Finite element modelling of tennis racket impacts to predict spin generation

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Finite element modelling of tennis racket impacts to predict spin generation

By

David Weir MEng

A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

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Abstract

Over the last 20-30 years the subject of spin in tennis has become increasingly important. A great deal of work has been done to establish the effects which, increased levels of spin have, on shots. The most prominent effect of increased spin in a tennis shot is the resulting deviation in flight which allows players to, amongst other things, strike the ball harder with top-spin in the knowledge that it will still fall inside the court due to the extra aerodynamic downward force. With such significant advantages available racket manufacturers are naturally keen to maximise spin generation. That being said, very little research has been performed into the subject of spin generation in tennis and the affecting factors.

This thesis details the development of a finite element model which is to be used to allow a greater understanding of spin generation and how varying properties such as string density (the number of strings in a string-bed), gauge and orientation affect its magnitude. The primary aim, or goal, of this research is to create an FE model which can be used to model oblique impacts and measure the resulting spin. Whilst considerable focus was placed on developing novel, modelling techniques to create the FE model, a great deal of emphasis was also placed on its validation. The validity of the model was examined under static loading conditions, such as that experienced during stringing. The dynamic performance was also validated using a combination of modal analysis and high speed video of dynamic impacts. Each of the validation methods provided assurance of the models performance, with all error margins less than 5%.

The two areas of the FE model which required the most attention were the interaction properties (specifically coefficient of friction (COF)) and material properties. Previous studies have sought to obtain a single value for the COF of a tennis racket/ball system but this study examines how the COF varies as the strings interact first, with themselves and secondly with the ball.

Each of the validation methods (dynamic and static) were deemed successful as they provided concise data which could be readily compared with the results produced by the FE model. Having validated the model’s performance, with respect to predicting outbound spin, a number of oblique impact angles were modelled to allow a greater understanding of how the mechanisms of spin generation change with the inbound trajectory of the ball. This analysis showed that for the impact conditions studied the contact time of the impact was
reduced from 6.2 milliseconds to 5.7 milliseconds when the angle was increased from 32 degrees to 40 degrees. Furthermore, a number of novel string-beds were modelled, with varying string orientations (between 30 degrees and 60 degrees relative to the rackets frame) and subjected to a similar analysis procedure, with their results providing the concluding section of the thesis.
Acknowledgements

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I also wish to thank Dunlop Slazenger, in particular Martin Aldridge and Dr. David Barrass, for all the support they have offered, both in terms of materials for testing and their insightful knowledge of the tennis industry.

Lastly I would like to thank my wife, my parents and my brother Charles for providing me with the support and security to fully focus on my PhD over the last three years as well as my Undergraduate degree.
“The one thing that cycling has taught me is that if you can achieve anything without a struggle it’s not going to be satisfying”

Greg Lemond
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Nomenclature

α = inbound angle (degrees)
α_i = angle between points C and P (degrees)
α_T = coefficient of thermal expansion
β = racket tilt angle (degrees)
ξ = critical coefficient of damping
μ = coefficient of friction
μ_s = coefficient of sliding friction
μ_r = coefficient of rolling friction
ρ = density (kg/m^3)
σ = stress (N/m^2)
Φ = angle 1 between cross and main strings (degrees)
φ = angle 2 between cross and main strings (degrees)
ν = poisson’s ratio
ω = frequency (Hz)
ω_n = natural frequency (Hz)
ω_1 = inbound spin (rev/min)
ω_2 = outbound spin (rev/min)
λ = rebound angle (degrees)
a = acceleration (m/s^2)
A = cross-sectional area (m^2)
COF = coefficient of friction
COP = centre of percussion
COR = coefficient of restitution
$C_{xi} = x$ co-ordinate of point C at time increment i

$C_{yi} = y$ co-ordinate of point C at time increment i

$d =$ diameter (m)

$E =$ Young’s modulus (N/m$^2$)

$E_i =$ internal Young’s modulus (N/m$^2$)

$F =$ force (N)

$FE =$ finite element

$f_i =$ internal Force (N)

$k =$ stiffness (N/m)

$k_b =$ ball stiffness (N/m)

$k_s =$ string stiffness (N/m)

$K_{Ei} =$ elastic component of the stiffness matrix

$K_{Gi} =$ geometric component of the stiffness matrix

$l_i =$ elongated length (m)

$l_{0i} =$ original length (m)

$m =$ mass (kg)

$m_e =$ effective mass (kg)

$m_b =$ mass of the ball (kg)

$m_s =$ mass of the strings (kg)

$MOI =$ moment of inertia (kg•m$^2$)

$r =$ radius (m)

$P_{xi} =$ x co-ordinate of point P at time increment i

$P_{yi} =$ y co-ordinate of point P at time increment i

$R =$ normal force (N)

$t_d =$ dwell time (s)
\[ t = \text{time (s)} \]
\[ t_i = \text{time at increment } i \text{ (s)} \]
\[ T = \text{torque (Nm)} \]
\[ v_b = \text{velocity of the ball (m/s)} \]
Glossary

Coefficient of restitution – ratio of outbound velocity to inbound velocity.

Dwell/contact time – The period of time for which the tennis ball is in contact with the string-bed

Finite element model – a computer generated model which allows the user to simulate loading of a structure

Oblique impacts – Impacts where the flight path of the ball is not normal to the plane of the string-bed

Normal impacts – Impacts where the flight path of the ball is normal to the plane of the string-bed

Racket – A strung racket frame

Racket frame – The frame which, the strings are woven into

Spin – The angular velocity of the ball about its centre

String-bed – The woven section of strings within the racket frame

String-bed density – The number/spacing of the strings within the string-bed

String-gauge – the cross-sectional diameter of the string

String orientation – the angle a string is strung across the racket frame
String-tension – This term refers to the magnitude of force applied to a string during stringing of a racket frame and thus dictates the overall stiffness of the string bed.

Sweet-spot – There are several technical definitions of this term but from a playing point of view it is the point of the strings which provides the most pleasant feeling impact.

Top-spin – This term describes the state of motion where a ball spins around its own centre in the same direction as the translational motion.
1 Introduction

1.1 History of tennis

Tennis has been documented, in one form or another, from as early as Egyptian times, when two players would propel the ball towards one another whilst straddling the back of a partner. When the players performed a “fault”, they would switch positions with their respective partners and support them (Clerici, 1976). Exactly what a “fault” entailed, however, is not entirely clear as the first record of court dimensions and rules in tennis did not appear until the twelfth century in France (Clerici, 1976).

From the twelfth to the fourteenth century, there are a series of documents which tell of holy-men playing longue paume or courte paume. The word paume in French literally translates as palm, with the game later becoming known as jeu de paume, which translates literally as, “the game of the hand”. Jeu de paume soon became the game of the upper classes in France and when the area of the Louvre closest to the Seine was built in the 16th century, Henry II of Valois ordered that it be built with adequate space for his favourite pass-time (Clerici, 1976).

By the mid-fifteenth century the game of paume was beginning to contradict its own name, and, the white leather gloves depicted by most artists were being replaced by wooden paddles and, soon after, strung rackets. The earliest rackets were strung obliquely, as opposed to the traditional method seen today, whereby the main strings are placed along the axis of the handle and the cross strings perpendicular to said axis. Developments in the racket were relatively limited for the next few hundred years and the only major changes were in the shape of the head. By the early 1900s, the experimentation with the shape of the racket had all but ceased, with the oval shaped head becoming widely accepted as standard (Clerici, 1976).
By the early 20th century the rules of the game had also traversed through several versions to become much more recognizable as the game we know today. The size of the court had been standardized and was now completely free of any walls, unlike earlier courts which were enclosed and often involved the walls as playing surfaces (much like in the modern game of squash) (Clerici, 1976).

With the invention of television and the subsequent boom in televised sports, there was a huge increase in the number of professional tennis players and the level of skill with which the game was played. Suddenly the margins for error were narrowing and the cost of being on the wrong side of that margin was increasing. As a result, the investment into the development of new equipment was substantial and exploration into alternative materials for the construction of rackets began.

The first commercially successful racket made from a material other than wood was the Wilson T2000 steel framed racket, produced in the late 1960s and popularized by American world number one Jimmy Connors. The development of tubular aluminium rackets allowed for the introduction of the first over-sized rackets, increasing the area of the head from 65 square inches to 110 square inches and providing the blue-print for the modern racket. An example of such a racket is shown in Figure 1.1, along with annotations which describe the tennis racket’s main features.
Figure 1.1: Annotated image of a modern over-sized racket

The increases in head-size was well-received by both professionals and amateurs as it increased the “sweet-spot” of the racket (there are several definitions for this term but the simplest is the area which offers the least vibration during ball impact and results in the most accurate rebound (Brody, 1981)) and reduced the chances of playing a bad shot or missing the ball altogether. In short, oversized rackets increased the quality of tennis across the sport.

By the 1980s the focus on racket performance was almost entirely on weight and racket stiffness, hence, the introduction of carbon-fibre. Carbon-fibre came to the fore
of tennis technology when John McEnroe won the 1985 Wimbledon Championships with the Dunlop Max 200G. Since then there have been a variety of graphite-composites used including boron, titanium, Kevlar and fibre-glass. Other, more innovative, approaches were also taken to increase the stiffness, such as changing the profile of the racket. Wilson, adopted this approach with their “Profile” racket where they increased the thickness of the racket at the top of the head, the area which is required to provide the most reaction force when the strings deflect due to impact, whilst reducing the thickness elsewhere (Cooper, 2003).

Despite the long history of tennis and the conception of rules in the game dating back to as early as the mid-fifteenth century, it was not until the 1970s (when an East-German horticulturist designed a new method of stringing rackets) that the ITF were forced to define the term “Tennis Racket”. The method of stringing that brought about this change is known as “Spaghetti stringing” and involved three sets of non-intersecting strings, two sets of mains, which are fed through rollers, and one set of crosses. The physics behind the increased levels of spin generated by “Spaghetti Strung” rackets (Goodwill et al., 2002) will be discussed later.

However, the newly introduced regulations on rackets from the ITF were still extremely lenient. The rules regarding tennis rackets can basically be summed up in a few words: the racket and its strings must be uniform. The current ITF ruling on rackets can be viewed in Appendix 1.
1.2 Background

Since the inception of metal rackets in the 1970s, and as racket technologies have advanced, there has been a steady increase in the capabilities of tennis rackets and subsequently the speed at which the game has played has increased. The main fear of the tennis industry is that this increased speed may lead to a downturn in spectator levels and thus, a reduction in the level of income which can be generated through sponsorship. The governing body of the sport, the International Tennis Federation (ITF), is extremely concerned about the effect this may have on the game at all levels and recently commissioned research into the development of a novel test machine which could “investigate racket performance under realistic service conditions” (Kotze, 2005).

To attribute the increase in performance solely to improvements in technology would be dismissive of the improved coaching and conditioning of athletes. These factors however, are out of the ITF’s remit, leaving them no option but to focus on the equipment if they wish to control the manner in which the game is played. The mounting pressure on the ITF to regulate the performance of rackets has not just been restricted to the issue of racket power. Some leading experts in the game, including former Wimbledon and US Open champion John McEnroe, have publicly called on the ITF to impart regulations on the amount of spin which can be generated by today’s modern rackets. The effects of such regulations could be extremely harmful to the major tennis manufacturers who would be restricted in terms of product development in an already near optimized market. Dignall et al. (2004) showed that an infinitely stiff tennis racket could only increase the power output of today’s top-end rackets by 1.8%.
1.3 **Aims and objectives**

With spin becoming such an integral part of tennis the primary aim, or goal, of this research is to create an FE model of the tennis racket which can be used to model oblique impacts and measure the resulting spin of different string bed configurations. A secondary aim is use the FE model to model novel string-bed designs and also obtain a greater understanding of how spin is generated. In order to create such models, a good database of material and the interaction properties is required and hence the need to develop appropriate means of acquiring such properties. Validation of the model is also a fundamental requirement to establish confidence in results of the models and therefore establishing good validation techniques will form a significant part of the work. Project goals are presented in tabular form in Table 1.1.
<table>
<thead>
<tr>
<th>Primary aim</th>
<th>Secondary aims</th>
<th>Tertiary aims</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obtaining a greater understanding of spin generation of resulting from oblique impacts</td>
<td>Obtaining material properties</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obtaining interaction properties</td>
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<tr>
<td></td>
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<td>Static validation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dynamic validation</td>
</tr>
</tbody>
</table>

Table 1.1: Project Goals
1.4 Thesis outline

The approach taken to creating an FE model of tennis racket impacts was first to create an initial model to then build on. The strategies adopted in creating this model are detailed in Chapter 3.

The next step is to validate the model’s prediction of the racket frame and string-bed under static loading conditions. The experimental work and corresponding modelling undertaken to validate the racket’s static performance, including the displacement of the racket after stringing and the tension profile of the string-bed, is detailed in Chapter 4.

In order to validate the fundamental model of the frame which is used in modelling dynamic impacts, a series of modal analysis experiments are performed and these experiments are detailed in Chapter 5. The experimental results of the modal analysis of the racket are used to validate the vibrational characteristics of the model.

Chapter 6 outlines the approach used to obtain a comprehensive materials properties library for a variety of tennis strings. This work is necessary to determine the most appropriate materials model to apply.

In any dynamic analysis of two contacting interfaces the contact model is crucial to providing accurate results. Chapter 7 describes the contact mechanisms which occur during tennis racket impacts, the most suitable way to computationally model them and the acquisition of the necessary experimental values.

In order to manipulate string-bed configurations such that they will generate an increased level of outbound spin, a good understanding of the mechanisms which generate spin is required. Chapter 8 describes a series of FE experiments where a
ball is projected at a number of oblique angles and the change in impact mechanism, due to the inbound angle, is observed.

Once the model has been validated under static conditions the next step is to examine its dynamic performance. This meant the introduction of a ball to the model to create dynamic impacts and high speed video of experimental impacts as a validation tool. The procedure is performed using a variety of ball impact velocities and impact angles, all of which are examined in Chapter 9.

The concluding chapter in this thesis, Chapter 10 integrates all of the work done in the previous chapters to realise the overall goals of the project, observing changes in spin generation due to variations in the string pattern through FE modelling. Such changes included changes in the string density, orientation and gauge (cross-sectional area).
2 Literature review

2.1 Finite element modelling

There have been several attempts to use finite element analysis (FEA) to model tennis rackets, balls and their impacts over the years. The first such attempt was by Bitz-Widing et al. (1989), who used a tennis racket as an example application of stiffening a frame with tension members. The strings were modelled using isoparametric non-linear elements and the analysis produced displacements, forces and stresses for the entire structure. The elements used to model the strings were referred to by the author as “cable members” and have a stiffness that depends on their internal force, $F_i$. The internal force of the members, were found using the relationship show in Equation 2.1:

$$F_i = \frac{E_i A_i}{l_{0i}} (l_i - l_{0i})$$  

Equation 2.1

Where $E_i$ is the modulus of the string, $A_i$ is the cross-sectional area, $l_{0i}$ is the original length and $l_i$ is the elongated length. The stiffness matrix of the elements consisted of an elastic component, $K_{Ei}$ (equations 2.2) and a geometrical component, $K_{Gi}$ (equation 2.3).

$$K_{Ei} = \frac{E_i A_i}{l_{0i}} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$  

Equation 2.2
The above matrices will combine to produce a complete stiffness matrix, subject to all translational degrees of freedom, for the element (Equation 2.4). Bitz-Widing et al. decomposed it into the above components, however, as she reasoned that some areas of the element would behave elastically and some would not.

The local element stiffness matrix, $K_L$, is obtained through the summation of these components and using standard finite element analysis practice (Rockey et al., 1983) the forces within the element can be calculated by multiplying out the matrix shown in Equation 2.5, where $F_i^{(j)}$ is the Force at node “$j$” in the “$i$” direction and $u_i^{(j)}$ is the displacement at node “$j$” in the “$i$” direction.
\[
\begin{bmatrix}
  F_1^{(1)} \\
  F_2^{(1)} \\
  F_3^{(1)} \\
  F_1^{(2)} \\
  F_2^{(2)} \\
  F_3^{(2)}
\end{bmatrix} = \begin{bmatrix}
  \frac{E_i A_i}{l_{oi}} & 0 & 0 & -\frac{E_i A_i}{l_{oi}} & 0 & 0 \\
  0 & \frac{F_i}{l_{oi}} & 0 & 0 & -\frac{F_i}{l_{oi}} & 0 \\
  0 & 0 & \frac{F_i}{l_{oi}} & 0 & 0 & -\frac{F_i}{l_{oi}} \\
  -\frac{E_i A_i}{l_{oi}} & 0 & 0 & \frac{E_i A_i}{l_{oi}} & 0 & 0 \\
  0 & -\frac{F_i}{l_{oi}} & 0 & 0 & \frac{F_i}{l_{oi}} & 0 \\
  0 & 0 & -\frac{F_i}{l_{oi}} & 0 & 0 & \frac{F_i}{l_{oi}}
\end{bmatrix} \begin{bmatrix}
  u_1^{(1)} \\
  u_2^{(1)} \\
  u_3^{(1)} \\
  u_1^{(2)} \\
  u_2^{(2)} \\
  u_3^{(2)}
\end{bmatrix}
\]

Equation 2.5

The more advanced materials models used in today’s finite element packages (such as the Neo-Hookean or Ogden material models available in Abaqus) will use a similar approach to calculate the stiffness matrix with an increased number of components.

Bitz-Widing et al. modelled the impact of the ball on the string bed as a static parabolic load, as shown in Figure 2.1. The displacements, strains and stresses were then calculated for each element, showing that the majority of stress propagation due to the ball force occurred in the lower half of the racket.
Figure 2.1: Parabolic load representing the impact force of the ball against the string bed (Bitz-Widing et al., 1989)

Due to the lesser capabilities of FE packages at the time, the slippage between the strings was not modelled and the strings were considered to be rigidly fixed at their intersection points.

Another example of FE modelling of a tennis racket is that of Hambli et al. (2006), who modelled the racket frame as a rigidly clamped structure, around a network of truss elements, representing the string-bed. The author’s use of truss elements somewhat limited the performance of the model, since truss elements are unable to simulate bending. His assumptions with regard to the boundary conditions, that the racket was best modelled rigidly clamped, is also contrary to the accepted wisdom within tennis communities that the best representation of a handheld racket is freely-suspended (Brody et al., 2002). Hambli also neglected the work of Cross (2000a), which found the coefficient of friction (COF) between tennis balls and strings to be between 0.27 and 0.43, choosing instead to impose a value of 0.1

The most recent attempt at modelling of a tennis racket string-bed is that of Allen et al. (2008). The author’s decision to use the relatively geometrically incompatible 8 noded brick elements meant that over 37,000 elements were required to provide a suitable representation of the racket geometry. The author did not expand on his reasons for using such a high volume of such computationally expensive elements;
therefore one would have to assume it was deemed necessary from an accuracy point of view. Unlike other authors, Allen et al. neglected to model the string-bed as part of a racket frame, instead choosing to fix the end of the strings to rigid cylinders. The author claimed that to model the tensioning of the strings a force of 150 Newtons was imposed on each cylinder, in order to provide an overall tensioning force of 300 Newtons to each string. Interestingly, the author chose not to validate his model using rackets strung to this tension, instead using rackets strung to 200 Newtons and 289 Newtons.

FE modelling has also been applied to other areas of sports equipment. For example, Penrose et al. (1999) used FE modelling in order to study the modal and impact characteristics of a cricket bat. By increasing the stiffness of the bat model the author was able to show that the contact time during impact decreased. As stiffness increased, a similar conclusion was reached by Wiezel et al. (2004), where a numerical model was used to show that increasing the stiffness of the string-bed in a tennis racket will increase the contact time. The numerical model was developed using basic mechanics and yielded Equation 2.6:

$$ T_d = \pi \sqrt{\frac{K_b+K_s}{K_bK_s}} \sqrt{\frac{m_b m_e}{m_b+m_e}} \sqrt{\frac{1}{1-\xi^2}} $$

Equation 2.6

Where $T_d$ is the dwell time, $K_b$ and $K_s$ are the stiffness of the ball and the strings respectively, $m_b$ and $m_e$ are the ball mass and effective mass of the racket respectively and $\xi$ is the critical coefficient of damping.

Vedula et al. (2004) used FEA to investigate the “sweet-spot”, which was defined by Vedula as the position giving maximum rebound velocity, in a baseball bat. Vedula used modal analysis to obtain the modes of the bat and their corresponding nodes.
Pendulum tests were also performed to obtain the moment of inertia (MOI) of the bat and hence, centre of percussion (COP) using Equation 2.7:

\[ \text{COP} = \frac{\text{MOI}}{m \times r} \]  

\[ \text{Equation 2.7} \]

Where \( m \) is the mass of the baseball bat and \( r \) is the distance from the axis of rotation to the centre of gravity. Two models of the bat were created, one wooden (solid) and one aluminium (hollow). The wooden bat was modelled using eight noded brick elements whilst, due to its hollow structure, the aluminium bat was modelled using four noded shell elements. Due to the anisotropic nature of wood it was necessary to assign it the properties of an orthotropic-elastic material (a material model which accounts for directionality), whilst the aluminium bat was modelled using isotropic properties.

By changing the mass distribution of the model, the author found that weighting the mass towards one end of the bat resulted in the COP moving in the same direction. Changes in material elasticity had no effect on the natural frequency node or the COP, however, the natural frequency of the bat did change.

Casolo et al. (2000) also used FEA to model a tennis racket in order to investigate several features of the tennis racket impact, such as the ball’s rebound velocity, the impulse force imparted on the hand and the vibration modes and the position of their nodes. The tennis racket was modelled using simple 2D beam elements which were assigned material properties obtained from tensile testing of a dissected racket. The stringing process was actually carried out within the model to evaluate the evolution of stress and strain. The results showed a variation of 200 Newtons to 260 Newtons for a specified tension of 240 Newtons. The model of the racket frame was then validated by a series of modal frequency tests, which showed the natural frequencies
of the frame (134 Hertz and 399 Hertz for the first lateral and torsional modes respectively) correlated well between the model and the experimental analysis.
2.2 Tennis strings

Tennis string tensions are traditionally quoted in Pounds-force but in the interests of consistency all values will be specified in Newtons within this thesis. For ease of comparison a look-up table comparing the most commonly used string tensions in Pounds and Newtons can be found below in Table 2.1:

<table>
<thead>
<tr>
<th>Pounds</th>
<th>Newtons</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>178</td>
</tr>
<tr>
<td>45</td>
<td>201</td>
</tr>
<tr>
<td>50</td>
<td>223</td>
</tr>
<tr>
<td>55</td>
<td>245</td>
</tr>
<tr>
<td>60</td>
<td>267</td>
</tr>
<tr>
<td>65</td>
<td>290</td>
</tr>
<tr>
<td>70</td>
<td>312</td>
</tr>
</tbody>
</table>

Table 2.1: Equivalent SI values of pounds-force string tension

The physics surrounding the behaviour of a string-bed during an impact is extremely complex. The player’s perception of the sweet spot is defined by the “feel” or comfort of a shot and also the performance of the shot in terms of its translational velocities. From a performance point of view, the sweet spot is the area of the string-bed which will provide the best return of energy, thus the most powerful shot. From a comfort point of view, there are two areas which contribute towards the most pleasant feeling shot. The first is the centre of percussion, which, when struck avoids any out of balance rotations of the racket along the handle (see Figure 2.2). The second is simply the point of impact which will provide the least vibration, the fundamental node (Brody, 1981).
The presence of a sweet-spot is created by the string’s tendency to deflect equally regardless of where along its length it is hit. The result of this behaviour is that off-centre shots seem stiffer as the string is deflecting unequally on either side of the ball, yet the effective stiffness of the shorter side of the string is much greater and dictates the feel of the shot. It is for this reason that string-beds will appear softer when the string spacing becomes larger, as the length of the string which is able to deflect uninhibited is increased. Consequently, in the majority of cases, the sweet spot of the string bed will be where the longest main string crosses the longest cross string (Brody, 1981).

![Diagram of reaction forces](image)

**Figure 2.2: Reaction forces when the ball strikes the racket at (a) the COP (b) above the COP and (c) below the COP**

An early study performed by Groppel et al. (1992) into the performance of different tennis string materials found that synthetic strings (in this case nylon) produced a lower coefficient of restitution (COR) than natural gut. This is thought to be due to the increased natural stiffness of synthetic strings relative to gut. Groppel tested strings at tensions of 178 Newtons, 223 Newtons, 267 Newtons, 312 Newtons and 356
Newtons. He found that COR generally decreased with increasing tension (although the results were by no means linear) with the decrease in COR becoming less pronounced as the tension was increased. Groppel reasoned that COR decreasing as string tension increased is due to the increasing deformation of the ball, and hence increasing contact time, on the stiffer string-bed. Naturally this trend is not linear across all string tensions as the energy saved through decreasing contact time will eventually be overcome by the string-bed’s inability to return energy to the ball as the tension decreases.

The issue of reference stringing tension not being equal to the actual tension was explored by Cross (2001). It is well known to professional racket stringers that the reference tension of a string is not the actual tension, since the string will creep after clamping. This is because tennis strings are viscoelastic and during the tensioning of the string the elastic fibres will immediately react and stretch to allow the string to elongate. In the first 20 seconds to 30 seconds after tensioning, however, the viscous fibres will unfold and elongate in the string and partially unburden the elastic fibres, thus slowing the rate of elongation.

Cross used load cells attached to strings to investigate the change in tension of strings which were tensioned to 275 Newtons and 177 Newtons as shown in Figure 2.3. The loss in tension in the first 100 seconds was around 10 Newtons and then the rate of loss began to slow, showing a linear trend against the logarithm of time. During the impact testing it was shown that the strings briefly increased in stiffness during the impact before decreasing to a tension lower than that prior to impact. Of all the strings tested, gut’s increased elasticity meant that it was able to retain its stiffness for longer during the impact testing.
Figure 2.3: Reduction in string tension over time Cross (2001)

The initial tension loss in strings during stringing can be limited, however, by increasing the length of time at which the string is clamped before being released. For instance, a string pulled to 311 Newtons of tension and held for 3 seconds will have the same tension as one pulled to 223 Newtons or 267 Newtons in tension for 55 seconds, 60 seconds after being released. Cross et al. found that the string tension of a racket is generally 30% to 40% lower than the reference tension. He also found that the tension in the mains was generally higher than in the crosses. This was thought to be due to the fact that the shorter cross strings created a greater deformation of the racket in their direction than the longer mains. As a result the racket would be reduced in length in the direction of the crosses and increased in the direction of the mains, further increasing the tension of the main strings.

