An investigation of efficient control strategies for a PWM inverter driven induction motor

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AN INVESTIGATION OF EFFICIENT CONTROL STRATEGIES FOR A PWM INVERTER DRIVEN INDUCTION MOTOR.

by

RIHMAN HILLAL ISSA, B.Sc., M.Sc.

A Doctoral Thesis
Submitted in Partial Fulfilment of the Requirements for the Award of the Degree of Doctor of Philosophy of Loughborough University of Technology.
M AR. 1987
Supervisors: Professor I. R. Smith, B.Sc.,PhD., D.Sc., C.Eng., F.I.E.E.
S. Williams, B.Sc.,PhD., C.Eng., M.I.E.E.
I dedicate this thesis to

my Mother
I would like to take this opportunity to express my special gratitude to Professor I. R. SMITH, the Head of the Department of Electronics and Electrical Engineering, Mr J. G. KETTLEBOROUGH and Dr S. WILLIAMS for their invaluable guidance, advice, encouragement and patience throughout the course of research and the preparation of this thesis.

Thanks are also due to my colleagues in the Power Electronics Research group for good humour.

The assistance given by the technical staff is greatly appreciated.

Thanks also to Mrs J Brown for typing this thesis.

Finally, my great thanks and appreciation are extended to my parents and my brother Mr Adnan H. Issa for their endurance and financial support they have readily given to me during the period of study, their generosity and long suffering are sincerely acknowledged and will always be remembered.
SYNOPSIS

Recent developments in power electronics switching devices have led to significant improvements in AC drives which, coupled with the obvious advantages of squirrel-cage induction motors, have generated a customer-led demand for an increase in AC drive performance.

This thesis describes the design and construction of a 3-phase pulse-width modulated inverter using gate turn-off (GTO) thyristor switching devices, which drives a 0.75 kW 3-phase squirrel-cage induction motor. The inverter control circuit comprises a purpose-built large-scale integrated circuit, which generates the 3-phase PWM drive signals and allows the output voltage and frequency to be varied independently. When operating in open-loop, the drive system is capable of reverse operation, and the maximum rate of acceleration and deceleration of the motor may be controlled. Compensation for resistive voltage drop is provided when the motor is running at low speed.

An analogue closed-loop proportional-integral-derivative speed controller is described, and for efficient operation under both no-load and on-load conditions torque feedback is also included. This provision both reduces the no-load losses in the motor and improves the torque-speed characteristic under load conditions. The improved closed-loop performance also includes power factor correction when the motor is lightly loaded, together with an automatic boost to the motor voltage when loads are applied at low speed. A comparison is made between the performance of the analogue system and a digital real-time control implemented using a microcomputer. A series of computer programs are presented which
simulate the performance of the drive system and which are suitable for running on the University mainframe computer. The programs enable the effects of the modulation technique and the inverter frequency on the pwm inverter steady-state output to be studied, and the performance of the induction motor to be investigated.

Throughout the work, the theoretical predictions are supported by considerable experimental results.
### List of Principal Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_s$</td>
<td>Synchronous speed</td>
<td>(r/min)</td>
</tr>
<tr>
<td>$n_r$</td>
<td>Motor Speed</td>
<td>(r/min)</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Synchronous frequency</td>
<td>(Hz)</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Rotor frequency</td>
<td>(Hz)</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Carrier frequency</td>
<td>(kHz)</td>
</tr>
<tr>
<td>$f_m$</td>
<td>Reference frequency</td>
<td>(Hz)</td>
</tr>
<tr>
<td>$s$</td>
<td>Slip</td>
<td></td>
</tr>
<tr>
<td>$V_s$</td>
<td>Supply voltage</td>
<td>(V)</td>
</tr>
<tr>
<td>$V_r$</td>
<td>Rotor induced voltage</td>
<td>(V)</td>
</tr>
<tr>
<td>$V_{DC}$</td>
<td>Inverter supply D.C. input voltage</td>
<td>(V)</td>
</tr>
<tr>
<td>$X_s', X_r$</td>
<td>Reactances per-phase of the stator and rotor circuit, respectively</td>
<td>(Ω)</td>
</tr>
<tr>
<td>$R_s, R_r$</td>
<td>Resistances per-phase of the stator and rotor circuits, respectively</td>
<td>(Ω)</td>
</tr>
<tr>
<td>$L_s, L_r$</td>
<td>Leakage inductances per-phase of the stator and rotor circuits, respectively</td>
<td>(H)</td>
</tr>
<tr>
<td>$I_s, I_r$</td>
<td>Stator and rotor currents, respectively</td>
<td>(A)</td>
</tr>
<tr>
<td>$I_m$</td>
<td>Magnetizing current</td>
<td>(A)</td>
</tr>
<tr>
<td>$L_{sm}$</td>
<td>Mutual inductance between stator phases</td>
<td>(H)</td>
</tr>
<tr>
<td>$L_{rm}$</td>
<td>Mutual inductance between rotor phases</td>
<td>(H)</td>
</tr>
<tr>
<td>$M_{sr}$</td>
<td>Maximum mutual inductance between stator and rotor circuits</td>
<td>(H)</td>
</tr>
<tr>
<td>$P_{co}$</td>
<td>Stator winding losses per-phase</td>
<td>(W)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Power input per-phase to the rotor</td>
<td>(W)</td>
</tr>
<tr>
<td>$\phi_g$</td>
<td>Flux/pole</td>
<td>(Wb)</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Electromagnetic torque developed</td>
<td>(Nm)</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Mechanical torque applied</td>
<td>(Nm)</td>
</tr>
<tr>
<td>$J$</td>
<td>Moment of inertia</td>
<td>($kg.m^2$)</td>
</tr>
<tr>
<td>$P$</td>
<td>Number of pole pairs</td>
<td></td>
</tr>
<tr>
<td>$k_f$</td>
<td>Rotor friction coefficient</td>
<td>($kg.m^2/s$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Relative position angle of the rotor with respect to stator</td>
<td>(Elec.Rad.)</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Synchronous angular velocity</td>
<td>(Elec.Rad./s)</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Angular velocity of the rotor</td>
<td>(Elec.Rad./s)</td>
</tr>
<tr>
<td>$h$</td>
<td>Time step</td>
<td>(s)</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>(s)</td>
</tr>
<tr>
<td>$T$</td>
<td>Sampling time</td>
<td>(s)</td>
</tr>
<tr>
<td>$F$</td>
<td>$d/dt$ operator</td>
<td></td>
</tr>
<tr>
<td>$A,B,C$</td>
<td>Suffixes denoting direct phase variables</td>
<td></td>
</tr>
<tr>
<td>$\alpha,\beta$</td>
<td>Suffixes denoting transformed 2-phase variables</td>
<td></td>
</tr>
<tr>
<td>$d,q$</td>
<td>Suffixes denoting 2-axis variables</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>The modulation index</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Switching angle</td>
<td></td>
</tr>
<tr>
<td>$R_L$</td>
<td>Frequency changing ratio</td>
<td></td>
</tr>
<tr>
<td>$e_r$</td>
<td>Error signal</td>
<td>(V)</td>
</tr>
<tr>
<td>$e_d$</td>
<td>Reference signal</td>
<td>(V)</td>
</tr>
<tr>
<td>$e_a$</td>
<td>Feedback signal</td>
<td>(V)</td>
</tr>
</tbody>
</table>
\(k_p\) Proportional coefficient
\(k_i\) Integral coefficient
\(k_d\) Derivative coefficient

All other symbols are defined as they appear
# CONTENTS

## ACKNOWLEDGEMENTS

## SYNOPSIS

## LIST OF PRINCIPAL SYMBOLS

## CHAPTER 1: INTRODUCTION

1.1 Technical Background of Squirrel-cage Motor 1

1.2 Mathematical Analysis of Induction Machines 5

1.3 Thesis Objective 8

## CHAPTER 2: VARIABLE SPEED INDUCTION MOTOR DRIVE USING STATIC INVERTERS

2.1 Motor Characteristics for Constant Supply Frequency 10

2.2 Motor Operation at Variable-Frequency 14

2.3 Static Inverters 15

2.4 Effect of Non-sinusoidal Excitation on Motor Losses 16

## CHAPTER 3: INVERTER A.C.-DRIVE MODULATION TECHNIQUES

3.1 Types of Inverter 25

3.1.1 Quasi-squarewave voltage source inverter 25

3.1.2 Quasi-squarewave current source inverter 26

3.1.3 PWM-voltage source inverter 27

3.2 PWM-Modulation Techniques 27

3.2.1 Level set-modulation 27

3.2.2 Squarewave-modulation 28

3.2.3 Sinusoidal-modulation 28
CHAPTER 4: OPEN-LOOP INVERTER DRIVE

4.1 Power Circuit
   4.1.1 Power supplies 45
   4.1.2 Power switches 46
   4.1.3 GTO and its snubber circuit 47
   4.1.4 The inverter bridge 50

4.2 Control Circuit
   4.2.1 HEF4752, PWM-IC modulator 51
   4.2.2 Speed reference circuit 53

4.3 GTO Gate-Drive Circuit 54

4.4 Current Limit Circuit 55

4.5 Adjustment of Modulation Process 56

CHAPTER 5: IMPROVEMENTS TO THE SPEED CONTROL SYSTEM

5.1 Bi-directional Speed Reference Circuit 76

5.2 IR-Voltage Drop Compensation Circuit 78

5.3 Inverter Output Waveforms 79

CHAPTER 6: MATHEMATICAL MODEL OF INVERTER-INDUCTION MOTOR DRIVE

6.1 Simulation of the Regular Switching Strategy 103

6.2 Induction Motor Model 105

6.3 Derivation of Stationary 2-axis Model 106
CHAPTER 7: CLOSED-LOOP SPEED AND TORQUE CONTROLLED DRIVE

7.1 Control Techniques 162
7.2 Implementation of Speed and Torque Scheme 163
7.3 System Development 165
   7.3.1 Speed reference circuit 165
   7.3.2 Torque regulating circuit 166
7.4 Experimental Configuration 167
7.5 Experimental Results 167

CHAPTER 8: CLOSED-LOOP SPEED CONTROL USING A MICROCOMPUTER

8.1 Introduction 182
8.2 Implementation of the Digital PID Algorithm 183
   8.2.1 Analogue PID 183
   8.2.2 Digital PID 185
8.3 Proposed Digital Speed Controller 186
8.4 System Hardware Developments 187
   8.4.1 The Microcomputer 187
   8.4.2 Motor speed monitoring circuit 188
   8.4.3 Digital output data 188
8.5 System Software 189
8.6 Experimental Results 191
CHAPTER 9: CONCLUSION

9.1 Conclusion and Remarks 209
9.2 Suggestions for Further Work 211

REFERENCES: 212

APPENDICES:

Appendix A: Inverter d.c. supply voltage 222
Appendix B: Motor specification 223
Appendix C: Computer program listing for the combined system 224
Appendix D: Listing of minicomputer software 235
CHAPTER 1

INTRODUCTION

1.1 Technical Background of Squirrel-cage Motor

1.2 Mathematical Analysis of Induction Machines

1.3 Thesis objective
This thesis is concerned with an investigation into the speed control of a squirrel-cage induction motor using a pwm-GTO inverter. The introduction presents the background to the investigation and outlines the aims and objectives of the work.

1.1 Technical Background of Squirrel-cage Motor

This section of the thesis is concerned with a review of the most important induction motor speed-control systems. Each system is described briefly and many references are provided, so that a detailed study of any particular system may be undertaken if required.

Historically, the first electric drive system was patented by Ward-Leonard in the 1890's\(^1\). This consisted of a DC motor driving a DC generator, which in turn supplied controlled power to a DC motor. The development of electric drives proceeded from this arrangement to include various improvements, aimed at controlling the speed in a more linear fashion\(^2\). Beginning with the development of power semiconductors in the late 1950's\(^3\), a new era of controllable devices opened up, and the use of the 3-phase induction motor as a variable-speed drive became a possibility. Although many variable-speed drives still use DC machines, due to the ease with which their speed can be controlled, their limitations, namely the need for regular maintenance in the form of brush replacement, the problem of sparking in hazardous environments and the creation of carbon dust, may preclude their use. Considering the motor only, the advantages\(^4,5,6\) of the squirrel-cage machine, such as ruggedness of construction, low maintenance, high starting torque and low cost are well known.

The standard squirrel-cage motor is designed to operate from a 3-phase fixed frequency sinusoidal supply voltage and at a speed that it closely determined by:

\[
n = \frac{f}{p}
\]
where \( f \) and \( P \) are, respectively, the supply frequency and the number of pole pairs of the motor. The formula suggests immediately two basic methods for controlling the motor speed.

1. Changing the pole number:

   This can be subdivided into:

   (a) Direct methods: The simplest means of changing the pole number is by reversing the second half of each phase winding. This produces a 2:1 change in pole number and hence a 2:1 change in the synchronous speed.

   (b) Pole amplitude modulation (PAM) \(^{7,8}\): PAM alters the number of poles in an electrical machine, by a technique which implies a modulation of the amplitude of the mmf produced by each phase of the stator winding. If an appropriate modulating waveform is chosen, motor operation is possible with pole numbers which may be relatively close together, e.g. 18/22, or far apart, e.g. 4/8 are possible. Externally, a PAM induction motor is quite standard, and it can readily replace a conventional induction motor, with little cost and circuit complexity penalties.

2. Changing the frequency:

   A variable-frequency supply to a conventional squirrel-cage motor provides continuously variable-speed operation. There are two types of frequency converter that can provide efficient and wide-range speed control for induction motors.

   (a) A rotating frequency converter \(^{5}\)

      In the past, variable-frequency supplies were often obtained
using a combination of rotating machines. An example of this is the DC motor/alternator set, in which the speed of the DC motor is controlled by variation of the motor field excitation and armature voltage. The driven alternator produces an output supply at a controlled frequency, which can then be used to drive the induction motor. The advantage of rotating frequency converters is that they produce sinusoidal output waveforms, in contrast to the chopped waveforms of an electronic inverter which is explained next. Their limitations lie however in the capital cost of extra machines, the increased maintenance and the limited range of output frequency.

(b) Static converters

With the advent of power semiconductor devices, the motor-alternator set has largely fallen out of favour, as static inverters have been developed to provide a variable-frequency supply which is both accurate and reliable\(^{(9,10)}\). The advantage of static inverter drives can be summarised as:

(i) The output frequency is independent of both load and transient conditions.

(ii) Continuous variable speed control is possible over a wide range of frequencies.

(iii) The motor power factor is almost constant over a wide operating range.

(iv) Inverters can easily be included in a closed-loop control scheme\(^{(11)}\), leading to more accurate control of the motor speed, torque and power, as well as better control of the transient performance.
Because of these advantages, static inverters are used in many trial drives, and thus form the basis of the variable-frequency systems which will be considered in this thesis.

There are two types of static converters, the first being the cycloconverters \((12,13)\), in which mains frequency is converted directly into A.C. of variable frequency. An arrangement of switching elements selectively connects the load to the supply, so that a low-frequency output voltage waveform is fabricated from segments of the supply voltage waveform. The disadvantage of this kind of converter is that the highest output frequency is limited to about one-third of the mains frequency. The second type of converter is the D.C.-link/3-phase bridge inverter \((14-20)\). In this case, the A.C. supply is first rectified to D.C., before subsequently being inverted to A.C. of variable frequency. The main switching elements of the inverter are triggered sequentially, such that a rectangular or stepped voltage waveform is generated at the output. Also in this category are PWM inverters \((21-25)\), which aim to synthesise pseudo (or quasi) sinusoidal waveforms from the D.C.-link voltage. In contrast to the cycloconverter, the output frequency of the D.C.-link inverter can range from a few hertz up to several kilohertz. For these reasons, D.C.-link inverters have found wide application in industrial variable-speed A.C. drives, and they will continue for many years to play a significant role in the overall variable speed applications.
1.2 Mathematical Analysis of Induction Machines

The transient and steady-state performance of induction machines has been the subject of extensive study, using both experimental and mathematical models \((26-34)\). While the experimental models of Waygandt and Charp \((29)\), Wood, Flynn and Shanmugasundaram \((31)\), and Smith and Sriharan \((33,34)\) have provided valuable insight into the operation of induction motors, the complexity of the experimental investigations has made their use expensive. In recent years, especially following the advent of fast digital computers, the emphasis in induction motor investigations has shifted towards the direct solution of the machine equations. While these equations are complicated, and exhibit certain non-linear characteristics, they can be solved quite rapidly on a digital computer if sensible simplifications are adopted. The models developed quickly give quantitative information which may be of direct use in either design or operation.

Stanley \((26)\) has derived general differential equations for several A.C. machines, using the stationary-axis method introduced by Park \((27)\) for the analyses of salient-pole synchronous machine. Stanley's equations for a 3-phase machine have been solved with the aid of a differential analyser, with special reference to plugging by Silfillan and Kaplan \((28)\). The parameters of an induction motor were assumed, and transient torques were predicted as functions of time. However, since no actual motor was considered, no measure of the accuracy of the theoretical results could be inferred. Waygandt and Charp \((29)\), solved Stanley's differential equations for the case of a 2-phase servomotor, again using a differential analyser. They obtained both current transient and speed response curves, which
were shown to compare well with experimental results obtained from an actual servomotor. Maginniss and Schultz\(^{(30)}\) carried out similar work to that of Gilfillan and Kaplan. They predicted the motor behaviour during the transient conditions following plugging, again using a differential analyser, and they assumed a linear change in the acceleration of the machine when studying the transient performance following either a sudden change in the voltage or plugging at various speeds and switching instants. The study was however, entirely mathematical. Wood, Flynn and Shanmugasundaram\(^{(31)}\) obtained experimental results for the starting transients in a 3-phase squirrel-cage motor on application of the supply voltage at different switching angles, and also during reconnection to the supply at different speeds. Some time later, as an alternative to the use of a differential analyser, various analogue computer simulations of the motor equations in \(d,q\) form were undertaken. In particular, Hughes and Aldred\(^{(32)}\) considered variable speed effects, and presented theoretical results for both a 2-phase servomotor and a 3-phase industrial motor under starting conditions. Some experimental verification of the work was given in the case of the starting transients of the 3-phase motor.

Following the development of fast digital computers, considerable attention was directed to numerical solutions of the machine equations. Smith and Sriharan\(^{(33,34)}\) used a digital computer to solve the machine equations in \(d,q\) form, including the effect of speed variations. They also computed the torque transmitted to a coupled load in terms of the electromagnetic torque developed by the machine and the mechanical coefficient of the load. The transient performance of the induction
motor following reconnection to the same supply or to a different supply (i.e. star/delta, plugging and D.C. dynamic braking), at different speeds for various lengths of supply interruption was also investigated. Computed results compared well with those obtained from experimental work. Another digital computer model was used by Slater, Wood and Simpson\(^{(35)}\) to analyse the torque transients following connection of a 3.5 kW squirrel-cage motor to the supply at zero speed and at 90\% of synchronous speed, and for different switching angles of the supply. In a number of studies, a common approach has been to assume that the motor voltage has a precisely defined waveform and analytical solutions have been developed using a number of advanced mathematical techniques\(^{(36-44)}\).

Many authors have analysed induction motors driven by a D.C.-link inverter. The solutions are obtained either with the aid of a digital computer or from simulations using an analogue computer. Lipo and Turnball\(^{(45)}\) have used the state-variable formulation of the machine equations to study two widely used drive systems incorporating square-wave inverters with 180° and 120° conduction angles. Steady-state characteristics with each inverter supplying three motors were obtained with computed results being compared to experimental results for an actual system. Other aspects of the dynamic performance of the inverter-fed induction motor drive have been considered by a number of authors\(^{(46-48)}\), with most of these studies relating to start-up conditions. However, in parallel with these analyses, Al-nimma and Williams\(^{(49,50)}\) developed a digital computer model for studying a much wider range of operating fault conditions using tensor techniques. Inverters with 120° and 180° conduction modes were considered, and computed results were compared to test results from a laboratory-scale system.
As an alternative to much of the above work analogue computation is still being developed (51-56).

Most of the papers mentioned in this section have used the familiar $d,q$ form of the motor equations. It is well-established that this model can provide excellent predictions of both the transient and steady-state behaviour of a drive system. For this reason much of the analyses in this thesis are undertaken using a $d,q$ model.

1.3 Thesis Objective

The speed of an induction motor can be controlled using a variable supply frequency which could be provided by a fully-controlled rectifier-inverter combination. A well designed system should include the following basic requirements:

(i) Adjustable output frequency to achieve the desired motor speed.

(ii) Adjustable output voltage, so as to maintain the induction motor air-gap flux

(iii) An ability to provide full rated current at any frequency within the desired constant torque output range.

Allowance can also be made to boost the motor voltage at low speed or during acceleration, to overcome stator resistance voltage drop.

This thesis presents an analytical and experimental investigation of several control strategies for a pwm-inverter/induction motor drive. A 3-phase GTO-thyristor inverter was constructed and used to drive a 0.75 kW, 3-phase squirrel-cage motor, which could be loaded electrically using a D.C.-generator or mechanically via a disc brake.
Chapter 2 develops the theoretical concepts of variable-speed drives, as a suitable starting point for the subsequent numerical analysis. Chapter 3 summarises the PWM-inverter switching strategies, and Chapter 4 describes the construction and testing of the inverter. Chapter 5 details the improvements made to the speed control system, to include facilities for speed reversal, together with control over the maximum rates of acceleration, and low speed IR-compensation. Chapter 6 describes a set of digital computer programs, developed for the analysis of the drive system and its accompanying control scheme. Theoretical results relating to various inverter modulating techniques, switching frequency, waveforms harmonic content etc. are presented. Chapters 7 and 8 are devoted to the presentation and discussion of experimental results obtained for the closed-loop drive system.

