Effective teacher training for the improvement of mathematics education in the Bahamas

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EFFECTIVE TEACHER TRAINING FOR THE IMPROVEMENT OF
MATHEMATICS EDUCATION IN THE BAHAMAS

By

E R STORR  BSc(UWI), Cert Ed(Southampton)

A doctoral thesis submitted in partial fulfilment of
the requirements for the award of Doctor of Philosophy
of the Loughborough University of Technology, 1983

Supervisor:  Professor A C Bajpai
Director of CAMET and Head of the
Department of Engineering Mathematics
Loughborough University of Technology

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ABSTRACT

Effective Teacher Training for the Improvement of Mathematics Education in the Bahamas

by E R Storr

(Centre for Advancement of Mathematical Education in Technology)
Loughborough University of Technology

This research is primarily concerned with the on-going development of the secondary mathematics curriculum in The Bahamas. In order to determine whether pupils assimilated mathematical ideas into a schematic structure, and could retrieve and use them as final behaviours outlined in the syllabuses, data from two diagnostic tests administered to pupils of 13+ and 15+ years respectively, were analysed. For the former age range, 377 pupils completed an Objective Test containing 50 items and for the latter age range, 241 pupils attempted a Choice-Type Test with the instruction to choose any 10 of the total of 14 questions.

On each of these tests the following indices per question are presented: popularity, facility, mean ability, and discrimination. Reliability and validity, were calculated and determined by KR20 formula and the syllabuses and pilot visits to schools respectively, according due recognition for sources of error and precautions in interpreting results. The main corrective measure suggested by this survey is that methods of teaching mathematics should include opportunities for:

(i) exposition by the teacher
(ii) discussion between teacher and pupils and among pupils themselves
(iii) appropriate practical work
(iv) consolidation and practice of fundamental skills and routines

(v) problem solving to include application of mathematics to daily situations as well as in physics, chemistry, biology and agriculture

(vi) investigational work with games and physical models in order to discover mathematical principles, formulae, etc.

The most important finding of this research is that a massive percentage of underachievement is occurring in the High Schools thus reflecting instructional ineffectiveness as well as inefficiency at learning tasks. The strength and clarity of the evidence compels the inescapable conclusion that a well-thought-out programme of In-Service Education for Teachers of Mathematics Education must be mounted in The Bahamas as a matter of urgent necessity. Initial teacher education and training for primary schools must continue but very strong emphasis must be placed on recruiting teacher trainees for the 11-16 age group. In order to ensure that present and future requirements of the Bahamian High Schools are met, a set of recommendations for continued improvement of mathematics education is being suggested to The Ministry of Education and Culture for implementation through the College of The Bahamas.

Key Words: Mathematics Education - Bahamas, Helical curriculum, Mixed ability teaching, Discovery learning strategy, Mathematical ideas, Applications of mathematics, Content - behaviour grid, Feedback, Validation, Repeated trial, Maths item banking.
ACKNOWLEDGEMENTS

I am deeply grateful to Professor A C Bajpai who, in 1978, transmitted to me his infectious enthusiasm for mathematics education. After visiting schools in New Providence and the Family Island, he highlighted the Commonwealth's need and, in his unique but persuasive style, put a challenge in terms of national priority which I could not refuse. At Loughborough, I have had the benefit of the use of the excellent facilities in CAMET (Centre for Advancement of Mathematical Education in Technology), a subset of the Department of Engineering Mathematics. I also had the chance to travel along with postgraduate secondary mathematics teachers, who were following one year in-service training courses in the All India Mathematics Education at CAMET (AIMEC) Project, to places and institutions of educational interest. In addition to these many opportunities which Professor Bajpai has so generously provided over three and a half years, I am honour bound to record my gratitude not only for his invaluable advice and assistance in the preparation of this thesis, but also to indicate that without him it could never have been written.

I acknowledge, with profound appreciation, the Commonwealth Scholarship awarded by The Association of Commonwealth Universities and administered through the good offices of The British Council. In the context of The British Council, I am grateful to Mr Bryan Wilson, Head of Education, Mathematics, Science and Medicine Consultancy, for influencing my ideas through his Seminars on "Mathematics Education in Developing Countries", given to the AIMEC Project Members.

Mr Emmanuel Apea, Chief Project Officer (Education) of The Commonwealth Secretariat, who has a comprehensive knowledge of the educational problems in The Bahamas deserves my gratitude.

I must also thank the Government of The Bahamas, who, through the Council of the College of The Bahamas, has provided generous conditions of in-service leave to permit me to carry out this
research at Loughborough University of Technology. At Council level, the policy of the Government has been carried out by Miss Marjorie Davis, Director of Education; previous Principals of the College; Dr (Mrs) Keva M Bethel, Principal of the College; and Mr Roger Brown, Secretary and Registrar of the College, who have all duly earned my sincere gratitude.

Many Principals of schools and mathematics teachers have taken me into their confidence; they gave me permission to use their classrooms; provided information about their schools and helped in the invigilation of my diagnostic tests. Pupils were also most cooperative often at short notice. I record my thanks to them all.

Sometimes the quality of a research thesis suffers from want of up-to-date and reliable background information. I am therefore profoundly grateful to The Commonwealth Secretariat who gave permission, through Mr Apoa, for me to quote from their Unpublished Confidential Report [5].

In addition, I thank Loughborough University of Technology for making available to me many services and facilities.

My father always encouraged me to pursue an academic career. But with deep feeling, I acknowledge the quiet contribution of my dear wife Mary, son Merrit and daughter Elvia whose constant sacrifice and encouragement, loyal support and untiring cooperation have gone a long way in making the research for this thesis possible.
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6.1.1 Pilot visits to schools
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CHAPTER 1

CONTEXT OF THE INVESTIGATION
AND ESSENTIAL BACKGROUND INFLUENCES
Figure 1.1: The Islands of The Bahamas and Neighbouring Countries
Education, unlike a bridge, cannot be measured accurately from one point in time to another because it is a continuous process affected by many social and natural influences which can be extrapolated backwards into history. Firstly, local geographical conditions provide the background conditions of topography and climate to which people must adapt for survival through time, as well as determine the lines of communication in an international community and within the country itself.

The Bahamas are a chain of islands, cays, rocks and reefs stretching south easterly, forming the northern group of the West Indian archipelago, and lying between 20°50' N, 27°25' N and 72°37' W, 80°32' W (Figure 1.1) [1]. The islands therefore have a subtropical climate with temperatures ranging from 60°F (15.5°C) in winter to 95°F (35°C) in summer. They also sit on a limestone base, are low and flat, the highest elevation on Cat Island being only 206 ft (62.79 m) above sea level (Table 1.1), and cover a total surface area of 5382 mls² (13943 km²) (Table 1.2).
<table>
<thead>
<tr>
<th>Island/Cay</th>
<th>Highest Point (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cat Island</td>
<td>206</td>
</tr>
<tr>
<td>Long Island</td>
<td>178</td>
</tr>
<tr>
<td>Eleuthera</td>
<td>168</td>
</tr>
<tr>
<td>Crooked Island</td>
<td>160</td>
</tr>
<tr>
<td>Acklins</td>
<td>142</td>
</tr>
<tr>
<td>Mayaguana</td>
<td>131</td>
</tr>
<tr>
<td>Exuma Cays</td>
<td>130</td>
</tr>
<tr>
<td>Rum Cay</td>
<td>130</td>
</tr>
<tr>
<td>Exuma</td>
<td>125</td>
</tr>
<tr>
<td>New Providence</td>
<td>123</td>
</tr>
<tr>
<td>San Salvador</td>
<td>123</td>
</tr>
<tr>
<td>Ragged Island</td>
<td>116</td>
</tr>
<tr>
<td>Inagua</td>
<td>109</td>
</tr>
<tr>
<td>Long Cay</td>
<td>108</td>
</tr>
<tr>
<td>Abaco</td>
<td>100</td>
</tr>
<tr>
<td>Andros</td>
<td>100</td>
</tr>
<tr>
<td>Berry Islands</td>
<td>80</td>
</tr>
<tr>
<td>Grand Bahama</td>
<td>34</td>
</tr>
<tr>
<td>Bimini</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 1.1: Elevations of the Bahama Islands.
1.2 Population Trends

<table>
<thead>
<tr>
<th>Island</th>
<th>Land area $m^2$</th>
<th>Population, 1970</th>
<th>Population per $m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Providence</td>
<td>80</td>
<td>101,503</td>
<td>258.7</td>
</tr>
<tr>
<td>Grand Bahama</td>
<td>530</td>
<td>25,859</td>
<td>48.8</td>
</tr>
<tr>
<td>Abaco</td>
<td>649</td>
<td>501</td>
<td>10.0</td>
</tr>
<tr>
<td>Acklins Island</td>
<td>192</td>
<td>936</td>
<td>4.8</td>
</tr>
<tr>
<td>Andros</td>
<td>2300</td>
<td>845</td>
<td>3.8</td>
</tr>
<tr>
<td>Berry Islands</td>
<td>12</td>
<td>443</td>
<td>36.9</td>
</tr>
<tr>
<td>Biminis, Cay Lobos and Cay Sal</td>
<td>11</td>
<td>1,503</td>
<td>136.6</td>
</tr>
<tr>
<td>Cat Island</td>
<td>150</td>
<td>2,657</td>
<td>17.7</td>
</tr>
<tr>
<td>Crooked Island</td>
<td>84</td>
<td>689</td>
<td>8.0</td>
</tr>
<tr>
<td>Eleuthera, Harbour Island, Spanish Wells</td>
<td>200</td>
<td>468</td>
<td>47.3</td>
</tr>
<tr>
<td>Exuma and Cays</td>
<td>112</td>
<td>767</td>
<td>33.6</td>
</tr>
<tr>
<td>Inagua</td>
<td>599</td>
<td>11,109</td>
<td>1.9</td>
</tr>
<tr>
<td>Long Cay</td>
<td>9</td>
<td>26</td>
<td>2.9</td>
</tr>
<tr>
<td>Long Island</td>
<td>230</td>
<td>861</td>
<td>16.8</td>
</tr>
<tr>
<td>Mayaguana</td>
<td>110</td>
<td>581</td>
<td>5.3</td>
</tr>
<tr>
<td>Ragged Island</td>
<td>14</td>
<td>208</td>
<td>14.9</td>
</tr>
<tr>
<td>San Salvador and Rum Cay</td>
<td>90</td>
<td>856</td>
<td>9.5</td>
</tr>
<tr>
<td>Total with other cays added</td>
<td>5,382</td>
<td>168,812</td>
<td>Average density</td>
</tr>
</tbody>
</table>

Table 1.2: The Bahamas - Land Area, Population in 1970, Population per $m^2$, and Average Density per $m^2$. 
Table 1.2 shows a breakdown of the total population by island for 1970. However the census of population and housing during 1980 favours a total of 169,534 (Table 1.3) and further shows that in the decade ending 1980, the population of the islands increased by 39,971 persons, representing a mean annual growth rate of 2.14 per cent. In fact the birth and death rates in 1974 were 7.1 per 1000 (2.1 per cent) and 5.6 per 1000 (0.56 per cent) respectively, giving a natural increase of 15.4 per 1000 (1.54 per cent). This rate of natural increase in a developing territory which lacks a substantially diversified industrial base is a very sobering statistic indeed since Kingsley Davis [2, p.46] says that

The peak of the industrial nations' natural increase rarely rose above 15 per 1000 population per year; ...

Looking closely at the distribution of population among the larger islands (Table 1.3) it becomes apparent that whereas in 1970 New Providence had 60.17 per cent of the population of The Bahamas, in 1980 it had 64.65 per cent of the people in the country. Further examination of the statistics shows that in ten years the population density per m² in New Providence climbed from 1275 to 1693 persons.
People are therefore moving away from the remote islands to New Providence in search of a better life but employment opportunities are not coping with the demand. This implies that a social service such as education must expand to meet other real and growing demands. In fact it is possible to
identify the primary and secondary schools which are experiencing the increased pressure for space. From Table 1.4 [3], Pinedale, South Beach, Bamboo Town, Yellow Elder and Carmichael are the host areas of this population shift.

<table>
<thead>
<tr>
<th>DISTRICT</th>
<th>1970</th>
<th>1980</th>
<th>CHANGE</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Montagu</td>
<td>5630</td>
<td>10213</td>
<td>+4583</td>
<td>+81</td>
</tr>
<tr>
<td>Fox Hill</td>
<td>4496</td>
<td>8384</td>
<td>+3888</td>
<td>+86</td>
</tr>
<tr>
<td>Anns Town</td>
<td>5718</td>
<td>5228</td>
<td>-490</td>
<td>-9</td>
</tr>
<tr>
<td>Salem</td>
<td>4209</td>
<td>4429</td>
<td>-220</td>
<td>5</td>
</tr>
<tr>
<td>Pinedale</td>
<td>3391</td>
<td>7368</td>
<td>+3977</td>
<td>+117</td>
</tr>
<tr>
<td>Shirlea</td>
<td>5128</td>
<td>4144</td>
<td>-984</td>
<td>-19</td>
</tr>
<tr>
<td>Centreville</td>
<td>5564</td>
<td>5593</td>
<td>+29</td>
<td>+1</td>
</tr>
<tr>
<td>Englerston</td>
<td>5510</td>
<td>7639</td>
<td>+2129</td>
<td>+39</td>
</tr>
<tr>
<td>South Beach</td>
<td>4173</td>
<td>9819</td>
<td>+5646</td>
<td>+135</td>
</tr>
<tr>
<td>Fort Fincastle</td>
<td>7278</td>
<td>7016</td>
<td>-262</td>
<td>-4</td>
</tr>
<tr>
<td>Grants Town</td>
<td>6253</td>
<td>5702</td>
<td>-551</td>
<td>-9</td>
</tr>
<tr>
<td>St Michael</td>
<td>5416</td>
<td>5756</td>
<td>+340</td>
<td>+6</td>
</tr>
<tr>
<td>St Barnaba</td>
<td>7225</td>
<td>7950</td>
<td>+425</td>
<td>+6</td>
</tr>
<tr>
<td>Bamboo Town</td>
<td>3263</td>
<td>8925</td>
<td>+5662</td>
<td>+174</td>
</tr>
<tr>
<td>St Agnes</td>
<td>6202</td>
<td>5051</td>
<td>-1151</td>
<td>-19</td>
</tr>
<tr>
<td>Bain Town</td>
<td>6362</td>
<td>5837</td>
<td>-525</td>
<td>-8</td>
</tr>
<tr>
<td>Fort Charlotte</td>
<td>5107</td>
<td>5323</td>
<td>+216</td>
<td>+4</td>
</tr>
<tr>
<td>Yellow Elder</td>
<td>2194</td>
<td>5011</td>
<td>+2817</td>
<td>+128</td>
</tr>
<tr>
<td>Delaporte</td>
<td>5394</td>
<td>6528</td>
<td>+1134</td>
<td>+21</td>
</tr>
<tr>
<td>Carmichael</td>
<td>3192</td>
<td>9521</td>
<td>+6329</td>
<td>+198</td>
</tr>
<tr>
<td>TOTAL</td>
<td>102005</td>
<td>135437</td>
<td>+33432</td>
<td>+33</td>
</tr>
<tr>
<td>NEW PROVIDENCE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.4: New Providence - Population Change per District

One can also identify the islands and sections of New Providence from which people are migrating. The highest net losses of population were experienced from Andros (Kemp's Bay), Acklins Island, Ragged Island and Crooked Island (Table 1.5).
<table>
<thead>
<tr>
<th>DISTRICT</th>
<th>1970</th>
<th>1980</th>
<th>CHANGE</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooper's Town</td>
<td>3257</td>
<td>3942</td>
<td>+ 685</td>
<td>+ 21</td>
</tr>
<tr>
<td>Marsh Harbour</td>
<td>2219</td>
<td>2206</td>
<td>- 13</td>
<td>- 1</td>
</tr>
<tr>
<td>Sandy Point</td>
<td>1031</td>
<td>1176</td>
<td>+ 145</td>
<td>+ 14</td>
</tr>
<tr>
<td>Acklins</td>
<td>936</td>
<td>616</td>
<td>- 320</td>
<td>- 34</td>
</tr>
<tr>
<td>Kemp's Bay</td>
<td>3364</td>
<td>2163</td>
<td>- 1201</td>
<td>- 36</td>
</tr>
<tr>
<td>Fresh Creek</td>
<td>1984</td>
<td>2801</td>
<td>+ 817</td>
<td>+ 41</td>
</tr>
<tr>
<td>Nicholl's Town</td>
<td>3541</td>
<td>3433</td>
<td>- 108</td>
<td>- 3</td>
</tr>
<tr>
<td>Berry Island</td>
<td>443</td>
<td>509</td>
<td>+ 66</td>
<td>+ 14</td>
</tr>
<tr>
<td>Biminis</td>
<td>1533</td>
<td>1432</td>
<td>- 101</td>
<td>- 7</td>
</tr>
<tr>
<td>Cat Island</td>
<td>2658</td>
<td>2143</td>
<td>- 515</td>
<td>- 19</td>
</tr>
<tr>
<td>Crooked Island</td>
<td>689</td>
<td>517</td>
<td>- 172</td>
<td>- 25</td>
</tr>
<tr>
<td>Governor's Harbour</td>
<td>3154</td>
<td>3162</td>
<td>+ 8</td>
<td>.....</td>
</tr>
<tr>
<td>Rock Sound</td>
<td>3118</td>
<td>3800</td>
<td>+ 682</td>
<td>+ 21</td>
</tr>
<tr>
<td>Harbour Island</td>
<td>3229</td>
<td>3638</td>
<td>+ 409</td>
<td>+ 13</td>
</tr>
<tr>
<td>Exuma and Cays</td>
<td>3777</td>
<td>3672</td>
<td>- 105</td>
<td>- 3</td>
</tr>
<tr>
<td>West End</td>
<td>9195</td>
<td>9173</td>
<td>- 22</td>
<td>.....</td>
</tr>
<tr>
<td>Pineridge</td>
<td>5223</td>
<td>7486</td>
<td>- 2263</td>
<td>+ 43</td>
</tr>
<tr>
<td>High Rock</td>
<td>11525</td>
<td>16443</td>
<td>+ 4918</td>
<td>+ 43</td>
</tr>
<tr>
<td>Inagua</td>
<td>1109</td>
<td>939</td>
<td>170</td>
<td>- 15</td>
</tr>
<tr>
<td>Long Cay</td>
<td>26</td>
<td>33</td>
<td>+ 7</td>
<td>+ 27</td>
</tr>
<tr>
<td>Long Island</td>
<td>3869</td>
<td>3358</td>
<td>- 511</td>
<td>- 13</td>
</tr>
<tr>
<td>Mayaguana</td>
<td>584</td>
<td>476</td>
<td>- 108</td>
<td>- 18</td>
</tr>
<tr>
<td>Ragged Island</td>
<td>208</td>
<td>146</td>
<td>- 62</td>
<td>- 30</td>
</tr>
<tr>
<td>San Salvador and</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rum Cay</td>
<td>857</td>
<td>804</td>
<td>- 53</td>
<td>- 6</td>
</tr>
<tr>
<td><strong>TOTAL FAMILY ISLANDS</strong></td>
<td>67529</td>
<td>74068</td>
<td>+ 6539</td>
<td>+ 10</td>
</tr>
<tr>
<td><strong>TOTAL BAHAMAS</strong></td>
<td>169534</td>
<td>209505</td>
<td>39971</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 1.5  
Family Islands: Population Change per District with Totals for The Bahamas
1.3 Social and Economic Conditions

As elsewhere in the West Indies, there exists great disparity in conditions when comparing New Providence and Grand Bahama with the rest of the islands: more explicitly it is possible, although not a common occurrence, to find luxury and squalor on opposite sides of the same street. On many Family Islands the population is engaged in either subsistence farming or fishing, or both. Opportunities to earn substantial amounts of money over a year, say, are very few and far between. In fact if and when they do come, the type of employment is fixed: unskilled labouring. This is not because people prefer to be bone idle but rather because the educational diet given in all-age schools, and indeed through the rest of the country is not formulated to equip people with employable skills beyond the elements of reading, writing and arithmetic. This, according to Broomes [4, p52] seems a common characteristic of education in rural and urban areas of developing countries and even if schools did provide pupils with marketable skills, people would have to migrate to New Providence or Grand Bahama – where Bajpai and Bajah [5], found the only operational technical centre – to find steady paid employment.

Culturally, the country has copied institutions, inherited language and the Christian religion from Britain but Florida and New York have great influences on the day-to-day activities – professional, commercial, social, educational – in the country. Indeed by agreement between the United States Central Bank and its counterpart in The Bahamas, the US dollar has a fixed parity with the Bahamian dollar. A common thread through all activities is the heritage of servitude and slavery, so education has the responsibility to create responsible, thoughtful and creative [4] individuals in a small country which has many peculiar problems.

In Section 1.2 of this Chapter, a population growth rate of 1.54 per cent for the year 1974 was given. Correspondingly per capita Gross National Product was $3080.70, and the year before it was $2703.90, giving a 12.2 per cent increase (see Appendix 1).
Allowing for statistical unreliability, it seems that GNP per capita was in excess of population growth. Subsequently, there has been political stability, but simultaneously inflation in the United States and the United Kingdom was 13.6 and 19.2 per cent respectively, to which The Bahamas paid import costs far in excess of export earnings for goods and services, added to which of course, inflation in New Providence kept increasing steadily until 1978 [6]. In effect, although the amount of GNP per capita looked substantial, the real value of the dollar was eroded by inflation.

Furthermore, per capita sums do not represent a true spread of the wealth especially when consideration is given to leakage to shareholders, managers and executives of off-shore corporations and other categories of foreign professionals. Still less does this per capita income represent a true indicator of the level of indigenous economic activity and development in an open economy. In fact the ratio of import payments to Gross Domestic Product gives a rough approximation of the percentage of income leaking out of the country. (Appendix 1).

Having pointed out some of the dangers of accepting a per capita sum at face value, a return is made to look for a consequence of two facts: population growth and GNP for the decade 1970 to 1980. Now the former was decidedly less than the latter, a salutary and healthy state of affairs since Davis [2, p.36] reports that:

In everyone of the industrialising countries of Europe economic growth outpaced population growth.

Whilst it is true that Davis was discussing an aspect of development constrained by different societal and geographic parameters, the researcher feels that a crucial prerequisite for the mental and physical health of any people is that economic growth must be in excess of population growth.
<table>
<thead>
<tr>
<th></th>
<th>Harrold Road</th>
<th>6</th>
<th>Carmichael Village</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Independence Drive</td>
<td>7</td>
<td>Bay Street</td>
</tr>
<tr>
<td>3</td>
<td>East Street</td>
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<td>John F Kennedy Drive</td>
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<td>Baillou Hill Road</td>
<td>9</td>
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</tr>
<tr>
<td>5</td>
<td>Carmichael Road</td>
<td>10</td>
<td>Luke Killarney</td>
</tr>
</tbody>
</table>

* College of The Bahamas

![Map of the Island of New Providence showing line of hills, south of which is densely populated area.](image)

Figure 1.2: The Island of New Providence Showing Line of Hills, South of which is Densely Populated Area
All industrially developed countries owe their prosperity and high per capita income to a select group of highly trained and experienced professional managers and executives, along with engineers, technicians and scientists who have played a pivotal role in propelling the economic growth in their countries. In Europe and the United States of America, the curriculum of colleges and universities for the education and training of this level of manpower has a strong bias towards mathematical and statistical analyses; consequently, it seems obvious to the researcher that, provided the data are available, there should be a way of quantifying a positive relationship between the learning and teaching of mathematics and economic growth.

As mentioned earlier, the subdivisions of New Providence which are experiencing a net gain in population - Pinedale, South Beach, Bamboo Town, Yellow Elder, Carmichael (Table 1.4) - are located south of the east-west ridge which runs a mile from the northern coastline (Figure 1.2). North of this ridge and strung out on the western coastline are to be found large hotels which operate throughout the year to accommodate tourists who escape the perishing cold weather in the northern United States of America and Canada. About 35 years ago tourists were wealthy visitors: these days many travel on hire purchase.

Communication between New Providence and the Family Islands is maintained by telephone, radio, aircraft and engine-driven boats, which take freight and some passengers at least once per week. A good network of roads exist on most islands - 260 mls (418.3 km) on New Providence, 125 mls (201.1 km) on Eleuthera and 130 mls (209.2 km) on Grand Bahama - consequently transport on land is, in the main, by private motor car. Internationally, connection is maintained with the outside world by jet aircraft, containerised ships, luxury cruise liners, direct distance dialling, operator assisted calls and telex services. Direct colour television transmission commenced in New Providence during mid-1977 and whilst it is possible to watch American programmes, the reception on Grand Bahama is excellent since the island is linked with the American mainland by direct cable.
The economy of The Bahamas depends almost entirely on tourism, which now earns about two-thirds of the country's revenue, employs the same proportion of the labour force and supplies half the foreign currency needs. The main tourist centres are New Providence and Grand Bahama which hosted 57.3 per cent and 42.7 per cent respectively of the visitors in 1980 [7], but together the islands have only 4.6 per cent of the labour force in farming. The researcher feels that precaution must be taken against the perpetuation of a people the majority of whom develop a domestic service mentality. Since these people acquire a very low level of general education and marketable skills, secondary education must be organised so that the tourism sector of the economy may develop at the same rate whilst absorbing technology but attracting a decreasing proportion of the labour force [5, p.24, paragraph 85].

The absence of any taxation, coupled with economic and political stability, have attracted generous amounts of foreign investment, much of which has been used to finance further expansion of the tourism sector together with related public utilities and communications. These conditions helped to raise The Bahamas to the status of one of the world's leading financial centres, and international banking, including Eurodollar and Trust business, is now the second main industry. But as part of a total strategy to broaden the economic base, research in crop and livestock farming is developing in North Andros, and with United Nations assistance a similar exercise is in progress in fishing. With regard to mineral resources, aragonite, limestone and salt are all being commercially exploited.

Foreign investment has been particularly attracted to Grand Bahama, the development of which began in 1955 with an agreement between The Bahamas Government and a private company financed by American, Canadian and British capital; and which, since its inception, has benefited from additional incentives in the form of guaranteed tax exemptions, duty-free import of construction materials along with operating equipment. Of course, and rightly so, since 1970 similar incentives have been made available throughout the Family Islands of the Commonwealth. There are a modern port and
bunkering facilities, an international airport, holiday resort and commercial centre on Grand Bahama which is also The Bahamas' foremost industrial centre with a large cement and pharmaceutical plant, a major oil refinery with a daily output capacity of 500,000 barrels per day and a large crude oil trans-shipment terminal operated by Burmah Oil Company on behalf of the Government. Foreign investment and the enormous invisible receipts from tourism have given The Bahamas a favourable overall balance of payments which, in turn, have put the country in a position from which it can finance much of its development from revenue.

Concerning social and economic development, the researcher believes that financing can be made available for educational projects; but crucially, as in other developing countries, it is more urgent to have a minimal cadre of dedicated qualified mathematics teachers who take pride in seeing the country (ie future citizens) develop and who themselves not only can engender respect for public property [5, p.28 paragraph 109] but also use initiative, creative talents and the ability to persevere without waiting to be supported in every venture by public funds. Given the will to succeed and the right attitude to human problems, there is a certain amount of progress which can be made without any financial outlay. This kind of selfless outlook is a critical element in the formula for success in improving mathematics education in particular.

Three important conclusions derive from this Chapter so far:

(i) infrastructure exists for a modest start to be made in distance learning

(ii) financing could be made available for local research projects in mathematics education

(iii) a minimum cadre of professionally qualified mathematics teachers urgently need to volunteer their services in the interest of the development of the country.

What follows in Section 1.4 is an attempt to peer in perspective at the influence of events in the colonial past and their possible consequences in shaping the future.
1.4 The Evolution of Education in the Bahamas

The Bahamas shares with the rest of the West Indies, as Figueroa [8] believes,

... a common history of growing out of a European push into the New World ....
It was the Trade Winds that first brought Columbus, in the name of Spain, on his voyage of discovery. He met Indians inhabiting the islands when he landed on Guanahani, one of the Bahamian Islands in 1492.

Spanish policy, as Whyte [9] reports, was to create replicas of the towns at home for the Indian colonists (who called themselves Ceboynas) and at the same time to christianize the natives through the Roman Catholic religion. It is therefore, natural to assume that the Roman Catholic Church and its Orders were first off the mark in providing educational services for the pre-literate Arawaks. Although writing about the religious traditions of Europe and clearly outside the context of the colonial adventures, Hans [10] casts the Catholic Church in a pioneering role when he states:

'In the West the Catholic Church was the cultural leader and initiator and founder of all educational institutions.'

But, as in Jamaica where the effects of the colonial experience are obviously seen and intensely felt [9], the influence of the Roman Catholic Church had already weakened when, in the mid-seventeenth century, the British came to the Bahamas, introducing Protestantism in the form of the Anglican Church, the administration of which was used to control the schools.

As would be expected, the coming of the British to the West Indies implied certain common consequences for the region.

The first and the most important, and the most obvious is that they share the English language, and a Creole
based on the English language. This presence of the English language immediately removes the islands of the West Indies from direct contact not only with their fellow island peoples who speak Spanish and French, but also from South and Central America, and from France and Spain; each island has tended to concentrate separately or London and New York.

The English language is also a sign of a close association with the United Kingdom, its social, spiritual and literary traditions, its economic policies, its educational practices, traditions and policies [8].

The author feels very strongly that this assessment by Figueroa [8] gives a most incisive and factual background to the historical development of every aspect of life in The Bahamas precisely because the islands were, until 1973, part and parcel of the once vibrant Empire which was controlled from Westminister and Whitehall. Admittedly, the use of the word 'Empire' does cause many people to become rather upset because from a typically unbalanced emotional point of view, the genuine advantages of the colonial past escape conscious evaluative meditation. Thus from an objective stance, parliamentary democracy based on the Westminster model, the judiciary, the education system, the English language (as already mentioned) and the Anglican Church are fundamental institutional pillars of the Bahamian colonial heritage which are most definitely destined to be perpetuated.

Accordingly, attention is now directed towards the contribution of the Anglican Church and its societies to the historical development of education in the Commonwealth of the Bahamas.

Because the Anglican church had a monopoly in education from the early seventeenth to the early nineteenth century in England, much of the spade work was done by three English Missionary Societies, namely The Society for the Promotion of Christian Knowledge (SPCK), The Society for the Propagation of the Gospel in Foreign Parts (SPG) and The Associates [10].
The SPCK, founded in 1698, co-ordinated several charitable bodies which founded schools in England, the objective of which was moral improvement of the poor through Christian indoctrination. Outlining the aim of the SPCK, Hans [10] writes that its charity schools took the view that the children of the poor needed training in the habit of labour and industry and proper humility towards the ruling class. The SPG, successor to the SPCK, had a policy of propagation of orthodox religion through the activities of the missionaries who, in turn, supervised the work of Headmasters and Catehists among settlers, natives and slaves in the colonies. The fact that Woodes Rodgers, the first Royal Governor, was instrumental in having the first SPG missionary to arrive in The Bahamas in 1733 indicates the level of esteem in which the movement was held. The local House of Assembly put its stamp of approval on the Governor's initiative by passing legislation two years later creating one parish out of the inhabited islands and effectively providing for the services of the clergyman. Sadly, however, the same House after having refused to remunerate a schoolmaster, the SPG voted money for this purpose; a teacher was recruited in Carolina and a school opened in 1739. This action was clear acknowledgement by the state that responsibility for education was the province of the church. Another school for the early period, recorded in a letter from the Governor to the Bishop of London in 1725, was one with a religiously dominated curriculum in Harbour Island. The third society, the Associates, was specifically formed to educate Negroes (who always got a different education from the local whites) and maintain libraries in the West Indies and America. One sees that class consciousness was transferred from the metropolitan country and this social barrier was perpetuated through the system of education. As late as the 1950's the then Inspector of Schools told an assembly of pupils at St Andrews, an independent school which was started by a group of wealthy politicians and businessmen, that they were being educated to manage the affairs of the country; the same inspector told a similar Negro assembly at Boys' Central School that they should endeavour to become good fishermen and farmers, even though at that time, and as is the case now, the school
was not providing an education which was either scientifically or technologically based to ensure that these were enjoyable and rewarding occupations.

The historical record shows that the Associates, an active society in America by 1760, came on the educational scene after the Catholics and Anglicans. Actually, it was not until 1793 that it opened its first school in New Providence and the teachers were ministers of the gospel [11]. As would be expected their priority was to assist the most needy and mainly poor free Negro children for whom no legal provision was made owing to the expiration of the 1772 Act. However, slave owners could afford to pay for education either at private schools or in their own plantation schools, it being mandatory to provide religious instruction. It follows therefore, that the school became an institution giving charity to free Negro children until 1844 when the Society transferred its support and teacher to the school at Carmichael near the southern coast of New Providence.

During the same period whilst The Associates were opening schools, the Non-Conformists - mainly Baptists and Methodists - arrived from America and were founding churches under the leadership of Negro missionaries. Both groups organised schools as well as Sunday Schools attached to churches for children and adults. With regard to Sunday Schools the point is made by Whyte [9] that they came into being in England for children who worked in factories during the Industrial Revolution but had Sundays off. Barnard and Lauwerys [12] concur with Whyte but add that their role is now strictly confined to religious instruction. Whereas Baptist converts were mainly Negro, the Methodist church had a significant number of white members whose Loyalist background enabled them to be utilized as teachers in schools in New Providence, Harbour Island, Eleuthera and Abaco.

Other organisations also took responsibility and initiative in founding schools: communities - whole settlements or social groups; private individuals as a means of livelihood; and commercial enterprise such as agricultural and property developers. Some of these were eventually absorbed into the
state system, late though it was in establishing a prominent presence in education in the country.

For upwards of fifty years, the Established, Catholic and non-Conformist churches shared responsibility unequally for formal elementary education. This parallels early educational development in England where, according to Lester Smith [13] the church volunteered to provide education throughout the middle ages until the great conflict about control was settled by the Elementary Education Act of 1870. When the State did commence its support in the Bahamian society, steps were very tentative, to say the least, since the annual financial allocation was not always voted. The consequence of this spasmodic support was that schooling was offered on an intermittent basis until in 1746 when the first Education Act was passed. This Act established a legal basis for the levy of an annual tax of one shilling and sixpence on every individual in the age range 16 to 60 years for the purpose of building a schoolhouse on New Providence and to pay a teacher's salary of £60 per annum.

On the Out Islands, as they were called then, there was only an SPG school on Harbour Island. A Bill was introduced in 1770 for the purpose of establishing more schools, but owing to intense disagreement over finance, was not passed until 2 years later. This piece of legislation gave statutory support for 2 men's and 2 women's schools along with a free school for New Providence and ones for Rock Sound, Savannah Sound and Harbour Island. The demand for primary education increased with the settling of the Loyalists who petitioned in 1789 for funds to establish schools in Abaco, Long Island, Great and Little Exuma and Cat Island. Unfortunately, without support in the Assembly, the petition died.

The pattern of alternate prosperity and financial retrenchment had the effect of postponing the re-enactment of the Act of 1772 and the closure of most of the existing schools. Fortunately in 1795 the Loyalists won representation in the Assembly during which time a new Act was passed, accepting state responsibility for providing and maintaining public schools organised through a central administration. Expenditure was to be greatly increased
and schools were to provide 160 free places as well as admit fee paying pupils, but again a period of economic difficulties followed and little of the Act was implemented.

Consequent upon the improvement in economic conditions 2 Acts were passed in 1804, the first of which was to establish a high school to avoid the necessity of sending children abroad. The subjects in the curriculum were to be English, Latin, Greek, French, Spanish, Writing, Arithmetic and Book-keeping. However, it was not long before problems of staffing and finance forced a sale of Dunmore House, the school building, to a Board of Trustees for public use. The second of the Acts was firstly, to revive and regulate the parish schools of St Matthew and Christ Church on New Providence; St John on West Eleuthera; St Patrick on East Eleuthera; St Salvadore on Cat Island; St Andrew on Exuma; St Paul on Long Island; and St David on Crooked Island, but in the absence of central authority, schools were to be controlled by visitors consisting of Rectors, Church Wardens and Justices of the Peace. Secondly, the state endorsed the qualifications of teachers by legally insisting that they became registered at the office of the Secretary to the Governor and 30 pupils per school was the maximum enrolment. Thirdly, the first form of grant-in-aid was established in order to help settlements remote from these schools to claim £50 towards the remuneration of teachers.

The latter of the 1804 Acts was re-enacted in 1880 and again 6 years later. Among other things, the second re-enactment stipulated that the maximum enrolment should be increased to 40 pupils, with the proviso that all teachers were to be members of the Churches of England and Scotland. It seems that the state was not only effectively ensuring that the Church remained in control of elementary education, but also that those who benefited from the teaching were not influenced by the doctrine of the non-conformists. A very important innovation in the management of elementary education, which was simultaneous with the legal increase of the teacher-pupil ratio, was the introduction of the Madras System. This sub-managerial category, otherwise called the Monitorial System, came into being to help
the school to meet the needs of more poor (Negro) children of New Providence. In fact, although the statutory maximum was 40 pupils, there were many areas where more children had to be admitted and equally, the reverse was also true. It is instructive to record that the monitorial system came into prominence in England principally in response to the dual needs of money and trained teachers. But on deeper reflection, aided by scanning the literature, the author had to reconcile the prevailing and oftentimes conflicting philosophies of the time. Whereas Hans [10], in quoting an authoritative representative of the Society of Bettering the Conditions of the Poor, says:

In the ornamental branches of the fine arts—in painting, sculpture and music, in literary attainments and in professional science, education must be as various as the condition, situation and talent of man. But in the acquisition of the alphabetic and numerical language, the poor have as good a right to the instruction which illumines and directs their path through life as the greatest and most elevated of their fellow subjects (p. 134)

he also begins on the same page the following quotation from Dr Andrew Bell (1753-1832), the inventor of the Madras System:

It is not proposed that the children of the poor be educated in an expensive manner or even taught to write and cipher... there is a risk of elevating, by an indiscriminate education, the minds of those doomed to the drudgery of daily labour above their conditions, and thereby... render them discontent and unhappy in their lot.

In order to focus clearly the underlying cause and course of the evolution of the elementary education which the present 85 per cent (classified as the poor) have inherited in The Bahamas, another extract from Hans [10] should be compared and contrasted with the previous two:
... the only education which could be firstly and safely given to the poor was a religious education, which renders them patient, humble and moral, and relieves the hardship of their present lot by the prospect of bright eternity (p.135).

Whyte [9], concurring with Hans and identifying the source of this less liberal philosophy which sought to deprive the poor of education, says that the missionaries faced opposition from the planters because they (the planters) felt that instruction would make the slave aware of his human worth, and prevent him from being a good slave.

Another justification of the Madras system, in which this researcher had a six-year apprenticeship, was that using learning largely by rote, a teacher could, with the help of monitors, cope with as many all-age pupils as a single room could accommodate. However, it is interesting to record that there is evidence to show that training of some kind was given to teachers for three New Providence schools, Rock Sound, Eleuthera and Harbour Island. In fact, the Appropriations Act of 1821 authorised the reimbursement of costs to two Out Island teachers and the one from Creeks settlement for attending the course. No mention is made, however, of the duration, content and level of the training or even of the experience and qualifications of those responsible for the tuition.

The next batch of legislation showing the support of the State for elementary education [10], [13] covers the fourteen-year period 1821-35. Of some significance is the fact that the Education Act of 1821 attempted to create a central administration, made up of five members from among the clergymen of Christ Church Cathedral, St Matthew's and the Presbyterian along with a magistrate, and five others appointed by the Governor, to control the schools. The historical record (the source and accuracy of which is sometimes open to question), suggests that this central administration was in fact the embryonic Board of Education, which Burgess [14] says was established in England in 1899. Its members, some of whom
served in the Legislature along with people who possessed (it was said) special skills, were appointed by the Governor. But it was unfortunate that in 1823 all of the Acts of 1816 had to be suspended owing to lack of funding. However, in the same year provision was made to give a grant to the Anglican school founded in 1817 and for 11 years afterwards remained the only one supported by the Government. It was Governor Smyth who thought that he would partly fill the vacuum created by withdrawal of State support by spending Crown Funds to open schools for liberated slaves at headquarters and in the new settlements of Carmichael and Adelaide on New Providence. But as would be expected, these schools closed when the Governor departed. However, one assumes that a source of funding was found by the Government because an Act of 1834 established the "Central School" which was to admit up to 140 poor children. Alternatively, it may have been pressure from the abolitionists which forced the coming into being of one public school (offering free education) in New Providence and four in the Out Islands. For whatever reasons, it was estimated that over 2000 of the 2800 children of primary school age (6-14) were unable to attend school but the roll of the Central School was to increase to 200.

Twenty-one years after the Emancipation Act took effect, Governor Colebrook encouraged an awareness in popular education and harnessed legislators' interest in founding a high school as a focus of their enthusiasm. The School Commission, presumably an outgrowth of a new burst of zeal, produced a plan which advocated a system open to all children without discrimination, with infant schools for children between 2 and 6 years old and schools of industry for training older pupils. It supported the idea of a high school and that training was required to raise the standards of teachers. It is very sad however that the execution of these ideas was unsuccessful with the consequent firm establishment of the Madras System, which lingered on well into the late 1960's.

With regard to higher education, the Commission envisaged a King's College School affiliated with its counterpart at the
University of London. Provision was made in the Appropriations of 1835 to pay the travel expenses of two teachers if they were recruited by the college, and to help defray salary expenditure if school fees were insufficient. One year later a group of shareholders was constituted as a corporation because the school was for the children of the wealthier classes. Although it was given considerable financial help, the corporation was unable to continue operations so in 1845 an Act was passed taking it over and vesting Public Works with the premises on what is now Parliament Street.

During the American Civil War, the Bahamas experienced a good deal of prosperity, one consequence of which was increased votes of expenditure on education. But despite this the standard not only failed to improve, it even declined. By 1854 only one-third of the children attended school and of these 2000 it was estimated that over half had not even begun the first processes in arithmetic. In 10 years' time, the high cost of living together with the opportunities to earn high wages had resulted in many of the pupils and better teachers leaving schools. The scale of the exodus was such that the average enrolment fell to less than 1000 and the number of schools from 27 to 23. The real ramifications of the decline however can only be imagined, in the absence of the statistics, before the downward trend took effect.

Over the period 17 to 28 years before 1864, there were changes in the organisation of the Board of Education along with constant manoeuvring for its control by the various religious denominations. To circumvent further quarrels, a new Act gave control to the Governor along with 5 members of the Legislature and the link between this new structure and the schools, was provided by an ex-officio Inspector of Schools who in fact became substantively Secretary to the Board of Education. The holder of this job along with teachers for the Boys' Central and schools in Harbour Island and Inagua were to be recruited in London. Further, financial support for 5 students to be given teacher-training at the Central School was instituted and a system of classification of teachers was introduced to attract better staff. Also, contained in the same package was the decision to peg school fees
at 2 pence per week, there being no legislative control over private schools. One year later detailed bye-laws were made public, covering in particular the training course for teachers and a Mr George Cole arrived from the United Kingdom destined for Harbour Island, from where he was transferred to Boys' Central School in Nassau.

In the late nineteenth century a prolonged economic depression caused such a serious decline in educational activity that enrolments fell by half and the average of 40 weeks per year to 33 weeks per year by 1878. Re-enactment in 1875 of a 10 year-old piece of legislation brought into being a 'Revised Code' to replace the bye-laws. It was based on an English counterpart of 1861 with the intention of improving the standards of school teachers. Moreover, there was another classification of teachers along with a system of payment by results, though the latter was never applied to the same extent as it was in the United Kingdom. It is reported that a more rigid curriculum than that of 1865 was introduced with the set up of school committees who were to help control the schoolteachers and assist them in their relations with the communities.

A compulsory Education Act was passed in 1877 for New Providence but extended to the other islands 2 years later. As a part of this package, compulsory attendance at school applied to children who had reached the age of 6 but under 14 years in New Providence. Enrolment, as a consequence, rose bringing with it the need for the appointment of a special constable to enforce the Act which was not extended to the other islands.

Grant-in-Aid teachers, who were badly paid, were allowed to augment their income by conducting school over a four-day week. These schools were envisaged only as a temporary expedient but by 1925 over half of them were
supported in this way. Fortunately, when they were finally taken over by the Board under the Act of 1962, only 19 still existed.

The 1886 Education Act, as well as stipulating that membership of the Board of Education should increase from 5 to 12 (8 of whom were to be in the Legislature, including 5 from the House of Assembly) also made provision for the abolition of school fees and extended the application of Compulsory Attendance Clauses to the Out (Family) Islands.

By the middle of the nineteenth century, the independent schools had a deliberate policy of erecting schools where none belonging to the state existed. In fact in 1883, the Anglican Church had 4 schools on New Providence and 27 on the Out Islands which compared with the Board's 24 schools. The Anglicans administered 5 of the 6 Exuma schools and 7 of the 8 on Andros. The Wesleyans had 1 school in Nassau and the Baptists had 2 in the Family Islands. There were also private schools in Harbour Island, Savannah Sound (Eleuthera) and Rum Cay. On New Providence, Anglican schools had a total enrolment of 485 pupils.
At the turn of the century, the 1902 Special Report of the Board of Education indicated:

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<td>Out Islands</td>
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<td>14</td>
<td>New Providence</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>9</td>
<td>Out Islands</td>
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Table 1.6 Distribution of Schools in New Providence and the Out (Family) Islands and Controlling Authorities

Just prior to the turn of the century, the school leaving age was fixed at thirteen but under the 1908 Education Act, it was raised to fourteen years of age. This Act, amongst other things, also had clauses on compulsory attendance as well as made provisions for inspection of independent schools. But its most outstanding feature was its lack of provision for secondary, technical and further education together with any ancillary services. Despite this lack of legislative direction in secondary education, the Government High School, which opened in 1925, was given statutory permanence by the Secondary School Act of 1927. It is important to note that this school was not a part of the Board System because it was administered through the Governor-in-Council. A similar Act which was passed the year before, instituted Grant-in-Aid for independent schools in which in excess of 50 per cent of the pupils were receiving secondary education. Subject to certain unspecified conditions, payment was based on the average attendance, the number and qualifications of the staff together with the examination results. Prior to 1945, the only independent secondary school that qualified to benefit was Queen's College, which is still run by the Methodists.
Eight years before the turn of the century a training institute for teachers was opened with one master who was sent from the British and Foreign Schools Society. Since only 5 of the first 12 students passed, the quality of further applicants was considered poor and after many disagreements, the institute closed in 1894.

Not only did the 1908 Primary Education Act control the work of the Board during the last 54 years of its existence, but it also reduced its strength to 5 members (2 of whom were from the House of Assembly) as well as detailed its commitments and responsibilities. Evidence from the archives shows that:

The Board of Education was a statutory corporation, capable of holding and disposing of property for educational purposes and of suing and being sued in its corporate name and capacity. It was entrusted with the superintendence, direction and control of all matters relating to public elementary education and having the duty, 'so far as the monies from time to time at its disposal admit', of providing free primary instruction and accommodation for all children between the ages of 6 and 14, and had the power to maintain schools or make grant-in-aid. [11]

It would appear that throughout its history, the capabilities of the Board were limited by financial difficulties. Although the successive Education Acts and their amendments stipulated fixed annual grants, the amounts of which rose slowly over the years, they were several times reduced and seldom adequate for the service. However, the legislature had to be relied upon to appropriate further sums. But the restricting disadvantages were that salaries could not be fixed, long term planning was impossible and it was difficult to utilise appropriations for capital development in the short time available.

The period prior to and during the First World War was again, one of financial difficulty and modest advance but it was followed by the sudden prosperity of the Prohibitions' period.
Consequently annual grants increased, teachers' salaries doubled and rapid expansion in the number of new buildings and schools took place. In fact, during the interval 1919-25 the number of schools increased to 44, four of which were operated by the Board. Although teacher training was re-organised at the Central School, the numbers entering the profession were small and the post-war expansion took place without any large increase in the number of trained teachers. It was this decrease in the supply of trained personnel which caused the status of grant-in-aid schools to remain unchanged.

The Board was, during 1926-31, in a position from which it had no other alternative but to recruit teachers for the Out (Family) Islands through the Colonial Office in London. But none of the 7 elected to renew their contracts so the recruitment of agricultural instructors from other Caribbean colonies was more successful.

The Government High School, one of the functions of which was also to train teachers, was housed in buildings vacated by the Boys' Central School (Nassau Court) out of which grew Eastern and Western Central Schools. The coming into being of this school was to provide secondary education for 11-18 year olds, entrance being made competitive by the 11+. The fees were £3. 10s per year but there were only 7 free scholarships, 2 for New Providence and 5 for the Out (Family) Islands. Places were made available for future teachers who were supported by a grant although the Board did not feel that this arrangement was entirely satisfactory since professional training had to be given later. Consequently the plan was scrapped during the period of financial retrenchment between 1930-5. The last intake of students to enter the school on these terms was in 1965.

Guided by the directives of the 1926 British Hadow Report, the Board of Education, presided over by the Attorney General, re-organised the New Providence schools in 1930 with a view towards attaining a continuous, graduated system of elementary education where all age schools were replaced by junior and senior schools. In each of the Eastern and Western Districts,
created in 1928 on the line of Victoria Avenue, the schools were separated as Table 1.7 shows below:

<table>
<thead>
<tr>
<th>TYPE</th>
<th>CLASS/GRADE</th>
<th>AGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preparatory</td>
<td>Class I</td>
<td>6-7</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>7-8</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>8-9</td>
</tr>
<tr>
<td>Junior</td>
<td>Grade I</td>
<td>9-10</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>10-11</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>11-12</td>
</tr>
<tr>
<td>Senior</td>
<td>Grade IV</td>
<td>12-13</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>13-14</td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>14-15</td>
</tr>
<tr>
<td></td>
<td>(VII)</td>
<td>15+-16+</td>
</tr>
</tbody>
</table>

Table 1.7 The 1930 Re-organisation of Schools as Dictated by the British Hadow Report of 1926.

As a means of relieving the over-crowded classroom conditions, and to act as an incentive to the pupils to improve their standards, the regulations were enforced making those who had reached the age of fourteen leave school unless they performed well or could, in the opinion of the Board, benefit from further education.

Quite apart from changes of name and the teaching of higher grades, the structure of the school system was, until the school year 1968/9, the same as that introduced in 1930 [11]. At that point in time primary schools replaced the infant and junior schools; also the formation of classes by teaching levels gave way to the correlated age-grade relationship which is shown in Table 1.8.
<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>GRADE</th>
<th>AGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>1</td>
<td>5-6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6-7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7-8</td>
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<tr>
<td></td>
<td>4</td>
<td>8-9</td>
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<tr>
<td></td>
<td>5</td>
<td>9-10</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>10-11</td>
</tr>
<tr>
<td>Secondary</td>
<td>7</td>
<td>11-12</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>12-13</td>
</tr>
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<td></td>
<td>9</td>
<td>13-14</td>
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<tr>
<td></td>
<td>10</td>
<td>14-15</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>15-16</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>16-17+</td>
</tr>
</tbody>
</table>

Table 1.8 Age-Grade Relationship in Government Schools, 1968/9.

In consequence, school buildings had to be larger because children were staying on longer, the demand for more teachers shot up, and there was automatic promotion from one class to another.

The sum appropriated to education in 1930 was 36 per cent less than the previous year, and immediately salaries had to be cut by 50 per cent, the better teachers were lost to the profession, and the recruitment of suitable students came to a halt.

The large and rapid turnover of teachers had always been a problem: the 50% decrease in salaries exacerbated the situation. A general exodus started since 1915 so that by 1926 only 14 Board teachers had 10 or more years of service. Moreover, apart from the obvious lack on continuity in the schools, coupled with the shortage of experienced teachers, their (the teachers') general qualifications were extremely low. In 1930, out of 16 regular teachers only 3 had Cambridge School Certificates and only 11 had passed the Cambridge Junior Examinations (which have now become the Bahamas Junior Certificate Examinations).
The Chairman, in the Annual Report for that year, severely criticised the existing system of training through pupil-teachers and monitors. Since a training institution stood no chance of success, summer schools and evening classes were put forward as a means of raising the standards of the profession. The annual Teachers' Conferences in the 1930's were largely concerned with improving the efficiency of the teachers. In addition, summer schools were started in 1943 to help Out (Family) Islands' teachers who had been taking correspondence courses conducted by certain teachers in Nassau under the direction of the Inspector of Schools. Even these make-shift arrangements for helping teachers' performances could not be relied upon because for the years 1940-4, the House of Assembly withdrew its grant. Prior to 1943, the only summer school organised previously was in 1906 when agriculture was being introduced into the curriculum and a practical course was given by the curator of the Botanical Gardens. The training of teachers seemed to have had a strong bias towards agriculture because over the period 1933-41, 8 teachers offered one year courses and 32 attended summer courses at Booker T Washington Institute, Tuskegee, Alabama. These included 25 in agriculture, 2 in domestic science and 1 in woodwork.

Early efforts to commence vocational training are recorded in a special report produced 2 years after the turn of the century, according to which tailoring and shoe-making were formally taught in some schools. But the first successful training venture was the Dundas Civic Centre founded in 1930 to provide training for hotel and domestic service workers. This venture was well supported by the state, its grant rising from £200 in 1933 to £750 per annum after the war. Supplementing the grant was a fee of 1 shilling per week. The annual output was between 100 and 250 from training which lasted for 4 weeks, the centre thus becoming an employment agency especially for the hotels.

One year after a report made by the Booker T Washington Institute in the United States of America, an Amendment Act authorised the provision of vocational training for students up to 18 years of age and during the same year, 1939, a building, now Columbus Primary School, was completed but efforts to operate the school
were unsuccessful. This matter was discussed by the House of Assembly in 1943 with the result that a recommendation was made that the Government High School should be expanded to offer commercial, agricultural and technical subjects.

It was as late as 1946 when the first Director of Education took office. At that time the provision of education was still inadequate: more money needed to be invested, an increase in the number and quality of teachers was necessary and legislation was urgently required to extend the responsibility of the Board to more than just elementary education. The extent of the difficulty facing education can be gauged from the following:

(i) Board schools in Nassau and the Out (Family) Islands were overcrowded

(ii) only 7 institutions offered secondary education:

- Government High School 130
- Queen's College 342
- St Francis Xavier 75
- St Augustine's 24
- Nassau Evening School 21
- Johnson's Private School 35
- St Vincent's, Harbour School 28

and (iii) there was no centre for teacher training.

With regard to the provision of teachers, a Select Committee of the House of Assembly recommended that 6 teachers be recruited in Canada but only 2 were obtained. By 1947, more Bahamians were studying abroad but it was very difficult to persuade the better qualified people to take up teaching. In fact, during this time 2 teachers went to Britain and the year before 2 had gone to Canada but even so, no idea is given of the level to which they were educated. Anyway, this Assembly took the decision to recruit 4 staff in the United Kingdom to help with raising the standard in the Out (Family) Islands and 2 for training teachers to work in New Providence.
These 4 new expatriate teachers were asked to supervise the schools in the islands to which they were appointed and in 1951 the post of Supervisory Headteachers placed in the estimates.

In order to meet the anticipated expansion, land had to be acquired. Accordingly, the Board requested that the Oakes Field Military Compound be purchased and drew up plans for its use for teacher and technical training, hostel and teachers' quarters. At the same time a Bill was tabled in the Assembly to extend the jurisdiction of the Board to include responsibility for commercial, vocational, adolescent and adult education, teacher training, hostels for Out (Family) Islands' students, a circulating library and school welfare services. Unfortunately, however, this piece of legislation together with another to place the Government High School under the Board was not tabled for discussion.

Educators who were keenly interested in the professional growth and progress of teachers would have been relieved that a step was made in the right direction when, in 1948, a training mistress was appointed. Following this appointment, a provisional training scheme, in the form of visits to Nassau schools and evening broadcasts, was started. As a prelude to a greater stride, a training master took office in the following year and accordingly in 1950 a Training College opened in Oakes Field with 25 teachers withdrawn from Nassau schools and 5 new student teachers. Annexed to the college in similarly renovated buildings was a training school with 147 pupils where the students spent 2 terms of the two-year course. One hour demonstration lessons at the school were broadcast each morning for the benefit of other schools, with educational talks for teachers and the general public given each afternoon. [15].

What started in 1948 as the fulfilment of great expectations, had begun to be dashed in 1955 when the broadcasts were discontinued. More seriously worrying was that this abandonment of the broadcasts for teachers was a prelude to a breakdown in the organisation and relationships within the college, resulting
in its closure in 1957 by the House of Assembly. The feeling of the legislators was that the money could be better spent in the provision of scholarships for study abroad.

Just 4 years prior to initiating action in the education and training of teachers, a proposal for a new Government High School to include a vocational training centre was adopted but, as with most other plans for education, implementation was postponed. Instead, 2 years later, it was agreed that the vocational school buildings erected in 1939 should be utilised as a technical school for boys and Southern Senior School (now Columbus Primary School) should occupy new accommodation on adjoining land. By 1948, although the proposals put forward for the opening of a technical school were regarded as practical and sensible, only a limited start in implementation could be possible owing to other large financial commitments. Two buildings were set aside for conversion to teachers' quarters, the appointment of a headteacher and a science teacher were approved and money was granted for equipment and materials. The following year, the new school commenced, offering boys from the senior schools 1 day per week for the study and practice of technical drawing and woodwork. Girls remained in their senior schools and studied domestic science. By 1950 science and agriculture were being taught as well.

Evening classes began and continued during the period 1951-5 with 75 students, stabilising to 21 in the last year. Courses offered telecommunication engineering, electrical engineering; and practical and mechanical engineering design along with City and Guilds examinations were taken. Some classes for pharmaceutical apprentices were also held but all of these extra educational activities were suspended when the three expatriate teachers who taught them left the country.

During the four-year period, 1957-61, the technical school functioned as a full-time day school offering two-year courses leading to the Cambridge Overseas School Certificate examinations at 'Ordinary' level. The percentage pass was possibly around 55 per cent but enrolment, indicated by the figures below (Table 1.9), was very small.
Table 1.9  Enrolment at Nassau Technical School, 1957-61.

At the request of the Bahamas Government which was under pressure to provide more school places, a report on the organisation of education in the country was submitted by the Deputy Educational Adviser to the Secretary of State for the Colonies. Among other things, the following changes were recommended:

<table>
<thead>
<tr>
<th>Before report</th>
<th>After report</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Inspector of Schools</td>
<td>2 Education Officers</td>
</tr>
<tr>
<td>4 Supervisory Headteachers</td>
<td>Out (Family) Islands' Inspectorate</td>
</tr>
<tr>
<td>Technical School</td>
<td>Technical Institute, and Bahamas Teachers' College</td>
</tr>
</tbody>
</table>

Other recommendations with far reaching implications were:

(i) Establishment of an evening institute at the Government High School for adult education

(ii) Links were to be forged with the University of the West Indies via office of Tutor for Extra Mural Studies

(iii) Sixth form work was to commence at the Government High School

(iv) School broadcasts were to be reintroduced

(v) Legislation, based on the United Kingdom 1944 Act 15 passed in 1962, was to become the legal basis of the present system of education (until the College of The Bahamas Act, 1974 shifted responsibility for all post-secondary education).
The Education Act of 1962 contains provisions regarding the Central Administration of Education under a Board of Education, the statutory system of education, independent schools, and certain general provisions. Under the Education Amendment Act, 1963, which came into effect with the New Constitution for The Bahamas on 7 February 1964, the Minister and a Department of Education replaced the Board of Education as the Central Education Authority. Four years later, the Ministry added 'Culture' to its title and activities. A corresponding change had to be made to the designation of the Minister who under the Act has responsibility for the control, direction and superintendence of all primary, secondary and further education. The Minister also determines educational policy and objectives and makes policy and programme decisions in conformity with the overall national policies and priorities fixed by the Cabinet. Subject to his approval, the officers of the Ministry frame, co-ordinate and carry out the programmes and measures required to implement his policy. In the interest of accuracy Statute Law [16] outlines the following responsibilities:

There shall be established a Department of Education which shall comprise a Permanent Secretary, a Director of Education and such other officers as may from time to time be authorised by the Legislature.

The Permanent Secretary shall be responsible for the organisation and administration of the Department and the Director shall be the professional advisor and technical executive officer of the Minister. (pp. 477, 478)

The statutory education system shall be organised in three progressive stages to be known as primary education, secondary education and further education; and it shall be the duty of the Minister, so far as his resources permit, to contribute towards the spiritual, moral, mental and physical development of the community by ensuring that efficient education throughout these stages shall be available to meet the needs of the population. (p. 479)
At the moment the country is suffering from the output of inadequately trained primary and lower secondary teachers in method and content. Section 41-1 of the Act (1962) is crystal clear about the duty of the Minister on this point:

In execution of the duties imposed on him by this Act, the Minister shall, in particular, make such arrangements as he considers expedient, and as his resources permit for securing that there shall be sufficient facilities for the training of teachers for service in schools and other establishments maintained by the Minister [16,p.496]

This however was not intended to be a criticism of the way in which the Minister deploys the manpower made available to him. In fact he can do no more than support the continued employment of those who are already in the teaching service, except the need arises for their transfer to other sectors of the public service but with the binding proviso that status remains comparable. As the researcher views the dilemma in education, more teachers need to volunteer to make sacrifices in order to become qualified, competent and efficient specialists in the disciplines of the curriculum so that the country may have a good chance of achieving the high expectations it has for balanced human development. But the kernel of the problem is that more articulation and enthusiasm must be shown in the professional leadership of education in the country. In the market place, for instance, a taste for consumer goods and services is created partly by advertising the products. In a similar way a demand must be created for education and training of local personnel. Education is an instrument of power in the sense that it gives individuals human dignity as well as true freedom and responsibility to chart their destiny. The country has independence but the only way to mount a strong challenge to the relic-colonial social and economic structures to change, and to usher in a new more dynamic and relevant system, with the flexibility for responding to continuous increments of change, is through education. It is this researcher's fervent belief that any chance of achieving these goals must be made
a possibility at the fundamental level of the classroom through curriculum development. It will not be easy and cannot be achieved overnight: but in the final analysis, it is in the classroom where a people prepare and make ready for change.
1.5 The Present Structure of Education

Previously it was shown how public authority in The Bahamas came around to accepting full responsibility for general education. In Europe there were centuries of tradition of schools and universities [17, pp 75-112]; in the United States of America there was also a long tradition of education organised largely by the religious bodies in semi-public fashion. So too, in The Bahamas the state did not commence with a blank sheet: there was much educational activity in being by the Churches. In fact, it was also pointed out that there were schools conducted by private persons - as there still are - for profit. The pattern that The Bahamas now has, therefore, is the resultant of these traditional conservative forces and any newer ones consequent upon state action because it is an accepted principle of human life that in solving existing problems new ones are created. For whatever reason the historical record shows no obvious attempts at assimilation between public and private educational bodies but a co-existence, even if sometimes unhappy, continues. As mentioned previously, there was a time when private educational institutions had a greater physical presence in the country than the state. However, by 1977 there were 228 schools in The Bahamas, 81.6 per cent of which were fully maintained by the government and 18.4 per cent were independent schools. Included under the umbrella of the state are special schools, which are all age in character, catering to the needs of pupils with varying degrees of learning disabilities.

The 1970's were a period of real reform and expansion [5, p 12, para 32] in the sense that more schools were built and therefore more pupils stayed on at school beyond the compulsory leaving age [5, p 11, para 28], and of course the important prerequisite of effecting change and social response was that an expansion in state educational policies took place especially where it was needed most - in secondary
education [18, pp 44-46]. School fees for attendance at Government High School, an important brake on social mobility for the under-privileged, were abolished [18, p 32]; subsidies were made to independent schools so that a decrease in fees and other benefits could be passed on to parents, and in partial return, these institutions relaxed their employment policies so that more local teachers, if qualified, may be recruited.

The College of The Bahamas, incorporating the sixth form studies offered at the Government High School, to the horror of those who cherished deep-seated loyalties, came into being not only as the institution at the apex of the structure but also to offer a second chance to those who may have missed their way. Commensurate with this expansion was the challenge to teachers to improve their competence in the skills of the curriculum so that correspondingly, parallel to an expansion in mental activity for enthusiastic and creative minds - and teachers do find it a challenge to be creative - which stem from increased opportunities, would be society's equal admiration and respect for the teacher in the private as well as the public sector, with the salutary result that both types of schools would be able to deliver the same academic and social product.

Instead of the one secondary school from which the majority was debarred by short-sighted policies prior to and including the mid-1960's, more secondary schools were created if only in name rather than quality of output. The researcher feels that these schools now need to diversify their curriculum offerings so that youngsters with a technical or vocational bent may develop alongside those with more academic inclinations in general science and arts. Although Bajpai and Bajah [5, p 14, para 36] found that

The attitude of the primary school children to Science can therefore be described as positive and favourable
the coming into bloom of a Bahamian scientist may be a vain hope for sometime yet because of "teacher attitudes", the effect of "chalk and talk" and

Lack of provision of equipment and laboratory facilities ..... [5, p 32, para 123]

However for general science it seems more of a possibility to mount teams of teachers who are not either specialists or nationals to train pupils if only in the elementary concepts and skills, and by so doing give heed to the advice of Harbison [2, pp 127-8]:

.... in its educational investment a country must adopt a balanced programme suited to its own needs and stage of development, or it may run into trouble. It will have a number of choices to make, and for its educational programme it will have to find the best compromise among: (1) quality and quantity, (2) science and the humanities, (3) vocational training in school and on the job, (4) regulation of salary incentives by the state and by the market and (5) the needs of the individual and the needs of the state. (This author's italics)

Although writing for technologically and industrially advanced societies, but in perfect harmony with Harbison and elaborating the needs of the individual vis-à-vis the needs of the state, Svennislon et al [19, p 19] suggest

Educational policy, within general national policy, has two main objects: to meet the demands of individuals for their own development, and to meet the needs of society for its general development. In a democracy that respects individual freedoms the first object is served by making education available to all citizens, irrespective of class and income, according to their individual gifts and desires. The second object, for which
governments have assumed responsibility, is served by seeing to it that industry, as well as cultural and public institutions, are provided with persons having the requisite general education and skills.

The basic structure of the system, fashioned initially by an overseas metropolitan power, is now predicated by 6 years of primary school followed by 3 + 3 years of secondary education. A social and, by implication economic, bottle-neck was created where there were too few places at one secondary school to cater for the aspirations of the majority who could neither qualify for scholarships nor pay their own way. The traditional élitist notion that only a few - and those from selected social strata - could benefit from secondary education had to be knocked on the head in preference to the comprehensive system. Decisions of the kind which limit the availability of the second stage of education to the majority are usually for administrative expediency rather than based on unassailable educational philosophy. At secondary level children want and need the opportunity to find and develop their potential so that they may qualify for selection to jobs in industry and thereby earn economic independence. A limited curriculum or teaching capacity or both will not help the achievement of this ambition but rather frustrate both parents - who have high expectations of their children - and the pupils themselves [5, p 33, para 126].

The researcher now attempts an identification of the connections between the structure of education in The Bahamas and its general social structure. The structure of education reflects the social structure although not precisely so for, importantly, this unquantified margin of difference gives room for manoeuvre. The historical evidence suggests that The Bahamas has copied an irrelevant pattern of secondary education which was created to perpetuate an élite on the one hand - a minority of the population partly of intelligence but largely of income and status - and on the other hand to qualify a few of the local majority population as clerks and
sub-managers in the public service. Few, like the present leaders, who saw what opportunities could become available to them if they had further education, made sacrifices with parental support, to succeed in the corridors of academic activity, mainly in the United Kingdom. On their return to the country in the mid-1950's, they expressed their civic interests in the general welfare and future of their homeland from a common platform. It was this coming together of minds in the fashion of the early Greek philosophers which charted a new direction in education for the country. The researcher therefore concurs with Elvin [20, p 174] when he says

Now the admitted need is for the extension of secondary education; both to more young people and to more studies than in the past.

In this way status or wealth is not taken from the already established members of society: instead, those of humble beginnings become a meritocracy [21, p 43] by earning, through disciplined hard work, greater social mobility [22, p 24], even though this opening of the doors is feared by existing high-status members of any society. The second level of education in The Bahamas must open the possibility to its citizens to develop their talents to the extent that the highest offices in the country will open to them. In his paper on Education for Development in four underdeveloped countries - Nyasaland, Colombia, China and Egypt - Harbison [2, p 118] touches a very sensitive nerve when he very appropriately advocates

The progress of a nation depends first and foremost on the progress of its people. Unless it develops their spirit and human potentialities it cannot develop much else - materially, economically, potentially, or culturally. The basic problem of most of the underdeveloped countries is not a poverty of natural resources but an underdevelopment of their human resources. Hence their first task
international English speaking world, 'O' level success is highly recognised but teaching mainly for examination success at any level does restrict the kind of classroom activity that would generate the understanding of concepts, pattern and structure in subjects like mathematics and science. While discussing some of the effects of examinations, it is expected that alternative papers, to cover a considerable range of ability, which 16 year olds will offer in 1986 in England and Wales would be some form of compromise between 'O' levels and the Certificate of Secondary Education (CSE) [4, p 62]. In the light of this development, the Government of The Bahamas has already informed Parliament [23, p 17] that it had under consideration the introduction of a National Senior Secondary School Certificate (NSC) to approximate the quantity of curriculum content assessed by the London GCE 'O' levels in individual subjects. Farther afield in the region, the Caribbean Examinations Council (CEC) [24, p 356] has come into existence to perform this same duty. As regards the debate about whether The Bahamas should join the CEC, the researcher feels that the country may have representation on that body but should not abandon plans to introduce its NSC. For the sake of parity of standards, the University of the West Indies (UWI) may be approached to mediate any discussions between the CEC and The Bahamas' Ministry of Education with a view to accepting its NSC as the equivalent of any examinations at a comparable level which the CEC may institute.

Another by-product of a rigid educational structure is that many people become resigned to, what the researcher calls, the fatalist philosophy that missing the boat at 11+ or at 'O' level implies a ruined future. If individual will can be generated, education must offer an opportunity for a way to be found out of the proverbial wilderness. Sadly, even in some European countries educational structures are especially defective in this regard.
FIGURE 1.3 ORGANISATION OF EDUCATION IN THE BAHAMAS
Very often appendaged to an independent school is a nursery school for children between the ages of 3 and 5. Especially and only on the island of New Providence, a few of these schools with licence from the Ministry of Education, operate for profit under private individual ownership. These schools develop, in the main, children's verbal, physical and number readiness skills together with positive attitudes and feed the maintained and independent primary or all age schools which combine primary and junior high grades. But most Bahamian children start formal education at age 5 in a primary school, or as is the case in many Family Islands, in the primary section of an all age school (Figure 1.3) and continue until age 11. Promotion from primary to junior high, or junior/senior high since 1974 has become automatic with the official abolition of the 11+ examination. It was a useless cultural borrowing because the country never had a tripartite system of secondary education [25, p 38] as in England and Wales, where this evaluative instrument was born out of administrative convenience instead of a thoroughly researched psychological principle.

At age 11 children transfer to a junior high school or move up to the junior high section of an all age school until age 14, the statutory upper limit of compulsory education [16, p 486, para 26-1]. At the end of junior high, the majority of pupils in independent and state schools offer Bahamas Junior Certificate (BJC) Examinations in conformity with the policy of the Ministry of Education. This is a subject-based examination and in practice, some pupils offer subjects as early as age 12. However, no child is refused entry into the next level of education because of failure to produce evidence of having passed BJC subjects. This examination is a carry-over from the mid-1950's prior to which time pupils offered the University of Cambridge Junior Certificate Examinations. Appendix B of [5] shows the results of 21 subject entries made by the maintained and independent
schools during the summer, 1977. Concerning the relationship between the structure of education and the economy, it is important to indicate that this qualification has relatively little currency on the employment market [5, p 30, para 118], thus showing that however well pupils have performed in securing BJC subjects, they command neither knowledge nor skills which can find immediate utility in the economy. Of course it could be stated in mitigation that at age 14, they are too young to be looking for employment but counter to this is the fact that much teaching time is spent in senior high schools preparing 15- and 16-year olds for BJC. It was as Assistant Chief Examiner for BJC mathematics in the summer of 1968 that the researcher first encountered and got a grasp of the desire for this research — an attempt to resolve the difficulties experienced in the learning and teaching of mathematics at secondary level in The Bahamas, but either never expected or had the vision that the opportunity would indeed come.

Prior to the advent of recent changes in the structure of secondary education in The Bahamas, as seen from Figure 1.3, the all age school — as the researcher knows very well indeed — did not stimulate a desire in pupils or prepare them for an extension of their education. In a well-balanced discussion of the system in Jamaica, Jervier [26, p 35] writes

> Historically, all age schools have been terminal and did not lead to any other type of school within the educational organisation.

Now, at the age of 14 pupils transfer to senior high or the equivalent section of a junior/senior high school. However, as obtains in some Family Islands where the number of pupils in individual communities cannot justify the existence of buildings for the second phase of secondary education, children of post-primary age are transported to central high schools where they remain until age 17+. This last category of The Bahamian system represents
... a major structural development within the maintained system, viz: the rapid development of central high schools on the Family Islands [18, p 33].

Senior high school climaxes in independent schools with pupils offering London GCE 'O' levels, structured to cater for the top 25 per cent of the secondary population in the UK [4, p 62], but very few from the maintained sector accomplish this feat. Having examined the matrix of subjects offered in BJC examinations and results [5, Appendix B], the researcher can assert that not enough is done in senior high schools to build on success accomplished in

(i) cookery, needlework and handicraft so that training in catering, dressmaking, carpentry, masonry and tailoring for example, equip young people with skills for the employment market

(ii) technical drawing, art and mathematics in anticipation of apprenticeships in architecture, draughtsmanship and boat-building

(iii) general science and biology so that some may become interested in fish and agricultural farming

(iv) general science, mathematics and technical drawing to generate interest in motor mechanics and branches of engineering

All of the potential employment categories in the catalogue above require at least basic proficiency in calculation. Those who show sufficient motivation and aptitude could be encouraged to pursue further studies. It seems to the researcher that today's secondary pupils need higher levels of general and specialist education and training as well as a longer time in preparation for employment because methods of production as well as social needs have changed. Also, a
part of the dilemma is that children, the parents of whom have only had primary education, fall victim to the temptation of easy money and abandon their studies to go after dead-end employment. But, as Bajpai and Bajah [5, p 24, para 85] rightly advise

Whilst this must be the right course of action for some, it would be quite wrong for a large body of the pupil population.

It is patently obvious, therefore, that the present structure of education in The Bahamas lacks the links with the economy it purports to serve because after six years of secondary education young people are still unemployable. The crucial question most surely is: can educators distinguish between what should be learned in school and what should be learned on the job? Unhesitatingly, the researcher points an accusing finger at the content of the curriculum of the maintained secondary education sector as the document which must embody far-reaching structural change and correspondingly, this transformation must be supplemented by improvement in the quality of primary education and teacher attitudes.

Since September 1974, the system makes provision for pupils (now students) to enter the College of The Bahamas at 17+. This institution combines the curricula previously offered by the teachers' training colleges, the technical college and the sixth form of the Government High School. In theory entry to the college is via 'O' levels but those without this qualification are permitted to offer college preparation courses, in mathematics and English in particular, over one year so that entry may be made to college level courses. But this consumes teaching time which can be otherwise used. As a solution Bajpai and Bajah [5, p 39, para 23] make the following suggestion:

Decrease the numbers of College Prep students and allow the Senior High Schools to take the bulk of the responsibility for the teaching of students
up to GCE 'O' level. This will:

(i) increase the academic quality of students entering the College

(ii) decrease the number of years spent in obtaining an Associate Degree, Diploma or Certificate

(iii) decrease the number of lecture hours used in upgrading students, thus enabling more time to be concentrated in lectures at the Associate Degree level

(iv) ease the problem of staff shortage - fewer College Prep courses - fewer lecturers needed in any one Division - especially Humanities and Natural Science Divisions

(v) give existing staff more time to develop more marketable and relevant courses, eg lab technicians, Certificate in Mathematics for existing teachers, Certificate in Agriculture for existing teachers, etc

(vi) reduce the severe space problem at the Oakes Field Campus

This suggestion if taken seriously, as it should, implies that Bahamian teachers must upgrade their mathematics content of knowledge and become more aware of, possibly through workshops [5, p 36, para 7], the current strategies in the learning and teaching of mathematics in order to meet the challenge of teaching successfully for 'O' level success or its equivalent. Likewise, the other consequence of this suggestion is that running in parallel should occur exercises in curriculum development [5, p 37, para 13].

Two years after entry to lower sixth courses, students qualify for an Associate Degree and some offer 'A' level examinations as well so as to improve their chances of entry
to universities and colleges overseas, although there are universities in the United States of America, Canada and the West Indies which admit students on the basis of work done for the Associate Degree. Those who are accepted by educational institutions overseas may be awarded scholarships provided by business and industry, government, the Association of Commonwealth Universities, in-service awards as well as they may be privately financed. For administrative convenience, the College was originally divided into seven teaching divisions [5, p 13] but the Applied Science and Technical and Vocational Studies Divisions have been combined to become the Division of Technology. In fact, Parliament was informed since 1975 [18] that the ultimate aim was for faculties to grow out of the existing institution. Maraj et al [27] have suggested the following:

(i) Faculty of General Studies: English, modern languages, social studies, and fine arts

(ii) Faculty of Commerce and Administrative and Management Studies: secretarial sciences, accounting, banking, business administration, and public administration

(iii) Faculty of Education: with minimum of overlap in subject expertise in this and other faculties

(iv) Faculty of Technology: applied sciences – mathematics, physical sciences, life sciences, building and civil engineering, mechanical and electrical engineering, marine sciences, agriculture

This suggested development of the College into faculties will challenge the institution to raise the level of its teaching and find a haven under the umbrella of an established institution of higher learning in preparation for awarding its own degrees.
The next task is to investigate the existence of relationships between the structure of education and the general administrative structure within the country. It is incumbent upon the administrative machinery to support the functioning of the schools. But it is quite wrong to place more importance on the administration than the teaching staff for on their shoulders rests the responsibility for the day-to-day operation of time tables, including the many classroom chores, together with the interpretation and teaching of the curriculum. It is admitted that the full quality and quantity of content is not delivered to some classrooms because educational decisions have to be taken in New Providence [5, p 19, para 54] but the researcher is inclined to the view that responsibility will have to remain centralised for balanced development and growth in a country that is, to a great extent, "distance and water" [5, p 7, para 17], until such time as more citizens can be prepared and made ready for local decision making in educational administration. The powerful argument of substance against this of course is that any school run from a distance is detached from the life of the local community since it has no say in establishing and running the school and makes no contribution towards its expenses. The real difficulty, however, is that when standards of academic achievement in the whole country need to be upgraded, the full weight of reason rests heavily on the side of central initiative and responsibility for ensuring that standards on the individual islands are comparable. It can be seen, therefore, that the only link between the structure of education and general administrative structure within The Bahamas is bureaucratic centralisation emanating from the seat of power in New Providence, which geographically is the only means of obtaining balanced growth and development.
1.6 Rationale for Improvement of Teacher Education and Training

A study of the evolution of education, and teacher education in particular, in The Bahamas revealed that there was no legal basis for supporting the structure and function of education: it was a charity and not a fundamental right. Whereas legal provisions do not necessarily guarantee that appropriate action would have been taken, the consequence of this deficiency was that the House of Assembly was not morally committed to give any form of priority to the budgeting of funds for education. Since the early 1960s however, the law gives the Minister responsibility for primary, secondary and further education, with a companion duty to secure facilities for the training of teachers for service in the schools. It was therefore incumbent upon the researcher to acquire competence in The Teaching of Further Mathematics as shown in Section 4.2.3 so as to be able and ready to tackle the problems in mathematics education at secondary level through a professional approach.

The earliest attempts to train teachers in The Bahamas were through a structure which had the Madras (Monitorial) System at its base. Supplementary methods were broadcasting, then subsequently in the later 1940s, through a training college. But these methods, excluding the 'Monitorial System', were not continuous through time. For example, in the mid 1950s the training college closed, and started again in the early 1960s as The Bahamas Teachers' College. These piecemeal methods of training teachers made a contribution but legally, the Board of Education was responsible for elementary education only. In effect, standards were kept at a depressingly low level whilst a post-war increase in school population took place, with no commensurate increase in the number of trained teachers. Paradoxically, one of the functions of the Government High School was to train teachers but it was never equipped to carry out this function. As late as the 1970s, the San Salvador Teachers' College and the Bahamas Teachers' College,
both with a very high proportion of expatriate staff, were training teachers but the majority of these were for primary schools.

When secondary education started in the country it was the Church which took the lead. The State made no attempt to introduce secondary education in the Family Islands before the early 1970s, and at the fundamental level of the syllabus, arithmetic was emphasised since the 17th century with no incentive for the better pupils and teachers to remain at school. The system of education therefore, had all of the characteristics for the perpetuation of a static society in the sense that social, political and economic advance were limited by lack of adequate provision of secondary education. Moreover, elementary education such as it was, perpetuated learning by rote which deprived the individual of any real value to, and human dignity in, society since training for hotel and domestic service workers was more successful than the training of teachers.

Against this background, the College of The Bahamas was established not only to continue the training of primary teachers but also to raise academic standards and train teachers for secondary schools also. Simultaneously, it is incumbent upon the College to initiate action in conducting workshops and other curriculum development exercises for the benefit of in-service teachers. Development of the four faculties, mentioned in the previous section, would ensure that the institution maintains high standards but it is essential to create the right perspective between the administration of the institution and the academic staff. That is to say, roles must be sharply defined so that the administration does not dictate to the academic staff.

In Chapter 2, the author discusses the various ways of tracing the problem to the level of the curriculum, and formulates a hypothesis the validity of which will be tested.
CHAPTER 2

THE MAGNITUDE OF THE PROBLEM
AND METHODS OF IDENTIFICATION.
The Magnitude of the Problem and Methods of Identification

2.1 The Influence of Historical Events on the Output of the Present Structure

In Chapter 1 an attempt was made to telescope the principal events in education from the landing of Columbus in the late 15th century to the late 1970's. Over this period all of the shortcomings of the European models of education were copied without critically examining their relevance either to local conditions or need. The ultimate aims of education have always been deposited in the hands of educators after they had been formulated by the state. Subsidiary to the definition of national needs individual goals, which are essential to the pursuit of national needs, are then determined. Discussing the role of the state in Caribbean education Jervier [26, p.10] suggests

When consideration is given to the forces which determine the character of an educational system, the nature of the state cannot be ignored. When the state is totalitarian, education reflects these qualities in aim and method of control. The state enters all institutional life. In democratic societies the educational patterns reflect many ideals of democracy. When a society is a colony, its culture demonstrates many features, ideas, ideals, and methods of operation which arise from its relationship with an imperial power. The education offered in colonial societies may be described as colonial education. (Author's italics)

Education in The Bahamas has always been associated with low standards of efficiency because primarily, since the days of the ancient Greeks, anyone who qualified for nothing else could teach and of course payment by results reinforced rote learning of curriculum content. Schools were classified on the basis of one day's inspection of the quality and quantity of the work done by pupils of six or seven classes.
One of these subjects was arithmetic, which remained an important subject of the curriculum for a long time. As long ago as 1952 there were two papers in an examination on arithmetic for elementary schools, the second of which was to test the pupils' ability to arrange his arithmetical knowledge. The following excerpt from an inspector's report [15, p.46] conveys the standard in that branch of the subject:

Arithmetic: The statement and setting out of a problem in logical order remains the main cause of weakness in this subject. Insufficient practice in "model example" technique, is evident, and requires immediate remedy. Correction of work is still inadequate and hap-hazard.

There was dependence on the mother country for curriculum and textbooks to the extent that pupils in Hong Kong, India, Ghana and the Caribbean wrote the same examinations without regard for local conditions. Papers originated and were corrected in English universities. The same is still true of GCE for overseas candidates and the fact that three papers are set as Arithmetic and Trigonometry, Algebra, and Geometry forces corresponding fragmentation in the teaching of the one subject mathematics. Coupled with this of course is the Modern Mathematics Syllabus C for which a particular independent school trains its pupils but the maintained senior high schools do not offer this syllabus.

Educational policy, when there was any, had been inextricably bound to the legacy of colonialism and however adversely the output of the education system was affected, there were advantages in the form of language and the Christian religion as previously mentioned. But a very strong psychological factor was that in large classes with no textbooks, inappropriate curricula, and teachers, the limits of whose knowledge was not far removed from that of the children's,
the products of the system were destined to be subjects in a colony instead of productive responsible citizens contributing to the development of the community of which they were a part. The awareness of the importance of education to The Bahamian majority is a recent phenomenon. After 6 years of primary education, the output of the first phase of the system emerged with the enthusiasm possessed on entry all but dissipated. This may not have been the case for the products of the independent schools; nor was it the case for every government primary school. However, because independent schools are usually very well equipped, classes are smaller and teachers more qualified, the result was that a more humane environment was presumed to exist in a setting where there was likely to be a practical approach to class teaching combined with less chalk and talk.

Historically the child went to the government schools to learn passively, with no opportunity to discuss and question, and the teacher, not far removed from the child in knowledge and teaching skills, to teach with unchallenged authority. This kind of environment, where children were sponges soaking up learning and 'teaching was telling', encouraged anything but alert minds. This was another aspect of an historical legacy where primary education was enough for the labouring classes to be able to read simple instructions and carry them out. Although it is a fact that today's primary child leaves school better equipped than counterparts in the 1950's because more teachers have had some training, yet since the standard of general education for admission to teacher training is usually two years below Ordinary level standard in England and Wales, teachers would not be as confident and relaxed with large classes and would tend to teach as they were taught. These problems multiply in an all-age school such as Red Bays, Andros which was visited by Bajpai [5,p.3] where in a combined class of 23 pupils, say, there would be three grade levels containing quite a heterogeneous mix of ability for a subject like mathematics.
In the distant past, primary school was not a preparation for secondary education in The Bahamas. In fact prior to the 1960's secondary education for the Family Islands was never even considered by the state so those who could benefit, and could pay, had to find board and lodgings in New Providence. Even there places were limited by the selection process and the number of scholarships to the old Government High School was far too few to meet the diversified needs of a growing country. In the early 1970's the pendulum swung to the other extreme: non-selection made way for automatic promotion throughout the system [5, pp. 18-20, paragraph 57], until 1981 when a policy statement (to be seen in Chapter 6, Section 6.1.2) announced that promotion from one class to another should be based on attainment and not on age. This was similar to what Wilson [24, p.356] says he found in the Eastern Caribbean:

The JSS's* were intended to provide a three-year employment-oriented education for those not selected for an academic secondary school, with no entrance qualification other than age, and no terminal examination. *It was the establishment of these JSS's that provided the opportunity for the major attempt to modernise the teaching of mathematics ...