Knudson (1991) carried out research into the effect which string tension had on vertical angle of rebound in tennis ball impacts. Three midsized rackets were strung at 223 Newtons, 267 Newtons and 311 Newtons using 15L gauge nylon and gut strings. Twelve balls were fired from a distance of 1.49 metres at a mean velocity of
19.1 metres per second and a mean angle of 25.6 degrees to the horizontal. The nylon strings exhibited little change in rebound angle throughout the range of stiffness values, whilst the guts strings varied significantly. At a tension of 223 Newtons they provided a more accurate rebound ratio (the ratio of the outbound angle over the inbound) than nylon (1.4 compared to 1.5). However, whilst nylon maintained its performance for the higher tensions of 267 Newtons and 311 Newtons, the ratio of rebound for gut deteriorated to 1.6 and 1.7 respectively. The same trend applied for COR, with gut originally outperforming nylon at 0.51 compared to 0.5 for 223 Newtons but decreasing significantly to 0.43 for 311 Newtons whilst nylon only decreased to 0.49.

Knudson (1993) also investigated the effect which impact location had on rebound angle accuracy for nylon strings. Central impacts were shown to give a rebound angle of 1.1 degrees consistently, whilst impacts 80 millimetres off-centre gave rebound angle ratios varying between 1.25 degrees and 1.65 degrees. This difference may seem relatively insignificant but when the length of the shot (which could be up to 24 metres) is taken into consideration such a difference in initial projection could result in an extra 2 metres in shot length.

Further work on rebound angle accuracy in relation to string tension was carried out by Bower et al. (1999). The author reasoned that lower string tension’s more accurate rebound angle in experimental analysis is compromised in practice due to a longer dwell time. The reason for this being, most ground-strokes are struck off-centre and when this happens the racket becomes unstable and rotates about the axis of the handle, altering the angle of the ball’s rebound plane. For looser strings there is an increase in dwell time, meaning that the effect of the racket’s rotation will be amplified. Bower quantifies this effect theoretically, stating that a decrease in string tension of 45 Newtons would increase the dwell time by 0.5 milliseconds and increase the difference in rebound angle by 1 degree.
Despite the broad range of literature showing the profound effect of changes in string tension Bower et al. (2003) showed that player’s perception of changes in string tension is not nearly as accurate as they would have academics believe. Bower strung five identical rackets at 177 Newtons, 206 Newtons, 215 Newtons, 234 Newtons and 261 Newtons and the participants of the study were asked to differentiate between the tensions of two rackets during play. To reduce the risk of someone guessing correctly they were tested on two different levels of tension. The first set of rackets they were tested on had a difference of 49 Newtons (177 Newtons to 226 Newtons and 226 Newtons to 275 Newtons) and the last a difference of 20 Newtons (206 Newtons to 226 Newtons and 226 Newtons to 246 Newtons). If the players could not correctly identify the higher tension of rackets with a 49 Newtons difference they work backwards to a difference of 98 Newtons until they successfully identified a racket with the higher tension. The results showed that most of the participants could not differentiate between tension differences less than 49 Newtons, whilst out of 30 participants only 2 were successful in every test. The author also expresses concerns at the similarity of the results to that of random guess work, suggesting that the actual results were even poorer than indicated.

2.3 Modal analysis

In the context of this project, modal analysis can be used as a multi-purpose tool. Its primary function is to act as a validation tool for the finite element model but a convenient by-product of this validation is an insight into how the vibrational characteristics vary from racket to racket (Mohr, 2008). The vibrational characteristics of the racket play a large role in the player’s perception of the shot quality since they dictate how the shot actually feels to the player. As a result the word “feel” is commonly used as a descriptive for the quality of a racket, with a racket providing little vibration termed as having good “feel”.

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By performing a modal analysis of a tennis racket one obtains the fundamental node of the racket, which is commonly used as a scientific definition of the area on the string-bed defined as the “sweet spot”. The sweet spot is a term used by players to define the area of the string bed which provides most pleasant feeling shot. As eluded too earlier, there are several definitions commonly used for the sweet spot (Brody, 1981), the most common of which are:

- The area which of the string bed which gives the highest COR.
- The centre of percussion of the racket.
- And, the one which is of most interest to this study, the fundamental node of the string bed.

Whilst none of these areas/points fall exactly within each other, they are generally found in the same area of the string bed, to the point where a player can often encounter all three within one shot. As a result players identify this area as being a large single region, rather than the collection of three smaller regions that it is.

In a study examining the sweet spot Hedrick et al. (1979) mounted an accelerometer on the handle of a freely suspended racket to measure the impulse and oscillation through the handle when the string bed was struck at various locations. Hedrick found that, as expected, there was negligible impulse when the racket was struck on its centre of percussion and there was minimal vibration when the node was struck.

In a study which focused on vibration damping of tennis rackets Brody showed a higher level of damping when in the hand held position than when freely constrained. This conclusion was drawn from an experiment where a small vibration sensor was taped to the throat of the racket to measure the vibrations transmitted to the handle when the string bed was struck. Brody also showed that, whilst the introduction of a string damper had very little effect in reducing the vibrations of the racket, it did have considerable effect in reducing the sound vibrations created by the strings.
Numerous authors (Axe et al., 2002, Mohr et al., 2008 and Hocknell et al., 1998) have used modal analysis as a technique of validating their models and exploring the changes in vibrational characteristics which result from altering certain parameters of their model. One such example is that of Penrose et al. (1999) who used the finite element package ANSYS to model two cricket bats, one representing a traditional design and the other representing a less conventional design. For the fundamental frequency the results were not particularly significant with the traditional design having a frequency of 93.9 Hertz compared to 94.1 Hertz of the unconventional design.

2.4 Spin generation during tennis impacts

The generation of top-spin in tennis is perhaps the most significant contributor to the increased pace at which the game is played. Tennis players can generate top spin by sliding the racket up and over the ball in a diagonal direction to the normal of the ball’s incoming trajectory. This has the effect of imparting a forward rotational velocity which, due to an increased drag force on the upper side of the ball, allows the ball to decrease in height at a much faster rate (as shown in Figure 2.4). Consequently, the ball can be hit much harder and still fall within the baseline, as demonstrated by the “high top spin” trajectory in the figure below.
The defining moment of spin quantification in tennis was instigated by the introduction of the “Spaghetti Strung” (SS) racket (see Figure 2.5). Goodwill et al. (2002) carried out an investigation into the characteristics of the racket, which consists of two non-intersecting planes of strings and a third set of strings which are tied across the main strings (the pink strings in Figure 2.5).

A traditional SS racket was compared to four conventionally strung rackets, strung with nylon and gut strings at tensions of 178 Newtons and 312 Newtons. A “BOLA” ball firing machine was used to fire balls at the rackets, which were clamped at an inbound angle, \( \alpha \), of 39 degrees to the flight path of the ball as depicted in Figure 2.6. The inbound ball spins imparted on the ball by the BOLA machine were in the range of 0 radians per second to 420 radians per second for the conventionally strung rackets and 0 radians per second to 300 radians per second for the SS racket. The author was not clear as to why a consistent level of inbound spin was not used for both racket types. The spin imparted on the ball by the BOLA machine was backspin (negative angular velocity) whilst the resulting spin after impact was topspin (positive angular velocity). The inbound and outbound angular and translational velocities of the balls were measured using high speed video footage, from which the ball’s logo was used as a reference point for spin measurements.
The rebound angle, $\lambda$, of the conventionally strung rackets varied from 23 degrees to 5 degrees for a tension of 312 Newtons, and 23 degrees to 10 degrees for a tension of 178 Newtons. The SS rackets had a markedly different trend with the rebound angles varying from -5 degrees to +15 degrees over a smaller range of spins (300 radians per second compared to 420 radians per second). The amount of spin imparted on the ball was in the region of 100 radians per second to 150 radians per second greater for the SS rackets and, for some inbound spin rates, almost twice as great. This is thought to be due to the fact that the string bed is not interwoven and, hence, is allowed to deflect in the direction of the ball before returning the energy in the reverse direction of the inbound angular velocity.
Goodwill et al. (2006) carried out a more recent study into the ball spin generated by different types of tennis strings. Identical tennis rackets were strung at 270 Newtons with 30 different types of string and allowed to “settle” for 24 hours. Balls were fired at each racket at a velocity of 25 metres per second, angles of 40 degrees and 60 degrees to the normal, and spin rates ranging up to 400 radians per second. The impacts were filmed at 1,000 frames per second and three mutually perpendicular lines were drawn on the ball to calculate the rate at which the ball was spinning.

It was observed that polyester strings gave higher outbound spin rates for all impacts at 40 degrees and only at inbound spin rates of 400 radians per second did polyester give less outbound spin than any other string for 60 degrees. Goodwill argued that the frictional properties of the strings would have no effect on the level of spin imparted on the ball since all strings possessed frictional coefficients high enough to initiate rolling and any increase in the COF above this level would not increase spin. He also reasoned that the increased stiffness of the polyester strings would allow them to recover faster from the deflection caused by impact, thus rebounding earlier and losing less energy than the other strings. This is a statement which many authors disagree with; including Cross (2000) who stated that the COF was “an important parameter since it determines the dynamics of the collision in a
direction parallel to the string bed and... the amount of spin which can be imparted on the ball”.

The difference in trend associated with the steeper, 60 degree angle is accredited to the fact that the ball is more likely to slide along the string bed, resulting in a different mechanism of spin production. The polyester strings would be less able to deflect in the direction of the ball as it impacts, hence the ball will be more likely to slip over them, rather than rebound in the opposite direction, as in the case of more elastic strings.

Although Goodwill deemed friction to have a negligible effect of the generation of spin Ashcroft et al. (2002) carried out a number of experiments with the intention of identifying a relationship between the two parameters. Using high speed video footage, outbound spin rates were obtained for impacts, without inbound spin, at angles of either 30 degrees or 35 degrees. Three different racket types, strung at 225 Newtons and 315 Newtons, were clamped between two plates and allowed to travel only in the plane of the racket itself (i.e. the racket can only move along the axis of its handle). The average value of the friction in this scenario was found to be 0.498, compared to a value of 0.39, obtained from dragging a weighted tennis ball along a string-bed. The author reasoned that the vast difference in these values is likely to be due to the secondary factor of friction: mechanical interlocking. Ashcroft stated that, whilst the majority of the friction in a drag test is due to the inherent surface properties of the materials, the friction resulting from an impact test will be subject to mechanical interlocking of the materials due to a higher normal force.

However, Bao et al. (2003) carried out a similar study using the same process of dragging a weighted tennis ball over a surface and deduced the COF from the required force. Contrary to Ashcroft et al.’s findings, during this experiment Bao found that the COF reduced as the contact force was increased. This would lead one to conclude that the weighted sled test is not an accurate enough representation of a dynamic impact to provide reliable values of the COF.
As well as exploring the effect which string tension has on the production of post-impact angular velocity, Bower et al. (2007) also looked at the role played by racket stiffness, which was altered by clamping the racket at various positions along its length. The rackets were strung at tensions of 180 Newtons, 225 Newtons and 270 Newtons using a conventional stringing machine and balls were fired at varying velocities of 16 metres per second, 20 metres per second and 24 metres per second at an angle of 45 degrees to the plane of the racket. The angular velocity measurement system used was similar to that used by Goodwill et al. (2006). The spin rates of the balls increased with string stiffness and impact velocity but seemed to be unaffected by the stiffness of the racket. The spin rate also increased as the impact speed was elevated, with a difference of 34 radians per second between high (24 metres per second) and medium (20 metres per second) speed impacts and 16 radians per second for the low (16 metres per second) and medium speed impacts. Bower also identified a small, yet significant relationship between string tension and the level of spin imparted by the ball.

Contrary to this Goodwill found that there was no relationship between experimentally obtained results and argued that the difference in spin at higher tensions must be due to the excess force players exert during the shot to compensate for lack of power (i.e. the player's subconsciously feel that a higher tension will reduce their power and strike the ball harder). However, Goodwill does observe that dwell time of the ball increases by 20% when the string tension is increased from 178 Newtons to 312 Newtons and the author assumed that the dwell time must have some effect on the ability to impart spin on the ball since, reasoning that the longer the ball lies on the string-bed the more its rebound characteristics are likely to be effected by the motion of the racket through mechanical interlocking. The reason for the dwell time of the ball increasing as the tension is increased is that the strings account for only 2% to 4% of the energy loss in a ball-racket impact (Hatze, 1993) and therefore, although higher string tensions lead to a reduction in string deformation, they also lead to an increase in contact time by increasing the deformation of the dominant energy loss component within the system; the ball.
In general, most high outbound spin rates are produced as a result of oblique impacts, the mechanism of which was explored by Cross (2003). Cross found that most experimental analyses of tennis racket impacts were performed with the racket fully clamped, effectively rendering the frame infinitely stiff in every plane. This is especially important when investigating the effects of friction, as a rigidly clamped racket will not allow for the movement of the racket which occurs as a result of the frictional force between the racket and ball. To eradicate this effect Cross mounted the racket on rollers, meaning that the racket is only infinitely stiff in the plane of the racket, whilst free to deflect in all other directions. The rollers also measured the perturbation of the racket, which, combined with the mass of the racket, allows the frictional force exerted by the ball to be calculated. The conclusion of this work was that for a ball impacting at around 25 degrees a frictional coefficient of 0.43 plus or minus 0.02 was obtained.

Cross (2005) further examined the role of friction in a tennis racket impact with particular emphasis on the variation with sliding speed. In order to draw a comparison between low and high sliding speed, two separate experimental set-ups were utilised. The first involved a small weighted sled with its underside lined with tennis cloth being dragged along a smoothed table top at relatively low speeds (0.001 metres per second to 1 metres per second) (Figure 2.7) whilst the second involved firing tennis balls at an angle of 17 degrees and speeds ranging from 1 metre per second to 20 metres per second (Figure 2.8). Cross essentially discovered that for low sliding speeds the COF decreased as mass was added to the sled whilst it increased with the apparent contact area.
Figure 2.7: Variation in COF with sliding speed for different normal forces and contact areas (Cross, 2005)

Figure 2.8: Variation in COF for a tennis impact at 17 degrees to the normal at various speeds (Cross, 2005)
Similar investigations into the role of friction in the creation of spin have been carried out in golf by Chou et al. (1994). Initially, Chou created an FE model of a club face which showed that for club face angles less than 30 degrees spin occurs mainly as a result of the ball “sticking” to the club face (i.e. mechanical interlocking of the ball and the club-face). A secondary analysis was carried out involving balls being fired against a steel block at a variety of angles. Each of the impacts was filmed using a high speed camera and, using dots on the ball, the spin rates for each impact were calculated. The results validated the FE model, showing that higher impact angles resulted in less spin due to the ball slipping across the surface and the interlocking effect not being initiated.

All of this previous research into the friction associated with tennis racket impacts is beneficial on two levels. Firstly it demonstrates potential methods for the acquisition of friction properties which can be used in a finite element model. Secondly, although each tennis string will have its own COF, previous studies provide an insight into the range in which one should expect values to fall when carrying out original testing. This will provide confidence in the results of this testing, assuming the values acquired are of a similar magnitude.
3 Generation of the initial finite element model

Like many processes, the development of a finite element model can often be an iterative process. This project has taken the approach of developing certain aspects of the model; such as material properties, interaction properties and energy dissipation, individually rather than trying to perfect them all at the same time. In order to do this however, it was necessary to have a base model which can be used as a platform for assessing the performance of these individual aspects. This chapter will detail the development of this basic, but crucial, part of the modelling process.

3.1 Equipment and software packages

3.1.1 Co-ordinate measuring machine

To obtain the geometry of the racket, an “LK Ultra” co-ordinate measuring machine (CMM) was used. The CMM uses a measurement probe mounted on a bridge which is free to move in Y and Z, whilst the probe can move along the length of the bridge allowing motion in the X direction. The machine has a measuring volume of 1000 millimetres by 800 millimetres by 600 millimetres (X, Y and Z respectively). The machine was calibrated to UK Accreditation Services (UKAS) standard and is accurate to 0.0018 millimetres over 1 metre of measurement length (Singh, 2012).
3.1.2 Abaqus

Abaqus is a commercially available suite of finite element analysis softwares. Abaqus contains two main solver options; Abaqus/Standard and Abaqus/Explicit. Abaqus/Standard uses an implicit solving method and is used for static or long duration analyses whereas Abaqus/Explicit uses an explicit solving method and is more suited to high speed or small duration analyses. Both define equilibrium using the relationship defined in Equation 3.1, where \( P_F \) is the external applied force, \( I_F \) is the internal element forces, \( m \) is the mass matrix and \( a \) is acceleration:

\[
m \times a = P_F - I_F \quad \text{Equation 3.1}
\]

Both methods solve for the nodal accelerations at each time step but use different approaches in the way they do so. The method used by Abaqus/Explicit allows it to calculate the nodal accelerations for large displacement, dynamic analyses in a far more efficient manner than Abaqus/Standard and, as such, will be the most commonly used method in this thesis.

Abaqus provide their own "Complete Abaqus Environment" (CAE) environment which provides everything the user requires to perform an analysis. Using the pre-processor the user can mesh geometry, define load-cases and submit the model for analysis. Once complete, the results of the job can then be reviewed in Abaqus/Viewer; the dedicated post-processing package in CAE.

3.1.3 Altair Hypermesh

Hypermesh is one of many software packages available as part of the finite element analysis software suite, Altair Hyperworks. Hypermesh allows users to mesh geometry in a largely similar way to Abaqus CAE. Hypermesh, however, provides a
larger number of meshing tools than most other finite element packages, making it more effective and quicker when it comes to meshing complex geometry. Hypermesh can also export meshed geometry in various formats, including Abaqus input (.inp) files, meaning that geometry can be meshed in Hypermesh and then exported to other softwares, such as Abaqus.

3.1.4 Siemens NX 5.0

NX 5.0 (also known as Unigraphics) is an advanced computer aided design (CAD) software package. Like most other CAD packages, NX 5.0 allows the user to generate virtual geometry which can be used in a variety of applications, although it is predominantly used as a design tool. NX 5.0 allows geometry to be generated and exported in a file format which is compatible with most finite element software packages.

3.2 Racket frame modelling

3.2.1 Modelling of racket frame geometry

Although the project is primarily concerned with the modelling of the string-bed, a satisfactory model of the racket frame is a fundamental requirement. In order to obtain the geometry of the racket, a co-ordinate measuring machine (CMM) was used. Using this machine, a series of points from around a tubular section “Dunlop 200” aluminium racket’s outer and inner edges were captured and translated into the CAD system NX 5.0. The points were measured at five millimetre intervals along the length of the handle, around the head of the racket and then along the yoke of the racket. This resulted in a data set of 252 measurement points.

From this series of points, a spline was created to represent the outline of the racket. The racket’s outline was then projected onto a solid block, the thickness of the
racket, and then extruded outwards to create a solid three dimensional profile of the racket.

The cross sectional area of the racket is uniform throughout the majority of the frame and consequently can be represented by the profile extracted from the block. The handle however differs from the frame both in magnitude and profile and must be represented by a separate entity. This entity is created by sketching the profile on the bottom face of the frame and then extruding this sketch to form the handle (see Figure 3.1).

![Figure 3.1: Creation of the racket handle in NX 5.0](image)

As well as capturing the points which represent the outline of the frame the CMM was also used to measure the position of the string holes on the racket. The holes were created using the imported points as their centre points. The extrusion representing
the frame was selected as the body from which the holes would be cut and the global X and Y axis were assigned as the vector along which the holes for the cross and main strings were created, respectively. Each of the holes were assigned a uniform diameter of 1.6 millimetres.

3.2.2 Racket head mesh

The hollow cross section of the racket head is not modelled in NX 5.0, instead the geometry is imported into the meshing package Altair Hypermesh as an “.iges” file and the solid geometry is modelled as being hollow, using shell elements with a specified thickness.

Unlike early carbon fibre rackets, which commonly possessed numerous irregularities in their cross-sections, aluminium is largely consistent throughout and can be modelled with constant property and a uniform thickness (which can be represented using shell elements).

The elements used to model the racket head are known as S3Rs, where S denotes that the elements are shell elements which can be used to model stress and displacement, 3 denotes the number of nodes within the element and R denotes that the element is reduced integration.

The introduction of reduced integration within the elements has benefits in terms of computational efficiency but can also give rise to a phenomenon known in finite element analysis as “Hour-glassing”. Hour-glassing is a term defined by Abaqus (2010) and occurs due to only one integration point being used, which can lead to all strains at said integration point being zero, which in turn leads to excessive distortion of the element. Hour-glassing can however, be avoided by the distribution of point
loads and boundary conditions over a number of nodes, a technique which will be described in further detail at a later stage.

Typically, shell elements can be used in situations where the wall thickness of the structure is $1/10^\text{th}$ of the overall dimension or less and the stress in the thickness direction is negligible (Abaqus, 2007). Therefore, by assuming that the stress in the thickness direction is negligible it is possible to dramatically reduce the number of nodes per element and, often reduce the number of elements.

Unlike three dimensional continuum elements, however, conventional shell elements can be used to model displacement and rotational degrees of freedom, i.e. they are able to simulate rotation of their nodes. The benefit of having elements which can model rotational degrees of freedom is that a greater number of outputs are available to the user, e.g. nodal rotation/rotational velocity/rotational acceleration.

Using the “Automesh” feature in the “2D” sub-panel of Altair Hypermesh, an element size of 0.5 millimetres was specified along with a maximum aspect ratio of 5. This resulted in the generation of 262,380 S3R elements. The resulting mesh is shown below in Figure 3.2.

![Figure 3.2: Mesh of the racket head (262,380 S3R elements)](image-url)
The main area of difficulty when generating the mesh was the string holes within the frames. Given their relatively small diameter, capturing their geometry required a much finer mesh than the rest of the frame. The result of using such a coarse mesh when modelling the holes can be seen below in Figure 3.3. It was therefore, necessary to increase the number of nodes and, hence the number of elements around the hole from 3 to 6. The resulting mesh used in the analysis is shown in Figure 3.4.

Figure 3.3 Coarsely meshed hole
The difficulty of meshing arises from the need to generate as uniform a mesh as possible. A good mesh will consist of uniform elements with consistent angle sizes. Figure 3.5 shows an exaggerated example of desirable and undesirable elements. The left-most element, containing similar angle sizes and edge lengths, is the more desirable of the two, whilst the right most element would be likely to artificially alter the stiffness of the structure, reduce the overall accuracy of the model and decrease the stable time increment, thus increasing the computation time of the model.

Figure 3.5: Desirable and undesirable elements
In order to avoid the presence of poor quality elements, the quality was checked using the “check elems” option in the “Tool” sub-panel. The “check elems” feature allows the user to specify targets for the elements - such as minimum angle or length – and highlights any elements within the mesh that do not meet these targets. Using this feature, the model was checked for triangular 2D elements with angles outside the range of 30 degrees to 130 degrees.

The failed elements were saved and re-meshed using the “elem cleanup” option in the “2D” sub-panel. The “elem cleanup” option is a predefined subroutine, which allows users to specify targets for element variables such as minimum/maximum angle, aspect ratio, element length etc. Once the user has specified the desired targets, the software will identify any failing elements and refine the mesh accordingly. This process was repeated several times until all unsatisfactory elements were eliminated.

When meshing the frame the element type in the automesh panel was set to “trias”, which generated a group of S3R elements, as shown in Figure 3.5. The element type was changed to the desired S3R by editing the .inp file, which was exported from Hypermesh, in a text editor and simply editing the file to read:

“Element type = S3R”

Rather than,

“Element type = S3”

Where S refers to the fact it is a shell element, 3 refers to the number of nodes. By adding the R to the element the user is specifying that there will be a reduced number of integration points (or Gauss points) in the elements (Figure 3.6). The integration points within an element are necessary to perform the integration quadrature necessary for finite element analysis. By reducing the number of integration points, the speed of the analysis is improved but the accuracy can, in some circumstances, be diminished (Abaqus, 2007).
Figure 3.6: Position of integration points
3.2.3 Racket Handle Mesh

In order to represent the solid geometry of the handle, solid four noded tetrahedral (C3D4) elements were selected (these elements will be described in more detail in section 3.2.2). As specified in Section 3.2.1, the geometry of the handle was represented as a simple solid extrusion. As a result, a far less detailed mesh was required to capture the geometry of the handle. Within Hypermesh’s “Tetramesh” sub-panel, the solid handle geometry was selected as the target to be meshed and an element size of 1 millimetre was specified. The resulting mesh of 65,699 tetrahedral elements can be seen below in Figure 3.7.

![Figure 3.7: Mesh of the racket frame (65,699 C3D4 elements)](image)

3.3 Modelling of strings

3.3.1 Modelling of string geometry

As with the acquisition of the frame geometry, the geometrical features of the string-bed (the points at which the strings intersect) were obtained using the CMM. The intersection points were obtained by placing the measurement probe at each intersection which, for a string-bed of eighteen main strings and twenty cross strings, resulted in a data set of three hundred and sixty points.
Two slightly differing approaches were taken to modelling the strings. For initial analyses the string bed was modelled simply as a 2D set of intersecting lines with no cross sectional area. For this model the points were imported and used to create a series of line representing the string-bed, the results of which can be seen in Figure 3.8.

The second model was more complex, in that it was a 3D model which captured the weave of the strings, and required two sets of points to be imported. The first set of points was offset to a height of -0.8 millimetres and the second to a height of +0.8 millimetres. The main strings were created using a spline through the points with the same X value and alternating between -0.8 millimetres and +0.8 millimetres whilst the cross strings were created using a similar strategy with the points of an equal Y value.

