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A STUDY OF AERO ENGINE FAN FLUTTER
AT HIGH ROTATIONAL SPEEDS USING HOLOGRAPHIC INTERFEROMETRY

BY

PHILIP ADRIAN STOREY

A Doctoral Thesis

Submitted in partial fulfilment of the requirements
for the award of

Doctor of Philosophy of the Loughborough University
of Technology

May 1983

Supervisor: Professor J. N. Butters

Department of Mechanical Engineering

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ABSTRACT

Aero-elastic instability is often a constraint on the design of modern high by-pass ratio aero engines. Unstalled supersonic flutter is an instability which can be encountered in shrouded fans, in which mechanical vibrations give rise to unsteady aerodynamic forces which couple further energy into the mechanical vibration. This phenomenon is particularly sensitive to the deflection shape of the mechanical vibration.

A detailed measurement of the vibrational deflection shape of a test fan undergoing supersonic unstalled flutter was sought by the author. This measurement was required in order to assess the current theoretical understanding and modelling of unstalled fan flutter. The suitability of alternative techniques for this measurement was assessed. Pulsed holographic interferometry was considered optimum for this study because of its full field capability, large range of sensitivity, high spatial resolution and good accuracy. A double pulsed holographic system, employing a mirror-Abbe image rotator, was built specifically for this study. The mirror-Abbe unit was employed to rotate the illuminating beam and derotate the light returned from the rotating fan. This therefore maintained correlation between the two resultant holographic images. The holographic system was used to obtain good quality interferograms of the 0.86m diameter test fan when rotating at speeds just under 10 000rpm and undergoing unstalled flutter. The resultant interferograms were analysed to give the flutter deflection shape of the fan. The study of the fan in flutter was complemented by measurement of the test fan's vibrational characteristics under non-rotating conditions. The resultant experimental data were in agreement with the current theoretical understanding of supersonic unstalled fan flutter. Many of the assumptions employed in flutter prediction by calculation of unsteady work were experimentally verified. The deflection shapes of the test fan under non-rotating and flutter conditions were compared with those predicted by a finite element model of the structure and reasonably good agreement was obtained.
The work described in this thesis was carried out by the author at Rolls-Royce Ltd, Derby, during the period 1978 to March 1983. The finite element vibrational modelling of the test fan was performed by the Stress Office of Rolls-Royce. The image rotator and optical system were manufactured by the technical staff of the Advanced Research Laboratory, Rolls-Royce, to the author's design.
ACKNOWLEDGEMENTS

The author is grateful for the encouragement and helpful advice given by Prof. J. N. Butters (Loughborough University of Technology), Dr. D. G. Jones, Dr. C. J. Moore and Dr. R. B. Price (Advanced Research Laboratory, Rolls-Royce). The author wishes to thank colleagues at Rolls-Royce in the Advanced Research Laboratory, Aeroelastic Group, Stress Office and Altitude Test Facilities who gave technical support for this work.

Thanks are also given to Rolls-Royce Ltd for the facilities to carry out this research. This work was funded by the Ministry of Defence (Procurement Executive).
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NOMENCLATURE

A list of the nomenclature employed is summarised below. In all cases, a full description is given locally.

**English lower-case**

**a** Radius of rotator apertures

**b** Distance between off-axis mirror and optical axis for mirror-Abbe rotator

**c** Half chord length

**d, d₁, d₀** Distance between optical system and an optical plane

**e** Mean apparent speckle diameter

**f** Fringe spacing

**g** Tangential displacement

**h, h₀, h₁, h₂** Axial displacement

**l** Aperture diameter

**p** Displacement parallel to chord

**q** Displacement perpendicular to chord

**r, r₀, r₁, r₂, r_{TIP}** Radial position

**s, s₀** Laplace parameter

**s₁, s₂** Transverse coordinates

**t** Time

**u** Amplitude of shock movement

**u₁, u₂, u₃** Components of displacement vector \( \mathbf{u} \)

**v** Distance between rotator and object point

**w** Pathlength

**x₁, x₀, x₁, x₂** Coordinates

**y₁, y₀, y₁, y₂** Coordinates

**z₁, z₂, z** Coordinates

**Vectors**

**b₁, b₂** Vibration vectors

**e₁, e₂** Unbalance vectors

**k** Sensitivity vector
\[ K_i, K_r \]
\[ u, u_r \]
\[ \chi, \chi_r \]

**English upper-case**

A, \( A_2 \), A, A,
\[ A_{11}, A_{12}, A_{21}, A_{22} \]
B, B, B,
\[ B_p, B_q, B_r \]
\[ B, B_i \]
C, C, C,
\[ C_p, C_q, C_r \]
\[ C, C_{d1}, C_{d2}, C_{d3}, C_{d4} \]
\[ C_m \]
D
\[ E, E_p, E_q, E_r, E_i \]
\[ E, E_{d1}, E_{d2}, E_{d3}, E_{d4} \]
\[ E_{d5}, E_{d6}, E_{d7} \]
\[ E_m \]
\[ F, F_{d1}, F_{d2}, F_{d3}, F_{d4} \]
\[ F_m \]
\[ G, G_{d1}, G_{d2}, G_{d3}, G_{d4} \]
\[ G_{d5}, G_{d6}, G_{d7} \]
\[ G_{m} \]
H
\[ I_{max}, I_{min} \]
\[ J_1 \]
\[ K, K_r, K_i \]
\[ K_{d1}, K_{a1}, K_{m1}, K_{o1} \]
\[ K_{d2}, K_{a2}, K_{m2}, K_{o2} \]
M
\[ N \]
\[ O \]
\[ P, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8 \]
\[ Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8 \]
\[ Q_{d1}, Q_{d2}, Q_{d3}, Q_{d4}, Q_{d5}, Q_{d6}, Q_{d7}, Q_{d8} \]
\[ Q_{m} \]
\[ S \]
\[ T_{d}, T_{m} \]
\[ V_{d}, V_{m} \]
\[ V_{d}, V_{m} \]
\[ V_{d1}, V_{d2}, V_{d3}, V_{d4}, V_{d5}, V_{d6}, V_{d7}, V_{d8} \]
\[ V_{m} \]
\[ W \]

Illumination vector
Viewing vector
Displacement vectors
Velocity vectors

Unsteady aerodynamic coefficients
Influence coefficients
Unsteady aerodynamic coefficients
Blade number
Unsteady aerodynamic coefficients
Gladstone-Dale coefficient
Diametral number
Energy
F-number
Unsteady blade forces
Autocorrelation of impulse response transfer function
Average fraction of incident energy radiated towards the receiving aperture per unit solid angle
Intensity
Bessel function
Constants
Gain factors
Magnification
Number of blades
Amplitude
Fringe order
Transmission coefficient
Spatial phase
Transfer function
Velocity
Voltage
Work
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<td>$\alpha$</td>
<td>Blade torsion</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Rotator mirror angle</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Blade stagger angle</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Logarithmic decrement</td>
</tr>
<tr>
<td>$\zeta, \eta$</td>
<td>Damping</td>
</tr>
<tr>
<td>$\theta, \theta_1, \theta_2$</td>
<td>Rotational position</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength of light</td>
</tr>
<tr>
<td>$\nu, \xi$</td>
<td>Geometrical angles</td>
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<tr>
<td>$\pi$</td>
<td>3.142</td>
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<tr>
<td>$\rho, \rho_1, \rho_2$</td>
<td>Air density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Geometrical angle</td>
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<tr>
<td>$\tau_M, \tau_F$</td>
<td>Time constant</td>
</tr>
<tr>
<td>$\phi_1, \phi_0$</td>
<td>Temporal phase</td>
</tr>
<tr>
<td>$X$</td>
<td>Angle between illuminating and viewing directions</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Angle of tilt</td>
</tr>
<tr>
<td>$\omega, \omega_n, \omega_1, \omega_2, \omega_0$</td>
<td>Frequency</td>
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<td>$\mathcal{V}$</td>
<td>Fringe visibility</td>
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<tr>
<td>$N, N_R, N_I$</td>
<td>Number of passes through a beam splitter</td>
</tr>
<tr>
<td>$A_R, A_H, A_S$</td>
<td>Area</td>
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<td>$\Omega_R, \Omega_H$</td>
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INTRODUCTION

Since its conception in the 1920s, the turbojet engine has been developed to the level where it is now the major form of aircraft propulsion. This approach to propulsion has produced cheaper, quieter, cleaner and more convenient air travel.

Early applications of the turbojet were mainly in military aircraft, where the high thrust to weight ratio and the high speed capability were attractive. It produced faster and more manoeuvrable aircraft than was possible using diesel driven propellers. However, the overall efficiency of the early turbojets was lower than that of the diesel engine.

A step increase in the fuel efficiency of gas turbine engines was achieved with the introduction of high bypass ratio turbofans. The generation of large, relatively high speed passenger aircraft was made economically attractive by this generation of aero engine. Following their introduction, turbofans for civil aircraft have been developed to give better and better fuel efficiency. This has been achieved in parallel with reductions in noise and gaseous pollution and while still maintaining a high reliability. The increase in overall performance of aero engine gas turbines has been obtained by the use of improved materials, manufacturing techniques and knowledge. An example of better materials is the use of higher temperature, high strength metals for turbine blades. An example of improved manufacturing techniques is the use of air-cooled laminated porous sheet (Transply) for combustor applications. Improved understanding of the aerodynamic and mechanical processes occurring within aero engines has been vital in improving performance. There has been a move away from empirical approaches towards designs based upon understanding of the fundamental physical phenomena. This thesis describes a study of one aeroelastic process which can occur within an aero engine. The work served to improve confidence in our fundamental understanding of the process.
Good design of the fan is an important ingredient in a fuel efficient high bypass ratio aero engine. In a typical modern turbofan such as the Rolls-Royce RB211-524 approximately 60% of the propulsive force is derived from the fan. One of the constraints on fan design is flutter. This undesirable phenomenon is the coupling of mechanical vibration and unsteady aerodynamics which results in large amplitude vibrational deflection and hence high stressing of the fan blades.

It is important for aero engine manufacturers to have a reliable flutter prediction capability. A fan which has an over-conservative flutter margin is likely to have an unnecessarily low efficiency. Conversely, a fan which undergoes flutter within its operating cycle must undergo expensive redesigning and rebuilding.

Early predictions of the onset of fan flutter were based upon empirical evidence. This technique was not adequate for advanced fan designs. It did not provide a sufficiently accurate extrapolation beyond previous experience e.g. Jeffers and Meece (1975). It was thus considered less desirable than a method based upon fundamental scientific principles. Since the late 1960s, flutter prediction has moved towards this approach. However, currently, such a prediction approach employs many simplifications and assumptions, due to the complexity of the flutter phenomenon. This can result in inaccuracy of flutter prediction.

An important aspect of fan flutter prediction is the determination of the flutter vibrational shape. There have been few published experimental studies of the flutter vibrational response of rotating fans. In this thesis, a detailed study of the vibrational response of a high performance aero engine fan undergoing unstalled flutter is
presented. The fan was rotating at supersonic tip speeds and was studied using holographic interferometry. The experimental technique was described in a recent publication (Storey, 1982) which is reproduced in Appendix 2 of this thesis. The experimentally determined vibrational response was compared with predictions based upon the current theoretical understanding of supersonic unstalled flutter. The experimental results were used to test some of the assumptions employed in current flutter prediction methods.
CHAPTER 1

FAN FLUTTER: CURRENT THEORETICAL UNDERSTANDING
AND AIMS OF THIS STUDY

Summary of Chapter

The current theoretical understanding of fan flutter is described. Flutter modelling by calculation of unsteady work is a preferred prediction method. This approach relies heavily on accurate determination of the vibrational mode shapes of the fan. Mode shape prediction methods are discussed.

There has been little previously published experimental measurement of the detailed vibrational shape of fans undergoing flutter. The aim of this study was to obtain a detailed measurement of the vibrational deflection shape of a fan undergoing supersonic unstalled flutter and to use this to evaluate the theories and assumptions used currently for fan flutter prediction. This study was centred upon measurements made on a particular test fan. The physical details of this fan are given.

1.1 Introduction

Flutter is any self-excited oscillation of a structure exhibiting aerodynamic lift. It is an instability in which mechanical vibrations give rise to unsteady aerodynamic forces which feed further energy into the mechanical vibration. Energy is supplied to the fluttering structure from the moving airstream.

Flutter has been experienced with different structures including aircraft wings, compressor and turbine rotors, propeller blades and helicopter rotor blades. The flutter mechanism and hence theoretical
flutter modelling can however vary considerably from structure to structure. The analysis applicable to aircraft wings, that is two-dimensional flutter of an isolated aerofoil, was developed by Theodorsen (1935). This subject was extensively treated by Bisplinghoff et al (1955), Fung (1955) and Scanlan and Rosenbaum (1951). These analyses show that the mechanical vibrational frequency is considerably affected by the air velocity, and that the flutter instability results from coalescence of the fundamental bending and torsional mode frequencies.

The flutter mechanisms are somewhat different for a typical compressor rotor in an aero engine gas turbine. This is mainly due to two factors. First, the relatively high structural density of a compressor aerofoil makes the vibrational mode shapes almost independent of aerodynamic effects. Second, the unsteady aerodynamic coupling forces between rotor blades are much more complex than the unsteady aerodynamics of an isolated aerofoil.

Experience has shown that the fan stage of an aero engine is particularly susceptible to flutter. This is due to the relatively high structural flexibility of the fan assembly which results from its high tip to hub radius.

In the next section of this chapter, the reported experimental observations of fan flutter and the different operating regimes in which it is found are described. The current theoretical understanding and modelling of fan flutter are then presented, and the important ingredients for successful flutter prediction are emphasised. The aims of this study and their relationship to the current understanding of fan flutter are then given. This study was centred upon measurements made on a particular test fan. The physical details of this fan are given in the last section of the chapter.
1.2 **Types of Fan Flutter**

At least four types of flutter have been observed in fans. These were summarised by Snyder and Commerford (1974) and Mikolajczak et al (1975). These different flutter types occur in different fan operating regimes and different unsteady aerodynamic forces are associated with each. However, each flutter type is characterised by a very sharp increase in measured blade stress as the fan is driven over a "boundary" in its operating characteristics. A graph of a fan's characteristics showing the boundaries for the four types of flutter is shown in Figure 1.1. This figure shows the mean fan stage pressure ratio plotted against a relative weight flow rate. In addition to the flutter boundaries, four constant speed lines and two typical operating or working lines are shown. The four flutter types, that is supersonic unstalled, subsonic and supersonic stalled and choked flutter, are now described in turn.

Supersonic unstalled flutter is a particularly important phenomenon because it sets a top speed limit on the operation of a fan. A successful fan design has the unstalled flutter boundary beyond the maximum operating speed for all flow conditions and for all nominally identical production units. This flutter occurs at supersonic flow velocities when measured relative to the blade tips but at subsonic axial flow velocities. Snyder and Commerford (1974), Mikolajczak et al (1975) and Hallidie (1976) reported blade strain gauge measurements made on fans undergoing supersonic unstalled flutter which showed that all the fan blades were fluttering at the same frequency and with a constant inter-blade phase angle. It was deduced that the flutter vibrational response was that of a coupled blade mode which rotated with respect to the fan. In this type of vibrational mode, mechanical coupling between blades occurs via the part-span shrouds and fan disc (as shown in Figure 1.2). Unstalled supersonic flutter is typically detected at design conditions. This implies that the unsteady
FIGURE 1.1 DIAGRAM OF A FAN'S CHARACTERISTICS SHOWING BOUNDARIES FOR DIFFERENT FLUTTER TYPES.

FIGURE 1.2 PHOTOGRAPH SHOWING A TYPICAL SHROUDED FAN.
aerodynamic forces are not related to an off-design aerodynamic effect.

Stalled flutter occurs as a fan's pressure ratio is increased towards the surge condition. The occurrence of stalled flutter at both subsonic and supersonic blade tip relative flow velocities was reported by Mikolajczak et al (1975). They suggested that the aerodynamic coupling was associated with separated flows. The flutter vibrational response has been observed in coupled blade modes e.g. Jeffers and Meece (1975). This is in agreement with unpublished measurements on shrouded fans at Rolls-Royce Ltd.

Choked flutter was reported by Carter (1953) and Carter and Kilpatrick (1957) at low pressure ratios and part fan speeds. It was associated with negative flow incidence angles which resulted in high in-passage flow velocities. This phenomenon has received little attention because its boundary is typically well away from the operating conditions of an aero engine fan.

1.3 **Flutter Prediction by Calculation of Unsteady Work**

The use of an unsteady work method for fan flutter prediction was pioneered by Carter (1967). With this approach, the work done on the fan per cycle of motion by aerodynamic forces is calculated for each mode of vibration. The onset of flutter is predicted when, for any mode of vibration, the aerodynamic work is positive and sufficiently large to overcome the fan's mechanical losses due to material and frictional damping. This basic approach to flutter prediction has, where possible, now replaced the previously employed and less reliable empirical methods. The salient steps of the unsteady work approach are now considered in greater detail.

The time dependent unsteady forces and moment acting on a fan blade are related to vibrational translation and torsion by the use of unsteady
aerodynamic coefficients. The relevant forces, moment, translations and torsion for a fan blade section are shown in Figure 1.3. The complex forces perpendicular and parallel to the chord and the complex moment are,

\[ F_n = -\pi \rho \omega^2 (A_p + A_q + A_\alpha) \] \hspace{1cm} Eq 1.1

\[ F_h = -\pi \rho \omega^2 (B_p + B_q + B_\alpha) \] \hspace{1cm} Eq 1.2

\[ F_o = -\pi \rho \omega^2 (C_p + C_q + C_\alpha) \] \hspace{1cm} Eq 1.3

where, \( A_p, A_q, A_\alpha, B_p, B_q, B_\alpha, C_p, C_q \) and \( C_\alpha \) are the unsteady aerodynamic coefficients, \( \rho \) is the air density, \( \omega \) is the vibrational frequency, \( c \) is the half chord length and \( p, q \) and \( \alpha \) are the displacements defined in Figure 1.3. The unsteady aerodynamic coefficients and the displacements are complex, thus allowing representation of the phase differences between the unsteady aerodynamic forces and the displacements. The force and translation parallel to the blade chord were neglected by Carta (1967) because they were considered of second order importance. He evaluated the unsteady aerodynamic coefficients by using the isolated aerofoil theory of Theodorsen (1935). This was later improved upon by Mikolajczak et al. (1975) who used the blade cascade analysis of Vernon (1973) and Vernon and McCune (1975) and by Halliwell (1980) who reported the use of the alternative cascade analyses of Nagashima and Whitehead (1974) and Goldstein et al. (1977).

The work done on a blade by the unsteady aerodynamic forces and moments is obtained by computing the product of the in-phase components of force and displacement perpendicular to and parallel to the chord and of moment and torsion. This is obtained by integrating over one vibrational cycle. Thus, the total work done on a blade per cycle is given by the following expression.
FIGURE 1.3 DIAGRAM SHOWING THE UNSTEADY FORCES, MOMENT, TRANSLATIONS AND TORSION FOR A FAN BLADE SECTION.

FIGURE 1.4 DIAGRAM SHOWING THE ELEMENTS USED TO MODEL THE BLADES OF THE TEST FAN.

- 28 -
\[ W = \int (-\int P_a \, dp - \int F_\theta \, dq + \int F_o \, dx) \, dr \]  
Eq 1.4

where \( r \) is the radial position on the blade. In general, the instantaneous displacements and unsteady aerodynamic coefficients and hence unsteady aerodynamic forces vary as a function of radial position.

The aerodynamic damping for the rotor system is obtained using the below relationship.

\[ \delta_{\text{aero}} = -\frac{N \omega}{45} \]  
Eq 1.5

where \( \delta_{\text{aero}} \) is the aerodynamic logarithmic decrement of the system, \( N \) is the number of blades and \( \bar{E} \) is the average kinetic energy of the vibrational mode.

Flutter is predicted when the total damping for any vibrational mode is less than zero i.e.

\[ \delta_{\text{total}} = \delta_{\text{aero}} + \delta_{\text{mech}} < 0 \]  
Eq 1.6

where \( \delta_{\text{mech}} \) is the logarithmic decrement due to mechanical damping.

This logical approach based upon the calculation of unsteady work is the current preferred method for flutter prediction. It relies heavily on accurate calculation of aerodynamic coupling and vibrational mode shapes. This study is particularly concerned with the later. It is important to accurately determine the relative magnitude and the phase between the blade translations and torsion. These must be obtained as a function of radial position along the blade for each mode of vibration.

This, thus, provides accurate values for \( p, q \) and \( \alpha \) for use in Equations 1.1 to 1.3. Calculation methods for the prediction of vibrational mode shapes are described in the next section.
1.4 Vibrational Mode Shape Prediction

An important part of the design of gas turbine engines has been the prediction of the vibration characteristics of the bladed rotor stages including the fan. This has been so for two reasons: first, to prevent excessive forced vibrational stressing of the rotors within their operating speed range and, second, for flutter prediction. For the first application, it has been important to calculate the distribution of blade stress within a mode. This has been less important for flutter prediction where the distributions of blade translations and torsion have been primarily required, for the reasons given in the previous section. For both applications, it has been important to calculate the natural frequencies of the rotor modes of vibration.

Historically, the most common approach to the modelling of a complete bladed disc has been to consider the structure as an assembly of separate components. Each component has been first analysed individually. Then the characteristics of the whole assembly have been obtained by joining the constituent parts using the boundary receptances at each of the joints. Typically, use has been made of the nominal symmetry of a bladed disc in order to reduce the computation time required for the analysis.

The vibrational characteristics of individual rotor blades have been obtained by a number of alternative mathematical approaches. For instance a model based on formulation of differential equations of motion from energy considerations was used by Carnegie et al (1966), and two alternative techniques based on a lumped parameter approach and on the use of finite elements were compared by Anderson (1975).

The disc is typically easier to analyse than the rotor blades. A lumped parameter approach, suitable for non-uniform discs, was given by Ehrich (1956). A model having annular elements and based on classical
thin disc theory was described by Ewins (1973). The use of finite elements was described by Kirkhope and Wilson (1972).

An early mathematical model which joined identical blades and the disc using a receptance coupling procedure to allow prediction of the coupled-blade modes of a complete rotor without shrouding was developed by Armstrong (1955). In this model, the spatial distribution of point forces and couples on the blades at the blade-disc boundary was assumed to be a sinusoidal function of circumferential position. Armstrong considered a rotor having a large number of blades, and thus the forces acting on the rim of the disc were approximated as being continuously (and sinusoidally) distributed. The analysis was kept to a compact form by using this approach. This model was extended to include a shroud connecting the blades by Cottney (1971). A more accurate model using finite-element description of the disc and blades, but still based on many of the assumptions used by Armstrong, was described by Kirkhope and Wilson (1971).

In a practical aero engine rotor, small differences exist between the blades due to manufacturing tolerance and this can have a significant effect on the vibrational characteristics of that rotor. This detuning effect has been modelled by several authors including Whitehead (1966 and 1976), Wagner (1967), Dye and Henry (1969), Ewins (1973 and 1975) and El-Bayoumy and Srinivasan (1975). These analyses showed that a detuned rotor has more modes of vibration than the corresponding tuned assembly. Also, each mode shape is more complex, each mode shape having more circumferentially distributed Fourier components than its tuned equivalent. Several authors noted that detuning often results in the splitting of a diametral mode into two modes having the same number of nodal diameters but having slightly differing natural frequencies.
Little attempt has been made to include the effects of detuning into flutter prediction models. This is partially due to the unpredictability of the detune-phenomenon for production rotors. However Ford and Foord (1979) and Ford (March 1980, Sept. 1980) used a simplified aerodynamic model to study the effects of detuning on fan flutter characteristics. Their theoretical model incorporated twin coupled blade-disc modes having slightly differing natural frequencies. They considered a flutter mechanism in which the response of one mode generated aerodynamic forces which drove the twin mode, and vice-versa. They predicted that detuning produces unequal blade amplitudes, variation from blade to blade in the temporal phase between torsion and flap within each individual blade, and a deflection shape which is not sinusoidal circumferentially. They concluded that detuning has a favourable inhibiting effect on flutter.

1.5 Aims of this Study

Supersonic unstalled fan flutter is typically predicted using a calculation of unsteady work which employs several approximations or assumptions. These include,

(a) the use of independent unsteady aerodynamic coefficients,
(b) the exclusion of any radial unsteady aerodynamic coefficients,
and (c) the exclusion of detuning effects.

Approximations such as these can result in inaccuracy of flutter prediction. This study is particularly concerned with the latter of the assumptions described above.

Carta (1967), Mikolajczak et al (1975) and Halliwell (1976) showed that accurate determination of the flutter vibration mode shapes is an important prerequisite for reliable flutter prediction based upon the calculation of unsteady work. A symmetrical coupled blade mode which rotated uniformly with respect to the fan was assumed in the prediction models for supersonic unstalled fan flutter which were used by these authors. The presence of a travelling assembly vibration mode was ascertained from limited time-resolved point measurements made using strain gauges attached to fan blades and casing mounted probes.

The detailed deflection shapes of the forced modes of vibration of non-rotating fans have been experimentally studied using techniques such as holography e.g. Hockley et al. (1978). However, there has been little published experimental study of the detailed vibrational shape of fans undergoing flutter. Previously published studies have been limited to measurements made at specific points e.g. blade tip deflection measurements were reported by Chivers (1980), and deflection measurements made at a limited number of blade radial stations were reported by Stargardter (1977).

The aim of this study was to obtain a detailed measurement of the vibrational deflection shape of a fan undergoing supersonic unstalled flutter and to use this to evaluate the theories and assumptions used currently for fan flutter prediction. Ideally this study of supersonic
unstalled fan flutter would have extended to the investigation of many alternative fan types. However, due to limited resources, only one test fan, which was of importance to Rolls Royce Ltd., was studied. The test fan was a typical modern high efficiency, high bypass ratio shrouded aero engine fan.

The specific aims of this study were,

(a) to experimentally determine the detailed flutter vibration shape of the test fan as a function of the temporal flutter cycle;

(b) to use these experimental data to determine how well the flutter response of a real fan approximates to a symmetrical uniformly travelling coupled blade mode;

(c) to compare the experimentally determined flutter shape with that predicted by a sophisticated finite-element calculation for the test fan, and to test any assumptions made in that finite-element calculation.

In addition to the above flutter studies, an experimental determination of the forced vibration mode shape of the non-rotating test fan was sought. This would allow comparison of the non-rotating mode shape with that predicted by the finite-element calculation, which would complement the comparison described in (c) above.

1.6 Test Fan: Description and Vibrational Modelling

The test fan was of a design which was typical for a modern high efficiency high bypass ratio aero engine. The fan was made of titanium, had a diameter of 0.86m and had thirty-three shrouded blades. The 100% design speed of the unit was 10 100 rpm, which corresponded to a tip speed of 457ms⁻¹. Further details of the test fan are given in Table 1.1.
<table>
<thead>
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<th>Characteristic</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan radius</td>
<td>0.43</td>
<td>m</td>
</tr>
<tr>
<td>Number of blades</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Radius of shroud</td>
<td>0.307</td>
<td>m</td>
</tr>
<tr>
<td>Radius of blade root platform</td>
<td>0.126</td>
<td>m</td>
</tr>
<tr>
<td>Blade chord at tip</td>
<td>0.102</td>
<td>m</td>
</tr>
<tr>
<td>Blade stagger angle at tip at zero speed</td>
<td>63</td>
<td>°</td>
</tr>
</tbody>
</table>

Table 1.1. Summary of the mechanical description of the test fan.
The shapes of the coupled blade-disc vibrational modes of the test fan were predicted by the finite element method. The mode shapes were calculated for both zero and full speed conditions. A symmetrical structure having 33 identical blades was assumed. Finite element models for both the blades and the disc were obtained. These were then combined to predict natural frequencies and mode shapes for the whole fan assembly.

