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### *A study of the effect of particulate deposit upon fibrous filter efficiency*

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# A study of the effect of particulate deposit upon fibrous filter efficiency.

Sarah J Dunnett<sup>1</sup>, Charles F Clement<sup>2</sup>

<sup>1</sup>Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leics. LE11 3TU, U.K.

<sup>2</sup>15 Witan Way, Wantage, Oxon, OX12 9EU, U.K.

[s.j.dunnett@lboro.ac.uk](mailto:s.j.dunnett@lboro.ac.uk)

**Abstract.** The aim of this work is to develop numerical methods to model the effects that particle deposit collected by fibrous filters has upon the flow field within the filter and hence upon further deposition. A single fibre model has been developed with the deposit modelled as a porous layer on the fibre surface. Using mathematical techniques the flow field outside and within the porous layer are determined. Once the flow field for a particular deposit has been obtained the equations of motion of the particles are solved and the feedback effects of the deposit upon further deposition investigated. The mechanisms of interception and diffusion are considered. It is found that for the smaller particles the porosity of the deposit has an insignificant effect upon the flow field. However the porosity becomes increasingly important as the relative size of the particle to the fibre increases.

## 1. Introduction

Fibrous filters are widely used to protect employees from exposure to airborne particulates in the form of face masks or larger scale filtration units. Such filters generally consist of many threadlike fibres which are positioned more or less normal to the direction of the air flow. The air passes through the regions between the fibres and some of the particles within the flow will be deposited onto the fibres surface and hence removed from the airflow. Neglecting the influence of electrical effects the main mechanisms by which particles are removed by the fibres are interception, impaction, diffusion and gravitational settling. The dominant mechanism for any application will depend upon the size of the particles in the flow. The collected particles accumulate forming complex structures which influence the fluid flow and further deposition. Eventually the filter will clog leading to regeneration or replacement.

In order to fully understand the performance of a filter it is necessary to have a detailed understanding of the flow field within the filter. Due to the importance of filters much work has been performed on acquiring this understanding, in particular for clean filters where no significant particle deposition has taken place. See for example, [1-6]. As for the fluid flow, extensive research has been undertaken into particle removal for clean fibres, see for example [7-12]. Due to this earlier work the performances of filters in their clean state are fairly well understood. However, in the case of a loaded filter where significant particle deposition has occurred on the fibres, the situation is more complex and less work has been performed in this area. In this case the deposited material will affect the air

flow through the filter and hence affect particle deposition. Previous work which has been performed includes the experimental work of [13,14] where the penetration and pressure drop of various filters were considered under loading. The semi-empirical models of [15] and the studies of the dendritic structures formed on the fibres by the deposited material by [16]. These studies were limited as they did not include the effects of the structures upon the flow field. Work which did take into account the effect the deposited material has upon the flow field includes that of [17-20]. However the models developed were restricted due to their considerable use of computing time and resources. More recently work has been undertaken to study the structure of deposit on single fibres and the effect the deposition has upon efficiency see [21-23].

In the work described here a numerical model has been developed which models the particulate deposit on the fibres as a porous media. The air flow and further particle deposition on the fibres is then investigated. In this way the deposit structure formed as loading increases can be determined. This has been investigated for various values of the relevant parameters. This model is an extension of that developed previously by the authors [24] where in that case the deposit was modelled as a smooth solid layer on the fibre surface.

