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by

Iain David Charles Tullis

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Abstract

Torque measurement of a rotating shaft is a method of monitoring machine performance. Steady transmission of mechanical power from the prime mover to the load is vital to avoid gear and bearing wear, shaft fatigue failure, bearing and coupling failure and noise. Mean and fluctuating torque are fundamental quantities of interest.

The laser torquemeter is capable of providing a non-contact measurement of time-resolved torque through a driven system. The laser torquemeter depends upon laser speckle produced from coherent light illuminating a diffuse object and the speckle pattern may be used in determining the angular position of a rotating object. When the object rotates the backscattered speckle pattern, which changes continuously but repeats exactly with every revolution, is sampled by a suitably positioned photodetector. The photodetector output signal is periodic and one period is recorded in memory as a reference and the angular position of a shaft can then be determined by a comparison of this recorded reference signal with the current photodetector output signal. The speckle pattern from two axially separated points on the shaft are monitored and under low or, ideally, zero torque the photodetector outputs are recorded into the laser torquemeter electronics. The laser torquemeter then tracks the live photodetector output and determines the angle at the two points on the shaft. Relative angular displacement in the two angle outputs appears when torque is applied and the shaft twists.
When the shaft is displaced, for example by vibration, the backscattered speckle pattern changes on the photodetector and the similarity between the recorded, reference signal and the live, current signal is reduced. In this thesis, the cross-correlation of the real-time photodetector output signal and the recorded reference signal as a function of shaft position is examined. The effects of various shaft motions – rotation, axial translation, pitch and yaw, and radial translation are theoretically and experimentally examined and the results can then be used in the design of an optical head for the laser torquemeter.

A review of the current torquemeter technology allows for discussion of the broad spectrum of typical torquemeter operating conditions. The optical head of the laser torquemeter may vary significantly for various torque measurement scenarios. A design procedure for the optical head of the laser torquemeter is summarised.

The holy grail of torquemeter manufacturers is to produce a cheap, easy to use, robust, accurate, reliable and non-contacting torquemeter. The laser torquemeter has great potential to meet these requirements.
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Contents

Abstract ii

Acknowledgements iv

List of Figures xiv

List of Tables xvi

Nomenclature 1

1 Introduction 8

1.1 Torque Measurement ........................................... 9
  1.1.1 Motor, Pump, and Compressor Efficiency ................. 10
  1.1.2 Condition Monitoring in the Oil and Gas Industry ...... 10
  1.1.3 Shear Modulus of Elasticity .............................. 11
  1.1.4 Polar Second Moment of Area .............................. 12
  1.1.5 A Review of Torquemeter Technology ................... 13
  1.1.6 Strain Gauge Based Torquemeters ......................... 15
  1.1.7 Magnetoelastic Torque Sensing ............................ 19

1.2 Measurement of the Twist of a Rotating Shaft ............ 19
  1.2.1 Shaft Twist Transmission Torquemeters ................ 19
  1.2.2 Torsion Bar Torquemeter ................................. 21
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2.3</td>
<td>Moiré Type Torquemeter</td>
<td>22</td>
</tr>
<tr>
<td>1.2.4</td>
<td>Phonic Wheel Torquemeter</td>
<td>22</td>
</tr>
<tr>
<td>1.2.5</td>
<td>Torsional Variable Differential Transformers</td>
<td>25</td>
</tr>
<tr>
<td>1.2.6</td>
<td>Commercially Available Torquemeters</td>
<td>26</td>
</tr>
<tr>
<td>1.2.7</td>
<td>The Torquemeter World Market</td>
<td>27</td>
</tr>
<tr>
<td>1.3</td>
<td>The Laser Torquemeter</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>Speckle</td>
<td>32</td>
</tr>
<tr>
<td>2.1</td>
<td>Fundamental Properties of Speckle</td>
<td>32</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Introduction</td>
<td>32</td>
</tr>
<tr>
<td>2.2</td>
<td>Single Point Statistics</td>
<td>35</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Speckle Contrast</td>
<td>40</td>
</tr>
<tr>
<td>2.3</td>
<td>Second Order Properties of Speckle</td>
<td>42</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Mean Speckle Size</td>
<td>42</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Gaussian Beams</td>
<td>44</td>
</tr>
<tr>
<td>2.4</td>
<td>First Order Statistics of Integrated Speckle Patterns</td>
<td>46</td>
</tr>
<tr>
<td>2.5</td>
<td>Effect of Scattering Surface</td>
<td>47</td>
</tr>
<tr>
<td>2.5.1</td>
<td>Effect of Surface Roughness</td>
<td>47</td>
</tr>
<tr>
<td>2.5.2</td>
<td>Effect of Retro-Reflective Tape</td>
<td>49</td>
</tr>
<tr>
<td>3</td>
<td>Speckle Decorrelation</td>
<td>53</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>53</td>
</tr>
<tr>
<td>3.2</td>
<td>Fundamental Principle of Laser Torquemeter Operation</td>
<td>54</td>
</tr>
<tr>
<td>3.3</td>
<td>Motions Causing Decorrelation of the Photodetector Output Signal</td>
<td>56</td>
</tr>
<tr>
<td>3.4</td>
<td>The Space-Time Correlation Function</td>
<td>59</td>
</tr>
</tbody>
</table>
3.4.1 Approximation of a Triangular Function by a Gaussian Function ................................................. 62
3.4.2 Space-Time Correlation Function of the Spatially Integrated Speckle Intensity ........................................ 66

3.5 Speckle Intensity Decorrelation due to Target Rotation ................................................................. 69
3.6 Translational Decorrelation ................................................................. 73
3.7 Tilt Motion Decorrelation ................................................................. 74
3.8 Radial Motion Decorrelation ............................................................... 75

3.9 Equivalence Between Autocorrelation and Cross-Correlation Functions ............................................. 76
3.10 Accuracy of the Autocorrelation ............................................................. 81
3.11 Maximum Width of Photodetector ......................................................... 83
3.12 Experimental Verification of Speckle Statistics ..................................................... 86
3.13 Measurement of Laser Beam Parameters ...................................................... 91
3.14 The Temporal Bandwidth of the Photodetectors ................................................ 92
3.15 Stability of Correlation of Spatially Integrated Speckle Pattern Over Time .............................................. 93
3.16 The Verification of the Speckle Statistics from Retro-Reflective Tape 94
3.17 Experimental Validation of Speckle Decorrelation Due to Shaft Rotation ................................................. 98
3.18 Experimental Validation of Speckle Decorrelation Due to Axial Translation of the Shaft ............................ 101
3.19 Experimental Validation of Speckle Decorrelation Due to Shaft Tilt 108
3.20 Experimental Investigation: Decorrelation Due to Radial Motion . 112
4 The Laser Torquemeter

4.1 Operating Environment of the Laser Torquemeter

4.1.1 Range of Measured Torques

4.1.2 Speed variation of the Rotating Shaft

4.1.3 Vibrational Motions of Typical Shafts

4.2 Excitation Torques and Expected Shaft Twists

4.3 Windup

4.4 Coupling to Torquemeter Shaft

4.5 Hybrid Digital-Analogue Torquemeter

4.5.1 Operation of the Prototype Torquemeter

4.6 Comparison with Traditional Torquemeters

5 Signal Processing for the Laser Torquemeter

5.1 Methods of Estimating Time Delay

5.2 Application of Time Delay Estimation

5.3 The Effect of Speckle Decorrelation on the Signal Processing

5.4 Correlation Methods

5.4.1 Open Loop Correlators

5.4.2 Closed Loop Correlators

5.4.3 Other Factors Influencing the Torquemeter

5.4.4 Anti-aliasing Filters

5.5 Optical Design

5.6 Torquemeter Optical Head Design Example

6 Conclusions

6.1 Recommendations for Further Work

6.1.1 Digital Signal Processor based Laser Torquemeter
6.1.2 Calculating the Processing Power Required for a Prototype Torquemeter ........................................... 166
6.1.3 DSP Laser Torquemeter Evaluation System ........... 167

A ............................................................................. 169
A.1 Measurement of Autocorrelation Functions .............. 169
A.2 Statistical Errors in Cross-Correlation Estimates ......... 171
A.3 Gaussian Cross-correlations .................................. 173
A.4 Correlation and Autocorrelation Using the FFT .......... 174
List of Figures

1.1 Diagram of the geometry for computing the polar second moment of area. 12
1.2 Bridge arrangement of strain gauges. .......................... 16
1.3 Dedicated shaft profiles for low capacity torque transducers. 17
1.4 Diagram of a magnetoelastic torquemeter .......................... 20
1.5 Diagram of an optically read-out torsion bar torquemeter. 21
1.6 Schematic diagram of a Moiré torquemeter. ...................... 22
1.7 Diagram of a phonic wheel torquemeter. .......................... 23
1.8 Schematic diagram of a LVDT. ................................. 25
1.9 Schematic diagram of a TVDT ................................. 26
1.10 Category of application in the projected 1997 torquemeter market. ... 28
1.11 Schematic diagram of the Laser Torquemeter. ...................... 29
1.12 Typical signals and their temporal relationships. .................. 30
2.1 Intensity distribution typical of a speckle pattern. .................. 33
2.2 The coordinate system for examining the statistical properties of speckles produced in the diffraction field by the random phase screen. ... 37
2.3 Speckle formation geometry. Constructive interference causes a bright region and destructive interference results in a dark region. ........... 38
2.4 A plot of the intensity distribution of the speckle pattern in Figure (2.1). 41
2.5 A log plot of the adjusted intensity distribution of the speckle pattern in Figure (2.1). ........................................ 41
2.6 Formation of three dimensional speckles in space from Yoshimura and Iwamoto. ............................................ 43
2.7 Image of simulated speckle pattern with superimposed autocorrelation function contours plotted. ...................... 45
2.8 The Gaussian beam. The beam waist radius (denoted $w_0$) is at location $z = 0$. The beam diameter $D$ as a function of range $z$ is defined in Equation (2.11). The radius of curvature of the beam is labeled as $r$. 46
2.9 The speckle pattern produced by retro-reflective tape and Airy disk of many overlapping speckle patterns ............... 51
3.1 Schematic diagram of the Laser Torquemeter. .................. 55
3.2 Schematic diagram of the optical arrangement showing the coordinate axes. ................................................. 56
3.3 The coordinate system for describing the vibrational motion of a shaft. 57
3.4 Typical configuration for one channel of a laser torquemeter. .... 59
3.5 Object and observation planes. .................................... 60
3.6 The speckle pattern produced by 'Scotchlight' retro-reflective tape and Airy disk of many overlapping speckle patterns ............ 64
3.7 Autocorrelation functions of a selection of apertures ........... 65
3.8 Reflection of paraxial rays from a cylindrical shaft illustrating an effective origin for the backscattered rays. ............... 71
3.9 Cross-correlation between row 10 and other rows. Matrix S size is 21 rows $\times$ 4096 columns. Note that the peak of the cross-correlations are not synchronized. ........................................... 80
3.10 Peak row-by-row cross-correlations plotted against row number.

3.11 Comparison of mean row-by-row cross-correlations and the mean autocorrelation of columns.

3.12 The accuracy of the autocorrelation function plotted against lag. $B = 500\text{kHz}$, $16384$ samples, $F_s = 1\text{MHz}$

3.13 Approximate signal to noise ratio of the photodetectors plotted against the number of speckles on the photodetecting aperture. $R_f = 100\text{k}\Omega$, $B = 1\text{MHz}$, and with a typical received optical power of $\sigma_I^2 = 2.5\mu\text{W}$.

3.14 The experimental apparatus used in determining the verification of speckle statistics from a rotating shaft.

3.15 Photodetector output signal for approximately 6 revolutions.

3.16 Spectrum of the photodetector output signal showing strong periodicity at the rotation rate of the shaft ($\approx 6\text{Hz}$).

3.17 Photodetector output signal from one revolution of the shaft. The period of rotation is $158.6\text{ms}$ ($\approx 6\text{Hz}$).

3.18 Bode plot of the photodetector output from one revolution.

3.19 Laser beam profile.

3.20 Stability of the cross-correlation function of the speckle pattern produced by a rotating shaft over nearly 45 minutes.

3.21 Speckle Intensity Distribution, $Q = 3.76$

3.22 Speckle intensity distribution, $Q = 4.44$

3.23 Speckle Intensity Distribution, $Q = 8.46$

3.24 Plot of the autocorrelation function with shaft rotation angle. $z_0=25.6\text{cm}$, $z=34\text{cm}$, $(\sigma_0) = 0.2187\text{mm}$, $D = 0.984\text{mm}$

3.25 Experimental apparatus used for determining the resistance to rotational decorrelation.
3.26 Plot of the theoretical rotational correlation angle $\theta_{cc}$ shows agreement with the measured rotational correlation angle. Error bars represent ± one standard deviation from the mean experimental value for 3 measurements.

3.27 Autocorrelation function of the photodetector output from 6 revolutions of the shaft.

3.28 Schematic diagram of optical configuration for measuring decorrelation due to shaft translation.

3.29 Plot of peak normalised cross-correlation between reference signal and signal from axial translated shaft with a Gaussian 'least squares' best fit.

3.30 Plot of peak normalised cross-correlation between reference signal and signal from axial translated shaft with a Gaussian 'least squares' best fit.

3.31 Plot of axial correlation width $x_c$ versus beam diameter.

3.32 Plot of the theoretical axial correlation width $x_c$ versus actual axial correlation width.

3.33 Schematic diagram of optical configuration for measuring decorrelation due to shaft tilt.

3.34 A Gaussian curve fitted to plot of the decorrelation due to shaft tilt.

3.35 Tilt cross-correlation experiment 'e'.

3.36 Plot of the theoretical tilt correlation width $\theta_{tc}$ versus actual tilt correlation width.

3.37 Plot of the apparent motion of the correlation peak due to radial shaft motion.

4.1 Schematic diagram of the optical arrangement to determine shaft tilt and axial float.
4.2 Displacement of the driveshaft flange of a diesel engine. 

4.3 Peak to peak axial motion of driveshaft of a diesel engine.

4.4 Peak to peak tilt motion of driveshaft of a diesel engine.

4.5 A schematic diagram of the twin loop torquemeter system.

4.6 Schematic diagram of a Common-Mode Difference-Mode laser torquemeter system.

5.1 Signal to Noise Ratio (SNR) versus the normalised correlation coefficient.

5.2 Autocorrelation function of a typical photodetector output signal and its S curve.

5.3 System diagram of a closed loop correlator.

5.4 Use of a lens to produce an effective photodetector aperture much larger than the aperture of the photodiode.

5.5 Uncertainty in laser torquemeter reading due to finite size of probe spots.

A.1 Typical data from photodetector output. Left portion: Probability density function. Right - typical speckle signal.

A.2 Typical autocorrelation function.

A.3 Correlation widths normalised to correlation width at $1/e^2$.
List of Tables

1.1 Values of shear modulus of elasticity for various materials. 11
1.2 The polar second moment of area for some common shaft geometries. 13
1.3 Range of dominant torque meters on the market 27
1.4 World market for torque meters 27
1.5 Prices of some commonly available torque meters. 28

2.1 Typical surface roughness levels for a selection of common manufacturing processes. 48
2.2 Normal range of surface finish for shaft finishing operations. 48

3.1 Width of laser beam as a function of range from laser. 91
3.2 Frequency response of the photodetectors used. 92
3.3 Summary of the optical configurations showing the angular correlation width of the photodetector output for a variety of optical configurations. 100
3.4 Summary of the optical configurations showing the effectiveness in resisting decorrelation of a photodetector output due to axial motion of the shaft. 105
3.5 Summary of the optical configurations showing the effectiveness in resisting decorrelation of a photodetector output due to shaft tilt. The mean and standard deviation (std) of the estimate of $\theta_{hc}$ computed from 5 experiments. 111
Nomenclature

Roman Symbols

\( a_k \) amplitude contribution of the \( k^{th} \) scatterer to the speckle pattern

\( Aa \) true aperture autocorrelation function

\( A(x, y) \) aperture function of the photodetector

\( A_0 \) aperture constant Iwai et al [1]

\( B \) bandwidth [Hz]

\( c \) speed of light: \( 2.99792458 \times 10^8 \) [m/s]

\( c_B \) bandwidth factor \( c_B = B/\omega \) [1/rad]

\( c_T \) sampling factor or shaft angle between torque measurements \( c_T = T\omega \) [rad]

\( C \) speckle contrast

\( C_s \) coefficient of speed variation in revolution

\( d \) shaft diameter [m]

\( D \) diameter of illuminated spot on scattering surface – equivalent to \( 2w \) [m]

\( D_A \) diameter of evenly illuminated disk [m]

\( E \) modulus of elasticity; electric-field complex amplitude [V/m]

\( f \) focal length of a lens [m]; also, frequency [Hz]

\( g(\Phi) \) restoring signal to lock DLL

\( G \) shear modulus of elasticity [Pa] or [N /m²]

\( Ga \) Gaussian approximation of aperture autocorrelation function

\( HB \) Brinell hardness [MN m⁻²]
\( H(f) \) frequency content of photodetector output signal

\( h_x, h_y \) radii at which the Iwai et al aperture function drops to \( 1/e^2 \) in the x and y directions

\( I \) optical intensity [W/m\(^2\)]

\( I_A \) spatially integrated intensity [W/m\(^2\)]

\( \langle I \rangle \) mean intensity [W/m\(^2\)]

\( \Delta I \) intensity fluctuation \( I - \langle I \rangle \) [W/m\(^2\)]

\( \Delta I_A \) intensity fluctuation integrated over aperture area [W/m\(^2\)]

\( J \) polar second moment of area [m\(^4\)]

\( j \) \( \sqrt{-1} \)

\( k \) wave number \( 2\pi/\lambda \) [m\(^{-1}\)]; index into summation

\( k_B \) Boltzmann's constant \( 1.3807 \times 10^{-23} \) [JK\(^{-1}\)]

\( k_0 \) Gaussian term for function approximation

\( K \) a 'twist calibration factor' \( K = (GJ)/\Delta L \) [Nm]

\( \Delta L \) axial distance between illuminated points A and B on shaft [m]

\( l_c \) laser coherence length [m]

\( L_x \) dimension of detecting aperture in x direction [m]

\( L_y \) dimension of detecting aperture in y direction [m]

\( M_{\text{dyn}} \) dynamic bending moment [Nm]

\( M_{\text{sat}} \) static bending moment [Nm]

\( M_s \) number of rows in matrix \( S \);

\( N \) number of scattering elements,

\( N_s \) number of columns in matrix \( S \)

\( N_{\text{safety}} \) safety factor

\( n \) rotation rate [rpm]

\( n \) an integer
\( n \) number of experimental runs

\( n(t) \) noise

\( P \) power [W]

\( q \) electron charge \( 1.602 \times 10^{-19} \text{ C} \)

\( Q \) number of speckle cells in a detecting aperture

\( R \) radius [m]

\( \mathfrak{A} \) photodiode responsivity [A/W]

\( R_a \) roughness factor [\( \mu \text{m} \)]

\( R_f \) photodetector feedback resistance [\( \Omega \)]

\( R_T \) shaft radius with curvature convention of Takai et al. [2] [m]

\( r \) radius of beam curvature [m]

\( r' \) effective beam curvature radius \( r' = \frac{-rR}{(R - 2r)} \) [m]

\( s(t) \) ideal photodetector output signal [V]

\( s_{mn} \) \( m \)th row, \( n \)th column element of matrix \( S \)

\( S \) effective area of photodetector [m\(^2\)]

\( S \) matrix for data collection

\( S_c \) estimate of speckle area [m\(^2\)]

\( S_{\text{end}} \) limit fatigue strength [Pa]

\( S_{\text{ult}} \) ultimate strength [Pa]

\( S_{\text{yld}} \) yield strength [Pa]

\( S_m \) detector aperture area [m\(^2\)]

\( S(\tau) \) S curve of delay locked loop

\( T \) temperature [K]

\( t \) time [s]

\( t_{\text{slip}} \) mean time between loss of lock events for a delay locked loop [s]

\( T \) torque [Nm]
\( T_{\text{dyn}} \) dynamic torque [Nm]
\( T_{\text{sat}} \) static torque [Nm]
\( v \) velocity of scatterer [m/s]
\( v(t) \) target motion velocity as a function of time [m/s]
\( v_{\xi}, v_{\eta} \) velocity of scatterer in directions \( \xi \) and \( \eta \) [m/s]
\( W \) work [J]
\( w(z) \) radius of Gaussian beam [m] – The beam intensity assumes its peak value on the beam axis and drops by the factor \( 1/e^2 \approx 0.135 \) at the radial distance \( w \)
\( w_0 \) waist radius of a Gaussian beam [m] = the minimum value of \( w \)
\( x \) vector in the detecting plane \([x, y]\)
\( x, y \) unit vectors defining axis in detector plane
\( x_l(t) \) live version of the photodetector output signal
\( x_s(t) \) recorded version of the photodetector output signal
\( X, Y \) spatial delay in the \( x \) and \( y \) directions: \( X = x_2 - x_1 \) and \( Y = y_2 - y_1 \)
\( x_c \) decorrelation displacement required to decorrelate speckle by \( 1/e^2 \) [m]
\( z \) range from scattering object to detector plane [m]
\( z_o \) range from beam waist to scattering object [m]

**Greek Symbols**

\( \alpha \) constant of proportionality
\( \beta \) constant of proportionality
\( \Gamma_A \) spatially integrated correlation function
\( \Gamma_{\Delta I} \) speckle intensity fluctuation correlation function
\( \Gamma_{xx} \) autocorrelation function of \( x(t) \)
\( \Gamma_{xy} \) Cross-correlation function of \( x(t) \) with \( y(t) \)
\( \Gamma_{nn}(a) \) discrete autocorrelation of row \( n \) as a function of axial delay
\( \Gamma_{m(m+a)}(\theta) \) discrete cross-correlation between row \( m \) and row \( m + a \) at angular delay \( \theta \)

\( \bar{\Gamma}_{m(m+a)}(\theta) \) mean discrete cross-correlation between row \( m \) and row \( m + a \) at angular delay \( \theta \)

\( \gamma_{nm}(a) \) mean discrete normalised autocorrelation of row \( n \) as a function of angular delay

\( \gamma_{zz}(\tau) \) Normalised temporal portion of the space-time autocorrelation function:

\[ \gamma_{zz}(\tau) = \frac{\Gamma_{zz}(0,0,\tau)}{\Gamma_{zz}(0,0,0)} \]

\( \gamma_{xy} \) normalised cross-correlation function of \( x(t) \) with \( y(t) \)

\( \Delta \nu_c \) spectral width [nm]

\( \epsilon \) strain

\( \zeta \) unit vector defining axis of rotation of the shaft

\( \eta_{\text{air}} \) refractive index of air \( \approx 1 \)

\( \theta \) angle, commonly shaft twist angle

\( \theta_{\text{misalign}} \) misalignment between rotation axis of shaft \( \eta \) and a line connecting the illuminated points on the shaft

\( \theta_s \) source half angle

\( \theta_{\zeta c} \) shaft rotation angle required to decorrelate live signal relative to recorded signal by \( 1/e^2 \)

\( \theta_{\zeta}, \theta_{\eta} \) rotation angle about the \( \zeta \) and \( \eta \) axes

\( \theta_{te} \) tilt angle required to decorrelate live signal relative to recorded signal by \( 1/e^2 \)

\( \vartheta_o \) angle between illuminating beam and target velocity vector

\( \lambda \) wavelength [m] (HeNe laser \( \lambda \approx 632.8 \text{nm} \))

\( \mu \) Poisson's ratio

\( \xi \) a vector in the object plane \( [(\xi, \eta)] \)

\( \xi, \eta \) unit vectors defining the object plane axis shaft

\( \rho \) wavefront curvature term: \( 1 + z/r \)

\( \langle \sigma_0 \rangle \) mean speckle size [m]
\( \sigma_I \) standard deviation of speckle intensity

\( \sigma_x, \sigma_y \) stress in the \( x \) and \( y \) directions [N/mm\(^2\)]

\( \tau \) lag time [s];

\( \tau_s \) shear stress [Pa]

\( \hat{\tau} \) estimate of lag time [s]

\( \nu \) (optical) frequency [Hz]

\( \phi_k \) phase component of the \( k^{th} \) contribution to speckle pattern – see \( a_k \)

\( \varphi \) angle

\( \omega \) angular velocity [rad/s]

\( \omega_D \) Doppler shift

\( \omega_R \) angular velocity of the target shaft [rad/s]
Special Functions

Rectangle Function \( \text{rect}(x) = \begin{cases} 1, & |x| \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \)

Sinc Function \( \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \)

Triangle Function \( \triangledown(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases} \)

Error Function \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-t^2)dt \)

Bessel Function of the First Kind \( J_1(x) = \frac{1}{\pi} \int_{0}^{\pi} \cos(\phi - x \sin \phi) d\phi \)
Chapter 1

Introduction

Many of the machines that we use in everyday life employ rotating shafts to transmit mechanical power. Manufacturers are faced with competitive pressure to improve efficiency as well as product quality. This drives the desire to measure and control this mechanical power in the industrial environment. A means to measure the power transmitted is to measure the torque and the rotational velocity of the shaft. Derived from *torquere*, from Latin meaning ‘to twist’, torque is part of our everyday life.

This thesis describes how the characteristics of a speckle pattern from a rotating target can be used to measure the torque produced by a driven shaft.

Chapter 1 discusses why it is important to measure torque, what current methods are used to measure torque, and concludes with a description of a new instrument, the laser torquemeter.

The laser torquemeter uses the ability of the coherent nature of laser light to produce speckle patterns and these speckle patterns are used to measure torque. Chapter 2 provides background material on laser speckle phenomena and explores the statistics of laser speckle patterns.

Chapter 3 is an in-depth analysis of how target motions give rise to different
types of speckle behaviour. Attention is paid to consequent changes in the output of a photodetector positioned to sample a particular part of a speckle pattern and to explicitly quantify the effects of these changes to the laser torquemeter performance. The method of data capture provides the opportunity to exploit the data set in order to ease the computational burden.

Chapter 4 re-examines the operating environment of a laser torquemeter in light of the knowledge gained in Chapter 3. A measurement of vibrations of a driveshaft from a diesel engine is examined. The vibrations of a driveshaft are important in the design of a laser torquemeter. The expected shaft twists are compared with the shaft twists of various types of torquemeter.

Chapter 5 examines methods of estimating the time delay between a recorded signal and a live photodetector signal. Motion of the shaft causes speckle decorrelation which, in turn, modifies the live photodetector signal and influences the accuracy of the time delay estimate. The effect of speckle decorrelation on the accuracy and robustness of the laser torquemeter is explored. Other factors influencing the signal quality are examined. A design example for the optical head of the laser torquemeter is presented.

Chapter 6 provides conclusions and suggestions for further work in developing the laser torquemeter.

1.1 Torque Measurement

Torque measurement is required to quantify and/or improve the performance of rotating machines. A range of typical applications for torquemeters might include: performance measurement; research, development and testing of prototype machines; continuous duty monitoring to provide a log of actual shaft
power transmitted and provide indication of a reduction in performance; control of speed for optimum performance of pumps, fans, propellers and turbines in order to maintain peak efficiency at various loads and speeds; production testing – steady state and transient testing of machines; and for acceptance testing.

1.1.1 Motor, Pump, and Compressor Efficiency

Torque measurement allows for the monitoring of efficiency of motors, pumps, and compressors. Mechanical power is the product of torque and angular velocity. The mechanical power output of a motor may be compared with the electrical energy input to assess the motor's efficiency. There are estimates that a decrease as small as 1% in the efficiency of a 100HP range electric motor, typically found in commercial heating ventilation and air conditioning (HVAC) equipment, can increase energy consumption costs by up to USD10,000 per annum [3]. Monitoring the efficiency may also allow for the prediction of failures and the low cost replacement of worn bearings and chain drives. Energy efficiency is becoming more important in product design.

1.1.2 Condition Monitoring in the Oil and Gas Industry

In the oil and gas industry, measurement of torque is commonly required for machine monitoring and commissioning. Currently the torque is measured either by attaching strain gauges on the shaft or by mounting a striped tape for an optical phonic wheel torquemeter. Shaft design for the gas turbine compressors usually allows for a principle surface strain due to torsion of 20-40 $\mu$strain at full power (power transmitted is on the order of 900MW [4]) and the transmitted torque is calculated from knowledge of the material parameters of the shaft.
1.1.3 Shear Modulus of Elasticity

Shaft twist transmission torquemeters as well as strain gauge-based torquemeters are dependent on the value of the shear modulus of elasticity of a shaft in order to determine torque from twist. The shear modulus of elasticity $G$, a material property of the shaft, is related to the modulus of elasticity $E$ by the following expression

$$G = \frac{E}{2(1+\mu)}$$

(1.1)

where $\mu$ is Poisson's ratio (0.25–0.3 for steel and $\approx 0.33$ for most other metals). Table (1.1) itemises the shear modulus of elasticity $G$, Young's modulus $E$, and Poisson's ratio $\mu$ for some common engineering materials. The maximum shearing stress imposed on the shaft must not exceed the yield stress as then the shaft material would not experience elastic deformation (often with catastrophic results) and Equation (1.1) would be invalid.