The blades were modelled using triangular finite elements as shown in Figure 1.4. The disc was modelled using annular elements. These models of the individual components were combined by assuming a sinusoidal circumferential distribution of forces and deflections. The blade model was coupled to that of the disc using one finite element node having four independent degrees of freedom. The blade to blade coupling at the shroud was modelled assuming a locked shroud ring. Coupling at the shroud was performed using three finite element nodes each having six degrees of freedom. The nature of the model produced a shroud ring which was very stiff in the circumferential direction.

The effects of centrifugal load, aerodynamic load and temperature distribution were included in the calculation of the mode shapes at speed. Centrifugal load, which was proportional to the square of the rotational speed, was the dominant effect. This was primarily because of the resultant increased stiffness of the structure. The combined centrifugal, aerodynamic and thermal effects were modelled,

(a) by including appropriate additional stiffness terms into the prediction calculation, and

(b) by accounting for blade untwist (decrease in blade stagger angle) resulting from the combined effects of centrifugal and aerodynamic load.
The predicted vibrational mode shapes for the test fan obtained using this finite element model are given in Chapters 3 and 7.
CHAPTER 2

FLUTTER DEFLECTION SHAPE DETERMINATION:

ASSESSMENT OF MEASUREMENT TECHNIQUES

Summary of Chapter

The suitability of alternative techniques for measuring the detailed flutter vibration shape of the test fan was assessed. Pulsed holographic interferometry was considered optimum for study of the test fan because of its full-field capability, large range of sensitivity, high spatial resolution and good accuracy. The large range of sensitivity allowed the same basic technique to be used for study of both the high amplitude response of the rotating fluttering fan and the low amplitude forced response of the non-rotating fan. The special considerations and limitations for application of holography to study of the fluttering test fan were considered in detail. It was concluded that use of a pulsed holographic system employing an image rotator and sensitive to only axial and radial deflections was a preferred measurement technique.

2.1 Introduction

Techniques capable of measuring the vibration of a rotating fan were considered, and they are discussed in this chapter. The suitability of the alternative techniques for measuring the detailed flutter vibration shape of the test fan was assessed. Deflections over a large area of the test fan were required, including measurements at all radii between the blade roots and tips. Peak deflections in the range from 0.1mm to several mms were expected. These measurements were considered likely to be a difficult application for any technique. This was due to the fan's high rotational velocity of 450 m s\(^{-1}\) at the tip, the high centrifugal loads
on the blades of up to $5 \times 10^5 \text{ ms}^{-2}$ (50 000g) and the high inter-blade flow velocities of several hundred metres per second.

The use of conventional vibration measuring techniques was first considered. The limitations of these techniques were assessed and they are discussed in the next section of this chapter. Optical techniques were considered to have several advantages for the required flutter measurements. These advantages and a review of candidate optical techniques is given in Section 2.3. Of the several promising optical measurement techniques, holographic interferometry was considered optimum for this investigation of the test fan. The remainder of the chapter is then devoted to an assessment of the limitations and capabilities of holographic interferometry.

2.2 Review of Conventional Techniques

The most commonly used device for vibrational measurement of gas turbine rotor blades is the strain gauge. Strain gauges are typically resistive elements which are bonded to a blade to give a measure of surface strain. Electrical signals are typically obtained from the rotating gauges using either mechanical slip-rings or radio frequency telemetry units, as reported by Worthy (1980). Strain gauges have been used extensively to measure point strains on rotating fan blades in flutter. However, determination of the detailed flutter vibration shape of the test fan would have required an impracticably large number of gauges. The attempted measurement of the amplitude and phase distributions of the different components of deflection of a fluttering fan using strain gauges was reported by Mikolajczak et al (1975). They reported that little success was obtained.

Piezoelectric accelerometers (Ewins, 1976) have been used extensively for vibrational impedance measurements on non-rotating gas turbine
components including non-rotating fans. They have not been greatly applied to measurements on rotating structures due to their unsuitability for use at high centrifugal loads.

The successful measurement of tip deflections of rotating vibrating blades has been reported by several authors, including Chivers (1980). The timing of the blade tip leading and trailing edges was used to determine the blade tip vibrational deflections. The timing of the blade passing was accurately obtained using casing mounted proximity detectors. This timing technique was considered by this author to be inappropriate for measurements other than at the blade tip where the casing provided a unique non-intrusive rigid mount for the proximity probes.

The above brief description covers the commonly used techniques for measuring rotor blade vibration. None of these techniques was considered appropriate for attaining the detailed flutter shape required of the test fan.

2.3 Candidate Optical Techniques

Several point measurement optical systems have been reported. These include the beam bouncing technique which was used by Stargardter (1977) for fan flutter deflection measurements; a laser-Doppler technique evaluated by Cookson and Bandyopadhyay (1979) in which points on a rotating structure were sampled once per revolution; and a further laser-Doppler technique suggested by Fagan and Beeck (1979) in which one point on a rotating structure was continuously sampled by employing an image rotator. The use of a point measurement technique, such as these, for obtaining the flutter deflection shape of the test fan would have required either the use of an impractically large number of systems operating in parallel or a scanning system. The idea of scanning such a system over the fluttering fan's surface was rejected due to the likely difficulty of maintaining
constant flutter conditions throughout the scan. Thus, the use of a point measurement technique was considered inappropriate for the required detailed flutter deflection shape determination.

The use of a full-field optical measurement technique was preferred and was considered to have several advantages for the required flutter studies. The prime advantage was the capability for making instantaneous deflection measurement over the whole or large areas of the visible surface of the fluttering test fan. The envisaged optical techniques were non-contacting and thus had the potential of making measurements without affecting the flutter response of the fan. Except for any surface preparations, they would be unaffected by the high centrifugal and aerodynamic forces acting on the blades.

The requirement for a measurement method capable of determining the flutter vibration shape at selected points in the flutter cycle, suggested the use of a pulsed optical technique. A time-averaged technique would not meet this requirement and was thus considered inappropriate.

The full-field optical techniques, which were assessed, fell into the broad categories of holographic, speckle or moiré methods. The relative advantages of these alternative categories were assessed and are described below.

The use of holographic interferometry for deflection measurement has been reported extensively e.g. in texts by Erf (1974) and Vest (1979). Pulsed holography provides a means of studying vibrations having amplitudes in the range from less than 1μm to several mms. For the case of double pulsed holography, this large range of sensitivity can be achieved by varying the laser pulse separation. For a holographic system employing a single illuminating beam, the holographic sensitivity vector is in the direction of the angular bisector of the illuminating and viewing directions. Thus, the holographic technique is particularly well suited to the
measurement of the out of plane component of vibration. It is a technique which is capable of high spatial resolution and good accuracy, and it can be applied to objects having a large depth of field. The holographic technique has the disadvantage that it is limited by decorrelation constraints, as discussed in Sections 2.4 to 2.6. The use of holographic interferometry for vibrational study of rotating objects has been reported by several authors, as described in Section 2.7.

Electronic speckle pattern interferometry (ESPI) was developed by Butters and Leendertz (1971). The use of pulsed lasers with ESPI was described by Cookson et al (1978). ESPI can be considered as a form of image plane holography in which the spatial frequencies of light at the recording plane are reduced sufficiently to allow the conventional photographic processing to be replaced by a television system. This, therefore, gives the convenience of real time viewing. ESPI has decorrelation limitations and a sensitivity equal to those of its holographic counterpart. It has the disadvantage of reduced spatial resolution when compared with holography.

Speckle photography was first introduced by Archbold et al (1970). It has been applied to measurement of in-plane translation and tilt of engineering structures e.g. DeBacker (1975) and Gregory (1978), respectively. Defocussed speckle photography was used by Stetson (1978) for the measurement of tilt of a rotating structure. Measurement of both in-plane translation and tilt requires the simultaneous use of focused and defocused speckle photography. The application of such a system required very careful experimental technique (Stetson, 1977) and analysis (Gregory, 1978). The minimum sensitivity of the technique is defined by the requirement for separation of corresponding speckles between exposures. The minimum sensitivity for in-plane translation is thus equal to the mean apparent speckle diameter on the object, which is approximately equal to $1.2 \frac{\lambda}{D}$.
where $\lambda$ is the wavelength of light and $\phi$ is the full angle subtended by the optical system at the object. Likely practical values of $\lambda$ and $\phi$ for a system applied to the test fan were considered to be 0.69$\mu$m and 0.02 radians, respectively. This corresponds to a minimum sensitivity of 40$\mu$m.

The use of double-exposure and time-averaged moiré techniques to measure surface deformation and vibrational displacement was first demonstrated by Abramson (1968), using a coherent fringe projection system. This technique was developed for vibrational studies by others including Vest and Sweeney (1972). The alternative and equivalent fringe projection technique using incoherent illumination was first reported by Der Hovanessian and Hung (1971). The extension of this technique to the study of a rotating component was reported by Sikora (1981). He used a pulsed incoherent fringe projection system to measure large amplitude deformation of a rotating propeller at speeds up to 2000 rpm. Projected fringe moiré has the advantage that it is not subject to decorrelation constraints, as are holographic and speckle techniques. It does however have a relatively large minimum sensitivity. The minimum sensitivity is determined by the depth of field required and by the practicable range of illumination and viewing angles. For application to the test fan, a depth of field in excess of 50mm was required. In addition, an angle between the illumination and viewing directions of no greater than 30° was required in order to prevent unacceptable shadowing of the fan blades. It was estimated that these constraints would have imposed a minimum sensitivity of the order of 4$\mu$m per moiré fringe, thus limiting any projection moiré technique to the measurement of high level flutter amplitudes of the test fan.

After assessment of the above full-field optical techniques for application to the test fan, double pulsed holographic interferometry was
chosen as the most appropriate for the required studies. The large range of sensitivity, allowing the study of submicrometre to millimetre vibration amplitudes, was considered an important advantage of pulsed holography. The holographic technique could thus be used for both the low amplitude vibrational studies of the non-rotating fan and the high amplitude fan flutter studies. The high spatial resolution and accuracy of the holographic technique were seen as two further important advantages. The convenience and advantage of real-time and remote viewing of the holographic interferograms was considered achievable using thermoplastic recording techniques.

2.4 Special Considerations for the Application of Holography to a Rotating Fluttering Fan

The special requirements of the double pulsed holographic technique for application to the fluttering test fan are described in this section. These special considerations arose due to the relatively high rotational velocity of the fan. They are now described in turn.

The first requirement was that each individual laser pulse should record a bright holographic image of the rotating fan. This problem was considered by Smith (1969) who stated that a bright holographic image is obtained when any phase change at the hologram due to movement of the object is small over the duration of the exposure i.e. when

$$\int V \cdot k \, dt < 1$$

Eq 2.1

pulse duration

where $V$ is the velocity vector of a point on the object and $k$ is the holographic sensitivity vector given by
\[ k = \frac{1}{2} (k_I - k_V) \]

where \( k_I \) and \( k_V \) are the wave vectors of light in the illuminating and viewing directions, respectively, both having magnitude \( \frac{2\pi}{\lambda} \).

This condition was made no more demanding than for a non-rotating object by arranging for the sensitivity vector to be orthogonal to the rotational vector for all object points, i.e. when \( V \cdot k = 0 \), where \( V \) is the rotational velocity component. This was achieved by illuminating and viewing the object from on-axis.

A second requirement was that the interference fringes due to vibrational movement should not be distorted by the fan's rotation. This was also achieved by ensuring that the sensitivity vector was orthogonal to the rotational displacement of the object, i.e. for \( U \cdot k = 0 \), where \( U \) is the displacement of a point on the fan between the two laser pulses due to whole-body rotation. This requirement prevented the measurement of the tangential component of the vibration deflection, restricting measurement to the axial and radial components.

A third requirement was that the rotational movement of the fan's image between exposures should be small in order to produce correlated light fields and thus high visibility interference fringes. Any relative movement of the two holographic images which was greater than the width of the autocorrelation function of the image plane light amplitude distribution would have resulted in an unacceptable reduction in fringe visibility.

Thus, the first two requirements could be satisfied by ensuring that the holographic sensitivity vector was orthogonal to the rotational movement of the fan. The third requirement set a limit on the use of a conventional double pulsed holographic system. This limit was quantified and is described in the next two sections of this chapter.
2.5 **Determination of Fringe Visibility as a Function of Fan Rotation**

Decorrelation of the two holographic images due to fan rotation and its effect on fringe visibility were quantified by both theoretical and experimental investigations. These investigations were performed for a general rotating object and are described in this section. The results of this work were used, as described later in this chapter, to determine whether any special technique would be needed to perform the required flutter studies of the test fan using pulsed holography.

The variation of holographic fringe visibility as a function of object movement was analysed in two recent publications by Dändliker (1980) and Celaya and Tentori (1980). This analysis called upon an earlier determination of the image plane autocorrelation function for coherent light by Lowenthal and Arsenault (1970). The conditions, assumptions and results of these analyses are presented below.

The reconstruction system which was considered is shown in Figure 2.1. The hologram position was not defined because it just acted as an intermediate storage medium, allowing two states of the object to be simultaneously present. As long as the hologram was correctly reconstructed and did not aperture the reconstruction, its position was unimportant.

The impulse response function of the imaging system was assumed invariant over the imaging field. The optically rough object was assumed to have a surface roughness autocorrelation peak which was much narrower than the resolution of the imaging system.

The fringe visibility was defined as,

\[ V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]  

Eq 2.3

where \( I_{\text{max}} \) and \( I_{\text{min}} \) are the maximum and minimum intensities, respectively, of the holographic interference pattern.
Fringe visibility as a function of image plane translation was shown to be given by the following relationship.

\[ \mathcal{V} = \left| G(\Delta S_i) \right| \]  
Eq 2.4

where \( G \) is the autocorrelation of the impulse response function of the imaging system and \( \Delta S_i \) is the mutual transverse shift of the image plane speckle patterns, where \( (\Delta S_i)^2 = (\Delta x_i)^2 + (\Delta y_i)^2 \) and \( x_i \) and \( y_i \) are shown in Figure 2.1. For the case of a circular aperture of diameter \( l \), the autocorrelated function, \( G \), and visibility, \( \mathcal{V} \), are of the form

\[ G = \frac{2J_1(\pi l \Delta S_i / d_i)}{(\pi l \Delta S_i / d_i)} \]  
Eq 2.5

and

\[ \mathcal{V} = \left| \frac{2J_1(\pi l \Delta S_i / d_i)}{(\pi l \Delta S_i / d_i)} \right| \]  
Eq 2.6

where \( J_1 \) is the first order Bessel function, and \( d_i \) is the distance between the lens and the image plane. The function for \( G \) given in Equation 2.5 is shown in Figure 2.2. From the full analysis of Dändlikier and Celaya and Tentori, a 180° phase reversal, converting bright fringes into dark fringes and vice-versa, was predicted in the regions where \( G \) is negative. The first zero of the expressions for \( G \) and \( \mathcal{V} \) occurs at

\[ \Delta S_i = \frac{1.22d_i}{l} \]  

which corresponds to the mean speckle size in the image.

The Equation 2.6 was used to obtain the holographic fringe visibility in terms of object coordinates for the case of a rotating object. Pure rotation of the object, through an angle \( \Delta \theta \) about an out of plane axis, results in

\[ \Delta S_i = Mr \Delta \theta \]  
Eq 2.7

where \( M \) is the magnification of the optical system, and \( r \) is the distance
FIGURE 2.1 DIAGRAM OF THE HOLOGRAM RECONSTRUCTION SYSTEM.

FIGURE 2.2 GRAPH SHOWING $G$, THE AUTOCORRELATION FUNCTION, PLOTTED AGAINST $\Delta \xi l/\lambda d_{\xi}$. 
of the object point from the axis of rotation.

The visibility is, thus, given by

$$\nu = \left| \frac{2J_1(\pi r \Delta \theta / \lambda d_o)}{(\pi r \Delta \theta / \lambda d_o)} \right|$$

Eq 2.8

where $d_o$ is the distance between the lens and the object plane.

The holographic fringe visibility is given in terms of easily determined experimental parameters, by this relationship. A visibility drop to zero is predicted for object rotational movement equal to the mean apparent speckle size on the object.

The theoretical analysis presented by Dandliker and Celaya and Tentori, described above was not substantiated with any experimental evidence. Thus an experimental determination of fringe visibility, $\nu$, as a function of object rotation and aperture of the reconstruction system was undertaken by this author and is described in Appendix 1. The experimental results were in good agreement with those predicted by Equation 2.8, thus verifying this theoretically determined expression.
Limitations of Standard Double Pulsed Holography for Flutter Measurement

Decorrelation limitations of the standard double pulsed holographic technique for study of the fluttering test fan were considered. These limitations are discussed in this section and are based upon Equation 2.8.

In the last section, it was shown that changing the f-number of the holographic reconstruction system changes the visibility of interference fringes on a rotating object. Increasing the f-number, increases the maximum angle of rotation between exposures which can be used without causing unacceptable loss of fringe visibility. Of course, the f-number of the reconstruction system can not be increased without limit. Increasing the f-number produces an increase in the image speckle size and hence a decrease in the image resolution. The maximum reconstruction system f-number which can be employed is limited by the resolution required to visualise the interference fringes. This concept was quantified for conditions of the fluttering test fan, as is described below.

The minimum acceptable fringe visibility on holographic interferograms of the fluttering test fan was set, at this author's discretion, as approximately 0.2. Therefore from Equation 2.8,

\[
0.2 < \left| \frac{2J_1(\pi r \Delta \theta / \lambda d_0)}{(\pi r \Delta \theta / \lambda d_0)} \right| \quad \text{Eq 2.9}
\]

thus,

\[
r \Delta \theta < 0.98 \frac{\lambda d_0}{\lambda} \quad \text{Eq 2.10'}
\]

and thus,

\[
V_{TTF} \Delta t < 0.98 \frac{\lambda d_0}{\lambda} \quad \text{Eq 2.11'}
\]
where $V_{\text{TIP}}$ is the rotational velocity at the blade tip and $\Delta t$ is the laser pulse separation.

A fringe resolution of one full fringe per 5mm at the fan, or better, was sought. It was estimated that a mean speckle diameter of less than approximately one tenth of the fringe spacing was required in order to clearly resolve the holographic interference fringes. Thus, an apparent mean speckle diameter of less than approximately 0.5mm was needed to achieve the required fringe resolution. The apparent mean speckle size, $e$, can be controlled by the size of the viewing aperture and is given by

$$e = 1.22 \frac{\lambda d_0}{\lambda}$$

Thus, for $e < 5 \times 10^{-4} \text{m}$,

$$\frac{\lambda d_0}{\lambda} > \frac{5 \times 10^{-4}}{1.22}$$

Combining inequalities 2.11 and 2.13 gives,

$$\Delta t < \frac{0.98 \times 5 \times 10^{-4}}{1.22 V_{\text{TIP}}}$$

$V_{\text{TIP}}$ at the design speed of the test fan was 450ms$^{-1}$. Substitution into Equation 2.14 indicates that a pulse separation less than 0.9µs was required in order to meet both the fringe visibility and resolution requirements.

It was estimated that an interferogram, having fringe orders up to approximately ten, was needed to achieve the required fringe density and hence the required measurement resolution. The measured component of deflection, $h$, occurring between laser pulses is approximately related to the fringe order, $P$, by

$$h \sim \frac{P \lambda}{2}$$

Eq 2.15
Therefore, for $P = 10$,

$$h \sim 5\lambda$$ \hspace{1cm} \text{Eq 2.16}

Thus, from Equations 2.14 and 2.16, the peak velocity of the minimum measurable flutter level was determined as

$$\frac{5\lambda}{0.9 \times 10^{-6}} \text{ ms}^{-1}.$$ 

At a wavelength of 0.7$\mu$m, this corresponds to a peak velocity of approximately $4\text{ms}^{-1}$. The expected flutter frequency was of the order of 4000 radians $s^{-1}$. Thus, the peak amplitude of the minimum measurable flutter wave was $\sim 4/4000 \sim 1\text{mm}$.

Thus it was estimated that, for achievement of the fringe visibility and resolution requirements, the standard double pulsed holographic technique was limited to flutter amplitudes greater than 1mm. This corresponded to fairly high levels of flutter.

### 2.7 Review of Special Techniques for Preventing Holographic Image Decorrelation

As a result of the analysis presented in Section 2.6, it was concluded that the use of holographic interferometry to study the test fan at low levels of flutter would require some special technique which suppressed or compensated for the fan's rotation. Historically, several methods have been proposed and tested for operation in this regime. These methods fall into three basic categories which have been termed by MacBain (1980) as stroboscopic, rotated plate, and image derotated holographic interferometry. This subject was reviewed by MacBain in 1980 and this section serves to summarise and update that review.

The stroboscopic category consists of techniques in which two states of the object having the same angular position are compared interferometrically. Waddell et al (1970), Waddell (1972), Smith (1974) and Kawase et al (1976) reported techniques in which a rotating object
was compared holographically with its static state. Alternatively, two conditions, an integral number of revolutions apart, could be compared. These techniques are limited to the measurement of only small deformations, typically much less than 100 μm, and thus were clearly inappropriate for study of the fluttering test fan.

The second approach is that of rotating the holographic recording material at the same speed as the object. This can be achieved by attaching the recording film or plate to the object as was reported by Tsuruta and Itoh (1970), Sikora and Mendenhall (1974) and Morozov et al (1981). A more convenient alternative was suggested by Beeck and Kreitlow (1977), which used a separate shaft to rotate the recording material. In either case, this approach was considered by this author to be impracticable for remote rig running due to the difficulty of film reloading.

The third category consists of techniques employing an image rotator, synchronised to half the speed of the object, to optically compensate for its rotation. This technique was pioneered by Stetson (1978) who used a transmissive folded-Abbe type rotator and a pulsed ruby laser. Initial interferograms were obtained on a disc at speeds up to 9200 rpm. This system was then applied to the study of a 0.8l m diameter model aero engine fan operating at speeds up to 7500 rpm, as reported by Erf and Stetson (1980). Stetson's holographic system was used by MacBain et al (1979) and Stange and MacBain (1981) for extensive vibration analysis of a rotated disc. It was also used by Bearden and Clarady (1980) for study of a bladed turbine disc rotated at speeds up to 7500 rpm in a vacuum chamber. MacBain et al (1981) extended the use of Stetson's image rotator to the study of the real-time response of a rotating disc with a holographic system employing a strobed argon-ion CW laser. An alternative approach was taken by Beeck, Fagan and Kreitlow who developed a holographic system employing a reflective Porro prism. They obtained interferograms of a
0.43m diameter automobile fan at speeds up to 2850rpm and of a 0.25m diameter disc at speeds up to 13000rpm. This work was reported in papers by Beeck and Fagan (1980) and Fagan et al (1981). Their holographic system was used by Haupt and Rautenburg (1982) for the study of a radial impeller of unspecified diameter at speeds up to 13000rpm.

This third approach, employing an image rotator, was considered the most easily applied to study of the test fan in the environment of a compressor test facility. The use of such a technique would allow study of the test fan at all flutter conditions including at low vibrational amplitudes during flutter onset.

2.8 Concluding Remarks

Double pulsed holographic interferometry was considered the most appropriate technique for study of the test fan,

(a) when undergoing unstalled flutter at high rotational speeds.

and also

(b) when undergoing forced vibrational excitation in the laboratory at zero speed.

The reasons for choosing this technique are described fully in Section 2.3. The limitations of the holographic technique were considered in detail, and it was concluded that a system having the holographic sensitivity vector orthogonal to the fan's rotational vector was required. This limited the technique, for application to the rotating fan, to measurement of the axial and radial components of the flutter vibration. Following a detailed evaluation of the holographic decorrelation constraints, it was concluded that a special technique was required in order to allow study of the rotating test fan at all flutter amplitude levels. The use of
an image rotator to optically compensate for the fan's rotation in order to prevent image decorrelation was considered the most suitable approach for study of the rotating test fan.
CHAPTER 3

MODE SHAPE MEASUREMENT OF THE NON-ROTATING TEST FAN AND COMPARISON WITH THE FINITE ELEMENT PREDICTION

Summary of Chapter

The vibrational mode shape and general forced vibrational characteristics of the test fan under non-rotating conditions were determined using mechanical impedance measurements and pulsed holography. This was undertaken (a) so as to allow comparison with the corresponding mode shape obtained using the finite element (FE) model and thus test the model before extension to the rotating case, and (b) so as to determine the effect of any detuning due to manufacturing tolerances. Effort was concentrated on the 3D2F coupled blade mode, because, this was the mode in which the fan fluttered during holographic study.

Detuning resulted in a discernible frequency split between twin orthogonal modes, but the effect was small. The circumferential distribution of deflection was sinusoidal to a good approximation. This was an assumption in the FE model which was thus verified. Detailed comparison of the measured and predicted 3D2F mode shapes was made. This revealed that the FE model gave a predicted 3D2F mode shape which was in reasonable agreement with the measured shape. However poor agreement between measured and predicted natural frequencies for the 2D2F to 5D2F modes cast doubt on the over-all reliability and accuracy of the FE model.

3.1 Introduction

A detailed measurement of the forced vibrational mode shape of the non-rotating test fan was performed. This was undertaken, primarily, so as to allow comparison with the predicted mode shape obtained using the FE model described in Section 1.6. The theoretical model was for the fan under corresponding conditions, i.e., zero centrifugal loading
and at a uniform temperature of 20°C. This was, therefore, a useful check of the finite element model under relatively simple conditions. The experimental study of the non-rotating fan also allowed the effects of vibrational detuning, due to manufacturing tolerances, to be assessed.

Chronologically, this experimental measurement was performed after the study of the fan in flutter, however it is presented here, in an early chapter, so as not to interrupt the descriptive flow of the thesis.

The vibrational mode shape of the non-rotating fan was measured using double pulsed holographic interferometry. The approach described by Hockley et al (1978) was employed. Holograms were recorded from two directions, thus allowing measurement of both axial and tangential components of deflection. Effort was concentrated on analysis of the three diameter, second family (3D2F) coupled blade-disc mode of the fan, because this was the mode in which the fan fluttered.

The preparation and mounting of the test fan is described in the next section. This is followed by a description of the vibrational excitation technique which was employed. Vibrational impedance measurements on the fan are described and the results are presented. The holographic technique is then described and some of the resultant interferograms are given. The procedure used for quantitative analysis of the interferograms is described. Finally, the resultant 3D2F mode shape deflections and the comparable finite element predictions are presented and discussed.

3.2 Fan Preparation

The blade roots were bonded to the disc, and the blade shrouds were bonded together at their interfaces. This was done in order to simulate centrifugal locking of the structure. The bonding was achieved using an
epoxy cement.

The fan, with its axis horizontal, was then clamped to a sturdy pillar. Attachment was via the disc, and engine mounting conditions were simulated as closely as possible.

The fan was sprayed with a white powder, Ardrox 9D6, so as to improve its light scattering properties. In order to aid hologram analysis, the centre-chord position at the tip of each blade was defined using an edge of applied PVC tape.

3.3 Vibrational Excitation and Impedance Measurement

3.3.1 Experimental Technique

The fan was vibrationally excited using a single small 8W electromagnetic shaker which was mechanically connected to a blade via a metal drive-rod. The force applied to the fan was measured using a miniature piezo-electric force gauge which was incorporated into the drive-rod. The vibrational response of the fan at a point was measured using a miniature piezo-electric accelerometer having a mass of 0.5g. A photograph of the shaker and monitoring transducers installed on the test fan is shown in Figure 3.1.

