On instantaneously adaptive delta modulation and encoding of video signals

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ON INSTANTANEOUSLY ADAPTIVE DELTA MODULATION

AND ENCODING OF VIDEO SIGNALS

BY

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A Doctoral Thesis submitted in partial fulfilment of
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the Loughborough University of Technology.

December 1977.

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Department of Electronic and Electrical Engineering.

© by Fernando Toshinori Sakane, 1977.
This Thesis is dedicated to my Family, in particular
to Kumiko - my wife.
SYNOPSIS

ON INSTANTANEOUSLY ADAPTIVE DELTA MODULATION AND ENCODING OF VIDEO SIGNALS

Conventional pulse-code and differential pulse-code modulators for encoding video signals are difficult to realise economically. To alleviate this problem, a technique which divides the modulators into two stages is proposed. The first stage is a two-bit instantaneously adaptive delta modulator operating at a high clock rate and using low-precision components. Two-bit signals conveying polarity and magnitude information are produced by this delta modulator and used as the input to the second stage, a code converter. The code converter transforms, digitally, delta modulated signals into Pulse Code Modulation (PCM) or Differential Pulse Code Modulation (DPCM) signals.

The resolution of the final PCM or DPCM encoder depends on the performance of the delta modulator used as the input stage. For that reason, the performance of the Two-bit Instantaneously Adaptive Delta Modulation (2BIADM) encoder is evaluated. This evaluation is made in two steps. First, a semi-empirical analysis of the High Information Delta Modulation (HIDM) is made, because the 2BIADM system is derived from the HIDM. Then the performance of the 2BIADM is derived considering the HIDM as a reference. For the HIDM and 2BIADM modulators operating at the same sampling frequency, the 2BIADM presents an improvement in peak signal-to-noise ratio (SNR) of 6 to 8 dB. Expressions are established to enable SNR to be calculated for the HIDM as a function of the encoding parameters. The expressions also apply to Constant Factor Delta Modulation, and represent the only known method of estimating numerically the SNR for instantaneously adaptive delta modulators.

The 2BIADM was tested, built and operated at a low sampling rate. This gave an insight into the operation of the proposed system, and complemented the computer simulation analyses.

The principles for the code conversion from the 2BIADM to PCM or DPCM
are fully discussed. The 2BIADM does not impose restrictions on the values that the coefficients of the digital low-pass filter required in the code converter can assume. For low bandwidth expansion rates, it was verified that a 2BIADM-to-PCM conversion filter with 5 stages performs better than a HDM-to-PCM conversion with a filter having 256 stages (both encoders operating at the same word-rate).

A generalization of the 2-bit encoder to a N-bit adaptive DPCM system is outlined.
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<td>A/D</td>
<td>analogue-to-digital</td>
</tr>
<tr>
<td>ADM</td>
<td>adaptive delta modulation</td>
</tr>
<tr>
<td>ADPCM</td>
<td>adaptive differential pulse code modulation</td>
</tr>
<tr>
<td>ALU</td>
<td>Arithmetic Logic Unit</td>
</tr>
<tr>
<td>BBC</td>
<td>British Broadcasting Corporation</td>
</tr>
<tr>
<td>BPO</td>
<td>British Post Office</td>
</tr>
<tr>
<td>CCIR</td>
<td>International Consultative Committee of Radio</td>
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<tr>
<td>CFDM</td>
<td>Constant Factor delta modulation</td>
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<tr>
<td>CODEC</td>
<td>coder + decoder</td>
</tr>
<tr>
<td>CONFRAVISION</td>
<td>BPO conference television system</td>
</tr>
<tr>
<td>CRT</td>
<td>cathode ray tube</td>
</tr>
<tr>
<td>D/A</td>
<td>digital-to-analogue</td>
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<td>DICE</td>
<td>Digital Intercontinental Standards Converter</td>
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<tr>
<td>DM</td>
<td>delta modulation</td>
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<tr>
<td>DPCM</td>
<td>differential pulse code modulation</td>
</tr>
<tr>
<td>DR</td>
<td>dynamic range</td>
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<tr>
<td>ECL</td>
<td>emitter coupled logic</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>HIDM</td>
<td>High Information Delta Modulation</td>
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<tr>
<td>HODM</td>
<td>Higher Order Delta Modulation</td>
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<tr>
<td>IADM</td>
<td>instantaneously adaptive delta modulation</td>
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<tr>
<td>LDM</td>
<td>linear delta modulation (single integration)</td>
</tr>
<tr>
<td>LSB</td>
<td>least significant bit</td>
</tr>
<tr>
<td>LSI</td>
<td>large scale integration</td>
</tr>
<tr>
<td>MADPCM</td>
<td>modified adaptive differential pulse code modulation</td>
</tr>
<tr>
<td>MSB</td>
<td>most significant bit</td>
</tr>
<tr>
<td>MSI</td>
<td>medium scale integration</td>
</tr>
<tr>
<td>NTSC</td>
<td>National Television Systems Committee</td>
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<tr>
<td>PAL</td>
<td>Phase Alternation Lines</td>
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<td>PCM</td>
<td>pulse code modulation</td>
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<tr>
<td>PCDM</td>
<td>pulse group delta modulation</td>
</tr>
<tr>
<td>PGDSM</td>
<td>pulse grouping delta-sigma modulation</td>
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<tr>
<td>ROM</td>
<td>read-only memory</td>
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<tr>
<td>S+H</td>
<td>sample + hold</td>
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snr, SNR : signal-to-noise ratio
TTL : transistor-transistor logic
TV : television
2BIADM : Two-bit instantaneously adaptive delta modulation
VTR : video tape recorder.

LIST OF PRINCIPAL SYMBOLS

\(a_1, a_2, a_3, a_4\) (Chapter III) : constants for the calculation of slope-overload noise \([26]\)

\(a_n, A_n\) (Chapter VI) : transversal digital filter coefficients

\(b_n\) : delta modulation binary output (high/low levels)

\(b_n\) : polarity of the delta modulation error signal (±1)

\(\beta_u\) : mean power of the n-th derivative of \(x(t)\), or, spectral moments

c : digital integrator leaky constant

\(\text{DOWN}\) : logic command to decrease the step sizes in IADM

\(\Delta_u, \Delta_r\) : ADPCM quantizer (uniform) step size

\(e_n, e(nT)\) : DM error signal just before (or at) the sampling instant

\(\epsilon_i\) : DM error signal after quantization of the signal (just after sampling)

\(\epsilon_i'\) : DM filtered error signal

\(E_s\) : input signal peak amplitude

\(f_{sb}\) : (HIDM) frequency when companding starts

\(f_{sbu}\) : (HIDM) frequency when companding becomes restricted because of limitation in number of step sizes

\(f_z\) : expected number of zero crossings in a Gaussian signal

\(f_o\) : expected number of extremals in a Gaussian signal

\(f_p\) : sampling frequency

\(f_s\) : signal frequency

\(f_c\) : signal bandwidth cut-off frequency

\(f_{sc}\) : colour subcarrier frequency

\(\gamma\) : LDM step size, IADM minimum step size

\(\gamma_{av}\) : IADM average step size

\(\gamma(i)\) : step size i
\( \gamma_i \): step at time \( iT \) after the start of encoding

\( \Gamma \): \( \gamma_{av} \) normalized by \( \sigma \)

\( g_n \): transversal filter coefficient

HOLD: logic command to maintain step size in IADM

\( h_k \): low-pass filter impulse response samples

\( H \cdot \): low-pass filter operation

\( K_q \): DM empirical formula constant \([19]\)

\( m(1) \): HIDM/2BIADM step multiplier 2,1 or 1/2

\( m(2) \): 2BIADM step multiplier 1 or 4

\( M(P_{r-1}), M_1, M_2, M_3, M_4 \): ADPCM adaptation multipliers \([95]\)

\( M'_1, M'_2, M'_3 \): MADPCM adaptation multipliers

\( N, M \): number of stages in a transversal digital filter

\( N_t \): total DM noise

\( N_{or} \): DM slope-overload noise

\( N_{dq} \): DM granuler or quantization noise

\( N_p \): DM error power

\( p_i(\cdot) \): probability of occurrence of step size \( i \)

\( R \): DM sampling frequency/Nyquist frequency, i.e. the bandwidth expansion ratio

\( S^2 \): signal power

\( S(x) \): digital representation of step size \( s(N) \) or \( Y(N) \)

\( \text{snr} \): signal-to-noise ratio

\( \text{SNR} \): signal-to-noise ratio in dB

\( S \): slope-overload factor \([26]\)

\( s_{n}, Y_n \): step size (analogue)

\( S_n \): digital representation of \( Y_n \)

\( T_p \): DM sampling period

\( T_{rn} \): 2BIADM threshold for generation of second list \( T_n \)

\( U_p \): logic command to increase step size in IADM

\( x'(t) \): time derivative of \( x(t) \)

\( \tilde{x}(t), \tilde{x}_{jR}, \tilde{x}(jRT_p) \): reconstructed signal (filtered \( y(t) \)).

\( y(t), y_n, y(nT_p) \): feedback signal in DM

\( \xi = x'(t)/f_p \): first-order difference signal \([99]\)
CHAPTER I

THESIS OVERVIEW

1.1. Introduction.

Initially the proposed research program was named "Bandwidth Compression and Data Reduction of High Quality Television Signals". The objective was quite straightforward: to seek for an efficient way to digitally encode broadcast quality colour television (TV) signals for transmission. However, as it will be subsequently described, the final achievements departed considerably from the original aims. In this section, we briefly analyse the reasons for this departure and then proceed to outline the main topics investigated during the course of this research. This chapter ends by describing the arrangement of the remainder of the thesis and the principal results that we believe to be original contributions.

1.2. Motivation for the work.

The underlying question throughout this research was "why digitize television signals?". This question is quite appropriate, as the existing analogue TV systems already provide a high quality service to the public. Also, the capital investment in the present analogue systems is of such a magnitude\(^1\) that, in the years to come, analogue TV techniques will still be predominant\(^1-8\). This is particularly

\( ^{1} \) Throughout this Thesis, the references cited are not necessarily the primary sources. However, the secondary sources referred to - particularly in the next chapter - should provide adequate entries to the relevant literature, and to the primary source.
true in the domestic (i.e., receiving) front where the mass produced receivers are designed to process signals transmitted by means of analogue techniques.

The answer to the question "why go digital" as applied to TV signals has much in common with the reasons that make Pulse Code Modulation (PCM) techniques attractive. In short, some of the most important reasons can be resumed as the "three R's" of digital technology:

(a) ruggedness of the signals - digital systems operate with two-level (binary) signals and as a consequence the signals can be accurately regenerated whenever the "digital" waveform is distorted. Because of this, the quality of the system is independent of the distance the signals are transmitted and of the topology of the network which in turn can operate with any digitally encoded information such as speech, data, etc. (compatibility of traffic).

(b) repeatability - this is a characteristic that is derived from the exactness with which digital operations can be performed. In other words, operations with digital signals produce predictable performances that can be associated with numerical calculations. Consequently, digital equipments can be easily reproduced and the use of digital devices and components provide scope for application of medium and large scale integration (MSI and LSI) techniques.

(c) reliability - the reliability of digital systems stems mainly from the above two factors, as they allow the incorporation of error
detection and correction\textsuperscript{13} capabilities into the system. Also, automated control and monitoring\textsuperscript{*} techniques\textsuperscript{3,14} can be readily employed with the inclusion of computers and microprocessors into the system. F.H. Steele\textsuperscript{1}, for instance, speculates that the number of controls set up by engineers between a TV camera and a transmitter 500 miles away could be reduced from 200 (in an analogue link) to a mere 10.

Digital equipments, such as the Digital International Standards Converter (DICE)\textsuperscript{18}, are more reliable and easy to set up for operation than its analogue equivalents\textsuperscript{8}. DICE converts broadcast TV standards from the NTSC\textsuperscript{+} to the PAL\textsuperscript{++} system and vice-versa. A bibliography on the principles and applications of digital techniques to broadcast television has been recently published by Heynes\textsuperscript{4}.

However, a number of disadvantages exist. Technically, the major constraint is the large bandwidth required to transmit the video signals in the linear PCM format. The PAL-I system (see Appendix 1 for specifications) adopted in this country requires\textsuperscript{17} for transmission a channel capacity of approximately 120 Megabits per second (Mbits/s). This assumes sampling at three times the colour subcarrier frequency of \(~4.43\) MHz, each sample being encoded to 8 bits plus one additional bit used for parity check, giving \(3 \times 4.43 \times 9 = 119.7\) Mbits/s. From the economic point of view, M. Remy\textsuperscript{5}

\textsuperscript{*} Monitoring the validity of the signal, and not of the content or artistic value of the picture.

\textsuperscript{+} NTSC : National Television Systems Committee (U.S.A.) - CCIR (International Radio Consultative Committee) System M.

\textsuperscript{++} PAL : Phase Alternation Line - CCIR System I in the United Kingdom.
speculates that until the use of digital equipments becomes widespread - thus reducing costs - there could be an increase of 20-25% in the cost of a digital equipment relative to an equivalent analogue one. Also, because most of the signal processors in a studio are presently only available in analogue form (signal mixers, cameras, monitors, video tape recorders (VTRs)), there would be required a large number of coders and decoders (CODECs) during the transition period, with each CODEC pair costing over £2500. Cost and bandwidth are therefore two items that need to be reduced to ensure a more widespread use of digital techniques in the area of large-bandwidth television systems. Conversely, higher capacity channels and extensive mass-production of digital equipments would also contribute to the rapid development of digital TV systems.

The main points for and against digital television are summarized in Fig. 1.1. With these points in mind, the problem, as presented at the start of this project was to seek inexpensive, yet efficient, digital TV CODECs with full broadcast specifications. By efficient systems we meant those requiring transmission channel capacities of much less than 120 Mbits/s - say, 30 to 40 Mbits/s - for the same subjective quality of the received signals.

1.3. Background to the research and re-formulation of objectives.

Though digital TV research is a well established field of work (see next chapter), it was still in its embryonic stage in this Department*. Consequently, the main facility provided for the research,

* Department of Electronic and Electrical Engineering, University of Technology, Loughborough, Leicestershire, England.
application computers/microcomputers
application novel media (e.g. optical fibers)
application MSI, LSI, VLSI
Automation
Reliability

GENERAL

TV CODING WHY?

(a.)

TRANSMISSION

Compatibility
Traffic
Media
simple interfacing

Puggedness/Regenerability

independence of distance/topology

STUDIO

signal enhancement
Digital Recording
storage
Special Effects

TELETEXT/VIEWDATA

Sound in Syncs

SIGNAL CODING

No agreed format
too large storage/speed
not enough known for efficient coding

GENERAL

Training of personnel
cost of transition period
specialized maintenance

(b.)

TRANSMISSION

Large capacity channels

Encroachment of analogue systems

STUDIO

Analogue Instrumentation (costly to digitise)

monitors video processors cameras

FIGURE 1.01 - Digital Coding: (a) WHY, and (b) WHY NOT
in addition to supervision, was the use of digital computers: the Departmental Hewlett Packard HP2100A and the International Computers ICL1904A (of the University's Computer Centre).

Two research programs were started by the late 1973: one was based around the HP2100A minicomputer and the other was "hardware oriented". The first one had the objective of producing a computer based video encoding simulation bench. In this, a video signal from an actual source (a TV camera, for example) would be fed to the computer via a hardware interface, processed for data reduction or any other purposes, and the final results subjectively evaluated by displaying the processed signals on a TV monitor. The second project resulted in this thesis, and was originally intended to have produced real-time processors. Figure 1.2 shows the original outlines for these two projects.

The budget for the research was modest, of the order of hundreds rather than thousands of sterling-£, with the exception of computer usage time. This was, for all purposes, not limited. Economic considerations, coupled with availability of ancillary resources (such as signal sources and measuring equipments), provided the framework for much of the investigations in this research. The final achievements departed considerably from the original aims of the project, with the slow and sometimes painful realization that a Ph.D. research

* Project taken by Mr. D.M. Fenwick as part of a Ph.D. program (to be submitted).

+ The author wishes to make clear that the initial approach for full broadcast specifications was his own. This sub-section is not meant to be critical of the Department, even less of his supervisor. Both he and the Department provided me with the utmost of available facilities, able guidance and encouragement, for which the author is forever indebted.
FIGURE 1.02 - Outline of two research programs on video coding at Loughborough University.
is not governed by technical considerations alone. The principal
effect of the compromise solutions adopted during the course of
the investigation was the shifting of the emphasis from the bandwidth
compression and data reduction of digital video signals to the
digitization of television signals. Also, we have removed the rather
stringent framework of "full broadcast specifications". A secondary,
but important, effect was that we have devoted much of the efforts
to subjects that apparently have no connections with video encoding.
Instances are the heuristic analysis of noise for a class of
instantaneously adaptive delta modulators (Chapter III), and a
hardware model of the system that we propose in Chapter IV, built
with standard transistor-transistor logic (TTL) devices (Chapter V).

The reformulated objective was to seek inexpensive techniques
of digitizing video signals, with the emphasis now placed on the
analogue-to-digital (A/D) conversion process rather than efficient
transmission. One of the most inexpensive and simple techniques
for digitization of analogue waveforms is that of Delta Modulation
(DM)\(^1\) and its variations. Consequently the emphasis was given to
the application of DM techniques to video coding, and related problems.

1.4. Overview and arrangement of the thesis.

We outline briefly each of the following chapters in this thesis.

Chapter II is also an introductory chapter, in which we scan
through some of the coding techniques as applied to video signals
with emphasis on the application of DM to colour video encoding.
The survey presented is not intended to be exhaustive and is basically
conceptual only. To introduce some of the specialized terms in the
field of television signal processing, we briefly describe the video signal and its main components. We also outline some important characteristics of the human visual perception system that are relevant to television engineering. The reason for the inclusion of this chapter in the thesis is two-fold: (1) to acquaint the non-specialized reader with the terminology in TV and with some of the techniques of digital encoding (or coding) of video signals, and (2) to establish the framework for the investigations in this thesis. Part of Chapter II (section 2.1-2.5.C) can be skipped by specialized readers without detriment to the understanding of the thesis. However, in the remainder of that chapter we establish the directions taken by this thesis.

From the early stages of the research, we have concentrated our attention on encoders whose ultimate goal is to reproduce the original analogue waveform signal as close as possible: the so called waveform coding techniques. The approach that has emerged as the most suitable to pursue considering the available resources was that of code conversion of DM to PCM and related techniques (see Chapter VI). However, the results provided by Goodman in his classical paper indicated that the linear, single integration, delta modulation (LDM) encoder would necessitate excessively high sampling frequencies to achieve even modest performances when encoding video signals. As an example, a signal-to-noise ratio (SNR) of 30 dB is obtained when the sampling frequency of the LDM is over 40 times the Nyquist frequency, defined as twice the maximum frequency present in the signal. This figure of 30 dB applies to flat-bandlimited Gaussian signals, and the conversion to PCM carried out with a 5-stage digital filter.
The main limitation of LDMs stems from its narrow dynamic range\(^1\) and, consequently, we propose in this thesis the use of instantaneously adaptive delta modulators in replacement to the linear delta modulator in such code converters. We review some representative types of IADM systems in Chapter II, section 2.5.D. Then, in Chapter III we give a closer look at one IADM system, the High Information Delta Modulation (HIDM)\(^{21,22}\) with the help of computer simulations. Based on the results obtained from these computer simulations we present a heuristic approach to estimate the SNR for the HIDM. Jayant's one bit memory adaptive delta modulator\(^{23}\), or First Order Constant Factor Delta Modulation (CFDM) encoder\(^{24,19}\), is also briefly analysed. We present computer simulation results for Gaussian and sinewave inputs.

In Chapter IV we introduce the concept of a new type of IADM encoder: the Two-bit Instantaneously Adaptive Delta Modulation (2BIADM) system. As the name implies, it produces two-bit words per sample. However, it is not characterized as a Differential Pulse Code Modulation (DPCM) encoder because it operates with sampling frequencies well in excess of the Nyquist frequency and, consequently, does not require a sample and hold (S+H) circuit at the input. A simple comparative analysis by computer simulation, with the HIDM as reference basis, is presented.

A low-speed hardware model of the above coder is described in Chapter V, with SNR measurements for narrow-bandlimited noise and sinewave inputs. The circuit described is a versatile one, which allows the 2BIADM encoder to be compared with three HIDM encoders and if so desired with LDM encoders with up to 63 step sizes.
The 2BIADM encoder has been developed with the aim of producing a simple and inexpensive A/D converter, whose output is locally applied to a digital filter. This filter rejects the out-of-band noise to allow the re-sampling of the digital stream at near the Nyquist frequency. According to the filter design, its output can be PCM or DPCM words. Chapter VI describes two digital filter structures that can be used in a 2BIADM-to-PCM code converter. The techniques described are based on existing DM-to-PCM and DPCM converters, which are briefly described in that chapter.

Finally, in Chapters VII and VIII we present a generalization of the scheme proposed in Chapter IV and the main results are analysed and criticized where deemed necessary, with suggestions for further work.

The over-all arrangement of the Thesis is shown schematically in Figure 1.3.

1.5. Summary of main results.

Two almost independent results are presented in this Thesis, i.e., they can be separately appraised. First, in Chapter III we show that by defining a statistical average step size $\gamma_{av}$, we can, to a reasonable approximation, estimate the noise generated by the HIDM and the 1st order CFDM encoders. The expressions used to estimate the noise, when encoding Gaussian signals, are those applicable to LDM. These are the "granular" (or quantization) noise and the slope-overload noise components. Some discrepancies from computer simulated results are observed and an attempt is made to analyse the reasons for these deviations.
CHAPTER I
Thesis Overview

CHAPTER II
Digital Encoding of Video Signals
an overview

CHAPTER III
Estimation of SNR for HIDM and CFDM

CHAPTER IV
2BIADM (new system)

CHAPTER V
Low-speed Hardware Model

CHAPTER VI
Code Conversion DM-to-PCM

CHAPTER VII
Suggestions for further research (generalization)

CHAPTER VIII
Recapitulation & Review

FIGURE 1.03 - Thesis lay-out
Then, in Chapters IV to VI we propose the use of the 2BIADM encoder followed by a digital filter and re-sampler as an alternative to the straightforward PCM encoder constituted by a sample and hold circuit followed by a multi-level quantizer. The results obtained by computer simulation and low-speed hardware model of the 2BIADM are analyzed relative to video encoding, and a critique of this procedure is given. The results show an improvement of up to 8 dB over existing HIDM encoders. It is also shown that the proposed system presents better frequency response under certain circumstances and a generally improved transient response. We also conclude that because of the maximum speed that a digital filter can operate, there is no possibility of achieving broadcast quality specifications with the use of the 2BIADM-to-PCM code converters. However, the extension of the technique to adaptive DPCM-to-FQM code conversion on the same lines suggested in this thesis appears to be promising.
CHAPTER II

DIGITAL ENCODING OF VIDEO SIGNALS - AN OVERVIEW

2.1. Introduction.

Of the many areas of research in image processing\(^{27}\), we concentrate on waveform encoding\(^{11}\) of television signals. Because most of the thesis revolves around DM related techniques, the emphasis is placed on a review of the application of DM to video encoding.

For the benefit of the non-specialized reader, we give initially an outline of some of the most important macroscopic characteristics of the human eye. We also present a brief description of the analogue video waveform, which constitutes the input signal to most digital encoders.

A more mathematically rigorous presentation of much of the subject that is presented in section 2.4 can be found in a book by Pearson\(^{28}\). In his book, Pearson presents concisely the process of video signal formation as a three-dimensional sampling operation on the image of the scene one wishes to transmit. He also introduces the principles of monochrome (i.e. "black and white") and colour television transmission and reception, the properties of the eye important to television systems design, and principles for subjective assessment of picture quality. However, digital television constitutes only a minor item in Pearson's book.

For a general introductory survey of digital picture coding, a paper by Habibi and Robinson\(^{29}\) offers an overview of this field of
research that is suitable to the non-specialized reader. The proceedings of a conference on bandwidth compression of picture signals have been published as a book\textsuperscript{30} and it has a number of review/survey types of papers on the most important aspects of picture bandwidth compression: the human observer, some of the theory behind data compression, and the major categories of video coding techniques.

Other specialized review papers have been published\textsuperscript{31-38}, of which references \textsuperscript{[31]} to \textsuperscript{[35]} concentrate on systems developed in the U.S.A. Those in Germany are reviewed in \textsuperscript{[36]} and a short listing of video CODEC designs in Japan is given in \textsuperscript{[37]}. The paper by Limb, Rubinstein and Thompson\textsuperscript{38} presents a comprehensive review of coding of colour TV signals, giving also the background of colour signal representation and colour perception. We introduce the reader to them by presenting a schematic survey of digital encoding of video signals in Section 2.5.

There is also a large number of papers on the subject of picture coding in special issues of IEEE journals\textsuperscript{39-42} and books\textsuperscript{11,30,43,44}. A bibliography\textsuperscript{45} on picture bandwidth compression and data reduction has been published, though limited to the period pre-1971.

2.2. The Television system.

The digital television system can be represented as in Fig. 2.1. Comparing this structure with Pearson's extension of Fano's communication system model\textsuperscript{46}, it can be seen that the source encoder has been split into two, and the importance of the encoding noise (or distortion) is emphasized. As with Pearson's model, the importance
FIGURE 2.01 - Video Communication System - a simplistic model

(a) Shannon model; (b) Video system
of the human viewer as an integral part of the system is highlighted by providing a quality or fidelity measure feedback from the human receptor to the various encoder stages. We indicate between brackets the basic components of the generalized model of a communication system by Shannon 47.

The objective of a television system is to convey to the viewer some sort of visual information. The signal source is primarily, but not always, a time dependent three-dimensional visual scene. The source analogue encoder in Fig. 2.1 is a stage that produces an analogue video signal \( W(t) \) that can be used to reconstruct a two-dimensional picture of the original scene in a television display. \( W(t) \) is usually in the form of the existing analogue television video signal (see section 2.4).

The criterion for the design of the source analogue encoder, indicated as (1) in Fig. 2.1, is that of determining the encoder specifications based on compromises between the system parameters and a "best" reproduction of a TV picture. Once the system parameters are fixed, the distortions introduced by the encoding process are essentially independent of the signal, provided the latter is within the ranges of amplitude and frequency defined for the system. The source analogue encoder performance can be objectively evaluated by means of secondary assessment techniques derived from subjective tests 48.

In the analogue TV system, \( W(t) \) would be fed to an analogue channel encoder, normally a VHF or UHF carrier modulator. In a digital TV system, \( W(t) \) constitutes the signal source for a source digital encoder. The latter produces a digital representation \( U(t) \)
of \( W(t) \). The source digital encoder can assume various forms, such as PCM, DPCM, DM encoders, or their variations (see section 2.5), and transform coders.

The design of the digital encoder can be separated in two groups. In the first, the aim is to obtain an "exact" reproduction of the input waveform signal, such as the linear DPCM or PCM system. In such cases the criterion (1) can be used. In the second and more common group, the objective is to obtain a representation of the input signal that produces a picture that is subjectively similar to the original one. The distortions, when objectively measured, would depend greatly on the signal. The quality or fidelity criteria measure in this case, indicated as (2) in Fig. 2.1, cannot be made into objective measurements at the present except for some particular cases. The tapered quantizer DPCM is one example.

The source encoders, analogue and digital, are usually represented in the literature as one stage, named simply source encoder or source-receiver encoder. The separation into two elements, however, facilitates the visualization of digital television systems, particularly in the case of colour systems. In such systems, there may be considerable signal manipulation before digitization and there may be a case for not adopting existing analogue formats.

The channel encoder is a processor that transforms \( U(t) \), reversibly, to a format \( X(t) \) that is more convenient for transmission. The criterion (3) for the channel encoder is the amount of disturbances (jitter, transmission errors, etc.) that can be tolerated for a satisfactory reception of TV signals and is also based on subjective tests.
The channel decoder and the analogue and digital source decoders perform the inverse operations to those of the corresponding encoders at the transmitter. With the exception of the channel CODEC, the remaining stages introduce non-reversible distortions, specially when non-linear processes are involved.

2.3. The Eye - macroscopic characteristics.

In the system of Fig. 2.1, the only fixed element is the human receiver, in particular the response of the eye and brain to visual stimulation. We summarize here the properties that are commonly exploited in television encoder design.

To keep the review short, we assume that the fundamental photometric concepts and units are known. The non-initiated reader can refer for instance to Chapter 2 of reference [28] for a concise review.

We first summarize some of the most important characteristics of achromatic vision, viz. (a) contrast resolution, (b) spatial resolution, (c) temporal resolution, and (d) how they can effect television coding.

Contrast resolution relates to how the brightness or lightness differences are perceived. It is at the simplest described by the Weber-Fechner law

$$\frac{\Delta B}{B} = \text{constant} \quad (2.1)$$

where B is the brightness\* of the background field and (B + \Delta B) is the brightness of a sharp-edged circular target in the center of the field, as shown in Fig. 2.2a. For \Delta B producing a just noticeable

* The brightness B can be related to the luminance L of the source by the relation $B = a L^\gamma$, where a and $\gamma$ are constants\textsuperscript{28}.
FIGURE 2.02 - Brightness contrast sensitivity (after Schreiber [31])

FIGURE 2.03 - Brightness contrast sensitivity with background as parameter (after Schreiber [31])
difference, a curve of contrast sensitivity can be plotted \(^3\) (Fig. 2.2b) for \(B\) varying.

The results indicate that the Weber ratio is practically constant over a wide range of brightness levels (about 10,000 to 1). However, if the contrast sensitivity is measured against a constant background brightness \(B_0\), used as parameter for the measurements (Fig. 2.3a), the curves of Fig. 2.3b are obtained. The curves now indicate much lower dynamic ratios because of a "masking" effect on the target by the background. Masking is defined \(^3\) as the influence of the signal surrounding a point in a picture on the perception of the signal at that point.

The Weber ratio indicates that if the background brightness is increased, a larger change in the brightness difference \(\Delta B\) can occur before the change becomes visible. It hints that encoding distortions can be larger in areas of increased brightness. Fig. 2.3b indicates that the visibility of the changes depends greatly on the adaptation of the eye to the background field and, as the eye presents a finite time to respond to excitation changes (persistence of vision), this hints that the picture contents, such as detail and movement, should have a strong influence on the perception of any distortion or noise. Therefore, one can expect that there should be an interaction between the spatial and temporal responses of the eye.

The spatial and temporal resolutions of the eye can be measured with sinusoidally modulated luminance signals. For the spatial frequency response, the signal is constituted by vertical bars whose luminance changes in the horizontal direction, \(x\), as given by Eq. 2.2

\[
L = \overline{L} + \Delta L \cos(2\pi ux) \tag{2.2}
\]

where \(u\) is the spatial frequency in cycles per degree subtended to the eye, \(\overline{L}\) is the average (or "dc") luminance and \(\Delta L\) is the peak luminance deviation.

Similarly, for the temporal frequency response,

\[
L = \overline{L} + \Delta L \cos(2\pi ft) \tag{2.3}
\]
The visibility of temporal changes in luminance levels caused by Eq. 2.3 originates the sensation of "flicker", and the temporal contrast sensitivity is also known as "flicker sensitivity".

When $\Delta L$ gives the just noticeable visibility (i.e. $\Delta L$ is the threshold modulation amplitude) the curves $L/\Delta L$ for varying frequencies constitute the contrast sensitivity curves that are shown in Fig. 2.4a and b for the spatial and temporal cases, respectively.

The response of the eye to horizontal bars is similar to that for vertical bars, hence the eye can be viewed as a three-dimensional low-pass filter with cut-off bandwidths of about 60 cycles/degree in the vertical and horizontal spatial frequencies and about 70 Hz in the temporal frequency. These values are for a bright display, and decreases with its average display luminance. The spatial response also varies with the orientation of the bars, and at 45° the sensitivity is decreased by 10-20%.

The visibility of random noise therefore decreases with increasing frequencies both in time and space domain, and increasing luminance levels. In short, with reservations:

1. noise is less visible in pictures with higher amount of detailed areas
2. random noise is less visible, and less annoying, than correlated noise and regular patterns are more noticeable than random ones
3. sharp changes in luminance levels in space or time reduce the visibility of small luminance changes in their vicinity (masking)
4. the higher the amount of movement (high temporal frequencies), the less noticeable are the details (high spatial frequencies)
5. at high spatial frequencies, the contrast resolution is reduced
Figure a. Contrast sensitivity of the eye for sine wave gratings (after Campbell and Robson, 1968). The pupil diameter was 2.5 mm.

Figure b. Flicker sensitivity of the eye for a 2 degree field at various values of retinal illuminance (after de Lange, 1958). As a very rough approximation for display viewing, 100 trolands is equivalent to about 10 cd/m² in the display.

FIGURE 2.04 - Resolution sensitivity (after Pearson [28])
(a) spatial; (b) temporal
(6) at high luminance levels, the eye is more sensitive to flicker but less to luminance changes (spatial).

(7) flicker is less noticeable in highly detailed areas.

Efficient design of video CODECS can therefore take advantage of the exchanges between spatial (detail), contrast, and temporal (motion) resolutions.\(^54,33\)

The properties of the eye that we have so far mentioned are related to the achromatic vision. Apart from brightness, another feature that can be incorporated into a television system is that of colour.

The description of a colour is made in terms of three notional parameters: hue, brightness or lightness, and saturation. Hue defines the colour as red, blue and so on, viz. it gives the name to the colour. It is related to a dominant wavelength, which is the wavelength of the spectral colour that matches (i.e. "equals" in terms of perceived colour) the fully saturated colour under consideration. Brightness determines how strongly the colour excites the eye and is quantified in terms of "luminance" of the colour. Brightness refers to self-luminous objects whereas lightness is the equivalent notion for non-self-luminous objects. Saturation gives a measure of how much the colour is diluted in white, and is related to the excitation purity, or simply purity. A 100% purity means fully saturated, i.e. it refers to the spectral colours, and a 0% purity refers to the reference white.

The perception of colour has an important property that is fundamental to an economical realization of colour television systems: the trichromaticity of colour vision. Its most important characteristic
is that most colours can be reproduced as a linear combination of three primary colours, or simply primaries. Any three colours such that none is the result of the combination of the other two can be used as primaries, but in practice the primaries are chosen so that a large, bright, gamut of colours can be reproduced, and that the primaries themselves are easily reproducible. The television primaries are particular shades of red (R), green (G), and Blue (B).

Most colours can therefore be described by the following equation:

\[ L_C = L_R + L_B + L_G \]  

(2.4)

where \( L_C \) is the luminance of the colour and \( L_R, L_B, \) and \( L_G \) are the luminances of each of the primaries to match the perceived colour \( C \).

For the purposes of this brief review, we will not examine how colours can be numerically represented, and interested readers can refer for instance to references [16] and [28] for detailed accounts, and to [38] for a concise presentation.

It suffices to know here that colour television systems can transmit the luminance and two colour-difference or chrominance signals; the luminances of the three primaries, or linear transformations of these. The luminance signals in the colour television system can be received and processed by monochrome receivers to display black and white pictures, and monochrome TV signals can be received by colour TV receivers and processed as luminance signals to display black and white pictures in the colour kinescope (the television cathode ray tube - CRT).

* luminance and luminance signal should not be confused. Following established practice in the field of television, luminance signal refers to the electrical waveform that conveys the brightness information in TV, and luminance is a psychometric quantity that describes the perceived brightness or lightness of an object.
The chrominance signals are effectively independent of the luminance signal, i.e. changes in the amplitude of the chrominance signals do not affect the luminance display but only the chromaticity, viz. hue and saturation. Changes in the luminance display do not affect the chromaticity, but only the brightness of the picture.

When the luminance signal and chrominance signals are used, the response of the eye is such that a sharp transition of luminance signal levels masks a smooth (blurred) change of chrominances and vice-versa. However these effects are strongly dependent of the colours, luminance levels and types of transitions involved. A random noise added to the chrominance signal has its visibility reduced for increased luminance levels.

Bushan presents results of subjective measurements of PCM quantization noise when the three primaries are used for picture transmission. The results are shown in Fig. 2.5, and it is clear that each primary has different responses to PCM quantization noise, and so has their combination and the monochromatic signal, with the sensitivity to noise being least to the blue signal and largest to the luminance only case.

In conclusion, it is never too often to remind ourselves that not enough is known about colour and monochromatic vision, and specially how different types of distortions and noise affect our perception of TV pictures. The overview given here only scratches the surface of the subject of the response of the human observer to visual stimulation, but it should be sufficient to understand most of the encoding techniques described in literature, and we hope also
COMMENTS
1. NOT PERCEPTIBLE
2. JUST PERCEPTIBLE
3. DEFINITELY PERCEPTIBLE,
   ONLY SLIGHT IMPAIRMENT
4. IMPAIRMENT BUT NOT
   OBJECTIONABLE
5. SOMEWHAT
   OBJECTIONABLE
6. DEFINITELY
   OBJECTIONABLE
7. EXTREMELY
   OBJECTIONABLE

FIGURE 2.05 - Subjective effects of PCM random noise (after Bushan [31])
to use this knowledge to justify later chapters in this thesis.

2.4. **The analogue video waveform.**

We present here a short and qualitative description of the formation and format of the analogue video waveform. Interested readers can refer to the many text-books on the subject for a more complete analysis. In particular the book edited by Fink\(^{56}\) is comprehensive in all aspects of analogue TV systems, including colour, but confined to NTSC systems as the book was published in 1957; the two books by Carnt and Townsend\(^{16}\) cover all major colour TV systems, with a large part devoted to colour representation; and Sims\(^{15}\) is a useful introductory book because of its concise block-diagrams approach. The book by Pearson\(^{28}\) differs from most other text-books because of its emphasis on signal formation and analysis as a three-dimensional sampling process (with Fourier expansion), aspects of digital transmission and reception, and subjective assessment of pictures. The presentation given here should suffice to acquaint the non-specialized reader with the terminology employed throughout the thesis.

A. Monochrome TV.

A two-dimensional electronic image of the scene to be transmitted is formed by a distribution of electronic charges whose density is proportional to the brightness of the light falling onto the opto-electronic *target* of the picture tube\(^{56, 57}\). This distribution of charges is "read" by a scanning beam of electrons. This reading, i.e. the information retrieving process, consists of the electronic
beam scanning the target from left to right and from top to bottom. The visual information is then extracted as a series of vertically spaced and nearly horizontal lines, in much the same way as words are arranged and read in this page. When the scanning beam is returning (flyback) from right to left and from the bottom to the top of the picture, the information retrieval process is inhibited, i.e. blanked. The process is illustrated in Fig. 2.6a.

One complete scanning from top to bottom of the image constitutes one field. After a field is produced, the scanning beam is returned to the top of the picture to produce another field, and so on. This process is equivalent to sampling a two-dimensional image in the vertical direction by a series of horizontal lines that covers the entire image. The higher the number of horizontal lines, the higher is the spatial resolution in the vertical direction.

The frequency of repetition of the fields is determined by the need to provide continuity of brightness in the television pictures, i.e. pictures without "flicker" effects. In the system I adopted in the U.K., the field frequency is set to 50 Hz. However, continuity of movement can be achieved if a sequence of frames is presented to the eye at about half that rate. Therefore, to accommodate efficiently the requirements of continuity of movement and brightness, each frame is made up by two interlaced fields (Fig. 2.6b). The two fields are said to be interlaced as the lines that constitute each field are spacially interlaced. The scanning process that generates such fields are said to be line interlaced scanning. In the non-interlaced scanning, the fields and frames coincide. This separation of frames into two fields follows closely the process in motion pictures where
amplitude proportional to brightness

horizontal scanning

video information

agest scanning ("horizontal") lines

Target

horizontal flyback (no reading)

vertical scanning:
field frequency = 50 Hz

frame frequency = 15,625 Hz

FIGURE 2.06 - Scanning in TV
(a) principle; (b) interlaced scanning
24 frames per second are recorded in a film, to provide continuity of movement, and each frame is then projected twice (i.e. at a rate of 48 projections per second) to give continuity of brightness. In both cases, to produce 50 frames per second in TV or 48 different frames per second in motion pictures would be redundant for the average moving scenes. In motion pictures, this redundancy would be reflected by the doubling of the film physical length and in TV it would double the video signal bandwidth\(^{56}\) (other parameters kept constant).

To summarize, the important points in video signal generation are illustrated in Fig. 2.7, where we show that a TV picture is constituted by frames, each frame by two fields and each field by lines. In the system I, there are 625 lines per frame and consequently 312\(\frac{1}{2}\) lines per field. This half line off-set is necessary for the interlacing process.

The nature of a line and a field in the context of television signals will now be described. When the electronic image in the picture tube target is scanned by an electron beam, an electrical signal (current) whose amplitude at any given time is proportional to the brightness of the spot where the beam is focused on is extracted\(^{56,57}\) from the picture tube as a voltage signal (resistive load). The voltage amplitude of the electrical signal that corresponds to a given brightness is often referred to as level of that particular brightness, such as black level, grey level, and so on. When the scanning electron beam is returned from the right to the left of the picture, there is no output from the picture tube and the corresponding level in the video signal is known as horizontal blanking.
FIGURE 2.07 - Principle of TV signal formation
level, and similarly for the flyback from the bottom to the top of the picture there is the vertical blanking level.

In the system I, the blanking level is also the black level, but in some other systems, such as M, adopted in the U.S.A. and Brazil (for example), they can differ. The time interval during which the television waveform is kept in the blanking level is known correspondingly as horizontal and vertical blanking intervals.

As the reconstruction of the picture in the receiver requires that the horizontal lines, the fields, and the frames be synchronized, horizontal and vertical synchronizing pulses (in short sync pulses) are added to the video waveform. To distinguish the sync pulses from the visual data, they are added towards the blacker levels of the video waveform, providing blacker than black levels. To distinguish horizontal sync pulses from the vertical sync pulses, their durations are made different. Horizontal sync pulses are also provided during the vertical flyback so that the horizontal line oscillator in the receiver is always in synchronism with the transmitter one. As the timing of the horizontal lines relative to the vertical flyback differs by half a line period between two consecutive fields due to interlacing, equalizing pulses are provided to compensate for this difference, around and during the vertical sync pulse duration. The complete video signal waveform is known as the composite video signal, and is shown in Fig. 2.8.

