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Current hysteresis and memory effect in a molecular quantum dot with strong electron-vibron interaction

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Theory of current hysteresis for tunneling through a molecular quantum dot (MQD) with strong electron-vibron interactions and attractive electron-electron correlations is developed. The dot is modeled as a \(d\)-fold degenerate energy level weakly coupled to the leads. The effective attractive interaction between polarons in the dot results in a "switching" phenomenon in the current-voltage characteristics when \(d > 2\), in agreement with the results for the phenomenological negative-\(U\) model. The degenerate MQD with strong electron-vibron coupling has two stable current states in certain interval of the bias voltage below some critical temperature.

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I. INTRODUCTION

Strongly correlated transport through mesoscopic systems with repulsive electron-electron interactions has received considerable interest in the past (see, for example, Refs. \cite{1,2,3}), and continues to be the focus of intense experimental and theoretical investigation \cite{4,5,6,7,8}. The Coulomb interaction suppresses tunneling for certain range of applied voltages, leading to what is commonly called the Coulomb blockade. There is now a growing interest in molecular nanowires and quantum dots used as the "transmission lines" \cite{13,14} and active molecular elements \cite{13} in molecular-scale electronics \cite{13}.

A few experimental studies \cite{14} provide evidence for a molecular switching effect, when the current-voltage (I-V) characteristics show two branches with high and low current for the same voltage. The effect exists in simple molecules too \cite{13}. It is important to identify the actual mechanism of switching.

Recently we have proposed a negative-\(U\) Hubbard model of a \(d\)-fold degenerate quantum dot, with an intrinsic non-retarded switching mechanism when \(d > 2\) \cite{16}. We argued that the attractive electron correlations could be caused by a strong electron-phonon (vibron) interaction in the molecule, and/or by the valence fluctuations.

It has been recently demonstrated that the low-bias conductance of molecules is dominated by resonant tunneling through coupled electronic and vibration levels \cite{17}. Different aspects of the electron-phonon/vibron (e-ph) interaction effect on the tunneling through molecules and quantum dots (QD) have been studied by several authors \cite{18,19,20,21,22,23,24,25}. In particular, Glazman and Shekhter, and later Wingreen et al. \cite{18} presented the exact resonant-tunneling transmission probability fully taking into account the e-ph interaction on a nondegenerate resonant site. Phonons produced transmission sidebands but did not affect the integral transmission probability. Li, Chen and Zhou \cite{4} studied the conductance of a double degenerate (due to spin) quantum dot with Coulomb repulsion and the e-ph interaction. Their numerical results also showed the sideband peaks and the main peak related to the Coulomb repulsion, which was decreased by the e-ph interaction. Kang \cite{20} studied the boson (vibron) assisted transport through a double-degenerate QD coupled to two superconducting leads and found multiple peaks in the I-V curves, which originated from the singular BCS density of states and the phonon sidebands. Ermakov \cite{21} calculated the I-V curves of a four-fold degenerate dot including both the onsite Coulomb and e-ph interactions. He obtained a switching effect in the numerical I-V curves, similar to that in the negative-\(U\) Hubbard model discussed by us recently \cite{16}. However, using a Hamiltonian averaged over the phonons, Ermakov missed all phonon sidebands and obtained an unphysical population \((n = 1)\) of each QD state. More recently Gogolin and Komnik \cite{25} have analyzed a nondegenerate QD, \(d = 1\), coupled with a single phonon mode. They found a switching effect in the Born-Oppenheimer approximation similar to that in our negative-\(U\) Hubbard model, but surprisingly for a non-degenerate case with \(d = 1\). However, we have to mention that the Born-Oppenheimer approximation does not apply to a non-degenerate level, since there are no "fast" (compared to the characteristic phonon time \(1/\omega_0\)) electron transitions within the dot. Despite differences in the models and approximations, Refs. \cite{18,19,20,21,22,23,24,25} pointed to a novel mechanism of the volatile molecular memory caused by the e-ph or any other attractive electron correlations.

Here we develop the analytical theory of a correlated transport through a degenerate molecule quantum dot (MQD) fully taking into account both Coulomb and e-ph interactions. We show that the phonon sidebands significantly modify the switching behavior of the I-V curves
in comparison with the negative-$U$ Hubbard model [16]. Nevertheless, the switching effect is robust. It shows up when the effective interaction of polarons is attractive and the state of the dot is multiply degenerate, $d > 2$.