The various electronic units used for vibrational excitation and monitoring are shown in Figure 3.2. The electromagnetic shaker was driven using a stable oscillator and amplifier. The oscillator produced either a random noise or sinusoidal voltage output. The force gauge and accelerometer electrical outputs were amplified and conditioned using two matched charge amplifiers. The outputs from the charge amplifiers were fed to a monitoring oscilloscope and a digital signal analyser (Hewlett-Packard 5420A). The amplified accelerometer signal was also monitored using a digital DC voltmeter.
FIGURE 3.1  PHOTOGRAPH OF THE ELECTROMAGNETIC SHAKER AND MONITORING TRANSUCERS INSTALLED ON THE TEST FAN.
FIGURE 3.2 BLOCK DIAGRAM SHOWING THE INTER-CONNECTION OF THE VARIOUS VIBRATIONAL EXCITATION AND IMPEDANCE MEASUREMENT UNITS.
The experimental arrangement described above was used to measure the mechanical impedance of the test fan. To be more specific, inertance, which is a point acceleration divided by the point force, was measured as a function of frequency. These measurements were performed in order to,

(a) determine the natural frequency of the 3D2F mode for subsequent holographic analysis;

(b) determine the best excitation position for isolation of a 3D2F mode;

(c) measure the mechanical damping of the structure for the 3D2F mode;

(d) measure any detuning effects.

Inertance was measured using random noise excitation and the transfer function capability of the signal analyser. The analyser employed simultaneous analogue to digital conversion on the force gauge and accelerometer signals and correlation processing of the resultant digital data to measure both the magnitude and phase of the inertance over a selected frequency range. Graphical output of the measured inertance was available in terms of either magnitude and phase as a function of frequency (Bode plot) or real versus imaginary parts of inertance (Nyquist plot).

3.3.2 Resultant Impedance Measurements

Initially, inertance was measured over the frequency range 200 to 1000Hz. This range encompassed the expected natural frequencies of most of the second family coupled blade-disc modes of the test fan. A resultant plot of inertance magnitude against frequency is shown in Figure 3.3. It was measured with the excitation and accelerometer at the blade shroud. The number of diametral nodes associated with each
**Figure 3.3** Inertance magnitude measured over the frequency range 200 to 1000 Hz. Excitation force and acceleration were measured at near coincident points on the fan shroud.

**Figure 3.4** Narrow bandwidth inertance measurement of the 3D2F resonance. Excitation was at the shroud between blades 15 and 16. Acceleration was measured at the tip of blade 16.
mode was determined using sinusoidal excitation and manual scanning of the accelerometer. The diametral number thus determined is indicated in Figure 3.3. This figure shows that each mode was well spaced from its neighbouring modes of different diametral number.

Narrow bandwidth, well resolved, inertance measurements were made for the 2D to 5D resonances identified in Figure 3.3. Measurements were made at several exciter positions and most emphasis was placed on study of the 3D2F resonance. Bode and Nyquist representations of inertance for the 3D2F resonance, for one exciter position at the shroud, are shown in Figure 3.4. This figure shows the presence of a double resonance. The 3D2F mode shape was associated with each resonant peak, however the corresponding antinodes were displaced by a quarter of a wavelength or 30° for this 3D mode. This is illustrated in Figure 3.5. This was typical of the other modes which were investigated and is in agreement with the report of Hockley et al (1978). It was found that, to some extent, the orientation of these split modes was fixed with respect to the fan. It was found possible to excite one of the modes in isolation by exciting exactly on a node of its twin mode. An inertance measurement obtained at such an excitation position for one of the 3D2F modes is shown in Figure 3.6.

The narrow bandwidth inertance measurements were used to determine the natural frequencies, twin-mode frequency split and damping for the 2D2F to 5D2F modes. The procedures advocated by Ewins (1976) and based upon analysis of the Nyquist plots were employed. The natural frequency of a mode, $\omega_0$, was identified where the rate of change of phase of inertance with respect to frequency was a maximum. This was identified on the Nyquist diagram as the frequency at which the spacing between equi-spaced frequency points was maximised. Damping was
FIGURE 3.5 DIAGRAM ILLUSTRATING THE RELATIVE ORIENTATION OF FREQUENCY SPLIT TWIN MODES.

estimated by drawing a best-fit circle through the points on the
Nyquist diagram and measuring the two frequencies, \(\omega_1\) and \(\omega_2\), which were
orientated at \(\pm 90^\circ\) with respect to \(\omega_0\) on the circle. The damping, \(\eta\),
was then related to \(\omega_1\), \(\omega_2\) and \(\omega_0\) by
\[
\eta = \frac{\omega_2 - \omega_1}{\omega_0}
\]
Eq 3.1
This technique is illustrated in Figure 3.7. The theoretical basis for
the above analysis procedure and the assumptions employed were described
by Ewins (1976).

The results obtained are summarised in Table 3.1. The natural
frequencies which were predicted by finite element calculation are also
given in this Table. There was a small variation in the measured
frequency difference between split twin modes for the various excitation
positions. This was possibly due to a small contribution to detuning
from the exciter and accelerometer. These results are discussed further
in Section 3.7.

3.4 Holographic Technique

3.4.1 Basic Approach

The basic approach was to sinusoidally excite a stationary mode
in isolation and then record pulsed holograms to measure the resultant
vibrational deflection. Pure stationary waves were produced and
analysed for this study of the non-rotating test fan. This contrasts
with the flutter vibrational shape. Strain gauge data suggests that,
in flutter, the vibrational pattern rotates relative to the fan. The
travelling vibrational pattern found in flutter may be considered as the
sum of two stationary twin modes which are excited with an appropriate
temporal phase difference. This approach to flutter was considered
FIGURE 3.7  NYQUIST DIAGRAM ILLUSTRATING THE MEASUREMENT OF NATURAL FREQUENCY AND DAMPING FOR A MODE.

FIGURE 3.8  DIAGRAM SHOWING THE OPTICAL ELEMENTS FORMING THE PULSED RUBY LASER.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured Natural Frequencies (Hz)</th>
<th>Measured Twin Mode Frequency Split (Hz)</th>
<th>Measured $Q$ ($= \frac{f}{\Delta f}$)</th>
<th>F.E. Prediction of Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D2F</td>
<td>500, 502</td>
<td>2</td>
<td>1500</td>
<td>427</td>
</tr>
<tr>
<td>3D2F</td>
<td>593, 594</td>
<td>0.6 to 1.8</td>
<td>1500</td>
<td>572</td>
</tr>
<tr>
<td>4D2F</td>
<td>645, 645</td>
<td>0.4</td>
<td>1000</td>
<td>637</td>
</tr>
<tr>
<td>5D2F</td>
<td>700, 702</td>
<td>2.4 to 2.8</td>
<td>900</td>
<td>703</td>
</tr>
</tbody>
</table>

**TABLE 3.1 Summary of the Mechanical Impedance Measurements for the 2D2F to 5D2F Modes**

Stationary rather than travelling modes were studied on the non-rotating test fan because,

(a) they were more easily reproducibly obtained, and
(b) the shape of the travelling waves, if required, could have been mathematically constructed from the deflection shapes of the two stationary twin modes.

Holography was chosen for measurement of the vibrational mode shape for the reasons given in Chapter 2. Pulsed holograms were recorded using near coincident illuminating and viewing directions. Holograms were obtained from two positions of view, one being on the fan's projected axis and the other being well off-axis. This allowed measurement of both the axial and tangential components of blade deflection.

3.4.2 Two Complementary Hologram Types

Pulsed rather than continuous wave holography was employed because this considerably eased the experimental fan stability requirements. Double pulsed operation of a Q-switched ruby laser was used to obtain quality double exposure holographic interferograms showing the whole of the test fan. However, there was nothing to distinguish the zero order interference fringes, representing no movement, from other fringe orders. Thus, the fan disc, which was a known stationary point, was used as a datum to determine fringe order. Also, a complementary form of pulsed hologram was recorded so as to aid fringe order identification. The pulsed ruby laser was operated in an unswitched mode giving a series of relaxation type pulses over a period of approximately 0.5 ms. This was used to produce a quasi-time-averaged hologram, which identified nodal
areas with a bright zero order fringe. It was also useful for checking that a purely stationary vibrational wave was present. In this thesis, this form of interferogram is referred to as an 'open-lase' hologram. It is intended by the author, that a full description of this open-lase holographic technique will be the subject of a separate future publication.

3.4.3 Pulsed Ruby Laser

The primary requirements of the pulsed laser for this application were as follows:

(a) The duration of the individual pulses of the Q-switched output were required to be sufficiently short to 'freeze' the fan's vibrational motion. It was required that the fan's movement during the pulse was considerably less than the wavelength of the laser light.

(b) The longitudinal coherence length was required to be sufficient for the recording of a bright hologram having unbiased interference fringes over the full depth of field presented by the test fan. This requirement was most arduous for the open-lase operation of the laser with off-axis viewing of the fan.

(c) It was required that the laser output power was adequate for illumination of the full area of the fan and optimum exposure of the hologram recording film.

These requirements were met using the pulsed ruby laser configuration shown in Figure 3.8. This diagram shows the configuration used for operation in the double pulsed mode. The laser operated at a wavelength of 0.694\(\mu\)m. The oscillator employed a ruby rod of 100mm length, pumped using a helical flash tube. A plane parallel mirror oscillator cavity configuration was used. Q-switching was achieved using a KD\(^*\)P
Pockel cell and polariser. The Pockel cell was pulsed twice to its half-wave retardation potential. The delay between the two Q-switch pulses was variable between 2μs and greater than 400μs. A good longitudinal mode stability was achieved using two quartz intra-cavity Fabry-Perot etalons. The oscillator output was amplified using two further ruby rods.

The laser was converted for operation in its open-lase mode by simply inserting a half wave retardation plate into the cavity at "A" as shown in Figure 3.8 and also disabling the switching of the Pockel cell. This, therefore, maintained a constant high Q within the oscillator cavity.

In the double pulsed mode, the individual pulse duration was approximately 25ns, which was more than adequate for 'freezing' the vibrational movement of the fan. The fan moved typically less than 10^{-3} \mu m during this exposure duration. The laser reliably produced a coherence length in excess of 0.7m in both the double pulsed and open-lase modes. This was just adequate for the off-axis view of the fan. The total output energy of the laser was up to 0.5J which was sufficient for recording holograms of the full area of the test fan.

3.4.4 Holographic System

A diagram giving the optical arrangement of the holographic system is shown in Figure 3.9. The output from the ruby laser was passed through two beam-splitters. The first was used together with a photo-detector for monitoring the optical output power of the laser. The second beam-splitter was used to form the reference and object beams. The object beam was expanded using a diverging lens and ground-glass-diffuser and used to
RUBY LASER SHOWN IN FIGURE 3.8

REFERENCE BEAM

RUBY LASER

KEY
BS: BEAM SPLITER
-L: NEGATIVE LENS
+L: POSITIVE LENS
G: GROUND GLASS DIFFUSER
D: PHOTODETECTOR
P: HOLOGRAPHIC PLATE

FIGURE 3.9 A DIAGRAM SHOWING THE OPTICAL ARRANGEMENT OF THE HOLOGRAPHIC SYSTEM USED FOR STUDY OF THE NON-ROTATING TEST FAN.

FIGURE 3.10 A PHOTOGRAPH OF THE HOLOGRAPHIC SYSTEM POSITIONED IN FRONT OF THE TEST FAN.
illuminate the test fan. The reference light was folded using mirrors and expanded using two lenses to produce a path-matched, large diameter, parallel beam which was directed onto the hologram recording plate. Holograms were recorded on Agfa Gevaert 8E75 photographic plates.

The object beam expanded from a point which was very close to the holographic plate. Thus, the direction of the holographic sensitivity vector was very near to the viewing direction.

The laser oscillator and amplifiers together with the other optical components forming the holographic system were mounted on one base-board and were housed in a light-weight transportable enclosure. This unit was mounted on a sturdy tripod. A photograph of it, positioned in front of the test fan, is shown in Figure 3.10.

3.4.5 Experimental Technique

The mode of interest was excited using a sinusoidal point force at the mode's natural frequency. An exciter position was chosen which suppressed the twin mode, thus generating a pure stationary vibrational wave.

Holograms were recorded from the two positions indicated in Figure 3.11. Double pulsed and open-lase holograms were recorded from each position. The laser was triggered from the accelerometer signal, such that the laser pulses were centred about a point of peak vibrational velocity. All double pulsed holograms shown in this chapter were recorded with a pulse separation of 400μs. The holograms were reconstructed using a helium-neon laser operating at a wavelength of 0.6325μm.

The experiments were performed at an ambient temperature of 20(±2)°C which corresponded to that used for the FE model.
Q is the point equispaced between the point illumination and the viewing aperture.

Figure 3.11 Diagram showing the position of the holographic system with respect to the fan for the two recording positions.
3.4.6 Resultant Holograms

Double pulsed and open-lase interferograms of one of the 3D2F modes, recorded from the two directions of view, are shown in Figures 3.12 to 3.15. These interferograms were recorded under identical vibrational conditions (to within 1% on accelerometer amplitude) within a period of 15 minutes. Excitation was at the shroud between blades 17 and 18, at a frequency of 593 Hz.

The relative circumferential orientation of twin modes is illustrated in Figure 3.16 which shows double pulsed interferograms of the twin 3D2F modes.

Most effort was concentrated on the 3D2F mode, however, interferograms were recorded of several of the other modes. Double pulsed interferograms of the 2D2F, 4D2F and 5D2F modes, recorded from an on-axis direction, are shown in Figure 3.17.

3.5 Quantitative Analysis of the Interferograms

The interferograms of the 3D2F mode shown in Figures 3.12 to 3.15 were analysed in detail to obtain the distribution of the axial and tangential components of centre-line deflection and of blade torsion. The open-lase interferograms were used to assign the correct fringe order to the double pulsed interferograms, which were then used for mode shape determination. Computer aids were employed where applicable for fringe position determination and for calculation.

For a structure vibrating linearly in a single stationary mode, the ratio

\[
\left( \frac{\text{deflection occurring over a time interval}}{\text{peak deflection}} \right) \text{ at a point}
\]

is the same for all points on the structure. The double pulsed
FIGURE 3.12 DOUBLE PULSED INTERFEROGRAM OF THE LOWER FREQUENCY 3D2F MODE RECORDED FROM THE ON-AXIS POSITION. THE EXCITATION FREQUENCY WAS 593Hz.
FIGURE 3.14 DOUBLE PULSED INTERFEROGRAM OF THE LOWER FREQUENCY 302F MODE
RECORDED FROM THE OFF-AXIS POSITION. THE EXCITATION FREQUENCY
WAS 593Hz.
FIGURE 3.15 OPEN-LASE INTERFEROGRAM OF THE LOWER FREQUENCY 3D2F MODE RECORDED FROM THE OFF-AXIS POSITION. THE EXCITATION FREQUENCY WAS 593Hz.
FIGURE 3.16 DOUBLE PULSED INTERFEROGRAMS OF THE TWIN 3D2F MODES.
FIGURE 3.17 DOUBLE PULSED INTERFEROGRAMS OF THE 2D2F, 4D2F AND 5D2F MODES.
interferograms were used to measure the deflection which occurred between the two laser pulses, which was thus representative of the fan's mode shape. The absolute magnitudes of the measured deflections were of little importance. Thus all measured deflections were normalised.

All deflection calculations were based upon the assumption that the radial component of blade deflection was insignificant. This was reasonable because the test fan had a very high stiffness in the radial direction.

The axial component of deflection was obtained using only the double pulsed interferogram recorded from on-axis and the relationships,

$$P = \frac{1}{\pi} (\mathbf{u} \cdot \mathbf{k})$$

Eq 3.2

and

$$k = \frac{1}{2} (k_I - k_v)$$

Eq 3.3

where $P$ is the fringe order,

$\mathbf{u}$ is the vibrational deflection vector,

$k$ is the holographic sensitivity vector

and $k_I$ and $k_v$ are the wave vectors of light in the illuminating and viewing directions, respectively, both having magnitude $\frac{2\pi}{\lambda}$.

Consideration of the geometry leads to the scalar relationship,

$$h = \frac{P\lambda}{2} \sec(\tan^{-1} \frac{r}{d}) \sec \frac{\phi}{2}$$

Eq 3.4

where $h$ is the axial displacement which occurred between holographic exposures,

$\lambda$ is the wavelength of light,

$r$ is the radial position of the point on the fan,

$d$ is the axial distance between the point on the fan and the holographic system.

and $\phi$ is the angle between the illuminating and viewing directions.
The tangential component of deflection was obtained using the double pulsed interferograms recorded from both the on-axis and off-axis positions. The geometry used for determination of the tangential deflection is shown in Figure 3.18. In this figure, the deflection vector $\mathbf{u}$ is shown in terms of orthogonal components $u_1$, $u_2$ and $u_3$ where $u_2$ lies in the plane of the fan and $u_1$ and $u_2$ lie in the plane $PQR$. Use of Equations 3.2 and 3.3 and consideration of the geometry was shown by Hockley et al (1978) to lead to the scalar relationships,

$$u_2 = \left( \frac{P_2}{\cos \phi} - \frac{P_1}{\cos \phi} \right) / (\tan \psi + \tan \xi) \quad \text{Eq 3.5}$$

$$u_1 = \frac{P_1}{\cos \phi} + u_2 \tan \psi \quad \text{Eq 3.6}$$

$$u_3 = u_1 \tan \sigma - u_2 \tan \theta / \cos \sigma \quad \text{Eq 3.7}$$

and

$$g = \frac{\lambda}{2 \cos \phi / 2} (u_1 \sin \sigma \sin \theta + u_2 \cos \theta - u_3 \cos \sigma \sin \theta) \quad \text{Eq 3.8}$$

where $P_1$ and $P_2$ are the measured fringe orders for the point under consideration for the on-axis and off-axis views, respectively, $\theta, \phi, \psi$ and $\sigma$ are the angles defined in Figure 3.18 and $g$ is the tangential displacement which occurred between holographic exposures.

A measure of blade torsion, $\alpha$, was obtained by calculation of the angle which a point on the leading edge rotated relative to the centre line. This is given by

$$\alpha = \frac{h_L - h_c}{c \sin(\gamma)} \quad \text{Eq 3.9}$$

where $h_L$ and $h_c$ are the axial displacements obtained at the leading edge and centre line, using Equation 3.4, $c$ is the half chord.
FIGURE 3.18 GEOMETRY USED FOR DETERMINATION OF THE TANGENTIAL DISPLACEMENT.

FIGURE 3.19 DIAGRAM SHOWING THE RELATIVE SIGN CONVENTION USED FOR AXIAL AND TANGENTIAL DISPLACEMENT AND TORSION.
length, \( \gamma \) is the stagger angle of the blade, and these are, generally, all variables of radial position. Making the approximation that the term \( \sec(\tan^{-1} \frac{r}{d}) \) is equal for the points on the leading edge and centre line leads to,

\[
\alpha = (P_L - P_C) \frac{\sec(\frac{\gamma}{2}) \sec(\tan^{-1} \frac{r}{d})}{2d \sin(\gamma)} \quad \text{Eq 3.10}
\]

where \( P_L \) and \( P_C \) are the fringe orders at the leading edge and centre line respectively.

The values of axial displacement, tangential displacement and torsion, \( \alpha \), were normalised with respect to the amplitude of the 3D Fourier component of centre line axial displacement at the shroud which was given the value 1.0. This feature of the mode shape was chosen because it could be measured very accurately. Thus, from Equations 3.4 and 3.8, the normalised axial and tangential displacements were,

\[
(h)_{\text{NORM}} = \frac{P \sec(\tan^{-1} \frac{r}{d})}{P_0 \sec(\tan^{-1} \frac{r_0}{d_0})} \quad \text{Eq 3.11}
\]

and \( (g)_{\text{NORM}} = \frac{u_1 \sin \sigma \sin \theta + u_2 \cos \theta - u_3 \cos \sigma \sin \theta}{P_0 \sec(\tan^{-1} \frac{r_0}{d_0})} \quad \text{Eq 3.12} \)

where, \( P_0 \) is the amplitude of the 3D Fourier component of the shroud centre line fringe count, \( r_0 \) is the radius of the shroud, and \( d_0 \) is the axial distance between the shroud centre line and the holographic system.
A multiplicative term, equal to the fan radius \( r_{TIP} \) was included in the normalisation of the torsion, \( \alpha \), so as to make it unaffected by the units of measurement. Thus normalised torsion was given by

\[
(\alpha)_{NORM} = \frac{r_{TIP} \sec(\tan^{-1} \frac{r}{d})}{\sin(\gamma) P_0 \sec(\tan^{-1} \frac{r_0}{d_0})}
\]

Eq 3.13

The relative sign convention employed for these normalised deflections is given in Figure 3.19.

3.6 Resultant 3D2F Mode Shape and Comparison with Finite Element Prediction

Selected circumferential and radial scans of deflection were used for comparison of the holographically measured 3D2F mode shape and the finite element prediction.

3.6.1 Circumferential Scans of Centre line Axial Displacement and of Torsion

The circumferential distributions of centre line axial displacement and of torsion were measured from the interferogram shown in Figure 3.12 at three radial positions. Measurements were made at the shroud, the blade tips and at a position approximately equispaced between the two. These measurement positions corresponded to centre line radii of 307mm, 424mm and 365mm, respectively. The distributions of torsion at \( r = 307 \text{mm} \) and 365mm were made along lines of constant radius i.e. both \( P_L \) and \( P_C \) were measured at the same radius. However, at the tip, the radius of the leading edge (\( r = 428 \text{mm} \)) was slightly greater than the centre line (\( r = 424 \text{mm} \)) and torsion was measured along the blade hade.
The unscaled fringe distributions from which these deflections were calculated are shown in Figure 3.20. Plots of both $P_L$ and $(P_L - P_C)$ are shown. It can be seen from this figure that the fringe distributions were predominantly sinusoidal. A discrete Fourier transform (Cochran et al., 1967) was performed upon each set of fringe counts in order to obtain accurate values of the magnitude and spatial phase of the 3D component of movement. The resultant values were normalised as described in Section 3.5 to obtain the normalised axial displacements and torsions given in Table 3.2. The normalised circumferential orientation of these distributions was measured in a clockwise direction. The corresponding normalised finite-element predictions are also presented.

3.6.2 Radial Scans of Torsion and Centre Line Displacement

The radial distribution of axial and tangential translation was measured along the centre line of blade 29. The double pulsed interferograms shown in Figures 3.12 and 3.14 were used. Blade 29 was chosen because it had very little torsion at the shroud. This was an identifying feature which allowed comparison with the FE results. The resultant normalised deflections, together with the corresponding finite-element predictions, are shown in Figure 3.21.

The radial distribution of torsion was measured for blade 26. The interferogram shown in Figure 3.12 was used and torsion was measured along lines of constant radius. Blade 26 was chosen because it had the identifying feature of very little axial deflection at the shroud. The resultant normalised distribution of torsion is shown in Figure 3.22, together with the corresponding finite-element prediction.
FIGURE 3.20 UNSCALED CIRCUMFERENTIAL FRINGE DISTRIBUTIONS, OBTAINED AT THREE RADIi. $R_1$ [+] AND $R_2 [\circ]$ ARE SHOWN.
### TABLE 3.2

<table>
<thead>
<tr>
<th>Radial Position</th>
<th>Deflection Type</th>
<th>Holographically Determined Value</th>
<th>FE Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>307mm (shroud)</td>
<td>Normalised Axial Displacement</td>
<td>1.0 /0°(a)</td>
<td>1.0 /0°(a)</td>
</tr>
<tr>
<td></td>
<td>Normalised Torsion</td>
<td>4.3 /-98°</td>
<td>4.6 /-93°</td>
</tr>
<tr>
<td></td>
<td>Torsion/Axial Displacement</td>
<td>4.3 /-98°</td>
<td>4.6 /-93°</td>
</tr>
<tr>
<td>365mm</td>
<td>Normalised Axial Displacement</td>
<td>1.0 /2°</td>
<td>1.14 /2°</td>
</tr>
<tr>
<td></td>
<td>Normalised Torsion</td>
<td>6.9 /-72°</td>
<td>7.4 /-92°</td>
</tr>
<tr>
<td></td>
<td>Torsion/Axial Displacement</td>
<td>6.9 /-72°</td>
<td>6.5 /-92°</td>
</tr>
<tr>
<td>424mm (Tip)</td>
<td>Normalised Axial Displacement</td>
<td>0.78 /172°</td>
<td>1.18 /180°</td>
</tr>
<tr>
<td></td>
<td>Normalised Torsion</td>
<td>11.1 /-125°</td>
<td>15.4 /-142°</td>
</tr>
<tr>
<td></td>
<td>(Along Rade)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Torsion/Axial Displacement</td>
<td>14.2 /63°</td>
<td>13.0 /38°</td>
</tr>
</tbody>
</table>

footnote: (a) Obtained by definition of normalisation.

TABLE 3.2 Circumferential Distributions of Axial Centre Line Displacement and Torsion for 3D2F Mode: Comparison of Holographically Measured Distributions and Those Predicted by FE Computation.

FIGURE 3.22  PLOT OF THE HOLOGRAPHICALLY DETERMINED TORSION ALONG BLADE 26 FOR THE 3D2F MODE AS SHOWN IN FIGURE 3.12. THE CORRESPONDING FINITE ELEMENT PREDICTION IS ALSO SHOWN.
3.7 Discussion

It can be seen from the mechanical impedance measurements and the 3D2F mode shape deflections presented in Section 3.3.2 and 3.6, that there were some differences between the experimental results and the finite element predictions. The extent and implications of these differences are discussed in this section.

The difference in experimentally measured and computed natural frequencies varied between 73Hz for the 2D2F mode and only 1Hz for the 5D2F mode (Table 3.1). The magnitude of the difference for the 2D2F mode suggested that there was a significant error in the finite element model which would probably produce measurable errors in the predicted mode shapes.

The mechanical impedance measurements indicated that detuning of the fan due to manufacturing asymmetries was small. Detuning resulted in a discernible frequency split between twin orthogonal modes, but the effect was small with a measured worse-case of 2.8Hz (in 700Hz) for the 5D2F mode. Thus, manufacturing asymmetry did not explain the variance between the measured and computed natural frequencies.

The interferograms shown in Figures 3.12 to 3.17 and the circumferential distribution of fringe counts presented in Figure 3.20 showed that the circumferential distribution of deflection was sinusoidal to a good approximation. This was an assumption employed in the finite element model which was thus experimentally verified.

The comparison of the experimentally measured 3D2F mode shape and that predicted by finite element calculation is summarised in Table 3.2 and Figures 3.21 and 3.22. These mode shape deflections were normalised with respect to axial displacement at the shroud. With a comparison such as this, the choice of normalising feature affects the relative
prominence of any disparities. However, with this in mind, the extent of agreement between measured and predicted deflections was assessed, as is described below.

Application of the analysis given in Section 1.3 for a typical modern high performance shrouded fan, such as the test fan, (e.g. Halliwell, 1976) indicates that flutter onset is particularly sensitive to,
(a) the radial position of any circumferential nodes, and
(b) the relative magnitude and phase of blade torsion and centre line displacement, particularly towards the tip.

