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Two Phase Flow and Cavitation in Centrifugal Pump
A Theoretical and Experimental Investigation

by

Andreas POULLIKKAS

A Master's Thesis submitted in partial fulfilment of the
award of Master of Philosophy of the Loughborough
University of Technology.

October 1992

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ABSTRACT

Existing designs of centrifugal pumps can only handle very modest amounts of gas before they deprime. A normal centrifugal pump cannot usually handle any liquid having more than 7-9 percent gas content (by volume). Even this small percentage creates drop in head on the order of 20 percent as compared with 100 percent liquid. Furthermore the cavitation phenomenon is one of the most important problems to be considered in designing and operating a centrifugal pump. Especially when the uprating of pumps and driving at higher speeds to reduce size, its importance becomes greater. The physical mechanism of both two phase flow and cavitation remains to a great extend unexplained. In this investigation an attempt has been made to study these phenomena. A control volume method was employed to derive a mathematical model for a centrifugal pump under two phase flow conditions. The model combines the ideal (Euler's) two phase head of a centrifugal pump with additional losses due to compressibility and condensation of the gas phase. Systematic tests were carried out on a centrifugal pump of conventional design in order to established the causes of two phase head degradation and support the derived mathematical model. The excellence of the agreement obtained between the mathematical model and the experimental results gives confidence that it will be possible in the near future to predict the two phase head-capacity curve for any centrifugal pump. High speed video observations identified the basic mechanism leading to flow breakdown when pumping two phase mixtures and the flow similarities between two phase flow and cavitation.

Two Phase Flow and Cavitation in Centrifugal Pump
M.Phil Thesis by A. POUILLIKKAS
ACKNOWLEDGEMENTS

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Last but not least, my deepest gratitude goes to my wife. Without her support and love this work would have been impossible; to her I dedicate this work.

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SYMBOLS USED

A  Area  \( m^2 \)
b  Passage height  \( m \)
B  Torque  \( N.m \)
d  Pipe diameter  \( m \)
D  Diameter  \( m \)
F_c  Centrifugal force  \( N \)
F_p  Pressure force  \( N \)
F_e  External force  \( N \)
g  Gravitational constant  \( m/s^2 \)
H  Total head  \( m \)
H_c  Compression head loss  \( m \)
H_e  Eulers (ideal) pump head  \( m \)
H_v  Kinetic energy head  \( m \)
k_a,k_b  Constants
m  Mass  \( Kg \)
\dot{m}  Mass flowrate  \( Kg/s \)
M  Moment of momentum  \( N.m \)
NPSH  Net positive suction head  \( m \)
NPSH_a  Available net positive suction head  \( m \)
NPSH_r  Required net positive suction head  \( m \)
n  Curvilinear coordinate
P  Static pressure  \( N/m^2 \)
Q  Volume flowrate  \( m^3/s \)
q  Volumetric gas content
r  Radial distance of the control volume  \( m \)
R  Radius  \( m \)
Symbols Used

R Specific gas constant KJ/KgK
s Curvilinear coordinate
r Time s
T Temperature °K
V Absolute velocity m/s
Vn Normal velocity m/s
Vp Fluid velocity within the pipe m/s
Vr Radial velocity m/s
Vt Tangential velocity m/s
V' Volume m³
u Peripheral velocity m/s
W Relative velocity m/s
x Quality
z Level m

α Void fraction
β Blade angle
β' Local geometric angle of the control volume
γ Local geometric angle of the control volume
ε Fluid absolute velocity
ζ Velocity ratio
λ Zakem nondimensional parameter
ξ Slip between the phases m/s
ρ Density m³/kg
σb Blade cavitation coefficient
ω Rotational speed rad/s
**SYMBOLS USED**

**SUBSCRIPTS & SUPERSCRIPTS**

* Separated two phase condition

i Impeller suction section

2 Impeller discharge section

D Discharge flange section

g Gas phase

H Homologous parameter

l Liquid phase

s Suction flange section

IN Inlet conditions to the system

sp Single phase flow

TP Two phase flow

Numbers in brackets indicate reference.
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CHAPTER ONE

INTRODUCTION
CHAPTER ONE

INTRODUCTION

The inherent simplicity of the centrifugal pump and its broad range of application have made it a vital link in virtually every aspect of life today, from water supply to nuclear power generation. The centrifugal pump is also the workhorse of the chemical and process industries where it handles a variety of process fluids and meets many operational requirements.

It is clear that existing designs of centrifugal pumps can only handle very modest amounts of gas before they deprime. A normal centrifugal pump cannot usually handle any liquid having more than 7-9 percent gas content (by volume). Even this small percentage creates a drop in head on the order of 20 percent as compared with 100 percent liquid.

The majority of previous work on two phase head degradation sprang from the American Nuclear Power Program where there was a specific need to simulate Loss of Coolant Accident conditions in reactor circulating pumps. Today the science of two phase pumping is powered by the needs of a number of industries including oil, chemical and biotechnology sectors.

The methods which have been developed, in order to describe
the behaviour of centrifugal pumps in two phase flow, have proved to be effective in certain areas of pump operation but have not proven to be universally applicable. Most of these methods approach the problem from outside the pump, that is, they do not look into physical and phenomenological processes taking place inside the pump as two phase mixture passes through the pump. These empirical methods are expressed in terms of simple correlations obtained by curve fitting the test data that were measured at the inlet and outlet of the pump. However, it is not easy to define a universally acceptable definition for the two phase pump head because each phase is associated with its own velocity and density. A physical consistent definition of two phase head is necessary to reduce test data properly.

Furthermore the cavitation phenomenon is one of the most important problems to be considered in designing and operating a centrifugal pump. Especially when the speed-up of pumps is highly demanded, as is today, its importance becomes greater. High speed pumps are preferred because they are economical in size, in energy use and capital cost. The physical mechanism of cavitation remains to a great extent unexplained notwithstanding a long history of intensive investigation. The phenomena are governed by seemingly intractable mathematics, rich of non linear behaviour.
In this investigation an attempt has been made to study both phenomena of two phase pumping and cavitation. The objectives drawn from the beginning of the research were:

(a) To determine the basic gas handling capability of a centrifugal pump.
(b) To establish the causes of two phase head degradation and improve the existing theories by comparing experimental results with analytical mathematical models.
(c) To develop an analytical method for the design of a two phase pump.
(d) To identify any similarities between two phase flow and cavitation by detailed examination of the behaviour of gas bubbles in the impeller passages.
(e) To examine the behaviour of a centrifugal pump subject to low suction pressures.

A control volume method was employed to derive a mathematical model for a centrifugal pump under two phase flow conditions. The model combined the ideal (Euler's) two phase head of the centrifugal pump with additional losses due to the presence of the gas phase. This model is a further step towards a complete theoretical study for two phase pumping. The excellence of the agreement so far obtained with the mathematical model and experimental results gives confidence that it will be possible in the near future to predict the two phase head characteristics for any pump of conventional...
design.

Systematic tests were carried out on a centrifugal pump of conventional design using four different impeller designs. Air-water was used as the two phase test mixture. During tests a High Speed Video equipment was used to analyze high speed motions occurred both in two phase flow and cavitation.

The next chapter (CHAPTER TWO) gives a review in the two phase and cavitation literature. An attempt has been made to give a concise critical review in order the reader to understand the basic mechanism of two phase pumping and cavitation and the problems associated with such conditions. CHAPTER THREE describes in detail the theoretical and experimental work done during the investigation and in CHAPTER FOUR a discussion of the findings and suggestions for further work are given. The discussion ends with CHAPTER FIVE where the main conclusions of this study are presented.

Two Phase Flow and Cavitation in Centrifugal Pump
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CHAPTER TWO

LITERATURE REVIEW
CHAPTER TWO

LITERATURE REVIEW

The science of single phase flow in pipelines and pumps is well established and researched. Industry is happy to pump single phase liquid or to blow single phase gas and the prediction of the performance of most components from scale models and the calculation of pressure losses through piping systems is quite easy. With single phase accurate measurement of the basic characteristics of the flows is possible, characteristics such as density, viscosity, specific heat and compressibility. But when it comes to two phase flow everything changes.

Furthermore cavitation, being a two phase flow in nature, is an unpleasant phenomenon leading to harmful effects especially in hydraulic machinery. However the principal mechanism of cavitation remains to a great extent unexplained.

In the following pages a concise critical literature review of relevant previous work in two phase flow and cavitation is presented. First the principles of two phase flow are presented followed by the analysis of two phase pumping research and the various methods developed to date for the prediction of the head degradation. Then the principles of cavitation together with relevant work in centrifugal pumps will be discussed.
2.1 TWO PHASE FLOW.

The science of two phase flow is regarded by many as black art, [1], and there is much to justify that comment. Still little is known about the way in which two phase flow propagates along a pipeline yet alone know how to pump a mixture of gas and liquid or to measure its flowrate. But the science of two phase flow is advancing rapidly, powered by the needs of industry, particularly in the nuclear and oil sectors.

2.1.1 WHAT IS TWO PHASE FLOW.

A phase is simply one of the states of matter. A Physicist will explain that there are only four states of matter, these being solid, liquid, gas and plasma.

Multiphase flow is the simultaneous flow of several phases. Two phase flow is the simplest case of multiphase flow. There are many common examples of two phase flows. Some such as fog, smog, smoke, rain, clouds and snow occur in nature. Several every day processes involve a sequence of different two phase flow configurations or flow patterns. In a coffee machine, for example, the water is first boiled to form steam bubbles, alternate slugs of liquid and vapour then rise through the central tube, the hot water percolates through the coffee grounds and eventually drips down into the pot. When coke is poured from a bottle, the rate of discharge is limited by the rise velocity of slug flow bubbles in the neck. Subsequently
bubbles nucleating from defects in the walls of the glass rise to form a pleasing foam at the surface.

Biological systems contain very few pure liquids. Body fluids, such as blood, semen and milk, are all multiphase, containing a variety of cells, particles or droplets in suspension.

Examples are equally prolific in the industrial field. Over half of all chemical engineering is concerned with multiphase flow systems. Many industrial processes such as power generation, refrigeration, and distillation depend on evaporation and condensation cycles. The performance of desalination plants is limited by the state of the art in two phase flow technology. Oil extraction depends on the proper functioning of multiphase devices.

Generally speaking in engineering applications the solid phase, except in the case of slurries, is seldom experienced in flowing systems while the plasma phase only exists on Earth in electrical discharges or man-made nuclear reactors. However the main concern is the flowing of the liquid and gaseous phases.

Two phase flows can be of several types. Firstly there is the type where a mixture of gas and liquid flow together such as steam and water or air and water. Next is the type where a
mixture of two immiscible liquids such as oil and water flow together which strictly speaking should be termed a single phase, two component flow. Mixtures of gases flowing together usually, unless there is a large density difference and little turbulence, diffuse together and can be treated as a single homogeneous phase.

Gas-liquid flows can be categorised, [3,4], into two main types. Flows where:
(a) The gas and the liquid are of the same substance, such as steam and water, known as single component flows
(b) Flows where the gas and the liquid are of different substances, such as air and water, known as two component flows.

The major difference between these two types is that single component (or condensable) two phase flow can have significant mass transfer between the phases and consequently the quality, (defined as the ratio of gas mass flowrate to total mass flowrate), is variable, whereas two component (or non-condensable) flows have negligible mass transfer between the phases and consequently the quality can be considered constant. In industrial applications a single component flow would occur in a boiler or condenser system and a two component system would occur in biological water treatment plants. It should not be forgotten that in some applications there is a possibility of a multi-component condensable two phase flow. This mainly occurs in aircraft fuel feed systems.
Two phase flows obey all of the basic laws of fluid mechanics. The equations are more complicated or more numerous than in the case of single phase flows. The techniques for analyzing two phase flows fall into several classes which can conveniently be arranged in ascending order of sophistication, depending on the amount of information which is needed to describe the flow as follows.

Correlation of experimental data in terms of chosen variables is a convenient way of obtaining design equations with a minimum of analytical work. The crudest correlations are mere mathematical exercises, readily performed with computers, while more sophisticated techniques use dimensional analysis or grouping of several variables together on a logical basis. A virtue of correlations is that they are easy to use. As long as they are applied to situations which are similar to those which were used to obtain the original data they can be quite satisfactory within statistical limits which are usually known. However, they can be quite misleading if applied indiscriminately to a variety of ways in which performance can be improved or accuracy of prediction increased.

Very simple analytical models which take no account of the details of flow can be quite successful, both for organising experimental results and predicting design parameters. For example in a homogeneous model the components are treated as a
pseudo-fluid with average properties, without bothering with a detailed description of the flow pattern. A suspension of droplets in a gas, a foam or stratified flow of a gas over a liquid are all treated exactly alike. In the separated flow model the phases are assumed to flow side by side. Separate equations are written for each phase and the interaction between the phase is also considered.

An one dimensional integral analysis starts from the assumption of the form of certain functions which describe, for example, the velocity or concentration distributions in a pipe. These functions are then made to satisfy appropriate boundary conditions and the basic fluid mechanics equations in integral form. Similar techniques are quite commonly used for analyzing single-phase boundary layers.

In a differential analysis the velocity and concentration distributions are deduced from suitable differential equations. Usually following the one dimensional flow idealisation, the equations are written for time-averaged quantities, as in single phase theories of turbulence. More sophisticated versions of the theory may even consider temporal variations.

Some common problems experienced with two phase, gas-liquid, flow include; transport of a gas and liquid flow along a pipeline, especially a pipeline that has different
orientations and gradients along its length, can accumulate large volumes of liquid which issued intermittently from the far end of the pipe in the form of slugs and can cause extensive surge and vibration. Conventional flowmeters can be sometimes give an error of up to 50 percent with the addition of only 5 percent of gas to a liquid flow and since the fluid is suitable for pumping as a liquid or for blowing with a fan or compressor as a gas it must be remembered that pumps are difficult to prime and seals and fluid lubricated bearings and other features suffer from damage.

2.1.1.1 Characteristics of Two Phase Flow in Pipes.

When a gas and liquid flow together in a pipeline the familiar single phase concepts, such as turbulence and fully developed velocity profiles, are no longer applicable. Instead a whole range of flow regimes are manifest, the exact character of which depends on the relative ratios of gas and liquid and of the velocities of each phase relative to the other. The flow regimes differ depending on the orientation of the pipe through which the phases flow and for this reason most experience is confined to horizontal or vertical flows. Flows through pipes aligned in intermediate orientations produce a mixture of the two while a transition from horizontal to vertical can produce severe slug flow. Referring to Fig 2.1 for a horizontal pipe the various two phase flow regimes will be, bubbly flow when air leaks slowly into a pipeline, the buoyancy of the air tends to stratify all the
air bubbles towards the top of the pipe and plug flow which occurs when the individual bubbles of air gather together in larger volumes which travel down the pipeline as discrete plugs. Stratified flow which occurs when the gas flowrate is such that it flows separately from the liquid phase and usually the gas velocity greatly exceeds the liquid velocity, which can result in wavy flow when the higher velocity of the gas tends to produce waves on the surface of the liquid phase, the same way that wind produces waves on the surface of a stretch of open water. This can lead to slug flow when the gas velocity becomes so high that the waves generated on the liquid interface become large and start to break up, just as ocean waves on a shore. When this occurs the gas velocity is so high that the little remaining liquid can be supported as a suspended spray of liquid droplets which tend to run against the walls before being whipped into the gas again, known as annular flow.

For a vertical pipe the two phase flow regimes will be, bubbly flow as in horizontal flow, but here there is no stratification of the bubbles and slug flow when discrete volumes of gas rise up through the liquid phase. One interesting phenomenon in this case is that as bubble of gas rises upwards, the liquid can briefly be forced downwards to let it through. In Churn flow, the gas flows much faster than the liquid flow and annular flow which occurs when as in the horizontal flow, the gas velocity is now so great that it
sweeps the liquid upwards as a spray and blows it along the walls of the pipe.

Historically, [3,10], pipeline design methods for a two phase flow have been inaccurate by comparison to single phase system with uncertainties of ± 50 percent being typical. The reason for the large discrepancies is related to the number of variables involved these being; liquid mass flowrate, density, viscosity and surface tension, gas or vapour mass flowrate and viscosity and the flow pattern. Of these variables it is the flow pattern which is the most difficult to evaluate. The flow pattern is the variable which describes the distribution of one phase relative to another.

2.1.1.2. Two Phase Flowrate Measurements.

The measurement of two phase flowrates is not easy. The simple problems associated with single phase flow measurements, [5,6], such as velocity, profile and installation effects become formidable obstacles when two or more phases flow together. Extreme variations in viscosity, density and velocity between the phases occurs not only spatially but also temporally across the pipe. Any device hoping to meter such flows, must contend not only with the physical extremes of the character of the flow but also with the severe mechanical stresses such flow imposes on the system used. The measurement of two phase flowrates thus presents measurement technology with one of its most difficult
problems. However, the need for such measurements is widespread, ranging over many industries, from the food process industry to the processing of nuclear fuels and from the brewing industry to the design and manufacture of boilers, condensers, heat exchanges and steam generators.

Methods of achieving two phase flow measurement currently lag well behind other aspects of two phase flow technology. The traditional metering methods of orifice plates, venturis, turbine meters and positive displacement meters all suffer from a variety of drawbacks, but principally the inability to calibrate meters in either the fluids being monitored or a representative fluid analogue system. Without this information it is impossible to know whether and how meter factors or discharge coefficients will vary with changes in the individual phase flowrates.

Early approaches to the problem [2,7,8,9] used a sample from the flowing two-phase mixture. However, the time for a complete measurement is around from 8 to 15 minutes. More recent methods vary from Doppler ultrasonic methods to differential pressure devices. The latter has been studied in more depth and essentially involves applying a "correction function" to the output of a conventional meter, the behaviour of which is known in single phase flow. However the technique, [5], is very complex.
Since measuring of two phase flowrate is very difficult so the alternatives must be examined. Examination of the total process or system to see if flowrate of the individual phases could be metered upstream before mixing. And examination of the total process or system to see if the flowrate of the individual phases could be metered downstream by completely separating the phases with existing single phase technology and then recombining if required. The latter certainly offers the most accurate method at present. However, its main disadvantage is the cost, space and weight of auxiliary separating equipment.

2.1.2 WHY USE A CENTRIFUGAL PUMP IN TWO PHASE PUMPING.

It is well known that existing designs of centrifugal pump can only handle very small amounts of gas before they deprime. Why therefore should effort be put into their development in gas liquid pumping, which is the traditional domain of the positive displacement machine? In a word, reliability.

All reciprocating pumps and many of the rotating types, are more complex than centrifugal pumps and so are inherently less reliable. The first cost of positive displacement equipment, [11], is also generally significantly higher. Furthermore using a centrifugal pump less maintenance is required, `down-time is lower and more continuous processing can be provided.

The majority of previous work, [12,13,14,15,16,17,18], on
intrinsic two phase head degradation, sprang from the American Nuclear Power Program in the early 1960s where there was a specific need to simulate Loss Of Coolant Accident, (LOCA), conditions in reactor circulating pumps.

An important aspect of nuclear energy is the question of safety. Though nuclear reactors have proven to be safe from operational experience, there is always a chance of accidents leading to radioactive leakage, like the recent Chernobyl accident. There are several hypothetical accident situations, which may give rise to core disruption and radioactive leakage. The emphasis in the nuclear industries and research organizations is on the understanding of the reliability of safety mechanism and accident situations. One of the most severe accidents is the LOCA, which may occur due to leakage of coolant in any of the reactor system components or break in any pipe section of the coolant loop. During such situations the recirculating coolant of a pressurized water reactor, (PWR), is reduced and may result in overheating and subsequent melting of the fuel elements.

One of the important components is the pump. Circulating pumps are used to force the water around the coolant circuit. PWRs typically have two, three, or four steam generators for each reactor and two, three, or four independent cooling loops. There are therefore up to four circulating pumps. Centrifugal pumps are usually employed in the U.S. reactors,
while mixed flow or axial pumps are more often used in Europe.

During the normal operation, there is only liquid flow and single phase analysis and performance curves for the pump are used. However, during accident conditions, the pump will have a two phase mixture coming into it. Such situations may arise due to leakage in the secondary loop which will prevent the heat exchanger from removing the heat from the primary coolant, causing an increase in the temperature and finally vaporization. It may also arise due to a break in the primary loop. A rupture may conceptually occur in either the suction or discharge pipe of a pump. The pressure difference between the fluid in the pipe and the atmosphere is large compared with the pressure rise through the pump. Therefore the pressurized water will flow out of both legs at the break, at least in the early stages of depressurization, regardless of break location. The water will flash into steam at the throttling point, which could be at the pipe break location or could be within the pump.

If the break is in the suction pipe to a pump, the flow will rapidly reverse through the pump, so that for a short time the pump will still be rotating forwards while the flow, in liquid or liquid/vapour phases, is reversed. The control system is normally arranged to maintain electrical power to the motor when it detects a depressurisation failure. In addition, reactor coolant pumps are fitted with anti-rotation devices.
If a rupture occurs in the discharge pipe of a coolant pump, the flow will force itself through the pump in the normal flow direction, causing the pump to operate in a turbine mode. The pump will be overspeeded in the forward direction.

In all these situations a pump designed for single phase operation will have degraded performance. From a PWRs safety point of view, it is therefore to anticipate essential such phenomena and develop an analytical capability for accurately predicting the performance of PWRs cooling pump under two phase flow conditions.

Oil and gas production creates demanding conditions for equipment operating both on and offshore. Wherever their location all mechanical installations share the common need for maximum reliability and minimum downtime. Similar two phase conditions also exist in submersible pumps for deep oil well pumping, [12,19], where a fair amount of gas is contained in oil. The head of the pump is substantially lower than that of single phase flow, thus resulting in performance reduction. The same head degradation phenomena is prevalent but in this case the two phase flow media consist of oil and non condensable gas unlike the above cooling systems.

Perhaps the oil and gas industry is the largest and most immediate market in two phase pumping. The composition of the fluid coming out from production wells is difficult to define.
and varies over time reaching gas volume percentages as high as 80–90 percent. The intake pressure is high; up to 100–200 bar. The presently available pump designs are not adequate due to their inability to pump highly gassed media and currently, large and costly separation devices are used upstream from the pumps. Moreover, the separated gas is either flared off or reinjected in the system, thus requiring additional equipment and cost.

Most of the early oil and gas discoveries in the U.K. continental shelf region of the North Sea were contained in large reserves generally located in relatively shallow water. To date, such reserves have been exploited using a series of fixed production facilities, such as the steel jacket platform. The majority of recent North Sea discoveries, [11, 20] earmarked for development in the near future are of smaller capacity than current fields and many are located in deeper water. The construction of large steel platforms with extensive topsides processing facilities is uneconomic under these circumstances. Exploitation of marginal reservoirs therefore depends on the development of new production techniques and facilities with lower capital and operating costs.

The Poseidon Subsea Scheme, being carried out jointly by the French Petroleum Institute, TOTAL and the Norwegian firm STATOIL, [21], is a new offshore production system that is
very particularly geared to the development of oilfields situated in severe environments (deep water, rough seas e.t.c.) well away from the coast (up to several hundred kilometres). The principal feature of Poseidon is that all equipment and facilities are underwater. The other main characteristics is that no processing is done on the site. As it comes out of the wells all the production is directly pumped to shore through one pipeline, by way of multiphase pumps.

Another major market is biotechnology where aerated mixtures form an essential part of processes like sewage treatment plants, biological water treatment plants, fermentation plants for bio-protein production, yeast suspension with a high CO₂ content when producing ethyl alcohol and methanol, foam suction for vinegar production, CO₂ separation in the production of ammonia e.t.c. With gas volume percentages varying form 5-30 percent.

2.1.3 CENTRIFUGAL PUMP PERFORMANCE UNDER TWO PHASE CONDITIONS.

It is known from pipeline theory that losses increase with the gas content in the liquid. Thus a loss increase even in the case of otherwise efficient pumps must be expected. The performance of a pump under two phase flow conditions is dependent on the behaviour of the gas liquid interaction in the impeller. If this interaction can be quantified and flow regimes identified, there is a chance that the two phase
performance can be explained and correlated on a sound physical basis. Previous work, [14,16,22,23], showed that a gas bubble entering an impeller is acted upon by a number of forces, due to local pressure gradients, centrifugal and coriolis effects, flow path curvatures or viscous drag between the bubble and the surrounding liquid.

Since the density of the gas is small compared with that of the liquid, the pressure increase in the gas bubbles due to the centrifugal action of the pump will be very small in comparison to that of the liquid. The result is a deceleration of the bubble in either the flow direction and/or normal to the flow path. The physical effect is to produce a slip velocity between the two phases (gas travelling at velocities less than the liquid velocity). There will therefore be a buoyancy acting on the gas bubble attempting to force it towards the suction side of the impeller blades. Opposing this will be a drag force exerted on the bubble by the velocity of water moving towards the outside of the impeller.

Providing the buoyancy forces are less than the drag forces, the gas will continue to flow through the pump. The gas velocity, however, which is less than that of the liquid, produces an accumulation of gas in the impeller and possibly the separation of the phases in the impeller channels. As void fractions are increased large amounts of gas accumulate in the impeller and a reduction of effective flow area due to this
accumulation results in an increase in the liquid velocity. Hence two phase flow conditions in pumps can be considered as the reverse of that in pipes since for the latter as void fractions are increased the velocity of the gas phase is increased.

The drop in the pump head, as indicated in Fig 2.2, can be explained by the different compressibility behaviour of the liquid/gas mixture as compared with pure liquid. Furthermore from visual observations, [23,24,25], of the flow it was discovered that the gas laden flow shows a greater separation tendency than a pure liquid flow. Thus the compressibility and increased separation tendency of the two phase flow require a smaller increase of the flow area than a pump dealing with pure liquid. Moreover it is known from pipelines that with increasing gas content, due to phase friction, the energy losses in the flow of gas containing fluid flow increase. Compared with the flow in pipelines complicated and often turbulent flow conditions exist in turbomachinery with ever changing mixture conditions and energy transfer in a rotating channel also the progress of phase changes from static into rotating and back into static systems.

A considerable amount of experimental work in two phase pumping, [18,19,22,23,24,25,27,28,29,30], has been conducted to date. Most of the experiments used steam-water or air-water as the two phase mixture. All investigations showed that
conventional centrifugal pumps are capable of handling only a limited amount of gas entrained in the liquid flow (somewhere between 5-7 percent of gas by volume). However observations showed, [26], that large unshrouded impellers of sewage pumps, with few blades and large cross-sectional area, are capable of pumping liquids with a higher content of gas (around 15-20 percent gas by volume).

A large difference exists in performance when the pump is passing condensable and non-condensable mixtures. It seems that the condensable mixture at the inlet entirely condensed through the blades, due to pressure rise, and did not appear at the outlet for small gas flowrates. It means that any adverse effect due to the existence of bubbles will only appear around the blade inlet area and then disappear towards the blade exit for low inlet void fraction cases. In general, pump performance degradation is more severe with air/water than steam/water mixtures.

The suction pressure also seems to have a significant impact on the head degradation characteristics of two-phase flow pumps. High suction pressures reduce gas compressibility effects through the pump, increase the condensibility effect of the gas phase and change the density of the gas.

Rotodynamic pumps are classified into three categories, centrifugal, mixed, and axial. Test results of these three
different types of pumps at design conditions showed larger head degradation for the centrifugal flow pumps and smaller head degradation for mixed and axial flow pumps. This appears to suggest that the blade-through flow dynamics have a significant contribution to make to the basic mechanism of pump head degradation under two phase conditions.

The average bubble size increases as the rotational speed reduces and becomes a critical parameter for two phase head and efficiency performance. High speed impellers have a homogenising effect on gas liquid flows thereby being suitable for high gas contents.