II. STEADY CURRENT THROUGH MQD

We apply the Landauer-type expression for the steady current through a region of interacting electrons, derived by Meir and Wingreen [7] as (in units $\hbar = k_B = 1$)

$$I(V) = -\frac{e}{\pi} \int_{-\infty}^{\infty} d\omega \left[ f_1(\omega) - f_2(\omega) \right] \mathrm{Im} \mathrm{Tr} \left[ \hat{\Gamma}(\omega) \hat{G}^{(1)}(\omega) \right],$$

(1)

where $f_1(\omega) = \left\{ \exp[(\omega + \Delta - eV/2)/T] + 1 \right\}^{-1}$, $T$ is the temperature, $\Delta$ is the position of the lowest unoccupied molecular level with respect to the chemical potential. $\hat{\Gamma}(\omega)$ depends on the density of states (DOS) in the leads and on the hopping integrals connecting one-particle states in the left (1) and the right (2) leads with the states in MQD, Fig. 1. This formula includes, by means of the Fourier transform of the full molecular retarded Green’s function (GF), $\hat{G}^{(1)}(\omega)$, the e-ph and Coulomb interactions inside MQD and coupling to the leads. Since the leads are metallic, electron-electron and e-ph interactions in the leads, and interactions of electrons in the leads with electrons and phonons in MQD can be neglected. We are interested in the tunneling near the conventional threshold, $eV = 2\Delta$, Fig.1, within a voltage range about an effective attractive potential $|U|$ caused by phonons/vibrations (see below).

The attractive energy is the difference of two large interactions, the Coulomb repulsion and the phonon mediated attraction, of the order of 1eV each. Hence, $|U|$ is of the order of a few tens of one eV. We neglect the energy dependence of $\hat{\Gamma}(\omega) \approx \Gamma$ on this scale, and assume that the coupling to the leads is weak, $\Gamma \ll |U|$. In this case $\hat{G}^{(1)}(\omega)$ does not depend on the leads. Moreover we assume that there is a complete set of one-particle molecular states $|\mu\rangle$, where $\hat{G}^{(1)}(\omega)$ is diagonal. With these assumptions we can reduce Eq.(1) to

$$I(V) = I_0 \int_{-\infty}^{\infty} d\omega \left[ f_1(\omega) - f_2(\omega) \right] \rho(\omega),$$

(2)

allowing for a transparent analysis of essential physics of the switching phenomenon. Here $I_0 = e\Gamma$ and the molecular DOS, $\rho(\omega)$, is given by

$$\rho(\omega) = -\frac{1}{\pi} \sum_{\mu} \mathrm{Im} G_{\mu}(\omega),$$

(3)

where $G_{\mu}(\omega)$ is the Fourier transform of $G_{\mu}(t) = -i\theta(t) \langle \{ c_{\mu}(t), c_{\mu}^\dagger \} \rangle$, $\{ \cdots, \cdots \}$ is the anticommutator, $\hat{G}^{(1)}(\omega)$ is the Fourier transform of $G^{(1)}(t) = -i \theta(t) \langle \{ \hat{c}_{\mu}(t), \hat{c}_{\mu}^\dagger \} \rangle$, and $\hat{c}_{\mu}$ annihilates an electron in molecular state $|\mu\rangle$.

FIG. 1. Schematic of the energy levels and phonon side-bands for molecular quantum dot under bias voltage $V$ ($eV/2\Delta = 0.75$) with the coupling constant $\gamma^2 = 11/13$. The level is assumed to be 4-fold degenerate ($d = 4$) with energies $\Delta + rU$, $r = 0, \ldots , (d - 1)$ (thick bars). Thin bars show the vibron side-bands with the size of the bar proportional to the weight of the particular contribution in the density of states (see text) in the case of one vibron with frequency $\omega_0/\Delta = 0.2$ at $T = 0$. Only the bands in the energy window $(eV/2, -eV/2)$ (shown) will contribute to current at zero temperature.

$$c_{\mu}(t) = e^{iHt}c_{\mu}e^{-iHt}, \theta(t) = 1 \text{ for } t > 0 \text{ and zero otherwise.}$$

We calculate $\rho(\omega)$ exactly in Section III in the framework of the Hamiltonian, which includes both the Coulomb $U^C$ and e-ph interactions as

$$H = \sum_{\mu} \varepsilon_{\mu} \hat{n}_{\mu} + \frac{1}{2} \sum_{\mu \neq \mu'} U_{\mu \mu'}^C \hat{n}_{\mu} \hat{n}_{\mu'} + \sum_{\mu, q} \hat{n}_{\mu} \omega_q (\gamma_{\mu q} d_q + H.c.) + \sum_q \omega_q (d_q^d d_q^d + 1/2).$$