<table>
<thead>
<tr>
<th>Material</th>
<th>Shear Modulus of Elasticity $G$ [GPa]</th>
<th>Young's Modulus $E$ [GPa]</th>
<th>Yield Stress $S_{Yld}$ [MPa]</th>
<th>Poisson's Ratio $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel (SAE 4140)</td>
<td>78.8</td>
<td>205</td>
<td>375</td>
<td>0.3</td>
</tr>
<tr>
<td>Eustenitic Stainless Steel</td>
<td>74 – 86</td>
<td>185 – 220</td>
<td>255</td>
<td>0.25 – 0.29</td>
</tr>
<tr>
<td>Aluminium (2024)</td>
<td>28</td>
<td>75</td>
<td>395</td>
<td>0.33</td>
</tr>
<tr>
<td>Titanium</td>
<td>46</td>
<td>125</td>
<td>100 – 225</td>
<td>0.36</td>
</tr>
<tr>
<td>Iron</td>
<td>82</td>
<td>212</td>
<td>80 – 100</td>
<td>0.29</td>
</tr>
<tr>
<td>Tungsten</td>
<td>116</td>
<td>298</td>
<td>550</td>
<td>0.28</td>
</tr>
<tr>
<td>Nylon 6</td>
<td>16.8</td>
<td>0.7</td>
<td></td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 1.1: Values of shear modulus of elasticity for various materials. [5] [6] [7]
1.1.4 Polar Second Moment of Area

The shaft twist is not only dependent on material properties but also on the shaft geometry. The polar second moment of area of a plane surface in relation to point $O$ within the plane is the sum of the products of the area-elements $dA$ and the squares of their distances $r$ from point $O$. Figure (1.1) is a diagram describing the geometry used for calculating the polar second moment of area and may be computed using the following integral:

$$J = \int_A r^2 dA$$

(1.2)

Table(1.2) tabulates $J$ for some common shaft geometries. Accurate knowledge of the polar second moment of area and the shear modulus of elasticity is required because the torque reading is proportional to the shear modulus and it is common for a shaft of known material properties to be inserted into the drive train. For the performance monitoring of machines the absolute value of the material properties is not critical as all measurements are made relative to the past history of the machine and the polar second moment of area and the shear modulus of elasticity
The polar second moment of area, \( J \), for some common shaft geometries is given by:

- Solid circular shaft radius: \( R \) or diameter: \( D \)
  \[ J = \frac{\pi R^4}{2} = \frac{\pi D^4}{32} \]

- Hollow circular shaft, inner radius: \( R_{\text{inner}} \), outer radius: \( R_{\text{outer}} \)
  \[ J = \frac{\pi (R_{\text{outer}}^4 - R_{\text{inner}}^4)}{2} \]

<table>
<thead>
<tr>
<th>Geometry</th>
<th>polar second moment of area, ( J )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid circular shaft radius: ( R ) or diameter: ( D )</td>
<td>[ J = \frac{\pi R^4}{2} = \frac{\pi D^4}{32} ]</td>
</tr>
<tr>
<td>Hollow circular shaft, inner radius: ( R_{\text{inner}} ), outer radius: ( R_{\text{outer}} )</td>
<td>[ J = \frac{\pi (R_{\text{outer}}^4 - R_{\text{inner}}^4)}{2} ]</td>
</tr>
</tbody>
</table>

Table 1.2: The polar second moment of area for some common shaft geometries.

Do not vary with time. The polar second moment of area and the shear modulus of elasticity do vary with temperature and so the temperature needs to be closely monitored or controlled.

1.1.5 A Review of Torquemeter Technology

The ideal torquemeter is an intrinsically safe, robust, easy to use instrument that requires a minimum of training to use and is not influenced by environmental conditions. The use of the torquemeter should not require the stopping of the machine to install, nor insertion of any special shaft or instrument into the drive shaft and the setup time should be negligible.

Mechanical shaft power is equal to the product of torque and rotational speed and therefore torque is an important parameter in measuring machine efficiency. Power-torque relationship is derived from power and is defined as the rate of doing work. The work done \( W \) by torque \( T \) in \( n \) revolutions is

\[
W = T(2\pi n). \quad (1.3)
\]
Power $P$ is the derivative of work with respect to time

$$P = \frac{dW}{dt} = T\omega$$

(1.4)

where $\omega$ is defined as the derivative of shaft angle ($\omega = \dot{\theta}$). There are three main approaches to torque measurement: absorption, reaction and transmission torquemeters.

Absorption systems replace the driven load with a dynamometer where all the mechanical energy is absorbed. The rotational energy is converted to heat by the dynamometer - by dry friction (Prony brake), by fluid friction (Froude brake), by air friction (fan) or by electrical means (the eddy current brake or DC dynamometer). The power absorbed by the dynamometer $P$ and the rotational velocity $\omega$ are measured and torque $T$ can then be calculated by $T = P/\omega$. Absorption torquemeters are reliable and robust instruments but the large rotational inertia of the brake limits accuracy in measuring fluctuating torques and the requirement that the load be replaced by the torquemeter is often undesirable. Absorption torquemeters are most suited to purpose-built laboratory or test bed measurements.

Reaction torquemeters measure the reaction force between the prime mover and the load. The prime mover and the load are mounted on a rigid surface and the reaction force is measured. These torquemeters eliminate the need for any rotating parts and are therefore very reliable devices. However, because of the inertia of the machine housings, they are unable to measure small fluctuating torques.

In electrical machines, torque may be measured via an indirect method of measuring the electrical power consumed by the machine. Frequency inverters,
which condition the electrical power to the motor, are commercially available which provide outputs of motor speed and torque [8].

In this thesis, absorption dynamometers, reaction torquemeters, and indirect power measurement torquemeters are not discussed in further detail because the laser torquemeter is a transmission torquemeter.

Transmission torquemeters are inserted in-line between the power source and sink and measure torque under actual operating conditions. Transmission torquemeters may be divided into two main types: those that measure strain in the shaft and those that measure shaft twist.

The measurement of a small deformation of a rotating shaft is a difficult task to perform and in seeking a solution to this measurement problem a variety of methods have been devised.

1.1.6 Strain Gauge Based Torquemeters

The strain in an object may be measured by an electrical resistance strain gauge which is a thin metal foil grid that may be adhesively bonded to the surface of the component of interest. When the component is loaded, strains develop and are transmitted to the foil grid and thus electrical resistance of the grid changes in proportion to the load-induced strain. The strain may be measured directly by strain gauges bonded to the shaft if the level of torsional shear strain is adequate – that is, if the shaft is not dramatically oversized. A gauge mounted at +45° to the shaft axis experiences the maximum tensile strain +ε and a gauge at -45° experiences the maximum compressive strain −ε. Strain is related to torque in a
solid shaft of radius $R$ by

$$\varepsilon = \frac{2T}{\pi GR^3}, \quad (1.5)$$

where $G$ is the shear modulus of elasticity. Four strain gauges are usually arranged as a fully balanced bridge because the bridge arrangement allows bending moments to be balanced out and reduces sensitivity to temperature changes. Figure (1.2) shows a schematic diagram of a strain gauge bridge. An excitation voltage is applied across the bridge and the output is the difference in voltage between the two voltage dividers. The bridge is arranged so that the strain that causes an increase in resistance in SG1 also causes an increase in SG4, which then reduces the voltage on the left side of the bridge and increases the voltage on the right side.

The strain gauges may also be bonded to a dedicated shaft which is then inserted into the drive line because knowledge of the shaft’s shear modulus of elasticity is required to calculate torque. In practice, the dedicated shaft for low capacity torque transducers is a ‘hollow cruciform’ or a four torsion bar frame in
which each bar experiences a combination of torsion and bending when loaded and provides a flat surface on which to mount the strain gauges [10] [11] [12]. Figure (1.3), reproduced from the LeBow Catalogue [10], shows typical shaft sections and strain gauge positions for commercially available torquemeters. For higher capacity transducers, a square shaft is used to facilitate the attachment of the strain gauges. The corners of a square shaft are stress free and the connection between the strain gauge and lead wire is made here to avoid fatigue failure in the connections. Strain gauges require a small area (typically 10mm²) to mount and only a short length of shaft is required to provide a valid measurement surface.

The strain for a full scale reading of a dedicated shaft torquemeter is on the order of 200μstrain – 500μstrain and the driveshaft for large machines is usually designed to have a surface strain in the region of 20 – 40 μstrain[4]. Strain gauge accuracy is on the order of ±1 μstrain and the nominal strain gauge sensitivity is just adequate to calculate shear strain (and hence torque) with sufficient accuracy
for meaningful efficiency measurements for an undedicated shaft. Calibration of a strain gauge torquemeter is almost always required[10] [13].

A major difficulty in using strain gauges to measure torque is that the shaft on which the strain gauges are mounted is rotating and the observer is stationary. The electrical bridge requires two connections to the instrument in order to supply a driving voltage and two connections are required to measure gauge resistance. Batteries are sometimes used to supply power but have limited life due to the high power consumption of the gauges. The recommended power density for a strain gauge on steel shaft is roughly 5mW mm\(^{-2}\)[14] and with four gauges in the bridge, typical battery life is on the order of 24 hours. Batteries also suffer from electrolyte loss as the electrolyte tends to be centrifuged out of the cell.

Power may be supplied directly by a slip ring or inductively by a rotating transformer where the primary coil is stationary and the secondary coil is mounted on the shaft. The strain gauge bridge output may be determined by connecting a slip ring to the bridge; the maximum rotational speed of the slip ring connection is limited by the maximum surface speed of brush to ring interface. The slip ring connection is sensitive to brush wear and contact dirt which can increase or produce fluctuations in the connection resistance. The gauge resistance is low (usually only 150Ω – 350Ω ) and therefore sensitive to parasitic resistance in the strain gauge bridge.

An alternative to slip rings is to use the output of the sensor bridge to modulate an oscillator and then detect the modulated signal with a stationary antenna. Power may be supplied to the strain gauge bridge without a slip ring using an inductive coupling but the coupling transformer contains fragile ferrites.

Strain gauge-based torquemeters have a large market share because they are relatively cheap and the technology is simple and easily understood. Strain gauge
torquemeters require medium to large amounts of maintenance which limits their usefulness. Torque sensors are commercially available with built in slip rings and speed sensors. A series of torque sensors cover the range from 5Nm –12kNm with a full scale output signal of 50mV. Individual torquemeter units have a torque range of 100:1. The maximum rotational velocity is in the range 24000 – 40000 rpm. The non-linearity over the full scale of the torquemeter is ≈ 0.1%. [10].

1.1.7 Magnetoelastic Torque Sensing

Certain materials exhibit a change in magnetic properties when they change shape and these materials are known as magnetoelastic materials. A common magnetoelastic material used in torquemeters is a titanium nickel alloy [3]. The stress in the shaft is measured via a magnetic circuit – a primary coil forces a magnetic flux through an air gap to the shaft material and then the magnetic flux returns from the shaft to the primary coil via another air gap. Figure (1.4) shows a diagram of a magnetoelastic torquemeter. The stress in the shaft varies the impedance of the coil and, from the stress, torque is calculated. This method requires the magnetoelastic shaft material to be inserted in the driveshaft under test. The response time of magnetoelastic torque sensors are on the order of 0.1ms with a torque range of 6 Nm to 120 Nm.

1.2 Measurement of the Twist of a Rotating Shaft

1.2.1 Shaft Twist Transmission Torquemeters

Torque may be measured by determining the twist produced in a section of shaft and the twist angle per shaft length is linearly proportional to torque. This method of operation is well-known and is based on the reversible linear elastic
properties of metals under small deformation. The shaft will twist by an angle $\theta$ over a length $\Delta L$ under a torsional load of $T$ according to the relation

$$T = \frac{GJ}{\Delta L} \theta$$

(1.6)

where $G$ is the shear modulus of elasticity and $J$ is the polar second moment of area.

If the torsion shaft in a torque meter has been calibrated then Equation (1.6) may be simplified by combining the terms of length $\Delta L$, shear modulus $G$, and polar moment of inertia $J$ into a 'twist factor' $K$ and, providing that the elastic limit of the shaft material is not exceeded, the torque may be expressed as

$$T = K \theta$$

(1.7)

The twist angle $\theta$ can be measured using a variety of techniques including
phase shift, variable optical transmission (Moiré), or with torsional variable differential transformers (TVDT)[16].

1.2.2 Torsion Bar Torquemeter

A bar capable of elastic deformation is inserted in the shaft between the driving and driven machines. A pointer is attached to one end of the bar and a scale to the other and this section is inserted into the shaft under test. The relative angular displacement in the bar can be read because of the stroboscopic effect and persistence of vision. A variety of optical configurations have been devised to read the scale [17] [18], an example of which is presented in Figure (1.5). These instruments are unable to provide reliable measurements when torque variations are present and may misrepresent the true torque transmitted because torque measurements are synchronised with the rotation of the shaft. These fundamental limitations in the read-out of the torquemeter usually require a torsional damper to be included in the system.

Figure 1.5: Diagram of an optically read-out torsion bar torquemeter [19].
1.2.3 Moiré Type Torquemeter

The Moiré type torquemeter is similar to the torsion bar torquemeter but the scale of the torsion bar torquemeter is replaced by two disks with alternating opaque/transparent sectors. Light is passed through both disks and the change in output intensity is detected by a photodetector with the intensity proportional to shaft twist. Sensitivity to shaft vibration is minimised with even illumination of the optical path (around the shaft) and by averaging the signal output from a large area detector or by having a number of detectors. Moiré type torquemeters require that the optical path be enclosed to eliminate the detrimental effects of stray illumination. See Figure (1.6) for a schematic of a Moiré optical torquemeter.

![Moiré Torquemeter Diagram](image)

Figure 1.6: Schematic diagram of a Moiré torquemeter. From Gindy 1985 [16]

1.2.4 Phonic Wheel Torquemeter

A phonic wheel measures the shaft twist by examining the phase relationship between pulses produced from axially separated sensors detecting notches on the shaft. Usually these pulses are produced by changes in reactance due to the magnetic properties of the teeth of a gear wheel but they may also be produced
by attaching a striped tape to the shaft and detecting the bright dark variation with a photodiode. The phonic wheel does not require any electronic circuits to be on the shaft. A diagram of a phonic wheel torquemeter is presented in Figure (1.7). A simple phonic wheel with only one pair of sensors is sensitive to shaft vibration and whirl but this may be minimised by distributing the sensors about the shaft.

The twist of the shaft is derived by the phase relationship of the output signal but if the shaft is not rotating there is no phase relationship and the phonic wheel cannot measure the torque of a stationary shaft. This limitation may be overcome by mounting pickups on a sleeve and rotating the sensors at a known speed but this reduces the torquemeter’s inherent simplicity and greatly increases the size of the torquemeter. Torquemeters are usually calibrated statically by suspending known masses on a lever arm. The inability to statically calibrate a phase shift torquemeter results in making an initial calibration a difficult procedure. The torsion bar is separately statically calibrated (hence determining shear modulus $G$) and the torquemeter is calibrated by combining the knowledge of
the shear modulus and separation between sensors, $\Delta L$. The typical torsion bar twist at full scale applied torque results in the shaft twisting by about one or two degrees. The common rule of thumb for permissible shaft twist is $1^\circ$ over a shaft length of 20 shaft diameters [20]. To obtain this amount of twist (on an undisturbed driveshaft) the torsion bar may have to be 20 to 100cm long in a typical installation [15]. The long length of shaft needed to obtain a reasonable amount of twist might prove to be a disadvantage in certain applications where space is at a premium. An early phonic wheel torquemeter was installed on the liner Queen Mary and this torquemeter required 4 metres of final drive shaft.

The range accuracy of a phonic wheel torquemeter is approximately 1–5% of the full scale reading. Error in torque measurement is mainly due to temperature variations (which alter the shear modulus of elasticity) and the misalignment of the twist detector and shaft axes.

The upper frequency response is limited by the spacing between marks or teeth on the phonic wheel – fewer detector output cycles per revolution results in a lowering of the upper frequency cutoff and consequently in an instrument with poor transient response.

Standard phonic wheel torquemeters are available in the range: 3Nm to 75kNm at rotational speeds of 100rpm to 80000rpm (Torquetronic). These torquemeters have a shaft diameter of 9 – 140mm and the shaft twist is made over a length of shaft 194 – 685mm. The frequency response is up to 50kHz and the overall accuracy (errors due to phase measurement error, shaft properties and hysteresis) is $\pm0.1\%$ full scale deflection in R & D applications and $\pm1\%$ when permanently installed [21].
1.2.5 Torsional Variable Differential Transformers

Torsional variable differential transformers (TVDT) measure shaft twist by measuring variations in a magnetic field.

TVDTs are closely related to linear variable differential transformers. LVDTs are composed of two secondary coils connected in reverse series and a primary coil. Figure (1.8) shows a schematic diagram of a LVDT. The magnetic coupling between the primary and the secondary coils is varied by the motion of a high-permeability core between them. When the primary coil is driven with an ac voltage a corresponding ac voltage is induced in the two secondary coils. The voltage output is linearly proportional to the core displacement and the phase of the output provides directional information. The phase of the output voltage changes by 180° when the core passes the null in centre position. The primary coil is typically excited by a sinusoidal current in the frequency range 60Hz –

Figure 1.8: Schematic diagram of a LVDT. The LVDT is excited by an ac source and the amplitude of the output voltage is linearly proportional to the core position. [9]
A TVDT is a pair of LVDTs arranged to measure the torsional strain of shaft. Figure (1.9) shows the configuration of a TVDT. TVDTs offer linearity over a large range and they have a higher sensitivity than strain gauges. TVDTs are not influenced by the dielectric properties of their environment but they are influenced by proximity to external magnetic fields and magnetic materials and they require that the shaft be made of a non-magnetic material [22].

1.2.6 Commercially Available Torquemeters

A selection of the torque ratings and performance of commercially available torquemeters is listed in Table (1.3). This selection shows the broad range of torquemeters manufactured with maximum torque capacity from a fraction of a Nm to several thousand Nm.
slip ring: high speed
slip ring: low capacity
slip ring: mid capacity
slip ring: high capacity
rotary transformer
rotary transformer
Moiré

<table>
<thead>
<tr>
<th>Description</th>
<th>Torque capacity [Nm]</th>
<th>maximum [rpm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>slip ring: high speed</td>
<td>0.07 - 7</td>
<td>20 000</td>
</tr>
<tr>
<td>slip ring: low capacity</td>
<td>15 - 1200</td>
<td>9 000</td>
</tr>
<tr>
<td>slip ring: mid capacity</td>
<td>8 500 - 68 000</td>
<td>4 000-2 000</td>
</tr>
<tr>
<td>slip ring: high capacity</td>
<td>100 000 - 340 000</td>
<td>1 125</td>
</tr>
<tr>
<td>rotary transformer</td>
<td>0.35 - 7</td>
<td>20 000</td>
</tr>
<tr>
<td>rotary transformer</td>
<td>5 - 11 300</td>
<td>15 000 - 4 000</td>
</tr>
<tr>
<td>Moiré</td>
<td>0.01 - 10 000</td>
<td>35 000</td>
</tr>
</tbody>
</table>

Table 1.3: Range of dominant torquemeters on the market

1.2.7 The Torquemeter World Market

It can be difficult for new products to enter the torquemeter market because the market is mature, saturated and not very profitable; however, a new torquemeter may become profitable if it fills a niche market. A user friendly in situ non-contacting time-resolved torquemeter is novel enough to stimulate the market for torquemeters. Demand for torquemeters will increase as product liability issues push manufacturers to improve reliability, quality and safety. [23] Strain gauges are expected to become less expensive as improved production techniques drive the cost of strain gauges down but because mounting a strain gauge is manually intensive a large reduction in strain gauge torquemeter prices is unlikely.

Table (1.4) shows the world-wide revenue in US dollars generated by torquemeter sales in the years 1991-1995.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Revenue [§ millions]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>21.0</td>
</tr>
<tr>
<td>1992</td>
<td>22.0</td>
</tr>
<tr>
<td>1993</td>
<td>23.8</td>
</tr>
<tr>
<td>1994</td>
<td>24.0</td>
</tr>
<tr>
<td>1995</td>
<td>25.2</td>
</tr>
</tbody>
</table>

Table 1.4: World Market for Torquemeters [23]

Torquemeter market share is roughly divided equally between automotive,
aeronautical and process control applications as shown in Figure (1.10).

![Figure 1.10](image)

Figure 1.10: Category of application in the projected 1997 $28 million torquemeter market. [23]

The price of a strain gauge torquemeter is obviously linked to the size and rating of the torquemeter and the associated readout and interface options. Table (1.5) lists the prices of a selection of torquemeters

<table>
<thead>
<tr>
<th>Type</th>
<th>Output Type</th>
<th>Range [Nm]</th>
<th>Price [US$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain Gauge</td>
<td>Digital Meter</td>
<td>0 – 12</td>
<td>1600</td>
</tr>
<tr>
<td>Strain Gauge</td>
<td>0–10V analogue</td>
<td>0 – 1000</td>
<td>2800</td>
</tr>
<tr>
<td>Strain Gauge with thrust bearings</td>
<td>0–10V analogue</td>
<td>5 – 200</td>
<td>6000</td>
</tr>
<tr>
<td>Strain Gauge with thrust bearings</td>
<td>0–10V analogue</td>
<td>200 – 5000</td>
<td>7200</td>
</tr>
<tr>
<td>Moiré</td>
<td>analogue</td>
<td>$3.5 \times 10^{-3}$ – 15</td>
<td>3000</td>
</tr>
<tr>
<td>Moiré</td>
<td>analogue</td>
<td>15 – 600</td>
<td>2500</td>
</tr>
<tr>
<td>Moiré</td>
<td>analogue</td>
<td>600 – 3000</td>
<td>8000 – 9000</td>
</tr>
</tbody>
</table>

Table 1.5: Prices of some commonly available torquemeters [24] [25].

1.3 The Laser Torquemeter

The laser torquemeter is a non-contacting instrument that measures twist, and hence torque, of a rotating shaft. The laser torquemeter was developed in the
late 1980’s and early 1990’s [26] [27] [28]. The laser torquemeter uses two probe beams, axially separated, illuminating a shaft with known mechanical properties as shown in Figure (1.11). When the coherent light illuminates the optically rough surface of the shaft it scatters light which forms an interference pattern in space. This takes the form of an array of light and dark regions due to constructive and destructive interference of wavelets scattered from the surface. This is called a speckle pattern. When the shaft rotates the speckle pattern incident on the photodetector aperture changes causing a continuous variation in the photodetector output. The photodetector output is periodic and recurs with revolution of the shaft. A periodic ‘speckle’ pattern is sampled by suitably positioned photodetectors and these periodic photodetector output signals are recorded under no-load conditions. The photodetector output signals are continuously compared with the recorded signals and for each channel the peak of the cross-correlation between the recorded signal and live signal is tracked. Relative angular displace-
ment between the peak cross-correlations of each channel is determined from the relative time delays \( \tau_a \) and \( \tau_b \) and allows calculation of shaft torque. Typical signals are shown in Figure (1.12). In channel A the live signal leads the recorded signal (or expected position of the shaft) and in channel B the live signal lags the recorded signal. The laser torquemeter is dependent on the repetition of the speckle pattern with revolution of the shaft and the similarity between the live photodetector output signal and the recorded signal is important in the robustness of the torquemeter.

The torquemeter electronics require, for a reliable low noise estimate of torque, that the ‘recorded’ signal be highly correlated with the live signal. This thesis investigates the correlation properties of the photodetector output signals and how they relate to various motions of the shaft. To achieve the aim of keeping the live signal correlated with the recorded signal the following parameters may be altered:

![Figure 1.12: Typical signals and their temporal relationships.](image-url)

- Channel A
- Channel B
$z$ detector – shaft range

$z_0$ laser beam waist – shaft range

$D$ size of illuminated spot on shaft

$L_y$ photodetector length

$L_x$ photodetector width

and if any optical elements are included. A lens may be positioned between the shaft and detector and its focal length and position may be altered. A lens may also be positioned between the shaft and laser and its focal length and position may be altered.

The design of the optical head of the laser torquemeter is complicated because many of these parameters are interrelated and cannot be varied independently.

This thesis addresses the issues related to the correlation of the live photodetector output signal and the recorded signal and how these available parameters may be utilised to produce a laser torquemeter.
Chapter 2

Speckle

2.1 Fundamental Properties of Speckle

When optically rough surfaces are illuminated with coherent light, the scattered beams have a random spatial variation of intensity. This 'speckle pattern' bears no obvious relationship to the illuminated object and appears chaotic and unordered and is best described by methods of probability and statistics. The speckle pattern formed in space is due to the self-interference of scattered waves from a number of scatterers and requires a coherent source for generation. The structure of the speckle pattern depends on the coherence properties of the illuminating beam and on the surface characteristics of the diffuse object. Figure (2.1) shows a typical laser speckle pattern.

This chapter gives an overview of speckle research including applications of speckle and methods of describing a speckle pattern.

2.1.1 Introduction

The discovery of the speckle phenomenon predates the discovery of the laser by over 100 years; in the late 1870s, a speckle pattern was sketched by Exner [29].
When observing the central region of a pattern diffracted by a large number of small particles, a radially fibrous structure was observed under white light. When a red filter was inserted the fibrous quality was replaced by a fine granular structure. Exner recognised that earlier ideas about the superposition of a large number of identical waves with random phase were wrong and that this superposition must lead to large fluctuations in local intensity [30]. The physical theory behind the phenomenon is well discussed in the literature. Von Laue felt that Rayleigh’s theory for the addition of $n$ waves with the same frequency but random phases was applicable to the problem of speckle and was able to show that the probability density of the intensity is a negative exponential and that the contrast is unity [31]. Von Laue was able to derive an expression for the joint probability density of intensity.

The knowledge of speckle then lay relatively dormant throughout the middle
of the 20th century until the development of the laser in the early 1960s. A flurry of papers followed the invention of the laser and the number of papers relating to the theory and applications of speckle increase each year.

At first speckle was perceived as a nuisance and something to be eliminated and, in 1971, Gabor [32] wrote:

'It is not really noise; it is information which we do not want, information on the microscopic unevenness of the paper in which we are not interested.'

This comment marked a turning point; now speckle is not just regarded as an annoyance but can be exploited to advantage in practical applications. The current major interests and applications of laser speckle may be broadly divided into the following categories:

• research into the fundamental statistical properties of speckle patterns [29] [33] [34] [35]

• research into the reduction of speckle in holography & optical systems [36]

• measurement of surface roughness via speckle [37] [38] [39]

• applications in metrology (which may be subdivided) [40]

  – measurement of displacement & strain
    * ESPI [41]
    * speckle photography [42] [40]
    * correlation linear and rotary encoders [43] [44] [45]

  – velocity measurement
* 'time of flight' velocity measurement by speckle correlation [46] [47]
  
* speckle statistic velocity measurement [48] [46] [1] [49]
  
  - measurement of deformation (e.g. angular tilt) [50] [44]
  
  - vibration measurement [51]
  
  - surface roughness profiling [52]

• stellar speckle interferometry [53] [54]

There is research into other closely related phenomena where the results of speckle statistics directly apply; for example, electromagnetic radiation at non-optical wavelengths and non-electromagnetic waves in other media. Topics where speckle may be analogous to these research areas are as follows:

• radar clutter [55] [56]
  
• acoustic speckle [57] [58]
  
• application of classical coherence theory [59] [60]

2.2 Single Point Statistics

As the exact microscopic structure of the scattering surface is unknown, statistics are required to describe the resulting speckle pattern produced by the surface. Several assumptions are made to clarify the analysis:

• light sources with a high degree of temporal and spatial coherence are easily available and in this analysis we need not worry about speckle patterns formed by partially coherent light. The coherence length \( l_c \) of the laser must be greater than the surface roughness and for most common surface finishes
and lasers this criterion is easily met. The coherence length of diode lasers is at least a few centimetres and for multimode HeNe lasers approximately 20cm. Singlemode HeNe lasers have a coherence length of 300m [61]. This means that the light source is almost perfectly monochromatic as the spectral width $\Delta v_c$ is related to the coherence length and the speed of light $c$ by [61]

$$\Delta v_c = \frac{c}{l_c} \quad (2.1)$$

for a rectangular spectral density.

- polarised light illuminates the target and no depolarisation occurs as a result of the scattering process at the surface of the object.

- the scatterers are randomly distributed over the area of the scattering surface.

- all scattering takes place at the surface of the object (i.e. there is no volume scattering).

A normal speckle pattern is considered to be due to a highly coherent beam incident upon a large area of an optically rough surface and where all the above conditions apply [29].