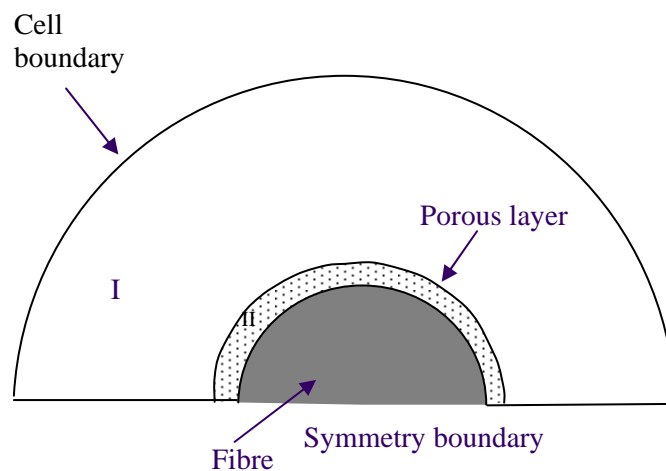
## 2. Formulation

### 2.1 Flow model.

As in [24] the fibres are modelled as infinitely long cylinders with their axis perpendicular to the flow. Hence the flow is two dimensional and the motion takes place in the plane perpendicular to the cylinder axis. A single fibre is considered and a cell model adopted. This is different from an isolated fibre as the presence of the other fibres is accounted for in the boundary conditions on the cell. The size of the cell is determined by the packing fraction of the filter considered,  $c$ , where the packing fraction is defined as the fraction of the perceived volume of the filter that is actually occupied by fibres. It is assumed that the Reynolds number of the flow,  $Re$ , is small and creeping flow is modelled.

Where  $Re$  is defined by,  $Re = \frac{d\rho U_0}{\mu}$ ,  $d$  is the diameter of the fibres making up the filter,  $\rho$  is the

density of the air,  $U_0$  is the air velocity and  $\mu$  the air viscosity. This is a reasonable assumption as generally for flow through a fibrous filter  $Re < 1$ . The particulate deposit collected on the fibre is modelled as a porous media and hence the situation modeled is shown in figure 1. Due to symmetry only half the fibre is considered.



**Figure 1.** Cell model.

In the model there are two regions to consider, region I the area in the filter between fibres where the air flows and region II which is the porous region made up of deposited particles. In region I the flow is described by the biharmonic equation for the stream function  $\psi$ , i.e.

$$\nabla^4 \psi = 0 \quad (1)$$

In region II the flow motion is described by Darcy's equation:

$$\nabla p = -\frac{\mu}{k} \underline{U} \quad (2)$$

Where  $k$  is the intrinsic permeability of the porous medium.

In terms of  $\psi$  equation (2) becomes

$$\nabla^2 \psi = 0 \quad (3)$$

Using the same approach as [24] the Boundary Element Method (BEM) was adopted to solve for  $\psi$  in regions I and II. In order to do this an extension of the Beavers-Joseph condition developed for fluid flow adjacent to a porous medium was adopted, see [25],

$$\left( \frac{\partial u_T}{\partial N} + \frac{\partial u_N}{\partial T} \right)_I = \frac{\alpha_{BJ}}{\sqrt{k}} \left( (u_T)_I - (u_T)_{II} \right) \quad (4)$$

Where  $N$  and  $T$  are the normal and tangent to the boundary,  $u_N$  and  $u_T$  are the components of fluid velocity in the normal and tangential directions respectively and the subscripts I and II denote the regions in which the terms are taken. The term  $\alpha_{BJ}$  is a dimensionless parameter which depends on the material parameters that characterize the structure of the permeable material.

Once  $\psi$  can be obtained anywhere in regions I and II the flow velocities in those regions can be determined using

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \text{and} \quad u_\theta = -\frac{\partial \psi}{\partial r} \quad (5)$$

Once the flow velocities are known the motion of particles within the flow can be determined.

## 2.2 Particle Motion.

In this work we are considering the motion of smaller particles as these are the particles of most interest when considering health effects. The major mechanism by which such particles are captured is diffusional deposition, in which the combined effect of the air motion and the Brownian motion of the particle brings it into contact with a fibre. In this case, for filtration problems it can be shown that, see [24], the particle concentration,  $n$ , is described by

$$u_r \frac{\partial n}{\partial r} + \frac{u_\theta}{r} \frac{\partial n}{\partial \theta} = \frac{2}{Pe} \left( \frac{\partial^2 n}{\partial r^2} + \frac{1}{r} \frac{\partial n}{\partial r} \right) \quad (6)$$

Where all terms in the equation are non-dimensional and  $Pe$  is the Peclet number which is a measure of the relative magnitude of the diffusional motion of the particles and the convective motion of the air past the fibre, given by;

$$Pe = \frac{U_0 d}{D} \quad (7)$$

Where  $D$  is the coefficient of diffusion of the particles. The particle concentration has been non-dimensionalised with respect to  $n_0$ , the particle concentration at the cell boundary.

