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FINITE ELEMENT CALCULATION OF ELASTO-DYNAMIC FIELDS GENERATED BY A POINT SOURCE IN THE LAYERED GROUND

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1. INTRODUCTION

Generation and propagation of elasto-dynamic fields and waves in a layered ground occur in many practical situations either naturally, e.g. during earthquakes, or as a result of technically-induced processes, such as ground vibrations generated by passing trains. During the last decades, a number of researchers paid attention to different aspects of the theory and applications of wave propagation in multi-layered elastic structures, aiming to understand and predict the vibration levels and frequency contents of elastic waves generated by different sources (see, e.g. [1-7]).

Both analytical and numerical approaches can be used for solving problems of elastic wave generation in layered soils. Exact analytical solutions exist only for a limited number of idealised layered structures, whereas different numerical techniques, especially finite element method, can be applied to almost unlimited range of complicated boundary value problems. In the light of this, they provide a researcher with the unique opportunity to validate approximate analytical solutions being developed for different practical situations.

The present paper examines the application of finite element method to the problems of elastic wave generation in a layered ground by a vertical time-harmonic point force acting on a free surface. In particular, such a force simulates the dynamic load applied from an individual sleeper to the ground under the impact of passing railway trains. Rectangular 4-node axi-symmetric elements are used for discretising the structure and studying the effects of different factors on the resulting displacement and stress amplitudes on the ground surface. These factors are load frequency, thickness and elastic parameters of each soil layer, material damping, etc. The results of calculations are compared with the existing analytical and numerical solutions.

2. OUTLINE OF THE NUMERICAL APPROACH

In the finite element method, the elastic continuum is replaced by a discretised mesh which is subdivided into a number of discrete subregions - finite elements. These elements are connected to each other by nodal points (joints). Each node has a certain number of degrees of freedom to represent the admissible displacements of the structure. Having defined a displacement field within the element, one can set up an integral form for the governing differential equation of the system. This integral equation is then represented by a set of linear equations expressed in a matrix form which can be solved by any computer-based numerical approach, e.g. by Gauss elimination. Full explanation of finite element formulation for elastic bodies can be found in several references, such as [8-9].

When wave generation problem is tackled by finite element method, the general equation of motion for one-, two- or three-dimensional body may be defined in matrix form as:
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\[ [K] \{ \delta \} - \omega^2 [M] \{ \delta \} + i \omega [C] \{ \delta \} = \{ P \}, \]

where

- \([K]\) is the global stiffness matrix of the structure,
- \([M]\) is the mass matrix of the structure,
- \([C]\) is the damping matrix of the structure,
- \(\{ \delta \}\) is the unknown displacement vector of the nodes,
- \(\{ P \}\) is the applied load vector,
- \(\omega\) is the natural frequency of the system, and
- \(i\) is the imaginary root of -1.

Note that three-dimensional finite element techniques for wave propagation in elastic media are quite expensive in terms of computer processing time and memory. However, it is often possible to reduce the problem to the approximate two-dimensional idealisation, such as plane stress, plane strain, or axi-symmetric geometries, depending on the nature of the problem. The case of elastic half space subjected to a point load at the surface is axi-symmetric with respect to the vertical axis. Therefore, axi-symmetric finite element method has been adopted in this paper to solve the problems of wave generation and propagation in multi-layered soils. Rectangular 4-node axi-symmetric elements were used for discretising the structure. Each node had two degrees of freedom, namely vertical and horizontal (radial) displacements. A ground layer of 60 m radius and 20 m depth was considered for the finite element solution. The discretised mesh consisted of 4800 elements and 4961 nodes, with each element sized 500 x 500 mm\(^2\). A typical finite element mesh for the structure under consideration is presented in Fig. 1.

A standard commercial finite element package LUSAS was employed to carry out the analysis. It is one of the most common packages available in the market at affordable price, and it can be installed and used successfully in any PC having DOS and Windows environment. Note that LUSAS has not got infinite elements in its element library which means that standard finite elements should be used for modelling an infinite domain. This can be achieved by increasing the mesh size. Also, as will be demonstrated below, one should consider up to 80 modes of vibration and take into account certain amount of soil damping to minimise the effect of wave reflection at the boundaries. Due to the symmetry of the problem, only half of the structure may be analysed. The vertical displacements of all the nodes along the axis of symmetry were permitted, whereas the horizontal displacements were suppressed to maintain the symmetry of the system.

