The application of particle image velocimetry to high speed flows

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THE APPLICATION OF PARTICLE IMAGE VELOCIMETRY TO HIGH SPEED FLOWS

by

Nicholas J. Lawson
B.Eng. (Hons)

A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of the Loughborough University of Technology

March 1995

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NOMENCLATURE

Chapter 2,3,4,5

L  Spatial resolution
D  Interrogation region length
M  System magnification at object plane
s_o, s_i  Object and image distances respectively
W  Light sheet width
N_v  Average number of particles in fluid/interrogation volume (fluid seeding density)
N_i  Average number of particle images/interrogation region after first exposure (particle image density)
N  Number of particle image pairs/interrogation region
N_u  Number of unpaired particle images/interrogation region
d_p  Particle diameter
x, y, z  Cartesian coordinates in interrogation volume
a  Particle position vector
x_i, y_i, z_i  Cartesian coordinates of initial particle positions in fluid volume
a_i  Particle position vector of initial particle position in fluid volume
\rho(l)  Uniform rectangular probability distribution
\alpha_i, \beta_i, \chi_i  Random independent variables from \rho(l)
t  Time
V(a, t)  Fluid velocity
u, v, w  Components of fluid velocity
V_0(a, t)  Fluid velocity in the centre of the interrogation volume
u_o, v_o, w_o  Components of fluid velocity in the centre of the interrogation volume
\Delta s  Particle displacement
\Delta t  Laser pulse separation
\Delta S  Particle image displacement
X, Y  Cartesian coordinates in transparency plane
\Delta X_o, \Delta Y_o  Particle image displacement at the centre of the interrogation region
\Delta X_i, \Delta Y_i  Particle image displacement due to velocity gradient perturbation
\Delta X_o, \Delta Y_z  Particle image displacement due out of plane velocity components
K  Transparency transmissivity constant
d  Particle image diameter
P(X, Y)  Intensity function of single particle image
I(X, Y)  Intensity distribution of transparency region
R_{II}(\xi, \eta)  Autocorrelation function
Chapter 4.5

Autocorrelation of single particle image

Autocorrelation dc peak function

Autocorrelation signal peak function

Autocorrelation noise function (paired particle images)

Autocorrelation noise function (unpaired and paired particle images)

Autocorrelation noise function (total noise)

Estimated signal peak position

Signal to noise ratio (dB)

Dynamic range

Maximum fluid velocity

Minimum fluid velocity

In plane loss of particle pairs function

Out of plane loss of particle pairs function

Velocity gradient strength

Velocity gradient components for a forced vortex flow

Maximum signal peak spread

Signal peak degradation factor

Signal peak degradation function

Baseline signal to noise ratio

Signal to noise loss functions

Upper limit of signal to noise ratio

Vector error in measurement

Mean initial particle position vector corresponding to a N particle image pairs

Cartesian coordinates of \( \bar{a}_N \) in fluid volume

Random error in measurement

Standard deviation of \( x_N, y_N, z_N \) from a sample of size N

Interrogation region pixel resolution

Focal lengths of lenses in sheeting optics

Separation between lenses

Depth of field of recording system

f number of recording system

Wavelength of light

Particle density

Dynamic viscosity

Particle slip velocity
$W_e$ Required exposure energy

$d_a$ Lens aperture

$h_s$ Laser sheet height

$\bar{\varepsilon}$ Exposure rating of film

Chapter 5

$x_i, y_i$ Cartesian coordinates in image plane

$I(x_i, y_i)$ Intensity distribution in image plane

$U_g(x_i, y_i)$ Geometrical optics prediction of image amplitude

$h(x_i, y_i)$ Impulse response function

$\tilde{x}, \tilde{y}$ Spatial frequencies in the plane of the pupil

$P(\lambda s_i \tilde{x}, \lambda s_i \tilde{y})$ Pupil function

$D_s$ Aperture separation

$r$ Cylindrical function radius

$A_0, B_0$ Constants

$U(x_o, y_o)$ Image labelled transparency amplitude function

$\Delta x_i, \Delta y_i$ Components of image labelled transparency particle image displacement

$f$ Focal length of transform lens

$\alpha, \beta$ Cartesian coordinates in Fourier plane

$U_f(\alpha, \beta)$ Fourier plane amplitude function

$U_t(\alpha, \beta)$ Fourier plane mask transmission function

$I_1(X, Y)$ First exposure intensity distribution

$I_2(X, Y)$ Second exposure intensity distribution

$R_{CC}(\xi, \eta)$ Cross correlation function

$d_s$ Aperture diameter for single aperture

$I_1$ Fringe peak intensity of labelled images

$I_s$ Peak intensity of unlabelled images

$A_b$ Polaroid absorption factor

$D_{max}$ Diameter of dual aperture limits
ABSTRACT

Particle Image Velocity (PIV) is now a well established, non-intrusive technique for the two dimensional measurement of fluid velocity from a single plane of interest within a fluid flow. This thesis presents new work into the application of the double pulsed PIV technique to high speed flows. The areas of work can be split into three major areas.

The first area of work involved a comprehensive study into data reduction using autocorrelation. Results from the study allowed the development of an optimisation method which provides a consistent basis for experimental design. Further work validated this method by comparing equivalent results from sets of PIV transparencies processed using a system developed from commercially available image processing equipment.

The second area of work involved supersonic flow studies of a de Laval expansion nozzle. PIV results were recorded from both inside and outside the nozzle. Inside the nozzle the PIV results resolved a normal shock and allowed comparisons with a 1D theoretical model, a CFD prediction and Schlieren photographs. Outside the nozzle the PIV data permitted overexpanded jet shock cell structures to be resolved and compared to a shock cell model.

The final area of work involved development of an image labelling system for high speed flows by changing the transfer characteristics of the recording optics between exposures. A general theory of this technique was developed and a system designed and tested which can be applied to flows of arbitrarily high speed.
ACKNOWLEDGEMENTS

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1.0 INTRODUCTION AND LITERATURE SURVEY

1.1 Introduction
Dramatic advances in computing power in recent years have firmly established Computation Fluid Dynamics (CFD) as a key design tool in fluid mechanics. Whilst providing invaluable data, even the most advanced CFD model is at best a close approximation to a given fluid flow and generally requires experimental validation. In recent years, therefore, much research effort has been invested in the development of suitable measurement techniques.

In the past experimental fluid dynamists relied on pitot tubes and hot wire anemometers for single point velocity measurements. These techniques, however, are intrusive and in certain applications their use becomes impractical. The advent of the laser in the 1950's [1.1] and its application to flow measurement in the 1960's using Laser Doppler Anemometry (LDA) [1.2] allowed non-intrusive point measurements in a fluid for the first time. In addition to this, the LDA technique offered high spatial resolution, rapid response and gave experimental fluid dynamicists velocity data of increased accuracy.

The disadvantage of LDA is apparent when large flow structures need to be mapped since high rig running costs prohibit the use of scanning measurement systems. Further to this, transient flows cannot be mapped satisfactorily. In these cases experimentalists generally rely on qualitative data obtained from flow visualisation techniques such as Schlieren photography.

Particle Image Velocimetry (PIV) is now a well established non-intrusive technique for recording instantaneous two dimensional fluid velocity maps from a single plane of interest. Like Schlieren and flow visualisation techniques, it allows large areas to be examined. It can be set up to record large quantities of data in a short period of time and furthermore, like LDA, the data is in a quantitative form.

Although PIV has been established now for over ten years, application of the technique to transient high speed flow regimes has been limited. Furthermore, only limited work has been carried out to produce a generalised optimisation method for experimentalists in these flow regimes. The objective of the work presented in this thesis is to address these problems through theoretical modelling and practical measurement of transonic flow structures. The remainder of the chapter is in the form of a literature review divided into three sections. In the first, the development and
practical application of PIV to fluid flow measurement is reviewed. Secondly, the range of PIV data processing techniques which have been developed is examined. In both cases, the relevance of the work to a high speed flow PIV system is discussed. Finally, in the last section the most recent innovations in PIV techniques are outlined.

1.2 Development of the Basic PIV technique

PIV is a development of the solid mechanics technique, laser speckle photography (LSP). This technique was first proposed by Burch and Tokarski [1.3] in 1968 and involves taking a double exposure photograph of an optically rough surface which is illuminated by coherent light. The optically rough surface causes a speckle effect with the pattern of the speckle being unique to the observed surface. Hence with an in plane displacement between exposures, the recorded pattern will move by a corresponding amount and the recorded photograph will consist of pairs of speckle images. The principle of this technique was extended to fluid flows by Dudderar and Simpkins [1.4]. In this case the speckle pattern from the surface was simulated by placing a laser sheet into a fluid flow seeded with small particles. This is illustrated in figure 1.1. By pulsing the laser sheet for two short intervals a similar image to LSP is recorded with pairs of particle images. Because discrete particle images are recorded, the application of LSP to fluid flows is termed particle image velocimetry (PIV), as proposed by Pickering and Halliwell [1.5].

In a typical PIV set-up the pulse separation of the laser is used to limit the particle image pair separation such that the resultant transparency can be broken down into a regular grid of smaller regions called interrogation regions. The size of the interrogation region will determine the spatial resolution of the final vector map. In each interrogation region there may be one or more particle image pairs, the number of pairs being dependent on the original seeding density in the fluid volume. In the final stage, individual interrogation regions are processed to determine the local particle displacement in the corresponding region of fluid. In general, seeding density and pulse separation are adjusted in relation to the required spatial resolution and data processing technique. The following sections will discuss the development of the PIV technique in more detail.

1.3 Application of PIV to Different Flow Regimes

The initial uses of PIV involved very controlled flow conditions. For example Poiseuille flow has been studied by Grousson and Mallick [1.6], Barker and Fourney [1.7] and Simpkins and Dudderar [1.8]. Application to slow flows and non-Newtonian fluids has been demonstrated by Binnington et al [1.9]. The first measurements in a
turbulent flow were by Meynart [1.10] using a double pulsed ruby laser to study a low speed turbulent air jet. Similar work proceeded this from various researchers [1.11-1.13]. Recent low speed work (<10m/s) has involved investigating complex flow structures such as junction and tip vortices [1.14-1.15], unsteady flow past an airfoil [1.16], acoustic streaming [1.17], water wave studies [1.18], two phase flows [1.19], oscillatory flows [1.20,1.21] and simple flow structures such as laminar flame measurements [1.22]. Now that PIV in low speed flows has been successfully established, it is increasingly being used to verify CFD, especially where complex flow structures are concerned [1.14,1.15,1.21]. The laminar flame work by Reuss et al [1.22] concentrated on the unburned regions as a first stage before attempting to apply PIV to the harsh environment of combustion regions. Further work by Reuss et al [1.23] involved the use of PIV inside an internal combustion engine. Work by Reeves et al [1.24] has combined a comprehensive study of flow structures inside a cylindrical bore glass engine with CFD analysis of the same engine design. This was the first time a realistic IC engine bore/cylinder/thead configuration was studied using PIV.

Application of PIV in the discussion so far has involved low speed flows (<10m/s). In turbomachinery such as gas turbines, any quantitative velocity measurements are likely to involve high speed flows (>100m/s). LDA work has shown that particle sizes in the region of 1µm will follow transonic flows [1.25] therefore giving confidence in PIV seeding requirements for this flow application. Work by Bryanston-Cross et al [1.26] has confirmed this by applying PIV to an enclosed turbine cascade. Seeding densities, in this case, were the major limiting factor at around one particle pair/interrogation region and resulted in restricted spatial resolution and the use of a manual data extraction technique. More recent work [1.27-1.29], which investigated open jet supersonic flows, achieved higher seeding densities and consequently permitted the application of an automated correlation data processing technique. In all these cases [1.27-1.29] seeding densities could be optimised allowing for millimetre scale spatial resolution. The previous work [1.26-1.29] has demonstrated the potential for the application of PIV to high speed flows and has shown PIV can achieve the spatial resolution and accuracy required for CFD code validation. Further work in this field will be presented in chapter 4 of the thesis.

1.4 Development of PIV Data Processing Techniques
A significant amount of PIV related research has involved data processing techniques. This is partly due to photographic PIV records containing a large amount of data. For example a typical 5"x4" PIV transparency may contain as many as 10000 data points.
Therefore the trend in PIV data processing research has been in reducing processing times to a minimum without any loss in accuracy. The following section outlines the type of systems that have been used to analyse PIV data and also work carried out to optimise these processing systems.

The two most commonly used formats for recording particle image data are photographic film and CCD's. The digital CCD format, although having the advantage of requiring no wet processing stage, is limited by the maximum information content of the medium. For example if a 1000x1000 pixel CCD camera is used, one megabyte of information will be stored per frame. A PIV 35mm film format, however, such as Kodak Technical Pan with a typical resolution 150 line pairs/mm [1.30], will store around 80 megabytes of information. Therefore because the maximum number of vectors in the vector map is dependent on the amount of stored information, film format has a significant advantage over CCD format and for this reason the majority of high speed PIV work has been recorded using photographic methods [1.26-1.28].

A double exposure PIV transparency will contain pairs of particle images. Therefore the requirement of any PIV data processing system is to reduce this information into point displacement vectors thus allowing a velocity map to be created. If more than one particle pair exists in any observed region a statistical analysis will be required to extract the displacement information. Consequently different seeding densities may result in different approaches to the analysis.

The first attempt at a statistical type of analysis was by Burch and Tokarski [1.3] who recorded LSP images. They demonstrated that the displacement information could be recovered by analysing the far field diffraction pattern when interrogating the transparency point by point with a low power laser beam. Because of the paired nature of the speckle images, the orientation and separation of the fringe pattern for each point contained the corresponding direction and magnitude information of the displacement vector at that point. This technique is called Young's fringe analysis and can be applied directly to PIV transparencies providing the seeding density is high.

Improvements in this process have included reduction of film noise by contact printing [1.31-1.33] and increase of fringe pattern sharpness by multiple exposures [1.34]. Fully automated Young's fringe analysis systems have also been developed [1.35-1.37]. These systems digitise the Young's fringe pattern and generally use fringe tracing algorithms to calculate the fringe frequency and orientation. Because, however, only small samples of data are used relative to the total fringe data available, these
systems are sensitive to noise and so rely on high quality fringes to produce acceptable results.

A second method to analyse regions of PIV data in a statistical manner is to take the autocorrelation of the intensity distribution transmitted by each region. The autocorrelation is essentially a spatial statistic which reflects the spatial frequency content of the Young's fringe pattern. Hence a typical PIV autocorrelation result will contain three distinct peaks. These are the central peak and two peaks symmetrically spaced about the central peak which have a position proportional to the displacement vector. The remaining smaller peaks are noise peaks due to spurious correlations. Therefore the autocorrelation is able to separate the information from the noise in the Young's fringe data and so has an immediate advantage over the Young's fringe method.

Comprehensive statistical pieces of work investigating the fundamental parameters of autocorrelation based PIV analysis have been completed [1.38-1.43]. The work has involved investigating the effects of variables such as seeding density [1.38-1.40], particle image diameter to interrogation region diameter ratio [1.38-1.39], in plane movement [1.40], out of plane movement [1.40], velocity gradients [1.40], turbulence [1.41,1.42] and resolution [1.43] on the accuracy and reliability of the autocorrelation output data. The majority of this analysis has involved computer simulated input data to ensure that the interrogation region parameters are precisely controlled and immediately analysed. Results from these pieces of work indicated that the data recorded for a PIV system must be optimised in order to produce a high percentage of reliable output data from the autocorrelation. Also fundamental measurement limits exist in terms of allowable in plane velocity, out of plane velocity and velocity gradient if a high percentage of valid output data is required. Similar work involving multiple pulsed PIV data has been completed [1.44]. In chapter two of the thesis, a new generalised approach for autocorrelation optimisation is presented which allows the user to predict the system performance given certain \textit{a-priori} information about the flow.

The simplest method to perform an autocorrelation is to digitise the Young's fringe pattern onto computer via a CCD and image grabber, and then perform a 2D Fast Fourier Transform (FFT). Commercially available systems such as the TSI system can use a pixel resolution range from a 512\(^2\) point transform to 128\(^2\) typically performing a 512\(^2\) at four seconds, a 256\(^2\) at one second and a 128\(^2\) at a quarter of a second with a loss in accuracy as the resolution is reduced. A 10000 point 5"x4" transparency will
therefore take from three quarters of an hour to twelve hours depending on the resolution and accuracy required. Research groups such as Reuss et al [1.22,1.23] have based their data processing systems on this technique because the hardware is reliable, readily available and cost effective.

An alternative autocorrelation method is to directly digitise the particle images and then perform two FFT stages. This technique is known as digital image plane autocorrelation. It has advantages such as image binarisation and directly controllable interrogation region resolution to allow easy optimisation of the autocorrelation. There is the additional advantage of only requiring incoherent illumination. Consequently, the introduction of phase noise which can be a problem in Young's fringe analysis [1.31-1.33] is avoided. Work by Prasad et al [1.43] has shown a reduction in the processing resolution to 128x128 in conjunction with centroid analysis does not result in any significant loss in accuracy. Using this result together with state of the art digital parallel processing technology, Meinhart et al [1.45] has assembled a digital image plane autocorrelation system which can perform up to 200 autocorrelations a second at 128x128 resolution. The system performance is now limited by the speed at which data can be downloaded from the input CCD.

Alternative, faster methods of performing the autocorrelation have in the majority of cases involved optical computing techniques. Coupland [1.46] has shown an optical system based on a BSO photo refractive crystal can perform an autocorrelation in a single operation. Taking into account the remaining computer storage requirements of the autocorrelation image, each region was interrogated in less than one second. This was faster than most computer based systems at the time. More recent optical autocorrelation systems have been based on liquid crystal Spatial Light Modulators (SLM) [1.47-1.49]. Again the autocorrelation was performed in a single optical operation. The limiting factor of all these optical autocorrelators was the last stage in the operation which involved digital storage of the autocorrelation image followed by peak finding algorithms. This would limit the overall interrogation time to the order of half a second a region. A more recent development by Mao et al [1.50] has overcome this problem by scanning the autocorrelation output from the SLM optical computer onto a fixed photo diode detector by using an x-y electrooptic device. This has allowed interrogation periods as low as 1ms/region to be attained with around 2% of full scale error. This result has demonstrated the speed potential of optically based computing devices. Further research, however, is needed to improve the reliability of the SLM devices before a commercially available system is produced.
Alternative techniques to autocorrelation have been demonstrated by Wernet and Edwards [1.51] and Grant and Liu [1.52,1.53]. These methods are computationally intensive and rely on locating the particle image centres followed by finding the most common displacement vector. Also the spatial resolution available from these systems is limited in comparison to a correlation based system. As with digital image plane autocorrelation these techniques only require incoherent light for imaging.

1.5 Development of Ambiguity Removal Techniques
The PIV data processing systems discussed so far retrieve velocity data from particle image pairs. Since, no information is present to distinguish the order of the first and second exposure images, the final result will always give a displacement vector subject to $180^\circ$ ambiguity. In the majority of cases this is not a problem since other established qualitative or theoretical data may be available. In more complex flows, additional information is required to obtain an unambiguous displacement vector. The following describes several techniques which have been used to overcome this problem.

In work by Shekarriz et al [1.14] the pulse duration of the laser was varied for both exposures, the second pulse being significantly longer than the first. This produced standard particle images for the first exposure followed by particle streaks for the second exposure. The interrogation regions were directly analysed by digitising the particle images and applying algorithms to distinguish the first and second exposures. Sparse seeding was required since only single particle pairs could be analysed at one time. This technique was applied to low speed flows, i.e. less than 10m/s and is not appropriate to the relatively short pulses required for high speed flow measurement.

A significant amount of recent PIV data processing work has concentrated on applying various tracking and filtering algorithms to multiple exposure digitised images [1.54-1.57]. These systems rely on directly recording the region of interest at video rates and then applying either tracking algorithms or digital convolution filtering techniques [1.58] to find the particle trajectories. Because the video pictures are stored in sequence, any ambiguity problems can be overcome. At present, however, all these techniques are limited by the video frame rate and can only record low speed flows (<10m/s).

Adrian [1.59] and Landreth and Adrian [1.60] have presented a system for ambiguity removal termed 'The Image Shifting Technique'. It involves either a rotating mirror [1.59] or an electrooptic device [1.60] which is used to add an additional image
displacement between laser pulses. This shift displacement must be larger than the greatest negative velocity and is subtracted from the interrogation displacement during data processing. Because, however, the greatest positive velocity is also increased by the image shift, applying this technique results in a decrease in spatial resolution when compared to a conventional autocorrelation system. The maximum flow speed to which this system can be applied will depend on the mirror rotation speed or the electrooptic frequency. The PIV system developed by Hocker and Kompenhans [1.27] has the potential to image shift from a range -500m/s to +500m/s.

Improvements over image shifting in terms of signal to noise, dynamic range and spatial resolution can be achieved by applying the cross correlation to regions of the particle image field. In contrast to autocorrelation, cross correlation will output only one signal peak which ensures ambiguity removal. Also the absence of a dc peak will ensure an increase in the dynamic range available. In order to cross correlate, two independent particle image fields are required. This can be achieved through holographic techniques [1.61], colour coding [1.62] or digital techniques [1.63]. In the first case [1.61] image plane holography is used to record two or more separable holograms on a single holographic plate. Each image is recorded using a different reference beam. This allowed separation of two exposures through image reconstruction by applying the two original reference beams in the required order. Separation of the two images allows cross correlation analysis to be carried out. In the second case [1.62], the transparency is recorded onto colour film using a double pulsed laser system which has a different colour for each pulse. The two exposures are then simply separated by placing a corresponding colour filter over the transparency and storing the two exposures using a CCD camera. In the last case [1.63], a region from a double exposure transparency is digitised and two independent interrogation windows cross correlated. The second window is shifted by a distance from the first window using displacement information obtained from the adjacent correlation or a priori knowledge.

The maximum flow speed possible using the holographic [1.61] technique is limited by ensuring that the maximum allowable particle movement during exposures is much less than one wavelength. The restriction on particle movement is less severe for the case of colour encoding [1.62] and the digital technique [1.63]. To avoid streaked images, however, flow speeds must be restricted to less than 1000m/s when using a high specification YAG laser.

Alternative methods of performing cross correlation have been achieved by relatively
simple image labelling techniques allowing the first and second exposure images to be separated. These techniques are described in chapter 5 of the thesis.

1.6 Recent PIV Techniques
The following section reviews the latest techniques to be proposed at the time of writing. Their potential for application to a high speed flow system will be discussed.

The advances in recent years in video and image processing techniques have encouraged the development of fully digital PIV techniques. These techniques directly record the PIV images using CCD cameras and frame grabbers. The systems will either immediately process the data before the next set of images is recorded, or store the data from a sequence of images and process the data after the recording has finished. The previous section described a number of data processing methods used by these techniques [1.54-1.58]. Cho [1.64] presented a detailed analysis of the parameters to be considered in such a system and Willert and Gharib [1.65] applied a fully digital system to a vortex ring low speed flow. In the system by Willert and Gharib [1.65] displacement information was retrieved by applying local cross correlation to two sequenced images in areas of the observed region. The images were recorded at video rates (30Hz) and hence limited the maximum flow speed to approximately 4cm/s.

