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ANALYSIS AND DESIGN OF MULTILAYER FREQUENCY SELECTIVE SURFACES

by

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A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy of the Loughborough University of Technology

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# TABLE OF CONTENTS

Abstract v
Acknowledgement vii

CHAPTER 1.0 INTRODUCTION 1

1.0 Introduction to Frequency Selective surfaces 1
1.2 The Driving force for Multilayer FSS Studies 3
1.3 Research Objectives and the proposed Approach 8
1.4 Organisation of the Thesis 10
References 13

CHAPTER 2.0 MODAL ANALYSIS OF SCATTERING FROM MULTILAYER FSS STRUCTURES 16

2.1 Introduction 16
2.2 Formulation of scattering from multilayer FSS structures 18
2.2.1 The geometry and Floquet expansion of the fields 18
2.3 Analysis of scattering from double layer FSS structures with Arbitrary lattice periodicities of the arrays 27
2.3.1 Concept of mutual periodicity between two periodic arrays with dissimilar lattice periodicities 30
2.3.2 Super-resolution of the spectral components of tangential Field vectors by means of correlation function 32
2.4 Super-resolution approach to modal analysis of scattering from Two layer FSS 44
2.4.1 Field expansion at different boundaries in the absence of the scatterers 44
2.4.2 Scattered field expansion in terms of the Floquet modes sets of mutual periodicity 52
2.4.3 The coupled integral equation formulation 61
2.5 Method of solution 65
2.5.1 Method of Moment solution to the integral equations 65
2.5.2 Optimised Integral Equations Expressed in Terms of the Floquet modes of the two arrays 73
2.5.3 Determination of Reflection and Transmission coefficients 76
2.5.4 The computer model and a summary of all the relevant Equations used 83
2.5.5 Summary 90
References 92
### CHAPTER 3.0 PLANE WAVE PERFORMANCE OF MULTILAYER FSS STRUCTURES WITH ARBITRARY LATTICE PERIODICITIES

#### 3.1 Introduction 95

#### 3.2 Experimental Set-up 97

- 3.2.1 Measurement Technique 97
- 3.2.2 Anechoic Chamber 97

#### 3.2.3 Plane Wave Measurement Set-up 99

- 3.2.4 The FSS Sample and the Array Geometry 100

#### 3.3 Results from representative arrays 105

- 3.3.1 Computed and Measured Plane Wave Transmission / Reflection Responses of Single Arrays Used in the Double Layer Structure 108
- 3.3.2 Analysis of results for Double Layer FSS of Tripoles With Arbitrary Lattice periodicities of the arrays Periodicities 112
- 3.3.3 Sensitivity of Band centre Frequency of the double layer FSSs with the Angle of Incidence 123

#### 3.4 Design of a Close-coupled Arrays of Crossed Dipoles with Wide band response 126

#### 3.5 Summary 129

References 131

### CHAPTER 4.0 PLANE WAVE PERFORMANCES OF MULTILAYER FSS STRUCTURES WITH MIXED ELEMENT GEOMETRIES AND LATTICE PERIODICITIES OF THE ARRAYS

#### 4.1 Introduction 132

#### 4.2 Analysis of the Plane Wave Response for Double layer FSS with Mixed element Arrays and Dissimilar Periodicitis 134

- 4.2.1 Analysis of the Results of Plane Wave Response for Double layer FSS of Tripoles and Rings 147

#### 4.3 Analysis of Results from Double-sided FSS of Tripoles and Rings 152

- 4.3.1 Double-sided Surfaces with Arbitrary Lattice Periodicitics and element Geometries of the two arrays 152
- 4.3.2 Double-sided FSS Structures with Integer Multiple Periodicity Ratio 161

#### 4.4 Wide band Design of Double-sided FSS 163

#### 4.5 Summary 168

References. 170
CHAPTER 5.0 RADIATION PATTERN MEASUREMENTS IN REFLECTION OF MULTILAYER FSS

5.1 Introduction 172
5.2 Radiation Pattern Measurement in Reflection 173
5.3 Experimental Set-up for Radiation pattern Measurements in Reflection 174
5.4 Radiation pattern Measurement Technique 177
5.5 Measurements at Various Planes of Scan 182
5.5.1 H-plane Incident Results 183
5.5.1.1 Horizontal (i.e. Zero degree) Scan Plane Measurements 183
5.5.1.2 45 Degree Plane of Scan Measurements 187
5.5.1.3 Vertical Scan Plane Measurements 193
5.5.2 E-plane Incidence Measurements 194
5.5.3 Variation of Average Peak Cross-polar Levels of the Two Types of FSS at Various Planes of Scan for Different States of the Incident polarised Wave 197
5.6 Summary 204
References 206

CHAPTER 6.0 CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK 207

APPENDICES

APPENDIX A Modal Analysis of Scattering from Single Layer FSS A.1
A.1 Formulation of the FSS Scattering problem A.1
A.2 The Floquet Harmonics A.3
A.2.1 The Grating Lobe Diagram A.7
A.3 Tangential Field Expansion in Terms of TE and TM Vector Floquet Modes A.10
A.4 Boundary Conditions and the Integral Equation A.12
A.5 Method of Moment Solution to the Integral Equation A.14
References A.18

APPENDIX B Rational and Irrational Numbers B.1
B.1 Rational Numbers B.1
B.2 Irrational Numbers B.1

APPENDIX C Correlation Functions Relating two Vector Floquet Sets C.1
Structures that include more than one layer of Frequency Selective Surfaces (FSS) are an attractive feature in applications with stringent performance requirements. They provide extra flexibility with regard to the adjustment of the transmission responses which can in principle be insensitive to angle of incidence because of the increased bandwidth. This thesis deals with computer simulation and experimental assessment of a variety of multilayer FSS structures. The plane wave vector modal analysis technique is adapted for analysing the scattering from these multilayer FSS structures. The novelty in the plane wave modal analysis method lies in the fact that they can be applied to arbitrary lattice and element geometries.

A novel super-resolution approach of analysing the scattering from FSS in cascade, with arbitrary lattice geometries of the two arrays is outlined. These type of structures exhibit multiresonant responses in a controlled manner. The problem of assigning different lattice geometries to the structure is addressed here by assigning the periodic fields adjacent to the arrays a common (or mutual) periodicity and by employing the convolution theorem to the modal (Floquet) sets that expands the tangential fields in each array. As a result, the spectral components of the Floquet mode coefficients from the various adjacent arrays are related to those of the common periodicity by means of a correlation function. This correlation function enables the spectral components of the Floquet mode sets expanding the tangential fields from any two adjacent arrays to be super-resolved from those of their common periodicity set. Once the convolution has been executed, application of electromagnetic boundary conditions are utilised, thus obtaining the coupled electric field integral equations. These integral equations relate the spectrums of the surface current densities to the various Floquet mode coefficients. The integral equations are in turn solved by the Method of Moment (MoM) technique for the unknown current coefficients from which the unknown transmission and reflection coefficients from the entire structure are obtained. A major assumption that is made in this technique for assigning a common periodicity lattice is that the ratio of lattice periodicities of any two adjacent arrays must be a rational number.
The importance of the proposed technique lies in unlocking the complexities that exist when the scattered Floquet modal coefficients from the arrays are related to the spectral components of the currents induced on the surfaces of the arrays, in the integral equation formulation. Furthermore, the proposed approach offers a computational advantage when applied to multilayer FSS structures, as it is invariant to the distance separating the arrays.

A computer model based on this technique is developed for obtaining the prediction results. Various double layer FSS structures with arbitrary element types and lattice geometries of the arrays and with variable separation distances between the two layers are studied. The plane wave transmission coefficients of these multiresonant structures are computed with a view to predict their radiation parameters. Extensive measurements are performed by using a purpose built experimental jig for mounting the structures, in an indoor anechoic chamber. The validity of the theoretical model is assessed by comparison with measurements from a variety of multilayer structures.
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CHAPTER One

INTRODUCTION

1.1 Introduction to Frequency Selective Surfaces
Frequency Selective Surfaces (FSS) are being widely used as filters for microwaves and optical signals. In recent years they have been the subject of extensive research. An FSS basically consists of periodic arrays of elements printed on a dielectric substrate. The prime feature of an FSS is the dependence of the transmission coefficients on the frequency of the incident wave. At certain frequencies, they are transparent to electromagnetic waves while at others they are strongly reflective. A typical geometry of free-standing FSS consisting of arbitrary periodic elements is shown in Figure 1.1 where the element spacings (known as the periodicities), $D_u$ and $D_v$, lie in arbitrary $(u, v)$ coordinate axes. Also shown in Figure 1.1 is the polar coordinate system with incidence polar angles $\theta$ and $\phi$. $\theta$ is the angle of incidence and $\phi$ is the angle that the x-axis makes with the plane of incidence. A plane electromagnetic wave is incident on the surface of the structure with amplitude $E_i$. Assuming a lossless structure, these screens exhibit total reflection or transmission in the vicinity of the element resonance at a single wavelength. This implies that an FSS will ideally behave as a solid metal surface over its resonance frequency band, while allowing transmission of RF energy through it essentially unattenuated, at other frequencies. The wavelength as well as the associated resonant band are mainly influenced by: the element configuration of the patch, within an infinite array of periodic cells; element spacing (i.e. the periodicity); thickness and permittivity of any dielectric support that may be a part of the screen; and the polarisation of the incident field. Because different element configurations drastically change the characteristic spectral response of the screen, the geometry may be used to tailor the response for a given application.

Incorporating FSS into the antenna assembly will enable the system to operate at more than one frequency band simultaneously [1]. They have often been considered for the reflector antenna applications. A typical plane wave spectral response curve of an FSS
FIGURE 1.1: Geometry of a free-standing FSS of arbitrary array of periodic elements (the front view). Also shown is the polar coordinate system with polar angles ($\theta$ and $\varphi$) where $\varphi$ is the angle between the plane of incidence and the x-axis.
is shown in Figure 1.2. Below the frequency $f_T$, the surface basically transmits, with a loss of less than 0.5 dB below the maximum power level. Around the frequency $f_R$, the surface is highly reflective. The geometry of a microwave reflector antenna that incorporates FSS therefore becomes frequency dependent, hence providing a possibility of multiband, multimission operation. By implementing an FSS in a dual reflector configuration, one may eliminate the need for the design of a single broad band feed [2]. Typically, an FSS is employed as the main *subreflector* and the different frequency feeds are optimised independently and placed at primary focus feed and cassegrain feed locations, as in the case of a cassegrain antenna system [2]. Hence, only a single main reflector is required for the multifrequency operation. For example, the *Voyager* FSS [3] was designed to separate the S and X bands. In that application, the S-band feed is placed at the prime focus of the main reflector, and the X-band feed is placed at the cassegrain focal point. Note that only one main reflector is required for this two band operation. Consequently, a considerable reduction in volume, weight, and most importantly, the cost of the antenna system is achieved with the FSS subreflector. Lee C.K. et al [4], proposed a system where a single reflector antenna is used at multiple frequencies by employing these FSS surfaces to separate the individual bands. One such configuration operating at three frequency bands is shown in Figure 1.3. Here the bands $f_1$, $f_2$ and $f_3$ are separated by two subreflectors FSS1 and FSS2. In Figure 1.3, FSS2 is required to separate two frequency bands. That is, reflect $f_2$ and transmit $f_3$. FSS1, however, must transmit two bands $f_2$ and $f_3$, while reflecting $f_1$. In addition, FSS1 must also be designed for a wide range of reflection/transmission band ratios varying from 1.3 to 3:1.

### 1.2 Driving Force for Multilayer FSS Studies

The current trend in satellite communication systems indicates increased use of antenna systems such as FSS employing multiple frequency channels operating simultaneously. Over the last few decades a number of analytical and numerical techniques have been developed for designing a variety of frequency selective surfaces. However, due to the increase in complexity of the specifications for the frequency response characteristic of these surfaces, many designers have been led to investigate *novel* FSS configurations.
Figure 1.2: Typical plane wave transmission response of a frequency selective surface. $f_R$ is the reflection band centre frequency.

Figure 1.3: Three frequency band reflector antenna incorporating two Frequency Selective Surfaces FSS1 and FSS2.
These in turn, have given rise to new, and yet some *unsolved* modelling problems.

FSS designs require properties meeting a range of specific characteristics that can be achieved flexibly using more than one layer of element [5], where they may have different shapes and lattices. These properties include multiresonant, very close, or very wide frequency band spacing ratios, with wide or narrow bandwidths. Referring to figure 1.2, the bandwidth of the response curve in the reflection band for example, is defined as the percentage ratio of the difference between the frequencies at the -10 dB point, to the reflection band centre (i.e. the reflection resonance) frequency. In other words, \( \text{B.W\%} = \frac{\Delta f}{f_R} \), where \( \Delta f \) is the difference between the -10 dB points). Also the frequency band spacing or the band separation ratio is defined as the ratio between the transmission band centre \( f_T \) and the reflection band centre \( f_R \) frequencies. A sharper transition from the upper edge of the transmission band to the lower edge of the reflection band would normally be desired, enabling a close band spacing ratio to be achieved. The breadth of achieving such diverse properties is dominated by the choice of element and lattice geometries in each layer without neglecting the interaction region between the arrays.

In contrast with single layer FSS design [4, 5, 6] where a systematic trade-off between the spacing of the reflection and transmission bands, the bandwidth of both bands and the proximity of them to grating lobe [7] may be involved, by considering multilayer surfaces the band separation criteria can be decoupled from the bandwidth and the grating lobe conditions. This is achieved by using arbitrary element geometries of the various arrays to control the grating lobe occurrence in the pass band and hence improving the bandwidth of the whole system. An enhanced frequency response can therefore be achieved as shown in figure 1.4 below, allowing for a controlled band pass performance important in many system applications where a broadband or multiband operation of the antenna are required.

In the context of FSS, the grating lobes are defined as energy transmitted or scattered in
Figure 1.4: Plane wave transmission response of a typical double-layer frequency selective surface.

undesirable directions when the ‘element periodicity’ (i.e. the spacing between the adjacent elements of the arrays) become electrically large. The onset of the grating lobe occurs when the element spacing in the FSS array is equal to one free-space wavelength. As the periodicity increases, a narrower reflection band will result since grating lobe begins to emerge within the pass band region. A general rule is that the spacing between adjacent elements should be less than one wavelength for normal incidence. For large incident angles, such as $45^\circ$, the spacing should be kept below half free-space wavelength. In general, as long as the above rules are obeyed, the peak of the grating lobe is prevented from entering real space. To avoid wasted energy, not even the shoulder region of the grating lobe should enter real space.

Furthermore, by exploiting the interference effects between the two close-by adjacent arrays, adjustment of the bandwidth and the spacing between the transmission and reflection bands has been proved to be achieved [5].

Multilayer FSS with both arrays having identical lattice periodicities may be used [5,8],
to produce multiresonance responses with higher bandwidths as compared to single layer FSS structures. However, these types of multilayer structures do not allow for much more effective control over the entire bandwidth and particularly over the band spacing ratio. Since the periodicities of the various arrays in this case are fixed and identical, then the emergence of any grating lobe due to the lattice periodicity of one array can strongly affect the bandwidth of the next layer, thereby reducing the overall bandwidth of the system.

A Multilayer FSS with the arrays having non-similar size lattice periodicities and element geometries however can produce multiresonance structures in a controlled manner [9] as will be demonstrated in our studies in the following Chapters. They provide flexibility regarding the adjustment of transmission / reflection band by including arrays that have dissimilar lattice geometries as well as exploiting the interaction regions separating the arrays [5, 9]. Deriving an analytical model for predicting the plane wave transmission / reflection responses of these types of structures therefore becomes a challenge.

The strength of multilayer FSS structures is in their various applications. Amongst many other, frequency scanning and reconfigurable FSS are becoming increasingly widespread. One application is beam steering by frequency scanning where the lattice periodicity of each array is selected independently to scan the beam on both sides of the pattern boresight (with the pattern boresight normally defined as the position of the maximum intensity in the radiated field amplitude pattern). For this application, periodic FSS structures are designed [11] by choosing the array geometries such that both the fundamental (0, 0) order Floquet modes [10] and the higher (-1, 0) order diffracted modes are propagating whilst the remaining modes are evanescent (i.e. non-propagating). In Appendix A (section A.2) the conditions under which the propagating and evanescent Floquet modes occur are explained. Frequency scanning is achieved by dimensioning the array geometries (i.e. the element spacing, the element length and the distance between the array and the ground plane if any) for power conversion from the incident field to the higher order diffracted wave that is to serve as a frequency-scanned beam. In this way, the
direct reflection is minimised. Since there are only two propagating modes the optimisation can be performed by minimising the power in the reflected zero order mode. The propagation direction of the \((-1, 0)\) diffracted signal is frequency sensitive \([11]\) and this signal is therefore used to scan the range as the frequency is varied. It has been shown \([12]\) that one of the most sensitive lattice parameters is the periodicity between the elements and the performance of the frequency scanned grating surfaces is dependent on how effectively these periodicities along with other lattice parameters are optimised. Johanson et al \([11]\) performed numerical investigation by computer optimising the most essential parameter such as (the length of the array element and the electrical distance between the array and the ground plane), while the remaining parameters such as (the element periodicity, the element width and the dielectric constant of the supporting substrate) are selected properly. Experimentally, the measurements are carried out by measuring the suppression of the basic reflected wave instead of measuring conversion loss to the linear diffracted wave, as this is not a practical option. Another important and novel application of double layer structures currently under extensive research, is in \textit{reconfigurable FSS}, where two single layer FSS are closely coupled for the purpose of obtaining a significant shift in the location of the resonance frequency. The key to this technique \([13]\) is to introduce high coupling between the arrays by a relative displacement of one array with respect to the second array. Vardaxoglou et al \([14]\) have found that the surface would resonate at a far lower frequency than under normal circumstances.

\section*{1.3 Research Objectives and the Proposed Approach}

The primary objective of the research work is centred on developing formalism for analysing the scattering from multilayer FSS structures with dissimilar lattice geometries of the arrays. This is based on the Floquet mode expansion \([10]\) of the tangential fields adjacent to the various arrays. Applications of The boundary conditions \([\text{Appendix A, section A.4}]\) result into a set of coupled integral equations system in the spectral domain for the induced currents on the surface of the arrays. These equations can then be solved for the unknown induced currents by the Method of Moment (MoM) \([15]\) technique. To the knowledge of the author, some techniques described here for the analysis of dissimilar lattice arrays of FSS are \textit{novel}. The validity
of the analytical procedures presented will be assessed by comparison with the measured results.

The important aspect that need to be addressed in analysing the scattering from the above types of multilayer structures are the couplings which exist between dissimilar Floquet mode sets that expand the fields adjacent to various arrays. The Floquet modes in the various arrays are non-similar due to the fact that the lattice periodicities are dissimilar. In particular, a combination of two different sets of vector Floquet modes (section A.2, Appendix A) are considered in the region separating two dissimilar adjacent arrays. At the time when the research studies were being conducted, the existing modal analysis techniques [8,16,17] of analysing the scattering from multilayer layer FSS surfaces were based on the assumption that the lattice periodicities of the various arrays in the structure are all identical. Vardaxoglou and Parker [8] have analysed such surfaces by employing modal analysis technique [18] applied to pairs of plane periodic parallel arrays. However, using the existing modal analysis technique to analyse FSS arrays with dissimilar lattice periodicities would result in complexities when the integral equations for the induced surface currents are being formulated. This is due to coupling between the two dissimilar sets of Floquet modes that expand the tangential fields. The main objective in the research studies was therefore to outline a formulation that is capable of unlocking these complexities [9], thereby enabling formulation of the integral equations that are needed for the unknown surface currents.

In order to achieve the above mentioned objectives, a novel approach [5,9,19] of coupled integral equation formulation for analysing multilayer FSS with arbitrary lattice geometries of the arrays is described. This is primarily a further extension to the existing [8] well established modal analysis technique of analysing multilayer FSS with identical lattice periodicities of the arrays. For the purpose of demonstrating the analysis and the method of solution that will be obtained, a variety of double layer structures are chosen. Measured results for these structures are obtained for a range of angles of incidence as well as separation distances. The plane wave transmission coefficients of multiresonant structures, which have triangular and square lattices of the
Chapter One

arrays, are also computed and their validity are assessed by comparing with measured results. Finally the properties of various multilayer FSS and their potentials in the design of dichroic subreflectors and similar applications are investigated. This will be accomplished mainly by computer simulation and analysis of practical results obtained from measurement of these multilayer surfaces.

1.4 Organisation of the Thesis

The work outlined in this thesis is organised in the following manner: Chapter 2 will form the core of the research works and the basis on which the results presented in the following chapters are analysed. In this Chapter an outline is presented of an efficient method based on the modal analysis technique, for analysing the scattering from multilayer FSS structure with arbitrary lattice geometries. This is outlined in Section 2.3. The concept of a common periodicity (section 2.3.1) is used where the field in any region is expressed in terms of a periodicity that is common between the lattice periodicities of the adjacent arrays. The approach adapted in this technique is that, the tangential field vectors describing each array are super-resolved according to the convolution theorem [9,18]. The technique allows the field expansion at various boundaries to be carried out in terms of the Floquet modes of a common periodicity, thereby reducing complexities that exist when formulating the integral equations. In order to demonstrate the potential of the novel technique described, the analysis is first performed in terms of the Floquet mode sets of the common periodicity as outlined in Section 2.4.2. Then in Section 2.5.2 it is shown how the integral equations for the surface currents expressed in terms of the common periodicity, are reduced (i.e. optimised) to a set of two integral equations expressed in terms of the Floquet mode of the individual arrays. The efficiencies of the two integral equation formulation methods are then discussed in terms of the computing times required. A (MoM) solution of the integral equations for the unknown current coefficients is outlined in section 2.5.1. Having obtained the induced currents, the total transmission and reflection coefficients from the entire structure are then calculated. This is given in section 2.5.3. A computer model is developed (section 2.5.4) based on the novel
technique described. This is used to generate the predicted results for plane wave transmission coefficients of the various FSS structures that will be studied.

In Chapter 3, results are presented from an study which examines the plane wave transmission/reflection responses of the two types of multilayer FSS structures consisting of arrays of tripoles and cross-dipoles. The main theme of this chapter will be to assess the validity of the computer models developed from the novel method of integral equation formulation described in Chapter 2. Measurement of various multilayer FSS of tripole element arrays with arbitrary lattice geometries are compared with the predictions. The bandwidth performance of these multilayer surfaces and also their crosspolar behaviour in reflection are discussed.

Chapter 4 describes a study of multilayer FSS with dissimilar lattice geometries of the arrays with mixed elements. In particular, the plane wave transmission/reflection responses of these surfaces measured in an anechoic chamber are assessed in comparison with the predicted results obtained from the computer models developed in Chapter 2. The plane wave responses of close-coupled arrays (i.e. double-sided surfaces) with mixed elements in both arrays such as tripole/ring, cross-dipole/ring are also examined. The validity of the prediction models with regard to predicting the location of reflection resonances and the pass band response for extremely close-coupled arrays are assessed where the two arrays are printed on either side of a single substrate. Particular attention is given to double-sided FSS with thin dielectric substrates where the coupling between the arrays becomes very significant. Another type of double-sided structure considered is the one that the periodicity of one array is an integer multiple of the periodicity of the second array.

Chapter 5 presents the radiation pattern measurements (both in copolar and crosspolar directions) of various double layer FSS with arbitrary lattice geometries. The measured radiation pattern responses of double layer FSS of tripoles with identical and dissimilar lattice geometries are examined. The profiles of both crosspolar and copolar patterns of such structures at various planes of scanning are assessed. The measured boresight
values of the patterns are compared with the computed boresight values predicted by the plane wave modal analysis technique. Performances such as peak crosspolar levels and losses within the reflection band for surfaces of varying separation distances between the two arrays are extensively assessed.

Chapter 6 gives a summary and concluding remarks of the research work carried out, and outlines suggestions for any further works.
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2.1 Introduction

In this Chapter the scattering from a frequency selective surface that consists of two parallel periodic arrays of elements with arbitrary lattice geometries, is analysed using the modal technique [1]. All the evanescent modes (i.e. the higher order Floquet modes [section A.2, Appendix A] that do not propagate and decay away from the aperture (z >0)) as well as the propagating modes are taken into account in the modal analysis. It can be applied to the case of multiple layers: the complexities lie in the calculation of the currents on the surface of the conducting elements. The reader is referred to Appendix A where an outline of modal analysis of scattering from a single layer FSS array of infinite extent is given. When the periodicities of the two arrays are identical, the modal analysis technique with the resulting equations, although lengthy and complicated, has previously been analysed by various authors [2,3,4]. However the prime objective in here is to formulate the analysis of scattering from multiple layer FSS with non-similar lattice periodicities of the arrays. A novel method of deriving the integral equation formulation in the spectral domain for analysing such FSS structures is described. The approach being adapted here for obtaining the scattered fields from the structure, is based on the Floquet expansion [5] of the tangential fields and where the periodic nature of the Floquet modes [5] describing the tangential fields are exploited. In Appendix A, the scalar and vector field expansion of Floquet modes of an infinite periodic array are defined where the TE (Transverse Electric) and TM (Transverse Magnetic) components of the modes are also described.

Conceptually, the Floquet theorem is an extension of the Fourier series theorem for periodic functions. This allows the modal description of any field or function that
repeats itself periodically. Such periodic function is an appropriate description for the field adjacent to an infinite periodic array [5] such as in FSS, excited uniformly in amplitudes but with a linearly varying phase. In the context of FSS, the Floquet modes are defined as a set of discrete components of plane waves whose spacing are function of the periodicity of a particular array. In Appendix A (section A.2), the physical meaning of the Floquet modes are further illustrated by means of a grating lobe [6] diagram.

At the time when the research studies were being carried out, the major drawback of the existing modal technique [2,3,4] of analysing the scattering from multilayer FSS arrays was the assumption that the various arrays in the structure must have identical lattice geometries. As a result the Floquet modes that are used to expand the tangential fields adjacent to various arrays are the same, thereby enabling straightforward modal analysis technique to be applied. In the case of multilayer FSS structures with arbitrary lattice periodicities of the arrays, which were the subject of our research studies, the Floquet modes that expand the tangential fields adjacent to the various arrays are dissimilar. As a result of coupling between these dissimilar Floquet modes, particularly when the arrays are closely spaced, new scattered Floquet modes are generated. This consequently leads to extra complexities being introduced into the expressions for the fields, when the usual boundary conditions [Appendix A, section A.4] are applied. These complexities inevitably block any simplifications that are needed in formulating the required integral equations for the unknown surface currents. In order to overcome these complexities, a technique is outlined in here where the concept of common periodicity is introduced [7]. It is suggested that the field expansion at each array boundary be carried out in terms of the Floquet modes of a lattice periodicity that is common between the lattices of the two arrays. It is proposed that provided the ratio of the lattice sides of the adjacent arrays is a rational [Appendix B] number, then the tangential field vectors at various boundaries may be expanded by re-defining their Floquet modes according to the convolution theorem [8]. The problem of assigning dissimilar geometries to the structure is therefore addressed by employing the convolution theorem.
to modal (Floquet) sets, which expand the tangential fields in each array. Once the convolution has been executed (section 2.3.1), the application of the boundary conditions (as outlined in section A.4 of Appendix A) is implemented in the usual manner thus obtaining the required integral equations for the unknown currents (section 2.4.3). The advantages in redefining the Floquet modes in this manner are to unlock the complexities, which exist when expressing the modal coefficients in terms of the Fourier transform of the surfaces’ currents in the integral equation formulation. These integral equations relating the spectrum of the unknown current, to the modal Floquet coefficients (either transmitted or reflected) are then solved (section 2.5.1) by the Method of Moment (MoM) technique [9]. The method effectively reduces these integral equations to a linear system of matrices (i.e. simultaneous equations) from which the unknown current coefficients are computed. Alternatively, the integral equations expressed in terms of the Floquet modes set of the common periodicity can be optimised in terms of the Floquet mode sets of the individual arrays. This is outlined in section 2.5.2. Induced unknown surface currents are obtained with a view to determining the plane wave transmission/ reflection coefficients. Having obtained the unknown current coefficients, it is described in section 2.5.3 how the complex transmission and reflection coefficients can then be calculated for given incident field coefficients. A computer model is then developed (section 2.5.4) to compute the element currents and for predicting the plane wave transmission / reflection coefficients.

2.2 Formulation of Scattering from Multilayer FSS Structures

2.2.1 The Geometry and Floquet Expansion of the Fields

Figure 2.1(a) shows the side view of two parallel infinite arrays geometry and Figure 2.1(b) shows the z dependence of the reflection and transmission modes in the two parallel periodic arrays. The Figure also shows the electric field of a linearly polarised plane wave incident in an arbitrary direction. A linearly polarised plane electromagnetic wave is incident on side a of array #1. Each screen consists of doubly periodic array; with arbitrary shaped elements, mounted on a dielectric substrate. The tangential fields in the dielectrics, adjacent to each array are expanded in a Fourier series form according
Figure 2.1: (a) Side view of two Parallel Layers

(b) Two Parallel periodic Arrays with Dissimilar Lattice periodicities.

Arrows show the sense of Z-dependance of the modes with amplitudes R and T.
to Floquet's theorem. These are expressed as a linear combination of two (TE and TM) vector Floquet modes in a similar fashion to those outlined in the analysis of single layer structures described in Appendix A.

Care must be taken when considering the separation region between the two screens where the evanescent fields are of great importance and the coupling between the two arrays become high, particularly when the layers are closely spaced. Unlike the dielectric region, in the separation region a combination of two different sets of TE-TM vector Floquet modes are considered (equation (2.28)). The field expansions are performed by distinguishing the coefficient of the tangential fields in terms of the corresponding array parameters. The presence of the two layers existing simultaneously has influence on fields and currents, in which their evanescent modes are included inherently.

In the analysis outlined in this Chapter, the superscript 1 and 2 denote layers 1 and 2 respectively. The letters $a$ and $b$ in the superscripts stand for the sides of the different media. The subscript $mpq$ corresponds to layer 1 with dielectric thickness $S_1$ and dielectric constant $\varepsilon_r^1$. Similarly indices $m'ln$ corresponds to layer 2 with dielectric thickness $S_2$ and dielectric constant $\varepsilon_r^2$. As in the case of single layer in Appendix A, we adapt the convention that fields with coefficients $R$ correspond to waves with positive $z$-dependance (i.e. they would travel in -ve $z$ direction if propagating). The fields with coefficients $T$ corresponds to waves with negative $z$ dependence (i.e. they would travel with the +ve $z$ component of direction if propagating). Hence referring to figure 2.1(b), the coefficients $R$ represents the amplitude of the wave propagating in the -ve $z$ direction and $T$ the complex amplitude of the wave in the +ve $z$ direction. It must be noted that the dielectric substrate and the arrays both lie in the $x$-$y$ plane. The incident field is taken to be the field produced by the incident plane wave, in the absence of the metallic grid (i.e. the element array) but with the presence of the dielectric substrate and the ground plane. The incident lies in the $y$-$z$ plane as shown in figure A.2 (a) of Appendix A. The source of the scattered field is the surface current induced on the array elements by the incident
field. Each Floquet mode of the scattered and incident field must satisfy the boundary conditions outlined in section A.4 of Appendix A.

Assume that each array in the double layer structure is spaced periodically along the arbitrary $\hat{u}$ and $\hat{v}$ unit vectors and the lattice in each array has vectors as shown in Figure 2.2. They all lie in the x-y plane. Figure 2.2 shows a small section of the infinite array geometry. The lattice vectors $\mathbf{D}_u$ and $\mathbf{D}_v$ specify the periodicity axes and therefore the position of the arbitrary shaped elements in the surface. If $\alpha$ is the angle between the lattice vectors $\mathbf{D}_u$ and $\mathbf{D}_v$ and $\alpha_1$ is the angle $\mathbf{D}_u$ makes with the x-axis, then the lattice vectors $\mathbf{D}_u$ and $\mathbf{D}_v$ can be written as:

$$\mathbf{D}_u^i = \mathbf{D}_u \cos \alpha_1 \hat{x} + \sin \alpha_1 \hat{y}$$

$$\mathbf{D}_v^i = \mathbf{D}_v \cos \alpha_2 \hat{x} + \sin \alpha_2 \hat{y}$$

(2.1)

where $\alpha_2 = (\alpha_1 + \alpha)$ as shown in Figure 2.2 and where the superscript $i$ ($i = 1$ or 2) indicates the various arrays.

Figure 2.2: Infinite geometry of an FSS array on an arbitrary lattice
From the Floquet expansion technique which involves solving the scalar Helmholtz wave equation [5] in rectangular co-ordinates for a periodic array, the scalar Floquet's phasor (i.e. the phase variation of the field) [Appendix A] with position \( \mathbf{r} = x\hat{x} + y\hat{y} \) is written as:

\[
\theta^i_{\sigma\beta}(\mathbf{r}, z) = \psi^i_{\sigma\beta}(\mathbf{r}) \exp(+_j \gamma^i_{\sigma\beta} z)
\]

(2.2)

where \( \psi^i_{\sigma\beta}(\mathbf{r}) \) is given in (Appendix A) as:

\[
\psi^i_{\sigma\beta}(\mathbf{r}) = \exp(-j \frac{1}{k_{T\sigma\beta}} \mathbf{r})
\]

(2.3)

\[
\theta^i_{\sigma\beta}(\mathbf{r}, z) = \exp(-j \frac{1}{k_{T\sigma\beta}} \mathbf{r}) \exp(+_j \gamma^i_{\sigma\beta} z)
\]

where

\[
i = 1 \text{ or } 2
\]

\[
\sigma, \beta = 0, \pm 1, \pm 2, \pm 3, \ldots
\]

\( k^i_{T\sigma\beta} \) is the transverse propagation vector of the \((\sigma, \beta)\)th Floquet mode and \( \gamma^i_{\sigma\beta} \) represents the modal or \( z \)-directed propagation coefficients with respect to the normal of the surface. Following the same notations as in Appendix A, these are given by:

\[
K^i_{T\sigma\beta} = K^i_{T\sigma\beta} + \sigma K^1_{12} + \beta K^1_{22}
\]

or

\[
k^i_{T\sigma\beta} = k^i_x \hat{x} + k^i_y \hat{y}
\]

(2.4)

where

\[
k^i_{T\sigma\beta} = k_{\sigma\beta} \hat{x} + k_{\sigma\beta} \hat{y}
\]

\[
k_{\sigma\beta} = k_{0x} \sin \theta \sin \varphi
\]

\[
k_{\sigma\beta} = k_{0x} \sin \theta \cos \varphi
\]

(2.5)

with \( k_0 = \frac{2\pi}{\lambda} \), i.e. the free-space propagation constant.
In the above equation (2.5), \( \theta \) is the incident angle to the normal of the surface (Figure A.2(b) of Appendix A) and \( \varphi \) the angle between the plane of incidence and the x-axis, with the plane of incidence being the plane that includes the incident field and the normal to the surface.

\[
k_x^i = (k_0x + \sigma k_lx + \beta k_2x) \hat{x}
\]

\[
k_y^i = (k_0y + \sigma k_ly + \beta k_2y) \hat{y}
\]

\[
k_1^i = (-2\pi / A_i) \hat{z} \times \hat{D}_v^i, \quad k_2^i = (2\pi / A_i) \hat{z} \times \hat{D}_u^i
\]

where

\[
A_i = \left| \hat{D}_u^i \times \hat{D}_v^i \right|
\]

Using the relationships in equation (2.1), \( k_1^i \) and \( k_2^i \) can be written as:

\[
k_1^i = \frac{-2\pi}{D_u^i D_v^i \sin \alpha} \hat{z} \times (D_v^i \cos \alpha_2 \hat{x} + D_v^i \sin \alpha_2 \hat{y})
\]

\[
k_2^i = \frac{-2\pi}{D_u^i D_v^i \sin \alpha} (D_v^i \cos \alpha_2 (\hat{z} \times \hat{x}) + D_v^i \sin \alpha_2 (\hat{z} \times \hat{y}))
\]

(2.8)

carrying out the vector product in the above equation and using the vector product rule, \((\hat{z} \times \hat{x}) = \hat{y}\), and \((\hat{z} \times \hat{y}) = -\hat{x}\), the above equation for \( k_1^i \) becomes:

\[
k_1^i = -\frac{2\pi}{D_u^i D_v^i \sin \alpha} (D_v^i \cos \alpha_2 \hat{y} - D_v^i \sin \alpha_2 \hat{x})
\]

The above equation for \( k_2^i \) becomes:

\[
k_2^i = -\frac{2\pi \cos \alpha_2}{D_u^i \sin \alpha} \hat{y} + \frac{2\pi \sin \alpha_2}{D_u^i \sin \alpha} \hat{x}
\]

(2.9)

\[
k_{1y}^i = -\frac{2\pi \cos \alpha_2}{D_u^i \sin \alpha}
\]

(2.10a)

\[
k_{1x}^i = \frac{2\pi \sin \alpha_2}{D_u^i \sin \alpha}
\]

(2.10b)

Similarly using the relationship in equation (2.1), the expression for \( k_2^i \) in equation (2.7) becomes:
Chapter Two

\[ \vec{k}_2^i = \frac{2\pi}{D_u \sin \alpha} (D_u^i \cos \alpha \hat{z} \times \hat{x}) + \frac{2\pi}{D_v \sin \alpha} (D_v^i \sin \alpha \hat{z} \times \hat{y}) \] (2.11)

By carrying out the vector product operations the above equation is reduced to:

\[ \vec{k}_2^i = \frac{2\pi \cos \alpha}{D_v \sin \alpha} \hat{y} - \frac{2\pi \sin \alpha}{D_v \sin \alpha} \hat{x} \]

It can therefore be deduced that:

\[ \vec{k}_{2y}^i = \frac{2\pi \cos \alpha}{D_v \sin \alpha} \hat{y}, \quad \vec{k}_{2x}^i = -\frac{2\pi \sin \alpha}{D_v \sin \alpha} \hat{x} \] (2.12)

The z-directed propagation constant of the \((\alpha, \beta)\) the Floquet mode, as in equation (A.13) of Appendix A, is given by:

\[ \gamma_{\sigma\beta} = (K^2 - \vec{k}_T^i \sigma \beta \cdot \vec{k}_T^i \sigma \beta)^{1/2} \] (2.13)

where \((K)^2\) is the constant \(K\) raised to the power of 2 and is given by:

\[ K = K_0 \sqrt{\epsilon_r}, \text{ with } K_0 = \frac{2\pi}{\lambda}. \]

Also, the unit field vector \(\vec{k}_{m\sigma\beta}^1\) which lies in the \((x,y)\) plane is given by:

\[ \vec{k}_{m\sigma\beta}^1 = \vec{k}_T^1 \sigma \beta / |\vec{k}_T^1 \sigma \beta| \] (2.14)

where \(m=1\) or 2 denotes TM and TE components respectively.

Therefore for the two arrays in the double layer structure, the TM and TE components of the unit field vectors can be written as:

\[ \hat{\vec{k}}_{1pq}^1 = \frac{\vec{k}_{1pq}^1}{|\vec{k}_{1pq}^1|}, \quad \hat{\vec{k}}_{2pq}^1 = \hat{\vec{z}} \times \hat{\vec{k}}_{1pq}^1 \] (2.15)

The x and -y components of the unit field vectors in above are written as:

\[ \vec{k}_{1pq}^1 = \frac{\vec{k}_{1pq}^1}{|\vec{k}_{1pq}^1|}, \quad \text{i.e. the TM components} \] (2.16)

\[ \hat{\vec{k}}_{1pq}^1 = \frac{\vec{k}_{1pq}^1}{|\vec{k}_{1pq}^1|} \]

and

24
\[ \kappa_{2pq}^1 = \hat{z} \times \kappa_{1pq}^1 = -\kappa_{1pq}^1 \quad \text{i.e. the TE components} \] (2.17)

\[ \kappa_{2pq}^1 = \hat{z} \times \kappa_{1pq}^1 = \kappa_{1pq}^1 \] i.e. the TE components

Similarly, the x- and y components of the unit field vectors in the second arrays, i.e. \( \nu_{2ln}^x \) are written as:

\[ \nu_{1nx}^2 = \frac{k_{Tln}^2}{k_{Tln}^2} \] 
\[ \nu_{1ny}^2 = \frac{k_{Tln}^2}{k_{Tln}^2} \]

i.e. the TM components (2.18)

and

\[ \nu_{2nx}^2 = \hat{z} \times \nu_{1nx}^2 = -\nu_{1nx}^2 \] i.e. the TE components (2.19)

\[ \nu_{2ny}^2 = \hat{z} \times \nu_{1ny}^2 = \nu_{1ny}^2 \] i.e. the TE components

The TM and TE components of the modal admittance (section A.3, Appendix A) in the air (i.e. in the incident field region) are given by:

\[ \eta_{pq}^{\text{air}} = \frac{k_{\text{air}}}{\gamma_{pq}^{\text{air}}} \] 
\[ \eta_{2pq}^{\text{air}} = \frac{\gamma_{pq}^{\text{air}} \eta_{pq}^{\text{air}}}{k_{\text{air}}} \] (2.20)

where \( k_{\text{air}} \) and \( k_{\text{air}}^{\text{air}} \) are given by:

\[ \eta_{\text{air}} = \sqrt{\frac{\varepsilon}{\mu}} \quad \text{and} \quad k_{\text{air}} = \frac{2\pi}{\lambda} \sqrt{\frac{\varepsilon}{\mu}} \] (2.21)

Similarly, the modal admittances in the dielectric regions of the two arrays are given by:

\[ \eta_{pq}^1 = \frac{k^1}{\gamma_{pq}^1} \] 
\[ \eta_{2pq}^1 = \frac{\gamma_{pq}^1 \eta_{pq}^1}{k^1} \] (2.22)

and

\[ \eta_{1ln}^2 = \frac{k^2}{\gamma_{ln}^2} \] 
\[ \eta_{2ln}^2 = \frac{\gamma_{ln}^2 \eta_{pq}^2}{k^2} \] (2.23)

with \( \eta_{pq}^1 \) and \( \eta_{pq}^2 \) referring to the modal admittance of first and second layers.
respectively. \( k^1 \) and \( k^2 \) refer to the propagation constants of first and second layers respectively as defined in equation (2.21). \( \gamma^1_{pq} \), \( \eta^1_{mpq} \) and \( \gamma^2_{ln} \), \( \eta^1_{m\ln} \) refer to the substrates 1 and 2 respectively.

We now introduce the correlation functions \( \phi_{m'n}^{pq} \) and \( \phi_{m'ln}^{pq} \), which involve integration of the product of two vector Floquet modes set. These correlation functions are generally denoted as the inner products calculated on the unit cell area \( A_1 \) and \( A_2 \) which represent the area of the unit cells in layers 1 and 2 respectively. These correlation functions are given as:

\[
\phi_{m'ln}^{pq} = \begin{cases} 
A_1 \delta_{m'm} \delta_{l'p} \delta_{n'q} & \text{iff } D_{u,v}^1 = D_{u,v}^2 \\
\langle \psi^n \psi^n \rangle \hat{k}_{m'ln}^{2} \hat{k}_{mpq}^{1} & \text{otherwise}
\end{cases} 
\]

(2.25)

\[
\phi_{m'ln}^{pq} = \begin{cases} 
A_2 \delta_{m'm} \delta_{l'p} \delta_{q'n} & \text{iff } D_{u,v}^2 = D_{u,v}^1 \\
\langle \psi^1 \psi^2 \rangle \hat{k}_{mpq}^{1} \hat{k}_{m'ln}^{2} & \text{otherwise}
\end{cases} 
\]

(2.26)

\( D_{u,v}^1 \) and \( D_{u,v}^2 \) are the periodicities of the second and first layer arrays respectively within the structure, in the \( u \)- and \( v \) directions.

The inner product is defined as:

\[
a < C, D > = \int C(D^\dagger) \, da
\]

(2.27)

It is shown in Appendix C that the inner products \( \phi_{mpq}^{l'm'ln} \) and \( \phi_{m'l'n}^{mpq} \) are proportional to the product of two (sinc) functions, with each term in the (sinc) functions representing the difference of the tangential wave vectors \( (k^1_{Tpq} \text{ and } k^2_{Tln}) \) of the two arrays. It is further shown in Appendix C that when the two arrays have identical lattice periodicities, the
above two correlation functions collapse to a delta (δ) function where (δ) is the kronecker delta as defined in section A.2 of Appendix A.

2.3 Analysis of Scattering from Double Layer FSS Structures with Arbitrary Lattice Periodicities of the Arrays

Referring to Figure 2.1, the two layers are separated by a free space of distance d. The lattice geometries in the two are assumed to be arbitrary with periodicities $D_u$ and $D_v$ along the u and v axes respectively, as was shown in Figure 2.2. Concentrating on the separation region, the tangential electric field $E_{ti}^{ab}(r,z)$ is expressed as the superposition of the reflected and transmitted fields. Note that the standard tangential electromagnetic field expansion as a Fourier series of TE and TM vector Floquet modes is outlined in section A.2 of Appendix A. From Figure 2.1, in the separation region (i.e. at $(z_1 < z < z_2)$) a combination of two different sets of TE-TM Floquet modes is considered. Therefore the electric field expansion in the region in terms of the Floquet modes of the first and second layer arrays is given by:

$$E_{ti}^{ab}(r,z) = \sum_{mpq} T_{mpq}^{ih} \psi_{pq}^1(\xi) \exp(-j\gamma_{pq}^1 z) \hat{k}_{mpq}^1 + \sum_{m'ln} R_{m'ln}^{2a} \psi_{ln}^2(\xi) \exp(j\gamma_{ln}^2 z) \hat{k}_{m'ln}^2$$

(2.28)

Similar expression for the magnetic field expansion in the separation region can be obtained. The transmitted and reflected fields in the separation region are both periodic with respect to lattice of the first layer and second layer, respectively. Hence equation (2.28) provides a general account of the resultant wave which, in the absence of the numerical ambiguities in the solution of the coupled integral equations, provides an exact account of the propagating as well as of the evanescent modes in the separation region.

Since the two arrays have dissimilar element lattice periodicities then the two different sets of Floquet modes that expand the fields adjacent to the two arrays are dissimilar as can be seen from equation (2.28) above. Hence superposition of the two periodic (reflected and transmitted) fields in the separation region would generate scattered field with periodicity that is different from those of the two periodic fields. The modal analysis
technique would involve application of the boundary conditions (i.e. the continuity and
discontinuity of the electric and magnetic fields) at different interfaces of the structure in
Figure 2.1 (b) as well as making use of the orthogonality (section A.2, Appendix A) of
Floquet’s expansion \( \psi_{pq}^1 \) and \( \psi_{ln}^2 \). Hence if the fields are matched say at the two
interfaces, \( z = z_1 \) and \( z = z_2 \) of the structure in Figure 2.1 (b) then new set of scattered
Floquet harmonics would result as the two sets of Floquet modes \( \psi_{pq}^1 \) and \( \psi_{ln}^2 \) are no
longer orthogonal. These therefore lead to expressions that can not be simplified, due to
the extra complexities that are introduced from the coupling between the two unequal
Floquet mode sets. Consequently, it would be impossible to obtain integral equations that
relate only one set of coefficient (either transmitted or reflected) to the spectrum of the
currents (equation (2.151) and (2.152).