(* Junior Secondary Schools)

(Italics are this author's.) The differences between the system in The Bahamas and that which prevailed in the Eastern Caribbean were a terminal examination at age 14, the certificate for which had no value for employment purposes, and the first phase of secondary education was not employment-oriented. But the basic philosophy - making some secondary education available to more young people and simultaneously offering the opportunity for local teachers to become specialists - was the same. Later in this Chapter further reference will be made to the modernisation of the teaching of mathematics because the effect of The Bahamian historical experience was to limit the teaching to chanting tables, axioms, proving theorems, the generalised arithmetical approach to real number algebra and when trigonometry did get a look in, as will be discussed in detail later, it was via rote-learning formulae with no practical graphical
demonstration of their fundamental structural links with algebra. It is this failure to demonstrate the fundamental cohesive relationships within the subject which perpetuates inertia, and even retrogression, in the teaching of mathematics.

The 11th stage in Bahamian education was one legacy of the past. The next is the Bahamas Junior Certificate Examination (BJC) at age 14 which replaced the Cambridge Overseas Junior Certificate in the 1950's. It was a qualification for entry to teaching as a pupil teacher or the public service as a junior clerk but the examiners had to be satisfied in certain groups of subjects, including English language and mathematics at the same sitting, in order to obtain a certificate. BJC has now become a subject examination for assessing the first nine years of formal education. The statutory age beyond which education was no longer compulsory was 14 years but many more pupils were staying for senior high school partly to complete BJC subjects and partly because it was felt that their chances of finding employment would improve. But among these there were those who accepted evening employment in hotels to the detriment of homework assignments. Theoretically the senior high school should be preparing pupils for London University 'O' level (mathematics: Syllabus A360) examinations and in fact the private schools do (mathematics: Syllabuses A360 and C362) but very few from the maintained sector succeed at these papers.

Deriving from Figure 1.3, are horizontal and vertical movements of pupils but students migrate overseas to complete further education. The implication here is that prior to the late 1960's the policy makers did not permit the system to produce a completed product: value had to be added elsewhere at great cost. So the determinant of the quality of life in the country is gauged by the standard of academic skills which are transmitted by secondary schools to the pupils who become representatives of the new majority.

It will be shown in Chapters 5 and 6 that there was a staggering degree of underachievement in the learning of mathematics. Khouj [28, p.8], in his paper on the relationship between
students' test scores in mathematics and science and corresponding scores in other subjects, concludes that making mathematics programmes more efficient and initiating good study habits can not only generate the cognitive responses essential to the successful learning of other subjects but also, attainment in mathematics and science is a pretty good indicator of attainment in other subjects. A better educated people, the author believes, can demonstrate a stronger sense of loyalty and give more to the building of their country.

The evidence available, such as it was, suggested that The Bahamian attitude to education was largely influenced by the historic procrastination of the state in assuming a positive policy for mass secondary education and failure to present the case for education as the avenue of human development. The consequences of this failure were lack of internal demand for education and dependence on external assistance. Educational policy was decided overseas by the Colonial Office Advisory Committee but the local administration was never obliged to implement it on the grounds that local conditions did not warrant implementation. Moreover, attention given to education in colonial territories was always spasmodic and piecemeal. Statements of policy in colonial development hinted at awareness of local independent management, yet the colonial education department never decided whether it was educating pupils for first-class or second-class citizenship in their own country. The author feels however, that well-defined policies, though necessary, are not sufficient to foster educational development. The real problem in The Bahamas was that administrators adopted English methods and aims without either supportive personnel or resources.

Traditionally, there is a current of opinion which refuses to conceive of social organisation outside the confines of slash-and-burn agriculture; limiting religious beliefs; line, sponge-hook and fish-pot methods of fishing; or the rearing of seven children per family, but paradoxically the majority want to enjoy the benefits of technology and economic development without the realisation that traditional beliefs and habits are geared to
a static instead of a dynamic society. Bain [29, p.18] grapples with the fundamental historical problem where he says:

As life offered no serious challenge similar to the frontier problems of America, the need for hard work, ingenuity and skill on land was not pressing. The important occupations were seamanship and shipbuilding, neither of which could be learned better than by experience and practice. No schooling was really necessary so there was no need for an educational policy to meet local demand arising naturally out of enterprise and development. Nor was there a strong religious motive for education as in the New England Colonies. Schools were a moral and social embellishment; not an economic or social necessity.

Bain's view was true only in so far as the philosophy expressed was a part of the strategy of the local elite to perpetuate itself by clutching onto colonialism as its very life blood and by so doing delayed national self-determination. The need for hard work is certainly not pressing: it is crushing since the mortgage and utility bills must be paid and imported goods and services are expensive. Schooling up to secondary level is now an urgent necessity - in fact, it is the germ from which human development springs. The majority, because of Western influence, want development: consequently, the ultimate solution is to educate people. But the curriculum needs to be transformed to meet the needs of reality in The Bahamas.

This traditional attitude to social organisation inherited from the past as well as the low academic and professional preparation of teachers put a brake on accelerated modification of the cognitive processes of pupils who leave the schools to take their places in an adult community. Why is it necessary to modify the outlook and attitude of The Bahamian society with regard to curriculum development in education? The author believes that in the world of the 1980's and beyond the only constant parameter in society will be change. Mead [30, p.253] answers the question in this way:
... particularly because of contact with Western culture, the function of education has necessarily changed. The need now is to move away to new knowledge and skills, to a new place in a new social order; education is now not for the maintenance of the old, but for change. Whether these cultures have sought Western contact or not, whether they want to change or not, the fact is that they have felt the effects of Western contact and must now be taught how to cope with these effects.

(Italics are the researcher's.) The Bahamas' contact with the West came partly because of proximity to the United States of America and as mentioned before, partly through being a colony of Britain. Now that the status of the country has changed, an additional function of education is to raise the level of skills and knowledge of the people so that they may not only take their place in the community of nations but also enjoy the benefits of science and technology by improving their standard of living. Indeed, for a developing country such as The Bahamas to experience the better life, a start will have to be made at the crucial level of curriculum development in general and in science and mathematics education in particular. These were, and in a very real sense still are, the problems in social transformation faced by samples of developing countries represented at Southampton Institute of Education during 1965/6 from the Caribbean in the west to Sierra Leone on the other side of the Atlantic, through Kenya in East Africa to Malaysia in the East Indies. But there is yet another dimension to the problem. Though put by a representative of Zambia, it is nevertheless experienced by all of these emerging third world countries. Mwanza [31, p.3] expresses it this way:

The type of education which has been and is still being provided to some extent, is that which prepared children for 'white-collar' jobs like clerical, and office work in general. This has brought resentment for outdoor work like farming, carpentry, bricklaying etc. These jobs are considered inferior and only fit
for the 'uneducated'. Unfortunately, the white-collar jobs have long since been filled and it is a very pathetic sight to see many educated young men loitering in the streets of cities and towns, unemployed.

Policy makers must therefore adopt a balanced approach to curriculum development: technical, vocational, scientific, humanities, managerial, sub-professional, and teacher-training all need attention. In a very real sense, therefore, the mathematics curriculum of the secondary section, as well as preparing the top quartile of the population for GCE 'O' level and equivalent examinations, needs to be modified to reflect biases towards these specific employment opportunities for the majority of the pupils who should have day release courses to complete the first phase of their training in the College of The Bahamas. What affects the quality of the output of the secondary system most however, is failure to train more teachers beyond the 'general practitioner' (primary) level. It is now the clear duty of the College of The Bahamas to abandon the practice of having tutors supervise students in the teaching of subjects about which they (the tutors) know nothing. No effort must be spared to bring into being the type of teacher who has sufficient self-confidence and ability to perform, and who can contribute to all aspects of mathematics education, especially curriculum development. For as Wain [32, p.144] says - and this is especially true of The Bahamas:

> When they (teachers) have little freedom, are poorly trained and ill-qualified, and are responsible mainly for conducting classes through well-defined procedures from set textbooks it is difficult to see how they can play the role of the professional.

Concurring with Wain, Griffiths and Howson [33, p.62] assert:

> The potential of an education system is directly related to the ability of its teachers. Hence, the more qualified and better trained teachers are, the
easier it is to effect curriculum development. No matter how distinguished the members of a project team are, how carefully structured a new course is, how brilliantly the various educational media have been exploited, the success or failure of any innovation ultimately hinges on the receptiveness and flexibility of the classroom teacher.

In retrospect, the developed countries have evolved philosophies, as a result of their societies' demands on education, by which teachers respond to the challenges of on-going curricula renewal; whether in a centralised system - as in Scotland where there is one mathematics curriculum leading to the Certificate of Sixth Form Studies - or in any school in England with complete freedom in curricula matters. In America the behaviourist school initiated by Dewey's pragmatic child-centredness [13, p.54], got impetus to move forward in coming terms with that society's demands and filtered as far afield as Western Europe, although questions of hierarchical organisation of objective goals were left to be sorted out by Bloom's taxonomy [34, pp.75-77], Wood's taxonomy [35, pp.166-171], a derivative of Bloom's, and Gagné's types of learning [36, pp.73-179]. The 'New-Math' approach emphasised renewal of curriculum content and its advocates suggested pre- and in-service education which were totally content oriented. Dieudonné's address [37, pp.31-45] to the Organisation for Economic Co-operation and Development seminar at Royaumont in 1959 was a classic example of the utility of the 'New-Math' philosophy in curriculum development. Contrasting with the behaviourist model is the structuralist approach (sometimes called the spiral or helical approach) the main proponents of which were Bruner [38, p.68-70] and Dienes [39, pp.112-115]. The principal characteristics of this school of thought were that it sought to solve the problems of low achievers by an examination of content and method through the strategy of discovery learning. Unlike the helical (spiral) approach, the integrated approach dispenses with hierarchic order in preference for the real context of mathematical ideas as the subject of the teaching and learning
processes. Finally, the *formative* conception takes into account developmental (Piagetian) psychology which is especially relevant to primary mathematics education. A return will be made to this discussion in Chapter 3 with a view to revealing how these philosophies find utility in present day curriculum development exercises in developing and developed countries. However, in the light of the foregoing view of five important approaches to the building of curricula, the author believes that the majority of Bahamian mathematics teachers are at what Beeby [40, pp.60-62] hypothesised as Stage II and this teacher-product of the system is in dire need of a higher standard of general and specialist education in addition to in-service training, proposals for which will be made in Chapter 7.
2.2 Social Dissatisfaction with Public Examination Results

Various organs exist in The Bahamas for ventilating public views on summative achievements of formal education at ages 14 and 16, which are very important signal posts in children's secondary education. Bahamas Junior Certificate (BJC) and London GCE 'O' level successes over the years 1972-78 indicate that pupils have anything but a firm grasp of the mathematical knowledge at the depth required. At the administrative level, the Ministry of Education supplied the schools with individual subject reports of general performance. In 1968 when the researcher was Assistant Chief Examiner, the report on BJC mathematics to the schools was based on the marking of 829 scripts. 170 candidates, 20.5 per cent of the total marked scripts, passed the examination and passes were allowed if any candidate achieved 39.0 per cent in either Algebra and Arithmetic or Geometry and Arithmetic. Of those who passed, 15 candidates, 1.8 per cent of the total entry, scored more than 60 per cent. The failure rate was a colossal 79.5 per cent. At that time it was thought that, among other inadequacies, very little homework was set in the non-selective schools (which were only senior schools as opposed to senior and junior high schools) and the examiners suggested that where the teacher-pupil ratio was 1:40, pupils could be trained to assist in marking their own work. On reflection, the author thinks that it would have been more helpful if the schools were given specimens of the behavioural errors made in the solutions of problems under examination conditions and models of expected behaviours with marking schemes attached. The most constructive piece of criticism arising out of the whole examiners' report on mathematics was that 2 candidates out of 800 managed to draw parallel but unequal chords of a circle. To state in a report that meaningless calculations which bore no relation to the questions asked or the right answers appeared after the wrong working is too vague, without specific supporting evidence, to be of use to anyone.
At the same level, the mathematics report for 1973 indicated that examination scripts reflected inadequate coverage of the syllabuses. In construction exercises and scale drawing, geometrical instruments (rulers, compasses, protractors, dividers and set squares) were not skilfully, and accurately used. In addition, the four operations in algebra were not carefully taught and practised; more time was needed for teaching children how to construct and use graphs; and answers to questions were to be logically set out. Whereas these were sensible comments, they do not target the misunderstandings derived from conceptual weaknesses or the inability to transform the written word into mathematical symbols or inefficiency at calculation. Models should be given to teachers at which both they, and their pupils may aim, because in the final analysis, it is the content of the trains of thought and deciding whether links between the components are logical or random which are decisive characteristics of sound mathematics education. Excellent examples are to be found in Section 6.4.1 of Chapter 6 where the author goes beyond the formative evaluation stage [33, p.46] to establish the research function of the total test.

The following year's report indicated that the objective items investigated the security of the behaviours recall and skill at computing. Section B had problems to elicit the degree of understanding achieved as well as the potential to apply mathematical thinking and logical reasoning to new problems. Section C, it is said, required the pupils to employ the highest form of behaviour - inventiveness - which should have brought together relevant aspects of the pupils' total mathematical experience.

At the Pilani Mathematics Workshop on curriculum development, the British team of consultants in mathematics education emphasised to the Indian secondary school teachers that examination papers should be set in such a way that the mathematically gifted would answer questions from the third section but it happens very infrequently indeed, given the constraint of time, to have questions set to test inventiveness. The researcher did not see the examination paper and can only question to what extent this happened at BJC level. The view

* mentioned in more detail at the end of this section.
expressed in the Bahamas examination report was therefore at variance with the views of Wood and Skurnik [35,p.171] who suggest:

This is the highest level of behaviour under review. If we have to try to define it, we might say that it is the assembling of elements and parts so as to form a pattern or structure not clearly visible before. Where it differs from Application and Comprehension is in its unique nature; the individual is encouraged to strive beyond previous endeavours and make discoveries which to him, although not necessarily to anyone else, are original and unique. Whereas in an Application problem the pupil can carry and tug at the material until it assumes a familiar shape, in a problem designed to tap creativity the material is much more intractable and the pupil is forced to improvise a problem-solving strategy by co-ordinating all his relevant skills and knowledge. The kind of situation which might induce Inventiveness is this:

Say how you would measure the diameter of the moon. Give actual numbers where possible.

In practice, Inventiveness was rarely cited by the teachers who used the classification because it was generally felt that it was beyond the capacity of the majority of their pupils. It was regarded throughout as an experimental category and needs a lot more attention before it can be confidently used.

The other fact of interest from the report was that there were 3038 entries but only 1866 passed, a pass rate of 61.4 per cent. However the pass mark was not given so no comparison or contrast can be made with the previous year's results. For the separate paper, Arithmetic, the pass rate was 54.1 per cent, 1716 candidates having sat. Modern Mathematics however had a pass rate of 88.9 per cent, 117 pupils having entered for the paper. As mentioned before,
pupils from the state maintained junior high schools did not offer this paper and it is a fortuitous coincidence that the sample offering the author's diagnostic test (in Chapter 5) from the junior high section of Queen's College, an independent institution, was 117 strong.

Modern mathematics gained in popularity in 1975 but still only attracting 394 entries, 308 of whom passed giving a pass rate of 78.2 per cent. The syllabus and copies of the year's examination papers are mentioned later in Chapter 5 and given as Appendix 5.2. But the society's apparent index of need, in contradistinction to its real index of need, in mathematics education can be gauged from the number of entries - 3805 candidates - to the traditional papers. Of this number, 1951 passed thus giving a pass rate of 51.3 per cent. The author fervently believes that society's real need would be adequately reflected in, and met by, a combination of modern and traditional mathematics since automation is making its presence felt in The Bahamian society. In this sense, the methods and techniques in education in general always seem to be lagging behind those employed to produce output by the factors of production. From the increased deployment of technology comes the clearest pointer towards in-service education of teachers with a view to the development of a curriculum containing a judicious blend of modern and traditional topics in mathematics, the teaching of which should always be done with the pupils' desire for further studies in mind.

Without specifying instances and frequency of occurrence of misuse of concepts, algorithms and operations, teachers are told, for example, that the process of solving simple equations needs attention. Küchemann [41, p.105-116] gives different meanings children assign to the letters and divide understanding into distinct levels but Wain and Woodrow [42, pp.78-81] by examining pupils' scripts, raise fundamental constructive criticisms, the most important of which is the analysis of techniques used by pupils, to interpret the depth of understanding achieved. For example, they (Wain and Woodrow, p.80) advise:
Cross-multiply is a dangerous isolated technique. It is surely frowned upon by nearly all mathematical educators since it applies only to a limited range of situations. Too many pupils, unfortunately still hearing it used perhaps in science classes, apply it blindly to many inappropriate situations.

The better technique is to use a device to unify the structure of the mathematics.

Multiplying through both sides of the equation by the lowest common denominator not only copes with:

$$\frac{6}{x} = 5 \quad \text{and} \quad \frac{6}{x} + 3 = 5$$

but also opens the way for taking inverses and to solving, for example, matrix equations:

$$Ax = b \Rightarrow x = A^{-1}b$$

Teachers were admonished to keep the performance level in arithmetic high while they build up the other two branches, clearly postponing an introduction to trigonometry and covertly giving official encouragement to a trichotomy in mathematics teaching. The same pressure comes from GCE 'O' levels where Syllabus A papers are Arithmetic and Trigonometry, Algebra, and Geometry, thus showing how examinations influence teaching styles. It was most gratifying to read, however, that junior high schools have been moving in the direction of adopting hybrid syllabuses incorporating modern and traditional approaches to mathematics, and the Ministry of Education and Culture was asked to encourage this trend by offering courses to help teachers to become proficient in this teaching mode. Statistics, probability and transformation geometry were to be introduced in the 1980 syllabuses and Chapter 5 of this research thesis gives data analysis of items of statistics and transformation geometry among others. The analysis also shows that 50 items with 5 options per item represented a more comprehensive sampling of knowledge of the curriculum and a better instrument in terms
of reliability and validity as opposed to 40 items with 4 options per item on papers set by The Bahamas' Ministry of Education and Culture.

The 1979 BJC examination report showed that the majority of the candidates were 14-year olds but a substantial number were 16-and 17-year olds. 3058 pupils offered the Traditional Mathematics papers but only 22.0 per cent passed; 5214 sat the Arithmetic paper but 38.0 per cent passed; and Modern Mathematics was offered by 228 pupils of whom 64.0 per cent were successful. On the strength of the performance in the examinations, the Testing and Evaluation Unit [43] suggests

... in our basic subject areas, the trend is downward. We are very concerned that results in Arithmetic, Traditional Mathematics, ... continue to remain at a very low level of achievement.

From the University of London GCE 'O' level reports for the years 1974 and 1977, it was recorded that 150 and 273 candidates offered Syllabus A papers in the summer examinations, 13 and 31 candidates passed, producing a pass rate of 8.7 per cent and 11.4 per cent respectively. In 1979, 311 entries were made for the same syllabus and 51 passed the examinations, a pass rate of 16.4 per cent.

In developed countries, England for instance, the press occasionally provided parents with help so that they, in turn, can help their children. During the year 1981, the magazine of The Sunday Times ran a ten-part serial prepared by a Deputy Headmaster giving worked examples and discussions of number bases, methods of subtraction, multiplication as extended addition and division as extended subtraction, decimals, fractions, ratios and percentages, sets, functions, and transformation geometry. Also, complaints were made through the press, particularly from industry, about the shortage of mathematics teachers, children's negative attitude towards mathematics - as Bajpai and Bajah [5, p.23, paragraph 76] found in The Bahamas - and the mismatch between the needs of industry and the syllabuses used, as reflected in public examinations - CSE and 'O' level - which children offer. The
researcher asserts that every time a sector of society criticises the level of school attainment, it is also challenging teacher trainers to examine the content and aims of their training programmes. In introducing the many-faceted nature of the problem a Royal Society/Council of Engineering Institutions Joint Education Committee working party [44, p.486] says:

Many people in industry and in further education have expressed great concern that there has been a decline over the past 5 years or so, of mathematical (mainly arithmetical) performance shown by the 16-year old school leavers who apply for the craft and technician apprenticeships. There have also been a number of articles in the national press which draw attention to this very serious position. We appreciate that the anxiety shown indicates a problem of considerable significance in the national industrial situation.

The problem was similarly identified in The Bahamas by Bajpai and Bajah [5, p.25, paragraph 94] at industries which they visited. In one instance prospective employees had BJC's but demonstrated a very low level of attainment in basic arithmetic.

Another factor contributing to the problem was the dependence on expatriate staff and social relationships which existed between them, local teachers and pupils [5, p.29, paragraph 14]. In a discussion of similar aspects of this problem, Ale [45, p.485] writes:

... many mathematics teachers are expatriates, therefore there is the problem of such teachers not really understanding the society they work in and also society not understanding them...

They also lack the ability to cite local examples, which students are familiar with, to illustrate
what they teach. Thus they often make mathematics more abstract than it need be for students. Most expatriate teachers, including some indigenous ones, believe that the Nigerian of African culture is alien to mathematics and science.

(Italics are the researcher's.) The consequence of this lack of confidence by teachers in children's ability to do well at mathematics was that children developed a negative attitude towards the subject as is manifested in their unjustified belief that mathematics and science are difficult subjects. In a country where dissimilar teaching conditions were prevalent, standards of general education were low for entry into teacher training institutions which themselves had staff of low academic and professional standards, poor progress of the junior and senior high school pupil cannot necessarily be an index of little ability. Supporting the researcher's claim, Krutetskii [46, p.60] admonishes that mathematics ability cannot be inborn and provides an explanatory key construct of psychological theory which helps to explain low achievement in mathematics in 15-year old pupils in developing societies:

Abilities are always the result of development. They are formed and developed in life, during activity, instruction and training.

Also corroborating this viewpoint was evidence submitted to the Plowden Committee in England [47, p.23]

... some psychologists are tending more and more to adopt a dynamic approach to the development of abilities. These hold that, strictly speaking, no child can be said to be born with a given ability; he is born with what may be described as the anatomical-physiological prerequisites for the development of this or that ability (for instance, mathematical or musical). These abilities can only be developed in practice - in the process of the child's education. Abilities are, therefore,
beginning to be seen as the resultant of a complex process of formation, often involving a series of stages, each of which is essential to the formation of the final ability (or mental operation).

Gerdes [48, p.466], in a paper outlining change in mathematics education in Mozambique, touches a key to the solution of the problems in developing countries where he observes:

There is no instructor specialised in Mathematics Education in the whole country.

This fact correctly implies that mathematics educators would look with disfavour on current practices in teaching strategies and learning methods which together explain low performance levels in mathematics in third world countries. But even in advanced countries teaching strategies can go badly amiss. Her Majesty's Inspectors of Schools [49, p.136], having had a long period of exhaustive inquiry, in describing a popular teaching approach in secondary schools in England have also encapsulated what was the norm in Bahamian government schools:

A common pattern, particularly with lower ability pupils, was to show a few examples on the board at the start of the lesson and then set similar exercises for the pupils to work on their own. There were few questions encouraging wider speculation or independent initiative. At its best, and given pupils who were sufficiently capable, this style of teaching achieved what it set out to do. At the worst it became 'direct telling how' by the teacher, followed by incomprehension on the part of the pupils. What was lacking in this approach, even at its best, was a sense of genuine enquiry, or any stimulus to curiosity or appeal to the imagination. There was little feeling that one can puzzle out an approach to fresh problems without having to be given detailed instructions.
The Cockcroft Committee [50, p.71], in a discussion of classroom practice, also gives some very poignant suggestions with regard to creating an atmosphere for optimum learning by children of average intelligence. Reiterating the clarion call that Bahamian pupils are equal to the challenge Bajpai and Bajah [5, p.40, paragraph 32] rest their case in the words:

Today's children are tomorrow's parents and teachers, and every effort should be made for the myth of science and mathematics being beyond the normal Bahamian child of average intelligence to be exploded. Better and more effective primary education is therefore something to strive for.

It was easy to find psychological support expressed by Krutetskii [46, pp.3-4] for the optimistic view of Bajpai and Bajah:

Soviet psychologists are unanimous in the opinion that all children are capable of being taught; that every normal, mentally healthy pupil is capable of obtaining a secondary school education, capable of mastering the school material within the limits of the curriculum; and the teacher should see to it that all pupils do so. As A. A. Budarnyi, The Moscow methodologist, observed, "It was not possible to find a single unit in the various school subjects that would prove inaccessible to the pupils [at a low level of development in ability - V.K.]. We did not encounter a single pupil whose level of ability was so low that he could not succeed in ordinary school [meaning normal children undergoing instruction in the mass schools]"

(63, pp.6-7) Authoritative testimony from the well-known mathematician and academician A. N. Kolmogorov reads: "The necessity of special aptitude for the study and understanding of mathematics is often exaggerated ... Ordinary,
average human abilities are quite sufficient for mastering — with good guidance or good books ... — the mathematics that is taught in secondary school" (180, pp 8-9)*.

(* Krutetskii's references)

In response to public debate, The Bahamas Government hosted a curriculum development workshop, at which the author was a resource person, from 28 October to 1 November 1974. It was conducted by a team from UNESCO Regional Project in conjunction with UWI's School of Education for teachers in junior high schools but the interest and enthusiasm generated were allowed to dissipate although Grades 10 and 11 syllabuses in Appendix 5.2 are evidence that some follow-up work did indeed take place. But since the UNESCO-UWI Workshop there was continued social dissatisfaction with the level of achievement of high (secondary) school pupils in mathematics. The problem, initially put to the Commonwealth Science Council, led to the decision that a series of case studies of the teaching of mathematics and science in different parts of the Commonwealth should commence with a pilot study in The Bahamas. Ultimately, and fortunately for the researcher, Professor A C Bajpai, assisted by Dr S T Bajah, led a Pilot Study of the problem during May 1978 and an unpublished confidential report [5] was prepared for the Commonwealth Secretariat. It was in recognition of work done as a local counterpart on the Pilot Study Team that the author earned a place in the Centre for Advancement of Mathematical Education in Technology (CAMET)* at Loughborough University.

On arrival at Loughborough the author found that Professor Bajpai was Director of a new project. Annually—23 postgraduate secondary teachers of mathematics come from various states in India for a year's in-service training in the All India Mathematics Education at CAMET (AIMEC) Project. At the end of their year these AIMEC Project Members are counted amongst other colleagues of previous years as linchpins: in a curriculum development strategy designed to generate multiplier effects

* A brochure giving the aims, philosophy and academic content of the in-service training in AIMEC was written by Professor Bajpai
in India through school- or college-based workshops and teachers' centres. As a result of his three years of research in CAMET, the author was fortunate enough to work on a British Team, led by Professor A C Bajpai, that visited India to conduct a series of Workshops organised by CAMET under the auspices of The British Council and state governments of India. Four one-week workshops were conducted in Pilani, Darjeeling, Hyderabad and Ahmedabad in the summer 1982. This practical experience has given this researcher a new vantage point for approaching curriculum development in The Bahamas. From Chapter 3, the reader would be able to trace the stages of development of a philosophy with respect to

**Effective Teacher Training for the Improvement of Mathematics Education in The Bahamas**

that the researcher has undergone as a student in CAMET. Social and economic problems are problems of education and mathematics education in particular, has a unique role in any heuristic strategy for solution.
2.3 The Relationship Between Expenditure on Education and General Government Expenditure

In an earlier section of this Chapter, it was stated that secondary education with a vocational bias should be continued at the College of The Bahamas. But a careful examination of the sectors of the economy is necessary in order to select the most likely one with the maximum idle potential which could be transformed into employment opportunities, e.g., land. Simultaneously, it is essential to recognise that in certain other sectors, opportunities for employment would be increasing at a slower rate than that at which secondary pupils are entering the market, thus causing educational inflation - i.e., too few jobs being chased by too many educated people. This was a very real problem on a small developing island such as New Providence, for instance, where the factors of production — land and capital — have severely restricted potential for further development whilst labour not only increased each year: it was also very expensive.

The main sectors of the economy offering employment were the hospitality industry, banking and other commercial activities on the one hand and traditional farming on the other. Quite apart from the incidence of recession, there comes a time when physical expansion on a small island must decrease until the limit is reached. So jobs in the hospitality industry, for example, would be increasing at a slower rate than the demand made for them. The same would be true of banking. It stands to reason, then, that the sector offering the most hope is agriculture (farming and fishing) and mathematics education has a secure and important contribution to make to this aspect of diversifying the economy. There will always be strong local demand for food in addition to some demand for export, especially to the southern United States of America where tastes for sub-tropical dishes are similar. The prospect of export would make import substitution of agricultural products a reality, thereby improving foreign currency earnings and partially stemming the flow of factor payments abroad for goods and services. But the prevailing traditional attitude suggests
that those who lacked motivation towards academic work were forced into 'slash-and-burn' farming which generated a very meagre income owing to inadequate knowledge of mechanisation, marketing and modern farm management techniques, use of fertilisers, application of plant and animal genetics and the use of co-operatives for raising capital and securing overseas markets. The College of The Bahamas offered courses in soil science, livestock science, crop protection, agricultural economics, agricultural botany, marine biology as well as chemistry and plant physiology. Also, mathematics education does have a role to play in educating young people for change in rural agricultural methods. Bacchus [51, p.3], in a paper delivered at the 4th International Conference on Higher Education at Lancaster University, puts the problem succinctly thus:

The banking, hospitality and other commercial activities comprise the modern high wage sector while small scale farming mainly comprise the low wage sector. This factor is reflected in the marked differences in income levels between New Providence and Grand Bahama on one hand and the remaining Family Islands on the other. For example, a 1973 Survey of Household Expenditure (5)* conducted by the Government, indicated that the average household expenditure of Bahamians in New Providence and Grand Bahama were 72 per cent and 142 per cent respectively higher than that for Bahamians in the rest of the Commonwealth. Secondly, in 1975, the mean household income of males in New Providence and Grand Bahama was 102 per cent and 143 per cent higher than that of males in the rest of The Bahamas (6).* Further, looking at the distribution of incomes by occupational groups, one finds that in 1975, they varied from $3259 for households whose heads were engaged in agriculture and fishing, to $15 225 for those in financing, real estate, insurance and business service - a difference of about 367 per cent (7).*

(* are references in Bacchus' paper)
At this juncture, it is apparent that increased expenditure on education may be necessary but certainly not sufficient for the output of secondary schools to diversify and reach the quality which society needs. The researcher's contention is that this is exactly what was happening in The Bahamas.

<table>
<thead>
<tr>
<th>YEAR</th>
<th>BUDGET*</th>
<th>EDUCATION*</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td>52.5</td>
<td>6.1</td>
<td>11.6</td>
</tr>
<tr>
<td>1967</td>
<td>77.1</td>
<td>10.5</td>
<td>13.6</td>
</tr>
<tr>
<td>1968</td>
<td>88.7</td>
<td>13.7</td>
<td>15.4</td>
</tr>
<tr>
<td>1969</td>
<td>98.6</td>
<td>15.2</td>
<td>15.4</td>
</tr>
<tr>
<td>1970</td>
<td>117.5</td>
<td>22.5</td>
<td>19.1</td>
</tr>
<tr>
<td>1971</td>
<td>118.3</td>
<td>25.7</td>
<td>21.7</td>
</tr>
<tr>
<td>1972</td>
<td>119.4</td>
<td>24.5</td>
<td>20.5</td>
</tr>
<tr>
<td>1973</td>
<td>117.1</td>
<td>29.3</td>
<td>25.0</td>
</tr>
<tr>
<td>1974</td>
<td>140.7</td>
<td>34.8</td>
<td>24.7</td>
</tr>
<tr>
<td>1975</td>
<td>157.7</td>
<td>30.7</td>
<td>19.5</td>
</tr>
</tbody>
</table>

* in B $10^6 = US $10^6 [52, p.144]

Table 2.1: Percentage of National Budget Spent on Education, 1966-75

Over the years 1966-75, as shown in Table 2.1 expenditure on education has been dramatic but mathematics education, for instance, was neither self-perpetuating nor making a balanced contribution to advancing the knowledge required in a modern developing economy as Broomes, Rees and Vockenberg suggest [4, pp.44-59; pp.68-79; pp.79-83]. It is likely that the period over which Table 2.1 spans was not long enough to train mathematics educators, and depending on where on the educational ladder time was reckoned to begin, that is fair comment. Further, at the time the researcher administered the diagnostic instruments for this research, 15 years had gone, time being reckoned from 1966. If time were rescheduled to begin at the start of primary school, at least an additional year was required to complete degree qualifications in mathematics on top of which due allowance had to be made for both teacher training and experience. Alternatively, assuming a good primary
education - and that is a very generous assumption even for some developed countries - then 15 years are long enough to produce qualified and experienced mathematics educators.

A further dimension to the discussion was that as expenditure on education increased, population was also increasing (see Table 2.2) so that total enrolments increased in addition to the effect of an extension in the duration of education. Also, equipment and operating costs were also on the rise; since increased enrolments meant inflated costs per pupil to which further allowance must be made for rises in teachers' salaries. In effect therefore, although an increased vote of educational funds should imply improvement in output with regard to mathematics education, many allowances had to be made for miscellaneous rising costs whilst the quality of education delivered in the classroom was constant. No attempt was made to introduce international comparisons because a prerequisite to sensible analysis at that level was that data on currency values had to become standardised in order to be meaningful. Moreover, the comparisons would have been further complicated by the fact that in some countries funding for education also derives from sources other than public finances.

Built into the system of financing education after funds had been allocated - and this anomaly cannot be seen from perusal of Table 2.1 - was a braking effect on the release of the money for the purpose for which it was budgeted. Having encountered this phenomenon, which was by no means peculiar to The Bahamas, Bajpai and Bajah [5, p.30, paragraph 119] warn:

"... the system whereby money is allocated, but not actually spent, in a given financial year is lost, exacerbates the general overall shortage of everything. Once a need has been recognised and money for it allocated, this should be left available for use as soon as possible, even if it is actually spent in the ensuing financial year. It appears that red tape and bureaucracy are fostered at the expense of educational efficiency."
Although Table 2.1 shows that expenditure on education has been increasing, the net effect was to swell the unemployment ranks, the tide of which can only ebb if greater employment prospects are created in another sector of the economy, possibly agriculture, because the annual rate of new employment opportunities decreased in the modern sector as those who sought employment increased. Any prospect of balancing the forces, let alone reversing the trend, would depend partly on mounting effective careers guidance in the secondary schools and partly on the Ministry of Agriculture and Fisheries demonstrating the rewards that can accrue to those who risk an education with an agricultural bias. Again, the philosophy of redirecting those who become newly employable to a new frontier is linked with change at a fundamental level - change in curriculum and attitude. In this context relating to the third world scene, Beckford's [53, p 233] considered opinion is:

We submit that the precondition of all preconditions for change is the structuring of the minds of people to accommodate the change ... Because of the historical legacy, all the people of plantation society see themselves as inferior and incapable of carrying out major schemes.

Whilst Beckford is right in saying that people have to be convinced of the advantages of change, he undoubtedly exaggerated the strength of feeling of inferiority but his general observation is nevertheless an important one.
2.4 Scope and Efficiency of Secondary Education in Terms of Enrolments and Success at Public Examinations

Section 2.3 of this Chapter showed that the crucial determinant of change in the aims of the curriculum and learning strategies was that a limit was being approached in the annual rate of acceptance of new recruits for employment in the modern sector of The Bahamian economy. Looking now at the binding conditions on attendance, it was legally compulsory to attend school between the ages of 5 and 14 years [16, p.486, paragraph 26] but since the early 1970's many children remain at school beyond the statutory leaving age up to age 17. This has also necessitated complete overhaul of all of the subjects of the curriculum. For comparative purposes, pupils in Zambia [31, pp. 8-10] were obliged to accept nine years, from age 7 to 16, of comprehensive education. Nearer home in the Eastern Caribbean, Wilson [24, p.356] says:

Education is for the most part nominally compulsory from the age of 5 or 6 up to 14 or 15, but in practice it is impossible to enforce this.

During the school year 1975-6, in the age range 10-14 years, there were 25 836 pupils attending schools in the Commonwealth where the estimated population for that range was 27 200. This meant that 94.9 per cent of the age band was involved in education. Similarly, for the same year 7834, or 36.6 per cent of the 15-19 year-olds were attending school. Subsequent to a study of seventy-five countries ranging from the world's most underdeveloped to the most advanced nations, Harbison[2,p.127] finds:

...the coefficient of correlation between educational level and gross national product per capita is 0.888. The best single indicator of a country's wealth in human resources is the proportion of its young people enrolled in secondary schools.

Although it was possible to calculate gross national product from Appendix 1 for the years 1973-77, the educational statistics
The increment in the junior high total enrolment for 1976 over 1975 was 5.3 per cent and that for the senior high over the same year was 3.7 per cent. These statistics show that more pupils opted to remain in secondary schools. The reasons for this option were not far to seek: selection at 11+ was abandoned in 1974; secondly, for the poorer families education was the only passport to mobility up the economic and social ladder; and finally, it was well known that the government had a policy of Bahamianisation designed to replace qualified—more often not so qualified—expatriates with qualified Bahamians when contractual commitments expired.

The domain of the curriculum for junior and senior high schools included 21 disciplines of which English (language and literature) and traditional mathematics are status subjects with very secure places in the programme of studies. The other subjects were history, geography, general science, art, handicraft, needlework, health science, religious studies, biology, Spanish, French, Latin, cookery, arithmetic, music, social studies, modern mathematics and technical drawing. The curriculum for Zambian schools [31,p.7] was similarly structured but, in contrast, had a better bias towards vocational subjects among which were automotive servicing, agricultural studies, tailoring and simple repairs of electrical appliances.

In the estimation of society the only determinant of the efficiency with which learning strategies were deployed in the classrooms of the high schools in The Bahamas was the performance of pupils at BJC and GCE 'O' level examinations. In Section 2.2 of this Chapter, reference has already been made to some examination performances but in order to judge efficiency in delivery and uptake of curriculum knowledge and skills, the records showed that in the summer of 1976, 225 candidates entered for GCE 'O' level Mathematics (Syllabus A) of whom 27 passed, giving a pass rate of 12.0 per cent. It was indeed a sobering statistic that in that year 6363 pupils were registered in senior high schools. (See Table 2.2). The statistics in Table 2.2 lack sufficient detail for the separate total registration of pupils in the independent and state junior high schools for 1977 but the following Table 2.3 shows the success
rates in mathematics for that year [5, p.44, Appendix B].

<table>
<thead>
<tr>
<th>Subject</th>
<th>Entry: State</th>
<th>Pass</th>
<th>%</th>
<th>Entry: Independent</th>
<th>Pass</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>2533</td>
<td>387</td>
<td>15.3</td>
<td>829</td>
<td>420</td>
<td>50.7</td>
</tr>
<tr>
<td>(Trad)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arithmetic</td>
<td>4232</td>
<td>1654</td>
<td>39.1</td>
<td>1264</td>
<td>982</td>
<td>77.7</td>
</tr>
<tr>
<td>Mathematics</td>
<td>131</td>
<td>99</td>
<td>75.6</td>
<td>327</td>
<td>253</td>
<td>77.4</td>
</tr>
<tr>
<td>(Mod)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: BJC Success Rates in Mathematics for State and Independent Junior High Schools, 1977

Obviously the independent schools were outperforming the state schools and entries in modern mathematics from that sector were more than those from the maintained schools by a factor of 2.5. In both sectors, as the statistics show, pupils regard traditional mathematics to be a difficult subject so the majority opt for arithmetic, in which in state schools the pass rate was 39.1 per cent. Performance was even worse in traditional mathematics: the pass rate was 15.3 per cent. These statistics reveal not only the chronic nature of the problem in mathematics education but also that there was a scandalous waste of human talent. Herein lies the reason for the popularity of some American schools with Bahamians. It was possible for them to secure higher education without as strong a secondary education background as those who went to the United Kingdom to study. Moreover, the lower socio-economic groups could meet their financial obligations in the United States by finding part-time employment with the consequence that The Bahamas became infiltrated by what Bacchus [51, p.4] calls "qualifications with low standards". The inefficiency of the state maintained junior and senior high schools may be gauged by this revelation from Bajpai and Bajah [5, p.32, paragraph 124]:

Last year in mathematics there were two graduates and a total of four in physics and chemistry. This indicates the seriousness of the present situation in science and mathematics.
A similar state of affairs prevailed in Mediterranean countries [20, p.62] in the 1960's, where a person 7 years of age began school with a lower probability of reaching the university than his peer in other regions belonging to the Organisation for Economic Co-operation and Development.

It has been conclusively shown, therefore, that more pupils were remaining in independent and state maintained high schools but owing to the inefficiency of the system, far too few were obtaining BJC and GCE 'O' level qualifications to meet the country's manpower needs and individuals' needs for satisfaction in personal ambitions. The result of this inadequacy in secondary education was that many Bahamian young people were gaining admission to some American schools and acquiring qualifications which were not recognised. An attempt is made in Section 2.5 to investigate the degree of quality in secondary education.
2.5 Quality in Mathematics Education

Apparently an international yardstick for assessing quality in secondary education in general and mathematics education in particular, does not exist. In any event, the researcher estimates that, apart from the inherently difficult nature of the exercise, quite a few criteria already considered in this Chapter have a direct bearing on what is now contemplated. Here concern is not with BJC and GCE 'O' level performance per se but primarily with those - the teachers - who deliver the input which in turn is transmitted via the pupils into those performances. A search is therefore being made to elicit evidence for the quantity, educational ability and pedagogic skills of the teaching staff as determined by the statistics, and qualifications they hold for their specialised task. Statistics for the maintained high schools in New Providence for the school year 1975-6 show that 68.0 per cent of the teaching staff were trained, with two years of teacher training as a basic minimum. But further investigation reveals that for mathematics education - the subject of this research - this majority held at most GCE 'O' level mathematics prior to training. In effect, the majority of teachers are guiding young minds in mathematics at secondary level which they themselves did not complete. In fact during the pilot visits to the five government senior high schools in the survey for this research, only one subsample was taught by a recognised graduate who claimed to have studied successfully second year university mathematics. Even so the classroom conditions under which the teaching was done did not give the researcher reason to believe that the full potential of this teacher was given to the job. Where teaching staff do not hold adequate professional and academic qualifications, each member counts as a deficit because poor teaching is the product of this type of teacher. This large fund of human potential - 32.0 per cent of the teaching staff - certainly needs to be buttressed by a well-managed and efficient in-service education and training programme to improve the quality of
their work because the stronger this important supplementary source of strength, the greater the probability that the quality of the output in mathematics education would rise beyond pre-independence levels. For that matter, in-service education and training are also essential to the professional growth and development of trained personnel.

This scarcity of trained specialist mathematics teachers limits the generation (self-sustaining growth) of more professionals, which means continued dependence on overseas supplies of motive power that are not necessarily equipped to function in subtropical cultures. The researcher does not wish to imply that primary mathematics teachers must be able to solve second order differential equations by the method of Laplace transforms, which effectively circumvents separate determinations of the complementary functions and the particular integrals, linear combinations of which give general solutions. Rather, what is required are primary teachers who have themselves successfully completed secondary education and acquired a sufficiently enlarged and diversified grounding in mathematics content and pedagogy to bring about changes necessary in inducing the high school system in The Commonwealth to output a better product. Beeby [40, pp 57-58], in an analysis of relevant problems, introduces a hypothesis on educational stages in this fashion:

What is at issue here is not the attitude towards change of either parents or teachers but the actual ability of the teachers to bring about the changes necessary to raise the school system to a higher stage ... My thesis is that there are two strictly professional factors that determine the ability (as distinct from the willingness) of an educational system to move from one stage to a higher one. They are: (a) the level of general education of the teachers in the system, and (b) the amount and kind of training they have received.
The author realises firstly, that Beeby is discussing general education but what he says about raising standards has application to mathematics education. Secondly, though he does not say it, a positive attitude towards change is both desirable and essential in the fight against crippling under-achievement produced by teacher ineffectiveness. Thirdly, for any hope of breaking the vicious circle in which poor primary mathematics is producing worse secondary mathematics, a transformation must begin at the primary level so as to filter upwards by a capillary effect. [5, p 39, paragraph 27]

In any discourse on quality in mathematics education, quite apart from mathematical and pedagogical knowledge and skills, the teacher-pupil ratio, about which great care must be exercised, is of sufficient importance to merit consideration. The usual premise is that given physical facilities, the smaller the class the more effective the teacher can be in delivering subject content, with the added proviso that the class is sufficiently homogeneous in ability. In 1976, for instance, on the island of New Providence there were 68 streams containing a total of 2323 pupils in Grade 9 and 76 streams with 2265 pupils in Grade 11, giving teacher-pupil ratios of 1:34 and 1:30 at the top ends of junior and senior high schools respectively. The policy of the Ministry of Education and Culture was that these ratios should be 1:30 and 1:25 respectively but they were hard to enforce especially in the face of a pupil population increase in excess of 5.0 per cent and 3.0 per cent per annum at lower and upper secondary levels correspondingly. The practice of streaming too early violates the spirit and purpose of comprehensive education which is based on the philosophy of mixed ability teaching. Some very powerful and incontrovertible arguments can be generated in answers to the pointedly relevant questions posed by the discussion paper in the Students' Material [54, pp 135, 136]. Likewise perusing some of the references in the Tutor's Guide [42, pp 91, 92], in particular [55, pp 16-23] and [56, pp 47-51], give points of view deriving directly out
of the classroom environment. The author was part of a visiting AIMEC team to Moray House College of Education, Edinburgh, where among other things it was demonstrated in the Mathematics Department how pre-service teachers were trained to cope with mixed ability teaching in the first two years of secondary school. This teaching strategy is especially relevant to The Bahamas where there is an acute shortage of textbooks and professionally trained and qualified teachers.

Although the teacher-pupil ratio is a worthwhile guide to determining quality in education yet, like other global statistics, it has its limitations. In senior high schools there were classes with anywhere from 31 to 40 pupils. Large classes, in consequence, aggravate the "drop out" rate which, in turn, encourages qualification-seeking despite weak backgrounds in secondary education. These qualifications are not only low in status but also bear no relevance to the employment for which the holders either seek selection or are already in jobs (See discussion on p. 90 of this Chapter).
2.6 The Effect of Independence

Sovereign status was granted to The Commonwealth of The Bahamas in 1973 but mainly because of the geography of the country, education was still centrally organised with headquarters in New Providence. Links with the periphery were maintained through Senior Education Officers and District Education Officers. New Providence, the site of the survey for this research, had 25 primary schools, 7 junior high schools, 5 senior high schools and the College of The Bahamas, the apex of the state system of education. The most far-reaching change in the organisation of education was the abolition of the selection examination at 11+, which meant that theoretically the system became comprehensive. Naturally along with this change were large enrolments at secondary level but accompanied by a correspondingly large 'drop out' rate at the senior high stage. The opening of secondary education to more underprivileged people was a means of leavening the old élite and modifying social stratification patterns. Clearly the intention was that a greater number should have a chance to become qualified for participation in development at the social, political, economic, professional and technical levels. As a consequence of short training courses for civil servants administrative functions in education in particular, and the public service in general, were reallocated. More importantly, independence stimulated the implementation of long-range planning and policies in education. For example, Parliament was told in 1972 that the College of The Bahamas was being planned and accordingly, it came in 1974. During the same year the first one-week workshop in curriculum development in mathematics education for teachers in the 7 junior high schools was mounted under the co-sponsorship of the United Nations Educational, Scientific and Cultural Organisation (UNESCO) and the Government of The Bahamas. One of the fruits of this workshop was the present mathematics curriculum for junior high schools, which has subsequently been extended to Grades 10 and 11 of the 5 state senior high schools as shown later.
in Appendix 6.8. The workshop was co-ordinated by a Senior Education Officer for personnel development within the Ministry of Education and Culture in New Providence; administered by UNESCO mathematics specialists based in Trinidad and Tobago and Jamaica; consultants from Trinidad and Tobago, Barbados and Mico Teachers' College, Jamaica; UNESCO specialist in administration based in Antigua; and there were four resource persons from The Bahamas, one of whom was the author, who was then lecturer in mathematics (content and methods) at The Bahamas Teachers' College.

The aim of curriculum development, Parliament was told, was to effect improvements in standards of learning and enable appropriate modifications to be made to The Bahamas Junior Certificate Examinations (BJC). Obliquely secondary teachers were being asked to become content specialists in their chosen disciplines. Still in the planning stage was a national examination to take the place eventually of GCE 'O' level at the end of senior high school. Primary teacher-training facilities were to be extended and secondary teacher-training facilities were to be developed. The need for expatriate teachers was expected to remain but the ultimate objective was self-sufficiency with qualified Bahamian teachers throughout the schools. Opportunity was for shaking off old patterns, traditions and habits; for finding new types of solutions to current problems; a general raising of the human level and shaking loose those social and intellectual attitudes which impede human growth and development; a breaking of conformity to old standards, obsolete concepts of social organisation and personal relationships. Speaking about the road to development Curle [57, p 16] suggests

A most powerful impetus towards development comes from the achievement of national independence, but how to keep up the pace once the original excitement has abated; what fixative should be applied, so to speak, to pride, ambition, and hope? But one part of the answer we do know can apply is education.
Later, out of the accumulated wisdom of experienced scholarship, he (Curle) advises

... the curriculum must be firmly grounded in reality (p 27).

In developing countries national independence may be viewed as the principal catalyst in the organisational and structural transformation of the curriculum, the ultimate weapon in the implementation of planned and steady social revolution. This fundamental ambition stands a greater chance of success if teachers volunteer to seek higher specialist academic skills and more intensive training in pedagogy. Despite the many variables inherent in the process, the resulting efficient primary education would have spill over effects in the form of better quality human output from secondary education. But this transformation which seems easily achievable in theory, will not be either an easy or an instant process in The Bahamas: it will span many years of toying with education and human variables in order to arrive at the right balance of expenditure on primary, secondary and college education (including teacher and vocational training together with continuing adult education). In every section of this Chapter the common denominator - curriculum development in mathematics education - seems inescapable because basic pre-independence assumptions about the purpose of secondary education in The Bahamas need to be challenged: some crucially important questions about the intrinsic value of the human being need to be answered. In this respect Lasley and Applegate [58, p 6] admonish before embarking on a sustained inquiry:

We need to ask some critical questions and challenge some fundamental assumptions.

Questions needing attention include: What do we want from our secondary schools? What do we expect from the teachers in those schools? What responsibility do we have in colleges of teacher education to affect the practices in
our institutions and in those institutions where new teachers will go? What pedagogical and academic skills do teachers need in order to be effective in a secondary school classroom? What are schools doing to accommodate adolescent needs and to deal with individual differences? What....?

The raison d'être of national independence therefore is that it creates the opportunity to sort out priorities in primary and secondary classrooms wherein lie the human potential to be developed and through whom further development can be effected. Surely, in the final analysis, national development is for the good and benefit of human beings.
2.7 The Scientific Process Applied to Educational Research

On the basis of

1. The qualifications in mathematics for entry to the College of The Bahamas (Appendix 5.2)

2. The content of the methods curriculum for teacher trainees (Appendix 6.7)

3. Achievement of most trained teachers when they leave the College of The Bahamas

4. Performance of secondary pupils in other school-based and external mathematics examinations

5. 19 years classroom experience of the author


The problem for this research and its aims were located and defined as follows:

| PROBLEM | Low attainment in BJC and London University GCE 'O' level mathematics as defined by Syllabus A360 for overseas candidates |
| LOCATION | In the learning-teaching situations provided by state-maintained junior and senior high (secondary) schools in The Bahamas |
| AIM | (i) To make recommendations for improvement in the attainment of curriculum behaviours agreed for BJC and GCE 'O' level mathematics  
      (ii) To suggest directions for further research |
The next step in the research procedure was to examine certain conclusions put forward by Bajpai and Bajah [5] and restate them as propositions in an attempt to arrive at a tentative but comprehensive derivative which could stand as a testable statement — a hypothesis — for verification or refutation by the evidence uncovered in the field survey. Furthermore, the evidence had to be examined against the background of existing verified psychological and psychometric theory concerning knowledge of mathematical abilities in secondary school children. Accordingly the researcher extracted the following seven constructs from the unpublished Report [5]:

CONSTRUCT 1  Paragraph 2  ... students entering the College seem to do poorly at mathematics

CONSTRUCT 2  Paragraph 47  Children find mathematics difficult, ... and they have unfortunately been used to hearing about the difficult nature of the subject from their parents

CONSTRUCT 3  Paragraph 60  ... when children leave the primary school they should be able to perform the basic operations of arithmetic, know their multiplication tables, etc

CONSTRUCT 4  Paragraph 72  ... low standards of achievement in the basic skills acquired by children when they enter the junior high and senior high schools from government primary schools

CONSTRUCT 5  Paragraph 76  Mathematics they (the pupils)... all said was a difficult subject... (See also Paragraph 24)

CONSTRUCT 6  Paragraph 91  They (the parents) were all conscious of the importance of science and mathematics but felt that these were difficult subjects ...
Paragraph 112 In mathematics the children exhibit a lack of basic knowledge and skills, including computation and place value, that they should have acquired in earlier years.

Using these constructs as the background to the aim of this research, the author derived the following hypothesis for the purpose of guiding the investigations of numerical values for the key dependent variables, which were crucial to the suggestion of partial solutions to the problem under study.

**HYPOTHESIS:**
The Bahamian secondary school pupil in the state-maintained sector of education has not acquired as full an understanding of the basic mathematics concepts as a prerequisite to enjoyment, progress and success at further studies in mathematics, and in particular for success in:

1. Bahamas Junior Certificate Mathematics (Appendix 5.2)
2. University of London GCE 'O' level mathematics as defined by Syllabus A360 for overseas candidates

Partial solutions are advisedly suggested because in a behavioural science such as education, many variables operate in a social problem space so that any proposed solution will not suffice for all time. Herein lies the justification for further research which, in turn, would begin to erect a useful compendium of coherent educational theory. But this is a very slow process which, for The Bahamas, has only just begun.

At this juncture, thought was given to assembling the evidence for establishing causality. Scientific process, therefore is
used in the sense of a type of investigation, intentionally narrow in scope but penetrating in depth thus offering the chance to reveal objective empirical evidence for the extension of the existing horizon of knowledge. With this point of view Nisbet [59, pp.4,5] disagrees: his opinion is that relatively minor problems can be resolved empirically whereas major issues require judgement of values. He does agree however, that fact-finding survey type research provides an adequate base of knowledge on which to support any judgement of value. All recent major educational reports in England – Mathematical Development [60], Mathematics Counts [50] and Aspects of Secondary Education in England [49] – have been accompanied by extensive surveys, and are much the better for them.

However, in education as in other behavioural sciences where a multitude of variables operate to influence any social outcome, the effectiveness of the survey method cannot therefore be a major determinant in the understanding, prediction and building of educational theory. But the researcher had to make a conscious choice of the best method from among many[61], given his distance from the field of investigation and the kinds of instruments that were most likely to yield useful data; the constraints of money, time and travel; and the chance of high validity and reliability of the instruments. Despite its disadvantages, the scientific method has extended the horizons of knowledge of human individual and group behaviour in the educationally associated disciplines of psychology and sociology.

Educational research – what is it? There is no entirely satisfactory existing definition so the author had an attempt at inventing one to suit the purpose of this study – the evidence for on-going curriculum renewal in mathematics education, certainly the area of research with the largest single growth in the United Kingdom and the United States of America since 1960. With the benefit of experience as a mathematics teacher and teacher educator together with varying
viewpoints from perusal of the literature on the state of the art, the author conceives of educational research as:

A scientific endeavour the objective of which is to create and enlarge current knowledge of theory which, in turn, would find immediate utility in practice to the ultimate good and benefit of, in this instance, a developing society.

Research in any discipline pre-supposes the existence of at least one route to a desired social goal. This requires information about the key variables which bear relevance to the achievement of the goal in view. As regards the variables operating in the domain of this research problem, the influential ones were:

1. The effect of the maintained junior and senior high schools on any sample of pupils on the island of New Providence
2. Sex of pupils of ages 13 and 15 years attending the schools
3. Mean performance of each pupil
4. Mean performance of each school
5. Standard deviations
6. Items 001-050 on junior high test (Appendix 5.1)
7. Questions 001-014 on senior high test (Appendix 6.4)
8. Home background

The independent variables constructed for the survey were the items and questions of 6 and 7 above and the key dependent variables, values for which were sought in the survey, were 2, 3, 4 and 5. Although speculative inferences may be made about the behaviour of variables 1, 8 and 9 on the basis of evidence obtained in other cultures, these remain prime candidates for further research in The Bahamas.

The details of the application of the scientific method in the survey are left for discussion in Chapter 4.
CHAPTER 3

REVIEW OF THE LITERATURE
3 Review of The Literature

3.1 Introduction

Prudence dictated that the existing corpus of publications should have had the function of uplifting into high relief the problems of mathematics education in The Bahamas. An avid perusal generated introspection about what the urgent need of the immediate future should be: the identification of a variable which, given appropriate positive change in operative power, could improve the motivation, enjoyment and attainment of secondary pupils in mathematics with the ultimate effect of generating a desire for further studies in the subject. The researcher had an intuitive notion that the answer to the problem - effective learning and teaching of mathematics - lay in the variable, approach, which needed a combination of experience, literature review and the findings of relevant research in mathematics education to identify. The nature of this approach would dictate an effective method or style of teaching mathematics to serve the approach.

The starting point was determined by reorganising the constructs in Chapter 2 into the following hierarchical model (Figure 3.1) to represent an aspect of the sociocultural experience of Bahamian pupils.

Constructs 2 & 6: Children acquired notions of the difficulty of mathematics from their home background.

Construct: 3: Pupils left the primary school with incompetence in basic arithmetic operations and tables.

Figure 3.1: The Influence of the Sociocultural Experience of Bahamian Pupils on the Learning of Mathematics
Construct: 4: Pupils transferred from State-maintained junior and senior high schools to corresponding Independent schools exhibited low standards of achievement in basic skills.

Constructs: 5 & 7: Pupils found mathematics difficult and consequently exhibited a lack of basic knowledge and skills.

Construct: 1: Students who entered College did poorly at mathematics.

The Second International Congress on Mathematical Education [62, p.63] puts the perspective in this way:

In many developing countries ... the following factors arising from the social and cultural background are relevant to the reform of mathematics teaching:

(a) instruction has been oral rather than visual; not many generations have elapsed since instructors were illiterate;

(b) reliance has been placed on memory of traditional patterns of interpretation of history, customs and techniques; it has been important for survival to maintain a well-tried and long-established system rather than to make possibly disastrous new experiments;

(c) the method of instruction has been didactic; where the oldest is the most experienced and respected, the duty of the young is to listen;

(d) spatial experiences have been quite different from those of the Western world and, in particular, representation of spatial relationships may be almost unknown;

(e) dynamical experiences will have been few or absent, with consequent difficulty in the alignment of
scales and visualisation of movement, and a general lack of mechanical facility. These factors inhibit many of the methods of investigation and discussion which are prevalent in modern teaching methods in Western countries.

Although there existed a cultural milieu of negative attitudes towards mathematics yet the secure status of the subject was acknowledged by the State through the curriculum and endorsed by the society despite the latter's view that the discipline was difficult. The key question therefore was: where should a start be made and how could the literature help in charting a course? The researcher sought to identify elements of knowledge which, when assembled, generated an understanding rather than an explanation. The cause-effect relationship was an exponent of explanation as opposed to understanding with its insights into motives and intentions. Ultimately understanding was the terminus of an heuristic operation on which the proposed hypothesis at the end of Chapter 2 would stand or fall.

Bajpai and Bajah [5, p.32, paragraph 121] say, after three weeks of pilot visits and exhaustive enquiries,

> It became quite evident that ... mathematics education in the Commonwealth is on the whole in a very poor state

but they balance this view by stressing (p.8, paragraph 22) that

> ... the problems we have identified as a team are not being seen for the first time.

Supporting this evaluation, Skemp [63, p.248], whilst asserting that teaching in the primary schools is often inefficient, also claims that:

> ... the teaching of mathematics is such that many - probably a majority - acquire a lifelong dislike of the subject, together with lack of confidence in their ability to understand it.
Consequently, by implication, the problem was not peculiar to developing countries and was partially institutional insofar as the pedagogy given to teachers in training in The Bahamas was a synthesis based on the history, philosophy, psychology and sociology of education. There was no central thoroughly researched theory of mathematics education tailored specifically to produce intelligent learning - the heart of mathematics education - as the result of co-operative effort between teacher and learner. Effective learning depends entirely on the teacher taking the lead in establishing the kind of relationships with pupils which help diagnosis, the prerequisite to the prescription of methods of effective learning. Skemp [63, p.262] distinguishes between two types of learning:

The distinction between relational learning and instrumental learning is a very general one. It relates to qualitative differences in learning which are independent of the subject matter, are deep rather than superficial, and thus can only be diagnosed, not directly observed. They can be identified most easily in low-noise examples such as mathematics, physics, chemistry, biology, where the underlying conceptual structure is strong relative to the detailed content, and its presence or absence is therefore easier to discern. But once the distinction has been conceptualized, one can begin to discern either relational or (regrettably, more often) instrumental learning taking place in almost every subject area: which is to say, intelligence being called into action strongly, or hardly at all.

This introduction has raised four important themes - mathematics learning, conceptual structure, intelligence and approach - which served as directives to the perusal of the literature on educational and psychological theory, and mathematics education.

3.2 Mathematics Learning

As a prerequisite to teaching mathematics successfully it is absolutely vital, indeed axiomatic, to know how mathematics is
learnt. Endorsing this view, Skemp [64, p.13] suggests:

Problems of learning and teaching are psychological problems, and before we can make much improvement in the teaching of mathematics we need to know more about how it is learnt.

That is to say, theories are needed to explain and predict (1) what happened or was likely to happen when the act of learning occurred, and (2) the conditions under which it was most likely to be optimised. Discussing this psychological point Dienes [65, pp.21, 22] believes that

Learning anything new, and in particular learning mathematics, consists

(1) sorting events into classes or categories, so that any event is immediately recognised as either belonging or not belonging to a class or category, or of course as being irrelevant to it;

(2) becoming aware of the relationship to each other of the classes or categories constructed.

But how are we to be satisfied that a person has formed a particular class or category? We might say that the class 'red' has been learnt if a person invariably selects a red object. One must of course, be reasonably certain that the subject is co-operating, and where human behaviour is concerned we can hardly ever speak of 100 per cent certainty. The only part of the sequence of events leading to the statement 'A has formed the class of red objects' which is verifiable is whether A does or does not pick red objects when called upon to do so. When we say that therefore he has learnt about red (just because we have seen him picking out red objects), we are doing no more than manipulate a model, a theory. It would be reasonably certain that a successful performance with coloured counters would enable someone to learn to stop at
traffic lights when they turn red. This is a prediction which the model, however simple, has enabled us to make.

Manipulating relevant data - from cues to be seen on and around them - forms a 'cognitive map' to which a learner refers on subsequent occasions. From sorting out his playthings or comparing his parents and brothers and sisters or friends, ideas such as 'the same as', 'greater than' and 'smaller than' are fixed on the cognitive map. Although Watson [66, p.31] is at variance with a teaching order predicated by a Utopian concept map

... though there seems little prospect of producing a uniquely determined teaching order based on some idealised conceptual map, the approach does seem a fruitful way of attacking problems of this nature, especially insofar as hypotheses put forward may be subjected to experimental test.

he agrees firstly, that especially at primary level, insights from psychology tend to legitimate exploratory work in mathematics and commends the concrete materials produced by Dienes; secondly, the patient and painstakingly thorough work of Piaget in the development of the child's capacity to grasp fundamental concepts of conservation (number, length, quantity and weight) makes a positive contribution to the problem under study; and thirdly, so does Skemp's [64] use of educational psychology to bear explicitly on problems of order in learning mathematics. For example, the chance of pupils reaching an adequate understanding of the sine ratio without some prior grasp of the idea of similarity, proportion and angle is an unlikely event. But at foundation level the basic concept is natural number, $N$ which in turn, permits the extended generalisations-integers, $Z$ rational numbers, $Q$ and real numbers, $R$. Later a return to this model extends the concept to the complex numbers, $C$. Following exercises on ordering and matching, the operations addition ($+$) and multiplication ($\times$) on $N$ generate five properties:
(1) \( + \) is commutative: \( a + b = b + a \)

(2) \( + \) is associative: \( a + (b + c) = (a + b) + c \)

(3) \( \times \) is commutative: \( a \times b = b \times a \)

(4) \( \times \) is associative: \( (a \times b) \times c = a \times (b \times c) \)

(5) \( \times \) is distributive over addition: \( n \times (a + b) = (n \times a) + (n \times b) \)

on which algebra, with its basic concept, \textit{variable} is built up. Associated with function in algebra are the geometric transformations \textit{reflection}, \textit{rotation} and \textit{translation} in which the basic concept is \textit{point}. Included among fundamental concepts is a notion of \textit{angle}. In The Bahamas pupils found the learning of fractions, directed numbers, and ratio and proportion very difficult. This was partly because the formation of concepts depends first and foremost upon experiences in which \textit{perception} is evoked. As a consequence of perception, abstracting of similar experiences occurs and ends in \textit{abstraction}, the final product of which is \textit{concept} formation. Accordingly Skemp [64, p. 22] claims

\begin{quote}
A concept therefore requires for its formation a number of experiences which have something in common. Once the concept is formed, we may (retrospectively and prospectively) talk about examples of the concept.
\end{quote}

Bajpai and Bajah [5, p.23, paragraph 77] found that secondary pupils had problems with 'recall' in doing mathematics. Skemp [64] addresses this problem by showing that schematic learning is twice as efficient (as indicated by recall) as rote learning when tested immediately after. Schematic learning is not only better learnt but better retained since over a period of four weeks this type of learning is seven times more efficient, and this \textit{active} use of intelligence in learning is crucial to the ease or difficulty with which later topics are mastered. Polya [67, pp.100-104] supports the philosophy of efficient and active (\textit{discovery}) learning by enunciating three principles, which he admits are not new since they are derivatives of the
experience of the ages, endorsements by the judgements of great men, and more importantly the suggestions of psychological study of learning:

(1) The principle of active learning ... The best way to learn anything is to discover it by yourself.

(2) The principle of best motivation ... the learner should be interested in the material to be learned and find pleasure in the activity of learning.

(3) The principle of consecutive phases ... an exploratory phase should precede the phase of verbalization and concept formation and, eventually the material learned should be merged in, and contribute to, the integral mental attitude of the learner.

In all of his work and in agreement with Polya, Bruner [68, p.39] a practising psychologist and resolute defender of learning by discovery, regretfully reflects:

... the high school student meets Euclidean geometry for the first time, as a set of axioms and theorems without having had some experience with simple geometric configurations and the intuitive means whereby one deals with them. If the child were earlier given the concepts and strategies in the form of intuitive geometry at a level that he could easily follow, he might be far better able to grasp deeply the meaning of theorems and axioms to which he is exposed later.

The author entirely agrees with Bruner's viewpoint on imparting to pupils at the early stages an intuitive feeling especially for the basic algebraic and geometric concepts, because these serve as a secure foundation on which axiomatisation can meaningfully stand later. For example, in Euclidean geometry it is very difficult to define a point and a line without
appealing in the introductory stages to abstraction. This amounts to no more than a futile attempt at presenting a formal explanation based on a logic of feeling and action that is distant from the pupil's mode of thought and sterile in its implications for him. The appeal to intuition, the author is convinced, is so powerful that assignment of an absolute level of difficulty to any particular topic is deferred. This modus operandi not only gives great pleasure in working with pupils and rewards the expenditure of enthusiasm as well as gives enjoyment in organizing and guiding learning; but more importantly it sets the scene for the authority of the teacher to be replaced by the authority of the truth. The employment of the Cartesian plane to represent Euclidean space can be used to good effect by giving pupils lots of experience in plotting points and joining some to form lines in a systematic way in the four quadrants of the plane. It would then be easy to advance through a series of steps to the abstraction that a point and a line are completely defined by the ordered pair \((x, y)\) and the equation \(ax + by + c = 0\) (a set of ordered pairs) respectively. Subsequent to supplementary exercises in freely rotating objects in the classroom, an angle may be abstracted as a rotation of a line \(ax + by + \theta = 0\) about a point \((x, y)\) on another line \(Ax + By + C = 0\) in this plane. Point, line and angle are absolutely fundamental geometrical concepts which lead naturally into the intuitive definition of a triangle.

Some of the advantages to be gained by this strategy are:

1. It serves as an appropriate method that is consistent with the philosophies of the spiral, behaviourist, integrated and formative approaches to curriculum development.

2. It offers the best chance of reverting to 'basics' through a smooth transition in mathematics teaching and learning without creating a traumatic experience for pupils who had generous doses of the *New Math method* in the learning and teaching of mathematics, which Lamon [69, p.8] dismisses as a failure in the words...

... it should not be surprising that people are expressing dissatisfaction. Mathematics professors,
education and psychology professors, as well as practising teachers, question seriously the validity of current pedagogy and the wisdom of teaching youngsters the so-called New Math.

(3) real number algebra is not presented as generalised arithmetic and is of great assistance in the study of vector algebra.

(4) the path is cleared for graphs to be used throughout the entire teaching of trigonometry in which equations of the form $a \cos x + b \sin x + c = 0$ derive from analytic experience.

(5) trigonometry remains a part of algebra and geometry so that later, in satisfying the fundamental requirement of the spiral approach, it naturally becomes integrated into analysis.

(6) this dynamic approach to geometry obviates the necessity for proving a large number of theorems and offers a choice of introduction via:

(i) vectors
(ii) coordinates
(iii) symmetry
(iv) transformations (reflection, rotation, translation, enlargement)

(7) it is the most convincing strategy for introducing hyperbolic functions (see Bajpai et al. [70, frames 48-67, pp. 1: 36 - 1: 48])

(8) permits the introduction of limits, continuity, sequences, series, differentiation and integration as one package for examining analytical behaviour and arriving at definitions intuitively. This offers the most plausible method of introducing integration as the limit of a sum instead of as an antiderivative. The author, as one of a team of mathematics education consultants, used this...
approach as a synthesis of ideas from Bajpai et al [71, pp. 106-142] and Waismann [72, pp. 123-152] in Hyderabad and validated it in Ahmedabad in India under the admiring and approving attention of professors of mathematics and mathematics education along with competent experienced mathematics teachers and lecturers. In conjunction with offering the best chance for communicating clarity in every essential concept, it satisfies the demands of discovery learning in mathematics and acquaints pupils with the theory behind the psychological premise of mathematical structure.

Griffiths & Howson [33, pp.229, 230], in supporting the views of the author, claim that pedagogically:

... the axioms of Euclidean geometry are all represented by theorems of coordinate geometry in the following sense. If we allow words 'point', 'line', 'on', etc, of Euclidean geometry to correspond to 'number pair (x, y)', 'equation \( ax + by = c \)', satisfies equation', etc, then the axiom 'there is exactly one line passing through two given points' corresponds to the statement 'there is exactly one equation \( ax + by = c \) which is satisfied by two given number pairs \( (x_1, y_1), (x_2, y_2) \)', and the latter is provable in coordinate geometry, using our knowledge of solving equations in algebra. Now coordinate geometry is easier to describe than a satisfactory set of axioms for Euclid; and, ... it contains all the axioms (and hence the results) of Euclid. Therefore we may gain a great deal pedagogically, and lose nothing mathematically, by working with coordinate geometry rather than with its isomorph, Euclidean geometry.

Much to their credit, educators in The Bahamas repeatedly ask "How should geometry be taught?" for there the Definition - Theorem - Proof syndrome is pervasive but sterile. Freudenthal [73, p.451] disagrees with axiomatic teaching and learning of geometry at school level because geometric axiomatics are
extremely complicated. The British treat all mathematical subjects in a spiral manner. That is to say, topics are introduced in the lower school and encountered repeatedly thereafter, so that each time the pupil is more experienced from previous encounters. This method of teaching is isomorphic to, and therefore serves, the approach to curriculum development. In contrast, in the United States of America, 'Tenth Grade Geometry' is concentrated in a single year. Dieudonné [37] advocated that the only possible way to teach geometry was the study of a vector space with a scalar product. But Dieudonné, a staunch advocate of the Bourbaki philosophy of mathematics, taught only university students. Approaches prepared in this spirit were therefore seriously and intensively questioned by Griffiths and Howson [33 pp.239,240] on pedagogical and mathematical grounds - it favoured nurturing an intellectual as opposed to the utilitarian value - and Lamon [69, p.8] adjudged it a failure for early secondary schools although it was brilliantly done for 15-18 year olds. Freudenthal [73, p.443] is also at variance with Dieudonné since linear algebra, though useful, is restricted by the geometry to which it lends itself and is by no means the type that would inspire or interest young pupils. Of course algebra is vitally necessary for some problems in geometry but its spirit is different from that of linear algebra. In defence of this stance Freudenthal [73, p.455] says:

The reader will have noticed that with linear algebra one gets rather artificial foundations of geometry, artificial because angles are absent among the fundamentals and it takes enormous trouble to reconstruct them. The angle concept is one of the precious gifts of geometry, a gift which should not be refused, a "transcendent" tool by which extraordinary results are easily obtained, much more easily than by algebraic - analytic methods. It is true that every proof can be translated into analysis, but before doing so one has to possess a proof, and to find a proof geometry is needed.
The British approach to geometry teaching places less emphasis on axioms and logical development of the subject. Expectedly modern projects fought shy of any attempts to teach geometry axiomatically. The School Mathematics Project (SMP) were greatly influenced by attempts of the Germans and Swiss to base the study of geometry on that of transformations. Up to 'O' level this is done empirically which seems to many to be physics rather than mathematics. Transformation geometry is then developed from the assumed properties of a vector space with a scalar product. The Scottish Mathematics Group (SMG) provide a formal treatment which combines vector and transformation approaches. Though the axioms are never stated, the development is a logical one.

Apparently an attempt, begun in this section, at identifying the theories primarily aimed at explaining how mathematics is learnt has now digressed into the communication of mathematics content. And Skemp [64, p.36] makes a most relevant observation:

"Now, to know mathematics is one thing, and to be able to teach it - to communicate it to those at a lower conceptual level - is quite another; and I believe it is the latter which is most lacking at the moment. As a result, many people acquire at school a lifelong dislike, even fear, of mathematics."

This translation from 'learning' to 'teaching' is occasioned because it is the writer's experience that a teacher's organisation of, and enthusiastic clarity in, teaching mathematics stimulates productive learning especially if varied learning activities are themselves attractively and carefully organised in a sequence to take account of the philosophy that every child has a peculiar style and rate of learning mathematics. Obviously, intervention by the teacher is axiomatic and the problem assumes psycho-pedagogical dimensions in the sense that this study is attempting to
find, for The Bahamian secondary school system, an input of teaching-learning which makes the function (teaching-learning) assume a maximum qualitative value. Symbolically, the specific objective is to find

\[
\begin{align*}
\text{DOMAIN} & \quad \text{RANGE} \\
\text{f: teaching-learning} & \quad \text{improved achievement}
\end{align*}
\]

with reference to mathematics education in The Bahamas. From an examination of the function shown above, it appeared that improved efficiency lay in manipulation of the input to improved achievement through mastery of mathematics content by the learner. These materials have to be altered in various compensating ways by trial and error in the classroom so as to set up optimum conditions of recall of content in such a manner that memory is not unduly burdened. Variety in the presentation of learning tasks in combination with good teacher-pupil relationships should motivate the learner to maximum output of learning outcome. Analytically this input of teaching-learning defies quantitative definition but situations can be carefully monitored for identification of a method or combination of strategies which tends toward a limiting maximum qualitative value.

3.3 **Summary I**

Published literature suggested that efficient learning of mathematics could be guaranteed by experience of primitive, simple but essential concepts of number, variable, order, point, angle, line, operation, and equality, through guided discovery by manipulating concrete materials. On this basic network of concepts others are built and the structural approach to curriculum building permits frequent reference to and use of the cognitive 'structure' of concepts. Structure was used in two contexts:

(1) the inter-relationships of primitive concepts and their development up to upper sixth form level into a mathematics matrix, and

(2) the hierarchical building of a curriculum which frequently doubles back onto itself.
Four other approaches to curriculum building were mentioned; of these New Math approach was, by consensus, a failure. Alternative strategies for introducing geometry were tried since the definition-theorem-proof syndrome (axiomatisation through the method of congruence) was a failure in The Bahamas. In particular, one strategy was found which offered the possibility of integrating algebra, geometry and trigonometry, thus serving the behaviourist, structural, formative and integrated approaches to curriculum building. Although intelligence was assumed to operate, the only reference to its existence was through the concept 'recall'.

For a contribution to informed classroom practice, attention is directed to an in-depth study of developmental, cognitive and educational psychology, learning theories and current approaches to curriculum development.

3.4 An Integrated Viewpoint

The merit of an integrated viewpoint is that it permits discussion and possible adoption of other ideas in the decision-making process as they bear on mathematics education. In the literature 'theory' had at least three different shades of meaning:

(1) Most commonly, it was used in the context of explaining a group of related scientific phenomena where the elements of the group comprised a set of interconnected hypotheses, the purpose of which was to describe and explain the particular series of natural phenomena.

(2) Moreover it was used to refer to a collection of prescriptions intended not to explain but to guide action. For instance, Marxist theory: the embodiment of a coherent set of opinions, a 'philosophy'.

(3) Finally 'theory' was used in an intermediate sense which did not refer to either an explanatory system or any particular set of views: it merely indicated the relation between certain kinds of problems.
There seemed no consensus on which of the above categories to place educational theory. Where it was seen as a kind of scientific theory, psychology and sociology were invoked to explain educational problems thus producing an unbalanced and inadequate view of any sound practice. In conjunction with its use for describing and explaining, a scientific theory may even be used as a basis of prediction. This last function is fraught with danger in a behavioural science such as education. There is a better chance for balanced development of an educational system when educational theory is informed by practical classroom realities in addition to relevant contributions from scientific theory.

The school of opinion which was guided by the view of educational theory as collections of more or less coherent prescriptions as to what teachers and schools ought to be doing has only managed to generate 'beautiful thoughts'. It has been rejected therefore in favour of more rigorous and differentiated study of the 'contributory' disciplines to education - philosophy, psychology and sociology. But the merit of the prescriptive approach lay in the sense of purpose it brought to bear in discussions on education; an awareness that education was indeed a practical activity and some ideas of what teachers and schools ought to be doing. Again this quality of slant in the debate can never emanate from the sciences of psychology or sociology or from the analytical processes of the philosopher. In fact Downey and Kelly [74] believe, and Musgrove [75, p.223] agrees, that educational theory does have elements of all three kinds of theory already cited: practical problems that teachers and others concerned with the practice of education need to give thought to and make decisions about; it needs to offer a well defined set of views that may help to decide on the directions to which these decisions point; and it must also contain a proper and rigorous scientific basis to ensure that these decisions are theoretically sound and consistent with whatever evidence is available. Stemming directly from this belief Downey and Kelly [74, p.3] say:
... it must be firmly rooted in the school and the classroom and must have a direct practical relevance to all aspects of the teacher's work...Secondly, if it is to deal adequately with issues of this kind, its approach must not lean too heavily on other disciplines or bodies of theory devised for other purposes. We have already commented on the disastrous consequences of basing educational decisions only on psychological and sociological considerations... It is not what each individual discipline or subject has to contribute to any particular issue that is interesting, important and useful; it is the combined effect of them that is needed, along with something more, a full account of and allowance for the aims and purposes of our activities.

Learning is associated with growth which in turn needs assistance along socially desirable pathways. Bruner [76] believes that in effect a theory of instruction is therefore a theory of how growth and development are assisted by various means. Whereas the subject of intellectual growth is not yet well understood by developmental psychologists, Bruner [76, p.7] gives Piaget a prominent place in the field of cognitive development.

Piaget, however, is often interpreted in the wrong way by those who think that his principal mission is psychological. It is not. It is epistemological. He is deeply concerned with the nature of knowledge per se, knowledge as it exists at different points in the development of a child... These descriptions are couched in terms of the logical structure that informs children's solutions to problems, the logical presuppositions upon which their explanations and manipulations are based. What he has done is to write the implicit logical theory on which the child proceeds in dealing with intellectual tasks.
Bruner goes on to state that there were definite planes of weaknesses in Piaget's formal descriptions which logicians and mathematicians have highlighted. In part, these faults stem from certain cultural constraints - ie language - that have influenced the actual conduct of the experiments and transliteration of the findings. However, the utility and power of the research conclusions are beyond doubt but cannot, by any stretch of the imagination, constitute a psychological description or explanation of the process of growth. But his reference to an internal schema attracts very naturally the work of Skemp [64, p.45] into the discussion:

One of the most basic mathematical schemas which we learn is that of the natural number system - the set of counting numbers, together with the operation of addition and multiplication ... Adding single-figure numbers, with the help of concrete materials such as Unifix, is soon learnt. Extending this to the addition of two-figure numbers requires, first, an understanding of our system of numeration based on place value ... Throughout the process assimilation predominates over accommodation.

It is another matter when fractional numbers are encountered. These constitute a new number system, not an extension of one that is known already... Before fractional numbers can be understood, a major accommodation of the number system is required ...

Indeed, the teaching of all categories of fractions and their structural relationships is a problem of far-reaching influence in mathematics teaching in The Bahamas. Also some interesting examples are contained in the history of mathematics highlighting the difficulty of accommodation presented by new number systems. For example, Bell [77, p.391] says when negative numbers first appeared in experience, as in debits instead of credits, they as numbers were held in the same
abhorrence as 'unnatural' monstrosities as were later the 'imaginary' numbers \( \sqrt{-1}, \sqrt{-2} \), etc. arising from the formal solution of equations such as \( x^2 + 1 = 0, \ x^2 + 2 = 0 \), etc.

The problem of accommodation, that learners experience, seems to suggest that the teacher is a mediator between the learner and mathematics, holding high expectations of his pupils (largely of average ability in The Bahamas) and passing on his contagious sympathy for, and understanding of, mathematical relationships. In support of this viewpoint Krutetskii [46] demonstrates that mathematical abilities are not innate but are properties acquired in life that are formed on the basis of certain inborn inclinations. These inclinations are minimal in instances of the growth of ordinary abilities in mathematics but outstandingly great in the development of the rare giftedness of research mathematicians. But Krutetskii [46, pp.74, 75] comes to the kernel of useful psychological information when he says

> By ability to learn mathematics we mean individual psychological characteristics (primarily characteristics of mental activity) that answer the requirements of school mathematical activity and that influence ... success in the creative mastery of mathematics as a school subject - in particular, a relatively rapid, easy and thorough mastery of knowledge, skills and habits in mathematics.

Krutetskii's reference to the psychological characteristics of the individual learner draws Matthews [78, p.21], a consistent disciple of the Geneva school, into the discussion. He (Matthews) indicates that, on an individual basis, children in middle schools at ages 8 to 12 or 9 to 13 do not automatically change their method of thinking from 'concrete operational' to 'formal operational' just because they have arrived at secondary school. This is evidence that the practical mathematical activity of the primary school should be continued in the early years of secondary school - (see
also Cockcroft [50, pp. 72,73, paragraph 247]) - so that there is a gradual transition from the concrete operational to the formal operational. In fact Rowland[78, p.29], who echoes Matthews' sentiments, gives an account of how learning is organised to cater for individual differences - the guiding philosophy of comprehensive education:

The main part of my teaching, or perhaps I should say the children's learning, is done through mathematical topics, which they investigate in groups. If the class can suggest a topic that we can profitably investigate I am delighted, but I usually find that I have to build up an interest first. I do this by introducing charts or models and asking the children to collect pictures or information. In this way I can stimulate interest before we start work. This may sound a little forced, but anyone who has dealt with this particular age group will know that it is not difficult to arouse interest in anything new ... As soon as we have decided on a topic we discuss it and consider what mathematics is involved. This is often a suitable time to introduce a new skill that is going to be needed. I then run off assignment sheets and the children divide into groups ready to start work.

This method, which serves the formative approach, requires reorganisation of classroom furniture, availability of basic items of a mathematics kit, storage of finished and unfinished work, discussions of difficulties at the individual level, ensuring a child knows what he is doing and why, encouragement of generalisations and conclusions from manipulation of concrete objects, and discovery and correction of basic weaknesses. It is out of an environment of this kind that motivation is provided for tables of various forms and it is not unknown for a study of tessellations to progress to the isometries - translation, reflection, rotation - which leave shape and size invariant. Motivation for a variety of topics is stimulated within an atmosphere where learning is maximised in mixed ability teaching organisation. Encourage-
ment in mathematical exploration is cajoled to terminate in
discovery but the teacher must be supremely confident of
content and pedagogy to derive pleasure from, and give
pleasure to, the pupils. The basic philosophy of mixed
ability teaching is individualised learning [55, pp.16-25],
where ideally children take different routes through topics.
One route, for instance, could guide bright children on to
work that would extend them whereas another could concentrate
on more basic work. Workcards, and duplicated sheets take the
place of textbooks although they could be available for
reference. Some class teaching, films, chalkboard work, and
class discussions are not eliminated although tedium enters
through varying levels of marking, assessment and record
keeping. But the problems are outweighed by the educational
advantages coupled with enjoyment for both enthusiastic teacher
and pupils eager to learn. In the literature, mixed ability
teaching is strongly supported on psychological grounds.
Dienes [69, pp.62, 63] claims that

... different rates of progress are evidenced in
different areas of the same concept structure by
different children. This means that collective
instruction is highly impractical and inefficient. If
thirty different children in the classroom were treated
in exactly the same way, some would be wasting their
time, others would receive presentations unsuitable to
their ways of thinking, and others would not follow at
all.

Berrill and Sampson [56, pp.47-51] in their paper, describe a
method of teaching mathematics to mixed ability classes of a
similar type to that found in United Estates, San Salvador,
say, where the author had six years non-professional teaching
experience. Whilst the basic philosophy contains a solution
to mixed ability teaching of mathematics in an all age school
in a typical Family Island of The Bahamas, the physical
environment and materials culture are not the same. Similarly,
for maximum learning effect, mathematics exercises designed
for overseas schools must always be modified to give a local
cultural bias. Whittaker [79, p.8] also makes helpful comments.
In the literature on psychology and mathematics education certain key phrases comprised a recurring theme. 'Different rates of progress' emanated from Dienes [69, pp.62,63]; 'characteristics of mental activity' was offered by Krutetskii [46, pp.74,75]; the acquisition of 'a major accommodation of the number system' was postulated by Skemp [64, p.45]; 'the logical structure that informs children's solutions to problems' was put forward by Bruner [76, p.7]; and 'concrete operational' and 'formal operational', epistemological language used by Matthews [78, p.21], an exponent of the Geneva school, were all references to the important concept of intelligence. A very popular philosophy encountered in the theory of mathematics education was that for optimum efficiency in learning mathematics, its teaching should be done in such a way that the structure of mathematics becomes integrated into the structure of intelligence. Skemp [63, p.264] puts the point very convincingly in this alternative fashion.