Work by Nishino et al [1.66] used three synchronised TV cameras to record a three dimensional decaying turbulent flow. Data was stored on a laser disc and post-processed. Flow speeds measured were in the order of 1mm/s. All the discussed digital systems have a promising future in low speed flow regimes and have advantages over double pulsed photographic PIV such as the elimination of wet processing and translation stages, removal of the ambiguity problem and simple application of cross correlation techniques. At present, however, the maximum flows speeds recorded are limited by video rates and resolution which is significantly less than current photographic emulsions.

The application of stereoscopic techniques to record three dimensional PIV flows has been demonstrated by Arroyo and Greated [1.67]. A novel camera design involving two sets of mirrors was used to record two simultaneous images of the same PIV field onto a single piece of film. With two images of the same field viewed from different angles, a third velocity component can be calculated. The camera was applied to an acoustic streaming flow. The advantage of the camera design was that it could be applied to any standard PIV photographic set-up.
Three dimensional PIV measurements have also been demonstrated using holographic methods by Coupland and Halliwell [1.68]. After holographically recording a simulated three dimensional PIV flow, the displacement vector data was extracted by applying optical autocorrelation techniques. The three components of velocity were simply obtained by focusing on the two outer peaks of the autocorrelation which, due to the nature of the original holographic data, are positioned in front and behind the output plane. The maximum flow speed possible using the technique is limited by the maximum allowable particle movement during exposures. Using a laser with a 10-15ns pulse duration would give an upper limit in the order of 100m/s. Also, because a single holographic PIV exposure may contain as many as one million interrogation points, optical processing techniques [1.50] with their speed advantages would be applicable to this technique.

1.7 Closure
The literature survey has highlighted the rapid developments in the last five years of PIV related technology such as image processing hardware, double pulsed lasers and CCD technology. This progress has created new opportunities to improve processing speed and accuracy and extend the technique to more demanding flow regimes.

The following thesis presents work which utilises a number of the most recent PIV developments to advance the application of PIV to high speed flows inside turbomachines. The project involved a comprehensive study of the complete PIV system from the optimisation of a correlation based data processing system to the problems of recording high speed PIV images from inside turbomachine environments.

In the first part of the thesis, chapter 2 presents a new data processing optimisation technique developed from a theoretical autocorrelation model and a Monte Carlo simulation. The technique is shown to provide a consistent basis for experimental design and in chapter 3 is extended to include the effects of recording PIV data onto transparency format.

In the second part of the thesis, chapter 4 addresses the practical problems of recording high speed flows in turbomachines through the application of PIV to a simple supersonic nozzle. The PIV results are directly compared to CFD and theoretical models as a method of validation. Following this, chapter 5 proposes a practical method of ambiguity removal for high speed flows using a polarisation based image labelling technique. Finally chapter 6 concludes and suggests a programme of further work.
2.0 THEORETICAL CONSIDERATIONS

2.1 Introduction
The PIV technique has distinct recording and data processing stages. For a given flow field, the objectives are to produce the highest percentage of valid measurements with the greatest accuracy and spatial resolution at the highest processing speed. In order to achieve these objectives the technique must be optimised during both the recording and data processing stages by adjusting a number of user controllable variables. The following chapter will consider the effect of these variables and produce an optimisation strategy for a generalised PIV system.

To optimise the technique a-priori information will be required about several fluid flow parameters which affect the performance of the PIV system. The most important fluid parameters required will be

Fluid Flow Parameters
1) Dynamic Range (Dr)
2) Velocity Gradient Strength (φ)

These parameters relate to the range of velocities that can be measured and the spatial rate of change of velocity which can be tolerated by the technique. From this fluid flow information predictions of system performance can be made in terms of the following parameters:

Performance Parameters
1) The Percentage of Valid Vectors
2) Correlation Signal to Noise Ratio (S)
3) Accuracy (E)
4) Processing Speed

To optimise system performance a number of user controllable variables can be adjusted. These variables can be split into two groups:

User Controllable Variables
1) Recording stage: Mean Fluid Seeding Density (Nv)
   Pulse Separation of Laser (Δt)
   Particle Size/ Particle Image Size (dp/d)
2) Data processing stage: Spatial Resolution (L)
Interrogation Region Pixel Resolution (RN)

The degree of adjustment of these variables will depend on the fluid flow parameters, the accuracy and the processing speed requirements of the system.

In the past [2.1,2.2], a limited amount of work has addressed the problem of PIV optimisation through computer simulation. Due to the complex relationships between fluid flow parameters, user controllable variables and system performance parameters it is fair to say that this work forms a limited set of look up tables which can only be used for a finite number of flow cases. The following work addresses this problem by proposing a novel optimisation strategy. First, the PIV recording and data processing stages are mathematically modelled. From this work, the mechanisms which degrade the performance of PIV are identified. By linking these mechanisms to the fluid flow parameters a window of operation is defined for the technique. This work provides the basis for the optimisation strategy and is demonstrated through example.

2.2 PIV System Model

With reference to figure 2.1, each interrogation region of a PIV transparency will correspond to a volume in the fluid flow which we will refer to as the interrogation volume. If the magnification of the recording system at the centre of the light sheet or object plane is \( M \), let the length of the volume define the spatial resolution of the system, \( L \), such that if the corresponding transparency interrogation region has a length \( D \) then

\[
L = \frac{D}{M} \tag{2.1}
\]

where \( M = \frac{s_y}{s_0} \). Assuming the laser sheet has constant intensity inside and zero intensity outside and the fluid volume has an average seeding density throughout the fluid of \( N_v \) particles/unit volume, then the mean number of initial particle images known as particle image density, \( N_i \), recorded on the corresponding interrogation region after the first laser pulse will equal

\[
N_i = N_v L^2 W \tag{2.2}
\]

If the interrogation volume has Cartesian co-ordinates \((x, y, z)\) with the origin located at the centre of the volume, let the initial position of a single particle at time \( t=0 \) be represented by the position vector \( \mathbf{a}_i \) where
\[ \mathbf{a}_i = [x_i] \hat{i} + [y_i] \hat{j} + [z_i] \hat{k} \]  

(2.3)

and \( \hat{i}, \hat{j} \) and \( \hat{k} \) are unit vectors in the x, y and z axes respectively. The variables \( x_i, y_i \) and \( z_i \) are random independent variables such that

\[ x_i = \mathbf{L} \times \alpha_i \]  

(2.4)

\[ y_i = \mathbf{L} \times \beta_i \]  

(2.5)

\[ z_i = \mathbf{W} \times \chi_i \]  

(2.6)

where \( \alpha_i, \beta_i \) and \( \chi_i \) are random values drawn from the uniform rectangular probability density function, \( p(I) \), shown in figure 2.2.

With reference to figure 2.3, if the fluid velocity in the volume is a slowly varying spatial function, the initial velocity of an arbitrarily positioned particle at time \( t=0, \mathbf{V}(\mathbf{a}_i, t) \), can be represented by the first order Taylor expansion

\[ \mathbf{V} = \mathbf{V}_o + (\mathbf{a}_i, \mathbf{V}) \mathbf{V}_{a=0,\mathbf{z}=0} \]  

(2.7)

where \( \mathbf{V}_o \) represents the velocity in the centre of the interrogation volume at a time \( t=0 \) and \( (\mathbf{a}_i, \mathbf{V}) \) represents the scalar operator

\[ x_i \frac{\partial}{\partial x} + y_i \frac{\partial}{\partial y} + z_i \frac{\partial}{\partial z} \]  

(2.8)

The displacement of an arbitrary particle can be found by integrating the function in equation 2.7 with respect to time over the pulse separation \( \Delta t \) of the laser. This function, however, is a complex integral which can be evaluated more simply by using a Taylor method. This solution is in the form of a Taylor series expanded about \( a=a_i \) and \( t=0 \) such that the position of an arbitrary particle at time \( t \) will equal

\[
\begin{align*}
\mathbf{a}(t) &= \mathbf{a}_i + \left\{ \mathbf{V}_o + (\mathbf{a}_i, \mathbf{V}) \mathbf{V}_{a=0,\mathbf{z}=0} \right\} t + \left\{ \left[ \left( \mathbf{V}_o + (\mathbf{a}_i, \mathbf{V}) \mathbf{V}_{a=0,\mathbf{z}=0} \right) \mathbf{V} \right] \mathbf{V}_{a=0,\mathbf{z}=0} \right\} \frac{t^2}{2} + \\
&+ \left\{ \left[ \left( \mathbf{V}_o + (\mathbf{a}_i, \mathbf{V}) \mathbf{V}_{a=0,\mathbf{z}=0} \right) \mathbf{V} \right] \mathbf{V}_{a=0,\mathbf{z}=0} \right\} \frac{t^3}{6} + \ldots
\end{align*}
\]

(2.9)

From this the displacement of the particle \( \Delta \mathbf{a} \) will equal
\[ \Delta s = a(\Delta t) - a(0) \] (2.10)

Therefore to a third order approximation \( \Delta s \) will equal

\[
\Delta s = \left[ \nabla_0 + (a_1 \cdot \nabla) \nabla \right]_{t=0} \Delta t + \left[ \left( \left[ \nabla_0 + (a_1 \cdot \nabla) \nabla \right]_{t=0} \right) \nabla \right]_{t=0} \Delta t^2 + \left[ \left( \left[ \left[ \nabla_0 + (a_1 \cdot \nabla) \nabla \right]_{t=0} \right) \nabla \right]_{t=0} \right) \nabla \right]_{t=0} \Delta t^3 \frac{\Delta t^3}{6}
\] (2.11)

In the following section particle displacements are modelled using the first order terms in equation 2.11. This approximation results in displacement errors of between 1-2% for the flow regimes studied. This error is acceptable because only mechanisms which degrade the signal to noise ratio (SNR) performance of the velocimeter are considered. In subsequent accuracy analysis, however, a third order approximation is required for modelling and details of this are found in section 2.3.4. Therefore expanding the first order terms in equation 2.11 in the unit vector directions \( i, j \) and \( k \) gives

\[
\Delta s = \left[ \left( u_0 + x_i \frac{\partial u}{\partial x} \right)_{t=0} + y_i \frac{\partial u}{\partial y} + z_i \frac{\partial u}{\partial z} \right] \Delta t \] i + \\
\left[ \left( v_0 + x_i \frac{\partial v}{\partial x} \right)_{t=0} + y_i \frac{\partial v}{\partial y} + z_i \frac{\partial v}{\partial z} \right] \Delta t \] j + \\
\left[ \left( w_0 + x_i \frac{\partial w}{\partial x} \right)_{t=0} + y_i \frac{\partial w}{\partial y} + z_i \frac{\partial w}{\partial z} \right] \Delta t \] k
\] (2.12)

where \( u_0, v_0, w_0 \) and \( u, v, w \) are components of \( \nabla_0 \) and \( \nabla \) respectively in the \( i, j \) and \( k \) directions. With reference to previous derivations [2.2-2.3], the coordinates in the fluid volume \( x, y \) and \( z \) map to transparency coordinates \( X, Y \) as

\[
[X]_i + [Y]_j = \left( \frac{-s_i}{s_o - z_i} \right) ([x]_i + [y]_j)
\] (2.13)
and will result in a corresponding particle image displacement in the transparency plane, \( \Delta S \), equal to

\[
\Delta S = [\Delta X_o + \Delta X_i + \Delta X_z]i + [\Delta Y_o + \Delta Y_i + \Delta Y_z]j
\]  

(2.14)

where

\[
\Delta X_o = Mu_o \Delta t
\]  

(2.15)

\[
\Delta Y_o = Mv_o \Delta t
\]  

(2.16)

\[
\Delta X_i = M \left[ \left( x_i \frac{\partial u}{\partial x} \right)_{t=0}^{i=0} + y_i \frac{\partial u}{\partial y} \left|_{t=0}^{i=0} \right. + z_i \frac{\partial u}{\partial z} \left|_{t=0}^{i=0} \right. \right] \Delta t
\]  

(2.17)

\[
\Delta Y_i = M \left[ \left( x_i \frac{\partial v}{\partial x} \right)_{t=0}^{i=0} + y_i \frac{\partial v}{\partial y} \left|_{t=0}^{i=0} \right. + z_i \frac{\partial v}{\partial z} \left|_{t=0}^{i=0} \right. \right] \Delta t
\]  

(2.18)

\[
\Delta X_z = \left( \frac{-x_i}{s_o - z_i} \right) \left[ w_o + x_i \frac{\partial w}{\partial x} \left|_{t=0}^{i=0} \right. + y_i \frac{\partial w}{\partial y} \left|_{t=0}^{i=0} \right. + z_i \frac{\partial w}{\partial z} \left|_{t=0}^{i=0} \right. \right] \Delta t
\]  

(2.19)

\[
\Delta Y_z = \left( \frac{-y_i}{s_o - z_i} \right) \left[ w_o + x_i \frac{\partial w}{\partial x} \left|_{t=0}^{i=0} \right. + y_i \frac{\partial w}{\partial y} \left|_{t=0}^{i=0} \right. + z_i \frac{\partial w}{\partial z} \left|_{t=0}^{i=0} \right. \right] \Delta t
\]  

(2.20)

In the following analysis paraxial recording is assumed where \((s_o - z_i) >> x_i\) and \((s_o - z_i) >> y_i\). Therefore the variables \(\Delta X_z\) and \(\Delta Y_z\) will tend to zero and out of plane particle displacements can be ignored.

Evidently the particle image displacement, \( \Delta S \), is proportional to the velocity in the centre of the volume together with a small perturbation due to velocity gradients. The degree of perturbation is dependent on the velocity gradient characteristics which are described through the gradient components \((\partial u/\partial x, \partial u/\partial y, \partial u/\partial z)\) and \((\partial v/\partial x, \partial v/\partial y, \partial v/\partial z)\). In order to understand how this perturbation affects velocity measurement it is necessary to model the random distribution of particles within a given interrogation region as follows.
Assuming \( N \) particles remain within the interrogation volume to form \( N \) particle image pairs and \( N_u \) particle images are unpaired, an intensity distribution of a typical positive recorded transparency can be modelled. If \( P(X, Y) \) represents the intensity function of a single particle image, the intensity distribution \( I(X, Y) \) of a typical transparency region will equal

\[
I(X, Y) = K P(X, Y) \ast \sum_{i=1}^{N} \left[ \delta(X - X_i, Y - Y_i) + \delta(X - [X_i + \Delta X_o + \Delta X_c], Y - [Y_i + \Delta Y_o + \Delta Y_c]) \right]
\]

\[
+ K P(X, Y) \ast \sum_{k=1}^{N_u} \delta(X - X_k, Y - Y_k)
\]

(2.21)

where \( K \) is a constant, \( \ast \) represents the convolution function, \( \delta(X, Y) \) represents the delta function, \( N \) particle image pairs are at initial co-ordinates \((X_i, Y_i)\) and \( N_u \) unpaired particle images are at coordinates \((X_k, Y_k)\). In the majority of cases the image of a single seeding particle is set by the point spread function of the recording optics. This is a bessel function which for reasons of mathematical simplicity can be approximated [2.4] to a Gaussian function with diameter, \( d \), at the \( 1/e^2 \) point.

The autocorrelation function, \( R_{II}(\xi, \eta) \), of the intensity distribution, \( I(X, Y) \), is defined as

\[
R_{II}(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(X, Y)I(X - \xi, Y - \eta)dXdY
\]

(2.22)

Substituting equation 2.21 into equation 2.22 and applying the Weiner-Khintchine theorem [2.7] leads to

\[
R_{II}(\xi, \eta) = K^2 \left[ R_{de}(\xi, \eta) + R_a(\xi, \eta) + R_{as}(\xi, \eta) + R_{al}(\xi, \eta) \right]
\]

(2.23)

where

\[
R_{de}(\xi, \eta) = R_{pp}(\xi, \eta) \ast \left[ 2N \delta(\xi, \eta) + N_u \delta(\xi, \eta) \right]
\]

(2.24)

\[
R_a(\xi, \eta) = R_{pp}(\xi, \eta) \ast \left[ R_q(\xi, \eta) \ast \left\{ \delta(\xi - \Delta X_o, \eta - \Delta Y_o) \right\} \right]
\]

(2.25)
\[ R_c(\xi, \eta) = \sum_{i=1}^{N} \delta(\xi - \Delta X_i, \eta - \Delta Y_i) \] (2.26)

\[ R_{nn}(\xi, \eta) = R_{pp}(\xi, \eta) \ast \left[ \sum_{i,j} \sum_{k,l} \delta(\xi - (X_i - X_j) + (\Delta X_i - \Delta X_j), \eta - [(Y_i - Y_j) + (\Delta Y_i - \Delta Y_j)]) + \delta(\xi + [\Delta X_o + \Delta X_i + (X_i - X_j)], \eta + [\Delta Y_o + \Delta Y_i + (Y_i - Y_j)]) + \delta(\xi - [X_i - X_j], \eta - (Y_i - Y_j)) + \delta(\xi + [X_i - X_j], \eta + (Y_i - Y_j)) \right] \] (2.27)

\[ R_{nl}(\xi, \eta) = R_{pp}(\xi, \eta) \ast \left[ \sum_{i,j} \sum_{k,l} \delta(\xi - (X_i - X_j), \eta - (Y_i - Y_j)) + \delta(\xi + [\Delta X_o + \Delta X_i + (X_i - X_k)], \eta + [\Delta Y_o + \Delta Y_i + (Y_i - Y_k)]) + \delta(\xi - [X_i - X_k], \eta - (Y_i - Y_k)) + \delta(\xi + [X_i - X_k], \eta + (Y_i - Y_k)) \right] \] (2.28)

and \( R_{pp}(\xi, \eta) \) represents a single correlated particle image.

If the autocorrelation function is examined in detail it is clearly symmetrical about the \( \xi \) or \( \eta \) axes and can be split into four distinct sections. These are the d.c. peak function \( R_{dc}(\xi, \eta) \), the two noise functions \( R_{ns}(\xi, \eta) \) and \( R_{nl}(\xi, \eta) \) and the signal peak function \( R_{s}(\xi, \eta) \). Figure 2.4 illustrates a typical autocorrelation function with velocity gradients present in the fluid volume. Each of the autocorrelation functions \( R_{dc}(\xi, \eta) \), \( R_{s}(\xi, \eta) \), \( R_{ns}(\xi, \eta) \) and \( R_{nl}(\xi, \eta) \) consists of a set delta functions with \( R_{pp}(\xi, \eta) \) convolved onto each delta function to construct the correlation plane. The function, \( R_{pp}(\xi, \eta) \), is Gaussian and has a \( 1/e^2 \) diameter of \( \sqrt{2}d \) (see Appendix 1).

Examining the d.c peak function \( R_{dc}(\xi, \eta) \), shows this is a central peak with a height dependent on the number of paired, \( N \), and unpaired, \( N_u \), particle images respectively.
The function $R_{dc}(\xi, \eta)$ represents the correlation of the interrogation region at $\xi=0$, $\eta=0$. This peak is the highest in the correlation plane and must be masked off to allow detection of the signal peak. The peak mask dimensions in the x and y directions will determine the lowest velocity which can be measured by the system and hence the maximum measurable dynamic range in terms of the maximum and minimum measurable velocities. These variables will be defined in more detail in the following section.

The two noise functions, $R_{ns}(\xi, \eta)$ and $R_{nl}(\xi, \eta)$, represent correlation noise generated from spurious correlations between particle images in the interrogation region and both are superimposed onto the correlation plane. The functions $R_{ns}(\xi, \eta)$ and the $R_{nl}(\xi, \eta)$ are generated by spurious correlations between paired and unpaired particle images respectively. If the functions $R_{ns}(\xi, \eta)$ and the $R_{nl}(\xi, \eta)$ are expanded they generate a quantity of spurious correlations which are proportional to the square of the particle image density, $N_i$. The statistical properties of the noise functions will directly affect the SNR characteristics of the autocorrelation. These will be investigated in more detail in the following sections using a Monte Carlo simulation.

The signal peak function, $R_s(\xi, \eta)$, contains information on the mean particle image displacement from which the two components of interrogation volume velocity can be determined. If this function is examined it contains two signal peak terms each with a height $N$ and distributed about the position $(\pm \Delta X_0, \pm \Delta Y_0)$. The position is generally estimated by applying centroid analysis to the signal peak structure. The number of particle image pairs, $N$, is dependent on the mean velocity in the interrogation volume since a proportion of particles leave the volume between exposures. The signal peak height is also dependent on the distribution of the peak structure which is related to the random variables $(\Delta X_0, \Delta Y_0)$ as defined in equations 2.17 and 2.18. These variables are directly dependent on the velocity gradient characteristics in the interrogation volume.

Evidently the system performance can be determined from the characteristics of the autocorrelation of a general particle image field. These characteristics are dependent on the mean velocity and velocity gradient strength within the interrogation volume. They are also dependent on the autocorrelation noise statistics and will determine the system accuracy, the system SNR and the percentage of valid vectors. The following section will define these parameters and form a set of performance degradation functions which will be investigated in detail by using a Monte Carlo simulation. These degradation functions will form the basis of the optimisation strategy.
2.3 PIV System Performance

As stated in the introduction the performance of a PIV system can be measured in terms of the four parameters, percentage of valid vectors, signal to noise ratio, processing speed and accuracy. The following section considers the relationship between the percentage of valid vectors, the SNR and the fluid flow parameters.

2.3.1 Signal to Noise Ratio and Valid Vector Characteristics

A major consideration in PIV system design is to optimise the percentage of valid vectors obtained from a given flow recording. This parameter is a complex function of several variables and as such is difficult to predict even with substantial a-priori information about the flow characteristics. In the past, researchers have approached this problem by considering the characteristics of the autocorrelation function, \( R_{ll}(\xi, \eta) \), as derived in the previous section with reference to the SNR [2.1,2.2]. In the following this parameter will be discussed and a method to optimise the percentage of valid vectors will be proposed.

The SNR, \( S \), is defined as the ratio of signal peak height to the next highest peak in the autocorrelation plane, excluding the central dc peak. The next highest peak in the plane will be the highest noise peak. Therefore in general terms using the dB scale, the SNR \( S \) can be defined

\[
S = 20\log_{10}\left(\frac{\text{Height of the signal peak}}{\text{Height of the highest noise peak}}\right)
\] (2.29)

Clearly, for a given measurement to be valid, the signal peak must have a height greater than the highest noise peak (i.e. \( S>0 \)) otherwise a random velocity vector corresponding to the highest noise peak will be substituted for the correct measurement. With reference to the previous section the SNR is a function of the number of paired particle images and the spread of the signal peak due to the influence of spatial velocity gradients. This effect can be quantified in terms of the dynamic range and velocity gradient strength parameters which are defined as follows.

