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Discrete Choice Modelling for Traffic Densities with Lane-Change Behaviour

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Abstract

This paper investigates the modelling for traffic densities with lane-change behaviour using the information provided by loop detectors. The existing studies on traffic density estimation for multi-lane roadways mainly focus on the scenario where either vehicles’ lane-change manoeuvres are not common or the lane-change pattern is time-invariant. This research, however, takes into consideration the time-varying nature of drivers’ lane-change manoeuvres, and models the lane-change probabilities using a number of discrete choice models. These lane-change models are then embedded into a state space model to capture the dynamics of traffic flow. The extended Kalman filter is used to update the estimated traffic densities of multi-lane motorways. A numerical study is carried out to investigate the performance of the developed approach.

Keywords: discrete choice analysis; Kalman filter; lane-change behaviour; Markov chain; traffic flow

1. Introduction

Advanced traffic management systems play an important role in managing congested traffic. They fuse traffic-surveillance-related information from a variety of sensors deployed across roadway networks, and operate on the basis of some key traffic parameters that are crucial for the understanding of the time-varying nature of traffic flow. Li (2009, 2010, 2012) has recently developed fast algorithms to update the estimates of some fundamental parameters such as vehicle speed, vehicle length, and time headway. In this paper we will consider the estimation of another important parameter required by the advanced traffic management systems, i.e. traffic density.

Traffic density differs from most of the other traffic parameters as it is a range concept and thus it cannot be measured directly using a point sensor (Singh & Li, 2012). Traffic density is defined to be the number of vehicles in a lane within a unit length of roadway segment. In theory, traffic density can be worked out using a snapshot photo of traffic on the roadway by an aerial camera. In practice, however, it is very difficult to continuously monitor the traffic in real time due to the lack of sufficiently large number of aerial cameras in traffic network systems. Although dense point sensor systems (e.g. inductive loop detectors) could approximate continuous measurements in space, the cost would be prohibitive in general.

The situation becomes more difficult for multi-lane motorways due to drivers’ lane-change manoeuvres. Lane-change behaviour can have great impact on traffic flow for some roadway sections and

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it has attracted considerable attention in the traffic literature in general. However, because of its technical
difficulties, the impact of lane change on traffic density estimation was not well investigated in the
transport and traffic literature. Gazis & Knapp (1971) estimated densities where lane change was assumed
to be insignificant. They applied the Kalman filter technique to estimate vehicle counts in a roadway
segment within which lane changes are in small numbers and hence are treated as part of system noise
with known statistical properties. Chang and Gazis (1975) improved this approach by explicitly
considering a few lane-change models when investigating the traffic density estimation problem.
However, their method assumes the availability of aerial data that has restricted its applications due to
potential high cost. More recently Coifman (2003) has developed an interesting approach to estimating
traffic densities by explicitly modelling inflow. This method, however, requires information from the
sparse vehicle reidentification system. Gazis & Liu (2003) presented a general research framework for
traffic density estimation. In their study, the traffic density over the time is modelled using a state space
equation, and the measurements on traffic flow are provided by the phenomenological relationship
between speed and density (see, e.g. Drake et al., 1967). However, the traffic density was estimated in
Gazis & Liu (2003) under the assumption that lane-change movements are not common in roadways and
can thus be ignored. Recently, Singh & Li (2012) used a Markov chain process to describe lane-change
behaviour. They assumed that each vehicle in a roadway segment has a certain probability to stay in the
current state (lane) or to change from one state (lane) to another. However, they considered drivers’ lane-
change behaviour by assuming that the traffic flow under investigation is stable so that the transition
probabilities of the Markov chain for the lane-change behaviour remain approximately constant over the
time. Although they suggested that the time period of interest (say a day) be split into a number of shorter
sub-periods to ensure the traffic flow is approximately stable over each sub-period, this assumption is
questionable in general due to the non-stationary nature of traffic flow.

The purpose of this paper is to extend the model developed in Singh & Li (2012) by taking into
account the time-varying nature of lane-change behaviour. Specifically unlike Singh & Li (2012), we will
relax the assumption of stable traffic condition and time-invariant nature of lane-change behaviour.
Instead, drivers’ lane-change decisions will be directly modelled using a number of traffic variables such
as vehicle speed, time headway and traffic density. Consequently, the lane-change probabilities at each
time point are directly linked to the current traffic condition, and hence are able to better characterize
vehicles’ movements between lanes.

