter.

![Figure 6.7 - Flexural modes of a baseball bat from Russell [110]](image1)

![Figure 6.8 - Hoop modes of a baseball bat from Russell [110]](image2)
Russell described previous commercial attempts to reduce the thickness of the barrel wall of the single wall aluminium baseball bat. The reason was to reduce the fundamental hoop frequency which was significantly higher than the optimum value demonstrated by the model. The alterations led to durability concerns that eventually gave rise to new constructions employing multiple thin layers of aluminium and bats made from composite materials. The author suggested that the multiple thin layers lower the effective spring constant whilst the strength comes from the sum of the layers and these bats tend therefore to have lower hoop frequencies. The best performing bats in the group were of composite construction, the author attributed their performance to the anisotropic properties of the composite material which can be orientated to give low radial stiffness but high longitudinal stiffness. Correct orientation results in a low hoop frequency of 1000-1150 Hz which is largely independent of a bending frequency.

Yamaguchi [9] used a lumped mass model to investigate mechanical impedance matching between various types of golf ball and golf club. Greatest enhancement was achieved with modern golf balls and club heads whose faces gave a fundamental bending frequency of 1000 Hz. The lumped parameter model used in this study varied considerably from those models already reviewed, in that the ball and club where both divided into between 9 and 11 smaller masses coupled to each other with linear spring and damper components. Despite the differences, the model results gave good agreement with previous studies [10-12].

6.2.1 Issues Particularly Relevant to Hollow Bat Design

The study by Johnson [11] illustrated that the appropriate levels of stiffness required to achieve enhanced rebound occurred at a thickness of striking face that gave concern over face durability and this is an important consideration in deciding upon an appropriate level of bat face stiffness. The concept of minimising the sprung mass whilst maintaining durability is equally important, as energy loss through subsequent vibration of the face following impact has been noted in previous studies [6, 12]. Similarly, minimising mass may give rise to durability issues. Russell [110] described the benefit of laminated and composite baseball bats. Wood is anisotropic and the blade construction is laminated, therefore the concept of multiple thin layers to improve durability and the effect of material orientation upon the structural stiffness of the striking face are appropriate to this study.
6.3 LUMPED PARAMETER CRICKET BALL MODEL

6.3.1 Generation of Cricket Ball Model

In order to investigate the possibility of impedance matching the face of the bat and the ball, an accurate model of the ball was required. The ball was modelled in similar fashion to the tennis ball by Cottey [109], with the lumped parameter model consisting of a linear spring and linear damper coupled to one mass as shown in Figure 6.3. The values of damping constant $c$ and spring stiffness $k_b$ were calculated as in Equations 6.11 and 6.13 using experimental values for inbound and rebound velocity and contact time, measured at five discrete inbound velocities ranging from 11-37 ms$^{-1}$. Balls were fired from a cricket ball launcher with no significant spin and impacted normally onto a fixed, rigid steel plate. A high speed camera was used to capture the impact and the data was digitised. Full details of all experimental work are discussed in Chapter 9.

Five leather cricket balls and one synthetic ball were tested and for each impact, values of $V_{in}$, $V_{out}$ and $t$ were measured and values $k_b$ and $c_b$ were calculated. Since a linear spring was used in the model, the relationship between $k_b$ and inbound velocity was plotted to give a polynomial expression that related the two parameters for all values of inbound velocity. Figure 6.10 illustrates the change in model spring stiffness in relation to the inbound ball velocity together with the polynomial trend line. Similarly, the relationship of $c_b$ to $v_{in}$ was expressed as a polynomial as shown in Figure 6.11.

The state matrix method outlined by Inman [108] was used to solve the general Equation of motion (Equation 6.9) to determine the displacement and velocity of the ball against the impact surface at any given time for a range of inbound velocities. MathCAD$^\circ$ was used to perform the calculations and software code for the leather cricket ball model is given in Appendix 4.
6.3.2 Evaluation of Ball Model

For a given inbound velocity, the MathCAD® model produces a graphical representation of the numerical solution for displacement and velocity as shown in figures 6.12 and 6.13, respectively.

The measurements of $d_{\text{max}}$, $t_{\text{contact}}$ and $v_{\text{out}}$ for an impact are interpreted as shown in the Figures 6.12 and 6.13. For a range of inbound velocities, these parameters were calculated and compared to values obtained from experiment. The experimental results were fitted to second order polynomials. Figure
6.14 illustrates the experimental and model data for outbound velocity against inbound velocity. Similarly, Figures 6.15 and 6.16 illustrate experimental and model data for inbound velocity against contact time and deformation respectively. Table 6.1 summarizes the accuracy of both models with respect to these impact metrics.
### Leather Cricket Ball – Difference in Values Between Experimental and Model Data

<table>
<thead>
<tr>
<th>Difference at 12.5 m/s $v_{in}$</th>
<th>Outbound Velocity (m/s)</th>
<th>Contact Time (m/s)</th>
<th>Lateral Deformation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24</td>
<td>0.03</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>0.78</td>
<td>0.11</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>1.61</td>
<td>0.27</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>2.26</td>
<td>0.38</td>
<td>3.20</td>
<td></td>
</tr>
</tbody>
</table>

### Synthetic Cricket Ball – Difference in Values Between Experimental and Model Data

<table>
<thead>
<tr>
<th>Difference at 12.5 m/s $v_{in}$</th>
<th>Outbound Velocity (m/s)</th>
<th>Contact Time (m/s)</th>
<th>Lateral Deformation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>0.21</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>0.78</td>
<td>0.08</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>1.39</td>
<td>0.04</td>
<td>3.34</td>
<td></td>
</tr>
<tr>
<td>2.04</td>
<td>0.12</td>
<td>6.18</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1 - Differences between experimental and numerical data for different inbound velocities

### 6.4 BALL AND FIXED BAT FACE MODEL

#### 6.4.1 Generation of Model

Previous studies [6, 9-12, 28, 110] have illustrated the usefulness of the lumped parameter model in determining how properties of the striking face affect ball rebound velocity. The model was developed by adding a second mass together with linear spring and damper components between the ball and the fixed wall to represent a ball impacting a fixed bat face. Figure 6.17 shows the model arrangement, the wall is present to eliminate gross translation whilst allowing for the deflection of the face. This impact scenario was repeated experimentally by clamping the supporting edges of a face to a fixed position whilst impacting the central face region, full details of this experiment are given later in chapter 9.

![Schematic of ball and fixed bat face model](image)

Figure 6.17 - Schematic of ball and fixed bat face model

As there are two masses, two general Equations of motion describe the system, they are determined by calculating the force balance around each mass in turn. Calculating the force balance around the ball mass $m_b$ gives:
\[ m_y \ddot{x}_b (t) + c_y \dot{x}_b (t) - c_y \dot{x}_j (t) + k_y x_b (t) - k_y x_j (t) = 0 \] \quad [6.14]

and the force balance around the 'sprung' mass of the face \( m_f \), gives:

\[ m_f \dot{x}_j + (c_f + c_y) \dot{x}_j - c_f \dot{x}_b + (k_f + k_y) x_j - k_f x_b = 0 \] \quad [6.15]

Combining the two second order differential Equations 6.14 and 6.15 in matrix form gives:

\[
\begin{bmatrix}
  m_y & 0 \\
  0 & m_f
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_b \\
  \dot{x}_j
\end{bmatrix}
+ \begin{bmatrix}
  c_y & -c_y \\
  -c_f & (c_f + c_y)
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_b \\
  \dot{x}_j
\end{bmatrix}
+ \begin{bmatrix}
  k_y & -k_y \\
  -k_f & (k_f + k_y)
\end{bmatrix}
\begin{bmatrix}
  x_b \\
  x_j
\end{bmatrix}
= 0
\] \quad [6.16]

The state matrix method was to solve the general Equations of motion using the Runge-Kutta integration to obtain an approximate solution in MathCAD. The MathCAD code for the leather ball and fixed deformable striking face model is given in Appendix 5.

As previous studies [6, 9-12, 28, 110] have highlighted, a model that includes the striking face of the implement requires a value for the portion of the implement directly coupled to the ball. The striking face of a cricket bat is substantially larger than the local area of contact between the ball and the face and it is likely that the value of mass representing the coupled portion of the implement is a percentage of the total mass of the face. However, there is no criterion to determine this percentage and additionally the value is likely to change with impact speed, impact location and face design. Therefore total mass of the face was used to represent the sprung mass.

Similarly, the damping characteristics of the face are unknown. The study by Cochran [12] used three values of damping which were termed 'zero', 'light' and 'heavy' to model the golf club face. This gave an appreciation of the sensitivity of the ball rebound velocity to the degree of face damping. The sensitivity of ball rebound to striking face parameters is worthy of investigation in this study and therefore in addition to assessing the effects of different damping coefficients, the sensitivity of the model to different values of sprung mass, face stiffness and ball inbound velocity have been examined.

**6.4.2 Results of Model**

Figures 6.18 to 6.20, illustrate the affect on COR due to changes in values of face damping, sprung mass and inbound velocity, respectively. COR is plotted against face stiffness in order to illustrate the characteristic relationship between the two parameters.
Figure 6.18 - Ball COR vs. face stiffness for different values of face damping with 128 g sprung mass at 44 m/s $v_{in}$

Figure 6.19 - Ball COR vs. face stiffness for different values of ball inbound velocity with 10 N/ms$^{-1}$ damping and 128 g sprung mass

Figure 6.20 - Ball COR vs. face stiffness for different values of sprung mass with 10 N/ms$^{-1}$ damping and 44 m/s $v_{in}$
6.4.3 Discussion

Figure 6.18 illustrates the importance of minimising viscous damping of the face. Following the relationship from the far right of the graph, it can be seen that the COR is purely a function of the ball as the face stiffness is too high to play a significant role in the impact. As the face stiffness lowers, the first optimum COR is reached with the greatest face damping but, the value is lower than that for faces with less damping. As the face stiffness reduces further, the effects of high damping become more significant as the COR rapidly decreases with lower stiffness. Low damping is therefore significant in improving the ball rebound performance, particularly if the value of face stiffness is limited by material or construction method.

A value of face damping and a sprung mass approximately equal to the mass of the ball were used to illustrate the relationship between face stiffness and ball rebound performance for different values of inbound velocity in Figure 6.19. As previously demonstrated in the ball and rigid plate model, increasing the inbound ball velocity reduces the ball COR due to viscous damping. The optimised stiffness also increases with higher inbound velocities. The increase in stiffness required to produce optimal results at 44 m/s is approximately 1.6 times that required at 20 m/s. Therefore the model illustrates that inbound ball velocity is an important parameter in determining a suitable face stiffness. During a game the bat and ball are likely to impact over a range of speeds. Typical bowling deliveries vary between 17 m/s from a spin bowler delivery to 44 m/s from a fast bowler. A study by Stretch [15] recorded bat impact speeds between 3.5 m/s to 11.8 m/s and so the potential range of contact velocities is between 20 m/s and 55 m/s (45-125 mph) . At an inbound velocity of 55 m/s the optimum stiffness calculated from the model is $5.6 \times 10^6$ N/m, this value is 1.8 times the optimum stiffness at 20 m/s. Fortunately, over this range of speeds there is a significant overlap of 'near' optimum COR values and so it is likely that one value can be chosen to successfully accommodate the range of impact speeds encountered.

Figure 6.20 illustrates the effect of sprung mass on the rebound performance of the ball. Increasing the value of sprung mass has a similar effect on ball rebound to increasing the face damping, but whereas higher damping dissipates more impact energy, a greater sprung mass results in more impact energy being transferred to the face. A lower sprung mass results in a lower optimum face stiffness which in turn reduces the level of impact energy stored and dissipated in the ball. This results in a higher ball rebound velocity, provided that the level of face damping is lower than ball damping. This agrees with the study by Nathan [6] who suggested that sprung mass and stiffness should be minimised to reduce the energy transferred to the baseball bat whilst minimising the storage and loss of energy in the ball.

The tuning of the model to accurately represent experimental data was achieved retrospectively following experimentation. Figures 6.21 to 6.24 illustrate the measured performance of two face constructions used in the prototype two bats together with the results of the tuned spring damper
model using the tuned component values, shown in Table 6.2, the results of the experimental testing are detailed in chapter 10.

![Figure 6.21 - Numerical and experimental performance data: face deflection vs. inbound velocity](image1)

![Figure 6.22 - Numerical and experimental performance data: ball deformation vs. inbound velocity](image2)

![Figure 6.23 - Numerical and experimental performance data: contact time vs. inbound velocity](image3)
Face 'sprung' mass \( (m_f) \) | 0.035 kg  
Face stiffness \( (k_f) \) | \( 7.1 \times 10^5 \) N/m \( \sim 10^5.85 \) N/m  
Face damping \( (c_f) \) | 200 N/m s\(^{-1}\)  

Table 6.2 - Tuned spring damper component values

### 6.5 BALL AND COMPLETE BAT MODEL

#### 6.5.1 Generation of Model

The fixed wall used in the fixed face model was replaced with a third mass representing the effective mass of the bat at point of impact. The results of the model are used to assess the performance of hollow faces constructed in this study, discussed in chapter 10.

Figure 6.25 illustrates the model schematic where \( m_e \) is the effective mass of the bat, \( m_f \) is the sprung mass and \( m_b \) is the mass of the ball. To obtain the general Equations of motion for the model, the force balance around each mass was determined as follows:

\[
\begin{align*}
\text{force balance around } m_e: \\
m_e \ddot{x}_e + k_f x_e - k_f x_f &= 0 
\end{align*}
\]  

\[\text{(6.17)}\]
Force balance around $m_f$:

$$m_f \ddot{x}_f - k_f x_f + (k_f + k_s) \dot{x}_f - k_s x_s = 0 \tag{6.18}$$

Force balance around $m_s$:

$$m_s \ddot{x}_s + k_s x_s - k_s x_f = 0 \tag{6.19}$$

Combining the three, second order differential Equations 6.17, 6.18, and 6.19 in matrix form gives:

$$\begin{bmatrix} m_s & 0 & 0 \\ 0 & m_f & 0 \\ 0 & 0 & m_s \end{bmatrix} \begin{bmatrix} \ddot{x}_s \\ \ddot{x}_f \\ \ddot{x}_s \end{bmatrix} + \begin{bmatrix} c_f & -c_f & 0 \\ -c_f & (c_f + c_s) & -c_s \\ 0 & c_s & c_s \end{bmatrix} \begin{bmatrix} \dot{x}_s \\ \dot{x}_f \\ \dot{x}_s \end{bmatrix} + \begin{bmatrix} k_f & -k_f & 0 \\ -k_f & (k_f + k_s) & -k_s \\ 0 & -k_s & k_s \end{bmatrix} \begin{bmatrix} x_s \\ x_f \\ x_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{6.20}$$

The state matrix method was to solve the general Equations of motion (Equations 6.17 to 6.19) using the Runge-Kutta integration method to obtain an approximate solution in MathCAD®. The MathCAD® code for the leather ball and freely constrained bat model is given in Appendix 6.
CHAPTER 7

COMPUTATIONAL FINITE ELEMENT MODELS

7.1 INTRODUCTION

FEA is a computational, numerical method commonly used to give approximate solutions to engineering and scientific problems. The type of FEA used in this study concerns modelling the strength and behaviour of solids under various loading conditions.

Factors which concern impact performance, such as bat vibration, deflection of the striking face and mass properties, etc. which have been described with basic mathematical models can be simulated in more detail with FEA computer simulation.

In FEA a complex problem is divided into smaller and more simple units. Subdivision continues until the behaviour of each unit is simple enough to fit a mathematical model appropriate to the required detail of the solution. In modelling the strength and behaviour of structures, the geometric computer models representing the physical objects are broken down or discretised into many small simple units, known as elements. The behaviour of a single element can be described by a relatively simple set of algebraic Equations and just as the elements are joined together to represent the whole structure, the Equations describing the behaviour of each individual element are used to produce a set of Equations that describe the behaviour of the whole structure.

Each element representing a discrete portion of the physical structure is defined and connected to neighbouring elements by nodes and the elements and nodes define the FEA mesh. Material definitions are assigned to each element and remain associated to the element throughout the analysis. The movement and distortion of the discrete model in space due to applied loads results from the translations and rotations at each node within the mesh.

The nodes of structural elements each have six degrees of freedom (DOF) corresponding to translation along and rotation about three orthogonal axes and the node DOF are the fundamental variables that are calculated during the analysis [111]. If the mesh density is increased then not only does the discrete model reflect more accurately the geometric shape of the physical structure but the total DOF within the model increases, potentially resulting in a more accurate and detailed solution. However, the size of the computational task is also increased and so a compromise between model detail and computational cost exists.
The accuracy of the solution is also dependent upon the accuracy of material definitions which are assigned to the elements and the accurate representation of applied loads and other external conditions or constraints that exist in the real structure.

It is important therefore that the model is validated by comparing the discrete solution against real observation. Difficulties associated with obtaining accurate constitutive material definitions and damping properties at the appropriate rates of strain for a bat and ball impact were overcome by modifying the initial elastic and damping properties in order that the solution accurately represented observed impact behaviour in terms of defined metrics such as COR and contact time.

ABAQUS FEA software was used in this study and two numerical solvers are provided. ABAQUS/Standard is generally used for problems in which the inertia properties of the objects in the analysis do not significantly affect the outcome of the solution. However, ABAQUS/Explicit was predominately used in this study as it caters for dynamic problems in which the inertia properties of the objects are significant in determining their behaviour [111].

7.2 SUMMARY OF FEA INVESTIGATIONS

A cricket ball FE model was initially developed and validated against the results of experimental ball testing to establish an accurate ball model which was subsequently used in the bat and ball impact simulations.

The experimental bat testing procedure discussed in chapter 9, involving the impact between an initially stationary bat and a moving ball, was simulated to assess the accuracy of a FE bat and ball impact model. The model provided a greater understanding of the behaviour of the bat and ball during impact and was subsequently modified to isolate and examine various determinants in bat performance, such as material damping and blade and handle stiffness.

Spring damper models, detailed in chapter 6, were initially developed to investigate the relationship between the dynamic properties of the face and impact performance. FEA is used to investigate the relationship between those dynamic properties and the design of the face.

7.3 THE BALL MODEL

This section describes the process of developing and validating the ball and bat FE models describing the discretisation of bat and ball geometry and the determination of material properties.

7.3.1 Ball Geometry

Figure 7.1 illustrates the geometry of the ball model, consisting of 2246 modified tetrahedral elements which represent a sphere of diameter 71 mm.
7.3.2 Determination of Material Properties

An isotropic ball material definition was applied to the element mesh to represent the bulk stiffness characteristics of the cricket ball. This approach was used by Smith et al. [38] in modelling a baseball. A general damping factor was also applied to account for the internal energy loss observed during a real ball impact. ABAQUS provides a damping mechanism, termed ‘β-Rayleigh damping’ (β) which when applied to the elements generates a stress proportional to both material stiffness and strain rate. This stress is added to the constitutive stress response at the element integration points and can be expressed as follows [112]:

\[
\sigma_d \propto \beta \cdot E \cdot \dot{\varepsilon}
\]

where \(\sigma_d\) = damping stress, \(E\) = the material’s current elastic stiffness and \(\dot{\varepsilon}\) = the material strain rate.

Simulation of the ball impact test study detailed in chapter 9 was conducted and the results from both studies compared. Measurements of COR, contact time and lateral ball deformation were used to assess the effect of changes in \(E\) and \(\beta\) for a range of incident ball velocities. A general relationship between applied material attributes and the impact characteristics of the FE ball model was established. The aim of these simulations was primarily to establish values of \(E\) and \(\beta\) which accurately represented the impact characteristics of a real cricket ball in terms of COR, contact time and lateral deformation.

Figures 7.2 to 7.5 illustrate the effect of changes in values of ball \(E\) and \(\beta\) upon values of COR, maximum lateral ball deformation and contact time, for a ball impact against a flat rigid surface. COR was strongly dependent upon changes in value of \(\beta\) and less dependent upon changes in \(E\). In contrast, lateral deformation and contact time were strongly dependent upon change in \(E\) and less dependent upon \(\beta\). Therefore, the value of \(E\) was adjusted in the model to produce approximately correct values of maximum lateral deformation and contact time. \(\beta\) was applied to the ball element mesh to establish a correct relationship between COR and impact velocity. Several iterations involving the minor
adjustment of $E$ and $\beta$ were necessary to develop an accurate FE ball model using values of $E = 90$ MPa and $\beta = 1.22 \times 10^{-4}$.

![Graph](image1)

**Figure 7.2** - Ball COR vs. $\beta$ at 12 m/s

![Graph](image2)

**Figure 7.3** - Ball COR vs. incident ball velocity for different values of $E$
7.3.3 Accuracy of Ball Model

Figures 7.6 to 7.8 compare the results of COR, maximum lateral deformation and contact time against incident ball velocity for the FE ball model and experimental ball impacts. The relationship between contact time and incident ball velocity is on average ~0.2 ms less in the FE model than observed experimentally which maybe partly attributed to error in experimental measurement. Both ball deformation and COR are accurately represented by the model.

Figure 7.9 illustrates the deformed shape of the FE ball model (a) and a leather covered cricket ball (b), at point of maximum lateral deformation for a 31 m/s impact. The deformed geometry of the real
ball demonstrates a greater apparent contact area relative to the FE model. A possible explanation is the separation of the leather cover in the immediate area of contact which is not represented by the homogenous FE model. Since the impact metrics bear good agreement, it was anticipated that the observed difference in deformed geometry was not significant in this study.

Figure 7.6 - Ball COR vs. incident ball velocity

Figure 7.7 - Maximum lateral ball deformation vs. incident ball velocity
7.4 BAT MODELS

7.4.1 Solid Bat Element Mesh

The solid bat CAD geometry, detailed in chapter 5 was used as the basis of the FE solid bat model. All secondary radius blends between primary features were removed to reduce the generation of numerous tiny elements during discretisation. The handle splice was modified by increasing the apex angle to improve the mesh quality in this region and the blade geometry was modified to accommodate this change. The modified CAD model of the handle and blade were imported separately into the Hypermesh software package and a free meshing algorithm was used to discretise the model geometry. The blade mesh consisted of 2781 modified tetrahedral elements and the handle, represented as one homogenous part, consisted of 1638 modified tetrahedral elements.
Blade and handle mesh parts were imported separately into the ABAQUS FEA environment and assembled by constraining the mating surfaces along the splice feature. Figure 7.10 illustrates the solid bat FE model assembly.

Figure 7.10 - FE solid bat model of template willow bat

7.4.2 Hollow Bat Element Mesh

The hollow bat CAD geometry, detailed in chapter 5 was used as the basis of the FE hollow bat model. An identical procedure to that described for the solid bat was used to eliminate secondary blends and to modify the splice geometry to facilitate an acceptable mesh quality. The blade element mesh consisting of 3103 modified tetrahedral elements was constrained to the handle mesh along the splice region. Figure 7.11 illustrates the hollow bat FE part model assembly.
7.4.3 Determination of Material Properties

Material definitions describing the bulk elastic properties of the handle and blade were experimentally determined. The application of $\beta$ to the blade material definition was investigated to account for energy loss in the bat local to the area of ball contact and to provide a mechanism to attenuate bat modal excitation. However, the addition of $\beta$ to the blade and handle had a significant effect upon the computational cost of the analysis. The ABAQUS damped bat model simulations were $\sim$200 times more expensive than an equivalent simulation in which $\beta$ was limited to the ball model. This resulted in a computation time $\sim$400 hrs per millisecond of simulation which was considered excessive. Consequently, $\beta$ was not incorporated within the blade and handle material definitions. It was anticipated that since the bulk elastic modulus defined for the cricket ball was approximately 2.5 times
less than the value of $E_f$ for willow (220 MPa), the majority of deformation in the region of impact was likely to occur in the ball. Therefore the damping present in the ball would be the predominant mechanism describing local energy losses.

7.4.4 Solid Blade Material Properties

An orthotropic elastic material definition was applied to the blade element mesh to represent the material properties of willow. Experimental determination of the elastic constants for willow was performed at the Forest Products Research Centre, Buckinghamshire Chilterns University College, according to BS EN 408:1995 [40]. Three and four point bending tests were performed in the $L$ direction with specimens machined to rectangular cross section and length in accordance with BS EN 408:1995 [40] and equivalent to the proportions of the willow blade. The three point bend test was used to generate both shear and bending stresses in the specimen from which a value of 'apparent' modulus could be determined. The four point bend test generated 'pure' bending within a region of the specimen from which the elastic modulus could be determined. Values of Shear modulus were then derived using the values of apparent modulus and elastic modulus determined from both tests. Values of elastic modulus were measured in the $L$ direction with specimen orientation as illustrated in Figure 7.12c. However, only the apparent modulus was determined in the $T$ and $R$ directions, since specimen size would not accommodate a four point test procedure.

