Effects of track properties on ground vibrations generated by high-speed trains

This item was submitted to Loughborough University's Institutional Repository by the/an author.

Citation: KRYLOV, V.V., 1998. Effects of track properties on ground vibrations generated by high-speed trains. Acustica [Acta Acustica united with Acustica], 84 (1), pp.78-90.

Additional Information:

- This article was accepted for publication in the journal, Acustica, and the definitive version is available from: http://www.ingentaconnect.com/content/dav/aaua

Metadata Record: https://dspace.lboro.ac.uk/2134/12352

Version: Accepted for publication

Publisher: © S. Hirzel Verlag / EAA

Please cite the published version.
This item was submitted to Loughborough’s Institutional Repository (https://dspace.lboro.ac.uk/) by the author and is made available under the following Creative Commons Licence conditions.

For the full text of this licence, please go to:
http://creativecommons.org/licenses/by-nc-nd/2.5/
Effects of Track Properties on Ground Vibrations
Generated by High-Speed Trains

V.V. Krylov

Centre for Research into the Built Environment,
Nottingham Trent University,
Burton Street, Nottingham NG1 4BU, UK
SUMMARY

Increase in speeds of railway trains is usually accompanied by higher levels of generated ground vibrations, which are especially large if train speeds exceed the velocities of Rayleigh surface waves in the ground. Therefore, it is vitally important to consider possible methods of protecting the built environment against intensive ground vibrations associated with high-speed trains. The present paper investigates the effects of some geometrical and physical properties of the track, i.e., those of track bending waves and distance between sleepers, on railway-generated ground vibrations. It is shown that the effect of track bending waves may cause increase in amplitudes of ground vibrations generated by trains travelling at speeds higher than the velocities of Rayleigh surface waves in the ground (trans-Rayleigh trains) if the train speeds approach the minimal phase velocity of track bending waves. This implies that tracks with low minimal phase velocities of bending waves should be avoided. The reduction of a sleeper period results in decrease of generated ground vibrations for trains travelling at speeds below the velocities of Rayleigh surface waves in the ground (sub-Rayleigh trains). This effect can be used for reduction of generated ground vibrations in selected parts of the rail route. For trans-Rayleigh trains, the reduction of a sleeper period does not affect the vibration levels. Theoretical results are illustrated by numerically calculated frequency spectra of ground vibrations generated by single axle loads travelling at different speeds and by TGV, Eurostar and British high-speed trains.
1. Introduction

Over the last few years, the railways have become one of the most advanced and fast developing branches of transportation (Banister & Hall, 1994; Streeter, 1994). The reasons are relatively low air pollution per passenger, compared to road vehicles, and very high speeds achievable at the most advanced modern trains, e.g., French TGV-trains for which maximum speed of more than 515 km/h was recorded in May 1990. Prospective plans for the year 2010 assume that the New European Trunk Line will have connected Paris, London, Brussels, Amsterdam, Cologne and Frankfurt by high-speed railway service that will provide fast and more convenient passenger communications within Europe.

Unfortunately, the increased speeds of modern trains are accompanied by increased levels of associated noise and vibration that are significant even for conventional railways (Remington et al., 1987; Walker & Ridler, 1991; Newland & Hunt, 1991).

Although a number of experimental and theoretical investigations of generated ground vibrations have been carried out for conventional passenger and heavy-freight trains travelling both above- and underground (Dawn, 1983; Volberg, 1983; Melke, 1988; Flemming, 1993; Ford, 1990; Heckl et al., 1987; Jones, 1994; Krylov & Ferguson, 1993, 1994; Krylov, 1994), very little has been done so far with regard to vibrations from high-speed trains. Theoretical investigations of such kind have been recently undertaken by the present author (Krylov, 1994, 1995a). Three-dimensional ground vibration fields from high speed trains have been studied using the Green’s function formalism taking into account generation of ground waves by each sleeper of the track subject to the action of the carriage wheel axles. The deflection of a track under applied vertical axle forces have been calculated in quasi-static approximation using a simple model of a track as an Euler-Bernoulli beam lying on Winkler foundation. It has been
shown (Krylov, 1994, 1995a) that high-speed trains are generally accompanied by higher levels of generated ground vibrations. Especially large increase in vibration level (more than 70 dB, as compared to conventional trains) may occur if train speeds $v$ exceed the velocity of Rayleigh surface waves in the ground $c_R$ (we suggest to call such trains "trans-Rayleigh trains"). The condition $v > c_R$, which is similar to that of supersonic jets, can be met e.g., by French TGV-trains travelling along tracks placed on relatively soft grounds.

In the present paper, we examine the effects of some geometrical and physical properties of the track, i.e., the effect of track bending waves freely propagating in the system track/ground and the effect of changing distance between sleepers in the track on railway-generated ground vibrations. The main aim of this investigation is exploring ways of possible use of these effects for protecting the built environment against intensive ground vibrations associated with high-speed trains.

In the following sections we describe the outline of the theory of generating ground vibrations by high speed trains and analyse the effects of track bending waves and inter-sleeper spacing on generated ground vibrations. Finally, we discuss the results of the numerical calculations of ground vibration frequency spectra generated by single axle loads travelling at different speeds and by complete TGV, Eurostar and British high-speed trains.

2. Outline of the theory

2.1. Generation mechanisms

As has been demonstrated experimentally for conventional trains (Remington et al., 1987; Dawn, 1983; Volberg, 1983; Melke, 1988; Flemming, 1993), there are several mechanisms of
generating ground vibrations which may contribute to the total ground vibration level in
different frequency bands. Among these mechanisms one can mention the wheel-axle pressure
onto the track, the effects of joints in unwelded rails, the unevenness of wheels or rails (all
these mechanisms cause vibrations at trainspeed-dependent frequencies), and the dynamically
induced forces of carriage- and wheel-axle bending vibrations excited mainly by unevenness of
wheels and rails (these occur at their natural frequencies). The most common generation
mechanism is a pressure of wheel axles onto the track. It always persists, whereas all other
mechanisms may be eliminated (at least in theory) if rails and wheels are ideally smooth and no
carriage or wheel-axle bending vibrations occur. For very high quality tracks and wheels, the
wheel-axle pressure mechanism is probably a major contributor to trainspeed-dependent
components of the low-frequency vibration spectra (up to 50 Hz), including the so called main
passage frequency \( f_p = v/d \), where \( v \) is train speed and \( d \) is the distance between sleepers. In
this paper we consider contribution of the wheel-axle pressure mechanism only, assuming that
rails and wheels are ideally even and no carriage or wheel-axle vibrations are excited.

2.2. Statement of the Problem

Let us consider a train having \( N \) carriages and moving at speed \( v \) on a perfect welded track
with sleeper periodicity \( d \) (Figure 1,a). The wheel-axle pressure generation mechanism being
considered results in downward deflections of the track beneath each wheel axle (Figure 1,b).
These deflections produce a wave-like motion along the track moving at speed \( v \) and resulting
in distribution of the axle load over the sleepers involved in the deflection distance (Krylov &
Ferguson, 1993, 1994; Krylov, 1994, 1995a). Thus, each sleeper acts as a vertical force
applied to the ground during the time necessary for a deflection curve to pass through the
sleeper. These result in generation of ground vibrations by passing trains. Since in the
considered frequency band (up to 50 Hz) the characteristic wave-lengths of generated elastic waves are much larger than the sleeper dimensions, each sleeper can be regarded as a point vertical force. The problem then requires superposition of the elastic fields radiated by all sleepers due to the passage of all wheel axles (Figure 1,c).