A series of planes were then created at the left most end of each cross string and the lower most end of each main string. A sketch of a circle was then created on the plane with a diameter of 1.27 millimetres (the diameter of a 15 gauge string when pulled to 223 Newtons tension (Racket Tech Publishing, 2005). A 3D representation of the string was then created by using the “sweep along guide feature”, with the spline acting as the guide string and the circle acting as the section string. The resulting woven string-bed can be seen below in Figure 3.9. As in the case of the frame an .iges file was then exported to Hypermesh.
Figure 3.8: Unwoven String-bed

Figure 3.9: Woven String-bed
3.3.2 String mesh

One of the main reasons for initially modelling the string-bed as a 2D structure is that it allows a CAE user to make use of line elements (essentially one dimensional), which, are far more convenient at an early stage. There are a variety of line elements available in Abaqus, the simplest of which is the two noded truss element (Figure 3.10). Truss elements are able to move in 3-space, with each of the nodes having three degrees of freedom, thus giving the truss element 6 degrees of freedom overall. The degrees of freedom for a truss element are commonly displayed in matrix form, as shown in Equation 3.2, where the subscript represents the node number and the superscript represents the degree of freedom to.

\[
\begin{bmatrix}
    u^{(1)}_1 \\
    u^{(1)}_2 \\
    u^{(1)}_3 \\
    u^{(2)}_1 \\
    u^{(2)}_2 \\
    u^{(2)}_3
\end{bmatrix}
\]

Equation 3.2

Figure 3.10: 2 noded truss element
A truss element, however, is only able to sustain axial forces in its local basis and in order to model the bending of the strings required an element which can be subjected to transverse forces. This requirement leads us to the beam element which introduces rotational degrees of freedom. The displacement matrix for a beam element can be seen below in Equation 3.3, where the $u$ components within the matrix are translational and the $\theta$ components are rotational.

$$\begin{bmatrix}
  u_1^{(1)} \\
  \theta_1^{(1)} \\
  u_2^{(1)} \\
  \theta_2^{(1)} \\
  u_3^{(1)} \\
  \theta_3^{(1)} \\
  u_1^{(2)} \\
  \theta_2^{(2)} \\
  u_1^{(2)} \\
  \theta_3^{(2)}
\end{bmatrix}$$  

Equation 3.3

For the purposes of contact, beam elements can be assigned a cross sectional area, which must be kept constant during the analysis (Abaqus does offer a beam element which can be assigned a constant change in cross-sectional area but the strain for the element is constant in all directions). The fundamental assumption which must be made when using beam elements is, therefore, that the cross sectional area of the element will remain constant during the analysis.

The beam elements are satisfactory for initial approximations of the model but their underlying shortcoming is an inability to plot stress contours due to their lack of a third dimension. For this reason it is therefore necessary to model the structure as a
3D interwoven structure in the more advanced analyses. Therefore an appropriate 3D element must be selected. The default element, for geometrical performance as much as any other reason, within most meshing packages is the four noded tetrahedral (C3D4) element.

![Figure 3.11: Four node tetrahedral element](image)

Despite their ability to easily model a wide range of geometrical shapes, tetrahedral elements, and indeed their 2D counterparts, can be overly stiff compared to elements containing a higher number of nodes. However, given the number of elements which will be required to model the curvature of the strings, artificial stiffness will not be an issue in this case.

Once again, the 3D “Tetramesh” function was used to generate the mesh of the strings. Due to the level of detail required to capture the geometry of the woven string-bed, an element length of 0.2 millimetres was specified. The resulting mesh of 203,682 elements can be seen below in Figure 3.12.
Figure 3.12: The string mesh (203,682 C3D4 elements)
3.4 Modelling contact interactions

The modelling of contact within finite element simulations is an extremely complex area. The two main methods of modelling contact with an Abaqus/Standard simulation are:

- Surface based contact.
- Contact element based contact.

Surface based contact requires the definition of surfaces in either the part of assembly module of Abaqus/CAE, and can be defined simply by the selection of elements, either individually or by selecting elements which lie along the same angle.

Contact elements are available for analysis situations where surface contact is not appropriate. Fortunately, the need for contact elements tends to be restricted to analyses involving pipelines, axisymmetric simulation or heat transfer, none of which are features of this study.

A third method of modelling contact, available exclusively in Abaqus/Explicit, is “all with self” (AWS). AWS contact is the simplest contact simulation to define as it does not require the definition or selection of any specific surfaces. AWS contact is however, computationally expensive as it involves the entire model within the simulation rather than a specified area.

Due to its simplicity, with respect to definition, the AWS contact method was initially used to define the contact between the strings of the interwoven string-bed. For unknown reasons however, the system was unable to correct initial element over-closures, which had arisen from slight distortions of the string geometry during meshing, and resulted in the strings slipping over one another.
For this reason the alternative approach of “Surface-to-Surface” (STS) contact, where individual contact relationships are defined between two surfaces, was adopted. Using the “select elements by angle” option, the six faces which define each string were selected to define its surface. A STS contact interaction is then defined for each of the 18 main strings with each of the 20 cross strings, resulting in a total of 360 interactions.

In order to define an interaction in Abaqus one must first define the properties of that interaction. The four main properties of an interaction which can be defined in Abaqus are:

- Mechanical contact properties
- Thermal contact properties
- Electrical contact properties
- Pore fluid contact properties

Although this study may look at the thermal effects of the contact between the strings at a later time, at present the only one of the four properties which is of interest is the mechanical contact.

The default tangential contact formulation within Abaqus is “frictionless” which prohibits tangential interactions and allows infinite slipping between surfaces. Conversely, the “rough” formulation does not allow any slip once the points defined in the contact interaction are in contact. The contact formulation which is used in this model is the “penalty” formulation. This formulation can be defined in terms of:

- Friction coefficient
- Slip rate
- Contact pressure
• Temperature

Although of the above options only the friction coefficient is compulsory. Although some research has been carried out with respect to the friction coefficient between the ball and the string-bed (Cross 2000a), extensive research performed during the initial phase of this thesis did not uncover any work looking into the friction between the strings. Accurately defining the friction coefficient of different strings could be particularly crucial to modelling their performance with respect to spin generation and therefore, research will be carried out to ascertain whether coefficients can be accurately obtained. Until this research has been completed a default value of 0.1 shall be used for the friction between the strings.

3.5 Materials properties

One of the main parameters easily altered in a tennis racket is the strings’ material properties. Tennis strings are available in a variety of materials and cross-sectional areas (gauge), both of which are easily altered in a finite element model. The material properties of strings vary widely. Strings made of materials such as cow intestine (also known as “natural gut”) and nylon have relatively good elastic properties which allow them to deform more when the ball impacts, thus providing the player with more power (since the ball deforms less).

The initial stretch of a nylon string is greater when compared to a gut string but this value is irrelevant since it is the performance of the string when strung to tension which is crucial. In this respect gut performs well, as, like nylon, it possesses a relatively low modulus and, moreover, it retains its tension for longer than other strings. The third most commonly used material in tennis strings is polyester. Polyester possesses a similar strength to nylon strings but is much stiffer and also more brittle. Polyester is, however, less expensive and therefore a better option for player’s who play less or simply do not hit the ball quite so hard (Brody, 2002).
In order to make full use of the finite element model a series of string materials properties were obtained in Chapter 5 to compare their performance. However, for the initial stages of the modelling a basic Young’s modulus test was used to obtain a smaller sample of strings materials.

In order to ascertain the properties of the strings a series of tensile tests (the experimental method is described in detail in chapter 6) were carried out at the following speeds:

- 5 millimetres per minute
- 25 millimetres per minute
- 50 millimetres per minute
- 100 millimetres per minute

And for the following string types:

- Gut (Wilson)
- Kevlar/Polyester (Prince Problend)
- Nylon (Prince Tournament)

The results can be viewed below in Figure 3.13.
Figure 3.13: Young’s modulus of tennis strings

The value used for the strings in the model was the average of the gut string, \(2.483 \times 10^3\) Megapascals.

For simplicity, within the early stages of modelling the racket frame was modelled as aluminium. Whilst carbon fibre rackets are designed to have varying material properties throughout the frame aluminium rackets are considerably more homogenous. As a result the racket was assigned a single material section with the following values taken from the Abaqus example problems within the documentation (Abaqus, 2007):

\[
E = 70 \times 10^3 \text{ MPa} \\
\rho = 2700 \text{ kg/m}^3 \\
\nu = 0.3
\]

Where \(E\) is the Young’s modulus, \(\rho\) is the density and \(\nu\) is the Poisson’s ratio.
3.6 Modelling string tension

A number of strategies were explored when modelling the effect of tension within the stringbed. In the case of the 2D stringbed, each string was attached to the frame using a “tie” constraint. Tie constraints require the selection of master and slave nodes, which in this case, were the end node of the string and its corresponding node on the frame, respectively. Once the master and slave nodes have been defined the slave node is then constrained to follow all displacements of the master node during simulation. As a result of utilising the tie constraint method, simply imposing a force on the end of the strings would not be possible, since any displacement of the string due to force would be experienced by the frame.

3.6.1 Predefined field method

As a result the stress state of the tension within the strings had to be modelled without altering the geometry of the strings. This was achieved by creating a predefined stress field assigned to the string-bed, which was changed to a stress field using the keywords editor (which is accessed by right-clicking on the current model in the model tree) of the model. It was necessary to first create the predefined field in Abaqus CAE as a temperature field and later change it to a stress field using the model keywords editor as the creation of a stress related predefined field is not supported in Abaqus CAE. The predefined stress field was assigned a value of 112 Megapascals and “instantaneous” amplitude, meaning the stress would immediately take effect.

The result of introducing tension can be seen below in Figure 3.14. The contours in Figure 3.14 (a) and (b) represent the deflection of the string-bed due to the impacting of the ball and Figure 3.14 (a) shows the more evenly distributed deflection one would expect from a taut set of strings, whilst the deflection in Figure 3.14 (b) is far more localised.
Although this technique gives a good representation of the effect the tensioning has on the strings themselves it does not simulate the strain placed on the racket, due to the fact there is no geometrical contraction from the strings. As a result an alternative approach was adopted for the 3D string-bed, where the material property of the string was assigned a thermal expansion coefficient and a temperature related predefined field was defined.
The coefficient of thermal expansion can be defined as the strain divided by the temperature drop. Since the diameter, d, is known to be 0.0016 metres, one can calculate the string area from Equation 3.4:

\[ A = \frac{\pi d^2}{4} = \frac{\pi \times 0.0016^2}{4} = 2 \times 10^{-6} \text{m}^2 \quad \text{Equation 3.4} \]

Where, if one assumes the strings are tensioned to a force, F, of 250 Newtons, the stress, \( \sigma \), can then be calculated from Equation 3.5:

\[ \sigma = \frac{F}{A} = \frac{250}{2 \times 10^{-6}} = 1.25 \times 10^2 \text{MPa} \quad \text{Equation 3.5} \]

Since the Young’s modulus, E, of the gut strings is known to be \( 2.483 \times 10^3 \) Megapascals the strain, \( \varepsilon \), can be calculated from Equation 3.6:

\[ \varepsilon = \frac{\sigma}{E} = \frac{1.25 \times 10^2}{2.483 \times 10^3} = 0.05 \quad \text{Equation 3.6} \]

Thus, if a temperature drop of 10 Kelvin is introduced during the simulation, the desired strain can be achieved from the following coefficient of thermal expansion (derived using Equation 3.7):

\[ \alpha = \frac{\varepsilon}{\Delta T} = \frac{-0.05}{-10} = 0.005 \text{K}^{-1} \quad \text{Equation 3.7} \]

A temperature predefined field is then created with a magnitude of -10 and assigned to the strings via an assembly set, containing the strings.
Although this method did have the desired effect of contracting the strings, it resulted in a very uneven string pattern. As the strings were pulled to tension uneven gaps began to open up between the strings. The uneven nature of the strings meant that there was continuing movement of the strings as they tried to obtain a state of equilibrium. It was hoped that given a “settling” period that the strings could obtain this state of equilibrium but this proved not to be the case, as the strings position simply continued to oscillate. Therefore, it was concluded that this would not be a suitable method of modelling the string tension accurately.

3.6.2 Connector element method

Having discarded the previous method it was concluded that a method of modelling the string tension was required whereby the strings could be fixed to the racket whilst they were being loaded, without the load having the effect of expanding the racket. With this in mind “connector” elements were considered.

Connector elements can be used in applications where two points are connected in some way but the relationship is not necessarily linear (Abaqus, 2010). Unlike other elements, connectors can be assigned behaviour dependencies such as locking or failure mechanisms whereby certain degrees of freedom will be locked or freed when a certain criteria is reached.

Rather than define the elements from geometry within a meshing packager as with previous cases, connector elements are created by assigning a “connector section” to a “wire” feature, which is defined between two points. The points which were used to define the wire features representing the connector element geometry were the centre nodes at each end of the strings and a reference point which was defined at the centre of the hole through which the string will pass. The connector elements were fixed to the racket, constraining the reference point, which defined its end point, to the racket frame using a “coupling” constraint. Coupling constraints are similar to
“tie” constraints but they allow a single control point; in this case the reference point which acts as the end point of the wire feature, to be coupled to a series of points; in this case the points surrounding the hole.

Defining the wire features relative to a manually defined reference point allows the length of the connector element to be readily adjusted. Once the wire feature has been defined it must then be assigned a connector section, which defines the connector behaviour. The connector behaviour can be defined as the available degrees of freedom of the connector. For example, an axial connector section has only one available degree of freedom and is only able to simulate motion along the 1st axis. Conversely, beam connector sections have no available degrees of freedom and are used to model rigid connections between two points.

In order to model the uni-directional motion which will represent the tensioning of the strings, “connector elements” were used. Connector elements are one dimensional line elements that can be used to define a connection between two nodes. They can be used to transfer displacements, rotations and forces between nodes but cannot be used to model contact.

There are a number of connector element types within the Abaqus element library, but in this case the most basic type was used, the axial connector. Axial connectors can only be used to model displacement along the 1 direction of the element’s local co-ordinate system.

A beneficial feature of connectors is that they can be rigidified or “locked” when certain limits such as maximum displacement or force are reached. This characteristic makes them ideal for modelling string tension as it allows the strings to be pulled to a certain tension before being “locked” in contact with the racket frame.
One end of each string was attached to the racket, using a tie constraint. An axial connector element was then created between the other end of the string and the frame.

The connector elements were loaded using a feature known as “connector loads”. Connector loads behave in the same way as conventional loads and can possess a magnitude in any direction in 3-space. However, for an axial connector section the only direction in which a connector load can have any meaningful contribution is the 1 direction, since all other directions are fixed and any loading would be meaningless.

In order to create symmetric loading around the racket, the end of the string which was loaded and the end which was assigned a connector was alternated. This resulted in the loading pattern seen in Figure 3.15.
A disadvantage of axial connector elements is that displacements and forces can only be modelled in the 1 direction. In the case of the main strings this is not an issue since their loading direction is, in fact, the 1 direction. However, in the case of the cross strings, where the strings are loaded along the 2 axis, it is necessary to define a local axis for the connector section. The local axis is defined with the 1 axis along
the line of the connector element by defining the reference point as the origin and the end of the string as a point which lies on the 1 axis.

In order to fix the strings in place when they have been loaded to the appropriate tension, the connector lock application is used. Connector locking criteria are defined as part of the connector section and can be associated to either the displacement or force experienced by the connector element. For example, by setting a position locking criteria of plus or minus 0.1 millimetres, the connector will lock in the specified degrees of freedom once this displacement is reached. Similarly, in this case, a force criterion was defined whereby the connector element was locked once a force of 180 Newtons was reached.

In order for the connector to realise this force it was assigned a connector load with a magnitude of 200 Newtons in the 1 direction and “smooth step” amplitude. The smooth step amplitude was used, since the default “instantaneous” amplitude would immediately impart a force of 200 Newtons on the connector and activate the locking criteria immediately.

For the smooth step amplitude, the magnitude of the force \( F \) between two points in time, \( t_i \) and \( t_{i+1} \), is calculated from Equation 3.8:

\[
F = F_i + (F_{i+1} - F_i) \delta^3 (10 - 15\delta + 6\delta^2)
\]

Equation 3.8

Where the component \( \delta \) is given by Equation 3.9:

\[
\delta = \frac{(t - t_i)}{(t_{i+1} - t_i)}
\]

Equation 3.9

The connector lock status can be verified by creating a history output for the geometry set which defines the wire. The connector lock status history output is
available in each of the three translational and three rotational degrees of freedom. For the axial connectors the only degree of freedom that is of interest is the 1st translational degree of freedom. Figure 3.16 below shows the effect of the smooth step amplitude, with the connector reaching its upper limit of 180 Newtons at 0.78 seconds. The transition from “unlocked” to “locked” then occurs during the next time increment with the connector reaching lock status at a time of 0.8 seconds (where 1 represents a lock status of “ON”).

Figure 3.16: Connector Lock Status
3.7 **Concluding comments**

A satisfactory representation of the strung racket has been created using finite element analysis. The model uses a novel method of applying a force between two structures to model the string tension. In order to validate the model it must be subjected to several stages of validation. The ultimate purpose of this model is to study high speed ball-racket impacts but to apply the model to such a study without first validating its correlation with experimental data under static load would be ill advised. For this reason the racket will be subjected to a series of static tests which will be recreated in Abaqus to validate the finite element model.
4  Static validation of the racket frame model

Although finite element models often appear realistic, it is far from advisable to simply assume that the results which they produce are completely accurate. Whilst FEA is a powerful tool it is often misused in this context, with many users simply creating impressive animations for marketing purposes which, if taken as an accurate representation of the system can be dangerous.

Given that this study is concerned with the modelling of dynamic impacts, the first method of validation should be dynamically impacting a ball against the string bed. This, however, would be a rather large leap to make without any intermediate steps and, as a result, a series of static validations of the racket model were carried out to ensure that the racket model is accurate under stationary loading conditions.

4.1  Three point bend testing

The most common method of quality assurance used by racket manufacturers is the three point bend test (Woodward, 2010). The position of the load and the support is often varied to test the bending properties of the racket at various points along its length. The most common bend test uses supports positioned just inside the string-bed at either end, with the load applied half way between the two, as shown in Figure 4.1.
The same individual aluminium racket frame was tested unstrung and strung to 223 Newtons tension with nylon strings. During the test, a load of 150 Newtons was applied at a rate of 1,000 millimetres per minute. The strung and unstrung racket was tested five times each and averaged to give a stiffness of 116.7 (plus or minus 1.7) kilonewtons per metre in the case of the unstrung racket and 129 (plus or minus 2.0) kilonewtons per metre in the case of the strung racket, where the experimental error was assumed to be one standard deviation (unless otherwise stated all other experimental errors in this thesis are calculated as one standard deviation).

The model of the racket was subjected to similar experimental conditions. Two rigid cylinders were introduced to represent the supports and a third cylinder used to apply the force. Two concentrated point loads were assigned to the nodes at either end of the applicator cylinder and given smooth step amplitudes in order that the load would...
be applied gradually, thus avoiding excessive distortion of the elements. The results of the FE bend test were 112 kilonewtons per metre and 124 kilonewtons per metre for the unstrung and strung rackets respectively. Both results were within 5% of the experimental values. An error margin of this magnitude can be attributed to a number of factors, such as the variation in the material properties due to the room temperature at the time of the experiment.

4.2 Validation of the racket frame and string bed model under tension

An effective way to validate the static deformation of the model is by observing the strung and unstrung states of the racket. When strung, the racket undergoes a deformation considerable enough that it is visible to the naked eye. Observing the deformation of the racket frame under string tension allows one to validate two characteristics of the model:

1. The magnitude of the frame’s deformation under load from the strings
2. The reaction force being imparted to the strings by the frame and the effect this will have on the overall string-bed tension

Validation of the second point is particularly important since it is the application of a novel, previously untested method. Validation of FE models of structures under load has traditionally been performed with the use of strain gauges placed on the structure’s surface (Carreira et al., 1985, Vollmer et al., 2000, Stolk et al., 2002). A less intrusive approach is now being adopted by many authors (Mosse et al., 2006, Ivanov et al., 2009) known as image correlation photogrammetry.
4.2.1 GOM Aramis photogrammetry system

The validation of the frame deformation due to string tension was performed using an “Advanced 3D Image Correlation Photogrammetry System” (ICPS) named “Aramis”, (GOM, 2010) which, like all ICPSs, tracks the changes in a pattern (in this case a random micro-pattern) which is applied to the surface of the structure being placed under strain.

Aramis obtains the change in the surface pattern via camera images captured during loading. Aramis tracks the deformation of the random micro-pattern on the target’s surface and processes the information to produce a real time strain contour of the target during loading.

The system can be used with a variety of cameras but the higher the resolution of the camera the more accurate the result will generally be. The cameras used in this case had a resolution of 1,280 pixels by 1,024 pixels. For this measurement set-up the Aramis system is capable of measuring a strain range of 0.01% to 100% to an accuracy of plus or minus 0.01% (GOM, 2012). The process flow used in conjunction with the Aramis system is shown in Figure 4.2 and described in the next section. A list of the performance specifications for the system configuration used in this analysis can be viewed in Appendix 2.
Figure 4.2: Process flow for measuring strain of the racket frame due to string tension
4.2.2 Stage 1

Before the random pattern was applied the racket was coated with a white Halfords primer paint over the surface to increase the definition of the pattern. A black spray paint (CRC Industries, Graphit 33) was then applied randomly to the entire surface of the racket. The quality of the pattern is particularly important as:

- If the pattern is too sparse, the system will not be able to identify enough points to track the motion of the surface between images.
- If the pattern is too dense, the system will not be able to distinguish between the individual speckles.
- If the individual speckles are too large/small then the system will not be able to identify them.

Examples of speckle patterns which performed poorly and well during the analysis procedure can be seen in Figure 4.3 and Figure 4.4 respectively.
Figure 4.3: Example of a speckle pattern which performed poorly

Figure 4.4: Example of a speckle pattern which performed well
Figure 4.3 and Figure 4.4 also show a difference in lighting condition. Figure 4.4 shows the more desirable condition, with good contrast between the racket and its background. Poor lighting, such as that in Figure 4.3, can lead to the system becoming confused and recognising features in the background as speckle pattern.

The improved lighting condition in Figure 4.4 was achieved by decreasing the frame rate from 250 frames per second to 50 frames per second and introducing additional lighting using an Arri 650 Watt lighting unit. Another method of increasing the light would have been to increase the aperture but, given the need for a certain level of focus, a relatively narrow aperture was required. Had a still frame camera been used the shutter speed could have been altered but because video cameras were more readily available, they were used; with the first frame of each sequence used.

4.2.3 Stage 2

Two high speed, high definition Photron SA5 cameras were positioned at an angle of 70 degrees with respect to the plane of the string-bed. The orientation of the system can be seen below in Figure 4.5. The system was calibrated using a calibration board with a unique sequence of symbols, which were photographed at a series of orientations. The calibration images were taken with the board a horizontal distance of 440 millimetres from the beam which supported the two cameras. Once the system has been initially calibrated at a distance of 440 millimetres from the target image, the user is then free to adjust the target distance in order to provide a significant field of view for their measurements. In the case of this study, a distance of 1,000 millimetres was chosen.
4.2.4 Stage 3

With the system set up and calibrated, the pre-painted racket was strung to a tension of 223 Newtons and left to “settle” for a period of 24 hours.

Figure 4.5: Orientation of the cameras with respect to the string-bed.
4.2.5 **Stage 4**

After this period the racket was placed into a specially constructed steel clamp - fabricated by the Loughborough University Sports Technology Institute technicians - to ensure that it was in exactly the same position when both the strung and unstrung images were captured.

An image of the clamp holding a racket can be seen below in Figure 4.6. The racket was clamped simply by placing the handle flat against the steel plate and tightening a fixing plate over the free side of the handle. The fixing plate contained four holes through which bolts were passed into the main body of the clamp. The plate was then secured by tightening the nuts shown visible in Figure 4.6.

Figure 4.6: Racket clamp
The first image captured was that of the racket in the strung state. Although the cameras used were video cameras rather than still cameras, the Photron Fastcam Viewer software which was used allows the user to specify the number of frames he or she wishes to capture. Therefore, the first and last frame specified for automatic download was frame 1. The auto download function was used, meaning the image was instantly downloaded to a specified file the moment it was captured. Following the successful download of the image of the strung racket, the strings were removed and the procedure was repeated for the unstrung racket.

4.2.6 Stage 5

Once captured, the images were imported into the Aramis analysis software as part of a “project”. Upon creating a new project one is prompted to specify the “facet” size and facet step which will be used when computing the images. A facet is a small area of the surface which the software will process for each image before stepping on to another area on the surface. Therefore, for optimum accuracy it is desirable to have a large facet size, which covers a large area of the surface, and a small facet step, allowing the system to move slowly over the surface, thus being less likely to lose its position with regards to the other images.

Naturally, the larger the facet size and smaller the facet step, the longer the computation time and the more memory is required. However, given that this analysis only requires 2 images, the luxury of a higher accuracy can be afforded. An example of different facet size and step values is shown below in Figure 4.7 and Figure 4.8. For clarity, only one in every five facets is displayed and the size of the facets is not to scale. It is still clear, however, that there is a much larger overlap for the more accurate procedure represented in Figure 4.8.
Having settled on facet size and step values 25 and 5 respectively the next stage in the process was to define a stage using the images that were collected. Due to the order in which the images were taken, with the strung racket (stressed state) captured first it was necessary to define two sets of two stages and then delete the first and last stages. This left the image of the unstrung racket as the first stage and
the image of the strung racket as the second stage, which allows the system to calculate the strain due to the racket being strung and display it as a contour field on the image of the strung racket.

4.2.8 Stage 7

Having defined the series of images which will form the basis of the analysis, the area of the images to be analysed was specified. This was achieved using the software's “mask” function. Using this software the entire image was masked and only the area of the image containing the frame and around 5 millimetres either side was unmasked. The resulting image can be seen below in Figure 4.9.
4.2.9 Stage 8

Once the two stages had been created from the image series, the computation of the project was initiated. After the software has completed the computation it automatically opens a post-processor window displaying the strain contours. The
resulting strain contours for the unstrung and fully strung racket can be seen below in Figure 4.10 (a) and Figure 4.10 (b).

The same racket was restrung and tested five times, both for the fully strung and mains only condition. There was no measured difference between either the magnitude of the strain or the position of the peak values.

4.2.10 Results/Discussion

4.2.10.1 Fully strung racket

Figure 4.10: Strain contour of the (a) unstrung and (b) fully strung racket

Figure 4.10 shows the strain contour of the fully strung racket, where Figure 4.10 (a) is the reference unstrung state and Figure 4.10 (b) is the loaded strung state. The
highest strain concentration can be seen along the inner edge of the frame and also along the outer edge of the corner points of the frame.

The corresponding model of the strung racket is shown in Figure 4.11. The image is similar in terms of the strain levels and distribution to that of the experimental data. As in the case of the experimental data, the highest strain is along the inside edge of the racket and the maximum value was 3%. The finite element model did not, however, display the same level of strain as the Aramis results along the outer edge of top end the racket. This could be due to a number of factors but given the level of accuracy achieved throughout the remainder of the racket the model can be considered satisfactory.
4.2.10.2 **Racket strung with mains strings only**

To further validate the deformation performance of the racket frame model, an image of the racket, strung solely with vertical main strings was also captured, and the resulting strain contour can be seen below in Figure 4.12.
To validate the deformation of the racket with only the main strings, a model was created in which the axial connector elements used to model the loading of the-crosses were defined as rigid beam elements, hence removing the tension loading from the cross strings. The contact interactions between the mains and crosses were also removed to ensure that the mains strings could elongate freely – as if the cross strings were not present. The resulting strain contour can be seen below in Figure 4.13.
Figure 4.13: Finite element model of the racket strung with mains only
It can be seen from Figure 4.10 (b), that not all of the frame surface area has been captured during the analysis. One of the main reasons for this is the curved, and relatively small, surface area of the racket and the angle at which the cameras view the frame from. The set-up is depicted in Figure 4.14 and shows the field of view of the two cameras. The curvature of the racket means that only a small area of the racket’s cross-sections is visible to both cameras.

