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THE EFFECT OF A CAUSTIC PHASE SHIFT ON REFLECTION OF A RAYLEIGH WAVE FROM THE RIB OF A WEDGE

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The reflection of a surface acoustical Rayleigh wave from the rib of an acutely angled elastic wedge with slanted incidence is theoretically investigated. An approximate approach is used for solving the problem which is based on geometrical optics (acoustics). One of the most interesting results of the performed analysis is establishment of the fact of existence of a substantial effect of the phase shift of the symmetrical mode of the field, which forms with its reflection from the caustic (the caustic phase shift), on the angular relation of the modulus of the reflection coefficient.

According to the known tenets of geometrical optics (acoustics), when a beam touches the caustic of a wave field the wave which corresponds to this particular beam acquires a shift in phase $\Delta \phi_k$, called the caustic phase shift \[1,2]. In particular with reflection off a simple caustic, $\Delta \phi = -\pi/2$. However, this shift has no practical significance, since, effecting only the phase of the reflected field, it is in no way manifested in experiments. This work analyzes the case when the caustic phase shift becomes substantial, determining not only the phase, but also the absolute magnitude of a field at the observation point.

Such a situation comes up in the problem about reflection of a surface acoustical Rayleigh wave which is incident at a slant to the rib of an acutely-angled elastic wedge which is of great independent interest for ultrasonic deflex-toscopy, acoustoelectronics, and seismology [3-5]. There is no precise solution for this problem even in the case of perpendicular incidence of a surface wave to the rib of the wedge [4,5]. As far as slanted incidence is concerned, it essentially has not been discussed in the literature. An approximate approach to solution of the examined problem [6,7] is used below. It is valid for acutely angled wedges (with wedge aperture angles less than 50-60°) and is based on representation of the Rayleigh wave incident to the rib of the wedge in the form of the sum of the two lower symmetrical (longitudinal) and antisymmetrical (bending) Lamb modes of identical amplitude for a plate with an evenly changing thickness of $h$. According to this approach, the propagation of the two modes to the rib and from it is examined separately using known solutions of problems about reflection of each of them from the free edge of an infinitely thin plate. Then, the fact that in the vicinity of the rib of the wedge the symmetrical and antisymmetrical Lamb modes are propagated at different velocities is taken into consideration. As a result, with propagation from the point of excitation to the point of reception, there is a difference in the carry over of phases, which is

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Fig. 1. Geometry of the problem: antisymmetrical (1) and symmetrical (2) modes.

Fig. 2. Dependence of the modulus of the coefficient of reflection \[ r_0 \] on the incidence angle \( \alpha \) of a Rayleigh wave onto the rib of an aluminum wedge with an aperture angle of \( \theta = 10^\circ \): 1) the theory and 2) the experiment.

A function of the aperture angle \( \theta \) and which, within the approximations used, determines both the phases and the amplitudes of the coefficients of reflection and transmission of a Rayleigh wave in the wedge.

In the case of slanted incidence of a Rayleigh wave of interest here, an approximation of geometrical optics (acoustics) of layer-heterogeneous media \(^{1,2}\), which are a generalization of the Wentzel-Kramers-Brillouin (WKB) approximation used in works \(^{5,7}\) for the case of slanted incidence are naturally used for calculating the above referenced phase carry overs. In this case, it must be considered that with approach to the rib, both the symmetrical and the antisymmetrical modes endure refraction, the nature of which for each mode is different due to the difference in the laws of change in their velocities near the rib \(^{15}\). The velocity of the antisymmetrical (bending) mode is reduced from the velocity of a Rayleigh wave \( c_R \) to zero \( (\sin \alpha = 1) \) and, therefore, this particular mode approaches the rib in an essentially perpendicular manner (Fig. 1). The velocity, however, of the symmetrical (longitudinal) mode with a reduction in \( \alpha \) is increased from \( c_R \) to the so-called "Plate" velocity \( c_m = \sqrt{c_0 (1 - \epsilon^2 a^2)} \), where \( c_0, c' \), \( c'' \) are the velocities of the longitudinal and shift volumetric waves. As a result, the symmetrical mode is incident at a more oblique angle to the rib (see Fig. 1) and beginning with a certain critical incidence angle \( \alpha = \alpha_0 \), a turn point appears on it. Thus, the rays which are incident at angles of \( \alpha > \alpha_0 \) form a simple caustic \(^{2}\), with reflection from which the symmetrical mode acquires a caustic phase shift \( \Delta \psi = -\pi/2 \).