Equation (6) was solved numerically to give values of  $n$  at finite points in the domain of interest, the region outside the surface formed by previously deposited particles. It is assumed that particles are attached to the porous boundary if they impact with it. The interception of particles with fibres has been accounted for by using the appropriate boundary conditions

The rate at which particles are deposited on a surface by diffusion is proportional to the concentration gradient  $\frac{\partial n}{\partial N}$  on the surface. Hence in this work  $\frac{\partial n}{\partial N}$  on the deposit surface is investigated for

various situations. Also once  $\frac{\partial n}{\partial N}$  is known the new surface formed by depositing particles can be determined for a certain time interval. This new surface then defines the boundary of the porous layer and the flow field can be recalculated using the process described in the previous section. In this way the shape of the surface can be investigated as deposition increases.

### 3. Results

It has been found in earlier work, [24], that for small particles and for a certain range of the parameters  $\kappa$  and  $Pe$ , the surface formed by the early stages of deposit on the fibres can be approximated by

$$r = r_0 \left[ 1 + \lambda (1 - a\beta^p)^b \right] \quad (8)$$

where  $r_0$  is the radius of the clean fibre,  $\lambda$  is the thickness of the deposit on the fibre front divided by  $r_0$  and  $a$ ,  $p$  and  $b$  are fitted constants. The angle  $\beta$  is measured from the front of the fibre. The parameter  $\kappa$  is the ratio of particle and fibre diameter and  $Pe$  is the Peclet number given by equation (7). Hence in this work equation (8) has been taken to define the boundary between the porous deposit and the air flow.

In order to determine the flow field it is necessary to impose a value for  $\alpha_{BJ}$ , the empirical constant in the boundary condition (4). From earlier work [25] and [26], for the situations considered here, it appears that  $\alpha_{BJ}$  will lie in the range  $2 \leq \alpha_{BJ} \leq 4$ . From an investigation of the flow field it appears that the effects of varying  $\alpha_{BJ}$  within the relevant range are insignificant hence in all results shown here  $\alpha_{BJ}$  has been taken to be 3.

In equation (2) the parameter  $k$  is introduced which is a measure of the permeability of the particulate deposit. This has been shown to be related to the porosity of the deposit,  $\phi$ , by equation (9), see [25]

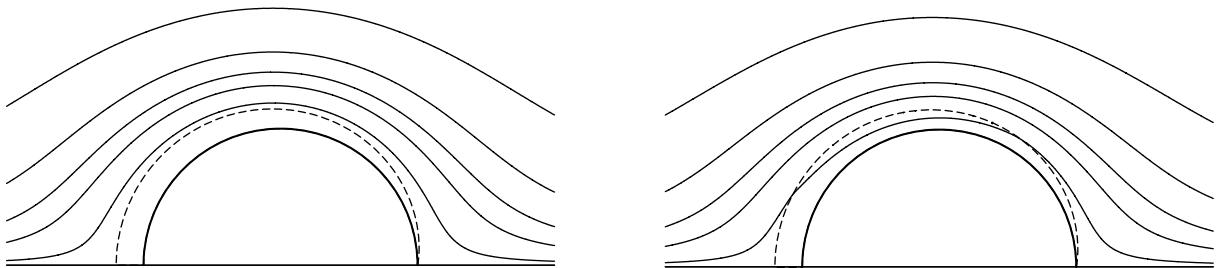
$$k = \frac{\phi^3 d_p^2}{180(1-\phi)^2} \quad (9)$$

Where  $d_p$  is the characteristic diameter of a particle in the porous deposit. In non-dimensional terms this becomes

$$k' = \frac{k}{r_0^2} = \frac{\phi^3 \kappa^2}{45(1-\phi)^2} \quad (10)$$

a)

b)