The agreement of measured and predicted mode shapes with regard to these features was considered. There was very good agreement between the measured and predicted position of the outer circumferential node for the axial component of centre line displacement, as seen in Figure 3.21. However, there was some disagreement between the measured and prediction ratio of torsion and centre line axial displacement as seen in Table 3.2. Discrepancies of 9% in magnitude and 25° in spatial phase were obtained at the tip. This was well outside the experimental error. An agreement in magnitude of within 9% was considered reasonable, but the variance of 25° in phase was considered an error which might significantly affect a flutter prediction analysis based upon this FE model.

Thus, in summary, the use of a prediction model which neglected detuning and assumed a sinusoidal circumferential variation of deflections was reasonable, at least for the second family modes of low diametral order. The finite element model used for the test fan gave a predicted 3D2F mode shape which was in good agreement in many respects with the measured shape. However, there was a significant error in the predicted spatial phase between torsion and centre line axial displacement.
Also, poor agreement between the measured and predicted natural frequencies for the 2D2F to 5D2F modes cast doubt on the overall accuracy of the FE model.
CHAPTER 4

AN IMAGE ROTATOR FOR HOLOGRAPHIC STUDY OF

FAN FLUTTER

Summary of Chapter

A mirror-Abbe image rotator, intended specifically for inclusion in a double pulsed holographic system for vibrational studies of the fluttering test fan, was successfully designed and constructed. The mirror-Abbe configuration was chosen because it gave minimal aberrations and allowed rotation of the high intensity illuminating beam. The mirror-Abbe optics were rotated using an electric motor having a hollow shaft. The speed of the image rotator was maintained at exactly half that of the fan using a phase locked loop control system. The top speed (12 000rpm) and speed following capability and accuracy were within the estimated requirements for study of the test fan.

4.1 Introduction

An image rotator, which was built specifically for holographic study of the flutter vibrational response of the test fan, is described in this chapter. The image rotator was designed to rotate at exactly half the speed of the fan and hence optically compensate for the fan's rotation. This, therefore, prevented decorrelation of the holographic images of the fan. In the next section, the performance requirements for this application are presented. The choices and a description of the design with regard to, in turn, the optical configuration, the mechanical design and the speed control system are then given. The optical alignment and balancing of the unit are described. Finally, an evaluation of the performance of the unit, making a comparison with the requirements, is presented.
4.2 Performance Requirements

The image rotator was intended for incorporation in a double pulsed holographic system to be used for vibrational studies of the rotating test fan. The main requirements of the image rotator for this application were as follows.

(a) A top speed of at least half that of the test fan was required. The maximum likely speed of the test fan was 11 100rpm, which was 110% of the design speed. Thus, the maximum speed required of the image rotator was 5550rpm.

(b) It was required to be capable of following typical speed fluctuations of the test fan. Previous tests of similar fans indicated that speed fluctuations were likely to be less than \( \pm 0.07\% \) in the range 1 to 10Hz and less than \( \pm 0.1\% \) in the range 0.1 to 1Hz.

(c) The rotator was required to have a speed following accuracy which was sufficient to maintain correlation between the two holographic images. Thus, any relative displacement of the two images resulting from control system errors was required to be less than the correlation distance for all parts of the image.

This last requirement concerning the speed following accuracy is now considered in greater detail. The variation in interference fringe visibility as a function of in-plane image rotation between holographic exposures is described in chapter 2. The maximum tolerable angular movement of the image is obtained from Equation 2.10 and is given by

\[
\Delta \theta < 0.98 \frac{\lambda \Delta \theta}{1T_{\text{TIP}}}
\]

Eq 4.1

where \( \Delta \theta \) is the angular movement of the image between exposures,

\( \lambda \) is the wavelength of light employed,
\[ d_0 \] is the distance between the fan and an imaging lens, \\
\[ l \] is the effective diameter of the imaging lens, \\
and \[ r_{\text{TIP}} \] is the radius of the fan.  

For a system making full use of the field of view of the image rotator and having an imaging lens close to its apertures,

\[
F = \frac{d_0}{r_{\text{TIP}}}
\quad \text{Eq 4.2}
\]

and \[ l = 2a \]

\[
\text{Eq 4.3}
\]

where \( F \) is the f-number of the image rotator, equal to its effective optical length divided by the diameter of its apertures, and \( a \) is the radius of the image rotator's entrance and exit apertures.

Thus,

\[
\Delta \theta \leq 0.49 \frac{\lambda F}{a}
\quad \text{Eq 4.4}
\]

For a laser pulse separation, \( \Delta t \), much smaller than the time constant of the image jitter, the tolerable angular velocity of any jitter is given by

\[
\frac{d\theta}{dt} \leq 0.49 \frac{\lambda F}{a \Delta t}
\quad \text{Eq 4.5}
\]

The above relationship was used to quantify the maximum tolerable angular velocity of any image jitter in terms of the rotator's parameters and the laser pulse separation.

Two further qualitative requirements were considered. First, an image rotator configuration which allowed maximisation of the energy density at the holographic film was an important requirement. Second, an image rotator having minimal aberrations was required.
4.3 Choice and Design of Optical Configuration

An image rotator optical configuration was chosen which fulfilled the requirements given in Section 4.2. As an extension to these requirements, an optical configuration was sought which allowed the rotation of the illuminating beam, in addition to derotation of the returned light from the fan. This desirable feature allowed the available laser light to be conveniently concentrated into an off-axis portion of the fan, thus increasing the energy density at the holographic film. The advantages of using a rotating illuminating beam are fully described in the next chapter. A further constraint on the optical configuration, which followed from the requirement for the efficient use of the available laser light, was that it should not be limited to use with only parallel light.

An excellent survey of image rotators was given by Swift (1972), in which he compared the salient features of the more commonly used devices. A summary, reproduced from Swift's paper, of the principal characteristics of seventeen image rotator types is shown in Table 4.1. The first design choice was between a transmissive and reflective rotator type. A holographic system using a transmissive rotator required fewer passes through a beam splitter than an equivalent system using a reflective rotator. Therefore, a higher object beam energy density at the holographic film could be achieved using a transmissive device, all other things being equal. It was for this reason that a transmissive optical configuration was chosen. Of the many alternative designs, an Abbé type rotator, constructed using dielectrically coated mirrors, was used. This configuration is shown diagramatically in Figure 4.1. It was chosen because it was the simplest transmissive rotator which has the whole of its optical path in air. Thus, it gave minimal aberrations and was capable of rotating
The figures in brackets refer to footnotes below.

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<th>No. of reflections</th>
<th>Quarter</th>
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<th>Optical path length (mm)</th>
<th>Parall. light use only?</th>
<th>Mechanical balance difficulty</th>
<th>Difficult to manufacture</th>
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<td>A</td>
<td></td>
<td></td>
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<td>2.02–1.58</td>
<td>(6) 2.40–2.28</td>
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<td>3</td>
<td>–</td>
<td>–</td>
<td>3.00 (19)</td>
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<td></td>
<td>A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes

(1) P indicates suitable for use in parallel light only.
(2) I indicates suitable for use in image plane only.
(3) Increasing turbulence indicated by more astersisks (no astersisks = balanced).
(4) Increasing manufacturing difficulty and cost indicated by more astersisks.
(5) A indicates suitable in principle for array. (Not necessarily practical.
(6) P indicates suitable for use in parallel light only.
(7) Two different optical path lengths.
(8) P indicates suitable for use in image plane only.
(9) I indicates suitable for use in image plane only.
(10) Ideal limit. Practical limit set by cost and production difficulties.
(11) Assumes all unnecessary glass removed. Diameter for normal configuration = 2.05.
(12) Dependent upon design.
(13) Angle.
(14) Reflection rotator.
(15) Excess length over 90° prism = 2.00.

Figure 4.1 Diagram showing the mirror-Abbe image rotator configuration.

Figure 4.2 Diagram of the geometry of the Abbe rotator as used to determine the optimum configuration.
the illuminating beam without producing unwanted Fresnel back-reflections. It was for these reasons that a modified mirror-Abbé configuration was chosen by Smart (1980) for laser anemometry applications. The mirror-Abbé configuration did, however, have the disadvantage that a relatively high inertia was produced due to the off-axis mirror.

The optimum design of a mirror-Abbé rotator was considered. The relationship between f-number (optical path length to pupil diameter ratio) and the angle $\beta$ shown in Figure 4.1 was obtained as follows. From Figure 4.2, the position of the off-axis mirror is defined by triangles HWV and HWI as

$$b = a \left( \frac{\tan 2\beta}{\tan \beta} \right)$$

Eq 4.6

The optical path length, $w$, through the image rotator along its axis, is equal to $GH + HI + IJ + JK$. Thus,

$$w = 2a \cot \beta + 2b \cosec 2\beta$$

Eq 4.7

From Eq 4.6 and Eq 4.7,

$$w = 2a \cot \beta (1 + \sec 2\beta)$$

Eq 4.8

Thus, $F$, the f-number of the rotator, equal to $\frac{w}{2a}$ is given by

$$F = \cot \beta (1 + \sec 2\beta)$$

Eq 4.9

This relationship is plotted in Figure 4.3. The minimum value of $F$ is 5.2 at $\beta = 30^\circ$ and $b/a = 3$. Thus, an optical configuration employing $\beta = 30^\circ$ and $b/a = 3$ was selected.

The optical elements of the mirror-Abbé rotator were constructed using a separate off-axis mirror and a solid glass prism to form the substrate for the two on-axis mirrors. These two components are shown
FIGURE 4.3 GRAPH SHOWING THE MIRROR ABBE F-NUMBER AS A FUNCTION OF THE PRISM ANGLE.
in Figure 4.4. This allowed complete alignment of the optical system with a minimum number of degrees of adjustment.

4.4 Mechanical Design

In this section, the important basic design alternatives are considered, and then the final mechanical design is described in detail.

Having decided upon a transmissive rotator, the most basic design choice was between a one and two shaft mechanical configuration. A one shaft design, having the optical system mounted in the centre of a hollow motor, was preferred to a two shaft design. This precluded any possible coupling errors such as belt slippage or mechanical chatter which might have arisen with a belt or gear coupled two shaft arrangement. On considering the types of motor, an electric unit was preferred to alternatives, such as an air driven turbine, because it gave ease of speed control. Commercially available electric motors were capable of the top speed and mechanical time constant requirements. Externally pressurised air bearings were chosen because of their good centering properties, smooth running and low vibration.

The mirror-Abbe optics were mounted in a steel shaft which passed through the centre of a Vatric-Mavilor 600 d.c. motor. The motor was modified to produce a hollow shaft of 30mm internal diameter. This motor had a light ironless dish-shaped armature which gave low inertia, near-constant torque from rest to high speeds and a high power to length ratio. It was capable of operation at speeds up to 6000rpm.

A photograph of the image rotator and a diagram of a section through it are shown in Figures 4.5 and 4.6, respectively. The prism forming the on-axis mirrors was mounted in the shaft on four screw adjustable
FIGURE 4.4 PHOTOGRAPH OF ON-AXIS PRISM AND OFF-AXIS MIRROR.

FIGURE 4.5 PHOTOGRAPH OF IMAGE ROTATOR.
FIGURE 4.6 DIAGRAM SHOWING SECTION THROUGH THE IMAGE ROTATOR.
pillars and was secured in position using a bar which passed through a hole in the prism. The off-axis mirror was held in a steel mount which was held against a steel cradle, attached to the shaft. The cradle incorporated a balance weight which countered the off-axis mirror. The mirror mount was held in the cradle using spring loading against four screws. The screws allowed adjustment of the mirror angle and of the axis to mirror distance. When rotating, centrifugal loading increased the force with which both the prism and mirror were held against their mounts. The rotor assembly was supported in externally pressurised air bearings. Two journal bearings and a double acting thrust bearing were employed. An optical tachometer was incorporated which provided 120 pulses per revolution (ppr) and 1 ppr outputs. It consisted of a chrome-on-glass radial grating and two LED-photodiode detectors.

The complete image rotator had entrance and exit apertures of diameter 27 mm and had an effective f-number of 7.2. The effective f-number of the unit was larger than the value of 5.2, which would be expected for the optics alone, due to constraints imposed by the size of the motor on the mechanical design.

4.5 Control System

The image rotator was maintained at half the fan speed using a control system which enabled it to follow speed fluctuations. The lack of a commercially available stepper motor, suitable for modification to produce a large internal diameter hollow shaft, precluded the use of a stepper motor control system as employed by Waddell (1973). A phase locked loop as described by Gardner (1966) and employed by Stetson (1978) was used because of its simplicity and compatibility with a d.c. motor.

A simplified block diagram of the control system is shown in Figure 4.7. The phase comparator compared the phase of the fan tachometer
FIGURE 4.7 DIAGRAM OF THE PHASE LOCKED LOOP CONTROL SYSTEM.
signal with that of the rotator, and gave an output which was a measure of the phase difference between these two inputs. This difference signal was filtered, amplified, and then used to drive the motor. The motor control voltage changed the motor speed in a direction which reduced the phase difference between the two tachometer signals. When the loop was 'locked', the motor control voltage was such that for every fan tachometer pulse there was one, and only one, tachometer pulse from the rotator.

During the acquisition of 'lock', there were large voltage swings at the output of the phase comparator. Saturation of the amplifier was prevented by using a low amplifier gain during 'lock' acquisition. Once 'lock' was obtained, the gain of the amplifier was automatically increased so as to maximise the speed range over which 'lock' was held (hold range).

The phase locked loop was modelled in terms of the transfer functions for the phase comparator, filter, motor and tachometer. This analysis is given below.

The loop was assumed 'locked'. The phase comparator output voltage, \( V_d' \), which was proportional to the difference in phase between its inputs, is given by

\[
V_d' = K_d (\phi_1 - \phi_o)
\]

Eq 4.10

where \( K_d \) is the phase comparator gain factor,

\( \phi_1 \) is the phase of the signal from the fan tachometer

and \( \phi_o \) is the phase of the signal from the rotator tachometer.

The phase error voltage, \( V_d' \), was filtered and amplified. The filter transfer function and amplifier gain are represented by \( T_F(s) \) and \( K_a \), respectively.

The motor with its tachometer was considered as a low-pass filter, having a transfer function \( T_M(s) \), followed by a voltage controlled
oscillator, having a transfer function $G_m(s)$. Using Laplace notation,

$$T_m = \frac{1}{1 + s\tau_m} \quad \text{Eq 4.11}$$

where $\tau_m$ is the motor time constant, and

$$G_m = \frac{K_m}{s} \quad \text{Eq 4.12}$$

where $K_m$ is the motor gain constant and has dimensions radians $s^{-1}V^{-1}$.

The phase of the tachometer signal is given by,

$$\phi_o = \frac{V_d T_F(s) K_a T_m(s) G_m(s)}{\phi_i(s)} \quad \text{Eq 4.13}$$

Combination of Equations 4.10 to 4.13 gives the basic loop equation,

$$\frac{\phi_o(s)}{\phi_i(s)} = \frac{K_o T_F(s)}{s(1 + s\tau_m) + K_o T_F(s)} \quad \text{Eq 4.14}$$

where $K_o$ is the loop gain, given by

$$K_o = K_m K_a K_d$$

Thus the loop control characteristics were determined by the selection of $K_o$ and $T_F(s)$.

The final circuit employed a passive filter whose transfer function, in the frequency range of interest, approximated to

$$T_F(s) = 1 + s\tau_F \quad \text{Eq 4.15}$$

where $\tau_F \ll \tau_m$

This resulted in a basic loop equation of the form,
\[
\frac{\phi_o(s)}{\phi_i(s)} = \frac{K_o}{s(1+sr_m)}
\]

Eq 4.16

This was thus a second order loop having lead-lag compensation.

Equation 4.16 can be re-written as,

\[
\frac{\phi_o(s)}{\phi_i(s)} = \frac{s\omega_n (2\zeta - \omega_n) + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

Eq 4.17

where, drawing on servomechanism terminology, \(\omega_n\) is the natural frequency given by,

\[
\omega_n = \sqrt{\frac{K_o}{r_m}}
\]

and \(\zeta\) is the damping factor given by

\[
\zeta = \frac{1}{2} \sqrt{\frac{K_o}{r_m} (r_f + \frac{1}{K_o})}
\]

This form of second order loop allowed the independent selection of bandwidth and damping. The values of \(\omega_n\) and \(\zeta\), which were employed in the final circuit, were 80 rad/s and 1.3, respectively. This produced a control system having a bandwidth sufficient to follow fan speed fluctuations, good transient response and good noise rejection properties (Section 4.7).

4.6 Alignment and Balancing

The on-axis prism and off-axis mirror were aligned relative to the rotational axis of the rotor, as described below.

(a) The prism and mirror were removed from the rotor assembly and
two collinear helium-neon (hene) laser beams, of opposite direction were aligned to the rotational axis. This was achieved using the configuration shown in Figure 4.8. A 400 μm pin hole was mounted in the rotor at the entrance aperture B and was centred on the rotational axis. A diffuse paper screen was mounted in the rotor at C and the position of the rotational axis was marked on it. The mirrors 1 and 2 and the pin hole at A were adjusted such that the diffracted Airy pattern from pin hole A was centred on the pin hole at B and that the resultant Airy pattern from the pin hole at B was centred on the rotational axis at C. The pin hole and screen at B and C, respectively, were then removed. At this stage of the alignment, the diffraction pattern from A was symmetric about the rotational axis. The pin hole at D was then centred on the diffraction pattern from A. Mirrors 3 and 4 were adjusted such that the centre of the diffraction pattern from the pin hole at D fell on the pin hole at A. Thus, the diffraction pattern from D was also made symmetric about the rotational axis of the rotor.

(b) The two collinear hene beams were used to define the position of the on-axis prism. The prism was adjusted for minimum skew i.e. the apex of the prism was adjusted to lie in a plane normal to the rotational axis of the rotor. All other degrees of adjustment were much less critical, because they could be compensated for by the position of the off-axis mirror. Minimum skew was achieved by seating and clamping the prism such that the two diffracted Airy patterns from A and D were reflected from the on-axis mirror surfaces and exactly over-lapped at some position above the apex of the prism.

(c) The off-axis mirror was mounted in its cradle and adjusted such that
FIGURE 4.8 DIAGRAM OF THE CONFIGURATION USED FOR ALIGNMENT OF THE MIRROR ABBE OPTICS.

FIGURE 4.9 DIAGRAM SHOWING TYPICAL PROJECTED BEAM LOCI WITH THE ROTATOR OPTICS ALIGNED AND MISALIGNED.
its surface was at the position of overlap of the two Airy patterns, as formed in step (b). The orientation of the mirror was then adjusted such that the reflected diffracted beams were collinear with the incident beams.

The above procedure made use of Airy diffraction patterns which were generated by passing the hene laser beams through circular apertures. This allowed more accurate edge definition of the beams than would have been possible with the otherwise Gaussian beam profile. Thus an accurate alignment of the mirror-Abbé optics was achieved.

Having optically aligned the image rotator, the rotor was balanced. First, a static balance was performed with the rotor in situ. It was held in its externally pressurised air bearings and the motor brushes were removed so as to reduce friction on the rotor. Balance weights were added or removed from one of the two balance planes shown in Figure 4.6 until the rotor was in indifferent equilibrium i.e. showed no tendency to turn under gravity when placed in any orientation.

Following a successful static balance of the rotor, the motor brushes were replaced and the unit was dynamically balanced. This was performed using the influence coefficient method described by Den. Hartog (1934). The complete rotor unit was mounted on a swing, thus allowing movement only in a horizontal direction. An accelerometer, sensitive in the horizontal radial direction, was mounted on each of the journal bearings. The rotor assembly was rotated at approximately 1000rpm and the vibration vector was determined from the accelerometer signals phased relative to the lppr tachometer signal. Any dynamic unbalance of the rigid rotor was corrected by adding appropriate weights in the two balance planes. The appropriate weights were determined using the following relationships.

\[ e_1 = A_{11} b_1 + A_{12} b_2 \]  

Eq 4.18
where, $e_1$ and $e_2$ are the unbalances at the two balance planes, $b_1$ and $b_2$ are the vibration vectors at the two bearings, and $A_{11}$, $A_{12}$, $A_{21}$ and $A_{22}$ are the complex dynamic influence coefficients for the system.

The influence coefficients were determined experimentally by adding known additional unbalance weights at the two balance planes and determining the change in the vibration vectors. Having achieved minimal unbalance at this low speed, the procedure was repeated at top speed to achieve maximum sensitivity to unbalance.

Finally a check of the optical alignment of the mirror-Abbé optics was performed at the operational speed of the unit. A converging laser beam was passed through the rotator and the locus of the rotated beam was viewed on a screen placed at the beam's focus. A perfectly aligned system resulted in either a circular or point locus for all screen to rotator distances. However, misalignment typically resulted in a locus having two circles of unequal radii as shown in Figure 4.9. Any misalignment was corrected by fine adjustment of the position of the off-axis mirror.

Using the above procedures, the optical plane of the mirror-Abbé optics and the rotational axis of the rotor were aligned to within $\pm$ 10 seconds of arc and balance was achieved within $\pm$ 0.5 g mm at each of the balance planes.

4.7 Evaluation of Performance

The image rotator and its control electronics worked well at object speeds up to 12 000 rpm. This speed limit was set by the motor amplifier but was close to the rated maximum speed for the motor. The system was set up to view a rotating 150 mm diameter disc to show its image derotation.
possibilities. The disc, which was rotating at 10 000 rpm, was photographed through the image rotator using an exposure of 3 seconds, during which it rotated 500 times. The resultant photograph is shown in Figure 4.10.

The alignment of the optical plane of the mirror-Abbé optics relative to the rotational axis of the rotor assembly was monitored as a function of rotor speed. This was done by passing a converging laser beam through the rotator and monitoring the beam locus at the beam focus on a distant screen. The optical axis moved by approximately 50 seconds of arc between static and top speed conditions. This change in alignment was considered due to asymmetric deformation of the cradle supporting the off-axis mirror. The mirror could be aligned to better than ±10 seconds of arc for operation at a selected speed.

The ability of the control system to follow fluctuations in fan speed without dropping out of lock was determined by using a simulated fan tachometer signal having a variable frequency modulation. The total range of fan speed over which lock was maintained having once set a centre speed (hold range) was measured as 800 rpm. The ability of the system to follow rapid fluctuations in fan speed was measured by sinusoidally modulating the fan tachometer frequency at 1Hz and then at 10Hz. The maximum peak to peak variation in fan speed before the system dropped out of lock was determined as 300 and 150 rpm, respectively.

The speed following accuracy of the control systems was determined by monitoring the error signal at the output of the phase comparator while frequency modulating the simulated fan tachometer signal. As an example, the speed following was determined for a frequency modulation at 1Hz having a modulation depth of equivalent to a peak to peak variation in fan speed of 300 rpm i.e. just before drop out of lock. This speed fluctuation was
FIGURE 4.10 PHOTOGRAPH OF A DISC ROTATING AT 10000 RPM VIEWED THROUGH THE IMAGE ROTATOR.
very much worse than would be expected for a typical fluttering fan. At this condition, the maximum instantaneous frequency difference between the fan and tachometer signals was determined as 0.025 Hz. This was equivalent to jitter on the derotated image having a maximum angular velocity of 0.16 radians s\(^{-1}\). Substituting into Equation 4.5 for this value of angular velocity, \(F = 7.2\), \(a = 13.5\) mm and \(\lambda = 0.694\) \(\mu\)m, showed that pulse separations up to 1.1 ms could be used without any possibility of unacceptable fringe visibility. This indicated that the speed following capability of the rotator was perfectly acceptable for flutter studies of the test fan.

The performance of the image rotator is summarised in Table 4.2.

It was concluded that all of the predicted performance requirements, as listed in Section 4.2, were met by the rotator and its control electronics.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum object speed</td>
<td>12000 rpm</td>
</tr>
<tr>
<td>Optical f-number</td>
<td>7.2</td>
</tr>
<tr>
<td>Optical aperture diameter</td>
<td>27mm</td>
</tr>
<tr>
<td>Hold range (a)</td>
<td>800 rpm</td>
</tr>
<tr>
<td>Maximum peak to peak variation in object speed at 1Hz before drop out(b)</td>
<td>300 rpm</td>
</tr>
<tr>
<td>Maximum peak to peak variation in object speed at 10Hz before drop out(b)</td>
<td>150 rpm</td>
</tr>
</tbody>
</table>

(a) Hold range is the range of fan speed over which lock is maintained, having once set a centre speed.

(b) These figures show the capability of the control system to maintain 'lock' when presented with a sinusoidally varying fan speed.

TABLE 4.2 Summary of the Mechanical, Optical and Control Characteristics of the Rotator
Summary of Chapter

A double pulse holographic system, employing the mirror-Abbé image rotator, was built specifically for flutter vibrational studies of the test fan. The rotator was employed in a double pass configuration to rotate the illuminating beam and derotate the light returned from the fan. This therefore maintained correlation between the two resultant holographic images. The system was developed and initial trials were performed using a laboratory fan rig. Interferograms were successfully obtained of the 0.56m diameter laboratory fan at speeds up to 5840 rpm. The optical efficiency and the effects of misalignment of the system were considered.

5.1 Introduction

The design of, and initial results from a double pulse holographic system are described in this chapter. The holographic system was built specifically for flutter vibrational studies of the test fan. The mirror-Abbé image rotator, described in Chapter 4, was employed to optically compensate for the fan's rotation and, thus prevent decorrelation of the two holographic images.

In the next section of this chapter, the factors affecting the light collection of holographic systems in general, which employ image rotators, are considered. The holographic system design choices and a detailed description of the system are then given. The holographic system required accurate alignment with respect to the projected rotational
axis of the fan. The affects of misalignment and the alignment procedure are then described. Before the holographic system was applied to the study of the fluttering test fan, it was used in the laboratory in order to evaluate its performance and to gain some operating experience. This laboratory application of the system was to the study of a 0.56m (22 inch) diameter aero engine fan which was rotated in a vacuum chamber. A description of this work together with some of the resultant interferograms is given in the last two sections of this chapter.

5.2 Light Collection Considerations

The factors affecting the object beam energy density at the holographic film in a generalised system employing an image rotator are described in this section. The expressions given here were used, as described in later sections of this thesis, in order to choose the optimum holographic system and to estimate the object beam energy density in particular applications.

The generalised object beam receiving optics shown in Figure 5.1 were considered. The effective area of the receiving optics was considered to be limited by the diameter of the image rotator apertures. The total energy, \( E_R \), incident upon a simple receiving aperture, was determined as,

\[
E_R \sim \frac{HA_R}{d^2} E_S
\]

Eq 5.1

where, \( E_S \) is the total energy incident on the object,

\( H \) is the average fraction of the incident energy radiated per unit solid angle by points on the object surface towards the receiving aperture,

\( A_R \) is the area of the receiving aperture,

and \( d \) is the distance between the object and the receiving aperture.
FIGURE 5.1 GENERALISED OBJECT BEAM RECEIVING OPTICS.

FIGURE 5.2 DIAGRAM OF THE HOLOGRAPHIC SYSTEM.
The total energy at the holographic film in a real system was obtained from Equation 5.1 by including terms to account for vignetting and absorption and reflection losses. Thus, the below expression was obtained.

\[
E_H \sim \left(\frac{1}{2}\right) N_R K_R Q_R H A_R E_S \frac{d^2}{d^2} \quad \text{Eq 5.2}
\]

where \(E_H\) is the total object beam energy at the film,

\(N_R\) is the number of passes through a 50% beamsplitter present in the receiving optics,

\(K_R\) is a constant which accounts for any absorption losses and any other reflection losses in the receiving optics,

\(Q_R\) is a constant which accounts for any vignetting in the receiving optics and equals one for a system without vignetting.

