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Variable Amplitude Fatigue
-of-
Adhesively Bonded Joints

by

Serhat Erpolat

A Doctoral Thesis
Submitted in Partial Fulfilment of the Requirements
for the Award of Doctor of Philosophy of
Loughborough University

Wolfson School of Mechanical and Manufacturing Engineering

August 2004

© 2004 S. Erpolat
To the members of my dear family,

Vasfiye & Nuri Erpolat and Rana & Sevket Kaya, who endured me when I was around and missed me whilst away...
Adhesive bonding is the most attractive joining technique for many aerospace applications. One of the most important reasons for this is the superior fatigue performance of bonded joints compared to traditional joining techniques, fatigue being considered as one of the most important design concerns for aerospace structures. Previously, efforts have been made to develop lifetime prediction methods for bonded joints under constant amplitude (CA) fatigue. Although CA fatigue conditions can be assumed for some real structures, a more complex load history is likely to be expected for many aerospace applications. However, a problem arises from the fact that materials can behave very differently under variable amplitude (VA) loading than they do under CA conditions. Cycles with different stress levels can interact and thus lead to acceleration or retardation of the fatigue process. Despite this, there is very little work on the VA fatigue of bonded joints. Therefore, in this work, the effect that VA fatigue has on the initiation and propagation of damage in bonded composite joints was studied and a predictive methodology for joints subjected to complex loading regimes was developed.

In this study, both double lap joints (DLJ) and double-cantilever beam (DCB) specimens were tested. The applicability of the Palmgren-Miner (P-M) rules and numerical crack growth integration (NCGI) to bonded joints subject to a block-loading VA spectrum were investigated. The P-M rules failed to predict the fatigue life of
double lap joints and NCGI failed to predict both the experimentally observed cohesive and interlaminar crack growth in bonded composite joints under VA loading. In all cases, severe fatigue crack growth acceleration was reported. This is obviously of some concern as traditional predictive methods will tend to overestimate the actual fatigue life of bonded components. In order to improve the ability to predict VA fatigue in bonded joints two novel useful predictive methodologies were developed: the ‘Linear Cycle Mix’ (LCM) model for uncracked joints and the ‘Damage Shift’ model for double cantilever beam (DCB) joints. The LCM method is based on the observation that the mean stress variations, i.e. transitions from a CA stage to another stage having a higher mean stress value, can be responsible for fatigue crack growth accelerations (i.e. the ‘cycle mix’ effect). This method proved to be a considerable improvement on traditional cumulative damage laws. The Damage Shift model requires the modification of NCGI to incorporate the effect of the damage zone induced by the overloads. This study showed that the method can be used to explain unusual sudden crack jumps during the initial stages of VA cycling and fatigue crack growth acceleration due to overloads. It is suggested that the Damage Shift model may be applicable to a variety of complex fatigue spectra.
ACKNOWLEDGEMENTS

The author wishes to express his deep gratitude and indebtedness to his supervisor Dr. Ian Ashcroft for his supervision, invaluable suggestions and constructive criticism and his friendship throughout this study. The author also would like to thank Dr. Memis Acar for his encouragement for a postgraduate research degree.

The author would like to acknowledge QinetiQ® for kindly providing material used in this research and the Wolfson School of Mechanical and Manufacturing Engineering at Loughborough University for their support of this research through the award of the Departmental Postgraduate Studentship. The author would like to thank technical staff in the Wolfson School, particularly John Jones, Andy Sandaver, Dave King, Steve Retter and also Karl Johnson from Zwick® for his assistance in setting-up the testing equipment.

The author wishes to acknowledge his fellow researcher Dr. Abdulhakeem Al-Ghamdi for his wonderful companionship during those long hours spent in the testing laboratories. The author expresses gratitude to Dr. Yasar Alper Ozkaya, who he shared the same house with over three years, for his patience and support.

Finally the author expresses gratitude to his family who had to put up with his absence year after year, for their patience and their support, which makes success more enjoyable and disappointments more bearable.
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NOMENCLATURE

ROMAN LETTERS

\( a \)  crack length
\( C \)  compliance
\( C \)  Miner's sum
\( C_p \)  Paris law constant
\( CM \)  cycle mix factor
\( D \)  damage
\( E \)  Young's modulus of elasticity
\( E_a \)  adhesive modulus
\( EI \)  flexural stiffness of the beam (where \( I \) is the second moment of area)
\( f \)  fatigue test frequency
\( G \)  strain energy release rate
\( G_a \)  strain energy release rate associated with crack arrest in a quasi-static test
\( G_{AC} \)  strain energy release rate associated with unstable crack growth in a variable amplitude test
\( G_{arr} \)  strain energy release rate associated with crack arrest in a variable amplitude test
\( G_c \)  critical strain energy release rate (fracture energy)
\( G_{int} \)  critical strain energy associated with fast crack propagation in a quasi-static test
\( G_{th} \)  threshold strain energy release rate
\( J \)  J-integral
\( K \)  stress intensity factor
\( K_c \)  critical stress intensity factor (fracture toughness)
\( L \)  load
Nomenclature

\( m \) empirical exponent in Paris law

\( N_f \) number of cycles to failure

\( N_B \) number of blocks to failure

\( N_C \) number of cycles to unstable crack growth in variable amplitude test

\( P \) load

\( r \) distance from crack tip

\( R \) R-ratio (i.e. stress, load or displacement ratio)

\( R \) residual strength

\( S_e \) endurance limit (fatigue limit)

\( S_u \) static strength

\( U \) crack opening ratio

\( v \) crack opening displacement

\( W \) strain energy density (energy of new crack surfaces)

GREEK LETTERS

\( \alpha, \beta \) cycle mix constants

\( \sigma \) stress

\( \sigma_y \) yield stress of adhesive

\( \varepsilon \) strain

\( \nu \) Poisson’s ratio

\( \Gamma \) clockwise contour in J-integral

\( \psi \) damage shift parameter

SUBSCRIPTS

\( B \) block

\( f \) final, at fracture

\( i \) instantaneous

\( int \) initiation

\( mn \) mean
### Nomenclature

<table>
<thead>
<tr>
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<tr>
<td>max</td>
<td>maximum</td>
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<tr>
<td>min</td>
<td>minimum</td>
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<tr>
<td>o</td>
<td>initial</td>
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### MAIN ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
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<tr>
<td>AF</td>
<td>Adhesive failure (used for DCB joints with cohesive cracks)</td>
</tr>
<tr>
<td>ASME</td>
<td>American Society of Mechanical Engineers</td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>BEF</td>
<td>Beam-on-elastic-foundation</td>
</tr>
<tr>
<td>BEPF</td>
<td>Beam-on-elastic/plastic-foundation</td>
</tr>
<tr>
<td>BS</td>
<td>British Standards</td>
</tr>
<tr>
<td>CA</td>
<td>Constant amplitude</td>
</tr>
<tr>
<td>CF</td>
<td>Composite failure (used for DCB joints with interlaminar cracks)</td>
</tr>
<tr>
<td>CG</td>
<td>Crack growth</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon fibre reinforced plastic</td>
</tr>
<tr>
<td>COD</td>
<td>Crack opening displacement</td>
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<td>DCB</td>
<td>Double-cantilever beam</td>
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<tr>
<td>DLJ</td>
<td>Double lap joint</td>
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<tr>
<td>EPFM</td>
<td>Elastic-plastic fracture mechanics</td>
</tr>
<tr>
<td>FCGR</td>
<td>Fatigue crack growth rate</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite element analysis</td>
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<tr>
<td>EC</td>
<td>Experimental compliance</td>
</tr>
<tr>
<td>LCM</td>
<td>Linear Cycle Mix</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear elastic fracture mechanics</td>
</tr>
<tr>
<td>L-N</td>
<td>Load vs. number of cycles to failure</td>
</tr>
<tr>
<td>MD</td>
<td>Multi-directional</td>
</tr>
<tr>
<td>NCGI</td>
<td>Numerical crack growth integration</td>
</tr>
<tr>
<td>P-M</td>
<td>Palmgren-Miner’s</td>
</tr>
<tr>
<td>PTFE</td>
<td>PolyTetraFluoroEthylene (Teflon®)</td>
</tr>
<tr>
<td>RMS</td>
<td>Root-mean-square</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
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</tr>
<tr>
<td>SBT</td>
<td>Simple beam theory</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning electron microscopy</td>
</tr>
<tr>
<td>SLJ</td>
<td>Single lap joint</td>
</tr>
<tr>
<td>$S-N$</td>
<td>Stress vs. number of cycles to failure</td>
</tr>
<tr>
<td>UD</td>
<td>Unidirectional</td>
</tr>
<tr>
<td>VA</td>
<td>Variable amplitude</td>
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CHAPTER ONE
INTRODUCTION

1.1 Background

Adhesives have been used in demanding structural applications for the last fifty years. They have found many uses in a great variety of industries, particularly aerospace industry and they are becoming increasingly popular. This is because of the many advantages they offer compared to traditional joining techniques such as mechanical fastening, welding, etc. Some of these advantages are:

- The ability to form light-weight, but strong and stiff structures.
- The ability to join dissimilar materials such as metals (including metals like aluminium and zinc-coated steels that are known as difficult to weld), plastics, fibre composites, etc.
- The ability to join thin-sheet materials (both metallic and non-metallic).
- An improved stress distribution in the joint resulting in an enhanced fatigue performance.
- An improved corrosion resistance (Adhesive bonding does not require holes or welds which may facilitate the start of corrosion. They can act as a good seal against moisture and increase the structure's resistance to corrosion. The insulating properties of adhesives prevent electrolytic corrosion between the different metals joined together)
- An improved appearance of the joined structure.
- Cost effective and convenient to be automated.

However, there are also some disadvantages:

- There is a lack of trusted non-destructive testing methods for monitoring bonded joints.
- Although the strength of bonded joints under shear is usually more than adequate, they are quite sensitive to cleavage and peel stresses.
- The resistance to elevated temperatures and fire are poor. Even high-strength structural polymeric adhesives limit the upper surface temperature of the joints to around 300 °C. The impact resistance is considerably lowered at low temperatures (-30 °C).
- Adhesive joints are prone to environmental degradation. Surface pretreatments for the substrates to be joined and good process control are essential to ensure long-term durability.

Nonetheless, adhesive bonding is generally reckoned to be the most attractive joining technique for many aerospace applications in terms of weight, cost and performance. It is used in commercial aircrafts in a great variety of structures, e.g. horizontal and vertical tail surfaces, wing panels, rudders, nose radomes, etc. The wings are almost entirely bonded. It is popularly used in helicopters as well. Helicopter rotor blades are now completely adhesively bonded. Adhesive bonding is also well suited for bonding carbon fibre reinforced plastic (CFRP), which is a lightweight and strong material that defence and aerospace designers have been using heavily since the late 1970's.
Although adhesive bonding has a very good fatigue performance, an aircraft structure is subject to a variety of mechanical loading and, as for many engineering materials, fatigue is a primary concern for bonded composite joints. Although there have been many studies on the fatigue of bonded joints and composites recently, the level of understanding of fatigue in adhesives joints is not as advanced as that for many metals.

1.2 Aims of the Present Research

Although adhesive bonding has a very good fatigue performance, there is still a concern that the aggressive environmental and loading conditions experienced in aerospace structures may lead to the deterioration and failure of bonded joints within the long service life, typically more than 20 years, expected of such structures. In fact, many primary aerospace structures still do not rely on an adhesive to carry or transfer 100% of the local structural loads; in many cases mechanical fasteners are also installed through the joints (bolted-bonded joints). In order to address these problems, efforts have been made to develop lifetime prediction methods for bonded joints under constant amplitude (CA) fatigue loads. Although CA loading condition can be assumed for many real structures, such as pressure vessels, a more complex load history is likely to be expected for many aerospace applications. Static loads, working loads, vibratory and accidental loads in an aerospace structure together comprise the fatigue spectrum. For example in an aircraft, a static load will be associated with self weight, working loads will arise from standard manoeuvres such as taking off and landing. Vibratory loads are high frequency loads that are superimposed on the working loads and an example would be the vibrations due to the interaction of aeroplane tyres with the runway. Accidental
loads are caused by events outside of normal operating practice. The above loads will be to some degree irregular but may conveniently considered as a simplified variable amplitude (VA) fatigue spectrum of loads seen in a standard flight. Many standardized service-simulation load histories, which are characteristic to a specific aircraft structure, can be found in the literature, e.g. FALSTAFF loading spectrum for a fighter lower wing skin or HELIX/FELIX loading spectrum for helicopter main rotor blades, etc. The problem arises from the fact that materials can behave very differently under VA loading than they do under CA conditions. Cycles with different stress levels can interact and thus lead to acceleration or retardation of the fatigue process. Despite these facts, there is very little work on the VA fatigue of bonded joints.

The current project was developed from fatigue work started at QinetiQ® (formerly, DERA) Farnborough, in which the fatigue performance of bonded CFRP joints under constant amplitude fatigue has been studied in various environments. This work has now been extended to include more complex fatigue loading situations, including the effect of fatigue overloading and crack growth in a joint subjected to a fatigue spectrum that is representative of the loading on a joint in a composite wing. The main aims of this work are first:

- To study the effect that VA fatigue has on the initiation and propagation of damage in bonded composite joints

and then

- To use this data to develop a predictive methodology for joints subjected to complex loading regimes.
1.3 Outline of the Thesis

This thesis incorporates 9 further chapters. These chapters will now be briefly summarized.

CHAPTER 2 "Constant Amplitude Fatigue Analysis": Some of the basic concepts of fatigue will be discussed in this chapter. Stress methods, strain methods, fracture mechanics and damage mechanics are the most important methods currently used in fatigue analysis. These methods will be summarised in this chapter.

CHAPTER 3 "Cumulative Damage Rules": This chapter will focus on some of the stress and strength based methods used to predict variable amplitude (VA) fatigue life using constant amplitude (CA) data. The methods that will be described include the RMS-method and various cumulative damage rules including stress and stiffness-wearout models. Load interaction effects will be discussed at the end of the chapter.

CHAPTER 4 "Fracture Mechanics Approach": This chapter will focus on fracture mechanics based methods used in CA and VA fatigue analysis. Basic concepts of fracture mechanics and calculation of strain energy release rate in bonded joints will be described. Methods used to predict VA fatigue life using CA data will be discussed. Load interaction effects will be discussed at the end of the chapter.

CHAPTER 5 "Experimental Work": This chapter introduces the materials and joints studied in this project. The main experimental techniques and loading spectra used will also be discussed.
CHAPTER 6 "Variable Amplitude Fatigue of Uncracked Specimens": In this chapter, results from double lap joints tested under constant and variable amplitude loading will be reported. The main attention will be given to cumulative damage rules, the most popular methods for predicting VA fatigue life using the data obtained from CA. Palmgren-Miner's rules will be assessed. The RMS method will also be briefly addressed. The critical load interactions will be defined and a model incorporating a 'cycle mix' factor will also be proposed. Finally, the mechanism of failure, crack initiation and propagation in bonded joints will be discussed at the end of this chapter.

CHAPTER 7 "Variable Amplitude Fatigue of Cracked Specimens": In this chapter, results from double cantilever beam (DCB) joints tested under constant and variable amplitude loading will be reported. The main attention will be given to numerical crack growth integration (NCGI), the most popular fracture mechanics based method for predicting VA fatigue life using the data obtained from CA.

CHAPTER 8 "Discussion": This chapter will present a discussion of the experimental and theoretical results.

CHAPTER 9 "Conclusion": This chapter will present the main conclusions drawn from the work.

CHAPTER 10 "Future Work": This chapter will present the suggested areas for future work.
CHAPTER TWO

CONSTANT AMPLITUDE FATIGUE ANALYSIS

2.1 Introduction

Some of the basic concepts of fatigue and methods used in fatigue analysis will be discussed in this chapter. Stress methods, strain methods, fracture mechanics and damage mechanics are the most important methods currently used for that purpose. These methods will be summarised in this chapter. The fracture mechanics approach will be discussed in full detail in Chapter 4.

2.2 Basic Concepts of Fatigue

It is often convenient in laboratory investigations to consider fatigue in terms of a sinusoidal wave form. Fig. 2.1 schematically shows a sinusoidal waveform with a constant stress amplitude and Table 2.1 lists a number of key parameters used to describe fatigue spectra. These parameters can be defined in terms of load or displacement in the same manner. For instance, the stress ratio \((S_{\text{min}}/S_{\text{max}})\) can be defined in terms of load or displacement (i.e. as \((L_{\text{min}}/L_{\text{max}})\) or \((v_{\text{min}}/v_{\text{max}})\)) and is then referred to as the 'load ratio' or 'displacement ratio'.

Only two of these stress parameters and frequency (i.e. number of cycles in one second) are required to characterise a constant amplitude (CA) spectrum. This means that it is impossible to investigate each of these parameters independently, e.g. with a
Figure 2.1 Constant amplitude sinusoidal waveform

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum stress</td>
<td>$S_{\text{max}}$</td>
</tr>
<tr>
<td>Minimum stress</td>
<td>$S_{\text{min}}$</td>
</tr>
<tr>
<td>Stress amplitude</td>
<td>$S_a = \frac{S_{\text{max}} - S_{\text{min}}}{2}$</td>
</tr>
<tr>
<td>Mean stress</td>
<td>$S_{\text{mn}} = \frac{S_{\text{max}} + S_{\text{min}}}{2}$</td>
</tr>
<tr>
<td>Stress range</td>
<td>$\Delta S = S_{\text{max}} - S_{\text{min}}$</td>
</tr>
<tr>
<td>Stress ratio</td>
<td>$R = \frac{S_{\text{min}}}{S_{\text{max}}}$</td>
</tr>
<tr>
<td>Period</td>
<td>$T$ (sec)</td>
</tr>
<tr>
<td>Frequency</td>
<td>$f = \frac{1}{T}$ (Hz)</td>
</tr>
</tbody>
</table>

Table 2.1 Parameters used to describe fatigue spectra
constant stress amplitude it is impossible to investigate the effect of the mean stress without also affecting the maximum stress.

2.3 Stress-life Approach

2.3.1 S-N curves

The stress-life approach to fatigue was first introduced in the 1860s by Wöhler (1860, 1863). In the stress-life approach, the 'fatigue life', i.e. number of cycles to failure, is characterized as a function of the stress level. Here the fatigue life implies the number of cycles to initiate fatigue cracks (initiation phase) plus the number of cycles to propagate the dominant crack to final failure (propagation phase).

The primary design tool in the stress-life approach is an 'S-N curve', which can be obtained by conducting a series of fatigue tests at different values of \( S_a \) for a given \( R \)-ratio. The relation between the stress amplitude, \( S_a \), and number of cycles to failure, \( N_f \), in a typical S-N curve, is schematically presented in Fig. 2.2. As shown in this figure, in semi-log scale, this relation is usually linear only until the fatigue limit. Therefore, a typical S-N curve can be defined as:

\[
S_a = C_{SN} + m_{SN} \log(N_f)
\]  

(2.1)

where \( C_{SN} \) & \( m_{SN} \) are regression coefficients. \( N_f \) increases as \( S_a \) decreases and, considering the log scale, small decreases in stress amplitude can result in large increases in the fatigue life. It should be noted that in some cases a polynomial can also be fitted to \( S_a \) vs. \( N_f \) data in a semi-log scale.
Materials commonly exhibit a fatigue limit, below which a nominally defect-free specimen will not undergo fatigue failure. This is called the ‘fatigue limit’ or ‘endurance limit’. As shown in Fig. 2.2, after a large number of cycles, the $S$-$N$ curve leads to an infinite life at a stress level of $S_e$. This is seen in many materials, including many steels and plastics. Where this is not seen, such as in aluminium alloys, $S_e$ is defined as the stress amplitude at some arbitrary, large number of cycles (typically $10^7$). In adhesive joints, $S_e$ is typically quoted at between 20 and 50% of the quasi-static failure stress.

![Graph](image)

**Figure 2.2** A schematic of a model behaviour of an $S$-$N$ curve

Generally, the fatigue data for a material is generated for a single $R$ value, most commonly for $R = -1$, i.e. for a fully reversed loading cycle. However, $S$-$N$ curves are usually a strong function of the $R$-ratio. This is related to the fact that an increase in the maximum load, load amplitude or the presence of a mean-stress component will reduce the fatigue life. At constant amplitude, a decrease in $R$ indicates a decrease in the
maximum load and mean load. Therefore, a reduction in $R$ at constant amplitude results in an increase in the fatigue life of a material. There are a number of analytical methods that can be used to construct $S$-$N$ curves associated with any $R$-ratio based on a single $S$-$N$ curve. This is particularly useful for variable amplitude (VA) loading, where loading spectra are composed of stages with a variety of load ratios. Some of the most popular of these methods are those proposed by Gerber (1874), Goodman (1930), Morrow (1968) and Manson (1979). The Goodman and Gerber approaches are popularly used due to their simplicity and give satisfactory results for many metals.

The Goodman and Gerber constant-life diagrams are shown in Fig. 2.3. $S_u$ is the static strength and $S_{fr}$ is the stress amplitude for a fully reversed cycle ($R = -1$) for that particular life. Curves connecting point A (i.e. $S_a = S_{fr}$; $S_{mn} = 0$) and B (i.e. $S_u = 0$; $S_{mn} = S_u$) depict different combinations of the stress amplitude and mean stress ($S_a$ vs. $S_{mn}$) resulting in the same constant amplitude fatigue life as $S_{fr}$. Goodman (1930) fitted a straight line between point A and B, such that:

$$\frac{S_a}{S_{fr}} + \frac{S_{mn}}{S_u} = 1$$

(2.2)

whereas Gerber (1874) suggested a second order polynomial fit:

$$\frac{S_a}{S_{fr}} + \left(\frac{S_{mn}}{S_u}\right)^2 = 1$$

(2.3)
Similarly, if the experimental S-N curve is constructed using an $R$-ratio other than $R = -1$, $S_f$ can be calculated using Eq. 2.2 and then substituted into Eqs. 2.2 & 2.3 to obtain Goodman and Gerber diagrams, respectively. As shown in Fig. 2.3, point C and D correspond to the same fatigue life. However, point D is at a higher stress level, thus Goodman is a more conservative approach and is often used as a design criterion. It should also be noted that these curves are often not accurate for compressive mean stresses. In this case, a conservative estimate may be made by assuming that compressive mean stresses provide no benefit. The Stress-life approach is the foundation of many cumulative damage rules (see Section 3.3) used in variable amplitude fatigue analysis, thus it is an important part of this research. The Stress-life approach will be used in the analysis of double lap joints (DLJ) in this study.