Figure (2.2) shows the coordinate system for examining the statistical properties of speckles produced in the diffraction field by the random phase screen. With reference to Figure (2.2) the Cartesian coordinates of position at the object plane are denoted by $\xi = (\xi, \eta)$ and of position at the detecting plane by $x = (x, y)$ which is at an axial distance $z$ from the object plane.
Figure 2.2: The coordinate system for examining the statistical properties of speckles produced in the diffraction field by the random phase screen.

Following Goodman [62], speckle may be described by a complex-valued analytic signal: \( u(x, y, z; t) \), at the observation point \((x, y, z)\) and at time \(t\).

\[
\begin{align*}
    u(x, y, z; t) &= E(x, y, z) \exp[j 2\pi vt], \\
    E(x, y, z) &= E(x, y, z) \exp[j \theta(x, y, z)].
\end{align*}
\]  

(2.2)

(2.3)

where \(v\) is the optical frequency and where the complex electric field phasor amplitude \(E(x, y, z)\) is given by

The electric field strength at a point in space is due to the sum of electric fields contributed by each portion of the scattering surface. At the frequencies involved with visible light, the electric field from each contribution cannot be detected directly but a photodetector is used because it responds to the optical power incident upon it. The optical power (or intensity) is proportional to the square of the total electric field and the photodetector produces an output voltage that
is proportional to the incident intensity. Thus the intensity \( I(x, y, z) \) at a point is proportional to the square of the electric field intensity

\[
I(x, y, z) = \alpha \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |u(x, y, z; t)|^2 \, dt = \alpha |E(x, y, z)|^2. \tag{2.4}
\]

We would like to determine the probability distribution of the intensity \( I \); this distribution is equivalent to the problem of the random walk studied by Lord Rayleigh [63]. The electric field of the speckle pattern at a point in space may be regarded as the sum of the contributions from all the illuminated scattering elements on the rough surface

\[
E(x, y, z) = \sum_{k=1}^{N} |a_k| \exp[j \phi_k], \tag{2.5}
\]

where \( N \) is the number of scattering elements and \( a_k \) and \( \phi_k \) are the amplitude and phase of the contribution due to the \( k \)th scatterer.
In order to use the results of Rayleigh for classical random walk problems and then link these results to the speckle pattern problem, the following assumptions are required: the amplitude and phase of each elementary phasor contribution must be statistically independent of each other and all other elementary phasors. If the surface variations are large compared to the optical wavelength, the associated phase fluctuations are uniformly distributed between \((-\pi, \pi)\).

The Central Limit Theorem states that a random variable resulting from the sum of a large number of independent random variables results in a Gaussian distribution in the limit when the number of terms approaches infinity [64]. Provided that the number of scatterers is large, the application of the Central Limit Theorem results in the following conclusions: a) the real and imaginary components of the field at \((x, y, z)\) are independent, zero mean, identically distributed Gaussian variables and b) the intensity \(I(x, y, z)\) has a negative exponential probability density distribution

\[
p(I) = \begin{cases} 
\frac{1}{\langle I \rangle} \exp \left[ \frac{-I}{\langle I \rangle} \right], & \text{when } I \geq 0 \\
0, & \text{otherwise},
\end{cases}
\]  

(2.6)

where \(\langle I \rangle\) is the mean intensity and \(p(I)\) is the probability density distribution. The intensity distribution of Figure (2.1) (512 \(\times\) 480 pixels, pixel size 8.6\(\mu m \times 8.3\mu m\)) is plotted in Figure (2.4). Background light and dark bias of the CCD causes some intensity signal when there should be none and shifting the histogram to the right [65]. The minimum pixel intensity value is 16 and the maximum 255. The camera has an intensity range of 0 to 255. The quantity \(\sigma_I/\langle I \rangle\), which should be 1 for a negative exponential, is 0.52. When the first two bins of the histogram are removed, the quantity \(\sigma_I/\langle I \rangle\) increases to 0.95. Plotting the same data with
a logarithmic scale in the y axis results in negative exponential distributions producing a straight line plot. The adjusted intensity distribution of the speckle pattern is plotted in Figure (2.5). The intensity 'blip' in the most intense bin is due to saturation of the CCD detector and this bin contains a count of all pixels with intensity greater than the maximum value.

The phase of the resultant field is uniformly distributed in \((-\pi, \pi)\) [66]. When a large number of independent scatterers introduce a phase change, with a uniform distribution over \(2\pi\), \(E(x, y, z)\) is then a circular complex Gaussian variable. Speckle patterns that satisfy these assumptions are commonly referred to as Gaussian speckles or fully developed speckle patterns.

### 2.2.1 Speckle Contrast

The ratio of the standard deviation \(\sigma_I\) of intensity to mean intensity may be written as

\[
C = \frac{\sigma_I}{\langle I \rangle} = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle}
\]

and the ratio \(C\) is used as a measure of the contrast of the speckle pattern [66]. The contrast of a fully developed polarised speckle pattern is always unity.

These results are not true if the phase distribution scattering surface is not evenly distributed over \((-\pi, \pi)\), nor if the number of scattering elements is small (often because the scattering elements are large), nor if the scattering surface depolarises the incident illumination on scattering[67] [68] [69] [70] [71]. If a large number of scatterers are illuminated and there is no depolarisation by the surface but the resulting phase variation is less than \((-\pi, \pi)\) then the resulting statistical distribution carries information about the surface roughness. This departure in
Figure 2.4: A plot of the intensity distribution of the speckle pattern in Figure (2.1).

Figure 2.5: A log plot of the adjusted intensity distribution of the speckle pattern in Figure (2.1). Ideal negative intensity distribution is plotted as a solid line.
the statistics of the speckle pattern from a fully developed speckle pattern may be utilised to make a surface roughness meter and this is explored in Section (2.5.1).

2.3 Second Order Properties of Speckle

The first order statistical properties describe the intensity only at a single point in space and are unable to describe the spatial structure of the speckle pattern. Viewed at many points, a speckle pattern consists of peaks and nulls of intensity with many scale sizes and an appropriate method of expressing the coarseness of the speckle pattern is to determine its spatial frequency distribution. Figure (2.6) from Yoshimura and Iwamoto [72] shows the spatial structure of a typical speckle pattern at an arbitrary plane in space. The speckles are 'cigar' shaped volumes in space and point away from the centre of the scattering surface and become more elongated with increasing distance from the scattering surface. An extension of the longest axis of the speckles crosses the centre of the scattering surface and the speckle pattern in space appears to diverge from this common origin [59] [73] [74] [75] [76] [77] [78].

2.3.1 Mean Speckle Size

An obvious characteristic of the speckle pattern is the average width of a speckle. A variety of estimates of speckle size are reported in the literature and this is due to differing diffuse object illuminating conditions and alternative definitions of width. The speckle width \( \sigma_0 \) is usually defined as the first minimum of the normalised spatial autocorrelation function or the width to the point where the intensity drops by a factor of \( 1/e^2 \) if autocorrelation has a Gaussian form.

In the case where the illuminated spot is uniform and circular with a diameter
Figure 2.6: Formation of three dimensional speckles in space from Yoshimura and Iwamoto [72]

$D_0$, the separation from the scattering surface to the detecting plane is $z$, the mean speckle width is defined as the first zero of the autocorrelation function and is given by [33]

$$\langle \sigma_0 \rangle = 1.22 \frac{\lambda z}{D_0}. \quad (2.8)$$

If the illuminated spot is square, with sides $D$ and the speckle width defined as the first zero of the autocorrelation function in the $x$ or $y$ direction, the speckle width may be given by [66]

$$\langle \sigma_0 \rangle = \frac{\lambda z}{D}. \quad (2.9)$$

Illuminating the surface with a Gaussian spot of diameter $D$ (measured to the $1/e^2$ of peak value) and autocorrelating the resulting speckle pattern results in
another estimate of speckle width. If the speckle width is interpreted as the $1/e^2$ width of the autocorrelation function, an estimate of the speckle width, $\langle \sigma_0 \rangle$, in the detecting plane is:

$$\langle \sigma_0 \rangle = \frac{2\sqrt{2}}{\pi} \frac{\lambda z}{D}$$

(2.10)

This equation most accurately describes the experimental configuration although for rough calculations Equation (2.9) is used ($2\sqrt{2}/\pi \approx 1$).

A comparison of the various interpretations of speckle width are shown against a simulated speckle pattern in Figure (2.7). In this plot, a speckle pattern is simulated by applying a 2-dimensional fast Fourier transform (FFT) to a complex array containing a circular central region with constant amplitude and random phase. Each element in the speckle pattern array is then squared, the result autocorrelated, and contour lines plotted at $1/e$, $1/e^2$ and 0.

### 2.3.2 Gaussian Beams

The spatial distribution of the light output of a laser depends on the geometry of the laser resonator and active medium. A laser with spherical mirrors supports a Gaussian beam. The beam power is mostly concentrated around the beam axis. The intensity distribution is a circularly symmetric Gaussian function centred on the beam axis. The wavefronts of a Gaussian beam are planar near the beam waist (at $z = 0$) and spherical far from the origin ($z \to \infty$).

The beam diameter (to where the beam intensity drops by a factor of $1/e^2$)
may be determined using the following formula [61]

\[ D = 2w_0 \left[ 1 + \left( \frac{z\lambda}{\pi w_0^2} \right)^2 \right]^{1/2} \]  

(2.11)

where \( w_0 \) is the beam waist radius as shown in Figure (2.8).

In addition, the wavefront curvature of a Gaussian laser beam is described by

\[ r = z_0 \left[ 1 + \left( \frac{\pi w_0^2}{z_0\lambda} \right)^2 \right]. \]  

(2.12)

The sign notation with respect to the wavefront curvature is applied in the following way: \( r \) is positive when the beam waist is to the left of the wavefront. The beam parameters are shown in Figure (2.8).
The preceding statistical results are interesting but they relate to single point detecting apertures. Practical photodetectors have a finite size, usually with a circular or rectangular shaped sensitive area. Their output is proportional to the intensity of the speckle pattern spatially integrated over the detecting surface. In order to construct a practical speckle-based instrument, the statistical properties of integrated speckle need to be understood. The spatially integrated intensity $I_A$ may be given by

$$I_A = \frac{1}{S} \iint_{-\infty}^{\infty} A(x, y) I(x, y) \, dx \, dy,$$  \hspace{1cm} (2.13)
where $A(x, y)$ is the aperture function and, for a uniformly sensitive detector, is given by

$$A(x, y) = \begin{cases} 
1, & \text{inside aperture} \\
0, & \text{outside aperture}
\end{cases}$$

(2.14)

and $S$ is the effective area of the aperture.

Goodman [66] provides an estimation of the contrast of the integrated intensity as

$$\frac{\sigma_{IA}}{\langle I_A \rangle} = \frac{1}{\sqrt{Q}}$$

(2.15)

for a circular Gaussian shaped intensity pattern incident on the scattering surface and where $Q$ is interpreted as the number of speckle correlation cells within the measurement aperture. $Q$ is given by

$$Q = \left( \sqrt{\frac{S_c}{S_m}} \operatorname{erf} \left( \sqrt{\frac{\pi S_m}{S_c}} \right) - \frac{S_c}{\pi S_m} \left( 1 - \exp \left[ -\frac{\pi S_m}{S_c} \right] \right) \right)^{-2}$$

(2.16)

where $S_c = \frac{1}{4} \pi \langle \sigma_0 \rangle^2$ is the speckle area and $S_m$ is the area of the integrating aperture. Experimental measurements by McKechnie in 1974 confirmed the validity of this approximation [34].

2.5 Effect of Scattering Surface

2.5.1 Effect of Surface Roughness

The surface roughness is dependent upon a number of factors including machining speed, tool geometry, tool wear, tool feed and machining pattern. Surface rough-
ness may be specified and measured using the parameter $R_a$ which is defined as the mean departure of profile from a reference line. In the United Kingdom this parameter is referred to as the CLA (centre line average) value. Table (2.1) summarizes the range of surface roughness for some common manufacturing processes and Table (2.2) summarizes the range of surface roughness for shafts.

<table>
<thead>
<tr>
<th>Surface Process</th>
<th>Roughness value $R_a$ [(\mu m)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand casting</td>
<td>12.5 – 25</td>
</tr>
<tr>
<td>Die casting</td>
<td>0.8 – 1.6</td>
</tr>
<tr>
<td>Grinding</td>
<td>0.1 – 1.6</td>
</tr>
<tr>
<td>Finish machining</td>
<td>0.5 – 1.0</td>
</tr>
</tbody>
</table>

Table 2.1: Typical surface roughness levels for a selection of common manufacturing processes [79].

<table>
<thead>
<tr>
<th>Shaft Finishing Process</th>
<th>Roughness value $R_a$ [(\mu m)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turned, Ground and Polished</td>
<td>0.2 – 0.5</td>
</tr>
<tr>
<td>Turned and Polished</td>
<td>0.4 – 1</td>
</tr>
<tr>
<td>Cold Drawn</td>
<td>1.25 – 3.2</td>
</tr>
</tbody>
</table>

Table 2.2: Normal range of surface finish for shaft finishing operations [20].

When illuminating a surface with a non-monochromatic source the radial structure of the speckle patterns produced depends upon the temporal coherence of the light source and the surface roughness. Also the contrast of the speckle pattern at the detector depends upon the temporal coherence of the light source, the position of the detector in the detection plane and the surface roughness [37] [80] [81] [82] [83].

In order to create a speckle pattern which may be utilised by the torquemeter the following conditions are required:
• Surface height variation must be uniformly distributed in at least the range $[0, \lambda/2]$. If the surface is smooth there are two effects which reduce the viability of the torquemeter:

1. there is insignificant scattering of the incident radiation and the incident optical energy may be reflected away from the detecting aperture
2. the resulting speckle pattern has low contrast and hence the photodetector signal does not vary significantly with revolution of the shaft

• The surface should not have a regular pattern or lay which produces a regular pattern in the diffraction field. The backscattered pattern may go through several repetitions per revolution resulting in multiple correlation peaks between the recorded signal and the live signal. An ambiguity in which correlation peak to track results in an unreliable torquemeter.

Application of a thin layer of paint or tape can simply eliminate the above problems with negligible interference to the mechanical system being measured. Retro-reflective tape was applied to the experimental shafts in this study in order to improve the intensity of the backscattered light.

2.5.2 Effect of Retro-Reflective Tape

Retro-reflective tape is used to increase the intensity of backscattered light from the scattering surface and the tape also provides variations in optical path length due to surface roughness which produces optical phase changes with a uniform distribution. The retro-reflective tape consists of one layer of small (40-60\(\mu\)m) spherical, high refractive index beads impregnated in a reflecting base material. Incident radiation is concentrated by the beads into the base material, where it is
scattered and then collected and directed back towards the source of the incident radiation. The scatter from retro-reflectors is commonly referred to as enhanced backscatter [84].

Interference of light occurs only when the electric fields are aligned. If the light scattered from the surface of an object is partially depolarized, two orthogonally polarized speckle patterns may be seen. When the speckle pattern formed is observed without a polarisation analyzer it may be regarded as the sum of two speckle patterns [66] and the contrast of the depolarized speckle pattern may be reduced by up to \(1/\sqrt{2}\). Illuminating a surface covered in retro-reflecting tape with linearly polarised light and examining the backscattered radiation with an analyser shows that the retro-reflective tape is good at preserving the polarisation properties of the incident beam. There is a good extinction ratio as the analyser is rotated. The polarization preserving properties of the retro-reflective tape removes the need for a polarization analyzer to block one of the orthogonally polarised speckle patterns from photodetectors. Fine particles of white magnesium dioxide or 'Developer' (a common surface coating used to increase the backscattered intensity) applied to the surface does not preserve polarisation.

The retro-reflective tape acts as a non-periodic array of apertures resulting in the backscattered speckle pattern having an intensity envelope corresponding to a diffraction pattern produced by an aperture on the order of the effective area of the beads. An Airy disk is the diffraction pattern produced by a circular aperture and varies with wavelength, \(\lambda\); aperture–detecting plane range, \(z\); and aperture size, \(D_A\). The radius of the disk is \(1.22\lambda z/D_A\). (Figure 2.9 shows an image of an Airy disk pattern.) The far-field diffraction pattern due to the effective apertures in the tape does not form an ideal Airy disk pattern because a number of slightly different sized apertures are illuminated and the resulting speckle pattern has an
Airy disk envelope. The speckle pattern is due to the longitudinal and lateral variation of the beads embedded in the medium.

Retro-reflective tape was applied to a shaft which was then rotated at a constant angular velocity. The shaft was illuminated by a HeNe laser and the backscattered light observed on a screen several metres away. Recording the range to the screen from the surface of the shaft and the width of the first null in the diffraction pattern enables an estimate of the effective aperture of the retro-reflective tape to be made. For the retro-reflective tape used (3M Scotchlite high gain sheeting type 7610) in the following experiments the effective aperture is \(52 \pm 4 \mu m\).

The above described diffraction effect is significant because it places an upper size limit on the dimensions of the photodetector for a given range. Any linear dimension of the aperture of the photodetector must be significantly smaller than the Airy disk central region to justify the approximation of a ‘top hat’ aperture transmission function otherwise a soft Gaussian aperture transmission function is more appropriate approximation of the Airy function. In Section (3.4.1) a more
detailed explanation is given of how the aperture transmission function modifies the aperture autocorrelation function.
Chapter 3

Speckle Decorrelation

3.1 Introduction

As described in Section (2.1), a speckle pattern is produced in an observing plane when coherent light is scattered from an optically rough surface. The speckle pattern changes when the surface moves. Speckle can be used to make non-contacting measurements of moving objects; this method has been utilised in many instruments. The rotational velocity of the surface may be determined [85] by auto-correlating the photodetector output signal. Rotation angle of a shaft may be determined by measuring speckle displacement caused by shaft rotation [43] [86] [87] [88]. This method requires an image sensor which tracks the movement of the speckles across the sensor to determine speckle displacement from which knowledge of the optical configuration then allows the computation of shaft rotation. In plane translational motion (velocity and displacement) may be measured by cross-correlating the output signals from two spatially separated detectors or by simply autocorrelating the output from one photodetector if the optical configuration is fixed and calibrated [44] [45] [89] [90] [91].

Speckle decorrelation is of key importance to the operation of the laser
torquemeter (as introduced in Section 1.3). The laser torquemeter is based on the fundamental observation that the speckle pattern appears to change (translate or boil [92] [46]) with each revolution of the shaft. The speckle pattern translation is not at the same velocity as the surface. The speckle pattern repeats with revolution of the shaft. Sampling the backscattered speckle pattern with a stationary photodetector produces an electrical output signal which is periodic with the revolution of the shaft. In order to measure the twist of a rotating shaft, and hence torque, the output from a pair of photodetectors may be utilised to produce a torquemeter. In this chapter the influences of the rotational and vibrational motions of the shaft and their effects on the repeatability of the photodetector output signal are explored theoretically and verified experimentally.

3.2 Fundamental Principle of Laser Torquemeter Operation

The laser torquemeter is a non-contacting instrument that measures twist, and hence torque, of a rotating shaft [27]. The working principles of the torquemeter are briefly described here while the operation of the torquemeter is more rigorously explored in Chapter 4. Figure (3.1) shows a schematic diagram of a laser torquemeter, which uses two probe beams, axially separated, illuminating a shaft with known mechanical properties. The backscattered speckle pattern is sampled by photodetectors and the periodic photodetector output signals are recorded under no-load conditions while the shaft is rotating. In order to increase the amount of light back-scattered to the photodetector the target surface is covered in retro-reflective tape. Once acquisition of the no-load photodetector signal is achieved, load can be applied and the photodetector output signals are
Figure 3.1: Schematic diagram of the Laser Torquemeter. The illuminated spots are separated by an axial distance $\Delta L$ and the photodetectors are at range $z$ from the shaft surface.

continuously compared with the recorded signals and for each channel the peak of the cross-correlation between the recorded signal and live signal is tracked. The temporal difference of the live and recorded photodetector output signals from each illuminated point is determined and allows the calculation of shaft twist, and hence torque. The laser torquemeter is dependent upon the repetition of the speckle pattern with the revolution of the shaft. The decorrelation between the live photodetector output signal and the recorded signal is important in determining the robust operation of the torquemeter.
3.3 Motions Causing Decorrelation of the Photodetector Output Signal

Motion of the shaft will change the speckle pattern at the detecting aperture and therefore change the photodetector output signal with reference to Figure (3.2). Rotational motion of the shaft about its \( \zeta \) axis provides a periodic photodetector output signal and is fundamental to the operation of the laser torquemeter. An understanding of the effects of this motion is required as this motion is linked to the minimum resolvable twist of the shaft and to the torque range to be measured by the instrument. Other motion of the shaft is undesirable and affects the periodicity of the photodetector output signal. If not adequately controlled this motion will result in the inability of the instrument to track the current signal and the torquemeter will cease to operate. Figure (3.3) shows the orientation of the various motions. A summary of the four shaft motions follows:

![Figure 3.2: Schematic diagram of the optical arrangement showing the coordinate axes.](image-url)
Rotational Motion Rotation of the shaft about its axis of symmetry $\zeta$ causes a change in the speckle pattern characteristics and hence a change in the photodetector output signal. This change in the speckle pattern is due to different scattering sites being illuminated as the shaft rotates – a slight rotation results in a few new scatterers being illuminated and results in only a small variation in photodetector output whereas a larger angular displacement results in a large variation. The angular displacement about $\theta_z$ required to cause a prescribed reduction in the autocorrelation of the photodetector output signal is referred to as the rotational correlation angle $\theta_{kz}$. In this thesis, a reduction to $1/e^2$ of the peak value is used as a convenient measure of decorrelation. The sensitivity of the photodetector outputs (and hence the torquemeter) to rotational motion is fundamental to its operation – too few fluctuations in the photodetector output per revolution means that small shaft twists cannot be resolved while the upper frequency response of the electronics, detector and processing equipment limits the maximum number of fluctuations per revolution. The number of signal fluctuations per second is linked to the rotational speed of the shaft and increasing the
rotational correlation angle reduces the bandwidth of the photodetector output.

**Axial motion** Axial motion (along the $\xi$ direction) of the rotating shaft, commonly called shaft float, will cause only a proportion of the original population of scatterers to be re-illuminated in each subsequent rotation and hence generates a different periodic speckle pattern. The amount of change in the detected speckle pattern is dependent on the amount of shaft axial motion. The photodetector outputs become dissimilar from those recorded in memory. The ability of the laser torquemeter to track the shaft twist is therefore compromised by axial vibration of the shaft.

**Tilt Motion** If the shaft tilts – yaw motion $\theta_\eta$ – as it rotates, the backscattered speckle pattern moves at a velocity proportional to twice the shaft tilt rate and the speckle pattern on the photodetector changes significantly and correlation between the photodetector output signal and the recorded signal is likewise reduced. A pitch $\theta_z$ motion of the shaft appears similar to vertical $\eta$ translation of the shaft and is addressed below. Tilt of the shaft around the $\eta$ axis causes large changes in the detected photodetector output signal whereas tilt about the $z$ axis causes minor changes in the speckle pattern.

**Radial Motion** Translational motion of the shaft perpendicular to the rotational axis and perpendicular to the optical axis can modify the detected speckle pattern and compromise the operation of the torquemeter. Motion in direction $z$ produces subtle changes in the speckle characteristics while motion in the direction $\eta$ causes changes in the speckle pattern as well as changing the position of the speckle pattern.
3.4 The Space-Time Correlation Function

The most appropriate tool used to investigate the dynamic properties of speckles is the space-time correlation function of the detected speckle intensity. This function relates properties of dynamic speckle at a point in space \((x_1, y_1)\) and at time \(t\) with the speckle pattern at another point \((x_2, y_2)\) and time \(t + \tau\).

An intuitive approach to reducing the sensitivity of the torquemeter to vibrations, is to allow the detecting aperture to span a large number of speckles. This allows for a small vibrational motion of the target to displace some of the speckles off the aperture and introduce a few new speckles onto the aperture without significantly affecting the total photodetector output signal. In the experimental measurement of the intensity of a speckle pattern, the detector aperture must be of finite size and the space-time correlation function of the spatially integrated speckle intensity is applied.

A typical configuration for one channel of a laser torquemeter is shown in Figure (3.4). A photodetector for the laser torquemeter would be ac-coupled to avoid a slowly changing background intensity becoming incorporated into the reference signal and thus it is prudent to consider only speckle intensity fluctuations.
defined by $\Delta I(t) = I(t) - \overline{I(t)}$. The space-time correlation function $\Gamma_{\Delta I}(X, \tau)$ of the speckle intensity fluctuation in the detecting plane is defined by:

$$\Gamma_{\Delta I}(X, \tau) = \langle \Delta I(x, t) \Delta I(x - X, t + \tau) \rangle$$  \hspace{1cm} (3.1)

where $x$ is the position $(x, y)$ and the quantity $X = (X, Y)$ describes the spatial delay between the position of a detecting aperture at time $t$ and the position of a detecting aperture at time $(t + \tau)$ and where $t$ and $\tau$ are respectively time and temporal delay. The symbols $\langle ... \rangle$ denotes the ensemble average. Introducing this delay into the aperture function leads to a space-time correlation function rather than simply a time correlation function.

![Figure 3.5: Object and observation planes. The vector $x$ describes a position in the detecting plane and the vector $X$ describes the spatial delay of the detecting aperture at time $t$ and time $t + \tau$.](image)

If speckles are integrated over a detecting aperture, the integrated speckle intensity fluctuation space-time correlation function, $\Gamma_A(X, \tau)$ becomes:

$$\Gamma_A(X, \tau) = \langle I_A(x, t) I_A(x - X, t + \tau) \rangle$$  \hspace{1cm} (3.2)
where the intensity is integrated over the aperture function, $A(x)$, by

$$ I_A(x, t) = \int_{-\infty}^{\infty} A(x) \Delta I(x, t) \, dx. \quad (3.3) $$

The integrated speckle intensity fluctuation space-time correlation function becomes

$$ r_A(X, T) = \left\langle \left[ \int_{-\infty}^{\infty} A(X_1) \Delta I(X_1, t) \, dX_1 \int_{-\infty}^{\infty} A(X_2 - X) \Delta I(X_2, t + \tau) \, dX_2 \right]^2 \right \rangle. \quad (3.4) $$

In this equation, each integral describes the summation of speckle intensity across the detector's active area and the spatially integrated speckle intensity is then cross-correlated. Reordering the integration results in

$$ r_A(X, T) = \int \int A(X_1) A(X_2 - X) \left( \Delta I(X_1, t) \Delta I(X_2, t + \tau) \right) \, dX_2 \, dX_1. \quad (3.5) $$

The vector quantities representing spatial positions may be substituted as follows: $x = X_1$ and $\Delta x = X_2 - X_1$. Moving the ensemble average within the integral, and reordering the integration, results in a useful expression for the spatially integrated correlation function:

$$ \Gamma_A(X, \tau) = \int \int A(x_1) A(x_2 - X) \left( \Delta I(x_1, t) \Delta I(x_2, t + \tau) \right) \, dx_2 \, dx_1. \quad (3.5) $$

The inner bracketed term is recognised as the spatial autocorrelation of the aperture function as a function of $\Delta x - X$. If the aperture is rectangular with dimensions $L_x$ by $L_y$ then the autocorrelation function of the aperture is a prod-
uct of triangle functions. The ensemble term is recognised as the space-time correlation function identical to Equation (3.1) in terms of $\Gamma_{\Delta t}(\Delta x, \tau)$.

### 3.4.1 Approximation of a Triangular Function by a Gaussian Function

In the analysis to be presented here, a Gaussian expression is used to approximate the autocorrelation function of the aperture. If the transmission of the aperture is a Gaussian function the aperture is considered to be 'soft' as opposed to a rectangular aperture function which is considered 'hard'. This method involves approximating the triangular aperture autocorrelation function with a Gaussian function governed by $L_x$ and $L_y$ which are the dimensions of the rectangular aperture in the $x$-$y$ plane.

An appropriate expression for the aperture autocorrelation function, with the vector $(\Delta x - X)$ expanded as $(\Delta x - X, \Delta y - Y)$, is:

$$
\int_{-\infty}^{\infty} A(x)A(x + \Delta x - X)dx = L_x \sqrt{\frac{\Delta x - X}{L_x}} L_y \sqrt{\frac{\Delta y - Y}{L_y}}
$$

$$
\approx L_x \exp \left[ -k_0 \left( \frac{\Delta x - X}{L_x} \right)^2 \right] L_y \exp \left[ -k_0 \left( \frac{\Delta y - Y}{L_y} \right)^2 \right]. \tag{3.7}
$$

Numerical analysis was used to calculate the total square error between the actual aperture autocorrelation function and its Gaussian approximation. The calculation was performed across a widening range of spatial delay from the aperture width up to a range very much wider than the actual aperture width. An area error term is defined as

$$
\text{Area Error} = \frac{\sum_i |Aa_i - Ga_i|}{\sum_i Aa_i} \tag{3.8}
$$
where $A\alpha_i$ is the true aperture autocorrelation function and $G\alpha_i$ is the Gaussian approximation of the autocorrelation function. For a rectangular aperture, the value of $k_0$ converges quickly to 2.85 with an Area Error of 10.8%.