**Figure 2.** Streamlines of the flow for a)  $k'=0.01$  and b)  $k'=0.05$

Considering the flow field past the fibre for different values of  $k'$ , figure 2 shows the flow streamlines for  $k'=0.01$  and  $0.05$ . In the results shown a typical shape for the deposit, as determined in earlier work, [24], has been assumed and the non-dimensional thickness of the deposit layer at the front of the fibre has been taken to be  $0.2$ . Due to symmetry, only half of the fibre is shown and the porous region is enclosed within the dotted curve. Only the flow in the region close to the fibre with deposit is shown. As can be seen, in the case of low permeability of the porous layer, which corresponds to the smaller value of  $k'$ , the streamlines are shifted away from the deposit with very little flow passing through. As  $k'$  increases more flow passes through the layer, as seen in figure 2b with the porous layer offering less resistance to the flow. Considering the corresponding values of  $\kappa$  and  $\phi$  to the values of  $k'$  taken in figure 2, as small particles are being considered  $\kappa$  is restricted to the range  $\kappa \leq 0.05$ . Hence for  $k'=0.01$ , for  $\kappa=0.05$  then  $\phi=0.933$  and for  $\kappa=0.01$  then  $\phi=0.985$ . For  $k'=0.05$ , shown in figure 2b) for  $\kappa=0.05$  then  $\phi=0.968$  and for  $\kappa=0.01$  then  $\phi=0.993$ . It appears therefore that for the creeping flow through filters, for these small particles considered when diffusion is the main mechanism of deposition, the porosity of the deposit must be high for any significant flow in pass through the deposit. In figure2, in general, over 90% of the volume of the deposit is void space.

In order to consider the effect deposited material has upon further deposit the value of  $\frac{\partial n}{\partial N}$  on the surface of the porous deposit is investigated, as the rate at which more particulate is deposited is proportional to this term. The way this has been achieved here is by, for a particular value of  $Pe$  and  $\kappa$ :

i) determining the flow field for a clean fibre and solving for  $n$ . This enables  $\frac{\partial n}{\partial N}$  to be determined on the fibre surface.

ii) The coefficients  $a, b$  and  $p$  which give the best fit to the equation

$$\frac{\partial n}{\partial N} = \left[ \frac{\partial n}{\partial N} \right]_{\beta=0} (1 - a\beta^p)^b.$$

are then determined.

iii) As the rate at which particles deposit is proportional to  $\frac{\partial n}{\partial N}$  the non-dimensional equation of the surface of the porous material on the fibre is written as

$$r = 1 + \alpha \frac{\partial n}{\partial N} = 1 + \lambda (1 - a\beta^p)^b$$

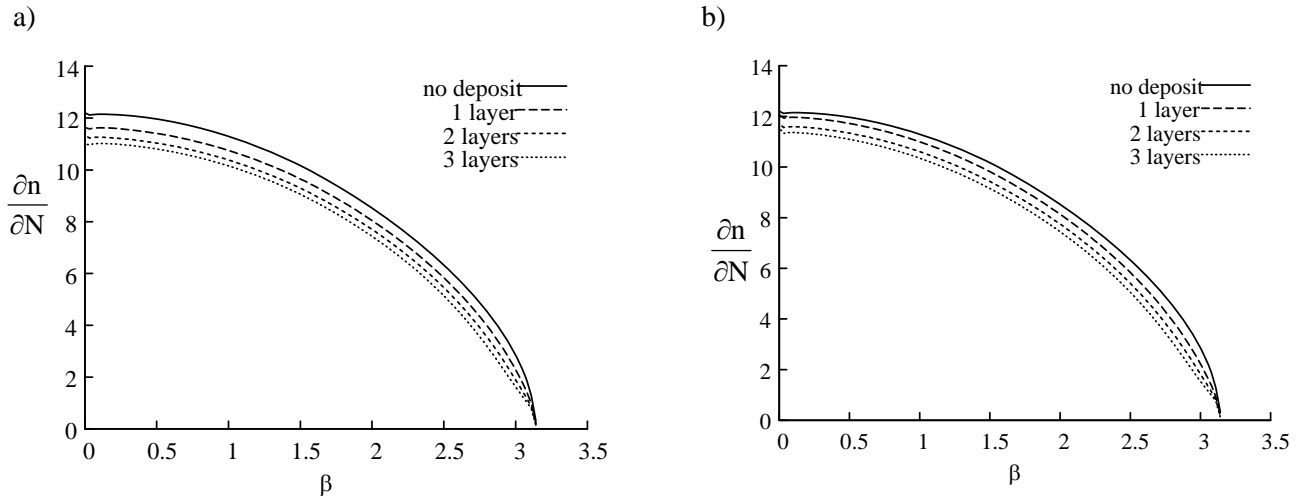
iv) The new flow field and particle concentration can then be obtained and  $\frac{\partial n}{\partial N}$  on the porous surface determined.

v) Step ii) is then repeated and more deposit added with the new surface of deposited porous

$$\text{material given by } r_{\text{new}} = r_{\text{old}} + \alpha \frac{\partial n}{\partial N} = r_{\text{old}} + \lambda (1 - a\beta^p)^b$$