Figure 4.14: Aerial schematic of the cameras’ field of view
The colour contour around the left side of the racket in Figure 4.12 shows that, as with the fully strung racket, a strain of around 3% was experienced. Along the top and bottom of the racket, however, there is an increased level of strain compared to the fully strung racket. Whereas, Figure 4.10 (b) displayed a maximum strain of 3%, an increased level of 7% was observed in Figure 4.12, along the middle of the frame on both the upper and lower edges.

The level of the maximum strain in the mains only case (Figure 4.12), at 7%, is not reflected in the finite element model (Figure 4.13). The poor performance of the mains only model, compared to the fully strung model, could be due to the higher deformation the mains only model experiences. Whilst, in the case of the fully strung racket some of the deformation in the direction of the mains’ load is neutralised by the cross strings’ load, in the case of the mains only strung racket, the racket experiences a much higher deformation, in the afore-mentioned direction. In the secondary mains only analysis the model clearly struggles to capture this more severe loading procedure. The full results of the strain levels at the inner, outer and mid sections of the racket at its top, bottom and side (described in Figure 4.15) can be seen in Table 4.1.
<table>
<thead>
<tr>
<th>Section</th>
<th>Fully Strung Experimental</th>
<th>Fully Strung Model</th>
<th>Mains Only Experimental</th>
<th>Mains Only Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top-Mid (1)</td>
<td>2%</td>
<td>2%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>Top-Outer (2)</td>
<td>3%</td>
<td>3%</td>
<td>7%</td>
<td>5%</td>
</tr>
<tr>
<td>Top-Inner (3)</td>
<td>3%</td>
<td>3%</td>
<td>-</td>
<td>3%</td>
</tr>
<tr>
<td>Bottom-Mid (4)</td>
<td>2%</td>
<td>2%</td>
<td>3%</td>
<td>5%</td>
</tr>
<tr>
<td>Bottom-Outer (5)</td>
<td>-</td>
<td>-</td>
<td>7%</td>
<td>5%</td>
</tr>
<tr>
<td>Bottom-Inner (6)</td>
<td>3%</td>
<td>3%</td>
<td>7%</td>
<td>5%</td>
</tr>
<tr>
<td>Side-Mid (7)</td>
<td>2%</td>
<td>2%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>Side-Outer (8)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2%</td>
</tr>
<tr>
<td>Side-Inner (9)</td>
<td>3%</td>
<td>3%</td>
<td>-</td>
<td>2%</td>
</tr>
</tbody>
</table>

**Table 4.1: Strain Values for the strung racket in FE model and experimental analysis**
The positions of high strain are perhaps not where one would intuitively expect them to be found. As the racket is strung, assuming equal deformation in the lateral and longitudinal directions, the racket would experience the highest levels of strain around the “corners”. However, due to the non-uniform shape of the racket the deformation is not equal in both directions, which leads to areas of high strains which are more difficult to capture. The high levels of strain experienced along the inside edge of the frame can be explained by two factors. The first is that there are a greater number of cross strings therefore, they will exert a greater force and, as a result, deformation on the racket during stringing. The second is that the inner edge of the frame is smaller than the outer edge and as a result, a greater strain will occur for the same deformation.
Another possible explanation for discrepancies between the experimental and computational results could be the fact that the experimental method is only capable of capturing the racket deformation in two dimensions. Therefore, any changes in the depth of the racket, as shown in Figure 4.16 will be neglected.

Figure 4.16: Deformation of the tubular racket frame’s cross section
4.3 Validation of the string-bed stiffness profile

4.3.1 Experimental measurement

4.3.1.1 Instron 3365 force measurement machine

The stiffness profile of the string-bed was acquired using an Instron 3365 force measurement machine which is part of the Loughborough University Sports Technology Institute equipment pool. The machine was fitted with a circular stainless steel applicator which was ten millimetres in diameter.

According to the manufacturer, the machine is accurate to within plus or minus 0.25% of the indicated force and plus or minus 0.5% of the indicated displacement (Instron, 2012). The machine had also been calibrated within the two year period which the manufacturer deems necessary (Farrand, 2012)

4.3.1.2 Methods

Using the “Instron” force measurement machine, the stiffness profile of the string-bed was acquired. An adjustable jig, similar to that which may be used in a 3 point bend test, was constructed to support the racket during testing. A diagram of the fixture is shown in Figure 4.17.
The jig was bolted on to the base of the Instron machine with the racket positioned below the compression tool, depending on the area of the string-bed which was being tested. The steel beam used in section 4.1 was removed and replaced with a small circular applicator, which was eight millimetres in diameter. The applicator size was chosen as it was small enough to focus on specific string intersections without creating stress concentrations which may lead to string failures.

Using Instron’s own software, “Bluehill”, a compression test procedure was defined. During the procedure, a 150 Newtons load, $F$, was applied at a rate of 5 Kilonewtons per minute. This procedure was followed for the intersection of each main and cross string, resulting in a total of 358 tests. From the test, the Bluehill software produced a plot of the compression force against displacement.

The maximum value of compressed displacement, $d_{0-150}$, was taken from the plot and, along with Equation 4.1, used to calculate the stiffness, $K_{0-150}$, of that particular string intersection.

$$K_{0-150} = \frac{F}{d_{0-150}}$$

Equation 4.1
Due to the number of tests to be carried out, there was not sufficient time to perform multiple tests at each string intersection. As a result the statistical accuracy of the test was determined by performing five tests for every tenth string intersection. This approach resulted in the multiple test locations being distributed as shown in Figure 4.18. The results of the multiple tests can be seen in Table 4.2, with the Standard Deviation less than plus or minus 3% in all cases.
Figure 4.18: Illustration of which intersection were tested multiple times
As can be seen in Table 4.2, the sting bed stiffness profile was obtained for string tensions of 178 Newtons, 223 Newtons and 267 Newtons. This was done in order to observe the difference in stiffness profile for varying string tensions.

**Table 4.2: Standard deviation of intersections subjected to multiple tests**

<table>
<thead>
<tr>
<th>Intersection</th>
<th>178 N String Tension</th>
<th>223 N String Tension</th>
<th>267 N String Tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.46</td>
<td>0</td>
<td>2.05</td>
</tr>
<tr>
<td>20</td>
<td>2.66</td>
<td>2.46</td>
<td>2.25</td>
</tr>
<tr>
<td>30</td>
<td>2.66</td>
<td>2.46</td>
<td>2.25</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>1.48</td>
<td>2.66</td>
</tr>
<tr>
<td>50</td>
<td>1.54</td>
<td>0</td>
<td>1.02</td>
</tr>
<tr>
<td>60</td>
<td>2.00</td>
<td>1.48</td>
<td>1.38</td>
</tr>
<tr>
<td>70</td>
<td>1.55</td>
<td>0</td>
<td>1.18</td>
</tr>
<tr>
<td>80</td>
<td>1.71</td>
<td>0</td>
<td>1.38</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>0</td>
<td>1.80</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>110</td>
<td>0</td>
<td>1.39</td>
<td>2.75</td>
</tr>
<tr>
<td>120</td>
<td>1.74</td>
<td>1.39</td>
<td>2.75</td>
</tr>
<tr>
<td>130</td>
<td>1.71</td>
<td>1.48</td>
<td>2.82</td>
</tr>
<tr>
<td>140</td>
<td>1.80</td>
<td>1.69</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>1.71</td>
<td>1.48</td>
<td>1.15</td>
</tr>
<tr>
<td>160</td>
<td>2.36</td>
<td>1.02</td>
<td>0.90</td>
</tr>
<tr>
<td>170</td>
<td>2.72</td>
<td>0</td>
<td>1.55</td>
</tr>
<tr>
<td>180</td>
<td>2.25</td>
<td>1.79</td>
<td>1.50</td>
</tr>
<tr>
<td>190</td>
<td>2.24</td>
<td>2.66</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>2.23</td>
<td>2.66</td>
<td>0</td>
</tr>
<tr>
<td>210</td>
<td>0</td>
<td>1.36</td>
<td>2.94</td>
</tr>
<tr>
<td>220</td>
<td>2.16</td>
<td>1.64</td>
<td>0</td>
</tr>
<tr>
<td>230</td>
<td>2.25</td>
<td>0</td>
<td>1.24</td>
</tr>
<tr>
<td>240</td>
<td>1.85</td>
<td>2.09</td>
<td>1.85</td>
</tr>
<tr>
<td>250</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>260</td>
<td>2.16</td>
<td>1.64</td>
<td>1.63</td>
</tr>
<tr>
<td>270</td>
<td>2.16</td>
<td>1.59</td>
<td>2.59</td>
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<tr>
<td>280</td>
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<td>2.94</td>
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<td>1.75</td>
<td>2.16</td>
<td>2.44</td>
</tr>
<tr>
<td>300</td>
<td>1.74</td>
<td>1.18</td>
<td>1.93</td>
</tr>
<tr>
<td>310</td>
<td>1.71</td>
<td>2.24</td>
<td>2.77</td>
</tr>
<tr>
<td>320</td>
<td>0</td>
<td>1.23</td>
<td>2.88</td>
</tr>
</tbody>
</table>
For each string tension the strings were left to elongate for 24 hours before being tested. Once the data had been obtained it was processed in Microsoft Excel and an “Area” plot of the stiffness for each string-bed was generated.

A number of formatting strategies were explored in order to achieve the best visualisation of the stiffness contour. The chosen method used increments of 20 kilonewtons per metre along the “depth” axis, as this gave the clearest image of how the different string tensions varied. The resulting plots for the three string-beds can be seen below in Figures 4.19, 4.20 and 4.21 respectively.

Figure 4.19: Stiffness profile of a Racket strung to 178 Newtons tension
Figure 4.20: Stiffness profile of a racket strung to 223 Newtons tension

Figure 4.21: Stiffness profile of a racket strung to 267 Newtons tension
4.3.2 Finite element model

To validate the stiffness profile of the racket generated in Chapter 3, a simulation was created similar to the experimental analysis. In order to carry out the analysis on the model of the tensioned string-bed (20 crosses and 18 mains) without having to rerun the analysis, the restart analysis function was used.

The restart function effectively begins the analysis from the end-point of an existing output database (.odb) file of a previous job, in this case that of the tensioning analysis. To perform a restart analysis a new step, containing the additional conditions, must be added to the model. New loads, boundary conditions, interactions etc. can all be added to the model in this new step but the previous steps’ attributes must remain constant.

To replicate the experimental conditions, a 150 Newtons load was created which was defined as a concentrated force of 10 Newtons applied to the 15 nodes which surround the intersection of the strings (see Figure 4.22).

![Figure 4.22: Load at string intersection](image-url)
When this function was initially used, an unexpected error message, which referenced “ErrNodeDomainDecompReStrt” was encountered. The error message can be viewed in Appendix 3.

Reviewing the set “ErrNodeDomainDecompRestrt” showed that the nodes in question were those being subjected to the connector lock condition. The domain decomposition, which the error message refers too, refers to the method used to break down the model when multiple processors are used to run the simulation. After consulting the Abaqus support service it was discovered that the error was not a problem with the model, rather it was a bug in the package. The problem occurs due to the software’s inability to distinguish the connector lock from boundary conditions. During the pre-processing stage of the restart analysis the software identifies the connector locks as boundary conditions and assumes that the user has violated the procedure by changing the boundary conditions of the previous analysis.

As a result, all jobs intended to be used as part of a restart analysis, and indeed the restart analyses themselves, had to be performed using a single processor, greatly increasing the computation time for all jobs. For the restart analyses themselves, however, there were numerous jobs to be submitted, meaning that several jobs could be run on a single processor simultaneously. Thus, there was little disruption imposed on the computational efficiency of the restart analyses through having to run each job on a single processor.

As for the experimental analysis, each intersection was subjected to a force of 150 Newtons (10 Newtons at 15 nodes in the case of the FE model). To obtain the deflection at the node a history output request was created for the displacement in the direction of the force (U3) at the centre node of the intersection. The stiffness profile of the string-bed strung to 178 Newtons can be seen below in Figure 4.23.
Figure 4.23: Stiffness profile of the finite element model string bed

4.4 Discussion

The first observation to be drawn from each of the stiffness profiles is that they are all unique both in shape and magnitude. The profile of the FE model should ideally be similar to that of the experimentally obtained profile of the racket strung to 178 Newtons. It is clear, however, that it is of a reduced stiffness and also a different profile. In the case of the FE model, the string stiffness decreases at the opposite end of the racket from the handle which is not the case for the experimental results. This could be a result of the short step time in which the loading of the string simulating tensioning is performed (around 10 milliseconds for the FE model compared with 6 milliseconds for the experimental analysis).
It is, however, interesting to note the continuity with which the experimental profiles change as string tension is increased. As the tension increases, so too, as one would expect, does the stiffness. The interesting feature is, however, the way in which the stiffness is distributed around the string bed. For increasing string tensions the profile becomes much steeper approaching the racket frame (this is illustrated by the increasing variety of colours around the edge of the profile and the larger area they occupy). It could be assumed, therefore, that if the magnitude of stiffness within the FE string-bed is simply not high enough then an increase in string tension would have the effect of increasing the stiffness whilst also creating a steeper gradient profile around the edge of the racket.

4.5 Concluding comments

The values provided in Table 4.1 show that the magnitude and location of the significant strain experienced by the racket due to stringing are of a good likeness. For the fully strung racket the strain magnitudes all correlate to the nearest percentage point. For the more extreme deformation associated with the mains only loading the model did not predict the magnitude as accurately (5% was the maximum for the model compared to a maximum experimental value of 7%) but it did still predict the location at which the highest strains occur.

A more complete image of the racket’s strain contour would have been desirable but due to the nature of the target surface this was difficult to obtain. Had more time been available it may have been possible to gain a more complete contour by performing several analyses on the racket which focused in on smaller areas of the frame, rather than trying to capture the entire racket. These individual contours could then be “stitched” together to form a more comprehensive contour than was possible with a single experiment.
Although the static deformation profiles of the string-bed and racket were not entirely similar to the experimental, within the central area of the bed the values showed a level of magnitude similar to the experimental values. Both the experimental and finite element model results gave a stiffness in the range of 20 plus or minus 5 kilonewtons per metre within the most central area of the racket. Since this study is predominantly focusing on the most commonly used hitting area, the fact that the model did not capture the higher stiffness around the edge of the string-bed is not of great concern. However, if future studies were to look at the performance of rackets with respect to off-centre impacts, a more sophisticated string model would be required.

This testing has demonstrated that the behaviour of the FE model of the racket frame and string-bed are similar to that of a racket under experimental loading. The experimental testing shows that the FE model predicts the areas of maximum deflection during loading, which allows confidence to then apply the model to more dynamic conditions.
5 Modal analysis

5.1 Introduction

The vibration characteristics of a tennis racket are of particular importance since they play a significant role in the sensations experienced by the player during competition. When a tennis player strikes the ball they will generally aim to do so at the “sweet-spot” of the racket. There are several definitions of a racket’s “sweet-spot”, the most scientific definition of which is the area which corresponds to the node of the racket’s first natural frequency (Brody, 1981). This node is generally located at the centroid of the string-bed, (Mohanty et al., 2002), which is also the area that offers consistent outbound flight trajectories. As a result this area will offer the most accurate shot (since the rebound angles outside of this area vary too widely to be reasonably controlled by the player) and hence the level of vibration produced in this instance has positive connotations for the player. Simply stated, rackets which vibrate less give the player the impression they are playing better.

As well as the connotations of a more effective shot, the lack of vibrations emanating from the fundamental node also have a more evident advantage, that of comfort. Not only can these vibrations be uncomfortable, they are also perceived to cause serious injury in the form of lateral epicondylitis, more commonly known as “tennis elbow” (Roetert et al., 1995, Pluim, 2000, Jobe et al., 1994).

In addition to providing a guide to the performance of the tennis racket, modal analysis can also be used as a means for validation of finite element (FE) models. Modal analysis has been used by various authors to help relate the characteristics of sports equipment to quantifiable parameters. Brody (1995), Cross (2001b) and Mohanty et al. (2002) have all used modal analysis as a means of explaining such characteristics in tennis rackets. In the case of Brody and Mohanty it was the so called “sweet-spot”, whilst Cross used modal analysis to examine the change in
racket stiffness due to stringing. Similar analysis has also been carried out on cricket bats (Brooks et al. (2006), baseball bats (Noble et al., 1994), golf clubs (Merkel et al. (1999), Thomas et al. (1995)) and even on soccer and golf balls (Ronkainen et al. (2007) and Axe et al. (2002), respectively).

Despite modal analysis being used as a means of validating finite element models of sports equipment, little has been done to understand how vibrational characteristics vary between different models of rackets and indeed, between rackets of the same model. The aim of the research work presented here is to offer an insight into how the vibrational characteristics of aluminium rackets of the same model vary between two rackets and how the same frame will vary when strung at varying string tension. Future work may also provide information the variation between different models of racket.

### 5.2 Methods

Instead of the more traditional methods used in modal analysis this study has chosen to use a technique which has been used rarely within the sports industry. With the exception of Ronkainen et al. (2007), most previous studies have been carried out using more traditional experimental equipment such as accelerometers and mechanical exciters. In the work presented here a 3D laser doppler vibrometer (LDV) is used to provide the signal to drive the excitation of the racket and also obtain the measurements of the system’s vibration.

The major difference between laser doppler vibrometry and more conventional modal analysis techniques is that, in conventional modal analysis the amplitude of displacement and frequency is determined simply by the direct measurement of the subject’s acceleration from the accelerometer at a specific point. An LDV, however, will extract the measurements from the Doppler shift of the laser beam frequency when the laser in question is reflected from the subject. As a data acquisition tool the
obvious benefits of LDVs are that, unlike accelerometers, they do not have to be attached and unattached for each measurement point. Moreover, the more advanced LDVs are able to scan multiple points rapidly, significantly reducing data acquisition time.

Another significant advantage of using LDVs is the elimination of mass-loading the structure during analysis. Attaching accelerometers to the structure can have the effect of changing the frequency response function (FRF) and continually moving the accelerometers around the structure will mean that one may not receive the same structural response for each point. The FRF of a system is represented in Figure 5.1 and can be defined as:

“the ratio of the Fourier transform of an output response \(X(\omega)\) divided by the Fourier transform of the input force \(F(\omega)\) that caused the output” (Schwarz et al., 1999)

\[
F(\omega) \times [H(\omega)] = X(\omega)
\]

Figure 5.1: Transformation of input force to output response for a mechanical system

One drawback of more basic LDVs is that they only consist of a single scanning head (i.e. one laser). As a result, they are unable to take measurements in three dimensions. However, the development of LDVs has led to the introduction of 3 dimensional systems which consist of three scanning heads. As a result the system is able to acquire the motion of the subject in all 3 axes and can directly acquire the
mode shapes of the subject and display them without the need for any intermediate software.

The system used for this series of experiments was the Polytec PSV-400-3D Scanning Vibrometer. The system’s performance specifications can be seen below in Table 5.1 (taken from the manufacturer’s web-site):

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>0 kHz – 80 kHz</td>
</tr>
<tr>
<td>Velocity range</td>
<td>0.01 µm/s ... 10 m/s</td>
</tr>
<tr>
<td>Working distance</td>
<td>&gt; 0.4 m</td>
</tr>
<tr>
<td>Laser wavelength</td>
<td>633 nm (red)</td>
</tr>
<tr>
<td>Laser safety class</td>
<td>Class II He-Ne laser, 1 mW per sensor, eye-safe</td>
</tr>
<tr>
<td>Scanner resolution</td>
<td>± 0.002°</td>
</tr>
<tr>
<td>Sample size</td>
<td>Several mm² up to m² range</td>
</tr>
</tbody>
</table>

Table 5.1: Performance specification of the Polytec PSV-400-3D Scanning Vibrometer ([http://www.polytec-ltd.co.uk/uk/products/vibration-sensors/scanning-vibrometers/psv-400-3d-scanning-vibrometer](http://www.polytec-ltd.co.uk/uk/products/vibration-sensors/scanning-vibrometers/psv-400-3d-scanning-vibrometer))

The generic process for performing a natural frequency extraction using the Polytec system described above is detailed below and supported by the process flow in Figure 5.2.
5.2.1 Stage 1

The first stage in the test process was to position the three laser heads so that they are facing the target which is to be measured. Ideally, the set-up should be similar to that in Figure 5.3, with the central scanning head lower to the ground than the other heads. Once the scanning heads are in position, the user must then choose a series of calibration points on the target. The user must then manually position each of the three laser heads on one of the calibration points before “locking” the point into the system. Once each of the three heads has been “locked” on a calibration point the user must then repeat this process for at least two more points.

5.2.2 Stage 2

The second stage in the process is to specify a number of sample points around the target. The more sample points that are specified, the longer the analysis will take but the more detail will be provided in the depiction of the mode shapes. To specify the test points, the user must manually position the laser heads (after calibration the laser heads will all move in synchronicity so the user need only move the central head) at each of the desired sample points.

5.2.3 Stage 3

Before extracting the mode shapes for the natural frequencies, the user must first perform a “white noise” excitation of the system, which is a random excitation, to identify where along the frequency spectrum the natural frequencies occur. During this analysis the system will take measurements at each of the specified sample points and use them to produce a Frequency Response Function (FRF) plot.
5.2.4 Stage 4

Having performed the “white noise” excitation, the user must then identify the natural frequency peaks on the FRF plot. Once the user has identified the natural frequencies of the desired modes, they must then perform an excitation of the system at this frequency. During this excitation, the system will again measure the response at each of the sample points, from which the mode shapes of the natural frequencies will be generated.
Stage 1
Calibrate the system for the target

Stage 2
Specify the sample points.

Stage 3
Perform a white noise analysis.

Stage 4
Perform an excitation at the desired natural frequency to obtain the mode shape.

Figure 5.2: Process flow for modal analysis using a Polytec 3D LDV
5.2.5 Racket Testing

Two tubular aluminium unstrung tennis racket frames (racket frame 1 and 2) of the same model were analysed in the first instance. Aluminium rackets were chosen due to the relative ease with which they can be modelled in finite element analysis. The unstrung analysis was performed on two rackets to identify the variability between rackets.

Then, in order to assess the effect of string tension on the vibrational characteristics of the racket, the same racket (racket frame 1) was strung to tensions of 182 Newtons, 223 Newtons and 267 Newtons and analysed. In order to assess the experimental error each analysis was performed 5 times and the average value obtained. Given the system is capable of measuring frequencies up to 80 kilohertz (Table 5.1) it was assumed that any significant error would be identified by the repeatability testing. Having identified the variability between racket frames in the unstrung state it was not deemed necessary to analyse each racket in the strung states, therefore only frame 1 was tested in the strung states. The number of tests performed is shown in Table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>Tested Unstrung 5 times</th>
<th>Tested Strung to 178 Newtons 5 times</th>
<th>Tested Strung to 223 Newtons 5 times</th>
<th>Tested Strung to 267 Newtons 5 times</th>
<th>Number of tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Racket Frame 1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>20</td>
</tr>
<tr>
<td>Racket Frame 2</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5.2: Rackets tested
During the analysis, an aluminium racket was suspended from a horizontal beam using string and elastic bands. The elastic bands were attached to the upper edge of the head and handle at either end of the racket and the string was passed through the bands to support the racket from above (see Figure 5.4). The inclusion of the elastic bands minimises any reaction forces created by supporting the racket in this manner, thus providing a more realistic representation of the freely suspended condition. This is particularly beneficial as it not only removes the need to model additional boundary constraints but also provides a better representation of the racket in the hand held condition (Brody, 1995).

Figure 5.3: Schematic of the LDV experimental set-up
The LDV control unit not only controls the data acquisition but also the excitation of the racket. Initially, a loudspeaker was used to excite the racket and stringbed, similar to the method used by Ronkainen et al. (2007) but this did not provide an adequate level of force to excite the natural frequencies. As a result a Brüel and Kjaer mechanical exciter (model number 4824) with a force rating of 100 Newtons was chosen and was connected to the control unit via a power amplifier. The exciter drove an axial stinger which was fixed to the end of the rackets (slightly above centre to avoid exciting the racket on the torsional node) using a general purpose sealant. The axial stinger simply consisted of a metal rod around 0.3 millimetres in diameter and 100 millimetres in length which could be fixed to the exciter at one end and the racket at the other. The exciter was hung from a gantry using string in order to reduce any reaction forces from the exciter.

For the unstrung racket, 96 equally spaced data collection points were assigned to the elliptical frame section. In the case of the strung racket, the strings and the frame were analysed individually. The data points for the frame were positioned as in the
case of the unstrung racket whilst the data points for the string-bed were assigned to the intersections between the cross and main strings, resulting in the analysis area shown in Figure 5.5 and a total of 360 points.

![Figure 5.5: The area of the string-bed analysed](image)

The natural frequencies of the system were obtained from the frequency response function (FRF) plot which, was obtained from a white noise excitation (frequency range 0 Hz - 1,200 Hz).
Once the spectrum of the FRF had been obtained the positions of the peaks representing the natural frequencies were noted and the rackets were excited and scanned at these frequencies to obtain the mode shapes. The resolution of the resulting spectrum was primarily dependent on two factors; the sample time (which itself is dependent on the bandwidth of the analysis and the number of Fast Fourier Transform lines used) and the sample frequency. The relationship between these factors is shown below in Equation 5.1 and Equation 5.2.

\[
sample\_time = \frac{FFT\_Lines}{Bandwidth}
\]

Equation 5.1

\[
resolution = \frac{Sample\_time}{Sample\_Frequency}
\]

Equation 5.2

For the majority of experiments the acquisition variables were as follows:

- FFT lines = 1600
- Sample frequency = 2.56 Hz
- Bandwidth = 1000 Hz
- Sample time = 1.6 s

These values were found to give a suitable definition for the spectrum without greatly increasing the sampling time.

Once the rackets had been re-scanned at the natural frequencies the LDV software created an animation of their mode shapes, the validity of which is discussed later.
5.3 Finite element model

An FE model of the aluminium racket was created by running a “restart” analysis of the model described in Chapter 3. In Abaqus, a “restart” analysis takes the final state of a previous analysis and starts a new analysis from that point.

In this case a linear perturbation frequency analysis step was initiated from the point at which the string tensioning process had been completed. Abaqus offers several forms of frequency analysis, the default of which is “Lanczos” frequency extraction as it is the quickest to run and, in most cases, the most accurate.