2.3. Quasi-static Calculation of Track Deflection Curves

An important aspect of analysing the above discussed wheel-axle pressure mechanism is calculation of the track deflection curve as a function of the elastic properties of track and soil and of the magnitude of the axle load. The form of the deflection curve determines ground vibration frequency spectra generated by each sleeper. In turn, these spectra strongly affect the resulting ground vibration spectrum generated by a passing train.

Since the track deflection distance is usually greater than the distance between sleepers, one can ignore the influence of rail periodic support by sleepers in the problem of track deflection under the impact of a moving load. Instead we treat a track (i.e., two parallel rails with periodically fastened sleepers) as an Euler-Bernoulli elastic beam of uniform weight \( p = m_0 g \) lying on an elastic or viscoelastic half space \( z > 0 \). For simplicity we assume that the uniform mass \( m_0 \) of the beam is formed entirely by the track (i.e., by rails and sleepers only), although in practice an adhered layer of ballast may form an additional mass which might result in increase of axle critical loads in the nonlinear deflection problem (see below) and in reduction of track wave velocities.

Firstly we recall a quasi-static approach to the solution of this problem. The classical solution starts with the static beam equation that models the response of an elastic half space as a force reaction of an elastic (Winkler) foundation which is proportional to the beam deflection magnitude \( w \). If \( E \) and \( I \) are Young's modulus and the cross-sectional momentum of the beam,
α is the proportionality coefficient of the elastic foundation, \( x \) is the distance along the beam and \( T \) is a vertical point force applied to the beam at \( x=0 \), then the static equation has the form (Timoshenko, 1942; Weitsman, 1970)

\[
EI \frac{\partial^4 w}{\partial x^4} + \alpha w = T \delta(x) + p,
\]

where \( \delta(x) \) is the Dirac’s delta-function. The solution of (1) may be written as

\[
w_{st} = \left(\frac{T}{8EI\beta^3}\right) \exp(-\beta|x|) \left[ \cos(\beta x) + \sin(\beta|x|) \right] + \frac{p}{\alpha}.
\]

where \( \beta = (\alpha/4EI)^{1/4} \). Index “\( st \)” in eqn (2) and in following expressions indicates values obtained in the quasi-static approximation. According to eqn (2), one can take \( x_{st} = \pi/\beta \) as the effective quasi-static track deflection distance.

The constant \( \alpha \) in (1) and (2) depends particularly on the stiffness of the ballast layer, of the ground and of the rubber pads inserted between rail and sleepers and under sleepers. In further consideration we assume that a well-compacted ballast layer is always present and is largely responsible for the proportionality coefficient \( \alpha \) of the equivalent Winkler foundation corresponding to the combined system ballast/ground. The results of static track deflection tests show (Brockley, 1992) that, e.g., for British Rail tracks lying on a well-compacted ballasted roadbed, the coefficient \( \alpha \) is determined mainly by the ballast layer. Typically \( EI = 4.85 \, MNm^2 \) and \( \alpha = 52.6 \, MN/m^2 \) (cited value of \( \alpha \) also includes contribution of rubber pads characterised by distributed over a sleeper spacing coefficient of proportionality \( \alpha_{rp.} = 357 \, MN/m^2 \)). This results in \( \beta = 1.28 \, m^{-1} \). For a typical distance between sleepers, \( d = 0.7 \, m \), about seven sleepers are involved in the deflection distance \( x_{st} = \pi/\beta \) associated with each axle (see Figure 1,b).
A more advanced approach to an analogous problem (Weitsman, 1970) acknowledges that for the type of loading under consideration tensile stresses cannot be transmitted between the beam and the elastic foundation. Such a model is more appropriate for track-soil contacts as they can respond only to compressive stresses. Thus, in this approach the contact nonlinearity of a real boundary between track and ground is taken into account.

Analysis of this model shows (Weitsman, 1970) that for values of the axle load $T$ less than a certain critical value $T_{cr}$, i.e., $T \leq T_{cr} = (2p/\beta)\exp(\pi)$, the simple classical solution (2) which describes a continuous contact between track and foundation is valid. However, for $T > T_{cr}$ the solution becomes more complicated and involves peripheral bulges of the track with loss of contact between track and soil. In this case the problem can be solved numerically versus the coordinates $x_0^{st}$ and $x_1^{st}$ of the deformed track where it intersects the ground level ($z = 0$), and for the five coefficients describing the shape of the deflection curve as a function of applied load. For passenger trains, however, it is always $T < T_{cr}$, and the classical solution (2) is sufficient.

After the track deflection curve having been determined, each sleeper may be considered as a vertical concentrated force applied to the ground surface $z=0$, with time dependence determined by the passage of the deflection curve through the sleeper. For a sleeper resting on Winkler foundation and located at $x = 0$ the general expression for this force, which is valid for both quasi-static and dynamic regimes, may be written in the form

$$P(t) = T[2w(vt)/w_{max}^{st}](d/x_0^{st})$$

where $w_{max}^{st}$ is the maximal value of $w(vt)$ in quasi-static approximation. To derive (3), one should take into account that $P(t)$ is proportional to the track deflection $w(vt)$ and to the sleeper width $\Delta d$: $P(t) = \alpha w(vt)\Delta d$, where $\alpha$ is a constant of Winkler foundation. To
exclude $\alpha$ and $\Delta d$ from this expression, one should integrate the quasi-static equation (1) over $x$ and neglect $p$ in comparison with $T$. The integration, which takes into account that contact takes place only between sleepers and the ground, results in the formula $\alpha w_{\text{max}}^{\text{st}} \Delta d N_{\text{eff}}^{\text{st}} = T$ which, combined with the previous one, gives the following expression for $P(t)$: $P(t) = T[w(vt)/w_{\text{max}}^{\text{st}}N_{\text{eff}}^{\text{st}}]$. Here $N_{\text{eff}}^{\text{st}}$ is the effective number of sleepers equalising the applied quasistatic axle load $T$:

$$
\frac{\sum_{m=-N_{\text{eff}}^{\text{st}}/2}^{N_{\text{eff}}^{\text{st}}/2} T} {N_{\text{eff}}^{\text{st}} w_{\text{max}}^{\text{st}}} = T
$$

(4)

where $m$ denotes a number of a current sleeper. Numerical solution of eqn (4) shows that for $\beta$ within the range of interest (from 0.2 m$^{-1}$ to 1.3 m$^{-1}$) the value of $N_{\text{eff}}^{\text{st}}$ may be approximated by a simple analytical formula $N_{\text{eff}}^{\text{st}} = \pi/2 \beta d = x_0^{\text{st}}/2d$. Using this formula in the expression for $P(t)$ gives eqn (3).