B. Colour TV.

The fundamental feature of colour that is exploited for the development of chromatic television is that most colours can be
Fig. 1. The waveform of a typical line showing synchronising signals. Pulse duration is measured at half-amplitude points. Blanking duration is measured at half-amplitude points with a white level signal of line duration and for this reason the picture signal has been shown starting and finishing at the white level.

Fig. 2. Vertical synchronising and blanking waveforms for a typical signal. Lines 7–14 and 320–327 have been omitted. Rise and fall times are from 10% to 90% of the pulse amplitude and for the field blanking are 300±100 ns, and for the field sync pulses and the equalising pulses the rise and fall times are ±50 ns.

Note 1: Lines 16–20 may contain identification control or test signals.

Note 2: The first and second fields are identical with the third and fourth in all respects except burst blanking.

FIGURE 2.08 - Composite video waveform (monochrome).[58]

(a) line period; (b) even and odd fields
represented as a linear combination of three primary colours. The main features that allows the colour TV system to be compatible with a monochrome TV system are:

(1) the brightness information used in monochrome TV systems is
the luminance information in the colour system, and the luminance signal is the result of a linear combination of the luminances of the three primaries

(2) the colour information can be transmitted within a bandwidth that is much smaller than that of the luminance signal

(3) the power spectrum of the monochrome TV signal present gaps as the spectral components are clustered around the harmonics of the line frequency, forming a line spectrum. The colour information can be inserted in these gaps

(4) apart from the luminance of the colour, there are only two more features that need to be transmitted to characterize the colour: the hue and saturation. These can be transmitted as one vector whose amplitude and phase (relative to a reference signal) convey the saturation and hue data, respectively.

Taking advantage of these features, in colour TV the colour source is first split optically into its three primary components, from each of which electrical signals are derived following a process similar to that for monochrome TV. The three colour components are usually referred to in literature as the R, G and B signals after the colours used as the primaries in TV, viz. red, green, and blue, respectively. From these three colour components, three other signals are derived for transmission, by a linear matrixing process: a
luminance \( Y \) and two colour-difference or chrominance signals \( C_1 \) and \( C_2 \). These are linear combinations of the differences \( R-Y, B-Y, \) and \( G-Y \), and the weights for the combinations depend on the particular standard of colour transmission. For the PAL system, the luminance and chrominance signals are\(^5\):

\[
Y = 0.299R + 0.587G + 0.114B \\
C_1 = U = \frac{B - Y}{2.03} \\
C_2 = V = \frac{R - Y}{1.14} 
\]

The chrominance signals modulate one colour subcarrier in quadrature. A quadrature modulation is accomplished by \( C_1 \) modulating in amplitude, with suppressed carrier, the subcarrier signal at a certain phase and \( C_2 \) does similarly with a subcarrier of exactly the same frequency as for \( C_1 \), but with its phase varied by \( \pi/2 \) (thus, quadrature). The resulting signal produces a subcarrier modulated in phase and amplitude. The phase carries the information about the hue of the colour, whereas the amplitude carries the saturation information.

The frequency of the subcarrier is carefully chosen (see for instance \( [16] \)) so that the chrominance signal spectrum falls within the gaps left vacant by the luminance spectrum. In the NTSC system, this happens if the subcarrier frequency is equal to an odd multiple of half the line frequency (half line off-set). However, in the PAL system the \( V \) component is switched by \( 180^\circ \) at each consecutive line, and this introduces a half-line offset in the spectrum of the \( V \) signal relative to that of the \( U \) signal. To avoid the coincidence of the \( V \) component spectrum lines with those of the \( Y \) spectrum, the
offset employed in the PAL colour systems is close to a quarter of the line frequency. Unlike in the NTSC case, the spectrum of the two chrominance signals in the PAL system do not coincide with each other. The subcarrier frequency for the PAL-I system is

\[ f_{sc} = (284 - \frac{1}{4})f_L + 25 \text{ Hz} \]  

(2.6)

The 25 Hz added to the quarter line offset frequency is intended to minimize the visibility of the subcarrier signal in monochrome reception. With

\[ f_L = 15.625 \text{ kHz}, \quad f_{sc} = 4.43361875 \text{ MHz}. \]

The signal formation and the composite colour TV signal is summarized in Fig. 2.9. In Appendix 1 we give some specifications required for broadcast TV (analogue).

2.5. Digital encoding of video signals - an outline.

A. Introduction.

The field of image processing for efficient transmission encompasses many types of visual communication systems, as sketched in Fig. 2.10a and b. The classification presented here is to some degree arbitrary, but should serve to give an indication of the types of services commonly encountered in practice. The two groupings real/non-real time picture processing and colour/monochrome systems are not mutually exclusive, i.e. moving pictures can be in colour or monochrome, colour pictures can be processed in real or non-real time, and so on.

In Fig. 2.10a, CCTV stands for Closed Circuit Television, and its specifications may vary greatly with particular applications.
FIGURE 2.09 - Colour TV system

(a) principle of signal formation; (b) composite waveform (line)
FIGURE 2.10 - Coding WHAT?: (a) What I ; (b) What II
CONFRATION is a British Post Office (B.P.O.) trade name for the service that provides two or three way vision and sound communication links between two or three different cities (not necessarily in the same country). It allows conferences to be held without requiring that all participants be physically present in the same conference room. The remaining types of services indicated in Fig. 2.10 do not require further explanations.

The main area of research that we outline in this section is that of waveform coding of the large bandwidth (4.2 to 6 MHz) public television signals. From here onwards by "television", we refer to this type of system.

Some of the techniques that we will refer to have been applied only to video-telephone coding. They are included here as the experience gained from the encoding of the small bandwidth (1 MHz) video-telephone signals may provide valuable insight for the development of television coders. It should be noted, however, that most of the investigations on video-telephone coding have been on monochrome signals. To the knowledge of the author, when colour is involved only component coding, i.e. the coding of the luminance and two chrominance signals or of the three primaries, has been used with video-telephone signals.

Though in principle television and video-telephone systems follow the same principles to produce the electrical signals that are to be encoded, the statistics of these signals differ because

* The Bell Labs/AT&T system is known as Picturephone, and the B.P.O. system as Viewphone. Abridged specifications of both systems can be found in reference [28].
of the different types of services that they offer. The visual source in the video-telephone service is usually a "head and shoulders" picture whereas in public TV the picture contents depend on particular programs. Therefore, great care must be taken when video-telephone coding techniques are contemplated for application with television signals.

B. Linear PCM representation.

So far, the application of digitally encoded broadcast television signals has been confined to the studio. All the local applications of digital television are with the linear, 8-bit PCM representation of the signal.

The principles of PCM have been extensively analyzed by Cattermole and other authors, therefore here we only present briefly one popular technique employed for the A/D conversion of television signals.

Because of the large bandwidth of the signal base-band, 5.5 MHz in the PAL-I system, the compromise solution between cost and hardware determined by the conversion speed considerations is that of cascading two stages of all-parallel 4-bit encoders. The basic principle for this type of structure is shown in Fig. 2.11a. The video signal sampled at around the Nyquist rates is first encoded by an all-parallel 4-bit A/D converter. This quantizes the input sample into 16 levels. Then, the difference between the input sample and the decoded quantized sample is applied to a second all-parallel 4-bit PCM encoder that resolves this difference into a further 16 possible levels. The first A/D converter produces the four most significant bits (MSBs) and the
FIGURE 2.11 - PCM Analogue-to-digital converter parallel/series

(a) principle; (b) BBC system (after Fletcher [60])
second, the four least significant bits (LSBs). The total resolution
of the encoder is $16 \times 16 = 256$ levels.

Although the 4-bit A/D converters quantize the samples into 16
levels, this quantization has to be achieved with an 8-bit quantizer
resolution, i.e. with an accuracy of one in 256 levels. This places
stringent requirements on the quantizers, the D/A converter and
subtractor used. To ease these requirements, the British Broadcasting
Corporation (BBC) Research Department has developed the encoder of
Fig. 2.11b where correction stages are inserted into the arrangement
of Fig. 2.11a.

Firstly, detection facilities for positive and negative overflows
that may occur because of quantization inaccuracies in the first 4-bits
stage. When such overflow occurs, the four MSBs stored in an
intermediary store are corrected in the following way:

1. If a positive overflow occurs, the second encoder quantizes the
difference between the input and the maximum amplitude (1/16 of
the over-all quantizer range) expected for the second stage,
and a "1" is added to the MSBs.

2. If a negative overflow occurs, the second encoder outputs the
value 16 minus the number of overflow increments and subtract
"1" from the MSBs.

Secondly, the subtractor is provided with a gain of 10, in this
case, to make the second quantizer less sensitive to encoding noise.
A muting device eliminates spurious transients in the subtraction
and artificially reduces the settling time of the subtractor amplifier.
This is achieved by re-sampling the output of the subtractor after a
delay long enough to let the transients subside to within 3% of the final value of the sample.

Sixteen voltage comparators are used for each of the 4-bit quantizers and 0.1% tolerance resistors for the generation of the reference voltages. The typical acquisition time for the sample and hold circuitry is of 20 ns and the maximum aperture uncertainty of less than 0.2 ns. Typically, the sampling period is 75 ns (approximately three times the colour subcarrier frequency), and the total conversion time is about 240 ns. A conversion time longer than the sampling interval is possible because of the use of a production line technique in which, by carefully timing the various encoding stages, one stage processes a sample while the other stages are processing adjacent samples.

It is easy to see that the speed and component precision requirements are rather stringent, making the cost of such converters of the order of £2,000 (1976/77 price).

PCM encoders introduce uniform distortion within the signal pass-band and the design is based primarily on the maximum frequency component in the signal and the resolution required. The first requirement determines the sampling frequency and the parameters of the sample and hold circuitry. The second requirement determines the quantizer resolution, viz. the number of bits per sample, and the components precision specifications. The PCM quantization ignores the correlation that might exist between the samples in the signal, or the differences in the perception of noise or distortions by the eye. Consequently, more efficient digital encoders can be designed by considering such factors. In the overview that
follows, we limit our scope to the main ideas behind coding schemes that can be found in the literature 29-38.

C. Bandwidth compression and data reduction.

Bandwidth compression and data reduction in television can be accomplished by processing directly at the source, on the analogue video waveforms, or on their digital equivalents in linear PCM format. The vast majority of the systems described in literature falls within the last two approaches for two main reasons. First, the elaborate process of the analogue video formation, specially in the case of colour television, is based already on many of the considerations that will govern the development of digital data reduction techniques. As an example, the choice of the chrominance signals produces results that are nearly as efficient as the more elaborate techniques that can be used to de-correlate the three primary components 38. Secondly, it is a matter of convenience as existing video signal generators, such as TV cameras, can be used without modifications. Therefore, techniques that operate on the standard analogue video waveforms, or their PCM equivalents, take advantage of the considerable data reduction and bandwidth compression already achieved for the existing analogue television signal.

Television encoding systems can be broadly classified into two major groups: orthogonal transform 34,35 and video waveform 31-33 coding techniques.

In the orthogonal transform coding systems, the video signal is first mapped into a new domain where redundancies are better revealed than in its original time waveform. The redundancy reduction is
performed on the transformed data. Examples of transform coders are those that employ \(^{34,35}\) Fourier, Walsh-Hadamard, Karhunen-Loève and Slant Transforms.

With the video waveform encoding systems, the objective is to reconstruct the TV picture from its time samples, or from a selected number of such samples.

Before proceeding further, we note that many video encoding systems can be better visualized by assuming that the video signal samples are spatially arranged in the positions that would correspond to in a television display, i.e. along lines forming fields and frames. When the video signal is sampled at near the Nyquist rates, each sample can be called \(^{32,34}\) a picture element, in short pel or pixel, as in analogue TV systems where these terms refer to the smallest resolvable detail in the picture \(^{56}\). The samples that constitute one frame can be arranged in a two dimensional geometrical space and the succeeding frames along a third, temporal, dimension. Therefore, although strictly speaking all the samples are taken sequentially from a voltage waveform that is a one-dimensional function of time, television encoders can be said to operate on one, two, or three dimensional spaces to describe systems that use samples from one line, more than one line but within one field, and more than one field, respectively.

For video waveform coding systems, the term intraframe \(^{33}\) is used to describe one and two dimensional coders, and interframe \(^{32}\) for three dimensional coders, though these terms can be equally employed for transform coders.

The work on the data reduction of digital television signals
applying any of the techniques (Fig. 2.12a) discussed above aims basically to take advantage of:

(1) the statistics of the signal\(^{63, 64}\), in what Schreiber\(^{31}\) calls a "pure statistical coding". The technique envisages to de-correlate the samples in the signal by either linear transformations or differential quantization\(^{70, 19}\) based solely on the signal statistics;

(2) the properties of the eye as reviewed in section 2.3, where the limitations and properties of the eye are exploited. This technique is called psychovisual or psychophysical coding, as distortions in the signal are allowed to occur as long as they are not subjectively noticeable, or at least not objectionable.

The two approaches are not mutually exclusive, and it is not hard to see that efficient encoding techniques would involve both the statistics of the signal and the eye properties, in what Schreiber calls "psychostatistical" coding. Fig. 2.12b illustrates the main points discussed above.

Of the two major classes of video encoding techniques, viz. transform and video waveform coding techniques, we concentrate our attention on the latter. The majority of those falls in the category of differential encoders\(^{70, 19}\) and related systems. Differential quantizers take advantage of the correlation that exists between the video signal samples by transmitting the difference between the actual input signal and an estimate, or prediction, of that signal based on its past. A short historical note is given by O'Neal\(^{71}\) in his classical paper about predictive quantization of television signals, and several
FIGURE 2.12 - Approaches for data reduction in video signals
(a) waveform coding ; (b) design approaches
possibilities for the arrangement of predictive systems are given
by Noll\textsuperscript{72}, though his encoders are analysed for speech signals only. Predictive encoders are usually arranged in a closed-loop structure, which automatically compensates for errors in quantization\textsuperscript{70}. Closed loop encoders, however, need not be viewed only as predictive encoders. One example where the analysis follows different lines is that of a noise-feedback encoder\textsuperscript{73}, where instead of an estimate of the signal the quantization noise is fed-back to the input.

The other group of video waveform encoders is constituted by open loop structures, i.e. without any feedback from later stages to earlier ones. Examples are PCM with dither\textsuperscript{74,17}, sub-Nyquist sampling\textsuperscript{75} and dual mode systems\textsuperscript{31,32}. Most transform coders would be in this category also. However, this thesis is confined to the closed loop encoders, and in particular to the application of DM techniques to broadcast video encoding.

In the next sub-section we review the application of DM to colour video encoding.

D. Application of DM techniques to video encoding.

Delta Modulation is a type of predictive, waveform tracking, differential encoder and has attracted attention because of the simplicity of its basic operation principles.

A comprehensive study of DM systems, including historical developments and general applications, is given in the book by Steele\textsuperscript{19}. At its simplest, a delta modulator is constituted by a subtractor, a two-level (one-bit) quantizer and a linear integrator in the feedback path (Fig. 2.13). With such an encoder, an analogue input signal $x(t)$
FIGURE 2.13 - Linear Delta Modulation principle
(a) Block diagram; (b) waveforms
is approximated by the tracking feedback signal \( y(t) \). This signal is built up by integrating the sampled output of the quantizer. To simplify the description, we assume the sampled output to be constituted by impulses of unitary amplitude and sign given by:

\[
L(nT_p) = \text{sgn}[e(nT_p)]
\]

and

\[
e(nT_p) = x(nT_p) - y(nT_p)
\]

where \( n \) is an integer number and \( T_p = 1/f_p \) is the sampling period.

If the integrator is ideal, with gain \( \gamma \), the feedback signal \( y(t) \) will be constructed by adding or subtracting a constant step size of amplitude \( \gamma \) at each consecutive sampling instant. Fig. 2.13b illustrates the waveforms that occur in a DM system with ideal elements.

O'Neal presents results of computer simulations for the performance of linear delta modulators for video-telephone and broadcast TV (monochrome) signals. The performance criterion used is that of S/N ratio, and it can be seen that the DM sampling frequencies have to be much higher than the Nyquist frequency to achieve large values of S/N ratios. Two other characteristics can be observed: the S/N ratios are highly dependent on the statistics of the signal and there is only one optimal operating point given the encoder parameters (sampling frequency relative to signal bandwidth and step size normalized by the input signal standard deviation \( \sigma \)). The results show the main disadvantage of the linear non-adaptive DM encoders, viz. their inability to follow signal variations without deterioration of performance. In practical terms, the linear, single
integration, non-adaptive DM encoders operates with maximum SNR at only one set of encoding conditions.

To overcome this major limitation, DM encoders whose step sizes are varied according to the signal have been proposed in literature. The adaptation algorithms are derived from the signal statistics such as past history, average power, and so on. Adaptive delta modulators can be generally classified as syllabically and instantaneously companded systems. Syllabically companded modulators have step sizes that vary relatively slowly compared to the sampling frequency, i.e. the adaptation of the step sizes occur at rates that are much slower than the instantaneous signal variations. They are mainly used with speech or speech-like signals. On the other hand, instantaneously adaptive DM systems have step sizes that vary considerably from one sampling instant to the next. They are normally used with video signals.

Steele presents a large survey of instantaneously adaptive delta modulators in Chapter 8 of his book, with a short analysis of the performance of some of them for television encoding. We therefore abstain from emulating that survey. Instead, we present here a brief summary on the application of DM techniques to colour video encoding, which has not been treated by Steele.

To the knowledge of the author, there has been three major contributions to the use of DM to colour television encoding: by Bhushan, Hawksford, and Oshima and Ishiguro.

Bhushan compares DM and PCM for different sampling frequencies and coding resolutions when the three primary components (R, G, and B) of the American NTSC colour TV are encoded. One set of results
for PCM has been introduced in this thesis, in section 2.3, Fig. 2.5. The corresponding results for DM are given in Fig. 2.14.

By means of subjective tests, Bhushan finds that PCM and quantization noise requires much higher (8 to 9 dB) SNR's than DM and random noise for the same apparent performance. He points out the unlike PCM, where the quantization noise manifests itself as contours for low resolution encoders, DM noise exhibits itself as granular (or grainy) noise, slope-overload noise, contouring and edge business. However, he notes that observers tend to judge the television picture by its most severe degradation rather than by their joint effect. As a consequence, he speculates that this gives an advantage to DM over PCM for lower bit rates, when DM systems are more efficient than PCM. The results of Fig. 2.14 shows that as with PCM, the SNR required for the same subjective quality vary with the colour. Bhushan employs linear delta modulators.

Hawksford's work, where it refers to the application of DM to colour TV encoding, differs on two major accounts. First, the DM encoder proposed by Hawksford is an instantaneously adaptive system. Secondly, it operates on the luminance and the two chrominance signals. Unfortunately, Hawksford had to work with signal components derived from the composite colour video signal (system PAL-I), with the consequent signal degradations introduced by the circuitry other than the delta modulator. Nevertheless, he was able to show that it is possible to multiplex the delta modulated chrominance signals, one at a time, into the horizontal blanking intervals of the luminance signal.
COMMENTS

1. NOT PERCEPTIBLE
2. JUST PERCEPTIBLE
3. DEFINITELY PERCEPTIBLE, ONLY SLIGHT IMPAIRMENT
4. IMPAIRMENT BUT NOT OBJECTIONABLE
5. SOMewhat OBJECTIONABLE
6. DEFINITELY OBJECTIONABLE
7. EXTREMELY OBJECTIONABLE

FIGURE 2.14 - Subjective Effects of DM random noise (after Bushan [51])
The adaptation algorithm used by Hawksford has been described in a generally available publication as an adaptive delta-sigma modulation using pulse-grouping techniques. A DM version is summarized by Steele under the name Pulse Group (PG) DM.

A few results are summarized here. Hawksford used signals obtained off-air from public broadcast TV for subjective analysis, and reports "extremely small" differences between the direct and the signals encoded at 100 MHz for the luminance channel and 20 MHz for the chrominance. However, he reports small changes in the saturation and moderate-level streaking in horizontal bars across the picture. An interesting result is that the encoding of the chrominance signals results in the green signal being most accurately encoded, followed by the red and then blue. This effect suits well the requirements for the encoding of the individual components (see also the overview in section 2.3).

Oshima and Ishiguro have developed a modified double integration delta modulator called Higher Order DM (HODM) for encoding NTSC colour television signals with non-adaptive DM. It presents frequency shaping circuits to match the delta modulator frequency response to the NTSC colour TV spectrum. As the paper referenced is in Japanese, we present here a brief summary of the circuit configuration. The arrangement of the system is shown in Fig. 2.15a.

The video signal, whose spectrum has approximately the shape indicated in Fig. 2.15b, is first de-emphasized around the colour subcarrier frequency $f_{sc}$ (3.58 MHz in the NTSC system). This spectral shaping reduces the possibility of the DM encoder overloading when colour signals are present in the composite signal. If the
FIGURE 2.15 - Higher Order Delta Modulation (after Oshima [78])
de-emphasized signal were processed by a conventional double integrator DM\textsuperscript{19}, the signal-to-noise ratios around the colour subcarrier would be degraded. To compensate for the attenuation of the signal, the second integrator is placed in the forward path of the DM coder and a resonant circuit is added to it, with the central frequency around the colour subcarrier frequency. Now, Oshima and Ishiguro show that because the quantising noise spectrum shape of such a DM CODEC is given by the inverse of the forward path frequency response of the DM coder, the DM quantising noise is also attenuated around $f_{sc}$. Consequently the S/N ratio is increased around that frequency by the addition of a resonant circuit to the second integrator.

The results are reported to be of sufficient quality for commercial broadcast applications, when the sampling frequency is 80 MHz (base-band of 4.5 MHz). The step size used is $V_{p-p}/15.9$ ($V_{p-p}$ is the video signal peak-to-peak amplitude), the emphasis/de-emphasis filter gain $g = 2.8$ and the resonant circuit gain $\rho = 10$ (see Fig. 2.15).

We conclude this brief survey on the application of DM techniques to colour TV signals coding by mentioning some results obtained by a finalist student\textsuperscript{*} in this Department with linear and adaptive DM coders, built with Emitter Coupled Logics (ECLs). The adaptive DM coder was the Hawksford's PGDM\textsuperscript{79} with memory length two. The sampling frequencies used were 27 and 92 MHz, and the input

signal was extracted "off-air". The composite signal was encoded rather than the luminance and chrominance components used in [77]. Unfortunately the circuitry built were not stable enough to allow prolonged and reproduceable tests (subjective tests included). Moreover, only a simple modified domestic colour TV receiver, with its inherently large signal distortions and poor convergence and picture controls, was used as the viewing monitor. It was, nevertheless, possible to verify that with the 92 MHz sampling frequency both the LDM and PGDM gave only moderate, though clearly noticeable, degradation compared with the direct signal. The degradations were mainly hue changes in highly saturated colours suggesting very large phase distortions in the high frequencies. Some loss of spatial and contrast resolution, coupled with de-saturation of colours were noticeable with the LDM. When the sampling frequency was reduced to 27 MHz, the LDM was giving a definitely poor performance with loss of colour and spatial and contrast resolutions. The PGDM was still capable of reproducing colours, with sharper but with more grainy noise pictures than with the LDM. The distortions were quite noticeable and objectionable.

2.6. Approaches to digital television research.

In the evaluation of television signal processing systems, it is necessary to take into consideration the human observer as the ultimate receiver. This means that subjective evaluations are usually necessary.

For analogue television systems, there are some secondary assessment techniques [48], derived from subjective tests, that can
be used to take objective measurements for the performance of the system. Also purely objective measurements such as differential phase and gain, frequency response and SNR can give a good indication of the quality of the signal. An important measurement of quality is the SNR, which is usually defined in television engineering as the peak signal power to rms noise power. Because the visibility of noise decreases for higher frequencies, it is also usual to weight the measurements with a network that attenuates the high frequency noise components.

An approximate evaluation of the picture quality can be easily and speedily carried out by observing special test cards. These test cards are carefully chosen to show geometrical distortions and some frequency and amplitude responses of the television system under analysis.

The main disadvantage of these test pictures is that only still pictures are examined with a limited class of contents, which at times can have little significance in practical conditions. For instance, fully saturated colour bars are used as test signals, though in average the amplitude of the colour subcarrier is only 11% of the peak amplitude allowed.

When the television signals are digitally processed, signal-to-noise ratio is a measure of quality. However, the value of such measurements have their significance reduced as the redundancies in the signal are removed. If for example the contrast and spatial resolutions are reduced in areas where the eye is less likely to perceive these reductions, the SNR would fall without necessarily degrading the picture quality as subjectively perceived.
If the SNR is high, say above 35dB unweighted in monochrome, the SNR measurements can be a good indication of quality\textsuperscript{80}. Below that value, the quality depends on the type and location of the distortion. Therefore, if SNR measurements give low values, a high degree of speculation is necessary to relate the measurements with picture quality, unless subjective tests are carried out. The same reasoning applies to other objective evaluation methods.

Basically there are five approaches to the investigation of television encoders:

(1) "pure" computer simulation
(2) all hardware
(3) computer simulation with hardware interface with the visual world
(4) computer controlled hardware experiments
(5) low-speed modelling.

"Pure" computer simulation\textsuperscript{76} operates only with signals generated by the computer itself, such as random signals with its frequency spectrum shaped to approximate that of the signal being simulated. Commonly SNR's are used as quality measure\textsuperscript{80,76}. Also responses to special signals such as step, narrow pulse and sinewaves can be analysed\textsuperscript{19}. However, this approach can give only an approximate indication of the expected performance of the system with actual signals. The main advantages lie in the use of the computer, with complete flexibility for changing signal and system parameters, and fully reproduceable measurements. To the knowledge of the author, there are no effective means of simulating the viewer reactions to different types of signals and distortions. The withdrawal of the
viewer and of the actual source signals are the major disadvantages of this approach.

All-hardware experiments on the other hand allow both objective and subjective evaluations. It has the psychological advantage that the hardware itself is a proof of the viability of the system, and the actual system operating conditions can be examined from source to the human receiver. In most cases the channel encoding and transmission are studied separately, and in the development of digital television CODECs they are usually part of separate investigations. The major disadvantage is the commitment towards the original design and configuration. It is less flexible for substantial changes in the design and in the case of video coding it may involve sizeable time and hardware investment.

A compromise approach is that of processing actual video signals in a computer, but at much slower rates (non-real time processing) than those in which they are generated, and displaying the processed picture on a video monitor or photographic film. This is a popular approach because it provides the flexibility of computer simulation of video CODECs, and allows a degree of subjective evaluation (observation of a processed picture). The hardware involved is limited to interfacing the visual word to and from a computer. The major disadvantage is that in most cases only still frames can be processed, or moving pictures of very short duration (10 seconds). Also, because the process is in non-real time more sophisticated techniques that take advantages of spatio-temporal resolution exchanges cannot

* including colour television signals.
be fully exploited. In the case of simulating moving pictures, the intermediate storage devices such as video disks \(^{81}\) may introduce additional distortions and mask the results of digital processing.

More sophisticated computer aided techniques may use the computer to link signal processing modules to "built up" different encoders, or to control various elements in a hardware processor. The advantages of such an approach are the computer related flexibility and the real-time processing of video signals. Such a system has been recently disclosed by Brown et al. \(^{82}\). The main disadvantage is the initial hardware investment needed to provide enough flexibility to the system. A secondary and not serious disadvantage is that the control function exerted by the computer has to be based on slowly varying data because of the speed limitation of the computer.

The last approach is that of a slow-speed model \(^{83}\). This is a poor alternative to any of the above approaches as it still cannot provide a picture display for subjective analysis, and lacks the flexibility of computer simulation facilities. However, it allows very inexpensive models to be built, and these may serve to give an insight to the operation of a system proposed and analysed by pure computer simulations. It can be built speedily without worries about component lay-out and speed, interconnections, and circuit noise. As a consequence, however, there is the disadvantage that it is not possible to simply change the clock frequencies to higher values and use faster components to process video signals. This is particularly true if the slow-speed model is built with conventional TTL and linear integrated circuits, and the actual model has to be built with much faster devices such as ECLs and discrete amplifiers. Notwithstanding the many
disadvantages, it is still one possibility that can be considered as a last resort.

In table 2.1 we summarize the five approaches. In the cases (1) and (5) the only evaluation method that can be used is that of waveform reproduction, viz. SNR, frequency response, and waveform tracking. The alternatives (2) and (4) allow all required objective and subjective assessments and should be preferred if time and capital are available. Alternative (2) is perhaps the best compromise if movement is not of paramount consideration.

To finalize, we note that a good waveform reproduction and high SNR (say above 35 dB) are sufficient conditions for good quality picture reproduction, but they are not necessary requirements. The sufficient conditions have been always the base for objective quality measurements adopted for analogue TV systems. The only necessary condition is probably the subjective judgement and acceptance of the quality provided by the system.

2.7. Final remarks.

We have broadly examined what constitutes the source signal (section 2.4) and the ultimate receiver or sink (section 2.3) in a digital television communication system (section 2.2). Also, we have seen how the investigations can be carried out (section 2.6) and how the encoding of video signals can be approached (section 2.5).

We will not digress about the circumstances that established the directions taken by this research. However, the outcome of the many compromise solutions adopted throughout this investigation will be briefly indicated now.
<table>
<thead>
<tr>
<th>(1) Pure Simulation</th>
<th>Advantages</th>
<th>Disadvantages</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>- flexibility to change algorithms</td>
<td>- non-real time processing</td>
</tr>
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<td></td>
<td>- fast and inexpensive</td>
<td>- viewer (receiver) and visual source out of the system</td>
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<td></td>
<td>- reproduceability of results</td>
<td>- results do not reflect accurately the performance of the system</td>
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<table>
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<th>(2) All-hardware</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td></td>
<td>- real-time processing</td>
<td>- commitment to original configuration</td>
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<td></td>
<td>- subjective &amp; objective evaluation</td>
<td>- costly</td>
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<td></td>
<td>- realistic results</td>
<td>- less flexible</td>
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<th>(3) Computer simulation of Picture Coding</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<td></td>
<td>- subjective &amp; objective evaluation</td>
<td>- non-real time processing</td>
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<td></td>
<td>- flexibility</td>
<td>- no movement and no interframe coding schemes</td>
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<th>(4) Computer controlled or aided hardware</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tr>
<td></td>
<td>- subjective &amp; objective evaluation</td>
<td>- less flexible than (1) and (3) under certain conditions</td>
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<td>- realistic models</td>
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<td>- real time processing</td>
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<td>- some flexibility to changes</td>
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<th>(5) Low-speed model</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tr>
<td></td>
<td>- inexpensive</td>
<td>- non-real time</td>
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<td></td>
<td>- simplicity of implementation</td>
<td>- only 'objective' measurements</td>
</tr>
<tr>
<td></td>
<td>(relatively to high speed model)</td>
<td>- viewer and visual source out of the system</td>
</tr>
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<td></td>
<td>- insight to operation of hardware</td>
<td>- unreliable and not recommendable extrapolation of results.</td>
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There are many advantages associated with performing the bandwidth compression in the digital domain, particularly when adaptive schemes are employed. First of all, there are the stability and reproduceability of digital circuits, and therefore of digital adaptation algorithms. Secondly, there is the possibility of their implementation with large scale integrated circuits with consequent reductions in size and cost. However, before the circuit integration is achieved digital systems may require much larger amount of hardware than their analogue or hybrid implementations. For example, analogue integrators are of much simpler implementation than their digital equivalents, specially when leakage is involved. Another point, apparently trivial, is that digital compressors require digital input signals.

With signals such as video, linear PCM A/D converters may be not only complex but also expensive to implement (section 2.4.B). Consequently, a search for alternative means of digitizing video signals is of great interest. One technique that is conceptually simple for performing analogue to digital conversions is that of DM, and we have seen that DM techniques can be applied to encode colour video signals (section 2.5.D) with some success. We should note that it is not possible to compare effectively the techniques proposed because of the different methods used by the different authors to evaluate their systems, but the published results indicate that the performance of DM to encode colour video signals are generally inferior to those by straight PCM, for the same transmission bit rates. Also, most digital signal operations are performed with the linear PCM representation.
A technique that combines the ease of A/D conversion of DM and the linear PCM representation of signals is that DM-to-PCM code conversion described in detail by Goodman. Examining the results presented by Goodman, it can be seen that the use of linear delta modulators necessitates relatively large over-sampling factors (ratio between the encoder sampling and the Nyquist frequencies), if high SNR's are to be achieved with PCM signals obtained by code conversion. In Fig. 2.16 we reproduce the performance curves presented in reference [20] for flat bandlimited Gaussian signals, having as parameters the digital filter length, N, used for the DM-to-PCM code conversion.

The main reason for the high sampling frequencies required of LDMs in such code converters is its limited dynamic range because, to ensure a given SNR for over a certain range of signal input powers, the peak SNR has to be very large. Companded DM systems circumvent such limitation by adapting their step sizes according to the signal power or slope, effectively increasing the dynamic range of the encoder. In the chapters that follow we study the use of an instantaneously adaptive delta modulator as the A/D converter used in the DM-to-PCM code converter structure.

During the course of these investigations, we have resorted to extensive computer simulation studies, and one result was the proposal of an empirical description of the SNR as function of signal power that approximates the simulated results for two types of instantaneous adaptive DM systems. The presentation of the results in this thesis does not follow the chronological development of our investigations, and indeed the empirical description is presented first (Chapter III)
FIGURE 2.16 - Performance curves of LDM-to-PCM code conversion, flat bandlimited Gaussian inputs (after Goodman [20])
followed by an extension of the HIDM encoder to a two-bit per sample system and finally by the code conversion structure. In fact, the analysis of the HIDM was carried out after the extension to a two-bit encoder was made, based on a coarse analysis of encoding parameters required for processing the composite colour signal. The results of the empirical analysis of noise in HIDM are independent of the remainder of the thesis, but is presented first because of a possibly broader implication of the approach adopted to achieve the results described. In Fig. 2.17 we present a rough outline of the main guidelines that produced this thesis, with the major deciding factors (which are not by any means the only ones considered) indicated curtly.
FIGURE 2.17 - Research directions considered in the thesis - an outline
CHAPTER III

ANALYSIS OF SIGNAL-TO-NOISE RATIO
OF AN ADAPTIVE DELTA MODULATOR

3.1. Introduction.

With instantaneously adaptive delta modulators, the fastest adaptation algorithm that allows the output to always decay to an oscillation between the smallest step sizes, for a steady input, is that of the HIDM. Candy, however, prefers encoders that present a slightly slower adaptation as a compromise between fast response and sufficiently damped overshoot characteristics. Other IADM systems (see Steele for survey) such as Hawksford’s Pulse Grouping Delta Sigma Modulation, Jayant’s One-Bit Memory ADM, and Kyaw’s second order Constant Factor DM do not present as fast responses for abrupt changes in signal amplitudes or, when they do, the overshoots are less damped, i.e. the transients last longer than with the HIDM. The disadvantage of the latter is the large amount of overshoots that may occur while tracking the signal because of the fast adaptation algorithm.

Amongst the IADM systems mentioned above, the HIDM and Jayant’s ADM (or first order CFDM) with multipliers set to 2 and 1/2 would generate step sizes that are only integer powers of two. This characteristic is desirable in view of fast digital filter implementations that will be analysed in later chapters. Jayant’s CFDM encoder has been investigated by Boyce, Cummisskey, and Steele and Kyaw, not to mention its inventor. On the other hand, the HIDM has had little analysis in literature apart from the original papers.
that introduced its principles. In the next sections we present an analysis of the response of the HIDM to Gaussian and sinusoidal inputs in what we believe to be the first semi-empirical description of the SNR as a function of encoding parameters by means of mathematical expressions for such an IADM.

The results for Gaussian inputs are also extended for the CFDM.

3.2. The approach.

Because of the difficulties in analysing mathematically the instantaneously adaptive DM systems, Boyce has investigated the response of Jayant's ADM for step inputs only. Kyaw has compared the responses of the HIDM and the first and second order CFDM systems for step and narrow pulse inputs, and calculated the SNR curve as a function of the input signal power for Gaussian inputs in the case of the CFDM encoders by means of computer simulations.

To investigate the HIDM system, we follow a somewhat different approach. The mainstay of our investigation is the intuitive notion that if the encoder is tracking a signal, the average slope of the tracking signal should equal the average slope of the input signal. If this did not happen the encoder would be either slope-overloaded or oscillating around the signal because of a too fast increase in the tracking signal slope. The tracking signal is built up by integrating the feedback step sizes and, consequently, the system performance should depend on the distribution of the step sizes that occur during the encoding process.

Further to this intuitive notion, Newsy has investigated a Syllabically Companded And Logically Encoded (SCALE) delta-sigma
modulator and obtained some success in expressing its SNR as a function of encoding parameters for Gaussian inputs. Newsry's approach was to substitute the fixed step size in the expression of S/N ratio derived by de Jager\textsuperscript{90} for the LDM case by a variable step size $H$. This variable step size was effectively an average step size whose amplitude $H$ depended, amongst other parameters, on the variance of the input signal which had a Gaussian amplitude distribution with mean zero.

Newsry's work was an extension of the calculations by Cartmale and Steele\textsuperscript{89A} for a syllabically companded delta-sigma modulator, in which the feedback step size $H$ was related to the amplitude of a sinusoidal input. This sinusoid was the approximation for a narrow band-limited random noise signal.

Our approach is to investigate the relation that might exist between the average step size $y_{av}$, defined as a function of the probabilities of occurrence of each possible step size in the HIDM feedback path, and the system performance measured in SNR's, with

$$y_{av} = \frac{1}{N} \sum_{i=1}^{N} p_i(\cdot) y(i), \quad i = 1, 2, \ldots, N \quad (3.1)$$

where $p_i(\cdot)$ is the probability of occurrence of the step size $y(i)$ for a given slope of the input signal. $N$ is the number of possible step sizes.

$$\text{SNR (dB)} = 10 \log_{10} \left( \frac{S_x^2}{N_q^2} \right) \quad (3.2)$$

where $S_x^2$ and $N_q^2$ are the signal and noise powers, respectively.
All our results are based on empirical analyses by computer simulations, encouraged by the promising results presented by Newsry and Cartmale and Steele for syllabically companded delta-sigma modulators.

3.3. The HIDM encoder.

The High Information Delta Modulation System, invented by Winkler, can be represented as in Fig. 3.1a. A bandlimited input signal $x(t)$ is tracked by the feedback signal $y(t)$. At any one given clock instant, i.e. $t = nT_p$, where $T_p$ is the sampling period and $n$ is integer, the error signal $e(nT_p)$ has its sign examined and a binary (digital) output $B_n$ is produced such that:

$$\begin{cases} B_n = 1 & \text{if } b_n = \text{sgn}(e_n) \geq 0 \\ B_n = 0 & \text{if } b_n = \text{sgn}(e_n) < 0 \end{cases}$$

(3.3)

In Eq. 3.3, $e_n = e(nT_p) = x(nT_p) - y(nT_p) = x_n - y_n$.

The current output $B_n$ and the two immediately preceding it, $B_{n-1}$ and $B_{n-2}$, are used by the HIDM adaptation logic to determine the current step size $\gamma_n$ as a function of the previous one, $\gamma_{n-1}$. The HIDM adaptation algorithm doubles the step sizes for every third consecutive pulse of same polarity, halves them when there is sign reversal in $e(nT_p)$, and keeps the step size unchanged in other cases. Representing the logic commands to double, half and maintain constant the step sizes by UP, DOWN and HOLD, respectively, we can write:
FIGURE 3.01 - High Information Delta Modulation
(a) block diagram; (b) waveforms
Eqs. 3.4 are logical (boolean) expressions and describes the HIDM step adaptation behaviour. In practice, the range of possible step sizes is limited by a maximum, $\gamma(N)$, and minimum, $\gamma(1)$, step sizes.

If a binary signal $S(N)$ is produced such that $S(N) = 1$ when $\gamma(N)$ is being used and 0 otherwise, and similarly for $S(1)$ corresponding to $\gamma(1)$, we can re-write Eq. 3.4 to incorporate the limitation in the range of step sizes as

$$
\begin{align*}
\text{UP} &= B_n \cdot B_{n-1} \cdot B_{n-2} + B_n \cdot B_{n-1} \cdot B_{n-2} \cdot S(N) \\
\text{DOWN} &= B_n \cdot B_{n-1} + B_n \cdot B_{n-1} \cdot S(1) \\
\text{HOLD} &= \overline{\text{UP}} \cdot \text{DOWN}
\end{align*}
$$

Eq. 3.5 indicates that the UP (DOWN) command is inhibited when the step size is already the maximum (minimum) permitted, and the HIDM step adaptation algorithm can be re-formulated as:

(a) if the current and previous two outputs are of the same polarity, the step size is doubled, unless $\gamma_{n-1}$ is already the maximum step size $\gamma(N)$ allowed;

* the indices of $\gamma$ normally indicates clock instants except when they are bracketed, in which case they refer to step amplitudes.
(b) if the last two outputs are of opposite polarities, the step size is halved, unless $\gamma_{n-1}$ is already the minimum step size $\gamma(1)$ allowed;

(c) in all other cases the step size is kept unaltered.

The feedback signal is then formed as

$$y_n = y_{n-1} + b_n \cdot \gamma_n = \sum_{i=-\infty}^{n} b_i \cdot \gamma_i$$

(3.6)

where $\gamma_i$ can assume one value in the set $\{\gamma(1), \gamma(2), \ldots, \gamma(N)\}$.

The step size is calculated at every sampling instant according to:

$$\gamma_n = m_n \cdot \gamma_{n-1}$$

(3.7)

where $m_n$ assumes the values 2, 1/2, or 1 when the UP, DOWN, or HOLD commands are present, respectively.

Eq. 3.7 describes an ideal integrator and consequently the feedback signal $y(t)$ changes by $\gamma(i)$ at every clock instant and remains constant between clocks. Fig. 3.1.b illustrates the HIDM encoding an arbitrary input waveform $x(t)$, the tracking signal $y(t)$, and the reconstructed signal $\hat{x}(t)$. The latter is obtained by low-pass filtering $y(t)$, i.e. by rejecting the out-of-band noise introduced by the quantization process.