The energy incident on the object, \(E_S\), was related to the laser output energy, \(E_L\), by using the following relationship.

\[
E_S \sim \left(\frac{1}{2}\right) N_I K_I Q_I E_L \quad \text{Eq 5.3}
\]

where \(N_I\) is the number of passes, through a 50% beamsplitter, present in the illumination optics,

\(K_I\) is a constant which accounts for any absorption and reflection losses in the optical system and losses due to mismatch of the illumination to the object geometry,

and \(Q_I\) is a constant which accounts for any vignetting of the illumination beam.

By combination of Equations 5.2 and 5.3, the following relationship, between the total object beam energy at the film and the laser output energy, was obtained.
where \( N \) equals \( N_R + N_I \) and is thus the total number of passes through a 50% beamsplitter present in the optical system,

\[ K \text{ equals } K_R K_I, \]

and \( Q \) equals \( Q_R Q_I \).

The energy density at the holographic film, assuming a spatially uniform exposure, was obtained from Equation 5.4, producing the following relationship.

\[ \frac{E_H}{A_H} \sim (\frac{1}{2})^N \frac{K Q H A_R E_L}{d^2 A_H} \quad \text{Eq 5.5} \]

where \( A_H \) is the area of the object beam at the holographic film.

A maximum limit to the angular spread of the object beam at the film was set by the spatial frequency response of the holographic recording material. This defined a minimum value of \( A_H \), and therefore a maximum energy density at the film. The energy density at the film was considered in terms of solid angles by using the below relationship which applies to any non-aberrating imaging system.

\[ A_R \cdot \Omega_R \sim A_H \cdot \Omega_H \quad \text{Eq 5.6} \]

where \( \Omega_R \) is the solid angle subtended by the illuminated area of the object at a point on the receiving optics collection aperture,

and \( \Omega_H \) is the solid angle subtended by the optical system at a point on the film.

These angles are shown in Figure 5.1.
Thus from Equations 5.5 and 5.6,

\[ \frac{E_H}{A_H} \sim \left(\frac{1}{2}\right)^N \frac{K Q H \Omega_H E_L}{d^2 \Omega_R} \]  

Eq 5.7

Also,

\[ \Omega_R \sim \frac{A_s}{d^2} \]  

Eq 5.8

where, \( A_s \) is the illuminated area of the object.

Thus, from Equations 5.7 and 5.8,

\[ \frac{E_H}{A_H} \sim \left(\frac{1}{2}\right)^N \frac{K Q H \Omega_H E_L}{A_s} \]  

Eq 5.9

It was seen from Equation 5.9 that the energy density was maximised when \( \Omega_H \) was also maximised within the spatial frequency limits of the film. For a system with \( \Omega_H \) thus optimised, increasing the area of the receiving aperture of the image rotator would not increase the energy density at the film. It would, however, increase the area of the object beam at the film and would increase the resolution of the system. Similarly, shortening \( d \) would have the same effect.

5.3 Holographic System Design Choices

5.3.1 Laser

The pulsed double exposure holographic technique was chosen because of its suitability for studying the large amplitude vibration found in supersonic unstalled flutter. A ruby laser, operating at a wavelength of 0.694\( \mu \text{m} \), was chosen by this author for the following reasons. Commercially available lasers of this type were capable of producing...
suitably spaced pairs of short duration pulses having high mutual coherence and a relatively high energy. The wavelength of 0.694μm was compatible with high resolution and high sensitivity hologram recording materials. Conventional Q-switched ruby lasers could produce individual pulses having a duration of approximately 25ns. Reference to Equation 2.1 shows that this was adequate for freezing the expected test fans flutter velocities of up to a few metres per second. Pulse separations up to 1ms, coherence length in excess of 1m and individual pulse energies greater than 1 J were possible from commercially available ruby lasers.

Recently, double pulse frequency-doubled neodymium-YAG lasers having comparable attributes have become commercially available. Use of such a laser for holographic interferometry was reported by Decker (1982).

5.3.2 Rotating Illumination

Reports of holographic systems using image rotators for the vibrational study of rotating objects are reviewed in Section 2.7. For all of the systems reported by other authors, a stationary illuminating beam was employed and only the light scattered from the rotating object was passed through the image rotator. With these systems, the illuminating wavefront was required to be symmetrical about the rotational axis of the object.

An optical configuration which had a double-pass through the image rotator was used by this author. In addition to derotation of the light scattered from the fan, the illuminating beam was rotated. Thus, there was no need to have a symmetrical illuminating wavefront. The tolerance on the spatial quality and alignment of the illuminating beam was consequently relaxed and even diffuse illumination could be used. The use of rotating illumination allowed the available laser light to be
concentrated into an off-axis portion of the fan, and, when used in this mode there was the operational convenience of being able to rotate the illuminated area on the fan by simply phase-locking the rotator at a different orientation with respect to the fan. Concentrating the available laser energy into a smaller area of the object resulted in an increased energy density at the film, as seen from Equation 5.9.

5.3.3 Holographic Recording Medium

Hologram recording was performed using either silver halide film or thermoplastic film, these being the most sensitive techniques which were commercially available. The silver halide recording was performed using Agfa Gevaert 8E75 or 10E75 film which was processed to produce an amplitude hologram. The thermoplastic recording was performed using a Rottenkolber thermoplastic camera, model HSB100, and thermoplastic film type PT100. The thermoplastic camera had the important advantage that the film could be developed and reconstructed in situ in the camera, and the resultant holographic image could then be viewed remotely using a video camera and monitor. This feature made it very attractive for remote rig running.

Some of the other properties of these recording media are given in Table 5.1. The thermoplastic phase hologram was capable of a considerably higher diffraction efficiency than was achievable using silver halide amplitude recordings. However, this increased diffraction efficiency was accompanied by a more than proportionate increase in unwanted surface scatter. This resulted in a signal to noise ratio which was less than for silver halide amplitude holograms. In addition, the diffraction efficiency and surface scatter often varied over the surface of the film to an extent which made it unsuitable for image-plane recordings.
<table>
<thead>
<tr>
<th>Properties</th>
<th>Photographic Emulsion</th>
<th>Thermoplastic Film</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8E75</td>
<td>10E75</td>
<td>PT100</td>
</tr>
<tr>
<td>Type of hologram</td>
<td>Amplitude</td>
<td>Amplitude</td>
<td>Phase</td>
</tr>
<tr>
<td>Energy density for optimum exposure (a)</td>
<td>11^{-21}(c)</td>
<td>3^{-6}(c)</td>
<td>4(d)</td>
</tr>
<tr>
<td>Maximum Diffraction Efficiency</td>
<td>4(c)</td>
<td>3(c)</td>
<td>30(b)</td>
</tr>
<tr>
<td>Spatial frequency response</td>
<td>0 to 3000(b)</td>
<td>0 to 2800(b)</td>
<td>500 to 1200(b,e)</td>
</tr>
</tbody>
</table>

(a) The effect of reciprocity failure at 25nS is included.
(b) Manufacturers data.
(c) Obtained from Collier et al (1971).
(d) Measured.
(e) For diffraction efficiency > 10%

TABLE 5.1 Properties of the Recording Media Used in the Holographic System
It can be seen from Table 5.1 that the spatial frequency response range for the thermoplastic film was less than for the silver halide photographic emulsions. Thus, an optical system subtending a larger solid angle at the film could be used with photographic emulsions. All else being equal, this would give a higher maximum energy density for silver halide film, as shown by Equation 5.9.

5.4 Description of the Holographic System

A diagram of the optical system is shown in Figure 5.2. The image rotator both rotated the illuminating beam and derotated the light scattered from the rotating fan before it was relayed onto the hologram recording plane. The viewing and illumination were from a point on the fan's projected rotational axis, thus ensuring that the holographic sensitivity vector was orthogonal to the fan rotation. This prevented unwanted optical path-length variation due to fan rotation which would have produced distorting bias fringes on the resultant interferogram and possibly even prevented hologram formation, as discussed in Section 2.4. The fine adjustment of the optical system relative to the fan's rotational axis was performed using two alignment mirrors, positioned between the image rotator and the fan.

Apart from the above features, the optical configuration was that of a conventional double pulse holographic system. The Q-switched pulsed ruby laser described in Section 3.4.3 was used. The laser was operated in its double-pulse mode. The longitudinal coherence length of 0.7m or greater was adequate for the generation of holograms of fans which were several metres in diameter.

The laser output was divided at a wedged beamsplitter where 4% of the light was reflected to produce a path-matched reference beam. The
majority of the light formed the object beam. This was projected through the image rotator onto the fan using illumination optics and a 50% beamsplitter. Where a large angle of illumination, approaching the full angle of view of the image rotator, was required, the illumination optics consisted of a diverging lens, a weak diffuser and a converging lens, as shown in Figure 5.2. The two lenses focused the beam within the rotator, and the diffuser prevented air breakdown by reducing the beam intensity at the focus. When a smaller angle of illumination was required, such as when the available light was concentrated into an off-axis portion of the fan, the illumination optics simply consisted of a negative lens which diverged the beam through the rotator onto the fan.

When thermoplastic film was used, the interferogram could be viewed remotely only a few seconds after recording. This was achieved by employing a video camera to view the holographic image and a reconstruction system employing a remotely operable shutter as shown in Figure 5.3.

The relay optics between the image rotator and the hologram recording film were used to match the angular spread of the returned light to the spatial frequency characteristics of the film, thus maximising the energy density at the film. None of the spherical optics, either associated with fan illumination or relaying the image to the film, appeared on the same side of the rotator as the fan, as this would have demanded fine manufacturing and positional tolerances of any such components.

A triggering system was used which ensured that the laser pulses occurred at a coincidence of selected points in the fan's rotational cycle and the vibrational cycle. A fan once per revolution signal and the output from a strain gauge on the fan were used as inputs to the
FIGURE 5.3 DIAGRAM OF THE RECONSTRUCTION SYSTEM EMPLOYED WITH THE THERMOPLASTIC CAMERA.
triggering system. The triggering system not only provided a pulse which Q-switched the laser at the required coincidence, but it also predicted this coincidence, approximately 1ms before it occurred, and provided a pulse which fired the laser flash-tubes. This prediction was performed assuming constant rotational velocity and vibration frequency.

The holographic system was housed in a compact two tier frame, having the ruby laser mounted on the lower level and the rest of the optical system mounted on the upper level. The complete frame was mounted on screw-jacks, thus providing a coarse vertical adjustment on the position of the optical axis of the system. Photographs of the system are shown in Figure 5.4. The approximate size of the unit was 1.3m long, 1m wide and 1.2m high.

5.5 Effects of Misalignment

The holographic system was optically aligned when the projected fan's axis passed through the centre of the image rotator apertures. At this condition, the holographic sensitivity vector was perpendicular to the fan's direction of rotation at all points, and thus no biasing of the interference fringes occurred due to rotation. When this was not the case, the fringe order at each point on the fan was modified or biased.

An expression was derived for the bias fringe field generated by misalignment. The stationary coordinate system and geometrical configuration which was used is shown in Figure 5.5. For simplicity, the source of illumination and the collection aperture were approximated to coincident points on the rotator axis at \( Q \), having coordinates \( x_Q, y_Q, z_Q \). \( R_1 \) was a general point on the fan at \( x_1, y_1, z_1 \) at the time of the first exposure. Rotation of the fan of \( \Delta \theta \) between exposures moved this
FIGURE 5.4 PHOTOGRAPHS OF THE HOLOGRAPHIC OPTICAL SYSTEM.
FIGURE 5.5 DIAGRAM SHOWING THE GEOMETRICAL CONFIGURATION USED FOR CALCULATION OF BIAS FRINGES DUE TO MISALIGNMENT.
point to \( R_2 \) at \( x_2, y_2, z_2 \). The rotation of the rotator served to maintain correlation between the two images of the fan, but did not itself introduce any pathlength change. Thus, the total pathlength change was \( 2(R_2 Q - R_1 Q) \). This would introduce an error of \( \Delta P \) full fringes in the resultant interferogram given by:

\[
\Delta P = \frac{2}{\lambda} (R_2 Q - R_1 Q) \tag{Eq 5.10}
\]

where \( \lambda \) is the wavelength of light.

From the geometry,

\[
(R_K Q)^2 = (x_Q - x_K)^2 + (y_Q - y_K)^2 + (z_Q - z_K)^2
\]

for \( K = 1,2 \) \tag{Eq 5.11}

\[
(R_K Q)^2 = (x_Q - r \cos \theta_K)^2 + (y_Q - r \sin \theta_K)^2 + (z_Q - z_K)^2
\]

for \( K = 1,2 \) \tag{Eq 5.12}

where \( r = OR_1 = OR_2 \) and \( \theta_1, \theta_2 \) are defined in Figure 5.5.

From an expansion of Eq 5.12 and using \( z_1 = z_2 \),

\[
(R_2 Q)^2 - (R_1 Q)^2 \approx 2x_Q r (\cos \theta_1 - \cos \theta_2) + 2y_Q r (\sin \theta_1 - \sin \theta_2) \tag{Eq 5.13}
\]

Assuming small \( \Delta \theta = \theta_2 - \theta_1 \) and letting \( \theta = \theta_1 \),

\[
(R_2 Q)^2 - (R_1 Q)^2 \approx 2 \Delta \theta x_Q r \sin \theta - 2 \Delta \theta y_Q r \cos \theta \tag{Eq 5.14}
\]

Letting \( x_R = x_1 \) and \( y_R = y_1 \),

\[
(R_2 Q)^2 - (R_1 Q)^2 \approx 2 \Delta \theta (x_Q y_R - y_Q x_R) \tag{Eq 5.15}
\]
For small misalignments, $R_2Q + R_1Q = 2v$, thus

$$R_2Q - R_1Q \approx \frac{(R_2Q)^2 - (R_1Q)^2}{R_2Q + R_1Q} \quad \text{Eq 5.16}$$

Thus from Eq 5.10,

$$\Delta P \approx \frac{2\Delta \theta}{\lambda v} (x_Q y_R - y_Q x_R) \quad \text{Eq 5.17}$$

Equation 5.18 is a useful relationship which quantified the bias fringe error for any point on the fan. For the case where the fan to image rotator distance was large compared with the fan radius, $v$ was approximately constant for all points on the fan and the bias fringe distribution could be approximated by a set of parallel equispaced fringes.

From the above analysis it was seen that tilting of the rotator axis about $Q$ (the point illumination and collection aperture) did not affect the bias fringes. However, excessive tilting of this axis resulted in decorrelation of the two holographic images and an attendant reduction of fringe visibility.

Measurements were made to validate Equation 5.18. The holographic system was set up to record double pulse interferograms showing the centre of a rotating fan. The rotator and fan axes were deliberately misaligned, and the spacing and orientation of the resultant bias fringes, at the rigid hub of the fan, were measured. A typical resultant interferogram is shown in Figure 5.6. The optical axis of the rotator was translated in a horizontal direction, perpendicular to the fan's projected axis, and holograms were recorded at various translation positions, i.e., at various positions of $x_Q$. The number of bias fringes parallel with the horizontal
FIGURE 5.6 RECONSTRUCTION OF A HOLOGRAM SHOWING BIAS FRINGES DUE TO MISALIGNMENT ON THE HUB OF A ROTATING FAN.

FIGURE 5.7 GRAPH SHOWING THE MEASURED NUMBER OF BIAS FRINGES PLOTTED AGAINST $X_0$. 
axis is shown plotted against $x_Q$ in Figure 5.7. The change in the number of bias fringes, per unit change in $x_Q$, was approximately constant and the measured value of 2.4 fringes/mm agreed with the value calculated from Equation 5.18 to 2 significant figures.

Substitution into Equation 5.18 was used to estimate the likely error due to misalignment of the system when used for flutter measurement on the test fan. A typical maximum error of ± 1 full fringe was estimated. This analysis is described in detail in Section 7.2.

5.6 Alignment Procedure

Fine alignment of the rotator relative to the fan's rotational axis was performed using the two output alignment mirrors shown in Figure 5.8. The relative angle between the fan and the rotator axes was changed by adjustment of two orthogonal tilt controls, and the relative displacement between these axes was adjusted by means of vertical and horizontal displacement controls. The two displacement controls were driven by stepper motors thus allowing remotely controlled adjustment. The alignment was performed in two parts. First, the optical axis of the rotator was made closely collinear with the rotational axis of the fan, as determined at very low speed. Second, the optical system was aligned interferometrically to the fan's axis at the operational speed of the fan.

The first part of the alignment procedure was performed as follows, using the configuration shown schematically in Figure 5.9.

(a) A slightly converging helium-neon laser beam was aligned to the optical axis of the spinning rotator by adjustment of mirrors A and B. This was achieved when the loci defined by the rotating beam in the near and far field of the output of the rotator were spots of minimum diameter.

(b) A small adjustable mirror, C, attached to the centre of the fan was aligned so as to be normal to the fan's rotational axis. This was obtained by monitoring the locus of a reflected laser beam at a distant screen when the fan was slowly rotated by hand.
FIGURE 5.8 PHOTOGRAPH OF THE OUTPUT ALIGNMENT MIRRORS.

FIGURE 5.9 DIAGRAM SHOWING CONFIGURATION USED FOR ALIGNMENT AT VERY LOW SPEED.
(c) The output alignment mirrors were then adjusted such that the beam aligned to the axis of the rotator was incident on the centre of the fan and was collinear with the resultant reflected beam.

(d) The mirrors A, B and C were then removed and the holographic system was reassembled.

Following this procedure, the projected fan axis was typically within a few millimeters of the centre of the rotator apertures.

The second part of the alignment was performed with the fan at its operational speed prior to taking vibrational measurements. It required the remote viewing and near real-time capability of the thermoplastic camera for successful operation in a compressor test rig environment. In addition it required a visible rigid hub at the centre of the fan. Alignment was achieved by recording double pulsed holograms of the fan with the rotator synchronised. The output alignment mirrors were adjusted, using the remotely operated vertical and horizontal translation controls, so as to minimise the number of bias fringes on the hub of the fan. This could be performed to a high accuracy by employing relatively long pulse separations. Use of Equation 5.18 made this a relatively short iterative procedure.

5.7 Initial Trials: Description of Laboratory Fan Rig

Development and initial trials of the holographic system were performed with the aid of a laboratory fan rig. The aero engine fan had a diameter of 0.56m (22 inches) and was of a shrouded design. The fan was rotated in a vacuum using the rig shown in Figure 5.10. The fan shaft was belt driven by a 11.2kWatt (15HP) electric motor. The rig was capable of rotating the fan at speeds up to 6000rpm. An
FIGURE 5.10 PHOTOGRAPH OF THE LABORATORY FAN RIG.
optical tachometer giving 60 pulses per revolution, was used to phase-lock the image rotator to half the fan speed. The fan was viewed through a 0.52m diameter glass window which formed the front of the vacuum chamber. The window was perpendicular to the fan axis and its surfaces were parallel to within 3 minutes of arc. Both window surfaces had a single layer dielectric coating so as to reduce unwanted back reflections. The fan was coated with retro-reflective paint in order to greatly increase the light returned to the holographic system.

Individual vibration modes of the fan could be excited using piezo-electric crystal exciters which were attached to the fan blades and powered via a slip ring unit. The crystals were positioned so as to excite the two nodal diameter, second family (2D2F) coupled-blade mode of the fan with a relatively high amplitude. Individual fan blades were also susceptible to excitation from the shaft's roller-element journal bearings. This resulted in a large response in first flap and first torsion blade modes, at certain fan speeds. The vibrational response of the rotating fan was monitored using several strain gauges attached to the blades and connected via slip rings.

Personnel were not allowed into the fan rig room when the fan was at high speeds, due to safety considerations, and the rig was controlled from a separate room. However, operation of the rig was relatively easy, and it was common practice for the author alone to operate the fan and holographic system.

Estimates were made of the total energy collected by the holographic system and the maximum achievable energy density at the holographic film, for illumination of the whole of the fan. The estimates were made using Equations 5.4 and 5.9 and values for the parameters used in these equations are given in Table 5.2. The resultant estimate of the maximum
<table>
<thead>
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<tr>
<td>K</td>
<td>Constant representing losses in the optical system</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Number of passes through the 50/50% beamsplitter</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>AR</td>
<td>Area of the rotator receiving aperture</td>
<td>5.7x10^{-4} m^2</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>Constant representing vignetting in the optical system</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Average fraction of incident energy radiated by points on the object surface towards the receiving aperture, per unit solid angle</td>
<td>10</td>
<td>For Scotchlite retro-reflecting print.</td>
</tr>
<tr>
<td>d</td>
<td>Fan to image rotator distance</td>
<td>3.5 m</td>
<td></td>
</tr>
<tr>
<td>EL</td>
<td>Laser output energy</td>
<td>0.5 J</td>
<td>This was the maximum laser output</td>
</tr>
<tr>
<td>H</td>
<td>Solid angle subtended by the optical system at a point on the film</td>
<td>0.2 Sterad</td>
<td>For thermoplastic film. This was the maximum value limited by the spatial frequency response of the film.</td>
</tr>
<tr>
<td>AS</td>
<td>Illuminated area of the object</td>
<td>0.25m^2</td>
<td>This is for full illumination of the fan.</td>
</tr>
</tbody>
</table>

**TABLE 5.2 Parameters for the Holographic System when Used with the Laboratory Fan Rig**
total energy collected was 4μJ, and the estimate of the maximum achievable energy density at the film was 0.08Jm⁻². This was ample for recording a hologram with an optimum exposure on both thermoplastic and silver halide film.

5.8 Results from Fan Rig

Initial holograms of the rotating fan were recorded using Agfa Gevaert 8E75HD holographic plates in an image plane mode. The image rotator was approximately 3.5m from the fan. The whole of the fan was illuminated and the resultant image at the holographic plate was 27mm in diameter. The holograms were recorded using laser output total energies of approximately 400mJ for the two pulses. Interferograms were successfully recorded at fan speeds up to 5840rpm, this being near the limit of the fan rig. The holograms were reconstructed using a filtered mercury arc lamp. Two examples of the resultant reconstructions are shown in Figures 5.11 and 5.12. The interferogram shown in Figure 5.11 was recorded when the fan was rotating at 3670rpm and was excited using piezo-electric exciters at 334Hz, which was the average natural frequency of the first flap blade resonances. Figure 5.12 shows an interferogram of the fan when it was rotating at 4125rpm and was excited at 712Hz, the natural frequency of a 2D2F mode. Both of these holograms were recorded with a pulse separation of 18μS. The presence of bias fringes, due to misalignment was evident by the broad fringes in the centre of the fan. This was particularly pronounced on the hologram shown in Fig. 5.11.

The plate holder was then replaced with the thermoplastic camera. Initially this was operated in an image plane mode, but this was found unsatisfactory due to irregular scatter from the film surface on
FIGURE 5.11  INTERFEROGRAM RECORDED AT A FAN SPEED OF 3670 RPM USING A PULSE SEPARATION OF 10 μS. MOST OF THE BLADES WERE VIBRATING IN THEIR FIRST FLAP MODE.
FIGURE 5.12  INTERFEROGRAM RECORDED AT A FAN SPEED OF 4125 RPM USING A PULSE SEPARATION OF 18 μS. THE FAN WAS VIBRATING PREDOMINANTLY IN A 202F ASSEMBLY MODE.
reconstruction. The optical system which relayed the returned light from the rotator to the film was modified so as to produce an image of the fan approximately 100mm in front of the film plane. The diameter of the object beam at the film plane was 15mm. Two examples of the interferograms recorded using the thermoplastic camera in this non-image plane mode are shown in Figures 5.13 and 5.14. Figure 5.13 shows an interferogram of one of the 2D2F assembly modes recorded with the fan stationary and using a pulse separation of 44μs. Figure 5.14 shows an interferogram of a portion of the fan which was recorded at a fan speed of 4030rpm and a pulse separation of 44μs, with the fan vibrating in a 2D2F mode. It can be seen from the fringe distribution on the hub, that misalignment resulted in approximately 15 bias fringes. The interferogram gave an indication of the fringe resolution, there being up to 11 resolvable fringes along the blade tips. This corresponds to a fringe frequency of 0.3 fringes/mm.

The illumination optics were then modified by using a single negative lens to concentrate the laser output into an off-axis portion of the fan, as described in Section 5.4. Holograms were recorded on Agfa Gevaert 10E75 plates of a 2D2F mode under stationary and rotating conditions. Figure 5.15 shows an interferogram which was recorded with the fan and image rotator stationary. It was recorded using a total laser energy of 210mJ and a pulse separation of 15μs. Figure 5.16 shows an interferogram which was recorded with the fan rotating at 4000rpm and the rotator phase locked to it at half speed. It was recorded with a pulse separation of 15μs, using a total laser energy of 250mJ. As can be seen from these two hologram reconstructions, there was no noticeable change in image quality and fringe contrast between holograms recorded under stationary and rotating conditions.
FIGURE 5.13  INTERFEROGRAM OF THE STATIONARY FAN VIBRATING IN A 202F MODE. THE HOLOGRAM WAS RECORDED ON THERMOPLASTIC FILM WITH A PULSE SEPARATION OF 44μS.

FIGURE 5.14  INTERFEROGRAM RECORDED AT A FAN SPEED OF 4830 RPM WITH THE FAN VIBRATING IN A 202F MODE. THE HOLOGRAM WAS RECORDED ON THERMOPLASTIC FILM WITH A PULSE SEPARATION OF 44μS.
FIGURE 5.15  INTERFEROGRAM OF THE STATIONARY FAN VIBRATING IN A 202F MODE. THE ILLUMINATING LASER LIGHT WAS CONCENTRATED INTO AN OFF-AXIS PORTION OF THE FAN.

FIGURE 5.16  INTERFEROGRAM RECORDED AT A FAN SPEED OF 4000 RPM WITH THE FAN VIBRATING IN A 202F MODE. THE ILLUMINATING LASER LIGHT WAS CONCENTRATED INTO AN OFF-AXIS PORTION OF THE FAN.
CHAPTER 6

APPLICATION OF THE HOLOGRAPHIC SYSTEM TO THE STUDY
OF THE FLUTTERING TEST FAN

Summary of Chapter

The holographic system was used to obtain interferograms of the fluttering test fan. The fan was driven at high speed under aerodynamic load using an 11 MW air turbine powered compressor test rig. Prior to the holographic study of the test fan, the tilt of the compressor test rig's axis was measured, and it was concluded that this did not pose any serious limitations on the experiment. After some initial problems, approximately 100 useful holograms were recorded. As a datum, a few holograms were recorded at speeds just below the onset of flutter. The majority, however, were recorded in steady unstalled flutter. These interferograms showed the fan to be fluttering with a 3D2F mode shape with a peak velocity of approximately 1ms⁻¹.

6.1 Introduction

The use of the holographic system to obtain interferograms of the fluttering test fan is described in this chapter. These measurements were the first reported use of holography for vibrational study of a rotor undergoing supersonic unstalled flutter. The holographic experiment was designed to provide measurements of the vibrational deflection shape of the fluttering test fan and to use these measurements to evaluate the theories and assumptions used currently for fan flutter prediction. The aims of the experiment are described in detail in Section 1.5.

The fan was driven at high speed under aerodynamic load into a flutter
condition using an air turbine powered compressor test rig. Details of the compressor test rig are given in the next section of this chapter. This test rig was chosen because it provided a well instrumented, easily controlled facility for study of the test fan. This particular rig was in great demand and had high running costs. Thus emphasis was placed on good operational planning of the experiment so as to minimise rig running time and the total duration of the test. In order to assess the suitability of the rig for this flutter study, a preliminary measurement of fan shaft tilt was performed. This measurement is described in Section 6.3.