A very peculiar property of two phase flow media is the large influence of the gas content on the speed of sound in the flowing mixture. The wave velocity in air water mixtures drops far below its value for either water (1000 m/s at 20°C) or air (340 m/s at standard conditions) phase and reaches values of about 25 m/s for inlet void fractions about 10 percent. Normal values of peak relative velocities around the blade leading edge are in this range, especially at off-design conditions. Therefore under two phase conditions pumps operate at transonic or supersonic local flow. It is not surprising that a blade design for incompressible, single phase flow is not very effective and produces choking for relatively highly gassy liquids. Theory, [19], shows that the dramatic variation of the speed of sound at low percentages is very much related
to the large difference in density between the two phases. The sonic velocity in two phase mixtures is also pressure dependent whereas in both the separate phases is independent of pressure, exactly in the case of gas and approximately in the case of the liquid. According to Fig 2.3, which is valid for isothermal homogeneous bubbly flow, a pump operating at a suction pressure of one bar and eye speed of 20-25 m/s would be close to or at "vapour locking" conditions for air content around 10 percent. In fact the peak velocity on the blade can be 20 percent or more higher than the inlet velocity depending on the blade geometry while the local blade surface pressure is lower than the suction pressure. In several compressor blading systems, sonic conditions are achieved in the blade throats when the peripheral tip speed at the eye is about 75 percent of the inlet speed of sound. From the above considerations it is clear that the Mach number effect cannot be excluded in pumps operating with gas content 10 percent or above depending on the suction pressure. The main effects produced by high Mach numbers can be summarised as, "sonic" blockage of the throat (chocking), narrow range of operating conditions depending on blade geometry and additional shock loss which also depends on the blade geometry and increases with the inlet relative velocity.

Recent investigations have been conducted on tandem bladed impellers, [25,28,29], in order to determine the performance of such impellers under two phase flow conditions. In a tandem
bladed impeller a liquid jet flows from narrow passages between the inner and outer blades and evacuates the gas cavity in the passage. An improvement in the two phase pump performance was observed with gas handling capability of 20-25 percent by volume.

Vogelbusch a Vienna, Austria engineering firm, [31], researched a variety of pumps when developing its VB-IZ fermentation process. The basis for the VB-IZ system's pump is the Worthington MN series mixed-flow sewage pump, which was modified to handle the fermentation system needs. The VB-IZ pump is a combination of an inducer and a mixed-flow pump which can handle up to 50 percent gas/liquid mixtures. As the fluid enters the impeller section, an inducer serves to reduce the net positive suction head and to separate gas from liquid. In the main impeller section, there are degasser holes drilled in the suction side of the vanes at the site of lower pressure. Gas collects here and is vented off by means of holes in the back shroud behind the impeller, thus reducing, the gas content in the blade passages.

Side channel pumps, [26,32], which constitute a special type of self priming centrifugal pump, are capable of pumping large gas flows with the liquid, Fig 2.4, when operating in the steady state condition. In the extreme case during evacuation of the suction pipe, side channel pumps can handle gas exclusively. However, compared with other fluid flow machines...
the efficiency of the side channel type of pump is bad. Large centrifugal pumps achieve efficiencies of 90 percent. The maximum efficiencies quoted for side channel pumps manufactured by different companies are 20 to 40 percent. The ability to handle gases is not identical for all the side channel pump designs as minor changes of the shape of the construction elements affect the air flow handled and the efficiency.

At least two British manufacturers are actively involved in positive displacement research, [11], for two phase pumping. Weir Pumps Ltd have announced (1987) the launch of a consortium support from the Offshore Supplies Office. A laboratory model has been tested and it was claimed that a proven system could be installed in about five years time (1992). Stothert and Pitt offer a multiphase screw pump for well-boost and flow line duties. The preliminary trials and general construction of this machine were described at the 1986 European Petroleum Conference held in London. Finally, BHr Group, [11,20,33], is currently undertaking a major research program into the behaviour of rotodynamic pumps with gas liquid mixtures. The project is being run on a consortium basis of three oil companies and seven pump manufacturers with additional funding being produced by the Department of Trade and Industry.
2.1.4 TWO PHASE HEAD DEGRADATION MECHANISM.

To attempt to develop an exact theoretical model of two phase flow in centrifugal pumps seems to be unreasonable at this time because no accurate model of the flow in centrifugal machines is available due to the complexity of the real flow in both impeller and casing. Attempts in the past to predict the head degradation from low to high gas contents are basically semi-empirical, correlating methods of experimental data which only partial attention to the physics of the problem.

There are six principal methods, [13,14,15,16,17,18,35], which have been developed, in order to describe the behaviour of centrifugal pumps in two phase flow. These methods have demonstrated effective in certain areas of pump operation but have not proven to be universally applicable. Most of these methods approach the problem from outside the pump, that is, they do not attempt to look into physical and phenomenological processes taking place inside the pump as two phase mixture passes through the pump. These models are expressed in terms of simple correlations obtained by curve fitting the test data that were measured at the inlet and outlet of the pump.

The Semiscal Method, [15,16,17], is based on the test data obtained by Aerojet Nuclear Company. The pump model developed was adopted in the LOCA analysis computer programs such as RELAP, RETRAN, and TRAC. The model implemented in these
packages employs a simple concept of two phase head and torque degradation multipliers that are assumed to depend solely on void fraction:

\[
H_\alpha = H_{\text{single phase}} - M_\alpha (H_{\text{single phase}} - H_{\text{twophase}})
\]

\[
B_\alpha = B_{\text{single phase}} - M_\alpha (B_{\text{single phase}} - B_{\text{twophase}})
\]

where \(H_{\text{single phase}}\) and \(B_{\text{single phase}}\) are the pump head and torque respectively, from homologous single phase curves and \(H_{\text{twophase}}\) and \(B_{\text{twophase}}\) represent the pump head and torque respectively from fully degraded homologous two phase curves. \(\alpha\) is the average void fraction through the pump and \(M_\alpha\) and \(M_\beta\) are empirical two phase head and torque degradation multipliers based on steam/water tests.

The Babcock and Wilcox method, \([15,17,18]\), uses a correlation which develops a series of head and torque multipliers. These multipliers are expressed as polynomial functions of the pump average void fraction over various ranges of homologous flow parameters. The series of these multipliers is defined as:

\[
M_i(\alpha) = \frac{(C)_{\text{single phase}} - (C)_{\text{twophase}}}{(C)_{\text{single phase}}} \quad i = 1, 2, 3, \ldots
\]

where \((C)_{\text{single phase}}\) represents the single phase characteristic variable such as homologous head and torque parameters, and \((C)_{\text{twophase}}\) is the two phase characteristic variable such as homologous head or torque parameters. Each multiplier \(M_i(\alpha)\) was obtained by fitting curves through the air/water test data as a function of the pump average void fraction, with the
boundary conditions $M_t = 0$ at $a = 0$ and $a = 1$. This method of formulating the multiplier by comparing the two phase data directly with single phase data also treats the pump as a "black box" without considering the pump configuration, design and point of operation.

The empirical correlation in the Creare Method; [15,35], homologous head and torque parameters are directly expressed as fourth order polynomials in void fraction. The coefficients of the polynomials were obtained for air/water and steam/water mixtures respectively by fitting the curves through the test data.

The Method of Pump Prediction, [17], developed at the National Aeronautics and Space Administration (NASA), is based on detailed compressible flow relationship and evaluations of heat transfer between the two phases. The principal application is to pumps handling liquid oxygen or liquid hydrogen. The method required as inputs a large number of details of the flow physics and the fluid properties.

A principal problem in the correlation of two phase pump performance is the treatment of the flow density, particularly in those flow regimes where there is considerable slip (the relative velocity between the liquid and vapour phases). In the Westinghouse Equivalent Density Method, [17], the desired head characteristics can be derived from the measured pressure.
characteristics via the isentropic outlet enthalpy and the two phase head characteristics are correlated with the single phase head characteristics by means of an equivalent density. Equivalent density is that density which, when used with the known two phase mass flowrate, provides an equivalent volumetric flow which can be used with the single phase pump curves to find the true two phase pump head. It is therefore a form of two phase degradation multiplier because it would permit the two phase head to be obtained from the single phase head curve together with the inlet conditions.

When applying this method to experimental results a considerable scatter was found to exist when plotting equivalent density against inlet void fraction. It is not clear how well this correlation and degradation multiplier would be effective when applied to new pump designs. As the method is not based on the fundamental flow physics occurring within the pump it may turn out not to be of general application.

The M.I.T. Method, [14,15,17], was originally to develop an analytical model based mainly on first principles. However, due to the unavailability of detail pump performance measurements of high quality the analytic modelling became more empirical. Thus this model employs a semi-empirical approach to analyzed two phase operation and incorporating experimental data for single phase and two phase flow to
produce what is termed head loss ratio. The so-called head loss ratio employed in the model is defined as:

\[
\text{Head Loss Ratio} = \frac{\text{Two Phase Theoretical Head} - \text{Two Phase Actual Head}}{\text{Single Phase Theoretical Head} - \text{Single Phase Actual Head}}
\]  

(2.3)

and is a function of the void fraction, a parameter representing a combination of a geometrical constant for any particular pump and a function of flow regime, a ratio of best efficiency volumetric flow rates to two phase volumetric flowrates, a function of void fraction and phase density ratio and the slip factor. However the slip factors used were those measured for pipe flow and are inverse to those expected in a pump.

The Zakem mathematical model, [14], represents the beginning of an attempt to analyze two phase flow phenomena in pump impellers. He analyzed gas bubble-liquid interaction in an impeller. Zakem identified a nondimensional parameter, \( \lambda \), describing the interaction between the momentum of the fluid trying to drag the bubbles through the impeller and the centripetal effect trying to force the bubbles to return toward the impeller inlet. In a mathematical form:

\[
\lambda = \frac{\alpha (\rho_l - \rho_g) u^2}{\rho V^2}
\]

(2.4)

where \( \rho \) is the fluid density, \( u \) the peripheral velocity and \( V \) the fluid absolute velocity. Fig 2.5 is a graph of this parameter versus loss in pressure rise due to two phase flow. The plot shows a reasonable but unrefined trend.
Kosmowski, [23,34], using the basic equations of continuity, energy and impulse, which take into account compressibility effects of the gas phase, derived the two phase pump head, $H_{TP}$, as:

$$H_{TP} = \frac{(1-q_1)(P_2-P_1)+q_1P_1\ln(P_2/P_1)+\frac{V_2^2-V_1^2}{g[l(1-q_1)\rho_1RTe+q_1P_1]}+z_2-z_1}{2g} \tag{2.5}$$

with $P$ being the static pressure, $g$ the gravitational constant, $q$ the volumetric gas content to the system and subscripts 1 and 2 were used to indicate the suction and discharge sections of the pump respectively.

Furuya, [12], based his approach on the one dimensional control volume method for rotating machinery and developed an analytical method to determine the performance of pumps operating under two phase flow conditions. The analytical method has incorporated pump geometry, void fraction, flow slippage and flow regime into the basic formula but neglected the compressibility and condensation effects of the gas phase. The two phase pump head suggested by Furuya can be expressed as:

$$H_{TP} = \frac{P_2-P_1}{\rho_{TP}g}+(1-x)\frac{V_{2L}^2-V_{1L}^2}{2g}+x\frac{V_{2g}^2-V_{1g}^2}{2g} \tag{2.6}$$

with $x$ being as the quality of the two phase mixture.

Finally Furuya proposed the following relation:

$$H_{TP} = H_{EP}-H_{W}+H_{E}-H_{G} \tag{2.7}$$

where $H_{W}$ is the loss due to the increase of the relative speed.
of liquid portion at pump exit caused by two phase flow condition, \( H_a \) is attributable to the slip velocity between the liquid and gas phases and \( H_a \) is due to the variation of void fraction along the flow passage between the blades. Equation (2.7) can then be written as:

\[
\frac{H_{TP}}{H_{SP}} = 1 - \frac{H_{w} + H_{o} + H_{a}}{H_{SP}}
\]  

(2.8)

This relation provides the ratio between the two phase flow pump head and single phase flow pump head. It must be pointed out that if no slip exists \( H_w = H_o = H_a = 0 \). For such a homogeneous two phase flow case \( H_{TP}/H_{SP} \) becomes unity indicating no head degradation within the framework of the assumptions used.

In order to calculate the quantities \( H_w \), \( H_o \) and \( H_a \) in equation (2.7) the detailed data for the relative velocities for both gas and liquid should be known as well as the void fraction along the path of blade through flow. This seems to be a very complicated method with a differential equation containing only one variable.

2.1.5 SUMMARY.

The literature review of relevant previous work in two phase pumping proved the complexibility of the process. Most of the analytical methods developed approaches the problem from outside the pump and there is a need of a universally acceptable definition for two phase head. However due to the current needs of the industry it will be possible in the near
future to develop a universally acceptable analytical method for the two phase pump.
2.2 CAVITATION.

Professor R.T. Knapp in the 1950s, [36], described cavitation as:

"Cavitation is a most unpleasant hydrodynamic phenomenon, whose harmful effects are both widespread and obvious seriously handicap many phases of science and engineering. Conversely, its basic nature has long been veiled in mystery and only recently is it beginning to be understood".

The physical mechanism of cavitation remains to a great extent unexplained notwithstanding a long history of intensive investigation. The phenomena are governed by seemingly intractable mathematics, highly nonlinear. Observation and measurement techniques are challenged by the three dimensionality and short time duration of events. The decade of the 1970s saw understanding of the role of viscosity and the decade of 1980s brought an awareness of the role of the dynamic nuclei spectrum. Almost surely research in the 1990s will elaborate on the interactions between the viscous flow and the nuclei content to reveal the process of cavitation.

The cavitation phenomenon is one of the most important problems to be considered in designing and operating a centrifugal pump. Especially when the speed-up of pumps is highly demanded, as is today, its importance becomes greater. High speed pumps are preferred by process plant designers because they are economical in size, in energy use and capital.
2.2.1 WHAT IS CAVITATION.

When a body of liquid is heated under constant pressure or when its pressure is reduced at constant temperature by static or dynamic means, a state is reached ultimately at which gas bubbles or cavities become visible and grow. The bubble growth may be at a nominal rate if it is by diffusion of dissolved gases into the cavity or merely by expansion of the gas content with temperature rise or pressure reduction. The bubble growth will be explosive if it is primarily the result of vaporization into the cavity. This condition is known as boiling if caused by temperature rise and cavitation (Latin: cavus=vacuous, empty, hollow) if caused by dynamic-pressure reduction at essentially constant temperature.

The phenomenon of cavitation was first observed, [36,37,38], in the early 1890s when a new British destroyer, the Turbinia, did not achieve the British Admiralty's specified speed. In the resulting investigation in 1895, it was concluded that the "cavities around the propellers" were the cause of the limited thrust. In spite of numerous and diverse investigations carried out in various hydraulic laboratories and hydro-electric plants, it is still not possible to explain in a completely convincing manner the mechanism of cavitation. In the various attempts to formulate a theory of cavitation, [36,37,39,40,41], chemical, mechanical and electro-chemical influences have
been taken into account.

One of the early theories, [36,37], tried to attribute the phenomenon of cavitation to the action of oxygen dissolved in water. When air dissolves in water, the oxygen content is considerably greater than the nitrogen content, hence, the air bubbles forming when the pressure drops are richer in oxygen than atmospheric air and when they impinge on the blade surfaces, it was thought that they produced chemical corrosion.

During the past twenty years research on cavitation has been characterized by progressive development. The decade of the 1970s chiefly produced research directed at uncovering the real fluid (viscous) effects in the cavitation process such as laminar separation and turbulence. In the early 1980s those processes were examined closely and found to be more complex than previously assumed. However, the major thrust of research in the 1980s was directed at uncovering the effects of both size distribution and the concentration of active nuclei. Now we have a vast store of information, [39,40,41], about cavitation and its early stages (cavitation inception) is well understood. However, the physics of the advanced stages still remains mostly descriptive.

It is understood that a cavitation bubble is formed from a small bubble consisting of vapour and non-condensable gas,
expand and collapses according to the surrounding pressure. It is presumed that very small gas and vapour filled voids (microbubbles) exists in the fluid, either in the interior or on a boundary, and that they become unstable at inception. The role of solid particles and organisms in producing tensile weakness in the fluid is recognized, but not considered dominant in most flows of interest. Rapid growth of a microbubble that remains stable is called gaseous cavitation. Gaseous cavitation may or may not involve significant mass transfer of gas between the liquid and the bubble. The mass transfer is a relatively slow process, observed perhaps in tip vortices but probably not significant for gaseous bubble expansion in processes of shorter duration. Sometimes the term "pseudo cavitation" for gaseous cavitation due to volumetric changes in a constant gas mass is used. The term "vaporous cavitation" for processes where there is significant mass transfer is extensively used. For both vaporous and gaseous cavitation, cavitation refers to the individual event, the growth and collapse of a single microbubble however short the lifetime of that microbubble.

There are a number of ways of classifying cavitation, [36, 37, 39], into types. For example one method is related to the conditions under which cavitation takes place, that is, cavitation in a flowing stream, cavitation on moving immersed bodies and cavitation without major flow. Another possible method is to classify according to the principal physical
characteristics. Using a combination of these two methods cavitation can be classifying into the following groups. Travelling cavitation, composed of individual transient cavities or bubbles which form in the liquid and move with the liquid as they expand, shrink and then collapse. Fixed cavitation which refers to the situation that sometimes after inception, in which the liquid flow detaches from the liquid boundary of an immersed body or flow passage to form a pocket or cavity attached to the boundary. Vortex cavitation here the cavities are found in the cores of vortices which form in zones of high shear. Vibratory cavitation where the forces causing the cavities to form and collapse are due to a continuous series of high-amplitude, high-frequency pressure pulsations in the liquid.

2.2.2 WHY CAVITATION IS IMPORTANT.

Cavitation is important as a consequence of its effects. These may be classified into three general categories. Effects that modified the hydrodynamics of the flow of the liquid, effects that produce damage on the solid-boundary surfaces of flow and extraneous effects that may or may not be accompanied by significant hydrodynamic flow modifications or damage to solid boundaries.

Unfortunately for the field of applied hydrodynamics the effects of cavitation, with very few exemptions are undesirable. Uncontrolled cavitation, [42], can produce
serious and even catastrophic results. The necessity of avoiding or controlling cavitation imposes serious limitations on the design of many types of hydraulic equipment. In the field of hydraulic machinery it has been found that all types of turbines, from a low specific speed (Francis) to the high specific speed (Kaplan) are susceptible to cavitation. Centrifugal and axial pumps suffer from its effects and even the various types of positive-displacement pumps may be troubled by it. Although cavitation may be aggravated by poor design, it may occur in even the best-design equipment when the later is operated under unfavourable conditions.

2.2.2.1 Hydrodynamic Effects.

The various hydrodynamic effects of cavitation have their source in the interruption of the continuity of the liquid phase as cavities appear. As cavity volume displaces liquid, the flow pattern is modified and the dynamic interaction between the liquid and its boundaries is affected.

The presence of cavitation increases the overall resistance to the flow. Usually the effect of cavitation on the guidance of the liquid by the boundary surface is to limit or lessen the force that can be applied to the liquid by the surface. Thus the two hydrodynamic results, of overall resistance to flow and reduced turning effect, combine to lower the performance of the equipment involved. In hydraulic machinery there is a drop in both the head and efficiency. Decrease in
power and head are indications of cavitation causing a decrease in guidance and hence effective momentum transfer between liquid and impeller. The decrease in efficiency is a measure of the increased losses.

2.2.2.2 Cavitation Damage.

Cavitation damage is the most spectacular and the most widely recognized effect of cavitation. Cavitation damage is so closely related to the cavitation phenomenon that such damage is often simply, [36], referred to as cavitation. Cavitation damages solid flow boundaries by removing material from the surface. It has been found that cavitation can damage all types of metallic solids. Thus all metals, hard or soft, brittle or ductile, chemically active or chemically inert, can be damaged by cavitation. Rubber, plastic, glass, quartz, concrete and other non metallic solids are likewise susceptible to cavitation damage. There are four basic principles, [36,37], for cavitation damage. The smaller the molecular size of the liquid is and the lower the viscosity, the easier it is for the liquid to penetrate into the surface pores of the metal, i.e. the penetration of water into the surface of the metal is deeper than in the case of oils. The greater the pressure, the deeper and quicker the liquid penetrates into the pores of the material. The smaller the area of the pore the greater the pressure produced when the vapour bubbles collapse. The higher the frequency of vibration, the more intensive is the destruction of the
surface layer of the metal.

The phenomenon of cavitation should be distinguished from the corrosion and erosion of metals:
(a) Corrosion of metals is caused solely by chemical and electro-chemical processes, the similarity of this process to cavitation occurs in corrosion by a liquid which is subjected to high frequency pressure oscillations.
(b) Erosion of metals consists in the abrasion of the surface of the metal walls by solids carried by the flowing liquid or the washing away of the walls by particles of clean liquid moving at a high velocity e.g. in high-pressure boiler feed pumps.

2.2.2.3 Extraneous Effects.
Two of the most common effects of cavitation that may not involve major modification in the liquid flow or damage to the solid surfaces are cavitation noise and vibration induced by cavitation. It has been found experimentally, [44,58], that considerable noise is produced by the collapse of cavities. It is possible that noise is evolved during the entire process but, if so, the intensity is much lower than that produced during the collapse so that little effort has been made to isolate and identify it. The importance of cavitation noise depends largely on the individual installation. For example in a power-house of a factory in which the noise level from other sources is already high, the addition of cavitation noise may
be hardly recognized and may offer no problem. On the other hand, the universal opinion in naval circles, [38,43], is that the most serious wartime effect of cavitation on surface or subsurface craft is the production of noise. Its presence makes it impossible to preserve secrecy of movement and offers considerable assistance to the enemy in determining the exact location of the vessel on which it originates.

The cavitation process is inherently an unsteady one and may involve large fluctuating forces. If one of the frequency components of these fluctuations matches a natural frequency of a portion of the equipment, vibration may result. The vibration is usually of fairly high frequency ranging from several thousand cycles per second.

2.2.3 CAVITATION STUDIES.

One of the major difficulties that retarded the development of an adequate understanding of the physical nature of cavitation is the fact that cavitation is a high speed phenomenon. This complicates and limits the ability to investigate the processes of inception, growth and collapse in both flowing and non-flowing environments. The individual action takes place so rapidly that the details cannot be resolved by the human eye. The use of the so called "slow motion" photography results in only a relatively minor improvement because the maximum picture-taking rate available in this type of equipment is too low to make a significant
improvement in the resolution of the details of the phenomenon. Moreover there is the additional difficulty that cavitation usually occurs in relatively inaccessible places and therefore it is not visible unless special arrangements are made. As a result, a great deal of speculation concerning the nature of cavitation has been based more upon study of some of the various effects of cavitation than on direct observations of the phenomenon itself.

At present there are several categories of methods for detecting the presence of cavitation. Indirect observation by determining the effect of cavitation on the performance of a piece of equipment. Indirect observation by measuring the effect of cavitation on the distribution of pressure over the boundary at which cavitation occurs. Indirect observation by sensing the noise emitted by cavitation, [44]. Indirect observation by allowing cavitation to scatter laser-beam light into a photocell, [36]. Cavitation susceptibility meters, [41,45,46], and direct observation by visual and photographic means, [47,48].

The first of these has the limitation that it gives no information about the character of the hydrodynamic phenomenon itself. The second method gives information about the location of the cavitating zone and the mechanics of force and moment of inertia transmission between the liquid and the boundaries. However, until cavitation is well established and definitely
advanced beyond an incipient stage there is usually no measurable effect. The third, fourth, fifth and sixth methods offer the best promise for investigating inception conditions. Sound measurements are especially useful as they provide a very sensitive way of detecting amounts of cavitation that may be too small to be viewed with even the best optical equipment. Moreover, the use of direction-sensitive pick up systems permits focusing on the location of the noise source. The fourth method is also a very sensitive way to indicate the presence of extremely minute cavities. Cavitation susceptibility meters, the fifth method, can give information on the critical tension which makes each cavity unstable. The sixth method, using photographic and high speed video techniques, offers the only possibility for detailed study of the hydrodynamic phenomenon either at inception or for advanced stages of cavity development.

2.2.4 CAVITATION IN CENTRIFUGAL PUMPS.

In the suction zone of a centrifugal pump the rotational effect of the blades is increasingly imposed on the liquid as it nears the impeller. This gives rise to the tangential velocity component which together with the axial velocity component results in increasing the absolute velocity, [26, 42, 49, 53], and decreasing static pressure. The fluid reaches the leading edge and then moves on to either the pressure or the suction surface of the blades. Pressure will in most cases continue to fall on the suction surface until the liquid is in
the passage and the differential pressure across the blade, which is a function of the blade action, is established. If the suction surface pressure falls below vapour pressure, cavities form and are carried into the passage where they collapse. The collapse of the bubbles takes place at very high speed and it resembles an implosion. This process is affected by the influence of the pressure gradients, the initial deformation of the shape of the bubble, flow velocity changes, closeness to pump surfaces and the physical properties of the liquid.

The flow in the impeller passage is complex not only because of the three-dimensional effects produced but also because of the induced centripetal forces. Thus general analytical or numerical techniques, [50,51], have not been fully developed.

Tests, [26,47,48,52,53,54,55], have shown that local pressure pulsations caused by turbulence also have a large influence on cavitation, in particular on its inception. Recent research has proved that the bubbles do not collapse concentrically. At commencement of the implosion in the case of bubbles close to the wall, an indentation takes place on that side of the bubble remote from nearest wall. In the case of bubbles in the main stream, indentation takes place on the high pressure side. With increasing indentation a so-called micro-jet of liquid is formed which splits the deformed bubble into two or more parts. If bubbles are attached to the wall or...
if they are in the vicinity of the wall, this micro-jet hits the surface of the wall with high velocity (jet impact) and attacks the wall. Pressure surges are also to be considered the mechanical cause of cavitation damage. The cavitation of material leads to a porous sponge-like structure.

The impeller blades and diffuser vanes and also the inner walls of the casing bounding the liquid flowing through the pump may be subjected to corrosion, erosion and cavitation damage attack. These kinds of damage can easily be distinguished on the basis of observation of the places attacked and their relative positions in the flow passages. The places attacked by cavitation are somewhat displaced in the direction of flow, in relation to the place where the cavitation bubbles burst.

At the incipient stage of cavitation the collapse of the bubbles terminates within the impeller passages. In the fully developed stage of cavitation the main part of the vapour bubbles collapses in the impeller itself. The rest may carried out of the impeller into the diffuser ring and volute casing and even into the next stage in a multistage pump, where they collapse and can give rise to cavitation damage.

The damage caused by cavitation Fig 2.6, occurs not only on the blades but also on the shrouds. The places with the lowest pressure are situated on the back faces of the blades near the
inlet edge. In this region there is a sudden increase in velocity, which favours the development of cavitation. When cavitation is intense, the tips of the blades at the outlet, the diffuser vanes and the volute tongue may also be damaged.

Cavitation does not occur during normal pump operation, it can do so in a well designed pump through the pump trying to deliver too much owing to the pumping head having been overestimated, the pump having been arranged with insufficient depth of liquid above its inlet or the pressure at the pump inlet being reduced by extensive friction, clogging or incompletely opened valve in suction line. Unsuitable configuration of the approach and the speed being too high.

The remedy to the first effect lies in adjusting the pump to the actual head by throttling, impeller trimming or speed reduction and the second and third cases by rectifying the fault or deficiency, raising the supply level (and/or pressure) or lowering the pump if necessary while for the last case if the speed cannot be readily adjusted but the discrepancy is too great it may be overcome by producing more head at the suction otherwise a pump of lower speed or special design may be necessary.