Iwai et al [1] validate the use of the following Gaussian soft aperture as an approximation of a hard aperture:

$$A(x, y) = A_0 \exp \left[-2 \left(\frac{x^2}{h_x^2} + \frac{y^2}{h_y^2}\right)\right]$$  \hspace{1cm} (3.9)

where $h_x$ and $h_y$ are the radii where the normalised aperture transmission function drops to $1/e^2$ and $A_0$ is the peak transmission constant. The autocorrelation function of such an aperture is

$$\Gamma(\tau_x, \tau_y) = \frac{1}{16} A_0^2 \exp \left[-\left(\frac{\tau_x}{h_x}\right)^2\right] \exp \left[-\left(\frac{\tau_y}{h_y}\right)^2\right] \pi \text{erf} \left[\frac{\tau_x + 2x}{h_x}\right] \text{erf} \left[\frac{\tau_y + 2y}{h_y}\right]$$  \hspace{1cm} (3.10)

and the form of this autocorrelation resembles the form of equation (3.7).

The effective aperture of a photodetector is not a pure top hat function as expected but is ‘softened’ by the diffraction pattern made by the retro-reflective tape. The individual retro-reflectors in the tape act as small apertures each producing a $(J_1(u)/u)^2$ diffraction pattern and the Airy disk diffraction patterns overlap centered on the optical axis. Figure (3.6) shows the diffraction effect of the retro-reflector. When the photodetector aperture is much smaller than the diffraction pattern the top hat assumption applies, but when the detecting aperture is increased the effective aperture approaches the Airy pattern due to the diffraction pattern of the retro-reflector. Figure (3.7) shows a variety of aperture functions and a Gaussian approximation to their autocorrelation functions. The
Figure 3.6: Left: The speckle pattern produced by ‘Scotchlight’ retro-reflective tape. Right: The typical Airy disk consisting of the ensemble average of many individual speckle patterns formed by backscatter from retro-reflective tape attached to a rotating surface. CCD camera pixel size 8.6\,\mu\text{m} \times 8.3\,\mu\text{m}.

The autocorrelation function of the aperture is computed and Gaussian function is fitted in a least squares sense. Approximating the autocorrelation function of the aperture with a Gaussian allows the aperture autocorrelation function to be modeled as a single numerical parameter. The parameter $k_0$ increases rapidly when the aperture function approaches the Airy pattern.
Figure 3.7: Plot of various aperture functions (left column) and their autocorrelation functions and Gaussian approximation to the autocorrelation function (right column). a) Rectangular 'top-hat' aperture. b) Autocorrelation of 'top-hat' aperture and Gaussian approximation. $k_0 = 2.85$, area error = 10.8% c) Rectangular aperture with a diffraction function. d) Autocorrelation and Gaussian approximation to c. $k_0 = 2.91$, area error = 8.8% e) Aperture function rect $\times$ Airy pattern f) Autocorrelation and Gaussian approximation of the Airy function aperture. $k_0 = 6.08$, area error = 2.1%.
3.4.2 Space-Time Correlation Function of the Spatially Integrated Speckle Intensity

The space-time correlation function theory proceeds by combining all of the above individual components. The ensemble average within the outer integral in Equation (3.6) is recognised as the speckle intensity fluctuation correlation function in Equation (3.1) and this is given by [46]

\[
\Gamma_{\Delta I}(X, \tau) = \alpha \exp \left[ -\frac{4 |v|^2 \tau^2}{D^2} \right] \exp \left[ - \left( \frac{\pi D}{2\lambda z} \right)^2 |X - \rho v \tau|^2 \right]
\]

(3.11)

where \(\alpha\) is a scaling constant, \(v\) is a vector describing the velocity of the scatterers translating in the \((\xi, \eta)\) plane, \(D\) is the \(1/e^2\) diameter of the illuminating beam, \(\lambda\) is the laser wavelength, \(z\) is the target-detector separation and \(\rho\) is a wavefront curvature term given by \(1 + z/r\) where \(r\) is the wavefront curvature radius of the illuminating beam. Note that this equation describes the speckle intensity fluctuation correlation function for a point photodetector.

When Equations (3.6), (3.7) and (3.11) and Equation (2.10) for speckle size are combined, the resulting correlation function can be written as:

\[
\gamma_A(X, Y, \tau) = \Gamma_{\Delta I}(0, 0, 0) L_x L_y \exp \left[ -\frac{4 (v_x^2 + v_y^2) \tau^2}{D^2} \right] \times \int_{-\infty}^{\infty} \exp \left[ -k_0 \left( \frac{\Delta x - X}{L_x} \right)^2 \right] \exp \left[ -\frac{2}{\langle \sigma_0 \rangle^2} (\Delta x - \rho v_x \tau)^2 \right] d\Delta x
\]

\[
\times \int_{-\infty}^{\infty} \exp \left[ -k_0 \left( \frac{\Delta y - Y}{L_y} \right)^2 \right] \exp \left[ -\frac{2}{\langle \sigma_0 \rangle^2} (\Delta y - \rho v_y \tau)^2 \right] d\Delta y.
\]

(3.12)

When the above equation is normalised by \(\Gamma_A(0,0,0)\) the space-time cross-
correlation function is given by:

\[ \gamma_A(X, Y, \tau) = \frac{\Gamma_A(X, Y, \tau)}{\Gamma_A(0, 0, 0)} \]

\[ = \exp \left[ - \left( \left( \frac{2}{D} \right)^2 + 2 \left( \frac{\rho}{\langle \sigma_0 \rangle} \right)^2 \right) (v_x^2 + v_y^2) \tau^2 \right] \]

\[ \times \exp \left[ -k_0 \left( \left( \frac{X}{L_x} \right)^2 + \left( \frac{Y}{L_y} \right)^2 \right) \right] \]

\[ \times \exp \left[ \frac{(k_0 \langle \sigma_0 \rangle^2 X + 2\rho L_x^2 v_x \tau)^2}{L_x^2 \langle \sigma_0 \rangle^2 (k_0 \langle \sigma_0 \rangle^2 + 2 L_x^2)} \right] \]

\[ \times \exp \left[ \frac{(k_0 \langle \sigma_0 \rangle^2 Y + 2\rho L_y^2 v_y \tau)^2}{L_y^2 \langle \sigma_0 \rangle^2 (k_0 \langle \sigma_0 \rangle^2 + 2 L_y^2)} \right]. \] (3.13)

This equation describes the normalised space-time correlation function of the spatially integrated speckle intensity backscattered from a target with in-plane motion \((v_x, v_y)\) and can be applied provided that target shape and/or motion does not change and, hence, \(\langle \sigma_0 \rangle\) and \(\rho\).

Normalisation of the correlation function removes the dependence of laser power and scattering efficiencies from Equation (3.13) and allows for simple comparison of different optical configurations. The normalised space-time correlation function will prove to be a useful expression to study the effect of various target motions on the photodetector output signal.

The physical description for the individual terms in Equation (3.13) is difficult but some insight can be gained by examining special cases of the normalised space-time correlation function.

- Setting only the temporal delay to zero \((X \neq 0, Y \neq 0, \tau = 0)\) results in
the space correlation function

\[ \gamma_A(X, Y, 0) = \exp \left[ -k_0 \left( \frac{X}{L_x} \right)^2 + \left( \frac{Y}{L_y} \right)^2 \right] \times \exp \left[ \frac{(k_0(\sigma_0)^2)X^2}{L_x^2(k_0(\sigma_0)^2 + L_x^2)} \right] \times \exp \left[ \frac{(k_0(\sigma_0)^2)Y^2}{L_y^2(k_0(\sigma_0)^2 + L_y^2)} \right] \] (3.14)

and the resulting normalised correlation function depends only on speckle width \( \langle \sigma_0 \rangle \), spatial delays \( X \) and \( Y \) and the aperture dimensions \( L_x \) and \( L_y \). Note that the wavefront curvature term \( \rho \) has no influence on the normalised correlation function.

- Setting the spatial delays to zero (\( X = 0, Y = 0, \tau \neq 0 \)) results in the time correlation function

\[ \gamma_A(0, 0, \tau) = \exp \left[ - \left( \frac{2}{D} \right)^2 + 2 \left( \frac{\rho}{\langle \sigma_0 \rangle} \right)^2 (v^2 + v^2) \tau^2 \right] \times \exp \left[ \frac{(2\rho L_x v_x \tau)^2}{(\sigma_0)^2(2L_x^2 + k_0(\sigma_0)^2)} \right] \times \exp \left[ \frac{(2\rho L_y v_y \tau)^2}{(\sigma_0)^2(2L_y^2 + k_0(\sigma_0)^2)} \right] . \] (3.15)

This function depends principally on the beam diameter, speckle size and aperture dimensions as well as the in-plane displacements and the curvature term \( \rho \).

- In the case where the aperture dimensions approach zero (\( L_x \to 0 \) and \( L_y \to 0 \)) Equation (3.13) reduces to the space-time correlation function for
point detectors

\[
\gamma_A(X, Y, \tau) = \exp \left[ -\left( \frac{2}{D} \right)^2 + 2 \left( \frac{\rho}{(\sigma_0)} \right)^2 \left( v_x^2 + v_y^2 \right) \tau^2 \right] \\
\times \exp \left[ -\frac{2}{(\sigma_0)^2} \left( X(X - 2v_x\rho\tau) + Y(Y - 2v_y\rho\tau) \right) \right]
\]  

(3.16)

which is identical to Equation (3.11).

3.5 Speckle Intensity Decorrelation due to Target Rotation

The decorrelation of the photodetector output signal with the rotation of the shaft is of prime importance for the operation of the laser torquemeter. For the proposed method of operation a signal must be generated when the shaft is rotated and the number of variations in the signal per revolution may be measured by the angle required to decorrelate the photodetector output signal.

If the radiation is incident upon the surface at an angle \( \phi_i \) and is scattered off at an angle \( \phi_r \), then a small rotation of the surface through an angle \( \delta\phi \) about an axis lying within the surface causes the speckle pattern to rotate in the same direction by an angle \( \delta\varphi \), where:

\[
\delta\varphi = \delta\phi \left( 1 + \frac{\cos \phi_i}{\cos \phi_r} \right).
\]  

(3.17)

This equation can be verified by considering the path difference of light scattered from different parts of the surface. When the speckles are observed close to the backscatter direction, \( \phi_i = \phi_r \), we notice that all the speckles move like mirror
reflections regardless of the angle of incidence [93]. If the shaft is tilted by an angle $\theta$, around an axis, then the speckle pattern rotates at twice this angular rate [94].

The illuminated surface tilts and translates tangentially as it rotates and both these motions associated with rotation must be accounted for. In order to simplify the analysis of the effect of target rotation, Equation (3.13) can be written in terms of a rotation angle $\theta = \omega t$. The rotational decorrelation angle, $\theta_{cc}$, is then defined as the rotation angle at which the resulting normalised autocorrelation falls to $1/e^2$. The surface tilts at the same rate at which it rotates, $\omega$, giving rise to speckle translation in the detecting plane at a velocity $2\omega z$ [43] [88]. In Equation (3.13) this translational motion in the detecting plane is modelled as a photodetector displacement $X = 0, Y = -2\omega z t$. The tangential velocity of the shaft of radius $R$ is modelled by setting $v_\eta = R \omega$ and this accounts for the decorrelation due to the changing of the scatterers involved in the production of the speckle pattern.

Setting the velocity $v_\xi = 0$, the rotational decorrelation angle $\theta_{cc}$ can be determined by manipulating Equation (3.13):

$$
\theta_{cc} = \sqrt{2} \left[ \frac{2R}{D} \right]^2 + \frac{2k_0(2z + \rho R)^2}{k_0(\sigma_0)^2 + 2I_y^2} \right]^{-1/2}.
$$

The rotational decorrelation angle will be important in a later consideration of the minimum angular resolution of the proposed laser torquemeter.

The spatially integrated space-time correlation of a speckle pattern produced by a flat translating surface is well-understood [1]. For a rotating shaft, the approach resulting in Equation (3.18) above takes into account surface in-plane motion as $v_\eta = R \omega$ and surface tilt as $Y = 2\omega z t$. An alternative approach taken
to this problem [2] is to acknowledge the in-plane motion in the same way as above and then model surface tilt directly by modifying the radius of curvature as shown in Figure (3.8). With reference to Figure (3.8) a Gaussian beam observed far from the beam waist and near the beam centre has an approximately spherical wavefront originating at point $O$. At the shaft, this wavefront has radius $r$. Applying Newton’s law of reflection and the paraxial approximation to the reflection of light at each point on the shaft surface (shaft has a radius of $R$) an effective radius of curvature $r'$ may be calculated by ignoring the microscopic variations of the surface of the shaft. The effective radius of curvature $r'$ can be determined from the shaft radius, $R$, and illuminating beam radius of curvature, $r$, by the following expression

$$\frac{1}{r'} = \frac{2}{R} - \frac{1}{r}. \quad (3.19)$$

The following sign notation is applied to determine the effective radius of curvature; the positive direction is from centre $O$ to reflecting surface. Convex surfaces have negative radii of curvature and concave surfaces have positive curvature. The
effective radius $r'$ is then

$$r' = -\frac{rR}{R - 2r}.$$  \hfill (3.20)

For practical configurations of the torquemeter, $r \gg R$ consequently $r' \approx R/2$ and the effective radius of curvature is dependent only upon the shaft radius.

Takai et al [2] give a theoretical value for the temporal autocorrelation of the photodetector output due to speckle backscattered from a rotating shaft as

$$\tau_c = \sqrt{2} \frac{1}{\omega|RT|} \left[ \frac{1}{(\frac{1}{2}D)^2} + \frac{1}{L_d^2 + 4/\pi^2(\sigma_0)^2} \left( \frac{z}{r} + 1 - \frac{2z}{RT} \right)^2 \right]^{-1/2}$$  \hfill (3.21)

where $L_d$ is the $1/e^2$ diameter of the Gaussian soft detector aperture $A(x,y)$, defined by

$$A(x,y) = \exp \left[ - \frac{2(x^2 + y^2)}{L_d^2} \right].$$  \hfill (3.22)

The first occurrence of shaft radius $R_T$ in Equation (3.21) requires the modulus because Takai et al. interpret the radius of curvature of a convex surface as having a negative radius rather than positive radius as used elsewhere in this thesis. When Equation (3.21) is adjusted to represent the angular $1/e^2$ autocorrelation width, the mean speckle size, the angular $1/e^2$ autocorrelation width is

$$\omega \tau_c = \sqrt{2} \left[ \left( \frac{2R}{D} \right)^2 + \frac{2(2z + \rho R)^2}{8/\pi^2(\sigma_0)^2 + 2L_d^2} \right]^{-1/2}$$  \hfill (3.23)

which is consistent with the form and terms of Equation (3.18) but for the definition of the detecting aperture. This method is useful in determining the temporal correlation of the spatially integrated speckle pattern backscattered from a rotat-
ing shaft; however, the full cross-correlation function is required to model target tilt-induced speckle motion.

3.6 Translational Decorrelation

Translation motion of the shaft along its rotational axis (ie float) also influences the detected speckle pattern. The sensitivity to this axial shaft motion may also be investigated with the comprehensive space-time correlation relationship of Equation (3.13). For study of the decorrelation of the photodetector output due to axial motion at velocity $v_\xi$, the target rotation can be neglected. Hence $\omega = 0$, and in this special case, Equation (3.13) reduces to:

$$\gamma_A(\tau) = \exp \left[ - \left( \left( \frac{2}{D} \right)^2 + \frac{2k_0\rho^2}{k_0(\sigma_0)^2 + 2L_z^2} \right) v_\xi^2 \tau^2 \right].$$  

(3.24)

If the decorrelation displacement, $x_c = v_\xi\tau_c$, is defined as the axial displacement required to reduce the normalised autocorrelation to $1/e^2$, then $x_c$ is given by the following relationship:

$$\frac{x_c}{D} = \left( \frac{k_0(\sigma_0)^2 + 2L_z^2}{2k_0(\sigma_0)^2 + 4L_z^2 + k_0\rho^2D^2} \right)^{1/2}.$$  

(3.25)

This ratio between decorrelation displacement and beam diameter, $x_c/D$, is proposed as a measure of the effectiveness of a configuration in resisting decorrelation. Importantly the ratio can be seen to take a maximum value of $1/\sqrt{2} \approx 0.7$ when:

$$(2k_0(\sigma_0)^2 + 4L_z^2) \gg k_0\rho^2D^2.$$  

(3.26)

This inequality dictates an optimum combination of, effectively, beam diameter,
speckle size, photodetector size (in the direction of speckle motion) and wavefront curvature for which the maximum resistance to decorrelation of the photodetector output can be achieved.

The wavefront curvature term $\rho$ may be reduced to zero by setting the radius of curvature $r = -z$ and this is easily accomplished with the use of a lens. The inequality of Equation (3.26) is then satisfied and the ratio between the decorrelation displacement and beam diameter reaches its limiting value of 0.7 for any combination of speckle size and aperture dimension $L_x$.

3.7 Tilt Motion Decorrelation

Rotation about the $\eta$ axis also influences the detected speckle pattern and such motion is likely as the shaft vibrates. This tilt motion acts in a fashion similar to a set of mirror facets on the surface of the shaft. The light incident upon, and reflected by, a tilting mirror moves at an angular rate of twice the angular rate of the mirror. If the shaft is tilted by an angle $\theta_\eta$ around the $\eta$ axis the optical system may be modelled by the space-time cross-correlation with a displacement $X = 2z\theta_\eta$. For the purposes of this study, rotational and axial motion of the shaft is neglected; in which case, $Y = 0$, $v_k = 0$, $w = 0$ and $v_\eta = 0$ and Equation (3.13) becomes

$$
\gamma_A(X, 0, 0) = \exp \left[ -k_0 \left( \frac{X}{L_x} \right)^2 \right] \exp \left[ -\frac{\langle \sigma_0 \rangle^2 (k_0 X)^2}{L_x^2 (k_0 \langle \sigma_0 \rangle^2 + 2L_x^2)} \right].
$$

(3.27)
If the decorrelation tilt $\theta_{nc} = |X/2z|$ is defined as the tilt angle required to reduce the normalised autocorrelation to $1/e^2$, then $\theta_{nc}$ is given by the following:

$$\theta_{nc} = \sqrt{\frac{k_0(\sigma_0)^2 + 2L_z^2}{4k_0z^2}}.$$  \hspace{1cm} (3.28)

Equation (3.28) allows the specification of an optical configuration for a laser torquementer that resists decorrelation due to a given shaft tilt vibration. Unlike resistance to decorrelation due to axial motion there is no limiting value.

### 3.8 Radial Motion Decorrelation

The correlation of the photodetector output signal with the recorded signal is also sensitive to radial ($\eta$) motion but a larger displacement in the $\eta$ direction is required than in the axial ($\xi$) direction to produce the same decorrelation of the live signal from the recorded signal. This is because the speckle motion resulting from the rotation of the shaft bears some similarity to the speckle motion resulting from the radial ($\eta$) motion of the shaft. If changes in the backscattered speckle patterns were the same in each of the above cases then the normalised cross-correlation, between one cycle of the photodetector output and a second cycle for which the position of the incident beam had been raised in the ($\eta$) direction, would show merely a change in the position of the cross-correlation peak without any change in its amplitude. An example of where this delaying effect in the photodetector output signal may occur is when the surface of the shaft is flat – for example a belt, travelling between two rollers. The speckle pattern repeats with rotation as the same scatterers periodically reappear with revolution and any motion of the rig (in the direction of belt motion) results in pure delay of the output signal. The signal delay due to radial motion is indistinguishable from the
signal delay caused by shaft twist and the radial motion does not result in a ‘loss of lock’ condition so further torque measurements are possible. As the radius of the shaft is decreased an increased sensitivity to radial motion is observed. The effect of radial (η) motion is therefore complicated by a dependence upon the curvature of the shaft.

Radial motion in the ξ direction (i.e. towards the photodetector) is considered insignificant because the length of speckles in the z direction is large compared to the maximum expected shaft motion in the z direction. In examining vibration criterion charts [20] [95] a reasonable assumption is that the maximum longitudinal shaft motion to be expected will have an amplitude of less than 1mm. The length of the speckles in the z direction is approximated by [74]

\[
\langle \sigma_{0z} \rangle = 8\lambda \frac{z^2}{D^2}
\]  

(3.29)

for a circular illuminating spot. There will also be an insignificant widening of the speckles due to increasing z. For practical configurations of the torquemeter optical system the speckle length is then at least an order of magnitude greater than the expected longitudinal shaft motion and no significant change in the speckle pattern and the photodetector output signal is expected.

3.9 Equivalence Between Autocorrelation and Cross-Correlation Functions

Experimental verification of the correlation widths for shaft axial motion and shaft tilt does not require the shaft to be rotating but a simple and elegant method to acquire the data and to confirm the theory which does use the rotation of the shaft is presented in this section.
The matrix \( S \) summarises the experimental data acquired when the laser beam illuminates the rotating shaft. In the matrix \( S \) each column corresponds to the integrated intensity variation with axial or angular position at a given angular location and each row corresponds to the integrated intensity variation with angular position at a fixed axial or angular position.

\[
S = \begin{pmatrix}
    s_{11} & s_{12} & \cdots & s_{1N_x} \\
    s_{21} & s_{22} & \cdots & s_{2N_x} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{M,1} & s_{M,2} & \cdots & s_{M,N_x}
\end{pmatrix}
\]  

The matrix is filled most efficiently by recording the photodetector output from (at least) one full shaft rotation at each axial or angular position. In this way only a small number of data captures, at an appropriate sample rate, produces a matrix with approximately 20,000 columns. The matrix \( S \) is very quickly filled row by row.

With data acquired in this manner, cross-correlation between rows is more convenient than autocorrelation of columns. In what follows, the equivalence of these two functions is shown.

To confirm the theoretical relationships for decorrelation, autocorrelation of the columns is required to determine the axial decorrelation displacement, \( x_c \) or the decorrelation angle \( \theta_{cc} \). The experimental value of \( x_c \) (or \( \theta_{cc} \)) is determined from the mean \( 1/e^2 \) width of the autocorrelation of every column in matrix \( S \). The discrete autocorrelation for the \( n \)th column as a function of axial delay \( a \) is
written $\Gamma_{nn}(a)$ where

$$\Gamma_{nn}(a) = \sum_{m=1}^{M_s} s_{mn}s_{(m+a)n}. \quad (3.31)$$

The mean of the discrete autocorrelations of $N_s$ columns may be written as

$$\bar{\Gamma}_{nn}(a) = \frac{1}{N_s} \sum_{n=1}^{N_s} \sum_{m=1}^{M_s} s_{mn}s_{(m+a)n}. \quad (3.32)$$

This method is sensitive to the synchronisation of the rows of data. It is useful to consider the rows of the matrix $S$ to be aligned but the rows of experimental data need not be (and were not) aligned when captured using this method. The peak value of the cross-correlation of a row with an arbitrarily chosen reference row is representative of the degree of similarity between the data in these two rows. With data acquired in this format, the cross-correlation is a straightforward function to evaluate and its peak value is not dependent on the synchronisation of the data. The cross-correlation between row $m$ and row $(m+a)$, written $\Gamma_{m(m+a)}(0)$, has its peak value at zero angular delay, $\theta = 0$, and this can be written as

$$\Gamma_{m(m+a)}(0) = \sum_{n=1}^{N_s} s_{mn}s_{(m+a)n}. \quad (3.33)$$

Averaging over $M_s$ cross-correlations of rows separated by axial delay $a$:

$$\bar{\Gamma}_{m(m+a)}(0) = \frac{1}{M_s} \sum_{m=1}^{M_s} \sum_{n=1}^{N_s} s_{mn}s_{(m+a)n}. \quad (3.34)$$

The original aim was to estimate the mean discrete autocorrelation of $N_s$ columns but this method of data acquisition is better suited to calculation of the cross-
correlation function. These two functions can now be related using Equations (3.32) and (3.34):

$$\Gamma_{mn}(a) = \frac{M_a}{N_a} \Gamma_{m(m+a)}(0),$$

(3.35)

and the mean discrete normalised autocorrelation is given by

$$\bar{\gamma}_{mn}(a) \equiv \frac{\Gamma_{mn}(a)}{\Gamma_{mn}(0)} = \frac{\hat{\Gamma}_{m(m+a)}(0)}{\Gamma_{mm}(0)}.$$  

(3.36)

Thus, the normalised autocorrelation of the integrated speckle intensity as a function of axial displacement (i.e., down the columns of $S$) is estimated conveniently as the peak value of the normalised cross-correlation between the rows of $S$ as a function of axial delay. Since the cross-correlation peak values are not affected by the synchronisation of the rows of $S$, there is no need to re-align the rows in this analysis. This would, of course, be necessary to calculate the autocorrelations of each column of $S$ directly. The equivalence of the row by row cross-correlation and the mean column autocorrelation function is thus demonstrated.

Figure (3.9) shows the normalised cross-correlation of experimental data from row 10 of matrix $S$ with neighboring rows in matrix $S$. The peak values of the cross-correlations do not occur at 0 lag because the rows were not synchronised. The peak values of the cross-correlations in Figure (3.9) may be plotted against relative row position. This process may be repeated for a collection of reference rows and the plot of the peak values against relative row position is shown in Figure (3.10).

A comparison of the mean column autocorrelation and the mean row by row cross-correlations is made in Figure (3.11). This plot was computed using simulated data generated by low pass filtering a vector of uniformly distributed random
Figure 3.9: Cross-correlation between row 10 and other rows. Matrix S size is 21 rows x 4096 columns. Note that the peak of the cross-correlations are not synchronized.

Figure 3.10: Peak row-by-row cross-correlations plotted against row number. □ reference row $N = 5$, ○ reference row $N = 10$, and ◇ reference row $N=15$. The errorbars represent ± one standard deviation, $n = 15$. 
values because the experimental data was not synchronized to a common point. The row by row cross-correlations method does not perform well when the peak of the cross-correlation is near zero because the peak is indistinguishable from noise.

\[ \text{Figure 3.11: Comparison of mean row-by-row cross-correlations, plotted with } \square \text{ and the mean autocorrelation of columns, } \circ \text{ using simulated data. The autocorrelation of the columns is not available for real data as the rows were not perfectly aligned. Matrix size } S \text{ is 128 rows by 128 columns. Error bars represent } \pm 1 \text{ standard deviation, } n = 128. \]

### 3.10 Accuracy of the Autocorrelation

The accuracy of the autocorrelation function may be estimated by following the procedure outlined in Bendat [96]. The photodetector output signal is low pass filtered white noise [97] and the variance of the autocorrelation estimate is

\[ \text{var}(\alpha, \beta) = \frac{2\beta - 1 + 2\exp[-2\beta] + [(2\alpha + 1)(2\beta - 1) - 2\alpha^2]\exp[-2\alpha]}{2\beta^2} \] (3.37)
where $\alpha$ and $\beta$ are

$$\alpha = \pi B\tau$$  \hspace{1cm} (3.38)

$$\beta = \pi BT$$  \hspace{1cm} (3.39)

and $B$ is the correlator bandwidth, $T$ is the measurement time and $\tau$ the autocorrelation lag. Because the data is acquired by sampling and then digitally correlated, the measurement acquisition system has an upper bandwidth $B$ of 1/2 its sampling rate.

The variance of the peak value may be given by

$$\text{var}(0, \beta) = \frac{2\beta - 1 + \exp[-2\beta]}{\beta^2}.$$  \hspace{1cm} (3.40)

The uncertainty in the autocorrelation function can be reduced to a negligibly small value by increasing the measurement time $T$ or by increasing the correlator bandwidth $B$ or both. Increasing the correlator bandwidth reduces errors proportionally and the increase in noise due to larger bandwidth increases with the $\sqrt{B}$. In the following experiments, the correlation bandwidth is approximately 1MHz and the measurement time 16–64ms and the resulting standard deviation of the peak autocorrelation function is less than 1%. Figure (3.12) shows how the error of the autocorrelation function varies with lag. The errors in autocorrelation due to finite length of the data set and due to the finite bandwidth in the measurement system used is less than 0.9% at the worst case (0 lag) and drops to 0.625% at large lags.
Figure 3.12: The accuracy of the autocorrelation function plotted against lag. $B = 500$kHz, 16384 samples, $F_s = 1$MHz

3.11 Maximum Width of Photodetector

The width of the photodetector $L_x$ is limited by the following reason: as the area of the photodetector increases the fluctuation of the photodetector output decreases and the shot noise component of the photodetector increases.