The experimental conditions and procedure for the holographic experiment are described in Section 6.4. Some operational problems were encountered and these, together with the contingency plans for the experiment, are then described. This is followed by presentation of some of the resultant interferograms obtained during this experiment. Analysis of the interferograms to obtain vibrational deflections is described in a subsequent chapter.

6.2 Compressor Test Rig Details

The flutter studies of the test fan were performed using the Rolls-Royce compressor test rig, CTRI, at the Altitude Test Facility in Derby. The main function of the altitude test plant was the testing of full scale aero engines over a wide range of aircraft altitude operating conditions. In order to achieve high utilization of the plant, several test rigs, including CTRI, were included in the facility. The plant was located on an 11 acre site and included engine and rig test cells, a compressor building, a cooling tower, a water treatment plant, a high voltage substation and an administration block.
A diagram of CTR1, as it was arranged for the holographic measurements, is shown in Figure 6.1. The fan was driven by an 11MWatt (15000hp.) air turbine which was capable of providing fan speeds of 3000 to 11000rpm. The turbine itself was powered from two electrically driven compressors. The compounded compressors were situated in a separate building and they provided air to the turbine at 10 atmospheres and 200°C. The fan rig was assembled with a short flared intake through which air was drawn from the compressor test hall. The internal dimensions of the test hall were approximately 35m by 10m by 10m. Passage of air from the atmosphere to the test hall was via a large door which provided a 5m by 1.5m opening. A screen was erected in front of the opening, so as to minimise eye hazard to passers-by due to any unintentionally specularly reflected laser light. The compressed output air from the fan was collected at the rear of the compressor test section and directed to the atmosphere via a discharge silencer.

The compressor test section is shown diagramatically in Figure 6.2. The flow was separated down stream of the fan so as to simulate the fan's performance in a high bypass ratio aero engine. The majority of the flow was passed through the outer bypass section, via a set of outlet guide vanes. The remainder of the flow was passed via a set of stators into a simulated engine section. Variable throttles were situated in both the bypass and engine sections, thus allowing independent control of the flow rates in each. The total flow rate to the fan was obtained from pressure measurements made within the calibrated air intake (airmeter). The flow within the engine section was passed through a Venturi tube, thus allowing the engine section flow rate to be obtained. The flow rate within the bypass section was determined by subtracting the engine section flow rate from the total flow rate. The efficiency
Figure 6.1 Sketch showing the compressor test rig as arranged for the holographic study of the test fan.
FIGURE 6.2 DIAGRAM SHOWING FLOW WITHIN THE COMPRESSOR TEST SECTION.

FIGURE 6.3 DIAGRAM OF THE TILT MEASURING SYSTEM.
of the fan, defined as the work done on the air divided by the 
mechanical energy supplied was measured as follows. The work done 
on the air was determined from measurements of pressure and temperature, 
obtained from arrays of transducers at and between the outlet guide 
vanes and the engine section stators, and from the measured mass flow 
rates. The mechanical energy supplied to the fan was determined using 
a torquemeter on the fan shaft.

6.3 Measurement of Fan Shaft Tilt

Accurate alignment of the image rotator with respect to the fan's 
axis was a requirement for successful application of the holographic 
system, as is described in Section 5.5. The angular movement of the 
fan shaft of the compressor test rig, CTR1, was measured several months 
before the proposed flutter studies of the test fan in order to assess the 
rig's suitability. The measurements were performed with a fan which 
was very similar to the test fan.

6.3.1 Measurement Technique

The tilt of the fan was measured by reflecting a converging laser 
beam from a small mirror attached to the fan's rotating nose-cone. 
Changes in the angular position of the reflected beam were used to measure 
changes of fan axis tilt. The use of an optical lever allowed the 
measurement of very small changes in the angle of the fan's axis.

A diagram of the tilt measuring system is shown in Figure 6.3. 
Diagrams of the source and monitoring units are shown in Figures 6.4 and 
6.5, respectively. A converging laser beam was obtained from the source 
unit by expanding the output from a helium-neon laser using a microscope 
objective and then converging this expanded beam using a multi-element
FIGURE 6.4 DIAGRAM SHOWING THE INTERNAL CONFIGURATION OF THE SOURCE UNIT.

FIGURE 6.5 DIAGRAM SHOWING THE INTERNAL CONFIGURATION OF THE MONITORING UNIT.
camera lens of focal length 400mm. A pin hole was placed at the focus of the output from the objective in order to spatially filter the beam, and thus minimise the spot size at the monitoring screen. The output from the source unit was directed onto a ground glass screen in the monitoring unit via the small mirror on the fan's axis, and the focus of the converging beam was formed at this screen. The position of the laser spot on the screen was viewed using a video camera and remote monitor.

The source and monitoring units were placed 8m from the fan and the light beams were each at an angle of approximately 15° to the fan's axis. A tilt of the fan's axis of \( \phi \) resulted in the reflected beam being rotated through \( 2\phi \). Thus a movement of the laser spot at the monitoring screen of 1mm represented a tilt of the fan's axis of 13 seconds of arc. The mirror on the fan's nose cone was deliberately positioned with the normal to its surface at a small angle to the fan's axis. A value of 9 minutes of arc was obtained. Thus with the fan rotating and without fan axis movement, the locus described by the laser spot at the monitoring screen was a circle of diameter 80mm. Thus, any steady shift of the angular position of the fan's axis was detectable as a displacement of the centre of the circle. Any angular vibration of the fan's axis was detectable as a modification of the shape of the locus.

The video camera output was displayed on a monitor in the control room and was also recorded on a video tape recorder together with the time, a fan speed signal and a voice log. Accelerometers were placed in the source unit, and these were monitored in the control room to ensure that the source unit was not subject to excessive vibration.

The fan was first accelerated from rest to 10 000rpm, where this speed was maintained for a few minutes, and then it was decelerated back
down to rest. On a second run, the fan was accelerated to 11,000 rpm with stops for several minutes at 3000, 9530 and 10020 rpm. Angular movement of the fan's axis was monitored and recorded throughout.

6.3.2 Results

The maximum steady angular deviation of the fan's axis between rotating and stationary conditions was 1 minute of arc.

The maximum angular vibration of the axis occurred at a speed of 11,000 rpm. At this speed, a third harmonic vibration having a peak angular deviation of 1.3 minutes of arc was detected, as is illustrated in Figure 6.6. Apart from at this condition, the peak observed vibrational tilt was 0.7 minutes of arc.

The vibration of the source unit was monitored throughout the tests and was found to be well below the level required to produce significant errors.

6.3.3 Implications for Holographic Measurements

The maximum steady angular deviation of the fan's axis defined the displacement adjustment required of the output alignment mirrors of the holographic system. The maximum observed value of 1 minute of arc would require a displacement of 1.7 mm on the output alignment mirrors, for a fan to holographic system distance of 6 m. This was well within the capabilities of the alignment mirror system.

The peak angular vibration of 0.7 minutes of arc, which was observed at fan speeds below 11,000 rpm, corresponded to a peak displacement of the fan's projected axis at the rotator of 1.2 mm. This equated to a possible peak error of approximately a half bias fringe for a holographic interferogram recorded with a pulse separation of 2 μs. This was considered an
FIGURE 6.6  LOCUS DESCRIBED BY THE LASER SPOT AT A FAN SPEED OF 11,000 RPM. THE ARROWS SHOW THE DEVIATION FROM A CIRCLE.

FIGURE 6.7  PHOTOGRAPH OF THE HOLOGRAPHIC SYSTEM MOUNTED IN THE COMPRESSOR TEST FACILITY, CTR1, PRIOR TO TAKING MEASUREMENTS IN FLUTTER.
acceptable error.

Thus, it was concluded that movement of the fan's axis on CTR1 did not pose any serious limitations in the use of the holographic system for study of the test fan.

6.4 Experimental Conditions and Procedure for the Holographic Measurements

The test fan was instrumented with strain gauges on six of its blades and was mounted on CTR1. Following an initial evaluation of its aerodynamic and flutter performance, the fan was sprayed with a commercially available retroreflecting paint (3M's silver) in order greatly to increase the light returned to the holographic system. The fan's aerodynamic and flutter performance was then re-evaluated. The paint was found to have significantly affected the performance of the fan. The total flow and efficiency were reduced by 4\% and 3\%, respectively. The flutter onset speed was reduced by 6\%. This was likely due to an increase in the blade surface roughness from approximately 0.8\µm (30 \microinches) to 6\µm (250 \microinches) CLA and a thickening of the leading edge of the blades, as a result of the application of the paint. Even though there was a significant change in the fan characteristics, the holographic experiment was valid and useful for study of the fundamentals of the flutter process and for comparison of the experimentally determined deflections with those predicted by finite element techniques.

The holographic system was installed in CTR1. The main optics unit was mounted approximately 6m from the fan and on its projected axis, as is shown in Figure 6.7. The laser power supplies and the amplifier for the image rotator were mounted within the main test hall. The
control, monitoring and triggering electronics for the system were situated in a separate control room, as shown in Figure 6.1. A block diagram showing the interconnections of these various electronic units is shown in Figure 6.8. The optics unit employed a reference beam which was folded within its top tier to give an optical path of 12m, thus path matching the object beam. The available laser light was directed onto approximately one quarter of the area of the fan, so as to increase the object beam intensity at the hologram recording plane. The illumination optics, as described in Section 5.4, consisted of a negative lens which diverged the beam through the rotator onto the fan. The parameters relevant to the optical efficiency of the system are given in Table 6.1. Using Equation 5.9, it was estimated that an object beam energy density of $0.8 \mu \text{Jcm}^{-2}$ was obtained at the film plane. The optical axis of the image rotator was made closely collinear with the rotational axis of the fan, as determined at low speed. This was performed using a helium-neon laser as described in the first part of the alignment procedure given in Section 5.6. Following this alignment the centre of the image rotator apertures was positioned to within ±2mm of the projected axis of the static fan. It was initially intended to perform a second stage of alignment, as described in Section 5.6, which used the remote viewing capability of the thermoplastic camera to align the holographic system with respect to the fan's axis at full speed. Unfortunately a fault developed in the thermoplastic camera in the early stages of the test, as described in the next section, and this second stage of alignment was not performed.

The compressor test rig was operated with a fully open bypass throttle to give the lowest flutter speed for the holographic studies. The flutter onset speed was at approximately 9850rpm. Analysis of signals
FIGURE 6.8: BLOCK DIAGRAM SHOWING THE INTERCONNECTIONS OF THE VARIOUS CONTROL, MONITORING AND TRIGGERING ELECTRONIC UNITS EMPLOYED IN THE HOLOGRAPHIC SYSTEM.
<table>
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<td>Area of the rotator receiving aperture</td>
<td>$5.7 \times 10^{-4}$ m$^2$</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>Constant representing vignetting in the optical system</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Average fraction of incident energy radiated by points on the object surface towards the receiving aperture, per unit solid angle</td>
<td>10</td>
<td>For Scotchlite retro-reflecting paint</td>
</tr>
<tr>
<td>d</td>
<td>Fan to image rotator distance</td>
<td>6m</td>
<td></td>
</tr>
<tr>
<td>El</td>
<td>Laser output energy.</td>
<td>0.5J</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Solid angle subtended by the optical system at a point on the film</td>
<td>0.13 sterad</td>
<td></td>
</tr>
<tr>
<td>AS</td>
<td>Illuminated area of the object</td>
<td>0.15 m$^2$</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 6.1** Parameters for the Holographic System When Used for Study of the Test Fan on CTRI
from the blade strain gauges and downstream pressure transducers revealed that the test fan was fluttering at a frequency of 686 Hz in a vibrational mode having three nodal diameters.

After some initial operational problems, which are described in the next section, approximately 100 useful holograms were recorded during 6 hours of running over a 2 day period. The majority of these were recorded in steady un stalled flutter. Holograms were recorded at selected fan orientations and alternating stress levels and at various phases in the flutter cycle. A few holograms were recorded at fan speeds just below the onset of flutter.

6.5 Contingency Plans and Operational Problems Encountered

Prior to the experimental study on CTRl, contingency plans in case of equipment failure were considered. Each of the major vulnerable sub-units within the holographic system was considered and alternative units were made available, where possible. These vulnerable units and the contingency alternatives are listed in Table 6.2. The 1 and 60 pulses per revolution (PPR) and strain gauge signals were important for synchronisation of the laser pulses and the image rotator. Contingency alternatives were available for each of these signals. In case of complete failure of the laser triggering electronics, alternative triggering systems of limited capability were available. The alternative triggering systems allowed triggering of the laser at any point in either the fan's rotational cycle or in the flutter cycle. However, unlike the main triggering unit, they did not allow triggering at a coincidence of pre-selected points in both cycles. A contingency option was not available for the case of total failure of the pulsed ruby laser. However, alternatives for key elements within the laser
Primary Unit | Alternative in Case of Failure
--- | ---
1PPR signal | Alternative 1PPR probes were available
Strain gauge signal | Alternative strain gauges on other blades were available
60PPR signal | Frequency multiplication of the 1PPR signal was possible
Laser triggering electronics | Alternative triggering systems of limited capability were available in case of total failure
Pulsed ruby laser | Key vulnerable elements were available including a spare ruby rod for two of the stages and spare flashlamps for all the laser stages.
Thermoplastic camera | A film transport for conventional silver halide film was available
Phase locked loop control electronics for the image rotator | No alternative was available
Image Rotator | No alternative was available

**TABLE 6.2** The Main Vulnerable Units Within the Holographic System and the Contingency Units in Case of Failure
were available, including a spare ruby rod for the oscillator and one amplifier stage and spare flashlamps for all stages. An alternative hologram recording system was made available in case of failure of the thermoplastic camera. This was a remotely operable film transport for 127mm (5 inch) wide conventional silver halide film. The two major items, for which contingency alternatives were not available, were the image rotator and its phase-locked loop control electronics. However, these units were given several tens of hours of running at simulated test conditions, in order to prevent an early-life failure during the flutter studies.

Several unexpected operational problems were encountered during the test. These are described in chronological order in the following paragraphs.

(a) The first problem resulted from mechanical deformation of the main optics unit housing which occurred when moving into the compressor test hall. This resulted in an ill-fitting cover on the unit which required slight twisting in order to make it fit. The fitting and strapping down of the cover prior to fan running resulted in a few millimeters movement of the folded 12m reference beam. This was sufficient to prevent formation of holograms. This problem was easily rectified by trimming the cover but proved difficult to diagnose and four days were lost in the testing programme because of it.

(b) A fault developed in the heating-plate voltage controller in the thermoplastic camera, and the complete camera was replaced by the contingency film transport. This fault occurred at an early stage in the test and the second stage of alignment described in Section 5.6 was not performed. All of the useful
holograms were recorded on Agfa Gevaert 10E75 film using this contingency system.

(c) The recording of the laser pulses relative to the phase of the strain-gauge signals was lost for most of the holograms due to two unexpected problems. First, there was a failure in the portion of the laser triggering electronics which was used to synchronise the laser pulses to a selected part of the flutter cycle. Second, the recording of the laser pulses and strain gauge outputs was not continuous due to human error.

(d) Finally, difficulty was encountered in obtaining laser pulse separations less than 2μs. This pulse separation was however close to the lower limit of operation of the pulsed laser.

6.6 Resultant Interferograms

The interferograms showed the fan to be fluttering with a 3D2F mode shape with a peak velocity of the order of 1ms⁻¹. Six examples of the interferograms recorded in flutter are shown in Figure 6.9. They were recorded at fan speeds in the range 9866rpm to 9915rpm, using pulse separations of either 2μs or 4μs. Full experimental conditions are given for each interferogram in Table 6.3. The stress levels quoted in this table are approximate values which were obtained from oscilloscope readings. They are for one strain-gauge position and give an indication of the relative flutter levels. The travelling nature of the flutter vibration can be seen by comparison of Figures such as 6.9(a) and 6.9(b) which show that there was a change in the circumferential position of the fringe pattern.

A few interferograms were recorded at fan speeds just below the
FIGURE 6.9 CONTINUED OVERLEAF.
FIGURE 6.9 CONTINUED OVERLEAF.
FIGURE 6.9 SIX INTERFEROGRAMS OF THE FLUTTERING FAN RECORDED AT THE CONDITIONS GIVEN IN TABLE 6.3
<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Fan Speed (rpm)</th>
<th>Stress Level (MPa)</th>
<th>Pulse Separation (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9(a)</td>
<td>9890</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>6.9(b)</td>
<td>9886</td>
<td>80</td>
<td>2</td>
</tr>
<tr>
<td>6.9(c)</td>
<td>9895</td>
<td>70</td>
<td>2</td>
</tr>
<tr>
<td>6.9(d)</td>
<td>9898</td>
<td>90</td>
<td>2</td>
</tr>
<tr>
<td>6.9(e)</td>
<td>9888</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>6.9(f)</td>
<td>9915</td>
<td>50</td>
<td>4</td>
</tr>
</tbody>
</table>

**TABLE 6.3** Experimental Conditions for the Interferograms Shown in Figure 6.9
onset of flutter. An example is shown in Figure 6.10 which was recorded at a fan speed of 9664rpm and with a pulse separation of 9μs.

There was some variation in the brightness of the holograms and in the fringe definition. This was probably due to variation in the laser pulse shape, which was critical for this short pulse separation application of the holographic system. On some of the interferograms, there was an additional low contrast widely spaced fringe set. This unexpected feature was due to the presence of a third laser pulse of low energy which occurred temporally close to one of the two main pulses. An example of such an interferogram is shown in Figure 6.9(c). (The additional fringe set is of very low contrast and can be seen on the hologram itself.)

6.7 Concluding Remarks

Many holograms showing good quality interference fringes were obtained during this experimental study of the test fan. Some operational problems were encountered during this experiment which resulted in limitations on two aspects of the study. First, the errors due to misalignment were larger than initially intended. Second, the position of the interferograms within the flutter cycle was not known, which thus prevented determination of the temporal variation of the flutter shape. However, considerable useful information was contained within the holographic interferograms. The detailed quantitative analysis of these interferograms, to determine the vibration shape of the fluttering test fan, is described in the next chapter.
FIGURE 6.10 INTERFEROGRAM OF THE FAN, AT A SPEED OF 9664 RPM, JUST BELOW THE ONSET OF FLUTTER.
ANALYSIS OF THE INTERFEROGRAMS OF THE ROTATING FAN AND
COMPARISON WITH FINITE ELEMENT PREDICTION

Summary of Chapter

The interferograms of the rotating test fan were quantitatively analysed to obtain vibration deflection shapes. Errors due to misalignment and unsteady aerodynamics were assessed. Analysis revealed the presence of random vibration superposed on the flutter vibration. This random vibration was at a different frequency to, and unconnected with, the un stalled flutter phenomenon. To some extent, the detail of the flutter deflection shape from any individual interferogram was masked by this random vibration. Thus, in order to reduce the relative effect of the random vibration, the deflections from several interferograms were normalised and averaged. The resultant averaged deflection shape was compared with finite element prediction and reasonable agreement was obtained.

7.1 Introduction

The interferograms of the fluttering test fan, which were presented in the last chapter, provided an easily determined qualitative assessment of the fan's vibrational shape. A detailed quantitative analysis of these interferograms was performed and is described in this chapter. Selected measured deflection distributions were compared with those predicted by the finite element model of the rotating fan described in Section 1.6.

In the next section, an estimate of the errors in the vibrational
deflection measurements is given. Interpretation of the interferograms obtained out of unstalled flutter is then discussed. This is followed by a detailed analysis of a selected interferogram obtained in flutter. Finally some averaged measured flutter deflections are compared with the corresponding finite element predictions.

7.2 Estimate of Errors

In addition to the errors present in conventional holographic interferometry, such as those incurred in determining the precise position of a fringe and its order, there were two further sources of error worthy of consideration for this case of a rotating fan under aerodynamic load. These sources of error were considered and are discussed here. First the effects of misalignment of the holographic system relative to the rotational axis of the fan were quantified, and second the path-length changes due to unsteady aerodynamics were considered. Steady aerodynamic effects, i.e. those which did not change in the fan's rotating coordinates, would not have produced any object beam path-length differences and hence would not have produced any biasing of the interference fringes.

7.2.1 Effects of Misalignment

An expression relating unwanted fringe bias due to any misalignment of the system is given in Equation 5.18. This expression was used to determine the likely error due to the estimated misalignment. The misalignment of the centre of the rotator's apertures with respect to the fan's projected axis was due to,

(a) error in aligning the rotator with respect to the static fan,
(b) change in the steady angular position of the fan between static and full speed conditions, and
(c) vibrational tilt of the fan's axis.
The standard error from (a) above was estimated at ± 2mm as described in Section 6.4. The peak errors from (b) and (c) above were ± 1.7mm and ± 1.2mm, respectively, as described in Section 6.3.3. However, the estimated standard errors (reference: Topping, 1962) were somewhat less at ± 1.2mm and ± 1mm, respectively. Thus, the combined standard error on the position of the projected fan's axis was estimated as ± 2.5mm. Substitution into Equation 5.18 for holograms recorded at a fan speed of 10 000rpm, a pulse separation of 2µs and with a misalignment of 2.5mm, predicts a standard error in the fringe order of ± 1 full fringe at the blade tips. It can be seen from Equation 5.18 that this error increased linearly with radial position, thus the blade tips were a worst-case.

7.2.2 Effects of Unsteady Aerodynamics

Some biasing of the interference fringe distribution was produced by unsteady aerodynamics and in particular by a moving intra-passage shock. The magnitude of that biasing was estimated, as is now described. The change in the fringe order resulting from a movement of the shock is given by the relationship,

$$\Delta P = \frac{\Delta w}{\lambda} (\rho_2 - \rho_1)$$

where $\Delta P$ is the change in the fringe order, due to shock movement,

- $\rho$ is the Gladstone-Dale constant, which equals $2.24 \times 10^{-4} \text{ m}^3 \text{kg}^{-1}$ for air,

- $\Delta w$ is the total optical path length over which the shock moves between exposures.
FIGURE 7.1 DIAGRAM OF MODEL USED TO CALCULATE ERRORS DUE TO UNSTEADY AERODYNAMICS.
λ is the wavelength of light

ρ₁ and ρ₂ are the pre- and post-shock air densities, respectively.

A normal shock moving with an amplitude u, at the flutter frequency ω, as shown in Figure 7.1, was considered. For pulse separations, Δt, which were short compared with the flutter cycle time, the maximum shock movement occurring between exposures was thus ωuΔt. Therefore, the maximum value of Δω is given by,

\[ Δω = 2ωuΔt \sec(γ) \]

Eq 7.2

where γ is the blade stagger angle and the factor 2 accounts for the double pass-optical system.

Thus, from Equations 7.1 and 7.2

\[ ΔP = \frac{2C ωuΔt \sec(γ)(ρ₂ - ρ₁)}{λ} \]

Eq 7.3

An estimate of the change in the fringe order due to unsteady aerodynamics was made by substituting appropriate values into Equation 7.3. A change in fringe order of approximately 0.01 of a full fringe was predicted for the following estimated typical conditions: \( ω = 4310 \text{ rad s}^{-1} \), \( u = 1\text{ mm} \), \( Δt = 2\mu s \), \( γ = 60^° \), \( (ρ₂ - ρ₁) = 1\text{Kg m}^{-3} \) and \( λ = 0.694\mu m \). Thus, it was concluded that errors due to unsteady aerodynamics were typically insignificant.

7.3 Interpretation of the Interferograms Obtained Out of Flutter

As described in Chapter 6, a few holograms were recorded at fan speeds just below the onset of unstalled flutter. An example of such a holographic interferogram is shown in Figure 6.10. This interferogram was recorded using a pulse separation of 9μs. This was longer than the
separation used for study of the fan in flutter, thus giving a higher sensitivity. Figure 6.10 shows that out of unstalled flutter there was vibrational activity, particularly along the leading edge and towards the tip of the blades.

Analysis of the interferogram shown in Figure 6.10 revealed that the peak fringe order was approximately 8. Thus, from Equation 3.4, this corresponds to a peak vibrational velocity of approximately 0.3ms⁻¹.

The nature of the fringe distribution shown in Figure 6.10, suggests that the blades were vibrating in a random uncoupled fashion. It also suggests that random flapping of blades and more localised leading edge activity were common features. The frequency of the leading edge activity was estimated by comparison with interferograms of the non-rotating fan obtained at known vibrational frequencies. Figure 7.2 shows double pulsed holographic interferograms of the non-rotating fan recorded at resonant frequencies of 1.332kHz and 3.175kHz, using the experimental technique described in Chapter 3. The spatial length between leading edge nodes is of the same order for the interferograms shown in Figures 6.10 and 7.2. This, therefore, implies that the vibrational frequencies were of the same order. Thus, the leading edge vibrational activity, shown in Figure 6.10, was at frequencies of a few kHz. The random blade flap probably corresponded to somewhat lower frequencies.

Supporting evidence for the above conclusions was obtained from auto spectra of the strain gauge signals. The auto spectrum of a strain gauge signal, recorded just below the onset of unstalled flutter is shown in Figure 7.3. The strain gauge was positioned on the blade centre-line, just outboard of the shroud, as indicated in Figure 7.3. The auto spectrum shows that there was relatively high alternating strain at frequencies up to approximately 6kHz where a cut-off occurred.
FIGURE 7.2 DOUBLE PULSED HOLOGRAPHIC INTERFEROMETER GRAMS OF THE NON-ROTATING FAN, RECORDED AT RESONANT FREQUENCIES OF 1.332KHz AND 3.175KHz.
Figure 7.3 An autospectrum of a strain gauge signal is shown in [A].
This was obtained with the fan operating just below the onset of unstalled flutter. The position of the strain gauge is shown in [B].
There were some peaks in the autospectrum, particularly below 2kHz, but no one of these was dominant. Most of these peaks corresponded to harmonics of the fan's rotational frequency (~160Hz). These observations are compatible with the hologram interpretation described above.

Thus in summary, a low level vibration having a peak velocity of approximately 0.3ms⁻¹, was observed at operating speeds below the onset of unstalled flutter. This low level vibration was in the form of a random uncoupled blade response, which was particularly 'lively' along the blade leading edges. The main content of the vibration was at frequencies up to approximately 6kHz.

7.4 Quantitative Analysis of the Interferograms

The interferograms were analysed to obtain the axial component of the flutter vibration shape, with particular emphasis on determining the spatial distribution and relative magnitudes of axial displacement and torsion of the blades. The distribution of blade torsion and displacement is important for modelling and prediction of the unstalled flutter phenomenon, as described in Section 1.3.

The first stage of the analysis was to determine the fringe order as a function of position. This was obtained by counting fringes from the blade root and recognising any inversion of the fringe count direction. The absolute deflections which occurred between the two laser pulses were then calculated using the expressions given in Section 3.5, which were used for analysis of the fan under non-rotating conditions. Equations 3.4 and 3.10 were employed for determination of axial deflection, \( h \), and blade torsion, \( \alpha \), respectively. For this case, the angle between the illuminating and viewing directions was equal to zero. Thus Equations 3.4 and 3.10 reduced to,
\[ h = \frac{PA}{2} \sec \left( \tan^{-1} \frac{r}{d} \right) \]  

Eq 7.4

where, \( P \) is the fringe order,

\( \lambda \) is the wavelength of light,

\( r \) is the radial position of the measured point,

\( d \) is the distance between the fan and the holographic system,

and

\[ \alpha = \frac{(P_L - P_C)}{2c \sin(\gamma)} \]  

Eq 7.5

where, \( P_L \) and \( P_C \) are the fringe orders at the leading edge and centre line, respectively.