(4)

Here $\varepsilon_{\mu}$ are one-particle molecular energy levels, $\hat{n}_{\mu} = \hat{c}_{\mu}^\dagger \hat{c}_{\mu}$ the occupation number operators, $c_{\mu}$ and $d_q$ annihilates electrons and phonons, respectively, $\omega_q$ are the phonon (vibron) frequencies, and $\gamma_{\mu q}$ are e-ph coupling constants ($q$ enumerates the vibron modes). This Hamiltonian conserves the occupation numbers of molecular states $\hat{n}_{\mu}$. Hence it is compatible with Eq.(3).
III. MQD DENSITY OF STATES

We apply the canonical polaron unitary transformation $e^{S}$ [20], integrating phonons out, as
\[ \hat{H} = e^{S} \hat{H}_e e^{-S}, \]  
where
\[ S = - \sum_{\mu q} \hat{n}_{\mu} [\gamma_{\mu q} \hat{d}_{q} - H.c.] \]
is such that $S^\dagger = -S$. The electron and phonon operators are transformed as
\[ \tilde{c}_{\mu} = c_{\mu} X_{\mu}, \]  
and
\[ \tilde{d}_{q} = d_{q} - \sum_{\mu} \hat{n}_{\mu} \gamma_{\mu q}^*, \]
respectively. Here
\[ X_{\mu} = \exp \left[ \sum_{q} \gamma_{\mu q} \hat{d}_{q} - H.c. \right]. \]
The Lang-Firsov canonical transformation shifts ions to new equilibrium positions with no effect on the phonon frequencies. The diagonalization is exact:
\[ \tilde{H} = \sum_{i} \tilde{\varepsilon}_{\mu} \hat{n}_{\mu} + \sum_{q} \omega_{q} (\tilde{d}_{q} \tilde{d}_{q} + 1/2) + \frac{1}{2} \sum_{\mu \neq \mu'} U_{\mu \mu'} \hat{n}_{\mu} \hat{n}_{\mu'}, \]
where
\[ U_{\mu \mu'} = U_{\mu \mu'}^{C} - 2 \sum_{q} \gamma_{\mu q}^* \gamma_{\mu q} \omega_{q} \]
is the interaction of polarons comprising their interaction via molecular deformations (vibrions) and non-vibrion (e.g. Coulomb repulsion) $U_{\mu \mu'}^{C}$. To simplify the discussion, we shall assume, without losing generality, that the Coulomb integrals do not depend on the orbital index, i.e. $U_{\mu \mu'} = U$. The molecular energy levels are shifted by the polaron level-shift due to a deformation well created by polaron,
\[ \tilde{\varepsilon}_{\mu} = \varepsilon_{\mu} - \sum_{q} |\gamma_{\mu q}|^2 \omega_{q}. \]
Applying the same transformation in the retarded GF we obtain
\[ G_{\mu}(t) = -i \theta(t) \left\langle \left\{ c_{\mu}(t) X_{\mu}(t), c_{\mu}^{\dagger}(t) X_{\mu}^{\dagger}(t) \right\} \right\rangle \]
\[ = -i \theta(t) \left\langle \left\{ c_{\mu}(t) c_{\mu}^{\dagger}(t) \right\} \left\langle X_{\mu}(t) X_{\mu}^{\dagger}(t) \right\rangle \right\rangle \]
\[ + \left\langle c_{\mu}^{\dagger}(t) c_{\mu}(t) \right\rangle \left\langle X_{\mu}^{\dagger}(t) X_{\mu}(t) \right\rangle, \]