The above can alternatively be explained by considering the complex coefficients of the
Floquet harmonics scattered from the double layer structure. For simplicity, let us assume
that both arrays have rectangular element lattice geometries with periodicity axes
\( D_x^i \) and \( D_y^i \) along the x and y axes respectively, where, as before i denotes the various
arrays. The complex Floquet harmonics are scattered into locations in k-space as
predicted by the Floquet’s theorem (equation (2.6)) as follows:

\[
\begin{align*}
\frac{k_x}{D_x}^1 &= \frac{2\pi}{D_x}^1 + k_{0x}^1 \quad \text{and} \quad \frac{k_y}{D_y}^1 &= \frac{2\pi}{D_y}^1 + k_{0y}^1 \\
\frac{k_x}{D_x}^2 &= \frac{2\pi}{D_x}^2 + k_{0x}^2 \quad \text{and} \quad \frac{k_y}{D_y}^2 &= \frac{2\pi}{D_y}^2 + k_{0y}^2
\end{align*}
\]

(2.29)

When the two arrays in the structure have the same x and y periodicities, \( D_x \) and \( D_y \),
then application of boundary conditions at the interfaces \( z = z_1 \) and \( z = z_2 \) say,
would result into expressions that contain scattered Floquet modes occupying the same
locations in k-space (i.e. in equation (2.29), \( k_x^1 = k_x^2 \) and \( k_y^1 = k_y^2 \)). These scattered Floquet modes all couple into each other. Therefore no new set of scattered Floquet harmonics are generated and as a result, the various Floquet harmonics from the arrays are compatible from the point of view of modal analysis. If however, we assume that the two arrays have dissimilar lattice periodicities along the x - and y-axes, then the interaction of the two dissimilar Floquet modes would generate complex scattered Floquet modes that have different locations in the k-space. Therefore new set of complex scattered Floquet mode coefficients are generated due to coupling of non-similar Floquet harmonics from the two arrays. It is these new sets of scattered Floquet modes, which would prevent any necessary simplification after the boundary conditions are applied.

A technique is proposed in here which, aims to reduce the complexities that are encountered when integral equations for the two currents are being formed. It is based on the concept of existence of a common periodicity between two dissimilar lattice periodicities of any two adjacent arrays and hence between the Floquet modes set that expand the tangential fields adjacent to the arrays. The major assumption is that the ratio \( D^2 / D^1 \) of any two adjacent arrays, in the x and y direction must be rational (Appendix B) number. Under these assumptions the superposition of the two periodic functions in the separation region of Figure 2.1 would result in another periodic function with a periodicity which is mutual (i.e. common) to both. The mutual periodicity is uniquely defined as different integer multiples of the individual periodicities. By defining a larger common periodicity, the field expansions may be carried out (section 2.3.2) in terms of the mutual Floquet harmonics. This inevitably reduces the complexity that exist when obtaining the required integral equations, since in this case a single set of (mutual) Floquet harmonics are used for field expansions. The problem therefore collapses to that of modal analysis of FSS arrays with identical lattice periodicities. This important concept is now presented in full in section 2.3.1 below.
2.3.1 Concept of Mutual Periodicity between Two Periodic Arrays of Dissimilar Lattice Geometries

One important condition that needs to be accounted and indeed forms the initial part of the analysis, is the mutual periodicity that results from adding two periodic functions together. In addition, the field in any region can be expanded either in terms of their own or the mutual periodicity [8]. In the rational number system the common periodicity is defined as integer multiples of the individual periodicities. For example, considering a one-dimensional case of two periods, $D_1$ and $D_2$ in which their ratio $R = D_2 / D_1$ is rational, then one can find integers $M$ and $N$ so that $D_3 = N D_1$ and $D_3 = M D_2$, where $D_3$ denotes the common periodicity. The new system formed by cascading the two arrays of dissimilar lattice periods will have a scattered spectrum which is in general, more densely packed than the spectra of any of the individual array components. The locations of the harmonics of the new composite spectrum is given by:

$$k^3 = \frac{2\pi}{D_3} + k_0$$

(2.30)

where $D_3$ is shown in the example of Figure 2.3 below as $D_c$ (the common period).

\[ \text{Figure 2.3: Illustration of the common periodicity for two arrays of dissimilar lattice periodicities with period ratios } \]
\[ R = D_2 / D_1 = N / M = 2 / 3, \text{ and } D_3 = N D_1 = M D_2 = 6 \]
Since the ratio of the periodicity of the two arrays is a rational number, then the resulting system has a spectrum, which is discrete. However as the ratio of the two periodicities becomes more and more irrational [section B.2, Appendix B] then the system's spectrum fills up and becomes more densely packed. Although the periodicity of the system in this case may be infinitely large, it is not however common to the periodicities of the two periodic arrays. Thus in our analysis scheme, the case is only considered when the ratio of the lattice periodicities of the two arrays is assumed rational, and by definition a mutual periodicity therefore exists between the two periods.

Figure 2.4 shows a plot of the ratio \( R = D^1 / D^2 = M / N \) in the range 1 to 4 at steps of 0.025 where the upper limits of \( N \) and \( M \) were 20 and 80 respectively. It can be seen that there exist a finite number of sets (A-G) of ratios that can be classified according to a specific sequence of numbers. As shown on the plot, the smallest integers \( M, N \) are obtained for integer and half-integer ratios, sets A and B. A large integer number is however required for ratios to more than one decimal place or fractions with denominators of 4, 5, 8, 10 and 20, corresponding to sets C-G. The sequence describing set F for example is \( \{ (j + 11) / 10 \} \) for \( j = 0, 2, 6, 8, 10, 12, 16, 18, 20, 22, 26, 28 \). The ratios corresponding to the remaining values of \( j \) appear in sets A, B, C, D and E. For example value of \( j = 14 \) representing the ratio \( (25/10) \), could fall into set B as \((5/2)\). It is also noted that the position of the ratio \( R \) can be identified to that of a fractal dimension. For computational purposes, efforts must be made to choose the smallest common integers when dividing \( M \) by \( N \). An example is the case when \( D^2 = 14 \) and \( D^1 = 10 \), giving a ratio of \( D^2 / D^1 = 14 / 10 = 7 / 5 = 1.4 \). This would imply that \( N = 7, M = 5 \), giving a mutual periodicity of 35 between \( D^1 \) and \( D^2 \). However if \( N = 14 \) and \( M = 10 \) were selected, a common periodicity of 70 would result with a computation time twice as long as the first. This is quite important, particularly when the two periodicities are given as real numbers, which could produce a ratio to more than 3 or 4 decimal places.
Therefore assuming a rational ratio of periodicities of the two dissimilar arrays, the tangential field vectors in the separation region adjacent to the two periodic arrays of figure 2.1 for example may be expanded in terms of the mutual Floquet mode sets. The periodic function $E(D^1)$ for example, when expanded in terms of the mutual Floquet mode produces the function $E(ND^1)$ with a very large number of spectral components in its spectrum. However a substantial majority of these components in the spectrum $E(ND^1)$ are redundant and would not be required in the computations. This is because the spectral components of $E(ND^1)$ is $N$ times the original. The novelty in here is that the spectral components of the periodic function $E(D^1)$ can be resolved from the spectrum $E(ND^1)$ at locations of Floquet harmonics $st = N(pq)$, where $st$ denotes the Floquet mode indices of the mutual periodicity. In effect, the above implies that it is required to compute the modal coefficients at these Floquet mode locations only, with the remaining components in the spectrum of the common periodicity being redundant. In the next section (2.3.2) it will be shown theoretically how these redundant components (i.e. very small values) in the spectrum of the common periodicity can be bypassed in the computation.

2.3.2 Super-Resolution of the Spectral Components of Tangential Field Vectors By Means of Correlation Functions

An analytical procedure is outlined in this section. The aim is to demonstrate the fact that the tangential field vectors adjacent to the dissimilar lattice arrays can be expanded in terms of the Floquet mode set of the mutual periodicity. Again the assumption being that the ratio of their lattice periodicities must be rational. Expressions are obtained which relate the spectrums of the tangential fields adjacent to the two arrays, to those of their mutual periodicities. The spectral components of the various tangential fields are super-resolved by redefining their Floquet modes according to convolution theorem (equation (2.53) and (2.54)). The result is, as will be shown a considerable reduction in computations by carrying out the expansions in terms of Floquet modes of the common. The convolution relationships that will be obtained can be applied to the analysis of
Chapter Two

Figure 2.4: Positions of the periodicity ratio (A-G) as a function of integers $M$ and $N$
scattering from any multilayer structure whose array surfaces are periodic and infinite in extent and with the major assumption that the ratio of the lattice periodicities of adjacent arrays are rational.

Figure 2.6 shows a cross-section of a double layer structure with separation distance \( d \) between the two arrays. \( D_u \) and \( D_v \) represent the periodicities along the arbitrary axes \( u \) and \( v \) respectively. The structure is illuminated by a plane electromagnetic wave incident on the side of the first layer as shown. In Figure 2.6, \( a^- \) and \( b^+ \) are the complex coefficients of the reflected and transmitted fields respectively. In the separation region the tangential electrical field \( E_d \) expanded as a superposition of the reverse field (\( E^- \)) and forward field (\( E^+ \)) is as follows:

\[
E_d = E^- + E^+ = \sum_{pq} a_{pq} e^{j\gamma_{pq}z} \psi_{pq}^1 + \sum_{ln} b_{ln} e^{-j\gamma_{ln}z} \psi_{ln}^2
\]

where as before, \( \psi_{pq}^1 \) and \( \psi_{ln}^2 \) are the vector Floquet modes in a doubly periodic fashion. The above expression is similar to equation (2.28) of section 2.3, except for the fact that for simplicity the mode index \( m \) denoting TE and TM are omitted in here. \( a_{pq} \) is the amplitude of the reverse wave \( E^- \) for the \( pq \) order mode and \( b_{ln} \) is the amplitude of the forward wave \( E^+ \) for the \( ln \) order mode. The reverse and forward waves are periodic with respect to the lattice of the first and second layers respectively. For simplicity, we assume that \( z=0 \). Equation (2.31) therefore becomes:

\[
E_d = E^- + E^+ = \sum_{pq} a_{pq} \psi_{pq}^1 + \sum_{ln} b_{ln} \psi_{ln}^2
\]

Let us consider the periodic function \( E(D^1) \), the reverse wave (i.e. the tangential reflected field adjacent to the first array, given below:

\[
E(D^1) = \sum_{pq} a_{pq} e^{-j(\frac{2\pi p}{D_k} x + \frac{2\pi q}{D_l} y)}
\]
For simplicity it has been assumed in the above equation that the lattice periodicities $D_X^1$ and $D_Y^1$ in the first array lie on square lattices along the x and y axes respectively instead of the triangular lattice shown in Figure 2.6. Similarly $E(D^2)$, the tangential field adjacent to the second array is given by:

$$E(D^2) = \sum_{l_n} b_{l_n} e^{j \left( \frac{2\pi l}{D_X^2} x + \frac{2\pi n}{D_Y^2} y \right)}$$

(2.34)

Again the assumption was made in the above equation that the lattice periodicities $D_X^2$ and $D_Y^2$ in the second array lie on a square lattice along the x and y-axes.

The two periodic functions $E(D^1)$ and $E(D^2)$ can also be expanded in terms of the Floquet modes of the common periodicity. If we denote the modal indices of the mutual set by $s_t$, then the function $E(D^1)$ expanded in terms of the mutual Floquet mode set $s_t$ is as follows:

$$E(ND^1) = \sum_{s_t} a_{s_t} e^{j \left( \frac{2\pi s}{D_X^1} x + \frac{2\pi t}{D_Y^1} y \right)}$$

(2.35)

In above $a_{s_t}$ are modal coefficients of the periodic function $E(ND^1)$ expanded in terms of the Floquet modes of the common periodicity, and where $D_X^3$ and $D_Y^3$ are the x-directed and y-directed lattices of the mutual periodic array $E(D^3)$, forming a square lattice. Similarly the periodic function $E(D^2)$ expressed in terms of the mutual Floquet mode set $s_t$ is given by:

$$E(MD^2) = \sum_{s_t} b_{s_t} e^{j \left( \frac{2\pi s}{D_X^2} x + \frac{2\pi t}{D_Y^2} y \right)}$$

(2.36)

where $b_{s_t}$ are the modal coefficients expressed in terms of the Floquet mode sets of the common periodicity.

The superposition of the two periodic functions $E(ND^1)$ and $E(MD^2)$ results into another periodic function $E(D^3)$ expressed in terms of the Floquet modes of the mutual periodicity with period $D^3$ as follows:
Figure 2.6: Definition of lattice geometry and the side view of the Double Layer showing symbolically the amplitude of the scattered waves.
\[ E(D^3) = \sum_{st} c_{st} e^{j\frac{2\pi s}{D_x} x + \frac{d\pi t}{D_y} y} \]  
\[ (2.37) \]

where \( c_{st} \) are the complex spectral components (i.e. the modal coefficients) of \( E(D^3) \) with the periodicity mutual to both sets. Thus:

\[ E(D^3) = E(ND^1) + E(MD^2) \]  
\[ (2.38) \]

\[ \sum_{st} c_{st} \psi_{st} = \sum_{st} a_{st} \psi_{st} + \sum_{st} b_{st} \psi_{st} \]
\[ = \sum_{st} (a_{st} + b_{st}) \psi_{st} \]  
\[ (2.39) \]

or

\[ \sum_{st} c_{st} e^{j\frac{2\pi s}{D_x} x + \frac{2\pi t}{D_y} y} = \sum_{st} \left( a_{st} + b_{st} \right) e^{j\frac{2\pi s}{D_x} x + \frac{2\pi t}{D_y} y} \]  
\[ (2.40) \]

The above equations can be further simplified by using the orthogonality (equation A.15, Appendix A) of the Floquet modes and the properties of Kronecker delta (\( \delta \)). Therefore from equation (2.40), as a result of taking inner product with \( \psi_{st}^* \) (i.e. multiply both sides of the equation by \( \psi_{st}^* \) and integrate over the unit cell area \( A_3 \)) and also applying Floquet orthogonality, the following equation is obtained.

\[ \sum_{st} (\delta_s \delta_t) (a_{st} + b_{st}) = \sum_{st} \delta_s \delta_t c_{st} \]  
\[ (2.41) \]

and using the property of Kronecker delta we obtain:

\[ c_{st} = a_{st} + b_{st} \]  
\[ (2.42) \]

It is seen from the above equation (2.42) that an excessive number of modes would be required in computing the modal coefficients \( a_{st} \) and \( b_{st} \), leading to a large amount of computation time. However, the mutual periodicity function \( E(D^3) \) is also written as:

\[ E(D^3) = E(D^1) + E(D^2) \]

\[ \sum_{st} c_{st} \psi_{st} = \sum_{pq} a_{pq} \psi_{pq}^1 + \sum_{ln} b_{ln} \psi_{ln}^2 \]  
\[ (2.43) \]
Multiplying both sides of equation (2.43) by $\psi_{st}^*$ and integrating over the unit cell area $A_3$ will result in the following expression.

$$
\sum_{st} c_{st} \left( \int_{A_3} \psi_{st} \psi_{st}^* dA_3 \right) = \sum_{pq} a_{pq} \left( \int_{A_3} \psi_{pq}^1 \psi_{st}^* dA_3 \right) + \sum_{ln} b_{ln} \left( \int_{A_3} \psi_{ln}^2 \psi_{st}^* dA_3 \right)
$$

or

$$
\sum_{st} c_{st} \delta_s \delta_t = \sum_{pq} a_{pq} \phi_{pq}^{st} + \sum_{ln} b_{ln} \phi_{ln}^{st}
$$

In the above equation (2.45), $\phi_{pq}^{st}$ and $\phi_{ln}^{st}$ are the correlation functions as defined previously which involve the integration of the product of two Floquet mode sets. In equation (2.43), substituting for $\psi_{pq}^1$, $\psi_{ln}^2$ and $\psi_{st}$ yields:

$$
A_3 \sum_{st} (\delta_s \delta_t) \ c_{st} = \\
\sum_{pq} a_{pq} \int_{-D_x^3/2}^{D_x^3/2} e^{2j\pi \left( \frac{s}{D_x^3} - \frac{p}{D_y^3} \right)x} \ dx \int_{-D_y^3/2}^{D_y^3/2} e^{2j\pi \left( \frac{t}{D_y^3} - \frac{q}{D_y^3} \right)y} \ dy \\
+ \sum_{ln} b_{ln} \int_{-D_x^3/2}^{D_x^3/2} e^{2j\pi \left( \frac{s}{D_x^3} - \frac{1}{D_y^3} \right)x} \ dx \int_{-D_y^3/2}^{D_y^3/2} e^{2j\pi \left( \frac{t}{D_y^3} - \frac{n}{D_y^3} \right)y} \ dy
$$

Carrying out the integration and simplifying with the fact that

$$D_x^3 = ND_x^1, \ D_y^3 = ND_y^1$$

and

$$D_x^3 = MD_x^1, \ D_y^3 = MD_y^1$$

one will arrive at the following simplified equation:

$$
A_3 \sum_{st} (\delta_s \delta_t) \ c_{st} = A_3 \sum_{pq} a_{pq} \ \text{sinc} \left( \pi \left( s - Np \right) \right) \ \text{sinc} \left( \pi \left( t - Nq \right) \right) \\
+ A_3 \sum_{ln} b_{ln} \ \text{sinc} \left( \pi \left( s - Ml \right) \right) \ \text{sinc} \left( \pi \left( t - Mn \right) \right)
$$

(2.47)
Equations (2.45) and (2.47) above shows that the modal coefficients $c_{st}$ are related to the spectral components $a_{pq}$ and $b_{ln}$, of the two periodic functions $E(D^1)$ and $E(D^2)$ respectively, by means of correlation functions which are proportional to the product of two (sine) functions. These (sine) functions include the difference of the tangential wave vectors of two arrays (equation (2.46)). An analogous function in the formulation of waves in quantum mechanics is presented in reference 10. For identical lattices these correlation functions collapse to a delta function, and the Floquet modes form a set of orthonormal functions. Furthermore, The (sine) functions in the summations of the above equation are given by:

$$\text{sinc} \left( \pi (s - Np) \right) \text{sinc} \left( \pi (t - Nq) \right) = \frac{\sin \left( \pi (s - Np) \right)}{\pi (s - Np)} \cdot \frac{\sin \left( \pi (t - Nq) \right)}{\pi (t - Nq)}$$

$$= \begin{cases} 1 & \text{when } s = Np \text{ and } t = Nq \\ 0 & \text{otherwise} \end{cases}$$

(2.48)

and

$$\text{sinc} \left( \pi (s - Ml) \right) \text{sinc} \left( \pi (t - Mn) \right) = \frac{\sin \left( \pi (s - Ml) \right)}{\pi (s - Ml)} \cdot \frac{\sin \left( \pi (t - Mn) \right)}{\pi (t - Mn)}$$

$$= \begin{cases} 1 & \text{when } s = Ml \text{ and } t = Mn \\ 0 & \text{otherwise} \end{cases}$$

(2.49)

Equation (2.47) is accordingly reduced to equation given below.

$$A_3 \, c_{st} = A_3 \left( \sum_{pq} a_{pq} \, \delta_{Npq}^{st} + \sum_{ln} b_{ln} \, \delta_{Mln}^{st} \right)$$

(2.50)

where

$$\sum_{pq} a_{pq} \, \delta_{Npq}^{st} = \sum_{pq} a_{pq} \, \text{sinc} \left( \pi (s - Np) \right) \text{sinc} \left( \pi (t - Nq) \right)$$

(2.51)

and

$$\sum_{ln} b_{ln} \, \delta_{Mln}^{st} = \sum_{ln} b_{ln} \, \text{sinc} \left( \pi (s - Ml) \right) \text{sinc} \left( \pi (t - Mn) \right)$$

(2.52)
Using the properties of the Kronecker delta function, the two summations involved in equation (2.50) become as:

\[ \sum_{pq} a_{pq} \delta_{N(pq)}^{st} = a_{N(pq)} \quad (2.53) \]

and

\[ \sum_{ln} b_{ln} \delta_{M(ln)}^{st} = b_{M(ln)} \quad (2.54) \]

Therefore, in order to resolve the expansion of the fields in terms of the common periodicity we have used the convolution of the fields of the first and second arrays with a set of delta functions, which have identical lattice spacing to that of the mutual periodicity. Furthermore it is seen from the above two equations (2.5.3) and (2.5.4) that the modal coefficients \( a_{pq} \) and \( b_{ln} \) can be resolved from the spectrum of their mutual periodicity functions \( a_{st} \) and \( b_{st} \) at Floquet mode locations \( st = N \cdot (pq) \) and \( st = M \cdot (ln) \) respectively.

It should be noted that the important replication property of the delta function has been utilised in above equations. This states that the convolution of any function with the delta function leaves that function unchanged, i.e., given a function \( g(t) \), then its convolution with the delta function \( \delta(t) \) defined as:

\[ \int_{-\infty}^{\infty} g(\tau) \delta(t-\tau) \, d\tau = g(t) \quad (2.55) \]

By definition convolution in the time domain is same as the multiplication in the spectrum domain. Therefore we have:

\[ g(t) \otimes \delta(t) = \text{FT}[g(t)] \cdot \text{FT}[^\delta(t)] \quad (2.56) \]

where the symbol \( \otimes \) denotes the convolution. Since the Fourier transform of a delta function is unity (its spectrum extends uniformly over the entire frequency interval), i.e.

\[ \text{FT}(\delta(t)) = 1 \]

Then

\[ \text{FT}[g(t)] \cdot \text{FT}[\delta(t)] = \text{FT}[g(t)] \quad (2.57) \]
From the relationships of (2.53) and (2.54), equation (2.50) is therefore written as:

\[ \text{c}_{st} = \text{a}_{st} + \text{b}_{st} = \text{a}_{N(pq)} + \text{b}_{M(ln)} \]  \hspace{1cm} (2.58)

Hence it is deduced that:

\[ \text{a}_{st} = \begin{cases} \text{a}_{N(pq)} = \text{a}_{pq} & \text{when } st = N(pq) \\ 0 & \text{otherwise} \end{cases} \]  \hspace{1cm} (2.59)

Similarly, for modal coefficients of the second array, we have:

\[ \text{b}_{st} = \begin{cases} \text{b}_{M(ln)} = \text{b}_{ln} & \text{when } st = M(ln) \\ 0 & \text{otherwise} \end{cases} \]  \hspace{1cm} (2.60)

The above two equations (2.59) and (2.60) finally demonstrate the fact that the modal coefficients \( a_{st} \) and \( b_{st} \) include a large number of zeros. Their non-zero values are those identical to \( a_{pq} \) and \( b_{ln} \) respectively. As an example, figure 2.7 below shows the amplitude of \( a_c \) (\( c \) being the compact form of the modal indices \( st \)), the spectral components of a simple cosine function of periodicity \( D^1 = 4 \text{mm} \) and pulse width \( 2 \text{mm} \).

![Figure 2.7: A Floquet modes set of the mutual periodicity](image-url)
If \( D^2 = 9 \text{mm}, (R = 2.25) \) then the smallest mutual period \( D^3 = 36 \text{mm} \) (\(|c| \leq 180\)). It can be seen from figure 2.7 that a lot of zero points exist in \( a_c \) since \( a_c \) is the spectrum of 9 times the original. The non-zero points, \( a_p \) (\( p \) being the compact form of the modal indices \( pq \)), require only 40 points (\(|p| \leq 20\)), but at a coarser sampling rate.

The validity of the important relationships in equations (2.53) and (2.54) which relate the mutual Floquet mode sets to either of the individual sets can also be verified as follows: The periodic function \( E(D^1) \), expressed in terms of the spectrum of the mutual periodicity function is written as:

\[
E(D^1) = \sum_{st} a_{st} \psi_{st}
\]  (2.61)

Inverse Fourier transformation of the above becomes:

\[
a_{st} = \frac{1}{A_3} \int_{A_3} E(D^1) \psi_{st}^* \, dA_3
\]  (2.62)

Substituting for \( E(D^1) \) gives:

\[
a_{st} = \frac{1}{A_3} \sum_{pq} \int_{A_3} (a_{pq} \psi_{pq}^1 \psi_{st}^*) \, dA_3
\]  (2.63)

Note that in the above equation it is assumed that summation and integration can be interchanged. It follows that:

\[
a_{st} = \sum_{pq} a_{pq} \phi_{pq}^{st}
\]  (2.64)

This relationship is similar to that of equation (2.51) with the correlation function being proportional to the product of two sinc functions, i.e.

\[
\phi_{pq}^{st} = \text{sinc} \left( \pi (s-Np) \right) \text{sinc} \left( \pi (t-Nq) \right)
\]

In a similar way, we can express \( b_{st} \) from equation (2.36)) as below:

\[
b_{st} = \sum_{ln} b_{ln} \phi_{ln}^{st}
\]  (2.65)

where
\[ \phi_{in}^{st} = \langle \psi_{in}^2, \psi_{st}^* > = \text{sinc} (\pi(s-M I)) \text{sinc} (\pi(t-M n)) \]

Note that the following relationship is also valid.

\[ a_{pq} = \sum_{st} a_{st} \phi_{pq}^{st} \]  

(2.66)

**Proof:** The right hand side of the above equation can be written as:

\[
\sum_{st} a_{st} \phi_{pq}^{st} = \frac{1}{A_3} \sum_{st} \int_{A_3} a \psi_{st}^* \psi_{st}^* \psi_{pq} \psi_{pq}^* dA_3 \]

\[
= \frac{1}{A_3} \sum_{st} \int_{A_3} \psi_{st} \psi_{st}^* dA_3 \int_{A_3} a \psi_{pq} \psi_{pq}^* dA
\]

Since

\[
\sum_{st} \int_{A_3} \psi_{st} \psi_{st}^* dA_3 = \sum_{st} \delta_{st} = 1 \quad \text{(by Floquet orthogonality)}
\]

then

\[
\sum_{st} a_{st} \phi_{pq}^{st} = \frac{1}{A_3} \int_{A_3} a \psi_{pq} \psi_{pq}^* = a_{pq}
\]  

(2.67)

Similarly it can be shown that the relationship below is also valid.

\[ b_{ln} = \sum_{st} b_{st} \phi_{ln}^{st} \]  

(2.68)

In summary therefore, we have outlined a common periodicity approach for analysing double layer FSS structures with dissimilar lattice geometries. The importance of the proposed approach is two fold. A super-resolution of the spectral components of the tangential fields was used and was achieved by assigning the fields a mutual periodicity. Second, the elimination of the superfluous components that were introduced was accomplished by the use of correlation functions and their orthogonality conditions.

Having established the concept of mutual periodicity, we apply the important convolution
relationships, for analysing the scattering from double layer structures such as the one depicted in Figure 2.1. This will be presented in section 2.4. The relationships enable the scattered field expansions at various boundaries to be carried out in terms of their own mutual Floquet mode sets with advantages described in this sections. The modal coefficients $a$ and $b$ are therefore easily determined from the electric and magnetic boundary conditions applied at the interfaces of the structure. It will be shown in section 2.4.2 that the number of resultant integral equations obtained is the same as the number of layers. Each integral equation contains the spectra of the unknown surface currents in a superimposed form.

2.4 Super-resolution Approach to Modal Analysis of Scattering from two layer FSS
Before proceeding to the expansion of the scattered fields there are some remarks which need mentioning with regards to the approach adopted here for obtaining the coupled integral equations. In order to reduce the length of the algebraic solutions, the analysis is divided into two parts. In the first (section 2.4.1) a solution to the problem is outlined in the absence of scatterers, but with the presence of the dielectric substrates. Next (section 2.4.2) the scattered field expansions are considered in order to obtain equations which relate the scattered Fourier coefficients to the surface current densities (equations (2.151) and (2.152)). The total tangential fields in the absence of the scatterers are then combined with the total tangential scattered fields under common boundary conditions, leading to the integral equations required. The resulting equations are then solved for the unknown currents and the reflected / transmitted fields.

2.4.1 Field Expansion at Different Boundaries in the Absence of Scatterers
Referring to Figure 2.1, the methodology adopted here is based on multiple reflection theory by considering the tangential electromagnetic fields and applying boundary conditions. Each transmitted or reflected field coefficient is the sum of multiple transmission or reflection coefficient at the interface considered.

The tangential field expansion in terms of the TE and TM vector Floquet modes of a
single layer FSS is outlined in section A.3 of (Appendix A). We now consider the tangential fields adjacent to the interfaces of the double layer structure in the absence of the scattering elements. In the absence of the scatterers, the only propagating modes are the dominant \((0,0)\) order modes. The field expansions are therefore carried out in terms of the \((0,0)\) order TE and TM vector Floquet modes. Application of boundary conditions at the various interfaces result into expressions for the reflected and transmitted coefficients from which the total tangential fields at the two boundaries are obtained. Referring to Figure 2.1(b), the process of field expansion at various interfaces are outlined as below.

At \(Z \leq 0\)

\[
E_t^{1s}(r) = \sum_{m=1}^{2} \eta_{moo}^{\text{air}} \psi_{oo}^{1}(r) e^{+j\gamma_{oo}^{\text{air}} z} \hat{\kappa}_{moo}^{1} + E_{t}^{1}(r) \tag{2.69}
\]

\[
H_t^{1s}(r) = - \sum_{m=1}^{2} \eta_{moo}^{\text{air}} R_{moo}^{1a} \psi_{oo}^{1}(r) e^{+j\gamma_{oo}^{\text{air}} z} \hat{\kappa}_{moo}^{1} + H_{t}^{1}(r) \tag{2.70}
\]

Where the incident electric and magnetic fields \(E_{t}^{1}(r)\) and \(H_{t}^{1}(r)\) respectively are given by the sum of their TE and TM components:

\[
E_{t}^{1}(r) = \sum_{m=1}^{2} b_{m}^{\text{inc}} \psi_{oo}^{1}(r) e^{-j\gamma_{oo}^{\text{air}} z} \hat{\kappa}_{moo}^{1} \tag{2.71}
\]

and

\[
H_{t}^{1}(r) = \sum_{m=1}^{2} \eta_{moo}^{\text{air}} b_{m}^{\text{inc}} \psi_{oo}^{1}(r) e^{-j\gamma_{oo}^{\text{air}} z} \hat{\kappa}_{moo}^{1} \tag{2.72}
\]

\(b_{m}^{\text{inc}}\) is the amplitude coefficient of the incident field. This amplitude depends on the polarisation, amplitude and direction of propagation of the incident wave.

Substituting for the incident fields into equations (2.69) and (2.70) will give:

\[
E_{t}^{1s}(r) = \sum_{m=1}^{2} \left( b_{m}^{\text{inc}} e^{-j\gamma_{oo}^{\text{air}} z} + R_{moo}^{1a} e^{+j\gamma_{oo}^{\text{air}} z} \right) \psi_{oo}^{1}(r) \hat{\kappa}_{moo}^{1} \tag{2.73}
\]

\[
H_{t}^{1s}(r) = \sum_{m=1}^{2} \eta_{moo}^{\text{air}} \left( - R_{moo}^{1a} e^{j\gamma_{oo}^{\text{air}} z} + b_{m}^{\text{inc}} e^{-j\gamma_{oo}^{\text{air}} z} \right) \psi_{oo}^{1}(r) \hat{\kappa}_{moo}^{1} \tag{2.74}
\]
Chapter Two

It should be noted that we are dealing here only with the propagating modes \( p=q=0 \). Higher order modes are included when the scatterers are introduced (section 2.4.2).

At \( 0 \leq Z \leq Z_1 \)

\[
E_{t}^{1\text{lab}} (t) = \sum_{m=1}^{2} \left( T_{\text{moo}}^{1\text{lab}} \psi_{\text{o}0}^{1} (t) e^{-j\gamma_{\text{o}0}^{2} z} \hat{e}_{z} + R_{\text{moo}}^{1\text{lab}} \psi_{\text{o}0}^{1} (t) e^{+j\gamma_{\text{o}0}^{2} z} \hat{e}_{z} \right) (2.75)
\]

\[
H_{t}^{1\text{lab}} (t) = \sum_{m=1}^{2} \left( \eta_{\text{moo}}^{1} \left[ T_{\text{moo}}^{1\text{lab}} e^{-j\gamma_{\text{o}0}^{2} z} - R_{\text{moo}}^{1\text{lab}} e^{+j\gamma_{\text{o}0}^{2} z} \right] \psi_{\text{o}0}^{1} (t) \hat{e}_{z} \times \hat{e}_{z} \right) (2.76)
\]

At \( Z_1 < Z < Z_2 \) i.e. the separation region

\[
E_{t}^{2\text{lab}} (t) = \sum_{m=1}^{2} T_{\text{moo}}^{2\text{lab}} \psi_{\text{o}0}^{1} (t) e^{-j\gamma_{\text{o}0}^{2} z} \hat{e}_{z} + \sum_{m=1}^{2} R_{\text{moo}}^{2\text{lab}} \psi_{\text{o}0}^{1} (t) e^{+j\gamma_{\text{o}0}^{2} z} \hat{e}_{z} \ (2.77)
\]

\[
H_{t}^{2\text{lab}} (t) = \sum_{m=1}^{2} \eta_{\text{moo}}^{2\text{lab}} T_{\text{moo}}^{1\text{lab}} \psi_{\text{o}0}^{1} (t) e^{-j\gamma_{\text{o}0}^{2} z} \hat{e}_{z} \times \hat{e}_{z} + \sum_{m=1}^{2} \eta_{\text{moo}}^{2\text{lab}} R_{\text{moo}}^{2\text{lab}} \psi_{\text{o}0}^{2} (t) e^{+j\gamma_{\text{o}0}^{2} z} \hat{e}_{z} \times \hat{e}_{z} (2.78)
\]

At \( Z_2 < Z < Z_3 \) (i.e. second dielectric region)

\[
E_{t}^{2\text{lab}} (t) = \sum_{m=1}^{2} \left( T_{\text{moo}}^{2\text{lab}} e^{-j\gamma_{\text{o}0}^{2} z} + R_{\text{moo}}^{2\text{lab}} e^{+j\gamma_{\text{o}0}^{2} z} \right) \psi_{\text{o}0}^{2} \hat{e}_{z} (2.79)
\]

\[
H_{t}^{2\text{lab}} (t) = \sum_{m=1}^{2} \eta_{\text{moo}}^{2\text{lab}} \left[ T_{\text{moo}}^{2\text{lab}} e^{-j\gamma_{\text{o}0}^{2} z} - R_{\text{moo}}^{2\text{lab}} e^{+j\gamma_{\text{o}0}^{2} z} \right] \psi_{\text{o}0}^{2} (t) \hat{e}_{z} \times \hat{e}_{z} (2.80)
\]

Finally at \( Z \geq Z_3 \) (i.e. the transmission region)

\[
E_{t}^{2\text{b}} (t) = \sum_{m=1}^{2} T_{\text{moo}}^{2\text{b}} e^{-j\gamma_{\text{o}0}^{2} z} \psi_{\text{o}0}^{2} (t) \hat{e}_{z} (2.81)
\]

\[
H_{t}^{2\text{b}} (t) = \sum_{m=1}^{2} \eta_{\text{moo}}^{2\text{b}} T_{\text{moo}}^{2\text{b}} e^{-j\gamma_{\text{o}0}^{2} z} \psi_{\text{o}0}^{2} (t) \hat{e}_{z} \times \hat{e}_{z} (2.82)
\]

In the absence of the scatterers the boundary condition that the tangential electric and magnetic fields are continuous across the different interfaces can now be enforced. The aim is to express the reflected field amplitude \( R_{\text{moo}}^{1\text{a}} \) in terms of the incident field amplitude. Applying the boundary conditions starting at the interface \( Z = Z_3 \), and working
towards the boundary $Z = 0$ and simultaneously eliminating the various transmitted fields, expression for the reflected field amplitude can be obtained. The following boundary conditions are implemented.

At $z = 0$ \[ E_t^{1a} = E_t^{1ab}, \quad H_t^{1a} = H_t^{1ab} \] (2.83)

At $z = z_1$ \[ E_t^{1ab} = E_t^{2ab}, \quad H_t^{1ab} = H_t^{2ab} \] (2.84)

At $z = z_2$ \[ E_t^{2ab} = E_t^{2ab}, \quad H_t^{2ab} = H_t^{2ab} \] (2.85)

At $z = z_3$ \[ E_t^{2ab} = E_t^{2b}, \quad H_t^{2ab} = H_t^{2b} \] (2.86)

Note that the condition that the tangential electric field $\mathbf{E}$ must vanish at the ground plane does not hold in here since there are no conductors.

Matching fields at $Z = Z_3$, taking inner product with $\psi_\infty$ from both sides of the equation

(i.e. multiplying both sides with $\psi_\infty$ and integrating with respect to the unit cell area $A$ where the symbol $\ast$ denote the complex conjugate) and applying Floquet orthogonality the following expressions are obtained:

\[ T_{moo}^{2ab} e^{-\mathcal{J}r_{oo}^2 z} + R_{moo}^{2ab} e^{+\mathcal{J}r_{oo}^2 z} = T_{moo}^{2b} e^{-\mathcal{J}r_{oo}^2 z} \] (2.87)

and

\[ \eta_{moo}^2 (T_{moo}^{2ab} e^{-\mathcal{J}r_{oo}^2 z} + R_{moo}^{2ab} e^{+\mathcal{J}r_{oo}^2 z}) = \eta_{moo}^{aur} T_{oo}^{2b} e^{-\mathcal{J}r_{oo}^2 z} \] (2.88)

From (2.87) and (2.88), we obtain:

\[ R_{moo}^{2ab} = R_{moo}^{2b} \] (2.89)

where

\[ R_{moo}^{2b} = \left( \frac{\eta_{moo}^2 - \eta_{moo}^{aur}}{\eta_{moo}^2 + \eta_{moo}^{aur}} \right) e^{-2\mathcal{J}r_{oo}^2 z_3} \] (2.90)

and
Matching fields at $Z = Z_2$ and taking inner product with $\psi_{oo}$ yields:

$$T_{la} e^{-j\gamma_{oo} z} + R_{2b} e^{j\gamma_{oo} z} =$$

$$T_{moo} e^{-j\gamma_{oo} z} + R_{2ab} e^{j\gamma_{oo} z}$$

and

$$\eta_{moo} T_{moo} e^{-j\gamma_{oo} z} - \eta_{moo} R_{moo} e^{j\gamma_{oo} z} =$$

$$\eta_{moo} (T_{moo} e^{-j\gamma_{oo} z} - R_{moo} e^{j\gamma_{oo} z})$$

Note that the orthogonality used at $Z = 0$ takes the form of a single layer since $\kappa^{1}_{moo}$ and $\kappa^{2}_{moo}$ are in the same direction (i.e. the Floquet modes are the same in both layers) and $\psi_{oo} = \psi_{oo}$. Substituting equation (2.89) into (2.92) and simplifying gives:

$$T_{2ab} = T_{la} e^{-j\gamma_{oo} z} + R_{2b} e^{j\gamma_{oo} z}$$

Substituting for $R_{2ab}$ from equation (2.89) into (2.93) and in the resultant equation substituting for $T_{la}$ gives the following:

$$R_{moo} = R_{moo}^{++} T_{moo}$$

where

$$R_{mpq}^{++} = \left( \frac{\eta_{mpq} - \eta_{mpq}^* \eta_{mpq}^{*}}{\eta_{mpq}^* + \eta_{mpq}^2} \right) e^{-2j\gamma_{pq} z}$$

and where

$$R_{mpq}^* = \frac{e^{-j\gamma_{ln} z}}{e^{-j\gamma_{ln} z} + R_{mpq} e^{j\gamma_{ln} z}}$$

and similarly

$$R_{mln}^* = \frac{e^{-j\gamma_{ln} z}}{e^{-j\gamma_{ln} z} + R_{mln} e^{j\gamma_{ln} z}}$$
Matching the fields at $Z = Z_1$, taking inner product with $\psi_{00}$, simplifying and substituting for $R_{moo}^{2b}$ from equation (2.95) will give the following:

\[ T_{moo}^{1ab} e^{-\gamma_{00 z}} + R_{moo}^{1ab} e^{\gamma_{00 z}} = \]
\[ T_{moo}^{1a} e^{-\gamma_{00 z}} + R_{moo}^{++} T_{moo}^{1a} e^{\gamma_{00 z}} \]  
(2.100)

and for the magnetic field:

\[ \eta_{moo}^{1} T_{moo}^{1ab} e^{-\gamma_{00 z}^1} - \eta_{moo}^{1ab} R_{moo}^{1\sigma} e^{\gamma_{00 z}^1} = \]
\[ \eta_{moo}^{1\sigma} T_{moo}^{1ab} e^{-\gamma_{00 z}^{1\sigma}} - \eta_{moo}^{1\sigma} R_{moo}^{1ab} e^{\gamma_{00 z}^{1\sigma}} \]  
(2.101)

Substituting for $R_{moo}^{2b}$ from equation (2.95) into the above equation (2.101) and also substituting for $T_{moo}^{1a}$ obtained from equation (2.100) into the resultant equation and simplifying will give:

\[ \eta_{moo}^{1} T_{moo}^{1ab} e^{-\gamma_{00 z}^1} - \eta_{moo}^{1ab} R_{moo}^{1\sigma} e^{\gamma_{00 z}^1} = \]
\[ \eta_{moo}^{1\sigma} T_{moo}^{1ab} e^{-\gamma_{00 z}^{1\sigma}} - \eta_{moo}^{1\sigma} R_{moo}^{1ab} e^{\gamma_{00 z}^{1\sigma}} \]  
(2.102)

where

\[ R_{mpq}^{**} = \frac{(e^{-\gamma_{pq}^{1\sigma}} - R_{mpq}^{++} e^{\gamma_{pq}^{1\sigma}})}{e^{-\gamma_{pq}^{1\sigma}} + R_{mpq}^{++} e^{\gamma_{pq}^{1\sigma}}} \]  
(2.103)

From equation (2.102), $R_{moo}^{1ab}$ is found as:

\[ R_{moo}^{1ab} = R_{moo}^{++} T_{moo}^{1ab} \]  
(2.104)

where

\[ R_{mpq}^{++} = \left( \begin{array}{c} \eta_{mpq}^{1} - \eta_{mpq}^{1\sigma} R_{mpq}^{**} \\ \eta_{mpq}^{1} + \eta_{mpq}^{1\sigma} R_{mpq}^{**} \end{array} \right) e^{-2j\gamma_{0b}^{1\sigma}} \]  
(2.105)

Finally, by matching the fields at $Z = 0$ and applying Floquet orthogonality, the following relationship is obtained.

\[ R_{moo}^{1a} + \psi_{m0}^{1\sigma} \delta = T_{moo}^{1ab} + R_{moo}^{1ab} \]  
(2.106)
where \( \delta \) is the Kronecker delta.

Substituting for \( R_{moo}^{lab} \) from (2.104), we obtain:

\[
R_{moo}^{la} + \mu_{m}^{inc} \delta = T_{moo}^{lab} (1 + R_{moo}^{+++}) \tag{2.107}
\]

Similarly, matching the magnetic fields at \( Z = 0 \) taking inner product and using equation (2.104) will give:

\[
- \eta_{moo}^{air} R_{moo}^{la} + \eta_{moo}^{air} \mu_{m}^{inc} \delta = \eta_{moo}^{1} T_{moo}^{lab} - \eta_{moo}^{l} R_{moo}^{+++} T_{moo}^{lab} \tag{2.108}
\]

Substitution using equation (2.107) into the above equation (2.108) and simplifying will result in the following equation:

\[
- \eta_{moo}^{air} R_{moo}^{la} + \eta_{moo}^{air} \mu_{m}^{inc} \delta = \eta_{moo}^{1} (R_{moo}^{la} + \mu_{m}^{inc} \delta) R_{moo}^{+++} \tag{2.109}
\]

where \( R_{mpq}^{+++} = \frac{1 - R_{mpq}}{1 + R_{mpq}} \tag{2.110} \)

From (2.109) in above, \( R_{moo}^{la} \), the reflection coefficient from the entire structure can be obtained as:

\[
R_{moo}^{la} = \mu_{m}^{inc} \delta R_{moo}^{ss} \tag{2.111}
\]

where \( R_{mpq}^{ss} = \frac{\eta_{mpq}^{air} - \eta_{mpq}^{1} R_{mpq}^{+++}}{\eta_{mpq}^{air} + \eta_{mpq}^{1} R_{mpq}^{+++}} \tag{2.112} \)

\( R_{moo}^{ss} \) is the reflection coefficient at \( Z = 0 \). Equation (2.111) gives the required amplitude of the field reflected from the whole surface (i.e. the reflection coefficient). Since \( \mu_{m}^{inc} \) is usually given, then the reflection coefficient can be calculated. The transmitted field amplitude \( T_{moo}^{2y} \) can be found in a similar way. In order to avoid lengthy derivation, this is not repeated in here. From equation (2.111), the coefficient \( R_{moo}^{la} \) is substituted into equation (2.107) giving the coefficient \( T_{moo}^{1ab} \). This is then substituted into equation (2.104), resulting in an equation for the reflection coefficient \( R_{moo}^{1ab} \). This in turn is
substituted into equation (2.102), resulting in an expression for $T_{moo}^1$. Inserting $T_{moo}^1$ into equation (2.95) will give the coefficient $R_{moo}^{2b}$ which when substituted into equation (2.94) will finally result in an expression for the transmission coefficient $T_{moo}^{2ab}$. Eventual substitution of the resultant expression into equation (2.87) will produce the required amplitude of the transmitted field, $T_{moo}^{2b}$.

Having obtained expressions for reflection and transmission coefficients, they can then be utilised to find the total tangential field at any region.

The total tangential field in the region $Z < 0$ is the sum of the incident field and the reflected field. Thus

$$
\xi_{t}^{1a} (r) = E_{t}^1 (r) + E_{t}^{1a} (r)
$$

$$
\xi_{t}^{1a} (r) = \sum_{m=1}^{2} b_{m}^{inc} \psi_{oo}^{1} (r) e^{-j \gamma_{oo}^{a} z} \kappa_{moo}^{1} + \sum_{m=1}^{2} R_{moo}^{1a} \psi_{oo}^{1} (r) e^{-j \gamma_{oo}^{a} z} \kappa_{moo}^{1}
$$

(2.113)

Using equation (2.111), we obtain

$$
\xi_{t}^{1a} (r) = \sum_{m=1}^{1} \left[ e^{-j \gamma_{oo}^{a} z} + R_{moo}^{s} e^{j \gamma_{oo}^{a} z} \right] b_{m}^{inc} \psi_{oo}^{1} \kappa_{moo}^{1} \psi_{oo}^{1} \kappa_{moo}^{1}
$$

(2.114)

Similarly the total tangential field in the region $Z \geq Z_3$ in the absence of the scatterer, is the sum of the incident field and the transmitted field. Note that in this case the incidence is taken to be in the transmission side (i.e. the same result is obtained whether the incidence is on the side of the first or the second layer array).

$$
\xi_{t}^{2b} (r) = E_{t}^1 (r) + E_{t}^{2b} (r)
$$

$$
\xi_{t}^{2b} (r) = \sum_{m=1}^{2} \left( b_{m}^{inc} + T_{moo}^{2b} \right) e^{-j \gamma_{oo}^{a} z} \psi_{oo}^{1} \kappa_{moo}^{1}
$$

(2.115)

The above total tangential (reflected and transmitted) fields in the absence of the
scatterers will be used in combination with total scattered tangential fields (to be obtained in the next section (2.4.2)), in order to formulate the required integral equations. The outline of these integral equation formulations will be given in section 2.4.3.