2.4. Dynamic Calculation of Track Deflection Curves

For high-speed passenger trains it may happen that train speeds $v$ become of the same order as the minimal phase velocity $c_{\text{min}}$ of dispersive bending waves propagating in the system track/ballast. In this case one can expect that dynamic effects may play a noticeable role in determining the track deflection curve. To calculate dynamic forces applied from sleepers to the ground in a rigorous way one should consider wave propagation in a coupled wave-supporting system comprising track and ground, one of which (track) being excited by a rapidly moving load. However, this rigorous approach is extremely complex. The traditional way of its simplification is the replacement of the elastic ground by Winkler foundation, as it’s usually being done for quasi-static loads (see the previous section). Strictly speaking, such a
replacement is valid only for slowly moving loads compared to the velocities of elastic waves in the ground, although some authors used it for calculating rail deflections for much higher load speeds (e.g., Fryba, 1973; Belzer, 1988). One should be aware that validity of the corresponding results for real ground is at best qualitative. Another pitfall can occur if the track wave velocity is close or equal to the Rayleigh wave velocity in the ground. In this case the coupling between the above mentioned two wave-supporting systems is the strongest, and determination of the dynamic forces applied to the ground using calculations based on wave propagation in the track alone may not be valid. To avoid this difficulty we will not consider the case when the track wave velocity is close or equal to the Rayleigh wave velocity in the ground.

Keeping all these in mind, we transfer from the static equation (1) to the dynamic equation of a beam on an elastic foundation (see, e.g., Belzer, 1988):

\[ EI \frac{\partial^4 w}{\partial x^4} + m_0 \frac{\partial^2 w}{\partial t^2} + \alpha w = T \delta (x-vt) . \]

In writing (5) we have neglected the influence of a uniform weight \( p \) in the right-hand side: this would be essential only in the case of heavy-freight trains for which axle loads may be larger than \( T_{cr} \).

It is useful first to consider free wave propagation in the supported beam, i.e., to analyse (5) with the right-hand side equal to zero. In this case, substitution of the solution in the form of harmonic bending waves \( w = A \exp(ikx - i\omega t) \) into (5) gives the following dispersion equation for track waves propagating in the system:

\[ \omega = (\alpha + Ek^4)^{1/2}/m_0^{1/2} . \]
In the quasi-static (long-wave) approximation \((k = 0)\) the dispersion equation (6) reduces to the well known expression for the so called track on ballast resonance frequency: 

\[ \omega_{tb} = \alpha^{1/2}/m_0^{1/2}. \]

For \(\alpha = 52.6\) MN/m\(^2\) and \(m_0 = 300\) kg/m this gives \(F_{tb} = \omega_{tb}/2\pi = 67\) Hz. The frequency \(F_{tb}\) represents the minimal frequency of propagating track waves. It follows from (4) that the frequency-dependent velocity of track wave propagation \(c = \omega/k\) is determined by the expression 

\[ c = (\alpha k^2 + EI k^2/m_0)^{1/2}, \]

which shows that at \(k = (\alpha EI)^{1/4}\) the velocity \(c\) has a minimum \(c_{\min} = (4\alpha EI/m_0^2)^{1/4}\). For the above mentioned typical track and ballast parameters \(c_{\min} = 326\) m/s \((1174\) km/h). 

The dynamic solution of (5) with the right-hand side different from zero has the form \((\text{Belzer, 1988})\)

\[ w(x-vt) = (T/8EI\beta^3 \delta \exp(-\beta \delta |x-vt|) [\cos(\beta \eta (x-vt) + (\delta/\eta) \sin(\beta \eta |x-vt|)] \]

which generalise the static solution (2). Here \(\delta = (1 - v^2/c_{\min}^2)^{1/2}\) and \(\eta = (1 + v^2/c_{\min}^2)^{1/2}\). 

It is seen from (8) that if the train speed \(v\) approaches the minimal phase velocity of free track waves \(c_{\min}\) then \(\delta \rightarrow 0\) and the amplitude \(w\) in (8) goes to infinity, demonstrating a resonance behaviour. The infinite value of track deflection at resonance reflects limitations of the linear elastic model considered. In this paper we discuss only the case \(v < c_{\min}\).

For typical parameters of track and ballast mentioned above the value of \(c_{\min}\) is essentially larger than even the highest train speed \((v = 515\) km/h). However, for specially designed vibro-isolated tracks or for soft marshy soils in the absence of ballast the value of \(c_{\min}\) may be much lower.
To calculate the forces applied from sleepers to the ground we should substitute the expression (8) into equation (3) which is valid for both quasi-static and dynamic regimes. Since the factor \( \delta = (1 - v^2/c_{\text{min}}^2)^{1/2} \) is present in the denominator of (8), these forces increase as the train speed approaches the minimal track wave velocity.

The forms of a dynamic deflection curve \( w(x) \) calculated according to eqn (8) for \( v < c_{\text{min}} \) are shown in Figure 2 for the axle load \( T = 100 \text{ kN} \) and for the values of train speed \( v \) equal to 0, 69, 138, 300 and 320 m/s (curves w1-w5 respectively). One can see that the curves corresponding to the first three values of train speed (i.e., up to 500 km/h) are almost indistinguishable. Only for train speeds approaching the minimal (critical) track wave velocity \( c_{\text{min}} = 326 \text{ m/s} \) a significant difference occurs. The corresponding forces \( P \) applied from each sleeper to the ground are shown in Figure 3 as functions of \( vt \) for the same parameters as in Figure 2 (curves P1-P5). As train speeds approach the minimal track wave velocity \( c_{\text{min}} \) these forces undergo a number of oscillations at the circular frequency \( \omega = \beta \eta v = \beta(1+v^2/c_{\text{min}}^2)^{1/2}v \). This may result in distortions in generated ground vibration spectra at the upper frequency bands.

2.5. Green’s Function Formalism

2.5.1. Generation of Ground Vibrations by Individual Sleepers

The physical meaning of the Green’s function for the problem under consideration is that it describes ground vibrations generated by individual sleepers which can be regarded as point sources in the low-frequency band. To derive the corresponding Green’s function one can make use of the results from the well-known axisymmetric problem for the excitation of an elastic half space by a vertical point force applied to the surface (at the low-frequency band considered we neglect the influence of a thin ballast layer on elastic wave propagation). The
solution of this problem gives the corresponding components of the dynamic Green's tensor (or, for simplicity, the Green's function) $G_{zi}$ for an elastic half space. This function satisfies the dynamic equation of elasticity for a half space and the boundary conditions on the surface. If an elastic half space is assumed isotropic and homogeneous, then the dynamic equation of elasticity is the well known Lame' equation:

$$
(\lambda + 2\mu) \text{grad} \text{ div} \mathbf{u} - \mu \text{rot} \text{ rot} \mathbf{u} - \rho_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} = 0.
$$

(9)

The boundary conditions on the ground surface, which take into account a vertical point source located at $x = x'$ and $y = y'$, have the form

$$
\sigma_{xz} = 2\mu u_{xz} = 0,
$$

$$
\sigma_{yz} = 2\mu u_{yz} = 0,
$$

$$
\sigma_{zz} = \lambda u_{nn} + 2\mu u_{zz} = -\delta(x-x')\delta(y-y')\delta(t-t').
$$

(10)

Here $\mathbf{u}$ is the particle displacement vector with the components $u_i$; $\lambda$ and $\mu$ are the elastic Lame' constants; $\rho_0$ is mass density of the ground; and $u_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ are components of the linearised strain tensor.