where now electron and phonon operators are averaged over the quantum state of the transformed Hamiltonian $\hat{H}$. There is no coupling between polarons and vibrions in the transformed Hamiltonian, and the electron and phonon averages are independent. The Heisenberg phonon operators evolve as
\[ d_{q}(t) = d_{q} e^{-i \omega_{q} t}, \]
so that we find after thermodynamic averaging of the phonon correlator over phonon occupation numbers (using the Weyl’s identity for exponential operators)
\[ \langle X_{\mu}(t) X_{\mu}^{\dagger}(t) \rangle \]
\[ = \exp \left[ \sum_{q} \frac{|\gamma_{\mu q}|^2}{\sinh \frac{\beta \omega_{q}}{2}} \left[ \cos \left( \omega t + \frac{\beta \omega_{q}}{2} \right) - \cosh \frac{\beta \omega_{q}}{2} \right] \right], \]
where $\beta = 1/T$. Repeating the calculations for $\langle X_{\mu}^{\dagger}(t) X_{\mu}(t) \rangle$ we find a simple useful relation
\[ \langle X_{\mu}^{\dagger}(t) X_{\mu}(t) \rangle = \langle X_{\mu}(t) X_{\mu}^{\dagger}(t) \rangle^*. \]
At low temperatures $T \ll \omega_{q}$ the phonon correlator [4] simplifies to
\[ \langle X_{\mu}(t) X_{\mu}^{\dagger}(t) \rangle = \exp \left[ \sum_{q} |\gamma_{\mu q}|^2 (e^{-\omega_{q} t} - 1) \right]. \]
Next, we introduce the multi-particle GFs, which will necessarily appear in the equations of motion for $\langle c_{\mu}(t)c_{\mu}^{\dagger}(t) \rangle$, as
\[ G_{\mu}^{(r,+)}(t) \equiv -i \theta(t) \sum_{\mu_1 \neq \mu \neq \mu_2 \ldots} \left\langle c_{\mu}(t) c_{\mu}^{\dagger} \prod_{i=1}^{r-1} \hat{n}_{\mu_i} \right\rangle, \]
and
\[ G_{\mu}^{(r,-)}(t) \equiv -i \theta(t) \sum_{\mu_1 \neq \mu_2 \ldots} \left\langle c_{\mu}^{\dagger}(t) c_{\mu} \prod_{i=1}^{r-1} \hat{n}_{\mu_i} \right\rangle. \]
Then, using the equation of motion for the Heisenberg polaron operator,
\[ i \frac{dc_{\mu}}{dt} = \left( \tilde{\varepsilon}_{\mu} + U \sum_{\mu' (\neq \mu)} \hat{n}_{\mu'} \right) c_{\mu}, \]
we derive the following equations for the multi-particle GFs,
\[ i \frac{dG_{\mu}^{(r,+)}(t)}{dt} = \delta(t)(1 - n_{\mu}) \sum_{\mu_1 \neq \mu_2 \ldots} \prod_{i=1}^{r-1} n_{\mu_i}, \]
\[ + [\tilde{\varepsilon}_{\mu} + (r-1)U] G_{\mu}^{(r,+)}(t) + U G_{\mu}^{(r+1,+)}(t), \]
and
\[ \frac{d G^{(r,-)}_{\mu}(t)}{dt} = \delta(t) n_\mu \sum_{\mu \neq \mu_2 \neq \ldots \neq \mu_1} \prod_{i=1}^{r-1} n_{\mu_i} - [\vec{\epsilon}_\mu + (r-1)U]G^{(r,-)}_{\mu}(t) + U G^{(r+1,-)}_{\mu}(t), \]  