Throughout the thesis, the analysis and investigations are supported by considerable experimental work, and the comparisons obtained between experimental and computed results always demonstrate good agreement.
CHAPTER 2

VARIABLE SPEED INDUCTION MOTOR DRIVE USING STATIC INVERTERS

2.1 Motor Characteristics for Constant Supply Frequency

2.2 Motor Operation at Variable-Frequency

2.3 Static Inverters

2.4 Effect of Non-sinusoidal Excitation on Motor Losses
This chapter presents an overview of the speed control of a squirrel-cage induction motor using a variable-voltage, variable-frequency static inverter. Expressions for the motor speed and developed torque are shown to be functions of both the input frequency and the supply voltage, so that, by control of the magnitudes of these quantities, any desired motor performance can be obtained. The final section of the chapter discusses the problems of increased motor losses associated with inverter drives.

2.1 Motor Characteristic for Constant Supply Frequency

When a 3-phase supply is applied to the stator windings of an induction motor, a constant-magnitude sinusoidally-distributed magnetic field is produced. This field rotates at a synchronous speed, given in terms of the supply frequency $f_s$ and the number of pairs of poles $P$ as

$$n_s = \frac{f_s}{P} \quad (2.1)$$

The stator field cuts the rotor conductors and induces currents in them, which in turn interact with the stator field to produce a torque. By Lenz's law, this causes the rotor to turn in the direction of the stator field, and it accelerates until it attains a constant speed $n_r$, slightly less than the synchronous speed given by equation (2.1).

An important quantity throughout induction motor theory is the slip, $s$ defined as

$$s = \frac{n_s - n_r}{n_r} \quad (2.2)$$

from which the rotor speed follows as

$$n_r = (1 - s) n_s \quad (2.3)$$
The frequency of the rotor voltages and currents is

\[ f_r = s f_s \] (2.4)

Among many important considerations in the steady-state performance of an induction motor are the variations of current, speed and losses as the load torque changes, together with the starting and maximum torque. All these quantities may be derived from the per-phase equivalent circuit for the motor shown in Figure 2.1(a). When the rotor is stationary, the machine acts as a transformer on short circuit and large stator and rotor currents at low power factor flow. The voltage induced in the rotor is

\[ V_r = k \phi g f_r \] (2.5)

where \( k \) is a constant and \( \phi g \) is the flux/pole established by the stator windings. The voltage \( V_r \) is a function of \( f_r \) and, as the motor accelerates from rest, both \( f_r \) and \( V_r \) decrease. At a slip \( s \) the induced rotor voltage becomes \( s V_r \), when the rotor current is

\[ I_r = \frac{s V_r}{R_r + j s X_r} \]

or

\[ I_r = \frac{V_r}{R_r/s + j X_r} \] (2.6)

The quantity \( R_r/s \) is an apparent rotor resistance, which may be thought of as the sum of the actual rotor resistance \( R_r \) and the so-called load resistance \( R_r (1-s)/s \), as shown in Figure 2.1(b). As the motor accelerates from rest \( R_r/s \) increases, leading to a reduction in the
motor line current. The power factor at first rises, before reaching a maximum and subsequently falling. As the motor approaches synchronous speed \( R_r/s \) becomes very large, reducing the rotor current almost to zero and producing negligible output torque. The torque/slip relationship may be derived from the per-phase equivalent circuit of Figure 2.1(a), in which the power input per-phase to the rotor is

\[
P_r = I_r^2 \frac{R_r}{s} \tag{2.7}
\]

The mechanical power developed per-phase is

\[
\text{P}_{\text{out}} = P_r - \text{rotor loss}
\]

or

\[
\text{P}_{\text{out}} = I_r^2 \frac{R_r}{s} - I_r^2 R_r = I_r^2 R_r \left( \frac{1 - s}{s} \right) \tag{2.8}
\]

The electromagnetic torque \( T_e \) corresponding to the output power is obtained by equating this power to the product of the torque and the angular velocity. Thus if \( \omega_s = 2\pi n_s \) is the synchronous angular velocity

\[
\text{P}_{\text{out}} = (1 - s) \frac{\omega_s}{s} T_e
\]

\[
\omega_r = \omega_s = 2\pi n_r
\]

where \( \omega_r = (1 - s) \frac{\omega_s}{s} \) is the angular velocity of the rotor. It follows from equations (2.8) and (2.9) that

\[
T_e = \frac{I_r^2 R_r (1 - s)}{2\pi n_r} \tag{2.10}
\]
and substituting equations (2.3) and (2.6) into equation (2.10) leads to

$$T_e = \frac{sV_r^2 R_x}{2\pi n_s (R_x^2 + (sX_r)^2)}$$  \hspace{1cm} (2.11)

Equation (2.11) shows that the torque is a function of the rotor voltage and frequency. Neglecting the effects of stator parameters, which infers that $V_r$ is constant, and differentiating this equation with respect to $s$, and equating the result to zero, gives the slip at which maximum torque is produced as

$$s_{\text{max}} = \pm \frac{R_x}{X_r}$$  \hspace{1cm} (2.12)

where the positive sign applies to motoring action (i.e. $1 > s > 0$), and the negative sign to generating action (i.e. $s < 0$). Substituting the positive value of $s_{\text{max}}$ into equation (2.11) gives the maximum torque produced by the motor as

$$T_{\text{max}} = \frac{V_r^2}{4\pi n_s X_r}$$  \hspace{1cm} (2.13)

The torque/slip relationship expressed by equation (2.11), is shown typically in Figure 2.2 with the motoring, generating and braking regions indicated. The starting torque is obtained by substituting $s = 1$ into equation (2.11), to give

$$T_s = \frac{V_r^2 R_x}{2\pi n_s (R_x^2 + X_r^2)}$$  \hspace{1cm} (2.14)
2.2 Motor Operation at Variable-Frequency

The squirrel-cage induction motor has historically been regarded as a constant-speed machine, since its speed is directly related to the supply frequency which is normally constant. With the advent of variable-frequency static inverters, the machine is however becoming increasingly used in variable-speed drives.

The supply frequency $f_s$ influences the magnetic flux per pole $\phi_g$ produced in the air-gap of the motor according to

$$\phi_g = \frac{V_s - I_z s s}{f_s} k$$

(2.15)

where

- $V_s$ = supply frequency
- $Z_s$ = stator impedance
- $k$ = machine constant.

Since the torque produced in the machine is a function of the air-gap flux, constant torque operation requires the voltage to frequency ratio to be maintained almost constant, showing that the supply voltage must be proportional to the supply frequency. If the operating frequency is low, the voltage drop due to the stator resistance becomes significant, resulting in a reduced gross mechanical torque. Under these conditions, it is therefore necessary to boost the supply voltage at low frequency, as shown in Figure 2.3, to ensure that the same maximum torque is achieved.
throughout the speed range. The effect of providing this boost is shown by comparing the torque/slip characteristics of Figures 2.4(a) and (b). Control of both the voltage and frequency of the motor supply are then necessary for efficient drive system operation, and this requires the need for some form of inverter supply.

2.3 Static Inverters

Most variable speed A.C. drives employ D.C.-link inverters. Figure 2.5 shows the elements of such a drive, where the A.C. input is first converted into D.C., by either a controlled or an uncontrolled rectifier, and then inverted to provide 3-phase voltages of variable magnitude and frequency for the induction motor. The three most common types of inverters are

(a) the quasi-square wave voltage source inverter
(b) the quasi-square wave current source inverter, and
(c) the pulse-width modulated (pwm) voltage source inverter.

There are many variations of these basic types, but the differences lie mainly in the method used for commutation. Both (a) and (b) require variable D.C.-voltage to provide voltage magnitude control, and they are usually fed from the output of a phase-controlled rectifier. In some cases, a diode rectifier and a chopper arrangement are used to replace the phase-controlled rectifier. A pwm inverter combines both frequency and voltage control in a single converter unit and it is therefore used typically in combination with a constant D.C.-voltage source, such as a diode rectifier. The basic power circuits, gate firing sequence and the output waveforms associated with each basic type of inverter are discussed in more detail in Chapter 3.
Operation of an induction motor connected to an inverter differs fundamentally from that when it is connected to a 3-phase supply (3), since the D.C.-link is unable to interchange stored magnetic energy with the power supply. The inverter must therefore provide the reactive power required by the induction motor, leading to the need for a method for exchanging energy between the phases at the motor terminals. In practice, this transfer is achieved via the line-to-line short circuit path across the D.C.-link provided by a voltage source inverter, or by the commutation of current from phase to phase in the current source inverter.

2.4 Effect of Non-sinusoidal Excitation on Motor Losses

All the loss components in an induction motor, except for friction and windage, are increased as a result of harmonics in the supply (57) voltage. These losses may conveniently be separated into the various components

(a) stator winding loss; this compromises the usual fundamental frequency component together with an additional term to account for the loss due to harmonic currents. The total stator winding loss $P_{co}$ is

$$P_{co} = m R_s \left[ I_s^2 + I_{har}^2 \right]$$  \hspace{1cm} (2.16)

where $m$ is the number of phases, and the harmonic current $I_{har}$ is

$$I_{har} = \sqrt{(I_{s5})^2 + (I_{s7})^2 + \ldots \ldots \ldots (I_{sk})^2}$$  \hspace{1cm} (2.17)

and $k$ is the harmonic order.
(b) Stator core loss; compromising the sum of the hysteresis and eddy current losses in the stator iron. This loss depends upon the magnitude and frequency of the harmonics in the stator flux density, produced by the non-sinusoidal excitation. Each harmonic produces its own iron loss. The increase in loss is generally only a small fraction of the total core loss and in a total loss evaluation it may often be neglected in comparison with the losses resulting from the inverter harmonics.

(c) Rotor copper loss; this is affected by harmonic currents in the same way as is the stator winding loss. In many cases the rotor harmonic copper loss is the largest component of the total loss.

(d) Although increased by the presence of harmonic current, the stray load loss is relatively small and it is normally taken as the same as with sinusoidal excitation.

The harmonic current supplied by a voltage source inverter is limited by the machine leakage reactance, and machines with a higher leakage reactance will have a lower harmonic current and lower harmonic losses. In contrast, the current-source inverter provides current harmonics, and a lower leakage reactance results in reduced harmonic voltages. A pwm inverter is best suited to a machine with a high leakage reactance, for the same reason as the voltage source inverter, and it is therefore suitable for driving small high-reactance machines. Since pwm inverters usually have large harmonic voltages at frequencies around the carrier frequency, skin effect in the stator and rotor conductors can be considerable, especially in large machines, and can lead to excessive harmonic losses.
Improved PWM modulation techniques \((58, 59)\) can however help to minimize this problem. The steady-state behaviour of an induction machine supplied by a static inverter \((60)\) may be satisfactorily predicted, using the equivalent circuit of Figure 2.6 to calculate each excitation harmonic separately. This method of analysis implies that the correct voltage, frequency and slip must be included in the equivalent circuit for each harmonic and the resultant current calculated. Since the harmonic frequencies are high in comparison with the fundamental, the speed of rotation of the harmonic slip approaches unity. It is adequate for most purposes to assume that the harmonic slip is in fact one, when the stator and referred rotor resistances become negligible in comparison with the reactances. Furthermore, the magnetizing reactance is much larger than the leakage reactances, which allows the stator magnetizing branch to be neglected in many calculations.
FIG. 2.1 INDUCTION MOTOR-EQUIVALENT CIRCUIT PER PHASE
Figure 2.2: TORQUE SPEED CHARACTERISTICS OF AN INDUCTION MOTOR
FIG. 2.3 TYPICAL VOLTAGE/FREQUENCY CHARACTERISTICS FOR MOTOR DRIVES
FIGURE 2.4 Steady state torque speed curves

(a) constant supply voltage to frequency ratio
(b) constant airgap flux
Fig. 2.5. SCHEMATIC DIAGRAM OF A D.C.-LINK INVERTER.
Fig. 2.6. INDUCTION MOTOR EQUIVALENT CIRCUIT PER PHASE FOR \( n^{th} \) ORDER HARMONIC.
CHAPTER 3

INVERTER A.C.-DRIVE MODULATION TECHNIQUES

3.1 Types of Inverter

3.1.1 Quasi-squarewave voltage source inverter
3.1.2 Quasi-squarewave current source inverter
3.1.3 PWM-voltage source inverter

3.2 PWM-Modulation Techniques

3.2.1 Level set-modulation
3.2.2 Squarewave-modulation
3.2.3 Sinusoidal-modulation

3.3 Sinewave Modulated PWM-Inverter

3.4 Sinusoidal Switching Strategies

3.4.1 Natural switching
3.4.2 Regular switching
3.1 Types of Inverter

The modulation techniques applicable to a voltage source inverter supplying a 3-phase start-connected squirrel-cage induction motor are summarised in this chapter. A review of the basic characteristics is given, with attention being focussed on the pwm-inverter. The three basic types of inverter, mentioned briefly in the previous chapter, are discussed in more detail.

3.1.1 Quasi-squarewave voltage source inverter

Early inverter designs used the quasi-squarewave principle, with a typical circuit configuration and thyristor triggering pattern being shown in Figure 3.1 (a) and (b) respectively. The term quasi-squarewave is applied to an inverter which has an output line voltage consisting of 60° dwell, 120° positive voltage, 60° dwell, and 120° negative voltage. Conduction is always through three switches: either two switches in the top row (1, 3 and 5) and one in the bottom row (2, 4 and 6), or vice versa. This process produces square wave inverter phase voltages with an equal mark-space ratio, as shown in Figure 3.1(c). The inverter output line voltage waveform shown in Figure 3.1(d) is obtained by subtraction of the corresponding phase voltages such that

\[
\begin{align*}
\bar{V}_{AB} &= \bar{V}_A - \bar{V}_B \\
\bar{V}_{BC} &= \bar{V}_B - \bar{V}_C \\
\bar{V}_{CA} &= \bar{V}_C - \bar{V}_A
\end{align*}
\]

(3.1)
when the inverter supplies a star-connected induction motor, the inverter line-to-neutral or motor phase voltage is as shown in Figure 3.1(e). The motor phase voltage obtained is referred to as a six-step waveform. Figure 3.1(f) shows a typical motor line current waveform. With this form of inverter, only the output frequency can be varied. However, in order to maintain constant motor flux, the motor phase voltage must be varied directly with the frequency. The amplitude of the D.C.-link voltage feeding the inverter must therefore be varied, which involves the use of either a phase-controlled rectifier circuit or some form of chopper arrangement.

3.1.2 quasi-squarewave current source inverter

A quasi-squarewave current source inverter provides a set of squarewave currents equal in magnitude to the D.C.-link current. The basic power circuit configuration is shown in Figure 3.2(a). The D.C.-link inductor, which replaces the capacitor in the voltage source inverter is large, to maintain the supply current constant and thus provide a current source. The feedback diodes in the voltage source inverter are omitted from the current source inverter, and the input-output constraint is therefore on current rather than on voltage. The gating sequence of the thyristors and the output current waveforms are shown respectively, in Figures 3.2(b) and (c). It is clear that the gating sequence results in $120^\circ$ conduction of each device, with only two devices conducting simultaneously. Commutation in a current source inverter is inherently slower than that of a voltage source inverter. This is however often an advantage, since conventional thyristors are satisfactory for current source inverters, whereas inverter grade thyristors are normally required voltage source inverters.
3.1.3 PWM-voltage source inverter

The PWM-inverter is a voltage source inverter which can provide both frequency and voltage control using the inverter switching devices, and it is often used with an uncontrolled bridge rectifier supply. Figure 3.3(a) shows the inverter power circuit supplied by a diode bridge, with a parallel smoothing capacitor to ensure a constant D.C.-link voltage. The thyristor gating sequence is shown in Figure 3.3(b) and the inverter output waveforms in Figure 3.3(c). Several switching techniques are possible and these are described in the following sections.

3.2 PWM-Modulation Techniques

Switching techniques have been the subject of intensive study in recent years, most notably by Green and Boys (23), Pollack (25), Bowes (58), Grant and Barton (59), Maria and Sciacocco (61), Bowes and Clement (62) and Bowes and Mount (63). The turn-on and turn-off of the switching devices (sometimes called the control strategy) may be adjusted so as to eliminate any significant harmonics in the inverter output, and methods of achieving this are now described.

3.2.1 Level set-modulation

Figure 3.4(a) illustrates the level set modulation method, in which a sinewave reference signal is compared with an adjustable voltage level $V_{set}$. Intersections of the sinewave with the levels $+V_{set}$, 0 and $-V_{set}$ all cause switching of the inverter output, such that $V_{set}$ may be used to adjust the value of the fundamental voltage, i.e. the pulse width varies with the level of $V_{set}$. Figure 3.4(b) shows the inverter
output phase voltage and Figure 3.4(c) the line voltage, obtained graphically by subtracting two inverter phase voltages as given by equation (3.1). The motor phase voltage (star-connected) is shown in Figure 3.4(d). Additional levels can be provided to improve the output waveform and to extend the lower end of the speed range. An induction motor supplied by this form of supply will develop a significant sixth-harmonic pulsating torque.

3.2.2 Squarewave-modulation

The squarewave-modulation technique is illustrated in Figure 3.5(a), where a triangular carrier waveform is compared with a square wave reference signal. The carrier frequency is locked to an integer multiple of the reference frequency and the amplitude of the squarewave determines the magnitude of the fundamental output voltage. The ratio of the carrier frequency to the reference frequency is used to control the harmonic content of the motor supply voltage. Figures 3.5(b), (c), and (d) present respectively waveforms of the inverter phase and line voltage and the motor phase voltage. Again, a significant sixth-harmonic pulsating torque will be produced, although reduced in amplitude from that with level-set modulation.

3.2.3 Sinusoidal-modulation

The harmonic content of an inverter output waveform may be decreased considerably by using sinusoidal modulation. This involves a comparison between a sinusoidal reference signal and a triangular carrier wave, as illustrated in Figure 3.6(a). The output waveforms are given in Figures 3.6(b), (c) and (d). Several variants of this technique are in use, including controllers which generate a variable
carrier-frequency over the inverter operating range, for improved performance. Sinusoidal modulation produces an acceptable harmonic content, with respect to both motor performance and losses, and it is therefore considered in more detail in the next section.

3.3 Sinewave Modulated PWM-Inverter

The method of achieving sinusoidal modulation is very important, and various schemes are available to change the output-voltage harmonic structure in order to achieve satisfactory performance. Three methods of modulation are feasible:

a) Trailing edge modulation, in which the leading edges occur at uniformly spaced intervals and the trailing edges are modulated.

b) Leading edge modulation, in which the trailing edges occur at uniformly spaced intervals and the leading edges are modulated and,

c) Double-edge modulation, in which both edges are modulated.

The type of modulation adopted is determined by the shape of the carrier waveform. For example, whereas leading edge modulation requires a positive-ramp waveform, trailing-edge modulation requires a negative ramp waveform and double-edge modulation requires a triangular waveform. The inverter output frequency is determined by the reference waveform, while the magnitude of the output voltage depends on the ratio of the amplitudes of the reference and the carrier signals, referred to as the modulation index. The ratio between the carrier and the reference waveform frequencies determines the number of pulses per cycle of output.
3.4 Sinusoidal Switching Strategies

There are three common sinusoidal PWM switching strategies, termed NATURAL, REGULAR, and OPTIMIZED. The choice of strategy depends on the application and, in particular, on the rationalisation between the inverter losses incurred by high frequency switching and the improved performance and reduced motor losses. A regular switching strategy was adopted for the present work, since it is easy to implement in a digital control scheme. Regular switching is a development of natural switching, and this is described in the next section.

3.4.1 Natural switching

The natural switching strategy is widely used, because of its ease of implementation using analogue techniques. It can be defined by comparing a triangular carrier waveform with a sinusoidal reference waveform. The intersections of the two waveforms shown in Figure 3.7(a) provide a number of pulses between the levels +1 and -1 which determine the inverter line-to-ground (phase) voltage waveform shown in Figure 3.7(b). The output voltage and frequency are controlled by adjusting the amplitude and frequency of the reference signal. If the amplitude of the reference is greater than that of the carrier, the number of pulses per output cycle is reduced. This results in over-modulation, which is characterised by the large pulse-widths in the centre of the cycle.

It is essential, at low output frequencies, to have a large number of switching pulses/cycle, to minimise the harmonic content. At high output frequencies, the number of pulses/cycle is limited to the switching speed of the power switching devices and a low number is required. This is achieved by adjusting the carrier frequency to reference frequency ratio.
Most analogue implemented PWM-control schemes have been based on natural sampling switching strategies. A practical implementation showing the general features of this technique is illustrated in Figure 3.7(c). The Figure shows that the method exhibits two important features

(a) The centres of the pulses are not regularly or uniformly spaced and

(b) The pulse-width cannot easily be expressed by simple analytical expressions.

However the width modulated pulse shown in Figure 3.7(c) may be defined by the transcendental equation \(^{(62)}\)

\[ t_p = t_2 - t_1 = \frac{T}{2} \left[ 1 + \frac{M}{2} (\sin \omega_m t_1 + \sin \omega_m t_2) \right] \quad (3.5) \]

where

- \(T\) is the carrier waveform period,
- \(t_1\) and \(t_2\) are the switching instants,
- \(\omega_m\) is the angular frequency of the reference signal, and
- \(M\) is the modulation index

Although natural switching is used mainly in analogue schemes, it may be implemented using digital techniques, when the generation and comparison of the waveforms is performed by microprocessor software. The technique is unacceptable for fast response drive applications, since any extension of the maximum operating frequency is limited by the reduction in the number of samples/cycle, which further increases the quantisation error associated with each sample value. These limitations can however be overcome using a sampling technique which has the potential for real-time PWM generation and is described in the next section.
3.4.2 Regular Switching

Regular switching\(^{(62,63)}\) is widely used in digital systems, and is defined as the comparison of a triangular carrier waveform with a stepped reference waveform, obtained by the regular or uniform sampling of a sinewave. Regular switching may be either asymmetric or symmetric, depending on the degree of modulation of each pulse edge with respect to a regularly spaced pulse position. In asymmetric modulation, illustrated in Figure 3.8, the leading and trailing edges of each pulse are generated using two different samples of the reference, and each edge is modulated by a different amount. Each sample is held for half a cycle of the carrier to produce the stepped waveform. In symmetrical modulation, illustrated in Figure 3.9, the same sample is used to generate both edges of the pulse and, consequently, both edges are modulated equally.