2.4.2 Scattered field expansion in terms of the Floquet mode set of mutual periodicity

Having obtained an expression for the fields in the absence of the scatterers, we now deal with the dielectric and the scattered field expansion. Application of boundary conditions will result in expressions relating the unknown induced surface currents to the scattered (reflected and transmitted) field coefficients. The total tangential scattered fields are then obtained by the same procedure as that outlined in the previous section. These are then combined with the total tangential fields without the scatterers obtained in the previous section 2.4.1, for the purpose of finding the integral equations for the unknown surface currents (in section 2.4.3).

The field expansions at various boundaries are initially carried out in terms of the Floquet modes of the two arrays. The fields adjacent to the two arrays are assigned a mutual periodicity. By using the concept of mutual (i.e. common) periodicity previously established, the superimposed fields are then expressed in terms of their own mutual periodicity sets and the analysis effectively reduces to that of a double layer structure with identical lattice periodicities [2.3] of the arrays. For simplicity, during the initial stages of the analysis, the modal indices are denoted in compact form by a single subscript \(p\) and \(l\) for the first and second layer arrays) and their mutual set is denoted by a single subscript as \(c\). Once the required expressions for the integral equations are obtained, these single compact subscripts are then replaced by their equivalent two dimensional modal indices \((pq)\), \((ln)\) and \((st)\) respectively. Furthermore, the index \(m\) denoting either TE or TM states of incidence is also omitted during the initial stages of analysis to avoid complexities.

The aim is to obtain expressions for \(R^{1a}\) and \(T^{2b}\), the scattered reflection and transmission coefficients respectively from the entire double layer structure. These coefficients will
then be used in formulating the integral equations required.

In the presence of the scatterers, the scattered fields at different boundaries are expanded as below. Again we refer to Figure 2.1(b).

At \( Z \leq 0 \)

\[
E_{t}^{1a} = \sum_{p} R_{p}^{1a} e^{j \gamma_{p}^{1a} z} \psi_{p}^{1} \bar{\kappa}_{p}^{1} \quad (2.116)
\]

\[
H_{t}^{1a} = -\sum_{p} \eta_{p}^{\text{air}} R_{p}^{1a} e^{-j \gamma_{p}^{\text{air}} z} \psi_{p}^{1} \hat{z} \times \bar{\kappa}_{p}^{1} \quad (2.117)
\]

For the remaining boundaries, the field expansions are the same as those outlined in Section 2.4.1 (equations (2.75)-(2.82)), except for the fact that the modal indices \((0,0)\) are replaced in here by the appropriate Floquet mode orders of the two arrays.

The continuity and discontinuity of the electric and magnetic fields (i.e. equations (2.83)-(2.86) of section 2.4.1) are now applied at various boundaries. Applying boundary condition for the electric field at \( Z=0 \) gives:

\[
E_{t}^{1a} = E_{t}^{\text{lab}}
\]

\[
\sum_{p} R_{p}^{1a} e^{j \gamma_{p}^{1a} z} \psi_{p}^{1} \bar{\kappa}_{p}^{1} = \sum_{p} (T_{p}^{\text{lab}} e^{-j \gamma_{p}^{1a} z} + R_{p}^{\text{lab}} e^{j \gamma_{p}^{1a} z}) \psi_{p}^{1} \bar{\kappa}_{p}^{1} \quad (2.118)
\]

The tangential fields are now assigned a mutual periodicity such that the ratio of the lattice periodicities of the two adjacent arrays is a rational number (i.e. \( D^{3} = D^{1}/D^{2} = M/N \), where \( M \) and \( N \) are integers as defined in section 2.3.2 and \( D^{3} = M D^{1} = N D^{2} \)).

In the above equation (2.118), taking inner product with \( \psi_{c} \) from both sides of the above equation over the unit cell area \( A_{3} \) results in the following equation:
\[
\sum \mathbf{R}_p \mathbf{\phi}^c_p = \sum \mathbf{T}_{p}^{\text{lab}} \mathbf{\phi}^c_p + \sum \mathbf{R}_p^{\text{lab}} \mathbf{\phi}^c_p
\]  \hspace{1cm} (2.119)

Applying the convolution relationships between the fields and their common periodicities (i.e. equations (2.53) and (2.54) of section 2.3.1), then:

\[
\sum \mathbf{R}_p \mathbf{\phi}^c_p = \sum \mathbf{R}_p \mathbf{\delta}^c_p = \sum \mathbf{R}_p \mathbf{\delta} (p-c)
\]

\[
= \mathbf{R}_c^1
\]

where \( \mathbf{\delta}^c_p = \begin{cases} 
1 & \text{at } c = N_p \\
0 & \text{otherwise} 
\end{cases} \)

Similarly

\[
\sum \mathbf{T}_{p}^{\text{lab}} \mathbf{\phi}^c_p = \mathbf{T}_c^1 \hspace{1cm} (2.120)
\]

and

\[
\sum \mathbf{R}_p^{\text{lab}} \mathbf{\phi}^c_p = \mathbf{R}_c^{\text{lab}}
\]

Equation (2.119) above therefore becomes:

\[
\mathbf{R}_c^1 = (\mathbf{T}_c^{\text{lab}} + \mathbf{R}_c^{\text{lab}}) \hspace{1cm} (2.121)
\]

Applying the boundary condition for the magnetic fields at \( Z=0 \) gives:

\[
\mathbf{H}^1 - \mathbf{H}^{\text{lab}} = \hat{\mathbf{Z}} \times \mathbf{I}
\]

i.e. the magnetic field is discontinuous.

\[
\sum \eta^a_p \mathbf{R}_p^1 e^{\text{i} p z} \psi^1_p \hat{\mathbf{Z}} \times \hat{\mathbf{K}}_p - \hat{\mathbf{Z}} \times \mathbf{I} = \sum \eta^1_c (\mathbf{T}_p^1 e^{-\text{i} p z} + \mathbf{R}_p^{\text{lab}} e^{\text{i} p z}) \psi^1_p \hat{\mathbf{Z}} \times \hat{\mathbf{K}}_p
\]

where \( \mathbf{I} \) is the current flowing on the surface of the first layer array.

Taking the inner products with \( \psi^c \) over the unit cell area \( A_3 \) from both sides of the above equation and applying the convolution relationships between the fields and their mutual periodicities, the following expression will be obtained.

\[
\eta^a_c \mathbf{R}_c^1 + \eta^1_c (\mathbf{T}_c^{\text{lab}} - \mathbf{R}_c^{\text{lab}}) = -\frac{1}{A_3} \mathbf{I}_c
\]  \hspace{1cm} (2.122)
where $\eta_c^a$ is the modal admittance in the air expressed in terms of the mutual Floquet mode set $c$, and $\eta_c^1$ is the modal admittance of the first layer dielectric. $\tilde{I}_c$ denotes the Floquet transform of the current $I$ in terms of the mutual Floquet modal set $c$ given by:

$$\tilde{I}_c = <I, \psi_c> \kappa_c$$

$$= \int_{A_3} \psi_c^* dA_3 \kappa_c$$

Substituting for $R_{c}^{1a}$ from equation (2.121) and further simplifying, one arrives at:

$$T_{c}^{lab} = (\eta_c^{1} - \eta_c^{a}) R_{c}^{1ab} - \frac{1}{A_3} (\eta_c^{1} + \eta_c^{a})$$

Next applying the boundary condition at $Z=Z_1$ for the electric field and taking the inner product with $\psi_c$ over the unit cell area $A_3$ from both sides of the equation gives:

$$E_{t}^{lab} = E_{t}^{ab}$$

$$\sum_p T_p^{ab} e^{-\eta_c^p Z_1} \phi_p^c + \sum_p R_p^{ab} e^{\eta_c^p Z_1} \phi_p^c = \sum_p T_p^{ab} \phi_p^c e^{-\eta_c^p Z_1} + \sum R_p^{2a} e^{\eta_c^p Z_1} \phi_i^c$$

Using the convolution of the fields with their mutual periodicities, the above is further reduced to:

$$T_{c}^{lb} e^{-\eta_c^2 Z_1} + R_{c}^{2a} e^{\eta_c^2 Z_1} = T_{c}^{lab} e^{-\eta_c^1 Z_1} + R_{c}^{1ab} e^{\eta_c^1 Z_1}$$

Similarly, applying boundary condition for the magnetic field at $Z=Z_1$, taking inner product with $\psi_c$ from both sides and applying the convolution of the fields, one arrives at:

$$H_{t}^{lab} = H_{t}^{ab}$$

$$\eta_c^a e^{-\eta_c^2 Z_1} T_{c}^{lb} - \eta_c^a e^{\eta_c^2 Z_1} R_{c}^{2a} = \eta_c^1 e^{-\eta_c^1 Z_1} T_{c}^{lab} - \eta_c^1 e^{\eta_c^1 Z_1} R_{c}^{1ab}$$

The aim is now to systematically eliminate the transmission coefficients in order to obtain
Chapter Two

expressions for the reflection coefficients. Multiplying both sides of equation (2.125) by \( \gamma_c^1 \) and adding with equation (2.126), and simplifying, an expression for \( T_c^{1b} \) can be obtained.

\[
T_c^{1b} = - \left( \frac{\eta_c^1 - \eta_c^a}{\eta_c^1 + \eta_c^a} \right) e^{2 \eta_c^a Z_1} R_c^2 a + \frac{2 \eta_c e^{i \eta_c^a Z_1}}{(\eta_c^1 + \eta_c^a) e^{-i \eta_c^a Z_1}} T_c^{1a b} \tag{2.127}
\]

Substituting for \( T_c^{1ab} \) from (2.123) gives:

\[
T_c^{1b} = - \frac{1}{R_c} R_c^a + \frac{2 \eta_c e^{-i \eta_c^a Z_1}}{(\eta_c^1 + \eta_c^a) e^{+i \eta_c^a Z_1}} \left( - \frac{1}{R_c} R_c^{1ab} - \frac{1}{A_3} \frac{\eta_c}{(\eta_c^1 + \eta_c^a)} \right) \tag{2.128}
\]

where

\[
\frac{1}{R_c} = \left( \frac{\eta_c^1 - \eta_c^a}{\eta_c^1 + \eta_c^a} \right) e^{2 \eta_c^a Z_1} \tag{2.129}
\]

At \( Z_1 < Z < Z_2 \), (i.e. the separation region), the fields are expanded as superposition of the forward and reverse waves (i.e. equation (2.77) of section 2.4.1) given by:

\[
E_t^{ab} = \sum_p T_p^{1b} e^{-j \gamma_p^a Z} \psi_p^1 \kappa_p^1 + \sum_l R_l^2 a e^{+j \gamma_l^a Z} \psi_l^2 \kappa_l^2 \tag{2.130}
\]

and

\[
H_t^{ab} = \sum_p \gamma_p^a T_p^{1b} e^{-j \gamma_p^a Z} \psi_p^1 \hat{Z} \times \hat{k}_p^1 - \sum_l \eta_l^a R_l^2 a e^{+j \gamma_l^a Z} \psi_l^2 \hat{Z} \times \hat{k}_l^2 \tag{2.131}
\]

Applying the boundary condition at \( Z=Z_2 \), taking the inner product with \( \psi_c \) over the unit cell area \( A_3 \) from both sides of equations and applying the convolution of the fields gives:

\[
E_t^{ab} = E_t^{2ab}
\]

\[
H_t^{ab} - H_t^{2ab} = \hat{Z} \times \mathbf{I}
\]

Where \( \mathbf{I} \) is the current flowing on the surface of the second layer array.

Hence

\[
T_c^{1b} e^{-j \gamma_c^a Z_2} + R_c^2 a e^{j \gamma_c^a Z_2} = T_c^{2ab} e^{-j \gamma_c^a Z_2} + R_c^{2ab} e^{j \gamma_c^a Z_2} \tag{2.132}
\]

56
and for magnetic field,

\[
\eta^a_c T^{1b}_c e^{-\nu^a_c z_3} - \frac{1}{A_3} j_c = \eta^a_c T^{2ab}_c e^{-\nu^2_c z_2} - \eta^2_c R^{2ab}_c e^{\nu^2_c z_2} + \eta^a_c R^{2a}_c e^{\nu^a_c z_2} \tag{2.133}
\]

where \( j_c \) is the Floquet transform of the current \( J \) given by:

\[
j_c = \langle J, \psi_c \rangle \hat{k}_c
\]

\[
= \int_A J \psi^*_c dA \hat{k}_c
\]

Finally matching the fields at \( Z=Z_3 \), taking the inner product from both sides with \( \psi_c \) and applying the convolution of the fields gives:

\[
T^{2ab}_c e^{-\nu^2_c z_3} + R^{2ab}_c e^{+\nu^2_c z_3} = T^{2b}_c e^{-\nu^2_c z_3} \tag{2.134}
\]

and

\[
\eta^2_c T^{2ab}_c e^{+\nu^2_c z_3} - \eta^2_c R^{2ab}_c e^{-\nu^2_c z_3} = \eta^a_c T^{2b}_c e^{-\nu^a_c z_3} \tag{2.135}
\]

Multiplying both sides of equation (2.134) by \( \eta^a_c \) and subtracting from (2.135) gives:

\[
R^{2ab}_c = R^+_c T^{2ab}_c \tag{2.136}
\]

where

\[
R^+_c = \left( \frac{\eta^2_c - \eta^a_c}{\eta^2_c + \eta^a_c} \right) e^{-2\nu^2_c z_3} \tag{2.137}
\]

The process of elimination begins by first eliminating \( T^{2ab}_c \). Substituting relationship (2.136) into expression (2.132) and (2.133) will result in:

\[
T^{1b}_c e^{-\nu^2_c z_3} + R^{2a}_c e^{+\nu^2_c z_2} = T^{2ab}_c \left( e^{+\nu^2_c z_2} + R^+_c e^{-\nu^2_c z_2} \right) \tag{2.138}
\]

and

\[
\eta^a_c T^{1b}_c e^{-\nu^2_c z_3} - \frac{1}{A_3} j_c = \eta^2_c T^{2ab}_c \left( e^{+\nu^2_c z_2} - R^+_c e^{-\nu^2_c z_2} \right) + \eta^a_c R^{2a}_c e^{+\nu^2_c z_2} \tag{2.139}
\]

Obtaining \( T^{2ab}_c \) from (2.138) and substituting into equation 2.139) gives:
\[ \eta_c T_c^{1b} e^{-j\gamma_c^2 Z_2} - \frac{1}{A_3} \tilde{J}_c = \eta_c^2 R_c^* (T_c^{1b} e^{j\gamma_c^2 Z_2} + R_c^{2a} e^{j\gamma_c^2 Z_2}) \]
\[ + \eta_c R_c^{2a} e^{j\gamma_c^2 Z_2} \]  
(2.140)

where

\[ R_c^* = \frac{e^{-j\gamma_c^2 Z_2} - R_c + e^{j\gamma_c^2 Z_2}}{e^{-j\gamma_c^2 Z_2} + R_c + e^{j\gamma_c^2 Z_2}} \]  
(2.141)

Simplifying equation (2.140), an expression for \( R_c^{2a} \) in terms of \( T_c^{1b} \) can be found as:

\[ R_c^{2a} = R_c^{+++} T_c^{1b} - \frac{1}{A_3} \eta_c^2 + R_c^* \eta_c \tilde{J}_c \]  
(2.142)

where

\[ R_c^{+++} = \left( \frac{\eta_c^a - R_c^* \eta_c^2}{\eta_c^a + R_c^* \eta_c^2} \right) e^{-2j\gamma_c^2 Z_2} \]  
(2.143)

Similarly by substituting relationship (2.136) into equations (2.132) and (2.135), but this time eliminating \( R_c^{2a} \), an expression for \( T_c^{2ab} \) can be found in terms of the current \( \tilde{J}_c \) only. This results in:

\[ T_c^{1b} e^{-j\gamma_c^2 Z_2} + R_c^{2a} e^{+j\gamma_c^2 Z_2} = T_c^{2ab} (e^{-j\gamma_c^2 Z_2} + R_c^* e^{+j\gamma_c^2 Z_2}) \]  
(2.144)

and

\[ \eta_c T_c^{1b} e^{-j\gamma_c^2 Z_2} - \eta_c^2 R_c^{2a} e^{j\gamma_c^2 Z_2} = \eta_c T_c^{2ab} \left( e^{-j\gamma_c^2 Z_2} - R_c^* e^{j\gamma_c^2 Z_2} \right) + \frac{1}{A_3} \tilde{J}_c \]  
(2.145)

From above two equations an expression for \( T_c^{2ab} \) is found as.

\[ T_c^{2ab} = \frac{2\eta_c^3 e^{j\gamma_c^2 Z_2} T_c^{1b} - \tilde{J}_c}{(\eta_c^a + \eta_c^2) e^{-j\gamma_c^2 Z_2} - (\eta_c^2 - \eta_c^a) R_c^* e^{j\gamma_c^2 Z_2}} \]  
(2.146)

It only remains now to find coefficients \( R_c^{1a} \) and \( T_c^{2b} \), i.e. the reflection and transmission coefficients from the entire double layer structure.
Chapter Two

An expression for $R_{c}^{1a}$ can be found from equation (2.121). $R_{c}^{1ab}$ and $T_{c}^{1ab}$ therefore need to be found. $R_{c}^{1ab}$ can be obtained as follows. In equation (2.125) multiply both sides by $\eta_{c}^{a}$ and subtract the resultant from equation (2.126). The result is the expression given below:

$$R_{c}^{1ab} = \frac{2\eta_{c}^{a} e^{\gamma_{c}^{ab} Z_{1}}}{(\eta_{c}^{a} + \eta_{c}^{1}) e^{\gamma_{c}^{b} Z_{1}}} R_{c}^{2a} + R_{c}^{+} T_{c}^{1ab}$$  \hspace{1cm} (2.147)

Substituting $R_{c}^{1ab}$ from above into equation (2.123) will result in an expression for $T_{c}^{1ab}$ in terms of the surface current on the first layer array only.

$$T_{c}^{1ab} = \frac{-1}{A_{3}} \frac{T_{c}}{(\eta_{c}^{1} + \eta_{c}^{a}) - (\eta_{c}^{1} - \eta_{c}^{a}) R_{c}^{++}}$$  \hspace{1cm} (2.148)

An expression for $R_{c}^{1a}$ is therefore obtained as given below:

$$R_{c}^{1a} = T_{c}^{1ab} (1 + R_{c}^{2}) + \frac{2\eta_{c}^{a} e^{\gamma_{c}^{ab} Z_{1}}}{\eta_{c}^{a} + \eta_{c}^{1}} e^{\gamma_{c}^{b} Z_{1}}$$  \hspace{1cm} (2.149)

Final substitution for $T_{c}^{1ab}$ and $R_{c}^{2a}$ into above equation (2.149) will result in an expression for the reflection coefficient $R_{c}^{1a}$, in terms of the current $\overline{I}_{c}, \overline{J}_{c}$ and the coefficient $T_{c}^{1b}$ given below:

$$R_{c}^{1a} = -\frac{1}{A_{3}} (1 + R_{c}^{2}) T_{c}^{1b} + \frac{\overline{I}_{c}}{(\eta_{c}^{1} + \eta_{c}^{a}) - (\eta_{c}^{1} - \eta_{c}^{a}) R_{c}^{++}} +$$

$$\left[ \frac{2\eta_{c}^{a} e^{\gamma_{c}^{ab} Z_{1}}}{(\eta_{c}^{a} + \eta_{c}^{1}) e^{\gamma_{c}^{b} Z_{1}}} (R_{c}^{++} T_{c}^{1b} - \frac{1}{A_{3}} \frac{e^{\gamma_{c}^{ab} Z_{2}}}{\eta_{c}^{a} + \eta_{c}^{2}} \overline{J}_{c} ) \right]$$  \hspace{1cm} (2.150)

Using equation (2.142), (2.147) and (2.148) we can eliminate $T_{c}^{1b}$. Substituting the
expression for $T_c^{1b}$ into equation (2.150) above subsequently results into an expression for $R_c^{1a}$ in terms of $\bar{I}_c$ and $\bar{J}_c$ only as given below:

$$
R_c^{1a} = \frac{1 + R_c^+}{A_3} \bar{I}_c - \left( \bar{I}_c \right)^+ + \frac{2\eta_c^a e^{j\eta_c^2z_1}}{(\eta_c^a + \eta_c^1) e^{j\eta_c^2z_1}} \left[ \frac{1}{A_3} \frac{\bar{I}_c (\eta_c^a + \eta_c^1) e^{j\eta_c^2z_1} (R_c^{++} - R_c^+)}{(\eta_c^1 + \eta_c^a) - (\eta_c^1 - \eta_c^a) R_c^{++} + \eta_c^a e^{j\eta_c^2z_1}} \right] 
$$

Equation (2.151) above is the required expression for the reflection coefficient $R_c^{1a}$ from the double layer structure expressed in terms of the two surface currents $\bar{I}_c$ and $\bar{J}_c$.

Similarly to obtain an expression for the transmission coefficient $T_c^{2b}$, we use the same procedure of algebraic manipulation that has been illustrated above for $R_c^{1a}$. This involves using equations (2.128), (2.134), (2.136) and (2.146). In order to avoid lengthy derivations, detailed manipulations are not shown in here. It can be shown that as a result of manipulations and simplifications, the expression for the transmission coefficient $T_c^{2b}$ related to the transforms of the currents $I_c$ and $J_c$ only is given as below.

$$
T_c^{2b} = \frac{(1 + R_c^+ e^{2j\eta_c^2z_2})}{e^{-j\eta_c^2z_3} e^{j\eta_c^2z_3}} \left( \frac{R_c^{+++} (\eta_c^a + \eta_c^1) e^{j\eta_c^2z_2} - R_c^{++} (\eta_c^a + \eta_c^1) e^{j\eta_c^2z_1}}{(\eta_c^1 + \eta_c^a) - (\eta_c^1 - \eta_c^a) R_c^{++} + \eta_c^a e^{j\eta_c^2z_2}} \right) \bar{I}_c 
$$
Having obtained expressions for the reflection and transmission coefficients from the entire structure, the integral equations required for the two surface currents $I_c$ and $J_c$ can now be formulated. This is outlined in the next section that follows.

### 2.4.3 The Coupled Integral Equation Formulation

Since the total tangential (transmitted and reflected) fields both in the absence and in the presence of the scattering elements have been determined, they are now combined under common boundary conditions applied on the surface of both array conductors (equations (2.153) and (2.154)).

We therefore examine the total tangential fields both at the boundaries $Z=0$ and $Z=Z_3=d$. These are expressed as a linear combination of the total scattered fields and the total field in the absence of the scatterers.

At $Z = 0$, the total tangential electric field is given by:

$$
E_t^T(r_T,0) = E_t^{1a}(r_T,0) + E_t^{la}(r_T,0)
$$

where $E_t^{la}(r_T,0)$, the total scattered reflected field is given by:

$$
E_t^{la}(r_T,0) = \sum_{m,e} R_{mc}^{la} e^{j\gamma_c z} \psi_c \kappa_{mc}
$$

and where $E_t^{1a}(r_T,0)$, the total tangential field in the absence of the scatterer was given by equation (2.114) of section 2.4.1 as:

$$
E_t^{1a}(r_T,0) = \sum_{m=1} e^{-j\gamma_{oo} z + R_{mo0} e^{j\gamma_{oo} z}} \beta_{m}^{inc} \psi_{oo}^{l} \kappa_{mo0}
$$

61
Hence substituting into equation (2.153) yields:

\[ E_t^T(r_T,0) = \sum_{mc} R_m^a \psi_c \hat{R}_{mc} + \]

\[ \sum_{m=1}^{2} \left[ e^{-j\eta_{oo} d} + R_{moo} e^{j\eta_{oo} d} \right] b_m^{inc} \psi_0^{1} \hat{R}_{moo} \]  
(2.154)

Similarly at \( Z = d \), the total tangential electric field is given by:

\[ E_t^T(r_T,d) = E_t^{2b}(r_T,d) + E_t^{2b}(r_T,d) \]  
(2.155)

where the total tangential field in the absence of the scatterers, \( E_t^{2b}(r_T,d) \), at \( Z = d \) was given by equation (2.115) of section 2.4.1 as:

\[ E_t^{2b}(r_T,d) = \sum_{m=1}^{2} (b_m^{inc} + T_{moo}^{2b}) e^{-j\eta_{oo} d} \psi_0^{1} \hat{R}_{moo} \]  
(2.156)

and the scattered field is given by:

\[ E_t^{2b}(r_T,d) = \sum_{mc} T_{mc}^{2b} e^{-j\eta_0 d} \psi_c \hat{R}_{mc} \]  
(2.157)

Hence at \( Z = d \), the total tangential field is the combination of equations (2.156) and (2.157), i.e.

\[ E_t^T(r_T,d) = \sum_{m=1}^{2} (b_m^{inc} + T_{moo}^{2b}) e^{-j\eta_{oo} d} \psi_0^{1} \hat{R}_{moo} + \sum_{mc} T_{mc}^{2b} e^{-j\eta_0 d} \psi_c \hat{R}_{mc} \]  
(2.158)

The only remaining boundary condition is that the total tangential electric fields over the conducting areas of the two arrays must vanish. This is now enforced.

\[ E_t^T(r_T,0) = 0, \]

and

\[ E_t^T(r_T,d) = 0 \]  
(2.159)

\[ E_t^T(r_T,0) = \sum_{mc} R_m^a \psi_c \hat{R}_{mc} + \sum_{m=1}^{2} (1 + R_{moo}) b_m^{inc} \psi_0 \hat{R}_{moo} = 0 \]  
(2.160)

The expression for the reflection coefficient \( R_{mc}^{1a} \) in above was given by equation (2.151) of section 2.4.2. It is rewritten here in a simplified form as:
\[ R_{mc}^{1a} = -\frac{1}{A_3} (w_{a}^0)_{mc} \bar{I}_{mc} - \frac{1}{A_3} (w_{a}^0 w_0^1 w_1^2)_{mc} \bar{I}_{mc} \]  

(2.161)

where it can be proved that:

\[ (w_{a}^0)_{mc} = \frac{1 + R_{c}^{+++}}{(\eta_{mc}^1 + \eta_{mc}^a) - (\eta_{mc}^1 - \eta_{mc}^a) R_{c}^{+++}} = \frac{e^{-\gamma_{c}^1 Z_1}}{(\eta_{mc}^1 + \eta_{mc}^a)} \]

(2.162)

and where it can be shown that:

\[ (w_{a}^0)_{mc} = (e^{-\gamma_{c}^1 Z_1} + R_{c}^{+++} e^{+\gamma_{c}^1 Z_1})/(1 + R_{c}^{+++}) \]

(2.163)

\[ (w_{1}^2)_{mc} = (e^{-\gamma_{c}^2 Z_2} + R_{c}^{+++} e^{+\gamma_{c}^2 Z_2})/(e^{-\gamma_{c}^2 Z_2} - R_{c}^{+++} e^{+\gamma_{c}^2 Z_2}) \]

(2.164)

Thus substituting for \( R_{c}^{1a} \) from (2.161) into equation (2.160) gives the first integral equation as below.

\[ \frac{1}{A_3} \sum_{mc} \left[ (w_{a}^0)_{mc} \bar{I}_{mc} + (w_{a}^0 w_0^1 w_1^2)_{mc} \bar{I}_{mc} \right] \psi_{c} \hat{\kappa}_{mc} = \sum_{m=1}^{2} (1 + R_{moo}) b_{m}^{inc} \psi_{oo} \hat{\kappa}_{moo} \]

(2.165)

Equation (2.165) above is further written in a simpler form as:

\[ \frac{1}{A_3} \sum_{mc} \left[ \Lambda_{mc} \bar{I}_{mc} + \Lambda'_{mc} \bar{I}_{mc} \right] \psi_{c} \hat{\kappa}_{mc} = \sum_{m=1}^{2} \Lambda_{moo} \psi_{oo} \hat{\kappa}_{moo} \]

(2.166)

where

\[ \Lambda_{mc} = (w_{a}^0)_{mc} \]

(2.167)

\[ \Lambda'_{mc} = (w_{a}^0 w_0^1 w_1^2)_{mc} \]

(2.168)

and

\[ \Lambda_{moo} = (1 + R_{moo}) b_{m}^{inc} \]

(2.169)

Similarly the total tangential electric field at \( Z=d \), i.e. \( E_{T}^{1T}(r, d) \), must also vanish over the conducting surface of the second array. \( E_{T}^{1T}(r, d) \) was given by equation (2.158) as:

\[ \sum_{mc} T_{mc}^{2b} e^{-\gamma_{c}^2 d} \psi_{c} \hat{\kappa}_{mc} + \sum_{m=1}^{2} (b_{m}^{inc} + T_{moo}^{2b}) e^{-\gamma_{oo}^2 d} \psi_{oo} \hat{\kappa}_{moo} = 0 \]

(2.170)
The expression for $T_{mc}^{2b}$ as given in equation (2.152) of section 2.4.2 is rewritten here in a simplified form as:

$$T_{mc}^{2b} = -\frac{1}{A_3} (w_2^3 w_1^2 w_0^1 w_n^0)_{mc} I_{mc} - \frac{1}{A_3} (w_2^3 w_1^2 \Delta)_{mc} J_{mc}$$  \hspace{1cm} (2.171)

where it can be shown that:

$$(w_2^3)_{mc} = \frac{e^{i \gamma_{cZ}^3} + R_c^+ e^{i \gamma_{cZ}^3}}{e^{i \gamma_{cZ}^3} - R_c^+ e^{i \gamma_{cZ}^3}}$$  \hspace{1cm} (2.172)

and

$$(\Delta)_{mc} = \left[ (w_1^2 w_0^1)_{mc} \left( (w_0^1 w_3^0)_{mc} + \frac{e^{i \gamma_{cZ}^1} - e^{-i \gamma_{cZ}^1}}{2 \eta_{mc}^1} \right) + \left( \frac{e^{i \gamma_{cZ}^3} - e^{-i \gamma_{cZ}^3}}{2 \eta_{mc}^3} \right) \right]$$  \hspace{1cm} (2.173)

An expression for $T_{moo}^{2b}$ (i.e. the amplitude coefficient of the incident field) can also be obtained from equation (2.87), (2.89) and (2.100) of the section (2.4.1). Detailed algebraic manipulation and simplification in obtaining an expression for $T_{moo}^{2b}$ is not given in here. It can be shown that $T_{moo}^{2b}$ is given by:

$$T_{moo}^{2b} = b_m^{inc} R_{moo} (w_2^3 w_1^2 w_0^1)_{moo}$$  \hspace{1cm} (2.174)

where $w_2^3$, $w_1^2$ and $w_0^1$ are as given previously except for the fact the Floquet mode index $c$ is replaced by the indices $(0,0)$ corresponding to the incident field contribution.

Thus substituting for $T_{mc}^{2b}$ and $T_{moo}^{2b}$ from (2.171) and (2.174) into equation (2.170) of total tangential field at $Z = d$ will result in:

$$\frac{1}{A_3} \sum_{mc} (w_2^3 w_1^2 w_0^1 w_n^0)_{mc} I_{mc} + (w_2^3 w_1^2 \Delta)_{mc} J_{mc} e^{-i \gamma_{cZ}^3} \psi_c \kappa_{mc} =$$

$$\sum_{m=1}^{2} (1 + R_{moo}) b_m^{inc} (w_2^3 w_1^2 w_0^1)_{moo} e^{-i \gamma_{oo}^a d} \psi_{oo} \kappa_{moo}$$  \hspace{1cm} (2.175)
The above equation (2.175) is further rewritten in a simpler form as:

\[
\frac{1}{A_3} \sum_{mc} \left[ \Lambda_{mc} \vec{I}_{mc} + \Lambda_{mc} \vec{J}_{mc} \right] \psi_c \kappa_{mc} = \sum_{m=1}^{2} \Lambda_{moo} \psi_{oo} \kappa_{moo}
\]  
(2.176)

where

\[
\Lambda_{mc} = (w_2^3 w_1^2 w_0^1 w_a^0)_{mc} e^{-\nu_2^d}
\]  
(2.177)

\[
\Lambda_{mc} = (w_2^3 w_1^2 \Delta)_{mc} e^{\nu_2^d}
\]  
(2.178)

and where

\[
\Lambda_{moo} = (1 + R_{moo}) b_t^{mc} (w_2^3 w_1^2 w_0^1)_{moo} e^{\nu_3^{aa} d}
\]  
(2.179)

Equations (2.166) and (2.176) are the two required integral equations for the surface currents \( I \) and \( J \). It is to be noted that these expressions for the two integral equations are very similar to those obtained in the analysis of scattering from double layer FSS with identical lattice geometries [2,3]. This is to be expected since the expansions at the two array boundaries in here were carried out in terms of same set of Floquet modes of the common periodicity.

It is seen that the number of the resultant coupled integral equations is the same as the number of layers. Each integral equation contains the spectra of the unknown currents. In the next section a method of solution of the two integral equations is outlined, from which the unknown current coefficients are computed.

2.5 Method of Solution

2.5.1 Method of Moment (MoM) Solution to the Integral Equations

The use of the method of moment for solving electromagnetic problems has been widespread as in references [5] and [9] where it is well documented. It is a general procedure for solving linear equations and owes its name to the process of taking moments by multiplying with appropriate weighting functions and then integrating.

The induced currents \( I \) and \( J \) are expanded in terms of a set of orthogonal basis functions...
in Cartesian form as:

\[ I(\mathbf{r}_T) = \sum_{\alpha=1}^{Q} C_{\alpha}^1 h_{\alpha}(\mathbf{r}_T) \quad \mathbf{r}_T \in A_1 \]  

\[ I(\mathbf{r}_T) = \sum_{\beta=1}^{R} C_{\beta}^2 g_{\beta}(\mathbf{r}_T) \quad \mathbf{r}_T \in A_2 \]  

where \( A_1 \) and \( A_2 \) are the areas of the unit conducting cells of the two arrays. \( C_{\alpha}^1 \) and \( C_{\beta}^2 \) are the complex amplitudes of the \( \alpha^{th} \) and \( \beta^{th} \) basis functions with \( Q \) and \( R \) being the total number of functions used to approximate as closely as possible the induced currents \( I \) and \( J \) respectively. The two integral equations are reduced to a system of linear equations by applying the Ritz-Galerkin's method, where the weighting functions are the same as the basis functions.

The demands placed on the basis functions are that they satisfy the boundary conditions and are orthogonal over the scatterers. For simple structures such as double ring elements [11], usually about 12 or less basis functions in practice are needed to represent the currents. More complicated elements such as Jerusalem crosses [12], gridded squares [13] and double squares [14] may require a larger number of basis functions to approximate the currents. The Ritz-Galarkin method mentioned above are used to obtain the desired matrix equations that contain the unknown coefficients \( C_{\alpha}^1 \) and \( C_{\beta}^2 \). Solving the matrix equations would result in computing the unknown current coefficients, which eventually lead to determining the reflection and transmission coefficients.

The efficiency with which the solution of the two integral equations can be derived for a desired accuracy depends on the choice of the basis functions. In order that the number of basis functions are kept to a minimal level and therefore the matrix size is left small, it is desirable that these basis functions satisfy the appropriate edge conditions [15]. Secondly, it is convenient to choose the basis functions that are analytically Fourier transformable so that the need to derive their transforms numerically is neutralised when using the operator equations in the transform domain.
In the context of the moment method, two types of basis functions are used, viz., the entire domain and the sub-domain basis functions. For the elements that are used in the various arrays of the double layer structures in our studies here, the entire domain basis function representations have found to be more appropriate. An important advantage of using the entire domain basis function is that the size of the resulting moment matrix is usually smaller than that of its counterpart, the sub-domain function [16]. Consequently, patterns for electrically large structures could be solved which could not otherwise be handled using the sub-domain functions. On the other hand, for treating FSS screens comprised of arbitrarily shaped apertures or patches, and for screens with finite conductivities, subdomain basis functions have been found to be more versatile than the entire domain function.

The Floquet transform of the currents $I$ and $J$ in terms of the Floquet modes of the mutual periodicity are given by:

$$\bar{I}_{mc} = <I(I_T), \psi_c> \kappa_{mc}$$  \hfill (2.182)

$$\bar{I}_{mc} = \int A_3 I(I_T) \psi_c^* dI_T \kappa_{mc}$$

$$= \int A_3 \left( \sum_{mp} I_{mp} \psi_p^1 \right) \psi_c^* dI_T \kappa_{mp}^1 \kappa_{mc}$$

$$= \sum_{mp} \bar{I}_{mp} \int A_3 \psi_p^1 \psi_c^* dI_T \kappa_{mp}^1 \kappa_{mc}$$

Hence

$$\bar{I}_{mc} = \sum_{mp} \bar{I}_{mp} \theta^c_p$$  \hfill (2.183)

which is in effect the convolution relationship of equation (2.64) given in Section 2.3.2.

Similarly the current $\bar{J}_{me}$ is given by:

$$\bar{J}_{me} = <J(I_T), \psi_c> \kappa_{me}$$

$$\bar{J}_{me} = \int A_3 J(I_T) \psi_c^* dI_T \kappa_{me}$$

$$= \int A_3 \left( \sum_{m1} \bar{J}_{m1} \psi_T^2 \right) \psi_c^* dI_T \kappa_{m1}^2 \kappa_{me}$$

67
\[ \sum_{m} \bar{J}_{m} \int A_{3} \psi_{1}^{2} \psi_{c}^{*} d \xi \kappa_{m}^{2} \kappa_{mc} \]

Hence

\[ \bar{J}_{mc} = \sum_{m} \bar{J}_{m} \phi_{1}^{c} \]  \hspace{1cm} (2.184)

Substituting for \( \bar{I}_{mpq} \) and \( \bar{J}_{m1n} \) from equations (2.180) and (2.181) into equations (2.183) and (2.184) will give:

\[ \bar{I}_{mc} = \sum_{mp} C_{\alpha} \left( \bar{h}_{\alpha} \left( k_{mp}^{1} \right) \kappa_{mp}^{1} \right) \phi_{p}^{c} \]

\[ \bar{I}_{mc} = \sum_{\alpha=1}^{Q} C_{\alpha} \sum_{mp} \left( \bar{h}_{\alpha} \left( k_{mp}^{1} \right) \kappa_{mp}^{1} \right) \phi_{p}^{c} \]

or

\[ \bar{I}_{mc} = \sum_{\alpha=1}^{Q} C_{\alpha} \left( \bar{h}_{\alpha} \left( k_{TC} \right) \kappa_{mc} \right) \]  \hspace{1cm} (2.185)

Similarly

\[ \bar{I}_{mc} = \sum_{m} \sum_{\beta=1}^{R} C_{\beta} \left( \bar{f}_{\beta} \left( k_{mc}^{2} \right) \kappa_{mc} \right) \phi_{1}^{c} \]

or

\[ \bar{I}_{mc} = \sum_{\beta=1}^{R} C_{\beta} \left( \bar{f}_{\beta} \left( k_{TC} \right) \kappa_{mc} \right) \]  \hspace{1cm} (2.186)

where \( \bar{h}_{\alpha} \left( k_{TC} \right) \) and \( \bar{f}_{\beta} \left( k_{TC} \right) \) are the Floquet spectrums of the basis function \( h_{\alpha}(r) \) and \( f_{\beta}(r) \) expressed in terms of the modal set \( c \) of the common periodicity. From (2.185) and (2.186), it is seen that:

\[ \bar{h}_{\alpha} \left( k_{TC} \right) = \sum_{p} \bar{h}_{\alpha} \left( k_{Tp}^{1} \right) \phi_{p}^{c} = \sum_{p} \bar{h}_{\alpha} \left( k_{Tp}^{1} \right) \delta_{p} \delta_{c} \]  \hspace{1cm} (2.187)

and

\[ \bar{f}_{\beta} \left( k_{TC} \right) = \sum_{l} \bar{f}_{\beta} \left( k_{T\Pi}^{2} \right) \phi_{l}^{c} = \sum_{l} \bar{f}_{\beta} \left( k_{T\Pi}^{2} \right) \delta_{p} \delta_{c} \]  \hspace{1cm} (2.188)

The above two relationships (2.187) and (2.188) indicate that the spectrums of basis
functions $h_\alpha (\mathbf{r}_T)$ and $f_\beta (\mathbf{r}_T)$ are super-resolved from those of their common periodicity function $\tilde{h}_\alpha (k_{TC})$ and $\tilde{f}_\beta (k_{TC})$ respectively, at spectral locations where Floquet mode $c = Np$ and $c = Mq$ (with $N = D^3 / D^1$ and $M = D^3 / D^2$).

To implement the moment method, we substitute for the basis functions $\tilde{I}_{mc}$ and $\tilde{J}_{mc}$ from equation (2.185) and (2.186), into the integral equation (2.165) and (2.175) that were obtained in the previous section. The resultant integral equations are then weighted according to Galerkin's method. This involves taking the inner products with $h_i (\mathbf{r}_T)$ and $f_j (\mathbf{r}_T)$ from both sides of each equation respectively. Note that in general one can take inner product (testing inner product) with a different set of basis functions. Here it is chosen however that the testing functions are the same as the basis functions. This is called the Ritz-Galerkin method. As a result the two integral equations given below are obtained.

$$\frac{1}{A_3} \left\{ \sum_{\alpha=1}^{Q} C_\alpha \left[ \sum_{mc} (w^0_\alpha)_{mc} (\tilde{h}_\alpha (k_{TC}) \hat{k}_{mc}) (\tilde{I}_{mc} (\mathbf{r}_T) \hat{k}_{mc}) \right] \right\}$$

$$+ \sum_{\beta=1}^{R} C_\beta \left[ \sum_{mc} (w^0_\beta w^1_\beta w^2_\beta)_{mc} (\tilde{f}_\beta (k_{TC}) \hat{k}_{mc}) (\tilde{J}_{mc} (\mathbf{r}_T) \hat{k}_{mc}) \right]$$

$$= \sum_{m=1}^{2} (1 + R_{moo}) b^\text{inc}_m (\tilde{h}_1 (k_{T0}) \hat{k}_{moo})$$  \hspace{1cm} (2.189)

and

$$\frac{1}{A_3} \left\{ \sum_{\alpha=1}^{Q} C_\alpha \left[ \sum_{mc} (w^3_\alpha w^1_\beta w^0_\beta)_{mc} e^{-\gamma^d_c d} (\tilde{h}_\alpha (k_{TC}) \hat{k}_{mc}) (\tilde{I}_{mc} (\mathbf{r}_T) \hat{k}_{mc}) \right] \right\}$$

$$+ \sum_{\beta=1}^{R} C_\beta \left[ \sum_{mc} (w^3_\beta w^2_\beta w^1_\beta)_{mc} e^{-\gamma^d_c d} (\tilde{f}_\beta (k_{TC}) \hat{k}_{mc}) (\tilde{J}_{mc} (\mathbf{r}_T) \hat{k}_{mc}) \right]$$

$$= \sum_{m=1}^{2} (1 + R_{moo}) b^\text{inc}_m (w^3_1 w^2_1 w^0_1)_{moo} e^{-\gamma^d_0 d} (\tilde{f}_1 (k_{T0}) \hat{k}_{moo})$$  \hspace{1cm} (2.190)

where $i = 1, \ldots, Q$.

69
and \( j = 1, \ldots, R \)

and where

\[
\tilde{h}_g^* (k_{Te}) = \int h_g^* (\tau_T) \psi_c \, d(\tau_T) \tag{2.191}
\]

\[
\tilde{f}_i^* (k_{Te}) = \int f_i^* (\tau_T) \psi_c \, d(\tau_T) \tag{2.192}
\]

It is to be noted that in order to achieve a correct numerical solution to the above integral equations system a proper set of basis functions has to be selected and the infinite Floquet mode spectrum must be truncated accordingly (i.e. the phenomenon of relative convergence [17]). A detailed discussion of the relative convergence phenomenon for sinusoidal bases is outlined in reference 17. A discussion on the choice of the basis functions and the truncation of the Floquet modes can also be found in section A.4 of Appendix A. Since truncation is applied the relative convergence can be examined. A sufficient number of Floquet modes are needed so that at least the main lobe of the current spectrum is covered. The recommendation is to increase the Floquet mode number, keeping the same number of current basis function until no difference is noticed in the results. A more convenient method would be to use the ratio \( \Omega = \frac{P'Q'}{R} \) as a figure of merit with \( P' \) and \( Q' \) representing the Floquet mode numbers and \( R \) representing the number of basis functions required. For increase in number of basis functions (i.e. increase \( R \)), then \( \Omega \) has to be checked again. It should however be noted that the convergence checks could be implemented as part of computer model. However, due to approximations in the solution of the integral equations it would be preferred to verify the predictions by experimental results.

Equations (2.189) and (2.190) constitute a linear system of two matrix simultaneous equations with two unknowns, \( C_\alpha \) and \( C_\beta \). In matrix form, they are written as:

\[
|L_4| = |G_{14}||C_\alpha| + |H_{15}||C_\beta|
\]

or

\[
\boxed{|L_4| = |G_{14}||C_\alpha| + |H_{15}||C_\beta|}
\]
where $i, \alpha = 1, \ldots, Q$.

The second matrix equation is

$$\left| L_j \right| = |G_j \beta| |C_\beta| + |H_j \alpha| |C_\alpha|$$

or

$$\begin{bmatrix} L_1 \\ L_j \\ L_R \end{bmatrix} = \begin{bmatrix} G_{11} & G_{1\beta} & G_{1R} \\ G_{j1} & G_{j\beta} & G_{jR} \\ G_{R1} & G_{R\beta} & G_{RR} \end{bmatrix} \begin{bmatrix} C_1 \\ C_\beta \\ C_\alpha \end{bmatrix} + \begin{bmatrix} H_{11} & H_{1\alpha} & H_{1Q} \\ H_{j1} & H_{j\alpha} & H_{jQ} \\ H_{R1} & H_{R\alpha} & H_{RQ} \end{bmatrix} \begin{bmatrix} C_1 \\ C_\beta \\ C_\alpha \end{bmatrix}$$

where $j, \beta = 1, \ldots, R$.