Normalizing the step sizes $\gamma(i)$ by the minimum step size $\gamma(1)$, the relative amplitudes of the steps in the HIDM encoder are $1, 2, 4, \ldots, 2^{N-1}$, where $N$ is the number of possible step sizes. These are, thus, integer powers of two but for a constant weighting factor $\gamma = \gamma(1)$.
3.4. Computer simulation outline.

The HIDM encoder, described in the previous section, plus LDM encoders have been simulated in an International Computers Ltd. digital computer ICL1904. The LDM encoders were simulated by simply by-passing the HIDM step adaptation logic and fixing the step size multiplier \( m \) to a constant value 1, i.e. feeding a constant step size \( \gamma \) through the +/- switch to the feedback integrator. The +/- switch in the LDM encoder was controlled by the 2-level quantizer output as with the HIDM.

The programming language used was the Extended FORTRAN IV known as FORTRAN 1900(91). We had access to all the facilities provided by the Loughborough University Computer Centre, which included the Centre's graph plotting package with subroutines UTP and the use of the collection of algorithms for the solution of numerical problems on computers known as the Nottingham Algorithms Group (NAG) Library93.

The overall simulation procedure is indicated in the diagram of Fig. 3.2. The input signal was generated locally by the computer, i.e. there was no interfacing between the computer and the actual signals such as speech or video. For the analysis described in this section, sinewaves and pseudo-random noise with a Gaussian amplitude distribution were used as the input signals to the encoder. The signal to be encoded was band-limited by convolving its samples with the impulse response samples of a sharp, quasi-ideal low-pass filter. The samples of the filter impulse response were also generated at the start of each program (details later).

---

* The UTP graph plotter package is being superseded now by a more powerful one known as GINO-F (ref. 92), which does not impose the restriction of JOBCORE 32K of the UTP package (Loughborough University Computer Centre Doc. SRIKE/170, Dec. 1970).
START

GENERATE LOW-PASS FILTER
IMPULSE RESPONSE SAMPLES

GENERATE PSEUDORANDOM
NOISES, GAUSSIAN DISTRIBUTION

GENERATE SINE WAVES

BAND-LIMITING

WEIGHT INPUT SAMPLES

CODEC SIMULATION

filter

Calculate error

error power

S/N (dB)

analysis

end

average step size

FIGURE 3.02 - Computer simulations - block diagram
The filtered signal samples were weighted before encoding to simulate different signal power levels. After coding and decoding, the error power in the message band was calculated and thence the S/N ratios. Usually the resulting curves of SNR (in dB) against input signal frequency (for sinewaves) and power (for Gaussian inputs) were plotted in graphs for analysis.

During the tracking of the input signals, the number of occurrences of each of the allowed step sizes was computed, and the average step size calculated according to Eq. 3.1 for each value of the input signal frequency or power.

In the sub-sections that follow, we discuss in a little more detail the main points of our computer simulation procedure.

A. Pre and post-encoder filter.

In the simulations, a band-limited input signal \( x(t) \) was processed by a HIDM (or LDM) encoder, and the local feedback signal \( y(t) \) was low-pass filtered to produce the signal \( \tilde{x}(t) \) that was an approximation of \( x(t) \). The filter used to band-limit the input and the reconstructed signals was a quasi-ideal low-pass filter with a sharp cut-off, a large out-of-band attenuation, and a linear phase response. Such a filter has been implemented as a linear transversal (non-recursive) digital filter to ensure the linear phase response.

The filtering process was achieved\(^\text{12}\) by convolving the signal to be band-limited with the impulse response of the filter, i.e.:

\[
y_n = \sum_{k=0}^{M-1} h_k \cdot x_{n-k}
\]  

(3.8)
where \( \{x_n\} \) was the sequence of input signal samples, \( \{y_n\} \) was the filter output sequence and \( \{h_k, k = 0,1,\ldots,M-1\} \) were the filter impulse response samples. These were obtained using the Frequency Sampling method of filter design\(^9_4\) at its simplest, i.e. by using the "cook-book" approach made possible by the extensive design data given in \([94]\).

Briefly, a uniformly sampled frequency response was specified in terms of the total number of samples, \( M \); the number of samples in the signal pass-band, \( BW \); and the number of samples in the transition band, which could be 1, 2 or 3 in the "cook-book" design. The \( M \) samples of the filter impulse response were then generated by finding the inverse Fast Fourier Transform (FFT).

The principle behind the technique is that defining a number \( M \) of equispaced samples of an idealized frequency response and assuming the number of samples in the impulse response to be equal to \( M \), then the continuous frequency response is exactly determined\(^9_4,12\). The Frequency Sampling technique described by Rabiner et al.\(^9_4\) requires the choice of a set of frequencies at which the sampled frequency response is specified, and the values of a few of them left as parameters for the optimization of the continuous frequency response of the filter. For the low-pass filter design, the samples in the pass-band are given the value "1", and those in the rejection band are given the value "0". The samples in the transition band are left as design parameters. Their values determine the amount of ripples in the pass-band and attenuation in the rejection-band. Ref. \([94]\) can be used as a "cook-book" for the design of many standard
filters, as it tabulates the optimal values for the transition samples. The tables are presented for 1 to 3 transition samples, for filters having $2^N (N = 4, \ldots, 8)$ coefficients. The optimization procedure to find these optimal values is fully described.

We have used only the tabulated data, and as a consequence the filters used in the simulations did not present the exact sampling frequency to bandwidth ratios corresponding to practical situations. In some cases, the desired number of samples in the filter pass-band was not given in the tables and the technique adopted was to linearly interpolate between the nearest two values given in the tables, as suggested by the authors of [24].

One filter designed with 256 samples, with 22 samples in the signal message band and two transition points was used more frequently. Its continuous frequency and impulse responses are illustrated in Fig. 3.3. Other filters eventually used had similar characteristics, but different cut-off frequencies.

In general, the rejection band presented an attenuation of at least $-65\text{dB}$ and the ripples in the message band did not exceed $0.3\text{dB}$. The cut-off transition band was very sharp, viz. only 3 sampling periods in 256. This filter was originally designed for the simulation of the encoding of a television signal, delta modulated at approximately 12 times the Nyquist frequency. The 256 coefficient digital filter of Fig. 3.3 had a nominal sampling frequency of about $66.5\text{MHz}$, consequently the message band (0dB) was approximately $5.45\text{MHz}$ which was slightly less than the $5.5\text{MHz}$ of the PAL-I system. The $-3\text{dB}$ point was about $5.65\text{MHz}$ reaching more than $-140\text{dB}$ at $6.2\text{MHz}$ (first
FIGURE 3.03 - Low-pass filter (quasi-ideal) response
(a) impulse response; (b) frequency response; (c) pass-band
zero in the ideal frequency response used for the design). A filter with such a sharp cut-off and linear phase responses is unrealizable in practice. The impulse response samples, which constitute the coefficients in the non-recursive digital filter design, were used in the simulations as obtained from the inverse FFT of the sampled frequency response, i.e. without limiting their word length or quantizing their values. The filter used is, thus, "quasi-ideal".

B. Encoder parameters.

As we have mentioned before, the analysis of the HIDM ensued from the calculations that were being made for the application of an extension of the HIDM encoder principle to a 2-bit DM system. The latter was proposed to operate on the composite PAL-I video signal. As a consequence, the parameters were determined by considerations primarily related to the application with the 2-bit system, rather than aiming at an exhaustive study of the HIDM. The results obtained are, however, quite general as we present it, though perhaps not as complete as if the research were concentrated solely for that purpose.

The encoder parameters frequently employed in the simulations are presented below. Because the integrator used in the simulation of the encoder was an ideal one (no leakage), the only parameters considered are the operating frequencies and step sizes.

The minimum step size used was approximately equivalent to the quantum step of a linear 5 or 7-bit PCM quantizers, with the range adjusted to encode the 100% colour bar test signal. Specifically,
the nominal step sizes used were 38.56 mV or 9.64 mV. Most HIDM encoders used in the simulations had 4 or 6 possible step sizes.

The signal bandwidth was theoretically 5.5 MHz, and the sampling frequency chosen to be at approximately 15 times the colour subcarrier frequency and 12.2 times the signal bandwidth. The nominal values employed were a sampling frequency of about 66.5 MHz and signal bandwidth of 5.45 MHz.

Other operation parameters were also employed, usually at closely related values, such as doubling or halving the sampling frequency.

In the sections that follow, the frequencies will be given as if wide-band television signals were being processed, keeping in mind the original aims of the simulations. However, because the results were obtained by computer simulations we could assign any value to the sampling frequency $f_p$ and the signal bandwidth would be changed accordingly. For instance, if $f_p$ is given the value of 40 KHz instead of 66.5 MHz, the signal bandwidth would be 3.28 KHz instead of 5.45 MHz.

C. Sinusoidal inputs.

For the analysis of the HIDM encoding sinusoidal signals, two simulation procedures were used to (a) assess the relationship between step sizes and sinewave frequency (and slope) and (b) determine the SNR vs. frequency curves.

For the case (a), a total of 3,000 samples were processed by the HIDM. The first 1,000 samples were discarded to allow the encoder to stabilize after the initialization of the tracking process. Then,
5 complete cycles were used for the calculation of SNR and average step sizes for any particular sinewave frequency. To ensure that there would be no integer relationship between the sampling and the signal frequencies that ranged between 0.19 to 5.5 MHz, the sampling frequency was expressed as $24.5^\times\exp(1)$ MHz. Only one sinewave amplitude of $15.5^\times\gamma(1)$ was considered, but the average step size and SNR's were calculated for the amplitudes $15.5^\times\gamma(1)+\gamma(1)^\times k/10$, for $k = 0, \pm 1, \pm 2, \pm 3$ because the response of instantaneously adaptive DM encoders may vary considerably with small changes in amplitudes. Notice that the maximum change in the sinewave amplitude was less than the minimum step size of the HIDM, $\gamma(1)$.

The program for the approach described above required a storage of 45K words, and a total of more than 10,000 mill units of computer run time. Thus, to obtain the SNR vs. frequency curves for several amplitudes and different step sizes, a much simpler procedure was used. Though less accurate, it allowed the use of less storage (32K) and considerably less processing time. The first simplification was to calculate the SNR ignoring the changes in the HIDM response due to small amplitude changes in the signal. Also, a smaller number of samples have been used for the calculations of SNR's. This procedure required the separation of the signal into low and high frequency regions. The division was made rather arbitrarily, with the simplistic aim of keeping the program storage requirements to less than 32K, which allowed the use of the graph plotting facilities. Briefly, the approach was as follows:

(a) for low frequency sinewaves (i.e. $f << f_p$) there would be a
large number of samples in one cycle precluding the use of many cycles to calculate the average SNR. Thus for sinewaves of less than 1 MHz ($f < f_p/66.5$) we have assumed that there would be tracking by the HIDM over at least part of the signal, viz. over the small slope regions around the peaks (Fig. 3.4a). This is a reasonable assumption because if at 1 MHz the signal were already heavily slope-overloaded, the higher frequency signals up to $f_c = 5.5$ MHz would be encoded with even worse slope-overload and consequently the system would be unusable. We recall that $f_p \approx 12.2f_c$. The SNR's were calculated over the first two cycles of the signal and, to ensure that it would be tracked by the HIDM, we have presented the encoder with the minimum slope of the signal at the start of the encoding. We have achieved this by using a rised cosine function (Fig. 3.4b). This avoids the slope-overload that might have occurred if a sinewave function were used (Fig. 3.4c). The SNR's were calculated after removing the d.c. level.

(b) for sinewaves of frequency larger than 1 MHz, the number of sample points within one cycle would not be too large (less than 66 samples per cycle), and the use of many cycles in the program would not necessitate large storage space in the computer. However, to maintain the same criteria of averaging the SNR's over two cycles of the signal as in (a), we have used a different signal as input. Fig. 3.4d illustrates the problem that could occur for high frequencies if the rised cosinewave were used. Consequently we have employed a simple sine function, but to
FIGURE 3.04 - Sinusoidal inputs - simulation rationale
(a) sine wave; (b) raised cosine wave;
(c) slope overload with sine wave; (d) slope overload with raised cosine wave;
(e) slope overload with cosine wave.
allow for the stabilization of the tracking pattern, we have ignored the first 2½ cycles and the SNR's were calculated over the next two complete cycles (Fig. 3.4e).

To summarize, the SNR's were calculated always over two cycles of the signal. Care has been taken to minimize the effects of the initial hunting of the signal by the encoder, and the SNR vs. frequency curves presented relates to the steady response only.

Because the performance of the encoder depends on the relative phases between the sampling clock and the sinewave, more accurate results could have been obtained by averaging the SNR's over a much larger number of cycles of the sinusoidal signal. In the simplified approach only two cycles were used but for each frequency two measurements were made, with the signal being presented to the encoder with phases 0° and 180° relative to the positive going step of the HIDM feedback signal in idling state. These situations corresponded to the "best" and "worst" attacking conditions by the tracking signal. The two measurements were then averaged and plotted in graphs, some of which are given later in this chapter.

We believe the simulation results to be within 3 dB, at the worst, from the actual steady state response of the HIDM that could have been obtained by averaging the SNR's over a much larger number of signal cycles and over a number of slightly changed amplitudes. Such measurements could have been obtained by simulating the technique used by Eggermont in his hardware experiments, where a sinewave is modulated in phase and amplitude to remove interferences between clock and signal frequency, and to obtain an average SNR to cater for the signal or step size fluctuations around the nominal amplitude.
This was not done, because of the large storage and computing time that would be required to find one point in the SNR vs. frequency curve, and because it was possible to ascertain that the simplified approach preserved the general character of the curves with reasonable accuracy by comparing the results obtained from the procedures (a) and (b) described above.

D. Gaussian inputs.

The procedure that we have adopted to simulate the encoding of random signals with Gaussian distribution is similar to the one described by O'Neal76, and his results for LDM encoders provide a reference basis for our simulations.

In short, Gaussian signals oversampled by a factor \( R = \frac{f_p}{2f_c} \) were simulated by generating pseudo-random numbers, each of which was considered to be one Nyquist sample taken at \( \frac{1}{f_p} \) intervals, and band-limiting them by a non-recursive digital low-pass filter with cut-off frequency at \( \frac{f_p}{2R} \).

Results by Jayant95 for a 2-bit adaptive quantizer (for speech signals) indicate that at least about 1,000 samples have to be used in the calculations of SNR's for the results to be independent of the starting conditions. The results for 1,000 samples are within 1 dB of the results for 10,000 samples. One-bit adaptive encoders, even fast ones like the HIDM, should require between 1,000 to 10,000 samples to obtain reliable results.

Again, considerations of computer storage and processing time have limited our simulations to a sub-optimal choice of the number of samples in the calculations of SNR's. In most measurements, we
have used only 512 samples, though in a few cases we have employed 2,048 samples. When 512 samples were used, several segments (up to 7) of different sample sequences were computed. The curves that are presented in this thesis are those that are closest to the average between the 7 curves. The response of a LDM encoder has been obtained for that particular sequence, and it was verified that the results are nearly equal to those reported by O'Neal. We are confident, therefore, that the results obtained are accurate to within 1 dB of the results that could be obtained by using more than 1,000 samples and this was verified by comparing the results with those for different sequences of 2,048 samples. The maximum deviation observed in any one sequence of 512 samples from the average was of about 3 dB.

The pseudo-random numbers were generated by calling a subroutine from the library of computer algorithms provided by the NAG. The subroutine that we have used (G05 ADF(x)) generates pseudo-random numbers with the standard normal distribution (mean zero and unity variance). The different power levels were simulated by weighting the samples prior to encoding. To ensure that the same sequence was called in the many simulation measurements, the pseudo-random numbers sequence was initialized by a call to another NAG library subroutine (G05 BAF(xo)).

E. The calculations.

The input signal power, $S_x^2$ in Eq. 3.2, was calculated by averaging the signal power over the observation window, i.e.
\[ S_x^2 = \frac{1}{N} \sum_{i=1}^{N} x_i^2 \]  

(3.9)

where \( x_i \) is the \( i \)-th sample amplitude, in a sequence of \( N \) samples. These samples were band-limited prior to the encoding and the signal power calculation.

The in-band error power \( N_p^2 \) was calculated in two ways, according to programming convenience:

(a) by first reconstructing the analogue signal \( \tilde{x}(t) \), as indicated in Fig. 3.1, and compensating for any phase delays in the coding/decoding process, then finding the error signal by subtracting \( \tilde{x}(t) \) from \( x(t) \) and finally the error power,

\[ N_p^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \tilde{x}_i)^2 \]  

(3.10)

(b) by calculating the sequence of the error signal \( \{e_i\} = \{x_{i-1} - y_i\} \) (not indicated in Fig. 3.1), which is not band-limited because the samples \( \{y_i\} \) are not. The sequence \( \{e_i\} \) is then low-pass filtered giving the error power

\[ N_p^2 = \frac{1}{N} \sum_{i=1}^{N} \tilde{e}_i^2 \]  

(3.11)

where \( \{\tilde{e}_i\} \) is the sequence of band-limited error samples.

The difference between the two methods is that with method (b) each filtered error sample is

\[ \tilde{e}_i = H[x_{i-1} - y_i] \]  

(3.12)

where \( H[\cdot] \) represents the band-limiting process.
Filtering by a linear non-recursive digital filter is a linear process, thus

\[ \tilde{e}_i = H[x_{i-1}] - H[y_i] = H[x_{i-1}] - \tilde{x}_{i-1} \quad (3.13) \]

Comparing the error signal given by Eq. 3.13 with the expression between brackets in Eq. 3.10, we notice that the input signal \( x_i \) is filtered twice in the approach (b). The delay by one sample period in Eq. 3.13 can be ignored in the argument (see also Fig. 3.2).

The noise power given by Eqs. 3.10 and 3.11 would be exactly equal if the low-pass filter were ideal (rectangular-like response). In the simulations carried out, the filter is quasi-ideal (see subsection A) and consequently there is a small difference between the two calculations that stems from a better rejection of the out-of-band "signal" in case (b) because of the double filtering of the input signal. The difference was minimal because of a relatively sharp cut-off and rejection characteristics of the filter used, and could not be detected in the program print-outs where the SNR's were given with an accuracy of ±0.1 dB.

This was only possible because of the use of the linear non-recursive (transversal) digital filter, which had a linear phase response that allowed the precise determination of the delay introduced by the filter (viz. half the filter length). This allowed the error between \( x_i \) and \( \tilde{x}_i \) to be determined exactly.

In the simulation results presented in this thesis, any phase shift between \( x(t) \) and \( \tilde{x}(t) \) has been computed as error in the reconstruction. We have felt that this approach is reasonable as phase distortions are important in the coding of video signals.
The SNR's were calculated by substituting Eqs. 3.9 and 3.10 or 3.11 in Eq. 3.2. In the HIDM system, the number of possible step sizes is fixed, and consequently to calculate the average step size we have simply counted the number of occurrences of each step size and calculated the weighted average according to

\[ \gamma_{av} = \frac{1}{N} \left[ \frac{M(1) \gamma(1) + M(2) \gamma(2) + \ldots + M(N) \gamma(N)}{M} \right] \]  

(3.14)

where \( M \) is the total number of samples in the observation window and \( M(i) \), \( i = 1, \ldots, N \) is the number of times each step size \( \gamma(i) \) occurs. \( N \) is the number of step sizes.

Eq. 3.14 is equivalent to Eq. 3.1 with \( p_i(*) = M(i)/M \), \( i = 1, \ldots, N \).

3.5. **HIDM response to sinusoidal signals.**

The results obtained with HIDM encoders indicated that the curve of S/N ratio as a function of the sinusoidal signal frequency may have three distinctive regions:

(a) a nearly flat response region, for low frequency and/or small amplitude signals,

(b) a segment falling at a rate of about \(-6 \text{ dB/octave}\), and

(c) a segment falling at about \(-12 \text{ dB/octave}\).

In Fig. 3.5 we show one of the SNR vs. normalized frequency curves obtained for a sinusoidal of amplitude \(15.5 \gamma(1)\) processed by a 4 steps HIDM encoder. The continuous curve was obtained by
FIGURE 3.05 - Signal-to-noise ratio vs. frequency response for sinusoid of amplitude $= 31f(1)^*$. (a) SNR curve, simulated; (b) see text in section 3.52
the simplified approach described earlier (sub-section 3.4C). The points of different shapes are the results obtained by slightly varying the sinewave amplitude in the more accurate method. The asymptotes of slopes 0, -6 dB/octave, and -12 dB/octave are shown with dashed lines.

We have verified that changes in the sinewave amplitude change the breaking points along the -6 dB/octave line, and that the distance between them is constant as long as the number of step sizes is maintained. This distance, measured along the vertical axis, is approximately 18 dB, reflecting the dynamic range of the HIDM given as the ratio between the maximum and minimum step sizes, viz.

$$20 \log_{10} \left[ \frac{\gamma(4)}{\gamma(1)} \right] = 20 \log_{10} 8.$$  

The flat portion of the SNR curve corresponds to the encoder operating as a simple LDM encoder with a fixed step size $\gamma(1)$. The higher frequency break-point corresponds to the point where a further step size increase is required, but the requisite is not satisfied because of the limitation in the range of allowed step sizes.

A. Analysis of simulation results.

The S/N ratio* response of a linear delta modulator with step size $\gamma$ for a sinusoid of amplitude $E_s$ can be expressed as

$$\text{snr} = \frac{1}{2K} \left( \frac{f_p}{f_c} \right) \left( \frac{E_s}{\gamma} \right)^2$$  \hspace{1cm} (3.15)

* snr represents the ratio between signal and noise powers, whereas its formulation in dB units is represented as SNR.
where \( K \) is a "constant" over a restricted range of the sampling frequency \( f_p \), the system bandwidth \( f_c \) and the step size \( \gamma \). The signal-to-noise ratio in dB is

\[
\text{SNR(dB)} = 10 \log_{10}(\text{SNR})
\]  

(3.16)

The constant \( K \) is usually\(^{19} \) of the order of 1/3. Indeed, for \( K = 0.33 \) and \( \gamma = \gamma(1) \), Eq. 3.15 can be used to estimate the SNR for the flat asymptote in Fig. 3.5. The encoding parameters are \( f_p/f_c = 12.19 \) and \( E_s/\gamma(1) = 15.5 \), for which the SNR is found to be approximately 36.5 dB.

In the following paragraphs, we try to analyse the HIDM response to sinusoids by relating it to the performance of LDM encoders, of which the HIDM is an extension.

First, let us consider the response of LDM systems to sinusoidal inputs and the encoder with a non-leaky (ideal) integrator. For as long as slope-overload does not occur, the snr can be calculated with the help of Eq. 3.15. For a constant amplitude low frequency signal, the snr is constant. As the frequency of the signal is increased, a point will be reached where the maximum slope of the signal, \( 2\pi f_s E_s \), is the maximum that the LDM encoder can track without slope-overloading, i.e.

\[
2\pi f_s E_s = \gamma f_p
\]  

(3.17)

At this frequency, the snr is, from Eqs. 3.17 and 3.15:

\[
\text{sdr} = \frac{1}{8\pi^2 K} \frac{f_p^2}{f_c^2 f_s}
\]  

(3.18)
For a sinewave of maximum slope $2\pi f E_s$, Eq. 3.18 gives the peak SNR as any increase in the signal power or frequency will place the encoder in slope-overload, at least partially, with the consequent fall in SNR.

Eqs. 3.18 and 3.16 give the loci of the points where sinusoids of different amplitudes "break" from a flat SNR vs. frequency response to a rapid fall due to slope-overload.

In Fig. 3.6 we show schematically the response of LDM encoders for constant amplitude sinusoids. Basically, a sinewave of amplitude $E_{sl}$ presents a constant signal-to-noise ratio value, represented by $(S/N)_l$ and given by Eqs. 3.15/16, for increasing frequencies up to $f_{sl}$, that can be calculated with Eq. 3.17. The SNR for that amplitude and frequency is on the straight line described by Eq. 3.18, represented by the dashed line. Similarly for the amplitude $E_{s2}$ and any other amplitudes. As soon as slope-overload starts, the SNR drops rapidly.

It should be noted that the SNR vs. frequency response outlined above is not the LDM performance characteristic presented by Johnson, where the overload characteristic is shown falling at the rate of -6 dB/octave with increasing frequencies. The curves presented in give the variation of the amplitude of the input sinewave as a function of its frequency for the condition in which the maximum slope of the signal is equal to the maximum slope following capability of the encoder. In the response outlined here, the amplitude of the sinusoid is kept constant throughout the frequency variation and the noise power is calculated considering the exact difference between the original and reproduced signals, phase differences included. Notice in ref. that the quantization noise is uniform, because
FIGURE 3.06 - SNR vs. frequency - schematic response for LDM

FIGURE 3.07 - Maximum slope allowed to avoid step adaptation in HDM
it is calculated after the correlation between the error and the wanted signal is removed (section IV of [96]). In the results that are presented here, the total error introduced in the encoding process is considered, with the effect that after slope-overload sets in, the SNR curve as a function of frequency falls at much faster rates than -6 dB/octave.

Now, to estimate the performance of the HIDM system we have already verified that before companding starts, Eq. 3.15 can be used to calculate the SNR. We recall that the step adaptation begins after, and including, three consecutive pulses of the same polarity. The beginning of the companding process occurs when the slope of the signal just exceeds the condition illustrated in Fig. 3.7, for which we can write

$$2\pi f_{sb} E_s = \frac{\gamma(1)f_p}{3}$$

(3.19)

where $f_{sb}$ is the frequency of the sinusoid whose maximum slope is given by Eq. 3.19.

Any sinusoid with maximum slope exceeding the value given by Eq. 3.19 will cause additional step sizes to be used by the HIDM encoder. That equation gives, therefore, the transition or break point from where the HIDM adaptation is operative.

If we vary the ratio $E_s/\gamma(1)$, then according to Eq. 3.15 the break-points and the snr's corresponding to them vary. Substituting $E_s/\gamma(1)$ (with $\gamma = \gamma(1)$) in Eq. 3.15 by using Eq. 3.19, we find

$$\text{snr}_b = \frac{1}{72\pi^2 k q} \frac{f_p^3}{f c_f^2 c_{sb}}$$

(3.20)
Eqs. 3.19, 3.20 and 3.16 give the loci of the break-points of
the SNR vs. frequency curves for the HIDM with four steps encoding
sinusoids of different amplitudes, i.e. different values of $E_s/\gamma(1)$.
In Fig. 3.8a we present the SNR responses to four different values
of input sinusoid amplitudes relative to the minimum step size.
The break-points calculated from Eq. 3.19 are shown as circles on
the line defined by Eqs. 3.20 and 3.16. It can be seen that the
estimates are good approximations to computer simulation curves.
In Fig. 3.8b we present the corresponding results obtained by computer
simulation of LDM encoders.

Before companding starts, the HIDM and LDM encoders show
similar performances. There are some deviations between the estimated
SNR's and those calculated by computer simulations, which are greater
for smaller values of $E_s/\gamma(1)$ and for lower frequencies. These results
are not unexpected because $K$ in Eq. 3.15 is only constant over a
restricted range of encoding conditions.

For the curves presented for LDM systems, the SNR's fall rapidly
once slope-overload ensues. This performance is due to increasingly
more correlated noise being introduced by the slope-overloading. It
is interesting to note, however, that for the HIDM system, once the
step adaptation starts the rate of fall in the SNR is approximately
$-6$ dB/octave, following closely the line described as the loci of
the break-points from non-adaptive to adaptive conditions. This rate
of fall is maintained for as long as larger step sizes are available,
but as soon as the condition where there are no additional larger
step sizes available, the rate of fall increases to about $-12$ dB/octave.
However, this value is still smaller than the rate for slope-overload
in LDM encoders.
FIGURE 3.08 - SNR vs. frequency curves for (a) HIDM, and (b) LDM. Curves 1, 2, 3, and 4 are for amplitudes 2, 4, 8, and 16 times the minimum step size, respectively.
In Fig. 3.9 we sketch the approximate response of the HIDM system for sinusoidal inputs in a similar way to that of the LDM case (Fig. 3.6). The first segment, of uniform response with frequency, corresponds to the HIDM still operating as a simple LDM. From \( f_{sb} \), which can be calculated from Eq. 3.19, the response can be represented by a line with slope \(-6 \, \text{dB/octave}\) up to a second break-point, \( f_{sbu} \), which can be calculated approximately as:

\[
f_{sbu} = f_{sb} \cdot 2^{(2N-1)/2}
\]

(3.21)

where \( N \) is the number of step sizes. In the case of Fig. 3.4, \( N = 4 \) and with \( f_{sb} = 0.041f_c \), Eq. 3.21 gives \( f_{sbu} = 0.46f_c \).

The 'distance' along the frequency axis between break-points is dependent only on the number of step sizes and it increases by one octave with the increase of one step size. For example, in Fig. 3.5, with four step sizes, the distance between \( f_{sb} \) and \( f_{sbu} \) is 3.5 octaves and with the SNR's falling at \(-6 \, \text{dB/octave}\), the 'distance' along the vertical axis is 21 dB. Increasing the number of step sizes by one increases the range of fall at \(-6 \, \text{dB/octave}\) by 6 dB in SNR.

Eq. 3.21 was verified to give satisfactory estimates for \( N > 2 \).

In Fig. 3.10 we present the results for \( N = 2 \), where it is interesting to observe that in this case the HIDM initially operates as LDM with a step size \( \gamma(1) \), before companding starts. The SNR curve "breaks" at a frequency that can be determined from Eq. 3.19 and follows the \(-6 \, \text{dB/octave}\) line, but soon starts to operate nearly as LDM with step size \( \gamma(2) \). It does not equal the performance of a LDM encoder.
FIGURE 3.09 - SNR vs. frequency - schematic response for HIDM
FIGURE 3.10 - SNR vs. frequency - HIDM with two step sizes. Amplitude = 31\gamma(1)
with step γ = γ(2) because each time the slope of the signal reverses polarities, the step size of the HIDM encoder returns to γ(1) and, for high frequencies, this is a disadvantage for the adaptive system.

It can be seen that by using two steps, there is a relatively large improvement that is almost equivalent to using two delta modulators of fixed step sizes γ(1) and γ(2) = 2γ(1), whose responses are also shown in the same figure.

During the transition interval there are losses in SNR due to overshootings that occur because of step size increase before it is actually required, viz. when the signal slope exceeds γf_p/3 but it is still less than γf_p.

The "flattening" of the SNR curve before slope-overload noise dominates was not clearly detected for N > 2, possibly because of over-corrections, i.e. too large or too small steps being used at times, which require a few clocks before the encoder is tracking properly. These overcorrections introduce additional noise.

The second break-point, at f_{sbu}, occurs because of the limitation in the number of step available. This is implied in Eq. 3.21.

B. Probing the results.

The slope of the SNR curve during companding indicates that SNR is proportional to the inverse of the square of the sinusoid frequency. As introduced earlier (section 3.2), our approach is to investigate the dependence of the snr on the average step size γ_{av} defined by Eq. 3.1. Consequently, we have computed γ_{av} as a function of the sinusoid frequency, and the results for the case illustrated in Fig. 3.5 are shown in Fig. 3.11.
Normalized average step size for HDM with 4 step sizes, as function of normalized frequency. The input is a sinewave of amplitude $E = (15.5 + k/10) \hat{y}_0$, $k = 0, \pm 1, \pm 2, \pm 3$. See section 3.46, pp.62/63 for details.
Newsrny substituted the fixed by the average step size in the expression of noise for LDM to estimate the noise of a syllabically companded delta modulator known as SCALE. Following the same approach, we have substituted \( \gamma \) by \( \gamma_{av} \) in Eq. 3.15. The resulting SNR curve is shown in Fig. 3.5 in dotted lines. It is, then, obvious that Eq. 3.15 alone cannot be used to describe the response of the HIDM during companding by using \( \gamma_{av} \) in place of the constant \( \gamma \). The deviation increases for increasing frequencies.

This result seems to invalidate the simplistic approach of making use of the expression of noise for LDM to estimate the noise for instantaneously adaptive systems. However, Figs. 3.5 and 3.11 suggest a strong correlation between the average step size and the signal-to-noise ratios, and indeed we can coarsely approximate the curve \( \gamma_{av} \) vs. frequency with three straight segments with the break-points at the same frequencies in the two graphs.

One possible explanation for the deviations is that the derivation of Eq. 3.15 assumes the noise to be random and uniformly distributed for all frequencies. Experimental measurements (to be described in Chapter V) have shown that this is not the case with HIDM systems, for which the noise power spectrum rises with increasing frequencies, thus the larger deviations for higher frequencies. Also, Eq. 3.15 is valid only before slope-overload occurs and only describes quantization (or granular) noise.

The responses of LDM and HIDM encoders for sinusoidal inputs shown in Fig. 3.8 are a clear indication of the sub-optimal operation of the HIDM step adaptation algorithm, as during companding there is a loss of up to 9 dB in SNR for the adaptive encoder. Steele
speculates that instantaneously adaptive systems could (should) present improvements in peak signal-to-noise ratios relative to the response of LDM encoders because of the ability of an instantaneous step size adaptation at each sampling instant. This improvement does not happen with the HIDM because its step size adaptation algorithm is a function of only the signs of the current and previous three error samples. i.e., it is not aimed at minimizing the error during encoding. Consequently, a fast adaptation is provided with possible overshootings because of too large steps being used, followed by a hunting period during which the feedback signal may oscillate around or lag behind the input signal. If such transients occur, there will be additional noise components that may fall within the signal pass-band thus reducing the SNR's. The loss in peak SNR in HIDM relative to LDM has been also detected by Eggermont, whose hardware measurements for an 800 Hz sinewave encoded by HIDM show an average deterioration of 2 to 5 dB in SNR compared to that encoded by LDM. Eggermont also shows that the SNR rises at about 9 dB/octave for increasing sampling frequencies, reflecting the dependence of the snr on $f_p^3$.

We recall once more that the performance characteristic presented in this thesis differs from that presented by Johnson, who calculates the SNR for uncorrelated noise only. Consequently, the response illustrated in Fig. 3.5 (for example) should not be compared with the LDM overload characteristic. The latter indicates that the amplitude of the input sinusoid is limited, decreasing with increasing frequencies. The fall in the SNR curve for HIDM does not mean that the amplitude of the reproduced signal decreases. In fact, we have
verified in the simulations that the amplitude of the reconstructed signal, after low-pass filtering, is approximately constant if the SNR's are of the order of (or larger than) 12 to 15 dB, though noticeably distorted for these values. The reduction in SNR's for increasing frequencies is due mainly to increased quantization noise (larger step sizes used more frequently). The integrators in the feedback path in the simulated LDM and HIDM encoders were ideal ones. As mentioned earlier, the major difference between Johnson's performance curves and the ones presented here is the inclusion in the latter of correlated noise generated by slope-overload and phase distortions. These type of noises are also usually ignored in hardware measurements with equipments such as the Marconi Distortion Factor Tester TF-2331.

The performance curves of Figs. 3.8a and b were computed using the same definition of snr's and consequently the results can be effectively compared. For sinusoidal inputs, the HIDM can process a wider band of frequencies relative to the LDM, though with poorer performance in some situation. For example, when $E_s/\gamma(1)$ is 2, the LDM system performs better than the HIDM in the upper end of the signal bandwidth as the SNR curve continues flat up to the cut-off frequency with the LDM, whereas the curve for the HIDM starts to fall by 6 dB/octave from about $0.4 f_c$ (in Fig. 3.8 $f_p/f_c = 12.2$). For $E_s/\gamma(1) = 4$, there is an increase in the operating band from about $0.65 f_c$ to $f_c$ when the HIDM is used instead of the LDM, but this gain is traded-off with a loss of up to about 8 dB in the range $0.2 f_c$ to $0.6 f_c$. Similarly for larger values of $E_s/\gamma(1)$, the increase in the range of frequencies that the adaptive system can
encode is traded-off with some losses over about 1.5 octaves.

It is interesting to note in Fig. 3.8 that the maximum frequency that the HIDM can encode does not vary with the signal amplitude as long as the number of step sizes is sufficient to cope with the change in the signal amplitude. For the LDM system, the maximum frequency that can be encoded is reduced with increasing amplitudes.

C. Summary and comments.

We have seen in this section that it is possible to estimate the response of the HIDM system to sinusoidal inputs (Eqs. 3.15/19-21), with an accuracy comparable to that for LDM systems. However, it was verified that the companding effect was not satisfactorily described by simply substituting $\gamma$ by a variable average step size $\gamma_{av}$, in the equation that gives only the quantization noise in LDM.

It remains the possibility that such an approach may give better results if expressions that quantify the total encoding error in LDM systems are used. Such expressions exist for Gaussian inputs (see Steele$^{19}$ and Greenstein$^{26}$ for short surveys) and consequently we exploit this avenue next.

3.6. HIDM response to Gaussian signals.

A. LDM response to Gaussian inputs.

We present here a brief review of the linear delta modulator response to Gaussian inputs, to establish the basis for the analysis of the response of the HIDM encoder. An idealized LDM system is shown in Fig. 3.12a. Its operation and performance have been studied by many investigators and, in particular, Steele$^{19}$ presents its history
FIGURE 3.12 - Linear Delta Modulation
(a) block diagram; (b) waveform tracking by LDM
(c) error waveforms: total error, granular and slope-overload

In short, a LDM encoder is a closed loop waveform tracking encoder, where the difference between an input signal $x(t)$ and the feedback signal $y(t)$ is quantized into two levels. At regular intervals $T_p$, a unit impulse is generated with polarity $b_n$ that is positive (negative) if the difference, or error signal, is positive (negative). The resulting binary impulse stream is applied to an ideal integrator with gain $\gamma$. The output of the ideal integrator is a staircase-like waveform $y(t)$ that changes by $\gamma$, the step size, at each $T_p$ seconds and approximates the input signal.

The maximum slope that such an encoder can follow is $\gamma/T_p$, which is called the LDM slope-following capacity. If the rate of change of the input signal $|dx/dt|$ exceeds $\gamma/T_p$, slope-overload occurs. Fig. 3.12b depicts the input and feedback signals in the encoding of an arbitrary signal $x(t)$. The error signal is shown in Fig. 3.12c, and it is obvious that its character changes when the encoder is slope-overloaded in the interval $t_1$ to $t_2$. Indeed, the total noise can be approximated by the sum of two noise components, the granular or quantization noise and the slope-overload noise. An accurate approximation for the quantization noise $N_q$ has been found by van de Weg 97 soon after de Jager 90 published the first paper in English describing DM, but the analytical formulation for the slope-overload noise $N_{ov}$ is of much later finding. Greenstein 26 presents a critical analysis of some of the most successful works and proposes an approximate expression estimated to be accurate to within 1 dB for all cases of practical interest.
In this thesis, we combine the expressions of quantization noise by van de Weg\textsuperscript{97} and slope-overload noise by Greenstein\textsuperscript{26} to approximate the total noise power $N_t$ produced by a LDM system encoding Gaussian signals, by writing

$$N_t = N_{ov} + N_q$$

(3.22)

where $N_{ov}$ represents the slope-overload noise contribution and $N_q$, the quantization noise. For a given set of encoder parameters and a Gaussian input signal the two noise components can be expressed as:\textsuperscript{97,26}

$$\frac{N_q}{\sigma^2} = \frac{8r^2}{7\pi^2 p} \left[ \frac{\pi}{12} + \sum_{n=1}^{\infty} \sum_{s=1}^{\infty} (-1)^{nt} \frac{\sin{2\pi n/F_p}}{2\pi n/F_p} \cdot \frac{1}{\xi^2} \cdot \exp\left(-\frac{2\xi^2}{\Gamma^2}(1-\alpha_n)\right) \right]$$

(3.23)

$$\frac{N_{ov}}{\sigma^2} = \left[1 + 2.753S + 2.952S^2\right] \cdot \exp(-.341S^2)$$

$$\cdot \exp \left\{ (a_1 - 2.753) S + a_2 \left[ \exp(a_3 S + a_4 S^2) - 1 \right] \right\}$$

(3.24)

where:

- $\sigma^2 = \text{variance of the Gaussian input signal } x(t)$
- $\Gamma = \frac{\gamma}{\sigma} = \text{step size normalised by } \sigma$
- $F_p = \frac{f}{f_c}$, where $f_p$ = sampling frequency and $f_c$ = signal bandwidth
- $\alpha_n = \text{autocovariance function of } x(t)$, which, for flat bandlimited (0 to $f_c$) Gaussian signals is:
\[ \alpha_n = \frac{\sin \frac{2\pi n}{F_p}}{2\pi n/F_p} \quad (3.25) \]

\[ S = \frac{\gamma_f}{(x'(t))_{\text{rms}}} \]

\( S \) is the slope overload factor defined as the ratio between the LDM slope-following capacity, \( \gamma_f \), and the rms value of the 1st derivative of \( x(t) \), represented as \((x'(t))_{\text{rms}}\). The smaller the \( S \), the more severe is the slope-overload. For flat bandlimited Gaussian inputs,

\[ (x'(t))_{\text{rms}} = \frac{2\pi c}{\sqrt{3}} \sigma \quad \text{thus,} \]

\[ S = \frac{\gamma_f}{2\pi c \sigma} \frac{\sqrt{3}}{\sqrt{3}} = 0.276 \frac{\gamma_f}{F_p} \quad (3.26) \]

\( a_1, a_2, a_3 \) and \( a_4 \) are coefficients that depend on the signal spectral moments, defined as

\[ \beta_n = \int_0^c \omega^{2n} X(\omega) \, d\omega \quad (3.27) \]

where \( X(\omega) \) is the power spectral density function of the input signal. \( \beta_n \) is the mean power of the \( n^{\text{th}} \) derivative of \( x(t) \).

The coefficients \( a_1 \) to \( a_4 \) are readily tabulated for several band-limited Gaussian spectra in Greenstein's paper, which eases the calculations.

For the case of flat bandlimited (uniform) Gaussian process, tables I and II of ref. 26 gives \( a_1 = 0.036, a_2 = 0.37, a_3 = 3.83 \).
and \( a_4 = -5.90 \).