Practical implementation of the generation of a single pulse using symmetrical modulation is shown in Figure 3.9(b), the amplitude of the modulating waveform at the sampling instant \(t_1\) is stored in a sample-and-hold circuit, which is synchronized to the carrier wave. The sample is held for the sample period \(T\) (i.e. from \(t_1\) to \(t_4\)) and the next sample is then taken. This produces a sample and hold version of the reference waveform, which is compared with the carrier waveform to define the switching instants \(t_2\) and \(t_3\) of the width modulated pulse. The widths of the output pulses are proportional to the value of the reference at each sampling instant, and hence the centres of the pulses are spaced uniformly in time.
With reference to Figure 3.9(b), Bowes and Clements\(^{(62)}\) have derived a simple trigonometric function to calculate the pulse widths of the pwm waveform as:

\[
  t_p = \frac{T}{2} \left[ 1 + M \sin(\omega_m t_1) \right] \quad (3.6)
\]

where

- \(T\) is the sampling time
- \(M\) is the Modulation index
- \(\omega_m\) is the reference angular frequency

The first term of equation (3.6) represents the unmodulated carrier frequency pulse width, and the second term the sinusoidal modulation required at time \(t_1\). The equation may be used to calculate the pulse width directly, and to generate the pwm output waveforms.

The switching angles required by the output waveform to switch between the two levels +1 and -1 may be defined as\(^{(62)}\):

- transition to +1
  \[
  \alpha_{2j-1} = \frac{\pi}{2R_t} \left[ 4j - 3 - M \sin(2j-1) \frac{\pi}{R_t} \right] \quad (3.7)
  \]
  and transition to -1
  \[
  \alpha_{2j} = \frac{\pi}{2R_t} \left[ 4j - 1 + M \sin(2j-1) \frac{\pi}{R_t} \right] \quad (3.8)
  \]

where \(R_t\) is the frequency-changing ratio defined as

\[
  R_t = \frac{f_c}{f_m} \quad (\text{carrier frequency})/\quad (\text{reference frequency})
\]

and

\(j\) is 1, 2, 3 ..., \(R_t\)
When $M$ is greater than unity, some of the pulses in the output waveform merge into their neighbours, and overmodulation occurs. When the inverter voltage/frequency ratio is to be maintained constant, the modulation index $M$ and the reference frequency $f_m$ are linearly related by the equation

$$M = k f_m$$  \hspace{1cm} (3.9)$$

Hence for a fixed frequency changing ratio $R_t$, equation (3.6) may be rewritten as

$$t_p = \left[ \frac{\tau}{2} + \left( \frac{k}{2R_t} \right) \sin (\omega_m t_1) \right]$$  \hspace{1cm} (3.10)$$
(a) POWER CIRCUIT.

(b) CONDUCTION SEQUENCE.

(c) OUTPUT WAVEFORMS.

Fig. 3.1 SQUARE WAVE VOLTAGE SOURCE INVERTER.
Fig. 3.1. CONTINUED.
FIG. 3.2. CURRENT SOURCE INVERTER.

(a) power circuit  (b) conduction sequence
(c) output waveforms
3-phase input

Uncontrolled rectifier

Inverter

(a)

(b)

(c)

FIG. 3.3. PWM-VOLTAGE SOURCE INVERTER

(a) power circuit  (c) output waveforms

(b) conduction sequence
FIG. 3.4 LEVEL SET PWM VOLTAGE CONTROL.
(a) Timing signals  (c) Output Line-to-Line voltage
(b) Output phase voltage  (d) Output Line-to-neutral voltage
FIG 3.5. VOLTAGE WAVEFORMS WITH SQUAREWAVE PWM

(a) timing signals  (b) output phase voltage  (c) output Line-to-Line voltage  (d) output Line-to-neutral voltage
FIG. 3.6 SINEWAVE PWM VOLTAGE WAVEFORMS
(a) timing signals (c) output Line-to-Line voltage
(b) output phase voltage (d) output Line-to-neutral voltage
FIG. 3.7. NATURAL SAMPLED PWM.

(a) Timing signals
(b) Output
(c) Single pulse generation
FIG. 3.8. ASYMMETRICAL SAMPLING
(a) Reference and sample—hold modulating signal.
(b) Timing waves.
(c) PWM output.
FIG 3.9. (a) SYMMETRICAL REGULAR SAMPLING PWM

(b) SINE PULSE GENERATED BY REGULAR SYMMETRIC SAMPLING
CHAPTER 4

OPEN-LOOP INVERTER DRIVE

4.1 Power Circuit
   4.1.1 Power supplies
   4.1.2 Power switches
   4.1.3 GTO and its snubber circuit
   4.1.4 The inverter bridge

4.2 Control Circuit
   4.2.1 HEF4752, PWM-IC modulator
   4.2.2 Speed reference circuit

4.3 GTO Gate-Drive Circuit

4.4 Current Limit Circuit

4.5 Adjustment of Modulation Process
A block diagram for the open-loop inverter drive is shown in Figure 4.1. The system comprises two main parts, the power circuit and the control circuit and these are described respectively in Sections (4.1) and (4.2). Experimental results, demonstrating the dynamic performance and the steady-state waveforms of the experimental drive system were recorded and are described in Section (4.5).

4.1 Power Circuit

The power circuit consists of the power supplies and the semiconductor inverter switches, together with their accompanying snubber circuits. The following subsections describe in some detail the various elements of the power circuit.

4.1.1 Power supplies

A circuit diagram for the various inverter power supplies is shown in Figure 4.2. These comprise a ±12 V supply for the control circuit, the high-frequency isolated supplies for the GTO gate drives and the 580 V D.C-link supply to the inverter.

The ±12 V supply is derived from a 240/15-0-15 V transformer (T1)/rectifier unit and the two integrated circuit voltage regulators IC1 and IC2, whose outputs supply the control circuit and the pulse transformer switching transistors in the GTO gate drives. The isolated supplies required by the GTO gate drives are shown in the block diagram of Figure 4.3. The drives for the upper three GTOs each require an isolated supply, whereas those for the lower three GTOs can
share a common supply, as shown in Figure 4.4. Each supply, which provides +8 V, 0 and -12 V rails, is obtained using a NE555 timer IC3 to switch TR1 on and off at 60 kHz. The isolating transformer T2 has a turns ratio of 1:3, and steps the voltage up to about 65 V peak-to-peak at the secondary. This is subsequently stepped down to about 22 V peak-to-peak by further isolating transformers T3 to T6. Transformers T3, T4 and T5 are for the three upper GTO gate drives and transformer T6 is for the lower GTO gate drives.

When TR1 is conducting, diodes D5 to D10 conduct, charging the capacitor connected to the positive supply in the GTO gate drives. TR1 is turned off, diodes D11 to D16 conduct and the energy stored in the cores of transformers T2 to T6 charges the capacitors connected to the negative supply in the gate drives. Zener diodes D17 to D20 limit the negative outputs to -12 V. In this way, an isolated smooth D.C. supply is provided for the GTO gate drives.

The high voltage supply for the D.C. link is obtained from the 3-phase 420 V 50 Hz supply, which is rectified by a full-wave diode bridge and smoothed. Resistor R1 of Figure 4.2 limits the peak rectifier current when the D.C. link capacitors C1 and C2 are being charged. The resistor is shorted out by contacts of relay B after an appropriate time delay of about 0.3 s, so that it does not dissipate power while the motor is running normally. As a safety measure, a second resistor R35 is used to discharge the D.C. link capacitors when the supply is removed.

4.1.2 Power switches

The drive efficiency depends partly on the inverter losses, which may be significant, particularly in low power drives of less than 5 kW.
Inverter losses are dependent on the choice of power semiconductor switches, the main requirements of which are:

a) The minimum forward blocking voltage must exceed the peak line-to-line voltage, to provide an allowance for regeneration.

b) A fast turn-off is essential for minimum switching losses and for the short delay times which are necessary for good wave-form definition.

c) The device must be capable of operating over a very wide range of duty cycle.

There are four main types of semiconductor switch which satisfy these requirements:

1) Bipolar Transistor
2) MOSFET
3) Conventional Thyristor (SRC)
4) Gate Turn-off Thyristor (GTO)

The properties of each device, summarised in Table 4.1, indicate that the GTO thyristor is the most appropriate choice for the PWM-inverter used in the present project.

4.1.3 GTO and its snubber circuit

The GTO thyristor has a 4-layer pn-pn structure, which has been developed in recent years from the basic-structure of the conventional thyristor. The structure and a transistor equivalent circuit are shown in Figure 4.5. Like the conventional thyristor, a GTO can block a high forward voltage while turned off, and it can pass a peak forward current far in excess of its average current rating while turned on. Typical operating
<table>
<thead>
<tr>
<th>Switching Device</th>
<th>Rating</th>
<th>Snubber Circuit Requirement</th>
<th>Switching Characteristic</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bipolar Transistor</td>
<td>Limited to low and medium power levels</td>
<td>Complex snubber circuit required</td>
<td>Fast switching</td>
<td>High voltage high current expensive</td>
</tr>
<tr>
<td>MOSFET</td>
<td>Generally available for low-voltage inputs and low powers (500V, 22 A). Medium power units are becoming available</td>
<td>Snubber circuit not required</td>
<td>High speed switching</td>
<td>Very expensive</td>
</tr>
<tr>
<td>Conventional thyristor SCR</td>
<td>High voltage and high current, but external circuit required for commutation</td>
<td>Snubber circuit required</td>
<td>Slow switching (turn-off)</td>
<td>Inexpensive</td>
</tr>
<tr>
<td>Gate-Turn-Off Thyristor GTO</td>
<td>High voltage and high current. No circuit required for turn off</td>
<td>Snubber circuit required</td>
<td>Fast switching (turn-off)</td>
<td>Moderately expensive</td>
</tr>
</tbody>
</table>

**TABLE 4.1 INVERTER SWITCH PROPERTIES**
characteristics are given in reference (65). The properties of the GTO device are well documented in the literature (65, 66) and only a brief description will therefore be given here.

Turn-on is achieved by applying a positive pulse of current to the gate, followed by a small gate current of about 1/3 of the pulse magnitude for the remainder of the on-period in order to minimise the on-state losses. Turn-off is achieved by withdrawing a current of about 1/5 of the anode current from the gate. A circuit which achieved both turn-on and turn-off is described in Section 4.3. In practice, it is necessary to connect a snubber circuit across the GTO, both to direct the anode current away from the device during turn-off and to limit the magnitude of dv/dt during turn off, so as to prevent unwanted turn-on.

The rate of decrease of anode current during turn-off may be sufficiently high to produce a large voltage spike across the GTO, due to the stray inductance of the snubber circuit, and this implies that the snubber must be connected as close as possible to the GTO leads. This voltage spike increases the turn-off losses and may possibly result in a breakdown of the GTO, although the turn-off loss can be minimised by using a fast turn-on diode with a low forward voltage across the GTO. In a bridge circuit, the snubber need only be a capacitor connected between the anode and cathode of each GTO, as shown in Figure 4.6. Because of its high surge current and di/dt ratings, the GTO can withstand the anode current pulse caused by this capacitor during the turn-on period. The size of the snubber capacitor C_s needed to prevent dv/dt from becoming excessive may be defined by the peak discharge current, which must not
exceed the maximum controllable anode current rating of the GTO. Good local decoupling of the D.C. supply is provided by capacitor C which effectively connects the upper and lower capacitors in parallel at the instant of switching.

4.1.4 The Inverter bridge

Figure 4.7 presents a block diagram for the inverter, which consists of three complementary legs, one for each of three output phases. The 580 V D.C.-link voltage and the inverter action produces a 3-phase output waveform of 1160 V peak-to-peak. A permitted rise of 150 V was assumed under regenerative braking conditions (580 + 150 = 730 V), and Mullard type BTV58-1000R GTO, with voltage and current ratings at 1000 V and 10 A were chosen for the drive.

Since the gates of the six GTO's are not all at the same potential, the control system was isolated from the gate drives by means of pulse transformers. The three lower GTO's have common cathode connections to the negative D.C.-link and share a single isolated supply. The three upper devices, however, have independent cathodes switching at the high-voltage levels of the output waveform. This requires gate drive isolation circuits, which can function correctly at high voltage levels and the upper devices must therefore have individually isolated supplies. The flywheel diodes across each GTO provide a path for inductive motor current as the inverter switches change their state. They also provide a regeneration path back to the D.C.-link when the motor frequency is suddenly reduced.
4.2 Control Circuit

The main function of the control circuit shown in Figure 4.8 is to respond to the control input setting $V_{\text{ref}}$ and to provide the pwm gate pulses in the correct sequence and at the correct frequency. The control circuit also contains the logic elements involved in the current limit circuit, which isolates the motor if a preset current limit is exceeded.

4.2.1 HEF4752, PWM-IC modulator

The main part of the control circuit is the purpose-designed integrated circuit IC8 of Figure 4.8. This is Mullard type HEF4752V, shown as a block diagram in Figure 4.9. The chip uses the regular switching pwm strategy described in Section (3.5). The main function of the pwm-IC, which is controlled by a frequency demand and a voltage controlled oscillator, is to provide three complementary pairs of output waveforms, which when applied to the inverter switches in an appropriate sequence produce the symmetrical 3-phase voltage waveforms given in Figure 4.10. Information on the internal organisation of the circuit, its operation and the relationships between the various control signals, clock inputs and the inverter output waveforms can be found in reference (67). The details of the main relationship are summarised in Table 4.2.
<table>
<thead>
<tr>
<th>Clock Input</th>
<th>Function</th>
<th>Relationship</th>
</tr>
</thead>
</table>
| FCT        | Set motor input frequency                     | $f_{\text{FCT}}\text{(kHz)} = 3.360 \times f_0\text{(Hz)}$  
             |                                               | $f_0$ - motor operating frequency |
| VCT        | Set motor volts/Hz                             | $f_{\text{VCT}}\text{(kHz)} = 6.720 \times f_0\text{(Hz)}$ |
| RCT        | Set the maximum switching frequency of the 3-phase inverter | $f_{\text{OCR}} = f_{s\text{ max}}\text{(kHz)} \times 280$  
             |                                               | $f_{s\text{ (max)}}$ - switching frequency rate |
| OCT        | Set inverter output switching delay period (the time delay between the start of turn-off of one half of an inverter bridge and the turn-on of the other half) | $f_{\text{OCT}}\text{(kHz)} = 16/T_d\text{(ms)}$  
             |                                               | where $T_d$ is a delay or dead space. |

**TABLE 4.2** Relationships between PWM-IC clock input frequencies and inverter outputs.
The FCT lock input which determines the output frequency of the inverter is controlled by \( V_{ref} \) as shown in Figure 4.8, via the speed reference circuit—described in detail in Section 4.2.2. The steady-state relationship between \( V_{ref} \) and FCT is approximately linear. The VCT clock input which sets the inverter output V/f ratio is controlled by the voltage controlled oscillator IC7. A constant VCT clock input frequency results in a constant V/f operation. Fine adjustment of VCT, RCT and OCT is obtained by means of potentiometer R26 of Figure 4.8. The CW input of pwm-IC8 determines the direction of rotation for the motor by changing the phase sequence, for example, to change the phase sequence from ABC to ACB (from forward to reverse) requires the CW input to be low. The four clock inputs FCT, VCT, RCT and OCT are routed to the pwm-IC so that the inverter operating conditions can be monitored.

4.2.2 Speed reference circuit

The speed circuit of Figure 4.8 was designed for unidirectional operation, with control over both the maximum rates of motor drive acceleration and deceleration. The input to the control board is a speed demand \( V_{ref} \) provided by a potentiometer Pt1. This voltage can vary from 0 to -10 V, giving motor speeds between standstill and to rated speed. It is applied to a comparator IC4(a) which forms the input signal to an integrator circuit IC5(b) giving a ramp output signal \( V_N' \). A step-wise variation of \( V_{ref} \) results in a linear increase or decrease of \( V_N' \). The output voltage appearing across \( R_{FCT} \) provides the frequency reference signal \( V_N' \) and is proportional to \( V_N' \). Adjustment of R27 provides
frequency control for the pwm-IC clock input FCT via the voltage controlled oscillator IC6.

This control determines the output frequency of the inverter, which in turn determines the synchronous speed of the motor. Clock inputs VCT, RCT and OCT are obtained from the multi-vibrator circuit IC7. The clock frequency of IC7 is set by C7, R11 and R26, with fine adjustment being provided by R26. The pulse amplifier IC9 ensures that the amplitude of the output waveforms from the pwm-IC are sufficiently large to drive the inverter GTO-gate drives. Logic signal CW is permanently connected to a logic high, so that a forward direction of rotation only is obtained. Forward and reverse operation requires an external circuit for automatic control of CW, and such a modification is discussed in the next chapter.

4.3 **GTO Gate-Drive Circuit**

A GTO latches on when a positive voltage pulse (typically 2 to 3 V for 10 μs) is applied to its gate, and it turns off when a negative gate voltage (-5 to -10 V, for 1 μs) is applied to withdraw about 1/5 of the anode current from the gate.

Figure 4.11 shows a gate drive circuit designed for use with Mullard GTOs. Isolation between the control and drive circuits is provided by the pulse transformer T7, energised by the switching transistor TR2 in the primary circuit. The transformer secondary voltage is a differentiated version
of the primary square waveform, and this is restored to the original shape using the inverter circuit IC16 which acts as a combined Schmitt trigger and memory circuit. The buffered output of this circuit controls the Darlington transistor TR4. When TR4 is turned off, TR3 is turned on, and the GTO is turned on by a positive pulse of gate current whose magnitude depends on the RC network, R33, C20 and R34. When C20 is fully charged, a lower steady-state current flows through R33 for the remainder of the on-period, to minimise the on-state losses of the GTO. Turn-off results when TR4 is turned on and current is withdrawn from the gate via diode D47 into the smoothing capacitor C22 connected to the isolated -12 V supply. The inductance of the loop formed by the GTO gate-cathode junction, D47, TR4 and C22 is kept below 1 μH to ensure rapid withdrawal of current from the gate.

4.4 Current Limit Circuit

The current limit circuit shown in Figure 4.12 monitors the D.C.-link current, and when this exceeds a preset value the outputs of the pwm-IC are inhibited to disconnect the motor from the supply.

The 0.1 Ω, 5 W resistor R12 in the negative side of the D.C.-link provides a voltage proportional to the D.C.-link current. This is applied to the differential amplifier IC17and, when the output of this stage exceeds the reference voltage set by R18, the output of the detector IC18 switches to high level, thus turning on the opto-isolator IC19. Isolation provided by the opto-isolator is necessary between the current detection amplifier and the control circuit, since the detection circuit is connected to the
negative D.C.-link and therefore floats at several hundred volts.

Once the preset current limit is exceeded and the light emitting diode conducts, the potential of the photo-transistor collector drops to about -12 V, causing the output of IC4(a) to switch to high level. This gives a low output to IC4(b), which turns off the pwm-IC at the start/stop input J (pin 24) of Figure 4.8. The flip-flop formed by IC4(c) and IC4(a) is in a stable-state, when the motor is off, since there is no D.C.-link current flowing and the collector of the photo transistor is at 0 V. The motor is restarted by connecting pin 1 of IC4(c) to -12 V, by press the current limit (reset) switch. This causes the flip-flop to change state and the motor to restart.

4.5 Adjustment of Modulation Process

Satisfactory operation of the drive system requires adjustments of both the modulation process and the inverter output voltage/frequency ratio. Table 4.2 of Section (4.2.1) details the various inputs to the pwm-IC, and the values of these inputs are now determined for the experimental rig under consideration.

Speed variation is achieved by varying the frequency applied to the FCT clock. The frequency required for maximum motor speed is given in Table 4.2 as

\[ f_{\text{FCT (max)}} = 3.36 \times f_o \text{ kHz} \]

where \( f_o \) is the rated motor frequency in Hz.
The rated frequency of the experimental drive is 50 Hz and \( f_{\text{FCT}}(\text{max}) \) is therefore 168 kHz. A variation in \( f_{\text{FCT}} \) from 0 to 168 kHz gives a motor speed variation between standstill and rated speed. The frequency applied to the VCT clock input \( f_{\text{VCT}} \) determines the inverter output voltage/frequency ratio. It has a fixed value calculated at the rated output frequency for a particular voltage/frequency ratio as:

\[
f_{\text{VCT}} = 6720 \times f_o \ \text{kHz}
\]

A constant value of \( f_{\text{VCT}} \) produces a constant inverter output voltage/frequency ratio. However, at low operating frequencies, the ratio must be increased to compensate for the motor IR-voltage drop.