This paper is organized as follows. In the next section we will develop a model for traffic flow where
the time-varying nature of lane-change behaviour is captured. The Kalman filter technique is then applied
to update the estimated traffic densities. A numerical study will be conducted to assess the performance of
the developed model. Finally discussion and conclusions are offered.

2. Methodology

2.1. Traffic density modelling

We first consider traffic density modelling. To investigate the time-varying nature of drivers’ lane-
change behaviour, we fuse three commonly used modelling techniques in traffic and transport research,
i.e. discrete choice analysis, Markov chain processes, and state space models, to capture the dynamics of
traffic flow in a segment of multi-lane roadway.
Let us consider a roadway segment with \( M \) lanes, where the segment is defined to be a detection zone with two single inductive loop detectors embedded, one upstream and the other downstream, plus a speed sensor. Following Gazis & Liu (2003) and Singh & Li (2012), we assume that the detectors are deployed about 500 meters to 1,500 meters apart. It is anticipated that the traffic condition may change substantially over a longer segment and thus may lead to unreliable results. The speed measurements can be provided by a separate speed detector (see, e.g. Li, 2010) or can be worked out using the two single loop detectors (see, e.g. Li, 2009).

Let \( \mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T \) denote the state vector where \( x_j(t) \) is the number of vehicles in lane \( j \) during time interval \( t \) (\( t = 0,1,2, \ldots \)). The state equation in the existing studies is formed on the basis of the traffic conservation equation where the number of vehicles in each lane is equal to the number of vehicles in the previous time interval, plus the net gain of vehicles that enter and leave the upstream and downstream loop detectors respectively, which is further adjusted for the vehicles’ lane-change movements within this roadway segment:

\[
x_j(t + 1) = x_j(t) + w_j(t) + G_j(t) + \xi_j(t),
\]

where \( w_j(t) \) denotes the net gain of vehicles entering and leaving the upstream and downstream detectors in lane \( j \). \( G_j(t) \) is the net gain of the vehicles’ lane-change movements in lane \( j \). \( \xi_j(t) \) denotes the corresponding error term.

Gazis & Liu (2003) assume that lane-change manoeuvres are not common so \( G_j(t) \) can be ignored. In this paper, to capture vehicles’ lane-change movements within the segment, we follow Singh and Li (2012) and use a Markov chain to characterize the lane-change manoeuvres. Specifically, we assume that each vehicle in the roadway segment has a certain probability to stay in the current state (lane) or to change from one state (lane) to another. Let \( p_{jk} \) denote the transition probability that a vehicle moves from state (lane) \( j \) to state (lane) \( k \). We then define the system matrix \( \mathbf{A} = [a_{jk}] \) of the state equation via the transition probabilities:

\[
a_{jk} = \begin{cases} 
p_{jk} & \text{if } j \neq k \\
1 - \sum_{l \neq j} p_{jl} & \text{if } j = k
\end{cases}.
\]

The net gain \( G_j(t) \) of the vehicles’ lane-change movements can thus be worked out using the transition probabilities \( p_{jk} \) and \( p_{kj} \) for all \( k \) (see Singh and Li (2012) for details). Consequently the state equation can be written as follows:

\[
\mathbf{x}(t + 1) = \mathbf{A} \mathbf{x}(t) + \mathbf{w}(t) + \mathbf{\xi}(t),
\]

where \( \mathbf{w}(t) = [w_1(t), w_2(t), \ldots, w_M(t)]^T \) and \( \mathbf{\xi}(t) = [\xi_1(t), \xi_2(t), \ldots, \xi_M(t)]^T \). Let \( \mathbf{Q} \) denote the covariance matrix of \( \mathbf{\xi}(t) \). Following Gazis & Liu (2003) and Singh and Li (2012), we have \( \mathbf{Q} = 2\sigma^2 \mathbf{I} \) where \( \mathbf{I} \) is an identity matrix and \( \sigma \) is the standard deviation of count error of each loop detector.