![Figure 7.12 - Specimen bending test orientations](image-url)
7.4.5 Hollow Blade Material Properties

A material definition was required to represent the mechanical properties of a birch ply laminate construction to be used in the manufacture of initial prototypes. Birch ply laminates were adhered together with a urea-formaldehyde (UF) resin and upon full curing, the blocks were machined to produce four test specimens of rectangular cross section, equivalent in size to a bat blade. During the test, specimens were orientated as shown in Figure 7.12b since ball impacts predominantly excited transverse bending modes which acted to deform the blade laminate material in this orientation. Determined values of elastic modulus and shear modulus were therefore appropriate to the observed mode of bat deformation. Since contiguous ply layers within the laminate were mutually perpendicular the values of elastic modulus obtained in the 2 direction were equivalent to those obtained in the 3 direction. Equally, the value of shear modulus in the $12$ and $13$ planes were equivalent. Only the apparent bending modulus was determined in the $1$ direction due to specimen size.

7.4.6 Handle Material Properties

An isotropic material definition was considered sufficient in representing the bulk properties of the handle. The handle assembly was of laminate construction and the orientation of the layers in relation to the plane of bending affected the bending stiffness in the 2 direction. The orientation illustrated in Figure 7.12a was used in determining a value of apparent bending modulus since ball impacts predominantly excited transverse bending modes acting to deform the handle in this orientation. Finished bat handles are generally oval and vary in cross section along their length. However, the bending test required specimens with a uniform circular or rectangular cross section. Therefore, specimens were machined from glued handle assemblies at the stage of manufacture prior to turning/shaping. The glued assemblies were turned to a circular cross section $\sim30$ mm diameter and sawn to a length $\sim500$ mm.

Figures 7.13 and 7.14 are photographic illustrations of the three and four point bend tests in the $L(2)$ direction, respectively. Figure 7.15 is a photographic illustration of the three point bend test used to determine apparent bending modulus in the $R(1)$ or $T(3)$ direction. Figure 7.16 is a photographic illustration of the handle three point bend test.
Figure 7.13 - Three point bend test to measure apparent bending modulus in $L_{(2&3)}$

Figure 7.14 - Four point bending test to measure bending modulus in $L_{(2&3)}$
7.4.7 Results and Determination of FE Elastic Constants

Table 7.1 summarizes the results of the material bend tests. The definition of an orthotropic elastic material requires 12 constants, nine of which are independent. A complete sets of elastic constants, published for birch [68] and approximated for willow [35] were used in conjunction with the results of testing to define elastic constants for the materials used in the solid and hollow bat FE models. Table 7.2 summaries the published elastic constants for birch and willow. The values of elastic modulus, shear modulus and Poisson's ratio taken from Bodig [68] were used to determine equivalent values for a birch ply multilayer laminate [68], with layer orientation equivalent to the birch ply specimen used in testing. Figure 7.17 illustrates the orthotropic model of the birch ply laminate, indicating the orientation of individual birch layers.
The published data provided was used to define the elastic constants which were not obtained from the bending tests. However, it was anticipated that the experimentally determined values of $E_I$ and $G_{LT}$ (for willow) and $E_I$, $G_{12}$ and $G_{13}$ (for the birch laminate) would be predominant in defining the flexural behaviour of the blade components in both the solid and (hollow) bat FE models, since impact excitation predominantly acts to create normal stress in the $L (2)$ direction and shear stress in the $G_{RL}$ ($G_{12}$, $G_{13}$) planes. The values used to define the FE materials are presented in bold, in Tables 7.1 and 7.2.

<table>
<thead>
<tr>
<th>Apparent Modulus (MPa)</th>
<th>Elastic Modulus (MPa)</th>
<th>Shear Modulus (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_R$</td>
<td>$E_{11}$</td>
<td>$E_{12}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Willow</td>
<td>7508</td>
<td>144</td>
</tr>
<tr>
<td>Birch Ply Laminate</td>
<td>501</td>
<td>78</td>
</tr>
<tr>
<td>Handle</td>
<td>700</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 7.1 - Results of material bending tests

<table>
<thead>
<tr>
<th>Elastic Modulus (MPa)</th>
<th>Shear Modulus (MPa)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_R$</td>
<td>$E_{11}$</td>
<td>$E_{12}$</td>
</tr>
<tr>
<td>Willow [35]</td>
<td>440</td>
<td>6670</td>
</tr>
<tr>
<td>Birch [68]</td>
<td>1256</td>
<td>15251</td>
</tr>
</tbody>
</table>

| Birch Ply Laminate | 1256 | 7946 | 7946 | 387 | 387 | 721 | 0.026 | 0.474 | 0.026 | 0.474 | 0.21 | 0.21 |

Table 7.2 - Elastic constants for willow [35], birch and a multilayer birch laminate [68]
7.5 COR ($c$) AND IMPACT EFFICIENCY ($e_d$)

FE impact simulations between bat and ball were solved for both the solid and hollow FE bat models. The post impact ball velocity and bat velocity at point of ball contact were extracted from the analysis to determine values of COR and $e_d$. Figures 7.18 to 7.21 illustrate the results together with those obtained experimentally.

The FEA results generally show good agreement with experimental values of COR and $e_d$. The relative difference between the experimental trends observed for the solid and hollow bat are reflected by the FE model results, with optimum values of COR and $e_d$ occurring further from the handle for the hollow bat. The greatest error $\sim 10\%$ COR between FE and experimental data occurs for hollow bat impacts at locations P1-P3. The greater values of FE COR imply that the FE model does not capture the extent of bat vibration in this region, since bat vibration is the predominant factor in modifying the value of COR along the striking region of the face. It is also possible that energy loss due to wood or bond failure, which gave rise to hollow bat durability concerns, resulted in low experimental values of COR compared to the FE model in which no failure mechanism was present.

Values of COR and $e_d$ are lower at 31 m/s than at 19 m/s inbound velocity, although the experimentally observed decrease in COR and $e_d$ is relatively less in the FE models. It is suggested that levels of bat vibration were more dependent upon impact speed than predicted by the FE model.
Figure 7.22 illustrates a visualisation of the FE solution compared to experimental video capture for a 19 m/s ball impact at bat location P1 and good agreement is evident.

Figure 7.18 - FEA and experimental results of bat and ball COR vs. impact location at 19 m/s $v_{rel}$

Figure 7.19 - FEA and experimental results of bat and ball COR vs. impact location at 31 m/s $v_{rel}$
Figure 7.20 - FEA and experimental results of bat and ball $e_d$ vs. impact location at 19 m/s $v_{ref}$.

Figure 7.21 - FEA and experimental results of bat and ball $e_d$ vs. impact location at 31 m/s $v_{ref}$. 
Figure 7.22 - FEA simulation and high speed video of bat and ball impact at 31 m/s $v_{rel}$
7.6 THE EFFECT OF HANDLE STIFFNESS AND BALL DAMPING UPON COR AND $e_d$

7.6.1 Introduction

A solid bat model was used to determine the significance of ball damping, bat vibration and handle rigidity on values of COR and $e_d$ along the striking region of the blade. Ball damping was eliminated from the simulation to isolate the effect of bat vibration upon impact COR and $e_d$ and to determine the approximate contribution of local damping effects. It was assumed that ball damping was the predominant factor in the overall value of damping local to the region of ball contact since the value of ball $E$ was a factor of $\sim 2.5$ less than $E_r$ for willow. This assumption was supported by the fact that good agreement in COR between experimental and FE bat models was obtained despite the definition of a perfectly elastic bat model.

It was suggested by Grant and Nixon [33] that increasing the stiffness of the handle would increase the third mode of transverse bat vibration above the excitation spectrum of the impact, thereby significantly reducing the energy loss coupled with this mode of vibration. Therefore the effect of handle vibration upon value of COR and $e_d$ were investigated.

7.6.2 Test Strategy

The $\beta$ component was removed from the ball material model and impact simulations were conducted using the established blade and handle material definitions. Values of COR and $e_d$ were recorded at impact locations P1-P9 for incident ball velocities of 19 m/s and 31 m/s. Undamped simulations were repeated with a rigid body constraint applied to the handle element mesh to represent a significant increase in handle stiffness since, it was anticipated that a rigid body definition would result in the greatest difference in bat behaviour.

Undamped simulations were also conducted with a rigid body constraint applied to the bat. The values of COR in this instance should equal unity and COR and $e_d$ were measured to assess possible error. These simulations were necessary to confirm that the observed reduction in COR and $e_d$ in the deformable and damped simulations were the result of deformation and damping rather than FE solution or measurement error.

7.6.3 Results

Figures 7.23 - 7.26 illustrate the COR and $e_d$ trends of the models with no material damping compared to the damped FE model simulating 'real' bat and ball impacts. The COR trend for the rigid bat model is constant for all impact locations and equal to 0.984. Ideally, for a rigid and elastic model COR = 1. Possible explanations for the observed error include the vibration of the ball following impact or a distortion of the ball element mesh resulting in a residual strain. However, the error 0.016 is consistent along the striking region and small in comparison to the changes in COR observed for the deformable models and is therefore ignored. The $e_d$ trend for the rigid bat model follows the change in effective bat mass along the striking region of the face as defined in Equation 4.23.
Both the 'real' damped model and the equivalent model without ball damping exhibit the same COR and $e_A$ trend characteristics. This suggests that the vibration properties of the bat are predominant in modifying the value of COR along the striking face. Whereas, the reduction in COR due to ball damping, which accounts for an additional ~50% reduction in COR, remains approximately independent of impact location at both inbound velocities.

The value of COR for the normal undamped model is closest to that of the rigid model in the region P7-P8, co-incident with the node locations of first three fundamental modes of bat vibration. A similar relationship between the rigid body model and the normal undamped model exists for values of $e_A$. At 19 m/s bat vibrations account for 7% (~P7) to 30% (P1) reduction in both COR and $e_A$. For a similar reduction in COR and $e_A$ to exist, an equal percentage reduction in the post impact velocities of bat and ball must occur. Therefore bat vibrations equally affect both the bat velocity and ball velocity following impact. At 31 m/s the reduction in COR and $e_A$ due to vibration increases only slightly in relation to values at 19 m/s, from 12% (P7) to 32% (P1) suggesting that bat vibration is weakly dependent upon $v_{ref}$ within the test velocity range. This relationship is similar to the weak dependency on velocity observed for ball damping.

The rigid body constraint has a significant effect upon the position and frequency of the bat transverse bending modes since it is normally the least rigid component of the bat. It is likely that the observed difference in COR and $e_A$ between bats with a normal and rigid handle is due to both differences in frequency and node position. A small improvement in COR $\leq$ 4% is apparent within the region P2 $\sim$ P8 and is reflected in a similar improvement in values of $e_A$. 
Figure 7.23 - FEA bat COR vs. impact location at 19 m/s $v_{rel}$

Figure 7.24 - FEA bat COR vs. impact location at 31 m/s $v_{rel}$
7.7 *FEA SIMULATION OF PROTOTYPE – 2 STRIKING FACE*

7.7.1 Introduction

A bat face was required which satisfied two important criteria. Firstly, to reduce the energy loss through local dissipation by utilising a 'trampoline' effect. Secondly, to exhibit structural integrity and consistent performance equivalent to or greater than a solid willow face. This face would be incorporated into prototype--2 bats. Since there were numerous possible designs for the construction of
the bat face. FE computer simulations were used to investigate the impact behaviour of different face configurations to establish their worth for further investigation and experimental tests. The following section describes a number of investigations that were undertaken using computer simulation to establish a basic understanding of how changes in face geometry and material orientation affected the value of face stiffness and levels of material strain.

The bat face was mated to a rigid support with a geometry equivalent to the shape of the internal support given by the hollow blade as shown in Figure 7.27. The rigid surface was fixed in space and the bat face constrained along the mating surfaces in order to isolate the appropriate area of the face.

![Figure 7.27 - Model of the impact face and underlying support geometry to represent the hollow bat face](image)

**7.7.2 The Effect of Material Orientation**

A model was developed using a single 4 mm thick planar face mated to the underlying geometry of the support. The orthotropic elastic constants for White ash wood, used in Major League baseball bats, were used in the material definition of the bat face due to its greater density, modulus of rupture and compression parallel to the grain in comparison to English willow, as shown in Table 3.3. A 44 m/s ball impact was simulated at an impact location co-incident with position PS and the centreline of the bat face. Figure 7.28 illustrates two identical ball impacts at maximum deformation using two different material orientations indicated by the orientation triads. In Figure 7.28a, the $L$ axis of the wood is parallel (0 degrees) to the support side edges and the level of deformation normal to the face is significant over the majority of the striking region. In Figure 7.28b, the orientation of the material is rotated so that the $L$ direction of the wood acts at 90 degrees to the supporting edges. This results in a stiffer load deflection response which exists over a more localised region. Figure 7.29 illustrates the load deflection response normal to the face for both models. Stiffness in both orientations is similar.
below a deflection of 8 mm however, with increasing deflection, the 90 degree material orientation exhibits a stiffer response.

Figure 7.28 - Effect of material orientation on face deformation
7.7.3 The Effect of Impact Position

FE simulations were conducted to investigate the uniformity of stiffness over the striking region of the face. Ball impacts were modelled at an inbound velocity of 44 m/s at 30 mm increments along the centreline of the bat face to determine whether the support conditions, particularly at impact positions near the handle splice and toe, would influence the load deflection behaviour of the face. The face was 4 mm thick and planar and the test was repeated with both 0 degree and 90 degree L axis material orientations.

Figure 7.30 illustrates the load deflection relationship for each impact location described using the distance of each impact from the top of the handle. The faces exhibit significantly consistent stiffness over the striking region of the face. A small increase in stiffness is observed at a deflection greater than 10 mm for the two impact positions closest to the underlying support geometry within the 0 degree material orientation model. With the material rotated at 90 degrees orientation, a consistent value of stiffness is maintained along the striking region including impact locations near the underlying handle and toe geometry, as shown in Figure 7.31.
7.7.4 The Effect of Face Curvature

An investigation was conducted to determine the effect of face curvature on the load deflection properties of the face. Two faces were modelled with a radius of curvature equal to 200 mm and 150 mm. Ball impacts were simulated at a distance of 640 mm from the handle top (P6) with an inbound ball velocity of 44 m/s and the results compared against those obtained for a planar face with identical thickness and material attributes. Figure 7.32 illustrates the load deflection relationship for the curved and planar geometries.

The trends can be characterised as having two distinct regions corresponding to significantly different values of stiffness. The transition to the stiffer region for the planar face occurs at approximately 8 mm whereas the transition is more gradual and occurs at the greater deflection of approximately 15 mm and 18 mm for the 200 mm and 150 mm radius faces, respectively. Therefore the 150 mm radius face
provides a consistent stiffness over the greatest range of face deflection. Further reductions in face radius were not investigated since difficulties in producing a curved face profile of radius less than 150 mm were apparent during manufacturing tests. Generally, commercial available cricket bats have a face curvature which is greater than 150 mm. The off-centreline impact study in chapter 10 established that a reduction in curvature would result in greater deviation in ball rebound angle for off-centreline impacts and therefore the use of a bat face significantly different in curvature to commercial bats was considered to be disadvantageous.

The spring damper models described earlier in the chapter used a linear spring to represent the stiffness of the bat face. The load deflection characteristics in Figure 7.32 illustrate that the value of stiffness is not constant but dependent upon the magnitude of deflection. However, within the anticipated range of deflection equal to approximately 10 mm, the trends are approximately linear. A possible explanation for the change in load deflection response concerns the change in the mode of deformation observed during ball impact. Figure 7.33 illustrates a cross section view of both the planar face and the 150 mm radius face at an identical stage during the impact. In both models the ball has travelled 15 mm in a direction towards the face from the initial point of contact. The strain contour plot of the curved face demonstrates the transition from compressive to tensile strain through the thickness of the face in regions near the edge support and at the mid-span. The variation of strain through the thickness of the face is indicative of bending, although the planar face shows a variation of stress through its thickness, most notably in the central region. The values of strains are predominantly tensile indicating a component of axial tensile stress in addition to bending stress. This axial component may contribute to the increase in structural stiffness experienced by the ball.

<table>
<thead>
<tr>
<th>Curvature</th>
<th>Material Orientation</th>
<th>200 mm Radius</th>
<th>150 mm Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
<td>90 degrees</td>
<td>90 degrees</td>
<td>90 degrees</td>
</tr>
</tbody>
</table>

Figure 7.32 - Load deflection response for different material orientations
The Effect of Face Thickness

FE simulations of a static compression test were used to determine a relationship between face thickness and stiffness. Therefore, faces of different thickness could be constructed based upon an anticipated change in value of stiffness. The test was repeated experimentally once a number of faces had been fabricated in order to obtain more accurate stiffness values.

The compression test model is illustrated in Figure 7.34. The face was modelled with a curvature of 150 mm and mated to the support geometry. A dome shaped indenter with a radius of curvature equal to a cricket ball (~35 mm) was pressed into the surface of the face to a depth of 10 mm at a location corresponding to 640 mm from the handle top (P6). Total reaction force and face deflection were used to determine a value of face stiffness for a range of face thickness, 0.5 mm-16 mm, and the results are shown in Figure 7.35.
Multiple Layer Constructions

Multiple layer constructions were considered for several reasons. The production of a curved face relied upon bending the wood and lower bending stresses were achieved using thin layers. Bat faces thicker than one layer were therefore constructed by adhering multiple layers together.

It was expected that more consistent mechanical properties would be achievable across different bat faces using a multiple layer construction since the bulk properties of the face are determined by the combined properties of each layer. As the properties of each wood layer are different due to the unique physical characteristic inherent in every wood specimen, it was anticipated that the combination of layers would promote a degree of averaging with respect to the resultant bulk properties of the face.

Wood is anisotropic and for white ash, the elastic modulus is greater in the $L$ direction than both $R$ and $T$ directions by a factor of 11 and 21, respectively [68]. A 27 m/s ball impact against a 6 mm thick, 150 mm radius face was simulated to determine different levels of strain in the $R$ and $L$ directions. The face was assigned an orthotropic elastic material definition with elastic constants for white ash taken from Bodig and Jayne [68]. Figure 7.36 illustrates the material orientation and the logarithmic strain in sections a-a and b-b. The maximum value of radial strain is approximately twice the longitudinal strain. The geometry of the deflection in section a-a indicates greater compliancy to the shape of the ball than observed in section b-b. Experimental impact tests on single and multiple layered faces with a uniaxial layer of equivalent orientation, exhibited poor durability. The propagation of cracks parallel to the longitudinal axis were apparent in the experimental impact testing of uniaxial face constructions. Further experimental tests showed that orientating contiguous layers differently within a multiple layer construction improved the durability of the face.

Computer simulation of ball impacts against faces with differing numbers of layers and layer orientation were conducted. Each layer in the model was 4 mm in thickness and assigned the elastic
constants for white ash [68]. Figure 7.37 illustrates the force deflection characteristics for 6 different face models. The longitudinal direction of the wood material is described as being either parallel to length of the bat (0 degrees) or perpendicular to the length of the bat (90 degrees). The face model named 'two layer 1-45deg, 2-135deg' describes a special case in which the material direction of each layer remained mutually perpendicular but was orientated such that the L direction of the material intersected the supporting edges at 45 degrees. The stiffness characteristics of this particular face model are similar to the two layer face model named 'two layer 1-90deg, 2-0deg' which also exhibits a mutually perpendicular layer orientation. The results of all simulations illustrate that the force deflection characteristics of the face can be significantly modified by the number and orientation of layers within the construction.

![Diagram](image)

Figure 7.36 - Effect of material orientation on deformation geometry
7.7.7 Segmented Face Constructions

Experimental impact tests highlighted concerns regarding the durability of both single and multilayer constructions. It was thought that the lower elastic modulus in the radial direction resulted in a greater degree of compliance during impact as illustrated in Figure 7.36 relative to the $L$ direction and that failure occurred due to excessive values of bending stress in the $R$ direction.

In an attempt to resolve the issue of poor durability without sacrificing the compliance of the face, faces were constructed using wood layers divided into multiple discrete segments as illustrated in Figure 7.38. An adhesive with a low value of $E$ was used to adhere multiple layers and join the segments within each layer. It was anticipated that the adhesive would not transmit significant values of shear or normal stress between segments or layers and therefore would reduce the likelihood of bond failure between layers and wood failure in the $R$ direction.

Figure 7.39 illustrates a contour plot of the shear strain at maximum deflection in the $RT$ plane. The peak values of shear strain in the segments are lower relative to values in the continuous face and the contour patterns do not cross the segment boundaries. Figure 7.40 illustrates a contour plot of the normal strain in the radial direction for both segmented and continuous faces. Lower values of normal strain are observed in the segmented face since the deflection of each segment is mutually independent. In this example the value of maximum logarithmic normal strain recorded in the segmented face is a factor of ten less than that observed in the continuous face. However, the independent behaviour of each segment results in an increase in gross deflection of the structure in the direction of ball travel, resulting in more significant levels of strain in the longitudinal direction of the wood as illustrated in Figure 7.41.
Segmented layers were arranged so that the longitudinal wood axis of contiguous segmented layers were mutually perpendicular. Since it was thought that the compliance observed in the radial direction would be reduced by the longitudinal stiffness of contiguous layers.

Figure 7.38 - Direction of segment division for mutually perpendicular faces

Figure 7.39 - Shear strain in the $RT$ plane of a single layer striking face
7.7.8 Layer Bonding

It was anticipated that the relative movement of segments and layers within the face would be dependent upon the properties of the adhesive and one with a low modulus of elasticity would facilitate greater relative movement. In addition, an adhesive with a low elastic modulus and high value of elongation to break would reduce the risk of bond failure during face deflection, since the adhesive would not transmit significant shear stress across the bond. However, it was anticipated that a low stiffness adhesive would also reduce the bulk stiffness of the face during impact.

To illustrate this point, two simple models of beam deflection under non-uniform bending, taken from Gere and Timoshenko [106] are shown in Figures 7.42 and 7.43. Beam 1 in Figure 7.42 is made from
two layers which are rigidly bonded together to eliminate the relative movement of the mating surfaces. Beam 2 in Figure 7.43 is built from two layers which are not bonded and therefore the layers are free to move independently. If the layers in beam 2 are treated as being entirely independent then bending will occur about the neutral axis for each layer. Whereas for beam 2 which acts as single layer, bending occurs about a single neutral axis. The corresponding normal and shear stress diagrams for both configurations are illustrated in Figures 7.42b and 7.43b, respectively. The Equations for maximum normal stress for both beams are given in Equations 7.2 and 7.3 [88].

maximum normal stress (beam 1) = \( \pm \frac{6M}{bh^2} \) [7.2]

maximum normal stress (beam 2) = \( \pm \frac{6M}{2h\left(\frac{h}{2}\right)^2} \) [7.3]

In this example, the maximum normal stress (compressive and tensile) in the beam 1 is half the value obtained in both independent layers of beam 2. The Equation for maximum shear stress is identical for both beams and is given in Equation 7.4 [88].

maximum shear stress = \( \frac{3P}{4bh} \) [7.4]

Therefore the maximum shear observed in beam 1 is equal to the maximum shear values in both layers of beam 2. Rectangular beam deflection under loading \( P \), can be expressed in the form of Equations 7.5 and 7.6 for both beams, respectively [88]:

maximum deflection (beam 1) = \( \frac{PL^3}{4Ebh^3} \) [7.5]

maximum deflection (beam 2) = \( \frac{PL^3}{8Eb\left(\frac{h}{2}\right)^3} \) [7.6]

Therefore, the deflection of the two independent layers will be four times that of the two bonded layers. Although the face impacts in this study are more complex than illustrated by this model of beam deflection. The example highlights the role of the adhesive in transferring shear stress between layer surfaces and the potential effect on face stiffness and maximum values of material stress.
Figure 7.42 - Beam deflection for two layers rigidly bonded together

Figure 7.43 - Beam deflection for two layers which act independently
CHAPTER 8

PROTOTYPE BAT MANUFACTURE

8.1 INTRODUCTION
The principal components of a cricket bat are the blade and the handle and they are glued together at the ‘splice’ to form the bat. This chapter describes the manufacturing process used to make hollow bat prototypes for testing and evaluation. A multilayer construction method was developed for the bat blade but the handles used in this study were of conventional construction as manufactured by DSI. Flowcharts of the DSI manufacturing process (2000) and the prototype manufacturing process used in this study are given in Appendix 7 and 8, respectively.

8.2 REASONS FOR LAMINATED CONSTRUCTION
Constructing the blade as a multilayer laminate gave advantages in areas of prototype design and manufacture which are described below. Though it should be noted that the processes employed for prototype manufacture may not be those used for production models.