The various elements in the above matrices are given by:

$$L_1 = \sum_{m=1}^{2} (1 + R_{\text{moo}}) b_m^\text{inc} (\bar{h}_1 (k_{\text{Teo}}) \bar{k}_{\text{moo}})$$

$$L_j = \sum_{m=1}^{2} (1 + R_{\text{moo}}) b_m^\text{inc} (w_2^3 w_1^{\frac{1}{2}} w_0^\frac{1}{2})_0 e^{-\gamma d} (\bar{f}_j (k_{\text{Teo}}) \bar{k}_{\text{moo}})$$

$$G_{1\alpha} = \frac{1}{A_3} \sum_{mc} (w_0^3 w_0^1 (\bar{h}_\alpha (k_{\text{TTe}}) \bar{k}_{mc}) (\bar{h}_1 (k_{\text{TTe}}) \bar{k}_{mc}))$$

$$H_{1\beta} = \frac{1}{A_3} \sum_{mc} (w_0^3 w_0^1 (\bar{f}_\beta (k_{\text{TTe}}) \bar{k}_{mc}) (\bar{f}_1 (k_{\text{TTe}}) \bar{k}_{mc}))$$

$$G_{j\beta} = \sum_{mc} (w_2^3 w_1^{\frac{1}{2}} w_0^2 e^{-\gamma d} (\bar{f}_\beta (k_{\text{TTe}}) \bar{k}_{mc}) (\bar{f}_j (k_{\text{TTe}}) \bar{k}_{mc}))$$

$$H_{j\alpha} = \sum_{mc} (w_2^3 w_1^{\frac{1}{2}} w_0^2 e^{-\gamma d} (\bar{h}_\alpha (k_{\text{TTe}}) \bar{k}_{mc}) (\bar{f}_j (k_{\text{TTe}}) \bar{k}_{mc}))$$
Chapter Two

The matrix equation (2.193) and (2.194) can be solved using a computer for the unknown current coefficients \( C_\alpha \) and \( C_\beta \). However, to obtain the value of current coefficients \( C_\alpha \) and \( C_\beta \), further matrix manipulations are needed in order to avoid matrix additions, multiplication, and more than one inversion. One way of solving the matrix equations (2.193) and (2.194) is to express them as a single matrix. Using the commutative law of matrices \( I A + I B = I B + I A \) keeping the same order of multiplication, the matrix equations above are reduced to one linear matrix equation:

\[
\begin{bmatrix}
L_i \\
L_j
\end{bmatrix} =
\begin{bmatrix}
G_{i\alpha} & H_{i\beta} \\
H_{j\alpha} & G_{j\beta}
\end{bmatrix}
\begin{bmatrix}
C_\alpha \\
C_\beta
\end{bmatrix}
\]

(2.201)

The above equation (2.200) is the desired matrix equation for the unknown current coefficients \( C_\alpha \) and \( C_\beta \). There is only one matrix inversion required for the solution of equation (2.200).

\[
\begin{bmatrix}
C_\alpha \\
C_\beta
\end{bmatrix} =
\begin{bmatrix}
G_{i\alpha} & H_{i\beta}
\end{bmatrix}^{-1}
\begin{bmatrix}
L_i \\
L_j
\end{bmatrix}
\]

The diagonal sub-matrices \( G_{i\alpha} \) and \( G_{j\beta} \) include a summation over the currents of individual layers only. The sub-matrices \( H_{i\beta} \) and \( H_{j\alpha} \) describe the coupling between the two different element currents, i.e., the interactions are formulated in terms of coupling between the spectra of basis functions on neighbouring arrays. The matrix size is therefore independent of the separation distance between the adjacent arrays and depends only on the total number of basis functions used. Indeed the matrix comprising the sub-matrices \( H \) and \( G \) is of the order \((J+J) \times (I+I)\). Clearly, the higher the number of basis functions used for the two surface currents, the bigger will be the size of the inverted matrix. As a result, a longer computation time will be required in computing the unknown coefficients \( C_\alpha \) and \( C_\beta \). The computer model written in Fortran 77, which is outlined in section 2.5.4, solves equation (2.201) for the current coefficients \( C \). The reflected and transmitted fields can in turn be computed from the equations involving coefficients \( C \), as described in section 2.5.3.
2.5.2 Optimised integral equations expressed in terms of the Floquet modes of the two Arrays

It will now be demonstrated that the two integral equations expressed in terms of the common periodicity can be optimised, by using the convolution relationships outlined previously. It will be shown that the original integral equations (2.166) and (2.176) will be optimised to a set of two integral equations expressed in terms of the Floquet modes of two individual arrays.

The compact modal indices \( p, l \) and \( c \) are now replaced by their two dimensional equivalence \( pq, ln \) and \( st \). Considering the first integral equation (2.166) of section 2.4.3, it is rewritten in here as:

\[
\frac{1}{A_3} \sum_{\text{mst}} (\Lambda_{\text{mst}} \bar{I}_{\text{mst}}) \psi_{st} \bar{k}_{\text{mst}} + \frac{1}{A_3} \sum_{\text{mst}} (\Lambda'_{\text{mst}} \bar{I}_{\text{mst}}) \psi_{st} \bar{k}_{\text{mst}} = \sum_{m=1}^{2} \Lambda_{\text{moo}} \psi_{oo} \bar{k}_{\text{moo}}
\]  

(2.202)

where \( \Lambda_{\text{mst}}, \Lambda'_{\text{mst}} \) and \( \Lambda_{\text{moo}} \) were given in section 2.4.3 by equations (2.167), (2.168) and (2.179).

Similarly, the second integral equation (2.176) of section 2.4.3 is also rewritten in here as:

\[
\frac{1}{A_3} \sum_{\text{mst}} (\Lambda''_{\text{mst}} \bar{I}_{\text{mst}}) \psi_{st} \bar{k}_{\text{mst}} + \frac{1}{A_3} \sum_{\text{mst}} (\Lambda''_{\text{mst}} \bar{I}_{\text{mst}}) \psi_{st} \bar{k}_{\text{mst}} = \sum_{m=1}^{2} \Lambda'_{\text{moo}} \psi_{oo} \bar{k}_{\text{moo}}
\]  

(2.203)

where \( \Lambda''_{\text{mst}}, \Lambda''_{\text{mst}} \) and \( \Lambda'_{\text{moo}} \) were given in section 2.4.3 by equations (2.177), (2.178) and (2.179).

Now the two summation terms in each of the above two integral equations can be expressed in terms of the Floquet modes of the two arrays by using the convolution expressions which, relate the modal coefficients of the two arrays to their mutual periodicities. Considering the integral equation (2.202) the first summation term is expressed as:
Chapter Two

\[ \frac{1}{A_3} \sum_{\text{mst}} (\lambda_{\text{mst}} \tilde{I}_{\text{mst}}) \psi_{\text{st}} \hat{k}_{\text{mst}} = \frac{1}{A_3} \sum_{\text{mst}} \sum_{\text{mpq}} (\lambda_{\text{mpq}} \tilde{I}_{\text{mpq}}) \phi_{\text{mpq}} \psi_{\text{st}} \hat{k}_{\text{mpq}} \]

Using the properties of the Kronecker delta, the above is reduced to:

\[ \frac{1}{A_3} \sum_{\text{mst}} (\lambda_{\text{mst}} \tilde{I}_{\text{mst}}) \psi_{\text{st}} \hat{k}_{\text{mst}} = \frac{1}{A_1} \sum_{\text{mpq}} (\lambda_{\text{mpq}} \tilde{I}_{\text{mpq}}) \psi_{\text{pq}} \hat{k}_{\text{pq}} \] (2.204)

The second summation term of equation (2.202) can also be expressed as:

\[ \frac{1}{A_3} \sum_{\text{mst}} (\lambda'_{\text{mst}} \tilde{I}_{\text{mst}}) \psi_{\text{st}} \hat{k}_{\text{mst}} = \frac{1}{A_3} \sum_{\text{mst}} \sum_{\text{mpq}} (\lambda'_{\text{mpq}} \tilde{I}_{\text{mpq}}) \psi_{\text{pq}} \hat{k}_{\text{pq}} \]

or

\[ \frac{1}{A_3} \sum_{\text{mst}} (\lambda'_{\text{mst}} \tilde{I}_{\text{mst}}) \psi_{\text{st}} \hat{k}_{\text{mst}} = \frac{1}{A_1} \sum_{\text{mpq}} (\lambda'_{\text{mpq}} \tilde{I}_{\text{mpq}}) \psi_{\text{pq}} \hat{k}_{\text{pq}} \] (2.205)

Therefore the first integral equation (2.202) expressed alternatively in terms of the Floquet modes of the two arrays will be as follows:

\[ \frac{1}{A_1} \sum_{\text{mpq}} (\lambda_{\text{mpq}} \tilde{I}_{\text{mpq}}) \psi_{\text{pq}} \hat{k}_{\text{pq}} + \frac{1}{A_1} \sum_{\text{mpq}} (\lambda'_{\text{mpq}} \tilde{I}_{\text{mpq}}) \psi_{\text{pq}} \hat{k}_{\text{pq}} = \sum_{\text{m=1}}^{\text{m=oo}} \psi_{\text{oo}} \hat{k}_{\text{m=oo}} \] (2.206)

Similarly, the second integral equation (2.203) can alternatively be expressed in terms of the Floquet spectrums of the two arrays by using the convolution relationships. If this is carried out, then the expression below for the second integral equation will be obtained.

\[ \frac{1}{A_2} \sum_{\text{m'=ln}} (\lambda'_{\text{m'=ln}} \tilde{I}_{\text{m'=ln}}) \psi_{\text{m'=ln}} \hat{k}_{\text{m'=ln}} + \frac{1}{A_2} \sum_{\text{m'=ln}} (\lambda'_{\text{m'=ln}} \tilde{I}_{\text{m'=ln}}) \psi_{\text{m'=ln}} \hat{k}_{\text{m'=ln}} = \sum_{\text{m=1}}^{\text{m=oo}} \psi_{\text{oo}} \hat{k}_{\text{m=oo}} \] (2.207)

The above two integral equations can be solved for the two unknown current coefficients using the method of moment outlined in the previous section 2.5.1. If this is carried out then the resultant integral equation will be as follows:
As expected the above two integral equation (2.208) and (2.209) for the unknown current coefficients $C_\alpha$ and $C_\beta$ expressed in terms of the Floquet mode sets of the two arrays, are equivalent to the original integral equation (2.189) and (2.190) of section 2.5.1 expressed in terms of the Floquet spectrum of the common periodicity. As before the above two integral equations may be written in a linear system of two matrix simultaneous equations with the two unknowns, $C_\alpha$ and $C_\beta$, in the same way as in section 2.5.1. Therefore the desired matrix equation for the unknown current coefficients will be identical to matrix equation (2.201) rewritten in here as:

$$
\begin{bmatrix}
L_A \\
L_J
\end{bmatrix} =
\begin{bmatrix}
G'_{1\alpha} & H'_{1\beta} \\
H'_{3\alpha} & G'_{3\beta}
\end{bmatrix}
\begin{bmatrix}
C_\alpha \\
C_\beta
\end{bmatrix}
$$

(2.210)
except for the fact that the summations in the diagonal sub-matrices $G'_{\alpha\alpha}$, $H'_{\beta\beta}$, $G'_{\beta\beta}$, and $H'_{\alpha\beta}$ above are carried out in terms of the appropriate Floquet modes of the two arrays according to equations (2.208) and (2.209). And the expressions for the sub-matrix $G'_{\alpha\beta}$, $H'_{\beta\alpha}$, and $H'_{\beta\beta}$ are exactly the same as those given in equations (2.197) – (2.200) except that the field expansion in these sub-matrices are carried out in terms of the Floquet modes of the individual arrays.

Computer program written in Fortran 77 which is outlined in section 2.5.4 is used to model both the matrix equation (2.201) and (2.210) for the unknown current coefficients. Using the computer model it has been established that the predicted values obtained for $C_\alpha$ and $C_\beta$ from modelling both the original and the optimised integral equations were exactly the same. Consequently, the computer program that models the optimised version of the integral equations will be used to generate the predicted results for the double layer surfaces that are being studied in Chapters 3 and 4.

2.5.3 Determination of Reflection and Transmission Coefficients

Since the current coefficients can be estimated from the previous section, the total reflected and transmitted fields can be calculated. The $z$-components can be easily found from Maxwell’s divergence theorem ($\nabla \cdot \mathbf{E} = 0$). Once the current coefficients $C_\alpha$ and $C_\beta$ are computed by solving the linear matrix equation (2.210) of the previous section 2.5.2, the transmission coefficient $T_{e}^{2b}$ and the reflection coefficients $R_{e}^{1a}$, are then computed from equations (2.171) and (2.161) respectively. In order to develop a full plane wave theoretical model, the co-polar and cross-polar components of the total reflection and transmission coefficients must be computed. These are defined and outlined in this Section.

In terms of the mutual periodicity set, the total transverse reflected and transmitted fields at $Z = 0$ and $Z = d$ respectively are given by equations (2.154) and (2.158) of section 2.4.3 as:
Chapter Two

At $Z = 0$,

\[ E^T_t (r, 0) = \sum_{mc} R^1_{mc} \psi^1 mc + \sum_{m=1}^2 (1 + R\mu_{\infty}) b^\text{inc}_m \psi^\text{inc}_{\infty} \kappa_{\mu_{\infty}} \]  

(2.211)

and at $Z = Z3 = d$,

\[ E^T_t (r, d) = \sum_{m=1}^2 (b^\text{inc}_m + T^2_{mc} \mu_{\infty}) e^{-j} \psi^1 \mu_{\infty} \kappa_{\mu_{\infty}} + \sum_{mc} T^2_{mc} e^{-j} \psi^1 \kappa_{mc} \]  

(2.212)

The field amplitudes are known since the current expansion coefficients have been estimated. The reflection and transmission coefficients can be written down for any propagating mode. The zero order mode is the dominant mode and always propagating. Higher order mode propagation depends on the lattice to wavelength ratio. Although the direction and amplitude of these mode differ from the basic $(0, 0)$ order mode their coefficient of reflection and transmission follow the same procedure. Hence for a given propagation mode, the reflection and transmission coefficients are defined as the projection of the total electric fields onto the total incident field direction. This result can also be identified as the co-polar component, $E^c_t (r, 0)$, or $E^c_e (r, 0)$.

In Cartesian co-ordinates, the total reflected field at $Z = 0$ is written as:

\[ E^R T (r, 0) = E^R_t (r, 0) + E^R_z (r, 0) \]  

(2.213)

\[ = E^R_x (r, 0) \hat{x} + E^R_y (r, 0) \hat{y} + E^R_z (r, 0) \hat{z} \]

where it can be proved from Maxwell's divergence theorem that the $z$-components of the field is given as:

\[ E^R_z (r, 0) = \frac{\text{Sin } \theta \text{ Cos } \phi \ E^R_x (r, 0) + \text{Sin } \theta \text{ Sin } \phi \ E^R_y (r, 0)}{\text{Cos } \theta} \]  

(2.214)

Similarly, in transmission the total transmitted field at $Z = d$ can also be written as the sum of the transversal components and the $z$-components, given by:

\[ E^T_e (r, 0) = E^T_x (r, 0) + E^T_z (r, 0) \]
\[ E_1^t (r,0) \hat{x} + E_2^t (r,0) \hat{y} + E_3^t (r,0) \hat{z} \] (2.215)

with

\[ E_{1x}^t (r,0) = \frac{\sin \theta \cos \phi \, E_{i1} (r,0) + \sin \theta \sin \phi \, E_{i2} (r,0)}{\cos \theta} \] (2.216)

In equation (2.211) above for the total reflected field at \( Z = 0 \), after substitution for \( R_{mc}^{1a} \) given by equation (2.161) of section 2.4.3 and in the subsequent equation substituting for \( \bar{\gamma}_{mc} \) and \( \bar{\beta}_{mc} \) (given by equations (2.185) and (2.186) of section 2.5.1), the expression given below will result.

\[ E_{1x}^{Tt} (r,0) = -\frac{1}{A_3} \left\{ \sum_{m_c} \left[ \left( w_0^0 w_0^1 w_1^2 \sum_{\alpha=1}^{Q} C_\alpha \left( \bar{\gamma}_\alpha \left( k_{Tc} \right) \right) \right) + \sum_{a=1}^{R} C_\beta \left( \bar{\gamma}_\beta \left( k_{Tc} \right) \right) \right] \psi_c \bar{\kappa}_{mc} \right\} \]

\[ + \sum_{m=1}^{2} \left( (1 + R_{moc}) b_{m}^{mc} \right) \psi_{oo} \bar{\kappa}_{moc} \] (2.217)

Similarly, in equation (2.212) for the transmitted field at \( Z = d \), after substitution for \( T_{mc}^{2b} \) from equation (2.171) of section 2.4.3 and the subsequent substitution for \( \bar{\gamma}_{mc} \) and \( \bar{\beta}_{mc} \), the following expression for the total transmitted field is obtained.

\[ E_{1x}^{Tt} (r,d) = -\frac{1}{A_3} \left\{ \sum_{m_c} \left[ \left( w_2^3 w_1^2 w_0^0 w_0^1 e^{-\eta_d^2 d} \sum_{\alpha=1}^{Q} C_\alpha \left( \bar{\gamma}_\alpha \left( k_{Tc} \right) \right) \right) + \sum_{a=1}^{R} C_\beta \left( \bar{\gamma}_\beta \left( k_{Tc} \right) \right) \right] \psi_c \bar{\kappa}_{mc}^2 \right\} \]

\[ + \sum_{m=1}^{2} \left( (1 + R_{moc}) b_{m}^{mc} \right) w_2^3 w_1^2 w_0^1 e^{-\eta_{oo}^2 d} \psi_{oo} \bar{\kappa}_{moc} \] (2.218)
In Cartesian form the total reflected field at \( Z=0 \) and the total transmitted field at \( Z=d \) as given by equations (2.213) and (2.215) respectively can also be expressed as:

\[
E_r^{T}(t,0) = (R_x \hat{x} + R_y \hat{y} + R_z \hat{z}) \psi_c(t)
\]

and

\[
E_t^{T}(t,d) = (T_x \hat{x} + T_y \hat{y} + T_z \hat{z}) e^{-j \gamma_d^2 \psi_c(t)}
\]

Now considering the \((0,0)\) order propagating mode for example, from equations (2.217) and (2.219) the \((x,y)\) components of the total reflected field at \( Z=0 \) are given by:

\[
R_x = \sum_{m=1}^{2} \left( (1 + R_{moo}) b_{m}^{inc} - F_{moo} \right) \hat{x}_{moo}
\]

\[
R_y = \sum_{m=1}^{2} \left( (1 + R_{moo}) b_{m}^{inc} - F_{moo} \right) \hat{y}_{moo}
\]

and the \(z\)-component of the total reflected field is given by:

\[
R_z = \sum_{m=1}^{2} \left\{ (1 + R_{moo}) b_{m}^{inc} - F_{moo} \right\} \left( \sin \theta \cos \phi \hat{x}_{moo} + \sin \theta \sin \phi \hat{y}_{moo} \right)
\]

with \( \hat{x}_{moo} = (\kappa_{moo} \hat{x} + \kappa_{moo} \hat{y}) \)

and the \(z\)-component of the total reflected field is given by:

\[
R_z = \sum_{m=1}^{2} \left\{ (1 + R_{moo}) b_{m}^{inc} - F_{moo} \right\} \left( \sin \theta \cos \phi \hat{x}_{moo} + \sin \theta \sin \phi \hat{y}_{moo} \right)
\]

where

\[
F_{mc} = \frac{1}{A_3} \left\{ \sum_{mc} \left[ \left( w_0^0 \sum_{\alpha=1}^{Q} C_{\alpha} (h_{\alpha} k_{Tc}) \hat{\kappa}_{mc} \right) \right] + \left( w_0^0 w_0^f \sum_{\beta=1}^{R} C_{\beta} (f_{\beta} (k_{Tc}) \hat{\kappa}_{mc} ) \right) \right\}
\]

Similarly the \((x,y,z)\) components of the total transmitted field at \( Z = 0 \), for the \((0,0)\) order propagating mode are found using equations (218) and (220) as:
\( T_x^t = \sum_{m=1}^{2} \left[ (1 + R_{moo}) b_{m}^{inc} w_2^3 w_1^2 w_0 \ e^{-j_\theta_{moo} d} - t_{moo} \right] \ k_{moo}^x \)  

(2.224)

\( T_y^t = \sum_{m=1}^{2} \left[ (1 + R_{moo}) b_{m}^{inc} w_2^3 w_1^2 w_0 \ e^{-j_\theta_{moo} d} - t_{moo} \right] \ k_{moo}^y \)  

(2.225)

and

\[ T_z^t = \sum_{m=1}^{2} \left[ (1 + R_{moo}) b_{m}^{inc} w_2^3 w_1^2 w_0 \ e^{-j_\theta_{moo} d} - t_{moo} \right] (\sin \theta \ \cos \phi \ k_{moo}^x + \sin \theta \ \sin \phi \ k_{moo}^y) \cos \theta \]  

(2.226)

where

\[ t_{mc} = \frac{1}{A_3} \left\{ \sum_{mc} \left[ \left( w_2^3 w_1^2 w_0 \sum_{\alpha=1}^{Q} C_\alpha (\tilde{n}_{\alpha} (k_{mc}) \ k_{mc}) \right) \right. \right. \]

\[ \left. \left. + \left( w_2^3 w_1^2 \sum_{\beta=1}^{R} C_\beta (\tilde{r}_{\beta} (k_{mc}) \ k_{mc}) \right) \right] \right\} \]  

(2.227)

Once the Cartesian components of the total reflected and transmitted fields are obtained; by projecting them onto the total incident field direction \( B_\theta \), their co-polar and cross-polar components can be computed. The co-polar components of transmitted and reflected fields are given by the projection of \( E^T_r (r,d) \) or \( E^T_r (r,0) \) onto the total incident field direction \( B^i \). Thus, in reflection:

\[ E^{rc} (r,0) = E^{rc} (r,0) B^i \]  

(2.228)

where

\[ E^{rc} (r,0) = E^T_r (r,0) \cdot B^i \]

Similarly, in transmission the co-polar component is given by:

\[ E^{lc} (r,d) = E^{lc} (r,d) B^i \]  

(2.229)

where
\( E_r^c (r,0) = E_{\text{inc}}^c (r,0) \hat{B}^1 \)

Note that in the above, \( \hat{B}^1 \) is given in terms of its (xyz) components as:

\[
\hat{B}^1 = (B_x^1 \hat{x} + B_y^1 \hat{y} + B_z^1 \hat{z})
\]

Hence the complex reflection and transmission coefficient in the co-polar direction, \( R_{\text{xm}} \) and \( \tau_{\text{xm}} \) respectively, are given by:

\[
R^c = \frac{E_{\text{rc}}^c (r,0)}{E_{\text{tr}}^c (r,0)} = (R_x B_x^1 + R_y B_y^1 + R_z B_z^1)
\]

(2.231)

and

\[
\tau^c = \frac{E_{\text{tc}}^c (r,d)}{E_{\text{tr}}^c (r,d)} = (T_x B_x^1 + T_y B_y^1 + T_z B_z^1)
\]

(2.232)

Derivation of the Cartesian (i.e. x, y and z) components of the total incident field \( E_{\text{tr}}^c (r,z) \) is given in reference [3], by solving the geometrical problem. The TE and TM components of the incident field amplitude \( b_{\text{inc}} \) outlined in reference [3] is given here in matrix form for convenience as:

\[
\begin{vmatrix}
 b_{1,\text{inc}}^1 \\
 b_{2,\text{inc}}^1 \\
\end{vmatrix} =
\begin{vmatrix}
 \kappa_{100}^x & \kappa_{200}^x \\
 -\kappa_{100}^y & \kappa_{200}^y \\
\end{vmatrix}
\begin{vmatrix}
 B_x^1 \\
 B_y^1 \\
\end{vmatrix}
\]

(2.234)

The Cartesian components of the incident field in the above matrix are normally obtained from the polar angles \((\theta, \phi)\) defining the polarisation direction. Hence the TE and TM components of the incident field amplitude \( b_{\text{inc}} \) can be computed from the determinant of the above matrix equation (2.234). Since all the components of the total incident field together with the TE and TM constituents are known, using equation (2.231) and (2.232) above the complex reflected and transmitted field coefficients in the co-polar direction can be then obtained.

The cross-polar component of the reflected and transmitted fields can be found by defining a unit vector, \( \hat{B}^1 \) say, which is perpendicular to \( \hat{B}^1 \), provided the unit ray path
vector is the unit normal to the plane of $\mathbf{B}^\|$ and $\mathbf{B}^\perp$. It should be noted that the definition vector is the unit normal to the plane of $\mathbf{B}^\|$ and $\mathbf{B}^\perp$. It should be noted that the definition adapted here for the cross-polarisation is “The polarisation orthogonal to a reference polarisation [18].” In the measurement, the reference and cross-polarisation are defined to be what one measures when antenna pattern are taken in the usual manner [19].

Determination of the (x, y, z)-components of $\mathbf{B}^\perp$ are outlined in reference [3]. The cross-polar components can be calculated by taking the projection of the total field onto the cross-polar direction of the incident field vector $\mathbf{B}^\perp$ as:

$$E_{rL}^{rL}(r,0) = E_{rL}^{rL}(r,0) \mathbf{B}^\perp$$ (2.235)

where

$$E_{rL}^{rL}(r,0) = E_{rT}^{rL}(r,0) \cdot \mathbf{B}^\perp$$ (2.236)

Similarly, in transmission the cross-polar component is:

$$E_{tL}^{tL}(r,d) = E_{tL}^{tL}(r,d) \mathbf{B}^\perp$$ (2.237)

where

$$E_{tL}^{tL}(r,d) = E_{tT}^{tL}(r,d) \cdot \mathbf{B}^\perp$$ (2.238)

Hence the coefficients of the complex reflected and transmitted fields in the cross-polar direction follow the same procedure as those obtained for co-polar components above (i.e. equations (2.231) and (2.232)). These are given by:

$$\left( R^\perp = \frac{E_{rL}^{rL}(r,0)}{E_{rT}^{rL}(r,0)} = (R_{r}^r B^r_x + R_{r}^r B^r_y + R_{r}^r B^r_z) \right)$$ (2.239)

where $\mathbf{B}^\perp = (B^\perp_x \hat{x} + B^\perp_y \hat{y} + B^\perp_z \hat{z})$

and

$$\tau^\perp = \frac{E_{tL}^{tL}(r,d) / E_{tT}^{tL}(r,d)}{(T_{t}^t B^t_x + T_{t}^t B^t_y + T_{t}^t B^t_z)}$$ (2.240)

It is worthwhile mentioning the fact that for pure TE incidence (i.e. $\phi = 0^\circ$) the co-polar
component of the reflected and transmitted field correspond to \( m = 2 \) only. The cross-polar components are those with \( m = 1 \). For pure TM incidence the opposite to above applies.

### 2.5.4 The Computer Model and a summary of all the relevant equations used

A computer program written in Fortran 77 has been developed based on the super-resolution modal analysis technique described in the present chapter. The model predominantly solves the linear matrix equation (2.201), i.e.

\[
\begin{bmatrix}
L_1 \\
L_j
\end{bmatrix} =
\begin{bmatrix}
G_{i\alpha} & H_{i\beta} \\
H_{j\alpha} & G_{j\beta}
\end{bmatrix}
\begin{bmatrix}
C_\alpha \\
C_\beta
\end{bmatrix}
\tag{2.241}
\]

or the linear matrix equation (2.210), i.e.

\[
\begin{bmatrix}
L_1 \\
L_j
\end{bmatrix} =
\begin{bmatrix}
G'_{i\alpha} & H'_{i\beta} \\
H'_{j\alpha} & G'_{j\beta}
\end{bmatrix}
\begin{bmatrix}
C_\alpha \\
C_\beta
\end{bmatrix}
\tag{2.242}
\]

for the complex current coefficients \( C_\alpha \) and \( C_\beta \).

Computer programs for modelling both the original and optimised version of the integral equations were developed for the purpose of comparing the results from the two methods. However, in generating the predictions for plane wave responses of the double layer surfaces that are to be studied in the following Chapters, it is chosen to utilise the computer model that is developed for the optimised version of the integral equations. This is due to the reductions obtained in computing time for evaluating the complex current coefficients of the two arrays as was explained previously. The two models use exactly the same equations except for the fact that in the program modelling the original integral equations, the expansions are carried out in terms of the Floquet modes of common periodicity.

The various steps involved in the development of the computer model were as follows:
The primary aim of the computer model is to first solve for the Floquet transform of the unknown complex current coefficients $C_\alpha$ and $C_\beta$. The total number of current coefficients $C_\alpha$ and $C_\beta$ computed depends on the number of basis functions $h_\alpha(r_T)$ and $g_\beta(r_T)$ used, to represent the surface current densities on the surfaces of first and second array elements respectively. In order to compute these unknown current coefficients, the matrix of excitation $\begin{bmatrix} L_i \\ L_j \end{bmatrix}$ and the matrix $\begin{bmatrix} G'_\alpha & H'_\beta \\ H'_\alpha & G'_\beta \end{bmatrix}$ are first formed in the computer model. Then by a simple matrix inversion the unknown coefficients can be computed. The components in the excitation matrix (i.e. the incident field components) are given by equation (2.195) and (2.196) of section 2.5.1 as:

$$L_i = \sum_{m=1}^{2} (1 + R_{moo}) b_m^{inc} (h_1^*(k_{Too}) r_{moo})$$  \hspace{1cm} (2.243)

and

$$L_j = \sum_{m=1}^{2} (1 + R_{moo}) b_m^{inc} (w_2^3 w_j^2 w_0^1)_{oo} e^{-j\mu_{oo}Z_3} (f_j^*(k_{Too}) r_{moo})$$  \hspace{1cm} (2.244)

where $i$ and $j$ are the number of basis functions used to represent the currents on the surfaces of first and second layer arrays respectively. The components of the main matrix of scattering coefficients were given by equation (2.197)-(2.200) of section 2.5.1 as:

$$G_{1\alpha} = \frac{1}{A_1} \sum_{m'pq} (w_a^0) (h_{\alpha}^*(k_{Tpq}) r_{mpq}) (h_1^*(k_{Tpq}) r_{mpq})$$  \hspace{1cm} (2.245)

$$H_{1\beta} = \frac{1}{A_1} \sum_{mpq} (w_a^0 w_0^1) (f_{\beta}^*(k_{Tpq}) r_{mpq}) (\bar{h}_1^*(k_{Tpq}) r_{mpq})$$  \hspace{1cm} (2.246)

$$G_{1\beta} = \sum_{m'n} (w_2^3 w_j^2 w_0^1 k_{Tpq}) e^{-j\mu_{ln}^d} (f_j^*(k_{Tpq}) r_{m'n}) (\bar{f}_j^*(k_{Tpq}) r_{m'n})$$  \hspace{1cm} (2.247)

$$H_{1\alpha} = \sum_{m'n} (w_2^3 w_j^2 w_0^1 k_{Tpq}) e^{-j\mu_{ln}^d} (h_{\alpha}^*(k_{Tpq}) r_{m'n}) (\bar{h}_1^*(k_{Tpq}) r_{m'n})$$  \hspace{1cm} (2.248)
The computation of the elements of both matrices \[
\begin{bmatrix}
L_x \\
L_y \\
\end{bmatrix}
\text{ and }
\begin{bmatrix}
G'_{1\alpha} & H'_{1\beta} \\
H'_{1\alpha} & G'_{1\beta} \\
\end{bmatrix},
\]
are carried out in a number of main do-loops in the program with many constituent nested do-loops. The four diagonal sub-matrices \(G'_{1\alpha}, G'_{1\beta}, H'_{1\beta}, H'_{1\alpha}\) in the matrix equation of (2.242) are evaluated in two separate main do-loops. The computations of the incident field components in the excitation matrix, i.e. \(L_x\) and \(L_y\), are also carried out inside these main do-loops.

The terms involving the dielectric coefficients \((W_2^0)\) and \((W_0^0 W_1^0 W_2^1)\) in the sub-matrix \(G'_{1\alpha}\) and \(H'_{1\beta}\) which were given by equations (2.162)-(2.164) of section 2.4.3 are computed in a separate main do-loop in terms of the Floquet modes of the first layer array. It should be noted that in this main loop, the computations of the individual (W) terms were carried out in various separate subroutines. In the main do-loop, these subroutines are then called. The TE and TM components of \((W_2^0)\) and \((W_0^0 W_1^0 W_2^1)\) terms are thereby computed. Also included in this do-loop are the computations of \((1 + R_{moo})\) and \(((1 + R_{moo}) (W_2^1 W_1^2 W_0^1))_{moo} e^{-j\phi_{moo} Z_3}\) involved in \(L_x\) and \(L_y\).

Similarly the term involving the dielectric coefficients \(((W_2 W_1^2 W_0^1)_{moo} e^{-j\phi_{moo} Z_3})\) and \(((W_1^2 W_0^1 \Delta) e^{-j\phi_{moo} Z_3})\) in the sub-matrix \(G'_{1\beta}\) and \(H'_{1\alpha}\) which were given by equations (2.177) and (2.1798) of section 2.4.3 are computed in another separate do-loop in terms of the Floquet modes of the second layer array. Again also involved in this main loop are a number of subroutines to evaluate the individual (W), \(\Delta\) and the propagation constant \(\gamma\) terms, which are then called in the main do-loop for computations of the dielectric coefficients.

In the computer model the computation of the TE and TM components of the incident
field amplitude coefficients $b_m^{inc}$ involved in the excitation matrix are obtained from the matrix equation (2.234) of section 2.5.3 given by:

$$
\begin{bmatrix}
    b_1^{inc} \\
    b_2^{inc}
\end{bmatrix} =
\begin{bmatrix}
    \kappa_{100}^x & -\kappa_{200}^x \\
    -\kappa_{100}^y & \kappa_{200}^y
\end{bmatrix}
\begin{bmatrix}
    B_x \\
    B_y
\end{bmatrix}
$$

(2.249)

where the Cartesian components of the incident field ($B_x$ and $B_y$ and $B_z$) are obtained from the polar angles ($\theta, \phi$) [3].

In the two main do-loops that compute the four diagonal sub-matrices $G'_{\alpha\alpha}, G'_{\beta\beta}, H'_{\alpha\beta}$ and $H'_{\beta\alpha}$, the Floquet transform of the two basis functions chosen to represent the current flow on the element arrays of the two layers, i.e. $\tilde{h}_\alpha(k_{TPq})$ and $\tilde{r}_\beta(k_{Tln})$ are called from separate subroutines in the main program. The way in which the Fourier transformation of the basis functions in these subroutines are carried out depends on the types of the elements used in the two arrays. The two subroutines return the x- and y-components of the Fourier transforms of the basis functions chosen for the two arrays. The TE and TM components of the current basis functions are then computed from the above Cartesian components. Therefore to form the sub-matrices $G'_{\alpha\alpha}, G'_{\beta\beta}, H'_{\alpha\beta}$ and $H'_{\beta\alpha}$ we multiply the TE components of the currents with the TE components of the relevant dielectric coefficients and similarly multiplying the TM components of the currents with the TM components of dielectric coefficient terms as is evident from equations (2.245-2.248) in above. The two TE and TM components are then added and the summations are performed for the number of Floquet modes chosen for the first and second layer arrays.

Having computed the individual sub-matrices, the overall main matrix is then formed in another separate main do-loop. By a simple inverse matrix transformation, the unknown complex current coefficients $C_\alpha$ and $C_\beta$ will be computed from equation (2.242) as:

$$
\begin{bmatrix}
    C_\alpha \\
    C_\beta
\end{bmatrix} =
\begin{bmatrix}
    G'_{\alpha\alpha} & H'_{\alpha\beta} \\
    H'_{\beta\alpha} & G'_{\beta\beta}
\end{bmatrix}^{-1}
\begin{bmatrix}
    L_1 \\
    L_j
\end{bmatrix}
$$

(2.250)

From these values of computed complex current coefficients, the TE and TM components
Chapter Two

of the Floquet transform of the unknown currents on the surface of the two arrays, i.e. $\tilde{I}_{mpq}$ and $\tilde{J}_{m'ln}$ will then be computed. These are obtained from the two equations below.

$$I(x_T) = \sum_{\alpha=1}^{Q} C_{\alpha} h_{\alpha}(x_T)$$

(2.251)

and

$$I(x_T) = \sum_{\beta=1}^{R} C_{\beta} f_{\beta}(x_T)$$

(2.252)

The above two equations are computed in Floquet transformation forms. The model computes both the TE and TM components of the two surface currents in a do-loop as follows:

$$\tilde{I}_{mpq} = \sum_{\alpha=1}^{Q} C_{\alpha} \sum_{mpq} (\tilde{h}_{\alpha}(k_{pq}) \kappa_{mpq})$$

or

$$\tilde{I}_{pq} = \sum_{\alpha=1}^{Q} C_{\alpha} \sum_{pq} [(\tilde{h}_{\alpha}(k_{pq})x_{1pq} + (\tilde{h}_{\alpha}(k_{pq})y_{1pq})]$$

(2.253)

$$\tilde{I}_{2pq} = \sum_{\alpha=1}^{Q} C_{\alpha} \sum_{pq} [(\tilde{h}_{\alpha}(k_{pq})x_{2pq} + (\tilde{h}_{\alpha}(k_{pq})y_{2pq})]$$

(2.254)

It should be noted that the x- and y- components of the Floquet transforms of the basis function for the first layer have been stored in separate arrays in a previous main do-loop and are being called in this do-loop.

Similarly, the TE and TM components of the Floquet transform of the current on the second layer array are computed in another do-loop in the computer model as:

$$\tilde{J}_{m'ln} = \sum_{\beta=1}^{R} C_{\beta} \sum_{m'ln} C_{\beta} (\tilde{f}_{\beta}(k_{ln}) \kappa_{m'ln})$$

The TE and TM components are therefore written as:
\[ \bar{J}_{1\text{ln}} = \sum_{\beta=1}^{R} C_{\beta} \sum_{\text{ln}} \left( \bar{r}_{\beta} (k_{T \text{ln} x} \kappa_{1\text{ln} x} + (\bar{r}_{\beta} (k_{T \text{ln} y} \kappa_{1\text{ln} y}) \right) \text{(the TM component)} \] (2.255)

\[ \bar{J}_{2\text{ln}} = \sum_{\beta=1}^{R} C_{\beta} \sum_{\text{ln}} \left( \bar{r}_{\beta} (k_{T \text{ln} x} \kappa_{2\text{ln} x} + (\bar{r}_{\beta} (k_{T \text{ln} y} \kappa_{2\text{ln} y}) \right) \text{(the TE component)} \] (2.256)

Having obtained the TE and TM components of the two currents, the model then computes the TE and TM components of the complex coefficients of the reflection and transmission coefficients \( R^{1a} \) and \( T^{2b} \) respectively from the two equations given below.

\[ R^{1a}_{mpq} = -\frac{1}{A_1} w_2^0 \bar{I}_{mpq} - \frac{1}{A_2} (w_2^0) (w_1^0) (w_1^0) \bar{J}_{m\text{ln}} \] (2.257)

and

\[ T^{2b}_{m\text{ln}} = -\frac{1}{A_1} (w_2^3 w_1^2 w_0^1) \bar{I}_{mpq} - \frac{1}{A_2} (w_2^3 w_1^2 \Delta) \bar{J}_{m\text{ln}} \] (2.258)

Both the TE and TM components of the reflection and transmission coefficients computed from the above two equations will then be used in the model to obtain the Cartesian components of the two coefficients from equations given below.

\[ R^{x}_{pq} = \sum_{pq}^{1a} R_{1pq} \kappa^x_{1pq} + R_{2pq} \kappa^x_{2pq} \] (2.259)

\[ R^{y}_{pq} = \sum_{pq}^{1a} R_{1pq} \kappa^y_{pq} + R_{2pq} \kappa^y_{2pq} \] (2.260)

with

\[ \kappa_{mpq} = (\kappa^x_{mpq} \hat{x} + \kappa^y_{mpq} \hat{y}) \] (2.261)

Similarly the Cartesian components of the complex transmission coefficient is computed from equation below.

\[ T^{x}_{ln} = \sum_{ln}^{2b} T_{1ln} \kappa^x_{1ln} + T_{2ln} \kappa^x_{2ln} \] (2.261)

and

\[ T^{y}_{ln} = \sum_{ln}^{2b} T_{1ln} \kappa^y_{1ln} + T_{2ln} \kappa^y_{2ln} \] (2.262)
with
\[ \kappa_{m \ln} = (\kappa_{m \ln}^x \hat{x} + \kappa_{m \ln}^y \hat{y}) \]  

(2.263)

Note that the z-component of the two coefficients is obtained from Maxwell's divergence equations outlined in section 2.5.3 by equation (2.222) and (2.226).

The computer model finally computes the copolar and crosspolar components of \( R_{1a} \) from the double layer structure by using equations below.

\[ R^c = \frac{E^{\text{ic}}(r,0)}{E^{\text{iT}}(r,0)} = (R_x^c B_x^i + R_y^c B_y^i + R_z^c B_z^i) \]  

(2.264)

and

\[ R^\perp = \frac{E^{\text{ix}}(r,0)}{E^{\text{iT}}(r,0)} = (R_x^{\perp} B_x^1 + R_y^{\perp} B_y^1 + R_z^{\perp} B_z^1) \]  

(2.265)

Similarly, the copolar and crosspolar components of \( T^{2b} \) are computed from equations given below.

\[ \tau^c = E^{\text{ic}}(r,d)/E^{\text{iT}}(r,d) = (T_x^c B_x^i + T_y^c B_y^i + T_z^c B_z^i) \]  

(2.266)

and

\[ \tau^\perp = E^{\text{ix}}(r,d)/E^{\text{iT}}(r,d) = (T_x^{\perp} B_x^1 + T_y^{\perp} B_y^1 + T_z^{\perp} B_z^1) \]  

(2.267)

The computer model described is used to generate the predicted results for the plane wave transmission and reflection coefficients of the various double layer structures that are studied in Chapters 3 and 4. These results are then compared with the measured values obtained from the experimental set-ups. The software written for the above computer model is easily modified to produce a computer program that solves the original integral equations. This is achieved by defining a common between the Floquet modes of the two arrays at the top of the program. In the software, all the expansions of the fields are then carried out in terms of the Floquet modes of common periodicity. In section 4.2 of Chapter 4 it is shown how the software for the common periodicity is modified for the general case when the arrays within the structure have dissimilar element geometries. However, the essential point to notice is that the computer model
outlined above for the optimised integral equations generate exactly the same predicted results as that of the software for common periodicity.

The software is run on the University of Manchester's mainframe system (i.e. MCC.cms). Despite the large number of main do-loops and the constituent nested do-loops that were present in the program, the total CPU time was very small, about 2 to 3 minutes, for a total of 169 Floquet modes being used to expand the fields adjacent to both arrays. The matrix inversions were performed by a Fortran routine from the Numerical Algorithms Group (NAG). It was found that by increasing the total number of Floquet modes the CPU run time did not increase considerably. However, as expected, an increase in the number of basis functions used for both arrays, inevitably had a considerable effect on the run time of the system since the matrix sizes are in this case larger. With Q and R are being the total number of basis functions used to represent the currents on the surfaces of arrays 1 and 2, then the dimensions of the square matrix which needs to be inverted is \((Q + R) \times (Q + R)\).

### 2.5.5 Summary

A method of analysing double layer FSS structures with dissimilar lattice geometries has been described. A novel technique was proposed for deriving the coupled integral equation system in the spectral domain. The importance of the proposed approach lies in unlocking the complexities in expressing the modal coefficients in terms of the Fourier transform of the surface's currents, and it is twofold. A super-resolution of the spectral components of the tangential fields was used and this was achieved by assigning the fields a mutual periodicity. Second, the elimination of the superfluous components that were introduced was accomplished by the use of the correlation function and its orthogonality condition.

It has been demonstrated that by assigning the periodicities of the two layers a mutual periodicity, then provided their ratio is a rational number, the tangential field vectors adjacent to the arrays can be super-resolved by redefining their Floquet modes according to convolution theorem. This meant that their spectral components are super-resolved.
with those of their mutual periodicity spectrums. The super-resolution of the fields has been achieved by means of a correlation function. This was defined as the integration of the product of two Floquet mode sets. This function in its simple form enabled a field to be taken from one layer to the next. Physically, it is equivalent to the product of two (sinc) functions, each (sinc) function including the difference of the tangential wave numbers of the two arrays. The correlation function becomes a delta (Kronecker delta) function when the two arrays have identical lattice periodicities and the Floquet mode sets in the two arrays are the same. The technique therefore enabled the scattered field expansions to be carried out in terms of the mutual periodicity set. The net result has been a substantial reduction in complexities in the modal analysis technique. Indeed, we have seen that the analysis is effectively reduced to that of the case for analysing the scattering from double layer FSS with identical lattice periodicities, with all the resultant matrix equations that relate the scattered coefficients to the spectra of the unknown currents, being similar.

The proposed approach offers a computational advantage since it is invariant to the distance separating the arrays. In a similar way, the analysis can be applied to more than two arrays including any associated dielectric substrates / superstrates. However, increasing the number of layers increases the matrix size as the square of that number with a corresponding increase in computing time.
REFERENCES

1. MONTGOMERY J.C. 'Scattering by an Infinite Periodic Array of Thin Conductors on a Dielectric Sheet', IEEE Trans. on Ant. and Prop., Vol AP-23, No 1, 1975, pp 70-75.


3.1 Introduction
In order to demonstrate the analysis and the method of solution described in Chapter 2, a variety of double layer FSS structures are studied with salient lattice geometries of the arrays. Measured results are obtained for a range of angles of incidence as well as the separation distances between the two arrays in the structure. These are obtained from a plane wave transmission performed in an anechoic chamber in the frequency range 10 to 40 GHz in which the surfaces are illuminated by a uniform plane wave at normal and oblique angle of incidence. Measured results are compared with the predictions that are obtained using the vector modal analysis technique described in Chapter 2.

In Section 3.3 the frequency band spacing ratio of various double layer FSS structures with arbitrary lattice periodicities of the arrays incorporating tripole elements are examined, with the particular aim of obtaining closer band-spacing. The effect of lattice geometry on the bandwidth and the band spacing ratio for these double layer surfaces are fully assessed in Sections 3.3.1 and 3.3.2. Particular attention is paid to the region separating the arrays in which the evanescent mode coupling becomes important for close separations. A variety of separations are studied for double layer structures with tripole elements. The effect of the inter-layer separation distance on the transmission response of the double layer structure will also be examined.

Although studies in the past [1,2] have shown that single layer arrays of tripoles satisfy the criteria for use as FSS, a less obvious feature is their cross-polar performance. In this Chapter (section 3.3.2) results are presented from a general study which, assesses the cross-polarisation of planar tripole arrays in a 45° incidence diplexer arrangement (Figure 5.1 of Chapter 5). The effect of the lattice geometry and the separation distance between
the two arrays on the cross-polarisation and in particular the peak cross-polar levels of single and double layer tripole arrays are briefly examined. Cross-polar levels at the boresight of the patterns are investigated for arrays with elements on square and triangular lattices. The studies will show that multilayer structures give a degraded cross-polar performance [3] and that this degradation is further increased for structures with dissimilar lattice periodicities. This will be illustrated in Section 3.3.2. In addition, assuming uniform incidence over the FSS (i.e. \( \theta = 45^\circ \)) the plane wave modal analysis technique outlined in Chapter 2 gives a guidance to the boresight cross-polar levels. The predicted cross-polar values in boresight position of the pattern will therefore be compared with those of measured boresight level results. The measured boresight values will be obtained by using the experimental radiation pattern set-up described in section 5.3 of Chapter 5.

The results obtained for the plane wave transmission response are presented in tabular forms and by various plane wave transmission responses. These are given in Section 3.3.2, for different states of polarised incident wave and (element) lattice orientations (i.e. \( \text{TM}_a + \text{TE}_b \), and \( \text{TE}_a + \text{TE}_b \)). The effect of the state of incidence to the transmission coefficient is also discussed in tabular form. It will be shown that there is good correlation between the predicted and measured results. In Section 3.3.3, the stability of the (reflection) band centre frequency to the angle of incidence for double layer structures of arbitrary lattice geometries are examined.