The maximum and minimum measurable velocities will determine the dynamic range \( D_r \) of the system. The dynamic range of the system, \( D_r \), is defined in terms of the maximum measurable velocity, \( [V_0]_{\text{max}} \) and the minimum measurable velocity, \( [V_0]_{\text{min}} \), in the fluid volume such that
The minimum measurable velocity of the system, \([V_0]_{\text{min}}\) is determined from the autocorrelation dc peak mask size. Because a single correlation peak diameter is \(\sqrt{2d}\) (Appendix 1), in practice the minimum measurable velocity, \([V_0]_{\text{min}}\), will be restricted such that an equivalent signal peak movement of one particle image diameter is observed. That is

\[
[V_0]_{\text{min}} = d/\Delta t
\]  

The maximum measurable velocity, \([V_0]_{\text{max}}\), will determine the number of particle image pairs, \(N\), which defines the signal peak height. Consequently this parameter influences the SNR directly. The number of particle image pairs, \(N\), is related to the volume of fluid retained within the original interrogation volume boundary between exposures. If the particle image density is uniform then \(N\) can be written

\[
N = N_i F_i(u_o, v_o) F_o(w_o)
\]  

where the functions \(F_i(u_o, v_o)\) and \(F_o(w_o)\) represent the fractional loss of pairs from the in and out of plane components of \(V_o\) such that

\[
F_i(u_o, v_o) = (1 - u_o \Delta t/L)(1 - v_o \Delta t/L) \quad \text{and} \quad F_i(u_o, v_o) \leq 1
\]  

\[
F_o(w_o) = (1 - w_o \Delta t/W) \quad \text{and} \quad F_o(w_o) \leq 1
\]  

As mentioned in the previous section the action of a spatial velocity gradient within the interrogation volume is to spread the power in the signal peak. It is convenient to define a parameter which we call velocity gradient strength, \(\phi\), in terms of the maximum velocity change across the interrogation volume as a proportion of the full scale velocity such that

\[
\phi = \frac{[V_2 - V_1]}{[V_o]_{\text{max}}}
\]
where

\[ V_1 = \text{Minimum velocity in the interrogation volume} \]
\[ V_2 = \text{Maximum velocity in the interrogation volume} \]

The degree of signal peak degradation will depend on the components of velocity gradient present in the interrogation volume. The aim of the following analysis is to determine the worst case which will cause the greatest degradation for a given value of velocity gradient strength. To this end, let us consider an in plane forced vortex flow regime. For this type of fluid flow the velocity is represented by

\[ \mathbf{V} = \left[ u_o + y_1 \frac{\partial u}{\partial y} \right] i + \left[ v_o + x_1 \frac{\partial v}{\partial x} \right] j \]  
\[ (2.36) \]

where \( +\partial u/\partial y = -\partial v/\partial x \) evaluated at \( a=0, t=0 \). For this flow regime to obtain the correlation signal peak distribution, \( R_g(\xi, \eta) \), the random variables \( (\Delta X_i, \Delta Y_i) \) must be defined from equations 2.17 and 2.18. In this case the velocity gradient components \( (\partial u/\partial x, \partial u/\partial z) \) and \( (\partial v/\partial y, \partial v/\partial z) \) will be zero and \( (\Delta X_i, \Delta Y_i) \) will be random independent variables such that

\[ \Delta X_i = \beta_i M L \Delta t \frac{\partial u}{\partial y}_{a=0, t=0} \]  
\[ (2.37) \]
\[ \Delta Y_i = \alpha_i M L \Delta t \frac{\partial v}{\partial x}_{a=0, t=0} \]  
\[ (2.38) \]

Since \( \Delta X_i \) and \( \Delta Y_i \) are independent, the signal peak will spread evenly in both axes across an area of the correlation plane. It should be noted that for the case of a simple shear, where for example \( \Delta X_i = \alpha_i M L \Delta t (\partial u/\partial x) \) and \( \Delta Y_i = \alpha_i M L \Delta t (\partial u/\partial y) \), a similar value of velocity gradient strength may result through the definition of \( \phi \) (equation 2.35). In this case, however, \( \Delta X_i \) and \( \Delta Y_i \) are not independent and consequently the signal peak will spread one dimensionally (i.e. in a line). Hence for a given velocity gradient strength the forced vortex represents the worst case scenario in terms of signal peak height degradation.
Therefore with reference to equations 2.25 and 2.26, it can be seen that for the case of a forced vortex the signal peak can be modelled as a sum of identical Gaussian functions each with centres randomly distributed within a square of side length, \( L_s \), such that

\[
L_s = ML\Delta t \left| \frac{\partial u}{\partial y} \right|_{x=0,t=0} = ML\Delta t \left| \frac{\partial v}{\partial x} \right|_{x=0,t=0} \quad (2.39)
\]

This result assumes all initial particles are paired in the interrogation volume. In practice any fractional loss of particle pairs from the interrogation volume will adjust the distribution limits. For the following theoretical analysis, however, a square distribution will allow acceptable signal peak characteristics to be identified. Consequently the mean height of the signal peak will be given by the maximum value of a convolution of the Gaussian function, \( R_{pp}(\xi, \eta) \), with a uniform two-dimensional Rectangular (Rect) function of side length \( L_s \) normalised by its area. With reference to Appendix 2 it can be shown that to a good approximation the signal peak will diminish by a factor \( f_v \) given by

\[
f_v = \frac{1}{1 + \left[ ML\Delta t/d \right]^2} \quad (2.40)
\]

where \( \omega = +\partial u/\partial y = -\partial v/\partial x \) evaluated at \( a=0, t=0 \). Clearly this factor shows a strong dependence on particle image diameter, \( d \), which must be increased for a given increase in velocity gradient strength, \( \phi \), to maintain signal peak height. Any particle image diameter increase will, however, reduce dynamic range, \( D_r \), by reducing the minimum measurable velocity, \( [V_o]_{\text{min}} \). This interdependence between \( \phi \) and \( D_r \) can be represented by re-defining the factor, \( f_{\text{pp}} \), as a function of the dimensionless variable \( \phi D_r \) which by definition will equal

\[
\phi D_r = \frac{[V_2 - V_1]}{[V_o]_{\text{max}}} \times \frac{[V_o]_{\text{max}}}{[V_o]_{\text{min}}} = \frac{[V_2 - V_1]}{[V_o]_{\text{min}}} \quad (2.41)
\]

Substituting \( [V_o]_{\text{min}} \) from equation 2.31 leads to
With reference to equation 2.36 for a forced vortex,

\[ V_2 = \left( u_o + \frac{L \omega}{2}, v_o + \frac{L \omega}{2} \right) \]  

(2.43)

\[ V_1 = \left( u_o - \frac{L \omega}{2}, v_o - \frac{L \omega}{2} \right) \]  

(2.44)

Hence,

\[ |V_2 - V_1| = \sqrt{2L \omega} \]  

(2.45)

Therefore \( \phi D_r \) is given by

\[ \phi D_r = \frac{\sqrt{2ML \omega \Delta t}}{d} \]  

(2.46)

Consequently the factor \( f_v \) can be re-written as the function \( F_v(\phi D_r) \) such that

\[ F_v(\phi D_r) = \frac{1}{1 + (1/2)[\phi D_r]^2} \]  

(2.47)

Therefore the reduction in signal peak height can be conveniently written as a function of the product \( \phi D_r \) where in the worst case scenario, when \( \phi D_r >> 1 \), the mean signal peak height will statistically diminish as an inverse of the square of the velocity gradient strength for a given dynamic range.

The functions \( F_i(u_o, v_o) \), \( F_o(w_o) \), and \( F_v(\phi D_r) \) represent independent mechanisms which reduce the expected height of the signal peak due to in plane velocity, out of plane velocity and the product of velocity gradient strength and dynamic range respectively. In terms of the SNR these mechanisms can be written as loss factors. With reference to equation 2.29 the SNR can be written

\[ S = 20 \log \left[ \frac{F_i(u_o, v_o) F_o(w_o) F_v(\phi D_r) N_1}{[R_n(\xi, \eta)]_{max}} \right] \]  

(2.48)
where with reference to equations 2.27 and 2.28, the total correlation noise $R_n(\xi, \eta)$ is given by

$$[R_n(\xi, \eta)]_{\text{max}} = [R_{\text{ns}}(\xi, \eta) + R_{\text{n}}(\xi, \eta)]_{\text{max}}$$  \hspace{1cm} (2.49)

and max denotes the maximum value of the argument.

Clearly, the SNR can be written,

$$S = S_b - S_i - S_o - S_v$$  \hspace{1cm} (2.50)

where $S_b$ is the baseline SNR which represents the maximum theoretical value of SNR for a given particle image size and interrogation region length and $S_i$, $S_o$ and $S_v$ represent loss factors corresponding to in plane displacement, out of plane displacement and velocity gradient mechanisms respectively, such that

$$S_b = 20\log\left[N_i/[R_n(\xi, \eta)]_{\text{max}}\right]$$  \hspace{1cm} (2.51)

$$S_i = 20\log[1/F_i(u_o, v_o)]$$  \hspace{1cm} (2.52)

$$S_o = 20\log[1/F_o(w_o)]$$  \hspace{1cm} (2.53)

$$S_v = 20\log[1/F_v(\partial \phi)]$$  \hspace{1cm} (2.54)

Evidently, the SNR, $S$, will equal the baseline value, $S_b$, when $S_i = S_o = S_v = 0$ which corresponds to a hypothetical interrogation region with zero particle image displacement.

Since equation 2.50 represents the expected value of SNR, given a baseline value of SNR the loss factors $S_i$, $S_o$ and $S_v$ can be used to define a window of operation for the PIV system such that the maximum measurable in plane velocity, out of plane velocity and velocity gradient strength can be defined at the point where 50% of the vectors will be valid. The latter point is defined by $S>0$ and in analysis will guarantee 50% valid vectors from a given number of interrogation regions. In order to implement this process it is necessary to estimate the expected value of the baseline SNR for a given particle image size and interrogation region length. This has been achieved using a Monte Carlo simulation as follows.
2.3.2 Monte Carlo Simulation

To predict the baseline SNR, a Monte Carlo simulation was created and used to measure the SNR characteristics for a range of seeding densities, particle image sizes, mean velocities and velocity gradient strengths. Details of the Monte Carlo simulation can be found in Appendix 3. The Monte Carlo model was designed to run at interrogation region resolutions between 32x32 and 256x256 pixels in order to assess the performance in terms of processing speed and accuracy. This work is described in section 2.3.4. For the purpose of measuring the baseline SNR all results were taken at the maximum interrogation region resolution, $R_N$, of 256x256.

Figure 2.5 shows the baseline SNR results from the Monte Carlo simulation for a range of particle image densities and particle image sizes. In each case the trend initially follows a curve representing the situation when the likelihood of coincident noise peaks is small. With reference to equation 2.51, this gives us an upper limit of SNR, $S_{\text{max}}$, which corresponds to $S_i=S_o=S_v=0$ and $R_n(\xi, \eta)=2$ such that

$$S_{\text{max}} = 20\log\left[\frac{N_i}{2}\right]$$  \hspace{1cm} (2.55)

In practice any overlapping noise correlations will increase with increasing particle image size and particle image seeding density resulting in a lower baseline SNR.

Figure 2.5 also shows no significant gains in SNR are obtained when particle image density is increased above $N_i=12$. This trend is related to the mean noise and signal peak characteristics of the autocorrelation function. The mean autocorrelation function is shown in figure 2.6. The mean noise floor is dependent on the number of spurious correlations which predominantly increases with $N_i^2$ (See Appendix 4). Therefore as a simple area argument, across the correlation plane the mean noise level will increase with $N_i$. The mean signal peak height also increases with $N_i$ as proven in the previous section (equation 2.32). This results in the SNR tending towards a constant value at high particle image densities ($N_i>12$). In general then, particle image density levels consistent with $N_i>12$ should be adopted.

If the results in figure 2.5 are re-plotted as shown in figure 2.7, they will define the minimum expected baseline SNR for optimised particle image density levels i.e. $N_i \geq 12$. In the following sections, together with the experimental work of chapters 4 and 5, figure 2.7 will be used as a look up table for the optimisation process where the ratio $D/d$ in conjunction with magnification, $M$, will define the spatial resolution, $L$, of the system and the baseline level of SNR, $S_b$. 

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In addition to finding the expected baseline SNR's above, the Monte Carlo simulation was also used to validate the SNR loss mechanisms derived theoretically and to demonstrate that these mechanisms are independent of particle image size. Figure 2.8 shows the in plane SNR loss function, $S_i$, predicted for a range of particle image sizes where the theoretical trend has been included for comparison. The deviations from the theoretical trends at higher in plane movements are attributed to sampling window edge effects of the Monte Carlo simulation. The results in general show the loss of SNR is independent of particle image diameter.

Figure 2.9 shows the out of plane SNR loss function, $S_o$, predicted for a range of particle image sizes and as with the in plane results the loss of SNR is independent of particle image size. For both functions a 50% movement results in a 6dB loss in SNR which corresponds to half of the initial particles leaving the volume between exposures.

Figure 2.10 shows the velocity gradient SNR loss function, $S_v$, predicted for a range of the product $\phi D_f$ and particle image sizes where the theoretical trend is included for comparison. The Monte Carlo results show the function $S_v$ to be independent of particle image size. The deviations in the theoretical trends at higher values of $\phi D_f$ can be attributed to $F_v(\phi D_f)$ reducing the signal peak value below a height equivalent to a correlation of a single particle pair. In practice at high values of $\phi D_f (\phi D_f>2)$ the signal peak becomes splintered into individual peaks of a height equivalent to a correlation of a single particle pair. With this function a 6dB loss occurs at $\phi D_f<1.5$ which corresponds to a signal peak spread of one correlation peak diameter.

The Monte Carlo simulation has been used to find the baseline SNR and has demonstrated the independence of the three SNR loss functions for a range of particle image sizes. The following section will describe the application of these functions in an optimisation process. Results from the Monte Carlo simulation will also be used to validate the technique by direct comparison with the predictions from the optimisation method.

### 2.3.3 Optimisation Method

The following optimisation method will allow the user to calculate a window of operation of the PIV system in terms of the spatial resolution, $L$, and the maximum allowable velocity gradient strength and dynamic range, $\phi$ and $D_f$ that will ensure a minimum of 50% valid vectors is achieved from the measured flow field. The method
allows an iterative loop to be carried out where the system limitations of $L$, $\phi$ and $D_r$ are adjusted in conjunction with the pulse separation, $\Delta t$, until the required measurement specification is obtained. The calculations are made using data from the graphs in figures 2.7-2.10.

The method requires the following parameters to determine the optimisation variables $D/d$, maximum in plane movement, maximum out of plane movement and $\phi D_r$. These parameters are listed as follows:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimisation Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Spatial resolution ($L$)</td>
<td>$D/d$</td>
</tr>
<tr>
<td>2) Maximum Fluid Velocity ($\left</td>
<td>V_{o} \right</td>
</tr>
<tr>
<td>3) Dynamic Range ($D_r$)</td>
<td>$\phi D_r$</td>
</tr>
<tr>
<td>4) Maximum Velocity Gradient ($\phi$)</td>
<td>$\phi D_r$</td>
</tr>
</tbody>
</table>

In the first stage of the method the required spatial resolution, $L$, is used to determine the variable $D/d$. Particle image size, $d$, can also be adjusted but this is generally set by the recording optics and available laser pulse energy. The value of $D/d$ allows the baseline SNR, $S_b$, to be obtained from the graph in figure 2.7.

In the next stage the maximum in plane and out of plane movements must be calculated in terms of a percentage of $D$ and a percentage of $W$. This allows the SNR loss from these mechanisms to be subtracted from the baseline level, $S_b$. This SNR loss is obtained from the graphs in figures 2.8 and 2.9.

In the final stage the remaining value of SNR is used to obtain the product $\phi D_r$ from the graph in figure 2.10. This product ensures the interdependence of velocity gradient and dynamic range where if the required value of $\phi$ is large, the maximum allowable $D_r$ will be small or vice versa. If at this stage the product $\phi D_r$ does not allow the required measurement values of velocity gradient and dynamic range, the user must repeat the calculations with a reduction in spatial resolution or an adjustment of the intended laser pulse separation. This iterative process can be repeated until a satisfactory compromise is found.

An example of the process is found in the Appendix 5 where comparisons are made to results from the Monte Carlo simulation. Results show the optimisation method
predicts the maximum allowable velocity and velocity gradient strength for 50% valid vectors to within 2% of the Monte Carlo prediction.

The previous section has addressed the performance of the PIV system in terms of the number of valid vectors. In the following the performance will be assessed in terms of accuracy and interrogation region resolution. Since a higher resolution demands more computing power, the interrogation region resolution will determine the processing speed of the system.

2.3.4 Accuracy

Estimation of measurement accuracy has been attempted by Keane and Adrian [2.2]. They have suggested that any PIV measurement is subject to both random and systematic error. The random error is attributed to the random distribution of particle positions within the interrogation volume and perturbations in the correlation signal peak position caused by local noise peaks. A random error is also introduced at the transparency analysis stage if the correlation pixel resolution is insufficient. Therefore the random error is expected to depend on seeding density, velocity gradient strength, interrogation region resolution and the correlation noise function. The systematic error is attributed to loss of particle image pairs which is more likely for high velocity particles within the interrogation region. Consequently the systematic error is expected to depend on the mean velocity and velocity gradient components in the interrogation volume.

For an arbitrary interrogation volume the error vector in velocity measurement, $\vec{E}$, will equal

$$\vec{E} = \frac{V_m - V_0}{|V_0|}$$  \hspace{1cm} (2.56)

where $V_m$ is the measured velocity and $V_0$ is the fluid velocity at the centre of the interrogation volume. With reference to equation 2.7, the measured velocity, $V_m$ will equal

$$V_m = V_0 + (\hat{a}_N . \nabla) V|_{z=0,i=0}$$  \hspace{1cm} (2.57)

where
Therefore with reference to equation 2.3, \( \bar{a}_N \) is a position vector which represents the mean initial position of \( N \) particles which remain in the interrogation volume between exposures to form \( N \) paired particle images in the interrogation region. It is necessary to consider paired particle images because they will define the correlation signal peak structure from which the particle displacement is estimated. Also it should be noted that only in plane components of \( \mathbf{V} \), described in equation 2.57, need to be considered since as shown in section 2.2 the out of plane component can be ignored.

Substituting the definition of \( \mathbf{V}_m \) into the definition of the vector error in equation 2.56 leads to

\[
E = \frac{(\bar{a}_N \cdot \mathbf{V})|_{\Delta=0,t=0}}{|\mathbf{V}_m|} \tag{2.60}
\]

This general definition of measurement error allows the error characteristics to be derived for a given flow regime and seeding density. In the following two sections, random and systematic error characteristics will be derived in each case and compared to results from the Monte Carlo simulation.

2.3.4.1 Random Error Characteristics
Equation 2.60 defines the measurement error in the form of a vector. To derive a random error quantity, which is scalar, we must firstly consider the magnitude of the vector error, \( |E| \). Therefore expanding equation 2.60 in the unit vector directions \( \hat{i} \), and \( \hat{j} \) and finding the modulus results in
From this definition the random error in measurement, \( \sigma_E \), can be estimated from the standard deviation of an ensemble average of \( |E|^2 \) about a mean of zero such that

\[
\sigma_E = \sqrt{\langle |E|^2 \rangle}
\]  

(2.62)

where \( \langle \rangle \) denotes the ensemble average. If the variances in the particle pair positions in the corresponding x, y and z axes are \( \sigma_{Nx}^2, \sigma_{Ny}^2 \) and \( \sigma_{Nz}^2 \) respectively, from equation 2.62 \( \sigma_E \) will equal

\[
\sigma_E = \frac{1}{|V_o|} \left[ \sigma_{Nx}^2 \left( \frac{\partial u}{\partial x} \right)_{t=0}^2 + \sigma_{Ny}^2 \left( \frac{\partial u}{\partial y} \right)_{t=0}^2 + \sigma_{Nz}^2 \left( \frac{\partial u}{\partial z} \right)_{t=0}^2 \right]^{1/2} + \left[ \sigma_{Nx}^2 \left( \frac{\partial v}{\partial x} \right)_{t=0}^2 + \sigma_{Ny}^2 \left( \frac{\partial v}{\partial y} \right)_{t=0}^2 + \sigma_{Nz}^2 \left( \frac{\partial v}{\partial z} \right)_{t=0}^2 \right]^{1/2}
\]  

(2.63)

The values of \( \sigma_{Nx}^2, \sigma_{Ny}^2 \) and \( \sigma_{Nz}^2 \) can be defined in terms of the interrogation volume dimensions \( L \) and \( W \). Therefore using standard statistical formulae [2.5] and with reference to equation 2.4 and the probability density function of figure 2.2, in the first case \( \sigma_{Nx}^2 \) for a population of size \( N \) is found from

\[
\sigma_{Nx}^2 = \frac{L^2}{N} \int_{-\nu/2}^{\nu/2} \rho(l)^2 \, dl
\]  

(2.64)

or
\[ \sigma_{nx}^2 = \frac{L^2}{12N} \quad (2.65) \]

Similarly the variance \( \sigma_{ny}^2 \) and \( \sigma_{nz}^2 \) in the y and z axes will equal

\[ \sigma_{ny}^2 = \frac{L^2}{12N} \quad (2.66) \]
\[ \sigma_{nz}^2 = \frac{W^2}{12N} \quad (2.67) \]

Substituting the definitions \( \sigma_{nx}, \sigma_{ny} \) and \( \sigma_{nz} \) into the definition \( \sigma_E \) gives

\[
\sigma_E = \frac{1}{|V_o| \sqrt{12N}} \left[ \frac{L^2}{\left[ \frac{\partial u}{\partial x} \right]_{t=0, z=0}}^2 + \frac{L^2}{\left[ \frac{\partial u}{\partial y} \right]_{t=0, z=0}}^2 + \frac{W^2}{\left[ \frac{\partial u}{\partial z} \right]_{t=0, z=0}}^2 \right] + \frac{L^2}{\left[ \frac{\partial v}{\partial x} \right]_{t=0, z=0}}^2 + \frac{L^2}{\left[ \frac{\partial v}{\partial y} \right]_{t=0, z=0}}^2 + \frac{W^2}{\left[ \frac{\partial v}{\partial z} \right]_{t=0, z=0}}^2 \quad (2.68) \]

This analysis shows as a percentage of the measured velocity, the random error is simply dependent on the number of particle pairs in the interrogation volume and the magnitude of the velocity gradient components for a given set of interrogation region dimensions. The particle image pair relationship will obey an inverse square root law and consequently for a given flow measurement, this suggests seeding levels would need to be quadrupled to halve the random error.

With reference to the SNR and valid vector analysis of section 2.3.1, if we now extend this analysis to consider the measurement error for a forced vortex, with reference to equation 2.68, \( \sigma_E \), in this case can be estimated from

\[
\sigma_E = \frac{L}{|V_o| \sqrt{12N}} \left[ \left| \frac{\partial u}{\partial x} \right|_{t=0, z=0} \right]^2 + \left[ \frac{\partial v}{\partial x} \right]_{t=0, z=0}\left[ \frac{\partial v}{\partial x} \right]_{t=0, z=0}^2 \quad (2.69) \]

If \( \omega=+\partial u/\partial y=-\partial v/\partial x \) evaluated at \( z=0, t=0 \), equation 2.69 is simplified to

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This result can be re-written in terms of the optimisation variables $D/d$ and $\phi D_r$ where with reference to equations 2.1 and 2.46 since

$$\omega = \frac{\phi D_r}{\sqrt{2(D/d)\Delta t}} \quad (2.71)$$

then

$$\sigma_E = \frac{L\phi D_r}{|V_o|\Delta t(D/d) \sqrt{12N}} \quad (2.72)$$

Also it is more convenient to represent $\sigma_E$ as a percentage of full scale particle displacement, $L$. Therefore replacing $|V_o|\Delta t$ by $L$ in equation 2.72 gives

$$\left[\sigma_E\right]_{\text{scale}} = \frac{\phi D_r}{(D/d) \sqrt{12N}} \quad (2.73)$$

This result shows for a forced vortex the random error, $\sigma_E$, as a percentage of full scale particle displacement is directly proportional to product $\phi D_r$ and inversely proportional to the ratio $D/d$. Therefore for a given seeding density and ratio of $D/d$ the random error would be expected to increase linearly with the product $\phi D_r$. Conversely for a given product of $\phi D_r$ and interrogation region length $D$, decreasing the particle image diameter $d$ is seen to decrease the random error.