Preparation for intelligent teaching includes an analysis of the subject matter in order to become aware of its conceptual structure; so that it can be taught in a way which enables the learner to re-synthesize this structure in his own mind as a schema.

This concept of structure has been a typical one amongst psychologists, mathematicians and mathematical educators because it is theoretically superimposed on primitive somatic structures thereby becoming more highly developed - insofar as it is partially a function of time and partially a function of mathematical activity - and leads to the only structure which develops onto itself: mathematical structure. In his well-informed and relevant discourse on mathematics and psychology, Gattegno [80, pp.38, 39] stakes the claim that:

The notion of structure in mathematics has provided the opportunity for an internal clarification of mathematics itself, a purification from which pseudo-problems were eliminated. It has similarly served to base mathematics on itself thus reducing it to an autonomous and
organised mental activity obeying its own laws at all levels ... The notion of mental structure has therefore served the double purpose of understanding a fundamental intellectual activity of humanity, and of enriching science which must attempt to explain all human activities.

In psychology, the notion of mental structures must be considered a primitive one.

In the same vein but no less eloquently, Dienes [69; p.62], contributing to the debate, suggests

The pedagogical problem is how to generate abstractive mathematics learning ... mathematics is extremely hierarchical; it is a network of all sorts of interwoven relationships, all built on top of and underneath each other. The learning of such an intricate set of patterns must naturally take into account the way in which the patterns fit together. This is one of the problems of getting any mathematical learning started. We must carefully take into account the subordinates and superordinates of the subject.

Alongside the preceding concept of the matrix of intelligence were two theoretical views which influenced classroom practice: (1) the psychometric view which was associated with the measurement of abilities and (2) the developmental view which was concerned with the structure of abilities.

On the basis of the first idea, intelligence was thought to be determined by innate factors, in which case children were sorted out by the 11+, and taught together in homogeneous groups. It was this interpretation which influenced the Norwood Committee [25, pp.37, 38] when its recommendations were made for tripartite secondary education in England and Wales in 1943. Unfortunately, environmental influences on test performance and measured intelligence, among other influences, were not brought to account. The effect of this view on classroom practice was
The developmental view of intelligence however, suggests a very different model of education: one the function of which is to guide the developing child through the stages of intellectual growth, providing an appropriately structured curriculum in terms of learning sequences, yet permitting pupils freedom to discover and structure knowledge for themselves. With regard to teaching method, the developmental (formative) approach—preferred by the British and adopted by the Nuffield Mathematics Project—emphasised individual teaching and learning. Writing about the merits of this approach, Downey and Kelly [74, p.40] advise

To try to teach a whole class of children as if they were all at one stage in everything is to deny the importance of individual rates of growth as revealed by Piagetian research: for those children who are not yet ready to cope with the concepts involved the content will mean little and they will make little or no progress; for those who have already mastered those concepts it will mean nothing but repetition of what they already know, with little or no novelty to activate their minds. Learning on this model of intelligence must by its very nature then be individualized; and since the classroom is the environment in school with which the child interacts to structure his intelligence, teaching methods must allow for this interaction. Discovery and inquiry methods are best suited to this but to offer optimum opportunities for intellectual growth, the material must be geared to the child's own stage of development.

This interpretation expressed by Downey and Kelly [74, p.40] harmonises perfectly with that postulated by Dienes [69, pp.62, 63], thus indicating how a classic example of coincidence of educational theory with psychological theory finds immediate application in current classroom practice. Moreover, stemming from this marriage of educational and psychological theory is the suggestion that discovery and inquiry methods (strategies)
of teaching should themselves be aligned to the stage of psychological - epistemological preparedness. Throughout his writing Bruner [38], [68], [76], [81] - like Matthews [78, p.21] and Krutetskii [46, p.43] - shows his deep commitment to the three hypothesised stages of intellectual development proposed by the Geneva school after empirical investigations, and uses them adroitly in a discussion of readiness for learning. Whereas the first stage provides useful background information, the stages of concrete development and formal operations (age 10-14) are of immediate relevance to the work of teachers in the sense that they can guide and shape the choice of teaching strategy. For Bruner [68, p.35]

The second stage of development - and now the child is in school - is called the stage of concrete operations. This stage is operational in contrast to the preceding stage, which is merely active. An operation is a type of action: it can be carried out rather directly by the manipulation of objects, or internally, as when one manipulates the symbols that represent things and relations in one's mind. Roughly, an operation is a means of getting data about the real world into the mind and there transforming them so that they can be organised and used selectively in the solution of problems.

It must not be thought, however, that Piaget's postulates were unquestionably accepted. Gattegno [82, pp. 2, 3] indicates that he volunteered to propagate the thinking of Piaget from 1945 to 1951 at the end of which time by force of evidence, he articulated a viewpoint which suggested he had found the limiting conceptual constraints in Piaget's work, thus making them less tenable as a contribution to the improvement of the teaching of mathematics. Nevertheless Gattegno [82, p.41] does believe in the prevailing psychological theory of the formation of mental structures.

Although the whole philosophy of learning by discovery has been the subject of intense debate and although experimental studies
undertaken to demonstrate the superiority of one method over the other have been generally inconclusive, the idea has had a profound effect on proposals for curriculum development. Bruner makes no claim to have discovered the discovery approach but his passionate and eloquent espousal of it strengthened its theoretical foundations. For, says he (Bruner [68, p.20]):

... it would seem that an important ingredient is a sense of excitement about discovery - discovery of regularities of previously unrecognised relations and similarities between ideas, with a resulting sense of self-confidence in one's abilities. Various people who have worked on curricula in science and mathematics have urged that it is possible to present the fundamental structure of a discipline in such a way as to preserve some of the exciting sequences that lead a student to discover for himself.

It is indeed remarkable, yet not surprising, that Bruner links discovery learning with 'the fundamental structure of a discipline' simultaneously. Among mathematics educators and mathematicians 'structure' is a dominant theme so a return will be made to consider its relevance to the research problem. But pursuing the philosophy of discovery learning, Bruner [76, pp10, 11] suggests that ideas may be represented in the understanding of the learner on three levels: enactive, where pupils as a result of manipulation, conserve past experience in a model and invent mental rules to govern storage and retrieval of information from this model; ikonic, in which mental images of objects derive from neuromuscular coordination of psychomotor skills; and symbolic, where symbols are substituted for the images. Fischbein [62, p.228], in a paper delivered at the 2nd International Congress of Mathematical Education, agrees with Bruner. The theme of mathematical investigations is certainly not new but was elevated to prominence particularly through the activities of members of the Association of Teachers of Mathematics (ATM) in England. A signal contribution by ATM [83] to the debate on 'how mathematics is learnt' and the approaches that teachers should
adopt is to be found between the red soft covers of the compendium of utterances by many practising mathematics educators prepared for The 1969 International Congress of Mathematicians held at Lyon, France. Its model on mathematical education [83, p.15] focuses the problems of the developing countries so very well that it is reproduced here. (Figure 3.2)

Figure 3.2: A Model Showing Elements of Mathematical Education
But the flavour of the underlying philosophy has also been well put by the Association of Teachers in Colleges and Departments of Education (ATCDE) [84, p.3]:

Although it is true that the learning of mathematics has always been considered to involve the solving of problems, there is a total difference of approach between problems set as exercises on specific ideas that are being learned, of which the aim is to establish the ideas securely, and problems which exist in their own right, for their own intrinsic interest, and for the solving of which no particular method is specified. It is the exploration of these more open problems which we feel to be the essential characteristic of real mathematical activity.

But any method or combinations of methods used will not reduce the barrier created by interference factors which lie between the pupil and learning, and between the pupil and the teacher. In fact, in an attempt at solving these problems others may be created. Bruner [81, pp.71, 72], one of the chief protagonists of the strategy of discovery learning, sees six sub-problems deriving from the principal problem of how to teach a child in such a way that he can be relied upon to use knowledge appropriately.

(1) How shall learning be contrived so as to encourage the child to extrapolate from information given and interpolate unconnected data?

(2) How shall a pupil be influenced into integrating and storing new material being learnt into an already cognitive schema of associations, subdivisions, categories, and frames of reference for retrieval on demand?

(3) How can a pupil be stimulated into awareness of his capacity to solve problems and thereby feel compensated for the self-imposed task of thought?
(4) How can the teacher arrange for each pupil to have adequate practice in using information processing and problem solving?

(5) How can teachers organise practice in learning by doing coupled with articulation of what was done?

(6) What can be done by teachers to train pupils to become efficient information processors who readily apply information gained in problem solving?

Dienes [65, pp.24-26], with some of these problems under study, sees that a distinction must be made between artificial and natural learning. The latter provides important clues to the comprehension of children's difficulties when they have to sit through sterile classroom learning situations - with visual aids, models and other mathematical apparatus (cuisenaire rods, etc) outside their experience. Natural learning, which is aided by manipulable objects, is not invariably preferable to artificial learning; it is however a priori probable that it will be more effective. This method greatly assists mathematical structures in taking cognitive roots. He (Dienes) believes that

... the artificial learning now taken for granted in the mathematical lesson does entail a very large failure rate. There is a virtual lack of understanding of the mathematical structures. In the large majority of cases, what students communicate by writing down or uttering mathematical signs is merely the signs themselves and not the structures for which the signs are supposed to act as symbols... One way of overcoming this has been demonstrated by the Leicestershire Mathematics Project, ... In this project, mathematical situations were set up from which children learned mathematical structures in much the same way as they learn about structures in the real world; that is by manipulating actual objects.
In the context of the developmental view of intelligence perusal of the English and North American psychological and educational literature showed that the words *ability*, *capacity*, and *aptitude*, each of which has its nuances, were used synonymously whereas there was a strong preference amongst Soviet psychologists to use *giftedness* because, for them, it logically constituted a universe of discourse for a well defined set of elements. Mathematical giftedness was a unique combination of mathematical abilities that opened up the possibility for individuals to acquire a creative mastery of the discipline. Stemming from this shade of opinion, the fundamental task of the organiser of learning was to establish the conditions in which the formation, cultivation and development of mathematical abilities could be realised. Further, by implication it seemed possible according to Dienes [65, p.31] to undertake a *retroactive analysis* of this integral property of the mind into the constituent elements that occupy an essential place in its structure. Specifically the researcher got impetus to seek a clarification of the features that characterise the mental activity of individually different pupils as revealed in their solutions of mathematical problems requiring the application of proof, calculation, simple interpolation, transformation (reflection) and construction using ruler and compasses. This research offered hope of providing a partial intuitive understanding of how mathematical structure is integrated into mental structure (intelligence) through creative mathematical activity. The psychological literature, and in particular Krutetskii [46], gave the author the distinct intuition of *ability* as a dynamic concept: it not only showed up and existed in an activity but was also created and even developed in it. Accordingly, because mathematical abilities exist in a dynamic state, they have potential to be cultivated. As regards the constituent elements (structure) of mathematical abilities, Krutetskii [46, pp.87,88], having completed exhaustive empirical studies, postulates:

(1) An ability to formalise mathematical material, to isolate form from content, to abstract oneself from concrete numerical relationships and spatial forms, and to operate with formal structure - with structures of relationships and connections.
An ability to generalize mathematical material, to detect what is of chief importance, abstracting oneself from the irrelevant, and to see what is common in what is extremely different.

An ability to operate with numerals and other symbols.

An ability for "sequential, properly segmented logical reasoning" ... which is related to the need for proof, substantiation, and deductions.

An ability to shorten the reasoning process, to think in curtailed structures.

An ability to reverse a mental process (to transfer from a direct to a reverse train of thought).

Flexibility of thought - an ability to switch from one mental operation to another; freedom from the binding influence of the commonplace and the hackneyed. This characteristic of thinking is important for the creative work of a mathematician.

A mathematical memory. It can be assumed that its characteristics also arise from the specific features of the mathematical sciences, that this is a memory for generalizations, formalized structures, and logical schemes.

An ability for spatial concepts, which is directly related to the presence of a branch of mathematics such as geometry (especially the geometry of space).

The child's use of mathematical ideas is an index of his ability and Bishop [85, p.120] admiring the contribution of Krutetskii advises:

The shift in construct from 'ability' to 'abilities' is significant and seems to convey a change from something that differentiates between pupils (high and low ability) to something that they all share in some form, from something which may be inherent, to something which is
potentially developable in all pupils. Some of Krutetskii's 'abilities' are a striving for clarity, simplicity, economy and rationality of solutions, the ability to curtail the process of mathematical reasoning, and the ability to switch from a direct to a reverse train of thought.

Further exploration of the common ground of debate between mathematicians cum psychologists and mathematical educators resulted in the identification of an informed school of opinion, for which Fischbein [62, p.226] stands proxy. This school advocates:

Piaget has defined structure in the broadest sense as a system and this system is a totality that has laws and properties that are characteristic of it as a totality [4].

A structure is constituted of a set of elements between which there are certain relations. It is well known that for Bourbaki there are three types of structures: (a) algebraic, (b) order, and (c) topological. With the help of these structures the diverse branches of mathematics can be brought together to form an architectural unity.

This excursion into various theoretical views of intelligence brought out substantial authoritative support for the more tentative suggestion (5) on page 115, which derived partly from experience (subjective judgement) and partly from perusal of the literature: namely, that pedagogically, mathematics should be treated as a unified subject instead of as four disjoint subsets arithmetic, algebra, geometry and trigonometry as is done at the secondary school level. But one has to make haste slowly in enforcing this as a policy of mathematics education in The Bahamas although it must be kept in mind as a guiding philosophy.

There was uncompromising agreement between Piaget and Bourbaki on the three fundamental structures on which knowledge of
mathematics rests. Having examined carefully the correspondence between mathematical structures and the postulated structures of thought (intelligence), Piaget was convinced, says Krutetskii [46, p.44], that

... in teaching mathematics a distinctive synthesis should occur between the mathematical structures revealed by mathematicians and the operative structures of thought revealed by psychologists.

Griffiths and Howson [33, pp.214, 215] credit Bourbaki with 'mathematical insight.' By analogy they (Griffiths and Howson) demonstrate that a student, who writes down the function

\[
\frac{2}{\sqrt{(-2-x^2)}} \quad \text{in answer to} \quad \frac{d}{dx} [\sin^{-1}(1 + x^2)],
\]

using algebraic manipulations of the calculus without the motivating graphical model of inverse trigonometric functions, shows no insight. Psychologists hypothesise that a natural leap beyond insight is intuition, an important prerequisite element in the matrix of intelligence for appreciation and construction of formal proof. Fischbein [62, p.225] says that natural (primary) intuitions, formulated during the ontogenesis of mathematics and outside the pale of systematic influence, give birth to the exercise of secondary intuitions which become especially active over the period of concrete operations. These are so essential in learning mathematics that Fischbein [62, p.225, 226] advises

... This process of refining and correcting the intuitive basis should not be allowed to go on in a haphazard manner (or not to happen at all) but should be an integral part of the teaching process. If an intuitive basis is lacking, as can frequently happen, for example, in the field of probability, it is vital that these intuitions should be constructed with the help of well-programmed exercises spread over a long period whilst intelligence is developing.
Piaget himself [86, pp.121, 126] is at one with the Bourbaki school, until further research proves otherwise, that the mathematical edifice rests on the three fundamental structures—algebraic, order and topological—and emphasises that

In reality, if the edifice of mathematics rests on 'structures' that correspond to the structures of the intellect, it is on the progressive organisation of these operational structures that mathematical teaching must be based.

Making his contribution to the debate, Skemp [64, p.16], who offers intelligence 'B' in Vernon's definition as

... the cumulative total of the schemata or mental plans built up through the individual's interaction with his environment, insofar as his constitutional equipment allows

contends that psychologists, with an interest in intelligent learning, conclude that studying the learning and understanding of mathematics is

studying the functioning of intelligence in what is at once a particularly pure, and also a widely available form.

Skemp [64, p.17] also argues that mathematics is an eminently good example of intelligence 'B' because (1) the learning of mathematics abounds in many clear instances which constitute intelligence 'B' as portrayed by Vernon and (2) the applications of mathematics in different spheres of knowledge are so predominant that it manifests an image of indispensability as a tool for dealing with our physical environment. Producing a crucial evaluation of the findings of psychology, Skemp [64, p.39] suggests that:

The study of the structures themselves is an important part of mathematics; and the study of the ways in which they are built up, and function, is at the very core of the psychology of learning mathematics.
Fischbein [62, pp.226, 227], an accredited disciple of Skemp's view, says that the polemic generated in the search for a unifying philosophy (the structure) within mathematics and psychology polarises into two main schools of opinion:

I The general schemes of thought should be left to form themselves gradually, by a sort of natural generalisation, after the pupil has assimilated a fairly considerable amount of mathematical knowledge. This school believes that:

(1) Knowledge develops naturally from the particular to the general; from the concrete to the abstract. However, there is no claim to originality because Pestalozzi and Froebel were the prototype of this model - the inductive method - in curricula construction.

(2) Characteristically mathematical structures are concepts having the potential for the acquisition of an enormously high degree of abstraction and, in consequence their comprehension is beyond utility until the intelligence has reached its zenith in the stage of formal operation, i.e. the state of final equilibrium (at age 15 or 16 years).

(3) These structures originate out of the confrontation between varied mathematical domains. The pupil must therefore have the opportunity and capability of understanding on a fairly broad mathematical basis in arithmetic, algebra and geometry.

In essence the view of this school was that acquaintance with mental structures following a good exposure to mathematics would foster assimilation of generalisations (mathematical structures). Obviously the members of this school were so fossilised in their views that their credibility would be threatened outside the haven of 'traditional mathematics'.

II It is better for children to be brought up in an environment which encourages manipulative mental activities (growth of
mathematical structures). This aids the establishment of a storehouse for the development of more complex mathematical thought. This school believes that these structures:

1. manifest the fundamental structures and general schemas of intelligence and are not just a method of giving information or a manner of proceeding in a particular example

2. must be allowed to operate during the period of the development of intelligence

3. should be introduced in the earliest period - during preoperational growth if possible - so as to permit maximum efficiency of operation of mathematical thought during the stage of final equilibrium in formal operations.

This second school is linked very closely with contemporary developmental psychology, two advocates of which are Krutetskii [46, p.85] and Bruner [.68, p.46]. Quoting from Inhelder's memorandum, the latter says:

One wonders in the light of all this (introduction of early probabilistic intuition through games) whether it might not be interesting to devote the first two years of school to a series of exercises in manipulating, classifying, and ordering objects in ways that highlight basic operations of logical addition, multiplication, inclusion, serial ordering, and the like. For surely these logical operations are the basis of more specific operations and concepts of all mathematics and science. It may indeed be the case that such an early science and mathematics 'precurriculum' might go a long way toward building up in the child the kind of intuitive and more inductive understanding that could be given embodiment later in formal courses in mathematics and sciences. The effect of such an approach would be,
we think, to put more continuity into science and mathematics and also give the child a much better and firmer comprehension of the concepts which, unless he has this early foundation, he will mouth without being able to use them in any effective way.

Subsequent to affirming Soviet psychologists' support for the specialised use of information in the sense used by Fischbein [62, p.227] and disclaiming originality of its use, Krutetskii [46, p.184] distinguishes the following states of mental activity in the process of solving mathematical problems:

(1) Receiving information about the problem (related to an initial orientation towards its terms, an attempt to understand it).

(2) Processing (transforming) the obtained information for the purpose of solving the problem, and obtaining the desired result.

(3) Retaining information about the problem.

The writer thinks that these disjoint stages in problem solving are only enunciated separately for convenience: in the literature called the perceptual, intellectual and mnemonic links in mental activity, they should naturally interpenetrate one another especially if one imagines the feeling of conquest a pupil experiences upon encounter with a mathematical problem the solution of which falls within the competence of his mental mathematical structures, which develop in conformity with the stimulus-response (S-R) theory of learning. S-R theory, which thrives on discovery learning, is admirably suited to traditional mathematics teaching which, as a philosophy, uses the behaviourist approach to curriculum building as a vehicle of expression. Similarly, the behaviourist psychology, as promulgated by Howson et al [87, pp. 182-205], Tyler [88, pp. 29-36], Glaser [88, pp. 37-51], and Wiseman and Pidgeon [88, pp. 60-67], is ideally appropriate for curriculum evaluation and validation of content, materials and methods. An exemplar of
the function of evaluation is discussed between pp. 158-160 but no attempt will be made there to compare and contrast it with research. Davis [89, pp. 14, 15], beyond giving a very thorough treatment of the teacher's contribution to effective curriculum evaluation in the rest of his book, argues that

...the main differences between evaluation and classic research methodology has been made by Henderson (1978), who concentrates on the following six considerations;

(i) problem selection and definition in research is the responsibility of the researcher, whereas in evaluation the context of the study almost completely defines the problems for investigation.

(ii) research hypotheses are usually derived by deduction from theories or by induction from an organised body of knowledge. In evaluation, precise hypotheses can rarely be generated, and the task more usually becomes that of testing generalisations derived from previous knowledge and experience.

(iii) whereas research can be replicated, given the statement of the problem and the hypothesis, every evaluation study is unique.

(iv) research design involves the control and manipulation of selected variables, with the systematic elimination of other variables by randomisation. Evaluations have to be conducted in the presence of a multitude of variables which could have relevance in the interpretation of results, with randomisation generally impossible or impractical to accomplish.
(v) the data required in a research study are determined largely by the problem and the hypothesis. In evaluation, the data to be collected are heavily influenced, if not determined, by feasibility.

(vi) and perhaps most crucial, the researcher tries to limit his value judgements to those implicit in the selection of the problem: thereafter his aim is objectivity. The evaluator cannot escape value judgements, both in his own and those of the people involved in the study, at every stage - in the definition of the problem, in the formulation of the generalisations to be tested, in the selection of variables for the study, and in the choice of the data to be collected.

But the similarities and differences between research and evaluation listed above, as Davis himself points out, are pegged to the restricted perspective of research as an empirical enquiry carried out by the hypothetico-deductive method. However, for positive change in the quality of schooling and the curriculum it would be far more profitable if less fuss were made of comparison and contrast between research and evaluation. In fact, in the interest of a balanced view, Davis [89, p.15] continues:

...there is an equally great variety of approaches, techniques and methods of procedure for evaluation as for research. In terms of actual practice, a strong case could in fact be made for placing some - evaluation techniques at one end of a scale, with action-research somewhere in the middle and laboratory-type controlled experiments at the other end. Various kinds of evaluation, action-research, classroom trials, ethnographies and social science research could be selected to fill the rest of this scale. Henderson himself has said that 'Research must always be involved
in an evaluative endeavour and some evaluative activity is often involved in research'.

Within the limits of the parameters defined above, this research therefore qualifies as part of an evaluative exercise to investigate the effectiveness, or ineffectiveness depending on one's viewpoint, of the secondary mathematics curriculum of The Bahamas education system up to the beginning of the school year 1981-82, and Davis [89, p. 17] corroborates the author's view by showing, in a diagram taken from Elliot et al 1979, the structural links between evaluation, measurement, assessment and description. The author is duty-bound to point out, however, that the serious limitation on the utility of findings derived from the present research is that variables which are not amenable to mathematical or statistical measurement (using 'or' in the logical sense) tend to go unnoticed.

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Figure 3.3 Structural Links Between Evaluation, Assessment, Measurement and Description (From Davis [89, p.17])
3.5 Summary II

Structure, the current notion of the psychology of cognitive functions (perception and intelligence), was found to be very relevant to this research problem

f: teaching-learning \rightarrow improved achievement

with respect to mathematics. The primary concepts were the natural number, \( N \) together with the operations of multiplication \((\times)\) and addition \((+); \) but conservation of length, weight, and capacity (volume) were also very important. The concept of \( N \) was extended to higher order concepts: the integer, \( Z \); the rational number, \( Q \); and the real number, \( R \). However the problem of assimilation was a perennial one in mathematics learning. The history of mathematics shows for instance, that similar difficulties were experienced when the concept of the natural numbers was extended via an example such as the solution of \( x^2 + 3 = 0 \) to accommodate the representation of the complex number, \( C \). The Cartesian plane was shown to be isomorphic to Euclidean space, consequently the definitions of point, line and angle - primary geometrical concepts - could be represented without a premature appeal to abstraction. This offered several other practical approaches to the teaching of geometry (coordinates, vectors, symmetry and transformations) instead of the congruence method which has been completely sterile in The Bahamas. Some of the advantages of these other approaches were that algebra was not introduced as generalised arithmetic and it was possible to show the crucial function of a variable in algebras - matrix, Boolean, vector and real number. The potential of matrix algebra lay in its unique suitability for developing the concept of transformation as one aspect of function thereby paving the way for the development of higher order concepts. Further, geometry, algebra and trigonometry became inter-related as one entity thus giving embodiment to Bourbaki's [86, p 121] postulate that the mathematical edifice rests on algebraic, order and topological structures. By keeping trigonometry as part of
a unity of mathematical structure, it then became easy to extend the concepts of circular functions to the concepts of hyperbolic functions. Moreover, the intuitive approach via analytical geometry of the Cartesian plane clarified certain fundamental aspects of functions: namely, limits and continuity which underlie the basic concepts of the calculus.

\[
\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}
\]

and

\[
\lim_{\Delta x \to 0} \sum_{x=a}^{b} f(x) \Delta x = \int_{x=a}^{x=b} f(x) dx
\]

where \(\frac{dy}{dx}\) is the limiting value of a sequence of quotients and \(\int_{x=a}^{b} f(x) dx\) is the limit of a sum rather than an antiderivative. A key advantage of this approach was that the necessity to prove many theorems was obviated. In fact, visual perception is such a powerful concept in learning that Skemp [63, p 37] says it is the state in which

..... inner reality and actuality fuse into a combined experience in which each is modified by the other.

So an intuitive feel preceded by insight illustrated the theoretical operation of intelligence in conceptual learning. But the spiral curriculum, which doubled back onto itself many times permitted constant reference to, and use of, previously formed concepts that were acquired initially by the algorithm:

\[
\text{Mathematical experience} \rightarrow \text{Formation of concepts} \rightarrow \text{Symbolic representation}.
\]

In fact, this order of acquisition suggested that mathematics is a structured system of concepts and not a collection of facts. Consequently, pupils should not be told mathematics as they can be told that Nassau is the capital city of
The Bahamas. Because of the prevailing theory that mathematical structure gets integrated into the structure of intelligence, Soviet developmental psychologists, with some justification, emphasise mathematical abilities instead of mathematical ability owing to the dynamics of concept formation and development.

A model of mathematical education postulated by the ATM in England was used only as a guide in an exercise for selecting the problems in The Bahamas. It seemed that any hope of partial solution must lie in espousal and application of a practical philosophy arrived at through study of prevailing models of curriculum strategies and an evaluation of the existing Bahamian curriculum, giving high priority to the primary stage. In this context Biggs [90, pp. 9-20] offers some useful suggestions which can trigger off other ideas.
3.6 Some Approaches to Curriculum Building

Now that the views of psychologists favour the structure of mathematical abilities; indicate how these are integrated dynamically into the structure of intelligence through activity: and mathematical educators and mathematicians-cum-psychologists have elucidated how mathematical structure is best developed and taught, the writer had to peruse the published literature for the influence of the theory by psychologists and mathematicians on the methods and techniques of curriculum design and development in mathematics education. It seemed axiomatic that those mathematics educators who advocated the approach should have been aware of how the techniques, by which information is communicated and processed, carry out a method that is consistent with any given approach.

Mention has already been made of the discovery or activity method and its influence on the development of mathematical structure and the structure of intelligence. The educational theory behind discovery learning has a well established place in the history of education - from Rousseau (French), Pestalozzi (Swiss), Froebel (German) to the Nuffield Project (English). It seemed, however, that the real reasons for the partial lack of influence of this theory on classroom practice are:

1 lack of suitably qualified teachers
2 interference in completion of syllabuses for external examinations.
3 demands of organisation and preparation on teachers' time

Theories deriving from the Geneva school strongly support discovery learning although they are silent on teaching methods. Further support comes from Gagné [69, p 171]:

The discovery method of instruction is another kind of instructional variable that can be reasonably investigated in educational settings.
Actually there have been a number of studies, many of them on mathematical subject matter. On balance, I believe they tend to show an advantage for what has come to be called "guided discovery" ... Cronbach [16] has reviewed these studies critically, and believes they do not provide dependable evidence for or against discovery ... A recent study by Worthen (1968) indicates that discovery methods of classroom ... teaching promote improved retention and transfer of learning. (* Gagné's reference)

The Nuffield Mathematics Project - a disciple of the formative approach - has devised methods of checking on children's learning in co-operation with the Piaget Institute, which has tremendous influence on the concrete operational stage of primary education that transitonally passes into the formal operational stage of the secondary school. From the literature it seemed that the theory of learning is still in an embryonic stage of development although Skinner's stimulus-response (S-R) theory [91, pp 26-28], despite opposition by Tolman [91, p 24] who says

..... student reactions may be understood in wider terms than those associated with the stimulus-response schemes of some psychologists

has not only found centre stage but branched into programmed learning [92, p 170]. S-R theory, used extensively throughout the teaching of traditional mathematics, rewarded pupils by reinforcement if responses were correct but ignored incorrect responses. Over time, responses were directed towards mastery of content by continuous rehearsals (the behaviourist approach).

Published work - for example Wilson [93, p 141], Griffiths and Howson [33, pp 125-153], and Bruner [68, pp vii-xvi] are representatives of a vast literature which shaped and
aided the writer's identification of problems - indicated that the trumpeters of curriculum change were individuals, mathematics teachers' associations and mathematics projects. Indeed Bruner, a leader of the *structuralist approach* [68, p vii] informs:

Major efforts in curriculum design had been launched by leading physicists, mathematicians, biologists, and chemists, and similar projects were in prospect in other fields of scientific endeavour. Something new was stirring in the land.

But it is Griffiths and Howson [33, p 146], although themselves university mathematics lecturers, who outline the logistics for initiation of action:

Irrespective of the source of money - whether private or public - it is essential that the initial appeal for assistance should be accompanied by some plan of action outlining the aims of the project and the methods it would use to attain them. The writing of such a plan is now frequently treated as a separate little project - the prelude to the main work - and is termed a 'feasibility study'. Such a study is generally carried out by one or two persons who will spend up to a year visiting schools and other educational establishments. There they collect opinions on suitable objectives for the project and on appropriate methods, and, in particular, they look out for existing work which can be taken as a starting point for the work of the project. For, if the project can build on existing work, it is more easily seen to be a practical proposition and not just a theoretical exercise. The study will probably conclude with proposals for staffing and housing the project, together with estimates of costs involved.
Projects in mathematics education seemed to emphasise co-operation among teachers in

1. the selection of suitable content areas for which work cards, etc are produced

2. managing trials of materials in pilot schools in which project teachers work

3. adjustments to original materials as dictated by feedback, with due allowance for total failure of impact of materials

4. training of teachers in the use of the materials, with teachers' guides to give support

5. establishing teachers' centres for on-going communication between teachers from pilot schools in both primary and secondary sectors

6. nominating project committees to sit on examination boards to ensure suitable examinations for pupils who use project materials

7. putting the view to primary schools that in order to deepen understanding of mathematics basic concepts must extend beyond arithmetic so that ultimately preparation is made for solving mathematical problems

8. assessing pupils' abilities on an on-going basis

9. seeing that dissatisfaction is not so much with organisation of content but the way in which mathematics is taught

10. realizing that the danger in university dominated reform is the prospect of training a new generation of professional mathematicians rather than providing mathematics courses for the average child as is expected of a schoolteacher dominated project
Examples of projects with international influence are SMSG (USA) and SMP and SMG (UK). On continental Europe [37] is a representative product of curriculum development activities.

Change in approach to methods and syllabuses was dictated by the sterility of traditional methods of axiomatic teaching. The predominant method emphasised by the projects of the 1960's was discovery which generated mathematical ideas rather than imposed technical skill. This method of teaching emphasised algebraic structure and permitted definitions and axioms to stem from concrete illustrations with which the pupil was familiar. Deductive treatment was deferred for later in the school experience especially since psychological findings supported allowance for group work and discussions of findings from physical models. It was the Geneva school of psychology which demonstrated that there was a correspondence between mathematical structure and the elementary structure which in turn aided a grasp of mathematics. Feedback from the classroom of project schools showed that increased excitement and enthusiasm went far beyond the novelty of the approach and the less able than average pupil to cope with the conventional traditional syllabus had kept up with the new syllabus although few pupils did find the new approach uncongenial. The aims:

1 to make mathematics a more attractive, enjoyable and exciting subject at school

2 to make mathematics more stimulating to teach

3 to impart knowledge of the nature of mathematics and its uses in the modern world to pupils of a wider ability range

4 to convey the nature of various algebraic concepts rather than impart a definite body of knowledge

were all achieved.
Although Bruner [68, p 12] agreed with structure as an important teaching-learning concept, he conceded that too little was known about how to teach fundamental structure effectively or how to provide learning conditions that fostered it. Nevertheless, having looked at all probabilities in the face of insufficient research he (Bruner [68, p 20]) asserts

..... it would seem that an important ingredient is a sense of excitement about discovery-learning of regularities of previously unrecognised relations and similarities between ideas, with a resulting sense of self confidence in one's abilities. Various people who have worked on curricula in science and mathematics have urged that it is possible to present the fundamental structure of a discipline in such a way as to preserve some of its exciting sequences that lead a student to discover for himself.

But a point which scores heavily against discovery teaching is that it is time-consuming to the extent that certain aspects of syllabus content and parts of the pupils' training by the teacher may have to be foregone. Anticipating these problems and, at the same time putting his observations in balance, Bruner [68, p 21] informs

They (the Committee on School Mathematics and the Arithmetic Project of the University of Illinois) have been active in devising methods that permit a student to discover for himself the generalization that lies behind a particular mathematical operation, and they contrast this approach with the "method of assertion and proof" in which the generalization is first stated by the teacher and the class asked to proceed through the proof. It has also been pointed out ... that the method of discovery would be too time-consuming for presenting all of what a student must cover in
mathematics. The proper balance between the two is anything but plain, and research is in progress to elucidate the matter, though more is needed.

Thwaites [94, p 17], reporting that one of the main changes to be found in new syllabuses was increased emphasis on algebraic structure, indicated that pupils must be helped to perceive a distinction between assumptions and definitions and consequences, and between intuitive feeling and proof. Furthermore, time for introduction of new ideas was gained from reducing the complication of the examples in which those ideas were applied rather than elimination of large parts of the existing syllabus. Thus, although it was expected that a pupil would acquire reasonable facility in solving simple and simultaneous linear equations, factorising, etc, the emphasis was on his understanding the process involved rather than his ability to cope with complicated applications. What Thwaites [94, pp 18, 19] says about geometry is of crucial importance.

Our proposals in geometry are a natural consequence of the rejection of axiomatic teaching in algebra. We do not dispute that the training in deductive reasoning offered by the formal geometry of the conventional syllabus has been of great value to some pupils (though such pupils would in all likelihood benefit equally from an axiomatic treatment of algebra). But for the majority of pupils, formal geometry offers little training in logical reasoning and emphasizes, instead, practice in the memorising of theorems and proofs of no particular worth. In place, therefore, of a 'watered-down Euclid' approach we have substituted the study of Euclidean space by means of the geometrical transformations of rotation, reflection, translation and enlargement, and we hope that in this way the child will come to have a feeling for such spatial relationships.....
Co-ordinates, vectors and displacements are all introduced much earlier than is now usual and throughout the course the interplay between algebra and geometry is emphasized. For example, it is shown how the geometrical transformations can be expressed in matrix form and yield simple examples of groups.

In the process of further elaborating the fundamental philosophy of SMP with reference to approach, Thwaites [94] showed that emphasis in syllabuses had shifted from manipulative technique which, according to Wilson [24, p 357], occurred ad nauseam in the English-speaking islands of the Eastern Caribbean, towards mathematical ideas. This transfer of emphasis however was not to strip manipulative technique of its importance in utility by scientists, mathematicians and engineers: informed and well considered opinion was that manipulative techniques were best left for the post-O-level stage where its need would then have become apparent. An added reason was that it minimised the learning of those pupils who proposed to leave school at the end of the O-level stage. But a philosophy does not come into being on day one: it evolves. Griffiths and Howson [33, pp 184-186], who recount the stages in development in SMP thinking, conclude with a terse statement of the final crystallisation of philosophy:

"The work on vectors, in particular, was greatly extended and some elementary topology included (SMP Book 2). Pupils were now explicitly required to know about rational, irrational and prime numbers and to find the solution sets of equations in various domains. In trigonometry, the general angle was now considered. Greater coverage was given to statistics, probability and other applications of mathematics.

On the other hand - and significantly for mathematical educators - some of the more abstract work contained in the earlier course had disappeared. Although the idea of structure
and of a group was retained (SMP Book 5), the more abstract concept of isomorphic structures (SMP Book T4) was omitted. Again, the frontal attack on 'proof' (SMP Book T) was dropped in favour of a more restricted attempt to show how certain conclusions in geometry can be reached from given data (SMP Book 4). This does not reflect any change in the overall aims of the SMP, but rather is a reframing of specific objectives in an attempt to bring them into line with what experience has shown to be feasible. Thus we have examples of feedback being used to determine changes not only in approach (method) but in specific objectives (content).

During the perusal of the literature, it appeared to the researcher that whichever model of curriculum development is employed - central dissemination or the periphery model - the change in content and method in the classroom is dictated by the verification or rejection of the original objectives of the initiators of curriculum modification. According to Cundy [95, p 43], the Caribbean Mathematics Project (CMP) adopted the latter model and the project's achievements, as evaluated by him indicates:

The Caribbean Mathematics Project certainly tried to change both content and method in the teaching of mathematics. Using the peripheral approach to development, it was dependent on the co-operation and adaptability of teachers in both respects... It... set out to do a number of things simultaneously: to determine what should be taught by diagnosis of pupils' difficulties and appropriate prescription; to move the content of the syllabus away from traditional 'O' level mathematics towards that of the Joint Schools Project texts which it introduced; to advocate group teaching and discovery methods through the use of individual workbooks. The first
objective was attained with considerable success so long as the content remained familiar to teachers. The second objective was achieved only partially: the early material was confused in the minds of some teachers with the American 'new math' in which they had in some cases been trained, and the latter material remained unfamiliar and largely untaught in consequence. The third objective was achieved at first but there are signs of its fading from view under the constraints of the high cost of workbooks and the pressure of examinations.

In this summative evaluation of CMP, the variables took responsive (educational) values as opposed to preordinate ones although there was some indication of a predetermined standard expected of the project. The evaluation views in perspective:

1 the project’s activities rather than its original intent

2 the qualitative values of the variables as being directly proportional to the different opinions of teachers as well as lay people

3 teachers' requirements for new injections of vitality into classroom activities

Furthermore, it was implied that the underlying theme of the evaluative study was that good mathematics education results in measurable outcomes in terms of pupils' performances, mastery, ability and attitude. However it is not always advisable to think of the instrumental value of mathematics education as a basis for evaluating it because the 'payoff' may be diffuse and long delayed, demonstrating a time-bomb effect. In a very real sense the impact may even be beyond the scrutiny of an evaluator in terms of pleasure and enjoyment derived by and given to various participants and consumers. Stake's definition of evaluation [59, p 75] in which he says
Evaluation is an Observed Value compared to some Standard. It is a simple ratio but this numerator is not simple. In program evaluation it pertains to the whole constellation of values held for the program. And the denominator is not simple, for it pertains to the complex of expectations and criteria that different people have for such a program harmonises with Cundy's findings for CMP.

An analysis of the nature of knowledge approach to curriculum planning (theoretical versus common sense knowledge), some educationalists found, led to no fundamental concern with the psychology of the child, although it did view the school as a part of society. Consequently, changes in society must be reflected in the school curriculum. This 'traditional' view of the curriculum was not only found in the best traditions of English education but also stems from Plato's objectively true knowledge type of curriculum, and it was this Platonic-Christian tradition in educational practice that Rousseau inveighed against in his *Emile*. Rousseau's plea to start from the child in making educational provision instead of the subject matter has created learner-centred education. At root, it was an attempt to plan the curriculum on the basis of the needs and interests of the child, thus seeking a justification of the content of education neither in the nature of knowledge nor even the nature of society but in the nature of the child. But taken to extreme, this outlook tends towards curriculum abolition because in effect, it says that the child must be withdrawn from the corrupting influence of society and left to develop on his own. So in one fell swoop the raison d'etre of the teacher ceases to exist. However John Dewey [74] is credited with bridging the discontinuous gap in curriculum theory and practice by suggesting that all knowledge of the world is hypothetical and that learning was a matter of framing and testing hypotheses. Accordingly the teacher directed (or managed) this process as opposed to inflicting indoctrination and inhibition of learning. Support for this interpretation came from Piaget
[86, p 120] and Bruner [81, p 127], who derived empirical evidence for the process of concept development which led to a new perspective on intelligence in which cognitive growth was promoted rather than propositional knowledge acquired. This philosophy placed the emphasis on the guided discovery method of learning and suggested that it is the task of education to enable children to acquire a range of concepts which they can use in their continuous interaction with the environment. The teacher's job, in the context of this philosophy, is to structure activities and experiences in such a way as to facilitate cognitive growth and conceptual development irrespective of the adopted approach to curriculum construction.

Piaget claimed that there was no substitute for the provision of appropriate concrete learning activities in the stage of the development of concrete operations. The author has arrived at a position that could not be anticipated prior to perusal of the literature: namely, the behaviourist philosophy initiated by Dewey has the support of both Piaget and Bruner, who are leading crusaders of the respective formative and structuralist psychological theories which find application in classroom practice. Support in this context did not mean that these theories have a common thread although, surely a valid inference was clear: that an awareness of existing curriculum approaches in mathematics could lead to adoption of what is appropriate in them all to creating a new strategy for an approach aimed at modifying teaching-learning input into the secondary system in The Bahamas for improvement in output (achievement) in mathematics education.

ie f: teaching-learning → improved achievement

is dependent upon a positive change in the independent variable methodology that is informed by local Bahamian cultural and classroom needs. The curriculum, therefore, becomes a proper educational plan when (and this is of crucial importance) it is conceived to take account of the competing claims of the child, the society and the subject matter.
According to Downey and Kelly [74, pp 155, 156], the theory that all knowledge - physical, social, cultural and aesthetic - was of a scientific kind and therefore acquired by the scientific method of framing and testing hypotheses, was upheld on epistemological grounds from Plato's systematic 'dialectic' to Whitehead's 'seamless coat of learning'. This method of curriculum planning emphasises the interdisciplinary nature of knowledge (called the integrated approach) which could generate intrinsic motivation in pupils. An heuristic approach to knowledge such as this was preferred by many teachers to more didactic methods and was given tremendous support in recent reports: namely, Cockcroft [50, pp 95-96] and Department of Education and Science [49, pp 148-154], thus making the learner an active participant in genuine learning experiences rather than a mere passive recipient. Marjoram [96, pp 134 - 163] devotes a whole chapter to the integrated curriculum in terms of pedagogical details and structure of the mathematics for the secondary school in particular. Mathematics Teacher Education Project (MTEP) very strongly supports the teaching of mathematics which uses modelling and provides a context for the practice of routine technique. It also introduces to student teachers in training materials of three curriculum projects - Schools Council Sixth Form Mathematics Project, Mathematics for the Majority Continuation Project and The Continuing Mathematics Project - which share the common philosophy of teaching mathematics through applications. With regard to the requirements of other subjects, Wain and Woodrow [42, p 132] say

The mathematics teacher has a 'service' rôle in providing pupils with the concepts and skills they will need not only in everyday life or in their future jobs, but also in the study of other subjects.

This rôle is becoming more vital and more difficult to fulfil, as more and more areas make use of an increasingly wide range of mathematical techniques - a development due largely to the advent of the computer.
Activities, which can be modified to suit any cultural environment, for pre- and in-service training of teachers are to be found in Wain and Woodrow [54, pp 175-196]. According to Howson et al [87, p 121], the integrated-teaching approach (sometimes called integrated studies) manifested itself simultaneously with, and on the same mental-hypothetical pretext as the formative approach. Beyond enunciation of statements on methods, it requires that the content of learning should be directed towards the interests and needs of learners. In fact they (Howson et al [87, p 122]) say

..... this approach does away with division into subjects of instruction; instead it integrates them according to requirements of the problem at hand. Progress in problem solving, in the sense of discovery learning, is enhanced by the application-oriented means of cognition and by the procedures of various disciplines. In this approach the contribution of mathematics is mathematisation and the provision of models which serve to relate mathematical systems to real situations. The real context of a mathematical idea becomes the subject of the teaching and learning process.

The integrated approach has unlimited popular support in mathematics education in the United Kingdom. The Assistant Masters Association [97, pp 207-208], convinced of the advantages of this approach, claims

..... knowledge is only compartmentalized to assist the presentation of the material; understanding would be improved and time saved if learning cut across subject barriers and the breakdown into subject areas was less rigid. Some degree of cross-reference between subjects is a definite help in the learning process, but teachers of all subjects should be prepared to adopt syllabuses and methods in an endeavour to present knowledge as a unity.
Physics

The link between physics and mathematics is a strong one and consequently the subject is rich in applications of mathematics. Almost every physical experiment requires mathematical calculation to obtain the result, and mathematical techniques such as finding maximum and minimum values of a function find frequent application.


In the comprehensive literature, method was transliterated variously as style, way or more popularly, over the last 20 years in Britain and the United States of America, as strategy to serve an approach. Its pervasive significance lay in its linkages among teacher-training, classroom practice and curriculum development; its unifying feature was the automatic and compulsive implication to impose the economy of the three aspects of educational practice through the same approach. Reference has already been made to the integrated, formative and behaviourist approaches. The psychological theory which informs the structural approach (sometimes called spiral or helical approach) is postulated by Howson et al [87, pp 107, 108] in this way:

..... cognitive structures are combinations of acquired concepts and thinking abilities. Simple structures, made of a few concepts, are developed into more elaborate ones through the addition of new concepts. At their highest stage of development, cognitive structures correspond to the structures of the sciences ..... These structures are so complex that they include all insights, concepts and procedures of the sciences, but they are simultaneously so easy to formulate that they can be transmitted at a lower level of cognition.
The purpose of the transmission of the structures of the scientific disciplines is ... displaying their process character. This ... is the basis for the correspondence between scientific and cognitive structures, and a means of promoting cognitive processes in pupils. Operating within the structures of the sciences reinforces the processes by which new concepts are assimilated by a given stage of cognitive development. The learner is thus given the opportunity to gain familiarity with the structures and to obtain a better grasp of their complexity which, in turn, will further the acquisition of new concepts, etc, until a complete correspondence between the learner's cognitive structures and those of the science is achieved, when the learner will have become a scientist.

Although it was shown previously how mathematical structure was integrated into intelligence, the quotation above from Howson et al goes further to indicate that psychologically the structures of the intelligence resemble the structures of the sciences. Bruner's strength of feeling [68, p 18] concerning the importance of structure is perhaps best depicted by:

The first and most obvious problem is how to construct curricula that can be taught by ordinary teachers to ordinary students and that at the same time reflect clearly the basic or underlying principles of various fields of inquiry.

Howson et al [87, p 108] agree with this author because their evaluation following analysis of the crucial tenets of Bruner's psychology is that:
The critical point of this theory is how to transmit these scientific structures to pupils endowed with lower cognitive structures.

This evaluation, contrary to popular belief in The Bahamas, is profound in its implication that pupils of median abilities are capable of success in secondary mathematics. Bajpai and Bajah [5, p 41, paragraph 32] concur with the conclusion arrived at deductively by Howson et al hence they say:

Today's children are tomorrow's parents and teachers and every effort should be made for the myth of science and mathematics being beyond the normal Bahamian child of average intelligence to be exploded.

The theory of Soviet psychologists also acquiesce in this viewpoint for Krutetskii [46, p 4] postulates

Soviet psychologists are unanimous in the opinion that all children are capable of being taught; that every normal, mentally healthy pupil is capable of obtaining a secondary school education, capable of mastering the school material within the limits of the curriculum; and that the teacher should see to it that all pupils do so... Authorative testimony from the well-known mathematician and academician A N Kolmogorov reads: "The necessity of special aptitude for the study and understanding of mathematics is often exaggerated... Ordinary, average human abilities are quite sufficient for mastering - with good guidance or good books ... - the mathematics that is taught in secondary school" [180, pp 8-9]* (* Krutetskii's reference)

Abilities are not something foreordained once and for all; they are formed and developed through instruction, practice, and mastery of an activity.
For Bruner [68, pp 11-13], one of the most prominent advocates of the structuralist philosophy, a crucial subject of thought in learning and teaching is the function of structure. Elaborating this point with reference to the spiral curriculum which is retrospective at higher levels, he (Bruner) says:

The teaching and learning of structure, rather than simply the mastery of facts and techniques, is at the center of the classic problem of transfer. There are many things that go into learning of this kind, not the least of which are supporting habits and skills that make possible the active use of materials one has come to understand. If earlier learning is to render later learning easier, it must do so by providing a general picture in terms of which the relations between things encountered earlier and later are made as clear as possible.

Dienes [39, pp 112-115] believes that if pupils progress through his Six Stages in the Process of Learning Mathematics, they would have a fertile background of practical mathematical experience and knowledge from which to axiomatise and create proofs for theorems deriving from the rules of games played. Out of this fund of mathematical wealth, the average pupil can effortlessly retrieve facts to satisfy the demands of the definition - theorem - proof syndrome although currently less emphasis is placed on this method of requesting feedback from pupils.

Whilst warming to his discourse on The Importance of Structure, Bruner [68, pp 23-26] appropriates the following four claims:

(1) ... understanding fundamentals makes a subject more comprehensible
(2) Perhaps the most basic thing that can be said about human memory, after a century of intensive research, is that unless detail is placed into a structured pattern, it is rapidly forgotten.

(3) ... an understanding of fundamental principles and ideas appeared to be the main road to adequate "transfer of training".

(4) ... by constantly re-examining material taught in elementary and secondary schools for its fundamental character, one is able to narrow the gap between "advanced" knowledge and "elementary" knowledge.

In (1), (2) and (3) above, Bruner raises what Skemp would call the functions of concept, schema and relational understanding respectively. In fact, Skemp [64, p 93] credits Bruner with describing mathematics as a 'calculus of thought'. However in (4), Bruner underlines the fundamental philosophy of the helical approach to curriculum building in mathematics: that is to say, it (the curriculum) replicates itself. The clearest pointer to the application of the helical curriculum in classroom practice is Brown's research [4, pp 23-40] which identified different levels of understanding of all mathematical concepts among secondary pupils.

Finally, the new-math approach, according to Howson et al [87, pp 100, 101], was a sequel to the treatise on mathematical structure by the Bourbaki school emphasising uniformity of language with accurate terminology. However, it was geared to university level whereas school mathematics seemed to lack both accuracy and systematic organisation. Incentive was thereby created to transform traditional school mathematics to give more emphasis to set theory and on structural aspects such as the associative, commutative and distributive laws. Also Euclidean geometry and advanced trigonometry were jettisoned to be replaced by probability, statistics and computer science. The fundamental stipulation of Bourbaki,
the deduction of content from axioms, became prominent in
the teaching of mathematics. In contrast to the behaviourist
approach, the new-math approach concentrated on better
preparation of potential students who could benefit from
university studies in mathematics: it was more concerned
with content than teaching practice, and its chief spokesman
was Dieudonné [37, pp 31-45]. Lamon [69, p 5], like
Fischbein [62, p 226], not only agrees with the Bourbakian
definition of the three basic mathematical structures;
namely,

1 algebraic structures based on the concept of operation
2 ordering structures based on relations
3 topological structures based on such concepts as
   continuity and proximity

but are also at one in advocating that these structures offer
the best chance of unifying the diverse branches of mathematics.
He (Lamon) suggests also that structured learning experiences
should receive priority in emphasis from the primary to the
end of the secondary school. But although to Lamon
traditional approaches to mathematics have proved to be
inappropriate, modern approaches demonstrate distinct
inadequacies in preserving the accomplishments past methodologies
did produce and improving past failures. In fact mathematics
professors, education and psychology professors as well as
practising mathematics teachers questioned (to put it
mildly) the wisdom of teaching youngsters the so-called
New-Math.

The views of the protagonists and antagonists of the
behaviourist approach will appear in discussion of objectives.
But it suffices to say that the behaviourist model is most
appropriate for a back to basics philosophy in mathematics
teaching since it is easy to apply task analysis (Gagné
[36, pp 263-270] and Davies [92, pp 35-39]) to the exercises
arising out of its content, thus permitting evaluation of integral learning steps. The content of New-Math however, is no less amenable to this kind of analysis.

In the state-maintained schools in The Bahamas, mathematics is taught as bits and pieces of isolated and sterile calculations alongside some set theory with rote learning of associative, commutative and distributive laws, etc for good measure. Linear programming and inequalities, for instance, as taught via the 'New Math' approach, have a secure place in any 'back to basics' movement in the teaching of mathematics. This is to say, new approaches to the teaching of mathematics must adopt a pedagogy emphasising clarity, precision and organisation of content within the presentation phenomenon. Moreover, the researcher emphasises the point - and this is absolutely vital - that the teaching of mathematics within a 'back to basics' approach must present a structural and integrated perspective to algebra, arithmetic, geometry and trigonometry. The purpose of this prescription is to facilitate the understanding of analysis which, beyond the secondary stage, presupposes that the various branches of mathematics had an integrated treatment at secondary level. Also, by any diversified approach it would be possible to treat highly demanding domains of basic mathematics at the high (secondary) school stage without having to make concessions to the pupils' understanding. In addition, chances of success would be further guaranteed if any diversified approach - developed as a synthesis of ideas from the present behaviourist, integrated, formative and helical approaches - should beam the content at the median ability with a strong bias towards the applications of mathematics.
3.7 The Span of the Curriculum

3.7.1 Introduction

The essential elements in planning a curriculum are prerequisite knowledge, inherent structure and sequence of content, psychological pacing of reinforcement materials, and evaluative feedback information but whether the concern is with textbook, lesson plan, module of instruction, or programmed learning, preparation should be jointly informed by the mathematician-mathematics educator, psychological findings and teachers of mathematics. Himself a practising mathematician—psychologist, Bruner [76, p 72] suggests that:

..... a curriculum reflects not only the nature of knowledge itself but also the nature of the knower and the knowledge getting process. It is the enterprise par excellence where the line between subject matter and method grows necessarily indistinct. A body of knowledge, enshrined in a university faculty and embodied in a series of authoritative volumes, is the result of much prior intellectual activity. To instruct someone in these disciplines is not a matter of getting him to commit results to mind. Rather it is to teach him to participate in the process that makes possible the establishment of knowledge. We teach ... to get a student to think mathematically for himself, ... to take part in the process of knowledge getting. Knowing is a process, not a product.

According to Griffiths and Howson [33, p 156] a curriculum embodies:

(a) **Purpose:** a statement or an assumption (for purposes usually exist even when they are not explicitly stated) of the aims of one's teaching
(b) **Content**: a statement of the content of what students are to learn and experience, and of the choices they will be offered.

(c) **Method**: a statement of the method or methods which are considered to be most likely to achieve the aims set out in (a).

(d) **Assessment**: a statement of how the course and work of pupils are to be evaluated.

By comparison, Downey and Kelly [74, p 135] say that curriculum means

'all the learning which is planned and guided by the school, whether it is carried on in groups or individually, inside or outside the school'.

Despite this all-embracing definition, it would be a mistake to assume that it is the same as a syllabus or the sum of all the syllabuses of any school since this limits it to content. But in the author's evaluation a more precise and relevant viewpoint, providing the trunk from which an appropriate definition of curriculum naturally and immediately stems, is that put forward by Howson [62, p 68]:

In many countries the educational system is so constructed that teacher-training, curriculum development and classroom practice are three separate activities...... change can only be effective when the three are seen as aspects of a single process...... It is naive to define curriculum as the content of the text book, or its development as the introduction of a new one. It is both more realistic and more constructive to define curriculum as what actually takes place in the classroom. This immediately gives the teacher a key role in curriculum innovation.
On the basis of this unified association of teacher-training, curriculum development and classroom practice the writer has chosen to keep the discussion in a corresponding style. But over the past 20 or more years, similar debates have emphasised the following four aspects, identical to the ones put previously by Griffiths and Howson [33, p 156], within the domain of the curriculum:

1. **Objectives**: qualities of mind, skills, kinds of knowledge to be developed in pupils

2. **Content**: subject matter by which it is hoped these objectives will be achieved

3. **Procedures**: the programmes of activities or work in which pupils and teachers will be engaged upon to attain these goals

4. **Evaluation**: the extent to which success has been achieved both in terms of the curriculum itself and the progress of individual pupils.

### 3.7.2 Objectives

Many definitions of objectives were given in the literature but the author prefers to commence with one given by Wood [98, p 83]:

"... objectives are essentially tactical in character. They are made up of what is called in the jargon of programmed learning, the 'terminal behaviours' of the pupil, that is to say, an account of what he should be able to do at the end of a course of study in terms of remembering, thinking and understanding with respect to certain subject matter areas."
Educators agree that curriculum planning must begin at the level of objectives but Downey and Kelly argue that Bloom's specification [35, p 165] has been too precise and specific and in the absence of any definition of 'education', it provides no criteria for evaluation between objectives. As Bloom himself knew, it was possible only at the conceptual level to distinguish objectives with so fine a degree of detail. Every activity in which pupils will be engaged will have a range of objectives traversing the three 'domains' - affective, cognitive and psychomotor. Wood [98, p 83] disagrees; although writing about the statement of objectives for teaching mathematics, he commends the exercise as one worthy of being done at all levels:

In the June 1965 issue of this journal, Lewis reviewed various lists of teaching objectives which might commend themselves to science teachers. This article attempts to provide a similar service for mathematics teachers. Although the arguments advanced for the desirability of stating objectives hold good for curricula at all levels of learning, the objectives recorded are only meant to be suitable for CSE and GCE mathematics courses.

Later in the same paper however, Wood states that the British have been more amenable to stating aims than objectives, while American educationalists have demonstrated a willingness to convert aims into objectives. This, he suggests, may be a result of the psychologists' penetration into education.

Curzon [91, p 86], in advancing points of disagreement with Bloom, states

Criticisms of Bloom's taxonomy have tended to be based on three grounds; first, that since it is formulated in behavioural terms, it is derived
from the "fallacious view" of learning inherent in behaviourism; second, that it is derived from a naive and therefore inadequate theory of human knowledge; third, that its cognitive/affective dichotomy is invalid.

In elaborating the objectives, Curzon thinks that one may, quite conceivably learn without being able to convey to another evidence of having learned. It is argued also that assessed behaviour should not be viewed as the only reliable indicator of the attainment of those goals set by the teacher. With regard to 'knowledge' he inquires whether a student can be said to have "knowledge" of Pythagoras' theorem if he is able to recall the precise wording of the theorem, but is unable to use it so as to solve a simple problem requiring the calculation of the length of the hypotenuse.

In support of objectives Rowntree [99, pp 35-36] suggests four principal areas of benefit:

1. Objectives enable you to communicate about the intentions of the teaching and learning. Students who know the objectives of a course, and who are given appropriate references and resources, can teach themselves in half the time taken by normal classroom methods. Objectives also enable you to discuss educational intentions with your colleagues.

A second benefit of objectives is that they help you to select and structure the content of your teaching.

Thirdly, objectives help you decide on appropriate learning activities and teaching media. Only if you know just what the students should be able to do as a result of the course can you decide what experiences they should have during it.
The fourth and final value of objectives is that they help you to decide on appropriate means of evaluation and assessment: on ways of testing the effectiveness of your course - or the success of your students if you still insist on looking at it that way.

On the other side of the coin, Rowntree [99, p 36] indicates some objections to objectives:

Objectives are too difficult to formulate (especially in arts-based subjects); they put too much stress on trivial and easily-measured behaviours; not all desirable results can be specified in advance; it is undemocratic to pre-specify; objectives are difficult for the teacher to work with.

Looking at the process of articulating goals, then striving towards them, then assessing progress, then re-examining goals, amending and re-stating them in the light of experience and data, together constitute what the researcher would call a dynamic and responsive style of teaching. Having looked at the positive side of each objection to objectives, Rowntree [99, p 41] concludes:

In general, then, it seems to me that 'objectors' do not identify fundamental weaknesses of objectives but merely warn of possible pitfalls in their formulation and use. Certainly, we cannot afford to ignore the warnings - they indicate the need for vigilance. But the dangers are minimal compared with those already rampant in content-based curricula presented to students in the hope that 'something useful's bound to rub off on them'. At any rate, the concept of behavioural objectives has surely generated
the most powerful approach to curriculum development that has yet become available to us.

Curzon [91, pp 87, 88] concurs with Rowntree [99] by stating:

..... It has warned against ambiguity in the definition of teaching objectives and has assisted in the construction of statements of goals which enable teachers to evaluate attainment by reference to a yardstick - albeit not perfect. Its detailed analysis of the outcomes of instruction has provided a new vantage point from which the content of the syllabus and the examiner's scheme might be surveyed and scrutinised in detail.

Bloom's taxonomy may assist the teacher in answering the fundamental question: "In what ways will my students have changed as the result of my teaching?" The very posing of this question constitutes a step towards the provision of some of the conditions for effective learning.

The author believes that entwined in the matrix enclosing objectives is not only a fundamental meaning of 'education' - changing the behaviour (thinking, feeling, acting) of a pupil so that he can, when faced with a particular need for decision-making, bring into operation a behaviour which hitherto he did not exhibit - but also a covert indication of the teacher's function: which is to help the pupil to learn new behaviours and determine where and when they are appropriate. In fact, spotting this meaning of education and function of the teacher has naturally occasioned a look at the behaviourist approach to instruction and curriculum development. Indeed it was this school of psychology which was credited with the rationale for teaching machines and programmed instruction. To continue the depth of progression, Wood [98, p 85] tells us
It is not surprising, then, that a renewed emphasis on educational objectives resulted from the development of programmed learning. The careful specification of a step-by-step procedure for the learner calls for clearly understood objectives specified at a level of detail far beyond that usually attempted. It is interesting to note that the two movements have actually been fused together in a programmed learning text (Mager, 1962), written to help teachers to define educational objectives.

The researcher made liberal use of this teaching-learning strategy of behavioural objectives in defining marking schemes for answers to the diagnostic test questions in Chapter 6. This technique also formed the fundamental basis of the analysis to each question, the aim of which was to trace pupils' thought patterns, thereby establishing whether the objectives of the curriculum had been achieved. Perusal of any CSE or GCE syllabus shows that the content is arranged as a list of topics from the basic elements to the most difficult topics with no indication of the qualitative differences in learning experiences offered by each topic. (In fact the writer used this same technique in traversing content from concepts of natural numbers up to differential and integral calculus in Section 3.4 of this Chapter). For example, when a syllabus mentions 'simple probability' there is no suggestion of the subtlety of the basic ideas which have to be grasped before this concept can be accommodated and finally assimilated by pupils. On the face of it, there seems a fondness for the logical to the neglect of the psychological as indicated by the unwillingness to depart from the traditional arrangement of topics.

Many would counter the objective approach in syllabus development by saying that it is not intended to be more than a catalogue of cues, each one suggestive of a number of teaching
possibilities for the teacher's translation into practice. This is fair enough where teachers are adequately trained and qualified. The mere act of preparing objectives will not bring about effective teaching, but a situation where objectives are seldom if ever articulated suggested to the researcher that mathematics was being taught without an awareness on the part of teachers of its impact on pupils' behaviour.

Musgrove [75, p 218] says of the curriculum:

..... it is an instrument for changing student behaviour; its objectives are statements of ways in which the knowledge, cognitive abilities, skills, interests, values and attitudes of students should change if the curriculum is effective... It is the contrived activity and experience - organised, focused, systematic - that life, unaided, would not provide.

Having followed some of the views for and against the statement of objectives the question may be asked: is the behaviourist model worth emulation in building a curriculum? If the answer is negative, the impetus to action could be ambiguous in so far as tasks for presentation may be unidentifiable. If the answer is affirmative, there is a possibility that the content may be fragmented [69, p 141]. However, Wood [98, p 94] resolves the dispute by arguing very strongly that if latitude for doubt exists, then

The best corrective in this situation is for the teacher or examiner to spell out exactly what he means by achievement, that is to say, what the students are expected to do as a result of a course of instruction. Once the objectives of instruction are established, tests can be devised to identify them and to assess, as objectively as
possible, the extent to which they have been attained. The results of assessment can then be reported, not as a single score, but as a profile of differential performance.
Some aspects of the Psychology of Learning and Teaching Mathematics

A proper study of mathematics infers an analysis of the logical development of its algebraic, order and topological structures. The teacher of pupils embarking on this study should develop, through activities, the pupils' ability to recognise and articulate the basic concepts; generalisations and principles that constitute the substance and pattern of mathematics. But this presupposes that the teacher himself knows the subject at sufficient depth; has acquired appropriate and relevant pedagogy to design and manage miscellaneous learning activities; and then subsequently evaluates the learning outcome in terms of knowledge, understanding, application, thinking and performance skills, and attitudes from feedback. This evaluation then becomes the criterion for advancement to further studies and/or individualised revision exercises. But individualised learning, with its problems of writing and production, has attracted the attention of Bajpai and Calus [100, p.456] who, though writing in the context of higher mathematics - beyond 'A' level - nevertheless advise:

As we see it, the writing of programmed texts ... must be done by someone:

1 who has a thorough understanding of the topic programmed,
2 who has the ability and desire to communicate it to others,
3 who has experience of teaching the topic at that level,
4 whose knowledge of the subject area extends beyond the bounds of the programmed material.

These, surely, are the attributes of the successful teacher.

Bruner is renowned for the hypothesis that pupils of almost any age and level of abilities can develop a grasp of the
nature of mathematics, provided the emphasis in teaching is not on isolated facts but on the fundamental concepts and unifying patterns (ie the structure) of the subject. Mathematics presents a way of thinking about order and topics should be redeveloped (ie the spiral approach) in later grades. Underlying this way of thought are connected generative axioms and rules (of association, commutation and distribution) of analysis. In the teaching of mathematics emphasis should be placed on the use of these rules rather than knowledge of the formal names. Nothing is more central to mathematics than its way of thinking so effective teaching should emphasise the structure of the subject. Bruner [68, p.9] endorses this viewpoint by arguing that:

Good teaching that emphasizes the structure of a subject is probably even more valuable for the less able student than for the gifted one, for it is the former rather than the latter who is most easily thrown off the track by poor teaching.

But Bruner [68, pp.9, 10] does not stop there. He balances his argument and underscores the utility of mathematics as a service discipline by saying:

This is not to say that the pace of the content of courses need be identical for all students - though ... "When you teach well, it always seems as if seventy-five per cent of the students are above the median"...One thing seems clear: if all students are helped to the full utilization of their intellectual powers, we will have a better chance of surviving as a democracy in an age of enormous technological and social complexity.

The power and economy of mathematical thought are well illustrated by the structure of the formula, for example

\[ S = \frac{1}{2}gt^2 \]

which summarises the characteristics of freely falling bodies, starting from rest under the earth's gravity, g. The mathe-
mathematics says:

the distance fallen, $S$ is proportional to the square of the time, $t$

$\Leftrightarrow$ the distance, $S$, varies as the square of the time, $t$.

The value of $g$ may be calculated if any one value for the pair $(t, S)$ is known. This example was introduced with the intent of demonstrating how the teaching of mathematics can be effected to convey an understanding of power and economy in the structure of mathematical thought: the kind of understanding which supersedes rote memory. But it also admits that mathematical structure is communicated and clothed in a symbolic language which is unlike ordinary prose. This answers the phenomenon maintained secondary pupils in The Bahamas experience with recall in mathematics. Coincidentally, the instance chosen straddled the integrated studies approach so evincing the pervasive clarity in the matrix of functional patterns within the architecture of mathematics. The other important truth which surfaced from this analysis was that mathematics educators should be aware of all current philosophies of methodology which inform the span of classroom practice. In the theory of structuralist psychology, Bruner [68, p.25] claims that what cropped up via the formula $S = \frac{1}{2}gt^2$ ...

...is to have learned not only a specific thing but also a model for understanding other things like it that one may encounter.

He (Bruner [76, p.44]) further postulates that for any learner:

Any problem within the domain of knowledge can be represented in three ways: by a set of actions appropriate for achieving a certain result (enactive representation); by a set of summary images or graphics that stand for a concept without defining it fully (iconic representation); and by a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws for forming and transforming propositions (symbolic representation).
Methodology in mathematics in The Bahamas reflected a premature reliance on abstraction, with very little if any opportunity for concrete manipulations in guided discovery. This *modus operandi* has led to a consummate absence of current acceptable pedagogy by which traditional mathematics content was communicated in the better schools of the developed countries: what Lamon [69, p. 4] calls

...a regrettable misunderstanding of what the learning and teaching of mathematics is really all about.

Alternatively, pupils can never be expected, through guided discovery, to arrive at all of the mathematical principles that would be necessary and sufficient for problem solving exercises. Accordingly Lamon [69, p. 5] asserts:

I, for one, have never had the privilege of noticing any student "discover" a mathematical abstract without being prompted by an enormous number of clues.

It occurred to the author while reviewing the literature that in good pedagogy in mathematics teaching,

(1) the presentation of tasks is designed to motivate learning and the content is relevant to the pupils;

(2) activity methods in mixed ability grouping and programmed learning allow individuals to learn at different rates;

(3) strategies employed are guided by sound principles of mathematics learning; and

(4) any task must be designed to allow each pupil to discover the concepts involved by:

(i) reflective thinking
(ii) problem solving
(iii) experimentation
(iv) analysis, and
(v) generalisation
with due allowance being made for self evaluations. Furthermore, enhancing the practice of good pedagogy is effective use of the overhead projector, electronic calculator, a tape recorder, and film strip projector in addition to the geometrical instruments designed for use on the chalkboard. Good organisation of tasks on the chalkboard is also an important element in the effective teaching of mathematics.

3.7.4 The Textbook

Various textbooks presented differences in content, sequencing of topics, and tasks which together contributed to variable pupil achievement across the population of the maintained secondary schools on New Providence. Traditionally, the sequencing of topics in the curriculum was a carbon copy of the table of contents of textbooks written for metropolitan cultures overseas, notwithstanding they were often quickly revised and sometimes replaced in overseas educational environments in response to classroom trials and feedback. If this organisation of subject content enhanced learning modestly despite obsolescence in the country of origin, the printed word increased in value and authority. However, it is realised now that the textbook should be assembled from units [101, pp.82-104] and modules [95] which were tried in the classes that would finally use it. Also, exercises in geometry and trigonometry should relate, for instance, EITHER
to the journey of a mail boat travelling from Potter's Cay Dock to Cockburn Town, San Salvador via Ship Channel Cays, East End Eleuthera and the principal local ports of call on Cat Island
OR
Bahamasair travelling from Nassau International Airport on New Providence to Freeport International Airport on Grand Bahama.
Concerning the routes quoted above, supplementary exercises should use, as points of reference, distances measured in kilometres from Halifax Road, Stapledon Gardens to Potter's Cay or the airport on New Providence. Similarly, bus routes on New Providence and local air and sea routes within the archipelago of The Commonwealth are meaningful geographical referents out of which to initiate elementary exercises in topology. Having got a comprehensive grounding in mathematics from local motivating models, pupils should then be in a position from which they can cope with mathematical imagery from other cultures. The author's contention is that pupils find mathematics difficult partly because it is taught in a way (methodology) which makes it foreign to local culture and partly because the communication of the discipline lacks updated pedagogy and depth of content. The textbook consequently is a key independent variable for improving attainment in mathematical tasks insofar as the teacher uses its sequencing and organisation of content as well as its approach to problem solving; and the pupils use them in class (when they are available) and take them home as a source of reference for the presentation of tasks and private study. Ideally, therefore, the sequencing of topics (ie the ordering of fractions, ratio and proportion, variation, series, ...) in the textbook needs consultative collaboration among psychologist (in whose absence the findings of psychological theory would suffice), mathematician-mathematics educator and teacher. The influence of the textbook in teaching-learning situations may in part, be gauged from the experience of Yaseen* [102, pp.4-20, 4-21]:

Inspector: Look! I once contributed to writing a textbook in mathematics. A number was mis-printed in one of the problems in the text. The correct number was 0.3 and the misprint 3. Though the results from the number 3 do not work for the problem, some teachers in one secondary school interpreted these results in an unbelievable situation, since the text is 'sacrosanct' and therefore it should be followed literally...

* a mathematics inspector from Kuwait
Teacher: The examinations are compatible with the material contents in the texts. I want my students to pass, and therefore the text is the best.

It appears that the good textbook:

(1) presents topics in a manner that builds understanding of concepts, structure, problem solving and computations: it is a tool to be used in attaining the objectives of the course.

(2) provides the exercises, experiences, directions for attaining mastery through practice, review, applications, and thought-provoking questions.

(3) provides a compact reference source which is useful in building the structure of mathematics. Tables, definitions, formulae, graphs, sample problems, theorems, and proofs are available to make problem solving an efficient heuristic.

(4) forms the basis of classroom instruction, which may and should often follow a different but essentially parallel development so that pupils are offered various approaches to any topic.

3.7.5. Assessment

The span of the curriculum also includes assessment and Matthews [103, pp.7,8] in a paper for the ATM, puts a case 'Against Examinations' in which he suggests that:

The purpose of the proposed 'check-ups' is two-fold:

(1) to give the teacher a good indication of how far her children have got in their understanding,

(2) to provide statistical evidence of progress.

However, Agazzi [104, p.74], in justifying the functions and uses of examinations, suggests that they:
provide teachers with periodical information about their pupils, thus enabling them to give more enlightened and effective help;

provide pupils with an indication of their own progress or otherwise;

provide useful data for pupil guidance;

provide parents and the public with reliable information on the overall results of the educational system;

provide a sound basis on which to adjust curricula to meet the needs of a constantly changing society.

It was mainly to satisfy this last function that the author researched the construction and use of:

(1) objective tests, in which each item was followed by five options, one of which was the key, and

(2) choice type tests.

A prototype of each instrument was assembled and pretested, and Chapters 5 and 6 are reports of their field performances. In classroom practice, however, some other forms of assessment would be homework exercises, quick classroom quizzes, and over a longer term, profile assessment.

3.8 Summary III

On balance, guided discovery learning was considered most effective in mathematics teaching and was acclaimed in all successful approaches - structural, integrated, formative and behaviourist - to curriculum building. These approaches were adopted by various projects and the author himself did research in association with the AIMEC Project, all of whose members through micro-teaching exercises, practice a distilled philosophy which overlap the various approaches owing to the peculiar needs and priorities relevant to the participating
developing countries - India, The Bahamas and St Lucia. The achievements of various projects were mentioned and, in particular, an evaluation of CMP by Cundy [95] was studied at depth. Because classroom practice, curriculum development and teacher training [62, p.68] constitute a single process, no attempt was made to fragment the discussion.

The presentation phenomenon - by the textbook and by the teacher - affects the type of questions pupils are best able to answer and the method of solution they employ in so doing. But mathematics content in textbooks written for secondary pupils, however the topics were sequenced, is constant. Therefore, the independent variable in the presentation phenomenon in which positive change must occur is the teacher. This positive change is expected both in terms of pedagogy and the acquisition of greater depth of content. It follows that the tentative hypothesis

\[ f: \text{teaching-learning} \rightarrow \text{improved achievement} \]

would be satisfied if the independent variable methodology was given a positive change, though justified, is only partially true. Teachers in The Bahamas, both pre- and in-service, must also be taught more mathematics content in order to appreciate the significance of the pedagogy. Teaching mathematics to emphasise the structure of the discipline would help Bahamian secondary pupils to acquire relational understanding which obviates the necessity for rote memory and illuminates the abstraction inherent in symbolic representation. To this end examples in problem solving should emphasise elements of local culture - geography, games (hopscotch, ring play, etc). Perforce of reason the author asserts the applicability of a pedagogy which serves the integrated studies approach. The 1968 Congress on the Integration of Science Teaching [105,p.20] patronises the utility of integrated studies by saying that:

For the teaching of mathematics ... to be lively, it is best to begin with concrete examples...

Such examples must not consist of an artificial mathematisation of any subjects whatever: it is not
merely a question of showing the pupils models (cubes, pyramids, cones, etc). This would only lead to a passive perception and would not stimulate any intellectual effort.

On the contrary, the practical aspects of mathematics must in different ways stimulate the intuition, the imagination and thus the mathematical activity of the child. From a general phenomenon such as shadows, the balancing of scales, an image in a mirror, it is possible to develop a scientific analysis of laws and structures in different situations, the pupil will be led to compare, to classify, and thus to synthesise.

Further, the Congress claimed that training of this kind ultimately leads to familiarity with the scientific method. Also, in developing countries the integrated studies approach allows more effective use of available teachers and teaching facilities, and the subject matter offers greater relevance and interest, thus increasing the chance of deeper understanding, enjoyment in learning and instant recall.

The presentation of this chapter reflects a critical appraisal of various current theories as they bear relevance to existing classroom practice in mathematics education in The Bahamas. In the author's evaluation, this attitude was justified in the interest of well-informed and reasoned conclusive judgement.

The following chapter illustrates how objectives are used in the presentation phenomenon and outlines the specifications of the research instruments.
CHAPTER 4

ELEMENTS IN THE DESIGN
OF RESEARCH INSTRUMENTS
4 Elements in the Design of Research Instruments

4.1 Methods of Acquiring Knowledge

Chapters 2 and 3 have forewarned jointly that the applications of the scientific method, the behaviourist philosophy in lesson preparation and the two-dimensional blueprint grid in research-instrument preparation are to be considered in this chapter because, having postulated the problem under study, they form fundamental elements in the design and procedure of the research prior to analysis of the field results, which are meticulously presented in Chapters 5 and 6.

Essentially the objective of research is the acquisition of knowledge which informs the decision-making process. Supporting this idea, Kallos [59, p.142] suggests that

The task of the educational researcher is not only to detect and describe the state of affairs but also to provide the remedy.

The author must point out however that Kallos's suggestion carries the implication that a remedy put forward for change in classroom practice would suffice for all time. Classroom practice has to be kept under constant review because new evidence is always informing modifications in strategy. But there are times when, as Becher [59, p.65] puts it,

The difficulty is not that the teacher wilfully refuses to listen to the researcher, but that however carefully he listens the researcher has little of interest to tell him.

This latter quotation immediately has two interpretations. Firstly, that the researcher uses a means of communication which either (1) the teacher does not understand: that is to say, the fruits of research are only interpretable to fellow researchers and other scientists, or (2) engages tactless language. Cane and Schroeder [106, pp. 39-41] give further reasons for the gulf between researcher and teacher. Secondly, that the problems on which educational research and development
engage are not defined by close study of educational practice. (Incidentally, development in the context of this thesis means, using a definition suggested in [106, p.2], the introduction

...of new method or material into the classroom without necessarily postulating its exact effects, though in the hope, of course, that it will be beneficial.)

This second interpretation not only contradicts what Kallós [59, p.142] says but is also at variance with Bruner [59, p.26] who suggests that the competence of conventional educational research lies in its capacity

... to evaluate practices as they exist.

The general consensus therefore is that the educational researcher studies practices as they exist at classroom level, reports his findings and recommendations for improvement in a way which stimulates and guides the teacher to more effective performance. If the researcher himself is a practising teacher, as the author most surely is, then he has an obligation to lead the way in showing, even if by making haste slowly, how change, referred to in Section 1.5 of Chapter 1 and Section 2.3 of Chapter 2, may be effected for the better.

The literature, in particular Lovell and Lawson [107] and Burroughs [61], gives knowledge as a derivative of five sources:

(1) experience
(2) authority
(3) deductive reasoning
(4) inductive reasoning, and
(5) the scientific method.

Experience is familiar and often, if not well used, its fertility is only functional if it derives from systematic and logical roots. Authority is useful for providing the accumulated
wisdom of informed scholarship. But the worst exercise of authority is that which forces meek acceptance in its own name. Aristotle used syllogistic reasoning in demonstrating the power and utility of the deductive method but those who use it should guard against beginning with false premises. However, Rosenbloom [69, p.87] suggests that

...mathematics is the primary vehicle for teaching one of two principal methods of obtaining new knowledge first-hand - the method of deductive reasoning. (The other is the method of inductive reasoning).

Dieudonné [69, p.100], in concurrence, says that the goal mathematicians seek in modern civilisation is

...to teach them to order and link their thoughts according to the method mathematicians use, because we recognize in this exercise an excellent way to develop a clear mind and a rigorous judgment. It is then the essence of the mathematical method that ought to be the object of this teaching, the subject matter being only well-chosen illustrations of it.

It was Sir Francis Bacon [1561-1626] who formally postulated that direct observation of many elements of a class should lead to a conclusion. This form of reasoning came to be called the inductive method. The disadvantage here is that it is practically impossible to observe every element in a given universe of objects or events for perfect induction. Moreover, the induction based on the regularity of structural behaviour in mathematics becomes unpredictable in educational research which deals with the legion of variables influencing erratic human behaviour. The researcher, therefore, had no alternative but to opt for a method - the scientific method - a combine of the strengths of induction and deduction in order to derive valid, reliable and therefore conclusive knowledge of high utilitarian value in The Bahamian secondary school system. This method is by no means perfect but by making use
of the combined strengths of induction and deduction, it is more reliable than both experience and authority which are too open to the influence of subjectivity; and also has the advantage, says Downey and Kelly [74] for instance, of having stood the test of time. Lovell and Lawson [107, p.9] suggest that despite a researcher's frequent shifts between collecting information, hypothesising to explain his data, finding out what are the logical consequences of hypotheses, and obtaining more data to test the truth of the hypotheses,

This general method of acquiring knowledge is more flexible than the other methods described, and it encourages doubt and experiment until evidence is obtained which is consonant with the hypothesis advanced. Moreover, if fresh evidence arises at a later date which is no longer congruent with the deduced consequences of a particular hypothesis, that hypothesis must be modified or abandoned. What has been broadly called the scientific method has been very valuable in establishing new knowledge in the natural sciences, and it has also helped social scientists, including educationists, to gain insight into their problems. But it cannot answer questions involving moral or value judgements. Indeed, man has no one method for acquiring the knowledge necessary to answer all his questions.

The binding reason for the author's choice of the scientific method was that despite its limitations, it offered the best chance of arriving at impartial conclusions although it was conceded that over time in behaviouristic terms, elements of uncertainty would occasion readjustments. To the casual observer this may seem a contradiction in terms: namely, that valid and reliable conclusions ultimately form only partial solutions. In fact, it seemed to the author that philosophy, although unable to indicate with precision what is the truth about doubts which it raised, has the potential to suggest many possibilities which enlarge thoughts and emancipate them from the shackles of tradition such as are inherent in an authori-
tarian Bahamian society. Moreover, what on the surface seemed a disadvantage was actually the reverse since any valid pedagogy must employ a distillation of many strategies instead of the customary one best method in many developing countries. Furthermore, quite apart from its utility in identifying unsuspected possibilities, philosophy has a value - probably its superlative value - through the supremacy of its objective contemplation, and freedom to pursue broad global directions as a consequence of this impersonal but diligent introspection.

4.2 The Work of Mathematics Teachers

The act of contemplation during research is fired by various points of view expressed by other researchers and introspection concerning the blend of elements which hopefully add up to effectiveness in the performance of mathematics teachers. Teachers spend a very high proportion of their working life

(1) preparing lessons, and
(2) assessing pupils - the criteria for judging their own effectiveness;

but a teacher's total impact cannot be completely captured in terms of marks or grades. Linking (1) above with the fundamental consideration of Chapter 3 - curriculum development - it can be seen as part of the strategy which serves an approach through the presentation phenomenon, and (2) as well as ranking pupils, also guides further learning and teaching. But what is the purpose of preparing mathematics lessons? Kanellopoulou [108, p. 2] suggest that

...the way in which the teacher conducts conversations with his class during the development of the new topic is of prime importance ...for the quantity and quality of learning acquired by the class... these dialogues are never provided by text books. They have to be invented by the teacher on the spot, to be carefully stepped so as to lead gradually from the easy parts of the topic to the difficult ones, not to leave gaps, never to disappoint anyone, often to reward the class
in order to keep interest high... Still, the mathematics teacher is neither a psychologist nor an actor nor a Socrates. The dialectic task is... much more demanding than the mathematics itself, and therefore he often fails. Especially when he is angry or tired he conducts such poor dialogue that he frustrates the class and produces negative attitudes, particularly among girls. This results in an utterly unstabilized quality of teaching across lessons for the same teacher and across teachers for the same lesson.

Kanellopoulou comes to the key to this research - stability in the relationship between teacher, or his substitute, and pupils so that a uniformly high (or as near to uniform as is humanly possible) degree of communication can be seen to take place. Each method of communication teachers employ, Kanellopoulou [108, p 2] continues, has its disadvantages:

(a)... every single method (expository, discovery, programmed, learning-by-doing) has its own pros and cons; (b)... text-books using only one method cause saturation and monotony, and (c) that hitherto no text-book that we had come across ever included dialectic work.

As a crucial part of this research exercise, a prototype of lesson preparation, which applies if one is teaching anywhere from primary school to university level, will be studied prior to the display of content-process grids from which the research instruments would be erected. But it is worthy of mention that in the process of assessment, teachers most frequently use the affective and cognitive constructs interest and confidence; and class participation and mathematics ability respectively. In the literature, mathematics ability correlated very highly with analytical approach and intelligence: in fact, the correlation in one instance was 0.93 and 0.89 pro rata, thus giving credence to the psychological theory that mathematical structure integrates with the structures of the intelligence.
Up to this juncture, the following topics, considered essential to this research, have been discussed in part if not fully:

1. the nature of mathematics
2. the nature of mathematical abilities
3. methods of learning and teaching mathematics, and
4. approaches to curriculum development in mathematics

which, along with Figure 3.2 give an intuitive feeling of what research in mathematics education is about. But the goal is surely to arrive at the core of research in mathematics education: namely, the basis of its existence. Rosenbloom [69, p.86] argues that

...the functions of a mathematics curriculum seem to be dependent ... on how we organize mathematical learning experiences and how we perceive the nature of mathematics and its related pedagogy.