The above calculations for both \( f_{\text{FCT}} \) and \( f_{\text{VCT}} \) give a frequency ratio \( \frac{f_{\text{FCT}}}{f_{\text{VCT}}} = 0.5 \) and are based on 100% modulation. To ensure normal modulation, the frequency ratio must be less than 0.5. If the ratio exceeds 0.5, the number of switchings per output cycle reduce and over-modulation occurs. If the ratio is further increased, the output eventually becomes a squarewave. The effect of changing the frequency ratio is illustrated experimentally in Figure 4.13(a) to (d) for frequency ratios of 0.4, 0.5, 1.0 and 2.0. Figure 4.14 shows an experimentally obtained line-to-line voltage waveform when operating at 50 Hz and a frequency ratio of 2.0, and this clearly exhibits a quasi-squarewave shape with an induction motor having the parameters given in Appendix (B) connected to the inverter.
The current limit was adjusted by loading the motor until the motor line current waveform was 10 A peak-to-peak and R18 of Figure 4.12 was adjusted to trip out pwm-IC at this current level.

The inverter voltage waveforms shown in Figure 4.15 are at 50 Hz operating frequency and a frequency ratio of 0.45. The corresponding motor voltage waveforms are shown in Figure 4.16. The motor phase voltage and line current waveforms of Figure 4.17 clearly shows that line current lags the phase voltage.
FIG. 4.1 GTO-PWM MOTOR DRIVE SYSTEM
Fig. 4.2. SYSTEM POWER SUPPLIES

To control circuit

To gate drives

To control board

- ve dc

DC LINK

+ ve dc

12 V

0 V

+ ve d.c.

- ve d.c.

To control circuit

Fig. 4.2
FIG. 4.3  GENERAL LAYOUT OF THE INVERTER.
FIG. 4.4 MULTIPLE-OUTPUT ISOLATED POWER SUPPLY
Fig. 4.5. GTO STRUCTURE & TRANSISTOR EQUIVALENT CIRCUIT.

Fig. 4.6 SLOW RISE CIRCUIT
FIG. 4.1 BLOCK DIAGRAM OF THE GTO-INVERTER
FIG. 4.8 THE CONTROL CIRCUIT

IC4  IC5  MC1458N
IC6  NE566
IC7  HEF4047B
IC8  HEF4752V
IC9  HEF40174B
IC11-14  HEF4093B
IC15  HEF4016B
FIG. 4.9. BLOCK DIAGRAM OF THE HEF 4752V.
FIG. 4.10  SYSTEM WAVEFORMS

a. Carrier Waveform
b. Inverter Phase Voltages
c. Inverter line-to-line voltage
Fig. 4.11 GTO THYRISTOR MODULE CIRCUIT.
4.12. CURRENT LIMIT CIRCUIT.
FIG. 4.13 INVERTER PHASE VOLTAGE WAVEFORMS RECORDED AT DIFFERENT FREQUENCY CHANGING RATIOS
FIG. 4.14 EXPERIMENTALLY OBTAINED STEADY-STATE INVERTER LINE-TO-LINE VOLTAGE WAVEFORM
FIG. 4.15 EXPERIMENTALLY RECORDED 3-PHASE INVERTER VOLTAGE WAVEFORMS AT 50 Hz
FIG. 4.17 EXPERIMENTALLY OBTAINED STEADY-STATE MOTOR PHASE VOLTAGE AND LINE CURRENT WAVEFORMS
CHAPTER 5

IMPROVEMENTS TO THE SPEED CONTROL SYSTEM

5.1 Bi-directional Speed Reference Circuit
5.2 IR-Voltage Drop Compensation Circuit
5.3 Inverter Output Waveforms
This chapter describes improvements made to the open-loop speed control system described in Chapter 4. The improvements are

a) The implementation of bidirectional speed control, together with control of the maximum rates of increase and decrease of the motor supply frequency.

b) Voltage drop compensation for the stator resistance at low supply frequencies by adjustment of the V/f ratio.

5.1 Bi-directional Speed Reference Circuit

A speed reference circuit was developed to provide bi-directional operation of the drive, as well as to control the maximum rates of increase and decrease in the motor supply frequency. The circuit, as shown in Figure 5.1, has a single speed reference voltage $V_{\text{ref}}$ provided by the potentiometer $P_t1$, the output of which may vary between $-10$ and $+10V$, to provide speed variation between rated speed in the reverse and forward directions. The reference voltage is applied to the comparator $I_{C1}$, whose output is limited to a predetermined value corresponding to a motor slip below that for maximum torque. This is achieved by the acceleration/deceleration limiting potentiometers $P_t2$ and $P_t3$, such that the maximum rate of speed change is limited by control of the maximum positive and negative values of $V_{\text{lim}}$. The maximum positive value $V_{\text{lim}}(\text{max})$ is determined by the setting of potentiometer $P_t3$ and the inverting operational amplifier $I_{C3}$, such that

$$V_{\text{lim}}(\text{max}) = - VR3 \times \frac{R7}{R6}$$

where $VR3$ is the voltage set by $P_t3$. Similarly, the maximum negative value of $V_{\text{lim}}$ is controlled by potentiometer $P_t2$ and the inverting operational amplifier $I_{C2}$. When the motor accelerates $V_{\text{lim}}$ will be
positive and if it exceeds $V_{\text{lim}}^{\text{(max)}}$ diode D3 will conduct, clamping $V_{\text{lim}}$ to $V_{\text{lim}}^{\text{(max)}}$. If $V_{\text{lim}}$ exceeds the preset maximum negative value while decelerating, diode D1 will conduct, clamping $V_{\text{lim}}$ to the maximum negative value. In this way control of acceleration/deceleration is achieved. The resultant speed signal forms the input to the integrator IC4, giving a ramp output voltage signal $V_N$. The value of the integrator capacitor C determines the rate-of-rise of the ramp voltage. The output of the integrator circuit is fed to the full-wave precision rectifier formed by IC5 and IC6, to produce a negative voltage reference signal $V_{\text{RFCT}} = -k|V_N|$ irrespective of the sign of $V_N$. The voltage reference signal $V_{\text{RFCT}}$, proportional to $V_N$, controls the inverter output frequency via the voltage-controlled oscillator of Figure 5.2. Any change in the sign of $V_N$ causes the reference polarity detector IC7 to switch the output of NAND gate IC8 from high to low level or vice versa, thereby changing the direction of rotation of the motor.

The response of a system in the frequency domain may be expressed by its frequency-response transfer function relationship between the output and the input of the system. Figure 5.3 shows ultra-violet recordings of the system responses when input sinewave test signals of specified magnitudes are applied to the speed reference circuit for frequencies respectively of 0.1, 0.125, 0.142, 0.20, 0.25 and 1.0 Hz. The Figures show clearly the distorted output sinewave at the frequencies of operation, the distortion being due to some non-linear characteristic of the system, the phase shift between the input and the output (motor speed) signals, and the change in amplitude of output as a consequence of the changing input frequency. From these experiments, the corresponding Bode-diagram of Figure 5.4 (showing the system gain and phase variation with input frequency) is obtained.
Figure 5.5 presents ultra-violet recordings of the speed and current waveforms following a step input voltage change from 0 to +10 V. This shows clearly that the motor speed is linearly accelerated from standstill to full speed in the forward direction in about 9 s. Motor speed, current and voltage waveforms for equal acceleration and deceleration times are shown in Figure 5.6. A negative step input signal to the system results in a build up motor speed in the reverse direction, as illustrated by Figure 5.7, for a step input from 0 to -10 V.

5.2 IR-Voltage Drop Compensation Circuit

Operation of an induction motor at a constant V/f ratio results in a low applied voltage at low input frequencies. Since the voltage drop across the motor stator resistance becomes relatively large at low frequency, this results in a reduced air-gap flux and a consequent low starting torque. This undesirable feature may be eliminated by increasing the V/f ratio at low frequencies. The value of the inverter output voltage at a given output frequency for the experimental scheme is determined by the clock input VCT of the PWM-IC. Reducing the frequency of this clock increases the inverter output voltage and vice versa. The frequency of VCT is determined by its voltage reference signal VR_VCT, so that stator IR-voltage drop compensation requires modification to this signal at low input frequencies. A diagram of the circuit developed to provide this modification is shown in Figure 5.8(a), with the characteristic of the circuit being shown in Figure 5.8(b).
An experimentally obtained torque-speed relationship for a constant V/f ratio is shown in Figure 5.9(a). It will be seen that the experimentally obtained motor rated torque, is not maintained for operating frequencies below 20 Hz. However, with implementation of the IR-voltage drop compensation, motor rated torque is achieved at all operating frequencies, as shown in Figure 5.9(b).

A step input voltage signal from 0 to +10 V, applied to the speed reference circuit, accelerates the 60% loaded drive (with IR-compensation) from standstill to rated speed in about 10 s. Consideration of Figure 5.10, shows that the amplitude of the motor line current during starting is increased in comparison with that of the uncompensated system of Figure 5.5. The difference is of course available to produce an increased acceleration in the load. Results for reversal of the drive from forward full speed to reverse full speed for an un-loaded drive system with IR-voltage drop compensation are presented in Figure 5.11. It is clear from the Figure that after deceleration to standstill in about 8 s, the motor pauses for about 1 s before restarting and accelerating to rated speed in the reverse direction, again in about 8 s.

5.3 Inverter Output Waveforms

Experimentally obtained ultra-violet recordings of the steady-state phase voltage waveforms of the inverter are shown in Figure 5.12. These waveforms relate to an operating frequency of 50 Hz and a frequency changing ratio \( \frac{f_{CT}}{f_{VCT}} \) of 0.5. The corresponding three line-to-neutral (or motor phase) waveforms are shown in Figure 5.13; these are of course identical in form but with a 120° phase shift.
Figures 5.14 - 5.17 show experimentally obtained ultra-voilet recordings of the steady-state inverter phase voltage $V_A$ and the inverter line-to-line voltage $V_{AB}$ waveforms, are respectively related to output frequencies of 20, 30, 40 and 50 Hz.

The inverter output line current waveform depends on the impedance of the motor windings, which attenuates considerably the higher frequency components of the current to result in a waveform which depends mainly on the modulation process. Figures 5.18 (a-d) show experimentally-obtained, steady-state line current waveforms at different operating frequencies, which clearly contain a ripple at the switching frequency. Figure 5.19 shows the input phase voltage and line current waveforms to the fully-loaded motor at the rated frequency of 50 Hz.
Fig. S.1.
BIDIRECTIONAL SPEED REFERENCE CIRCUIT.
FIG. 5.2. VCO CIRCUIT DIAGRAM.
FIG. 5.3 OPEN-LOOP SYSTEM SPEED RESPONSE
FIG. 5.3 CONTINUED
FIG. 5.4. BODE diagram of the open-loop Drive System.
FIG. 5.5  MOTOR SPEED AND CURRENT FOR ACCELERATION AND DECELERATION TIME - NO-LOAD
FIG. 5.6  MOTOR SPEED CURRENT AND VOLTAGE WAVEFORMS FOR EQUAL ACCELERATION AND DECELERATION TIME - NO-LOAD
FIG. 5.7 MOTOR SPEED AND CURRENT FOR REVERSE ROTATION
Fig. 5.8(a)  IR - COMPENSATION CIRCUIT
FIG. 5.8(b) CHARACTERISTICS OF IR-COMPENSATION CIRCUIT
FIG. 5.9(a) TORQUE-SPEED CHARACTERISTICS OF THE MOTOR DRIVE WITH CONSTANT V/F RATIO
FIG. 5.9(b) TORQUE-SPEED CHARACTERISTICS OF THE MOTOR DRIVE WITH THE IMPLEMENTATION OF IR-VOLTAGE COMPENSATION
FIG. 5.10 MOTOR SPEED AND CURRENT WAVEFORMS RESPONDING TO STEP INPUT SIGNAL - WITH IR - VOLTAGE COMPENSATION - 60% LOAD
FIG. 5.11  FULL REVERSAL OF THE DRIVE SPEED
FIG. 5.12  RECORDED INVERTER OUTPUT VOLTAGE WAVEFORMS  4 ms/cm
FIG. 5.13  RECORDED MOTOR DRIVE 3-PHASE VOLTAGE WAVEFORMS
FIG. 5.14  STEADY-STATE INVERTER VOLTAGE WAVEFORMS AT 20 Hz
FIG. 5.15 STEADY-STATE INVERTER VOLTAGE WAVEFORMS AT 30 Hz
FIG. 5.16  STEADY-STATE INVERTER VOLTAGE WAVEFORMS AT 40 Hz
FIG. 5.17 STEADY-STATE INVERTER VOLTAGE WAVEFORMS AT 50 Hz
FIG. 5.18 MOTOR DRIVE LINE CURRENT WAVEFORMS SHOWING DIFFERENT SWITCHING MULTIPLES (PULSE NUMBER)
FIG. 5.19  MOTOR PHASE VOLTAGE AND LINE CURRENT WAVEFORMS RECORDED AT FULL LOAD
CHAPTER 6:

MATHMATICAL MODEL OF INVERTER-INDUCTION MOTOR DRIVE

6.1 Simulation of the Regular Switching Strategy
6.2 Induction Motor Model
6.3 Derivation of Stationary 2-axis Model
   6.3.1 Direct phase model
   6.3.2 3-phase/2-phase transformation
   6.3.3 D,Q transformation
6.4 Computer Program
6.5 Combined Inverter/Induction Motor System Model
6.6 Harmonic Analysis
6.1 Simulation of the Regular Switching Strategy

It is shown in this section how the equations for the regular sampled-switching strategy, derived in Section (3.5.2), may be used in a computer program to generate the inverter output waveforms. A flowchart for the program is presented in Figure 6.1. As given previously (in equation (3.7) of Section (3.5)), the angles at which the output voltage is switched between the positive and negative of the D.C. supply are:

Transition from $-V_{DC}/2$ to $+V_{DC}/2$

$$\alpha_{2j-1} = \frac{\pi}{2R_t} \left[ 4j - 3 - M \sin(2j - 1) \frac{\pi}{R_t} \right]$$  \hspace{0.5cm} (6.1)

and transition from $V_{DC}/2$ to $-V_{DC}/2$

$$\alpha_{2j} = \frac{\pi}{2R_t} \left[ 4j - 1 + M\sin(2j-1) \frac{\pi}{R_t} \right]$$  \hspace{0.5cm} (6.2)

where $j = 1, 2, \ldots$ $R_t$, $R_t$ is the frequency changing ratio and $M$ the modulation index.

For constant torque applications, the reference frequency $f_m$ and the modulation index $M$ are related linearly by

$$M = kf_m$$  \hspace{0.5cm} (6.3)
where \( k \) is constant, except at low frequencies, when the modulation index must be increased to compensate for the motor stator resistance voltage drop.

The computation process begins with the reading in of initial parameters such as the operating frequency, carrier frequency, the D.C.-link voltage, and the time step. The modulation index and the frequency changing ratio are then calculated and substituted in equations (6.1) and (6.2), to give a series of values for the switching angles \( (a_1, a_2 \ldots a_n) \) corresponding to the rising and falling edges of the PWM waveform. By means of a comparison between a pair of corresponding switching angles, i.e. the rising and falling edges, the pulse width can be generated. The program can also be used to generate the 3-phase inverter output waveforms, by defining the \( \frac{2\pi}{3} \) rad. phase shift between inverter phases in the sine terms of equations (6.1) and (6.2). The inverter line-to-line voltage waveform may then be obtained by subtracting two of the three inverter phase waveforms, to give

\[
\begin{align*}
\overline{V}_{AB} &= \overline{V}_A - \overline{V}_B \\
\overline{V}_{BC} &= \overline{V}_B - \overline{V}_C \\
\overline{V}_{CA} &= \overline{V}_C - \overline{V}_A
\end{align*}
\] (6.4)

Output waveforms provided by the program for 100% modulation at the system rated frequency, i.e. \( f_m = 50 \) Hz, \( M = 0.9 \) and a carrier frequency of 1050 Hz are presented in Figures 6.2(a) and (b), which give respectively the inverter 3-phase voltage waveforms, and the system 3-phase line-to-line waveforms. It is clear from the phase voltage waveforms that a carrier
frequency of 1050 Hz results in 21 switching pulses/cycle of output. The program output for \( f_m = 30 \) Hz is given in Figures 6.3(a) and (b), which show that the number of switching pulses/cycle has increased to 30, as a consequence of the reduced output frequency. To demonstrate the validity of the computer simulation, a number of comparisons between the simulated and experimental steady-state inverter output phase and line voltage waveforms were obtained. Figures 6.4-6.7 present respectively these waveforms at frequencies of 20, 30, 40 and 50 Hz. In all cases the two sets of waveforms are in close agreement and have the same number of switching pulses/cycle.

6.2 Induction Motor Model

The three most popular methods for the mathematical modelling of an induction motor are based on the direct 3-phase, the rotating 2-axis (αβ) and the stationary 2-axis (d-q) reference frames. Each of these is subject to approximations to differing extents, and the choice of reference frame is dependent on the computer power available and the required degree of accuracy and operating conditions of the system to be studied.

Induction motor models based on the direct 3-phase reference frame require the most computing time, due to the time-varying nature of the various inductance coefficients, but the operating conditions to which it may be applied are not restricted to the same extent as is the case with the other two models. However, the rotating 2-axis α-β has been found to be more convenient under certain unbalanced conditions, although
its inductance coefficients are still time-varying. The d-q model\(^{(26,27)}\) offers considerable computational simplicity when compared with the other two approaches, due to the absence of time-varying inductance coefficients. The corresponding differential equations are linear with the constant coefficients, provided that the rotor speed is constant. Models based on the d-q equations have been extensively applied to the study of the dynamic performance of induction motors supplied from both sinusoidal and non-sinusoidal voltage sources, and they have been particularly valuable for predicting the harmonic content of the machine stator current. For this reason, the \(d,q\) model was used in this present investigation, in conjunction with the pwm-inverter model of section (6.1), to form the complete drive model, described in section (6.4).

6.3 Derivation of Stationary 2-axis Model

The transformation from the direct phase model of an induction motor to a stationary 2-axis d-q model is developed in the following sections.

6.3.1 Direct phase model

An induction motor may be represented by a number of interacting coils for which a set of differential equations may be generated. The following assumptions\(^{(27,35)}\) simplify the analysis:

a) The rotor is perfectly cylindrical and the air-gap is uniform.

b) The mutual inductance between any stator and rotor windings is a cosine function of the electrical angle between the axes of the two windings.

c) The effect of saturation, hysteresis, and eddy currents are negligible.
Based on the above assumptions, the matrix differential equation, relating to the machine is

$$\begin{bmatrix}
V_A \\
V_B \\
V_C \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
R_A + pL_A & pM_{AB} & pM_{AC} & pM_{Aa} & pM_{Ab} & pM_{Ac} \\
pM_{BA} & R_B + pL_B & pM_{BC} & pM_{Ba} & pM_{Bb} & pM_{Bc} \\
pM_{CA} & pM_{CB} & R_C + pL_C & pM_{Ca} & pM_{Cb} & pM_{Cc} \\
pM_{aA} & pM_{aB} & pM_{aC} & R_a + pL_a & pM_{ab} & pM_{ac} \\
pM_{bA} & pM_{bB} & pM_{bC} & pM_{ba} & R_b + pL_b & pM_{bc} \\
pM_{cA} & pM_{cB} & pM_{cC} & pM_{ca} & pM_{cb} & R_c + pL_c
\end{bmatrix}
\begin{bmatrix}
i_A \\
i_B \\
i_C \\
i_a \\
i_b \\
i_c
\end{bmatrix}
$$

where suffices with capital letters and small letters denote respectively stator and rotor quantities. In equation (6.5)

$$L_A = L_B = L_C = L_a = L_b = L_c = M_{sr} + L_s$$

$$L_a = L_b = L_c = M_{sr} + L_r$$

where \(L_s\) and \(L_r\) are the leakage inductances of the stator and the rotor winding, and \(M_{sr}\) is the mutual inductance between them when their magnetic axes coincide. Equation (6.5) may be written in the abbreviated form

$$[V] = [R][I] + p[L][I]$$

(6.6)

where

- \([V]\) is the voltage vector \([V_A, V_B, V_C, V_a, V_b, V_c]^t\)
- \([I]\) is the current vector \([i_A, i_B, i_C, i_a, i_b, i_c]^t\)
- \([R]\) is the machine resistance matrix

$$\text{diag} [R_A, R_B, R_C, R_a, R_b, R_c]$$

and \([L]\) is the machine inductance matrix.
It is obvious from equation (6.6) that \( p \) operates on the time varying inductance term, as well as the currents so that the equation may be re-arranged as

\[
[V] = [R + G][I] + L_p[I]
\] (6.7)

where \( G \) is the rate of change of inductance matrix \( \frac{dL}{dt} = \frac{dL}{d\theta} \cdot \frac{d\theta}{dt} \) and \([V], [I] \) and \([R]\) are as defined previously. The inductance matrix \([L]\), given in full, in terms of its angle varying coefficients is

\[
[L] =
\begin{bmatrix}
L_s & M_s & M_s & M_{sr}\cos\theta & M_{sr}\cos\theta + \frac{2\pi}{3} & M_{sr}\cos\theta - \frac{2\pi}{3} \\
M_s & L_s & M_s & M_{sr}\cos\theta - \frac{2\pi}{3} & M_{sr}\cos\theta & M_{sr}\cos\theta + \frac{2\pi}{3} \\
M_s & M_s & L_s & M_{sr}\cos(\theta + \frac{2\pi}{3}) & M_{sr}\cos(\theta - \frac{2\pi}{3}) & M_{sr}\cos\theta \\
M_{sr}\cos\theta & M_{sr}\cos(\theta - \frac{2\pi}{3}) & M_{sr}\cos(\theta + \frac{2\pi}{3}) & L_r & M_r & M_r \\
M_{sr}\cos(\theta + \frac{2\pi}{3}) & M_{sr}\cos\theta & M_{sr}\cos(\theta - \frac{2\pi}{3}) & M_r & L_r & M_r \\
M_{sr}\cos(\theta - \frac{2\pi}{3}) & M_{sr}\cos(\theta + \frac{2\pi}{3}) & M_{sr}\cos\theta & M_r & M_r & L_r
\end{bmatrix}
\] (6.8)

where suffices \( s \) and \( r \) denote respectively stator and rotor.
The rate-of-change of inductance matrix $G$, given in full is

$$
[G] = M_{sr} \frac{d\theta}{dt} \begin{bmatrix}
0 & 0 & 0 & \sin\theta & \sin(\theta + \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) \\
0 & 0 & 0 & \sin(\theta - \frac{2\pi}{3}) & \sin\theta & \sin(\theta + \frac{2\pi}{3}) \\
0 & 0 & 0 & \sin(\theta + \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & \sin\theta
\end{bmatrix}
$$

The equations may be re-arranged and the time variation of current vector $[I]$ may be obtained using numerical integration.