2.2.5 CENTRIFUGAL PUMP CAVITATION CHARACTERISTICS.

To avoid or to limit cavitation in centrifugal pumps a pressure reserve in relation to the vapour pressure of the
fluid must be available at the impeller inlet. This difference between the absolute pressure and the vapour pressure is referred to as Net Positive Suction Head, NPSH, and is measured in meters. In algebraic terms:

\[
\text{NPSH} = \frac{P_{a1} - P_v}{\rho g}
\]  

(2.9)

where subscripts 1 and v were used to indicate suction and vapour pressures respectively.

NPSH is the criterion for the suction condition provided by an installation with what is needed by a pump to work satisfactorily. As such it has two aspects, the NPSH available from the system and the NPSH required by the pump. The latter must not exceed the former.

A pump in operation will create a vacuum in its suction branch and as a result of the atmospheric pressure acting upon the suction fluid level the liquid will rise in the suction pipe and enter the impeller. The NPSH available, \( \text{NPSH}_A \), is a measure of the energy available in the fluid as it enters the pump from the installation.

Further for every pump there is for every throughput and speed of rotation a particular head of liquid depending on the detail of the pump design, which is needed at the pump inlet to keep the liquid from cavitating in entry to the impeller. In pump terminology this factor is term the NPSH required,
NPSH\textsubscript{R}, and is a matter of establishment by actual running test. It may also be predicted using the empirical equation suggested from many researchers [49,60]:

\[ NPSH = k_a \frac{V_{RI}^2}{2g} + k_b \frac{u_t^2}{2g} \] (2.10)

where \( V_R \) is the radial velocity. Typical values for the constant \( k_a \) are 1.0 to 1.2 and for the constant \( k_b \) 0.1 to 0.3.

Theoretical work by Pearsall, [58], showed how to get optimum cavitation performance for a pump with a given duty and that the main factors in giving good cavitation performance are choice of correct size, speed and geometry rather than detailed blade design. He suggested an analytical method of how to calculate the optimum inlet diameter. The equation proposed by Pearsall for the calculation of the NPSH\textsubscript{R} of the impeller for any point on the blade inlet is:

\[ NPSH = \frac{V_{RI}^2}{2g} (\sigma_b + 1.04) + \frac{u_t^2}{2g} \] (2.11)

where \( \sigma_b \) is the blade cavitation coefficient. This corresponds to the empirical equation (2.10). He also proposed another equation giving the optimum inlet diameter of the impeller as:

\[ D_{e1} = 1.37 \left[ \left( \frac{2(1+\sigma_b)}{\sigma_b} \right)^{1/2} \frac{Q}{\omega} \frac{1}{(1-\lambda^2)} \right]^{1/3} \] (2.12)

with \( D_{e1} \) being the inlet tip diameter of the impeller, \( \lambda \) the hub to tip ratio, \( Q \) the flowrate and \( \omega \) the rotational speed.

Thus optimum diameters for pumps of low suction pressures
will have larger eye diameters (i.e. as $\alpha_b$ increases $D_{el}$ will increase) than those for ordinary conditions and these diameters are larger than the inlets of many standard pumps. However, as will be discussed later, the possibility of internal recirculation, at flows below 100 percent of rated conditions, must also be accounted for due to the large inlet diameters.

This theoretical method of determining the cavitation break down point, has the great advantage over the empirical techniques of enabling the effect of the various variables to be predicted whereas the empirical techniques tend to combine both good and bad designs together.

Over the years various test techniques have been used to determine and define the cavitation characteristics of centrifugal pumps. Two approaches, [56,57], have been used in defining the cavitation limit of a pump, Fig 2.7, the upper limit of capacity for a given NPSH and speed of rotation and the lower limit of NPSH at a given capacity and speed of rotation. The second definition has already been preferred. A change in performance such as a drop in head or in efficiency is considered to be an indication of cavitation. Because of the difficulty of determining the exact condition when this change takes place, it is often the practice to define required NPSH as that value where a drop of 3 percent in head will have taken place. This is illustrated in Fig 2.7b. The
NPSH at given capacity and speed is NPSH$_2$, which is that NPSH at which the head will have been reduced by 3 percent.

Cavitation has two effects on pumps, when developed it causes a drop in performance (head, flow and efficiency) whilst prolonged operation under cavitation conditions may cause cavitation damage on the blades. Observations, [47, 48, 50] showed that incipient cavitation occurs at the point A in Fig 2.6 where the peripheral velocity is maximum and hardly occurs at nominal capacity and by further increase of flow the cavitation is apt to occur at the pressure side of impeller blades. When the flowrate is small, cavitation appears on the suction side and with further decrease of capacity the cavitation point moves towards the hub. Referring to Fig 2.7a, generally speaking, if capacity is increased under a certain value of NPSH, head deviates and drops from standard curve which is not attended with cavitation. And any further increase in the discharge valve opening the capacity reaches a limit and the head drops straight downward.

A cavitation characteristic where the flow is kept constant and the absolute pressure or NPSH reduced until cavitation occurs is shown in Fig 2.8. On this characteristic curve cavitation starts (both visually and acoustically) at a point A, it then gradually increases until at point B, the performance is affected and finally at C the cavitation is so extensive that performance breakdown occurs. The normal or
critical criterion of the performance breakdown, as mentioned earlier, is represented by the point where the head (or efficiency) drops by 3 percent. It is not however a completely satisfactory criterion as the exact values depend very much on the shape of the characteristic curve, which can also be influenced by experimental errors, [57,58].

There is a confusion where the performance starts to be affected and point B is commonly used as the inception point. There is also a sense of security that cavitation does not occur in region A and B where in fact it does. Indeed the maximum likelihood of cavitation damage occurs somewhere near B and this is illustrated by the noise trace shown in Fig 2.8. The noise characteristic demonstrates both the initial inception point, by the sudden rise in high frequency noise corresponding to A, and the likely cavitation damage point that seems to occur at the maximum of the noise curve D. The reason for the variation of noise level is not immediately evident, [44,58], but a suggested interpretation of this variation is given. When cavitation commences in a pump from a change in NPSH the high frequency levels rise as a result of the growth and collapse of the corresponding range of small vapour bubbles. As the pressure is reduced the range of bubble radii and the number of bubbles increases and the noise levels continue to rise in this frequency band. The high frequency noise levels then decrease as the bubbles tend to grow bigger and absorb sound as well as radiate less noise in the higher
frequencies. At the same time it is also possible that the presence of the cavitation cloud, at these conditions effectively reduces the characteristic impedance of the fluid medium resulting in smaller radiated sound pressures for a given power source. Further reductions in NPSH to the breakdown region sometimes results in cavities from the suction faces being conveyed into the next passage-way and collapsing in the pressure field of the adjacent blade giving rise to increase noise levels. Therefore where the pump is required to be completely free of cavitation, it should operate at above the inception point A in Fig 2.8.

It is a common practice to assume that the NPSH required from 100 percent flow on up to as 150 percent varies as the square of the capacity. When it comes to flows below 100 percent of rated conditions, [50,56,59], the situation becomes quite complex. Because a centrifugal pump can be designed only for one capacity at a given head and speed, the geometry of the impeller can be ideal only for these rated conditions. At all flows below that rated capacity, the distribution of velocity and the flow angles are distorted, also pressure pulsations may be exhibited. The presence of these pulsations prevents obtaining any accuracy in the use of the accepted definition of cavitation limits, since neither the total head developed by the pump nor the suction pressure can be determining accurately in the capacity region where this phenomenon occurs. The required NPSH no longer follows the
“square of flow” rule but increases at a lesser rate. For this reason NPSH curves extended below 50 percent of best efficiency flow cannot be considered to be accurate. In general, however, these will resemble those in Fig 2.9, where the deviation from the square law is clearly indicated. As a matter of fact at a certain reduce flow (which varies with impeller design) a new phenomenon arises, that of internal recirculation, [47,48]. This phenomenon leads to disturbances in performance, hydraulic pulsations and even destructive action on the impeller metal.

At the capacity where incipient internal recirculation occurs at the suction eye of an impeller, the flow at the outer eye diameter has a tendency to reverse itself and develops a vortex as Fig 2.10 indicates. It is the collapse of these vortices which creates a form of cavitation leading to impeller damage. While it is impossible to calculate or measure the exact value of NPSHR to prevent this cavitation, it can be indicated qualitatively as shown in Fig 2.11, which conditions prevailing in two different impellers are indicated:

(a) An impeller with very low NPSH and extremely oversized inlet diameter

(b) An impeller with moderate NPSH requirements.

It becomes evident that impeller (b) develops an internal recirculation at flows considerably lower than impeller (a) and is less apt to be subject to cavitation damage.
There are special problems concerned with cavitation in handling chemicals or hydrocarbons. Cavitation prediction and testing of pumps is usually done in cold water, [55,61,62]. Roughly such data can be applied to other fluids, taking into account vapour pressure in the pump NPSH. There is a thermodynamic effect, however, due to the different latent heats and specific volumes of many liquids. Fortunately the effect is small and on the side of safety, for instance the pump NPSH in butane may be 0.61-0.82 m lower than in water. Thus if the plant is designed using data from cold water tests it will be safe in other liquids. There are factors, however, that may offset this simplification. If fluid has a large change in vapour pressure for small changes in temperature, it will be very important to make sure the design suction pressures and temperatures are exact.

There are some cases in the process industry where low NPSH is required. One way of achieving it is to reduce the speed of the pump, but this may mean increasing the number of stages if high pressures are needed. However, the improved design and manufacture of modern conventional centrifugal pumps can lead to marked improvements in cavitation performance. Another method is to raise speeds to produce smaller cheaper units in a single stage. Where such pumps have insufficient cavitation performance, inducers and supercavitating pumps are being used to give a low NPSH.
The inducer, \([63,64,65]\), is an axial type impeller or propeller shaped like a male conveyor screw, which is placed in the eye area of a conventional impeller. Both inducer and impeller are set on the same shaft and rotate at the same speed. The inducer itself has a low NPSH\(_R\). It generates enough head to provide suction pressure for the conventional impeller, thus avoiding problems of cavitation where the NPSH available is low.

Liquid enters the inducer at suction pressure. Because of the small pressure drop at the inducer entrance, in most cases, pressure never gets below vapour pressure. The inducer subsequently generates enough head to provide adequate suction pressure for the conventional impeller. In spite of the large pressure drop at the conventional impeller inlet, the fluid never again approaches vapour pressure, avoiding any vaporization in the impeller. By increasing the generated head of the inducer a large safety margin between the vapour pressure and the lowest pressure in the impeller can be achieved.

Basically, the inducer is a very small booster pump, lifting the suction pressure to a sufficient level for the main conventional impeller. With this design the booster pump and main pump are incorporated into one casing and mounted on a single shaft.
Investigations showed, [66,67,68,69,70,71], that although pressure within the inducer is normally in excess of vapour pressure, however, if it dips below the result in cavitation is minimal and does not damage the inducer. When vaporization occurs at the inducer inlet, the liquid is vaporized into a lot of small bubbles which will eventually collapse. But, since the inducer is designed for a very small and steady pressure rise all along its passages, the bubble implosions are spread over the entire passage length. As a result there is no big shock. The inducer continues to run smoothly and vibration and cavitation damage do not occur. However cavitation damage may occur towards the end of the passages.

The supercavitating pump, [62,72], makes use of the cavitation by employing special hydrofoil blade sections designed to give acceptable performance in the fully cavitating region. The cavities form as one large bubble on the suction surface of the blade so that they remain undamaged. In this way cavitation is deliberately used instead of being avoided. It is essential in the supercavitating impeller that the blade are passages not chocked with cavitation and hence a low blade spacing or solidity is necessary. For this reason an axial flow type appears to be the most suitable.

Both inducer and supercavitating pump have typical pump characteristics. The inducer produces a greater head and tends
to be of a lower specific speed. The extra head could be useful when matching to centrifugal pump, although it is not necessary as it can be shown that supercavitating pump will provide sufficient head to satisfy the NPSH need of the centrifugal impeller.

The inducer head and efficiency are unaffected by decreasing NPSH until the cavitation is extensive and choking occurs, when the head and flow fall suddenly. The supercavitating pump, on the other hand, has a falling head and efficiency characteristics with reducing NPSH and no lower limit at which choking occurs exists to the same extent. The inducer efficiency is higher, as only partial cavitation exists. However, the efficiency at off design conditions, where the higher suction specific speeds occur, is perhaps lower. In the case of a supercavitating inducer, where with a conventional pump, this efficiency forms only a small part of the overall efficiency.

2.2.6 SECONDARY CAVITATION.

In the previous discussion attention has been focused on what may be called primary cavitation i.e. cavitation that affects the main flow. However in many cases it is possible to design a piece of equipment so that no primary cavitation occurs over the entire operating range. It does not necessarily follow from this that there will be no cavitation since secondary cavitation may occur in interference zones, at
discontinuities in the guiding surface such as occur between various parts of the machine in regions of secondary flow, or in other regions of minor disturbance.

Fig 2.12 shows two guiding surfaces separated by a gap at the inlet section of a centrifugal pump. If such a gap exits in a low pressure region, secondary cavitation may develop. This is usually one of the lowest pressure areas in the machine. Cavitation has been observed frequently in this zone and if the intensity is sufficient, cavitation damage results.

It is possible for cavitation to develop in secondary flows, that is, in recirculating flows in clearance areas which are induced by the primary flow. Usually the maximum velocity of such a flow is not sufficient to induce cavitation. However, the possibility should not be ignored. Lastly, if the characteristic dimension of the roughness is a small fraction of the local boundary layer thickness, it may have little effect on cavitation. On the other hand, if the roughness protrudes far enough into the boundary layer it can have the effect of a series of small surface discontinuities and produce local cavitation.

2.2.7 SUMMARY.

The literature review of relevant previous work in cavitation proved the complexity of the process. It is a requirement for the design engineer to understand and model...
such situations for better design. Although the formulation of cavitation leads to nonlinear mathematics understanding of the cavitation mechanism will undoubtedly lead to an acceptable cavitation model.
Fig 2.1 Two phase flow regimes in pipes.

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(a) Head vs Flowrate

(b) Head vs Volumetric Gas Content

Fig 2.2 Typical head degradation curves
Fig 2.3 Influence of the speed of sound in a two phase (gas/liquid) mixture, [19].

Fig 2.4 Side channel pump two phase head characteristics, [26,32].
Fig 2.5 Zakem dimensionless two phase flow pump performance loss correlation, [14].

Fig 2.6 Cavitation damage regions in a centrifugal pump, [37,48].
Fig 2.7 Typical cavitation characteristics for a centrifugal pump.

(a) Head characteristic curve.

(b) NPSH characteristic curve.

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Fig 2.8 Cavitation mechanism in centrifugal pump, [44,58].

Fig 2.9 Required NPSH for centrifugal pump, [47,48].
Fig 2.10 Internal recirculation during low capacities in centrifugal pump, [48,51,56].
(a) Oversized eye diameter

(b) Moderate NPSH requirements
Fig 2.11 Internal recirculation characteristics, [56].

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Fig 2.12 Typical section of a centrifugal pump with clearance gap near the inlet, [36].
CHAPTER THREE

RESEARCH DESIGN
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RESEARCH DESIGN

Pump head in single phase flow can be defined unambiguously. In two phase flows, it is not easy to define a universally acceptable pump head because each phase is associated with its own velocity and density. If one defines a total head based on the homogeneous mixture density and velocity one can show such a definition can sometimes lead to a negative head rise. Thus a physically consistent, universally acceptable definition of head is needed to reduce the test data properly. The main work of the research concentrated on the two phase head definition and equally distributed between theoretical and experimental investigations described in this chapter. At the end of the chapter an error analysis is performed to understand the cause of uncertainty occurred in some of the test data and a detailed plan of the research is discussed.

3.1 THEORETICAL WORK.

In this section, in order to accomplish the third objective, described in CHAPTER ONE, a mathematical model is proposed, [73], for the calculation of the pump two phase head based on the one dimensional control volume method. The analytical method has incorporated separated two phase flow conditions, pump geometry, compressibility of the gas phase and condensation effects. In the following pages, the principles of the single phase pump head characteristics will be
presented following with a detailed derivation of the proposed two phase mathematical model for the head of a centrifugal pump.

3.1.1 SINGLE PHASE FLOW.

3.1.1.1 Euler Equation for Single Phase Flow.

The starting point of any calculation of the mean flow through a turbomachine is the Euler equation, which combines Newton's Law with the first law of thermodynamics to express the change of enthalpy of, or the work done by, the fluid flowing through the machine to change the moment of momentum of the flow into and out of the rotor. Euler's equation is a general equation, applying in all circumstances, including those when two phase flow passes through the impeller. For a single phase flow it can be stated that the Euler's equation is given by:

\[ gH_{esp} = u_2V_{u2} - u_1V_{u1} \]  \hspace{1cm} (3.1)

where \( H_{esp} \) is the Euler's (ideal) head of the single phase pump, \( g \) the gravitational constant and subscripts 1 and 2 are used to indicate the suction and discharge section of the impeller respectively. This is one form of Euler's equation.

A typical velocity triangle for a centrifugal pump is indicated in Fig 3.1 where \( V \) is the absolute velocity of the fluid, \( W \) the relative velocity, \( u \) the peripheral velocity, \( V_r \) the radial velocity, \( V_\theta \) the tangential velocity, \( \beta \) the blade angle and \( \epsilon \) the fluid absolute angle, (both referred to the
tangential direction). From the velocity triangle many forms of the Euler's equation can be derived. The tangential component of the absolute velocity can be written as:

\[ V_u = u - \frac{Q}{\pi b D \tan \beta} \quad (3.2) \]

where

\[ \beta = \frac{Q}{\pi b D} \quad (3.3) \]

is used, with \( Q \) being the liquid flowrate, \( b \) the height of the passage and \( D \) the diameter as in Fig 3.1. By substitution of equation (3.2) into (3.1) the following can be obtained:

\[ H_{E_{EP}} = \frac{u_2^2 - u_1^2}{g} + \frac{Q}{g \pi} \left( \frac{u_1}{b_1 D_1 \tan \beta_1} - \frac{u_2}{b_2 D_2 \tan \beta_2} \right) \quad (3.4) \]

The above gives the single phase Euler's head involving the geometry of the impeller and the liquid flowrate. Also the tangential component of the absolute velocity can be expressed as:

\[ V_u = V \cos \epsilon \quad (3.5) \]

Substitution of the above into equation (3.1) yields:

\[ g H_{E_{EP}} = u_2 V_2 \cos \epsilon_2 - u_1 V_1 \cos \epsilon_1 \quad (3.6) \]

This form of Euler's equation involves the absolute velocity of the liquid at the inlet and outlet of the impeller.

### 3.1.1.2 Total Pump Head for Single Phase Flow

The work done by a pump represents an increase in the total energy of the fluid between the suction and delivery connections. This increase is equal to the sum of the increase in pressure head, the increase in velocity head and the increase in geometrical head in the pump itself. These
increases are expressed in meters of pumped liquid.

Pressure gauges and manometers do not recognise the velocity head of the liquid at their measuring points and this must be added to the indicated reading. It may calculated from:

\[ V_o = \frac{4Q}{\pi d^2} \]  

where \( V_o \) is the velocity in the pipe and \( d \) is the pipe diameter.

A typical pump configuration is indicated in Fig 3.2. Applying Bernoulli's equation from the suction to the delivery tapping planes referred to the pump:

\[ \frac{P_e}{\rho g} + \frac{V_{oe}^2}{2g} + z_o + H_{sp} = \frac{P_d}{\rho g} + \frac{V_{od}^2}{2g} + z_D \]  

where \( P \) is the static pressure, \( \rho \) the density of the fluid and subscripts \( e \) and \( d \) indicate the suction and discharge planes of the pump. Thus the total single phase pump head will be:

\[ H_{sp} = \frac{P_d - P_e}{\rho g} + \frac{V_{od}^2 - V_{oe}^2}{2g} + z_D - z_o \]  

For a relatively small pump the geometrical head term is negligible and for equal suction and delivery pipe diameters the kinetic energy head term will give zero. Furthermore the suction and delivery pressures are normally monitored at the inlet and outlet flanges of the pump just before and after the inlet and outlet of the impeller. Hence the pressure
difference $P_0 - P_a$ can be considered as the difference $P_2 - P_1$ of the impeller (i.e. $P_0 - P_a = P_2 - P_1$). And the single phase total head generated by a relatively small pump and equal suction and discharge pipe diameters can be expressed as:

$$H_{\text{isp}} = \frac{P_2 - P_1}{\rho g} \quad (3.10)$$

### 3.1.1.3 Impeller Head for Single Phase Flow.

A typical section for the impeller of a centrifugal pump during operation is given in Fig 3.3. Applying Bernoulli's equation from the inlet to the outlet of the impeller:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + R_1 + H_{\text{ISP}} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + R_2$$

or

$$H_{\text{ISP}} = \frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + R_2 - R_1 \quad (3.12)$$

where $H_{\text{ISP}}$ is the impeller single phase head.

On the right hand side of the above equation the first term is the pressure head which is the single phase total head as given in equation (3.10). The second term is the kinetic energy head and the third term is the geometrical head, which can be neglected for a small level difference. Hence equation (3.12) can be written as:

$$H_{\text{ISP}} = \frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g} \quad (3.13)$$

or

$$H_{\text{ISP}} = H_{\text{Ep}} + H_{\text{ISP}} \quad (3.14)$$

with $H_{\text{Ep}}$ being the single phase kinetic energy head.
In order to calculate the absolute velocities, just before the blade inlet edges, ignoring blade interference, from the velocity triangle, Fig 3.1, the following relation can be obtained:

\[ v = \frac{Q}{\pi bD \sin \epsilon} \]  

Thus for the suction point 1 the fluid absolute velocity is:

\[ v_1 = \frac{Q}{\pi b_1 D_1 \sin \epsilon_1} \]  

(3.16a)

and for the discharge point 2, if blade blockage is ignored:

\[ v_2 = \frac{Q}{\pi b_2 D_2 \sin \epsilon_2} \]  

(3.16b)

Substitution of equation (3.16) into equation (3.13) will give:

\[ H_{\text{inlet}} = \frac{P_2 - P_1}{\rho g} + \frac{Q^2}{2g\pi^2} \left( \frac{1}{b_2^2 D_2^2 \sin^2 \epsilon_2} - \frac{1}{b_1^2 D_1^2 \sin^2 \epsilon_1} \right) \]  

(3.17)

The fluid absolute angle \( \epsilon \), can also be determined with the aid of the velocity triangle. From the triangle the following relationships can be obtained:

\[ v_u = \frac{V_R}{\tan \epsilon} \]  

(3.18)

and

\[ v_u = u - \frac{V_R}{\tan \beta} \]  

(3.19)

Equating the above relationships and using equation (3.3) will give:

\[ \epsilon = \tan^{-1} \frac{Qtan \beta}{u \pi bD \tan \beta - Q} \]  

(3.20)

Therefore for the suction point 1 the fluid absolute angle is:
and for the discharge point 2:

\[ \epsilon_2 = \tan^{-1} \frac{Q \tan \beta_2}{u_2 \pi b_2 D_2 \tan \beta_2 - \alpha} \]  (3.22)

3.1.2 TWO PHASE FLOW.

3.1.2.1 Euler Equation for Two Phase Flow.

Here an attempt will be made to derive the Eulers equation under two phase conditions. In order to do that the following assumptions, as in single phase flow, will be made:

(a) The fluid moves without friction.

(b) The paths of the liquid and gas phases are parallel to the impeller blades (one dimensional flow)

(c) Infinite number of blades with zero thickness

(d) Liquid and gas phases enter and leave the impeller with the same relative angle (there may in fact be some difference, but the effect is very small).

A typical two phase stream surface is examined which intersects the suction edge at point 1 and the discharge edge at point 2 as indicated in Fig 3.3. Since momentum changes in the tangential direction, give rise to a torque and thus work, moment of momentum equations for each individual phase for elemental areas of flow at the point of suction and discharge can be written down:

a. For the liquid phase:
dM_{1L} = (\rho_{1L} V_{n1L} dA_{1L}) V_{u1L} R_1 \tag{3.23a}
dM_{2L} = (\rho_{2L} V_{n2L} dA_{2L}) V_{u2L} R_2 \tag{3.23b}

b. For the gas phase:

dM_{1G} = (\rho_{1G} V_{n1G} dA_{1G}) V_{u1G} R_1 \tag{3.23c}
dM_{2G} = (\rho_{2G} V_{n2G} dA_{2G}) V_{u2G} R_2 \tag{3.23d}

where M is the momentum, \( V_n \) is the normal velocity, \( dA \) is the elemental area of flow and subscripts \( L \) and \( G \) are used to represent the quantities of liquid and gas respectively. The mass flowrates in the liquid and gas phases \( d\dot{m}_L \) and \( d\dot{m}_G \) are given as:

\[ d\dot{m}_L = \rho_L V_{nL} dA_L \tag{3.24a} \]
\[ d\dot{m}_G = \rho_G V_{nG} dA_G \tag{3.24b} \]

Thus on substitution of equation (3.24) into equation (3.23), the total moment of momentum change will be:

\[ M_1 + M_2 = \int d\dot{m}_{2L} R_2 V_{u2L} - \int d\dot{m}_{1L} R_1 V_{u1L} + \int d\dot{m}_{2G} R_2 V_{u2G} - \int d\dot{m}_{1G} R_1 V_{u1G} \tag{3.25} \]

Application of the theorem of momentum to the fluid between suction and discharge sections with respect to the axis of revolution (the impeller axis) will give:

TORQUE ABOUT THE CONSIDERED AXIS = RATE OF INCREASE OF ANGULAR MOMENTUM OF BOTH PHASES ABOUT THE AXIS.

In algebraic terms:

\[ B = \int d\dot{m}_{2L} R_2 V_{u2L} - \int d\dot{m}_{1L} R_1 V_{u1L} + \int d\dot{m}_{2G} R_2 V_{u2G} - \int d\dot{m}_{1G} R_1 V_{u1G} \tag{3.26} \]

In two phase flow it is often convenient to have a measure of fraction of the total mass flow across a given area which is composed of each component. The total mass flowrate of a gas-liquid mixture is the sum of the mass flowrates of the individual phases:
\[ \dot{m}_{TP} = \dot{m}_L + \dot{m}_G \]  \hspace{1cm} (3.27a)

where subscript \( TP \) is used to represent the quantity of the two phase mixture. The quality, \( x \), of the mixture can be defined as:

\[ x = \frac{\dot{m}_G}{\dot{m}_{TP}} \]  \hspace{1cm} (3.27b)

also

\[ (1-x) = \frac{\dot{m}_L}{\dot{m}_{TP}} \]  \hspace{1cm} (3.27c)

The peripheral velocity \( u \) is given as:

\[ u = \omega R \]  \hspace{1cm} (3.27d)

where \( \omega \) is the angular velocity. Substituting the above equations into the equation (3.26) and integrating will give:

\[ B\omega = [(1-x_2)u_2V_{u2L}+x_2u_2V_{u2g}]-[(1-x_1)u_1V_{u1L}+x_1u_1V_{u1g}] \]  \hspace{1cm} (3.28)

Transformation of equation (3.28) to give work done per unit mass will be:

\[ gH_{ETP} = [(1-x_2)u_2V_{u2L}+x_2u_2V_{u2g}]-[(1-x_1)u_1V_{u1L}+x_1u_1V_{u1g}] \]  \hspace{1cm} (3.29)

with \( H_{ETP} \) being the Euler two phase (ideal) head for a pump.