Rosenbloom further shows that these functions are viewed in terms of their relevance and importance to the individual and society. For on the one hand certain objectives of the curriculum must present the nature of mathematics as a science, an art and a language. Conversely some objectives are

1. cognitive: a pupil must be able to compute; an adult must be able to translate simple problems into mathematical language
2. affective: mathematics can give enjoyment and reveal its beauty
3. individual: a person should learn the mathematics he needs for the vocation he chooses
4. social: schools must produce people with the distribution of mathematical competencies and interests required in the various trades and professions.
Paiget [69, p.136] contends that

In reality, if the edifice of mathematics rests on "structures" that correspond to the structures of the intellect, it is on the progressive organization of these operational structures that mathematics teaching must be based.

Here Piaget, in response to a paper on mathematical structure by Dieudonné, is not only attempting to provide a firm foundation for the theory of learning by the use of logic and mathematics, but also he is showing that mathematical connections are formed through activity of the intellect and that psychologists are in a unique position to furnish teachers with data on how rigorously high standards can be obtained and maintained in mathematics teaching. But the researcher reiterates that for effective classroom practice, the teacher of mathematics is well advised to synthesize his own views with those of the psychologist, mathematician, mathematics educator and sociologist. Gagné [69, p.165] postulates that

...investigations are conducted to answer some very basic questions about the nature of learning, the conditions required for learning, the basis of memory, or the variables affecting learning transfer...regardless of the degree of specificity expected of the results, it is reasonable to suppose that they will...throw light on the problem of designing effective instruction in mathematics.

...It does not seem unreasonable to expect that learning research will...provide the systematic knowledge and theory that will make improvements in mathematics instruction possible.

Gagné's view of the purpose of 'learning research' is abundantly clear but before the researcher states a conclusion, the views of another mathematics educator would create a wider spectrum of professional judgement on which to draw. Wain [32, p.2] concludes that
...many teacher training institutions have been giving more to what is often called 'mathematical education', which has been defined as 'a study of aspects of the nature and history of mathematics and the psychology of its learning and teaching which contribute to the teacher's understanding of his work with children, together with a study and analysis of mathematical curricula for schools, the principles underlying their developments and the practice of their use in the classroom.'

Wain's definition of mathematics education is a comprehensive and therefore useful one for any practising mathematics educator to have. But in another paper Wain [32, p.142] suggests that

...the quality of the mathematical education is ultimately determined by the teacher's work in the classroom and the effect of an individual teacher may far outweigh the effect of the particular way that the syllabus has been determined.

Mathematical education has become more complex in recent years... It is becoming increasingly important ...to have knowledge of the psychology of learning, the nature of mathematics in society at all levels, techniques of resource management, curriculum evaluation and assessment procedures; the mathematics educator is essentially concerned with the maintenance and extension of knowledge in these areas as well as of the subject matter of mathematics itself at a variety of levels of difficulty. The transmission of this knowledge is mainly in the hands of those who train teachers, teach in-service courses and publish the results of the relevant research.

It seems clearly conclusive to the researcher that the raison d' être for research in mathematics education is to improve mathematics teaching and learning. It is as simple as that.
4.2.1 The Presentation Phenomenon: Some Suggestions

During their year in AIMEC, Arora et al [109, pp.9-15] prepared a package of 8 lessons which were demonstrated and approved. Supplementary information and classroom activities have also been checked. A part of Lesson 1 is reproduced here, with the permission of the authors, as evidence of how the behaviourist philosophy is applied in the training programme for in-service teachers. Heywood [110, p.143] advocates a similar format in teacher training.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Geometry of a circle: some elementary properties of a circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>Standard VII, Age 12+</td>
</tr>
<tr>
<td>No. of students</td>
<td>30</td>
</tr>
<tr>
<td>Date</td>
<td></td>
</tr>
<tr>
<td>Location</td>
<td>Classroom</td>
</tr>
<tr>
<td>Duration of lesson</td>
<td>40 minutes</td>
</tr>
<tr>
<td>Resources</td>
<td>See worksheet</td>
</tr>
</tbody>
</table>

**PREREQUISITE KNOWLEDGE**

Basic concepts related to symmetry, circle, diameter, centre, chord, angle, parallelism and perpendicularity, perpendicular bisectors of line segments.

**GENERAL OBJECTIVE**

The expected learning outcome is that the student

1 states the basic symmetry properties of a circle

**SPECIFIC OBJECTIVES**

The expected learning outcome is that the pupil

1.1 establishes every diameter is an axis of symmetry and vice versa

1.2 proves intuitively through transformation that a line passing through the centre of a circle and
the mid-point of a chord will be perpendicular to the chord and conversely, a line passing through the centre and perpendicular to a chord bisects the chord.

1.3 proves intuitively through transformation that any perpendicular bisector of a chord passes through the centre of a circle.

Class activities and follow-up exercises appear next in the plan.

On the syllabus for Grade 8 in The Bahamas, one of the content areas is multiplication of fractions. Similarly, the behaviourist model may be used to plan a guided discovery lesson in which unitary fractions are represented on graph paper, geoboards, by cuisenaire rods and Dienes multibase blocks, etc to lead to results like

\[ \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \]

but with some real life application such as the area of a table top in \( \text{m}^2 \). Acetate sheets may be used to check areas of figures and an intuitive notion of the limit of an area is sure to arise when an exact number of squares cannot be counted. Rigour is not required in these exploratory exercises in which mathematical experience is being acquired for formalisation later in the spirit of the philosophy of the helical curriculum. On this subject Gattegno [82, p.55] says that:
Rigour demanded too early indicates a lack of understanding of the real situation. Our task is to make our pupils first acquire experience in mathematics, and as the awareness of relationships increases, we progressively bring in a spontaneous demand for more precision in communication, required because of the ambiguities of speech.

One could rote learn \( \frac{1}{2} \times \frac{3}{16} = \frac{3}{32} \) with the usual rule that the numerators and denominators are multiplied to give the answer. The author passed Cambridge Overseas School Certificate in 1959 armed with rote-learnt rules to the exclusion of understanding. At that time he was unaware that multiplication of one unitary function by another makes the resulting fraction smaller than each of the two which were multiplied. In classrooms where concrete materials are used in lessons, there will always be a basis for discussion. In this context, Cockcroft [50, p.71, paragraph 243] suggests that

Mathematics teaching at all levels should include opportunities for

(1) exposition by the teacher;

(2) discussion between teacher and pupil and between pupils themselves;

(3) appropriate practical work;

(4) consolidation and practice of fundamental skills and routines;

(5) problem solving, including the application of mathematics to everyday situations;

(6) investigational work.

These six points are never in balance in any classroom: too much emphasis is given to (1) and (4) and not enough to any of the other four points. The ATCDE [84, p.13] warns that:
Unless a teacher of mathematics - and this applies at all levels - is conscious of his responsibility for continuously deepening and extending his own knowledge and powers, his teaching will become sterile, irrelevant and antiquated within a very short space of time.

Further, the activity arising out of multiplication of unitary fractions may be linked not only with finding areas of triangles, parallelograms, trapezia and circles; the opportunity ought never be missed to apply unitary fractions to simple exercises in probability using language such as 'the chance that ...'. The topic could be developed by appealing to intuition backed up with scientific evidence in the form of experiments tossing dummy coins in free play as a starting point. The ultimate objective of these experiments, of course, would be to develop the language of the topic and its definition, \( 0 \leq p \leq 1 \). Later it would be appreciated that one of the applications of the multiplication of unitary fractions falls naturally in simple exercises on probability. Here some general objectives could be stated as follows:

At the end of this topic the pupil will

1. define probability, relative frequency, event, trial and outcome
2. calculate experimental probability from data found in simple experiments
3. calculate expected frequencies from a knowledge of the possible events and the number of trials
4. predict probabilities in simple cases and design an experiment to test predictions
5. place an event on the probability spectrum

It seems a necessity that teachers should be trained to be creative and enterprising in forging these links concretely. With a geoboard, it is possible to demonstrate relational understanding by using the knowledge of the area of a triangle to deduce the areas of a parallelogram and a trapezium;
similarly, using the area of a parallelogram it is possible to approach the area of a circle. And in these exercises no extraordinary degree of dexterity in manipulating two-dimensional models is required. Success depends on careful guided discovery planned thoroughly by the teacher to tap the intuition and insight of the pupil. Rosenbloom [69, p.88] puts it this way:

Mathematical work consists largely of observation and experiment, guessing at what might be true, feeling what ought to be true, testing hypotheses, looking for analogies, building mental pictures, and trying out ideas without any certainty of success.

But the key here is that the teacher must build the kind of relationship with his class which encourages the adventurous and enterprising activity in mathematics: it requires the exercise of freedom with responsibility on the part of pupils, a challenge to which they make adequate response provided they know that the teacher holds high expectations of them. About 'feeling what ought to be true' as put forward by Rosenbloom [69, p.88], Bruner [38, p.613] suggests

Intuition implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytic apparatus of one's craft... It precedes proof; indeed it is what the techniques of analysis and proof are designed to test and check.

Gattegno [82, p.56] further enlarges our understanding of intuition by offering the following definition:

For the psychologist... the word intuition means an undifferentiated contact with situations. It means contact of the whole being, open to the demands contained in the situation, perceptive with all its senses, active with all its means, intelligent with all its experience...
To penetrate a complex situation, to allow oneself to be penetrated by it, and to extract from it what is appropriate is, for me, accomplished with the act of intuition, which is more than reason, intelligence etc., but which is also less than these since it is less specialised and less adequate for the reaching of particular goals.

In the spirit of the helical curriculum, the teacher returns to the area of the trapezium in post 'O' level classes to give an intuitive introduction to integration as the limit of a sum, where analytically (and this is very important) the exact area of trapezium

\[
ABCD = \lim_{\Delta x \to 0} \sum_{x=a}^{b} y \Delta x
\]

ie exact area of \(ABCD = \int_{a}^{b} f(x) \, dx\).

However, if the elements of area, \(y \Delta x\) lie under a curve, \(y = x^2 + 3\), say, then progress to the result would be impeded by inductive proof of the formula

\[
1^2 + 2^2 + \ldots + n^2 = \frac{n(n + 1)(2n + 1)}{6}
\]
on which the right answer depends. The teacher should establish an intuitive proof of this formula as a separate exercise before attempting to demonstrate its use in problems on integration. Intuition, therefore, is a powerful psychological weapon and the resourceful teacher must use it to advantage in the teaching of mathematics, which is not 'telling how' but using techniques that spring naturally out of an understanding derived from manipulation in concrete activities. Lasley and Applegate [58, p.4] support the author by arguing that

Teachers who are locked into the teaching-as-telling syndrome, are severely limited in the learning environment they create for students. This does not mean that teachers should structure a different learning experience for each student, but it does suggest that teachers must be able to use a variety of instructional skills. Teachers who possess a large repertoire of teaching strategies become what Medley (1981) terms professional decision-makers who use what they know to structure what students learn.

4.2.2. Some Views on Assessment and the Use of Specification Grids

The purpose of the diagnostic tests, which are analysed in Chapters 5 and 6, was specifically to ascertain the extent to which the final behaviours of the syllabuses had been achieved. Supporting this perspective, Bishop [85, p.126] says that

...the APU(1980) distinguishes concepts, skills, applications and problem solving, largely from the viewpoint of testing, but with an implication that there are likely to be differences in teaching methods relating to these different components of the curriculum.
But in a study of this kind, the aim of which is to prepare the researcher for further teacher-training responsibilities, it was considered necessary to take a wider view of assessment. It is said, for instance, that schools are primarily selection and certification agencies which measure and label people at the expense of legitimising social opportunities and equalities. This argument implies that assessment is part and parcel of the apparatus by which schools perpetuate the existing hierarchical structure of society, for it creates and applies labels which determine pupils' opportunities in further education and life itself; their social status, privilege and power and even their value to society. Also, a capitalist society, on which The Bahamas is modelled, has an authority structure which depends for its continued existence on the identification of an élite to occupy its most influential positions. If the school system in The Bahamas can output people with the diversity of skills, knowledge and professional competences required by the society, the structure of the élite will keep in a state of constant change.

Alternatively some have dared to suggest that if teachers would discontinue assessments, the way would be paved for a better social order. But even if this objection is permitted to stand, some form of assessment would be necessary to protect the public from the practice of professions which require special skills and knowledge by those least qualified by lack of training to deliver certain services. Assessment of some form is therefore essential for the well being of society, and schools, by themselves in a democracy, cannot bring about changes in the social order. They only transmit what, by social consent, are worthy values for perpetuation. This point of view can only prevail in totalitarian states where the education system is specifically structured to propagate a single ideology. Practices of assessment therefore seem destined to continue as long as they serve functions required by the majority opinion as expressed by the democratic process. Schools have an obligation to use assessment to plan and guide learning. In any event the authority of the state in a
democracy will prevent teachers from perpetuating a model of society which is postulated by the school. In The Bahamas assessment, by majority feeling should serve to produce a circulating élite so that no one social, economic or hereditary group dominates the positions of influence, authority and power in the country. This offers the best chance of equality of opportunity and the opportunity to be equal in a society which had overt expressions of inequality on the invalid psychological assumption that certain outwardly expressed physical characteristics of a particular racial group, who comprise upwards of 85.0 per cent of the total population, implied inferior intelligence.

Curzon [91, p.207], although writing about post-secondary education, argues that

A syllabus may be determined by an external examining body, after which it becomes a central task of the teacher to work to its requirements. His freedom to decide on the treatment or weight of subject matter is unduly limited and the learner's liberty of exploring the many side paths in the subject area are curtailed, but in defending the value of examinations suggests

...if the objectives of a learner's course of study are worthwhile, if the syllabus which is to assist his attaining those objectives is sound and if the course of study is to result in a systematic movement to increasingly higher levels of attainment, then the repeated measuring and assessing of the learner's skills and the teacher's effectiveness become essential and therefore valuable... In Bruner's words, examinations can be "allies in the battle to improve curricula and teaching."
Rowntree [99, p.99] defends testing on the ground that it
...should become a teaching rather than a
policing device.

Heywood [111, p.23], in harmony with Rowntree and at the same
time attesting to his discipleship in the behaviourist school
of psychology and approach to curriculum development, argues
forcefully that:

Criticisms of examinations have concentrated on
negative aspects of assessment. It is not
possible to resolve many of the issues which
have been raised about the curriculum and
student learning in the absence of detailed
statements of objectives and carefully designed
procedures for their assessment. If this is
done examinations are likely to have a powerful
and beneficial influence on learning. To
achieve this goal evaluation must become an
integral part of the teaching activity.

In another of his publications, Heywood [110] adopts a crucial
posture: not only does he show how examinations and assessment
courage the development of learning along specific and valued
directions, but also [110, p.138] graphically depicts the inter-
locking relationships among the curriculum, teaching strategies
and methods of assessment. Here he (Heywood) says:

The consequences of this approach for the decision
of the curriculum are that its aims and objectives
have first to be determined. Appropriate teaching
strategies have then to be devised to meet these
objectives. At the same time methods of assess-
ment appropriate to particular objectives will be
developed. In devising the teaching strategies
the designer will take into account the concepts,
principles and values associated with the
curriculum.
It seems therefore that the benefits of relating the assessment procedure to the teaching strategy, and of showing the pupils quite clearly what final behaviour is wanted in their performance cannot be over-emphasised. But assessment is not always in the form of proposing standard-type answers to problems or mechanical-type questions. Brissenden [112, pp. 75-76] informs that

Mathematical investigations and projects are in fact steadily gaining ground both as a topic of interest and a form of assessment. In use for a number of years in some CSE courses and the mathematics course at Southampton University, they form part of the assessment of a new mathematics in-service qualification for teachers...

This mode of assessment however will not find application in The Bahamas because the teachers who operate it need a good deal of independence in curriculum matters. To continue the defence of assessment Heywood [111, p. 21], having stated that the desirability of examinations arises from the need to select and therefore to predict, quotes Pidgeon and Yates (1969) in suggesting that

'...For certain kinds of occupation and for promotion within them, there are more claimants than can be accommodated. In this kind of situation there would be no acceptable alternative in the use of tests or examinations of some kind. We do not have to ask, for example, what it would be like to try to staff the Administrative Branch of the Civil Service without recourse to these devices. We have tried it. The result was patronage, nepotism and inefficiency. In the ordering of public life, examinations have demonstrably promoted public justice and helped us make progress towards the goal of equality of opportunity.'
In the discussion of assessment under the scope of the curriculum, Matthews [103, pp.7,8] puts a case against examinations; but it is clear however that he does not propose a substitute. His real feeling on the matter may be gauged by the convincing assertion [78, p.36]

Assessment is certainly essential, but must it necessarily be by the 'big bang' of examination rather than through 'continuous creation' of confidence and ability measured by some other device, perhaps spread over a considerable period?

The effect of the metaphor is to suggest that Matthews wants instead, a more humane approach to assessment by spreading it over a longer period.

Having looked at some views representing a sample of feeling in the debate about assessment, it appeared that, for the sake of public justice and progress towards equality of opportunity as well as the opportunity to be equal, institutions of learning would continue to assess their pupils and students. In effect, Bahamian pupils would still be offering 'O' level mathematics alongside any equivalent local examination which may develop in the future. It is important therefore to consider the terminal behaviours that the 'O' level examiners expect of pupils and to investigate the degree to which Bahamian junior and senior high schools are achieving those performance outcomes.

But of 'O' level mathematics Brown [78, p.42] reports

Some of the more enlightened chief examiners... have set questions designed to test understanding of concepts and the ability to apply them in a new situation, rather than simply to test the memorising of techniques.

The content - behaviour specifications in Tables 4.1 and 4.3 have been designed so as to give pupils in the junior and senior high schools the opportunity to demonstrate knowledge of the basic concepts and techniques and skill in their use. But in the
light of what Brown says, it is realised already that the balance in question-setting will have to shift to permit the performance of an increased percentage of higher order behavioural skills - comprehension and application - so as to guarantee success at 'O' level. Concerning the distribution of percentages to behaviour types, the junior high school should commence development to the stage where in Table 4.1 the percentages under

(1) knowledge and applications can swap places

(2) techniques and skill is reduced by 10.0 per cent and redistributed 3.0 per cent, 5.0 per cent and 2.0 per cent respectively among translation, interpretation and extrapolation in comprehension.

These medium-term goals are consonant with a model of teacher effectiveness which envisages an input of pedagogy and mathematics content into the system via the teacher for improved attainment in mathematics. Begle [113, p.31], asserts that

...the only satisfactory way we have of defining teacher effectiveness is in terms of student learning.
Table 4.1: Specification Grid for 50-Item Diagnostic Test
(See Appendix 5.1 for Test Items and Appendix 5.5 for Grade 8 Syllabus)

<table>
<thead>
<tr>
<th>Content</th>
<th>Knowledge</th>
<th>Techniques &amp; skill</th>
<th>Comprehension</th>
<th>Interpretation</th>
<th>Application</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Percentages</td>
<td>25, 26, 27</td>
<td>30</td>
<td>28</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio</td>
<td></td>
<td></td>
<td>10</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Place Value</td>
<td>2, 22</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Approximation</td>
<td>23, 24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtraction</td>
<td>2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplication</td>
<td>3</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division</td>
<td>4</td>
<td></td>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fractions</td>
<td>17, 18, 19, 20, 21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decimals</td>
<td>8, 12, 13, 14, 15, 16, 29</td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Multi &amp; Add.</td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factors</td>
<td>49</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Directed</td>
<td>31, 32, 33, 34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>39</td>
<td>1</td>
</tr>
<tr>
<td>(3) Congruence</td>
<td>37</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Shapes-3D</td>
<td>38</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shapes-2D</td>
<td>40</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angles</td>
<td>41</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Straight line</td>
<td></td>
<td></td>
<td></td>
<td>45, 46</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Curvilinear</td>
<td>47</td>
<td></td>
<td></td>
<td>48</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Histogram</td>
<td></td>
<td></td>
<td></td>
<td>43, 44</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>(5) Length</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td>42</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td></td>
<td></td>
<td></td>
<td>11</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Equilateral $\Delta$</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Equal Angles</td>
<td>35</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal Sides</td>
<td>36</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>6</td>
<td>33</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td>PER CENT</td>
<td>12.0</td>
<td>66.0</td>
<td>4.0</td>
<td>10.0</td>
<td>8.0</td>
<td>100</td>
</tr>
</tbody>
</table>
Viewed in this sense, the specification of content - behaviour matrices in Tables 4.1 and 4.3 were constructed to direct the sampling of pupils' learning in the junior and senior high schools in The Bahamas for the purpose of deriving a relative, as opposed to an absolute, appraisal of teacher effectiveness as it existed in the latter half of 1981. The following catalogue, adapted by Wood and Skurnik [35, pp.166-171] from Bloom's *Taxonomy of Educational Objectives, Cognitive Domain*, details the expected terminal behaviours and Tables 4.2 and 4.4 display the percentage distribution for each content area.

(1) Knowledge: recall of definitions, notations and concepts

(2) Techniques and skill: computation, manipulation of formulae, using measuring instruments to stipulated accuracy, making simple constructions with ruler, compasses, protractor, etc.

(3) Comprehension (Translation): translation of illustrations, model, tables, diagrams, and graphs to verbal form and vice-versa. Given geometric concepts in verbal terms, translate them into spatial form.

(4) Comprehension (Interpretation): making inferences from data presented in tabular or graphic form. Making a deduction, given a set of restraining conditions. Deciding on the validity of a chain of reasoning.

(5) Application: of appropriate concepts to unfamiliar mathematical situations.

<table>
<thead>
<tr>
<th>Content</th>
<th>N</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Number (concepts)</td>
<td>9</td>
<td>18.0</td>
</tr>
<tr>
<td>(2) Number (operations)</td>
<td>25</td>
<td>50.0</td>
</tr>
<tr>
<td>(3) Space</td>
<td>4</td>
<td>8.0</td>
</tr>
<tr>
<td>(4) Graphs</td>
<td>6</td>
<td>12.0</td>
</tr>
<tr>
<td>(5) Geometry</td>
<td>6</td>
<td>12.0</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>50</td>
<td>100.0</td>
</tr>
</tbody>
</table>

*Table 4.2: Breakdown of Items in Table 4.1 by Content*
Table 4.1 indicates that the content areas in Table 4.2 are further divided into 27 specific categories. This classification in Table 4.2 was attempted with a view to presenting as simple and clear a picture as possible but there are items which could be justifiably classified under more than one content category or behaviour type. For instance, Items 26 and 27 which are classified under 'percentages' could equally well be under 'fractions' and 'decimals' respectively. Also, Item 45, classified under 'translation' could also be under 'interpolation' (which, incidentally, is not mentioned by Wood and Skurnik [35, p.166] as a behaviour within comprehension). This classification grid must therefore be interpreted with some reservation. But the real value of constructing content - behaviour grids lies in their potential use in starting an item bank to serve the needs of national examinations. To this end Wood [114, p.119] advises that

The blueprint is a double-pronged analytical tool. Firstly and most importantly it can be used to improve curriculum planning by providing a medium through which teachers can articulate

(continued on p. 218)
<table>
<thead>
<tr>
<th>Content</th>
<th>Techniques &amp; Skill</th>
<th>Comprehension &amp; Skill</th>
<th>Comprehension &amp; Skill</th>
<th>Application</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALGEBRA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real numbers</td>
<td>1</td>
<td></td>
<td></td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Difference of two squares</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction</td>
<td>5</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simple equation</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simultaneous equation</td>
<td>11</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Multiplication: binomials</td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>GEOMETRY/TRIG</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction: parallelogram</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Trigonometric ratio or Area</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>or Circle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isosceles triangle</td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>and Parallel lines</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARITHMETIC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle: perimeter</td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Standard form</td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Percentages</td>
<td></td>
<td></td>
<td></td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Ratio</td>
<td></td>
<td></td>
<td></td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>Fraction</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>PERCENT</td>
<td>35.7</td>
<td>21.4</td>
<td>14.3</td>
<td>28.6</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.3: Specification Grid for 14 Choice-Type Diagnostic Test Questions. (See Appendix 6.4 for Test Questions and Appendix 6.8 for Grade 10 Mathematics Syllabus)
### Table 4.4: Breakdown of Questions in Table 4.3 by Content

<table>
<thead>
<tr>
<th>Content</th>
<th>N</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>6</td>
<td>42.9</td>
</tr>
<tr>
<td>Geometry/Trig</td>
<td>3</td>
<td>21.4</td>
</tr>
<tr>
<td>Arithmetic</td>
<td>5</td>
<td>35.7</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>14</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 4.4: Breakdown of Questions in Table 4.3 by Content

The behaviours they wish their pupils to develop. At the same time it enables the teacher, in his role of examination architect, to compile a suggestive document which he or any other exam 'builder' can use to create items designed to measure how far the objectives of instruction have been attained.

Begle [113, pp. 97,99] in agreement with Wood on both points, asserts

The best predictor of mathematics achievement is previous mathematics achievement

and with reference to the curriculum suggests

The status of student knowledge about mathematics is well worth knowing, both as a guide for curriculum developers and as a means of monitoring mathematics programs.

While preparing the blueprint in Table 4.3, the author had to scan the Grade 10 syllabus shown in Appendix 6, and the result of that close scrutiny is manifested in the distribution produced between content and behaviour over the whole paper, which does not reflect too many questions on mechanical computational skill that teachers recurrently overemphasise in The Bahamas. The problem is that, despite initial training, many teachers tend to teach the way they were taught. Further, pre-testing questions prescribed by content-behaviour grids not only makes them sound and efficient; it also gives national credibility to any public
examinations based on questions from a bank. The author's evaluation of the blueprint (Table 4.3) produced for the diagnostic test in senior high schools however is that 30.0 per cent of the questions in the techniques and skill behavioural cell would have been adequate. The 5.7 per cent extra should be transferred to the applications cell. This would go a long way in satisfying Bishop's [115, p.47] thesis

...that the types of questions used in mathematics examinations at present fail to do justice to some of the ideas emphasized in the modern approach to mathematics teaching. It is inevitable that...

the type of terminal examination largely determines the style of teaching.

This approach to teaching the mathematics syllabuses would better prepare secondary pupils to perform efficiently in an economy and society which is undergoing structural change. The behaviour inventiveness, already discussed in Section 2.2 of Chapter 2, does not appear in the content-behaviour grid partly because in the author's experience there is never enough time to sample it along with other behaviours and more importantly, because more experienced practitioners Wood and Skurnik [35, p.171], have evidence from teachers that it was outside the competence of the majority of their pupils. But, in a paper on mathematical modelling, Burghes [116, p.114] introduces a demonstration of the empirical utility of Bishop's [115, p.47] suggestion by asserting

Mathematical modelling is a unifying theme for all applications of mathematics... The left had circle represents the real world, where problems must be solved, decisions taken or designs made. These real problems are put into a mathematical form by introducing variables and making assumptions about the laws connecting these variables. This is the mathematical model, and in most cases, this translation stage takes the problem from its real context into a mathematical problem... we are now in the mathematical world, the right hand circle. Using our mathematical
tools, we solve the mathematical problem and translate the solutions back into the original context. If the model is not adequate for its purpose, the assumptions in the model must be revised and the cycle traversed again; probably with a more sophisticated model. This of course means that the mathematical problem will be more complex. The art of the expert applied mathematician is to achieve the optimum balance between the complexity of the model and the manageability of the resulting mathematics.

Formulation

REAL WORLD

MATHEMATICAL WORLD

Interpretation

Mathematical Modelling (From Burghes[116, p.114])

Here Burghes is not only applying the behaviourist psychology in classroom activity but is also an advocate for the integrated studies philosophy in the mathematics curriculum. However he does not see any novelty in his views because in concluding his paper he (Burghes [116, p.130]) concedes:

Mathematical modelling is not a new concept. We have been applying mathematics for many centuries to help solve practical problems. We teach mathematics, not only for its beauty, but also for its usefulness for man and society. With the increasing use of mathematics in other disciplines, the idea of mathematical modelling provides a suitable framework for all applications of mathematics.

In order to implement a shift in style of teaching mathematics, it is necessary to encourage less 'chalk and talk' and
emphasise precision in the writing of specific objectives to accompany a mixed strategy. The fact is that the test constructor, curriculum maker, textbook writer and classroom teacher all work from a common base - the behaviourist psychology.
4.2.3 The Teaching of Further Mathematics

The following items of evidence suggest that the College of The Bahamas will function in the main as a sixth form college.[27]

1 The present teaching divisions are developing into four faculties:

(i) General Studies
(ii) Commerce, Administrative and Management Studies
(iii) Education
(iv) Technology

2 Communication to Parliament [23, p 20] directs the College to:

(i) offer courses which would include
   (a) ... GCE 'A' level studies formerly offered at the Government High School
   (b) academic courses of pre-university level
   (c) sub-professional and professional courses related to various sections ... of the Bahamian economy
   (d) university level courses as appropriate

   and

(ii) engage in research that is directed towards the optimal utilization of the country's natural resources

3 Bajpai and Bajah [5, p 39, paragraph 23] advise The Bahamas Government to

   ... allow the Senior High Schools to take the bulk of the responsibility for the teaching of students up to GCE 'O' level
Now Bajpai and Calus [100, p 456] postulate that a teacher must demonstrate professional and academic competence at a level beyond that at which he is required to teach. Begle [113, p 28], though considering elementary school teachers, shows

..... that it is important for a teacher to have a thorough understanding of the subject matter being taught.

Support for Bajpai and Calus comes also from Freudenthal [73, pp 163-165] who not only argues

..... just as long as there have been mathematics teacher examinations, candidates have been required to know more mathematics than they would have to teach afterwards

but also goes beyond to say

..... that the teacher should also be able to teach according to a programme that vastly differs from the programme that was in vogue when he went to school.

Unequivocal concurrence is volunteered by Théron [37, p 94] out of the experience of a survey where he recommends conditions for successful recruitment of a teacher, one of which is that he

..... must be trained in science to a level higher than that which he is going to teach. A secondary-school teacher must, for example, have a university degree.

Théron does not say so but the binding proviso is that this degree should include studies in the subject which the teacher
is required to teach. Enlarging understanding of what is expected of a secondary mathematics teacher in terms of pedagogy and content, Howson [62, p 47] advises:

The secondary school teacher ..... should be a mathematician capable of creating and organising mathematical ideas as well as disseminating them. His mathematics courses during training should reflect the changes in the subject as a contemporary body of knowledge as well as developing deductive processes and the creative aspects of mathematical discovery. His course should also include didactic analysis of a variety of teaching and learning approaches and the methodological consideration of classroom procedures.

Aware of the strength of the evidence presented above and its total justification for improvement in mathematics education at all levels in The Bahamas, the researcher gleefully accepted the challenge to deliver six lectures on Partial Differentiation and Ordinary Differential Equations to first year students who were offering the joint honours degree course in Education and Mathematics within the Department of Engineering Mathematics but jointly administered by the Department of Education. The AIMEC/CAMET brochure [117, p 4], mentioned earlier in Section 2.2, says:

The course in Education and Mathematics is carefully designed to suit the needs of young men and women who are entering the teaching profession and who wish to offer a subject or subjects closely related to the conditions of present-day industrialised society. The main objective of the course is to provide qualified graduate teachers of mathematics for schools and other educational establishments.

Before setting out the format of the first lecture, it must be mentioned that at secondary level, it is advisable not to
plan more than three, or at most four, objectives per lesson. In lecturing higher mathematics where one could expect a higher degree of motivation and prerequisite knowledge — although expectations are sometimes sadly dashed — the first lecture on a new topic usually involves establishing connections with previous knowledge, the derivation of definitions and the introduction of new symbols. The author was therefore justified in having five specific objectives but in the second lecture, the number was reduced to four. For the reasons given, the content of the first lecture may be deduced from the specific objectives which follow.

<table>
<thead>
<tr>
<th>Date</th>
<th>24 November 1980</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic</td>
<td>Partial Differentiation</td>
</tr>
<tr>
<td>Level</td>
<td>1st year university</td>
</tr>
<tr>
<td>No of students</td>
<td>20</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>(i) Revision of GCE 'A' level syllabus of any UK examination board or equivalent course of study, giving emphasis to elementary ideas of sets, number systems and inequalities, and relations and functions</td>
</tr>
<tr>
<td></td>
<td>(ii) Formulae booklets should be made available to students for reference during lectures and private study</td>
</tr>
<tr>
<td>General Objectives</td>
<td>At the end of this series of six lectures the students will:</td>
</tr>
<tr>
<td></td>
<td>(i) differentiate a function of several variables and use the notations for partial derivatives correctly</td>
</tr>
<tr>
<td></td>
<td>(ii) understand the total differential and use it to solve problems involving small changes or errors</td>
</tr>
<tr>
<td></td>
<td>(iii) differentiate a function of a function</td>
</tr>
</tbody>
</table>
Specific Objectives

At the end of the first lecture any student will

(i) define the partial derivatives \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) for \( z = f(x, y) \)

(ii) determine partial derivatives for simple functions of two variables

(iii) use alternative notations

\[ f_x = \frac{\partial f}{\partial x} = z_x \] correctly

(iv) determine second order partial derivatives for simple functions of two variables

(v) verify that \( z_{xy} = z_{yx} \) at all points at which \( z \) and its derivatives are continuous

Wherever intuition and insight in secondary school mathematics are discussed in this study, the author makes constant reference to limits and continuity because they are fundamental concepts which must be accommodated and assimilated for success in higher mathematics. The fact is that functions, which are continuous over a specific neighbourhood and have one point of accumulation (limit) are differentiable. But the converse is not true; namely, a point of accumulation is not necessarily a limiting value. Waismann [72, pp 123-140] details the empirical contributions made by Euler, Bernoulli and Descartes in organising mathematical knowledge on this topic after many initial frustrations. He (Waismann [72, p 148]) also reports

There arose the problem to develop a universal method by which the behaviour of tangents could be studied for any curve whatever. This problem, which Descartes had bequeathed to posterity, was solved a generation later by Newton and Leibniz and they did this at about the same time. Today the solution of this problem is called differential calculus.
In the applications of mathematics, the limit of a continuous function is of crucial importance to the success of mathematicians, engineers and scientists.

The content of the second lecture is reproduced here not only to emphasise the power and economy within the structure of the symbolic language of mathematics, but further to demonstrate a strategy of teaching - problem solving - which necessitates the recall and effective utility, through deductive logical reasoning, of the terminal behaviours knowledge, techniques and skills, comprehension (translation, interpretation, extrapolation) and application.
Specific Objectives

At the end of the second lecture any student will:

(i) solve problems using higher partial derivatives

(ii) state the formula for the total differential of \( z = f(x, y) \) and extend it to \( z = f(\phi, x, y) \), etc

(iii) calculate small changes or small errors and percentage changes in \( z = z(x, y) \), \( z = f(\phi, x, y) \), etc

(iv) verify given partial differential equations using the technique of differentiating function of a function.

To illustrate further the technique of higher partial differentiation, consider the following problem.

Find the values of the parameter \( n \) so that

\[ V = r^n(3 \cos^2 \theta - 1) \] satisfies

\[
\frac{\partial}{\partial x} \left( r^n \frac{\partial V}{\partial x} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) = 0
\]

Solution:

\[ V = r^n(3 \cos^2 \theta - 1) \]

\[ \frac{\partial V}{\partial x} = n r^{n-1} (3 \cos^2 \theta - 1) \]

\[ \therefore r^2 \left( \frac{\partial V}{\partial x} \right) = n r^{n+1} (3 \cos^2 \theta - 1) \]

\[ \therefore \frac{\partial}{\partial x} \left( r^2 \frac{\partial V}{\partial x} \right) = n(n + 1) r^n (3 \cos^2 \theta - 1) \]

Also \( \frac{\partial V}{\partial \theta} = -r^n (3 \sin 2\theta) \)

\[ \therefore \left( \sin \theta \frac{\partial V}{\partial \theta} \right) = -3 r^n \sin 2\theta \sin \theta \]

\[ \therefore \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) = -3 r^n \cos \theta \sin 2\theta - 6 r^n \cos 2\theta \sin \theta \]

and \( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) = -6 r^n \cos^2 \theta - 6 r^n (2 \cos^2 \theta - 1) \)

\[ = -6 r^n (\cos^2 \theta + 2 \cos^2 \theta - 1) \]

\[ = -6 r^n (3 \cos^2 \theta - 1) \]
Hence
\[ \frac{3}{\partial r} \left( r^2 \frac{3}{\partial r} \right) + \frac{1}{\sin \theta} \frac{3}{\partial \theta} \left( \sin \theta \frac{3}{\partial \theta} \right) \]
\[ = n(n + 1)r^n(3 \cos^2 \theta - 1) + \{-6r^n(3 \cos^2 \theta - 1)\} \]
\[ = (3 \cos^2 \theta - 1) \{n(n + 1)r^n - 6r^n\} \]
\[ = r^n(3 \cos^2 \theta - 1)(n(n + 1) - 6) \]

If \( r^n(3 \cos^2 \theta - 1)(n(n + 1) - 6) = 0 \), then \( (n + 3)(n - 2) = 0 \)
\[ \therefore \quad n = 2 \text{ and } n = -3 \]

Try this one during your private study.

Show that the equation \( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \) is satisfied by
\[ z = \ln \sqrt{x^2 + y^2} + \frac{1}{2} \tan^{-1} \left( \frac{y}{x} \right) \].

The volume, \( V \) of a cylinder or radius, \( r \) and height, \( h \) is given by \( V = \pi r^2 h \) and we may write \( V = f(r, h) \).

So \( \left( \frac{\partial V}{\partial r} \right)_h = 2\pi rh \) and \( \left( \frac{\partial V}{\partial h} \right)_r = \pi r^2 \).

What happens if \( r \) and \( h \) both change simultaneously?

If \( r \) becomes \( r + \delta r \), and \( h \) becomes \( h + \delta h \) then \( V \) should become \( V + \delta V \).

Note the difference between "\( \partial \)" and "\( \delta \)". The new volume would be given by
\[ V + \delta V = \pi (r + \delta r)^2 (h + \delta h) \]
\[ = \pi (r^2 + 2r \delta r + (\delta r)^2) (h + \delta h) \]
\[ = \pi (r^2 h + 2 rh \delta r + h(\delta r)^2 + r^2 \delta h + 2r \delta r \delta h + \delta h (\delta r)^2) \]
\[ \therefore \quad \delta V = \pi (2 rh \delta r + h(\delta r)^2 + r^2 \delta h + 2r \delta r \delta h + \delta h (\delta r)^2) \]
\[ \therefore \quad \delta V = 2\pi rh \delta r + \pi r^2 \delta h \]

since \( \delta r \) and \( \delta h \) are small and all the remaining terms are of a higher degree of smallness.
\[ \therefore \quad \delta V = \frac{\partial V}{\partial r} \delta r + \frac{\partial V}{\partial h} \delta h \]

Generally speaking, if \( f(x, y) \) is a continuous function defined on a region \( R \) of the xy-plane, and both \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) are continuous in this region, then
\[
\delta f = \frac{\partial f(x, y)}{\partial x} \delta x + \frac{\partial f(x, y)}{\partial y} \delta y
\]

Put another way, what we are saying is that if \( u = f(x, y) \)
\[
\delta u = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y
\]

We call the quantity \( \delta u = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y \) the TOTAL DIFFERENTIAL for the increments \( \delta x, \delta y \). It will be applied later, but at the moment we use the approximate formula to estimate errors.

Example 1:

If \( I = \frac{V}{R} \), and \( V = 250 \) volts and \( R = 50 \) ohms, find the change in \( I \) resulting from an increase of 1 volt in \( V \) and an increase of 0.5 ohm in \( R \).

Solution:

\[
I = f(V, R)
\]
\[
\therefore \delta I = \frac{\partial f}{\partial V} \delta V + \frac{\partial f}{\partial R} \delta R
\]

But \( \frac{\partial f}{\partial V} = \frac{1}{R} \) and \( \frac{\partial f}{\partial R} = -\frac{V}{R^2} \)

\[
\therefore \delta I = \frac{1}{R} \delta V + (-\frac{V}{R^2}) \delta R
\]
\[
= \frac{1}{50} (1) - \frac{250}{2500} (0.5)
\]
\[
\therefore \delta I = -0.03
\]

ie, \( I \) decreases by 0.03 amperes

Example 2:

If \( y = \frac{\omega s^3}{d^4} \), find the percentage increase in \( y \) when \( \omega \) increases by 2\%, \( s \) decreases by 3\% and \( d \) increases by 1\%.

Solution:

\[
y = f(\omega, s, d)
\]
\[
\therefore \delta y = \frac{\partial y}{\partial \omega} \delta \omega + \frac{\partial y}{\partial s} \delta s + \frac{\partial y}{\partial d} \delta d
\]
\[
\begin{align*}
\frac{d^3 s}{ds^4} \delta s &+ \frac{3\omega s^2}{ds^4} \frac{d}{ds^5} \delta s - \frac{4\omega s^3}{ds^5} \delta d \\
&= \frac{s^3}{ds^4} \cdot \frac{2\omega}{100} - \frac{3\omega s^2}{ds^4} \cdot \frac{3s}{100} - \frac{4\omega s^3}{ds^5} \cdot \frac{d}{100} \\
&= \omega s^3 \left( \frac{2}{100} - \frac{9}{100} - \frac{4}{100} \right) \\
&= y \left( - \frac{11}{100} \right) \\
\end{align*}
\]

\[\therefore \delta y = -11\% \text{ of } y\]

ie, \( y \) decreases by 11\%

**Example 3:**

Find the total differential \( du \) given that

\[ u = f(x, y) = e^{(x^2+y^2)} \]

**Solution:**

\[ du = \frac{3u}{\partial x} \, dx + \frac{3u}{\partial y} \, dy \]

\[ \therefore du = 2xe^{(x^2+y^2)} \, dx + 2ye^{(x^2+y^2)} \, dy \]

ie \( e^{(x^2+y^2)} \, (xdx + ydy) \)

**Example 4:**

Show that \( (3x^2y - 2y^2)dx + (x^3 - 4xy + 6y^2)dy \) can be written as an exact differential of a function \( \phi(x, y) \) and find this function.

**Solution:**

Suppose that \( (3x^2y - 2y^2)dx + (x^3 - 4xy + 6y^2)dy = d\phi \)

\[ \text{But} \quad d\phi = \frac{\partial \phi}{\partial x} \, dx + \frac{\partial \phi}{\partial y} \, dy \]

\[ \therefore \frac{\partial \phi}{\partial x} = 3x^2y - 2y^2 \quad (i) \]

and \[ \frac{\partial \phi}{\partial y} = x^3 - 4xy + 6y^2 \quad (ii) \]

Integrating (i) w.r.t \( x \) and keeping \( y \) constant, we have

\[ \phi = x^3y - 2xy^2 + F(y) \]

where \( F(y) \) is the constant of integration.
Substituting for \( \phi \) in (ii) we get

\[
\frac{\partial}{\partial y} (x^3 y - 2xy^2 + F(y)) = x^3 - 4xy + 6y^2
\]

\[\therefore \quad x^3 - 4xy + F'(y) = x^3 - 4xy + 6y^2\]

\[\Rightarrow F'(y) = 6y^2\]

\[\Rightarrow \int F'(y) \, dy = \int 6y^2 \, dy\]

\[\Rightarrow F(y) = 2y^3 + c\]

Hence the required function is \( \phi = x^3 y - 2xy^2 + 2y^3 + c \), where \( c \) is an arbitrary constant.

We know that if \( f = f(u) \) and \( u = u(x) \), then \( \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} \). This is a FUNCTION OF A FUNCTION.

This result may be immediately extended to the case when \( f \) is a function of two or more independent variables. For the case where \( f = f(u) \) and \( u = u(x, y) \).

\[x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{df}{du} \cdot \frac{du}{dx}\]

Example:

If \( z = xf(\frac{y}{x}) + g(\frac{y}{x}) \), where \( f \) and \( g \) are arbitrary functions, show that

\[x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z - g\]

and deduce that \( x \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0\)

Solution:

\[\frac{\partial z}{\partial x} = f(\frac{y}{x}) \cdot \frac{y}{x} + xf'(\frac{y}{x}) \left( -\frac{y}{x^2} \right) + g'(\frac{y}{x}) \left( -\frac{y}{x^2} \right)\]

\[\therefore \quad x \frac{\partial z}{\partial x} = xf(\frac{y}{x}) - yf'(\frac{y}{x}) - (\frac{y}{x}) g'(\frac{y}{x})\]
\[
\frac{\partial z}{\partial y} = xf'(\frac{y}{x}) \left(\frac{1}{x}\right) + g'(\frac{y}{x}) \left(\frac{1}{x}\right)
\]

\[
\therefore\ y \frac{\partial z}{\partial y} = yf'(\frac{y}{x}) + g'(\frac{y}{x})
\]

\[
\therefore\ x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = xf(\frac{y}{x})
\]

But it is given that \( z - g(\frac{y}{x}) = xf(\frac{y}{x}) \)

\[
\therefore\ z - g = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}
\]

Differentiating this result w.r.t \( x \) we get

\[
g'(\frac{y}{x}) \left(\frac{1}{x}\right) = y \frac{\partial^2 z}{\partial x \partial y} + x \frac{\partial^2 z}{\partial y^2}
\]

(i)

Differentiating the same result w.r.t \( y \) we get

\[
-g'(\frac{y}{x}) \left(\frac{1}{x}\right) = x \frac{\partial^2 z}{\partial x \partial y} + y \frac{\partial^2 z}{\partial y^2}
\]

(ii)

Multiplying (i) by \( \frac{x^2}{y} \): \( g'(\frac{y}{x}) = \frac{x^2 \partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial x \partial y} \)

Multiplying (ii) by \( -x \): \( g'(\frac{y}{x}) = -x^2 \frac{\partial^2 z}{\partial x \partial y} - xy \frac{\partial^2 z}{\partial y^2} \)

\[
\therefore\ \frac{x^2 \partial^2 z}{\partial x^2} + 2x^2 \frac{\partial^2 z}{\partial x \partial y} + xy \frac{\partial^2 z}{\partial y^2} = 0
\]

Multiplying by \( \frac{y}{x} \) : \( x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0 \)
In the next lecture the following topics would be discussed, using appropriate examples to illustrate the theory:

1 Rate of change of \( z = f(x, y) \)
2 Maxima, minima and saddle points for \( z = f(x, y) \)
3 Maxima or minima involving constraints by the method of Lagrange multipliers

During the one hour presentation, it was not necessary to exhibit on the chalkboard or overhead projector all minute details as shown in the preparation so that the content slotted nicely into the time allocation. Also, these lectures were supported by tutorial discussions of selected problems requiring methods, techniques, definitions, symbols and axioms in students' behaviour. Encouragement was given to students, mainly by good researcher-student relationships, to bring any doubts to the attention of tutorial meetings or the researcher's desk. Furthermore, examples for practice were circulated to students not only to be kept as part of their notes but also for prior study in anticipation of tutorial meetings and lectures. In this way, they had enough work to do in their subject area between meetings for lectures and tutorials.

The author makes the point that elements of the behaviourist, integrated studies, formative and helical philosophies of curriculum development when skilfully applied in classroom practice produce clarity in communication within the presentation phenomenon even when that communication is of an expository nature but supplemented by positive personality traits, effective use of the chalkboard, overhead projector, geometrical instruments and thorough prior preparation. The criterion of success was gauged by students' invitation for the researcher to continue at the end of the sixth lecture. Permission was granted and what started as a six-lecture series developed into an exciting and enjoyable eight-lecture
course by both researcher and students. The content of presentation and tutorial problems over the eight lectures were a synthesis of ideas from Bajpai et al [71, pp 382-399], Bajpai et al [70, pp 1:209 - 1:234], Boas [118, pp 121-159], Jeffrey [119, pp 119-231] and Spiegel [120, pp 101-133]. It has been justified therefore that the content-behaviour (specification) grids, used for the construction of the instruments in this research, may be erected for assessing attainment at any level of teaching extending from primary school to university.
4.3 Official Attitude to Data Collection in The Bahamas

The apprehension of the administrative/technical arm of the education system revealed itself in the form of an unyielding negative attitude towards classroom investigations, the aim of which is to inform and improve the process of decision making in teaching practice. The researcher felt that he was regarded as one of them instead of one of us. Educators in decision-making roles in developing countries must resist the temptation to feel threatened in the face of new approaches to obtaining knowledge for curriculum improvement. In contrast, it would never have been possible to produce recent reports - [49], [50] and [60] - in England, for example, without the cooperation of schools, industry and commerce. The systems in developing countries cannot benefit by fresh ideas, let alone respond to change, if it is impossible for experienced, qualified, local nationals to objectively examine the systems in order to arrive at a rationale for suggesting improvement.

Originally it was decided to administer the diagnostic tests two weeks before Easter in 1981 and to get teachers' input as well as feedback from pupils: repeated field trials were to be conducted in the autumn 1981 for fresh feedback to be used in a validation exercise. Acting on the advice of the Director of Education, the Senior Education Officer for Curriculum Development was consulted, whereupon it was agreed that technical staff would support any survey carried out by this researcher in the name of CAMET. But since there was no written agreement, the Ministry of Education could not be asked to honour its promise. For the benefit of future researchers, this author sees the attitude of authority as a powerful braking force on curriculum development in developing countries, so perpetuating inertia, and even regression, in a vital area influencing human development - classroom practice. Until this psychological barrier is broken, education systems in developing countries will refuse to respond to change, in defiance of reasonable democratic demands made through the machinery of the State.
4.4 **Summary**

 Appropriately, the author draws attention to a retrospective view of progress so as to link the preceding aspects of this study with what is yet to come. The research *problem* - low attainment in BJC and London University GCE 'O' level mathematics (Section 2.7 of Chapter 2) - has been specified and a hypothesis, which remains to be tested, has been formulated. A résumé of current international views on curriculum development, teacher training and classroom practice, as synthesised from practical classroom experience, psychological and sociological theory and research in mathematics education were discussed in Chapter 3 with due acquiescence having been made for the nature of mathematical abilities alongside the views of mathematicians. In the present chapter, matters of pertinence to the *design* of this research and its weaknesses were outlined, and, taking a forward perspective, the following two chapters present the *procedure* deployed in pretesting items and questions over samples of junior and senior high school pupils on the island of New Providence, coupled with *analysis* of data so obtained.
CHAPTER 5

FIELD SURVEY RESULTS:
JUNIOR HIGH SCHOOLS
Field Survey Results: Junior High Schools

The purpose of this chapter was to examine the feedback obtained from the items constructed according to the specification grid laid out in Chapter 4. Further, the author had to ascertain whether certain key facts and operations in the junior high mathematics syllabus were known and performed accurately so that further work which is dependent on this knowledge could be successfully attempted. Also the state of this knowledge was required to establish or negate the truth value of the hypothesis proposed in the last section of Chapter 2. Downey and Kelly [74, p.155], concurring with the research method of the author, offer the reminder:

... John Dewey, who saw all knowledge as ultimately of a scientific kind, a result of the experience that each individual has as he solves the problems presented to him by his environment, physical, social, cultural and aesthetic, problems which are solved in all of these spheres by the application of the scientific method of framing and testing hypotheses.

This diagnostic test not only sought to measure the levels of mastery of factual knowledge as well as comprehension but also to look at the feedback with a view to commencing in-service training of teachers through curriculum development and selecting items for an item bank. This exercise was indeed the beginning of a process of determining whether samples of secondary pupils have a qualitative grasp of mathematical concepts, and if so, their depth and kind of understanding as educational theory is informed by the empirical psychological findings of Skemp [63] and the psychometric findings of Wood [121], to cite two examples. To this end statistics were necessary. However, these statistics were not motivated by probabilistic assumptions but mainly by dichotomous items scored 0 or 1. For each item, raw response data consisted of the frequency counts of the number of individuals choosing each item, together with the number not answering the item at all (omits). The author
has not been unduly concerned about calculating item 
difficulty as opposed to item facility, although psychometrically this is a very important distinction. In fact 
Wood [121, p. 241] uses the delta ($\Delta$) statistic, defined by 

$$\Delta = 4 \phi^{-1}(p) + 13$$

where $p$ is the item facility and $\phi^{-1}$ is the inverse normal transformation. But teachers who have no confidence in their mathematical ability would be happier using a definition which uses simple arithmetic in the calculation of item difficulty. Satterly [122, p. 108] uses such a definition:

$$\text{Difficulty index } = 1 - \frac{R}{T}$$

where $R =$ number of correct responses and $T =$ total number of pupils. Put another way,

$$\text{Difficulty index } = 1 - \text{Facility index},$$

which does in fact indicate how difficult the item was. For dividing the survey population into ability groups, mean test score seemed the most obvious criterion for observing response patterns. This statistic was used extensively in the analysis but more balance and objectivity could have been brought to bear on the entire exercise if teachers' views of each candidate were obtained, and if the candidates themselves were interviewed after the test was marked. This would have generated a more comprehensive picture of ability in combination with degree of motivation shown by the pupils. These limitations in the research findings are consequent upon distance from the problem space as well as lack of cooperation from the administrative and technical leadership within the rigid hierarchical structure of education.

Discrimination is regarded a key statistic because it is itself partially a function of mean values and partially a function of facility. It was therefore expected that each item would discriminate the same way as the test in the sense of separating those of low ability from those of high ability. According to Wilmut [123], for the vast majority of items it should lie between 0 and 0.59, its minimum value being -0.1
and maximum, 0.75 if the point biserial coefficient of correlation is used. Psychometric theory puts the view that the more candidates offering an item, the better the discrimination; also, the discrimination is very low if a large subsample get an easy question right. In order to inform the process of decision-making in classroom matters, the writer will also look for findings which compare and contrast with those of Ebel [124 p.565], who expects the reliability index to increase as the number of choices per item increases; and Dunn and Goldstein [125, p.178], who believe that further research is needed before it could be upheld that numerical values of the variables difficulty, validity and reliability vary with adherence to:

1. use of incomplete statements as item leads
2. avoidance of specific determiners or cues to the correct alternative
3. alternatives of equal lengths
4. consistency in grammar between lead and alternatives.

Dunn and Goldstein do not define 'difficulty' but the writer will interpret it to mean 'facility'. Also 'alternative' will be interpreted 'option'. Every care was taken to make certain that the sample from each school was at least 10.0 per cent of the cohort so a useful pattern should emerge despite the unpredictable and less obvious effects of powerful behavioural variables.

5.1 Pilot Visits to Schools, Administration of Test and Marking of Answer Books

The previous chapter discussed the thinking, planning and preparation (mainly consulting the literature) put into the construction of the process-content grid out of which, in turn, the research instruments stemmed in final form, after revisions of the items and meticulous proof reading. These activities took place in the resource-rich environs of the Centre for Advancement of Mathematical Education in Technology, but in The Bahamas the logistics of the execution
of the plan to use the instruments had to be decided. The first in a sequence of steps was permission from the Ministry of Education and Culture (as it was then) to use its schools. Leave to approach the schools was first formally sought through the bureaucracy since March 1979 but when no written reply was received up to the summer 1981, a personal request to the Director of Education met an apologetic reply for previous tardiness in responding to correspondence. The researcher was referred to the Deputy whom it was impossible to see.

5.1.1 Pilot Visits to Schools (Autumn 1981)

On arrival at all of the schools the author found eager co-operation which easily translated into dialogue with Heads of Mathematics Departments. Dates were fixed for pupils to offer the test at 6 maintained junior high schools and 3 similar independent schools. Prior to fixing dates, mathematics teachers expressed satisfaction that the content was aimed at the previous year's Grade 8 syllabus, the implication being that Grade 9 were suitable subjects for the test. Two independent junior high schools had not abandoned the form system so the parallel classes for them was Form 3.

Except for a few broken window louvres, schools were in good repair. For those that were not, an extensive renovation programme was in effect. Inside the classrooms pastel colours were bright, there was one desk per child but it seemed that classrooms in the independent schools had more space even though the mean teacher-pupil ratio was 1:37 as opposed to 1:34 in a state-maintained school. In the interest of effective mathematics education and better management of learning the researcher suggests a teacher-pupil ratio of 1:30 at this level. Over the full teaching week mathematics was taught for between 14.3 per cent of the teaching time allocated in two independent schools and 20.0 per cent of the time in state-maintained schools. At one maintained school, where the teaching week had 35 periods of 40 minutes each, mathematics for the class being tested was divided into two double periods, one of which was at the end of an
afternoon, and one single period per week. This allocation of teaching time was probably decided upon by the wishes of the teaching staff, the lay-out (geography) of the school and the constraints of other subjects. It appeared to the author however, that it was more prudent to have mathematics periods scattered over different days and times of the week so as to avoid 40.0 per cent of the teaching time allocated to mathematics to occur at the end of a single afternoon in a badly ventilated classroom. In their paper on 'The Evolution of Mathematics Curricula in the Arab States', Jurdak and Jacobsen [4, pp.141, 142], say:

Compared to other subjects, mathematics is given a substantial weight in the total school curriculum. This is reflected in the high percentage of the number of weekly periods allotted to mathematics. In the elementary school, mathematics is allotted 15 to 21 per cent of the usual thirty teaching periods per week of 40 to 50 minutes each. It is worth noticing that mathematics ranks third (after Arabic language and religion) in terms of time allotment. In the intermediate school, the percentage varies between 13 and 17 per cent of the thirty to forty weekly periods of 50 to 60 minutes each. And in the secondary school, the percentage varies between 16 and 26 per cent of the 30 to 42 weekly periods of 55 to 60 minutes each.

The weight given to mathematics has not appreciably changed since the survey of 1969. Such weight is still comparable to that of other developed countries.

There was no form of school-based in-service training in evidence. Schemes of work were well prepared, in one case, for a whole year but not detailed enough to show what was expected to be taught and learned during each week of the three terms as was seen at an independent junior/senior high school. From these schemes in the maintained schools, it seemed that the
teaching was destined to be pretty routine, didactic and axiomatic - chalkboard examples followed by class exercises in the various streams, with no preparation made for individual differences among pupils. Also there was no evidence of consultations, let alone links, with other subject disciplines in the maintained schools although such evidence was obvious in two of the independent schools which participated in the survey.

The syllabus in all schools was that provided by the Ministry of Education and Culture. One of the independent schools used SMP Books C, half of D and E supplemented by Caribbean Mathematics I, II, III published by Longman. At least in one of the maintained schools, SMP Books C and D were used for angles, directed numbers, and percentages in conjunction with Mathematics Book I by L H Clarke, and Discovering Mathematics Book I by Shaw and Wright. Although Shaw and Wright do show pictures of practical applications, no attempt has been made to adapt the suggestions perhaps owing to shortage of apparatus, unwillingness to improvise and pressure to complete the syllabus in time for BJC examinations. Otherwise the latter books are traditional in philosophy, providing no help to the teacher in motivating the pupils towards mathematics. With regard to school libraries, one independent school had a well stocked and managed classroom where there was ample evidence that the books were read. However, schools are not in the habit of providing author's name, year of publication, title of book and publisher when required to communicate the textbooks they were using.

The author found that classes had already begun primarily for culmination in public examinations at the end of the year. But noticeably absent from the BJC syllabus (Appendix 5.2) were standardised books for mathematics, model answers to questions which revealed an insight into behaviours expected, specimen papers, marking schemes, methods of teaching, apparatus to be used in teaching and teacher's guides. Whilst the reports on past examination scripts were helpful in a general way, no examples were given of the types of mistakes which occurred and teaching techniques for putting them right. Nothing was said
about the use of models in teaching or the kinds of models which can be fashioned from inexpensive used materials like egg cartons, scraps of hessian cord, cardboard, various used containers which had preassigned weights and measures of liquid and dried foods. A used resources corner can be a generous contributor to experiment and discovery for pupils to arrive at concepts of number, length, weight and capacity. There was no evidence of scales for weighing in metric or Imperial units. Essentially mathematics at this level should be very practical and there is no substitute for continuing and extending the methods of the primary school. In particular the first two years of junior high school should be a transition from primary to secondary education. It is commendable however that the mathematics syllabuses are developed on the behaviourist model of curriculum development but this does not ensure that the behaviourist learning-teaching psychology is effected in classroom practice. It was evident also that some attempt had been made to marry 'traditional' with 'modern' mathematics in the presence of transformation geometry but, although measuring and drawing angles of given sizes are final behaviours, no mention was made of the kinds of activities which would give children an intuitive notion of what a point, line and an angle were - fundamental geometrical concepts upon which to build other concepts for ultimate appreciation and use of mathematical structure. Too often in the author's experience, children were pushed into exercises on triangles when they had no idea what an angle was. This work must be put on a footing such that it becomes easy for pupils to understand angles larger than 360°. The prerequisite knowledge for every part of the syllabus was assumed but not stated; the same was true of specific objectives although the ultimate behaviours are excellent. There was, therefore, too much dependence on the textbook, the contents of which were a mismatch with the curriculum and certainly were not sequenced. There was no standard textbook for use in all classrooms at the same grade level and the balance was in favour of those which stressed sterile calculations without reference to utility in The Bahamian society. Appropriate traditional content areas amenable to the application of set language and notation were in the sections of the syllabus on powers and roots,
<table>
<thead>
<tr>
<th>Primary School</th>
<th>Junior High School</th>
<th>Senior High School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Willard Patton</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Woodcock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T A Thompson</td>
<td>→ T G Glover</td>
<td></td>
</tr>
<tr>
<td>T Gibson</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Naomi Blatch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chippingham</td>
<td></td>
<td>+ C C Sweeting</td>
</tr>
<tr>
<td>Oakes Field</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mabel Walker</td>
<td>→ H O Nash</td>
<td></td>
</tr>
<tr>
<td>Wilton Albury</td>
<td></td>
<td></td>
</tr>
<tr>
<td>William Sayles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gambier</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uriah McPhee(⅓)*</td>
<td>→ C I Gibson</td>
<td></td>
</tr>
<tr>
<td>Claridge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>William Phipps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>William Gordon(⅓)*</td>
<td>+ R M Bailey</td>
<td></td>
</tr>
<tr>
<td>Palmdale</td>
<td>→ D W Davis(⅓)*</td>
<td></td>
</tr>
<tr>
<td>Centreville</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columbus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stephen Dillet</td>
<td>→ C H Reeves</td>
<td>→ Government High</td>
</tr>
<tr>
<td>E P Roberts</td>
<td>D W Davis(⅓)</td>
<td></td>
</tr>
<tr>
<td>William Gordon(⅓)*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandilands</td>
<td>→ L W Young</td>
<td>→ L W Young</td>
</tr>
<tr>
<td>Uriah McPhee(⅓)*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yellow Elder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ridgeland</td>
<td>→ S C McPherson</td>
<td>→ A F Adderley</td>
</tr>
<tr>
<td>C W Sawyer</td>
<td>Adelaide</td>
<td></td>
</tr>
<tr>
<td>Carmichael</td>
<td>(Primary/Junior High)</td>
<td></td>
</tr>
</tbody>
</table>

*At the end of 6 years Uriah McPhee and William Gordon feed half of their top primary classes into the different schools shown. Similarly, at the end of 3 years R M Bailey and Government High receive top pupils from D W Davis.*
Samples from all junior high schools, except S C McPherson and the upper school of Adelaide, offered the diagnostic test. A separate test, described and analysed in Chapter 6, was offered by a sample from each senior high school. The table below shows the schools which cooperated in the conduct of the survey of mathematics achievement, the dates on which their pupils offered the test, the identification of the scripts and the size of the subsample from each school. The total population was 377 pupils (called cases in the computer

<table>
<thead>
<tr>
<th>DATE</th>
<th>SCHOOL</th>
<th>COMPUTER SUBFILES</th>
<th>ANSWER BOOKS</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.9.81</td>
<td>H O Nash (G)</td>
<td>1</td>
<td>1-31</td>
<td>31</td>
</tr>
<tr>
<td>15.9.81</td>
<td>L W Young (G)</td>
<td>2</td>
<td>32-66</td>
<td>35</td>
</tr>
<tr>
<td>15.9.81</td>
<td>Queen's College (I)</td>
<td>3</td>
<td>67-183</td>
<td>117</td>
</tr>
<tr>
<td>16.9.81</td>
<td>C H Reeves (G)</td>
<td>4</td>
<td>184-217</td>
<td>34</td>
</tr>
<tr>
<td>17.9.81</td>
<td>C I Gibson (G)</td>
<td>5</td>
<td>218-252</td>
<td>35</td>
</tr>
<tr>
<td>17.9.81</td>
<td>D W Davis (G)</td>
<td>6</td>
<td>253-282</td>
<td>30</td>
</tr>
<tr>
<td>18.9.81</td>
<td>L W Young (G)</td>
<td>0</td>
<td>283-298</td>
<td>16</td>
</tr>
<tr>
<td>21.9.81</td>
<td>Prince Williams (I)</td>
<td>7</td>
<td>299-320</td>
<td>22</td>
</tr>
<tr>
<td>22.9.81</td>
<td>St John's College (I)</td>
<td>8</td>
<td>321-352</td>
<td>32</td>
</tr>
<tr>
<td>25.9.81</td>
<td>T G Glover (G)</td>
<td>9</td>
<td>353-377</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
<td></td>
<td></td>
<td>377</td>
</tr>
</tbody>
</table>

Table Details of Junior High Survey Population (G) Government; (I) Independent

file created on 30.10.81) and the data for each subpopulation were organised in a separate subfile numbered from 1 to 0.* The statistics stored in File 0 were the results of the top stream Grade 11 pupils who also offered the diagnostic test analysed in the next chapter.

5.1.2 Administration of Test

Complete physical and material preparations were always made prior to the arrival of the pupils at the room in which the

* (See column 3 above)
test was to be written. Roughwork paper, question and answer books were placed on the desks but turned the wrong way up until everyone was asked to follow the reading of the instructions, which were modified - as will be seen in the next section - to cover eventualities for which no allowances were made when the test was compiled at Loughborough. All of the candidates were given the instruction to begin at the same time. At Queen's College where 117 pupils offered the paper, the Head of Mathematics and six mathematics teachers helped the author with the distribution and collection of the item and answer books. So did at least one mathematics teacher or Head of Department at other schools. The practice of numbering the answer books immediately after each sitting for identification of the school, and giving the Head of Mathematics a question book was strictly followed. There were no incidents of irregular conduct or disruptive behaviour by the pupils during the writing of the test: the standard of personal conduct was very high. It was found however, that after each administration, some answer books had been defaced by mistake but these were not many and they were usually found before the next run of the test. The test was very popular with both teachers and pupils. (See Appendix 5.1).

5.1.3 Marking of Answer Books

Instructions to the candidates were well carried out but there were a few cases where, examinations being what they are, directives were not fully heeded but these cases were anticipated and allowances were made for them in the supplementary verbal instructions just before the test began. For example, if a candidate circled one of the five options to a question and subsequently decided that another option was correct, the first choice was cancelled thus (c) and the second choice (e) was permitted, if in fact, it was the key. There was another permissible variant. Suppose the choice (a) was made but option d was calculated to be the right answer. Then the answer book was marked (a) b c d e. But if, on second check, it happened that A was in fact the correct option owing
to a genuine mistake in calculation, then (A) B C (D) E was accepted. Although it was not mentioned in the instructions, candidates were expressly forbidden to use erasers.

For the purpose of computer marking the following data were copied from left to right on fortran coding forms:

(i) answer book number (001-377)
(ii) sex (B or G)
(iii) date of birth (eg. 25 06 67)
(iv) total correct answers (eg. 15)
(v) 50 sets of options on answer books (each option being 0, 1, 2, 3, 4, 5 or 9).

Between each element of the set of data, a vacant space was left and candidates from individual schools could be identified by the number who offered the test and the numeral at which any school's answer books began. The data for each school was therefore stored in a subfile. In describing the options for the computer, numerals 1-5, in (v) above, represented the options A-E respectively. 0 meant that no option was circled and 9 indicated a multiple response (ie. an attempt with at least two options circled). The data from each answer book was punched onto individual computer punch-cards and this information from the 377 cards was stored in a file for future reference. Using a manual, the author punched a set of cards, put them in the right order, sent them to the computer centre with an instruction to get the file and select the correct indicated option for each question. For example, the computer was told on one card to select option 3 for variable (item) 050, and to print out the mean, standard error and standard deviation for that question. Each time a new set of statistics was required, instructions had to be set out on punch-cards and sent to the computer centre. Having marked and stored the information, the same method was used to produce most of the raw statistics relevant to this study, including a cumulative frequency graph from which the mark/grade distribution for the junior high school sample was determined. For the calculation of various means and

* Two specimen fortran coding forms are contained in Appendix 5.3.
percentages, and especially reliability and the point biserial coefficient of correlation, the writer used his 'Casio fx-120 scientific calculator' which was purchased in Loughborough especially for the research and mainly iterative problems for tutorial classes.

Prior to computer marking of the scripts, the author made a template and, while scoring the answer books in The Bahamas, discovered that there were two answer books from an independent school on which the pupils did not give their sex. This does not affect the mean performance of the schools but it does make the detailed analysis between the achievements of boys and girls slightly imprecise.
5.2 Performance of Survey Population on All Items

The author did not wish to impute that a score on one test has decided, for all time, a candidate's level of performance mainly because mathematical ability was seen by psychologists as a dynamic entity which, therefore, has the potential to improve in response to diverse, stimulating, and practical experiences. However, the only and the best numerical value for quantifying the variable 'ability' was the mean criterion of the candidates [121, pp. 241, 242], which Table 5.1 provides for a typical individual, a boy, and a girl. But great care should be exercised in interpreting the mean performance on the basis of sex if only because two answer books did not identify whether the respondents were both boys, or both girls, or one of each. From the statistics, their mean mark of 15.5 was 58.1 per cent of the size of the mean, 26.7 for the entire survey population. There is not a lot of difference (0.8) between the mean score of the boys, 26.2 and that of the girls, 27.0. But from a scan of the marks, the boys seemed to have outperformed the girls and have shown a performance which is only 0.5 of a mark less than the performance of the entire population. The evidence for saying that the boys were of a higher ability than the girls on the test was contained in the standard deviations. The higher standard deviation of the boys' scores indicates that their marks were more widely dispersed from the mean. The same reasoning holds true if the standard deviations of boys' and girls' means are compared with the standard deviation of the mean for the entire population.

<table>
<thead>
<tr>
<th></th>
<th>TOTAL</th>
<th>MEAN</th>
<th>SD</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>31.0</td>
<td>15.5</td>
<td>3.5</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>3618.0</td>
<td>26.2</td>
<td>9.4</td>
<td>138</td>
</tr>
<tr>
<td>G</td>
<td>6404.0</td>
<td>27.0</td>
<td>7.9</td>
<td>237</td>
</tr>
<tr>
<td>EP*</td>
<td>10 053.0</td>
<td>26.7</td>
<td>8.5</td>
<td>377</td>
</tr>
</tbody>
</table>

Table 5.1: Total mark broken down by sex
* EP - Entire population
The following table shows suitably detailed data obtained after the computer was asked to print out the information in each subfile (SF) to show a breakdown of the criterion variable total by the variable sex. However, for tabulation, it was more convenient to record the variables mean, standard deviation (SD) and number of cases (N) [126, p.7], so as to unveil the full significance of the mean scores.

<table>
<thead>
<tr>
<th>SF(SCH)</th>
<th>MEAN</th>
<th>SD</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>HON 1(G)</td>
<td>22.7</td>
<td>24.2</td>
<td>22.0</td>
</tr>
<tr>
<td>LWY 2(G)</td>
<td>18.3</td>
<td>15.5</td>
<td>19.5</td>
</tr>
<tr>
<td>QC*3(I)</td>
<td>25.2</td>
<td>21.9</td>
<td>27.9</td>
</tr>
<tr>
<td>CHR4(G)</td>
<td>33.5</td>
<td>37.2</td>
<td>32.0</td>
</tr>
<tr>
<td>CIG5(G)</td>
<td>29.0</td>
<td>30.7</td>
<td>28.2</td>
</tr>
<tr>
<td>DWD6(G)</td>
<td>24.7</td>
<td>25.6</td>
<td>24.2</td>
</tr>
<tr>
<td>PW 7(I)</td>
<td>28.5</td>
<td>29.0</td>
<td>28.2</td>
</tr>
<tr>
<td>STJC8(I)</td>
<td>37.6</td>
<td>41.4</td>
<td>35.9</td>
</tr>
<tr>
<td>TGG9(G)</td>
<td>23.1</td>
<td>23.6</td>
<td>22.9</td>
</tr>
<tr>
<td>LWY O(G)</td>
<td>28.7</td>
<td>31.6</td>
<td>26.4</td>
</tr>
</tbody>
</table>

SF - subfile, SCH - school, EP - entire population, SD - standard deviation, N - number

Table 5.2: Mean Mark for Each School Broken Down by Sex

* 2 scripts of unnamed sex had a mean = 15.5 and SD = 3.5

G - Government, I - Independent

The purpose for the display of this information was mainly to compare the mean attainment of a typical candidate at each school with that of a similar candidate from the entire population, the mean of which was 26.7 and standard deviation 8.5. A more precise indicator of performance was derived by applying Morrison's [127, pp.153-155] expected theoretical distribution to the ogive (Figure 5.1) plotted from the cumulative frequency data in Appendix 5.4 for the 377 candidates, in order to find the partition points for
the grades. There was excellent agreement between the valid and the theoretical distribution of candidates as shown by Table 5.3.

<table>
<thead>
<tr>
<th>MARK RANGE</th>
<th>GRADE</th>
<th>DISTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Valid</td>
</tr>
<tr>
<td>41 and above</td>
<td>A</td>
<td>28</td>
</tr>
<tr>
<td>32-40</td>
<td>B</td>
<td>75</td>
</tr>
<tr>
<td>20-31</td>
<td>C</td>
<td>189</td>
</tr>
<tr>
<td>14-19</td>
<td>D</td>
<td>72</td>
</tr>
<tr>
<td>13 and below</td>
<td>E</td>
<td>13</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>377</td>
</tr>
</tbody>
</table>

† round off error

Table 5.3: Expected and Valid Percentage of Entire Population in each Grade

The mark range was determined by applying the theoretical percentage - grade distribution for any year's cohort to the ogive in Figure 5.1. This grade specification was based on the performance the author expected in an average year, but for future use, due allowance has to be made for variations in ability from year to year. If 5.0 per cent of the candidates merit grade A, then 95.0 per cent of the survey class lay below the A/B boundary. The point (0, 95.0) was located on the CF-axis and traced along the line CF = 95.0 until it intersected the ogive at (41.0, 95.0). Consequently any candidate who scored 41 out of 50 or more earned an A grade. Also, 75.0 per cent of the candidates lay below the B/C boundary, which was defined by tracing the horizontal line CF = 75.0 to (32.0, 75.0) on the ogive. Similarly the C/D boundary had 25.0 per cent of the pupils below it, so the horizontal line, the equation of which was CF = 25.0, intercepted the ogive at (20.0, 25.0). Finally, below the D/E boundary lay 5.0 per cent of the candidates, so CF = 5.0 was followed to its intercept (14.0, 5.0) with the ogive. Consequently, the Mark Range - Grade Table was uniquely defined by the following inequalities:
The algorithm applied here is very useful for the teacher because it does not require the scaling of the original mark distribution nor does it depend on the distribution being normal. It was out of the exercise of applying Cartesian coordinates to Euclidean space that the partitioning of the valid distribution of the junior high candidates was erected in Table 5.3. But on close examination, the lattice of the table seemed fragmented by discontinuities at the partitions. These discontinuities are, in fact, very real owing to the non-linearity of the scale interval between the grades. That is to say, the difference in ability between D and C is not the same as that between B and A. Consequently, the researcher is at variance with the common practice in The Bahamas of assigning numbers 1 to 5 to a 5-point non-linear scale and operating on them to find a mean grade for potential teachers. A possible solution to the problem of assessing students' performances is to give more 'weight' to the teaching practice than the other components, say, psychology, sociology, method, philosophy and educational measurement, which have to be assessed simultaneously.

Clearly any mark less than 20 out of 50 (Table 5.3) was unsatisfactory. If the diagnostic test was a public examination, pupils obtaining grades D and E, 22.5 per cent of the entire population, would fail, having been deemed not to have had a secure grasp of basic concepts required for future progress and success. It would be mandatory for them to undertake a programme of individualised learning in order to improve their chance of success.

The performance of the candidates shows that the Anglican independent school (STJC 8), in Table 5.2, recorded a mean
performance of 37.6 out of a possible 50 marks, with a standard deviation of 6.4; maintained school, CHR 4, scored a mean of 33.5 with a linear spread of 5.9; and another school from the maintained sector, CIG 5, earned a mean of 29.0 with a linear spread of the marks about the mean measured by 6.2 standard deviations units. Obviously, for all practical purposes there was not a lot to choose between School PW 7(1) with a mean of 28.5 and linear spread of 6.2, and School CIG 5(G) - mean 29.0 and standard deviation 6.2 - assuming a linear relationship between the means. School LWY 0(G) showed that its top Grade 11 stream had a grasp of the behavioural objectives of the test at a level which is 7.5 per cent higher than the mean performance for the entire population.

On the basis of the gross performance of the whole field class, it seemed that the test was pitched at the right level of difficulty for eliciting information about basic mathematics concepts. But it must be said that Items 1, 2, 3, 4, 7, and 14 (see Appendix 5.1) were very easy, although in mitigation, it was necessary to have some easy items in order to build the confidence of the pupils. The range of marks over the whole survey population was 45; thus, the standard deviation was only one unit in excess of one-sixth of the range. In effect, one-sixth of the range was very nearly equal to the linear scatter of the scores from the mean. By implication, from this objective test the scatter of the marks expressed faithfully the range of ability which existed in the entry. Furthermore, the mean was slightly greater than central location. So the entire population was a fairly balanced heterogeneous group of candidates spread over the ability range. A very tentative judgement was that the diagnostic test seemed a valid one with marking free of error, as would be expected on an objective test. But confirmation or denial of this judgement would be forthcoming after calculation of the reliability and an investigation of validity. Moreover, reliability was needed to determine whether $2 \leq \text{SE}_m \leq 3$, where $\text{SE}_m$ is the standard error of measurement [128 p.139]. This last statistic was very important to set the limits within which any child's score could be reasonably interpreted.
5.3 Item Analysis

Candidates' scores on any test can only be meaningfully considered within upper and lower limits set by the Standard Error of Measurement which is a function of the reliability of the scores obtained and the variance of those scores [129, pp. 29-32]. In this way psychometric theory arising out of research can make a useful contribution to relevant educational practice in the classroom. Reliability will forge a link between the aim of this research and the rationale for the rest of this investigation the raison d'etre for which is - to partially restate the problem:

The Bahamian society has complained of low attainment in Bahamas Junior Certificate (BJC) mathematics as defined in the syllabus and past examination paper laid out in Appendix 5.2. This problem has been located in the learning-teaching situations provided by the state maintained junior high (secondary) schools.

Although reliability of the data obtained was important, any definitive recommendations for improvement must stem from data, the validity of which is beyond doubt. Tables 5.4 - 5.53 contain feedback data from a survey of the final behaviours in the Grade 8 mathematics syllabus prescribed with the input of practising teachers for that part of the junior high school population on the island of New Providence. (See Appendix 5.5). The following analysis attempts to show that the data collected were both reliable and valid. The instrument constructed by the researcher to investigate the dimensions of this problem was a 50-item diagnostic objective test. The items were the completion type with one possible correct answer (key) among five plausible options for each stem. The stems, each concerned with one mathematical concept only, comprised 40 incompleted statements and 10 questions - the simplest completion type items apart from true-false items. The four distractors accompanying each key
were constructed so as to have a positive appeal to candidates who lacked the knowledge and skill which it was intended to test. No clues were given to direct the candidates to the correct options.

The items scored 1 if they were correct or 0 if they were wrong. From the raw response data it was immediately possible to calculate the percentage or proportion of individuals who preferred the right answers. This statistic psychometricians have called the facility because, according to Wood [121 p.241],

... the higher the proportion correct, the easier is the item.

Each item, which generated data, is followed by a table showing the choices of option made by the 377 respondents and the percentage of the population attracted to each option. Another very important statistic for each question was the discrimination coefficient, indicating the degree to which the item separated the respondents out into varying cognitive levels of operation. In order to give a comprehensive picture of item performance, an Item Analysis Chart was constructed (Table 5.54).
5.3.1. **Performance of Individual Items**

**Item 1**

$$476 + 25 + 908 + 675 =$$

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A</td>
<td>8024</td>
<td>0.3</td>
</tr>
<tr>
<td>B</td>
<td>4280</td>
<td>2.1</td>
</tr>
<tr>
<td>C*</td>
<td>2084</td>
<td>95.2</td>
</tr>
<tr>
<td>D</td>
<td>8204</td>
<td>1.1</td>
</tr>
<tr>
<td>E</td>
<td>2048</td>
<td>1.3</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.4: **Response Analysis for Item 1.**

The correct option, alternative C, was scored by 359 pupils who formed 95.2 per cent of the sample field survey. These pupils, in the exercise of this percentage preference, have therefore decided the facility index of this item. The very strong preference for the key limited distractors B, E, D and A to very weak performances.

Every school found this item very easy. The mean score on the test by those who got the item right was only 1.1 per cent higher than the corresponding measure of mental power for the entire group. Also the spread of marks was over almost the same linear range from the mean in both cases. The item had, consequently, no role to play in separating pupils of varying ability and this was reflected in the correlation coefficient, 0.17 in Table 5.54.

† correct options are starred* in this Section
Item 2

The right answer, option B, was identified by 356 pupils who constituted 94.4 per cent of the field class. Again, distractor weaknesses were shown by A, E and C, in that order, with D making no appeal. The facility of this item therefore was, by definition, decided by the percentage who chose option B. Only three pupils omitted the item and one answer book had at least two options circled.

Also, for all schools this question was very easy. The mean mark and spread of the marks about the mean for those who got the item correct, and for the entire entry, are near enough to being identical (Table 5.54), but since the number who got the item correct has come down by 3 pupils, the correlation coefficient had improved by one in one hundred.

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>A</td>
<td>9</td>
<td>2.4</td>
</tr>
<tr>
<td>B*</td>
<td>356</td>
<td>94.4</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>D</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>6</td>
<td>1.6</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
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</tr>
</tbody>
</table>

Table 5.5: Response Analysis for Item 2
Item 3

\[ 203 \times 9 = \]

<table>
<thead>
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<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A</td>
<td>8721</td>
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<tr>
<td>B</td>
<td>7812</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>2817</td>
<td>4</td>
</tr>
<tr>
<td>D*</td>
<td>1827</td>
<td>366</td>
</tr>
<tr>
<td>E</td>
<td>1728</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td></td>
</tr>
</tbody>
</table>

†round off error

Table 5.6: Response Analysis for Item 3

366 candidates who represented 97.1 per cent of the survey class, elected alternative D, the right option. This overwhelming selection of option D, not only dictated weak performances for distractors C, A, B and E but also identified the facility index for the item. Only two answer books had at least two options circled in each.

Table 5.6 shows that this item was also very easy. However, the spread of marks about the mean for those who got the item correct was slightly less than that recorded for the entire group, so the correlation coefficient goes up to 0.25 (Table 5.54).
**Item 4**

\[2051 \div 7 =\]

<table>
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<th>Respondents</th>
<th>%</th>
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<td>8</td>
<td>2.1</td>
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<tr>
<td>A* 293</td>
<td>338</td>
<td>89.7</td>
</tr>
<tr>
<td>B 923</td>
<td>9</td>
<td>2.4</td>
</tr>
<tr>
<td>C 932</td>
<td>8</td>
<td>2.1</td>
</tr>
<tr>
<td>D 329</td>
<td>8</td>
<td>2.1</td>
</tr>
<tr>
<td>E 392</td>
<td>5</td>
<td>1.3</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Table 5.7: Response Analysis for Item 4

The correct answer, option A, was selected by 338 pupils who made up 89.7 per cent of the voluntary survey class. The facility index for this item was therefore decided by the percentage who chose the correct option. Distractors B, C and D were of almost equal appeal to the weaker pupils and E appealed to 5 - 1.3 per cent - of this ability range. 8 pupils, or 2.1 per cent of the survey class, omitted the item and only one submitted an answer book with a multiple response.

This was another very easy item (Table 5.7). But the mean ability has gone up slightly more, the number who got the item correct has decreased and so has the spread of marks about the central tendency. Consequently, the potential of the question to distinguish and sort out individuals of varying giftedness was pegged at 0.32.

*The significance of omitting an item is discussed in Section 5.4.*
Item 5

In three days Baillou Hill Corner Shop sold 2026 eggs. On the first day 689 were sold and on the second day 105 more were sold than on the first day. How many eggs did the grocer sell on the third day?

<table>
<thead>
<tr>
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<th>Respondents</th>
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</tr>
</thead>
<tbody>
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<tr>
<td>A 345</td>
<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>B 354</td>
<td>27</td>
<td>7.2</td>
</tr>
<tr>
<td>C 453</td>
<td>39</td>
<td>10.3</td>
</tr>
<tr>
<td>D 534</td>
<td>47</td>
<td>12.5</td>
</tr>
<tr>
<td>E* 543</td>
<td>181</td>
<td>48.0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.1</td>
</tr>
</tbody>
</table>

+ round off error

Table 5.8: Response Analysis for Item 5

The key, option E, was chosen by 181 candidates who not only made up 48.0 per cent of the sample population but also decided in percentage terms the facility of the item. Distractor D appealed to 47 candidates - 12.5 per cent of the population - and the order of performance for the others was C, B and A. Only one answer book had a multiple response to this item but 64 candidates, or 17.0 per cent of the population omitted the item.

5 Government schools and the 3 Independent schools recorded facility indices which indicated that the item varied from acceptable to easy. The mean achievement by those who chose the correct option was 16.1 per cent better than the corresponding measure for the entire sample, with the linear measure of spread depressed by 7.1 per cent. Also the number who got the item correct was considerably reduced (Table 5.8). As a result the sorting power of the item reached 0.49
Item 6

When a certain number is divided by 17 the answer is 26 and there is a remainder of 15. What is the number?

<table>
<thead>
<tr>
<th>Options</th>
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<th>%</th>
</tr>
</thead>
<tbody>
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<td>54</td>
<td>14.3</td>
</tr>
<tr>
<td>A 754</td>
<td>11</td>
<td>2.9</td>
</tr>
<tr>
<td>B 745</td>
<td>27</td>
<td>7.2</td>
</tr>
<tr>
<td>C 574</td>
<td>35</td>
<td>9.3</td>
</tr>
<tr>
<td>D* 457</td>
<td>239</td>
<td>63.4</td>
</tr>
<tr>
<td>E 547</td>
<td>11</td>
<td>2.9</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.9: Response Analysis for Item 6

Option D, the right answer, was identified by 239 pupils who formed 63.4 per cent of the field survey class. This percentage of the survey class also decided the facility of this item. Distractor C turned 35 pupils - 9.3 per cent of the entry - away from the right answer with B, A and E (the latter two of equal strength) in descending order of popularity. However 54 pupils - 14.3 per cent of the sample - omitted the item.