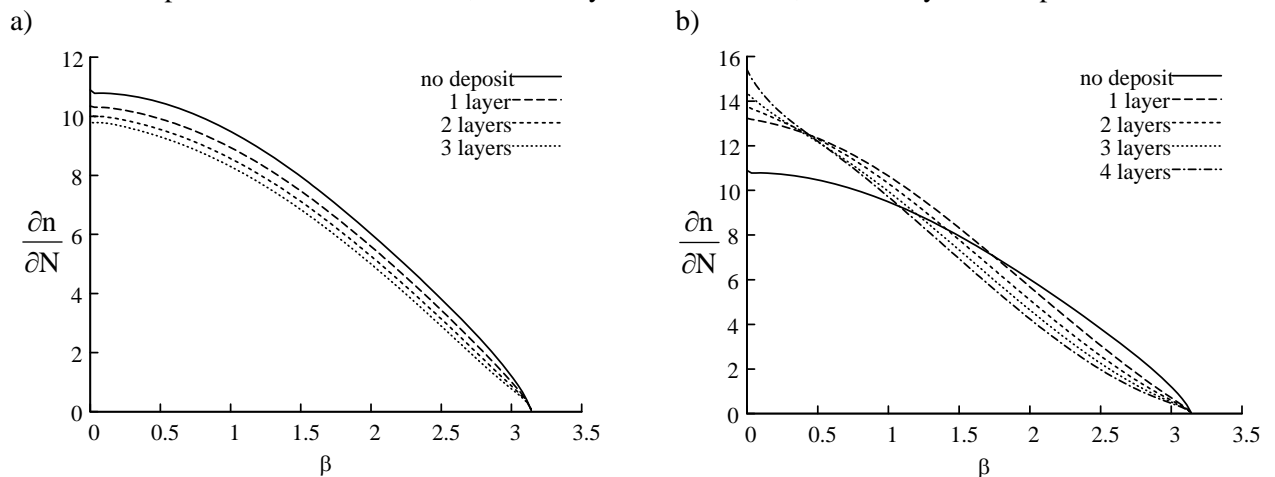
Steps iv) and v) are then repeated as deposit grows.

In figure 3  $\frac{\partial n}{\partial N}$  is shown as a function of the angle  $\beta$  for  $Pe=5891$  and  $\kappa=0.0025$ ,  $\beta=0$  correspond to the front of the fibre and  $\beta=\pi$  is at the back. The amount of deposited material added before the flow field was recalculated was taken to be such that the non-dimensional width of the porous layer added was  $0.2$  at the front of the fibre. In figure 3a) the deposit has been assumed to form a solid layer on the fibre and hence no flow passes through it, in figure 3b) the deposit is assumed to be highly porous with  $\phi=0.95$ . The values of  $Pe$  and  $\kappa$  shown correspond to a fibre of  $20\mu\text{m}$ , particles of size  $0.05\mu\text{m}$  and a flow velocity of  $0.1\text{m/s}$ . As can be seen for these parameters, even when the deposit has a high porosity, the effect upon  $\frac{\partial n}{\partial N}$ , and hence further deposit is insignificant.



**Figure 3.**  $\frac{\partial n}{\partial N}$  as a function of  $\beta$  for  $Pe=5891$ ,  $\kappa=0.0025$  when deposit is assumed to be a) solid, b) porous,  $\phi=0.95$ .

Considering  $Pe=1473$ ,  $\kappa=0.05$  figure 4 shows  $\frac{\partial n}{\partial N}$  as a function of  $\beta$  for a) solid and b) porous,  $\phi=0.9$ , deposit. In this case the parameters correspond to a fibre of  $1\mu\text{m}$  diameter, particles of size  $0.05\mu\text{m}$  diameter and a flow velocity of  $0.5\text{m/s}$ . It can be seen that for this larger value of the relative size of particles considered,  $\kappa$ , the porosity of the deposit does have a significant effect upon the results. As can be seen the effect of the deposit is to increase the value of  $\frac{\partial n}{\partial N}$ , and hence the amount of deposit, at the front of the fibre, around  $\beta=0$ , and decrease it at the back. This effect continues as the amount of deposited material increases, as seen by the curves for 2, 3 and 4 layers of deposit.