**Figure 2.3** Constant-life curves for fatigue
2.3.2 Lap shear joints

Lap shear joints (i.e. single-lap [SLJ] and double-lap joints [DLJ]) are some of the most commonly occurring joints in current engineering practice and are often used for testing adhesives (e.g. ASTM D1002-01-‘Standard Test Method for Apparent Shear Strength of Single-Lap-Joint Adhesively Bonded Metal Specimens by Tension Loading (Metal-to-Metal) Adhesives’, ASTM D5868-01-‘Standard Test Method for Lap Shear Adhesion for Fiber Reinforced Plastic (FRP) Bonding’). S-N curves are the main tool for fatigue analysis of this type of joints. Here, an average shear stress is used. The applied load is divided by the total nominal bonded area to calculate the average shear stress. Although this approach is recommended in the standards (e.g. ASTM D1002, ISO 4587-‘Determination of tensile lap-shear strength of rigid-to-rigid bonded assemblies’), it may be misleading. For instance, Oplinger (1975) showed that there is a length of bondline beyond which no load capacity increase occurs (ineffective length), i.e. failure load does not increase although average shear stress is reduced by increasing the overlap area. This is because of the non-uniform nature of the shear stresses and the existence of significant peel stresses in bonded joints. Fig. 2.4(a) and Fig.2.4(b) show a typical adhesive stress distribution in a single lap joint and a double lap joint, respectively. This is one of the reasons why an S-N diagram for one sample geometry is difficult to apply to a different sample geometry. Another difficulty is the large number of failure modes that can be identified for bonded joints, particularly bonded composite joints. For instance, a detailed classification is given in ASTM D5573-94-‘Standard Practice for Classifying Failure Modes in Fibre-Reinforced-Plastics (FRP) Joints’. Figure 2.5 is given to describe different failure modes defined by this standard. These are (1) adhesive failure (interfacial failure), (2) cohesive failure, (3) thin-layer cohesive

![Diagram of overlap and shear stress](image)

**Figure 2.4** Typical variation of adhesive shear stress in (a) a single lap joint and (b) double lap joint.
Adhesive Failure  
(Interfacial Failure)

Cohesive Failure

Thin-Layer Cohesive Failure  
(Interphase Failure)

Fibre-Tear Failure

Light-Fibre-Tear Failure

Stock-Break Failure

Figure 2.5 Failure modes in bonded FRP joints (ASTM D5573-94)

To sum up, instead of using shear stress, load can be used, thus generating ‘L-N curves’ as a design tool. In spite of its empirical nature, the S-N curves is a commonly accepted design criterion, e.g. ASTM D671- ‘Standard Test Method for Flexural Fatigue of Plastics by Constant Amplitude Load’. The testing methods of adhesive lap joints to generate S-N curves is also covered by the standards BS EN ISO 9664:1995- ‘Adhesives. Test methods for fatigue properties of structural adhesives in tensile shear’ and ASTM D3166-99- ‘Standard Test Method for Fatigue Properties of Adhesives in Shear by Tension Loading (Metal/Metal)’. Double lap joints will be used in this research.

2.4 Strain-life Approach

Coffin (1954) and Manson (1954) independently proposed a strain-based approach to fatigue. The strain-life approach characterizes the life expectancy of a material as a
function of the strain fluctuation it experiences and hence can take the plastic deformation into account. Here the fatigue life implies the number of cycles to initiate fatigue cracks (initiation phase) plus the number of cycles to propagate the dominant crack to final failure (propagation phase) as in the case of the stress-life approach.

When a material is deformed under a fluctuating loading, it may exhibit cyclic strain hardening or softening behaviour. For example, for a strain hardening material in a load-controlled test, the strain amplitude, $\Delta \varepsilon/2$, becomes smaller and smaller, and finally reaches a stabilized state forming a stabilized hysteresis loop. The stabilized strain range, $\Delta \varepsilon$, can be calculated by means of a correlation derived by Morrow (1968):

$$\Delta \varepsilon/2 = \frac{S_a}{E} + \varepsilon'_f \left( \frac{S_a}{S_f} \right)^{1/n'}$$  \hspace{1cm} (2.4)

where $S_a$ is the stress amplitude, $S_f$' the fatigue strength coefficient, $E$ the Young’s modulus, $\varepsilon'_f$ the fatigue ductility coefficient and $n'$ the cyclic strain hardening exponent.

$S_f'$ and $\varepsilon'_f$ are defined as the critical values of stress and strain, respectively, that cause fracture of a specimen at the first reversal of cyclic loading. However, they can be approximated by the true fracture strength (corrected for necking), $S_f$, and true fracture ductility, $\varepsilon_f$, respectively, for most metals. $n'$ can be determined by the relation between the stress amplitude, $S_a$, and the total cyclic strain, $\Delta \varepsilon$, by using the equation above and the cyclic stress-strain curve, which can be obtained by testing a number of
specimens under constant amplitude loading or incremental step strain testing of only one specimen.

The fatigue life, $N_f$, corresponding to strain fluctuation $\Delta \varepsilon$ can be calculated by solving Eq. 2.5 implicitly:

$$\Delta \varepsilon / 2 = \frac{S'}{E} (N_f)^b + \varepsilon_f' (N_f)^{\frac{b}{n}}$$

(2.5)

where $b$ is the fatigue strength exponent. $b$ is the slope of the S-N curve in the finite-life region, i.e. the linear region. This equation is based on the observation that the relation between the total strain amplitude (i.e. elastic strain plus plastic strain) and the number of cycles to failure, in a log-log scale, is linear for most metals (Coffin, 1954; Manson, 1954).

All the equations above are defined for a fully reversed cycle, i.e. for $R = -1$, and need to be modified for non-zero mean stress. A simple approach to incorporate mean stress effects is to assume that a tensile mean stress, $S_{mn}$, reduces the fatigue strength coefficient, $S_f'$, to $(S_f' - S_{mn})$ (Morrow, 1968). However, this approach is quite simplistic and the effect of mean stress on fatigue strength may be more complicated than this.

Stress concentrations complicate the analysis even further. In this case, the fatigue life is determined for the local stress and strain histories at the tip of the notch. This is usually a rather challenging task. For example, although the stress and strain concentration factors applied to the nominal stress and strain ($K_\sigma$ and $K_\varepsilon$, respectively)
are equal to the theoretical stress concentration factor ($K_t$) for the elastic case, they deviate from each other and $K_t$ in the case of a local plastic deformation. An empirical equation relating these three concentration factors was proposed by Neuber (1961):

$$K_\sigma K_\varepsilon = K_t^2$$

(2.6)

Therefore, $\Delta \varepsilon$ can only be calculated by solving Eqs. 2.4 and 2.6 together.

As seen, the strain-life approach is more difficult to implement than the stress-life method, particularly for non-homogenous material systems such as bonded joints. Adhesively bonded joints have stress singularities, such as the embedded corner of a substrate in the fillet. This leads to a complex stress-strain state through the adhesive. Another difficulty is that it is not possible to describe the coefficients, such as $n'$ and $b$, for the whole system (the joint). Instead, the fatigue properties of the bulk material, i.e. fatigue properties of the adhesive and the substrates, are needed. That is why this method has seen little application to adhesively bonded joints and will not be used in this study.

**2.5 Fracture Mechanics Approach**

Fracture mechanics has been successfully used in the characterization of fatigue crack propagation since the 1960s. Unlike the stress and strain-life approaches, the fracture mechanics approach only deals with the crack propagation phase. Materials are assumed to have crack initiation during the early stages of the fatigue cycling or a pre-existing crack as a material flaw. Then the number of cycles for a dominant crack to propagate
to a critical size is estimated using the correlation between the fatigue crack growth rate, \( da/dN \), and an appropriate fracture mechanics parameter, such as the strain energy release rate, \( G \), or stress intensity factor, \( K \). The initial crack size used in the analysis can be measured using a non-destructive inspection method. If the inspection does not reveal any dominant cracks, the initial size of the dominant crack is usually assumed to be the largest possible crack size undetected by the inspection method.

Adhesive joints are very likely to contain crack-like flaws such as air-filled voids, cracks/scratches, inhomogeneities, etc. Similarly, composite substrates are very likely to contain pre-existing cracks, particularly interlaminar ones, introduced during the preparation of the laminates. Therefore, fracture mechanics approach seems very useful for the fatigue crack propagation analysis in bonded joints and has been used by many researchers successfully since Williams (1966) discussed it for the first time in 1966. The fracture mechanics approach will be discussed in full detail in Chapter 4.

### 2.6 Damage Mechanics Approach

The damage mechanics approach is mostly based on the work undertaken by Lemaitre (1984, 1985) and Kachanov (1986). Damage mechanics requires a damage variable, \( D_n \), to be defined as a measure of the severity of the material damage. Although it must be a vectoral function defined by the normal vector of the cross-section on which it is defined, the general approach is to assume an isotropic distribution of cracks and voids throughout the material. This assumption reduces \( D_n \) to a scalar variable, \( D \) and simplifies the problem a great deal.
If \( A \) is the overall cross-sectional area and \( A_D \) is the area of the surface intersections of the micro cracks and voids, the overall load-bearing area is \((A - A_D)\). This is schematically shown in Fig. 2.6. An effective resisting area, \( A_{\text{eff}} \), is calculated by correcting the overall load-bearing area for the micro stress-concentrations and interactions between closed surfaces. The damage variable, \( D \), can be defined in terms of \( A_{\text{eff}} \), such that:

\[
D = \frac{A - A_{\text{eff}}}{A}
\]

(2.7)

It is assumed that \( D \) is equal to 0 for a virgin material and \( D = 1 \) implies the complete rupture of the material. Between those two extremes, there is another critical value for the damage variable, \( D_c \). \( D_c \) characterises the macro crack initiation and is usually between 0.2 and 0.8 depending on the material.
Damage variable, $D$, is difficult to determine physically, since it is almost impossible to monitor all the micro-cracks and voids in a material. However, a simple estimation is usually made using the stiffness degradation:

$$D = 1 - \frac{E_D}{E}$$

(2.8)

where $E$ and $E_D$ are the Young’s modulus of the undamaged and damaged material, respectively. Once the damage variable is defined, damage equivalent effective stress, $\sigma_{eff}^*$, can be defined as:

$$\sigma_{eff}^* = \frac{\sigma^*}{(1 - D)}$$

(2.9)

where $\sigma^*$ is the damage equivalent stress. This is defined as:

$$\sigma^* = \sigma_{eq} \left[ \frac{2}{3} (1 + \nu) + 3 (1 - 2 \nu) \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2 \right]^{1/2}$$

(2.10)

where $\sigma_{eq}$ is the von Mises equivalent stress and $\sigma_H$ is the hydrostatic stress. $\sigma_{eff}^*$ can be used in a quasi-static failure criterion.

In order to apply the damage mechanics approach to fatigue, Lameitire (1984, 1985) derived the following equation for the variation of damage variable per cycle, $\delta D / \delta N$:

$$\frac{\delta D}{\delta N} = \frac{2 B_0 \left[ \frac{2}{3} (1 + \nu) + 3 (1 - 2 \nu) \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2 \right]^{1/2}}{(\beta_0 + 1) (1 - D)^{\beta_0 + 1}} \left( \sigma_{eq,max} - \sigma_{eq,min} \right)$$

(2.11)
where $s_o$ is a material and temperature dependent coefficient. $B_o$ and $\beta_o$ are the functions of $s_o$, Young’s modulus, $E$, and hardening coefficients, $K$ and $M$, in the Ramsberg-Osgood equation:

$$\varepsilon = K \sigma^M$$

(2.12)

where $K$ and $M$ are material-dependent constants.

Equation 2.11 can be integrated for a constant amplitude loading. Using the boundary conditions ($N = 0 \rightarrow D = 0$) and ($N = N_R$ [number of cycles to rupture] $\rightarrow D = 1$), the following expression is obtained for $N_R$ for a CA loading corresponding to a maximum and minimum von Mises equivalent stresses of $\sigma_{eq. \ max}$ and $\sigma_{eq. \ mins}$ respectively:

$$N_R = \frac{(\beta_o + 1)\left(\sigma_{eq. max}^{\beta_o+1} - \sigma_{eq. min}^{\beta_o+1}\right)^{-1}}{2(\beta_o + 2)\beta_o \left[2(1 + \nu) + 3(t - 2v)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^{2}\right]}$$

(2.13)

As seen, Eq. 2.13 considers the damage mechanics theory with respect to the entire fatigue process without distinguishing the initiation and propagation phases. This is called a coupled approach. There are also uncoupled approaches that utilize a fracture mechanics approach once a dominant crack is initiated (i.e. when $D > D_c$).

Damage mechanics can also be applied to the variable amplitude fatigue of materials. By integrating Eq. 2.11 numerically between the boundaries ($N = 0 \rightarrow D = 0$) and ($N = N_R \rightarrow D = 1$), a fatigue damage accumulation curve, $D$ against $N$, can be obtained.
It is worth noting that extensive damage zones ahead of the crack tip have been observed in fatigue tested composite joints. Figure 2.7 (Ashcroft et al., 1997) shows an example of such a damage zone in a unidirectional CFRP lap-strap joint tested under constant amplitude loading using PEXR (i.e. penetrant-enhanced X-radiography). As seen, the damage zone can be distinctly identified. Although there are just a few studies on this subject (Wahab et al., 2001; Chow & Lu, 1992), the damage mechanics approach is a very promising method for the analysis of bonded joints. Although damage mechanics approach described by Lemaitre (1984, 1985) will not be used in this study, we will refer to the damage zone concept frequently throughout this study and try to incorporate it in some of our models.

Figure 2.7 X-radiograph showing an interlaminar crack in a lap-strap joint (Ashcroft et al., 1997)
3.1 Introduction

The following chapter will focus on some of the stress and strength based methods used to predict variable amplitude (VA) fatigue life using constant amplitude (CA) data. The methods described include the RMS-method and various cumulative damage rules including stress and stiffness-wearout models. Load interaction effects will be discussed at the end of the chapter.

3.2 RMS Method

In this method, the average stress level is assumed to be the representative of the whole spectrum by taking the root-mean-square (RMS) of the spectrum. Thus, the VA fatigue analysis takes a simpler form and the fatigue life can easily be determined using one of the CA fatigue analysis methods, such as the stress-life or strain-life approach. The root-mean-square stresses and $R$-ratio (min. stress / max. stress) are defined as:

$$S_{\text{rms max}} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (S_{\text{max, } i})^2}$$  \hfill (3.1)

$$S_{\text{rms min}} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (S_{\text{min, } i})^2}$$  \hfill (3.2)
Chapter 3. Cumulative Damage Rules

\[ R_{rms} = \frac{S_{\text{rms}}}{S_{\text{rms}}} \]  

(3.3)

where \( S_{\text{max}} \) and \( S_{\text{min}} \) are the maximum and minimum stresses, respectively, of a loading cycle, \( M \) is the total number of \( S_{\text{max}} \) or \( S_{\text{min}} \).

As seen, the RMS approach does not take into account any load interaction effects, such as the effect of overloads, load sequencing, etc. It is a statistical method without a solid physical base and is only useful for the narrow-band random loading.

3.3 Cumulative Damage Rules

Cumulative damage rules are the most popular methods of predicting variable amplitude fatigue life using the data obtained from constant amplitude testing. They have been proven to be quite satisfactory for many metals and composites and are relatively easy to apply.

The main concept of cumulative damage rules is to define damage, \( D \), as a linear or non-linear function of the ratio of the number of stress cycles imposed on a component to the total number of cycles of the same amplitude necessary to cause failure, \( n/N_f \). Cumulative damage rules can be classified into three groups on the basis of the assumed relationship between these two variables, as proposed by Owen & Howe (1972):

(i) Linear, stress-independent

(ii) Non-linear, stress-independent

(iii) Non-linear, stress-dependent
These classifications can be explained using Figure 3.1. Figure 3.1(a) shows a single relationship between damage and \( n/N_f \) for different stress levels, \( S \). This relation can be linear as in the case of the Palmgren-Miner's rule or nonlinear as in the case of the Modified Palmgren-Miner's rule. Although the effect of stress level is implicitly included by \( N_j \), which is directly related to the stress level, such models are classified as stress-independent.

Figure 3.1(b) shows a relationship between damage and \( n/N_f \) dependent on the stress levels. This is called a stress-dependent model. As seen in the figure, such a relationship is intrinsically non-linear. A popular example is the Marco & Starkey model (1954), where the progress of damage depends on applied stress level.

**Figure 3.1 Classification of cumulative damage rules**

Stress-independent models do not take any load sequencing effects into account. It is also notable that stress-independent models, either linear or nonlinear,
predict the same variable amplitude fatigue life in spite of their different damage accumulation procedures. This is explained in Section 3.4.2. However, stress-dependent models take the sequencing into account as shown in Figure 3.2(a). When the stress level changes, the progress of damage continues as if all previous damage had been accumulated at the second stress level. However, like stress-independent models, they also ignore any load interaction effects, such as the effect of overloads, understress cycles, mean stress variations, etc. The most realistic approach will be as shown in Figure 3.2(b) because the damage accumulation can follow a completely different path during the second stress level due to load interactions that occur during the transition from one cycle to another with a different stress level.

The most important cumulative damage models are described in the following sections. An extensive review can also be found in Reference (Hwang & Han, 1986).
3.3.1 Palmgren-Miner’s (P-M) rule and its variations

The most widely used cumulative damage rule is the Palmgren-Miner’s (P-M) rule. This was initially proposed by Palmgren (1924) and then developed by Miner (1945). In the P-M rule, damage is considered as accumulating in a linear manner without consideration of the load interaction effects. For a block-loading spectrum, Palmgren-Miner’s rule can be defined as:

\[ N_B \sum_{i=1}^{n_B} \frac{n_i}{N_i} = C \]  

(3.4)

where \( N_B \) is the number of loading blocks to failure; \( n_B \) is the number of constant amplitude stages in a block; \( n_i \) is the number of cycles in a stage with a stress level corresponding to a fatigue life of \( N_i \), and finally \( C \) is called the Miner’s sum or damage sum. \( C \) would be equal to 1.0 for one hundred percent damage, assuming no load interactions.

Schutz et al. (1989) suggested that if \( C \) significantly deviates from unity and it is sought to utilize the Miner’s rule without any significant modifications, \( C \) could be set to another value. This value can be estimated intuitionally or with the knowledge of the \( C \) values associated with a similar loading spectrum. If \( N_B \) is the known fatigue life for spectrum B and \( N_{b,\text{pred}} \) is the predicted fatigue life, the value of \( C \) used for the spectrum A, that is similar to spectrum B, can be modified, such that:

\[ C_{u} = \frac{N_B}{N_{b,\text{pred}}} \]  

(3.5)
This is called the Relative Miner’s rule. The similarity between two spectra is usually defined in terms of the magnitude and sequence of overloads. For instance, Schutz et al. (1989) suggested that the Relative Miner’s rule does not produce reliable results if the peak stresses of the two spectra differed by more than 20 or 30 percent.

Another limitation of the P-M rule is that all the cycles applied must lie above the fatigue limit, as originally stated by Miner (1945). However, the effect of understress cycles on the fatigue life is not clearly understood. Schutz et al. (1989) recommended that S-N curves should be extended below the fatigue limit with a slope equal to the slope of the high cycle fatigue line (Elementary Miner’s rule) or with a decreased slope (Extended Miner’s rule) to take the loading below the fatigue limit into account, as shown in Fig. 3.3.

![Figure 3.3 Modifications to Palmgren-Miner’s rule](image)

In one of the few works on the VA fatigue of adhesively bonded joints, Jones & Williams (1989) studied bonded metal box sections subjected to block-loading spectra using the P-M rule. They observed severe fatigue crack growth retardation, with
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$C$ being much greater than unity. However, it should be noted that the structure they considered was very different from the samples considered in the present study.

3.3.2 Non-linear cumulative damage models

There are numerous non-linear damage rules proposed as alternatives to the linear Palmgren-Miner's (P-M) rule. As well as being non-linear, these approaches differ from the Miner's rule in that some of them contain parameters dependent upon the applied stress level. The biggest handicap in using the non-linear equations is that they require material and shaping constants that have to be determined from a large number of tests. Furthermore, stress-dependent models require the monitoring of damage accumulation. For a composite material, damage can be defined microscopically, in terms of the void density, length of cracks, number of fractured fibres, the delamination area, etc. However, fatigue damage is a combination of several damage types and difficult to monitor. Instead, damage can be defined physically by monitoring the residual strength or stiffness degradation with fatigue cycling.

- Modified Palmgren-Miner's rule

The most widely used non-linear cumulative damage rule is the Modified Palmgren-Miner's rule (Leve, 1969). In this method, damage accumulation procedure is defined as:

$$ D = \left( \frac{n}{N_f} \right)^\nu $$

(3.6)
where $\kappa$ is a constant independent of the applied stress level. $\kappa$ is generally considered greater than 1 indicating an accelerating damage accumulation, as shown in Figure 3.4.

![Figure 3.4 Modified Palmgren-Miner's rule](image)

**Figure 3.4 Modified Palmgren-Miner's rule**

- **Marco-Starkey's model**

  Marco & Starkey (1954) proposed a model similar to the Modified P-M rule. However, they suggested that $\kappa$ is a stress-dependent constant.

- **Shanley's model**

  Shanley (1952) modified the linear P-M rule as follows:

  $$D = c S^{kb} n$$

  (3.7)

  where $b$ is the shape parameter for the central part of the $S$-$N$ curve (slope of the central portion), $S$ is the applied stress level, $c$ and $k$ are two material constants ($k>1$).
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- Corten-Dolan’s model

Corten & Dolan (1956) defined damage accumulation procedure as:

\[ D = r n^a \]  

(3.8)

where \( r \), the coefficient of damage growth rate, is a function of the stress level and \( a \) is a constant. Shanley and Corten-Dolan models are similar to the P-M and Modified P-M rules, respectively. However, unlike them, they define \( D \) as a function of the number of cycles applied rather than the \((n/N_f)\) ratio.