Singh and Li (2012) assume that the entries of matrix \( \mathbf{A} \) are all time-invariant. Due to the dynamic nature of traffic flow, however, this assumption is in general not realistic. The lane-change probability \( p_{ij} \) between two lanes normally depends on the current traffic condition that varies from time to time. To
capture drivers’ lane-change manoeuvres, we now link the lane-change probabilities to the current traffic condition.

In transport research, a decision-maker’s choice among all possible available alternatives is usually modelled using discrete choice theory (Train, 2003). Logit is one of the most widely used discrete choice models. It was originally derived by Luce (1959). Marschak (1960) has showed that the logit model is consistent with utility maximisation.

Now consider a driver travelling in lane $i$. Let $z_j$ be a vector of attributes relating to an alternative (lane) $j$. The closed-form logit choice probability is (Train, 2003):

$$p_{ij} = \frac{\exp(V_{ij})}{\sum_{k=1}^{M} \exp(V_{ik})}, \quad (j=1,\ldots,M),$$

where $p_{ij}$ is the probability that a driver travelling in lane $i$ chooses lane $j$. $V_{ij} = \beta_i^T z_j$ is the corresponding utility. $\beta_i$ is a vector of unknown parameters. Li (2011) has extended Eq. 2 to a semi-parametric logit model. For simplicity, we restrict our interest to the logit model Eq. 2 in this paper.

Now we turn to consider the attribute vector $z$ for lane change. Chang and Kao (1991) carried out an empirical investigation of lane-changing characteristics on multi-lane motorways. They have found that drivers’ lane-change behaviour is affected by some measurable traffic variables such as traffic density, time headway, vehicle speed, etc. In this paper, we treat each driver as an individual decision-maker and follow the approach of Chang and Kao (1991) to model the lane-change probabilities.

Let us first consider the simplest scenario where the roadway has two lanes only. Following Chang and Kao (1991), drivers’ lane-change decision-making depends on several traffic variables, including time headway, traffic density and vehicle speed. Hence the probability $p_{ii}$ that a vehicle stays in lane $i$ is

$$\log\left\{\frac{p_{ii}}{1-p_{ii}}\right\} = V_{ii} - V_{ij} \quad (i,j=1,2),$$

where $V_{ij} = \beta_i^T z_j$ with $z_j = [v_j(t), x_j(t), h_j(t)]^T$. $v_j(t)$ and $h_j(t)$ are the average vehicle speed and average time headway in lane $j$ during time interval $t$ respectively. Note that the average time headway can be worked out using single loop detectors (see, e.g. Li, 2012). Clearly the probability that a vehicle in lane $i$ changes to the next lane $j$ is $p_{ij} = 1 - p_{ii}$. It is assumed here that no relevant information on drivers themselves is available online so the variables associated with such information are unable to enter into the model. In practice, the coefficient vector $\beta_i$ needs to be estimated in the stage of system identification.

For more general scenario where the roadway has three or more lanes, drivers have more alternatives to choose as described in Eq. 2. In this research, however, we restrict our interest to the scenario where each time interval is short (e.g. 20s) so that the lane change of each vehicle can only take place once and no subsequent lane changes are possible within the same time interval. Consequently, each driver travelling in a middle lane has three alternatives, i.e. staying in the same lane or changing to the left-side lane or to the right-side lane. The logit choice probability is given by:

$$p_{ij} = \frac{\exp(V_{ij})}{\sum_{k=i-1}^{i+1} \exp(V_{ik})}, \quad (i=2,\ldots,M-1; j=i-1, i, j+1).$$

On the other hand, each driver travelling in the outermost or innermost lane has only two alternatives, i.e. staying in the same lane or changing to the next lane:
\[ p_{1j} = \frac{\exp(V_{1j})}{\sum_{k=1}^{2} \exp(V_{1k})} \quad (j=1,2), \quad (3b) \]

\[ p_{Mj} = \frac{\exp(V_{Mj})}{\sum_{k=M-1}^{M} \exp(V_{Mk})} \quad (j=M-1, M). \quad (3c) \]

The series of choice models (3a)-(3c) describe the lane-change behaviour. The state equation that takes into consideration the time-varying nature of lane-change behaviour and the dynamics of traffic densities includes both Eq. 1 and Eq. 3:

\[ \mathbf{x}(t+1) = \mathbf{A}(\mathbf{x}(t), t)\mathbf{x}(t) + \mathbf{w}(t) + \mathbf{\xi}(t), \quad (4) \]

where the system matrix \( \mathbf{A}(\mathbf{x}(t), t) \) directly depends on the current state vector \( \mathbf{x}(t) \).