3. EXCITATION OF A HOMOGENEOUS HALF SPACE

3.1 Effects of Observation Distance and Load Frequency

Vertical surface displacements of an homogeneous elastic half space were calculated as functions of the distance from the load and compared with the results obtained analytically from the approximate formula taking into account only Rayleigh branch of the corresponding Green's function [1, 5]. The following soil parameters were used in calculations: Young modulus \( E = 4 \times 10^7 \) N/m\(^2\), Poisson's ratio \( \nu = 0.35 \), mass density \( \rho = 2000 \) kg/m\(^3\), and soil damping ratio \( \xi = 0.05 \). It was assumed that a vertical time-harmonic load was applied at the surface with the amplitude \( P_o = 10^5 \) N and frequency \( f = 10\)Hz. The results are shown in Fig. 2.

One can see from Fig. 2 that the dependence of generated vibrations on distance shows an excellent matching between the numerical and analytical results, apart from very close distances from the load application. This proves that the finite element model adopted in the present work is correct. Also, there are no visible effects of wave reflection from the remote boundary. This means that consideration of 60 m radius for the application of standard rectangular axi-symmetric elements to model an infinite domain proves to be sufficient for the soil damping used.

Vertical vibrations of the ground surface as functions of load frequency were calculated for the following ground parameters: \( E = 106 \times 10^6 \) N/m\(^2\), \( \nu = 0.33 \), \( \rho = 2000 \) kg/m\(^3\), and \( \xi = 0.025 \). The load amplitude \( P_o \) was \( 10^5 \) N. The results are shown in Fig. 3. One can see that the amplitudes of generated vibrations have peaks at frequencies around 15 Hz, in agreement with the theory.
3.2 Effect of Material Damping

Wave generation in an homogeneous elastic half space of the same material, as described in section 3.1, was analysed by finite element method for the following values of damping ratios: 1, 5, 10 and 20% (see Fig. 4). This was done in order to investigate the effect of material damping on surface displacement excited by a concentrated harmonic load with the same parameters, as in the previous example. It can be seen from Fig. 4 that the displacement curve for 1% damping has some oscillations, as if the waves are reflected at the boundary. Whereas, the displacement curves for the cases of 5, 10 and 20% damping have rather smooth decaying trend with little effect of wave reflection. Therefore, a relatively large value of soil damping, say 5 – 10%, should be assumed to give more accurate results.

3.3 Effect of number of modes

The same problem of the elastic half space excitation by a point source was considered for different numbers of mesh modes being taken into account, namely 20, 40, 80 and 100 modes. The results in terms of vertical displacements of the surface are shown in Fig. 5. It can be seen that taking into account 20 and 40 modes do not produce accurate results. Whereas, the results for the cases of 80 and 100 modes are very close to each other. One can conclude that the higher the number of modes the more accurate the results are. However, considering more modes means that the size of the problem in terms of memory and computer processing time will increase. Therefore, on the basis of the results shown on Fig. 5, it is advisable to take about 80 modes whenever wave propagation in semi-infinite media is analysed using standard LUSAS software.

4 EXCITATION OF A TWO-LAYERED HALF SPACE

4.1 Effect of Observation Distance

In this section, a two-layered soil was analysed by finite element method, and the results were compared with the approximate analytical Green’s function for layered elastic media suggested in [5]. The following material characteristics were assumed:

Bottom layer (layer 1): \( E_1 = 236 \times 10^6 \text{ N/m}^2, \quad v_1 = 0.25, \quad \rho_1 = 2000 \text{ kg/m}^3, \quad \xi_1 = 0.05 \) and \( h_1 = 15 \text{ m} \)
Top layer (layer 2): \( E_2 = 92 \times 10^6 \text{ N/m}^2, \quad v_2 = 0.25, \quad \rho_2 = 2000 \text{ kg/m}^3, \quad \xi_2 = 0.05 \) and \( h_2 = 5 \text{ m} \)

A vertical harmonic load was applied at the surface with the amplitude \( P_0 = 10^5 \text{ N} \) and frequency \( f = 10 \text{ Hz} \). The number of modes of vibration was taken as 80. The finite element result for the vertical displacement as function of distance is shown in Fig. 6. One can see that the agreement with the analytical model [5] is quite good. This means that the approximate analytical expressions for the Green’s function [5] can be used in practice for analysing elastic wave generation and propagation in two-layer systems.