The developed motor torque is

$$
T_e = \frac{1}{2} P \ [I]^t \ \frac{dL}{d\theta} \ [I]
$$

where $P =$ pairs of poles, and the mechanical equation is

$$
\frac{J}{p} \ p^2 \ \dot{\theta} + k_f \ p \ \dot{\theta} + T_m = T_e
$$

where $k_f$ is the rotor friction coefficient and $J$ is the inertia.

Since $p \ \dot{\theta} = \omega$ (the motor angular velocity) then

$$
p \omega = \frac{p (T_e - T_m - k_f \omega)}{J}
$$
6.3.2 3-phase/2-phase transformation

The 3-phase model of an induction motor is shown in Figure 6.8(a), and that of its equivalent 2-phase model is shown in Figure 6.8(b). With the A-phase stator winding of the 3-phase machine coincident with that of stator phase a of the 2-phase machine, the mmfs developed in two models may be equated, and assuming that the turns/phase of the two machines are identical, the relationship between their currents in the abbreviated form are:

\[
[I_{αβ}] = [C]^t [I_{ABC}] \tag{6.13}
\]

where

\[
[C]^t \text{ is the transpose of } [C], \text{ and}
\]

\[
[C] = \sqrt{\left( \frac{2}{3} \right)} \begin{bmatrix}
1 & \frac{1}{2} & -\frac{1}{2} \\
0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix} \tag{6.14}
\]
For invariance of power through the transformation,

\[ [V_{\alpha\beta0}]^t [I_{\alpha\beta0}] = [V_{ABC}]^t [I_{ABC}] \]  

(6.15)

substituting

\[ [I_{ABC}] = [C][I_{\alpha\beta0}] \]

\[ [V_{\alpha\beta0}]^t [I_{\alpha\beta0}] = [V_{ABC}]^t [C][I_{\alpha\beta0}] \]

or

\[ ([V_{\alpha\beta0}]^t - [C]^t [V_{ABC}]^t) [I_{\alpha\beta0}] = 0 \]

since \([I_{\alpha\beta0}] \neq 0\)

hence

\[ [V_{\alpha\beta0}]^t = [C]^t [V_{ABC}]^t \]

or

\[ [V_{\alpha\beta0}] = [C]^t [V_{ABC}] \]  

(6.16)

A similar argument may be presented for the rotor voltage and current transformations and a complete current transformation may be defined as,

\[
\begin{bmatrix}
I_{ae} \\
I_{bs} \\
I_{ar} \\
I_{ar}
\end{bmatrix} = [C_1]^t
\begin{bmatrix}
I_A \\
I_B \\
I_C \\
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

(6.17)

where \([C_1]^t\) is the transpose of \([C_1]\), and

\[
[C_1] = \begin{bmatrix}
[C] & 0 \\
0 & [C]
\end{bmatrix}
\]
and a voltage transform as

$$[v_{\alpha\beta}] = [c_1]^t [v_{ABC}]$$  \hspace{1cm} (6.18)

when equations (6.16) and (6.17) are substituted in equation (6.6), the impedance of the new system \([z_{\alpha\beta}]\) may be written in terms of \([z_{ABC}]\) as

$$[z_{\alpha\beta}] = [c_1]^t [z_{ABC}] [c_1]$$ \hspace{1cm} (6.19)

which gives

$$[z_{\alpha\beta}] = \begin{bmatrix}
R_s + pL_s & 0 & p \cos \theta & -p \sin \theta \\
0 & R_s + pL_s & p \sin \theta & p \cos \theta \\
p \cos \theta & p \sin \theta & R_r + pL_r & 0 \\
-p \sin \theta & p \cos \theta & 0 & R_r + pL_r
\end{bmatrix}$$ \hspace{1cm} (6.20)

where \(M = \frac{3}{2} M_{sr}\).

6.3.3 \textbf{\(d,q\) transformation}

A second transformation is required to eliminate the time dependent inductance coefficients inherent in both the 3-phase and 2-phase models. Bearing in mind that the stator coils of the 2-phase and the \(d,q\) models, shown respectively in Figures 6.8(b) and (c), are coincident, and that
the α-coil is at an angle $\theta$ to the d-axis, the relationship between the currents in the two machines is

$$[I_{dq}] = (C_2) [I_{a\beta}]$$  \hspace{1cm} (6.21)

where

$$[C_2] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \cos \theta & -\sin \theta \\
0 & 0 & \sin \theta & \cos \theta
\end{bmatrix}$$  \hspace{1cm} (6.22)

Assuming power invariance during the transformation, the impedance matrix $[Z_{dq}]$ may be obtained from

$$[Z_{dq}] = [C_2]^t [Z_{a\beta}] [C_2]$$  \hspace{1cm} (6.23)

which gives

$$[Z_{dq}] = \begin{bmatrix}
R_s + pL_s & 0 & pM & 0 \\
0 & R_s + pL_s & 0 & pM \\
pM & \delta M & R_r + pL_r & \delta L_r \\
-\delta M & pM & -\delta L_r & R_r + pL_r
\end{bmatrix}$$  \hspace{1cm} (6.24)

which may be re-structured in terms of resistances,

$$[R_{dq}] = \text{diag} \{ R_s', R_s', R_r', R_r' \}$$

and inductances
Thus, the matrix differential equation, relating to the 2-axis machine may be written as

\[
\begin{bmatrix}
V_{sd} \\
V_{sq} \\
V_{rd} \\
V_{rq}
\end{bmatrix} =
\begin{bmatrix}
R_s + pL_s & 0 & pM & 0 \\
0 & R_s + pL_s & 0 & pM \\
pM & M_s & R_r + pL_r & L_r \\
-M_s & pM & -L_r & R_r + pL_r
\end{bmatrix}
\begin{bmatrix}
I_{sd} \\
I_{sq} \\
I_{rd} \\
I_{rq}
\end{bmatrix}
\]

or in the abbreviated form

\[
[V_{dq}] = [R_{dq}] [I_{dq}] + [G_{dq}] p\theta [I_{dq}] + [L_{dq}] p [I_{dq}]
\]

(6.27)

where

\[
[I_{dq}] = [I_{sd}' I_{sq}' I_{rd}' I_{rq}]^t
\]

and

\[
[G_{dq}] =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & M & 0 & L_r \\
-M & 0 & -L_r & 0
\end{bmatrix}
\]

(6.28)

Since the above equations are functions only of the motor speed \(p\theta\), they can be solved analytically when the speed is considered constant.

However, for variable-speed application, equation (6.27) may be rearranged in the form
and a step-by-step solution for the current vector \( I_{dq} \) may be obtained using numerical integration. The electro-magnetic torque developed by the motor is

\[
T_e = \frac{3}{2} (p) \left[ I_{dq} \right]^t \left[ G_{dq} \right] I_{dq}
\]  

(6.30)

and the mechanical equation for the drive is defined by equation (6.12)

6.4 Computer Program

A computer program (dq-1) was written in Fortran 77, to predict the induction motor behaviour using a d,q model. A simplified flow chart for the program is given in Figure 6.9. The program starts by reading the parameter matrices, together with the initial machine conditions. Equation (6.29) are solved on a step-by-step basis using numerical integration to give new values for the current vector \( I_{dq} \). These new currents, together with the new voltages obtained from equation (6.18), form the initial condition for the next step. At each step, the electro-magnetic torque developed by the motor is calculated using equation (6.30). After substituting this new torque into equation (6.12), a solution is obtained for the motor speed. The program runs until steady-state conditions are attained.
6.5 **Combined Inverter/Induction Motor System Model**

The computer program (pwm-1) for the pwm-inverter of section (6.1) is combined with that for the d,q-model of the induction motor given in section (6.4) to form a program called (pwm/dq), for the prediction of the complete system performance. The full program listing is given in Appendix (C).

The program may be run for a number of steps beyond the start-up transient, in order to achieve a steady-state solution for the system. Figures 6.10(a) and (b) illustrate the computed steady-state motor terminal voltages, respectively for operating frequencies of 50 Hz and 30 Hz. Apart from the \( \frac{2\pi}{3} \) rad. phase shift, the three voltages at the same frequency are identical and, as before, the number of switching pulses is seen to be increased as the operating frequency is decreased. Figures 6.11(a) and (b) present the simulated steady-state d-q voltage waveforms, \( V_D \) and \( V_Q \), again for operating frequencies of 50 Hz and 30 Hz. It is clear from the Figure that the waveform of \( V_D \) has the same shape as that of the motor phase voltage (Figures 6.11(a) and (b)), whereas that for \( V_Q \) is a scaled version of the inverter line voltage waveform (Figure 6.2(b)).

The dynamic performance of the drive system is illustrated in Figure 6.12. This gives computed waveforms of voltage, current, torque, and speed as the unloaded motor accelerates from rest to rated speed at rated voltage and rated operating frequency (50 Hz) following the direct-on-line
switching of the inverter system. The initial oscillatory nature of the developed torque (Figure 6.12(h)) causes dips in the speed (Figure 6.12(g)) and also results in starting currents (Figure 6.12(b)) with rising and falling amplitudes. Corresponding results obtained for an operating frequency of 30 Hz are given in Figure 6.13 and these show clearly that the maximum torque of Figure 6.13(h) is the same as that for 50 Hz operation shown in Figure 6.12(h).

The simulated motor performance at 20 Hz is presented in Figure 6.14. It is clear from Figure 6.14(d) that the maximum motor torque is less than that for 50 Hz (Figure 6.12(h)), which is expected due to the stator voltage drop at lower frequencies. With 12% IR-voltage drop compensation, the maximum motor torque is raised to the 50 Hz value, shown in Figure 6.15(d). Figure 6.15 shows the 20 Hz performance with IR-compensation, when the motor is accelerated from direct-on-line switching with constant load torque of half the rated value. The starting time during acceleration for 20 Hz operation is improved from 0.24 s (Figure 6.14(c)) to 0.22 s (Figure 6.15(c), with IR-voltage compensation.

The motor drive performance following the application of load is shown in Figure 6.16, for an operating frequency of 50 Hz. After the unloaded motor has achieved its no load steady-state speed, a sudden short 5.0 Nm pulse of load torque is applied to the shaft. The motor speed is thereby reduced (Figure 6.16(g)), the developed torque (Figure 6.16(h)) is increased and the shape of the motor current waveform (figure (6.16 (b-f)), is slightly changed.
Figures 6.17-6.20, compare the simulated and experimentally obtained steady-state input voltage and current waveforms for one phase of the motor at frequencies of 20, 30, 40 and 50 Hz respectively, and these are seen to generally be in good agreement.

6.6 Harmonic Analysis

When the pwm voltage and current waveforms obtained from the inverter/induction motor program are supplied to the harmonic analysis program, listed in Appendix (C), the absolute magnitude of the real-part of the harmonic coefficients are obtained as a percentage of the fundamental. The analysis program uses the Fast Fourier Transformation (FFT).

At any given frequency the spectrum is naturally the same for each phase. Figures 6.21-6.24 present a comparison between the predicted and experimental harmonic content of the inverter phase voltage waveforms for various operating frequencies. The generally good agreement between corresponding results gives confidence in the mathematical model, and in each case, it is seen that the amplitudes of the harmonics fall off inversely as their order increases. As the waveform is half-period symmetric all even harmonics are absent, although the significant 3rd-harmonic and its odd multiples are clearly visible. The obvious frequency bands present in these figures, are centred on the carrier frequency and its odd multiples, and comprise upper and lower side band components of approximately equal amplitudes and displaced by even multiples of the reference frequency. Figures(6.25-6.32) are respectively spectra of the motor phase voltage and line current for the operating conditions of Figures(6.21-6.24). Cancellation of the 3rd harmonic in the 3-phase floating neutral system is obvious and the lower order harmonic components are shown greatly reduced or even eliminated, but high frequency components usually centred at the reference frequency and its multiples are introduced.
Fig. 6.1 FLOW CHART FOR THE INVERTER ANALYSIS.
FIG. 6.2 SIMULATED INVERTER OUTPUT VOLTAGE WAVEFORMS RELATED TO 50 Hz

(a) - SIMULATED INVERTER PHASE VOLTAGE WAVEFORMS

(b) - SIMULATED LINE-LINE INVERTER VOLTAGE WAVEFORMS
(a) SIMULATED INVERTER PHASE VOLTAGE WAVEFORMS

FIG. 6.3 SIMULATED RESULTS FOR 30 Hz OPERATING FREQUENCY

(b) SIMULATED LINE-LINE INVERTER VOLTAGE WAVEFORMS
FIG. 6.4: SIMULATED AND EXPERIMENTAL RESULTS OF THE INVERTER VOLTAGE WAVEFORMS AT 20Hz.
FIG. 6.5. SIMULATED AND EXPERIMENTAL WAVEFORMS OF INV. PHASE VOLTAGE AT 30 Hz.
FIG. 6.6. SIMULATED AND EXPERIMENTAL RESULTS OF THE INVERTER VOLTAGE WAVEFORMS AT 40Hz.
FIG. 6.7. SIMULATED AND EXPERIMENTAL RESULTS OF THE INVERTER VOLTAGE WAVEFORMS AT 50Hz.
FIG6.8: TRANSFORMATION FROM 3-PHASE TO d,q-AXIS
READ IN SYSTEM PARAMETERS AND INITIAL CONDITIONS.

CALCULATE THE INVERSE OF THE INDUCTANCE.

CALCULATE MOTOR PHASE VOLTAGES.

CALL RUNGE-KUTTA NUMERICAL ROUTINE AND OBTAIN NEW VALUES OF CURRENTS AND SPEED.

CALCULATE NEW DEVELOPED TORQUE.

IS THE TIME OF INTEGRATION OVER?

YES

PRINT RESULTS.

NO

STOP

Fig. 6.9. FLOW CHART FOR THE MOTOR ANALYSIS USING STATIONARY 2-AXIS MODEL.
FIG. 6.10 SIMULATED MOTOR VOLTAGE WAVEFORMS AT (a) 50 Hz and (b) 30 Hz OPERATING FREQUENCY
FIG. 6.11 SIMULATED D-Q VOLTAGE WAVEFORMS AT FREQUENCIES OF
(a) 50 Hz and (b) 30 Hz
FIG. 6.12  SIMULATED RESULTS FOR START-UP OF THE MOTOR DRIVE AT
50 Hz OPERATING FREQUENCY
Fig. 6.12 continued

(c) D-axis Stator Current

(d) Q-axis Stator Current
FIG. 6.12 CONTINUED
FIG. 6.13  SIMULATED RESULTS FOR START-UP OF THE MOTOR DRIVE RELATED AT 30 Hz OPERATING FREQUENCY.

(a) - MOTOR PHASE VOLTAGE

(b) - MOTOR PHASE CURRENT
FIG. 6.13 CONTINUED

(a) d-AXIS ROTOR CURRENT

(b) q-AXIS ROTOR CURRENT
FIG. 6.13 CONTINUED

(c) D-AXIS STATOR CURRENT

(d) Q-AXIS STATOR CURRENT
FIG. 6.14 SIMULATED RESULTS FOR START-UP OF THE MOTOR DRIVE AT 20 Hz OPERATING FREQUENCY
FIG. 6.14 CONTINUED
FIG. 6.15 SIMULATED RESULTS FOR START-UP OF THE MOTOR DRIVE AT 20 Hz,
WITH IR-VOLTAGE DROP COMPENSATION.

(a) MOTOR PHASE VOLTAGE.

(b) MOTOR PHASE CURRENT
(c) MOTOR SPEED

(d) MOTOR DEVELOPED TORQUE
WITH IR-COMPENSATION, 1/2 FULL LOAD

FIG. 6.15 CONTINUED
Fig. 6.16. MOTOR DRIVE PERFORMANCE ON SUDDEN APPLICATION AND REMOVAL OF THE LOAD

(a) MOTOR PHASE VOLTAGE.

(b) MOTOR PHASE CURRENT.
FIG. 6.16 CONTINUED

(c) D-AXIS STATOR CURRENT

(d) Q-AXIS STATOR CURRENT
(e) d-AXIS ROTOR CURRENT

(f) q-AXIS ROTOR CURRENT

FIG. 6.16 CONTINUED
FIG. 6.16 CONTINUED
FIG. 6.17 EXPERIMENTAL AND SIMULATED MOTOR INPUT WAVEFORMS AT OPERATING FREQUENCY OF 20 Hz
FIG. 6.18

EXPERIMENTAL AND SIMULATED MOTOR INPUT WAVEFORMS AT OPERATING FREQUENCY OF 30 Hz.
MOTOR PHASE VOLTAGE WAVEFORM AT 40-HZ

SIMULATED MOTOR CURRENT WAVEFORM AT 40-HZ

FIG. 6.19 EXPERIMENTAL AND SIMULATED MOTOR INPUT WAVEFORMS AT OPERATING FREQUENCY OF 40 Hz
FIG. 6.29 EXPERIMENTAL AND SIMULATED MOTOR INPUT WAVEFORMS AT OPERATING FREQUENCY OF 50 Hz

MOTOR PHASE VOLTAGE WAVEFORM AT 50-HZ

SIMULATED MOTOR CURRENT AT M=0.9, f_m=50 Hz
FIG. 6.21  SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF INVERTER PHASE VOLTAGE WAVEFORM AT 50 Hz OPERATING FREQUENCY.
FIG. 6.22  SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF INVERTER PHASE VOLTAGE WAVEFORM 40 Hz OPERATING FREQUENCY
Fig. 6.23  Simulated and experimental harmonic spectrum of inverter phase voltage waveform 30 Hz operating frequency.
FIG. 6.24  SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF INVERTER PHASE VOLTAGE WAVEFORM AT 20 Hz OPERATING FREQUENCY
FIG. 6.25 SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF THE MOTOR PHASE VOLTAGE AT 50 Hz OPERATING FREQUENCY
FIG. 6.26   SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF MOTOR LINE CURRENT WAVEFORM AT 50 Hz OPERATING FREQUENCY
FIG. 6.27 SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF THE MOTOR PHASE VOLTAGE AT 40 Hz FREQUENCY
FIG. 6.28 SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF MOTOR LINE CURRENT WAVEFORM AT 40 Hz OPERATING FREQUENCY
FIG. 6.29  SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF THE MOTOR PHASE VOLTAGE AT 30 Hz FREQUENCY
FIG. 6.30 SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF MOTOR LINE CURRENT WAVEFORM 30 Hz OPERATING FREQUENCY
FIG. 6.31 SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF THE MOTOR PHASE VOLTAGE AT 20 Hz FREQUENCY
FIG. 6.32  SIMULATED AND EXPERIMENTAL HARMONIC SPECTRUM OF MOTOR LINE CURRENT WAVEFORM 20 Hz OPERATING FREQUENCY
CHAPTER 7

CLOSED-LOOP SPEED AND TORQUE CONTROLLED DRIVE

7.1 Control Techniques
7.2 Implementation of Speed and Torque Controller
7.3 System Development
   7.3.1 Speed reference circuit
   7.3.2 Torque regulating circuit
7.4 Experimental Configuration
7.5 Experimental Results
This chapter presents an experimental investigation into the closed-loop operation of the motor drive system described in Chapter 5, in which the motor performance can be substantially improved under lightly-loaded conditions by controlling the applied motor voltage simultaneously with the input frequency.

7.1 Closed-loop Techniques

Stable operation of an induction motor is normally limited to the speed range between maximum torque and synchronous speed. If the slip is constrained and controlled to a value below that corresponding to maximum torque, (Figure 7.1), high efficiency and high power factor operation can be achieved under all load conditions.

Closed-loop techniques for improving the performance of a motor, whose speed is controlled by variation of the output frequency of an inverter, are well established. The choice of the particular scheme depends on the controller, the motor and the characteristics of the load. Typical control methods such as slip control, flux control and phase-locked loop control are widely reported in the literature \(^\text{(68, 69, 70)}\). A number of these take advantage of the availability of a slip-speed signal for further improving the dynamic performance of the drive, i.e. during changes in speed and torque in, for example, flux control schemes using

(a) direct flux sensing

(b) voltage sensing, or

(c) current-slip control.

The last method is that most commonly used, due to the ease of setting of both the controlled motor current and the slip. The required relationship between the current and slip can be most easily obtained by referring to the steady-state characteristic.
7.2 Implementation of Speed and Torque Controller

An induction motor is normally designed to maintain a high efficiency when used close to its full-load condition. As the load reduces the motor efficiency decreases considerably, as the motor losses (particularly the iron losses) become an increasing proportion of the input. A reduction in the motor supply voltage can however lead to more efficient operation, and this principle forms the basis of many power factor controllers and energy saver schemes \(^{(71)}\).

The control scheme presented here is based on maintaining the motor speed constant at the desired value and regulating the motor torque according to the load conditions, in order to improve the performance (a) of a lightly-loaded motor drive, and (b) during starting, when a loaded drive at low speeds requires constant air-gap flux rather than a constant \(V/f\) ratio, because of the predominating influence of the stator winding resistance.