Substituting these coefficients values and \( S \) from Eq. 3.26 into Eq. 3.24 an approximate formula for the slope-overload noise as function solely of \( \Gamma = \frac{Y}{c} \) and \( F_p = f_p/f_c \) is obtained:

\[
N_{ov} = \left[ 1 + 0.76 \Gamma F_p + 0.225 \Gamma^2 F_p^2 \right] \exp \left[ -0.026 \Gamma F_p^2 \right] .
\]

\[
\exp \left[ -0.77 \Gamma F_p + 0.37 \left[ \exp \left( -1.057 \Gamma F_p - 0.449 \Gamma^2 F_p^2 \right) - 1 \right] \right]
\]

(3.28)

Using Eqs. 3.28, 3.23 and 3.22, we have calculated the curve of SNR vs. input power for the LDM encoder with \( F_p = 12.2 \) and \( \gamma = 0.03856 \) (volts)*. The result is depicted in Fig. 3.13, curve (1). It can be compared with curve (2), obtained by simulating the LDM system processing Gaussian signals as described in section 3.4D, and it can be seen that the results closely agree.

The procedure employed to obtain the curves in Fig. 3.13 is analogous to that adopted by O'Neal*", but the results are more accurate in the region of slope-overload because of the improved approximation of \( N_{ov} \).

B. HIDM response.

The objective now is to analyse the effects of substituting the

* see section 3.4b for an explanation for the choice of these parameters. For the arguments in this section, they can be considered arbitrary values.
FIGURE 3.13 - SNR vs. input power (dBm) for LDM; $F_p = 12.2, \gamma = 0.03856$, flat band-limited Gaussian input. Curves (1): theoretical; (2) simulation results; (3) $\rho_{xe}$.
fixed step size $\gamma$ in the expressions that describe the total noise in LDM systems by the average step size $\gamma_{av}$, which is a function of the rms power $\sigma$ of the input signal (Eqs. 3.1 and 3.14). If by doing so the calculated curves are in close agreement to the simulated results, it will mean that it is possible to estimate analytically the performance of the HIDM, provided $\gamma_{av}$ is known.

The HIDM system has been simulated for different values of encoding parameters, and the SNR's and step size distributions computed. The results for $\gamma(1) = 0.03856$, 4 step sizes and $P_p = 12.2$ are presented in Fig. 3.14.

Fig. 3.14a shows the probabilities of occurrence of each of the four step sizes as functions of the input signal rms power $\sigma$, and the normalized average step size, $\gamma_{av}/\gamma(1)$, is depicted in Fig. 3.14b. To keep the thesis short, expanded scale graphs and computer print-outs are not reproduced, as the essential points in the curves can be observed in the figures presented in this thesis. In short, before companding starts only $\gamma(1)$ is present. As $\sigma$ increases, $\gamma_{av}$ increases fast at the beginning but tends to a "saturation" value which depends only on the last two largest steps, in our example $\gamma(3)$ and $\gamma(4)$.

The substitution of $\gamma_{av}$ in Eqs. 3.28, 3.23 and 3.22 produces curve (1) of Fig. 3.14c. This estimated curve of SNR as function of $\sigma$ can be compared with the simulation results, plotted as curve (2) in the same figure. Although there are some discrepancies between the calculated and computer simulated curves in Fig. 3.14c, it can be seen that there is close agreement over a large range of input signal levels. In particular, before companding starts - which is obvious as the HIDM is operating as a LDM in that region - and when
FIGURE 3.14 – Performance of HIDM; $F_p = 12.2$, 4 step sizes, $\gamma_{(1)} = 0.01856$, flat bandlimited Gaussian input
(a) Probability of occurrence of step sizes
(b) Normalized average step size (relative to the minimum)
(next page)
(c) SNR vs. input power; curves (1) theoretical;
(2) simulation results; (3) $\gamma_{(0)}$
FIGURE 3.14 - Performance of HIDM; \( F_p = 12.2 \), 4 step sizes,
\( \gamma(1) = 0.03856 \), flat bandlimited Gaussian input

(a) Probability of occurrence of step sizes
(b) Normalized average step size (relative to the minimum)
(c) SNR vs. input power; curves (1) theoretical;
(2) simulation results; (3) \( \rho_{xe} \)
the SNR curve starts to fall due to slope-overloading.

This agreement between simulation and analytical curves was maintained with some changes in the encoding conditions, supporting the initial assumption that the formulae of noise developed for LDM can be extended to application with the HIDM. However, we did not proceed to do an exhaustive run of tests with widely varied changes in encoding conditions, as there is an obvious obstacle in the approach, viz. the need for an exact knowledge of the probabilities of occurrences of the step sizes. The use of computer simulation to find these values would also allow the direct calculation of the SNR's. We propose, therefore, to simplify the problem by seeking an empirical equation for $\gamma_{av}$ as a function of $\sigma$, based on the computer simulations, but that once defined will allow the computation of the SNR curve without the need to simulate the encoder processing the Gaussian inputs.

Before exploiting this avenue, we analyse the significance of the above results. By producing a different step size ($\gamma_{av}$) for each value of $\sigma$ and using it in the equations of noise for LDM, we are effectively modelling the HIDM encoder as a combination of a large number of LDM encoders, each of which has a different step size for each input signal power level. This is not an unexpected result, as this "model" is implicitly assumed to be valid when the dynamic range of an adaptive encoder is given as the ratio (in dB) between the maximum and minimum step sizes$^{19,23,88}$. The results are also indirectly corroborated by published results on ADM systems$^{23,88,89}$ that show that the peak SNR obtainable with adaptive encoders are generally close to that obtainable with a LDM system$^{19}$. 
C. Empirical equation for $\gamma_{av}$.

The exact calculation of $\gamma_{av}$ using Eq. 3.1 requires knowledge of $p_1(\sigma)$ and these are not easily obtainable. Therefore, an empirical approximation of $\gamma_{av}$ as function of $\sigma$ is sought, and such that it can be calculated without using $p_1(\sigma)$.

To establish the empirical representation of $\gamma_{av}$, we approximate the curves of $\gamma_{av}$ obtained by computer simulation by means of simple functional descriptions. Fig. 3.14B is representative of the general shape of such curves, and basically four regions can be identified and described by simple expressions:

(a) before companding starts, when $\gamma_{av} = \gamma(1)$

(b) around the region at the start of companding, when the HIDM algorithm of Eq. 3.4 is restricted by the necessity in practice of limiting the decrease of the step sizes to a minimum value $\gamma(1)$, where

$$\gamma_{av} = \gamma(1)[1 + \exp(B_1\sigma)]$$

(c) during most of the companding and until the slope-overload is so severe that the feedback signal is a sawtooth or triangular-like waveform that nevertheless follows most of the major changes in the signal direction, where

$$\gamma_{av} = A_1(1 - \exp(-B_2\sigma))$$

(d) and finally, a region where the feedback signal changes directions at about the same rate of zero crossings of the signal. In this region $\gamma_{av}$ approaches a saturation value $A_2$. The transition between $A_1$ and $A_2$ occurs very slowly.
Fig. 3.15 illustrates a condition that represents the limit of validity of the representation given in (c), and where $\gamma_{av}$ is approximately equal to $A_1$. It can be observed that the number of extremals in the tracking signal closely approximates the number of extremals in the input signal. The tracking signal is constituted mainly of the two largest step sizes, $\gamma(N)$ and $\gamma(N-1)$.

The expected number of extremals in the random waveform is

$$f_o = \frac{1}{2\pi} \sqrt{\beta_2/\beta_1}$$  \hspace{1cm} (3.29)

and this is also approximately the number of positive (and negative) peaks in the triangular-like tracking waveform whose frequency, in an average sense, is also $f_o$.

By assuming that the feedback signal is a uniform triangular waveform of frequency $f_o$ - in the average sense - it is possible to deduce the step sizes pattern produced by the HIDM encoder according to its adaptation algorithm, which requires that in a triangular-like waveform each time the polarity of the slope reverses there should be two consecutive steps of half the amplitude of the step before the change in the direction of the slope. Fig. 3.16 shows the pattern for $F_p = 10$ (arbitrary value).

In general, the average number $M$ of samples in one period of the triangular waveform of frequency $f_o$, sampled at $T_p = 1/f_p$ intervals, is

$$M = \frac{f_p}{f_o}$$  \hspace{1cm} (3.30)

Of the $M$ samples, $(M - 4)$ of them will have the largest size
FIGURE 3.15 - Input and tracking waveforms for severe slope-overload.
HIDM encoder. $S = 0.19$; average step size $= 6.95\gamma$;
flat-bandlimited Gaussian signal with power $\approx 31.5$ dBm.
FIGURE 3.16 - "Average" tracking pattern for HIDM in severe slope-overload condition. $F_p = 10$. 
\( \gamma(N) \) and the remaining 4 have the magnitude \( \gamma(N-1) = \gamma(N)/2 \). Hence

\[
\gamma_{av} = \frac{(M - 4)\gamma(N) + 4\gamma(N-1)}{M}
\]

and this equals \( A_1 \).

As \( \gamma(N-1) = \gamma(N)/2 \)

\[
A_1 = \frac{\gamma(N) \cdot (M - 2)}{M}
\]

By using Eqs. 3.32, 3.30, 3.29 and 3.27, the value of \( A_1 \) for flat band-limited Gaussian signals can be expressed as

\[
A_1 = \gamma(N) \cdot \left(1 - \frac{2}{\pi} \mu_5/5\right)
\]

Now, Fig. 3.17 shows an example of the condition described in (d) in which the number of times the feedback signal changes directions is approximately equal to the number of zero crossings by the signal. Consequently, the average frequency of the feedback waveform is equal to the expected number of positive (and negative) zero crossings:

\[
f_z = \frac{1}{2\pi} \sqrt{8_1/8_2}
\]

Again, the knowledge that only the two largest step sizes constitute the triangular waveform allows the calculation of the average step size according to Eqs. 3.31 to 3.33, with \( A_1 \) substituted by \( A_2 \) and
FIGURE 3.17 - Input and tracking waveforms for severe slope-overload. HIDM encoder. $S = 0.013$; average step size $= 7.1\gamma$; $\sigma = 55$ dBm.
the number $M$ of samples in one period of the feedback waveform given now by

$$M = \frac{f_p}{f_z}$$

Fig. 3.18 shows an example in which $A_1$ and $A_2$, calculated as described above, are indicated in the graph of $\gamma_{av}/\gamma(1)$ plotted as a function of $\sigma$ and of the slope-overload factor $S$ defined as

$$S = \frac{\gamma_{av}}{f_p} \frac{f_p}{f_c} \sqrt{\frac{3}{2\pi}}$$

(3.25)

It can be seen that the change in $\gamma_{av}$ from $A_1$ to $A_2$ occurs very slowly, and, moreover, for $\gamma_{av} > A_1$ the SNR is less than 1 dB because the slope-overload is such that most amplitude information about the input signal is lost. For most practical applications, the situation described by the fourth condition (d) can be ignored. Consequently a three-segment representation of $\gamma_{av}$, described by conditions (a) to (c) could be adopted as the proposed empirical expression for the average step size as function of $\sigma$.

A further simplification is, however, carried out to avoid the need for the determination of one of the two remaining unknown "constants" $B_1$ and $B_2$. Figs. 3.14b and 3.18 provide the grounds (see the region where companding starts) for this simplification, which allows $\gamma_{av}$ to be represented as

$$\gamma_{av} = A(1 - \exp(-B_0)) \quad \text{if} \quad A(1 - \exp(-B_0)) \geq \gamma(1)$$

and

$$\gamma_{av} = \gamma(1) \quad \text{if} \quad A(1 - \exp(-B_0)) < \gamma(1)$$

(3.36)
$A_2$ calculated with $f_z$

$A_1$ calculated with $f_o$

**FIGURE 3.18** - Normalized average step size as function of $\sigma$, and $S$. 

<table>
<thead>
<tr>
<th>$f_{m}/K_0$</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_2/K_0$</td>
<td>6.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1/K_0$</td>
<td>3.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

rms power($\sigma'$)

slope-overload factor $S$
With this approximation, the empirical \( \gamma_{av} \) deviates most from the actual average step size around the region where companding starts. This point is further examined in sub-section F.

The "constant" A can be calculated using Eqs. 3.29 to 3.33 with \( A_1 \) replaced by A in the relevant equations. Because A is the value towards which \( \gamma_{av} \) moves to when the HIDM system is in a severe slope-overload condition, Eq. 3.33 was easily verified to be applicable.

Unlike A, we were unsuccessful in estimating the value of B by theoretical considerations alone. However, from Eq. 3.36 we know that the product AB gives the initial slope of \( \gamma_{av} \). Consequently, we have analysed by computer simulations the effects of changes in the encoding parameters on the initial slope of the curve that gives the average step size as a function of \( \sigma \). We have found that AB varies almost linearly with changes in the sampling frequency, but to a large extent it is independent of the amplitude of the steps used. At first this is surprising, as one would expect that if the minimum step size is increased, the rate of increase of \( \gamma_{av} \) would be larger. On closer analysis, however, we should observe that \( \gamma_{av} \) is being presented as a function of \( \sigma \) and not of \( \sigma/\gamma(1) \). Thus if \( \gamma(1) \) is increased but \( \sigma \) is kept unchanged, there will be less need to employ the larger step sizes. And for increasing \( \sigma \), the rate at which the larger step sizes are called upon by the HIDM algorithm will also be reduced. According to Eq. 3.14, \( \gamma_{av} \) is a function of \( \gamma(1) \) and of the number of times the larger steps are used. Consequently, a reduction in these and an increase of \( \gamma(1) \) compensate each other.

The dependence of the average step size on the sampling frequency
is intuitively justifiable. If the sampling frequency, relative to the signal bandwidth, is increased there will be less need for step adaptation and consequently the rate of increase in $\gamma_{av}$ should be inversely proportional to $f_p/F_c$, which by definition is $F_p$.

From the above comments, $AB = k/F_p$, where $k$ is a constant of proportionality. Unfortunately, computer simulations show that $k$ varies slightly with $F_p$, and also that Eq. 3.36 is not always a satisfactory approximation. A compromise expression that holds over a limited range of encoding conditions was found empirically:

$$AB \sim 0.08F_p + 6.4 \quad \text{for } F_p < 32$$  \hspace{1cm} (3.37)

Summarizing, to represent $\gamma_{av}$ by Eq. 3.36, it is necessary to determine the "constants" $A$ and $B$, which are functions of the encoding parameters. Once these are fixed, they are indeed constants. The value of $A$ is first found using Eq. 3.33 and, then, Eq. 3.37 is used to obtain $B$.

Using this procedure, the empirical representation of the average step size as function of $\sigma$ for the example of Fig. 3.14c is found to be

$$\gamma_{av} \approx 0.269 \left(1 - \exp(-2.26\sigma)\right) \quad , \quad \sigma \geq 0.069$$

and

$$\gamma_{av} \approx 0.03856 \quad , \quad \sigma < 0.069$$  \hspace{1cm} (3.38)

In Fig. 3.19a we depict the curve $\gamma_{av}/\gamma(1)$ as function of $\sigma$ as calculated according to Eq. 3.38 and as obtained by computer simulation. Differing from Eq. 3.14b, $\sigma$ is plotted in dB, as this expands the graph in the region of small values of $\sigma$, where most
FIGURE 3.19 - Results of simulations and calculations: curves (1) and (2), respectively. HIDM encoding flat-bandlimited Gaussian signal; $F_p = 12.2; \gamma(1) = 0.03856; 4$ step sizes. (a) normalized average step size; (b) SNR vs. input power.
of the deviations are concentrated. The empirical formula produces curve (2) which is a very close approximation to the simulated curve (1) over most of the range of the input signal power (Fig. 3.19a).

The estimated curve of SNR as function of $\sigma$ (dB) is shown as curve (2) in Fig. 3.19b, where it can be compared with the simulated curve (1). Most deviations occur around the region where companding starts.

D. Changing encoder parameters.

The approach described in the previous section was verified for a few different encoder parameters and two examples are presented here. In Fig. 3.20 the estimated SNR's are compared with the values obtained by computer simulation. Fig. 3.20 depicts the case where $\gamma(1) = 0.00964$, $F_p = 12.2$, and the number of step sizes is 6.

Under these conditions, Eqs. 3.33 and 3.37 give $A = 0.269$ and $B = 2.26$. The noise is calculated through Eq. 3.22 in conjunction with Eqs. 3.23 and 3.24, and thence the SNR's.

Comparing the above results with the example of Fig. 3.14c, where $\gamma(1) = 0.03856$ and 4 step sizes were used, we can observe that the values of $A$ and $B$ are unchanged even though the minimum step size is now reduced to a quarter and the number of step sizes increased by two. This is because $A$ depend only on the maximum step size and $F_p$, according to Eq. 3.33. For $A$ constant, $B$ will be constant according to Eq. 3.37.

Fig. 3.20b shows the case where $\gamma(1) = 0.02$, $F_p = 32$ and the number of step sizes is six. In this case $A = 0.609$ and $B = 0.46$.

Both results can be seen to agree over a large range of input
FIGURE 3.20 - HINN response with different encoding parameters.
(a) six step sizes, $r_{(1)} = 0.00964, \frac{P}{P} = 12.2$
(b) as for (a) but $P = 32$
signal powers, though deviations of about 5 dB occur on some points. These deviations are not due to a poor representation of \( \gamma_{av} \), but rather to the application of the expressions of noise for LDM to estimate the noise in HIDM. They show that the implicit modeling of the adaptive encoder as an ensemble of LDM encoders of different step sizes \( \gamma_{av} \), for each input signal power level is not always valid. This conclusion is expected, as the quantized noise in linear DM has a noise spectrum that is essentially flat. The only other noise component during encoding by LDM is the slope-overload noise, which is highly correlated with the input signal. In adaptive systems, over-corrections introduce an additional correlated noise component that is not present in LDM, and is not considered in the calculations described here.

In Fig. 3.21a we present the feedback signal of a HIDM tracking an arbitrary segment of a random signal, and Fig. 3.21b illustrates the same signal processed by a LDM encoder whose step-size is the average step size of the HIDM of Fig. 3.21a. The distortions are seen re-distributed, and consequently although the calculations indicate similarity in performance measured as SNR's, there should be differences in subjective perception if these waveforms represent speech signals, for instance. The most striking difference is the concentration of errors around the peak transitions for the HIDM case and around the small variation segments for the "equivalent" LDM, where the quantization or granular noise appears to be relatively large. Apparently, the amount of noise in the peaks are exchanged by noise in the smoother parts of the signal in the calculations proposed for estimating noise in HIDM. Deviations in the estimations
FIGURE 3.21 - Waveform tracking by (a) HIDM, and (b) LDM with step size equal to the average step size in (a).
can be attributed to inequalities in these exchanges.

The distortion introduced by the overshootings are clearly correlated with the signal and this suggests our next step, which is presented in the following sub-section.

E. Cross-correlation between input and error signals.

To understand better the nature of the noise components generated by the HIDM, we examine the cross-correlation coefficient \( \rho_{xe} \) between the input and error signals, given by

\[
\rho_{xe} = \frac{\frac{1}{N} \sum_{i=1}^{N} x_i e_i}{\sigma_x \sigma_e}
\]

(3.39)

where \( \{x_i\} \) is the sequence of \( N \) samples \( x_i \) of the band limited input signal \( x(t) \), and \( \{e_i\} = \{x_i - y_i\} = \) error sequence, where \( \{y_i\} \) is the filtered output (and feedback) sequence.

\[
\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} x_i^2}, \quad \sigma_e = \sqrt{\frac{1}{N} \sum_{i=1}^{N} e_i^2}
\]

(3.40)

The variation of the cross-correlation coefficient as a function of the input power has been calculated by computer simulating the HIDM and computing Eq. 3.39 for different values of the input signal power. The results are plotted as curve (3) in Figs. 3.13 and 3.14c for LDM and HIDM, respectively. We note the following:

(a) for large values of \( \sigma_x \) when slope-overload occurs, \( \rho_{xe} \) is positive and approaches unity for increasing slope-overload conditions.

The reason is that in slope-overload \( y(t) \) cannot follow \( x(t) \),
i.e. \(|y_i| < |x_i|\) and consequently \(e_i\) has the same sign of \(x_i\) and Eq. 3139 produces positive answers. Fig. 3.22a illustrates this encoding condition, for a LDM encoder.

(b) when the HIDM is operating prior to slope-overload and \(y_i\) increases more than \(x_i\), overshooting the signal, the error signal as defined earlier will have a sign that is opposite to that of \(x_i\), consequently \(\rho_{xe}\) is negative. Fig. 3.22b illustrates roughly this condition.

(c) if the step sizes are too large compared to the signal variations, generally \(|y_i| > |x_i|\) as illustrated in Fig. 3.22c, and again \(e_i\) has the opposite sign to that of \(x_i\) and as consequence \(\rho_{xe}\) is negative.

From (b) and (c) above, it can be seen that as far as the cross-correlation coefficient is concerned, the noise due to too large steps and to overshoots are similar as both have negative coefficients. This corroborates the analysis made in conjunction with Fig. 3.21, where we speculated that the contribution of the overshoot noise is partly exchanged with the contribution of too large quantization noise.

Also, three noise components can be identified in HIDM, compared to two in LDM. The two noise components of the latter, which are present in HIDM are:

(i) quantization (granular) noise, which is to a large extent white and almost uncorrelated with the input signal, except when the steps are too large compared to signal variations. The cross-
FIGURE 3.22 - Types of encoding conditions that produce correlated noise and error waveforms.

- Curve (1): input
- Curve (2): tracking waveform
- Curve (3): unfiltered error
- Curve (4): filtered error

(a) slope-overload (LIM), $\rho_{60} > 0$; (b) overshoots (HIDC), $\rho_{60} < 0$;
(c) too large step sizes, $\rho_{6e} < 0$. 
correlation coefficient was found to be less than 0.2 in the simulations with LDM (Fig. 3.13).

(ii) slope-overload noise, which can be highly correlated with the input signal and has a positive coefficient. The more severe the slope-overload, the larger the coefficient.

The third noise component, present in the HIDM and in other fast adaptive DM systems, is the overshoot noise, which is partially correlated with the input signal. The cross-correlation coefficient is negative as described above in (b).

The exclusion of this noise component is possible the one key factor in the deviations between the estimated and actual SNR's for the HIDM system.

Observing Figs. 3.13 and 3.14c, it is possible to note that there is some correspondence between changes in the SNR's and the changes in $\rho_{xe}$. In particular, when $|\rho_{xe}|$ increases towards 1 the SNR curve decreases to zero dB.

F. Discussions.

We have assumed that the HIDM noise can be determined by replacing the step size $\gamma$ in the well established formulae for quantization and slope-overload noise in LDM by a statistical average step size $\gamma_{av}$. This was verified to produce estimates that were close to within few dB's of the simulated values, and thus to a first approximation this approach is applicable. The inclusion of overshoot noise component may improve the estimate and we believe that $\gamma_{av}$ is one important parameter that can be used to derive expressions for estimating noise
in HIDM and other instantaneously adaptive delta modulators.

In this thesis, $\gamma_{av}$ was approximated by a functional description, with two "constants" that are functions of the encoder parameters. One of them, $A$, can be found analytically (Eq. 3.33) by examining the average tracking signal pattern. It is interesting to note that Eq. 3.33 can be derived also by adopting Greenstein's two-band process model for representing Gaussian inputs when the encoder (LDM) is slope-overloaded. In short, Greenstein has used the observation that slope-overload noise in LDM is basically dependent only on the zero-th, first and second spectral moments $\beta_0$, $\beta_1$, and $\beta_2$, respectively.

He then approximated the Gaussian process by a sinusoid of frequency $f_o$ plus a d.c. component. The frequency $f_o$ is the one given by Eq. 3.29. The amplitude of the sinusoid has a Rayleigh probability density function (p.d.f.) with mean square value $2\beta^2_1/\beta_2$ the d.c. component has a mean of zero, a variance of $(\beta_0 - (\beta_1^2/\beta_2))$ and a Gaussian p.d.f. The phase of the sinusoid, $\phi$, is uniformly distributed over the interval $(-\pi, +\pi)$. Greenstein notes that the d.c. component and the phase do not influence the calculation of slope-overload noise, which is then calculated as function of the sinusoid amplitude and frequency alone.

In a severe slope-overload condition, the feedback signal of an HIDM encoder tracking a sinusoid is a triangular waveform with the same frequency $f_o$ and the amplitude of the triangular waveform is independent of the amplitude of the input signal. This is so because given a number $M$ of samples in one period of the sinusoid, the HIDM algorithm determines what amplitudes they can assume, viz. $(M - 4)$ with amplitude $\gamma(N)$ and 4 with amplitude $\gamma(N-1)$.
This approach is not given to calculate A in section C because Greenstein shows that this model is valid for a large slope-overload factor S, defined by Eq. 3.35. The value of A is calculated for S of the order of 0.4 to 0.2, when the SNR's are of the order of 2 to 1 dB. As a matter of reference, a LDM encoder reaches peak SNR's with S around 2 to 3.

The value of B cannot be found analytically as A, and as a consequence it was determined empirically based on computer simulation results. From Eq. 3.36 we can determine the rms value of the input signal when companding starts, by equating $\gamma_{av}$ to $\gamma(1)$:

$$\gamma_{av} = A(1 - \exp(-B\sigma_1)) = \gamma(1)$$

from which

$$\sigma_1 = -\frac{1}{B} \ln(1 - \gamma(1)/A) \quad (3.41)$$

This point $\sigma_1$ is marked in Fig. 3.19. It defines the transition from a LDM response to that of HIDM. As $\sigma_1$ is a function of B, an error in its determination will affect this change-over point and may cause significant errors in the estimation of SNR's over the companding range. For $F_p < 32$, the determination of B from Eq. 37 produced expressions for $\gamma_{av}$, which used with Eqs. 3.22 to 3.24 resulted in SNR's that were within 5 dB from the simulated curves (see Fig. 3.20b for example).

The proposed method has limitations, the biggest of which is the exclusion of overshoot-noise from the estimates. This exclusion can explain the lower estimate of noise in Fig. 3.20a and in the first half of Fig. 3.20b, which causes the calculated SNR's to be higher.
than the values measured by simulation. However, it does not explain the better simulated results just before slope-overload in Fig. 3.20b, which is most likely caused by a break down in the trade-off between types of noise as described with Fig. 3.21.

3.7. **CFDM response to Gaussian signals.**

The approach described in the previous section was tried on Jayant's\(^2^3\) adaptive delta modulation (ADM) system with one-bit memory, called first order Constant Factor DM by Steele.\(^1^9\) The CFDM, in short, has two multiplicative factors \(P\) and \(Q\) that are selected when the polarities of the current and previous outputs are equal or of opposite signs, respectively. We have examined only the case \(P = 1/Q = 3/2\). This ADM system was selected because of the simplicity of its step adaptation algorithm, and because the step sizes vary by one constant multiplicative factor, which is to a certain extent the rule for the HIDM system whose step sizes vary by 2.

The procedure adopted was substantially the same to that used for the HIDM and it will not be repeated here. A short description is however given next.

The CFDM encoder is simulated in a digital computer, with Gaussian inputs, and the SNR's as function of the input power determined. The average step is also calculated, and a functional approximation to describe it in terms of the input signal rms power \(\sigma\) is found, with parameters that are functions of the encoding conditions. It has been found that Eq. 3.36, which describes \(\gamma_{av}\) for HIDM, can be used with the CFDM. This is expected because both encoders are single integration delta modulators with the step sizes adjusted before
integration by a constant multiplicative factor.

The calculations of the constants $A$ and $B$ are made in similar lines to that described in section 3.6C, viz. $A$ is calculated by examining the average distribution of step sizes when the encoder is heavily slope-overloaded, and $B$ is calculated by examining the initial slope of the exponential curve by computer simulation. Because the encoder increases and reduces the step sizes by a constant factor, $3/2$ in this case, only two step sizes make up the average feedback signal, which is a triangular waveform of frequency $f_o$ determined by the expected number of extremals in the signal. Fig. 3.23 illustrates the pattern determined by the CDFM step adaptation algorithm. This pattern occurs only because in practice the maximum amplitude that the feedback step can assume is limited and fixed. From that figure, we observe that there are $(M - 2)$ samples with amplitude $\gamma(N)$ and 2 samples with amplitude $\gamma(N-1) = (2/3)\gamma(N)$, which occurs when the slope reverses polarities.

Following the procedure described for HIDM (Eqs. 3.30 to 3.33), we obtain:

$$A = \gamma(N) \left[ 1 - \frac{2}{3(F^p)^{5/3}} \right]$$  \hspace{1cm} (3.42)

The empirical formula for the calculation of $B$ is

$$ABF_p = 8.9$$  \hspace{1cm} (3.43)

The noise in CFDM can be estimated using $A$ and $B$ calculated from Eqs. 3.42 and 3.43 in conjunction with Eq. 3.36 for $\gamma_{av}$ and the expressions (Eqs. 3.23 and 3.24) for quantization and slope-overload.
FIGURE 3.23 - "Average" tracking pattern for CFDM in severe slope-overload condition. $F_p = 10$
noise in LDM. In Fig. 3.24 and 3.25 we present two estimates, with computer simulated results plotted on the same graphs.

Fig. 3.24a is the case for $F_p \approx 12.2$, $\gamma(1) = 0.03856$ and 6 step sizes. For these encoding conditions, $A \approx 0.28$ and $B \approx 2.64$ in Eq. 3.36 for $\gamma_{av}$. Curve (1) is the estimate, curve (2) is the result of computer simulations and curve (3) gives the cross-correlation coefficient $\rho_{xe}$ as function of the input signal power. These curves for the CFDM can be compared with those of Fig. 3.14 that are the corresponding responses for HIDM, which are reproduced in Fig. 3.24b.

It is interesting to note two points:

(a) the cross-correlation coefficient in the CFDM is small during companding, and appears to be almost independent of the signal. This indicates that there are less overshoots with the CFDM, which is expected because of a slower adaptation constant ($3/2$ compared to 2) in the CFDM. The HIDM encoder was designed to present a faster step input response, which is not reflected accurately by SNR measurements. As Candy remarked, there is a trade-off between fast response and damped overshoots.

(b) there is some correspondence between the slopes of the cross-correlation and SNR curves, and this may indicate that $\rho_{xe}$ can be used to advantage to estimate the noise in instantaneously adaptive systems. We have not advanced this avenue for lack of time.

The LDM response has been checked against published results, and serves as anchor for the simulation of the HIDM and CFDM encoders, whose waveform tracking characteristics were verified by examining
FIGURE 3.24 - Simulated (curve (1)) and calculated (curve (2)) results of SNR as function of input power. (a) CDFM with \( P_p = 12.2 \), \( \gamma_{(1)} = 0.03856 \), 6 step sizes. (b) HNFM - same as Fig. 3.14c. Curve (3) gives the cross-correlation coefficient as function of input power.
the step adaptation sequences for a variety of input waveforms. The SNR curves for H1DM can only be referred to established approximations that\textsuperscript{19} give the dynamic range as the ratio (in dB) between the maximum and minimum step sizes, and the heuristic knowledge that the peak SNR is close to that of LDM, with losses between 2 and 5 dB for sinusoidal inputs\textsuperscript{88}. For the CFDM more information is available and, in particular, Kyaw\textsuperscript{24,19} gives the SNR vs. input power response for the case where the sampling frequency is 40 KHz, and the system bandwidth is flat (0 dB) to 2.5 KHz and -3 dB at approximately 3.1 KHz. We have, therefore, tested the empirical formulae developed in this thesis relative to the simulation measurements\textsuperscript{24} by Kyaw, which are reproduced in Steele's book\textsuperscript{19}. The ratio between the sampling frequency and the flat response bandwidth is $F_p = 16$, $\gamma(1) = 0.03$, and $\gamma(\text{max}) = 5.0$. The number of step sizes was not determined and has not been used in our calculations.

Using Eqs. 3.42 and 3.43 we have found $A \approx 4.83$ and $B \approx 0.116$. Using the expressions for noise given by Eqs. 3.23 and 3.34, the SNR curve was found and is given as curve (2) in Fig. 3.25. The results of Kyaw's simulations are shown as crosses (1).

It can be seen that the estimate and computer simulation results generally agree, for both peak SNR and dynamic range. These results are the more reassuring because Kyaw's simulation technique differed substantially from the one adopted in this thesis. Briefly, he has used a different random generator routine\textsuperscript{*} and more significantly

\* the random number generator used by Kyaw has been discontinued and superseded by a NAG library subroutine [93]
FIGURE 3.25 - SNR vs. input power (dBm) for CFDM. \( P_p = 16 \), \( y(1) = 0.03V \), \( y_{(max)} = 5.0V \). Results (1) are after Know [24]; (2) for calculated curve; (3) give simulated values.
the calculation of SNR was made in the frequency domain compared to the time domain approach in this thesis. Basically, the frequency domain calculation of signal-to-noise ratio requires the calculation of the signal and error power spectra by means of Fast Fourier Transformation (FFT), after windowing to remove the effects of using limited number of samples. Only the error samples in the message pass-band are considered for the noise power calculation. The signal was band-limited prior to the calculation of the FFT in a similar way to the one adopted here, viz. by convolving the input signal samples with the filter impulse samples, arranged as coefficients of a non-recursive digital filter. Kyaw used a filter with 64 coefficients.

To verify Kyaw's measurements, we have also simulated the CFDM and the SNR was calculated as described in section 3.4, and the results are given as circles (3) in the same figure.

3.8. Comments.

We have verified that the SNR's for HIDM and CFDM can be estimated by using an empirically derived average step size, which is a function of the input signal rms power and the encoder parameters, in the equations that have been developed for LDM. The virtue of the expression that we have proposed is its simplicity. More accurate descriptions of $\gamma_{av}$ were not sought because the approach ignores a third noise component caused by over-corrections introduced by the adaptation algorithms, i.e. step sizes made too large or too small. The errors introduced by ignoring the true effects of the step size adaptations are greater than those caused by inaccurate representation of the
average step sizes.

After these studies were finalized, we came across a paper by Tazaki et al.\textsuperscript{99} in which a method is proposed for the calculation of SNR in discrete adaptive DM systems. They describe the total noise as composed of three components with each dominating over part of the input signal power range. The three components are granular, first class slope-overload, and second class slope-overload noises. These noise components are given as functions of the first order difference between two samples of the signal, i.e. as function of $\xi = x'(t)/f_p$.

The granular noise is defined as the component introduced when the error signal $e_n$, at the $n$-th sampling instant, is smaller than the step size $\gamma_n$ produced at that epoch and the feedback signal crosses the input signal. The first class slope-overload noise is the one generated when $e_n$ is greater than $\gamma_n$ whilst the encoder has still the capability of correcting its step sizes over the next few clocks, i.e. the feedback signal does not cross the input signal though its slope is still smaller than the maximum that the encoder can track. The second class slope-overload noise is the component introduced when the slope of the signal exceeds the maximum that the feedback signal can follow.

Tazaki et al. quantify the second class slope-overload noise using Greenstein's expression\textsuperscript{26} for slope-overload noise (Eq. 3.24 in section 3.6A), with $\gamma$ substituted by $\gamma_{(\text{max})}$. Both the granular and first class slope-overload noise components are approximated by trinomials with $\xi$ as variable and whose coefficients are dependent on particular encoding algorithms. The definition of first class
The slope-overload condition is somewhat peculiar in that it is defined in terms of sample amplitudes rather than the slopes of the input and feedback signals as usual. Their results present larger deviations than the ones observed in the approach presented in this thesis, particularly in respect to the dynamic range of the encoders. For Gaussian signals, we believe that the use of the maximum average step size (as defined by Eqs. 3.33 and 3.42) is a better approximation because each time the signal slope changes signs, a step size that is smaller than the maximum is inevitably used according to all discrete step adaptation algorithms. If $f_p \gg f_o$, the two approaches should provide similar results (Eq. 3.31) for severe slope-overload conditions. A source of error in the approach by Tazaki et al. is the approximation of the granular and first class slope-overload noises by the trinomial, which introduces errors in the description of the relevant noise powers by more than 10% in average (table 1 of ref. [99]), with consequent deviations in the estimation of the total noise. It is possible that the approach described in [99] to quantify the first class slope-overload noise can be used to estimate the overshoot noise defined in this thesis (section 3.6E), though the definition for these noise components differ. Overshoot noise can be identified with over-corrections (see Fig. 3.22b) whereas the first class slope-overload noise, as defined by Tazaki et al. [99], is more easily identified with the delay or slowness of the step increases when the error signal exceeds the current step size.

Many avenues for further research are left open. Two of these are:
(a) the exact calculation of $\gamma_{av}$ as per Eq. 3.1. Fig. 3.13a, which shows the probabilities of occurrence of each step size in the HIDM encoder, suggests that $p_i(o)$ are either exponentials or combinations of exponentials. Functional descriptions of $p_i(o)$ may yield better representation of $\gamma_{av}$. An alternative approach is to investigate the probabilities of sequences of consecutive pulses having a certain distribution of 1's and 0's as the step sizes are varied according to them.

(b) derivation of $\rho_{xe}$ as function of the input signal rms power, and possibly of $\rho_{yx}$, which is the cross-correlation coefficient between the feedback and input signals. They could be used, for instance, to identify the type of probable noise present.

The major asset of the HIDM encoder is the simplicity of the adaptation algorithm, which produces a fast and stable system whose step sizes rapidly decay to the minimum amplitude when the signal becomes constant or varies little after a large d.c. transition. The major disadvantage is the rigidity of the adaptation rule (multiply or divide by two except when two pulses of same polarity occurs after a sign reversal), which may cause large over-corrections. Steele's observation\(^\text{19}\) that instantaneously adaptive systems may provide higher SNR's than LDM systems breaks down because of such over-corrections in adaptive encoders that found the step adjustments on past history alone. It is possible that encoders that make use of the statistics of the signal in addition to the past few samples may provide some gains in peak SNR's compared to LDM. Alternatively, some measures can be taken to avoid such over-corrections by, for instance, delayed
encoding and error threshold control. To conclude, we note that the method proposed has limitations, but presents better estimates than the only other approach known to the author for estimating S/N ratio as function of the input signal power for instantaneously adaptive DM. Cummiskey presents calculations to estimate the peak SNR for speech-like inputs, assuming that the feedback prediction by the encoder is optimal and that the encoder is a single integration adaptive delta modulator. Goldstein and Liu have also calculated the peak SNR as function of the sampling frequency for ADPCM systems, and presents Jayant's ADM as a particular case for the multipliers P having the values 1.25 and 2, for input variance of unity. As Cummiskey, they have not studied the effects of varying the input signal power. Boyce analyses the stability problems for step inputs and other studies are computer simulations or hardware measurements. A similar method, with some simplifications, has worked for a syllabically companded delta modulator. Although we have not tried the method proposed here for other instantaneously adaptive systems which have been described in literature, we are hopeful that this method may give satisfactory estimates or, at least, give a better understanding of the operation of the encoder.

3.9. Note on publication.

A paper entitled "Estimation of Signal-to-Noise Ratio in High Information and Constant Factor Delta Modulation Systems", in co-authorship with Dr. R. Steele (thesis supervisor), has been accepted for publication in the IEEE Transactions on Communication and is scheduled for the December (1977) issue. The paper is an abridged version of sections 3.6 and 3.7.
4.1. Introduction.

There is a growing interest in the development of digital encoders that do not require high precision components and are, therefore, suitable for implementation by large scale integration. One technique that has attracted considerable attention\textsuperscript{20,84,89,101-107} generates PCM and DPCM signals in two stages. In the first stage, a simple analogue-to-digital converter operating at sampling frequencies higher than the Nyquist, LDM for instance, is used. The digital signal so produced is then processed by an all-digital code converter to generate signals that are more convenient for transmission or processing\textsuperscript{12}. Such techniques take advantage of the trade-off that exist between word-rate and component precision requirements\textsuperscript{20}, because signals that are sampled at frequencies in excess of the Nyquist frequency (oversampling) require smaller number of bits per sample than those encoded at near the Nyquist rates\textsuperscript{9,19}. The poorer the resolution (number of bits per sample) of the encoder, the less rigorous are the requirements for high precision components\textsuperscript{65-67}.

The use of a single integration linear DM encoder in the first stage necessitates very high sampling frequencies\textsuperscript{20} to conform with the SNR and dynamic range requirements of a communication system conveying speech or video signals. The sampling frequencies can be reduced by employing modified double integration DM\textsuperscript{101}, delta-sigma modulation\textsuperscript{104}, instantaneously companded DM\textsuperscript{105} (HIDM), or DPCM\textsuperscript{103}
encoders as analogue-to-digital converters. Most encoders that use the DM-to-PCM code conversion principle are applied to speech signals\textsuperscript{89,104-107}. The systems by Candy\textsuperscript{103} and Ishiguro et al.\textsuperscript{101} are for application with video-telephone signals. Hawksford\textsuperscript{77} refers to the possibility of code conversions to PCM in his work on component coding of colour TV signals with Pulse Grouping Delta-sigma Modulation.

The all-digital code converters from DM to PCM or DPCM can be viewed\textsuperscript{20} as digital filters, estimators or interpolators. The design can be approached on any of these three lines\textsuperscript{101,20,103}, but the implementation is usually in the form of digital filters\textsuperscript{20,101,104-106}. As such, the maximum speed of operation is limited by the speed of the digital arithmetic operations in the filter, of which multiplications - if any - are the dominating factor in determining the total processing time\textsuperscript{12,108}. The multiplications can be avoided if the samples\textsuperscript{105} or the coefficients\textsuperscript{109} are integer powers of two. Thus, very high speed filters can be implemented by restricting either the sample (the step size) or the coefficient values, or both. From the point of view of the filter response characteristics, it is advantageous to restrict the sample rather than the coefficient values because the response of the filter is highly dependent on the coefficients used\textsuperscript{12}.