\( c \) is the half chord length,

and \( \gamma \) is the stagger angle of the blade.

Absolute values of deflection were of little use, in themselves, because they were a function of the laser pulse separation, \( \Delta t \), which was a measurement variable. Thus, Equations 7.4 and 7.5 were used to obtain values for \( \frac{h}{\Delta t} \) and \( \frac{\alpha}{\Delta t} \), which, for the short pulse separations employed, were good approximations to the instantaneous axial velocity and blade torsional velocity, respectively.

The analysis of the interferograms to obtain the radial distribution of axial velocity was performed with the aid of a computer linked television system. A photographic negative of each of the interferograms was viewed using a video camera and the resultant image was displayed on a cathode ray tube video screen. The blade tip, blade root and fringe positions in the image field were defined by the operator. These data were used by the computer system to define fringe positions in the fan's coordinates and to calculate axial deflections using Equation 7.4.

Distributions of blade torsion were obtained using a less automated
approach. The distributions of $P_L - P_C$ were measured by hand from large photographic prints of the interferograms. These were then converted to distributions of torsional velocity, $\frac{\alpha}{\Delta t}$, using Equation 7.5, with the aid of a desk top computer.

7.5 Detailed Analysis of a Selected Interferogram

7.5.1 Analysis and Results

A detailed analysis of the interferogram shown in Figure 6.9(c) was performed. This particular hologram was recorded at a fan speed of 9895 rpm using a pulse separation of 2 μs. As described in Section 6.6, this hologram was recorded with an additional third laser pulse, thus producing a low contrast fringe set which helped confirm the fringe order.

The interferogram was analysed to give the distribution of centre line axial displacement and torsion, $\alpha$, along two selected radial lines. Also, the circumferential distributions of axial displacement and torsion were obtained at three selected radial stations.

A photographic negative of the interferogram recorded from the visual display of the computer-television analysis system is shown in Figure 7.4. Superimposed on the interferogram is the fringe order distribution along the centre-line of a blade (number 27) having minimal torsion at the tip. This fringe order distribution was used to compute the centre line axial velocity as a function of radius, shown in Figure 7.5. A plot of blade torsional velocity as a function of radial position along blade 29 is shown in Figure 7.6. This blade was selected because it had the identifying feature of minimal centre line displacement at the tip. Plots of the circumferential distribution of torsional and centre-line axial velocity are shown in Figure 7.7. These circumfer-
Figure 7.4 Interferogram of the fluttering fan, recorded from the visual display of the analysis system.

Figure 7.5 Measured axial velocity along the centre-line of blade 27, obtained from the interferogram shown in Figure 7.4.
FIGURE 7.6 MEASURED TORSIONAL VELOCITY AS A FUNCTION OF RADIAL POSITION FOR BLADE 29, OBTAINED FROM THE INTERFEROGRAM SHOWN IN FIGURE 7.4.
FIGURE 7.7 PLOTS SHOWING THE CIRCUMFERENTIAL DISTRIBUTION OF TORSIONAL (○) AND AXIAL CENTRE-LINE (I) VELOCITY, OBTAINED FROM THE INTERFEROGRAM SHOWN IN FIGURE 7.4.
ential distributions were obtained at radii corresponding to the tip, shroud and a radius halfway between the two. The error bars in Figure 7.5 and 7.7 represent the standard error in the measured values due to misalignment effects.

7.5.2 Discussion

A single travelling coupled blade-disc mode having a sinusoidal circumferential distribution of deflection is assumed by current unstalled flutter modelling. The deflection distributions for the forced 3D2F mode of the non-rotating test fan were verified as being sinusoidal by the author, as described in Chapter 3. However, large deviations from sinusoidal deflection distributions were observed on the rotating fluttering fan, as shown in Figure 7.7. The circumferential distributions of axial displacement and torsion at the shroud were sinusoidal to a good approximation, but this was not the case outboard of the shroud.

Similarly, there was considerable variance between the measured radial distribution of torsional velocity shown in Figure 7.6 and that predicted for the pure 3D2F mode. The normalised measured radial distribution of torsion, obtained from Figure 7.6, and the corresponding prediction, obtained using the finite element model described in Section 1.6, are shown in Figure 7.8. The measured down turn in torsion towards the tip was not predicted. The normalised measured radial distribution of axial velocity shown in Figure 7.5 is compared with the predicted distribution in Figure 7.9. In this case, there was good agreement between measured and predicted deflections.

The extent of the variance between measurement and prediction, obtained with this selected interferogram, was typical of the other
**Figure 7.8** Normalised torsion as a function of radial position. Measured values and those predicted by finite element calculation are compared.

**Figure 7.9** Normalised axial deflection as a function of radial position. Measured values and those predicted by finite element calculation are compared.
interferograms recorded in flutter. However, the sign and exact form of the variance varied randomly from interferogram to interferogram. On average, the variance was most marked outboard of the shroud.

In order to understand the above observations, the strain gauge signals recorded in unstalled flutter were studied. An autospectrum of such a signal is shown in Figure 7.10. It was obtained from the same gauge as used for Figure 7.2. Comparison of Figures 7.2 and 7.10 showed that a strong peak appeared in the autospectrum at 686Hz in unstalled flutter. This corresponded to the travelling 3D2F mode. The level on the autospectrum of the broadband strain associated with the low-level vibration seen out of flutter ('operating vibration') did not vary on the onset of unstalled flutter. Therefore, this suggested that the random 'operating vibration' was present in unstalled flutter to the same extent as found out of unstalled flutter, i.e. with a peak velocity of approximately 0.3ms\(^{-1}\).

Thus, it was concluded that the variance between the holographically measured deflections in unstalled flutter and those predicted was due primarily, to the superposition of deflections due to the random 'operating vibration'. At the shroud, the individual blade response of the 'operating vibration' was preferentially inhibited by the restraining effect of the shroud ring, and thus the deflections at the shroud were predominantly sinusoidal and due to unstalled flutter.

Thus, in summary, it was concluded that the shot to shot variations in the normalised deflection shape obtained in unstalled flutter were primarily due to the presence of random vibrations. These random vibrations were at a different frequency to, and unconnected with, the unstalled flutter phenomenon.
FIGURE 7.10 AUTOSPECTRUM OF A STRAIN GAUGE SIGNAL OBTAINED WITH THE FAN OPERATING IN UNSTALLED FLUTTER. THE STRAIN GAUGE AND THE NORMALISATION COEFFICIENT WERE IDENTICAL TO THOSE FOR FIGURE 7.3.
7.6 Technique for Averaging the Flutter Deflections

As described in the last section, deflections due to random vibrations were superposed on the measured deflections due to unstalled flutter. A reduction in these random perturbations was sought in order to allow comparison with the deflection shape predicted by finite element calculation. This was achieved by averaging several interferograms. The deflections from individual interferograms were normalised and then averaged so as to constructively add the deflections due to unstalled flutter. The relative contribution due to random vibrations was thus reduced.

Some means of normalising the deflections was required because, in general, the interferograms were recorded at different flutter amplitude levels and at different phases within the flutter cycle. The strain gauge signals provided a measure of the flutter amplitude and phase. However, the timing of the laser pulses relative to the strain gauge signals was not known, for the reasons described in Section 6.5. Thus, it was not possible to use the strain gauge signals for normalisation. The circumferential distribution of axial displacement at the shroud was sinusoidal to a good approximation, as seen in Figure 7.7. This consistent feature was used to normalise the measured deflections before averaging. In order to improve accuracy for the holographic analysis of the non-rotating fan, the 3D Fourier component of displacement was used for normalisation, as is described in Section 3.5. However, for these interferograms of the rotating fluttering fan, insufficient measurement points were available to allow a discrete Fourier transformation. Thus, in lieu of this option, a best sine wave was fitted to the circumferential distribution of axial deflection at the shroud.
for each of the interferograms. The fitting function was of the following form,

\[ h = 0 \sin \left( \frac{2\pi D}{33} (B + S) \right) \]

Eq 7.6

where \( h \) is the axial displacement
\( D \) is the diametral number of the mode (3 in this case),
\( B \) is the blade number
\( 0 \) is the amplitude of the best-fit sine wave
and \( S \) is the spatial phase of the best fit sine wave, measured in blade numbers.

An iterative least squares fitting computer program was employed to obtain values of \( 0 \) and \( S \) which provided a minimum standard deviation of the measurement points from the fitting function. Thus, a minimum value to the following expression was obtained.

\[
\sum_{i=1}^{n} \left( h_i - 0 \sin \left( \frac{2\pi D}{33} (B_i + S) \right) \right)^2
\]

where \( h_i \) and \( B_i \) are the individual measurements of axial displacement and blade number and \( n \) is the number of measurements.

The values of \( 0 \) and \( S \) obtained for a particular interferogram were used to normalise all deflections obtained from that interferogram. The normalisation was achieved as follows.

(a) The magnitude of the normalised deflection was obtained by dividing the actual deflection by \( 0 \), and

(b) the normalised circumferential position was obtained by adding \( S \) to the actual circumferential position.

Following this normalisation, the deflection data was in a form such that data from all the interferograms could be constructively combined.
7.7 Averaged Flutter Deflections and Comparison with Finite Element Prediction

Deflections obtained from the six interferograms shown in Figure 6.9 were normalised and constructively combined. Averaged circumferential distributions of torsion, \( \alpha \), and axial centre-line displacement, \( h \), were obtained at radii of 307mm (the shroud), 365mm and 424mm (the tip).

The measured distributions of axial displacement at the shroud and the corresponding sine waves having the best least-squares-fit are shown in Figure 7.11 for the six interferograms. The values of 0 and \( S \) obtained for the best-fit sine waves were used to normalise measured \( \alpha \) and \( h \) at radii of 307mm, 365mm and 424mm from all six interferograms.

These normalised data were combined to give the plots of normalised \( \alpha \) and \( h \) at the three radii, as shown in Figure 7.12. A best sine wave was then fitted to these new combined sets of data. The function defined by Equation 7.6 was fitted to the data points using the same least squares fitting routine which was employed for normalisation. The resultant best-fit sine waves are also shown in Figure 7.12.

The values of 0 and \( S \) which defined these sine waves were used to obtain the relative amplitudes and spatial phases of torsion and axial displacement at the three selected radii. The resultant values are given in Table 7.1. The standard deviations of the normalised combined data points from the best-fit sine waves were used to assess the likely measurement errors. The calculated standard error associated with each measurement is shown in Table 7.1.
FIGURE 7.11 MEASURED FRINGE DISTRIBUTIONS (b) AT THE SHROUD WHICH WERE OBTAINED FROM THE INTERFEROGRAMS SHOWN IN FIGURE 6.9, [a] TO [f] RESPECTIVELY. THE BEST-FIT SINE WAVE IS ALSO SHOWN (+++) FOR EACH DISTRIBUTION.
FIGURE 7.12 COMBINED NORMALISED DEFLECTIONS OBTAINED FROM THE SIX INTERFEROMETERS SHOWN IN FIGURE 6.9. A BEST-FIT SINE WAVE IS ALSO SHOWN FOR EACH DEFLECTION DISTRIBUTION.
<table>
<thead>
<tr>
<th>Radial Position</th>
<th>Deflection Type</th>
<th>Holographically Determined Value</th>
<th>FE Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>307mm (Shroud)</td>
<td>Normalised Axial Displacement</td>
<td>1.00 ( /0^\circ ) (a)</td>
<td>1.00 ( /0^\circ ) (a)</td>
</tr>
<tr>
<td></td>
<td>Normalised Torsion</td>
<td>4.2(\pm 0.1)/-80(\pm 2)(^\circ)</td>
<td>4.7 ( /-92^\circ )</td>
</tr>
<tr>
<td></td>
<td>Torsion/Axial Displacement</td>
<td>4.2(\pm 0.1)/-80(\pm 2)(^\circ)</td>
<td>4.7 ( /-92^\circ )</td>
</tr>
<tr>
<td>365mm</td>
<td>Normalised Axial Displacement</td>
<td>0.78(\pm 0.03)/9(\pm 2)(^\circ)</td>
<td>0.77 ( /4^\circ )</td>
</tr>
<tr>
<td></td>
<td>Normalised Torsion</td>
<td>9.4(\pm 0.5)/-109(\pm 3)(^\circ)</td>
<td>9.9 ( /-95^\circ )</td>
</tr>
<tr>
<td></td>
<td>Torsion/Axial Displacement</td>
<td>12.1(\pm 0.7)/-118(\pm 4)(^\circ)</td>
<td>12.9 ( /-99^\circ )</td>
</tr>
<tr>
<td>424mm (Tip)</td>
<td>Normalised Axial Displacement</td>
<td>0.84(\pm 0.07)/172(\pm 4)(^\circ)</td>
<td>0.94 ( /175^\circ )</td>
</tr>
<tr>
<td></td>
<td>Normalised Torsion (Along Hade)</td>
<td>12.6(\pm 1.9)/-119(\pm 8)(^\circ)</td>
<td>16.8 ( /-116^\circ )</td>
</tr>
<tr>
<td></td>
<td>Torsion/Axial Displacement (Along Hade)</td>
<td>15.0(\pm 2.5)/70(\pm 9)(^\circ)</td>
<td>17.9 ( /+69^\circ )</td>
</tr>
</tbody>
</table>

footnote: (a) Obtained by definition of normalisation

Table 7.1  Circumferential Distributions of Averaged Normalised Axial Centre-line Displacement and Torsion for the Fluttering Fan. The Corresponding Finite Element Predictions are also Presented
The finite element prediction of normalised deflections, corresponding to those measured, is also given in Table 7.1. The FE prediction was for the 3D2F mode, obtained using the model described in Section 1.6. The effects of centrifugal load, aerodynamic load and temperature distribution were included in this FE calculation. The agreement between the measured and predicted deflections is fair. In particular, the important ratio of torsion to axial displacement shows reasonable agreement at the three radial stations, with variance in amplitude and spatial phase of up to 19% and 19°, respectively. The predicted natural frequency of the 3D2F mode at 10 000rpm was 695Hz which compares well with the measured flutter frequency of 686Hz.

The results given in this section are discussed further in the next chapter.
CHAPTER 8

DISCUSSION, CONCLUSIONS AND
FUTURE DEVELOPMENTS

Summary of Chapter

The results of this study are discussed and the main conclusions are drawn. Future developments of the holographic technique are also discussed.

The experimental results were in agreement with the current understanding of supersonic unstalled fan flutter. Many of the assumptions employed in flutter prediction by calculation of unsteady work were experimentally verified. The particular finite element model which was used to predict the exact form of the flutter mode, was shown to give a shape which was in reasonable agreement with the measured shape. However, this particular finite element model had some inadequacies, as was shown by measurements of the fan's vibrational characteristics under non-rotating conditions. The experimental data on the test fan, provided in this thesis, should aid the development of improvements to the finite element model.

8.1 Introduction

In this final chapter of the thesis, the results of the study are discussed, the main conclusions are drawn and future developments are outlined. The results are discussed particularly with respect to the initial aims of the study which are given in Section 1.5. The conclusions are drawn under two headings: first, those concerning the flutter study of the test fan, and second, those concerning the holographic technique.

8.2 Discussion of Results

The main initial aim of this study was to perform a detailed measurement of the vibrational shape of a fan undergoing supersonic unstalled flutter and use this data to evaluate the current understanding
and modelling of the unstalled flutter phenomenon. With this in mind, various aspects of the experimental results are now discussed.

One unexpected feature of the study was the high level of random vibrations which were observed during normal operating conditions and unstalled flutter. These random vibrations were unconnected with, and at a different frequency to the unstalled flutter phenomenon. Thus, it was concluded, from the analysis of Section 1.3, that the onset of unstalled flutter was unaffected by the presence of this 'operating vibration'.

One of the specific aims of the work was to study the flutter vibration shape in detail as a function of the temporal flutter cycle. This was prevented due to operational problems which were encountered, as is described in Section 6.5. However, the flutter vibrational response was observed to be that of a coupled blade-disc mode. It was seen that the orientation of the vibration shape varied from interferogram to interferogram. These observations were in agreement with the current understanding of unstalled flutter which predicts a rotating coupled blade-disc mode.

A further specific aim of the study was to measure any deviation of the actual flutter vibration shape from that of an axisymmetrical and circumferentially sinusoidal distribution. The random superposed deflections of the 'operating vibration' masked, at most spatial positions, any deviations of the unstalled flutter response from a sinusoidal distribution. However at the shroud, the circumferential distribution of axial deflection was sinusoidal to a good approximation. This was the location which appeared least affected by the random vibrations due
to 'operating vibration'. Thus, at this radial position, at least, the deflection shape was sinusoidal, as is assumed by current flutter modelling.

The finite element model which was used to predict the exact form of the flutter mode (3D2F) gave a mode shape which was in reasonable agreement with the measured shape. Agreement was particularly good with respect to the ratio between torsion and axial displacement at the blade tips (Table 7.1). This concurred with over-tip probe measurements which had shown good agreement with the FE prediction on similar previous fan tests.

Study of the fan under non-rotating conditions revealed that the FE model had some inadequacies. In particular, there was poor agreement between the measured and predicted natural frequencies for the second family coupled blade-disc modes. The experimental data provided by this study should aid development of improvements to the FE model.

Although inadequacy was detected in the specific finite element model used for the test fan, the basic assumptions employed in the flutter vibration shape prediction were experimentally verified to a large extent. Thus, an improved finite element model, e.g. employing improved boundary conditions at the blade-blade and blade-disc interfaces and more or better elements, should be capable of accurately and reliably predicting the unstalled flutter deflection shape. This together with accurate understanding and modelling of the unsteady aerodynamics should provide reliable prediction of unstalled flutter allowing improved aero engine fan design.

8.3 Conclusions Concerning the Flutter Study
(a) Detailed full field measurements of the vibrational response of an aero engine fan undergoing supersonic unstalled flutter were performed. This is the first report of such detailed measurements. The measurements were performed at high rotational speed (~10 000rpm) using pulsed holographic interferometry. The resultant holographic interferograms were quantitatively analysed to obtain useful deflection data.
(b) The study was confined to the detailed study of one test fan. However, this rotor was typical of shrouded fans employed in modern, high efficiency, high bypass ratio aero engines, and thus many of the conclusions drawn from this study are likely to have wide applicability.

(c) An unexpectedly high level of random uncoupled blade vibration (~0.3ms⁻¹) was observed at normal fan operating conditions. This random 'operating vibration', was also present during unstalled flutter of the test fan. It had a response which was particularly lively along the leading edge of the blades and outboard of the shroud. It was deduced that this random vibration was broad band in frequency content with response at frequencies up to approximately 6kHz. It was concluded that it did not affect the onset of unstalled flutter.

(d) The response of the test fan in unstalled flutter was that of a 3D2F coupled blade-disc mode. The mode shape was rotating with respect to the fan. This is in agreement with the current understanding of supersonic unstalled fan flutter.

(e) Any asymmetry in the unstalled flutter shape was masked by perturbations due to superposed 'operating vibration'. However, at the shroud, where the effects of the 'operating vibration' were minimal, the circumferential distribution of axial deflection was sinusoidal to a good approximation. This is an assumption employed in the finite element model of the test fan which was thus verified experimentally. This assumption was further verified by holographic mode shape measurement of the fan under non-rotating conditions.
Study of the non-rotating test fan using mechanical impedance measurements revealed a measurable but very small degree of detuning. Detuning resulted in a small frequency split between twin orthogonal modes.

The holographically measured vibrational mode shapes of the fluttering fan and of the fan under non-rotating conditions were compared with those predicted by finite element calculation. The prediction was in reasonable agreement with the measured mode shapes for the 3D2F mode. However, doubt was cast on the over-all reliability and accuracy of the FE model by comparison of the measured and predicted natural frequencies for the 2D2F to 5D2F modes for the non-rotating fan. (Table 3.1). Poor agreement was obtained, particularly for the 2D2F mode. The development of improvements to the FE model should be aided by the experimental data provided by this study.

8.4 Conclusions Concerning the Holographic Technique

(a) The application of holographic interferometry for the vibrational study of aero engine fan flutter at high rotational speeds was demonstrated.

(b) The measurement system had a relatively high cost and complexity. However, detailed quantitative measurement of vibrational deflection was successfully performed at all radii between the blade roots and tips, and this was data which was not obtainable using simpler and more conventional techniques.

(c) A mirror-Abbe image rotator was successfully employed in the holographic system. This provided minimal aberrations and prevented any unwanted Fresnel back-reflections.

(d) The use of a rotating illuminating beam was demonstrated. This allowed concentration of the available laser light into an off-axis portion of the rotating fan and, when used in this mode, provided operational advantages.
(e) Errors due to misalignment and aerodynamic effects were quantified for this test fan. It was concluded that likely errors due to aerodynamic effects were negligible.

(f) 'Open-lase' pulsed interferometry was a useful adjunct to conventional double pulsed holography for mode shape measurement of the non-rotating fan.

6.5 Future Developments

The following improvements are currently being made to the holographic system.

(a) The optical system is being modified to incorporate a polarisation-selective beam splitter. This will increase the optical efficiency of the system by a factor of nearly four, which will allow larger areas of the object to be illuminated.

(b) The thermoplastic camera is being replaced with a remotely controlled 35mm film transport for use with conventional silver halide film. This modification will reduce the cost and complexity of the measurement system.

(c) A computer-controlled data logging system is being incorporated to log all salient system parameters. This will make the system easier to use and one-man operation will be possible.

The above improvements should increase the efficiency and reliability of the measurement system for future applications. These improvements were not implemented for the flutter study described in this thesis, as the system was considered adequate for this first major application.
Future applications of the system may include study of alternative fan flutter mechanisms, such as that of supersonic and subsonic stalled flutter. Advanced aero engine fan designs, such as the new generation of wide-chord fans, may exhibit vibrational or flutter response, and, if so, this may be a further area of application for the system. Similarly, the study of aircraft propeller vibration is a potential future application. Aircraft propulsion using advanced turbine driven propellers is currently of great interest because of its potential for high fuel efficiency. The holographic system described in this thesis would be ideally suited to propeller studies, because the unique rotating illumination would allow matching of the available laser light to the propeller geometry.
REFERENCES


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Vest C. M. (1979), "Holographic interferometry", John Wiley and Sons Inc.


APPENDIX 1

EXPERIMENTAL DETERMINATION OF HOLOGRAPHIC FRINGE VISIBILITY AS A FUNCTION OF OBJECT ROTATION

An experimental determination of fringe visibility as a function of object rotation and aperture of the reconstruction system is described in this Appendix. This study was undertaken in order to substantiate the theoretical analysis described in Section 2.5.

Double exposure holographic interferograms were recorded of a disc which was given a small in-plane rotation between exposures. The holographic sensitivity vector was arranged to be directed towards a point which was off-set a small amount from the disc's axis of rotation. Thus, interference fringes due to the disc rotation were produced. The visibility, \( V \), of these interference fringes was used to check Equation 2.8.

The experimental configuration which was used is shown diagrammatically in Figure A.1. Double exposure holograms were recorded of an aluminium disc, of diameter 153.5mm, having a matt-white spray-painted surface finish. The disc was mounted on a precision rotational stage and it was rotated a known amount between the two holographic exposures. The angle of rotation was measured using a dial gauge at a known radial station. An argon-ion laser, operating at a wavelength of 0.5145\( \mu \)m, was employed to form the holograms. The point illumination and centre of the holographic plate were positioned on the disc's projected axis of rotation. This was achieved using a 50\% beam splitter. The holograms were recorded on
FIGURE A.1 DIAGRAMS OF THE EXPERIMENTAL CONFIGURATION USED FOR THE DETERMINATION OF FRINGE VISIBILITY FOR A ROTATED OBJECT.
Agfa Gevaert 8E56 photographic plates using an off-axis parallel reference beam.

The processed holograms were illuminated, in turn, using a reconstruction beam which was identical to the original reference beam. The resultant virtual holographic images of the disc were photographed using a conventional camera employing a plano convex imaging lens placed close to the hologram and having a focal length of 160mm. The reconstruction camera was positioned so as to be off-set from the projected rotational axis of the disc. This resulted in the formation of a set of approximately parallel and equispaced interference fringes in the holographic image. Stopping of the reconstruction system was performed using one of several circular apertures placed between the hologram and the lens. Photographic reconstructions were recorded from each hologram at several selected f-numbers.

Two holograms, 01 and 02, were recorded, having angular rotations of $2.11 \times 10^{-3}$ radians and $1.06 \times 10^{-3}$ radians, respectively. Examples of the resultant reconstructed interferograms obtained from these two holograms at several apertures are shown in Figures A.2 and A.3. These interferograms show a circular central region on the disc having high visibility fringes. Outer annuli having fringes of reduced visibility can also be seen on many of the interferograms. It can be seen that there is a phase reversal in the interference fringes in the first annulus. The diameter of the central high visibility region increased for an increase in the f-number of the reconstruction system and for a decrease in the angle of rotation between exposures. These qualitative observations were in complete agreement with the theoretical analysis given in Section 2.5 and summarised in Equation 2.8.

The radial positions of the annuli of zero fringe visibility were then used to make a quantitative comparison with the theoretical prediction.
FIGURE A.2 PHOTOGRAPHIC RECONSTRUCTIONS OF HOLOGRAM Ø1 OBTAINED AT FOUR DIFFERENT f-NUMBERS.
FIGURE A.3 PHOTOGRAPHIC RECONSTRUCTIONS FROM HOLOGRAM B2 OBTAINED AT FOUR DIFFERENT f-NUMBERS.
The zeros of \( \mathcal{H} \) predicted from Equation 2.8 occur at \( J_1(\pi r \Delta \theta / \lambda d_0) = 0 \). Thus, the predicted first zero occurs at a radius, \( r_1 \), given by

\[
r_1 \Delta \theta = 1.22 \frac{\lambda d_0}{\lambda}
\]

Eq A.1

Similarly, the predicted second zero occurs at a radius, \( r_2 \), given by

\[
r_2 \Delta \theta = 2.23 \frac{\lambda d_0}{\lambda}
\]

Eq A.2

In total, 11 photographic interferograms were recorded from the two holograms at various reconstruction apertures. Measurements were made to allow comparison of experimentally determined values of \( r_1 \Delta \theta \) and \( r_2 \Delta \theta \) with those predicted from Equation 2.8. These measurements are summarised in Table A.1. The measured values of \( r_1 \Delta \theta \) and \( r_2 \Delta \theta \) are shown plotted against \( \frac{\lambda d_0}{\lambda} \) in Figure A.4. The solid lines represent the theoretical relationship between \( r \Delta \theta \) and \( \frac{\lambda d_0}{\lambda} \). Thus, it can be seen from Figure A.4, that there was reasonable agreement between the measured positions of zero fringe visibility and those predicted by Equation 2.8. The mean experimental values of \( r_1 \Delta \theta / (\frac{\lambda d_0}{\lambda}) \) and \( r_2 \Delta \theta / (\frac{\lambda d_0}{\lambda}) \) were 1.13 \pm 0.03 and 2.16 \pm 0.06, respectively. These compared with the theoretically predicted values of 1.22 and 2.23, respectively. Thus, there was a small variance between experimental and theoretical values, particularly in the value of \( r_1 \Delta \theta / (\frac{\lambda d_0}{\lambda}) \). This was possibly due to lens aberrations resulting in a small deviation from the diffraction limited impulse response of the lens.

In summary, the experimental work described in this appendix verified Equation 2.8 to the accuracy required for this study of fan flutter.
### Table A.1 Summary of Measurements Obtained for the 11 Photographic Reconstructions.