where \( n_\mu = \langle \epsilon_i^\mu \rangle_c \) is the expectation number of electrons on the molecular level \( \mu \).

We can readily solve this set of coupled equations for MQD with one \( d \)-fold degenerate energy level and with the e-ph coupling \( \gamma_{\mu q} = \gamma_q \), which does not break the degeneracy. At zero bias voltage the empty level will lie by some energy \( \Delta \) above the Fermi levels of the electrodes (Fig. 1). Assuming that \( n_\mu = n \), Fourier transformation of the set yields

\[
G^{(1,+)}_{\mu}(\omega) = (1 - n) \sum_{r=0}^{d-1} Z_r(n) \frac{Z_r(n)}{\omega - rU + i\delta}, \tag{22}
\]

\[
G^{(1,-)}_{\mu}(\omega) = n \sum_{r=0}^{d-1} Z_r(n) \frac{Z_r(n)}{\omega - rU + i\delta}, \tag{23}
\]

where \( \delta = +0 \), and

\[
Z_r(n) = \frac{(d-1)!}{r!(d-r)!} n^r(1 - n)^{d-1-r}. \tag{24}
\]

### A. MQD Green’s function at low temperatures

\( (T \ll \omega_q) \)

It is the easiest to find the total Green’s function at low temperatures. Indeed, transforming back to real time and using Eqs.(22), (23) and (10), (13) we arrive at

\[
G_{\mu}(t) = -i \theta(t) Z_0 \sum_{r=0}^{d-1} Z_r(n) e^{-irUt} \times \left[ (1 - n) \exp \left( \sum_q |\gamma_{\mu q}|^2 e^{-i\omega_q t} \right) + n \exp \left( \sum_q |\gamma_{\mu q}|^2 e^{i\omega_q t} \right) \right], \tag{25}
\]

where \( Z_0 = \exp \left[ - \sum_q |\gamma_q|^2 \right] \). This is an exact solution with respect to correlations and e-ph interactions which satisfies all sum rules. Expanding the exponents in the brackets we find its Fourier component as

\[
G_{\mu}(\omega) = Z_0 \sum_{r=0}^{d-1} Z_r(n) \left[ \frac{1}{\omega - rU + i\delta} \right. \left. + \sum_{l=1}^{\infty} \frac{1}{l!} \sum_{q_1,\ldots,q_l} |\gamma_{q_1} \cdots \gamma_{q_l}|^2 \frac{1 - n}{\omega - rU - \sum_{k=1}^l \omega_{v_k} + i\delta} \right. \left. + \frac{n}{\omega - rU + \sum_{k=1}^l \omega_{v_k} + i\delta} \right]. \tag{26}
\]

If the e-ph interaction is weak, \( |\gamma_q| \ll 1 \), the essential contribution comes only from the first (phonon-less) term, and we recover the result of the Hubbard model [16]

\[
G_{\mu}(\omega) = \sum_{r=0}^{d-1} \frac{Z_r(n)}{\omega - rU + i\delta}. \tag{27}
\]

At finite \( |\gamma_q| \gg 1 \), the phonon side-bands become important in Eq.(26), which is obviously in the form of the multi-polaron expansion. If one neglects the correlations, \( U = 0 \), a standard polaron GF [27] is recovered:

\[
G_{\mu}(\omega) = Z_0 \left[ \frac{1}{\omega + i\delta} + \sum_{l=1}^{\infty} \prod_{q_1,\ldots,q_l} |\gamma_{q_1} \cdots \gamma_{q_l}|^2 \right. \left. \times \left( \frac{1 - n}{\omega - \sum_{k=1}^l \omega_{v_k} + i\delta} + \frac{n}{\omega + \sum_{k=1}^l \omega_{v_k} + i\delta} \right) \right], \tag{28}
\]

where we have applied the sum rule

\[
\sum_{r=0}^{d-1} Z_r(n) = 1. \tag{29}
\]

### B. MQD Green’s function at finite temperatures, single vibron mode

By applying the same method, as in the case of \( T = 0 \), and going over back to real time with the use of Eqs.(22), (23) and (10), (13) we arrive at

\[
G_{\mu}(t) = -i \theta(t) Z \sum_{r=0}^{d-1} Z_r(n) e^{-irUt} \times \left[ (1 - n) \exp \left( \sum_q |\gamma_{\mu q}|^2 \frac{\sinh \frac{\beta \omega_q}{2}}{\sinh \frac{\beta U}{2}} \cos \left( \omega t + i \frac{\beta \omega_q}{2} \right) \right) + n \exp \left( \sum_q |\gamma_{\mu q}|^2 \frac{\sinh \frac{\beta \omega_q}{2}}{\sinh \frac{\beta U}{2}} \cos \left( \omega t - i \frac{\beta \omega_q}{2} \right) \right) \right], \tag{30}
\]

where now

\[
Z = \exp \left[ - \sum_q |\gamma_q|^2 \frac{\coth \frac{\beta \omega_q}{2}}{\frac{\beta \omega_q}{2}} \right]. \tag{31}
\]

In approximation, where we retain a coupling to a single mode with the characteristic frequency \( \omega_0 \) and \( \gamma_q \equiv \gamma \), we can expand the exponents in the temporal Green’s function (26) in powers of \( \exp \left( \omega t + i \frac{\beta \omega_q}{2} \right) \). It is then trivial to find the Green’s function in the frequency domain as

\[
G_{\mu}(\omega) = Z \sum_{r=0}^{d-1} Z_r(n) \sum_{l=0}^{\infty} I_l(\xi) \]
\[
\begin{align*}
&\times \left[ e^{\frac{\beta \omega l}{2}} \left( \frac{1 - n}{\omega - rU - l\omega_0 + i\delta} + \frac{n}{\omega - rU + l\omega_0 + i\delta} \right) \\
&+ (1 - \delta_0)e^{-\frac{\beta \omega l}{2}} \left( \frac{1 - n}{\omega - rU + l\omega_0 + i\delta} \\
&+ \frac{n}{\omega - rU - l\omega_0 + i\delta} \right) \right],
\end{align*}
\]