Next we turn to consider the observation equation. In traffic engineering, the relationship between traffic speed \( v \) and density \( K \) is given by (see Drake et al. 1967) \( v = v^* \exp[-0.5(K/K^*)^2] \), where \( v^* \) is the free flow speed and \( K^* \) is the density corresponding to the maximum flow in a lane of a roadway segment. This relationship forms the basis of the observation equation in the existing studies such as Gazis & Liu (2003) and Singh & Li (2012). Specifically, by taking into account noise, the speed measurement in lane \( j \) is assumed to follow the equation below:

\[ v_j(t) = v_j^* \exp\left[-0.5 \left( \frac{x_j(t)}{K_j^*L} \right)^2 \right] + \eta_j(t), \quad (5) \]

where \( L \) is the length of the segment so the density in each lane \( j \) is given by \( x_j(t)/L \). \( \eta_j(t) \) is the corresponding measurement error. All measurement errors are assumed to be independent of each other with a common variance \( \tau^2 \). The parameters \( v_j^* \) and \( K_j^* \) can be treated as tuning parameters if they are unknown (see Gazis & Liu 2003). Now let \( \mathbf{h}(\mathbf{x}(t)) \) denote the vector of the nonlinear functions given by the first term of the right-hand-side of Eq. 5. Let \( \mathbf{\eta}(t) = [\eta_1(t), \eta_2(t), ..., \eta_M(t)]^T \) and \( \mathbf{R} = \tau^2 \mathbf{I} \) denote the covariance matrix of \( \mathbf{\eta}(t) \). The observation equation may be rewritten in a matrix form

\[ \mathbf{v}(t) = \mathbf{h}(\mathbf{x}(t)) + \mathbf{\eta}(t). \quad (6) \]

### 2.2. Density estimation

The estimate of the traffic density vector can be updated on the basis of the nonlinear state space model (4) and (6) via the extended Kalman filter which allows the system to be nonlinear. During each time interval, the nonlinear system (4) and (6) are linearized about the current estimate of the state vector. Specifically, let \( \hat{\mathbf{x}}(t|t) \) denote the estimate of the state vector in each time interval \( t \). Then for equation (6), the Jacobian evaluated at the current estimate \( \hat{\mathbf{x}}(t|t) \) is \( \mathbf{H}(t) = \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}|_{\mathbf{x} = \hat{\mathbf{x}}(t|t)} \). By some algebra it can be shown that matrix \( \mathbf{H}(t) \) is diagonal with each diagonal entry equal to \( -v_j(t)x_j(t)/(K_j^*L)^2 \). Likewise, the system equation also needs to be linearized with the Jacobian \( \mathbf{G}(t) = \frac{\partial \mathbf{A}(\mathbf{x}, t)}{\partial \mathbf{x}}|_{\mathbf{x} = \hat{\mathbf{x}}(t|t)} \). The analytical form of the Jacobian \( \mathbf{G}(t) \) is quite complicated. In practice, it can be evaluated via a suitable numerical approach. On the basis of \( \mathbf{G}(t) \) and \( \mathbf{H}(t) \), the extended Kalman filter (see e.g. Simon, 2006) can be applied to update the estimate of the traffic density vector of a multi-lane roadway.
3. Simulation Study

The developed state space model for traffic density estimation will be illustrated via a simulation study. One advantage of simulation study is that the ‘true’ values of the parameters of interest are known so it is straightforward to assess the performance of an approach in terms of accuracy.

3.1. Simulation description

The experiment settings were similar to that outlined in Singh & Li (2012). See Singh & Li (2012) for the details on how the relevant traffic variables were simulated. However, unlike Singh & Li (2012), the ‘true’ lane-change probabilities here were assumed to be time-varying and depend on the current traffic condition via the lane-choice probabilities. In the subsequent simulation, they were simulated using the choice probabilities in Chang and Kao (1991). These choice probabilities were then fed into the system matrix in state equation Eq. 1, which in turn generated the ‘true’ traffic densities in the simulation.