As a result a “Lanczos” frequency extraction step was specified for frequencies ranging from 0 Hertz to 1,200 Hertz. This frequency range was chosen since it was shown from the experimental work that all natural frequencies fell comfortably within 1,200 Hertz, Brody (1995).
5.4 Results

The experimental results of the unstrung aluminium racket frames 1 and 2 are shown in Table 5.3, where rackets 1 and 2 are the same model.

<table>
<thead>
<tr>
<th>Racket-Frame</th>
<th>Lateral Mode (Hz)</th>
<th>Torsional Mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>130 +/- 0.4</td>
<td>353 +/- 1.1</td>
</tr>
<tr>
<td>2</td>
<td>132 +/- 0.5</td>
<td>352 +/- 0.7</td>
</tr>
</tbody>
</table>

Table 5.3: Natural frequencies of unstrung rackets (experimental)

The natural frequencies for the frame and stringbed for a strung racket (racket 1) are shown for both the experimental case (Table 5.4 and Table 5.5) and the natural frequencies of the finite element model (Table 5.6 and Table 5.7).

<table>
<thead>
<tr>
<th>Tension</th>
<th>Lateral Mode (Hz)</th>
<th>Torsional Mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>182 N</td>
<td>137 +/- 1.8</td>
<td>369 +/- 2.7</td>
</tr>
<tr>
<td>223 N</td>
<td>140 +/- 1.9</td>
<td>367 +/- 1.9</td>
</tr>
<tr>
<td>267 N</td>
<td>136 +/- 2.1</td>
<td>366 +/- 1.6</td>
</tr>
</tbody>
</table>

Table 5.4: Average natural frequencies of Racket 1 strung at varying tensions (experimental)
### Table 5.5: Average natural frequencies of the string-beds of Racket 1 when strung at varying tensions (experimental)

<table>
<thead>
<tr>
<th>Tension</th>
<th>1st String Mode</th>
<th>2nd String Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>178 N</td>
<td>576 +/- 2.6</td>
<td>853 +/- 2.9</td>
</tr>
<tr>
<td>223 N</td>
<td>574 +/- 1.8</td>
<td>856 +/- 1.4</td>
</tr>
<tr>
<td>267 N</td>
<td>576 +/- 1.3</td>
<td>856 +/- 1.9</td>
</tr>
</tbody>
</table>

### Table 5.6: Natural frequencies of finite element model of an unstrung aluminium tennis racket

<table>
<thead>
<tr>
<th>FE Model</th>
<th>1st Lat</th>
<th>1st Tors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>131 Hz</td>
<td>357 Hz</td>
</tr>
</tbody>
</table>

### Table 5.7: Natural frequencies of a finite element model of a string-bed tensioned to 178 Newtons

<table>
<thead>
<tr>
<th>FE Model</th>
<th>1st String</th>
<th>2nd String</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>560 Hz</td>
<td>768 Hz</td>
</tr>
</tbody>
</table>
(a) 1\textsuperscript{st} Lateral mode of the finite element model (132 Hz)

(b) 1\textsuperscript{st} Lateral mode of the experimental analysis (Racket 1, 130 Hz)

**Figure 5.6: Lateral Modes (Unstrung Racket)**
The lateral mode shapes for the FE model and experimental analyses are displayed in Figure 5.6 (a) and Figure 5.6 (b) respectively. The figures show similar mode
shapes have been achieved for both the model and experimental data. Figure 5.6 (b) does, however, display a minor discrepancy at the tip of the racket in the form of a slight “kink”. This is also the case for the experimental torsional mode in shown in Figure 5.7 (b), but again it is otherwise consistent with the finite element model mode shape shown in Figure 5.7 (a).
(a) 1\textsuperscript{st} String mode of the experimental analysis (576 Hz) superimposed onto the image of the racket

(b) 1\textsuperscript{st} String mode of the finite element mode (560 Hz)

\textbf{Figure 5.8: 1\textsuperscript{st} string modes}
(a) 2\textsuperscript{nd} String mode of the experimental analysis (853 Hz)

(b) 2\textsuperscript{nd} string mode of the finite element model (768 Hz)

\textbf{Figure 5.9: 2\textsuperscript{nd} String Modes}
Figure 5.8 (a) and Figure 5.9 (a) show the string-bed mode shapes associated with the natural frequencies of the aluminium rackets obtained via the LDV, whilst Figure 5.8 (b) and Figure 5.9 (b) show their finite element counterparts.
5.5 Discussion

5.5.1 Unstrung racket frames

The natural frequencies of the FE model of the aluminium racket frame, displayed in Table 5.6 showed good correlation with the experimental results shown in Table 5.3, with the fundamental lateral value of 131 Hertz falling exactly between the experimental values and the torsional value yielding a 1.5% difference.

The discrepancy at the tip of the racket shown in Figure 5.6 (b) was a common problem, caused by the calibration of the laser positioning. Each time a new racket was tested the lasers had to be repositioned at the centre of the racket and each one calibrated at a series of fixed points around the racket, as described in section 5.2.1. Naturally the laser positioning at the centre of the racket was easily refined, however, the positioning of the lasers on measurement points at the extreme ends of the racket was difficult. It is for this reason that only the head and sections which join the handle to the racket were included in the LDV scans.

A similar discrepancy to that witnessed in Figure 5.6 (b) can be seen for the torsional mode in Figure 5.7 (b), where the expected smoothness at the curved tip of the racket is not apparent. The correlation between the FE and experimental modes is, however, still very good and further validates the FE Model.

5.5.2 Strings

It is widely accepted (Brody 1995) that the frequency range for a racket-ball impact in tennis is within the range of 100 Hertz to 200 Hertz. However, the results of the modal analysis of the tennis strings shows that the value for the fundamental mode is 576 Hertz (see Table 5.5), which, indicates that the role of the natural frequency of the strings may have little effect on the experience of the player during competition.
This can be concluded from the contact time of a typical ball-stringbed interaction, which lasts around 5 milliseconds Brody (1987) and, hence, results in excitations at a frequency of around 100 Hertz. In order for the fundamental frequency of the string-bed to be excited an impact of around 2 milliseconds would be required. The 1\textsuperscript{st} mode shape, shown in Figure 5.8 is similar to that of a trampoline and is the shape one would associate with with a clamped circular plate, as shown by Reddy (2007) in Figure 5.10. Like the first mode, the experimental results of the second mode are concurrent with the mode shapes obtained from the FE model.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.10}
\caption{1\textsuperscript{st} Mode shape of a clamped circular plate}
\end{figure}

5.5.3 Strung racket frames

The analysis of the strung rackets also revealed higher natural frequencies than those obtained when the frames were scanned unstrung. This was concurrent with the results of the three point bend tests obtained in section 4.1, where the stiffness of the strung racket were greater than the unstrung rackets. The ratio of the natural frequencies of the strung and unstrung rackets can normally be obtained via the results of a three point bend test using the relationship derived below, where \( \omega_n \) is the fundamental mode, \( k \) is the stiffness and \( m \) is the mass (Reddy, 2007).

\[ \omega_n^2 = \frac{k}{m} \]  
\text{Equation 5.3}
\[
\left( \frac{\omega_{\text{strung}}}{\omega_{\text{unstrung}}} \right)^2 = \left( \frac{k_{\text{strung}}}{m} \right) \left( \frac{k_{\text{unstrung}}}{m} \right)
\]

\[
\left( \frac{\omega_{\text{strung}}}{\omega_{\text{unstrung}}} \right)^2 = \left( \frac{k_{\text{strung}}}{k_{\text{unstrung}}} \right)
\]

This relationship, of course, assumes that the mass of the strings is negligible. For the case of the experimental results obtained from this study, this relationship becomes:

\[
\left( \frac{138}{131} \right)^2 \approx \left( \frac{129}{116.7} \right)
\]

\[
1.11 \approx 1.1
\]
The mode shapes for the 2 strung aluminium rackets tested were similar to one another and what one would expect from a structure which can be loosely approximated as a beam. The mode shapes also showed good similarity to that of the FE model.

Table 5.4 shows the results of analysis carried out on racket frame 1 strung to several different string tensions. For each analysis the racket was strung the previous evening and left to “settle” overnight. The environmental conditions, such as ambient temperature, humidity etc. were not measured during this analysis so it cannot be stated whether slight changes in the conditions had any effect on the results. It can be stated, however, that the analyses, which are discussed in this section were all performed within a five day window and that any changes in room conditions would have been relatively small.

The results of this procedure show little difference in the results and do not display any particular trend. It is more reasonable to assume that the 2 Hertz swing in the results is due to uncontrollable changes in measurement conditions. It can be concluded, therefore, that the fundamental frequency is 138 Hertz across all the tension range used by most players. It is more than likely that the increase in racket frequency that can be seen between the strung and unstrung rackets occurs significantly over the lower tension ranges and the the increase in frequency across the tension range of 178 Newtons to 267 Newtons is relatively negligible, as indicated in Figure 5.11.
As a result, it can be concluded that changing the string tension, in the range which most players will string their rackets, has no significant impact on the fundamental natural frequency of the racket. The change in frequency due to stringing the racket could be explained by a combination of several factors. The first factor is the increased stiffness of the racket due to stringing. As stated in Equation 5.3, the natural frequency, $\omega_n$, of a system is proportional to the square root of the stiffness, $k$.

\[ \omega_n \propto \sqrt{k} \quad \text{Equation 5.3} \]

The second factor is the deformation of the racket caused by stringing. When rackets are strung they tend to deform more under the load of the cross strings than the mains strings which leads to an increase in the rackets length and reduction in its width. The bending stiffness, $k_{\text{bend}}$, of the frame is the product of $I$, the second moment of area and $E$, the Young's modulus (Equation 5.4 (Reddy, 2007)).

\[ k_{\text{bend}} = EI \quad \text{Equation 5.4} \]
If we neglect the handle of the racket, which is not subject to deformation during stringing and can be assumed to remain constant, the unstrung head of the racket can be treated as a square cross section. Once strung, the racket will become more oval-like in shape or, adopting a more basic assumption, like a rectangular cross section as shown in Figure 5.12:

![Figure 5.12: Second moment of area dimensions](image)

For a rectangular cross section the Second moment of area is represented by Equation 5.5, Reddy (2007):

\[
I = \frac{b_o \cdot h_o^3 - b_i \cdot h_i^3}{12}
\]

Equation 5.5

For this system, as \(b_o\) and \(b_i\) decrease and \(h_o\) and \(h_i\) increase, assuming a constant thickness, \(I\) will increase. Hence, if we assume that the tennis racket will behave in a similar manner then it can also be assumed that the bending stiffness, and as a result \(\omega_n\), will increase.
5.6 Concluding comments

A 3D laser doppler vibrometer (LDV) has been used to obtain the natural frequencies of aluminium tennis rackets, both in their strung and unstrung states. The first two natural frequencies and modes shapes were obtained for the racket. The values for the fundamental frequency of the racket were similar to those obtained by Brody (1995) (between 100 Hertz and 150 Hertz). A comparison could not be drawn on the values for the string modes, since the extremely small mass of a stringbed had meant that conventional modal analysis – adding an accelerometer – would have rendered results meaningless. This non-contact measurement of this study, however, did not encounter this problem.

The mode shapes observed experimentally are consistent with what would be expected from basic consideration of vibration theory, with the racket being represented as a free beam whilst the string bed behaved as a clamped circular plate.

The experimental variation of the results obtained for the racket frame using the LDV was less than 3% in all cases. Such a low level of variation gave good confidence in the results and allowed for their use as a correlation method for the FE models. Although an accuracy of 3% is sufficient for the validation of the FE model, it was not accurate enough to identify any variation in the natural frequency of the rackets or the strings due to variation in string tension. Although it is disappointing not to identify a relationship between string tension and natural frequency, it is not the main priority for the use of modal analysis in this study.

The experimental values for the natural frequencies of the aluminium tennis racket were used to validate the finite element model created for the same racket. The model showed good similarity with the experimental results (less than three percent
variation between the experimental and FE results for both modes), both in terms of the values of the natural frequencies and their corresponding mode shapes.

The FE model did not perform as well at the higher values of natural frequency found for the string-bed. The two results were within 3% of one another for the first string mode but the FE model produced a second string mode with a difference of 10% when compared to the experimental results. Having said that, it is very ambitious to expect the FE model to remain accurate at such a high frequency which, furthermore, is well beyond what is likely to ever be experienced by a tennis player.

The validation of the mode shapes was somewhat subjective and was performed purely by visual inspection. More robust methods of comparison, such as use of a Modal Assurance Criteria, are possible through commercially available softwares (LMS Virtual Lab, 2012). If a higher level of accuracy was required then this would be an appropriate approach but for the purposes of this study it would be unnecessarily costly. As a result, the use of the LDV to correlate the model can be considered to be very successful and the model itself can be deemed fit for further development.

Having validated the dynamic characteristics of the racket, the next step in this process is to validate its performance in contact with the ball during an impact. Before this can take place however, it is necessary to obtain the materials properties of a number of strings in order that the models performance can be validated for different string types.
Section 2.2 reviewed the laboratory work carried out by various authors with regards to the material properties of tennis strings. Although a great deal of research has been carried out into the resultant performance of different string materials when strung in a racket, relatively little has been done to understand how the fundamental properties, such as Young's modulus, affect the performance.

One author, Cross (2000a, 2000b and 2003), has done more than any other to examine the fundamental properties of tennis strings. In the specified research Cross focussed mainly on the dynamic stiffness, $k_d$; the stiffness of the string after it has been tensioned. Cross's main findings were that, when pre-tensioned at 196 Newtons:

- The stiffness of all the tested strings (Natural gut, Nylon, Polyester, Kevlar) increases.
- The force transmitted through the tennis string to its clamp increases.

The relationship between the Peak force experienced at the clamp and the displacement of the strings relative to one another is summarised in Figure 6.1. Cross found that Kevlar transmitted the highest force whilst experiencing the least displacement whilst Gut absorbed more force while experiencing less displacement.
Figure 6.1: General trend of peak force and displacement experienced by pre-tensioned tennis strings when struck by a swinging hammer (Cross, 2000b)

Although the dynamic stiffness is interesting in terms of string performance it is not a parameter which is compatible with the Abaqus materials models. To define a materials model within Abaqus one requires some form of stress-strain data, the simplest form of which is the Young’s modulus.

6.1.1 Equipment

The same Instron 3365 force measurement machine was used as described in Section 4.3.1.1. In this instance the circular compressive applicator and racket support were replaced with vice grips, as shown in Figure 6.2. The vice grips contained a textured surface to ensure that slip did not occur during testing.
6.1.2 Methods and Results

To obtain the Young’s modulus of a variety of tennis strings, a procedure similar to that used in Section 3.5 was adopted. In this set of experiments, however, the Young’s modulus was measured for a variety of string tension ranges rather than the single range (100 Newtons - 150 Newtons) used in the initial experimentation. The Young’s modulus of the strings, were measured at a strain rate of 100 millimetres per minute in the following ranges:

- 178 Newtons - 228 Newtons
- 223 Newtons - 273 Newtons
- 267 Newtons - 317 Newtons
These tensions were chosen as the lower limits; 178 Newtons, 223 Newtons and 267 Newtons, represent the most commonly used string tensions. The upper limit was simply an additional 50 Newtons, assumed to represent an impact force similar to that of a tennis ball impact. The gauge of all the strings tested was 15, meaning a cross sectional diameter of 1.3 millimetres. The cross sectional diameter of the strings, post-tensioning was obtained from Racket Tech Magazine (Racket Tech Publishing, 2005), the final diameter of the string was fed into the Bluehill software to account for changes in cross sectional area and hence allow a more accurate modulus to be calculated.

Figure 6.3: Young's modulus of tennis string for different tension ranges

Figure 6.3 shows the results of the testing and some interesting trends across the tension ranges. The first point which is apparent from Figure 6.3 is the variation in the
trend between the nylon (Prince Synthetigut, Prince Tournament Nylon) and polyester (Babolat Pro Hurricane, Prince Tournament Poly Luxilon Big Banger) strings across the tension range. The Young’s modulus is shown to increase with tension for the nylon strings as opposed to the polyester strings, which decrease.

It is also apparent that the variation in Young’s modulus across the different tensions is at least 10% for all strings, which is significant enough to have an effect on an impact simulation. Therefore, when defining the material data for the finite element model it would be prudent to adjust the Young’s modulus depending on which string tension condition is being modelled.

6.2 Variation in Young’s modulus due to extension rate

As with any dynamic analysis, the strain rate of the material data can play a significant role in the accuracy of the results. In a tennis impact the string-bed will typically deflect by around 5 millimetres to 10 millimetres in around 5 milliseconds to 8 milliseconds (meaning the maximum deformation occurs at around 2.5 milliseconds to 4 milliseconds). The length of the elongated string can be calculated using Pythagoras, as in Equation 6.1, where \( l_0 \) is the initial string length, \( l_1 \) is the elongated length and \( d \) is the string deformation.

\[
((0.5l_0)^2 + (d)^2) = \left(\frac{l_1}{2}\right)^2
\]

\[
((0.5 \times 0.2)^2 + (0.01)^2) = \frac{l_1^2}{4}
\]

\[
l_1^2 = 4 \times 5.05 \times 10^{-3}
\]

\[
l_1 = 0.201m
\]

This results in a strain of 0.001 and a strain rate of 0.4 metres per second (24,000 millimetres per minute). Recreating strain rates of this magnitude, whilst measuring
the string response, was beyond the capabilities of this study. Efforts were taken, however, to examine the variation in Young's modulus due to lower strain rates (25 millimetres per minute to 100 millimetres per minute).

6.2.1 Methods and Results

Three strain rates were examined, 25 millimetres per minute, 50 millimetres per minute and 100 millimetres per minute, at the three previously mentioned tension ranges for the nylon, titanium and polyester strings. The results of these tests can be seen below in Figure 6.4, Figure 6.5 and Figure 6.6.

![Figure 6.4: Young's modulus for 178 Newtons - 228 Newtons at various strain rates](image)

**Figure 6.4: Young's modulus for 178 Newtons - 228 Newtons at various strain rates**
Figure 6.5: Young’s modulus for 223 Newtons - 273 Newtons at various strain rates
Reviewing the results of the strain rate experiments shows that the titanium polymer string’s modulus tends to vary for the lower applied force of 178 Newtons to 228 Newtons but remains relatively constant for the higher forces. Conversely, the nylon and polyester strings show little variation in modulus across strain rates for the lower forces but increase with strain rate for the higher force.

Another point which is interesting to note is that whilst the titanium string decreases in modulus as the strain rate is increased, the polyester and nylon strings increase in modulus.

It is perhaps, no coincidence that the value of tension (223 Newtons) which the strings’ modulus is most consistent in, with respect to strain rate, for all the strings is also the most commonly used string tension. Noting the consistency of the results at
this value one can also assume that the value of the modulus is unlikely to be changed at the higher strain rates achieved in an impact during play. With this in mind the decision was taken to use 223 Newtons as the default tension when analysing the change in output variables, such as spin and COR, due to changes in material properties.

6.3 Coefficient of Restitution

The COR of a tennis racket impact is dependent upon a variety of factors:

- Energy loss through deformation of the ball and strings during impact
- Energy loss through oscillation of the ball post-impact
- Energy transformed into internal vibration of the frame

Hatze (1993) quantified the contribution of each of these factors through a mathematical model. He stated that in a typical impact with a COR of 0.8, around 2%-4% of the energy would be lost through the strings during impact, 15% would be lost due to the deformation of the ball whilst the majority of the energy loss (56%-65%) would take place post-impact in the form of ball oscillation and racket vibration.

As a result, the critical role played by the string-bed in an impact, with respect to COR, is not the amount of energy it absorbs, it is the speed with which they return to their original position and the ball rebounds. Therefore, the string’s ability to deflect and return to its initial position is the most significant factor when considering its merits.

In order to quantify the string’s contribution to the COR of the impact system the most significant variable is therefore, the contact time. In order to form a comparison of the contribution made to the COR by different strings a procedure was put in place to
model the impact of a solid tennis ball. The reason for using a solid representation of the ball was that this would remove the effect of the energy dissipation of the ball and isolate the effect of the strings.

6.3.1 Methods

To obtain the desired experimental set-up an INSTRON 9250G Drop-test tower, shown in Figure 6.7, was used. The "G" in the system's title stands for gravity as this variation of drop-tester uses gravity to generate the impact force. This is achieved by raising a weighted carriage to a specified height above the test sample and releasing it. The 9250G is available with a light, medium and heavy carriage. The model used in this instance contained the medium carriage which was capable of generating kinetic energy in the range of 4.6 Joules to 300 Joules. At the time these experiments were performed, the system was less than six months old and the initial calibration certificate was still valid.
The user can define the initial position of the carriage by specifying one of the three following variables:

- Impact velocity
- Kinetic energy
- Drop height

This action is performed via a control unit (typically a PC) which is installed with Instron’s own interface software, "Impulse". According to the manufacturer the final impact condition is accurate to within 2% of that specified by the user (Instron, 2011).
Since the weight of the carriage can be altered at any point, the system requires the user to perform a weight calibration before each measurement. This is achieved by moving the carriage to a position where it is not in contact with the measurement specimen and selecting the "Weigh" option within the Impulse interface. Until this action is performed, all measurement options will appear greyed out within the interface.

Inside the drop-test tower, two light gates are positioned just above the string-bed to measure the inbound and outbound velocity. The manufacturer states that the light gates will give a velocity measurement which is accurate to within 0.25% (Instron, 2011).

Once the impact has been performed, the Impulse interface will produce a Force-Time plot. An example of such a plot is shown in Figure 6.8. Furthermore, the Impulse interface will also provide the inbound and outbound velocity, measured by the light gates.

![Figure 6.8: Example of the data output from the drop-test for a Polyester string-bed](image-url)
A half-hemisphere, 67 millimetres in diameter, was machined from aluminium to represent the shape of a tennis ball, attached to the under-side of the carriage and positioned above a custom made square string-bed, as can be seen in Figure 6.9.

![Figure 6.9: Experimental set-up for string COR testing](image)

The string-bed was fabricated by the technicians within the Loughborough University Sports Technology Institute, specifically to fit within the drop test tower. It consisted of ten millimetre diameter solid steel tubing, with holes strung at five millimetre intervals along the length of the tubing. The frame was sixty millimetres in length and breadth from the inner edges. The string-bed was constructed along with special fixtures which allowed it to be strung on a Babolat racket stringing machine, as shown in Figure 6.10.
In order to deflect the string-bed to a similar magnitude as in a ball-bed impact some general calculations were made to find the energy required. If the kinetic energy, $E_k$, is given by Equation 6.2:

\[ E_k = \frac{1}{2} m_b v_b^2 \]  

Equation 6.2

\[ E_k = \frac{1}{2} \times 0.057 \times 30^2 = 25.65 \text{ Joules} \]

Where, $m_b$ and $v_b$ are the mass and velocity of the ball, respectively (Matsuhisa, 2004). Initial trials of with this kinetic energy, however, resulted in consistent failure of the string-bed. This was thought to be due to the absence of ball deformation and an increased stress concentration in the string-bed (normally the ball would deform and spread out during contact). Some ballistics experts (Dr. Ron Thomson, Department of Mechanical Engineering, University of Glasgow) state that the different peak stresses between a high-mass/low-velocity impact and a low-mass/high velocity impact may result in material failures even if $E_k$ remains constant.
As a result, a reduced energy level of fifteen Joules was specified. This level of impact will still provide enough deformation of the different string types to highlight any variation in their characteristics.

This value of potential energy resulted in an impact velocity of 2 metres per second. For completeness the drop tests for impact velocities of 1 metre per second and 1.5 metres per second were also performed. These drop tests were completed five times for gut, polyester and nylon string-beds, each of which was strung to a tension of 223 Newtons.

As well as recording the force measured from the load cell and the velocities via the light gates the impacts were also recorded using a Photron Fastcam SA5 high speed video camera at a frame rate of four thousand frames per second. The manufacturer claims that the timing of the system is accurate to within plus or minus 0.005% (Photron, 2012), which is not a significant enough error to be of concern.

The contact time of each impact was taken from the high speed video by counting the number of frames from when the impacter first contacted the string bed to when it rebounds and contact ceases. If we assume a potential error of half a frame due to contact occurring slightly before it is captured and half a frame due to it ceasing slightly before it is captured, this gives a potential experimental error of plus or minus 0.00025 seconds.

The average COR and contact time and the corresponding standard deviation were calculated for each of the string types and each impact velocity. In the case of the contact time, there was no measured variability detected i.e. the experimental variation was too small to be identified with a frame rate of four thousand frames per second. Therefore the potential error for the contact time is 0.00025 seconds. For the COR the variability was found to be less than 0.5% in all cases. When added to the
potential error of the initial velocity measurement this gives a maximum potential error of 0.75% across all impact conditions.

The results of the experiments for COR and contact time can be seen in Figure 6.11 and Figure 6.12 respectively. Figure 6.11 shows that for each string material the COR reduces as the impact velocity is increased. Conversely, an opposing trend, in the case of the contact time can be seen in Figure 6.12.

![Figure 6.11: Coefficient of restitution of various string materials](image1)

![Figure 6.12: Contact time for various string materials](image2)
It is interesting to note that, whilst the COR for the different strings shows some level of variation at the higher impact velocities, there is very little difference between them for impacts at one metre per second. This may suggest that for lower velocity impacts, such as drop shots and volleys, there would be a noticeable variation in the COR for different strings but for high velocity impacts such as serves and baseline forehands there would be little difference.

However, as previously noted, the most significant contribution of the strings in terms of the overall COR of the ball racket system is how quickly the strings return to their original position, which dictates the dwell time of the ball. By looking at the contact times in Figure 6.12 it can be seen that the contact times of the strings and the rigid ball actually shows the greatest variation at the higher velocity of two metres per second. This would suggest that, since the ball will have more time to deform on the bed, nylon strings would have a reduced COR at higher velocities, relative to the other strings.

6.4 Concluding comments

The materials properties testing obtained the desired results in terms of Young’s modulus required for the finite modelling whilst also providing insight into string performance. It was shown that the tension which provided the most consistent value of modulus for a varying strain rate was 223 Newtons. It was also shown that whilst all the strings performed similarly in terms of COR with an impact with a solid hemisphere, nylon provided the longest contact time, which when in contact with a real tennis ball, would lead to a lesser COR as more deformation of the ball, and hence more energy dissipation, would occur.
7 Modelling of friction

The following section deals with the role of friction in oblique tennis impacts and seeks to compile a library of data which can be used to define the contact between the ball and strings and strings with themselves, within the finite element model.