The expression for the number of speckle cells in a detecting aperture $Q$ may be approximated by [66]

$$Q = \begin{cases} 1, & S_m < 0.1S_c \\ \frac{S_m}{S_c}, & S_m > 10S_c \end{cases}$$

(3.41)

The value of $Q$ is related to the reciprocal of the contrast of the spatially inte-
grated speckle pattern by

\[ \frac{\langle I_A \rangle}{\sigma_{I_A}} = \sqrt{Q} \]  \hspace{1cm} (3.42)

as seen in Section (2.4). The signal to noise ratio of the signal input to the
cross-correlator may be estimated by examining the noise sources present in the
photodetector. The photodetector consists of a pin photodiode followed by a
transimpedance amplifier and the major sources of noise in the photodetector are
due to shot noise in the photodiode and thermal noise in the amplifier. The rms
shot noise, \( i_{\text{shot}} \) is associated with current flow and is given by [98]

\[ i_{\text{shot}} = \sqrt{2qi_{\text{dc}}B} \]  \hspace{1cm} (3.43)

where \( q \) is the electron charge \((1.602 \times 10^{-19} \text{ C})\), \( B \) is the correlator bandwidth, and
\( i_{\text{dc}} \) is the current flowing through the photodiode. The current in the photodiode
is related to the mean intensity by

\[ i_{\text{dc}} = R \langle I_A \rangle \]  \hspace{1cm} (3.44)

where \( R \) is the responsivity of the photodiode at the illuminating wavelength (for
a Si photodiode \( R \approx 0.4 \text{A/W @ 632.8nm} \)). The thermal noise generated by the
transimpedance resistor is given by [98]

\[ V_n = \sqrt{4k_BTBR_f} \]  \hspace{1cm} (3.45)

where \( k_B \) is Boltzmann’s constant \((1.38 \times 10^{-23} \text{ J/K})\), \( R_f \) is the amplifier feedback
resistance and \( T \) is the temperature of the resistor. The signal to noise ratio of
the system may be written by combining Equations (3.43 - 3.45):

\[
SNR = \frac{\Re \sigma_{IA} R_f}{\sqrt{2 B q \sqrt{Q} \Re R_f^2 \sigma_{IA} + 4 B k_B R_f T}}
\]  

(3.46)

The signal to noise ratio (SNR) for the photodetectors is plotted against number of speckles on the detector in Figure (3.13). A SNR of approximately 15dB is obtained when there are approximately 100 speckles on the detector.

Increasing the responsivity of the photodiode and reducing the bandwidth and the number of speckles incident improve the SNR of the photodetectors. The responsivity of the Si diode may be increased by increasing the wavelength of the laser to its maximum value at 850nm but this wavelength is in the infra-red making easy alignment of the Laser Torquemeter difficult.
Ambient light incident on the photodetector decreases the SNR by increasing the shot noise in the photodiode. Baffles may be used to reduce the amount of background light reaching the photodetector and an interference filter transmitting light only at the laser wavelength rejects almost all stray light from sources other than the laser.

The maximum width of the photodetector is limited by the ability of the torquemeter electronics to process the signal. A SNR of 15dB is a good signal to noise ratio and this corresponds to approximately 100 speckles.

3.12 Experimental Verification of Speckle Statistics

In order to proceed with the experimental validation of the theory for speckle decorrelation due to shaft rotation, some preliminary experiments are required. The speckle statistics need to be verified to ensure that what is observed is, in fact, a fully developed speckle pattern (see Section 2.2). The repeatability of the speckle signal also needs to be verified because this is fundamental to the operation of the laser torquemeter as described in Section (3.2).

Figure (3.14) is a photograph of the optical system used to produce speckles on the detector aperture – this is essentially 'half' a torquemeter. This simple apparatus consists of a rotating shaft, a beam splitter and a He-Ne laser. A schematic of the optical arrangement was shown in Figure (3.4). The shaft is illuminated by a Gaussian laser beam whose waist size is ascertained by measuring the beam’s intensity profile. The method of determining the parameters of the incident laser beam are described in Section (3.13). The shaft was covered with retro-reflective tape (3M Scotchlite high gain sheeting type 7610) to increase the back-scattered signal intensity and the incident beam was aligned
Figure 3.14: The experimental apparatus used in determining the verification of speckle statistics from a rotating shaft.

to be coincident with the rotational axis of the shaft. A non-polarising beam splitter allows the back-scattered radiation to be detected by a photodetector on the optical axis. A baffle was introduced to prevent the reflection of the speckle pattern off the sides of the beam splitter from reaching the photodetector. The lights in the laboratory were switched off to prevent 100 Hz background light (2×50 Hz from the fluorescent lamps) from reaching the photodetectors. The beam splitter was adjusted so the internal reflections of the incident laser beam were not incident on the detecting aperture. A dc motor rotated the shaft and the resulting photodetector output signal was recorded on a digital oscilloscope. The sampling rate of the oscilloscope was chosen such that it was more than twice the frequency response of the system (i.e. sampling above the Nyquist rate). More than one revolution of the shaft was captured on the oscilloscope. The recorded
signal was downloaded to a computer where any remaining mean voltage was removed resulting in a signal proportional to $\Delta I(t)$.

Figure (3.15) shows the photodetector output for several revolutions of shaft. Figure (3.16) is a plot of the power spectral density of the photodetector output for many rotations which shows strong periodicity at the rotation rate of the shaft. The photodetector output signal is a series of random signals that repeat exactly with revolution of the shaft. The spectrum of a pseudo-random signal such as this has strong peaks at multiples of rotation frequency. Figure (3.17) shows a typical output from the photodetector for one revolution of the shaft from Figure (3.15). The power spectral density of the photodetector output signal due to speckle motion across the photodetecting aperture is plotted in Figure (3.18) which is one rotation (or 158ms) of data from Figure (3.17). The frequency content of the photodetector output signal rolls off (-3dB point) at approximately 20kHz. The roll-off frequency is mainly dependent upon the speckle size, photodetector size, and shaft rotation rate.
Figure 3.15: Photodetector output signal for approximately 6 revolutions. 
($z_0 = 25.6\text{cm}$, $z = 34\text{cm}$, detector aperture $\varnothing 2.5\text{mm}$, $D = 0.98\text{mm}$, $\langle \sigma_0 \rangle = 0.22\text{mm}$). The rotational frequency of the shaft is $6.3\text{Hz}$ ($378.3\text{rpm}$).

Figure 3.16: Spectrum of the photodetector output signal showing strong periodicity at the rotation rate of the shaft ($\approx 6Hz$).
Figure 3.17: Photodetector output signal from one revolution of the shaft. The period of rotation is 158.6ms ($\approx 6Hz$).

Figure 3.18: Bode plot of the photodetector output from one revolution of the shaft. The photodetector output rolls off at approximately 20kHz.
3.13 Measurement of Laser Beam Parameters

The profile of the beam from a 5mW UniPhase 65776 He-Ne laser was measured at several ranges with a Reticon RL256G linear image sensor having a 25μm pixel spacing. The results confirm the manufacturer's specifications for beam parameters.

<table>
<thead>
<tr>
<th>Range from laser [m]</th>
<th>Beam width to 1/e² points [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.53</td>
<td>0.829</td>
</tr>
<tr>
<td>2.61</td>
<td>1.319</td>
</tr>
<tr>
<td>3.17</td>
<td>2.510</td>
</tr>
</tbody>
</table>

Table 3.1: Width of laser beam as a function of range from laser.

Figure 3.19: Ideal Gaussian beam profile and observed beam profile. Range for beam waist z=153cm.

waist (0.48mm) and divergence (1.5mrad). The measured mean half angle (θ, in Figure (2.8)) to 1/e² point is 0.613± 0.156 mrad. The laser is operating near its diffraction limit and the beam profile is a Gaussian function as can be seen...
in Figure (3.19). From the measurement of beam divergence the beam waist is approximately 1cm inside the front aperture of the laser. The measured output power of the laser was 3.5mW.

3.14 The Temporal Bandwidth of the Photodetectors

The transfer function of the photodetector was measured by illuminating the active area with light from a fast red LED. The specifications of the Thor Labs PDA50 photodetector far exceeded the frequency response of the other photodetectors and the frequency response was confirmed to be flat in the range 20Hz – 800kHz. The photodetector frequency response is limited by the response of the amplifier-photodiode circuit. The low and high -3dB cutoff points for the photodetectors are tabulated in Table (3.2). The photodiode area has a significant effect on the frequency response of the photodetector as the junction capacitance is linked to surface area. Photodiodes with smaller sensitive areas have lower junction capacitance and result in faster photodetectors. A practical torquemeter photodetector is likely to have a photodiode with a small sensitive area (<1mm²) with a lens and external aperture.

<table>
<thead>
<tr>
<th>Photodetector</th>
<th>Photodiode</th>
<th>Photosensitive area [mm \times mm]</th>
<th>-3 dB cutoff frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mk I</td>
<td>Philips BPX 65</td>
<td>0.981 × 0.981</td>
<td>20 Hz – 736 kHz</td>
</tr>
<tr>
<td>Mk II</td>
<td>Hamamatsu S1337-16</td>
<td>5.9 × 1.1</td>
<td>22 Hz – 330 kHz</td>
</tr>
<tr>
<td>Thor Labs PDA50</td>
<td>–</td>
<td>2.5</td>
<td>DC – 10MHz</td>
</tr>
</tbody>
</table>

Table 3.2: Frequency response of the photodetectors used.

The finite bandwidth of the amplifier in the photodetectors places an inherent range on the speckle size and on the rate at which speckle motion may be detected. If the speckles are too small relative to the photodetector aperture or the speckle translation rate is too fast, the signal output may have significant
frequency content above the cutoff frequency of the detector. The temporal re-
response of the photodetector distorts the signal so that the minimum width of
the photodetector output autocorrelation function is governed by the cutoff fre-
quency of the photodetector rather than speckle size and translational velocity.
For valid measurements of the decorrelation due to scattering motion and the
correct operation of the laser torquemeter the bandwidth of the photodetector
must be greater than that of the integrated speckle signal. In the following ex-
periments care was taken to ensure that the integrated speckle signal bandwidth
was less than the bandwidth of the photodetector.

3.15 Stability of Correlation of Spatially Integrated
Speckle Pattern Over Time

The laser torquemeter depends on the speckle pattern being stable with time. The
stability of a static speckle pattern is easily observed by examining the backscat-
tered speckle pattern produced by a stationary object. The speckle pattern is
constant unless the optical configuration is disturbed.

The cross-correlation between the recorded photodetector output signal and
the live signal is also stable over time. Intuitively there should be no change to
the speckle pattern, and therefore the cross-correlation signal; however, over time
small movements of the motor shaft and of the optical components may radically
change the cross-correlation output. A simple trial was performed to confirm that
the speckle pattern is indeed stable over the duration required to perform the
experiments. Data was collected from one revolution of the shaft at various times
and cross-correlated with data from the first revolution. Figure (3.20) shows that
the peak of the normalised cross-correlation function is unity to within ±1.5%
over an extended period.

Figure 3.20: Stability of the cross-correlation function of the speckle pattern produced by a rotating shaft over nearly 45 minutes.

3.16 The Verification of the Speckle Statistics from Retro-Reflective Tape

Distribution of speckle intensity was measured by illuminating a first piece of retro-reflective tape mounted on a flat surface which could be translated by a linear translation stage and a second piece of retro-reflective tape mounted on a rotating shaft. The surfaces were illuminated by a continuous wave He-Ne laser and the surfaces set in motion – translated or rotated for the respective arrangements. Various sized apertures were produced by an adjustable leaf aperture reducing the effective light sensitive area of the detector. The dimensions of the leaf aperture were measured by examining the far field diffraction pattern of the aperture. Some typical speckle intensity distributions are shown in Figures (3.21–3.23). The error bars in the figures throughout this thesis indicate
± one standard deviation, with the number of experiments \( n = 3 \), unless otherwise indicated. The solid line on the intensity distribution plots are the predicted

![Intensity Distribution Graph](image)

Figure 3.21: Speckle intensity distribution for \( z = 23.2 \text{cm}, L_x = 0.37 \text{mm} \) \( D = 0.10 \text{mm} \) \( \langle \sigma_0 \rangle = 1.40 \text{mm} \). Least squares fit estimate for \( Q = 3.76 \). Error bars ± one standard deviation, number of experiments \( n = 5 \).

Probability distribution functions of fully developed spatially integrated speckle and this is given by [66]

\[
P(I) = \frac{1}{\text{gamma}(Q)} \left( \frac{Q}{\langle I \rangle} \right)^Q I^{Q-1} \exp \left( -Q \frac{I}{\langle I \rangle} \right) \tag{3.47}
\]

for \( I \geq 0 \) and where \( Q \) is interpreted as the number of speckle correlation cells within the measurement aperture and where \( \text{gamma}(Q) \) is a gamma function of argument \( Q \) defined by

\[
\text{gamma}(x) = \int_0^\infty \exp[-t]t^{x-1} \, dt \quad x > 0 \tag{3.48}
\]

These intensity distributions confirm that the retro-reflective tape produces a
Figure 3.22: Speckle intensity distribution, $z = 41.3\text{cm}$, $D = 0.34\text{mm}$, $(\sigma_0) = 0.78\text{mm}$, $L_x \times L_y = 1\text{mm} \times 1\text{mm}$. Least squares fit for $Q = 4.44$, $n = 3$.

Figure 3.23: Least squares fit for $Q = 8.46$, $z = 21.7\text{cm}$, $(\sigma_0) = 1.3\text{mm}$, $D = 0.10\text{mm}$, $L_x = 5.9\text{mm}$, $L_y = 1.1\text{mm}$, $n = 3$. 

96
'fully developed speckle pattern' for a variety of illuminating spot sizes.

Other Effects

When an illuminating beam is focused down onto the retro-reflective tape two distinct phenomena occur: the speckle pattern is no longer fully developed as only a few scattering elements contribute to the speckle pattern and an intensity pattern with regular peaks is seen. This effect is identical to that of illuminating a grid of apertures and the intensity distributions of dot arrays with Gaussian random spatial fluctuations has been explored [99]. This effect would be disastrous for a laser torquemeter attempting to track shaft twist because the photodetector output would be highly sensitive to axial float and no measurements of torque would be possible. The laser torquemeter measures shaft twists along a length of shaft and in order to localise the illuminated spots, small spots are desirable; however, these spots need to be large enough so a speckle pattern is produced.

There is also evidence that multiple images of the speckle pattern are visible superimposed on the detecting plane; this is caused by internal reflection on the inner surfaces of the beam splitter. Internal reflections of the speckle pattern in the beam splitter may be minimised by the introduction of apertures which limit the extraneous speckle patterns and allow only the primary speckle pattern to reach the photodetector. This effect would alter the speckle pattern incident on the photodetecting aperture and would alter the operation of the laser torquemeter.
3.17 Experimental Validation of Speckle Decorrelation Due to Shaft Rotation

The autocorrelation function of the photodetector output signal from a purely rotating shaft is important to the operation of the laser torquemeter. The narrower the width of the autocorrelation function of the photodetector output signal the higher the potential resolution of shaft twist; however, this comes at the expense of the robustness of the tracking system. In order to develop the torquemeter the theory of photodetector output signal decorrelation due to shaft rotation, as seen in Section (3.5), needs to be experimentally verified.

A typical autocorrelation plot is illustrated in Figure (3.24). A series of experiments were conducted where the principal parameters (laser-shaft range, $z_0$; shaft - detector range, $z$; aperture dimension, $L_y$; shaft radius $R$ and spot diameter $D$) were adjusted. Figure (3.25) shows the experimental apparatus. The photodetector output signal was recorded for six rotations of the shaft for each

![Figure 3.24: Plot of the autocorrelation function with shaft rotation angle. $z_0=25.6\,\text{cm}$, $z=34\,\text{cm}$, $(\sigma_0) = 0.2187\,\text{mm}$, $D = 0.984\,\text{mm}$](image)
Figure 3.25: Experimental apparatus used for determining the resistance to rotational decorrelation.

experiment and the autocorrelation function of the signal was computed. The temporal lag which reduces the autocorrelation to $1/e^2$ of the peak value was determined and the decorrelation angle $\theta_{\theta\text{e}}$ was calculated from the temporal autocorrelation width and the rotational velocity.

Table (3.3) presents the results of these experiments and Figure (3.26) shows agreement to within ±17% between the theoretical and experimental results. (The data points in the figure and the table have been labelled with the letters a-j to facilitate comparison).

In general, increasing the aperture in the $y$ direction increases the angular autocorrelation width and reducing the shaft radius has the same effect. The temporal correlation function of Takai et al [2] and Equation (3.18) are consistent.

The angular correlation width is greatly influenced by the curvature of the rotating shaft and a device to measure the radius of curvature of the rotating shaft may be constructed. The width of the autocorrelation peak is mainly de-
<table>
<thead>
<tr>
<th>Data Point</th>
<th>R [mm]</th>
<th>$z_0$ [m]</th>
<th>$z$ [cm]</th>
<th>$L_y$ [mm]</th>
<th>Correlation Width [mrad]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>measured ± one std theory</td>
</tr>
<tr>
<td>a</td>
<td>40.0</td>
<td>8.54</td>
<td>18</td>
<td>1.0</td>
<td>1.973 0.051 2.059</td>
</tr>
<tr>
<td>b</td>
<td>40.0</td>
<td>2.76</td>
<td>18</td>
<td>1.0</td>
<td>1.606 0.005 2.052</td>
</tr>
<tr>
<td>c</td>
<td>40.0</td>
<td>0.60</td>
<td>54</td>
<td>5.9</td>
<td>3.781 0.043 4.282</td>
</tr>
<tr>
<td>d</td>
<td>40.0</td>
<td>2.51</td>
<td>54</td>
<td>5.9</td>
<td>3.688 0.099 4.361</td>
</tr>
<tr>
<td>e</td>
<td>40.0</td>
<td>8.52</td>
<td>54</td>
<td>5.9</td>
<td>3.283 0.380 4.401</td>
</tr>
<tr>
<td>f</td>
<td>40.0</td>
<td>0.52</td>
<td>54</td>
<td>1.0</td>
<td>0.885 0.025 0.776</td>
</tr>
<tr>
<td>g</td>
<td>40.0</td>
<td>2.46</td>
<td>54</td>
<td>1.0</td>
<td>0.831 0.016 0.742</td>
</tr>
<tr>
<td>h</td>
<td>40.0</td>
<td>8.52</td>
<td>54</td>
<td>1.0</td>
<td>0.740 0.013 0.736</td>
</tr>
<tr>
<td>i</td>
<td>12.5</td>
<td>8.64</td>
<td>54</td>
<td>5.9</td>
<td>4.226 0.203 4.520</td>
</tr>
<tr>
<td>j</td>
<td>12.5</td>
<td>8.64</td>
<td>54</td>
<td>5.9</td>
<td>4.149 0.129 4.518</td>
</tr>
</tbody>
</table>

Table 3.3: Summary of the optical configurations showing the angular correlation width of the photodetector output for a variety of optical configurations.

Figure 3.26: Plot of the theoretical rotational correlation angle $\theta_{cc}$ shows agreement with the measured rotational correlation angle. Error bars represent ± one standard deviation from the mean experimental value for 3 measurements.
dependent upon the shaft radius, aperture $L_y$ and the rotational velocity. The rotation velocity of the shaft may be estimated by the lag between peaks in the autocorrelation function. Figure (3.27) shows the autocorrelation function of the photodetector signal plotted in Figure (3.15).

![Figure 3.27: Autocorrelation function of the photodetector output from 6 revolutions of the shaft.](image)

3.18 Experimental Validation of Speckle Decorrelation Due to Axial Translation of the Shaft

The shaft and motor may be translated by a micrometer stage in the direction parallel to the rotational axis of the shaft to model shaft float. The rotational axis is arranged to be perpendicular to the incident laser beam. Figure (3.28) shows the experimental configuration. The backscattered signal, from one revolution of the shaft, was captured and down-loaded to a computer. This signal, consisting of $N$ samples, was recorded in a matrix $S$ for later processing (see Equation (3.30)). The shaft was then displaced in discrete steps in the axial direction by a linear
translation stage until the total displacement exceeded the $1/e^2$ diameter of the beam illuminating the shaft. For each of the displaced axial shaft positions the data capture process was repeated, each time adding another row to matrix $S$. A middle row of matrix $S$ was selected as a reference row and all the other rows were cross-correlated with this reference row.

The normalised autocorrelation function was plotted versus the axial distance from the reference position. A Gaussian curve was then fitted (minimising the least squares error function) to the set of peak values to simplify the estimate of the axial decorrelation displacement. The whole experiment was repeated and plotted with error bars indicating ±1 standard deviation for $n = 3$ experiments. Some typical plots of correlation coefficient as a function of axial displacement are presented in Figures (3.29 and 3.30).

An intuitive approach leads one to expect the axial displacement required to reduce the normalised autocorrelation to $1/e^2 (x_c)$ to increase with an increase
Figure 3.29: Plot of peak normalised cross-correlation between reference signal and signal from axial translated shaft with a Gaussian 'least squares' best fit. Reference signal from position: $x = -2\text{mm}$. $z_0 = 124\text{cm}$, $z = 57\text{cm}$, $(\sigma_0) = 0.2495\text{mm}$, $D=1.45\text{mm}$, $n=3$. 
in beam diameter because of the larger number of illuminated scatterers. Shaft translation results in a smaller proportion of scattering elements being lost and proportionally only a few new scattering elements being gained, so the resulting speckle pattern is essentially unchanged. An opposite argument may be presented concerning speckle size – a few large speckles on the detector result in the photodetector signal decorrelating when the shaft is displaced because the integrated speckle intensity is significantly changed by the disappearance and appearance of individual speckles on the detector.

Table (3.4) summarises the optical configurations for which the resistance to decorrelation of the photodetector output was investigated. The configurations are listed in ascending order of number of speckles across the width of the photodetector ($L_x/(\sigma_0)$). Table (3.4) also lists likely configurations for practical
Table 3.4: Summary of the optical configurations showing the effectiveness in resisting decorrelation of a photodetector output due to axial motion of the shaft.

<table>
<thead>
<tr>
<th>Data Point</th>
<th>$L_x/\langle\sigma_0\rangle$</th>
<th>$x_c/D$</th>
<th>$D$</th>
<th>$z$</th>
<th>$L_x$</th>
<th>$L_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2.1</td>
<td>0.355</td>
<td>1.45</td>
<td>107.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>b</td>
<td>3.9</td>
<td>0.372</td>
<td>1.45</td>
<td>57.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>f</td>
<td>5.29</td>
<td>0.327</td>
<td>1.94</td>
<td>57.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>g</td>
<td>6.8</td>
<td>0.269</td>
<td>2.51</td>
<td>57.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>h</td>
<td>11.2</td>
<td>0.206</td>
<td>3.54</td>
<td>49.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>j</td>
<td>11.7</td>
<td>0.628</td>
<td>1.14</td>
<td>91</td>
<td>5.9</td>
<td>1.1</td>
</tr>
<tr>
<td>d</td>
<td>13.3</td>
<td>0.420</td>
<td>1.45</td>
<td>17.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>k</td>
<td>17.9</td>
<td>0.653</td>
<td>1.75</td>
<td>91</td>
<td>5.9</td>
<td>1.1</td>
</tr>
<tr>
<td>c</td>
<td>21.4</td>
<td>0.476</td>
<td>1.18</td>
<td>16.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>e</td>
<td>28</td>
<td>0.721</td>
<td>0.882</td>
<td>30.0</td>
<td>5.9</td>
<td>1.1</td>
</tr>
<tr>
<td>i</td>
<td>44</td>
<td>0.716</td>
<td>1.43</td>
<td>30.0</td>
<td>5.9</td>
<td>1.1</td>
</tr>
</tbody>
</table>

setups, particularly configurations for which $(x_c/D)_{\text{max}} \approx 0.7$ is achieved. Axial displacements greater than $0.7D$ cannot be accommodated, for example, by increasing the detector size, as the scatterers and the resultant speckle patterns are now too different from those which created the original set of speckle patterns.

A general trend in the data indicates increasing $x_c/D$ with increasing $L_x/\langle\sigma_0\rangle$ but this relationship is also influenced by beam diameter which acts, not surprisingly, to reduce $x_c/D$. Ideally, for optimum performance, $(x_c/D) \approx 0.7$ is required which suggests the use of a small beam diameter but large $x_c$ in absolute terms is also desirable which requires a large beam diameter. In reality, a beam diameter will be chosen based on knowledge of the expected axial motion and optimum performance is then ensured by the appropriate choice of optical configuration. Figure (3.31) shows a plot of decorrelation displacement $x_c$ against beam diameter. The resistance to decorrelation initially increases with beam diameter but a limit is reached where larger beam diameters produce no improvement in decorrelation displacement $x_c$. These experimental observations support Equation (3.24) but agreement with theoretical analysis is best illustrated by Figure (3.32) which
Figure 3.31: Plot of axial correlation width $x_c$ versus beam diameter $D$. A theoretical line is plotted for two values of photodetector aperture width: $\circ L_x = 1\text{mm}$, $\Box L_x = 5.9\text{mm}$.
Figure 3.32: Plot of the theoretical axial correlation width $x_c$ versus actual axial correlation width. Photodetector aperture width (in the $x$ axis): $\circ L_x = 1\text{mm}$, $\square L_x = 5.9\text{mm}$.
shows good agreement in a comparison between the experimental and theoretical values of $x_c$. (The data points in Figure (3.31) and Table (3.4) have been labelled with the letters a-i to facilitate comparison).

3.19 Experimental Validation of Speckle Decorrelation Due to Shaft Tilt

The apparatus to examine the decorrelation due to shaft tilt is similar to the equipment used in the previous experiments. The procedure for gathering data is identical to the procedure for the translational decorrelation except that instead of translating the shaft, it is displaced angularly about the $\eta$ axis in discrete steps and at each of the tilted shaft positions the data capture process is repeated. Figure (3.33) shows the system used to determine the normalised cross-correlation

![Schematic diagram of optical configuration for measuring decorrelation due to shaft tilt.](image)

Figure 3.33: Schematic diagram of optical configuration for measuring decorrelation due to shaft tilt.

as a function of tilt angle. With reference to this figure, the rotating shaft may
be tilted about the $\eta$ axis in precise discrete steps by incrementing a micrometer acting on a lever. The shaft rotational axis $\zeta$ is perpendicular to the optical axis and the axis of tilt as shown in Figure (3.2). Speckles are detected by a photodetector with a finite sized aperture at a distance $z$ from the shaft. The data gathering process is repeated for several different values of $z_0$, $z$ and $L_x$ dimensions. Figures (3.34 and 3.35) show plots of the peak of the normalised temporal cross-correlation function for a series of angular positions but with the same optical configuration. The tilt decorrelation angle $\theta_{pc}$ is determined by the following procedure: the photodetector output signal is recorded for several revolutions of the shaft at a series of shaft tilt angles and entered as a column into matrix $S$; then the data from one tilt angle is selected as the reference tilt angle and cross-correlated with the data from all of the other tilt angles; the maximum value of the cross-correlation function is then plotted against the tilt angle, a Gaussian curve fitted to the data set and the $1/e^2$ width of the Gaussian is

![Figure 3.34: A Gaussian curve fitted to plot of the decorrelation due to shaft tilt. Tilt cross-correlation experiment 'as'. $z_0 = 12.5\, \text{cm}$, $z = 21.7\, \text{cm}$, shaft $2.54\, \text{cm}$, $L_x = L_y = 1\, \text{mm}$, $\theta_{pc} = 137.4\, \text{mdeg}$, $n = 3$.](image)
determined. The data processing is identical to the process used in Section (3.17) except the columns here vary by angular position $\theta'$. Figure (3.36) compares the experimental decorrelation widths from Table (3.5) with the theoretical prediction of Equation (3.28).