The consideration of this equation suggests a close similarity to the Euler equation form for single flow, equation (3.1), simply by putting \( x_1=x_2=0 \). Therefore the ideal two phase head developed by a pump can be expressed as:

\[ H_{ETP} = \frac{1}{g} \left[ [(1-x_2)u_2V_{u2L}+x_2u_2V_{u2g}]-[(1-x_1)u_1V_{u1L}+x_1u_1V_{u1g}] \right] \]  \hspace{1cm} (3.30)

It should be noted that the above is valid for a compressible, condensable two phase mixture. For a non-condensable mixture, since there is no mass transfer between
the phases i.e. \(x_1 = x_2 = x\) then:

\[
H_{ETP} = \frac{1}{g} \left[ \frac{(1-x)u_2 V_{u2l} + x u_2 V_{u2o} - [(1-x)u_1 V_{u1l} + x u_1 V_{u1o}]}{[(1-x)u_2 V_{u2l} + x u_2 V_{u2o} - [(1-x)u_1 V_{u1l} + x u_1 V_{u1o}]} \right] (3.31)
\]

For two phase flow conditions, the simple (one dimensional) velocity triangles at the suction and discharge sections for the liquid and gas phases of a general centrifugal pump are shown in Fig 3.4. From the velocity triangles, the tangential velocity component can be written as:

a. For the liquid phase:

\[
\begin{align*}
V_{u1l} &= u_1 - \frac{Q_L}{\pi b_1 D_1 \tan \beta_1} \\
V_{u2l} &= u_2 - \frac{Q_L}{\pi b_2 D_2 \tan \beta_2}
\end{align*}
\] (3.32a, 3.32b)

b. For the gas phase:

\[
\begin{align*}
V_{u1o} &= u_1 - \frac{Q_{1o}}{\pi b_1 D_1 \tan \beta_1} \\
V_{u2o} &= u_2 - \frac{Q_{2o}}{\pi b_2 D_2 \tan \beta_2}
\end{align*}
\] (3.33a, 3.33b)

where equation (3.3) is used. Returning now to the original equation (3.31) by substitution of equations (3.32) and (3.33) the following can be obtained:

\[
gH_{ETP} = \left[ \frac{u_2^2 - u_2 [(1-x)Q_L - xQ_{2o}]}{\pi b_2 D_2 \tan \beta_2} \right] - \left[ \frac{u_1^2 - u_1 [(1-x)Q_L - xQ_{1o}]}{\pi b_1 D_1 \tan \beta_1} \right] \]

(3.34)

The above can be considered as another form of two phase Euler equation involving the geometry and the flowrates of both phases at the inlet and outlet of the impeller.

Furthermore from Fig 3.4 the tangential component of the absolute velocity can be written as:
a. For the liquid phase:

\[ V_{u1L} = V_1 \cos \epsilon_{1L} \]  
\[ V_{u2L} = V_2 \cos \epsilon_{2L} \]  

(3.35a)
(3.35b)

b. For the gas phase:

\[ V_{u1G} = V_1 \cos \epsilon_{1G} \]  
\[ V_{u2G} = V_2 \cos \epsilon_{2G} \]  

(3.35c)
(3.35d)

Returning now to equation (3.31) by substitution of equation (3.35) the following can be obtained:

\[ H_{ETP} = \frac{1}{g} \left\{ \frac{[(1-x)(u_2V_2L \cos \epsilon_{2L} - u_1V_1L \cos \epsilon_{1L})]_1}{[x(u_2V_2G \cos \epsilon_{2G} + u_1V_1G \cos \epsilon_{1G})]} \right\} \]  

(3.36)

This form of Euler's equation involves the absolute velocities of both phases at the inlet and outlet of the impeller.

3.1.2.2 Total Head for Two Phase Flow.

The method used here is somewhat similar to that of Zakem, [14], and Furuya, [12], but the solution methods are quite different. Zakem applied the results of two phase flow equations only to a non-dimensional parameter without investigating the basic mechanism of degradation. Furuya solved the two phase flow equations with the compressibility and condensation effects neglected.

A control volume method is employed for the derivation of Bernoulli equation for the flow going through the blades of a pump under two phase flow conditions. However, in order to
present more accurately the behaviour of the centrifugal pump under two phase flow the compressibility and condensation of the gas phase are included. Application of such a method requires the following assumptions:

(a) The trajectory of the liquid is identical to that of the gas so that we can use a control volume bounded by two streamlines.

(b) The gas phase is treated as a perfect gas undergoing isothermal process.

The control volume used in the analysis is shown in Fig 3.5 where the streamline, s, and the normal to the streamline, n, are used as the curvilinear coordinates. The control volume is composed of area dA with a unit thickness in the direction normal to the stream surface.

The total cross section area is the sum of the cross section areas occupied by the phases:

\[ A_{TP} = A_L + A_g \]  \hspace{1cm} (3.37)

with A being as the cross section. The void fraction, \( \alpha \), can be defined as the ratio of the gas flow cross section to the total cross section:

\[ \alpha = \frac{A_g}{A_{TP}} \]  \hspace{1cm} (3.38a)

also

\[ (1-\alpha) = \frac{A_L}{A_{TP}} \]  \hspace{1cm} (3.38b)

Hence the masses in the liquid and gas phases \( dm_L \) and \( dm_g \)
respectively contained in this control volume are:

\[ dm_L = dsdA(1-\alpha)\rho_L \]  
(3.39a)

\[ dm_g = dsdA\alpha\rho_g \]  
(3.39b)

where \( \rho_L \) and \( \rho_g \) are the densities of liquid and gas, respectively.

The momentum balance applied to the control volume in the direction of flow, that is the \( s \) direction, will be as follows:

\[ \frac{d}{dt}(dm_LW_L+dm_gW_g) = F_e \]  
(3.40)

where \( F_e \) is an external force acting on the control volume in the \( s \)-direction and \( W_L, W_g \) are the relative flow velocities of liquid and gas respectively in the \( s \) direction. The external force, \( F_e \), consists of two components. The centrifugal force, \( F_c \), and the pressure force, \( F_p \).

Thus the external force \( F_e \) can be written as:

\[ F_e = F_c + F_p \]  
(3.41)

where:

\[ F_c = dm_Lr^2\sin\beta'\cos\gamma \]  
(3.42)

\[ F_p = \frac{\partial P}{\partial s}dsdA \]  
(3.43)

\[ dm = dm_L + dm_g \]  
(3.44)

and \( r \) is the radial distance of the control volume centre of rotation, \( \beta' \) and \( \gamma \) are the local geometric angles of the stream surface shown in Fig 3.5 and \( P \) denotes static pressure.

Substitution of (3.42) and (3.43) into (3.40) assuming steady state conditions so that \( dm_L \) and \( dm_g \) do not change along the streamline we find:
\[ \frac{dm_w}{dt} + dm_{\theta} \frac{dW_{\theta}}{dt} = dm_r \omega^2 \sin \beta \cos \gamma - \frac{\partial}{\partial s} dV \quad (3.45) \]

Also from Fig 3.5:
\[ dV' = dV = dsdA \quad (3.46) \]
\[ \frac{dr}{ds} = \sin \beta \cos \gamma \quad (3.47) \]
with \( V' \) being as the volume of the control volume.
Substituting equation \((3.46)\) and \((3.47)\) into \((3.45)\):
\[ \frac{dm_w}{dt} + dm_{\theta} \frac{dW_{\theta}}{dt} = dm_r \omega^2 \frac{dr}{ds} - \frac{\partial}{\partial s} dV' \quad (3.48) \]

Based on the assumption that all the quantities at equation \((3.48)\) change along the streamline direction \( s \) we will be able to write:
\[ \frac{dW_w}{dt} = \frac{\partial W_w}{\partial t} + W_L \frac{\partial W_w}{\partial s} \quad (3.49a) \]
and
\[ \frac{dW_{\theta}}{dt} = \frac{\partial W_{\theta}}{\partial t} + W_{\theta} \frac{\partial W_{\theta}}{\partial s} \quad (3.49b) \]
For steady state conditions:
\[ \frac{\partial W_w}{\partial t} = 0 \quad \text{and} \quad \frac{\partial W_{\theta}}{\partial t} = 0 \quad (3.49c) \]
Thus equations \((3.49a)\) and \((3.49b)\) can be written for steady state conditions:
\[ \frac{dW_w}{dt} = W_L \frac{\partial W_w}{\partial s} \quad (3.50a) \]
and
\[ \frac{dW_{\theta}}{dt} = W_{\theta} \frac{\partial W_{\theta}}{\partial s} \quad (3.50b) \]
Substitution of the above equations into \((3.48)\) yields:
\[ dm_w \frac{\partial W_w}{\partial s} + dm_{\theta} \frac{\partial W_{\theta}}{\partial s} = dm_r \omega^2 \frac{dr}{ds} - \frac{\partial}{\partial s} dV' \quad (3.51) \]
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Noting that:
\[
\frac{\partial W_L}{\partial s} = \frac{d}{ds} \left( \frac{W_L^2}{2} \right) = \frac{d}{ds} \left( \frac{\partial W_L}{\partial s} \right) \quad \text{dm}_L = \text{const. (3.52a)}
\]

and
\[
\frac{\partial W_g}{\partial s} = \frac{d}{ds} \left( \frac{W_g^2}{2} \right) = \frac{d}{ds} \left( \frac{\partial W_g}{\partial s} \right) \quad \text{dm}_g = \text{const. (3.52b)}
\]

Substituting equation (3.52) into the equation (3.51) the later will become:
\[
\frac{d}{ds} \left( \frac{\partial W_L}{\partial s} + \frac{\partial W_g}{\partial s} \right) = \frac{d m_r}{ds} \frac{dr}{ds} - \frac{\partial P}{\partial s} \frac{d V'}{d s} \quad (3.53)
\]

Also:
\[
u = \omega \frac{dr}{ds} \quad (3.54)
\]

Substitution of the above equation into equation (3.53) will give:
\[
\frac{d}{ds} \left( \frac{\partial W_L}{\partial s} + \frac{\partial W_g}{\partial s} - \frac{\partial m_r}{\partial s} \right) = - \frac{\partial P}{\partial s} \frac{d V'}{d s} \quad (3.55)
\]

The volume of the control volume \(dV'\) is at any time equal to the sum of the two phases volume, \(dV_L'\) and \(dV_g'\) for the liquid and gas respectively. Thus the volume \(dV'\) can be written as:
\[
dV' = dV_L' + dV_g' \quad (3.56)
\]

Therefore we can write that:
\[
\frac{\partial P}{\partial s} \frac{d V'}{d s} = dV_L' \frac{\partial P}{\partial s} + dV_g' \frac{\partial P}{\partial s} = \frac{d}{ds} (dV_L'P) + dV_g' \frac{\partial P}{\partial s} \quad (3.57)
\]

Since \(dV_g'\) is not constant due to compressibility effects the derivative of the gas phase on the above equation will be:
\[
dV_g' \frac{\partial P}{\partial s} = \frac{d}{ds} (dV_g'P) - P \frac{\partial}{\partial s} \left( dV_g' \right) \quad (3.58)
\]

where:
\[
P \frac{\partial}{\partial s} \left( dV_g' \right) = \frac{d}{ds} (PdV_g') = P \frac{d}{ds} (dV_g') \quad (3.59a)
\]

thus:
\[
dV_g' \frac{\partial P}{\partial s} = \frac{d}{ds} (dV_g'P) - P \frac{d}{ds} (dV_g') \quad (3.59b)
\]

Substitution of equation (3.59b) into equation (3.57) gives:
\[
\frac{\partial P}{\partial s} dV' = \frac{d}{ds}(PdV') - \frac{d}{ds}(dV_p') \tag{3.60}
\]

The assumption of the gas undergoing isothermal changes will allow us to write:

\[PdV_p' = \Delta m_o RT \quad (T=\text{constant}, \ R=\text{constant}) \tag{3.61}\]

where \(R\) is the specific gas constant and \(T\) the temperature of the two phase mixture. Solving equation (3.61) for \(dV_p'\) the derivative will be:

\[\frac{d}{ds}(dV_p') = - \frac{\Delta m_o \ RT}{P^2} \frac{dP}{ds} \tag{3.62}\]

Multiplication of both sides with \(P\) yield:

\[P\frac{d}{ds}(dV_p') = - \frac{\Delta m_o \ RT}{P} \frac{dP}{ds} \tag{3.63}\]

Also

\[\frac{1}{P} \frac{dP}{ds} = \frac{d}{ds} \log P \tag{3.64}\]

Hence equation (3.63) can be written as:

\[P\frac{d}{ds}(dV_p') = - \frac{\Delta m_o \ RT}{P} \frac{d}{ds} \log P \tag{3.65}\]

Equation (3.60) then becomes:

\[\frac{\partial P}{\partial s} dV' = \frac{d}{ds}(PdV') + \frac{\Delta m_o \ RT}{ds} \frac{d}{ds} \log P \tag{3.66}\]

Returning now to our original equation (3.55) by substitution of the above equation the following can be obtained:

\[\frac{d}{ds} \left( \frac{d \left( \Delta m_o \ W_L^2 \right)}{2} + \frac{d \left( \Delta m_o \ W_g^2 \right)}{2} - \frac{d \left( u^2 \right)}{2} \right) = - \frac{d}{ds} (PdV') - \Delta m_o \ RT \frac{d}{ds} \log P \tag{3.67}\]

Rearrangement of the above equation will give:

\[\frac{d}{ds} \left( \frac{d \left( \Delta m_o \ W_L^2 \right)}{2} + \frac{d \left( \Delta m_o \ W_g^2 \right)}{2} - \frac{d \left( u^2 \right)}{2} + PdV' + \Delta m_o \ RT \log P \right) = 0 \tag{3.68}\]

Dividing the above equation by \(\Delta m\) and \(g\):
We consider as the average density in the separated two phase condition and \( x \) the quality. Integration of equation (3.69) from the suction side of the impeller to discharge side denoted by subscripts 1 and 2 respectively yield the relation:

\[
\int_1^2 \frac{d}{ds} \left[ (1-x) \frac{W_L^2}{2g} + x \frac{W_a^2}{2g} - \frac{u^2}{2g} + \frac{1}{\rho_{TR^*} g} P + x \frac{RT}{g} \log P \right] ds = 0 \tag{3.70}
\]

The relation obtained here is considered to be the Bernoulli equation for the rotating machinery operating under two phase flow conditions. There exist compressibility and condensation effects, thus the void fraction and quality of the mixture is allowed to change. It must be pointed out that by putting \( x_1 = x_2 = 0 \) then the above equation gives the familiar Bernoulli equation for a single phase conditions.

We now determine the total (actual) head of the pump operating under the two phase flow condition. According to Pythagoras theorem, from Fig 3.1, the following relation can be written:

\[
W^2 = u^2 + V^2 - 2uV \cos \epsilon \tag{3.71}
\]

also \( V_u = V \cos \epsilon \tag{3.72} \)

thus \( W^2 = V^2 + u^2 - 2uV_u \tag{3.73} \)
Replacing in equation (3.70) the relative velocity $W$ with the aid of equation (3.73) the above relationship will give:

$$\frac{1}{g} \left[ \left( 1-x_z \right) u_2 V_{2L} + x_2 u_2 V_{2g} \right] - \left[ \left( 1-x_1 \right) u_1 V_{1L} + x_1 u_1 V_{1g} \right]$$

$$= \frac{P_z}{\rho_{2TP} g} - \frac{P_1}{\rho_{1TP} g} + \frac{(1-x_2)V_{2L} - (1-x_1)V_{1L}}{2g} + \frac{x_2 V_{2g} - x_1 V_{1g}}{2g} + \frac{RT}{2g} \frac{P_2}{P_1} x_2 + \frac{\log}{g} \frac{P_2}{P_1} x_1$$  \hspace{1cm} (3.74)

It is clear that the left hand side of equation (3.74) is the ideal (Euler) two phase pump head $H_{ETP}$ derived before in equation (3.30). Therefore equation (3.74) can be written as:

$$H_{ETP} = \frac{P_z}{\rho_{2TP} g} - \frac{P_1}{\rho_{1TP} g} + \frac{(1-x_2)V_{2L} - (1-x_1)V_{1L}}{2g} + \frac{x_2 V_{2g} - x_1 V_{1g}}{2g} + \frac{RT}{2g} \frac{P_2}{P_1} x_2 + \frac{\log}{g} \frac{P_2}{P_1} x_1$$  \hspace{1cm} (3.75)

The above equation is valid for both compressible and condensable two phase mixture. For a compressible non condensable mixture, where $x_1 = x_2 = x$ due to no mass transfer between the phases, the two phase ideal pump head can be reduced to:

$$H_{ETP} = \frac{P_z}{\rho_{2TP} g} - \frac{P_1}{\rho_{1TP} g} + \frac{(1-x)V_{2L} - V_{1L}}{2g} + \frac{x V_{2g} - V_{1g}}{2g} + \frac{RT}{2g} \frac{P_2}{P_1} x + \frac{\log}{g} \frac{P_2}{P_1}$$  \hspace{1cm} (3.76)

or

$$H_{ETP} = H_{TP} + H_{VL} + H_{Vg} + H_c$$  \hspace{1cm} (3.77)

with the total two phase pump head as:

$$H_{TP} = \frac{P_z}{\rho_{2TP} g} - \frac{P_1}{\rho_{1TP} g}$$  \hspace{1cm} (3.77a)

the liquid kinetic energy head as:
the gas kinetic energy head as:

\[ H_{VL} = (1-x) \frac{V_{2L}^2 - V_{1L}^2}{2g} \]  (3.77b)

and the compression head loss as:

\[ H_{c} = x \frac{RT}{g} \log \frac{P_2}{P_1} \]  (3.77d)

therefore:

\[
\text{Eulers Two Phase Head} = \text{Total Two Phase Head} + \text{Liquid Kinetic Energy Head} \\
\quad + \text{Gas Kinetic Energy Head} + \text{Compression Head Loss} \]  (3.78)

### 3.1.3 ADDITIONAL SUPPORTIVE RELATIONSHIPS.

#### 3.1.3.1 Gas Phase Flowrate.

The flowrate of a gas in two phase flow depends on the system pressure. The flowrate at the suction side is different at discharge. The difference in the volume is proportional to the isothermal change of the volume of the gas because of the change in pressure. For isothermal processes:

\[ PV' = \text{constant} \]  (3.79)

which can also be written as:

\[ PQ = \text{constant} \]  (3.80)

If the inlet conditions of the gas to the system are, \( P_{\text{Ino}} \) the inlet pressure and \( Q_{\text{Ino}} \) the inlet flowrate, the gas condition at any point will be:

\[ P_{\text{Ino}}Q_{\text{Ino}} = PQ_e \]  (3.81)
and the total two phase capacity will be:

\[ Q_{TP} = Q_L + Q_0 \]  

(3.83)

Therefore for the suction point 1 the flowrate will be:

\[ Q_{10} = \frac{P_{INL}}{P_1} Q_{INL} \]  

(3.84a)

and

\[ Q_{1TP} = Q_L + Q_{10} \]  

(3.84b)

And for the discharge point 2:

\[ Q_{20} = \frac{P_{INL}}{P_2} Q_{INL} \]  

(3.85a)

and

\[ Q_{2TP} = Q_L + Q_{20} \]  

(3.85b)

3.1.3.2 Fluid Absolute Velocity,

For two phase flow the simple (one dimensional) velocity triangles at the suction and discharge sections for the liquid and gas phases of a general centrifugal pump are shown in Fig 3.4. From the velocity triangles, using equation (3.3), the absolute velocities can be written as:

a. At the inlet section:

\[ V_{1L} = \frac{Q_L}{\pi b_1 D_{1} \sin \epsilon_{1L}} \]  

(3.86a)

\[ V_{10} = \frac{Q_{10}}{\pi b_1 D_{1} \sin \epsilon_{10}} \]  

(3.86b)

b. At the outlet section:

\[ V_{2L} = \frac{Q_L}{\pi b_2 D_{2} \sin \epsilon_{2L}} \]  

(3.87a)

\[ V_{20} = \frac{Q_{20}}{\pi b_2 D_{2} \sin \epsilon_{20}} \]  

(3.87b)
3.1.3.3 Fluid Absolute Angle.

From the two phase velocity triangles, Fig 3.4, the absolute fluid angle $\epsilon$ and the blade angle $\beta$ can be written as:

\[ V_u = \frac{V_R}{\tan \epsilon} \quad (3.88) \]

and

\[ V_u = u - \frac{V_R}{\tan \beta} \quad (3.89) \]

Equating equations (3.88), (3.89) and using equation (3.3) then:

\[ \epsilon = \tan^{-1} \frac{Q \tan \beta}{u \pi b D \tan \beta - Q} \quad (3.90) \]

Thus at the suction and discharge sections for the liquid phase the absolute angle will be:

\[ \epsilon_{1L} = \tan^{-1} \frac{Q_L \tan \beta_1}{u_1 \pi b_1 D_1 \tan \beta_1 - Q_L} \quad (3.91a) \]

\[ \epsilon_{2L} = \tan^{-1} \frac{Q_L \tan \beta_2}{u_2 \pi b_2 D_2 \tan \beta_2 - Q_L} \quad (3.91b) \]

and for the gas phase:

\[ \epsilon_{1G} = \tan^{-1} \frac{Q_G \tan \beta_1}{u_1 \pi b_1 D_1 \tan \beta_1 - Q_G} \quad (3.92a) \]

\[ \epsilon_{2G} = \tan^{-1} \frac{Q_G \tan \beta_2}{u_2 \pi b_2 D_2 \tan \beta_2 - Q_G} \quad (3.92b) \]

3.1.3.4 Relative Velocity.

From the velocity triangles Fig 3.4 the relative velocity can be expressed as:

\[ W^2 = u^2 + V^2 - 2uV \cos \epsilon \quad (3.93) \]

Therefore for the liquid phase the relative velocity will be:
3.1.3.5 Velocity Ratio and Slip.

The velocity ratio, $\zeta$, is the ratio of the relative gas velocity to the liquid velocity expressed as:

$$\zeta = \frac{W_\text{G}}{W_\text{L}}$$  (3.96)

Therefore at the suction point 1 the ratio will be:

$$\zeta_1 = \frac{W_{1G}}{W_{1L}}$$  (3.97a)

and for the discharge point 2:

$$\zeta_2 = \frac{W_{2G}}{W_{2L}}$$  (3.97b)

Slip is defined as the difference between the mean velocities of the gas and liquid phases:

$$\xi = W_\text{G} - W_\text{L}$$  (3.98)

Hence for the suction point 1 the slip will be:

$$\xi_1 = W_{1G} - W_{1L}$$  (3.99a)

and for the discharge point 2:

$$\xi_2 = W_{2G} - W_{2L}$$  (3.99b)
3.1.3.6 Void Fraction.

The void fraction was defined in equation (3.3B) as:

\[ \alpha = \frac{A_g}{A_{tp}} \]  

(3.3B a)

The area occupied by the gas phase can be expressed as:

\[ A_g = \frac{Q_g}{W_g} \]  

(3.100)

and the area occupied by the two phase mixture is the area of the impeller passage. Hence:

\[ A_{tp} = \pi b D \]  

(3.101)

Returning now to the original equation (3.3B a), substituting equations (3.100) and (3.101) yields:

\[ \alpha = \frac{Q_g}{\pi b D W_g} \]  

(3.102)

Hence for the suction point 1 the void fraction will be:

\[ \alpha_1 = \frac{Q_{1e}}{\pi b_1 D_1 W_{1e}} \]  

(3.103a)

and for the discharge point 2:

\[ \alpha_2 = \frac{Q_{2e}}{\pi b_2 D_2 W_{2e}} \]  

(3.103b)

3.1.3.7 Density of the Mixture.

The average density in the separated two phase condition is:

\[ \rho_{TP}^* = \frac{\dot{m}_{TP}}{Q_{TP}} = \frac{\dot{m}_L + \dot{m}_g}{Q_L + Q_g} \]  

(3.104)

The above equation can also be written as:

\[ (1-a)\rho_L W_L + a\rho_g W_g = \rho_{TP}^*[1(1-a)W_L + aW_g] \]  

(3.105)
Liquids containing gas can be compressed which is not possible with pure liquids, thus under compressible conditions the density of the gas phase undergoing an isothermal compression will be:

\[ \rho_a = \frac{P}{RT} \]  (3.106)

with \( R = \text{constant}, \ T = \text{constant.} \) Rearrangement of equation (3.105) by substituting equation (3.106) yields:

\[ \rho_{\text{TP}} = \frac{(1-a)\rho_L RT + aW_a P}{[(1-a)\rho_L + aW_a]RT} \]  (3.107)

Dividing by the liquid relative velocity, \( W_L \), yields:

\[ \rho_{\text{TP}} = \frac{(1-a)\rho_L RT + a\zeta P}{[(1-a) + a\zeta]RT} \]  (3.108)

where \( \zeta \) is the velocity ratio given by equation (3.96). Hence for the suction point 1 the two phase density is given by:

\[ \rho_{1\text{TP}} = \frac{(1-a_1)\rho_{1L} RT + a_1\zeta_1 P_1}{[(1-a_1) + a_1\zeta_1]RT} \]  (3.109a)

and for the discharge point 2:

\[ \rho_{2\text{TP}} = \frac{(1-a_2)\rho_{2L} RT + a_2\zeta_2 P_2}{[(1-a_2) + a_2\zeta_2]RT} \]  (3.109b)

3.1.3.8 Quality of the Mixture.

The quality has been defined in equation (3.27) as the ratio of the gas mass flowrate to the total two phase mass flowrate:

\[ x = \frac{\dot{m}_g}{\dot{m}_{\text{TP}}} \]  (3.27)

The above can also be written as:

\[ x = \frac{\rho_a Q_a}{\rho_{\text{TP}} Q_{\text{TP}}} \]  (3.110)
With the assumption of the gas undergoing an isothermal process, substitution of equation (3.106) into (3.110) will give:

\[ x = \frac{P}{RT} \frac{Q_0}{Q_{TP}} \frac{1}{\rho_{TP}} \]  

(3.111)

Thus for the suction point 1 the quality will be:

\[ x_1 = \frac{P_1}{RT} \frac{Q_{10}}{Q_{1TP}} \frac{1}{\rho_{1TP}} \]  

(3.112a)

and for the discharge point 2 the quality will be:

\[ x_2 = \frac{P_2}{RT} \frac{Q_{20}}{Q_{2TP}} \frac{1}{\rho_{2TP}} \]  

(3.112b)

For a non condensable mixture the quality at the suction must be equal the quality at the discharge allowing to write that:

\[ x_1 = x_2 = x \]  

(3.113)

Therefore:

\[ \frac{P_1}{\rho_{1TP}} \frac{Q_{10}}{Q_{1TP}} = \frac{P_2}{\rho_{2TP}} \frac{Q_{20}}{Q_{2TP}} \]  

(3.114)

3.1.4 SUMMARY.

In two phase flows, a universally acceptable definition, for two phase pump head is needed to reduce test data properly. In this section a mathematical model was proposed for the calculation of the pump two phase head, incorporated two phase flow conditions, pump geometry and compressibility and condensation effects:

\[ H_{TP} = H_{TP} + H_{VL} + H_{Vg} + H_c \]  

(3.77)
It is therefore possible to use the above relationship, for the calculation of two phase pump total head, for the reduction of the two phase test data.
3.2 EXPERIMENTAL WORK.

Systematic tests have been carried out on a centrifugal pump of conventional design in order to identify the pump design parameters with most remarkable impact on two phase flow and cavitation performance. In the following pages the test program, instrumentation and procedures will be presented following by the data reduction methods used in order to give an accurate and detailed presentation of the test data.

3.2.1 TEST PROGRAM.

The work scope for the test program established a detailed schedule for measuring pump head and torque, for four different impeller designs, at selected steady state flowrates of air-water mixtures and cavitating conditions. To accomplish the objectives, presented in CHAPTER ONE, a test program consisting of three connected studies was defined; each of these phases is described below.

Study 1 established approximately 200 different pump, steady state operating conditions by varying, impeller geometry, flowrate, volumetric gas content and system suction pressure. All selected test conditions were constrained by the allowable range of parameters available in the air-water test facility. Study 2 established approximately 150 different pump cavitating conditions by varying impeller geometry, flowrate, and NPSH of the system and study 3 used high speed video, (HSV), recording where required to assist the development of
the analytical model and to identify any similarities on the behaviour of bubbles during cavitation and two phase conditions.