Also briefly examined in Section 3.4, will be the bandwidth and band-spacing ratios of a close-coupled arrays of crossed dipoles with wide-band characteristics. A distinctive feature of this double layer structure is that the two array elements are printed on both sides of thin dielectric substrate, where the fields from the two arrays are highly coupled. The wide bandwidth nature of the surface is fully explored by means of plane wave responses, both theoretical and measured. Section 3.5 gives a brief summary outline of the results obtained in this chapter.
3.2 Experimental Set-up

3.2.1 Measurement Technique
The method used is a free space measurement technique, which more closely approximate the environment in which the surfaces are designed to operate. Apart from the obvious connection with antenna measurements, the technique has the advantage that it is immediately extendable to curved geometries. Despite problems due to diffraction and other extraneous factors, which can arise and influence the outcome, the technique is able to produce results that, within experimental accuracy, verify the theoretical results.

3.2.2 Anechoic Chamber
Measurements in the chamber offers advantages in terms of the variety of geometrical configurations and scan planes and angles of incidence that can be measured. A schematic diagram of the measurement set up in the chamber is shown in Figure 3.1. The transmitting and receiving antennas are pyramidal horns. The transmitting horn is connected to a microwave sweep oscillator (HP 8350B) and the receiving horn to a scalar network analyser (HP 8757A) via a microwave detector capable of detecting microwave signals between the range of 8-40 GHz. The transmitting antenna is mounted on a table and the receiving horn positioned on a purpose built jig. The FSS sample to be measured is taped onto low-loss, low dielectric constant, support (about 1 cm thick, foam support material), whose dielectric constant is very close to unity, thus having no significant effect on the characteristic response of the FSS. The FSS assembly is then placed on an aperture cut through a flat microwave absorbing sheet to minimise edge diffraction from the FSS. The entire FSS assembly with the absorbing sheet is then inserted on an aperture cut through the middle of the wooden frame, which is permanently positioned halfway between the transmitting and receiving horns. The rest of the region in the wooden frame, apart from the aperture, is covered with microwave absorbers. This wooden frame can be rotated about its vertical axis. The centre of the aperture lines up with that of the transmitting and receiving horns. The aperture holding the FSS assembly measures approximately 80 x 80 cm. The whole configuration, including the adjacent walls, the ceiling and the floor surrounding the horns are covered by microwave absorbers in order to achieve an anechoic environment.
Chapter Three

Figure 3.1: Experimental set up in the chamber for plane wave measurement
The microwave sweep oscillator shown in Figure 3.1, was used to generate microwave signals of the required band. The sweep oscillator must be used with an RF plug-in unit operating in the desired frequency range in order to have a complete operating unit. This plug-in unit is capable of producing signals from 2.3 GHz to 40.70 GHz with a maximum output power of up to 15 dBm.

The scalar network analyser (HP 8757A) used in the set up is a microwave based receiver capable of making scalar reflection and transmission measurements over a frequency range determined by the external detectors used. It is programmable over the HB-IB (Hewlett-Packard Interface Bus). The unit can control a specified plotter through the 8757 system's interface terminal. The analyser unit has remote programming interfaces using the HP-IB, which provides a remote operator that controls the instruments. With the aid of a computer that sends commands to and receives data from the HP 8350B via the HP-IB interface, the entire measurement is automated and the results stored on a floppy disc for later processing.

3.2.3 Plane Wave Measurement Set Up

The plane wave measurements as was depicted in Figure 3.1 were conducted in the Anechoic chamber in order to obtain the transmission responses of the various FSSs that are designed. The most useful data in FSS measurements for design purposes is the response of the sample to a swept frequency as the angle of incidence is varied, for a fixed plane of polarised incident wave. Since frequency is usually the independent variable, it is expedient to take advantage of a network analyser. To carry out angular response measurements, the wooden frame as shown in figure 3.1 is rotated about its vertical axis, in steps of 5 degrees, while the position of the transmitter and receiver horns are fixed. The received data detected, are displayed on the network analyser screen, the response plotted in two dimensions is then displayed on the analyser screen.

Prior to measuring the response of the FSS sample, the wooden frame is positioned normal to the angle of incidence. The network analyser is then normalised with respect to the maximum power at the boresight. The FSS is then placed in position on the aperture, and the received power is recorded. The set up therefore enables the incident wave to
illuminate the FSS surface at an angle, which gives information of the transmitted power as the angle of incidence changes. Measurements are performed for both incidences (TM and TE incidence), and the transmission response of various surfaces for a swept frequency are obtained at various angles of incidence.

3.2.4 The FSS Samples and the Array Geometry

In order to verify the predicted results, a number of periodic surfaces were constructed. The construction of the samples begins by first producing a negative or a positive film depending on whether a slot or a patch element array respectively is required. This is carried out by a process known as photo masking which is normally computer controlled. The arrays are then printed on a copper coated dielectric substrate (a polyester type) using a flexible printed circuit board technique, a process normally known as photolithography. Different size samples were possible by the above technique, with sample sizes varying from 20 cm square to somewhat larger linear dimensions.

The parameters associated with the elements and the array lattices are marked on Figure 3.2 (a). They are the element arm length and the lattice vectors $D_1$ and $D_2$ in arbitrary $u$ and $v$ directions respectively, together with the lattice angles $\alpha_1$ and $\alpha_2$. The parameters $D_1$ and $D_2$ measure the element separations, and are the magnitude of the lattice vectors in the directions defined by the angles $\alpha_1$ and $\alpha_2$. The conductor width, W, throughout this study is 0.2 mm. The elements are printed on a substrate 0.037 mm thick with $\varepsilon_r = 3.00$. In addition to reducing the reflection band centre frequency as well as reducing its sensitivity to the angle of incidence, the main effects of the substrate thickness are, to increase the upper edge of the lower transmission band, and to broaden the reflection band. The effects are common to many resonant element surfaces and have been described elsewhere [1,4].

The elements chosen to form the arrays were tripoles. It simply consists of three linear dipoles separated by $120^\circ$ and connected together at the element centres as shown in Figure 3.2 (b). Tripoles were used originally as elements for slot arrays in metallic radomes [5]. They proved to be useful elements giving reflection bandwidth of about 10% and band spacing of about 3:1 or greater, [1,2]. Two types of FSS structures have been
manufactured. In one of them, double layer FSS of tripoles, the two arrays within the structure had the same lattice periodicities and the elements were arranged on equilateral triangular lattices as shown in Figure 3.2a, with $\alpha_1 = 30^\circ$ and $\alpha = 60^\circ$. In another type, double layer FSS with dissimilar lattice periodicities, the elements in the two arrays were arranged on more symmetrical lattices with $\alpha_1 = 15^\circ$ and $\alpha = 60^\circ$. For both FSS structures the tripoles had equal length of 2.5 mm. For double layer FSS with identical lattice periodicities, both arrays within the structure had element lattice periodicities 3.5 mm in $u$ and $v$ directions. On the other hand, for the double layer FSS structure with different lattice periodicities the lattices in both $u$ and $v$ directions had dimensions 4.6 mm for one array and 3.5 mm for the second array. The arms of tripoles were separated by an angle of $120^\circ$ as shown in Figure 3.2b. Figure 3.3a shows a sample of the FSS mounted on a dielectric support and Figure 3.3b shows the FSS being supported by the absorbing aperture on the wooden frame. Expanded polystyrene was used to separate the arrays, with the two arrays being sandwiched together by low loss tapes, to form the double FSS structure.

It was important for the whole structure to remain in a rigid planar configuration. To achieve this, the two array surfaces on both layers of the structures were mounted flat on sheets of expanded polystyrene. These polystyrene sheets had very little effect on the FSS's characteristics as their dielectric properties are fairly close to those of free space. Such an FSS structure mounted on an aperture of the absorbing screen in the anechoic chamber are shown in Figures 3.3c and 3.3d. The structure is illuminated by a plane wave incident on the first layer. The feeds illuminating the surface were of rectangular types and were distanced from the FSS surface so that measurements in far field are carried out, ensuring a plane wave illumination. For the arrays to be used as a dual polarised diplexer (as in reflection measurement), the incident was divided into two orientations and hence two principal orientations relative to the incident plane wave were considered. Figure 3.4 shows the orientation of the fields (both electric and magnetic) in two planes of incidence (TE and TM). Note that for TE incidence, $\phi = 0^\circ$ ($\phi$ being the angle between the plane of incidence and the x-axis as was shown in figure 1.1 of Chapter 1 and $\phi = 90^\circ$ defines the
Figure 3.2: 
(a) Geometry of planar tripole arrays
(b) Orientation of tripole element arms
(c) Geometry of the rotated axes
Figure 3.3: (a) The FSS sample mounted on a polystyrene sheet  
(b) Absorbing screen with the mounting FSS
Figure 3.3: Anechoic chamber showing the FSS support being mounted on an Aperture of a wooden frame for
(c) : Normal incidence
(d) : Oblique incidence
TM incidence. There are only two orientations investigated here. In orientation \( a \), the tangential component of the incident electric field is parallel to one arm of the tripole (i.e. parallel to the y-axis in Figure 3.5). Orientation \( b \) is defined when the array is rotated through 90° in its own plane so that the incident field is perpendicular to one arm of the tripole (i.e. perpendicular the y-axis in Figure 3.5). Since the tripole array is used in a diplexer arrangement for reflection measurement as discussed in section 5.2 of Chapter 5, the common lower transmission and reflection bands are considered for two pairs. The state of incidence was paired with alternate orientations: TE in orientation \( a \) with TM in orientation \( b \), and TM in orientation \( a \) with TE in orientation \( b \). This definition of incidences is described geometrically in Figure 3.5. Hence in our discussion of the results that are obtained from plane wave measurements and predictions, \((\text{Tea} + \text{TMb})\) and \((\text{TEb} + \text{TMa})\) are considered in pairs. Measurement tests have shown that for all the possible states of TE and TM polarisation, the results obtained for TEa were the same as for TMb at normal incidence. Similarly, the response for TEb gave the same result as the TMa at normal incidence.

3.3 Results From Representative Arrays
The aim is to validate the predicted results obtained from the theoretical models developed in the previous Chapter. To fully assess the reliability of the computed results obtained from the modal analysis technique a series of double layer of periodic arrays of tripole elements with salient values of lattice parameters and separation distances have been measured in the chamber. The measured responses of these periodic surfaces to a plane wave incident normally and at an oblique angle are compared with the predicted results. The reflection and transmission coefficients from various double layer structures obtained from the theoretical models are assessed and compared with the measured values.

It will be demonstrated that wider bandwidths together with closer band spacing ratios can be obtained by using double layer FSS structures as compared with single layer FSS [3]. The results obtained are illustrated by various plane wave transmission curves and are then summarised in a tabular form. Also assessed will be effect of varying the separation
Direction of propagation of incident field

Figure 3.4: Orientation of the Fields (E and H) in
(a) H-plane (TE) incidence
(b) E-plane (TM) incidence
Figure 3.5: Definition of Various Incidences for Tripole Arrays
distance between the two layers on the bandwidth of the entire structure. We have chosen to define the reflection band centre as the midpoint of the -10 dB points on the transmission response curve and the transmission band centre is defined as the midpoint of the -0.5 dB loss level on the plane wave transmission response curve. The frequency band spacing ratio is then defined as the ratio of the reflection band centre to the transmission band centre frequencies. It will be shown that the frequency band spacing ratio is very much affected by the change in the separation distance between the two arrays and it is also dependant on how the lattice geometries are set in each array.

In designing the various arrays within the double layer structures, the reflection resonant frequencies are set by properly dimensioning the array element lengths. The desired transmission band is obtained by varying the separation distance between the two arrays. The element spacing (i.e. the periodicities) are then used to obtain a trade-off between the bandwidths of the transmission and reflection bands.

3.3.1 Computed and Measured Plane Wave Transmission/Reflection Responses of Single Arrays of Tripoles Used in the Double-Layer Structure

In order to demonstrate the choice of the array parameters (i.e. element and lattice dimensions) that are used in our studies to form the double layer arrays, a summary of plane wave transmission/reflection characteristics of eight tripole arrays (single layers) are examined. The arrays are arranged on square and triangular lattices. The study of such arrays were previously carried out by Vardaxoglou [6], where these were examined in order to establish the influence of the lattice geometries on the cross-polar pattern and the amount of band spacing that is available. Table 1 shows these array types. In total eight different arrays of tripole elements are considered. Table 2 shows the summary of the transmission/reflection characteristics of these resonant arrays, common for all TEa/TMb incidences at 45°. The results were broadly similar for TEb/TMb incidence, with the exception that there were no common bandwidth at 45° when the arrays are arranged on a square lattice (i.e. arrays 1 and 2). The lattice periodicity D is varied giving two ratios of L/D, 0.5 and 0.55, for the square lattice and three ratios for the triangular lattice, 0.49, 0.54 and 0.71.
Chapter Three

<table>
<thead>
<tr>
<th>Array</th>
<th>$\alpha_1 ^\circ$</th>
<th>$\alpha^\circ$</th>
<th>D (mm)</th>
<th>L/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>90</td>
<td>5.2</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>90</td>
<td>4.6</td>
<td>0.55</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>60</td>
<td>5.2</td>
<td>0.49</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>60</td>
<td>4.6</td>
<td>0.54</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>60</td>
<td>3.5</td>
<td>0.71</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>60</td>
<td>5.1</td>
<td>0.49</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>60</td>
<td>4.6</td>
<td>0.54</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>60</td>
<td>3.5</td>
<td>0.71</td>
</tr>
</tbody>
</table>

TABLE 1: Element and lattice dimensions of eight different tripole arrays

<table>
<thead>
<tr>
<th>Array</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection frequency (GHz)</td>
<td>-</td>
<td>27.62</td>
<td>28.85</td>
<td>28.5</td>
<td>29.2</td>
<td>28.5</td>
<td>28.1</td>
<td>28.5</td>
</tr>
<tr>
<td>Bandwidth (%)</td>
<td>-</td>
<td>4.5</td>
<td>5.3</td>
<td>7.0</td>
<td>11.6</td>
<td>4.1</td>
<td>4.6</td>
<td>10.5</td>
</tr>
<tr>
<td>Transmission Frequency (GHz)</td>
<td>-</td>
<td>17.3</td>
<td>20.5</td>
<td>15.6</td>
<td>10.0</td>
<td>20.0</td>
<td>17.1</td>
<td>10.0</td>
</tr>
<tr>
<td>Band spacing ratio</td>
<td>-</td>
<td>1.6</td>
<td>1.38</td>
<td>1.82</td>
<td>2.92</td>
<td>1.4</td>
<td>1.64</td>
<td>2.85</td>
</tr>
</tbody>
</table>

TABLE 2: Reflection/transmission characteristics of the arrays for TEa / TMb incidence at 45°

The results shown in the table are experimental. The computed values are obtained from the computer model that was developed in Chapter 2 based on the plane wave modal analysis. In the computer model the lattice geometries in the two arrays are set to be identical and the separation distance between the two arrays are set to zero. This effectively reduces the structure to that of a single layer FSS. The predictions obtained were closely related to measurements for all states of incidence. Where the triangular lattice is concerned, the common -10 dB reflection bandwidth up to 45° increases from 4.1% to 10.5 as the ratio L/D increases from 0.49 to 0.71. However, a noticeable drift to
the lower frequency edge of the transmission band has been observed as can be seen from
the table. This leads to the bands pacing ratio being increased from about 1.4 to about 2.9.
This increase has a significant influence on the cross-polar levels of the array. The cross-
polar patterns in reflection were obtained in conjunction with the radiation pattern set-up
outlined in section 5.3 of Chapter 5.

In the case of a more symmetrical triangular lattice, i.e. when the angle $\alpha_1$ is reduced to $\alpha_1$
$= 15^\circ$, the bandwidth increases from 5.3% to 11.6%, as L/D is increased from 0.49 to
0.71, again at the expense of the band centre frequency ratio. A trade-off between the
bandwidth and the band centre frequency ratio is therefore required in practice. However,
it will be shown in section 3.3.2 that by considering multilayer FSS (in our case double
layer FSS), the band separation criteria can be de-coupled from the bandwidth criteria by
controlling the grating lobe occurrence within the pass-band. This is achieved by
adjustment of the periodicities of the two arrays and the separation distance between the
two arrays. The result will be lower ratios of reflection to transmission band centre
frequencies, at the same time obtaining a reasonable percentage of bandwidth as will be
shown.

As far as the cross-polar performances of arrays above are concerned, measurements in
the 45° incidence diplexer has revealed that for a more symmetrical (square) lattice, the
cross-polar levels at the boresight are quite high. These cross-polar levels however
improve when the elements are arranged on a close packed triangular lattice, albeit at the
expense of the band centre frequency ratio. Reflection Measurement in the 45° incidence
diplexer recorded a maximum cross-polar level of $-8$ dB at the frequency near the band
centre, for TEa and TMb incidences. The peak cross-polar level appeared on the boresight
position. For this (TEa + TMb) lattice orientation, it appears that major depolarisation
appeared near the centre of the array where the illumination is high. No peak cross-polar
level has been measured at off-boresight positions of the patterns. The high level of
cross-polarisation at boresight, for square lattice geometries is effectively due to the fact
that the transmission bands for TM- and TE -incident are not coincident. As a result the
phase difference between TM- and TE-transmission coefficients remains nearly
unchanged. The implication is that when the incident field is a combination of TM and
Chapter Three

TE, the phase difference causes a change in the polarisation of the transmitted field. This inevitably results in an increased polarisation loss and a higher cross-polarisation level. The worst case of degeneration of the transmission properties is when the phase difference between the TM- and TE-transmission coefficients has the highest effects.

For arrays with triangular lattice geometries (i.e. arrays 3 and 4), for the two pairs of incidence, the mean cross-polar levels were about -17 dB and again the location of the peak levels was found to be near or at the boresight position. For these arrays there was a maximum of the cross-polar levels near the centre of the reflection band of 28 GHz. When the ratio of L/D was increased to 0.71 (i.e. close-packed lattice), the cross-polar levels were reduced by about 3 dB. This reduction appears to be a bandwidth effect and is related to the TE, TM reflection bands being more coincident. This reduction is larger by a factor of two when compared to the array with L/D = 0.49. For triangular lattice arrays with $\alpha_1$ increased to $\alpha_1 = 30^\circ$, a further reduction of the cross-polar levels was observed. This is shown in Figure 3.6 (a). Peak cross-polar levels of -35 dB and below for all states of incidence have been recorded throughout the reflection band. The computed cross-polarised field at the boresight position, using the plane wave modal analysis ($\theta = 45^\circ$), were closely related to those of the measured values for TE incidence in both orientations. For TM incidence however, the prediction levels were higher by about 5 dB to those measured.

Figure 3.6 (b) shows the calculated contour plots of the angular distributions of the cross-polar component of the scattered field at 28 GHz for incidence TEa and TMb. The location of the peak appeared on the 90° scanning plane for TMb incidence but near the 35° plane for TEa incidence. Figure 3.6 (b) shows that there is difference in the shape of the dominant contours. It shows that the lattice geometry has altered the distribution of the cross-polar components of the reflected field. The effects of the induced element current to the transmission responses can be conveniently explained by plotting the x- and y-directed resultant current distribution on the unit cell of the array. The amplitude of the x-directed surface current for TEa at 45° and at 28 GHz is shown in figure 3.6 (c). The asymmetrical behaviour of the current about the x-z plane is apparent from the figure.
Also noticeable is the smooth behaviour of the current distribution. An edge discontinuity is also observed across the width of tripole element for the x-directed current, but at lower amplitude. Although its amplitude was small compared to the y-directed current, it nevertheless had an influence on the predominant features of the cross-polar pattern. The x-directed current was higher for TM-incidence, reaching a maximum near the array’s centre.

In the next section that follows, it will be shown that by having two single layers in parallel, a closer band spacing ratio is achieved with an improved peak cross-polar level. This is obtained by carefully selecting the lattice geometries of the two layers and by choosing a suitable value of separation distance between the two layers.

3.3.2 Analysis of the Results for Double Layer FSS of Tripoles with Arbitrary Lattice Periodicities

From Table 1, array 8 was chosen to form the double layer FSS of tripoles with identical lattice periodicities (i.e. DLTR2), while arrays 4 and 5 were used to form the double layer FSS with dissimilar lattice periodicities (i.e. DLTR1). In the measurement of the surfaces, a polystyrene sheet separated the two layers and there were no dielectric substrates to the left of either array as was sketched in Figure 2.1a of Chapter 2. For both types of FSS (DLTR1 and DLTR2), three different surfaces with separation distances of \( d = 2, 4 \) and 6 mm were constructed and measured. The transmission/reflection responses of the types of FSS’s were measured by illuminating the surfaces with wide angle feed placed at the far field of FSS. Plane wave transmission measurements on all three surfaces for both types of FSS are now compared with predictions obtained from the computer models employing a full wave modal analysis by including higher order mode interaction between the arrays.

In the theoretical model, in view of the element considered to form the periodic structures, the basis functions used to expand the current distributions on the tripole arms were *entire domain* sinusoidal basis functions. These functions span the entire support of the unknowns, and are typically tailored for the specific geometry of the region over which
Figure 3.6: (a) Cross-polar peaks in reflection. Triangular lattice, $L/D = 0.71$
(••, calculated values; ---, plane wave approximation).
Bars show measured values
(b) Computed cross-polar patterns at 28 GHz
(c) $x$-directed surface current distribution for TEa incidence at 28 GHz
the unknowns are being expanded. Fourier components were used as the basis functions with the current flowing along the arm form the tripole centre. A suitable set of basis functions for the tripole elements used is the appropriate Fourier expansion given by:

$$h_n(v) = \sqrt{\frac{2W}{L}} \sin \left[ \frac{n\pi}{2L} v \right] \phi$$

for odd values of $n$

$$h_n(v) = \sqrt{\frac{2W}{L}} \cos \left[ \frac{n\pi}{2L} v \right] \phi$$

for even values of $n$

(3.1)

for the current flow on the element directed along the $\hat{v}$ direction. $W$ is the width of the conducting elements, and $L$ is the element length and where $(2W/L)$ is a normalisation factor arising from the orthogonality of these basis functions. In equation (3.1) above, $h_n(v)$ are the expansion functions required to estimate the induced current, with $n$ being the number of basis functions.

Owing to the angular separation of the three tripole arms it is convenient to calculate the transform of the current induced along one arm only, with the arm being at an arbitrary angle, $\psi$, to the $y$-axis as was shown in Figure 3.2 (c). Then $\psi$ can be set to the specific angle required to form the tripole elements. This can be quite useful for constructing elements of more than 3 arms. If $(u',v')$ are the rotated coordinates (Figure 3.2 (c)) with respect to $(x,y)$ axes, then the $x$ and $y$ components can be derived by simple geometrical projection. These will be required for the calculation of the field component tangential to the array. The transform of the basis function as defined in equation (2.182) of Chapter 2 on the array surface of the first layer for example, can be calculated analytically, provided the bases are in a simple integrable form. Therefore in integration over the unit cell area, the integral taken over an arm incident at an angle $\psi$ to the $y$-axis for example is carried out in the rotated coordinates $(u',v')$ in Figure 3.2(c), and then resolved into $x$-and $y$-components, i.e. in integrating over $v'$,

$$\tilde{h_n}(x) = -\sin \psi \tilde{h_n}(v')$$

(3.2)

$$\tilde{h_n}(y) = \cos \psi \tilde{h_n}(v')$$

(3.3)

where $\tilde{h_n}(v') = \int h_n(v') \exp(ikTp'q'x) dx$ 

(3.4)
and \( L = u' \hat{u} + v' \hat{v}, \) and where \( k_{TP'Q'} \) is the tangential wave number of the Floquet mode. The integration is taken over the area of one tripole arm, \((0,L),\) and is easily solved in closed form.

Thus in general the symmetrical current component induced in the tripole arm along the \( v \)-direction for example, is approximated satisfactorily by the term below:

\[
I(v) = \sum_n C_n \sqrt{(2W)/L} \cos [(2n-1)\pi v/2L] \quad n = 1, 2, \ldots
\]  

(3.5)

This current has a maximum at the junction of the tripole arms and is zero at the ends of the element. \( C_n \) are the unknown current coefficients. The current on each arm has, therefore, a simple sinusoidal form, with a maximum value at the junction with the other two arms.

Equation (3.5) implies that the elements are assumed narrow enough so that the components of the current parallel to the width of the element can be neglected. Hence the surface current density is constant (i.e. uniform) across the width of the arm and flows longitudinally along the arm with the sinusoidal functional dependance given by equation (3.5). Indeed, for the surfaces that are considered here, the width of the elements are chosen to be negligibly small compared to the length.

Careful and comprehensive inspection of the computed complex current amplitudes \( C_n \) and Floquet modes coefficients were carried out. As a result, for the triangular lattice geometries considered here a total of three current modes, one \( \cos (\pi v/2L) \) per tripole arm were found to be sufficient for both orientations \( a \) and \( b \) up to \( 45^\circ \) incidence. For both FSS types (DLTR1 and DLTR2), 169 Floquet modes were found adequate to expand the field adjacent to the array. The ratio of the Floquet modes to the number of functions, \( \Omega \), is about 56. For all the tripole element geometries and spacings that were considered in here, it was found that adequate convergence in predicting the scattered field is obtained. This has been achieved by using the basis function mentioned and by using 169 Floquet modes (i.e. 13 \( p' \)-modes and 13 \( q' \)-modes for each array) in the field expansion.
Chapter Three

It should be noted that for the double layer FSS DLTR1, the ratio of the periodicities of the two arrays gives an irrational number of \( \frac{4.6}{3.5} = 1.3142 \ldots \). To be able to utilise the super-resolution technique to analyse the scattering from this double layer structure, as described in the previous Chapter, it was found necessary to slightly reduce the size of the lattice in one of the arrays, to ensure a rational ratio of the two periodicities. Hence it was decided to choose the periodicity of one of the arrays as \( D = 4.55 \text{ mm} \) instead of 4.6 mm, so that the ratio \( \left( \frac{4.55}{3.5} \right) \) is a rational number of 1.3 or \( \frac{13}{10} \). The common periodicity between the two arrays was then 45.5 mm (i.e. \( 10 \times 4.55 \) or \( 13 \times 3.5 \)). The number of Floquet modes used to expand the fields, in terms of the mutual periodicity function was \( 13 \times 13 = 169 \), in the first and second arrays respectively. This is in contrast with \( 130 \times 130 = 16900 \) (smallest number) modes that would have been required if the entire mutual set was considered in the computation. In other words, due to the larger number of small values present in the spectrum of the mutual periodicity set, only the non-zero harmonics corresponding to those of the first and second arrays need to be considered. It must be noted that reducing the periodicity from 4.6 mm to 4.55 mm had been found to have no significant effect on the values of bandwidths and band spacing ratios of Table 2. Indeed, in practice, a compromise figure for the periodicity (\( D \)) can be obtained when selecting the array parameter, in order to maintain the ratio of the periodicities of the two arrays as a rational number. This therefore enable one to use the novel modal analysis technique that has been outlined in our studies.

The plane wave transmission response of the two types of FSS (DLTR1 and DLTR2) are illustrated in Figure 3.9 for TE incidence in orientation \( a \). The effect of the separation distance on the bandwidth of the two FSS types is clearly observed. Figure 3.10 show that for TM incidence in orientation \( b \), the reflection bands are narrowed as expected. The -0.5 dB reflection bandwidth common to all angles up to 45° incidence is, as often the case, governed by the TM incidence. The rapid transition from the reflection to transmission region is defined by the TE incidence as was depicted by Figure 3.9. The transmission losses fall to -0.5 dB at about 16 GHz for separation distance of \( d \geq 4 \text{ mm} \). However for a separation distance of \( d = 2 \text{ mm} \), there seems to exist no common bandwidth within the
transmission region at -0.5 dB level. The reflection resonance frequency for all the separation distances and for both DLTR1 and DLTR2 FSS occurs mainly at $f_r = 29$ GHz, and the transmission bands centre at $f_t = 18$ GHz. This enables a band spacing ratio of about 1.6 or less to be obtained. Similar responses for TEb and TMa incidences at 45° were observed. A distinguishing feature of the plane wave responses for both FSS types are the loss in the upper portion of the transmission band at the separation distance of 2. These predicted losses have been validated by measurement. These may be due to misalignment of the two array's centres when they are sandwiched together to form the double layer structures, and it may also be due to mismatch between the arrays when they are close-coupled. Losses in the reflection band for TE-response of both FSS at a separation distance of 6 mm are also observed. The -10 dB reflection bandwidth for this case is not affected, since the band is mainly governed by the TM response. However the loss within the reflection band for TM-response of DLTR1 FSS (Figure 3.8b) at a close separation distance of $d = 2$ is of concern. This maybe a lattice effect, and also due to the fact that the two arrays are close-coupled.

In Table 3, the influence of separation distances $d$ on the stability of the reflection band centre frequency at 45° incidence and the effect on the frequency band spacing ratio are summarised. The values given in the table are predicted results. $f_{R1}$ and $f_{R2}$ define the extreme edges of the -10 dB reflection band. $f_{R}$ and $f_{T}$ are the reflection and transmission band centre frequencies respectively. It should be noted that for the structures with separation distance of 2mm and 6mm, it was found to be unrealistic to obtain the transmission band between the -0.5 dB loss levels. The -1 dB loss in the transmission region was therefore chosen in determining the bandwidth between the upper and lower edges of the transmission band.

Table 3 shows that the reflection band centre frequencies of double layer FSS are sensitive to changes in the distance of the separation between the two layers. Improved bandwidths are obtained at smaller distance (not < 2 mm) together with reasonable band centre frequency ratio, as compared to the single layer FSS. Furthermore, tables 3 (a) and 3 (b) show that Double layer FSS with identical lattice periodicities of the arrays tend to
Chapter Three

Figure 3.7a

Plane wave transmission response of DLTR1 FSS for various separation distances.
(TEa) Incidence at 45 degrees

Figure 3.7b

Plane wave transmission response of DLTR2 FSS for various separation distances.
(TEb) Incidence at 45 degrees
Figure 3.8a

Figure 3.8b
exhibit transmission responses that produce a reasonable percentage of bandwidth throughout the separation distances, with a closer band spacing between the transmission and reflection band. This bandwidth tends to remain fairly stable as the separation distance is increased but at the expense of the band spacing ratio. On the other hand, by arranging the lattice periodicities of the two arrays to be dissimilar, the percentage of bandwidth that are obtained appear to be lower as compared with those that are obtained with identical lattice FSS. Unlike double layer FSS with identical lattice periodicities, the bandwidth is sensitive to the changes in separation distance.

The theoretical model was capable of predicting very successfully the plane wave transmission responses obtained from the measurements. The predictions were closely related to the measurement with the exception of a discrepancy at the higher frequencies and for the smaller separation distances. Again this may be due to the close coupling between the layers considered, where higher order propagating and evanescent modes become significant. It should be noted that in the prediction models, when computing the transmission and reflection coefficients the evanescent modes were not taken into considerations and the transmission/reflection coefficients of the dominant (0,0) order only were computed. For the case of DLTR2 FSS however, although the gratings may be closely spaced, most of the higher order non-propagating modes excited by the first array are almost completely damped out before incidence on the second array. This implies that the interactions between the two arrays can, with good approximation be described using the dominant (0,0) order mode and maybe a few higher order modes.

Finally for the purpose of comparison between theoretical and experimental results the predicted and measured plane wave responses of both DLTR1 and DLTR2 FSSs for the separation distance of 4 mm only are summarised in here. Table 4 shows the transmission / reflection characteristics for the DLTR1 and DLTR2 both at (TEa + TMb) and (TEb + TMa) incidences at 45°. Figure 3.9 and 3.10 show the predicted and measured plane wave responses of the two FSS DLTR1 and DLTR2 respectively for (TEa + TMb) incidence. Inspection of the two plane wave responses shows that the predictions are very closely related to the measurements although there is a slight discrepancy at frequencies beyond 30 GHz. As expected, for the DLTR1 FSS, the common bandwidth is increased to 19%
when compared to that of single layers used individually (i.e. Table 2). The response for DLTR2 FSS behaves similarly to DLTR1 with the exception of an increased reflection bandwidth especially for TE incidence (52%) as shown in table 4. This increase is evident from the close element spacing of array 8 ($L/D = 0.71$) used to constitute the double layer. Although the band spacing available from DLTR2 FSS is slightly higher than that of DLTR1 FSS, the effect of having the same lattice geometries for both layers means that an improved cross-polar level is achieved.

<table>
<thead>
<tr>
<th>Separation distance</th>
<th>TEa + TMb (45°)</th>
<th>DLTTR2 FSS</th>
<th>DLTTR1 FSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>fr1</td>
<td>fr2</td>
<td>fr</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
<td>33</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>31</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>31</td>
<td>27</td>
</tr>
</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>Separation Distance</th>
<th>TMa + TEB (45°)</th>
<th>DLTTR2 FSS</th>
<th>DLTTR1 FSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>fr1</td>
<td>fr2</td>
<td>fr</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>4</td>
<td>30</td>
</tr>
</tbody>
</table>

(b)

**TABLE 3:** Reflection/transmission characteristics of the two double layer FSSs, DLTTR1 and DLTTR2 at various coupling distances for:
(a) (TEa + TMb) incidences at 45°
(b) (TEb + TMa) incidences at 45°
Figure 3.9: Plane wave transmission response of DLTR1 FSS at 4 mm Separation distance for (TEa + TMb) incidence.

Figure 3.10: Plane wave transmission response of DLTR2 FSS at 4m separation distance for (TEa + TMb) incidence.
Chapter Three

<table>
<thead>
<tr>
<th>Arrays</th>
<th>(TM^a + TE^b) 45°</th>
<th>(TE^a + TM^b) 45°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Refl BW %</td>
<td>f_e (GHz)</td>
</tr>
<tr>
<td>Double layer DLTR1</td>
<td>21</td>
<td>28.5</td>
</tr>
<tr>
<td>Double layer DLTR2</td>
<td>26</td>
<td>28.5</td>
</tr>
</tbody>
</table>

**TABLE 4:** Reflection/transmission characteristics of DLTR1 and DLTR2 FSS at \( d = 4 \) mm for (TE^a + TM^b) and (TE^b + TM^a) incidences

The effects of the separation distance on the peak cross-polar level in reflection were more significant. For DLTR1 FSS the peak cross-polar level increases as the separation distance \( d \) is increased reaching a value of -8.76 dB at 6 mm separation distance. For DLTR2 FSS, measurements in the 45° incidence diplexer recorded a peak cross-polar level at boresight of -28 dB and below for the separation distance of \( d = 4 \) mm. As with DLTR1 FSS this peak cross-polar was affected with the change in distance separating the arrays.

A more detailed discussion on the influence of the separation distance to the boresight cross-polar radiation patterns at principal and 45° scan planes for both of these FSS will be outlined in Chapter 5.

### 3.3.3 Sensitivity of Band Centre Frequency of the double layer FSSs with the Angle of Incidence

Whereas it is difficult to achieve stability of the transmission response to high angles of incidence with single layers FSS array, the study in here has shown that multilayer FSS are capable of providing this effect. For some applications the instability of transmission response with the angle of incidence could have serious consequence when a range of angles of incidence are used simultaneously. For example, when the FSS is being illuminated by a diverging beam and in applications such as radome, where surfaces are illuminated over a range of angles of incidence simultaneously. Investigations, both
Chapter Three

Figure 3.11 Plane wave transmission response of DLTR1 FSS for various angles at (TEb) incidence.

sep.distancs (d) = 4mm

Figure 3.12 Plane wave transmission response of DLTR2 FSS for various angles at (TEb) incidence.

sep.distancs (d) = 4mm
theoretical and experimental have shown that for both types of double layer FSS of tripole elements considered in here, the band centre frequencies remain generally unchanged with relatively good insensitivity to the changes in the angles of incidence. There is only a maximum of 1 GHz drift in the reflection band centre frequency over the incidence range from normal to 45° and this small change is not dependent on the separation distance between the two layers considered. Figure 3.11 shows that for the DLTR1 FSS for the separation distance of 4 mm, as the angle of incidence is changed from θ = normal to 45°, the reflection band centre frequency drifts from 29 GHz to 28.4 GHz, a drift of about 3.8%, at TEb incidence. The response for the DLTR2 FSS for the same separation distance and at TEb incidence is shown in Figure 3.12. The reflection band centre frequency $f_c$ in this case drifted from 31 GHz to 30.2 GHz for varying from normal to 45°, a drift of approximately 2.6%. Similar responses were observed for TM incidence in orientation b and TE incidence in orientation a, with a much lower drifts in the band centre frequency as the angle of incidence changes. However a distinctive feature of this incidence plane is, a decrease in reflection coefficient where the lower edge of the -10 dB reflection band drifts by approximately 2 GHz. This is evident from table 5 below.

<table>
<thead>
<tr>
<th>$\theta^\circ$</th>
<th>DLTR1 FSS</th>
<th>DLTR2 FSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>$f_1$</td>
<td>$f_2$</td>
</tr>
<tr>
<td>0</td>
<td>24</td>
<td>33</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>33</td>
</tr>
<tr>
<td>45</td>
<td>26</td>
<td>31.5</td>
</tr>
</tbody>
</table>

125
Chapter Three

Table 5 shows the (measured) plane wave transmission/reflection characteristics of the two types of FSS considered. The results outlined are for TE\textsubscript{b} and TM\textsubscript{b} incidence only, for the two double layer FSSs with the separation distance of 4 mm. The decrease in the reflection bandwidth for both DLTR\textsubscript{1} and DLTR\textsubscript{2} at TM\textsubscript{b} incidence are clearly noticeable as the angle of incidence changes from normal to 45°. Also shown in the table are the small amounts of centre frequency drift at the reflection band.

It is interesting to note that during the plane wave measurement of the DLTR\textsubscript{1} and DLTR\textsubscript{2} FSS, a large amount of ripples in the lower transmission region were noticeable, especially for TM\textsubscript{a} and TM\textsubscript{b} incidences at 45°. However these losses did not exceed below -0.6 dB. It is understood that these ripples might have been due to misalignment of the centres of the two arrays considered when they have been sandwiched to form the double layers structures. The result would be a lateral shift in the x and y directions of one of the arrays and these were not taken into considerations in the prediction models.

### 3.4 Design of a Close-Coupled Array of Crossed Dipoles with Wide band response

To further demonstrate the potential of the method of analysis and the solution used, a double-layer structure is now considered with integer multiple ratios of the two array periodicities. The elements forming the two arrays were both cross-dipoles and the two arrays share a single dielectric substrate of thickness 37 μm, thereby forming very close-
coupled arrays of double-sided FSS. The results obtained demonstrate the wide pass band characteristics of the structure. An array geometry of such double-sided structures is shown in Figure 3.13, where the element periodicity of one array is shown to be twice that of the second array. It should be noted that Figure 3.14 is a snapshot of the manufactured double-sided FSS used for measurements.

In the structure the cross-dipole elements of the first array had length of 5.14 mm both along the x-directed and y-directed arms, and a width of 0.15 mm. For the second array, the element length was 11.6 mm and the width of 1 mm. Both arrays were arranged in symmetrical square lattices of sides 6.3 mm and 12.6 mm for first and second layers respectively, thus giving a periodicity ratio of 2 and a common periodicity of 12.6 mm. In the prediction model, a total of five current functions (2 along the x-directed arm and 3 along the y-directed arm) were used to expand the current induced on the elements of both arrays. A total of $27 \times 27 = 729$ Floquet modes were found to be sufficient in expanding the fields adjacent to both arrays. The same number (minimum) of modes would be required if the entire mutual set was considered in the computations as the ratio of the two periodicities is 2:1 (i.e. integer multiple of the two periodicities).

Figure 3.14 shows the plane wave transmission response for the H-plane incidence in orientation b and for TMb incidence, at normal and 45°. Stability of the higher reflection band centre with the angle of incidence for both incident planes are the prime features of the two responses, with a drift of approximately 0.5 GHz in the band centre frequency. The same is true for the lower reflection resonance where the drift is again 0.5 GHz for TE incidence only. There is excellent correspondence between the measured and predicted values particularly, at the location of the resonances and the pass band for both incident planes at normal and 45°. Similar responses were observed for (TEa + TMb) at normal and 45° incidence. The -0.5 dB loss at the transmission band, centres at 18 GHz with a bandwidth of 56%. The -10 dB reflection bandwidth common between TEa and TMb at 45° is 7.4%. With the (upper) reflection band centring at 27 GHz a frequency band spacing ratio of 1.5 is obtained.
Figure 3.13: Element array geometry of a double-sided FSS of cross-dipoles with integer multiple periodicities ratio of the arrays, printed on either side of a 0.037 mm thickness substrate.
The important features of this design are therefore, the stability of the reflection band centres with the angle of incidence which could be an important requirement for applications where a range of angles of incidence are used. Secondly, there exists no shift in the reflection band centre frequency as the polarised incidence wave is changed from TE to TM. This implies the possibility of the structure being used for dual-polarised applications.

3.5 Summary

Two types of double layer FSS structures with arbitrary lattice periodicities of the arrays have been studied. The aim has been to validate the predicted results that were generated using the theoretical model developed. Excellent correspondence between the predicted and measured results was obtained for both FSS types. The study has demonstrated that by having two arrays in parallel, in this case arrays of tripole elements, an increased bandwidth together with closer band spacing can be achieved depending on the lattice geometries of the two arrays and the separation distance between them. Broadly speaking, the analysis has shown that in order to achieve a wide transmission band with small reflection losses, the two periodic arrays ought to have identical lattice geometries. The reflection band is still wide enough for most applications and significantly wider than in the case of single layer arrays. The effect of the lattice geometry on the plane wave transmission responses and on the peak cross-polar levels in reflection of the two FSS types was examined. The prediction model reproduced measurements of these parameters. It has been established that for the FSS with identical lattice geometries there were no significant differences between the bandwidths for both principal orientations of the lattices. These suggest the possibility of dual polarisation being obtained. By having the two arrays printed on either side of a thin dielectric substrate, it has been established that wide band responses are obtained with greater stability of the reflection band centre greater stability of the reflection band centre frequency with the change in the angle of incidence. Finally, the study has shown that the (peak) cross-polarisation levels are predominantly affected by the lattice geometries and the distances separating the two arrays. It has been observed that double layer structures with dissimilar periodicities of the array lattices tend to exhibit much higher cross-polar levels at closer coupling distances than the structures with identical lattice periodicities of the arrays.
Figure 3.14: Transmission response of double-sided FSS of x-dip / x-dip
for (a): H-plane incidence (b): E-plane incidence
Thickness of the substrate = 0.037mm
REFERENCES


CHAPTER 4.0

PLANE WAVE PERFORMANCES OF MULTILAYER FSS STRUCTURES
WITH MIXED ELEMENT GEOMETRIES AND LATTICE
PERIODICITIES OF THE ARRAYS

4.1 Introduction

The plane wave transmission coefficients of a number of multiresonant structures which have triangular and square lattices with integer and non-integer multiple ratio of their lattices, are computed and their validity are tested by comparisons with the measured results. In particular, the plane wave performances of double layer structures with the two arrays having dissimilar element shapes and geometries as well as arbitrary lattice periodicities are investigated. The aim here is to show that a more flexible transmission response with regards to the bandwidths (both transmission and reflection) and the band spacing ratios can be obtained. In particular, the occurrence of grating lobe interference [1,2] due to the periodicity of the first array, within the pass band region can be more effectively controlled for wide ranging angle of incidence, at the same time ensuring reflection resonances centred further apart. One of the periodic screens provides the lower reflection band while the second array exhibits the upper reflection band.

In some applications where the FSS may be used as sub-reflectors, the propagated wave may encounter a large variation of incident angle. It is essential therefore that the FSS response does not change significantly with the incident angle. It has been demonstrated elsewhere that the circular ring elements used as FSS [1,3,4,5] are less sensitive to variations in incident angle as will be explained in section 4.2. Metal conducting rings have therefore been chosen as array elements of one layer, with the tripoles forming the array of the second layer. Double-layer structures are considered, where the elements lattice periodicities of the two arrays lie on arbitrary geometries (i.e. triangular lattice for one array and square lattice for second array).
In Section 4.3 results are discussed from a study that assesses the validity of the theoretical model for predicting the transmission/reflection responses of double-layer FSS of tripole and ring elements. The region separating the two arrays is explored where the effects of Floquet modal interaction from the arrays are assessed. The effects of coupling distance between the two arrays on the performances such as the bandwidths and band centre frequency ratios are discussed. In one type, the arrays of tripole elements are arranged on equilateral triangular lattices whereas the ring elements of the second layer are arranged on square lattices. In the second type, the elements (tripoles for first and rings for the second) are both arranged on equilateral triangular lattices.

In Section 4.4, the design and development of very close-coupled arrays of double layer structure are examined where both arrays are being printed on either side of a thin dielectric substrate. In order to fully assess the validity of the theoretical model, various substrate thickness are considered. Particular attention is given to double-sided FSS with extremely thin dielectric substrates (about 37 \textmu m thickness) where the field interactions from the two arrays are very strong. Three types of double-sided FSS structures are being considered. In one type, the element (tripole for first and ring for second) periodicities of the two arrays are arranged on triangular and square lattices respectively. In the second type, again double-sided FSS of tripoles and rings, the lattice vectors form equilateral triangles for both arrays. In the third type the elements are cross-dipoles for the first and ring for the second array respectively, and both are arranged on symmetrical square lattices with integer ratio of the periodicities.

In the penultimate section of the chapter, Section 4.4.3.1, the design and development of a specific type of double-sided FSS of tripoles and ring elements where the two arrays share an extremely thin dielectric substrate, is proposed. The aim is to demonstrate that wide band characteristic responses can be obtained with the two reflection resonances centred further apart. Possible applications of this proposed design would include satellite antenna systems and radar where wide pass-bands are required.
4.2 Analysis of the Plane Wave Response for Double Layer FSS with Mixed Element Arrays and Dissimilar Lattice Periodicities

The choice of the array elements in the two layers are primarily governed by their transmission response performance and the degree of insensitivity of their plane wave responses with changes in the angle of incidence. In particular, the stability of the reflection band centres at moderate angle of incidence is a prime factor in the choice of array elements. Circular ring element arrays are known to be less sensitive to variations in incident angles (i.e. less $\theta$ dependant). This is due to the fact that their responses can produce wider reflection band and greater band separations, by simply adjusting their element spacing, expressed by the parameter $(2r/D)$. $r$ is the radius of the ring element and $D$ is the spacing between the centres of the adjacent circular elements. The larger the value of this parameter (i.e. the closer the spacing), the wider the reflection band and the greater the band separation [5]. The reflection band centre frequencies of their responses also remain fairly stable at various planes of incidence. Cahill et al [6] in a study of plane wave illumination of ring FSS have demonstrated that reducing the element periodicity has the effect of decreasing the displacement of the reflection band centres whereby they are almost coincident at various planes of incidence (mainly TE and TM). Further demonstration of wider bandwidth with close packed rings have been carried out by Parker et al [5], where between the points that the reflection coefficients fall to $-0.5$ dB, the reflection bandwidth common to all angles of incidence up to $45^\circ$ in both principal planes were approximately 26%. They showed that the insensitivity to angle of incidence of the reflection band centres were far greater than those of the ring element arrays with widely spaced lattice. For a wide spaced square lattice, i.e. $2r/D = 0.62$ compared to $2r/D = 0.79$ for the close packed rings, the resonance of the reflection band decreases by about 3.5 GHz as the angle of incidence $\theta$ increased from normal to $45^\circ$ in the H-plane. This is in contrast with the closely packed triangular arrays where the band centre was relatively insensitive to changes in incidence.