The solutions of this and related problems of an elastic half space excitation by different load forces, whether using the terminology of Green’s function or otherwise, have been considered by many authors beginning with H. Lamb (Lamb, 1904). Different methods of solution of the Lamb’s problem, both in time and frequency domains, are reviewed in the books (Ewing et al., 1957; Graff, 1975).
In what follows only Rayleigh surface waves (the Rayleigh part of the Green's function) are considered since Rayleigh waves transfer most of the vibration energy to remote locations. For these waves the spectral density of the vertical vibration velocity at $z=0$ may be written in the form (see also Krylov & Ferguson, 1994)

$$v_z(\rho, \omega) = P(\omega)G_{zz}(\rho, \omega) = V(\omega)(1/\sqrt{\rho}) \exp(ik_R\rho - \gamma k_R\rho),$$  \hspace{1cm} (11)

where

$$V(\omega) = (\pi/2)^{1/2}P(\omega)(-i\omega)q k_R^{1/2} k_t^2 \exp(-i3\pi/4) / \mu F'(k_R).$$  \hspace{1cm} (12)

Here \(\rho = [(x-x')^2 + (y-y')^2]^{1/2}\) is the distance between the source (with current coordinates \(x', y'\)) and the point of observation (with coordinates \(x, y\)), \(\omega = 2\pi F\) is a circular frequency, \(k_R = \omega/c_R\) is the wavenumber of a Rayleigh surface wave, \(c_R\) is the Rayleigh wave propagation velocity, \(k_l = \omega/c_l\) and \(k_t = \omega/c_t\) are the wavenumbers of longitudinal and shear bulk elastic waves, where \(c_l = [(\lambda + 2\mu)/\rho_0]^{1/2}\) and \(c_t = (\mu/\rho_0)^{1/2}\) are longitudinal and shear propagation velocities, and \(q = (k_R^2 - k_t^2)^{1/2}\). The factor \(F'(k_R)\) is a derivative of the so called Rayleigh determinant

$$F(k) = (2k^2 - k_t^2)^2 - 4(k^2 - k_t^2)^{1/2}(k^2 - k_l^2)^{1/2}$$  \hspace{1cm} (13)

taken at \(k = k_R\), and \(P(\omega) = (1/2\pi) \int_{-\infty}^{\infty} P(t) \exp(i\omega t) dt\) is a Fourier transform of \(P(t)\).

In writing (11) we have accounted for attenuation in soil by replacing \(1/c_R\) in the exponential of the Green’s function by the complex value \(1/c_R + i\gamma/c_R\), where \(\gamma = 0.001 - 0.1\).
is a constant describing the "strength" of dissipation of Rayleigh waves in soil (eqn (11) implies a linear frequency dependence of soil attenuation, in agreement with the experimental data (White, 1965; Gutovski & Dym, 1976; Jones & Petyt, 1991). The factor $1/\sqrt{\rho}$ in (11) describes cylindrical spreading of Rayleigh waves with propagation distance.

It is seen from (11) and (12) that the Fourier transform $P(\omega)$ plays an important role in determining spectra of radiated waves. Taking the Fourier transforms of (8) and (3) at $x = 0$, one can easily obtain the corresponding formula for $P(\omega)$ which takes into account the effect of track bending waves:

$$P(\omega) = \left(\frac{T}{\pi d^2} \delta\right) \left[\frac{\beta \delta v + (\beta \eta v + \omega)}{((\beta \delta v)^2 + (\beta \eta v + \omega)^2)} + \frac{\beta \delta v + (\beta \eta v - \omega)}{((\beta \delta v)^2 + (\beta \eta v - \omega)^2)}\right].$$

(14)

In the quasi-static limit, $v/c_{min} \rightarrow 0$ resulting in $\delta \rightarrow 0$ and $\eta \rightarrow 1$, the equation (14) goes over to the corresponding quasi-static expression for $P(\omega)$ (Krylov & Ferguson, 1994; Krylov, 1995a). Comparison of the dynamic and the quasi-static spectra $P(\omega)$ for three values of train speed, $v = 69$ (curves P1, P2), 138 (curves P3, P4) and 300 m/s (curves P5, P6), is shown in Figure 4. It is seen that significant difference between quasi-static and dynamic curves occurs only for the value of $v = 300$ m/s which is close to the minimal track wave velocity.

To describe the spectrum for successive passage of two axle loads separated by the distance $a$ (the case of a bogie), $P_b(\omega)$, one should use the following obvious relationship between $P_b(\omega)$ and $P(\omega)$ (Krylov & Ferguson, 1994):

$$P_b(\omega) = 2P(\omega) \cos(\omega a/2v).$$

(15)
2.5.2. *Ground Vibrations from Single Axle Loads and Complete Trains*

To calculate the ground vibration field radiated by a train requires superposition of fields generated by each sleeper activated by all axles of all carriages, with the time and space differences between sources (sleepers) being taken into account (Figure 1,c).

Using the Green's function this may be written in the form (Krylov & Ferguson, 1994)

\[
v_z(x,y,\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x',y',\omega) G_{zz}(\rho,\omega) dx' dy', \quad (16)
\]

where \( P(x',y',\omega) \) describes the space distribution of all load forces acting along the track in the frequency domain. This distribution can be found by taking a Fourier transform of the time and space dependent load forces applied from the track to the ground.

It is useful firstly to consider a single axle load moving at speed \( v \) along the track lying on the elastic ground. Then the load force which makes a wave-like motion along the track may be written in the form

\[
P(t, x', y'=0) = \sum_{m=-\infty}^{\infty} P(t-x'/v) \delta(x'-md) \delta(y'), \quad (17)
\]

where \( \delta(x'-md) \) takes the periodic distribution of sleepers into account.

The Fourier transform of (17) is being written as follows

\[
P(x',y',\omega) = (1/2\pi) \sum_{m=-\infty}^{\infty} P(t-x'/v) \exp(\iota \omega t) \delta(x'-md) \delta(y') dt \quad (18)
\]
which, after integration over $t$, yields

$$P(x', y', \omega) = P(\omega) \exp(i \omega x'/v) \sum_{m=-\infty}^{\infty} \delta(x'-md) \delta(y'). \quad (19)$$

Substituting (19) into (16) and using the properties of integrating delta-functions, we have, after taking eqns (11), (12) into account, the following formula for the vertical vibration velocity of Rayleigh waves generated at $z = 0, x = 0, y = y_0$ by a single axle load moving along the track at speed $v$:

$$v_z(x=0, y=y_0, \omega) = V(\omega) \sum_{m=-\infty}^{\infty} \exp\left[i(\omega/v)md + (i-\gamma)(\omega/c_R)\rho_m\right]/\sqrt{\rho_m}, \quad (20)$$

where $\rho_m = [y_0^2 + (md)^2]^{1/2}$. Formula (20) shows that a single axle load moving at conventional speeds ($v << c_R$) generates a quasi-discrete spectrum with frequency peaks close to $f_p(s)$, where $f_p = v/d$ is the so-called main passage frequency, and $s = 1, 2, 3, \ldots$ Deviation from perfect discreteness results from the $i(\omega/c_R)\rho_m$ term in eqn (20) which takes into account path-length differences of waves propagated from each sleeper to the point of observation.

To take account of all axles and carriages one needs a more complicated load function:

$$P(t, x', y'=0) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n[P(t - (x'+nL)/v) + P(t - (x' + M + nL)/v)] \delta(x'-md) \delta(y'). \quad (21)$$

where $N$ is the number of carriages, $M$ is the distance between the centres of bogies in each carriage and $L$ is the total carriage length. Dimensionless quantity $A_n$ is an amplitude weight-
factor to account for different carriage masses. For simplicity we assume all carriage masses to be equal \((A_n=1)\).