- Owen-Howe’s model

Owen & Howe (1972) mainly focused on the fatigue of glass-reinforced plastic (GRP). They proposed the following non-linear, stress-independent equation for damage accumulation:

\[ D = A \left( \frac{n}{N_f} \right) - B \left( \frac{n}{N_f} \right)^2 \]  

(3.9)

where \( A \) and \( B \) parameters are determined by a curve fitting process such that the resulting life predictions lie close to the experimental trendline. Substitution of failure criteria (i.e. \( n/N_f = 1 \rightarrow D = 1 \)) into Eq. 3.9 shows that \((A - B = 1)\). This is used to reduce the number of coefficients to one.

- Subramanyan’s model

Subramanyan (1976) studied VA fatigue of steels and proposed a non-linear cumulative damage model based on the fatigue limit. He defined damage accumulation procedure as:
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\[ D = \frac{\log(N_e) - \log(N_f)}{\log(N_e) - \log(n)} \]  \hspace{1cm} (3.10)

where \( N_e \) is the number of cycles at the fatigue limit (i.e. \( N_f \) corresponding to fatigue limit, \( S_f \)). Srivatsavan & Subramanyan (1978) later modified this model by considering fatigue limit reduction during fatigue damage accumulation.

- Bond’s model

Bond (1999) studied fatigue of glass-reinforced plastics (GRP) and proposed a modified version of the Owen & Howe’s model:

\[ D = B \left( \frac{n}{N_f} \right) - C \left( \frac{n}{N_f} \right)^A \]  \hspace{1cm} (3.11)

where \( A, B, C \) parameters are determined by an iterative curve fitting process such that the resulting life predictions lie close to the experimental trendline. Substitution of failure criteria (i.e. \( n/N_f = 1 \rightarrow D = 1 \)) into Eq. 3.10 shows that \( (B - C = 1) \). This is used to reduce the number of coefficients to two.

- Henry’s model

Henry (1955) defined fatigue damage as the ratio of the reduction of the fatigue limit to the original fatigue limit of undamaged material:

\[ D = \frac{S_e - S_{e,i}}{S_e} \]  \hspace{1cm} (3.12)
where $S_e$ is the fatigue limit for undamaged material and $S_{e,i}$ is the reduced fatigue limit after the application of the $i$th loading cycle. He proposed the following expression for the damage accumulation procedure:

$$D_i = \frac{n_i/N_i}{1 + \left( \frac{S_{e,i-1}}{S_i - S_{e,i-1}} \right) \left( 1 - \frac{n_i}{N_i} \right)}$$

(3.13)

where $S_i$ is the stress level associated with the $i$th loading cycle. Henry's rule should be applied by using Eqs. 3.12 and 3.13 successively. It should be noted that $S_e$ in Eq. 3.12 should be updated after the application of each stress amplitude, such that:

$$S_{e,i} = S_{e,i-1} - S_{e,i}$$

(3.14)

Henry's model differs from the other non-linear models in that it actually incorporates a physical definition of the damage, thus resembling the strength and stiffness-based wearout models that will be described in the next section.

### 3.3.3 Strength and stiffness-based wearout models

#### 3.3.3.1 Introduction

Strength-based or stiffness-based wearout models are a variation of the cumulative damage models. They seem to be the most common approaches for the variable amplitude fatigue analysis of composite materials (Yao & Himmel, 2000; Lee & Jen, 2000; Whitworth, 1990; Yang et al., 1990, Wang & Chim, 1983). Broutman & Sahu (1972) were the first to assume monotonically decreasing residual strength behaviour of
composite materials with cyclic loading. They proposed that final failure occurs when the residual strength decreases down to the maximum load in the spectrum applied, i.e. peak load. Although it will not be proven that such a decrease in residual strength really exists in the joints studied here, previous work on bonded composite-to-metal joints (Yang & Shanyi, 1983; Wolff & Lemon, 1975) showed that residual strength does decrease during fatigue cycling although the mechanisms of this residual strength degradation was not investigated. The strength degradation may probably be attributed to damage accumulation (i.e. formation of voids, micro-cracks, etc.) and fatigue crack growth during fatigue cycling.

Degradation of stiffness is also considered as a measure of damage accumulation in a similar manner. Dibenedetto & Salee (1979) were the first to define the rate of damage in composites by the rate of change of compliance. Although the residual strength is a more meaningful measure of the fatigue damage than the residual stiffness, unlike the stiffness degradation approach, it does not allow non-destructive evaluation.

3.3.3.2 Strength Degradation under Constant Amplitude Loading

The residual strength of the material is defined as a function of the number of cycles applied, \( R(n) \). For virgin material, it is initially equal to the static strength, \( S_0 \). However, it decreases continuously as the damage accumulates during the fatigue cycling. Failure occurs when the residual strength equals the maximum stress of the spectrum, i.e. when \( R(N_f) = S_{\text{max}} \).
The rate of strength degradation mainly depends on $S_u$, $S_{\text{max}}$ and $R$-ratio, $R_L$ (i.e. $S_{\text{min}}/S_{\text{max}}$). Using a function $f$, which describes the rate of strength loss associated with cycling loading, the residual strength can be written as:

$$R(n) = S_u - f(S_u, S_{\text{max}}, R_L) n^\kappa$$

where $\kappa$ is the strength degradation parameter. Substitution of the failure criterion (i.e. $R(N_f) = S_{\text{max}}$) into Eq. 3.15 above gives the following expression for $f(S_u, S_{\text{max}}, R_L)$:

$$f(S_u, S_{\text{max}}, R_L) = \frac{S_u - S_{\text{max}}}{N_f^\kappa}$$

and the residual strength, $R(n)$, can be defined as:

$$R(n) = S_u - (S_u - S_{\text{max}}) \left( \frac{n}{N_f} \right)^\kappa$$

### 3.3.3.3 Strength Degradation under Variable Amplitude Loading

For simplicity, the method will be first described for a two-stage block-loading spectrum. As schematically shown in Fig. 3.5, the loading block is composed of two constant amplitude stages with $n_1$ cycles at a maximum stress of $S_{\text{max},1}$ and $n_2$ cycles at $S_{\text{max},2}$, respectively. The residual strength at the end of the first stage, $R(n_1)$, is:

$$R(n_1) = R_1(n_1) = S_u - (S_u - S_{\text{max},1}) \left( \frac{n_1}{N_{f,1}} \right)^{\kappa_1}$$
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Figure 3.5 Two-stage block-loading spectrum

Figure 3.6 illustrates how to determine the residual strength after the second stage, \( R(n_1+n_2) \). In this figure, curves ABE and ACD represent the strength degradation of a material subjected to a constant amplitude loading at stress levels of \( S_1 \) and \( S_2 \), respectively. Point B represents the strength after \( n_1 \) cycles at stress level \( S_1 \). Point C represents a location of equivalent strength as that of point B on the residual strength curve associated with stress level \( S_2 \). The number of cycles corresponding to point C is called the effective number of cycles, \( n_{\text{eff}} \), and can be calculated by equating \( R_1(n_1) \) and \( R_2(n_{\text{eff}}) \), such that:

\[
n_{\text{eff}} = \left[ \frac{S_u - R_1(n_1)}{S_u - S_{\text{max},2}} \right]^{1/2} N_{f,2}
\]  
\( (3.19) \)
Using the effective number of cycles, the residual strength after \((n_1 + n_2)\) cycles, \(R(n_1 + n_2)\), can be written as:

\[
R(n_1 + n_2) = R_2(n_{\text{eff}} + n_2) = S_u - \left( S_u - S_{\text{max},2} \right) \frac{n_{\text{eff}} + n_2}{N_{f,2}} \]

(3.20)

**Figure 3.6** Residual strength degradation during two-stage loading block

The method described above for a two-stage block loading spectrum can easily be extended to a multi-stress block-loading spectrum. The following generalized equations derived by Schaff & Davidson (1997a, 1997b) is applied on a stage-by-stage basis and describes the residual strength degradation for a material subjected to such a loading spectrum:
where \( j \) represents the current stage and \( n_j \) is the number of cycles elapsed in stage \( j \).

### 3.3.3.4 Stiffness Degradation under Constant and Variable Amplitude Loading

As mentioned in Section 3.3.3.1, an alternative approach to the strength-based wearout models is to associate damage accumulation with stiffness degradation. Like strength degradation, the stiffness degradation rate is considered a power function of the number of load cycles (Dibenedetto & Salee, 1979; Yang et al., 1990; Whitworth, 1990). This power relation can be formulated in many different ways depending on the system considered. For instance, it was defined by Yang et al. (1990), such that:

\[
E(n) = E(0) - E(0)(d + a_2 BS)n^{a_1 + a_3 S} \tag{3.23}
\]

\[
d = a_1 + a_2 a_3 \tag{3.24}
\]

where \( E(0) \) is the initial stiffness and \( S \) is the stress level, \( a_1, a_2, a_3 \) and \( B \) are stress-independent parameters. Whitworth (1990) proposed an alternative model:
\[ E^a \left( \frac{n}{N_f} \right) = E^a(0) - H \left[ E(0) - C \right]^b \left( \frac{n}{N_f} \right) \] (3.25)

where \( C \) is a stress-dependent parameter, whereas \( a \) and \( H \) are independent of the applied stress level.

The failure criterion for a stiffness-based wearout models is not as straightforward as that for the strength-based wearout models. However, it is well-known that higher stress levels result in higher failure stiffness, \( E(N_f) \), i.e. greater amount of stiffness reduction occurs at the lower stresses. This leads to the useful assumption that

\[ \frac{E(N_f)}{E(0)} = \frac{s_{\text{max}}}{s_u} \Rightarrow \overline{E_f} = \overline{s_{\text{max}}} \] (3.26)

which can be used as a failure criterion for the stiffness-based wearout models Whitworth (1990).

In one of the few works on the VA fatigue of adhesively bonded joints, Yang & Shanyi (1990) studied VA fatigue of CFRP-to-metal and metal-to-metal adhesively bonded single lap joints subjected to a random flight-by-flight loading spectrum and suggested a statistical strength based wearout model, which gave fairly good results.

### 3.4 Load Interaction Effects

#### 3.4.1 Mechanisms of load interaction

Non-interactive cumulative damage rules, either linear or non-linear (see Section 3.3), become unreliable when there are significant load interaction effects, such as the effect
of overloads, changes in mean stress and load amplitude, load sequencing, etc. (schematically shown in Fig. 3.7). If the load interaction effects are significant, the prediction based on such an analysis will be either conservative due to crack growth retardation or unconservative due to acceleration.

**Figure 3.7** Possible load interactions in a block-loading spectrum

In fact, crack growth retardation is the more commonly reported phenomenon. For example, it is well known that overloads generally retard fatigue crack propagation in metals. Some of the proposed mechanisms to account for this phenomenon are the compressive residual stresses in the vicinity of a crack tip as well as crack tip closure and crack tip blunting (Chan & Nisitani, 1991). This will be discussed further in Sec. 4.6.3.2. Although most of the studies in this field indicate retardation behaviour, there is also some work in the literature reporting crack growth acceleration for both metals (Chang & Szamossi, 1979; Agerskov, 2000; Nisitani & Nakamura, 1982) and composites (Farrow, 1989; Schaff & Davidson 1997a, 1997b). These studies report
several different mechanisms accounting for the acceleration behaviour. These will be summarised in this section.

Chang & Szamossi (1979) considered the effect of compressive stresses following an overload in an aluminium alloy. They suggested that compressive loading in the spectrum somehow annihilates the compressive residual stresses built up in the vicinity of the crack tip and thus had an acceleration effect. However, several cases were reported showing the existence of crack growth acceleration even when there is no compressive loading at all.

Nisitani & Nakamura (1982) studied the crack growth behaviour of steel specimens tested under a VA fatigue spectrum composed of a very small number of overloads and a very large number of cycles below or near the fatigue limit. They observed that the application of a linear cumulative damage rule resulted in extremely unconservative results. A possible explanation of this phenomenon is that although understress cycles cannot initiate a crack, they can contribute to existing fatigue damage created by the overloads. In other words, understress cycles might cause considerable damage just after the initiation of a crack by an overload or when the crack reaches a certain size. However, there may also be other factors at work. For instance, Nisitani & Nakamura (1982) also observed that overloads enhance the damaging effect of understress cycles for physically short cracks (≤1 mm), even when they do not cause a substantial change in the crack size. The crack growth acceleration for such physically short cracks has been observed by several other investigators (McClung & Sehitoglu, 1988; Vormwald & Heuler, 1993; Frost et al., 1974; Gurney, 1979). This is a very important phenomenon considering the fact that even nominally defect-free specimens
are likely to have short cracks that are initiated during the early stages of the fatigue cycling or pre-exist as a material flaw. The short crack behaviour, which is usually associated with the crack closure concept, will be discussed further in Sec. 4.6.3.3.

Farrow (1989) and Schaff & Davidson (1997a, 1997b) studied fatigue crack growth acceleration for composite materials. Farrow (1989) found that the fatigue life of composite laminates subjected to small block loading is shorter than that of laminates subjected to large block loadings when the blocks have different mean stress levels. He called this phenomenon the “cycle mix effect”. Schaff & Davidson (1997a, 1997b) suggested that the cycle mix effect occurred during the transition from a CA stage to another stage having a higher mean stress value, although they did not discuss the reason for the strength degradation during transition. The cycle mix effect will be discussed further in Sec.3.4.3.

As mentioned before, there is very little work on the VA fatigue of bonded joints (Jeans et al., 1981; Jones & Williams, 1989; Yang & Shanyi, 1990). In one of them, Jones & Williams (1989) studied bonded metal box sections subjected to block-loading spectra with different characteristics including overloads, mean stress jumps, etc. Based on the P-M results they observed severe fatigue crack growth retardation in all cases. They attributed this to blunting of the crack tip due to yielding. However, it should be noted that the structure they considered was very different from the samples considered in the present study. In a more relevant work, Yang & Shanyi (1990) studied VA fatigue of CFRP-to-metal and metal-to-metal adhesively bonded single lap joints subjected to a random flight-by-flight loading spectrum and observed crack growth acceleration for both combinations.
3.4.2 Sequencing effect

As mentioned before, stress-independent models do not take any load sequencing effect into account and either linear or nonlinear, they predict the same variable amplitude fatigue life in spite of their different damage accumulation procedures. This can be shown analytically, such that: for a block-loading spectrum, the Modified P-M rule (Leve, 1969), the most widely used non-linear cumulative damage rule, can be defined as:

\[
\left( N_B \sum_{i=1}^{n_B} \frac{n_i}{N_i} \right)^{\kappa} = D
\]

where \( N_B \) is the number of loading blocks to failure; \( n_B \) is the number of constant amplitude stages in a block; \( n_i \) is the number of cycles in a stage with a stress level corresponding to a fatigue life of \( N_i \), and finally \( D \) is the damage sum. \( D \) would be equal to 1.0 for one hundred percent damage, thus turning the Modified P-M rule into the linear P-M rule (see Eq. 3.4), such that:

\[
\left( N_B \sum_{i=1}^{n_B} \frac{n_i}{N_i} \right)^{\kappa} = 1 \quad \text{for failure} \quad \iff \quad N_B \sum_{i=1}^{n_B} \frac{n_i}{N_i} = 1
\]

Failure criterion for the P-M rule

Unlike stress-independent models, stress-dependent ones take the sequencing into account. For instance, Lee & Jen (2000) applied the non-linear cumulative damage rules to the fatigue of AS4/PEEK (PEEK, i.e. polyetheretherketone, is a polyether high-performance thermoplastic polymer) composite laminates subjected to a two-stage block loading and observed that the damage sum is smaller than unity in the case of a
high-damage to low-damage two-stage fatigue loading and is greater than unity in the case of a low-damage to high-damage one. The effect of stacking sequence can be seen schematically by comparing Figures 3.8(a) and 3.8(b).

Although this is a general case, there can be some exceptions. In fact, the empirical data Lee & Jen (2000) used in their analysis was taken from Broutman & Sahu (1972) and in this work the exponent of the damage function of E-glass/epoxy was in inverse proportion to the maximum loading. In other words, the $S_1$ curve in Fig. 3.8 corresponds to a lower stress level than that of $S_2$ in the case of E-glass/epoxy composite.

![Figure 3.8 Effect of stacking sequence](image)

3.4.3 The ‘cycle mix’ effect

Farrow (1989), Schaff & Davidson (1997a, 1997b) studied fatigue crack growth acceleration for composite materials. Farrow (1989) found that the fatigue life of composite laminates subjected to small block loading is shorter than that of laminates...
subjected to large block loadings when the blocks have different mean stress levels. He called this phenomenon the “cycle mix effect”.

Schaff & Davidson (1997a, 1997b) reviewed the study made by Farrow and tried to develop a strength-based wearout model. They suggested that the cycle mix effect occurred during the transition from a CA stage to another stage having a higher mean stress value, i.e. mean stress jumps, although they did not discuss the reason for the strength degradation during this transition. They incorporated a cycle mix factor, $CM$, in their damage accumulation model and proposed that the cycle mix effect should be taken into account when the stages in a spectrum are such that the ratio of cycles in a stage to cyclic life for that stage were less than 0.001 (as is the case for the spectra used in the present study). A good correlation between theory and experiment was obtained.

In Section 3.3.3.3, a strength-based wearout model was formulised for continuous residual strength degradation due to fatigue cycling (Eqs. 3.21 & 3.22). This can be modified to incorporate degradation due to mean stress jumps using the cycle mix factor, $CM$:

$$R(n) \rightarrow R(n) - CM \quad \text{for } \Delta S_{mn} > 0$$

(3.28)

The cycle mix factor, $CM$, is defined as:

$$CM = C_m S_a \left[ \frac{\Delta S_{mn}}{R(n)} \right]^{(\Delta S_{mn}/\Delta S_{max})}$$

(3.29)

where $\Delta S_{mn}$ and $\Delta S_{max}$ are the changes in the mean and maximum load values, respectively, during the transition from one stage to another and $C_m$ is the cycle mix
Chapter 3. Cumulative Damage Rules

constant dependent on material and geometry. \( C_m \) can be determined by comparing variable amplitude fatigue lives under different spectra, e.g. two spectra with and without mean stress jumps.

Using the cycle mix factor, Equation 3.21 can be modified to:

\[
R \left( \sum_{i=1}^{j} n_i \right) = S_u - \left( S_u - S_{\text{max},j} \right) \left( \frac{n_j + n_{\text{eff},j}}{N_{f,j}} \right)^{\kappa_j}
\]  
(3.30)

and Equation 3.22 to:

\[
n_{\text{eff},j} = N_j \left( S_u - \left( R \left( \sum_{i=1}^{j-1} n_i \right) - CM_{j-1 \rightarrow j} \right) \right)^{\frac{1}{\kappa_j}} / \left( S_u - S_{\text{max},j} \right)
\]  
(3.31)

where

\( j \): Current stage

\( n_j \): Number of cycles elapsed in stage \( j \)

\( \kappa_j \): Strength degradation parameter for stage \( j \)

\( S_{\text{max},j} \): Maximum stress of stage \( j \)

\( N_{f,j} \): Number of cycles to failure for stage \( j \)

\( CM_{j-1 \rightarrow j} \): Cycle mix factor for the transition from stage \( j-1 \) to stage \( j \) if \( \Delta S_{mn} > 0 \)

Cycle mix effect will be an important part of this study and the approach we will use will be similar to the one described in this section.
4.1 Introduction

This chapter will focus on fracture mechanics based methods used in constant and variable amplitude fatigue analysis. Basic concepts of fracture mechanics and calculation of strain energy release rate in bonded joints will be described. Methods used to predict variable amplitude (VA) fatigue life using constant amplitude (CA) data will be discussed. Load interaction effects will be discussed at the end of the chapter. Finite element analysis (FEA) applied to bonded joints will also be briefly discussed in this chapter.

4.2 Basics Concepts of Fracture Mechanics

4.2.1 Introduction

Unlike the stress and strain methods, the fracture mechanics approach mainly deals with the crack propagation phase, not the initiation phase. Materials are assumed to have a crack initiated during the early stages of fatigue cycling or a pre-existing crack. The growth of this crack is then defined in terms of some fracture parameter. There are two main fracture mechanics criteria commonly used: the energy criterion and the stress intensity factor approach.
4.2.2 Strain energy release rate \((G)\)

The energy criterion is based on Griffith's theory of fracture. Griffith (1920) stated that fracture can only occur when sufficient energy is released by the growth of a crack to supply the energy requirements of the new fracture surfaces. This energy release comes from stored elastic or potential energy of the loading system. However, a great deal of this energy can dissipate because of the plastic and viscoelastic processes which occur in real materials. Griffith (1920) defined the energy criterion in terms of the energy released per unit crack area \((J/m^2)\), which is called the strain energy release rate, \(G\). The critical value for crack growth, \(G_c\), is referred to as the fracture energy.

4.2.3 Stress intensity factor \((K)\)

Although it is not so different from the energy criterion in essence, Irwin (1958) suggested a different approach. He introduced the stress intensity factor, \(K\), which defines the stress distribution around a crack. \(K\) is a function of the applied stress, size of the crack, and position of the crack, as well as the geometry of the solid piece in which the crack is located. Fracture occurs when \(K\) reaches a critical value, the fracture toughness, \(K_c\), which is considered a material property.

4.2.4 J-integral \((J)\)

\(G\) was originally developed for brittle materials and is based on linear elastic fracture mechanics (LEFM). For the elastic/plastic fracture of materials, the most widely recognized fracture parameter is the J-integral introduced by Rice (1968). J-integral
is a path-independent line integral taken along any line path $\Gamma$ surrounding the crack tip. It is equal to the strain energy release rate, $G$, for linear elastic behaviour. However, it increasingly deviates from $G$ as the plastic zone size increases. There are two concepts that should be introduced first in order to define the $J$-integral: the strain energy density, $W$, and the traction vector, $T$. An infinitesimal change in strain energy density, i.e. the strain energy per unit volume, is given by:

$$dW = \sum_{i,j} \sigma_{ij} d\varepsilon_{ij} = \sigma_{11} d\varepsilon_{11} + 2\sigma_{12} d\varepsilon_{12} + \sigma_{22} d\varepsilon_{22}$$

(4.1)

where $\sigma_{ij}$ (i.e. $\sigma_{11}$, $\sigma_{12}$, $\sigma_{22}$) and $\varepsilon_{ij}$ (i.e. $\varepsilon_{11}$, $\varepsilon_{12}$, $\varepsilon_{22}$) are the 2-dimensional stress and strain tensors, respectively. The total strain energy density corresponding to such a stress-strain state can be determined, such that:

$$W = \int_{0}^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$$

(4.2)

The traction vector is defined as:

$$T_x = \sigma_{xx} n_x + \sigma_{xy} n_y$$

$$T_y = \sigma_{yx} n_x + \sigma_{yy} n_y$$

(4.3)

where $n_x$ and $n_y$ are the x and y components, respectively, of the unit vector normal to the path $\Gamma$. Using these two concepts, the $J$-integral along any line path $\Gamma$ surrounding the crack tip counter clockwise (Fig. 4.1) can be defined as:
where \( \mathbf{u} \) is the displacement vector with components \( u_x \) and \( u_y \) and \( ds \) is the differential distance along the path.