When adaptive DM systems are used, at least 7 bits are necessary\textsuperscript{105} to represent the step sizes. For each bit in the step size, a shift-and-add operation is required for the operation of multiplication, unless the step size is an integer power of two. In this case only one shift-and-add operation is required, with the number of shift positions determined by the power of two. Thus, instantaneously adaptive systems that generate steps that are powers of two are
desirable when speed considerations are important. Goldstein and Liu\textsuperscript{110} have disclosed recently\textsuperscript{*} (Aug. '76) and ADPCM realization of non-recursive digital filters, in which the A/D conversion is carried out by an ADPCM quantizer whose levels are varied by a factor of two at each clock instant according to the magnitude of the level used in the previous sampling instant\textsuperscript{95}. By using standard Transistor-Transistor-Logic integrated circuits (TTL IC's) and read-only memories (ROM's), they achieve throughputs of 40 MHz, with an all-parallel, pipe-lined structure. This speed was limited by the slowest of three operations: memory access, multibit shift, or addition, which was the ROM access time. With faster logic components, such as Emitter Coupled Logic (ECL) IC's, higher speeds can be obtained.

The sampling frequency of the DM encoder that can be used with a digital filter to generate PCM, DPCM, or even ADM\textsuperscript{102} signals, is limited by the speed of available components. By adopting the HIDM encoder because of its step variation by powers of 2, the multiplications in the filter are avoided and therefore the speed of the digital filter, and consequently of the encoder, is limited by the maximum speed with which ECL memories, adders and shifters can operate. ECL arithmetic logic units (ALU's) have throughputs of around 7 ns and ECL memories have access times of around 10 ns. Its gates have propagation delays of under 1 ns if the fastest of the ECL series is used (ECL-III).

\textsuperscript{*} though introduced here, the paper by Goldstein and Liu was available to us only after the computer simulations of our proposed system were completed and the work was concentrated on the analysis of noise in HIDM, which as explained earlier, was an off-shoot of the computer studies on the extension of the HIDM encoder, the 28IADM. The sampling frequency was determined considering only the speed (estimated) that could be obtained with accumulators built with ECL's.
Based on these speeds, the HIDM can operate only at speeds that are less than 100 MHz. We have adopted, somewhat arbitrarily, the sampling frequency of approximately 15 times the colour subcarrier frequency. This gives around 66.5 MHz, which is nearly 12 times the signal bandwidth. A second choice was around 80 MHz, i.e. approximately 18 times the colour subcarrier, but that seemed to leave little room for any additional delays that could be introduced in the circuitry. An integer multiple of three times the colour subcarrier was sought because the delta modulated data would be re-sampled at the customarily adopted PCM sampling rate for colour video coding.

The 66.5 MHz sampling frequency indicates that there is no possibility of obtaining broadcast quality with the simple HIDM-to-PCM code conversion that we have proposed to adopt. We have verified with computer simulations that if 6 steps were used with the HIDM, the SNR vs. frequency response* would present a curve falling at a rate of -6 dB/octave over most of the signal bandwidth, for the input sinusoids of amplitude approximately equal to 46 times the HIDM minimum step size (Fig. 4.1). This corresponds to a sinewave amplitude of 0.4425 V, which is the maximum peak amplitude of the colour subcarrier in a 100% colour bar, and the minimum step size of the HIDM corresponding to the quantum step of a linear 7-bit PCM quantizer. If a linear 8-bit PCM quantum step were used, the encoder would be overloaded. If larger step sizes were used, the SNR at very low frequencies would have been reduced by about 6 dB for each doubling of the minimum step size (see Fig. 3.7a).

The average colour picture is represented by a composite colour TV signal with a colour subcarrier with an amplitude that is around 11%...

* For constant input signal amplitude, as discussed in section 3.5B.
FIGURE 4.01 - SNR vs. frequency response for HIDM with 6 step sizes. Amplitude 46 times minimum step size.
of the peak subcarrier amplitude\(^{38}\) (average value for 295 studio scenes and slides). Thus, for non-broadcast applications, it is perhaps more important to ensure that the low frequencies are reproduced with high SNR's, and allow for some distortion to occur with fully saturated colours. This is the approach that we have adopted, and though not decisive the experiments carried out in this Department by a finalist student, and described in section 2.5D, showed that most off-air signals processed by a second-order Pulse Group DM, operating at 92 MHz, where sufficiently good to the effect that the distortions introduced by the delta modulator were not easily detected, unless the unprocessed original (off-air) pictures were also shown.

However, initial simulations with the HIDM encoder showed that the use of 6 step sizes would introduce occasionally large overshoots (maximum of 3\(\gamma\)\(_y\) before filtering), and for fast and large amplitude transitions there was a delay introduced by the build up sequence of step sizes (viz., 1,1,2,4,8,16,32,32). For a sampling frequency of 66.5 MHz, the total delay of about 7 clock intervals is of the order of 105 ns. This delay is approximately equal to the duration of a picture element (which is \(\approx 91\) ns in a 5.5 MHz system), and consequently it may be noticeable as edge business in vertical contours. Though the overshoots are smoothed by low pass filtering, the fast transients would introduce high frequency components in the message band that would interfere with the colour subcarrier. These overshoots can be corrected by either introducing a threshold control\(^{85}\) or overshoot suppressors\(^{111}\), but neither would improve the speed of response to fast step inputs. One possible solution is delayed encoding\(^{19}\) and a related solution is proposed in this thesis.
4.2. **The Two-bit Instantaneously Adaptive Delta Modulator (2BIADM).**

We recall that we have considered the use of a HIDM-to-PCM code converter to generate PCM or DPCM signals for application with colour (non-broadcast) TV. Thus, the delta modulator is used only locally. An increase in its output word length (usually one bit per sample) will incur only in an increase in storage requirements of the digital filter used as code converter, and it will not influence the output PCM or DPCM bit rate. We can trade such storage increases with improved performance, and this is the avenue that we follow by providing a side-information stream that will convey data on possible overshoot or occurrence of a sudden large error. The increase in the number of bits per sample generated by the delta modulator is limited by the need to keep the encoder as insensitive to component variations as the HIDM, from which our system is derived, and more importantly, it should still generate step sizes that are powers of two.

Such an encoder is proposed in this thesis and is called Two-bit Instantaneously Adaptive Delta Modulation (2BIADM) encoder. It is characterized as a type of delta modulator, and not as a 2-bit adaptive DPCM, because the system is designed to operate at frequencies much larger than the Nyquist and the polarity bits are processed as a stream of the HIDM encoder.

As with other instantaneously adaptive systems, the techniques for mathematically analysing the performance of the 2BIADM are unknown. To understand its operation, we carry out a simple comparative analysis with the HIDM encoder as the reference basis.

The block diagram of the 2BIADM is shown in Fig. 4.2a. The binary output \( b_n \) at the \( n \)-th sampling instant from the quantizer \( Q_1 \) contains
\[ B_n \leftarrow b_n = \text{sgn}(e_n) \]

**FIGURE 4.32** - Two-bit Instantaneously Adaptive Delta Modulator

(a) block diagram of encoder; (b) quantization characteristic
information about the polarity of the error signal, i.e., about \( b_n = \text{sgn}(e_n) \). This error is obtained by subtracting the reconstructed signal \( y_n \) from the input signal \( x_n \). The unbracketed subscripts\* indicate the values that the time signal has at the \( n \)-th sampling instant, i.e. for \( t = nT \), where \( T \) is the encoder sampling period. The magnitude of the error signal is monitored by the quantizer \( Q_2' \), whose binary output \( T_n \) is taken at instants \( t = nT_p + \tau \), where \( \tau \) is a delay smaller than \( T_p \).

The feedback step size \( \gamma_n \) is produced in two stages: first, an intermediate step \( s_n \) is produced according to Winkler's HIDM adaptation rule, described by Eqs. 3.3 to 3.5. The rule is repeated here for convenience:

\[
 s_n = m(1) s_{n-1}
\]

where \( s_{n-1} \) is the magnitude of the intermediary step at the previous sampling instant, and the multiplicative factor \( m(1) \) can assume the value 2, 1/2 or 1 depending on which output is high, respectively

\[
\text{UP} = B_n \cdot B_{n-1} \cdot B_{n-2} + \overline{B}_n \cdot \overline{B}_{n-1} \cdot \overline{B}_{n-2}
\]

\[
\text{DOWN} = B_n \cdot \overline{B}_{n-1} + \overline{B}_n \cdot B_{n-1}
\]

\[
\text{HOLD} = B_n \cdot B_{n-1} \cdot \overline{B}_{n-2} + \overline{B}_n \cdot \overline{B}_{n-1} \cdot B_{n-2}
\]

As discussed in section 3.3, in practice there is a limit to the increase and reduction in the step sizes. In the example illustrated in Fig. 4.2a, only four step sizes are allowed. We can represent the step size \( s_n \) digitally as:

\* we recall that the magnitudes are represented by bracketed subscripts.
or, in general,

\[ S_n = \left[ \sum_{j=1}^{N} S(j)2^{j-1} \right] \]

or, in general,

\[ S_n = \left[ \sum_{j=1}^{N} S(j)2^{j-1} \right] \]

where \( N \) is the number of permissible step sizes and \( S(j) \) can be 1 or 0, and only one out of the \( N \) coefficients \( S(j) \) can be unity at any clock instant. In other words the magnitude of \( S_n \) can only assume the values 1, 2, 4 and 8 in the example of Fig. 4.2a. The analogue step size magnitude is related to its digital representation by:

\[ S_n = \gamma S_n \]

where \( \gamma \) can be considered to be the digital-to-analogue (D/A) conversion factor, e.g. \( \gamma = 1 \) for \( j = 1 \), \( \gamma = 4 \) for \( j = 3 \), and so on.

With \( \gamma_n \) determined as in HIDM, \( S_n \) is temporarily stored in a "step steering logic block" while the error signal \( e_n \) is compared in \( Q_2 \) with a variable threshold signal \( T_{r_n} \), where

\[ T_{r_n} = 2\gamma_n \]

The binary output of \( Q_2 \) determines a second multiplicative factor \( m(2) \), such that the 2BIADM feedback step size \( \gamma_n \) is calculated as

\[ \gamma_n = m(2)\gamma_n = \gamma S'_n \]

where \( S'_n = \) output of the "step steering logic" and

\[ m(2) = \begin{cases} 
1 & \text{if } T_n \text{ is low, i.e. } |e_n| < T_{r_n} \\
4 & \text{if } T_n \text{ is high, i.e. } |e_n| > T_{r_n}
\end{cases} \]
The determination of $T_n$ depends on the calculation of Eq. 4.5, in which $s_n$ is produced from, amongst other parameters, the knowledge of $B_n$. Thus, $T_n$ has to be produced after a delay $\tau$ relative to the instant $B_n$ is generated, because of the circuit delay taken to produce $s_n$.

From Eqs. 4.6 and 4.1, we can write

$$\gamma_n = n(1)m(2)s_{n-1} = (m(1)m(2)s_{n-1})\gamma$$

(4.8)

where it is implicit that the feedback step size $\gamma_n$ is a function of the current and two previous polarity bits because both $m(1)$ and $m(2)$ are functions of $B_n$, $B_{n-1}$, and $B_{n-2}$.

Assuming an ideal integrator in the feedback loop of the 2BIADM, $\gamma_n$ is

$$\gamma_n = \sum_{i=-\infty}^{n} b_i \gamma_i = \gamma_{n-1} + b_n \gamma_n$$

(4.9)

which is similar to Eq. 3.6. The difference between Eqs. 3.6 and 4.9 resides in the way the step size $\gamma_n$ is produced at each clock instant.

From Eq. 4.8, observe that if the encoding conditions are such that $|e_n|$ is always smaller than $Tr_n$, $m(2) = 1$ and the 2BIADM operates as a HIDM with step sizes $s_n = m(1)s_{n-1}$. In the example of Fig. 4.2a, these step sizes range from $\gamma$ to $8\gamma$. Alternatively, if $|e_n|$ is always larger than $Tr_n$, $m(2) = 4$ and the 2BIADM would operate as a HIDM with step sizes $4s_n$, which in our example means step sizes in the range 4 to $32\gamma$.

For the normal operating conditions, at each sampling instant two bits are generated by the 2BIADM, viz. $B_n$ and $T_n$. The polarity bit describes the sign of the error signal, and the bit $T_n$ indicates
which of the two possible magnitudes \( s_n \) or \( 4s_n \) is used to quantize the error signal. Thus, a total of four levels is available for each sample, resulting in the quantization characteristic of Fig. 4.2b.

The quantizer is adapted according to the HIDM adaptation algorithm of Eq. 4.2, which expands, reduces, or keeps unchanged its decision thresholds and its output levels. The result is a "state diagram", depicted in Fig. 4.3, which illustrates the case when the adaptation is limited to four step sizes at the output of the HIDM step size logic. The transitions between states are ruled by Eq. 4.2.

The 2BIADM system employs only four quantization levels, but the encoder could be extended for a larger number of quantization levels and the multipliers \( m_{(1)} \) and \( m_{(2)} \) could be given different values. The main consideration for the determination of the threshold and \( m_{(2)} \) was the ease of implementation, i.e. multiplication factors that are powers of two (2 and 4 respectively).

The proposed encoder differs from the one described by Jayant in that the step multipliers in ref. [95] are functions of the magnitude of the quantization level used in the previous sampling instant. Consequently, the ADPCM of [95] has an up-dated quantizer already defined at the moment the new sample \( x_n \) is taken. In the 2BIADM, the step multiplier is a function of the three most recent polarity bits and the magnitude bit is produced after the sign of the error signal \( e_n \) has been examined and then the quantizer \( Q_2 \) up-dated. It is possible to combine the two strategies, for instance by employing syllabic adaptation with the polarity bits and instantaneous adaptation derived from the magnitude bits. In this thesis, however, we only analyse the simple case of the 2BIADM on its own.
γ is the encoder minimum step size

FIGURE 4.03 - Variation of quantization characteristics presented as a state diagram, for 2BIADM with 4 states
4.3. Computer simulation of waveform tracking capability.

A. Waveform reproduction.

The 2BIADM tracking performance can be better visualized with an example and a brief description. For that, we show the encoding of a sinusoid, illustrated in Fig. 4.4. The 2BIADM response is compared with an HIDM with a similar range of step sizes, i.e. \( \gamma_{(\text{max})}/\gamma_{(\text{min})} = 32 \). The sinusoid frequency is 1/12-th of the sampling frequency.

In the following description, the step sizes are described relative to \( \gamma_{(\text{min})} \). At clock instant 1, there are two possible step sizes for the 2BIADM, 1 and 4. The threshold \( T_{n} \) at this instant is 2 and is obviously exceeded by the error signal. Consequently, step size 4 is allocated. The HIDM encoder uses the only step available, which is 1. For the clock instants 2 to 4 both encoders have their step sizes varied according to Eq. 3.7 and 4.8, i.e. the step sizes are doubled from, and including, the third consecutive pulse of same polarity.

At clock 5, the 2BIADM has already reached its maximum step size whereas the HIDM steps are still increasing. At clock 6, the error signal is smaller than the threshold at this instant, which is 16, and consequently step size 8 is used. The overshoot that would normally have occurred with an HIDM encoder in similar conditions is avoided.

To illustrate the point, we note clock 19, where a condition as described earlier occurs for the HIDM, and the resulting relatively large overshoot that impairs the tracking for a further 4 clock periods. Clock instants 1 and 13 illustrates the faster response of the 2BIADM to signal changes (relative to the HIDM).

For slowly varying signals, the 2BIADM has little or no advantages over the HIDM, but it should never perform worse than the latter.
FIGURE 4.04 - Sine wave tracking by 2BIADM (curve (a)) and HIDM (curve (b))

with similar ratio between maximum and minimum step sizes.

Sine wave amplitude = 65 \gamma(t), F_p = 12.
One way to visualize the operation of the 2BIADM is to imagine that the input waveform is being tracked simultaneously by two HIDM quantizers, whose minimum step sizes are related by the ratio of 1:4. One of the quantizers produce step sizes in the range 1 to 8, and the other, from 4 to 32. Therefore, at each sampling instant there are two step sizes generated and the choice between them is governed by the magnitude of the error signal at that instant.

In Fig. 4.5a we show an arbitrary waveform constituted by a slow rising ramp, a short impulse, and a step. The signal is band-limited by the sharp low-pass filter described in section 3.4A. The encoding conditions are signal bandwidth = 5.5 MHz

   sampling frequency = 66.5 MHz
   slope of the ramp = \( \gamma(1) \frac{f_p}{10} \)
   minimum step size of the 2BIADM = \( \gamma(1) = 0.00964 \) (quantum step of a linear 7-bit PCM quantizer)
   impulse: rising \( 45\gamma(1) \) and falling \( 32\gamma(1) \), approximately
duration of the impulse = 12 clock intervals
   step input: amplitude = \( 85\gamma(1) \)

In Fig. 4.5b we show the response of the HIDM encoder. The input and reconstructed (low-pass filtered) signals are shown super-imposed, and the filtered error signal is also illustrated. The peak error amplitude is about \( 12.5\gamma_{(\text{min})} \) and each error pulse lasts for about 12 clock periods. It should be noted that these errors are introduced by overshoots and delays due to the build up of step sizes, and not strictly by slope-overload. By changing slightly the phase of the waveform relative to the sampling instants, or changing the signal amplitude, it has been verified that the encoder has the capability
FIGURE 4.05 - Slow rising ramp + impulse + step waveform reproduction
(a) original waveform
(b) HIDM (1-32)
(c) ZB1ADM
(d) HIDM (1-8)
(e) HIDM (4-32)
(b) HIDM {1-32}

(c) 2BIADM
(d) HIDM (1-8)

(e) HIDM (4-32)
to properly encode the signal, i.e. given favourable attacking conditions the HIDM encoder can track the signal without significant errors.

Fig. 4.5c illustrates the same signal processed by the 2BIADM, and it can be seen that the errors are greatly reduced. The peak error signal is now only a quarter of the corresponding error in HIDM. Changing the attacking conditions does not significantly alter the waveform tracking response.

The response of HIDM encoders with step sizes range of 8:1 instead of 32:1 are also presented in Figs. 4.5d and e. The first figure covers the range $\gamma$ to $8\gamma$, corresponding to an encoder that we represent in short as HIDM {1-8}. The second figure is for HIDM {4-32}, with step sizes ranging from $4\gamma$ to $32\gamma$. These figures show that if the range of step sizes is not enough, either slope-overload or large quantization errors (in the slowly-rising slope) can ensue. However, as Fig. 4.5b shows, the simple increase in the range of permissible step sizes is not a good enough solution because of the relatively slow build-up of step magnitudes from 1 to 32, for example. To reduce the errors due to these delays, one could increase the sampling frequency. This is not done here because we are assuming that the maximum sampling frequency is limited by component speed restrictions in the following digital filter stage. The solution provided by the 2BIADM is to allow the use of a larger step size whenever a given threshold is exceeded by the error signal, as described in the previous section.

B. SNR vs. frequency response.

The curves of SNR as function of frequency for sinusoidal inputs of constant amplitude have been found by the simulation approach
described in section 3.4C, viz. by averaging the SNR's over two cycles and for two 180° dephased attacking conditions.

In Figs. 4.6a to d, we present the results for 4 sinusoid amplitudes: 46, 24, 12, and 8, relative to $\gamma_{(1)}$ which is the 2BIADM minimum step size. The 46$\gamma_{(1)}$ amplitude corresponds approximately to the maximum amplitude that the colour subcarrier can assume in a 100% colour bar signal, viz., 0.4425 V. In the first figure, we also depict the peak SNR that can be achieved with LDM (Eq. 3.18), by a dashed, straight line. In each figure, four curves are shown, corresponding to the 2BIADM encoder, and three HIDM encoders with the ranges of step sizes {1 to 32}, {1 to 8}, and {4 to 32}. The last two curves are presented only as illustrations, and we confine the qualitative analysis to the comparison between the 2BIADM response and the HIDM encoder with step sizes range {1 to 32}.

For large amplitudes and high frequencies, the 2BIADM presents a clear improvement over the HIDM. In Fig. 4.6a and b, it can be noticed that the SNR curves for the 2BIADM present 3 regions that can be identified by the rate of fall in SNR. The first (A to B) occurs before the start of the 2BIADM adaptation algorithm, i.e. the encoder is operating as a simple HIDM, because the signal slope is still small. The rate of fall in that region is approximately -6dB/octave. The second region (B to C) is where the additional bit in the proposed encoder plays a role, and the rate of fall is reduced to about half that of HIDM {1-32}. However, as the slope (i.e. frequency) of the signal is increased, the 2BIADM tends to operate with only the larger step sizes, and the SNR curve drops rapidly. For very small amplitudes (Fig. 4.6d), a fourth, flat
FIGURE 4.06 - SNR vs. frequency response for HDM and 2BIADM
(a) amplitude = 46 $\gamma(1)$
(b) amplitude = 24 $\gamma(1)$
(c) amplitude = 12 $\gamma(1)$
(d) amplitude = 8 $\gamma(1)$

next pages
(b) amplitude = \(24 \gamma(1)\)
(c) amplitude = 12 $\gamma_{(1)}$
(d) amplitude = 8 \gamma(1)
The region where both the 2BIADM and HIDM encoders are operating as simple LDM because the slope of the signal is so small that the step adaptation never occurs (frequency and/or amplitude of the sinusoid very small).

The region of approximately -3dB/octave can be extended by increasing the number of step sizes. The improvement obtained with the additional bit per sample is reduced as the signal slope (frequency and/or amplitude) is decreased. The better response of the 2BIADM for higher frequencies will be reflected in faster transient responses.

C. SNR vs. input power for flat band-limited Gaussian inputs.

The simulation technique used to obtain the SNR vs. input power for Gaussian signals has been described in section 3.4D. The curves shown in Fig. 4.7 are the result of one of 5 segments of 512 sample points. The encoding parameters used are: ratio of sampling frequency to message bandwidth = 12.2, and the ratio between the maximum and minimum step sizes is 32.

Before companding starts, the HIDM and 2BIADM encoders operate as simple LDM. After companding starts, the 2BIADM operates as a HIDM with step sizes range {1 to 8} until the errors start to become large enough to require the use of the second bit, when its influence improves the performance of the proposed system over the HIDM by providing additional quantization levels. This improvement is over the range of approximately 18 dB, viz. \(20\log_{10} 8\).

It is interesting to compare the 2BIADM with an ADPCM with the adaptive quantizer described in [95]. The dynamic range of both is given by the range of variation of the quantizer, but the 2BIADM...
FIGURE 4.07 - Results of computer simulations for flat-bandlimited Gaussian signals. SNR vs. power (0 dB fixed arbitrarily).

Curves 1, 2, 3 and 4 are for 2BIADM, HIDM (1-32), HIDM (1-8), and HIDM (4-32), respectively.
present an additional platform for low signal powers, corresponding to the region where the error signals rarely exceeds the threshold $T_{th}$, and the encoder is operating as HIDM (Fig. 4.7). The ADPCM does not present such behaviour, i.e. it does not change the mode of operation from ADPCM to ADM at any stage, and consequently it does not show a 2-platform SNR curve. The comparisons between 2BIADM and ADPCM are given in more detail in Chapter VII.

4.4. Comments.

We have proposed here a simple extension to the HIDM by providing one additional bit per sample. This improves the peak signal-to-noise ratio with Gaussian inputs by about 5 dB, according to computer simulation results.

The encoder is apparently similar to usual DPCM encoders with a four level quantizer, which is made adaptive. However it differs in performance as for very low amplitudes it operates as LDM, then as HIDM, and finally it presents the performance expected from a DPCM encoder. An ADPCM system with the quantizer that is adaptable in the range 8:1 (see Fig. 4.3) would show a dynamic range of approximately 19 dB, and before the flattening of the SNR curve, it would present a response rising at a constant rate of unit slope. The 2BIADM on the other hand show two levels of flattening, one when it starts to operate as HIDM and finally the one that corresponds with the ADPCM response (Fig. 4.7). The improvement in peak SNR is, however, of less than the expected 6 dB by the doubling of the number of levels. This is probably due to the non-optimal quantizer output levels, which are in the ratio of 4:1 instead of the optimal
3:1 for the 2-bit quantizers.\textsuperscript{11}

The proposed system is a combination of ADM and ADPCM. Its disadvantage relative to the latter is that the 2BIADM is designed to operate only at sampling rates higher than the Nyquist. The disadvantage relative to ADM is the use of two bits per sample. Consequently, the 2BIADM show advantages only when its output is not intended for transmission, as in the case for which it has been devised.

Relative to the HIDM, the higher SNR response at high input power levels reflects the improvement in overshoot suppression and better response to fast, large transients.

The component precision requirements are largely defined by the quantizer resolution\textsuperscript{65,66}, and the coarser the quantization, the more relaxed are the requirements. Also, the requirements are eased when the signals are over-sampled and a feedback arrangement is provided\textsuperscript{19,103}, so that the quantization errors are compensated over a number of clock periods. This is the case with HIDM encoders. Relative to these, the 2BIADM has two additional levels, and as consequence the component precision requirements should be tighter for the latter. The main source of deviations in the 2BIADM, which is not present in the HIDM, is the threshold determination for the transition between the two possible step magnitudes. Thus, we have verified the effects of some changes in the magnitude of the threshold $T_{n}$. The tests consisted simply in changing the threshold by $+10\%$, $-10\%$, and asymmetrically for positive and negative errors by, respectively, $+10\%$ and $-10\%$ from the theoretical value given by Eq. 4.5. Although the unfiltered feedback signal showed some noticeable differences in the unfiltered
tracking pattern, the filtered output did not show significant differences because the feedback arrangement tended to correct for the deviations over a few clock instants. Also, there were no significant differences in the SNR's. This would not be the case if the signal were sampled at near the Nyquist rates, in which case more accurate representation of the samples would be required. The increase in the number of levels from 2 to 4 does not increase prohibitively the tolerance requirements, mainly because of the oversampled operation and the feedback arrangement.

The 2BIADM is stable as the step adaptation algorithm ensures that the step sizes decay to a minimum when the input signal level is constant, as it happens with the HIDM, i.e. it will idle with a pattern 101010... and with the minimum step size.

The effects of transmission errors were not investigated because of its intended local application. Nevertheless, it is possible to presume that should errors occur, the effect will be as serious as with HIDM because of its high dependence on the past polarity bit patterns. The error should propagate until a long sequence of 1010... idling pulses occur, or alternatively by the use of leaky integrators.

We have mentioned in section 4.1 that there would be no possibility of achieving broadcast quality encoding with the HIDM operating at a sampling frequency of around 66.5 MHz. This is still the case with the 2BIADM, although the latter show some improvements in performance relative to HIDM.

In Fig. 4.8 we show the encoding of a sinusoid of amplitude .3V (i.e., $A/\gamma(1) = 31$) by the HIDM (a,b) and 2BIADM (c,d), both with the minimum step size corresponding to the quantum step of a
FIGURE 4.08 - Sinusoid reproduction for (a)(b) HDM (1-32), and (c), (d) 2BIACK
amplitude of signal = 0.30V (i.e. $= 31 \gamma_1$), $F_0 = 15$.
Corresponds to colour burst signal. 2 phases 180° apart.
7-bit linear PCM quantizer. The amplitude of the sinusoid correspond to that of the colour subcarrier burst signal, and the encoding is shown for two phases 180° apart corresponding to a favourable and unfavourable initial encoding conditions. The figures show the input and reconstructed signals super-imposed, and it can be observed that the signal is processed with reasonable accuracy by the 2BIADM. At the zero crossings, there may be a maximum instantaneous phase delay of around 10° in one period, but in average the signal is reproduced without delays. There is also a small amplitude variation from period to period. These distortions are greater for the HIDM, with about three times larger phase delays.

Signals of smaller amplitudes are encoded more accurately, and consequently, the 2BIADM should be able to process the composite colour signals for the average colour picture, in which there are no saturated colours present.

If monochrome TV signals are encoded by the 2BIADM, we believe that the resulting pictures will be of high quality. This is because the low frequency signals are encoded with a 7-bit linear quantizer accuracy, and as the frequency of the signal increases, the encoding accuracy decreases but the eye also becomes less sensitive to the noise.
CHAPTER V

LOW SPEED HARDWARE MODEL OF THE 2BIADM

5.1. Introduction.

A technique that is not very frequently used to evaluate high speed encoders is that of building a low speed model. To the knowledge of the author, a simulation in a hybrid computer is the closest approach to it. Such a technique falters mainly because there is no visual display generated and consequently the human element is not included in the system. However, some information can be gained by examining its waveform reproduction performance.

The main advantages are:
(a) easy availability of components (linear and digital IC's)
(b) simple interconnections, facilitating rapid changes in circuitry
(c) no worries with the lay-out, grounding and shielding
(d) an over-all ease in handling the model.

The main disadvantages are:
(a) exclusion of the viewer and visual source from the system, thus precluding the study of psychovisual or psychostatistical coding methods
(b) although digital, TTL devices do not always have direct equivalents in the ECL series and consequently the design and implementation for the low-speed model cannot be used for the ECL prototype without modifications, i.e. it is not just a question of scaling up the circuit clocks
(c) it is not possible to estimate precisely what effects will have some distortions on the subjective evaluation of picture quality
Nevertheless, with awareness of the shortcomings, only a low-speed model has been built using standard TTL and linear IC's. No particular case has been taken for the design, which was intended to be simply an intermediate experiment to take full advantages of the available instrumentation. To gain an insight into the operation of the 2BIADM, the low-speed model was built with facilities to change its operation to HIDM and LDM by means of switches or external wiring connections.

We test the proposed system against the HIDM encoder, and present SNR measurements for sinusoidal and random noise inputs. The waveform tracking performance is illustrated with some photographs of time waveforms and from the display of a spectrum analyser.

5.2. Hardware description.

The circuit of Fig. 4.2a (redrawn in Fig. 5.1a), with the analogue integrator substituted by a digital accumulator, has been built to evaluate the performance of the 2BIADM relative to that of HIDM. Fig. 5.1b shows a more detailed diagram with the essential logic blocks indicated functionally in table 5.1. The use of a digital accumulator instead of an analogue integrator has the advantages of the added stability of digital circuits over their analogue equivalents, and of more uniform step sizes over the whole range of input signal levels. The one major disadvantage is the loss in the flexibility of changing the integration parameters, such as leakage, additional poles and zeros, and time constants, which with analogue circuitry can be achieved simply by adding or substituting
FIGURE 5.01 - Block diagram of the low-speed hardware model.

(a) simplified

(b) detailed
<table>
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<th>TABLE 5.1</th>
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| **a** | HIDM LOGIC | **UP** = $B_n \cdot B_{n-1} \cdot B_{n-2} + \overline{B}_n \cdot \overline{B}_{n-1} \cdot \overline{B}_{n-2}$  
**DOWN** = $B_n \oplus B_{n-1}$  
**HOLD** = $\overline{UP} \cdot \overline{DOWN}$  
($\oplus$ = EX-OR.) |
| **b** | SHIFT LEFT/RIGHT LOGIC | **LEFT** = $UP \cdot CLOCK$  
**RIGHT** = $DOWN \cdot CLOCK$  
**HOLD** = $\overline{LEFT} \cdot \overline{RIGHT} + S_{(max)} \cdot UP + S_{(min)} \cdot DOWN$ |
| **c** | STEP STEERING LOGIC | $S'(1) = \overline{T}_n \cdot S(1)$  
$S'(2) = \overline{T}_n \cdot S(2)$  
$S'(3) = T_n \cdot S(1) + \overline{T}_n \cdot S(3)$  
$S'(4) = T_n \cdot S(2) + \overline{T}_n \cdot S(4)$  
$S'(5) = T_n \cdot S(3)$  
$S'(6) = T_n \cdot S(4)$ |
| **d** | SWITCHES | SW2 : selects $S_{(max)} = S(4)$ OR $S(6)$  
SW3 : able/disable output $T_n$  
SW4 : manually set $T_n = 1$ OR $0$  
SW1 : set $S(1) = \overline{S}(j)$ , $j = 2, \ldots , 6$ |
few components. This loss in flexibility was not considered serious because the model was not built to operate with any particular input signal, but merely to complement the computer simulated comparisons between the 2BIADM and HIDM.

The 2BIADM was basically structured as a HIDM system with an additional stage where the step sizes normally generated by the HIDM step generator are kept unchanged (when \(|e_n| < |Tr_n|\)) or multiplied by 4 \((|e_n| \geq Tr_n)\). Digitally, multiplication by four means shift-two-to-the-left and, consequently, the number of step sizes available to the 2BIADM is equal to the number that exists at the output of the HIDM step generator block plus two. The model that we built has a total of six step sizes available to the 2BIADM, and as consequence the HIDM step generator could have had only four step sizes. However, the HIDM step generator was also provided with six step sizes, to allow comparisons between two systems with a similar range of step amplitudes.

The description that follows relates to Fig. 5.1b. The input signal \(x(t)\) and the feedback signal \(y(t)\) are compared with each other, and a binary output \(B_n\) is provided according to the sign of the difference between them. The three most recent bits are fed to the "HIDM logic block" that outputs the commands UP, DOWN or HOLD to a "shift left/right register", where a solitary "1" is moved around according to the command input applied to the shift register. Switch SW1 and a preset circuitry ensures that only the first of the shift register stages is a "1" at

* representing the binary words with the most significant bits at the left. The step sizes are positive numbers, thus the numbers are represented only by the magnitude bits.
the beginning of the operations. Because the number of step sizes is limited, the UP/DOWN/HOLD instructions are not applied directly to the shift register, but through a "shift left/right logic block" which disables the UP and DOWN instructions when the maximum and minimum step sizes are used, respectively, in the previous sampling instant.

Switch SW2 allows the choice between four and six step sizes in the encoder. The step sizes thus generated would constitute the feedback steps in the HIDM system, and if switch SW3 is open, this is the case. For the system to operate as ZBIADM, only four steps are used at the output of the shift register, which is D/A converted to produce a variable threshold signal $T_n$. The latter is compared with the error signal and a binary output $T_n$ indicates whether the threshold is exceeded or not. For SW3 closed, $T_n$ controls a "step steering logic". If the threshold is not exceeded, the step sizes $S(i), i = 1, \ldots, 4$ are presented as it is to the accumulator. If the threshold is exceeded, these steps are shifted by two to left before presentation to the accumulator. When the switch SW3 is open, switch SW4 can be used to manually control the step steering logic, thus allowing two HIDM encoders with four steps or one with six steps to be produced (the choice is made with SW2).

Also, if the preset switch SW1 is on, only the first stage of the shift register is active, thus only $S(1)$ is present at the input of the step steering logic block. SW2 in this case is inoperative, and with SW3 open, SW4 can be used to input $S(1)$ into the first or third stages of the accumulator, which makes the system operate as LDM with step size 1 or 4. The system is built in such a way that
any one, or a combination of, the inputs to the accumulator can be kept constant to produce LDM's of different step sizes. However, this can be done only with external wire leads.

The polarity of the most recent output, $B_n'$, indicates whether the steps $S_i', i = 1, \ldots, 6$ are to be added to or subtracted from the contents of the accumulator, whose output is D/A converted to provide the feedback signal $y(t)$.

The D/A converters were constructed using low precision resistors and poor quality switches. The only restriction placed on the converter was that its output signal $Y_n$ was required to increase monotonically as the magnitude of the binary words in the accumulator was progressively increased. 5% tolerance resistors were used, which resulted in non-uniform step sizes throughout the 256 levels, but the required monotonicity was, nevertheless, easily obtained. The non-uniformity in the step sizes is allowed because of the feedback configuration of the encoder, which ensures that errors eventually introduced in the encoding process are automatically corrected in subsequent clock instants.

The switches used in the R-2R network in the D/A converter are not as good as say FET switches, but simply open-collector TTL inverters (type 7416). The inverter's leakage currents does not significantly affect the encoder's performance but it requires that the d.c. offsets shown in Fig. 5.1b are used. The off-set in the outer loop is also used to ensure that the coder idles with its minimum step size. However, the adjustment of the d.c. off-set for the DAC used to generate the variable threshold voltage, $T_n'$, is somewhat critical as it introduces asymmetries in the quantizer $Q_2$. If this d.c. off-set is too large, the output $T_n$ is permanently in one of the logical states,
independently of $T_n$; i.e., of $s_n$. The experimental model of Fig. 5.1b was constructed with 30 conventional TTL digital integrated circuits, excluding timing circuits, and 6 linear integrated circuits (comparators and operational amplifiers).

The one major drawback of the proposed system is the limited capacity of the digital accumulator, which has 8 stages. This restricts the range of amplitude variations of the input signal to 256 levels. For such a resolution, however, the use of 5% tolerance resistors is not recommended, but because of the feedback arrangement this was done and the results were still satisfactory. The main consequence of the use of poor performance D/A converters is that the encoder departs occasionally from its idealized operation described in section 4.2.

5.3. Waveform tracking.

In Figs. 5.2 to 5.4 we show the waveform tracking obtained with the 2BIADM, for sawtooth, square-wave and sinusoidal inputs, respectively. Fig. 5.2 depicts the original signal (trace in the middle) and the unfiltered feedback signals for the HIDM (top trace) and 2BIADM. It is clear that the overshoots are practically eliminated. Also, after the fast transition the HIDM response overshoots and it oscillates over a few clocks before resuming the tracking of the signal.

Fig. 5.3a depicts the response of the HIDM for a square-wave input signal, whose fundamental frequency is $f_p/27$ and the peak-to-peak amplitude is approximately $65\gamma_{(1)}$. The top trace shows the feedback signal, and the lower one the error signal. It is quite clear that the output signal will be greatly distorted. The same
FIGURE 5.02 - Tracking of a sawtooth by HIDM \{1-32\} (top trace) and 2BIADM (bottom trace). $V_{p-p} = 120 \gamma_1$, $F_p = 42$
FIGURE 5.03 - Tracking of a square-wave
(a) HINM and (b) 281ADM for $F_p = 27$. Feedback and error signals
(c) HINM and (d) 281ADM for $F_p = 23$. Feedback and reconstructed signals.
signal encoded by the 2BIADM is depicted in Fig. 5.3b, and the reduction in overshoots and a more symmetrical square-wave reproduction is clearly noticeable.

To emphasize the better performance of the 2BIADM over the HIDM for fast transitions, Fig. 5.3c and d depicts the feedback signal and its smoothed waveform when the input is a square-wave signal of fundamental frequency equal to approximately $f_p/23$. The smoothing filter used was a 3-stage active low-pass filter with cut-off frequency (-3 dB point) at about $f_c = f_p/12$. Fig. 5.3c and d shows the HIDM and 2BIADM responses, respectively.

The amplitude of the square-waves in Fig. 5.3 was selected to highlight the overshoot reduction in 2BIADM, relative to HIDM. The frequencies have been chosen to illustrate the faster response possible with the proposed encoder.

The response to sinusoidal inputs is shown in Fig. 5.4, where in (a) and (b) we show the feedback and error signals for the 2BIADM and HIDM, respectively. It could be argued that because of the over-sampling the overshoot transients would be filtered out, thus we show in Fig. 5.4c the low-pass filtered outputs, and it is clear that, in this particular case, the overshoots are not entirely rejected. The ratio between the sampling and sinusoid frequencies is approximately 110, and the amplitude approximately $110\gamma(1)$. Because of the low frequency of the signal, the periodicity of the occurrence of overshoots falls in the message-band.

The better performance of the 2BIADM is obvious and expected because of the doubling in the number of bits per sample. We do not advocate the use of the 2BAIDM for transmission because of this.
FIGURE 5.04 - Tracking of a sinusoid of amplitude $220 \gamma (\tau)$ and frequency $f_p/110$. (a) 2BIADM; (b) HDM (1-32); (c) filtered outputs For (a) and (b): top traces for feedback signal and bottom trace for error signal. For (c): top trace for 2BIADM and bottom for HDM (1-32)
Indeed, the comparison of the responses of the HIDM and 2BIADM encoders obtained by computer simulations for the same bit-rate shows the HIDM to perform better than the 2BIADM for small amplitudes (all frequencies), and for large amplitudes but low frequencies. The performance is about equal for larger amplitudes and high frequencies (see Fig. 4.6). We recall, therefore, the reasons behind the development of the proposed encoder, which are local application and limitations in the increase of the sampling frequency (but not of the number of bits per sample). Comparing the two systems for the same bit-rate, the 2BIADM maintains the overshoot suppression (or rather, reduction) and the speed in response to the transients becomes comparable between the two systems, but the HIDM operating at twice the sampling frequency presents less granular noise in the message band. Because the overshoots and fast transitions occur only occasionally, the HIDM is in balance better than the 2BIADM for the same bit-rate, but worse for the same sampling rate.

The photographs in Figs. 5.2 to 5.4 were taken from the display of a storage oscilloscope, "freezing" one period scan. We show in Fig. 5.5 several periods displayed super-imposed. The faster adaptation of the 2BIADM show one additional advantage in these pictures, viz. the signal jitter is reduced. Fig. 5.5a shows the case when \( f_p / f_s = 50 \), and Fig. 5.5b the case \( f_p / f_s = 12.5 \) (close to the end of the message bandwidth), and in each of them the top trace corresponds to the response of a HIDM encoder and the bottom trace to that of the 2BIADM. These traces should be viewed only on a comparative basis for the two systems, and not as evidence for the evaluation of jitter on an absolute basis. This is so because the
FIGURE 5.05 - Illustration of jitter in the feedback signal while tracking a sinusoid of constant frequency. (a) $F_p = 50$, amplitude 120 $\gamma(1)$
(b) $F_p = 12.5$, amplitude 120 $\gamma(1)$
For (a) and (b) : top trace for HIDM, bottom for 2BIADM
signal itself jitters, and more significantly, the traces thicken because of the quantization.

These pictures also indicate a disadvantage of fast adaptive systems relative to LDM. With the linear delta modulator, the relative phases of the signal and sampling instants are not as significant as with the adaptive encoders, to which the attacking phase* affects the step adaptation patterns. We can presume, therefore, that a LDM encoder with a sinusoidal input will perform better than both the HIDM and 2BIADM as far as jitter is concerned (dynamic range considerations apart).

It is also clear from Fig. 5.4c that high frequency components are introduced by the HIDM, and to a lesser extent by the 2BIADM, and this may be an indication that the encoding of a composite colour signal by an IADM system may show interference patterns in colour, as the colour information is carried in the high frequencies region of the signal pass-band. It is possible, therefore, that the use of linear delta modulators with its spectral response shaped to match the input signal spectrum (such as delta-sigma-modulators with frequency response shaping networks added) will show a better performance when the colour subcarrier is present.

5.4. Spectrum analyzer display for sinusoidal input.

In Fig. 5.6a we show the feedback signal for LDM and in Figs. 5.6b and c the feedback and error signals for HIDM and 2BIADM.