Taking the Fourier transform of (21), substituting it into (16) and making simple transformations similar to the above, we obtain the following expression for the frequency spectra of vertical vibrations at \(z=0, x = 0\) and \(y = y_0\) generated by a moving train:

\[
v_z (x=0, y=y_0, \omega) = V(\omega) \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} \left[ \exp(-\gamma \omega \rho_m / c_R) / \sqrt{\rho_m} \right] \left[ 1 + \exp(i M \omega / v) \right] \cdot \exp(i (\omega / v)(md + nL) + i(\omega / c_R) \rho_m).
\]

(22)

The summation over \(m\) in (20) and (22) considers an infinite number of sleepers. However, the contribution of remote sleepers is small because of soil attenuation and cylindrical spreading of Rayleigh waves, and a few hundred sleepers are adequate for practical calculations.

3. Spectra of Generated Ground Vibrations

3.1. Conventional Trains

For conventional trains \((v \ll c_R)\) it follows from eqn (22) that spectra of generated ground vibrations are quasi-discrete, with the maxima at frequencies determined by the condition \((\omega / v)(md + nL) = 2\pi l\), where \(l = 1, 2, 3, \ldots\). Obviously, \(n=0\) corresponds to the passage frequencies \(f_p\), determined by the sleeper period \(d\). Other more frequent maxima are
determined either by the carriage length \( L \) (\( m=0 \)) or by a combination of both parameters (for \( n\neq0, \ m\neq0 \)).

There are also many zeros present in the train vibration spectra which may be used in practice for suppressing vibrations from conventional trains at chosen frequencies (Krylov & Ferguson, 1994). The most important zeros are those which do not depend on a number of sleepers or carriages and are determined only by the geometrical parameters of a carriage. For example, one of these zeros is determined by the distance \( a \) between the wheel axles in a bogie (see eqn (15) for the spectrum \( P_b \)). Setting \( P_b \) to zero, one can obtain \( f_z = (v/a)(n + 1/2) \) for zero-frequencies. If, for instance, we want to use this condition to suppress one of the train passage frequencies \( f_p\delta \), we should choose \( f_z \) to be equal to \( f_p\delta \). It follows from this that the value of \( a \) should be selected as

\[
a = (d/s)(n + 1/2) .
\]

For British Rail freight carriages \( a = 2.2 \ m \) usually. Therefore, to suppress the main passage frequency (\( s=1 \)) one can choose \( a= 2.45 \ m \) corresponding to \( n=3 \) in eqn (23).

Other important zero frequencies reflect the distance \( M \) between bogies in a carriage. Condition (23) is also valid for this case if \( a \) in eqn (23) is replaced by \( M \). The value of \( M \) providing suppression of the main passage frequency which is closest to the British Rail standard for freight carriages (\( M=4.88 \ m \)) is \( 4.55 \ m \), corresponding to \( n=6 \).

### 3.2. Trans-Rayleigh trains

The general expressions (20) and (22) derived above are applicable to trains moving at arbitrary speeds. However, for the specific case of "trans-Rayleigh trains", i.e., trains travelling at speeds higher than the Rayleigh wave velocity in the ground, an additional analytical
treatment is useful to elucidate the special features of the problem and to clarify the time and space distributions of radiated waves.

We first consider the vibration field generated by a single load \((N = 1, M = 0, L = 0, a = 0)\) moving at speed \(v\) through a part of a track having a small number of sleepers \(2Q + 1\). Let the point of observation be arbitrarily located on the ground surface, i.e., \(\rho_m = \sqrt{y^2 + (x-md)^2}\). Then, for far-field distances \((R >> Qd, \text{ where } R = \sqrt{y^2+x^2})\) the expression for \(\rho_m\) can be simplified as follows

\[
\rho_m \approx R-md \cos \Theta,
\]

where \(\cos \Theta = x/R\) (here \(\Theta\) is the observation angle). Substitution of eqn (24) into (20), with a limited number of sleepers being taken into account, gives the following expression for the vertical component of ground vibration velocity:

\[
v_z(x, y, \omega) = \frac{V(\omega)}{\sqrt{R}} \exp\left[i - \gamma(\omega/c_R)R\right] \sum_{m=-Q}^{Q} \exp\left[i(\omega/v)md - (i - \gamma)(\omega/c_R)md \cos \Theta\right],
\]

where we have neglected the small term \(md \cos \Theta\) in the denominator.

It is seen from (25) that maximum radiation of ground vibrations takes place if the train speed \(v\) and the Rayleigh wave velocity \(c_R\) satisfy the relation

\[
\cos \Theta = c_R/v,
\]

which is similar to the conditions of Mach or Cherenkov radiation. This relation means that elastic fields radiated by all sleepers activated by a moving load are combined in phase at the point of observation. Since the radiation angle \(\Theta\) should be real \((\cos \Theta \leq 1)\), the train speed \(v\) should be larger than Rayleigh wave velocity \(c_R\). In this case the ground vibrations are generated as cylindrically attenuated Rayleigh surface waves (factor \((R)^{1/2}\) in the denominator) symmetrically propagating at angles \(\Theta\) with respect to the track, and with amplitudes much larger than for "sub-Rayleigh trains".
All principal features of the above remain valid also for tracks with an infinite number of sleepers. As was shown in the earlier paper (Krylov & Ferguson, 1994), dissipation of Rayleigh waves in the ground and their geometrical attenuation (factors $\rho_m^{1/2}$ in the denominator of (22)) mean that normally only about 200 sleepers need to be considered. However, since in this case we usually deal with the near-field of radiating track, the analytical description is very bulky (like in the Fresnel zone of classical flat radiators), and it is preferable to use direct numerical calculations of formula (22) with the exact expression for the distances $\rho_m$.

The amplitudes of railway-generated ground vibrations for $v > c_R$ are determined by two features. The first is that under this condition the surface waves radiated by different sleepers are combined in phase. Therefore, an increase by the number of effectively radiating sleepers of the track, i.e., about 200 times, can be expected compared to the average vibration level for conventional trains. The second feature is the dependence of the function $P(\omega)$, determined by eqn (14), on train speed $v$. The analysis shows that function $P(v,F)$, where $F = \omega/2\pi$, provides an average increase of about 10 times for $v = 138.8$ m/s (500 km/h), compared with $v = 13.88$ m/s (50 km/h). Thus, a total increase of ground vibration amplitudes by 1000-2000 times (60-66 dB) can be expected for the case of trans-Rayleigh trains.

3.3. Effect of Inter-Sleeper Spacing

According to eqn (25), the amplitudes of the generated vibration field radiated at angles $\Theta = \arccos(c_R/v)$ depend neither on the periodicity of sleepers $d$ nor on their number $2Q+1$. They are determined only by the track distance considered. In fact, since the summation in (25) gives $2Q+1$ in this case and $V(\omega)$ is proportional to $d$, the value of $v_z(x,y,\omega)$ is proportional to the distance $S = (2Q+1)d$.

Note that this dependence remains valid also in the limiting case $d \to 0$, for a constant track distance $S$. This is easy to prove by replacing the sum in eqn (25) by the integral (for simplicity we neglect the ground attenuation):
\[ v_z(x,y,\omega) = \lim_{d \to 0} \frac{V(\omega)}{\sqrt{R}} \frac{1}{d} \exp\left[i(\omega/c_R)R\right] \int_{-S/2}^{S/2} \exp\left[i(\omega/v)\xi - i\cos\Theta(\omega/c_R)\xi\right] d\xi, \quad (27) \]

where discrete distance \( md \) has been replaced by \( \xi \) and sleeper spacing \( d \) by differential \( d\xi \). Since \( V(\omega) \sim d \), we obtain that for \( \cos\Theta = c_R/v \) the value of \( v_z(x,y,\omega) \) is proportional to \( S \).