The relationship between motor input frequency, voltage and air-gap flux \(\phi_g\) is given approximately by:

\[
\phi_g = k \frac{V_r}{f_s} \quad (7.1)
\]

where \(V_r = V_s - I_s Z_s\) \quad (7.2)
$V_r$ is the rotor e.m.f. generated per phase when the slip is $s$ and the rotor frequency is $f_r = sf_s$.

$V_s$ is the supply voltage.

$f_s$ is the supply frequency.

$Z_s$ is the stator impedance.

$k$ is a machine constant.

In addition, the motor torque is a function of air-gap flux, given by

$$T_m = k_1\phi_g^2 f_r$$

(7.3)

where $f_r$ is the rotor frequency, and $k_1$ is a constant.

Substituting for $\phi_g$ from equation (7.1) into equation (7.3), gives

$$T_m = k_2\left(\frac{V_s}{f_s}\right)^2 f_r$$

where $k_2$ is a constant

(7.4)

A block diagram of the control scheme is shown in Figure 7.2. The scheme maintains the motor speed constant at all load conditions, by monitoring the shaft speed and regulating the available torque. This requires the load torque to be sensed and summed with a shaped voltage reference signal, derived from the speed-error signal $\omega_{er}$, to produce direct $V/f$ control. If for example the load is increased by $\Delta T_m$, the motor speed will tend to decrease, and the speed-error between the demanded and the actual speed will be increased to provide an increase in the motor stator frequency. However, by the action of the speed
controller, the motor speed is regulated to the demanded value \( \omega_d \).

Due to the increased speed error, a voltage error signal is generated, leading to a consequent increase in terminal voltage. The new V/f ratio leads to an increase in the available torque and to a corresponding decrease in the slip. In this way, a lightly-loaded motor operates under reduced motor terminal voltage conditions. If an increase in load torque is demanded, the rms value of the motor terminal voltage is increased in proportion to the load torque, and hence a family of new improved motor torque-speed curves at any given drive speed is achieved as illustrated in Figure 7.3. It can be seen that the intersection of a load-torque curve \( T_m \) and the motor torque curve determines the point for which the required motor supply conditions are obtained. This leads to an improved motor performance at every load point, especially for light load conditions.

7.3 **System Development**

The following subsections present details of the closed-loop speed and torque controlled drive system.

7.3.1 **Speed reference circuit**

The speed of the motor drive is determined by an external reference signal. By monitoring the speed and comparing this with the reference, a speed-error signal is formed. A simplified diagram of a closed-loop speed control circuit developed for this purpose from Figure 5.3 is shown in Figure 7.4. The circuit has two inputs, the reference speed signal \( V_{\text{ref}} \) and the feedback signal \( V_{\text{TACHO}} \). The speed dependent-feedback signal is initiated by a D.C. tachogenerator coupled to the
motor shaft. After filtering, the tachogenerator voltage is inverted, summed with the adjustable reference value, and supplied to the speed (proportional + integral, PI) controller formed by IC1, so as to produce a controlled speed-error signal. This error signal is added to the speed feedback signal, in order to maintain constant motor speed when the load conditions are changed.

7.3.2 Torque regulating circuit

A circuit developed for the purpose of torque regulation is shown in Figure 7.5. The shaped voltage reference \( V_{\text{REF}} \) derived from the speed-error signal and obtained using the absolute value circuit of Figure 7.6(a) provides a positive output voltage, irrespective of the change in sign of the speed-error signal. The circuit-input/output characteristic is illustrated in Figure 7.6(b), with the minimum output voltage \( V_x \) adjusted by the potentiometer CR5, as necessary for no-load operation, and the maximum value limited by the zener voltage \( V_z \). The torque dependent feedback signal \( V_{\text{set}} \) is obtained from the strain-gauge bridge of Figure 7.7, mounted on a tie-bar connected to the friction brake calipers. The output of the bridge is amplified as shown in Figure 7.7 and the amplifier output is summed with the shaped reference voltage \( V_{\text{REF}} \) given in Figure 7.5, before being fed to the PI-controller. Any load change generates a torque error signal from the PI-controller, which is fed to the VCO circuit, whose output is the clock input frequency \( V_{\text{CT}} \) of the pwm-IC. This in turn controls the inverter output voltage and hence the motor terminal voltage.
7.4 Experimental Configuration

Figures 7.8(a) and (b) show photographs of the experimental drive. The motor is directly coupled to a disc friction brake as shown in Figure 7.8(b), to provide mechanical retardation. The brake structure is hinged, so that the braking force is transmitted to a tie-bar, on which a strain-gauge bridge is mounted to produce a torque dependent feedback signal. The motor speed feedback signal is obtained from the D.C.-Tachogenerator mounted on the motor shaft, shown also in Figure 7.8(b). Figure 7.8(c) shows the complete system, taken when the drive was operating on load; and the corresponding motor phase voltage and line current wave-forms are displayed on the oscilloscope screen.

7.5 Experimental Results

Figure 7.9 shows a number of experimentally obtained torque/speed relationships for the closed-loop drive, and these demonstrate well the function of the control circuit in maintaining the motor speed almost constant when the load conditions are changed. As explained in Section (7.3), the motor voltage is controlled as a function of the load, so that when the motor is lightly loaded it operates at a reduced voltage. This decreases the motor losses and leads to the improved low-load performance at rated speed evident in Figure 7.10. Figure 7.11 presents experimental speed and current waveforms following a step variation in the speed reference signal from 0 to $+10 \text{ V}$. The motor speed changes smoothly from zero to the set value corresponding
to the motor rated speed, in an acceleration time of about 7 s. The motor current is limited to a peak value of just under 1.5 A during acceleration. The drive performance following application and removal of the load is shown in Figures 7.12 and 7.13 respectively, for a 50 Hz operating frequency. After the unloaded motor has achieved steady-state no-load speed, a sudden full-load torque is applied to the motor shaft, which results in the fast speed response shown in Figure 7.12. Figure 7.13 shows results for load rejection. In this case the motor speed tries to rise, but the reduction in the drive frequency and the motor current causes it to be held constant.
FIG. 7.1. CHARACTERISTICS OF CONSTANT SLIP MOTOR DRIVE.
Fig. 7.2. CONTROLLED SPEED & TORQUE DRIVE SYSTEM.
FIG. 7.3  TORQUE-SPEED CURVES BY v/f CONTROL
FIG. 7.4. SPEED CONTROLLER

ICs - 741
Diodes 1 - 7 OA202
- 8 BZX81
C8V3
FIG. 75. TORQUE REGULATING CIRCUIT.
FIG. 7.6  ABSOLUTE VALUE UNIT

(a) Circuit Diagram.

(b) Characteristics
FIG. 7.7 STRAIN GAUGE BRIDGE AND GAIN AMPLIFIER
Fig. 7.8  Photographs showing the combined inverter/induction motor system (a) the inverter, (b) the motor, (c) the complete system
Fig. 7.9  TORQUE–SPEED CHARACTERISTICS UNDER CLOSED LOOP CONTROL SYSTEM
FIG. 7.10. INDICATION OF POWER SAVING: USING REDUCED VOLTAGE.
FIG. 7.11 RECORDED MOTOR SPEED AND CURRENT WAVEFORMS FOLLOWING A 10 V CHANGE IN THE REFERENCE VOLTAGE
FIG. 7.12  RECORDS MOTOR SPEED AND LINE CURRENT WAVEFORMS FOLLOWING THE SUDDEN APPLICATION OF LOAD
FIG. 7.13  RECORDED MOTOR SPEED AND LINE CURRENT WAVEFORMS FOLLOWING REMOVAL OF LOAD

- 1000 r/min
- 500 r/min/cm
- 1.5 A/cm
- 50 cm/min
CHAPTER 8

CLOSED-LOOP SPEED CONTROL USING A MICROCOMPUTER

8.1 Introduction

8.2 Implementation of the Digital PID Algorithm
   8.2.1 Analogue PID
   8.2.2 Digital PID

8.3 Proposed Digital Speed Controller

8.4 System Hardware Developments
   8.4.1 The Microcomputer
   8.4.2 Motor speed monitoring circuit
   8.4.3 Digital output data

8.5 System Software

8.6 Experimental Results
This chapter presents a description of the digital closed-loop speed controller, incorporating a PID algorithm and implemented by a microcomputer. Experimental results which demonstrate the validity of the proposed control system are given and discussed.

8.1 Introduction

Analogue controllers have a number of important disadvantages, amongst which are:

(a) the effect on the system performance of variations in the controller properties,
(b) the properties themselves are hardware based and as such are inconvenient and difficult to change,
(c) any change which is required is expensive to implement.

Not surprisingly, these disadvantages have stimulated work on digital techniques, which has been coupled with a number of advances in semiconductor technology (in particular LS1 technology), and the advent of the microprocessor. In recent years, one important outlet for this activity has been the application of microcomputer to the control of A.C. drives (72-73).

In digital controllers, the hardware is part of the computer and is clearly never changed. The complexity of the control algorithm is unimportant, since it is performed using software and can be as complex as is necessary and implemented at a lower cost than an equivalent analogue arrangement. The principal disadvantage of a digital controller is, however, that the computation is sequential and the control algorithm processing must be stopped and a hold (latch) circuit must be implemented which retain the previous information until the updated results are received. In electromechanical systems, including motors, some of the time constants
are long, possibly several seconds. Computation times however are of the order of milliseconds and for this type of application the use of a microcomputer is justified.

8.2 Implementation of the Digital PID Algorithm

The following subsections will describe implementation of the PID controller in both analogue and digital schemes.

8.2.1 Analogue PID

The general form of an analogue controller is illustrated in block diagram form in Figure 8.1. The input $e_d(t)$ represents the demanded value of the controlled variable $p(t)$. The value of $p(t)$ is sensed and the feedback element produces a voltage signal $e_a(t)$ proportional to $p(t)$, which, when compared with $e_d(t)$, produces an error signal defined as

$$e_r(t) = e_d(t) - e_a(t)$$

(8.1)

This signal, after modification in various ways, produces the signal $e_m(t)$ used to drive the system so as to reduce the error, and to optimise the performance. These modifications may, for example, be employed to provide the demanded response in minimum time, to minimise the steady state error, or to achieve any other required performance.

Figure 8.2 is a block diagram of an analogue PID controller acting on the error signal $e_r(t)$. The voltage signal $e_m(t)$ includes a term $k_p e_r(t)$ proportional to the error signal, where $k_p$ is a proportional gain constant chosen with regard to the properties of the elements in the control system.
Although the output of the P-controller is at all times proportional to the input variable, the controller suffers from the disadvantages of a permanent steady-state error between the demand input and the actual output. However, by the inclusion of an integral term $k_i \int e_r(t) dt$ the error may be eliminated, since the integral term produces a controlling effect which leads to changes in the system output in such a way that the error is eventually reduced to zero.

The derivative term $k_d \frac{de_r(t)}{dt}$ provides an anticipatory action and reduces any overshoot in the response, thereby reducing the maximum difference between the transient and steady-state conditions.

The manipulated variable of a PID-controller includes all three terms, and the equation for a PID analogue controller is thus

$$e_m(t) = k_p e_r(t) + k_i \int e_r(t) dt + k_d \frac{de_r(t)}{dt}$$  \hspace{1cm} (8.2)

The coefficients $k_p$, $k_i$ and $k_d$ are all chosen to obtain the best performance from the controlled system and in general $k_p > k_d > k_i$. The response of a typical system when containing P, PI and PID controllers is illustrated in Figure 8.3. A proportional only controller leads to the sizeable steady-state error evident in response (1). Elimination of this when an integral term is included is shown by response (2). With the further inclusion of the derivative term the overshoot is substantially reduced as shown in curve (3).
8.2.2 Digital PID

The principle of an analogue PID controller, expressed by equation (8.2), may be applied in digital control form and implemented in a number of different ways. As seen in Figure 8.4, the variable to be controlled $P(t)$ is sensed via a transducer, and the feedback signal $e_a(t)$ is sampled at discrete intervals of time $T$ using an analogue-to-digital converter (ADC). As a consequence, the digital system error signal is known only at these discrete times, and is defined by

$$e_{rn} = e_r(nT)$$

(8.3)

where

$$e_r(nT) = e_d(nT) - e_a(nT)$$

and

$$e_d(nT)$$

is the reference value (demand) and

$$e_a(nT)$$

is the feedback value (actual).

The error signal is then applied to the PID controller.

On this basis, equation (8.2) may be expressed in digital form as

$$P_n = k_p e_{rn} + k_i \sum_{r} e_{rn} \Delta T + k_d \frac{\Delta e_{rn}}{\Delta T}$$

(8.4)

where $\Delta T$ is the sampling time interval. For a discrete system, having a block diagram as in Figure 8.4, the corresponding digital PID expression is:

$$e_m(nT) = k_p e_r(nT) + \sum_{k=1}^{n} k_i e_r(kT) + k_d \frac{e_r(nT) - e_r((n-1)T)}{\Delta T}$$

(8.5)

where $e_r(nT)$ is the error at time $nT$ and

$$e_r((n-1)T)$$

is the error at the previous sampling time.
The controller output is converted to an analogue signal using a digital-to-analogue converter (DAC) and maintained by a zero-order-hold until the next sampling instant, to produce a piecewise continuous signal which is summed with the load state for use as the plant control input.

8.3 Proposed Digital Speed Controller

A block diagram for the proposed controller is shown in Figure 8.5. The motor speed is monitored by an analogue D.C.-tachogenerator and this signal is digitized to produce an 8-bit code \(\omega_r(nT)\) approximating to the rotor speed at the instant of sampling. This is compared with the digital reference speed signal \(\omega_d(nT)\), set by the microcomputer keyboard, to give the speed error signal

\[
\omega_{er}(nT) = \omega_d(nT) - \omega_r(nT) \tag{8.6}
\]

which may be modified using the digital PID algorithm described in Section (8.2.2). The output of the controller is compared with a preset slip value corresponding to the maximum torque of the motor. The output error signal \(\omega_{er}(nT)\) is summed with the feedback signal as;

\[
\omega_s(nT) = k_p \omega_{er}(nT) + \sum_{k=1}^{n} \frac{k_i}{\Delta T} \omega_{er}(kT) + \frac{k_d}{\Delta T} \omega_{er}(nT) - \omega_{er}((n-1)T) + \omega_r(nT) \tag{8.7}
\]

The digital output code \(\omega_s(nT)\) of the controller has to be converted to an analogue signal for use as the drive system frequency and voltage control inputs.
8.4 System Hardware Developments

The following subsection describes the system hardware developed for the closed-loop speed controller.

8.4.1 The Microcomputer

The computer used in the proposed control scheme is required to do more than simply compare and implement the speed control algorithm given by equation (8.7). The particular application requires, in addition, specific operational features, such as enabling and disabling of the ADC and DAC and initialisation of the ADC start conversion signal. The microcomputer used was a Commodore PET-32K IBM(74) which consists of three basic parts:

(a) the central processing unit (CPU), a 6502-microprocessor which performs all the necessary arithmetic and logic operations,

(b) memories - a read-only memory (ROM) and a read/write or random access memory (RAM),

(c) the peripheral interface adapter, (6522-Versatile Interface Adaptor (VIP)), and the group of devices that serve as inputs and outputs. These are often referred to as Input/Output (I/O) devices. In order to exchange data between the microcomputer and the drive system, the bi-directional I/O signal lines must be interfaced to the controlled system via eight-bit ADCs and DACs, and these are described in the following subsections. Figures 8.6(a) and (b) respectively are block diagrams of 6502 and 6522.
8.4.2 Motor speed monitoring circuit

The motor speed is sensed by a D.C-tachogenerator, with an output of 2.5 V/(r/min). As Figure 8.7 shows, the tacho-voltage is filtered, using an active filter, and buffered (Figure 8.8(a)), before it is fed to the eight-bit monolithic ADC shown in Figure 8.8(b). The output code generated by the converter represents the motor speed at the sampling time instant. The ADC requires a start-conversion signal (SC) which initiates the conversion process. When the conversion is completed the ADC produces an output control signal EOC indicating the end of the conversion process. At this instant, and at every subsequent sampling instant, the digital output represents the analogue signal present at that input. For correct operation of the ADC, the SC signal (Figure 8.8(b)) requires synchronization to the digital process in the computer. Generation of the required signal is performed by the circuit shown in Figure 8.9, which produces an output pulse coincident with, and of the same duration as, a negative-going clock pulse. Generally, the ADC will be connected directly to the computer bus. In these circumstances it is usual to have tri-state output circuits, so that when the ADC is active there is no loading of the system bus, but when the computer requests data from the converter the output circuits change to a low impedance state in order to drive an appropriate digital pattern into the system bus. The output enable signal is provided for these purposes.

8.4.3 Digital output data

After the output data from the microcomputer is read from the data-bus at the I/O user port, it is interfaced to the drive system using a circuit
incorporating an eight-bit DAC. The latch action is controlled by an ENABLE input signal, which is provided by one of the microcomputer control signals. When the ENABLE signal is held low, the data input drives the device directly. Otherwise, the input data is held in the data latch and the output remains unaffected by the state of the data-bus. In this condition, the DAC appears transparent to the microcomputer. The circuit arrangement which provides this data interfacing is shown in Figure 8.10, where IC2 is included to provide both amplification and a degree of isolation for the DAC. The circuit uses a 741 operational amplifier. Gain control is provided by VR2. The DC supplies for both interfacing units are provided by a +5V regulator IC4, and a ±15V encapsulated DC-DC converter module, IC3. This configuration is illustrated in Figure 8.10.

8.5 System Software

The software required for the control system has to perform the following functions:

(a) programme the microcomputer Input/Output port to be either an input or an output
(b) input the speed demand, sample the motor speed and calculate the speed error signal.
(c) perform the digital computation to implement a PID-controller acting on the speed-error signal that controls the motor speed.
(d) produce control signals for enabling and disabling both the ADC and DAC and for controlling the ADC conversion process.
The overall aim of the system software is to bring the motor speed to the desired value during starting, to maintain it constant against load changes and to ensure stability of the drive system under all operating conditions. The source program is written entirely in low level assembly language to ensure speedy processing. A simplified software flowchart is shown in Figure 8.11, with the program listing being given in Appendix D. The software enables the microcomputer to read the speed reference signal \( \omega_d(nT) \) demanded from the keyboard as a number ranging between 0 and 1000 r/min. Using an eight-bit system, a speed resolution of about 4 r/min is obtained. The digitized motor speed-dependent signal \( \omega_r(nT) \) is read into the microcomputer from the output of the ADC. This has an integral value between 0 and 255 representing the motor speed. The conversion time for the ADC and the subsequent processing time produces a sampling time \( T \) of approximately 10 ms. The digital error-signal \( \omega_{er}(nT) = \omega_d(nT) - \omega_r(nT) \) forms the input to be manipulated using the PID control algorithm given by equation (8.7). The microcomputer then performs the PID controller calculation.

The sampling time is synchronized with the ADC conversion process, which is about 10 ms in this case. The digital output of the PID controller is an eight-bit number, whose magnitude depends on the size of the proportional, integral and derivative constants. This number must not exceed a pre-set value corresponding to the maximum torque of the motor.

The demanded speed signal is presented to the controller at all times. However the ADC only carries out a conversion of the motor speed feedback signal \( \omega_r(nT) \) once within each cycle, so that the ADC includes output
latches to hold the \( \omega_r(nT) \) signal until the next sampling time. At an appropriate time, the numbers \( \omega_d(nT) \) and \( \omega_r(nT) \) are read and the digital processing may begin.

8.6 Experimental Results

Figure 8.12 shows a photograph of the experimental closed-loop speed control drive system which was subjected to a series of practical tests. The demanded speed and the controller coefficients were entered from the computer keyboard and the controller coefficients were carefully selected to meet the required performance. Figures 8.13(a) and (b) show experimental recordings of the motor speed control during start-up with proportional (P) control only. These results were obtained for a desired speed of 500 r/min and proportional gains respectively of 2.0 and 3.0, all values being set up via the keyboard. Figure 8.13(a) shows that a small overshoot of speed occurred and that a small steady-state error of about 120 r/min is obtained. Figure 8.13(b) shows the system speed response when the gain is increased to 3.0. The steady-state error has now been reduced as expected and is about 75 r/min, but the starting time is increased from 2 s in Figure 8.13(a) to 2.5 s in Figure 8.13(b).

All controllers with a proportional term only suffer from the defect of a steady-state error. With the inclusion of the integral term, this error is eliminated and no overshoot appears, as recorded in Figure 8.14. The proportional and integral gains here are respectively 1.0 and 0.6 and the starting time is about 3 s.
The results given in Figure 8.14 confirm that the system response does not over-shoot, so that the derivative action can be omitted from the control algorithm process without affecting the system performance. In the final form a controller with only the two terms (P+I) was preferred to the three term PID controller. Speed and current waveforms illustrating the drive system response (with PI controller) following the application and rejection of load were obtained experimentally. The results presented in Figure 8.15 for a desired speed of 1000 r/min, (with $K_p = 1.0$ and $k_i = 0.6$), clearly show that the starting time is about 6 s. The motor speed during the steady-state duration is maintained constant, with only a very short transient speed change following a change in the load conditions.
FIG. 8.1. ANALOGUE CONTROLLER

FIG. 8.2 THE ANALOGUE PID CONTROLLER
FIG. 8.3. TYPICAL SYSTEM RESPONSE TO THE 3-TERM CONTROLLER.
(1) P (2) PI (3) PID
FIG. 8.4 DIGITAL PID CONTROLLER
Fig. 8.5  PROPOSED DIGITAL SPEED CONTROLLER.
Fig 8.6(a): 6502 Block Diagram
Fig 8.6 (b) 8522 Block Diagram
Fig 8.7. Low Pass Filter
Fig. 3.8. A D C CIRCUIT DIAGRAM SHOWING INPUT AND OUTPUT SIGNALS.
Fig. 8.9. TIMING CIRCUIT FOR THE ZM427 ADC.