Fig. 1. Simulated lane-change probabilities in one run of the simulation study
Figure 1 displays the choice probabilities $p_{11}(t)$ and $p_{22}(t)$ over 360 time intervals, each having a duration of 20s, in one simulation run. The choice probabilities characterize drivers’ choice behaviour in the two-lane system over a two-hour period. In particular, the upper and lower graphs in Figure 1 show the probabilities that vehicles driving in lane 1 and lane 2 will stay in the same lane in time interval $t$. It can be seen that the drivers exhibited a range of choice probabilities which varies from 0.2 to 1 at different time points. This was due to the dynamic nature of traffic flow that impacts on the drivers’ choice over the time.

3.2. Simulation results

A number of settings for the parameters were explored in the analysis. Here we first focused on the situation where there were eight vehicles per 20s time interval entering the given segment of motorway. The count error ($\sigma$) from the loop detectors was set as one vehicle per time interval, and the speed error ($\tau$) of the speed detectors was set as 1 km/hr. The upper left and upper right panels of Figure 2 display the simulated traffic densities (broken lines) and the estimated values using the developed model (solid lines) in one run of the simulation experiment. Overall, it can be seen that the estimated traffic densities are close to the ‘true’ values.

![Figure 2](image-url)

Fig. 2. Simulated traffic densities (broken line) and the estimated traffic densities (solid line) using the developed model for lane 1 (upper left) and lane 2 (upper right), and using Gazis & Liu’s method for lane 1 (lower left) and lane 2 (lower right) respectively.
The same data were also analysed using the method of Gazis & Liu (2003). The results are displayed in the lower left and right panels of Figure 1. It can be seen that when lane changes are substantial within a roadway segment, this method performed poorly as it does not take into account lane-change behavior.

We also investigated the performance of the developed approach in various scenarios by setting the parameters equal to different values in the state space model: \( \tau = 1, 2 \) and \( \sigma = 1, 2 \) respectively when traffic is relatively light (\( \lambda = 3 \)) and congested (\( \lambda = 8 \)). In the simulation study each experiment was repeated 100 times and the evaluation of method was based on the RMSE between the ‘true’ and estimated vehicle counts. The averaged RMSE over the 100 runs was calculated as displayed in Table 1.

It can be seen that the developed model performed well: the estimated vehicle counts were fairly close to the ‘true’ values. When the parameter \( \sigma \) and \( \tau \) were small, the average estimation error was just above one vehicle per 20s for light traffic. When the two parameters became larger, the average estimation error increased (in particular for congested traffic) but overall they were still at an acceptable level.

Table 1. Average RMSEs of vehicle counts over 100 simulation runs using the developed model

<table>
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<tr>
<th></th>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>Lane 1</th>
<th>Lane 2</th>
<th>( \tau = 1 )</th>
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<td>1.70</td>
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<td>( \lambda = 8 )</td>
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<td>Lane 2</td>
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<td>3.44</td>
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</tr>
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</table>

4. Discussion and Conclusions

In this paper, we have investigated traffic flow modelling and the application of the Kalman filter to traffic density estimation. We have used a Markov chain approach developed by Singh & Li (2012) to describe lane-change behaviour so that the state equation can better reflect the movements of vehicles between lanes. To take into account the time-varying nature of drivers’ lane-change behaviour, we have modelled the lane-change probabilities using discrete choice theory that makes use of the current traffic information including headway, density and speed.

The developed approach has important practical implications. In practice, as inductive loop detectors are widely deployed (as compared to the aerial cameras) on motorways, the developed methods can be applied to provide an effective approach to traffic surveillance. The estimated traffic density can facilitate traffic management of networks, and also provide inputs for both transport planning and traffic control.

The research in this paper has several limitations which need to be addressed in the future research. First we note that logit model Eq. 2 for lane-change probabilities may not be able to capture the potential nonlinearity/heterogeneity in lane-change manoeuvres. In the future research it will be extended using the
The developed approach will also be evaluated using real traffic data to assess robustness when it is applied to different roadway layouts and real traffic conditions. In addition, in this research we restrict our interest to the scenario where each time interval is short so that the lane change of each vehicle can only take place once. In the future research, we shall explore an approach to relax this restriction.

References