A good first order design procedure for close packed rings is to equate the resonant wavelength (near the centre of the reflection band) to the ring circumference (i.e. $2\pi r$).
Another important reason that makes ring attractive as FSS array elements is for the simple fact that the number of basis functions needed to represent the current induced by the incident field are very small [7] compared with the requirement of other more complex elements. As a result, some of the constraints on the computer resources encountered during the current studies are eased. Parker et al [7] in a study of plane wave illumination of ring FSS have shown that the observed transmission response can be predicted over a range including both the transmission and lower reflection band, if only the fundamental current mode terms (cos $\omega$) are included in the current expansion equation (4.30). The angle $\omega$ expresses the position of a point on the circumference of the ring element (Figure 4.1).

In the first of our design studies, an array of tripoles [8], each arm having a length of 2.58 mm and a width of 0.3 mm was chosen for the first layer. For the array in the second layer, circular ring elements of diameter 3.75 mm and of width 0.25 were chosen. Element array geometries of such double-layer surfaces is shown in Figure 4.1. The elements in the first layer were arranged on equilateral triangular lattices of sides 3.5 mm along the u and y directions with $\alpha_1 = 30^\circ$ and $\alpha = 60^\circ$. $\alpha_1$ is the angle that lattice vector along the u-direction makes with the x-axis. $\alpha_2$ is the angle that the lattice vector along the y-direction makes with x-axis and with $\alpha_2 = \alpha_1 + \alpha$. In the second layer, the lattice geometry was rectangular with lattice sides of 9.8475 mm in the x-direction and 9.8 mm in the y-direction. The substrate onto which the elements were printed had a thickness of 0.037 mm (i.e. a polystyrene type) with the dielectric constant $\varepsilon_r = 3.0$. Expanded polystyrene sheets of low density were used to separate the arrays, thus forming the double layer structures. In total three different surfaces with separation distances of 2, 4 and 6 mm were constructed. Using the plane wave experimental set up described in Chapter 3, the TE and the TM responses of the surfaces are measured, and the results compared with the predictions obtained from the theoretical model.

The predicted results can be obtained by the plane wave theoretical model described in
Figure 4.1: Element lattice geometries of the two arrays for the double-layer structures
section 2.5.4 of Chapter 2. The model solves the reduced version of the integral equations where the fields adjacent to the arrays are expanded in terms of the Floquet modes of the two arrays. These predictions could also be obtained from the common periodicity model that solves the original integral equations outlined in Chapter 2 and where the fields are expanded in terms of the Floquet modes of common periodicity. In our design in here, since the lattice vectors in the two arrays are situated along different geometries (i.e. one array having triangular lattice geometry while the second array has an square lattice geometry), a common periodicity lattice between the two arrays is not easily available. In order to establish a common between the two arrays, the lattices of one array needs to be transformed so that they are in the same directions as those of the second array. It should be noted that in order for the common periodicity to exist, the periodic fields adjacent to the arrays must be of the same amplitude as well as the ratio of the periodicities of the two adjacent arrays being rational.

Referring to Figure 4.1, to establish the common for the first array requires the lattice along the arbitrary \( u \)-axis to be projected into \( x \)- and \( y \)-directions so that the projected lattices are in the same directions as those of the second layer lattices. The common for the first array along the \( x \)-axis is straightforward and is given by \( \mathbf{D}_u \cos \alpha_1 \), where \( \mathbf{D}_u \) denotes the lattice periodicity vector along the \( u \)-axis. However along the \( y \)-axis there exists two lattices. One of them being the projection of the lattice vector along the \( u \)-direction onto the \( y \)-axis, given by \( \mathbf{D}_u \sin \alpha_1 \). The second is the lattice vector \( \mathbf{D}_y \), the lattice periodicity along the \( y \)-axis. The first objective therefore will be to establish a common between the two lattices along the \( y \)-axis.

A side view of the double layer structure was shown in Figure 2.1 (a) of Chapter 2. Assume that FSS # 1 of the Figure 4.1 form the first array of the structure with FSS # 2 forming the second array. Assuming also that a plane wave is incident normally (\( \theta = 0 \)) on the side of the first array, then the tangential wave number (i.e. the transverse propagation constant) for the common in the first array denoted by \( K_{TPQ'} \), is given by:
\[ K_{T'p'q'} = P K'_{1} + q K'_{2} \] (4.1)

where
\[ K'_{1} = \frac{-2\pi}{A} \hat{z} \times \mathcal{D}_{y}^{1} \] (4.2)
\[ K'_{2} = \frac{2\pi}{A} \hat{z} \times \mathcal{D}_{u}^{1} \]

with \[ \mathcal{D}_{u}^{1} = D_{u}^{1} (\cos \alpha_{1} \hat{x} + \sin \alpha_{1} \hat{y}) \] (4.3)
and \[ \mathcal{D}_{y}^{1} = D_{y}^{1} (\cos \alpha_{2} \hat{x} + \sin \alpha_{2} \hat{y}) \] (4.4)

\[ A \] is the area of a unit cell on the first array given by:
\[ A = D_{u}^{1} D_{y}^{1} \sin \alpha \]

Equation (4.1) therefore becomes:
\[ K_{T'p'q'} = P \left( \frac{-2\pi}{A} \hat{z} \times \mathcal{D}_{y}^{1} \right) + q \left( \frac{2\pi}{A} \hat{z} \times (\mathcal{D}_{u}^{1} \cos \alpha_{1} \hat{x} + \mathcal{D}_{u}^{1} \sin \alpha_{1} \hat{y}) \right) \] (4.5)

Assume that the new transformed (i.e. projected) lattices along the x and y axes are \( D_{cx}^{1} \) and \( D_{cy}^{1} \) (i.e. the common periodicity lattice vectors along the x - and y - axes), then \( K_{T'p'q'} \) is given by:
\[ K_{T'p'q'} = - \frac{2\pi}{A} \hat{z} \times (P' D_{cy}^{1} \hat{y} - q' D_{cx}^{1} \hat{x}) \] (4.5a)

where \( P' \) and \( q' \) represents the Floquet mode numbers of the common lattice for the first array.

By substituting for \( K_{T'p'q'} \) from equation (4.5a) into equation 4.5 above, we obtain:
\[ -\frac{2\pi}{A} \hat{z} \times (P' D_{cy}^{1} \hat{y} - q' D_{cx}^{1} \hat{x}) = -\frac{2\pi}{A} \hat{z} \times [(P D_{cy}^{1} - q D_{u}^{1} \sin \alpha_{1}) \hat{y} - (q D_{u}^{1} \cos \alpha_{1}) \hat{x}] \] (4.6)

By using the same vector product rule as that outlined in section 2.2.1 of Chapter 2, the above equation (4.6) becomes:
\[ \frac{2\pi}{A} (P' D_{cy}^{1} \hat{x} + q' D_{cx}^{1} \hat{y}) = \frac{2\pi}{A} [(P D_{cy}^{1} - q D_{u}^{1} \sin \alpha_{1}) \hat{x} + (q D_{u}^{1} \cos \alpha_{1}) \hat{y}] \] (4.6a)
From the above equation the common lattice vectors for the first array, i.e. $D_{cy}^1$ and $D_{cx}^1$ can be obtained by equating the $x$ and $y$ components. Equating the $x$ components gives:

$$\frac{p' D_{cy}^1}{A'} = \frac{1}{A} \left( P D_{y}^1 - q D_{u}^1 \sin \alpha_1 \right)$$  \hspace{1cm} (4.7)

Equating the $y$ components gives:

$$\frac{1}{A'} q' D_{cx}^1 = (q D_{u}^1 \cos \alpha_1) \frac{1}{A}$$ \hspace{1cm} (4.8)

From (4.7) we have:

$$\frac{p' D_{cy}^1}{D_{cx}^1 D_{cy}^1} = \frac{1}{D_{u}^1 D_{y}^1 \sin \alpha} \left( P D_{y}^1 - q D_{u}^1 \sin \alpha_1 \right)$$ \hspace{1cm} (4.9)

From which we obtain:

$$\frac{p'}{D_{cx}^1} = \left( \frac{P}{D_{u}^1 \sin \alpha} - \frac{q \sin \alpha_1}{D_{y}^1 \sin \alpha} \right)$$ \hspace{1cm} (4.10)

or

$$p' = \left( \frac{D_{cx}^1}{D_{u}^1 \sin \alpha} \right) p - \left( \frac{D_{cx}^1 \sin \alpha_1}{D_{y}^1 \sin \alpha} \right) q$$ \hspace{1cm} (4.11)

or

$$p' = N' P - N'' q$$ \hspace{1cm} (4.12)

where

$$N' = \frac{D_{cx}^1}{D_{u}^1 \sin \alpha} , \quad N'' = \left( \frac{D_{cx}^1}{D_{y}^1 \sin \alpha} \right) \sin \alpha_1$$ \hspace{1cm} (4.13)

and where $N'$ and $N''$ are both taken to be integers.

Hence the common along the $p'$ is obtained from equation (4.13) as:

$$D_{cx}^1 = N' (D_{u}^1 \sin \alpha)$$ \hspace{1cm} (4.14a)

or

$$D_{cx}^1 = \left( \frac{D_{y}^1 \sin \alpha}{\sin \alpha_1} \right) N''$$ \hspace{1cm} (4.14b)
Similarly, the common along the q' lattice can be obtained from equation (4.8) as follows:

\[
\frac{q'}{D_{cy}} = \left( \frac{\cos \alpha_1}{D_y \sin \alpha} \right) q
\]

(4.15)

or

\[
q' = \left( \frac{D_{cy}}{D_y} \right) \left( \frac{\cos \alpha_1}{\sin \alpha} \right) q = M'q
\]

(4.16)

where

\[
M' = \left( \frac{D_{cy}}{D_y} \right) \left( \frac{\cos \alpha_1}{\sin \alpha} \right)
\]

(4.17)

Where M' is taken to be integer. Therefore D_{cy}, the common along the q' is therefore given by:

\[
D_{cy} = M' \left( \frac{D_y \sin \alpha}{\cos \alpha_1} \right)
\]

(4.18)

The common in the first layer has therefore been established with the two common periodicity lattices in the first layer array given by equations (4.14) and (4.18). It is seen from the above that the Floquet harmonics p' and q' of the common periodicity lattice for the first layer has triangular geometry given by equations (4.12) and (4.16). The overall common periodicity lattice between the two arrays can be obtained in a straightforward manner, for a particular surface geometry.

Referring to the design of the double layer structure of tripoles and rings with the values for element length and periodicity as outlined at the onset of the present section, the common periodicity lattice for the first layer array is first obtained. The overall common periodicity lattice follows thereafter. The lattice corresponding to p and q in the first layer array is triangular, with \( \alpha_1 = 30^\circ \), \( \alpha = 60^\circ \) and with \( D_x^1 = D_y^1 = 3.5 \text{ mm} \). Hence p' and q' corresponding to the common for the first array are easily obtained from these values.

From equation (4.11), p' is given by:
\[ p' = \left( \frac{D_{cx}^1}{3.5 \sin 60} \right) p - \left( \frac{(D_{cx}^1 \sin 30)}{3.5 \sin 60} \right) q \]  

(4.19)

However from (4.13), we have:

\[ N' = \left( \frac{D_{cy}^1}{3.5 \sin 60} \right) \text{ and } N'' = \left( \frac{D_{cy}^1}{3.5 \sin 60} \right) \sin 30 \]  

(4.20)

It can be deduced from above equation (4.20) that:

\[ N'' = N'/2 \]  

and the smallest integer values for \( N'' \) and \( N' \) are:

\[ N' = 2 \text{ and } N'' = 1 \]

Hence from equation (4.14), \( D_{cx}^1 \) is obtained as:

\[ D_{cx}^1 = 2 \times 3.5 \times \sin 60 = 6.06 \text{ mm} \]  

(4.21)

\( p' \) is obtained from equation (4.19) as:

\[ p' = 2p - q \]  

(4.22)

In order to obtain the common along \( q' \), \( M' \) is first obtained from equation (4.17) as:

\[ M' = \frac{D_{cy}^1}{D_y^1} \cos 30 = \frac{D_{cy}^1}{D_y^1} \]  

(4.23)

The smallest integer value for \( M' \) is \( M' = 1 \). Hence

\[ D_{cy}^1 = D_y^1 = 3.5 \text{ mm} \]  

(4.23)

hence from equation (4.16), \( q' \) is found as:

\[ q' = \left( \frac{D_{cy}^1}{D_y^1} \right) q = q \]  

(4.24)

We have therefore established the common lattice for the first array with lattice sides of

\[ D_{cx}^1 = 6.06 \text{ mm} \]

\[ D_{cy}^1 = 3.5 \text{ mm} \]

and where the lattices \( D_{cx}^1 \) and \( D_{cy}^1 \) lie along the projected Floquet modal direction of \( p' \) and \( q' \) given by equation (4.22) and (4.24) respectively.

All that remains now is to establish an overall common periodicity lattice between the
common lattice of the first array and that of the second array. For the lattice in the second layer array, the dimensions were given as $D_{x}^2 = 9.8$ mm, $D_{y}^2 = 9.8475$ mm in the x and y directions respectively. It should be noted that a decimal fraction chosen for $D_{y}^2$ in above is, to ensure rational ratio of the periodicities as will soon become clear.

The lattice vectors for the overall common periodicity array are denoted by $D_{3x}$ and $D_{3y}$ in the x and y directions respectively. The assumption is made that the ratios of the periodicities of the first and second arrays are rational, i.e.

$$D_{3x} = \frac{D_{x}^2}{D_{cx}^1} = \frac{M_x}{N_x}$$ \hspace{1cm} (4.25)

$$D_{3y} = \frac{D_{y}^2}{D_{cy}^1} = \frac{M_y}{N_y}$$ \hspace{1cm} (4.26)

Substituting values from our design into equations (4.25) and (4.26) gives:

$$D_{3y} = \frac{9.8}{3.5} = 2.8$$ \hspace{1cm} (i.e. a rational number)

then

$$D_{3y} = \frac{D_{y}^2}{D_{cy}^1} = 2.8 = \frac{28}{10} = \frac{14}{5} = \frac{M_y}{N_y}$$ \hspace{1cm} (4.27)

Hence $M_y = 14$ and $N_y = 5$, which imply that:

$$D_{3y} = 9.8 \times N_y = 9.8 \times 5 = 49 \text{ mm}$$

and

$$D_{3y} = 3.5 \times M_y = 3.5 \times 14 = 49 \text{ mm}$$

Similarly $D_{3x}$ is obtained as:

$$D_{3x} = \frac{9.8475}{6.06} = 1.625$$

The above value for $D_{3x}$ is by definition a rational number since the decimal point is terminated and can by the definition of rational number be expressed as the ratio of two integers.
With \( D_{3x} = \frac{1625}{1000} = \frac{65}{40} = \frac{13}{8} = \frac{M_x}{N_x} \). Then \( D_{3x} \) is obtained as:

\[
D_{3x} = 6.06 \times M_x = 6.06 \times 13 = 78.78 \text{ mm}
\]

and

\[
D_{3x} = 9.8475 \times 8 = 78.78 \text{ mm}
\]

The value of 78.8 mm was chosen for \( D_{3x} \), the lattice vector of the common periodicity array in the x-direction.

The overall common periodicity lattice between the lattices of the two given arrays have therefore been established with lattice values, \( D_{3y} = 49 \text{ mm} \) and \( D_{3x} = 78.8 \text{ mm} \).

In the theoretical model for the common periodicity, a simple routine can be incorporated for establishing the overall common periodicity according to the brief analysis outlined above. The software can therefore be generalised to take into account any type of lattice geometry of the two arrays. This generalisation is normally carried out in the main loop of the program with a small routine, according to the equations outlined in above. Although the computations are carried out in terms of the Floquet harmonics of the common periodicity, very fewer modes are required since only the non-zero values in the spectrum of the common periodicity are considered as was postulated in Chapter 2.

The brief analysis in this section has shown that the Floquet modes of the common periodicity for the first layer array are along a triangular geometry in the direction \( p' \) and \( q' \). Consequently, in the computer model care needs to be taken when computing the components in the Floquet spectrums of the common periodicities. For the common in the first array, it is required to compute the components at Floquet mode locations:

\[
S = M_x \times p'
\]

and

\[
t = M_y \times q'
\]
where \( s \) and \( t \) denotes the Floquet mode indices of the common periodicity.

Similarly, for the common in the second array the components at Floquet mode location \( S = N_x \times 1 \) and \( t = N_y \times n \) need computing where \((ln)\) are the Floquet mode indices of the second array.

To illustrate the above, consider computation of the Floquet spectrum of function \( h_r(\mathbf{r}) \), the basis function used to represents the current on the surface of the first layer array. Then according to the convolution relationship of section 2.3.2 of Chapter 2 we have,

\[
\tilde{h}_{pq} = \sum_{st} \tilde{h}_{st} \delta_{pq} = \sum_{p'q'} \tilde{h}_{M(2p-q),M(q)}
\]

indicating that in the spectrum of the common periodicity the computation of the components at Floquet mode locations \( M \times (2p-q) \) and \( M \times (q) \) in the directions \( s \) and \( t \) respectively are required.

Let us consider the example of double layer of tripole / ring given above where the integers \( M_x \) and \( M_y \) were found to be 13 and 14 respectively. A simple table of Floquet mode locations of the common periodicity as shown in table 1 can further illustrate equation (4.29). For simplicity, 49 (i.e. \( p \times q = 7 \times 7 = 49 \)) Floquet modes of the first layer array are considered.

The triangular geometry of the Floquet mode \( s \) and \( t \) of the common periodicity can be observed from table 1. It is seen that a total of 49 Floquet modes only of the common periodicity are required in the computation. This is in contrast with 8918 (i.e. \( st = (13 \times 7) \times (14 \times 7) = 8918 \)) number of modes that would have been required if the entire Floquet modes of the common periodicity was considered. The remaining components in the Floquet spectrum of the common for the first layer array are redundant and are therefore
bypassed in the computation. This enables a considerable amount of optimisation in the software model that is developed for the common periodicity. The result is a drastic reduction in computation time.

\[
s = m_r(2p-q)
\]

\[
t = m_r q
\]

<table>
<thead>
<tr>
<th></th>
<th>P = 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>q = 0</td>
<td>(0,0)</td>
<td>(26,0)</td>
<td>(52,0)</td>
<td>(78,0)</td>
<td>(104,0)</td>
<td>(130,0)</td>
<td>(156,0)</td>
</tr>
<tr>
<td>1</td>
<td>(-13,14)</td>
<td>(13,14)</td>
<td>(39,14)</td>
<td>(65,14)</td>
<td>(91,14)</td>
<td>(117,14)</td>
<td>(143,14)</td>
</tr>
<tr>
<td>2</td>
<td>(-26,28)</td>
<td>(0,28)</td>
<td>(26,28)</td>
<td>(52,28)</td>
<td>(78,28)</td>
<td>(104,28)</td>
<td>(130,28)</td>
</tr>
<tr>
<td>3</td>
<td>(-39,42)</td>
<td>(-13,42)</td>
<td>(13,42)</td>
<td>(39,42)</td>
<td>(63,42)</td>
<td>(91,42)</td>
<td>(117,42)</td>
</tr>
<tr>
<td>4</td>
<td>(-52,56)</td>
<td>(-26,56)</td>
<td>(0,56)</td>
<td>(26,56)</td>
<td>(42,56)</td>
<td>(78,56)</td>
<td>(104,56)</td>
</tr>
<tr>
<td>5</td>
<td>(-65,70)</td>
<td>(-39,70)</td>
<td>(-13,70)</td>
<td>(13,70)</td>
<td>(39,70)</td>
<td>(65,70)</td>
<td>(91,70)</td>
</tr>
<tr>
<td>6</td>
<td>(-78,84)</td>
<td>(-52,84)</td>
<td>(-26,84)</td>
<td>(0,84)</td>
<td>(26,84)</td>
<td>(52,84)</td>
<td>(78,84)</td>
</tr>
</tbody>
</table>

**Table 1:** The locations of the Non-zero Components in the Floquet Spectrum of the common periodicity Function for the first layer array. Each entry in the table represents the locations of the Floquet mode \( st \) (in terms of both \( p \) and \( q \)) at which the computations are carried out.
In computing the predicted results for the plane wave transmission response of the double layer tripoles and rings above, a total of 169 (i.e. $13 \times 13$) Floquet modes were used to expand the fields adjacent to the two arrays. A total of 9 entire domain current basis functions, were used to represent the current flow on the surface of the first array; three assigned on each arm of the tripole (i.e. $\cos(\frac{\pi y}{2L})$, $\cos(\frac{3\pi y}{2L})$ and $\cos(\frac{5\pi y}{2L})$). For the ring elements of the second layer the widths of the ring elements were assumed to be small as compared to the wavelength and the ring diameter. Hence the induced currents were assumed to be in circumferential direction of the ring. These currents on the rings were expressed in the computer model as a series of sinusoidal and cosine modes of the form;

$$J(\omega) = \sum_n (a_n \cos(n\omega) + b_n \sin(n\omega)) \phi$$  \hspace{1cm} (4.30)

where $n = 0, 1, 2, 3, \ldots$.

In above, $\omega$ is the angular position of a point on the circumference of the ring as shown in Figure 4.1, centred on $x = 0$ and $y = 0$, and $\phi$ is a unit vector in the direction of increasing $\omega$. Hence sinusoidal and cosine functions are used along $\omega$ where the distribution along the radial direction of the ring is assumed to be constant. In total 9 current expansion functions were included for each array element in the above equation (4.30); 5 cosine functions of ($\cos\omega$, $\cos\omega$, $\cos3\omega$, $\cos5\omega$, and $\cos7\omega$) and 4 sine functions of ($\sin\omega$, $\sin2\omega$, $\sin4\omega$, and $\sin7\omega$). Studies in the past [3,5] have shown that these types of basis functions proved effective for dichroic surfaces consisting of ring elements giving accurate results when compared with experimental values.

The plane wave transmission/reflection responses of a number of double-layer FSS structures of tripole and rings with various separation distances between the arrays have been measured and their values are then compared with the predicted results. These are outlined in the following sections.
4.2.1 Analysis of the Results of Plane Wave Responses for Double Layer FSS of Tripoles and Rings

The primary aim in this study is to verify the predictions that are obtained from the computer model described previously. In designing a specific double layer FSS structure, a systematic design procedure, consisting of the following steps has been developed. First, the geometry of each array, including the adjacent dielectric sheet is determined to obtain the desired resonant frequency. The required transmission band is obtained by then dimensioning the distance between the arrays and the thickness of dielectric sheet included in each array. Note that the performance of the transmission is mainly dependent upon the polarisation of the incident field. Finally, a trade-off between the bandwidths of the transmission and reflection bands is obtained by varying the element spacing (i.e. the periodicity) of the arrays. In all of the designs outlined in this chapter, the FSS can be assumed to be planar with the distance between the feed horns and the subreflector being sufficient in relation to the periodicity to justify locally plane wave incidence. The performance of the FSS structures can be judged from the evaluation of the transmission and reflection properties for plane wave incidence on the infinite planar structures.

The dimension of the arrays in the double layer structures considered in here were as given in previous section 4.2. For the first double layer FSS of tripoles and rings structure the two arrays were sandwiched together by means of two expanded polystyrene sheets. The surfaces of the two arrays were therefore in close contact with each other. A separation distance of 0.25mm between the two arrays was then assumed as a realistic value.

With the incident electric field being parallel to the y axis (i.e. TE incidence), the plane wave transmission response of the structure at normal incidence is shown in Figure 4.2. Also shown in Figure 4.2 is the response for (TEa + TMb) at 45° incidence. Although the Pass bands in between the two reflection bands are not wide enough, nevertheless the close agreement between predictions and measurements are clearly noticed. The loss in
Chapter Four

Amplitude transmission response of double-layer Tr/Ring for H-plane incidence at normal (TEa)
Thickness of the air gap = 0.015 mm

Fig. 4.2a

Amplitude transmission response of double-layer Tr/Ring for (TEa+TMb) incidence at 45 deg.
Thickness of the air gap = 0.025 mm

Fig. 4.2b
the pass-band regions suggests that no useful bandwidth is obtained for this configuration with no meaningful band spacing ratio associated with this response. Theoretical model has successfully reproduced the measured responses, particularly with regard to the locations of lower and upper reflection frequencies. The difference between the responses for TE and TM at off-normal incidence is clearly observed. The peak of the reflection band for TE incidence occurs at about 27 GHz, whereas for TM incidence the peak occurrence is at the frequency of around 29 GHz. This may suggest therefore that the structure may not be useful for applications where dual polarisation is required. However, studies carried out in the past [9], have shown that the performance of the transmission band can be improved by properly selecting the thickness of dielectric backing. This forces the transmission band for TM- and TE-incidence to coincide.

Next double layer arrays of tripole/ring, with wider coupling distances between the arrays are considered. The main objectives in this case are to firstly assess the validity of the predicted model and secondly to examine the effect, if any that the separation distances might have on the transmission/reflection response and on the bandwidth of the structure. In total three surfaces with separation distances 2, 4, and 6 mm were measured. In measurement expanded polystyrene sheets of thickness 2, 4, and 6 mm have been used to separate the two arrays.

In Figure 4.3, the predicted and measured plane wave transmission responses of the double layer structure for the separation distance of 2, 4 and 6 mm for TE incidence at 45° (in orientation a) are shown. The predictions using the proposed theoretical technique are in good agreement with the measured values with regard to the location of the reflection resonance as well as the pass band, with the exception of some discrepancies at the higher frequencies for the smaller separation distance of 2 mm. This may be due to the close coupling between the two arrays, where the higher order propagating and evanescent modes become significant. Similar response for (TEb + TMa) incidence at 45° were observed. Various investigations were carried out in order to identify the origin of these discrepancies. Parameter optimisations were considered as options. Amongst all, the
number of current functions (CF) on the tripole elements were increased, along with an increase in the number of current functions for ring elements from 9 to 12 in total. The total number of Floquet modes (FM) used were also increased accordingly from 139 (13 x 13) to 1225 (35 x 35) modes of TE and TM types. As a result, a very small improvement on the predicted response was observed. Other parameter variation considered was a small amount of shift being introduced on the lattice co-ordinates of one array with respect to the co-ordinates of the second array. These lattice co-ordinate shifts are realistic since measurement errors can occur if the two FSS boards are not properly sandwiched together when forming the double layer structures, i.e. a shift in the origin of the lattices in one array with respect to that of the second array. As a result of this optimisation, a further slight improvement on the predicted responses for all coupling distances was gained.

The losses within the pass band regions of the responses in Figure 4.3, for all separation distances suggest that no useful bandwidth is gained for these configurations of tripole and ring element structures. However these surfaces may be useful for applications where two reflection bands centred far apart are required. The origins of these losses may be due to the mismatch between the two arrays. Hence some of the energy are reflected back from the first array while most of them are transmitted through the second array. The losses may also be due to the dielectric losses, although with the thickness of the substrate that is considered in here this seems less probable. Ideally, the presence of the second screen minimises any partial reflection that may be present from the first screen. However the exact spacing between the arrays in terms of multiple of wavelength is important in ensuring that the partial reflection signals from the two screens cancel each other. The second layer array therefore serves as a transmission impedance-matching layer to tune out the unwanted reflection.

In another type of structure considered, the lattice geometry of the ring element array in the second layer was changed from square to triangular lattice. Hence both arrays had the same equilateral triangular lattice geometries with $\alpha_1 = 30^\circ$ and $\alpha = 60^\circ$, but with dissimilar lattice periodicities. The lattice sides of the ring and tripole array were 9.8 mm
Figure 4.3: Amplitude transmission response of double-layer Tripole/Ring for H-plane incidence (TEa, 45 deg.), for sep. dist. of (a): 2mm, (b): 4mm, (c): 6mm.
and 3.5 mm respectively, resulting in a ratio of 2.8 (i.e. 14/5) and a common periodicity of 49 mm (5 × 9.8 or 14 × 3.5). In the theoretical model a total of 225 (15 × 15) Floquet modes of TE and TM types were used to expand the fields adjacent to both arrays. In total 18 current functions were used, three assigned on each tripole arm and nine around the circumference of the ring.

Figure 4.4 shows the response for TMb incidence at 45°, for double layer tripole and rings with separation distances of 2 and 4 mm. Observation shows that the prediction using the proposed theoretical method is in good agreement with the measured values. The locations of the reflection resonance as well as the pass band are well reproduced by the prediction model. Similar agreements were observed for (TEb + TMa) at 45° incidence with the exception of one GHz difference at frequencies beyond 25 GHz. As in the previous case of tripole and ring with triangular/square lattice geometries, losses appear in the pass band region of the response for all separation distances.

In the theoretical model, further increase in the number of basis functions used to expand the currents on the ring and tripole elements had no significant effect on the location of the higher resonance, where discrepancies exist between measured and predicted values for TE incidence only. A possible cause as to the discrepancies for small separation distance of 2 mm might be due to interactions of the higher order propagating and evanescent modes from the two arrays, where the coupling between these modes from the two arrays are high. In the computation, the higher order propagating and evanescent modes coupling were not taken into account and the coefficients of dominant (0,0) order mode only were computed.

4.3 Analysis of Results from Double-Sided FSS of Tripoles and Rings

4.3.1 Double-Sided FSS Structures with Arbitrary Lattice Periodicities and Element Geometries of the Arrays

Results are now presented from a study of double-sided (i.e. close-coupled) surfaces of arrays of tripole and ring elements, arranged on arbitrary lattice. A single substrate is used
Figure 4.4: Plane wave transmission response of double layer FSS of tripole/ring with triangular lattices of the arrays TMb incidence at 45 deg, for interlayer separation distances of (a) 2 mm, (b) 4 mm
to separate the two array elements. Figure 4.5 shows the surface geometry of a typical
double-sided FSS consisting of dissimilar gratings of tripole and ring elements printed on
either sides of an RT-DURIOD substrate of thickness 165 µm and of dielectric constant \( \varepsilon_r = 2.35 \). The element arrays of the tripoles were on triangular lattice geometry whereas
those of the second array of rings had square lattice geometry as can be seen in Figure 4.5.
Note also that Figure 4.5 is a snap photo shot of the actual double-sided surface used in
the measurement. With such small thickness of the substrate, the results obtained are quite
satisfactory as will be shown in here, with the location of the higher frequency resonance
accurately predicted. The location of the lower frequency null is almost predicted with a
slight discrepancy. However the shape of the response between the two results are very
similar.

In total 225 (i.e 15 x 15) Floquet modes of the TE and TM types were used in computing
the field adjacent to the arrays. For the results obtained here, the propagating (i.e. the (0,0)
order) mode is dominant, the evanescent mode interaction contributing marginally to the
far field result. In the computation, the reflection and transmission coefficients of this
dominant mode only were considered. Six orthogonal basis functions were used to
express the current induced on the tripole elements of the first array (i.e. 2 per tripole arm,
\( \cos\left(\frac{\pi}{2L} v\right) \), \( \cos\left(\frac{3\pi}{2L} v\right) \). For the second layer array, 7 basis functions were used to
represent the current on the circumference of the ring element (mixture of symmetrical
cosine and anti-symmetrical sine functions were utilised).

The plane wave transmission response for TM incidence in orientation \( \alpha \) at normal and 45
degrees are shown in Figure 4.6. A slight shift in the location of higher resonance is seen
as the incidence is changed from normal to 45°. The drift on the band centre frequency of
the higher reflection band was about 1 GHz, as the incidence angle is varied from normal
to 45°. The band centre of the upper reflection region is located at 28 GHz. The loss in the
transmission band, determined in this case by -1 dB level centres at about 15 GHz, thus
giving a band spacing ratio of about 1.86. For the low frequency reflection band, there
Figure 4.5: Array geometry of a double-sided FSS of tripole and rings using RT-DURIOD substrate of thickness 168 μ. 
Frequency Response of Double Sided FSS Tripole/Ring (with RT-DURIOD substrate)
(dielectric thickness = 0.165)

Figure 4.6.a

Frequency Response of Double Sided FSS Tripole/Ring (with RT-DURIOD substrate)
(dielectric thickness = 0.165)

Figure 4.6.b
was no noticeable drift in the band centre frequency. Similar responses have been found for TEa, TEb and TMb incidence at 45°. Based on the above results, the outlook for designing surfaces with double-sided configurations exhibiting wide band response between the two reflection bands, seems very promising. These are achieved by carefully selecting the two array element dimensions and periodicities as well as the thickness of the substrate in between the two arrays. Design of such structures using RT-DURIOD substrate of various thickness will be outlined in the latter part of this chapter.

To further assess the validity of the theoretical model and demonstrate the potentials of the proposed prediction model, the (reflection/transmission) characteristics of a double-sided surface using a very thin dielectric substrate with thickness of 37 μm as a support, are next examined. The substrate is of polyester type with conducting rings and tripoles printed on both sides. The tripole elements were arranged on triangular lattices and the rings on square lattices. A very significant aspect of this structure is strong mutual coupling between the two arrays due to the small thickness of the substrate where the higher order and evanescent modes from the two arrays may strongly interact.

Figure 4.7 shows the amplitude transmission response for TM incidence in orientation a at normal and 45°. It is seen that at normal incidence, the location of the higher resonance frequency is well predicted, whereas at the location of the lower resonance, the predicted and measured results differ by 1 GHz. At oblique (i.e. 45°) incidence, the resonance frequency of both the lower and upper reflection bands have drifted and discrepancies exist between predictions and measurements at both resonance frequencies. In Figure 4.8 the plane wave response for (TEa + TMb) at 45° incidence is shown. As expected the performance of the transmission band is dependent upon the polarisation of the incident field. It can be seen from Figure 4.8 that at the lower edge of the upper reflection band, the measured and predicted values differ by about 1.5 GHz, with the bandwidth being wider for measured response.

For this structure the prediction model has not been able to accurately reproduce the plane
Frequency Response of Double Sided FSS Tripole/Ring, E-plane incidence at Normal
(thickness of the substrate = 0.037mm)

Figure 4.7.a

Frequency Response of Double Sided FSS Tripole/Ring, E-plane incidence at 45 deg.
(thickness of the substrate = 0.037mm)

Figure 4.7.b
Figure 4.8: Plane wave transmission response of double-sided FSS of tripole/ring with substrate thickness of 0.037mm, for (TEa + TMb) incidence at 45° (crosses and circles show the measured values)

wave transmission response. The reasons may well be due to the fact that since elements of the two arrays are closely coupled, then the usual cosine and sinusoidal basis functions chosen to express the current on the tripole elements of the first layer for example, may not be accurate. This is because the shape of the tripole elements are somewhat affected by the nearby ring elements of the second array. The same is also true of the basis functions chosen to represent the currents on the ring elements. As a consequence, in the theoretical model a common basis function between the two set of basis functions is not accurately obtained. This is in contrast to the case of double-sided FSS with RT-DURIOD substrate, which had a thicker substrate with lesser or no interactions between the elements of the two arrays. The mutual coupling between the two array elements in that
case were not as significant as in the structure with thinner dielectric substrate. For the double-sided structure with thinner substrate due to the interactions of the elements from the two arrays, elements of arbitrary shape and geometry are subsequently generated. This implies that entire domain basis function representation of the current flow on the surfaces of the two arrays may be inaccurate since these currents are presumed to flow in certain directions. Furthermore, the close coupling of the arrays may result in greater numbers of higher order and evanescent modes to propagate and for the scattered field coefficients due to these modes to become more significant. In our computation of transmission and reflection coefficients, these evanescent and higher order modes were not taken into account and the coefficient of reflection and transmission for the dominant (0,0) order mode only were computed. In general, the one-mode interaction approximation is valid only when the inter-layer spacing is large compared with the wavelength. However with spacing smaller than $\frac{\lambda}{4}$ (in average) the coupling due to evanescent modes may appreciably alter the transmission/reflection coefficients.

As a method of improving the discrepancies between the theoretical and measured response, it is suggested that sub-domain basis functions [10] representation could be employed instead to calculate the current distribution on the elements of the two arrays. These functions allow arbitrary shaped elements. Such basis functions representation may result in closer approximation of the current flows on the complicated periodic elements that result from close coupling of the elements of the two arrays as encountered in the above double-sided FSS. Functions such as the "roof-top" basis functions [10] are very common where each element within a periodic cell is segmented into a number of element sections, and the surface current density within the unit cell is approximated by a finite number of current elements having roof-top spatial dependence [11]. Since the conductor shape is defined through the locations of sub-sectional current elements, then arbitrary shaped elements such as those which may result from very close-coupling surfaces can be accommodated. This is however beyond the scope of current studies in this thesis, but could form the basis for further studies of double-sided FSS with arbitrary lattice geometries, important in many applications such as reconfigurable FSS [12]
structures and frequency scanning surfaces.

In another study of double-sided FSS with the dielectric substrate thickness of 0.037 mm, the two array elements both had triangular lattice geometries. The results obtained were very similar to those of double-sided FSS of tripole/ring with triangular/square lattice geometries of the two arrays. Again discrepancies existed between the predicted and measured results at the location of both resonances.

Despite the uncertainties that exist in the analysis of the above double-sided structures, the theoretical model in its present form has produced results that are not far away from the measured values. The location of the higher reflection resonance is almost reproduced, with discrepancies existing at the lower reflection resonance (up to 1.5 GHz). The shape of the measured and predicted responses in between the two nulls are however quite similar.

4.3.2 Double-Sided FSS Structures with Integer Multiple Ratio of Periodicitites

As part of further studies into aspects of close-coupled array FSS, structures are now considered where the periodicity of one array is chosen to be integer multiple of that of the second array. The elements selected were cross dipoles for first layer array and circular rings for second layer. Both arrays were printed on either side of a 0.037 mm thickness substrate (a polyester type). The length of the cross-dipole element was 9.5 mm along both symmetrical arms, with a width of 0.3 mm. The inner radius of the ring element was 1.73 mm and width 0.3 mm, giving an effective radius of 1.91 mm. The lattice sides of the ring and crossed dipole array were 5.00 mm and 10 mm respectively, resulting in a periodicity ratio of 2 and a common periodicity of 10 mm (2 × 5 or 1 × 10). In the theoretical model a total of six current basis functions were used for cross-dipole, two anti-symmetrical (sine) functions along the x-directed arm and four cosine functions along the orthogonal arm. For the ring element, a total of five current basis functions (2 cosines and 3 sines) were used for representing the induced current. A total number of \((27 \times 27 = 729)\) Floquet modes used were used to expand the fields adjacent
Figure 4.9: Array element geometry of a double sided FSS of cross-dipoles and rings with integer multiple of the periodicities, printed on both sides of a polyester type substrate of thickness 0.037 mm
to the two array respectively. The same number (minimum) of modes would be required if the entire mutual set was considered in the computations when the fields are expanded in terms of Floquet modes of the common periodicity. This is due to the fact that the ratio of the periodicities is 2:1.

Figure 4.9 shows the surface array geometry of such a double-sided structure. Figure 4.9 is the exact photo shot of the manufactured FSS used for measurement. The axis coordinates of one of the arrays are shifted with respect to the co-ordinates of the second array as can be seen from Figure 4.9. Hence in the computation model a shift of 2.35 mm in both the x and y directions were introduced for the first layer array. The theoretical model has very successfully predicted the measured response for all states of incidence. Figure 4.10 shows the transmission response for (TMa + TEb) incidence at 45°. The results obtained are very interesting for although the arrays are closely coupled separated by only 0.037 mm of substrate thickness, the effects of coupling between the two arrays seem to be minimal. This is primarily due to the fact that the array's element periodicity ratio are integer. This effectively eliminates the element interactions that existed for the previous double-sided FSS studied, where the periodicity ratios were non-integer, but rational. In addition, the elements of the two arrays lie on the same lattice geometries, square lattice for both arrays, as was shown in Figure 4.10. In the next section wide band designs are outlined of double-sided surfaces using crossed dipole and ring elements for the two arrays, with integer ratio of the two arrays element periodicities.

4.4 Wide band Designs of Double-Sided FSS
Based on the results obtained from the studies of double-sided arrays, with integer multiple of the array periodicities, outline of a design is now presented of double-sided surfaces where the pass band responses in between the two reflection resonances are examined. With the two reflection band centre frequencies fixed, the periodicities of the two arrays are chosen such as to obtain a wide pass band in between the two reflection bands (by adjusting the L/D ratio) as well as ensuring no grating lobe interference within the pass band region. As previously described, the location of the two reflection
resonances are determined by the length of the elements in the two arrays. The width of the reflection band can be altered by adjusting the element spacing in the two arrays. However the periodicity of the first layer array must be chosen such that any grating lobe occurrence from this array falls beyond the resonant frequency of the second layer array.

The design procedure is as follows. The onset of grating lobe will occur at $\lambda_g = \frac{1}{D}$.
where $\lambda_g$ is the grating lobe wavelength and $D$ is the periodicity of the first array. By ensuring that the element spacing $D_1$ is kept below at least one half free space wavelength (i.e. the resonance frequency), grating lobe can then be pushed away from the reflection region of the second array. Note that since element periodicity of one array is integer multiple of the second array, we therefore choose the element periodicity of the second array to be one half of that of the first array. Thereby, ensuring that any grating lobe from the second array falls beyond the upper edge of the frequency response.

In the design that follows RT-DURIOD substrate has been chosen as dielectric support for the two arrays, and integer ratio of element periodicities of the arrays is ensured for the reasons explained in the previous section. The elements of the first arrays were of cross-dipoles of length 4.2 mm and width 0.25 mm. The elements forming the arrays of the second layer were circular rings of inner radius 3.6 mm and width of 0.3 mm, giving an effective radius of 3.78 mm. The lattice sides of the rings and cross-dipoles were 9 and 4.5 mm respectively, resulting in a periodicity ratio of 2 and a common periodicity of 9 mm (i.e. $2 \times 4.5$ or $1 \times 9$). Both arrays were arranged on symmetrical square lattices. In the prediction model, nine current basis functions were used for the cross-dipoles (3 sine along the x-directed arm and 6 cosine along the orthogonal arm). A total of six current functions were used for the ring element (3 cosines and 3 sine). The number of Floquet modes used to expand the fields adjacent to the arrays were $(27 \times 27 = 729)$. The arrays were printed on either side of the dielectric substrate of thickness 0.13 mm and of dielectric constant $\varepsilon_r = 2.2$. From the above design values it is clear that the ring element array exhibits the lower reflection band and that the upper resonance frequency of the response is due to the cross-dipole elements of the first layer array.

For both principal planes of incidence at normal, the higher reflection resonances were at 35 GHz and the lower reflection band centred at 11 GHz. Plane wave measurements have shown that as the incidence changes from normal to 45°, there is a drift of 1 GHz on the
Chapter Four

Plane wave transmission response of double-sided FSS of xdip/ring for E-plane incidence. Thickness of the substrate = 0.13mm

Figure 4.11.a

Plane wave transmission response of double-sided FSS of xdip/ring for (TEa + TMb) at 45 deg inc. Thickness of the substrate = 0.13mm

Figure 4.11.b
higher reflection band centre frequency, meanwhile the lower reflection resonance remains stable as shown in Figure 4.11. The -10 dB reflection band is governed by the TM response. Figure 4.11 also shows the response for (TEa + TMb) incidence at 45°. The wide pass band in between the two reflection bands is clearly noticed. A reflection bandwidth of 8.3% at -10 dB points is obtained. The transmission bandwidth at the -0.5 dB loss level is 64%. With the pass band centring at 23.5 GHz and with the higher reflection resonance of 36.5 GHz, a band centre frequency ratio of 1.55 is obtained. The same bandwidths and centre frequency ratios were obtained for (TEb + TMa) incidence at 45°. It is important to note that the response due to ring element array, especially the lower reflection band centre frequency at 11 GHz, is quite insensitive to the wide variation of the incidences. The position of the first null hardly ever changes as the incidence is changed. The upper reflection band on the other is quite sensitive to changes in angle of incidence.

The design suffers from two disadvantages. Firstly, the higher reflection band centres are sensitive to changes in incidence angle. The responses have shown that for both planes of incidence, the higher band centre frequency drifts by 1 GHz. Secondly, there are no common bands between TM and TE responses at the upper reflection band region. The band centre frequencies at the two incidence planes differ by about 2 GHz.

The results presented are theoretical only. However, owing to confidence established in the software model, it is expected that the correspondence between predicted and measured results would be satisfactory. This has been demonstrated by a second design which was outlined in Section 3.4 of Chapter 3. For that design, cross-dipole elements were chosen for both arrays. The two arrays were printed on either side of a thin dielectric substrate of thickness 37 μm. The integer ratio of the two periodicities were again chosen. The results that followed in that section demonstrated that the structure exhibited a reasonably wide pass band together with close band centre frequency ratios. The predicted results very closely followed those of the measured values with excellent agreement between them as was seen from Figure 3.14 of Chapter 3.
4.5 Summary

In this chapter a study has been presented where the plane wave transmission response of double-layer FSS structures with arbitrary element and lattice geometries were examined both experimentally and theoretically. The validity of the theoretical analysis has been tested for these structure types. Satisfactory results were obtained for surfaces with wider separation distances between the two arrays. For close-coupled arrays however, particularly when the two arrays are printed on either side of a dielectric substrate, the prediction model has not successfully reproduced the measured values. It has been shown that provided the thickness of the supporting substrate onto which the two array elements are printed, is not very small (i.e. > 0.16 mm) then the theoretical model reproduces the measured results. The studies have shown that for double-sided FSS when the arrays are separated by a thin substrate, as a result of higher order propagating and evanescent mode interaction between the two arrays, discrepancies exist between the predicted and measured results, particularly in the lower reflection region. These evanescent and higher order propagating mode couplings were not taken into account in the analysis when computing the transmission and reflection coefficients. It should be noted that higher order modes could be propagating or evanescent. The higher order Floquet mode (|p| > 0, |q| > 0), that do not propagate and corresponds to the grating lobes are called evanescent modes. The higher order mode propagation depends on the lattice periodicity to wavelength ratio. Although the direction and amplitude of these higher order modes differ from the fundamental (0,0) order mode, their coefficient of reflection and transmission follow the same procedure.

A second possible cause of discrepancies between the predictions and measurements for close-coupled arrays were as follows: due to the manner in which the elements of the two arrays interact with each other, it was found that entire domain basis function representation of the currents appear to be unsuitable. Hence it is proposed that for these type of structures representation of the surface currents by sub-domain basis functions would be far accurate.
A special case of double-sided structure was examined where the ratio of the periodicities of the two arrays was integer multiple. Even with a very thin dielectric support (0.037 mm) separating the two layers, the proposed theoretical model has very successfully reproduced the measured results with good correspondence between measurements and predictions. Based on this design, transmission and reflection responses of a variety of double-sided FSS structure with cross-dipoles and rings forming the array's elements were assessed.
REFERENCES


Chapter Five

CHAPTER 5

RADIATION PATTERN MEASUREMENTS IN REFLECTION
OF MULTILAYER FSS

5.1 Introduction

In this chapter the far-field amplitude radiation pattern response of double layer FSS structures with arbitrary lattice geometries of the arrays are examined both in copolar and crosspolar directions. Since predictions were not available experimental results only are discussed here, except for the values at the boresight position of crosspolar and copolar patterns. These boresight predictions were obtained by the plane wave theoretical model that has already been described in Chapter 2. The predicted boresight values will be compared with the measured results where possible.