By including a sufficient number of single phase test points into the selection of test conditions for studies 1 and 2 and repeating each test five times, the first, second and fifth objectives were satisfied. The first objective was to determine the basic gas handling capability of a centrifugal pump, the second to establish the causes of two phase head degradation and improve existing theories by comparing experimental results with analytical models and the fifth objective was to examine the behaviour of a centrifugal pump during low suction pressures. The third study, using HSV recording complied with the fourth objective, which was to identify any similarities between two phase flow and cavitation by detailed examination of the behaviour of the gas bubbles in the impeller passages.

A detailed test schedule was prepared that included the requirements for the variation of water and air flowrates and system pressure. Generally, the actual test program proceeded according to the schedule, but two modifications occurred due to the following unforeseen events:

(a) Two phase pump testing was performed for volumetric gas contents between 0 and 50 percent, but further testing was eliminated due to insufficient gas flowrate from the
Compressed air workshop line.

(b) Addition of an extra 53 test points to define two phase and cavitation performance more thoroughly.

<table>
<thead>
<tr>
<th>Block No</th>
<th>Test Data No</th>
<th>Test Impeller</th>
<th>Water Flowrate $(10^{-3} \text{ m}^3/\text{s})$</th>
<th>Air Flowrate $(10^{-3} \text{ m}^3/\text{s})$</th>
<th>System Suction Pressure (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
<td>A</td>
<td>1.0 - 5.5</td>
<td>0 - 0.20</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>51</td>
<td>A</td>
<td>1.0 - 5.5</td>
<td>0 - 0.50</td>
<td>1.50</td>
</tr>
<tr>
<td>3</td>
<td>51</td>
<td>A</td>
<td>1.0 - 5.5</td>
<td>0 - 0.50</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>A</td>
<td>1.0 - 5.5</td>
<td>0 - 0.08</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>B</td>
<td>1.0 - 5.5</td>
<td>0 - 0.23</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>29</td>
<td>C</td>
<td>1.0 - 5.5</td>
<td>0 - 0.23</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>27</td>
<td>D</td>
<td>1.0 - 5.5</td>
<td>0 - 0.23</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>A</td>
<td>1.5 - 4.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>B</td>
<td>1.5 - 4.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>C</td>
<td>1.5 - 4.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>48</td>
<td>D</td>
<td>1.5 - 4.44</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1 Operational program for two phase flow and cavitation tests.

A total of 11 blocks of two phase and cavitation data was obtained. The specific ranges of water and air flowrates, impeller geometry, and system pressure for the entire test program are exhibited in Table 3.1.

3.2.2. Description of Test Facility and Procedure.

The two phase flow test facility, designed and constructed at the Mechanical Department, Loughborough University of Technology, used a test pump supplied by HMD Seal Less Pumps Ltd with design performance values as displayed in Fig 3.6.
The system allows air water flowrates to be adjustable over a wide range of volumetric gas contents, is able to create low NPSH\textsubscript{a} when required, and will operate stably. In addition to these considerations a transparent casing was designed through which observation of the flow phenomena was made possible.

3.2.2.1 Test Loop Description.

A plan view of the general arrangement of the closed circuit loop is displayed in Fig 3.7. Water from the vented storage tank was circulated through pipes of PVC design. The tank system was designed to allow air to be evacuated from the vented storage tank when cavitation and two phase runs occurred, and provided a reasonable residence time. Where connections were necessary i.e. bends, valves e.t.c. care was taken to ensure that the joint was totally sealed so that no water leakage from the system and no air leakage into the system occurred. A valve downstream of the pump, controlled the flowrate and a control valve upstream of the pump, before the air inlet, was used to vary the suction pressure (or NPSH). A booster pump driven by a 6HP motor at a constant speed of 2900 rpm was also incorporated into the system. This smaller pump was arranged on the suction side of the pump to assist flow from the main tank and to increase suction pressure when required.

Air supply was obtained from compressed air workshop line. The air was injected into and mixed with the water in the air-
water mixer, which uses a pipe located on the centerline of the suction passage with a cylindrical perforated end, followed by an orifice. The orifice was included to generate a degree of turbulence and to obtain a more uniform mixture of air and water. The discharge of the air-water mixture into the storage tank allowed the water to release the entrained air, and an approximate two-minute settling time transpired before the water again entered into the suction piping.

The assembly of the test loop, was simply supported on large blocks of wood and rigidly constrained only at the storage tank, the booster pump and the test pump. This arrangement helped to maintain accurate alignment and eliminated vibrations.

3.2.2.2 Test Pump Description.

The test pump was an HMD end suction centrifugal pump driven by a 12.5 HP motor at a constant speed of 2900 rpm with vertical discharge. The pump was of single stage design with a volute collector and a suction and discharge diameters of 50 mm. The inlet pipe and casing of the pump was transparent through which observations of the flow phenomena were made possible by means of strobo-light and HSV recordings.

Previous work on similar pump designs, pointed out the influence of the gas accumulation on the impeller passages as discussed in CHAPTER TWO. The selected four impeller
parameters are indicated in Table 3.2. Fig 3.8 shows the impeller profiles used in the experiments. Impeller A was the original, 1 DMS HMD design, with a duty flowrate and total head of $4.44 \times 10^{-3} \text{ m}^3/\text{s}$ and 34 m respectively. The other three impellers were of straight bladed design with no specified operation parameters since they were produced arbitrarily but with the same inlet and outlet diameters. The idea behind the straight bladed impeller designs was that gas accumulation at the impeller passages may be eliminated by the high level of turbulence produced of such design.

<table>
<thead>
<tr>
<th>Description</th>
<th>No of blades</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$H_0$</th>
<th>$Q_p$</th>
<th>$N_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 1DMS</td>
<td>4</td>
<td>7</td>
<td>26</td>
<td>50</td>
<td>180</td>
<td>25</td>
<td>6</td>
<td>34</td>
<td>$4.44 \times 10^{-3}$</td>
<td>2900</td>
</tr>
<tr>
<td>B 40 straight bladed</td>
<td>4</td>
<td>40</td>
<td>50</td>
<td>180</td>
<td>23</td>
<td>10</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>C 60 straight bladed</td>
<td>4</td>
<td>60</td>
<td>50</td>
<td>180</td>
<td>30</td>
<td>10</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>D 90 straight bladed</td>
<td>4</td>
<td>90</td>
<td>90</td>
<td>50</td>
<td>180</td>
<td>30</td>
<td>10</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 3.2 Test impellers design parameters.

3.2.2.3 Test Loop Instrumentation.

Instrumentation was incorporated into the test loop to provide the capability for measuring air and water flowrates, system temperatures and pressures and several pump performance values. The instruments were satisfactory for measuring steady
state test conditions on the test pump. To develop pump head, torque and NPSH values for each specific steady state test condition it was necessary to obtain measurements of the following parameters:

- Water flowrate
- Air flowrate
- Pump inlet pressure
- Pump outlet pressure
- Differential pressure across the booster pump
- Air pressure
- Pump inlet temperature
- Pump outlet temperature
- Air temperature
- Pump torque

The instrumentation required to make these measurements was installed, as in Fig 3.7, and is described in Table 3.3.

Water flowrate was measured by a turbine meter upstream from the test pump before the air-water mixer. A second turbine meter was also employed downstream the test pump for reference during single phase tests. Air flowrate was measured by a rotameter before injection into the system. The pressure and temperature prior to injection were also monitored in order to be able to calculate the density and the volume of air flowing through the pump.

At the inlet and outlet of the test pump the pressure and
temperature of the water or air-water mixture were measured to calculate the pump head and the NPSH values. Pressure loss across the booster pump were also monitored to calculate the NPSH by the system when the booster was not in operation. Pressures measured by displacement transducers with a digital readout indicating gauge pressures and temperatures were monitored by thermocouples connected to a multi channel digital readout.

<table>
<thead>
<tr>
<th>Description</th>
<th>Range</th>
<th>Manufacturer &amp; Model No</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WATER FLOWMETER:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turbine flowmeter</td>
<td>0-100 gpm</td>
<td>RHODES &amp; SONS Model A/4, No 8210</td>
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<tr>
<td><strong>AIR FLOWRATE:</strong></td>
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<td></td>
</tr>
<tr>
<td>Rotameter</td>
<td>0-100 l/min</td>
<td>ROTAMETER CO LTD Model MFG</td>
</tr>
<tr>
<td><strong>INLET PRESSURE:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure Transducer</td>
<td>0-5 bar</td>
<td>DRUCK LTD Model DPI260,No 406/87-12</td>
</tr>
<tr>
<td><strong>OUTLET PRESSURE:</strong></td>
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<td></td>
</tr>
<tr>
<td>Pressure Transducer</td>
<td>0-10 bar</td>
<td>DRUCK LTD Model DPI260,No 406/87-15</td>
</tr>
<tr>
<td><strong>DIFFERENTIAL PRESSURE ACROSS BOOSTER PUMP:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differential Pressure Transducer</td>
<td>0-10 bar</td>
<td>DRUCK LTD Model DPI260,No 9784/92-2</td>
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<tr>
<td><strong>TEMPERATURES:</strong></td>
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<td></td>
</tr>
<tr>
<td>Microprocessor</td>
<td>0-150 ºC</td>
<td>COMARM ELECTRONICS Model 6200, No 11847</td>
</tr>
<tr>
<td>Thermometer</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TORQUE:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamometer</td>
<td>0-20 lb</td>
<td>SALTER No 235</td>
</tr>
</tbody>
</table>

Table 3.3 Experimental instrumentation.

The pump torque was measured by a swinging field dynamometer.
so that measurement of the force exerted on the spring balance by the motor at a fixed radius allowed the torque values to be calculated.

3.2.2.4 Test Procedure.

Before the pump was operated it was ensured that all the control valves were open. With all the power switches turned to the ON position, the test pump was started. After a few seconds when the pump had taken hold and was running smoothly, it was set to constant running speed. If the booster pump was necessary, this was started in a similar fashion. The test pump was started before the booster pump was activated. If this was not done, pressure built up at the impeller eye could cause damage to the casing.

For single-phase tests, having the upstream valve fully open and the air valve fully closed, the water flowrate was altered from the downstream control value. Readings were taken for the inlet and outlet pressures and temperatures, water flowrate and torque. This was done for several water flowrates. For two-phase tests, the same procedure was followed as for the single-phase studies but with the aid of the air valve air was injected into the system. Readings were taken for the inlet and outlet pressures and temperatures, water and air flowrates and air pressure and temperature. With the inlet valve slightly opened at the desired flowrates, during cavitation tests, the discharge valve was opened gradually, having
flowrate constant until the pump lost its suction. Readings were taken for the inlet pressure and temperature, flowrate and torque. This was done for several flowrates. These tests were repeated five times for each specific test condition to ensure the most accurate results would be obtained.

3.2.3 HIGH SPEED VIDEO

Two phase flow in the pump and cavitation are fluctuating processes and these fluctuations do not have a constant frequency. The one powerful photographic technique that has been found to be very useful in studying high speed constant-frequency phenomena in "slow motion" has not been applicable to two phase flow and cavitation. This is the point by point process, which moving pictures are created from a series of individual photographs of the repeated event taken once every cycle or once every given number of cycles, with the phase varying slightly between each photograph. Since this technique depends entirely upon the cyclic process being constant, it cannot be applied to the irregular fluctuations of two phase flow and cavitation. Four general types of photography records have been found useful. Individual photographs taken with short enough exposure times to stop the fluid motion and thus obtain sharp pictures free from blur. The use of relatively long time exposure to define average shapes of two phase bubbles and cavitation regions. Streak photographs on a moving strip of film to obtain envelops of the motion of individual cavitation elements. High speed motion pictures to study in
detail the dynamic of the process.

The use of high speed video, (HSV), equipment during tests was necessary because of the inherent inability of the human eye to analyze high speed motion, in other words, to resolve small time differences with respect to moving bodies. This is very similar to the inability of the unaided eye (and brain) to resolve very small differences in position. The optical tool that overcomes the latter difficulty is the microscope. High speed photography, which may be classed as another optical tool, serves a similar purpose with regard to time and may well be thought of as a time microscope in that it resolves small differences in time as a microscope resolves small differences in space.

HSV recordings were conducted on seventeen specific steady state conditions, eleven for air-water mixtures and six for cavitation. The HSV assisted in the development of the analytical model and identified the bubbles motions into the impeller passages for both two phase flow and cavitation.

3.2.4 DATA REDUCTION.

Pump performance data can be presented in several different forms ranging from a simple curve, (head and power versus capacity), to a nondimensional form. For single phase flow through the pump, the information presented in the two kinds of curves is summarized in Fig 3.9 where the complete
performance characteristics are exhibited. In two phase flow through the pump, as was discussed in Section 3.1, it is not easy to define universally acceptable performance parameters because each phase is associated with its own velocity and density. However by normalizing the pump operating parameters, it is possible to develop homologous parameters for pump head, pump torque and flowrate.

3.2.4.1 Homologous Parameters.

The concept of homologous curves for two phase pump performance was first introduced by Wings and Parks, [18], and widely used from many researchers. The conditions for the dynamic similarity of behaviour of a particular pump at two operating conditions can be stated in terms of the impeller speed $N$, the volume flowrate $Q$, the pump head $H$ and the torque $B$ as follows:

$$\text{if } \left(\frac{Q}{N}\right)_1 = \left(\frac{Q}{N}\right)_2, \text{ then } \left(\frac{H}{N^2}\right)_1 = \left(\frac{H}{N^2}\right)_2 \text{ and } \left(\frac{B}{N^2}\right)_1 = \left(\frac{B}{N^2}\right)_2$$

(3.115)

These similar (homologous) states can be expressed in terms of nondimensional quantities as follows:

$$\text{if } [Q/Q_0]=[Q/Q_0] \text{ then } \left[\frac{H/H_D}{(N/N_D)^2}\right] = \left[\frac{H/H_D}{(N/N_D)^2}\right] \text{ and } \left[\frac{B/B_D}{(N/N_D)^2}\right] = \left[\frac{B/B_D}{(N/N_D)^2}\right]$$

(3.116)

where subscript $D$ indicates design conditions. Therefore the three homologous parameters are:

Homologous flow parameter:

$$Q_H = \frac{Q/Q_0}{N/N_D} \quad (3.117a)$$
CHAPTER THREE • RESEARCH DESIGN

Homologous head parameter:
\[ H_H = \frac{H/H_D}{(N/N_D)^2} \]  \hspace{1cm} (3.117b)

Homologous torque parameter:
\[ B_H = \frac{B/B_D}{(N/N_D)^2} \]  \hspace{1cm} (3.117c)

The advantage of these homologous curves is that they deal only with measured parameters (e.g. torque, speed flowrate), and not with hydraulic power or efficiency which are not easy to be defined in two phase flow. However in two phase flow it is also difficult to define pump head and confusion exists (see CHAPTER TWO). Thus a universally acceptable definition, for two phase pump head, is needed to reduce test data properly.

3.2.4.2 Data Calculation.

A computer database program has been developed in QBASIC, called CENTRIF, for data calculation. CENTRIF has the capability to calculate and print pump performance values for any set of the following criteria:

- Single Phase Flow
- Two Phase Flow
- Cavitation Characteristics

CENTRIF calculates Euler's head and total head for single and two phase flow conditions for a given pump. It also calculates the NPSH of the pump during low suction conditions. The set of
equations used and the CENTRIF printout are given in APPENDIX A. Generally speaking for single phase and cavitation conditions the universally accepted equations were used. However for two phase flow calculations the equations derived in Section 3.1 were used.

3.2.4.3 Data Presentation.

Performance data of a pump can be presented in many forms. Pump capacity is widely used as a parameter plotted against various pump characteristics (e.g. head, power, torque, efficiency e.t.c.). Pump capacity is usually designated by flowrate, that is the medium volume flowing through the pump per unit time:

\[ Q = \frac{V'}{t} \]  

(3.118)

This defined capacity is a characteristic parameter of the pump performance principally because it is constant throughout the pump. However when it comes to two phase flow the flowrate is variable mainly due to the compressibility behaviour of the gas phase, (i.e. flowrate at pump inlet is different from the outlet). For an air-water medium the total flowrate of the mixture is the sum of the flowrates of the individual phases:

\[ Q_{TP} = Q_L + Q_o \]  

(3.119)

Hence the term inlet flowrate to the system was introduced in order to describe the total two phase capacity flowing through the system:

\[ Q_{INTP} = Q_{INL} + Q_{ING} \]  

(3.120)
The flowrate of the individual phases was measured at the air-water mixer section before mixing.

The void fraction has been defined in equation (3.38) as the ratio of the gas flow cross section to the total cross section:

$$\alpha = \frac{A_g}{A_{TP}}$$  \hspace{1cm} (3.38a)

However in the two phase pump literature, [12,13,17,18,23], values of void fraction were calculated with the following equation:

$$\alpha = \frac{Q_g}{Q_{TP}} = \frac{Q_g}{Q_L+Q_g}$$  \hspace{1cm} (3.121)

Strictly speaking the above is valid only for homogeneous flowrate conditions where the velocities of both phases are equal. For separated flow conditions the above equation is not valid and the introduction of the volumetric gas content parameter, $q$, is required for the analysis of test data:

$$q = \frac{Q_g}{Q_{TP}} = \frac{Q_g}{Q_L+Q_g}$$  \hspace{1cm} (3.122)

For the presentation of the two-phase test data, homologous curves of head and torque parameters versus homologous flow parameter were plotted as well as head and torque homologous degradation curves versus volumetric gas content. The typical shapes of such curves are presented in Fig 3.10 and a detailed
presentation and analysis of the test data is given in CHAPTER FOUR. For the presentation of the cavitation performance, pump head values were plotted against NPSH values and the required NPSH curve was determined from these. Typical shape of these curves are indicated in Fig 3.11 with detailed analysis in CHAPTER FOUR.

3.2.5 SUMMARY.

In this section a detailed description of the test facilities and instrumentation has been presented. Furthermore, two phase data reduction suggests the development of homologous parameters for pump head, torque and flowrate in order to present two phase test data properly. Cavitation test data can be reduced with the common parameters like NPSH.
3.3 ERROR ANALYSIS.

The range of conditions for the tests varied over the span of instrumentation used to record the test data, hence, accuracy of individual test points must be determined. Graphs of homologous pump head, homologous torque and NPSH, exhibit scatter in some of the test data, but it is not consistently equal throughout the data. An error analysis was performed to understand the causes of uncertainty.

3.3.1 INSTRUMENTATION ERRORS.

The accuracy of measurement of parameters displayed and recorded for each test condition is described in Table 3.4. Five sets of values were obtained at each test condition, and the average was calculated to ensure the most accurate results would be obtained. Though the calibration accuracy of pressure transducers is usually less than 1%, the accuracy of the measurement has been assumed to be 2% due to the frequent oscillatory movement of the digital readout and acknowledging some human error in reading the value.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Range</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid flowmeter, $Q_L$, gpm</td>
<td>0 - 100</td>
<td>± 1%</td>
</tr>
<tr>
<td>Gas flowmeter, $Q_g$, l/min</td>
<td>0 - 100</td>
<td>± 1%</td>
</tr>
<tr>
<td>Pump inlet pressure, $P_1$, bar</td>
<td>0 - 5</td>
<td>± 2%</td>
</tr>
<tr>
<td>Pump outlet pressure, $P_2$, bar</td>
<td>0 - 10</td>
<td>± 2%</td>
</tr>
<tr>
<td>System temperatures, $T$, °C</td>
<td>0 - 150</td>
<td>± 3%</td>
</tr>
<tr>
<td>Pump torque, $B$, lb</td>
<td>0 - 20</td>
<td>± 1%</td>
</tr>
</tbody>
</table>

Table 3.4 Accuracy of test loop instrumentation.
3.3.2 ERROR ANALYSIS OF CALCULATED PERFORMANCE.

An error analysis was performed on the equations derived in Section 3.1 to express pump performance for two phase mixtures flowing through the pump. The investigation shows that significant errors may exist in the calculated values of pump head over a certain ranges of air and water flowrate.

To bound the many possible dimensions of the error analysis, due to the large number to variable test conditions, the density of the liquid was assumed to be constant throughout the test loop flow path. (No significant temperature changes were occurring in the test loop.)

The general relationship for calculating the error in any parameter is:

$$\delta G = \sqrt{(\frac{\partial G}{\partial x})^2(\delta x)^2 + (\frac{\partial G}{\partial y})^2(\delta y)^2 + \ldots + (\frac{\partial G}{\partial z})^2(\delta z)^2} \quad (3.123)$$

where $G$ is any function represented by measurements $x$, $y$, ... and $z$.

The two phase flowrate equation is of the general form:

$$Q_{TP} = Q_L + Q_a \quad (3.120)$$

then:

$$\delta Q_{TP} = \sqrt{(\frac{\partial Q_{TP}}{\partial Q_L})^2(\delta Q_L)^2 + (\frac{\partial Q_{TP}}{\partial Q_a})^2(\delta Q_a)^2} \quad (3.124)$$
By expressing the uncertainty in each calculated parameter in relation to the parameter, the errors in any computed parameter can be displayed as a fraction of the parameter. For example, $Q_{TP}$ is the value of two phase flowrate. $\delta Q_{TP}$ is the error calculated in that flowrate and $(\delta Q_{TP}/Q_{TP})$ is the uncertainty in $Q_{TP}$ expressed as a fraction of the liquid flowrate. By plotting all errors as a fraction or percentage of the measured parameter, the uncertainties of different parameters can be compared and their significance evaluated.

The fractional error in the two phase flowrate shown in Fig 3.12 was determined with:

$$\frac{\delta Q_{TP}}{Q_{TP}} = \sqrt{\left(\frac{\delta Q_{L}}{Q_{L}}\right)^2 + \left(\frac{\delta Q_{G}}{Q_{G}}\right)^2} \quad (3.125)$$

Errors in the liquid flowrate were very small compared to the error in gas flowrate but at partial flowrates the uncertainty increases dramatically.

For the flowrate of gas assuming isothermal process the equation used was:

$$Q_{G} = \frac{P_{INS}}{P} \cdot Q_{INS} \quad (3.82)$$

Therefore the error will be:

$$\delta Q_{G} = \sqrt{\left(\frac{\partial Q_{G}}{\partial P_{INS}}\right)^2 (\delta P_{INS})^2 + \left(\frac{\partial Q_{G}}{\partial P}\right)^2 (\delta P)^2 + \left(\frac{\partial Q_{G}}{\partial Q_{INS}}\right)^2 (\delta Q_{INS})^2} \quad (3.126)$$

and the fractional error in the gas flowrate shown in Fig 3.13 was determined with:
where for volumetric gas contents below 6 percent the fractional error becomes too large.

Volumetric gas content, q, was defined as the volumetric flowrate of gas divided by the total two phase flowrate:

\[ q = \frac{Q_o}{Q_{TP}} = \frac{Q_o}{Q_L + Q_o} \]  

where the error in q is given below as:

\[ \delta q = \sqrt{\left(\frac{\delta q}{\partial Q_L}\right)^2 (\delta Q_L)^2 + \left(\frac{\partial q}{\partial Q_o}\right)^2 (\delta Q_o)^2} \]  

By dividing the error by the calculated value of volumetric gas content, the fractional uncertainty in the volumetric gas content becomes:

\[ \frac{\delta q}{q} = \sqrt{\left(\frac{\delta Q_L}{Q_L}\right)^2 + \left(\frac{\delta Q_o}{Q_o}\right)^2} \]

It is apparent from test data that the range of gas flowrate was varied from zero to maximum. Thus Fig 3.14 shows the percent error in volumetric gas content where the error is in the range of 2-3 percent.

The calculated values of pump head and torque (used to develop the dimensionless homologous values of pump head and torque) depend on the average density in the separated two phase condition and the mixture quality. The separated two phase density, at any point, was defined as:
\[ \rho_{TP}^* = \frac{(1-a)\rho_{RT} + a\rho}{[(1-a) + a\zeta]RT} \]  \hspace{1cm} (3.108)

In this analysis the liquid density, \( \rho_l \), is constant at 20°C. Thus the expression for the error in \( \rho_{TP}^* \) is:

\[ \delta \rho_{TP}^* = \sqrt{ \left( \frac{\partial \rho_{TP}^*}{\partial P} \right)^2 (\delta P) + \left( \frac{\partial \rho_{TP}^*}{\partial \rho} \right)^2 (\delta \rho) + \left( \frac{\partial \rho_{TP}^*}{\partial Q_{IN}} \right)^2 (\delta Q_{IN})^2 + \left( \frac{\partial \rho_{TP}^*}{\partial T} \right)^2 (\delta T)^2 } \]  \hspace{1cm} (3.130)

Fig 3.15 displays the fractional error in the calculated two phase density given by:

\[ \frac{\delta \rho_{TP}^*}{\rho_{TP}^*} = \sqrt{ \left( \frac{\delta P_{IN}}{P_{IN}} \right)^2 + \left( \frac{\delta \rho}{\rho} \right)^2 + \left( \frac{\delta Q_{IN}}{Q_{IN}} \right)^2 + \left( \frac{\delta T}{T} \right)^2 } \]  \hspace{1cm} (3.131)

Here the fractional error in separated two phase density increases steadily as the volumetric gas content becomes large.

The quality of the mixture at any point was defined as:

\[ x = \frac{P}{RT} - \frac{Q_g}{Q_{TP}} - \frac{1}{\rho_{TP}^*} \]  \hspace{1cm} (3.111)

de the error in quality, \( x \), is given below as:

\[ \delta x = \sqrt{ \left( \frac{\delta x}{\rho_{IN}} \right)^2 (\delta P_{IN})^2 + \left( \frac{\delta \rho}{\rho} \right)^2 (\delta \rho)^2 + \left( \frac{\delta Q_{IN}}{Q_{IN}} \right)^2 (\delta Q_{IN})^2 + \left( \frac{\delta T}{T} \right)^2 (\delta T)^2 + \left( \frac{\delta Q_L}{Q_L} \right)^2 (\delta Q_L)^2 } \]  \hspace{1cm} (3.132)

Dividing the error by the calculated value of quality the fractional error becomes:

\[ \frac{\delta x}{x} = \sqrt{ \left( \frac{\delta P_{IN}}{P_{IN}} \right)^2 + \left( \frac{\delta \rho}{\rho} \right)^2 + \left( \frac{\delta Q_{IN}}{Q_{IN}} \right)^2 + \left( \frac{\delta T}{T} \right)^2 + \left( \frac{\delta Q_L}{Q_L} \right)^2 } \]  \hspace{1cm} (3.133)
where Fig 3.16 displays the above quality fractional error and appears to be constant over the volumetric gas content range tested.

The definition of the total two phase pump head derived in Section 3.1 was used to calculate the data presented in this report.

\[
H_{TP} = \frac{P_2}{\rho_{TP2}^* g} - \frac{P_1}{\rho_{TP1}^* g}
\]

(3.78)

then:

\[
\delta H_{TP} = \sqrt{\left(\frac{\partial H_{TP}}{\partial P}\right)^2 \delta P^2 + \left(\frac{\partial H_{TP}}{\partial \rho_{TP1}^*}\right)^2 \delta \rho_{TP1}^{*2} + \left(\frac{\partial H_{TP}}{\partial \rho_{TP2}^*}\right)^2 \delta \rho_{TP2}^{*2}}
\]

(3.134)

and the percent error in pump total head in terms of errors in \(\delta P\) and \(\delta \rho_{TP}^*\) is:

\[
\frac{\delta H_{TP}}{H_{TP}} = \sqrt{\left(\frac{\delta P}{P}\right)^2 + \left(\frac{\delta \rho_{TP1}^{*}}{\rho_{TP1}^{*}}\right)^2 + \left(\frac{\delta \rho_{TP2}^{*}}{\rho_{TP2}^{*}}\right)^2}
\]

(3.135)

Fig 3.17 shows the calculated percent error on pump two phase head and Fig 3.18 indicates fractional error versus volumetric gas content to the system where the error in calculated head increases steadily above 12 percent of volumetric gas content.