The above analysis, however, does not account for the additional contributions to random error caused by correlation noise, inadequate interrogation region pixel resolution, $R_N$ and loss of particle pairs. To include these effects it is necessary to rely on the Monte Carlo simulation as described in Appendix 3. The results in figure 2.11 show the Monte Carlo predictions of random error for a range of $\phi D_r$ and interrogation region pixel resolutions at $D/d=16$ and $N_l=12$. It should be noted, however, that the theoretical trends predicted by equation 2.73 are confirmed by the simulation. This is illustrated in figure 2.12 which shows the linear dependence and the
inverse dependence of the random error on the product $\phi D_r$ and $D/d$ respectively for a constant particle image density ($N_i=12$).

To understand the random error mechanisms in more detail the measurement error distribution curves from the Monte Carlo simulation must be examined. Figure 2.13 shows the x component distribution curves for a range of $\phi D_r$ at $D/d=16$, $N_i=12$ and $R_N=256$. The increased spread in the distribution as $\phi D_r$ increases occurs through the signal peak broadening effects initially described in section 2.3.1. Clearly the greater the signal peak is spread in the correlation plane the greater the random error. Figure 2.14 shows the x component distribution curves for a range of interrogation region pixel resolutions, $R_N$, at $D/d=16$, $N_i=12$ and $\phi D_r=0$. The increase in random error is clearly evident when the interrogation region pixel resolution is reduced below 64x64. With inadequate pixel resolution, the centroid of the signal peak will become more sensitive to correlation noise which accounts for the increase of random error.

The results from this section have firstly shown that the magnitude of the random error ranges between 0.4-0.76% of full scale particle displacement, $L$, for $\phi D_r=0-1.0$ at $R_N=256$. For $D/d=16$, when measuring the minimum allowable particle displacement which is limited in the correlation plane to a single particle image diameter, $d$, the random error of 0.76% of full scale, $L$, corresponds to a velocity measurement error of 12%. This is a significant error and therefore to reduce the magnitude of these velocity measurement errors the user must optimise the velocimeter to allow the greatest possible particle displacement within the interrogation volume. Secondly it is clear that in the presence of velocity gradients where $\phi D_r>0$ if interrogation region pixel resolution is reduced below 64x64 which corresponds to a 4x4 particle image pixel array at $D/d=16$, then the random error will significantly increase. Therefore an interrogation region pixel resolution of 64x64 pixels with a particle image pixel resolution of 4x4 pixels is the minimum requirement to sufficiently suppress random errors caused by this mechanism. This result is consistent with previous recommendations [2.8].

2.3.4.2 Systematic Error Characteristics

With reference to the general definition of measurement error in equation 2.60, the systematic error in measurement will be defined as the average vector error, $E_v$, for a given value of $\mathbf{V}_0$ calculated from a sample of interrogation regions such that
\( \langle \mathbf{E} \rangle = \frac{\langle (\mathbf{a}_N) \cdot \nabla \rangle \mid_{a=0,t=0}}{|\mathbf{V}_o|} \) \hspace{1cm} (2.74)

where \( \langle \rangle \) denotes the ensemble average. The average value of \( \langle \mathbf{a}_N \rangle \) is dependent on the fractional loss of particles from the interrogation volume between exposures. It should be noted that particle arrivals into the interrogation volume between exposures are not considered since only particles which form pairs will define the signal peak structure. A derivation of \( \langle \mathbf{a}_N \rangle \) can be found in terms of the fluid velocity in the centre of the interrogation volume, \( \mathbf{V}_o \). If the vector \( \langle \mathbf{a}_N \rangle \) is separated into separate components \( \langle x_N \rangle, \langle y_N \rangle \) and \( \langle z_N \rangle \) in the corresponding x, y, and z axes, let us firstly consider the x component in terms of the fluid velocity \( u_0 \). By taking a first order approximation of particle displacement from equation 2.11 and with reference to figure 2.15 it can be seen that \( \langle x_N \rangle \) will equal

\[ \langle x_N \rangle = -u_0 \Delta t/2 \] \hspace{1cm} (2.75)

Similarly

\[ \langle y_N \rangle = -v_0 \Delta t/2 \] \hspace{1cm} (2.76)

\[ \langle z_N \rangle = -w_0 \Delta t/2 \] \hspace{1cm} (2.77)

Consequently, to a first order approximation the position vector \( \langle \mathbf{a}_N \rangle \) will equal

\[ \langle \mathbf{a}_N \rangle = -\mathbf{V}_o \Delta t/2 \] \hspace{1cm} (2.78)

Substituting this definition into equation 2.74 gives

\[ \langle \mathbf{E} \rangle = -\frac{\bigg[(\mathbf{V}_o \cdot \nabla)\mid_{a=0,t=0}\bigg]\Delta t}{2|\mathbf{V}_o|} \] \hspace{1cm} (2.79)

Expanding 2.79 in the unit vector directions \( \mathbf{i} \) and \( \mathbf{j} \) gives
\[
\langle E \rangle = -\frac{\Delta t}{|V_o|} \left[ \left( u_o \frac{\partial u}{\partial x}|_{x=0, t=0} + v_o \frac{\partial u}{\partial y}|_{y=0, t=0} + w_o \frac{\partial u}{\partial z}|_{z=0, t=0} \right) i + \left( u_o \frac{\partial v}{\partial x}|_{x=0, t=0} + v_o \frac{\partial v}{\partial y}|_{y=0, t=0} + w_o \frac{\partial v}{\partial z}|_{z=0, t=0} \right) j \right]
\] (2.80)

Therefore the first order model shows the systematic error in measurement, \( \langle E \rangle \), is dependent on the components of velocity \( u_o, v_o, w_o \) and the velocity gradient components \( (\partial u/\partial x, \partial u/\partial y, \partial u/\partial z) \) and \( (\partial v/\partial x, \partial v/\partial y, \partial v/\partial z) \) in the centre of the interrogation volume. The error in measurement will be zero through the term \( (V_o \cdot \nabla) V \) when the gradient components are perpendicular to the velocity, \( V_o \), or when no velocity gradients are present in the interrogation volume.

With reference to the SNR and valid vector analysis of section 2.3.1, if we now extend this analysis to consider the systematic error for a forced vortex, using the first order model in equation 2.80 and with reference to equation 2.36, the systematic error \( \langle E \rangle \) for a forced vortex will equal

\[
\langle E \rangle = -\frac{\Delta t}{2|V_o|} \left( v_o \frac{\partial u}{\partial y}|_{y=0, t=0} + u_o \frac{\partial v}{\partial x}|_{x=0, t=0} \right) i + \left( v_o \frac{\partial v}{\partial y}|_{y=0, t=0} + u_o \frac{\partial v}{\partial x}|_{x=0, t=0} \right) j
\] (2.81)

This vector result shows the systematic error will have a direction dependence on the components of velocity in the centre of the interrogation volume. The magnitude of this error, \( |\langle E \rangle| \), is given by

\[
|\langle E \rangle| = \frac{\alpha \Delta t}{2|V_o|} \sqrt{u_o^2 + v_o^2}
\] (2.82)

where \( \alpha = \frac{\partial u/\partial y - \partial v/\partial x}{2} \) evaluated at \( a=0, t=0 \). Since

\[
|V_o| = \sqrt{u_o^2 + v_o^2},
\] (2.83)

then \( |\langle E \rangle| \) will equal
\[ |\langle E \rangle| = \alpha \Delta t / 2 \]  

(2.84)

This result can be re-written in terms the optimisation variables \( \phi D_r \) and \( D/d \) where with reference to equation 2.71,

\[ |\langle E \rangle| = \frac{\phi D_r}{2 \sqrt{2} (D/d)} \]  

(2.85)

Hence the first order model shows that for a forced vortex the magnitude of the systematic error in measurement is simply dependent on the product \( \phi D_r \) for a given value of \( D/d \). Results from the Monte Carlo simulation are shown in figure 2.16 for a range of \( \phi D_r \) and velocity \( V_0 \) and confirm this characteristic. This result also agrees with the theoretical prediction of Keane and Adrian [2.2].

The systematic error, however, is of the same order as that introduced by the Taylor series expansion of the particle displacement given in equation 2.11. This suggests that the simulation based on a first order model has oversimplified the true particle displacement and is therefore unable to distinguish the systematic error. To investigate this problem in more detail the Monte Carlo simulation was adjusted to include the third order terms of the particle displacement model (equation 2.11). Results from this modification are shown in Figure 2.17 and are compared to results from the first order model. They show a reduction in the systematic error in this case to a level where the magnitude of error does not appreciably change with increasing velocity gradient strength. Therefore the application of the third order particle displacement model shows the systematic error for a forced vortex to be maintained at around 0.5% and hence is independent of velocity gradient strength.

Although a first order model of fluid velocity was specified in the work of Keane and Adrian [2.2] the method by which the particle displacement was obtained was not clear. The above result, however, indicates that when a first order Taylor series approximation is used to calculate the displacement similar errors resulted. Clearly, further work is necessary to refine this process of error estimation to conclude whether or not systematic errors are significant.
2.4 Closure

A theoretical model of the complete PIV system has been derived using a set of Fluid Flow Parameters, User Controllable Variables and Performance Parameters. Analysis of the theoretical model has shown the mean SNR characteristics can be reduced to a baseline SNR function and three SNR loss functions which represent the fluid flow parameters and the autocorrelation function. This has allowed an optimisation method to be developed which permits the user to define the worst case fluid flow scenario which will ensure 50% of valid vectors. Validation of the optimisation method was carried out by using a Monte Carlo simulation based on the theoretical model of the system. Comparisons between the optimisation predictions and Monte Carlo predictions has shown a match to within 2%. This important new optimisation method provides a consistent basis for experimental design.

In addition, Monte Carlo simulation results show that the accuracy of PIV measurements results from three identifiable mechanisms:

Firstly, a random error is observed which is due to both noise in the vicinity of the signal peak and the fact that the particle image pairs only represent a finite sample of the flow statistics within a given region. Errors due to this mechanism are of the order of 0.4% of full scale particle displacement, \( L \), with zero velocity gradients rising to approximately 0.8% for significant gradients \( (\phi D_r = 1.0) \). With \( D/d = 16 \), this random error corresponds to an error in velocity measurement of around 6% and 13% respectively when measuring a particle displacement equivalent to a single particle image diameter. This is a significant error in velocity measurement and therefore to reduce its magnitude the user must optimise the velocimeter to allow the greatest possible particle displacement within the interrogation volume.

Secondly, substantial increases in random error are observed when the resolution of the interrogation region is decreased. It is shown that each particle image must be covered by a least 4x4 pixels to suppress this additional noise mechanism. Experimentally this means autocorrelation resolutions of between 64x64 to 256x256 pixels are required according to the given choice of dynamic range and velocity gradient restrictions.

Thirdly, a systematic error in the velocity measurement is observed which is directly dependent on the product \( \phi D_r \) and has a direction dependent on the components of velocity being measured. This error is also dependent on the particle displacement model used in the simulation. The work has shown if the particle displacement model
is oversimplified by using only first order terms from a Taylor series, the systematic error measured will be due to the approximations in the model and will not be due to the PIV system characteristics. For the case of a forced vortex, results from a particle displacement model using higher order terms has shown no significant increases in error for an increase in velocity gradient strength. Therefore in the presence of strong velocity gradients ($\Phi D_r = 1.0$) this error can be ignored when compared to the magnitude of the corresponding random error in velocity measurement.

Finally it should be noted that by calculating the necessary experimental parameters through consideration of the SNR provides an elegant solution which can be extended to include SNR loss mechanisms originating from a photographic and digitising stage. The following chapter will apply these concepts to the optimisation method.
3.0 TRANSPARENCY ANALYSIS

3.1 Introduction
The principal objective of the work described in this chapter is validation of the theoretical work and Monte Carlo simulation described in the previous chapter. This was achieved by analysing a comprehensive set of PIV transparencies representing well defined flows with varying mean velocities and velocity gradient strengths. The results from the transparencies also allowed an investigation into further SNR loss mechanisms which take into account the transparency recording and digitisation effects. Quantifying these mechanisms permits an extension of the optimisation method of the previous chapter to include experimental effects.

The chapter is divided into two main parts. In the first part the transparency recording technique is described along with analysis of the transparency particle image density and particle image characteristics. The second part of the chapter compares results from the processed transparencies to results from the theoretical work and Monte Carlo simulation. From these comparisons a number of SNR loss mechanisms are proposed.

3.2 Simulated PIV Transparency Data
The following section describes the technique developed to record sets of PIV transparency data. The technique allowed the recording of in plane, out of plane and velocity gradient data onto transparency format for a range of particle image sizes and particle image densities. The transparency format introduced imperfections such as film grain noise and particle image distortion that are normally present on PIV photographic recordings and which were not present in the Monte Carlo model. The technique allowed the parameters in plane movement, out of plane movement and velocity gradient strength to be pre-determined by the user so that direct comparisons with the Monte Carlo simulation and theoretical work could be made.

The data was recorded onto 35mm film by photographing hollow glass spheres suspended in solid optical araldite blocks and illuminated by a laser light sheet. Figure 3.1 shows a schematic of the system. The araldite blocks were manufactured by permanently setting Ciba Geigy Optical Araldite MY753+XD716 into open top glass containers. The glass containers were constructed from sheet glass pieces adhered together using silicone sealant. This method gave acceptable viewing properties with good non-scratch characteristics. Air bubbles which formed in the resin during mixing with the hardener were removed by allowing the resin to gel under low vacuum.
Particulate contaminants in the resin were removed from each constituent before moulding by using a 0.5\,\mu m polycarbonate filter system.

The particles chosen to seed the araldite blocks were Potters Ballotini Sphericel\textsuperscript{TM} hollow glass spheres with the smallest size range available having a diameter of 45-100 \,\mu m. They were chosen because of their excellent scattering properties in the resin. The original 45-100\,\mu m diameter range was wire mesh sieved down to a 45-60\,\mu m size range before seeding the resin to improve the homogeneity of the sample. Approximate seeding densities were initially determined by the amount of seeding weight added to the resin before moulding.

To obtain the relevant in plane and out of plane movements the block was simply translated in the corresponding plane between exposures. For velocity gradient data, a forced vortex flow was recorded by rotating the seeded block on a rotary stage as shown in figure 3.1. To record the transparencies, a Nikon F3 camera system and 105mm Micro-Nikor lens was used in conjunction with 35mm Kodak Technical Pan film developed in D19. All images were taken at a demagnification of 6:1 to reduce particle image size to approximately 20\,\mu m. Subsequent visual inspection under the microscope of 35mm test photographs showed well resolved particle images with low levels of film noise. Figure 3.2 shows an examples of double exposed regions taken from the transparencies.

3.3 Transparency Seeding Statistics

Preliminary analysis was completed on single exposure transparencies to determine particle image diameter, particle image intensity and particle image density characteristics. The analysis was carried out using the digital image plane system illustrated in figure 3.3. The system was PC based and consisted of an ITI Overlay Frame Grabber (OFG), an ITI Image Processing Accelerator board (IPA) and a Photon Control x-y traverse.

Two analysis algorithms were developed to obtain the particle image statistics diameter, particle image density and peak intensity. The first algorithm was used to size particle images at lower image densities ($N_i<30$) and the second algorithm was used to estimate particle image density for the remaining high density transparencies. Both algorithms were designed to scan the complete negative at pre-set increments using the image plane data processing system described previously.
The first algorithm consisted of a particle sizing method based on scanning the profiles of individual particle images and calculating their diameters by taking a $1/e^2$ threshold along each profile. The routine stored the particle image size, particle image intensity and particle image count for each region. Because this technique relies on discrete particle images to obtain an accurate sizing and particle count, when the routine was applied to dense seeding situations ($N_i > 30$) unsatisfactorily results were obtained. Further work showed this to be caused by insufficient separation of particle images. Therefore to allow particle image density estimates to be obtained from the higher seeding density transparencies a second routine was developed using the same image processing set-up.

The second algorithm estimated the particle image density by analysing binarised images of each interrogation region. The algorithm assumed that the average pixel value of a binarised interrogation region was directly proportional to the number of particle images in the region. To apply this assumption the average particle size in the region must be known and a binarisation threshold must be set. In the first case the mean particle size obtained from the previous algorithm was used. In the second case the binarisation threshold was adjusted until average particle image densities obtained from the first algorithm matched with results from the second algorithm for the same transparency. Using both routines it was found that the manufactured blocks had particle image densities in the range $N_i = 3$ to 120 particle images/mm$^2$.

Figure 3.4 shows a typical particle image size distribution where the range of mean diameters from the blocks was found to be between 18-24µm which is a realistic size for high speed PIV experiments. Figure 3.5 shows an example of a particle intensity distribution obtained from the analysis. The threshold for the minimum particle image intensity corresponds to a 165 limit in the intensity scale which was obtained from the noise statistics of the digitised image. The wide range of intensities can be attributed to the particle size distribution and non-uniform shell thickness of the particles in the resin. Figure 3.6 shows a typical particle image density distribution which is Poisson in nature and reflects expected flow field particle image density statistics.

In all subsequent results the particle image density, $N_i$, will refer to the number of first exposure particle images/mm$^2$.

3.4 Transparency Analysis

The following section describes the main transparency analysis which was completed using autocorrelation analysis. The section is split into three parts. In the first part the
autocorrelation analysis system is outlined. Following this the results from the analysis are compared to results from the Monte Carlo simulation and in the final part of the section these comparisons are discussed.

3.4.1 Transparency Analysis System

The main transparency analysis was completed using the digital image plane system outlined in figure 3.3. The analysis of each transparency consisted of retrieving the signal peak and noise peak values from a sample of interrogation regions. The following algorithm completed this analysis:

1) Increment transparency to required position
2) Grab image and download from frame store to IPA
3) Apply forward Fast Fourier Transform (FFT) to image
4) Convert output to power spectrum
5) Mask off dc pixel
6) Apply reverse FFT to obtain autocorrelation
7) Mask off dc peak
8) Store maximum signal peak value and mask off signal peak
9) Store maximum noise value in autocorrelation plane

To ensure the transparency analysis had comparable SNR and valid vector criteria as defined in chapter 2 section 2.3, the correlation plane analysis was based on the Monte Carlo simulation. Therefore with reference to the Appendix 3, section 2.2, for analysis of the in plane and out of plane sets of data, the search for the signal peak value was limited to within a circular area of diameter $\sqrt{2}d$. The centre of this search area was at a fixed position in the correlation plane corresponding to the expected signal peak position. This position was pre-determined from the known transparency particle image displacement. The noise peak was found by searching the remaining half of the correlation plane. For the velocity gradient analysis the signal peak search area could not be placed at a fixed position. Therefore the vortex centre on the transparency was initially located to within two pixels by autocorrelating several interrogation regions in a known x-y grid around the centre of the vortex. From the grid data several best fit lines were used to determine the vortex centre. The signal peak search area was then incremented around the correlation plane during the analysis using the known velocity gradient strength and interrogation position from the vortex origin. Also with a known value of $\phi D_r$ from the transparencies, the search area in this case was defined by the area limits of equations A3.5 and A3.6 in Appendix 3.
All the following transparency results were analysed at a pixel resolution of $R_N=256$ to allow direct comparisons with the Monte Carlo results.

### 3.4.2 Results and Comparisons

Figure 3.7 shows the percentage of valid vectors obtained for 10% in plane displacement as a function of particle image density, $N_i$. The results show that the expected number of valid vectors does not increase appreciably when the initial number of particle images, $N_i>12$. This trend matches well with those of the Monte Carlo simulation.

Figure 3.8a and 3.8b show the percentage of valid vectors as a function of in plane displacement for a range of particle image densities, $N_i$. For the higher particle image densities ($6<N_i<28$) the experimental trends follow that of the Monte Carlo simulation. The $N_i=3.1$ experimental result, however, is significantly lower than that predicted by the Monte Carlo simulation. This trend is again replicated in the out of plane results which are shown in figures 3.9a and 3.9b. The reasons for this will be discussed in the next section.

Figure 3.10 shows the percentage of valid vectors as a function of $\phi D_r$ for a fixed dynamic range of $D_r=4$ and a particle image density of $N_i=12$. These results show a degradation in the level of experimental valid vectors from the Monte Carlo results as the velocity gradient is increased from zero.

These discrepancies will be discussed in the following section.

### 3.4.3 Experimental Valid Vector Loss

The valid vector loss from the transparency analysis can be attributed to three SNR loss mechanisms. These are characteristics of the transparency recording, CCD camera noise and the autocorrelation function itself.

The most significant SNR mechanism originated from the noise characteristics of the CCD camera. The noise from the camera, unfortunately, was found to be highly correlated in the vertical direction and occupied up to four bits of the digitised data. Figure 3.11 shows an autocorrelation function at a particle image density of $N_i=3.1$ and an in plane movement of 10% of $D$ where the d.c. peak has been removed. The correlated noise appears in the correlation plane as a decaying periodic profile running along the complete $\xi$ axis. The signal peak in this case sits on top of this profile. The periodic profile originates from the autocorrelation algorithm where at the power
spectrum stage, the single dc pixel is set to zero. This causes the noise structure in the next transform to take a cosine type profile.

The effect of the CCD noise structure is significant at the lower particle image densities (N_i<6) or in the presence of velocity gradients (\phi_{D,>}0). In these situations the signal peak height is small in relation to the noise structure and the noise peak will always be measured along the centreline of the CCD correlation noise structure therefore reducing the SNR and the percentage of valid vectors. At higher particle image densities (N_i>6) the signal peak will be significantly higher than the CCD noise structure resulting in a good match to the number of Monte Carlo valid vectors.

To illustrate the magnitude of the CCD noise, additional autocorrelation analysis was carried out using the lower four bits of digitised data. An autocorrelation function from this analysis is shown in figure 3.12 and is taken from the same interrogation region used for figure 3.11. The result shows removal of the CCD correlation noise structure. The resulting increase in SNR is illustrated in figure 3.13 where results from a three bit transparency analysis are compared to results from an 8 bit transparency analysis and the Monte Carlo simulation for a range of particle image densities. It can be seen at the lower particle image densities the three bit result matches more closely to Monte Carlo result.

Results in figure 3.13 show, however, there is still a 3dB loss in SNR at the higher particle image densities from the transparency analysis relative to the Monte Carlo simulation. This 3dB loss can be attributed to transparency recording characteristics such as film noise and variations in particle image intensity. The film noise will be constant for all transparencies and simply increases the correlation noise floor level by a fixed amount resulting in a decrease in mean SNR when compared to equivalent Monte Carlo results. The variation in particle image intensities will also affect the correlation noise characteristics and will be random in nature. The Monte Carlo simulation did not account for these variations and further work is needed to quantify the loss of overall SNR as a result of these affects.