* relative phase between the tracking waveform and signal at the start of encoding.
FIGURE 5.06 - Tracking of a sinusoid by (a) LDM; (b) HIDM (1-32); (c) 2BIAADM.
For (b) and (c): top curve is the feedback waveform, bottom is error waveform.
respectively. The input signal is a sinusoid of frequency \( f_s = 800 \text{ Hz} \) (i.e. \( f_p / 51 \)) and amplitude that is approximately \( 7\gamma \) for LDM and \( 40\gamma(1) \) for the adaptive encoders. The input signal amplitude is in fact constant for all three encoders but the LDM encoder step size \( \gamma \) was chosen so that the system is just before entering the slope-overload condition.

In Fig. 5.7 we show the spectrum of the feedback signal (unfiltered) for the LDM, HIDM, and 2BIADM encoders. In Fig. 5.8 we show the spectra for HIDM and 2BIADM super-imposed to provide a more direct comparison basis. The photographs of the spectra were taken from the display of the Hewlett-Packard Spectrum Analyzer 8556A + 8552B + 141T-display combination, in the store mode and the 100 Hz video filter. This filter effectively smoothes out the traces conserving only the main features of the spectra, and consequently the SNR should not be inferred from these photographs.

The over-all features are briefly described. The noise spectrum in LDM is essentially flat but there is some degree of line structuring, possibly due to the periodicity that can be noticed in the tracking waveform of Fig. 5.6a. The frequency component (spike) near the end of the trace in Fig. 5.7a appears to be related to the sawtooth-like tracking on the ascending and descending slopes of the sinusoid, as the frequency of the spike is about \( 4.6f_s \) and cannot be related to any harmonic of the input signal. The spectrum of the signal processes by the HIDM (Fig.5.7b) rises for the higher frequencies, reflecting

* The low bandwidth "video" filter in the HP spectrum analyzer effectively averages the noise for any one frequency.
FIGURE 5.07 - Spectrum of the signals in Fig. 5.06. (a) LDH; (b) HTHM, and (c) 2BIADN.

FIGURE 5.08 - Spectrum of the feedback signal for HTHM and 2BIADN, super-imposed.
the existence of overshoots. The slope of the noise spectrum is about 6 dB/octave. Finally, the noise spectrum for the 2BIADM is virtually flat and more randomized than for LDM (no clear structures).

Comparing the time waveforms of Figs. 5.6a and c, it can be inferred that the line formation in the LDM spectrum arises from a relatively large step size used in LDM and the near slope-overload condition that creates an effect that could appear in a picture as contouring, because of the 101 or 010 patterns that occur at around 6 clock interval spaces (on the slopes).

Fig. 5.7b is a clear evidence that high frequency noise is introduced within the signal pass-band. Fig. 5.8 shows that the overshoot reduction by the 2BIADM provides a spectrum that is flat as in LDM.

Referring back to Chapter III, where we carried out a heuristic analysis of HIDM encoders, it was found by computer simulations that the overshoots introduced noise correlated with the input signal, and this is noticeable in the time waveform (Fig. 5.6b) but not in the spectrum display (Fig. 5.7b), where although rising for increased frequencies, the noise appears to be random. An explanation to this apparent paradox can be deduced from Fig. 5.5a, where the same signal is shown with several periods superimposed. It is clear then that the location of overshoot points vary with time, breaking up the line formation in the spectrum.

5.5. **SNR measurements.**

A. Sinusoidal inputs.

The SNR measurements for sinusoidal inputs were taken with the
Marconi Distortion Factor Meter TF 2331, and the main results are shown in Figs. 5.9 and 5.10.

Fig. 5.9 depicts the curves of SNR vs. frequency for sinusoids of two different constant amplitudes. As the input signal amplitudes are decreased, the range over which the 2BIADM present higher SNR's decreases. Overall, the 2BIADM system show improvements for high amplitude, high frequency signals. The measured improvements are at the maximum around 6 to 7 dB.

Fig. 5.10 shows the variation of SNR as function of the sampling frequency for a sinusoid of constant frequency (800 Hz). The effects here are similar to those noticed in conjunction with Fig. 5.9, viz. the improvement obtained with the 2BIADM is more noticeable for larger amplitudes. The relative improvement decreases with increasing sampling frequencies because as the ratio between sampling and signal frequencies increases, there is less need for step adaptation, and indeed there is a point when even the HIDM adaptation also stops to occur. The SNR curves tend to a 9 dB/octave slope for very high sampling frequencies, but for low sampling frequencies, the slope is closer to 3 dB/octave. This is in apparent contradiction with the measurements reported by Eggermont, whose curve of SNR vs. sampling frequency for a sinewave modulated in phase and amplitude presents a 9 dB/octave slope. It should be noted that the measurement techniques are different, with Eggermont measuring the SNR over the speech band only (300 to 3400 Hz) with the encoder fully loaded. Eggermont defines "fully loaded" as the signal level that is 6 dB lower than the input level at which the signal is coded as a triangular wave for the first time.

In the measurements presented in this thesis, the SNR's are
**FIGURE 5.09 - SNR vs. input signal frequency for sinewaves of amplitudes 240 $\gamma_1$ and 60 $\gamma_1$, for HIDM (1-32) and 2BIADM.**

$\gamma_0 = 40$ kHz

**FIGURE 5.10 - SNR vs. sampling frequency for sinusoids of constant amplitudes 240 $\gamma_1$ and 60 $\gamma_1$, for HIDM (1-32) and 2BIADM.**
measured for constant amplitude inputs, and the Marconi meter measures the distortion over a 20 KHz bandwidth (or 100 KHz, though the latter was not used). We note that for LDM, the accepted notion of "9dB/octave increase in SNR for increasing sampling frequencies" hold for the peak SNR (or fully loaded encoding condition), as indicated by Eqs. 1.8 to 1.10 of Steele's book. For constant amplitude and step size, the SNR increases by 3 dB/octave according to Eq. 1.9 of [19]. The "fully loaded" measurements were not made because of the limited capacity of the accumulator, which limited the system by peak-clipping rather than by slope-overload.

However, a reference point can be found for the measurements by Eggermont and the curves presented in this thesis, viz. the SNR for the sampling frequency of 40 KHz, for which the amplitude of $240\gamma(1)$ is close to the fully loaded condition described in [88]. Our measurements give approximately 18 dB, whereas Eggermont's value is about 24 dB. Considering the different bandwidths over which noise is measured, we can add to our measurement approximately 7.7 dB (i.e. $10\log_{10}(20/3.4)$), resulting in a SNR of 25.5 dB. The difference in 1.5 dB could be accounted for by the fact that Eggermont's measurements are an average SNR as the signal is modulated in phase and amplitude, and the inequality in the loading conditions of the encoders.

We do not try to relate the results of practical measurements with those obtained by computer simulation, because of the differences in the definitions of SNR. In the computer simulations, the SNR values are calculated by considering as "noise" the difference, on a sample-to-sample basis, between the input and reconstructed signals, with
all phase delays compensated for. In practical measurements, as described by Eggermont and in the Marconi TF-2331 operating instructions, the SNR's are calculated from the measurements of the rms power of the signal plus distortions (i.e., of the reconstructed waveform) before and after removing the fundamental frequency of the input signal with a notch filter. Thus, distortions that occur at the fundamental frequency of the input signal are not taken into consideration. Such distortions (amplitude attenuation and phase delay relative to the input waveform) occur when the encoder is slope-overloaded. As an absolute measurement of the error introduced by the digital processing, the computer simulation results give a better description. However, in systems where phase distortions are unimportant and the high frequency components have amplitudes that are much smaller than those of lower frequencies (speech, for instance), the results obtained from the practical measurements reflects well the performance of the system, as far as objective evaluations are concerned. For systems where phase and amplitude distortions are important, measurements obtained with equipment such as the Marconi TF-2331 should be considered with reservations if correlated noise at the frequency of the input signal is present, or suspected to be.

The calculations of SNR by computer simulation do not, normally, follow the procedure adopted in practical measurements, because of the relatively simple sample-to-sample difference calculations, compared to notch filtering. Practical measurements do not follow the computer simulation procedure, because of the difficulties in compensating for phase delays that are introduced by the encoding/decoding throughput time and the smoothing filter.
B. Bandlimited noise inputs.

The SNR curves for narrow band-limited with noise, as functions of the input signal power level, are given in Fig. 5.11, with the sampling frequency as a parameter. The measurements were taken with the Marconi Quantization Distortion Tester TF 2343A. This instrument generates pseudo-random white noise band-limited from 450 to 550 Hz, and the SNR is measured over the speech bandwidth (300 to 3400 Hz). It again ignores amplitude and phase distortions for the input signal fundamental frequencies. The consequence of this technique of calculating (measuring) SNR's can be seen in the SNR curve for 8 KHz. Before slope-overload ensues, the measurements are reliable as only quantization noise is present. When slope-overload starts to become significant, the SNR curve is seen rising, instead of the expected fall. This happens because as the measurements ignore amplitude and phase distortions, in severe slope-overload the instrument tends to measure the power of the fundamental frequency of a triangular waveform relative to the power of its harmonics within the message pass-band as the SNR's. Consequently, as quantization noise becomes negligible compared to slope-overload, the SNR increases towards the ratio (in dB) between the powers of the fundamental frequency of the triangular waveform and its harmonics in the pass-band, i.e., around 17 dB.

The measurements shown in Fig. 5.11 are limited by peak-clipping for higher sampling frequencies because of the limited number of stages in the digital accumulator used as integrator, which limits the increase in the signal power level. It is, however, possible to ascertain improvements of up to about 8 dB maximum in the SNR's obtained with the 2BIADM relative to those with HIDM.
FIGURE 5.11 - SNR vs. input power (dBm) for different sampling frequencies
5.6. Comments.

Notwithstanding the differences in computing methods to find the SNR's by experimental and computer simulation measurements, the results presented in this Chapter generally corroborates the findings in the previous Chapter.

The waveform tracking responses give some indication of the performance of the 2BIADM to process video signals, but the extrapolations can be viewed only as speculations.

With regard to the constructed encoders, no special care was taken in its design, as the objective was to simply gain an insight into the operation of the encoder before proceeding to the design of a system using ECL devices. Some difficulties that were apparent even with the low speed model built with conventional TTL components are:

(a) the use of 5% resistors for the D/A conversion. The HIDM step adaptation algorithm is such that after a step input, the tracking signal rapidly falls to a 1010.... pattern. However, deviations in the step sizes caused the constructed encoder to occasionally track the signal in a 111000.... pattern because the step sizes were not related exactly in a 2:1 ratio. This behaviour was noticed only to a few amplitudes of square-wave inputs, and in general the tracking was satisfactory. For signals other than square-waves, the deviations from the theoretical tracking were barely noticeable.

(b) the d.c. off-sets shown in Fig. 5.1b interact with each other and may up-set the adjustment of the comparators used for detecting when the error signal exceeds the thresholds.
(c) in the particular implementation with a digital accumulator as the feedback integrator, care has been taken to detect over and under-flows and to set and reset the accumulator, respectively. The 8-bit accumulator limits the range of input signal levels by peak-clipping.

One of the difficulties that can be expected to occur for the design of a high speed model is the speed of D/A conversion. In the model built, open-collector TTL inverters were used as switches, and a 741 linear operational amplifier used as summing amplifier and to provide d.c. off-sets. For the encoder operating over 66 MHz, the amplifier and switch settling times could become crucially important. Most probably, analogue integrators will have to be used instead. A second difficulty may lie in adjusting the timing for the threshold determination and steering the step sizes.

The hardware model showed that the 2BIADM can be built without high precision components, and in particular, that it is tolerant to deviations in the determination of the thresholds for the error signal and that some improvements are still present when the timing is such that the previous step size \( s_{n-1} \) is used to generate the threshold \( T_{n} \). In such case, the main consequence was that the overshoots were not as effectively damped and it was noticeable with square-wave inputs. Measurements of SNR with Gaussian inputs did not show marked differences (within 1 to 2 dB difference).

With regard to the TTL model, its usefulness can be improved by providing a larger capacity accumulator. If enough care is taken to improve the over-all stability of the system, it should provide a good teaching aid to introduce DM, for it is able to produce LDM encoders
with a large number of step sizes, and it has a "staircase-like" tracking waveform close to the theoretical.
CHAPTER VI

CODE CONVERSION FROM DM TO PCM

6.1. Introduction.

Analogue signals such as speech and video signals can be represented by means of binary numbers for transmission and digital signal processing purposes. For the latter, PCM with linear quantization is usually employed, although DM and DPCM-based signal processors are currently making inroads. The advantages of digital representation and processing of signals have been summarized in Chapter I, Section 1.2, and are described in detail by Cattermole (particularly for transmission) and Rabiner and Gold (for signal processing).

Usually, PCM analogue-to-digital converters comprise an input signal band-limiting filter, a sample and hold operating close to or at the Nyquist rates (although not necessarily), a quantizer, a number assignment stage, and an output register or buffer from which the digital signals can be extracted serially or in parallel. These stages are schematically represented in Fig. 6.1. An example of an 8-bit PCM quantizer structure is given in Chapter II, Section 2.5B, for application with video signals. In general, a N-bit binary word can represent \(2^N\) levels, and the tolerance requirements of the components used in the realization of N-bit quantizers are directly dependent on the necessity to resolve 1 level in \(2^N\).

In most practical systems, the successive signal samples are correlated to each other, and this correlation can be exploited (e.g. with differential encoding) to reduce the average number of bits per quantized sample to represent the signal with a certain quality, such
1 Low-pass filter, to reject out-of-band frequency components
2 Sample and hold
3 QN: quantizer and binary number assignment system
4 Bi: output register (buffer). The contents can be taken in parallel or sequentially

(a)

(b)

FIGURE 6.01 - Analogue-to-digital conversion to PCM
(a) principal stages of a conventional PCM encoder
(b) principle of indirect conversion to PCM
as SNR. This correlation between samples can be further increased by increasing the sampling frequency over the Nyquist rate. Thus, the number of bits required to represent a sample can be traded-off with the sampling frequency, to the extreme of necessitating only one bit per sample, as in delta modulation. Because this latter encoder requires a simple two-level quantizer, the precision requirements are more relaxed than for the realization of multi-level quantizers used in PCM and DPCM. However, for SNR larger than about 15 dB obtained with bandlimited white noise inputs, linear delta modulators require higher transmission bit rates than PCM.

The analogue complexity in conventional PCM encoders can be traded-off with digital complexity by a process of DM-to-PCM code conversion, which has been first described in detail by Goodman. The advantages of this trade-off lie mainly in the possibility of employing large scale integration techniques to implement complex digital processors, while keeping the analogue stages simple. This results in more economical realizations. Goodman describes the LDM-to-PCM conversion process alternatively as digital estimation, interpolation, and filtering, but with the emphasis in the estimation point of view. Ishiguro et al., Potter and Thurlow, and Eggermont present the conversion as a filtering process, viz. the rejection of out-of-band noise before re-sampling at near Nyquist rates. Candy uses as the input stage a low-resolution (4 bits) direct feedback quantizer and generates 8-bit resolution PCM signals as an interpolation process. Pennington and Steele deviate from the above by delta modulating signals that are sampled and held at near the Nyquist rate. However, a common factor to all these techniques is the exchange between
the tight requirements for high precision components and the higher encoding rates that require low resolution A/D converters. The trade-offs that are exploited are mainly between: (a) component precision requirements and sampling rates; (b) analogue and digital complexities; and (c) encoding speed and storage requirements. The general structure of these indirect coding methods is depicted schematically in Fig. 6.1b.

The DM-to-PCM converters have two main stages: a DM encoder and a digital filter. Individually they have received considerable attention\textsuperscript{19,12} but their combination is still relatively new\textsuperscript{20,101-107}. Thus, in the next section we review the basic principles before introducing two filter structures for the conversion from 2BIADM to PCM. Readers familiar with DM-to-PCM code conversion techniques, and digital filtering, can skip Section 6.2 to 6.5.

6.2. DM-to-PCM code converters - basic principles.

A. General principles.

The DM-to-PCM code conversion process can be viewed in the frequency or time domains. In the frequency domain, we recall that the LDM signal (for high SNR's) are obtained by over-sampling the signal. Thus, as part of the code conversion to PCM, the signals have to be re-sampled at near the Nyquist frequency. In the re-sampling process, the components of the noise spectrum that extend above the message pass-band\textsuperscript{19} will be folded back into the baseband. To reduce such aliasing distortions, the LDM signal has to be band-limited before re-sampling and this low-pass operation constitutes the principal process in the DM-to-PCM code conversion. The visualization of the process in the time domain,
is more difficult. Thus, a more detailed outline is given next, by considering the trivial DM-analogue-PCM encoder introduced by Goodman, and which is shown in Fig. 6.2.

Basically, it consists of a LDM encoder that produces a binary stream $\{b_n\} = \{b(nT_p)\}$, which is a sequence of ±1 pulses generated at $T_p = 1/f_p$ intervals. The subscript $n$ indicates the number of sampling intervals from the start of the encoding process. The sequence $\{b_n\}$ is used in a local decoder, which consists of an integrator of gain $\gamma$, to generate the feedback signal $y(t)$. For an ideal feedback integrator:

$$y(nT_p) = y_n = \gamma \sum_{k=-\infty}^{n} b_k$$

(6.1)

where $\gamma$ is the LDM step size. The LDM receiver is the local decoder followed by a low-pass filter, i.e. the signal is reconstructed by low-pass filtering $y_n$. Representing the LDM receiver impulse response by $y_g(\cdot)$, the reconstructed signal $\tilde{x}(t)$ can be represented by the convolution of the impulse response of the receiver and the sequence of DM pulses $\{b_n\}$, i.e.

$$\tilde{x}[(n+M)T_p] = \gamma \sum_{k=0}^{\infty} g(kT_p)b[nM-k]T_p$$

(6.2)

where $MT_p$ corresponds to the delay introduced by the low-pass filter, and the sequence $\{b[nM-k]T_p\}$ for $(n+M-k)T_p > (n+M)T_p$, i.e. $k < 0$, has been excluded from the convolution equation because at the time $(n+M)T_p$ the LDM output sequence consists only of the pulses that have occurred prior to that instant.

The re-sampling process at $RT_p$ intervals, where $R > 1$ is the ratio
FIGURE 6.02 - LDM-to-analogue-to-PCM conversion process. After Goodman [20]
between the DM sampling frequency and the Nyquist frequency (viz. the bandwidth expansion ratio), can be represented by taking the samples at \( n_T = jRT \), which are, from Eq. 6.2:

\[
\tilde{x}_{jR} = \gamma \sum_{k=0}^{\infty} g(kT_p) b \left( (jR+M-k)T_p \right)
\]

or with the simplified notation:

\[
\tilde{x}_{jR} = \gamma \sum_{k=0}^{\infty} g_k b_{jR+M-k} ; \quad j \text{ integer positive} \quad (6.3)
\]

Eq. 6.3 can be approximated to an arbitrary accuracy by writing

\[
\tilde{x}_{jR} = \gamma \left[ \sum_{k=0}^{N-1} g_k b_{jR+M-k} + \sum_{k=N}^{\infty} b_{jR+M-k} \right]
\]

such that \( N = 2M + 1 \). The larger the \( N \), the closer the results of Eqs. 6.4 and 6.3. The digital realization of Eq. 6.4 constitutes the main process in the DM-to-PCM code conversion. The second term between brackets can be realized as a simple accumulator or up-down counter, whereas the first term can be implemented as an \( N \)-coefficient digital transversal filter, whose output is added to the output of the accumulator.

By writing \( n = k-M \), Eq. 6.4 becomes

\[
\tilde{x}_{jR} = \gamma \left[ \sum_{n=-M}^{M} g_{n+M} b_{jR-n} + \sum_{n=M+1}^{\infty} b_{jR-n} \right]
\]

(6.5)

To understand the meaning of the above equation, consider the simple case \( N = 1 \), i.e. \( M = 0 \). In such a case, Eq. 6.5 is reduced to

\[
\tilde{x}_{jR} = \gamma \left[ g_0 b_{jR} + \sum_{n=1}^{\infty} b_{jR-n} \right]
\]

(6.6)
and if \( g_o = 1 \) and a change in variables is made by writing \( n' = jR - n \), Eq. 6.6 becomes

\[
\bar{x}_{jR} = \gamma \sum_{n' = \infty}^{jR} b_{n'}
\]

(6.7)

which means that the PCM sample taken at instant \( jRT_p \) is equal to the output of an up-down counter (or accumulator), whose input is the LDM output sequence. From Eq. 6.1, this is also the unfiltered output of the LDM decoder with an ideal integrator.

For the general case, Eq. 6.5 indicates that the PCM sample at time \( jRT_p \) is equal to the unfiltered output of the DM decoder (which is also the feedback signal) at the instant \( (jR - M - 1)T_p \) plus the weighted average of the following \( 2M + 1 \) samples in the sequence \( \{b_n\} \), which are the most recent DM outputs. To show that the second term within the brackets in Eq. 6.5 is equal to \( y_{jR - M - 1} \), simply substitute the variable \( n \) by a new variable \( n' = jR - n \) and use Eq. 6.1. Fig. 6.3a illustrates schematically the transversal filter arrangement, plus the up-down counter, for the implementation of Eq. 6.5. The single lines indicate 1-bit binary signal paths, whereas the double lines indicate \( n \)-bit word paths.

It can be shown (see appendix 4) that Eq. 6.5 is equivalent to

\[
\bar{x}_{jR} = \sum_{k=-M}^{M} a_k y_{jR-k}
\]

(6.8)

where the coefficients \( a_k \) are such that

\[
\sum_{k=-M}^{M} a_k = g_{n-M} \quad ; \quad -M \leq n \leq M
\]

\[
\sum_{k=-M}^{M} a_k = 1
\]

(6.9)
FIGURE 6.03 - Digital low-pass filter for DN-to-PCM conversion
(a) filtering \( b_n \); (b) filtering \( y_n \)
single lines indicate 1-bit paths; double lines indicate binary words

FIGURE 6.04 - Filter structure for \( N = 5 \), after Goodman [26]
and \( a_k = 0 \) for \( k > M \)

Eq. 6.8 indicates that the PCM sample at instant \( jRT_p \) is equal to the weighted average of \( 2M+1 \) output samples of the DM decoder (i.e. of the DM feedback samples). Fig. 6.3b illustrates the implementation of Eq. 6.8 with a transversal filter.

From the above analysis, it can be observed that:

(a) a digital DM decoder with a transversal filter (non-recursive) filter with \( N \) coefficients and an up-down counter can be used to approximate the analogue decoder given by Eq. 6.3, with an accuracy that increases with \( N \) (Eq. 6.4). The limit, for \( N \to \infty \), is the result that can be obtained by using an analogue filter, and consequently the PCM signal generated with the LDM-to-PCM code conversion cannot exceed the quality of the original delta-modulated signal;

(b) the filtering operation can be applied to the decoded samples (Eq. 6.8), or to the quantized error signals, i.e. the sequence \( \{ b_n \} \) (Eq. 6.5);

(c) the code conversion can be viewed in the time domain as a problem of estimation or interpolation (Eq. 6.5 and 6.8), in which the PCM sample is calculated from the evidence provided by the observation of \( 2M+1 \) delta modulated samples. With this approach the coefficients \( \{ a_k \} \) are found by minimizing the mean square estimation error between the input sample, \( x_{jR} \), and \( \hat{x}_{jR} \).

(d) alternatively, in the frequency domain the code conversion process can be viewed as a low-pass filtering operation, which is carried
out before re-sampling to avoid the aliasing of the out-of-band noise into the pass-band. The coefficients of the filter are found in this case considering the out-of-band rejection characteristics required \cite{101,105}. This approach allows the use of conventional filter design techniques.

For a better visualization of the code conversion process as described by Goodman, we present two examples.

**Example (1).**

Let us consider the filter derived by Goodman in [20] with five coefficients. From Eqs. 6.5 and 6.1:

\[
\tilde{x}_{jR} = \gamma \left[ \sum_{n=-2}^{1} g_{n+2} b_{jR-n} + \sum_{n=3}^{\infty} b_{jR-n} \right]
\]

thus

\[
\frac{\tilde{x}_{jR}}{\gamma} = g_0 b_{jR+2} + g_1 b_{jR+1} + g_2 b_{jR} + g_3 b_{jR-1} + g_4 b_{jR-2} + y_{jR-3}
\]

(6.10)

The coefficient values derived by Goodman are:

\[
g_0 = \frac{1}{8}
\]

\[
g_1 = \frac{3}{8}
\]

\[
g_2 = \frac{5}{8}
\]

\[
g_3 = \frac{7}{8}
\]

\[
g_4 = 1
\]

(6.11)

with which Eq. 6.10 becomes

\[
\frac{\tilde{x}_{jR}}{\gamma} = \frac{1}{8} b_{jR+2} + \frac{3}{8} b_{jR+1} + \frac{5}{8} b_{jR} + \frac{7}{8} b_{jR-1} + y_{jR-2}
\]

(6.12)
In Eq. 6.12, the term $b_{jR-2}$ has been combined with $y_{jR-3}$ to give $y_{jR-2}$. The implementation of such a filter is schematically shown in Fig. 6.4.

As an example of the encoding process, let us consider two arbitrary curves (Fig. 6.5). The staircase-like waveform is the un-filtered LDM decoder output, with each horizontal section corresponding to the output levels of the up-down counter (ideal integrator or accumulator) held constant between DM clocks. The inputs to the up-down counter are the ±1 pulses generated by the delta modulator tracking the input signal (full lines). Let us consider the instant indicated by $A$ in curve $a$, which corresponds to the time $jR=A$. At that instant, $b_A = -1$ and $y_A = 6$. At the previous clock, $b_{A-1} = -1$ and $y_{A-1} = 7$, and so on. Eq. 6.12 is used to calculate the PCM sample at that instant, with $jR = A$:

$$\tilde{x}_A \div y = \frac{1}{8} (-1) + \frac{3}{8} (+1) + \frac{5}{8} (-1) + \frac{7}{8} (-1) + 8 = 6.75 \quad (6.13)$$

This point is indicated by a dot in Fig. 6.5, at time $A$.

If the PCM sample were taken from the output of the counter alone, as given by Eq. 6.7, $\tilde{x}_A$ would be equal to $y_A$, i.e. $\tilde{x}_A = 6y$, instead of 6.75$y$ as calculated in Eq. 6.13. It is clear that the use of the filter produced additional levels between those existent in the DM tracking (unfiltered) waveform. In this instance, the level 6.75, between 7 and 6. The filter process for the code conversion can, then, be viewed as an interpolation process, as mentioned before.

The next PCM sample is taken at time $B = A + R$. In the example of Fig. 6.5, curve $a$, $R = 8$, and
EACH 8-th SAMPLE CONSTITUTES ONE PCM SAMPLE

FIGURE 6.05 - Response to an arbitrary signal with filter of Fig. 6.04 (dots) and of Fig. 6.06 (crosses). (a) fast varying signal; (b) slow varying signal
\[
\frac{\tilde{x}_B}{\gamma} = \frac{1}{8} (+1) + \frac{3}{8} (+1) + \frac{5}{8} (-1) + \frac{7}{8} (+1) + 6 = 6.75.
\]

Similarly, at \( C = B + R = A + 2R \), \( \tilde{x}_C/\gamma = 12 \), and so on. In Fig. 6.5 we present the interpolated points (dots), according to Eq. 6.12, to all DM sampling instants, to illustrate the approximation that can be obtained to the original signal.

A more slowly varying signal is also shown in Fig. 6.5, curve b. In both cases the interpolation achieved is clearly noticeable.

Example (2).

In Example (1), Eq. 6.5 has been employed. As another example, we use Eq. 6.8, i.e. the up-down counter outputs are used as the inputs to a transversal filter:

\[
\tilde{x}_{jR} = \sum_{k=-M}^{M} a_k y_{jR-k}
\]

For simplicity, let \( a_k = 1/(2M+1) \), \(-M \leq k \leq M\), with which the d.c. gain of the filter is unitary. With \( M = 2 \),

\[
\tilde{x}_{jR} = \frac{1}{5} (y_{jR-2} + y_{jR-1} + y_{jR} + y_{jR+1} + y_{jR+2})
\]  

(6.14)

Considering the same signal as in the previous example, the PCM sample at time \( A \) is now given by

\[
\frac{\tilde{x}_A}{\gamma} = \frac{1}{5} (8 + 7 + 6 + 7 + 6) = 6.8
\]

Similarly,

\[
\frac{\tilde{x}_B}{\gamma} = \frac{1}{5} (6 + 7 + 6 + 7 + 8) = 6.8
\]
and

\[ \frac{\bar{x}_c}{y} = 12 \]

and so on. These points are indicated by crosses in Fig. 6.5, where again the interpolated values are shown for all DM sampling instants, although only each R-th sample constitutes a PCM sample. In Eq. 6.14, \( \bar{x}_{jR} \) can be viewed as an estimate from the knowledge of \( y_{jR} \) and the two samples that precede and that succeed it.

The filter for the realization of Eq. 6.14 is shown in Fig. 6.6a, with its frequency response shown in Fig. 6.6b. Albeit poor, it represents a low-pass filter, and the conversion process can be viewed also as the process of reducing the out-of-band noise before re-sampling. In this particular example, when the output of the filter is re-sampled at Nyquist rates (i.e. at \( f_p/R = 0.125f_p \)), there will be quantizing noise being folded back into the message pass-band (shaded area). Improving the filter design (i.e. the out-of-band attenuation characteristics), this amount of noise that is folded back is reduced. The "best" PCM signal is obtained with this process when all the out-of-band noise is removed before re-sampling. The "poorest" case occurs when the DM feedback signal is sampled at Nyquist rates without filtering, when all the out-of-band noise is folded back into the message band.

B. Comments.

Examples (1) and (2), with Fig. 6.5, illustrate how the code conversion process can be viewed in the time domain, when an ideal integrator (up-down counter) is used to reconstruct the tracking signal.
FIGURE 6.06 - Unitary coefficients filter

(a) filter structure; (b) frequency response
First, if the signal is re-sampled directly at the output of the up-down counter, the quantization resolution of the PCM output obtained is equal to the DM step size. This step size is basically a function of the DM sampling frequency and the rms power of the input signal, thus the resolution of the PCM signal obtained with the DM-to-PCM code converter can be increased with an increase in the DM sampling frequency. However, for a given DM sampling frequency, the resolution of the PCM quantizer can also be improved by the use of a digital filter between the up-down counter and the re-sampler, as the filter produces additional levels between the output levels provided by the up-down counter. In example (1), for instance, if the output of the up-down counter were re-sampled directly to produce PCM signals, these would require only 4 bits for the representation of the 16 levels needed to encode the signal shown in Fig. 6.5. Maintaining the sampling frequency, the use of a filter of 5 stages before re-sampling has produced additional levels, and a PCM signal with a resolution of 6 bits can now be produced* (there are about 60 levels possible at the output of the filter).

The same effect (better approximation to the original signal) could have been obtained without the use of the filter, by increasing the sampling frequency, thus there is a trade-off between the sampling frequency and the use of filters. This can also be seen from the performance curves for the LDM-to-PCM code conversion presented by Goodman, one of which is reproduced in Chapter II, Fig. 2.16, and in the next page. The response for \( N = 1 \) corresponds to the case

* as long as DM slope-overload is avoided. This increase from 4 to 6 bits output is only possible if the PCM word generator has the capability of increasing the output word length, otherwise the use of the filter would be unnecessary.
FIGURE 2.16 - Performance curves of LDM-to-PCM code conversion, flat bandlimited Gaussian inputs (after Goodman [26])
where only the up-down counter is used. As example, if a SNR of 25 dB is required for flat spectrum Gaussian inputs, corresponding approximately to a 6-bit PCM encoder, if only the up-down counter is used, the delta modulator would have to operate at a sampling rate of over 75 times the Nyquist frequency. Alternatively, lower sampling frequencies can be used if a filter is used before re-sampling. The limit is for $N = \infty$, which corresponds to the case where an ideal low-pass filter is used to smooth the output of the DM decoder. In this case, the DM encoder can be operated at a sampling frequency of about 12 times the Nyquist frequency. The peak SNR for flat spectrum Gaussian signals encoded by linear delta modulators, operating at a sampling frequency of approximately 12 times the Nyquist frequency (24 times the bandwidth), is close to 25 dB\textsuperscript{76,19}, which shows that the "best" performance of the DM-to-PCM code converter is limited by the performance the delta modulator. This particular example illustrates a case when PCM requires a lower transmission bit rate (6 bits PCM, thus $12f_c$ bits per second, where $f_c$ is the signal bandwidth) than linear DM (approximately $24f_c$), and as consequence DM-to-PCM code converters may be useful.

We re-iterate the point that these converters are designed to trade-off analogue and digital complexities, to take advantage of large scale integration techniques. If these are not available, or not used, the generation of PCM signals from delta-modulated signals would probably be more costly than the conventional realization of PCM encoders, because of the need of generating different clock signals (for DM, re-sampling, digital filtering) and because of the additional hardware for the realization of the digital filter. However, apart
from cost considerations, such code converters would also find applications when systems operating with DM and PCM are linked together.

To conclude this introduction to DM-to-PCM code conversions, we summarize some of the results reported by Goodman. As an interpolator (of levels), the digital filter can have smaller number of stages as the sampling frequencies are increased, because the LDM step sizes can be reduced for higher \( f_p \) thus necessitating less interpolating levels. As an estimator, the larger the observation window (i.e. number of filter stages), the more accurate can be the estimate. Finally, as digital filter, the higher the number of stages, the sharper the cut-off and the better is the out-of-band noise attenuation. From any of the three approaches to the design, there is the trade-off between speed (sampling frequency) and storage (filter length) requirements, and between the filter length and resolution. Roughly, the use of one filter with 3 stages improves the SNR by about 7 to 7.5 dB in relation to a DM-to-PCM code converter made up of simply an up-down counter. There is a further improvement of about 2 dB when \( N \) is increased to 5 and a further 2.5 dB is obtained by increasing \( N \) to 9. The improvements are, therefore, increasingly smaller with the increase in the number of filter stages and for \( N = 33 \), the improvement in SNR is of about 15 dB compared with the converter with only the up-down counter. For the bandwidth expansion \( R \) small (less than 10) there is practically no improvements for \( N \) increasing above 9. However, the larger the \( R \), the more improvements can be achieved with the increase in the number of filter stages.
In this thesis, we adopt the filter design approach, to take advantage of the advances in digital filter design techniques. The estimation approach as described by Goodman requires the knowledge of the statistical parameters of the signal and the encoding process, viz. the auto-covariance function of the signal and the cross-covariance between \( \bar{x}(t) \) and \( x(t) \). Because the function of the transversal filter is mainly to reject the out-of-band noise before re-sampling at lower rates, it is not surprising to find that the coefficient values are largely independent of the signal spectrum and encoder parameters, as Goodman has observed.

6.3. Digital operations in code conversion by filtering.

Some of the digital operations that can be used in DM-to-PCM conversions are briefly recapitulated in this sub-section. They are given only as illustrations rather than as an exhaustive study of digital processes, which can be found in detail in, for instance, [12].

A. Digital integration.

A leaky digital integration can be represented as

\[ y_n = cy_{n-1} + x_n \tag{6.15} \]

where \( c \leq 1 \) is the leakage constant. Fig. 6.7 shows the basic digital integrator. If the constant \( c \) is given as:

\[ c = 1 - 2^{-p} \]

with \( p \) integer, the integration process is greatly simplified because the multiplication in the right hand side of Eq. 6.15 is simplified as:
FIGURE 6.07 - Representation of a digital, leaky integrator
(a) structure; (b) poles and zeros in z-plane

FIGURE 6.08 - Direct realization of a transversal filter

FIGURE 6.09 - Inversion of integration and filtering
(a) integrator + transversal filter
(b) transversal filter + integrator
\[ c y_{n-1} = y_{n-1} - 2^{-p} y_{n-1} \quad (6.16) \]

which means that the multiplication can be implemented by a shift-right by \( p \) positions and subtract from itself, instead of shifts and additions in the number of bits in the multiplicand. An example will clarify this point:

Let \( y_{n-1} = 1101_2 = 13_{10} \) and \( c = .1111_2 = .9375_{10} = 1 - 2^{-4} \).

First consider a conventional multiplication. This is realized by the following operation:

\[
\begin{array}{ccccccc}
1 & 1 & 0 & 1 & \leftrightarrow & 13 \\
\times & .1 & 1 & 1 & 1 & \leftrightarrow & \times .9375 \\
\hline
1 & 1 & 0 & 1 & & & \\
1 & 1 & 0 & 1 & & & \\
1 & 1 & 0 & 1 & & & \\
1 & 1 & 0 & 1 & & & \\
\hline
1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & \leftrightarrow & 12.1875
\end{array}
\]

In this example, \( y_{n-1} \) has to be shifted 3 times and an addition of four numbers has to be performed (or three additions of two numbers).

In general, if \( B \) is the number of bits representing \( c \), there has to be \( B-1 \) shifts and additions. However, the implementation of Eq. 6.16 is:

\[
\begin{array}{ccccccc}
1 & 1 & 0 & 1 & 0 & 0 & 0 & \leftrightarrow & 13.0000 \\
- & .1 & 1 & 0 & 1 & \leftrightarrow & -.8125 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & \leftrightarrow & 12.1875
\end{array}
\]

and only one shift by \( p \) positions and one subtraction are required.

There is a considerable saving in storage and speed of operation compared to the conventional implementation. Thus, if the digital

\[ ^{*} \] indicates number notation in base \( b \). However, the indication of bases are omitted when there is little likeliness of confusion.
integrators are to be made leaky, it is convenient to use \( c = 1 - 2^{-p} \).

If \( c = 1 \), we have the case of an ideal integration and its transfer function can be given as:

\[
I(z) = \frac{1}{1 - z^{-1}}
\]  

(6.17)

where \( z^{-1} = \exp(-j\omega T_p) \) represent a unit delay operator. \( T_p \) is the sampling period. In the \( z \)-space, \( I(z) \) has one pole at \( z = 1 \) and a zero at \( z = 0 \). If the integrator is leaky, the pole moves to inside the unit circle at \( z = c \).

Eq. 6.17 will be used in later sub-sections.

B. Digital transversal (non-recursive) filters.

Fig. 6.8 shows the conventional direct convolution realization of a digital filter, which is the implementation of:

\[
y_n = \sum_{k=0}^{N-1} a_k x_{n-k}
\]  

(6.18)

and \( a_k, k = 0,1,\ldots,N-1 \) are the \( N \) coefficients of the filter.

C. Inversion of the filtering and integration.

In the system of Fig. 6.2, the filter follows the integrator. As both operations are linear, their order can be interchanged, e.g. suppose that \( y_n \) of Eq. 6.15 is filtered by a transversal filter with coefficients \( a_k, k = 0,\ldots,N-1 \), yielding:

\[
z_n = \sum_{k=0}^{N-1} a_k y_{n-k}
\]  

(6.19)

i.e.

\[
z_n = \sum_{k=0}^{N-1} a_k (c y_{n-k-1} + x_{n-k})
\]  

(6.20)

where \( c \) is the leakage constant.
The digital realization of Eq. 6.20 is schematically represented in Fig. 6.9a.

Eq. 6.20 can be re-arranged as:

\[ z_n = c \sum_{k=0}^{N-1} a_k y_{n-k-1} + \sum_{k=0}^{N-1} a_k x_{n-k} \]  

(6.21)

where the first summation term is equal to \( z_{n-1} \) according to Eq. 6.19.

Thus, Eq. 6.21 can be written as

\[ z_n = cz_{n-1} + \sum_{k=0}^{N-1} a_k x_{n-k} \]  

(6.22)

The digital realization of the above equation is depicted in Fig. 6.9b, and comparing it to the realization shown in Fig. 6.9a, it can be seen that the principal difference is that the filter operation (convolution with the sequence \( \{a_k\} \)) is carried out with the integrated samples \( \{y_n\} \) in one and with \( \{x_n\} \) in the other case.

In the application of the operations above to DM-to-PCM code conversions, the integrator is part of the DM decoder and its inputs are the DM feedback step sizes. In the LDM case, they are the +1 or -1 binary outputs of the two-level quantizer, i.e. \( x_n = \pm 1 \). In this particular case, the savings in hardware are considerable if the integration is performed after the filtering operation, as in Fig. 6.9b, because the multipliers can be eliminated and substituted by add/subtract modules and only 1-bit delays are required. For the implementation indicated in Fig. 6.9a, \( y_n \) is a B-bit word, thus B-bit delays are required, and if the coefficients are represented by B'-bit words, the multipliers have to process inputs of B and B' bit-long words. (B-by-B' bit multipliers).
In the case of companded delta modulators, the feedback step sizes will be represented by 7 to 8-bit words. However, savings in hardware can still be attained with the inversion of the orders of integration and filtering if the values that the step sizes can assume are such that multiplications can be avoided. This is the case with step sizes that are integer powers of two, in which case the multiplications can be achieved by one shift and add operation. The savings with syllabically companded systems will be to a smaller degree, because the step sizes have to be represented by 7 to 8 bit-words, and consequently it is not possible to dispense with the multipliers unless the filter coefficients are powers of two.

D. Integration and sample rate reduction.

In the DM-to-PCM conversion process, the filtered DM signal has to be re-sampled at Nyquist rates. Let the output of a leaky integrator (as given by Eq. 6.15) be taken at RT intervals, i.e. at time $jRT_p$, for $j$ integer positive. The samples at these instants are given by:

$$y_{jR} = cy_{jR-1} + x_{jR}$$ (6.23)

The calculation of $y_{jR}$ according to Eq. 6.23 indicates that the output of the integrator at time $(j-1)T_p$ has to be known, and similarly to calculate $y_{jR-1}$, the output at time $(j-2)T_p$ has to be known. Thus, the digital implementation of Eq. 6.23 requires integration at every $T_p$ seconds.