According to Equation (3.28), the resistance to decorrelation due to tilt may be improved by reducing the spot diameter $D$, reducing the photodetector range $z$ or by increasing the width of the photodetector $L_x$; however, the selection of the value of $D$ is limited by the need to avoid decorrelation due to axial translation of the shaft. The optimal choice of these adjustable parameters is made by considering the expected tilt and translation vibrations.
<table>
<thead>
<tr>
<th>Data Point</th>
<th>( z ) [cm]</th>
<th>Shaft Ø [cm]</th>
<th>( L_x ) [mm]</th>
<th>( D ) [mm]</th>
<th>( \langle \sigma_0 \rangle ) [mdeg]</th>
<th>( \theta_{qc} )</th>
<th>± 1 std [mdeg]</th>
<th>( \theta_{qc} ) [mdeg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>27.5</td>
<td>2.5</td>
<td>0.98</td>
<td>0.97</td>
<td>0.18</td>
<td>108.37</td>
<td>5.60</td>
<td>87.96</td>
</tr>
<tr>
<td>l</td>
<td>50.0</td>
<td>2.5</td>
<td>0.98</td>
<td>0.97</td>
<td>0.33</td>
<td>63.96</td>
<td>2.60</td>
<td>50.82</td>
</tr>
<tr>
<td>m</td>
<td>67.5</td>
<td>2.5</td>
<td>0.98</td>
<td>0.97</td>
<td>0.44</td>
<td>10.28</td>
<td>2.60</td>
<td>39.68</td>
</tr>
<tr>
<td>y</td>
<td>14.8</td>
<td>2.5</td>
<td>0.28</td>
<td>0.97</td>
<td>0.10</td>
<td>52.06</td>
<td>3.14</td>
<td>49.46</td>
</tr>
<tr>
<td>u</td>
<td>14.8</td>
<td>2.5</td>
<td>0.59</td>
<td>0.96</td>
<td>0.10</td>
<td>79.52</td>
<td>2.81</td>
<td>96.73</td>
</tr>
<tr>
<td>i</td>
<td>23.2</td>
<td>2.5</td>
<td>0.30</td>
<td>0.96</td>
<td>1.40</td>
<td>121.11</td>
<td>5.09</td>
<td>175.72</td>
</tr>
<tr>
<td>o</td>
<td>23.2</td>
<td>2.5</td>
<td>0.37</td>
<td>0.10</td>
<td>1.40</td>
<td>159.42</td>
<td>7.80</td>
<td>177.01</td>
</tr>
<tr>
<td>a</td>
<td>21.7</td>
<td>2.5</td>
<td>0.98</td>
<td>0.10</td>
<td>1.31</td>
<td>137.43</td>
<td>9.50</td>
<td>204.28</td>
</tr>
<tr>
<td>d</td>
<td>21.7</td>
<td>2.5</td>
<td>5.90</td>
<td>0.10</td>
<td>1.31</td>
<td>679.52</td>
<td>20.18</td>
<td>674.99</td>
</tr>
<tr>
<td>f</td>
<td>21.7</td>
<td>2.5</td>
<td>5.90</td>
<td>0.10</td>
<td>0.41</td>
<td>790.76</td>
<td>46.97</td>
<td>654.73</td>
</tr>
<tr>
<td>g</td>
<td>21.7</td>
<td>2.5</td>
<td>0.98</td>
<td>0.34</td>
<td>0.41</td>
<td>114.07</td>
<td>1.35</td>
<td>121.58</td>
</tr>
<tr>
<td>h</td>
<td>21.7</td>
<td>2.5</td>
<td>0.43</td>
<td>0.34</td>
<td>0.41</td>
<td>48.56</td>
<td>1.38</td>
<td>71.70</td>
</tr>
<tr>
<td>j</td>
<td>41.3</td>
<td>2.5</td>
<td>0.98</td>
<td>0.34</td>
<td>0.41</td>
<td>75.62</td>
<td>1.11</td>
<td>63.90</td>
</tr>
<tr>
<td>k</td>
<td>41.3</td>
<td>2.5</td>
<td>5.90</td>
<td>0.34</td>
<td>0.78</td>
<td>358.80</td>
<td>2.62</td>
<td>347.07</td>
</tr>
<tr>
<td>z</td>
<td>56.5</td>
<td>8.7</td>
<td>5.90</td>
<td>0.34</td>
<td>0.78</td>
<td>236.60</td>
<td>9.64</td>
<td>253.69</td>
</tr>
<tr>
<td>x</td>
<td>29.0</td>
<td>8.7</td>
<td>5.90</td>
<td>0.34</td>
<td>1.06</td>
<td>476.20</td>
<td>12.85</td>
<td>499.45</td>
</tr>
</tbody>
</table>

Table 3.5: Summary of the optical configurations showing the effectiveness in resisting decorrelation of a photodetector output due to shaft tilt. The mean and standard deviation (std) of the estimate of \( \theta_{qc} \) computed from 5 experiments.
Figure 3.36: Plot of the theoretical tilt decorrelation angle $\theta_{nc}$ versus actual tilt correlation width. A linear 'best fit' is made through the measured points. Photo detector aperture width (in the $x$ axis): $\circ - - L_x = 1\text{mm}$, $\Box - - L_x = 5.9\text{mm}$.

### 3.20 Experimental Investigation: Decorrelation Due to Radial Motion

The experimental technique for measuring the decorrelation due to radial motion is similar to that of the other experiments except that the shaft is moved in discrete steps in a direction normal to the axis of rotation and the incident laser beam. Data is collected and processed in the same way, but because the correlation peak occurs when roughly the same scatterers are illuminated there is a temporal lag in the correlation peak. The processing ignores the shift in the peak location by only examining the relative height of the cross-correlation function.
Figure 3.37: Plot of the apparent motion of the correlation peak due to radial shaft motion.

Figure (3.37) shows the normalised cross-correlation variation as the incident beam is moved in the ($\eta$) direction. The shift in the position in the cross-correlation peak is very clear but at the same time there is a significant reduction in the amplitude of the peak. This shows that the photodetector output is influenced by changing the position of the incident beam in the ($\eta$) direction and the change in the backscattered speckle patterns is almost a delay or advance in the cycle of the continuously changing speckle pattern. The rotational motion of the shaft cannot bring exactly the same scatterers at the same angle relative to the illuminating beam back to the same spatial orientation and this difference in location alters the resultant speckle pattern and hence the photodetector output signal with attendant reduction in the correlation peak.
Chapter 4

The Laser Torquemeter

4.1 Operating Environment of the Laser Torquemeter

In this chapter, the static and dynamic performance of the torquemeter is addressed and compared with competing torque measurement instruments. The laser torquemeter is intended as a practical measuring instrument and its operating range is limited by the motion of the shaft. The minimum twist observable is perhaps the most important of the parameters in quantifying the performance of the laser torquemeter because this is related to the torque resolution, but in order to design a reliable laser torquemeter other parameters need to be characterised. These parameters are the torsional vibration, axial and tilt vibrations of the rotating shaft and expected frequency content of these vibrations as well as the shaft geometry.

4.1.1 Range of Measured Torques

In order to develop a useful torquemeter an overview of the variation of torques produced by a variety of machines is required. A survey of the torquemeter market indicates that most torquemeters cover a torque range of 20:1. From
Table (4.1) there is a large variation in the range of torques that may be measured spanning nine orders of magnitude. Very small torques have not been included in Table (4.1) as they are not typically recorded by in-line rotary torquemeters. The

<table>
<thead>
<tr>
<th>Device</th>
<th>Rotational speed [rpm]</th>
<th>Maximum Torque</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Largest practical steam turbine</td>
<td>1800, 3600</td>
<td>136, 68 MNm</td>
<td>1300 MW</td>
</tr>
<tr>
<td>Power generation for oil industry</td>
<td>3000</td>
<td>2.5 MNm</td>
<td>40 MW</td>
</tr>
<tr>
<td>Steel slabbing mill</td>
<td>50</td>
<td>3 MNm</td>
<td>2.5 MW</td>
</tr>
<tr>
<td>McLaren Formula 1 Engine (1998)</td>
<td>16500</td>
<td>2100 kNm</td>
<td>580 kW</td>
</tr>
<tr>
<td>Washing Machine</td>
<td>700 – 15000</td>
<td>0.1 – 0.5 Nm</td>
<td>25 W</td>
</tr>
</tbody>
</table>

Table 4.1: Typical power and rotational speeds for various equipment. [4] [100] [101]

design of the laser torquemeter optics and cross-correlators requires a minimum twist to be specified because twist is related to the torque applied and shaft material properties. Commonly quoted maximum permissible shaft twists are $1^\circ$ twist for a shaft length of 20 times the shaft diameter [20].

4.1.2 Speed variation of the Rotating Shaft

The speed variation and amplitude of the transient torque of a rotating shaft are important parameters in the design of a torquemeter because the sampling rate of the tracking cross-correlators needs to be adequate to continuously track the photodetector output signal. The size and frequency of torsional variations of a rotating shaft for a variety of sources are presented in Table (4.2). The coefficient of speed variation in revolution, $C_s$ is given by

$$C_s = \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\omega_{\text{avg}}} \quad (4.1)$$

and describes the variation in speed as a machine progresses through its cycle where $\omega_{\text{max}}$, $\omega_{\text{min}}$, and $\omega_{\text{avg}}$ are the maximum, minimum and average shaft speeds.
respectively. In machine design, the reduction in speed variation per revolution is desirable and constant torque throughout the entire revolution of the shaft allows for the maximum transfer of power to the load for the minimum size shaft. A qualitative description of speed variation is presented in Table (4.3).

Table 4.2: Sources of excitation of torsional vibration. (n = 1, 2, 3...).[6]

<table>
<thead>
<tr>
<th>Source</th>
<th>Max Torque / Rated Torque</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear runout</td>
<td>–</td>
<td>n× rpm</td>
</tr>
<tr>
<td>Coupling unbalance</td>
<td>–</td>
<td>rpm</td>
</tr>
<tr>
<td>Generator (load fault)</td>
<td>–</td>
<td>n× rpm, n× electrical supply frequency</td>
</tr>
<tr>
<td>Synchronous motor startup</td>
<td>5 – 10</td>
<td>2× slip frequency</td>
</tr>
<tr>
<td>Induction motor start-up</td>
<td>3 – 10</td>
<td>power line frequency (50Hz/60Hz)</td>
</tr>
<tr>
<td>Centrifugal pump</td>
<td>0.1 – 0.4</td>
<td>n× number of vanes × rpm</td>
</tr>
<tr>
<td>Engine geared system</td>
<td>0.15 – 0.5 (soft – stiff coupling)</td>
<td>n×rpm and $\frac{1}{3}$n× rpm</td>
</tr>
<tr>
<td>Shaft vibration</td>
<td>–</td>
<td>n× rpm</td>
</tr>
</tbody>
</table>

Table 4.3: Suggested values of Coefficient of Speed Fluctuation [95].

4.1.3 Vibrational Motions of Typical Shafts

An estimate of the typical shaft vibration amplitudes is required to design the optical system of the laser torquemeter. As discussed in Section(3.3), the decorrelation of the speckle pattern can be related to the axial and radial displacement of the shaft as well as to shaft tilt as examined in Chapter 3. Actual measurements on a diesel engine and machine design tables provide data on the size and frequency content of these speckle decorrelating motions.
The axial float and shaft tilt motions were measured on a medium size four cylinder diesel engine. A pair of laser vibrometers were aligned with their probe beams parallel to the rotational axis of the driveshaft, such that the points of incidence on a flange were equidistant from the rotational axis, as shown in Figure (4.1). With the engine running at full throttle and the load adjusted for a variety of engine speeds, the vibrometer outputs are captured by a dual channel digital oscilloscope. The raw flange displacement is measured by integrating the velocity output from the vibrometers and is plotted in Figure (4.2). Estimates of the axial shaft motion are made by taking the mean of the peak to peak flange displacement and an estimate of driveshaft tilt is made by dividing the difference in flange axial displacement by the separation of measurement points. These estimates of vibration peak to peak axial displacement are plotted in Figures (4.3, 4.4). The maximum displacement due to radial motion may also be approximated using typical values from a machine design table.

Maximum axial vibration and tilt amplitudes may be set for torquemeter

Figure 4.1: Schematic diagram of the optical arrangement to determine shaft tilt and axial float.
Figure 4.2: Displacement of the driveshaft flange of a diesel engine (1500 rpm at full torque 110Nm).

design purposes at 1mm peak to peak and a shaft tilt motion of 1° peak to peak.

### 4.2 Excitation Torques and Expected Shaft Twists

For reciprocating engines, the excitation torques are a function of engine type (two/four stroke & diesel/petrol) and are a function of bore, stroke and gas properties. Significant transient torques usually arise from fault conditions and invari-

<table>
<thead>
<tr>
<th>Declared Engine Speed [min⁻¹]</th>
<th>Rated power output of Generating Set [kW]</th>
<th>Axial Vibration Displacement [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 – 3600</td>
<td>10, 50</td>
<td>1.27, 0.95</td>
</tr>
<tr>
<td>1000 – 2000</td>
<td>10, 50, 200</td>
<td>0.64, 0.48, 0.45</td>
</tr>
<tr>
<td>720</td>
<td>1000</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 4.4: ISO 8528-9:1995/BS7698-9:1996 Reciprocating internal combustion engine driven alternating current generating sets: Part 9 Measurement and evaluation of mechanical vibrations. [102] [103] [104]
Figure 4.3: Peak to peak axial motion of driveshaft of a diesel engine.

Figure 4.4: Peak to peak tilt motion of driveshaft of a diesel engine.
ably occur when electric generators or motors are part of the power transmission train. Large transient torques occur when induction motors or synchronous machines are started while pumps and compressors tend to generate cyclical torque fluctuations. Induction motors are able to produce 2.0 – 2.8 rated torque during acceleration. Synchronous machines can provide up to 6 times rated torque when mis-synchronised.

The design of a driveshaft for most applications usually follows from previous experience and a cursory analysis or rule of thumb (1° twist over 20 shaft diameters [20].) This is because the designer considers, from past experience, the major loads on the shaft and designs the driveshaft with a relatively large margin of safety. The exceptions to the case are when the application requires minimum weight or a large driveshaft or the driveshaft is critical to the safety of the machine.

Power transmission driveshafts are usually round (solid or tubular) except for some small machinery where the shaft may be square. Square profiles are used where length compensation is involved or to simplify the connecting of auxiliary equipment.

Typically design of a solid cylindrical driveshaft proceeds by computing the shaft diameter considering static and dynamic bending and torque. This is given by:

$$d = \left[ \frac{32N_{\text{safety}}}{\pi} \left( \sqrt{\left( \frac{M_{\text{dyn}}}{S_{\text{end}}} \right)^2 + \frac{3}{4} \left( \frac{T_{\text{dyn}}}{S_{\text{end}}} \right)^2} + \sqrt{\left( \frac{M_{\text{stat}}}{S_{\text{ult}}} \right)^2 + \frac{3}{4} \left( \frac{T_{\text{stat}}}{S_{\text{ult}}} \right)^2} \right) \right]^{1/3}$$

(4.2)

where

$$d = \text{shaft diameter [m]}$$
\( N_{\text{safety}} = \) safety factor

\( M_{\text{dyn}} = \) dynamic bending moment [Nm]

\( M_{\text{stat}} = \) static bending moment [Nm]

\( S_{\text{end}} = \) fatigue shaft strength from Table (4.5). [Pa]

\( S_{\text{ult}} = \) ultimate shaft strength from Table (4.5). [Pa]

\( T_{\text{dyn}} = \) dynamic torque [Nm]

\( T_{\text{stat}} = \) static torque [Nm]

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Ultimate Strength</th>
<th>Fatigue Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI Steel Grade</td>
<td>( S_{\text{ult}} ) [MPa]</td>
<td>( S_{\text{end}} ) [MPa]</td>
</tr>
<tr>
<td>1010 – 1020</td>
<td>345</td>
<td>172</td>
</tr>
<tr>
<td>1030 – 1040</td>
<td>483</td>
<td>241</td>
</tr>
<tr>
<td>1040 – 1050</td>
<td>586</td>
<td>293</td>
</tr>
<tr>
<td>4130 – 4140</td>
<td>879</td>
<td>440</td>
</tr>
<tr>
<td>4140 – 4150</td>
<td>931</td>
<td>465</td>
</tr>
<tr>
<td>4340</td>
<td>1034</td>
<td>517</td>
</tr>
</tbody>
</table>

Table 4.5: Approximate strengths for various shaft materials. [105]

The recommended amount of shaft twist for a typical material (BS817M40) depends on the shear stress. The shear stress for a solid shaft is given by

\[
\tau = \frac{16T}{\pi d^3}
\]  

(4.3)

and following a design example from Plint [106], a shaft with a \( \varnothing 100 \text{mm} \) has a maximum torque rating of 19600Nm. For this shaft, there is a twist of 0.145° between two points separated by 10cm.

Table (4.6) lists typical shaft diameters and nominal torque loadings for a variety of internal combustion engines. The shaft twist is calculated over a 20cm
length for the larger diameter shaft with solid section, shear modulus of elasticity of 78.8GPa and nominal torque.

<table>
<thead>
<tr>
<th>Engine size [cm³]</th>
<th>Shaft Diameter [mm]</th>
<th>Torque Capacity Static Failure [Nm]</th>
<th>Nominal Torque Maximum Drive for calculated service life [Nm]</th>
<th>Twist over 20cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td>19.1 - 21</td>
<td>1900</td>
<td>1040</td>
<td>220</td>
</tr>
<tr>
<td>1800</td>
<td>22.3 - 25.7</td>
<td>3000</td>
<td>1660</td>
<td>330</td>
</tr>
<tr>
<td>2800</td>
<td>25.5 - 31</td>
<td>4500</td>
<td>2530</td>
<td>450</td>
</tr>
</tbody>
</table>

Table 4.6: Table of typical shaft diameters and torque loadings for a variety of internal combustion engines. Twist over 20cm length calculated for largest diameter shaft. [107]

In a commercially available series of torquemeter (Moiré Type) the full scale torsional deflection is 0.5° for the lower torque range units (0–10mNm) and 0.25° for the higher torque range (0–10000Nm) over a 48mm long shaft [108] [25].

4.3 Windup

Windup is a non-reversible deformation of the torsion element and this non-linear effect limits the maximum torque on a length of shaft for which the laser torquemeter will reliably operate. The design of high performance systems often allow for a degree of windup – at the end of a race, a Formula One driveshaft often has 30° of windup over its 50cm length [109] – and the windup of a shaft limits the ability of a laser torquemeter to accurately measure shaft twist.

4.4 Coupling to Torquemeter Shaft

A practical torquemeter may require that a shaft be inserted in the drive train in order to obtain practical shaft twists because the shaft is overly rigid or because the shaft is composed of an unknown material and the values of $G$ and/or $J$ are
unavailable. A shaft manufactured of a composite material with non-linear elastic properties is unsuitable for direct torque measurement and also requires the insertion of a shaft of known $G$ and $J$ and linear elastic properties. When a different material shaft is inserted a shaft coupling is required and the selection of an appropriate coupling is important. The torsional stiffness, end float, parallel and angular misalignment, and speed rating are all parameters of the coupling that need to be addressed. Often a torque measurement will need to be made across a coupling and the torsional stiffness of the coupling will need to be accounted for.

Machines can be set up accurately at installation, but thermal gradients, pressure differences in the system media (steam, air etc) and structural movements cause misalignment of coupled shafts.

Table (4.7) summarizes a range of parameters of a variety of different shaft couplings that may be used to couple a driveshaft to the torquemeter shaft.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Straight gear</th>
<th>High Performance Gear</th>
<th>Spindle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Torque [MNm]</td>
<td>4.5</td>
<td>1.5</td>
<td>0.45</td>
</tr>
<tr>
<td>Max speed [rpm]</td>
<td>12000</td>
<td>40000</td>
<td>30000</td>
</tr>
<tr>
<td>Max bore [mm]</td>
<td>900</td>
<td>300</td>
<td>600</td>
</tr>
<tr>
<td>Angular Misalignment [°]</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Parallel offset [mm/mm]</td>
<td>0.008</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>Axial travel [mm]</td>
<td>3.2 – 25</td>
<td>3.2 – 25</td>
<td>12 – 80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Elastomeric element</th>
<th>Metallic element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Torque [MNm]</td>
<td>shear</td>
<td>compression</td>
</tr>
<tr>
<td>Max speed [rpm]</td>
<td>5000</td>
<td>8000</td>
</tr>
<tr>
<td>Max bore [mm]</td>
<td>250</td>
<td>600</td>
</tr>
<tr>
<td>Angular Misalignment [°]</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Parallel offset [mm/mm]</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Axial travel [mm]</td>
<td>±8</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 4.7: Maximum shaft coupling capacities. From Mancuso [110]
4.5 Hybrid Digital-Analogue Torquemeter

The hybrid digital-analogue laser torquemeter was developed in the late 1980's and early 1990's [26] [27] [28]. A prototype torquemeter signal processor was constructed using approximately 50 digital devices and 30 analogue devices and included a simple interface to BBC computer to provide a means to control the various operating modes of the torquemeter.

In the prototype torquemeter a phase-lock circuit preserves the alignment between the recorded photodetector output signal and the live photodetector output signal. The shaft twist is computed from the relative angles between the two phase-lock circuits. Subsequent improvements of the torquemeter electronics developed various feedback configurations to reduce susceptibility to 'whole body' vibrations of the shaft, to increase dynamic range and to reduce limit-cycle oscillations due to the restricted angular resolution and quantisation of the twist output signal caused by the memory size of the recorded signal [26].

4.5.1 Operation of the Prototype Torquemeter

In order to make a torque measurement the torquemeter enters several modes of operation. These operating modes are controlled by a simple computer and are described below:

**Acquire Mode** Adjusts the internal clock of system so that the photodetector output signal due to one revolution of the shaft output would fill the available memory. There is a possible ambiguity because the torquemeter may acquire two or more periods of photodetector output into memory.
**Record Lock Phase**  In this mode, the torquemeter electronics seek to load into memory the photodetector output signal for one revolution of shaft. The shaft twist between the two illuminated points is the reference shaft twist so the shaft should ideally be unloaded. The delay estimate $\hat{\tau}$ in the peak tracking correlator is adjusted so that the tracking correlator locks the recorded photodetector output signal onto the live signal.

**Run Lock Phase**  The live signal is locked to recorded signal. The shaft twist signal is output and provided the polar second moment of inertia $J$ and shear modulus $G$ are known, an estimate of torque may be made. Shaft twist is measured relative to when the data was recorded during the Record Lock Phase.

The laser torquemeter electronics also have several versions of control loop. These systems may be selected by the computer and are summarised below:

**Twin Loop System:** The torquemeter operates as a pair of independent shaft encoders and outputs the difference between the encoder measurements as the shaft twist. Figure (4.5) shows a schematic diagram of a twin loop torquemeter system. This system configuration is susceptible to common mode shaft angle fluctuations [26].

**Common-Mode Difference-Mode System:** In this configuration the common-mode shaft angle is tracked by one control loop and another control loop tracks the 'difference mode' or shaft twist as shown in Figure (4.6). Record Phase: Under no load conditions, the photodetector output signals are input into memory which is clocked by the voltage controlled oscillator (VCO) such that the time taken to cycle through all the memory addresses is the same as one revolution of the shaft. Run Lock Phase: The peak tracking correlator outputs a control signal to the VCO which clocks the memory addresses of the recorded shaft in an attempt
Figure 4.5: A schematic diagram of the twin loop torquemeter system where the angle at each measurement point is independently monitored and the twist along the shaft length is simply the difference in the outputs of the angle encoders.

to maintain the position of the recorded shaft identical to the photodetector outputs from live shaft. This forms the common mode part of the control loop and provides a degree of insensitivity in the output of the laser torquemeter to variations in the rotational speed of the shaft. A difference mode control loop tracks the twist in the shaft. Separating the control loops allows for different loop dynamics to be applied to the control loop tracking the shaft speed variations and measurement of twist.

The hybrid laser torquemeter operating in the Common-Mode Difference-Mode offers a shaft twist tracking range of $3^\circ$ with the shaft rotating at 1800rpm and a torque fluctuation frequency of 100Hz [27].

A typical measurement for this torquemeter is 100 torque readings per revolution with an applied twist signal of 17.5mrads$_{p-p}$ at 60 Hz and with 300 speckle fluctuations recorded per shaft revolution with a shaft speed of 1800rpm (60Hz). The output twist signal had a noise spectral density of 17μrads/$\sqrt{Hz}$ [111].
Figure 4.6: Schematic diagram of a Common-Mode Difference-Mode laser torquemeter system [28].
4.6 Comparison with Traditional Torquemeters

The laser torquemeter is a genuine non-contact torque measurement method. There is no contact with rotating parts and in many cases no special device needs to be inserted into or attached to the power transmission system. It is an easy 'stand and point' operation with little setup time. The operation of the laser torquemeter is not limited by shaft dimensions, cross section, or rotational speed. The laser torquemeter could easily be manufactured to be intrinsically safe.

The current laser torquemeter implementation requires rotation to measure torque and cannot be statically calibrated. Although other torque measuring devices suffer from the same problem, strain gauge based torquemeters do not. It is possible to measure static torque for calibration purposes via an optical speckle angle encoder which could be included in the laser torquemeter [86] [87] [43] [88]. The laser torquemeter is sensitive to vibration – axial, tilt and radial – which if the tracking of the correlation peak is lost requires the resetting of the load to the low load baseline torque. Typical twists on reasonable sections of shaft are very small requiring a large separation between detectors.
Chapter 5

Signal Processing for the Laser Torquemeter

Real time tracking of a time varying parameter in a noisy environment is one of the most challenging problems in signal processing. The observations consist of a wanted signal component and an unwanted noise component. The signal component contains information about the desired parameter. If this signal component could be perfectly isolated, the desired parameter could be measured with deterministic accuracy. [112])

The function of the torquemeter is to measure the torque of a rotating shaft and this requires an electronic system that correlates the live signal with a recorded signal taken under a condition of low (ideally no) load. The temporal lag between the live and recorded signals from two axially separated positions on the shaft may be used to determine shaft twist. One approach for obtaining an estimate of shaft twist is to attach angle encoders to the shaft and output the difference in angle between the two encoders. The required angle encoders need to be high precision as the twist of the shaft is small; therefore the encoders are expensive. A lower cost and non-contacting torquemeter may be constructed if the angle encoders are replaced by a laser, two beam splitters, retro-reflective tape, a pair of
photodetectors and signal processing electronics. The recorded signal from both channels is aligned to the live signal from the photodetectors – the delay between the live and recorded signals is minimised with a feedback loop which controls an index into the recorded signal. The angle encoder concept is a simplified model of the actual signal processing, which includes a control loop to reject common mode fluctuations in angle, or rotational speed fluctuations, from being included in the torque output signal [27].

In the torquemeter an estimate of the time delay between the recorded signal and the live signal is required for each photodetector in order to determine the shaft angle and determine the shaft twist, and hence torque. As the torque on a rotating shaft is a time varying parameter a tracking system is required to estimate the shaft twist. The task of the tracking system is to estimate the variable time delays with high precision and to produce reliable measurements in the presence of signal disturbances. The process of maintaining alignment between the live signal and the recorded signal is performed by a peak tracking correlator.

5.1 Methods of Estimating Time Delay

The signal processing problem of tracking a pair of periodic signals from the photodetector outputs and extracting (or estimating) the delay due to the applied torque is closely related to the problem of velocity estimation by cross-correlation of two spatially separated sensors. The sensors may be optical but other sensors such as capacitive, inductive, thermal, acoustic and electro-magnetic may be used. Butterfield et al [113] use an optical sensor and a tracking cross-correlator to measure the velocity of strip steel.
In speedometer type applications the sensors are mounted on a vehicle in motion \[91\] \[114\] \[115\] and in flowmeter type applications the sensors are stationary \[116\] \[117\] \[118\]. The first cross-correlation flow measurement, using radioactive tracers, was developed in the 1950s. A similar procedure also exists in radar and sonar range finding where a known signal is transmitted towards the target and the range is determined by the round trip time delay of the signal. An improvement in temporal resolution is gained by replacing pulsed signals with random or pseudo-random signals that are then cross-correlated with the returned signal. *A priori* knowledge of the transmitted signal allows the design of a matched filter to perform the cross-correlation. A filter whose impulse response equals the transmitted signal reflected back in time is a matched filter. When the input to a matched filter is the intended signal, the output is the cross-correlation between the transmitted signal and the received signal.

### 5.2 Application of Time Delay Estimation

The noise in the live signals is not mainly from the electronic sources but due to the properties of the speckle pattern. The translational motion of the rotating shaft influences the photodetector output signals as different scatterers are illuminated which were not illuminated when the recorded photodetector signal was recorded. Shaft tilt influences the photodetector output signal as a different part of the speckle pattern is detected. The distorted live photodetector output signal caused by shaft motion or tilt may be modelled as the sum of the original photodetector signal and a noise component. The bandwidth of the noise component is identical to that of the photodetector output signal as determined primarily by the size of the speckle pattern, the dimensions of the detecting aperture and the
rotational speed of the shaft as set out in Chapter 3. The electronic noise of the photodetector also appears as band-limited white noise but the cutoff frequency is determined by the bandwidth of the photodetector circuit. With careful design and construction, the electronic noise of the photodetectors can be made to be insignificant when compared to the noise due to decorrelation of the speckle pattern. Ignoring electronic noise, the recorded photodetector output signal may be simply modelled as the reference signal $x_1(t)$ and the live photodetector signal as $x_2(t)$

\begin{align}
  x_1(t) &= s(t) \\
  x_2(t) &= C(s(t + \tau) + n(t))
\end{align}

where $n(t)$ is the band-limited noise component due to speckle decorrelation due to non-rotational motion and $s(t)$ is the true signal component. The torquemeter records one signal, $x_1(t)$, and attempts to find the delay between the recorded signal and the live signal, $x_2(t)$. Any possible attenuation or gain of the detected signal is accommodated in the gain term $C$. The signal and noise components can be assumed to be stationary (at least in the short term), uncorrelated baseband processes. The signal $x_2(t)$ is a leading/lagging version of the general reference signal, $x_1(t)$, because of the time varying delay. The photodetector signals in this section are assumed to be ergodic, that is to say that the process generating the photodetector output signal is a stationary random process with the property that the time averages (over an infinitely long time) and ensemble averages are equal [119]. This is not true for the laser torquemeter as the properties of the random signals are governed by the rotational speed of the shaft, and in a practical instrument, this is free to vary. When the rotational speed is changed and
the photodetector output signal is within the bandwidth of the amplifiers, the current photodetector signal is a time compressed/expanded version identical to the original recorded signal. If the system tracks this change in speed then small changes in the differential delay may then be considered ergodic.