**Figure 4.**  $\frac{\partial n}{\partial N}$  as a function of  $\beta$  for  $Pe=1473$ ,  $\kappa=0.05$  when deposit is assumed to be a) solid, b) porous,  $\phi=0.9$ .

An important characteristic used to evaluate the performance of a filter is the particle collection efficiency;  $\eta$ , which is the measure of the efficiency of a single representative fibre in the



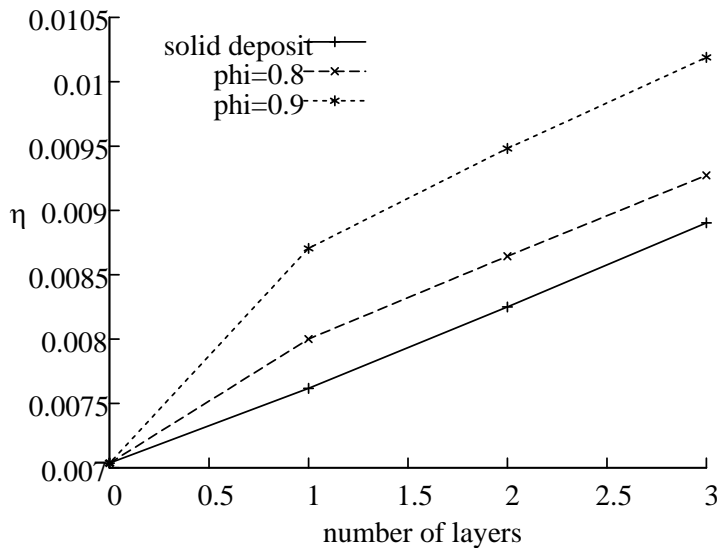
filter. This is defined to be the ratio of the number of particles that are actually removed by the fibre to the number entering the 'cell'. As shown by [24] this can be written as

$$\eta = \frac{2I\sqrt{c}}{Pe} \quad (11)$$

where I is the integral

$$I = \int_{\theta=0}^{\theta=\pi} \left[ \frac{\partial n}{\partial N} \right]_{r=r_1} ds \quad (12)$$

$r_1$  is the interception radius which is given by  $r_s + \kappa$ , where  $r_s$  is the non-dimensional distance from the fibre centre to the surface formed by the fibre with deposit. This radius will change as deposition of particles occurs. As an example of the effect of deposit, and its porosity, upon efficiency figure 5 shows  $\eta$  as a function of the number of layers of deposit for  $Pe=1473$  and  $\kappa=0.05$ . The cases of a solid deposit and the two values of porosity 0.8 and 0.9 have been considered. As can be seen the effect of deposit is to increase efficiency as the area upon which particles can deposit increases. Also as the porosity of the deposit increases and more flow is able to pass through it, the efficiency of the filter also increases. Other values of  $Pe$  and  $\kappa$  were considered and the same behaviour was found to occur.



**Figure 5.** Filter collection efficiency as a function of layers of deposit for  $Pe=1473$ ,  $\kappa=0.05$ .

#### 4. Conclusions

When using fibrous filters to protect people from exposure to particulate it is important to understand how the filters will perform. One area in which we do not have a detailed understanding is how the filters perform when they are loaded with particulate deposit. The aim of the work, some of which is described here, is to develop a numerical model of a filter which describes the loading process and the effects the porosity of the deposit has upon filter efficiency. Such knowledge could then be used to understand the changes occurring in real filters due to particle deposition. In developing the model certain assumptions have been made, such as the neglect of electrostatic forces. These assumptions do not affect the results obtained describing the effects of the particulate deposit.

In this work a relatively straightforward mathematical model has been described that considers the particulate deposit on the fibres to be a porous medium. The model enables the airflow around and through the porous medium to be obtained in an efficient manner. In the preliminary results obtained it appears that for many of the small particles, which are of interest for occupational health, the porosity

of the deposit does not have a significant effect upon the flow behaviour within the filter. In this case a simpler model with a solid deposit appears adequate. However for larger particles the effect of the porosity is more significant causing the particle build up to increase on the front of the fibres and decrease on the back.

In future work these effects will be investigated in more detail and larger size particles will be considered where the mechanism of impaction becomes important for deposition.

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