4.2 Effect of a Soft Upper Layer on Surface Displacements

A two-layered system was considered with the parameters of the top layer \( h_2 = 5 \text{ m} \) and \( E_2 = 40 \times 10^6 \text{ N/m}^2 \) and of the bottom layer \( h_1 = 15 \text{ m} \) and \( E_1 = 120 \times 10^6 \text{ N/m}^2 \). Both layers had \( v = 0.35, \rho = 2000 \text{ kg/m}^3 \) and \( \xi = 0.05 \). The load characteristics and the number of modes considered were the same as in the section 3.3. The results were compared with the case of homogeneous soil which had \( E = 120 \times 10^6 \text{ N/m}^2 \). Fig. 7 shows the vertical displacement along the depth for the two above mentioned cases. It can be noticed that the existence of a soft layer significantly increases the displacements near the surface. It can be also seen that the curve for the two-layered system has a steep gradient up to a depth of 5 m (which is the thickness of the top soft layer). In the same time, for a homogeneous half space, the curve is found to be smooth all the way into the depth.
4.3 Effect of Load Frequency on the Excitation of a Two-Layered Half Space

The following material characteristics were considered:

Bottom layer (layer 1): \( E_1 = 106 \times 10^6 \text{ N/m}^2, \) \( v_1 = 0.33, \) \( \rho_1 = 2000 \text{ kg/m}^3, \) \( \xi_1 = 0.025 \) and \( h_1 = 18 \text{ m} \)
Top layer (layer 2): \( E_2 = 53 \times 10^6 \text{ N/m}^2, \) \( v_2 = 0.33, \) \( \rho_2 = 2000 \text{ kg/m}^3, \) \( \xi_2 = 0.025 \) and \( h_2 = 2 \text{ m} \)

A vertical harmonic load was applied at the surface with the amplitude \( P_0 = 10^6 \text{ N} \), and the number of modes of vibration was taken as 80. The finite element results for the vertical displacements at various points along the surface as functions of the load frequency are shown in Fig. 8. One can see that all the curves have peak values at load frequencies around 25 Hz. This means that the effect of soft upper layer results in the increase of peak frequencies in comparison with the case of homogeneous half space with the parameters of the bottom layer, where these frequencies were around 15 Hz (see Fig. 3).

5. CONCLUSIONS

In this paper an attempt has been made to apply finite element approach to the problems of wave generation and propagation in a layered elastic ground subjected to a vertical harmonic point load acting on the free surface. Rectangular 4-node axi-symmetric elements were used for discretising the structure. A standard finite element package LUSAS was employed to carry out the numerical analysis. Several examples have been considered and the obtained results compared with the analytical ones. A good agreement has been observed, indicating that both the numerical model considered and the analytical results have been validated. The damping characteristics of the soil and the number of modes required for obtaining accurate results have been investigated as well. It was shown that values of the damping ratio between 5 – 20% provide good results. It was also found that 80 – 100 vibration modes of the finite mesh were sufficient to get a required accuracy in the problems of wave generation and propagation in unbounded layered media.

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REFERENCES

Fig. 1. Finite element mesh for an elastic half space.

Fig. 2. Comparison between analytical and numerical results for vertical displacement on the surface due to dynamic loads with frequency $f = 10$ Hz.
Fig. 3. Vertical displacements as functions of frequency for a homogeneous and two-layered elastic half spaces.

Fig. 4. Effect of soil damping on vertical displacements on the surface.
Fig. 5. Effect of number of modes on vertical displacements on the surface.

Fig. 6. Comparison between analytical and numerical results for vertical surface displacements in the two-layered system due to a dynamic load with frequency \( f = 10 \text{ Hz} \).
Fig. 7. Vertical displacements as functions of depth in a homogeneous half space and in a two-layered system.

Fig. 8. Vertical displacements as functions of frequency in a two-layered elastic half space.