The author analyzes the effect of the referenced phase shift on the reflection of a Rayleigh wave. To do this, the known relation of geometrical acoustics for the phase of a wave propagated in a medium nonuniform in one direction is used \(^{1,2}\) (in this particular case in the direction perpendicular to the rib of the wedge), and the expression for the difference in the carry over of phases \( \Delta \psi = \psi_{ss} - \psi_{as} \) of the antisymmetrical and symmetrical modes is written as:

\[
\Delta \psi = -2kR \left\{ \int_0^{\pi} \left[ \frac{k_R^2(x)}{k_R} - \sin^2 \alpha \right] \frac{dx}{s(x)} - \int_{s(\alpha)}^{\pi} \left[ \frac{k_R^2(x)}{k_R} - \sin^2 \alpha \right] \frac{dx}{s(x)} \right\}.
\]

(1)

Here \( k_R, s(x) \) are the wave numbers of the symmetrical and antisymmetrical modes.
which are a function of the coordinate $x$ read off from the tip of the wedge, $k_p$ is the wave number of the Rayleigh wave in the semispace $(a)$ and $x_a(a)$ is the coordinate of the turn point of the symmetrical mode. With consideration of the fact that $k_2(x, 0) = 2x(\theta/2)$, expression (1) is conveniently rewritten in the form:

$$\Delta \phi = -25(\theta)/\sin(\theta/2),$$

$$\delta(x) = \frac{k_p}{2} \left[ \int_{0}^{x} \frac{k_2^2(h)}{k_p^2} \sin^2 \alpha \right]^{1/2} dx - \int_{x_a(a)}^{0} \left[ \frac{k_2^2(h)}{k_p^2} \sin^2 \alpha \right]^{1/2} dh.$$

According to formulas (2), the value of the local thickness of the wedge $k_p(x)$ which characterizes the turn point, is determined from the equation:

$$k_p^2(h)/k_p^2 - \sin^2 \alpha = 0.$$  \hspace{1cm} (3)

But the value of the critical angle of incidence $a_0$, beginning from which this point is manifested, corresponds to replacement of $k_p(h)$ with $k_p(0) = k_p = \omega/\sqrt{c_p}$. Thus, $a_0 = \arcsin(\epsilon_s/c_p)$. In particular, $a_0 = 32^\circ$ for aluminum. Acting further in an analogy with works [6,7], an expression is acquired for the value of practical interest — the modulus of the coefficient of reflection of a Rayleigh wave from the tip of a wedge with slanted incidence:

$$|R| = \left| \sin \left[ \frac{\delta(x)}{15(\theta/2)} - \frac{\pi - \theta}{4} + \frac{\Delta \phi}{2} \right] \right|.$$  \hspace{1cm} (4)

Here $\Delta \phi$ is the caustic-phase shift equal to zero at $a < a_0$ and $-\pi/2$ at $a > a_0$. At $\alpha = 0$ expression (4) coincides with the formula for the coefficient of reflection with perpendicular incidence [6,7]

$$|R| = \left| \sin \left[ \frac{\delta}{15(\theta/2)} - \frac{x - \theta}{4} \right] \right|$$

(where $\delta = \delta(0) = \frac{1}{2} \int_{0}^{x} \left[ k_2(h) - k_1(h) \right] dh$ ), which describes the experimentally [3] observed multiple oscillations of $|R|$ with a change in $\alpha$. Since the relations $k_p(x)$ have no analytical expressions, it is convenient in the calculations to use approximations of the corresponding dispersion curves calculated by numerical methods (see, for example, [6]). In particular, works [6,7] used approximations which allow analytical calculation of the value of $\delta(0)$ and of the coefficient of reflection $|R(\theta)|$, accordingly.

At $\alpha \neq 0$ the analytical solution cannot be found even with the use of simpler approximating relations and numerical calculation of $\delta(\alpha)$ and $|R(0, \alpha)|$ must be resorted to. Such calculation was performed on a computer for an aluminum wedge with approximations of $k_p(x)$ taken from works [6,7]. In order to compare the acquired solution with the results of the experiment performed by S. V. Korolev and the author, the dependence of $\delta(x)$ acquired in the calculation process was multiplied by the correcting factor $\gamma$, determined from a condition of precise coincidence between the theoretical value $|R|$ at $\alpha = 0$ and its experimental values for this case [3]. In particular, a factor $\gamma = 0.925$ was used for a wedge with an aperture angle of $\alpha = 30^\circ$, which corresponded to replacement of the value of $\delta(\alpha) = 2.75$, calculated in works [6,7], with $\delta(\alpha) = 2.52$. Such correction was caused by the necessity for establishing the initial correspondence between the theory and the experiment, without which their comparison of arbitrary incidence angles $\alpha$ would make no sense.
Figure 2 presents the relation $|R(a)|$, calculated using formulas [2, 3], for an aluminum wedge with an angle of $a$ equal to $30^\circ$. This same figure presents the corresponding experimental relation. As the calculations show, the presence of a caustic phase shift in the symmetrical mode (at $a > 32^\circ$) has a substantial effect on the course of the theoretical curve of $|R(a)|$. In particular, at $a = 32^\circ$ the value of $|R(a)|$ changes abruptly. A similar anomaly is also clearly observed in the behavior of the experimental points. However, unlike the theory, the experiment produces a more even drop in the coefficients of reflection at $30^\circ < a < 35^\circ$. This may be partially explained by the fact that the plane surface waves are figured in the theory, while in the experiment all of the examined processes were simulated for actual wave beams which have a finite angular spectrum.

Thus, the work shows that in the problem about reflection of a Rayleigh wave from the slit of an acutely angled wedge, there is a natural caustic phase shift of the symmetrical mode of the field which has a substantive effect on the behavior of the angular relations of the modulus of the reflection coefficient. It should be noted that with respect to the role of the caustic phase shift, the above examined case of reflection of a Rayleigh wave is apparently unique. This uniqueness is caused by the presence in the problem of a reference wave (the antisymmetrical mode), which is not reflected from the caustic.

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