### 4.2.5 Discussion

Although the stress-intensity approach is widely used for the analysis of metals, it is more complicated to apply to bonded joints, where constraint effects of the substrates on the adhesive layer make it difficult to define the stress distribution around the crack tip. Therefore, \( G \) is often used as the governing fracture parameter in the analysis of bonded joints in preference to \( K \).
4.3 Double Cantilever Beam (DCB) Joints

Fracture mechanics requires a great deal of experimental effort. The double-cantilever beam (DCB) joint, schematically shown in Fig. 4.2, is the most popular experimental test specimen used to generate mode I fracture data. It is easy to manufacture and to test. Furthermore, it requires relatively simple analysis methods.

As shown in Fig. 4.2, the DCB specimen has an induced crack and is loaded by applying a symmetrical opening tensile force (mode I) at the cracked end of the beam. The load is usually applied directly to the substrates (Fig. 4.2), or through pins inserted into load-blocks (Fig. 4.3(a)), or through hinges attached to the specimen (Fig. 4.3(b)). All configurations should allow rotation of the specimen end.

![DCB joint](image)

**Figure 4.2** DCB joint

Unlike pins, hinges do not require special fixtures. They can be simply held by grips. In bonded composite joints, load values are quite low and substrates are
relatively thin. Therefore, hinges are well-suited for the testing of composite joints and in the present study, light alloy hinges were used to provide a point to apply the load.

![Application of the load to the DCB](image)

Figure 4.3 Application of the load to the DCB

### 4.4 Determination of $G_I$ in Bonded DCB Joints

#### 4.4.1 Introduction

Several methods can be used to calculate the strain energy release rate, including those based on experimental compliance (EC) measurements, those based on beam theory and finite element based methods. These will be discussed in detail in this section.

#### 4.4.2 Experimental compliance (EC) method

The experimental compliance method requires the monitoring of load, crack opening displacement and crack length, i.e. the horizontal distance between the point of
application of the load and the crack tip, to determine the variation of compliance with crack growth. The mode I strain energy release rate is calculated from the following equation (Irwin & Kies, 1954):

\[ G_I = \frac{P^2}{2b} \frac{dC}{da} \]  

(4.5)

where \( P \) is the load, \( b \) the specimen width, \( a \) the crack length and \( C \) is the compliance, which is defined by:

\[ C = \frac{v}{P} \]  

(4.6)

with \( v \) being the crack opening displacement, assuming that crack closure effects are negligible. In order to estimate \( dC/da \) a plot of compliance, \( C \), against crack length, \( a \), is curve fitted either using an appropriate polynomial function or a power relation as suggested in the Berry Method (Davies, 1992):

\[ C = Ka^n \]  

(4.7)

where \( K \) and \( n \) are regression coefficients. These can be determined by straight line fitting to the logarithmic plot of \( dC/da \).

### 4.4.3 Analytical methods

\( G \) can be calculated using Eq. 4.5 once the variation of compliance with crack growth, \( dC/da \), is known. \( dC/da \) can be determined either experimentally (i.e. the EC
method) or analytically. Some of the most important analytical methods to determine strain energy release rate will be discussed in this section.

4.4.3.1 Simple Beam Theory (SBT)

Mostovoy et al. (1967) treated each arm of the specimen as a linear cantilever-beam with a rigidly supported built-in end as shown in Fig. 4.4 and derived the following equation for the compliance of the beam:

$$C = \frac{8a}{bEh^3 (a^2 + h^2)}$$  \hspace{1cm} (4.8)

where $E$ is the Young’s modulus of the substrate, $b$ the specimen width, $a$ the crack length and $h$ is the thickness of the substrate.

Using the assumption that $a >> h$, strain energy release rate, $G$, is defined as:

$$G = \frac{12P^2a^2}{b^2h^3E}$$  \hspace{1cm} (4.9)

The SBT method does not account for joint rotation or the thickness of the adhesive, thus assuming a stiffer joint. Therefore it usually underestimates the strain energy release rate for specimens tested under load control and overestimates it when the testing is carried out under displacement control.
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4.4.3.2 Experimental Corrections to the SBT Model

As mentioned in the previous section, the SBT method does not account for the joint rotation or thickness of the adhesive. This can be a considerable source of error. Some empirical corrections to the simple beam theory have been suggested to correct this effect (Hashemi et al., 1989). The SBT equations are modified by an assumed increase in crack length (i.e. $a_{corr} = a + \Delta$) for the deflection due to the beam-end rotation.

The assumed increase in crack length, $\Delta$, can be interpolated from a plot of $C^{1/3}$ vs. $a$ where $C$ is the compliance and $a$ is the crack length. If no length correction term was necessary, this linear relationship would pass through the origin. However this line usually proves to have a negative intercept as shown in Fig. 4.5.

![Figure 4.4 The SBT model](image-url)
In this case, the correction factor is the absolute value of this intercept and equation should be modified to:

\[ C = \frac{8(a + \Delta)^3}{bEh^3} \]  \hspace{1cm} (4.10)

Williams (1987) introduced two more corrections to the beam theory equations: the load-block correction, \( N \), and the large displacement correction, \( F \). \( N \) is only necessary when load-blocks are used to provide a point to apply the load. For hinges and for loading holes drilled directly to the substrate it is simply equal to 1. The effect of \( F \) can be significant if the ratio of the displacement to the crack length, \( \frac{w}{a} \), is greater than 0.4. Since hinges will be used to provide a point to apply the load in the present study and \( \frac{w}{a} \) ratio is usually lower than 0.4, \( N \) and \( F \) will not be discussed further here. As for \( \Delta \), although it is a sensible approach, it should be noted that if a reliable compliance history is already available, the EC method can be applied, negating the requirement for an analytical method of computing \( G \).
4.4.3.3 Models with a Compliant Crack Front

More recently, analytical models with a compliant crack front, based on Kanninen's (1973) beam-on-elastic-foundation model, have been proposed. These truly analytical models not only take into account the contribution of the rotation at the assumed built-in end, but also the effect of the adhesive and the region beyond the crack. Therefore, they do not require any experimental correction.

There are two main models with a compliant crack front: The beam-on-elastic-foundation model (the BEF) developed by Kanninen (1973) and the beam-on-elastic/plastic-foundation model (the BEPF) developed by Yamada (1987) by extending Kanninen's model to include plasticity at the crack tip.

- Beam-on-elastic-foundation (BEF) model

The beam-on-elastic-foundation model (the BEF model) was originally developed by Kanninen (1973). As shown in Fig. 4.6, in the BEF model each half of the beam is considered as a beam partly free and partly supported by an elastic foundation, which represents the interaction of the two beams along the bonded length. \( a \) and \( d \), shown in the figure, represent the lengths of the cracked and uncracked parts of the beam (the crack length and the length of the foundation), respectively. The spring constant of the distributed spring (per unit length of the beam), \( k \), is defined by:

\[
k = \frac{E_a b}{t} \tag{4.11}
\]
where $E_a$ is the Young's modulus of the adhesive, $b$ is the specimen width and $t$ is the thickness of the foundation (i.e. half the thickness of the adhesive).

\[ \sigma = \frac{P}{t} \left( \frac{a}{b} \right) \]  

\[ \varepsilon \equiv \frac{1}{E} \]  

\[ \varepsilon \equiv \frac{1}{E} \]  

\[ \varepsilon \equiv \frac{1}{E} \]  

**Figure 4.6** The beam-on-elastic-foundation model

Using the BEF model, Kanninen (1973) derived the following equations for the crack opening displacement, $v$:

\[ v = 2 \left( \frac{Pa^3}{3EI} - R_1 a + R_2 \right) \]  

(4.12)

where

\[ R_1 = -\frac{P}{2EL^2} D_1 \]  

(4.13)

\[ R_2 = \frac{P}{2EL^2} D_2 \]  

(4.14)

\[ \Delta^4 = \frac{1}{\lambda^4} = \frac{4Et}{E_o b} \]  

(4.15)
where $EI$ is the flexural stiffness of the beam and $\Delta$, i.e. $\lambda'$, serves as a length scale.

The equations for $D_1$ and $D_2$ incorporate hyperbolic cosine and sine functions and are defined as:

$$D_1 = \frac{\sinh^2(\lambda d) + \sin^2(\lambda d)}{\sinh^2(\lambda d) - \sin^2(\lambda d)} + 2a\lambda \frac{\sinh(\lambda d)cosh(\lambda d) + \sin(\lambda d)cos(\lambda d)}{\sinh^2(\lambda d) - \sin^2(\lambda d)}$$

(4.16)

$$D_2 = \frac{\sinh(\lambda d)cosh(\lambda d) - \sin(\lambda d)cos(\lambda d)}{\sinh^2(\lambda d) - \sin^2(\lambda d)} + a\lambda \frac{\sin^2(\lambda d) + \sin^2(\lambda d)}{\sinh^2(\lambda d) - \sin^2(\lambda d)}$$

(4.17)

However, they can be simplified to:

$$D_1 \approx (1 + 2a\lambda)$$

(4.18)

$$D_2 \approx (1 + a\lambda)$$

(4.19)

by using the assumption that $\lambda d$ is comparable to or larger than $2\pi$ (when $\lambda d$ is greater than approximately $2\pi$, $\sinh \lambda d \approx \cosh \lambda d$ and $\sinh \lambda d > \sin \lambda d$, $\cos \lambda d$). Since both $R_1$ and $R_2$ are directly proportional to $P$, once $v$ is defined $P$ can be calculated explicitly from Eq. 4.12.

Chang et al. (1976) and Chow et al. (1979) independently derived equations for the strain energy release rate in a bonded DCB joint using Kanninen's BEF model. Differentiating the work done by the applied load with respect to the crack length, Chang et al. (1976) obtained the following equation for $G$: 


Chow et al. (1979) made the useful assumption that $\lambda d$ is larger than $2\pi$. However, unlike Chang et al. (1976) they included the shear deformation of the beam in their formulation and gave the following equation for $G$:

$$G_I = \frac{12P^2 D_2^2}{b^3 h^3 E \lambda^2}$$  \hspace{1cm} (4.20)

where $G_m$ is the shear modulus of the adherend. Yamada (1988), who analytically calculated the J-integral for the DCB specimen within the framework of the strength of materials, obtained exactly the same expression when no plasticity was assumed. It should be noted that for a typical DCB joint $\lambda d$ is usually larger than $2\pi$ and the shear energy in the beam is usually negligible compared with the bending energy. Therefore, it is sensible to use the two assumptions together in our study.

**- Beam-on-elastic/plastic-foundation (BEPF) model**

Yamada (1988) extended Kanninen's (1973) method to consider the plasticity effects at the crack tip and developed the beam-on-elastic/plastic-foundation model shown in Fig. 4.8, and by using this model he calculated the $J$-integral analytically.

In the BEPF model, the foundation is assumed to be elastic/perfectly plastic, while the cantilever beams remain elastic. The spring constant of the distributed spring (per unit length of the beam), $k$, is defined by:
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\[ k = \frac{E_a b}{t} \]  \hspace{1cm} (4.22)

where \( E_a \) is the Young’s modulus of the adhesive, \( b \) is the specimen width and \( t \) is the thickness of the foundation (i.e. half thickness of the adhesive). The idealized behaviour of the adhesive is shown in Fig. 4.7. Therefore, within the yield zone, a uniform uniaxial stress is assumed, and springs reaching the elastic limit around the crack tip are replaced by a uniformly distributed load equivalent to the yield strength of the adhesive, \( \sigma_y \).

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{elastic_perfectly_plastic_stress_strain.png}
\caption{Elastic/perfectly plastic stress-strain behaviour}
\end{figure}
Yamada (1988) derived the following equation for the plastic zone size, \( l_p \):

\[
\frac{1}{\lambda} \left( \alpha_1 - 1 + \sqrt{\alpha_1^2 + 2\alpha_2} \right)
\]

\( l_p = \frac{1}{\lambda} \left( \alpha_1 - 1 + \sqrt{\alpha_1^2 + 2\alpha_2} \right) \tag{4.23} \]

\[
\alpha_1 = \frac{P\lambda}{\sigma_y b}
\]

\( \alpha_1 = \frac{P\lambda}{\sigma_y b} \tag{4.24} \)

\[
\alpha_2 = \frac{Pa\lambda^2}{\sigma_y b}
\]

\( \alpha_2 = \frac{Pa\lambda^2}{\sigma_y b} \tag{4.25} \)

\[
\alpha_3 = \frac{\sigma_y b}{EI\lambda^2}
\]

\( \alpha_3 = \frac{\sigma_y b}{EI\lambda^2} \tag{4.26} \)

The following relationship between displacement, \( v \), and load, \( P \), were derived:
\[ v = 2 \left( \frac{Pl_f^3}{3EI} - R_1l_f + R_2 \right) \]  \hspace{1cm} (4.27)

\[ d = l_p + l_e \quad \quad a = l_f - l_p \]  \hspace{1cm} (4.28)

\[ R_1 = \frac{\alpha_3}{6} (l_p \lambda)^3 + K_1 \]  \hspace{1cm} (4.29)

\[ R_2 = \frac{\alpha_3}{24 \lambda} \left( (l_p \lambda)^4 + 6 \right) \]  \hspace{1cm} (4.30)

\[ K_1 = -\left( K_3 + \frac{K_2}{2 \lambda^2} \right) \lambda \]  \hspace{1cm} (4.31)

\[ K_3 = \frac{1}{4 \lambda} \alpha_3 \]  \hspace{1cm} (4.32)

\[ K_2 = \alpha_3 \lambda \left( \alpha_2 - \frac{1}{2} \lambda^2 l_p^2 + \alpha_1 l_p \right) \]  \hspace{1cm} (4.33)

where \( l_e \) is the length of the part of the beam on the elastic foundation, \( l_f \) is the total length of the free beam part and the plastic zone.

It is notable that unlike Eq. 4.12 derived for the BEF model, Eq. 4.27 should be solved implicitly to find the load \( P \) corresponding to a displacement \( v \). The main drawback of the BEPF model against the BEF model lies here. However, these non-linear equations can be readily solved using such tools as Solver in Excel and this procedure can be automated by writing a simple Excel macro.
Finally, Yamada (1988) obtained Eq. 4.34 for the calculation of the $J$-integral for a DCB joint.

$$J = -2\frac{p}{b}\left(\frac{Pd}{2EI} + R_1\right)$$

(4.34)

Once again the assumption that $\lambda d$ was comparable to, or larger than, $2\pi$ was used to simplify the equation into this final form. It is important to note that if $l_p$ turns out to be negative according to Eq. 4.23 the plastic zone does not exist, and the simpler BEF model can be used to calculate $G$.

4.4.4 Finite element methods applied to fracture mechanics

4.4.4.1 Finite Element Method

The finite element (FE) method, pioneered by Zienkiewicz (1971), is now a well-established means for mathematically modelling stress analysis problems and many other problem types. The stress in a body of almost any geometrical shape under load can be determined using finite element analysis (FEA). Therefore, FEA is a very useful tool for the analysis of bonded joints. It is often used to determine the stress-strain state throughout a joint. However, it also enables us to calculate fracture mechanics parameters for bonded joints containing cracks. Wooley & Carver (1971) and Harrison & Harrison (1972) were among the first to apply FE to a single-lap joint. However, their analyses were simplistic, ignoring fillets and deformation of adherends. Adams & Peppiatt (1974) made a more complicated analysis using plane strain assumption and modelled single and double lap joints with an assumed 45°
triangular fillet. They also compared their results with an experimental stress analysis (Adams et al., 1973) and obtained a very good agreement. Since then, FEA has been used by many researchers in the analysis of bonded joints for both stress-strain and fracture mechanics analysis (Wright, 1978; Harris & Adams, Adams & Harris, 1987; Pickett & Hollaway, 1985; Groth (1988); Crocombe et al., 1990; Pradham et al., 1995; Guild et al., 2001; Abdel Wahab et al., 2004).

In a finite element model, a structure is divided into small elements over which behaviour is described by differential governing equations. The elements are then assembled using the requirements of equilibrium and continuity between neighbouring elements. Correct element selection is important in accurate and efficient modelling. The main features in element selection are element shape and order of interpolation. A number of 2-dimensional continuum elements are shown below. These are 3-noded triangular, 4-noded quadrilateral, 6-noded triangular and 8-noded quadrilateral continuum elements, respectively. The 8-noded element is usually the most effective choice for modelling bonded joints.

Figure 4.9 Conventional 2-D continuum elements
In addition to the conventional 2-D elements shown in Fig. 4.9, there are also special crack tip elements, used to model stress singularities. These are mainly triangular and quadrilateral quarter-point singularity elements. They are shown in Figure 4.10. These elements are used only at the crack tip and should be mixed with the conventional continuum elements.

As can be seen in Fig. 4.10, above, the mid-side nodes are moved to the quarter points to produce a singularity at the crack tip. This simple modification allows the stress near the crack tip to vary as the inverse of the square root of the distance from the crack tip, \((1/r)^{1/2}\), as suggested for the stress field around a crack in a homogenous linear-elastic material. The triangular crack tip element is more effective than the quadrilateral one. This is because of the fact that the radial strain component, \(e_r\), for all rays emanating from the node corresponding to the crack tip, possesses a \((1/r)^{1/2}\) singularity. However, for the quadrilateral quarter-point element \((1/r)^{1/2}\) singularity is achieved only along the element edges. Another advantage of
the triangular crack tip elements over quadrilateral ones is that they enable more effective mesh refinement.

In the case of an elasto-plastic material, the stress singularity is considered a function of the strain hardening exponent, $M$, in the Ramberg-Osgood equation (See Eq. 2.12). The stress near the crack tip varies as $(1/r)^{1/(M+1)}$. Similarly, for a non-homogenous material, such as for a crack terminating at a bi-material interface, the order of stress singularity may differ from $(1/r)^{1/2}$. In this case, the order of stress singularity depends on the material properties of the two constituents and the angle that crack makes with the interface.

As far as adhesive joints are concerned, these two cases are quite common. Adhesive plasticity is often taken into consideration and interfacial cracks are quite common. In such cases, variable order singularity elements should be used in order to obtain accurate results. Here, the concept of shifting the mid-side nodes of a quadratic, isoparametric element is extended to generate an approximated variable order singularity. Nevertheless, the quarter point elements, i.e. square-root stress singularity, are often used as an approximation to these kinds of problems and give acceptable results.

In the case of a more complex geometry, it might be quite difficult to define a crack tip field using continuum mechanics. In these cases, finite element methods can be very useful. The strain energy release rate components for mode I (opening mode) and mode II (shearing mode), i.e. $G_I$ and $G_{II}$, can be computed by using
Irwin's crack closure integral (1958) or using the J-integral concept introduced by Rice (1968).

The modified crack closure methods are relatively easy to apply in comparison with the J-integral method. They allow calculation of $G_I$ and $G_{II}$ independently in a mixed mode problem. However, it should be noted that the total energy release, i.e. $(G_I + G_{II})$ is often considered the critical parameter for crack propagation (Johnson & Mall, 1985). The modified crack closure methods are based on linear elastic fracture mechanics and are valid when the plastic zone is very small. Therefore, they do not account for any plastic deformation. However, the J-integral method is applicable to any stress-strain relationship and therefore takes into account the size of the plastic zone at the crack tip. These two methods will be described in the following sections.

4.4.4.2 The Modified Crack Closure Method

This method is also called the virtual crack closure method. Rybicki & Kanninen (1977) were the first to calculate the strain energy release rate by using the virtual crack method. They used simple 4-noded quadrilateral elements around the crack tip and followed an approach based on nodal forces and displacements at the crack tip. A schematic presentation of the elements and nodes at the crack tip is shown in Figure 4.11.
In Fig. 4.11, nodes b & b' define the actual crack tip and Δc is the virtual closure of the crack. Therefore, a & a' define the new crack tip after the assumed closure. Rybicki & Kanninen (1977) proposed the following expressions for the mode I and mode II strain energy release rates, $G_I$ and $G_{II}$, respectively:

$$G_I = \frac{1}{2\Delta c} F_{y,a} u_{y,a}$$  \hspace{1cm} (4.35)

$$G_{II} = \frac{1}{2\Delta c} F_{x,a} u_{x,a}$$  \hspace{1cm} (4.36)

where $u_y$ and $u_x$ are the relative opening and sliding displacements of corresponding nodes, respectively (e.g. $u_{y,a} = v_a - v_{a'}$, where $v$ is the displacement in the $y$ direction). $F_{y,a}$ and $F_{x,a}$ are the nodal forces that must be applied to close and slide the crack surfaces $ab$ and $a'b'$ in the $y$ and $x$ direction, respectively. Since it is not feasible to calculate the nodal forces and displacements at the same node, Rybicki & Kanninen
(1977) suggested that nodal forces holding node \( b \) and \( b' \) together, i.e. \( F_{y,b} \) and \( F_{x,b} \), can be used to estimate \( F_{y,a} \) and \( F_{x,a} \) provided that length \( l \) is set to equal to \( \Delta c \) and \( \Delta c \) is sufficiently small. These nodal forces can be measured in FEA by placing a very stiff spring between nodes \( b \) and \( b' \). This is schematically presented as \( Z \) in Fig. 4.11. Therefore, Eqs. 4.35 & 4.36 can be modified to:

\[
G_I = \frac{1}{2\Delta c} F_{y,b} u_{y,a} 
\]

\[
G_{II} = \frac{1}{2\Delta c} F_{x,b} u_{x,a} 
\]

4-noded quadrilateral elements are not the most effective elements for modelling crack tips. Because of this, the method has been extended to include 8-noded quadrilaterals and singularity elements, i.e. quarter-point elements (Krishnamurthy et al., 1985; Sethuraman & Maiti, 1988). This is presented schematically in Figure 4.12. On the analogy of Rybicki & Kanninen's model (1977), nodes \( c \) & \( c' \) define the actual crack tip and \( \Delta c \) is the virtual closure of the crack. Therefore, \( a \) & \( a' \) is the new crack tip after the assumed decrease in crack length. Very stiff springs can be placed between nodes \( c \) and \( c' \) and between \( d \) and \( d' \) in order to estimate \( F_{y,a} \) & \( F_{x,a} \) and \( F_{y,b} \) & \( F_{x,b} \), i.e. the nodal forces that must be applied to close and slide the crack surfaces abc and \( ab'c' \) in the \( y \) and \( x \) direction, respectively.