6.2 MHz crystal oscillator

CE2 signal from the Minicomputer I/O port

R = 330 Ω
C = 220 pF
Fig. 8.10.

OUTPUT SECTION OF THE SPEED CONTROLLER.
READ SET VALUE AND PID CONTROLLER CONSTANTS.

ENABLE A/D. CONVERTER.

INITIALISE THE CONVERSION PROCESS AND INTEGRATING TIME AND SAMPLE THE MOTOR SPEED.

PRODUCE THE ERROR SIGNAL
\[ \omega_{e}(nT) = \omega_{d}(nT) - \omega_{f}(nT) \]

DO THE CONTROL CALCULATIONS (PID)

\[
\begin{align*}
\omega_{e}(nT) &> \text{SLIP LIMIT} \\
\omega_{s}(nT) & = \omega_{e}(nT) + \omega_{r}(nT)
\end{align*}
\]

Fig. 8.11. SIMPLIFIED FLOWCHART.
Fig. 8.11. CONTINUED.
FIG. 8.12. EXPERIMENTAL SET-UP
FIG. 8.13. EXPERIMENTAL MOTOR DRIVE SPEED RESPONSE TO 5000/min FROM THE KEYBOARD - P CONTROLLER.
Fig 8.14. SYSTEM RESPONSE WITH PI-CONTROLLER.
FIG. 8.15. MOTOR SPEED AND CURRENT WAVEFORMS FOR SUDDEN APPLICATION AND REJECTION OF LOAD
CHAPTER 9

CONCLUSIONS

9.1 Conclusions and Remarks

(a) The experimental and theoretical investigation of the inverter/induction motor drive developed in the research work, and in particular the results presented in Chapters 4, 5 and 6, show clearly the following:

(i) the regular switching strategy is very effective and offers the advantage of relatively high-quality inverter output waveforms and consequently a significant reduction in harmonic content.

(ii) since the operating frequency and fundamental output voltage are both obtained by straightforward inverter switching, the system is inherently extremely flexible. It is easy to introduce modifications leading to an improved low-speed performance, rapid speed reversal and closed-loop operation as demonstrated in Chapters 5, 6 and 7 respectively.

(b) The use of GTO thyristors as power switches in the inverter greatly simplifies the power circuit. It eliminates the need for forced commutation and results in a more compact and lighter unit than when using conventional thyristors.

(c) The use of the d,q model in conjunction with the regular switching strategy model, greatly simplifies the analysis of the combined inverter drive and enables accurate predictions to be made of the current waveforms and their harmonic contents. The simulated
dynamic performance of the drive operating at frequencies of 50, 30
and 20 Hz was presented in Chapter 6. These results immediately
suggest that at a frequency of 20 Hz, 12% voltage drop compensation is
needed to achieve the same maximum torque as at 50 Hz.

The simulated results presented in Chapter 6 for inverter output
voltage and current waveforms, and their harmonic contents were
obtained for operating frequencies of 20, 30, 40 and 50 Hz. These
results were confirmed by experimentally obtained results, and the
good agreement obtained established the validity of the combined
system model developed.

(d) the design of a closed-loop controller using speed and torque feedback
was presented in Chapter 7. Experimental results for this drive are
also presented in Chapter 7, and demonstrate well the function of
the control circuit in

(i) improved motor performance at every load intersection point
    with the motor torque curve, especially for light load conditions,
    and

(ii) maintaining the motor speed almost constant when the load
    conditions are changed.

(e) One advantage of a microcomputer, when used as the supervisory element
in a control scheme, is that it enables the efficiency of any proposed
scheme to be rapidly evaluated. The design of a digital closed-loop
speed control scheme using a microcomputer is presented in Chapter 8.
In this, the hardwire logic circuitry presented in Chapter 7 is replaced by a proportional and integral control algorithm implemented by the microcomputer for the speed control of the drive. Experimental results showed well the function of the speed controller in controlling the motor speed during start-up and in maintaining constant motor speed following a change in the load condition.

9.2 Suggestions for Further Work

Since digital control can be advantageously used for motor drives, it is anticipated that microcomputer and microprocessor control systems will be widely used for more sophisticated control of electrical drives in the near future. A microcomputer control system suitable for multi motor control schemes is recommended.
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Inverter d.c. supply voltage

The maximum r.m.s. voltage that the inverter can provide to the motor is determined by the mains supply voltage. In general, a motor may be used which has a rated voltage equal to or less than the mains supply voltage. For an A.C. supply voltage of $V_{ac}$ line-to-line, the average D.C. voltage of the 3-phase uncontrolled rectifier is

$$V_{dc}(nom) = \frac{3\sqrt{2}}{\pi} V_{ac}$$

where $V_{dc}(nom)$ is the highest continuous value of D.C. supply voltage.

Assuming a 3-phase mains supply of 420 V ($\pm$ 10%), then when rectified, the nominal continuous D.C. supply voltage is 570 V ($\pm$ 10%).

When the pwm inverter is supplied at $V_{dc}(nom)$, it gives a maximum output fundamental rms line voltage of

$$V_o(line) = \frac{\sqrt{6}}{\pi} V_{dc}(nom) = 444 \text{ V}$$

and a maximum fundamental rms phase to neutral (motor phase) voltage of

$$V_o(phase) = \frac{\sqrt{2}}{\pi} V_{dc}(nom) = 250 \text{ V}$$
Motor specification

1 hp, 6-pole, 50 Hz, 380/420 V (for star connection).

moment of inertia

\[ R_s, R_r = 5.09 \, \Omega \]

\[ L_s, L_r = 0.499 \, H \]

\[ L_{sm}, L_{rm} = -0.233 \, H \]

\[ L_{sl}, L_{rl} = 0.034 \, H \]

\[ L_m = 0.697 \, H \]

\[ M_{sr} = 0.465 \, H \]

All values are referred to the stator.

J = 0.045 kg.m\(^2\)

resistance per phase of the stator and rotor circuits, respectively.

Self inductance per-phase of the stator and rotor circuits, respectively.

Mutual inductance between stator phase and rotor phase respectively.

Leakage inductance per-phase of the stator and rotor circuits, respectively.

Magnetising inductance

Maximum mutual inductance between stator and rotor circuits.
APPENDIX C

Computer program listing for the combined system
PARAMETER (NI=100)
INTEGER IA,N,NN,IUNIT,IFAIL
INTEGER IW1,IW2
INTEGER I,J,IK

INTEGER DL
REAL*8 VA(NI), VB(NI), VC(NI), VP(3), VT(NI,3)
REAL*8 JR, IPP, CFF, PI, W, R120
REAL*8 X01AAF
REAL*8 RS, RR, LS, LR, LSS, LRR, LM, LSM, LRM, MSR
REAL*8 A(4,4)
REAL*8 T
REAL*8 FM, TMAX
REAL*8 H
REAL*8 U(6,7)
REAL*8 UNIT(4,4), WKSPCE(7)
REAL*8 F(6), Y(6)
REAL*8 RP(4,4), G(4,4)
REAL TIME(NI)

REAL YVAL(NI)
REAL*8 VMAX, TORQ, TORQM
REAL*8 AT(NI), YRES(NI,6), TMARR(NI)
COMMON/BLK1/UNIT, RP, G, VMAX, W, JR, PI
COMMON/BLK2/CFF, IPP, TORQ, TORQM, R120
COMMON/BLK3/VA, VB, VC
COMMON/BLK4/AT, TMARR
COMMON/BLK5/VT
COMMON/BLK8/YRES
EQUIVALEN E (Y(1), THETA), (Y(2), SPEED), (Y(3), IP(1))

SUBROUTINE REFERENCES
D02YAF, FCN, F01AAF
FO1AA(A, IA, NN, UNIT, WKSPCE, IFAIL)
D02YAF(X, H, N, Y, FCN, U, IW1, IW2)

EXTERNAL FCN

CALL GINO
CALL T4010
call piccle
call movto2(0.0,0.0)
PI=4.0*ATAN(1.0)
R120=2.0*PI/3.0
TMAX=.650
DL=100
H=TMAX/DL
DO 10 I=1,6
    Y(I)=0.0
10  IW1=6
    IW2=7
    N=6
    IA=4
    IUNIT=4
IFAIL=0
NN=4

C INPUT DATA
C ============
RS=5.09
RR=5.09
LS=0.034
LR=0.034
LSS=0.499
LRR=0.499
LSM=-0.233
LRM=-0.233
LM=0.697
MSR=0.465
VMAX=314.173
FM=20.0
W=2.0*PI*FM
IPP=3.0
JR=0.045
CFF=0.0015
TORQM=0.0

C DO 40 I=1,4
DO 40 J=1,4
40 RP(I,J)=RS
WRITE(*,700)
700 FORMAT(/'RPC1        RP(1)        RP(2)        RP(3)        RP(4)'/) WRITE(*,700)((RP(I,J),J=1,4),I=1,4)
701 FORMAT(3X,4F10.4)

C DO 49 I=1,7
49 WKSPCE(I)=0.0

C A(1,1)=LSS-LSM
A(1,2)=0.0
A(1,3)=1.5*MSR
A(1,4)=0.0
A(2,1)=0.0
A(2,2)=LSS-LSM
A(2,3)=0.0
A(2,4)=1.5*MSR
A(3,1)=1.5*MSR
A(3,2)=0.0
A(3,3)=LRR-LRM
A(3,4)=0.0
A(4,1)=0.0
A(4,2)=1.5*MSR
A(4,3)=0.0
A(4,4)=LRR-LRM

C C WRITE(*,130)
C130 FORMAT(/' A(1)  A(2)  A(3)  A(4)'/) WRITE(*,130)((A(I,J),J=1,4),I=1,4)
C150 FORMAT(3X,4F10.4)
CALL FO1AAF(A, IA, NN, UNIT, IUNIT, WKSPCE, IFAIL)
IF (IFAIL.EQ.0) GO TO 20

C 20 WRITE(*,120)
120  FORMAT(/'   UNIT(1)    UNIT(2)    UNIT(3)   UNIT(4)' /)
     WRITE(*,140)((UNIT(I,J),J=1,4),I=1,4)
140  FORMAT(3X,4F10.4)
     G(1,1)=0.0
     G(1,2)=0.0
     G(1,3)=0.0
     G(1,4)=0.0
     G(2,1)=0.0
     G(2,2)=0.0
     G(2,3)=0.0
     G(2,4)=0.0
     G(3,1)=0.0
     G(3,2)=1.5*MSR
     G(3,3)=0.0
     G(3,4)=(LRR-LRM)
     G(4,1)=-1.5*MSR
     G(4,2)=0.0
     G(4,3)=-(LRR-LRM)
     G(4,4)=0.0
     C  WRITE(*,197)
C197  FORMAT(/'   G(1)    G(2)    G(3)    G(4)' /)
     C  WRITE(*,199)((G(I,J),J=1,4),I=1,4)
C199  FORMAT(3X,4F10.4)
     C
     T=0.0
     IK=0
     DO 100 IK=1,DL
     CALL FCN(T, Y, F)
     DO 201 I=1,6
     U(I,1)=F(I)
     CALL D02YAF(T, H, 6, Y, FCN, U, IW1, IW2)
     TORQ=1.5*1.5*IPP*MSR*(Y(4)*Y(5)-Y(3)*Y(6))
     AT(IK) = T
     TMARR(IK) = TORQ
     YRES(IK,1) = Y(1)
     YRES(IK,2) = Y(2)*(10.0/PI)
     YRES(IK,3) = Y(3)
     YRES(IK,4) = Y(4)
     YRES(IK,5) = Y(5)
     YRES(IK,6) = Y(6)
     CALL PWM(T,VP)
     VT(IK,1)=VP(1)
     VT(IK,2)=VP(2)
     VT(IK,3)=VP(3)
     T=T+H
     C  WRITE(*,51) T,TORQ,Y(1), Y(2), Y(3), Y(4), Y(5), Y(6)
     C  WRITE(*,51) T,Y(4)
C51  FORMAT(8(3X,E12.5))
     IF(MOD(IK,50).EQ.0)THEN
       PRINT*,IK,' POINTS HAVE BEEN CALCULATED SO FAR'
     ENDIF
     IF(MOD(IK,100).EQ.0)WRITE(*,*))''
100  CONTINUE
     CALL DBLSNG(TIME,AT,NI)
     CALL DBLSNG(YVAL,VT(1,1),NI)
     CALL GRAPHS(TIME,YVAL,NI,1)
     CALL DBLSNG(YVAL, VT(1,2), NI)
     CALL GRAPHS(TIME,YVAL,NI,1)
     CALL DBLSNG(YVAL, VT(1,3), NI)
CALL GRAPHS(TIME,YVAL,NI,1)
   CALL DEVEND
   STOP
   END

SUBROUTINE FCN(T, Y, F)
   INTEGER I,J
   REAL*8 JR,IPP,PI,CFF,W,R120
   REAL*8 T
   REAL*8 RP(4,4),G(4,4),UNIT(4,4),VP(3),VDQ(4)
   REAL*8 FF(4),F(6),Y(6)
   REAL*8 VMAX,TORQ,TORQM
   COMMON/BLK1/UNIT,RP,G,VMAX,W,JR,PI
   COMMON/BLK2/CFF,IPP,TORQ,TORQM,R120

   CALL PWM(T,VP)
   VDQ(1)=(2.0*VP(1)-VP(2)-VP(3))/3.0
   VDQ(2)=(VP(2)-VP(3))/SQRT(3.0)
   VDQ(3)=.0
   VDQ(4)=.0
   F(1)=Y(2)
   F(2)=(TORQ-TORQM-CFF*Y(2))/JR*IPP
   DO 220 I=1,4
    FF(I)=VDQ(I)
   DO 220 J=1,4

   220  FF(I)=FF(I)-Y(J+2)*RP(I,J)-Y(J+2)*G(I,J)*Y(2)
   DO 30 I=1,4
    F(I+2)=0.0
   DO 30 J=1,4

   30  F(I+2)=F(I+2)+UNIT(I,J)*FF(J)
   RETURN
   END

SUBROUTINE PWM
SUBROUTINE PWM(T,VP)

RT IS THE FREQUENCY RATIO
M THE MODULATION DEPTH
FM MODULATING FREQUENCY

PARAMETER (NI=100)
REAL*8 ALFA(3000)
REAL*8 AT(NI),VA(NI),VB(NI),VC(NI),VP(3)
C REAL*8 Va(NI),Vb(NI),Vc(NI)
REAL*8 T,M,K,V,IU,TN,D
COMMON/BLK3/VA,VB,VC
PI=4.0*ATAN(1.0)
FM=50.0
R120=2.0*PI/3.0
RT=21
M=FM*0.018
K=PI/RT
V=540.00/2
WM=2.0*PI*FM
DO 999 KI=1,3
WRITE(*,55)
55 FORMAT(/' ALFA(2*J-1) ALFA(2*J)'/)
DO 10 J=1,RT
  IF (KI.EQ.1) GO TO 100
  IF (KI.EQ.2) GO TO 200
  IF (KI.EQ.3) GO TO 300
100 ALFA(2*J-1)= (K/2.0)*(4*J-3-M*SINC(2.0*J-1)*K))
  ALFA(2*J)= (K/2.0)*(4*J-1+M*SINC((2.0*J-1)*K))
  GOTO 10
200 ALFA(2*J-1)= (K/2.0)*(4*J-3-M*SINC((2.0*J-1)*K-2.0*PI/3.0))
  ALFA(2*J)= (K/2.0)*(4*J-1+M*SINC((2.0*J-1)*K-2.0*PI/3.0))
  GOTO 10
300 ALFA(2*J-1)= (K/2.0)*(4*J-3-M*SINC((2.0*J-1)*K+2.0*PI/3.0))
  ALFA(2*J)= (K/2.0)*(4*J-1+M*SINC((2.0*J-1)*K+2.0*PI/3.0))
C
11 WRITE(*,90) ALFA(2*J-1),ALFA(2*J)
90 FORMAT(2(F10.4,2X))
C
10 CONTINUE
C
IU=-V
DO 30 J=1,RT
  IF(ALFA(2*J-1).LE.D.AND.D.LT.ALFA(2*J)) GO TO 35
30 CONTINUE
  GO TO 40
35 IU=V
40 CONTINUE
C
ASSIGN CALCULATED VALUES TO ARRAYS
  IF(KI.EQ.1) VA(IK)=IU
  IF(KI.EQ.2) VB(IK)=IU
  IF(KI.EQ.3) VC(IK)=IU
999 CONTINUE
C
RETURN
C
END
C
C***********************************************************************
C
C SUBROUTINE GRAPHS(X, Y, NPTS,DEV)
INTEGER DEV
INTEGER LENT, STLENG
DIMENSION X(NPTS), Y(NPTS)
CHARACTER *60 YTITLE, GTITLE

DO 901 I = 1, NPTS
  PRINT *,I,X(I), Y(I)
901 CONTINUE
C
PRINT *,,' '
PRINT *,,' '
PRINT *,,' Input the TITLE of the graph'
READ 1,GTITLE
PRINT *,,' ',GTITLE
PRINT *,,' '
PRINT *,,' '
C
PRINT *, 'Input the Y axis title'
READ 1, YTITLE
PRINT *, 'Input '
FORMAT(A)

IF(DEV .EQ. 2.0 .OR. DEV .EQ. 3) CALL DEVPAP(297., 210., 0)
IF(DEV .EQ. 4) INK = 0
IF(DEV .NE. 4) INK = 1

CALL PAPENQ(XPAP, YPAP, IPAPTY)
XS = XPAP/297.
YS = YPAP/210.
CHSX = 3.0*XS
CHSY = 3.0*YS

CALL WINDOW(2)

CALL PENSEL(INK, 0, 0)
CALL CHASIZ(CHSX, CHSY)

C Set axis parameters

XLEN = 200.0
YLEN = 100.0
XO = 65.0
YO = 50.0
NINTSX = 10
NINTSY = 5

XMIN = X(1)
XMAX = X(NPTS)
YMAX=0.0
YMIN=0.0

DO 5 I=1, NPTS
  IF(Y(I) .GT. YMAX) YMAX=Y(I)
  IF(Y(I) .LT. YMIN) YMIN=Y(I)
5 CONTINUE

YMAX = 1.05 * YMAX
YMIN = 1.05 * YMIN

CALL PICCLE
CALL WINDOW(2)
ICURX = 1
ICURY = 2
  CALL MOVTO2(0.0, 0.0)
  CALL LINTO2(0.0, 210.*YS)
  CALL LINTO2(297.*XS, 210.*YS)
  CALL LINTO2(297.*XS, 0.0)
  CALL LINTO2(0.0, 0.0)

CALL AXIPOS(1, XO* XS, YO* YS, XLEN * XS, ICURX)
CALL AXISCA(1, NINTSX, XMIN, XMAX, ICURX)
CALL AXIDRA(0, 0, ICURX)
CALL AXIPOS(1, XO* XS, YO* YS, YLEN * YS, ICURX)
CALL AXISCA(1, NINTSY, YMIN, YMAX, ICURY)
CALL AXIDRA(0, 0, ICURY)
CALL GRID(-2, 1, 1)

CALL GRAPOL(X, Y, NPTS)
XX = XO+XLEN-48.0
CALL MOVTO2(XX*XS, 40.*YS)
CALL CHAOL('*UT*LIME SEC *.')
XL = ((XO ) - 15.0 ) * XS
NN = STLENG(YTITLE)
X1 = (YLEN/2.0) + YO
X2 = (NN/2.0)*3.0
YL = X1 - X2
CALL MOVTO2(XL, YL*YS)
YL = YL*YS
CALL CHAANG(90.0)
CALL MOVTO2(XL, YL)
LENT = STLENG(YTITLE)
YTITLE(LENT+1:) = '*.'
CALL CHAOL(YTITLE)
YTITLE(LENT+1:) = '.
CALL CHAANG(0.0)
CALL PTITLE(GTITLE, XS, YS, XO, YO, XLEN)

CALL CHAMOD
CALL MOVTO2(0.0, 0.0)
CALL PICCLE
PRINT *, ' >..'
READ(*,*)
CALL MOVTO2(0., 150.*YS)
CALL CHAMOD
RETURN
END

C***************************************************************
C FUNCTION RETURNS THE LENGTH OF THE STRING
C

INTEGER FUNCTION STLENG(A)
INTRINSIC LEN
CHARACTER * 60 A
INTEGER N,I,LENGTH

N=LEN(A)
DO 10,I=N,1,-1
   LENGTH=I
   IF(A(I:I).NE.' ') GOTO 20
10 CONTINUE
20 STLENG=LENGTH
RETURN
END

C***************************************************************
C Subroutine to plot TITLE

SUBROUTINE PTITLE(TITLE, XS, YS, XO, YO,XLEN)
CHARACTER *60 TITLE
REAL XS, YS
INTEGER LENT, STLENG

NN = STLENG(TITLE)
X1 = (XLEN/2.0) + XO
X2 = (NN/2.0)*3.0
XL = X1 - X2
CALL MOVTO2(XL*XS, 25.0*YS)
LENT = STLENG(TITLE)
TITLE(LENT+1:) = '*.'
CALL CHAHOL(TITLE)
TITLE(LENT+1:) = ''
CALL CHAMOD
CALL MOVTO2(0.0, 0.0)
RETURN
END
SUBROUTINE DBLSNG(X,Y,NPTS)
REAL X(NPTS)
REAL *8 Y(NPTS)
DO 5 I=1,NPTS
X(I) = SNGL(Y(I))
5 CONTINUE
RETURN
END
C INVENTER OUTPUT WAVEFORMS
C HARMONIC ANALYSIS