The use of orthogonal polarisation to provide two communication channels for each frequency has led to interest in the polarisation purity of antenna patterns. It is therefore imperative that FSS used as a sub-reflector, exhibits low crosspolarisation particularly in the reflection band if they are to operate with two orthogonal polarisations as in the case of frequency re-use. Sufficient channel isolation is therefore provided. The measured results in this chapter are accordingly restricted to measurements in the reflection band only in the range 25 to 40 GHz, with the feed used at the transmitter operating at Q-band (i.e. 26-40 GHz) region. It is important to assess the crosspolarisation across the whole reflection frequency range since this may have profound effects on the frequency band centre ratios, which would be accepted. In Sections 5.2-5.3, experimental set up and measurement procedures for pattern measurements in the various scanning planes are discussed with a pictorial view of the chamber and the equipment set up in reflection.

It is interesting to examine the influence of the lattice periodicities of the arrays in the FSS structure and the inter-layer coupling distance (i.e. the separation distance between the two arrays), on the copolar and crosspolar patterns. The peak crosspolar levels in reflection (at boresight position of the patterns only) for double layer with arbitrary lattice geometries

172
was considered briefly in Section 3.3 of Chapter 3. In this chapter however, detailed analysis of the experimental results are carried out. The profile of the crosspolar patterns over a range of observation angles in various scanning planes (i.e. the zero degree, 45° and 90° planes) of the pattern are obtained. The measurements are performed both at boresight and off-boresight positions of the feed. In all the measurements discussed in this chapter the performance of the FSS antennas are assessed with reference to a metal mirror. In total, three different plane cuts of the patterns, namely the zero degree, 45° and the 90° scan planes are considered. The profile of the radiation patterns for two different types of double layer structure, namely the double layer of tripole / tripole with identical lattice periodicities of the arrays (ILFSS) and double layer arrays of tripole / tripole with dissimilar lattice periodicities (DLFSS) are examined.

In Section 5.5 the profiles of copolar and crosspolar patterns of the two FSS type at three different scan planes are examined with reference to the patterns of a perfect metal plate of the same size as the FSS's. The variation of peak crosspolar levels with the inter-layer coupling distance between the arrays, at the three scan planes are examined for some frequencies across the reflection band.

Section 5.6 gives a summary of the overall variation of the average peak crosspolar level of the two FSS types at various scan planes, for two states of polarised incident waves. It illustrates in tabular forms how differently the peak crosspolar levels of the two FSS behave across the reflection band, in particular at the two extreme edges of the band. Finally, in Section 5.7, a summary of the whole results is outlined.

5.2 Radiation Pattern Measurements in Reflection

The objectives of the measurements here are to observe how the copolar and crosspolar patterns of a corrugated conical horn are affected by various multilayer FSS configurations with arbitrary lattice geometries of the arrays, for various planes of the polarised incident field.

The method adopted here to measure the amplitude distribution of the field patterns of an
FSS antenna is to vary the angular position between the antenna being tested and a receiving probe located at a fixed distance. This is in effect equivalent to moving the receiving probe over an arc of an imaginary spherical surface centred on the antenna. The amplitude distribution patterns are therefore measured by a scanning field probe over a pre-selected surface of a sphere.

With the transmitting and the receiving feeds placed in reflection mode and the FSS arranged in a 45° incidence diplexer as shown in figure 5.1, the surface of the FSS are illuminated by linearly polarised EM waves. Note that in Figure 5.1, T represents the transmission mode of the feed with the feed at 45° to the plane of the array. R is the position of the feed in reflection mode. The patterns of the reflected scattered fields from the FSS are measured using the radiation pattern measurement set-up of Figure 5.2. Their amplitude radiation patterns over a set of observation angles are then plotted for the purpose of comparison with the predictions. The plane wave modal analysis technique that was used to produce the theoretical results, provides information about only the boresight crosspolar and co-polar levels. It does not however provide off-boresight information of the radiation pattern. In the measurement, for a particular plane of pattern scan, the copolar and crosspolar components of the reflected fields are measured for two polarisation states of the incident field, mainly TE and TM. For Comparison purposes, the patterns measured for a perfect metallic surface are also plotted. Comparison of the FSSs amplitude patterns obtained, with those of reference metal plate, enable their characteristic responses in reflections to be investigated.

5.3 Experimental Set-up for Radiation Pattern Measurements in Reflection

Figure 5.2 illustrates the equipment set up for reflection measurement in the chamber. The transmitting conical feed horn is held in position on a sliding (Darvic) plate by means of a number of supporting necks, with the (sliding) plate in turn resting on a movable and rotating resting table. Figure 5.3 (a) shows the close-up view of the transmitting feed assembly and the FSS support, and which also shows the FSS (covered with polystyrene sheet) in position in a 45 degree incidence diplexer arrangement. The entire feed assembly
Chapter Five

Figure 5.1: Geometry of the FSS Array in 45° incidence diplexer
(The geometry shows the position of the feed in transmission and reflection modes)
Figure 5.2: Equipment set up for amplitude radiation pattern measurement in reflection
and the FSS mounting is fixed on the resting table by means of a wooden plate. The FSS was supported in front of the feed horn by a circular polystyrene sheet held in position by two (Darvic) rods (shown in Figure 5.2 as FSS mounting support). These are in turn attached to the plate onto which the feed assembly is resting. Note that due to the manner, in which the Double layer FSS structures were prepared for measurement, polystyrene sheets were placed on either side of the FSS. In figure 5.3a therefore, the actual FSS array is not visible. The set up is such that by tilting (as shown in 5.3 (a)) the entire feed assembly together with the FSS attachments from 0° to 90°, enables measurement of the field patterns at scan planes between 0° and 90° to be performed. Figure 5.3 (b) shows the view from the receiver side where the whole assembly is tilted (in this case by 90 degrees). Intermediate planes of scan between the 0° and 90° plane is obtained by appropriate tilting of the entire assembly. The surrounding area of the feed assembly together with the movable table onto which the feed assembly is resting is being covered with Microwave absorber to reduce reflections and scattering. Although the support had little effect on copolar patterns the crosspolar patterns were found to be extremely sensitive to the type of supporting structure used. Hence Darvic materials of very low reflection coefficients were used for the feed and the FSS support mechanism.

5.4 Radiation Pattern Measurement Technique

To perform measurement in reflection, both the transmitting and receiving antenna were first placed in the reflection mode of the array as was shown in Figure 5.1. Note that whereas a pyramidal rectangular horn was used for the transmitting feed in the case of plane wave measurement, in the reflection measurement in here it is desirable to use a corrugated horn feed at the transmitter. This is to ensure a certain degree of rotational pattern symmetry and crosspolarisation suppression in reflection; a necessary pre-requisite if frequency re-use is required. This is also to ensure lower crosspolarisation from the FSSs under test. The conical feed horn had a crosspolar performance of better than -35 dB. In order to obtain accurate far-field measurements, the range i.e. the far field distance between the antenna under test and the receiving feed need to be sufficiently large. This range was taken to be $>> 2D/\lambda$, where $D$ is the largest dimension of the antenna, in our case $D$ was the aperture diameter of the feed used which was measured to be 8 cm, $\lambda$ is the
Figure 5.3: Experimental set-up in the Anechoic Chamber showing:

(a) the Feed assembly and the FSS mounting in the 45° incidence diplexer.

(b) the view of the set-up from the receiver side.
wavelength. Due to constraint of chamber a range of 1 m was chosen for the far field Distance.

With the corrugated conical feed operating between 26 and 40 GHz which is the reflection region of the tripole FSSs being measured, spot frequency pattern measurement, i.e. pattern measurement at a single frequency is carried out. For a fixed plane of scan, the polarised incident wave illuminates the surface of the FSS. Referring to figure 5.3 (b), the propagating scattered wave is received by a pyramidal rectangular horn (i.e. the receiving probe) as shown. The signal via the receiving probe is fed to the scalar network analyser by a microwave detector as was shown in Figure 5.2, where the patterns are displayed on the analyser screen. Due to the constraints of the anechoic chamber, a pyramidal rectangular horn feed has been used at the receiver, instead of a more desirable corrugated conical horn feed that was used at the transmitter. Nevertheless, for the purpose of the measurements in here this was found to be sufficient with negligible degradation in the pattern symmetry and crosspolarisation of the received signal pattern. Angle calibrations of the measured amplitude radiation patterns displayed on the network analyser screen were provided by a linear potentiometer. A fixed voltage from a stabilised power supply excites the input to the potentiometer. The output of the potentiometer is fed into the ADC input of the network analyser. For ease of complexities in the diagram, this potentiometer is not shown in figure 5.2. The potentiometer was designed to calibrate the movement of the scanning probe (i.e. the receiving probe) as it scans the radiated pattern. A horizontal beam (parallel to floor of the chamber) that supports the scanning probe shown in figure 5.3 (b) is attached at its pivoting point to the linear potentiometer. The movement of the scanning probe therefore prompts the changes on the potentiometer output voltage. As the pattern of the FSS is scanned by the probe, the output voltage of the potentiometer changes which when fed to ADC input of the network analyser is converted to a digital readout on the analyser screen. This change in the output voltage prompts movement of the cursor position on the network analyser screen to a new scan angle position. The amplitude of the pattern at each scan position is then displayed on the analyser screen as dot. As the pattern of the FSS is scanned from one side of the pattern boresight to a point on the other side, the cursor on the analyser screen moves from left to right or vice versa depending on the
direction of the movement of the scanning probe. After a complete movement of the
scanning probe the amplitude radiation pattern of the FSS distributed over a set of
observation angles is displayed on the analyser screen. Owing to the problems that were
encountered in the prescribed set up and possible reflections from the supporting structure,
measurements for radiation patterns were obtained for observation angle ranging from -50
to +50 degrees only. The measurement is repeated for frequencies throughout the reflection
band region of the FSS.

The above experimental set up was improved in the later stages of the measurement by
computer-controlled software written in Quick Basic language. An HP-IB interface card in
the PC shown in figure 5.2 allows computer control of the sweep oscillator (i.e. the power
source) so that at each scan angle position (i.e. at every movement of the scanning probe),
measured amplitude data are obtained for a range of reflection frequencies. This implies
that at every position of the scanning probe, the software programming of the HB-IB card
in the computer prompts the sweep oscillator to sweep through a set sequence of
frequencies within the reflection band of the FSS. The amplitude response value at every
frequency of the polarised incident wave is then fed to the analyser via the detector, as the
receiving probe scans the patterns. The computer is therefore programmed such that after a
complete cycle of the scanning probe movement from one side of the boresight position to
the other, the complete amplitude patterns at a number of reflection frequencies are
obtained.

The technique above enables more rapid measurements to be taken over a wider frequency
range at every observation angle when compared to spot frequency scanning technique that
was used prior to automation. In order to perform multiple frequency measurements, the
software programming of the HP-IB card requires angle calibration files to be obtained
prior to scanning of the amplitude pattern of a particular surface. Also required are the
power level calibration files for all the frequencies at which pattern measurements are
required. These power level calibrations are performed at the boresight position of the
feed. The calibrations were carried out only once without the FSS in position in the
diplexer. When the scanning of the FSS patterns commences, the software in the PC calls
the angle as well as the appropriate power level calibration files. It is to be noted that in the software programming of the HB-IB card, provisions are allowed for sufficient amount of delay between successive frequencies swept at each position of the scanning probe. The primary aim of this delay is to enable stabilisation of the cursor level on the network analyser screen to be obtained at a particular frequency before the next frequency is swept. This therefore allows sufficient time for the averaging of the cursor value to be achieved. This is particularly important when the pattern at noise level is scanned, i.e. at the extreme ends of the observation angle. The method hence ensures that far accurate and reliable measured radiation patterns are obtained. Notice also that the operator chooses the amount of delay between successive scanning probe positions. Furthermore, during the measurement it has been observed that a better and smooth amplitude pattern is obtained if the scanning is performed over more frequent observation angles. Hence on average, amplitude radiation patterns of the various FSS were scanned for every 2 to 5 degrees movement of the scanning probe.

The above-improved technique allowed amplitude radiation patterns for the whole frequency range in reflection to be scanned after one complete movement of the receiving probe.

The conical corrugated horn used as the transmitting feed has been designed to operate in the Q-band from 26 to 40 GHz. It was designed to exhibit low crosspolarisation with symmetrical feed pattern. However during the initial stages of the measurement it was found necessary to design a polarising grid [1,2] consisting of periodic strips printed on a dielectric substrate, which was placed at the aperture of the feed. The main objective was to further minimise the peak crosspolar levels of the corrugated horn feed in order that accurate and reliable crosspolar level measurements of the FSS are obtained in reflection. This polariser primarily suppresses (i.e. minimises) the field in the crosspolar direction. Note that the crosspolarised field is obtained by rotating the aperture of the receiving probe by 90°, i.e. the field orthogonal to that of the polarized incident field assuming linear polarization. The polariser having been attached to the feed aperture is then rotated so that a minimum crosspolar level at boresight is obtained. It is then kept at this position of
minimum crosspolar level throughout the frequency range at each state of the polarized
incident wave. In the copolar direction, this position of the polarizing grid would
correspond to minimum power loss at the boresight. It should however be noted that each
and every time the state of the polarized incident field is changed i.e. from TE to TM or
vice versa, the position of the polariser is readjusted accordingly for minimum crosspolar
level.

5.5 Measurement at Various Planes of Scan
The radiation pattern of an antenna is normally three-dimensional. However in practice it
is not feasible to measure a three-dimensional pattern. A number of two-dimensional
pattern cuts are therefore measured. They can be used to construct a three-dimensional
pattern. These two-dimensional pattern cuts are obtained by keeping one of the angles (θ
or φ) constant while varying the other, where (θ, φ) are the polar angles.

In here, radiation patterns were mainly measured in the 0 degree, 45 degrees and 90
degrees scan planes with some also in the intermediate planes in order to check for any
crosspolarisation peaks. As far as we can tell, peak crosspolarisation occurred in the 45°
plane. Radiation patterns were only measured to the third up to forth side-lobe angles. In
the measurement technique considered here, the various scan planes are obtained by
appropriate rotation of the (transmitting) feed and the tilting movement of feed assembly
together with the FSS attachment shown in Figures 5.3. This will be explained further later
in the chapter. At each plane of scan, a complete movement of the scanning probe from
one extreme end to another gives the radiation pattern distributed over a set of observation
angles. It should be noted that for every scan plane, the centre of the receiving (or
scanning) probe is aligned with the centre of the FSS array as is the centre of the
transmitting feed aperture with that of the FSS array. The distance between the centre of
the receiving probe and FSS array was measured to be about 1 m within the constraints of
the anechoic chamber.

When measuring the radiation pattern of an antenna, the location of the phase centre [3] of
the transmitting feed is important particularly for phase measurements. When used as a
feed for reflector antennas in conjunction with the FSS it is essential that its phase centre is known and it is located at the focal point of the reflector [4]. For the purpose of the measurements in here determining the location of the phase centre is not highly critical since only amplitude measurements are being considered. Nevertheless, it is appropriate to define this parameter. The phase centre is essentially a point along the horn where the fields radiated by the antenna are spherical waves with identical wavefronts. This phase centre of a horn is not usually located at its throat (i.e. the junction of circular waveguide and corrugated waveguide) or at its aperture but in between the two. The exact location of the phase centre of a horn very much depends on the dimensions of the horn especially on its flare angle. As the flare angle is increased, the phase centre moves inwards from the aperture towards the throat of the horn. However, it quite never reaches this point, and eventually its direction of motion is reversed. As the flare angle of the horn becomes smaller, the phase centre moves towards the aperture of the horn. Experimental technique [5,6] are available for locating the phase centre of an antenna. However, for amplitude pattern measurements in here, we have assumed the location of phase centre to be at feed aperture. Also a Q-band feed operating between 26-40 GHz and of 8 cm aperture diameter is being used for the transmitting feed as opposed to much larger aperture of J-band or the K-band feed. For a particular plane of scan, the radiation patterns (both in copolar and crosspolar directions) are scanned about the phase centre of the feed.

5.5.1 H-plane Incidence Measurements

5.5.1.1 Horizontal (i.e. Zero Degree) Scan Plane Measurements

The conical transmitting feed together with the FSS assembly is positioned horizontally on the test table. The set up is arranged in the reflection mode, with the FSS to be measured placed on the 45° incidence diplexer (i.e. at the boresight direction of the feed the local angle of incidence on the FSS array was 45°). The amplitudes of the pattern on both sides of the boresight position is then scanned by the receiving probe.

Two types of FSS structure are considered, namely double layer FSS of tripoles with both arrays having identical periodicities (ILFSS) previously called (DLTR2) in Chapter 3 and double-layer FSS of tripoles with dissimilar lattice geometries of the two arrays (DLFSS),
i.e. DLTR1 of Chapter 3. The plane wave transmission/reflection responses and the crosspolar patterns at boresight of both of these FSS were studied in Chapter 3. Hence the dimensions (i.e. the element length and width and the lattice periodicities) of ILFSS and DLFSS in here are exactly the same as those given for DLTR2 and DLTR1 respectively in Chapter 3. For each FSS type, three different surfaces with separation distances of 2, 4 and 6 mm were constructed and measured.

The metal plate used as reference was of the same size as the FSS, measuring 20 x 20 cm square. With the metal plate placed in the diplexer, the pattern is first normalised at the boresight position on the network analyser. Note that the normalisation procedure is only carried out once when the metal plate is being measured and therefore care must be taken not to offset the alignment when the FSSs are being measured. One source of error that can result, especially for crosspolar measurement, is the incorrect positioning of the polarising grid at the feed aperture. It should be ensured that the grid is positioned permanently for minimum peak crosspolar level at boresight prior to normalisation of the reference plate pattern, and this position is kept fixed for a particular plane of incidence. Radiation patterns of the reference plate both in the copolar and crosspolar directions are first measured by scanning the field pattern on both sides of the boresight position. Having measured the patterns of the reference plate across the whole frequency range in the reflection band, the FSS are then placed in the diplexer and measurements of their patterns both in crosspolar and copolar directions are performed.

Figure 5.4 shows the representative normalised H-plane copolar and crosspolar patterns at the lower frequency edge of the reflection band. The figures show the patterns for both types of FSS with interlayer separation distance of 2 mm together with the patterns of the reference metal plate for comparison purposes. The incident electric field is taken to be polarised along the y-axis parallel to one arm of the tripole array (i.e. TEa incidence). For both FSS at 2 mm, the frequency of 29 GHz represents the band centre frequency. 25 GHz represents the frequency at the lower edge of the reflection band. It is seen that at 25 GHz (shown in Figure 5.9a) the energy is mainly concentrated in the main lobe with symmetrical pattern with respect to the boresight position, with a loss of 1 dB at boresight
occurring for both FSS types. It can also be observed that the profiles of the copolar patterns for the two FSS moderately agree with that of the reference metal plate across the reflection band with the exception of deeper nulls and slightly narrower main lobes.

The measured crosspolar levels at the boresight positions are at maximum peak at the lower edge of the reflection band, for both types of FSS as can be observed from Figure 5.4a. Measurement results have indicated that these peaks decrease as the frequency departs from the lower edge of the reflection band and stabilises between the band centre frequency and the frequency at the upper edge of the band. As expected the measured crosspolar level for DLFSS structures are quite higher at the boresight position as compared to the levels obtained for ILFSS. In the prediction model a peak crosspolar level at boresight of about -15 dB was predicted. The measured value as seen from Figure 5.4a is about -16 dB. This peak boresight value decreases to -24 dB at the reflection band centre frequency of 29 GHz, compared to -23 dB obtained by the plane wave model.

The peak crosspolar levels occur at or near boresight position of the feed within the main lobe of the copolar radiation pattern where the de-polarisation is maximum at the centre of the array. The array's lattice geometries clearly have major influence on the crosspolar performances of the FSS. Figure 5.5 shows the representative normalised H-plane copolar and crosspolar patterns at the reflection frequencies of 25 and 31 GHz for both FSS at separation distance of 4 mm. The peak crosspolar levels for both FSS at this separation distance occur at the boresight position. The peak at boresight gradually increases as the frequency deviates from the lower edge of the reflection band. Measurement recorded a peak value of -26 dB for the ILFSS at 33 GHz, although not shown here. The corresponding peak for the DLFSS occurred towards the upper edge of the reflection band with a value of -13 dB at the frequency of 33 GHz.

Similar measurements were obtained for TE incidence in orientation b. As with the case of TEa incidence, the plane wave theoretical model successfully reproduced the measured peak level at the boresight position only, for both DLFSS and ILFSS at frequencies across
Chapter Five

Figure 5.4.a: H-plane Co-polar and X-polar Radiation Patterns in reflection of DLFSS and ILFSS at the lower edge of the reflection band.

Figure 5.4.b: H-plane Co-polar and X-polar Radiation Patterns in reflection of DLFSS and ILFSS near the band centre frequency.

186
the reflection band. The peak crosspolar levels at boresight for both ILFSS and DLFSS appeared at the edges of the reflection band for 2 mm and 4 mm separation distances. For surfaces with 6 mm separation distance, the peak crosspolar level occurs near the band centre frequency.

The effects of the separation distance between the arrays on the peak crosspolar level in reflection at boresight are summarised in Figure 5.6 for DLFSS and ILFSS FSS at (TEa + TMb) incidence. Figures 5.7 shows the sensitivity (i.e. the variation) of peak boresight crosspolar levels in reflection with the separation distance between the two arrays. As can be seen from Figure 5.7, for DLFSS the crosspolar peak appears to be high at the two extreme ends of the separation distance \( d \) and slightly lower peaks are obtained in between the two separation distance at \( d = 4 \) mm. On the other hand, for structures with identical lattice periodicities of the arrays the peak boresight crosspolar level is higher for small separation distance. The computer model has reproduced the measured values at the boresight position. The profiles of the measured peak crosspolar levels at boresight for both FSS types generally follow those of the predicted results obtained by the plane wave model.

5.5.1.2 45° Scan Plane Measurements

This plane is obtained by first rotating the transmitting feed by 45° such that the polarised electric field is in the direction of 45° to the reference polarised field of the horizontal plane. The receiving probe is also rotated by 45° (with respect to the reference Polarised field). Next the feed assembly together with the FSS attachment is tilted by 45° with respect to the horizontal position such that the \( y \)-axis of the feed makes an angle of 45° with the \( y \)-axis of the receiving probe. Since as a result of this tilting movement of the feed assembly the centre of the FSS array is being tilted by 45°, the height of the receiving probe needs to be altered accordingly. This is to ensure that the centre of the receiving probe is in line of sight with the centre of the array. The normalisation procedure is first performed with the metal plate positioned at the diplexer. The polarising grid at the feed aperture is repositioned for minimum crosspolar level at boresight. Having scanned the field pattern of the reference plate in both copolar and crosspolar directions, FSSs are then
Figure 5.5.a: H-plane Co-polar and X-polar Radiation Patterns in reflection of DLFSS and ILFSS at the band centre frequency of 28 GHz.

Figure 5.5.b: H-plane Co-polar and X-polar Radiation Patterns in reflection of DLFSS and ILFSS at the upper edge of the reflection band.
Figure 5.6: The Variation of the Boresight Crosspolar component of the reflected field
(a): DLFSS at (TEa + TMb) incidence.
(b): ILFSS at (TEa + TMb) incidence.
Figure 5.7: Sensitivity of peak crosspolar levels with the distance separating the two layers, at the reflection band centre frequency for:

(a) Double layer FSS (DLFSS)

(b) Double layer FSS (ILFSS)
placed on the diplexer and their patterns measured. The H-plane copolar patterns at the 45° plane show similar trends to patterns obtained at the zero degree plane of scan. In Figures 5.8 the normalised H-plane copolar and cross-polar patterns for both FSS types at the frequency of 30 GHz (near the reflection band centre) can be observed. The variations of both the copolar and the cross-polar patterns with the change in the separation distance between the two layers can be observed.

The obvious features of the patterns in this scan plane are that at the boresight position for both FSS types, the crosspolar and the copolar levels are exactly the same as the levels in the zero degree plane. This is as expected since at the boresight position all the scan plane pass through the same point. The copolar patterns for both FSS types show that the loss at the boresight for all the FSSs are the same as those measured in the zero degree scan plane. Measured results have shown a loss at the boresight of -6 dB for the DLFSS with separation distance of 6 mm, as shown in figure 5.8. This loss was also obtained (although not shown in here) for the same FSS at the zero degree scan plane.

The measured crosspolar patterns for this plane show that the peak crosspolar levels for all of the surfaces occur at off-boresight positions. Indeed this is the main feature of the 45(scan plane as it is a plane in between the two principal scan planes. The level of the crosspolarisation for any FSS in this plane is a measure of how efficiently the surface behaves specially when used in dual polarisation applications where minimum crosspolar levels are required.

Figure 5.8 shows that for both FSS types at a frequency near the reflection band centre, the boresight crosspolar peaks are virtually unaffected by the change in separation distance. These were well predicted by the plane-wave theoretical model. Measured results have also shown that for both DLFSS and ILFSS at a smaller separation distance, the peak crosspolar level at boresight occurs at the lower edge of the reflection band and stabilises across the reflection band. On the other hand, For both FSS with bigger separation distance of 6 mm, the peaks at boresight occur at a frequency near the upper edge of the reflection band at 29.5 GHz. Measurements have recorded a peak crosspolar level of -10 dB for the
Chapter Five

45 degree plane of scan, at TEa incidence

**DLFSS**

30 GHz

**Figure 5.8.a:** 45 degree scan plane Co-polar and X-polar Radiation patterns of DLFSS at 30 GHz.

45 degree plane of scan, at TEa incidence

**ILFSS**

30 GHz

**Figure 5.8.b:** 45 degree scan plane Co-polar and X-polar Radiation patterns of ILFSS at 30 GHz.
Chapter Five

DLFSS. The corresponding peak value for the ILFSS at the same frequency was measured to be -28 dB. Similar results have been obtained at other incidence planes.

5.5.1.3 Vertical (90°) Scan Plane Measurements

To perform measurements at this plane, the transmitting feed is first rotated by 90° so that the polarised field obtained is orthogonal to the field polarised at the zero degree plane. The whole feed assembly together with the FSS attachments is then tilted by 90° from the horizontal position such that the y-axis of the transmitting feed is orthogonal to the y-axis of the receiving probe. The height of the receiving probe is lowered accordingly so that its centre of aperture is in line with the centre of the FSS array. The normalisation procedure is carried out with the perfect reflecting metal plate.

The results obtained show similar trends to the results from the horizontal scan plane already discussed. An important point to notice here is that the field incident at TE in orientation $a$ would be at TM state of incidence in orientation $b$ at the zero degree scan plane. Hence for comparison purposes, it would be appropriate to discuss the results for TMb incidence at the 90° plane as compared to those of TEa incidence already obtained for the zero degree scan plane.

Representative E-plane (TMb incidence ) normalised copolar and crosspolar patterns for both DLFSS and ILFSS in their reflecting modes in the 45° TMb incidence diplexer are plotted in Figure 5.9 at the reflection frequency of 25 GHz. Also shown are the patterns at frequency near the reflection band centre (Figure 5.10). The variation in the profile of copolar and crosspolar patterns with the separation distance is clearly observed. It is seen from Figure 5.9 that the profiles of the copolar patterns for DLFSS at the separation distances of 2 and 4 mm are similar to those of zero degree scan plane for TEa incidence (shown in Figures 5.4 and 5.5). In particular the amount of loss in the copolar patterns for various DLFSS surfaces at the boresight level are the same as those depicted in the zero degree. The same can be argued about the various ILFSS surfaces, whose copolar patterns at the 90° planes shown in Figure 5.9b and 5.10b are similar to their pattern profiles at 45° and the zero degree plane.
Chapter Five

The off-boresight peak crosspolar levels for all the various surfaces at 25 GHz as shown in Figure 5.9 also seem to follow those in the zero and 45° scan plane. In particular when compared to those at the zero degree plane, the two-lobal pattern for the crosspolar at separation distance where maximum crosspolar level occurs are noticed.

Similar results have been obtained for copolar and crosspolar patterns of the various surfaces in their reflecting modes in a 45° TMa incidence diplexer. These were similar to results obtained for the TEb incidence of the zero degree scan plane. Generally the crosspolar levels follow the profiles of the patterns for TEb at the zero degree plane. However the one exception is that the peak level at boresight for the DLFSS with 6 mm separation distance was found to be higher for TMa at the 90° plane than it is for TEb at the zero degree plane. The average peak of the crosspolar level at off- or near-boresight for the DLFSS at all separation distances were found to be slightly higher (by about 3 dB) for the TMa at the 90° plane than they were for TEb incidence in the zero degree scan plane.

5.5.2 E-Plane Incidence Measurement

The E-plane incidence radiation patterns of the two types of FSS at various coupling distances in both copolar and crosspolar directions are briefly outlined in this section. For each FSS type, the copolar and crosspolar patterns at all scan planes are assessed. In Figure 5.11, the representative E-plane copolar and crosspolar patterns for the DLFSS surface in its reflecting mode in a 45° TMb incidence diplexer is plotted for the zero degree, 45° and 90° planes at the reflection frequency of 33 GHz. It can be seen that the profile of the copolar patterns for a particular separation distance are similar at all three scan planes, except for the fact the main lobe of the normalised copolar patterns at the 45° scan plane averages between the horizontal and the 90° scan planes. This is expected since the 45° scan plane is diagonally divided between the other two planes. As expected, the copolar and crosspolar patterns for all three scan planes have the same values at the boresight position. This can be observed from Figure 5.11.

The off-boresight peak crosspolar levels for all the FSS boards, are slightly higher in the 45° scan plane than they are in the zero and 90° scan planes. The DLFSS with coupling
Chapter Five

Figure 5.9a: E-plane Normalised Co-polar and X-polar Radiation Patterns of DLFSS of various copl.distances at 25 GHz.

Figure 5.9b: E-plane Normalised Co-polar and X-polar Radiation Patterns of ILFSS of various copl.distances at 25 GHz.
Figure 5.10a: E-plane Normalised Co-polar and X-polar Radiation Patterns of DLFSS at various copl.distances at 29.5 GHz.

Figure 5.10b: E-plane Normalised Co-polar and X-polar Radiation Patterns of ILFSS at various copl.distances at 29.5 GHz.
distance of 4 mm exhibits the highest peak crosspolar level at boresight at this reflection frequency of 33 GHz. The same surface also gave a peak value for the H-plane incidence although not shown in here. Measurement in the 45° incidence diplexer has recorded an average off-boresight peak crosspolar level of -15 dB at the 45° plane.

The patterns for the ILFSS surfaces are plotted in Figure 5.12 for all three scan planes, for the frequency at the edge of the reflection band (i.e. 25 GHz). It is seen that for this FSS type the surface with coupling distance of 2 mm exhibits the highest peak at off-boresight position. Again the peak crosspolar level occurs at the 45° plane of scan. The peak boresight crosspolar level at all scan planes was measured to be about 25 dB below the copolar level at the centre of the main beam. However an off-boresight peak crosspolar value of -21 dB at the azimuth scan angle of -11° has been recorded for the 45° scan plane. The corresponding off-boresight value for the zero degree plane was measured as -23 dB at a scan angle of +10° and -24.5 dB was measured for the off-boresight peak at the 90° scan plane.

Measurements of copolar and crosspolar patterns for the two types of FSS were also carried out although not shown in here, at other frequencies throughout the reflection bands. The results that were obtained have shown that at some frequencies the reflected copolar beams were narrower. Although the losses at the beam centre of the copolar patterns were insignificant compared to the patterns of the reference plate, these were however far greater towards the edges of the FSS (i.e. at scan angle of 20° or over)

5.5.3 Variation of Average Peak Crosspolar Levels of the Two Types of FSS at Various Planes of Scan for Different States of the Incident Polarised Wave

The variation of average peak crosspolar levels, for the two types of the structures at three different planes of scan are summarised in here. These are summarised in tabular form in Table 5.1 and 5.2 for the DLFSS and ILFSS respectively. The values given in the tables are measured results only. Note also that these peak crosspolar levels for each surface are the average crosspolar values for every whole pattern and not the levels at the boresight position of the pattern.
Chapter Five

0 degree plane of scan, at TMb incidence

Figure 5.11 (a)

45 degree plane of scan, at TMb incidence

Figure 5.11 (b)
Figure 5.11(c)

Figure 5.11: Normalised E-plane (TMb incidence) Radiation pattern of DLFSS at various scan planes, in the co-polar and cross-polar directions at the reflection frequency of 33 GHz.

(a) 0 degree Plane of Scan.
(b) 45 degrees Plane of Scan.
(c) 90 degrees Plane of Scan.
Chapter Five

Figure 5.12(a)

zero degree plane of scan, at TMb incidence

Relative power, dB

Scan angle, degrees

25 GHz

ILFSS

Figure 5.12(b)

45 degree plane of scan, at TMb incidence

Relative power, dB

Scan angle, degrees

25 GHz

ILFSS
Figure 5.12: Normalised E-plane (TMb incidence) Radiation of ILFSS at various scan planes, in the co-polar and cross-polar directions at the reflection frequency of 25 GHz.

(a) 0 degree Plane of Scan.
(b) 45 degrees Plane of Scan.
(c) 90 degrees Plane of Scan.
Table 5.1: Variation of Peak Boresight Crosspolar level across the reflection band for DLFSS at various scan planes, for:

(a) TEa incidence

(b) TMb incidence
### Table 5.2: Variation of Peak Boresight Crosspolar level across the reflection band for ILFSS at various scan planes, for:

(a) TEa incidence

(b) TMb incidence

<table>
<thead>
<tr>
<th>Reflection Frequency (GHz)</th>
<th>Coupling distance (mm)</th>
<th>Plane of scan (deg)</th>
<th>Peak Crosspolar Level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0°</td>
<td>45°</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>-24</td>
<td>-21.5</td>
</tr>
<tr>
<td>28</td>
<td>2</td>
<td>-30</td>
<td>-29</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>-29.8</td>
<td>-31</td>
</tr>
<tr>
<td>31</td>
<td>2</td>
<td>-29.5</td>
<td>-27</td>
</tr>
<tr>
<td>33</td>
<td>2</td>
<td>-32</td>
<td>-28</td>
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<table>
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<tr>
<th>Reflection Frequency (GHz)</th>
<th>Coupling distance (mm)</th>
<th>Plane of scan (deg)</th>
<th>Peak Crosspolar Level (dB)</th>
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<tr>
<td>33</td>
<td>4</td>
<td>-34</td>
<td>-31</td>
</tr>
</tbody>
</table>

(a)

(b)
Chapter Five

The results in Tables 5.1a and 5.1b show that the average crosspolar peaks for DLFSS at 6 mm separation distances between the arrays, occurs at a frequency near the reflection band centre. For the DLFSS with 2 mm separation distance, a peak average crosspolar level of -13 dB (at the 45° scan plane) is observed at the lower edge of the reflection band, whilst for the surface with 4 mm separation distance a value of -13.5 dB (at 45° scan plane) is recorded at the upper edge of the band. The fact that the highest peak occurs at the 45° plane for all surfaces is clearly noticeable from the table.

The results in Tables 5.2a and 5.2b illustrate that the highest average peak crosspolar levels for the ILFSS surfaces occur at the two edges of the reflection band, when a distance of 2 mm separates the two arrays. For the surface with 6 mm coupling distance between the two arrays, the crosspolar levels follow the usual two-lobal pattern, with the peak occurring near the band centre frequency at 30 GHz.

The tables suggest that in general the peak crosspolar patterns of the two types of FSS at all scan planes behave in a similar fashion. However the peak crosspolar levels for the ILFSS surfaces are far lower than those of DLFSS are. The fact that these peak crosspolar levels are far higher than the acceptable levels of -40 dB to -50 dB noise level, implies that for applications where frequency re-use is required, these surfaces will not be suitable. Alternatively some method of crosspolar control could be used which may necessitate for example modifying the shapes of the FSS. This is beyond the scope of this thesis.

5.6 Summary
The copolar and crosspolar radiation pattern measurements in reflection, of two types of double layer structures of tripole elements have been investigated. The computer model was used to reproduce the boresight values of the pattern for the various FSS structures. The broad agreement between the measured and predicted copolar and peak crosspolar levels at boresight were highly noticeable from the results. Results have shown that for these structure types, both experimentally and theoretically there were no significant differences between the peak crosspolar levels at the TE and TM states of incidences. This is an important factor for frequency re-use, since a surface in orientation a, for one plane of
polarisation is in orientation $b$, for the orthogonal polarisation.

The peak crosspolar levels for ILFSS were relatively higher than those of the feed horn alone (with the reference metal plate). Measurements have shown that the peak crosspolar levels are largely dependent upon the interlayer coupling distance between the two arrays. For the DLFSS, surfaces with larger coupling distance (i.e. 6 mm) tend to exhibit higher peak crosspolar levels within the reflection band. Measurements in the 45° incidence diplexer have recorded peak levels of about -10 dB at the 45 degrees plane, for this FSS. In contrast, for the ILFSS surfaces, highest peak crosspolar levels are introduced when the two arrays in the structure are close-coupled (i.e. 2 mm separation between the arrays). This peak occurs at the lower edge of the reflection band. The crosspolar levels for the ILFSS with 2 mm coupling distance were found to be typically 21 dB below the copolar level at the centre of the main beam. These would not be low enough for some antenna applications, particularly in cases where frequency re-use is applied. Measured results have also shown that near the band centre frequencies (i.e. 27-28.5 GHz), the crosspolar level from both FSS types are relatively independent of the coupling distance between the arrays, although the levels are slightly sensitive to the polarisation state of the incident field.

Although FSS with dissimilar lattice geometries offer certain advantages in terms of bandwidths and band centre frequency ratios, their poor crosspolar levels at boresight are of obvious drawback for applications such as frequency re-use. FSSs with similar lattice geometries of the arrays, on the other hand, offer reasonable bandwidth performance with moderate crosspolar levels at the centre of the main beam however not very satisfactory.
REFERENCES


CHAPTER 6.0

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

The quest for developing a novel analytical technique to analyse multilayer FSS structures consisting of arbitrary array lattice geometries stems from the increase in complexity of the specifications for the frequency response characteristic of these surfaces. The study in this thesis has shown that the flexibility that multilayer FSS provide regarding adjustment of transmission / reflection characteristic can be exploited by including arrays that have arbitrary element lattice geometries, without neglecting the effects of the separation distance between the two arrays. Towards this aim, the study has concentrated on extending a novel theoretical technique of analysing such FSS structures. This has been achieved by developing computer models to predict the scattered coefficients from these structures. Experimental studies have been carried out on the effects of inter-layer separation distance and the lattice geometry to the bandwidth and the crosspolarisation performances of such structures.

An efficient technique of coupled integral equations formulation in the spectral domain has been employed for analysing the scattering from multilayer FSS with dissimilar lattice geometry of the two arrays. The tangential field vectors describing the two arrays were super-resolved by redefining their Floquet modes according to convolution theorem. These were achieved by assigning the fields in the adjacent arrays a mutual periodicity. Therefore central to the analysis technique was the concept of common (or mutual) periodicity between the lattice periods of the two arrays, with the major assumption that ratio of the lattice periodicity’s of the two arrays are rational. The strength of the proposed approach in its simplistic form was to unlock the complexities in expressing the modal coefficients in terms of the Fourier transform of the surface currents. The spectral components of the tangential fields adjacent to the two arrays were related to those of the mutual periodicity array. These were conveniently related by a set of correlation functions that were found to be reduced to a delta function due to orthogonality of the Floquet mode sets. As a result, the tangential field vectors
describing the two arrays were super-resolved with a set of delta functions that had identical lattice periodicity to that of the mutual periodicity field. This meant that the spectrum of the field expanded in terms of the Floquet modes of common periodicity array contain a large number of redundant terms with very negligible values due to the properties of the Kronecker delta. The non-zero information carrying components in the spectrum of the mutual periodicity field corresponds to those of two individual fields. Therefore the importance of the novel technique described was twofold. A super-resolution of the spectral components of the tangential fields was accomplished by assigning the fields a mutual periodicity with the assumption that the ratio of the two periodicities is rational. Second, the elimination of the superfluous non-information carrying components were achieved by the use of a set of correlation functions that relate the Floquet mode sets of the common periodicity array to those of the two individual arrays.

A computer model based on the super-resolution approach has been developed for predicting the plane wave transmission responses of double layer FSS structures with arbitrary lattice geometries of the two arrays. Despite the fact that the scattered field expansions were carried out in terms of the Floquet modes of the mutual periodicity, nevertheless considerable optimisation of the modes were obtained in our computer model. These have been achieved by avoiding the calculations of the large number of spectral components with a substantial proportion of zeros. The way, in which the convolution was applied in conjunction with the boundary conditions, these non-information carrying components were bypassed thereby greatly reducing the number of Floquet mode coefficients being computed. The process substantially reduced the amount of computing time that would otherwise be needed had the computation been carried out in terms of the full spectrum of the mutual periodicity array. An optimised version of the above computer model was therefore developed from which the measured plane wave response of a number of double layer structures were examined.

Validity of the theoretical mode based on the super-resolution technique has been assessed, by examining the plane wave transmission response of a number of double
layer FSS structures with dissimilar lattice geometries and with varying inter-layer separation distances. The plane wave responses of these surfaces were extensively measured in the chamber. Generally, the model was able to reproduce the measured results with the exception of some discrepancies at higher frequencies for TE incidence. Measured plane wave responses of double layer FSS of tripoles with various separation distances between the two arrays have shown that both the bandwidth and band centre frequency ratio are sensitive to changes in the separation distance. In general the percentage of the bandwidth is reduced as the separation distance is increased, for some incidences at oblique angle. The band centre frequency ratio on the other hand, improved with the increase in the separation distance depending on the lattice geometries of the two arrays. In another structure studied, conducting element of cross-dipoles were printed on either side of a single substrate of small thickness, thus forming very close-coupled arrays. For this structure the prediction model was able to successfully reproduce the measured results, particularly with regards to the locations of reflection nulls and the pass band in between the nulls. The computer model has verified the wide bandwidth response, together with good stability of the reflection band centre to changes in angle of incidence for this close-coupled structure. Also well predicted was the stability of reflection band centre frequency with the changes in the polarised state of incident wave, with excellent correspondence with the measured results.

Investigations on the crosspolarisation performances of double layer FSS with dissimilar lattice geometries revealed that the crosspolar levels were predominantly effected by the distance separating the two arrays and by the lattice geometries of the two arrays. The computer model has predicted that the FSS with dissimilar lattice geometries exhibited higher boresight crosspolar level in reflection as compared to the surfaces with identical lattice geometries of the arrays. For double layer FSS of tripoles with dissimilar lattice geometries of the two arrays the crosspolar levels at boresight, peak near the band centre frequency. Depending on the Polarisation State of the incident wave the computer model predicted the peak crosspolar levels at boresight to be high at two extreme ends of the separation distance. Lower peak boresight levels
were obtained at the separation distance in between the two extremes. The agreement between the measured values and the corresponding peak crosspolar values at pattern boresight predicted by the plane wave model were found to be excellent. The measured values were however higher by about 1.5 dB near the reflection band centre frequency.

The plane wave transmission responses of double layer FSSs with mixed array elements and with arbitrary lattice geometries have also been extensively studied in order to further examine the validity of the theoretical model. Plane wave responses of double layer structures consisting of tripole / rings elements and cross-dipole / rings elements with arbitrary lattice geometries of the two arrays and for various separation distances between the arrays have been measured. Generally, the agreement between the measured and the predicted results were found to be good. For extremely small separation distance between the two arrays, the prediction model has successfully reproduced the measured values. The locations of upper and lower reflection band centre and the pass bands in between were well predicted. The effects of separation distance on the plane wave responses of double layer FSS of tripole / rings with arbitrary element geometries were investigated. The results that were obtained have shown that for both TE and TM incidence, the locations of resonances are virtually unaffected as the separation is increased. The theoretical model has also predicted the loss within the pass band and the upper reflection band regions. These losses were due to the mismatch between the two dissimilar lattice arrays.

The plane wave transmission / reflection responses of double-sided FSS structures of mixed element arrays were considered in which the two array elements were printed on either side of a single substrate. Plane wave responses of both tripole / ring and cross-dipole / ring forming the arrays of the double-sided structure were examined. The results that were obtained illustrated that the pass bands in between the two reflection resonances were optimally improved by using dielectric substrates of small thickness (0.037 mm thickness in our studies), and by careful arrangement of array element geometries (i.e. the lattice periodicities and the separation distance). Results obtained have shown that for close-coupled arrays separated by an very thin dielectric substrate,
discrepancies existed between the measured and the predicted values at the locations of resonances. These were thought to be due to the presence of high couplings that were introduced between the elements of two arrays. As a result, higher order propagating and evanescent modes interaction from the two arrays become significant. In our theoretical model these effects were not taken into account when computing the reflection and transmission coefficients. In contrast when the arrays of tripole and ring elements were printed on either side of a single RT-DURIOD substrate of larger thickness (165 µm in this case), the computer model based on super-resolution technique however reproduced the measured plane wave response of the structure. The exceptions have been slight discrepancies at frequencies beyond 30 GHz for both planes of incidences. Further investigations into plane wave responses of double-sided structure with thinner dielectric backing (of thickness 0.037mm) have shown more improved pass bands together with closer band centre frequency ratio between the reflection and transmission bands as compared to those obtained from double layer structures with wider separation distances. These were due to lower dielectric losses that may be introduced. Discrepancies of up to 1.5 GHz between the predicted and measured responses at the location of the lower reflection band for these structures illustrated the deficiency of the theoretical mode. The root of these discrepancies was assumed to be due to the close proximity of the elements from the two arrays as they both shared an extremely thin substrate. This meant that in the theoretical model, the entire domain basis function representations of the currents induced on the surfaces of the two arrays would be inaccurate. It was therefore suggested that in our theoretical model, sub-domain basis function representations of the currents would have been more accurate as the conductor shape in this case would be defined through the locations of sub-sectional current elements. An investigation into this aspect would obviously be helpful as part of further studies. As part of future studies it is further suggested that close examinations into the behaviour of the higher order propagating and evanescent mode interactions from the two close-coupled arrays would be of an interest. The effects of these scattered
Chapter Six

Floquet modes coefficients were not taken into account in our prediction model for computations of transmission and reflection coefficients.

Results from double-sided FSS structure with integer multiple ratio of the array periodicities demonstrated some useful responses. Despite the fact that two arrays shared a thin dielectric substrate of (thickness 0.037 mm) thereby forming extremely close-coupled arrays structure, there were excellent correspondence between the measured and the predicted values. The theoretical model had very successfully reproduced the measured plane wave responses, with the locations of both resonances as well as the pass band well predicted. Further investigation into design of double-sided surfaces based on these array geometries has demonstrated their wide band characteristics. These were made possible through the careful selection of the two lattice periodicities such that the grating lobe of the first array was outside the pass band region (i.e. beyond the higher reflection band centre frequency of the response). The predicted results very closely followed the measured values.

Measurement of the radiation patterns in reflection, of various FSS structures with arbitrary lattice geometries were assessed both in copolar and crosspolar directions. The theoretical models based on the plane wave modal analysis technique gave the copolar and crosspolar values of these surfaces at boresight position only. The models in general reproduced the measured copolar and peak crosspolar levels at boresight position for the two types of the FSS structures, with good correspondence between the measured and predicted values. Measurements of the patterns at various planes of scanning have shown that the peak crosspolar levels exhibited by the FSS's with arbitrary lattice geometries were much higher than those for FSS with identical lattices.