7.1 The role of friction in tennis racket impacts

The role of friction in tennis rackets impacts is a complex topic and opinions vary amongst academics who study the sport. Some authors (Daish, 1972, Goodwill, 2002, Brody, 2002, Allen et al., 2010b) have taken the simplistic view that all tennis strings have a large enough COF to instigate rolling and that any increase in COF above this critical point has no effect. Cross (2000), however, disagreed and adopted a more detailed approach and analysed the COF ($\mu$) as two separate components $\mu_s$ and $\mu_R$, the coefficients of sliding and rolling friction respectively.

The underlying fault with the assumption that the COF has no effect on the generation of spin is that, unlike the work of Cross, it assumes a constant COF throughout contact. In reality $\mu_s$ will be significantly different from $\mu_R$ and a system which has a similar value of $\mu$ for the rolling section of an impact may vary greatly during the sliding section. As a result the ball may slide for longer and at a higher velocity and the resulting level of spin would surely be affected.

Using a series of mechanical formulas Cross calculated the effect which increasing the value of $\mu_s$ would have over the outbound spin, $\omega_2$. The results can be viewed below in Figure 7.1, where the tilt angle, $\beta$, (Figure 7.2) represents the angle of the head. Cross showed that for a COF equal to 0.2 the spin rate will increase as the tilt angle of the racket, $\beta$, is increased.
Figure 7.1: Variation in spin vs head angle for $\mu_s = 0.2$ (Cross, 2000)
It seems reasonable to assume, therefore, that if \( \mu_s \) has such a profound effect on the generation of spin then the point at which sliding occurs will also be a significant factor in the level of spin generation. Whilst sliding motion is relatively simple to simulate, rolling motion, requires rather more consideration.

### 7.2 Coefficient of sliding friction

#### 7.2.1 Tribology

Obtaining the COF for a system through experimental data can be difficult since the magnitude is dictated by a number of factors. COF is very much a function of the entire system and a value of \( \mu \) obtained for a ball sliding at 10 metres per second would not be representative of a system containing a ball spinning at 200 revolutions per minute. It is therefore necessary to create an experimental setup where the conditions of the system in question are reproduced as closely as possible.
The most commonly used type of apparatus in friction measurement is known as a tribometer. One of the most prevalent manufacturers of Tribometers, CSM Instruments (CSM, 2012) describes the basic principle of tribology (the science of tribology) as follows:

“Tribometers determine the magnitude of friction and wear as two surfaces rub together. In one measurement method a flat or a spherical probe is placed on the test sample and loaded with a precisely known weight. The sample is either rotating or reciprocating in a linear track. The resulting frictional forces acting between the probe and the sample are measured.”

7.2.2 Measurement Equipment Set-up

7.2.2.1 Tribometer

Cotton (2008) performed significant research into the measurement of sliding friction of soccer balls against synthetic turf. A schematic of the system used by Cotton is shown below in Figure 7.3. At one end of the lever system lies a steel drum which is attached to a motor turning at a constant speed. At the opposite end is a weight stack which is used to apply a known force to the drum via the sample tray.
As Cotton was investigating the relationship between soccer balls and turf the drum was lined with soccer ball panel material approximately one millimetre thick and the drum was placed above a sample tray containing synthetic turf. Once the motor is up to full speed the sample tray is pressed against the drum and, since the tangential resistance applied via the tray is known from the weight stack, the resistance measured via the drum can be converted to COF.

When the ball impacts the string-bed it meets with a frictional force, \( F \), which is the product of the COF, \( \mu \), and the normal force, \( R \), as shown in Equation 7.1.

\[
F = \mu R
\]  

Equation 7.1
Since the radius of the drum, \( r \), and the applied normal force, \( R \), are known it is possible to calculate \( \mu \) from the difference in torque, \( T \), experienced by the motor when the force is applied to the rotating drum, as shown in Figure 7.4. Inserting Equation 7.2 into Equation 7.1 yields Equation 7.3:

\[
F = \frac{T}{r} \quad \text{Equation 7.2}
\]

\[
\mu = \frac{F}{R} = \frac{T}{rR} \quad \text{Equation 7.3}
\]

**Figure 7.4: Free-body diagram of the tribometer in contact with the test surface**

As stated previously, in order for this procedure to give as accurate a value of \( \mu_R \) as possible the conditions of the system must be closely replicated during the experimental analysis. One such condition is the rotational velocity of the drum. A typical tennis ball impact would occur with an initial rotational velocity of around 600 radians per second. Unfortunately, the drum was only capable of rotating at speeds of around 150 radians per second, so the maximum conditions experienced during an impact could not be investigated. It was felt however, that the speeds produced by
the drum would offer valuable insight into the behaviour of friction under a variety of conditions.

The other significant variable in this procedure was the normal force applied via the sample tray. In order to ascertain the level of applied force required it was first necessary to obtain the level of deformation experienced by the ball during an oblique impact.

7.2.2.2 High Speed Camera

The apparatus used to obtain the deformation of the ball was similar to that used by Cottey (2002) to capture the COR of a tennis racket impact. To fire the balls Cottey used a pneumatic cannon, which was specially designed and built by the Loughborough University Sport Technology Institute for the analysis of high speed ball impacts. The velocity of the balls can be determined by varying the pressure of the cannon. The relationship between the pneumatic pressure of the cannon and the initial velocity generated for a tennis ball is shown below in Figure 7.5:
7.2.2.3 Compressive force experienced by the ball during impact

This setup differed from that used by Cottey, however, in that the high speed video of the oblique impact was acquired using a Fastcam SA5 camera (Cottey used a Kodak Ektapro 4540). The impact was filmed at 4,000 frames per second with a pixel ratio of 1,280 by 768.
In order to obtain an impact of as close to thirty metres per second as possible, the air cannon was set to 0.276 Megapascals. A set of light gates were placed at the exit point of the cannon to measure the initial velocity and the cannon was fired several times before an impact of 29.8 metres per second was achieved.

The footage used (shown in Figure 7.6) was of a racket tilted to an angle of 29 degrees (or 61 degrees off normal) relative to the flight path of the ball. As stated, the ball impacted the racket at a velocity of 29.8 metres per second, with negligible, assumed zero, angular velocity. The deformation of the ball was measured using the image processing software Image Pro-Analyzer® and plotted against time. The results can be viewed below in Figure 7.6.

Given that the deformation of the ball was measured from the images of the ball during impact it was necessary to ascertain the clarity of the images to understand the potential error. It is shown below in Figure 7.7 that the surface of the ball can be
identified to within plus or minus two pixels. The standard diameter of a tennis ball is known to be 0.067 metres (Appendix 1) and, using Image Pro-Analyzer, was measured at 154 pixels. Thus, using Equation 7.4 the potential measurement error associated with this method is plus or minus 0.0008 metres.

**Figure 7.7: Potential measurement error from the image of a tennis ball**

\[
\text{Error}_{\text{metric}} = \text{Error}_{\text{pixels}} \frac{d_{\text{metric}}}{d_{\text{pixels}}} 
\]

\[
= \pm \left( 2 \times \frac{0.067}{154} \right) 
\]

\[
= \pm 0.0008 \, m 
\]

Using the footage of the impacts, a plot of the ball’s deformation against time was generated and is shown below in Figure 7.8.
The stiffness of the tennis ball was assumed to be 16 kilonewtons per metre, as stated by Sissler et al. (2010), since the same make and model of ball was used in both studies. Applying this value of stiffness, $K$ and the maximum deformation, $d$ (taken from Figure 7.8) to Equation 7.5 resulted in a calculated force, $F$ of 176 Newtons being applied to the drum via the sample tray.

\[
F = K \times d
\]

\[
= 16,000 \times (11 \pm 0.8)
\]

\[
= 176 \pm 13.6 N
\]
7.2.2.4 Experimental method using Tribometer

Having determined the applied force required, the experiment was carried out and the results logged via the acquisition unit. Once the drum is in motion, the acquisition unit gives a read-out of the torque measured from a transducer within the drum. A torque value, \( T \), was measured by the tribometer during the test and, using the applied force, \( F \), and drum radius, \( r \), converted into \( \mu \).

The British Standard for measuring the friction of rubber, BS ISO 15113:1 1999, suggests that each sample should be tested five times and each time should consist of three cycles, as shown in Figure 7.9, the average of which is then taken to be the true value. As such, the sample was loaded three times and an average of the three was calculated using Equation 7.6.

![Figure 7.9: Coefficient of friction vs. time](image)

**Figure 7.9: Coefficient of friction vs. time**

\[
\text{Mean } (\mu) = \frac{F_1 + F_2 + F_3}{3R}
\]

Equation 7.6
In order to convert the tribometer to one suitable for testing tennis string/ball cloth friction the drum was wrapped tennis cloth and the sample tray was replaced with the small square string-bed previously mentioned.

The variation in COF for several variables was investigated using the Tribometer, including string gauge, string-bed density and orientation. Each variable was tested for Babolat Hurricane mono and multi-filament strings.

7.2.3 Testing procedures and results

The conditions altered during testing were:

- String gauge: 1.25 millimetres, 1.30 millimetres and 1.35 millimetres
- String spacing (mains and crosses simultaneously): 5 millimetres, 10 millimetres, 15 millimetres and 20 millimetres (for a string gauge of 1.30 millimetres)
- String orientation: Conventionally strung and diagonally string (45 degrees to the normal of the frame) (for a string gauge of 1.30 millimetres)

The results were processed as described in the previous section and can be viewed in Figures 7.10, 7.11 and 7.12. The first conclusion to be drawn from each of the figures is that the multifilament string has a consistently higher COF.

Figure 7.10 shows that, for both string compositions, there is an increase in COF by around 7%-8% as the string diameter is increased. This could be explained by the fact that thinner strings tend to be more elastic and thus would deform more readily to the shape of the wheel under load.
The theory that a more deformable string-bed leads to a reduction in COF is reinforced by the results in Figure 7.11, where the COF for different string densities are displayed. Naturally, a less densely strung strung-bed would lead to a less inhibited, more deformable bed which, the results seem to suggest, leads to lower level of friction.
Although it was difficult to compare the string-densities of the conventional and diagonally strung beds it was visibly noticeable that the diagonally strung pattern was more open. It seems logical therefore, to assume that the change in COF due to string orientation is again due to the change in string-bed stiffness.
The mechanism of sliding friction between two contacting interfaces is well documented in a variety of fields. As earlier stated, the value of sliding COF tend to be system specific and there have been studies that have attempted to ascertain values for the sliding phase of a tennis racket impact. Cross (2000) and Bao et al. (2003) attempted to measure the sliding COF of a tennis ball over a string-bed by placing weights on half a tennis ball to form a sled and measuring the force required to drag the sled along the bed at a constant velocity, resulting in COF values of 0.2-0.49.

Although attempts to quantify the sliding COF between tennis ball cloth and racket strings have been made, little work has been done with respect to the interaction between the strings themselves. The interaction between the strings is of importance because as the ball impact the string-bed there will inevitably be some movement of the strings relative to one another and the magnitude of this movement will be, to some extent, dependent upon the COF between the strings.
With this in mind, an experimental set-up was created, similar to that of Cross, which could be used to measure the interaction properties of the strings with themselves as well as the strings with the ball. The main difference with the experimental set-up compared to the apparatus used by Cross is that rather than dragging a sled across a string-bed, a hanging weight stack was pulled between two pulleys, with a known friction, by tennis string.

To measure the COF of the string against the ball, a platform covered in tennis cloth was placed under the string. In order to measure the COF for the string against the string the platform was simply replaced with a tennis string pulled to a tension of 178 Newtons.

The Instron 3365 Force-Displacement machine described in section 4.3.1.1 was used to control the velocity at which the string was pulled (0.45 metres per second) and measure the required force. The normal force, $N$, applied to the tennis cloth and string was calculated from the mass of the weight stack, $m$, using Equation 7.7:

$$N = (mg) \times 0.5 \quad \text{Equation 7.7}$$

Where $g$ is the gravitational acceleration exerted on the weight stack. The factor of 0.5 in Equation 7.7 is a result of assuming that the force created by the weight stack is shared evenly between the pulley which feeds in the tennis string and the tennis ball cloth or string being tested.

The mass was varied to provide a normal force of 2.25 Newtons and 75.8 Newtons in the case of the string against the ball cloth and 35.9 Newtons and 75.8 Newtons in the case of the string against string. The normal force values were restricted by the experimental set-up, which was only able to support a certain number of weights.
The COF, $\mu$, was calculated using Equation 7.8, where $F$ is the force required to pull the string across the tennis ball cloth or string.

$$\mu = \frac{F}{N}$$  \hspace{1cm} \text{Equation 7.8}

As with all experimental set-ups measuring frictional force, there will be a component of the force which should be attributed to the inherent friction within the testing apparatus. An initial test was performed, therefore, to establish the force required to freely pull the sled along the testing apparatus without any tennis cloth or string impeding its progress (Figure 7.13). Having established this force, it was then subtracted from all subsequent values (Equation 7.9) in order to give the true value of frictional force for the interaction between the string and the ball (Figure 7.14) and the strings themselves.

$$F = \text{Measured Force} - \text{Frictional Force of Rig}$$  \hspace{1cm} \text{Equation 7.9}
Figure 7.13: Experimental set-up for testing sliding friction of rig

Figure 7.14: Experimental set-up for dragging a tennis string over tennis cloth
The four test conditions (two string tensions against the tennis cloth and two string tensions against the tennis string) were performed five times for a mono-filament (Babolat Hurricane Pro Tour) and multi-filament (Babolat Hurricane Tour). The data points displayed in Figures 7.15 and 7.16 display a distinct pattern of COF variation with string travel. For the multi-filament string against cloth (N = 75.8 Newtons) the standard deviation varied across the five tests as shown in Figure 7.17. For consistency, the maximum standard deviation from each test condition was assumed the truest value and presented on Figures 7.15 and 7.16 as such, e.g. the maximum standard deviation on Figure 7.17 was 8% of the mean average and presented as the most reliable value across all values in Figure 7.15.

The results of the string against cloth experiments can be seen below in Figure 7.15 along with a tabular comparison of values obtained from the previously mentioned studies, in Table 7.1.

![Figure 7.15: Sliding COF of different string against tennis ball cloth](image)
Table 7.1: Comparison of COF for tennis string against tennis ball from various studies

<table>
<thead>
<tr>
<th>Author</th>
<th>COF</th>
<th>Pressure</th>
<th>Sliding Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross, 2000</td>
<td>0.27 – 0.45</td>
<td>30 KPa</td>
<td>&lt;1 ms(^{-1})</td>
</tr>
<tr>
<td>Bao, 2003</td>
<td>0.20 – 0.49</td>
<td>5 – 20 KPa</td>
<td>0.2 ms(^{-1})</td>
</tr>
<tr>
<td>This study</td>
<td>0.19 – 0.53</td>
<td>n/a</td>
<td>0.45 ms(^{-1})</td>
</tr>
</tbody>
</table>

The main observation to be drawn from Table 7.1, is that the results are in a similar range to that of Cross' and Bao's. Figure 7.15 shows that the multi-filament tour string has a higher COF than the mono-filament and also suggests that this difference is amplified at a greater string tension.

Figure 7.16: Sliding COF of string against string

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In the case of the string against string experiments it was necessary to raise the lower normal force from 21.6 Newtons to 347.3 Newtons. The upper string tension, however, remained constant at 347.3 Newtons.

Figure 7.16 shows the sliding COF of the string against string testing for the mono and multi-filament strings. The results show a similar trend to the string against ball with the COF reducing as the string tension is increased. The differential between the mono and multi-filament strings is also amplified at higher normal force.

As suspected, however, there is a significant difference between the sliding COF results for the two experimental set-ups, with the string against string values proving to be significantly less in all cases. This reduction in COF is likely to be due to the string containing significantly less surface asperities than the tennis cloth, thus
reducing the level of mechanical interlocking which will take place between the interfaces during sliding.

Two conclusions can be immediately drawn from viewing these results:

- The magnitude of sliding COF is significantly different between the ball and the string and the strings themselves and, thus, independent contact definitions should be assigned to the two interactions.
- The magnitude of COF for the strings sliding against the strings is highly dependent upon the tension of the string. Thus, assigning a single interaction property across all values of string-bed tension may not be appropriate.

7.4 Friction properties of different strings

As well as indicating the deviation in the COF across the string-bed, examining the variation in the contact properties of different strings also allows an insight into the stringing process. During stringing a uniform tension is applied to all of the strings but as the stringer reaches the end of the stringing process it becomes more difficult to weave the cross strings through the mains and, as a result, less extension of the string is required to pull it to tension (since a greater level of tension already exists from the string having been woven through a stiffer bed than the previous string).

Therefore, an experiment is required, where the different friction levels experienced by mains strings during the stringing process is identified. Performing this experiment will not only provide insight into possible pitfalls of a global interaction property assignment, it will also indicate if a similar concern exists with regards to material property assignment.
This test was performed in a similar manner to the previous tests with the exception that the racket was strung to various points and then the next string was pulled to tension using the Instron 3365 force measurement machine described in Section 4.3.1.1 (as shown in Figure 7.18) and the other end of the string was tied off and effectively clamped. The same set-up was used as described in Section 6.1.1 but in this instance the lower vice grip was used to hold the racket in place whilst the upper, mobile, vice grip was used to elongate and tension the cross string.

The string’s Young’s modulus was calculated from the resulting force displacement curve. This procedure was carried out for the string without the racket to provide a reference value and for the 1st, 4th, 8th, 12th and 16th cross strings.

Figure 7.18: Experimental set-up for measuring the variation in frictional properties/Young’s modulus of different cross strings
The results of the experiment can be seen in Figure 7.19. The results show a very slight reduction in modulus for the cross strings relative to the reference value (approximately 6%). Furthermore, a very slight downward trend is present in the modules of the measured string as the strings are added. This is contrary to the expected outcome. A possible explanation could be due to the fact that more force is required to elongate the string than to simply “straighten” it out by displacing the mains strings. As it is difficult to tell at what point the force is acting to elongate the string and not simply “straighten” it by displacing the mains, however, it is difficult to prove or disprove this theory. Given the very slight magnitude of the difference between the strings (>3% from the 1st to the 16th string) it was assumed that a global property assignment for the string on string COF would be appropriate.
7.5 Concluding comments

It has been shown that assigning a global interaction property to the model is not suitable if the different interactions between ball and strings are to be modelled accurately. The results of the experiments carried out in this section have shown that the values of COF between the mains and cross strings and the ball and the strings are significantly different.

It has also been shown that the COF varies with string tension, and as such, when modelling different string-bed tensions; it would be prudent to alter the interaction property assignment accordingly. As a result, subsequent models will contain a COF value for the strings against the strings and the strings against the ball and the values will be altered depending on the string material being used.
In the previous chapter, it was shown that the level of friction produced by a string-bed is dependent upon several characteristics. Friction is of interest to this study because of the role it plays in spin generation, since high levels of friction will cause the ball to “bite” sooner and slide less. Although it is accepted that these factors have an effect on spin generation, little knowledge of the “effect” is available. In this chapter, a series of simulations will be performed to study the behaviour of the ball during impacts of different inbound angles. It is hoped that by studying these varying impacts a better knowledge of the following points can be obtained:

- How the ball behaves during an oblique impact (i.e. does the ball start spinning as soon as it impacts the string-bed or does it slide first?)
- How this behaviour might change with the inbound angle (i.e. is the potential sliding phase of the impact lengthened or shortened?)
- What affect this all has on outbound spin.

8.1 Dynamics of oblique impacts

As discussed previously, in Chapter 7, there are two main components to the oblique impact; the sliding component and the rolling component. Figure 8.1 is taken from Brody et al. (2002) and depicts the various velocity components of a ball sliding (a-b) and biting (c-d). In the case of the sliding ball a point at the top of the ball is rolling around the axis at 6 metres per second while the axis itself is travelling at 6 metres per second. The result of this motion is an overall forwards velocity of 14 metres per second for the point at the top of the ball.

Similarly the point at the bottom of the ball is travelling at a resultant forward velocity of 2 metres per second, as is the case for every point on the bottom of the ball in
contact with the string-bed. The ball’s most forward point has a forward velocity of 8 metres per second whilst also rotating downwards at a velocity of 6 metres per second giving a resultant velocity of 10 metres per second (using Pythagoras $10^2 = 8^2 + 6^2$) in a downward/forward direction. The point to the rear has a similar resultant velocity with an upward/forward direction. When viewing Brody’s graphical interpretation of this mechanism in Figure 8.1, it is important to remember that the velocity vectors around the edge of the ball are relative to the centre of the ball, whereas the velocity vector at the ball’s centre is relative to the surface it is impacting.

The frictional force experienced by the section of the ball in contact with the string-bed, however, has the effect of reducing the forwards velocity and increasing the rotational velocity of the ball. When this happens, a critical point is reached where the velocity of the bottom of the ball and the centre have equal and opposite velocity components and the ball is translationally at rest. This is known as the point at which the ball “bites”. As the ball continues to rotate there is a build-up of energy at the front, which is returned to the rear and transformed to an upwards translational velocity, causing the rear of the ball to become “unstuck” from the string-bed, thus causing it to rebound.
A simpler way of thinking about oblique impacts is to compare the ball impacting the string-bed to a car during heavy braking. The frictional force experienced by the front of the ball can be compared to the braking force applied to the front wheels of the car. As the car brakes, the car slides forward whilst also rising up at the rear end and lowering at the front end as the momentum from the rear of the car seeks to overcome the braking force at the front. The same principal applies to the ball during impact. As the ball slides along the string bed, the increasing friction force acts like the braking force on the front wheels of the car and, just like the car, the rear of the ball will begin to rise up. When the friction force is significant enough the front of the ball begins to move even slower than the rear of the ball, the ball begins to pivot around its front and rolling occurs.
8.2 FE modelling of oblique impacts

8.2.1 Creation of the model

Although several authors (Ashcroft et al., 2002, Bao, 2003, Cross, 2003) have looked at the results of changing the ball’s inbound angle, in terms of outbound spin and velocity, little research has been carried out into how the mechanism of the impact changes (i.e. when do the sliding/rolling phases begin/end). In order to obtain a more complete understanding of how the mechanisms of the impact change with inbound angle a simple model of a ball obliquely impacting a square string-bed was created. This model was similar in dimensions to the string-bed mounted on the tribometer. The strings’ geometry and mesh were generated using the strategy described in chapter 3. The resulting model can be seen in Figure 8.2.

An initial translational velocity of 30 metres per second (zero rotational velocity) was assigned to the ball and the trajectory of the ball was defined by altering the Y and Z velocity components, where Y and Z are the vertical and horizontal directions respectively. The ball model used was developed by Sissler et al. (2010) and required the inclusion of an inflation step to obtain a suitable internal pressure before the application of the velocity. The velocity boundary condition was created in a short dynamic/explicit step lasting 0.05 milliseconds before being deactivated in the impact analysis step; a dynamic/explicit (described in Section 3.1.2) step lasting 7 milliseconds. This procedure was repeated for the following impact angles, $\alpha$ (for a depiction of the impact angle see Figure 8.3):

- 32 degrees
- 34 degrees
- 39 degrees
- 45 degrees
The ball was fired against a string-bed smaller than that of a normal racket, in order to decrease the number of elements used, and hence computation time, whilst retaining a good geometrical accuracy of the strings.

The impact angle, $\alpha$, is the angle between the velocity vector and the plane of the string-bed (see Figure 8.3). These angles were chosen as they are amongst the most commonly used in experimental examination of oblique tennis impacts (Goodwill et al., 2002, Goodwill et al., 2004). The material and interaction properties used were that of synthetic gut, obtained in the previous chapters.

![Figure 8.2: Model used to examine oblique impacts.](image-url)
8.2.2 Results analysis

Once the analysis had been completed, a plot of the nodal force (Field output option NFORC) within the ball experienced during impact was plotted in Abaqus/Viewer. NFORC is defined in the Abaqus user’s manual (Abaqus, 2010) as “the force at the node from the regular deformation modes of the element”. These nodal forces are obtained from an element stiffness matrix similar to that shown in Equation 2.5, although the matrices generated by a finite element solver such as Abaqus for a dynamic analysis such as this will be much more complex.

The plot of the nodal forces was achieved using the “create XY data” option in Abaqus/Viewer (N.B. “XY data” is a term used to refer to the data plot, which has X and Y axes, and can be used for data of any sort, not just data in the X or Y planes). The data plot for the 32 degree impact can be seen in Figure 8.4.
Figure 8.4: Total nodal NFORC experienced by the ball for a 32 degree impact at 30 metres per second

As described in section 8.1, there are three distinct points during an impact where sliding and rolling are present. The force contour of the ball at each of the peak points can be seen below in Figures 8.5-8.7, where the grey area indicates stress “hot-spots”. Figures 8.5-8.7 show the ball at various stages of contact with the string-bed during an oblique impact. The images were obtained by “blanking” the string-bed from the animation such that the deformation created by the strings (the grey area) was visible. Comparing Figure 8.5 and Figure 8.6 shows the propagation of NFORC due to the ball deformation during sliding. The grey area on these plots represents the areas where the NFORC experienced by the ball exceeds the maximum value of 55.62 Newtons on the contour legend.
Figure 8.5: NFORC contour of the ball at the point where pure sliding is initiated

Figure 8.6: NFORC contour of the ball at the point where rolling is initiated
During the impact the ball experiences significant deformation. As the ball travels through the sliding phase it stretches in the Z direction while compressing in the X direction. As the frictional force brings the ball to rest translationally the ball begins to return to its original shape and in doing so initiates the rolling phase of the impact. This is also the point at which the maximum negative NFORC is experienced, as shown in Figure 8.4.

Figure 8.7: NFORC contour of the ball as it begins to rebound
Figure 8.8: Total NFORC experienced by the ball for a 34 degree impact

Figure 8.9: Total NFORC experienced by the ball for a 36 degree impact
Figure 8.10: Total NFORC experienced by the ball for a 40 degree impact at 30 metres per second

Comparing Figure 8.4, Figure 8.8, Figure 8.9 and Figure 8.10 shows the different rates at which the NFORC experienced by the ball accumulates for the different impact angles and the point in time at which it reaches its peak. In each case there is a visible plateau around the peak negative force. This negative force is generated by the compression of the ball during impact and the plateau represents the sliding phase of the impact. The point at which the ball begins to return to its original shape and rolling is initiated is identifiable from the plots as the point when the force begins to increase.

The most noticeable change in the NFORC plots is the peak positive value experienced by the ball. This indicates that for increasingly normal impacts the ball experiences a greater impact force and hence greater deformation. The second point of note is that the time between the maximum compressive (negative) force and the force beginning to return to zero is far greater for the 32 degree impact than the 40
degree impact. During this phase, where the NFORC holds relatively constant around its maximum, the ball experiences its maximum deformation and the dominant motion is sliding. It should be noted that the NFORC does not actually cease to increase as the ball begins to slide, its rate of increase simply slows.