This means that radiation of ground vibrations by trans-Rayleigh trains may take place also on tracks without sleepers. However, for conventional low-speed trains \( (v \ll c_R) \), the exponential function inside the integral in (27) oscillates quickly and for large \( S \) and \( \omega/v \) the integral value is close to zero, indicating that ground vibrations in the form of waves are almost not generated. This agrees with the well known result of the elasticity theory (Cole & Huth, 1958) that, for loads moving along a free surface of an elastic semispace at speed \( v < c_R \), radiated wave-fields do not exist (only localised quasi-static fields can accompany the moving load). Thus, the presence of sleepers is essential for generating ground vibrations by conventional trains due to the mechanism of wheel-axle pressure considered here. In real situations change of \( d \) to a smaller value results in noticeable reduction in high-frequency components of generated ground vibration spectra. In case of conventional trains, this effect may be used as a mitigation measure to protect the built environment in selected parts of the existing railway route. However, for trans-Rayleigh trains this method is not effective and alternative mitigation methods should be used.

4. Numerical Calculations and Discussion

4.1. General Information

Calculations of ground vibrations generated by high-speed trains have been carried out according to eqns (20) or (22) for different values of train speed \( v \), track wave critical velocity \( c_{\text{min}} \), number of sleepers \( 2Q+1 \), and for different geometrical parameters of both track and train. Summation over \( m \) in eqns (20) and (22) was carried out from \( m=-Q \) to \( m=Q \). In the
majority of calculations the chosen value of $Q$ in eqn (9) ($Q=150$) was such that the corresponding length of track $(2Q+1)d$ was greater than the total train length $NL$ and the attenuation distance of Rayleigh waves at the frequency band considered. The Poisson's ratio of soil was set at 0.25, and the mass density of soil $\rho_0$ was 2000 kg/m$^3$.

4.2. Ground Vibrations from Single Axle Loads

Figure 5 shows the spectra of ground vibration velocity (in linear units, relative to the reference level $10^{-9}$ m/s) generated by a single axle load $T = 100$ kN moving along the track at sub-Rayleigh speeds, from 2.5 m/s to 100 m/s (quasi-static and dynamic approaches reveal no difference in this case). The results are shown for the frequency band 0 - 50 Hz for the values of ground attenuation $\gamma = 0$ (a) and $\gamma = 0.05$ (b). Units of calculation were $\Delta v = 2.5$ m/s and $\Delta F = 1$ Hz. The Rayleigh wave velocity $c_R$ was equal to 125 m/s. Other parameters are: $\beta = 1.28$ m$^{-1}$, $y_0 = 30$ m.

One can see that with increase of train speed the ground vibration level generally grows. For relatively low train speeds, the peaks corresponding to the train passage frequencies are clearly seen (especially in Figure 5,a), indicating linear dependence of the corresponding frequencies on $v$. Second harmonics of the main passage frequencies are also visible. Note that in the absence of attenuation (Figure 5,a) for train speeds above 12.5 m/s the phenomenon of broadening and splitting of peaks for the main passage frequencies takes place (see also the earlier calculations (Krylov & Ferguson, 1994; Krylov, 1995a). This phenomenon can be explained by the effect of phase shifts between waves radiated from different sleepers. These shifts are obviously more pronounced for higher speeds and higher frequencies, when contribution of the second term in the sum of eqn (20) is greater.

In Figure 6, the spatial distributions of ground vibration fields generated by a single moving load at the spectral component $F = 31.4$ Hz (vibration velocity or vertical surface displacement in arbitrary linear units) are shown for sub-Rayleigh (a) and trans-Rayleigh (b) speeds. The area of the ground surface considered is 48 m x 48 m. To demonstrate the formation of wave fields for both speeds, a small part of the track with just 10 sleepers located
in the centre of the area has been considered. The Rayleigh wave velocity $c_R$ was set as 125 m/s, other parameters being the same as in Figure 5.

It is clearly seen that at low train speeds (a) the waves are radiated in almost all directions, whereas at trans-Rayleigh speeds (b) the generated wave-field is concentrated mainly in the direction of a train movement, occupying the sector roughly determined by the angles $\Theta = \arccos(c_R/v)$ with respect to the track. The amplitudes of generated waves are approximately 1000 times larger in (b) than in (a) as can be seen from the vertical scales on the figures.

The spatial distribution of the ground vibration field generated by an axle load moving at trans-Rayleigh speed along a part of the track with 100 sleepers (instead of 10) is shown on Fig.6,c. It is seen that the effect of a larger number of radiated sleepers is to produce a very high directivity of ground vibration radiation. The vibration field consists of almost perfect plain waves propagating at the angles $\Theta = \arccos(c_R/v)$ with respect to the track.

The ground vibration spectra (in linear units, relative to the reference level $10^{-9}$ m/s) generated by a single axle load moving at speeds ranging from 10 m/s to 320 m/s are shown in Figures 7 and 8 respectively for quasi-static and dynamic calculations in the form of surface graphs. The value of ground attenuation was $\gamma = 0.05$. These calculations extend the results of Figure 5,b to higher values of train speed $v$. The results are shown for the frequency band 2-50 Hz; the units of calculation are $\Delta v = 10$ m/s and $\Delta F = 1$ Hz.

Note that the details of the spectra for relatively low train speeds shown in Fig. 5 are almost invisible in Figures 7 and 8 because of the huge increase of vibration level in the trans-Rayleigh range ($v \geq c_R$). Comparison of Figures 7 and 8 shows that effects of track dynamics occur only for train speeds approaching the critical track wave velocity ($c_{\text{min}} = 326$ m/s) and reveal in the increase in spectral amplitudes of generated ground vibrations (see Figure 8).

4.3. Ground Vibrations from Conventional Trains

Ground vibration spectra generated by French TGV trains or Eurostar trains comprising $N=5$ equal carriages of length $L = 18.9$ m are shown in Figure 9 at the frequency band 2-50 Hz for three sub-Rayleigh values of a train speed: $v = 50$ km/h (curve Vz1), $v = 150$ km/h (curve
Vz2) and \( v = 250 \text{ km/h} \) (curve Vz3). The Rayleigh wave velocity in the ground was \( c_\text{R} = 125 \text{ m/s} \) \((450 \text{ km/h})\), and the soil attenuation coefficient has been set as \( \gamma = 0.05 \). Since the bogies of TGV and Eurostar trains have a wheel spacing of 3 m and are placed between carriage ends, i.e., they are shared between two neighbouring carriages, to use the eqn (22) one should consider each carriage as having one-axle bogies \((a = 0)\) separated by the distance \( M = 15.9 \text{ m} \). Other parameters used in the calculations were \( T = 100 \text{ kN}, \ \beta = 1.28 \text{ m}^{-1} \), and \( y_0 = 30 \text{ m} \).

The behaviour of the curves Vz1-Vz3 shows that, although for higher values of train speed \( v \) the peaks of ground vibrations corresponding to sleeper passage frequencies go out of the frequency band considered, the averaged level of vibration increases with increase of \( v \). This is also seen in Figure 10 which displays the amplitude of 1/3-octave spectral component of ground vibrations generated by the same TGV train at the central frequency of 25 Hz as a function of train speed \( v \). The sharp peak in Figure 10 around \( v = 20 \text{ m/s} \) relates to the sleeper passage frequencies.