3.5 Closure
Work has been completed which attempts to validate the Monte Carlo simulation of the previous chapter. To achieve this, a novel method of producing realistic PIV transparencies has been demonstrated which involves photographing solid seeded araldite blocks using a low powered laser sheet for illumination. The method has
allowed sets of in plane, out of plane and velocity gradient data to be recorded such that direct comparisons can be made to equivalent theoretical autocorrelation results.

The combined effect of recording particle images photographically and subsequently digitising each interrogation region causes a drop of approximately 4-5 dB in SNR compared to results obtained from Monte Carlo simulation. With reference to the chapter 2, figure 2.5, since baseline values of SNR are typically 12dB this is a significant degradation. In practical terms with reference to the calculations of Appendix 5, this loss in SNR can be likened to the effect of a 60% drop in dynamic range.

The majority of the SNR degradation at low particle image densities and strong velocity gradients was shown to be caused by CCD camera noise which in a digitised form was up to four bits of data. Therefore any digital based PIV system must aim to use the highest quality CCD to reduce these effects or methods must be used to thresholding out the CCD noise. At higher particle image densities degradation was thought to mainly originate from recording effects of the PIV data onto transparency such as film grain noise and particle intensity variation. Further work investigating these effects, however, must be carried out.
4.0 SUPERCSONIC FLOW STUDIES

4.1 Introduction
The following section outlines work concerning in the application of PIV to an enclosed high speed flow. The main objective of the work was to assess the feasibility of applying supersonic PIV to a turbomachine environment.

In order to assess the performance of the technique data was recorded from a Mach 1.4 de Laval expansion nozzle. An expansion nozzle design was chosen because it would allow a simple, enclosed supersonic flow regime to be obtained which would replicate a turbomachine environment. The nozzle would also allow the investigation of a single normal shock of varying strength. One of the flow study objectives was to compare the PIV nozzle results with CFD and with this nozzle configuration CFD modelling would be straightforward. Finally, by moving the laser sheet and recording system, the open exhaust jet could be analysed for a range of plenum chamber pressures. This would provide a more complex supersonic flow structure to compare with the internal nozzle flow.

The first part of the chapter will describe the nozzle/rig design, experimental set-up and data analysis system. In the second part an analysis of practical considerations of the recording system is completed. Following this the optimisation method is used to optimise spatial resolution, SNR and valid vectors. Data obtained from a CFD prediction of the nozzle flow field is used in this analysis. Finally the PIV results obtained from the rig are assessed and conclusions made.

4.2 Rig Design/Experimental Set-up
Figure 4.1 illustrates the set-up used for the recording stage and the following section will describe the design and implementation of the PIV set-up in more detail.

The two major parts of the rig consisted of a nozzle and a plenum chamber. The nozzle was a scaled down version of a Plint & Partners Mach 1.4 supersonic wind tunnel and consisted of a rectangular section de Laval expansion nozzle with a throat size of 10mm square. This scale is representative of minimum sizes that are found inside aero gas turbines. The nozzle was an all glass design constructed by sandwiching two identical preformed sides between two glass plates and then cementing them together with epoxy. The shape of the preformed sides was accurately produced by bending glass strips at 800°C over a CNC machined high temperature ceramic former. The nozzle was then clamped down to a plenum chamber containing a
converging exit in the form of a V-type slot. This gave a combined plenum exit to throat contraction ratio of 30:1.

The plenum chamber design consisted of a high pressure cylinder of diameter of 200mm and height 500mm with an air line feed at one end and the nozzle inlet at the other end. The initial design was seeded by direct injection into the plenum chamber, where after mixing it would pass though the nozzle. This design was found to cause significant problems with particles coating the nozzle walls. Coagulation of the particles rapidly formed droplets which moved up the walls, causing aberrations of the PIV images and therefore a new type of seeding system was required. The new design consisted of a seeding plenum chamber within the main plenum chamber incorporating a 3mm diameter injector tube centrally protruding into the glass nozzle to a point 30mm upstream of the throat. This design is illustrated in figure 4.2. By injecting seeding into the centre of the nozzle, it was found that the wall coating problem was reduced sufficiently to allow several PIV pictures to be taken before the nozzle required cleaning. The disadvantage of this design was that the whole flow field could not be investigated. Figure 4.3 illustrates a typical PIV transparency with central seeding injection.

The seeding system was a commercially available TSI six jet atomiser which allowed seeding to be injected under pressure into the plenum chamber. By choosing one to six jets on the seeder, seeding densities could be easily adjusted. Analysis in the following sections shows a 0.4\mu m diameter particle to be suitable for the accelerations encountered in the flow. To obtain this diameter a mineral oil (OLNA DS15) was used which gave a mean particle size of around 0.4\mu m \[4.1\].

The recording device was a 35mm Nikon F3 professional system with a 105mm Micro-Nikor lens. In this case a demagnification was chosen of between 1.5 and 1.0. Focusing of the lens with respect to the laser sheet was achieved by using a Nikon DW-4 12x eyepiece attached above a clear ground glass D type focusing screen. This system was found to be sufficient for f numbers of f8 and above after careful alignment of the camera and the laser sheet optics relative to the laser table mounts.

The laser was a Spectra Physics double pulsed YAG system with a frequency doubler giving output light of \lambda=532nm. Maximum output power available was 150mJ per pulse. Pulse duration was the order of 10ns and pulse separations of 0.15\mu s to infinity were possible with the driver circuits used.
Data processing was carried out using a commercially available digital image plane autocorrelation system called Visiflow produced by AEA. The system has centroid peak analysis and selectable interrogation region pixel resolution. No time constraints were present during data processing and therefore a pixel resolution of 256x256 was chosen to provide high accuracy. Using this resolution one transparency was processed in 45 minutes to one hour to yield 800 data points. In the presence of time constraints, however, the minimum allowable resolution of 64x64 could be used resulting in a processing time of under 10 minutes for the same transparency. The magnitudes of expected accuracy will be discussed in section 4.4.

In addition to the PIV experimental analysis, Schlieren work was carried out on the nozzle and open jet to define the shock locations and geometry. The pictures were exposed for an 8ms period.

4.3 Practical Considerations
The following section outlines the calculations used to design the experimental set-up. The study will be based on a light sheet thickness of 0.4mm. This estimate is related to the expected two dimensional nature of the flow field to be investigated inside the nozzle. Therefore out of plane components are considered negligible.

4.3.1 Optical Requirements
Consider the optical requirements of producing a light sheet of collimated thickness 0.4mm. In this case, the system was based on an arrangement shown in figure 4.4. In this configuration L1 is a plano convex cylindrical lens and L2 a plano concave lens. The system will collimate the light sheet width in the axis perpendicular to the recording device and expand the sheet in the axis parallel or in plane to the recording device. If the initial YAG laser beam has a typical diameter of around 6mm a contraction ratio of 15:1 is required to reduce the beam to 0.4mm thickness. With the arrangement shown in figure 4.4, the contraction ratio is equivalent to \(-f_1/f_2\) [4.2]. Therefore if \(f_1=400\)mm then the second plano concave lens will have a focal length of \(f_2=-25\)mm.

The separation of the two lenses \(s_1\), can be calculated from standard Gaussian Optics formulae [4.2]. Using the standard formulae for a combined lens system where \(f\) is the combined focal length of the system from L1 then,

\[
f = \frac{f_2(f_1-s_1)}{f_1+f_2+s_1}, \tag{4.1}
\]
To collimate the output beam, $f_1 + f_2 + s_1 = 0$. Therefore the separation required is $s_1 = 375\text{mm}$.

With a light sheet thickness of 0.4mm, the depth of focus of the recording device must be matched to this dimension. Using PIV based formulae [4.3], the depth of field $\delta z$ can be found from

$$\delta z = 4\left(1 + M^{-1}\right)^{3/2} f_n^2 \lambda$$

(4.2)

where $M$ is the magnification of the system, $f_n$ is the f number of the system and $\lambda$ is the wavelength of light. Using YAG laser light at $\lambda=532\text{nm}$ and a system demagnification of 1.5, the required f number for sufficient depth of field will be $f_n=8$. With a demagnification of 1.5 an area 52x37mm can be recorded which is sufficient for the area of interest inside the nozzle.

4.3.2 Particle Tracer Requirements

The non-intrusive velocimeter technique LDA has already established suitable tracer materials for supersonic flows [4.4]. A suitable particle must sufficiently follow the strongest velocity gradients encountered in the flow and this choice is based on the ability of a particle to respond to a sudden acceleration such as is found in a normal shock region. Previous work [4.5] has indicated an estimate of the particles response to step change in velocity, as is found in a shock, can be made using the formula

$$\frac{|U|}{|U_0|} = \exp\left(-\frac{18\mu}{d_p^2 \rho_p} \frac{t}{\rho_f}\right)$$

(4.3)

where $d_p =$ particle diameter
$\rho_p =$ particle density
$\mu =$ dynamic viscosity
$V_p =$ particle velocity
$V =$ fluid velocity
$|U| =$ $|V_p - V|$ (slip velocity)

If the minimum allowable slip velocity is defined as 5% of the step change, the relaxation time $t$ taken for a particle diameter $d_p=0.4\mu\text{m}$ will be $1.23\mu\text{s}$ where the

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initial step change $|U_0| = 150 m/s$, $\rho_p = 800 kg/m^3$ and $\mu = 17 \times 10^{-6}$ Pas. This corresponds to a relaxation distance of 0.3 mm. Therefore any spatial resolution greater than the relaxation distance will result in a velocity measurement equal to the relaxation profile average across the shock front.

4.3.3 Film Requirements

The film chosen must have high contrast, a low noise base, sufficient speed and acceptable resolution. To find the minimum acceptable film resolution the particle image diameter must be estimated. Because the particle size is sub-micron at $M = 0.67:1$ the particle image will be diffraction limited and can be found using the formulae from reference 4.3 where

$$d = 2.44(1 + M)f_n\lambda.$$

With $M = 0.67:1$, $f_n = 8$ and YAG laser light at $\lambda = 532 nm$ the recorded diameter $d = 17 \mu m$. Therefore the required resolution of the film in terms of the modulation transfer characteristic is approximately $(1/d) = 60$ line pairs/mm. Kodak Technical Pan has a resolution of 150 line pairs/mm [4.6] and acceptable base and contrast characteristics.

4.3.4 Laser Requirements

An estimation of the required laser power to expose the particles on Kodak Technical Pan high contrast film is now considered. Typical ratings of the film are 25-125 ASA with 0.3-1 mJ/m² for an acceptable exposure [4.6]. Applying the PIV based formulae [4.3] an estimate of the required exposure energy $W_e$ can be calculated using the exposure rating of the film $\bar{e}$. For a 0.4 mm diameter particle $W_e$ is estimated from

$$W_e = \frac{2.44^2 \lambda^4 s_o^2 s_i^2 h_s \bar{e} W}{d_p^4 d_a^4}$$

(4.5)

where $s_o$ and $s_i$ are object and image distances respectively, $h_s$ and $W$ are the sheet height and sheet width respectively, $d_a$ is the lens aperture diameter, $d_p$ is the particle diameter and $\lambda$ is the wavelength of light. Using YAG at $\lambda = 532 nm$, with $d_a = 13 mm$, $s_o = 160 mm$, $s_i = 105 mm$, $h_s = 200 mm$, $W = 0.4 mm$ and $\bar{e} = 1 mJ/m^2$, the estimated required exposure energy $W_e = 15 mJ$. This is the minimum acceptable level of light to illuminate the particle. In practice the actual laser power required could be 2-4 times higher depending on losses in the optics and the nozzle.
Finally a criterion must be stated concerning the pulse duration of the laser. If the pulse duration is too long the particle images will become distorted in the direction of the flow. Therefore the criterium adopted will be

\[ \left| [\dot{V}]_{\text{max}} \right| \Delta t < d/2 \]  \hspace{1cm} (4.6)

where \([\dot{V}]_{\text{max}}\) is the maximum fluid velocity. Consequently for a mainstream velocity of 400m/s the minimum acceptable pulse duration will be 20ns for a mean particle image diameter of 17\(\mu\)m. The YAG laser system outlined in the previous section fulfils this requirement.

4.4 Data Processing Optimisation

Work in the previous two chapters has concentrated on the optimisation of processing PIV data in relation to variables controlled by the user at the recording stage. These criteria will now be applied in the following section to maximise the spatial resolution and the percentage of valid vectors given a-priori information about the expected dynamic range and velocity gradients in the flow.

4.4.1 Theoretical Analysis of Fluid Flow

To obtain the required a-priori information a CFD prediction was produced for the flow inside the nozzle. This work was completed by Dr Marios Christoudoulou at The Applied Science Laboratories of Rolls Royce plc, Derby. The package used was called Phoenics and was applied to the nozzle area profile and plenum/atmospheric conditions. The software was based on an iterative solution to the Navier Stokes equation and included K-\(\varepsilon\) turbulence modelling. The output data could be in a velocity, Mach or pressure format as requested.

Additional theoretical work was completed to produce a 1D theoretical prediction for comparison with the PIV results. Also outside the nozzle, simple shock cell theory was used for comparison with the PIV data. Details of this work are outlined in Appendix 6.

4.4.2 Optimisation of PIV Variables for Data Processing

To apply the optimisation method several parameters are required to determine the variables \(D/d\), maximum in plane movement, maximum out of plane movement and the dynamic range and velocity gradient strength product \(\phi D_r\). The fluid flow parameters are taken from the CFD prediction and the spatial resolution is the minimum required for a comparison with the CFD results. These parameters are listed as follows:
i) Maximum Fluid Velocity 
\[ V_{0,\text{max}} = 450 \text{ m/s} \]

ii) Dynamic Range 
\[ D_T = 2 \]
Outside shock region \( \phi = 5\% \)
Inside shock region \( \phi = 40\% \)

iii) Maximum Velocity Gradient:
\( L = 0.5 \text{ mm} \)

iv) Required Spatial Resolution
To maximise spatial resolution the smallest particle image must be used which is fixed
at \( d = 17 \mu \text{m} \) by recording optics characteristics, available laser power and particle tracer
requirements. With a demagnification of 1.5, this establishes \( D/d = 20 \). Also from the
work in the previous two chapters the particle image density required will be \( N_p \geq 12 \).

In order to apply the optimisation method the \( S_B \) curve (Figure 2.7), the in plane loss
of SNR curves (Figure 2.8) and the velocity gradient loss of SNR curves (Figure 2.10)
are used. Also it will be assumed that a 4dB loss occurs through digitising and the use
of a transparency format. The following now outlines the optimisation calculation.

1. Obtain value of \( S_B \) from Figure 2.7 \( S_B = 9.0 \text{ dB} \)
2. Specify full scale in plane movement \( 30\% \) of \( D \)
3. Obtain in plane loss in dB from Figure 2.8 \( 2.7 \text{ dB} \)
4. Subtract transparency/digitisation loss in dB \( 4.0 \text{ dB} \)
5. Look up \( \phi D_T \) from Figure 2.10 \( \phi D_T = 0.5 \)
6. Calculate \( \phi \) from \( D_T \) \( \phi = 25\% \)

Therefore the maximum allowable velocity gradient strength to ensure 50\% of valid
vectors will be \( \phi = 25\% \). Because the CFD predicts a maximum velocity gradient in the
shock region of 40\% the PIV technique will be unable to achieve 50\% of valid vectors
in the shock region. Outside the shock region, however, providing particle image
density levels are sufficient (\( N_p \geq 12 \)), significantly greater levels of valid vectors should
be possible since the maximum velocity gradient strength in this area of the flow is
of the order, \( \phi = 5\% \).

To estimate the accuracy of the PIV data we must consider both the random and
systematic error in measurement. If it is assumed that the mechanisms which determine
accuracy in the following PIV application are similar to those of chapter 2, section
2.3.4, then estimates of random and systematic errors can be made. Therefore with
reference to figure 2.11, if \( R_N = 256 \) the random error in measurement outside the
shock region, where from CFD data $D_r=0.1$, is expected to be 0.5% of full scale length, $L$. The dynamic range of the technique from the SNR optimisation corresponds to particle movements of between 15% and 30% of $L$ which results in the random errors in velocity measurement of between 3.3% and 1.7% outside the shock region. In the shock region where CFD data predicts a value of $D_r=0.8$, the expected random error will be 0.8% of full scale length, $L$. The particle displacement in this region corresponds to 22% of $L$ which results in a random error in velocity measurement of 3.6%. For the systematic error, with reference to figure 2.16, using the third order model estimate, the magnitude of the expected error in velocity measurement will be 0.5% anywhere in the flow.

Therefore it should be noted that although the estimate of random error in the shock region is similar to the majority of the PIV data, the number of valid vectors in this region is expected to be unacceptably low for CFD validation. Outside the shock region, however, the PIV data provides an acceptable basis for comparison and validation.

4.5 Results

The following section presents results obtained from the nozzle and compares these with the CFD prediction, the theoretical models outlined in Appendix 6 and previous work [4.10-4.13].

4.5.1 Internal Nozzle Results

Obtaining results from inside the nozzle was always restricted by window contamination and poor mixing of the seeding which resulted in a reduction in the number of valid vectors. An illustration of typical window contamination is illustrated in figure 4.5 where it can be seen in the centre section of the nozzle. This figure is a Schlieren photograph of the nozzle with the normal shock clearly visible. The shock positions obtained from these photographs were used for direct comparison with the PIV results. Figure 4.6 shows a PIV vector map obtained from inside the nozzle with central seeding injection and a plenum chamber gauge pressure of 62.0kPa. A global mean of $200\text{ m/s}$ has been subtracted from the vector magnitudes to facilitate the shock location. Gaps in the central areas of the vector map are due to poor mixing of the seeding and window contamination.

Previous work [4.10] has attempted to avoid window contamination by using a short duration blow down facility and low seeding densities. The low seeding density, however, significantly restricted the spatial resolution possible to around 2mm/point.
with 200 vector points/map. In this case the spatial resolution is 0.5mm/point with between 600-800 vector points. This improvement is due to the increased seeding levels which were achieved in the rig but is also at the cost of significant window contamination.

The vector map shown in figure 4.6 covered approximately 25% of the total central cross sectional area of the nozzle. The exploded view of the vector map illustrates the shock area which can be clearly seen from the change in vector magnitude. This result shows that PIV can resolve the shock across one interrogation region which is 0.54mm wide. The number of valid vectors in the shock region, however, is significantly less than the remaining area of the vector map. This confirms the optimisation method prediction and illustrates the effects of the strong velocity gradients on data output in this region of the flow.

Figure 4.7 illustrates the u component of velocity with the vector gaps filled by averaging the four nearest surrounding valid vectors. This method is applied to a result at 62.0kPa and 41.3kPa plenum chamber pressure. The result shows the 41.3kPa plenum pressure produces a significantly weaker shock which is located approximately 10mm upstream of the stronger 62.0kPa plenum pressure shock position.

4.5.1.1 One Dimensional Theoretical Comparisons

Figure 4.8 is a graph containing results from the PIV analysis, the 1D theoretical analysis and the Schlieren analysis all at a plenum chamber pressure of 62.0kPa gauge. The PIV result is in the form of a single streamline taken in the central area of the vector map. The Schlieren and PIV shock locations agree well to within one PIV interrogation region or 0.54mm.

The 1D theory at 62.0kPa predicted the shock wave outside the nozzle. The shock location only compared favourably with the PIV results at 45.5kPa. There is a significant difference, however, in the shock strength of approximately 50m/s and after the shock this discrepancy is maintained. There are also fluctuations in the PIV data of around ±10% of the measured velocity downstream of the shock which are significantly greater than the random error estimate of section 4.4.2 of ±3.3%. This result indicates possible boundary layer separation downstream of the shock. Further discrepancies between the 1D prediction and PIV results are attributed to the presence of moist air in the air supply which was not cleaned or dried. Moist air would through condensation processes affect the expansion process and hence the shock position.

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4.5.1.2 CFD Comparisons

Figure 4.9 is a graph comparing results from the CFD prediction with PIV and Schlieren results. At 62.0kPa plenum the CFD predicts a shock front approximately 12-15mm downstream of the PIV and Schlieren results. The magnitude of velocity change across the shock is comparable to the PIV results. Downstream of the shock the CFD prediction matches well with the mean value of the PIV measurements. At 55.1kPa plenum the CFD shock location is a closer prediction to the PIV result, but there is a greater discrepancy in the magnitude of velocity downstream of the shock. These discrepancies are attributed to mechanisms outlined in the previous section.

4.5.2. Open Jet Results

The open jet results had significantly greater spatial resolution and data quantity in comparison with the internal nozzle results. These improvements were possible because of the absence of window contamination. This allowed the seeding density to be increased to obtain greater spatial resolution from the full width of the jet flow. For example the maximum attained spatial resolution for the internal results was 0.54mm/point with around 800 vector points per map while this was increased to 0.34mm/point for the open jet results with around 3000-4000 vector points/map. This spatial resolution is considerably greater than previous open jet work [4.11-4.13] which achieved a spatial resolution of 1-2mm/point respectively. Figure 4.10 shows the x component results for two plenum chamber pressures. It can be seen from these results that a shock cell structure exists in both cases which is highlighted by the presence of a decaying wave train trend in the central velocity profile. If the wavelengths from these two cases are taken and compared to the predictions from Pack's formulae, (see Appendix 6) results match well as shown in table 4.1.

<table>
<thead>
<tr>
<th>Pressure (kPa)</th>
<th>PIV $L_{CELL}$ (mm)</th>
<th>Pack's prediction $L_{CELL}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>144.7</td>
<td>7.1</td>
<td>6.4</td>
</tr>
<tr>
<td>186.0</td>
<td>8.9</td>
<td>8.1</td>
</tr>
</tbody>
</table>

A similar comparison can also be made by contrasting the time averaged Schlieren picture with the instantaneous x component PIV picture. This comparison is shown in figure 4.11 and clearly shows a relationship between shock cell structure in both results.
4.6 Closure

The PIV technique has been successfully applied to an enclosed high speed flow regime representative of the scale that would be encountered in a gas turbine. A PIV vector map was obtained in the central 25% of the nozzle area allowing location of a normal shock to an accuracy of one interrogation region (0.54mm).

Applying the optimisation method described in the previous two chapters allowed spatial resolution to be optimised outside the shock region to 0.5mm/point. This was a significant improvement over previous enclosed high speed work [4.10]. In the shock region, however, because of the significant velocity gradients the optimisation method predicted less than 50% valid vectors and this was confirmed from the results.

Seeding levels of at least 12 particles images per interrogation region recommended by the optimisation studies were achieved when seeding both the complete and central section of the nozzle. Because of the high mass flows and corresponding seeding levels involved in running the supersonic rig, significant window contamination took place for full field seeding. Injecting the seeding into the centre of the flow reduced the problem but limited the area which could be mapped. Therefore further work must be carried out to reduce window contamination problems before true full field PIV analysis at this scale is possible.

The limitations of the enclosed flow system in terms of window contamination were highlighted by analysing the open jet from the same nozzle. In this case spatial resolution was improved by 60% and the vector maps obtained allowed complex shock cell structures with a scale of less than 10mm to be observed across the complete jet structure.

The PIV results inside the nozzle were obtained with sufficient resolution to allow direct comparisons with 1D supersonic theory and CFD modelling. Both comparisons showed significant errors in both shock location and velocity magnitudes with the CFD giving a more acceptable prediction.
5.0 FURTHER METHODS

5.1 Introduction
The following chapter will demonstrate an alternative PIV data recording and processing technique developed during the research programme. This technique has two important benefits. First the ambiguity problem described in Chapter 1 of the thesis is overcome. Second the technique can be shown to result in a substantial increase in SNR. Practically this translates to significant benefits in dynamic range and velocity gradient tolerance and the technique can be justified for this reason alone.