However, the above equation can be expanded in the following way:
\[ y_{jR} = c y_{jR-1} + x_{jR} = c^2 y_{jR-2} + c x_{jR-1} + x_{jR} = \]
\[ = c^3 y_{jR-3} + c^2 x_{jR-2} + c x_{jR-1} + x_{jR} \]
and so on, until we arrive at
\[ y_{jR} = c^R y_{jR-R} + \sum_{i=0}^{R-1} c^i x_{jR-i} ; \text{ } j \text{ integer positive} \]

(6.24)

The first term in the right-hand side of the above equation is the output of the integrator at the previous re-sampling instant, i.e. \((j-1)T_p\). The second term is a weighted sum of the input samples to the integrator at times \((jR-i)T_p, i = 0,1,...,R-1\), and can be implemented by a transversal filter stage with coefficients \(c^i\).

Eqs. 6.23 and 6.24 are equivalent, but while Eq. 6.23 can be realized as indicated in Fig. 6.10a, Eq. 6.24 can be implemented as indicated in Fig. 6.10b, i.e. with an integrator with delay \(R T_p\) (instead of \(T_p\) as for Eq. 6.23) and preceded by a transversal filter that is the realization of the summation term in the right-hand side of Eq. 6.24. Thus, the structure of Fig. 6.10b requires more storage, because of the non-recursive section at the input of the integrator, but the integration has to be performed only at \(R T_p\) intervals. There is, then, a trade-off between storage requirements and the integrator speed.

Digital integrators (Fig. 6.7a) are realized in a recursive structure, thus a fast recursive stage (Fig. 6.10a, feedback delay of \(T_p\)) is traded-off with a lower speed recursive stage (Fig. 6.10b, integrator with feedback delay \(R T_p\), \(R > 1\)) preceded by a transversal filter stage.
FIGURE 6.10 - Sample-rate reduction and integration time constant
(a) Integration + resampling is equivalent to
(b) transversal filter + integrator (at lower rates)

FIGURE 6.11 - Combination of two digital transversal filters
(a) two stages in series
(b) the combination of both into one single transversal filter
(c) application of (a) and (b) to simplification of a filter + sample-rate reduction in a digital integrator.
E. Combining two non-recursive stages.

In the previous sub-section (D) we have seen that if the order of integration and sample rate reduction operations are interchanged, a non-recursive section has to be added to the circuit. The effect of this on the storage requirements can be found by writing the equations for two cascaded transversal filters (Fig. 6.11a).

\[
y_n = \sum_{j=0}^{R-1} d_j x_{n-j} \quad (6.25)
\]

and

\[
z_n = \sum_{i=0}^{N-1} a_i y_{n-i} \quad (6.26)
\]

where \(d_j\) are the R coefficients of the transversal filter whose inputs are \(\{x_n\}\); and \(a_i\) are the N coefficients of the transversal filter whose inputs are \(\{y_n\}\). From Eqs. 6.25 and 6.26:

\[
z_n = \sum_{i=0}^{N-1} a_i \left[ \sum_{j=0}^{R-1} d_j x_{n-j-i} \right] = \sum_{k=0}^{N+R-2} a_k' x_{n-k} \quad (6.27)
\]

where

\[
a_k' = \sum_{j=0}^{R-1} d_j a_{k-j} \quad (6.27a)
\]

Eq. 6.27 becomes:

\[
z_n = \sum_{k=0}^{N+R-2} a_k' x_{n-k} \quad (6.28)
\]

Eq. 6.28 indicates that two non-recursive sections, of lengths R and N, cascaded produce one non-recursive section of R+N-1 coefficients, as illustrated in Fig. 6.11b. Thus, the transversal filter used in code conversion has to be increased from N to R+N-1 stages, if the order of integration and sample-rate reduction operations
is exchanged (Fig. 6.11c), and the coefficients have to be modified according to Eq. 6.27a.

F. The conversion filter.

Viewing the principal operation in DM-to-PCM code conversion as a low-pass filter process to reject the out-of-band quantization noise before re-sampling at lower rates, the design problem becomes that of a digital filter\textsuperscript{12,94,108,109}. Thus, in this thesis we have only outlined what refinements can be introduced into a conventional filter design because of the particular nature of the signals to be processed, (sub-sections A-E). In a later sub-section we present a general structure for the filtering of a 2BIADM signal for code conversion to PCM.

Most filters used in DM-to-PCM code converters are structured as simple accumulators and up-down counters\textsuperscript{77,84,89} or combinations of accumulators\textsuperscript{104}, which are basically filters with unitary coefficients. For more complex filters, the direct convolution realization shown in Figs. 6.8 to 6.11 is not the actual hardware implementation because of the large number of multipliers and adders required. One usual digital transversal filter implementation employs\textsuperscript{12} recirculating shift registers for the data and coefficients, or alternatively the coefficients can be stored in read only memories\textsuperscript{108,101}. Other high speed implementations are subject of active research\textsuperscript{110,112,12} but we will not advance the subject further in this thesis.

6.4. DM-to-DPCM conversion by filtering.

Returning to the basic principle of code conversion as presented
in Fig. 6.2, if only the integrator is used between the DM encoder and the PCM sampler, the transfer function of the "code converter" is that of the integrator, which when ideal (no leakage) is represented by Eq. 6.17. To generate DPCM signals in such a situation, we recall that in DPCM the differences between two Nyquist samples are transmitted. The transfer function of such a process can be given by:

\[ D(z) = \frac{1}{1 - z^{-1}} - \frac{z^{-R}}{1 - z^{-1}} \]  

(6.28)

which means that the current output of the integrator is subtracted by output of \( R \) sampling periods before.

The poles and zeros of Eq. 6.28 indicate that \( D(z) \) represents a low-pass filter with zeros at multiples of the frequencies \( f_\text{p}/R \), thus by cascading a number of similar sections, it is possible to achieve the out-of-band attenuation required for the re-sampling. Ishiguro et al.\(^{101}\) use this principle and observing that the higher frequencies in the message band are also attenuated, they use a non-recursive section \( A(z) \) to adjust the frequency response in the message band, i.e. to compensate the in-band attenuation introduced by \( D(z) \). The general form given in \( \text{[101]} \) is:

\[ G(z) = K \ A(z) \left( \frac{1 - z^{-R}}{1 - z^{-1}} \right)^N \]  

(6.29)

and

\[ A(z) = \left( \sum_{i=-M}^{M} A_i z^{-i} \right) z^M \]  

(6.30)

And, in particular, to generate DPCM signals for application with video-telephone signals, Ishiguro et al.\(^{101}\) employ \( R = 8 \), \( N = 3 \), and \( A(z) = -z^8 + 5z^4 -1 \).
The resulting filter can be appreciated by observing the frequency responses of the recursive and non-recursive sections, which can be estimated from the distribution of poles and zero-s in the $z$-plane. Fig. 6.12 illustrate schematically the frequency response shaping by the addition of the section $A(z)$.

The signal used to illustrate the basic LDM-to-PCM encoding in Fig. 6.5 is used again to verify the DM-to-DPCM encoding as given by Eq. 6.29/6.30, and the resulting waveforms are shown in Fig. 6.13. The output of the filter is shown as dots for each $T_p$ intervals, though in practice only each 8-th pulse is read out. To reconstruct the tracking signal (crosses), the output of the filter at a given clock instant (dots) has to be added to the tracking signal samples of 8 clocks in the past. Again the reconstructed signal samples are shown at each $T_p$ intervals though only each 8-th should be considered. It is obvious that (a) the DPCM samples occupy a smaller range of amplitudes compared to the signal (about half), and (b) the original signal is more closely approximated by the samples reconstructed from the output of the filter than by the DM feedback signal, showing the interpolation that has occurred in the conversion process by digital filtering.

6.5. Summary.

We have seen in the previous sections that the process of code conversion from DM signals to PCM can be realized as a process of digital low-pass filter operation followed by a re-sampler. As such, the converter design can be approached as a digital filter design, which can follow known techniques. However, because DM systems
$G'(z) = \frac{1-z^{-3}}{1-z^{-1}}$  \hspace{1cm} (a)

$A(z) = -z^8 + 5z^4 - 1$  \hspace{1cm} (b)

$G(z) = A(z)G'(z)$  \hspace{1cm} (c)

**FIGURE 6.12** - DM-to-DPCM converter filter (not in scale)

(a) 3rd order low-pass section

(b) compensation section for un-wanted attenuation in the message-band

(c) combined response
reconstructed signal from the digital filter output for all DM clock instants. DPCM samples are only each 8-th integrated DM output.

FIGURE 6.13 - DM-to-DPCM conversion waveforms for arbitrary input. (a) fast varying signal; (b) slow varying signal. Dots represent the output of the digital filter for DM clock instants.
operates with over-sampled signals and these are subsequently re-sampled at near Nyquist rates, there may be a need to modify the filter coefficients as described in section 6.3D/E. This is necessary if the re-sampling is performed before the integration required to reconstruct the DM signal. The use of a digital integrator for code conversion may require the introduction of a leakage, for instance if the DM encoder is built with analogue, leaky, integrators. In such cases the hardware can be simplified by choosing the leakage constants carefully (section 6.3A). Also, DPCM signals can be generated instead of PCM by re-setting the digital integrator after re-sampling (section 6.4).

Because the order of the operations of integration and low-pass filtering can be exchanged, the out-of-band noise reduction is performed on the signals that are present at the input of the integrator in the DM encoder feedback loop. For adaptive systems, they are the variable step sizes. If these are integer powers of two, the coefficients can assume any values, allowing flexible design\textsuperscript{12} of digital filters and yet their implementation is kept simple by avoiding the use of multipliers\textsuperscript{109,110}.

6.6. 2BIADM-to-PCM code conversion filter.

We describe two possible 2BIADM-to-PCM code conversion digital filters. The first is a general structure which is applicable when the number (N) of filter coefficients is larger than the bandwidth expansion ratio R. The implementation is described for a serial structure and consequently the maximum speed of operation is limited by the internal clock of the filter. The coefficients, however, can
assume any values. The second structure is given for a filter whose coefficients are powers of two, and it is shown that for small values of $R$, this simple structure is almost as good as the more complex one.

A. Filter structure for $N$ large.

We recall that the feedback signal in the 2BIADM encoder is given by:

$$y_n = y_{n-1} + \gamma b_n S'_{n}$$  \hspace{1cm} (6.31)

Eq. 6.31 is the same as Eq. 4.9, with $y_n$ substituted by its digital representation $S'_n$ which can assume one of the values $[S'(1), S'(2), \ldots, S'(6)]$, for the 2BIADM having six possible step sizes, such that $S'_n = y_n / \gamma$.

The basic scheme for the 2BIADM-to-PCM code conversion is illustrated in Fig. 6.14(a), where the output of the digital filter $\hat{y}_n$ is given by:

$$\hat{y}_n = \sum_{k=0}^{N-1} a_k y_{n-k}$$  \hspace{1cm} (6.32)

where the sequence $\{a_k\}$, $k = 0, \ldots, N$, constitutes the $N$ coefficients of a linear transversal digital filter. The use of this type of filter is convenient for its linear phase response if the coefficients are symmetrical.

Because integration and low-pass filtering are linear processes, the order of their operations can be interchanged (Figure 6.14b, see also section 6.3). From Eqs. 6.31 and 6.32

$$\hat{y}_n = \sum_{k=0}^{N-1} a_k (y_{n-1-k} + \gamma b_{n-k} S'_{n-k}) = \sum_{k=0}^{N-1} a_k y_{n-1-k} + \gamma \sum_{k=0}^{N-1} a_k b_{n-k} S'_{n-k}$$
FIGURE 6.14 - 2BIA DM-to-PCM conversion principle
(a) "direct" realization
(b) inverting the order of integration and filtering
\[ \hat{y}_n = \hat{y}_{n-1} + z_n \]  \hspace{1cm} (6.33)

where

\[ z_n = \gamma \sum_{k=0}^{N-1} a_k b_{n-k} s'_{n-k} \]  \hspace{1cm} (6.34)

Eq. 6.34 shows that the 2BIADM step sizes are filtered in place of the reconstructed samples \( y_n \). Instead of storing the expanded step sizes, the delta modulated 2-bit words \( \{h_n, i_n\} \) can be stored, saving storage space. The step sizes are, then calculated just before the multiplication by the appropriate filter coefficients.

Further savings in hardware can be made by using only one multiplier and an adder and calculating sequentially the weighting of the step sizes by the filter coefficients\(^{12}\). Then, for each \( z_n \) output, the \( N \) multiplications and additions required by the transversal filter have to be performed within the encoder sampling interval \( T_p \). Because the 2BIADM system generates step sizes that are integer powers of two, the multiplications can be implemented as simple binary shift operations.

The \( z_n \) samples thus obtained are integrated to generate the filtered digital version of \( y_n \), viz., \( \hat{y}_n \), and each \( R \)th sample can be taken as a PCM sample. The digital filter accumulator \( E_1 \) and the 2BIADM integrator \( E_2 \) can be combined into one accumulator.

The implementation of the filter of Fig. 6.14b can be made as in Fig. 6.15, which we now describe.

At the start of one conversion cycle of duration \( RT \), shift registers \( SR_1 \) and \( SR_2 \) hold \( N+2 \) bits each, where \( R = f_c T_p \) is the sample-rate reduction factor, \( f_c \) is the cut-off frequency of the
SR1 and SR2 are clocked synchronously to re-sampler and PCK word-generator.

**FIGURE 6.15** - Filter structure for N large, and exceeding the bandwidth expansion ratio R.
signal pass-band, and $T_p$ is the DM sampling interval. SR1 holds the sequence of bits $\{B_n\}$ that conveys the polarity information of the error signal, while SR2 stores the sequence of bits $\{T_n\}$ which are related to the magnitude of this error. The address generator for the filter coefficients store (CS) is set to its starting address. The initial step size for that cycle is fed to the HIDM step expander (SE). Switches SW1 and SW2 are in position 1. The converter is ready for operation.

The shift registers SR1 and SR2 are clocked synchronously, and they are equivalent to one shift register with parallel two-bit stages. The outputs of stages $N$, $N+1$ and $N+2$ of SR1 correspond, respectively, to the 'present' and the 'past two' polarity bits and are taken in parallel by the HIDM step expander SE to determine the step multiplication factor $m(1)$ of 2, 1 or $\frac{1}{2}$. This factor multiplies the step size employed in the previous clock instant to generate $S_n = m(1) S_{n-1}$, except at the beginning of the conversion cycle when the initial step size from the memory ISS is used instead. The output of SE is fed to a shifter SH4 controlled by the sequence $\{T_n\}$. Whenever $T_n$ indicates that the error magnitude is large, there is a shift to the left by two, i.e. it multiplies the HIDM step size by $m(2) = 4$. Otherwise, the output of SE remains unchanged ($m(2) = 1$). The output $S'_n$ of SH4 is given by $m(2) S_n$.

These step sizes are now filtered using Eq. 6.34, to give the sequence $\{z_n\}$ with the $N$ filter coefficients $a_k$ stored in the memory CS. The coefficients are addressed in sequence as the bits $\{B_n\}$ and $\{T_n\}$ are shifted in SR1 and SR2. Each term of the right hand side of Eq. 6.34 is calculated sequentially, and because $S'_{n-k}$
are integer powers of two, the multiplication is performed by shifting in SC the binary representation of $a_k$ by the number of positions given by the step size. Only the magnitude bits of $a_k$ are fed to the shifter. The sign bit of $a_k$ is added modulo-two (EX-ORed) to the sign bit $B_{n-k}$ to determine whether the product $a_k \cdot S'_{n-k}$ is to be added or subtracted from the contents of the accumulator $\Sigma_l$.

After the first $R$ sampling clock instants, switches SW1 and SW2 are moved to position 2 and at the same time the step size at this instant is stored in ISS. This provides the initial step size for the next cycle of code conversion. With SW1 and SW2 in position 2, the contents of these shift registers are recirculated.

At the end of $N$ clock instants, Eq. 6.34 has been processed. The first $R$ stages of SR1 and SR2 are empty and bits $B_{n+N-1}$ and $T_{n+N-1}$ are in stage $N$. The shift registers are then clocked, while the coefficients shifter is disabled, until $B_{n+N-1}$ and $T_{n+N-1}$ are in stage $R+1$, at the top of the recirculating portions of SR1 and SR2. At the same time, the next $R$ bits of sequencies $\{B_n\}$ and $\{T_n\}$ are taken in parallel to fill the first $R$ stages of the shift registers. The initial step size is fed into the HIDM step expander SE and switches SW1 and SW2 are moved to position 1. The whole process is then repeated.

The output $\{z_n\}$, of the filter, is fed to an accumulator which is the realization of Eq. 6.33 and corresponds to the 2BIADM integration. 
To obtain the PCM samples, the output of $\Sigma_2$ is re-sampled at $RT_p$ intervals.

This structure is similar to the one described by Eggermont to produce a HIDM-to-PCM code converter. The digital filter used
by Eggermont has 75, 13-bit coefficients. It operates with an internal clock of about 800 KHz, for the HIDM sampling frequency of 64 KHz (R = 8). In the converter of refs. [105, 106], the accumulator is made "leaky" because the HIDM encoder is built with a leaky integrator in the feedback path. It also has the order of operations of integration and sample-rate reduction interchanged (as described in section 6.3D/E). The main differences between the HIDM-to-PCM and 2BIADM-to-PCM code converters are due to the use of two bits per sample in the input stages of the latter. Briefly, they are a stage for multiplication by a factor of 1 or 4 and the use of shift registers having 2 bits per stage instead of 1 bit per stage. With the same filter coefficients, the results in Chapters IV and V suggest an improvement of up to 8 dB when the 2BIADM is used instead of the HIDM in the code conversion input stage.

The high frequency of the internal clock used in the HIDM-to-PCM code converter is due to the serial implementation of the filter. If the sampling frequency of the input modulator is increased, say for video encoding, faster filter structures have to be sought, such as parallel or pipe-lined implementations. Any of the latter techniques would increase substantially the storage requirements for the converter filter. Some savings in storage requirements could be achieved by reducing the word-length of the coefficients, but this would limit the amount of attenuation that can be obtained outside the message bandwidth. Consequently message band-limiting must be made by the analogue pre-sampling filter, instead of combining this function with the conversion noise rejection, as in Eggermont’s filter.

The improvement that was observed in Chapters IV and V with the
use of the 2BIADM instead of HIDM is transferred to the PCM output, but the use of two bits instead of one bit per sample means that an increase in storage requirements will be needed. Edwards and Eggermont present an estimate of the chip area that their HIDM-to-PCM code converter would occupy if realized as a LSI circuit, and the input shift register occupy less than 15% of it. Thus, the doubling in the number of bits per stage in these shift registers will not increase prohibitively the chip area required.

B. Filter for small N.

With the bandwidth expansion ratio small, the performance curves given by Goodman indicate that there is not much to be gained with the increase in the number of filter stages. The structure presented in sub-section A, above, would be useful for high values of $R$. When analyzing the 2BIADM, for application with large bandwidth video signals, we have mentioned that the maximum value of $R$ is limited by component speed limitations. In such cases, much simpler structures could be used, and as an example we present the filter of Fig. 6.16a. The filter coefficients $a_k (k = -2, -1, \ldots, 2)$ are $1/8$, $1/4$, $1/4$, $1/4$ and $1/8$ (d.c. gain of unity), and can be realized using only delays, shifters and adders. In Fig. 6.16b we present its frequency response. The design of this filter followed roughly the guidelines given by Tomozawa, which in short consist of using two basic filter sections which are

$$H_1(z) = \sum_{i=0}^{N-1} z^{-i}$$

(6.35)

and

$$H_2(z) = a + z^{-m} + az^{-2m}$$

(6.36)
FIGURE 6.16 - Filter structure for N = 5

(a) direct realization

(b) frequency response of the filter in (a)
Eq. 6.35 represents a basic low-pass filter whose frequency response presents zero-s at \( f_p/N \) and at its integer multiples. Because of the attenuation that exists for higher frequencies in the pass-band, the section described by Eq. 6.36 is added to compensate for that distortion, as illustrated by Fig. 6.12; section 6.4. The required frequency response is approximated with combinations of basic sections. The knowledge of their frequency responses allows the rough estimate of the number of stages in \( H_1(z) \), and how many of these to be cascaded. \( H_2(z) \) is then used to shape the frequency response in the pass-band, as its response is also known\(^{109}\). The frequency response is then adjusted by trial and error.

The filter whose coefficients were given earlier was designed by restricting the number of stages, \( N \), to a maximum of 5, which is the bandwidth expansion ratio used. This avoids the use of re-circulating shift registers that would be required if \( N \) exceeded \( R \) (see subsection 6.5A). Also, the d.c. gain of the filter was restricted to a power of two, which facilitates the scaling by hardware\(^{109}\). The best result was achieved by cascading two 3-stage type 1 filters, and without using the type 2 section, i.e. the filter was produced as:

\[
H(z) = (1 + z^{-1} + x^{-2})^2 = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}
\]

which was subsequently approximated by:

\[
H(z) = 1 + 2z^{-1} + 2z^{-2} + 2z^{-3} + z^{-4}
\] (6.37)

and whose d.c. gain is 8 and as consequence it can be easily scaled to give unitary gain.

These coefficients constitute the impulse response samples of
the filter, and the low-pass filtering operation of the delta
modulator step sizes is carried out by convolution, i.e.

\[ z_n = \gamma \sum_{k=-2}^{2} a_k b_{n-k} S'_{n-k} \]  

(6.38)

where \( b_{n-k} \) is +1 or -1 and gives the polarity of the error signal
at that instant. \( a_k \) are the filter coefficients as defined before,
and \( S'_{n-k} \) are the step sizes.

These filtered step sizes are then used to approximate the input
signal \( z_n \) to the delta modulator according to:

\[ \hat{y}_n = \hat{y}_{n-1} + z_n \]  

(6.39)

Each 5-th sample is taken as a PCM sample, and the error in
the conversion process is \( (x_n - \hat{y}_n) \).

C. HIDM and 2BIADM-to-PCM code conversion filter - simulations.

To verify the PCM encoding using the HIDM and 2BIADM followed
by a digital filter, we have calculated by computer simulation the
SNR curve as function of the input power for Gaussian inputs with a
flat spectrum. The simulation technique was as described in Chapter III,
where a sequence of 2050 DM samples were generated, from which 410 PCM
samples produced according to Eqs. 6.38 and 6.39 with \( n = 5j \),
j integer.

The SNR for the PCM samples was used as a quality measure, and
defined as

\[ \text{SNR} = 10 \log(\text{snr}) \]
where

\[
\text{snr} = \frac{\sum_{j=1}^{410} x_{5j}^2}{\sum_{j=1}^{410} (x_{5j} - \hat{y}_{5j})^2}
\]  

(6.40)

For comparison purposes, a "quasi-ideal" filter with 256 coefficients \( \{a_k\} \) calculated by the frequency sampling technique was also used for the code conversion. This filter was originally employed to digitally band-limit the pseudo-random numbers used as the input samples, and presents about 60 dB attenuation in the rejection band (see Chapter III, section 3.4A).

Fig. 6.17 shows the SNR responses for the PCM outputs of the filters with 2BIADM and HIDM inputs. Curves (a) and (b) are for the conversion by the quasi-ideal filter, and curves (c) and (d) for the conversion by the simple 5-coefficients digital filter.

The close results between the responses obtained with the two filters (within 3 dB) are expected as discussed in section 6.2. These results corroborate the assumption that we made earlier, that the gains observed for the 2BIADM over HIDM are transferred to the PCM outputs.

For this particular example (\( R = 5 \)), the 2BIADM with the simple 5-stage filter presents a better performance than the HIDM with the 256-stage filter. This happens because, as discussed in section 6.2, the improvements that can be gained by using more complex filters is only effective for large values of \( R \). The performance curves presented by Goodman, and reproduced for Gaussian signals with flat spectrum in Chapter II - Fig. 2.16, suggest that the HIDM-to-PCM code converter filter has to have at least 33 stages to show results comparable to
FIGURE 6.17 - SNR vs. input power for PCM samples from 2BIADM (curves (a) and (c)), and HIDM (curves (b) and (d)) by digital filtering. Curves (a) and (b) are for \( N = 256 \) and curves (c) and (d) for \( N = 5 \).
those obtained by 2BIADM-to-PCM code converter filter with 5 stages, because the increase from 5 to 33 stages in the filter length improves the output performance by about 5 dB, whereas the 2BIADM show more than 5 dB gain over the HIDM (Chapters IV and V).

Thus the increase in storage requirements for the input shift-registers, in the case of the filter discussed in section 6.6A, is traded-off with a need of shorter length filters, and as the speed of filtering is dependent on the number of filter coefficients, there is also a trade-off with speed.

Unfortunately, due to circumstances beyond our control, we could not proceed to consider one important aspect of the code conversion process: that of the actual PCM word generation. When LDM encoders are used at the input, linear PCM signals are obtained\(^{20,84}\). However, in the case of adaptive encoders with a finite set of step sizes such as HIDM and 2BIADM, there will be non-linear distortions in the analogue-to-digital conversion process by the delta modulator, with errors that are larger for increasing signal power and slope. Not all PCM words will be present at the output, and this can be illustrated with an example. Suppose that the conversion filter simply averages two consecutive samples. Then, if the inputs are HIDM samples (which are only powers of two) such as 8 and 16, the average is 12 and the remaining words do not appear. The larger the window of observation, i.e. the filter length, the more levels in between are generated (see section 6.2), but the encoding errors will continue to be larger for larger amplitudes.

Because the interpolation levels are generated by weighted averages of the step sizes over a certain number of sampling instants
(determined by the filter length), the 2BIADM will produce more interpolation levels because the use of two bits per sample provides more step sizes for the same number of samples than the HIDM. However, even then the distortions will not be uniform for varying encoding conditions. This and the effects of finite word-length arithmetic have not been considered in this thesis.

Another point is that, as mentioned earlier, it may be necessary from a speed point of view, to employ analogue integrators in the feedback path instead of using digital accumulators and D/A converters as an integrator. If that is the case, the accumulator in the code converter will have to be made leaky, as described in section 6.3A.

Thus, the performance of the 2BIADM-to-PCM code conversion can, in this thesis, be evaluated only with reference to the HIDM-to-PCM code conversion. And, from that point of view, it shows significant improvements under the conditions specified in Chapters IV and V.

6.7. Note on publication.

A paper entitled "Two Bit Instantaneously Adaptive Delta Modulation for PCM Encoding" has been accepted (Nov. 1977) for publication by the IERE journal The Radio and Electronic Engineer. The paper is an abridged version of Chapters IV to VI.
CHAPTER VII

SUGGESTIONS FOR FURTHER RESEARCH

7.1. Introduction.

In this chapter, we briefly outline a number of topics that may be of interest for further research. Some approaches believed to be promising are suggested, but we leave open the options for the investigations that we hope to inspire and encourage here. A generalization of the 2BIADM encoder, which combines its step adaptation algorithm with that of an ADPCM, is dealt with in some detail.

7.2. On SNR calculations for IADM's.

In Chapter III, a technique has been proposed to estimate SNR's as function of the input signal power for HIDM and CFDM, when the inputs are band-limited Gaussian signals. The procedure investigated allows the calculation of SNR as a function of a statistical average step size $\gamma_{av}$. Although the results obtained showed relatively large deviations at times (e.g. in Fig. 3.20b), it was possible to verify that $\gamma_{av}$ is a useful parameter that can be used in the estimation of the performance of IADM's for varying signal levels.

It has been verified that improving the empirical estimation for the actual $\gamma_{av}$ does not improve significantly the estimates of noise. The principal cause for the deviations between the estimates and the actual SNR's is believed to be a noise component that is cross-correlated with the input signal. As a consequence, the analysis of the dependence on $\gamma_{av}$ of the cross-correlation coefficient between the input and error
signals, $p_{xe}$, could prove fruitful for the calculation of noise in IADM's,

A second possible line of attack is to investigate the probabilities of occurrences of patterns of 1's and 0's as function of the input power (or $\gamma_{av}$). These probabilities, if found, could be used to estimate the number of occurrences of different step sizes. Flood and Hawksford\textsuperscript{79}, for instance, have investigated the number of occurrences of consecutive pulses of the same polarity for given d.c. inputs, when analyzing their Pulse Grouping Delta-sigma Modulator (PG-DSM). With HIDM and CFDM, the problem is rather more complicated because the step adaptation depends not only on consecutive pulses of the same polarity (as does the PG-DSM), but also on the reversals of polarities. Thus, theoretically, a study of all possible patterns of 1's and 0's from the time the encoding operation starts is required. However, in practice, the number of step sizes is limited, thus effectively restricting the observation window to a finite number of sampling instants.

As an example, let us consider the HIDM whose adaptation algorithm is such that the step size doubles from, and including, the third consecutive sample of same polarity, halves at each polarity reversal, and is constant in all other cases (see Chapter III, section 3.3). If only two step sizes are used, $\gamma_{(1)}$ occurs whenever there is a sign reversal, and as long as there are less than three consecutive pulses of the same polarity. Thus, to calculate the probability of occurrence of $\gamma_{(1)}$, it would be necessary to examine only the occurrence of output sequences 01, 10, 011, and 100. The occurrence of $\gamma_{(2)}$ can be calculated from the knowledge that with two step sizes, $p_1 + p_2 = 1$, \[1\]
where $p_1$ and $p_2$ are the probabilities of occurrence of step sizes $\gamma_1$ and $\gamma_2$. For three step sizes, $\gamma_3$ occurs when at least four consecutive pulses are of the same polarity, and for sequences such as $1111000$ and $0000111$ (with the arrows indicating where the step size under consideration is used). The step size $\gamma_2$ could occur for $11110$, $111100$, $010111$ and their opposites (e.g. $00001$). It is possible to observe that only a finite number of pulses has to be considered, as assumed earlier. Apparently, this number increases exponentially with the number of step sizes. As a consequence, the study would be limited to a fairly small number of step sizes (e.g. 3), to begin with.

We feel that this approach is potentially rewarding, as it would not only allow the exact calculation of $\gamma_{av}$, but also of the estimation of occurrences of overshoots. This is because we know that they are maximum when the next larger step size occurs for the first time. Computer simulations would simplify greatly the calculations of such probabilities, but if the patterns of step sizes are related to slopes, it would be possible to calculate mathematically the occurrence of given slopes for Gaussian inputs. Notice that slopes can be related to $\gamma_{av}$.

7.3. On 2BIADM, code conversion, and related techniques.

With respect to the subject covered in Chapters IV to VI, an obvious line of research is to carry out the hardware implementations for application to actual television signals. It would be advantageous

* the assumption here is that there is no capital available for the purchase of high quality colour TV equipment, and that whoever undertakes the project has no previous experience with colour TV and high speed circuit design.
to restrict the initial work to monochrome TV to gain an understanding of the design with high speed devices, and to concentrate the efforts on details such as D/A converters, lay-out, earthing, shielding, and interconnections. This approach is also attractive from the point of view of cost, as black and white monitors and signal generators (say, VTR's and cameras) are less expensive than those for colour TV. We feel that it is preferable to operate with high quality monochrome monitors instead of with low quality colour receivers adapted as monitors. The disadvantage of the latter is that the distortions introduced by the poor display would mask distortions introduced by digital processing. Also, the operation with monochrome TV would ease the requirements for very high quality analogue circuitry such as low-pass filters, black or sync pulse level clampers, amplifiers, signals distributors and possibly phase-locked loops.

From the point of view of subjective assessments, the over-all procedure would be simplified with monochrome systems (no need for colour balance adjustments, for instance, and the number of distortion variables to be observed is reduced). It seems desirable that work on subjective assessment of pictures should be a separate project to establish the guide-lines and the setting up of test apparatus and conditions, not forgetting the proper statistical analysis of the data obtained. Once such experience is gained, there could be inroads to colour TV coding whose results would be widely acceptable on the grounds of subjective assessments.

In the absence of such facilities and experience for subjective and objective measurements, the simplest solution - as, incidentally, repeatedly advocated by the research supervisor - is to resort to
better equipped and qualified laboratories after the initial tests have been carried out under existing conditions. The tests would be carried out on rigorous grounds only when any proposed system appears to produce promising results on the less exacting settings.

On code conversion, the realization of ultra-high speed digital filters, i.e., filters operating at word-rates in the order of tens of MHz, is still largely untouched. Although the theory on digital filters has seen already great advances, their realization is becoming possible only now due to the recent advances in digital circuit technology such as high speed, low-power dissipation, LSI devices. This is because of the large amount of digital hardware (unless realized with LSI technology) needed for the implementation of digital filters, compared to relatively simple and well established analogue equivalents. For instance, Goldstein and Liu\(^{110}\) estimate that a 32-coefficients digital filter, operating with samples taken at Nyquist-rates and with word-lengths of 8 bits would require 800 IC's for a parallel PCM implementation, with a word-rate of 14MHz and power dissipation of 360 watts. A serial implementation would require 16 RAM's, 1 ROM, 29 IC's, for a 450 KHz word-rate and a power dissipation of 22 watts.

Known techniques can be applied to devise digital filters for application in code conversion, but there is a big step from there to the actual implementation, which in itself could well be a new project.

The process of code conversion would not be complete without the detailed analysis of the noise produced in the generation of PCM words of limited length from the low-pass filtered (digitally) delta modulated signals. Because of the non-linear nature of HIDM and 2BIADM signals, the studies would have to include an analysis of the nature of the
noise in PCM signals generated from such non-linear encoders.

For the encoding of video signals by the 2BIADM, it has been assumed that there would be no need to employ an analogue sample and hold stage. This assumption may require a review in face of the jitter that is uncomfortably noticeable in Fig. 5.5. If a sample and hold is required, one favourable point in the use of 2BIADM is diluted. This is so because one of the reasons that led to the choice of oversampling and using delta modulators was the dispensation of the sample and hold stage, which would reduce costs.

Finally, one point that has been raised in Chapter IV, viz. the extension of the 2BIADM principle to a n-bit system, can be the subject of further investigations. For that, one approach is to examine the HIDM, 2BIADM and the Bell Laboratories one-word memory ADPCM. In Fig. 7.1 we depict the ADPCM system, with the adaptive quantizer representation by Goodman and Gersho. In it, the input to a fixed, unit range, uniform quantizer is shown to be weighted by a factor that depends on the quantizer level occupied in the previous sampling instant. This is equivalent to the system in which the quantizer step size is adapted.

Representing the code-word magnitude transmitted in the previous sampling instant by $|P_{r-1}|$, the weighting factor is given by:

$$m_r = m_{r-1} M(|P_{r-1}|)$$

(7.1)

where $M(\cdot)$ is a time invariant multiplier whose magnitude depends on the quantizer level occupied in the previous sampling instant.

To better visualize the system, the quantizer characteristic for a 3 bit system is illustrated in Fig. 7.1b, reproduced from [114].
FIGURE 7.01 - Adaptive DPCM quantizer

(a) Goodman and Gersho [113] quantizer representation
(b) quantizer characteristics, after [114]
The code-words magnitude $|P_r|$ are the binary numbers in that figure, and to each of them corresponds a multiplier $M(|P_r|)$, which are represented simply as $M_1$, $M_2$, $M_3$ and $M_4$. The quantizer is uniform with step size $\Delta_r$, which varies according to:

$$\Delta_r = \Delta_{r-1} \cdot M(|P_{r-1}|)$$  \hspace{1cm} (7.2)

The relationship between Eqs. 7.1 and 7.2 is that

$$\Delta_r = \Delta_u \cdot m_r$$  \hspace{1cm} (7.3)

where $\Delta_u$ is the step size of a unit range scale quantizer.

Notice that the quantizer is shown to vary in Fig. 7.1b and Eq. 7.2, whereas in Fig. 7.1a the quantizer is fixed and the input to it is weighted by $1/m_r$. This is a standard technique that is used for varying the quantizer.

To verify, let us examine the system of Fig. 7.1a.

At a sampling instant indicated by the subscript $r$, the error signal $e_r$ between the input and feedback signals is weighted by $1/m_r$, where $m_r$ is the weighting factor of the previous clock instant multiplied by a factor that is function of the most recent code-word transmitted, $|P_{r-1}|$, i.e. of the quantizer level occupied at time $(r-1)$. The resulting signal, $e_r$, is quantized by the unit-scale quantizer producing the quantized output $\hat{e}_r$. Its magnitude is allocated the code-word $|P_r|$, which will define the multiplier $M(\cdot)$ for the next sample and is therefore stored in a unit delay memory. The quantized error signal of the DCPM encoder, $\hat{e}_r$, is obtained by multiplying $\hat{e}_r$ by $m_r$, and the feedback signal is obtained by integrating $\hat{e}_r$.

The unit-scale quantizer output at time $r$ can be expressed for
the 3-bit system as

\[ \hat{e}_x = p_x \cdot \frac{\Delta u}{2} ; \pm p_x = 1, 3, 5, \text{ or } 7 \]

(7.4)

thus,

\[ \hat{e}_x = p_x \cdot \frac{\Delta u}{2} \cdot m_x \]

(7.5)

From Eqs. 7.5 and 7.3:

\[ \hat{e}_x = p_x \cdot \frac{\Delta r}{2} ; \pm p_x = 1, 3, 5, \text{ or } 7 \]

(7.6)

which is one of the output levels of the variable quantizer depicted in Fig. 7.1b, as stated.

The particular case of a 2-bit ADPCM is shown in Fig. 7.2a, where the magnitude and sign quantizers are shown separately. There are only two multipliers, \( M_1 \) and \( M_2 \), corresponding to the two possible magnitudes. The operation is as explained for the general case of Fig. 7.1. Observe that only the magnitude output controls the choice of the multiplier factor, and that it is the previous occupied level that determines the choice.

The HIDM encoder can be depicted on similar lines, as shown in Fig. 7.2b. This figure is the same as that of Fig. 3.1a. In short, the current and the two previous outputs determine the multiplier, which can be 2, 1/2 or 1. The detailed explanation of the operation of the adaptation is given in Chapter III, section 3.3, and will not be repeated here. Observe that only one bit is produced, and it conveys the information about the polarity of the error signal. The polarity bits control the step adaptation.

The weighting of the error signal is unnecessary as it does not
FIGURE 7.02 - Two-bit adaptive systems
affect its polarity. Nevertheless, to emphasize the relationship between the HIDM and ADPCM systems, the weighting that would be present is indicated in dashed lines.

The 2BIADM encoder is depicted in Fig. 7.2c. The step adaptation is basically that of HIDM, but there is also a two-level magnitude quantization. As in the case of ADPCM, the magnitudes quantizer is represented by one which is invariant, but whose input is varied instead. As explained in Chapter IV, section 4.2, the weighting factor is derived from the HIDM logic. The output of the fixed error magnitude quantizer is 1 or 4, and it multiplies $r_y$ before integration, i.e. the input to the feedback integrator is $r_y$ or $4r_y$. Observe that two bits are produced, but the polarity bits control the multiplier factor, instead of the magnitude bits as in the ADPCM.

Comparing Figs. 7.1a and 7.2c, there is an obvious extension which combines the quantizer adaptation modes of HIDM and 2BIADM with that of ADPCM. Namely, by using both the polarity and the magnitude outputs to control the choice of multipliers. The combination is generically indicated in Fig. 7.3, where the multiplier determined from the analysis of the magnitude bits is subsequently modified by an additional multiplying factor derived from the sequence of polarity bits. In the following sub-section (A) we give one example. One possible information that can be extracted from the polarity bits is the slope overload condition, which may occur when the quantizer is small and a sudden large transition that lasts over many sampling instants occur. Examples of these conditions are the transition from unvoiced to voiced in speech, and for a step transition with video signals.
\[ m_x = m_{x-1} \cdot M(|p_{x-1}|) \cdot M'(B_x, B_{x-1}, B_{x-2}) \]

**FIGURE 7.03** - Generalized adaptive DPCM system (MADPCM)
For speech, there may not be much (if any) improvement. Jayant states that a step-size decrease of 20 dB is needed for adaptation to an idle-channel situation in speech. For a step multiplier of 0.95, the time needed for such an adaptation is 45 samples, which for Nyquist sampled speech, represent only about 5 ms. Jayant holds that the ADPCM multipliers derived in [95] are fast enough for typical speech applications. For Nyquist-sampled video signals, however, one Nyquist sample delay between step transitions in two adjacent lines, corresponding to a vertical boundary, may cause edge-business. Thus any step response improvement may be of some significance.

A. Step response of a modified ADPCM.

As an example, we have simulated in a computer one 3-bit modified ADPCM (MADPCM) system incorporating Jayant's one-word memory quantizer, whose multipliers where modified by an additional factor $M'(\cdot)$ as follows:

$$\Delta_r = \Delta_{r-1} \cdot M(|P_{r-1}|) \cdot M'(B_r, B_{r-1}, B_{r-2}) \quad (7.7)$$

The multiplier factors $M(|P_{r-1}|)$ are functions of the magnitude of the code-word used at the previous sampling instant, $|P_{r-1}|$, and can assume one of the values $M_1$ to $M_4$ as indicated in Fig. 7.1b, and:

$$M_1 = M_2 = 0.9$$

$$M_3 = 1.25$$

$$M_4 = 1.75 \quad (7.8)$$

In the hardware implementation of the ADPCM encoder, the range
of the step sizes is limited by a maximum and minimum step sizes, \( \Delta_{\text{max}} \) and \( \Delta_{\text{min}} \), and furthermore, a finite number of step sizes (i.e. a finite dictionary) is used. It would be convenient, therefore, that the additional multipliers \( M'(\cdot) \) would generate step sizes that are part of the existing dictionary. This can be achieved by using new multiplier factors that are similar to those used in the ADPCM, in addition to the trivial unitary case:

\[
\begin{align*}
M'_1 &= 0.90 \\
M'_2 &= 1 \\
M'_3 &= 1.25
\end{align*}
\]  

(7.9)

These multipliers correspond to the HIDM adaptation factors \( 1/2, 1, \) and \( 2 \), respectively, according to the same rule: the step sizes are decreased when there is polarity reversal in the error signal, increased for each third consecutive pulse of the same polarity, and kept unchanged for all other cases (see section 3.3 of Chapter III).

In Fig. 7.4a and b we show the results for a step input of amplitude \( 145\Delta_{\text{min}} \), and in Fig. 7.4c and d for a step of amplitude \( 46\Delta_{\text{min}} \). In both figures, (a) corresponds to the ADPCM and (b) to the MADPCM.

The improvement is small, but noticeable, for the build up of the step sizes, and for the damping after the transition. The encoder was started with step size \( \Delta_{\text{opt}} \), and the transition in the step input occurs within one sampling period. The encoder idles with the minimum step size.