8.2.1 Complex Geometry
Wood laminates describe the internal geometry through a section of the blade as each layer is machined separately prior to assembly. If the bat design required complex internal geometry such as cross bracing or a honeycomb pattern then a laminated construction facilitates the manufacture of such designs. The internal geometry observed in successful sports products such as golf club driver heads and aluminium baseball bats, exhibit an internal construction which often reflects the shape and size of the external shape resulting in a shell like structure, and a laminate construction would facilitate such a design.

8.2.2 Modification of Mechanical Properties
Wood can be modelled as an orthotropic material possessing a degree of internal organisation that accounts for distinctly different properties in three orthogonal directions. As a conventional blade is made from a single piece of willow, the fibre orientation is predominantly in one direction (aligned to the longitudinal axis of the bat), where as fibre orientation within a laminated bat is a function of layer orientation. The layers can be organised within the blade to exhibit conventional fibre orientation or be modified so that fibre orientation is not uni-directional giving rise to modified bulk properties. The blade design can employ layers of different wood species in order to obtain different mass or mechanical properties. For instance, if the cricket bat requires very tough edges to increase durability, layers of a more durable wood can be used along the edges of the bat. Similarly, it would be possible to arrange the layers so that a less expensive wood was used to build the non striking portion of the
blade. In addition, each layer can be inspected prior to manufacture facilitating a degree of control over the apparent quality of the wood used in the construction of the blade.

8.3 MANUFACTURE OVERVIEW

8.3.1 Prototype 1 Bat
Wood laminates were cut into rectangles and the internal geometry describing the hollow through the depth of the blade was laser cut in each layer. The layers were bonded and pressed together to form a rectangular block. The outside shape of the blade was CNC machined from the block and a handle was added to the blade in the conventional manner.

8.3.2 Prototype 2 Bat
A similar procedure was used in prototype 2 bat manufacture except that the external geometry of the front face was not machined in the same manner. Instead, a curved recess was machined along the length of the blade to accept a bat striking face which was constructed separately. The face was bonded to the blade and the machining process was completed.

8.4 WOOD SELECTION

8.4.1 Prototype 1 Bat
A material was required that facilitated a degree of experimentation in the development of the manufacturing process. A plywood was selected due to its greater availability, robustness and geometric stability in comparison to thin wood veneer. Plywood is usually made from layers of rotary cut veneer bonded together. The common adhesives used in their manufacture are phenol-formaldehyde (PF) and urea-formaldehyde (UF) based resins and the primary difference is their resistance to moisture. UF resins are typically suitable for internal use whilst PF resins offer greater resistance to 'weathering'.

A plywood made from three layers of rotary cut Finnish birch, with a density ~820 kg/m³, was selected for prototype 1 manufacture. Contiguous layers in the plywood were mutually perpendicular, bonded with a PF resin, with sufficient strength and dimensional stability to facilitate manufacturing trials. Plywood sheets of dimensions 1828 mm x 2438 mm x 3.2 mm were cut to provide a number of rectangular bat layers.

8.4.2 Prototype 2 Bat
White ash equivalent to that used in MLB bat manufacture was selected for prototype 2 blades. Layers of ~4.5 mm thick ash were sawn from sections of solid wood and sanded to a thickness of ~4.0 mm at the FPRC. A sawing process was used since laminates can be produced with less ruptured fibre or cracks within each layer in comparison to rotary cut veneer which undergoes high stress as the layer is 'peeled' from the trunk. Although a greater percentage of the wood is wasted during the sawing process, it was considered worthwhile since these surface features can act as stress raisers under applied loading [113, private communication FPRC]. Layers were sufficiently stiff and robust to
facilitate manufacture and the $L$ direction of each rectangular layer was oriented predominantly along the $y$ axis of the bat in order to enhance prototype-2 structural rigidity.

8.5 ADHESIVE SELECTION

8.5.1 Prototype 1 Bat

A UF resin (Aerolite KL liquid resin with L38 powder hardener), supplied by Dynochem, was selected to bond the prototype 1 laminates. The risk assessment for a urea based resin was within the scope of safe working practice at the place of prototype manufacture.

The hardener determined the resin penetration into the wood and the temperature dependent rate of curing and under the recommended part ratios, a 'pot-life' and pressing time of $\sim$30 minutes at 20 deg. Celsius was available. The possibility of heat application during pressing was investigated for the reason that the assembly of the laminated block would be a manual process taking an estimated time of 25 minutes. It was anticipated that premature curing of the glue line due to higher room temperatures or errors in the mix ratio would add difficulty to the later stages of assembly. A resin and hardener combination that initiated curing at a higher glue line temperature was considered since significant hardening would only take place on addition of heat. However, the laminated blocks were $\sim$710 mm in thickness and the time required for heat to conduct from hot platens to the innermost glue line was estimated at 45 minutes. This pressing time was beyond the limit of the press machine used in this study. Radio or microwave frequency heating can be used in conjunction with heat conduction to generate quicker and more even heat distribution through thick laminate sections. However, it was anticipated that these heating methods would require a considerable amount of time to establish and were beyond the scope of bat manufacture in this study.

8.5.2 Prototype 2 Bat

A PVA (W3001), supplied by Bondrite Adhesives Ltd., was used in the manufacture of prototype 2 blades. It is a one part adhesive which cures at a slower rate in comparison to the UF resin which simplified the application procedure and allowed for greater assembly time. Since it was anticipated that full bond strength would not be achieved within the allowable pressing time of 30 min. Clamps were used to apply a compressive force on the blade lay-up for an additional 24 hrs period after removal from the press machine.

8.6 PREPARATION OF BAT BLADE GEOMETRY

8.6.1 Internal Geometry

The CAD model of the hollow bat blade was used to generate the required machining geometries. In Unigraphics, the shape of the hollow cavity was subtracted from a rectangular prism representing the laminate block from which the blade would be machined. The model was exported into VisCAM RP software in which it was sliced into discrete layers of a thickness equal to the wood laminate thickness. 2-D CAD 'slice' profiles of the hollow shape were exported into AlphaCAD software and internal profile
cutter paths were defined for each layer. The paths were then post processed to be read by a computer controlled laser cutting machine.

8.6.2 External Geometry
A 3-axis Wadkin CNC milling machine was used to machine the external bat geometry. To facilitate the machining of the blade, a total of four separate surface geometries and corresponding CNC programs were generated. Each surface geometry required a roughing/step-cut stage to remove the majority of material followed by a finishing/contour stage using a smaller cutter to achieve the desired surface finish.

8.7 PROTOTYPE MANUFACTURING PROCESSES
8.7.1 Laser Cutting
Figure 8.1 illustrates the CO₂ laser cutting machine used to create the hollow geometry in each layer of the laminated block. A 2-axis moving bed was computer numerically controlled and cutting was achieved through a stationary laser aperture positioned above the bed. A jig was manufactured to support the layers so they remained flat and in the correct position with respect to the machine datum using two locating pins. An investigation into the correct laser characteristics was undertaken to optimise the cutting performance through the wood. Satisfactory cutting was achieved by pulsing a 75 watts source at 1 kHz through a 1 mm air gap at 1 mm/min bed feed rate. A wood laminate with the laser cut hollow profile is shown in Figure 8.2.

![Laser cutting machine](image)

Figure 8.1 - Laser cutting machine
8.7.2 Layer Assembly

Layers were sanded and rubbed with a cloth \(\sim 1\) hr before assembly to ensure layer surfaces were clean and had high surface activation. The adhesive was applied by a hand roller to one surface of each layer. Glued layers were mated to contiguous layers using two pins which located through two holes drilled from each layer. Figure 8.3 illustrates the UF resin and the application roller and Figure 8.4 illustrates the positioning of glued layers. Figure 8.5 illustrates the shape of the internal bat geometry constructed from the hollow section profiles.
8.7.3 Pressing

The press used in this study, shown in Figure 8.6, was designed to test the compressive strength of concrete blocks and could apply a maximum load of 300 tons. However, it was not designed to apply a constant load for more than a few minutes. The value of compressive force necessary in this study was approximately 1-6% of the full machine capacity and consequently, a 30 minute press time was safely achieved. In addition, the press platen surface areas were smaller than the laminated block. Therefore, two 35 mm thick steel platens were 'sandwiched' between the laminate block and the press to transfer the compressive force over the required area. Two holes were machined into the bottom steel platen to house the two locating pins which were used during the laser cutting and the layer assembly stages. During the pressing operation, the pins ensured that each layer remained correctly aligned.
8.7.4 CNC Machining

Figure 8.7 illustrates the CNC Wadkin machine used to machine the external geometry of the blade from the laminated block. The spindle speed operated from 20-5000 rpm with an axis feed rate of 10 m/min. A tungsten carbide tipped (TCT) 20 mm slot drill and a TCT 15 mm ball nose cutter were selected to perform the roughing stage and the finishing stages, respectively. The laminated block size allowed sufficient region for clamping around its edge to avoid cutter or machine head interference. The block was clamped to a steel plate and located using the steel pins employed during previous stages of production. The back face of the blade was machined first, after which a plastic film was adhered securely to the newly created surface of the blade and the machined walls of the block, shown in Figure 8.8 and wax/plaster was poured into to the cavity, shown in Figure 8.9. Once the wax/plaster had solidified, the steel plate and block were re-mounted onto the Wadkin machine and the wax/plaster was machined flat and flush with the remaining face of the block. The block was turned over and re-clamped and the wax/plaster provided the support for machining the top face of the blade. The blade was be removed from the mould by hand upon completion of the top face.

Figure 8.7 - CNC machining of external geometry

Figure 8.8 - A thin plastic film bonded to the back surface of the blade protects against wax penetration into the wood
8.7.5 Finishing
The blade required smoothing by hand with a wood rasp where the four machining geometries intersected near the shoulder region and all surfaces were given a light sanding. The handle was fitted to the blade by Dunlop Slazenger to complete the prototype hollow cricket bat.

8.8 CONSTRUCTION OF PROTOTYPE 2 BAT FACES
Bat faces were constructed using different wood species, adhesives, numbers of layers, and layer orientation to investigate their effect on impact performance. Faces were manufactured and tested in groups in order that the results from initial tests could be used to influence successive designs.

The first set of faces were made from layers of 1.5 mm birch ply adhered together using a PVA adhesive. The layers were curved to approximately 150 mm radius, prior to gluing using a steaming and rolling process. Four multilayer faces were constructed with two, three, four, and five layers, respectively. Figure 8.10 is a photographic illustration of the birch ply faces constructed in this study.

To investigate a greater range of face thickness and to simplify the bending and laminating processes, the second set of faces were made from layers of 4 mm white ash. Layers were bonded using an MS polymer, with an elastic modulus of 1 MPa, significantly less than for wood, and with an elongation to break of 150-350%. Each layer in every face was orientated at 90 degrees with the wood fibre axis perpendicular to the side support edges resulting in a uniaxial arrangement. Four 150 mm radius faces were constructed with one, two, three and four ash layers, respectively, and two planar faces were constructed each with two layers of 4 mm ash. Figure 8.11 is a photographic illustration of the uniaxial ash face constructions.
The third set of faces were made from segmented layers of 4 mm white ash bonded together with the MS polymer adhesive. Layers were oriented so that the longitudinal axis of contiguous layers were mutually perpendicular resulting in a biaxial arrangement. Figure 8.12 is a photographic illustration of the biaxial ash faces constructed in this study.

Figure 8.10 - Faces constructed from 1.5 mm birch ply laminates and PVA

Figure 8.11 - Uniaxial faces constructed from 4 mm ash layers and MS polymer
8.9 FINISHED PROTOTYPE BATS

Figure 8.13 illustrates the prototype 1 bat made from birch ply laminate and UF resin. A section has been removed from the blade to expose the hollow cavity. Figure 8.14 illustrates the prototype 2-1 bat made from ash veneer and PVA adhesive, which employed the 8 mm biaxial face construction, shown in Figure 8.12. Figure 8.15 illustrates the prototype 2-2 bat made from ash veneer and PVA adhesive which employed the 12 mm-a biaxial face construction, shown in Figure 8.12.
Figure 8.13 - Prototype 1 bat with blade section removed to expose hollow cavity
Figure 8.14 - Prototype 2-1 bat

Figure 8.15 - Prototype 2-2 bat
CHAPTER 9

CRICKET BAT IMPACT TEST PROCEDURE

9.1 INTRODUCTION

An experimental test procedure was developed to investigate the impact between a bat and ball in order to evaluate and compare the performance of hollow and conventional cricket bats. The test was required to reproduce bat and ball impact conditions observed during play and facilitate the measurement of important impact parameters which could be used to determine a measure of bat performance. This chapter begins by presenting performance metrics used in baseball and softball with a description of the standard test procedures used to determine these metrics. Following this review, a cricket bat test method is established and the design of a laboratory rig is described together with the bat testing procedure.

Additional tests have also been conducted using a modified rig to determine the elasticity of a cricket ball and to investigate the performance of different hollow bat face constructions. These tests were developed in conjunction with the spring damper models in chapter 6 and the computer simulation of cricket bat impact behaviour in Chapter 7.

9.2 PERFORMANCE METRICS

In the absence of standardised cricket bat performance metrics, a review of all metrics used to define the striking performance of baseball and softball bats has been undertaken. These sports are of most interest as the implements of the game are similar to cricket as are the conditions in which the two implement come into contact. Furthermore, both sports have well established performance standards and standardised test procedures, which have been adopted by the sports regulatory bodies in order to ensure fair play. The following section describes the performance metrics and the fundamental principles surrounding their derivation and use.

9.2.1 Bat Performance Factor (BPF)

This value is the ratio of COR between a bat and a ball to the COR (e) between ball and a rigid surface, measured using identical ball properties, shown in Equation 9.1. The BPF ratio highlights the effect the bat has upon the elasticity of the impact by indicating an improvement (BPF > 1) or reduction (BPF < 1) in elasticity.

\[
BPF = \frac{e_{\text{ball and bat}}}{e_{\text{ball and rigid surface}}} \quad [9.1]
\]
BPF as a comparative metric for different bats since the value of BPF is dependent upon the ball elasticity. If two identical bats were tested with balls of different COR then the BPF values would be different. To illustrate this point Nathan et al. [6] describes a 'super ball' with a COR = 1 and a 'dead ball' where COR = 0. Impact between a perfectly elastic bat and the 'super ball' would result in a BPF value of 1, whereas with the 'dead ball' the BPF would be infinite.

9.2.2 Ball Exit Speed Ratio (BESR)

The performance factor currently used by the National Collegiate Athletic Association (NCAA) is the Ball Exit Speed Ratio (BESR) and is related to $e_A$, defined in equation 4.22, as:

$$\text{BESR} = e_A + \frac{1}{2}$$  \[9.2\]

This Equation can be re-arranged to find BESR in the form of Equation 9.3, [114].

$$\text{BESR} = \frac{v_{\text{ball}} + \frac{1}{2}(u_{\text{ball}} - u_{\text{bat}})}{u_{\text{ball}} + u_{\text{bat}}}$$  \[9.3\]

where, $u = \text{inbound velocity}$ and $v = \text{outbound velocity}$. Nathan [31] refers to $e_A$ and BESR as model independent metrics since the inertia properties of bat and ball are not required. The values can be used to directly determine the rebound velocity of the ball given that the velocities of both bat and ball are known prior to impact. In addition, both $e_A$ and BESR facilitate the use of a range of laboratory testing procedures since their values are independent of the absolute velocities prior to impact. A variety of testing procedures involving a stationary or moving bat may therefore be entertained provided that the relative speed of contact remains constant.

9.2.3 Batted Ball Speed (BBS)

The rebound ball velocity or 'batted ball speed' following an impact during play can be calculated by determining the performance of the bat in the laboratory ($e_A$ or BESR) and using values of bat and ball speed prior to impact which have been established to represent conditions during play. BBS could also be tested directly in the laboratory however; the speed of the bat and ball prior to impact would have to be equivalent to the established field values in order to produce a valid BBS measurement.

Equation 4.22 can be rearranged to give BBS, a more symmetrical form of Equation 9.4 can be made by substituting the value of BESR, in Equation 9.2, into Equation 9.4, in the form of Equation 9.5.

$$\text{BBS} = e_A u_{\text{bat}} + (1 + e_A) u_{\text{bat}}$$  \[9.4\]

$$\text{BBS} = (\text{BESR} - \frac{1}{2}) u_{\text{ball}} + \frac{1}{2} \text{BESR} u_{\text{bat}}$$  \[9.5\]
COR and BPF are measures of the elasticity of impact between bat and ball. To determine BBS from these metrics requires values for the inertia of the bat and ball. Substituting Equation 4.23 into Equation 9.4 results in a relationship for BBS in terms of COR.

\[
BBS = \left( \frac{e-r}{1+r} \right) \cdot \frac{1}{u_{ball}} + \left( \frac{e+1}{1+r} \right) \cdot \frac{1}{u_{bat}} \tag{[9.6]}
\]

It is worth noting that BBS is more sensitive to bat speed than ball inbound speed because the bat is initially travelling in the same direction as the post impact direction of the ball. This is apparent from Equations 9.4 and 9.5 and the effect is enhanced if the collision efficiency is low. In the extreme case where \( \varepsilon_a \to 0 \) the BBS approaches a value equivalent to the bat speed and the influence of ball speed is negligible. Therefore, BBS is not solely dependent upon the efficiency of impact but is significantly affected by the bat swing speed.

### 9.3 BASEBALL AND SOFTBALL BAT PERFORMANCE STANDARDS

The goal of a performance test is to undertake measurements in a laboratory that can be used to accurately predict the playing performance of the bat in the field. A realistic comparison between bats can therefore be made and the maximum bat performance can be measured against performance limits defined by the rules of the game.

The American Softball Association (ASA) have been the national governing body of softball in the United States since 1993. Its duties include regulating competition and upholding universally accepted rules of play. The ASA first adopted a bat performance standard in 2000 based upon the American Society for the Testing of Materials (ASTM) standard F1890 [50] to determine both BPF and BBS. In 2004 the ASA adopted a more recent standard but, other softball associations that also regulate the game including the United States Specialty Sports Association (USSSA), National Softball Association (NSA) and International Softball Federation (ISF) continue to use ASTM F1890.

#### 9.3.1 ASTM F1890 - Softball Bat Performance

ASTM F1890 describes an experiment to measure the inbound ball speed and rebound bat speed. COR of bat and ball impact is determined without measuring an outbound ball speed. Instead, the rule of conservation of momentum is used to derive the ball rebound velocity from the velocity of the bat by substituting the expression for \( v_{ball} \) in Equation 9.7 into the conservation of linear momentum expressed in Equation 9.8 resulting in a solution for COR independent of \( v_{ball} \) as shown in Equation 9.9.

\[
v_{ball} = e \cdot \frac{u_{ball} - v_{bat}}{m}\tag{[9.7]}
\]

\[
m \cdot u_{ball} = m \cdot v_{ball} + m \cdot v_{bat}\tag{[9.8]}
\]

\[
\varepsilon = \left( 1 + \frac{1}{m \cdot R^2} \right) \left( \frac{u_{bat}}{u_{ball}} \right) - 1\tag{[9.9]}
\]
where:

\[ m_e = \frac{I_{\text{pivot}}}{R^2} \quad \text{(for pivoted bat)} \]  

\[ R = \text{Distance between pivot and impact position (conjugate pair)} \]

\[ I_{\text{pivot}} = \text{Bat MOI about the pivot} \]

\[ u_{\text{ball}} = \text{Initial ball velocity} \]

\[ v_{\text{bat}} = \text{Bat rebound velocity at location of impact} \]

The experiment is arranged as shown in Figure 9.1. A cannon is used to accelerate the ball to 22.8 m/s (60 mph) which impacts the softball bat clamped 154 mm (6 in.) from the end of the handle to a pivot assembly. The location of the assembly is adjusted relative to the cannon so that the ball impacts the bat at the COP determined prior to testing. This ensures that the pivot is co-incident with the free body point of rotation of the bat and does not significantly affect its behaviour. Light/speed gates are mounted to record the inbound ball velocity at the exit of the cannon and also the angular velocity of the bat which is later used to determine the speed of the bat at the location of impact. BPF is solved as in Equation 9.1 using a predetermined value of COR, the measurement of which is standardised in ASTM F1887 [47]. BBS is also calculated for established pre-impact conditions using Equation 9.6.

9.3.2 ASTM F1881 - Baseball Bat Performance

Prior to 2004 the NCAA, responsible for certifying baseball bats as legal for use in high school and intercollegiate baseball, certified bats based upon ASTM F1881 [48]. This standard is similar to ASTM F1890 [50] and was used to determine BPF and BBS for baseball bats. A further testing development was the Baum Hitting Machine™ used to swing a bat at 68 mph towards the inbound ball thus increasing the relative contact speed.
9.3.3 ASTM F2219 - Updated Bat Performance

In 2004 ASTM F2219 [115] was developed following concerns over the validity of both softball and baseball certification standards F1890 and F1881. The concern related to the impact position defined as the conjugate percussion point to the bat pivot when mounted in the pivot assembly, 6 inches from the knob. Brody's work [27] had determined that a COP impact point relative to an arbitrary point on the handle does not constitute the point of maximum performance, a fact supported by Russell [116]. In addition, it was discovered that minor alterations to the mass distribution prior to testing could significantly affect the outcome of the testing result [116]. The testing procedure detailed in ASTM F2219 is similar to previous standards except that it provides two alternatives for the location of the bat and ball collision in addition to the COP location. The first alternative location is 154 mm (6 in.) from the barrel end of the bat and remains fixed, independent of COP position for all bats. The second requires that multiple impact locations are used to identify the maximum value of performance. This is achieved by using the initial impact location 154 mm from the barrel end and repeating the test at 25.4 mm intervals. The repeated tests are pursued in a direction along the length of the barrel that results in an increase in performance value until a local maximum is found [51]. Finally, the test is repeated halfway between the two intervals corresponding to the two highest values obtained in order to establish, more accurately, the location and value of the greatest performance. It is interesting to note that the standard does not require the adjustment of the bat pivot point location to the COP for each impact location. Throughout the test the pivot point remains fixed at 154 mm (6 in.) from the handle knob of the bat.

9.3.4 ASTM F1888 - Baseball/Softball Compression

Since the measured performance of the bat is dependent upon the properties of the ball. The ASTM bat performance standards require that all balls used in testing conform to specific standards relating to mass, size, stiffness and COR properties. The latter two properties are measured using ASTM standards. ASTM F1888 [49] describes a procedure to measure the 'stiffness' of the ball using a compression test. The ball is placed between two flat platens in a force-displacement test machine and the force required to achieve a reduction in diameter of 6.35 mm (0.25 in.) is recorded. The force required for an approved ball must be between limits defined within the bat performance standards. According to ASTM F2219 the values are 1335-1668 N for baseball and 1557-1668 N for softball.

9.3.5 ASTM F1887 - Baseball/Softball COR

ASTM F1887 describes the procedure for measurement of ball COR using a ball launcher to propel the ball towards a rigidly held target. The inbound and outbound velocities for 6-12 impacts that rebound normally within a specified tolerance are recorded and averaged to obtain a value of COR. The standard permits the use of either steel or northern white ash to act as the impact surface of the rigidly held target. The COR of an approved ball must be between limits defined within the bat performance standard. According to ASTM F2219 the values are 0.525-0.550 for baseball and 0.430-0.440 for softball, for an inbound velocity of 26.82 m/s (60 mph).
9.3.6 BS 5993:1994 - Specification for Cricket Balls

This British Standard (BS) specifies the requirement for various grades and sizes of leather covered cricket balls and is intended to provide guidance to the purchaser as to the acceptability of the grades for given participation levels or uses within the game [117]. The standard specifies materials and methods of construction, static performance specifications, such as ball circumference, mass and seam dimensions and the dynamic performance characteristics including COR, 'hardness', impact resistance and wear resistance. The 'height of bounce' test determines ball elasticity by measuring the ball rebound height from a 2 m drop test (6.26 m/s $v_{in}$). The allowable rebound height is 28%-38% of the initial drop height, which corresponds to a COR range of 0.530-0.617. This range is 3.5 times the allowable range for baseball and due to the dependency of COR on $v_{in}$, the values may not be representative of field conditions. 'Hardness' is measured as the deceleration g, of a flat steel striker which is dropped from a height of 1.1 m onto the ball placed upon a solid concrete base. Upon impact, the ball is effectively 'sandwiched' between two bodies and the resulting deceleration, measured with an accelerometer attached to the striker, provides an indirect measurement of the combined dynamic stiffness and damping properties of the ball.