Pump shaft torque was measured with a dynamometer thus the instrument error, Table 3.4, in all the measured torque values was ± 1 percent.
3.3.3 SUMMARY.

An error analysis has been performed to understand the cause of uncertainty in some of the test data. It should be noted that the calculated uncertainties in the pump performance values are only estimates of the true uncertainty due to the assumptions used in the derivation of the equations.
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3.4 RESEARCH PLAN.

The objectives drawn from the beginning of the research, presented in CHAPTER ONE, were:

(a) To determine the basic gas handling capability of a centrifugal pump.
(b) To establish the causes of two phase head degradation and improve the existing theories by comparing experimental results with analytical mathematical models.
(c) To develop an analytical method for the design of a two phase pump.
(d) To identify any similarities between two phase flow and cavitation by detailed examination of the behaviour of the gas bubbles in the impeller passages.
(e) To examine the behaviour of a centrifugal pump subject to low suction pressures.

In order to accomplish these objectives a detailed research plan was prepared as indicated in Fig 3.19 together with the time scales. The work of the research was equally distributed between theoretical and experimental investigations. Generally, the actual plan proceeded according to the schedule. However one modification occurred in the literature survey since it was extended from 4 to 7 months in order to give a concise critical literature review in the field of two phase flow and cavitation and to assist in the development of the analytical model.
Initial tests involved the selection and calibration of the instrumentation and as a next stage the identification of the problem by observations and the applicability of the various theories. The literature survey analyzed a considerable amount of relevant papers and textbooks on two phase flow and cavitation and a detailed review was given in CHAPTER TWO. Theoretical work contributed in the development of an analytical model in determining the total two phase head of a centrifugal pump and detailed experimentation assisted the above model.
Fig 3.1 A typical velocity triangle and impeller section for a centrifugal pump.

Fig 3.2 A typical layout of a centrifugal pump system.
Fig 3.3 Fluid stream paths at the inlet and outlet points for a centrifugal pump.

Fig 3.4 Simple (one dimensional) two phase velocity triangles for a centrifugal pump.
Fig 3.5 Control volume method for rotating machines and force diagram.

\[ dV' = ds \, dn \, 1 = ds \, dA \]

\[ ds \, \sin \beta \, \cos \gamma = dr \]

\[ \frac{dr}{ds} = \sin \beta \, \cos \gamma \]
MODEL NO. CS 10 N SYNCHRONOUS

IMPELLER DIAMETER (max/min) 180-130mm

MOTOR SPEED 2900 rpm

CONNECTIONS

FLOWRATE (US GPM)

FLOWRATE (m^3/hr)

P.S.H (m)

Differential Head (m)

POWER (kW)

Differential Head (ft)

POWER (hp)

Fig 3.6 Original HMD performance characteristics.

CURVES ARE FOR S.G. OF 1.0

MIN. SAFE FLOW IS BASED ON WATER. FOR OTHER LIQUIDS MULTIPLY BY:

S.G. LIQUID X SP. HT. LIQUID

S.G. WATER X SP. HT. WATER

Two Phase Flow and Cavitation in Centrifugal Pump

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Fig 3.7 Plan view of the test loop.

Fig 3.8 Plan view of the test impellers.
Fig 3.9 Single phase pump performance characteristics.
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(a) Two phase head characteristics

(b) Two phase head degradation.
Fig 3.10 Typical homologous two phase head degradation curves.
(a) NPSH characteristic.

(b) Required NPSH curve.

Fig 5.11 Typical cavitation characteristic and NPSH curves.
Fig 3.12 Calculated uncertainty in two phase flowrate.

Fig 3.13 Calculated uncertainty in gas flowrate.
Fig 3.14 Calculated uncertainty in volumetric gas content.

Fig 3.15 Calculated uncertainty in separated two phase density.
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Fig 3.16 Calculated uncertainty in mixture quality.

Fig 3.17 Calculated uncertainty in total two phase head.
Fig 3.18 Calculated uncertainty in total two phase head for various volumetric gas contents.
<table>
<thead>
<tr>
<th>1. RESEARCH PROPOSAL &amp; PROJECT DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. INITIAL TESTINGS</td>
</tr>
<tr>
<td>3. LITERATURE SURVEY</td>
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<tr>
<td>4. THEORETICAL MODELS</td>
</tr>
<tr>
<td>5. DETAILED EXPERIMENTATION</td>
</tr>
<tr>
<td>6. DATA SELECTION</td>
</tr>
<tr>
<td>7. REPORT (WRITING UP)</td>
</tr>
<tr>
<td>8. THESIS SUBMISSION</td>
</tr>
</tbody>
</table>

Fig 3.19 The plan of the research.
CHAPTER FOUR

DISCUSSION
CHAPTER FOUR

DISCUSSION

In the previous chapters a literature review on the two phase flow and cavitation as well as the description of the work done during the investigation was presented. Here a discussion on both theoretical and experimental investigations performed during the research will be given. Suggestions for further work is also included where necessary.

The excellence of the agreement so far obtained between the analytical model and experimental results give confidence that the present investigation will contribute in the two phase pumping literature. It is believed that the theoretical method developed appears to be the most rigorous and versatile one ever developed and that in the near future it will be possible to predict the two phase pump characteristics for any centrifugal pump.

4.1 ANALYTICAL MODEL

There are two principal methods of flow analysis in turbomachinery, the streamline curvature method and the matrix throughflow method. Both methods rely on certain assumptions of flow pattern and conditions (e.g. axysymmetry of flow through the machine and non-detachment of the boundary layer). However, the separation of the flow in two phase
centrifugal pumps, the separation of the phases and different flow regimes at different locations in the impeller passages make the application of the analytical methods more complicated and in some cases, controversial.

The new analytical model for two phase centrifugal pump, which has been developed in Section 3.1, uses the streamline curvature method taking into account the separation of the phases and compressibility and condensation of the gas phase. This model is a further step towards a complete theoretical work of two phase pumping. It is believed that the method is the most rigorous and versatile one ever developed to date in this field. Furthermore, the final equation obtained from this new analytical model conveniently reveals several important features for understanding the basic mechanism of two phase flow through a centrifugal pump. These features also serve as pertinent information concerning what type of new two phase flow pump tests are to be conducted in the future.

The main purpose of the investigation was to model analytically the two phase flow performance of centrifugal pumps and to use experimental data to validate and refine the analytic modelling effort. Although this has not been achieved in full, the excellence of the agreement so far obtained with the mathematical model gives confidence that it will be possible in the near future to predict the two phase head-
capacity curve for any pump of conventional design, for any specified value of volumetric gas content with far greater accuracy and confidence than has been possible before.

The two assumptions used in the derivation of the mathematical model were that the trajectory of the liquid was identical to that of the gas in order to be able to use a control volume bounded by two streamlines and the gas phase was treated as a perfect gas undergoing an isothermal process. The isothermal process was assumed because the liquid serves as a cooling medium for the gas phase and thus the heat transfer between the phases was not negligible. Because the compression of the gas phase in the pump is similar to the compression in a blower, an isothermic change of state can be assumed.

The final equation obtained from the suggested mathematical model was given in Section 3.1 as:

\[
\text{Euler's Two Phase Head} = \text{Total Two Phase Head} + \text{Liquid Kinetic Energy Head} + \text{Gash Kinetic Energy Head} + \text{Compression Head Loss} \tag{3.78}
\]

where the Euler's two phase head can be expressed as:

\[
H_{\text{TP}} = \frac{1}{g} \left[ (1-x)u_2 V_{u2L} + xu_2 V_{u2g} \right] - \left[ (1-x)u_1 V_{u1L} + xu_1 V_{u1g} \right] \tag{3.31}
\]

the total two phase pump head may be stated as:

\[
H_{\text{TP}} = \frac{P_x}{\rho_{z\text{TP}} g} - \frac{P_1}{\rho_{z\text{TP}} g} \tag{3.77a}
\]

the liquid kinetic energy head as:
the gas kinetic energy head as:

\[ H_{g} = x \frac{V_{2g}^2 - V_{1g}^2}{2g} \]  

(3.77c)

and the compression head loss as:

\[ H_{c} = x \frac{RT}{g} \log \frac{P_2}{P_1} \]  

(3.77d)

Consideration of the above equations suggests a close similarity to the equivalent single phase equations simply by putting \( x_1 = x_2 = 0 \) and \( p_1 = p_2 = p \). Then equation (3.31) gives the Eulers single phase head, equation (3.77a) the total single phase pump head and equation (3.77b) the single phase kinetic energy head. Equations (3.77c) and (3.77d) become zero indicating single phase flow conditions through the pump passages.

Fig 4.1 is representative of typical values of the parameters involved in the derived equation as fractions of Eulers head. These values were calculated from the experimental data obtained during tests. The total two phase pump head degrades dramatically at a volumetric gas content, \( q \), around 12 percent and remains almost constant at low values for higher \( q \) values, whereas liquid kinetic energy head remains relatively constant in all the \( q \) range. Gas kinetic energy head increases as \( q \) increases but this increase is
negligible. However, compression head loss values increases with higher rate indicating the significance of the compressibility effect of the gas phase.

For the calculation of the parameters involved in the suggested model, a set of equations were derived in Section 3.1.3 taking into account separation of the phases. These equations make use of the superficial phase velocities, where "superficial velocity" is the velocity the phase would have if it flowed alone. These are the developments of the homogeneous set of equations suggested at the early stages of the investigation, [74], as given in Appendix B. The comparison of these two theories is given in Fig 4.2 indicating that at low flowrates the separated model gives lower head values and at higher flowrates, near the designed values, the two models give the same values. At these flowrates the homogenization of the phases takes place due to the mixing effect of the impeller.

As mentioned earlier, originally the research was to analytically model the two phase flow performance of a centrifugal pump. However, this has not been possible, mainly due to the time constraints, but the excellence of the agreement so far obtained with the mathematical model, indicates that with further investigation it will be possible to predict the two phase head-capacity curve for any given
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Calculations showed that the derived mathematical model can predict the two phase performance of centrifugal pump as indicated in Fig 4.3. Eulers two phase head was calculated from the derived equation (3.31) and plotted against flowrate. Then, the kinetic energy heads of both phases and compression head loss values were subtracted from the Eulers curve as equation (3.78) suggests. The star points around the experimental total two phase head curve indicate the validity of equation (3.78) and suggest that it is possible to predict the total two phase head of any pump.

However the main problem that remains to be solved is the prediction of the discharge pressure. Discharge pressure is necessary for the calculation of the gas conditions at the outlet of the pump. If this can be verified then the Euler, liquid and gas kinetic energy heads as well as the compression head loss can be determined leading to the prediction of the total two phase head. A relation may be obtained by careful consideration of equations (3.79) to (3.114) and by using trial and error techniques then the discharge pressure may be evaluated.

In two phase flows it is not easy to define a universally acceptable definition for the pump total head, because each
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phase is associated with its own velocity and density. Thus, a physically consistent, universally acceptable definition of head is needed to reduce the test data properly. The formulation of the total head across the pump working with two phase flow has been attempted more or less differently in several reports as discussed in CHAPTER TWO. Winks and Parks, [18], in order to reduce their test data, used the following equation:

\[ H_{TP} = \frac{P_2 - P_1}{\rho_{pump}g} \]  

(4.1)

and they defined the \( \rho_{pump} \) term as the average homogeneous density of the mixture:

\[ \rho_{pump} = \rho_\ell - \frac{q_1}{2}(\rho_\ell - \rho_{1e}) - \frac{q_2}{2}(\rho_\ell - \rho_{2e}) \]  

(4.2)

Kosmowski, [23,24], suggested an equation taking into account compressibility effects of the gas phase by:

\[ H_{TP} = \frac{(1-q_1)(P_2 - P_1) + q_1P_1ln(P_2/P_1)}{q[(1-q_1)\rho_\ell RT_e + q_1P_1]} \]  

(2.5)

and finally, Furuya, [12], in his paper gave the following equation:

\[ H_{TP} = \frac{P_2 - P_1}{\rho_{tp}g} \]  

(2.6)

with the homogeneous two phase density determined from:

\[ \rho_{tp} = (1-q)\rho_\ell + q\rho_g \]  

(4.3)

It should be mentioned here that the pressures indicated in
the above equations are static and the dynamic term should be added in order to get the total pressures. For single phase conditions this term can be eliminated for equal area suction and discharge pipes. In two phase flows this is not the case due to the pressure variation across the pump. The flowrate at the pump inlet is different from the outlet leading to unequal velocities at the two tapping planes. However, the dynamic term is much smaller than the static pressure term differences and can be neglected.

Fig 4.4 is the comparison of the above equations with the derived equation (3.77a). It can be seen that Kosmowski's equation is overestimating the two phase head leading to higher value for two phase head in all the range of flowrates. On that equation, (although the derivation is not included in his paper), the compression head of the mixture was taken into account, which is the second term of the numerator. However, compression head is not a useful head but a head loss and should not be added to the total head of the pump at any time.

Winks and Parks and Furuya's equations give head values close to the current theory. These values are slightly greater and this can be explained by the two phase density variation through the pump. In single phase the density of the liquid is relatively constant during the pumping action, whereas in two phase flow varies according to the system pressure. Fig 4.5
show this density variation indicating that at the outlet section the density decreases at a higher rate and around 50 percent of volumetric gas content the outlet density of the mixture becomes almost half of the inlet. The fact that both of the previous equations use an average homogeneous density through the pump gives rise to total head. However equation (3.77a) takes into account density variation and phase separation indicating an acceptable definition for two phase total pump head, established by application of the momentum theory. It is believed that this equation is a better two phase total pump head definition and will be useful for the proper reduction of the test data.

In the theoretical model account was taken for condensation, compressibility and phase separation effects. For small gas contents these losses seems to be dominant in pump head degradation. However additional losses may take place that may be indispensable for higher volumetric gas contents, leading to the break of the flow, as it will be seen later in Section 4.3. Thus these possibilities should also be examined in order to have a better understanding of the two phase flow during pumping.

It is known from single phase flow through an impeller that the steep pressure gradient at the low suction side of the blades leads to the separation of the streamlines and the
formation of a turbulent wake. In a centrifugal pump this separation occurs mainly at the impeller discharge, universally known as slip. The separation leads to a deflection of the flow and a reduction in the outlet fluid angle $\beta_z$. A number of theoretical approaches have been made to the "correction" of this angle to give an actual change in the fluid velocity. However when it comes to two phase flow through a centrifugal pump the phases leave the impeller with different absolute velocities, leading to a deflection of the flow and a reduction of the active outlet area, which is much more pronounced than in the case of pure liquid pumping operation. Gas moving with velocities much lower than liquid is trapped in the separation zone and the developing gas concentration in the separating area can lead to a space containing only pure gas as Fig 4.6 suggest. An important question is whether the theoretical approaches developed for single phase are valid in two phase flow conditions. It is obvious that for the separated model derived these approaches could not considered as valid due to the velocity difference between the phases. Therefore the slip concept should be accounted for in equation (3.78) by replacing the phase outlet angles by $\beta'_{zL}$ and $\beta'_{zG}$ instead of the blade angle $\beta_z$ as Fig 4.7 suggests.

As a gas bubble enters the impeller the centrifugal effect tends to separate it from the liquid and to keep it at the
suction side of the blades. However the flow of the liquid has a drag effect, Fig 4.8, that tends to carry the bubble through the impeller. Normally the drag effect will carry along a reasonable amount of gas. But when the gas content becomes large or the liquid flow is small the drag effect of the liquid in turn becomes small indicating that gas accumulation prevents the passage of any more liquid and as a result the reduction in liquid velocity.

This drag effect, although, at low volumetric gas contents seems to assist the flow, causes a reduction in liquid velocity, and thus a reduction in pump head. These drag losses were roughly examined in the model derived by taking into account the phase slip. They should be examined in greater detail combined with the gas concentration areas in the impeller in order to get a better view of gas-liquid interaction in the passages.

Friction losses are also very important in pumping. However in single phase flow a completely acceptable definition for these losses has not been reached and all the relationships developed are dependent on dimensional analysis. Lockhart and Martinelli successfully correlated the pressure drop in two phase flow through pipes. Therefore it might be feasible to correlate these losses using the Lockhart and Martinelli method but caution must be exercised since in two phase
pumping the reverse phenomena occurs (i.e. gas velocity is lower than liquid in pumping whereas gas velocity is greater than liquid in pipes).

4.1.1 SUMMARY.

The derived analytical method proved to be successful and the excellence of the agreement obtained so far, indicates that it will be possible to predict the two phase characteristic curve for any centrifugal pump in the near future. Comparison of the equation (3.77a) with previous relevant equations indicated the validity of the equation as an acceptable definition for the two phase pump head. Recommendations for further investigations were also given including flow separation, drag and friction losses.
4.2 EXPERIMENTAL INVESTIGATION

Systematic tests, [75], have been carried out on a conventional centrifugal pump using four different impeller designs. The scope of the tests was mainly to identify the pump design parameters with most remarkable impact on two phase flow and cavitation performance. In the following pages the single phase, two phase and cavitation characteristics of the impellers tested will be discussed in detail.

4.2.1 SINGLE PHASE PERFORMANCE CHARACTERISTICS

The four impellers' design parameters used in the experimental investigations were discussed in Section 3.2. These being the original 1 DMS, HMD design and three of straight bladed design with outlet blade angles of 40°, 60° and 90° degrees. The complete single phase performance characteristics for all the impellers are exhibited in Fig 4.9 to 4.11.

Fig 4.9 is the graphical presentation of the head characteristics for the four impellers. Also the modified 1 DMS without the front shroud is included for comparison. The three straight bladed impellers give higher head values than the original one. But high velocities within the passages take place leading to higher losses. The modified impeller (no shroud) indicates almost half head of the original one and for an increase in flowrate there is high reduction in head.
Power curves, indicated in Fig 4.10, suggest that the modified impeller gives the smaller power dissipation. On the other hand the three straight bladed impellers show high power dissipation values which may lead to lower efficiency. Fig 4.11 exhibits the efficiency curves and indicates that the 1DMS is the most efficient with an efficiency at design flowrate (0.00444 m³/s) around 55 percent. It must be added that the 90° straight bladed impeller shows a sudden reduction in the performance values for a flowrate around 0.0045 m³/s indicating partial cavitation.

4.2.2 TWO PHASE PERFORMANCE CHARACTERISTICS.

Presentation of the two phase characteristic values are given in terms of homologous parameters as discussed in Section 3.2. Unfortunately in two phase pumping it is not possible, yet, to define the hydraulic power and thus efficiency parameters, due to the variation of the two phase density in the system. Therefore, the selected parameters are head, torque and flowrate. Two phase head, as mentioned earlier, was calculated using the derived equation (3.77a).

The four impellers were tested in detail for two phase flow pumping conditions. The selection of a large number of test conditions was considered necessary to accurately define the expected variation in single phase performance characteristic curves as volumetric gas content increases. Maximum volumetric
gas content before break of the flow appeared to be around 9 percent for all impellers. Volumetric gas contents up to 50 percent were achieved during tests with suction pressure of 1.5 bar.

For each of the four impellers, data points with volumetric gas contents to the system of:

(a) 1 to 3 percent
(b) 4 to 6 percent
(c) 7 to 9 percent

were segregated into groups. Fig 4.12 to 4.19 indicate the two phase characteristics of the impellers. Although the 90° straight bladed impeller tested in detail unfortunately the calculation of the two phase head was not possible. The derived equation (3.77a) takes into account the geometry of the impeller and thus the use of the cosine and tangent of the blade angles leads to no solution. This can be considered as a weak point to the analytical model. This problem is not significant since radial bladed impellers are not commonly used. If calculation of the two phase head for a radial impeller is necessary, then it can be determined from the homogeneous model given in Appendix B. For design conditions this model gives values similar to the separated model as discussed earlier in Section 4.1.

Fig 4.12 and 4.13 are the graphical presentations of head
and torque degradation for the 1 DMS impeller. It can be noticed that the deterioration of head is already remarkable at volumetric gas content of 1 to 3 percent and it is much more pronounced at off design conditions especially at part flows indicating an unstable head-capacity curve. Unstable head-capacity curve is defined as one where a range of flowrate occurs in which the head increases with increasing flowrate. With an unstable curve, two or more values of flowrate can be associated with a single value of head. This must be taken into consideration when designing a two phase pump in order to avoid variations of the flowrate and unpleasant surges.

Fig 4.14 to 4.17 exhibit the two phase characteristics of the two straight bladed impellers. Here, the same head degradation phenomena occurs as before but are more pronounced than the 1 DMS. Moreover, torque degradation curves indicate higher torque deterioration. This higher degradation may be due to the high absolute velocities developed by these impellers. High velocities lead to higher friction and drag losses. Also "sonic" blockage at the impeller outlet may occur as discussed in CHAPTER TWD. In addition to these experimental and calculation errors may have taken place as discussed in the error analysis presented in Section 3.3. The modified impeller indicated head and torque degradation in a similar manner. But for volumetric gas content of 1 to 3 percent, Fig
4.18 to 4.19, the same characteristics as with the single phase can be observed.

The effect of suction pressure on head and torque degradation versus the volumetric gas content, q, for the 1 DMS impeller was examined. Fig 4.20 to 4.27 exhibit the head degradation for suction pressures, $P_s$, of 1 bar and 1.5 bar from part flows to the design one. All the graphs indicate that as $P_s$ increases the head degradation tends to be reduced, i.e. the two phase pump delivering higher head and the head degrades at higher volumetric gas contents. At part flowrates, Fig 4.20 and 4.21 both 1 bar and 1.5 bar $P_s$ give similar two phase head values. For higher flowrates, Fig 4.22 to 4.27, the head degradation between the two suction pressures suggests that for $P_s$ of 1.5 bar the head degradation is less than for $P_s$ of 1 bar.

This can be explained by the flow visualisations performed during tests. High Speed Video observations determined that gas is dispersed circumferentially and uniformly at the entry region of the impeller. Independent of the conditions in the pipe (e.g. bubbly flow, plug flow, wavy flow e.t.c.) bubbly flow occurs just before the blade edge. This bubbly flow shows a finer bubble structure than the bubbly flow in the pipe. An increase of the suction pressure tends to reduce the average diameter of the gas bubbles and therefore minimize their
blockage effect and the drag loss due to the internal motions of the bubbles inside the impeller blades decreases.

4.2.3 Cavitation Characteristics.

The cavitation phenomenon is one of the most important problems to be considered when designing and operating a centrifugal pump. However, the physical mechanism of cavitation remains to a great extent unexplained and only its early stages are well understood. An attempt has been made in this investigation to identify any similarities between two phase flow and cavitation. Also, examination of the behaviour of a centrifugal pump during cavitation conditions has been made using the existed facilities and impellers used for two phase flow investigations. These two aspects were considered as secondary investigations due to the time constraints. In the following pages, a description of the findings together with suggestions for further work are presented.

HSV observations showed that cavitation can briefly be considered as a two phase flow with the same flow phenomena in the impeller passages. However, the volumetric gas contents associated with cavitation are close to zero particularly at the inlet and outlet sections of the pump. It can be theorised that if the volumetric gas contents at the inlet and outlet of the impeller are known by means of measurement at prediction, then the head degradation can be calculated using the derived

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equation (3.77a) for the two phase total pump head. During cavitation the density of the fluid changes due to the presence of gas bubbles thus the above equation can give accurate results. However, further investigation is needed for the identification of the similarities between two phase and cavitation since for the later although the same flow phenomena may occur, completely different dynamic phenomena take place leading to the damage of the impeller surfaces.

The test results of performance variation due to cavitation are shown in Fig 4.28 and 4.29. Fig 4.28 shows the 90° straight bladed impeller trying to deliver too much. Total head, shaft power and efficiency were plotted against flowrate. It can be seen that as capacity increases, head deviates and drops from the standard head-capacity curve which is not attended with cavitation. And any further increase in the discharge valve opening the capacity reaches a limit that the head drops almost straight downwards. This was proved by many experiments, [47,48,49,50,56], as discussed in CHAPTER TWO. An interesting phenomenon is that the shaft power shows a slight increase near the point of head drop, but it decreases with a further drop in the head. The efficiency variation shows a similar property to that of the head.

Fig 4.29 indicates the required NPSH, NPSH\textsubscript{m}, of the four impellers at different flowrates. The NPSH\textsubscript{m} values of the Two Phase Flow and Cavitation in Centrifugal Pump
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impeller were taken from the head-NPSH curves as explained in CHAPTER TWO. The criterion where the head drops by 3 percent was used in order to determine the NPSH₄ values. On the same graph, the available NPSH, NPSH₄, from the system is plotted. These were calculated using Bernoulli's equation from the tank level to the suction tapping point including pressure losses due to bends, valves etc. The pressure losses in the booster pump were also monitored and used in the calculations. It can be seen that the 40° straight bladed impeller shows the lower NPSH₄ whereas the other three impeller NPSH₄ curves are almost of the same values. At this investigation no conclusion can be obtained due to the limitation in the geometry of the tested impellers.

A suggested cavitation experimental program will involve impellers with different diameters, angles and passage heights at the suction. Examination also of the internal recirculation at low capacities is absolutely instructive to be examined in detail by measurements of the velocities and pressures at various points just before inlet. Thus the effect of the above parameters may give a knowledge for better design.

4.2.4 SUMMARY.

The effects of volumetric gas content and suction pressure in two phase flow through a centrifugal pump were examined. For an increase of the gas content the head curve becomes
unstable and further increase around 9 percent break of the flow takes place. Also, for an increase in the suction pressure a reduction in head degradation occurs leading to higher head delivery. Cavitation investigations although not examined in detail due to time constraints, showed that in cavitation the same two phase flow phenomena occur but not the same dynamic phenomena. A suggested experimental program for cavitation tests was also given.
4.3 HIGH SPEED VIDEO

High Speed Video, HSV, recordings during tests were necessary for the development of the analytical model and for the identification of the bubbles motion into the impeller passages. A detailed analysis of these observations will be discussed in the following pages and suggestions for two phase pump design and modifications, [76], will be accounted too.

At the entry region of the impeller, corresponding to the significantly deviating densities of gas and liquid, gas bubbles are orientated in areas of relatively low pressure. HSV observations confirmed that at low gas contents the gas bubbles tend to concentrate at the impeller eye on the blade suction side.

The gas accumulation takes place adjacent to the back plate of the impeller as indicated in Fig 4.30 (lowest pressure field) and gradually as the gas content increases advances towards the passage length, width and height as in Fig 4.31. At "high" gas contents (around 9 percent) the gas occupies the space between the back plate and the shroud, as Fig 4.32 suggests, leading to passage blockage and the break in the pump flow. This is the primary reason for the break in the flow for conventional pumps operating with gas inclusions.

In addition to phase separation, flow separation, Fig 4.33, ...
at the discharge section may occur with gas concentration in the separating area. This leads to deflection of the flow and a reduction of the outlet area which is much more pronounced than in the case of pure liquid pumping operation. Corresponding to the deflection of flow and the increase of the liquid relative velocity, the direction changes due to the blades and the radial velocity is strongly influenced leading to a significant reduction of the impeller head.

Lastly, due to the rotating flow, a pressure field is generated between the impeller plates and casing walls. This separates phase of significantly different densities. Initially gas accumulations, in the case of low gas content, are concentrated at small radii. With an increase in the gas content two asymmetric gas rings, as in Fig 4.34, appear and expand in the space between the impeller plates and the casing walls. When these rings expand and reach the impeller outlet, gas accumulation at the impeller discharge is increased. This increase is due to the gas from the space between impeller plates and casing walls, (gas ring) leading to the break in the pump flow.