C LISTING OF SYSTEM
C SUBROUTINE CO6EAF TO CALCULATE THE HARMONIC ORDER
CHARACTER INFIL*128,OUTFIL*128
INTEGER IFAIL,J,N2,N,NJ,M
INTEGER TITLE(20)
REAL *8 A(11000),B(11000),X(11000)
COMMON/BLK1/Y,H,YA
COMMON/BLK2/A,B,X
DIMENSION Y(10010),HA(1000),YN(10010)
PRINT*
PRINT*, 'PLEASE ENTER NO. OF POINTS N AND NO. HARM. M'
READ(1,*)N,M
PRINT*
PRINT*, 'ENTER NAME OF INPUT FILE ','READ(1,*)INFIL
OPEN(5,FILE=INFIL,STATUS = 'OLD')
PRINT*, 'ENTER THE OUTPUT FILENAME',
READ(1,*)OINFIL
OPEN(6,FILE=OINFIL,STATUS = 'NEW')
IF (IN.LE.1) STOP
READ(5,*) (X(J),J = 1,N)
CLOSE(5)
IFAIL=0
CALL CO6EAF (X,N,IFAIL)
A(1)=X(1)
B(1)=0.0
N2=(N+1)/2
DO 60 J=2,N2
NJ=N-J+2
A(J)=X(J)
A(NJ)=X(J)
B(J)=X(NJ)
B(NJ)=-X(NJ)
60 CONTINUE
NMAX = NJ
IF (MOD(N,2).NE.0) GO TO 80
A(N2+1)=X(N2+1)
B(N2+1)=0.0
NMAX = N2 + 1
80 CONTINUE
DO 70 L=2,NMAX
J=L-1
Y(L)=DSQRT(A(L)**2+B(L)**2)
C H=1/DSQRT(1+(F*J/4200.0)**(2*6))
C Y(L)=H*Y(L)
C Y(NL)=DSQRT(A(NL)**2+B(NL)**2)
YN(L)=Y(L)/Y(2)*100
C YN(L)=Y(NL)/Y(2)*100
70 CONTINUE
C PRINT*, 'ENTER NAME OF OUTPUT FILE ',
C READ(1,*)OINFIL
C OPEN(6,FILE=OINFIL,STATUS = 'NEW')
C NORDER = MIN(NMAX,M)
DO 100 J=2,NORDER
HA(J)=J-1
100 CONTINUE
```fortran
WRITE(6,99994) HA(J),Y(J)
100 CONTINUE
C CLOSE(6)
99994 FORMAT (2X,2F10.5)
NPTS = NORDER - 1
CALL PLOT(HA(2),Y(2),NPTS)
STOP
END
C
SUBROUTINE PLOT(HA,YN,NPTS)
PARAMETER (NWORDS = 40, NFORM = 2)
REAL HA(NPTS),YN(NPTS),X(101),YNP(101)
INTEGER TITLE(NWORDS)
C HARMONIC ORDER PLOT
C
VYMAX = YMAX(YN,NPTS)
DO 10 I = 1,NPTS
X(I) = 0.0
YP(I) = YN(I)/VYMAX*100.0
10 CONTINUE
C
PRINT 150
150 FORMAT ('1*T4010 2*C1051N ', )
READ (*,*) KP
PRINT 1
1 FORMAT ('INPUT FREQUENCY: ')
READ (*,55) TITLE
55 FORMAT (40A2)
IF (KP.EQ.1) CALL T4010
IF (KP.EQ.2) CALL C1051N
CALL ERRMAX(100)
CALL DEVPAK (297.,300.,1)
CALL PICCLE
CALL WINDOW (2)
CALL MOVTO2 (35.0,225.0)
CALL CHASIZ (10.0,10.0)
CALL CHAHOL ('HARMONIC ANALYSIS*.' )
CALL CHASIZ (2.0,2.0)
C DRAW LINE
CALL MOVTO2 (45.0,220.0)
CALL LIINTO2 (270.0,220.0)
CALL CHASIZ (2.,2.)
CALL AXIPOS (1,45.0,40.0,100.0,2)
CALL AXIPOS (1,45.0,40.0,100.0,1)
CALL AXISCA (5,NPTS,1.0,REAL(NPTS),1)
CALL AXISCA (1,10,0.0,YMAX(YNP,NPTS),2)
CALL AXIDRA (1,1,1)
CALL AXIDRA (-2,-1,2)
CALL GRABAX(X,YNP,NPTS,0.0)
C
CALL MOVTO2 (120.,32.0)
CALL CHAHOL ('HARMONIC ORDER*.' )
CALL MOVTO2 (37.0,74.0)
CALL CHAANG (90.0)
CALL CHAHOL ('AMPLITUDE(%)*.' )
CALL CHAANG (0.0)
CALL CHASIZ (4.,4.)
CALL MOVTO2 (120.0,20.0)
```
CALL CHAARR(TITLE,NWORDS,NFORM)
C
DRAW LINE
CALL MOVT02 (45.0,10.0)
CALL LINT02 (265.0,10.0)
CALL DEVEND
RETURN
END

C
C
REAL FUNCTION YMAX(Y,NPTS)
C
THIS FUNCTION RETURNS THE MAX VALUE IN ARRAY Y
C
REAL Y(NPTS)
C
YMAX = Y(1)
DO 10 I = 2,NPTS
   YMAX = MAX(YMAX,Y(I))
10 CONTINUE
RETURN
END
APPENDIX D

Listing of microcomputer software
100 ST=26000
110 RN=ST REQUIRED SPEED
120 KP=ST+1 PROPORTIONAL GAIN
130 KI=ST+2 INTEGRAL GAIN
140 KD=ST+3 DERIVATIVE GAIN
150 AKI=ST+4 ADDITIONAL DIVISOR FOR INTEGRAL TERM
160 AKD=ST+5 ADDITIONAL DIVISOR FOR DERIV TERM
170 SE=ST+6 SIGN OF ERROR TERM RN-CN
180 E=ST+7 ERROR TERM RN-CN
190 SD=ST+8 SIGN OF DERIV TERM
200 D=ST+9 DERIV TERM CN-1 -CN
210 DSLO=ST+11 PID RESULT LOW BYTE
220 DSHI=ST+12 PID RESULT HIGH BYTE
230 SPID=ST+12 SIGN OF PID RESULT
240 MS=ST+14 SIGN OF M2B
250 MHI=ST+15 HI BYTE OF MULT
260 MLO=ST+17 LO BYTE OF MULT
280 TEMP=ST+18
290 NDS=ST+19 NEW DEMANDED SPEED; SPEED THIS SAMPLE; BEFORE THIS HOLDS NDS FOR
300 ; PREVIOUS SAMPLE
320 CI=ST+21 CN-1 PREVIOUS SPEED
330 CH=ST+22 CN ACT SPEED NOW T=TH
340 DIVR=ST+10 NO OF SHIFTS TO REDUCE GAIN INPUT TO MAX GAIN ALLOWED
350 IS=ST+23 SIGN OF INTEGRAL TERM
360 INTHI=ST+24 HIGH OF INTEGRAL TERM
370 INTLO=ST+25 LOW BYTE OF INTEGRAL TERM
390 DDRA=59459
400 IORA=59471
410 PCR=59468
420 PBS=59456
430 IFR=59469
440 **=26000
450 BYT 0,1,2,3,4,5,6,7,8,9,0,1,2,3,4,5,6,7,8
455 ; THE SYSTEM CONSTANTS ARE STORED
456 ; IN THE BYTES ABOVE
460 PID JSR FORMT GETS SPEED DIFFER-
465 ; -ENCES AND SIGNS
470 LDA #0
480 STA DSLO Initialise SUM
490 STA DSHI
500 STA SPID
510 ;ROUTINE TO PRODUCE INTEGRAL TERM
520 ;REGISTERS INTHI,INLO,IS TO BE
530 ;INITIALISED AT START
540 ;INTEGRAL TERM=SIGMA KI*ERROR/aki
550 ;SIGN IN IS
552 LDA KI
554 BEQ DER1
560 LDA SE
570 STA MS
580 LDA E
590 STA MHI
600 LDA KI
610 STA MLO
620 JSR M2B
630 I2 LSR AKI SHIFTS FOR DIVISION
640 BEQ I3 NO MORE SHIFTS REQU'D
650 LDA SE
660 BNE E1
670 CLC SE +VE THUS FEED 0'S
680 ROR MHI
690 ROR MLO
700 CLV
710 BVC I2
720 I1 SEC SE -VE THUS FEED 1'S
730 ROR MHI
740 ROR MLO
750 CLV
760 BVC I2
770 I3 CLC ADD: IN NEW RESULTS TO
780 LDA MLO PREVIOUS SUM: INCLUDE
790 ADC INTHI SIGNS TO UPDATE SIGN
800 STA INTHI
810 LDA MHI
820 ADC INTHI
830 STA INTHI
840 LDA MS
850 ADC IS
860 AND #1
870 STA IS INTEGRAL TERM NOW UPDATED
880 ;NOW ADD INTO FINAL RESULT
890 CLC
900 LDA INTHI
910 STA DSLO
920 LDA INTHI
930 STA DSHI
940 LDA IS
950 STA SPID
960 ;PRODUCE DERIVATIVE TERM
970 //=K*D/AKD AND ADD RESULT
980 ;INTO DSLO,DSHI AND SIGN SPID
982 DER1 LDA KD
984 BEQ PRO1
990 LDA SD
- 1000 STA M3
1010 LDA D
1020 STA MHI
1030 LDA KD
1040 STA ML0
1050 JSR M2B
1060 D2 LSR AKD
1070 BEQ D3
1080 LDA MS
1090 BNE D1
1100 CLC
1110 ROR MHI
1120 ROR ML0
1130 CLV
1140 BVC D2
1150 D1 SEC
1160 ROR MHI
1170 ROR ML0
1180 CLV
1190 BVC D2
1200 D3 CLC DERIV TERM COMPLETE; NOW
1210 ;ADD INTO D3LO,D3HI AND SIGN SPID
1220 LDA ML0
1230 ADC D3LO
1240 STA D3LO
1250 LDA MHI
1260 ADC D3HI
1270 STA D3HI
1280 LDA MS
1290 ADC SPID
1300 AND #1
1310 STA SPID
1320 ;NOW PRODUCE PROPORTIONAL TERM
1330 ;=KP*ERROR
1340 PRO1 LDA KP
1350 STA ML0
1360 LDA E
1370 STA MHI
1380 LDA SE
1390 STA MS
1400 JSR M2B
1410 ;NOW ADD INTO D3LO,D3HI AND SIGN
1420 CLC
1430 LDA ML0
1440 ADC D3LO
1450 STA D3LO
1460 LDA MHI
1470 ADC D3HI
1480 STA D3HI
1490 LDA MS
1500 ADC SPID
1510 AND #1
1520 STA SPID
1530 ;D3LO,D3HI AND SIGN SPID CONTAIN
1540 ;SUM; IF SPID=1 THEN DEMAND SPEED
1550 ;=0; IF D3HI>0 THEN DEMAND SPEED
1560 ;=255; OTHERWISE D3LO HAS NEW
1570 ;COMPUTED SPEED; WE CAN USE LOOK-UP
1580 ;TABLE TO COMPUTE THE DEMANDED
1590 ;SPEED
1600 LDA SPID
1392 BEQ P1
1393 LDA #0
1394 BEQ P2
1395 P1 LDA DSHI
1396 BEQ P3
1397 LDA #255
1398 BNE P2
1600 P3 LDA DSO
1610 P2 STA MDS; MDS HAS COMPUTED O/P
1620 TAX
1630 LDA LUT,X
1640 STA 137 137 HAS LOOK-UP TABLE O/P
1650 RTS
1700 M2S CLC SUBROUTINE TO MULT TWO
1710 LDA MS BYTES IN MHI AND MLO
1720 BEQ M3 ANSWER IS IN SAME TWO
1730 LDA MHI BYTES: PREDICT SIGN AND
1740 EOR #255 PUT IN M5; NUMBER PUT IN
1750 ADC #1 MLO IS ALWAYS +VE
1760 STA MHI
1770 M3 LDA #0
1780 LDX #8
1790 CLC
1800 M2 ROR A
1810 ROR MLO
1820 BCC M1
1830 CLC
1840 ADC MHI
1850 M1 DEX
1860 BPL M2
1870 STA MHI
1880 LDA M5
1890 BEQ M5
1900 LDA MHI
1910 EOR #255
1920 STA MHI
1930 LDA MLO
1940 EOR #255
1950 CLC
1960 ADC #1
1970 STA MLO
1980 BCC M5
1990 INC MHI
2000 M5 RTS
2010 FORMT SEC FORMS ERROR TERM IN E
2020 LDA RN AND DERIV TERM IN D,
2030 SBC CN IN TWO'S COMPLEMENT
2040 STA E SIGNS IN SE AND SD
2050 BCS F1
2060 LDA #1
2070 BNE F2
2080 F1 LDA #0
2090 F2 STA SE
2100 SEC
2110 LDA C1
2120 SBC CH
2130 STA D
2140 BCS F3
2150 LDA #1
2160 BNE F4
2170 F3 LDA #0
2180 F4 STA SD
2190 RTS
2240 K3 LDA CH
2250 STA C1
2260 LDA PCR
2270 ORA #224
2280 STA PCR CB2=1:START CON
2290 K1 LDA IFR
2300 AND #2 CHECK CON FINISH
2310 BEQ K1
2320 LDA #0
2330 STA DDRA
2332 LDA #E840
2334 AND #247
2342 STA #E840 PB3 OFF ENABLES A TO D
2370 LDA IORA
2380 STA CN
2385 JSR PID
2390 LDA #255
2400 STA DDRA PORT=O/P
2410 LDA 137
2420 STA IORA PUT RESULT ON BUS
2432 LDA #E840
2434 ORA #3
2436 STA #E840 PB3 ON ENABLES D TO A
2470 LDA PCR
2480 AND #223
2490 STA PCR MAKE CB2=0
2500 INC 136
2510 BNE K3
2520 LDY CN
2530 JSR #027C
2540 JSR #DCE3
2550 LDX #5
2560 K2 LDA #32
2570 JSR #FFD2
2580 DEX
2590 BNE K2
2600 LDY NDS
2610 JSR #027C
2620 JSR #DCE3
2630 LDA #10
2640 JSR #FFD2
2650 LDA #13
2660 JSR #FFD2
2670 JSR #FFE4
2680 CMP #64
2690 BNE K5
2700 RTS
2710 K5 JMP K3
20 LUT BYT 0,10,12,13,15,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103
30 BYT 35,37,38,40,41,43,44,45,46,47,48,49,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,68,70,71,73,75,77,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,104
40 BYT 58,60,61,62,63,64,65,66,66,68,70,71,73,73,74,75,77,79,80,81,81,82,83,84,1
50 BYT 83,84,85,86,87,88,89,90,91,92,94,94,95,96,97,97,99,99,100,100,101,101,102,104
60 BYT 104
80 BYT 118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,132,133
90 BYT 134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151
100 BYT 152,153,155,157,158,159,160,161,162,164,165,165,166,168,168,170
110 BYT 170,171,172,173,174,175,175,176,178,180,180,181,181,182,183,184,186
120 BYT 187,187,189,190,191,192,192,193,194,195,195,195,196,198,199,200,201,202
130 BYT 203,203,204,205,207,208,209,210,210,211,212,213,213,214,215,215
140 BYT 217,218,219,220,221,222,222,223,224,225,225,226,227,228,229,229,230
160 BYT 245,246,247,247,248,249,250,251,252,253,254,255
170 END
70 INPUT"ENTER REQUIRED SPEED (RPM) ":;SP=INT(S/4):POKE 26000,SP:PRINT
80 CL#:="
90 INPUT"ENTER MAX GAIN (.1,2,4,...,256) ":;G=DR=INT(LOG(256/G)/LOG(2)+.4)
100 POKE 26100,DR:POKE 136,0
10 PRINT"ENTER UNIT OF GAIN COEFFICIENT =1":;256/G
10 INPUT"ENTER PROPORTIONAL GAIN ":;PG:POKE 26001,PG
10 INPUT"ENTER DERIVATIVE GAIN ":;DG:POKE 26003,PG
10 INPUT"ENTER INTEGRAL GAIN ":;IG:POKE 30002,IG
10 PRINT"ENTER ADDITIONAL DIVISION FOR KI"
60 INPUT"(1,2,4,...,256) ":ADR
70 IF ADR=0 THEN PRINT"|";CL#:="PRINTCL#:PRINT"|":"GO TO 3150
80 POKE 26023,LOG(ADR):;LOG(2)
90 FOR N=26020 TO 26026:POKE N,0:FOR N=26011 TO 26013:POKE N,0
100 SYS26426:PRINT"PRINT REQUIRED SPEED =";S
10 INPUT"MAXIMUM GAIN =";G:POKE 2605:OVERALL PROPORTIONAL GAIN =;G/256*PG
10 INPUT"OVERALL DERIVATIVE GAIN =";G*DG/256
10 INPUT"OVERALL INTEGRAL GAIN =";G*IG/256:ADR
10 PRINT"PRINT GOTO 3070
10 PRINT"ENTER NEXT NO:";N1
10 INPUT"ENTER MAXIMUM GAIN ":;D1:=256/D1:DR=LOG(D)/LOG(2):=S=0
10 POKE 26024,0:POKE 26025,0:POKE 26026,0:ITERM=0
10 PRINT"ENTER ADDITIONAL DIVISION FOR KI":;INPUT ADR
10 POKE 29203,LOG(ADR):;LOG(2)
10 PRINT"ENTER C1,C2,KD,KP,KI,DISP":;INPUT C1A,C2A,KD,KP,KI,DP
10 C2A=C1A+C1A:INPUT"ENTER CH",CNA
10 POKE 26000,DP:POKE 26001,KP:POKE 26002,KI:POKE 26010,DR
30 POKE 26000,C2A:POKE 26001,C1A:POKE 26002,CH:POKE 26003,DR:SYS26029
40 ITERM=ITERM+KID*(DP-CNA)+ITERM+KID*(2*C1A-C2A-CNA)
50 FS=INT(S/3)
60 S1=256*PEEK(26012)+PEEK(26011):A$="
70 IF PEEK(26013)>12 THEN A$=";S1=256*256-S1
80 DD=PEEK(2019)
90 PRINT"INHI=";PEEK(26024):TAB(12)"SI=";PEEK(26006):TAB(21)"BASIC SUM=";FS
100 INPUT"IMD=";PEEK(26025):TAB(12)"SP=";PEEK(26004):TAB(21)"M/C SUM=";A$;"DD"
100 INPUT"ILO=";PEEK(26026):TAB(12)"SD=";PEEK(26008):TAB(21)"DEMANDED=";DD
120 PRINT"SLO=";PEEK(26011):TAB(12)"I=";PEEK(26007):TAB(21)"ITEM =";ITE
120 PRINT"SMD=";PEEK(26012):TAB(12)"P=";PEEK(26005)
140 PRINT"SHI=";PEEK(26013):TAB(12)"D=";PEEK(26009)
150 GOTO 3310
180 POKE 3310
210 PRINT"SPEED RANGE 0-1000 RPM";PRINT"SCALE FACTOR 4"
300 INPUT"ENTER REQUIRED SPEED (RPM) ":;SP=S=4:POKE SS,SP
120 PRINT"ENTER PROPORTIONAL GAIN (KP) ":";INPUT 1,2...,255 ";;PG=POKE SS+1,F
310 INPUT"ENTER INTEGRAL GAIN (KI) 0,1,...255 ";;IG=POKE SS+2,IG
140 PRINT"ENTER ADDITIONAL INT. GAIN DIVISOR":;INPUT (AKI) 1,2,4,128 ";;AI
10150 POKE SS+4,A1
10160 PRINT"OVERALL INTEGRAL GAIN =";IG/AI
10170 INPUT"ENTER DERIV GAIN <KD> 0.1..2.55 ";DG:POKE SS+3,DG
10180 PRINT"ENTER ADDITIONAL DERIV GAIN DIVISOR"
10190 INPUT"<KD> 1,2,4..128 ";AD:POKE SS+5,AD
10200 PRINT"OVERALL DERIV GAIN =";KD/AD
10210 POKE SS+23,0:POKE SS+24,0:POKE SS+25,0:REM ZEROES INTEGRAL REGISTER
10220 POKE SS+22,0:REM PUT SPEED NOW=0
10230 SYS26426:GOTO 10000
10300 END
20000 SS=26000:PRINT"CN":INPUT"ENTER CN ";CNX:POKE SS+22,CNX
20010 INPUT"ENTER CN-1 ";N1%:POKE SS+21,N1%
20020 INPUT"ENTER DEMANDED SPEED ";DDX:POKE SS,DDX%
20021 INPUT"ENTER PROPORTIONAL GAIN ";KP%:POKE SS+1,KP%
20022 INPUT"ENTER INTEGRAL GAIN ";KI%:POKE SS+2,KI%
20023 INPUT"ENTER ADDITIONAL DIVISOR AKI ";AI%:POKE SS+4,AI%
20024 INPUT"ENTER DERIVATIVE GAIN ";KD%:POKE SS+3,KD%
20025 INPUT"ENTER ADDITIONAL DIVISOR AKD ";AD%:POKE SS+5,AD%
20027 POKE SS+23,0:POKE SS+24,0:POKE SS+25,0:IT=0
20030 SYS26429:PRINT"NEW DEMANDED SPEED <PID> =";PEEK(SS+19)
20035 ER%=DDX-CN%:N1%=CNX:IT=IT+KI%*ER%/AI%
20037 PI=KP%*ER%+KD%*DDX+AD%*IT;
20040 A1=PEEK(SS+11):A2=PEEK(SS+12):A3=PEEK(SS+13)
20060 PRINT"NDS (BASIC) =";PI
20070 INPUT"ENTER NEW SPEED ";CNX:POKE SS+22,CNX;GOTO 20030
20080 READY.