9.4 IMPORTANT CONSIDERATIONS IN ESTABLISHING A TEST PROCEDURE

9.4.1 Characterising Performance Over the Striking Region of the Bat

ASTM F1890 and F1881 measure of performance of the softball and baseball bat under laboratory conditions at a single location on the bat. ASTM F2219 allows for several measurements to be undertaken at discrete positions along the length of the striking portion of the bat in order to obtain a maximum value. The recorded value is important since it is used to predict the performance of the bat during play conditions and to determine whether the bat adheres to performance limits. Figure 9.2 illustrates the general relationship between bat performance and impact location measured as a distance from the barrel end of the bat, for two bats (revised from Russell [116]).

![Figure 9.2 - Relationship between COR and impact location for two baseball bats](image-url)
The performance characteristic along the barrel for both bats is very similar. However, a small difference in the mass distribution of bat 'b' results in a difference in the location of its COP relative to bat 'a', as defined by Equation 4.17. If both bats are tested at the COP in accordance with ASTM F1890 or F1881, bat a would be deemed illegal whilst bat 'b' would be certified for play since the performance limit exists between the two values. The fact the maximum performance values for both bats are beyond the limit would only be discovered if the test was repeated at multiple positions along the length of the barrel as provided in ASTM F2219.

This example illustrates the limitation of a single impact position for measuring and predicting performance. The use of multiple impact positions not only results in a more accurate determination of maximum performance but also characterises the performance of the bat along the its striking region, providing a more valuable and comprehensive assessment.

9.4.2 Bat Support Conditions
The bat support condition affects the post impact response of the bat and can determine how performance is calculated. A determination of impact efficiency or elasticity does not require that both bat and ball velocities are measured after impact provided that the inertia properties of the both bat and ball are included in the calculation. In ASTM F1890 and F1881 the post impact velocity of the ball is not measured and therefore the speed of the bat is required together with its inertia properties in order to determine both efficiency and elasticity.

If ball rebound speed is not measured, the calculated value of performance is dependent upon the post impact behaviour of the bat and therefore upon its support condition. Measurement of the bat post impact velocity for any supported condition is problematic since a support, by definition, will affect the motion of the bat. For any support condition the measured value of COR is lower than that observed from a bat with no constraint and in the extreme case of a fixed support, only the vibration modes of the bat would remain resulting in a post impact velocity of zero.

The exception is a pivot support which is located at the conjugate percussion point to the impact. In this instance, there is no rigid body reaction force at the pivot and the support does not influence the rigid body motion of the bat. However, the flexural modes of the bat will remain affected. Using a pivoted arrangement, the testing of multiple impact positions on the face of the bat would require adjustment of the pivot point to the conjugate percussion position for each impact location in accordance with Equation 4.17 so that the rigid body response of the bat remains independent of the support. Figure 9.3 illustrates the upper and lower limits of the blade region that can be tested if the pivot mechanism was designed to accommodate the handle. Where:

\[ L = \frac{I_{\text{COM}}}{md} \]  

[9.11]
Figure 9.4 illustrates the location of the pivot point corresponding to impact locations outside of this region. To measure impacts illustrated in Figure 9.4a and b would require an external attachment to the bat. Whereas, the bat in Figure 9.4c would require a pivot point through the blade portion. Figure 9.5 illustrates the region of the striking face which could be tested using a pivot mechanism designed for the handle. However, it is only approximately 37% of the available striking region of the blade.

Figure 9.3 - Conjugate points on the striking face corresponding to the limits for pivoting about the handle.
9.5 DIFFERENCES IN BAT PERFORMANCE BETWEEN LABORATORY AND FIELD

9.5.1 Swing Speed

If bat impact performance during play is defined by the exit speed of the ball then, according the Nathan [31], there are three contributing factors which determine performance. They are the elasticity of the impact, the ball and bat inertia properties and the speed of the bat at the location of impact. However, the inertia properties of the bat also affect the player’s ability to generate swing speed and therefore the effect of bat inertia on field performance can be more complex than perceived under laboratory test conditions.
Nathan [31] suggests that if a player could generate a swing speed independent of bat swing MOI, the advantages of 'end loading' the bat, by distributing weight further towards the barrel end, would be significant. Optimum \( \theta_f \) for an 'end loaded' bat would be located further from the handle corresponding to a higher impact velocity, due to the rotation of the swing and this would result in a higher BBS.

Nathan suggests that in reality, swing speed is dependent upon MOI about the instantaneous pivot. As a result, players often achieve a slower swing speed with 'end loaded' bats reducing the effectiveness of this concept. Equally, a player would be able to generate faster swing speeds using a bat with a more even mass distribution (lower swing MOI). However, the location of greatest performance efficiency would also be located (more evenly) towards the handle end of the bat where the impact speeds are slower thus, reducing the effectiveness of this concept for improved bat performance. This example demonstrates that a measurement of bat performance undertaken in a laboratory may not translate directly into observed performance during play, due to the complex interaction between player and bat. Currently no baseball or softball regulatory bodies vary the certification inbound bat speed based upon the inertia properties of the bat.

### 9.5.2 Support and Grip

A further difference between laboratory and field conditions concerns the player's grip on the bat and its possible effect on bat performance. As briefly mentioned in Chapter 4, a study by Brody [46] investigated the importance of grip constraint on the response of the bat following impact. Brody used both wooden and aluminium baseball bats in the study which aimed to verify that a hand-held baseball bat behaves at impact as if it were a free body. This assumption facilitates the investigation of dynamic performance by negating the inclusion of grip variation in the analysis of results. The principle adopted to absolve the free-body model concerned the different vibration characteristics that can be observed between a bat whose handle is clamped and another whose ends are not constrained. Brody described unique modes of vibration dependent upon clamping condition, which he observed experimentally. Brody conducted a further experiment with a bat gripped in the hands. Modal analysis of consequent impacts illustrated similar vibration between a gripped bat and one that is unconstrained and the tightness of the grip appeared to have no effect upon the modal characteristics. However, a tight grip did increase the rate at which subsequent vibrations were attenuated. It was concluded therefore, that grip firmness should not influence the post impact velocity of the ball but speculated that grip is an important factor in terms of controlling the swing.

### 9.6 SUMMARY AND DEFINITION OF CRICKET BAT TEST PROCEDURE

The manner in which the bat is constrained and the method of achieving high speed contact with the ball experimentally are important as they determine how the performance metrics are measured as well as their validity in evaluating playing performance.
Both the 'pivoted handle' and the 'free handle' condition can be used to measure impact elasticity and efficiency directly from experiment provided that both bat and ball inbound and outbound velocities are measured. To simplify the test procedure, ASTM standards derive the performance metrics by including inertia properties in the calculation. Consequently, only one outbound velocity measurement is required and the ASTM standards measure the bat velocity. The pivot mechanism facilitates this measurement since the bat follows a fixed rotational path around which light/speed gates can be mounted. The speed of the bat at the location of contact can then be determined. In contrast, the direction of ball travel after impact is not constrained and so the measurement of velocity may prove to be more difficult.

A free condition allows for the entire region of the striking face to be tested using the same initial support conditions, since the location of the conjugate point to impact is not relevant to the design of the rig. With regard to computer simulation of bat impact performance, discussed in chapter 7. A bat with no constraint allows for a more simple comparison between simulated and experimental impact behaviour since it does not require the additional modelling of the constraint and its potential influence on bat motion.

Impact testing a bat with no constraint to motion after impact does generate its own difficulties. There are issues regarding how to physically create a free support condition whilst controlling bat alignment and impact position. As both bat and ball have unconstrained freedom after impact, there are issues regarding the safety of the experimental test in terms of potential damage to bat and ball and the test rig itself. The capability to increase the relative speed of impact with an initial bat speed is more difficult when a free condition is required at impact since a form of constraint be would necessary to accelerate the bat. The use of light gates to measure the post impact velocities of bat and ball may prove problematic as the motion is free and therefore susceptible to variation.

A stationary bat provides a simpler solution but the relative speed of impact, which should be representative of field conditions, is limited by the ability of the ball launching system. It is important since impact elasticity is dependent upon the relative contact speed. Results of experimental tests described later in this chapter have shown that COR decreases linearly with increasing $v_{rel}$. The strength of this dependency determines how exact laboratory contact speeds must reflect those obtained in the field to produce performance data that is equally valid in laboratory and field conditions. If bat COR and $e_d$ properties are measured at two or more discrete values of $v_{rel}$ then a linear relationship can be used to predict values of COR and $e_d$ for values of $v_{rel}$ other than those used in laboratory testing.

Despite practical difficulties in achieving a test method with no post impact constraints. It is argued that it can provide a greater depth of information than can be gathered using a constrained bat. The 'no constraint' method requires the use of a measurement device such as a high speed camera to elicit
the free body motion of the bat so that the speed at the location of impact can be derived. A camera can also provide visual information on the resulting bat vibration. Additionally, it can be used to measure the outbound velocity of the ball, eliminating the need to derive performance metrics using the inertia properties of the both objects.

Following a review of current performance standards and the consideration of important testing principles, the cricket bat test method used in this study adopted an initially stationary bat which was released from all constraints prior to impact with a ball propelled from a ball launcher. The velocity of the ball prior to impact and the velocity of both bat and ball after impact were derived from high speed video images, recorded over an appropriate duration. Performance metrics were then derived directly from the velocity values obtained. The test was conducted at 19 m/s and 31 m/s \( v_{\text{in}} \) and at nine discrete positions along the bat face to characterise the majority of the striking region. The following section describes the design of the test rig and the testing procedures.

9.7 TEST RIG DESIGN AND OPERATION

9.7.1 Impact Test Rig - Version 1

The first version of an experimental bat testing rig was developed to test and prove rig components and to assess the overall practicality in achieving the defined test procedure. Once the rig had been developed and provided an accurate and repeatable impact scenario, it was used to test one conventional willow bat and a prototype 1 hollow bat. In addition, initial ball impact testing and off-centreline bat spin tests were undertaken.

Figure 9.6 shows a picture of the first test rig in its entirety and Figure 9.7 illustrates a simplified side view. The rig was constructed from aluminium extrusion with polycarbonate sheeting to provide an enclosed but transparent cage. The bat was initially stationary, supported at the top of the handle by two adjustable swing arms shown in Figure 9.8. As both arms were pivoted, the attachment did not exert any lateral constraint and facilitated the alignment of the stationary bat prior to contact. The bat gantry was vertically adjustable to enable ball impact tests across a range of impact locations on the bat face.

A Bola machine was used to propel a cricket ball horizontally, at a fixed height, through a set of light gates to impact the bat within the enclosure. The Bola machine propelled the ball using two counter-rotating wheels whose rotational speed was independently controlled via two electric motors. The ball was fed between the wheels and the friction between the ball and the rubber wheel surfaces allowed for a transfer of kinetic energy to the ball. The ball entered the experimental viewing area via a circular opening in the polycarbonate sheeting. Upon impact, the bat was released from the support and travelled towards the right hand end of the cage, eventually making contact with the impact cushions. For the majority of bat impact locations, the ball rebounded towards the light gates. However, contact with the bat introduced small deviations in the normal rebound of the ball and as such, the ball would
not engage the circular opening with sufficient accuracy to pass back through and therefore the ball remained within the enclosure.

Figure 9.6 - Rig version 1: a photograph of experimental setup

Figure 9.7 - Rig version 1: a side view illustration
9.7.2 Impact Test Rig - Version 2

A second version of the rig was developed to re-address issues of bat support and alignment with a view to improving upon the accuracy and repeatability of the test. The second rig was used to complete the testing of the ball, to test bat hollow face constructions and measure the performance of two further willow bats and two prototype 2 hollow bats. Figure 9.9 is a picture of the second rig in its entirety and Figure 9.10 illustrates a simplified side view. The rig was split into two separate sections dividing the ball launcher and the impact enclosure. The impact enclosure section was bolted to a concrete floor to eliminate movement from impact and the ball launcher section was made independent to facilitate ball alignment and to isolate the enclosure from undesirable vibrations generated by the launcher mechanism. The ball launcher was developed as part of an ongoing Ph.D. by Laura Justham at Loughborough University, concerned with automatic training devices in cricket. The machine uses the same principle as the Bola to impart a ball velocity however, both wheels are computer numerically controlled. A barrel was incorporated into the rig design to improve upon the accuracy and consistency of the ball trajectory.

The bat handle support condition was improved with a solenoid actuated mechanism which released the bat a few milliseconds before impact, eliminating all support interference. The support mechanism was mounted to the bat gantry through a bearing which allowed the mechanism to align with the principal vertical axis of the bat, shown in Figure 9.11.

The term 'bat orientation' refers to the angle which the bat face presents to the inbound ball. For the purposes of the test, the impact should be 'normal' and so the face normal at point of impact should be perpendicular to the direction of inbound ball travel. Prior to testing, the barrel laser was used to align
the two halves of the rig. A flat plate was fixed square to the rolling gantry and a mirror attached to the plate surface reflected the laser beam onto a paper target that was used to adjust the lateral position and angle of the ball launcher section. The orientation laser and centreline laser were then aligned to assist in the correct orientation and positioning of the bat during the test.

Figure 9.9 - Rig version 2: a photograph of the experimental setup
Figure 9.10 - Rig version 2: a side view illustration

Figure 9.11 - Rig version 2: the bat support
9.8 MEASUREMENT OF OBJECT MOTION

9.8.1 Measurement Devices

The measurement of bat, bat face and ball motion prior to and subsequent to impact involved several devices which are briefly described in this section.

Light/Speed Gates

Light gates were used to directly measure the inbound velocity of the ball. The instrument consists of two light emitters and two receivers. Each gate includes one emitter which is paired to a receiver. They are aligned across an air gap to allow an object to pass in between and disrupt the light signal to the receiver. Both gates are mounted parallel to each other at a known distance apart (~200 mm) and the time taken for consecutive receivers to measure a disruption from their respective light source can be measured using a timer, enabling the ball velocity to be calculated to an accuracy ~0.01 m/s.

Vibrometer

A vibrometer was used to measure bat face deflection. The instrument uses the Doppler effect to measure the velocity and displacement of a moving target object by measuring the change in phase and frequency of a laser beam reflected off the vibrating object. The vibrometer is mounted on a tripod, in a similar fashion to a camera and the laser beam is positioned and focused on a single point of interest on the target. A decoder in the vibrometer outputs a voltage which is proportional to the measured velocity parallel to the axis of the beam. A LeCroy 9314CL oscilloscope was used to convert the analog voltage signal into a digital format at a sample rate of 50 kHz. Subsequently, a Matlab® program was used to analyse the digital signal to obtain values for face deflection.

High Speed Camera

Image analysis of high speed video (HSV) data was used to determine both the rigid body and deformable behaviour of bat and ball. The principles of capturing the image and extracting experimental data are similar for both areas of investigation and measurement of rigid body motion will be used to illustrate the principle involved. Two cameras were used because of availability problems.

Experimental test rig, version 1, was used in conjunction with a Kodak 4540 high speed video camera; a black and white digital camera with a maximum frame rate of 44,000 frames per second (FPS). A frame rate of 4500 FPS was sufficient to capture the rigid body motion of the bat and ball which was recorded at a maximum resolution of 352 x 288 pixels. Experimental test rig, version 2, was used in conjunction with a Photron Fastcam Ultima APX 120K camera capable of 100,00 FPS. Using this device, the rigid body modes were captured in black and white at 6000 FPS with a resolution of 512 x 512 pixels. Although there are differences in frame rate and image quality between the two cameras, the method used to extracting a measurement of object motion is the same.
9.8.2 Bat Impact Test Measurements

Bat markers were fixed to the bat to assist in the measurement of bat position and orientation. The markers were constructed using four black plastic spheres, 6 mm in diameter, fixed at the ends of two lightweight aluminium tubes. Two holes were drilled at known locations through the blade of the bat, normal to the front face, and the tubes were fixed through the holes so that their midpoints were coincident with the face.

A normal impact on the centreline of the bat face resulted in a planar motion of both objects. The camera was positioned so that the plane of motion was parallel to the captured image within the field of view. The motion of the objects could therefore be successfully described using 2-D measurement of the captured image.

Digital images were recorded in video format and imported into the imaging software package Image Pro Plus version 5.1 (IPP). The software was used to obtain the $x$ and $y$ pixel co-ordinate positions of the ball centre and the four bat markers. Figure 9.12 illustrates a high speed image shortly before impact. The software provides various tools that are available to assess the properties of individual or groups of pixels. In this example, a circumference tool has been used to determine the position of the centre of the ball and a single pixel tool has been used to measure the position of the bat markers.

Four images were extracted from the video and the position of the bat and ball features in each image were determined. Figure 9.12 is an example of the first image, in which the ball has just entered the impact enclosure. To determine ball speed using high speed video, a second image was required just prior to impact as shown in Figure 9.13. Similarly, to determine outbound motion, an image just after impact, Figure 9.14, and an image showing significant displacement, as in Figure 9.15, was required. The final image was selected from the video showing the maximum displacement of the bat and ball prior to interference or obstruction by the enclosure. For impacts locations near the bat COM, the ball motion is significantly greater than the bat and for impacts near the toe, the opposite is true. In either case, two final outbound images are required to maximise the displacement of each object.

The inbound and outbound velocities of the ball were calculated by dividing the ball inbound and outbound displacements in a direction normal to the bat face by the difference in time between the images. The relative and absolute positions of the four bat markers were used to determine the angular rotation and translation of the bat about the bat COM. The outbound velocity of the bat at point of impact was calculated as the resultant velocity of bat rotation and translation at the location of impact. In addition, the distance between the markers at either end of the same marker tube was used to derive a value of angular displacement through the polar axis of the bat. From this measurement, bat polar spin rate was calculated to assess the accuracy of ball impact location, described later in this section.
9.8.3 Ball Impact Test Measurements

For COR measurements, balls were normally impacted against a rigid and constrained steel plate. Values of inbound and rebound ball velocity were measured to determine COR. The camera was positioned so that the recorded image was parallel and in-line with the plate surface and images captured at 10,000 FPS and 512 x 256 pixel resolution. Additionally, contact time and maximum lateral deformation were measured to assist in the validation of computer simulation, discussed in chapter 7. Figure 9.16a-e shows a typical impact between a leather covered cricket ball and the plate and illustrates the measurements taken using the IPP software to determine ball properties. Inbound velocity was derived from the distance travelled by the ball between Figure 9.16a and b. Maximum ball deformation was determined from the measurement of ball diameter in Figure 9.16c. Contact time was measured as the difference in time between Figure 9.16b and d which illustrate the ball making contact and ending contact with the plate, respectively. Outbound velocity was determined from the distance travelled by the ball between Figure 9.16d and e divided by the difference in time.
9.8.4 Hollow Face Testing Measurements

The test measured the impact properties for different bat face constructions using the experimental setup shown in Figure 9.18. Bat faces were fixed to the 'bat face support' which was rigidly clamped to the frame of the test rig. The internal cut-out profile of the support was shaped identically to the internal support given by the hollow prototype 1 and 2 blade designs. This allowed for the realistic movement of the face from impact, based upon the correct support geometry. Since the face support was fixed, the impact properties of each face construction were assessed without the additional effect of the bat blade and handle. The impact position of the face relative to the underlying support was equivalent to the impact location P7 (698 mm from the handle top), equivalent to a location of enhanced hollow bat impact performance.

The high speed camera was set-up to capture a similar image to that used in ball testing and is illustrated in Figure 9.17a-e. The deflection of the face local to ball contact made measurements of maximum ball deformation and face deflection difficult to ascertain from the images alone since the region of ball contact clearly visible against a steel plate surface in Figure 9.16c, became partially obscured, as in Figure 9.17c. The measurement illustrated in Figure 9.17c is the product of ball deformation and face deflection. A vibrometer was setup as shown in Figure 9.18 to record the movement of the back surface of the face at a location co-incident with the point of impact and along the axis of inbound ball travel. A value of ball deformation was determined by calculating the difference between total ball and face deflection measured using the high speed camera and the vibrometer measurement of face deflection. Since the vibrometer reading was taken from the back face, the calculation was based on the assumption that the compression of the bat face material was significantly less than the value of ball deformation due to the relative values of material stiffness (Ball ~90 MPa, white ash $E_t \sim 620$ MPa).
Figure 9.16 - HSV: cricket ball impacting steel plate

Figure 9.17 - HSV: cricket ball impacting thin bat face
Figure 9.18 - Experimental setup for hollow face testing

9.8.5 Measurement of Off-Centreline Impacts

This experiment measured the motion of the bat and ball following a ball impact that was not coincident with the vertical centreline of the bat. The polar rotation rate, change in angle between ball inbound and rebound direction and the direction of post impact bat translation was measured as shown in Figure 9.19.

Initially, the height of the bat was adjusted to locate the ball impact at a point co-incident with the bat COM. The Kodak HSV camera was positioned to capture a side view of the impact enclosure as shown in Figures 9.12 to 9.15 and adjustments were made to the location of ball impact to eliminate bat rotation about the COM from the camera’s perspective. To measure the polar rotation of the bat, the camera was re-positioned directly above the stationary bat as shown in Figure 9.20. Figure 9.21 illustrates still images from video data identifying the important object features from which subsequent measurements were taken.

The test procedure involved translating the position of the stationary bat in the y direction to create an offset between the ball centre and the centreline of the bat. The offset was increased by increments of 10 mm until the ball centreline impacted beyond the edge of the bat. Five impacts were repeated for each offset at a ball inbound velocity of 13.4 m/s (~30 mph).
PRIOR TO IMPACT

AFTER IMPACT

Figure 9.19 - Measurements of bat spin and bat and ball rebound angle following an off-centreline impact.

Figure 9.20 - Setup of impact enclosure for measurement of bat spin.
Figure 9.21 - Off-centreline impacts: significant still frames taken from high speed video data (Kodak 4540 HSV)
9.9 MEASUREMENT ACCURACY AND RESOLUTION

The alignment of the bat, ball and measurement equipment is an obvious source of potential error. Significant effort was made to ensure the bat, ball and measurement devices were aligned prior to testing using laser pointing devices. In addition to error associated with alignment there are other issues that concern the ability to extract valid measurements from the tests described. The following section briefly explains the limitations to accuracy and resolution imposed by the equipment and describes the manner in which their effects were minimised.

9.9.1 Vibrometer Measurement

The vibrometer records the velocity of an object parallel to the measurement beam. During the cricket bat face tests, the measurement beam was aimed normal to the back surface of the hollow face, coincident with the intended ball impact location on the front surface. However, small deviations in the location of each impact were observed as a result of the ball firing mechanism. To quantify the accuracy of the machine, a thin coating of coloured wax was applied to the ball prior to testing. The area of contact on the bat face was identified from the transfer of wax material from the ball to a paper film which was adhered to the front surface of the bat face. The distance between the vibrometer target location and centre of each impact was then measured and used to assess the validity of each vibrometer reading based upon the proximately of the vibrometer target to the impact location.

9.9.2 HSV Camera Measurement

The HSV camera generates a 2-D video image from which distance measurement can be extracted. The validity of this method depends upon how accurately the distance determined from the 2-D image reflects the real distance travelled by the object. As previously discussed, the alignment of the camera image normal to the intended measurement plane is important. Equally, the bat and ball must remain co-incident to the intended measurement plane over the recorded duration. The following section describes the sources of camera measurement error and the manner in which their effects were minimised.

Object Deviation

The test rig was equipped with one camera providing a single 2-D recording. It was difficult to detect whether the bat or ball had deviated from the intended plane of motion since any movement perpendicular to the plane of image would not be measurable as shown in Figure 9.22. The experimental study of bat polar rotation highlighted that deviation perpendicular to the side view image occurred due to off-centreline impacts. Therefore the results of this study were used to determine a relationship between bat polar rotation rate and bat and ball deviation as shown in Figure 9.23. During the bat performance study, values of bat polar rotation were measured from the side image view and corresponding values of deviation were determined from the established relationship. Impacts which demonstrated a level of bat polar rotation corresponding to an impact location greater than 2 mm from the bat centreline were discarded prior to analysis due to error associated with bat and ball rebound angle and bat rotational energy. The second rig provided the greatest control over the impact location.
resulting in ~ 70% of all collisions satisfying the criterion.

Figure 9.22 - Measurement error due to post impact bat deviation

Figure 9.23 - The relationship between rate of bat polar rotation and the bat and ball deviation from the intended plane of motion

Image Distortion, Resolution, and Human Error

The image formed by a camera lens was not an orthographic projection of the intended plane of motion but included distortion due to perspective and the curved shape of the camera lens. Perspective, characterised by the positions of the lens relative to the intended view, was minimised by positioning the camera at the furthest distance from the enclosure within the confines of the laboratory (~3 m). Lens distortions were minimised by operating with the correct lens within a narrow field of view. Errors associated with the resolution of positional measurements were dependent upon the resolution of the digital image and it is conceivable that human error was introduced during the manual extraction of pixel co-ordinate data.