Conversion of Time Delay to Shaft Twist Angle
The laser torquemeter requires a measure of shaft twist rather than the tracking of a time delay. The time delay $\tau$ may be converted into an angle by the simple relation

$$\theta = \omega \tau$$  \hspace{1cm} (5.3)

where $\theta$ is shaft angle and $\omega$ is the angular velocity of the shaft. A circuit is required to maintain an estimate of the angular velocity and this may be accomplished using the peak tracking correlator. The value of $\omega$ is derived from the rate of change of the index into the recorded signal with respect to time; however, this method requires that only one revolution of photodetector output signal be recorded into memory.

5.3 The Effect of Speckle Decorrelation on the Signal Processing
The effect of motion of the shaft may be likened to adding noise to the photodetector output signal as discussed in Section (5.2) and the mean square signal-to-noise ratio may be related to magnitude of the peak cross-correlation between the recorded signal and live photodetector output signal. Equation (5.2) is now
rewritten as

$$x_2(t + \hat{\tau}) = C(s(t) + n(t))$$  \hspace{1cm} (5.4)$$

where $\hat{\tau}$ is an estimate of the delay $\tau$ which maximises $\Gamma_{z_1z_2}(\tau)$, the cross-correlation function of signals $x_1(t)$ and $x_2(t)$. Following the results of [116], the normalised cross-correlation of live signal $x_1(t)$ and the recorded signal $x_2(t)$ is given as

$$\gamma_{z_1z_2}(\tau) = \frac{\Gamma_{z_1z_2}(\tau)}{\sqrt{\Gamma_{x_1x_1}(0)\Gamma_{x_2x_2}(0)}}.$$  \hspace{1cm} (5.5)$$

The cross-correlation of the signals $x_1(t)$ and $x_2(t + \hat{\tau})$ results in

$$\Gamma_{z_1z_2}(\tau) = C(\Gamma_{z_1z_1}(\tau - \hat{\tau}) + \Gamma_{z_1n}(\tau - \hat{\tau}))$$  \hspace{1cm} (5.6)$$

and the autocorrelation of signal $x_2$ is

$$\Gamma_{x_2x_2}(\tau) = C^2(\Gamma_{z_1z_1}(\tau) + \Gamma_{z_1n}(\tau) + \Gamma_{nz_1}(\tau) + \Gamma_{nn}(\tau)).$$  \hspace{1cm} (5.7)$$

Assuming that the noise component $n(t)$ is uncorrelated with $x_2(t)$ then the cross-correlations with the noise $n(t)$ is zero (ie $\Gamma_{z_1n}(\tau) = 0$ and $\Gamma_{nz_1}(\tau) = 0$). This allows for the simplification to the above correlation equations with

$$\Gamma_{z_1z_2}(\tau) = C(\Gamma_{z_1z_1}(\tau - \hat{\tau}))$$  \hspace{1cm} (5.8)$$

$$\Gamma_{x_2x_2}(0) = C^2(\Gamma_{z_1z_1}(0) + \Gamma_{nn}(0))$$  \hspace{1cm} (5.9)$$

Substituting Equations (5.8) and (5.9) into (5.5), the normalised cross-correlation
of the photodetector output signals becomes

\[ \gamma_{x_1x_2}(\tau) = \frac{\Gamma_{x_1x_1}(\tau - \hat{\tau})}{\sqrt{\Gamma_{x_1x_1}(0)(\Gamma_{x_1x_1}(0) + \Gamma_{nn}(0))}} \] (5.10)

and when the estimate of the delay equals the actual delay (\(\tau = \hat{\tau}\)), Equation (5.10) becomes

\[ \gamma_{x_1x_2}(\hat{\tau}) = \frac{1}{\sqrt{1 + \Gamma_{nn}(0)/\Gamma_{x_1x_1}(0)}}. \] (5.11)

The signal to noise ratio (SNR) may be estimated by the normalised cross-correlation [116]:

\[ SNR = \frac{\Gamma_{x_1x_1}(0)}{\Gamma_{nn}(0)} = \frac{\gamma_{x_1x_2}^2(\hat{\tau})}{1 - \gamma_{x_1x_2}^2(\hat{\tau})}. \] (5.12)

The SNR ratio is of importance to the Laser Torquemeter because it relates the cross-correlation coefficient to a conventional measure of signal to noise ratio at the photodetector output.

Figure (5.1) is a plot of the SNR against the normalised cross-correlation.

### 5.4 Correlation Methods

The considerations in choosing a correlation algorithm and related parameters which allow for the acquisition of shaft twist, and hence torque, may be reduced to one question: how large can the noise component \(n(t)\) be for the time delay still to be estimated with reasonable precision after a fixed observational time? This is an important question and the result is applicable not just to torque measurement but to many fields including radar [120], astronomy, optical range finding,
synchronisation in digital systems [121], correlation speed measurement [113] and correlation flowmeters [116].

The methods of determining time delay may be summarised into two groups: Open Loop Correlators and Closed Loop Correlators.

5.4.1 Open Loop Correlators

The time delay of a system may be estimated by computing the cross correlation function between input and output signals and noting the lag at the peak correlation value. The cross-correlation between signals $x_1(t)$ and $x_2(t)$ is defined by

$$\Gamma_{x_1x_2}(\tau) = \int_{-\infty}^{\infty} x_1(t)x_2(t-\tau)dt. \quad (5.13)$$

The value of $\tau$ that maximises the cross-correlation provides an estimate of
the delay. In such cases, the delay can only be estimated because of the finite observational time of the signal. The cross-correlation method usually finds a good estimate of the true delay but it may be misled into selecting the incorrect maximum when the observation time is small or the signal to (uncorrelated) noise ratio is small. In this case the delay estimate output is *not* related to the true delay of the system. Improvements to the cross-correlation may be made by pre-filtering $x_1(t)$ and $x_2(t)$ to accentuate the signal passed at the frequencies where the signal to noise ratio (SNR) is the greatest. The selection of the filter response is made to optimise certain performance criteria and is always a compromise between good resolution and ability to find the true correlation peak. The open loop cross correlation algorithm is computationally expensive, requiring a large number of multiply and accumulate cycles to provide a result.

The characteristics of an open loop correlator are: a) no stability problems because there is no feedback loop, b) low sensitivity to variation in time delay when observations are made at small regular intervals, c) the location of the maximum of the cross-correlation function may not be correctly selected when observation times are short, and c) high resolution systems are expensive. The low sensitivity, the reduced ability to select the true correlation peak and the hardware cost make open loop cross-correlator unsuitable for a practical laser torquemeter [122].

### 5.4.2 Closed Loop Correlators

The closed loop correlator depends upon the generation of an error signal between the input signals or the live and recorded signals; this error signal is used as feedback to control a delay line. If some parameters about the measured system are obtainable, this knowledge may be applied to reduce the measurement
time while maintaining accuracy. When the structure of the dynamic system is known, and only the system parameters need to be derived, a comparison with a reference model is useful in determining the time delay. The output of the model is compared to the true output and the model parameters are adjusted to minimise the error between true and model outputs. The feedback loop which controls a delay element requires an odd function of the delay error; a function that is commonly used is the derivative of the cross-correlation function. The error function is frequently known as an S curve due to its shape [123].

The closed loop correlator is also referred to in the literature as a delay-locked loop (DLL) and many of its characteristics are derived from its similarity to the phase-locked loop.

**Importance of the Cross-correlation Shape**

In analysing the system, a reasonable assumption to make is that the temporal correlation function of the recorded photodetector signal and the live signal can be modelled as a Gaussian (as seen in Figure (3.24)) that may be written as

\[
\Gamma(\tau) = A \exp \left[ -2 \left( \frac{\tau}{\tau_e} \right)^2 \right]
\]  

(5.14)

with \(1/\epsilon^2\) width \(\tau_e\) and peak correlation coefficient \(A\).

An odd function with respect to delay \(\tau\) of the cross-correlation is required for the delay locked loop. The temporal derivative of the cross-correlation function between the recorded signal and live signal is suitable for the error function or S curve. Figure (5.2) shows a plot of an autocorrelation function of real speckle data and the derivative of the autocorrelation function.

A block diagram of the circuit which implements the closed loop correlator is
Figure 5.2: Autocorrelation function of a typical photodetector output signal and its S curve.

shown in Figure (5.3). The signals $x_1(t)$ and $x_2(t - \tau)$ are input and the $x_2(t - \tau)$ input is delayed by the estimate of the time delay by $\hat{\tau}$. The input $x_1(t)$ is subtracted from delayed component, resulting in an error signal $e(t)$. The error signal is multiplied by $x_1'(t)$ and the resulting signal is smoothed by an integrator, and then used to update the delay estimate $\hat{\tau}$ which is output. This peak tracking correlator structure offers better immunity to tracking of false peaks than simpler peak tracking correlator circuits [116].

The discriminator or error function characteristic of the delay-locked loop is the derivative of Equation (5.14) with respect to lag $\tau$

$$S(\tau) = \Gamma'(\tau) = -4A \frac{\tau}{\tau_0^2} \exp \left[ -2 \left( \frac{\tau}{\tau_0} \right)^2 \right]. \quad (5.15)$$

The linear range of the delay-locked loop is limited to small errors of a few percent
and for large errors of $\tau$ the error signal vanishes. If the assumption that the delay error is small is false then the non-linearity of the $S$ curve must be accounted for or the delay wanders off the peak correlation and the closed loop correlator falls 'out of lock'.

If the autocorrelation function of the photodetector output signal is not the assumed Gaussian function ($\Gamma(\tau) = \exp[-\frac{1}{2}(\tau/\tau_e)^2]$) but an exponential function ($\Gamma(\tau) = \exp(-|\tau|/\tau_e)$) or some other function where there is a discontinuity (usually at $\tau = 0$) where the derivative at the singularity is undefined then there is no range where the delay locked loop may track the speckle signal. Typical results indicate that this assumption is generally true, but some results do have a discontinuity at the origin. This is a serious problem for tracking the cross-correlation peak and a possible solution is to pre-filter the signals so that their autocorrelation functions become continuous at $\tau = 0$. The electronic filtering of the photodetector signals may be problematic as the shaft speed may vary over a wide range – a better solution may be to spatially filter the speckles. A soft aperture could provide a Gaussian-shaped autocorrelation function and hence no
discontinuity at the origin. The frequency response of the spatial filter would automatically match the frequency content of the photodetector output signals.

The closed loop correlator must have a high enough frequency response to maintain lock for a reasonable length of time (this is dependent upon the duration that continuous torque measurement is required) at the expense of the steady state error. The closed loop correlator may then lock on to some false local maxima or to no signal at all, because it calculates only one point of the correlation function at a time and cannot distinguish local maxima from the global maximum. A start up period is required to find the true global maximum in the correlation function (an open loop correlator) and a separate monitoring system is required to ensure that the global maximum is being tracked. The closed loop correlator can only track delays with limited dynamics (in the change of time delay) because to maintain stable operation the dynamical error must not exceed the linear range of the error restoring signal.

The steady state tracking error is traded-off against transient dynamic tracking errors. An understanding of the dynamics of the measurement required allows sensible decisions to be made in this trade-off. The signal to noise ratio of the input to the closed loop correlator is of concern because low SNR results in less time between 'loss of lock' events. If a 'loss of lock' event occurs, it is no longer possible to track the correlation peak and the laser torquemeter would be unable to provide a torque output. If the photodetector output is significantly changed, the mechanical load may have to be removed and the reference signal reloaded. An important question in applying a closed loop correlation concerns the duration over which the closed loop correlator can track the time varying delay. Knowledge of the expected SNR is required to answer this question and the main component of noise in the input signals is derived from the optical arrangement.
of the torquemeter and the vibrational motion of the shaft.

The characteristics of the closed loop correlator include: a) high precision of the delay estimate when the delay lock loop is 'locked', b) dynamic tracking errors may be reduced by changing the integration time and gain of the integrator in Figure (5.3), c) relatively easy implementation; however, extra hardware is required to acquire the initial peak of the correlation function and to detect when the delay loop is 'locked', d) the tracker can fall 'out of lock' if $\frac{dr}{dt}$ is too large due to high noise levels (which may be due to shaft motion) [122].

Mean Time to Loss of Lock

The closed loop correlator computes only a small portion of the correlation function and tracks the peak correlation value to provide a continuous stream of estimates of the time delay. If the tracking of the correlation peak between the live and recorded signals is corrupted by noise (due to decorrelation of the live photodetector output signal with respect to the recorded signal) so that the tracker wanders away from the true correlation peak, the closed loop correlator is said to have lost lock. In order for the system to maintain lock, the estimate of time delay must stay within the bounds of the $S$ function for which the difference between the real delay and the delay estimate tends to reduce the error. Once the delay estimate is outside the bounds where the error signal pushes the control signal towards the correct delay, the delay locked loop can no longer track the time delay. Common noise sources are Gaussian in nature and there always exists a finite probability that the noise will exceed the threshold and cause the loop to fall out of lock.

Delay locked loops require a monitoring circuit that signals the loss of lock condition; however, it is difficult to develop an unambiguous lock detector [123].
The mean time to lose lock is a problem similar to cycle slipping in phase locked loops with a major difference – the nonlinearity of the delay locked loop is aperiodic [121].

Variance of the Closed Loop Correlator Output

The output of the closed loop correlator is the estimate of the time delay between two signals. The sources of error in estimating the time delay using the cross-correlation method are due to the finite averaging time $T$ between estimates, the finite bandwidth $B$ of the signal and interfering noise (including both the decorrelation of the photodetector output signal and signal quantisation error) [124].

An instinctive approach suggests that the more time spent on determining the time delay the better this estimate is and when the level of noise increases more time is required to give an estimate of time delay with the same accuracy.

In the laser torquemeter, the speed of shaft rotation and the speckle characteristics place an upper limit on the signal bandwidth and the torquemeter frequency response is inversely proportional to the averaging time; these characteristics are fixed by the desired performance of the torquemeter.

The variance in the cross-correlator output signal may be determined by following the work of Hayes [124]. The cross-correlation between noise-free versions of the recorded signal and the recorded signal may be estimated by

$$\hat{\Gamma}_{x_1x_2}(\tau, T) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x_1(t) x_2(t - \tau) \, dt$$

(5.16)

where $\hat{\Gamma}_{x_1x_2}(\tau, T)$ denotes an estimate of the cross-correlation between signal $x_1(t)$ and $x_2(t)$ using records of duration $T$. Including signal corrupting noise $n(t)$ due to decorrelation, electrical noise and quantisation errors and the technique of
Section (5.3) results in

\[ \hat{\Gamma}_{z_1z_2}(\tau, T) = C \left( \Gamma_{z_1z_1}(\tau - \hat{\tau}, T) + \Gamma_{z_1n}(\tau - \hat{\tau}, T) \right) \]  \hspace{1cm} (5.17)

where the estimate of the time delay is denoted by \( \hat{\tau} \), which is the value of \( \tau \) that maximises \( \Gamma_{z_1z_1}(\tau, T) \). The cross-correlation between the signal \( x_1(t) \) and the noise \( n(t) \) is denoted by \( \Gamma_{z_1n}(\tau, T) \). For the peak delay, the derivative of the correlation function is zero so

\[ \frac{d}{d\tau} \Gamma_{z_1z_2}(\tau, T) = \Gamma'_{z_1z_2}(\tau, T) = 0. \]  \hspace{1cm} (5.18)

Differentiating Equation (5.17) results in

\[ \Gamma'_{z_1x_1}(\tau - \hat{\tau}, T) + \Gamma'_{z_1n}(\tau - \hat{\tau}, T) = 0. \]  \hspace{1cm} (5.19)

A Taylor series expansion of the correlation function \( \Gamma_{z_1x_1}(\tau - \hat{\tau}, T) \) results in

\[ \Gamma_{z_1x_1}(\tau - \hat{\tau}, T) = \Gamma_{z_1x_1}(0, T) + \frac{(\tau - \hat{\tau})}{1!} \Gamma'_{z_1x_1}(0, T) + \frac{(\tau - \hat{\tau})^2}{2!} \Gamma''_{z_1x_1}(0, T) + \ldots \]  \hspace{1cm} (5.20)

The first derivative term is zero because the correlation function is at a maximum when \( \tau - \hat{\tau} = 0 \) and if the difference \( \tau - \hat{\tau} \) is small then higher order terms may be assumed to be negligible. Then the correlation function may be approximated by

\[ \Gamma_{z_1x_1}(\tau - \hat{\tau}, T) = \Gamma_{z_1x_1}(0, T) + \frac{(\tau - \hat{\tau})^2}{2!} \Gamma''_{z_1x_1}(0, T) \]  \hspace{1cm} (5.21)
and the derivative of the above equation results in

$$
\Gamma'_{x_1,x_1}(\tau - \hat{\tau}, T) = (\tau - \hat{\tau}) \Gamma''_{x_1,x_1}(0, T).
$$

(5.22)

The relationship between the error of the time delay estimate and the actual time delay of the signal $(\tau - \hat{\tau})$ and the noise over the measurement time $T$ is given by combining Equation (5.22) with Equation (5.19) and may be written as

$$
\tau - \hat{\tau} = -\frac{\Gamma_{x_1,n}(\tau - \hat{\tau}, T)}{\Gamma_{x_1,x_1}(0, T)}
$$

(5.23)

and relates the error in the estimate of $\tau$ (i.e. $\tau - \hat{\tau}$) to the cross-correlation function of the signal $x_1(t)$ and the noise $n(t)$ made over a finite time $T$.

Assuming that $T$ is large enough so that $\Gamma_{x_1,x_1}(0, T)$ approaches the constant value $\Gamma_{x_1,x_1}(0)$, the variance of $N$ estimates of $\tau$ is given by

$$
\text{var}(\hat{\tau}) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (\tau_i - \hat{\tau})^2
$$

(5.24)

$$
= \lim_{N \to \infty} \frac{1}{N} \left( \frac{1}{\Gamma_{x_1,x_1}(0)} \right) \sum_{i=1}^{N} \left( \Gamma'_{x_1,n}(\tau - \hat{\tau}, T) \right)^2
$$

(5.25)

where the $i$th estimate is denoted by the superscript $i$.

In order to determine the mean square value of $\Gamma'_{x_1,n}(\tau_i - \hat{\tau}, T)$ the following assumptions are required: the signal $x_1(t)$ and the noise component $n(t)$ are uncorrelated, the functions $\Gamma'_{x_1,n}(\tau_i - \hat{\tau}, T)$ form a random process, and this process is stationary and ergodic. These assumptions allow Equation (5.25) to be written as a time average [124]

$$
\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left( \Gamma'_{x_1,n}(\tau - \hat{\tau}, T) \right)^2 = \lim_{T \to \infty} \frac{1}{T_N} \int_{-T_N/2}^{T_N/2} \left( \Gamma'_{x_1,n}(\tau, T) \right)^2 d\tau.
$$

(5.26)
Equation (5.25) can be rewritten as

\[
\text{var}(\hat{r}) = \lim_{T_N \to \infty} \frac{1}{T_N} \int_{-T_N/2}^{T_N/2} \frac{(\Gamma_{x_1 n}(\tau, T))^2}{(\Gamma_{x_1 x_1}(0))^2} d\tau.
\] (5.27)

The autocorrelation function and power density spectrum are related by the Wiener-Khinchin theorem [54]. The power density spectrum of the autocorrelation function $\Gamma_{x_1 x_1}(\tau)$ is given by

\[
\Gamma_{x_1 x_1}(\tau) = \int_{-\infty}^{\infty} S_{x_1 x_1}(f)e^{j2\pi f\tau} df
\] (5.28)

and the second derivative is

\[
\Gamma''_{x_1 x_1}(\tau) = \int_{-\infty}^{\infty} (j2\pi f)^2 S_{x_1 x_1}(f)e^{j2\pi f\tau} df.
\] (5.29)

When $\tau$ is set to zero, Equation (5.29) becomes

\[
\Gamma''_{x_1 x_1}(0) = -4\pi^2 \int_{-\infty}^{\infty} f^2 S_{x_1 x_1}(f) df.
\] (5.30)

The power density spectrum of the noise $n(t)$ signal is given by

\[
\Gamma_{nn}(\tau) = \int_{-\infty}^{\infty} S_{nn}(h)e^{j2\pi h\tau} dh.
\] (5.31)

Combining Equations (5.30, 5.31 and 5.27)

\[
\int_{-\infty}^{\infty} \Gamma''_{x_1 x_1}(\tau) \Gamma_{nn}(\tau) d\tau = -4\pi^2 \int f^2 S_{x_1 x_1}(f)S_{nn}(h)e^{j2\pi f(h+h)} df dh d\tau
\] (5.32)

\[
= -4\pi^2 \int f^2 S_{x_1 x_1}(f)S_{nn}(f) df.
\] (5.33)
The variance of the peak tracking correlator may be expressed in terms of power density spectra of the signal $x_1(t)$ and noise $n(t)$ as

$$\text{var}(\hat{\tau}) = \frac{1}{4\pi^2T} \int_{-\infty}^{\infty} f^2 S_{x_1 x_1}(f) S_{nn}(f) df \left( \int_{-\infty}^{\infty} f^2 S_{x_1 x_1}(f) df \right)^{-2}. \quad (5.34)$$

If the frequency content $H_{x_1 x_1}(f)$ of signal $x_1(t)$ is white noise limited by bandwidth $B$ and the frequency content given by

$$H_{x_1 x_1}(f) = \begin{cases} H_{x_1 x_1}(0) & \text{when } f < B \\ 0 & \text{when } f \geq B \end{cases} \quad (5.35)$$

and the noise process $n(t)$ is assumed to be white noise (i.e. $S_{nn}(f) = S_{nn}(0)$) then the variance of estimates of the delay parameter $\hat{\tau}$ is given by

$$\text{var}(\hat{\tau}) = \frac{1}{4\pi^2T} 2 \int_{0}^{B} f^2 H_{x_1 x_1}(0) H_{nn}(0) df \left[ 2 \int_{0}^{B} f^2 H_{x_1 x_1}(0) df \right]^{-2}.$$ (5.36)

$$= \frac{1}{8\pi^2T} \frac{H_{nn}(0)}{H_{x_1 x_1}(0)} \left[ \int_{0}^{B} f^2 df \right]^{-1} \quad (5.37)$$

$$= \frac{3}{8\pi^2TB^3} \frac{1}{\Gamma_{nn}(0)} \quad (5.38)$$

which may then be reduced to [125]

$$\text{var}(\hat{\tau}) = \frac{3}{8\pi^2TB^3} \left( \frac{1}{\gamma_{x_1 x_1}^2(\hat{\tau})} - 1 \right). \quad (5.39)$$

This result is important for the development of the laser torquemeter because it relates the quality of a time delay measurement to the bandwidth of the signal, the update rate of these time delay measurements and the signal to noise ratio of the input signal. Equation (5.39) must be modified because the measurement
of interest for laser torquemeter is shaft twist $\theta$ and is related to time delay $\tau$ by rotation frequency $\omega$. The bandwidth of the photodetector signal $B$ is fixed to the rotation frequency by a fixed constant $c_B$ (otherwise the recorded and live signals are not alike) and the time between twist estimates $T$ is also fixed to the rotation frequency by $c_T/\omega$. The variance of the measured twist becomes

$$\text{var}(\hat{\theta}) = \omega^2 \text{var}(\hat{\tau}) = \frac{3}{8 \pi^2 c_B^3 c_T} \left( \frac{1}{\gamma_{x_1x_2}(\hat{\tau})} - 1 \right). \quad (5.40)$$

The most effective way of decreasing the variance of the torquemeter output is by increasing the bandwidth of the input signals because of the cube law dependence of variance on the bandwidth $B$ of the signals. The bandwidth of the input signal depends upon $c_B$ which is principally influenced by the speckle width $\langle \sigma_0 \rangle$, the aperture area $L_y$ and the photodetector range $z$. Reducing the number of twist measurements per revolution by increasing $c_T$ decreases the variance in the twist measurement in proportion with $1/c_T$. The variance of the estimate of twist $\theta$ is also reduced by increasing the normalised correlation between the signals $x_1$ and $x_2$.

The rotation frequency has no effect on the variance of the estimate of shaft twist angle.

In order to illustrate the variance of the laser torquemeter output the following hypothetical measurement scenario is presented: a solid steel driveshaft, 20mm in diameter, twists 1.68° at 220Nm (see Table (4.6)). If the shaft is rotating at 1500rpm, the bandwidth of the photodetector output signal is 50kHz (which is set by the optical configuration, the rotational velocity of the shaft and limited by speed of the signal processing electronics), the cross-correlation between the recorded signal and the live signal exceeds 50% and 100 torque measurements are
required per revolution, the rms twist noise is \( \approx 1 \text{mdeg} \) or 0.06%.

**Effect of Signal Quantisation**

Quantisation of the signal amplitude does not change the peak location nor the symmetry of the cross-correlation function, but signal quantisation decreases the SNR. This decrease in the SNR increases the variance of the delay estimate which then requires a longer averaging time to obtain the same accuracy and reduces the frequency response of the instrument. The amount of quantisation for common types of cross-correlators may be loosely divided into three groups: a) both signals in analogue or finely quantised form, b) one signal is binary quantised according to the sign of its deviation from its mean value and the other finely quantised or analogue which is known as the "Stieltjes correlation calculation", and c) both signals binary quantised - the "Polarity cross-correlation form" [116]. The ratio of the variance of the peak delay estimate for the direct, Stieltjes and Polarity cross-correlation forms, under a constant integration time \( T \), is [116]

\[
\text{var}(\hat{r}_{\text{direct}}) : \text{var}(\hat{r}_{\text{Stieltjes}}) : \text{var}(\hat{r}_{\text{polar}}) = 1 : 1.4 : 2.6. \tag{5.41}
\]

"Increasing the number of quantisation levels by only a small amount leads to a significant reduction of the degradation factor, and further reduction is obtained by sampling faster than the Nyquist rate." [126] In a typical modern cross-correlation system, with 16 bits of precision for both signals, and sampling at least at the Nyquist rate, the estimate of time delay has a variance approaching that of the direct cross-correlation form [116] [127].
Temporal Signal Quantisation

The sampling rate of the laser torquemeter has an effect on noise. The sampling rate limits the bandwidth of the signal due to the Nyquist limit and places an upper bound on the frequency content of the recorded signal. Any reduction of the bandwidth $B$ influences the variance of the output signal as described by Equation (5.38). The time between samples of the photodetector output signal must be much smaller than the width of the cross-correlation peak to maintain 'lock' of the peak tracking correlator. In a peak tracking correlator where the correlation peak width is only a few samples wide the tracking correlator attempts to find the true delay by oscillating between the adjacent (quantised) delays and the mean value of delay approaches $\tau$ [26].

5.4.3 Other Factors Influencing the Torquemeter

The torquemeter is sensitive to some factors that can be of significant influence if they are not considered early in the design process. These include the following:

Resolution and Rangeability

The minimum possible delay estimate is limited by the minimum delay produced by a one step change in the length of the variable delay line. The maximum delay is the length of the delay line. The range is then governed by the depth of the buffer of the delay line, and the sampling rate of the system. In general, faster sampling rates are desired as this reduces the variance of $\tau$, but this is at the cost of a larger buffer memory, necessary in order to store a complete set of samples in one revolution.
Unequal Speckle Dynamics between Detectors

The shaft motion may cause different responses for each of the photo detecting channels. This may be a frequently encountered problem as the torquemeter may be installed with its axis not parallel to the shaft's axis, resulting in a different $z$ and $z_0$ value for each channel. The torquemeter has a degree of cross-talk between each channel in order to reduce common body error but if the characteristics of the photodetector output signal are different, the cross-talk will result in artifacts in the torque output signal.

Influence of Modulation Depth

The photodetector and the detection circuits are ac-coupled in order to provide the cross-correlation inputs with a signal with a mean value of zero.

The modulation level of the photodetector output signal is reduced if there are a large number of speckles incident upon the detecting aperture. The photodetector aperture spatially filters the speckle pattern reducing the frequency content of the photodetector output signal. The reduction in modulation depth increases the variance of the torquemeter estimate by reducing the bandwidth $B$ of the signal input into the tracking correlator. Equation (5.39) shows that the variance changes with bandwidth by a factor of $1/B^3$ and this strong influence of bandwidth suggests that the laser torquemeter requires the smallest photodetector aperture that resists photodetector signal decorrelation due to shaft motion.