In double exposure PIV no information is present to distinguish the order of the first and second exposure images. Therefore the calculated vector will be subject to a 180° ambiguity. In the vast majority of practical cases this is not a problem since the flow may be biased in a certain direction or a-priori information may be available at one or more points allowing the flow direction to be traced using continuity arguments. Turbulent high speed flows, however, may exhibit regions of rapid recirculation. In these cases extra directional information is required and the basic double exposure technique often fails.

5.2 Image Labelling Technique
A simple solution to the ambiguity problem is to label the first and second exposure images in such a way that they can be addressed independently. This technique is called image labelling. By recording the transparency in this manner, the first and second images can be separated allowing the application of cross correlation techniques which give an unambiguous vector and improved SNR. The following section will theoretically model the image labelling system and determine the SNR benefits of the technique.

5.2.1 Theoretical Considerations
The method used to label the first and second images on the same transparency is achieved by changing the transfer characteristics of the recording optics between exposures. This idea was originally proposed by Francon [5.1] as a means of multiplexing laser speckle modulated images. Since the size of recorded particle images is generally set by the diffraction limit of the lens, this technique is also applicable to the labelling of PIV recordings. Therefore the simplest method of labelling the first and second exposures is to code the point spread function. This can be accomplished by introducing a rotating plate with dual apertures into the iris plane of the imaging lens as shown in figure 5.1.
For such a system, the intensity distribution, \( I(x_i, y_i) \), observed in the image plane can be written in terms of the geometrical optics prediction of the image amplitude, \( U_g(x_i, y_i) \), by the relation,

\[
I(x_i, y_i) = |h(x_i, y_i) * U_g(x_i, y_i)|^2 \tag{5.1}
\]

where \( x_i, y_i \) are position variables in this plane and \( * \) denotes convolution. In equation 5.1, \( h(x_i, y_i) \) is the impulse response function which is defined in terms of the pupil function, \( P(\lambda s, \bar{x}, \lambda s, \bar{y}) \) by the Fourier relation [5.2],

\[
h(x_i, y_i) = \int \int P(\lambda s, \bar{x}, \lambda s, \bar{y}) \exp[-j2\pi(x_i \bar{x} + y_i \bar{y})] \, dx \, dy \tag{5.2}
\]

where \( \bar{x}, \bar{y} \) are spatial frequencies in the plane of the pupil, \( \lambda \) is the wavelength of illumination and \( s_i \) is the image distance. Hence, if the geometrical image of the particles is narrow with respect to the bounds of the impulse response function, the particles can be considered as point sources and the shape of the recorded particle images will be determined uniquely by this function.

For the case of two circular apertures of diameter, \( d_a \), symmetrically located with respect to the optical axis and separated in the \( y \) direction by a distance, \( D_s \), the pupil function can be written,

\[
P(\lambda s, \bar{x}, \lambda s, \bar{y}) = \text{cyl} \left[ \frac{r}{d_s} \right] \ast \left\{ \delta(\lambda s, \bar{x}, \lambda s, \bar{y} + D_s/2) + \delta(\lambda s, \bar{x}, \lambda s, \bar{y} - D_s/2) \right\} \tag{5.3}
\]

where the cylinder function \( \text{cyl}() \), is defined such that [5.3],

\[
\text{cyl} \left[ \frac{r}{d_s} \right] = \begin{cases} 
1 & 0 \leq r < d_s/2 \\
1/2 & r = d_s/2 \\
0 & r > d_s/2 
\end{cases} \tag{5.4}
\]

and \( r = \lambda s \sqrt{\bar{x}^2 + \bar{y}^2} \). The intensity distribution corresponding to a centrally located particle image is then given by,
\[ I(x_i, y_i) = \left( \frac{\pi \rho_i^2}{2(\lambda s_i)^4} \right)^2 \times \left[ \frac{J_1(\pi \rho_i / (\lambda s_i))}{\pi \rho_i / (\lambda s_i)} \right]^2 \times 2 \left[ 1 + \cos \left( \frac{2\pi D_i y_i}{\lambda s_i} \right) \right] \] (5.5)

where \( J_1 \) is a Bessel function of the first kind and \( \rho = \sqrt{x_i^2 + y_i^2} \). Therefore the intensity distribution is an Airy pattern modulated by cosinusoidal fringes with orientation defined by that of the aperture plate. Hence by rotating the aperture plate between exposures the first and second exposure particle images can be coded by altering their spatial frequency content. Consequently, if the images are recorded photographically they can be addressed independently via a simple spatial filtering operation using an optical configuration similar to that shown in figure 5.2.

Let us consider a positive PIV transparency which has been recorded using a camera with such an aperture plate inserted into the iris plane of the objective lens and rotated through 90° between exposures. Now, consider a small region of this transparency which contains the first and second exposure, fringe modulated images, of a single particle which has been displaced a distance, \((\Delta x, \Delta y)\). In practice, due to dynamic range restrictions, only the part of each image which is located within the central peak of the Airy distribution is faithfully reproduced by the photographic process. In this case, to a good approximation, the Airy distribution can be replaced by an appropriately scaled Gaussian distribution \([5.3]\). Hence,

\[ \exp\left[-\pi(\rho d_s / \lambda s_i)^2\right] = \left[ \frac{2J_1(\pi \rho d_s / \lambda s_i)}{\pi \rho d_s / \lambda s_i} \right] \] (5.6)

Thus, if the transparency is inserted into the object plane \((x_o, y_o)\) of the Fourier filter shown in figure 5.2, the complex amplitude, \(U(x_o, y_o)\) transmitted by this region can be modelled as
where the $\Delta x_l, \Delta y_l$ are the components of the particle image displacement and $A_o$ is a constant of proportionality. According to the Fourier transforming properties of convex lenses, the complex amplitude in the rear focal plane of the imaging lens, $U_f(\alpha, \beta)$, describes the complex spectrum of the spatial frequencies recorded within the illuminated region and is given by,

$$
U(x_o, y_o) = A_o \left\{ \exp \left[ \frac{-\pi \lambda s_i^2}{(\lambda s_i)^2} (x_o^2 + y_o^2) \right] \times \left[ 1 + \cos \left( \frac{2\pi D_x y_o}{\lambda s_i} \right) \right] + \right. \\
\left. \exp \left[ \frac{-\pi \lambda s_i^2}{(\lambda s_i)^2} \left[ (x_o - \Delta x_1)^2 + (y_o - \Delta y_1)^2 \right] \right] \times \left[ 1 + \cos \left( \frac{2\pi D_x x_o}{\lambda s_i} \right) \right] \right\} 
$$

(5.7)

where $\Delta x_1, \Delta y_1$ are the components of the particle image displacement and $A_o$ is a constant of proportionality. According to the Fourier transforming properties of convex lenses, the complex amplitude in the rear focal plane of the imaging lens, $U_f(\alpha, \beta)$, describes the complex spectrum of the spatial frequencies recorded within the illuminated region and is given by,

$$
U_i(\alpha, \beta) = B_o \left[ \frac{\lambda s_i}{d_s} \right]^2 \times \exp \left[ -\pi \lambda s_i^2 \left( \alpha^2 + \beta^2 \right) \right] \times \left\{ \exp \left[ -j2\pi (\alpha \Delta x_1 + \beta \Delta y_1) \right] \right. \\
\left. \exp \left[ -\pi \lambda s_i^2 \left( \alpha^2 + \beta^2 \right) \right] \times \left( \alpha, \beta - \frac{D_s}{\lambda s_i} \right) \right\} \\
\left( \alpha, \beta + \frac{D_s}{\lambda s_i} \right)
$$

(5.8)

where a quadratic phase factor has been omitted for simplicity, $\alpha, \beta$ denote spatial frequencies scaled such that $\alpha = x \lambda f$ and $\beta = y \lambda f$ and $B_o$ is a complex constant. Equation 5.8 shows that the light transmitted by the transparency is diffracted into a central Gaussian peak plus four symmetrically arranged satellite peaks also with Gaussian profiles. This arrangement is shown in figure 5.3 where the satellite lobe dimensions are determined from a $1/e^2$ diameter in equation 5.8 and the dc peak diameter is determined at an equivalent amplitude as the $1/e^2$ amplitude of the satellite lobes.
Providing the peaks are sufficiently separated a spatial filter element consisting of a single aperture can be placed in this field in such a way as to isolate any of the satellite peaks. For instance if the peak described by the second term, peak A in figure 5.3, is isolated in this way, the field transmitted by the element, \( U_t(\alpha, \beta) \), is given by,

\[
U_t(\alpha, \beta) = B_0 \left[ \frac{\lambda s_i}{d_s} \right]^2 \times \left\{ \exp \left[ -\frac{\pi (\lambda s_i)^2}{d_s^2} \left( \alpha^2 + \beta^2 \right) \right] \ast \frac{1}{2} \delta \left( \alpha, \beta \frac{D_s}{\lambda s_i} \right) \right\} \tag{5.9}
\]

The intensity distribution in the image plane, \( I_c(X, Y) \), observed by the CCD array now corresponds to the power spectrum of \( U_t(\alpha, \beta) \) which can be written,

\[
I_c(X, Y) = |B_0/2|^2 \times \left\{ \exp \left[ -\frac{\pi d_s^2}{(\lambda s_i)^2} (X^2 + Y^2) \right] \right\}^2 \tag{5.10}
\]

Thus, with the spatial filter element in this position, the first exposure image is reconstructed onto the CCD array. Similarly, if the spatial filter element was used to isolate peak B, the second exposure image is reconstructed as desired. Therefore with two separated images cross correlation can be implemented.

To model the cross correlation stage of the process we must first refer to the theoretical model in section 2.2, chapter 2. If the first and second exposures are separable into the intensity distributions \( I_1(X, Y) \) and \( I_2(X, Y) \) respectively, each exposure can be modelled as

\[
I_1(X, Y) = KP(X, Y) \ast \sum_{i=1}^{N} \left\{ \delta(X - X_i, Y - Y_i) \right\} \tag{5.11}
\]

\[
I_2(X, Y) = KP(X, Y) \ast \sum_{i=1}^{N} \left\{ \delta(X - [\Delta X_0 + X_i + \Delta X_i], Y - [\Delta Y_0 + Y_i + \Delta Y_i]) \right\} \tag{5.12}
\]

where \( P(X, Y) \) represents the intensity function of a single particle image, \( \ast \) represents the convolution function, \( \delta(X, Y) \) represents the delta function, \( N \) particle images are
at initial coordinates \((X_i, Y_i)\) and the particle image displacement variables \((\Delta X_0, \Delta Y_0)\) and \((\Delta X_i, \Delta Y_i)\) are defined by equations 2.15-2.18 in chapter 2. The length of the transparency interrogation region will be \(D\) and the particle image diameter will equal \(d\). Also to simplify the model no unpaired particle images will be present in the interrogation region.

The theoretical cross correlation function of the two exposures can now be derived. The cross correlation function \(R_{cc}(\xi, \eta)\) of \(I_1(X, Y)\) and \(I_2(X, Y)\) is equal to

\[
R_{cc}(\xi, \eta) = \int \int I_1(X, Y)I_2(X - \xi, Y - \eta)dXdY
\]  

(5.13)

Substituting equations 5.11 and 5.12 into \(R_{cc}(\xi, \eta)\) leads to

\[
R_{cc}(\xi, \eta) = K^2 [R_s(\xi, \eta) + R_n(\xi, \eta)]
\]  

(5.14)

where

\[
R_s(\xi, \eta) = R_{pp}(\xi, \eta) * \left[ \sum_{i=1}^{N} \delta(\xi - [\Delta X_0 + \Delta X_i], \eta - [\Delta Y_0 + \Delta Y_i]) \right]
\]  

(5.15)

\[
R_n(\xi, \eta) = R_{pp}(\xi, \eta) * \left[ \sum_{j=1}^{N} \sum_{i=1}^{j} \delta(\xi - [\Delta X_0 + \Delta X_i + \Delta X_j], \eta - [\Delta Y_0 + \Delta Y_i + \Delta Y_j]) \right]
\]  

(5.16)

and \(R_{pp}(\xi, \eta)\) represents a single correlated particle image.

If the cross correlation function is examined it can be split into two distinct sections. These are the noise function \(R_n(\xi, \eta)\) and the signal peak function \(R_s(\xi, \eta)\). Figure 5.4 illustrates a typical cross correlation function. Each of the cross correlation functions \(R_n(\xi, \eta)\) and \(R_s(\xi, \eta)\) consists of a set of delta functions with a single correlated particle image, \(R_{pp}(\xi, \eta)\), convolved onto each delta function to construct the correlation plane.

The noise function, \(R_n(\xi, \eta)\), represents correlation noise generated from spurious correlations between particle images in the interrogation region and is superimposed
onto the correlation plane. The function \( R_\eta(\xi, \eta) \) is generated by spurious correlations between paired particle images and if expanded will contain \( N(N - 1) \) spurious correlations. When this is compared to the equivalent autocorrelation noise function, \( R_{ns}(\xi, \eta) \), in equation 2.27, this function will contain \( 4N(N - 1) \) spurious correlations. Therefore the cross correlation function will contain one quarter of the spurious correlations of an equivalent autocorrelation. Also the minimum noise level of the cross correlation noise function will correspond to \( R_\eta(\xi, \eta) = 1 \) from equation 5.16. With reference to equation 2.27, when this is compared to the autocorrelation minimum noise level of \( R_{ns}(\xi, \eta) = 2 \), this amounts to a significant benefit in SNR which will be quantified in the next section.

If the signal peak function, \( R_s(\xi, \eta) \), is examined and compared to the autocorrelation peak function (equations 2.25 and 2.26), it contains only one signal peak term with a height \( N \) and distributed about the position \((\pm \Delta X_0, \pm \Delta Y_0)\). There is also no dc peak term present. The signal peak structure described by the random variables \((\Delta X_i, \Delta Y_i)\), however, is identical to the autocorrelation signal peak structure (equation 2.24). This results in the signal peak systematic error characteristics described in section 2.3.4 being identical to the autocorrelation characteristics.

As with the autocorrelation function, the characteristics of the cross correlation function will determine the system performance from a general labelled particle image field. These characteristics result in direct benefits over the autocorrelation function which will be discussed in the following section.

5.2.2 System Performance Benefits

Because the cross correlation function contains a single signal peak, the immediate benefit of the image labelling system is removal of the ambiguity problem. Also the absence of a dc peak allows particle image displacements of less than one particle image diameter to be measured. This results in an increase in dynamic range which will now be limited by the accuracy to which the signal peak can be located. Moreover, because the signal peak characteristics are identical to the autocorrelation, the systematic error characteristics will be similar when measuring an equivalent flow. For the random error component, however, as described in section 2.3.4, because the number of noise correlations are one quarter of an equivalent autocorrelation, the cross correlation random error would be expected to be reduced. Further work, however, is required to quantify this reduction.
The SNR benefits of the labelling system are directly related to the SNR characteristics of the cross correlation function. With reference to section 2.3.1, the mean cross correlation SNR, $S$, for a given particle image density, $N_i$, can also be represented in terms of the mean velocity functions $F_i(u_o, v_o)$ and $F_o(w_o)$, the velocity gradient function $F_v(\phi D_r)$ and the correlation noise function $R_n(\xi, \eta)$ such that

$$S = 20 \log \left[ \frac{F_i(u_o, v_o)F_o(w_o)F_v(\phi D_r)N_i}{[R_n(\xi, \eta)]_{\text{max}}} \right]$$

(5.17)

where $F_i(u_o, v_o)$, $F_o(w_o)$ and $F_v(\phi D_r)$ are used to define identical SNR loss functions $S_i$, $S_o$ and $S_v$. If the baseline SNR, $S_b$, is considered, because the minimum level of correlation noise will correspond to $R_n(\xi, \eta)=1$ (see equation 5.16), the upper limit $S_{\text{max}}$ will equal

$$S_{\text{max}} = 20 \log [N_i]$$

(5.18)

With reference to equation 2.55, this provides the cross correlation technique with a 6dB SNR advantage over the autocorrelation technique. This is a significant gain in SNR considering the autocorrelation Monte Carlo baseline SNR is typically 12dB (section 2.3.2).

The benefits of this SNR gain in terms of measurable dynamic range or velocity gradient can be calculated using the optimisation method outlined in section 2.3.3. This is possible because the signal peak definition of the cross correlation function (equation 5.15) ensures identical SNR loss characteristics as defined by the SNR functions $S_i$, $S_o$ and $S_v$. Two adjustments must be made, however, for the calculations where the first is a 6dB addition to the baseline prediction $S_b$ and the second is an adjustment to the dynamic range variable $D_r$ in terms of the minimum velocity that can be measured. The second adjustment will depend on the accuracy to which the signal peak position can be located. An example calculation and comparison of the cross correlation and autocorrelation optimisation is given in Appendix 7 and shows the gains in dynamic range or velocity gradient to be highly dependent on the minimum measurable particle image displacement, but may be up to 18 times greater than the equivalent optimised autocorrelation.
5.3 High Speed Flow Image Labelling System

The following section will describe the design and testing of a system capable of image labelling flow speeds up to 500m/s, i.e. supersonic flow regimes. The system is based on the dual aperture plate technique which was theoretically modelled in the previous section.

5.3.1 System Design

The image labelling system consists of a recording stage and a data processing stage. To optimise the recording of labelled particle images, the aperture plate design must be considered in detail. The complete recording set-up will be described separately. For the data processing stage a digital processing system will be described although the image separation could be performed optically as described in the theoretical model of section 5.2.1.

5.3.1.1 Aperture Plate Design

In order to use the labelling technique in practice the aperture plate must be rotated during the time interval between exposures. Clearly, this requirement leads to an upper limit on the fluid velocity that can be recorded in this way. It is estimated that in practice engineering considerations restrict the maximum speed of plate rotation to around 10000 rpm which corresponds to a maximum flow velocity of the order of 0.2-0.3m/s. This is clearly unsuitable for use in high speed flow regimes (>100m/s). Therefore an alternative non-mechanical method must be found which will effectively rotate the dual aperture mask between exposures in the required time.

The mask design outlined in figure 5.5 can achieve the same result but without the need for mechanical rotation between exposures. This is accomplished by constructing a four aperture mask in which orthogonal aperture pairs are sensitive to the polarisation of side-scattered light from the seeding particles. The mask consists of two pairs of orthogonally oriented apertures, each pair of apertures having polarisation plane alignment at 0 and 90° respectively. If the polarisation of the illuminating laser sheet is rotated between exposures and seeding particles are used which preserve the illuminating polarisation in side scatter, the effect is that of a dual aperture rotated 90° between exposures. Rotation of the illumination polarisation can be achieved in the order of 5ns by using a Pockels cell placed in the path of the laser beam and synchronised with the laser driver circuits. Hence this technique allows particle images to be labelled in flows up to and exceeding supersonic speeds where pulse separations of the order of several hundred nanoseconds are required.
To optimise the image labelling system the smallest particle image size with the minimum number of fringes must be obtained. The minimum number of fringes will ensure the smallest aperture separation. Achieving the smallest possible aperture separation is the most important requirement because it allows the maximum possible aperture size to be used which will minimise particle image size and laser power requirements. The minimum particle image size will also ensure maximum use of the film resolution available.

For a given aperture size, the number of fringes/particle, \( N_f \), is set by the aperture diameter, \( d_a \), and the aperture separation, \( D_s \). With reference to equation 5.5, assuming a \( 1/e^2 \) particle image diameter leads to the number of fringes/particle equal to

\[
N_f = \frac{1.6D_s}{d_a}.
\]

The number of fringes will determine the spacing of the four satellite lobes in the Fourier plane during spatial filtering. Ideally, the satellite lobes should be completely separated from the dc peak in the centre of the Fourier plane to prevent any dc peak cross talk. The lobe dimensions estimated from the Gaussian approximation of equation 5.6 suggest four fringes are the minimum required to achieve sufficient separation from the dc peak. This is also confirmed by calculating the exact lobe dimensions which can be found from the autocorrelation of the aperture function. In practice, however, the labelling system will allow the use of three fringes/particle image without any significant effects from dc peak cross talk and this is illustrated in the experimental tests in section 5.3.2.

The number of fringes/particle allow the smallest possible particle image diameter to be defined from the corresponding fringe spacing and the maximum film resolution. For example Kodak Technical Pan film with a typical resolution of 150 line pairs/mm [5.5] can resolve a 5\( \mu \)m fringe. Therefore with three fringes the smallest possible particle image diameter that can be recorded is 35\( \mu \)m. From this particle image diameter the aperture diameter, \( d_a \), can be calculated using the standard formulae for a diffraction limited particle image (equation 4.4).

The requirement to resolve fringes on the labelled images will also lead to a greater laser power requirement than conventional unlabelled images due to the larger particle images. To determine this difference we must compare the ratio of power required for
unlabelled images using a single aperture with labelled images using a dual aperture for the same magnification M. For a recording system the particle peak intensity at the film plane for a single aperture $I_s$ is as found from reference 5.6 such that

$$I_s = Kd_s^4$$  \hspace{1cm} (5.20)

where $K$ is a constant dependent on the film characteristics and $d_s$ is the single aperture diameter. For the dual aperture plate the central fringe peak intensity of the labelled image, $I_l$, will be four times that of a single aperture of the same size (Appendix 8). Therefore by including the absorption factor of the Polaroid $A_b$,

$$I_l = 4A_b^2I_s$$  \hspace{1cm} (5.21)

Consequently the ratio of the unlabelled to labelled intensities can be represented by

$$\frac{I_s}{I_l} = \frac{d_s^4}{4A_b^2}$$  \hspace{1cm} (5.22)

To estimate the difference in power requirements consider the intensity ratio $I_l/I_s$ for a 35μm diameter labelled image and a 20μm diameter unlabelled image (a 20μm particle image is typical in high speed flow PIV). If Polaroid efficiency is 80%, i.e. $A_b=0.8$ [5.7], then $I_l/I_s=3.66$. Hence, around four times the power required for an unlabelled image will be required for the labelled image to attain the same film exposure. This is consistent with the experimental work described in section 5.3.2.

Finally the depth of field for a labelled system will be modified when compared to a conventional PIV system. If the depth of field is determined from conventional formulae (see equation 4.2) by using the diameter $D_{\text{max}}$ as shown in figure 5.5, in most cases $D_{\text{max}}$ will be greater than $d_a$ meaning the depth of field of the labelling system will be reduced. Previous Moire analysis [5.9] involving a double slotted aperture system has suggested, however, that if $D_{\text{max}}$ is used this will underestimate the depth of field and that a greater value should be expected.

5.3.1.2 Recording Set-up

Figure 5.6 is a schematic of the complete image labelling recording set-up. The polarisation plane sensitivity of the apertures was achieved by using two sets of photographic Polaroid cut into segments and mounted in the dual aperture mask.
shown in figure 5.5. The two diametrically paired segments were placed over the corresponding sets of dual apertures with their polarisation planes aligned orthogonally to one another. The segments were then cemented into the mask and the whole assembly placed approximately 20mm behind the iris of a \( f=80 \text{mm} \) medium format macro lens. The lens was operated at magnifications between 0.5 and unity.