A test was conducted in order to establish the magnitude of error incurred due to these factors. A calibrated screen image was captured with the HSV camera using the same distance, alignment and camera and lens settings used during the bat performance experimental study. The screen was made
to the same size as the intended field of view and five arrays consisting of nine markers were positioned at either corner and in the centre of the image. The image was captured and the manual extraction of every marker position from the screen was repeated five times. The measurement of distance between the points in each corner array was compared to the distance measured between the points from the centre array to establish the relative image distortion, since the effects of both perspective and lens distortions are greatest at the extremes of an image. In addition, the human and resolution errors incurred in extracting distance measurements from the image were determined by comparing values obtained to the real distance between the markers from the central array. Figure 9.24 illustrates a captured image of the test screen.

The test was repeated five times and the results were averaged. Measurements of the central array gave a point marker spacing of 100 mm, identical to the real distance. The spacing between marker points in the bottom right array generated greatest error with an average measured distance of 98 mm. Therefore, the magnitude of error incurred during the test was approximately 2% which was deemed an acceptable level for image analysis.

Figure 9.24 - HSV: test screen used to assess image distortion, resolution and measurement errors
10.1 BALL IMPACT PROPERTIES AND SELECTION

10.1.1 Test Strategy

It was necessary to choose a suitable ball for bat testing and the properties of five different balls were investigated using the ball impact test method described in the chapter 9. The range of balls included three leather covered cricket balls, marketed as having a different quality of material and construction, and two hockey balls. The hockey balls were of similar mass and size to the cricket balls but were of a moulded construction, were exempt of the seam feature, and exhibited greater sphericity than the cricket balls. The similarity of the hockey balls' dynamic properties to that of a leather covered cricket ball was investigated to ascertain whether they could provide a more consistent alternative for use within the bat performance testing procedure. Measurements of COR, contact time and maximum lateral deformation were determined for a range of inbound speeds (8 m/s - 36 m/s) and the results are shown in Figures 10.1 to 10.3.

10.1.2 Results and Discussion

Figure 10.1 illustrates a weak dependence of ball COR on impact speed which reduces at approximately $5.7 \times 10^{-3}$ COR per m/s for all balls. There is no significant difference between the COR properties of the different leather covered cricket balls. The hockey ball with a cork centre exhibits slightly greater COR over the range of impact speeds tested with an average increase of $8.6 \times 10^{-3}$ COR over the leather covered cricket balls. The plastic hockey ball exhibits significantly lower COR relative to the leather covered balls with an average difference of 0.06 COR present over the range of impact speeds tested.

The plastic hockey ball COR results were significantly lower than those for the leather cricket balls and so the ball was deemed unsuitable for bat testing. The hockey ball with a cork centre showed COR and contact times that were more representative of a leather cricket ball. However, values of maximum deformation at 31 m/s highlighted a difference in ball stiffness. The spring damper model results illustrated the importance of the relative stiffness between bat and ball to the outcome of impact elasticity. Therefore it was decided that leather covered cricket balls would be used during bat testing to ensure the validity of experimental results in predicting playing performance.

Figure 10.2 illustrates a significantly linear relationship between maximum lateral deformation and impact speed over the experimental range and Figure 10.3 shows that contact time is largely independent of inbound velocity. This data validates the use of a linear spring component to model the ball stiffness properties in chapter 6. Since, for a linear spring the force is proportional to amplitude of
displacement and the time period of an oscillating linear spring mass system is independent of its amplitude.

Experimentally determined relationships for deformation and contact time exhibit approximately linear relationships over the range of inbound velocity tested. Although a Hertzian model may provide a more accurate prediction of ball behaviour at low values of velocity. The observable trends from experimental results gives sufficient agreement to a linear model over the range of inbound speed tested. Since the impact speed between bat and ball observed during play is unlikely to be lower than the minimum value of inbound speed tested, it can be argued that a linear model is valid in predicting ball behaviour under field conditions.

![Figure 10.1 - Ball COR vs. inbound velocity](image-url)
10.2 PROTOTYPE – 2 BAT STRIKING FACE IMPACT PERFORMANCE

10.2.1 Striking Face Quasi-Static Stiffness

Prototype 2 striking faces were mounted on the bat face support and a quasi static compression test was conducted at impact position P6, using a spherical indenter with a radius of curvature equivalent to that of a cricket ball. The force displacement characteristic of each face was measured to a maximum displacement of 10 mm. A value of face stiffness was determined from the force deflection relationship.
The results in Figure 10.4, show that the majority of constructions are of a similar stiffness and the values are particularly close between the curved multilayer constructions made from ash. The stiffest face is the '8 mm planar ash - uniaxial' construction which is approximately seven times the stiffness of the '4 mm ash - uniaxial' construction. An increase in thickness by a factor of 2.5 between the 3 mm and 7.5 mm birch ply faces results in an approximately proportional increase in stiffness by a factor 2.6. The proportional relationship between face thickness and stiffness remains approximately true for all the birch ply faces suggesting that the experimental relationship between stiffness and thickness is similar to the relationship defined by FEA, illustrated in Figure 7.35.

**Figure 10.4 - Stiffness of striking face constructions**

### 10.2.2 Striking Face Impact Test Results

Measurements of face deflection, ball deformation, contact time and COR versus inbound ball velocity were recorded for all impacts during the experiment. However, only impacts with a centre located within 11 mm of the intended target point on the face are presented in order to provide a more consistent description of dynamic behaviour. The results are illustrated in Figures 10.5 to 10.8 and the data points used in each graph have a shape corresponding to the group in which the face belongs and are coloured according to their value of quasi-static stiffness.

**Face Deflection**

In Figure 10.5, constructions with greater quasi-static stiffness generally show relatively less deflection across the range of inbound velocities than faces with lower measured stiffness. The trends are approximately linear for the range of inbound ball velocities tested and the faces with similar stiffness exhibit similar values of face deflection.
Ball Deformation

In Figure 10.6, values of ball deformation generally do not exceed 7 mm within the range of inbound velocities tested and the trends are approximately linear. The increase in ball deformation with inbound velocity is not as significant as the increase in face deflection suggesting that the ball exhibits a stiffer response than for all faces tested. For example, at an inbound velocity of 33 m/s, the average ball stiffness is approximately 1.7 times the apparent dynamic stiffness of the '12 mm b ash - biaxial - segmented' face and three times the apparent dynamic stiffness of the '8 mm ash - uniaxial' face.
Contact Time
In Figure 10.7, contact time appears independent of ball inbound velocity over the range of velocities tested. The impact therefore demonstrates a significantly linear stiffness response in which the time period of oscillation (approximately equal to twice the contact time) is independent of deflection or impact velocity. The contact times for the two faces with relatively low stiffness are approximately 1.7 times greater than for the other faces. The linear trend illustrated in Figure 10.7 for a ball impact against a steel plate demonstrates the minimum contact time corresponding to an approximately rigid surface.

![Figure 10.7 - Contact time vs. inbound velocity](image)

COR
Figure 10.8 illustrates COR against inbound ball velocity for all hollow faces, a solid willow face and a steel plate. A small improvement in COR above that of the rigid steel plate is made by the majority of constructions made from ash whilst the birch ply constructions exhibit a COR which is equivalent or lower than values for the steel plate. Improvements are more significant at lower inbound ball velocities. The '8 mm ash - uniaxial' face demonstrates the most significant improvement equal to 0.06 COR relative to the steel plate at 10 m/s which reduces to a difference of 0.03 COR at 35 m/s. Willow COR deviates from an approximately linear relationship for velocities greater than 30 m/s resulting in a significant relative improvement in COR for the majority of hollow face constructions. However, it is likely that the reduction in willow COR can be attributed to structural failure of the solid willow face as a result of its rigid constraint. Failure in the willow specimen was apparent, upon inspection, after the completion of the test. However, there was no visible indication of failure with the biaxial faces constructions.
10.2.3 Discussion of Bat Face Performance

Following the results of the experimental testing, the '8 mm ash - biaxial - segmented' face and the '12 mm a - ash - biaxial - segmented' face were used in the making of the prototype 2 bats as they provided a small improvement in COR relative to a willow face whilst demonstrating no obvious signs of failure upon inspection.

In Chapter 6, the spring damper models provided an accurate description of the general dynamic behaviour of the face involving the interplay between ball and face stiffness, damping and mass. Figure 10.9 illustrates the results of the complete bat spring damper model which was developed. The solid line illustrates the relationship between face stiffness and COR at impact location P7 with values for effective mass, sprung mass and face damping tuned to the experimental results for both the faces used in the prototype 2 bats. It is clear from this relationship that the value of face stiffness derived from the tuned model, equal to $10^{5.85}$ N/m, is lower than the optimum value of $10^{6.5}$ N/m predicted by the model. The model predicts that a ball impact against a steel plate of stiffness $\sim 10^9$ N/m produces greater ball rebound relative to the tuned value of stiffness. However, experimental results illustrate that the performance of a rigid surface and the chosen face constructions are approximately equivalent. The model illustrates that an improvement in performance would result by increasing the face stiffness by a factor of four to five, provided the sprung mass and damping of face remain constant. Alternatively, a more significant improvement is predicted by reducing the value of damping. The dashed line in Figure 10.9 results from a reduction in the value of face damping which has been tuned so that the optimum performance of the face occurs at a stiffness equivalent to experimental values.

In Chapter 6, Figure 6.20 illustrates that reducing the sprung mass lowers the value of optimum face stiffness. However, the value of sprung mass derived from experimental data (0.035 kg) is relatively
low compared to the effective mass of the bat (0.64 kg) and the ball mass (0.154 kg) and therefore a further reduction in sprung mass has little influence upon the optimum value of face stiffness.

The optimum value of stiffness predicted by the model, for tuned values of damping and sprung mass, was not achieved by the faces constructed in this study despite significant variation in both materials and thickness. Therefore it was anticipated that an investigation into reducing the value of damping for a given value of stiffness would be worthy of further investigation. Values of mass, stiffness and damping can be changed independently within the spring damper models. In reality, it is likely that these properties are mutually dependent. For example, an increase in stiffness requires a change in the design of the face, such as an increase in thickness or change in material, which is then likely to affect values of sprung mass and damping. Therefore a stiffness to damping ratio may provide a more useful value as an indication of performance. It is speculated that metals such as aluminium and titanium facilitate the design of structures with a greater ratio of stiffness to damping and as such are commonly used in striking implements such as baseball bats and golf club driver heads.

![Spring damper model of complete bat: $\varepsilon_f$ vs. face stiffness](image)

**Figure 10.9 - Spring damper model of complete bat: $\varepsilon_f$ vs. face stiffness**

### 10.3 BAT CENTRELINE IMPACT PERFORMANCE

#### 10.3.1 Test Strategy

Three prototype bats and three willow bats were tested and the performance results compared. A 'template' willow bat was tested as this model originally provided the external geometry applied to the prototype bats. The 'traditional' and 'bowed' bats provided two further sets of results for willow bat performance. The difference in shape between the three willow bats is subtle and so Figure 10.10 exaggerates the key features of the blade portions in order to make their differences more apparent. All three prototype bats were of a hollow laminated construction. The prototype 1 bat was constructed from birch ply and UF resin, the prototype 2-1 bat was made from ash wood and PVA with a 12 mm thick face construction, and the prototype 2-2 bat has an identical ash wood and PVA blade but incorporates a 16 mm thick front face.
The bat performance test method described earlier in the chapter was undertaken using top grade leather covered cricket balls. The striking surface of each bat was tested at nine discrete positions (P1-P9) and the COR and $\varepsilon_A$ were calculated at impact velocities of 19 m/s and 31 m/s.

Figure 10.10 - Exaggerated bat blade profiles highlighting key differences in design

10.3.2 Bat COR

Figures 10.11 and 10.12 illustrate the centreline impact COR along the striking region of the blade. The general trends for each bat are described below. To assist in the performance comparison between bats, a region of bat considered to exhibit desirable elasticity has been characterised. The term 'optimum region' has been defined as the region in which the value of COR remains within 0.02 of the peak COR value.

**Willow Bats**

From the toe region, the COR of the willow bats rises steeply from approximately 0.5 to towards an optimum value. The highest COR value of 0.63 at 680 mm from the handle top, is achieved by the template bat which also demonstrates a relatively short optimum region of 49 mm. The bowed willow bat demonstrates the longest optimum region of 81 mm with a peak of 0.61. Impacts further towards the handle result in a gradual, approximately linear reduction in COR. For the template and traditional bats the gradient reduces to a more constant value of 0.41-0.44 in a region 530-560 mm from the top of the handle. The gradient of transition from the peak value towards the handle is less for the bowed bat. However, the bowed bat does not exhibit a reduction in the gradient and the COR continues to decline for impacts nearer the handle ending at value of 0.4 at a distance 440 mm from the top of the handle. The characteristic shape of the curves remains similar at an impact velocity of 31 m/s. However, the COR values are on average 11% lower with peak values of 0.52-0.55. The bowed bat maintains the longest optimum region equal to 91 mm.

**Prototype Bats**

The prototype and willow bats illustrate a similar elastic trend along the striking region. However, the prototype trends are shifted to the right of the graph relative to the willow bats. COR performance beyond 730 mm from the handle for prototype 2 bats is equal or greater than that for willow bats.
Peak performance is on average 43 mm further from the handle top than observed for willow bats and the peak values are 9% lower on average. The prototype 1 bat exhibits the highest peak equivalent to the traditional willow bat at a value of 0.60 and the shortest optimum region equal to 40 mm. The prototype 2 bats exhibit an average optimum region length of 79 mm. The gradient of reduction in elasticity from the peak value towards the handle is similar to the willow bats. The gradient lessens to a more constant value of COR further from the handle region relative to the willow bats, corresponding to a COR value of 0.40-0.42 for the prototype 2 bats and 0.34 for the prototype 1 bat.

Impact testing at 31 m/s gave rise to prototype bat durability concerns. Delamination of multiple layers in the prototype 1 bat occurred at the start of testing. However, the bat testing was completed to record the effect of delamination on performance. Testing of the prototype 2-2 bat was stopped at the first sign of delamination, which became apparent after 15 impacts. Because of the durability issues, the prototype 2-1 bat was not tested.

At a relative impact speed of 31 m/s, the prototype 1 bat exhibits an average COR reduction of 18% compared to values obtained at 19 m/s. The reduction in COR towards the handle occurs over a shorter region giving rise to a larger region in which the COR remains relatively constant.

![Figure 10.11 - Bat centreline performance at 19 m/s $v_{imp}$: impact elasticity vs. impact position](image)
10.3.3 Bat $e_A$

**Willow Bats**

In Figures 10.13 and 10.14, the bowed and traditional bats exhibit a similar trend in $e_A$ which increases steadily from the toe of the bat to a maximum of 0.35 at a distance of 610 mm from the handle top. From a value of peak efficiency the performance gradually reduces towards the handle ending near the handle splice at a value of 0.24-0.26. The template bat exhibits a flatter response from a peak value of 2.95 at 650 mm to the beginning of the handle splice at a value of 2.7. At impacts speeds of 31 m/s, all three willow bats demonstrate a flatter efficiency response which is approximately 20% lower than the performance measured at 19 m/s.

**Prototype Bats**

In Figures 10.13 and 10.14, the prototype bats exhibit a similar response to the template willow bat except the trends are shifted 40 mm further from the handle and are approximately 10% lower. Efficiency values beyond a distance of 708 mm from the handle top are a maximum of 50% greater than measured for the willow bats. The prototype 1 bat exhibits the greatest efficiency of 0.37 but exhibits a more dramatic reduction in performance either side of the peak value and demonstrates a lower $e_A$ of 0.19 near the handle splice. At an impact velocity of 31 m/s, $e_A$ was approximately 40% lower than that measured at 19 m/s. The prototype 1 bat demonstrates a broadly flat response which peaks at a value 38% lower than that at 19 m/s, at a distance 730 mm from the handle top. The transition between performance near the handle splice and the peak value is not gradual but dips to form a trough at a distance of 607 mm from the handle top which is 12% lower than the value of efficiency observed near the handle splice.
10.3.4 Discussion of General COR Trends

A value of COR less than one implies that some portion of the initial kinetic energy of the ball is not returned to the rigid body motion of the objects. The energy that is not returned may be lost during the contact period or stored and subsequently dissipated after the collision has taken place. Differences in the COR profiles for those bats tested are due to differences in the total amount of energy lost at each impact location. To explain the characteristic shape of the COR curve and to explain differences between the trends observed for each bat, it is necessary to explain how the level of energy dissipation is dependent upon impact location and upon the collision speed.
Energy Loss Through Local Deformation

FEA modelling of bat and ball impacts, in chapter 7, has shown local damping accounts for a significant portion of the total energy loss (~50%). However, there was no significant variation of energy loss due to local effects along the striking region of the face. This is in agreement with studies in baseball by Nathan [45] and Van Zandt [100] who suggest that the vibration characteristics of the bat are predominant in modifying the value of COR along the striking region of a baseball bat.

Energy Loss Through Vibration

As a general rule Goldsmith [96] suggests that objects with a high surface area to volume ratio exhibit a greater degree of vibration than those with a low surface area to volume ratio. Since a spherical shape exhibits the lowest surface area to volume ratio of any 3-D object, it is likely that the energy stored in the vibration modes of a solid cricket ball following impact will be negligible in comparison to the inelastic energy dissipation. Goldsmith expresses an Equation which defines the ratio of vibration energy to total energy for the collision of a solid sphere at moderate velocity $v_0$, as being:

$$\frac{\text{vibrational energy}}{\text{total energy}} = -\frac{v_0}{50 \cdot c_0} \quad \text{[10.1]}$$

where $c_0$ is the 'rod wave velocity' given by:

$$c_0 = \sqrt{\frac{E}{\rho}} \quad \text{[10.2]}$$

Where $E$ is the modulus of elasticity and $\rho$ is the density. From this model, the ratio of vibration energy to total collision energy of a cricket ball ~ 0.001 at 19 m/s $v_{rel}$ and ~ 0.002 at 31 m/s $v_{rel}$. However, it is likely that the model refers to an elastic object. In reality, the cricket ball is composed of many layers of different materials which permanently move and deform upon impact and it is likely that the impact energy is absorbed before vibration can be propagated. Therefore it is argued that the ratio described above tends toward zero for a cricket ball and as such the vibration of the ball subsequent to impact can be ignored.

In contrast, the length and cross section of the bat which approximates that of a beam suggests that transverse vibrations are significant following impact and the time period of excitation determines which frequencies are likely to propagate. For example, if the time period is large compared to the fundamental mode then several reflections of the wave occur during the contact period and the body may be considered to be a state of static equilibrium [96]. The contact period for bat and ball impact from Figure 10.3 is approximately 1.2 ms (for 19-31 ms$^{-1}$ $v_{rel}$). Therefore the contact impulse can be described as one half of a sinusoidal waveform with a frequency ~420 Hz. According to Goldsmith, it is likely that frequencies lower than ~420 Hz will propagate without hindrance whereas the propagation of higher frequencies will be hampered. A similar argument is proposed by Nathan [45] who suggested
that modes with an angular velocity much greater than the reciprocal of the contact time period would not be excited. Therefore, the first three modes of transverse vibration are taken into consideration as it is assumed that the higher modes are not significantly excited. In addition, the lower frequencies are associated with greater amplitudes coupled with greater levels of energy.

Table 10.1 illustrates FEA and experimentally determined frequencies for the first three flexural modes of bat vibration. Values were measured using an accelerometer attached near the toe of the blade and connected to a charge amplifier. The voltage time signal from the charge amplifier was processed using SignalCalc® software to determine the frequency content of excitation from a low velocity bat and ball impact located away from the node points. The greatest difference between experimental and FEA results were apparent for the first mode of vibration. It was anticipated that since the value of handle $E$ was determined for a specimen without the string and rubber grip components, an increase in the FEA handle $E$ definition would improve the accuracy for this mode. A handle stiffness ~2.5 times $E$ determined under quasi-static bend tests (identified by an asterisk in Table 10.1) resulted in equivalent values for the first mode of vibration. However, the second and third mode frequencies were also sensitive to the change in handle $E$ resulting in ~20% increase above experimental values for these two modes. The study by Grant and Nixon [33] described a 20% difference between FEA and experimental values for the first three modes of bat vibration and a 50% increase in FEA wood density was necessary in order to improve the accuracy of the model to within 4%. It is likely that simplification of material and geometric definitions within the bat FEA models accounts for the differences in observed mode frequencies. The materials may also demonstrate a degree of strain rate dependency and the measured values of quasi-static stiffness may need adjustment when used in loading conditions equivalent to higher rates of strain.

FEA frequency simulations were conducted to assess the effect on node location from modifying the FEA model to agree with the measured frequency values for the first mode. Table 10.2 illustrates the location of the nodes of the first three modes of vibration including values for each bat with increased handle stiffness. The difference between the first mode node locations resulting from the change in handle $E$ for all bats was between 3-8 mm. Therefore the position of these nodes is not significantly dependent upon the required change in handle stiffness. The general shift in node location further from the handle in the prototypes bats relative to the solid bat remains true for both values of handle $E$, but the relative distance increases slightly (~5 mm) for the case of increased handle stiffness.

Figures 10.15 to 10.17 graphically illustrate the FEA mode shapes for the first three modes of vibration for the template willow bat, the prototype 1 bat, and the prototype 2-1 bat, respectively. The normal value of handle $E$ is used although it has been shown that little difference exists if the model is adjusted to accurately represent the first mode frequency. The amplitude of vibration is given as a relative excitation displacement from zero to one. For all modes, the greatest displacement occurs at the end of the handle and therefore the relative value of displacement for all remaining locations is less
than one. Since the magnitude of excitation is relative for each mode, the Figures cannot be used to compare displacements between different modes or between the same modes for each bat. However, they illustrate a region where the nodes of the first three modes are concentrated within a relatively small distance along the blade. Several studies have shown that an impact at the node location of a mode will result in a minimal excitation for that particular mode and impact at an antinode (harmonic peak) will result in a maximum excitation [27, 30, 33, 34, 45, 118]. Therefore, the summation of vibration energy resulting from an impact in this region will be lower than that from other impact locations on the striking face. This agrees with the results of FEA bat impact modelling, in chapter 7, which showed that COR values were highest at the region corresponding to the node locations for the first 3 modes of flexural bat vibration.

<table>
<thead>
<tr>
<th></th>
<th>First Mode (Hz)</th>
<th>Second Mode (Hz)</th>
<th>Third Mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FEA</td>
<td>Exp.</td>
<td>FEA</td>
</tr>
<tr>
<td>Template Willow Bat</td>
<td>84</td>
<td>127</td>
<td>349</td>
</tr>
<tr>
<td>Template Willow bat*</td>
<td>127</td>
<td>127</td>
<td>441</td>
</tr>
<tr>
<td>Prototype 1 Bat</td>
<td>79</td>
<td>117</td>
<td>306</td>
</tr>
<tr>
<td>Prototype 1 Bat*</td>
<td>113</td>
<td>117</td>
<td>342</td>
</tr>
<tr>
<td>Prototype 2-1 Bat</td>
<td>82</td>
<td>125</td>
<td>357</td>
</tr>
<tr>
<td>Prototype 2-1 Bat*</td>
<td>121</td>
<td>125</td>
<td>433</td>
</tr>
</tbody>
</table>

* 2.5 times value of quasi-static handle $E$

Table 10.1 – FEA and experimentally determined frequencies of flexural vibration

<table>
<thead>
<tr>
<th>Striking Face Node Distance From Handle Top (mm)</th>
<th>First Mode</th>
<th>Second Mode</th>
<th>Third Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Template Willow Bat</td>
<td>611</td>
<td>671</td>
<td>701</td>
</tr>
<tr>
<td>Template Willow bat*</td>
<td>614</td>
<td>682</td>
<td>714</td>
</tr>
<tr>
<td>Prototype 1 Bat</td>
<td>626</td>
<td>708</td>
<td>738</td>
</tr>
<tr>
<td>Prototype 1 Bat*</td>
<td>634</td>
<td>718</td>
<td>756</td>
</tr>
<tr>
<td>Prototype 2-1 Bat</td>
<td>620</td>
<td>706</td>
<td>746</td>
</tr>
<tr>
<td>Prototype 2-1 Bat*</td>
<td>628</td>
<td>726</td>
<td>764</td>
</tr>
</tbody>
</table>

* 2.5 times value of quasi-static handle $E$

Table 10.2 – FEA node locations for solid and hollow bats with normal and modified handle stiffness
10.3.5 Differences in Bat COR Trends

The differences in bat COR trends are dependent upon differences in bat transverse vibration characteristics. The shift in the hollow bat node positions further from the handle corresponds to the relative shift in optimum COR. It is likely that the lower COR values observed for the hollow bats are due to lower flexural rigidity of the blade resulting in an increase in amplitude of transverse vibration and energy loss.
10.3.6 Discussion of General Bat $e_d$ Trends

Figure 10.18 compares measured values of $e_d$ with values derived using Equation 4.23 for the bowed willow bat and the prototype 2-2 bat. Good agreement between both sets of results reinforces the idea that outbound ball velocity relative to the bat is determined through the interplay between the elasticity of the impact and the inertia properties of the bat and ball.