It was reported, [26], that "unshrouded" impellers, especially of sewage pumps with few blades and large cross sectional area (so-called single or multi-channel unchockable impellers) are capable of pumping liquids with higher content
of gas.

Hence, it can be theorised that similar effects occur as in the case of conventional impeller except for high gas contents. Although the gas occupies the space between the back plate and the blade height, the pressure field in the clearance between the front casing wall and the blades produces mixing of the phases, as Fig 4.35 indicates. Also only one asymmetric gas ring appears and expands between the impeller back plate and casing back wall. Break in the pump flow occurs when the ring reaches the impeller outlet. This is the primary reason for the break in the flow of "unshrouded" impellers.

Generally, the mechanism of the flow breakdown when pumping two phase mixtures can be summarised as:

(a) Gas accumulation at the impeller passages
(b) Flow separation
(c) Asymmetric gas ring in the impeller-casing clearances

with (a) the primary reason for the conventional impeller and the combination of (b) and (c) for the "unshrouded" impeller.

Gas accumulation starts near the eye of the impeller, at the suction side of the blade adjacent to the rear plate, as shown in Fig 4.36. On that region the lowest pressure gradient
takes place due to the sudden change in the fluid direction and the rotation of the impeller. If the rear plate could be removed, as Fig 4.37 indicates, the bubbles moving towards the low pressure region will terminate in the clearance between the impeller blades and the casing back wall. There, gas and liquid will be mixed due to the mixing action generated by the impeller blades and the back casing wall interaction.

At this stage the effect of the blade angle should also be examined. The void fraction, at any point in the impeller passages, is given according to the suggested theory as:

$$a = \frac{A_\alpha}{A_{TP}} = \frac{Q_\alpha}{W_\alpha \pi b D}$$ (3.102)

where $a$ is the void fraction, $A$ is the area of the section, $Q$ is the flowrate, $W$ is the relative velocity, $b$ is the blade height and $D$ is the diameter from the impeller centre.

It was found when calculating the experimental results, that the void fraction at the inlet and outlet of the impeller was constant for each impeller. This is due to the influenced of the relative velocity factor which in turn, depends on the geometry of the blade angle.

At angles lower than 30° degrees, calculation of the void fraction, will give a value smaller than 50% e.g. 30%. However, calculation of the $(1-a)$ given as:
\( (1-a) = \frac{Q_L}{W_L \cdot \text{rbD}} \) \hspace{1cm} (4.4)

will give exactly the same result, i.e. 30\%. Addition of these two values will give only 60\% thus an area of 40\% is missing. This may be explained with the flow separation described before. Although the available area for the mixture to flow in is 100\%, due to flow separation, which depends on the geometry of the blade angle, only 60\% of the area is available allowing 40\% of the area for separation and gas concentration.

On the other hand, at angles higher than 30\° degrees, calculation of the void fraction will give a value bigger than 50\%. Also, calculation of \((1-a)\) will also give value bigger than 50\%. These will give sum higher than 100\% which is impossible. This may be the main reason why the impellers tested with blade angles higher than 30\° degrees showed higher losses and dramatic reduction in the head-capacity curve. Since high absolute velocities occur leading to high friction and drag losses and "sonic" blockage.

At the angle of 30\° degrees both \(a\) and \((1-a)\) give the same result of 50\%. This ensures (at least theoretically) that no flow separation will occur during two-phase flow as well as minimum gas concentration at the impeller passages. Fig. 4.38 indicates the void fraction variation with respect to blade angle.
In theory the combination of the above and the removal of the back plate of the impeller, gives an attractive solution to the problem. However, the disadvantage of the above design is the weak region between the shaft connection and the impeller blades as indicated in Fig 4.39. These strength considerations must be taken into account.

Lastly, the above design may also be used during low suction applications. Cavitation damage may be eliminated, as in Fig 4.40, or even avoided since the bubbles will collapse in the clearance region between the impeller blades and the casing back wall.

Modifications of conventional centrifugal pumps for handling two phase mixtures may lead to higher volumetric gas content through the impeller passages before deprime. However this increase may be of the order of 4 to 5 percent.

Fig 4.41 shows a portion of the flow diverted from the discharge pipe into a by-pass loop and re-injected back into the suction pipe in a high velocity jet. The jet may have geometry as in Fig 4.41 but many possible geometries can be considered. However if the jet velocity is too low to produce mixing then a small pump between the discharge and suction pipes in the by-pass loop may be installed. By this arrangement a number of jets can be positioned in the suction
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Pipe thus increasing the degree of mixing. But this arrangement indicates the use of an extra power source.

In Fig 4.42 another possible modification is presented by the use of vanes at the suction pipe. Although this technique may achieve the mixing of the phases, pressure losses can lead to cavitation of the pump. This problem can be overcome by the use of a rotating propeller instead of the vanes. The propeller running at high speed will produce mixing of the phases without pressure losses. However an external power source is required to drive the propeller.

4.3.1 SUMMARY.

The analysis of the HSV recordings suggested that the mechanism of the breakdown in two phase pumping is due to gas accumulation, flow separation, and asymmetric gas ring in the impeller-casing clearances. Furthermore a suggestion for a new impeller design introduces the removal of the impeller back blade thus generating a mixing area for the phases.
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1.2. LEUR8 TWO PHASE HEAD
0.014

LIQUID KINETIC ENERGY HEAD
0.008

TOTAL TWO PHASE HEAD
0.004

GAS KINETIC ENERGY HEAD
0.002

VOLUMETRIC GAS CONTENT

Fig 4.1 Typical values of the parameters involved in the suggested theory.

Fig 4.2 Homogeneous and separated flow models comparison.
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Fig 4.3 Two phase head prediction with suggested theory.

Fig 4.4 Two phase head comparisons with the suggested two phase head equation for $0.04 < q < 0.06$. 

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Fig 4.5 Inlet and outlet two phase density variations for different volumetric gas contents.

Fig 4.6 Flow separation area with gas accumulations at the suction side of the blades.
Fig 4.7 Outlet phase angles for two phase flow through a centrifugal pump.

Fig 4.8 Gas-liquid drag effect in the impeller passages.
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Fig 4.9 Single phase head curves comparisons.

Fig 4.10 Single phase power curves comparison.
Fig 4.11 Single phase efficiency curves comparison.

Fig 4.12 1 DMS two phase head characteristics.
Fig 4.13 DMS two phase torque characteristics.

Fig 4.14 40 straight bladed two phase head characteristics.
Fig 4.15 40 straight bladed two phase torque characteristics.

Fig 4.16 60 straight bladed two phase head characteristics.

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Fig 4.17 60 straight bladed two phase torque characteristics.

Fig 4.18 1 DNS two phase head characteristics (no shroud).

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Fig 4.19 DMS two phase torque characteristics (no shroud).

Fig 4.20 DMS homologous head curve for flow parameter of 0.22
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![Graph](image)

**Fig 4.21** DMS homologous head curve for flow parameter of 0.34

![Graph](image)

**Fig 4.22** DMS homologous head curve for flow parameter of 0.45
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Fig 4.23 1 DMS homologous head curve for flow parameter of 0.56

Fig 4.24 1 DMS homologous head curve for flow parameter of 0.68
Fig 4.25 1 DMS homologous head curve for flow parameter of 0.79

Fig 4.26 1 DMS homologous head curve for flow parameter of 0.90
Fig 4.27 1 DMS homologous head curve for flow parameter of 1.00 (design conditions)

Fig 4.28 90 straight bladed cavitation characteristics.
Fig 4.29 Required NPSH Curves for the four impellers.

Fig 4.30 Gas accumulation at low gas content for a conventional centrifugal pump.
Fig 4.31 Gas accumulation at medium gas content for a conventional centrifugal pump.

Fig 4.32 Passage blockage at high gas content for a conventional centrifugal pump.
Fig 4.33 Flow separation area with gas accumulations at the suction side of the blades.

Fig 4.34 Asymmetric gas ring between the impeller plates and the casing walls.
Fig 4.35 Mixing of the phases for an "unshrouded" impeller.

Fig 4.36 Initial gas accumulation for a centrifugal impeller.
Fig 4.37 Flow pattern in an impeller with the back plate missing.

Fig 4.38 Void fraction variation with blade angle.
Fig 4.39 The disadvantage of the proposed two phase impeller.

Fig 4.40 Cavities flow pattern in an impeller with the back plate missing.
Fig 4.41 Centrifugal pump modification for handling two phase mixtures with high velocity jet.

Fig 4.42 Centrifugal pump modification for handling two phase mixtures with vanes.
CHAPTER FIVE

CONCLUSIONS
CHAPTER FIVE

CONCLUSIONS

The following general conclusions may be drawn from this theoretical and experimental investigation of two phase flow, associated with the two phase pumping and cavitation in the centrifugal pumps.

5.1 ANALYTICAL MODEL.

(a) Based on the control volume method for rotating machines a mathematical model has been determined for the performance of centrifugal pumps operating under two phase flow (gas-liquid) conditions (equation 3.78). The model combines the ideal (Euler's) two phase head of a centrifugal pump with additional losses due to the presence of the gas phase. The analytical method developed has incorporated pump geometry, separation of the phases, condensation and compressibility effects of the gas phase. The excellence of the agreement so far obtained with the mathematical model gives confidence that it will be possible in the near future to predict the two phase characteristic head-capacity curve for any centrifugal pump.

(b) A physically consistent definition of two phase total head has been developed, (equation 3.77a). It is believed that this equation will be useful for the proper reduction of test data since it has incorporated density variation and phase
separation of the two phase mixtures.

(c) Additional losses may take place leading to the blockage of the passage and break of the flow. Possible additional losses are flow separation losses, drag losses and friction losses. These possibilities should be examined for better understanding of two phase pumping.

5.2 EXPERIMENTAL INVESTIGATION.

(a) When handling two phase mixtures head degradation takes place leading to an unstable head-capacity curve.

(b) The maximum volumetric gas content before break of the flow appeared to be around 9 percent for the four impellers tested.

(c) Impeller 1 DMS appeared to be the most "efficient" with low head degradation values.

(d) An increase of the suction pressure tends to reduce the average diameter of the gas bubbles and therefore minimize their blockage effect. Hence a reduction in head degradation is expected leading to higher delivery head.

(e) Cavitation can be considered as a two phase flow with the same flow phenomena take place but different dynamic phenomena. However the volumetric gas contents associated with cavitation are close to zero.
5.3 HIGH SPEED VIDEO.

(a) At low gas contents the gas bubbles tend to concentrate at the impeller eye on the blade suction side, adjacent to the back plate of the impeller. At medium gas contents gas accumulations advance towards the passage length, width and height of the impeller. At "high" gas contents (around 9 percent by volume) passage blockage and break in the pump flow occurs.

(b) The three basic mechanisms leading to flow breakdown when pumping two phase mixtures are gas accumulation, flow separation and an asymmetric gas ring in the impeller-casing clearances.

(c) The removal of the back plate of the impeller gives an attractive solution to two phase pumping. Modifications of conventional centrifugal pumps for handling two phase mixtures will result in a small increase of the order of 4-5 percent of volumetric gas content.
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Equations Used and the Computer Program.

The set of equations that transformed measured test loop parameters into consistent pump performance values and the database program developed to calculate these values are presented in this APPENDIX.

A computer database program has been developed in QBASIC, called CENTRIF for data calculation. When running the program calls the following subprograms to be set up:

- Introduction
- DisplayHeader
- GetMenu
- SinglePhase
- Cavitation

CENTRIF can be used for calculations for any centrifugal pump. This because in all sets of calculations the design parameters of the impeller should be input by the user. All units are in S.I. and appear at the end of each entry or result, except the dynamometer reading variable which is in lb (this is because of the dynamometer scale used during tests). An option at the end of each set of calculations allows the user to print both input data and results.
A.1 Single Phase Calculations

The SinglePhase subprogram, as the name implies, performs calculations with the universally known equations for the single phase performance for a given centrifugal pump. The subprogram is organised as follows:

- Input information:
  
Pump design parameters
  Experimental measurements

- Calculations:
  
Fluid absolute angle at inlet, \( \varepsilon_1 \), using equation . . . (3.21)
Fluid absolute velocity at outlet, \( \varepsilon_2 \), using equation . (3.22)
Absolute velocity at inlet, \( V_1 \), using equation . . . (3.16a)
Absolute velocity at outlet, \( V_2 \), using equation . . . (3.16b)
Eulers single phase head, \( H_{\text{ESP}} \), using equation . . . . (3.6)
Total single phase head, \( H_{\text{sp}} \), using equation . . . . (3.10)
Kinetic energy single phase head, \( H_{\text{vsP}} \), using equation (3.14)
Impeller single phase head, \( H_{\text{ISP}} \), using equation . . . (3.13)
Torque, \( B \), using equation mgl . . . . . . . . . (A.1)
Input power, \( P_{IN} \), using equation \( 2\pi NB/60 \) . . . . . . . (A.2)
Hydraulic power, \( P_H \), using equation \( p g h_{\text{ESP}} \) . . . . . . . (A.3)
Efficiency, \( \eta \), using equation, \( P_H/P_{IN} \) . . . . . . . (A.4)
Homologous flow parameter, \( Q_H \), using equation . . . . . (3.117a)
Homologous head parameter, \( H_H \), using equation . . . (3.117b)
Homologous torque parameter, \( B_H \), using equation . . . (3.117c)

A.2 Two Phase Calculations.

The TwoPhase subprogram calculates the two phase head and torque of a given centrifugal pump using the equations derived in Section 3.1. The subprogram is organized as follows:

- Input information:
  
Pump design single phase parameters
  Experimental measurements

- Calculations:
  
Inlet flowrate to the system, using equation . . . . . (3.170)
Gas flowrate at inlet, \( Q_{1G} \), using equation . . . . (3.84)
Gas flowrate at outlet, \( Q_{2G} \), using equation . . . . (3.85a)
A.3 Cavitation Calculations.

The Cavitation subprogram calculates the performance of a centrifugal pump during low suction conditions. The equations used for such calculations are the universally accepted. The subprogram is organized as follows:

- **Input information:**
  - Pump design parameters
  - Experimental measurements

- **Calculations:**
  - Total two phase flow at inlet, $Q_{1TP}$, using equation.
  - Total two phase flow at outlet, $Q_{2TP}$, using equation.
  - Liquid absolute angle at inlet, $\epsilon_{1L}$, using equation.
  - Liquid absolute angle at outlet, $\epsilon_{2L}$, using equation.
  - Gas absolute angle at inlet, $\epsilon_{1G}$, using equation.
  - Gas absolute angle at outlet, $\epsilon_{2G}$, using equation.
  - Liquid absolute velocity at inlet, $V_{1L}$, using equation.
  - Liquid absolute velocity at outlet, $V_{2L}$, using equation.
  - Gas absolute velocity at inlet, $V_{1G}$, using equation.
  - Gas absolute velocity at outlet, $V_{2G}$, using equation.
  - Liquid relative velocity at inlet, $W_{1L}$, using equation.
  - Liquid relative velocity at outlet, $W_{2L}$, using equation.
  - Gas relative velocity at inlet, $W_{1G}$, using equation.
  - Gas relative velocity at outlet, $W_{2G}$, using equation.
  - Velocity ratio at inlet, $\zeta_1$, using equation.
  - Velocity ratio at outlet, $\zeta_2$, using equation.
  - Slip at inlet, $\xi_1$, using equation.
  - Slip at outlet, $\xi_2$, using equation.
  - Void fraction at inlet, $\alpha_1$, using equation.
  - Void fraction at outlet, $\alpha_2$, using equation.
  - Density at inlet, $\rho_{1TP}$, using equation.
  - Density at outlet, $\rho_{2TP}$, using equation.
  - Quality at inlet, $x_1$, using equation.
  - Quality at outlet, $x_2$, using equation.
  - Eulers two phase head, $H_{ETP}$, using equation.
  - Total two phase head, $H_{TP}$, using equation.
  - Liquid kinetic energy head, $H_{KL}$, using equation.
  - Gas kinetic energy head, $H_{KG}$, using equation.
  - Compression head loss, $H_C$, using equation.
  - Torque, $B$, using equation.
  - Volumetric content to the system, $q$, using equation.
  - Homologous flow parameter, $Q_h$, using equation.
  - Homologous head parameter, $H_h$, using equation.
  - Homologous torque parameter, $B_h$, using equation.
Net positive suction head, NPSH, using equation ........ (2.9)
Head, H, using equation ........................................ (3.10)
Input power, $P_{in}$, using equation ........................... (A.2)
Hydraulic power, $P_{H}$, using equation ........................ (A.3)
Cavitation number, $a$, using equation NPSH/H' ............. (A.5)
Efficiency, $\eta$, using equation ................................. (A.4)

A.4 Printout of CENTRIF.

The following pages show the printout of CENTRIF developed to calculate all the test data.

LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY
MECHANICAL ENGINEERING DEPARTMENT

CENTRIFUGAL PUMP CALCULATIONS
-----------------------------

CENTRIF is a QBASIC, Database Program created to perform calculations for Single Phase, Two Phase and Low Suction Conditions through a Centrifugal Pump.

To RUN this program, press SHIFT+F5
To EXIT, press ALT, F, X

Programmer: A. POULLIKKAS
RESEARCH STUDENT

Date: Feb 1992

DECLARE SUB Introduction ()
DECLARE SUB DisplayHeader ()
DECLARE SUB GetMenu (choice%)
DECLARE SUB SinglePhase ()
DECLARE SUB TwoPhase ()
DECLARE SUB Cavitation ()

Introduction
DisplayHeader
DO
    GetMenu choice%
SELECT CASE choice%
    CASE 1
        LOCATE 3, 40: PRINT "SINGLE PHASE"
        SinglePhase
    CASE 2
        LOCATE 3, 40: PRINT "TWO PHASE"

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SUB DisplayHeader
' The DisplayHeader Subprogram displays the status information on the first three lines of the screen and the two dividing lines that set off program information window.
CLS
COLOR 13, 1
PRINT " CENTRIFUGAL PUMP CALCULATIONS "
COLOR 7
PRINT ""
PRINT "File: " ; " Mode: " ; " Time: "
PRINT STRING$(80, "-");
LOCATE 24, 1: PRINT STRING$(80, "-");
END SUB

SUB GetMenu (choice%)
' The GetMenu Subprogram gets a menu choice from the user and returns it to the main program in the choice% variable.
' Initialize choice% to zero
choice% = 0
VIEW PRINT
LOCATE 3, 11: PRINT "CENTRIF.BAS"
LOCATE 3, 40: PRINT "MENU "
LOCATE 3, 73: PRINT LEFT$(TIME$, 5)
COLOR 14
LOCATE 25, 1: PRINT "Type a number between 1 and 4 and press ENTER.....";
COLOR 7
VIEW PRINT 5 TO 23
CLS 2
PRINT
PRINT
COLOR 12
PRINT " SELECT AN OPTION"
COLOR 7
PRINT
PRINT " 1. Single Phase Head Calculations"
PRINT
PRINT " 2. Two Phase Head Calculations"
PRINT
PRINT " 3. Cavitation Calculations"
PRINT
PRINT " 4. QUIT CENTRIF.BAS"
PRINT
DO WHILE (choice% < 1) OR (choice% > 4)
COLOR 14
PRINT
INPUT " Please enter choice (1 - 4) : ", choice%
LOOP
COLOR 7
CLS 2
VIEW PRINT
END SUB

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SUB SinglePhase
' The SinglePhase Subprogram performs calculations for the
'Total, Impeller and Eulers Head of a centrifugal pump during
'single phase flow.
COLOR 13
VIEW PRINT 5 TO 23
LOCATE 7, 25: PRINT "SINGLE PHASE HEAD CALCULATIONS"
LOCATE , 29: PRINT "---------------------------------
COLOR 14
LOCATE 13, 10:
PRINT "An introduction for the SINGLE CALCULATIONS is
available."
LOCATE 25, 1: PRINT "Insert your choice..."
LOCATE 15, 10: INPUT "Would you like to view the introduction
(Y/N) :", ans$
IF ans$ = "Y" OR ans$ = "y" THEN
CLS 2
COLOR 13
LOCATE 7, 10: PRINT "SINGLE PHASE HEAD CALCULATIONS"
LOCATE , 18: PRINT "---------------------------------
COLOR 14
PRINT
PRINT
LOCATE , 6: PRINT "% The calculations are carried out with
the known equations"
LOCATE , 6: PRINT " for a Single Phase flow through a
centrifugal pump."
PRINT
LOCATE , 6: PRINT "% All units are in S.I. and appears at
the end of each entry."
PRINT
LOCATE , 6: PRINT "% At the end of each set of calculations
an option is available"
LOCATE , 6: PRINT " for the results to be printed."
LOCATE 25, 1: INPUT "Press ENTER to view next screen...",
dummy$
CLS 2
COLOR 13
LOCATE 6, 6: PRINT " Calculations available are for:"
PRINT
COLOR 14
LOCATE , 6: PRINT " 1. Fluid absolute angle at inlet"
LOCATE , 6: PRINT " 2. Fluid absolute angle at outlet"
LOCATE , 6: PRINT " 3. Absolute velocity at inlet"
LOCATE , 6: PRINT " 4. Absolute velocity at outlet"
LOCATE , 6: PRINT " 5. Eulers single phase head"
LOCATE , 6: PRINT " 6. Total single phase head"
LOCATE , 6: PRINT " 7. Velocity single phase head"
LOCATE , 6: PRINT " 8. Impeller single phase head"
LOCATE , 6: PRINT " 9. Torque"
LOCATE , 6: PRINT "10. Input power"
LOCATE , 6: PRINT "11. Hydraulic power"
LOCATE , 6: PRINT "12. Efficiency"

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PRINT
INPUT " Speed = ", N!
LOCATE 9, 45: PRINT "r.p.m"
INPUT " Flowrate = ", q!
LOCATE 10, 45: PRINT "cubic meters/second"
INPUT " Liquid density = ", Den!
LOCATE 11, 45: PRINT "Kg/cubic meter"
INPUT " Suction pressure = ", P1!
LOCATE 12, 45: PRINT "bar"
INPUT " Discharge pressure = ", P2!
LOCATE 13, 45: PRINT "meters"
INPUT " Dynamometer arm length = ", x!
LOCATE 14, 45: PRINT "meters"
INPUT " Dynamometer reading = ", m!
LOCATE 15, 45: PRINT "lb"
COLOR 14
LOCATE 25, 1: INPUT "Press ENTER to perform calculations......
       dummy$
COLOR 7
U1! = (PI! * N! * D1!) / 60
U2! = (PI! * N! * D2!) / 60
RBeta! = (Beta! * PI!) / 180
RBeta2! = (Beta2! * PI!) / 180
REpsilo1! = ATN(q! * TAN(RBeta!)) / (U1! * PI! * b1! * D1! * TAN(RBeta!) - q!))
REpsilo2! = ATN(q! * TAN(RBeta2!)) / (U2! * PI! * b2! * D2! * TAN(RBeta2!) - q!))
Epsilo1! = (180 * REpsilo1!) / PI!
Epsilo2! = (180 * REpsilo2!) / PI!
V1! = ABS(q! / (PI! * b1! * D1! * SIN(REpsilo1!)))
V2! = ABS(q! / (PI! * b2! * D2! * SIN(REpsilo2!)))
HESP! = (U2! * V2! * COS(REpsilo2!) - U1! * V1! * COS(REpsilo1!)) / G!
HSP! = ((P2! * 100000!) - (P1! * 100000!)) / (Den! * G!)
HVSP! = ((V2! ^ 2 - V1! ^ 2) / (2 * G!))
HISP! = HSP! + HVSP!
Torque! = G! * m! * x! * .45359237#
PowerIn! = (2 * PI! * Torque! * .001 * N!) / 60
PowerH! = Den! * G! * q! * HSP! * .001
EFFIC! = (PowerH! / PowerIn!) * 100
HQ! = ((q! / DQ!) / (N! / DN!))
HH! = ((HSP! / DH!) / (N! / DN!) ^ 2)
HTorque! = ((Torque! / DTorque) / (N! / DN!) ^ 2)
CLS 2
PRINT
COLOR 12
PRINT " Results :"
COLOR 7
PRINT
PRINT "Fluid absolute angle at inlet="; Epsilo1!, "degrees"
PRINT "Fluid absolute angle at ="; Epsilo2!, "degrees"
PRINT "Absolute velocity at inlet ="; V1!, "meters/second"
PRINT "Absolute velocity at outlet ="; V2!, "meters/second"
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PRINT "Eulers single phase head ="; HESP!, "meters of liquid"
PRINT "Total single phase head ="; HSP!, "meters of liquid"
PRINT "Velocity single phase head="; HVSP!, "meters of liquid"
PRINT "Impeller single phase head="; HISP!, "meters of liquid"
PRINT "Torque ="; Torque!, "Newton.meters"
PRINT "Input power ="; PowerIn!, "Kilowatts"
PRINT "Hydraulic power ="; PowerH!, "Kilowatts"
PRINT "Efficiency ="; EFFIC!, "X"
PRINT "Homologous flow parameter ="; HQ!
PRINT "Homologous head parameter ="; HH!.
PRINT "Homologous torque parameter="; HTorque!
COLOR 14
LOCATE 25, 1: INPUT "Would you like to print your results ? (Y/N) ;", ans$
COLOR 7
IF ans$ = "Y" OR ans$ = "y" THEN
LPINT
LPINT
LPINT
LPINT
LPINT "CENTRIF.BAS SINGLE PHASE CALCULATIONS"
LPINT "---------------------------------------------"
LPINT
LPINT
LPINT
LPINT
LPINT
LPINT
LPINT
LPINT
LPINT "Test Number = "; TNZ
LPINT
LPINT
LPINT
LPINT
LPINT
LPINT
LPINT "Design parameters :"
LPINT
LPINT "Design Speed ="; DN!; "r.p.m."
LPINT "Design Flowrate ="; DO!; "cubic meters/second"
LPINT "Design Head ="; DH!; "meters"
LPINT "Design Torque ="; DTorque!; "Newton.meters"
LPINT "Inlet diameter ="; D1!; "meters"
LPINT "Outlet diameter ="; D2!; "meters"
LPINT "Inlet blade angle ="; Betal!; "degrees"
LPINT "Outlet blade angle ="; Betau2!; "degrees"
LPINT "Inlet passage ="; b1!; "meters"
LPINT "Outlet passage ="; b2!; "meters"
LPINT
LPINT
LPINT "Experimental data :"
LPINT
LPINT "Speed = "; N!; "r.p.m."
LPINT "Flowrate = "; q!; "cubic meters/second"
LPINT "Liquid density = "; Dsn!; "Kg/cubic meters"
LPINT "Suction pressure = "; P1!; "bar"
LPINT "Discharge pressure = "; P2!; "bar"
LPINT "Dynamometer arm length= "; x!; "meters"
LPRINT "Dynamometer reading= "; m!; "lb"
LPRINT
LPRINT "Results :"
LPRINT
LPRINT "Fluid absolute angle at inlet= "; Epsilo1!; "degrees"
LPRINT "Fluid absolute angle at outlet= "; Epsilo2!; "degrees"
LPRINT "Absolute velocity at inlet = "; V1!; "meters/second"
LPRINT "Absolute velocity at outlet = "; V2!; "meters/second"
LPRINT
LPRINT "Eulers single phase head = "; HESP!; "meters of liquid"
LPRINT "Total single phase head = "; HSP!; "meters of liquid"
LPRINT "Velocity single phase head = "; HVSP!; "meters of liquid"
LPRINT "Impeller single phase head = "; HISP!; "meters of liquid"
LPRINT
LPRINT "Torque = "; Torque!; "Newtons.meters"
LPRINT "Input power = "; PowerIn!; "Kilowatts"
LPRINT "Hydraulic power = "; PowerH!; "Kilowatts"
LPRINT "Efficiency = "; EFFIC!; "%"
LPRINT
LPRINT "Homologous flow parameter = "; HQ!
LPRINT "Homologous head parameter = "; HH!
LPRINT "Homologous torque parameter = "; HTorque!
LPRINT Chr$(12)
COLOR 14
LOCATE 25, 1: INPUT "Your results are being printed. Press ENTER to continue...", dummy$
COLOR 7
END IF
20 COLOR 14
LOCATE 25, 1: INPUT "Press ENTER to return to main menu ...", dummy$
COLOR 7
END SUB