The sliding phase of the ball occurs when a force large enough to overcome the frictional force is achieved and ends, when the elastic energy stored at the rear of the ball, due to deformation, is transferred to the fore and the ball begins to roll and rebound. As this rebound phase begins, the NFORC tends towards zero and then increases in the opposite direction as the ball is propelled away by the strings and its own elastic energy. The positive peak which can be seen on the NFORC plots is as a result of the ball expanding back to its original shape and then slightly beyond. Whilst the other slight dip below zero is simply the ball experiencing a slight contraction in shape before returning to its original form.

The previously described process can be seen in Figure 8.11 where the left most images are the front of the ball and the right most images are the rear of the ball. As the impact develops the difference in NFORC between the front and rear of the ball can be seen as the ball slides along the strings and stress increases. After the ball overcomes the frictional force and begins to roll, the transfer of energy to the rear of the ball can be seen from the changing contours. The same stress “hot-spots” are not present on the rear of the ball as energy is transferred, rather, there is a much more even distribution of stress around the rear of the ball as it begins to expand and separate from the strings.
Energy builds at the front of the ball as it slides and begins to bite.

The energy is transferred to the rear of the ball which causes it to separate, rebound and roll.

Figure 8.11: NFORC contours of (a) the front and (b) the rear of the ball during an oblique impact.
### Table 8.1: Oblique impact data

<table>
<thead>
<tr>
<th>Impact angle (degrees)</th>
<th>32</th>
<th>34</th>
<th>36</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact time (ms)</td>
<td>6.2</td>
<td>6.0</td>
<td>5.7</td>
<td>5.6</td>
</tr>
<tr>
<td>Time to roll (ms)</td>
<td>3.36</td>
<td>2.94</td>
<td>2.73</td>
<td>2.68</td>
</tr>
<tr>
<td>Time to peak F (ms)</td>
<td>4.6</td>
<td>4.35</td>
<td>4.1</td>
<td>3.7</td>
</tr>
<tr>
<td>Peak F (N)</td>
<td>-744.6</td>
<td>-751</td>
<td>-802</td>
<td>-891</td>
</tr>
</tbody>
</table>

Table 8.1 lists some of the key measurables for the different impact angles. In the case of the 32 degree impact, the NFORC increases relatively gradually before beginning to slide at around 2.8 milliseconds. Unfortunately, it is not possible to identify a specific point in time at which the sliding phase is initiated, as the entire ball’s trajectory does not change simultaneously.

The area of the ball closest to the string-bed will naturally be the first to deviate from the original flight path as the frictional force is overcome with the string-bed and the rest of the ball will change its trajectory as the force is increased and the ball deforms until it is sliding uniformly along the bed. For the 32 degree angle, once the sliding phase has been fully initiated, it continues for a period of about 1.2 milliseconds.

An annotated example of the NFORC plot for the 34 degree impact angle can be seen below in Figure 8.12, in which the sliding and post impact oscillation phases are labelled. It can be seen from the NFORC plots that, as the impact angle is increased the time taken to initiate sliding is reduced as is the length of the sliding phase; to the point where it is of negligible length for impact angles of 40 degrees and above.
Figure 8.12: Example NFORC plot of a ball during an impact with a string-bed

Another trend visible from the plot is the increasing nodal forces experienced by the ball post impact. This represents the more severe oscillation experienced by the ball at more normal impact angles.

8.3 Concluding comments

By exploring how oblique impacts change with inbound angle, a useful method for investigating the mechanisms of spin generation has been discovered. It was shown that for angles lower than forty degrees there are two distinct phases during the
impact; the biting phase and the rolling phase. It was also shown that the initiation of
the sliding phase is accelerated as the impact angle is increased and that the peak
force experienced by the ball is increased.
9 Dynamic validation of spin generation

Before using the model to predict the performance of novel string-bed designs it must first be validated against experimental data of an existing string pattern. This section describes the methods used to validate the model of said existing string pattern.

9.1 Normal impacts

9.1.1 Experimental set-up/method

To validate the normal impacts, a series of impacts were filmed using a Photron Fastcam SA5 high speed camera, as described in section 7.2.2.3. The tennis racket was clamped around its handle, and mounted to a frame (similar to the set-up used by Allen et al., 2010a) and placed within a poly-carbonate case for safety. Unlike the oblique impact described in Chapter 7 the tennis balls were fired at the racket along a velocity vector normal to the string-bed.

The same racket frame was used for each analysis and was strung to 223 Newtons tension with polyester, nylon and natural gut strings. After each stringing procedure, the racket was left to settle for twenty four hours in order to avoid the immediate drop in tension found by Cross (2001a). Again, the same pneumatic cannon was used to impart velocity on the ball but this time the velocity was varied between 15 metres per second and 30 metres per second. The variation in ball speed was obtained by adjusting the pressure of the pneumatic cannon as per Figure 7.6, with nine impacts performed for each string material.

The camera was positioned, such that the line of sight captured was parallel to the string-bed as shown in Figure 9.1. The camera recording was remotely triggered at the same time as the ball was fired from the cannon and the high speed videos of
each impact were analysed to give the inbound and outbound velocities as well as the contact time.

The contact times, $t_c$, were obtained simply by counting the number of frames for which the ball was in contact with the string-bed and multiplying this number by the length of a frame, in this case, 0.00025 seconds.

Measuring the COR required an inbound and outbound velocity measurement. In both instances, this was achieved by counting the number of frames (which subsequently gives a value of time) that it takes the ball to travel the length of its own diameter. The COR was then calculated using Equation 9.1:

$$COR = \frac{No.\ of\ frames\ inbound}{No.\ of\ frames\ outbound}$$

Equation 9.1
9.1.2 Results/Discussion

The contact times for the impacts can be seen in Figure 9.2. As one would expect the synthetic gut strings, conventionally regarded as the string material with the largest dynamic stiffness, have the longest contact time, whilst the ball tends to dwell slightly less on the natural gut and polyester strings. The opposite trend for the COR can be viewed in Figure 9.3, with the less stiff natural gut and polyester strings providing a greater outbound velocity relative to the polyester strings.
Figure 9.2: Contact time for experimental normal (β=90°) tennis racket impacts

Figure 9.3: COR for experimental normal (β=90°) tennis racket impacts
Figure 9.4 shows a comparison of how the COR varies with the contact time. The results show that the stiffer materials produce a longer contact time than less stiff strings. What is interesting to note from these results, however, is that the synthetic gut strings produce a lower COR for a given contact time relative to the other strings. This is evidence of the relatively poor elasticity of the synthetic gut strings since they are unable to return the same level of energy to the ball during the rebound phase. The increased elasticity of the natural gut relative to synthetic, however, allows it to experience greater deformation whilst dissipating less energy.

![Figure 9.4: COR versus contact time](image)

Such characteristics can make synthetic strings desirable to players as they allow for a longer contact time during impact, which in turn leads to increased control for the player, without losing too much power (outbound velocity).
9.2 **Oblique Impacts**

9.2.1 **Experimental set-up/methods**

A similar approach to that described in section 9.1 was used to obtain oblique impact data. The only difference in the two experimental set-ups was that in the case of the oblique impacts the racket was turned around the axis along its handle to vary the angle of the ball’s inbound angle, $\beta$, relative to the plane of the string-bed, as shown in Figure 9.5. Also, in the case of the oblique impacts, due to time constraints only the synthetic gut strings were analysed.

![Figure 9.5: Racket position during oblique impact](image)

The different angles tested were similar to those used to examine oblique impacts in section 8.2, ranging from 29 degrees to 45 degrees rotation around the axis parallel to the handle. The specific values of the angles were dictated by the nature of the racket clamp. As in section 9.1, the rackets were strung using natural gut strings to a tension of 223 Newtons and the inbound velocity was altered by varying the pressure of the pneumatic cannon.
9.2.2 Results/Discussion

The results of the experiment can be seen below in Figure 9.6, where a decreasing trend in outbound rotational velocity, as $\beta$ is increased, is apparent. These results are in keeping with Cross (2000a) who also found a decreasing level of spin generation as the inbound angle tended towards normal.

![Figure 9.6: Spin rate for oblique impacts at ball various speeds](image)

Furthermore, the results show that as the inbound velocity is increased, so too is the resulting outbound spin rate. In order for this greater level of outbound spin to occur, an increased transformation of energy must take place. This would suggest that either less energy is lost during the impact (and transformed into rotational velocity) or a greater level of translational velocity is transformed into rotational velocity; or indeed a combination of the two.
The relationship between inbound translational velocity and outbound rotational velocity could be investigated (much in the same way as the relationship between rotational velocity and inbound angle was investigated in Chapter 8) using a finite element model if the time were available to run all of the necessary simulations. However, given the time taken to run a simulation of this type (over twenty four hours) the resources were not available to investigate how the energy transformation varies as a result of changing inbound velocity.

9.2.3 FE Model

An FE model was compiled to recreate the experimental oblique impacts at 29.8 metres per second. The ball was assigned a velocity vector of 30 metres per second at an angle of 28 degrees, 32 degrees, 36 degrees, 39 degrees and 40 degrees to the horizontal. The material and interaction properties used were those of the polyester multifilament string obtained in Section 7.2.

Quantifying the rotational velocity of the ball is challenging as the ball is constructed using 3D elements. The rotational velocity, VR, is not available as an output for 3D elements since only the translational degrees of freedom are available. Another method of measuring the ball’s spin rate is therefore required. The rotational velocity is available for the shell elements used to model the foundation layer of the ball’s cloth. This is not a good indicator of the spin rate, however, as the ball is deforming locally and the nodal values of UR at these points will only show how much the elements are bending at this point.

Although the rotational properties of the ball’s elements are not available, it is still possible to calculate the rotational velocity of the ball using the Cartesian co-ordinates of the nodes. The co-ordinates of a point on the ball’s circumference are sampled at two time increments, t₁ and t₂ and using the average co-ordinates of the ball’s centre C as a reference point, the change in angle can be calculated. The
changing position is shown below in Figure 9.7, where $P_i$ and $C_i$ are the position of $P$ (a point on the surface of the ball) and $C$ (the centre of the ball) with respect to $t_i$. Since the ball rotates predominantly in the XY plane, change in $P$ and $C$ along the $Z$ axis is omitted from the calculation.

![Diagram of a rotating ball with points $P_1$, $P_2$, $C_1$, and $C_2$.]

Figure 9.7: Position of a point on the ball’s surface during rotation

Once the analysis is complete, the coordinate position of each of the ball’s outer surface nodes is obtained via the field output option and, using the “Operate on XY
Data” option from within the “XY data” menu, these points are averaged to obtain, C, the coordinates of the ball’s centre. Using the Cartesian co-ordinates of P in the X and Y direction the distance from C is then calculated.

The angle, \( \alpha_i \) for a point in time “i”, of the vector can be calculated using Equation 9.2.

\[
\alpha_i = \tan^{-1}\left[ \frac{P_y - C_y}{P_x - C_x} \right]
\]

Equation 9.2

The terms of this equation are depicted in Figure 9.8:
Figure 9.8: Change in angle of P with respect to C due to rotation
Having calculated the angle, $\alpha$ for each time increment, the spin rate can then be obtained by dividing the resulting change in angle by the respective change in time, as shown in Equation 9.3.

$$\text{Spin Rate} = (\alpha_2 - \alpha_1)/(t_2 - t_1) \quad \text{Equation 9.3}$$

$$\text{Spin Rate} = \frac{\tan^{-1}\left[(P_{y2} - C_{y2})/(P_{x2} - C_{x2})\right] - \tan^{-1}\left[(P_{y1} - C_{y1})/(P_{x1} - C_{x1})\right]}{t_2 - t_1}$$

In order to eliminate the effect of ball deformation the spin rate is calculated 2 milliseconds after the ball has achieved separation from the string-bed and any oscillations have been damped.

The above process was carried out for eight sample points around the ball’s perimeter. The initial results of the FE analysis, shown in Figure 9.9, displayed a decreased level of spin compared to the experimental results (for the 29.8 metres per second impacts from section 9.2.2) and also did not follow the same trend; increasing as the angle incidence is increased as opposed to decreasing. Looking at the FE impacts objectively, it seems that the prolonged contact time of the shallower angles is creating a higher level of energy dissipation and thus reducing the energy returned to the ball and hence, the rotational velocity.
At this point of the investigation, the material damping being used for the ball is the same stiffness proportional damping used by Sissler et al. (2010). The most commonly used method of damping, within Abaqus is Rayleigh damping. Rayleigh damping is defined within the material model and consequently applied to any element to which that material has been assigned. As shown in Equation 9.4, the Rayleigh damping matrix, $C_m$, is equal to a linear combination of the mass and stiffness matrices, $M$ and $K$ respectively:

$$C_M = \alpha_R M + \beta_R K$$

Equation 9.4

Where $\alpha_R$ is the mass proportional damping factor and $\beta_R$ is the stiffness proportional damping factor.
The non-linear rubber model used by Sissler et al. to represent the core of the ball is defined in such a way that as the deformation of the ball increases, the stiffness, \( K \), also increases. As a result the stiffness proportional component, \( \beta_RK \), increases at a non-linear rate, thus generating an increased level of damping.

As a result the value of beta damping applied (\( \beta = 0.0032 \)) by Sissler to the rubber core was removed and replaced with an alpha damping value of 1,000. This value of \( \alpha \) was obtained by a process of trial and error.

Since the contact time for impacts of smaller inbound angle (e.g. 30 degrees) is greater, the deformation is far more prolonged. As a result, the level of energy dissipation is far greater and energy which ought to be transformed into rotational velocity is lost.

With this in mind, the \( \beta_R \) component was removed from the model and replaced with a value of \( \alpha_R \). The \( \alpha_R \) component operates on the mass matrix which, for an analysis of this type, remains constant. As a result the energy dissipation stands to be more representative of experimental analysis and will not increase excessively with increased contact time.

To alleviate this excessive energy loss the beta damping was removed from the ball and mass proportional alpha damping was introduced. The results of the experimental analysis are plotted against the results of the different models containing alpha and beta damping in Figure 9.9. Whilst the beta damping model displays the opposite trend previously described, the alpha damping results are much more in keeping with the experimental results.

### 9.3 Concluding comments

A series of experimental impacts were performed for both normal and oblique inbound trajectory. The relationship between inbound velocity, contact time and COR
was investigated for polyester, synthetic gut (nylon) and natural gut. As found by Brody et al. (2002) the contact time increased with inbound velocity as a result of the higher deformation associated with a greater impact velocity. It was also shown that the COR decreased with increasing inbound velocity, due to a greater level of energy dissipation occurring during the elongated contact time.

A similar method of experimental measurement was then used to obtain the outbound spin rate of oblique tennis impacts. The outbound spin rate was obtained for a variety of inbound angles (28 degrees to 40 degrees) and velocities (15.8 metres per second to 29.8 metres per second), with a view to ultimately using the data to establish the accuracy of oblique impacts simulated using the finite element model.

The experimental results showed that as the inbound angle tended away from normal (90 degrees) the outbound spin rate increases. For example, in the case of the 29.8 metres per second impacts the spin rates were 4,000 revolutions per minute and 3,582 revolutions per minute for inbound angles of 28 degrees and 40 degrees respectively. Furthermore, it was also shown that as the impact velocity was increased, so too is the outbound spin rate. In the case of the impacts with an inbound angle of 28 degrees impact velocities of 29.8 metres per second yielded an outbound spin of 4,000 revolutions per minute compared to just 2,448 revolutions per minute for a lower impact velocity of 15.8 metres per second.

Initially, when comparing the outbound spin rate experimental results for the inbound velocity of 29.8 metres per second with that of the model, the opposite trend was observed. In the case of the model the spin rate was decreasing as the inbound angle tended away from the normal. Taking into account the behaviour of the ball during normal impacts, where increased energy dissipation occurs as a result of increased contact time, it was felt that in the model the increased contact time associated with more oblique inbound angles could be having a similar effect.
As a result, the material model used to represent the damping within the system was altered from being a stiffness based feature (one which increases non-linearly with the deformation of the ball) to a mass based feature (one which will remain constant regardless of deformation). This method of modelling material damping proved to be far more effective for oblique impacts and resulted in a trend which was much more in keeping with the experimental results.

Having developed and correlated a model which can be used to simulate outbound spin rates for oblique impacts, the next step in the project was to use the model to investigate how spin rate changes as key variables such as string spacing, gauge and orientation are altered.
10 Modelling of different string patterns

This chapter of the thesis brings all of the previous work together in an attempt to model novel string-bed arrangements and measure their spin generation. In doing so it is hoped that a greater understanding of how various string-bed characteristics affect spin can be achieved.

10.1 Automation of string-bed geometry creation

As a number of string-patterns are to be modelled, steps are taken to partially automate the procedure for generating a string-bed mesh. Although it is not possible to fully-automate the process, due to the different programs used to create the mesh, there is vast room for improvement, which will be addressed in this chapter.

In the procedure described in Section 3.3.1, the points on the string-bed where the strings intersect are measured using a CMM and imported into the CAD package NX (both of which are described in Section 3.1.1). This method is convenient when creating geometry of an existing racket but is not viable when generating a novel design.

The string variables which are altered during the analyses are those which a manufacturer or indeed a player would be readily able to specify when selecting a racket, such as:

- String density
- String orientation
- String gauge
- Young’s modulus
Therefore, a system is required, whereby a set of points representing the string geometry can be generated at the touch of a button, whilst allowing each of these variables to be instantly modified.

10.1.1 Generating points using Excel®

As well as generating splines through a series of specified points, NX also provides the option of generating splines with “points from file”, where the file is a text file containing the Cartesian coordinates of a series of points. The task of generating such files is undertaken with the use of Excel®. Using Excel®, a spread-sheet is created where lines representing the strings are generated using a series of formulae.

10.1.1.1 Creating the X and Y position of the strings

One of the first things to consider when defining the geometry of the string-bed is the X and Y position of the string bed. The spread-sheet was set-up with the cells containing the main variables – string spacing and angle – position in the top left of the page. The format of these cells is shown in Table 10.1:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>strings</td>
<td>spacing</td>
<td>angle deg</td>
</tr>
<tr>
<td>2</td>
<td>Mains</td>
<td>20</td>
<td>10</td>
<td>Ф</td>
</tr>
<tr>
<td>3</td>
<td>Cross</td>
<td>20</td>
<td>10</td>
<td>φ</td>
</tr>
</tbody>
</table>

Table 10.1: String variable definition cells

The “Y” position of the mains strings is then determined using the formulae shown in Table 10.2, whilst the “X” position of cross strings is generated using the formulae displayed in Table 10.3.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>cross 1 = (B7+C$3)</td>
</tr>
<tr>
<td>7</td>
<td>cross 2 = (B8+C$3)</td>
</tr>
<tr>
<td>8</td>
<td>cross 3 = (B9+C$3)</td>
</tr>
<tr>
<td>9</td>
<td>cross 4 = (B10+C$3)</td>
</tr>
<tr>
<td>10</td>
<td>cross 5 = (B11+C$3)</td>
</tr>
<tr>
<td>11</td>
<td>cross 6 = (B12+C$3)</td>
</tr>
<tr>
<td>12</td>
<td>cross 7 = (B13+C$3)</td>
</tr>
<tr>
<td>13</td>
<td>cross 8 = (B14+C$3)</td>
</tr>
<tr>
<td>14</td>
<td>cross 9 = IF(B3=17,0+D3,IF(B3=16,(-(C3/2)+D3,IF(B3=18,(C3/2)+D3,B15+C3))))</td>
</tr>
<tr>
<td>15</td>
<td>cross 10 = IF(B3=19,0+D3,IF(B3=18,B14-C3,IF(B3&lt;18,B14-C3,B16+C3))))</td>
</tr>
<tr>
<td>16</td>
<td>cross 11 = IF(B3=22,(C3/2)+D3,IF(B3=21,0+D3,IF(B3=20,(-(C3/2)+D3,B15-C3))))</td>
</tr>
<tr>
<td>17</td>
<td>cross 12 = (B16-C$3)</td>
</tr>
<tr>
<td>18</td>
<td>cross 13 = (B17-C$3)</td>
</tr>
<tr>
<td>19</td>
<td>cross 14 = (B18-C$3)</td>
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<tr>
<td>20</td>
<td>cross 15 = (B19-C$3)</td>
</tr>
<tr>
<td>21</td>
<td>cross 16 = (B20-C$3)</td>
</tr>
<tr>
<td>22</td>
<td>cross 17 = IF(B$3&lt;17,&quot;&quot;,(B21-C$3))</td>
</tr>
<tr>
<td>23</td>
<td>cross 18 = IF(B$3&lt;18,&quot;&quot;,(B22-C$3))</td>
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<td>cross 19 = IF(B$3&lt;19,&quot;&quot;,(B23-C$3))</td>
</tr>
<tr>
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<td>cross 20 = IF(B$3&lt;20,&quot;&quot;,(B24-C$3))</td>
</tr>
<tr>
<td>26</td>
<td>cross 21 = IF(B$3&lt;21,&quot;&quot;,(B25-C$3))</td>
</tr>
<tr>
<td>27</td>
<td>cross 22 = IF(B$3&lt;22,&quot;&quot;,(B26-C$3))</td>
</tr>
</tbody>
</table>

Table 10.2: Formulae used to calculate the “Y” position of the cross strings

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>mains 1 = (E7+C$2)</td>
</tr>
<tr>
<td>7</td>
<td>mains 2 = (E8+C$2)</td>
</tr>
<tr>
<td>8</td>
<td>mains 3 = (E9+C$2)</td>
</tr>
<tr>
<td>9</td>
<td>mains 4 = (E10+C$2)</td>
</tr>
<tr>
<td>10</td>
<td>mains 5 = (E11+C$2)</td>
</tr>
<tr>
<td>11</td>
<td>mains 6 = (E12+C$2)</td>
</tr>
<tr>
<td>12</td>
<td>mains 7 = (E13+C$2)</td>
</tr>
<tr>
<td>13</td>
<td>mains 8 = IF(B2=16,IF(((C2/2)+D2)&lt;0,&quot;&quot;,C2/2+D2),E14+C2)</td>
</tr>
<tr>
<td>14</td>
<td>mains 9 = IF(B2=17,0+D2,IF(B2=18,(C2/2)+D2,IF(B2=16,(-(C2/2)+D2),E15+C2)))</td>
</tr>
<tr>
<td>15</td>
<td>mains 10 = IF(B2=20,C2/2+D2,IF(B2=19,0+D2,IF(B2=18,(-(C2/2)+D2),(E14-C2))))</td>
</tr>
<tr>
<td>16</td>
<td>mains 11 = E15-C2</td>
</tr>
<tr>
<td>17</td>
<td>mains 12 = E16-C2</td>
</tr>
<tr>
<td>18</td>
<td>mains 13 = E17-C2</td>
</tr>
<tr>
<td>19</td>
<td>mains 14 = E18-C2</td>
</tr>
<tr>
<td>20</td>
<td>mains 15 = E19-C2</td>
</tr>
<tr>
<td>21</td>
<td>mains 16 = E20-C2</td>
</tr>
<tr>
<td>22</td>
<td>mains 17 = IF(B2&lt;17,&quot;&quot;,E21-C$2)</td>
</tr>
<tr>
<td>23</td>
<td>mains 18 = IF(B2&lt;18,&quot;&quot;,E22-C$2)</td>
</tr>
<tr>
<td>24</td>
<td>mains 19 = IF(B2&lt;19,&quot;&quot;,E23-C$2)</td>
</tr>
<tr>
<td>25</td>
<td>mains 20 = IF(B2&lt;20,&quot;&quot;,E24-C$2)</td>
</tr>
</tbody>
</table>

Table 10.3: Formulae used to calculate the “X” position of the main strings
Since the centre of the string-bed is the origin it is necessary to construct the equations in such a way that the string closest to the origin can be changed depending on the number of strings (e.g. for a string-bed with 16 cross strings, strings 8 and 9 would be either side of the origin but if there were 20 cross strings, it would be strings 10 and 11). For this reason, conditional “IF” statements are included for the strings which will be at the centre of the string-bed and the equations for the other strings are defined with respect to the central strings.

A standard scatter plot was created using the values generated from the formulae listed in Table 10.3. The X value of the intersection was given by the main strings formulae and the Y value was given by the cross strings formulae. This resulted in a series of lines similar to that seen below in Figure 10.1:

![Figure 10.1: Scatter plot generated from the formulae in Table 10.2 and Table 10.3](image-url)
10.1.1.2 **Creating X and Y positions for angled string-beds**

The procedure for generating angled string-beds is largely similar to that described in the previous section. The main difference is that when a non-zero value of the angle $\Phi$ is specified, the lines generated from the mains formulae are divided by $\cos \Phi$. Similarly, when a non-zero value of $\varphi$ is specified the cross strings are divided by $\cos \varphi$. As a result, the angled lines are generated from the scatter plots and using the “line intercept” function in Microsoft Excel® the intersection points of the string-bed are extracted.

10.1.1.3 **Exporting the string intersection points from Excel®**

Having created a spread-sheet which can conveniently create the points for the strings, the next step in the automation process is exporting the data from Excel® and into NX. This is achieved, using a “macro” which allows the user to specify a series of actions which can be re-created at the touch of a button. In this case, the actions are selecting each of the columns which represented a single string and, individually writing them to a .dat file (a text file compatible with the “create points from file” feature in NX). The .dat file produced a number of points to represent the strings, one for each point where the string met the racket and one for each string intersection. The string intersection points are offset by the radius of the string in order to create a 3D woven bed without penetration. In order for this to be successful, the user must first save the spread-sheet to a convenient location, before running the macro, as this will define the location to which the .dat files (which contain the points define the strings) are saved when the program is run.

Due to incompatibilities in the software it is not possible to automate the stage of the process during which the splines are transformed from 1-dimensional lines to 3D parts. This process must be undertaken manually by sketching a circle at the end of the spline and using the “sweep along guide” feature within NX (Figure 10.2), where
the shape of the circle is swept along the length of the spline to create a solid geometry. Following this procedure, the user is then left with a 3D solid, as shown in Figure 10.3, which can be exported into the meshing package, Hypermesh, to be meshed before being analysed.

Figure 10.2: Splines through offset points to create woven bed

Figure 10.3: Circles swept along splines to create 3D strings
10.2 Spin of different string-beds

10.2.1 Different string density

Using the method outlined in section 10.1, a number of different string-beds, with varying string-density, are generated. The characteristics of the string-beds are detailed in Table 10.4.