The over-all step range used was 128:1, as in [114], with the central value \( \Delta_{\text{opt}} = 0.586 \), which is the optimal step size of a
Step response for 3-bit quantizer ADPCM (a and c) and MADPCM (b and d), for two step amplitudes: \( 145 \delta_{\text{min}} \) (a and b) and \( 46 \delta_{\text{min}} \) (c and d).

(Figures c and d: next page)
uniform quantizer to encode Gaussian signals with unit variance and zero mean. Thus, the minimum step size is approximately \( \Delta_{opt}/\sqrt{128} \).

The simulation presented is of an unrealistic nature, as the multiplier factors and the range of step sizes of 128:1 are those determined for a first-order Markov sequence and applied successfully for speech signals. One does not expect such step inputs for speech. However, it illustrates that it is possible to improve the performance of the ADPCM, to step inputs by adding a stage for using the polarity bits in addition to the magnitudes to control the step sizes.

It remains to be investigated the case where the multipliers are adjusted to encode video signals, and the study of non-uniform quantizers. In this section, we feel free to suggest the investigation of an adaptation where the polarity bits can be used to "syllabically" control the encoder by using them to compensate for the presence of the subcarrier, and the magnitudes for the "normal" ADPCM adaptation. It appears challenging to employ two-mode adaptation to encode signals made up by multiplexing two principal components: the luminance and the modulated colour subcarrier.

B. Response to a Gaussian input.

For the simulations we have employed the technique described by Jayant in [95], i.e. a first order Gauss-Markovian sequence of 10,000 samples generated by the recursive rule:

\[
x_r = \rho x_{r-1} + \sqrt{1-\rho^2} \cdot x_r ; \quad x_o = 0
\]  

(7.10)
where \( x_r \) are pseudo-random numbers with a Gaussian distribution of mean zero and unit variance. \( \rho \) is the correlation between adjacent samples.

The signal-to-noise ratio was calculated as a function of the initial step size, \( \Delta_0 \), and the number of samples \( N \) of the input sequence:

\[
\text{snr}(N, \Delta_0) = \frac{\sum_{r=1}^{N} x_r^2}{N \sum_{r=1}^{N} (x_r - y_r)^2}
\]  

(7.11)

and then averaged over different values of starting conditions and number of samples as:

\[
\text{snr}_{av} = \frac{1}{20} \sum_{N} \sum_{\Delta_0} \text{snr}(N, \Delta_0)
\]

(7.12)

The parameters used are \( N = 10, 100, 1000, \) and \( 10000 \), and \( \Delta_0 = [1/10, 1/\sqrt{10}, 1, \sqrt{10}, 10] \Delta_{\text{opt}} \). These values have been borrowed from Jayant's work, who justifies the choice of this peculiar performance criterion of calculating the average snr on the basis that it represents the varying encoding conditions encountered in speech systems.

In this short study, we have studied systems with a 3-bit quantizer and multipliers as per Eqs. 7.8 and 7.9, with \( \Delta_{\text{opt}} = 0.586 \). The average signal-to-noise ratio was calculated for \( \rho = 0.0, 0.5 \) and 0.99. The results are also given for the case \( \text{snr}(10,000; \Delta_{\text{opt}}) \), which is an asymptotic value practically independent of \( \Delta_0 \).
We can observe that the MADPCM presents losses relative to ADPCM, which are smaller than 1 dB, although the step response showed a small improvement. These results are expected on two accounts: (a) the multipliers used are those that have been optimized for the ADPCM and not for MADPCM; and (b) the faster adaptation algorithm in the MADPCM increases the probability of occurring over-corrections. This is just noticeable in Fig. 7.5, which shows that the overshoot in (b) is slightly larger than for (a).

The results presented in table 7.1 are such that the values given for $\rho = 0.5$ coincide with those given by Jayant in [95], are worse for $\rho = 0.0$ and better for $\rho = 0.99$. The reason for this is that Jayant present the simulation results for the quantizer alone (i.e. as a PCM encoder), whereas in this thesis the arrangement is as DPCM, with a single ideal integrator in the feedback path. With such encoders, there is an improvement relative to PCM when the correlation between consecutive samples is larger than 0.5, and if $\rho$ is less than 0.5, the differential arrangement works to disadvantage. In such cases, the "best" prediction is to assume that the next sample magnitude will be 0, i.e. a PCM quantization.

In the time available, we have tried a few other weighting factors,

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>ADPCM</th>
<th>MADPCM</th>
<th>ADPCM</th>
<th>MADPCM</th>
<th>ADPCM</th>
<th>MADPCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>9.0</td>
<td>8.6</td>
<td>11.5</td>
<td>11.1</td>
<td>26.5</td>
<td>25.5</td>
</tr>
<tr>
<td>0.5</td>
<td>10.0</td>
<td>9.6</td>
<td>12.6</td>
<td>12.0</td>
<td>29.6</td>
<td>28.5</td>
</tr>
<tr>
<td>0.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
without much success. One set of results is shown when $M'(\cdot)$ in Eq. 7.7 could assume two values: $M'_1 = 0.9$ when the polarity of the error signal changed, and $M'_2 = 1.25$ when the two most recent pulses had the same polarity. The values assumed by $M(\cdot)$ were kept unchanged from those of Eq. 7.8.

Table 7.2 (snr in dB), MADPCM

<table>
<thead>
<tr>
<th></th>
<th>$\rho = 0.0$</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{snr}_{av}$</td>
<td>8.3</td>
<td>10.0</td>
<td>24.4</td>
</tr>
<tr>
<td>$\text{snr}(10,000,\Delta_{opt})$</td>
<td>9.5</td>
<td>11.4</td>
<td>27.2</td>
</tr>
</tbody>
</table>

Clearly, the search for optimum multipliers should be made on systematic lines, as there is a trade-off between speed of response to step inputs and overshoots. These reduce SNR's.

7.4. Conclusions.

We have briefly outlined some of possible research topics that may be followed up. In the case of MADPCM, we have introduced only a crude combination of HIDM and Jayant's adaptive quantization algorithms. The results are indecisive, but reflect the possibility of combining different adaptation modes in one quantizer. Systematically searching for optimal adaptation algorithms that improve the step response and the snr's may prove fruitful. There is no obvious reasons to restrict the adaptation to the discrete multipliers as presented
in the previous section. Tapered quantizers could also be employed.

The MADPCM as presented here is not aimed at local applications, as was the 2BIADM. Consequently, effects of transmission errors and the possibility of forward error corrections are other possible areas of investigation.
CHAPTER VIII
RECAPITULATION AND REVIEW

8.1. Introduction.

We recall that the objective of the research reported in this thesis was the application of delta modulation and related techniques to the encoding of large bandwidth video signals. In the search for inexpensive digital encoders, our attention was drawn to the application of digital filters for the code conversion from delta modulated signals to a PCM format, which is the format most commonly used in digital signal processing.

Because of the high speed at which these filters would have to operate, two approaches were considered: one, the development of special purpose digital filter structures to operate with existing adaptive delta modulators and, secondly, the development of a novel digital encoder that would be used in conjunction with known high speed filter structures. Either way, the digital operations required for the code conversion would be considerably simplified if multiplications were avoided. We therefore decided to examine the performance of one of the fastest, yet stable, instantaneously adaptive delta modulators: the High Information Delta Modulation encoder. This encoder is attractive because of the simplicity of its step adaptation algorithm, which generates step sizes that are powers of two. The main objection to the use of the HIDM for transmission is the high hierarchy of the pulses used to generate the step sizes, which may cause large errors if one pulse is processed erroneously. Candy reduces the effects of error propagation by producing a delta
modulator with a more damped response, whose step adaptation is interrupted each time the polarity bit changes signs. If a delta modulator is used only locally as the input stage of a code conversion filter, the effects of transmission errors are unimportant and cease to be a deterrent to the use of encoders such as HIDM, which therefore can be used to its fullest advantages.

However, a cursory analysis has indicated that the HIDM could not be used for the purpose of encoding composite video signals, in a code converter arrangement, because of the speed limitations imposed by hardware considerations. An obvious solution in such a case is to increase the number of quantizer levels, i.e. to change from DM to DPCM or more specifically, from HIDM to some form of ADPCM coding. The component precision requirements and costs increase rapidly with the resolution of the encoder, and as a compromise solution we have limited the number of bits per sample to two, and sacrificed the quality that could be obtained when encoding large bandwidth video signals. Instead of using existing ADPCM systems with 2-bit quantizers, a new two-bit encoder has been developed and called "Two-bit Instantaneously Adaptive Delta Modulation" (2BIADM).

Because of the difficulties in analyzing mathematically such instantaneously adaptive encoders, the performance of the 2BIADM has been evaluated in relation to that of the HIDM. In the course of the many computer simulation measurements of the HIDM performance, part of the investigations has been devoted to a better understanding of this encoder, which has received relatively little attention in the literature, although it is frequently referred to as a classical example of an instantaneously adaptive delta modulator.
Thus, in this thesis, we have carried out a heuristic analysis of the HIDM in addition to the development of the 2BLADM, which was designed to be used in a DM-to-PCM code converter structure. Below we recapitulate the main results of our investigations, which can be divided in two main sections, viz. Chapter III and Chapters IV to VII.

8.2. Estimation of SNR for HIDM and CFDM\textsuperscript{115}.

In Chapter III, we reported on the analysis of the HIDM carried out by means of computer simulations and the procedures to estimate the SNR performance for sinusoidal and Gaussian inputs.

We have shown that the SNR vs. frequency response for sinusoidal inputs can be estimated by determining the asymptotes for the regions where the HIDM is operating as a LDM, as a "fully" adaptable encoder, and with restricted adaptation. In the first region, which corresponds to sinusoids with small slope, the response is flat with frequency. In the second, the SNR curve falls at a rate of approximately 6 dB/octave. Then, because the maximum step size is limited in practice, a point is reached when the step adaptation is restricted and the rate of fall increases to about 12 dB/octave. A fourth region exists, though not described in Chapter III, which corresponds to the situation where the HIDM is severely over-loaded and the feedback signal assumes a triangular-like waveform. In this region, the rate of fall should be as fast as for LDM, as most of the time the HIDM will be operating with only its maximum step size (the exceptions are when the feedback signal reverses the sign of the slope, in which case the next smaller step is used, twice).
The "duration" of the segment with the slope of -6 dB/octave increases by one octave for each additional step size in the HIDM, except for the first increase, from one to two step sizes, when there is a transition from the absence of such a segment to one lasting more than 1.5 octaves. This difference in the SNR response, when increasing the number of step sizes from one to two and thence to larger numbers, is due probably to the unequal step adaptation of the HIDM at the start of and during companding, caused by the restrictions in the variation of step sizes.

One interesting offshoot of the study of the HIDM response to sinusoids was the analysis of the relationship between its SNR response and the LDM "-6 dB/octave slope-overload characteristic". The latter corresponds to the SNR measured only for un-correlated noise (slope-overload avoided). For constant amplitude sinusoidal inputs, the response of a LDM encoder is flat until slope-overload and from that point the SNR curve falls at rates much greater than 6 dB/octave, if the slope-overload noise is taken into account in the calculations of SNR. This point is rarely mentioned in literature!

On the other hand, the response of the HIDM, during companding, has the SNR curve falling by 6 dB/octave for constant amplitude inputs, which implies a noise spectrum that raises by that amount. This could have been caused only by the changes in the step sizes, which is the variable parameter in the HIDM relative to LDM.

The SNR estimation for Gaussian inputs was investigated by considering the statistical average step size as function of the input power. We have achieved partial success employing the expressions derived for the quantization and slope-overload noise components in
LDM, and substituting the fixed step size in these expressions by the statistical average step size. The results were satisfactory as a first approximation, but the calculations were complicated by the need to know the probabilities of occurrence of each step size to obtain $\gamma_{av}$ (Eq. 3.1). To ease the calculations, an empirical expression was derived for the average step size, by functional approximations to the actual averages obtained by computer simulation of the encoder.

The deviations between the estimated and actual SNR's (obtained by computer simulation) are explained in terms of an additional noise component, which we have called "overshoot-noise". This noise component can be identified by calculating the cross-correlation coefficient between the input and band-limited error signals, as a function of input rms power.

The empirical formula for $\gamma_{av}$ in the calculations of SNR for the HIDM cannot, unfortunately, be used to all encoding conditions. However, for $F_p = f_p/f_c < 32$, the estimated SNR curves match the simulated ones to a large extent, though there may be points where the deviations are as great as about 5 dB.

The same technique has been applied to Jayant's one-bit memory ADM, i.e. Constant Factor DM, and the results were equally satisfactory. For the CFDM, the estimated curve has been verified in relation to a SNR curve obtained by another researcher, who had employed a substantially different simulation approach.

8.3. **2BIADM and Code Conversion to PCM by digital filtering**

In Chapter IV the concept of a new adaptive encoder, the 2BIADM, was introduced and evaluated by means of computer simulations against
the HIDM operating at the same sampling frequency.

The 2BIADM was devised to reduce the effects of excessive overshoots and delays in the tracking waveform of HIDM for sudden input signal variations. To achieve this, a delayed encoding structure was introduced into the HIDM, i.e. the magnitude of the error signal is examined and if found to exceed a given threshold, the feedback step size is made to increase. For that, side-information about the error magnitude is sent to the receiver and the local decoder, which doubles the number of bits per sample. However, as the 2BIADM is for local application its output words can be processed in parallel. Thus, instead of a trade-off between quality and sampling frequency, there is one between quality and storage requirements.

The comparisons with the HIDM have been made for sinusoidal and Gaussian inputs, and they have shown that the use of two bits per sample in the 2BIADM results in an overall better performance relative to that of a HIDM at the same sampling rates, and in some cases (large amplitudes and high frequencies) comparable with a HIDM operating at double the sampling frequency. We have seen that the 2BIADM has several modes of operation: first as LDM for low signal powers and slopes, and as these are increased, as HIDM and finally as ADPCM, when the side-information bit is frequently used.

The low-speed model that was built with standard TTL components is described in Chapter V. The main objective of its construction was to take advantage of the ease in designing and handling low-speed circuitry and test the computer simulations before proceeding to the more complex work with ECL integrated circuits for video encoding. As it turned out, unfortunately this second part of the investigations
could not be undertaken. However, the first part was successful and achieved by examining waveform reproductions and measuring the SNR's for HIDM and 2BIADM, with sinusoidal and narrow bandlimited white noise inputs. The low-speed operation enabled us to use available equipments for these measurements. The results showed that the proposed encoder was workable, and therefore its application for video encoding, if limited, would be mainly because of component speed considerations.

In Chapter VI we have reviewed the basic principles of DM-to-PCM code conversion, with emphasis on the digital filtering approach. Some of the refinements that can be introduced are also reviewed, for instance, the use of leaky integrators and generation of DPCM instead of PCM signals. Two filter structures without multipliers are briefly analyzed for application with 2BIADM signals. Performance curves presented by Goodman\textsuperscript{20} (for LDM-to-PCM code conversion) and results of computer simulations were used to show that the 2BIADM with a 5-stage digital filter has a performance that is superior to that of a HIDM with much larger filters.

Finally, an encoder that combines features of HIDM and one-word memory ADPCM\textsuperscript{95} was briefly described in Chapter VII. The encoder operation was not optimized, nor different adaptation algorithms tested, but it was possible to verify that the step response can be slightly improved, with a small loss (less than 1 dB) in SNR for Gaussian inputs. The improvements are in the speed of response and damping of transients. Relating this result with the second order CFDM improvement\textsuperscript{24} over the first order CFDM, we recall that ADPCM signals are encoded at Nyquist rates, where a delay of one sampling
period is more significant than a delay of many sampling periods in oversampled systems such as CFDM.

In Chapter VII, we have outlined some topics that are felt to be of more immediate research interest, of which two are: (a) hardware implementation of 2BIADM-to-PCM code conversion for application with video signals in real-time, and (b) investigations into the nature of noise in HIDM by analyzing either the cross-correlation coefficient between the filtered error and input signals or the distribution of the step sizes based on the HIDM adaptation algorithm.

8.4. Review.

The research reported in this thesis was partially successful in:

(a) deriving the HIDM response to sinusoidal signals (Chapter III.5),

(b) deriving an empirical formula for the average step size generated by HIDM and CFDM when encoding Gaussian signals, and estimating the SNR response as function of the input signal power (Chapter III. 6/7),

(c) developing a new adaptive encoder and showing it is workable and effective in reducing overshoots and delays in the response for fast transitions (Chapters IV and V),

(d) showing that a filter can be structured without multipliers to operate with the 2BIADM signals (Chapter VI),

(e) and finally, showing that an ADPCM system can be operated with two different adaptation algorithms for the polarity and magnitude bits (Chapter VII).
We feel the success to be partial because many areas have either been treated insufficiently, or omitted in this research. Briefly, the most important items that have been omitted are:

(a) 2BIADM hardware prototype to operate with real-time video signals, which precluded subjective assessment experiments. A major trouble spot may be found with D/A conversions, and it is likely that passive analogue integrators will have to be used instead of accumulators followed by D/A converters in the feedback loop, because of the short loop delay time of about 15 ns if the encoder is designed to operate at a sampling frequency of approximately 66.5 MHz. The non-uniform step sizes that will exist with analogue integrators may be a major factor that could compromise the design. If lower sampling frequencies are imposed by hardware considerations, the use of ADPCM (perhaps as suggested in Chapter VII) is felt to be inevitable.

(b) conversion filter hardware. We are confident that the filters described in Chapter VI are implementable to operate at word rates of 66.5 MHz, with ECL integrated circuits. Two factors would support this belief: that no multipliers are used, and secondly because DPCM filters described in literature can achieve a 40 MHz word rate with standard high-speed TTL circuitry. However, the problems associated with the hardware implementation of digital filters have not been studied. These constitute a major area of active research at present.

(c) thorough study of noise in HIDM and CFDM by considering all the noise components. Our approach effectively ignored how the noise
is distributed during coding, and how it relates to the step adaptation algorithms. We believe that $\rho_{xe}$ and $\gamma_{av}$ are the keys to the problem.

8.5. Closing remark.

The initial objective of the research reported in this thesis, viz. digital encoding of colour video signals, was constricted by hardware and cost considerations. To overcome these limitations, the avenue chosen was to investigate the application of DM-to-PCM code conversion techniques. This choice, though leading to a personally interesting research field, was on hindsight somewhat unfortunate. We believe that it has spread the problem over the perplexing grounds of designing ultra-high speed digital filters and of producing efficient delta modulators whose outputs are convenient for digital processing. As a consequence the activities were dispersed and the solutions reported in this thesis barely scratched the surface of these yet unproven grounds. However, it has also lead to the problem of estimation of the performance of IADM's, and if the results reported in this thesis are seen to stimulate the discussions and contribute in a modest way towards the understanding and representation of SNR's in those encoders, to the Author it will more than recompense for the efforts that have gone into this research. We would like to believe, nevertheless, that the work on 2BIADM and code conversion might find applications in non-broadcast TV situations, possibly in a modified form, and that it has laid the foundations for more fruitful research in a not too distant future.
Abridged Specifications, PAL-I System

- Number of lines per frame: 625; 2:1 interlaced scanning
- Colour subcarrier frequency: \( f_{sc} = 4.43361875 \text{ MHz} \)
- Line frequency: \( f_L = 15.625 \text{ KHz} \)
- Field frequency: \( f_f = 50 \text{ Hz} \)
- Video bandwidth: 5.5 MHz (0 dB)
- Chrominance bandwidth: -3 dB at 1.3 MHz, -20 dB at 4 MHz
- Colour burst signal: 10 ± 1 cycles in the back porch of horizontal blank, \( V_{p-p} = 300 \text{ mV} \) for 700 mV white level signal
- Tolerance: ± 11%
- Position: ± 100 ns from leading edge of sync pulse
- Luminance signal: \( E'_Y = 0.299E'_R + 0.587E'_G + 0.114E'_B \) (gamma corrected).

In the thesis we have mentioned the impossibility of achieving "broadcast quality" with IADM's operating at about 12 times the signal bandwidth. To give a sense of perspective to the measurements presented in the thesis, we show below the tolerances that are acceptable in one of the equipments commonly used in TV studios: a video tape recorder. Details about the measurements and other specifications can be found in [58]. In this appendix we only outline some of the specifications without further explanations.

- Luminance non-linear distortion: 10% for a 5 staircase signal with 140 mV steps. The error is defined as (largest - smallest step)/largest step.
- Total phase error in chrominance: ± 6° relative to burst phase.
- Noise: random, defined as \( S_{p-p}/\text{rms-Noise} \)
  - Weighted luminance: -50 dB
  - Weighted chrominance: -46 dB

Figs. A.1.1 and A.1.2 depicts the weighting networks for SNR measurements.
A.1.1 Response of luminance noise weighting network.

- 2T pulse response (sine-square pulse, half-amplitude width 200 ns)
  measured on specially calibrated mask: 2% K-rating
- delay between luminance and chrominance (measured with modulated
  sine-square pulse) ± 40 ns.

A.1.2 Response of chrominance noise weighting network.
A.2.1 - Introduction.

Numerous programs have been developed in the course of this research, but their structural details must be omitted due to space limitations. This is not a serious omission because most programs are straightforward interpretations of the algorithms and calculations that have been described in detail in the main text of this thesis. The details given there should suffice to allow the interested reader to originate the appropriate programs. The programs used in this research have been written in Extended Fortran 1900\textsuperscript{91}, with a particular computer set up in mind, namely that of the Loughborough University Computer Centre, based around the ICL1904 computer + ICL1934/4 graph plotter. Thus, complete listings of particular programs would be of no use to the general reader, or even to this Department's users, in view of constant up-gradings of computer facilities that made some of the programs out-of-date by the time of this writing. In this appendix, therefore, we only concentrate on the main aspects of some representative programs.

A.2.2 - Low-pass Filter.

The pre and post-encoding band-limiting operation was performed by convolving the signal samples with a quasi-ideal low-pass filter impulse response samples. These were generated by calling a subroutine named FILTER(PULSE,N,C), where

\begin{itemize}
    \item \textbf{PULSE} is a real array holding the impulse response samples
    \item \textbf{N} is the number of sample points
    \item \textbf{C} is a complex array holding the frequency response samples.
\end{itemize}

The frequency response samples of the low-pass filter are generated by assigning the value 1.0 (imaginary component = 0.0) to BW samples in the pass band (first sample for \( f = 0.0 \)); the value 0.0 to the samples in the stop-band, and the values of two samples in the transition band are derived from a table produced by Rabiner et al.\textsuperscript{94}, on whose
work the design of the filter is based. Fig. A.2.1 illustrates the frequency samples distribution for \( N = 16 \) and two transition points.

The basic relationships for the filter design are

\[
\text{bandwidth (0 dB)} f_c = (\text{BW -1}) \Delta f \\
\Delta f = \frac{f_p}{N}
\]

such that given the sampling frequency and bandwidth required, it is possible to find the number of samples in the pass-band. The sharpness in the cut-off response is determined by the number of samples in the transition band. The "cook book" tables by Rabiner et al. lists the optimal values for 1, 2, and 3 transition samples for which the out-of-band attenuation are of the order of 42 dB, 65 dB and 85 dB, respectively.

The impulse response is generated by calling a subroutine that performs the inverse FFT, called NLOGN(N,X,LX,DIR), where

\[
N = \log_2(LX) \\
LX \text{ is the number of samples for transformation (power of two)} \\
X \text{ is a complex array that holds the input for the transformation,} \\
\text{and on return to the main program the transformed values} \\
\text{DIR = +1 performs the inverse transform (frequency to time)} \\
-1 \text{ performs the direct transform (time to frequency)}
\]

The filter operation is performed by convolving the sequence of signal samples with the filter impulse response samples thus generated. The convolution operation is carried out by a call to a subroutine FOLD(LA,A,LB,B,LC,C), where

\[
A, B, \text{ and C contains the two input and the output sequences and} \\
LA, LB, \text{ and LC are the number of samples in each of the above} \\
\text{sequences, respectively.}
\]

The subroutines NLOGN and FOLD have been taken from a book by E.A. Robinson 117.

The listings for the subroutine FILTER, NLOGN and FOLD are presented in lists 1, 2 and 3, respectively.

A.2.3 - HIDM, LDM and 2BIADM simulation.

The above encoders are simulated by calling a subroutine named MHIDM(XINC,LB,ERR,YFB,SAVE,KL,KS,B), where
XINC is a real array of input samples
LB is the number of samples in XINC
ERR is a real array with the error samples
YFB is a real array with the feedback samples
SAV is the average step size
KI = 1, 2, or 3 determines the encoder
KSM = maximum step size, relative to the minimum.

JB is an integer variable (only required if SAV for different signal powers is to be stored for later use).

KI = KSM = 1 produces a LDM
KI = 1 and KSM = 8, for instance, produces HIDM {1-8}
KI = 2 indicates that the minimum step size is multiplied by 4, e.g.
with KI = 2 and KSM = 8, HIDM {4-32} is produced.
KI = 3 and KSM = 8 produces 2BIADM with 4 states

The principal variables in the subroutine listing (list 4) is schematically indicated in Fig. A.2.2. The detailed description of the adaptation algorithms can be found in Chapters III and IV for HIDM and 2BIADM, respectively.

A.2.4 - Example of application of subroutines FILTER, NLOGN, FOLD and MHIDM.

The program (main part only) to calculate the signal-to-noise ratio, the cross-correlation coefficient, and the average step sizes for different values of input signal powers (indexed by JB) is presented without further explanations, as the comments in the program extract indicate the various operations executed.
SUBROUTINE FILTER(PULSE,N,C)
  INTEGER BW
  COMPLEX C(N)
  REAL PULSE(N),TR(2)
  IF(2!=22)
    TP(1)=9.58121003
  DO 1 K=1,BW
  C(K)=CMPLX(1.0,0.0)
  C(BW+1)=CMPLX(TR(1),0.0)
  C(BW+2)=CMPLX(TR(2),0.0)
  DO 2 K=BW+3,N/2+1
  2 C(K)=CMPLX(0.0,0.0)
  DO 3 K=2,N/2
  3 C(N+2-K)=C(K)
  KN=N
  DO 5 K=1,25
  5 CONTINUE
  KN=KN/2
  IF(KN,EQ,1.0)GO TO 6
  CALL NLOGN(K,C,N,1.0)
  DO 4 K=1,N/2
  4 PULSE(K)=REAL(C(N/2+K))
  PULSE(K+N/2)=REAL(C(K))
  RETURN
END

PROGRAM LIST 1

SUBROUTINE FILTER(PULSE,N,C)
  INTEGER BW
  COMPLEX C(N)
  REAL PULSE(N),TR(2)
  IF(2!=22)
    TP(1)=9.58121003
  DO 1 K=1,BW
  C(K)=CMPLX(1.0,0.0)
  C(BW+1)=CMPLX(TR(1),0.0)
  C(BW+2)=CMPLX(TR(2),0.0)
  DO 2 K=BW+3,N/2+1
  2 C(K)=CMPLX(0.0,0.0)
  DO 3 K=2,N/2
  3 C(N+2-K)=C(K)
  KN=N
  DO 5 K=1,25
  5 CONTINUE
  KN=KN/2
  IF(KN,EQ,1.0)GO TO 6
  CALL NLOGN(K,C,N,1.0)
  DO 4 K=1,N/2
  4 PULSE(K)=REAL(C(N/2+K))
  PULSE(K+N/2)=REAL(C(K))
  RETURN
END

FIGURE A.2.1

---

PROGRAM LIST 3

SUBROUTINE FOLD(LA,A,LR,B,LC,C)
  C THIS SUBROUTINE CONVOLVE ELEMENTS OF ARRAYS A AND B TO PRODUCE C
  C LA IS THE LENGTH OF THE REAL INPUT A
  C LB IS THE LENGTH OF REAL INPUT2 ARRAY B
  C LC > UR= LA+LR-1, IS THE LENGTH OF OUTPUT ARRAY C
  DIMENSION A(LA),B(LR),C(LC)
  IC=1,LA+LR-1
  DO 1 I=1,IC
  1 C(I)=0.0
  DO 2 J=1,LA
  2 C(I)=C(I)+A(I)*B(J)
  RETURN
END

---

FIGURE A.2.1
PROGRAM LIST 2

SUBROUTINE INLOGN(N,X,LX,DIR)
COMPLEX X,UK,HOLD,Q
DIMENSION N(25),X(LX)
C DIR=1.0 GIVES INVERSE TRANSFORM; DIR=1.0 GIVES DIRECT TRANSFORM
C DATA TO BE TRANSFORMED IS HELD IN COMPLEX ARRAY X AND THE RESULT
C REPLACES IT IN THE SAME ARRAY X
C PROGRAM FROM E.A.ROBINSON'S BOOK 'MULTICHANNEL TIME SERIES ANALYSIS'
C WITH DIGITAL COMPUTER PROGRAMS,HOLDEN-DAY,1967
   DO 1 I=1,N
1   H(I)=2**(N-I)
   DO 4 L=1,N
   NBLOCK=2**(L-1)
   LBLOCK=LX/NBLOCK
   LBHALF=LBLOCK/2
   K=0
   DO 4 IBLock=1,NBLOCK
     FK=K
     FLX=LX
     VK=DIR*0.24318551*FK/FLX
     WK=CMPLX(COS(V),SIN(V))
     ISTART=LBLOCK*(IBLOCK+1)
     DO 2 I=1,LBHALF
     J=ISTART+I
     VH=J+LBHALF
     Q=X(JH)*WK
     X(JH)=X(JH)+Q
     X(J)=X(J)+Q
     2 CONTINUE
   DO 3 I=2,N
     II=I
     IF(K,LT,M(I))GO TO 4
     K=K+M(I)
     4   K=K+M(II)
     K=0
     DO 7 J=1,LX
     IF(K,LT,J)GO TO 5
     HOLD=X(J)
     X(J)=X(K+1)
     X(K+1)=HOLD
     5   DO 6 I=1,N
     II=I
     IF(K,LT,M(I))GO TO 7
     K=K+M(I)
     7   K=K+M(II)
     IF(DH.LT.0.0)RETURN
     DO 8 I=1,LX
     X(I)=X(I)/FLX
     8   CONTINUE
   RETURN
END
PROGRAM LIST 4

SUBROUTINE H1DM(XINC,LB,ERR,YFB,SAV,K1,KSM,JB)
INTEGER S1(40),S2(40),S4(40),S8(40)
INTEGER S16(40),S32(40)
LOGICAL BOUT(3),MU1,MU2,MD1,MD2,UP,DOWN
INTEGER YOUT,W,COUNT
DIMENSION XINC(LB),ERR(LB),YFB(LB)
REAL SAV(40)
ERR(1),ERR(2),XINC(1),XINC(2),YOUT,YFB(1),YFB(2)=0.0
S1(JB),S2(JB),S4(JB),S8(JB)=0
S16(JB),S32(JB)=0
NSS=2048
COUNT=1
U=4
BOUT(1),BOUT(2),BOUT(3)=.FALSE.
J=3

100 ERR(J)=XINC(J)-YFB(J-1)
BOUT(1)=BOUT(2)
BOUT(2)=BOUT(3)
IF(ERR(J).GE;0.0)GO TO 200
BOUT(3)=.FALSE.
GO TO 300
200 BOUT(3)=.TRUE.
300 CONTINUE
STEP=0.03
IF(KSH.EQ.1)GO TO 560
MU1=(BOUT(3),AND,BOUT(2),AND,BOUT(1))
MU2=.NOT.(BOUT(3),OR,BOUT(2),OR,BOUT(1))
UP=H1T.OR.MU1
MD1=BOUT(3),AND,.NOT.BOUT(2)
MD2=.NOT.BOUT(3),AND,BOUT(2)
DOWN=MD1.OR.MD2
IF(NOT,UP)GO TO 400
IF(COUNT,EQ,KSM)GO TO 500
COUNT=COUNT+1
GO TO 300
400 IF(NOT,DOWN)GO TO 500
IF(COUNT+.EQ.1)GO TO 500
COUNT=COUNT/2
500 CONTINUE
IF(COUNT+.EQ.1)S1(JB)=S1(JB)+1
IF(COUNT+.EQ.2)S2(JB)=S2(JB)+1
IF(COUNT+.EQ.4)S4(JB)=S4(JB)+1
IF(COUNT+.EQ.8)S8(JB)=S8(JB)+1
501 CONTINUE
IF(K1,EQ.1)GO TO 510
IF(K1,EQ.2)GO TO 520
IF(K1,EQ.3)GO TO 530
510 U=1
GO TO 760
520 U=4
GO TO 760
530 TR=2.*COUNT*STEP
IF(ABS(ERR(J)) .LT. TR)510,520,520
560 IF(BOUT(3))GO TO 600
YOUT=YOUT+U*COUNT
GO TO 700
600 YOUT=YOUT+U*COUNT
700 CONTINUE
YFB(J)=YOUT*STEP
ERR(J)=XINC(J)-YFB(J)
error between input and feedback signals (no output)

shift memory contents

error between input and reconstructed signals
FIGURE A.2.2 Notations used in the encoder simulation program
PROGRAM EXTRACT

MASTER ADFM
REAL SAV(40)

REAL COEF(40), PULSE(256), POWERIN(40), YFB(2050), XIN(2050)
REAL XDUT(2306), XINC(2050), ERR(2050), FILTER(2306), ERRROUT(2050)
REAL SNR(40), ZOUT(2050)
REAL SIGMA(40), SIGERR(40)
COMPLEX D(256)

CALL FILTER(PULSE, 256, D)  generate filter impulse response

L=8
N=1
KKS=2
STEP=0.03
B=0.02
IF(N.GT.1)B=0.009
JB=1

50 CONTINUE
CALL G05BAF(1.)
DO 60 K=1, 2050
XIN(K)=G05ADF(X)
CALL FOLD(256, PULSE, 2050, XIN, 2306, XOUT)  filter by convolution
IF(KT.EQ.2)GO TO 72
DO 70 K=3, 2050
XINC(K)=XOUT(K+128)
XINC(K)=XOUT(K+128)

AUTOC=0.0
FILTER=0.0
DO 60 XIN(K)=A*G05ADF(X)
CALL FOLDI256(PULSE, 2050, XIN, 2306, XOUT)  generate random numbers
generate random numbers

70 XINC(K)=XOUT(K+128)
AUX=3.2
POWER=POWER=0.
DO 90 J=10, 2050
S=SAV(JB)*0.03+12.19+0.276/SIGMA(JB)
SIGMA(JB)=SIGMA(JB)+0.001  encode

90 FILTER=FILTER+ERROR(J)  encode
POWERIN(JB)=POWER+ERROR(J)  encode
SNR(JB)=10.*ALOG10(Power(JB))  encode
SNR(JB)=10.*ALOG10(Power(JB))  encode
WRITE(2, 906) POWERIN(JB)*SNR(JB)*SIGMA(JB), COEF(JB), SAV(JB)
906 FORMAT(1HO, 5HSIGMA=, F8.4, 1X, 4HSNR=, F7.4, 1X, 6HSIGMA=, F7.4, 1X, 4HCOR=, F7.3, 1X, 4HSAV=, F8.6)
WRITE(2, 907) S
907 FORMAT(1HO, 3HS=, F8.5, 1X)
910 CONTINUE
JB=JB+1
B=1.65*B
IF(JB.LT.16)GO TO 50

repeat the calculation for the next layer of output power levels.
APPENDIX 3

LOW-SPEED HARDWARE MODEL

The low-speed hardware model constructed to test the performance of the 2BIADM compared to HIDM, has been described in Chapter V, section 5.2, Fig. 5.1 and table 5.1. The details of the hardware design and other characteristics are omitted from this thesis, as the main purpose of the model was to complement the computer simulation measurements and to check the validity of the simulation structure for the proposed encoder. A more important reason for omitting the detailed analysis of the hardware model is that the 2BIADM is proposed for application with video signals in a DM-to-PCM code conversion structure, and the speed with which the circuit has to operate (in the order of tens of MHz) means that the logic components have to be of the Emitter Coupled Logic family. This requires a totally different design approach from that for TTL families, as mentioned in the main text. As examples of the differences, there are no "open-collector inverters" in the ECL family and the shift-right shift-left register with ECL (ECL 10141) is arranged as an universal shift register with different modes of operation and controls compared to TTL SR's.

Therefore, in this appendix we only present, without further details, the complete circuit diagram of the 2BIADM/HIDM/LDM encoder, as described in Chapter V, in Fig. A.3.1. In Figs. A.3.2 to A.3.5 we show the several stages in a larger scale diagram. Table A.3.1 depicts the complete list of components used in the construction of the hardware model. Fig. A.3.6 depicts the DAC response, i.e. analogue output voltage vs. input binary words for a six-bit DAC. Observe that the response is monotonic, but deviations for large output voltages are larger than the step corresponding to the LSB. Finally, in Fig. A.3.7 we depict the simple 3-stage active low-pass filter used for the reconstruction of waveforms shown in Figs. 5.3 and 5.4.
FIGURE A.3.1 - Circuit diagram of the low-speed hardware model of 2BIADM with capability of operation as HIDM and LDM.
Figure A.3.2 - Input/quantizer

Figure A.3.3 - Digital-to-analogue converters

SW1: preset (LDM/ADM)
SW2: 4/6 steps
SW3: Tn fixed/variable
SW4: x1/4
Figure A.3.4 - Feedback step generator

Figure A.3.5 - Feedback "integrator" (digital accumulator + step steering)
<table>
<thead>
<tr>
<th>Representation</th>
<th>Component</th>
<th>Quantity IC's</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="SR" /></td>
<td>4-bit Right-shift Left-shift register</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>type : 7495</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4-bit Arithmetic Logic Unit</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>type : 74181</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="NAND gates" /></td>
<td>NAND gates</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>type : 7400 quad 2 i/p</td>
<td></td>
</tr>
<tr>
<td></td>
<td>type : 7410 triple 3 i/p</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="Inverters" /></td>
<td>inverters</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>for DACs type : 7416 open-collector (hex)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>for delay/inverter type : 7404 (hex)</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="EX-OR" /></td>
<td>EX-OR</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>type : 7486</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="D-flip-flops" /></td>
<td>D-flip-flops</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>type : 7474 (dual)</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="Comparator" /></td>
<td>comparator</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>type : linear, 710</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="Operational Amplifier" /></td>
<td>operational amplifier</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>type : linear 741</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="Not shown" /></td>
<td>clock delay monostables</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>type : 74121</td>
<td></td>
</tr>
<tr>
<td><strong>Supplies:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ = + 5 V (DAC reference)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>all logic boards : + 5 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>++ = + 14 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-- = - 14 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>not indicated : -6.8V for 710 comparators</td>
<td></td>
</tr>
<tr>
<td><strong>Resistors:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5% tolerance</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE A.3.6 - 6-bit D/A converter response

FIGURE A.3.7 - 3-stage active low-pass filter (3.4 kHz cut-off). 741 operational amplifiers; resistors in Ohms; capacitors in pico-faradays unless otherwise indicated.
APPENDIX 4

In Chapter VI we have assumed that:

\[ \tilde{x}_{jR} = \gamma \left[ \sum_{n=-M}^{M} g_{n+M} b_{jR-n} + \sum_{n=M+1}^{\infty} b_{jR-n} \right] \]  \hspace{1cm} [Eq. 6.5]

was equivalent to

\[ \tilde{x}_{jR} = \gamma \sum_{k=-M}^{M} a_k y_{jR-k} \]  \hspace{1cm} [Eq. 6.8]

with

\[ \sum_{k=-M}^{n} a_k = g_{n+M} \]

\[ \sum_{k=-M}^{M} a_k = 1 \]  \hspace{1cm} [Eq. 6.9]

and

\[ a_k = 0 \quad \text{for } k > M \]

To show the validity of the assumption, we can write from Eqs. 6.5 and 6.9:

\[ \tilde{x}_{jR} = \gamma \left[ \sum_{n=-M}^{M} \sum_{k=-M}^{n} a_k b_{jR-n} + \sum_{k=-M}^{M} \sum_{n=M+1}^{\infty} a_k b_{jR-n} \right] \]  \hspace{1cm} (A.4.1)

The first term between brackets in Eq. A.4.1 can be expanded as:

\[ \sum_{n=-M}^{M} \sum_{k=-M}^{n} a_k b_{jR-n} = \]

\[ = a_{-M} b_{jR+M} + (a_{-M} + a_{-M+1}) b_{jR+M-1} + (a_{-M} + a_{-M+1} + a_{-M+2}) b_{jR+M-2} + \ldots \]

\[ + \ldots + (a_{-M} + \ldots + a_{-M+M}) b_{jR-M} \]  \hspace{1cm} (A.4.2)
Eq. A.4.2 can be re-arranged as

\[ \sum_{n=-M}^{M} \sum_{k=-M}^{M} a_k b_{jR-n} = \]

\[ = a_{-M} (b_{jR+M} + \ldots + b_{jR-M}) + a_{-M+1} (b_{jR+M-1} + \ldots + b_{jR-M}) + \ldots + a_{M} b_{jR-M} = \]

\[ = \sum_{k=-M}^{M} a_k \sum_{n=k}^{M} b_{jR-n} \quad \text{(A.4.3)} \]

Thus, Eq. A.4.1 can be re-written as:

\[ \tilde{x}_{jR} = \sum_{k=-M}^{M} a_k \sum_{n=k}^{M} b_{jR-n} + \sum_{k=-M}^{M} a_k \sum_{n=M+1}^{\infty} b_{jR-n} = \]

\[ = \sum_{k=-M}^{M} a_k \sum_{n=k}^{\infty} b_{jR-n} \quad \text{(A.4.4)} \]

By putting \( n' = jR-n \), Eq. A.4.4 becomes

\[ \tilde{x}_{jR} = \sum_{k=-M}^{M} a_k \sum_{n'=jR-k}^{\infty} b_{n'} \quad \text{(A.4.5)} \]

Recalling that

\[ y_n = \gamma \sum_{k=-\infty}^{n} b_k \quad \text{[Eq. 6.1]} \]

Eq. A.4.5 can be written as

\[ \tilde{x}_{jR} = \gamma \sum_{k=-M}^{M} a_k y_{jR-k} \]

which is Eq. 6.8.
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