SUB TwoPhase
' The TwoPhase Subprogram performs calculations for the Total,
' Impeller and Eulers Head of a centrifugal pump during two
' phase flow.
COLOR 13
VIEW PRINT 5 TO 23
LOCATE 7, 25: PRINT "TWO PHASE HEAD CALCULATIONS"
LOCATE , 25: PRINT "-----------------------------"
COLOR 14
LOCATE 13, 10: PRINT "An introduction for the TWO PHASE
               CALCULATIONS is available."
LOCATE 25, 1: PRINT " Insert your choice..."
LOCATE 15, 10: INPUT " Would you like to view the
               introduction? (Y/N) : ", ans$
IF ans$ = "Y" OR ans$ = "y" THEN
  CLS 2
  COLOR 13

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LOCATE 7, 10: PRINT "TWO PHASE HEAD CALCULATIONS"
LOCATE , 10: PRINT "-------------------------------"
COLOR 14
PRINT
LOCATE , 6: PRINT "* The calculations are carried out with
the Two Phase equations"
LOCATE , 6: PRINT " derived by the programmer."
PRINT
LOCATE , 6: PRINT "* All units are in S.I. and appears at
the end of each entry."
PRINT
LOCATE , 6: PRINT "* At the end of each set of calculations
an option is available"
LOCATE , 6: PRINT " for the results to be printed."
LOCATE 25, 1: INPUT "Press ENTER to view next screen...",
dummy$
CLS 2
COLOR 13
LOCATE 6, 6: PRINT " Calculations available are for:" PRINT
COLOR 14
PRINT " 1. Inlet flowrate to the system"
PRINT " 2. Gas flowrate at inlet"
PRINT " 3. Gas flowrate at outlet"
PRINT " 4. Total two phase flow at inlet"
PRINT " 5. Total two phase flow at outlet"
PRINT " 6. Liquid absolute angle at inlet"
PRINT " 7. Liquid absolute angle at outlet"
PRINT " 8. Gas absolute angle at inlet"
PRINT " 9. Gas absolute angle at outlet"
PRINT "10. Liquid absolute velocity at inlet"
PRINT "11. Liquid absolute velocity at outlet"
PRINT "12. Gas absolute velocity at inlet"
PRINT "13. Gas absolute velocity at outlet"
LOCATE 25, 1: INPUT " Press RETURN to view next screen..."
, dummy$
CLS 2
COLOR 13
LOCATE 6, 6: PRINT "Calculations available are for
(continue):"
PRINT
COLOR 14
PRINT " 14. Liquid relative velocity at inlet"
PRINT " 15. Liquid relative velocity at outlet"
PRINT " 16. Gas relative velocity at inlet"
PRINT " 17. Gas relative velocity at outlet"
PRINT " 18. Velocity ratio at inlet"
PRINT " 19. Velocity ratio at outlet"
PRINT " 20. Slip at inlet"
PRINT " 21. Slip at outlet"
PRINT " 22. Void fraction at inlet"
PRINT " 23. Void fraction at outlet"
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PRINT " 24. Density at inlet"
PRINT " 25. Density at outlet"
PRINT " 26. Quality at inlet"
PRINT " 27. Quality at outlet"
LOCATE 25, 1: INPUT " Press RETURN to view next screen...", dummy$
CLS 2
COLOR 13
LOCATE 6, 6: PRINT " Calculations available are for (continue):"
PRINT
COLOR 14
PRINT " 28. Eulers two phase head"
PRINT " 29. Total two phase head"
PRINT " 30. Liquid velocity head"
PRINT " 31. Gas velocity head"
PRINT " 32. Impeller two phase head"
PRINT " 33. Compression head loss"
PRINT " 34. Torque"
PRINT " 35. Volumetric content to the system"
PRINT " 36. Homologous flow parameter"
PRINT " 37. Homologous head parameter"
PRINT " 38. Homologous torque parameter"
LOCATE 29, 1: INPUT "Press ENTER to start the application...", dummy$
ELSE GOTO 1
END IF
3 CLS 2
COLOR 14
LOCATE 25, 1: PRINT " Please enter data...";
COLOR 7
VIEW PRINT 5 TO 23
CONST G! = 9.81
CONST PI! = 3.14
PRINT
PRINT INPUT " Test Number = ", TN$
PRINT INPUT " Test details = ", details$
PRINT
COLOR 12
PRINT " Design Parameters:"
COLOR 7
PRINT INPUT " Design speed = ", DN!
LOCATE 13, 45: PRINT "r.p.m."
PRINT INPUT " Design single phase flowrate = ", DQ!
LOCATE 14, 45: PRINT "cubic meters/second"
PRINT INPUT " Design single phase head = ", DHSP!
LOCATE 15, 45: PRINT "meters"
PRINT INPUT " Design single phase torque = ", DTorque!
LOCATE 16, 45: PRINT "Newtons.meters"
PRINT INPUT " Inlet diameter = ", D1!

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LOCATE 17, 45: PRINT "meters"
INPUT "Outlet diameter = ", D2!
LOCATE 18, 45: PRINT "meters"
INPUT "Inlet blade Angle = ", Beta1!
LOCATE 19, 45: PRINT "degrees"
INPUT "Outlet blade Angle = ", Beta2!
LOCATE 20, 45: PRINT "degrees"
INPUT "Inlet passage = ", b1!
LOCATE 21, 45: PRINT "meters"
INPUT "Outlet passage = ", b2!
LOCATE 22, 45: PRINT "meters"
COLOR 14
LOCATE 25, 1: INPUT "Press RETURN to enter experimental data...", dummy$
CLS 2
COLOR 7
PRINT
PRINT COLOR 12
PRINT "Experimental Data :"
COLOR 7
PRINT
INPUT "Speed = ", N!
LOCATE 9, 45: PRINT "r.p.m."
INPUT "Liquid flowrate = ", QL!
LOCATE 10, 45: PRINT "cubic meters/second"
INPUT "Liquid density = ", DenL!
LOCATE 11, 45: PRINT "Kg/cubic meters"
INPUT "Gas inlet flowrate = ", QING!
LOCATE 12, 45: PRINT "cubic meters/second"
INPUT "Gas inlet pressure = ", PING!
LOCATE 13, 45: PRINT "bar"
INPUT "Gas specific constant = ", R!
LOCATE 14, 45: PRINT "Joules/Kg.K"
INPUT "Average temperature = ", T!
LOCATE 15, 45: PRINT "Kelvin"
INPUT "Suction pressure = ", P1!
LOCATE 16, 45: PRINT "bar"
INPUT "Discharge pressure = ", P2!
LOCATE 17, 45: PRINT "bar"
INPUT "Dynamometer arm length = ", x!
LOCATE 18, 45: PRINT "meters"
INPUT "Dynamometer reading = ", m!
LOCATE 19, 45: PRINT "lb"
COLOR 14
LOCATE 25, 1: INPUT "Press RETURN to perform calculations...", dummy$
COLOR 7
QIN! = QL! + QING!
q! = QING! / QIN!
U1! = (P1! * N! * D1!) / 60
U2! = (P1! * N! * D2!) / 60
Q1G! = (PING! * QING!) / P1!

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$Q_2G! = (P! * Q!)/P!$
$Q!TP! = QL! + Q!G!$
$Q!2TP! = QL! + Q!G!$
$RBetal! = (Beta1! * PI!)/100$
$RBeta2! = (Beta2! * PI!)/100$
$REpsilo1L! = ATN((QL! * TAN(RBeta1!))/ (U1! * PI! * b1! * D1! * TAN(RBeta1!) - QL!))$
$REpsilo2L! = ATN((QL! * TAN(RBeta2!))/ (U2! * PI! * b2! * D2! * TAN(RBeta2!) - QL!))$
$REpsilo1G! = ATN((Q1G! * TAN(RBeta1!))/ (U1! * PI! * b1! * D1! * TAN(RBeta1!) - Q1G!))$
$REpsilo2G! = ATN((Q2G! * TAN(RBeta2!))/ (U2! * PI! * b2! * D2! * TAN(RBeta2!) - Q2G!))$
$Epsilo1L! = (180 * REpsilo1L!)/PI!$
$Epsilo2L! = (180 * REpsilo2L!)/PI!$
$Epsilo1G! = (180 * REpsilo1G!)/PI!$
$Epsilo2G! = (180 * REpsilo2G!)/PI!$

$V!L! = ABS(QL! / (PI! * b1! * D1! * SIN(REpsilo1L!)))$
$V!2L! = ABS(QL! / (PI! * b2! * D2! * SIN(REpsilo2L!)))$
$V!1G! = ABS(Q1G! / (PI! * b1! * D1! * SIN(REpsilo1G!)))$
$V!2G! = ABS(Q2G! / (PI! * b2! * D2! * SIN(REpsilo2G!)))$

$W!L! = SQR(U1! ^ 2 + V!L! ^ 2 - 2 * U1! * V!L! * COS(REpsilo1L!))$
$W!2L! = SQR(U2! ^ 2 + V!2L! ^ 2 - 2 * U2! * V!2L! * COS(REpsilo2L!))$
$W!1G! = SQR(U1! ^ 2 + V!1G! ^ 2 - 2 * U1! * V!1G! * COS(REpsilo1G!))$
$W!2G! = SQR(U2! ^ 2 + V!2G! ^ 2 - 2 * U2! * V!2G! * COS(REpsilo2G!))$

$Zital! = W1G! / W!L!$
$Zita2! = W2G! / W2L!$
$X!1! = W1G! - W!L!$
$X!2! = W2G! - W2L!$

$Alpha1! = Q1G! / (PI! * b1! * D1! * W1G!))$
$Alpha2! = Q2G! / (PI! * b2! * D2! * W2G!)$


$QUAl! = (P1! * 100000 * Q!G! / (R! * T! * Q!TP! * Den1TP!)$
$QUA2! = (P2! * 100000 * Q!2G! / (R! * T! * Q!2TP! * Den2TP!)$

$HETP! = ((1 - QUA1!) * (U2! * V2L! * COS(REpsilo2L!)) - U1! * VIL! * COS(REpsilo1L!) + QUA1! * (U2! * V2G! * COS(REpsilo2G!)) - U1! * V1G! * COS(REpsilo1G!)) / G!$

$HTP! = ((P2! * 100000) / (Den2TP! * G!)) - ((P1! * 100000) / (Den1TP! * G!))$

$HVL! = ((1 - QUA1!) * (V2L! ^ 2 - V1L! ^ 2)) / (2 * G!)$
$HVG! = (QUA1! * (V2G! ^ 2 - V1G! ^ 2)) / (2 * G!)$

$Torque! = G! * m! * x! * .45359237#
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HQ! = ((QIN! / DQ!) / (N! / DN!))
HQH! = ((HTP! / DHSP!) / (N! / DN!) ^ 2)
HTorque! = ((Torque! / DTorque!) / (N! / DN!) ^ 2)
CLS 2
PRINT
COLOR 12
PRINT "Results :"
COLOR 7
PRINT
PRINT "Inlet flowrate to the system = " ; QIN!, "cubic meters/second"
PRINT "Gas flowrate at inlet = " ; QIG!, "cubic meters/second"
PRINT "Gas flowrate at outlet = " ; Q2G!, "cubic meters/second"
PRINT "Total two phase flow at inlet = Q1TP!, cubic meters/second"
PRINT "Total two phase flow at outlet = Q2TP!, cubic meters/second"
PRINT "Liquid absolute angle at inlet = " ; EpsiloIL!, "degrees"
PRINT "Liquid absolute angle at outlet = " ; Epsilo2L!, "degrees"
PRINT "Gas absolute angle at inlet = " ; Epsilo1G!, "degrees"
PRINT "Gas absolute angle at outlet = " ; Epsilo2G!, "degrees"
PRINT "Liquid absolute velocity at inlet = " ; V1L!, "meters/second"
PRINT "Gas absolute velocity at inlet = " ; V1G!, "meters/second"
PRINT "Liquid absolute velocity at outlet = " ; V2L!, "meters/second"
PRINT "Gas absolute velocity at outlet = " ; V2G!, "meters/second"
COLOR 14
LOCATE 25, 1: INPUT "Press RETURN to view next screen...",
dummy$

CLS 2
PRINT
COLOR 12
PRINT "Results (continue) :"
COLOR 7
PRINT
PRINT "Liquid relative velocity at inlet = " ; W1L!, "meters/second"
PRINT "Liquid relative velocity at outlet = " ; W2L!, "meters/second"
PRINT "Gas relative velocity at inlet = " ; W1G!, "meters/second"
PRINT "Gas relative velocity at outlet = " ; W2G!, "meters/second"
PRINT "Velocity ratio at inlet = " ; Zita1!
PRINT "Velocity ratio at outlet = " ; Zita2!
PRINT "Slip at inlet = " ; Xi1!, "meters/second"
PRINT "Slip at outlet = " ; Xi2!, "meters/second"
PRINT "Void fraction at inlet = " ; Alpha1!
PRINT "Void fraction at outlet = " ; Alpha2!
PRINT "Density at inlet = " ; Den1TP!, Kg/cubic meter"
PRINT "Density at outlet = " ; Den2TP!, Kg/cubic meter"
PRINT "Quality at inlet = " ; QUA1!
PRINT "Quality at outlet = " ; QUA2!
COLOR 14
LOCATE 25, 1: INPUT "Press RETURN to view next screen...",
dummy$

CLS 2
PRINT
COLOR 12
PRINT "Results (continue) :"
COLOR 7
APPENDIX A

PRINT "Euler's two phase head = "; HETP!, "meters"
PRINT "Total two phase head = "; HTP!, "meters"
PRINT "Liquid velocity head = "; HVL!, "meters"
PRINT "Gas velocity head = "; HVG!, "meters"
PRINT "Impeller two phase head = "; HITP!, "meters"
PRINT "Compression head loss = "; Hc!, "meters"
PRINT "Torque = "; Torque!, "Newtons.meters"
PRINT "Volumetric content to the system = "; q!
PRINT "Homologous flow parameter = "; HQ!
PRINT "Homologous head parameter = "; HH!
PRINT "Homologous torque parameter = "; HTorque!
COLOR 14
LOCATE 25, 1: INPUT "Would you like to print your results? (Y/N) "; ans$
COLOR 7

IF ans$ = "Y" OR ans$ = "y" THEN
  LPRINT LPRINT LPRINT LPRINT
  LPRINT "CENTRIF.BAS TWO PHASE CALCULATIONS"
  LPRINT "-------------------------------------"
  LPRINT LPRINT LPRINT LPRINT
  LPRINT "Test Number = "; TN7.
  LPRINT "Design parameters:"
  LPRINT LPRINT LPRINT LPRINT
  LPRINT "Design speed = "; DN!; "r.p.m."
  LPRINT "Design single phase flowrate=; DQ!; "cubicmeters/second"
  LPRINT "Design single phase head = "; DHSP!; "meters"
  LPRINT "Design single phase torque=" DTorque! "Newtons.meters"
  LPRINT "Inlet diameter = "; D1!; "meters"
  LPRINT "Outlet diameter = "; D2!; "meters"
  LPRINT "Inlet blade Angle = "; Beta1!; "degrees"
  LPRINT "Outlet blade Angle = "; Beta2!; "degrees"
  LPRINT "Inlet passage = "; b1!; "meters"
  LPRINT "Outlet passage = "; b2!; "meters"
  LPRINT LPRINT LPRINT LPRINT
  LPRINT "Experimental data:"
  LPRINT LPRINT LPRINT LPRINT
  LPRINT "Speed = "; N!; "r.p.m."
  LPRINT "Liquid flowrate = "; QL!; "cubic meters/second"
  LPRINT "Liquid density = "; DenL!; "Kg/cubic meter"
  LPRINT "Gas inlet flowrate = "; QING!; "cubic meters/second"
  LPRINT "Gas inlet pressure = "; PING!; "bar"
  LPRINT "Gas specific constant= "; R!; " Joules/Kg.K"
  LPRINT "Average temperature = "; T!; " Kelvin"

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LPRINT "Suction pressure = "; P1!; " bar"
LPRINT "Discharge pressure = "; P2!; " bar"
LPRINT "Dynamometer arm length = "; x!; " meters"
LPRINT "Dynamometer reading = "; m!; " lb"
LPRINT
LPRINT "Results :"
LPRINT
LPRINT "Inlet flowrate to the system=QIN!cubic meters/second"
LPRINT "Gas flowrate at inlet=": Q1G!; " cubic meters/second"
LPRINT "Gas flowrate at outlet=": Q2G!; " cubic meters/second"
LPRINT "Total two phase flow at inlet=Q1TP!cubic meters/second"
LPRINT "Total two phase flow at outlet=Q2TP!cubic meters/second"
LPRINT "Liquid absolute angle at inlet=":Epsil01L!"degrees"
LPRINT "Liquid absolute angle at outlet=": Epsil02L!"degrees"
LPRINT "Gas absolute angle at inlet=": Epsil01G!; "degrees"
LPRINT "Gas absolute angle at outlet=": Epsil02G!; "degrees"
LPRINT "Liquid absolute velocity at inlet=V1L!meters/second"
LPRINT "Liquid absolute velocity at outlet=V2L!meters/second"
LPRINT "Gas absolute velocity at inlet=":V1G!; "meters/second"
LPRINT "Gas absolute velocity at outlet=":V2G!; "meters/second"
LPRINT "Liquid relative veloc at inlet=":W1L!meters/second"
LPRINT "Liquid relative veloc at outlet=":W2L!meters/second"
LPRINT "Gas relative veloc at inlet=":W1G!; "meters/second"
LPRINT "Gas relative veloc at outlet=":W2G!; "meters/second"
LPRINT "Velocity ratio at inlet =": Zita1!
LPRINT "Velocity ratio at outlet =": Zita2!
LPRINT "Slip at inlet =": Xi1!; "meters/second"
LPRINT "Slip at outlet =": Xi2!; "meters/second"
LPRINT CHR$(12)
LPRINT
LPRINT "(results continue...)
LPRINT
LPRINT " Test number = ";TNZ
LPRINT
LPRINT " Void fraction at inlet = "; Alphal!
LPRINT " Void fraction at outlet = "; Alpha2!
LPRINT " Density at inlet = "; Den1TP!; "Kg/cubic meter"
LPRINT " Density at outlet = "; Den2TP!; "Kg/cubic meter"
LPRINT " Quality at inlet = "; QUA1!
LPRINT " Quality at outlet = "; QUA2!
LPRINT
LPRINT "Eulers two phase head= "; HETP!; "meters of mixture"
LPRINT "Total two phase head = "; HTP!; "meters of mixture"
LPRINT "Liquid velocity head = "; HVL!; "meters of mixture"
LPRINT "Gas velocity head = "; HVG!; "meters of mixture"
LPRINT "Impeller two phase head= HITP!; "meters of mixture"
LPRINT "Compression head loss = "; Hc!; "meters of mixture"
LPRINT "Torque = "; Torque!; "Newtons.meters"

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LPRINT
LPRINT "Volumetric content to the system = q!"
LPRINT "Homologous flow parameter = HQ!"
LPRINT "Homologous head parameter = HH!"
LPRINT "Homologous torque parameter = HTorque!"
LPRINT CHR$(12)
COLOR 14
LOCATE 25, 1: INPUT "Your results are being printed. Press return to continue... ", dummy$
COLOR 7
END IF
COLOR 14
LOCATE 25, 1: INPUT " Press ENTER to return to main menu... ", dummy$
COLOR 7
END SUB

SUB Cavitation
'The Cavitation subprogram performs calculations for the NPSH, Head and Efficiency of a centrifugal pump during low suction conditions.

COLOR 13
VIEW PRINT 5 TO 23
LOCATE 7, 25: PRINT "CAVITATION CALCULATIONS"
LOCATE , 25: PRINT "---------------------"
COLOR 14
LOCATE 13, 10: PRINT "An introduction for the CAVITATION
CALCULATIONS is available."
LOCATE 25, 1: PRINT "Insert your choice..."
LOCATE 15, 10: INPUT "Would you like to view the introduction
? (Y/N) ": ans$
IF ans$ = "Y" OR ans$ = "y" THEN
CLS 2
COLOR 13
LOCATE 7, 10: PRINT "CAVITATION CALCULATIONS"
LOCATE , 10: PRINT "---------------------"
COLOR 14
PRINT
PRINT
LOCATE , 6: PRINT "$ The calculations are carried out
with the known equations"
LOCATE , 6: PRINT "for Cavitating centrifugal pump."
PRINT
LOCATE , 6: PRINT "$ All units are in S.I. and appears at
the end of each entry."
PRINT
LOCATE , 6: PRINT "$ At the end of each set of
calculations an option is available"
LOCATE , 6: PRINT "for the results to be printed."
LOCATE 25, 1: INPUT "Press ENTER to view next screen... ", dummy$
CLS 2

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COLOR 13
LOCATE 6, 6: PRINT "Calculations available are for:"
PRINT
COLOR 14
LOCATE , 6: PRINT "1. Net positive suction head"
LOCATE , 6: PRINT "2. Head"
LOCATE , 6: PRINT "3. Input power"
LOCATE , 6: PRINT "4. Hydraulic power"
LOCATE , 6: PRINT "5. Cavitation number"
LOCATE , 6: PRINT "6. Efficiency"
LOCATE 25, 1: INPUT "Press ENTER to start application..."
, dummy$
ELSE GOTO 2
END IF
2
CLS 2
COLOR 14
LOCATE 25, 1: PRINT "Please enter data...";
VIEW PRINT 5 TO 23
COLOR 7
CONST G! = 9.81
CONST PI! = 3.14
PRINT
PRINT
INPUT " Test Number = ", TN!
INPUT " Test details : ", details$
PRINT
INPUT " Speed = ", N!
LOCATE 10, 45: PRINT "r.p.m."
INPUT " Liquid flowrate = ", q!
LOCATE 11, 45: PRINT "cubic meters/second"
INPUT " Liquid temperature = ", T!
LOCATE 12, 45: PRINT "Kelvin"
INPUT " Liquid density = ", Den!
LOCATE 13, 45: PRINT "Kg/cubic meter"
INPUT " Liquid vapour pressure = ", PV!
LOCATE 14, 45: PRINT "bar"
INPUT " Suction pressure = ", P1!
LOCATE 15, 45: PRINT "bar"
INPUT " Discharge pressure = ", P2!
LOCATE 16, 45: PRINT "bar"
INPUT " Inlet diameter = ", D1!
LOCATE 17, 45: PRINT "meters"
INPUT " Dynamometer arm length = ", x!
LOCATE 18, 45: PRINT "meters"
INPUT " Dynamometer reading = ", m!
LOCATE 19, 45: PRINT "lb"
COLOR 14
LOCATE 25, 1: INPUT "Press ENTER to perform calculations...",
dummy$
COLOR 7
H! = ((P2! * 1000000!) - (P1! * 1000000!)) / (Den! * G!)
V! = (4 * q!) / (P1! * D1! ^ 2)
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NPSH! = (PI! * 1000000! - PV!) / (Den! \* G!) + (V! / (2 \* G!))

Sigma! = NPSH! / H!

PowerIn! = (2 \* PI! \* N! \* G! \* m! \* 45359237# \* .001) / 60

PowerH! = Den! \* G! \* q! \* H! \* .001

EFFIC! = (PowerH! / PowerIn!) \* 100

CLS
PRINT
PRINT
COLOR 12
PRINT "Results :"
COLOR 7
PRINT " Net positive suction head =", NPSH!, "meters"
PRINT " Head =", H!, "meters of liquid"
PRINT " Input power =", PowerIn!, "Kilowatts"
PRINT " Hydraulic power =", PowerH!, "Kilowatts"
PRINT " Cavitation number =", Sigma!
PRINT " Efficiency =", EFFIC!, "%"

COLOR 14
LOCATE 25, 1: INPUT "Would you like to print your results? (Y/N) :", ans$
COLOR 7
IF ans$ = "Y" OR ans$ = "y" THEN
  LPRINT
  LPRINT
  LPRINT
  LPRINT
  LPRINT "CENTRIF.BAS CAVITATION CALCULATIONS"
  LPRINT "-------------------------------"
  LPRINT
  LPRINT
  LPRINT
  LPRINT
  LPRINT "Test Number = "; TN!
  LPRINT "; details$"
  LPRINT
  LPRINT
  LPRINT
  LPRINT
  LPRINT " Given data :
  LPRINT "Speed ="; N!; "r.p.m."
  LPRINT "Liquid flowrate ="; q!; "cubic meters/second"
  LPRINT "Liquid temperature ="; T!; "Kelvin"
  LPRINT "Liquid density ="; Den!; "Kg/cubic meter"
  LPRINT "Liquid vapour pressure ="; PV!; "bar"
  LPRINT "Suction pressure ="; P1!; "bar"
  LPRINT "Discharge pressure ="; P2!; "bar"
  LPRINT "Inlet diameter ="; D1!; "meters"
  LPRINT "Dynamometer arm length ="; x!; "meters"
  LPRINT "Dynamometer reading ="; m!; "lb"
  LPRINT
  LPRINT

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LPRINT
LPRINT
LPRINT "Results :"
LPRINT
LPRINT "Net positive suction head ="; NPSH!; "meters"
LPRINT "Head ="; H!; "meters of liquid"
LPRINT "Input power ="; PowerIn!; "Kilowatts"
LPRINT "Hydraulic power ="; PowerH!; "Kilowatts"
LPRINT "Cavitation number ="; Sigma!
LPRINT "Efficiency ="; EFFIC!; "%"
LPRINT CHR$(12)
COLOR 14
LOCATE 25, 1: INPUT "Your results are being printed. Press RETURN to continue...", dummy$
COLOR 7
END IF
COLOR 14
LOCATE 25, 1: INPUT "Press ENTER to return to main menu...", dummy$
COLOR 7
END SUB
Homogeneous Two Phase Model.

In the homogeneous two phase model the velocities of the two phases, were treated to be equal to every point (i.e. $W_L = W_o = W$). Hence the relations leading to the calculation of void fraction, two phase density and quality are different from the separated flow model.

The key parameters for the calculations are the void fraction and two phase density. The average density in the separated two phase condition have been defined as:

$$\rho_{TP^*} = \frac{\dot{m}_{TP}}{Q_{TP}} = \frac{\dot{m}_L + \dot{m}_o}{Q_L + Q_o} \quad (3.104)$$

The above equation can also be written as:

$$(1-a) \rho_L W_L + a \rho_o W_o = \rho_{TP^*} [(1-a) W_L + a W_o] \quad (3.105)$$

For a homogeneous two phase flow the density of the combined flow is equal at every point and the relative velocities of the two fluids are equal also equal (i.e. $W_L = W_o = W$). Hence equation (3.105) under homogeneous flow conditions will give:

$$\rho_{TP} = \rho_{TP^*} = (1-a) \rho_L + a \rho_o \quad (8.1)$$

with $\rho_{TP}$ being as the homogeneous two phase flow density. Also the void fraction will become:

$$\alpha = \frac{A_o}{A_{TP}} = \frac{W A_o}{W A_{TP}} = \frac{Q_o}{Q_{TP}} \quad (8.2)$$
Rearrangement of equation (B.1) and substituting equation (B.2) assuming isothermal process yields:

\[ \rho_{TR} = \left(1 - \frac{Q_B}{Q_{TR}}\right) \rho_L + \frac{P}{RT} \frac{Q_B}{Q_{TR}} \]  

(B.3)

The density of the homogeneous two phase pseudo-fluid, therefore, depends only on the system pressure. The quality of the mixture can then be calculated with equation (3.111).