<table>
<thead>
<tr>
<th>String-bed</th>
<th>Mains/Crosses</th>
<th>String spacing</th>
<th>String diameter</th>
<th>Modified E (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>String-bed 1</td>
<td>16/16</td>
<td>12 mm</td>
<td>1.38 mm</td>
<td>$2.800 \times 10^3$</td>
</tr>
<tr>
<td>String-bed 2</td>
<td>16/17</td>
<td>12 mm</td>
<td>1.38 mm</td>
<td>$2.760 \times 10^3$</td>
</tr>
<tr>
<td>String-bed 3</td>
<td>17/18</td>
<td>10 mm</td>
<td>1.38 mm</td>
<td>$2.625 \times 10^3$</td>
</tr>
<tr>
<td>String-bed 4</td>
<td>18/20</td>
<td>10 mm</td>
<td>1.38 mm</td>
<td>$2.483 \times 10^3$</td>
</tr>
</tbody>
</table>

Table 10.4: Characteristics of the string-beds

Initially, the string-beds are all assigned the material properties of natural gut at 223 Newtons tension, the properties of which were as follows:

\[
E = 2.483 \times 10^3 \text{MPa} \\
\rho = 1301 \text{kg/m}^3 \\
\nu = 0.35
\]

An explicit analysis is performed in which the ball impacts the string-beds on a velocity vector of 30 metres per second, 32 degrees from parallel to the string-bed (i.e. 58° to normal). An oblique impact is used in order to generate post-impact spin.

As the contact time of an impact is increased, so too is the potential for energy, to be dissipated through ball deformation (which otherwise could be transformed into
rotational velocity). Given that spin, therefore, could be dependent on contact time, which itself is dependent on the stiffness of the string-bed, a secondary analysis is performed where the Young’s modulus of the strings is altered such that all the string patterns are of the same stiffness.

To obtain the global stiffness of the string-bed a concentrated load of 10 Newtons is applied to the fifteen most central nodes – thus giving a cumulative load of 150 Newtons - within the string-bed along the Z axis (the direction normal to the string-plane). The stiffness of the bed is then calculated using the Z-displacement, (Abaqus field output, U3), and the modulus of the each string-bed is adjusted such that all string patterns gave the same value of U3 under load as the 18 by 20 bed. The modified values of E can be seen in Table 10.4.

10.2.2 Different string gauge

In terms of the variability of performance characteristics of different string gauges Brody et al. (2002) states that; since the elasticity of a string is proportional to the inverse of its cross sectional area thinner strings are more desirable. This is because thinner strings will deform elastically more under a given load and provide the player with increased control and power. However, Brody makes little reference to the spin generation performance of different string gauges.

One would expect that increasing the string diameter of a string-bed would have a similar effect to increasing the string density, as the gaps between the strings will decrease and the string-bed will become stiffer. However, as has been observed in the previous chapter, the relationship between string-bed stiffness and mechanical interlocking can lead to varying levels of spin. It seems prudent, therefore, to perform a similar set of experiments to those in the previous section, in which the variation in spin for the three most conventional string gauges (1.25 millimetres, 1.3 millimetres and 1.38 millimetres) is examined.
The model used to perform the analysis is similar to String-bed 4 used in section 10.2.1, the only variation being the string gauge. This is altered by changing the cross-sectional area of the strings at the CAD stage of the model development such that three different string beds are generated for the diameters previously specified.
10.2.3 Different string orientation

Having observed the variation in spin due to changes in string density, another interesting question may be: how is spin generation affected by the string orientation? It can be assumed that increased mechanical interlocking will result from the increased surface asperities associated with a higher string density. It is unclear, however, how spin levels would react to significant changes in the magnitude and orientation of those surface asperities which would result from the orientation of the strings being altered.

Using the string-bed generation system described in section 10.1, a series of string beds are created with different string orientations. The string-beds all contain 20 cross strings and 20 main strings and had a cross sectional diameter of 1.3 millimetres. The orientation of the strings is altered by varying the angle, \( \Phi \), of the cross and main strings relative to one another, displayed in Figure 10.4.

![Figure 10.4: Novel string-bed arrangement](image.png)
In total, seven string-beds (shown in Figure 10.5) are tested, with the value of $\Phi$ varying from 30 degrees to 60 degrees in 5 degree intervals. The string orientations of each of the string-beds are listed in Table 10.5 and shown in Figure 10.5. As in the previous section the stiffness of the string-beds are tested prior to performing a full impact analysis and the results of these analysis can be seen in Table 10.5. In this case, however, the overall stiffness’s of the string-beds did not vary significantly ($<1\%$), therefore it was not considered necessary to alter the Young’s modulus to eradicate the effect of the stiffness may have on spin generation.

<table>
<thead>
<tr>
<th>String-bed</th>
<th>$\Phi$ (degrees)</th>
<th>Stiffness (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>16.05</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>16.1</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>16.11</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>16.08</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>16.02</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>16.05</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>16.1</td>
</tr>
</tbody>
</table>

Table 10.5: String orientation of the modelled string-beds
As in previous analyses, a velocity vector with an overall magnitude of 30 metres per second at an angle of 32 degrees to the plane of the string-bed is imparted on the ball. Having completed the various simulations, the outbound spin rates are obtained using the procedure defined in section 9.2.3.
10.2.4 Spin generation of different string materials

In order to characterise the spin generation performance of different string properties, the different material properties obtained earlier for the titanium polymer and the polyester string are submitted to the model, to be compared with the results obtained for the synthetic gut string. The ball is subjected to a velocity vector with a magnitude of 30 metres per second at an incidence angle of 32 degrees perpendicular to the string bed. The different string materials and their corresponding properties can be seen below in Table 10.6. All strings had a cross-section diameter of 1.38 millimetres.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus (MPa)</th>
<th>Density (kg/m³)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titanium</td>
<td>(5.015 \times 10^3)</td>
<td>2056</td>
<td>0.35</td>
</tr>
<tr>
<td>Polyester</td>
<td>(3.886 \times 10^3)</td>
<td>1565</td>
<td>0.35</td>
</tr>
<tr>
<td>Synth Gut</td>
<td>(4.369 \times 10^3)</td>
<td>1670</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 10.6: Material properties for different strings

10.3 Results/Discussion

10.3.1 Different string-bed densities

![Figure 10.6: Spin rate for different string densities](image)
It can be seen from Figure 10.6 that the spin tends to increase with the number of strings. There are a number of factors which could lead to this increase. Firstly it could be the result of an increased level of mechanical interlocking between the strings and the ball due to the increased number of strings.

One of the most interesting points to note from this result is that increasing the main strings (conventionally considered to be the string with the most significant contribution to spin generation Brody et al. (2002)) has a lesser effect on the generation of spin than increasing the cross strings. Figure 10.7 shows that there is an increase in spin of around 11% when 1 cross string and 1 main string are added compared to an increase of only 2.5% when 3 main strings are added. This raises the question of whether there is a critical string spacing at which the increase in spin - due to reduction in the area between the strings - begins to plateau.

![Figure 10.7: Outbound spin for different string densities](image)

The respective spin rates for the normal and modified values can be viewed in Figure 10.7. It can be seen that the spin rates for the strings with a modified value of E (detailed in Table 10.4) are reduced relative to the strings with a normal value of E
(2.483 \times 10^3 \text{ Megapascals}). By viewing these results it can be seen that spin rate does increase with string-bed density.

As a point of reference, the ball is also subjected to a 30 metres per second impact at an angle of 32 degrees relative to the horizontal, against a rigid plate. This allows us to see the levels of spin obtained without the contribution of mechanical interlocking and effect on contact time due to string-bed deformation. Interestingly, the values of spin for the plate are consistently less than those for the normal values of E but in the case of the modified values of E, the most open string-bed (16 by 16) is actually less than the plate.

This raised the question of whether or not the two factors in question (mechanical interlocking and contact time) are independently variable with regards to the spin or if, indeed, they interact at some point to increase spin levels, i.e. for a string density less than 16 by 17 the increased contact time leads to an outright reduction in spin relative to a solid, rigid surface. This would explain why, for the unmodified value of E, the 16 by 16 string pattern still generates a greater level of spin than the rigid plate, since it is less stiff and has a shorter contact time for the reduction in energy due to mechanical interlocking to propagate.

As was stated in section 8, there are two mechanisms which are critical to the generation of spin, the sliding phase and the biting phase. If the sliding phase is prolonged, as would be the case for the rigid surface due to its lack of surface asperities, greater levels of energy dissipation occur which leads to a reduction in spin rates. In order to maximise spin, therefore, the sliding phase must be minimised and the biting phase accelerated. These results show that this is possible with mechanical interlocking, such as that introduced by a woven string bed. If the asperities become too large, however, the contact time will be inadvertently increased and all benefit lost.
10.3.2 Different string orientation

The variation in spin rate due to string orientation can be seen in Figure 10.8.

![Graph showing spin rates for different string orientations](image)

**Figure 10.8: Spin rates for different string orientations**

The first conclusion to be drawn from the results is that the spin rates are consistently lower than that of the conventional string-bed configurations. Initial suspicions that this reduction in spin could be due to the change in stiffness of the string-beds relative to the conventionally strung beds were dispelled by performing a stiffness test similar to that described in section 10.2, which showed a 22% reduction in stiffness in the case of the novel string arrangements. This reduction in the overall stiffness of the string-bed is due the fact that the distance between the string intersections is generally larger for the novel string-beds than for the conventional string-beds. As a result, deformation of the strings between their intersections (and therefore across the string-bed as a whole) occurs more freely for the novel string-beds.
Figure 10.8 does, however, display a generally increasing trend as $\Phi$ is increased, peaking at 50 degrees before decreasing slightly. It would seem, therefore, that the spin tends to increase as the angle of strings opens up to the flight-path of the ball, e.g. configuration (a) in Figure 10.9 and Figure 10.10 will generate more spin than configuration (b) (where configuration (a) represents a string configuration with a greater value of $\Phi$ than configuration (b)).

This can be explained by the fact that after the strings are deformed by the ball in the direction of its flight path, they will return this energy in the opposite direction, thus imparting rotational energy upon the ball. The reason why configuration (a) is more able to do this is that the component, L, which performs the role of the mains string – in terms of imparting rotational velocity on the ball – is greater than configuration (b). As a result, it is less resistant to deforming in the direction of the ball’s flight and hence has a greater potential to absorb translational energy as it experiences a greater level of elongation (Figure 10.10).

![Diagram showing ball impacting different string configurations](image)

(a)              (b)

Figure 10.9: Ball impacting different string configurations
This also explains why the conventional configuration produces more spin, since having the main strings normal to the flight path of the ball provides the optimum opportunity for the type of string deformation required for spin generation.

10.3.3 Different string gauges

The results of the analysis can be viewed in Figure 10.11. As expected, there is a slight reduction in spin for the largest diameter (1.38 millimetres), due to the increase in stiffness it creates. The minimal difference, however, makes it difficult to ascertain whether the increased surface area of a larger string diameter will have any effect. Even if one could ascertain the effect a larger diameter has on the difference in spin generation, the observed effect would likely be so negligible that it would be of little worth.
10.3.4 Different string material

The results of this analysis can be seen in Figure 10.12. As one would expect, the stiffest string material, the titanium polymer, gives the lowest level of spin. This is because, as previously stated, a stiffer string bed leads to higher levels of ball deformation which, in turn leads to a higher level of energy loss. Conversely, a lower level of ball deformation occurs on the synthetic string bed and, as a result, increased levels of spin occur.
10.4 Concluding comments

In order to allow novel string-bed designs to be generated quickly and efficiently an automated string-bed generation system was created. Using this system the spin generation of string-beds with different string orientation, density and gauge were examined. It was found that the most significant factor in spin generation was the stiffness of the string-bed and that, generally, any change in spin generation for differing string patterns is often, in part due to the resultant change in stiffness.

It was shown however, that due to an increased level of mechanical interlocking, outbound spin tended to increase with string density. It was also shown that for novel string-bed orientations, the outbound spin increased as the angle opposite the ball’s flight path increased. None of the novel string-beds tested, however, resulted in outbound spin greater than that of a conventional string bed.
11 Conclusions and Further Work

11.1 Conclusions

The primary objective of this study was to create an FE model of a tennis racket, for which suitable CAD geometry of the racket was a fundamental requirement. Not only was this achieved but a partial automation of the CAD generation was also accomplished. This was achieved through the creation of a Microsoft Excel spreadsheet based system which allowed the user to specify key parameters of a string-bed before exporting the fundamental geometry data. The Excel document produces a 2D line representing the string which the user can export to the CAD system NX5.0, whereupon they are required to create the 3D volume of the string. Some further work to fully automate the process would, therefore, still be beneficial.

Having created and meshed the racket geometry, several strategies of tensioning the strings whilst connected to the racket were examined. A method using contracting “connector” elements, attached to both the racket frame and strings, was adopted as it allowed the racket and the strings to be loaded simultaneously.

With a satisfactory finite element model of the racket in place, several static validation procedures were developed for the model before the ball was introduced. The racket was first validated statically, by testing the bending stiffness of the racket frame (strung and unstrung) and the stiffness profile of string-bed. The bending stiffness of the racket was obtained using a traditional 3 point bend test and the bending stiffness of the strung and unstrung rackets were found to be 117 kilonewtons per metre and 129 kilonewtons per metre respectively. Furthermore, the model was found to correlate well in both the strung and unstrung states yielding a variation of less than 5% in both cases.
The stiffness profile of the string-beds were obtained by creating a plot of the individual stiffness values for each intersection of a cross and main string. The stiffness of each intersection was obtained using a force measurement machine, after which the plots were created using Microsoft Excel. This was performed for string tensions of 178 Newtons, 223 Newtons and 267 Newtons with the resulting plots showing that as the string tension increases, so too does the variation in string stiffness across the string bed. For the lowest string tension the stiffness was between 20 kilonewtons per metre and 24 kilonewtons per metre for 95% of the intersections – with only the intersections closest to the frame outside this range.

A simulation of this testing was carried out with the FE model to enable the correlation of its string-bed stiffness profile – a method not previously used to validate finite element model of tennis rackets. The model showed a slightly lower magnitude of stiffness – 16 kilonewtons per metre to 19 kilonewtons per metre for the majority of intersections – but was still of a similar enough magnitude to allow the model to be used for dynamic impacts. To date, this technique has not been used by any other author for correlating finite element models of tennis rackets.

The technique of photogrammetry was also used to validate the deformation of the racket frame under the tension of the strings. The GOM Aramis system used for this work is traditionally used in the automotive industry and has not previously been applied to research of this kind. Two conditions were tested; the racket fully strung and the racket strung with mains strings only. The Aramis system captured images of the racket in its strung and unstrung state and by tracking a random speckle pattern on the racket’s surface, was able to produce a strain contour projected onto the image of the strung racket. This was then compared to the strain values given by the FE model when subjected to similar string loading. The fully strung model predicted the location and magnitude (2%-3%) of all the highest strains. The model did not predict the higher strains experienced by the racket when strung with mains strings only as accurately as in the fully strung racket. Although the model did predict the location of the highest strains it predicted values of 5% as opposed to the 7% yielded by the photogrammetry analysis.
In order to validate the vibrational properties of the racket 3D laser doppler vibrometry was used. Previous modal analysis studies have been carried out for tennis racket frames using contacting measurement techniques (Brody, 1995, Cross, 2001b and Mohanty et al., 2001). The non-contact approach used in this thesis, however, allowed for the extraction of the natural frequencies of the string-bed as well as the frame.

Using this technique, the lateral (131 Hertz) and torsional (353 Hertz) frequencies of the racket frame and their respective natural frequencies were obtained. These values were compared to those obtained from a simple analysis of the FE model and found to have a correlation error of less than 5% in both cases. The lateral values of the strung racket were also obtained for the racket strung at varying tensions of 178 Newtons, 223 Newtons and 267 Newtons and found to show no level of variation higher than that which, could be attribute to experimental error (<1%). The vibration modes of the string-bed tensioned to 178 Newtons were also acquired experimentally (576 Hertz and 853 Hertz) but were slightly less cohesive with the finite element model (560 Hertz and 768 Hertz), yielding an error of under 5% and 10% for first and second modes respectively.

Having validated the model’s static performance and vibrational characteristics a series of tests were performed to acquire and examine the properties of a variety of different string materials at varying tensions. The strings were subjected to a number of extension and impact tests to establish how their elongation and energy absorption properties varied under different loading conditions. It was found that the most consistent string tension over a variety of different loading speeds was 223 Newtons whilst nylon displayed the highest dwell time in impact testing which, in practice would lead to greater energy dissipation and hence a reduction in COR.

Previously, a number of authors have researched the issue of friction between the ball and the string-bed but to the author’s knowledge a value of the friction coefficient between the strings themselves has not been obtained. As a result a number of
experimental procedures were defined to establish the COF between the strings and
the ball and the strings themselves. It was shown ball/string COF varied significantly
from the string/string value, with the ball/string value producing values in the range of
0.19 to 0.53 depending on string construction, material and normal force, whilst the
string/string values were in the range of 0.12 to 0.23. A tribometer was also
developed in order to more accurately represent the scenario of a rolling ball’s
interaction with the string-bed. Using the tribometer the variation in friction for a
number of different string-bed configurations was explored and found to be at its
highest level for a densely strung, conventional string-bed.

Using the various properties obtained, a model was set-up to examine how the
mechanisms of oblique impacts changed with impact angle. It was shown that three
distinct phases; sliding, biting and rolling, exist within an oblique impact of a certain
angle range but as the impact direction moves towards normal the sliding phase
becomes less obvious and is eventually eradicated above angles of 40 degrees
(where 90 degrees is normal to the string bed).

Contour plots of the nodal forces due to element stress (NFORC) during these
oblique impacts were created. These plots showed how the nodal forces increased at
the front of the ball as the sliding phase began - with a number of nodes at the
central area of the leading edge experiencing forces as high as 66.1 Newtons. The
transfer of this energy was evident from following increments of the analysis, which
showed an immediate reduction of nodal forces at the leading edge of the ball, whilst
nodal forces at the rear of the ball increased to 55.6 Newtons. This transfer of energy
leads to the separation of the ball from the string-bed and the ultimately the initiation
of rotational velocity.

Oblique impacts of the ball impacting the string-bed were performed for varying
inbound angles (ranging from 29 degrees to 40 degrees at 29.8 metres per second)
experimentally and compared to those obtained from a similar simulation using the
FE model. Initially it was found that the FE model showed the opposite trend of the
experimental data, with spin decreasing as the angle tended away from normal. Upon further inspection, it became clear that this was due to the method used to model energy dissipation (stiffness proportional damping) over-damping the model.

As a result the energy dissipation was modelled using mass proportional damping in the strings. The beta damping component of 0.0032 applied the ball’s rubber core was removed and replaced with an alpha damping component of 1,000. This method of damping yielded computational values which correlated within 10% of the experimental values. The values of spin produced by the model varied from 4,000 revolutions per minute to 3,000 revolutions per minute for an inbound angle range of 29 degrees to 30 degrees respectively. This was compared to experimental outbound spin values ranging from 4,200 revolutions per minute to 3,300 per minute for the same range of inbound angles.

In the final chapter of the thesis the primary objective of modelling novel string patterns was achieved. Using the string-bed generation technique described earlier, a number of string-beds were created and using the properties which, had been validated earlier in the model, a series of analyses were performed. Among the analyses run were string-beds which varied in string pattern density (i.e. number of strings), string gauge, string orientation and string material. In the case of the different string pattern density and material it was found that the densest pattern (20 cross strings and 18 mains strings) and the synthetic gut strings gave the highest level of spin, whilst the novel string orientations were shown to offer no improvement on conventionally strung beds.

11.2 Future Work

Although this model achieved the desired goal, in terms of predicting spin generation, a great deal of work could still be done to reduce the time taken to perform simulations. The most straightforward way of reducing the computation time of the
model would be to refine and reduce the mesh. The mesh used in this study was chosen as it represented the geometry well and was uniform throughout the racket and string-bed. It would be possible to reduce the number of elements in the model by introducing a coarser mesh in less critical areas of the racket (e.g. away from the impact zone of the string-bed and on the racket handle).

Furthermore, there are a wide variety of interaction models available in Abaqus which could be used to model the contact properties of a tennis racket impact. The “All With Self” model was used because of its robustness and ease of application but a less computationally expensive model - which may be just as accurate - could be available. Another reason for not exploring other materials models in this thesis is that they generally require specific interaction property data, which would also give rise to further experimental work. However, if it was deemed worthwhile, it would be possible to acquire the necessary data, either through consulting an established company or by setting up specific equipment such as the Tribometer detailed in Chapter 7.

Another way in which the model could be improved is with the introduction of Hyper-elastic material models. Currently, a linear elastic material model is used to represent the behaviour of the strings. Although this proved effective for impact speeds of up to 30 metres per second it is likely that impacts of a higher velocity would require a non-linear material curve. The acquisition of such materials data would require specialist equipment, capable of testing samples at much higher strain rates than were possible for this study. If such equipment were available then it would be a worthwhile investment of a research student’s time to investigate how the string properties change at higher strain rates.

The model could also be improved by introducing non-uniform materials property assignment to the string-bed. It was shown in Section 4.3 that the stiffness profile of the model’s string-bed, although of a similar magnitude, differed from the experimental data. By applying a non-uniform materials assignment the user could
reverse-engineer the stiffness profile of the string-bed to match the experimental data more closely.

There is also potential for further work in the area of Photogrammetry. The strain contour plots acquired using the GOM system were not as comprehensive as they could have been, and values were missing for some areas of the racket. By analysing the sections of the racket which displayed the highest strains, rather than the entire racket, it may be possible to obtain a more detailed strain contour of the racket.

Although the values of the natural frequencies obtained using the LDV in Chapter 5 displayed a good repeatability (plus or minus 0.5 Hertz for the fundamental mode of the racket frame) the animation of the mode shapes did display some discrepancies at the extreme ends of the racket. These discrepancies could be removed by performing a series of more focussed analyses on different sections of the racket and “stitching” them together to form an animation of the whole racket.

The validity of this project has reached a good level in terms of the semi-dynamic (i.e. static racket and moving ball) validation data, given the validation data currently available. Spin data from an actual tennis stroke could be acquired but achieving the repeatability which would be needed to compare the performance of different string types and bed configurations is still unattainable at this time. Given the ever increasing technologies available in the sports industry however, it would not be inconceivable to perform repeatable tennis impacts against a moving racket mimicking the motion imparted on it by a player. In fact, such a study, relating to the foot-strike of a running shoe, already exists (Ronkainen et al., 2009) and could provide a useful template for this work. This would be the natural progression to this study and would yield further confidence in the spin generation predictions the model could provide.
An area which could potentially affect the performance of tennis rackets - and hence the reliability of this model - is the environmental conditions in which they are used. The conditions in which tennis is played can differ significantly, even at the same tournament (NDTV Sports, 2012), and the ability to predict how this will affect the performance of a given racket configuration would be greatly beneficial. Future work in this field could include modelling the effects of the frame and strings subjected to varying levels of temperature and humidity and comparing the results to experimental analysis performed within a controlled environment of similar conditions. If the model were to be used to predict the variation in performance due to environmental conditions it would be prudent to perform a further comprehensive validation of the model, given the extra level of uncertainty that varying environmental conditions would introduce. As a result, the model would need to be validated at various stages of complexity, as in this thesis, to give confidence and understanding of how the environment affects the racket frame and strings during play.
12 References


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13 Appendices

13.1 Appendix 1: ITF regulation for tennis equipment

a. The hitting surface, defined as the main area of the stringing pattern bordered by the points of entry of the strings into the frame or points of contact of the strings with the frame, whichever is the smaller, shall be flat and consist of a pattern of crossed strings connected to a frame and alternately interlaced or bonded where they cross. The stringing pattern must be generally uniform and, in particular, not less dense in the centre than in any other area. The racket shall be designed and strung such that the playing characteristics are identical on both faces. The racket shall be free of attached objects, protrusions and devices other than those utilised solely and specifically to limit or prevent wear and tear or vibration or, for the frame only, to distribute weight. These objects, protrusions and devices must be reasonable in size and placement for such purposes.

b. The frame of the racket shall not exceed 29.0 inches (73.7 cm) in overall length, including the handle. The frame of the racket shall not exceed 12.5 inches (31.7 cm) in overall width. The hitting surface shall not exceed 15.5 inches (39.4 cm) in overall length, and 11.5 inches (29.2 cm) in overall width.

c. The frame, including the handle, and the strings, shall be free of any device which makes it possible to change materially the shape of the racket, or to change the weight distribution in the direction of the longitudinal axis of the racket which would alter the swing moment of inertia, or to change deliberately any physical property
which may affect the performance of the racket during the playing of a point. No energy source that in any way changes or affects the playing characteristics of a racket may be built into or attached to a racket.
### Appendix 2: GOM Aramis technical data (GOM, 2012)

<table>
<thead>
<tr>
<th>System Configurations</th>
<th>High Speed Camera</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame Rate (Hz)</td>
<td>up to 500 (4000)</td>
</tr>
<tr>
<td>Camera Resolution (pixel)</td>
<td>1280 x 1024</td>
</tr>
<tr>
<td>Measuring Area</td>
<td>mm² to &gt; m²</td>
</tr>
<tr>
<td>Strain Measuring Range (%)</td>
<td>0.01 up to &gt; 100</td>
</tr>
<tr>
<td>Strain Measuring Accuracy (%)</td>
<td>up to 0.01</td>
</tr>
<tr>
<td>Image Memory</td>
<td>uses PC RAM</td>
</tr>
<tr>
<td>Tool Free Mounting</td>
<td>No</td>
</tr>
<tr>
<td>Positioning Pointers</td>
<td>1 or 3</td>
</tr>
<tr>
<td>Illumination</td>
<td>external</td>
</tr>
<tr>
<td>High-End PC</td>
<td>Yes</td>
</tr>
<tr>
<td>Notebook</td>
<td>N/A</td>
</tr>
<tr>
<td>Control Device</td>
<td>Sensor Controller</td>
</tr>
<tr>
<td>Operating Temperature</td>
<td>5-40°C</td>
</tr>
<tr>
<td>Specimen Temperature</td>
<td>typ. -100°C up to +1500°C</td>
</tr>
</tbody>
</table>
13.3 Appendix 3: Abaqus error message

“The Abaqus/Explicit restart job cannot proceed due to a violation of the domain decomposition of the original analysis. There are one or more nodes involved in constraint that were shared nodes in the original analysis but are now part of an implicit constraint system and hence are non-shared nodes in the restart analysis. This could be due to application of boundary conditions on these nodes in the restart step. A dummy step may be introduced in the original analysis with the same set of boundary conditions on these nodes as defined in the restart step, to prevent this error. A node set name “ErrNodeDomainDecompReStrt” has been created for use in Abaqus/Viewer to identify these nodes.”