Krishnamurthy et al. (1985) gave the following equations for \( G_I \) and \( G_{II} \) for 8-noded elements:
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\[ G_I = \frac{1}{2\Delta c} \left( F_{y,c} u_{y,a} + F_{y,a} u_{y,b} \right) \]  
\[ G_{II} = \frac{1}{2\Delta c} \left( F_{x,c} u_{x,a} + F_{x,a} u_{x,b} \right) \]

Figure 4.12 Virtual crack closure method using (a) 8-noded quadrilateral, 
(b) quarter-point elements
Similarly, Sethuraman & Maiti (1988) defined $G_I$ and $G_{II}$ for the quarter-point elements, such that:

\[ G_I = \frac{u_{x,b}}{\Delta c} (F_{y,d} + (1.5\pi - 4)F_{y,c}) \]  
\[ G_{II} = \frac{u_{x,b}}{\Delta c} (F_{x,d} + (1.5\pi - 4)F_{x,c}) \]  

4.4.4.3 J-integral Method

The J-integral was defined in Section 4.2.4. As mentioned before, the J-integral along any line path $\Gamma$ surrounding the crack tip counter clockwise (Fig. 4.14) can be defined as:

\[ J = \int_{\Gamma} \left[ W \, dy - \left( T_x \frac{\partial u_x}{\partial x} + T_y \frac{\partial u_y}{\partial x} \right) ds \right] \]  

where $W$ is the strain energy density (Eq. 4.2), $T_x$ & $T_y$ are components of the traction vector $T$ (Eq. 4.3) in the $x$ and $y$ direction, respectively with $u$ is the displacement vector with components $u_x$ & $u_y$ and $ds$ is the differential distance along the path. Although the J-integral was originally developed for homogenous materials, it can also be used for bonded joints. Smelser & Gurtin (1977) showed that the J-integral for a bi-material interface can be calculated accurately using Eq. 4.43 provided that surfaces are traction free and the interface is a straight line. The integration procedure for evaluating Eq. 4.43 will be explained using Figure 4.13.
First of all, in order to make the calculations easier, an integration path consisting of straight lines can be chosen. In this case, the traction vector, $T$, is constant along each straight line. After running an FE model, the stress and displacement variation along any straight line can easily be determined. The derivatives of displacements with respect to $x$, $du/dx$, can be determined numerically by shifting the path a small amount in the $x$-direction, as depicted for path $\Gamma_2$ in Fig. 4.14. The finite element mesh should be fine enough in order to minimize error. Mesh refinement may be particularly important when the $J$-integral is taken along a line where high variations in displacement can occur. The $J$-integral for a rectangular path symmetric around the crack tip, as shown in Fig. 4.13, can be calculated using Eqs. 4.44–47:

$$
\begin{align*}
    ds &= -dx \\
    ds &= -dy \\
    ds &= dy
\end{align*}
$$

**Figure 4.13** Calculation of $J$-integral around a rectangular path symmetric around the crack tip
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\[ J = 2(J_{AB} + J_{BC} + J_{CD}) \]  \hspace{1cm} (4.44)

\[ J_{AB} = \int_{A} W \, dy - \left( \sigma_{x} \frac{\partial u_{x}}{\partial x} + \tau_{xy} \frac{\partial u_{y}}{\partial x} \right) dy \]  \hspace{1cm} (4.45)

\[ J_{BC} = \int_{B} \tau_{xy} \frac{\partial u_{x}}{\partial x} + \sigma_{y} \frac{\partial u_{y}}{\partial x} \, dx \]  \hspace{1cm} (4.46)

\[ J_{CD} = \int_{C} W \, dy - \left( \sigma_{x} \frac{\partial u_{x}}{\partial x} + \tau_{xy} \frac{\partial u_{y}}{\partial y} \right) dy \]  \hspace{1cm} (4.47)

The J-integral method has many advantages over the virtual crack closure method. As mentioned before, unlike the virtual crack closure method, the J-integral method is applicable to any stress-strain relationship and thus can take plasticity into
account. Furthermore, the path independency of the J-integral expression allows the calculation along a contour remote from the crack tip. This makes modelling of the crack tip, a major challenge in the FE analysis, less critical. In fact, it is even possible to choose such a contour that contains only elastic loads and displacements. This allows an elasto/plastic strain energy release rate to be calculated from an elastic calculation.

4.4.5 Discussion

In Section 4.4 several methods for calculating the strain energy release rate, $G$, were described, including those based on experimental compliance (EC) measurements, those based on beam theory and FEA based methods. A comparison of these methods will be made in this section.

The EC method offers a number of advantages. First of all, it requires the determination of the incremental crack length, $\Delta a$, rather than the absolute value of the crack length (the horizontal distance between the point of application of the load and the crack tip) although $G$ is usually required as a function of crack length. Furthermore, it does not require a careful measurement of the specimen and bondline thicknesses or material properties. However, for practical reasons, there is still a need for analytical methods (i.e. those based on beam theory or finite element analysis). One of them is that the load values in a bonded joint, particularly in the case of composite joints, can be as low as 50 N. Therefore the load values are susceptible to error. Since the EC method requires an accurate monitoring of load in order to determine the compliance variation; the load cell should be very well calibrated and
the temperature in the testing laboratory carefully controlled. If an analytical method is used to determine $G$ from the test, it is sufficient to monitor either load or displacement.

It can also be seen from Eq. 4.5 that in the EC method, sufficient data points must be collected to determine the change in compliance as a function of crack length, $dC/da$. This can be difficult to achieve for a fast moving crack. Finally, even if the EC method is used to determine $G$ from tests, analytical methods are useful in predicting behaviour prior to testing. An analytical approach can also be very useful in variable amplitude fatigue analysis provided that the effect of crack tip closure can be ignored, i.e. the crack is assumed fully open as soon as the load is applied. This enables determination of the strain energy release rate corresponding to a single stage in a loading block that can be later used in the numerical crack growth integration. This is very difficult to achieve using the experimental compliance variation, particularly when the number of cycles in a stage is relatively small (In this case, the common practice is to define an average strain energy release rate for the whole spectrum using some statistical techniques, such as the RMS-method (See Sec. 4.6.1).

Several analytical methods were discussed in detail in the previous sections, including the SBT method, the BEF and BEPF models. The most commonly used analytical approach is the simple beam theory (SBT) method (Sec. 4.4.3.1). Although it is recommended in ASTM D5041-98-‘Standard Test Method for Fracture Strength in Cleavage of Adhesives in Bonded Joints’, it neglects the contribution of the adhesive and rotation at the assumed built-in end, which can be a considerable source
of error. Therefore the SBT usually underestimates the strain energy release rate for specimens tested under load control and overestimates it when the testing is carried out under displacement control. Some empirical corrections to the simple beam theory were also discussed (Sec. 4.4.3.2). However, it is noted that if a reliable compliance history is already available, the EC method can be applied, negating the requirement for an analytical method of computing $G$.

Unlike the SBT model, models with a compliant crack front (see Sec. 4.4.3.3), i.e. the beam-on-elastic-foundation model and the beam-on-elastic/plastic-foundation model, take into account the contribution of the rotation at the assumed built-in end as well as the effect of the adhesive and the region beyond the crack. Yamada (1987) showed an excellent agreement between a finite element analysis and the beam-on-elastic-foundation model. Williams (1995) even extended the method to calculate the $G$ value for asymmetric DCB joints used for investigating interfacial failure and a very good agreement with the finite element analysis. Abou-Hamda et al. (1998) also successfully employed the BEPF model in their analysis. It was, thus, decided to use the BEF/BEPF model to calculate the strain energy release rate, $G$, in the current study. However, there have not been any experimental justifications of any of these two methods so far. Therefore, an experimental justification of these two methods will be undertaken first.

### 4.5 Fracture Mechanics Applied to Fatigue

Paris et al. (1961) showed that the fatigue crack growth rate per cycle, $da/dN$, is a function of the stress intensity factor range, $\Delta K$ (i.e. $K_{\text{max}} - K_{\text{min}}$), and the $R$-ratio (i.e.
$S_{min}/S_{max}$). Since a crack is not a stress raiser in the case of a compression loading, it can be assumed to be only a function of $K_{max}$ in the case of a tension-compression cycle. This allows the problem to be handled as if the load was fluctuating between 0 and $S_{max}$. These important functional relationships can be written as:

$$\frac{da}{dN} = f_1(\Delta K, R) \quad \text{for } R>0 \quad (4.48)$$

$$\frac{da}{dN} = f_2(K_{max}) \quad \text{for } R<0 \quad (4.49)$$

The relation between $\Delta K$ and $da/dN$ are given by the Paris law (Eq. 4.50) or by a more complicated equation derived by Forman (Eq. 4.51). The latter also accounts for the effect of R-ratio:

$$\frac{da}{dN} = C_p \Delta K^m \quad (4.50)$$

$$\frac{da}{dN} = \frac{C_p \Delta K^m}{(1-R)K_c - \Delta K} \quad (4.51)$$

where $K_c$ is the fracture toughness, $C_p$ and $m$ are empirical constants dependent on the material properties, fatigue frequency, R-ratio, environment, etc. In the same way, $da/dN$ can be related to other fracture mechanics parameters, such as strain energy release rate, $G$ or $J$-integral, $J$. This choice is system dependent. As mentioned before, although $K$ is widely used for the analysis of metals, it is more complicated to apply to bonded joints, where constraint effects of the substrates on
the adhesive layer make it difficult to define the stress distribution around the crack tip. Therefore, $G$, is often used as the governing fracture parameter in the analysis of bonded joints in preference to $K$.

The maximum strain energy release rate, $G_{\text{max}}$, is usually used for the fatigue analysis of bonded joints, in preference to the strain energy release rate range, $\Delta G$, which is used for testing metals and ceramics; because the facial interference of the adhesives on the debonded surfaces may lead to the generation of surface debris, which may prevent the crack from fully closing, thus giving an artificially high value of $G_{\text{min}}$ (Martin & Murri, 1990). $G_{\text{max}}$ is directly proportional to $\Delta G$, such that:

$$\Delta G \propto (1-R^2) G_{\text{max}}$$  \hspace{1cm} (4.52)

Therefore, the Paris law (Eq. 4.50) for adhesively bonded joints can be defined as a function of $G_{\text{max}}$:

$$\frac{da}{dN} = C \cdot G_{\text{max}}^n$$  \hspace{1cm} (4.53)

In fact, a typical logarithmic (log-log) plot of crack growth rate, $da/dN$, against maximum fracture energy consists of three different regions as shown in Fig. 4.15. Region II is described by the Paris law, thus it is a linear region. Region I is defined by the threshold strain energy release rate, $G_{\text{th}}$. In this region, there is little or no crack growth. In Region III unstable fast crack growth occurs, because $G$ approaches the critical strain energy release rate, $G_c$. 
Unlike the Paris law, the Forman's equation not only describe Region II, but also Region III. All three regions, which approximate a sigmoidal curve, can be represented by the empirical equation below:

$$\frac{da}{dN} = C_p \cdot G_{\text{max}}^{n_1} \left[ 1 - \left( \frac{G_{\text{th}}}{G_{\text{max}}} \right)^{n_2} \right]^{n_3}$$

(4.54)

where $n_1$, $n_2$ and $n_3$ are material constants.
The control mode is also an important issue in fatigue testing. Crack behaviour is dependent on the control mode. In a DCB joint tested in load control, $G$ increases as the crack propagates. Therefore, the crack growth accelerates until $G_{\text{max}}$ approaches $G_c$, which causes fast fracture of the specimen. However, in displacement control, $G$ continuously decreases as the crack extends until the crack is arrested when $G_{\text{max}} \leq G_{\text{th}}$. This allows a number of different tests to be performed on the same specimen. Therefore, constant displacement testing seems to be more advantageous, allowing the crack growth curve and $G_{\text{th}}$ to be obtained from one specimen.

4.6 Fracture Mechanics Applied to Variable Amplitude Fatigue

4.6.1 RMS Method

This approach is similar to that described for cumulative damage rules in Section 3.2. It is based on defining an average strain energy release rate for the whole spectrum. Alternatively, an average stress level can be determined as the representative of the whole spectrum by taking the root-mean-square of the spectrum and the strain energy release rate corresponding to this equivalent stress level can be used in the analysis.

In fact, the equivalent values have been defined in many different ways in the literature depending on the system. To illustrate, Yamada et al. (2000) used the normalized root-mean-cube (RMC) value rather than RMS. He used the minimum-stress-constant and maximum-stress-constant loading spectra to simulate low and
high residual stresses, and defined an equivalent root-mean-cube stress, $\Delta \sigma_{RMC}$, for the whole spectra:

$$\Delta \sigma_{RMC} = \left( \frac{\sum_{i} d\sigma_{i}^{3}}{\sum_{i} n_{i}} \right)^{\frac{1}{3}}$$

(4.55)

and he defined the equivalent R-ratio for the maximum-stress-constant loading as:

$$R = \frac{\sigma_{\text{max}} - \Delta \sigma_{RMC}}{\sigma_{\text{max}}}$$

(4.56)

and for the minimum-stress-constant loading as:

$$R = \frac{\sigma_{\text{min}}}{\sigma_{\text{min}} + \Delta \sigma_{RMC}}$$

(4.57)

Although the RMS approach is frequently used, it should be noted that this is a statistical approach and is only useful for narrow-band random loading.

### 4.6.2 Numerical crack growth integration

After obtaining constant amplitude (CA) fatigue crack growth rate (FCGR) curves ($da/dN$ vs. $G$), an attempt to predict the variable amplitude (VA) fatigue life can be made. This can be accomplished using such well known techniques as the RMS-method or numerical crack growth integration (NCGI). Numerical crack growth integration, which has a solid physical base compared to the RMS-method, is suitable for block-loading spectra. It also allows for the consideration of load sequencing effects.
There is a number of different ways to perform the NCGL. The procedure will be described specifically for a block-loading spectrum. A block-loading spectrum has repeated blocks composed of a number of stages. Each stage is a CA load or displacement fluctuation of $n_i$ cycles. An example, a two-stage block loading spectrum, is shown in Fig. 3.5. In the first approach, the crack size, $a$, and corresponding strain energy release rate, $G$, is assumed to be constant throughout a stage. The maximum strain energy release rate associated with this crack length, $G_{max,i}$, can be calculated using one of the methods described in Section 4.4, such as the BEF model, the crack growth rate per cycle, $(da/dN)_i$, for this stage can be obtained using an appropriate correlation, such as the Paris law (Sec. 4.5). Multiplication of this rate by the number of cycles in the stage, $n_i$, gives the overall crack growth during the stage, $\Delta a_i$, that will be used to find the crack size for the subsequent stage (i.e. $a_{i+1} = a_i + \Delta a_i$). This procedure is repeated until $G_{max}$ exceeds the fracture energy, $G_c$, or in the case of displacement control until it becomes equal to the threshold value, $G_{th}$. This algorithm is summarized, as follows:

\[ a_{i+1} = a_i + n_i \cdot \frac{da}{dN}(G_{max}(S_i, a_i, \ldots)) \]

Until $G_{max,i+1} \geq G_c$ \quad \{ in load control \}

or

Until $G_{max,i+1} \leq G_{th}$ \quad \{ in displacement control \}

\[ a_{i+1} = a_i + n_i \cdot \frac{da}{dN}(G_{max}(S_i, a_i, \ldots)) \]
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It might be necessary to divide a stage into sub-stages, each having its own crack length to make a more precise analysis. Conversely, if the block size is small, crack size can be assumed to be constant throughout an entire block rather than a stage. In this case, \( G_{\text{max},i} \) values for each stage can be calculated using the average crack length and \( \sum \Delta a_i \) is used to estimate the crack size for the next loading block.

If the block size and the crack extension per block are small enough, it is preferable to use the first method, thus taking the sequencing in a block into account, and avoiding combining cycles as much as possible, even if they have equal amplitudes. In fact, the calculation procedure can easily be automated by writing a simple macro, thus a cycle-by-cycle NCGI is as easy to perform as a stage-by-stage or block-by-block integration.

It is important to note that if the load interaction effects are significant, the prediction based on such an analysis will most probably be either conservative due to crack growth retardation or unconservative due to crack growth acceleration. In this case, the numerical crack growth integration can be modified using one of two main approaches, namely models based on crack plasticity or those based on crack closure, which will be described in the following section. In the present study, NCGI will be implemented on a cycle-by-cycle basis by writing a macro in MS Visual Basic.
4.6.3 Load interaction effects

4.6.3.1 Introduction

If the load interaction effects are significant, the prediction based on a simple numerical crack growth integration will most probably be either conservative due to crack growth retardation or unconservative due to crack growth acceleration. In this case, the numerical crack growth integration can be modified using one of two main approaches, namely models based on crack plasticity or those based on crack closure. They are described in the following sections.

4.6.3.2 Models Based on Crack Tip Plasticity

Models based on crack tip plasticity are particularly useful to explain crack growth retardation following an overload. This is a well-known phenomenon due to the compressive residual stresses in the plastically deformed region in the vicinity of the crack tip. Overloads cause the stresses at the crack tip to exceed the yield point, thus creating a relatively large overload plastic region. The elastically deformed region around induces compressive residual stresses in the plastically deformed region, which tends to close the crack, retarding the crack propagation.

The Paris and Forman laws (Sec. 4.5) can be modified to take the retardation effect into account. Two approaches are commonly used: Wheeler's Model (Wheeler, 1972) and Willengborg's Model (Willengborg et al., 1971).
- Wheeler's Model

In Wheeler's (1972) model, the Paris or Forman's equations (Eqs. 4.37 & 4.38) are modified by using a retardation factor $A$. $A$ is a function of the crack size just after the overload, $a_{OL}$, the size of the overload plastic zone, $r_{OL}$, and size of the plastic zone induced by the subsequent loading, $r_p$. The overload plastic zone size (i.e. the extent of the plastically deformed region ahead of the crack tip) is given by:

$$r_{OL} = Z \left( \frac{K_{OL}}{\sigma_y} \right)^2$$

(4.58)

where $K_{OL}$ is the stress intensity factor corresponding to the overload, $\sigma_y$ is the yield strength of the material and $Z$ is a constant mainly dependent on the geometry of the plastic zone. Irwin (1960) assumed a circular plastic zone and proposed that $Z$ is equal to $(1/3\pi)$ for plain strain and $(1/\pi)$ for plane stress, whereas Dugdale (1961) assumed a long, slender plastic zone and gave the value of $(\pi/8)$ for $Z$ for plane strain condition.

Therefore, modification of Forman's equation (or the Paris eqn.) for retarded cycles:

$$\frac{da}{dN} = A_i \frac{C_{p,j} \Delta K_i^{n_i}}{(1 - R_i) K_c - \Delta K_i}$$

(4.59)

where

$$A_i = \left[ \frac{r_{p,i}}{r_{\text{max},i}} \right]^k$$

where $r_{\text{max},i} > 0$ (4.60)
where \( r_{p,i} \) is the plastic zone size associated with the \( i \)th (post-overload) stress cycle and \( r_{\text{max},i} \) is the distance from the current crack tip to the elastic-plastic boundary created by the overload (i.e. \( r_{\text{max},i} = (a_{\text{OL}} + r_{\text{OL}}) - a_i \)) and \( k \) is an empirical exponent chosen to obtain best overall crack growth prediction for a given material and loading spectrum. \( r_{p,i} \) can be approximated, such that:

\[
  r_p = Z \left( \frac{K_i}{\sigma_y} \right)^2
\]

(4.61)

- Willengborg’s Model

Willengborg’s model (Willengborg et al., 1971) is based on the assumption that the compressive residual stresses ahead of the crack tip induced by an overload cause a reduction in stress intensity factor by an amount \( K_{\text{red}} \). This is defined as the difference between \( K_{\text{req}} \), i.e. the stress intensity factor that must be exceeded to create a plastic zone that will extend to the border of the overload plastic zone, and the current value of \( K \). Thus, an effective stress intensity factor, \( K_{\text{eff}} \), can be used in the Paris or Forman’s equations (Eqs. 4.50 and 4.51). This procedure can be summarised, such that:

\[
  \frac{da}{dN} = \frac{C_{p,i} \Delta K_{\text{eff},i}^{n_i}}{(1 - R_{\text{eff},i})(K_e - \Delta K_{\text{eff},i})}
\]

(4.62)

where

\[
  K_{\text{max,eff},i} = K_{\text{max},i} - K_{\text{red},i}
\]

(4.63)

\[
  K_{\text{min,eff},i} = K_{\text{min},i} - K_{\text{red},i}
\]

(4.64)
Models based on crack tip plasticity are particularly applicable to metals at temperatures where yielding is significant rather than composite materials, which are generally quite brittle.

4.6.3.3 Models Based on Crack Closure

More recent crack growth models incorporate crack closure, a concept first introduced in the pioneering work of Elber (1970). Unlike the plasticity-based models, crack closure can also be used to explain crack growth acceleration.

Crack closure is based on the assumption that a crack under fatigue loading is fully open for only a part of the load cycle, even when the loading is tension-tension (i.e. $R > 0$). This is usually attributed to the plastic deformations in the wake of the propagating crack. This kind of crack closure is called the plasticity-induced crack closure. There are also oxide-induced and roughness-induced crack closure but at sufficiently high $\Delta K$ values crack closure is predominantly described in terms of the plasticity-induced closure.