Table 5.9 shows that this item required information well within the pupils' fund of knowledge. The mean competence of those candidates who identified the correct option was 9.4 per cent higher than the corresponding performance of the entire group and the linear spread of the scores about the mean competence was slightly less than that for the whole field class, consequently the potential to separate the pupils into those of high ability and those of low ability measured 0.39.
Item 7

Write in numerals:
seven thousand, two hundred and eighty six

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>A* 7286</td>
<td>355</td>
<td>94.2</td>
</tr>
<tr>
<td>B 700 286</td>
<td>6</td>
<td>1.6</td>
</tr>
<tr>
<td>C 70 286</td>
<td>7</td>
<td>1.9</td>
</tr>
<tr>
<td>D 72 086</td>
<td>5</td>
<td>1.3</td>
</tr>
<tr>
<td>E 72 806</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>G</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.10: Response Analysis for Item 7

355 pupils, who represented 94.2 per cent of the field class, not only elected the right answer, option A, but also at the same time determined the facility index of the item. This very strong choice of the right answer effectively limited distractors C, B and D to very weak performances and E to a non-event. Four pupils omitted the item.

All of the schools found this question very easy. The mean performance of those who picked the right alternative was only just higher than that for the entire field class and the linear spread of marks about this mean performance was virtually the same as that for the whole sample population. Consequently the potential of the item to discriminate between those of low ability and those of high ability was only 0.19.
**Item 8**

1.45 + 0.76 + 8.3 + 21.09 =

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<th>%</th>
</tr>
</thead>
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<td>19</td>
<td>5.0</td>
</tr>
<tr>
<td>A</td>
<td>14</td>
<td>3.7</td>
</tr>
<tr>
<td>B*</td>
<td>305</td>
<td>80.9</td>
</tr>
<tr>
<td>C</td>
<td>23</td>
<td>6.1</td>
</tr>
<tr>
<td>D</td>
<td>13</td>
<td>3.4</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>99.9 +</strong></td>
</tr>
</tbody>
</table>

+ round off error

**Table 5.11: Response Analysis for Item 8**

The correct answer, option B, appealed to 305 pupils, who constituted 80.9 per cent of the respondents. The facility of this item was therefore 80.9 per cent. Distractor C turned 23 pupils - 6.1 per cent of the respondents - away from the correct answer and A, D and E ensued. But 19 pupils - 5.0 per cent of the field class - omitted the item.

This was another very easy question (Table 5.11). Whereas the mean aptitude for those who preferred the correct option was 5.6 per cent in excess of the corresponding quantity for the entire survey group, the linear spread of scores about this mean aptitude was 3.5 per cent less than the corresponding size for the entire field class (Table 5.55). Consequently the capacity of the item to discriminate between levels of ability was 0.37.
Item 9

This sheet of paper is _______ centimetres (cm) wide.

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
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<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>A 17.5</td>
<td>19</td>
<td>5.0</td>
</tr>
<tr>
<td>B 18.3</td>
<td>9</td>
<td>2.4</td>
</tr>
<tr>
<td>C 21.0*</td>
<td>276</td>
<td>73.2</td>
</tr>
<tr>
<td>D 20.4</td>
<td>45</td>
<td>11.9</td>
</tr>
<tr>
<td>E 19.2</td>
<td>7</td>
<td>1.9</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.12: Response Analysis for Item 9

The precise answer, option C, was measured by 276 candidates who formed 73.2 per cent of the field class. In effect this percentage, by definition, was the facility of this item. 45 pupils - 11.9 per cent of the survey class - preferred distractor D. Thus A, B and E were left to compete for the remaining respondents. Only three answer scripts had at least two options each circled but 18 pupils - 4.8 per cent of the field class - omitted the item.

This was another easy item for the whole field class (Table 5.12). The mean capacity to cope with the item attained by those who selected the key was 6.7 per cent greater than that achieved by the entire sample population but the linear spread of scores about this mean capacity was slightly less (Table 5.54). Consequently, the potential of the item to distinguish and sort out ability was indicated by 0.36, the correlation coefficient.
Item 10

The distance from Trinidad to New York is 3550 kilometres (km).
If an aeroplane flies at 500 km per hour, how long will it take to fly from Trinidad to New York? (Give your answer to the nearest hour).

<table>
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</thead>
<tbody>
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<td>29</td>
<td>7.7</td>
</tr>
<tr>
<td>A 4</td>
<td>30</td>
<td>8.0</td>
</tr>
<tr>
<td>B 6</td>
<td>43</td>
<td>11.4</td>
</tr>
<tr>
<td>C 8</td>
<td>60</td>
<td>15.9</td>
</tr>
<tr>
<td>D 9</td>
<td>32</td>
<td>8.5</td>
</tr>
<tr>
<td>E* 7</td>
<td>178</td>
<td>47.2</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>1.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.13: Response Analysis for Item 10

178 pupils, who represented 47.2 per cent of the survey population, selected the key, option E. This percentage choice of the key pegged the facility index at 47.2 per cent. Distractor C (endorsed by 60 pupils, or 15.9 per cent of the sample) was the most popular, followed by B, D and A. 5 pupils each circled at least two options but 29 - 7.7 per cent of the population - omitted the item.

6 Government schools and the Independent schools had enough command of the curriculum skills required by the question to score facility indices within the range of acceptability. The mean competence of those who chose the correction option was 9.4 per cent better than that of the entire field class but the linear spread of the marks about this mean was only just less than that achieved by the entire field survey group. Consequently the discriminating power of the item was 0.28 (Table 5.54).
Item 11

1 litre (l) = 1000 cubic centimetres (cc).
How many litres of milk are contained in a box which measures 19 centimetres (cm) long, 10 cm wide and 20 cm high?

<table>
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<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>84</td>
<td>22.3</td>
</tr>
<tr>
<td>A 8.3</td>
<td>38</td>
<td>10.1</td>
</tr>
<tr>
<td>B 38</td>
<td>89</td>
<td>23.6</td>
</tr>
<tr>
<td>C 2.8</td>
<td>51</td>
<td>13.5</td>
</tr>
<tr>
<td>D 8.2</td>
<td>49</td>
<td>13.0</td>
</tr>
<tr>
<td>E* 3.8</td>
<td>65</td>
<td>17.2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.14: Response Analysis for Item 11

The correct answer appealed only to 65 candidates, who comprised 17.2 per cent of the respondents and chose option E. This choice, consequently, put the facility index at 17.2 per cent. Of the distractors, B was most popular (endorsed by 89 respondents, or 23.6 per cent of the entry), followed by C, D and A. Only one candidate submitted an answer book with at least two options circled; but 84 candidates, representing 22.3 per cent of the entry, omitted the item.

The facility at the individual school level shows that the field performance of the item varied from very difficult to difficult. The mean measure of capacity to cope with the item scored by those who endorsed the correct option was 18.0 per cent higher than the performance of the whole field class and the linear scatter of the marks about this mean capacity was 9.4 per cent, also higher than the corresponding quantity for the whole survey class (Table 5.54). As a consequence, 0.26 is the real indicator of the potential of the item to spot and divide the sample into levels of ability.
Item 12

2.63 \times 10^{-1} =

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<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
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<td>6</td>
<td>1.6</td>
</tr>
<tr>
<td>A* 26.3</td>
<td>186</td>
<td>49.3</td>
</tr>
<tr>
<td>B 263</td>
<td>12</td>
<td>3.2</td>
</tr>
<tr>
<td>C 0.263</td>
<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>D 2630</td>
<td>56</td>
<td>14.9</td>
</tr>
<tr>
<td>E none of these</td>
<td>98</td>
<td>26.0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.1\dagger</strong></td>
</tr>
</tbody>
</table>

\dagger round-off error

Table 5.15: Response Analysis for Item 12

Alternative A, the right answer, was identified by 186 candidates who comprised 49.3 per cent of the sample survey class. In exercising this percentage preference, these pupils have consequently decided the facility index of this item. Distractor E turned too high a percentage of the candidates away from the key but distractor D performed satisfactorily. Distractor B would have to be revised because it was ineffective whereas distractor C may be tried in a post-test before a decision is taken about possible modification.

Enough of the respondents from 5 Government and 3 Independent schools have selected the right solution so that the item ranged from moderately difficult to easy. The mean ability of those who chose the correct option was 13.9 per cent greater than the same index for the entire field class but the linear distribution of the marks about this mean was 2.4 per cent less than that achieved by the entire survey class (Table 5.54). The capacity of this item to discriminate ability, as expressed by the correlation coefficient of 0.43, was therefore well justified.
Table 5.16: Response Analysis for Item 13

The right answer, option C, was scored by 287 pupils who formed 76.1 per cent of the sample survey population. The facility index for this item was therefore pegged at 76.1 per cent. Distractor B, preferred by 51 pupils - 13.5 per cent of the survey class - captured most of the less able candidates and left A, D and E to compete for the majority of the remaining low ability pupils. Only one pupil offered a multiple response to this item and 10 pupils - 2.7 per cent of the sample - omitted it.

At the level of the schools' performances, the facility indices showed that candidates found the item varying from easy...to...very easy. The mean capacity of these candidates to retrieve the correct curriculum skill was 4.5 per cent beyond that achieved by the whole sample population and the linear scatter of those scores about this mean capacity was only just slightly greater than the standard deviation for the whole population (Table 5.54). The result was that the potential of the item to detect and separate candidates into ability groups was calculated to be 0.26.
Item 14

\[ 52.61 - 41.93 = \]

<table>
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<th>Respondents</th>
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</tr>
</thead>
<tbody>
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<td>1.3</td>
</tr>
<tr>
<td>A 1.068</td>
<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>B 10.68</td>
<td>328</td>
<td>87.0</td>
</tr>
<tr>
<td>C 106.8</td>
<td>5</td>
<td>1.3</td>
</tr>
<tr>
<td>D 1068</td>
<td>4</td>
<td>1.1</td>
</tr>
<tr>
<td>E 0.1068</td>
<td>17</td>
<td>4.5</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.17: Response Analysis for Item 14

The key, option B, was scored by 328 pupils who comprised 87.0 per cent of the sample field class. This percentage response determined the facility of the item. The distractor pairs A and E, C and D shared almost equally (the former pair performing better than the latter) the competing low ability respondents. 5 pupils - 1.3 per cent of the sample population - omitted the item.

All of the schools found this item to be very easy. Its mean criterion, scored by those who preferred the right option, was only 2.6 per cent higher than the corresponding figure for the entire entry but the linear spread of the scores about this mean criterion was slightly less than that secured by the whole field class (Table 5.54). In effect, since the content of the item did not challenge the knowledge of the field class, its sorting power was 0.23.
**Item 15**

\[ 2.7 \times 1.6 = \]

<table>
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<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
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<td>4.2</td>
</tr>
<tr>
<td>A 0.0432</td>
<td>7</td>
<td>1.9</td>
</tr>
<tr>
<td>B 0.432</td>
<td>15</td>
<td>4.0</td>
</tr>
<tr>
<td>C 43.2</td>
<td>119</td>
<td>31.6</td>
</tr>
<tr>
<td>D* 4.32</td>
<td>208</td>
<td>55.2</td>
</tr>
<tr>
<td>E 432</td>
<td>11</td>
<td>2.9</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.1</strong></td>
</tr>
</tbody>
</table>

†round-off error

Table 5.18: Response Analysis for Item 15

208 pupils, who together made up 55.2 per cent of the group under study, selected the key, thus giving this item a facility index of 55.2 per cent. Of the distractors, C (chosen by 119 pupils, or 31.6 per cent of the group), was the most popular followed by B, E and A. Only one pupil gave a multiple response to the item; however, 16 pupils - 4.2 per cent of the group who offered to be tested, omitted the item.

5 Government and 2 Independent schools found that the item varied from acceptable to very easy. The mean mental power exercised by those who preferred the correct option was 11.6 per cent greater than the corresponding performance for the whole field class but the scatter of the marks about the mean was just less than that secured by the whole entry. The result was that the potential of the item to spot individuals of like talent and sort them into ability groups was calculated to be 0.40.
Item 16

1.68 ÷ 1.4 =

<table>
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<th>%</th>
</tr>
</thead>
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</tr>
<tr>
<td>A 0.12</td>
<td>84</td>
<td>22.3</td>
</tr>
<tr>
<td>B* 1.2</td>
<td>182</td>
<td>48.3</td>
</tr>
<tr>
<td>C 12</td>
<td>57</td>
<td>15.1</td>
</tr>
<tr>
<td>D 0.012</td>
<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>E 120</td>
<td>9</td>
<td>2.4</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.1†</td>
</tr>
</tbody>
</table>

† round-off error

Table 5.19: Response Analysis for Item 16.

The 182 candidates who themselves consisted of 48.3 per cent of the sample, selected the right option, B and consequently pegged the facility index of this item at the same percentage. Distractor A, more than any other, turned away 84 candidates - 22.3 per cent of the sample - from the key and was followed by C, D and E. The attempt of only one candidate ended in a multiple response but 26 candidates, or 6.9 per cent of the sample, omitted the item.

4 Government and 2 Independent schools found that they could cope with the curriculum skill required by the item. The mean criterion (Table 5.54) attained on the item by those who got it right was 13.1 per cent better than the mean achievement of the entire survey population and the linear scatter of the marks about this mean was only slightly less than that attained by the entire entry. Considering the response pattern a discriminating power of 0.41 was quite good.
of those who selected the correct option was 12.7 per cent more than that recorded for the entire sample, but the linear distribution of scores about the mean was 4.7 per cent less than the corresponding measure for the whole sample (Table 5.54). A discrimination index of 0.45 was therefore an effective indicator of the potential of the item to separate the pupils into groups of similar ability.
Item 17

\[
\frac{2}{5} + \frac{5}{6} = \]

<table>
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<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
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<td>17</td>
<td>4.5</td>
</tr>
<tr>
<td>A 27/30</td>
<td>16</td>
<td>4.2</td>
</tr>
<tr>
<td>B 2 7/30</td>
<td>21</td>
<td>5.6</td>
</tr>
<tr>
<td>C 17/30</td>
<td>11</td>
<td>2.9</td>
</tr>
<tr>
<td>D* 1 7/30</td>
<td>208</td>
<td>55.2</td>
</tr>
<tr>
<td>E 7/30</td>
<td>102</td>
<td>27.1</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.20: Response Analysis for Item 17

The correct answer, option D, was scored by 208 pupils who consisted of 55.2 per cent of the population under study. In agreement with the definition, this percentage of correct answers was the facility index of the item. Distractor E, which appealed to 102 pupils - 27.1 per cent of the population - was very effective and was followed in order by B, A and C. Only two pupils circled at least two options whereas 17 - 4.5 per cent of the population - omitted the item.

4 Government and 2 Independent schools found that they had enough command of the curriculum skills required by the item to classify it in the range from acceptable to very easy. The mean competence
Item 18

\[
\frac{7}{8} - \frac{3}{4} = 
\]

<table>
<thead>
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<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>21</td>
<td>5.6</td>
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<tr>
<td>A (\frac{3}{8})</td>
<td>20</td>
<td>5.3</td>
</tr>
<tr>
<td>B (\frac{1}{4})</td>
<td>51</td>
<td>13.5</td>
</tr>
<tr>
<td>C (\frac{5}{8})</td>
<td>27</td>
<td>7.2</td>
</tr>
<tr>
<td>D (\frac{3}{4})</td>
<td>28</td>
<td>7.4</td>
</tr>
<tr>
<td>E* (\frac{1}{8})</td>
<td>230</td>
<td>61.0</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.21: Response Analysis for Item 18

Alternative E, the key, was identified by 230 pupils who formed 61.0 per cent of the survey group. This percentage of the pupils who calculated and identified the correct answer was also the facility of the item. 51 pupils - 13.5 per cent of the survey group - were distracted by option B. Distractors D, C and A followed in order of strength, however 21 pupils - 5.6 per cent of the group - omitted the item.

6 Government and 2 Independent schools were so adequately prepared for the item that their performances classified it as ranging from acceptable to very easy. The measure of central tendency (Table 5.54) scored by those who identified the correct option was 12.4 per cent better than that earned by the entire field class but the linear spread of the scores from this central mark was 5.9 per cent smaller than the entire class accomplished. The quality of the field performance of this item set the stage for the achievement of 0.49, a very good and effective discrimination index.
Item 19

\[
\frac{2}{3} \times \frac{5}{6} = \]

<table>
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<th>Respondents</th>
<th>%</th>
</tr>
</thead>
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<tr>
<td>0</td>
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<tr>
<td>A 7/9</td>
<td>28</td>
<td>7.4</td>
</tr>
<tr>
<td>B 2/3</td>
<td>50</td>
<td>13.3</td>
</tr>
<tr>
<td>C* 5/9</td>
<td>255</td>
<td>67.6</td>
</tr>
<tr>
<td>D 4/9</td>
<td>10</td>
<td>2.7</td>
</tr>
<tr>
<td>E 2/9</td>
<td>12</td>
<td>3.2</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Table 5.22: Response Analysis for Item 19

The right option C, was scored by 255 candidates who constituted 67.6 per cent of the entry. This percentage strength of choice of the key isolates the facility index for this item. Distractor B drew 50 candidates - 13.3 per cent of the field class - away from the key. It was followed in order of strength by A, E and D. Also 22 candidates - 5.8 per cent of the field class - omitted the item.

7 Government and 2 Independent schools deployed their curriculum skills in such a way that, on the basis of the facility indices achieved, the item was classified in the range acceptable to extremely easy. The mean attainment of those who preferred the key was 8.6 per cent higher than the parallel mark for the entire field class but the linear spread of the scores about this mean attainment was 4.7 per cent less than that obtained by the entire entry (Table 5.54). This performance deserved a discrimination index of 0.40, an effective indicator of the sorting ability of the item.
Item 20

\[
\frac{3}{5} \div \frac{9}{10} =
\]

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24</td>
<td>6.4</td>
</tr>
<tr>
<td>A* ( \frac{2}{3} )</td>
<td>209</td>
<td>55.4</td>
</tr>
<tr>
<td>B ( 1 \frac{1}{2} )</td>
<td>72</td>
<td>19.1</td>
</tr>
<tr>
<td>C ( 1 \frac{2}{3} )</td>
<td>42</td>
<td>11.1</td>
</tr>
<tr>
<td>D</td>
<td>16</td>
<td>4.2</td>
</tr>
<tr>
<td>E</td>
<td>14</td>
<td>3.9</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.1†</td>
</tr>
</tbody>
</table>

Table 5.23: Response Analysis for Item 20

† round-off error

The correct answer, option A, was scored by 209 of the field class who made up 55.4 per cent of the sample population. This percentage who chose the key demarcated the facility index for this item. Among the distractors, B was most popular (endorsed by 72 of the survey group or 19.1 per cent), followed by C, D and E. But 24 pupils, or 6.4 per cent of the field class, omitted this item.

7 Government schools and 1 Independent school, on the basis of their facility indices, have indicated that the content of the item varied from medium difficulty (42.9 per cent) to very easy (88.2 per cent) [56, p.110]. The mean ability index, for those who preferred the correct answer, was 10.9 per cent better than that for the entire survey group but the linear spread of the marks about this mean ability index was 2.4 per cent less than that scored by the entire entry (Table 5.54). The potential of this item, therefore, to select and separate individuals into high and low ability groups was calculated to be 0.39.
Item 21

\[ \frac{2}{3} + \frac{1}{4} - \frac{7}{8} = \]

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<th>Respondents</th>
<th>%</th>
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<td>46</td>
<td>12.2</td>
</tr>
<tr>
<td>A* 1\frac{1}{24}</td>
<td>217</td>
<td>57.6</td>
</tr>
<tr>
<td>B \frac{5}{24}</td>
<td>40</td>
<td>10.6</td>
</tr>
<tr>
<td>C \frac{11}{24}</td>
<td>34</td>
<td>9.0</td>
</tr>
<tr>
<td>D \frac{13}{24}</td>
<td>21</td>
<td>5.6</td>
</tr>
<tr>
<td>E \frac{19}{24}</td>
<td>17</td>
<td>4.5</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.24: Response Analysis for Item 21

The right solution, option A, was scored by 217 pupils who represented 57.6 per cent of the sample field class. This percentage who chose the right option also labelled the facility index for this item. Of the distractors, B was the most effective in diverting the attention of 40 pupils – 10.6 per cent of the field class – ensued by C, D and E. Only two answer books had multiple responses – an attempt at solution – but 46 pupils, or 12.2 per cent of the field class, omitted the item.

For 7 Government and 2 Independent schools the facility indices revealed that the skills and knowledge required by the item fell in a range which varied from medium difficulty to very
easy. The mean attainment achieved by those who chose the right alternative was 10.1 per cent higher than the mean score for the entire group but the linear spread of the scores about this mean attainment was 2.4 per cent less than the corresponding linear scatter for the entire field class. Consequently, the capacity of the item to discriminate ability, 0.37, was quite good.
### Item 22

In the number 2.83, the value of the '3' is

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</tr>
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</tr>
<tr>
<td>$\frac{3}{1000}$</td>
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<td>6.4</td>
</tr>
<tr>
<td>$\frac{3}{10}$</td>
<td>69</td>
<td>18.3</td>
</tr>
<tr>
<td>$\frac{3}{100}$</td>
<td>97</td>
<td>25.7</td>
</tr>
<tr>
<td>3 ones</td>
<td>140</td>
<td>37.1</td>
</tr>
<tr>
<td>none of these</td>
<td>31</td>
<td>8.2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.25: Response Analysis for Item 22

The correct alternative, option C, was scored by 97 candidates who represented 25.7 per cent of the field class. In consequence this percentage was the facility index of the item. Distractor D (preferred by 140 pupils, or 37.1 per cent of the field class) was unquestionably the most effective, followed by B, E and A. Only one pupil circled at least two options and 15 - 4.0 per cent of the field class - omitted the item.

Only 1 Government school and 1 Independent school had the correct option chosen with such strength that the item could be classified as being of medium difficulty. The mean criterion obtained by those who preferred the right option was 18.0 per cent above that accomplished by all of the
respondents. But the linear spread of this mean criterion about the marks increased by 17.6 per cent. The resultant effect was that the potential of the item to sense and separate pupils into ability groups was 0.34.
Item 23

5.06 rounded to the nearest whole number is

<table>
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<th>Respondents</th>
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</tr>
</thead>
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<td>22</td>
<td>5.8</td>
</tr>
<tr>
<td>A 50</td>
<td>39</td>
<td>10.3</td>
</tr>
<tr>
<td>B 6</td>
<td>63</td>
<td>16.7</td>
</tr>
<tr>
<td>C 506</td>
<td>121</td>
<td>32.1</td>
</tr>
<tr>
<td>D* 5</td>
<td>94</td>
<td>24.9</td>
</tr>
<tr>
<td>E 5.1</td>
<td>35</td>
<td>9.3</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>99.9†</td>
</tr>
</tbody>
</table>

† round-off error

Table 5.26: Response Analysis for Item 23

The correct answer was option D, chosen by 94 candidates, who were 24.9 per cent of the sample. The facility of this item was therefore 24.9 per cent. Distractor C however, chosen by 121 pupils (32.1 per cent of the entry) was more popular than the key, and it was followed in order of strength by B, A and E. 22 pupils, 5.8 per cent of the sample, omitted the item and 3 pupils offered multiple responses in their answer books.

Only 1 Government school had sufficient of its pupils select the right option to record a facility index which classified the item as being of moderate difficulty. The mean mental capacity of the pupils who selected the right answer was 17.6 per cent higher than that accomplished by the entire survey class but the linear scatter of the marks about this mean mental capacity was 9.4 per cent in excess of that realised by the whole field class (Table 5.54). Using these performances, the potential of the item to detect and separate individuals into ability groups was calculated to be 0.32.
Item 24

5.06 corrected to one decimal place is ...

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</tr>
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<td>5.8</td>
</tr>
<tr>
<td>A 5.0</td>
<td>86</td>
<td>22.8</td>
</tr>
<tr>
<td>B* 5.1</td>
<td>58</td>
<td>15.4</td>
</tr>
<tr>
<td>C 6.0</td>
<td>23</td>
<td>6.1</td>
</tr>
<tr>
<td>D 6.1</td>
<td>17</td>
<td>4.5</td>
</tr>
<tr>
<td>E none of these</td>
<td>168</td>
<td>44.6</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.27: Response Analysis for Item 24

The correct answer, option B, was chosen by 58 pupils thus giving the item a facility of 15.4 per cent. 3 pupils offered multiple responses to the item whilst 22 (5.8 per cent of the sample) omitted it. Distractor E, attracting 44.6 per cent of the entry, was most popular. It was followed by distractor A which was only just better than half as effective as E. Between them options C and D managed to distract 10.6 per cent of the entry, which, in statistical terms, was a representative sample.

The facility indices at the school level showed that 1 Government school and 1 Independent school had enough of their entry select the correct answer so that the item could be classified as being of moderate difficulty. The mean ability index of those who selected the correct option was 28.5 per cent more than the corresponding index for the whole field class but the linear scatter of the scores about that mean index was 7.1 per cent higher than that for the entire survey class. Using these pieces of information, 0.38 was calculated to be the sorting index for the item. Because the spread of the scores was relatively narrow, this was considered a good sorting index.
Item 25

30% written as a fraction in its simplest form is

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</tr>
</thead>
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</tr>
<tr>
<td>A (\frac{3}{1000})</td>
<td>7</td>
<td>1.9</td>
</tr>
<tr>
<td>B (\frac{300}{100})</td>
<td>19</td>
<td>5.0</td>
</tr>
<tr>
<td>C (\frac{3}{100})</td>
<td>96</td>
<td>25.5</td>
</tr>
<tr>
<td>D (\frac{300}{1000})</td>
<td>25</td>
<td>6.6</td>
</tr>
<tr>
<td>E* (\frac{3}{10})</td>
<td>210</td>
<td>55.7</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0.8</td>
</tr>
</tbody>
</table>

TOTAL 377 100.0

Table 5.28: Response Analysis for Item 25

The correct answer was option E, chosen by 210 candidates who represented 55.7 per cent of the entry. Consequently 55.7 per cent was the facility index for the item. Of the distractors, C was most popular (endorsed by 96 candidates, 25.5 per cent of the entry), followed by D, B and A. However, 17 candidates, 4.5 per cent of the entry, omitted the item and 3 candidates, 0.8 per cent of the entry, submitted multiple responses in their answer books.

6 Government and the 3 Independent schools had a sufficient number of their candidates to select the right option so that the item could be classified as having ranged from moderate difficulty to very easy. The mean competence of those who preferred the key was 10.5 per cent greater than that achieved by the entire survey class but the linear distribution of the marks ranged over a distance which was 2.4 per cent longer than the corresponding spread for the whole entry. A sorting power of 0.37 was therefore well justified for this item.
Item 26

$\frac{3}{4}$ written as a percentage is

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<th>%</th>
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</thead>
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<td>5.0</td>
</tr>
<tr>
<td>A</td>
<td>113</td>
<td>30.0</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>2.1</td>
</tr>
<tr>
<td>C</td>
<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>D*</td>
<td>217</td>
<td>57.6</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.1</strong>+</td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.29: Response Analysis for Item 26

The key was option D, chosen by 217 candidates, who were 57.6 per cent of the entry, this percentage indicating also the facility index for the item. Of the distractors, A was the most popular (endorsed by 113 candidates, or 30.0 per cent of the entry), followed by C and B since distractor E registered a most ineffective performance. 19 candidates, representing 5% of the entry, omitted the item and 1 candidate (0.3 per cent of the entry) submitted a multiple response to the item.

5 Government and 3 Independent schools offered pupils who posted facility indices on this item ranging from satisfactory to very easy but the mean facilities were 54.2 per cent and 69.0 per cent respectively – acceptable to easy.

The mean mental capacity on the test by those who got the item correct was 30.8 and this was 15.4 per cent higher than the corresponding measure of giftedness for the entire field
sample. Also, this mean had scores spread over a range that was 7.1 per cent shorter.

Those pupils who got this item correct formed almost 60 per cent of the entire field class thus giving the item a fair challenge to separate individuals into high and low ability groups. The correlation coefficient of 0.57 indicated that this was very sensitively done.
Item 27

25% as a decimal is

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<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
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<td>16</td>
<td>4.2</td>
</tr>
<tr>
<td>A* 0.25</td>
<td>182</td>
<td>48.3</td>
</tr>
<tr>
<td>B 2.5</td>
<td>105</td>
<td>27.9</td>
</tr>
<tr>
<td>C 0.025</td>
<td>16</td>
<td>4.2</td>
</tr>
<tr>
<td>D 25</td>
<td>51</td>
<td>13.5</td>
</tr>
<tr>
<td>E 2.05</td>
<td>7</td>
<td>1.9</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Table 5.30: Response Analysis for Item 27

The correct response was option A which 182 candidates, or 48.3 per cent of the sample chose, thereby selecting also the facility index of the item. The most effective distractor, B, was selected by 105 candidates, 27.9 per cent of the sample, and was followed in distracting candidates from the key by D (selected by 51 candidates or 13.5 per cent of the sample), C and E. 16 candidates, 4.2 per cent of the sample, omitted the item but no answer book was submitted with multiple responses.

3 Government and 2 Independent schools did sufficiently well on the item to record facility indices which ranged from 41.0 per cent to 87.5 per cent. But the mean facility for the schools was 47.3 per cent and 55.6 per cent respectively.

The mean competence of those who got this item correct (Table 5.54) was 30.5, which was 14.2 per cent higher than the corresponding measure of competence for the entire group,
but the linear spread of the marks from the mean was virtually unchanged. The discrimination index, 0.44, indicated that the subsample who got this item correct was reasonably well sorted into similar ability groups.
Item 28

30% of $50.00 is

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>33</td>
<td>8.8</td>
</tr>
<tr>
<td>A $150</td>
<td>31</td>
<td>8.2</td>
</tr>
<tr>
<td>B $1500</td>
<td>59</td>
<td>15.6</td>
</tr>
<tr>
<td>C* $15</td>
<td>190</td>
<td>50.4</td>
</tr>
<tr>
<td>D $1.50</td>
<td>30</td>
<td>8.0</td>
</tr>
<tr>
<td>E $25</td>
<td>32</td>
<td>8.5</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.31: Response Analysis for Item 28

The key, option C was chosen by 190 pupils, who were 50.4 per cent of the entry, thereby isolating the facility index for the item. Of the four distractors, B, selected by 59 pupils or 15.6 per cent of the entry, was the most effective followed by E, A and D, which were of almost equal popularity. Although only two answer books had multiple responses, 33 pupils - 8.8 per cent of the entry - omitted the item.

5 Government schools and 3 Independent schools offered candidates who achieved facility indices on the item ranging from satisfactory to easy but the mean facility for corresponding schools was 48.2 per cent and 60.8 per cent. The mean competence on the test by those who got the item right was 30.6, 14.6 per cent higher than the corresponding score for the entire group, and the linear spread away from the mean was 7.1 per cent less. A measure of the efficiency with which the item sorted out the candidates into ability groups was given by the correlation coefficient 0.46, which was quite effective.
Item 29

$\frac{1}{2}$ written as a decimal is

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
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<td>29</td>
<td>7.7</td>
</tr>
<tr>
<td>A 0.2</td>
<td>102</td>
<td>27.1</td>
</tr>
<tr>
<td>B 2.0</td>
<td>38</td>
<td>10.1</td>
</tr>
<tr>
<td>C 0.05</td>
<td>11</td>
<td>2.9</td>
</tr>
<tr>
<td>D 5.0</td>
<td>29</td>
<td>7.7</td>
</tr>
<tr>
<td>E* 0.5</td>
<td>167</td>
<td>44.3</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.1</td>
</tr>
</tbody>
</table>

*round-off error

Table 5.32: Response Analysis for Item 29

The correct (asterisked) answer was option E, chosen by 167 candidates who, as indicated in the response analysis table, were 44.3 per cent of the entry. The facility index of this item was therefore 44.3 per cent. A was the most effective of the four distractors (endorsed by 102 candidates, or 27.1 per cent of the entry), followed in order by B, D and C. Only one candidate offered an answer book with two options circled but 29 candidates, 7.7 per cent of the entry, omitted the item.

4 Government (G) and 3 Independent (I) schools answered the item so well that facility indices ranged from satisfactory to very easy as School 4 (G) found it. The overall facility scored by the whole subsample was 44.3 per cent but the mean facility for Government (G) and Independent (I) schools was 36.3 per cent and 58.2 per cent respectively.
The mean ability of the entire field class was 26.7 (Table 6.54), which was 21.0 per cent less than the mean ability for those who got the item correct. The degree of linear spread of the scores from the mean score for the entire group was 15.3% less than that for those who got the item correct: The discrimination index pegged at 0.59, very good indeed, showed that the question actually selected those of low ability from those of high ability.
Item 30

27 written as a percentage of 36 is

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>65</td>
<td>17.2</td>
</tr>
<tr>
<td>A 57%</td>
<td>60</td>
<td>15.9</td>
</tr>
<tr>
<td>B* 75%</td>
<td>99</td>
<td>26.3</td>
</tr>
<tr>
<td>C 76%</td>
<td>37</td>
<td>9.8</td>
</tr>
<tr>
<td>D 67%</td>
<td>67</td>
<td>17.8</td>
</tr>
<tr>
<td>E 77%</td>
<td>48</td>
<td>12.7</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Table 5.33: Response Analysis for Item 30

The key was option B, selected by 99 pupils who represented 26.3 per cent of the sample. This percentage was, therefore, the facility index of the item. The distractors were well near to being evenly balanced in their effectiveness with the percentage points separating D, A, E and C standing at 1.9, 3.2 and 2.9 respectively. Only one answer book had at least two options circled but against this 65 pupils, representing 17.2 per cent of the entry, omitted the item.

Pupils from 2 Government (G) schools and 1 Independent (I) school did the item so well that those who got it right recorded facility indices which were very satisfactory. But the mean facility for each type of school was 26.7 per cent and 34.2 per cent, neither of which was satisfactory. The overall facility for all those who got the item right in the whole field class was 26.3 per cent, which again was not good enough. It was observed too that distractor D may need to be changed since the percentage who chose it was too high.
The mean ability of the pupils who got the item correct was 32.2, which is 20.6 per cent in excess of the score on the test for the entire group, and the spread was 8.2 per cent higher than that corresponding to the entire group's performance (Table 5.54). This latter statistic means that the scores for the 99 pupils who got the item correct were spread out farther from 32.2 than the corresponding scores from the mean of 26.7. It was apparent that among the subsample who got the item correct, there were some who scored very well on the test. This was borne out by the discrimination coefficient which was 0.39.
Item 31

\((-4) + (+5) = \)

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13</td>
<td>3.4</td>
</tr>
<tr>
<td>A ((-9))</td>
<td>81</td>
<td>21.5</td>
</tr>
<tr>
<td>B ((-9))</td>
<td>53</td>
<td>14.1</td>
</tr>
<tr>
<td>C* ((+1))</td>
<td>169</td>
<td>44.8</td>
</tr>
<tr>
<td>D ((-1))</td>
<td>37</td>
<td>9.8</td>
</tr>
<tr>
<td>E cannot (\text{tell})</td>
<td>22</td>
<td>5.8</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>99.9*</td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.34: Response Analysis for Item 31

The correct answer was option C, preferred by 169 pupils who not only constituted 44.8 per cent of the entry but also decided the facility index for the item. Distractor A, selected by 81 pupils (21.5 per cent of the entry) attracted most respondents and was followed by B, D and E respectively in order of priority. Only two answer scripts had at least two options circled but, added to these were 13 pupils, 3.4% of the entry, who did not attempt the item.

Enough candidates from 4 Government and 3 Independent schools selected the correct option, thereby achieving facility indices varying from satisfactory in School 9(G) to easy in School 8(I). Whereas the facility for the entire survey group on this item was 44.8 per cent and the mean facility recorded by the Independent schools was 52.0 per cent, the mean facility posted by the Government schools was 28.6 per
However, the mean mental capacity of the candidates who chose the correct answer was 30.2, 13.1 per cent higher than the mean mental capacity for the entire field class, the linear spread remaining practically unchanged. But the improved percentage of the whole field survey who got the item correct offered sufficient scope for selecting individuals of similar mental capacity. The degree of efficiency by which this was done was measured by the satisfactory correlation coefficient of 0.38 but distractor A was overly effective as a plausible option and may have to be replaced.
Item 32

\((-4) - (+5) = \)

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19</td>
<td>5.0</td>
</tr>
<tr>
<td>A (+20)</td>
<td>10</td>
<td>2.7</td>
</tr>
<tr>
<td>B (+9)</td>
<td>59</td>
<td>15.6</td>
</tr>
<tr>
<td>C (-1)</td>
<td>97</td>
<td>25.7</td>
</tr>
<tr>
<td>D (+1)</td>
<td>93</td>
<td>24.7</td>
</tr>
<tr>
<td>E* (-9)</td>
<td>95</td>
<td>25.2</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Table 5.35: Response Analysis for Item 32

The key to this item was option E, preferred by 95 candidates (or 25.2 per cent of the entry), who simultaneously decided the facility index. Among the distractors C and D (endorsed by 97 and 93 candidates, representing 25.7 and 24.7 per cent of the entry respectively) were almost equal in their appeal and were followed in order by B and A. Four scripts had at least two options circled but 19 candidates (5.0 per cent of the entry) did not attempt the item.

The subsample from a single school (Independent) found the item within their knowledge of 'curriculum skills to the' extent required to have achieved the acceptable facility index of 46.9 per cent. The mean competence of the candidates who chose the right option was 31.0, which was 16.1 per cent in excess of that achieved by the whole survey population. But the linear spread of the marks about the mean was 3.5 per cent in excess of that corresponding to the achievement of the latter subgroup. Since the number of candidates who found the correct option was only just over one-quarter of
the entire survey group, there was a big and diverse enough subgroup from which to detect and pocket individuals of the same aptitude. In consequence, the discrimination coefficient was justifiably 0.30, a satisfactory level of performance by the item. Distractors C and D were too plausible as alternative keys but a decision to replace them should be properly left until validation is effected after a re-test exercise. The researcher took this view because the statistics deriving from this field survey were sample bound.
Table 5.36: Response Analysis for Item 33

The correct answer was option C, chosen by 63 pupils who not only represented 16.7 per cent of the sample but also decided the facility index of the item. Distractor A was by far the strongest of the four (having been chosen by 145 pupils, or 38.5 per cent of the sample), followed in order by B, E and D. Significantly 22 pupils, accounting for 5.8 per cent of the sample, omitted the item.

Only one school (Government) which offered this item had the mental capacity required by sufficient pupils to cope with the knowledge of curriculum content that was necessary and sufficient to achieve the satisfactory facility index of 44.1 per cent. The mean mental capacity of the pupils who found the key was 31.1 which was 16.5 per cent higher than that registered by the subgroup of the whole field class who got the item correct. But the linear spread of the marks

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
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<td>5.8</td>
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<tr>
<td>A (-9)</td>
<td>145</td>
<td>38.5</td>
</tr>
<tr>
<td>B (+9)</td>
<td>79</td>
<td>21.0</td>
</tr>
<tr>
<td>C* (+20)</td>
<td>63</td>
<td>16.7</td>
</tr>
<tr>
<td>E (+ 4/5)</td>
<td>26</td>
<td>6.9</td>
</tr>
<tr>
<td>E (- 4/5)</td>
<td>42</td>
<td>11.1</td>
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<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>
about the mean was 3.5 per cent in excess of the corresponding linear spread registered by the whole field class. The net effect, since the number who found the key was only 16.7 per cent of the whole field class, was to reduce the potential of the item to discern and pigeon hole pupils of comparable aptitude. However, this depressed index of discrimination, 0.23 (Table 5.54), qualified the item for further use although distractors A and B were too plausible as alternative correct options. (Incidently, psychometricians differ on the spelling of 'distractor': for example, this form is advocated by Wood [121,p 243] whereas Satterly [122,p 90] prefers an 'e')
Item 34

\((-4) - (-5) = \)

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>19</td>
<td>5.0</td>
</tr>
<tr>
<td>A (-1)</td>
<td>65</td>
<td>17.2</td>
</tr>
<tr>
<td>B* (+1)</td>
<td>74</td>
<td>19.6</td>
</tr>
<tr>
<td>C (-9)</td>
<td>121</td>
<td>32.1</td>
</tr>
<tr>
<td>D (+9)</td>
<td>71</td>
<td>18.8</td>
</tr>
<tr>
<td>E none of these</td>
<td>25</td>
<td>6.6</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>99.8+</td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.37: Response Analysis for Item 34

The key was option B, identified by 74 examinees who comprised 19.6 per cent of the sample. Consequently, this percentage indicated the facility index for the item. The most effective distractor was C (endorsed by 121 examinees, or 32.1 per cent of the sample), followed by D, A and E. Only two examinees had at least two options circled in response to this item but 19 others, representing 5.0 per cent of the sample, omitted the item.

Only the candidates from one Independent school—had command of the knowledge of curriculum content required to cope with the item. The mean measure of command of knowledge possessed by those who got the item correct was 28.5, merely 6.7 per cent in excess of that obtained by the whole entry, and extended over a scatter of marks from the mean which was 3.5 per cent less than that obtained by the whole entry. Also the item was quite insensitive at spotting and separating individuals.
of similar aptitude as indicated by a discrimination coefficient of 0.11 (Table 5.54).
The diagram alongside shows a mirror EF, standing on the arms of an angle, at P and Q. A' is the reflection of the point A in the mirror. If A' is the same distance behind the mirror as A is in front of it, answer Items 35, 36 and 37.
Item 35

If the angle at A is $39^\circ$, then the angle at $A'$ is

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>31</td>
<td>8.2</td>
</tr>
<tr>
<td>A $29^\circ$</td>
<td>15</td>
<td>4.0</td>
</tr>
<tr>
<td>B $19^\circ$</td>
<td>22</td>
<td>5.8</td>
</tr>
<tr>
<td>C $49^\circ$</td>
<td>22</td>
<td>5.8</td>
</tr>
<tr>
<td>D* $39^\circ$</td>
<td>271</td>
<td>71.9</td>
</tr>
<tr>
<td>E $59^\circ$</td>
<td>16</td>
<td>4.2</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>99.9+$^\dagger$</td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.38: Response Analysis for Item 35

The key, chosen by 271 candidates who represented 71.9 per cent of the sample, was option D. The choice made by these candidates decided the facility of this item. The distractors were very weak in their performance: B and C were of equal strength followed by E and A which were separated only by two-tenths of a per cent. However, 31 candidates who comprised 8.2 per cent of the sample, omitted the item.

The pupils from all schools had a grasp, ranging from satisfactory to excellent, of the syllabus content required by the item. However, the actual measure of competence, as achieved in the mean mark by those who got the item correct, was $29.1$, $9.0$ per cent in excess of that attained by the whole sample population. That mean was achieved with a spread of
marks that was only 5.9 per cent less than the range for the whole sample. Still, a discrimination index of 0.45 indicated that the item was quite effective in distinguishing and sorting out respondents of like aptitude.
Item 36

If AQ is 3.4 cm, then A'Q is

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>41</td>
<td>10.9</td>
</tr>
<tr>
<td>A* 3.4 cm</td>
<td>233</td>
<td>61.8</td>
</tr>
<tr>
<td>B 34 cm</td>
<td>48</td>
<td>12.7</td>
</tr>
<tr>
<td>C 0.34 cm</td>
<td>31</td>
<td>8.2</td>
</tr>
<tr>
<td>D 6.8 cm</td>
<td>21</td>
<td>5.6</td>
</tr>
<tr>
<td>E 8.6 cm</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.39: Response Analysis for Item 36

The correct option was A. Its selection by 233 pupils, 61.8 per cent of the entry, has pegged the facility index at the same percentage. Distractor B performed moderately well (endorsed by 48 pupils, or 12.7 per cent of the entry), followed by C, D and E, the performance of which was very poor. But in statistical terms, the omission of the item by 41 pupils, 10.9 per cent of the entry, was very characteristic of an underlying weakness.

5 Government schools and the Independent schools found that within their capacity to cope, the item varied from acceptable to easy. The overall mean facility of 61.8 per cent for the entire field class gives less information than 55.3 per cent and 79.8 per cent for the Government and Independent schools respectively. The actual mean measure of this capacity to cope with the item was 29.9, 8.2 per cent higher than that obtained for the whole field class over all of the items. Similarly, the spread of the marks from the mean
was 10.6 per cent less. Also, a discrimination index of 0.48 (Table 5.54) indicated that the item was quite effective in spotting and grouping individuals of like aptitude.
**Item 37**

Which of the following statements about the areas of triangle APQ and triangle A'PQ is correct?

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>52</td>
<td>13.8</td>
</tr>
<tr>
<td>A APQ is greater</td>
<td>30</td>
<td>8.0</td>
</tr>
<tr>
<td>B APQ is less</td>
<td>36</td>
<td>9.5</td>
</tr>
<tr>
<td>C* APQ is equal</td>
<td>216</td>
<td>57.3</td>
</tr>
<tr>
<td>D APQ is half of</td>
<td>22</td>
<td>5.8</td>
</tr>
<tr>
<td>E APQ is twice</td>
<td>19</td>
<td>5.0</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>99.9</strong></td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.40: Response Analysis for Item 37

The key was option C. 216 candidates, 57.3 per cent of the sample, preferred it, thus pegging the facility index at this percentage. 36 candidates, 9.5 per cent of the sample, endorsed distractor B which was followed in order by A, D and E. Only two answer books had at least two options circled for this item but 52 candidates, who represented 13.8 per cent of the sample, did not attempt it. This percentage of omits indicates a fundamental lack of knowledge of this aspect of curriculum content.

4 Government and the Independent schools found the item varying from acceptable to very easy as indicated in the facility attained by each school. The overall mean facility
of 57.3, though a useful statistic, masks the fact that whereas the mean facility of the Independent schools was 82.8 per cent, that scored by the Government schools was only 45.2 per cent. However, the mean ability of those who got the item right was 30.0, 12.4 per cent higher than the measure obtained for the entire sample, with the spread of the marks from that mean scattered over a slightly shorter range. Also the item, as indicated by the correlation coefficient of 0.46, was very effective in discerning and dividing those who got the item correct into subgroups of varying talents.
Items 38, 39, 40, 41 and 42 refer to the solid shown in the diagram alongside.
Item 38

This solid is

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
<td>5.3</td>
</tr>
<tr>
<td>A a sphere</td>
<td>22</td>
<td>5.8</td>
</tr>
<tr>
<td>B a cylinder</td>
<td>12</td>
<td>3.2</td>
</tr>
<tr>
<td>C a pyramid</td>
<td>17</td>
<td>4.5</td>
</tr>
<tr>
<td>D a cone</td>
<td>6</td>
<td>1.6</td>
</tr>
<tr>
<td>E* a cube</td>
<td>298</td>
<td>79.0</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>
| TOTAL     | 377         | 99.9+

+ round-off error

Table 5.41: Response Analysis for Item 38

The right answer was option E, selected by 298 pupils who represented 79.0 per cent of the sample. By definition, this percentage of the sample was also the facility index for the item. Distractor A, selected by 22 pupils - 5.8 per cent of the entry - was followed by C, B and D. Only two pupils submitted answer books with at least two options circled but 20 pupils - 5.3 per cent of the sample - omitted the item.

On the evidence presented by the facility index for each school, the mean facility for Government and Independent schools and the overall mean facility for the entire field survey class, this item varied from acceptable to very easy. Although the mean ability of the entry was 28.2 (Table 5.54), 5.6 per cent higher than the overall mean ability of the total population, and the spread of the scores from this mean was only just less than that for the total entry, the
A correlation coefficient, 0.35, indicated that the item was sensitive in detecting and separating the survey class into varying pockets of ability.
**Item 39**

The set \{faces, corners, edges\} corresponds, in order, to the set

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>68</td>
<td>18.0</td>
</tr>
<tr>
<td>A {6, 12, 8}</td>
<td>35</td>
<td>9.3</td>
</tr>
<tr>
<td>B* {6, 8, 12}</td>
<td>155</td>
<td>41.1</td>
</tr>
<tr>
<td>C {12, 8, 6}</td>
<td>64</td>
<td>17.0</td>
</tr>
<tr>
<td>D {8, 6, 12}</td>
<td>28</td>
<td>7.4</td>
</tr>
<tr>
<td>E {12, 6, 8}</td>
<td>26</td>
<td>6.9</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.42: **Response Analysis for Item 39**

The precise solution was option B, identified by 155 candidates who comprised 41.1 per cent of the sample. This percentage by definition was also the facility index of the item for this sample of candidates. Distractor C, chosen by 64 candidates, turned 17.0 per cent of the sample away from the key. It was followed by A, D and E. Only one candidate offered an answer book with at least two options circled but 68 candidates – 18.0 per cent of the sample – omitted the item.

4 Government and 2 Independent schools coped well with the item, scoring mean facilities of 39.2 and 43.6 per cent respectively. The mean achievement, 29.3, was 9.7 per cent better than the mean for the whole field class. But this mean has the other scores at a shorter radius from it. Looking at the coefficient of correlation, 0.26, the feeling conveyed was that many low scoring candidates were in this sample. However, the level of separation of ability was acceptable.
**Item 40**

Any face of the solid is called

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>31</td>
<td>8.2</td>
</tr>
<tr>
<td>A a rhombus</td>
<td>42</td>
<td>11.1</td>
</tr>
<tr>
<td>B a rectangle</td>
<td>17</td>
<td>4.5</td>
</tr>
<tr>
<td>C a parallelogram</td>
<td>30</td>
<td>8.0</td>
</tr>
<tr>
<td>D* a square</td>
<td>242</td>
<td>64.2</td>
</tr>
<tr>
<td>E a circle</td>
<td>11</td>
<td>2.9</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Table 5.43: Response Analysis for Item 40

The correct answer was option D, selected by 242 pupils who represented 64.2 per cent of the entry. The facility of this item were therefore, by definition, the percentage which selected the correct option. Of the distractors, A, having turned 42 pupils - 11.1 per cent of the entry - away from the key, was followed by C, B and E. 4 pupils submitted answer books with multiple responses to this item and 31 others - 8.2 per cent of the entry - omitted the item.

6 Government and the 3 Independent schools found the item well within their range of ability since the mean facility was 62.7 and 71.3 per cent respectively. The mean mark for those who got the item correct was 29.2, 9.4 per cent higher than the mean score on the test for the entire sample. This mean was also slightly less widely spread than that for the entire sample population. The extent to which the item separated the candidates into pockets of ability was indicated by the discrimination index, 0.40 (See Table 5.54).
Item 41

Faces and corners meet at ____ angles

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>33</td>
<td>8.8</td>
</tr>
<tr>
<td>A* right</td>
<td>220</td>
<td>58.4</td>
</tr>
<tr>
<td>B acute</td>
<td>45</td>
<td>11.9</td>
</tr>
<tr>
<td>C obtuse</td>
<td>35</td>
<td>9.3</td>
</tr>
<tr>
<td>D base</td>
<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>E reflex</td>
<td>25</td>
<td>6.6</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.1+</td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.44: Response Analysis for Item 41

The right answer, option A, selected by 220 candidates, was 58.4 per cent of the entry. This percentage of the candidates who selected the right answer defined the facility index of the item. Although it only turned 45 candidates - 11.9 per cent of the entry - away from the right answer, distractor B was the most popular, followed by C, E and D. Only one candidate submitted an answer book with a multiple response but 33 - 8.8 per cent of the entry - omitted the item.

8 Government schools and the 3 Independent schools recorded acceptable facility indices, with the mean facility at 54.0 and 67.8 per cent respectively. The mean mark on the question was 30.0, 12.4 per cent higher than that recorded for the entire field class, and the spread about the mean, was 0.8 of a standard deviation less than the overall target for the entire field class. The correlation coefficient, 0.47, indicated that the item effectively separated the varying ability levels.
Item 42

If an edge of the solid is 5 centimetres (cm), the area of any face in square centimetres (cm$^2$) is

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>52</td>
<td>13.8</td>
</tr>
<tr>
<td>A 20</td>
<td>92</td>
<td>24.4</td>
</tr>
<tr>
<td>B 50</td>
<td>46</td>
<td>12.2</td>
</tr>
<tr>
<td>C* 25</td>
<td>164</td>
<td>43.5</td>
</tr>
<tr>
<td>D 125</td>
<td>14</td>
<td>3.7</td>
</tr>
<tr>
<td>E 150</td>
<td>9</td>
<td>2.4</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.45: Response Analysis for Item 42

Option C, the correct answer, was the alternative picked by 164 examinees - 43.5 per cent of the sample - who simultaneously decided, in conformity with the definition, the facility index for this item. Of the distractors, A was the most popular (decided by 92 examinees, or 24.4 per cent of the sample), followed by B, D and E. But 52 examinees, who constituted 13.8 per cent of the sample, omitted the item.

3 each of Government and Independent schools recorded acceptable facility indices and the mean performance of the Independent schools indicated that the syllabus content was very secure. The mean mark achieved by those who got the item right was 30.5 or 14.2 per cent higher than that for the entire sample, with the spread of the marks scattered over a narrower range from the mean. The correlation coefficient of 0.39 was a good indicator that the item discriminated well between those who mastered the syllabus content and those who did not.
Items 43 and 44 refer to the bar chart below.
Item 43

How many policemen wear a size 9 boot or larger?

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>39</td>
<td>10.3</td>
</tr>
<tr>
<td>A 16</td>
<td>121</td>
<td>32.1</td>
</tr>
<tr>
<td>B 20</td>
<td>46</td>
<td>12.2</td>
</tr>
<tr>
<td>C 25</td>
<td>26</td>
<td>6.9</td>
</tr>
<tr>
<td>D 30</td>
<td>16</td>
<td>4.2</td>
</tr>
<tr>
<td>E* 34</td>
<td>128</td>
<td>34.0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.46: Response Analysis for Item 43

The correct answer (which required reading and an interpolation of the graph along with addition of three natural numbers) was option E, the preference of 128 pupils who constituted 34.0 per cent of the population. The facility of this item was, consequently, 34.0 per cent. The most effective distractor, A (endorsed by 121 pupils, or 32.1 per cent of the population), was followed by B, C and D. One answer book had a multiple response to this item but 39 pupils - 10.3 per cent of the population - omitted it.

Only 2 Independent schools reached an acceptable facility level and the mean facility for all Independent schools was 46.9 per cent. The mean mark achieved by those who chose the key was 32.3 which was 21.0 per cent higher than the mean for the whole field survey class. Also, the marks were over a slightly narrower range. Accordingly, as the discrimination index of 0.47 shows, the item separated out very well those pupils who mastered the relevant syllabus content from those who were having difficulties.
Item 44

Which size of boot is worn by most policemen?

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>40</td>
<td>10.6</td>
</tr>
<tr>
<td>A 6</td>
<td>10</td>
<td>2.7</td>
</tr>
<tr>
<td>B* 8</td>
<td>291</td>
<td>77.2</td>
</tr>
<tr>
<td>C 7</td>
<td>6</td>
<td>1.6</td>
</tr>
<tr>
<td>D 9</td>
<td>18</td>
<td>4.8</td>
</tr>
<tr>
<td>E 10</td>
<td>11</td>
<td>2.9</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.1+</td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.47: Response Analysis for Item 44

291 candidates, who represented 77.2 per cent of the sample population, chose the correct answer, option B and thereby decided the facility index for the item for this sample population. Distractor D, although it only managed to divert 18 candidates - 4.8 per cent of the population - away from the key nevertheless recorded the best performance, followed by E, A and C. There was one answer book with at least two options circled but 40 candidates, or 10.6 per cent of the sample, omitted the item.

The respondents who got this item right registered a mean performance of 28.8, 7.9 per cent better than the performance of the whole survey population, with a similar spread of ability. The facility indices of the various schools showed that both Government and Independent schools achieved a mean index which indicated that the item was easy: in fact, it was too easy for 2 of the Independent schools. The capacity of the item to separate the respondents into pockets of ability was expressed through a point biserial coefficient of correlation of 0.46 which was quite effective.
The answers to Items 45 and 46 are obtained from the graph below.

The following table shows the time taken by a car to travel various distances at the same speed.

<table>
<thead>
<tr>
<th>Time in minutes</th>
<th>0</th>
<th>15</th>
<th>25</th>
<th>30</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance in kilometres</td>
<td>0</td>
<td>12</td>
<td>20</td>
<td>24</td>
<td>44</td>
</tr>
</tbody>
</table>

This information is plotted on the axes below, in the form of a straight line graph.

Item 45

How far has the car gone in 45 minutes?

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>55</td>
<td>14.6</td>
</tr>
<tr>
<td>A</td>
<td>32 km</td>
<td>10.6</td>
</tr>
<tr>
<td>B</td>
<td>40 km</td>
<td>17.2</td>
</tr>
<tr>
<td>C</td>
<td>44 km</td>
<td>12.5</td>
</tr>
<tr>
<td>D*</td>
<td>36 km</td>
<td>34.2</td>
</tr>
<tr>
<td>E</td>
<td>48 km</td>
<td>10.3</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>99.9</td>
</tr>
</tbody>
</table>

† round-off error

Table 5.48: Response Analysis for Item 45.
From the graph the precise answer was option D, chosen by 129 pupils, who were 34.2 per cent of the entry. The facility index of this item was, therefore, 34.2 per cent. Each of the distractors diverted at least 10.0 per cent of the pupils away from the answer but distractor B, preferred by 65 pupils which were 17.2 per cent of the entry, was the most popular. There were only two answer books with at least two options circled but 55 pupils—representing 14.3 per cent of the entry—omitted the item.

The mean achievement of those who had this item correct, 32.7 was 22.5 per cent better than the performance of the entire field class although there was substantially the same spread. The sorting power of the item was expressed in a discrimination index of 0.51. In fact this high index of sorting reflected the plight of the Grade 9 classes from the Government schools on this item because every sample class found this item difficult unlike the classes from the Independent schools.
Item 46

How long does it take the car to travel 40 kilometres?

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>49</td>
<td>13.0</td>
</tr>
<tr>
<td>A* 50 min</td>
<td>158</td>
<td>41.9</td>
</tr>
<tr>
<td>B 45 min</td>
<td>56</td>
<td>14.9</td>
</tr>
<tr>
<td>C 40 min</td>
<td>43</td>
<td>11.4</td>
</tr>
<tr>
<td>D 35 min</td>
<td>36</td>
<td>9.5</td>
</tr>
<tr>
<td>E 30 min</td>
<td>33</td>
<td>8.8</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.49: Response Analysis for Item 46

The right answer was option A, selected by 158 candidates who comprised 41.9 per cent of the sample population. By definition, therefore, the facility index was 41.9 per cent. The best distractor was B (endorsed by 56 candidates, or 14.9 per cent of the population), followed in order by C, D and E. Only two answer books had multiple responses but 49 candidates - 13.0 per cent of the population - omitted this item.

The respondents who had this item right showed that their mean performance was 31.6, 18.4 per cent better than that for the whole field class, but the spread of their talents was over the same range. Clearly, the item discriminated in a very effective way, achieving an index of 0.49. 4 Government schools were at a level of competence which was lower than that demanded by the item, but all Independent schools showed an acceptable index of facility, with a mean of 52.2 per cent. The mean facility of the Government schools, 38.2 per cent, fell just short of what would be considered acceptable.
The answers to Items 47 and 48 are obtained by reading the graph below.

The graph below shows the number of one manufacturer's old and new alarm clocks in use during each year from 1957 to 1966.

Item 47

The largest number of old clocks in use during any one year over this period is:

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>63</td>
<td>16.7</td>
</tr>
<tr>
<td>A 10 000</td>
<td>60</td>
<td>15.9</td>
</tr>
<tr>
<td>B* 9000</td>
<td>156</td>
<td>41.4</td>
</tr>
<tr>
<td>C 8000</td>
<td>52</td>
<td>13.8</td>
</tr>
<tr>
<td>D 7000</td>
<td>23</td>
<td>6.1</td>
</tr>
<tr>
<td>E 6000</td>
<td>22</td>
<td>5.8</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.50: Response Analysis for Item 47

The precise answer was option B, preference for which was shown by 156 pupils who constituted 41.4 per cent of the
entry. The facility of this item was, therefore, 41.4 per cent. Of the distractors, A was most popular (preferred by 60 pupils, or 15.9 per cent of the entry), followed by C, D and E. There was only one answer book with at least two options circled but 63 pupils - 16.7 per cent of the entry - omitted the item.

The mean criterion on this item was 30.5 (14.2 per cent in excess of that for the whole sample survey) and the indication of spread was 8.9, which was not unlike the measure of spread for the whole sample survey. The field performance of the item in separating those of high ability from those of moderate and low ability was shown in the discrimination coefficient of 0.38. A feel for the manner in which this item worked as a separation instrument may be had from the fact that 4 Government schools had hardships in reaching an acceptable level of facility although their mean performance in a facility index of 40.8 per cent was only just acceptable as opposed to the Independent schools with a satisfactory mean facility of 53.7 per cent.
Item 48

In 1963, the total number of clocks (old and new) in use was

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>67</td>
<td>17.8</td>
</tr>
<tr>
<td>A 8000</td>
<td>70</td>
<td>18.6</td>
</tr>
<tr>
<td>B 7000</td>
<td>42</td>
<td>11.1</td>
</tr>
<tr>
<td>C 9000</td>
<td>48</td>
<td>12.7</td>
</tr>
<tr>
<td>D*11000</td>
<td>123</td>
<td>32.6</td>
</tr>
<tr>
<td>E 10000</td>
<td>26</td>
<td>6.9</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>377</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Table 5.51: Response Analysis for Item 48

Option D, selected by 123 pupils who constituted 32.6 per cent of the sample population, was the right answer. This percentage of pupils who selected the right answer was, by definition, the facility index of the item. The most effective distractor A, was endorsed by 70 pupils who represented 18.6 per cent of the population. Following in order were the performances of distractors C, B and E. There was only one answer book with a multiple response to this item but 67 pupils - 17.8 per cent of the population - omitted it.

The mean ability on this item 33.1 (24.0 per cent higher than the mean for the whole survey population) indicated quite clearly that those who got the item right were the better able in the entire population, although the spread was virtually the same. The discrimination index of 0.53 also indicated that the item was very effective at sensing and sorting out the different pockets of ability. The effect of this sorting power was reflected in the fact that 6 Government schools and 1 Independent school made heavy weather of the item. Although the mean facility, 23.2 per cent for the Government school did reflect this inability to cope with the item, the mean facility for the Independent
schools which stood at 48.3 per cent did show, and rightly so, that the difficulty was not so acute since the facility index of 38.5 per cent for one Independent school only just failed as being acceptable.
**Item 49**

Written in the form of prime factors $2520 = \ldots$

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>68</td>
<td>18.0</td>
</tr>
<tr>
<td>$A\ast 2^3 \times 3^2 \times 5 \times 7$</td>
<td>117</td>
<td>31.0</td>
</tr>
<tr>
<td>$B 2^2 \times 3^3 \times 5 \times 7$</td>
<td>57</td>
<td>15.1</td>
</tr>
<tr>
<td>$C 2 \times 3 \times 5^2 \times 7^3$</td>
<td>55</td>
<td>14.6</td>
</tr>
<tr>
<td>$D 2 \times 3 \times 5^3 \times 7^2$</td>
<td>46</td>
<td>12.2</td>
</tr>
<tr>
<td>$E 2 \times 3 \times 5 \times 7$</td>
<td>32</td>
<td>8.5</td>
</tr>
<tr>
<td>$9$</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>99.9</td>
</tr>
</tbody>
</table>

† round-off error

Table 5.52: Response Analysis for Item 49

Option A, the right answer, was the alternative picked by 117 candidates - 31.0 per cent of the entry - who at the same time decided, in conformity with the definition, the facility index for this item. As a distractor, B was only just better than C which was selected by 55 candidates who represented 14.6 per cent of the entry. To continue the order, D and E followed. Only two answer books had at least two options circled but 68 candidates, who together represented 18.0 per cent of the entry, omitted the item.

The mean achievement on this item by those who got it right was 31.4 and the spread of ability in the subsample had a standard deviation of 8.0 (Table 5.54). Whilst this spread of ability was not significantly higher than that for the whole field class, the mean achievement was 17.6 per cent better. Also, the power of the item to sort the various levels of ability was measured by a discrimination index of 0.37. Further probing showed that 4 Government schools and 1 Independent school found this item very difficult indeed although their mean facilities, 34.7 and 46.7 respectively, did not reflect this in the same way as the overall facility index of 31.0 per cent.
Item 50

O is the centre of the circle. AB is equal in length to the radius of the circle. The size of the angle AOB in degrees is

![Diagram showing a circle with points A, O, and B]

<table>
<thead>
<tr>
<th>Options</th>
<th>Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>63</td>
<td>16.7</td>
</tr>
<tr>
<td>A 75°</td>
<td>34</td>
<td>9.0</td>
</tr>
<tr>
<td>B 30°</td>
<td>62</td>
<td>16.4</td>
</tr>
<tr>
<td>C* 60°</td>
<td>93</td>
<td>24.7</td>
</tr>
<tr>
<td>D 90°</td>
<td>86</td>
<td>22.8</td>
</tr>
<tr>
<td>E 45°</td>
<td>39</td>
<td>10.3</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL</td>
<td>377</td>
<td>99.9</td>
</tr>
</tbody>
</table>

+ round-off error

Table 5.53: Response Analysis for Item 50

The correct solution was option C, identified by 93 pupils who comprised 24.7 per cent of the sample population. By definition this percentage was also the facility index for the question for this population. Distractor D, chosen by 86 pupils turned 22.8 per cent of the sample away from the key. It was followed by B, E and A but 63 pupils - 16.7 per cent of the entry - omitted the item.
The mean mark achieved by those who got this item correct was 30.4 and the range of ability in the subsample was expressed by a standard deviation of 8.6 (See Table 5.54). Closer observation showed that the mean ability on this item was 13.9 per cent higher than the overall mean, 26.7, for the whole sample. But the spread of ability, at 8.6, was practically unchanged. In addition, the sensitivity with which that ability was identified and separated was expressed by a discrimination index of 0.25. Government and Independent schools found this item quite difficult as reflected in their mean facilities, 24.4 per cent and 27.3 per cent respectively.
5.3.2 Determination of Facility

In Section 5.1.3 it was indicated that the computer was instructed to output the statistics mean, standard deviation and standard error for variable 050. In order to find the mean performance of those pupils who got the item right, the computer did a frequency count of the valid cases (93 for this item) of those who chose option C - for which 3 stood proxy - summeć their total marks, then divided by 93 to give 2.589.

In the calculation of a facility index for this item, the researcher read the 93 valid cases from the computer print-out. On the electronic calculator the numeral/decimal point keys [9], [3], [·], and [0] were punched, after moving the left side switch forward, to give 93.0 on the display panel. The key [·] was pressed whereupon the zero disappeared after [3] was punched. This was followed by punching the keys [7] twice, [·], [0], [=] immediately after which the read-out showed 0.24668435 in response to the instruction '93.0 ÷ 377.0'. Variations of this instruction, 93.0 ÷ 377, 93:377, 93÷377.0, were all electronically acknowledged to be the same in so far as the answer was the same decimal fraction to 8 decimal places. However, the degree of accuracy necessary for this exercise was 3 decimal places. Thus, 0.247 was the proportion of respondents who chose the correct option to variable 050. This proportion conveniently converted to 24.7 per cent, the usual way of expressing the Facility Index, F. By substituting this value of F and T = 377 in the formula quoted early in this chapter, the

\[
\text{Difficulty Index} = 1 - \frac{93.0}{377.0} = 75.3 \text{ per cent}
\]

thus indicating that the proportion of the entry who found the item difficult was 0.753.

The valid observations of absolute frequency and relative frequency per cent were printed out for each option per item, along with a running cumulative frequency per cent. It was from this information that a Response Analysis Table was prepared.
for each item in Section 5.3.1. The computer was justifiably necessary only because the sample was large, but for multiple choice classroom tests the calculator brings speed and efficiency to the calculations at very little cost and permits the teacher to choose the degree of accuracy required in deriving the descriptive statistics for all options.

5.3.3 Discrimination Coefficients

For the class teacher one of the best definitions of the function of a Discrimination Index is given by Purushothaman [130, p.17]:

Discrimination of an item is a measure of how well the item differentiates between the better and poorer pupils taking the test.

But there was fierce competition between the biserial coefficient and the point biserial coefficient for use in the calculation of this statistic. Owing to the time-consuming nature of the calculations involved, short-cut methods, admittedly less precise (see Macintosh and Morrison [131, pp.62-68], for example), have come into use. However for psychometricians and chief examiners of large public examination boards Macintosh [127,p.50] claims that:

The most rigorous statistic used for the discrimination index of an item is the point biserial correlation coefficient, \( r_{pbis} \): 

\[
    r_{pbis} = \frac{M_p - M_q}{\sigma \sqrt{pq}} 
\]

where, \( p \) is the proportion of candidates giving a correct response to the item and \( M_p \) is their mean score on the test, \( q \) is the proportion giving an incorrect response and \( M_q \) is their mean score on the test and \( \sigma \) is the standard deviation for all candidates' test scores.

Wood [121,p.243] concurs with Macintosh and introduces practically the concept of a gradient of difficulty as being...
synonymous with discriminating power which distinguishes between 'clever' and 'dull' candidates. Discussing the function of this index he (Wood) advises:

... The usual approach to obtaining a measure of item discrimination is to calculate the correlation between score on the item (1 or 0) and score on the test as a whole, the idea being that the higher the correlation between candidates' scores on an item and their score on the test, the more effective the item is in separating them. Naturally, this relationship is a relative one; when included in one test an item could have a higher item-test correlation than when included in another, yet produce poorer discrimination.

The correlation I am talking about has a special name, point biserial, and it is worth examining how it is calculated to see how much information ... it actually uses. The formula for the sample point biserial correlation can be set out in various ways, but the most convenient for my purposes is as follows:

\[ r_{pbis} = \frac{M_p - M}{S} \cdot \sqrt{\frac{p}{1-p}} \]

where \( M_p \) is the mean score on the test obtained by those who got the item correct, \( M \) is the mean score on the test for the entire group, \( S \) is the standard deviation of the test scores for the entire group, and \( p \) is the proportion getting the item right (the item facility).

As a consequence of comparing and contrasting the biserial and the point biserial correlation coefficients, Wood advises that the potential user should make a choice of one and study its behaviour. The writer used the latter and calculated the index for each question using the Casio fx-120 electronic calculator, taking the relevant valid observations from computer print-outs.
5.3.4 **Reliability**

In attempting to present a picture of the internal consistency of the objective diagnostic test, sympathy for and a working knowledge of the concepts *mean*, *variance* and *correlation* were absolutely vital to looking beyond the mechanical substitution of numerical values for variables in a formula. Also, even more than in physics experiments, the possible sources of error in scores on a test, in so far they varied both with individuals at different times as well as between individuals, had to be identified. It was impossible to be precise about whether scores by any two children on the test had 'real' or 'chance' differences. The problem of reliability, therefore, was that of differentiating these types of variance and the literature had to be thoroughly searched for a formula which consistently took them into account in tandem with the dichotomous nature of the test items (scored 0 or 1) in order to produce a meaningful coefficient. Satterley [122, pp.179-200], Nuttall and Willmott [129, p.67], Guilford and Fruchter [132, pp.427-430], and Macintosh [127, p.52] are unanimous in advocating that the Kuder-Richardson Formula 20 estimates agreement between all items within a single test thereby giving a measure of internal consistency. This formula:

$$R = \frac{\left[ \frac{n}{n-1} \right] \left( S_t^2 - \sum_{i=1}^{n} pq \right)}{S_t^2}$$

[132, pp.427-430],

where \( n \) = the number of items in the test, \( p \) = the proportion passing an item, \( q = 1-p \), and \( S_t^2 \) = total variance, yields \( 0 < R < 1 \) (\( R = 1 \) would be perfect reliability). For instance \( R = 0.88 \) says that 88.0 per cent of the differences in obtained scores from junior high schools in The Bahamas occurred because of differences in true scores. The error of measurement in this instance was quite obviously 12.0 per cent. Concerning the sources of error, candidates needed to read and think as opposed to think and write; were requested to draw a series of circles in response to elaborate instructions; had to answer 50 separate items in a comparatively short time instead of answering ten questions on a paper offering choice. The efficiency with which pupils completed these tasks depended to some extent on the amount of practice they had in responding to items of this type and in
part on the amount of revision - which could not have been much if any indeed - they had since the new school year commenced.

Determination of a numerical value of R required perusal of the computer print-out, from which q for each item was deduced directly by retrieving a value for p. Using the calculator, pq for each item was fed in, the result read from the display panel and organised in tabular form. Then deploying the statistics mode,

\[ \sum_{pq=1}^{50} (pq) = 9.941 \]

The keys [8], [·], [5], [1] and [3] were punched to show 8.513 on the read-out panel, immediately after which the [INV] and \( \chi^2 \) keys were pressed to give \( S_t^2 = 72.471169 \), which approximated to 3 decimals places, gave \( S_t^2 = 72.471 \).

Hence \[ R = \frac{50}{49} \left( \frac{72.471 - 9.941}{72.471} \right) \]

ie \( R = 0.88 \)

Despite a 12.0 per cent estimation of error (small in a behavioural science such as education) there was every likelihood that if this test was given to the same population within, or at most, one month after the first administration, the scores would be very similar. This is the function of item analysis, namely, to predict how items will behave on another occasion with a similar group of candidates. Herein lies the promise and importance of item banking as a library of examination questions and items.

The term \( \sum_{pq=1}^{50} (pq) \) was the sum of the variances of all items and \( S_t^2 \) was the total test variance. The difference between the two (72.471-9.941) - technically the sum of the covariances - was interpreted as the amount of overlap between the scores on the items. Now overlap between items was assumed to result
from the same general ability, which in effect was where the true variance was identified. It follows that this analysis exposed the structure of the research function of a coefficient of reliability. But, one may ask, how does knowledge of the research function of a coefficient of reliability inform practical classroom decisions? Satterly [122, p.201] claims that reliability is defined as the ratio of true variance and obtained variance:

\[ R = \frac{S_t^2}{S_o^2} \quad \ldots (1) \]

But, by simple algebra, the variance of obtained scores, \( S_o^2 \), can be shown to be the sum of true variance, \( S_t^2 \) and error variance, \( S_e^2 \);

\[ S_o^2 = S_t^2 + S_e^2 \quad \ldots (2) \]

where subscripts 0, t and e stand for obtained, true and error scores respectively.

\[ S_t^2 = S_o^2 - S_e^2 \quad \ldots (3) \]

Hence \( R = \frac{S_o^2 - S_e^2}{S_o^2} \), substituting (3) in (1).

\[ R = 1 - \frac{S_e^2}{S_o^2} \quad \ldots (4) \]

Rearranging (4) and extracting the square root:

\[ S_e = S_o \sqrt{1-R} \quad \ldots (5) \]

Equation (5) says that the Standard Error of Measurement, \( S_e \) is in fact the standard deviation of the error. In other words, \( S_e \) represents the linear spread of the error scores about the mean error score. In the literature on techniques of assessment \( S_e \), the standard error of measurement, is written \( S_{Em} \).
Suppose, for instance, that in a term of 14 weeks two mathematics tests had a mean of 50 and standard deviation 10. One of the tests had a reliability, $R = 0.96$ and the other $R = 0.64$. Further, assume that a girl had 56 on the first test and a boy had 56 on the second. How closely did these marks lie to true scores?

With reference to the girl’s score on the first test where $R = 0.96$, (the proportion of true variance), then $1 - 0.96$ or 4.0 per cent was error variance. The standard error of measurement was, therefore

$$SE_m = \frac{10}{\sqrt{0.04}}$$

ie $SE_m = 2$ points

Since about two-thirds (68.0 per cent) of the scores lay between true score + one standard deviation (from theory on the curve of normal distribution) the odds are 2 to 1 that the girl's obtained score would not exceed 56+2 nor fall below 56-2 points on repeated testing. For added confidence, use can be made of the fact that 95.0 per cent of the scores lay between the mean and +2 standard deviation units. The odds here were approximately 19 to 1 that the girl's obtained score did not exceed 60 nor fall below 52 points. Sometimes these are called the 95.0 per cent confidence limits of true score. In fact, the 95.0 per cent confidence interval was the obtained score $\pm 1.96 \ SE_m$ but it was easier to use 2 for practical purposes. Using the same reasoning the $SE_m$ for the boy’s score on the second test was 0.6 of the standard deviation (ie 6 points). The 68.0 per cent confidence interval, therefore was from 62 to 50 points (compared with 58 to 54 for the girl's) and the 95.0 per cent interval from 63 to 44 points (compared with 60 to 52).

The standard error of measurement therefore is a function of the reliability and the standard deviation of the test. What psychometricians have always known is that the higher the reliability coefficient the closer the obtained score is to the true score. Given the reliability, Satterly [122,p.205] claims
that it is possible to estimate a pupil's true score using the simple formula:

\[ T = M + R(x_i - M) \]

where

- \( T \) = true score
- \( M \) = mean score on the test for the entire group
- \( R \) = reliability coefficient
- \( x_i \) = raw score of \( i \)th pupil

But the algebraic structure of this formula shows a linear combination of \( T \) with \( x_i \). Hence the implication is that true scores have a linear relationship with raw scores which is in fact the theoretic basis of the least squares formula. (See Bajpai et al [71, p.384]).

It is interesting that Satterly [122, p.205] also claims:

The higher the reliability the more confident we can be that the difference between two scores in the same group of children is real.

In the instance of a boy from School HON 1(G) in Table 5.2, a computer print-out showed that his score was 42.0 on the diagnostic test the reliability for which was 0.88. His group's mean was 22.74 with a standard deviation of 5.42 points, so the least squares formula gave his true score

\[ T = 22.74 + 0.88 (42.0 - 22.74) \]

ie \( T = 40.0 \), to 2 significant figures.

But for the existence of 9 scores which were less than 20, the minimum partition in Table 5.3, the true score calculated above would be higher. Even so there has been no appreciable decrease in the raw score. However, the crucial importance of this instance is that it can generate a very instructive discussion since it lies in the neighbourhood of the B/A grade partition.
5.3.5 **Validity**

The criterion of validity is not amenable to statistical measurement yet in the final analysis the data collected by research instruments must be shown to have measured what they attempted to measure - in this instance the mathematical ability of the sample pupil population. It would appear that a feel for the concept of validity is best conveyed by discovering a qualitative relationship between test scores and changes in specific curriculum behaviours where changes mean the acquisition of new knowledge and skill that normally might not have been acquired over the same time scale without exposure to and direction by a teacher. On this point Macintosh and Morrison [131, p.15] observe:

Validity is essentially a matter of degree. There is no such thing as an absolutely valid or an absolutely invalid test, ... The validity of any test depends not only upon its reliability and relevance, but also upon the purpose for which it is to be used and the skill with which it is used to achieve this purpose ...

In its construction an objective test, ... requires the careful consideration of objectives and the drawing up of specifications.

With reference to 'reliability and relevance', the former was calculated to be 0.88, thus partially giving the degree to which validity had been achieved and the latter condition was satisfied by choosing the items from the syllabus agreed for the pupils who wrote the test. The 'purpose' of the test was to discover to what extent pupils had acquired a working familiarity with the basic mathematics concepts laid out in the syllabus in Appendix 5.2. How well the test sampled the content areas of the course was built into the specification grid, and Heads of Mathematics Departments, mathematics teachers and pupils approved the items. Content validity was therefore satisfied. The mark range-grade distribution table (Table 5.3) constructed from the cumulative frequency distribution curve (Figure 5.1) gave the extent to which predictive validity...
was achieved in that it shows the percentage of pupils who were considered eligible to benefit from further studies in mathematics. Those pupils who attained grades D and E have not satisfied the conditions necessary for further studies: 22.5 per cent of year's cohort must undergo some programme of individualised learning before they can be recommended to advance. Here construct validity was satisfied because the function of a diagnostic instrument is, in the main, to identify the pupils with learning difficulties. Although it appeared that all aspects of validity important to teachers' work in the classroom have been satisfied, it must be declared that in absence of an empirical method of quantifying the achievement of validity, experienced judgement came into operation. That is to say, allowances had to be made for a strong subjective element that crept into the decision which, ostensibly, should have been an objective conclusion. However, despite the tendency to subjective influence, the tentative judgement that the diagnostic test was a valid one with marking free of error now stands vindicated.
5.4 Discussion

Ideally if the proportion of respondents choosing the key to an item was 0.5 (F = 50.0 per cent) it would be deemed suitable for future use with a similar population. But psychometric theory advocates that the range 40.0 per cent ≤ F ≤ 60.0 per cent is a better criterion of qualification for item selection. Accordingly, for the sample under study, the writer thought that any item which secured a difficulty index of 0.8 (F = 20.0 per cent) was very hard for The Bahamian entry; 35.0 ≤ F ≤ 65.0 indicated a reasonable level of achievement; whereas F ≥ 90.0 per cent was classified very easy. The researcher, in the interest of making the analysis direct and simple, worked from the assumption that most pupils would only have selected options of which they were sure, thereby eliminating any major influence of guessing. Allowance for some guessing therefore meant that a five option item, on the balance of probability, attracted at most 20.0 per cent of the respondents to each option. In comparison with the upper limit of the suggested range of reasonable performance, any distractor would attract 8.75 per cent of the respondents and the lower limit gave 16.25 per cent distractor attraction. This compared most favourably with the proposition of psychometric theory which gives any distractor, d, a range of performance such that 10.0 per cent ≤ d ≤ 15.0 per cent in any try-out of items. But the researcher finally decided that the range sufficiently elastic to subsume guessing was 9.0 per cent ≤ d ≤ 20.0 per cent, which is quite adequate for classroom use. Actually the lower limit of this condition floated the requirement that any distractor attracting less than 9.0 per cent of the respondents needed modification since it reduced the original five option choice to a 4 or even 3 option item. Similarly, when d > 20 per cent, the option was tending towards plausibility as an alternative key. Any item that was not technically deficient, that is to say, badly worded or ambiguous, but attracted a low facility clearly indicated non-mastery of curriculum behaviours by almost all respondents. On the other hand, a few with 80.0 ≤ F ≤ 90.0 on the 50-item objective test were justified not only because they assisted
the pupils in settling to the test-taking routine, but also because the assessment was not intended to destroy the pupils' human dignity. Despite humane conditions, the psychology of testing and teaching demanded that easy items had to be balanced by a few difficult ones to challenge the most able pupils in the sample and to offer a basis for selecting them. It seemed that any objective test suitable for investigating the extent of achievement of basic mathematics concepts should comprise items where the majority fall within the range $30.0 \text{ per cent} \leq F \leq 70.0 \text{ per cent}$, although it was very difficult to legislate in advance the behaviour of test items. Repeated try-outs is the key to providing useful feedback for the validation of any test, which must contain about three or four items per concept to satisfy the diagnostic function.

Now item facility was a sample dependent parameter. The Rasch technique — held in high repute by Wilmut [123, pp.45-48] and Purushothaman [130, pp. 20-30] but criticised by Wood [121, pp. 250, 251] — which takes the ability of the individual and the difficulty of the item as fundamental elements in the model, was designed to overcome the sample-bound nature of facility indices. Given the varying motives for human behaviour, the decision to select or reject an item on the solo criterion of its facility-measuring capacity did not seem broadly based enough in scope. The 'omits' and 'multiple option' categories of response deserved consideration to extract their possible contribution to decision-making. The researcher reckoned that where the percentage of respondents electing these options was small, ie less than say, 10.0 per cent, then no important inference could be drawn except to say that where the omits were equal to or larger than 10.0 per cent, a fundamental lack of curriculum skill and knowledge of the particular objective in question was exhibited. As it happened, only 3.7 per cent of the sample (14 items) had 10.0 or more per cent omits as respondents. The multiple response category made no effective contribution to the decision-making process either because its highest percentage of attraction was only 8.8 per cent (Item 46).
However, embedded in the voluminous feedback from the try-out of the 50-item multiple choice diagnostic test was also the variable parameter discrimination index, $D(r_{pbis})$, the research function of which was to sort out the entire population into low and high ability groups. At this stage it was instructive to look at the contributions of the school of psychometric theory to classroom practice. Wilmut [123, pp.29,30] demonstrates quite convincingly that the range of achievable values of $D$ is $-0.1 \leq D \leq 0.75$. On this basis alone, without reference to the peculiarity of The Bahamian educational environment, this author estimated that $0.30 \leq D \leq 0.45$ was a satisfactory range of item potential in sensing and grouping individuals of similar mathematical abilities. In fact Wood [121, p.244] demonstrates by example that 0.49 is quite effective discrimination for the point biserial coefficient, $r_{pbis}$, as opposed to an ordinary product moment coefficient where $-1 \leq D \leq +1$. Furthermore, he (Wood) says that $r_{pbis}$, the discrimination coefficient $D$, is invariant from one testing situation to another, and this is a virtue the biserial definitely does not possess. Macintosh [127, p.50], also speaking of the point biserial coefficient in the context of utility, says that:

The vast majority of the items in practice have values between 0 and 0.6, ... In general, items would be considered acceptable if the facility index were satisfactory and the discrimination index exceeded 0.25, ...