where \( \xi = |\gamma|^2 / \sinh \frac{\beta \omega}{2} \), \( I_l(\xi) \) is the modified Bessel function, and \( \delta_{lk} \) is the Kronecker symbol. At low temperatures, where \( \beta \omega_0 \gg 1 \), \( \xi \ll 1 \) and \( I_l(\xi) \approx \left( \frac{\xi}{l} \right)^{l+1} / l! \), this expression gives (36) in the form

\[
G_\mu(\omega) = Z_0 \sum_{r=0}^{d-1} Z_r(n) \sum_{l=0}^\infty |\gamma|^2 \left( \frac{1 - n}{\omega - rU - l\omega_0 + i\delta} \\
+ \frac{n}{\omega - rU + l\omega_0 + i\delta} \right).
\]

The important feature of the DOS, Eq.(34), is its non-linear dependence on the occupation number \( n \), which leads to the switching effect and hysteresis in the I-V characteristics for \( d > 2 \), as will be shown below. It contains full information about all possible correlation and inelastic effects in transport, in particular, all the vibron-assisted tunneling processes (Fig. 2).

**IV. NONLINEAR RATE EQUATION AND SWITCHING**

Generally, the electron density \( n_\mu \) obeys an infinite set of rate equations for many-particle GFs which can be derived in the framework of a tunneling Hamiltonian including correlations [14]. In the case of MQD only weakly coupled with leads one can apply the Fermi-Dirac golden rule to obtain an equation for \( n \). Equating incoming and outgoing numbers of electrons in MQD per unit time we obtain the self-consistent equation for the level occupation \( n \) as

\[
(1 - n) \int_{-\infty}^{\infty} d\omega \left\{ \Gamma_1 f_1(\omega) + \Gamma_2 f_2(\omega) \right\} \rho(\omega) \\
- n \int_{-\infty}^{\infty} d\omega \left\{ \Gamma_1 [1 - f_1(\omega)] + \Gamma_2 [1 - f_2(\omega)] \right\} \rho(\omega) = 0
\]

where \( \Gamma_{1(2)} \) are the transition rates from left (right) leads to MQD. Taking into account that \( \int_{-\infty}^{\infty} \rho(\omega) = d \), Eq.(35) for the symmetric leads, \( \Gamma_1 = \Gamma_2 \), reduces to

\[
2nd = \int d\omega \rho(\omega) (f_1 + f_2),
\]

which automatically satisfies \( 0 \leq n \leq 1 \). Explicitly, the self-consistent equation for the occupation number is

\[
n = \frac{1}{2} \sum_{r=0}^{d-1} Z_r(n)[na_r + (1 - n)b_r],
\]

where

\[
a_r = Z \sum_{l=0}^\infty I_l(\xi) \left( e^{\frac{\beta \omega l}{2}} [f_1(rU - l\omega_0) + f_2(rU - l\omega_0)] \\
+ (1 - \delta_0)e^{-\frac{\beta \omega l}{2}} [f_1(rU + l\omega_0) + f_2(rU + l\omega_0)] \right),
\]

\[
b_r = Z \sum_{l=0}^\infty I_l(\xi) \left( e^{\frac{\beta \omega l}{2}} [f_1(rU + l\omega_0) + f_2(rU + l\omega_0)] \\
+ (1 - \delta_0)e^{-\frac{\beta \omega l}{2}} [f_1(rU - l\omega_0) + f_2(rU - l\omega_0)] \right).
\]

The current is expressed as
\[ j \equiv \frac{I(V)}{dI_0} = \sum_{r=0}^{d-1} Z_r(n)[na'_r + (1-n)b'_r], \] (40)

where
\[ a'_r = Z \sum_{l=0}^{\infty} I_l(\xi) \left( e^{\frac{\beta \omega_0}{2}} \left[ f_1(rU - \omega_0) - f_2(rU - \omega_0) \right] 
+ (1 - \delta_{00}) e^{-\frac{\beta \omega_0}{2}} \left[ f_1(rU + \omega_0) - f_2(rU + \omega_0) \right] \right), \] (41)
\[ b'_r = Z \sum_{l=0}^{\infty} I_l(\xi) \left( e^{\frac{\beta \omega_0}{2}} \left[ f_1(rU + \omega_0) + f_2(rU + \omega_0) \right] 
+ (1 - \delta_{00}) e^{-\frac{\beta \omega_0}{2}} \left[ f_1(rU - \omega_0) + f_2(rU - \omega_0) \right] \right). \] (42)