Elber defined a parameter called the crack opening ratio, $U$:

$$ U = \frac{K_{\text{max}} - K_{\text{op}}}{K_{\text{max}} - K_{\text{min}}} = \frac{\Delta K_{\text{eff}}}{\Delta K} $$ (4.67)
where $K_{op}$ is the crack opening stress intensity factor and $\Delta K_{efr}$ (although any fracture mechanics parameter can be used in the same way) is the effective stress intensity factor range that is used in the Paris law (Eq. 4.50), such that:

$$\frac{da}{dN} = C_p (U \Delta K)^n = C_p (\Delta K_{efr})^n$$  \hspace{1cm} (4.68)

Several researchers have tried to formulise $U$. For instance, Elber (1970) proposed that $U$ is a function of the $R$-ratio only and derived the following equation for thin sheets of aluminium:

$$U = 0.5 + 0.4R$$  \hspace{1cm} (4.69)

This was later modified by Schijve (1981) into:

$$U = 0.55 + 0.35R + 0.1R^2$$  \hspace{1cm} (4.70)

However, now it is well-known that $U$ depends not only on $R$ but also on the thickness of the material, crack length, etc. Thus, an accurate estimation of $U$ usually requires the experimental measurement of $K_{op}$. $K_{op}$ is usually determined by the compliance method. This is based on the fact that compliance changes sharply when the crack opens. Therefore, $K_{op}$ could be identified from the load, $P$, vs. crack opening displacement curve, $COD$, as the stress intensity factor corresponding to the load level where the slope changes abruptly. This is shown in Figure 4.16.
However, it should be noted that this point usually is not so distinct. There are a number of approaches to accomplish this. For example, Josefson et al. (2000) fitted a polynomial to the \( P \) vs. \( \text{COD} \) curve and searched for the point corresponding to the largest second derivative. As the second derivative gives the rate of change of the compliance, it should maximize during crack opening.

It is usually seen that \( K_{op} \) stabilizes at a certain level under constant-amplitude (CA) loading. The fact that \( K_{max} \) and \( K_{min} \) change in each stage of a variable amplitude (VA) loading implies that \( K_{op} \) must also change continuously, which complicates the problem. However, some of the studies on variable amplitude fatigue loading have shown that even under variable amplitude loading the crack closure level, somehow, stabilizes after a while. For instance, from results on the testing of steel specimens, Josefson et al. (2000) has shown that the crack closure...
level stabilizes at a certain level provided the maximum load is repeated often enough in a loading sequence. Lee & Song (1999) observed the same behaviour for short cracks using aluminium alloy plates.

Another interesting observation is that the stabilized $K_{op}$ during VA fatigue cycling usually proves to be lower than the one corresponding to CA loading, resulting in a higher effective stress intensity factor range, $\Delta K_{eff}$. This is particularly the case for physically short cracks ($\leq 1$ mm). Lee & Song (1999) showed that the primary factor governing fatigue crack growth of physically short cracks under random loading is the crack closure and they observed that the crack opening load of short cracks is much lower under random loading than under CA loading corresponding to the largest load cycle. This results in the crack growth acceleration under VA loading, which has been observed by several investigators (Nisitani & Nakamura, 1982; McClung & Sehitoglu, 1988; Vormwald & Heuler, 1993; Frost et al., 1974; Gurney, 1979). As mentioned in Sec. 3.4.1, this is a very important phenomenon considering the fact that even nominally defect-free specimens are likely to have such short cracks that are initiated during the early stages of the fatigue cycling or pre-exist as a material flaw.

In summary, crack closure is a very useful concept that can be used to explain both crack growth acceleration and retardation. However, as can be seen, unlike the plasticity models, such a model still entails some experimental VA data, specifically, $K_{op}$ under VA loading. Furthermore, the crack closure in bonded composite joints is usually ignored, i.e. it is assumed that the crack opens as soon as a positive load is applied. Although it is thought that crack opening might be an important factor
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governing the fatigue of bonded joints as well, due to the lack of any previous studies and precision equipment, the crack closure is ignored throughout this study.

4.6.3.4 Discussion

Although there have been a few previous studies on the VA fatigue of bonded joints (Jeans et al., 1981; Yang et al., 1983; Jones & Williams et al., 1989), (Sec. 3.4.1), these were based on cumulative damage rules, such as the Palmgren-Miner’s (P-M) rule.

Therefore, the main task to be undertaken in the present study will be to determine whether numerical crack growth integration is a valid method for predicting crack growth in bonded composite joints subjected to spectrum loading and to define load interaction effects.

4.7 Fatigue Crack Growth Measurement Techniques

There are a number of different techniques to measure the rate of fatigue crack growth, \( da/dN \), in a fracture test. These methods can be divided into two main categories: direct measurement techniques, where the crack is directly observed, and indirect methods, where a parameter (e.g. load or electrical resistance) dependant on the crack length is recorded. Some of the crack growth measurement techniques are described below.
- Optical Microscope

The simplest direct method is the use of an optical microscope. This is the most commonly used method for monitoring a crack in bonded joints. The specimen is viewed from one side (both sides if possible) using a traveling optical microscope fixed to a rigid base with a vernier scale (see Sec. 5.4.3). The side of the specimen is usually painted with white correction fluid or polished prior to testing to highlight the crack and the vernier scale usually allows incremental measurement of crack length to ±0.01 mm. This exceeds the required resolution of 0.1 mm (0.004 in.) in ASTM-647- ‘Standard Test Method for Measurement of Fatigue Crack Growth Rates’ by a factor of 10. Although this is an inexpensive and simple technique, it does not give accurate and reproducible results as the identification of the crack tip will vary with the user. The method cannot be automated, so it is very tedious and time-consuming for the researchers. Furthermore, specimen vibrations can make it very difficult to locate the crack. However, an optical microscope will be utilized in this study for calibration purposes (see Sec. 5.4.3).

- DCPD Method

One of the most commonly used indirect techniques is the direct current potential drop (DCPD) method. This involves monitoring changes in the electrical resistance of the specimen during crack growth.

In this method the specimen is supplied with a large direct current and the resulting potential drop across the crack site is measured. This potential drop increases as the crack grows. Therefore, the crack length can be evaluated by interpreting the voltage after a suitable calibration. The voltage readings are usually
very small and so it is difficult to ensure stability of the current produced and the amplified output signal. It is also important to note that most structural adhesives are not electrically conductive, thus act as an insulator between the bonded substrates.

- ‘Krak Gage’ and ‘Fractomat’ System

If the specimen tested is not electrically-conductive (e.g. bonded CFRP joints), metal foils adhesively-bonded to the surface of the sample can be used to utilize the DCPD method. There is a commercially available crack growth monitoring system based on this technique: the ‘Krak Gage’ and ‘Fractomat’ system from Rumul™. This system enables a very accurate monitoring of crack length. As the crack grows, the gauge bonded on one side of the sample along the bondline (see Section 5.3.3) tears and the increase in its resistance is measured by the ‘Fractomat’ and converted to a crack length. The Fractomat (Fig. 4.17) is a complete 2-channel system enabling the use of two Krak Gages at the same time. Therefore if asymmetry in crack growth is a matter of concern, Krak Gages can be bonded to both sides of the specimen and the crack-length on each side can be recorded simultaneously. Alternatively, a single output can be obtained taking the average of the values from the two channels. Although the infinite resolution of the Krak Gage guarantees an accuracy of about ±0.1μm, the Fractomat limits it to ±0.01 mm. This still exceeds the ASTM standard by a factor of 10. The Krak Gage and Fractomat system is considered to be the most reliable method for measuring the fatigue crack-length under ‘dry’ fatigue conditions.
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The indirect potential drop method can be utilized using alternative current (AC). In comparison to the DCPD methods, the effective resistance of the alternative current potential drop (ACPD) method is greater, thus smaller inputs are needed to produce the same surface electric field, which makes it particularly suitable for larger specimens. Furthermore, the percentage of the total current disturbed by the crack is much higher, giving greatly improved sensitivity. However, it is not very suitable for the inspection for buried defects and there is no commercially available AC system at the moment.

- **ACPD Method**

The indirect potential drop method can be utilized using alternative current (AC). In comparison to the DCPD methods, the effective resistance of the alternative current potential drop (ACPD) method is greater, thus smaller inputs are needed to produce the same surface electric field, which makes it particularly suitable for larger specimens. Furthermore, the percentage of the total current disturbed by the crack is much higher, giving greatly improved sensitivity. However, it is not very suitable for the inspection for buried defects and there is no commercially available AC system at the moment.

- **Non-destructive testing methods**

Non-destructive testing (NDT) methods such as ultrasonic inspection, acoustic emission and X-ray radiography identify the presence of a crack by observing the changes in reflected or transmitted signals. As mentioned above, the
optical microscope or the Fractomat system measure the crack length only at the side of the specimen, which may be different to that in the centre. In fact, it is well known that the crack front is usually a 'thumb nail' shape. An example of such a crack front is presented in Figure 4.18. The NDT methods enable the shape of the fatigue crack front to be observed. They produce a visible image of the voids, cracks buried in the specimen.

However, they usually fail to meet the required resolution of 0.1 mm and are very difficult to implement in situ. They also may require special treatment of the specimen prior to the inspection, e.g. saturating the sample in a penetrant for penetrant-enhanced X-ray radiography (PEXR). Therefore, in most cases they are not a convenient way of measuring fatigue crack growth.

![Diagram of crack front with labels for crack tip, crack front, and crack growth direction.](image)

**Figure 4.18** X-radiograph showing a 'thumb nail' crack front in a composite lap-strap joint (Ashcroft et al., 1997)

In the present study, the crack length variation during quasi-static testing and fatigue cycling will be determined using the commercial 'Krak Gage' and
'Fractomat' system. It is very suitable for bonded composite joints tested at relatively high frequencies (10 Hz) under dry conditions. The crack length data will be logged on a computer. A travelling microscope (shown in Fig. 5.9) will also be utilized on the opposite side of the sample as a measurement check.

4.8 Estimation of fatigue crack propagation rate, $da/dN$

The selection of $\Delta a$ increments between crack length measurements has a statistical significance. According to ASTM E647- 'Standard Test Method for Measurement of Fatigue Crack Growth Rates', the minimum value of $\Delta a$ is required to be 0.25 mm (0.01 in.) although it can be reduced below 0.25 mm for the data points in the near-threshold regime. However, the minimum $\Delta a$ is 10 times the crack length measurement precision. It is recommended that $da/dN$ data are nearly evenly distributed with respect to $G$.

Once an $a$ against $N$ curve is obtained, one of several reduction techniques can be used in order to estimate the fatigue crack propagation rate, $da/dN$. The most important of these are the secant method, the incremental polynomial method and the graphical method. The secant and incremental polynomial methods are recommended in the ASTM E647- 'Standard Test Method for Measurement of Fatigue Crack growth Rates'. All three methods are described below.
- Secant method

The secant method is a point-to-point method and involves calculating the slope of the straight line connecting two adjacent data points on the $a$ against $N$ curve. It is defined as:

$$FCGR = \left( \frac{\text{d}a}{\text{d}N} \right)_{\bar{a}} = \frac{a_{i+1} - a_i}{N_{i+1} - N_i}$$

where $(a_i, N_i)$ and $(a_{i+1}, N_{i+1})$ are two adjacent data points and the calculated crack growth rate is associated with the average crack length, i.e. $\bar{a} = (a_{i+1} + a_i)$. This method can give very large scatter and so has not been favoured by many researchers.

- Incremental polynomial method

In the incremental polynomial method, the crack growth rate is estimated by fitting a second order polynomial to sets of $(2n+1)$ successive data points, where $n$ is usually 1, 2, 3 or 4. The form of this equation is defined as:

$$\hat{a}_i = b_0 + b_1 \left( \frac{N_i - C_1}{C_2} \right) + b_2 \left( \frac{N_i - C_1}{C_2} \right)^2$$

where

$$-1 \leq \left( \frac{N_i - C_1}{C_2} \right) \leq 1$$

and

$$C_1 = (N_{i,n} + N_{i,n+1})/2 \quad ; \quad C_2 = (N_{i+1,n} + N_{i+1,n+1})/2$$
where $b_0$, $b_1$ and $b_2$ are the regression coefficients determined by the least squares method over the range $a_{i-n} \leq a \leq a_{i+n}$, $C_1$ and $C_2$ are scaling parameters to avoid any numerical difficulties in determining the regression parameters and $\bar{a}_i$ is the fitted value of crack length at $N_i$. The use of 7 successive points seems to be the most popular choice in curve fitting. Because of that, it is sometimes called the 7 point polynomial method.

- **Graphical method**

The graphical method is similar to the incremental polynomial method. However, the polynomial fit is made to the entire data points instead of a local fit and then this polynomial equation is differentiated to give $da/dN$ at any point. This method gives reliable results only when all the data points lie in a smooth curve.

In the current study, the 7-point polynomial method will be used to estimate the fatigue crack growth rates, $da/dN$. 
5.1 Introduction

The following chapter introduces the materials and joints studied in this project. The main experimental techniques and loading spectra used will also be discussed.

5.2 Materials

5.2.1 Composite substrate

The adherends used throughout this study were epoxy based carbon fibre reinforced (CFRP) composite, Hexcel® AS4/8552. Hexcel type AS4 is a continuous, standard modulus fibre with a Young's modulus of 228 GPa and tensile strength of 4150 MPa. It was supplied in 12000 (12K) filament count tows. HexPly® 8552 is a high performance tough epoxy matrix for use in primary aerospace structures. Multidirectional (MD) composite panels were produced from unidirectional (UD) prepreg tape. Laminate thickness was 0.25 mm and nominal fibre volume was %57.42. Laminates were laid up using a (0°, +45°, -45°, 90°)s stacking sequence. The composite panels were autoclave cured at 180°C for 130 minutes and then grit blasted and degreased prior to bonding. Material properties for UD tape determined by British Aerospace are presented in Table 5.1. The mechanical properties of the MD composite were estimated from the properties of the individual plies using laminate analysis. They are presented in Table 5.2.
5.2.2 Epoxy adhesive

The adhesive used throughout this study was a single part epoxy paste, Cytec® 4535A. The composite panels were secondary bonded by curing the adhesive in a bonding press at 135°C for 60 minutes with the panels shimmed to give a 0.5 mm bondline thickness. The Young's modulus of this adhesive was determined by testing 10 samples of bulk adhesive in accordance with ISO 527-2: 1996. A typical stress-strain curve is as shown in Figure 5.1. The modulus was found to be 1.1 GPa with a standard deviation of 81 MPa. Poisson's ratio for polymers well below the glass transition temperature can be assumed to be 0.33 (Adams et al., 1997). Therefore, the Poisson's ratio for the adhesive, \( \nu \), was assumed to be 0.33 in our analysis.
5.3 Adhesive Joint Preparation

5.3.1 Introduction

Two different types of specimen geometry were employed in the current investigation:

(i) Double lap joint (DLJ) (see Section 2.3.2)
(ii) Double-cantilever beam (DCB) (see Section 4.3)

Two types of DCB joint configuration were used in the test programme. These were termed AF and CF-type DCB joints. They will be described in Section 5.3.3.
Figure 5.2 Double lap joints tested in the present study

5.3.2 Double lap joints (DLJ)

Double lap joints with an overlap length of 12.5 mm were used in this study (Fig. 5.2). Specimens were made by Qinetiq® and end tabs were attached by us. The materials used were described in Section 5.2. Composite panels were secondary bonded ensuring a bondline thickness of $0.5 \pm 0.05$ mm. Aluminium tabs were bonded onto the specimen ends to provide a grip area using a two-component structural adhesive that exhibits high peel and tensile lap shear strength, Hysol® EA 9359.3 from Loctite. The adhesive was cured at 82°C for 60 minutes. Specimen geometry is shown in Figure 5.3.

Figure 5.3 Dimensions of double lap joints (DLJ)
5.3.3 Double-cantilever beam (DCB) joints

Double-cantilever beam (DCB) joints with a bondline thickness of 0.5 mm were used in this study (Fig. 5.4). Specimen dimensions are presented in Figure 5.5. Materials used were the same as those used in the double lap joints and were previously described in Section 5.2. Brass hinges were bonded to the specimens in order to provide a point to apply the load. ‘Krak Gages’ (see Section 4.7) were bonded to one side of the sample along the bondline, as shown in Fig. 5.4.

Two types of joint configuration were used in the test programme. The first type of joint incorporated a very thin sheet of PTFE as a starter crack at the adhesive bondline. These joints were produced by bonding cured panels of composite. The second sample type consisted of two composite panels joined with no adhesive layer and a starter crack between the 0° plies at the centre of the joint, i.e. by co-curing the CFRP laminates. By the use of these two types of sample it was possible to investigate crack growth through the adhesive (cohesive failure) and through the composite (interlaminar failure). These specimens were termed AF and CF-type, respectively.

Figure 5.4 ‘Krak Gage’ attached to bonded DCB joint
After the manufacture of the samples, brass alloy hinges were bonded to the specimens in order to provide a point to apply the load. They were bonded using a two-component structural adhesive that exhibits high peel and tensile lap shear strength, Hysol® EA 9359.3 from Loctite. The adhesive was cured at 82°C for 60 minutes. Finally, ‘Krak Gages’ to be used to measure crack length (see Section 4.7) were bonded to one side of the sample along the bondline, as shown in Fig. 5.4, using a two-component epoxy adhesive, M-Bond® AE-15 from Vishay. This adhesive is recommended for use with strain gages and special-purpose sensors. After solvent cleaning of the sides, the adhesive was applied and cured at 50°C for 6 hours. The next step was to wire the Krak Gages. This required soldering of the input/output leads of Krak Gages. Before soldering, the terminals on the solder pad on the Krak Gage were roughened using a glass pen. This also removed any loose adhesive. In addition, the Krak Gages were cut using a razor blade along the line between the input and output
terminals to prevent tearing of the solder pads during the initial cracking of the gauge. After soldering, the wires were loosely attached to the specimen using adhesive tape in order to prevent solders being broken during handling and testing.

5.4 Experimental Test Procedures

5.4.1 Introduction

The following tests were conducted on each type of specimen:

(i) Quasi-static tests
(ii) Constant amplitude (CA) fatigue tests
(ii) Variable amplitude (VA) fatigue tests

5.4.2 Testing of double lap joints (DLJ)

The testing was undertaken in the ambient laboratory environment. A servo-hydraulic, fatigue testing machine with computerised control and data logging was utilised for the testing. It is shown in Fig. 5.6.

- Quasi-static testing of DLJ specimens

The quasi-static testing was conducted using a constant displacement rate of 1 mm/min.
Figure 5.6 Fatigue testing machine used for testing DLJ

- Constant amplitude (CA) fatigue testing of DLJ specimens

In the CA test programme, the spectra under which the double lap joints were tested in load control had sinusoidal waveforms (see Section 2.2) with a frequency of 10 Hz and load ratios (i.e. min. load /max. load) of $R = 0.1$ and 0.5. The fatigue threshold was defined as the highest maximum load in a loading block at which a sample could survive $10^6$ cycles with no observable damage using an optical microscope.

In some polymers, cyclic loading can result in significant hysteretic heating. In order to check if the adhesive used in this testing programme was susceptible to this, the temperature of one of the joints was monitored using thermocouples attached to the
specimen during fatigue testing. One of the thermocouples was attached to the adhesive near the fillet area and the other was attached to the adhesive near the PTFE block. This experimental set-up used is shown in Fig. 5.7. No increase in temperature was observed with either thermocouple during fatigue cycling, even when a peak load of 10 kN and a frequency of 35 Hz was applied. This is probably because heat generation is localised at points of high stress in the adhesive and any heat generated is quickly dissipated in the surrounding material.

Figure 5.7 Set-up used for monitoring hysteretic heating of DLJ

- Variable amplitude (VA) fatigue testing of DLJ specimens

The VA tests were also carried out in load control, with the same frequency, 10 Hz, and ambient laboratory environment. The main spectrum input to the controller was
representative of the loading on a joint in a composite wing. For convenience, load levels in the VA loading spectra were identified by the highest load value in the spectrum (i.e. peak load), $L_{\text{peak}}$. All other loads are scaled accordingly. The main spectrum corresponding to $L_{\text{peak}}$ of 9 kN is shown in Figure 5.8.

![VA spectrum for a peak load of 9 kN](image.png)

**Figure 5.8** VA spectrum for a peak load of 9 kN

In this complex spectrum, one block composed of 100 cycles and 17 stages with a variety of maximum load, $L_{\text{max}}$, and $R$-values. This spectrum exhibits all the major features responsible for load interaction effects, e.g. overloads, changes in mean load and changes in load amplitude (see Section 3.4.1). Several other spectra were designed for the investigation of the ‘cycle mix’ effect. They will be described in detail in Section 6.7.2. Like CA loading, the fatigue threshold for VA loading was defined as
the highest maximum load in a loading block at which a sample could survive $10^6$ cycles with no observable damage using an optical microscope.

The test spectra were programmed in the test machine software using a complex function generator prior to the testing. The MIMICS (Multiple Input Minimal Integral Control Synthesis) control system (Stoten & Benchoubane, 1990) was used to improve control in the spectrum testing.

5.4.3 Testing of double-cantilever beam (DCB) joints

Testing was carried out in a temperature controlled laboratory environment (22±2°C) in accordance with ISO 291 using a servo-hydraulic fatigue testing machine with computerised control and data logging. Prior to each test, specimens were loaded at a constant cross-head speed of 1 mm/min until the crack was seen to move on the edge of the specimen in order to provide a sharpened fatigue crack of adequate size and straightness as suggested by Blackman & Kinloch (2001). The fracture mechanics test set-up will be described in the subsequent section.

- **Quasi-static testing of DCB joints**

  The quasi-static testing was conducted using a constant cross-head speed of 1 mm/min.

- **Constant amplitude (CA) fatigue testing of DCB joints**

  Testing was carried out under displacement control. The CA fatigue spectra used were sinusoidal waveforms with a frequency of 10 Hz and displacement ratios (i.e. min. disp. /max. disp.) of $R = 0.1$ and 0.5. The crack closure in bonded composite joints
is usually ignored, i.e. it is assumed that the crack opens as soon as a positive load is applied (see Section 4.6.3.3). Therefore, these represent absolute values.