The modulation depth of the photodetector output signal is also reduced when the scattering surface depolarises the incident beam or the illuminating beam is randomly polarised. The contrast ratio of the speckle pattern may be reduced by a factor of up to $1/\sqrt{2}$ depending upon the amount of depolarisation. A simple solution to improving the contrast ratio when using a depolarising target
is to place a polarisation analyser in front of the detecting aperture and to use a linearly polarised laser.

5.4.4 Anti-aliasing Filters

Anti-aliasing filters are not strictly required or desired as the photodetector output signal has a band-limited spectrum (the filtering is done in the spatial domain of the optical system – the detector aperture produces a cut-off filter and limits the bandwidth of the photodetector output signal). The signals’ natural band-limit is an advantage because this removes the requirement for an electronic anti-aliasing filter. If a lower bandwidth signal is desired for the correlation circuits, due to bandwidth limitations of the processing electronics, the cut-off frequency of the anti-aliasing filters needs to be matched with the rotation rate of the shaft. The sampling rate of the torquemeter should be at least twice the largest frequency component of the signal. The maximum sampling rate is limited by the speed of the electronic hardware.

5.5 Optical Design

The design of the optical head configuration is crucial to the reliable operation of the laser torquemeter. There are a variety of parameters that are free to vary –

Aperture Dimensions The aperture size is adjustable in the $x$ and $y$ directions.

Aperture Properties The aperture may be ‘hard’ where the transmission function is either 0 or 1 or the aperture may be ‘soft’ where the transmission function is free to vary in the range 0 to 1.

Beam properties Beam waist size $w_0$, position $z_0$, divergence, wavelength $\lambda$,
spot size on shaft $D$, and intensity profile.

**Detector Position** Range from shaft to detector $z$.

The optical head configuration to some extent depends on the measurement conditions. Large shafts present different demands over smaller shafts – for example the axial float of larger shafts tends to be lower (see Table (4.4)).

There are some other parameters that are not easily independently variable but are linked to the choice of the above parameters:

**Speckle size** which is dependent on the spot size $D$ and the photodetector – shaft range $z$.

**Radius of curvature** which is linked to the wavelength, beam waist and range to shaft as described by Equation (2.12).

**Modulation Depth** is linked to the speckle size and aperture size.

**Aperture Dimensions**

The photodiode aperture in the photodetector is likely to be much smaller than the desired aperture – fast photodiodes have small active areas to reduce the junction capacitance. A practical solution is to use an aperture and lens to provide a large effective sensitive area with a small photodiode sensitive area. Figure (5.4) shows an optical arrangement where a lens is used to enlarge the aperture of a photodetector. As seen in Section (3.11) the minimum sized aperture required to adequately tolerate axial float and tilt motions of the shaft is best for the laser torquemeter. A lens and aperture arrangement is an easy method of adjusting
Figure 5.4: Use of a lens to produce an effective photodetector aperture much larger than the aperture of the photodiode.

$L_x$ and $L_y$ for varying measurement scenarios and also allows for a cheaper, small area photodiode to be used.

$L_x$ – Axial Float and Tilt Motions

For reliable operation of the torquemeter, the width of the photodetector in the $x$ direction is dependent on the axial float and $\theta_\eta$ tilt of the shaft. The width should be the larger of the two estimates for $L_x$: one for which the conditions in Equation (3.26) are met and resistance to decorrelation due to shaft float is at a maximum value of $\approx 0.7D$ and the value of $L_x$ required to resist decorrelation due to expected shaft tilt as described by Equation (3.28).

$L_y$ – Rotational and Radial Motions

The aperture in the $y$ direction should be as small as possible as this increases resolution of the rotation angle $\theta_\xi$. A narrow aperture reduces the total area of the aperture and this increases the modulation depth. The sensitivity of the detector and the signal processing bandwidth both limit the minimum size of $L_y$. The bandwidth of both the photodetector and the peak tracking correlator circuitry must be larger than the bandwidth of the photodetector output signal to obtain maximum sensitivity to rotational motion $\theta_\xi$ and minimum variance in the peak tracking correlator output as seen in Section (5.4.2). There is little
point in reducing the size of $L_y$ much below that of the speckle size $\langle \sigma_0 \rangle$ because the speckle signal bandwidth rolls off when $L_y \approx \langle \sigma_0 \rangle$.

**Alignment of Torquemeter with Shaft**

It is important to have the axis of measurement (the line between the illuminated spots on the shaft) parallel to the axis of rotation of the shaft. This is to avoid cosine error caused when the axis between the illuminated spots and the axis of torque forms an angle of $\theta_{\text{misalign}}$. The error in the value of $\Delta L$ follows $1 - \cos(\theta_{\text{misalign}})$. For small angles the cosine error produces only small errors in the estimate of length – a misalignment of $1^\circ$ produces an error of 0.015% in torque.

**Illuminated Spot Size on the Shaft**

The size of the spots illuminating the shaft effectively sets a lower limit on the separation of spots on the shaft. A typical value of $\Delta L$ of 20cm results in a laser torquemeter output uncertainty of 1.4% as an uncertainty in the measurement of $\Delta L$. Figure (5.5) shows a plot of $\Delta L$ against uncertainty in the laser torquemeter output due to the finite size of the illuminating spots. A calibration procedure may be used to deduce a twist calibration factor $K = (GJ)/\Delta L$ and indirectly measure the effective beam separation $\Delta L$ as long as the incident beams are parallel.

**Cross-Talk Between Photodetectors**

Photodetector output signals should ideally have no common component, which in practice is impossible to achieve. Isolation can be achieved by using baffles to shield the photodetectors. If the back-scatterer preserves the polarisation of the illuminating light the detectors can be isolated from each other by having the
Figure 5.5: Uncertainty in laser torquemeter reading due to finite size of probe spots. $\Delta L = 1.4\text{mm}$

laser illuminating channel $A$ horizontally polarised and a horizontally polarised analyser on the detector and a vertical polarisation scheme on channel $B$. An alternative scheme is to use different wavelengths for each channel and narrow pass interference filters on the detectors. The interference filter also reduces noise in the photodetector by reducing background light.

**Divergence of Illumination Beam**

The divergence of the illuminating beam concerns how the spot size on shaft $D$ varies with range. Radial motion in the $z$ direction (in the optical axis) may vary the size of the spot on the shaft. If the illuminating beam is well collimated (with a divergence less than 50 mdeg) the spot size is constant to within 1\% at a range ($z$) of 20cm with a vibration amplitude of 1cm$_{p-p}$. 

156
Elliptical Spots on Shaft

Elliptical spots on the shaft allow for different speckle sizes in the $x$ and $y$ directions. The cost advantage of simple spherical optics or no optics at all overrides the advantage of elliptical spots; however, aspherical optics may be avoided because laser diodes emit light from a small aperture in an elliptical pattern. The mathematical treatment of the elliptical spot requires the substitution of spot diameter $D$ with chord dimensions $D_x$ and $D_y$.

Wavelength and Laser Power

It is convenient for alignment to use a visible laser beam. Common wavelengths are 442nm (He-Cd), 632.8nm (He-Ne), 670nm (Ga$_{0.5}$In$_{0.3}$P diode). Power output should be limited to less than 1mW in a visible wavelength to keep the optical source relatively safe for accidental momentary viewing (BS60825-1:1994 Class 2) [128].

5.6 Torquemeter Optical Head Design Example

A simple design procedure for a torquemeter optical head is presented:

1. Choose a laser type to make a choice of wavelength. The wavelength of a He-Ne laser is $\lambda = 632.8$nm – this is a good choice because this wavelength is visible and lasers are easily available. Semiconductor diode lasers are also available in this wavelength. The circular $\text{TEM}_{00}$ Gaussian beam profile of the He-Ne laser is preferable to the broad asymmetric spatial intensity distribution associated with diode lasers.

2. Noting that the decorrelation due to radial vibration is small, determine axial float and tilt for shaft from design tables, rule of thumb or measure-
ment. For this example: peak to peak axial float 1mm$_{p-p}$ and shaft tilt 1°$_{p-p}$.

3. Define amount of photodetector output signal decorrelation before tracking is lost. The amount of decorrelation is related to the mean time to loss of lock and for this example, loss of lock occurs when the peak of the cross-correlation function drops by a factor of $1/e^2$.

4. From Equation (3.25) on page 73, the spot size on the shaft $D$ should be set at least 1.4 times the axial float, provided the conditions in Equation (3.26) are met. $D = 1.4$mm

5. Set a safe range from the optical head to the shaft. The optical head – shaft range $z$ should not be too large as this increases the sensitivity to shaft tilt but if $z$ is too small then there is a risk that the optical head or any associated cables of the laser torquemeter becomes accidentally entangled in the shaft. A range of $z = 20$cm is a reasonable compromise and $z_0 = z$ because the photodetector and the laser are in the same unit.

6. Using Equation (2.11) on page 45, and knowing the beam waist to shaft range, the illuminated spot diameter $D = 2w$ and the wavelength $\lambda$, solve for beam waist $w_0$. A lens arrangement may be required to expand the incident beam to this value. $w_0 = 0.7$mm

7. A compromise between the material properties of the shaft and the space available to allow for a $\Delta L$ axial separation of illuminated spots define the maximum shaft twist. A length of shaft may be inserted in the driveline which allows the alteration of shear modulus of elasticity $G$ by changing the shaft composition and polar second moment of area $J$ may be altered
by necking the shaft down. The torsional stiffness of the inserted shaft is then compromised.

The variance in the torquemeter output is linked to the bandwidth of the speckle signal. The aperture $L_y$ dimension must be small enough to keep the variance in the output low at the slowest shaft speeds and the bandwidth of the processing electronics must be high enough at the highest shaft rotation speed. Use Section (5.4.2) to compute the bandwidth required and Equation (3.18) to calculate the height of the aperture $L_y$. Maximum torque produces a twist of $1^\circ$ over axial spot separation of $\Delta L$. $L_y = 0.9\text{mm}$

8. Compute the following parameters: $r = 30\text{m}$, $\rho = 1.0067$, $\langle \sigma_0 \rangle = 0.08\text{mm}$, $2k_0\langle \sigma_0 \rangle^2 = 0.0377\text{mm}^2$ and $k_0\rho^2D^2 = 5.66\text{mm}^2$. Use the criteria of Equation (3.26) to get an estimate of aperture width which adequately resists decorrelation due to shaft float. $L_x \geq 3.8\text{mm}$

9. Use Equation (3.28) to get an estimate of aperture width which resists decorrelation due to shaft tilt. $L_x \geq 8.3\text{mm}$

10. To resist decorrelation due to shaft tilt and float choose the larger of the computed values for $L_x$.

The design of the optical head for the laser torquemeter is mainly dictated by the measurement situation. The choice of the main parameters ($L_x$, $L_y$ and $D$) is made considering the three decorrelation widths due to the shaft rotational motions $\theta_\xi$, $\theta_\eta$ and axial motion. The mean time between loss of lock events is also dictated by the measurement situation and influences the decorrelation widths and hence all the main parameters. Aperture dimension $L_y$ and spot side $D$ are set by the angular resolution required and is $L_x$ is related to axial and
tilt decorrelation. The choice of $z_0$, $z$ and $\lambda$ is made by practical considerations. The errors in torquemeter output are mainly associated with uncertainty in the measurement of $\Delta L$ and may be improved upon by calibration.
Chapter 6

Conclusions

Torque measurements are mainly made to determine the power developed or absorbed by rotating machinery. The measurement of torque is a difficult parameter to measure and a variety of different methods have been developed. The most commonly marketed transmission torquemeter is based on strain gauges attached to the shaft and suffers from the requirement that the strain gauges are mounted on a rotating shaft and that the torquemeter shaft must be inserted into the driveshaft. The laser torquemeter offers a considerable advantage over traditional torquemeters by providing a true non-contacting torque measurement.

This thesis has addressed the issue of photodetector output signal decorrelation due to shaft motion in the development of the laser torquemeter – the spatially integrated speckle pattern and its space-time cross correlation are explored theoretically in Section (3.4). Space-time cross-correlation function widths of spatially integrated speckle patterns are investigated experimentally by examining the cross-correlation of a live photodetector output signal with a previously recorded photodetector output signal when the scattering surface is subjected to various motions [129] [130]. All decorrelation widths are measured to the point that the correlation function is reduced to \(1/e^2\) of the peak value. These cor-

161
relation widths may be easily adjusted to widths at other decorrelation levels – Figure (A.3) plots the decorrelation widths normalised to $1/e^2$.

Decorrelation due to shaft rotation about the $\zeta$ axis is theoretically explored in Section (3.5) and experimentally in Section (3.17). The decorrelation width due to rotation of the shaft is important for the operation of the laser torquemeter because this decorrelation width sets the minimum angular resolution of the torquemeter.

The variance of the torquemeter output is explored in Section (5.4.2) and is independent of the rotational velocity $\omega$ and inversely proportional to the cube of the bandwidth of the photodetector output signal – larger photodetector output signal bandwidths reduce the variance of the twist estimates. The photodetector output bandwidth is proportional to the rotational decorrelation angle and this is mainly influenced by aperture height, speckle size, the illuminated spot size, and the scattering surface – detector range [131]. The variance of the shaft twist estimates is also inversely proportional to square of the correlation between the recorded and live photodetector output signals and inversely proportional to the number of torque measurements required per revolution of the shaft.

The decorrelation due to axial shaft translation of a rotating shaft (in the $\zeta$ direction) as explored in Section (3.6) and experimentally confirmed in Section (3.18). The translational decorrelation width is the amount of axial float that a laser torquemeter could tolerate and is influenced by speckle size, aperture length and most importantly, the illuminated spot size. The ratio of the axial displacement to the illuminated spot diameter takes a maximum value of approximately 0.7 and therefore the minimum width of the illuminating spot on the shaft should be greater than approximately 1.4 times the expected maximum axial shaft displacement.
The effect of shaft tilt about the \( \eta \) axis displaces the speckle pattern at an angular rate of twice that of the tilt. This displaces the speckle pattern off the photodetecting aperture and disrupts reliable operation of the laser torquemeter. The amount of tilt required to decorrelate the photodetector output signal depends on the speckle size, the aperture dimension \( L_x \) and the range from the shaft to the photodetector. The link between the space-time correlation function and the effect of shaft tilt on photodetector output signal was described in Section (3.7).

Shaft motion in the radial direction (and perpendicular to the viewing direction) advances/retards the arrival of the live photodetector output signal. This appears to the laser torquemeter as a variation in rotational velocity of the shaft and is accommodated in the signal processing electronics. Shaft motion in the radial direction (and parallel to the viewing direction) produces no significant variation in the speckle pattern and does not influence operation of the laser torquemeter.

A survey of the operating environment of the laser torquemeter was made, including measurements of shaft tilt and axial float on a diesel engine driveshaft. The maximum shaft tilt of the driveshaft was estimated at \( 1^\circ \) and the maximum axial shaft float 1mm\(_{pp}\).

A design procedure for the laser torquemeter optical head was presented. An optimal set of optical head parameters for all measurement conditions is difficult to produce because of the wide ranging torque measurement situations. The shaft diameter and mechanical properties, vibrational motions, rotational velocity expected torque and length of shaft available for measurement may all vary significantly resulting in a variety of optical heads for different measurement scenarios. The requirement of a variety of optical heads is not a significant disadvantage.
for the laser torque meter because conventional torque meters come in a variety of torque, rotational velocity and shaft size ranges.

6.1 Recommendations for Further Work

The main practical developments for the laser torque meter that will be required in the future are as follows:

1. What maximum decorrelation between the recorded photodetector output signal and the live signal is possible while still maintaining tracking? The probability of loss of lock (or mean time to loss of lock) is related to the decorrelation between the output signals and this relation needs further investigation;

2. An evaluation of the possibility of an ‘on the fly’ updating of the recorded photodetector signal to reduce the probability of loss of lock by reducing the decorrelation between the recorded and live photodetector output signals.

3. An investigation into what algorithm to apply when the inevitable loss-of-lock condition occurs. Is there a procedure that avoids removing the load to the shaft and acquiring a new recorded photodetector output signal?

The laser torque meter is a viable instrument; however, further prototypes need to be made. A Digital Signal Processor based laser torque meter has several advantages over the hybrid analogue – digital laser torque meter.

6.1.1 Digital Signal Processor based Laser Torquemeter

The digital signal processor (DSP) based torque meter offers several advantages over a hybrid digital/analogue torque meter. These are due to the fact that the
DSP torquemeter is implemented in software and this allows the torquemeter design and configuration to be easily modified and additional prototypes may be made be simply purchasing more hardware and copying the software. A DSP implementation of the laser torquemeter has a predictable, repeatable behaviour which is not influenced by component variations or environmental changes. The existing laser torquemeter is a complex analogue circuit and it is very sensitive to variations in component values and environmental changes. Because the analogue torquemeter contains a significant number of digital circuits and both analogue to digital converters (ADC) and digital to analogue converters (DAC), it is proposed that a DSP torquemeter be implemented which would convert the raw photodetector output signal into a digital data stream and all the processing to convert this data stream into a torque output would take place in a DSP.

The purchase of a new DSP card was considered and the following issues were addressed:

- A ‘one off’ purchase of a DSP card is expensive but for a manufactured instrument the cost could be substantially reduced with increasing quantities. The cost of a ‘one-off’ system is increased by the development software required to program the DSP and hardware required to interface with a PC. The programming software need only be purchased once.

- The speed of the DSP is of prime concern to a commercially viable instrument as this influences measurement accuracy and operating range of the torquemeter. For higher range of shaft rotation speeds a faster DSP is required. Is it economical to purchase a cheaper, slower DSP card knowing that a faster signal processor is available (in the same family) and the torquemeter may be upgraded without modifying any of the software by
purchasing a faster DSP? An estimate of the processing power is required in order to correctly specify a DSP board.

6.1.2 Calculating the Processing Power Required for a Prototype Torquemeter

An algorithm needs to be developed which implements the functions of the DSP torquemeter and the time critical code identified. A peak tracking correlator is required to follow the live signal from photodetectors and the time-critical section of code for the laser torquemeter is the tracking correlator. For each of the two channels, live data from the photodetector must be acquired and the index to the previously recorded data adjusted so the live and the reference signals are synchronous and this delay-locked-loop requires at least 10 machine cycles per channel. The tracking correlator as shown only provides an output proportional to the shaft angle. In order to determine torque this needs to be compared with the instantaneous shaft angle at another point along the length of the shaft. The frequency content of the photodetector output signal may be between ten times and several thousand times greater than the frequency content of the expected torque fluctuations and an estimate of the torque is not required for every sample made by the tracking correlator. Computational load increases as the bandwidth of the torque increases. The torquemeter output is the low pass filtered difference between the encoder outputs and this filtering may be performed by the host PC.

The tracking correlators expect each sample to have a fixed change in the shaft angle with time (constant \( \frac{d}{dt} \)) and the sampling rate of the correlator is to be adjusted to keep the angle per sample constant. The rotation speed of the shaft varies only slowly compared to the sampling rate and requires a few tens
of machine cycles every 15 data samples. Some of this processing may be passed off on the host computer if the bus connecting the two machines is fast enough. Additional computation overhead is required to monitor the correlation tracker to determine if the system is locked. A routine is required in the record-lock mode to find the main correlation peak and adjust the tracker to lock the system.

Specifications for the proposed torquemeter are difficult to quantify because output noise, accuracy, immunity to decorrelation of the speckle pattern due to shaft motion and maximum shaft speed are all interlinked. Slower shaft speeds and smaller decorrelation widths (due to shaft rotation) are desirable parameters.

6.1.3 DSP Laser Torquemeter Evaluation System

A DSP based laser torquemeter system was proposed with the following characteristics:

- Entire system is digital except for the analogue photodetector circuitry.
- Maximum sampling rate of 100kHz – if 1000 samples are desired per revolution, measurements on a shaft rotating at 6000rpm are possible.
- Minimum applied shaft twist – on the order of 0.1 sample width, e.g. 36mdeg for the above example.
- Output signal is the weighted average of 10 instantaneous differential samples (100 averages in a revolution at 6000rpm) and is output as an analogue signal.
- Correlate / Record lock / Run lock phases implemented as in analogue version as described in Section (4.5.1).
The DSP Laser Torquemeter has the potential of a very low noise floor – on the order of 0.01 sec of arc. A twist of 0.01 arcsec is produced by a torque of 750μNm in a Φ25mm solid steel shaft with laser spots separated by 20cm (ΔL).

DSP based torquemeter is an attractive alternative to the analogue model and a Blue Wave Systems (formerly Loughborough Sound Images) PCI/C42 DSP card was purchased. The DSP card has two 60MHz TMS320C44 floating point processors and 10M words of memory. A dual channel 12 bit analogue to digital converter module that has a maximum sampling rate of 1MHz was also purchased.

The laser torquemeter is a promising non-contact device to measure transmitted torque. The amount of decorrelation between the live and recorded photodetector output signals due to shaft motion limit the performance of the laser torquemeter. The relationship between shaft motion and the decorrelation of the signals is well understood and allows the design and an estimation of the performance of the laser torquemeter for any particular measurement scenario. The development of a prototype DSP based laser torquemeter has started.
Appendix A

A.1 Measurement of Autocorrelation Functions

Typical data and its probability density function is shown in Figure (A.1).

![Figure A.1: Typical data from photodetector output. Left portion: Probability density function. Right – typical speckle signal.](image)

Usually ensemble average is unavailable because only one record is taken. The only available procedure is to calculate the time autocorrelation function for the finite time interval under the assumption that the process is ergodic

\[
\hat{r}_{xx}(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} x(t)x(t+\tau)dt \quad 0 \leq \tau \ll T \tag{A.1}
\]
The system is observed for the time 0 to $T$ seconds. It is possible to define the estimated autocorrelation function as an average taken over $T - \tau$ because only a portion of observed data is available for $x(t)$ and $x(t + \tau)$. It is usually not possible to perform the integral because $x(t)$ is not available analytically.

Approximation of the integral is possible by sampling the continuous time function at discrete intervals and calculating the discrete equivalent of the above. If sampling is at times $0, \Delta t, 2\Delta t, \ldots, N\Delta t$ and sampled values $x_0, x_1, x_2, \ldots, x_N$ then the estimate of the autocorrelation function is

$$\hat{R}_{xx}(n\Delta t) = \frac{1}{N - n + 1} \sum_{k=0}^{N-n} x_k x_{k+n} \quad n = 0, 1, 2, \ldots, M \quad M \ll N \quad (A.2)$$

Figure (A.2) is the autocorrelation function of the data in Figure (A.1). The autocorrelation width is defined, in this thesis, as the delay required to reduce the autocorrelation function to $1/e^2$ of its peak value.

In order to estimate the quality of the estimate of the autocorrelation function,
the mean and variance of $\hat{\Gamma}_{xx}(n\Delta t)$ is required. As the estimate is a random variable, the precise value depends on the function sampled and the set of samples taken. The mean is the true value of the autocorrelation function. The variance of the estimate must be smaller than

$$\text{var}[\hat{\Gamma}_{xx}(n\Delta t)] \leq \frac{2}{T} \int_{-\infty}^{\infty} \Gamma_{xx}^2(\tau) d\tau$$  \hspace{1cm} (A.3)

where $T$ is $N\Delta t$. In discrete form

$$\text{var}[\hat{\Gamma}_{xx}(n\Delta t)] \leq \frac{2}{N} \sum_{k=-M}^{M} \Gamma_{xx}^2(k\Delta t)$$  \hspace{1cm} (A.4)

and assumes that the $2M + 1$ estimated values of the autocorrelation function span the region in which the autocorrelation function has significant amplitude.

Typically the portion of the autocorrelation peak of interest is approximately 20 samples either side of the peak ($M = 20$) and a 8192 samples long ($N = 8192$). The variance in the autocorrelation function is then 5.6% – i.e. the standard deviation of the estimate should be no greater than $\sqrt{\frac{5.6}{100}}$ of the true mean value of the random variable. [132]

### A.2 Statistical Errors in Cross-Correlation Estimates

**Bias Error**

Bias error ($b(\hat{x})$) is the difference between the mean of the estimate of $\hat{x}$ and the true value of $x$.

$$b(\hat{x}) = E(\hat{x}) - x$$  \hspace{1cm} (A.5)
Variance

The variance (var($\hat{x}$)) of the output describes the random portion of the error with respect to the expectation of the sample measurement. The variance takes no account of the true value of $x$.

$$\text{var}(\hat{x}) = E(\hat{x}^2) - [E(\hat{x})]^2$$  \hspace{1cm} (A.6)

Mean Square Error

The mean square error describes the error between the actual sample values $\hat{x}$ and true values $x$.

$$\text{MSE}(\hat{x}) = E[(\hat{x} - x)^2]$$  \hspace{1cm} (A.7)

Root Mean Square Error

The RMS error is the square root of the mean square error.

Normalisation

Normalisation of the statistical results is achieved by dividing by the actual value of the quantity being measured. If the actual value of the quantity being measured is not known, then normalisation is based on the expectation $E(x)$ rather than the actual value $x$.

- Normalised bias error $\varepsilon_b = b(\hat{x})/x$
- Normalised variance $\varepsilon_{\text{var}} = \text{var}(\hat{x})/x$
- Normalised RMS error $\varepsilon_{RMS} = (E[(\hat{x} - x)^2])^{\frac{1}{2}}/x$

172
Variance of Correlation Estimates

The variance of ac coupled bandwidth limited white noise, with bandwidth $B$, integration time $T$. The variance of the cross-correlation between signals $x$ and $y$ is given by

\[ \text{var}(\hat{r}_{xy}(\tau)) = \frac{1}{2BT} \left( \Gamma_{xx}(0)\Gamma_{yy}(0) + \Gamma_{xy}^2(\tau) \right). \]  

(A.8)

The normalised mean square error is given by (for $\Gamma_{xy} \neq 0$) [133]

\[ \varepsilon^2 = \text{var}(\hat{r}_{xy}(\tau)/\Gamma_{xy}^2(\tau)) \]
\[ = \frac{1}{2BT} \left( 1 + \frac{\Gamma_{xx}(0)\Gamma_{yy}(0)}{\Gamma_{xy}^2(\tau)} \right) \]
\[ = \frac{1}{2BT} \left( 1 + \frac{1}{\gamma_{xy}^2(\tau)} \right). \]  

(A.9)

where the normalised correlation coefficient $\gamma_{xy}(\tau)$ is given by

\[ \gamma_{xy}(\tau) = \frac{\Gamma_{xy}}{\sqrt{\Gamma_{xx}(0)\Gamma_{yy}(0)}}. \]  

(A.10)

The mean square error reduces proportionally with the time-bandwidth product and increases with reduction of the normalised correlation coefficient.

A.3 Gaussian Cross-correlations

The work in this thesis is based around the correlation width at $1/e^2$ of the peak correlation. Table (A.1) relates the normalised correlation width $x_c$ at $1/e^2$ to widths at other factors of the peak correlation and Figure (A.3) provides an illustration of the table.

173
A.4 Correlation and Autocorrelation Using the FFT

The correlation function is similar to that of convolution and this similarity may be exploited to rapidly compute cross-correlation functions. The correlation between the continuous functions \( g(t) \) and \( h(t) \), denoted \( \Gamma_{gh}(\tau) \) is to be computed. The cross-correlation function may be obtained by the following equation

\[
\Gamma_{gh}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} g(t) h(t + \tau) dt,
\]

(A.11)

but this is not the most efficient way of computing the cross-correlation function.

The Fourier transform or spectral density of the function \( g(t) \) is given by

\[
G(f) = \int_{-\infty}^{\infty} g(t) \exp(-2\pi j f t) dt
\]

(A.12)

and the complex cross-spectral density function \( \Upsilon_{gh}(f) \) is given by

\[
\Upsilon_{gh}(f) = G(f)H^*(f).
\]

(A.13)

To obtain the cross-correlation function \( \Gamma_{gh}(\tau) \) the inverse Fourier transform is applied to the cross spectral density function [134]:

\[
\Gamma_{gh}(t) = \int_{-\infty}^{\infty} \Upsilon_{gh}(f) \exp(2\pi j f t) df.
\]

(A.14)

The continuous functions \( g(t) \) and \( h(t) \) may be represented by the discrete sampled

<table>
<thead>
<tr>
<th>Correlation Width Criteria</th>
<th>Correlation Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/e^2 )</td>
<td>1</td>
</tr>
<tr>
<td>( 1/e )</td>
<td>( \sqrt{2} )</td>
</tr>
<tr>
<td>( 0.5 )</td>
<td>( \sqrt{2}/2 \sqrt{\ln 2} )</td>
</tr>
</tbody>
</table>

Table A.1: Correlation widths normalised to correlation width at \( 1/e^2 \)
Figure A.3: Correlation widths normalised to correlation width at $1/e^2$.

functions $g_k, h_k$, and the cross-correlation computed using the fast Fourier transform (FFT).
References


181


[118] F. Mesch, R. Fritsche, and H. Kipphan. Transit time correlation - a survey on its applications to measuring transport phenomena. Transactions of the ASME