A further look at numerical values of the parameters discrimination, $D$ and facility, $F$, stimulated Purushothaman [130, p.19] to lay an item - evaluation table in which he suggests that for any value of $F$, the absolute minimum value of $D$ is 0.19 below which any item is deemed poor and consequently should be rejected. Essentially, Macintosh and Morrison [131, p.67] agree with this verdict by a margin for $D$ of 1 in 100 higher than that proposed by Purushothaman in their classification showing that for any value of $F$, and $D < 0.20$, an item should be rejected. From Table 5.54, the author evaluated Items 2 and 7 as marginals, in which case distractors need to be modified so that each makes a stronger appeal to the candidates who are less secure in the
concepts being tested. Hopefully this revision of distractors

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<td>44</td>
<td>28.8</td>
<td>8.0</td>
<td>77.2</td>
<td>0.46</td>
</tr>
<tr>
<td>20</td>
<td>29.6</td>
<td>8.3</td>
<td>55.4</td>
<td>0.39</td>
<td>45</td>
<td>32.7</td>
<td>8.0</td>
<td>34.2</td>
<td>0.51</td>
</tr>
<tr>
<td>21</td>
<td>29.4</td>
<td>8.3</td>
<td>57.6</td>
<td>0.37</td>
<td>46</td>
<td>31.6</td>
<td>8.4</td>
<td>41.9</td>
<td>0.49</td>
</tr>
<tr>
<td>22</td>
<td>31.5</td>
<td>9.8</td>
<td>25.7</td>
<td>0.34</td>
<td>47</td>
<td>30.5</td>
<td>8.9</td>
<td>41.4</td>
<td>0.38</td>
</tr>
<tr>
<td>23</td>
<td>31.4</td>
<td>9.3</td>
<td>24.9</td>
<td>0.32</td>
<td>48</td>
<td>33.1</td>
<td>8.0</td>
<td>32.6</td>
<td>0.53</td>
</tr>
<tr>
<td>24</td>
<td>34.3</td>
<td>9.1</td>
<td>15.4</td>
<td>0.38</td>
<td>49</td>
<td>31.4</td>
<td>8.0</td>
<td>31.0</td>
<td>0.37</td>
</tr>
<tr>
<td>25</td>
<td>29.5</td>
<td>8.7</td>
<td>55.7</td>
<td>0.37</td>
<td>50</td>
<td>30.4</td>
<td>8.6</td>
<td>24.7</td>
<td>0.25</td>
</tr>
</tbody>
</table>

M = 26.7, S = 8.5, I = Item, D(r_pbis) = Discrimination Index

M_p = mean score on the test by those who got the item right

σ = standard deviation of test scores by those who got the item right

M = mean score on the test for entire population

S = standard deviation of test scores for entire population

p% = item facility

Table 5.54: Item Analysis Chart
should result in $D > 0.20$. Items 1 and 34 however are prime candidates for outright rejection. (School PW 7(I) was the only one in the entire population to achieve a satisfactory facility ($F = 45.5$ per cent) on Item 34). The rationale on which these decisions were based was two-fold: firstly, the research instrument had already achieved a reliability of 0.88 in the pre-test stage; secondly the author was influenced by Wilmut's [123, pp.43, 44] finding that

... the continual pruning of a test in the quest for greater uniformity of items (their conformity to a test characteristic) will probably lead to low validity. Guildford (1956) points this out in a discussion of the relationship between reliability and validity and then says: "Maximal reliability requires items of equal difficulty: maximal validity requires items differing in difficulty". It may be that, in order to achieve a valid test, some poor items must be included.

Table 5.54 seems eloquent testimony to Wilmut's findings. But the authenticity of that tentative judgement will stand or fall after a validation exercise in the form of a post-test. For items 11, 24 and 33 (Table 5.54), $F < 20.0$ per cent but $D > 0.23$ so they may be retained. Obviously therefore, the discrimination index $D$ had a decisive role in the choice of an item for future use. The neighbourhood $0.11 \leq D \leq 0.59$, evinced from the first field trial of the author's 50-item diagnostic research instrument, was a well defined subset of the range of values for $D$ predicted by Wilmut [123] and Macintosh [127]. Naturally, this piece of corroborated evidence in psychometric theory constitutes a concrete pavement on which informed classroom practice may securely stand.

Just before the beginning of Section 5.1 of this chapter, the view of a school of psychometric thought was articulated: that the more candidates offering an item, the better the discrimination; and the discrimination is very low if a large subsample get an easy question right. On the premise of the indices exhibited in Table 5.54, the author contests the first
part of the philosophy under study. Consider Items 1 and 3. Although the mean achievement, \( M_p \) was the same, the facility per cent for Item 3 was greater than that for Item 1. But the standard deviation, \( \sigma \) for both items circumscribe more crucial information. The respondents to Item 3 were less linearly scattered from the mean and therefore more homogeneous in ability than the respondents to Item 1. Giving more force of authority to the author's contention: the mean achievement on each of Items 4 and 5 went up, so did the discrimination coefficient, \( D \) but the facility, per cent for each item plummeted and so did the standard deviation, \( \sigma \). That is to say, as the ability of the respondents increased, the decrease in standard deviation reflected the homogeneity of ability and the item thereby showed greater sensitivity in selecting and sorting candidates of similar ability into groups. Effectively therefore, the discrimination coefficient of an item varied directly as the ability of the respondents who chose to answer it. Although the second part of the philosophy under study was correct, it was incomplete. For instance, Item 34 did show that candidates of ability only just higher than that of their counterparts who attempted Item 4, with identical scatter of ability about the mean, only managed to score a facility of 19.6 per cent. Clearly, because they attempted a difficult item - one which requested them to demonstrate knowledge of curriculum content which was beyond their ability to cope - the resulting discrimination index, 0.11 was very poor. The amended second part of the statement of philosophy should now read: '..., the discrimination is very low if a large subsample get an easy question right and if respondents of comparable ability attempt a difficult item.'

Dunn and Goldstein [125, p.178] say in their summary:

The purpose of this study was to determine the effect on difficulty, reliability and validity of tests of four generally accepted principles of item construction: use of incomplete statements as item leads, avoidance of specific determiners or cues to the correct alternative, alternatives of equal lengths, and consistency in grammar between lead and alternatives.
The researcher observed these principles and found no cause
to disagree with the conclusions of Dunn and Goldstein, namely,
that test difficulty was not affected by the character of the
lead (incomplete statement or questions); and that no
quantifiable change in reliability or validity – which is very
difficult to quantify – occurred in contravention of the four
item-writing principles stated above. Their view that further
research is needed stands uncontested. However, the
reliability $r = 0.88$ yielded by data from a 50-item multiple
choice test of 5 options per item was comparable with $r = 0.874$
obtained by Ebel [124, p.565] on a 100-item objective test with
the same number of choices per item. But the formula he (Ebel)
suggests

$$r = \frac{k}{k-1} \left[ 1 - \frac{9(N+1)}{k(N-1)} \right],$$

where $k$ is the number of items on the test and $N$ is the number
of options per item, only predicts $r = 0.74$ which is 18.9 per
cent less efficient than the point biserial coefficient.

Despite this significant percentage disagreement, the advice
was clear: that a 50-item test was as reliable as a 100-item test,
each having 5 options per item. In practice however a 100-item
classroom test is much too long in The Bahamian context, for
one sitting.

The number of respondents, $n$ who got an item correct –
beginning with $n = 366$ for Item 3 ($M_p = 27.0$ in Table 5.54) –
was arranged in descending order down to $n = 58$ for Item 24
($M_p = 34.3$). The general pattern observed, with only very slight
exceptions, was that as the number, $n$ getting the items right
was decreased, the mean $M_p$ went up obviously because these were
the candidates of superior ability who performed very well on
the test as a whole.

Further, the mean mathematical ability of the Independent
schools was 27.9 with the marks having a linear dispersion of
7.2 standard deviation units from the mean ability. Similarly,
the corresponding ability for the Government schools was 25.6
with the scores having a linear scatter of 5.7 units about the
mean performance. Not only was the attainment of the Independent schools 9.0 per cent better than the State schools; their spread of ability about the mean attainment was also better by 26.3 per cent. Therefore, whereas the State schools were, in general, homogeneous in their weakness, the Independent schools had a wider range of ability.

Items 5, 10, 11, 15, 16, 22, 24, 28, 37, 39, 42, 43, 45, 48, 49, 50, representing 32.0 per cent of the basic mathematics concepts from the syllabus content tested, were selected because they were an embodiment of widely differing final behaviours of which many pupils had an insecure grasp. Their facilities, $F$ were displayed in matrix $F = \left[ f_{is} \right]$, where $1 \leq i \leq 16$, $1 \leq s \leq 7$, $i =$ item and $s =$ school. From the elements of matrix $F$, two very important pieces of evidence were extracted. Firstly, where the elements, $f$ along a row, $i$ of the matrix $\left[ f_{is} \right]$ revealed unsatisfactory values of $f(f < 40.0$ per cent) for the seven Government schools, $s$, a low ability population was implied. Secondly, for unsatisfactory values of $f$ along any groups of columns, $s$ (groups of different items, $i$ offered by different schools, $s$), ineffective teaching was implied.

5.5 Conclusion

Items 1 and 34 should be replaced but Items 2 and 7 require modification of the distractors before re-trial and final validation of the test. These measures are necessary to raise to 0.20 their potential to sort out respondents into ability groups, thereby validating the research instrument before filing the items into an item bank. The first field trial of the items as a whole was otherwise very successful using the reliability coefficient, $r = 0.88$ as the index of performance. Satterly [122, p.112] fortifies this view by suggesting that teacher-made objective tests should aim for a least upper bound of reliability, $r = 0.85$. The Grade 9 population of Government junior high schools had reached a lower ability measure (25.6) on the diagnostic mathematics test, itemised details of which are

+ Underlined items strongly supported conclusions below about teaching and learning mathematics
exhibited in Table 5.54, than the Independent schools (27.9). In order to interpret these mean performances within relevant confidence limits, the standard errors of measurement are $\text{SE}_m = 1.97$ and $\text{SE}_m = 2.49$ respectively, both occupying the neighbourhood $2 \leq \text{SE}_m \leq 3$ suggested by Beeching [128, p.139].

Furthermore, a matrix of item performance by school intimated that there was intractable evidence for both a low ability population and ineffective teaching of mathematics in the junior high schools on the island of New Providence in The Bahamas. Thus the first part of the hypothesis which related to the state of mathematics teaching and learning in these schools and enunciated at the end of Chapter 2, ie:

The Bahamian secondary pupil in the state-maintained sector of education has not acquired as full an understanding of the basic mathematics concepts as a prerequisite to enjoyment, progress and success at further studies in mathematics, and in particular for success in:

- Bahamas Junior Certificate Mathematics (BJC)
- (Appendix 5.2)

has now been rigorously proved. Taken together, the incidence of low ability and ineffective teaching constitute the basis of the suggestion that curriculum development should reflect 'a back to basics' philosophy, and that in-service teachers should be trained through writing material, editing it in groups, conducting trial-runs, and validation should be informed by fresh feedback from the classroom. The writer holds the view that curriculum content should be developed and up-dated constantly from the needs that are dictated by the demands of the classroom. But the proof of the hypothesis above should not imply that the teaching of the mathematics curriculum must be geared exclusively to training pupils to pass BJC mathematics, important though that is. Rather, pupils should have an intuitive and inductive introduction to mathematics concepts in a concrete, manipulative and informal atmosphere so as to encourage the helical integration of more complex concepts of
mathematical structure into cognitive mental structures. This condition is necessary and sufficient to ensure rich and varied mental exercises in the concrete operational stage of learning mathematics for smooth and easy transition to a fruitful plateau of formal operations, thus opening the possibility for easy retrieval of fundamental knowledge required to nurture the growth of relational understanding in the discipline of mathematics.
CHAPTER 6

FIELD SURVEY RESULTS:
SENIOR HIGH SCHOOLS
6.1 Pilot Visits to Schools, Administration of Tests and Marking of Scripts

6.1.1. Pilot Visits to Schools

The initial move in a chain of exercises, deliberately planned to result in curriculum development, was to investigate the degree of success which was being achieved in the transmission of the specific objectives of the mathematics curriculum. With this ultimate aim to pursue, the researcher approached Principals of the five state-maintained and three of the independent senior high schools and explained the purpose of a field survey in mathematics education and its value to The Bahamas. The result was a warm reception together with willing help and cooperation. The author was put in touch with Heads of Mathematics Departments who, after satisfying themselves with the content of the test paper, suggested possible dates, to which the author agreed willingly, on which the test could be held.

On these pilot visits prior to the test, it was found that classrooms had adequate space and buildings were in good repair (except in one school where extensive renovations were in progress to correct damage caused by wanton vandalism and in another where a few ceiling tiles were missing). Timetabling was so badly interrupted in the former school that a shift system had to be brought into effect to ensure that all children got a certain minimum of classes per week. In most cases, classrooms could have had more display space for pictures, charts, diagrams, teachers' visual aids and children's work. In every school chalkboards were fixed to classroom walls but often they were too short. There was also a need for bench space for teachers of mathematics to demonstrate model making and its uses. It was obvious that the design of classrooms did not include provision for practical mathematics.

It could be seen that methods of teaching mathematics and the organisation of learning as well as teacher qualifications were not uniform across the sample. The same was true of textbooks.
Further, the time spent on the teaching of mathematics varied from 2.9 hours to 4.0 hours per week in three independent schools in the sample as against 2.7 to 4.7 hours per week in the government schools. This meant that of the total amount of teaching time per week, mathematics was given anywhere from 12.4 per cent to 17.2 per cent of the total timetable allocation in the independent sector as opposed to 11.6 per cent to 20.2 per cent in the maintained schools. In the United Kingdom, it varied from 12.5 per cent to 15.0 per cent at the time of this survey in The Bahamas [133, p.17]. Discussing this matter, Cockcroft [50, p.149] believes that teaching mathematics for somewhere between one-eighth and one-seventh of the teaching week is adequate provided it is supplemented by homework assignments, but warns that the distribution of teaching time for mathematics should be carefully allocated in the timetable. In fact, this assumes more significance in subtropical countries. In Queen's College, for example, homework was set for Grade 11 on two evenings per week and each pupil was expected to spend 40 minutes per evening on this task. These assignments were usually carried out well, with parents showing interest and ensuring that the work was done. At the other extreme, however, home assignments were not treated with the same seriousness by pupils of the maintained schools. Bajpai and Bajah [5, p.20, paragraph 59] support this finding in their unpublished report where they say:

We are given the impression that children, particularly in government schools, did not treat self-study and self-discipline for doing homework as something considered "respectable". Their attitudes in the main were of laziness and everything being achieved by least effort and sacrifice.

Another vital determinant of the quality of mathematics education at the time of the survey was the teacher-pupil ratio. For the maintained senior high schools the mean was 1:35 but for the independent schools it was 1:20. In England over the years 1975-78 the full range comprehensive schools at a similar level had 1:28 [49, p.147]. Whereas in Queen's College, an independent school, in the first 3 years (Grades 7-9) the classes were organised as mixed ability groups and
within this organisation the pupils were arranged in 'sets' according to ability for English (language and literature), mathematics and French, in the state-maintained high schools a policy of streaming existed throughout the system. Teachers' loads varied because some of them helped in the teaching of accounts and general studies but normally a mathematics teacher's load was 31 periods (77.5 per cent of the full load) per week leaving 9 periods for preparation and marking. In this matter, the researcher did not find a clear policy in the government schools and therefore got the impression that not enough time was allocated for preparation of classes and marking of children's written work. With regard to schemes of work, these were usually very well done and seen regularly by Heads of Mathematics Departments. But the researcher was most favourably impressed by the scheme of work prepared by the Head of Mathematics at Prince Williams High School (independent). It was prepared for the 38 weeks of the academic year ending on 18 June 1982, whilst in other schools they were planned on a short-term basis. The writer conducted in Loughborough mathematics tutorials for first-year Chemical Engineering students to a year's scheme of work which was prepared by the lecturer. But, in a developing country, to have found a year's scheme of work laid out was a classic example of responsible leadership and complete commitment to the task of ensuring that the mathematics education, such as it was, of those whose future lay in the hands of the Head of Department, was taken very seriously indeed.

New Comprehensive Mathematics by Greer was used as a textbook by two government and one independent senior high schools for teaching the subject to 'O' level of London University. Two of those schools - one government and the other independent - were following syllabus A and the other was pursuing an amalgam of syllabuses B and D. At another independent school, Mathematics: An Integrated Approach by McMillan and the School Mathematics Project Books X, Y and Z were being used with Grade 11. It was found also that New General Mathematics books 1 - 4 by Channon, McLeish Smith and Head were used by two independent and two government schools. Used less frequently was 'O' Level
Mathematics by Clarke and Discovering Mathematics by Shaw and Wright. The author saw copies of these books which the teachers used in the maintained schools but was not told what the pupils used. In two independent schools however, the author saw enough copies of mathematics textbooks for each child to have one. This finding was also in harmony with what Bajpai and Bajah [5, p.28, paragraph 111] found in 1978:

In connection with text books, these are often not related to the local environment, there is often a lack of synchronisation between the curriculum and the contents of the books and there is a lack of sequential textbooks, particularly in mathematics.

6.1.2 Administration of Test

On the day of the test, the researcher arrived at each school in good time, with enough question books and foolscap paper, so that the room for the test could be prepared before the arrival of the candidates. Seating was adequate and well-spaced, rooms were well ventilated, and everyone listened carefully to instructions before writing began. But there were four answer scripts on which the space labelled 'date of birth' was not completed. At A.F. Adderley and Queen's College where the subsamples were larger than usual, teachers volunteered to help distribute and collect papers as well as invigilate. Directly after each sitting, the researcher had to number the answer scripts so as to avoid mistaking the schools from which they came and the Head of Department was given a copy of the question paper.

Concerning the attitude of the pupils to the test, everyone seemed enthusiastic and willing to work but, as was discovered later, this will to work was not usually matched by command of the curriculum skills and knowledge required by the questions. The fact that movement from one class to another in the government schools was independent of attainment partially explains the empty show of anxiety to work. However, as shown by the following extract from the Minister's speech [134] at
the first national conference of teachers, promotion on merit was to be substituted for automatic advance from one grade to the one above.

NASSAU GUARDIAN, Back to School Supplement, August 1981-11
Ringing in change ...

I can, as a layman, expect to understand and perhaps be more satisfied, that a much improved system of moving pupils through the school system will emerge. I can understand that it is perhaps best for most children to progress from grade to grade and from school to school within his age group. But I cannot be satisfied that this reason alone is good enough when children progress from grade to grade without any regard to whether or not they have mastered that level of work at that grade. To allow such a system to continue would not only be unfair to many Bahamian children, but a perversion of the true meaning of social promotion. I, therefore, now require and direct all supervisory staff and principals to institute systems to ensure that all pupils receive adequate instruction in basic knowledge and skills at every grade level, and that a satisfactory level of pupil-achievement be an important consideration in pupil progress from grade to grade. No student should be allowed to go forward to the next grade until he has mastered the present grade. Where in any school this policy causes overcrowding or other problems beyond the control of the principal and his staff, he should consult the Ministry for guidance ...

We should be aware that the world's most renowned educational institutions are not mere examination factories. They feel a positive responsibility to establish and sustain success. The failure of any student, especially in what is basic to earning a living and ensure survivals is, in this sense, the failure of those who had the responsibility to teach and to arrange his instruction.
6.1.3 **Marking of Scripts**

The papers were brought back to Loughborough for marking which was done a second time to adjust the marks where the author was either too lenient or too severe. They even had to be scanned a third time to identify the stages at which errors occurred in the thinking and calculations, and the more commonly occurring and the unusual instances of slips were selected to appear in the analysis later in this chapter. When the author was satisfied that the marks were allocated to all scripts fairly, the sample report sheets in Appendix 6.1 were completed and sent to each senior high school along with a letter of thanks, a specimen of which is attached in the same Appendix.
6.2 Introduction to Data Analysis

The key purpose of this survey was to document the mathematics achievement of a representative sample of senior high pupils in state-maintained and independent schools on the island of New Providence in The Bahamas using data collected in the autumn, 1981. The data presented indicate the extent to which the performance objectives of the syllabus for Grade 10 (age 14+-15) were transmitted to the pupils but since the researcher could not be in The Bahamas for the end of the school year in June, the test was administered in September at the beginning of Grade 11. The disadvantages to the pupils were that they were only just settling down to a new environment; they had very little, if any, revision of the previous year's work; and probably did not know what to study for the test.

The following table shows the schools which volunteered to participate in the survey, the date on which their candidates offered the test, the identification of the scripts and the size of the subsample for each school together with the total survey population.

<table>
<thead>
<tr>
<th>DATE</th>
<th>SCHOOL</th>
<th>SCRIPTS</th>
<th>NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9.81</td>
<td>Government High School, 1GHS(G)</td>
<td>1-32</td>
<td>32</td>
</tr>
<tr>
<td>10.9.81</td>
<td>C. C. Sweeting, 2 CCS(G)</td>
<td>33-50</td>
<td>18</td>
</tr>
<tr>
<td>14.9.81</td>
<td>A F Adderley, 3 AFA(G)</td>
<td>51-101</td>
<td>51</td>
</tr>
<tr>
<td>14.9.81</td>
<td>Queen's College, 4 QC(I)</td>
<td>102-142</td>
<td>41</td>
</tr>
<tr>
<td>16.9.81</td>
<td>L.W. Young, 5 LWY(G)</td>
<td>143-167</td>
<td>25</td>
</tr>
<tr>
<td>21.9.81</td>
<td>Prince Williams High, 6PWH(I)</td>
<td>168-185</td>
<td>18</td>
</tr>
<tr>
<td>22.9.81</td>
<td>R.M.Bailey, 7 RMB(G)</td>
<td>186-214</td>
<td>29</td>
</tr>
<tr>
<td>23.9.81</td>
<td>St. John's College, 8 STJC(I)</td>
<td>215-241</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
<td></td>
<td>241</td>
</tr>
</tbody>
</table>

Table 6.1 Details of Senior High Survey Population
It was a deliberate decision to exclude the names of pupils. Instead, each script was assigned a number. These numbers, sex of pupils, date of birth, marks for each question and total marks for the test were transferred to fortran statement forms, two of which appear in Appendix 6.2. A computer data card was made for each pupil. From these cards, the data were analysed on the computer and all of the necessary statistical information was copied from print out sheets. In order to economise on space, each school was assigned a number, its initial letters and (G) or (I) to indicate whether it was government (state-maintained) or independent.

Each question was assigned 5 marks, thus making it possible for each pupil to score 50 marks on a choice of 10 out of 14 questions. Marks were awarded in eleven categories from 0.0 going up by increments of 0.5 to the maximum for the question. The questions were numbered from 001 to 014 and on the fortran statement and computer cards a mark of 9.0 means that the question was omitted. Much more statistical information was provided by the computer than was necessary for this analysis but the most important criteria of the analysis were the mean achievement of the whole field class, the mean mark on each question and the mean performance of each school.
6.3 Performance of Field Class on All Questions

Table 6.2 sets out the mean values for the New Providence sample over the variable Mathematics Total, measuring the total score on the Diagnostic Test of 14 questions which permitted a choice of any ten questions.

<table>
<thead>
<tr>
<th>Sex</th>
<th>Total</th>
<th>Mean</th>
<th>SD</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>B and G</td>
<td>3597.5</td>
<td>14.9</td>
<td>12.8</td>
<td>241</td>
</tr>
<tr>
<td>B</td>
<td>1791.5</td>
<td>15.2</td>
<td>13.2</td>
<td>118</td>
</tr>
<tr>
<td>G</td>
<td>1806.0</td>
<td>14.7</td>
<td>12.5</td>
<td>123</td>
</tr>
</tbody>
</table>

Table 6.2 Total Mark Broken Down by Sex

The sample population of 241 was almost equally divided between boys (49 per cent) and girls (51 per cent) from all government senior high schools and three similar independent institutions. Care was taken to ensure that the subsample from each school was greater than or at least equal to 10 per cent of the entire year group [135, pp.230-235]. In fact, an independent school and a government school offered substantially more than this percentage of their respective year groups. This lower limit on sample population size had to be set in order to state conclusions about the whole senior high population which have a good chance of lying within reasonable limits of reliability.

Had it been available, the statistic showing the total Grade 11 population by sex across all senior high schools in the survey, when the test was administered, would have fortified a base for more securely optimistic deductive reasoning concerning the ability of the sample. But it was helpful - provided the statistics were correct - to know that the corresponding total population of 15 year olds for the senior high schools under study was 2230 in 1977 [136, pp.21-22]. However, in the absence of the relevant total population, the researcher found that the adult male - female ratio of 1:1.06 obtained in the census of population taken in The Bahamas during 1980 [3] . For this
sample of senior high school pupils the boy-girl ratio was 1: 1.04, indicating that for every 100 boys there were 104 girls. So, although extrapolations about the total adult population must be made cautiously, yet inferences from these test results must fall well within the limits of reliability.

Over the period 7-23 September 1981, the youngest pupils to offer the test were a boy and a girl who were each age 14 years 3 months, and the oldest was a girl of 18 years 8 months. From the information collected, the mean age of all but four of the sample was 15 years 6 months. The youngest boy and girl scored 37 out of 50 and 17.5 out of 50 respectively whilst the oldest pupil scored 24 out of 50. On further examination, not only must it be observed that the youngest boy's score was more than double the mean of all the boys' scores in the entire sample and that the oldest pupil's score was slightly less than double the girls' mean score in the entire sample; but also, these pupils are from independent schools. Furthermore, the youngest girls' score of 17.5 out of 50 was only marginally above the mean score achieved by the girls in the whole sample population and she attends a government school. An inference from a comparison of the mean scores of pupils from independent schools would be made later in this chapter.

From Table 6.1 the mean performance of the boys was only just better than the girls; their ability, as was indicated by a higher standard deviation, was spread over a wider band of the continuum; and their mean score was better than the mean of the whole sample by 0.25, which was itself spread over a narrower range of the continuum of scores as was indicated by the standard deviation of 12.8. Also, since the boys have a lower total score than the girls but a higher mean, it would appear that their attainment is marginally higher than the girls. In a psychological model which links academic choice to expectations of success and the subjective value of a particular course, Meece and Parsons [137] partially explain
the difference in achievement in mathematics by boys and girls but research has to be done in the Bahamas to decide the relevance of the assumptions and conclusions.

The graph of the cumulative frequency distribution curve (Figure 6.1) shows that it is displaced to the left with 10.8 per cent (26) of the candidates scoring no marks and 78 per cent (188) scoring one half or less of the marks. Appendix 6.3 gives the data from which the ogive was constructed: and was also the source for determining the numbers of candidates at the various grade levels. According to Morrison [127] it was expected that the marks of 241 pupils would be distributed so as to correspond very nearly to the following scheme:

<table>
<thead>
<tr>
<th>Grade</th>
<th>N</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>C</td>
<td>121</td>
<td>50</td>
</tr>
<tr>
<td>D</td>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>E</td>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.3 Expected Percentage of Sample in Each Grade

Instead, the researcher found, as shown by the ogive, the following valid observations:

<table>
<thead>
<tr>
<th>Mark range</th>
<th>Grade</th>
<th>N</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 and above</td>
<td>A</td>
<td>11</td>
<td>4.6</td>
</tr>
<tr>
<td>24-39</td>
<td>B</td>
<td>53</td>
<td>22.0</td>
</tr>
<tr>
<td>3-23</td>
<td>C</td>
<td>118</td>
<td>49.0</td>
</tr>
<tr>
<td>2 and below</td>
<td>D/E</td>
<td>59</td>
<td>24.4</td>
</tr>
</tbody>
</table>

Table 6.4 Valid Percentage of Sample in Each Grade
The most obvious observations are that there was no D/E boundary; the C mark range cut off at far too low a point even though the number and percentage of pupils in the range was around what should be expected; similarly, the lower boundary of the B range was much too low, but the A range was satisfactory.

In the interest of standards, the researcher has taken the enterprising decision to modify the mark range/grade table to the following:

<table>
<thead>
<tr>
<th>Mark Range</th>
<th>Grade</th>
<th>N</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 and above</td>
<td>A</td>
<td>11</td>
<td>4.6</td>
</tr>
<tr>
<td>30-39</td>
<td>B</td>
<td>24</td>
<td>9.9</td>
</tr>
<tr>
<td>20-29</td>
<td>C</td>
<td>48</td>
<td>19.9</td>
</tr>
<tr>
<td>19 and below</td>
<td>E</td>
<td>158</td>
<td>65.6</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td>241</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 6.5 Modified Mark Range/Grade Distribution

Cockcroft [50, p.13], having referred to the official pass mark that an average ability 16 year old pupil would have to achieve in order to be awarded a Grade 4 Certificate of Secondary Education in mathematics, says

> We cannot believe that it can in any way be educationally desirable that a pupil of average ability should, for the purpose of obtaining a school-leaving certificate, be required to attempt an examination paper on which he is able to obtain only about one-third of the possible marks ...

In any public examination, when more than 30 per cent of the examinees finish with an E grade a serious breach of policy is committed and the examination has to be scrutinised as well as the marking. However, when the ability of the candidates is low, as this is a case in point, there is no
teaching and efficiency of the learning which take place at tremendous public expense. In fact, the total entry from three government schools scored E's and these were, presumably, the best pupils in those schools.

With regard to the mean ability of the pupils offered by the schools, the Anglican independent school scored a mean of 31.3 out of a possible 50 marks, with a standard deviation of 8.8; the Methodist independent school scored a mean of 26.1 with a spread of 11.1; and the Baptist independent school scored a mean of 24.2 with a spread of 8.4. Of the government schools, the highest mean was 19.3 with a spread of 5.9.

The mean ability indices of individual schools gave the gross structure of the attainment of the sample 15 year olds on the island of New Providence. This picture, as reported by Bajpai and Bajah [5], was one which depicts the independent schools in high relief, towering over the comprehensive schools which are barely visible in the shadows of the background. Any extrapolation with reference to the ability of the entire population of 15 year olds on the island of New Providence would be highly reliable because every government senior high school was well represented in the sample.

Attention is now turned to the performance of the individual questions per school during the field survey.
6.4 DATA ANALYSIS

The purpose of the Tables 6.7 - 6.49 which follow is to display, for study, the mean total mark per school, the linear spread of the individual total marks about the mean, and the size of the subsample, broken down by sex and percentages, who offered each question. From each school, the size of the class or classes which participated in the survey is also given. When required, the mean mark per question and the standard error associated with each mean was obtained by the writer. It is most important to state that the overall mean mark for any question cannot be deduced by taking the mean of the means of all of the schools. That is a separate calculation, the details of which were worked out by the researcher. Tables also show the application of marking schemes and identification of levels of behavioural errors.

The fundamental pillar upon which the analysis was based were the mean marks because they are crucial, in the absence of a more precise quantifier of the attainment of the pupils. The overall mean total mark for the whole subsample offering any question was converted to a percentage and called the Mean Ability Index, $M_T$; similarly, the mean percentage mark on a question was designated $M_Q$. These two variables $M_T$ and $M_Q$ were plotted on independent and dependent axes respectively to give, as Morrison [138, pp 5-9] in his paper demonstrates, a Question Synoptic Chart from which the Facility Index for each question was extrapolated. Another very important parameter of the performance of any question was the Discrimination Index, $D$ which was found by calculating the correlation between the mean mark on the question and the mean total mark for those who attempted the question, using the formula for the Pearson Product - Moment Coefficient of Correlation.

Finally, to complete the Question Analysis Table (Table 6.7), the Choice Index per question was the cardinality of the subsample expressed as a percentage of the total sample survey population.

Although the question booklet is located in Appendix 6.4, each question is written separately as a part of the analysis, showing the behaviours tested and the marking scheme used.
Perusal of the test scripts from each school provided the source of behavioural errors and comments are made on the cognitive responses of the pupils along with a source of similar examples not only for routine demonstration, discussion and practice classwork, but more importantly, for discussing children's thinking, as reflected in the errors they commit, with teacher trainees.

6.4.1 Performance of Individual Questions by Schools

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>MEAN</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GHS(G)</td>
<td>32</td>
<td>B 15 G 17</td>
<td>46.9 53.1</td>
<td>19.3</td>
<td>6.0</td>
</tr>
<tr>
<td>2 CCS(G)</td>
<td>18</td>
<td>6 8</td>
<td>42.9 57.1</td>
<td>5.9</td>
<td>5.3</td>
</tr>
<tr>
<td>3 AFA(G)</td>
<td>51</td>
<td>15 16</td>
<td>48.4 51.6</td>
<td>5.2</td>
<td>5.7</td>
</tr>
<tr>
<td>4 QC(I)</td>
<td>41</td>
<td>20 17</td>
<td>54.1 45.9</td>
<td>26.7</td>
<td>10.9</td>
</tr>
<tr>
<td>5 LWY(G)</td>
<td>25</td>
<td>8 13</td>
<td>38.1 61.9</td>
<td>3.7</td>
<td>3.8</td>
</tr>
<tr>
<td>6 PWH(I)</td>
<td>18</td>
<td>6 11</td>
<td>35.3 64.7</td>
<td>24.4</td>
<td>8.9</td>
</tr>
<tr>
<td>7 RMB(G)</td>
<td>29</td>
<td>7 17</td>
<td>29.2 70.8</td>
<td>9.1</td>
<td>6.5</td>
</tr>
<tr>
<td>8 STJC(I)</td>
<td>27</td>
<td>17 7</td>
<td>70.8 29.2</td>
<td>32.4</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Table 6.7 Mean Achievement on Question 1.

As shown by Table 6.7 the mean competence of the pupils from every comprehensive school but one, School 1, was very weak. Schools 8, 4 and 6 scored means of 32.4, 26.7 and 24.4 out of 50 respectively. School 4 offered the largest sample, which showed that the linear scatter of their marks about the mean performance was larger than that for any other school. The overall mean competence of the 200 pupils who attempted the question was brought down to 16.7, with 12.8 units of linear spread of scores about the mean, presumably by the very low attainment of the government schools. 83 per cent of the entire senior high field class who attempted this question achieved a mean score of 2.5 out of 5 marks with 2 units of
linear spread about this mean. The range of marks on the question, as shown by Appendix 6.5, was 5. (A sample is appended)

Of those who scored full credit on this question 31 (54.4 per cent) were boys and 26 (45.6 per cent) were girls, who together constituted 28.5 per cent of those who attempted the question. 143 pupils had at least one mistake in their solutions. These errors can be classified under one of the following headings:

(i) inability to carry out arithmetic operations on pronumerals
(ii) unfamiliarity with removal of brackets
(iii) very weak knowledge of the arithmetic of directed numbers.

The question booklet is attached in Appendix 6.4. Question 1 is reproduced here with the distribution of the marks corresponding to the various expected behaviours (Table 6.8).

Q.1 Simplify $2(x + 3y - z) - 3(x + y - z) + 4(x - y)$

Solution: $2x + 6y - 2z - 3x - 3y + 3z + 4x - 4y$

$= 3x - y + z$

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x + 6y - 2z$</td>
<td>1</td>
</tr>
<tr>
<td>Multiplication and removal of first brackets</td>
<td></td>
</tr>
<tr>
<td>$-3x - 3y + 3z$</td>
<td>2</td>
</tr>
<tr>
<td>Multiplication and removal of second brackets</td>
<td></td>
</tr>
<tr>
<td>$4x - 4y$</td>
<td>1</td>
</tr>
<tr>
<td>Multiplication and removal of third brackets</td>
<td></td>
</tr>
<tr>
<td>Addition of terms: $3x - y + z$</td>
<td>1</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>5</strong></td>
</tr>
</tbody>
</table>

Table 6.8 Allocation of Marks to Expected Behaviours
Of the 200 (83% of the sample) who attempted Question 1, 143 (71.5%) need exercises to give confidence in the expected behaviours but the weakest behaviour, manipulation of directed numbers, occurred on 130 scripts, 65% of the total attempts.

<table>
<thead>
<tr>
<th>SOURCE OF NO. OF BEHAVIOURAL ERRORS</th>
<th>NO. OF BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2x + 6y - 2z</td>
<td>4</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>-3x - 3y + 3z</td>
<td>6</td>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>4x - 4y</td>
<td>2</td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>3x - y + z</td>
<td>15</td>
<td>13</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>45</td>
<td>91</td>
</tr>
</tbody>
</table>

Table 6.9  Percentage of Errors per Behaviour.

For this question the important cognitive responses were reading ability, comprehension and execution of the arithmetic operations as prerequisites to writing the answer in an acceptable form. Where the process skills required the use of multiplication facts in combination with removal of brackets, 20 per cent of the total amount of behavioural errors occurred. Since very often the operation of multiplication was not distributed over the second and third pronominals when the brackets were removed, it appeared that the existence of the brackets interfered with the execution of this arithmetical skill. At the stage in the execution of the process skills where, in addition to the behaviours mentioned above (Table 6.9), a negative sign was in front of the brackets, the percentage of the total errors increased significantly by 6 per cent to 26 per cent. Finally, faced with the simplifying processes of the arithmetic of directed number coupled with the abstraction of algebra just prior to encoding the result, 34 per cent of the total errors occurred. Appropriate exercises for discussion
with teacher trainees are to be found in Wain and Woodrow [54, pp 41-45].

Despite the slips, the respondents who earned top marks almost balance those at the other extreme of the eleven marks-catchment points of the range 0.0 to 5.0. At mark points 1.0, 2.0, 3.0, 4.0 as shown on the cross-tabulation of variable 001 by sex 7.5, 9.5 and 12.0 per cent of the respondents were attracted, thus showing a pretty good spread of the ability of those who answered the question. The discrimination index 0.72 is a good measure of the power with which the question separated the pupils into various ability bands.

The extent of the pupils' knowledge of the behavioural skills required to answer this question is revealed by the mean percentage marks for girls and boys which were 48.5 per cent and 53.5 per cent giving an overall mean percentage mark $M_Q = 51$ per cent. The other critical measure of attainment was the mean of the totals, $M_T = 33$ per cent, of all who attempted to answer this question.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>SCHOOL MEAN</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GHS(G)</td>
<td>32</td>
<td>B 6 G 4</td>
<td>B 60 G 40</td>
<td>17.7 6.3</td>
<td></td>
</tr>
<tr>
<td>2 CCS(G)</td>
<td>18</td>
<td>5 3</td>
<td>62.5 37.5</td>
<td>4.6 5.5</td>
<td></td>
</tr>
<tr>
<td>3 AFA(G)</td>
<td>51</td>
<td>8 2</td>
<td>80 20</td>
<td>4.1 4.9</td>
<td></td>
</tr>
<tr>
<td>4 QC(I)</td>
<td>41</td>
<td>2 4</td>
<td>33.3 66.7</td>
<td>30.3 10.3</td>
<td></td>
</tr>
<tr>
<td>5 LWY(G)</td>
<td>25</td>
<td>1</td>
<td>100</td>
<td>10.0 0.0</td>
<td></td>
</tr>
<tr>
<td>6 PWH(I)</td>
<td>18</td>
<td>2 2</td>
<td>50 50</td>
<td>25.5 1.9</td>
<td></td>
</tr>
<tr>
<td>7 RMB(G)</td>
<td>29</td>
<td>2 1</td>
<td>66.7 33.3</td>
<td>6.3 4.9</td>
<td></td>
</tr>
<tr>
<td>8 STJC(I)</td>
<td>27</td>
<td>18 6</td>
<td>75 25</td>
<td>31.5 9.4</td>
<td></td>
</tr>
<tr>
<td><strong>241</strong></td>
<td><strong>44</strong></td>
<td><strong>22</strong></td>
<td><strong>66.7</strong></td>
<td><strong>33.3</strong></td>
<td><strong>20.0</strong></td>
</tr>
</tbody>
</table>

Table 6.10 Mean Achievement on Question 2.

This was the most unpopular of the fourteen test questions, being offered by only 66 pupils (27 per cent) of the field class. But the mean performance was 20.0 out of 50 and the linear spread...
of scores about it was 13.9 standard deviation units. Schools 8, 4 and 6 respectively offered candidates of higher ability than the maintained schools. The linear spread of the marks (23.5, 24.5, 26.0 and 28.0, each out of 50) scored by the respondents from School 6 was 1.9 standard deviation units about their mean performance, 25.5 marks. Twice as many boys as girls attempted the question. From the detailed accomplishments of the field class, almost 1 in 3 of those attempting the question earned full marks. The fact that 76.2 per cent of these were boys and 23.8 per cent were girls was a noteworthy statistic since the curriculum skills investigated required the respondents to demonstrate their ability to use compasses, rulers and protractors.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>B</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GHS</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>2 AFA</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>4 QC</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>6 PWH</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8 STJC</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>TOTAL</td>
<td>16</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.11 School and Sex of Candidate Scoring Full Marks on Question 2

Perusal of the answer scripts showed that of the 24 respondents (18 boys, 6 girls) to the question from School 8, 16 of them (12 boys, 4 girls) earned full marks (Table 6.10). From the data obtained, a reasonably good fan out of the marks to the six distribution points was observed, thus explaining the discrimination index of 0.55.

The mean mark achieved by the boys was 2.9 out of 5.0 (58.0 per cent) and that of the girls was 2.3 out of 5.0 (46.0 per cent). Together these gave a mean percentage mark $M_Q = 54.0$ per cent for this question. However the mean ability indicator $M_T = 40$ per cent, with a linear spread of 27.8 per cent,
indicated the general level of achievement of curriculum skills in this mathematics content area.

Attention is now directed to the structure of the question, a plane geometrical figure (a parallelogram) representing the correct solution, the marks allocated to the expected behaviours in the solution, and an indication of the occurrence of errors per behaviour as they appeared from reading the answer scripts very carefully.

**Question 2 (See Appendix 6.4)**

Using ruler and compasses only, construct a parallelogram with sides 6 cm and 9 cm, the angle between these sides being 60°. Measure, in centimetres (cm), the length of any ONE of the diagonals.

![Drawn to Specification in Question 2.](image)

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment PQ = 9 cm</td>
<td>1</td>
</tr>
<tr>
<td>Segment PS = 6 cm</td>
<td>1</td>
</tr>
<tr>
<td>From P, Q two pairs of intersecting arcs of same radii, QPS = 60°</td>
<td>1</td>
</tr>
<tr>
<td>Measure PR = 13.1 ± 0.1 cm OR QS = 7.9 cm ± 0.1 cm</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
</tr>
</tbody>
</table>

Table 6.12 Allocation of Marks to Expected Behaviours
The answer scripts showed that there were pupils who did not know how to begin their attempts at solution; there were those who used no construction lines as well as those who had too many. Cases were also in evidence where construction lines did not demarcate either an angle of $60^\circ$ or parallel lines, and scripts where the radii of construction arcs were too short. Furthermore, some scripts indicated that too much pressure had been put on blunt pencil points which circumscribed arcs and drew straight line segments some of which, at the other extreme, were unclear.

<table>
<thead>
<tr>
<th>SOURCE OF BEHAVIOURAL ERRORS</th>
<th>BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line segment PQ=9cm</td>
<td>3 6 4 1 1 1 1 17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>PS=6cm</td>
<td>4 6 4 1 1 1 17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Angle QPS=$60^\circ + 1^\circ$;</td>
<td>intersecting arcs of equal radii</td>
<td>4 8 6 5 - 2 2</td>
<td>27 26</td>
</tr>
<tr>
<td>PR = $13.1 \pm 0.1$ cm</td>
<td>9 8 7 3 1 2 2 8</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>or QS = $7.9 \pm 0.1$ cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 28 21 9 2 4 6 11 101</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.13 Percentage of Errors per Behaviour

As indicated in Table 6.13 straight line segments 6 centimeters and 9 centimeters long each accounted for 17 per cent of the inaccuracies. In an extreme case, these segments were opposite sides of a rectangle; in another instance they formed two sides of a triangle with the third side of length 8 centimetres; yet again they appeared as segments of lengths 5.9 centimetres and 8.9 centimetres drawn along lines on the answer paper and intersected by a transversal 9 centimetres long.
26 per cent of the total mistakes were committed in the construction of an angle of $60^\circ$ including four scripts where the size of the angle was $58^\circ$, $62^\circ$, $63^\circ$ and even $66^\circ$ on each.

In response to the request to measure the length of one of the diagonals, an offer was as short as 6 centimetres whereas at the other extreme many parallelograms were drawn, including some which were correct, but had neither diagonal drawn nor measured. The errors at this part of the construction amounted to 40 per cent of the total. (See Table 6.13).

The execution of the process skills indicated that reading and comprehension of the language of the problem presented no great challenge. However, the high percentage of pupils whose attempts had neither of the diagonals may indicate that they needed practice in representing a diagonal on a geometrical plane figure. The choice index of this question was 27 per cent - the most unpopular question in fact - clearly indicating that more pupils urgently need motivation towards geometry. But despite the low popularity index, the performance on the field trial gave a facility$^+$ of 64 per cent thus showing that it was of the right level of difficulty for this ability range.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGE</th>
<th>SCHOOL MEAN</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GHS(G)</td>
<td>32</td>
<td>B 12 G 12</td>
<td>B 50 G 50</td>
<td>19.2</td>
<td>6.4</td>
</tr>
<tr>
<td>2 CCS(G)</td>
<td>18</td>
<td>5 11</td>
<td>31.3 68.7</td>
<td>6.6</td>
<td>5.5</td>
</tr>
<tr>
<td>3 AFA(G)</td>
<td>51</td>
<td>15 17</td>
<td>46.9 53.1</td>
<td>4.1</td>
<td>5.3</td>
</tr>
<tr>
<td>4 QC(I)</td>
<td>41</td>
<td>20 18</td>
<td>52.6 47.4</td>
<td>27.4</td>
<td>10.5</td>
</tr>
<tr>
<td>5 LWY(G)</td>
<td>25</td>
<td>9 10</td>
<td>47.4 52.6</td>
<td>4.1</td>
<td>3.8</td>
</tr>
<tr>
<td>6 PWH(I)</td>
<td>18</td>
<td>5 9</td>
<td>35.7 64.3</td>
<td>25.4</td>
<td>8.6</td>
</tr>
<tr>
<td>7 RMB(G)</td>
<td>29</td>
<td>8 15</td>
<td>34.8 65.2</td>
<td>9.2</td>
<td>6.5</td>
</tr>
<tr>
<td>8 STSC(I)</td>
<td>27</td>
<td>16 9</td>
<td>64 36</td>
<td>31.6</td>
<td>9.2</td>
</tr>
<tr>
<td>total</td>
<td>241</td>
<td>90 101</td>
<td>47.1 52.9</td>
<td>16.6</td>
<td>13.1</td>
</tr>
</tbody>
</table>

Table 6.14 Mean Achievement on Question 3

$^+$ See Table 6.51 on page 462
191 candidates (79 per cent of the sample) attempted Question 3 and owing to the presence of many low ability performers, the mean mark has plunged to 16.6 out of 50.0, with the standard deviation of 13.1 indicating how widely spread the attainment of the candidates was about the mean. It is worthy of note however that despite the downward trend in the overall mean performance, Schools 8, 4, 6 and 1 still hold their relative positions respectively. School 4 was still showing that its ability range was rather more widespread than the rest. School 8 was performing very well considering that its total entry was 27, and 16 boys and 9 girls have attempted the question scoring a mean of 31.6 out of 50 (Table 6.14).

The range of marks covered a span from 0.0 to 5.0, with every intermediate point on the scale represented. At the top end of the scale 13 boys and 12 girls (13.1 per cent of those who attempted the question) earned full marks. Provided that the question was not too easy - as indeed it was not since the facility was 53 per cent - this was a commendable performance considering that Morrison [127] suggests 5 per cent success as the statistical expectancy at this end of the ability range. Further, information obtained revealed that 12 boys and 8 girls (10.5 per cent of those attempting the question) scored 4.0 out of a possible 5.0 marks. At the top end, therefore, the boys have scored very well. Because of the fact that 45.5 per cent of the respondents fell into the bottom cell of the mark range and the other 54.5 per cent are well distributed among the other ten cells, a discrimination index of 0.73 was an excellent barometer of the potential of the question to sense and separate the candidates into pockets of similar aptitude.

The mean percentage mark, $M_Q$ was 35.5 per cent, the boys scoring 37.1 per cent and the girls 34.1 per cent. But the overall measure of achievement on this aspect of curriculum content by the subgroup who offered the question was given by a mean ability index, $M_T = 33.3$ per cent.

Immediately following is the structure and content of the question together with the marking scheme for the expected behaviours in Table 6.15.
Question 3 (See Appendix 6.4)

By how many is \((p-q)^2\) greater than \(p^2 - q^2\) when \(p = 3\) and \(q = -1\)?

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution: ((p-q)^2 = [3 -(-1)]^2)</td>
<td>2</td>
</tr>
<tr>
<td>(= 16)</td>
<td></td>
</tr>
<tr>
<td>(p^2 - q^2 = 3^2 -(-1)^2)</td>
<td>2</td>
</tr>
<tr>
<td>(= 8)</td>
<td></td>
</tr>
<tr>
<td>(\therefore (p-q)^2 - (p^2 - q^2) = 8)</td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.15 Allocation of Marks to Expected Behaviours

Having carefully studied the behavioural errors on the test scripts, the following categories, as indicated by Table 6.16 were identified.

<table>
<thead>
<tr>
<th>SOURCES OF BEHAVIOURAL ERRORS</th>
<th>NO. OF BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) ((p-q)^2)</td>
<td>14 3 1 4 2 7 - 2</td>
<td>33</td>
<td>21.3</td>
</tr>
<tr>
<td>(ii) (p^2 - q^2)</td>
<td>1 1 - 8 1 3 1 9</td>
<td>24</td>
<td>15.5</td>
</tr>
<tr>
<td>(i) and (ii)</td>
<td>9 12 30 3 16 2 21 5</td>
<td>98</td>
<td>63.2</td>
</tr>
<tr>
<td></td>
<td>24 16 31 15 19 12 22 16</td>
<td>155</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 6.16 Percentage of Errors per Behaviour

At the stage of substitution and calculation of the numerical values of the separate subtasks, 21.3 per cent and 15.5 per cent of the errors occurred. Similarly, where encoding the final
behaviour was wrongly done, 63.2 per cent of the total errors appeared. The diagnosis showed further that having substituted the numerical values, there were slips in the calculation as shown by the following typical examples:

1. $(3 + 1)^2 = 9 + 1$
2. $3^2 - (-1)^2 = 9 + 1$
3. $3^2 - (-1)^2 = 6 - 1$
4. $4^2 = 8$
5. $(3 + 1)^2 = 5^2$
6. $p^2 - q^2 = 9 - 2$
   \[= 7\]

Few candidates did the expansion $(p - q)(p-q)$ resulting in the loss of the middle term. Models and overhead projector transparencies do greatly assist pre-service teacher trainees and inservice teachers to appreciate how they can keep their classes alert and responsive while demonstrating the concept of multiplication of two binomials. To this end Wain and Woodrow [54, p.59] and Hope [139, pp 114-117] provide valuable assistance for teacher trainers and trainees. Furthermore, on five scripts, concluding behavioural statements such as "$(p-q)^2$ is $16-8 = 8$ times greater than $p^2 - q^2$" were used in this and other varying forms of incorrectness. In fact, on one paper the statement "It is twice as great" was used without thinking that the magnitude of the difference depended on the numerical values of $p$ and $q$. Far too many candidates cannot handle the removal of brackets when they are immediately preceded by a negative sign. A case in point is:

\[(p-q)^2 - p^2 - q^2 = 8\]

Despite the many errors in computation, the index of difficulty for the question is 53 per cent. It is therefore an ideal question for this ability range.
Table 6.17  Mean Achievement on Question 4

100 pupils, or 41.0 per cent of the sample population, chose to attempt Question 4 and achieved a mean of 19.1 out of a possible 50 marks. The standard deviation, 14.6, indicated that the ability range of the pupils was pretty widely dispersed about the mean. The overall mean performance on this question was not impressive and from Table 6.17 the reason was that there were too many pupils of very low ability offered by the maintained schools. In fact, although Schools 8, 4 and 6 continued to lead, in that order, (School 1 only just showed a satisfactory rating), the other government secondary schools continued to show that they were abysmally low in their achievement of the objectives of the mathematics curriculum.

The range of marks stretched from zero to full credit but slightly over half of the pupils failed to score. A look at the other extreme of the mark range showed that at the 4.0, 4.5 and 5.0 mark levels 12.0 per cent, 5.0 per cent and 14.0 per cent respectively of those who attempted the question scored, with equal numbers of boys and girls scoring 4 out of 5 but rather more boys than girls at the two top levels.
of the range. Further, the subsample who attempted the question were distributed over nine mark points with categories 0.5 and 3.5 missing from the strata. Also 53.0 per cent of those who attempted the question were placed in the bottom stratum of the ability range. The other 47.0 per cent were scattered in irregular fashion over the remaining eight catchment points. 12.0 per cent and 14.0 per cent of the respondents have clustered at the 4.0 and 5.0 mark points respectively. Thus, 0.68 was quite an effective measure of the sorting power of the question.

The mean percentage mark $M_Q$ was 35.0 per cent, the girls scoring 33.6 per cent and the boys 36.2 per cent. But the overall measure of ability, $M_T$, for this subsample on the question was 38.3 per cent.

Question 4, as indicated below, offered a choice between two aspects of content which are closely related and very important for further work in mathematics. The expected behaviours for each alternative, marking scheme and breakdown of errors per behaviour are also given for easy reference (Tables 6.18 and 6.19).

**Question 4** (See Appendix 6.4)

**EITHER** In the right angled triangle alongside, find the length of $y$ in centimetres (cm) to one decimal place

```
<table>
<thead>
<tr>
<th>angle</th>
<th>sine</th>
<th>cosine</th>
<th>tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>36°</td>
<td>0.5878</td>
<td>0.8090</td>
<td>0.7265</td>
</tr>
<tr>
<td>54°</td>
<td>0.8090</td>
<td>0.5878</td>
<td>1.3764</td>
</tr>
</tbody>
</table>
```

**OR** The radius of the circle alongside is 5 cm and the acute angle POQ is 60°.

Find the area, in cm², of the shaded sector POQ.

(Take $\pi = \frac{22}{7}$)
### Table 6.18 Allocation of Marks to Expected Behaviours

Perusal of the solutions offered to this question revealed mistakes at the sources indicated by Table 6.19.

### Table 6.19 Percentage of Errors per Behaviour

<table>
<thead>
<tr>
<th>SOURCE OF BEHAVIOURAL ERRORS</th>
<th>BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>EITHER</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) ( \frac{22}{7} \times 5 \times 60 ) ( \frac{360}{360} ) cm²</td>
<td>11</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>(ii) Cancelling to get ( \frac{275}{21} ) cm²</td>
<td>2</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>(iii) ( \frac{275}{21} ).cm²</td>
<td>3</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td>OR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) ( y = 10 \cos 36° ) ( y = 10 \sin 54° )</td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>(ii) ( y = 10 \times 0.809 )</td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>(iii) ( y = 8.1 )</td>
<td>1</td>
<td>-</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td>21</td>
</tr>
</tbody>
</table>
Although one alternative of the question asked for the area of the sector, candidates, because they were unsure of which of either $\pi r^2$ or $2\pi r$ would give the correct answer, mistakenly remembered which to choose. Even when $\pi r^2$ was chosen, 73.3 per cent of the mistakes concerned the substitution of the value of $r$ instead of $r^2$ and omitting to multiply by $\frac{60}{360}$.

Whilst it is true that every content area of the syllabus will not find ready examples of application, no pupil who attempted this question used

$$\text{Area of sector } = \pi \times \frac{d^2}{4} \times \frac{60}{360}$$

which would most likely be used by an engineer, after having measured the inside or outside diameter of a pipe say, using a pair of appropriate calipers.

At a later stage in the same calculation, those who made the correct substitutions committed 20.0 per cent of the total slips by cancelling wrongly. The remaining 6.7 per cent of the errors appeared on scripts from Schools 1 and 3 because the pupils could not perform the mechanical operation $275:21$ correctly. (Table 6.19).

The 34.4 per cent of total misunderstandings in the first or second statement of the solution of the trigonometry question indicated quite clearly that

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{and} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

were learnt instrumentally instead of by, for example, the scale factor method [140], [141] which engages similar triangles from enlargement of a right-angled triangle the hypotenuse of which is of unit length. This strategy of introducing trigonometry using similar triangles from everyday experiences attracts the intense interest and rapt attention of pupils because the researcher has used it with a group of 23 postgraduate teachers on the All India Mathematics Education in the Centre for Advancement of Mathematical Education in Technology
(AIMEC) Project. They claimed that beyond stimulating interest and generating the thoughtful responses, the style and strategy motivate respondents into a state of anxiety to use the apparatus and models in the process of learning trigonometry. The consequence was that recall of facts was from association instead of rote learning.

With a view to developing a syllabus in trigonometry for the government secondary schools in The Bahamas, the researcher got much advice and support from Professor Bajpai for testing the suitability of strategies and curriculum material with the AIMEC Project Members since they were preparing to mount in-service programmes for mathematics teachers in their own country [93, p.141]. Some lessons were tried from the syllabus shown in Appendix 6.δ, which progressed naturally to some more advanced problems in the applications of differentiation and integration [71], [70], [142] which would challenge graduates who have not kept abreast of current methods and ideas in degree mathematics. The point being made here is that the mathematics teacher at secondary level must be a practising specialist who knows the content of the subject matter at a level far beyond that at which he is required to teach. This is not stated as an opinion but as a substantiated philosophy [37], [58].

Returning to the analysis of errors in the solution of Question 4, a catalogue of the more common slips follows:

1. \( y = 10 \sin 36^\circ \)

2. \( \frac{y}{10} = \cos 8.09 \)

3. \( \frac{y}{10} = \tan 36^\circ \)

Whereas over the whole sample of respondents, 24.6 per cent of the mistakes were occasioned by pupils who could not perform the operation \( 10 \times 0.809 \) correctly, 41.0 per cent were made in approximating 8.09 to one decimal place. It is the author's experience that pre-service students, from the whole comprehensive
system of secondary education in The Bahamas, acquire no more than an instrumental understanding of this content area of the mathematics curriculum. From the test scripts it appeared that methods of teaching need to be diversified and supported by practical demonstrations as well as pupil-teacher and pupil-pupil discussions [50]. A word of warning however is that Bahamian parents expect that pupils should learn from teachers and not from other pupils. Any new strategies would, therefore, have to take into account prevailing social and cultural peculiarities. But the enterprising teacher can always adapt his teaching, with no loss in effectiveness, to these circumstances.

Finally, the 47.0 per cent facility index obtained in the field performance made it an excellent question for future trials.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>MEAN</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GHS(G)</td>
<td>32</td>
<td>B G</td>
<td>B G</td>
<td>17.5</td>
<td>5.3</td>
</tr>
<tr>
<td>2 CCS(G)</td>
<td>18</td>
<td>7 8</td>
<td>46.7 53.3</td>
<td>4.6</td>
<td>4.9</td>
</tr>
<tr>
<td>3 AFA(G)</td>
<td>51</td>
<td>21 17</td>
<td>55.3 44.7</td>
<td>3.3</td>
<td>4.4</td>
</tr>
<tr>
<td>4 QC(I)</td>
<td>41</td>
<td>18 11</td>
<td>62.1 37.9</td>
<td>24.5</td>
<td>9.9</td>
</tr>
<tr>
<td>5 LWY(G)</td>
<td>25</td>
<td>7 12</td>
<td>36.8 63.2</td>
<td>4.1</td>
<td>3.8</td>
</tr>
<tr>
<td>6 PWH(I)</td>
<td>18</td>
<td>3 5</td>
<td>37.5 62.5</td>
<td>20.7</td>
<td>9.4</td>
</tr>
<tr>
<td>7 RMB(G)</td>
<td>29</td>
<td>9 11</td>
<td>45 55</td>
<td>6.7</td>
<td>4.6</td>
</tr>
<tr>
<td>8 STJC(I)</td>
<td>27</td>
<td>9 7</td>
<td>56.2 43.7</td>
<td>30.9</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>241</td>
<td>82 83</td>
<td>49.7 50.3</td>
<td>12.9</td>
<td>12.0</td>
</tr>
</tbody>
</table>

Table 6.20 Mean Achievement on Question 5

165 pupils, 68 per cent of the senior high survey population, offered Question 5, achieving a mean competence of 12.9 out of a possible 50.0 marks, with 12.0 units of linear spread of scores about the mean. From available data, 3 girls (1.8 per cent of the subsample who attempted the question) scored 4.0 out of 5.0 possible marks; and 5 boys (3.0 per cent of the
respondents) scored full marks. The real weakness in command of the relevant curriculum skills was exposed at the bottom end of the mark scale where 141 pupils, 85.8 per cent of the respondents, scored no marks. Moreover, four catchment points on the mark range, 0.5, 1.5, 3.5 and 4.5, did not attract any respondents. The other 14.5 per cent who attempted the question were allocated to the remaining notches, giving the question a sorting power of 0.50.

The mean percentage mark for the question, $M_Q$, was 8.8 per cent. This substantiated the assertion that a large majority of the candidates were of extraordinarily low ability. This view was further supported by the fact that the girls scored a mean percentage mark of 5.5 per cent and the boys scored 12.1 per cent. The mean ability index, $M_T$, was 25.8 per cent with a linear spread of 24.0 per cent about this mean. (Table 6.20).

Following is the structure and content of the question along with the expected behaviours and a possible allocation of the marks (Table 6.21).

**Question 5 (See Appendix 6.4)**

Simplify $\frac{2}{5x+1} - \frac{1}{3x-1}$

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution $\frac{2}{5x+1} - \frac{1}{3x-1} = \frac{6x - 2 - (5x+1)}{(5x + 1)(3x -1)}$</td>
<td>1+1</td>
</tr>
<tr>
<td>$= \frac{x - 3}{(5x + 1)(3x -1)}$</td>
<td>1+1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.21 Allocation of Marks to Expected Behaviours
A scan of the answer scripts reveals that slips in the working were made at the positions indicated by the following table:

<table>
<thead>
<tr>
<th>SOURCE OF BEHAVIOURAL ERRORS</th>
<th>BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1 (\frac{2(3x-1)-(5x+1)}{(5x+1)(3x-1)})</td>
<td>15</td>
<td>15</td>
<td>38</td>
</tr>
<tr>
<td>2 (\frac{6x-2-5x-1}{(5x+1)(3x-1)})</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3 (\frac{x-3}{(5x+1)(3x-1)})</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>15</td>
<td>38</td>
</tr>
</tbody>
</table>

Table 6.22 Percentage of Errors per Behaviour

+ round-off error

This question, popular though it was, completely dashed the aspirations of 81.0 per cent of the subsample offering it at a stage where it was impossible to recover, and in their attempts amassed 89.3 per cent of the total amount of errors. Of the minority who made a promising beginning, three pupils made mistakes in the first line because they forgot the effect of a negative sign in the front of brackets \[40\]. Another pupil multiplied out the denominator, a product of two binomial factors, only to obtain another binomial \(15x+1\). Among those who circumvented behavioural errors in Stage 1 and progressed to Stage 2, 9.3 per cent of the total errors occurred because they had the numerator wrongly simplified. Included in this percentage are seven pupils who elected to commence their attempts at solution at this stage but came to grief with no hope of creating any semblence of order.
out of total chaos. At Stage 3, 1.3 per cent of the errors were a response different from the behaviour shown in Table 6.22. The most frequently occurring errors are indicated by the types below.

\[
\frac{2}{5x + 1} - \frac{1}{3x - 1} = \frac{2}{6x} - \frac{1}{2x} = \frac{1}{4x}
\]

\[
\frac{2}{5x + 1} - \frac{1}{3x - 1} = \frac{1}{2x}
\]

\[
\frac{2}{5x + 1} - \frac{1}{3x - 1} = \frac{2(3x - 1) - 1(5x + 1)}{(5x + 1)(3x - 1)} = \frac{1}{1}, \text{ after cancelling and subtracting 1 from 2.}
\]

\[
\frac{2}{5x + 1} - \frac{1}{3x - 1} = \frac{1}{2x + 2}
\]

The logic employed in these error-riddled examples is unassailable: the difficulty however, was that conceptualisation of the problem as an algebraic fraction and the procedure for solution, a part of which was to keep the denominator as a product of factors, have got lost presumably in a barrier between teacher and pupils across which communication was impenetrable. It seems to the researcher that the teacher needs an armoury of techniques, skills and knowledge in order to be aware of obstruction to communication and to employ varying methods of approach in transforming the barrier into a permeable membrane so that knowledge may diffuse across it. The frequency and seriousness of the slips in algebraic manipulation explain the low index of facility, pegged at 33 per cent, thus making the question a hard one in the field trial.
The following flow diagram details the stages in the algorithm for obtaining a correct solution to a more difficult but similar question. Accordingly Brand, Wade and Sherlock [143, pp. 41-44] suggest that a number alongside the solution should identify the corresponding behaviour on the flow diagram as shown below (Figure 6.2).

Q. 5 Solution:

\[
\frac{2}{5x + 1} - \frac{1}{3x - 1} = \frac{2(3x-1) - (5x+1)}{(5x + 1)(3x - 1)} = \frac{6x - 2 - 5x - 1}{(5x + 1)(3x - 1)} = \frac{x-3}{(5x + 1)(3x - 1)}
\]
Bring down the numerator in each case and multiply it by every other factor than the one it has as a denominator.

Multiply out the numerator of the new fraction.

Simplify the numerator.

Has numerator any factors?

Yes

Factorise the numerator.

Do any factors cancel?

Yes

Finish.

NO

Finish.

Figure 6.2 Flow Diagram Showing Stages in Simplification of Question 5 of Diagnostic Test
A mean mark of 15.1 out of 50, with individual marks having a linear spread of 12.7 about the mean, was achieved by 139 pupils, 58.0 per cent of the field class. For the first time the order of the top schools has changed to 8, 4, and 1. School 6 seemed to have had quite a number of low ability girls offering the question. The boys have scored a mean percentage mark of 21.0 per cent and the girls 14.5 per cent, thus giving an overall mean percentage mark, $M_Q = 17.8$ per cent. However, the real indicator of competence with which the respondents faced the question is gauged by the mean ability index, $M_T$, which was 30.2 per cent. (See Table 6.23 above).

At the top end of the mark distribution, as found by the author, 4 boys and 1 girl, together comprising 3.6 per cent of the total respondents, scored 4.0 marks out of a possible 5.0. Although five points on the mark scale attracted no respondents, the lowest graduation on the scale attracted 66.2 per cent of the candidates and another 18.0 per cent have earned a place just above the mid-point of the calibration. This potential of the question to sort the candidates into high, average and low scoring groups was well described by the discrimination index of 0.61.
Following is the structure and content of the question in addition to an allocation of marks to the expected behaviours. (Table 6.24).

**Question 6 (See Appendix 6.4)**

In the diagram alongside, find in degrees, the size of the angle marked 'a'.

![Diagram with angle 'a' and 96°]

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution:</td>
<td></td>
</tr>
<tr>
<td>$96^\circ + 2a = 180^\circ$</td>
<td></td>
</tr>
<tr>
<td>base $\angle^s$ of isos $\triangle$ are equal;</td>
<td></td>
</tr>
<tr>
<td>alt. $\angle^s$ are equal from // lines;</td>
<td></td>
</tr>
<tr>
<td>adj. $\angle^s$ on straight line are supplementary</td>
<td>2</td>
</tr>
<tr>
<td>$\Rightarrow 2a = 84^\circ$</td>
<td>2</td>
</tr>
<tr>
<td>$\Rightarrow a = 42^\circ$</td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.24 Allocation of Marks to Expected Behaviours

Table 6.25 identifies the points at which cognitive difficulties arose. 66 per cent of the mistakes were such that pupils gave no indication that they understood or had in their experience, the mental responses required of them after having studied the question. It was very important to identify the schools from which the scripts scoring no marks on this question have come as was done for an earlier discussion.
Table 6.25  Percentage of Errors per Behaviour

because this is a very important area of mathematical content since the study of trigonometry presupposes that the knowledge being tested here is secure. The category of pupils above have no idea of how to marshall the information given along with the arithmetical calculation in order to justify a conclusion. As instances:

(i) \[ \frac{180^\circ}{24^\circ} \times 24 \]

\[ \therefore \text{size of angle in degrees} = 12^\circ \]

(ii) \[ 96^\circ = \text{sum of 2 base angles (exterior angle property)} \]

\[ \therefore a = 96 \div 2 \]

\[ = 32^\circ \]

Far too many cannot use brackets in a way which dictates what part of the calculation was being done first. For example:

(i) \[ a = 180^\circ - 96^\circ \div 2 \]

(ii) angle 'a' = \[ 180^\circ - 96^\circ \times \frac{1}{2} \]

did not tell the reader that the operation of subtraction was the starting point. Others, though very few, used brackets
correctly but gave no geometric reasons for the algebraic statements which had arithmetic slips in the subtraction as in

\[ a = \frac{180^\circ - 96^\circ}{2} \]

\[ = \frac{88^\circ}{2} \]

\[ = 44^\circ \]

There were also cases where the given geometrical information and arithmetic operation were not sorted out in stages, thus giving rise to discontinuous thought patterns being strung together by the wrong use of the equal symbol as in

\[ \text{angle } a = 180^\circ - 96 = \frac{84}{2} = 42^\circ \]

At the other extreme of errors

\[ a = 96^\circ - 180^\circ = 84 \]

isos \( \Delta \) the base are equal

\[ a = \frac{84}{2} = 42^\circ \]

was encountered along with

\[ 96^\circ - a - 45^\circ = 39^\circ \quad a = 39^\circ \]

Admittedly no justification was given for the appearance of the 45\(^\circ\) in this last example, but it did seem that the logic behind the conclusion possibly ran thus:

"5 subtract 6 ...", then by equal addition or decomposition, the 9 resulted. This was followed by taking ... 5 from 9 if equal addition was used or 4 from 8 if decomposition was used to give 3, resulting in angle 'a' enclosing 39\(^\circ\).

In the rough work margin, pupils should have formulated the equation and thought through the solution after documenting the reasons for the validity of the equation [140]. As it happened in this solution, alternate angles being equal as a consequence of the existence of parallel lines was very rarely stated.
although an unjustified mental assumption (at best a guess) seemed to have occasioned equality between these angles. As the crosstabulation done by the author led him to believe that the majority of the pupils who earned reasonable marks, namely 2 out of 5 or 3 out of 5, got them after the first two lines of the expected behaviours if they were coupled with the geometrical reasons for initiating those mental responses. But even so, after the first two lines of expected behaviour a further almost 34.0 per cent of the total amount of errors appeared. (See Table 6.25).

The effect of all of the shortcomings in the attempts at solution was to justifiably give the field performance of the question a facility index of 38.0 per cent, thereby classifying it as being only just too difficult for this ability range. The researcher is convinced however, that given changes in approach to teaching and opportunities for practice exercises along with discussion, many of the same subsample of pupils would cope more successfully on a subsequent field trial of the same question.

Of fundamental importance is that in-service teachers and pre-service teacher trainees should be trained to take pupils systematically through the exercises [54, p. 73] in the process of marshalling their thoughts from premise to conclusion, using deductive reasoning. Mathematical Development [60, p. 60] shows that the problems in mathematics education encountered by the developing countries are not unique.
The index of popularity of the question was 73.0 per cent since it was selected by 176 members of the field class. Although Schools 8, 4, and 6 return to the order of their positions established on five of the previous questions, School 1 has also returned to the position after School 6. The overall mean of 17.4 out of a possible 50.0 marks is explained by the very weak performances of the government senior high schools. (Table 6.26).

At the top end of the mark scale, 19.5 per cent and 20.2 per cent of the boys and girls respectively, who offered the question earned full marks. Taken together, these were 19.9 per cent of the total attempts.

It is obvious to the researcher that this question performed very well in separating the respondents and distributing them to every point on the mark scale from 0.0 to 5.0. Having fixed 31.3 per cent at the lowest end of the scale, it was expected that at least between 6.0 per cent and 7.0 per cent should have been allocated to each of the ten other points, although theoretically 50 per cent was the statistical expectancy, with 5.0 per cent at each extreme [127]. The consequent percentage distribution was very good as reflected in the discrimination index of 0.67.
The mean percentage mark scored by the girls was 46.4 per cent and that of the boys 49.5 per cent, giving an overall mean percentage mark $M_Q = 47.7$ per cent on the question. The quality of the performance in terms of the cognitive level of operation on the question exhibited by the entry was indicated by the mean ability index $M_T = 34.7$ per cent.

An examination of the structure and content of the question, together with expected behaviours and the allocation of marks per behaviour, gave a good indication of the level of performance expected of the respondents. (Table 6.27).

**Question 7** (See Appendix 6.4)

Solve the equation $4(2x - 1) - 5(x - 2) = 1$

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution: $\frac{(8x - 4) - (5x - 10)}{1}$</td>
<td>1</td>
</tr>
<tr>
<td>$8x - 5x - 4 + 10 = 1$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$3x = 1 - 6$</td>
<td>$1\frac{1}{2}$</td>
</tr>
<tr>
<td>$3x = -5$</td>
<td>1</td>
</tr>
<tr>
<td>$x = -1\frac{2}{3}$</td>
<td>1</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>5</strong></td>
</tr>
</tbody>
</table>

Table 6.27  Allocation of Marks to Expected Behaviours

The numerals 1 to 5 catalogued under Behavioural Error in the following Table 6.28 indicate behavioural levels at which slips were made in the calculation by pupils from the eight schools which participated in the survey. At level 1 each term inside the brackets should be multiplied by the factor outside.
40 per cent of the total errors occurred here, 98 per cent of which led to the award of no marks because the respondents had no idea of how to begin their solutions. At the next level, where 25.0 per cent of the errors were committed, the pupils should have removed the brackets and changed the sign inside the second set of brackets since it was preceded by a negative sign. Further in the solution, 14.0 per cent of the attempts at solution went wrong because pupils should have collected like terms and added the inverse to each side at behavioural level 3. Of those who proceeded to level 4 where 1-6 had to be added on the right hand side of the equation, 16.0 per cent of all of the mistakes were recorded. Finally 5.0 per cent of all the mistakes occurred where pupils at level 5 were expected to multiply both sides of the equation

\[ 3x = -5 \]

by the multiplicative inverse \( \frac{1}{3} \). Graham and Sargent [144, pp 98-107] detail all of the teaching points which should be explained with the aid of a model in the routine of solving simple equations in one unknown. Brissenden [112, p.67] supports by creating the whole class phase of a teacher-led discussion of an instructive example.

Having found a numerical value for \( x \), pupils could have checked that this value satisfied the original equation thus:
\[
\begin{align*}
8\left(-\frac{5}{3}\right) &- 5 \left(-\frac{5}{3}\right) + 6 \\
&= -\frac{40}{3} + \frac{25}{3} + 6 \\
&= -\frac{15}{3} + 6 \\
&= -5 + 6 \\
&= 1,
\end{align*}
\]

and the original equation is satisfied. Even the best pupils did not carry out this check. Having mentioned that, the researcher admits that this additional behaviour was an option which earned no marks but it could have served as a confidence booster if the answer was correct. Alternatively, in the event of wrong answers, it could have encouraged perseverance to the right answer if pupils had sufficient confidence in their own ability to succeed.

The most obvious confusion about the method of solution occurred on one of the scripts of a pupil from an independent school, who wrote:

\[
\begin{align*}
8x - 4 &= 1 \text{ or } -5x + 10 = 1 \\
8x &= 5 \quad \text{or } -5x = -9 \\
x &= \frac{5}{8} \quad \text{or } x = \frac{9}{5}.
\end{align*}
\]

This pupil failed to notice that the power of the unknown categorises the equation as the simple linear type instead of quadratic.

The other obvious infringement of the convention for solution was the profuse use of the equal symbol as the following attempts show:

\[
\begin{align*}
&= 8x - 4 - 5x + 10 = 1 \\
(1) &= 3x + 6 = 1 \\
&= 3x = 1 - 6 \\
&= 3x = -5 \\
&= x = \frac{5}{3} = \frac{12}{3}
\end{align*}
\]
\[(2) \quad 8x - 4 - 5x - 2 = 1
= 8x - 5x - 4 - 2 = 1
= 3x + 6 = 1
\]
\[3x = -5
x = -\frac{5}{3}
= -1\frac{2}{3}
\]

\[(3) \quad 8x - 4 - 5x - 10 = 1
= 8x - 5x - 10 - 4 = 1
= 3x - 14 = 1
\]

\[(4) \quad 8x - 4 - 5x - 10 = 1 = 3x + 6 = 1
3x = 1 + 6 = 7 \quad x = \frac{7}{3} = 2.33
\]

\[(5) \quad 8x - 8 - 5x + 10 = 1
= 3x = 18 - 1
= 3x = 17
\]
\[x = 5\frac{2}{3}
\]

\[(6) \quad 8x - 4 - 5x - 10 = 1
= 3x + 6 = 1
\]
\[-6 = -6
\]
\[\frac{3x}{3} = \frac{-5}{3}
\]
\[x = -1\frac{2}{3}
\]
\[
\begin{align*}
(7) & \quad 8x - 1 - 5x + 10 = 1 \\
& \Rightarrow 8x - 5x = 3x \\
& \Rightarrow 3x - 1 + 10 = 1 \\
& \Rightarrow 3x + 9 - 9 = 1 - 9 \\
& \Rightarrow 3x = -8
\end{align*}
\]

In addition to the incompetent use of the equal symbol, the logical symbol \( \Rightarrow \) was also used. This was probably an indication that, at best, the independent school, which the pupil attended, advocated the teaching of the subject as a combine of traditional and modern mathematics. Among the seven examples listed, there were slips in the working which suggested that pupils need miscellaneous graduated reinforcement exercises in calculation with directed number. Example (1) for instance, implies a misconception that \( x \in \mathbb{Z}^+ \). Perhaps this is natural since any effort by the school to extend their horizon of natural numbers was unsuccessful. In fact in this example:

\[
\begin{align*}
8x - 4 - 5x + 10 &= 1 \text{ (which is quite correct)} \\
3x + 6 - 1 &= 0 \\
3x + 5 &= 0,
\end{align*}
\]

the pupil sensed from the second line that a negative number was creeping into the solution, so the right of the equation is put equal to zero so as to avoid the inevitable appearance of \( x \in \mathbb{Q}^- \). Pupils seem to take fright when their solution ends with a negative number. There were many other instances where pupils could not cope with \( 3x = 1 - 6 \) so one wrote \( 3x = 1+ 6 \), concluding with the result \( x = 2.33 \), in all likelihood from the display panel of a calculator since there was no instruction to prohibit its use.

In another example

\[
8x - 8 - 5x + 10 = 1
\]

the \(-8\) indicates a lapse in concentration resulting in a careless slip. Yet another pupil who could not handle \( 3x = 1-6 \) wrote instead \( 3x = 6-1 \) and eventually \( x = \frac{12}{3} \).
At another cognitive level, the reasoning proceeded thus:

\[
\begin{align*}
8x - 4 - 5x + 10 &= 1 \\
3x - 4 + 10 &= 1 \\
10 - 4 - 1 &= 3x \\
5 &= \frac{3}{x} \\
x &= \frac{3}{5}
\end{align*}
\]

Occurring less frequently were instances where pupils forgot to multiply \(-2\) by 5 in the second bracket although the sign was changed to \(+2\) when the bracket was removed. For example:

\[
\begin{align*}
8x - 4 - 5x + 2 &= 1 \\
3x &= 1 + 4 - 2 \\
3x &= 3 \\
x &= 1
\end{align*}
\]

Another pupil in a similar predicament wrote after the previous two lines:

\[
\begin{align*}
3x &= 3 \\
x &= 3 - 3 \\
x &= 0
\end{align*}
\]

At the other extreme, a script indicated \(-2\) multiplied by 5 correctly, but did not effect a change of sign in the product as shown in: \(8x - 4 - 5x - 10 = 1\) but, strangely enough the working continued:

\[
\begin{align*}
8x + 5x &= 1 + 4 + 10 \\
13x &= 15 \\
x &= \frac{15}{13} \\
x &= \frac{12}{13}
\end{align*}
\]

Another pupil proceeded in the same way but stopped at \(13x = 15\).

Very similar to this trend of thought was where the first line \(8x - 4 - 5x + 10 = 1\) was correct but the second line read:

\[
\begin{align*}
3x - 14 &= 1 \\
\text{followed by} \quad x &= 1 + 14 = 15 \\
x &= \frac{15}{3} \\
x &= 5 \text{ ans}
\end{align*}
\]

exhibiting an unwritten break in thought, and 'ans' presumably was an abbreviation for answer. There was a need for definite instruction in the tidy set out of solutions to problems.
Parallel in thought to the last error-riddled example was the case where a correct first line

\[ 8x - 5x - 4 + 10 = 1 \]

was followed by

\[ -3x + 6 = 1 \]

but going no further, perhaps because intuition suggested that both sides of the equation would have negative signs.

There were two scripts on which pupils progressed from \((8x - 4) - (5x - 10) = 1\) to

\[ 8x - 5x + 4 + 10 = 1 \]

in one case, giving

\[ 3x + 14 = 1 \]
\[ = 3x = 1 - 14 \]
\[ \text{ans } 3x = -13; \]

and in the other case directly to

\[ 3x - 14 = 1 \]
\[ 3x = 15 \]
\[ x = 5 \]

The solution of this equation required sustained thought in proceeding step by step through the routine of applying the skill required for successful conclusion [145, pp. 16-17]. The researcher saw this question as typical of many for which logical reasoning and use of inverses combine to produce a balancing act that is so fundamental to the success of further work. The same skill is required in manipulating and inventing identities, functions and formulae, and testing that they hold. The aim here surely is that pupils should understand the process involved in solving equations.

Despite the massive tally of errors, the facility index of the question was 63 per cent, which puts it within the expected competence of the sub-group of the sample which was tested.
Table 6.29: Mean Achievement on Question 8

This question, having been attempted by 91 pupils (38 per cent of the sample field class) was not only unpopular but also was attempted by pupils of exceptionally low attainment as indicated by the overall mean achievement of 13.7 out of 50 possible marks and a linear spread of the scores about this mean of 13.1. At the school level, Schools 8 and 4 have scored acceptable mean marks but others were very poor. Full marks were scored by 8.8 per cent of the subsample, 16.2 per cent of whom were boys and 3.7 per cent girls.

Though largely attempted by low ability respondents, the extent of the sorting into attainment levels at the top end, ie at points 4.0, 4.5 and 5.0 on the mark scale, was well executed. In consequence of the low cognitive level of operation of more than three-quarters of the respondents, a discrimination index of 0.56 was achieved.
Further evidence that the comprehension of the respondents was weak was indicated by the fact that the mean percentage mark of the girls was 6.9 per cent and of the boys 18.4 per cent, thus giving an overall mean percentage mark, $M_Q = 11.5$ per cent. The real barometer of how the subsample coped with the question was arrived at by converting the overall mean mark of the candidates who answered the question into a percentage. From Table 6.29 p. 404 this mean ability index $M_T = 27.4$ per cent had a standard deviation of 26.2 per cent.

Given below are the structure and content of the question along with an allocation of marks to the expected behaviours. (Table 6.30).

**Question 8** (see Appendix 6.4)

The minute hand of a clock is 6 cm long. How far does the tip travel in 30 minutes?

(Take $\pi = \frac{22}{7}$)

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance travelled $= \left(\frac{1}{2} \times 2 \times \frac{22}{7} \times 6\right)$ cm</td>
<td>2</td>
</tr>
<tr>
<td>$= \frac{132}{7}$ cm</td>
<td>2</td>
</tr>
<tr>
<td>$= 18 \frac{6}{7}$ cm</td>
<td>1</td>
</tr>
<tr>
<td>($= 18.857$ cm)</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>5</strong></td>
</tr>
</tbody>
</table>

Table 6.30: Allocation of Marks to Expected Behaviours
### Table 6.31: Percentage of Errors per Behaviour

The vast amount of errors occurred because pupils commenced the calculation with an insecure concept of both length and area, consequently either \( \pi r^2 \) or \( \frac{1}{4} \pi r^2 \) was used instead of \( \pi r \) in finding the length of the curved line. One pupil tried \( 2\pi r^2 \), crossed it out, then tried \( \pi d \) from which 37.5 resulted without stating any unit of length. For another, the reasoning was:

\[
\pi = \frac{22}{7}
\]

\[
= \frac{22}{7} \times 30 \times 6
\]

\[
= 565 \frac{5}{7} \text{ cm} \quad \text{(See Table 6.31)}
\]

It is inconceivable that 15-year-old pupils who used \( \pi r^2 \) should obtain a linear unit of length in the answer. Others circumvented the use of a standard formula by calculating:

\[
\frac{11}{22} \times 30 \times \frac{1}{6} = \frac{110}{7}
\]

\[
= 15.5 \text{ metres}
\]

\[
\frac{22}{7} \times 30 = \frac{660}{7} = 94 \pi r^2
\]

\[
60)94
\]

\[
1 \text{r}34+2=36
\]

The tip travels 1 min 36 secs
and

\[
\frac{6}{30} \times \frac{11}{7} \cdot \frac{22}{15} = \frac{66}{105} = 1 \text{ hr 39 secs}
\]

A large number of errors was committed at the start presumably because pupils had conceptual difficulty in combination with the inability to translate the language of the problem, through the application of the correct formula, into a symbolic operational state. In marking the scripts slips in the arithmetic were ignored because if the formula was wrong no credit could be given since it was a case of the misapplication of rote memory. In anticipation of these conceptual hurdles in his lessons, the writer always got some prospective teachers to make cardboard models like the ones demonstrated by Hope [109, pp 85-87] for discussions of how children's prior knowledge of area can be extended. Graham and Sargent [140, pp 3-8] have similar exercises which simultaneously explain how the electronic calculator can be used to bring efficiency and speed in routine practice examples. However the Assistant Masters Association [97, pp 98-101] go to the very heart of the problem and discuss the teaching points that should be raised with primary teacher trainees whose responsibility it is to introduce children to the concept of area. It is, consequently, ever so important for teachers in lower secondary schools to keep in touch with their primary colleagues so that there is continuity of methods and strategies of teaching in the early classes of the secondary school.

The majority of pupils who attempted this question relied entirely on the recall of a formula without seeking to ensure which conceptual picture was required. This led to the selection of the wrong process skill which, when applied, in turn encoded an inaccurate result. It was not surprising that
the measure of how difficult the respondents found the question was given by a facility of 34.0 per cent, thus making it rather difficult but nevertheless quite within their range of ability.

The analysis below refers to the statistical data in Table 6.32.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>MEAN</th>
<th>S.D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>G</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>GHS(G)</td>
<td>32</td>
<td>6</td>
<td>5</td>
<td>54.5</td>
<td>45.5</td>
</tr>
<tr>
<td>CCS(G)</td>
<td>18</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>100.0</td>
</tr>
<tr>
<td>AFA(G)</td>
<td>51</td>
<td>8</td>
<td>7</td>
<td>53.3</td>
<td>46.7</td>
</tr>
<tr>
<td>QC(I)</td>
<td>41</td>
<td>11</td>
<td>9</td>
<td>55.0</td>
<td>45.0</td>
</tr>
<tr>
<td>LWY(G)</td>
<td>25</td>
<td>4</td>
<td>4</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>PWH(I)</td>
<td>18</td>
<td>4</td>
<td>10</td>
<td>28.6</td>
<td>71.4</td>
</tr>
<tr>
<td>RMB(G)</td>
<td>29</td>
<td>6</td>
<td>9</td>
<td>40.0</td>
<td>60.0</td>
</tr>
<tr>
<td>STJC(I)</td>
<td>27</td>
<td>13</td>
<td>6</td>
<td>68.4</td>
<td>31.6</td>
</tr>
<tr>
<td></td>
<td>241</td>
<td>52</td>
<td>55</td>
<td>48.1</td>
<td>51.9</td>
</tr>
</tbody>
</table>

Table 6.32: Mean Achievement on Question 9

This was one of the less popular questions as indicated by the number of pupils, 107 (44.4 per cent of the field class) who offered it. The overall mean achievement of the subsample was 18.7 and the linear spread of the scores about that mean was 13.2. This overall blanket statistic was the basis for establishing a mean ability index, $M_T$, which for this question was 37.4 per cent. More girls offered the question than boys and the mean percentage marks were 35.8 per cent and 47.1 per cent respectively. Consequently the mean mark on the question, $M_Q$, was 41.4 per cent. In performance, Schools 8, 4 and 6
continued to lead but from the number of the entry from School 8 combined with the linear scatter of its individual pupils' scores about the mean, the conclusion that its entry was very competent was inescapable. Except for the performance of School 1, the government senior high schools performed very poorly.