The crack length variation during the fatigue cycling was determined using the commercial ‘Krak Gage’ and ‘Fractomat’ system from RUMUL. As described in Section 4.7, ‘Krak Gages’ (see Fig. 5.4) are thin strips of constantan, which are designed to tear as the sample cracks enabling accurate measurement of crack length by the ‘Fractomat’. An important aspect of operating in displacement control is that strain energy release rate, $G$, continuously decreases as the crack extends until the crack is arrested when $G$ is smaller than the threshold value, $G_{th}$. This allowed a number of different tests to be performed on the same specimen, utilizing the long Krak Gage length (100 mm). A travelling microscope (see Fig. 5.9) was also utilized on the opposite side of the sample as a measurement check. The fracture mechanics set-up is schematically shown in Figure 5.10.

**Figure 5.9** Traveling optical microscope used in this study
Figure 5.10 Set-up of the fracture mechanics tests

- Variable amplitude (VA) fatigue testing of DCB joints

The VA spectra were two-stage block loading spectra with displacement ratios of $R = 0.1$ (overloads) and $R=0.5$ (remaining cycles) as shown in Figure 5.11. The overloads were applied every 20 cycles. For convenience, displacement levels in the VA loading spectra are identified by the highest displacement value in the spectrum (i.e. peak displacement), $v_{peak}$. All other displacements are scaled accordingly.

For fatigue tests, accurate measurements of specimen dimensions are particularly important because analytical methods were used for calculating the strain energy release rate, $G$. This requires a careful measurement of the specimen and bondline thicknesses. Measurements of specimen dimensions were made as suggested by Blackman & Kinloch (2001):
The thickness of each substrate was measured using a micrometer prior to bonding. Measurements were made at three different points along the length of the beam (30 mm from either end, and at the mid length) and their average was obtained.

The thickness of each specimen was measured after bonding and a value of bondline thickness was determined by subtracting the substrate thicknesses from the total thickness of the joint.

The width of the specimen was measured after bonding using a vernier calliper. Measurements were made at three different points along the length of the beam (30 mm from either end, and at the mid length) and their average was obtained.

Figure 5.11 Two-stage block loading spectrum used in the testing of DCB joints
6.1 Introduction

In this part of the research, double lap joints (DLJ) were tested under constant and variable amplitude loading. The main attention was given to cumulative damage rules (see Section 3.3), the most popular methods for predicting variable amplitude (VA) fatigue life using the data obtained from constant amplitude (CA) tests.

In this chapter, Palmgren-Miner’s rules (see Section 3.3.1) will be assessed for bonded joints subjected to a representative aerospace fatigue spectrum. The RMS method will also be briefly addressed. The critical load interactions, i.e. the effect of overloads, understress cycles, mean stress variations, etc. will be defined and a model incorporating a ‘cycle mix’ factor will also be proposed. Finally, the mechanism of failure, crack initiation and propagation in bonded joints will be discussed at the end of this chapter.

6.2 Prediction Methods

6.2.1 Introduction

Two conventional methods of predicting VA fatigue life using CA data were investigated initially. These were the Palmgren-Miner’s rule (see Section 3.3.1) and the
RMS Method (see Section 3.2). A method incorporating a "cycle mix" factor (see Section 3.4.3) was also proposed and will be introduced in later sections.

### 6.2.2 RMS method

The RMS method was described in Section 3.2. The root-mean-square (RMS) values and corresponding R-ratios was defined in terms of load for the joints studied in this project. Therefore, Eqs. 3.1 - 3.3 were modified accordingly.

\[
L_{\text{rms}}^{\text{max}} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (L_{\text{max},i})^2}
\]

(6.1)

\[
L_{\text{rms}}^{\text{min}} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (L_{\text{min},i})^2}
\]

(6.2)

\[
R_{\text{rms}} = \frac{L_{\text{rms}}^{\text{min}}}{L_{\text{rms}}^{\text{max}}}
\]

(6.3)

where $L_{\text{max}}$ and $L_{\text{min}}$ are the maximum and minimum load, respectively, of a loading cycle in the spectrum, $M$ is the total number of $L_{\text{max}}$ or $L_{\text{min}}$.

### 6.2.3 Palmgren-Miner’s (P-M) rule

The Palmgren-Miner’s (P-M) rule is the most widely used cumulative damage rule. It was described in detail in Section 3.3.1. As previously mentioned, in the P-M rule, damage is considered accumulating in a linear manner without consideration of the load interaction effects. For a block-loading spectrum, Palmgren-Miner’s rule can be defined as:
where $N_B$ is the number of loading blocks to failure; $n_B$ is the number of constant amplitude stages in a block; $n_i$ is the number of cycles in a stage with a stress level corresponding to a fatigue life of $N_i$, and finally $C$ is called the Miner's sum or damage sum. $C$ would be equal to 1.0 for one hundred percent damage (i.e. failure) if there are no load interactions.

In this study, both P-M and Extended P-M rules were employed. The latter was used to take the loading below the fatigue limit into account, as shown in Fig. 3.3, and also described in Section 3.3.1.

6.3 Experimental

6.3.1 Joint configuration

Double lap joints (DLJ) were used in this study. The material used and specimen geometry was described in detail in Sections 5.2 and 5.3.2.

6.3.2 Test conditions

Specimens were tested under quasi-static, constant amplitude (CA) fatigue and variable amplitude (VA) fatigue loading. Testing conditions were described in detail in Section 5.4.2.
6.4. DLJ Testing Results

All joints failed predominantly in the composite adherends, in the 0°-ply adjacent to the bondline. The fracture path of a failed specimen is shown in Figure 6.1. On one side of the joint, failure was in the outer adherend, and on the other side, failure was in the middle adherend. The loci of failure will be discussed further in Section 6.8. The important thing at this point is that loci of failure were the same for the specimens tested under both constant amplitude (CA) and variable amplitude (VA) loading, an important condition that should be satisfied in order to predict VA fatigue life from CA data.

Figure 6.1 Fracture path for DLJ

$L-N$ curves (maximum load, $L_{peaks}$ against number of cycles to failure, $N$ [see Section 2.3]) for VA and CA spectra were constructed (see Fig.6.2). As mentioned before, the fatigue threshold for VA loading was defined as the highest maximum load in a loading block at which a sample could survive $10^6$ cycles with no observable
damage using an optical microscope. The results are presented in Table 6.1. It can be seen that the CA fatigue threshold in terms of $L_{\text{peak}}$ (i.e. the maximum load of peak cycle, $L_{\text{peak, max}}$) increases as the load ratio, $R$, increases. However, this depends on how the loading spectrum is characterised. If the threshold value is defined in terms of $\Delta L_{\text{peak}}$ instead of $L_{\text{peak}}$ it can be said that the threshold value decreases as $R$ increases.

<table>
<thead>
<tr>
<th>Static Failure Load (kN)</th>
<th>$R$</th>
<th>Fatigue Threshold $L_{\text{peak}}$ (kN)</th>
<th>Fatigue Threshold $\Delta L_{\text{peak}}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.8 ($\pm$ 1.4)</td>
<td>0.1</td>
<td>6.8</td>
<td>6.1</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>7.8</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td>VA</td>
<td>8</td>
<td>7.2</td>
</tr>
</tbody>
</table>

**Table 6.1 Summary of results for DLJ's**

Another observation is that the threshold value for the VA spectrum is almost the same as that for $R = 0.5$ CA loading. However, this also depends on how we compare the spectra. In this case we have defined the VA spectra in terms of the peak load in the spectra. In fact, most of the cycles had a much smaller maximum load value than this peak load, by at least 20%. Therefore, it can be said that the fatigue threshold value for VA is much smaller than expected. The $L$-$N$ curves for the VA spectrum and the CA spectra are presented together in Figure 6.2. As seen, it was possible to fit a linear curve to VA $L$-$N$ curve and CA $L$-$N$ curve corresponding to $R=0.5$. However, a third-order polynomial fit was required for $R=0.1$. 
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120

Figure 6.2 L-N plots for CA and VA fatigue of DLJ's

6.5 RMS Results

RMS values corresponding to the main spectrum (see Fig. 5.8) were calculated. As previously mentioned, in this study the variable amplitude (VA) spectra were defined in terms of the peak load in the spectra, $L_{\text{peak}}$. $L_{\text{max}, \text{rms}}$ and $L_{\text{min}, \text{rms}}$ were calculated in terms of $L_{\text{peak}}$ using Eqs. 6.1-6.3. The results are presented in Figure 6.3. Constant amplitude (CA) load cycles with $L_{\text{max}, \text{rms}}$ and $L_{\text{min}, \text{rms}}$ equivalent to the VA spectra proved to be much lower than the fatigue limit (see Table 6.1) estimating an infinite life for all $L_{\text{max}}$ values. This shows that the RMS method is not suitable for the VA spectra used in this study and should only be considered for narrow-band random loading.
6.6 P-M Rules Results

The P-M rule (see Section 3.3.1 & 6.2.3) was used next to predict the VA fatigue life from the CA results. Most of the cycles in the spectrum had a load ratio close to either 0.1 or 0.5. Therefore, CA data was used directly without any modification for R-ratio effect (see Section 2.3.1). The cycles below the fatigue limit, which are assumed not to contribute to the fatigue crack growth, were not taken into consideration in the basic P-M rule. At the end of the analysis, the total damage sum, C, which ideally should have been equal to 1.0, proved to be at most 0.25. This means that the P-M rule overestimates the fatigue life, leading to unconservative life prediction. The extended Miner's rule (see Section 3.3.1) was, therefore, applied in order to take the cycles below the fatigue limit into account.
limit into account. This had only a modest effect, $C$ increasing to at most 0.3. The Miner’s sums obtained by both methods are presented in Figure 6.4.

![Figure 6.4 Plot of $C$ vs. $L_{max}$ for VA loaded specimens](image)

**6.7 Cycle Mix Effect**

**6.7.1 Linear Cycle Mix model**

The cycle mix effect was described in detail in Section 3.4.1 & 3.4.3. As previously mentioned, the cycle mix effect occurs during the transition from one CA stage to another having a higher mean stress value, i.e. during mean stress jumps. In Section 3.4.3, a strength-based wearout model incorporating a cycle mix factor, $CM$, was
described. This was developed by Schaff & Davidson (1997a, 1997b). Based on this model, the Linear Cycle Mix (LCM) model has been proposed for the bonded lap joints in this project. The LCM model is a strength-based wearout model (see Section 3.3.3) and can be described as follows: The residual strength of a bonded joint is initially equal to the quasi-static failure load, \( L_u \). However, the residual strength continuously decreases during fatigue cycling. The final failure occurs when the residual strength, \( R \), equals the peak load of the spectrum, \( L_{\text{peak}} \), i.e. when \( R(N_f) = L_{\text{peak}} \). Although it has not been proven that such a decrease in residual strength really exists in the joints studied here, previous work on bonded composite-to-metal joints (Yang & Shanyi, 1983; Wolff & Lemon, 1975) showed that residual strength does decrease during fatigue cycling.

The first step in the LCM model is to calculate the total strength degradation during a single loading block. As shown in Figure 6.5, the residual strength is assumed to degrade by two different mechanisms: degradation due to cycles above the fatigue limit and degradation due to the transitions from one CA stage to another having a higher mean load value (\( \Delta L_{\text{mn}} > 0 \)), i.e. mean load jumps. Assuming a linear damage accumulation, residual strength degradation by each cycle above the fatigue limit, \( LD \), can be defined as:

\[
LD = \frac{L_u - L_{\text{OL}}}{N_{f,\text{OL}}} \tag{6.5}
\]
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Figure 6.5 Theoretical residual strength degradation during VA fatigue cycling

where $L_{OL}$ is the maximum load of the cycle and $N_{f, OL}$ is the fatigue life corresponding to this cycle. The residual strength degradation due to mean load jumps, $CM$, is defined as:

$$CM = \alpha (\Delta L_{mn})^\alpha L_{peak} \left( \frac{\Delta L_{max}}{\Delta L_{min}} \right)$$  \hspace{1cm} (6.6)

where $\Delta L_{mn}$ and $\Delta L_{max}$ are the changes in the mean and maximum load values during the transition and $L_{peak}$ is the peak load in the spectrum. $\alpha$ & $\beta$ are the cycle mix constants, which are dependent on material and geometry. They can be determined by comparing VA fatigue lives under different spectra, e.g. two spectra with and without mean load jumps. Assuming that in each loading block, there are a number of cycles above the fatigue limit ($OL_1, OL_2, \ldots$) and a number of mean load jumps ($CM_1, CM_2, \ldots$), the strength degradation during a single block, $\Delta R_S$, can be defined as:
\[ \Delta R_g = \alpha \left[ \left( \frac{\Delta L_{mn,1}}{\Delta L_{mn}} \right)^{\beta L_{peak}} \left( \frac{\Delta L_{mn,2}}{\Delta L_{mn}} \right)^{\beta L_{peak}} + \ldots \right] + \frac{L_u - L_{OL,1}}{N_{f,OL,1}} + \frac{L_u - L_{OL,2}}{N_{f,OL,2}} \right] \quad (6.7) \]

and the corresponding number of blocks to failure is:

\[ N_B = \frac{L_u - L_{peak}}{\Delta R_g} \quad (6.8) \]

The main difference between the Linear Cycle Mix model and the cycle-mix model proposed by Schaff & Davidson (1997a, 1997b) is the definition of the cycle mix factor, \( CM \) (see Eqs. 3.29 and 6.6). Schaff & Davidson assumed the exponent of \( \Delta L_{mn} \) to be proportional to the square of \( \left( \frac{L_{max}}{L_{mn}} \right) \) and independent of the maximum load of the overall spectrum, i.e. peak load (see Section 3.4.3). However, in the LCM method, \( L_{peak} \) was introduced in order to obtain a linear VA \( L-N \) curve, whose slope can be adjusted to match the experimental \( L-N \) curve. Testing with spectra having different \( \left( \frac{L_{max}}{L_{mn}} \right) \) ratios indicated that the exponent for the bonded CFRP joints tested in this study should be defined as a linear function of \( \left( \frac{L_{max}}{L_{mn}} \right) \) rather than a square one.

### 6.7.2 Investigation of cycle mix effect

An experimental testing programme was undertaken to investigate the existence of the cycle mix effect in the double lap joints. In this programme, double lap joints were tested using a number of modified fatigue spectra. They are shown in Figure 6.6 along
with the original variable amplitude (VA) spectrum (Fig. 6.6(a)) described in Section 5.4.2.

Figure 6.6 ‘Cycle mix effect’ testing programme

4-5 specimens were tested for each load spectrum. Specimens tested under constant and variable amplitude spectra (Fig. 6.6(a)) for an $L_{peak}$ of 9 kN resulted in
fatigue lives of 22.5K and 200K respectively. Given that only 2 out of 100 cycles were distinctly above the threshold value in the VA spectrum (see Section 5.4.2), it can be said that the fatigue life was significantly shorter than expected and thus the acceleration effect due to load interactions was high.

The first modification to this spectrum is shown in Fig. 6.6 (b), where the ratio of number of understress cycles to overstress cycles is 10 and the mean load is kept constant ($\Delta L_{mn} = 0$). The Miner’s sum for this spectrum was approx. 0.9, which suggested that the acceleration effect had practically disappeared, i.e. the acceleration in this case cannot be attributed to accelerated growth following overstressing.

In the second modified spectrum, Fig. 6.6(c), the understress cycles were at a lower maximum load level than those in the first modified spectrum and mean load jumps, $\Delta L_{mn}$, were introduced. For this case, the Miner’s sum turned out to be as low as 0.2. This gives a strong indication that the changes in mean load are more damaging than the overloads for fatigue crack growth.

The third spectrum used is shown in Fig. 6.6(d). This spectrum looks the same as the initial VA spectrum, except for one very important difference in terms of the ‘cycle mix’ effect. Some of the understress cycles were raised to a higher load level and hence all $\Delta L_{mn}$ values were reduced to half of their original values. The change in fatigue life was quite radical. The joint did not fail even after $10^6$ cycles. This confirms that $\Delta L_{mn}$ is one of the main factors leading to crack growth acceleration.

The last spectrum applied is shown in Fig. 6.6(e). In this spectrum, overload cycles were trimmed leaving only understress cycles with mean load jumps. Loading
was such that the understress cycles coincided with our original spectrum with an $L_{\text{peak}}$ of 9 kN. The elimination of the overloads reduces $L_{\text{peak}}$ and hence after $10^6$ cycles occasional overloads were applied to check if the residual strength had decreased to $L_{\text{peak}}$ associated with the overloads. Testing continued to $1.5 \times 10^6$ cycles with no failure. The samples were then examined using optical microscopy and there was no evidence of any sort of damage. This suggests that changes in $\Delta L_{mn}$ activate accelerated crack growth during the cycles slightly below the threshold or increase the damage of the overload cycles. However, $L_{\text{peak}}$, and hence overloads may be more important in initiating damage in an uncracked sample considering the fact that mean stress jumps alone, as shown in Fig. 6.6(e), could not result in fatigue failure.

These results were broadly in agreement with the observations of Farrow (1989) and Schaff & Davidson (1997a, 1997b) for polymer composite samples. It is clear that the cycle mix effect is crucial for the VA analysis of bonded joints as well and that a general predictive method should include $\Delta L_{mn}$ as well as $L_{\text{peak}}$ effects.

6.7.3 Experimental validation of the LCM model

In Section 6.7.2, it was shown that that the cycle mix effect is crucial for the VA fatigue analysis of double lap joints. Therefore, a quantitative analysis of the cycle mix effect was made. The Linear Cycle Mix (LCM) model was implemented and an experimental justification of the model was made.

The cycle mix constants in the LCM model (see Eq. 6.6), $\alpha$ & $\beta$, were determined by fitting a straight line between two points on the VA $L$-$N$ curve of the original spectrum (Fig. 6.6(a)). This procedure is show in Figure 6.7.
They were calculated as $2.348 \times 10^{-6}$ and 0.514, respectively. For these values of $\alpha$ and $\beta$, there was a very good agreement with the rest of the experimental points on the $L-N$ curve, as shown in Figure 6.7. The next step was to apply the method to different spectra to see if the experimentally determined values of $\alpha$ and $\beta$ were valid for any kind of block-loading spectra.

For joints tested under the spectrum shown in Fig. 6.6(c), the experimental fatigue life was 80K cycles. The fatigue life predicted by the Palmgren-Miner's rule was 390K cycles, i.e. almost 5 times overestimated. When the LCM model was applied, the predicted fatigue life proved to be 62K, i.e. the error was only about 20%.

The model also accounted for the drastic change in the experimental fatigue life when the stress cycles were raised to a higher load level to reduce the mean load.
jumps as shown in Fig. 6.6(d) as such a change increased the fatigue life to over 1M cycles, as predicted.

For a spectrum without any mean load variations, as shown in Fig. 6.6(b), the LCM model predicts a fatigue life almost the same as the P-M rule. These results are summarised in Table 6.2.

<table>
<thead>
<tr>
<th>Spectra</th>
<th>Fatigue life (cycles at $L_{max} = 9kN$)</th>
<th>Damage sum, $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>P-M</td>
</tr>
<tr>
<td>6.6 (a)</td>
<td>200K</td>
<td>$&gt; 10^6$</td>
</tr>
<tr>
<td>6.6 (b)</td>
<td>180K</td>
<td>200K</td>
</tr>
<tr>
<td>6.6 (c)</td>
<td>80K</td>
<td>390K</td>
</tr>
<tr>
<td>6.6 (d)</td>
<td>$&gt; 10^6$</td>
<td>$&gt; 10^6$</td>
</tr>
<tr>
<td>6.6 (e)</td>
<td>$&gt; 10^6$</td>
<td>$&gt; 10^6$</td>
</tr>
</tbody>
</table>

**Table 6.2** Comparison of the P-M rule and the LCM model

This indicates that the Linear Cycle Mix (LCM) model is a considerable improvement on traditional cumulative damage laws for predicting the fatigue life of bonded lap joints subjected to VA fatigue.
6.8 Initiation and Propagation of Crack in a DLJ

6.8.1 Introduction

It is important to understand how a material fails in order to develop more realistic predictive methods. For instance, if a fracture mechanics approach (Section 2.5) is applied, an understanding of failure mechanisms will allow us to locate dominant cracks within the joint and their orientations. In this section, the locus of failure for the double lap joints tested in this study will be described in detail and a likely failure scenario will be proposed. The results of a video microscopy study undertaken in order to determine whether the fatigue life is dominated by the initiation or propagation phases and their relative contribution to the fatigue life will be reported. A scanning electron microscopy (SEM) study will also be presented in order to understand micromechanisms of the fracture.

6.8.2 Locus of failure

As mentioned in Section 6.4, the loci of failure were the same for the specimens tested under both constant amplitude (CA) and variable amplitude (VA) loading including the modified VA spectra (see Fig. 6.6). All joints failed predominantly in the composite adherends, in the $0^\circ$-ply adjacent to the bondline. On one side of the joint, failure was in the outer adherend, and on the other side, failure was in the middle adherend. This is schematically presented in Fig. 6.8.
As shown in Fig. 6.1 and schematically in Fig. 6.8, there was usually a considerable asymmetry between the two fillets of a joint, one side having a smaller fillet. It is important to note that the interlaminar failure of the middle adherend always took place on the side with a smaller fillet, i.e. on side A in Fig. 6.8.

During the post-failure analysis some cracks were observed on side B at the edges of the specimens as the mirror images of the locus of failure on side A. This is shown in Figure 6.9. These cracks were relatively long, being almost one-fourth of the overlap length. This observation was used to substantiate the failure scenario that will be described in the subsequent section.
6.8.3 Finite element model

The commercial finite element package LUSAS was used to analyse the stresses in double lap joints. Geometrically non-linear models based on plane-strain assumption were constructed. 8-noded elements (see Section 4.4.4.1), which are the most effective choice for modelling bonded joints, were used for meshing. A typical mesh is shown in Fig.6.10. Boundary conditions were as shown in Figure 6.11. The composite was modelled as a linearly elastic, orthotropic solid, whereas the adhesive was modelled as an elastic isotropic material. Although a plane of symmetry dividing the specimen across its thickness is usually assumed to reduce the size of models, in this case, a complete model was constructed because of the considerable asymmetry between the fillets in the joints (see Section 6.8.2).
Cracked specimens were modelled in order to simulate crack growth as a part of 3-step finite element analysis. This analysis is based on the assumption that in the fillet of bonded joints with high-strength structural adhesives, cracks are formed approximately
at right angles to the directions of the maximum principal stresses predicted by finite element analysis. This procedure will be described in the subsequent section.