Figure 11 illustrates the ground vibration spectra (in dB, relative to the reference level \( 10^{-9} \text{ m/s} \)) generated by British High Speed Trains (maximal speed of 200 km/h) consisting of \( N = 5 \) equal carriages of length \( L = 23 \text{ m} \) for two sub-Rayleigh values of train speed: \( v = 50 \text{ km/h} \) (curve Vz1), and \( v = 200 \text{ km/h} \) (curve Vz2). This trains have usual two-wheel bogies with a wheel spacing \( a = 2.6 \text{ m} \) and a bogie spacing \( M = 16 \text{ m} \). Other parameters used in calculations were the same as in Figures 9 and 10. Again, it is clearly seen that the value of train speed \( v = 200 \text{ km/h} \) corresponds to a higher averaged level of generated ground vibrations.

Effect of reducing sleeper spacing from \( d = 0.7 \text{ m} \) to \( d = 0.2 \text{ m} \) on generating ground vibration spectra by 5-carriage TGV trains travelling at conventional speed \((v = 50 \text{ km/h})\) is shown in Figures 12 (curves Vz1 and Vz2 respectively). Summation over number of sleepers \( m \) in (22) was carried out from \(-150\) to \(+150\) for \( d = 0.7 \text{ m} \) and from \(-525\) to \(+525\) for \( d = 0.2 \), thus keeping the total track length \( S \) the same for both cases. The soil attenuation coefficient has been set as \( \gamma = 0.05 \); other parameters were the same as in Figure 9. It is seen that ground vibration level for \( d = 0.2 \text{ m} \) is lower then that for \( d = 0.7 \text{ m} \). This agrees with the earlier conclusions for a single moving load. We recall that changing sleeper spacing
should not have similar effect on generating ground vibrations by trans-Rayleigh trains (see below).

4.4. Ground Vibrations from Trans-Rayleigh Trains

Figure 13 illustrates the ground vibration spectra (in dB, relative to the reference level $10^{-9}$ m/s) generated by French TGV trains or Eurostar trains consisting of $N=5$ equal carriages for both sub-Rayleigh and trans-Rayleigh train speeds: respectively $v = 50 \text{ km/h}$ (curve Vz1) and $v = 500 \text{ km/h}$ (curve Vz2). The Rayleigh wave velocity in the ground was $c_R=125 \text{ m/s}$ ($450 \text{ km/h}$), the critical track wave velocity had a value $c_{\text{min}} = 326 \text{ m/s}$ ($1173.6 \text{ km/h}$), and the soil attenuation coefficient was $\gamma = 0.05$. Other parameters used in calculations were $T = 100 \text{ kN}$, $\beta = 1.28 \text{ m}^{-1}$, $d = 0.7 \text{ m}$ and $y_0 = 30 \text{ m}$.

One can see that the averaged ground vibration level from a train moving at trans-Rayleigh speed $500 \text{ km/h}$ (138.8 m/s) is approximately 70 dB higher than from a train travelling at speed $50 \text{ km/h}$ (13.8 m/s). This very large increase in ground vibration level agrees well with the general analytical estimates given in the previous sections and with the numerical results obtained for a single axle load (Figures 6-8).

Calculation of the ground vibration spectrum generated by the same trans-Rayleigh TGV train ($v = 500 \text{ km/h}$) travelling along the track with a smaller distance between sleepers, $d = 0.2 \text{ m}$, shows that it entirely coincides with the above mentioned spectrum calculated for $d = 0.7 \text{ m}$. This confirms that spectra of ground vibrations generated by trans-Rayleigh trains on tracks with different distances between sleepers are absolutely identical.

Influence of soil attenuation on ground vibration spectra generated by TGV trains travelling at speed $500 \text{ km/h}$ is shown in Figure 14 for three values of attenuation coefficient: $\gamma = 0.005$ (curve Vz1), $\gamma = 0.015$ (curve Vz2), and $\gamma = 0.045$ (curve Vz3). Other parameters are the same as in Figure 13.

According to Figure 14, for very low soil attenuation ($\gamma = 0.005$) the peak levels of ground vibrations generated by trans-Rayleigh TGV-trains can be as high as 140 dB (relative to $10^{-9}$
m/s). This corresponds to ground vibration velocities of about 10 mm/s. Vibrations of such a high level may even cause damage to nearby properties.

4.5. Expected Contributions from Other Effects

In general, we would like to mention that the theoretical model used in this paper, although being quite sophisticated, remains highly idealised. Here we discuss some possible reasons why the predicted ground vibration spectra from high-speed trains may differ from those observed.

First of all, we did not consider influence of layered geological structure of the ground which is present in most practical situations. To take account of the effects of layered structure on generated ground vibrations one should use the Green’s function for a layered elastic half space, instead of that for a homogeneous half space used above. Derivation of this function, which would contain information about the complex elastic field generated in the half space considered (including reflected and refracted bulk waves), is, as a rule, not available analytically. Nevertheless, for description of generated Rayleigh waves only, the problem can be simplified by considering an appropriate approximate solution which takes into account the effects of layered structure on the amplitudes and velocities of generated Rayleigh waves.

We recall that in layered media Rayleigh waves become dispersive, i.e., their phase velocity $c_R$ is a function of frequency: $c_R = c_R(\omega)$. Since a shear modulus of the ground $\mu$ normally has higher values at larger depths, this will cause an increase of Rayleigh wave velocities at lower frequencies associated with deeper penetration of surface wave energy into the ground. This may violate the trans-Rayleigh condition $v > c_R$ and cause significant reduction in the low-frequency components of ground vibration spectra generated by high-speed trains. The model of a homogeneous elastic half space, used above, is applicable for practical situations if the upper layers have thickness larger than the Rayleigh wavelength at given frequency. For example, if the upper layer of thickness 10 m is characterised by shear wave velocity of 140 m/s, the model of a homogeneous half space is applicable for frequencies above 11 Hz.
Another possible reason why the predicted ground vibration spectra may differ from experimentally observed is that the wheel-axle pressure mechanism of ground vibration generation may not dominate in the existing normal rolling stock. If other possible generation mechanisms mentioned in the section 2.1 do prevail, then the expected averaged increase in ground vibration level should be reduced roughly to \((70 - I)\) dB, where \(I = 20 \log(A_{oth}/A_{wp})\) determines the relation (in dB) between the ground vibration amplitudes due to the wheel-pressure mechanism \(A_{wp}\) and other \(A_{oth}\) mechanisms of generation.