![Figure 10.18 - Experimental and derived values of efficiency for a solid and a hollow bat at 19 m/s $v_{rel}$](image)

10.3.7 Differences in Bat $e_d$ Trends

The willow bats show a peak $e_d$ which is approximately 65 mm nearer the handle than location of peak elasticity and is therefore located between the bat COM and the location of peak elasticity. The combination of effective mass and the elasticity of the hollow bat results in a flatter response along the striking region suggesting that the potential increase in ball kinetic energy from an impact closer to the bat COM is cancelled by a similar increase in energy loss. Enhanced hollow bat performance occurs at either end of the striking region corresponding to improved elasticity in these regions.

10.4 BAT OFF-CENTRELINE IMPACT PERFORMANCE

10.4.1 Test Strategy

Off-centreline impact tests were conducted using three solid willow bats and a prototype 1 hollow bat. The traditional and template willow bats were the same models previously tested against centreline impacts. In order to broaden the range of bats tested, the 'slender' willow bat model was included as its polar MOI was the lowest measured from a range of 20 bats. This can be partly attributed to its relatively small blade and low mass. All bats were tested using the off-centreline impact test procedure outlined earlier in the chapter at a relative impact speed of 19 m/s.

10.4.2 Off-Centreline Impact Results

Off-centreline impacts can be understood using a similar mathematical model to that used for centreline impacts located away from the COM as they are both eccentric collisions. An additional complexity arises due to the curvature of the face which creates a contact angle that is not
perpendicular to the initial ball trajectory. A similar problem was modelled by Daish \cite{119} concerning the impact of a golf ball with a lofted club. The loft creates a contact angle which not perpendicular to the direction of club travel and the term 'loft' describes the angle of the club face to the vertical plane which is designed to lift the ball into the air and generate back spin.

Off-centreline bat and ball impacts have been investigated using a similar model to that described by Daish \cite{119} in order to understand the experimental results and the relative importance of several bat properties in determining the angle of ball rebound and the bat polar rotation. These are considered as two most important factors since ball rebound angle is associated with a player's control over the ball direction and bat polar rotation is associated with decrease in the change of ball momentum. Theoretical and experimental values of bat and ball rebound angle and bat polar rotation are illustrated in Figures 10.24 to 10.27.

**Ball Rebound Angle**

Figure 10.19 illustrates the rigid body model used to determine ball rebound angle. An effective mass is used to represent the portion of the total bat mass which is experienced by the ball during the impact. Figure 10.20 illustrates the relationship between effective bat mass and the distance of impact from the bat centreline for the range of bats tested, determined using Equation 4.10. To determine the contact angle presented at each impact location on the bat face, a measurement of the face curvature of each bat was recorded using a CMM machine. Five points were measured across the bat face and values of radius and error were calculated using a least squares fit, shown in Figure 10.21. Figure 10.22 illustrates the values of radius and circular deviation error associated with each bat face, measured at a distance from the handle top corresponding to the location of experimental ball impact.

Figure 10.23 shows the forces exerted on the ball during impact as components that are normal and parallel to the bat face. The normal component acts through the ball COM and causes ball deformation and relaxation. The parallel component results in sliding and rolling of the ball over the surface of the bat. The normal ball velocity after impact is determined using conservations of momentum and the value for COR in the form of Equation 10.3.

\[
V_{\text{normal}} = \frac{u_b \cos \phi (m_s + m_e \cdot \varepsilon)}{m_s + m_e} \quad [10.3]
\]

where, \(u_b\) is the initial ball velocity and \(m_b\) and \(m_e\) are the ball mass and effective bat mass, respectively. A value of COR was obtained from the results of the centreline tests for each bat at a location equivalent to the COM. Upon contact, the value of \(v_{\text{parallel}}\) is equal to \(u_b \sin \phi\), the component of initial ball velocity acting parallel to the face. The relative motion of the ball against the face generates a force, \(F_{\text{parallel}}\) which is equal to \(F_{\text{normal}}\) times \(\mu\) where \(\mu\) is the coefficient of friction between the bat and ball surfaces. \(F_{\text{parallel}}\) generates a torque about the ball COM which has the effect of reducing the
relative velocity between both surfaces. The angular velocity of the ball increases until the relative velocity between the contacting surfaces is zero, at which time the ball is in pure rolling and $F_{\text{parallel}}$ is defined only by the rolling resistance, which for relatively hard objects can be ignored over the duration of contact [119].

For a solid ball, rolling begins when the relative velocity between the face of the implement and the ball has been reduced by friction to five sevenths of its initial value and is independent of the value of friction [119]. Therefore $v_{\text{parallel}}$ was defined as follows:

$$v_{\text{parallel}} = \frac{5}{7}u_{b} \sin \phi$$

[10.4]

Values of $v_{\text{horizontal}}$ and $v_{\text{vertical}}$ for the ball are equal to the combined components of both $v_{\text{horizontal}}$ and $v_{\text{parallel}}$ in each respective direction as follows:

$$v_{\text{vertical}} = v_{\text{normal}} \sin \phi + v_{\text{parallel}} \cos \phi$$

[10.5]

$$v_{\text{horizontal}} = v_{\text{parallel}} \sin \phi - v_{\text{normal}} \cos \phi$$

[10.6]

The rebound angle of the ball relative to its inbound direction was determined from the horizontal and vertical velocity components.

![Rigid body diagram of off-centreline ball impact used to determine ball rebound angle](image)
Figure 10.20 - Effective mass determined about the polar axis of the bat

Figure 10.21 - Determination of bat face radius and roundness error
Bat Polar Rotation

Figure 10.23 illustrates the rigid body model used to determine bat polar rotation. $F_{\text{normal}}$ and $F_{\text{parallel}}$ are the components of force acting normal and parallel to the face of the bat. The angular rotation of the bat about the COM is generated by a torque equal to the component of the resultant force acting in a direction perpendicular to the COM multiplied by a distance $L$. Since impact force is not constant over the duration of contact, the change in ball momentum in both normal and parallel directions was calculated using the velocity values obtained from the previous model. The change in normal and
parallel ball momentum is equal to the impulse generated in each direction as shown the form of Equations 10.7 and 10.8, respectively.

$$\int F_{\text{normal}} \, dt = m_b (u_b \cos \theta - v_{\text{normal}})$$  \hspace{1cm} [10.7]

$$\int F_{\text{parallel}} \, dt = m_b (u_b \sin \theta - v_{\text{parallel}})$$  \hspace{1cm} [10.8]

The torque impulse is equal to the component of the combined normal and parallel impulses acting in a direction perpendicular to the COM multiplied by the distance $L$. Therefore:

$$\int F_{\text{torque}} \, dt = m_b (u_b \cos \theta - v_{\text{normal}}) \sin \theta + m_b (u_b \sin \theta - v_{\text{parallel}}) \cos \theta$$  \hspace{1cm} [10.9]

Since torque is equal to the MOI times the angular acceleration of an object. The value of bat angular velocity was determined as in Equation 10.10.

$$\int F_{\text{torque}} \, dt = I_{\text{polar}} \cdot \Delta \omega$$  \hspace{1cm} [10.10]

### 10.4.3 Model Limitations

The model concerns the collision of rigid bodies and does not account for the deformation of the objects during impact. However, it is likely that the curvature of the front face and therefore the contact angle between both objects, used within the Equations for both models, will be affected by the local deformation observed experimentally.

The value of COR used within the model remains constant. However, as the effective mass decreases away from the centreline of the bat, it is likely that the elasticity of the impact will also improve as contact force and levels of local deformation reduce.

The model assumes that the contact angle between bat and ball remains constant throughout impact. In reality, the force from impact will tend to rotate the bat about its polar axis resulting in a change in contact angle during the impact. The model error will increase with increasing polar rotation during contact, which is likely to occur from impacts further from the bat centreline and at higher collision speeds. The approximate change in bat polar rotation angle during contact can be determined as follows. The Equation for the change in angle for an average angular acceleration $\bar{\alpha}$ is:

$$\theta = \frac{1}{2} \bar{\alpha} t^2$$  \hspace{1cm} [10.11]

$\bar{\alpha}$ can be determined from the average moment generated from impact $\bar{M}$, divided by the bat polar MOI, where $\bar{M}$ equals the average linear impact force $\bar{F}$, times the perpendicular distance $L$, from impact to principal $\bar{y}$ axis.
Using an approximate value of polar MOI = $10 \times 10^{-4}$ kgm$^2$ (chapter 5) and a distance $L = 50$ mm (~half the bat width), Equation 10.12 returns a value of $\bar{a} = 50 \bar{F}$. The average force, $\bar{F}$ is equal to the change in ball momentum divided by the contact time, shown in Equation 10.13.

$$\bar{F} = \frac{m_b \Delta v_s}{t}$$  \hspace{1cm} [10.13]

where the ball mass $m_b \sim 0.16$ kg and the change in ball velocity $\Delta v_s$, determined from experimental tests, $\sim 13$ m/s. Therefore $\bar{a}$ equals:

$$\bar{a} = \frac{0.16 \times 13 \times 50}{1 \times 10^{-5}} \approx 1 \times 10^5 \text{rads s}^{-2} \hspace{1cm} [10.14]$$

Substituting the value of $\bar{a}$ into Equation 10.11 gives a change in bat polar angle during contact of 0.05 rads (3 degrees), which should not significantly affect the accuracy of the model. However, a change in ball velocity $\sim 50$ ms$^{-1}$, which is more representative of a game situation, results in an angular bat rotation during contact $\sim 10$ degrees. Therefore the model is satisfactory for the experimental impact conditions used in this study, but may need to include bat rotation during contact if significantly larger changes in ball momentum are being modelled.

10.4.4 Off-Centreline Impact Test Results and Discussion

Figure 10.24 illustrates a predominantly linear relationship between the change in bat angle and the distance of impact from the centreline. There is no significant difference in the results between different bats suggesting that the direction of the rigid body impulse experienced by a batsman from an off-centreline shot would be similar for each bat tested.

Figure 10.25 illustrates a predominantly linear relationship between both experimental and theoretical ball rebound angle and the distance of impact from the bat centreline. Both experimental and model data demonstrate that the sensitivity of ball rebound angle to the location of impact is similar for each bat tested. Good agreement exists between experimental and model data over the full range of impact locations for both the prototype 1 and template willow bats. Generally, the agreement between model and experimental data improves with an increase in impact distance from the bat centreline. Since the rotation of the bat during contact would lead to a greater difference in the value of rebound angle between experimental and model data as distance of impact increased from the centreline. The results tend to suggest that rotation during contact is not a significant factor in determining ball rebound for the range of bats tested.
Figure 10.26 illustrates a predominantly linear relationship between the rate of bat polar rotation and the distance of impact from the bat centreline. There is significant difference observed between the range of bats tested with the prototype 1 bat demonstrating an average polar rotation rate 34% lower than the willow template bat over the range of impact locations.

Figure 10.27 illustrates the rotational energy stored in the rigid body motion of the bat following an off-centreline impact which is equal to the rate of rotation squared, multiplied by the polar MOI. The rotational energy stored in the prototype 1 and template willow bats following an impact 50 mm from the bat centreline equals 5.8 J and 12 J respectively. The initial ball kinetic energy is approximately 53 J and therefore the percentage of the initial energy of impact stored in the hollow and solid bat equals 11% and 22%, respectively. Therefore in this example, the solid bat stores an additional 20% of the initial ball kinetic energy in post impact rotation relative to the hollow design for an impact near the edge of the bat.
10.4.5 Sensitivity of Model to Changes in Bat Properties

Figures 10.28 and 10.29 illustrate the sensitivity of the ball rebound angle and the rate of polar rotation to several bat properties to clarify which properties are significant in developing desirable off-centreline impact performance.

The curvature of the front face affects the relationship between ball rebound angle and distance of impact within the model. An increase in the radius of curvature reduces the ball rebound angle for impacts close to the centreline. However, towards the edge of the bat the change is minimal. It is doubtful that the effect of curvature observed in the model would be reflected experimentally since the face angle generated at the contacting surfaces will be altered by the local deformation of both objects.

Both rebound angle and rate of rotation have little dependency upon change in bat mass without an associated change in MOI. It is likely however, that a more significant dependence would exist if the bat was moving with an initial velocity opposing that of the ball. In this case Equation 10.3 would become:
\[ v_{\text{normal}} = \frac{u_x \cos \phi (m_y + m_z \cdot e) + m_z \cdot u_y (1 + e)}{m_y + m_z} \]  

Resulting in a higher normal rebound ball velocity for a given value of initial bat velocity \( u_x \). It is reasonable to suggest therefore that the rebound angle would reduce whilst the rate of rotation would increase under such conditions.

The strength of the dependency of ball rebound angle on change in bat MOI increases as the impact moves further from the bat centreline. Since the effective mass is dependent upon distance of impact squared. The differences in value of effective mass between bats of differing MOI will be greatest towards the edge of the bat and equally so will the change in normal ball momentum and ball rebound angle.

Both ball rebound angle and rate of bat rotation are strongly dependent upon the value of COR. Increasing the COR has the effect of increasing the change in normal ball momentum, as illustrated in Equation 10.3 and reduces the ball rebound angle due to a greater horizontal velocity component. In contrast, the rate of bat rotation increases with COR since a greater change in normal ball momentum is accompanied by an increase in the torque impulse imparted to the bat.

The model is relatively insensitive to changes in the position of the COM relative to the front face of the bat as the model does not account for a change in contact angle during impact. The position of the COM determines the change in contact angle associated with bat rotation. For a COM close to the front face the change in angle would be greater than for a COM positioned further behind the face for the same degree of angular rotation.

The position of COM is significant to the design of golf club driver heads since the rotation of the head about the COM provides an additional function which can affect the overall accuracy of the driving shot. An eccentric impact between ball and head results in a rotation which generates relative movement between the club head and the ball in a direction parallel to the face. This in turn can impart spin on the ball which has the effect of swerving the ball during flight. The curvature of the clubface affects the contact angle and can increase the component of the club angular rotation acting parallel to the face. Daish [119] suggests that the initial directional error incurred from an eccentric impact maybe countered to some degree by the swerved flight of the ball as a result of imparted spin, as shown in Figure 10.30. It is unlikely however, that this phenomena would be as significant in benefiting the batman as it does the golfer, since spin is not used in cricket to assist in control over the ball. Reasonable agreement between model and experimental data suggests that the change in contact angle during ball contact does not significantly affect the post impact properties of the ball under the impact conditions of the test. However, during play the inbound ball speed is typically greater than 19 m/s and the bat will exhibit an initial inbound velocity, imparted by the batsman's stroke, which will
affect both the rebound angle and polar rotation of the bat. The greater impact forces generated during play may also add greater significance to the rotation of the bat during contact and therefore increase the significance of the distance between the striking face and the COM.

Figure 10.28 - Dependency of ball rebound angle on bat properties derived from model
Figure 10.29 - Dependency of rate of bat polar rotation on bat properties derived from model
Figure 10.30 - The 'gear effect' observed in the eccentric impact of a golf club driver head.
CHAPTER 11

ASSESSMENT OF BAT PERFORMANCE

11.1 INTRODUCTION

The impact tests described in the previous chapter defined bat COR and $e_d$ under laboratory conditions. This section describes the process of translating the laboratory results into a description of bat performance under playing conditions to facilitate a more relevant comparison between the performance of solid and prototype hollow bats.

11.2 CHARACTERISING BAT PERFORMANCE

The term 'sweet spot' is commonly used to describe the point or region on the striking surface of the bat synonymous with enhanced performance. The term is used by both sporting and scientific communities which has led to some ambiguity concerning its definition. Noble and Walker [118] illustrate this point with the following statement:

'Sweet spot is a general term indicating the most desirable location on which to impact the ball. It is not clear if this term relates more to the effect of the impact on the ball (i.e., post impact velocity) or to the effect of the impact on the hitter (i.e., discomfort and pain).'

The term is currently used to describe both quantitative and qualitative values and several studies have attempted to reconcile both descriptions [26, 28, 29]. However, the principal objective for this study is to establish an objective measure of bat performance, in this case post impact ball velocity.

Kirkpatrick [90] suggested that for sports in which the hitting of a ball is central to the game, it is often the desirable to transfer as much momentum to the ball as possible with the additional issue of directional guidance or control. Scoring 'runs' in cricket will, in most cases, benefits from maximising the batted ball speed. Maximising ball speed increases the probability of the ball reaching the boundaries of the pitch, for which the batsman receives a score equivalent to four or six 'runs' as defined by the rules of the game. Therefore, it is likely that a bat which enhances the maximum batted ball speed for a shot of the same intention would be beneficial to the batsman.

Due to the direct significance of BBS in the game relative to other metrics such as COR or energy efficiency, BBS has been used to characterise bat playing performance for impacts along the striking region of the face.

In addition, the present understanding of the relationship between impact location and the perceived pleasantness of a batting shot were investigated in order that general observations could be made regarding perceived 'feel' in the hands for shots along the striking face.
11.3 Definition of Standard Game Conditions

To determine values of BBS, the incident bat and ball velocities and the impact efficiency are required as defined in Equation 9.4. To calculate realistic BBS values for bat comparison, realistic inbound parameters were required. Therefore the incident velocities of the bat and ball for a standard game condition were established based upon the results of two independent kinematic studies of batting in cricket by Stretch et al. [15] and Elliot et al. [14]. The results of the bat impact testing, described earlier in the chapter, were used to determine appropriate values of COR and $e_a$.

11.3.1 Ball Speed

As part of the kinematic study, Elliot et al. [14] recorded the inbound ball motion moments before impact with the bat. An elite fast bowler was used in the study and an inbound ball velocity of ~22 m/s prior to impact was recorded. The kinematics study by Stretch et al. [15] reported a mean horizontal velocity of ~21 m/s just before impact using an elite bowler of undisclosed bowling style. As both values are similar, either was suitable for the present study and as such, a value of 22 m/s was used as the incident ball speed in the definition of the standard game impact.

11.3.2 Bat Speed

Limb movements during the execution of a batting stroke introduces both angular and translational velocity components to the bat. This motion can be defined as a single component of angular velocity about an instantaneous centre of rotation (ICR). This concept was successfully used to describe the rotation of a tennis racket during a shot by Mitchell et al. [120]. The position of the ICR changes according to the relative values and direction of bat translation and rotation about the bat COM during the batting stroke.

Stretch et al. [15] describes the front foot off-drive as the most common attacking stroke observed during first class cricket and therefore the kinematics of this stroke are valid in this present study. The motion of players' limb joints and the bat at the COM were recorded for several shots using 14 elite batsmen. Similarly, Elliot et al. [14] investigated the kinematics of the off-drive and on-drive for cricket by recording the motion of the players' limb joints and the bat using ten elite batsmen.

In both studies [14, 15] bat motion was only measured at the bat COM. Therefore the motion of the batsmen's hands were used to provide the additional information required to calculate the position of the ICR. This approach was based on the assumption that the motion of the hands (at the wrist joints) would be very similar to the underlying motion of the handle.

Figure 11.1 illustrates the results of both studies [14, 15] and good agreement exists on the position of the ICR and bat angular velocity. The results from both studies were averaged, resulting in values of $\omega$ ~29 rads/s and the position of the ICR ~32 mm from the top of the handle. The location of the ICR
was used as the point of rotation about which the first moment of mass and swing MOI were calculated.

It is likely that the value of bat angular velocity \( \omega \) derived from the both studies [14, 15] would have been dependent upon the inertia properties of the bat in addition to the strength and technique of the batsmen. The effect of bat inertia upon the speed of a player's swing or stroke has been commented upon in several studies [22, 31, 86, 90, 121]. Since the solid willow and prototype bats in the present study have different mass properties. A relationship between bat inertia and swing speed has been included in the definition of bat incident velocity.

Nathan [31] also describes a method of accounting for the effect of bat inertia upon bat velocity prior to impact by defining two extreme cases which concern the ability of the player to impart kinetic energy to the bat. The resultant bat velocity obtained in each case can be compared and the significance of bat inertia can be assessed. Nathan describes the general relationship between bat angular velocity and bat inertia as follows:

\[
\omega \sim I^{-n}
\]  

[11.1]

The two limiting cases concern the value of \( n \). If the angular velocity of the bat generated by the player is independent of changes in bat MOI, then \( n = 0 \). However, if the player is only able to impart a constant kinetic energy to the bat, then \( n = 0.5 \). In reality, it is likely that the relationship between \( \omega \) and \( I \) exists between these limits. The mass of commercially available cricket and baseball bats, equal to \( \sim 0.8-1.5 \) kg, suggests that this is the case, since values of \( n = 0 \) and \( n = 0.5 \) would encourage the use of unrealistically large and small values of bat mass, respectively. Nathan suggests a value of \( n = 0.3 \) within a limited range of bat mass gives good agreement with experimental data and is equivalent to the player generating constant bat power [31]. Nathan defines an expression for bat angular velocity about the ICR as follows:

\[
\omega = \omega_{ref} \left( \frac{I_{ref}}{I} \right)^n
\]  

[11.2]

where \( \omega_{ref} \) and \( I_{ref} \) are reference values which enable a new value of angular velocity to be calculated based upon the value of bat inertia, \( I \). The average angular velocity determined from studies [14, 15] was used as the reference velocity \( \omega_{ref} \) and information regarding the bat used in the study by Stretch et al [15] was used to establish a corresponding value of \( I_{ref} \).

In the study [15], the bat was described as 'a short handle Slazenger cricket bat of standard size, with a mass of 1.156 kg'. In the present study the inertia of a short handle Slazenger cricket bat of
standard size, with a similar mass of 1.108 kg was measured to obtain a value of $I_{ref}$ approximately equivalent to the bat used by Stretch [15].

The ratio $\left[ \frac{I_{ref}}{I} \right]$ was then used to determine the effect of inertia upon $\omega$ for the two limiting cases of $n = 0$ and $n = 0.5$, and the case of constant bat power $n = 0.3$. Table 11.1 summarises the mass properties and resultant angular velocities for all bats under investigation.

![Figure 11.1 - Bat angular velocity about the Instantaneous centre of rotation, derived from Stretch et al [15] and Elliot et al [14]](image)

| Reference bat       | 1.156 | 29 x10⁴ | 28.5  | 28.5  | 28.5  |
| Willow - Template   | 1.196 | 30 x10⁴ | 28.1  | 28.3  | 28.5  |
| Willow - Traditional| 1.108 | 28 x10⁴ | 29.0  | 28.0  | 28.5  |
| Willow - Bowed      | 1.179 | 31 x10⁴ | 27.6  | 38.0  | 28.5  |
| Prototype 1         | 1.373 | 36 x10⁴ | 25.5  | 26.7  | 28.5  |
| Prototype 2-1 (thin face) | 1.333 | 33 x10⁴ | 26.8  | 27.4  | 28.5  |
| Prototype 2-2 (thick face) | 1.545 | 40 x10⁴ | 24.3  | 25.9  | 28.5  |

Table 11.1 - Mass properties and angular velocities of bats under standard game conditions

11.3.3 COR and $e_d$
As demonstrated previously in Figures 10.11 to 10.14, the values of COR and $e_d$ for the bat and ball impact vary according to the location of impact but also decrease with increasing relative impact speed. Since the standard game condition defines a variation of bat velocity along the striking face of the bat, the values of COR and $e_d$ used for the standard game condition must account for both location and the relative speed of impact.
To establish a relationship between $e_d$ and $v_{rel}$ along the striking region of the bat. The values of $e_d$ established for all bats in the laboratory impact tests at 19 m/s and 31 m/s were compared. The results illustrate that an average reduction of $\sim 22\%$ ($\sigma = 0.039$) in $e_d$ between 19 m/s and 31 m/s occurred for all impact locations on each bat used in the study. Therefore, a linear correction factor, illustrated in Figure 11.2, was applied to the value of $e_d$ measured at 19 m/s to correct for the difference in relative impact velocity between laboratory tests and the standard game condition.

Figure 11.2 - Relationship between collision efficiency $e_d$ and relative collision velocity $v_{rel}$

11.4 BATTED BALL SPEED UNDER THE STANDARD GAME CONDITION

Figures 11.3 to 11.5, illustrate BBS under the standard game condition for the three cases of swing speed sensitivity to MOI. The trend observed for the solid willow bats for each case demonstrate peak values of BBS between locations $P_6-P_7$ whereas the hollow bat trends are shifted further from the handle (to the right of the graph) resulting in the highest observed BBS equivalent to the limit of laboratory impact testing at $P_9$.