Rotation of the plane of polarisation of illuminating light was achieved in the order of 5ns by using a Pockels cell synchronised with the main laser driver circuits. An alternative, simpler, method is to have a dual oscillator double pulsed laser configured with orthogonal polarisation beams.

Because of the reduced depth of field, to aid focusing of the system during experimental tests, a small region of the camera film plane was imaged using a CCD array and microscope objective system as shown in figure 5.6. This allowed magnified particle images to be stored on a frame store and displayed on a video monitor for analysis. The microscope assembly could be traversed to view any portion of the film plane, allowing focusing adjustments as the experiment was running.

5.3.1.3 Data Processing Set-up

To process the labelled images two distinct stages must be carried out to obtain each interrogation region particle image displacement. The first is the separation of the two images using spatial filtering techniques. The second is to cross correlate both images. The design of the data processing system was based on the digital image plane configuration described in chapter 3 section 3.3 and is illustrated in figure 3.3. Further details of the processing system software is outlined in the Appendix 9.

One disadvantage of this system when compared to an equivalent autocorrelation system is the number of Fast Fourier Transforms (FFT's) required to obtain the cross correlation function. In this case 6 FFT's per interrogation region are necessary to spatially filter and cross correlate as against two FFT's for an autocorrelation function. Therefore the time penalty over the autocorrelation technique is a factor of three.

5.3.2 Experimental Tests

Experimental tests for the image labelling system were carried out in two parts. In the first part the polarisation preservation properties of the particle images were investigated and in the second part the complete image labelling system was tested.
Because the image labelling system relies on polarisation preservation of the scattered light from the particles, this property was tested before attempting to record any labelled PIV images. The tests consisted of measuring the mean intensity of side-scattered light collected by the camera lens from an illuminated region of a glass vessel filled with the olive oil mist with a mean size range of 1-2μm. This seeding has previously been used in high speed flow measurements [5.8]. Results were taken for polarisation states both in and out of the illumination plane and showed polarisation to be preserved in side scatter with a total extinction ratio better than 20:1.

The complete image labelling design was tested by recording PIV transparencies of flow reversals generated from a jet impinging on the bottom of a glass vessel. By scanning the film plane with the microscope/CCD system, it was found that labelled particle images of sufficient quality were achieved over the central 25% of the film plane area. The full field limitation was due to 3mm thick polarising element which caused aberrations and vignetting at the edge of the film plane. These effects could be reduced by using a thinner mask and placing the mask in the iris plane of the camera lens.

Figure 5.7a shows an interrogation region taken from the central portion of the transparency. The particle image diameters are around 40μm in diameter with a fringe width of 7μm. Consequently the fringe width is almost at the resolution limits of the film which accounts for the poor fringe visibility in the particle images. With this fringe quality, however, image separation was still possible using the digital image plane system. These are shown in figures 5.7b and 5.7c and show the first and second exposure images can be separated with negligible cross talk. Figure 5.7d shows the final cross correlation result. The centre of the cross correlation plane is indicated by a cross and the centre of bright lobe located beneath it corresponds to the average particle displacement (and hence velocity) in the original image.

5.4 Closure

A system has been described which allows image labelling to be carried out at high speed flows. By image labelling during the recording stages, separation of the first and second exposure images can be achieved by spatial filtering techniques during the data processing stage. At the time of writing the system presented has undergone preliminary tests and has shown 25% of the central film plane can be successfully image labelled. This is hoped to be extended to the entire film plane be using a thinner mask placed at the iris plane of the lens. Disadvantages of the technique are the
requirement of around four times more light than the equivalent unlabelled system and an increase by a factor of three in data processing time.

Separation of the first and second exposure images has been successfully demonstrated using a digital image plane system. This allows cross correlation to be implemented which eliminates the ambiguity problem and also gives a 6dB increase in baseline SNR over equivalent autocorrelation techniques. This 6dB advantage will result in significant increases of the maximum allowable dynamic range or velocity gradient over an equivalent autocorrelation application. These increases will be highly dependent on the minimum measurable particle image displacement, but may be up 18 times greater than the equivalent optimised autocorrelation.

The theoretical work in section 5.2 has been published as a full paper in Applied Optics 33 p4241-4247 (1994), entitled, "Particle Image Velocimetry: Image Labelling by use of Adaptive Optics to Modify the Point Spread Function" by N.J.Lawson, N.A.Halliwell and J.M.Coupland.
6.0 CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

PIV research has been completed to advance its industrial application to high speed turbomachinery flows and enable CFD code validation. The literature survey, which reviewed current PIV techniques, highlighted the need for research into the application of PIV in these environments. In particular the need for studies into the optimum choice of experimental parameters, the type of transparency analysis and practical problems encountered in the measurement of transonic flows were required. These studies have resulted in a specification of a PIV system for this flow regime and can be summarised as follows:

Digital image plane correlation techniques offer the most promising form of data processing for a high speed flow system. Correlation techniques offer high spatial resolution and high accuracy data with easy automation of the processing system. Digital based techniques are also more flexible over equivalent optical based techniques in terms of the type of correlation possible and optimisation of all the variables during processing. Optical techniques, however, have speed advantages which would be applicable to the processing of 3D PIV data on a holographic format. The fastest digital technique is currently one third of the speed of the fastest optical technique and uses digital parallel processing techniques. The factor limiting the speed of this technique is the fastest possible digital data transfer rate.

A comprehensive study of optimisation of the autocorrelation technique was carried out using a theoretical model and a Monte Carlo computer simulation. Conclusions from the study showed by considering the mean SNR, several SNR loss mechanisms which reduce the number of valid vectors can be isolated. Further by obtaining a baseline SNR level from the Monte Carlo simulation, these mechanisms can be used to form an optimisation method which allows predictions of maximum allowable dynamic range and velocity gradient to be made. Providing the seeding density is at least 12 particles/interrogation volume, this prediction ensures a minimum of 50% valid vectors will be obtained from the measured flow field for a specified spatial resolution. This important new method provides a consistent basis for experimental design.

In addition, Monte Carlo simulation results have shown that the accuracy of PIV measurements is dependent on a random error and a systematic error. For a fixed interrogation region length, the random error is dependent on the velocity gradient strength and dynamic range product $\phi D_p$, the particle image size and the number of particle image pairs. This error is of the order of 0.4% of full scale particle
displacement, \( L \), with zero velocity gradients rising to approximately 0.8% for significant velocity gradients \( (\Phi/D_r > 1) \). For an interrogation region of \( D/d = 16 \) with zero velocity gradients, when measuring a particle displacement equivalent to one particle image diameter this random error will correspond to an error in velocity measurement of 6%. Substantial increases in random error are also observed when the pixel resolution of the interrogation region is decreased. It is shown that each particle image must be covered by at least \( 4 \times 4 \) pixels to suppress this additional noise mechanism. Experimentally this means autocorrelation resolutions of between \( 64 \times 64 \) to \( 256 \times 256 \) pixels are required according to the given choice of dynamic range and velocity gradient restrictions.

The systematic vector error in velocity measurement is directly dependent on the velocity gradient and dynamic range product \( \Phi D_r \) and has a direction dependent on the components of velocity being measured. Analysis using the Monte Carlo simulation has also shown the systematic error to be dependent on the particle displacement model used. Results have demonstrated if the particle displacement model is oversimplified by using only first order terms from a Taylor series expansion, the systematic error measured will be due to the approximations in the model and will not be due to the PIV system characteristics. With reference to figure 2.17, for the case of a forced vortex results from a particle displacement model using higher order terms have shown no significant increases in systematic error for an increase in velocity gradient strength. Therefore in the presence of strong velocity gradients \( (\Phi D_r = 1.0) \) this error can be ignored when compared to the magnitude of the corresponding random error in velocity measurement.

The optimisation method was extended to include SNR degradation effects originating from the recording of PIV data onto transparency format and the digitisation stage. These predictions were made by analysing a set of PIV transparencies which were recorded from a set of solid, seeded araldite blocks. The recording technique allowed the PIV parameters of in and out of plane velocity, velocity gradient strength and particle image size to be carefully controlled. Results from the analysis showed a degradation of 1-5dB in baseline SNR over equivalent Monte Carlo levels where the magnitude of loss was dependent on the level of seeding density. This dependence was attributed to a correlated noise structure caused by the presence of CCD noise which was up to four bits of the digitised data. Removal of this noise resulted in a gain of up to 3dB in SNR. The remaining loss in SNR was attributed to transparency recording effects such as film grain noise and particle image intensity variation. The work highlighted the flexibility of the digital system in terms of CCD noise reduction which
allowed recovery of over half the SNR loss, i.e. 2-3dB's. This level of SNR recovery is significant considering baseline to noise levels are only around 12dB.

The PIV technique has been successfully applied to an enclosed high speed flow regime representative of the smallest scale that would be encountered in a gas turbine. A PIV vector map was obtained in the central 25% of the nozzle area allowing location of a normal shock to an accuracy of one interrogation region (0.54mm). Optimum seeding levels of at least 12 particles per interrogation volume were achieved and the optimisation method developed in the earlier part of the thesis was successfully applied to maximise system performance outside the normal shock region. In the shock region, however, the velocity gradient was too severe to allow optimisation of the complete flow.

The PIV results inside the nozzle were obtained with sufficient spatial resolution to allow direct comparison with a CFD model. This comparison showed significant errors in both shock location and velocity magnitudes but at present the exact mechanisms causing these discrepancies are unknown. The spatial resolution in this case was significantly improved over previous work due to the increased seeding densities achieved but was at the cost of a high level of window contamination. This contamination was caused by the high mass flows and corresponding seeding levels involved in running the supersonic rig for full field seeding. Injecting the seeding into the centre of the flow reduced the problem but limited the area which could be mapped. It is thought that a pulsed seeding system which provides seeding on demand is likely to resolve this problem.

The limitations of the enclosed flow system in terms of window contamination were highlighted by analysing the open jet from the same nozzle. In this case spatial resolution could be extended by 60% through increased seeding densities allowing complex shock cell structures with a scale less than 10mm to be observed across the complete jet structure.

A system has been demonstrated which allows image labelling to be carried out at high speed flows. By image labelling during the recording stages, separation of the first and second exposure images can be achieved by spatial filtering techniques during the data processing stage. This allows cross correlation to be implemented which eliminates the ambiguity problem and also gives a 6dB increase of baseline SNR over autocorrelation techniques. By extending the optimisation method to cross correlation, it was shown that this increase in baseline SNR would allow dynamic range or velocity gradient
strengths up to 18 times greater than an equivalent autocorrelation application, although this increase would depend on the accuracy to which the signal peak position could be located.

Disadvantages of the image labelling system are the requirement of around four times more light than the equivalent unlabelled system and an increase by a factor of three in data processing time. At the time of writing the system presented has undergone preliminary tests and has shown 25% of the central film plane can be successfully image labelled. This is hoped to be extended to the entire film plane by using a thinner mask placed at the iris plane of the lens.

There are a number of suggestions for further work as a result of the research carried out.

Firstly, further studies into the systematic error characteristics need to be completed. The work carried out in chapter 2 has shown significant discrepancies exist in the error predictions if a first order particle displacement model is used. This was established by comparison with results from a forced vortex simulation with higher order terms in the displacement model and for $\phi D_r \leq 1$. This simulation could be extended to greater values of $\phi D_r$ and to a number of different flow regimes in order to determine these error characteristics in more detail.

Additional studies could be made into alternative algorithms for signal peak position analysis. The current centroid algorithm could also be extended to account for the effects of signal peak distortion caused by the loss of pairs in the presence of strong velocity gradients.

Secondly, in relation to chapter 3, an extension of the transparency analysis could be carried out to investigate the exact mechanisms involved in SNR degradation originating from the recording process. At present these have been attributed to film grain noise and particle image intensity variations but as yet no precise relationships have been established. The araldite block recording process would allow a film grain noise mechanism to be isolated through the analysis of an additional set of transparencies of varying exposures. This investigation could be complemented by an extension of the Monte Carlo simulation which would incorporate the type of particle intensity variations found on the negatives.
Further work could also be completed investigating SNR recovery techniques. These would include binarisation, thresholding and filtering techniques. In addition, work could also be carried out investigating noise sources originating from the CCD camera and the digitisation stage.

Further work resulting from the supersonic flow studies would involve finding techniques to reduce window contamination. Suggested methods involve a window washing system combined with pulsed seeding systems or a technique which would discourage any seeding from contacting the window. Until these problems are solved full field high speed PIV at this scale will not be possible.

Finally an extension of the Monte Carlo simulation to include cross correlation would allow comparisons between equivalent autocorrelation SNR mechanisms. This would also allow an investigation into cross correlation measurement accuracy in the presence of strong velocity gradients and low interrogation region pixel resolutions. With reference to Appendix 7, quantifying this error in measurement is of great importance because of the direct dependence of cross correlation dynamic range on this parameter.
APPENDIX 1

Autocorrelation of Particle Image Function

With reference to section 2.2, the function of a particle image, \( P(X, Y) \), can be represented by a Gaussian function with a \( 1/e^2 \) radius of \( d/2 \) where \( d \) is the particle image diameter. Hence

\[
P(X, Y) = \exp\left( -\frac{8\left[ X^2 + Y^2 \right]}{d^2} \right) = \text{Gaus}\left( \frac{X}{\sqrt{\pi d/2}} \right) \text{Gaus}\left( \frac{Y}{\sqrt{\pi d/2}} \right)
\]  

\text{(A1.1)}

where \( \text{Gaus}(X) \) and \( \text{Gaus}(Y) \) represent one-dimensional Gaussian functions \[2.4\]. The autocorrelation of the particle image function \( R_{pp}(\xi, \eta) \) is represented by

\[
R_{pp}(\xi, \eta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(X, Y)P(X - \xi, Y - \eta) \, dX \, dY
\]  

\text{(A1.2)}

If the forward Fourier transform of \( f(X) \) is represented by \( F[f(X)] \) and the inverse transform of \( f(X) \) is represented by \( F^{-1}[f(X)] \) then by the Weiner-Khintchine theorem \( R_{pp}(\xi, \eta) \) can be re-written \[2.7\] as

\[
R_{pp}(\xi, \eta) = F^{-1}\left\{ |F[P(X, Y)]|^2 \right\}
\]  

\text{(A1.3)}

Consequently,

\[
F[P(X, Y)] = \left[ \frac{\pi d^2}{8} \right] \text{Gaus}\left( \frac{\alpha}{2\sqrt{2/\sqrt{\pi d}}} \right) \text{Gaus}\left( \frac{\beta}{2\sqrt{2/\sqrt{\pi d}}} \right)
\]  

\text{(A1.4)}

where \( \alpha \) and \( \beta \) represent the rectangular coordinates in the Fourier plane. From equation A1.4, \( |F[P(X, Y)]|^2 \) can be found such that

\[
|F[P(X, Y)]|^2 = \left[ \frac{\pi d^2}{8} \right] \text{Gaus}\left( \frac{\alpha}{2/\sqrt{\pi d}} \right) \text{Gaus}\left( \frac{\beta}{2/\sqrt{\pi d}} \right)
\]  

\text{(A1.5)}
Hence

\[ R_{pp}(\xi, \eta) = \left[ \frac{\pi d^2}{16} \right] \text{Gaus} \left( \frac{\xi}{\sqrt{\pi d/2}} \right) \text{Gaus} \left( \frac{\eta}{\sqrt{\pi d/2}} \right) \]

\[ = \left[ \frac{\pi d^2}{16} \right] \exp \left( -\frac{4[\xi^2 + \eta^2]}{d^2} \right) \]  

(A1.6)

By comparing \( P(X, Y) \) to \( R_{pp}(\xi, \eta) \) it can be seen the equivalent 1/e^2 diameter will be \( \sqrt{2d} \) in the correlation plane.
APPENDIX 2

Signal Peak Degradation Function

Let us consider the convolution, $R(x, y)$, of a two dimensional Gaussian function, $Gaus[(x/a), (y/a)]$, with a two dimensional Rectangular function, $Rect[(x/b), (y/b)]$ where $R(x, y)$ is defined as

$$R(x, y) = \int\int_{-\infty}^{\infty} Gaus\left(\frac{x}{a}, \frac{y}{a}\right) Rect\left(\frac{x}{b}, \frac{y}{b}\right) d\alpha d\beta$$  \hspace{1cm} (A2.1)$$

and the maximum value of this convolution is clearly at the origin $R(0, 0)$. The central value can be written

$$R(0, 0) = \int\int_{-b/2}^{b/2} Gaus\left(\frac{\alpha}{a}, \frac{\beta}{a}\right) d\alpha d\beta$$  \hspace{1cm} (A2.2)$$

Equation A2.5 is separable into functions of $x$ and $y$ giving

$$R(0, 0) = \int_{-b/2}^{b/2} \exp\left[-\pi\left(\frac{\alpha^2}{a^2}\right)\right] d\alpha \int_{-b/2}^{b/2} \exp\left[-\pi\left(\frac{\beta^2}{a^2}\right)\right] d\beta$$  \hspace{1cm} (A2.3)$$

The integrals $\exp\left[-\pi\left(\alpha^2/a^2\right)\right]$ and $\exp\left[-\pi\left(\beta^2/a^2\right)\right]$ can be solved using standard integral tables [2.5] such that

$$\int_{-b/2}^{b/2} \exp\left[-\pi\left(\alpha^2/a^2\right)\right] d\alpha = \int_{-b/2}^{b/2} \exp\left[-\pi\left(\beta^2/a^2\right)\right] d\beta = a \left[\text{erf}(b\sqrt{\pi}/2a)\right]$$  \hspace{1cm} (A2.4)$$

where erf($x$) represents the standard error function [2.5] defined by

$$\text{erf}(x) = \left[2/\sqrt{\pi}\right] \int_0^x \exp(-t^2) dt$$  \hspace{1cm} (A2.5)$$

Therefore the convolution $R(0, 0)$ can be written
For the purposes of modelling the signal peak function described in equations 2.25 and 2.26, the function \( R(0, 0) \) must be normalised such that the Rectangular function will have a constant area in the x and y axes represented by the function \( \frac{1}{b^2} \text{Rect}[(x/b), (y/b)] \) and the Gaussian function is defined in terms of a \( 1/e^2 \) diameter, \( c \), where with reference to A1.1, \( a = [\sqrt{\pi/8}] c \). Rewriting \( R(0, 0) \) in this form as the function \( R_{ac}(0, 0) \) leads to

\[
R_{ac}(0, 0) = \left[ \frac{\pi^2}{8b^2} \right] \left[ \text{erf}\left(\sqrt{2}(b/c)\right) \right]^2
\]  

\[\text{(A2.7)}\]

Figure A2.1 is a plot of the function \( R_{ac}(0, 0) \) for a range of \( b \) where the function \( 1/(1+2[b/c]^2) \) has been included for comparison. The function \( 1/(1+2[b/c]^2) \) has been used in previous work [2.2] to describe the spread of the autocorrelation peak in the presence of a forced vortex and as figure A2.1 shows this is a good approximation to \( R_{ac}(0, 0) \) with a maximum error of 8%.

Figure A2.1 - Characteristics of \( R_{ac}(0, 0) \) Compared to the Function \( 1/(1+2[b/c]^2) \) for a Range of \( b \)
APPENDIX 3

Monte Carlo Simulation
The following appendix will describe the development of the Monte Carlo simulation from the theoretical model of the velocimeter given in section 2.2. The computer simulation has a recording stage and a data processing stage.

1.0 Recording Stage
The recording stage of the simulation produces a data file in the form of a floating point array which represents the digitised data of an interrogation region. In practice this information would be obtained from a transparency by using a CCD, microscope objective and image processing board. To produce this data the simulation uses the theoretical model developed in section 2.2.

Results from the simulation will be quoted using the particle image density, \( N_i \).

1.1 Fluid Volume Seeding Simulation
The first part of the simulation sets up an interrogation volume as a three dimensional floating point array at the same pixel resolution as the corresponding interrogation region. Initial random positions \( x_i, y_i \) and \( z_i \) are assigned to \( N_v \) particles in the fluid volume. All three coordinates are independent of each other and follow the probability density function model described by equations 2.4-2.6 and illustrated in figure 2.2.

The fluid volume has dimensions \((2L, 2L, 2W)\) with the interrogation volume located at the centre with dimensions \((L, L, W)\). This enables loss of pair effects to be simulated by the model and hence allows particles to enter and leave the centrally positioned interrogation volume for a given displacement.

1.2 Velocity Gradient Model
In the second stage of the simulation, particle image displacements are defined by using the velocity gradient model of section 2.2. The velocity gradient model will depend on the magnitude and direction of the gradient components, \( \partial u/\partial x, \partial u/\partial y, \partial u/\partial z \) and \( \partial v/\partial x, \partial v/\partial y, \partial v/\partial z \) in the fluid volume. With reference to the SNR and valid vector analysis of section 2.3.1, the analysis will be based on a forced vortex. Therefore this case \( \partial u/\partial x=\partial u/\partial z=\partial v/\partial y=\partial v/\partial z=0 \) and \( \partial u/\partial y=-\partial v/\partial x \).

To simulate this flow regime, the particles in the fluid volume are initially assigned a velocity \( \mathbf{V} \) relative to the interrogation volume origin where \( \mathbf{V} \) is defined by equation
2.36. For a given laser pulse separation, $\Delta t$, each individual particle is then assigned a floating point displacement to the second exposure position. The displacement is calculated from the Taylor series expansion of equation 2.11. In the next stage of the simulation, the x and y coordinates within the boundaries of the interrogation volume are transferred to a separate floating point array which represents the transparency interrogation region.

1.3 Interrogation Region Model
In the final part of the simulated recording stage the floating point data file representing the transparency interrogation region is produced. This data file contains the particle images with positions which correspond to the interrogation volume data where the interrogation region has a pixel resolution between 32x32 to 256x256. The particle image is described using a two dimensional Gaussian function transferred onto a square array. The minimum allowable particle image pixel resolution is a 2x2 pixel array. The transparency region array is produced through a convolution of the particle image arrays onto the pre-determined particle image positions. First exposure particle image positions are at exact pixel locations. Because the particle image displacement is a floating point value, however, the second exposure particle images must be placed at a sub-pixel position. To allow this the particle image pixel array has an additional single pixel border and the Gaussian particle image profile, $P(X, Y)$, is placed onto the larger array about the sub-pixel centre. This correction is necessary to ensure realistic CCD and transparency properties.

The next stage in the model is to perform the data processing of the simulated interrogation regions.

2.0 Data Processing Stage
The data processing stage must calculate the autocorrelation of the interrogation region and extract the average particle image displacement from this result. The autocorrelation is theoretically described in equations 2.23-2.28 and its application will now be considered in the following section.

2.1 Calculation of the Autocorrelation Function
The simplest way of calculating the autocorrelation function is through the use of Fast Fourier Transform (FFT) functions. FFT based autocorrelation allows easy manipulation of processing pixel resolution and interrogation region size. For this reason it is the most commonly used method in digital PIV data processing systems [2.6].
The theoretical autocorrelation has been derived in section 2.2. By applying FFT
analysis such that if a forward 2D transform of \( f(X, Y) \) is represented by \( F\{f(X, Y)\} \)
and a reverse transform is represented by \( F^{-1}\{f(X, Y)\} \), the autocorrelation of \( f(X, Y) \)
can be found from the Weiner-Khintchine theorem such that

\[
R_\eta (\xi, \eta) = F^{-1} \left\{ \left| F\{f(X, Y)\} \right|^2 \right\}
\]

where \( I(X, Y) \) is the intensity description of the interrogation region [2.7].