Finally, it should be mentioned that we also did not consider possible influence of bulk shear and compressive elastic waves (S- and P-waves), radiating into the bulk of the ground, on the total level of ground vibrations generated by high-speed trains. Obviously, radiated S- and P-waves can also be significantly amplified if the train speeds are high enough and the conditions \(v > c_t\) or even \(v > c_l\) hold, in addition to the trans-Rayleigh condition \(v > c_R\) considered so far (we recall that \(c_R < c_t < c_l\)). In such cases these waves will be radiated into the ground as conical Mach waves propagating at the angles \(\Theta_t = \arccos(c_t/v)\) and \(\Theta_l = \arccos(c_l/v)\) relative to the track, in addition to Rayleigh waves radiated as quasi-plane waves along the surface at the angles \(\Theta = \arccos(c_R/v)\). The most likely contribution to the total ground vibration field might be that of radiated S-waves since their velocity \(c_t\), being only about 10% higher than the velocity of Rayleigh waves, can be easier achieved by moving trains than the velocity of compressive waves \(c_l\). In the presence of layered structure in the ground, the S-waves initially radiated into the bulk at the angles \(\Theta_t = \arccos(c_t/v)\) relative to the track, may experience total internal reflection on the layer bottom and return to the surface. Repeated reflections from the ground surface and from the layer bottom may cause a waveguide propagation of S-waves which will affect the total vibration field in relatively remote locations. Another possible influence of effectively generated shear waves on the total ground vibration field may occur for high-speed underground trains. In this case the contribution of bulk shear waves is often more essential than that of Rayleigh waves (Krylov, 1995b). Both these cases, however, need special attention and will be considered in a separate publication.
5. Conclusions

Increase in speeds of railway trains is generally accompanied by increased levels of generated ground vibrations. Especially large increase in ground vibration amplitudes occurs for trans-Rayleigh trains, i.e., for trains travelling at speeds larger than Rayleigh wave velocity in the ground. Calculations performed for French TGV trains or Eurostar trains show that the average increase of about 70 dB takes place as compared with conventional trains. Fortunately, soils with low Rayleigh wave velocities (around 100 m/s) are uncommon, the most typical range of $c_R$ values being 250-500 m/s. Nevertheless, the designers and builders of tracks for high-speed trains should be aware of the potential risk of excessive ground vibrations.

Effect of track bending waves may cause increase in amplitudes of ground vibrations generated by trans-Rayleigh trains if the train speeds approach the minimal phase velocity of track bending waves. Minimal phase velocities of bending waves propagating in the system track/ground are usually much higher than speeds achieved by modern high-speed trains. However, under certain circumstances, e.g., for marshy and sandy soils, these velocities may be comparable with train speeds.

The reduction of track inter-sleeper spacing does not affect the level of ground vibrations generated by trains travelling at speeds higher than Rayleigh wave velocities in the ground (trans-Rayleigh trains). However, it results in noticeable decrease in generated ground vibrations for trains travelling at speeds below the velocities of Rayleigh surface waves in the ground. Obviously, this effect could be used in selected parts of the rail route for suppression of ground vibrations generated by conventional trains.

REFERENCES


FIGURE LEGENDS

Figure 1. Geometrical parameters of track and train - (a); wheel-axle pressure mechanism of ground vibration generation - (b); superposition of ground vibrations generated by different sleepers at the point of observation \( \{x, y\} \) - (c)

Figure 2. Calculated track deflection curves taking into account effect of track bending waves. Curves w1, w2, w3, w4 and w5 correspond respectively to the train speeds 0, 69, 138, 300 and 320 m/s; critical track wave velocity \( c_{\text{min}} \) is 326 m/s, axle load \( T \) is 100 kN

Figure 3. Vertical forces applied from each sleeper to the ground as functions of \( vt \) for axle loads \( T = 100 \) kN moving along the track at speeds 0, 69, 138, 300 and 320 m/s (curves w1, w2, w3, w4 and w5 respectively); distance between sleepers \( d \) is 0.7 m; all other parameters are the same as in Fig.2
Figure 4. Comparison of dynamic and quasi-static force spectra $P(F)$, where $F = \omega/2\pi$, for three values of train speed: $v = 69$ m/s (curves P1 and P2), $138$ m/s (curves P3 and P4), and $300$ m/s (curves P5 and P6); all other parameters are the same as in Fig.3.

Figure 5. Spectra of ground vibration velocity (in linear units, relative to the reference level $10^{-9}$ m/s) generated by a single axle load moving along the track at sub-Rayleigh speeds, from $2.5$ m/s to $100$ m/s. The results are shown in the frequency band $0 - 50$ Hz for the values of ground attenuation $\gamma = 0$ - (a), and $\gamma = 0.05$ - (b). Mesh: $\Delta v = 2.5$ m/s and $\Delta F = 1$ Hz.

Figure 6. The spatial distributions of ground vibration fields generated by a single axle load running over small part of the track with 10 sleepers (vibration velocity or vertical surface displacement in arbitrary linear units) at the spectral component $F = 31.4$ Hz for sub-Rayleigh (a) and trans-Rayleigh (b) speeds. Fig. 6,c shows the corresponding spatial distribution of the ground vibration field generated by an axle load moving at trans-Rayleigh speed along a part of the track with 100 sleepers (instead of 10). The area of the ground surface considered is $48$ m x $48$ m. Mesh: $\Delta x = \Delta y = 1$m.

Figure 7. Spectra of ground vibration velocity calculated using quasi-static approximation (in linear units, relative to the reference level $10^{-9}$ m/s) for a single axle load moving along the track at speeds from $10$ m/s to $320$ m/s (these include trans-Rayleigh speeds). The results are shown in the form of surface graph for the frequency band $2$ - 50 Hz. Mesh: $\Delta v = 10$ m/s and $\Delta F = 1$ Hz.

Figure 8. Spectra of ground vibration velocity calculated using the dynamic approach (in linear units, relative to the reference level $10^{-9}$ m/s) for a single axle load moving along the track at speeds from $10$ m/s to $320$ m/s (these include trans-Rayleigh speeds). The
results are shown in the form of surface graph for the frequency band 2 - 50 Hz. 
Mesh: $\Delta v = 10$ m/s and $\Delta F = 1$ Hz

Figure 9. Ground vibration spectra (in dB, relative to the reference level $10^{-9}$ m/s) generated by French TGV or Eurostar trains comprising $N = 5$ equal carriages for three sub-Rayleigh values of train speed: $v = 50$ km/h (curve Vz1), $v = 150$ km/h (curve Vz2) and $v = 250$ km/h (curve Vz3)

Figure 10. Amplitude of 1/3-octave component of ground vibration spectra (in dB, relative to the reference level $10^{-9}$ m/s) generated by French TGV or Eurostar trains consisting of $N = 5$ equal carriages at the central frequency of 25 Hz as a function of train speed $v$

Figure 11. Ground vibration spectra (in dB, relative to the reference level $10^{-9}$ m/s) generated by British High Speed Trains consisting of $N = 5$ equal carriages for two sub-Rayleigh values of train speed: $v = 50$ km/h (curve Vz1) and $v = 200$ km/h (curve Vz2)

Figure 12. Effect of sleeper spacing $d$ on ground vibration spectra (in dB, relative to the reference level $10^{-9}$ m/s) generated by French TGV or Eurostar trains comprising $N = 5$ equal carriages and travelling at conventional speed $v = 50$ km/h. The results are shown for $d = 0.7$ m (curve Vz1) and $d = 0.2$ m (curve Vz2)

Figure 13. Ground vibration spectra (in dB, relative to the reference level $10^{-9}$ m/s) generated by French TGV or Eurostar trains consisting of $N = 5$ equal carriages for both sub-Rayleigh and trans-Rayleigh train speeds respectively: $v = 50$ km/h (curve Vz1) and $v = 500$ km/h (curve Vz2)
Figure 14. Effect of soil attenuation constant $\gamma$ on ground vibration spectra (in dB, relative to the reference level $10^{-9}$ m/s) generated by French TGV or Eurostar trains comprising $N = 5$ equal carriages and travelling at trans-Rayleigh speed $v = 500$ km/h. The results are shown for $\gamma = 0.005$ (curve $Vz1$), $\gamma = 0.015$ (curve $Vz2$) and $\gamma = 0.045$ (curve $Vz3$)
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5, b
Fig. 6, b
Fig. 6, c
Fig. 13
Fig. 14