<table>
<thead>
<tr>
<th>Hologram</th>
<th>Angle of rotation, $\Delta \theta$ (radians)</th>
<th>Diameter of circular aperture (mm)</th>
<th>Approximate Reconstruction system f-number</th>
<th>$\lambda \Delta \Phi / \lambda \Delta \alpha$ (um)</th>
<th>$r_1$ (mm)</th>
<th>$r_2$ (mm)</th>
<th>$r_1 \Delta \theta$ (um)</th>
<th>$r_2 \Delta \theta / \lambda \Delta \alpha$</th>
<th>$r_1 \Delta \alpha / \lambda \Delta \alpha$</th>
<th>$r_2 \Delta \alpha / \lambda \Delta \alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>$2.12 \times 10^{-3}$</td>
<td>15.93 25.03 13.2 27.9 1.12 (b)</td>
<td>11.33 35.19 17.9 35.0 1.08 2.10 (b)</td>
<td>8.19 47.47 25.6 52.4 110.8 1.14 2.34 (b)</td>
<td>8.4 48.69 25.2 50.2 106.2 1.10 2.18 (b)</td>
<td>5.59 71.33 38.7 81.9 1.15 (b)</td>
<td>5.2 76.68 43.4 91.9 1.20 (b)</td>
<td>3.96 117.62 58 122.7 1.22 (b)</td>
<td>15.93 10 25.03 13.2 27.9 1.12 2.10 (b)</td>
<td></td>
</tr>
<tr>
<td>02</td>
<td>$1.06 \times 10^{-3}$</td>
<td>15.93 25.03 26.4 49.8 1.12 2.10 (b)</td>
<td>11.33 35.19 35.1 68.7 1.05 2.06 (b)</td>
<td>8.19 48.69 52.2 55.2 1.13 (b)</td>
<td>5.59 71.33 (b) 52.2 (b) 55.2 (b) 1.13 (b)</td>
<td>5.59 71.33 (b) 52.2 (b) 55.2 (b) 1.13 (b)</td>
<td>5.59 71.33 (b) 52.2 (b) 55.2 (b) 1.13 (b)</td>
<td>15.93 10 25.03 13.2 27.9 1.12 2.10 (b)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Footnotes:
(a) Obtained from the aperture diameter, the aperture to object distance of 775 mm and $\lambda = 0.5145 \mu m$.

(b) Experimental data was not obtained.
FIGURE A.4  GRAPH SHOWING MEASURED VALUES OF $r_{\Delta B}$ PLOTTED AGAINST $\lambda d_{o}/1$. 
APPENDIX 2

AIAA PAPER 82-1271, "HOLOGRAPHIC VIBRATION MEASUREMENT OF A ROTATING FLUTTERING FAIR"

HOLOGRAPHIC VIBRATION MEASUREMENT OF A ROTATING FLUTTERING FAN

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Abstract

The use of holographic interferometry to determine the deformation of a rotating aero engine in an undergoing unstable supercritical flutter is described. A mirror-Abbe image rotator was employed in a double exposure holographic system to compensate for the fan's rotational motion and thus maintain correlation between the two resultant holographic images. The mirror-Abbe unit both rotated the illuminating beam and deviated the light returned from the rotating fan. Holographic interferograms were recorded of a 0.81 m diameter fan rotating at speeds just under 10000 rpm and undergoing unstable flutter. The amplitude and spatial distribution of blade torsional and axial deflections on the fluttering fan are obtained from the holograms. Errors due to misalignment of the system and unsteady aerodynamics are discussed.

Introduction

Aerelastic instability is often a constraint in the design of modern high by-pass ratio aero engines. Unstable supercritical flutter is an instability which can be encountered in clappered (shrouded) fans, in which mechanical vibrations (vibrational or noise) lead to unsteady aerodynamic forces which saple further energy into the mechanical vibrations. This phenomenon is particularly sensitive to the deformation shape of the mechanical vibrations. Successful computation of the flutter onset condition is similarly sensitive to this deformation shape. This paper describes the first reported use of holographic interferometry to determine the deformation shape of a rotating fan undergoing unstable flutter.

Conventional measuring techniques, such as strain gauges and accelerometers attached to the fan blades and on-tip proximity transducers used in the fan casing, provide information about the deformation shape of a rotating fan. An object at only a limited number of positions. Holographic interferometry allows the determination of an object's vibration deformation over its full visible surface. This feature has lead to the use of the technique for modal analysis of non-rotating bladed-disc assemblages.

The use of holographic interferometry to study the vibration of rotating structures often requires some form of rotation compensation. In the case of double exposure holography, this is a requirement where the rotational movement is greater than the mean speckle size in the holographic image, otherwise decorrelation between the two images results in loss of fringe visibility. Historically, several methods have been proposed and tested to prevent image decorrelation due to object rotation, and they fall into three basic categories. The first category consists of techniques in which two states of the object having the same angular position are compared interferometrically. Several reports have been made in which a rotating object has been compared holographically with its static state[4]. These techniques are limited to measurement of only small deformations, typically much less than 100μm, and are thus not suitable for determination of the much larger deformations found in unstable fan flutter. The second approach is that of rotating the holographic recording material at the same speed as the object. This is achieved by attaching the recording film or plate to the object[5,6] or alternatively could be achieved by rotating it using a separate shaft[7]. Both alternatives are fraught with practical difficulties for remote rig running.

This paper describes the design and special features of a holographic system employing a mirror-Abbe image rotator for measuring the axial component of vibration of a rotating fan. The system of the object to obtain holographic interferograms of a fluttering fan and the analysis of these interferograms is described. Some sample results which quantify the distribution of the axial deflection of the fluttering fan are then presented.

Image Rotator

Most image rotators fall into one of two categories[8], either transmissive or reflective. Holographic systems employing both types have been reported. Stetson's pioneering system[8,9] used a transmissive folded-Abbe rotator. This is the only previously reported holographic system which has been applied to the measurement of vibration of rotating aero engine fans. Interferograms, obtained with this system, of a 0.81 m diameter fan operating at speeds up to 7500 rpm were reported. More recently, a reflective Porro prism rotator has been used by Beek, Pagan and Kreitzlow[15,16] in a holographic system for measurements on a 0.43 m diameter automobile cooling fan at reported speeds up to 2850 rpm. This system has also been used on a 0.25 m diameter disc at speeds up to 13000 rpm.

1982 by Rolls-Royce Ltd.

Research Scientist
The author decided upon a transmissive image rotator because this simplified the holographic system and minimised the required number of passes of the object beam through a beam splitter, thus also minimising the attendant loss of light. A mirror-Abbe rotator configuration, as shown in Figure 1, was chosen. This is the simplest collinear transmissive rotator, having the whole of the optical path in air, thus making the rotator free from unwanted back-reflections and aberrations. These features together with the use of dielectric mirror coatings make rotation of the high intensity illuminating beam as well as derotation of the returned scattered light from the fan. The ability to rotate the illuminating beam increases the versatility of the holographic system as described in the following section. The mirror-Abbe configuration does have a high inertia due to its off-axis mirror, thus making very high speed operation difficult. However, a 27mm aperture unit capable of operation at object speeds up to 12000 rpm has been built, as described below. This speed capability is adequate for most aeroplane fan applications.

The mirror-Abbe optics are formed from two optical elements; a glass prism forms the two on-axis mirrors and a separate off-axis mirror is used. These are mounted in a steel housing which is held on the axis of a modified Varic-Havilor 600 d.c. motor. This has a light ironless dish-shaped armature which gives low inertia, near constant torque from rest to high speeds and a high power to length ratio. The motor was modified to produce a hollow shaft of 30mm internal diameter.

A section through the image rotator is shown in Figure 2. The rotor assembly is supported in externally pressurised air bearings; two journal bearings and a double-acting thrust bearing. Air bearings are used because of their good centering properties, smooth running and low vibration. An optical tachometer is incorporated which provides 60 pulses per revolution (ppr) and 1ppr outputs. This rigid rotor assembly was dynamically balanced at high speed using accelerometers in two planes and an influence coefficient method. A photograph of the unit is shown in Figure 3.

The image rotator is synchronised at half the object speed using a phase-locked loop control system, which enables the rotator to follow fluctuations in object speed. A simplified block diagram of the control system is shown in Figure 4. The phase comparator compares the phase of the 60ppr tachometer signal from the object with that of the 120ppr signal from the rotator, and gives an output which is a measure of the phase difference between these two inputs. This difference signal is filtered, amplified and then used to drive the motor. The motor control voltage changes the motor speed in a direction which reduces the phase difference between the two tachometer signals. When the loop is 'locked', the motor control voltage is such that for every object tachometer pulse there is one, and only one, tachometer pulse from the rotator, thus ensuring that the rotator is maintained at exactly half the speed of the object.

During the acquisition of 'lock' there are large voltage swings at the output of the phase comparator. Thus, saturation of the amplifier is prevented by using a low amplifier gain during 'lock' acquisition. Once 'lock' has been obtained,
The gain of the amplifier is automatically reduced so as to maximise the speed range over which 'lock' can be held (hold range).

The filter was designed to produce a control loop having second order characteristics and linear compensation. This allowed the independent action on loop gain, bandwidth and damping, thus producing a control system having a reasonable holding range, a bandwidth sufficient to follow speed fluctuations, good transient response and good noise rejection properties. The tracking performance of the control system, together with a summary of the mechanical and optical characteristics of the rotator is given in Table 1.

### Table 1 Summary of the mechanical, optical and control characteristics of the rotator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum object speed</td>
<td>12000 rpm</td>
</tr>
<tr>
<td>Optical f-number</td>
<td>7.2</td>
</tr>
<tr>
<td>Optical aperture diameter</td>
<td>27 mm</td>
</tr>
<tr>
<td>Hold range a</td>
<td>800 rpm</td>
</tr>
<tr>
<td>Maximum peak to peak variation in object speed at 12s before drop out b</td>
<td>300 rpm</td>
</tr>
<tr>
<td>Maximum peak to peak variation in object speed at 100s before drop out b</td>
<td>150 rpm</td>
</tr>
</tbody>
</table>

a. Hold range is the range of fan speed over which lock is maintained, having once set a centre speed.
b. These figures show the capability of the control system to maintain 'lock' when presented with a sinusoidally varying fan speed.

### Holographic System

A diagram of the optical system is shown in Figure 5. The image rotator both rotates the illuminating object beam and de-rotates the light scattered from the fan before it is relayed onto the hologram recording plane. This overcomes the decorrelation between the two holographic images which would otherwise be present. The use of rotating illumination relaxes the tolerance on the spatial quality and alignment of the illuminating beam and even allows the use of diffuse illumination. It allows the available laser light to be concentrated into an off-axis position of the fan, and when used in this mode, there is the operational convenience of being able to rotate the illuminated area on the fan by simply phase locking the image rotator at a different orientation with respect to the fan. Additionally, it ensures that the direction of polarisation of the object light at the hologram recording plane does not change between exposures.

**Figure 5 Diagram of the holographic system**

The viewing and illumination are from a point on the fan's projected rotational axis, thus ensuring that the holographic sensitivity vector is orthogonal to the fan rotation. This prevents unwanted optical path-length variation due to fan rotation, which would produce distorting bias fringes in the resultant holographic interferogram. The need for orthogonality of the sensitivity vector and the fan's rotation, limits the technique to the measurement of only the axial component of fan vibration. The fine adjustment of the optical system relative to the fan's rotational axis is performed using two alignment mirrors, positioned between the image rotator and the fan.

Apart from the above features, the optical configuration is that of a conventional double pulse holographic system. A Q-switched pulsed ruby laser giving up to 0.5 J in two 25 ns duration pulses is used. The pulse separation is variable from 2 µs to several hundred microseconds. The laser operates at a wavelength of 6943 Å and reliably produces a coherence length in excess of
Alignment Procedure

The fan's projected axis must pass through the centre of the rotor aperture, in order to prevent unwanted bias fringes on the resultant interferogram due to fan rotation. This fine alignment is performed using two output alignment mirrors. The relative angle between the fan and rotor axes is changed by adjustment of two orthogonal tilt controls, and the relative displacement between these axes is adjusted by means of vertical and horizontal displacement controls. The optical axis of the rotor is made closely collinear with the rotational axis of the fan, as determined at very low speed. This alignment is performed as follows, using the configuration shown schematically in Figure 6.

(i) A slightly converging helium-neon laser beam is aligned to the optical axis of the spinning rotator by adjustment of mirrors, A and B. This is achieved when the loci defined by the rotating beam in the near and far field of the output of the rotator are spots of minimum diameter.

(ii) A small adjustable mirror, C, attached to the centre of the fan is aligned so as to be normal to the fan's rotational axis. This is obtained by monitoring the locus of a reflected laser beam at a distant screen when the fan is slowly rotated by hand.

(iii) The output alignment mirrors are then adjusted such that the beam aligned to the axis of the rotator is incident on the centre of the fan and is collinear with the resultant reflected beam.

(iv) The mirrors A, B and C are then removed and the holographic system is reassembled.

Following this procedure, the projected fan axis is typically within ± 2 mm of the centre of the rotor apertures.

Laboratory Trials

The holographic system was used to record interferograms of a 0.56 m (22 inch) diameter clapper fan. The fan was rotated in a vacuum chamber and viewed through a large glass window. Individual modes of vibration of the fan could be excited using piezo-electric crystal exciters which were attached to the fan blades and powered via a slip ring unit. This fan rig proved to be an excellent laboratory facility for development of the holographic system and an example of the interferograms obtained is shown in Figure 7. This was recorded using a pulse separation of 18µs and with the fan rotating at 125rpm and vibrating predominantly in a second flexure assembly mode having two diametral nodes (2B2F).
The holographic system was used to measure the axial vibration shape of a fan undergoing supersonic unstalled flutter. The fan was 0.86 m (34 inches) in diameter and was installed in an air-turbine driven compressor test rig. The test rig had a variable choke core section and a separate bypass section. A flared intake, open to the atmosphere, was employed for these measurements. The instrumentation included strain-gauges on six of the fan blades. The fan was sprayed with retro-reflective paint in order greatly to increase the light returned to the holographic system.

The holographic unit was mounted on-axis, 6 m in front of the fan, as shown in Figure 8. The available laser light was directed onto approximately one quarter of the area of the fan, so as to increase the object beam intensity at the hologram recording plane. Holograms were recorded on Agfa Gevaert 10275 film using a remotely operated film transport system. It was originally intended to use thermoplastic film, which would have allowed remote viewing of the interferograms, but unfortunately a fault developed in this system during the early stages of the test. The reference beam was folded within the two-tier optics unit to give an optical path of 12 m, thus path matching the object beam. Full remote control of the system was employed including control of all the important laser parameters, the image rotator and the film transport. Up to 70 holograms were recorded before a film change became necessary.

Approximately 100 useful holograms were obtained during 5 hours of running over a 2 day period. The majority of these were recorded with the fan in steady unstalled flutter, at speeds just below 10000 rpm and with a flutter frequency of 686 Hz. Holograms were recorded at selected fan speeds.
rotating interferograms showed the fan to be fluttering with a 2P mode shape with a peak velocity of the order 2 500 in. Two examples of the interferograms recorded in flutter are shown in Figures 9 and 10. They were recorded at fan speeds of 9890 and 9886 rpm, respectively, using a pulse separation of 2 μS. Holograms were recorded at approximately the same fan orientation, but at different phases with respect to the flutter cycle. The change in the recurrent orientation of the fringe pattern is the travelling nature of the flutter vibration. A few interferograms were recorded at fan speeds just below the onset of flutter. An example shown in Figure 11, which was recorded at a fan speed of 9660 rpm and with a pulse separation of 5 μS. It can be seen that out of uninstalled flutter there is significant vibrational activity of individual blades, particularly at the leading-edge, with vibrational velocities of the order of 0.3 m/s. Similar individual blade response superimposed on the 3P2P mode shape can be seen on many of the interferograms recorded in flutter.

The holographic results were obtained with the fan operating with a wide-open bypass to give the lowest flutter speed. The use of retro-reflecting paint affected the aerodynamic and flutter performance of the fan. The total flow and the flutter onset were reduced by 5% and 6%, respectively. This is likely to be due to an increase in the blade surface roughness of approximately 30 to 250 μ inches CLA and a thickening of the leading-edge of the blades, as a result of the application of the paint. Even though there was a significant change in the fan characteristics, the holographic measurements are used for comparison with the predicted mode shape, as determined by finite element techniques and for study of the fundamentals of the flutter process.

Estimate of Errors in Fringe Position

In addition to the errors present in conventional holographic interferometry, such as those incurred in determining the precise position of a fringe and its order, there are two further sources of error worthy of consideration for the case of a rotating fan under aerodynamic load. These sources of error are discussed here. First the effects of misalignment of the holographic system relative to the rotational axis of the fan are quantified, and second the path-length changes due to unsteady aerodynamics are considered.

The holographic system is optically aligned when the projected fan's axis passes through the centre of the image rotator apertures. At this condition, the holographic sensitivity vector is perpendicular to the fan axis direction of rotation at all points, and thus no biasing of the interference fringes occurs due to rotation. When this is not the case, the fringe order at each point on the fan is modified according to the relationship

$$a = \frac{2\pi\lambda}{\lambda} (x_p y_q - y_p x_q)$$

where, a is the change in the fringe order at a point p on the fan due to misalignment, λ is the wavelength of light, Δθ is the angle of rotation occurring between exposures, l is the distance between the rotator and point p, x_p and y_p are the coordinates of a point p on the rotator axis as defined in Figure 12, and x_q and y_q are coordinates of a point Q on the rotator axis as defined in Figure 12.

The above relationship is derived in the Appendix for point illumination and observation at the point Q on the image rotator axis. For the case where the fan to image rotator distance is large compared with the fan radius, l is approximately constant for all points on the fan and the change in the fringe distribution can be approximated by a set of parallel equispaced bias fringes.

The likely errors in the flutter measurements due to misalignment are now considered. When installing the holographic system it was estimated that the centre of the image rotator apertures was aligned to within ± 3 mm of the projected axis of the static fan. Prior measurements suggested...
Unsteady aerodynamics, and in particular an intra-passage shock, will produce some bias of the interference fringe distribution. The nature of that bias is now estimated. The fringe in the fringe order resulting from a movement the shock is given by the relationship,

\[
\Delta F = \frac{C \Delta \rho}{\lambda} (\rho_2 - \rho_1)
\]

where, \( \Delta F \) is the change in the fringe order, due to shock movement, \( C \) is the Gladstone-Dale constant, which equals \( 2.24 \times 10^{-1} \) m kg\(^{-1}\) for air, \( d \) is the total optical path length over which the shock moves between exposures, \( \lambda \) is the wavelength of light, and \( \rho_1 \) and \( \rho_2 \) are the pre- and post-shock air densities, respectively.

Normal shock moving with an amplitude \( \Delta \), at the latter frequency \( f \), as shown in Figure 13, is illustrated. For pulse separations, \( \Delta t \), which are not compared with the flutter cycle time, the sinusoidal shock movement occurring between exposures thus \( \omega \Delta t \). Thus, the maximum value of \( d \) is given by

\[
d = 2 \omega \Delta t \sec (\alpha)
\]

where, \( \alpha \) is the blade stagger angle, 1 the factor 2 has been included because we have double pass optical system. Thus from equations 2 and 3 we obtain,

\[
\Delta F = \frac{2C \omega \Delta t \sec(\alpha)}{\lambda} (\rho_2 - \rho_1)
\]

An estimate of the change in the fringe order due to unsteady aerodynamics can now be made by substituting appropriate values into Equation 4. Change in the fringe order of approximately 0.01 full fringe is predicted for the following estimated typical conditions: \( \omega = 1310 \text{ rad} \cdot \text{s}^{-1} \), \( a = 1 \text{ m} \cdot \text{s}^{-1} \), \( \Delta t = 2 \text{ ms} \), \( \delta = 60^\circ \), \( (\rho_2 - \rho_1) = 1 \text{ kg} \cdot \text{m}^{-3} \) and \( \lambda = 0.65 \text{ mm} \).

Thus, careful alignment of the holographic system relative to the rotational axis of the fan is very important in order to minimise the errors in the resultant deflection data. Errors due to unsteady aerodynamics are typically much smaller.

Quantitative analysis of the interferograms

The interferograms were analysed to obtain the axial component of the flutter vibration shape, with particular emphasis on determining the spatial distribution and relative magnitudes of axial displacement and torsion of the blades. The distribution of blade torsion and displacement is important for modelling and prediction of the instigated flutter phenomenon. The analysis was performed with the aid of a computer linked television system which was used to define fringe positions relative to the fan coordinates and then calculate a deflection shape. The fringe order was determined by incrementing or decrementing, as appropriate, from the blade root and recognising any inversion in the fringe count direction. An additional low contrast, widely spaced fringe-set was present on some of the interferograms. This unexpected but useful feature was due to the presence during the recording of the hologram of a third laser pulse of low energy, which occurred a few hundred ns after one of the two main pulses. When present, this additional fringe-set helped to confirm the fringe order.

The absolute axial displacement which occurred between the two laser pulses at a point is calculable from the interferograms, by assuming that there is zero radial movement and using the relationship,

\[
P = \frac{1}{\lambda} (L \cdot \vec{n})
\]

where, \( P \) is the fringe order,
\( L \) is the displacement vector, and \( \vec{n} \) is the holographic sensitivity vector, having magnitude \( \frac{2\pi}{\lambda} \).

Consideration of the geometry leads to the scalar relationship,

\[
A = \frac{PA}{2} \sec (\tan^{-1} \frac{A_f}{s})
\]

where, \( A \) is the axial displacement which occurred between holographic exposures, \( \lambda \) is the wavelength of light, \( r \) is the radius of the point on the fan under consideration, and \( s \) is the distance between the fan and the holographic system.

A measure of blade torsion, \( T \), is obtained by calculation of the angle which a point on the leading edge rotates relative to the centre-line. This is given by,

\[
T = \frac{2(A_L - A_R)}{\sin(\delta) c}
\]

where \( A_L \) and \( A_R \) are the axial displacements obtained at the leading-edge and centre-
line, respectively, using Equation 6,  
\[ \delta \] is the stagger angle of the blade,  
\[ c \] is the blade chord length,  
these are, generally, all variables of radial  

direction.

By way of illustration, plots of blade torsion,  
and axial displacement, \( A \), obtained from one  
interferogram are presented. This particular  
interferogram was recorded at a fan speed of 9895 rpm  
using a pulse separation of 21s. A photographic  
print of the interferogram recorded from the  
visual display of the computer-televison analysis  
system is shown in Figure 11. Superimposed on  
is interferogram is the fringe distribution along  
the centre-line of a blade (number 27) having  
minimal torsion at the tip. This fringe distribution  
was used to compute the plot of centre-line  
displacement as a function of radius, shown  
Figure 15. A plot is shown in Figure 16 of  
the torsion as a function of radial position  
along the blade 29, which was selected because it  
has minimal centre-line displacement at the tip.  
There is considerable deviation in this plot of  
radial torsion from the expected distribution for the  
flutter mode. Plots of the circumferential distribution  
of torsion and centre-line axial displacement,  
at three radii, are shown in Figure 17.  
These plots also show some deviation from the sinusoidal  
distribution of torsion and bending which  
would be expected. This is due to response of the  
blades in their individual modes of vibration, in  
addition to the 302P assembly mode. However, from  
Figure 17, estimates of the spatial phase between  
torsion and axial displacement can be made, for  
instance, at the clapper this phase is approximately 90°  
as would be expected. The error bars in Figures 15 and 17 represent the uncertainty in  
the measurement due to the positional tolerance of  
the rotator with respect to the projected fan's  
axis. The average axial and angular velocities  
which occurred between the two laser pulses is  
obtained from the data shown in Figures 15 to 17,  
by simply dividing the values of \( A \) and \( T \) by the  
pulse separation \( \Delta t \). Thus, for instance, the  
average axial velocity at the tip of blade 27 was  

![Figure 11](image1.png)  
Interferogram of the fluttering fan,  
recorded from the visual display of  
the analysis system.

![Figure 15](image2.png)  
Measured axial displacement along  
the centre-line of a blade having  
minimal torsion at tip.

![Figure 16](image3.png)  
Measured torsion as a function of  
radius for a blade having minimal  
axial displacement at tip.

![Figure 17](image4.png)  
Plots showing the circumferential  
distribution of torsion (\( O \)) and  
centre-line axial displacement (\( I \)).
\[ 2 \times 10^{-6}/2 \times 10^{-6} = 0.62 \text{m/s} \text{ and the average angular velocity at the tip of the blade was } 1 \times 10^{-6}/2 \times 10^{-6} = 19 \text{ rad/s}^{-1}. \]

**Conclusions**

This paper demonstrates the use of holographic interferometry to quantitatively determine the normal vibration over a large area of a fan rotating approximately 10000 rpm and undergoing unstalled torsional flutter. In particular, the determination of the relative amplitude and phase of torsional and axial bending of the fan's outer characteristics can be compared. The termination of the distribution of torsional and axial bending is crucial for the modelling of a fan's outer characteristics.

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**References**


**Appendix - Expression for Bias Prindle Field**

An expression is derived for the bias fringe field generated by misalignment of the image rotor relative to the fan's axis. The stationary coordinate system and geometrical configuration is shown in Figure 12. For simplicity, the source of illumination and the collection aperture are approximated to coincident points on the rotor axis at \( r \), having coordinates \( x, y, z \). \( P_q \) is the general point on the fan at \( x_q, y_q, z_q \), at the time of the first exposure. Rotation of the fan by \( \Delta \phi \) between exposures moves this point to \( P_2 \) at \( x_2, y_2, z_2 \). The rotation of the rotor serves to maintain correlation between the two images of the fan, but does not introduce any pathlength change. Thus, the total pathlength change is 2(\( P_2 - P_1 \)).

This will introduce an error of a full fringe in the resultant interferogram given by

\[ m = \frac{2}{\lambda} (P_2 - P_1) \]

where \( \lambda \) is the wavelength of light.

From the geometry,

\[ (P_2 - P_1)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \]

for \( k = 1,2 \)
\[(P_2 Q)^2 = (x_q - r \cos \theta_k)^2 + (y_q - r \sin \theta_k)^2 +
(x_q - x_k)^2 \quad \text{for } k = 1, 2 \tag{43}\]

where \(r = OP_1 = OP_2\) and \(\theta_1, \theta_2\) are defined in figure 13.

On an expansion of (4) and using \(x_1 = x_2\),
\[(P_2 Q)^2 - (P_1 Q)^2 = 2x_q r(\cos \theta_1 - \cos \theta_2) + 2y_q r(\sin \theta_1 - \sin \theta_2) \tag{44}\]

Assuming small \(\Delta \theta = \theta_2 - \theta_1\) and letting \(\theta = \theta_1\),
\[(P_2 Q)^2 - (P_1 Q)^2 \approx 2\Delta \theta x_q r \sin \theta \tag{45}\]

At \(x_p = x_1\) and \(y_p = y_2\),
\[(P_2 Q)^2 - (P_1 Q)^2 \approx 2\Delta \theta (x_q y_p - y_q x_p) \tag{46}\]

\[P_2 Q - P_1 Q \approx \frac{(P_2 Q)^2 - (P_1 Q)^2}{P_2 Q + P_1 Q} \tag{47}\]

For small misalignments, \(P_2 Q + P_1 Q \approx 21\), thus
\[P_2 Q - P_1 Q \approx \frac{\Delta \theta}{2} (x_q y_p - y_q x_p) \tag{48}\]

Using \(\Delta \theta\) from (4),
\[a = \frac{2\Delta \theta}{21} (x_q y_p - y_q x_p) \tag{49}\]