2.2 Extraction of the Displacement Vector
Following calculation of the autocorrelation, the final stage involves the extraction of
the displacement vector. The symmetry of the correlation plane means only one half
needs to be analysed. The remainder is masked off. Before searching for the signal
peak, the central dc peak must also be masked off. To estimate the particle
displacement \((\Delta X', \Delta Y')\), the centroid of the signal peak structure must be found such
that

\[
\Delta X' = \frac{\int_{\xi_{\text{lim}}}^{\xi_{\text{lim}}} \xi R_\eta (\xi, \eta) d\xi}{\int_{\xi_{\text{lim}}}^{\xi_{\text{lim}}} R_\eta (\xi, \eta) d\xi}
\]

\[
\Delta Y' = \frac{\int_{\eta_{\text{lim}}}^{\eta_{\text{lim}}} \eta R_\eta (\xi, \eta) d\eta}{\int_{\eta_{\text{lim}}}^{\eta_{\text{lim}}} R_\eta (\xi, \eta) d\eta}
\]

The variables \((\xi_{\text{lim1}}, \xi_{\text{lim2}})\) and \((\eta_{\text{lim1}}, \eta_{\text{lim2}})\) describe the outer limits of the peak
area to be used for the centroid analysis. The minimum centroid limits are found from
the radius of \( R_{pp}(\xi, \eta) \) which from Appendix 1 equals \( \sqrt{2}d/2 \) and will represent the
signal peak dimensions. These are used when no velocity gradients are present. To
account for the velocity gradient signal peak broadening, however, the limits are
increased in proportion to the velocity gradient strength. Therefore with reference to
equation 2.39 which defines the magnitude of the signal peak broadening, the centroid
limits are increased by a factor of \( L_g/2 \). This factor can be re-written in terms of the
optimisation variable \( \phi D_r \) where with reference to equation 2.46

82
Therefore the centroid limits are now found from \( \sqrt{2 + (D_t/\sqrt{2})d/2} \) such that

\[
\xi_{\text{lim}} = \Delta X_o \pm \sqrt{2 + (D_t/\sqrt{2})d/2}
\]

\[
\eta_{\text{lim}} = \Delta Y_o \pm \sqrt{2 + (D_t/\sqrt{2})d/2}
\]
APPENDIX 4

Derivation of Correlation Noise Characteristics

The autocorrelation noise function, \( R_{\text{ns}}(\xi, \eta) + R_{\text{nll}}(\xi, \eta) \) (equations 2.27, 2.28) can be considered as a spatial map of all spurious correlations between paired and unpaired particle images. If \( R_{\text{ns}}(\xi, \eta) \) and \( R_{\text{nll}}(\xi, \eta) \) are expanded, the total number of spurious correlations, \( N_s \), will be equal to

\[
N_s = 4N(N-1) + N_u(N_u - 1) + 4NN_u \quad \text{(A4.1)}
\]

where \( N \) and \( N_u \) are the number of paired and unpaired particle images respectively. This can be expressed in terms of the particle image density \( N_i \) and a loss of particle image pairs factor, \( F_i(u_o, v_o)F_o(w_o) \). With reference to equations 2.32 and 2.33, the factor, \( F_i(u_o, v_o)F_o(w_o) \), is proportional to the percentage of mean in plane and out of plane particle displacement in the interrogation volume such that

\[
N = N_i F_i(u_o, v_o)F_o(w_o) \quad \text{(A4.2)}
\]

where

\[
F_i(u_o, v_o) = (1 - u_o \Delta t/L)(1 - v_o \Delta t/L) \quad \text{(A4.3)}
\]

\[
F_o(w_o) = (1 - w_o \Delta t/W_z) \quad \text{(A4.4)}
\]

Clearly the number of unpaired particles is given by

\[
N_u = 2(N_i - N) \quad \text{(A4.5)}
\]

Therefore

\[
N_u = 2N_i(1 - F_i(u_o, v_o)F_o(w_o)) \quad \text{(A4.6)}
\]

Substituting equation A4.6 into equation A4.1 and simplifying leads to

\[
N_s = 4N_i^2 - 2N_i\left[1 + F_i(u_o, v_o)F_o(w_o)\right] \quad \text{(A4.7)}
\]
Figure A4.1 illustrates the characteristics of this function for a range of $F_i(u_0, v_0)F_o(w_0)$ and shows the number of spurious correlations, $N_s$, is strongly dependent on particle image density, $N_i$, and has negligible dependence on the percentage of loss of pairs.
Figure A4.1 - Number of Spurious Correlation Characteristics for a Range of Seeding Densities and Loss of Pairs

Note: Points on the curve from the different series cannot be distinguished because of the similarity of the values.
APPENDIX 5

Comparison of Optimisation Method and Monte Carlo Simulation Predictions

The simulation was run for two cases such that

<table>
<thead>
<tr>
<th>Case</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Interrogation region to particle image size ratio D/d</td>
<td>64</td>
</tr>
<tr>
<td>ii) In plane displacement of 30% of D</td>
<td></td>
</tr>
</tbody>
</table>

If we assume a particle image diameter of d=20μm and a magnification of M=1 in both cases this will define the spatial resolution, L, from i) and the dynamic range, Dr, from ii) such that

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>iii) Spatial resolution L (mm)</td>
<td>1.3</td>
</tr>
<tr>
<td>iv) Dynamic range Dr</td>
<td>19</td>
</tr>
</tbody>
</table>

Figure A5.1 shows the valid vector predictions from the Monte Carlo simulation for both cases for a range of velocity gradients from φ=0 to φ=33%.

In order to calculate the velocity gradient prediction using the optimisation method, the baseline SNR curve (Figure 2.7), the in plane loss of SNR curves (Figure 2.8) and the velocity gradient loss of SNR curves (Figure 2.10) are required. The following outlines the calculation for both cases.

1. Obtain value of Sb from Figure 2.7
   - Case 1: Sb=12.8
   - Case 2: Sb=10.2
2. Obtain in plane loss in dB from Figure 2.9 for 30% of D
   - Case 1: 3.1dB
   - Case 2: 2.7dB
3. Calculate remaining SNR
   - Case 1: 9.7dB
   - Case 2: 7.5dB
4. Look up φDr from Figure 2.10
   - Case 1: φDr=2.6
   - Case 2: φDr=2.2
5. Calculate φ from Dr
   - Case 1: φ=13.0%
   - Case 2: φ=29.3%

Monte Carlo prediction for 50% valid vectors

For comparison the Monte Carlo predictions have been listed last and show the optimisation method can predict the maximum allowable gradient to within 1-2% of the computer simulation.
Figure A5.1 - Monte Carlo Predictions of Valid Vectors

![Graph showing Monte Carlo Predictions of Valid Vectors with two cases: Case 1 and Case 2. The x-axis represents Velocity Gradient (%), ranging from 0 to 35, and the y-axis represents Valid Vectors (%), ranging from 0 to 100. The graph includes data points for each case, indicating a decrease in valid vectors as the velocity gradient increases.](image-url)
1D Nozzle and Open Jet Theory

A 1D supersonic theoretical model was applied to an arbitrary central streamline inside the nozzle. Standard compressible flow theory [4.7] was used in the model. The following assumptions were necessary for the theory to be valid:

a) Curvature of the axis is zero and changes in flow area are gradual
b) The flow before and after the shock wave can be assumed to be isentropic
c) The shock can be treated as an adiabatic frictionless process
d) The flow is frictionless with no boundary layer effects
e) Exhaust static pressure will equal atmospheric pressure
f) The fluid used is dry air

A computer routine was written to read in the nozzle area profile and the atmospheric/plenum stagnation pressure and temperature data. The routine then applied data point by point to 1D theory, giving the velocity profile along the central streamline. Shock location was achieved by an iterative loop which increased plenum chamber pressure to a level where assumption e) was achieved.

Theoretical modelling of the open jet has been attempted by Tam et al [4.8] but its application is beyond the scope of this thesis. There has been work completed, however, on jet noise in relation to the shock cell structures which form on an imperfectly expanded jet. The shock cell spacing or wavelength $L_{CELL}$ for an overexpanded jet can be estimated from work by Pack [4.9] who suggested $L_{CELL}$ can be found using the expression

$$L_{CELL} = C . D_n . k$$  \hspace{1cm} (A7.1)

where $k = \sqrt{M_j - 1}$ and $M_j$ can be found from

$$\frac{P_s}{P_a} = \left[ 1 + \frac{(v-1)}{2} M_j^2 \right]^{\frac{v}{v-1}},$$  \hspace{1cm} (A7.2)

where $P_s$ is the supply pressure, $P_a$ is the ambient pressure, $D_n$ is the nozzle diameter and the constant $C=1.1$. 
APPENDIX 7

Comparison between Autocorrelation and Cross Correlation using the Optimisation Method

The comparison was based on the following case:

i) Interrogation region to particle image size ratio $D/d=25$

ii) Velocity Gradient Strength $\phi=30\%$

iii) In plane displacement of $30\%$ of $D$

If we assume a particle image diameter of $d=20\mu m$ and a magnification of $M=1$ this will define the spatial resolution in both cases from i) as $L=0.5\text{mm}$.

In the following we wish to illustrate the dynamic range advantages of the cross correlation technique over the autocorrelation technique by using the optimisation method. In order to calculate the dynamic range predictions the baseline SNR curve (figure 2.7), the in plane loss of SNR curves (figure 2.8) and the velocity gradient loss of SNR curves (figure 2.10) are required. For the cross correlation prediction, two adjustments must be made to the calculation when compared to an autocorrelation calculation. In the first case an additional $6\text{dB}$ of SNR is added to the baseline SNR value obtained from figure 2.7. In the second case a constant $k_d$ is introduced which represents the ratio of the minimum measurable particle image displacement with its diameter, $d$. The minimum measurable particle image displacement is dependent on the accuracy to which the signal peak position can be located. For the following comparison it is assumed the minimum measurable displacement will be one tenth of a particle image diameter, i.e. $k_d=0.1$. This assumption is consistent with previous work [5.4].

<table>
<thead>
<tr>
<th>Step</th>
<th>Autocorrelation</th>
<th>Cross Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Obtain value of $S_b$ from Figure 2.7</td>
<td>$S_b=10.2$</td>
<td>$S_b=16.2$</td>
</tr>
<tr>
<td>2. Obtain in plane loss in dB from Figure 2.8 and $30%$ of $D$</td>
<td>$2.7\text{dB}$</td>
<td>$2.7\text{dB}$</td>
</tr>
<tr>
<td>3. Calculate remaining SNR</td>
<td>$7.5\text{dB}$</td>
<td>$13.5\text{dB}$</td>
</tr>
<tr>
<td>4. Look up $\phi D_f$ from Figure 2.10</td>
<td>$\phi D_f=2.2$</td>
<td>$k_d \phi D_f=4.1$</td>
</tr>
<tr>
<td>5. Calculate $D_f$ from $\phi$</td>
<td>$D_f=7.5$</td>
<td>$D_f=137$</td>
</tr>
</tbody>
</table>
Therefore in this comparison, the maximum dynamic range possible from the cross correlation technique will be 18 times greater than the equivalent optimised autocorrelation.
Comparison of Peak Intensity Recorded on a Labelled and Unlabelled Image Using the same Aperture Diameter

The pupil function of the labelled system \( P_l(x, y) \) and the unlabelled system \( P_s(x, y) \) can be represented by

\[
P_l(x, y) = kA(x, y) \ast \left\{ \delta(x, y + D_s/2) + \delta(x, y - D_s/2) \right\}
\]

(A9.1)

and

\[
P_s(x, y) = kA(x, y)
\]

(A9.2)

where \( k \) is a constant, \( x, y \) are coordinates at the pupil plane, \( \delta(x, y) \) represents the delta function, \( \ast \) represents the convolution function, \( A(x, y) \) represents the aperture function, \( D_s \) is the diametrical separation of the apertures about the centre of the pupil function \( P_l(0, 0) \) for the labelled system and \( P_s(x, y) \) is a centrally located aperture for the unlabelled system. Applying the impulse response function as outlined in equations 5.1 and 5.2 of chapter 5 leads to the intensity prediction in the image plane \( I_l(\xi, \eta) \) for the labelled images and \( I_s(\xi, \eta) \) for the unlabelled images as

\[
I_l(\xi, \eta) = Kt(\xi, \eta)^2 \left\{ \exp\left[ +j\pi D_s \eta \right] + \exp\left[ -j\pi D_s \eta \right] \right\}^2
\]

(A9.3)

and

\[
I_s(\xi, \eta) = Kt(\xi, \eta)^2
\]

(A9.4)

where \( K \) is a constant, \( \xi, \eta \) are coordinates in the image plane and \( t(\xi, \eta) \) represents the impulse response of the aperture function. Simplifying equation (A9.3) leads to

\[
I_l(\xi, \eta) = Kt(\xi, \eta)^2 \left[ 2 \cos(\pi D_s \eta) \right]^2
\]

(A9.5)

Therefore the ratio of peak intensities of labelled to unlabelled images at \( \xi=0, \eta=0 \) is 4:1.
APPENDIX 9

Image Labelling Data Processing Software Description

With reference to the hardware description of section 3.3, the spatial filtering and cross correlation was carried out at each interrogation region by running the following computer routine:

1) Increment transparency to required position
2) Grab image and download from frame store to VIPA
3) Apply forward (Fast Fourier Transform) FFT to image to obtain Fourier plane
4) Carry out image separation by storing complex and real parts from the two orthogonal satellite peaks into separate VIPA areas of interest
5) Apply reverse transform FFT to both areas of interest to obtain first and second exposure images
6) Apply forward transform FFT to first and second exposure images
7) Multiply FFT output of first exposure image by phase conjugate of second exposure FFT output and store real and complex results in separate areas of interest
8) Reverse transform FFT to output cross correlation plane
9) Find signal peak position and send to data file
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Chapter 4


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5.3 Gaskill J.D. "Linear Systems, Fourier Transforms and Optics" Wiley, P40-98 (1978)
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5.7 Melles Griot/Photon Control "Optics Guide 5" ISSN 1051-4384
Figure 1.1 - An Example of a PIV Set-up

Sheeting Optics

Laser Beam

Transparency Plane
(interrogation region)

Particle Image Pair

Camera

Seeded Flow

Laser Sheet
Figure 2.1 - PIV Recording Set-up

Laser Beam

Sheeting Optics

Transparency Plane
Interrogation Region

Interrogation Volume

Recording System

\[ U \]

\[ V \]

\[ W \]

\[ L \]

\[ 

\[ s_i \]

\[ s_0 \]
Figure 2.2 - Interrogation Volume Particle Position Probability Density Function

\[ \rho(l) \]

Figure 2.3 - Schematic of Interrogation Volume
Figure 2.4 - Typical Autocorrelation with Velocity Gradients ($N_i=6$)
Figure 2.5 - Monte Carlo SNR Characteristics for a Range of Particle Image Densities and Particle Image Sizes (10% in plane movement)

Figure 2.6 - Mean Autocorrelation Function Normalised to Signal Peak Height
Figure 2.7 - Baseline SNR Characteristics for a Range of D/d (N_i=12)

Figure 2.8 - Loss in SNR Characteristics for a Range of In Plane Movements and Particle Image Sizes (N_i=12)
Figure 2.9 - Loss in SNR Characteristics for a Range of Out of Plane Movements and Particle Image Sizes (Ni=12)

Figure 2.10 - Loss in SNR Characteristics for a Range of the Product $\phi \times D_r$ and Particle Image Sizes (Ni=12)
Figure 2.11 - Random Error Characteristics for a Range of $\phi D_r$ and Interrogation Region Pixel Resolutions ($N_t = 12, D/d = 16$)

Figure 2.12 - Random Error Characteristics for a Range of $\phi D_r$ and $D/d$ ($R_N = 256, N_t = 12$)
Figure 2.13 - x Component Error Distribution Characteristics for a Range of $\phi$D_r
($N_I=12$, $D/d=16$, 40% in plane movement, $u_o=v_o$, 1000 samples, $R_N=256$)

- Velocity Gradient x
  Dynamic Range=0,
  Mean=0.003%, Random Error=0.297%

- Velocity Gradient x
  Dynamic Range=0.5,
  Mean=-0.045%, Random Error=0.454%

- Velocity Gradient x
  Dynamic Range=1.0,
  Mean=-0.084%, Random Error=0.676%

Figure 2.14 - x Component Error Distribution Characteristics for a Range of Interrogation Region Resolutions ($N_I=12$, $\phi$D_r=0, $D/d=16$, 30% in plane movement, $u_o=v_o$, 1000 samples)

- RN=32, Mean = -0.173%, Random Error=0.55%

- RN=64, Mean = -0.027%, Random Error=0.33%

- RN=128, Mean = 0.012%, Random Error=0.32%

- RN=256, Mean = 0.027%, Random Error=0.28%
Figure 2.15 - Illustration of x Component of Mean Initial Particle Image Position $\langle x_N \rangle$ Through Fractional Loss of Particle Image Pairs for a given Velocity Component $u_0$

- Unpaired Particle Area/Limits

- Interrogation Volume

- $x_i$ Probability Distribution

\[ \langle x_N \rangle = -u_0 \Delta t/2 \]
Figure 2.16 - Systematic Error Characteristics for a Range of $\phi D_r$ and In Plane Movements ($D/d=16, N_i=12, R_N=256, u_o=v_o$)

Figure 2.17 - Comparison of Systematic Error Characteristics for a First and Third Order Particle Displacement Model for a Range of $\phi D_r$ and In Plane Movements ($D/d=16, N_i=12, R_N=256, u_o=v_o$)
Figure 3.1 - Schematic of Velocity Gradient Data Recording Installation

- HeNe Laser Sheet
- Seeded Block
- Camera
- Axis of Rotation
- Rotary Translation Stage

Figure 3.2 - Typical Double Exposed Transparency Regions
a) 1x1mm region \( (N_1=14) \)
b) 2x2mm region \( (N_2=28) \)
Figure 3.3 - Schematic of Digital PIV Image Plane Data Processing System

Figure 3.4 - Particle Size Distribution (mean \(d=21.4\mu m\), sample of 2000, \(N_i=14\), 0.5x0.5mm sample regions)
Figure 3.5 - Particle Image Intensity Distribution (N_i=14, 0.5x0.5mm sample regions)

Particle Image Intensity (0-black, 255-white)

Figure 3.6 - Particle Image Density Distribution (N_i=14, 0.5x0.5mm sampling window, 500 samples)
Figure 3.7 - Comparison of Monte Carlo and Experimental Valid Vector Characteristics for a Range of Particle Image Densities (10% in plane movement, D/d=50)

Figure 3.8a - Experimental Valid Vector Characteristics for a Range of Particle Image Densities and In Plane Movements (D/d=50)
Figure 3.8b - Monte Carlo Valid Vector Characteristics for a Range of Particle Image Densities and In Plane Movements (D/d=50)

In Plane Movement (% of D)

Valid Vectors (%)

- Ni = 3
- Ni = 6
- Ni = 12
- Ni = 25

Figure 3.9a - Experimental Valid Vector Characteristics for a Range of Particle Image Densities and Out of Plane Movements (D/d=50)

Out of Plane Movement (% of W)

Valid Vectors (%)

- Ni = 3.1
- Ni = 7.5
- Ni = 14.12
- Ni = 28.1
Figure 3.9b - Monte Carlo Valid Vector Characteristics for a Range of Particle Image Densities and Out of Plane Movements (D/d=50)

Figure 3.10 - Comparison of Monte Carlo and Experimental Signal to Noise Characteristics for a Range of $\phi D_r$ (mean in plane movement=25%, $N_i=12$, D/d=25)
Figure 3.11 - Autocorrelation Function with dc Peak Removed (10% in plane movement, $N_i=3.1$, $D/d=50$)

Figure 3.12 - Autocorrelation Function from 3 Bit Image with dc Peak Removed (10% in plane movement, $N_i=3.1$, $D/d=50$)
Figure 3.13 - SNR Characteristics from Monte Carlo, Experimental and Experimental 3 Bit Analysis for a Range of Particle Image Densities (10% in plane movement, D/d=50)
Figure 4.1 - Nozzle/Plenum Chamber Configuration

- Laser Beam
- Sheet Flow
- Glass Nozzle
- Seeded Flow
- Camera
- Plenum Chamber
- Laser Sheet
- Sheet Flow Optics
Figure 4.2 - Seeding Injector Design

Figure 4.3 - PIV transparency with Central Seeding Injection
Figure 4.4 - Sheeting Optics Arrangement

![Sheeting Optics Arrangement](image)

Figure 4.5 - Schlieren Photograph Showing Normal Shock Position for de Laval Nozzle at a 62.0kPa Plenum Chamber Gauge Pressure

![Schlieren Photograph](image)
Figure 4.6 - PIV Vector Plot showing Normal Shock Position for de Laval nozzle at a 62.0kPa Plenum Chamber Gauge Pressure
Figure 4.7 - PIV Results Showing Comparison of x Component of Velocity at 62.0kPa and 41.3kPa Plenum Chamber Pressures

Plenum Pressure 62.0kPa

Plenum Pressure 41.3kPa

Air Flow Direction

Central Streamline

Axial Distance from Nozzle Inlet (mm)

>460m/s

<260m/s

>400m/s

<250m/s
Figure 4.8 - 1D Theory vs PIV Results for a Single Streamline

Figure 4.9 - CFD Prediction vs PIV Results for a Single Streamline
Figure 4.10 - PIV Open Jet Results showing Comparison of x Component of Velocity at 186.0kPa and 144.7kPa Plenum Chamber Pressures

- Plenum pressure 186.0kPa
- Plenum pressure 144.7kPa

Central Streamline

<table>
<thead>
<tr>
<th>Distance from Nozzle Exhaust (mm)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>350</td>
</tr>
<tr>
<td>10</td>
<td>400</td>
</tr>
<tr>
<td>15</td>
<td>450</td>
</tr>
<tr>
<td>20</td>
<td>500</td>
</tr>
<tr>
<td>25</td>
<td>550</td>
</tr>
</tbody>
</table>

- >515m/s
- <340m/s
- >460m/s
- <300m/s

Air flow direction: arrow pointing right.
Figure 4.11 - x Component Open Jet Result Compared with Equivalent Schlieren Result

Air Flow direction

>460m/s

<300m/s

Schlieren Results

Central Streamline

velocity (m/s)

Distance from Nozzle Exhaust (mm)
Figure 5.1 - Image Labelling Recording Set-up

Object Plane

Optical System (Camera Objective)

Point Source

Pupil Mask (Aperture Fn.)

Image Plane

2f

2f

Film
Figure 5.2 - Spatial Filtering Set-up

Figure 5.3 - Schematic of Amplitude Distribution Produced in the Fourier Plane from a Double Exposure Image Labelled Transparency
Figure 5.4 - Typical Cross Correlation with Zero Velocity Gradients ($N_j=6$)
Figure 5.5 - Details of Polarisation Sensitive Aperture Plate Design

Polaroid segments with axes in direction of arrows

Figure 5.6 - Experimental Set-up for High Speed Image Labelling System
Figure 5.7 a) Image Labelled Transparency Interrogation Region. b) First Exposure Separated Image. c) Second Exposure Separated Image. d) Cross Correlation of b) and c)