The theoretical models had predicted that these peak boresight crosspolar levels were influenced by the array geometries and changes in the separation distance between the two arrays. Parameter variations in our studies however indicated that the crosspolar performance is primarily determined by the element geometry and the incidence angles. Although small improvement can be made by varying the thickness of the separation
distance between the two arrays and the dielectric constant, these are usually dominated by the inter-grating crosspolar contributions. The reasons for the variation in crosspolar level as a function of array parameters are not very obvious. However lower levels of crosspolarisation are obtained for structures with higher insensitivity with the angle of incidence. This is due to the fact that the reflection band centre of the responses would be more stable to the angle of incidence. As a result the TE and TM responses are more coincident with smaller phase differences between the two responses.

Finally a full theoretical modelling of radiation patterns in reflection of double layer FSS with arbitrary lattice geometries could form part of the future study in assessing the predicted off-boresight variations in the crosspolar and copolar patterns as well as the boresight level variations. This is however beyond the scope of the present studies.
APPENDIX A

MODAL ANALYSIS OF SCATTERING FROM SINGLE LAYER FSS

A.1 Formulation of the FSS Scattering Problem

The scattering of electromagnetic waves by periodic structures is a classical problem which has received considerable attention. Both numerical and asymptotic methods have been devised to study this important and basic problem. All of these techniques however hinge upon the Floquet theory [1] which allows the decomposition of an infinite domain into fundamental cells.

The first step in formulating the problem of electromagnetic scattering from a frequency selective surface is to relate the fields scattered from the FSS to the surface currents, induced on the screen by the incident field. Throughout this thesis we will assume that the FSS is infinitesimally thin, an assumption which is usually valid for most applications. Here we consider the case of single conductive screens printed on a dielectric substrate. The formulation forms the basis needed for analysing multilayer FSS structures. To properly pose the problem of scattering from a periodic surface, the excitation field must be a function with constant amplitude and linear phase i.e. a plane wave. Therefore the field considered is a time harmonics plane wave and can be derived from the potential induced on the surface [2]. The incident field derived will be divided into components transverse electric (TE) and transverse magnetic (TM) to \( z \) as outlined in the next section.

There are various techniques available for analysing FSS structures. They all however arrive at the same electric field integral equation. The vector potential approach [3], the plane wave expansion technique [4] and the network circuit representation of the transverse electromagnetic fields [5] are some of the alternative methods to the one adapted here. Some of these may however present practical difficulties if the interface of
the dielectric substrate is taken into account. The modal analysis technique is applied here to the FSS problem where the induced current in the array is found using Floquet mode expansion [1] and mode matching technique [6]. The unknown fields, due to the periodic nature of the surface, are all represented by a series of vector Floquet modes [1] whose coefficients are determined by solving proper boundary conditions between two regions. The technique is basically a vector formulation with the coupling between the arrays element and the wave incident taken into account. The modal analysis for many years has proved to be a powerful tool in scattering and propagation problems. The modal description of the electromagnetic fields may be further enhanced by orthogonality [1] as well as spectral conditions [7]. These conditions improve convergence [8] and require less computation time. It is a strong technique when dealing with vector formulation of the fields and may be applied to currents flowing on an array of conducting elements and to fields in apertures. Standard electromagnetic boundary conditions and Floquet expansion lead to an integral equation, with the unknown induced current to be computed by the Method of Moment (MoM) [9]. The complex amplitudes of the current induced on the array elements are then computed, based on the assumption that an individual element behaves as though situated in an infinite plane array tangential to the local surface and illuminated with a plane wave. The computation of transmission and reflection coefficients then follow, since the unknown current coefficients have already been computed.

The analysis outlined in this section in brief is a derivation of an earlier work by Montgomery [10], later outlined by Vardaxoglou et al [11,12] in which they included the effect of the dielectric substrate on which the thin layer array of elements are printed. The notation used here are the same as in reference 10, where a detailed derivation of the integral equation and the remaining equations may be found.

In the modal analysis technique, we take the dielectric substrate to be at a distance d from the array as shown in Figure A.1, where the reflection coefficient due to the dielectric in
the presence of the array is outlined. Unlike in reference 10, the incident field is included from the beginning in the scattering equation for the fields and in the application of boundary conditions. This will result in an integral equation relating the total field (i.e. the incident plus the scattered fields), to the transform of the induced surface current.

A.2 The Vector Floquet Harmonics

As is well known, the modal analysis assumes that the structure is infinitely plane and illuminated by a plane wave. For an infinite and periodic array, the fields in the region adjacent to any element can be represented in terms of a complete orthogonal set of modes (i.e. scalar Floquet modes) by means of Floquet's theorem, due to the periodic nature of the array surface. The theorem provides a means of formulating the phase array boundary problem in the form of a Floquet integral equation.

The phase variation of the fields (scalar Floquet mode) with position \( \mathbf{r} = x \hat{x} + y \hat{y} \) may be written as:

\[
\theta_{pq}(\mathbf{r}, z) = \psi_{pq}(z) e^{i\gamma_{pq}z} = \sqrt{A} e^{iK_{pq}z} e^{i\gamma_{pq}z}
\]  

(A.1)

where A is the area of the unit cell, defined as: \( A = |D_1 \times D_2| \), with \( D_1 \) and \( D_2 \) representing the two lattice vectors.

\[ p, q = 0, +1, +2, \ldots \]  

(A.2)

Note that the time variation \( e^{i\omega t} \) is assumed. Each Floquet mode \((p, q)\) has a propagation constant \( \gamma_{pq} \) along the \( z \)-axis. The fields are expressed in terms of conventional waveguide modes \( \psi_{pq} \), with a characteristic modal admittance \( \eta_{pq} \). The propagation constants \( K_{pq} \) and \( \gamma_{pq} \) are both functions of the lattice geometry, the polar angles (\( \theta, \varphi \)) shown in Figure A.2(b) and the Floquet numbers \( p, q \). They are given as follows.

The transverse propagation constant (or the tangential wave number, as it is sometimes called) is given as:

A.3
Figure A1: Reflection ($R$) and Transmission ($T$) field amplitudes of the FSS showing their $Z$-dependence arrows.
Figure A2: (a) Geometry of an infinite periodic array.
(b) Polar co-ordinates, $\varphi$ is the angle that the plane of incidence makes with the x-axis.
Appendix A

\[ \mathbf{k}_{\text{Tpq}} = \mathbf{k}_{\text{Too}} + p\mathbf{k}_1 + q\mathbf{k}_2 \]
\[ = k_x \hat{x} + k_y \hat{y} \]  
\[ \text{(A.3)} \]

where

\[ \mathbf{k}_{\text{Too}} = k_\text{oo} \hat{x} + k_\text{oy} \hat{y} \]  
\[ \text{(A.4)} \]

\[ k_1 = \frac{-2\pi}{A} \hat{z} \times \mathbf{D}_2 \]  
\[ \text{(A.5)} \]

\[ k_2 = \frac{2\pi}{A} \hat{z} \times \mathbf{D}_1 \]  
\[ \text{(A.6)} \]

with

\[ k_\text{ox} = k_0 \sin \theta \cos \varphi \]  
\[ \text{(A.7)} \]

and

\[ k_\text{oy} = k_0 \sin \theta \sin \varphi \]  
\[ \text{(A.8)} \]

where

\[ k_0 = \frac{2\pi}{\lambda} \]

In the above, indices 1 and 2 denote TM (Transverse Magnetic) and TE (Transverse Electric) incidences respectively.

Alternatively, \( k_x \) and \( k_y \) in equation (A.3) can be written as

\[ k_x = k_\text{ox} + p k_{1x} + q k_{2x} \]  
\[ \text{(A.9)} \]

\[ k_y = k_\text{oy} + p k_{1y} + q k_{2y} \]  
\[ \text{(A.10)} \]

\( k_{\text{Tpq}} \) therefore becomes

\[ \mathbf{k}_{\text{Tpq}} = (k_\text{ox} + p k_{1x} + q k_{2x}) \hat{x} + (k_\text{oy} + p k_{1y} + q k_{2y}) \hat{y} \]  
\[ \text{(A.11)} \]

Using equations (A.5), (A.6), (A.7), and (A.8) in the above, \( k_{\text{Tpq}} \) can be written in the form below:

\[ \mathbf{k}_{\text{Tpq}} = k_0 \sin \theta (\cos \phi \hat{x} + \sin \phi \hat{y}) + \frac{2\pi}{|\mathbf{D}_1 \times \mathbf{D}_2|} (q \hat{z} \mathbf{D}_1 - p \hat{z} \mathbf{D}_2) \]  
\[ \text{(A.12)} \]

From the Floquet theorem, it follows that the propagation constant with respect to the \( \hat{z} \) axis of the \( pq \)th order mode (i.e. the \( z \)-directed propagation constant (or the modal propagation constant)) is given by:

\[ \gamma_{pq} = \left( k^2 - k_{\text{Tpq}}^2 \right)^{1/2} \]

\[ = \left( k^2 - (k_x \hat{x} + k_y \hat{y}) \cdot (k_x \hat{x} + k_y \hat{y}) \right)^{1/2} \]  
\[ \text{(A.5)} \]
Appendix A

\[ \gamma_{pq} = \sqrt{\left( k^2 - (k_x^2 + k_y^2) \right)} \]

(A.13)

where \( k = k_0 \sqrt{\epsilon} \) (propagation constant of the media)

It is seen that the propagation constants both \( k_{Tpq} \) and \( \gamma_{pq} \) are both functions of the lattice geometry (in particular the lattice periodicities), the Floquet mode numbers \((p,q)\) and the polar angles \( \theta \) and \( \phi \) (Figure A.2(a)).

Note that the propagating \((p,q = 0)\) or evanescent modes are given by positive real or negative imaginary values of \( \gamma_{pq} \) respectively, i.e. \( k^2 \geq (k_x^2 + k_y^2) \) then the wave propagates with \( \gamma_{pq} \) being real and positive.

This means that the propagating modes or those for which the magnitude of the transverse propagation constant \( k_{Tpq} \) is less than the propagation constant of the free-space \( k \).

\( K^2 < (k_x^2 + k_y^2) \) then the wave is evanescent with \( \gamma_{pq} \) being negative and imaginary.

It should be noted that the propagating waves for which \( p,q \neq 0 \) are often referred to as grating responses, with \( \gamma_{pq} \) being real.

Equation (A.1) represents an orthonormal set of scalar Floquet modes. An orthonormal set of "vector" Floquet modes can be derived from the scalar modes in much the same manner as vector TE and TM modes are derived from a scalar Hertzian potential in waveguide mode theory [13]. A complete set of vector TE and TM modes can be derived from the scalar Floquet mode \( \Psi_{pq}(t) \).

The vector Floquet mode can be expressed as:

\[ \Psi_{pq}(t) = \psi_{pq}(t) \hat{k}_{mpq} \]

\[ = (\sqrt{A e^{-j\gamma_{pq} t}}) e^{\pm j \pi \frac{x}{2}} e^{\pm j \pi \frac{y}{2}} \hat{k}_{mpq} \]

(A.14)
where the unit field vector $\hat{k}_{mpq}$ lies in the (x,y) plane and is given by equations (A.22) and (A.23) where $m=1$ denotes TM mode and $m=2$ denotes TE mode. The set of TE and TM free-space modes have the property
\[
\int_{\Omega_{cell}} \psi_{mpq}(r) \psi_{m'p'q'}^*(r) d(r) = A\delta_{mm'}\delta_{pp'}\delta_{qq'} \tag{A.15}
\]
where
\[
\delta_{\alpha\beta} = \begin{cases} 
1 & \text{for } \alpha = \beta \\
0 & \text{for } \alpha \neq \beta 
\end{cases}
\]
where the kronecker delta, $\delta_{mm'}$ indicates that the TE and TM vector modes are mutually orthogonal and where (*) denotes the complex conjugate. Equation (A.15) is known as the orthogonality relation of the Floquet mode.

Physically, the vector Floquet mode functions, $\psi_{mpq}(r)$ are TE or TM plane waves which propagate (or decay) away from the aperture plane ($z=0$). In Section A.3 it is shown that the tangential fields adjacent to the array can be expanded as a linear combination of two (TE and TM) vector Floquet modes.

A.2.1 The Grating Lobe Diagram

From the grating lobe equations (A.12) and (A.13) the propagating and non-propagating Floquet modes can be further illustrated by a grating lobe diagram. Consider the situation when the periodic array of Figure (A.1) is illuminated by a plane wave under the incidence angles $\varphi = 0$ and $\theta > 0$. Furthermore, assume a rectangular lattice geometry of the array with $D_1$ and $D_2$ as the two lattice vectors, along the x and y- axes respectively. Then the grating lobe equation (A.12) becomes:
\[
k_{npq} = (k_o \sin \theta \hat{x} \times \frac{2\pi}{D_1 D_2} (q \hat{z} \times D_1 \hat{x} - p \hat{z} \times D_2 \hat{y}))
\]
following the vector cross-product rule below i.e.
The equation for $k_{Tpq}$ becomes:

$$k_{Tpq} = (k_0 \sin \theta + \frac{2\pi p}{D_1}) \hat{x} + \frac{2\pi q}{D_2} \hat{y}$$

where

$$k_x = k_0 \sin \theta + \frac{2\pi p}{D_1}$$

and

$$k_y = \frac{2\pi q}{D_2}$$

From the grating lobe equation for the modal propagation constant of (A.13) we have

$$\gamma_{pq} = \sqrt{k^2 - (k_x^2 + k_y^2)}$$

where $k = k_0 \sqrt{\varepsilon r}$

then $k = k_0$.

For any mode $(p,q)$ to propagate, then

$$k_0^2 \geq (k_x^2 + k_y^2)$$

or

$$\left( k_0 \sin \theta_{p,q} \right)^2 = k_x^2 + k_y^2$$

where $(\theta_{p,q})$ is the direction of propagation of the mode $(p,q)$.

Substituting values for $k_x$ and $k_y$ from equation (A.16) into the above, gives

$$k_0^2 (\sin \theta_{p,q})^2 = \left( k_0 \sin \theta + \frac{2\pi p}{D_1} \right)^2 + \left( \frac{2\pi q}{D_2} \right)^2$$

we have
Hence from equation (A.17), a grating lobe diagram can be plotted for the region \( z \geq 0 \), as shown in Figure A.3 below.

Figure A.3: The grating lobe diagram

It is therefore seen from the above grating lobe diagram that the locus of any propagating mode is a unit circle of radius \( k_0 = \frac{2\pi}{\lambda} \), with the dominant \((0,0)\) order mode that is always propagating, falling inside the grating lobe diagram and with the non-propagating mode falling outside the circle. It is therefore deduced that any Floquet mode propagating will fall inside the grating lobe. As an example, if the higher order (-1,0) mode is to propagate, then the grating lobe equation becomes
Appendix A

\[
\left(\frac{2\lambda}{\lambda}\right)^2 \sin^2 \theta_{(-1,0)} = \left(\frac{2\pi}{\lambda} \sin \theta - \frac{2\pi}{D_1}\right)^2 + 0
\]

or

\[
\sin^2 \theta_{(-1,0)} = \left(\sin \theta - \frac{\lambda}{D_1}\right)^2
\]

Hence

\[
\sin \theta_{(-1,0)} = \pm \left(\frac{\lambda}{D_1} - \sin \theta\right)
\]

(A.18)

The above equation implies that in practice, a higher order mode can be made to be propagating by careful selection of the array lattice periodicity. It can be further deduced from the grating lobe diagram of (A.3) that as the frequency increases, then \(k_o = \frac{2\pi}{\lambda}\) becomes bigger thus implying that higher order modes may fall within the grating lobe and therefore be propagating.

A.3 Tangential Field Expansion in Terms of TE and TM Vectors Floquet Modes

Consider a plane TEM wave incident on an infinite periodic array of conductors on a dielectric slab of width d. Figure A.2(b) shows a small portion of the infinite array geometry and the electric field of a linearly polarised wave incident in an arbitrary direction.

For the different media, according to Floquet theorem, the tangential fields can be expanded in a Fourier series form. The tangential electromagnetic field in the plane of the array expressed as a linear combination of two Floquet vector modes (TE or TM) are given below. (Full proof of the derivation of such modes is given in reference 1).

a) Those with their entire magnetic vector mode parallel to the plane of the array, called transverse magnetic (TM) modes and with the electric vector mode tangential to the array given by:
where \( \theta_{pq} \) is given by equation (A.1). Thus (A.19) becomes:

\[
E_{pq}^{TM} = \psi_{pq}(t) \, e^{+\kappa_{1pq} z} \, \hat{\kappa}_{1pq} \quad \text{(A.20)}
\]

b) Those with their entire electric vector mode parallel to the plane of the array, called transverse electric (TE) modes and with the tangential magnetic vector mode to the array given by:

\[
E_{pq}^{TE} = \theta_{pq} \, \hat{\kappa}_{2pq}
\]

\[
= \psi_{pq}(t) \, e^{+\kappa_{2pq} z} \, \hat{\kappa}_{2pq} \quad \text{(A.21)}
\]

The two unit field vectors \( \hat{\kappa}_{1pq} \) and \( \hat{\kappa}_{2pq} \) lie in the (x,y) plane. They are given by:

\[
\hat{\kappa}_{1pq} = \frac{k_{1pq}}{|k_{1pq}|} \quad \text{(for TM mode)} \quad \text{(A.22)}
\]

and

\[
\hat{\kappa}_{2pq} = \hat{\mathbf{z}} \times \hat{\kappa}_{1pq} \quad \text{(for TE mode)} \quad \text{(A.23)}
\]

For a given mode the magnitudes of the tangential vector fields \( E_t \) and \( H_t \) are related by the modal admittance given by:

\[
\frac{|E_t|}{|H_t|} = \frac{1}{\eta_{mpq}}
\]

for TM mode \( \eta_{1pq} = \frac{k}{\gamma_{pq}} \frac{\epsilon \omega / \gamma_{pq}}{\gamma_{pq}} \quad \text{(A.24)} \)

for TE mode \( \eta_{2pq} = \frac{\gamma_{pq}}{k} \frac{\gamma_{pq}}{\omega \mu} \quad \text{(A.25)} \)

where \( \eta \) = \( \sqrt{\frac{\epsilon}{\mu}} \), with \( \epsilon \) and \( \mu \) being the permittivity and permeability of the media respectively. The tangential electromagnetic field can now be written as a Fourier Series of the vector TM and TE Floquet modes. In general, an electric field takes the following form

\[
A.11
\]
\[ E_i^\varepsilon (r, z) = \sum_{pq} g_{1pq}^\text{TM} E_{pq}^\text{TM} (r, z) + g_{2pq}^\text{TE} E_{pq}^\text{TE} (r, z) \]  

(A.26)

where \( g_{1pq}^\text{TM} \) and \( g_{2pq}^\text{TE} \) are the amplitude of TM and TE modes.

Using equations (A.20) and (A.21) and by introducing an extra subscript \( m=1,2 \), \( E_i^\varepsilon (r, z) \) becomes:

\[ E_i^\varepsilon (r, z) = \sum_{mpq} g_{mpq} \theta_{pq} (r, z) K_{mpq} \]

(A.27)

In the above equation, the positive sign in the exponential term corresponds to fields with amplitude \( R_{mpq} \) with positive \( z \) dependence, travelling in the negative \( z \) direction as shown in Figure A.1. The opposite to this applies for fields with amplitude \( T_{mpq} \).

A.4 The Boundary Conditions and the Integral Equations

By expanding the total tangential electromagnetic (EM) field as Floquet modes, for the different media, standard EM boundary conditions are applied at each interface of the structure, resulting in an algebraic equation that is easily solved for the unknown current. This is the process of matching the fields (both electric and magnetic) at the different interfaces. The process of obtaining this integral equation also involves application of orthogonality of Floquet expansion, leading to an expression relating the various amplitudes to the induced current. The Floquet orthogonality condition was given by equation (A.15) of the previous section.

There are basically two types of boundary condition that are considered in the integral equation formulation. They are:
Appendix A

a) The total tangential electric (E) and magnetic (H) fields are continuous at all dielectric or dielectric/air interfaces, i.e.
\[ \Delta \times E = 0, \quad \Delta \times H = 0 \]

b) At the FSS plane, the total tangential E field must be zero on the conducting part of FSS (i.e. the tangential components of the electric field are continuous across the boundary, and vanish on the surface of the conducting elements), and the tangential components of the magnetic field (H) are discontinuous at the conducting boundaries by an amount equal to the surface current density \( \mathbf{J} \), i.e.
\[ \Delta \times E = 0 \]
and \[ \Delta \times H = \mathbf{J} \]

Note also that the total tangential electric field (i.e. the scattered tangential field plus the incident field) must also vanish on the conducting surface.

The aims in applying the boundary conditions above are to relate the reflected coefficient \( R_{pq}^* \) and the transmitted coefficient \( T_{pq}^* \) to the unknown current \( \mathbf{I} \).

Expanding the fields by starting at the interface \( Z=d \) of Figure A.1 and working towards \( Z=0 \) and applying the boundary conditions, with continuous elimination of the transmitted field amplitudes, a relationship between \( R_{pq}^a \) and \( T_{pq}^b \) and current \( \mathbf{I} \) is obtained [10].

As a result, the following integral equation, with the induced surface current \( \mathbf{I} \) as unknown is obtained:

\[ \sum_{n=1}^{2} (1 + R_{moo}^*) b_{m}^{inc} \psi_{oo} (x) \hat{k}_{moo} \]
\[ = \sum_{n=1}^{2} \sum_{pq} A^{-1} \left[ (R_{mpq}^*) \hat{k}_{mpq} \int_{A} \frac{\psi_{pq}^* (x)}{\hat{k}_{mpq}} \mathrm{dA} \right] \psi_{pq} (x) \hat{k}_{mpq} \quad (A.28) \]

where \( b_{m}^{inc} \) is the complex amplitude coefficient of the incident field.
The above electric field integral equation has been obtained using the boundary condition that the total tangential electric field (i.e. the sum of scattered and incident fields) is zero over the conducting area of the unit cell.

Note that the notations used in the equations here are exactly as given in the reference where the integral equation procedure is outlined in detail.

It should be noted that propagating incident fields correspond to \( p=q=0 \) in the Floquet expansion. It can be shown \([10]\) that \( R^e_{mpq} \) (the equivalent modal admittance for the \( pq^{th} \) mode) in equation (A.28) is given by:

\[
R^e_{mpq} = (1 + R^+_{mpq})/\left[\eta^e_{mpq} (1 - R^+_{mpq}) - \eta^e_{mpq} (1 + R^+_{mpq})\right]
\]  
\[
(A.29)
\]

where \( R^+_{mpq} \) is given by:

\[
R^+_{mpq} = \frac{\eta^\text{diep}_{mpq} - \eta^e_{mpq}}{\eta^\text{diep}_{mpq} + \eta^e_{mpq}} e^{-2\pi f_0 z}
\]  
\[
(A.30)
\]

It can also be shown that \( R^e_{moo} \) is given by:

\[
R^e_{moo} = \left( \eta^e_{moo} (1 + R^+_{mpq}) - \eta^\text{diep}_{mpq} (1 - R^+_{mpq}) \right)/\left( \eta^\text{diep}_{mpq} (1 - R^+_{mpq}) - \eta^e_{mpq} (1 + R^+_{mpq}) \right)
\]  
\[
(A.31)
\]

### A.5 Method of Moment Solution to the Integral Equation

The numerical solution of the integral equation given by equation (A.28) is carried out by using the method of moment \([9]\), where the unknown current \( I \) is expanded in terms of a set of complete orthogonal basis functions. These may be either sub-domain or entire domain basis functions and have the form:

\[
I(x,y) = \sum_{n=1}^{N} C_n \ h_n(x,y)
\]  
\[
(A.32)
\]
where \( h_n(x,y) \) are the basis functions and \( C_n \), the complex amplitude of the \( n^{th} \) basis function and \( N \) is the number of functions used to approximate as closely as possible the actual induced current.

Since \( I(x,y) \) is of finite form, then the choice of the basis function must form a complete set. This completion implies that \( h_n(x,y) \) is convergent as \( n \to \infty \) and a finite limit therefore exists in the domain they are defined [9]. The resultant system of algebraic equations would therefore produce an infinite number of unknowns. In order to solve the system it is necessary to use an approximation. A finite number \( N \) say, of functions can be considered to approximate the unknown distributions. The system now is of finite order and has a non-trivial solution if the \( \{h_n\} \) is linearly independent. However, for convenience the set may be forced to be orthonormalised since an orthonormal set is also linearly independent. The method basically converts the integral equation into a linear algebraic equation that can be solved for the unknown coefficient. By weighting (i.e. testing) the resultant integral equation according to Galerkin's method, the following system of linear equations can be shown [6] to be obtained:

\[
\sum_{m=1}^{2} b_{m}^{n} \left(1 + R_{\text{Teloo}}^{\ell}\right) \hat{k}_{\text{Teloo}} \cdot \vec{g}_{j} (\kappa_{\text{Teloo}})
\]

\[
= \sum_{n=1}^{N} C_{n} \sum_{n=1}^{2} \sum_{pq} A^{1} R_{mpq}^{\ell} \hat{k}_{mpq} \left(\vec{g}_{n} (\kappa_{\text{Teloo}}) \cdot \hat{k}_{mpq}\right) \vec{g}_{j} (\kappa_{\text{Teloo}}) \tag{A.33}
\]

where \( j = 1, \ldots, N \) and

\[
\vec{g}_{n} (\kappa_{\text{Teloo}}) = \langle h_n(x,y), \psi_{pq}(\xi) \rangle, \hat{k}_{mpq} = \int_{A} h(x,y) \cdot \psi_{pq}^{*}(\xi) \, dA \cdot \hat{k}_{mpq} \tag{A.34}
\]

The above inner product (equation (A.34)) is the Floquet transform of the electric current basis function. The bar on the top signifies the Floquet spectrum of the current distribution. Note that the spacing of the discrete components of \( I(x,y) \) depends on the periodicity of the element array. One key factor in the numerical convergence of the solution of the integral equation is the inclusion of the adequate Floquet harmonics to cover at least the major lobe of \( I_{mpq} = \vec{g}_{n} (\kappa_{\text{Teloo}}) [8] \). It is advisable to increase the Floquet mode number, keeping the same number of current basic functions until there is little
difference in the result. The convergence checks [8] could always be implemented as part of the computer model at the expense of computer time. The choice of the bases however remains a key factor in the convergence.

Equation (A.33) is a matrix equation for the unknown coefficient of the current expansion and once the solution to the equation system is found, the scattered fields can be obtained [11]. The linear equation that results from equation (A.33) is written in the form:

$$\bar{L}_r = \bar{C} \bar{G}_r$$

or

$$L_r = \sum_{n=1}^{N} C_n G_m$$ for $n = 1, \ldots, N$$ \tag{A.35}$$

where we choose the same basis function for the weighting function (Galerkin's method). However in general one can take the inner product with a different set of basis functions. The equation is of infinite order for the unknown coefficients $C_n$'s. $G_r$ is a matrix of dimension $J \times J$ and is independent of the excitation; $C$ is a column vector of the unknown current expansion coefficient and $L_r$ is the excitation vector for the $r^{th}$ order mode. $L_r$ and $G_m$ are given by:

$$L_r = \sum_{n=1}^{2} (1 + R_{n00}) b_{n}^{\text{ste}} (\hat{g}_L (k_{\text{loc}}), \hat{k}_{n00})$$ \tag{A.36}

$$G_m = \frac{1}{A} \sum_{mpq} R_{mpq} (\hat{g}_n (k_{mpq}), \hat{k}_{mpq}) (\hat{g}_L (k_{mpq}), \hat{k}_{mpq})$$

$$C_n = \text{unknown coefficients of the basis function}$$ \tag{A.37}

The matrix equation of (A.35) is solved numerically by a computer for the unknown coefficients $C_n$, with the quantities in equation (A.36) defined and $h_n(x,y)$ chosen to represent the current density flowing in the unit cell. The square matrix $[G_m]$ is of the finite order and the solution is obtained by matrix inversion, i.e.

$$[C_n] = [G_m]^{-1} [L_r]$$ \tag{A.38}

where $[G_m]^{-1}$ denotes the inverse of the matrix $[G_m]$. Once the coefficients $C_n$ are computed, it is a relatively straightforward matter to obtain the reflection and
transmission coefficients. The derivation of plane wave transmission and reflection coefficients are omitted here, due to repetition in references [11,12], where the detailed manipulation of these can be found.

It should be noted that an increase in the number of basis functions (i.e. for a more complicated current distribution) would generally imply an increase in the number of Floquet modes used, in order to guarantee a reasonably accurate prediction. The result would be a larger computer run time.

It is understood that the approximation of the current density by N number of basis functions will improve as the number of functions N increase, provided these basis functions obey the conditions outlined before. However the inversion has to be carried out by a computer, therefore it can be costly in terms of memory and CPU time. Hence the choice of the basis functions and prior knowledge of the problem is paramount. In the solution of the integral equation so far, it is assumed that there is a finite number of Floquet modes. In practice however it is impossible to take the sum to infinity, and a truncation to a finite two dimensional number (p,q) in our equation, has to be considered.
REFERENCES


Appendix A


APPENDIX B

RATIONAL AND IRRATIONAL NUMBERS

A brief description and the properties of a simple rational number are now given as they are used in the concept of common periodicity functions applied to the analysis of double layer surfaces with dissimilar lattice periodicities in Chapter 2.

B.1 Rational Numbers

Rational numbers are numbers that can be expressed as quotients of two integers, such as -2/3, 1/5 and 2/7, or generally in a fractional form, a/b, where a and b represent integers and b is not zero. These numbers arose to permit solutions of equations such as bx=a for all integers a and b where b≠0. This leads to the operation of division or inverse of multiplications, and we write x=a/b or a+b where a is the numerator and b the denominator. The set of integers is a subset of the rational numbers, since integers correspond to rational numbers when b=1. It is important to remember the condition b≠0, because any integer divided by zero is undefined. Also the rational number 6/1 can be conceptually distinguished from the integer 6. However, it is more practical and convenient to identify 6/1 with 6, or consider 6/1=6. Thus the set of rational numbers includes all integers.

B.2 Irrational Numbers

These are numbers which are not rational, i.e. cannot be expressed as a/b (called the quotient of a and b) where a and b are integers and b≠0.

Irrational numbers have infinite, non-decaying and non-repeating decimal expansion, i.e. the decimal point neither terminates to zero nor does it repeat itself. The word "irrational" means "not rational", or a number that cannot be expressed as the ratio of two integers. Examples of irrational numbers are the \( \sqrt{2} \) (the ratio of the length of a diagonal of a square to a side) and \( \pi \) (the ratio of the circumference of a circle to its diameter). \( \sqrt{2} = 1.41421 ... \) and \( \pi = 3.1412927 ... \)
The two correlation functions $\phi_{mpq}^{m'in}$ and $\phi_{m'in}^{mpq}$, which involve integration of the product of the two vector Floquet mode sets are normally denoted as the inner products calculated on the unit cell areas $A_2$ and $A_1$ respectively. These inner products are given by:

$$\phi_{mpq}^{m'in} = \langle \psi_{pq}^{2} , \psi_{in}^{2} \rangle \hat{k}_{mpq}^{1} \hat{k}_{m'in}^{2}$$

$$= \frac{1}{A_2} \int_{A_2} \int \psi_{pq}^{2} (\psi_{in}^{2})^* \ dA_2 \hat{k}_{mpq}^{1} \hat{k}_{m'in}^{2}$$ (C.1)

and

$$\phi_{m'in}^{mpq} = \langle \psi_{in}^{1} , \psi_{pq}^{1} \rangle \hat{k}_{m'in}^{2} \hat{k}_{mpq}^{1}$$

$$= \frac{1}{A_1} \int_{A_1} \int (\psi_{pq}^{1})^* \psi_{in}^{1} \ dA_1 \hat{k}_{m'in}^{2} \hat{k}_{mpq}^{1}$$ (C.2)

where (*) denotes conjugate.

It will be shown that when the two arrays within the double-layer structure have the same lattice periodicities, then these correlation functions are reduced to delta function, otherwise they are proportional to the product of two sinc functions that represent the difference of the tangential wave vectors of the two arrays, i.e.

$$\phi_{mpq}^{m'in} = \begin{cases} A_2 \delta_{mm'} \delta_{pt} \delta_{qn} & \text{if and only if } D_{1,2}^{1} = D_{1,2}^{2} \\ A_2 \langle \psi_{pq}^{1}, \psi_{in}^{1} \rangle \hat{k}_{mpq}^{1} \hat{k}_{m'in}^{2} & \text{otherwise} \end{cases}$$ (C.3)

and

C.1
\[
\phi_{m'n'a}^{pq} = \begin{cases} 
A_1 \delta_{m'n'} \delta_{pq} & \text{if and only if } D_{1,2}^2 = D_{1,2}^1 \\
A_1 < \psi_{in}^2 \psi_{pq}^* > \hat{k}_{m'n'}^2 \hat{k}_{m'n'}^1 & \text{otherwise}
\end{cases}
\]  

(C.4)

where \(D_{1,2}^1\) and \(D_{1,2}^2\) denote the lattice periodicities of the first and second array in the double layer structure. The unit cell area \(A_1\) and \(A_2\) are given by:

\[
A_1 = |D_i^1 \times D_i^2| 
\]

and

\[
A_2 = |D_i^2 \times D_i^2| 
\]

(C.5)  
(C.6)

To ease the complexity in the equations that are followed, both superscripts for array one and two are omitted and the arrays are distinguished by their Floquet mode subscripts \(pq\) and \(in\).

A general case is considered where the lattice periodicities of the two arrays are along arbitrary lattice axes \(u\) and \(v\) with periodicities \(D_u\) and \(D_v\) as shown in Figure C.1. The figure also shows the cross-section of a double-layer structure.

Considering the correlation function \(\phi_{m'n'a}^{pq}\), we have

\[
\phi_{m'n'a}^{pq} = \frac{1}{A_1} \int_{A_1} \int_{A_1} \psi_{in}(r) \psi_{pq}^*(r) \, dr \, \hat{k}_{m'n'} \hat{k}_{m'n'} 
\]

(C.7)

where

\[
\psi_{in}(r) = e^{-iK_{in} \cdot r}
\]

and

\[
\psi_{pq}(r) = e^{-iK_{pq} \cdot r}
\]

with \(r = x\hat{x} + y\hat{y}\)
Figure C.1: Definition of Lattice Geometries and Cross-Section of the Double Layer Surface
and \[ dr = \hat{x}dx + \hat{y}dy \]

In order to obtain the complex harmonics \( K_{\nu p q} \) and \( K_{\nu n m} \) along the direction of the arbitrary axes \( u \) and \( v \), axis transformations are carried out using Jacobian, i.e.

\[ x = \cos \alpha_1 \hat{u} + \cos \alpha_2 \hat{v} \]

\[ y = \sin \alpha_1 \hat{u} + \sin \alpha_2 \hat{v} \]

with \( \alpha_2 \) being the angle that the lattice vector \( \hat{D}_v \) makes with the x-axis.

with \( dx dy = |J| \ dudv \), where \( |J| \) is the Jacobian given by:

\[
|J| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}
\]

(C.9)

therefore

\[
|J| = \cos \alpha_1 \sin \alpha_2 - \cos \alpha_2 \sin \alpha_1 \]

(C.10)

Hence the correlation function can be expressed in terms of the \( u \) and \( v \). From equation (C.7):

\[
\phi_{m'n'a'}^{mpq} = \frac{1}{A_1} \int \int_{A_1} e^{-ik\cdot\hat{u}} e^{ik\cdot\hat{v}} \ dr \ (\hat{k}_{mpq} \hat{k}_{m'n'a'})
\]

\[
= \frac{1}{A_2} \int \int_{A_2} e^{-j(K_{\nu p q} - K_{\nu n m})} e^{j(K_{\nu m'n'a'} - K_{\nu n'm'n'a'})} \ dx dy (K_{\nu p q} K_{m'n'a'} + K_{\nu n m} K_{m'n'a'})
\]

(C.11)

\[
\phi_{m'n'a'}^{mpq} = \frac{1}{A_1} \int \int_{A_1} e^{-j(K_{\nu p q} - K_{\nu n m})} e^{-j(K_{\nu m'n'a'} - K_{\nu n'm'n'a'})} \ dx dy (K_{\nu p q} K_{m'n'a'} + K_{\nu n m} K_{m'n'a'})
\]

(C.12)

Equation (C.12) can be expressed in terms of \( u \) and \( v \) as:

\[
\phi_{m'n'a'}^{mpq} = AS \int \int_{A_1} e^{-j(ux\cos \alpha + vy\sin \alpha)} e^{-j(uy\sin \alpha + vx\cos \alpha)} \ |J| \ dudv
\]

(C.13)
where \( k_x = k^x_{mn} - k^x_{pq} \)
and \( k_y = k^y_{mn} - k^y_{pq} \)  

and also

\[
AS = \frac{1}{A_1} \left( k^x_{mpq} k^x_{mn} + k^y_{mpq} k^y_{mn} \right)
\]

Equation (C.13) can be written as:

\[
\phi_{m'n'n} = AS \int_{A_1} \int e^{-j(k_x \cos \alpha_1 + k_y \sin \alpha_1)u} e^{-j(k_x \cos \alpha_1 + k_y \sin \alpha_1)v} |J| \ dudv
\]

Let \( k_x \cos \alpha_1 + k_y \sin \alpha_1 = ku \)
and \( k_x \cos \alpha_2 + k_y \sin \alpha_2 = kv \)

Then \( \phi_{m'n'n} \) becomes:

\[
\phi_{m'n'n} = AS \int_{A_1} \int e^{-jku} u e^{-jkv} v |J| \ dudv
\]

with

\[
A_1 = |D_1^1 \times D_2^1| = \begin{vmatrix} D_x^1 & D_y^1 \\ -\frac{D_y^1}{2} & \frac{D_x^1}{2} \\ -\frac{D_y^1}{2} & \frac{D_y^1}{2} \end{vmatrix}
\]

thus integrating between \(-\frac{D^1_1}{2}\) and \(\frac{D^1_1}{2}\) along both \(x\) and \(y\) axes, the equation for \( \phi_{m'n'n} \)

becomes
\[
\phi_{m'n}^{mpq} = AS \int_{D_x'} e^{(-jku)u} du \int_{D_y'} e^{(-jkv)v} dv \ |J|
\]

\[
= AS \left[ \frac{e^{(-jku)u} - D_x'}{(-jku) D_x'} \right] \left[ \frac{e^{(-jkv)v} - D_y'}{(-jkv) D_y'} \right] \ |J| \quad \text{(C.19)}
\]

Performing the above integration will result in the following expression for the correlation function \( \phi_{m'n}^{mpq} \).

\[
\phi_{m'n}^{mpq} = (AS) \frac{D_x'}{2} \left[ - \frac{e^{(-jku) D_x'} - e^{(-jku) D_x'}}{(-jku) D_x'} \right] \frac{D_y'}{2} \left[ - \frac{e^{(-jkv) D_y'} - e^{(-jkv) D_y'}}{(-jkv) D_y'} \right] \quad \text{(C.20)}
\]

Simplifying the above equation results in the following expression:

\[
\phi_{m'n}^{mpq} = (AS) \frac{D_x'}{2} \left[ \sin \left(\frac{(ku) D_x'}{2}\right) \right] \frac{D_y'}{2} \left[ \sin \left(\frac{(kv) D_y'}{2}\right) \right] |J|
\]

\[
= (AS) D_x' D_y' \text{sinc} \left(\frac{(ku) D_x'}{2}\right) \text{sinc} \left(\frac{(kv) D_y'}{2}\right) |J| \quad \text{(C.21)}
\]

Substituting for \( AS \) from equation (C.15) into equation (C.21), gives:

\[
\phi_{m'n}^{mpq} = \frac{|J|}{D_x' D_y' \sin \alpha} D_x' D_y' \text{sinc} \left(\frac{(ku) D_x'}{2}\right) \text{sinc} \left(\frac{(kv) D_y'}{2}\right) (k_{mpq} k_{m'n} + k_{mpq} k_{n'm}) \quad \text{(C.22)}
\]

C.6
Hence, the correlation function \( \phi_{m'n'ln}^{mpq} \) is equal to the product of two sinc functions, with each term in the two sinc functions corresponding to the difference between the two tangential wave numbers \( K_{\tau pq} \) and \( K_{\tau ln} \). The final expression for the correlation function is therefore given by:

\[
\phi_{m'n'ln}^{mpq} = \frac{|J|}{\sin \alpha} \text{sinc} \left[ \left( (K_{\tau ln} - K_{\tau pq}) \cos \alpha_1 + (K_{\tau ln} - K_{\tau ln}) \sin \alpha_1 \right) \frac{D_1}{2} \right] \\
\text{sinc} \left[ \left( (K_{\tau ln} - K_{\tau pq}) \cos \alpha_2 + (K_{\tau ln} - K_{\tau ln}) \sin \alpha_2 \right) \frac{D_1}{2} \right] \\
- (K_{mpq} - K_{m'n'ln})
\]  

(C.23)

The above expression for the correlation function is valid for the general case when the lattice periodicities of the arrays are along any arbitrary lattice axis. The special case is when the lattice periodicities of any of the two arrays form a more symmetrical square lattice. It can be deduced from (C.23) that for this square lattice, the correlation function reduces to the expression below, i.e. for \( \alpha_1 = 0, \alpha = 90^\circ \), then

\[
\phi_{m'n'ln}^{mpq} = \text{sinc} \ (K_{\tau ln} - K_{\tau pq}) \frac{D_1}{2} \text{sinc} \ (K_{\tau ln} - K_{\tau pq}) \frac{D_1}{2} \ (K_{mpq} - K_{m'n'ln})
\]

(C.24)

In the case when the two arrays periodicities are identical, then the correlation function given by equation (C.24), can be shown to be reduced to a delta function.

From equation (C.24) assuming TM incidence, then if \( D_1 = D_2 \) and \( D_1 = D_2 \), and for \( \alpha_2 = \alpha = 90^\circ, \alpha_1 = 0^\circ \), we obtain:
\[
(\phi_{EM}^{P})_{TM} = \frac{\sin (-z \frac{2\pi}{D_1^1} - (p \frac{2\pi}{D_2^1})) \frac{D_1^1}{2}}{(-z \frac{2\pi}{D_1^1} + p \frac{2\pi}{D_2^1}) \frac{D_1^1}{2}}
\]

\[
= \frac{\sin (-\pi z + \pi p z)}{\pi (p + z)} \quad (C.25)
\]

Note that equations (2.10a) and (2.10b) of Section 2.1.1 in Chapter 2 have been used in equation (C.24) in order to obtain equation (C.25) above.

Therefore:

\[
(\phi_{EM}^{P})_{TM} = \frac{\sin (\pi (p - z))}{\pi (p - z)} \quad (C.25)
\]

and since the lattices of the two arrays are identical, then \( p = l \) and hence

\[
(\phi_{EM}^{P})_{TM} = \delta^p \text{ i.e. a delta function} \quad (C.27)
\]

Similarly for TE state of incidence, the correlation function from equation (C.24) becomes:

\[
(\phi_{EM}^{P})_{TE} = \frac{\sin (\pi (q - n))}{\pi (q - n)} \quad (C.28)
\]

If the lattice periodicities are identical, then \( q = n \) and hence

\[
(\phi_{EM}^{P})_{TE} = \delta^n \quad (C.29)
\]

Again equations (2.10a) and (2.10b) of Section 2.2.1 have been utilised in equation (C.24) in order to obtain equation (C.28). For the case when the arrays periodicities lie on an arbitrary lattice (i.e. square lattice in one array and triangular lattice for second array, it can be proved that equation (C.23) becomes a delta function.
An expression for the second correlation function $\phi_{mpq}^{m'ln}$ given by equation (C.1) can be found in a similar fashion to $\phi_{mpq}^{m'ln}$ just described. From equation (C.1) we have:

$$\phi_{mpq}^{m'ln} = \frac{1}{A_2} \int_{A_2} \psi_{pq}^l (\psi_{lm}^2)^* dA_2 \left( \hat{k}_{mpq} \cdot \hat{k}_{m'ln} \right)$$

where the integration is performed on the unit cell area $A_2$. $\phi_{mpq}^{m'ln}$ is written as:

$$\phi_{mpq}^{m'ln} = \frac{1}{A_2} \int_{A_2} e^{-iK_{m'ln}^x x} \cdot e^{-iK_{m'ln}^y y} \ dx dy \ (k_{m'ln}^x k_{m'ln}^x + K_{m'ln}^y K_{m'ln}^y)$$

(C.30)

Performing the integration results in an expression for $\phi_{mpq}^{m'ln}$, similar to equation (C.23). This is given as:

$$\phi_{mpq}^{m'ln} = \frac{|J|}{\sin \beta} \ \text{sinc} \ ([((K_{rpq}^x - K_{rna}^x) \cos \beta_1 + (K_{rpq}^y - K_{rna}^y) \sin \beta_2) \frac{D_x^2}{2}]
\ \text{sinc} \ (((K_{rpq}^x - L_{rna}^x) \cos \beta_2 + (K_{rpq}^y - K_{rna}^y) \sin \beta_2) \frac{D_y^2}{2}) \ (K_{m'ln}^x K_{mpq}^x + K_{m'ln}^y K_{mpq}^y)

(C.31)

where $\beta_1$ is the angle that the lattice vector along the $u$-axis makes with the $x$-axis, and $\beta_2$ is the angle between the lattice vector along the $v$-axis and the $x$-axis. The Jacobian $J$ in this case is given by:

$$|J| = \cos \beta_2 \sin \beta_2 - \cos \beta_2 \sin \beta_1$$

(C.32)

For the case when the lattices of the two arrays are square, equation (C.31) reduces to:

\[ \text{C.9} \]
\[
\phi_{mpq}^{m'ta} = \text{sinc} \left( K_{R_{pq}}^x - K_{R_{ln}}^x \right) \frac{D_x^2}{2} \text{sinc} \left( K_{R_{pq}}^y - K_{R_{ln}}^y \right) \frac{D_y^2}{2}
\]

\[
(K_{m'^{x}_{ln}} K_{m^{x}_{pq}} + K_{m'^{y}_{ln}} K_{m^{y}_{pq}})
\]

(C.33)

Again if the lattice periodicities of the two arrays are identical, then it can be similarly shown that \( \phi_{mpq}^{m'ta} \) becomes a delta function. In this case \( \phi_{mpq}^{m'ta} \) can be shown to become as:

\[
(\phi_{pq}^{ln})_{TM} = \delta_i^2 \quad \text{for TM incidence}
\]

(C.34)

and

\[
(\phi_{pq}^{ln})_{TE} = \delta_n^3 \quad \text{for TE incidence}
\]

(C.35)

The above is true for both arbitrary lattice geometries of the arrays and for the more symmetrical square lattice, except for the fact that the terms that are multiplied by the \( \delta \) functions would be different.

The important properties of \( \phi_{mpq}^{m'ta} \) and \( \phi_{mpq}^{m'ta} \) described here will play effective roles when we attempt to analyse double-layer FSS structures with arbitrary lattice periodicities of the arrays. Depending how the lattices within the arrays are set, these correlation functions will aid to reduce the complexities involved in relating the transform of the currents induced on the array elements to the various Floquet coefficients.

C.10