Let us analyze the I-V curves for \( d = 1, 2, \) and \( 4. \)

**A. Absence of switching for nondegenerate and two-fold degenerate MQD**

There is one term in the sum over \( r, \) \( r = 0 \) with \( Z_0(n) = 1, \) if \( d = 1. \) Hence there is only one solution of the rate equation (47)
\[ n = \frac{b_0}{2 + b_0 - a_0}, \] (43)
and the current is single-valued at any voltage
\[ j = \frac{2b'_0 + a'_0b_0 - a_0b'_0}{2 + b_0 - a_0}. \] (44)

This is an exact result, which is valid for any \( e\)-\( ph \) coupling and any phonon frequency. We have to conclude that there is no switching of a nondegenerate MQD. The opposite conclusion reached in Ref. [25] might be due to the Born-Oppenheimer (static) approximation used by Gogolin and Komnik. In fact, the Born-Oppenheimer approximation does not apply to the non-degenerate level model, since there are no “fast”, compared to phonon times \( 1/\omega_0, \) electron transitions within the “molecule”.

In the case of a double-degenerate MQD, \( d = 2, \) there are two terms, which contribute to the sum over \( r, \) with \( Z_0(n) = 1 - n \) and \( Z_1(n) = n. \) The rate equation becomes a quadratic one
\[ n^2(a_0 + a_1 - b_0 - b_1) + n(2 - a_0 + b_0 - b_1) - b_0 = 0. \] (45)

with two solutions,
\[ n_{1,2} = \frac{2 - a_0 + b_0 - b_1}{2(a_0 + a_1 - b_0 - b_1)} \pm \left[ \frac{(2 - a_0 + b_0 - b_1)^2}{4((a_0 + a_1 - b_0 - b_1)^2 + b_0} + \frac{b_0}{a_0 + a_1 - b_0 - b_1} \right]^{1/2}. \] (46)

However, one of them is negative because \( 0 < b_r < a_r < 1 \) for any temperature and voltage. Therefore, we conclude that there is only one physical population of MQD, and the current is also single-valued at any voltage and temperature, in agreement with the Hubbard model [16].

**B. Switching of four-fold degenerate MQD**

In this case the rate equation is of the fourth power in \( n, \)
\[ 2n = (1 - n)[na_0 + (1-n)b_0] 
+ 3n(1-n)^2[na_1 + (1-n)b_1] 
+ 3n^2(1-n)[na_2 + (1-n)b_2] 
+ n^3[na_3 + (1-n)b_3]. \] (47)

In the limit \( |\gamma| \ll 1 \) we have \( b_r = a_r, \) \( Z = 1, \) the remaining interaction is \( U = UC \) [see Eq.(10)] and Eq.(47) is reduced to
\[ 2n = (1 - n)^3b_0 + 3n(1-n)^2b_1 
+ 3n^2(1-n)b_2 + n^3b_3. \] (48)

If we assume now that the non-vibron interaction \( UC \) is negative, for example due to valence fluctuations, then we recover the negative-\( U \) model [16], and the kinetic equation is reduced to
\[ 2n = 1 - (1-n)^3 \] (49)
in the voltage range \( \Delta - |U| < eV/2 < \Delta \) and \( T = 0 \) because \( b_0 = 0 \) and \( b_1 = b_2 = b_3 = 1 \) there, if \( |U| < \Delta/2. \) The current is simplified as
\[ j = 2n. \] (50)

Equation (49) has two physical roots, \( n = 0 \) and \( n = (3 - 5^{1/2})/2 \approx 0.38. \) Hence we obtain two stationary states of MQD with low (zero at \( T = 0 \)) and high current, \( j \approx 0.76 \) for the same voltage as we discussed earlier in Ref. [16]. The current-voltage characteristics show a hysteretic behavior for \( d = 4. \) When the voltage increases from zero, 4-fold degenerate MQD remains in a low-current state until the threshold \( eV_2/2 = \Delta \) is reached. Remarkably, when the voltage decreases from the value above the threshold \( V_2, \) the molecule remains in the high-current state down to the voltage \( eV_1/2 = \Delta - |U| \) well below the threshold \( V_2. \) This is a correlation mechanism of electronic molecular switching without retardation. Therefore, the negative-\( U \) degenerate molecular dot possesses the volatile memory originating from the many-particle attractive correlations.