Turning to the sorting power of the question, the information indicated that the sensitivity was such that candidates were spread over the mark range at all points except 0.5, 1.5, 3.5 and 4.5, and have clustered at 0.0, 1.0 and 5.0. At the top end of the mark range 29.9 per cent of the entry were grouped whereas at the bottom there were 28.0 per cent. The question was, therefore, very sensitive in separating the candidates into groups of similar abilities. This power to select and sort out was expressed by a discrimination index of 0.70.

Concerning Question 9, its structure and content appear below along with the expected behaviours and distribution of the marks. (Table 6.33).

**Question 9** (see Appendix 6.4)

Find, in square centimetres (cm²), the area of a rectangle which is (2x - 5) cm long and (x + 3) cm wide.

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution: Area of rectangle = (2x - 5)(x + 3) cm²</td>
<td>1</td>
</tr>
<tr>
<td>= (2x² + 6x - 15)</td>
<td>3</td>
</tr>
<tr>
<td>= (2x² + x - 15) cm²</td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.33: Allocation of Marks to Expected Behaviours
Following the marking scheme very closely is a matrix (Table 6.34) showing where the behavioural errors occurred.

<table>
<thead>
<tr>
<th>SOURCE OF BEHAVIOURAL ERRORS</th>
<th>BEHAVIOUR ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (2x-5)(x+3) cm²</td>
<td>3</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2 (2x²+6x-5x-15) cm²</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3 (2x²+x-15) cm²</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 6.34 Percentage of Errors per Behaviour

A scan of the test scripts from each school revealed that 43.5 per cent of the errors occurred because pupils had no idea of how to begin the solution. Four of the many unsuccessful attempts, the first of which was an entirely random approach, were:

(1) \[(2x - 5) = (x + 3)\]

\[= 2x - 5 = x + 3\]

\[= 2x - x = 5 + 3\]

\[x = 8\]

Area of rectangle is 8

Although no penalty has been imposed if a unit of length or area was omitted (see CSE model solutions, [146, pl.]), it would have reflected completeness of solution and that the pupil had at least a rudimentary, even if instrumental, understanding [64] of the concept of area.
(2) In $(2x - 5) \text{ cm} \times (x + 3) \text{ cm}$

$$(2x - x)(5 + 3) \text{ cm}$$

$$2 \times 8 \text{ cm} = 16 \text{ cm}^2$$

the initial step is correct but what follows reveals that the basic knowledge of the mechanics of the operation was lacking.

(3) Here, $(2x - 5)(x + 3)$

$$7x \quad 3x$$

area of rect $= L \times W$

$$= 7x \times 3x$$

$$= 21x \text{ cm}^2$$

the second line portrays the very core of the cognitive difficulty: it was mentally a challenge to operate with either $(2x - 5)$ or $(x + 3)$ as a multiplier because the variable and constants are separated by operation symbols. So a rule was invented to simplify each to a single integral coefficient times the variable. Even after inventing a rule, it clashed with other existing conventions because $7x \times 3x \neq 21x \text{ cm}^2$.

(4) $(2x - 5)(x - 3)$

$$2x + 6x - 5x - 15$$

$$8x + 20$$

What is important however is that the pupil did have the ability to think but simply lacked skill and knowledge in algebraic manipulation.

Firstly, the pupil copied the problem incorrectly: In $2x$, the power of $x$ is missing, $2x(-3) \neq +6x$, and $(-5)(-3) \neq -15$. In the simplification it seemed that "like signs were added", but further, in $-5x - 15$, the variable $x$ is mentally unacknowledged so that the simplified operation becomes $+20$.

More slips, 44.9 per cent of them in fact, were made where the pupils seemed to have remembered, though not necessarily understood practically the concept, that multiplication
of the lengths would result in the right answer but the mechanics of the operation went wrong [44]. The following attempts at solution highlight the nature of mental activity.

(1) \[(2x - 5) \times (x + 3) = 2x^2 - 6x - 5x - 15\]
\[
(2x^2 - 11x - 15)^2
\]

The multiplicative error here was \(2x)(3) = -6x\). Understandably this gave \(2x^2 - 11x - 15\) but the justification for squaring this result was unthinkable.

(2) \[
[((2x - 5)(x + 3))] \text{ cm}^2
\]
\[
= (2x^2 - 5x + 6x - 8) \text{ cm}^2
\]
\[
= (2x^2 + x - 8) \text{ cm}^2
\]

Here brackets are very well used but there was a lapse of concentration in finding the product of \((-5)\) and \((+3)\). It seemed unlikely that the directed number symbols were off-putting because the operation \((-5)(+x)\) was correctly carried out. However, the possibility that this was the source of error cannot be ignored.

(3) \[(2x - 5)(x + 3)\]
\[
= 3x^2 + 6x - 5x - 15
\]
\[
= 3x^2 + x - 15 \text{ cm}
\]

In this instance it appeared that \((2x)(x)\) and \((2x + x)\) have together produced \(3x^2\); but in

(4) \[(2x - 5)(x + 3)\]
\[
2x^2 + 6 - 5x - 15
\]
\[
2x^2 - 5x - 9
\]

there has been either a loss of concentration in multiplying \(2x\) by \(3\) or a slip in not writing variable \(x\) on the right side of the coefficient \(6\) in the second line. But for this slip the last line was correct.
In the following solution, which has an error in multiplication 
$(3 \times 2x \neq 3x)$, the reverse operation gives different factors from the ones with which the pupil began.

$$
(2x - 5)(x + 3) \\
2x + 3x - 5x - 15 \\
2x - 2x - 15 \\
(x - 3)(x - 5) \\
\therefore x = 3 \text{ or } 5
$$

The difficulty here was that the pupil could not distinguish between an expression and an equation.

Into another category fell the errors on a further 8 scripts, types of which follow. These, amounting to 11.6 per cent of the mistakes, either did not have middle terms or the answers were incorrect for reasons other than the multiplication going astray as in:

(1) 
$(2x - 5)(x + 3) = 2x^2 - 15$

(2) 
$2x - 5$

$$
x + 3 \\
2x^2 - 5x \\
6x - 15 \\
2x^2 - x - 15
$$

(3) 
$(2x - 5)(x + 3) = 2x^2 - x - 15 \text{ cm}^2$

Graham and Sargent [140, p 76] give a diagrammatic explanation of the meaning of binomial products together with a graphic illustration of how each element of the product was obtained. This is one of the approaches that Bahamian teachers of mathematics need to adopt in their attempts to promote understanding of manipulative skills in simple algebra. More appropriately, these diagrams may be modelled out of inexpensive cardboard to demonstrate in three dimensions the compelling logic of abstract reasoning [147]. But, for exposing prospective teachers
and in-service teachers to children's cognitive processes, the examples [54, p 59] of the students' material are most useful.

32 per cent of those who elected to offer the question selected the right arithmetic operation required to obtain a solution and were each awarded 1 out of the 5 marks. But the real impediment which obstructed the right solution was the inability of the pupils to perform the algebraic operation necessary for the task. In addition, 11.6 per cent of the errors occurred where the product had to be simplified into an acceptable form (Table 6.34). Despite the inability to execute the algebraic operation of multiplication, a facility of 56 per cent made it quite acceptable for this age and range of ability.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>MEAN</th>
<th>S.D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>G</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>1 GHS(G)</td>
<td>32</td>
<td>12</td>
<td>15</td>
<td>44.4</td>
<td>55.6</td>
</tr>
<tr>
<td>2 CCS(G)</td>
<td>18</td>
<td>7</td>
<td>11</td>
<td>38.9</td>
<td>61.1</td>
</tr>
<tr>
<td>3 AFA(G)</td>
<td>51</td>
<td>25</td>
<td>21</td>
<td>54.3</td>
<td>45.7</td>
</tr>
<tr>
<td>4 QC(I)</td>
<td>41</td>
<td>21</td>
<td>18</td>
<td>53.8</td>
<td>46.2</td>
</tr>
<tr>
<td>5 LWY(G)</td>
<td>25</td>
<td>10</td>
<td>11</td>
<td>47.6</td>
<td>52.4</td>
</tr>
<tr>
<td>6 PWH(I)</td>
<td>18</td>
<td>5</td>
<td>10</td>
<td>33.3</td>
<td>66.7</td>
</tr>
<tr>
<td>7 RMB(G)</td>
<td>29</td>
<td>9</td>
<td>19</td>
<td>32.1</td>
<td>67.9</td>
</tr>
<tr>
<td>8 STJC(I)</td>
<td>27</td>
<td>15</td>
<td>9</td>
<td>62.5</td>
<td>37.5</td>
</tr>
</tbody>
</table>

Table 6.35: Mean Achievement on Question 10
This was the most popular of the test questions, having been chosen by 218 pupils, 90.5 per cent of the total field class. The mean competence of this group was 15.0 and the scores, with a linear scatter of 12.8 about this mean, indicated that it was a very heterogenous collection of pupils. From this mean competence the mean ability index, $M_T$, was calculated to be 30.0 per cent. Also, boys and girls scored means of 29.6 per cent and 33.0 per cent respectively, from which a mean percentage mark, $M_Q$, for the question was 31.4 per cent. Schools 8, 4 and 6 still maintain their relative positions as top scorers but the government schools continue to perform very poorly. (Table 6.35).

The data shows how the question identified pupils of similar ability and distributed them to pockets on the mark scale. 22.5 per cent of the respondents were at the bottom end of the scale whereas 4.6 per cent were at the top. At the 2.0 mark point, however, there was a cluster of 43.1 per cent followed by 5.5 per cent at the 3.0 mark point above. No candidates were allocated to the 0.5, 2.5, 3.5 and 4.5 positions but the power to discriminate, expressed by the correlation coefficient of 0.60, was quite effective.

What follows are the question and an organisation of the expected behaviours together with a marking scheme. (Table 6.36).

**Question 10** (see Appendix 6.4)

Multiply 0.376 by 0.16 and write your answer in standard form.

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution: 0.376</td>
<td></td>
</tr>
<tr>
<td>$\times 0.16$</td>
<td></td>
</tr>
<tr>
<td>3760</td>
<td></td>
</tr>
<tr>
<td>2256</td>
<td></td>
</tr>
<tr>
<td>0.06016</td>
<td>2</td>
</tr>
<tr>
<td>Now 0.06016 = 6.016 $\times 10^{-2}$</td>
<td>3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.36: Allocation of Marks to Expected Behaviours
Among others, this topic in both algebra and arithmetic was amenable to much rote learning because it was possible to carry out the partial operations of multiplication and addition without having a mental grasp of the size of the numbers being manipulated or what magnitude of answer was to be expected. That 
\[ 0.376 \times 0.16 = 0.37 \times 0.16 \]
\[ = 0.0592 \]
and 
\[ 0.376 \times 0.16 = 0.38 \times 0.16 \]
\[ = 0.0608 \]
set lower and upper limits respectively to the answer, 'a', of this problem was an example of how estimation would give any pupil unshakeable confidence in his exploratory approach since the next stage
\[ 0.0592 = 5.92 \times 10^{-2} \]
\[ 0.0608 = 6.08 \times 10^{-2} \]
\[ \therefore 5.92 \times 10^{-2} < a < 6.08 \times 10^{-2} \]
However what put pupils off conceptually was that for \( \{xy = z: x, y \in N\} \) the result was always \( z \in N \) and \( x < z \), \( y < z \) whereas for \( \{ab = c: a, b \in Q\} \), \( c \in Q \) but \( c < a \), \( c < b \). Moreover, this degree of searching analysis gives the teacher confidence in his judgment that the pupil has a flawless understanding of logically correct mathematical thinking with regard to magnitude. The pupil could then have pursued vigorously the objective of calculating 'a', knowing its greatest lower bound and its least upper bound. Otherwise the exercise was merely executing operations and hoping that the answer was correct instead of being excited and jubilantly confident of success. The finish could have been (though other ways were permissible) as follows:

\[
\begin{array}{c}
376 \\
\times 16 \\
376 \\
2256 \\
6016
\end{array}
\]
Now \(376 \times 16 = 6016 \Rightarrow 0.376 \times 0.16 = 0.06016\)

\[0.06016 = 6.016 \times 10^{-2} = a\]

and the condition \(5.92 \times 10^{-2} < a < 6.08 \times 10^{-2}\) is satisfied.

The approach outlined above, if nothing else, achieves the relief of ignoring the decimal points until the very end.

Analysing the steps in the attempts at solution (Table 6.37) revealed that 24.0 per cent of the errors occurred in the positioning of the row of digits after multiplication by the tens digit in the multiplier as well as sometimes placing the decimal point somewhere in this row. Similarly, 6.0 per cent of the errors were made in the row below resulting from multiplication by the units (ones) digit 6 in the multiplier. At the immediate later stage in the calculation 16.0 per cent of the mistakes resulted from the subtask of addition and inserting the decimal point in the correct position. But by far the stage which presented a real hurdle was the transformation of the product into standard form. 54.0 per cent of the errors made in the whole calculation occurred at this point mainly because, whereas several remembered that if \(1 \leq x < 10\), then \(x\) is in standard form, they could not recall the rule for expressing a number less than one in standard form. 

*This is definite evidence of a crucial weakness in place value.* Still more were content to leave the working at the stage where the multiplication was complete because obviously they had either never heard what standard form was or had forgotten about it. Despite the many errors the facility of 51.0 per cent indicated that it was an excellent question for this weak mean ability range.
Table 6.37: Percentage of Errors per Behaviour

This was the most popular question perhaps, because pupils felt that they were well practised in the multiplication of decimals so it was not surprising that many mistakes appeared in the multiplication bonds, addition bonds and in each of the previous operations where there were slips in the carrying digit after multiplying by the tenths and hundredths digits in the multiplier 0.16. The most fundamental errors, however, were the wrong position of the decimal point and an overly generous use of the place holder, zero.

In \( 0.376 \times 0.16 = 0.06016 \)

the loosely repeated rule that the decimal points must lie under each other was retrieved obviously from rote memory.
Secondly, \(0.376\)

\[
\begin{array}{c}
0.16 \\
2.256 \\
0.376 \\
0.00 \\
0.06.016
\end{array}
\]

may have gone well if the pupil knew what was happening in the calculation.

\[
0.376 \\
\times 0.16 \\
\hline
2256 \\
00376 \\
00.39856
\]

and \(0.376\)

\[
\begin{array}{c}
0.16 \\
0.2256 \\
0.376 \\
0.6016
\end{array}
\]

are different ways in which the same misconception occurred. However

\[
\begin{array}{c}
0.376 \\
0.16 \\
2256 \\
0376 \\
0000 \\
0.05016
\end{array}
\]

showed correct positioning of the decimal point: it was the hundredth digit, though care was taken with the carrying in the operation of addition, which was 0.01 too small. The nearest attempt to the method outlined above by the researcher was the following:

\[
\begin{array}{c}
376 \\
16 \\
2226 \\
370 \\
05926
\end{array}
\]
but unfortunately in the ten's digit of the first row of multiplication, the 2 should be 5 and the badly made digit in the second row (i.e., multiplication by 1 ten) should be 6.

As indicated in [37] the researcher feels very strongly that there is no need for very long multiplication but crucially, society wants those who serve it to get decimal points correct, to use zero correctly and to acquire an understanding of the processes involved in effecting the operation of long multiplication prior to feeding the multiplier and multiplicand into an electronic calculator, pressing the $\times$ key between the entry of the numbers and pressing the $=$ key prior to reading the answer from the display panel. Question 10 illustrated prerequisite knowledge for a formal study of the use of logarithms using the calculator, which Graham and Sargeant [140, p 262-272] admirably demonstrate, with the theory for use in the concluding behaviour also explained.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>MEAN</th>
<th>S.D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 GHS(G)</td>
<td>32</td>
<td>14</td>
<td>14</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>2 CCS(G)</td>
<td>18</td>
<td>2</td>
<td>7</td>
<td>22.2</td>
<td>77.8</td>
</tr>
<tr>
<td>3 AFA(G)</td>
<td>51</td>
<td>13</td>
<td>7</td>
<td>65.0</td>
<td>35.0</td>
</tr>
<tr>
<td>4 QC(I)</td>
<td>41</td>
<td>20</td>
<td>15</td>
<td>57.1</td>
<td>42.9</td>
</tr>
<tr>
<td>5 LWY(G)</td>
<td>25</td>
<td>2</td>
<td>9</td>
<td>18.2</td>
<td>81.8</td>
</tr>
<tr>
<td>6 PWH(I)</td>
<td>18</td>
<td>6</td>
<td>10</td>
<td>37.5</td>
<td>62.5</td>
</tr>
<tr>
<td>7 RMB(G)</td>
<td>29</td>
<td>6</td>
<td>11</td>
<td>35.3</td>
<td>64.7</td>
</tr>
<tr>
<td>8 STJC(I)</td>
<td>27</td>
<td>9</td>
<td>5</td>
<td>64.3</td>
<td>35.7</td>
</tr>
<tr>
<td></td>
<td>241</td>
<td>72</td>
<td>78</td>
<td>48.0</td>
<td>52.0</td>
</tr>
</tbody>
</table>

Table 6.38: Mean Achievement on Question 11
As displayed in Table 6.38 earlier this question was quite popular, having been selected by 150 pupils who comprised 62.2 per cent of the senior high school field survey class. The measure of their mean command of curriculum skills on this question was 17.9 out of a possible 50 marks, and the linear spread of individual marks about the mean performance was 12.8 standard deviation units. Deriving from this mean competence was the mean ability index, $M_T$, which was 35.8 per cent. Another very important indicator of achievement was the mean percentage mark, $M_Q$, scored on the question: the girls scored 36.7 per cent and the boys scored 33.3 per cent, thus giving $M_Q = 35.1$ per cent.

At the school level, Schools 8, 4 and 6 continued to maintain their relative roles of leadership and School 1 persisted in its effort to achieve an acceptable mean attainment of between 20 and 25 out of a possible 50 marks. However, pupils from the other government senior high schools either could not retrieve (which was the reason given to Professor Bajpai in 1978 [5, p 23, para 77]) or were unfamiliar with enough relevant curriculum knowledge and skills to reach anywhere near to registering minimum acceptable performances.

Concerning the potential of the question to identify and distribute the candidates into catchments of like talent, eleven different pockets of ability as permitted by the mark scale, have been selected by the author. The presence of 38.7 per cent of the respondents at the 0.0 catchment point was more than a hinted implication for diversification of learning and teaching strategies at this level; particularly in the state-operated comprehensive system. Similarly, the accumulation of 14.7 per cent of those who elected to answer the question at the 1.0 mark-point was added evidence of weakness in acquisition of curriculum skills and knowledge. 8.7 per cent at the 2.0 mark-pocket was only just acceptable but in addition, 7.3 per cent, 6.0 per cent and 15.3 per cent of the respondents allocated to the 3.0, 4.0 and 5.0 positions on
the mark scale completed the investigation which categorised the question an effective and sensitive measuring instrument as the discrimination index of 0.73 indicated.

Attention is now focussed on a study of the question, varying methods of displaying the expected behaviours, the application of marking schemes and an examination of the sources of errors as they appear on the test scripts. (Tables 6.39 and 6.40).

Question 11 (See Appendix 6.4)

Find x and y by any method if

\[
\begin{align*}
5x + 2y &= 2 \\
2x + 3y &= -8
\end{align*}
\]  

\text{EXPECTED BEHAVIOURS} \quad \text{MARKS}

\begin{array}{|l|l|}
\hline
\text{Solution: EITHER} & \\
\begin{vmatrix}
5 & 2 \\
2 & 3
\end{vmatrix} &= 11 & 1 \\
\begin{vmatrix}
2 & 2 \\
-8 & 3
\end{vmatrix} &= 2 & 2 \\
\therefore \frac{11}{11} &= 2 & \\
\text{when } x = 2 \text{ in equation (ii),} & \\
4 + 3y &= -8 & 1 \\
3y &= -12 & \\
y &= -4 & \\
\text{TOTAL} & 5 \\
\hline
\end{array}

\text{OR } \begin{vmatrix}
5 & 2 \\
2 & 3
\end{vmatrix} = 11 = A, \text{ say.} & 1 \\
\end{array}
\[
\text{So } \frac{1}{|A|} \text{ adj } A = \frac{1}{11} \begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix} \\
\therefore A^{-1} = \begin{bmatrix} \frac{3}{11} & \frac{-2}{11} \\ \frac{-2}{11} & \frac{5}{11} \end{bmatrix} \\
\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{11} & \frac{-2}{11} \\ \frac{-2}{11} & \frac{5}{11} \end{bmatrix} \begin{bmatrix} 2 \\ -8 \end{bmatrix} = \begin{bmatrix} \frac{6}{11} + \frac{16}{11} \\ \frac{-4}{11} - \frac{40}{11} \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}
\]

**EXPECTED BEHAVIOURS**

<table>
<thead>
<tr>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>OR (i) \times 3: \quad 15x + 6y = 6</td>
</tr>
<tr>
<td>(ii) \times 2: \quad 4x + 6y = -16</td>
</tr>
<tr>
<td>By subtraction: \quad \frac{11x}{22} = 22</td>
</tr>
<tr>
<td>\quad x = 2</td>
</tr>
</tbody>
</table>

when \( x = 2 \) in (ii)

\[
4 + 3y = -8 \\
3y = -12 \\
y = -4
\]

| TOTAL | 5 |

OR From (ii ) \( x = \frac{-8 - 3y}{2} \) (iii) | 1 |
When $y = -4$ in (iii),

$$x = \frac{-8 + 12}{2}$$

$$x = 2$$

Table 6.39: Allocation of Marks to Expected Behaviours

The most popular method of solution was elimination but this usually encountered disaster at the point where, frequently, the variable $x$ was eliminated and the calculation of $y$ began decidedly because the second linear equation was equal to a negative integer. The process of finding the numerical value of $x$ was born out of the operation of subtraction which ended with $11x = -22$ or $-10$, giving $x = -2$ or on some other scripts $x = -\frac{10}{11}$. From this point onwards the algebraic method of reasoning was correct but obviously the arithmetic went wrong and became prohibitive when $-\frac{10}{11}$ was substituted for $x$. One pupil from School 2 who attempted to eliminate $y$ first, ended up with $9y = -36$ but did not know, possibly owing to the lack of enough enrichment operations on directed number, how to proceed beyond that point. From the positions where mistakes most frequently occurred, the researcher was left in no doubt that pupils were most uncomfortable when the algebra passed transitionally into the arithmetic of operations on directed number [144], [31, pp 56-59]. In order to transform the negative integer into a positive one on the RHS of equation (ii), one pupil bravely approached a solution in this manner:

$$5x + 2y = 2 \quad (i) \quad x = -3$$

$$2x + 3y = -8 \quad (ii) \quad x = -2$$
-15x - 6y = -6  
-4x - 6y = -16  
11x = 22  
x = 2

Sub-value of x into (i)

5x + 2y = 2  
2y = 2 - 10  
2y = -8  
y = -4

Check 5x + 2y = 2  
10 - 8 = 2

The apparent rationale for transforming -8 into a positive integer under the operation of multiplication by another negative integer did indicate the degree of confidence this pupil from School 4 had in his mathematical ability but

<table>
<thead>
<tr>
<th>SOURCE OF BEHAVIOURAL ERRORS</th>
<th>BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) × 3</td>
<td>15x + 6y = 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) × 2</td>
<td>4x + 6y = -16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>By subtraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11x = 22</td>
<td>27 2 10 11 3 6 59 47.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = 2</td>
<td>- - 1 - - - 1 0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>When x = 2 in (ii)</td>
<td>4 + 3y = -8</td>
<td>2 2 1.6</td>
<td></td>
</tr>
<tr>
<td>3y = -12</td>
<td>- - 1 - - - 1 0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = -4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|                               | 27 9 20 20 11 12 17 9 125 100.0 |}

Table 6.40: Percentage of Errors per Behaviour
Unfortunately the attempt at execution was brought to grief by a momentary lapse in concentration at the crucial time when \((-8)(-2)\) should have resulted in 16 instead of -16 as was mistakenly shown in the solution above. Another slip in the elimination of \(y\) by subtraction resulted in \(11x = 22\) when in fact \(-11x = -22\). At the end of the calculation the pupil checked, as did three others, that the values of \(x\) and \(y\) satisfied the same equation into which the value of \(x\) was substituted in order to find \(y\). The check would have been more rigorous if the other linear equation was used, although the researcher was convinced that confidence would have been boosted had the check revealed that both equations held. Unfortunately this decision by the pupil would not have been more efficient in time or effort even if the fateful slip \((-8)(-2) = -16\) was not made.

On pupil from School 4 attempted to solve the equations by a matrix method which is shown below.

\[
\begin{pmatrix}
5 & 2 \\
2 & 3
\end{pmatrix}
\begin{pmatrix}
2 \\
-8
\end{pmatrix}
= \frac{1}{19}
\begin{pmatrix}
3 & -2 \\
-2 & 5
\end{pmatrix}
\begin{pmatrix}
2 \\
-8
\end{pmatrix}
= 6 - 16 = -10 \quad \frac{10}{19}
\begin{pmatrix}
10 \\
44
\end{pmatrix}
\begin{pmatrix}
19 \\
19
\end{pmatrix}
= -40 = -44 \quad \frac{44}{19}
\]

In training pupils to set out their work, it seems absolutely essential to have rules like "one equal symbol per line" until they know and acquire efficiency in precisely what is being communicated when the symbol is used. Since an attempt was being made to find the vector \(\begin{bmatrix} x \\ y \end{bmatrix}\), it should have appeared somewhere in the first line of the algebraic lay-out as a basis from which the investigation proceeded.

\[
\begin{pmatrix}
5 & 2 \\
2 & 3
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
= \begin{pmatrix}
2 \\
-8
\end{pmatrix}
\]

\[
\begin{pmatrix}
5 & 2 \\
2 & 3
\end{pmatrix}
= 15 - 4
\]

\[
= 11
\]
So \[ \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}^{-1} = \frac{1}{11} \begin{pmatrix} 3 & -2 \\ -2 & 5 \end{pmatrix} \]

\[ = \begin{pmatrix} \frac{3}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{5}{11} \end{pmatrix} \]

\[ \begin{pmatrix} \frac{3}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{5}{11} \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{5}{11} \end{pmatrix} \begin{pmatrix} 2 \\ -8 \end{pmatrix} \]

\[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{6}{11} + \frac{16}{11} \\ -\frac{4}{11} - \frac{40}{11} \end{pmatrix} \]

\[ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \]

Wain and Woodrow [54] illustrate this method with true-to-life examples in their "Children's Thinking" exercises on pp 37-39.

Back to the pupil's attempt at solution, it seemed that the determinant of the matrix of coefficients of x and y was wrongly 19 for a reason which was not obvious but at the end -44 followed by \( \frac{44}{19} \) gave the most likely clue that another of this pupil's weaknesses was arithmetic operations on directed numbers, thus possibly explaining why the numerical value of the determinant was in error.

In general, the respondents did not like the negative integer in equation (ii) and no solution was carried out either by the method of substitution or the determinantal method better known as Cramer's Rule. With the benefit of hind sight, the researcher feels strongly that some graph paper should have been available although it was not requested by any candidate. Had it been requested by any examinee, a loan from one school
would have led the researcher to take some to other schools. Excellent exercises on graphical methods are given in [140 pp 163-165]. As a precautionary measure, the researcher checked that the determinant of the coefficients of $x$ and $y$ did not vanish. If it did, either the equations could have represented the same straight line resulting in the existence of an infinite number of solutions, or both straight lines could have had the same gradient, in which case parallelism implied no solution [71]. It was the non-vanishing determinant of the coefficients matrix of $x$ and $y$ which gave the assurance that the system of linear equations had a unique solution.

The use of a matrix method in solving this system of linear equations is certainly further evidence that the mathematics taught at School 4 is probably a combination of modern and traditional. Consequently it is possible not only to put forward valid and reliable inferences about the learning of mathematics but also about the teaching of the subject.

It seems to the researcher that lots of graph exercises in transformation geometry would give pupils a feel for the structure and behaviour of vectors and matrices under different operations, as well as practice in the correct lay-out of solutions to problems, and manipulation of the elements of matrices and vectors. Pupils must be taught to distinguish between a matrix and its determinant along with symbolic representation. Except they are asked for by the pupils, the technical terms adjoint, transpose and cofactor may be omitted until they can carry out the other procedural calculations successfully but they must know how to find and use the inverse correctly together with the awareness of its effect [71].

The researcher was convinced that the main hurdles in this question were transformation of the system of equations so that one unknown could be eliminated, and execution of the
routine process skills. The methods used were secure but slips in the execution of the algorithm as a consequence of insufficient enrichment practice exercises in directed number effected an incorrect encoding operation. However the facility index of 49 per cent makes this an excellent question for this age and ability range since ideally 40 per cent ≤ F ≤ 60 per cent [129] was one criterion for acceptability.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>MEAN</th>
<th>S.D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B G</td>
<td>B G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 GHS(G)</td>
<td>32</td>
<td>7 13</td>
<td>35.0 65.0</td>
<td>18.2</td>
<td>4.9</td>
</tr>
<tr>
<td>2 CCS(G)</td>
<td>18</td>
<td>3 6</td>
<td>33.3 66.7</td>
<td>6.6</td>
<td>6.3</td>
</tr>
<tr>
<td>3 AFA(G)</td>
<td>51</td>
<td>13 10</td>
<td>56.5 43.5</td>
<td>3.1</td>
<td>4.9</td>
</tr>
<tr>
<td>4 QC(I)</td>
<td>41</td>
<td>12 11</td>
<td>52.2 47.8</td>
<td>26.4</td>
<td>10.5</td>
</tr>
<tr>
<td>5 LWY(G)</td>
<td>25</td>
<td>6 7</td>
<td>46.2 53.8</td>
<td>2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>6 PWH(I)</td>
<td>18</td>
<td>1 3</td>
<td>25.0 75.0</td>
<td>26.9</td>
<td>10.9</td>
</tr>
<tr>
<td>7 RMB(G)</td>
<td>29</td>
<td>8 11</td>
<td>42.1 57.9</td>
<td>7.3</td>
<td>5.9</td>
</tr>
<tr>
<td>8 STJC(I)</td>
<td>27</td>
<td>4 4</td>
<td>50.0 50.0</td>
<td>24.1</td>
<td>6.7</td>
</tr>
<tr>
<td></td>
<td>241</td>
<td>54 65</td>
<td>45.4 54.6</td>
<td>13.2</td>
<td>11.8</td>
</tr>
</tbody>
</table>

Table 6.41: Mean Achievement on Question 12

The 54 boys and 65 girls who attempted this question (Table 6.41) represented 49.4 per cent of the sample survey class but their mean competence of 13.2 out of a possible 50 marks indicated that a very large majority of the candidates were of desperately poor ability. This was borne out especially by the mean marks of Schools 2, 3, 5 and 7, all of which were state-operated comprehensive senior high schools; and secondly by the 11.8 standard deviation units of linear spread of the marks about
the mean performance. Further, the quality of this mean attainment was expressed in a mean ability index, $M_T$, which was 26.4 per cent. Also, the mean percentage performance on the question by the boys was 3.7 per cent and that of the girls was 6.5 per cent, giving a combined mean percentage mark, $M_Q = 5.2$ per cent.

At the school level, the degree of security of the knowledge and skills of this aspect of curriculum content, as expressed in the mean achievement of Schools 6, 4 and 8, was acceptable, with School 6 only just managing to qualify for the top of the order. Even School 8 seemed to have had the mean competence of its volunteers greatly reduced either by pupils of low attainment or, more likely by those who were unfamiliar with the knowledge and skills required by the question. Yet a third alternative, and in fact the most plausible explanation of poor performance, was that the language of the problem was incomprehensible.

A scan of available data showed that the question distributed only 8.4 per cent of the respondents over the mark range, and those at the three points 1.0, 2.0 and 5.0. This meant that 91.6 per cent of those who attempted the question found it too hard and were left in the bottom compartment of the ability range, Whilst seven catchment points over the mark range were empty. The sensitising and distribution mechanism of the question was, therefore, pretty blunt but good enough to achieve 0.40 as a discrimination index.

Following is a careful look at the structure of the question. (Table 6.42) A possible organisation of the expected behaviours, a marking scheme and a tabulation of the number of mistakes adjacent to the behaviours in which they occur on the test scripts. (Table 6.43).
Question 12 (see Appendix 6.2)

If \( \frac{2}{3} \) of the children in a school are absent when there are 322 present, how many children are registered at the school?

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution:</td>
<td></td>
</tr>
<tr>
<td>( \frac{93\frac{1}{3}}{3} ) of a school = 322 pupils</td>
<td>1</td>
</tr>
<tr>
<td>( \therefore 100% ) of the school = ( \left{ 322 \times \frac{100}{93\frac{1}{3}} \right} ) pupils</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6.42: Allocation of Marks to Expected Behaviours

The author concluded from the data obtained that pupils who earned any marks would not have gone very far before disaster struck and for 91.6 per cent of those who attempted this question the start was completely false. The vast majority could not transform the communication into numerical symbols on which meaningful arithmetic operations could proceed. A good many candidates saw \( \frac{2}{3} \) per cent of the school = 322 pupils and could not proceed beyond that wrong statement. For others \( (100 - \frac{2}{3}) \) per cent of the school = 322 pupils was not only remotely correct - a language problem - but also difficult because a mixed number was being subtracted from a whole number. Those who managed to proceed beyond this point - the author found that they were very few - had to recall their knowledge of fractions as ratios and distinguish between them and their relative sizes given by their positions on the number line. Graham and Sargent [144 p 52] make a very crucial distinction between a
ratio and a fraction whilst simultaneously providing useful examples which illumine the concept of a ratio. Complementing these ideas are the three dimensional models [147, p.8], which are especially useful for teaching. What eluded the respondents was the fact that between $93\frac{1}{3}$ per cent and 100 per cent, two non-unitary ratios can be formed: $\frac{93\frac{1}{3}}{100}$ and $\frac{100}{93\frac{1}{3}}$, the former being less in magnitude than the latter. The link between these ratios and the information in the question was that when everyone was present, the total was greater than 322 pupils. In order to satisfy this condition, 322 had to be multiplied by the larger of the two ratios, $\frac{100}{93\frac{1}{3}}$. Unquestionably, the pupils were asked to multiply 322 by the factor $\frac{100}{93\frac{1}{3}}$; thereafter, applications of the rules for multiplication of fractions gave the required solution [97]. One pupil wrote down the correct answer alongside the question number but there was no way of telling whether the reasoning and arithmetic were understood so only one mark was awarded.

Fractions, percentages and ratios desperately need explanation by visual aids and models supplemented by reinforcement exercises, and the idea that a ratio is one integer divided by another non-zero integer must be communicated to the pupils.
Table 6.43 : Percentage of Errors per Behaviour

Whilst the reading of this question should not have presented difficulty, the point at which 96.4 per cent of the total errors occurred mirrored clearly that the lack of comprehension of the fundamental elements of the question prevented mathematical symbolic transformation. (Table 6.43). It was obviously difficult for pupils to perceive how this problem on percentages transforms itself into one requiring a knowledge of ratio. The researcher is confident that it was this link forged between the concepts of percentage and ratio which provides crystal clear evidence that these pupils had not acquired what Skemp [64] calls relational understanding. Finding support in Brissenden's words [112, p 13] when he writes in a similar vein

Unfortunately there is a tendency to blame the pupils rather than examine the teaching .......
the researcher must venture the adventurous and bold statement that the variety and flexibility in the approaches to teaching mathematics must ultimately help to elevate the pupil to the plane of relational understanding, provided that the balancing term in the equation is exposure to the varying conceptual disguises and constant practice of routine skills. A classical example of conceptual disguise is the phrase

an increase of 15 per cent

wherein the conventional multiplying factor is \( \frac{115}{100} \) and not \( \frac{15}{100} \) as pupils will confidently volunteer [139].

The performance of this question in the field trial was such that its facility index was 29 per cent, thus making it the most difficult on the whole test. The presence of one difficult question on the paper was a psychologically sound policy because, it not only challenged the more able in the sample, but also balanced the very easy Question 14 which gave confidence to the weaker pupils. Used on another field trial with the same age group, there is no guarantee that these questions would perform in a similar way. They are, consequently, as well as the other 50 test items, very good candidates for storage in an item bank. The onus is, therefore, on the mathematics teacher-trainer to be versatile in exposing teachers and potential teachers to the diversified techniques of teaching, knowledgeable in the content of the subject and positive and humane in the conduct of classroom activities - group work, experiment and discovery methods, exposition, activity methods, discussion, individual work and varying combinations of these.
Table 6.44: Mean Achievement on Question 13

The subsample of boys and girls (142) who chose to offer this question (Table 6.44) represented 58.9 per cent of the sample survey population. The mean mark achieved, 17.1 out of a possible 50 marks, however indicated that it was attempted by a fair number of pupils whose attainment of the relevant curriculum content was very low. In fact, the standard deviation of 13.2 units showed that the linear spread of individual attainment about the mean was quite wide. Stemming from this mean mark was the mean ability, $M_T$, of the respondents which was 34.2 per cent. Moreover, the mean mark on the question scored by the boys was 2.7 out of 5.0 (54.0 per cent) and that of the girls was 2.1 out of 5.0 (42.0 per cent), thus producing a mean percentage mark, $M_Q = 48.0$ per cent.

Schools 8, 6 and 4 have secured good mean performances. School 1, however, only just failed to qualify for an acceptable mean performance so the state schools, except one, continue to make a considerably poor showing.
Data collected showed that 40.1 per cent of the respondents were shifted to the top end of the mark range but 44.4 per cent remained at the bottom. This meant that 15.4 per cent of the respondents were left to be distributed over the remaining mark range which had no candidates assigned to the pockets labelled 0.5 and 3.5. Consequently, the discrimination index of 0.44 was only just better than acceptable.

The structure of the question and its curriculum content along with a display of the expected behaviours and an allocation of the marks follow for examination.

**Question 13 (see Appendix 6.4)**

Divide $1200 in the ratio 3:5

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution:</td>
<td></td>
</tr>
<tr>
<td>Total no. of parts</td>
<td>5 + 3 = 8</td>
</tr>
<tr>
<td>.` 1st share = $(\frac{3}{8} \times 1200)$</td>
<td>2</td>
</tr>
<tr>
<td>= $450.00</td>
<td></td>
</tr>
<tr>
<td>and 2nd share = $(\frac{5}{8} \times 1200)$</td>
<td>2</td>
</tr>
<tr>
<td>= $750.00</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.45: Allocation of Marks to Expected Behaviours

The attempts at answering this question were such that the initial recorded thoughts of 63 pupils were indicative of no real grasp of the premise and therefore fetched no marks. 3 other pupils, however, wrote commencing statements which communicated a basis for progress but did not demonstrate any
mental capacity for going beyond a mere beginning. The second stage in the calculation was to use the operation of multiplication of fractions to find either of the two shares and to repeat the process for the final stage (Table 6.45).

Table 6.46 resulted from a scan of the behavioural errors as they appeared on the test scripts.

<table>
<thead>
<tr>
<th>SOURCE OF BEHAVIOURAL ERRORS</th>
<th>BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total no. of parts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 8</td>
<td>15 7 15 2 10 3 12 2</td>
<td>66</td>
<td>83.5</td>
</tr>
<tr>
<td>1st share</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= $\left( \frac{3}{8} \times 1200 \right)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= $450$</td>
<td>-- -- 6 1 -- --</td>
<td>7</td>
<td>8.9</td>
</tr>
<tr>
<td>2nd share</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= $\left( \frac{5}{8} \times 1200 \right)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= $750$</td>
<td>1 1 3 -- -- 1 6</td>
<td>7</td>
<td>7.6</td>
</tr>
<tr>
<td></td>
<td>16 7 16 11 11 3 12 3</td>
<td>79</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 6.46: Percentage of Errors per Behaviour

However, 7 scripts showed that second stage errors impeded progress to the last stage. Of course two pupils, having got successfully beyond the second stage showed discontinuity of thought by the use of the unconformable method of subtracting the first share from the total to find the second share, and
four others, without any justification, wrote the answer down. The mistake in this procedure is better illustrated if the ratio of the shares were, say, 1:3:4 in which case the second and third shares could not be arrived at correctly by subtraction of the first from the amount to be shared. The method of tackling similar answers in [144] is highly recommended. The following four examples are illustrative and typical of the cognitive difficulties respondents had in marshalling their knowledge of this branch of curriculum content.

1. $1200 \text{ ratio } 3:5 = 8 \times 1200 = 150$
   
   $3:5 \text{ or } 450 + 750 = \$1200$

2. $1200 \text{ in the ratio } 3:5$
   
   $= \frac{8\times1200}{150}$
   
   $3 \times 150 = 450$

   $1200 - 450 = 750$

   Ratio $3:5 = 450:750$

3. $\frac{150}{8}\text{)1200} = 3:5$

   $= 450:750 \text{ ans}$

4. $\frac{1200}{8} = 150$

   $3 = 3 \times 150$

   $= \$450$

   $5 = 5 \times 150$

   $= \$750$

   Check $= 450 + 750 = \$1200$

Whereas the previous four examples indicated that the respondents were taught how to give organised cognitive and affected responses to similar problems but had not acquired the degree of proficiency in the use of mathematical language
in order to effect precision in communicating these thoughts and feelings, this last example also illustrates a lack of understanding of the cancelling process in the multiplication of fractions.

\[
\begin{align*}
3:5 & \quad 3 + 5 = 8 \quad $1200 \\
\frac{3}{8} \times 1200 & \quad 400 \\
\frac{5}{8} \times 1200 & \quad 240 \\
& \quad = $32.00
\end{align*}
\]

Apparently the pupil did have some idea of the magnitude of the quantities of money because the researcher suspected he perceived that \((8 \times 240)\) as one share - though entirely wrong - was less than the first and therefore discontinued the calculation - a decision based on the previous rationale but giving \((8 \times 240) = $19.20\).

Although there were comprehension, transformation and process skills' difficulties together with finding a way of writing the answer in an acceptable form, the facility index of 64 per cent substantiates that the question was pitched at the right level for this group.
<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>SAMPLE</th>
<th>VALID OBSERVATIONS</th>
<th>PERCENTAGES</th>
<th>MEAN</th>
<th>S.D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>G</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>GHS(G)</td>
<td>32</td>
<td>12</td>
<td>16</td>
<td>42.9</td>
<td>57.1</td>
</tr>
<tr>
<td>CCS(G)</td>
<td>18</td>
<td>6</td>
<td>7</td>
<td>46.2</td>
<td>53.8</td>
</tr>
<tr>
<td>AFA(G)</td>
<td>51</td>
<td>19</td>
<td>15</td>
<td>55.9</td>
<td>44.1</td>
</tr>
<tr>
<td>QC(I)</td>
<td>41</td>
<td>20</td>
<td>18</td>
<td>52.6</td>
<td>47.4</td>
</tr>
<tr>
<td>LWY(G)</td>
<td>25</td>
<td>7</td>
<td>13</td>
<td>35.0</td>
<td>65.0</td>
</tr>
<tr>
<td>PWH(I)</td>
<td>18</td>
<td>6</td>
<td>12</td>
<td>33.3</td>
<td>66.7</td>
</tr>
<tr>
<td>RMB(G)</td>
<td>29</td>
<td>10</td>
<td>18</td>
<td>35.7</td>
<td>64.3</td>
</tr>
<tr>
<td>STSC(I)</td>
<td>27</td>
<td>13</td>
<td>9</td>
<td>59.1</td>
<td>40.9</td>
</tr>
<tr>
<td></td>
<td>241</td>
<td>93</td>
<td>108</td>
<td>46.3</td>
<td>53.7</td>
</tr>
</tbody>
</table>

Table 6.47: Mean Achievement on Question 14

The total valid observations in Table 6.47 amounted to 201, a popularity of 83.4 per cent. Again the mean mark, 16.3 out of a possible 50 marks, indicated that many of the pupils were of impoverished ability. This was verified not only by 12.6 units of linear spread of the scores about the mean but also by looking carefully at the mean mark for the individual schools, four of which – namely Schools 2, 3, 5 and 7 – scored mean marks between 3.3 and 8.4 inclusive out of a maximum of 50.0. Obtained from this mean competence of the subsample who attempted the question was the mean ability, $M_T$, which was 32.6 per cent. Also the mean marks for boys and girls were 2.4 out of 5.0 (48.0 per cent) and 2.9 out of 5.0 (58.0 per cent) respectively, together giving a mean percentage mark for the question, $M_Q = 53.0$ per cent.
The figures showed that 23.4 per cent of those who offered the question did not move from the lowest position on the mark scale, whereas 31.3 per cent were selected and placed in the highest ability group at the top of the range. 45.3 per cent (less than half by 4.7 per cent) were left to be distributed over the nine catchment pockets on the scale from 0.5 to 4.5. The efficiency with which the question separated the remaining candidates into low, intermediate and high ability groups was determined by a discrimination index of 0.52.

Alternative layouts of behavioural responses together with marking schemes were completed in anticipation of perusal of test scripts from which the numbers and samples of mistakes in the answers were recorded. The sequence of thoughts and attempts at explaining misapplications are given as well as supportive references especially useful for guiding teachers and pre-service potential teachers of mathematics into investigations of children's cognitive patterns when faced with problem-solving exercises. These aspects of the analysis follow. (See Tables 6.48 and 6.49).

Question 14 (see Appendix 6.4)

Simplify \(3\frac{1}{5} - 1\frac{1}{7}\) + \(\frac{24}{25}\)

<table>
<thead>
<tr>
<th>EXPECTED BEHAVIOURS</th>
<th>MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution: EITHER</td>
<td></td>
</tr>
<tr>
<td>(\left(\frac{112 - 40}{35}\right) \times \frac{24}{25})</td>
<td>2</td>
</tr>
<tr>
<td>= (\frac{72}{35} \times \frac{25}{24})</td>
<td>2</td>
</tr>
<tr>
<td>= (\frac{1}{7})</td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5</td>
</tr>
</tbody>
</table>
Table 6.48: Allocation of Marks to Expected Behaviours

The data gave the percentage of the subsample which scored no marks and an idea of where mistakes were likely to have occurred in the arithmetic. A careful study of the scripts indicated that there was a wide variety of sources of error.

1 The equal symbol was wrongly used and subtasks were in error as in

\[
\frac{31}{5} - \frac{1}{7} = 2 \cdot \frac{7 - 5}{35} = \frac{2}{35}
\]

\[
= 2 \cdot \frac{2}{35} \div \frac{24}{25}
\]

\[
= \frac{72}{35} \times \frac{24}{25}
\]

\[
= \frac{24}{7}
\]

TOTAL 5

2 Inversion of divisor occurred but division symbol remained until at the end just before the right answer, as in the example above.
Inability to spot a high common factor before cancelling \( \frac{72}{35} \times \frac{25}{24} \) resulted in cramped and untidy presentation as well as loss of accuracy in the final answer.

Corresponding numerator and denominator multiplied together in subtraction of fractions:

\[
\frac{16}{5} - \frac{8}{7} \times \frac{24}{25} = \frac{128}{35} \times \frac{24}{25}
\]

The whole problem equal to fragmented subtasks yet sometimes candidates arrived at the right answer:

(i) \( \frac{31}{5} - \frac{1}{7} \times \frac{24}{25} \)

\[
= \frac{31}{5} - \frac{1}{7} = 3 - 1 = 2
\]

\[
\frac{1}{5} - \frac{1}{7} = \frac{7 - 5}{35} = \frac{2}{35} \times \frac{24}{25}
\]

\[
= \frac{25}{24} \times \frac{2}{35} = \frac{5}{84}
\]

(ii) \( \left( \frac{16}{5} - \frac{8}{7} \right) \times \frac{24}{25} = \frac{112 - 40}{45} = \frac{6}{25} \times \frac{24}{45} \)

\[
= \frac{15}{7} = 2 \frac{1}{7} \quad \text{(by cancelling 72 and 24; 25 and 45; then dividing the result by 7)}
\]

Concerning this latter instance, the pupil probably knew that for the smallest common multiple \( 5 \times 7 = 35 \) but wrote 45 in error. However, in the cancelling, \( 45 \div 5 = 7 \) suggested that perhaps the multiplication bonds were insecure.

(iii) \( \frac{112 - 40}{35} = \frac{72}{35} = \frac{24}{35} \times \frac{24}{25} \)

\[
= \frac{72}{35} \div \frac{24}{25} = \frac{72}{3} \times \frac{24}{25} = \frac{7}{8}
\]
Here one also found that $\frac{72}{35}$ was wrongly inverted, the multiplication operator was substituted for division whilst $\frac{24}{25}$ remained unchanged and $\frac{7}{3} \times \frac{1}{5} = \frac{7}{8}$. On another script the attempt at solving this problem showed the same errors but differed only in showing the final result as $\frac{35}{75}$.

(iii) $\frac{16}{5} - \frac{8}{7} = \frac{24}{25}$

\[ \frac{16}{5} - \frac{8}{7} = \frac{25}{24} \]

ie 5 cancelled itself and cancelled $\frac{25}{3}$ which became 5

\[ \frac{80}{21} = \frac{317}{21} \]

(iv) $\frac{11}{2} \times 40 = \frac{35}{35} \times \frac{5}{24} = \frac{15}{7}$, but did not simplify.

6 There were cases of no real understanding of subtraction and division of fractions as exhibited by

(i) $\frac{3}{5} - \frac{1}{7} = \frac{2}{2}$

(ii) $\frac{16}{5} - \frac{8}{7} = \frac{24}{25}$

\[ \frac{16}{5} - \frac{8}{7} = \frac{25}{24} \]

ie 5 cancelled itself and cancelled $\frac{25}{3}$ which became 5

\[ \frac{80}{21} = \frac{317}{21} \]

(iii) $\frac{16}{5} + \frac{8}{7} = \frac{25}{24} \]

\[ \frac{16}{5} + \frac{8}{7} = \frac{50}{21} = \frac{8}{21} \]

ie cancelling by 8 transformed 16 into 2 and 24 into 3. Similarly, 5 became 1 and 25 became 5.

Also $2 + 8 \times 5 = 50$ in the numerator but in the denominator $1 + 7 \times 3 = 21$.

(iv) $\left( \frac{3}{5} - \frac{1}{7} \right) \div \frac{24}{25}$

\[ \left( \frac{21}{35} - \frac{5}{35} \right) \div \frac{24}{25} \]

\[ \frac{16}{35} \div \frac{24}{25} \]

\[ \frac{35}{16} \times \frac{25}{24} \]

\[ = 410 \]

For the transformation of the bracketed subtask into $\left( \frac{21}{35} - \frac{5}{35} \right)$, it appeared that the pupil's thinking process operated thus:
"Denominator 5 divided into 35 gives 7, which when multiplied by whole number three gives 21. Similarly, denominator 7 divided into 35 gives 5, which when multiplied by whole number 1 gives 5".

As shown above, both divisor \( \frac{24}{25} \) and dividend \( \frac{16}{35} \) were inverted and multiplied to give 410 which, wrongly, was intended to be the product of 25 and 16 as shown on the test script. Another pupil did this problem by the same method but got

\[
\frac{2}{35} \times \frac{25}{24} = \frac{10}{21},
\]

thus indicating that only \( \frac{24}{25} \) should have been inverted.

7 The whole number was unaffected by cancelling as in

\[
\frac{2}{35} \times \frac{25}{24} = \frac{25}{84},
\]

However \( \frac{72}{35} \times \frac{25}{24} = \frac{90}{45} \) showed labourious cancelling to give \( \frac{90}{42} \) after a struggle, the details of which were impossible to show in this report. Finally the numerator was reduced on division by 2 but the denominator was not similarly treated. This gave \( \frac{45}{42} \) when \( \frac{45}{21} \) was in fact nearer the simplified answer.

8 Whereas there were pupils who had no slips in the arithmetic but lost credit because thoughts were incoherently set out, there were far too many scripts which showed the most elementary arithmetical slips.

Table 6.49 below gives a summary of where the errors in behaviour occurred. It differs from the cross-tabulation of the question by sex from the data because solutions sometimes showed more than one type of error.
<table>
<thead>
<tr>
<th>SOURCE OF BEHAVIOURAL ERRORS</th>
<th>BEHAVIOURAL ERRORS PER SCHOOL</th>
<th>TOTAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>EITHER</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>112 - 40 : 2 25 35 25</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>72 - 24 : 23 25 35</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1/7</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (7 - 5) : 7 35 25</td>
<td>9</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>2 (24 35 25) : 24 25</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>35 5 7</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>15 7</td>
<td>3</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>1/7</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>5</td>
<td>28</td>
</tr>
</tbody>
</table>

Table 6.49: Percentage of Errors per Behaviour
The language of this mechanical problem was in numerical and operational symbolic form, with brackets indicating which operation was to be performed first. But before the bracketed operation could be executed, the pupil had to recognise the symbols via the cognitive domain and represent the understanding on paper via the affective and psychomotor channels. Graham and Sargent [144, p 50] discuss the teaching points to be made in solving this type of mechanical problem.

In the first method of solution, whereas 38 per cent of the commencing behavioural slips were the consequence of faulty computation, 57 per cent were made in cancelling and inverting the operator, and 4.8 per cent in writing the answer in an acceptable conventional form. Almost in complete contrast, the second method of solution effectively showed that 58 per cent of the total amount of behavioural errors occurred in the execution of the first operation and the majority of these attempts were random responses. At the subsequent stage where the subtask comprised the operation of division on the result of the first subtask, 17 per cent of the mistakes arose from faulty computation. Where cancelling and inverting the operator – the process prior to encoding – were required, 19 per cent of the slips occurred. The actual encoding, where the answer was to be written in an acceptable form, accounted for 5 per cent of the wrong behavioural responses not unlike in the former method [148].

Despite the many errors – and this was one of the most popular questions – the facility index of 70 per cent classifies the question as being very easy. But there had to be an easy question to balance the effect of any difficult one such as Question 12, upon which comments have already been made.
### Percentage of Respondents Scoring in Low, Average and High Ability Ranges

<table>
<thead>
<tr>
<th>Q</th>
<th>0.0-2.0</th>
<th>2.5-3.5</th>
<th>4.0-5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47.5%</td>
<td>11.5%</td>
<td>41.0%</td>
</tr>
<tr>
<td>2</td>
<td>48.4</td>
<td>15.2</td>
<td>36.3</td>
</tr>
<tr>
<td>3</td>
<td>62.8</td>
<td>12.5</td>
<td>24.6</td>
</tr>
<tr>
<td>4</td>
<td>62.0</td>
<td>7.0</td>
<td>31.0</td>
</tr>
<tr>
<td>5</td>
<td>91.5</td>
<td>3.6</td>
<td>4.8</td>
</tr>
<tr>
<td>6</td>
<td>77.7</td>
<td>18.7</td>
<td>3.6</td>
</tr>
<tr>
<td>7</td>
<td>47.7</td>
<td>11.3</td>
<td>40.9</td>
</tr>
<tr>
<td>8</td>
<td>89.0</td>
<td>-</td>
<td>11.0</td>
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<tr>
<td>9</td>
<td>66.3</td>
<td>1.8</td>
<td>31.8</td>
</tr>
<tr>
<td>10</td>
<td>87.7</td>
<td>5.5</td>
<td>6.9</td>
</tr>
<tr>
<td>11</td>
<td>66.8</td>
<td>10.6</td>
<td>22.6</td>
</tr>
<tr>
<td>12</td>
<td>95.8</td>
<td>-</td>
<td>4.2</td>
</tr>
<tr>
<td>13</td>
<td>53.5</td>
<td>2.8</td>
<td>43.6</td>
</tr>
<tr>
<td>14</td>
<td>47.8</td>
<td>12.0</td>
<td>40.3</td>
</tr>
</tbody>
</table>

| ̄x | 67.5    | 8.0     | 24.5    |
| s  | 17.2%   | 5.7%    | 14.9%   |
| SE | 1.1     | 0.4     | 0.9     |

Table 6.50: Percentage of respondents scoring in specified Mark Range per question

The statistics in Table 6.50 show that a mean of 67.5 per cent of the pupils who attempted Questions 1 to 14 scored a mark in the range from 0.0 to 2.0 out of a possible 5 marks, including the end points of the range. This reflects almost perfect agreement with the percentage scoring Grade E in Table 6.5,
and underlines the researcher's objective consistency in analysing the test results. In fact, the difference of 1.9 per cent arose because the cut-off point in Table 6.5, p 365 was 19 out of 50 but was 20 out of 50 in Table 6.50, p.448. According to Newman's error hierarchy [148, pp 321-322], the author thought that the reading, comprehension and transformation stages of the attempts at answering the diagnostic questions combine to form an insurmountable hurdle since in his (Newman's) investigation these factors were the source of 47 per cent of the errors committed. In this simplified model, the statistics derived from the Bahamian sample show a significant variance from what Morrison [127, p. 153] expects in the performance of an average entry in the bottom and middle ranges of ability. However, 24.5 per cent of the sample who scored marks in the top of the range did accord very well with normal statistical expectancy and the very low percentages of pupils who scored top marks identify the questions which were found to be difficult (Table 6.50 ). The exception was Question 10, the high facility index for which was explained by the fact that 43.1 per cent of the subsample who offered the question scored 2.0 out of 5 marks. In this case only 12.4 per cent of the candidates scored a better mark.

It is well known that it does not matter how competent, enthusiastic and motivating the teacher, a percentage of the pupils would not benefit, or at least would benefit very little, from formal lessons in mathematics. The researcher was convinced, however, that 67.5 per cent, as indicated by Table 6.50 was far too high. With a strong force of academically and professionally qualified teachers (and this notion is perhaps overly idealistic for even developed countries), reasonably good physical surroundings, a good teacher/pupil ratio, say 1:25, 67.5 per cent can be considerably reduced and 8.0 per cent in the middle range of ability can be dramatically raised. This was a massive percentage of underachievement in the fundamentals of mathematics at senior high school level. Achievement was a dependent variable which was affected
principally by the input of teachers since the pupils in the
government schools had all come from an unselected background,
the mathematics curriculum was the same and motivation from
the home was very weak because a vast majority of parents
did not have secondary education themselves. Moreover, there
was minimal contribution from differential teaching since
mathematics teachers in many schools have not had 'A' level
content experience in mathematics let alone degree qualifications.

Granted that attainment was mainly a function of teaching
(minimal conditions of good health and intelligence being assumed on the part of the pupils), it stood to reason that
low attainment in Bahamian schools was a direct result of
instructional ineffectiveness and inefficiency at learning
tasks. One may hope that in the normal course of events, the
output of teachers from the College of The Bahamas would be
enough to eventually correct this deficiency. But the scale
of the problem was such that a supplementary programme to
pre-service training must be effected immediately as suggested
by the strength and clarity of the evidence. The magnitude
of this problem calls into question a critical examination of
and re-thinking in

1 writing objectives in lesson and syllabus preparation
2 methods and strategies of teaching
3 management of learning
4 learning styles
5 raising the level of content for practising mathematics
teachers
6 curriculum development in mathematics education with special
reference to the needs of Bahamian society
7 suitability of textbooks

which lie at the heart of this research.
6.4.3 Determination of Facility Indices

Although item analysis for objective type questions was well documented prior to this research - [149, pp.383-405], [50], [151, pp.273-303], [152, pp.13-34], to cite but a few references - preceding the paper by Morrison[138], there was no established technique of analysis of choice type questions in the United Kingdom to provide feedback on how questions performed. As part of its curriculum development thrust recommended by Bajpai and Bajah [5, p.37], The Bahamas needs the benefit of research into attainment and attempts by the author to initiate action can be seen as a datum line from which future progress may be measured.

Against this background of pressing need, a choice type diagnostic test (Appendix 6.4) was administered by the author in Bahamian senior high schools and their counterparts in the independent sector, the aim of which was to establish a quantitative and qualitative profile [46] of abilities of 15-year old pupils on a paper of 14 questions. 39.3 per cent of this paper was arithmetic, 42.8 per cent was algebra, 14.3 per cent was geometry and 3.6 per cent was trigonometry; but the pupils had a choice of any 10 of the questions. This diagnostic instrument could have been administered with the aim of determining the social rank order of the pupils. Whilst an exercise of that nature partially contributes to revealing another characteristic of the research function - the degree of reliability [153, pp.206-209] - the underlying aim was more far-reaching. The researcher wanted pupils to demonstrate the skills they acquired from mathematical activity, including the set out of deductive reasoning, calculation, transformation from language to symbols, and simple formulae and geometrical constructions in order to test a conclusion - an important hypothesis for this survey put forward by Bajpai and Bajah [5, p.29, paragraph 112]:

In mathematics the children exhibit a lack of basic knowledge and skills, including computation and place value, that they should have acquired in earlier years.
In testing this hypothesis, it was necessary to investigate what Krutetskii [46] calls the structure of ability perceived by looking objectively at the diagnostic and research functions of the solution process and ways of achieving results. Comment has already been made on the diagnostic function and suggestions offered for improvement in teaching and learning strategies. What follows then, is one method to evince the evidence for one aspect of the research function. The procedure for tying down the evidence was to investigate whether the survey population had adequate competence in the curriculum behaviours required by the questions.

Consider now the performance of the questions as revealed by two very important parameters derived from the mean total mark per pupil and the mean mark obtained on the questions. The mean percentage total mark, $M_T$, was found by converting the mean total mark for those attempting the question into a percentage and $M_Q$, the mean percentage mark per question was similarly found. Using a two dimensional coordinate system with $M_T$ as the independent variable and $M_Q$ as the dependent variable, these two values for each question were plotted on the $M_T$, $M_Q$ graph as shown in Figure 6.3. According to Morrison [129], this graph was called The Question Synoptic Chart because the cluster of question locations gave a synoptic picture of the performance of the questions in the survey.

$M_T$ was the mean ability index for each question and since $M_T < 40$ per cent for the whole Bahamian sample, then the entire sample was of low ability because all of the questions fell to the left of Figure 6.3. Questions 1, 3 and 14 plotted on the same vertical line and, therefore, had the same mean ability index. Further observations are to be made about the independent variable $M_T$ but it was essential to look in depth at the basis of the analysis.

In the objective test analysis, facility was a crucial index of performance. But whereas the facility of an objective item was defined as the percentage of a sample making the correct
response, since the number of omits was usually small, the element of choice depresses this parameter if the respondents are of low ability and vice versa for high ability respondents. Added to this of course was the subjectivity of the researcher which could raise the facility if the marking was lenient or lower it, if the marking was severe. In fact, marks had to be adjusted where both of these possibilities occurred. Consequently, it was fortunate for the author that Morrison [138] defined the facility of a question so that it was independent of both the ability of the particular subgroup electing to offer the question and marker unreliability. So, assuming the researcher was an ideal examiner - neither lenient nor severe - a homogeneous group of average ability pupils (which in practice is very difficult to find) and questions which were matched and discriminating effectively, the mean mark per question, $M_Q$ would be 50 per cent and the mean ability index, $M_T$, would be 50 per cent. Conceivably, $M_T$ may change to say, 35 per cent, but provided questions and examiner remained ideal, $M_Q$ would also change to 35 per cent. Similarly, $M_T = 55$ per cent would correspond to $M_Q = 55$ per cent. Consequently, given ideal questions marked by an ideal examiner, the locus of performance by different ability groups lay on a line of unit slope passing through the origin of 50 per cent, 50 per cent.

Proceeding to the effect of operating the choice variable instead of changing the mean ability of the entry, different $M_T$ values would emerge for each question and these values would be located at different points along the line of unit slope which passes through 50 per cent, 50 per cent. Importantly, since $M_T$ would vary along with choice, the facility of the question would remain 50 per cent.

Finally under ideal conditions but changing the question and using say Q. 2 from Table 6.51 (see p.462), where $M_T = 40$ per cent and $M_Q = 54$ per cent, as the ideally marked question. Firstly, the question was located at 40 per cent, 54 per cent and all questions such as this one under the influence of choice, would lie on a line of unit slope passing through 40 per cent, 54 per cent. Secondly, since an $M_T = 40$ per cent-entry obtained a
mean percentage mark of 54 per cent for this question, this line may be regarded as the facility index line for 54 per cent as was shown on the Question Synoptic Chart (Figure 6.3). For choice type questions therefore, the facility index, $F$, was found by drawing a line of unit slope through $(M_T, M_Q)$ and taking the value of $M_Q$ where this line intersected the $M_T = 50$ per cent axis. This procedure eliminated the effect of choice. As an independent check the formula

$$F = 50 + M_Q - M_T \text{ (per cent)} \ [127]$$

was used.
6.4.4 Calculation and use of discrimination coefficients

Table 6.51 also shows the discrimination index, D, for each question. D is the Pearson Product-Moment Coefficient of Correlation [154, pp 370-374] between the mean marks for a question and the corresponding mean total marks gained by the candidates on all other questions: i.e. the product-moment coefficient of correlation between the means of $X_Q$ and $(X_T - X_Q)$, where $X_Q$ was the total marks for a question and $X_T$ was the total marks for the test [138, p 5]. The data cards, the information bank derived from the examination scripts, were sent to the Computer Centre along with a programme from which the variables were calculated, followed by the correlation between the means of each ordered pair $(X_Q, (X_T - X_Q))$. During the same exercise, the significance for each coefficient was calculated at the 0.001 level [154, pp 191-194]. As performances on the individual questions showed, the sample was pretty heterogeneous in ability. Consequently, the research function of the correlation between the means of the ordered pairs $(X_Q, (X_T - X_Q))$ was to give a measure of the potential of the questions to sense differences in ability among the individuals in the sample and to separate the respondents into pockets of low, average and high ability. These indices, which are sample bound, are therefore unlikely to obtain for a similar age group or, for that matter even apply to the same group, offering the same test on a subsequent occasion [132, pp 86-88].

6.4.5 Validity

Essay-type tests suffer from a lower validity — especially decreased content validity — than any other form of assessment partly because they require pupils to demonstrate the ability to structure arguments [155, pp 209-212]. Unlike Yaseen [102, pp 2-36, 37], the researcher could not invite a panel of practising professionals to assist in a discussion of
content coverage but in order to go as far as possible towards satisfying content validity, the domain of behavioural objectives of The Bahamian Grade 10 syllabus formed a basis for the blueprint which was thoroughly contemplated so that all appropriate topics and behaviours, except one, were sampled in reasonable proportions. For, as Mehrens and Lehmann [151, p 111] suggest

If these grids are both carefully constructed and carefully followed in building the test, this will do much to ensure adequate content validity.

In addition, heads of mathematics departments as well as some teachers from senior high schools were shown the test paper on pilot visits prior to the administration of the instrument to ensure that questions were within the experiential capability of the sample and hence, a fair means of eliciting a high degree of efficiency in the learning of specific behaviours. Except at one government school, the test was thought to be fair and balanced.

Since there was no formula for establishing quantitative indices of construct validity, the researcher elected to interpret this aspect of the survey qualitatively with respect to some psychological findings of Bajpai and Bajah [5] since Ebel [149, p 437] says

*Construct validity is concerned with "what psychological qualities a test measures", and one evaluates it "by demonstrating that certain explanatory constructs account to some degree for performance on the test".*

The author acted on the suggestion of Professor Bajpai, that diagnostic test instruments should be administered in The Bahamas, in order to test his (this author's) claim that the Bahamian pupil acquired an inefficient, mechanical and sterile acquaintance with mathematics education in the secondary school. Also seven constructs from Bajpai and Bajah's (unpublished) Report had to be verified. These constructs are listed below.
CONSTRUCT 1  Para 2 ... students entering the College seem to do poorly in mathematics

CONSTRUCT 2  Para 47  Children find mathematics difficult,... and they have unfortunately been used to hearing about the difficult nature of the subject from their parents

CONSTRUCT 3  Para 60  ... when children leave the primary school they should be able to perform the basic operations of arithmetic, know their multiplication tables, etc

CONSTRUCT 4  Para 72  ... low standards of achievement in the basic skills acquired by children when they enter the junior high and senior high schools from government primary schools

CONSTRUCT 5  Para 76  Mathematics they (the pupils) all said was a difficult subject ... (See also Para 124)

CONSTRUCT 6  Para 91  They (the parents) were all conscious of the importance of science and mathematics but felt that these were difficult subjects ...

CONSTRUCT 7  Para 112.  In mathematics the children exhibit a lack of basic knowledge and skills, including computation and place value, that they should have acquired in earlier years

In a subsequent section, a decision will be taken on the outcome of testing the researcher's experience and the constructs.
6.4.6 Reliability

The reliability of this choice-type test was calculated using the modified Kuder-Richardson formula

\[ R = \frac{k}{k-1} \left( 1 - \frac{6 \left( \sum \left( \frac{F}{100} \right) - \frac{\sum \left( \frac{F}{100} \right)^2}{\sum D} \right)}{\sum D} \right) \]

where \( F \) and \( D \) are the facility and discrimination indices and \( k \) is the number of questions on the test paper. Since the number of questions on the paper, \( k \), was constant, the reliability of the test, \( R \), was a mathematical function of the two variables \( F \) and \( D \). But relatively arbitrary decisions, based partly on the author's experience of Bahamian pupils' performances and partly on exposure to research evidence, had already been taken on the level of discrimination, \( D \), below which a question was considered inadmissible and the parameters of facility, \( F \), outside which a question was deemed too difficult or too easy to merit retention (Table 6.51). That being the case, the author had an intuitive feeling that, despite other sources of error, the reliability, \( R \), would be a pretty dependable measure of the research function which was the internal consistency of the test. For quick calculation of this parameter, a teacher only requires familiarity with a calculator which has a statistics mode.

Errors in educational measurement can be relatively quite large and, as mentioned before in another context, originate from four main sources with consequent reflection in the reliability index. Firstly, the marking can be subjective to the extent that marks on the same answer could vary between different markers, but this was less likely to happen with the researcher alone marking the scripts. Secondly, the mental response to the language of the questions was not constant for every candidate. Thirdly, pupils have 'off' days as well as days when they feel more confident. In fact
some could also have had a run of good luck in their choice of questions. Finally, the method of sampling could have produced a mismatch in the level of ability from the different streams of the same year. Moreover, Morrison [127, pp 56-57], in his paper on Question Analysis and Item Validation, thinks that for $k < 15$, the reliability coefficient is not a trustworthy statistic. Although it occurred as an after thought that a question could have been included on quadratic equations, at the time of construction of the test paper, it was felt that an extra question would have required additional time for reading, thinking and decision-making by each pupil. However, the small standard error of 0.83 was consistent with the high reliability index of 0.78. If this reliability were untrustworthy, there would be much greater variation reflected in a larger standard error. One conclusion was that this high reliability gave confidence in the ranking of the candidates who may form a standardisation sample for senior high schools on New Providence.

6.4.7 Discussion

A thorough objective study of the question characteristics in Table 6.51 illuminated the following observations. Although Question 14 was very popular with that part of the subsample of average ability, it was very easy and quite effective at sorting out candidates within their range of ability. Questions 1, 2, 7 and 13 whilst also maintaining a very good sorting power, easily appealed to that part of the entry possessing average ($M_T = 33$ per cent) to slightly above-average ability ($M_T = 40$ per cent) for the total sample, and it appeared that the more popular a question in this group, the lower the ability of the subsample answering it and vice versa. For example, $M_T = 33$ per cent and the Choice Index, $C$, was 83 per cent for Question 1 whilst at the other extreme, $M_T = 40$ per cent for Question 2 but the Choice Index, $C$ was 27 per cent. But the apparent exception to this rule was
Question 7 where $M_T = 35$ per cent and in contrast the Choice Index was 73 per cent. Questions 3, 4, 9, 10 and 11 with coefficients 0.73, 0.68, 0.70, 0.60 and 0.73 respectively, discriminated best among the candidates and illustrated in no uncertain fashion that the least popular questions were attempted by the more able candidates since for each of these $M_T$ was correspondingly 33 per cent, 38 per cent, 37 per cent, 30 per cent and 36 per cent. Question 6, moderately difficult and chosen by over half of the total sample, achieved good discrimination although it was attempted by those of average ability ($M_T = 30$ per cent) for the sample. However, Questions 5 and 8 difficult though they were ($F = 33$ per cent and $F = 34$ per cent respectively) appealed to the below average part of the total entry - $M_T = 26$ per cent and $M_T = 27$ per cent respectively - with almost equally effective discrimination power as Question 6. Finally, Question 12 was very difficult ($M_T = 26$ per cent), appealed to almost half of the total entry (Choice Index, $C = 49$ per cent), and had a sorting power ($D = 0.40$) which was only just adequate. This minimum potential to discriminate, $D = 0.40$, was necessary and sufficient to withhold the decision to reject any of the 14 questions on the test. In effect, these questions may be banked for future use, which in fact was another objective of the survey exercise since The Bahamas needs practicing mathematics educators with experience in assessment techniques.

As far as was possible, the researcher aimed at making the questions of equal difficulty but a very popular subsample of about the average mean ability, $M_T$, for the total entry found Question 14 very easy. It does not follow that if higher ability candidates had chosen the question they would have found it easier. In fact, this index appears to be nothing more than an indication of the percentage of the level of ability of the subgroup choosing to offer this question. The main function of the diagnostic test was to find out what pupils do not know so that before proceeding to further work,
a teacher can use different methods to re-teach previously
taught lessons, provide graded enrichment revision exercises
and allow teacher-pupil and pupil-pupil discussions [50]
along with practical applications and demonstrations, since
mathematics is a hierarchical and practical subject. It must
be pointed out however that although practical applications of
mathematics do help to motivate pupils, it may not always be
possible to find them for all of the content areas of the
secondary syllabus. On the basis of the analysis and
performance of the questions in the field trial, the diagnostic
test had one very easy question (Question 14) balanced by a

<table>
<thead>
<tr>
<th>Q</th>
<th>Attempts</th>
<th>C%</th>
<th>$M_T$%</th>
<th>$M_Q$%</th>
<th>F%</th>
<th>D</th>
<th>Comments</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>200</td>
<td>83</td>
<td>33</td>
<td>51</td>
<td>68</td>
<td>0.72</td>
<td>Easy but very good sorting power</td>
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<td>66</td>
<td>27</td>
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<td>54</td>
<td>64</td>
<td>0.55</td>
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<td>191</td>
<td>79</td>
<td>33</td>
<td>36</td>
<td>53</td>
<td>0.73</td>
<td>Excellent performance</td>
</tr>
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<td>100</td>
<td>41</td>
<td>38</td>
<td>35</td>
<td>47</td>
<td>0.68</td>
<td>Good performance</td>
</tr>
<tr>
<td>5</td>
<td>165</td>
<td>68</td>
<td>26</td>
<td>9</td>
<td>33</td>
<td>0.50</td>
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</tr>
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<td>139</td>
<td>58</td>
<td>30</td>
<td>18</td>
<td>38</td>
<td>0.61</td>
<td>Moderately difficult</td>
</tr>
<tr>
<td>7</td>
<td>176</td>
<td>73</td>
<td>35</td>
<td>48</td>
<td>63</td>
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<tr>
<td>8</td>
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<td>38</td>
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<td>59</td>
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($M_T$ for total entry = 30%)
very difficult one (Question 12); three difficult questions (Questions 5, 6 and 8) and four easy ones (Questions 1, 2, 7 and 13); and five (Questions 3, 4, 9, 10 and 11) falling within the limits of acceptability. Having considered this feedback critically along with the structure and content of the questions, as well as the mark schemes against the background of the syllabus for the year group in addition to the test specification grid, the diagnostic test administered to the senior high school sample in The Bahamas was a very well-balanced research instrument. But it could be further argued that the limit of balance would be improved if Question 14 were modified to reduce the facility and Question 12 had its facility raised to appeal to a higher ability subgroup and simultaneously increase its power of discrimination.

The Question Synoptic Chart was a two dimensional grid with the independent variable $M_T$ on the horizontal axis, the dependent variable $M_Q$ on the vertical axis and the origin at 50 per cent, 50 per cent (Figure 6.3). Any question which matched the ability of the sample to whom it was administered plotted on the Facility Datum Line, with the boundary lines for easy and difficult questions set at 60 per cent and 40 per cent respectively. Since vertical separation of the questions was a function of facility, Question 12 plotted at the bottom and Question 14 at the top. In a similar way, horizontal separation was a function of the mean ability of those choosing to answer the questions. The lowest ability candidates opted for Questions 5 and 12 and those which plotted to the right, Questions 4 and 2, were elected by the candidates of higher ability in the sample.

Interesting features arising out of this sample are that Questions 2 and 13 plotted on the same line of unit gradient which passed through 64 per cent on the $M_Q$ axis, and consequently had a facility of 64 per cent (Figure 6.3). Questions 1, 3 and 14 plotted on the same vertical line and were therefore chosen by candidates of the same mean ability, $M_T = 33$ per cent.
So also were Questions 5 and 12 where $M_T = 26$ per cent. *Very significantly*, Questions 6 and 10 lay on the same vertical line where $M_T = 30$ per cent, which in fact turned out to be the mean ability of the whole senior high school sample and dovetailed with the mean already established in Figure 6.3.

All of the questions on this diagnostic test plotted to the left of $M_T = 50$ per cent, thus indicating that the questions were pitched above the mathematical ability range of the sample. Accordingly, the implication here was that either the marking was severe or the entry was rather poor. The marking scheme for each question was set adjacent to the behaviours expected in each solution. The researcher was consistently idealistic in marking even to the point of awarding half of a mark. The conclusion most assuredly was that on the basis of mean attainment, the sample from Bahamian senior high schools, since the language and mathematics content were within the grasp of 15 year olds, was rather poor. That remark however in no way must be understood to mean that the sample was ineducable since no intelligence test was administered. It could be argued that the test was constructed to assess purely academic concepts with no opportunity for the practically inclined to demonstrate employable skills. Whilst that is fair comment, the researcher was based in a research studentship on the other side of the Atlantic Ocean without the support of his country's Ministry of Education and Culture and had to prepare something on the outside which could be administered inside the country whilst, simultaneously, maintaining reasonably good human relations within the schools. At the root of this magnitude of under-achievement in mathematics was the fact that these pupils were not all taught in a system of uniform methods. If they were, then Krutelskii [46, p 5] in discussing the theoretical and practical importance of mathematical abilities, says that

Bogoyavlenskii and Menchinskaya are entirely right when, in discussing differences in the mastery of information by pupils under the same teaching conditions, they indicate that the explanation must be sought in the traits of the pupil himself.
So the conclusion that the teaching and learning strategies have to be modified with corresponding alterations in the mathematics curriculum is inescapable although it must be conceded that a percentage of any cohort of pupils will not benefit from improved instruction and better conditions of learning.

Deriving from the reliability, validity and discussion, the second part of the hypothesis, which related to the state of mathematics teaching and learning in senior high schools on the island of New Providence in The Bahamas, set out at the end of Chapter 2, namely:

The Bahamian secondary pupil in the state-maintained sector of education has not acquired as full an understanding of the basic mathematical concepts as a prerequisite to enjoyment, progress and success at further studies in mathematics, and in particular for success in:

(1) …

(2) University of London GCE 'O' level mathematics as defined by Syllabus A360 for overseas candidates

now stands uncompromisingly proved.

NOTE: For the benefit of the reader the Course Outlines of Mathematical Methods followed by the College of the Bahamas and the Mathematics Syllabuses for Grades 10 and 11 are also appended in Appendices 6.7&6.8 respectively.
6.5 Conclusions

The abysmally low mean performance of the sample senior high school population from The Bahamas demonstrated that:

1. the researcher's claim, which derived from 19 years of professional practice, was substantiated
2. the constructs, which emerged from an unpublished pilot study [5], did not contradict reality.

The combination of the researcher's past experience and the constructs from the pilot study were mutually exclusive findings which formed the hypothesis for the sample survey conducted in The Bahamas. Clearly, the convergent conclusion—that both the researcher's experience and the constructs were true—implies that the hypothesis has been proved by the data obtained from the diagnostic testing exercise. Indeed, Table 6.51 has already shown that the mean ability index, $M_T$, for the entire field class was 30 per cent. Thus, there were yawning gaps in both knowledge and understanding of curriculum skills, which must also mean that many pupils needed confidence in their own ability. This diagnosis of deficiencies was a signal to guide teachers into techniques of closer observation of the pupils and remedial teaching. Questions were selected because they covered knowledge that must be acquired if the pupils were to make progress in learning mathematics. They were a pre-condition for following a secondary mathematics course. Also, the policy of the Ministry of Education was that basic traditional mathematics should be taught in all state maintained secondary schools. However, the author feels that an integrated mathematics curriculum with a judicious blend of traditional and "modern" topics is better professionally and from the point of view of a broadly based, secure foundation in mathematics education.

Admittedly, pupil interviews before and after the test, indications from teachers of cognitive and affective attributes such as class participation and mathematical ability along with interest and confidence respectively, summative achievements from record cards all in conjunction with testing would have
provided a comprehensive profile of pupils' attainment at any given time in the learning experience; also it was uncertain whether the sample of senior high school pupils had achieved comparable mathematical ability or at least had the same learning opportunities backed by an equal degree of motivation towards the subject; it is therefore unwise to make too much of these test results in isolation. However, these test results do provide a basis for drawing conclusions about the effectiveness of both the learning and teaching of mathematics in Bahamian senior high schools from which the mean age of the sample was 15.5 years.

The strength and clarity of this evidence from the diagnostic test compels the ultimate and inescapable conclusions - the principal findings of this research - that

1. A well-thought out programme of in-service education and training in mathematics education, similar to the Nuffield Mathematics Teaching Project and the AIMEC Project must be mounted in The Bahamas as a matter of urgent necessity since initial training cannot and was not intended to give teachers all the skills they will need throughout their professional career. The quality of the teaching and learning depends on those who are already in jobs in the schools and who form the majority of the teaching force. Curriculum development with a project bias may be started as a joint venture between the College of The Bahamas and the School of Education at the University of the West Indies since the latter institution already endorses teacher training certificates issued by the college and runs an in-service B.Ed programme, in which the researcher participated as a tutor, for experienced teachers.

2. Initial teacher education and training for primary schools must continue but strong emphasis must be placed on recruiting teacher trainees for the 11-16 age group.
The academic level of individual teachers needs to be raised and up-dated in mathematical content in order to give them a firmer grasp of the varied developmental approaches in methodology. The author is confident that, should the decision that the College of The Bahamas prepares for the teaching of mathematics to degree level be taken in the interest of ensuring that present and future requirements of the senior high schools are met, he can, as a result of his experiences in CAMET, lead a team in formulating curriculum proposals and in teaching mathematics content, using current practices in methodology as were employed in institutions of higher learning in the United Kingdom.
CHAPTER 7

CONCLUSIONS, IMPLICATIONS, RECOMMENDATIONS AND FURTHER RESEARCH
Conclusions, Implications, Recommendations and Further Research

7.1 Summary

It seems to the author that the credibility of any piece of educational research rests in part on a well constructed framework which clearly defines the

(1) Problem
(2) Design
(3) Procedure
(4) Analysis, and
(5) Conclusions

and in part on the thoroughness with which the information gleaned within the domain of each of these concepts is treated. Consequently, not only would the study be easily submitted to objective evaluation but also of inestimable value to the general reader of research as well as to the specialist.

7.1.1 The Problem

Chapter 1, an introduction, described the interaction between the physical and ideological culture and evolutionary trends in space and through time under the umbrella of an overseas metropolitan power, thereby conveying an intuitive feeling of the significance of the problem and urgency of solution. But the problem itself was postulated in Chapter 2, at the end of which was put forward a hypothesis, a logical derivative of seven constructs. The constructs form a natural link between the work of Bajpai and Bajah [5] and this research. Incidentally this Report of a Pilot Study [5] is the only one of its kind on Mathematics and Science Education in The Bahamas.

7.1.2 Design

The main assumptions were that teachers were working to the spirit of the syllabuses for Grades 8 and 10 laid out in Appendices 5.5 and 6.8, and pupils had acquired the final
behaviours therein defined. The total population and samples have been fully described; the method of sampling was appropriate; and controls -

(i) that pupils should have been registered and attending classes regularly during the previous year

(ii) that the subsample from each school was greater than or at least equal to 10.0 per cent of the year's cohort -

were appropriate and enforced. Important terms either have been defined or more frequently, their context indirectly provided a definition. The research design was, therefore, fully described; was appropriate to the aim of the survey; and to the best of the author's knowledge, is free of structural weaknesses. However, although the catalogue above constitutes crucial circumscribing parameters defining the domain of definition of this research report, the elements of this catalogue do not guarantee immunity from limitations. The fundamental limitation was distance from the cultural problem space to which the field survey related. The subproblems stemming from this were

(i) expensive travel

(ii) lack of communication with mathematics teachers, educational administrators, and groups of pupils who offered the research instruments.

With reference to the pupils themselves, their performances were adversely affected by

(i) short notice of pending tests

(ii) inadequate time for revision

(iii) lack of acquaintance with the test-taking routine

(iv) home background

(v) religious traditions

(vi) teacher expectations

(vii) support services provided by schools eg libraries, etc.
7.1.3 Procedure

The data-gathering methods were appropriate, effectively used and described in every detail in Sections 5.1.1 - 5.2 and 6.1.1 - 6.3 of Chapters 5 and 6, for which reliability indices were 0.88 and 0.78 pro rata; and for each instrument the evidence in favour of predictive, content and construct validity was securely established.

7.1.4 Analysis

The difference in method was dictated by the type and number of items and questions on each research instrument. But each method was appropriate, applied so as to extract the maximum amount of useful information, and the results are clearly presented in Sections 5.3 - 5.3.3 of Chapter 5 and Section 6.4 - 6.4.4 of Chapter 6. However, in the analysis of some of the pupils' responses reproduced in Chapter 6, examples of problems equivalent in content have been cited from various sources, which could be used to trigger off discussions with teacher trainees. Where appropriate, it is advisable to adapt the language in which some of these examples are clothed to reflect aspects of local culture.

7.2 Findings

The following are findings derived from various parts of this study:

(1) reliability of 0.88 and 0.78 for objective and choice-type instruments pro rata

(2) high estimate of (i) content validity as indicated by content-behaviour grids (Tables 4.1 and 4.3 of Chapter 4); popularity of instruments with both pupils and teachers as well as conformity with the syllabuses (Appendices 5.2 and 6.8)

(ii) predictive validity satisfied by information derived from the cumulative frequency distribution curves, Figures 5.1 and 6.1, and manifested in Tables 5.3 and 6.4, the mark-range grade distributions
(iii) construct validity - identification of those pupils with learning difficulties (Grades D and E in Table 5.3 and in Table 6.5)

(3) high percentage of the behaviours, knowledge, and techniques and skill in content-behaviour grids (Tables 4.1 and 4.3) weigh heavily in favour of sampling for the acquisition of basic concepts

(4) mean achievement 26.7 with standard deviation 8.5 corresponding to mean age 13.5 years; and mean ability index $M_T = 30.0$ per cent corresponding to mean age 15.5 years for junior and senior high schools pro rata altogether add up to an uncontested and rigorous vindication of the hypothesis postulated in Section 2.7 of Chapter 2. That is to say, the hypothesis is true for $n_1 = 377$ and $n_2 = 241$ where $n_1$ and $n_2$ are the number of pupils offering the research instruments from junior and senior high schools respectively. At this juncture, without going into the mathematical structure of the inductive procedure, incomplete mathematical induction - simply called mathematical induction in the literature - permits the researcher to conclude that the hypothesis was also true of the whole junior and senior high school populations on the island of New Providence in The Bahamas during the beginning of the academic year 1981/82.