11.4.1 Constant Bat KE

Figure 11.3 illustrates the case of constant bat KE describing the greatest swing speed sensitivity to bat inertia. Impact locations $P_1-P_8$ correspond to a BBS approximately 3-4 mph greater for the solid bats than for the hollow bats. From impact locations $\sim P_8-P_9$, BBS is equivalent or greater for the hollow bats.
11.4.2 Constant Bat Power

The case describing constant bat power, illustrated in Figure 11.4 describes a relative bat performance between that of the other two limiting cases. Between P1-P7 the solid bats demonstrate \( \sim 2 \) mph improvement in BBS in comparison to the hollow bats. At P8 the values are equivalent and from P8-P9 BBS is approximately 0.2-2 mph greater for the hollow bats.

11.4.3 Constant Bat Swing Speed

Since the hollow bats exhibit greater values of MOI about the ICR, greater comparative performance is demonstrated for the case where swing speed is independent of MOI, shown in Figure 11.5. In this scenario, BBS is approximately equivalent for all bats (except prototype 1) along the striking region P1-P3. The region P3-P7 demonstrates a reduction in hollow bat performance of \( \sim 2 \) mph BBS in comparison the solid bats. The BBS for the hollow bats continues to increase for impacts in the region P7-P9 resulting 1-3 mph increase in BBS relative to the solid bats in this region.
Values of maximum BBS under the standard game condition are given in Table 11.2. The values of maximum BBS between all bats for all swing speed cases are relatively similar, with a largest difference of 2.2 mph observed between the traditional willow bat and the prototype 2 thick face bat, for the case of constant bat KE. The willow bats show increased maximum BBS for the two cases $n = 0.5$ (~5%) and $n = 0.3$ (~2%) relative to the hollow bats. Whereas the hollow bats show an increase in BBS relative to the solid bats of ~2% for the case of constant swing speed, $n = 0$.

<table>
<thead>
<tr>
<th>Maximum BBS (distance from handle top)</th>
<th>Constant bat KE ($n = 0.5$)</th>
<th>Constant bat power ($n = 0.3$)</th>
<th>Constant swing speed ($n = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Willow - Template</td>
<td>24.6 (680 mm)</td>
<td>24.7 (684 mm)</td>
<td>24.9 (682 mm)</td>
</tr>
<tr>
<td>Willow - Traditional</td>
<td>25.2 (667 mm)</td>
<td>25.1 (681 mm)</td>
<td>24.9 (667 mm)</td>
</tr>
<tr>
<td>Willow - Bowed</td>
<td>24.8 (676 mm)</td>
<td>25.1 (668 mm)</td>
<td>25.4 (674 mm)</td>
</tr>
<tr>
<td>Prototype 1</td>
<td>23.8 (696 mm)</td>
<td>24.5 (770 mm)</td>
<td>25.3 (694 mm)</td>
</tr>
<tr>
<td>Prototype 2-1 (thin face)</td>
<td>24.3 (770 mm*)</td>
<td>24.8 (770 mm*)</td>
<td>25.6 (770 mm*)</td>
</tr>
<tr>
<td>Prototype 2-2 (thick face)</td>
<td>23.0 (770 mm*)</td>
<td>24.1 (770 mm*)</td>
<td>26.0 (770 mm*)</td>
</tr>
</tbody>
</table>

*peak values determined within the limits of bat face testing

Table 11.2 - Maximum values of BBS under standard game conditions

### 11.5 THE EFFECT OF BAT SWING

Figures 11.6 and 11.7 illustrate the values of $e_A$, bat velocity, and ball velocity along the striking region of the template willow bat and the prototype 2 thin face bat under the condition for constant bat power. Without the variation of linear bat velocity along the striking region due to the swing, the $e_A$ trend would resemble that established for a constant impact velocity, illustrated in Figure 10.13. However, an increase in velocity towards the toe of the bat has the effect of reducing $e_A$ for impacts away from the ICR. Additionally, BBS would follow the $e_A$ trend if bat velocity remained constant.
However, the increase in bat velocity towards the toe results in a peak ball velocity ~40 mm further from the handle for the solid bat, than would be the case for constant bat velocity. The hollow bat demonstrates a peak velocity at least 115 mm further from the handle than would be the case for a constant bat velocity. These examples demonstrate the significance of a rotating bat swing in affecting both the value and location of maximum BBS.

![Graph of ball and bat velocity](image1)

**Figure 11.6 - Template willow bat, case of constant bat power: Values of $e_A$, bat linear velocity and BBS**

![Graph of ball and bat velocity](image2)

**Figure 11.7 - Prototype 2-1 (thin face) bat, case of constant bat power: Values of $e_A$, bat linear velocity and BBS**

### 11.6 BBS AND THE 'SWEET SPOT'

If the objective of the batting shot is to impart maximum velocity to the ball, then an impact equivalent to the location of maximum BBS would be the most desirable location for impact. The perceived feel in the hands however, must also be considered. For example, if an impact at the location of maximum BBS generates painful vibrations in the hands then it is likely the player will not favour this bat design.
Several studies have investigated likely causes of undesirable or painful vibrations in the hands resulting from bat and ball impact in baseball and two causes of unpleasant sensation are predominantly discussed [7, 26, 28, 29]. However, the relative contribution of each to the overall sensation felt in the batsman's hands is subject to a difference of opinion, which is summarised below.

Rigid body models describe the translation and rotation of the bat about the COM following impact. The bat centre of percussion, explained in chapter 4, is normally defined as an impact location on the striking region of the bat which results in zero net reaction force at a desirable location on the handle. Whether the ideal conjugate point should reside under the top or bottom hand or at some other location on the handle is not particularly relevant in this study as it is likely to be player dependent. A percussion zone, used by Cross [29] to define a region on the striking face, the limits of which correspond to conjugate percussion points at either end of the handle, was used in this study to illustrate the region in which rigid body modes in the handle are likely to be minimised.

In defining the location of a baseball bat sweet spot, Cross [28] suggests that the sensation felt in the batsman's hands results from an impulsive waveform reaching the hands shortly after impact which contains components of bat rotation, translation and vibration. Cross suggests that the sensation is minimised when the summation of energy from all three components in the handle region is minimised. Cross argues that for baseball bats, the energy coupled with the second mode of vibration is of the same order as that coupled with the fundamental mode of vibration and total vibration energy is lowest at a location of impact approximately halfway between the two respective nodes. If the percussion zone extends either side of the node locations then the summation of rigid body and vibration energy is minimised in the handle for an impact at this location.

Adair [26, 86] suggests that minimum hand sensation occurs from an impact equivalent to the node of the fundamental mode of vibration, and the location of the COP and the node of the second mode are of little significance. Adair argues the perceived sensation is dependent upon the sensitivity of human skin to the excited frequencies of vibration and that the fundamental frequency of a baseball bat ~170 Hz is detected sensitively by the hand whereas the second mode ~560 Hz and the rigid body modes are not.

Cross [29] suggests the rigid body modes are detected as they produce an impulsive spike composed of a broad spectrum of frequency components which would be detected by the hands in similar fashion to the 'sting' felt by a player catching a fast moving ball. Russell [116] suggests that the COP is irrelevant in baseball because the ICR is located ~64 mm beyond the end of the handle. A study by Noble and Walker [118] defined a general impact region between the COP and the node of the fundamental mode which would result in the most pleasant hand sensation but the study did not discriminate either point as being more or less significant.

Figures 11.8 and 11.9 illustrate the relationship between BBS and bat impact location for the template.
willow bat and the prototype 2 thin face bat. As several opinions exist regarding the location of the sweet spot in baseball, all likely locations are identified in Figures 11.8 and 11.9 in order that some general conclusions can be made. These include the conjugate percussion points corresponding to the top hand, the bottom hand, and the bat ICR, and the nodes of the first three modes of bat vibration. In addition, the PC region [29] encapsulating all conjugate percussion points along the length of the handle is indicated by the blue shaded areas and the red shaded areas identify the region between the nodes of the first two modes of vibration [26].

For both bats, the region between the nodes of the first two modes of vibration, corresponding to the region of minimum vibration (MV) [29], is located within the extent of the larger PC region of the striking face. The MV region is located centrally in the PC region for the solid bat corresponding to percussion points between the top and bottom hand. Whereas, the MV region for the hollow is located towards the limit of the PC region corresponding to conjugate points towards the bottom of the handle, including that of the bottom hand. The PC region is 18% larger for the hollow bat and its midpoint is the same distance from the handle top as for the solid bat. The MV region is 38% larger for the hollow bat and its midpoint is located 26 mm further from the handle. For both bats the MV region does not encompass the location of maximum BBS. However, for the solid bat the PC region encompasses the maximum BBS whereas the PC region for the hollow bat does not.

The location of minimum hand sensation as defined by Cross [28] as being the centre of the MV region (within the PC region) corresponds to a BBS of ~24 m/s for both bats and is ~4% lower than the maximum BBS. The percussion point corresponding the bottom hand for the hollow bat and between the hands for the solid bat coincides approximately with the centre of their respective MV regions. Under the definition of minimum sensation proposed by Cross, these location would result in minimal combined excitation.

The location of minimum hand sensation as defined by Adair [26, 86] as being the position of the fundamental node corresponds to a hit ball velocity 6% lower than the maximum BBS for the solid bat and 8% lower than the maximum BBS for the hollow bat. This location is 608 mm and 624 mm from the top of the handle for the solid and hollow bat, respectively.

The two definitions of impact location which result in minimum hand sensation [28, 86] are not equivalent to the location of maximum BBS for either bat. If the player attempted to minimise hand sensations through adjustment of impact location then a reduction of ~4-8% from the maximum possible value of BBS is anticipated. If a general region of minimal hand sensation is defined as the intersection between PC and MV regions, then minimal hand sensation is located further from the handle and exists over a larger area for the hollow bat than the solid bat.
Figure 11.8 - Template willow bat, case of constant bat power: BBS vs. impact location including location of percussion points and vibration nodes

Figure 11.9 - Prototype 2-1 (thin face) bat, case of constant bat power: BBS vs. impact location including location of percussion points and vibration nodes

11.7 PROTOTYPE 2 IMPACT PERFORMANCE

It is likely that a location of maximum BBS equivalent to the location of greatest perceived pleasantness would be of benefit to batsmen. However, published research concerned with the location of greatest perceived pleasantness for baseball suggests that neither the solid nor hollow bats tested in this study fulfil this criteria.
The current understanding has permitted an insight to a possible relationship between BBS and perceived pleasantness. However, the only reasonable conclusion that may drawn from the impact performance study is that maximum BBS under playing conditions for the solid and hollow bats are comparable but the hollow bat impact characteristics are such that maximum BBS occurs further from the handle than predicted for the solid bat.

It has been shown that the increase in $\overline{I}_m$ and $\overline{I}_p$ demonstrated by the hollow design leads to an increase in effective mass for eccentric ball impacts. It is anticipated that this will be of benefit to the batsman in changing the momentum of the ball.

The face constructions used in the hollow bats demonstrated a small enhancement in ball COR relative to willow, determined as part of the face impact performance study. However, the COR characteristic measured along the striking region of the bat suggests that the improvement in local impact elasticity was lost due to a reduction in bat flexural rigidity.

The predominant concerns regarding the current hollow design are durability of the laminated blade construction and the relatively low flexural rigidity observed upon impact. It is likely that these concerns are not independent. Although the laminated construction provided the manufacturing capability to generate internal geometries, permit greater control over wood quality, and investigate the effects of wood orientation, bat rigidity and durability may have suffered as a consequence.

11.7.1 Validity of Hollow Bat Concept
The hollow bat concept has been validated in terms of the ability to modify bat inertia properties and to produce comparable impact performance. Pending a relaxation of current MCC laws regarding the bat, it is anticipated that further development in hollow bat design could result in commercial designs which give the batman a greater choice in selecting ideal playing characteristics.
CHAPTER 12

CONCLUSIONS

This study investigated the science of cricket bat playing properties. Since striking the ball is central to the game of cricket, a significant amount of research was apportioned to investigating bat impact performance. The design and manufacture of a novel hollow bat concept was simultaneously developed based upon the success of hollow striking implements in other sports, such as golf and baseball, and the absence of a recent and significant change in cricket bat design. The main work and achievements of the study are as follows:

12.1 BAT GEOMETRY AND MASS DISTRIBUTION

The geometry of a hand crafted bat was digitised and an idealised representation of a conventional bat design was computer modelled. From the model, a hollow design was developed with sculptured internal geometry. The hollow bat CAD model demonstrated a significant increase in MOI about the bat COM about the three principal axes, achieved by using higher density wood, and an improvement in the performance from eccentric ball impacts was anticipated. The value of total mass, COM location and swing MOI about the handle remained largely unaltered since a significant difference in perceived bat mass properties was considered to be undesirable from the batsman's perspective.

12.2 DEFINITION OF BAT GRADING SYSTEM

A study was conducted to assess the sensitivity of a batsman's perception to changes in bat mass, first moment and the value of swing MOI, for which measurement instruments were developed. From the results of the study, a bat grading system was established based upon a measure of first moment about the handle. This grading system could used by manufacturers to assist the batsman in selecting an appropriate bat based upon mass properties. This method of grading is in contrast to the current system which is based upon the aesthetic qualities of the striking face of the bat and total bat weight.

12.3 MODELLING BAT IMPACT PROPERTIES

Rigid body models were initially used to investigate the effect of bat mass distribution on the rebound properties of the ball for eccentric impacts along the centreline of the bat and for off-centreline impacts.

Spring damper models were used to account for the deformable nature of the bat and ball. The models were specifically used to assist in the design of the hollow bat striking face by establishing relationships between COR and values of ball and face stiffness, damping and mass. FEA simulations provided
further understanding of the relationship between the geometry of the bat face and bat face stiffness which guided the development and production of prototype 2 faces.

FEA bat and ball impact models were developed for a template willow and prototype 1 hollow bat design. Impacts along the striking region of the face were simulated and good agreement between numerical and experimental impact properties was evident. The FEA models were used to assess the relative contribution of handle vibration, bat vibration and ball damping upon COR and $e_d$.

12.4 DEFINITION OF A BAT IMPACT TEST SPECIFICATION

A test specification was established following a review of current standards in baseball to determine the playing performance of bats tested under laboratory conditions. Impact test cages were designed and constructed to satisfy the test criteria. The COR and $e_d$ of several bats including three prototype hollow bats were determined by the tests and the results were used to predict the values of BBS during play.

12.5 LAMINATED HOLLOW BAT MANUFACTURING METHOD

A method of laminated hollow bat manufacture was established with the internal and external bat geometry realised through numerically controlled machining processes. The method was used to produce prototype 1 bats. Subsequently, the method was modified to accommodate curved laminate face constructions in the manufacture of prototype 2 bats.

12.5.1 Hollow Bat Concept

A significant part of the research effort was concerned with the development a hollow cricket bat made from wood. The centreline impact performance measured under standard game conditions was comparable to the solid willow bat with approximately equivalent values of maximum BBS. Off-centreline impact properties were significantly modified due to an increase in principal moment of inertia. The primary concerns were impact durability and the increased propensity to vibrate following impact. However, the study established the validity of the concept and proved a method of manufacture.
CHAPTER 13

RECOMMENDATIONS FOR FURTHER WORK

13.1 PLAYER TESTS

13.1.1 Player Perception of Cricket Bat Playing Properties

Research by Roberts [66] established a method to elicit the perceived characteristics of a golf shot by interviewing a number of elite players during ball hitting sessions. A method of analysing the data was established to obtain a perception map in which the important perceived characteristics of a shot and their inter-relationship were defined. A similar study in cricket would prove beneficial in furthering an understanding of the bat or shot characteristics that are important to batsmen.

13.1.2 Relationship Between Bat Mass Properties, Player Perception and BBS

A first attempt to relate measured and perceived bat mass properties was investigated in this study and the results were used to bring about a bat grading system. A paired comparison test method could also be used to determine the sensitivity of players to changes in bat mass property and would be a worthwhile study to compliment existing results.

Although a grading system may assist the player in appropriate bat selection. The system would be more meaningful if the player was aware of the value of bat mass property from which they would most benefit. Bahill [21, 22] developed the Bat Chooser™ for baseball in order to establish recommended bat mass for different players. The method was based upon an experimental determination of the relationship of swing speed and bat mass for an individual player and deriving the corresponding values of BBS using conservation of momentum laws. Additionally, the results of extensive tests have been used to establish a database from which general measurements of player height, age, weight and level of play and can be used to estimate a bat weight in the absence of measuring the individual’s swing speed and bat mass profile. More recently, the relationship between bat inertia [23] and swing speed has been investigated with a similar view to establishing an ideal bat MOI based upon BBS. It was recognised in the studies by Bahill [21, 22] that factors such as bat manoeuvrability and shot accuracy were also important to the batsman in addition to BBS and this is likely to be the case for cricket. However, a study in cricket to establish recommended bat mass properties based upon values of BBS would be a worthwhile investigation and would compliment the bat grading system.

13.1.3 Kinematics Analysis of Bat Motion

The definition of a standard game condition required a value of incident bat velocity. Bat velocity was determined from the results published in two independent studies [14, 15] which were predominately
concerned with the motion of the batsmen's limbs for different batting shots. An investigation focused upon the motion of the bat during a shot would be worthwhile in order to obtain more detailed information regarding bat motion during a shot. The study could be linked with the previous recommendation to investigate the relationship between bat mass properties and swing speed for a range of batsmen using a motion analysis system such as CODA.

13.1.4 Relationship Between Bat Characteristics and Impact Sensation

Several studies [26, 28, 29, 86, 118] have investigated the batsman's perception of pleasantness in a baseball batting shot. Cross [28] used a series of piezoelectric transducers to measure the relative acceleration for different locations along the bat and the force exerted on the hands from ball impact. Noble and Walker [118] used a single measurement of bat vibration near the hands and correlated values with player ratings of pain and discomfort. A study combining the level of instrumentation used by Cross [28] and the use of player response as in Noble and Walker [118] could be undertaken in cricket to investigate a relationship between the sensation perceived in the batsman's hands and measured values of handle motion. Results from this type of study could ultimately be used to minimise painful handle vibrations through bat design.

13.2 MODELLING BAT CHARACTERISTICS

13.2.1 FE Material Definitions

The bat and ball FEA models in this study were defined using elastic material definitions and therefore β-damping was introduced to the ball model to account for observed energy dissipation. The value of β-damping used in the model was obtained through experimental validation of observed model behaviour. It would be worthwhile investigating the use of an inelastic material definition in which the damping properties are defined as part of a material test procedure. This route would eliminate the computational cost of including β-damping in the model. The current version of ABAQUS (version 6.5) only permits the definition of isotropic inelastic materials and is therefore not entirely suitable for representing the anisotropic inelastic properties of wood. One solution maybe to perform the analysis in an FEA package which does include anisotropic inelasticity, such as ANSYS.

13.2.2 Detailed FE Bat Model

Defining the individual handle constituents would be beneficial to understanding their contribution to the overall behaviour of the handle and the bat. The definition of blade laminates would also provide further understanding of the effect of layer orientation and the adhesive in a multilayer blade construction. It has been noted in studies such as Grant and Nixon [33] for cricket and Van Zandt [100] for baseball that the handle is the least rigid bat component and a change in its geometry or material properties can have a significant effect upon the lower modes of excitation [100]. Although the behaviour of a rigid and deformable handle have been simulated in this study it would be beneficial to investigate further the role of handle design upon the node position and natural frequency of the lower modes of excitation.
13.2.3 Bat Flexibility and Transverse Vibrations
The results of this study support the generally accepted view that bat flexibility plays a significant role in determining ball rebound. Further studies have also suggested that resulting bat vibrations play a significant role in the perceived ‘feel’ of a shot [26-29, 86, 118]. In this study the vibration properties of the bat were determined using an eigenvalue extraction analysis provided in ABAQUS. Studies in baseball by Van Zandt [100] and Nathan [45] and in cricket by Gutaj [37] have used other numerical means based upon beam deflection theory to model the transverse vibration of the bat. It is argued by Gutaj [37] that this method provides a more efficient means of analysing the sensitivity of bat design parameters on the bat vibration in contrast to a FEA optimisation study which may involve numerous modelling and simulation iterations. These theoretical models could be used to compliment and focus FEA work on specific bat design parameters.

13.3 MEASURING BAT CHARACTERISTICS
13.3.1 Bat Performance Test Machine
The experimental impact test rigs used in this study enabled measurement of COR and $e_d$ using a high speed camera. There is an opportunity to develop a test machine similar in function to that described for baseball under ASTM standards [48, 50, 51] which can be used to measure maximum COR and $e_d$ without the use of an expensive camera system or the need to undertake image digitisation. Values of maximum BBS can be derived under standard game conditions from the results of testing, in order to quantify bat performance. It is anticipated that the ability to determine accurate values of maximum BBS for different bats will be of interest to both bat manufactures and the MCC.

13.4 HOLLOW BAT DESIGN
The potential for increased variety and enhanced playing characteristics using a hollow bat design are worth further investigation. Although a hollow bat does not constitute a direct breach of the game rules, prototype bats made in this study have been judged to contravene the current MCC bat design regulations due to their method of construction. The bat rule states that the blade must be ‘made solely of wood’ [3] and it is the adhesive between laminate layers to which the MCC, during an informal discussion, has taken exception.

The rules concerning the implements in other major sports have seen significant development following the introduction of new materials and technologies. In contrast, cricket has maintained an immutable position regarding the introduction of novel bat technologies, a point highlighted recently following the ruling to exclude the Kookaburra graphite covered bats from international and first class cricket [121]. The following recommendations are of academic interest unless a favourable change in the rules governing bat design generates a stronger commercial incentive to develop significant changes in bat design.
13.4.1 Construction

In this study the hollow blades, with the exception of the prototype 2 striking face, have been constructed with planar laminates through which the internal and external geometries have been machined. It would be worth investigating the forming of a contoured laminate structure in which the internal and external blade geometry are defined predominantly by the curvature assigned to continuous multilayer segments. In order to achieve the blade shape without significant machining, the blade geometry may need to be simplified to accommodate the limitation in forming complex curvatures.

The use of spring damper models in this study has highlighted the importance of minimising sprung mass and damping properties of the striking face in order to maximise the ‘trampoline’ effect. These properties are more readily minimised with the use of thin shell like structures as observed in titanium golf club driver heads and aluminium baseball bats. An effort to re-create similar performance in wood was partially successful in this study and a further investigation into the durability and stiffness of thin, light weight wood composite structures is necessary in order to realise significant enhancement in the local transfer of energy for a wooden cricket bat.

A hollow bat design is not limited to a multilayer construction. Although a blade made from at least two parts would be necessary to facilitate the machining of a sculptured hollow cavity. Using less constituent components may reduce the number of potential failure sites which exist at the mating surfaces and edges of layers within a multilayer system. Since the laminate constructions in this study demonstrated durability concerns, the use of less constituent parts may be worth further investigation; particularly since the concept of increasing the degree of isotropy facilitated by a multiple layer construction has been discounted in favour in orientating the wood to optimise longitudinal blade stiffness.

The handle which was incorporated in the prototype bats was of conventional design in order that the differences in bat impact behaviour between prototype and solid bats could be attributed predominantly to the blade portion. The FEA study illustrated that the handle has a significant effect upon the vibration properties of the bat and is therefore likely to affect the sensation felt in the batsman’s hands. Whether handle properties have a significant effect upon the ball rebound is less clear. A more detailed investigation to the sensitivity of handle design upon ball rebound and perceived feel of a shot is worthwhile in conjunction with hollow blade design. One possible area of study would be to investigate the potential benefits of a hollow handle as used in aluminium baseball bats and modern tennis rackets.
REFERENCES


APPENDICES
Willow Trees

Willow is the predominant wood used in the manufacture of cricket bat blades worldwide and two principal varieties are used. Kashmir willow as the name suggests is grown in Kashmir mostly in the Pulwama and Anantnag districts. Originally, the trees were grown for cattle fodder and charcoal production. However, the global popularity of cricket has provided further demand for this variety. Bats are also made from Salix alba coerlea, commonly termed 'English' or 'cricket bat' willow, this material is less dense and than Kashmir willow and is the preferred material of elite players. English willow trees are commercially grown in Southern England and regulation governing their planting and felling is overseen by the Forestry Commission. Cleft suppliers and bat manufacturers work closely with the Forestry Commission in obtaining the required volume and quality of willow material for worldwide bat production.