The \( e\)-\( ph \) coupling results in the phonon sidebands of DOS, which are fully taken into account in Eq. (43) for the self-consistent occupation of the molecular level \( n. \) In the
The I-V in the vibron case is much more complex too: as $U$ corresponds to larger voltage bias compared to negative-U case the threshold for the onset of bistability $\gamma$ is generated by them phonon side-band ladders with the step of $\Delta$ close to the same value of the attraction as we have used $U$. In the negative-U case formally corresponds to $0 = U$. The current hysteresis persist up to some critical temperature, which is low, $T/\Delta \approx 0.01$.

FIG. 3. The I-V curves for tunneling through molecular quantum dot (Fig. 1) with the electron-vibron coupling constant $\gamma^2 = 11/13$ and $\omega_0/\Delta = 0.2$. The up arrows show that the current picks up at some voltage when it is biased, and then drops at lower voltage when the bias is being reduced. The bias dependence of current basically repeats the shape of the current bistability exists in the vibron case, is lower temperatures. Indeed, the hysteresis loop almost approaches the limiting value $\Delta + \omega_0/\Delta = 0$. Since $\omega_0$ is not generally commensurate with $U$, we obtain pretty irregular picture of the steps on the I-V curve, Figs. 3 and 4. Those steps are generated by the phonon side-bands originating from correlated levels on the dot with the energies $\Delta$, $\Delta + U$, ..., $\Delta + (d-1)U$. The level occupation approaches the limiting value $n = 0.5$ at large bias voltages, Figs. 3 and 4.

There is an important difference between switching with vibron-mediated electron-electron attraction and the negative-U model. We show the numerical results for $\omega_0 = 0.2$ (in units of $\Delta$, as all the energies in the problem) and $U / \Delta = 0$ for two values of the coupling constant, $\gamma^2 = 11/13$ (Fig. 3) and $\gamma^2 = 13/11$ (Fig. 4). This case formally corresponds to $U = -2\gamma^2 \omega_0 \approx -0.4$, i.e. close to the same value of the attraction as we have used in the negative-U model (16) (we selected those values of $\gamma^2$ to avoid accidental commensurability of a ladder of the correlated level energies separated by $U$ and generated by them phonon side-band ladders with the step $\omega_0$). In the negative-U case the threshold voltages were $eV_1/2\Delta = 1 - 0.4 = 0.6$ and $eV_2/2\Delta = 1$. However, in the vibron case the threshold for the onset of bistability corresponds to larger voltage bias compared to negative-U case (at $eV/2\Delta = 0.86$ for $\gamma^2 = 11/13$ and $\omega_0 = 0.2$). The I-V in the vibron case is much more complex too: as one can see from Figs. 3 and 4, the current discontinuity at the threshold strongly depends on the value of the e-ph coupling constant. The inelastic tunneling processes through the level, accompanied by emission/absorption of the phonons (Fig. 2), manifest themselves as steps on the I-V curve, Figs. 3 and 4. Those steps are generated by the phonon side-bands originating from correlated levels on the dot with the energies $\Delta$, $\Delta + U$, ..., $\Delta + (d-1)U$. The level occupation approaches the limiting value $n = 0.5$ at large bias voltages, Figs. 3 and 4.

The bistability region shrinks down with temperature. In the specific example of the negative-U model with $U/\Delta = -0.4$ the bistability is over at $T/\Delta \approx 0.1$ (16). Importantly, in the vibron case this happens at much lower temperatures. Indeed, the hysteresis loop almost closes at $T/\Delta = 0.01$. The critical temperature, below which the current bistability exists in the vibron case, is suppressed by about an order of magnitude compared to the negative-U case. At finite temperatures, the overall I-V curve shows the smoothed out steps at the bias voltages coinciding with the voltage where the vibrons are
emitted/absorbed as at $T = 0$ (Figs.3, 4).

In conclusion, we have developed the multi-polaron theory of tunneling through a molecular quantum dot (MQD) taking phonon sidebands and strong electron correlations into account. The degenerate MQD with strong electron-vibron coupling shows a hysteretic volatile memory if the degeneracy of the molecular level is larger than two, $d > 2$. The hysteretic behavior strongly depends on electron-vibron coupling and characteristic vibron frequencies. The current bistability vanishes above some critical temperature. It would be very interesting to look for an experimental realization of the model, possibly in a system containing a certain conjugated central part, which exhibits the attractive correlations of carriers with large degeneracy $d > 2$.

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