The aim of this research has therefore been achieved: namely, to make a thorough study of final behaviours within curriculum content - mainly knowledge and techniques and skill but with a few items and questions on comprehension and applications - so as to identify the reasons for poor performance in BJC and GCE 'O' level mathematics. It seemed that up to 1981, the emphasis in teaching mathematics in the maintained high schools on New Providence was on sterile calculations, using 'chalk and talk' methods which largely consisted of 'telling how'. This artificial skeletal mathematical structure could do no more than encourage instrumental learning and further descent into a mathematical abyss where the Euclidean axiom - theorem - proof
syndrome was totally unproductive. Consequently the pupil population did not enjoy the subject and lacked motivation to pursue further studies in the discipline which gives meaning and purpose to a scientific and technological Western society on which The Bahamas is modelled. Mathematics has to be taught by methods from which pupils achieve a comprehensive grasp of the basic concepts, the utility of which they, in turn, demonstrate fluently in applications of mathematical thinking to real-life problem-solving situations.

It follows therefore that all conclusions are clearly stated and are substantiated by the evidence presented; generalisations are confined to the junior and senior high school populations from which the samples were drawn; and the report is logically organised and clearly written, with an impartial and scientific tone.

7.2.1 Some Observations

The author got direct exposure to the problems of mathematics education as Assistant Chief Examiner for BJC mathematics in the summer of 1968. His interest in these problems were kept fully alive by membership in the Association of Teachers of Mathematics, which dates back to early 1966 whilst a Commonwealth Bursar at the University of Southampton. But this thesis started as a follow-up to the Pilot Study reported by Bajpai and Bajah [5]. The core of this research is contained in Chapters 5 and 6, the high reliability of which is sufficient evidence for the whole field survey exercise in the summer 1981 to be justifiably regarded a success. The analysis contained in Chapter 5, when projected backwards, indicates not only that primary teachers need the support of in-service education and training in mathematics content and pedagogy: it also suggests that pre-service teachers need their coverage of mathematics content increased in depth and their scope of pedagogy diversified to emphasise mixed ability teaching, a form of individualised learning. This action is necessary to eradicate the weaknesses in the primary sector which are filtering upwards by a process akin to capillary transfer of liquids in physics, chemistry and biology.
Whereas the research instrument analysed in Chapter 6 did not provide an equivalent breadth of coverage of the Grade 10 syllabus as did the previous chapter for the Grade 8 syllabus, indicated by a lower percentage of one in ten but quite effective index of reliability, its yield of information has far-reaching importance in the sense that it explains in part, why so many pupils drop out of the system at the end of senior high school and also why, for those who manage to enter the College of The Bahamas, standards of attainment and motivation are so low. For increased motivation towards mathematics beyond the senior high school curriculum, a pedagogy must be applied which not only brings intuition, insight and assimilation into play but also gives pleasure and enjoyment to pupils. As a corollary, the teacher has to be supremely confident of mathematics content to a depth in excess of that at which he is required to teach.

Although successful use of the scientific method means that induction and deduction were logically and effectively used, unique and permanent solutions cannot be guaranteed: that is to say, successful use of this method does not imply infallibility in the implications one puts forward. All the scientific method does better than any other is to provide an impervious barrier against the penetration of any subjective interference. Yet however, this empirical knowledge so derived is especially useful in the sense that it can give hunches for further work as well as give guidance to teachers and policy makers for the improvement of mathematics teaching and learning.

Concerning learning in general, geographical distance combined with segmented land mass cause disparity with respect to educational support services, and the problem is not helped by lack of an equitable formula for distribution of incentives to encourage the better qualified mathematics teachers to work in the distant parts of The Commonwealth. This has the effect of limiting the potential pool of trained and educated manpower services on which the country can draw. Yet despite lack of incentive, the author worked successfully (measured against performances in local public examinations) and enjoyed his
appointments at Stanyard Creek, Andros; Rolletown, Exuma; and George Town, Exuma. Having followed some of its mathematics teaching telecasts, the Open University in the United Kingdom is a classic prototype; and, with the advent of satellite communication, The Bahamas comprise an ideal laboratory for investigations and trials in distance teaching and learning aimed at improving the quality of educational services offered to peoples who are similarly circumscribed by tradition, custom and convention. Some form of distance teaching is especially important because it is an effective means of propagation of educational technology thus serving to prevent teachers in isolated posts from resorting to methods by which they were taught as pupils. Supporting the author's viewpoint, Butts [156, p. 27], in an examination of the relationship between distance learning and educational technology, contends

...those who maintain that the application of educational technology can give distance learning a new lease of more effective life have strong arguments at their disposal. Any form of distance study which is backed by a tutorial organisation is essentially a learning system; and the design of learning systems is the basic concern of educational technology.

Now most distance teaching has as its chief target group mature citizens with professional, social and family responsibilities, but full-time students on and off campus can also make use of distance teaching. An important technological device with which mature employees will have to learn to cope for greater efficiency in their jobs is the electronic calculator. Edwards [157, pp 200-213] supports the author by offering empirical evidence that the calculator is an indispensable weapon in teaching the skills of basic numeracy. However the School Mathematics Project (SMP) goes further: in addition to producing an SMP Calculator Series of textbooks, it has also authored a Teacher's Guide for these publications. At this level also are Graham and Sargent [144], [140], written especially for use with a
To continue the ascending order or progression, Cheung [158], in a paper demonstrating how the calculator can be used as a good computational tool, shows also that it is an effective teaching and learning device in, for example, three content areas of mathematics - numbers, functions and equations. Her Majesty's Inspectorate [49] and Cockcroft [50] acknowledge the impact of electronic calculators on society and give more than a gentle hint that mathematics syllabuses will have to reflect it as a teaching and learning aid. Furthermore, Cornelius [159, p. 136] advocates that

Acceptance of the calculator as an essential element in external examinations at 16+ is a national necessity, both in itself and as a catalyst for CLAIM (Calculator Assisted Learning in Mathematics).

However some GCE boards have already given the green light on the use of calculators in their 'O' level mathematics papers. But optimism has to be tempered since there exists a school of mathematical opinion which suggests that the calculator is an alternative, but not essential, aid to calculation. The exception of course is that one syllabus has already been produced by SMP to feature the calculator as a very cheap but valuable learning aid. Most CSE pupils already own one, or at least have access to one at home, and often use it to check or do homework. Because they are generally accepted in solving problems in mathematics papers at GCE 'A' level and have become the means of carrying out all but the most trivial calculations for the vast majority of pupils in sixth forms, first year tutorial classes in the Department of Engineering Mathematics at Loughborough use them mainly for solving problems in Numerical Methods. In statistics, the author knows only too well that calculators speed up the achievement of answers, thus leaving more time for reflection upon the associated statistical principles and inferences. Aware of this pervasive impact, Cornelius [159, p.135] concludes:

Perhaps an educational system like our own, but operating during Napier's lifetime, would have
fossilised his 'bones' into its mathematical syllabuses and rejected his more potent invention! In direct contrast, the calculator has been welcomed with open arms by industry, commerce and the public at large. It is therefore reasonable to hope that this assimilation can be helped by what takes place in our schools during the 1980s.

From seminars conducted in AIMEC by comprehensive and grammar school teachers and mathematics educators, combined with a searching review of relevant literature, the author suggests that, since The Bahamas is in the mainstream of Western influence, age 13+ is about the most appropriate time to give formal lessons on the use of the electronic calculator in Bahamian junior high schools although some pupils would have used them earlier. But this suggestion presupposes familiarity with place value in the number system as well as the concepts of fractions, percentages and elementary operations on these. The real impact of the electronic calculator may be gauged from the fact that it has made the mechanical desk calculator, slide rule, logarithms and pencil-and-paper algorithms redundant. However, the key to effective learning is still the teacher.

7.2.2 Implication: Junior High School (Grade 8 syllabus)

On the 50-item diagnostic test, 68.0 per cent of which covered basic number concepts and operations (see Chapter 3, pp 111, 112), 34.0 per cent of the final behaviours displayed in the specification grid on p 214 had not been satisfactorily assimilated. This finding is justified by the results of the research function of the diagnostic test as communicated for

Techniques and skill: Items 22, 23, 24, 29, 31, 32, 33, 34, 49, 50

Comprehension - translation, interpretation: Items 45, 43, 48

Applications: Items 10, 11, 39, 42
in Table 5.54, p 345. As indicated by the catalogue of items above, 41.2 per cent of the syllabus content of which pupils had an insecure grasp were the higher order behaviours, comprehension (translation and interpretation) and application. The other 58.8 per cent of the syllabus content concerned basic concepts, the structure of which was not integrated into the structure of the pupils' intellect.

7.2.3 Implication: Senior High School (Grade 10 syllabus)

The 14-question choice-type diagnostic test had 42.9 per cent algebra, 21.4 per cent geometry/trigonometry, and 35.7 per cent arithmetic (see Table 4.4, p 218). Questions 4, 5, 6, 8 and 12 or 35.7 per cent of the final behaviours were not retrieved and used. Of these five questions, 80.0 per cent required the use of the higher order behaviours, comprehension and application, which, from the evidence in Table 6.51, p 462, had not been satisfactorily assimilated.

7.2.4 Conclusions

There was too much

(i) 'chalk and talk'

(ii) 'telling how'

(iii) of the 'definition-theorem-proof' (Euclidean) approach along with a high incidence of

(iv) sterile calculations without application to Bahamian culture

(v) inability to transform the vocabulary of mathematics into symbolic manipulable form

which together encouraged rote learning at the expense of relational understanding.
7.3 Recommendations

I. In order to meet present and future requirements in mathematics education, The Ministry of Education should ensure that the College of The Bahamas:

(i) conducts curriculum development workshops of two* weeks duration in content and methodology in

1 New Providence (two areas)
2 North Andros, Central Andros and Berry Islands
3 South Andros
4 Grand Bahama and Bimini
5 Abaco and surrounding Cays
6 Eleuthera
7 San Salvador, Cat Island and Rum Cay
8 Exuma, Long Island and Ragged Island
9 Mayaguana, Acklins, Crooked Island and Long Cay

[5, p 36]

(* The possibility of running one week workshops could also be explored)

(ii) initiates discussion with an institution in the United Kingdom with a view to making places available to primary and secondary teachers on a one year in-service training scheme similar to the AIMEC Project (Financial support for such a project should also be sought from aid-awarding bodies).

(iii) admits prospective mathematics teachers on the understanding that they follow a two year curriculum of post 'O' level content, at the end of which one year is to be spent in training as teachers
(iv) provides a core mathematics curriculum leading to 'A' level for all students who offer mathematics but with applications for those who will seek employment in technology, agriculture, business and commerce, and management.

(v) trains teachers to use the electronic calculator in secondary schools

(vi) helps schools to establish procedures for closer observation of pupils' progress and encourage their use to improve the management of learning

(vii) begins teaching mathematics beyond 'A' level as soon as qualified staff and demand permit.

II. Individual teachers must keep abreast of current ideas in relevant content and methodology. Teachers' centres should be set up to encourage this.

III. In the developed countries and many developing countries, Teacher Associations have been effective agents of change in curriculum development. These associations also give a suitable forum to both primary and secondary teachers for discussions and dissemination of ideas and experiences. An Association of Mathematics and Science Teachers should be started in The Bahamas as a matter of priority.

IV. Pupils, teachers and parents should be encouraged in setting up and participating in maths and science clubs. They should also be invited to set up special projects/mini-projects to demonstrate the relevance and applications of mathematics in the sciences and humanities.
7.4 Suggestions for Further Research

(i) Research along the lines of this thesis could be advantageously carried out in the primary mathematics curriculum.

(ii) Further research should be done to define levels of achievement in secondary pupils as started in 6.4.2 of this thesis.

(iii) There exists a need to research the effects of the organisational structure of mathematics in various schools on pupils' attainment.

(iv) Investigations should be started to determine whether curriculum content is appropriate to the ability and needs of secondary pupils.

7.5 Final Remarks

Efforts must be made to redress the balance between 'chalk and talk' and 'guided discovery learning'. Other methods of introducing geometry as suggested in (6) on p 115, must be vigorously pursued. It cannot be over-emphasised that effective mathematics teaching

(i) emphasises the underlying structure of the subject as well as the higher order behaviours eg application, which together help to prepare pupils for employment.

(ii) engages a diversified methodology which takes cognisance of the strategy of individualised learning.

It is this author's hope that this thesis provides a basis for making realistic decisions to meet present and future requirements of secondary schools in offering an improved content of mathematics education to Bahamian pupils.
REFERENCES AND BIBLIOGRAPHY

The Bahamas, CUP

Technology and Economic Development, Penguin

[3] Cabinet Office, Bahamas, 1980,
Census of Population and Housing (Preliminary Review), Dept of Statistics

Studies in Mathematics Education, UNESCO

Developing Effective Science and Mathematics Education in Third World Countries (Unpublished Confidential Report for The Commonwealth Secretariat)

The Meaning of Inflation, Dept of Statistics, Bahamas

Bahamas Handbook, Dupuch Publications

Society Schools and Progress in the West Indies, Pergamon Press

A Short History of Education in Jamaica, Hodder & Stoughton

[10] Hans, N., 1958,
Comparative Education, 3rd ed., Routledge & Kegan Paul


A Handbook of British Educational Terms, Harrap


[14] Burgess, T., 1964,
A Guide to English Schools, Penguin

Board of Education, Annual Report, Nassau Guardian

[16] Cabinet Office, 1955,
Statute Law, Ch. 23 Education (Revised ed), Bahamas Government Printing Dept.
<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
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</tr>
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<tbody>
<tr>
<td>17</td>
<td>Durkheim, E.</td>
<td>1977</td>
<td>The Evolution of Educational Thought, Routledge &amp; Kegan Paul</td>
</tr>
<tr>
<td>18</td>
<td>Ministry of Education &amp; Culture</td>
<td>1971-75</td>
<td>Annual Reports, Bahamas Gov't Printing Department</td>
</tr>
<tr>
<td>20</td>
<td>OECD</td>
<td>1962</td>
<td>Planning Education for Economic and Social Development</td>
</tr>
<tr>
<td>21</td>
<td>Bottomore, T. B.</td>
<td>1964</td>
<td>Elites and Society, Penguin</td>
</tr>
<tr>
<td>22</td>
<td>Vaizey, J.</td>
<td>1962</td>
<td>Britain in the Sixties: Education for Tomorrow, Penguin</td>
</tr>
<tr>
<td>23</td>
<td>Cabinet Office, Bahamas</td>
<td>1975</td>
<td>Communication to Parliament, Bahamas Gov't Printing Department</td>
</tr>
<tr>
<td>25</td>
<td>Pedley, R.</td>
<td>1963</td>
<td>The Comprehensive School, Penguin</td>
</tr>
<tr>
<td>26</td>
<td>Jervier, W. S.</td>
<td>1977</td>
<td>Educational Change in Postcolonial Jamaica, Vantage Press</td>
</tr>
<tr>
<td>28</td>
<td>Khouj, A. M.</td>
<td>1982</td>
<td>A Study of the Relationship of Student Test Scores on Mathematics and Science Subjects with their Scores in Other Subjects in a suburban School of Jeddah, Saudi Arabia</td>
</tr>
<tr>
<td>30</td>
<td>Mead, M.</td>
<td>1955</td>
<td>Cultural Patterns and Technical Change, The World Federation for Mental Health, UNESCO</td>
</tr>
</tbody>
</table>


[34] Bloom, B. S., 1956, Condensed version of the taxonomy of educational objectives in the cognitive and affective domains in I. K. Davies (ed), The Management of Learning, McGraw-Hill


[37] OECD, 1961, New Thinking in School Mathematics


[40] Beeby, C. E., 1966, The Quality of Education in Developing Countries, Harvard University Press


[53] Beckford, G. L., 1972, Persistent Poverty, O.U.P.

[54] Wain, G. T., & Woodrow, D., 1980 Mathematics Teacher Education Project, Students' Material, Blackie


[63] Skemp, R. R., 1979, Intelligence, Learning and Action, Wiley


[77] Bell, E. T., 1953, Men of Mathematics 2, Penguin


[89] Davis, E., 1980, Teachers as Curriculum Evaluators, George Allen & Unwin, Australia


[94] Thwaites, B., 1972, The School Mathematics Project: The first ten years, CUP
[95] Cundy, H. M., 1977, The Caribbean Mathematics Project: Training the Teacher as the Agent of Reform, IERS, UNESCO


[103] Matthews, G., 1968, Against Examinations in Examinations and Assessment, Mathematics Teaching Pamphlet No 14, ATM, pp 7, 8


[113] Begle, E. G., 1979, Critical Variables in Mathematics Education, Mathematical Association of America & National Council of Teachers of America, Washington, DC
[117] Bajpai, A. C., 1980, AIMEC Project/CAMET Brochure, Loughborough University of Technology
[122] Satterly, D., 1981, Assessment in Schools; Blackwell
<table>
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<th>Reference</th>
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<td>125</td>
<td>Dunn, T. F. &amp; Goldstein, L. G.</td>
<td>Test Difficulty, Validity, and Reliability as Functions of Selected Multiple-Choice Item Construction Principles in Educational and Psychological Measurement, Vol. 19, No. 2</td>
<td>1959</td>
</tr>
<tr>
<td>126</td>
<td>McIntosh, D. M.</td>
<td>Statistics for the Teacher, 2nd Ed.</td>
<td>1967</td>
</tr>
<tr>
<td>127</td>
<td>Macintosh, H. G.</td>
<td>Techniques and Problems of Assessment, Arnol'</td>
<td>1974</td>
</tr>
<tr>
<td>128</td>
<td>Beeching, C.</td>
<td>Testing the Mathematical Attainment of 7 to 15 year olds by the NFER/IMA Bulletin, Vol. 9, No. 5/6</td>
<td>1973</td>
</tr>
<tr>
<td>130</td>
<td>Purushothaman, M.</td>
<td>Secondary Mathematics Item Bank</td>
<td>1976</td>
</tr>
<tr>
<td>133</td>
<td>Department of Education &amp; Science</td>
<td>Aspects of Secondary Education in England, Supplementary Information on Mathematics</td>
<td>1980</td>
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<td>134</td>
<td>Nassau Guardian</td>
<td>Ringing in Change</td>
<td>1981</td>
</tr>
<tr>
<td>135</td>
<td>Reichmann, W. J.</td>
<td>Use and Abuse of Statistics</td>
<td>1964</td>
</tr>
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<td>137</td>
<td>Meece, J. L. &amp; Parsons, J. E.</td>
<td>Sex Differences in Math Achievement: Toward a Model of Academic Choice</td>
<td>1982</td>
</tr>
<tr>
<td>138</td>
<td>Morrison, R. B.</td>
<td>A Method for Analysing Choice Type Question Papers, Educational Measurement Research Unit, School of Education, University of Reading</td>
<td>1970</td>
</tr>
<tr>
<td>139</td>
<td>Hope, C.</td>
<td>Midlands Mathematical Experiment Bk. 2</td>
<td>1964</td>
</tr>
</tbody>
</table>


[146] East Midland Regional Examinations Board, CSE, 1980, Solutions and Marking Schemes, Mode 1, Mathematics Syllabus 1, Paper 3B, (S33)

[147] British Association for Commercial and Industrial Education 1962, Mathematics by Visual Aids


Further References Relevant to Mathematics Education but not Quoted in the Text

Brimer, A., Madaus, G. F., Chapman, B., Kellaghan, T., & Wood, R., 1978, Sources of Difference in School Achievement, NFER


NFER, 1975, Mathematics Attainment Test EF:
(i) Manual of Instructions
(ii) Answer Booklet
(iii) Answer Sheet
(iv) Marking Template

Rust, W. B., 1973, Objective Testing in Education and Training, Pitman
Skemp, R. R., 1965, Understanding Mathematics 2, University of London Press
APPENDICES
# APPENDIX 1

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<td>454</td>
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<td>94</td>
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<td><strong>Gross domestic expenditure</strong></td>
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<td>166</td>
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<td>567</td>
<td>591</td>
<td>636</td>
<td>658</td>
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<td><strong>Imports</strong></td>
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<td>564</td>
<td>497</td>
<td>536</td>
<td>557</td>
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<td><strong>GDP at market prices</strong></td>
<td>561</td>
<td>682</td>
<td>612</td>
<td>651</td>
<td>701</td>
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<td>-79</td>
<td>-62</td>
<td>-64</td>
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<td>-84</td>
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<td><strong>Factor receipts</strong></td>
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<td>(130)</td>
<td>(126)</td>
<td>(118)</td>
<td>(135)</td>
<td>(156)</td>
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<td><strong>Factor payments</strong></td>
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<td>(-209)</td>
<td>(-188)</td>
<td>(-182)</td>
<td>(-179)</td>
<td>(-240)</td>
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<td><strong>GNP at market prices</strong></td>
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<td>644</td>
<td>687</td>
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<td><strong>GDP at constant prices</strong></td>
<td>505</td>
<td>515</td>
<td>480</td>
<td>490</td>
<td>507</td>
<td>537</td>
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1/ Includes changes in stocks.

Table: Bahamas - National Accounts Estimates (in $10^6$).
<table>
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<th>Period</th>
<th>Population</th>
<th>% Change</th>
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<td>1970</td>
<td>168,812</td>
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<td>1971</td>
<td>176,502</td>
<td>4.6</td>
</tr>
<tr>
<td>1972</td>
<td>182,906</td>
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<td>1973</td>
<td>189,728</td>
<td>3.7</td>
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<tr>
<td>1974</td>
<td>196,708</td>
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<td>1975</td>
<td>203,946</td>
<td>3.7</td>
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<td>1976</td>
<td>213,944</td>
<td>4.9</td>
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<td>1977</td>
<td>219,510</td>
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<tr>
<td>1978</td>
<td>225,220</td>
<td>2.6</td>
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</tbody>
</table>

**NOTE:**

(i) Population estimates for the period 1970-1975 are based on Assumption 'A' which assumes that the net immigration figure for the period 1963-1970 to be the same for each year during the period 1971-1975.

(ii) Population estimates for the period 1976-1978 are based on Assumption 'B' which assumes that net immigration declines each year.

**SOURCE:** Department of Statistics.
INSTRUCTIONS
Five answers are given to each question. ONLY ONE IS CORRECT.

Choose the answer you think is correct and circle the corresponding letter ON THE ANSWER SHEET. For example, if you think that B corresponds to the correct answer, mark the answer sheet like this:

A B C D E

You may do any rough work on the extra sheets provided.

PLEASE DO NOT WRITE ON THIS QUESTION PAPER

1 476 + 25 + 908 + 675 =
A: 8024 B: 4280 C: 2084 D: 8204 E: 2048

2 614 - 268 =
A: 346 B: 346 C: 634 D: 642 E: 364

3 203 x 9 =
A: 1821 B: 7812 C: 2817 D: 1827 E: 1728

4 2051 + 7 =
A: 293 B: 923 C: 932 D: 320 E: 392

5 In three days Baillou Hill Corner Shop sold 2026 eggs. On the first day 689 were sold and on the second day 105 more were sold than on the first. How many eggs did the grocer sell on the third day?
A: 345 B: 354 C: 453 D: 534 E: 543

6 When a certain number is divided by 17 the answer is 26 and there is a remainder of 15. What is the number?
A: 754 B: 745 C: 574 D: 457 E: 547

7 Write in numerals: seven thousand, two hundred and eighty six.
A: 7286 B: 700286 C: 70286 D: 72086 E: 72806

8 1.45 + 0.76 + 8.3 + 21.09 =
A: 39.61 B: 31.6 C: 63.1 D: 13.6 E: 16.3

9 This sheet of paper is ___ centimetres (cm) wide
A: 17.5 B: 18.3 C: 21.0 D: 20.4 E: 19.2

10 The distance from Trinidad to New York is 3550 kilometres (km). If an aeroplane flies at 500 km per hour, how long will it take to fly from Trinidad to New York? (Give your answer to the nearest hour)
A: 4 B: 6 C: 8 D: 9 E: 7

11 1 litre (l) = 1000 cubic centimetres (cc). How many litres of milk are contained in a box which measures 19 cm long 10 cm wide and 20 cm high?
A: 8.3 B: 38 C: 2.8 D: 8.2 E: 3.8

12 2.63 x 10 =
A: 26.3 B: 263 C: 0.263 D: 2630 E: none of these

13 65 ÷ 10 =
A: 0.65 B: 65 C: 6.5 D: 650 E: 0.065

14 52.61 - 41.93 =
A: 1.06 B: 10.68 C: 106.8 D: 1068 E: 0.1068

15 2.7 x 1.6 =
A: 0.0432 B: 0.432 C: 43.2 D: 4.32 E: 432
16 \[1.68 + 1.4 =\]
A: 0.12 B: 1.2 C: 12 D: 0.012 E: 10

17 \[\frac{2}{5} + \frac{5}{6} =\]
A: \[\frac{27}{30}\] B: \[\frac{7}{30}\] C: \[\frac{17}{30}\] D: \[\frac{17}{30}\] E: \[\frac{7}{30}\]

18 \[\frac{7}{8} - \frac{3}{4} =\]
A: \[\frac{3}{8}\] B: \[\frac{1}{4}\] C: \[\frac{5}{8}\] D: \[\frac{3}{4}\] E: \[\frac{1}{8}\]

19 \[\frac{2}{3} \times \frac{5}{6} =\]
A: \[\frac{1}{3}\] B: \[\frac{2}{3}\] C: \[\frac{5}{9}\] D: \[\frac{4}{9}\] E: \[\frac{2}{9}\]

20 \[\frac{3}{5} \div \frac{9}{10} =\]
A: \[\frac{2}{3}\] B: \[\frac{11}{24}\] C: \[\frac{12}{7}\] D: \[3\] E: \[2\]

21 \[\frac{12}{3} + \frac{1}{4} - \frac{7}{8} =\]
A: \[\frac{1}{24}\] B: \[\frac{5}{24}\] C: \[\frac{11}{24}\] D: \[\frac{13}{24}\] E: \[\frac{19}{24}\]

22 In the number 2.83, the value of the '3' is
A: \[\frac{3}{1000}\] B: \[\frac{3}{10}\] C: \[\frac{3}{100}\] D: 3 ones E: none of these

23 5.06 rounded to the nearest whole number is
A: 50 B: 6 C: 506 D: 5 E: 5.1

24 5.06 corrected to one decimal place is
A: 5.0 B: 5.1 C: 6.0 D: 6.1 E: none of these

25 30% written as a fraction in its simplest form is
A: \[\frac{3}{100}\] B: \[\frac{300}{100}\] C: \[\frac{3}{10}\] D: \[\frac{300}{100}\] E: \[\frac{3}{10}\]

26 \[\frac{3}{4}\] written as a percentage is
A: 34% B: 73% C: 71% D: 75% E: 57%

27 25% written as a decimal is
A: 0.25 B: 2.5 C: 0.025 D: 25.0 E: 2.05

28 30% of $50.00 is
A: $150 B: $1500 C: $15 D: $1.50 E: $25

29 \[\frac{1}{2}\] written as a decimal is
A: 0.2 B: 2.0 C: 0.05 D: 5.0 E: 0.5

30 27 written as a percentage of 36 is
A: 57% B: 75% C: 76% D: 67% E: 77%

31 \((-4) + (+5) =\)
A: (+9) B: (-9) C: (-1) D: (+1) E: cannot tell

32 \((-4) - (+5) =\)
A: (+20) B: (+9) C: (-1) D: (+1) E: (-9)

33 \((-4)(-5) =\)
A: (-9) B: (+9) C: (+20) D: (+4\frac{1}{5}) E: \[\left(\frac{4}{5}\right)\]

34 \((-4)-(-5) =\)
A: (-1) B: (+1) C: (-9) D: (+9) E: none of these
The diagram alongside shows a mirror EF, standing on the arms of an angle, at P and Q. A' is the reflection of point A in the mirror. If A' is the same distance behind the mirror as A is in front of it, answer Questions 35, 36 and 37.

35 If the angle at A is 39°, then the angle at A' is
A: 29° B: 19° C: 49° D: 39° E: 59°

36 If AQ is 3.4 cm, then A'Q is
A: 3.4 cm B: 34 cm C: 0.34 cm D: 6.9 cm E: 8.6 cm

37 Which of the following statements about the areas of triangle APQ and triangle A'PQ is correct?
A: APQ is greater than A'PQ B: APQ is less than A'PQ C: APQ is equal to A'PQ D: APQ is half of A'PQ E: APQ is twice A'PQ

Questions 38, 39, 40, 41 and 42 refer to the solid shown in the diagram alongside.

(From NFER Test 263)

38 This solid is
A: a sphere B: a cylinder C: a pyramid D: a cone E: a cube

39 The set {faces, corners, edges} corresponds, in order, to the set
A: {6, 12, 8} B: {6, 8, 12} C: {12, 8, 6} D: {8, 6, 12} E: {12, 6, 8}

40 Any face of the solid is called
A: a rhombus B: a rectangle C: a parallelogram D: a square E: a circle

41 Faces and corners meet at angles
A: right B: acute C: obtuse D: base E: reflex

42 If an edge of the solid is 5 centimetres (cm), the area of any face in square centimetres (cm²) is
A: 20 B: 50 C: 25 D: 125 E: 150

Questions 43 and 44 refer to the bar chart alongside.

(From NFER Test 263)

43 How many policemen wear a size 9 boot or larger?
A: 16 B: 20 C: 25 D: 30 E: 34

44 Which size of boot is worn by most policemen?
A: 6 B: 8 C: 7 D: 9 E: 10

The answers to questions 45 and 46 are obtained from the graph below.

The following table shows the time taken by a car to travel various distances at the same speed.

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<th>Time in minutes</th>
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<td>0</td>
<td>.12</td>
<td>20</td>
<td>24</td>
<td>44</td>
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</table>
This information is plotted on the axes below, in the form of a straight line graph.

Distance in kilometres

Time in minutes

45. How far has the car gone in 45 minutes?
   A: 32 km  B: 40 km  C: 44 km  D: 36 km  E: 48 km

46. How long does it take the car to travel 40 kilometres?
   A: 50 min  B: 45 min  C: 40 min  D: 35 min  E: 30 min

The answers to questions 47 and 48 are obtained by reading the graph below.

The graph below shows the number of one manufacturer's old and new alarm clocks in use during each year from 1957 to 1966.

47. The largest number of old clocks in use during any one year over this period is
   A: 10 000  B: 9000  C: 8000  D: 7000  E: 6000

48. In 1963, the total number of clocks (old and new) in use was
   A: 8000  B: 7000  C: 9000  D: 11 000  E: 10 000

49. Written in the form of prime factors, 2520 =
   A: \(2^3 \times 3^2 \times 5 \times 7\)  B: \(2^2 \times 3^3 \times 5 \times 7\)
   C: \(2 \times 3 \times 5^2 \times 7^3\)  D: \(2 \times 3 \times 5^3 \times 7^2\)
   E: \(2 \times 3 \times 5 \times 7\)

50. O is the centre of the circle.
    AB is equal in length to the radius of the circle. The size of angle AOB in degrees is
    A: 75°  B: 30°  C: 60°  D: 90°  E: 45°
### Diagnostic Test

**Mathematics (1 1/2 hours)**

**Answer Book**

**Date of birth**

**Sex**

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<th>B</th>
<th>C</th>
<th>D</th>
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### CORRECT ANSWERS TO DIAGNOSTIC TEST

(To be administered in the Bahamas during September 1981)

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<td>19</td>
<td>C $\frac{5}{9}$</td>
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<tr>
<td>2</td>
<td>B 346</td>
<td>20</td>
<td>A $\frac{2}{3}$</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>D 1827</td>
<td>21</td>
<td>A $\frac{1}{24}$</td>
<td>39</td>
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<tr>
<td>4</td>
<td>A 293</td>
<td>22</td>
<td>C $\frac{3}{100}$</td>
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<tr>
<td>5</td>
<td>E 543</td>
<td>23</td>
<td>D 5</td>
<td>41</td>
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<tr>
<td>6</td>
<td>D 457</td>
<td>24</td>
<td>B 5.1</td>
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<td>7</td>
<td>A 7286</td>
<td>25</td>
<td>E $\frac{3}{10}$</td>
<td>43</td>
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<td>8</td>
<td>B 31.6</td>
<td>26</td>
<td>D 75%</td>
<td>44</td>
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<tr>
<td>9</td>
<td>C 21.0</td>
<td>27</td>
<td>A 0.25</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>E 7</td>
<td>28</td>
<td>C $15$</td>
<td>46</td>
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<tr>
<td>11</td>
<td>E 3.8</td>
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APPENDIX 5.2

INDEX NO.

MINISTRY OF EDUCATION AND CULTURE
BAHAMAS JUNIOR CERTIFICATE EXAMINATION
1975

MODERN MATHEMATICS – Paper I

(1 hour)

Attempt all questions.
Four possible answers are given to each question. Circle the one that is correct.

1. The best estimate of \(4\pi(18.7)^2\) is:
   A. 48,000
   B. 4,800
   C. 480
   D. 48

2. The formula \(T = \frac{3}{\sqrt{c}}\) rewritten with \(c\) as the subject is:
   A. \(c = \frac{3}{T}\)
   B. \(c = \frac{3}{T^2}\)
   C. \(c = \frac{9}{T^2}\)
   D. \(c = \frac{3}{\sqrt{T}}\)

3. If \(f : x \rightarrow 2x\) and \(g : x \rightarrow x - 1\) then \(f(g(4))\) is:
   A. 10
   B. 7
   C. 6
   D. None of these

4. \(3\frac{1}{2} \div \frac{3}{4}\) is:
   A. 5
   B. 12\(\frac{1}{2}\)
   C. 14
   D. \(\frac{2}{3}\)

5. \((x + y)(2 - x)\) is equal to:
   A. \(2x - x^2 - 2y + xy\)
   B. \(2x - x^2 + 2y - xy\)
   C. \(2x - x^2 - 2y - xy\)
   D. \(2x + x^2 - 2y + xy\)

6. If the matrix \(\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\) is applied to the unit square with co-ordinates \((0, 0), (1, 0), (0, 1), (1, 1)\); then the area of the image is:
   A. 2 sq. units
   B. 4 sq. units
   C. 5 sq. units
   D. 3 sq. units

7. If \(M_2 = \{\text{multiples of 2 less than 20}\}\) and \(M_5 = \{\text{multiples of 5 less than 20}\}\) then \(M_2 \cap M_5\) is:
   A. \(\{2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18\}\)
   B. \(\{10, 20\}\)
   C. \(\{5, 10, 15, 20\}\)
   D. \(\{10\}\)

8. The difference between the binary numbers 1101 and 11 is:
   A. 1010
   B. 1100
   C. 110
   D. 111

9. The determinant of the matrix \(\begin{pmatrix} 3 & -1 \\ 1 & 4 \end{pmatrix}\) is:
   A. -13
   B. 11
   C. 13
   D. \(\frac{1}{13}\)
10. The acute angle between the two hands of a clock at 10 o'clock is:
A. 10°  B. 60°  C. 30°  D. 300°

11. The decimal number 26 written as a base 5 number is:
A. 11  B. 15  C. 51  D. 101

12. 96 written as a product of primes is:
A. $2^5 \times 3$  B. $5^2 \times 3$  C. $12 \times 8$  D. $2 \times 3$

13. The value of $(1 \ 2) \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$ is:
A. $\begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$  B. $\begin{pmatrix} 13 & 16 \end{pmatrix}$  C. $\begin{pmatrix} 13 \\ 16 \end{pmatrix}$  D. $\begin{pmatrix} 3 & 4 \\ 10 & 12 \end{pmatrix}$

14. The shaded part of the diagram shows:
A. XUY  B. X∩Y  C. X'  D. X'∩Y'

15. The line $y = x + 1$ passes through the point:
A. (2, 2)  B. (4, 2)  C. (2, 4)  D. None of these

16. The shaded region is:
A. $x < 3$ and $y < 2$  B. $x < 3$ and $y > 2$  C. $y < 3$ and $x > 2$  D. $x > 3$ and $y < 2$

17. $3\frac{2}{3} + 3\frac{2}{3}$ is equal to:
A. $6\frac{2}{3}$  B. $\frac{2}{3}$  C. $\frac{1}{3}$  D. $\frac{2}{3}$
18. The matrix to describe this network is:

A. \[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]
B. \[
\begin{pmatrix}
0 & 1 \\
0 & 1
\end{pmatrix}
\]
C. \[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]
D. \[
\begin{pmatrix}
1 & 1 \\
1 & 1
\end{pmatrix}
\]

19. The probability of throwing a number greater than 4 on an ordinary die is:

A. \[\frac{4}{6}\]  B. \[\frac{5}{6}\]  C. \[\frac{1}{3}\]  D. \[\frac{1}{2}\]

20. The median of 5 7 2 8 8 is:

A. 7  B. 2  C. 8  D. 6

21. The height of a boy is 1 m and that of his father is 200 cm. The ratio of the height of the boy to that of his father is:

A. 1:200  B. 200:1  C. 1:20  D. 1:2

22. If \(2x - 4 = 10\) than \(x\) is equal to:

A. 3  B. 7  C. 28  D. 12

23. If the point P = (2, 1) is reflected in the line \(x = 0\) then the co-ordinates of its image are:

A. (2, -1)  B. (-2, 1)  C. (-2, -1)  D. (1, 2)

24. 1, 3, 6, 10, 15, . . . . . . The next two terms in this sequence are:

A. 21, 28  B. 20, 27  C. 21, 29  D. 20, 29

25. If \(A = (1, 0)\) and \(B = (2, 3)\) then the vector \(\overrightarrow{AB}\) is equal to:

A. \[
\begin{pmatrix}
3 \\
3
\end{pmatrix}
\]  B. \[
\begin{pmatrix}
3 \\
0
\end{pmatrix}
\]  C. \[
\begin{pmatrix}
1 \\
0
\end{pmatrix}
\]  D. \[
\begin{pmatrix}
3 \\
1
\end{pmatrix}
\]

26. 20\% written as a fraction is equal to:

A. 5  B. \[\frac{1}{5}\]  C. 2  D. \[\frac{1}{20}\]

27. If the length of each side of a square is 4.2 cm then its perimeter is equal to:

A. 16.8 cm  B. 16.8 cm²  C. 17.64 cm  D. 17.64 cm²

28. If \(P = \{\text{quadrilaterals}\}\) and \(Q = \{\text{triangles, squares, rectangles, pentagons, trapezia}\}\) the \(P \cap Q\) is:

A. \{\text{triangles}\}  B. \{\text{squares, rectangles}\}  C. \{\text{squares, rectangles, pentagons}\}  D. \{\text{squares, rectangles, trapezia}\}

29. If \(^{*}\) means 'multiply the first number by the second and then divide by 2' then \(10*7\) is equal to:

A. 6  B. 12½  C. 35  D. 70
30. If $S = \frac{u + v}{2} t$ then when $u = 0, v = 8, t = 5, S$ is equal to:
A. $\frac{9}{2}$ B. 9 C. 0 D. 20

31. The value of $\sqrt{160}$ is:
A. between 12 and 13 B. 40 C. 25600 D. none of these

32. If $P = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$ then $PQ$ is equal to:
A. $\begin{pmatrix} 1 & -1 \\ -6 & -12 \end{pmatrix}$ B. $\begin{pmatrix} -1 & -6 \\ 9 & -6 \end{pmatrix}$
C. $\begin{pmatrix} 7 & 5 \\ 9 & 13 \end{pmatrix}$ D. $\begin{pmatrix} 1 & -1 \\ -6 & -9 \end{pmatrix}$

33. If $P = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ the $P$ represents:
A. a rotation B. a reflection C. a shear D. a translation

34. $\frac{3}{4}$ of $57380$ is equal to:
A. 56569 B. 56560 C. 56568 D. 56060

35. $\frac{1}{7}$ expressed as a decimal is:
A. 0.45 B. 0.54 C. 0.8 D. 8.0

36. If $P$ and $Q$ are sets such that $n(P\cup Q) = 21$, $n(P\cap Q) = 4$ and $n(P) = 12$ then $n(Q)$ is equal to:
A. 37 B. 13 C. 9 D. 29

37. In the diagram (not drawn to scale) $a$ is equal to:
A. $5^\circ$ B. $53^\circ$ C. $132^\circ$ D. $79^\circ$

38. If $y = \frac{1}{2}x^2$ then the value of $x$ when $y = 8$ is:
A. 4 B. $\pm 4$ C. 2 D. $\pm 2$

39. The value of $53.59$ to the nearest dollar is:
A. $53.00$ B. $53.60$ C. $53.50$ D. $54.00$

40. If $\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ then the values of $x$ and $y$ are:
A. $x = 2\frac{1}{2}, y = 2\frac{1}{2}$ B. $x = 2\frac{1}{2}, y = 3$
C. $x = 3, y = 2$ D. $x = 2, y = 3$
MINISTRY OF EDUCATION AND CULTURE
BAHAMAS JUNIOR CERTIFICATE EXAMINATION
1975

MODERN MATHEMATICS—Paper II

(2 hours)

Answer eight questions.

1. (i) Solve the following equations:
   (a) \(2(x + 3) = -8\)
   (b) \(\frac{6}{x} + 1 = 3\)
   (c) \(4x - y = 9\) ............(i)
      \(2x + 3y = 43\) ............(ii)
   (ii) If \(V = u + at\), find \(u\) if \(V = 34\), \(a = 10\) and \(t = 3\).
   (iii) If \(D = \frac{a + b}{c}\), make \(a\) the subject of this formula.

2. (a) Work out (i) \(23_4 + 45_6\), (ii) \(123_4 \times 22_6\), (iii) \(100011_2 \div 101_2\), giving all answers in the given base.
   (b) Convert \(64_{10}\) to a base 5 number.
   (c) Convert \(202_4\) to a decimal number.

3. (a) Arrange in ascending order: \(\frac{3}{5}, \frac{4}{1}, \frac{7}{14}, \frac{3}{5}\)
   (b) Convert \(2.8\) to a fraction.
   (c) Convert \(\frac{2}{3}\) to a decimal.
   (d) Work out \(86.13 \div 2.7\).
4. (a) Write down the following sets:
   - \( E \) = \{counting numbers less than 20\}
   - \( T \) = \{triangle numbers less than 20\}
   - \( M_6 \) = \{multiples of 6 less than 20\}
   - \( M_3 \) = \{multiples of 3 less than 20\}

(b) Draw a Venn diagram to show the relationship between the sets.
(c) Is the following statement true: \( M_6 \subseteq M_3 \)?
(d) List the members, if any of \( M_6 \cap T \).

5. Plot and join the points A (2,1), B (7,1), C (7,4).
(a) Reflect the triangle ABC in the line \( x = 1 \) to give A'B'C'.
(b) Rotate A'B'C through 90° with centre (1,0) to A* B* C*. Write down the co-ordinates of A*.

6. Plot and join the points P(3,2) Q(6,2) R(5,4).
(a) Write down the area of PQR.
(b) Enlarge PQR to P'Q'R' with scale factor 2, centre of enlargement (1,0).
(c) Write down the area of P'Q'R'.

7. (a) Using values of \( x \) from -4 to +4 and values of \( y \) from -4 to +4, draw, on the same diagram the lines \( y = x - 2 \) and \( x + y = 3 \).
(b) Shade the regions: P, for which \( y < x - 2 \) and Q, for which \( x + y > 3 \). Label clearly the region \( P \cap Q \) and write down the co-ordinates of one point in this region.

8. The following table shows the number of mistakes made in one sentence by a group of 50 students learning to type:

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(a) Draw a frequency diagram to illustrate this information.
(b) What is the mode?
(c) What is the modal frequency?

9. Grace brought a bag of pens to school, it contained 3 red ones and 2 blue ones. She gave one pen to Basil.
(a) If Basil put his hand into the bag and took out a pen, what is the probability that he got a red one?
(b) Basil got a red one and Grace then decided to give two pens to her best friend Anne. If Anne put her hand into the bag and took out two pens:
   (i) Write down the set \( \epsilon \) of all possible combinations she could pick out.
   (ii) What is the probability that she picks a red and a blue one?
10. Compile the route matrix for the above network.
   (b) Is the network traversible?
   (c) Write down the least number of colours that would be necessary to colour all the regions of
       the diagram so that no arc is bounded on both sides by the same colour.

11. The operation $\oplus$ is defined on the set $\{0,1,2,3,4\}$ as addition around the illustrated clock face,
   e.g. $4 \oplus 2 = 1$

   (a) Copy and complete the operation table:

   $\begin{array}{c|cccc}
   \oplus & 1 & 2 & 3 & 4 \\
   \hline
   1 & \ & \ & \ & \\
   2 & \ & \ & \ & \\
   3 & \ & \ & \ & \\
   4 & \ & \ & \ & 1
   \end{array}$

   (b) Is the set closed under the operation?
   (c) Is there an identity element? If so, what is it?
   (d) Does 2 have an inverse element? If so, what is it?
EXAMINATION SYLLABUSES

(1) Bahamas Junior Certificate Arithmetic (Subject Number 23)

1. Four rules of numbers including series, HCF, LCM; averages
2. Four rules of fractions and decimals
3. Four rules of measurement, weight, currency
4. Areas of circles, triangles, quadrilaterals (including square, rectangle, parallelogram), volume of regular solids
5. Percentages; simple interest; profit and loss
6. Squares and square roots using prime factors
7. Wages, expenditure, bank, domestic, and consumer services
8. Graphs - pie, bar, line, formula, travel

(2) Bahamas Junior Certificate Mathematics (Subject Number 09)

1. (a) Four rules of numbers including series, HCF, LCM, averages
   (b) Four rules of fractions and decimals
   (c) Four rules of measurement including weights, currency, areas of circles, triangles and quadrilaterals (including Pythagoras' theorem), volumes of regular solids
2. Percentages including simple interest and profit and loss
3. Squares and square roots using prime factors
4. Wages, expenditure, bank and domestic services
5. Algebraic expressions, substitution, powers and roots, factorisation
6. Equations - simple
   - simultaneous
   - quadratic (by factors)
7. Graphs - pie, bar, frequency polygons
   - of equations
   - of travel
8. Angles, triangles including congruency and similarity
9. Polygons - including parallelograms
10. Constructions - bisectors and angles
    - of triangles and quadrilaterals
11. Sets: theory, union, intersection, universal set, Venn diagrams, identity, closed sets
12. Co-ordinates: graphs and regions
13. Symmetry including reflection
14. Number bases: base 2, base 3, and so on up to base 12

Modern Mathematics - Papers I and II (Subject Number 26)

1. Number bases
   (i) Four operations: -, *, +, \(^+\) in binary only
   (ii) Base 5, base 7, base 12
2. Sets: complement, universal set, Venn diagrams, closed sets
3. Co-ordinates, graphs and regions
4. Fractions
5. Angles, triangles, polygons, polyhedra - Euler's relationship
6. Decimals
7. Area - triangles, squares, rectangles, volume
8 Negative numbers
9 Symmetry
10 Topology
11 Statistics - bar charts, pie charts, line graphs, simple probability
12 Similarity and enlargement - scale factors
13 Algebra - equations, four operations
14 Transformation - reflection, translation, rotation matrices of transformation vectors
15 Matrices - addition, and multiplication
16 Circle - area, circumference
17 Ratios
18 Parabola, straight-line functions
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**Name:** E. Storr

**Program Title:** CAMEL, BAILHAME

**Department:** Eng Maths

**User Number:** 6310 (SCH.8) ST. JOHN'S COLLEGE

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### APPENDIX 5.4

Cumulative Frequency Data for Conversion of Marks to Grades

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JUNIOR/SENIOR HIGH SCHOOL MATHEMATICS CURRICULUM

BEHAVIOURAL OBJECTIVES

GRADE 7

I Sets of numbers

At the end of this topic the student should be able to identify and list:

1 The set of natural numbers
2 The set of whole numbers
3 The sets of odd, even, rectangular and square numbers
4 Prime numbers to 100 (Sieve of Eratosthenes)
5 The prime factors and multiples of a number
6 Identify multiples from the tables of multiplication
7 List the common multiples of two or more given numbers
8 Select the lowest common multiple of two or more given numbers from tables

II Operations

At the end of this topic the student should be able to:

1 Describe the origin of numbers
2 Recognise, identify and apply place value up to ten thousand
3 Find the sum with three, four and five figure addends resulting in six-figure totals
4 Read and solve addition word problems involving three, four and five figure addends
5 Perform the operation of subtraction up to six figures
6 Solve simple word problems involving six figures, including use of equal addition or decomposition methods
7 Recall from memory multiplication tables to 12
8 Find the product of three-digit multipliers and multiplicands
9 Solve division word problems with and without remainders
10 Solve any word problem involving up to three digits

III Laws and Properties of Arithmetic and Algebra

At the end of this topic the student should be able to:

1 Demonstrate that the commutative law applies to addition and multiplication of whole numbers
2 Demonstrate that the commutative law does not apply to subtraction and division
3 Demonstrate that the associative law applies to addition and multiplication and NOT to subtraction and division of whole numbers
4 Demonstrate that the associative law does not apply to solving statements that have mixed operations of whole numbers
5 Compute number expressions with grouping symbols using the laws
6 Use letters in algebra
7 Identify algebraic terms eg 5a = 5 x a
8 Identify algebraic expressions
9 Recognise coefficients
10 Simplify by grouping positive like terms

IV The Point and Line

At the end of this topic the student will be able to:

1 Define and name a point
2 Use the point to indicate position on a line
3 Use the point to indicate position in a plane
4 Represent and name a straight line
5. Use a ruler to draw lines of given lengths - inches, centimetres and millimetres
6. Use the ruler to measure lines in inches, centimetres and millimetres

V. Signed numbers
At the end of this topic the student should be able to:
1. Identify positive and negative numbers
2. Define the number line
3. Draw the number line and insert the signed numbers given
4. Add and subtract positive numbers on the number line

VI. Common Fractions
At the end of this topic a student should be able to:
1. Identify various shapes of \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \)
2. Identify various kinds of fractions (proper and improper)
3. Identify and write equivalent fractions
4. Reduce fractions to their lowest terms
5. Change improper fractions to mixed numbers and vice versa
6. Add simple fractions by inspection (like fractions)

VII. Decimal Fractions
At the end of this topic the student should be able to:
1. Write denary fractions (decimal fractions)
2. Identify denary fractions to thousandths (decimal places to thousandths)
3. Convert simple common fractions, denominators of which are powers of 10 (up to \( 10^3 \)), to decimal fractions
4. Convert decimal fractions up to thousandths to common fractions
5. Add and subtract decimals to thousandths
6. Multiply whole numbers by tens, hundreds and thousands
7. Divide whole numbers by tens, hundreds and thousands
8. Multiply decimals by one digit whole numbers
9. Solve word problems using decimals
10. Demonstrate that Bahamian/US currencies are based on the decimal system
11. Solve word problems involving Bahamian/US currency
12. Calculate shopping bills

VIII. Statistics
At the end of this topic the student should be able to:
1. Collect statistical information and represent it in a tally chart
2. Represent this data by a bar (column) chart
3. Represent this information in a pictogram
4. Interpret (i.e., analyse and extract information from) these diagrams
5. Determine the most frequently occurring event (mode)
6. Describe the information shown by these graphs and interpret the graphs

IX. Mensuration in the Imperial and Metric Systems
Metric units to be used:
(a) Length - centimetre, metre, kilometre
(b) Weight - gramme, kilogramme, tonne
(c) Capacity - litre, cubic centimetre

Imperial units
(a) Measurement - Length
At the end of this topic the student should be able to:
1. Describe the origins of units – imperial and metric historical background.
2. Identify units of measurement from the inch to a mile and centimetre to kilometre.
3. Identify place value in the metric system.
4. Draw shorter lines and physical objects accurately.
5. Measure shorter lines and physical objects accurately.
6. Use measuring equipment, eg ruler, yard stick, metre rule, parts of the body, trundle wheel.
7. Estimate lengths without the use of measuring aids.
8. Measure accurately quantities of liquid.
9. Convert from one unit to another within a system.

(b) Measurement – Weight
1. Describe the origins of units of weight.
2. Identify units of weight from ounce to ton (gm, kg, tonne).
3. Measure small amounts using scales.
4. Estimate weight and compare with balance.
5. Distinguish between lighter and heavier weights.
6. Solve weight problems using the four rules.
7. Convert from one unit to another within a system.

(c) Measurement – Time
1. Describe the origins of units of time.
2. Identify units of time from seconds to years.
3. Calculate time using 12-hour clock.
4. Estimate time (long and short periods).
5. Calculate time using 24-hour clock.
6. Convert from one unit to another.
7. Solve problems of time.

(d) Measurement – Capacity
1. Describe origins of units for capacity.
2. Identify units of capacity (pints to gallons).
3. Estimate quantities of liquid using everyday containers.
4. Measure accurately quantities of liquid.
5. Convert from one unit to another.
6. Solve problems of capacity.

X. Angles
At the end of this topic the student should be able to:
1. Define an angle.
2. Draw an angle.
3. Name an angle.
4. Name the parts of an angle.
5. Distinguish between acute, right, obtuse, straight and reflex angles.

XI. Triangles
At the end of this topic the student should be able to:
1. Define and name the triangle.
2. Classify triangles according to angles (acute, obtuse, right).
3. Name the SIDES and VERTICES of a triangle.

XII. Quadrilaterals and other Polygons
At the end of this topic the student should be able to:
1. Define a quadrilateral.
2 Recognize a square and rectangle
3 Recognize other polygons according to number of sides

XIII The Circle

At the end of this topic the student should be able to:
1 Recognize a circle
2 Distinguish between a circle and a sphere
3 Name the radius and centre
4 Draw circles of given radii

XIV Solids

At the end of this topic the student should be able to:
1 Identify different kinds of solids - prism, cube, cuboid, cylinder, cone, pyramid, sphere

XV Substitution

At the end of this topic the student should be able to:
1 Find the value of a simple term by substitution, eg \( X = 2, \ 2X = 4 \)
2 Find the value by substitution, of simple Algebraic expressions
3 Find the value by substitution, of Algebraic expressions involving subtraction
4 Find the value, by substitution, of expressions involving addition and subtraction

XVI Powers and Roots

At the end of this topic the student should be able to:
1 Recognize the squares of whole numbers (to 12)
2 Recognize the cubes of 2, 3, 4, 5, 10
3 Calculate power (to 4) of any whole number (to 12)
4 Recognize the square roots of perfect squares (to 144)
5 Recognize the cube roots of 8, 27, 64, 125, 1000

XVII Simple Equations

At the end of this topic the student should be able to:
1 Identify signs of equality and inequality (\( >, <, = \))
2 Use the above signs to show numerical relationships
3 Complete mathematical sentences by filling in blanks
4 Identify an equation

XVIII Sequences

At the end of this topic the student should be able to:
1 Complete sequences of numbers connected by simple relationships

XIX Ratio and Proportion

At the end of this topic the student should be able to:
1 Recognize the per cent sign
2 Identify that a 100% is equal to a 'whole'
3 Identify that a percentage, less than 100%, is a fractional part of a 'whole'
4 Identify that a percentage, greater than 100%, is greater than a 'whole'
5 Convert percentages to common fractions
6 Convert percentages to decimal fractions
At the end of this topic the student should be able to:

1. **Define ratio and rate**
2. Recognize and use the symbol ":" for the writing of ratios in sequence
3. Write ratios as fractions \( \frac{y}{x} \), \( \frac{x}{x} \)

**XX Simple Expansion and Factorizing**

At the end of this topic the student should be able to:

1. Expand simple algebraic terms of the form \( 3x = 3 \times x \), \( ab = a \times b \), \( x^2 = x \times x \) (to order 3)
2. Identify all factors of simple algebraic expressions (to order 3) eg: \( ab^2 \) has factors 1, a, b, \( b^2 \), ab, \( ab^2 \)

**XXI Number Bases**

At the end of this topic the student should be able to:

1. Recognize Roman numerals to 20
2. Convert from base 5 to denary
3. Convert from denary to base 5
4. Perform addition and subtraction in base 5
5. Perform multiplication in base 5 (using a multiplication table)

**BEHAVIOURAL OBJECTIVES**

**GRADE 8**

**I Operations and Brackets**

At the end of this topic the student should be able to:

1. Simplify both simple and compound operations with and without brackets
2. Compute number expressions with grouping symbols using the associative and commutative laws
3. Evaluate, by substitution, expressions involving brackets

**II Powers and Roots**

At the end of this topic the student should be able to:

1. Calculate the \( \text{HCF} \) of two or more numbers using prime factors
2. Calculate the \( \text{LCM} \) of two or more numbers using prime factors
3. Calculate \( \text{HCF} \) and \( \text{LCM} \) using the notation form
4. Find the \( \text{HCF} \) of algebraic terms
5. Find the \( \text{LCM} \) of algebraic terms

**III Fractions**

At the end of this topic the student should be able to:

1. Add fractions by use of equivalent fractions
2. Subtract fractions by use of equivalent fractions
3. Identify fractions in everyday use and perform common types of calculation
4. Show that any number can be mapped on the number line
5. Demonstrate the relative size of numbers on a number line
VI _ Triangles
At the end of this topic the student should be able to:
1 Classify triangles according to sides
2 Determine the sum of the interior angles of a triangle
3 Calculate the size of one of the angles of a triangle having been given the sizes of the other two angles
4 Identify the median as the line of symmetry of an isosceles triangle
5 Identify the lines of symmetry of the equilateral triangle

VII _ Graphs
At the end of this topic the student should be able to:
1 Identify positions and directions on maps and pictures
2 Locate in numerical form positions in their seating arrangements
3 Compare positions of others in pairs using positive numbers only
4 (Plot) mark exact positions of other students and self on provided paper
5 Evaluate and write distances of one object from another using routes along columns and rows
6 Apply from the above the idea of axes using positive quadrants only
7 Mark points using +x, +y axes only
8 Join these points to form rectangles, squares and triangles
9 (Plot) write co-ordinates of points from simple data

VIII _ Number Bases
At the end of this topic the student should be able to:
1 Explain the use of the binary system
Convert binary number to base 10
Convert a base 10 number to binary
Perform the operations of addition and subtraction in base 2
Perform the operations of multiplication in base 2
Perform the operations of division in base 2

IX Decimals
At the end of this topic the student should be able to:
1 Multiply any decimal by any other decimal
2 Divide any decimal by any other decimal
3 Express decimal numbers in significant figures
4 Express decimal numbers to required number of decimal places
5 Demonstrate that other systems such as the metric system, and the British currency are also decimal
6 Solve British simple number problems involving British currency and the metric system
7 Calculate telephone bill (including telegrams)
8 Solve word problems using decimals

X Substitution
At the end of this topic the student should be able to:
1 Find the values of terms with indices
2 Find the value of terms involving division signs
3 Find the values of terms involving brackets and powers
4 Apply rules of substitution to given formula eg: \( A = \pi r^2 \)

XI Simple Equations
At the end of this topic the student should be able to:
1 Solve simple equations by subtraction
2 Solve simple equations by addition
3 Solve simple equations by division
4 Solve simple equations by multiplication
5 Solve simple equations by involving the use of two or more operations
6 Solve simple equations by involving fractions by multiplying the term by a common multiple of the denominators
7 Construct simple equations from word problems

XII Quadrilaterals
At the end of this topic the student should be able to:
1 Name a quadrilateral
2 Identify different kinds of quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium, kite)
3 Calculate the sum of the angles of a quadrilateral
4 Identify lines of symmetry of given quadrilaterals
5 Identify points of symmetry of given quadrilaterals
6 Identify lines of symmetry of regular polygons
7 Identify points of symmetry of regular polygons

XIII Mensuration
At the end of this topic the student should be able to:
1 Define perimeter of irregular shapes
2 Estimate perimeter of irregular shapes
3 Calculate perimeter of rectangular shapes
4 Calculate perimeter of shapes formed from 2 or more rectangles
5 Apply the concept of area using the idea of cover
6 Calculate area of irregular shapes by continuing counting squares
7 Calculate area of rectangles and squares
8 Calculate area of a cuboid
9 Express square units as another square unit
10 Calculate area of border formed by two (2) rectangles

XIV Statistics
At the end of this topic the student should be able to:
1 Collect statistical information and represent it in a tally chart
2 Represent this data by a bar (column) chart
3 Represent this information in a pictogram
4 Interpret (ie analyse and extract information from) these diagrams
5 Determine the most frequently occurring event (mode)
6 Define and determine the means from given data
7 Describe the information shown by these graphs and interpret the graphs
8 Determine the class interval
9 Construct a bar chart involving class intervals
10 Determine the modal class
11 Represent data using lines instead of bars
12 Plot points for discrete variations (simple fractions)

XV Directed Numbers
At the end of this topic the student should be able to:
1 Identify positive and negative numbers
2 Define the number line
3 Draw the number line and insert the signed numbers given
4 Add and subtract positive numbers on the number line
5 Add negative numbers on the number line
6 Subtract negative numbers on the number line
7 Add and subtract negative numbers on the number line
8 Simplify mathematical sentences by grouping positive and negative numbers
9 Multiply and divide positive numbers
10 Multiply two (2) negative numbers
11 Multiply one (1) of each
12 Perform the above using letters

XVI The Circle
At the end of this topic the student should be able to:
1 Name the parts of the circle - centre, radius, diameter, circumference, semi-circle
2 Construct patterns using the circle

XVII Percentages
At the end of this topic the student should be able to:
1 Change decimals to percentages
2 Change percentages to decimals
3 Find percentages of various given quantities decimally and fractionally
XVIII Ratio and Proportion

At the end of this topic the student should be able to:

1. Solve problems involving direct variation
2. Solve problems involving inverse variation
3. Solve problems involving joint variation

XIX Construction of Solids

At the end of this topic the student should be able to construct:

1. A cube
2. A cuboid
3. A tetrahedron

BEHAVIOURAL OBJECTIVES

GRADE 9

I Simple Interest

At the end of this topic the student should be able to:

1. Calculate simple interest by the Unitary Method
2. State the simple interest formula
3. Solve problems involving simple interest
4. Distinguish between savings, current and fixed deposit accounts

II Percentages

At the end of this topic the student should be able to:

1. Change fractional percentages to fractions
2. Change decimal percentages to decimals
3. Change mixed number percentages to fractions
4. Change mixed number percentages to decimals
5. Recognise percentage increases and decreases
6. Use a decimal, a fraction and a percentage to describe the same situation
7. Find a quantity when a percentage of it is known
8. Solve problems involving percentages

III Consumer Mathematics

At the end of this topic the student should be able to:

1. Calculate the cost of running a car
2. Calculate profit and loss from data given in simple examples
3. Calculate discounts and no-claim bonuses for insurances
4. Calculate wages (hourly, weekly, commission), including National Insurance and Union Dues.
IV Constructions
At the end of this topic the student should be able to:
1 construct a perpendicular from a point in a given straight line
2 construct a perpendicular to a given straight line from a point outside the line
3 construct a perpendicular to a straight line from the point at the end of the line
4 draw lines of given lengths using compasses or dividers
5 divide lines of given lengths into a given number of parts eg halves (finding the mid-point) thirds, etc
6 bisect a given line using compasses and ruler

V Expansion and Factorisation
At the end of this topic the student should be able to:
1 Identify grouping ie: ( ), { }, [ ]
2 add and subtract monomials involving indices equal to or greater than 1
3 multiply and divide monomials involving indices equal to or greater than 1
4 recognise the rules of indices ie: \( x^n \times x^m = x^{n+m} \)
   \( x^n \div x^m = x^{n-m} \)
   \( (x^n)^m = x^{mn} \)
5 remove the common factors from expressions

VI Operations on Positive and Negative Monomials
At the end of this topic the student should be able to:
1 simplify by grouping positive and negative like terms
2 simplify by grouping positive and negative unlike terms
3 write factors in index notation form
4 simplify by multiplying positive unlike terms
5 simplify by dividing positive like terms
6 simplify by dividing positive unlike terms
7 simplify by multiplying positive and negative unlike terms
8 simplify by dividing positive and negative unlike terms
9 simplify by multiplying and dividing positive and negative terms with indices

VII Mensuration
At the end of this topic the student should be able to:
1 calculate the area of a triangle
2 calculate the area of a parallelogram
3 calculate the area of any border problem
4 calculate the area of a circle
5 calculate surface area of a cylinder
6 discover the formula for area of circle
7 use the formula \( A = \pi r^2 \) to calculate the area of a circle
8 distinguish between a circle and a sphere
9 draw circles of given radii
10 define volume
11 calculate volume of a cube
12 calculate volume of a cuboid

VIII Bearings and Clocks
At the end of this topic the student should be able to:
1 identify and list the e.g.1t major points of the compass
2 calculate the sizes of angles between given directions
3 determine bearing of one point from another from zero degrees
4 calculate the angles turned through by the hour and minute hands on clock face
IX Linear Graphs
At the end of this topic the student should be able to:
1 plot points using +x and +y axes only
2 join points to make geometrical shapes and find the areas of these shapes
3 distinguish between a line and co-ordinates eg: line \( y = 5 \), co-ordinate (4, 3)
4 recognise the lines \( x = 1 \), \( y = 3 \), \( x = -4 \)
5 shade regions like \( x < 1 \), \( y > 5 \), etc
6 plot co-ordinates and identify groups which join to give straight lines
7 plot graphs of \( y = x \) using change in values of \( x \) and identify points through which the connecting line passes

X Angles
At the end of this topic the student should be able to:
1 define adjacent angles
2 define supplementary angles
3 define adjacent supplementary angles
4 identify complementary angles
5 identify vertically opposite angles
6 identify the exterior angle of a triangle
7 determine that the exterior angle of a triangle is equal to the sum of the two interior opposite

XI Factors
At the end of this topic the student should be able to:
1 calculate, by using factors, the square root of any perfect square
2 calculate, by using factors, the cube root of any perfect cube

XII Proportion
At the end of this topic the student should be able to:
1 identify and write direct proportions
2 identify and write inverse proportions
3 distinguish between direct and indirect proportion
4 solve problems involving proportion

XIII Substitution
At the end of this topic the student should be able to:
1 substitute given values in all formulae covered in the course so far

XIV More Constructions
At the end of this topic the student should be able to:
1 construct a triangle having been given the lengths of three sides
2 construct a triangle having been given two sides and the included angle
3 construct a triangle having been given two angles and one side
4 construct a right-angled triangle having been given hypotenuse and one side
5 construct an equilateral triangle
6 construct an isosceles triangle

XV Standard Form
At the end of this topic the student should be able to:
1 write any number greater than 1 in the standard form eg: \( 34.6 = 34.6 \times 10^1 \)
2 write any positive number less than 1 in standard form eg: \( 0.0063 = 6.3 \times 10^{-3} \)
3 express decimal numbers correct to decimal places
4 express decimal numbers correct to significant figures
XVI  Line Graphs
At the end of this topic the student should be able to:
1  represent data using lines instead of bars
2  plot points for discrete variations
3  plot points for continuous variations
4  draw and label axes correctly
5  describe the information shown by these graphs and interpret the graphs

XVII  Travel Graphs
At the end of this topic the student should be able to:
1  analyse and extract information from a simple travel graph
2  determine uniform speeds and average speeds from graphs.
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Maximum Marks for Each Question 5

SIGNATURE OF RESPONSIBLE EXAMINER

AVERAGE MARK 31.28 (62.56%)

σ = 8.76

DATE: 2 November 1981
**APPENDIX 6.1**

Mean age of entire population: 15 years 6 months

**ST JOHN’S COLLEGE**

**Date of Test:** 23 September 1981. **Class:** Form V

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Standard Deviation = 8.76  
Mean = 31.28

---

**LOUGHBOROUGH LEICESTERSHIRE LE11 3TU**  
Tel: 0509 83791  
Telex 14199 Technology Loughborough

**CAMEET**  
(Centre for Advancement of Mathematical Education in Technology)  
Professor A.C. Dajpani  
Director

---

21 December 1981

Father H Ward  
Principal  
St. John's College  
PO Box N-4858  
Nassau  
BAHamas

**Dear Father Ward**

I write to express my most sincere thanks for the invaluable help and co-operation you gave me during the conduct of a field survey last summer. You would be pleased to know that the exercise was so successful that I brought back to Loughborough a total of 618 scripts which were offered by pupils of Junior and Senior High Schools as well as Independent High Schools in New Providence.

The 241 scripts offered by Senior High Schools and corresponding sections of Independent High Schools have been marked. The attached marks list indicates the performance of the pupils of your school.

Would you be most kind and thank your Grade 11 (Form V) pupils as well as the following members of staff for their willing co-operation and help:

- Head of Mathematics Dept., Mr I Osborne  
- Miss F Martin  
- Mr H Clayton

Season's greetings and happy new year.

Yours sincerely

E R Storr, Cert Ed(Southampton, UK) BSc (UWI)  
Commonwealth Scholar
SENIOR HIGH AND INDEPENDENT SCHOOLS

St John's College
(PO Box N-4858)
Principal: Father H Ward
Mr I Osborne (Head of Maths Dept)
Miss F Martin
Mr H Clayton

Queen's College
(PO Box N-7127)
Vice Principal: Mr P Cash
Mr R Nichols (Head of Maths Dept)
Mr R Martlew
Miss N Hendry
Mrs S Blong
Miss C Lewis
Mr A Roberts

Prince Williams High School
Principal: Rev. C W Saunders
Rev. S Hall, Vice Principal
Miss V Hoilett
Mr M Lundy (Head of Dept)
Mr J Wood (Senior Master)

Government High School
Principal: Mr E E Moncur
Miss C Verity (Head of Dept)
Mrs J A Wallace

R M Bailey
Principal: Mr E J Bowe
Vice Principal, Mrs M Ferguson
Mrs J Patterson (Head of Dept)

C C Sweeting
(PO Box N-8438)
Principal: A L Archer
Mr L L Price

A F Adderley
Principal: Mr E Bethel
Vice Principal, Mrs V Rouche
Mr P Moncur
Mrs E Reilley (Head of Maths Dept)
Mr R Taylor

L W Young
Principal: Mr R Adderley
Mr L R Bain (Head of Maths Dept)
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## APPENDIX 6.3

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CAMEF
(Centre for Advancement of Mathematical Education in Technology)
LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

DIAGNOSTIC TEST

MATHmanship
Time: 70 mins

Date of birth __________ day/month/year
Sex __________ Boy/Girl

PLEASE DO NOT WRITE ON THIS QUESTION PAPER

1 Answer the questions in any order but number them clearly
2 Mark off a rough work margin on each page
3 Set out your answers as carefully as you can
4 DO NOT copy any diagram that is given on this question paper

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Answer ANY TEN questions

1 Simplify $2(x + 3y - z) - 3(x + y - z) + 4(x - y)$

2 Using ruler and compasses only, construct a parallelogram with sides 6 cm and 9 cm, the angle between these sides being 60°. Measure, in centimetres (cm), the length of any ONE of the diagonals.

3 By how many is $(p - q)^2$ greater than $p^2 - q^2$ when $p = 3$ and $q = -4$?

4 EITHER

In the right angled triangle alongside, find the length of $y$ in centimetres (cm) to one decimal place

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OR

The radius of the circle alongside is 5 cm and the acute angle POQ is 60°. Find the area, in cm², of the shaded sector POQ.

(Take $\pi = \frac{22}{7}$)

5 Simplify $\frac{2}{5x + 1} - \frac{1}{3x - 1}$
7 Solve the equation \(4(2x - 1) - 5(x - 2) = 1\)

8 The minute hand of a clock is 6 cm long. How far does the tip travel in 30 minutes?

\((\text{Take } \pi = \frac{22}{7})\)

9 Find, in square centimetres \((\text{cm}^2)\), the area of a rectangle which is \((2x - 5)\) cm long and \((x + 3)\) cm wide.

10 Multiply 0.376 by 0.16 and write your answer in standard form.

11 Find \(x\) and \(y\) by any method if

\(5x + 2y = 2\)
\(2x + 3y = -8\)

12 If \(\frac{3}{4}\) of the children in a school are absent when there are 322 present, how many children are registered at the school?

13 Divide $1200 in the ratio 3:5.

14 Simplify

\[\left(\frac{3\frac{1}{5} - \frac{1}{7}}{\frac{24}{25}}\right)\]
## APPENDIX 6.5

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APPENDIX 6.6

TRIGONOMETRY

Age: 13++ 'A' Level

Prerequisites:
(i) Angles: Degree measure, use of protractor, triangles, fixing a position, rotations, circles
(ii) Enlargement of shapes, right-angled triangles
(iii) Graphical representation and use of scales

Visual aids/models: Circular protractor fixed to graph board (with rotating diameter), swing, pendulum, weights and elastic bands or springs, tuning fork, violin or guitar, three-figure tables, graph board for vectors, rope (12' - 15')

General objectives: At the end of this series of lessons, the student will:

1 Solve right-angled triangles for angles and lengths of sides using sine, cosine and tangent ratios
2 Sketch and identify sine and cosine curves (very good students would enjoy sketching the tan curve)
3 Define and use the six basic trigonometric ratios for angles of any magnitude
4 Use the formulae for sine rule, cosine rule and area of triangle
5 Apply trigonometry to the solutions of practical problems
6 Sketch graphs of functions involving sine, cosine
7 Derive relationships between trigonometric ratios
   eg \( \csc^2 A = 1 + \cot^2 A \)
8 Simplify and solve problems involving trigonometric relations using identities
9 Solve 3 dimensional triangulation problems
10 Plot graphs of trigonometric functions in terms of frequency
11 Use compound angle formulae for addition of sine and cosine functions

12 Apply the sum and product formulae

13 Derive trigonometric relationships between two angles

Specific Objectives: (Will be given for individual lessons)

NB On the London University 'A' level syllabus, "A knowledge of inverse trigonometric functions is not expected". However, schools do teach it in order to prepare their pupils adequately for university studies.

References

London GCE 'O' and 'A' level syllabuses
SMP Mathematics Books (ABCDE) FGH
The Midlands Mathematical Experiment (MME) Books 1, 2, 3
The Nuffield Foundation Mathematics Project (NFMF')
Contemporary School Mathematics (CSM)
Bank of Objectives, Technician Education Council
College of The Bahamas, Teacher Education Division,
Course Outline - Mathematical Methods II

Brief review of the psychology and teaching of elementary school mathematics

Historical antecedents of current mathematics teaching

Evaluation methods - testing skills and concepts; preparing and scoring examinations

Teaching Mathematics in Grades 4 - 6

Number and sets - The language of sets; set application to logic problems; addition and subtraction algorithms; place value and the exponential notation; basic rules of arithmetic as applied to whole numbers, fractions, decimals and percentages, early experience with negative numbers; number bases; opened and closed sentences; multiplication and division algorithms; ratio and proportion

Measurement - Brief history of measurement; direct and indirect measurements; the measures of length, area, volume, capacity, weight, time, speed, and temperature; the use of metric units; scale drawing and graphs

Geometry - The teaching of geometry in the elementary school; the properties of the basic shapes of two and three dimensions; the measure of angles; the use of coordinates and simple transformations

Practical - The preparation of units, lesson plans, activity cards and apparatus to accompany the above topics

Evaluation - 15% the making of a suitable apparatus kit
15% home assignments
15% mid-semester test
55% final post test
100%

Text Books - Primary Mathematics for Today/Teaching Elementary School Mathematics, K. Kramer, Allyn & Bacon

Reading List - Primary Mathematics Today, Williams & Shuard, Longman

Teaching the Child Mathematics, Schminkle & Maertens, Arnold
College of The Bahamas, Teacher Education Division.

Course Outline - Mathematical Methods III

The course is outlined in detail in the UNESCO Teachers' College Mathematics Project Booklets. The topics covered are:

- Sets
- Measurement
- Introduction to Algebra
- Statistics
- Graphs
- Transformation Geometry

Student content is from a variety of books but the recommended books for the course are Joint Schools Project Mathematics 1, 2, 3.

Professional Activities

To develop **curriculum skills** so that student teachers will be able to:

1. Select, write, design and create curriculum components suitable for junior secondary pupils

   **curriculum skills:** Selecting content
   Formulating objectives
   Listing/writing resource material

To develop **material skills** so that the student teacher will be able to:

2. Select/produce and use materials/aids which will help to teach above topics

   **material skills:** Using blackboard
   Duplication/photocopying
   Model making
   Chart making
   etc.

To develop **instructional skills** so that the student teacher will be able to:

3. Use the components and materials in 1 and 2 in an effective teacher/learning situation

Evaluation

Curriculum skills and material skills will be evaluated on the basis of the quality of pupil curriculum materials, aids etc, which the student produces. Instructional skills will be evaluated on the basis of lesson preparation and actual teaching of lesson concept.

In all cases evaluation will be on the basis of:

(a) self-evaluation
(b) group or class evaluation
(c) tutors final evaluation
BEHAVIOURAL OBJECTIVES

GRADE 10

I Quadratic Equations
At the end of this topic the student should be able to:
1. Identify a quadratic expression and equation
2. Recognise that in a quadratic expression, the highest power of the variable is 2
3. Given any quadratic equation re-write it in the form \( ax^2 + bx + c = 0 \)
4. Factorise a quadratic expression
5. Factorise a quadratic equation of the form \( ax^2 + bx + c = 0 \)
6. Identify the factors of zero property
   i.e. if \( a \) and \( b \) are real numbers and \( ab = 0 \) is true, then \( a = 0 \) or \( b = 0 \)
7. Apply the condition of zero property to the solution of quadratic equations by factors (including fractional coefficients)

II Graphs
At the end of this topic the student should be able to:
1. Identify the equation of a straight line graph
2. Tabulate values for linear equations in two variables
   eg \( y = x + 1, \quad y = 2x + 3, \quad y = -3x + 5 \)
3. Plot the graphs of linear equations
4. Give the intercept of the line and the y-axis
5. Calculate the gradient of a straight line graph
6. Identify the equation of straight line graph
7. From the equation, give the intercept of the line and the y-axis
8. From the equation, give the gradient and direction of the line

III Consumer Mathematics
Income
At the end of this topic the student should be able to:
1. Compute regular earnings given the hourly rate, and the number of hours worked
2. Calculate overtime earnings and total earnings, given the hourly rate and number of hours worked
3. Compute the commission earned, given the amount of sales and rate of commission
4. Compute net earnings, given gross earnings and the amount of deductions
5. Calculate amounts for each segment of a budget, having given each part expressed as a percent of the whole, and the total amount to be budgeted

IV Congruency
At the end of this topic the student should be able to:
1. Define congruent triangles
2. Identify congruent triangles
3 Use the concept of congruent triangles to solve problems
4 Define similar triangles
5 Identify similar triangles
6 Apply the properties of similar triangles in solving problems
7 Construct triangles, using scale drawing, to solve problems

V Quadrilaterals
At the end of this topic the student should be able to:
1 Identify special quadrilaterals
2 Determine the properties of:
   (a) Parallelogram
   (b) Rectangle
   (c) Square
   (d) Rhombus
3 Construct a quadrilateral from given information
4 Construct scale drawings of plans
5 Identify lines of symmetry of given quadrilaterals
6 Demonstrate that the area of a trapezium is the sum of the areas of the two triangles formed by cutting along a diagonal of the trapezium
7 Demonstrate with the aid of a drawing or a model that the formula for the area of a trapezium is: 
   \[ \frac{1}{2} (a + b)h \] where \( a \) and \( b \) are parallel sides and \( h \) is the distance between them
8 Apply the formula as stated in 2

VI Similar Triangles
At the end of this topic the student should be able to:
1 Identify corresponding parts of similar triangles
2 Write and use ratios derived from two similar triangles to find the missing parts of these triangles
3 Identify similar right-angled triangles and find their corresponding parts

VII Square Roots and Pythagoras' Theorem
At the end of this topic the student should be able to:
1 Use the Pythagorean theorem to compute the square of the third side of a right-angled triangle, given the other two sides. Then find the third side when it is an integer
2 Use the Pythagorean theorem to compute the square of the third side of a right-angled triangle when it is not a perfect square. Estimate the length of this third side
3 Determine from square root tables, the square root of any positive rational number
4 Identify the basic numerical properties of squares and square roots
   (a) That for every real number \( r \), \( r^2 \geq 0 \)
   (b) Every positive real number, \( 's' \) has two square roots: \( \sqrt{s} \) and \( -\sqrt{s} \)
   (c) If \( s = 0 \), then \( s \) has only one square root
   (d) If \( s < 0 \), then \( s \) has no real square root
5 Apply the facts that:
   (a) If \( a \) and \( b \) are real numbers, then
      \[ \sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \]
   (b) If \( a \) is non-negative and \( b \) is positive, then:
      \[ \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \]
6 Reduce a square root to its simplest form
   \( \sqrt{12} = 2\sqrt{3} \)

VIII The Circle
At the end of this topic the student should be able to:
1 Name the parts of the circle: quadrant, chord, tangent, sector, arc
2 Determine the chord properties of a circle:
(a) the line drawn from the mid-point of a chord through the centre of a circle, is perpendicular to the chord
(b) the line from the centre of a circle, perpendicular to the chord, bisects the chord
(c) equal chords are equidistant from the centre
(d) equal chords stand on equal arcs.

GRADE 11

1 Quadratic Functions
At the end of this topic, the student should be able to:
1 Solve a quadratic equation of the form \((x-r)^2 = s\) in which \(r\) and \(s\) \(\neq 0\). So that \(x-r = \pm \sqrt{s} \).
   eg \((x-2)^2 = 4\)
2 Recognise and solve a quadratic equation of the form \(ax^2 + bx + c = 0\) where the L.H.S. is a perfect square
   eg \(x^2 - 8x + 16 = 0\)
   \((x-4)^2 = 0\)
   \(x-4 = 0\)
   \(x = 4\) repeated
3 Identify the coefficients of the variables of the equation \(ax^2 + bx + c = 0\)
4 Recognise that the solutions of the equation \(ax^2 + bx + c = 0\) are given by the formula:
   \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
5 Apply the formula \(x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}\) to the solution of quadratic equations
6 Write down and solve quadratic equations from word problems

II Graphs
At the end of this topic the student should be able to:
1 Identify maximum and minimum points of a parabola
2 Sketch the graph of a curve, given its equation
   eg \(7 = 2x^2, y = x + x^2, y = 4-3x-x^2\)
3 Use the quadratic graph to solve equations of the type \(ax^2 + bx + c = 0\)

III Borrowing Money

At the end of this topic, the student should be able to:

1 Calculate the Hire Purchase charges, given the cash price, and the total amount of repayments (deposit + instalments)
2 Calculate the rate of interest, given the amount borrowed and the amount of interest
3 Calculate the interest, given the amount borrowed and the rate of interest
4 Calculate the interest (H.P. charges) given the cash price, down payment, monthly payment and number of payments
5 Compute the total amount paid for property, given the down payment and the number and amount of monthly mortgage payments
6 Calculate the total cost of running a car, given depreciation, mileage etc
7 Calculate monthly payments, given the down payment, rate of interest, balance and period of time

IV Rates and Taxes

At the end of this topic, the student should be able to:

1 Read a water meter
2 Compute the amount of water used, given the previous and present readings
3 Compute the charges using a given schedule of rates
4 Read an electricity meter
5 Compute the amount of electricity in kW hours given the previous and present readings
6 Compute the charges using a given schedule of rates

V Triangles

At the end of this topic, the student should be able to:

1 Use scale drawing to solve problems in simple navigation
2 Show the relationship between the sides of the right-angled triangle
3 Recognise the 3, 4, 5, triangle
4 Prove the properties of the isosceles triangle
   (a) base angles are equal
   (b) the perpendicular bisector of the base bisects the angle at the apex
   (c) the bisector of the angle at the apex is the perpendicular bisector of the base
   (d) the median divides the triangle into two equal parts
5 Identify the median as the line of symmetry of an isosceles triangle
6 Apply the properties of an isosceles triangle to the equilateral triangle
7 Identify the lines of symmetry of an isosceles triangle

VI Trigonometry

At the end of this topic, the student should be able to:

1 Construct a table of values of the sine ratio for angles between 0° and 90°, in 10° intervals, by drawing
2 Use the above values in the solution of problems involving the right-angled triangles
3 Construct a table of values of the cosine ratio for angles between 0° and 90°, in 10° intervals by drawing
4 Identify the relationship \(\sin x = \cos(90-x)\)
5 Use the above values in the solution of problems involving right-angled triangles
6 Determine from table the values of sine and cosine for any angle between 0° and 90°
7 Apply the two ratios to the solution of right-angled triangles
8 Construct a table of values of the tangent ratio for angles between 0° and 50° in 10° intervals
VIII The Circle

At the end of this topic, the student should be able to:

1. Determine the angle properties of the circle:
   (a) The angle at the centre = 2 x the angle at the circumference
   (b) The angle in a semi-circle is 90°
   (c) Angles in the same segment are equal
   (d) Opposite angles of a cyclic quadrilateral are supplementary
   (e) Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle

IX Surface Area and Volume

1. Given a prism, cone, pyramid or sphere, with dimensions indicated, the student will be able to apply appropriate formulae to calculate the surface area of each

2. Given a prism, cone, pyramid or sphere, with dimensions indicated, the student will be able to apply appropriate formulae to calculate the volume

X Area of an Annulus

At the end of this topic, the student should be able to:

1. Demonstrate that the area of an annulus (or ring) is the difference between the area of two concentric circles

2. Apply the property above in finding the area of an annulus given its dimensions

XI Arc Length and Area of a Sector

At the end of this topic, the student should be able to:

1. Use the formula: \( \text{Arc length} = \frac{\text{angle at centre}}{360} \times \text{Circumference} \)

2. Use the formula: \( \text{Area of sector} = \frac{\text{angle at centre}}{360} \times \text{Area of circle} \)

to calculate the area of a sector
At the end of this topic the student should be able to:

1. Recognise that in a histogram the frequency is illustrated by the area of the column
2. Construct a histogram from a frequency table
3. Read data from a histogram
4. Construct a running-total column in a frequency table
5. Draw a running total graph (using columns)
6. Construct a cumulative frequency column in a frequency table
7. Draw cumulative frequency curve
8. Read information from a cumulative frequency curve
9. Identify the median from the graph
10. Be able to compare the mean, median and mode of a set of data