Willow cuttings are taken from the felled tree and planted as 'sets' 9 m apart situated near running water, shown in Figure A2.1. The preferred size of a tree for felling is 1.3 m diameter with a 3 m trunk section void of branches and it can take 15 years for a tree to reach this size. Throughout its growth, the trees must be maintained as wood quality and associated value can deteriorate from afflictions such as Gaul's disease, fungi, lichen and the emergence of low branches. Also, adverse conditions such as stagnant water, harsh frosts and dry summers are detrimental to the final quality of wood. The cleft supplier or bat manufacturer will usually assess the health and size of each tree within the plantation prior to purchase.

The main plantation harvest is between August and December. During the winter months the ground is often too soft for the machinery used to fell and transport the wood. However, trees can be felled all year round to accommodate modern production demands.

Clefts

A section of the main tree stem from the ground to a height of 3 m is used in cricket bat production. Above this height, the branches which have been allowed to grow outwards form knots within the stem which may appear in the blade of the bat.

The trunks are then cross cut into sections no less than 760 mm in length, shown in Figure A2.2. These sections are known to the industry as 'Rounds' which are split further into four parts by driving a wedge positioned radially through the cross-section of the Round, shown in Figure A2.3. This has the effect of splitting the wood along the longitudinal grain boundaries and is important as it minimises the appearance of ruptured wood cells on the hitting face of the blade.
The clefts are 'green timber' with a moisture content of approximately 50%. In order to reduce the moisture content, they are stacked on pallets in rows of 9 to a height of 20 clefts, each layer separated to provide a gap for air circulation, shown in Figure A2.4. The clefts are air dried in a kiln under an evaporation and stabilisation cycle which is monitored daily for four weeks reducing the moisture content to 12%. The principal motives of this are to reduce the weight of the material and to ensure equilibrium with an atmosphere of a relative humidity to which the bat will be exposed during its life. Shrinkage principally occurs along the radial and tangential planes of wood and therefore the ends of the clefts are sealed with wax to reduce the number of cracks resulting from shrinkage which occurs most rapidly where the end grain is exposed.

After the drying process the waxed ends of the cleft are removed with a circular saw reducing their length to that required for the machining of both full size or junior sized bats.

A Wadkin five cutter profiling machine is used to machine a generic profile of uniform cross section along the length of each cleft, shown in Figure A2.5. The profile describes a convex curve on the front face and blend radii to the front edges, the edges are parallel and the back face is machined to a 'V' shape, shown in Figure A2.6. Prior to machining, an assessment of the most suitable side of the cleft to act as the face of the bat is made through visual inspection for surface defects and uniformity of wood grain and growth ring structure. Also, the position of the cutters can be adjusted to remove more material from one side of the cleft than the other and is used to exclude areas of 'heart wood' from the cleft. The clefts are made oversize to allow for a reduction in size that results from pressing.

The hitting face and edges of the bat are compressed to increase there density and improve bat life. Players also believe that the 'feel' of the bat is also improved by this process. The pressing operation involves passing the edges and the face of the bat through a series of rollers whose profile matches that of the cleft and ensures a uniform distribution of pressure, shown in Figure A2.7. The position of the rollers can be adjusted to regulate the compressive force acting upon the cleft. The faces of the cleft are first moistened to reduce the risk of brittle fracture as the wood undergoes densification. The cleft is clamped onto the machine in a jig which holds the apex of the bat firmly in a 'V' channel and the pressure level is set according to what is believed to be the minimum value of pressure to ensure acceptable durability and improved playing performance. Higher levels of pressure are avoided as they lead to a reduction in performance and material failure as the rollers transmit an excessive force through the body of the wood. The adjustment of the machine to a suitable level of wood compression is a subjective process which is achieved by the experience of the bat maker. Bouncing a cricket ball on the face of the cleft gives a first indication to the performance of the compressed face and whether further pressing is necessary. Ideally, this evaluation would be undertaken for every cleft on the basis that an optimal setting will be different for each cleft due to their inherently unique structure. However, time constraints of production result in entire batches processed using the same setting with an occasional 'bounce' test and adjustment during production.
The clefts are graded upon a visual inspection of the wood surface. It is widely accepted that features such as large knots and surface cracks which interrupt the continuity of tree stem structure are detrimental to the playing performance of the bat and therefore clefts with such features are often rejected.

The manufacturer decides on cleft quality using a visual grading system. Bats are graded 1-4 based upon the number and variety of defects observed and to broaden the cost range of bats to the consumer. Figures A2.8 and A2.9 illustrate the appearance of a grade 1 and grade four hitting face, respectively.

A decision is also made to the orientation of the cleft with regard to visible defects that could be masked behind brand stickers added later. A 'V' shaped wedge is then sawn from the most suitable end of the cleft to accommodate the handle. The cleft is mounted in a jig which swings through an angle equivalent to the apex angle of the desired wedge, therefore both sides of the wedge can be met by swinging the jig from one extreme to the other. The jig allows a consistent 'V' shape to be cut from every cleft and the cut out wedges are kept as a component for bat handle manufacture.

The Handle
The handle is predominantly made from rattan cane with strips of cork or rubber dividing the cane into sections running the length of the handle. This construction has been designed to give the handle suitable damping and stiffness properties. The cane for the junior handles is firstly sawn into 480 mm lengths and then three cuts are made. The first cut, straight down the middle of the cane is 190 mm long and the second and third cuts are made either side of this to a depth of 80 mm. A wedge is jammed into the middle cut allowing glue to be applied thoroughly. A piece of black rubber 115 mm long and 2 mm thick is then placed into the bottom of the middle cut, followed by a piece of fibreboard of 80 mm in length. Two more pieces of this fibreboard are then applied with glue and used to fill the other two cuts. The handle is tied up and left to dry for 24 hrs. Once dry the handle is untied, sawn to the required handle length and set aside for machining.

Those canes that present the largest diameter are used to make full size handles. The largest are sawn in half making up the outer sections of the handle. The others are sawn into rectangular strips of a width half the diameter of the handle and two strips are then glued together along their thinnest edge to make the central section of the handle, as shown in Figure A2.10. Sheets of 2 mm thick latex bonded cork are cut into rectangular strips and positioned within the handle composite. The final element to the handle is the wedge cut from the cleft and it is positioned within the centre of the handle 'sandwich' to approximately 80% of its length. The reason for including the wedge is to increase the depth of the handle towards its base as many bat models exhibit a deep cross section towards the shoulder of the bat. The handle needs to accommodate this deep cross section to ensure a continuous
surface over the back of the blade following machining. The complete handle assembly is shown in Figure A2.11. The freshly glued handles are stacked and wedges are used to separate the handles and provide some compression to aid the adhesion of all elements, shown in Figures A2.12 and A2.13.

Once the glue has cured the handles are cross cut to the exact length. The circular profile of the handles is machined between centres on a copy-shaping machine with a rotating cutter and the handle is rotated slowly to produce a round section, shown in Figure A2.14. Both full size and junior handles are turned to an appropriate size however, various profile patterns exist for the different sizes for example, the handles may be turned to produce either long or short handles. The base of the handle is sawn so as to fit the cleft, shown in Figure A2.15.

The handles are checked for fit against each cleft before they are glued together and aligned. The bat is left to cure for a minimum of 15 hours after which time the adhesive is strong enough for further machining.

**Bat Shaping and Finishing**

The shape of the back face, shoulders and toe as well as the weight and balance of the bat are traditionally fashioned with hand tools and is a highly skilled craft. CNC machining is a recent advance in cricket bat production at Slazenger UK, shown in Figure A2.16. Hand crafting is reserved for bats used by Slazenger professional contract players and exists in less technologically advanced factories as the predominant method of manufacture. The following section describes the method of CNC manufacture as used by Slazenger UK (2000).

The CNC router machine is capable of processing 20 bats every hour. The machine follows predetermined programs corresponding to different bat shapes and they incorporate all information required to operate the machine within a production cycle. The instructions include a decision on the choice of cutters, the order of machining and specific instructions governing cutter speeds and paths with respect to the machine datum. The program also controls the rotation of the work bed, extraction of arising material and the removal and release of machined bats. The machine operates four axes of movement including a bed that holds and rotates the bats along their major axis. A vacuum fixture clamps the bat face down onto the bed and two cutting heads machine simultaneously, each allocated to a separate bed. The machine has four beds in total and two bats are machined, whilst two different bats can be prepared or removed. The CNC machine can accurately and consistently remove a relatively large volume of material from the cleft to leave only small regions that require finishing. The tool profiles and machining bed give rise to limitations as the tools must move relative to the work bed without interference. The Wadkins machine is unable to blend the shoulders of the cleft to the handle because this feature of the bat is too close to the bed for the tool to approach and machine. Also, the assembly of the handle to the cleft is successfully achieved through aligning the two components before glue sets. Inevitability, as this alignment is achieved
visually, almost each handle is aligned to a cleft with slight difference to the next. The CNC program cannot account for these subtle variances and a slight discrepancy of handle position with respect to the datum would entail a degree of over or under machining if the shoulder blends were attempted.

The shoulder blending operation is therefore performed by hand using a draw knife to cut away the bulk of material and create the general shape, followed by a rasp to smooth the blends into the handle, shown in Figure A2.17.

Two sanding stages finalise the bat shape and provide a level of surface finish suitable for polishing. Sanding wheels, shown in Figure A2.18, are used with different grades of emery paper to ensure a high degree of blend symmetry.

Grade 4 bats are bleached after the rough sanding has corrected blend radii and removed machining marks. Bleaching is a procedure added to increase the visual appeal of low grade bats. The removal of undesirable surface colouration or marks is achieved by bleaching the surface of the blade so the appearance is more akin to a high grade bat. The bats are dipped into a bath of 50% hydrogen peroxide, promptly withdrawn and placed upon a rack to dry. The rack is situated in an ammonia atmosphere and bleaching of the wood cells occurs over three days, as shown in Figure A2.19.

The bare cane handles are covered with a layer of wound cotton string, the winding process begins at the top of the handle and the string is stapled upon reaching the shoulders of the bat. It is believed that the string adds structural support to the handle increasing flexural rigidity and also the tension applied during winding acts to compress the handle reinforcing the integrity of glued components.

Each full size bat is weighed together with the accompanying rubber handle grip and stickers to measure the mass of the finished product. The bats are stamped to register the length of handle and the colour of the stamp corresponds to the mass of the bat.

The rubber grip is added to the handle on top of the wound string, increasing the diameter of the handle and gives a surface with substantial grip against the batsman’s gloves. The tubular rubber grip has a diameter smaller than the handle and so must be strained to fit the handle. This is achieved through expanding the rubber tube over the inside of a larger diameter pipe by removing the air between the two. The handle is then positioned inside the pipe and the vacuum released, the rubber attempts to regain its original shape tightly enveloping the handle.

Stickers are co-ordinated and added to the blade according to the range and model of the bat. A final inspection is given to the bat before it is bagged and boxed. The sample of the finished bat products are shown in Figures A2.20 and A2.21.
Figure A2.1 - Willow trees planted in 'sets'

Figure A2.2 - Mature trees cut into Rounds

Figure A2.3 - Clefts cut from rounds
Figure A2.4 - Clefts stacked for drying

Figure A2.5 - Wadkins profiling machine

Figure A2.6 - Profiled clefts
Figure A2.7 - Cleft passing through front face press machine

Figure A2.8 - Typical example of a grade 1 cleft

Figure A2.9 - Typical example of a grade 4 cleft
Figure A2.10 - Orientation of thinner cross sections in full size handles

Figure A2.11 - Handle lamination including outer cane components, cork and cane strips and willow wedge

Figure A2.12 - Glued handle assembly
Figure A2.13 - Handles stacked and separated by wedges whilst adhesive cures

Figure A2.14 - Copy shaping machine with rotary cutters

Figure A2.15 - Handles after copy shaping
Figure A2.16 - Wadkins CNC router

Figure A2.17 - Using the draw knife to blend the shoulder to the handle

Figure A2.18 - Blending the toe feature
To lighten their colour, bats are dipped into a bath of aqueous hydrogen peroxide and are stored for three days in a room kept with an ammonia atmosphere.

Figure A2.20 - Slazenger 2001 product range – front view

Figure A2.21 - Slazenger 2001 product range – side view
1. **Width and length**

   The bat overall shall not be more than 38 inches / 96.5 cm in length. The blade of the bat shall be made solely of wood and shall not exceed 4 1/4 in / 10.8 cm at the widest part.

2. **Covering the blade**

   The blade may be covered with material for protection, strengthening or repair. Such material shall not exceed 1/16 in / 1.56 mm in thickness, and shall not be likely to cause unacceptable damage to the ball.

3. **Hand or glove to count as part of bat**

   In these Laws,

   (a) Reference to the bat shall imply that the bat is held by the batsman.

   (b) Contact between the ball and either

      (i) The striker's bat itself

      or (ii) The striker's hand holding the bat

      or (iii) Any part of a glove worn on the striker's hand holding the bat shall be regarded as the ball striking or touching the bat, or being struck by the bat.
APPENDIX 3

DUNLOP SLAZENGER CLEFT GRADES (2000)

Grade 1* Cleft (reserved for professional players)
There must be no amount of brown wood evident on the face. There must be at least four growth rings visible on the face which must be straight and parallel to the bat edge. Only one ‘butterfly mark’ is allowed on the face.

Grade 1 Cleft
There must be no amount of brown wood evident on the striking face, although some may be permitting on the bat edge. There must be at least four growth rings visible on the face which must be straight and parallel to the bat edge. There must be no obvious blemishes or discolouration on the striking face although two ‘butterfly marks’ are allowable and twig marks are allowable on back face.

Grade 2 Cleft
There must be at least four growth rings visible on the face which must be predominantly straight and parallel to the bat edge. The presence of ~15% brown wood is allowed on the face and small blemishes such as ‘pin knots’ and ‘specks’ also permitted.

Grade 3 Cleft
A Grade 3 cleft can have up to 40% of the face as brown wood provided the grain is predominantly straight. A limit of four ‘specks’ or ‘butterfly marks’ are allowed on the front face and several ‘pin knots’ are permitted.

Grade 4 Cleft
A Grade 4 cleft can have over half the face as brown wood, any number of growth rings are allowable and the face can exhibit discolouration or other blemishes. Grade 4 wood is normally bleached to reduce visible markings and discolouration.

Reject Cleft
A cleft exhibiting cracks, large knots greater than ¼ inch diameter, and other undesirable features which cannot be eliminated through further machining into a ‘boys’ bat.

| Figure A5.1 - Illustration of grading features |
|---|---|---|---|
| 'pin knot' | 'Speck' | 'Butterfly stains' | brown wood |
| 266 |
APPENDIX 4

STATE-MATRIX METHOD TO SOLVE A CRICKET BALL IMPACT ONTO A RIGID SURFACE

Adapted from section 4.10 Inman [108] and Cottey [109] and written in MathCAD®

\[
\begin{align*}
mb &:= 0.157 & \text{Ball mass and initial velocity} \\
v_{in} &:= 35
\end{align*}
\]

\[
\begin{align*}
ck &:= 0.1983 \cdot v_{in}^2 - 1.7212 \cdot v_{in} + 193.0336 \\
kb &:= 1081 \cdot v_{in}^2 + 1718 \cdot v_{in} + 1340655 \\
t &:= -0.0000109 \cdot v_{in} + 0.00113
\end{align*}
\]

\[
M := mb \quad C1 := cb \quad K1 := kb
\]

\[
\begin{align*}
ck &:= 2.725 \times 10^6 \\
t &:= 7.485 \times 10^{-4} \\
cb &:= 375.709
\end{align*}
\]

\[
\begin{align*}
O &:= 0 \\
I &:= 1
\end{align*}
\]

\[
A1 := \text{augmen} \left( \text{stack} \left( O, -M^{-1}K1 \right), \text{stack} \left( I, -M^{-1}C1 \right) \right)
\]

\[
Y := \begin{pmatrix} 0 \\ -v_{in} \end{pmatrix}
\]

\[
D(t, Y) := A1 \cdot Y
\]

\[
Z := \text{rkfixed} \left( Y, 0, 0.0020, 1000, D \right)
\]

\[
\begin{align*}
t &:= Z^{(0)} \\
yb &:= Z^{(1)} \\
vyb &:= Z^{(2)} \\
Y &:= \begin{pmatrix} yb \\ vyb \end{pmatrix}
\end{align*}
\]

Results to the numerical solution in graphical form

![Graph 1: Ball displacement (lateral deformation) vs. time](image1)

![Graph 2: Ball velocity vs. time](image2)
APPENDIX 5

STATE-MATRIX METHOD TO SOLVE A CRICKET BALL IMPACT ONTO A FIXED, DEFORMABLE BAT FACE

Adapted from section 4.10 Inman [108] and Cottey [109] and written in MathCAD®

\[
\begin{align*}
v_{in} &= 35 \\
m_b &= 0.157 \\
m_f &= 0.035 \\
c_f &= 200 \\
k_f &= 1 \times 10^5 \\
c_b &= 0.1983 - v_{in}^2 - 1.7212 - v_{in} + 193.0336 \\
k_b &= 1081 - v_{in}^2 + 1718 - v_{in} + 1340655 \\
k_b &= 2.725 \times 10^6 \\
e_r &= 375.709 \\
\end{align*}
\]

Ball mass and initial velocity

Face mass, damping and stiffness values

Experimental impacts tests used to determine relationships between ball properties and inbound velocity

Checking the values of ball stiffness, damping

Setting up zero matrix 0 and identity matrix 1

Setting up the matrix equations from the force balance on the cricket ball spring damper system

Formulation of the state-matrix

Setting the initial conditions

Setting the derivative function that describes the system in the first order form

Runge_kutta integration method to obtain an approximate solution

Assigning the columns of the solution matrix \( Z \) to \( y_b = \text{ball distance}, v_b = \text{ball velocity}, y_f = \text{face distance}, v_f = \text{face velocity} \)

Results to the numerical solution in graphical form
APPENDIX 6

STATE-MATRIX METHOD TO SOLVE A CRICKET BALL IMPACT ONTO A CRICKET BAT

Adapted from section 4.10 Inman [108] and Cottey [109] and written in MathCAD®

\[
\begin{align*}
\text{vin} & := 19 & \text{mb} & := 0.157 & \text{mf} & := 0.0001 & \text{me} & := 0.69 \\
\text{cf} & := 200 & \text{kf} & := 1 \times 10^5 \\
\text{cb} & := 0.1981 \text{vin}^2 - 1.7212 \text{vin} + 103.0336 & \text{kb} & := 101 \text{vin}^2 + 1715 \text{vin} + 1346655 \\
\text{cb} & := 1.764 \times 10^5 & \text{cb} & := 231.917 \\
\end{align*}
\]

\[
O := \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \quad I := \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
M := \begin{bmatrix}
0 & \text{mf} & 0 \\
0 & 0 & \text{mb}
\end{bmatrix} \quad C := \begin{bmatrix}
-\text{cf} & -\text{cf} & 0 \\
-\text{cb} & -\text{cb} & \text{cb}
\end{bmatrix} \quad K := \begin{bmatrix}
\text{kf} & -\text{kf} & 0 \\
0 & -\text{kb} & \text{kb}
\end{bmatrix}
\]

\[
A1 := \text{augment} (\text{stack} (O, -M^{-1} \cdot K), \text{stack} (I, -M^{-1} \cdot C))
\]

\[
Y := \begin{bmatrix}
\phi \\
\phi \\
\phi \\
\phi \\
\phi \\
\phi \\
\phi
\end{bmatrix}
\]

\[
D(I, Y) := A1 \cdot Y
\]

\[
Z := \text{rkfixed} (Y, 0, 0.002, 10000, D)
\]

\[
t := Z^0 \quad \text{ye} := Z^0 \quad \text{yf} := Z^0 \quad \text{ybf} := Z^0 \quad \text{vf} := Z^0 \quad \text{vbf} := Z^0
\]

\[
Y := \begin{bmatrix}
\text{yme} \\
\text{yf} \\
\text{yb} \\
\text{ve} \\
\text{vf}
\end{bmatrix}
\]

\[
ACOR := \frac{\text{max(vf)}}{\text{vin}} \quad \text{COR} := \frac{\text{max(vf)} + (\text{-min(ve)})}{\text{vin}}
\]

\[
\text{ye} := \text{yf} - \text{ye} \quad \text{vfe} := \text{vf} - \text{ve} \quad \text{ybf} := \text{yb} - \text{yf} \quad \text{vbf} := \text{vb} - \text{vf}
\]

Values for \text{COR, eA,}

- \text{ACOR} = 0.248
- \text{max(vf)} = 7.008
- \text{ball rebound velocity (m/s)}

Experimental impacts tests used to determine relationships between ball properties and inbound velocity

Checking the values of ball stiffness, damping

Setting up zero matrix 0 and identity matrix 1

Setting up the matrix equations from the force balance on the cricket ball spring damper system

Formulation of the state-matrix

Setting the initial conditions

Setting the derivative function that describes the system in the first order form

Runge_kutta integration method to obtain an approximate solution

Assigning the columns of the solution matrix \( Z \) to \( yb \) = ball distance, \( vb \) = ball velocity, \( yf \) = face distance, \( vf \) = face velocity, \( ye \) = ball effective mass distance, \( ve \) = bat effective mass velocity

- \text{ACOR} = 0.248
- \text{max(vf)} = 7.008
- \text{ball rebound velocity (m/s)}
Results to the numerical solution in graphical form

- **Ball displacement (lateral deformation) vs. time**
  - $y_{bf}$
  - $y_{bf} = -0.01$ to $0.01$ vs. $0$ to $0.002$
  - $\min(y_{bf}) = -2.79 \times 10^{-3}$

- **Ball velocity vs. time**
  - $v_{b}$
  - $v_{b}$ vs. $0$ to $0.002$
  - $\min(v_{b}) = -4.84 \times 10^{-3}$

- **Ball displacement (deflection) vs. time**
  - $y_{bf}$
  - $y_{bf}$ vs. $0$ to $0.002$

- **Face displacement (deflection) vs. time**
  - $y_{fe}$
  - $y_{fe}$ vs. $0$ to $0.002$
LAMINATED PROTOTYPE 1 BAT MANUFACTURE

APPENDIX 8

**CAD MODEL**
- CAD model of cricket bat blade is generated
- CAD model of rectangular blank is generated
- Internal bat geometry is subtracted from the blank
- 2 datum holes modelled at either end of the block
- CAD model of blade sliced into layers along the zy plane at correct thickness
- CAD model of block sliced into layers along the xy plane to correct thickness
- What is the orientation of the layers with respect to the bat face?
- Length and width of sheet sawn to x and y dimensions of CAD block with correct fibre orientation
- Length and width of sheet sawn to z and y dimensions of CAD block with correct fibre orientation
- What is the layer thickness?
- Does the sliced layer include internal geometry?
- Profile of internal geometry converted into 2D CAD drawing file for each layer
- 2D CAD drawing files converted to 2D cutter paths for each layer
- External blade geometry divided into front and back regions including an overlap
- The 2 regions are subdivided into a total of 4 diamond regions, including an overlap (since the length of the blade geometry exceeds the available CNC machine travel)
- Each region is revolved by a blank modelled to represent the relevant blade's volume to be machined
- Roughing and finishing cutter paths are computer generated and verified for each of the 4 regions

**MANUFACTURE**

**Form of material**
- Block of wood

**Layer Preparation**
- Large sheets of single veneer or ply

**Laser Cutting**
- 2 datum holes drilled at either end of each sheet

**Preparation of Internal Geometry**
- Holes are cut from each layer using a CO2 laser cutting machine following 2D cutter paths. The internal piece from each layer is discarded.

**Lay-Up and Pressing**
- Layers arranged in correct order
- Adhesive is prepared and applied to one surface of each layer using a hand-held roller
- The block is 'laid up', arranging each layer on top of the other, sheets are aligned using 2 steel pins located through the layer: datum holes
- The block is mounted onto a steel plate, the protruding steel pins are located through the plate datum holes
- A second plate is positioned on top of the block and the block is pressed at 0.26MPa

**Preparation of External Geometry**
- The 2 back regions are machined using a 3-axis CNC milling machine
- A PVC film is adhered to the newly machined geometry

**External Machining**
- A volume of wax/paste just exceeding the machined volume is poured into the machined cavity and allowed to solidify. Resin/thermoset gel is then placed in the cavity to machine the 'back face' geometry
- The 2 front regions are machined using a 3-axis CNC milling machine

**Machined blade is expanded from wax/plaster mould**
- Blade is sent to factory for handle fitting

**Handle fitting**
- PVC adhesive is applied to the mating faces of the handle and bat 'spice' and are joined at the desired angle
- Blade is sanded to blank 'shoulder' feature
- Handle is wrapped with string
- Rubber grip is applied to handle
- Bat is sanded to remove machining marks and polished

**Finished prototype**

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