6.8.4 Failure scenario for DLJ

A likely failure scenario is proposed according to the locus of failure predicted in an iterative finite element analysis (FEA). This is schematically presented in Figure 6.12.

According to this scenario, the fatigue crack initiates at the embedded corner of substrate A (i.e. substrate with a smaller fillet) in the fillet and progresses along a 45° line running through the substrate corner. This is supported by the fact that the FEA showed that the embedded corner of the adherend on side A is a more critical singularity point than that on side B, as shown in Figure 6.13. Stresses around the substrate corner in the smaller fillet proved to be approximately 15% higher than those around the fillet on side B. In addition to that it is possible to draw a 45° line approximately running through the substrate corner and normal to the maximum
principal stresses, as shown in Figure 6.14, which corresponds to the experimentally observed fracture path.

![Stress Contours of SMax](image)

Figure 6.13 Stresses in the adhesive layer around the fillets

After reaching the middle adherend, crack runs through the adherend causing interlaminar fracture of the composite. The crack reaches the PTFE strip. The PTFE can be considered a free surface; because the bond between PTFE and adhesive is very weak. Therefore, the overlap region on side A is released and no longer bears any load. Henceforth, only uncracked part of side B bears load, so the joint can be considered as a type of single lap joint. Since the load-bearing capacity of a single lap joint is much lower than that of a double lap joint, it fails rapidly. Stresses around the embedded corner of the PTFE proved to be much more critical than the stresses around the embedded corner of substrate B in the fillet. Therefore, the fracture continues at the corner of the PTFE strip, following a 45° line running through that corner. Another FEA
model of the specimen with a crack extending from point 1 to 2 was built, as shown in Fig. 6.15, and the results of this model were in accordance with the observed locus of failure.

![Crack propagation direction](image)

*Figure 6.14 3-step FEA (Step I- Fillet A)*

Finally, a model incorporating a crack extending from point 1 to 3, as shown in Figure 6.16, was built. Figure 6.16 shows that the fracture path determined using the assumption that a crack propagates along a line normal to the maximum principal stresses perfectly matches the observed fracture path.
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Figure 6.15 3-step FEA (Step II- PTFE)

Figure 6.16 3-step FEA (Step III- Fillet B)
There were some doubts about the fracture path because it is usually reckoned that both surfaces are generated by similar processes and the difference between the loci of failure is due to the asymmetry caused by the crack initiation starting earlier in one fillet than the other. However, as mentioned in Section 6.8.2, during the post-failure analysis some cracks were observed on side B at the edges of the specimens as the mirror images of the locus of failure on side A (see Fig. 6.9). These cracks were relatively long, almost one-fourth of the overlap length. Adams et al. (1997) studied the failure of single lap joints with metallic adherends with a locus of failure as shown in Fig. 6.17 and suggested that a crack in a SLJ runs along, or close to, the adhesive-adherend interface and meets a similar crack running in the opposite direction, thus resulting in such a fracture path.

![Image](image_url)

**Figure 6.17** A commonly reported locus of failure in SLJ

Similarly, it can be suggested that the arrested cracks on side B were the fatigue cracks lagging behind the cracks on side A and that they remained embedded within the specimen because they did not converge with the cracks coming from the opposite direction. Therefore, the presence of such cracks substantiates our failure scenario and also shows that the fatigue crack propagates a long distance on both sides of the joints before the total fracture.
6.8.5 Video microscopy

Since optical microscopy showed that the fatigue cracks were observable at the edges of the specimen and the fatigue crack propagates a long distance on both sides of the joints before the total fracture, a video microscopy study was undertaken to substantiate our failure scenario and to determine the relative contribution of the crack initiation and propagation phases in the fatigue life. However, the high-speed camera used for this purpose failed to create the image quality required to identify cracks due to the vibration of specimens during fatigue cycling. Therefore, an alternative procedure to video microscopy was undertaken. The specimen was viewed from one side using a traveling optical microscope fixed to a rigid base with a vernier scale. This is shown in Figure 5.9. Using this microscope, it was possible to check for any flaws without interrupting the test. Since the travelling microscope has a relatively low magnification (x10), the testing was interrupted occasionally and fillets were carefully examined under a higher powered optical microscope (x 10 – x 80) to see if any crack had initiated. Surprisingly, it was shown that significant edge cracking did not occur until the last couple of thousands of cycles to failure. Although a quantitative analysis was not made, it was obvious that initiation was the dominant phase, based on the observation of cracking at the edges. However, it should be noted that very different results may be obtained even when studying similar adhesive systems. To illustrate, Harris & Fay (1992) and Crocombe (1991) identified the initiation as being the dominant phase in the fatigue of steel/epoxy single lap joints using a stiffness loss technique and video microscopy, respectively. On the contrary, Taylor (1997) using the same adhesive system as Crocombe and utilizing the backface strain technique, pioneered by Imanaka (1995) and Zhang & Shang (1995), claimed propagation to be the dominant phase. Similarly, Little
(1999) using the same technique claimed the propagation phase to be the dominant one for aluminum epoxy single lap joints.

To sum up, this preliminary study suggested that the initiation phase is the dominant phase in the fatigue failure of double lap joints with composite adherends. However, a more extensive investigation using better optical equipment as well as X-ray radiography to see cracks buried in the specimen would be useful.

6.8.6 SEM

A fractographic study of the samples was undertaken by using scanning electron microscopy (SEM). This is one of the most popular techniques for fractographic studies. Here, fracture surfaces are scanned by a finely focused beam of high-energy electrons and an image builds up sequentially. Compared to optical microscopes, SEM has a very large depth of field, which allows us to go to very high magnifications without continuously adjusting the position of the specimen. For the SEM analysis of double lap joints, fractured surfaces were carefully sectioned using a diamond cutter and bonded to aluminium stubs. In order to examine a sample using SEM, the sample should be either conductive or coated with either carbon or a heavy metal for efficient charge transfer. To ensure a conductive surface, our samples were coated with a conducting layer of gold-palladium.
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Figure 6.18 Middle Adherend – Side A

Figure 6.19 Outer Adherend – Side B
Localized shear stresses in a bonded joint due to mode II loading results in the formation of shallow cusps. Shear loading a lamina leads to a principle stress state at an angle to the loading direction, which results in micro-cracks perpendicular to the maximum principle stresses. Coalescence of these micro cracks forms the shallow cusps. This is schematically shown in Fig. 6.20. Shallow cusps can be identified in Fig. 6.18 and 6.19. The number of cusps and their angle with respect to the fracture surface increases as the percentage of mode II loading is increased. As shown in the figures, the density of shear cusps in the double lap joints was not found to be high enough to suggest predominance of mode II failure. Therefore, according to SEM results, the composite failed under mixed mode loading (mode I-II).

It was also possible to observe: fibre fracture, cohesive resin fracture and fibre-matrix debonding. However, examination showed that two surfaces of the material (i.e. side A
and side B [see Fig. 6.8] are very similar, which makes it very difficult to identify two different mechanisms that generated them.

6.9 Conclusion

The Palmgren–Miner rule, which gives quite satisfactory results in many cases, failed to predict the fatigue life of bonded composite joints subject to a block-loading $VA$ spectrum. The results indicated a severe fatigue growth acceleration due to load interactions. This is of concern to analysts using CA fatigue data to predict variable amplitude fatigue life. It was shown that mean stress variations can be more damaging than overloads with respect to fatigue crack growth accelerations but that $L_{\text{max}}$, and hence overloads, may be important in fatigue crack initiation. It was thus considered that any general model for fatigue life prediction in bonded joints should incorporate both $\Delta L_{\text{mn}}$ and $L_{\text{max}}$. A simple predictive model incorporating a ‘cycle mix’ factor has been proposed. This method proved to be a considerable improvement on traditional cumulative damage laws for predicting the fatigue life of bonded lap joints subjected to variable amplitude fatigue.

The failure mechanisms in bonded joints were also investigated as a foundation to develop more realistic predictive methods in the future. The locus of failure of double lap joints was determined by optical microscopy and SEM. Then, a likely failure scenario was plotted using a 3-step finite element analysis. This new explanation was supported by further optical microscopy evidence. These were the fatigue cracks as the mirror images of the locus of failure on the side where complete failure initiates.
7.1 Introduction

In this chapter, results from testing double cantilever beam (DCB) joints under constant amplitude (CA) and variable amplitude (VA) loading will be reported. Both cohesive and interlaminar crack growth will be investigated using two different joint configurations (i.e. AF & CF-type joints). The main attention will be given to numerical crack growth integration (NCGI), the most popular fracture mechanics based method for predicting VA fatigue life (see Section 4.6.2). In addition, an experimental justification of analytical methods used to calculate strain energy release rate (G) in bonded DCB joints will be made before using them in VA fatigue analysis. Load interactions will also be discussed in this chapter and a model incorporating a ‘damage shift’ factor will be proposed. The methodology used is shown in Figure 7.1.

7.2 Experimental

7.2.1 Joint configuration

Double cantilever beam (DCB) joints were used in this study. The material used and specimen geometry was described in detail in Sections 5.2 and 5.3.3. As mentioned, two types of joint configuration were used in the test programme.
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FRACTURE MECHANICS BASED VA FATIGUE ANALYSIS

Testing of DCB joints under CA and VA loading
(AF-type joints for cohesive crack propagation;
CF-type for interlaminar crack propagation)

Construction of CA fatigue crack growth rate
(FCGR) curves corresponding to different R-ratios

Construction of crack growth (CG) curves
for VA fatigue crack growth
(crack length vs. number of cycles)

Application of Numerical Crack Growth Integration
(NCGI) to predict VA CG using CA FCGR-curves

Comparison of predicted and experimental CG-curves

If NCGI gives satisfactory results,
assume that load interaction effects are not significant

If not, load interaction effects are significant. Study their mechanisms and
modify the NCGI to incorporate a load interaction parameter

Application of fracture mechanics to more realistic joints, such as DLJ's, using FEA

Figure 7.1 Methodology used in fracture mechanics fatigue testing programme
The first type of joint (AF-type) incorporated a very thin sheet of PTFE as a starter crack at the adhesive bondline. The second sample type (i.e. CF-type) consisted of two composite panels joined with no adhesive layer and a starter crack between the 0° plies at the centre of the joint. By the use of these two types of sample, it was possible to investigate crack growth through the adhesive (cohesive failure) and through the composite (interlaminar failure). Only AF-type joints were studied in the first part of this testing programme, i.e. in the investigation of analytical beam theory equations.

7.2.2 Test conditions

Specimens were tested under quasi-static, constant amplitude (CA) fatigue and variable amplitude (VA) fatigue loading. Testing conditions were described in detail in Section 5.4.3.

7.3 Beam Theory Equations Used in the Analysis

7.3.1 Introduction

Several methods that can be used to calculate the strain energy release rate, $G$, in bonded DCB joints were presented in Section 4.4.1. These included models based on experimental compliance (EC) measurements (see Section 4.4.2) and those based on beam theory (see Section 4.4.3). As mentioned in Section 4.4.5, although the EC method has many advantages, there is still a need for analytical methods (i.e. those based on beam theory or finite element analysis). Furthermore, an analytical method is particularly useful in variable amplitude (VA) fatigue analysis. Analytical methods enable the determination of $G$ corresponding to a single stage in a loading block. Once
load or displacement levels of a cycle are known, \( G \) can be calculated based on the geometry and material properties using an analytical method. This value can then be used in the numerical crack growth integration (see Section 4.6.2). This is very difficult to achieve using only the EC variation, \( dC/da \), particularly when the number of cycles in a loading block is relatively small. In this case, a common practice is to define an average strain energy release rate for the whole spectrum using some statistical techniques, such as the RMS-method (see Section 4.6.1). However, the spectra used in this study is a block-loading spectrum with intermittent overloads (see Section 5.4.3 and Fig. 5.11) and the number of cycles are relatively low. A statistical technique is not very reliable for such a spectrum. That is why there is a requirement for a reliable analytical method in this work.

Simple beam theory (SBT) (see Section 4.4.3.1) is the most commonly used analytical approach. Although it is described in ASTM D5041-98-'Standarad Test Method for Fracture Strength in Cleavage of Adhesive in Bonded Joints', it has some major drawbacks, as discussed in Section 4.4.4. However, two relatively new methods are potentially promising alternatives. These are the beam-on-elastic-foundation (BEF) and the beam-on-elastic/plastic-foundation (BEPF) models, i.e. models with a compliant crack front (see Section 4.4.3.3). However, there has been little experimental justifications of these methods to date. Before using them in VA fatigue analysis, an experimental justification of these two methods is needed. This justification will be based on how well they correlate to the EC method for a CA fatigue test. The main attention will be given to the BEF model because the adhesive used in this study is quite
brittle at room temperature. A typical stress-strain curve for bulk adhesive is presented in Fig. 5.1. Subsequent sections will report the results of this analysis.

7.3.2 Theory

Methods used to calculate $G$ in bonded DCB joints were described in detail in Sections 4.4.2 & 4.4.3. Equations that should be implemented for the BEF and the BEPF models were given in Section 4.4.3.3. As mentioned in Section 4.4.3.3, Chang et al. (1976) and Chow et al. (1979) independently derived equations for the strain energy release rate in a bonded DCB joint using BEF models. Chow et al. (1976) made the useful assumption that $\lambda d$ is larger than $2\pi$, where $d$ is the length of the elastic foundation, i.e. the uncracked part of the beam, and $\lambda$ is a length scale defined by Eq. 4.15. Chang et al. (1976) included the shear deformation of the beam in their formulation. For a typical DCB joint $\lambda d$ is larger than $2\pi$. Furthermore, the shear energy in the beam is usually negligible compared with the bending energy. Therefore, these two assumptions can be used together and the following simplified form of Eqs. 4.20 & 4.21 obtained:

$$G_t = \frac{12P^2}{b^2h^3E}(a + \Delta)^2$$

(7.1)

where $P$ is the load, $a$ the crack length, $b$ the specimen width, $h$ the thickness of the substrate and $E$ the Young's modulus of the substrate.

Eq. 7.1 was used to calculate $G$ based on the BEF model, Eq. 4.34 to calculate $G$ based on the BEPF model and Eq. 4.7 was used in the experimental compliance (EC) method to determine the variation of compliance with respect to crack length, $dC/da$, throughout this study. The 7-point polynomial method, which is one of the methods
suggested in the ASTM E647-'Standard Test Method for Measurement of Fatigue Crack Growth Rates', was used to calculate the fatigue crack growth rate, $da/dN$. This method was described in detail in Section 4.8.

7.3.3 Finite element analysis

The commercial finite element package LUSAS was used to determine the variation of compliance with respect to crack length, $dC/da$, in bonded DCB joints. Geometrically non-linear models based on plane-strain assumption were constructed. 8-noded elements (see Section 4.4.4.1), which are the most effective choice for modelling bonded joints, were used for meshing. Mesh refinement around the crack tip was not critical, because the effect of stress singularity on overall compliance of the specimen was relatively low. A typical mesh is shown in Figure 7.2. Boundary conditions used were as shown in Figure 7.3. A plane of symmetry dividing the specimen across its thickness was assumed to reduce the size of models. The composite was modelled as a linearly elastic, orthotropic solid. The adhesive was only modelled as an elastic isotropic material because the adhesive used in this study is quite brittle at room temperature, as shown in Fig. 5.1.
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7.3.4 Variation of compliance with respect to crack length

A set of values for the crack length, $a$, and the maximum load, $L_{\text{max}}$, as a function of number of cycles, $n$, were obtained. They were used to determine the variation of compliance with respect to crack length, $C$ vs. $a$. A finite element analysis of the joints
was also undertaken for comparison with the analytical models. This model was described in detail in the previous section.

$C$ vs. $a$, was determined using FE analysis, the beam-on-elastic-foundation (the BEF) model and the simple beam theory (the SBT). The true analytical SBT was used without any modification to extend the crack length (see Section 4.4.3.2). $C$ vs. $a$ variation are presented in Fig. 7.4 along with two typical experimentally determined compliance variations.

![Figure 7.4 Compliance variation with respect to crack length](image)

As seen, although there is an excellent agreement between the FEA and BEF for moderate crack lengths, the BEF tends to assume a more compliant crack as the crack extends. Previous studies (Yamada, 1987; Abou-Hamda et al., 1988) revealed a closer agreement between the FEA and the BEF model. This is mostly due to the fact
that the beam theory equations were originally developed for isotropic materials. However the composite substrate used in this testing programme was modelled as an orthotropic material. Figure 7.4 also shows that in contrast with the BEF model the SBT predicts a significantly stiffer joint than the FEA.

The experimental compliance values proved to lie between the BEF and SBT curves, showing excellent agreement with the BEF curve for short crack lengths less than 30 mm but approaching the SBT curve as the crack extended.

7.3.5 Comparisons of FCGR-curves

Fatigue crack growth rate curves (FCGR-curves) were constructed by plotting log $G$ against log $da/dn$ for the CA fatigue tested DCB joints. The scatter level within each test was quite low and tests were generally consistent with each other, having the same mean behaviour (see Section 7.5.1). The FCGR-curves for a typical test obtained by calculating the strain energy release rate using 4 different methods (EC, BEF, SBT and BEPF) for a moderate crack length is presented together in Fig. 7.5 to enable a comparison to be made. As can be seen in this figure, there is an excellent agreement between the EC FCGR-curve and the BEF and BEPF curves, whereas the SBT results deviate from the rest of the curves. The SBT has overestimated $G$, resulting in an FCGR-curve to the right of the other curves. However, as the crack length increased, the BEPF/BEF models tended to underestimate the strain energy release rate and the EC FCGR-curve tended to approach the SBT curve giving a close agreement for long crack lengths. This is shown in Fig. 7.6.
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Figure 7.5 Comparison of the FCGR-curves from different methods for $a_0 = 23\text{mm}$

It should be noted that the adhesive used in this study is too brittle to be assumed an elastic/perfectly plastic (see Figure 4.7) material. Therefore, the BEPF model is not an appropriate choice for our analysis. It is shown in Fig. 7.5 & 7.6 that the BEF and BEPF results were almost identical for the threshold and the linear regions of an FCGR-curve. Therefore, even if the adhesive used was an elastic/perfectly plastic one, the more complicated calculation procedure involved in the BEPF model would thus only be justified in region III of the FCGR-curve, where unstable fast crack growth occurs or to determine the plastic zone size.
When all the FCGR curves obtained were investigated, the EC $G$, $G_{EC}$, values were found to lie between the values calculated using BEF ($G_{BEF}$) and SBT ($G_{SBT}$), generally being closer to the BEF results. Given the compliance variation behaviour shown in Fig. 7.4, it is no surprise that in displacement control, the BEF model slightly underestimates $G$ assuming that $G_{EC}$ is the correct value of strain energy release rate. This results in a higher crack propagation rate at a given $G$ value and predicts a lower threshold strain energy release rate, $G_{th}$. Therefore, it can be said that the BEF model is slightly conservative. On the other hand the SBT overestimates $G$ in displacement control and hence is unconservative, particularly at short crack lengths.

Figure 7.6 Comparison of the FCGR-curves from different methods for $a_0 = 38\text{mm}$
Since the $G$ values from the EC method appears to lie between the $G_{BEF}$ and $G_{SBT}$ values, it is useful to understand the relation between these two functions. It can be seen in Fig. 7.7 that $G_{BEF}$ and $G_{SBT}$ significantly deviate from each other as $G$ increases and that, as expected, this deviation increases with bondline thickness. As stated above, $G_{EC}$ lies between $G_{SBT}$ and $G_{BEF}$, which is the area between the dashed reference line ($G_{SBT} = G_{BEF}$) and one of the solid lines corresponding to the bondline thickness selected in Fig. 7.7.

![Figure 7.7 $G_{BEF}$ corresponding to $G_{SBT}$ for different bondline thickness, $t$ ($a=25$ mm)](image)

**Figure 7.7** $G_{BEF}$ corresponding to $G_{SBT}$ for different bondline thickness, $t$ ($a=25$ mm)

### 7.3.6 Conclusion

The BEF and BEPF models showed a very good agreement with the experimental compliance results at moderate crack lengths. It was seen that values of $G$ calculated
from experimental compliance measurements lie between $G_{SBT}$ and $G_{BEF}$ results, closer to $G_{BEF}$. It should be noted that in fatigue design it is the threshold region that is generally of most interest. The BEF model agrees well with the EC derived threshold strain energy release rate, whereas the SBT model predicts a higher threshold value and is hence unconservative. It is also noted that the use of the BEPF model for this study is physically wrong because the adhesive used in this study is too brittle to be assumed elastic/ perfectly plastic. It is suggested that BEF is the most appropriate analytical model for calculating $G$ in most cases. A simplified equation for calculating $G$ based on the BEF model has been proposed (Eq. 7.1) which allows calculation of this parameter to greater accuracy than the SBT model but more simply than using the existing BEF and BEPF expressions. It was used throughout this study.

7.4 Quasi-static Testing Results

7.4.1 Cohesive crack growth

5 specimens were tested to obtain quasi-static values for AF-type specimens. The results from different tests were consistent with each other. The critical strain energy release rate proved to be 664 J/m$^2$ with a standard deviation of 105 J/m$^2$. For the AF-type specimens, the load was seen to decrease with increasing crack length, as expected from any fracture mechanics specimen tested in displacement control because of the decrease in compliance. However, the crack growth consisted of a succession of rapid growth and arrest (very slow crack growth) phases. This can be seen in two typical test result shown in Fig. 7.8. It is also important to note that fracture was almost entirely cohesive. This is schematically presented in Fig. 7.9.
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Figure 7.8 Typical plots of load and crack length vs. time in the quasi-static testing of AF-type specimens
Such a crack growth regime has been reported previously in bonded joints (Yamini & Yang, 1977; Gledhill et al., 1978; Ashcroft et al., 2001). This behaviour is commonly referred to as stick-slip behaviour. A number of different mechanisms were used to explain this phenomenon, such as viscoelastic nature of polymers, plasticity ahead of the crack tip, crack tip blunting mechanisms, etc. In this case, the most likely explanation is the damage zone formation ahead of the crack tip and resulting decrease in the crack growth resistance. A typical plot of $G$ vs. $a$ in the quasi-static testing of AF-type specimen is shown in Fig. 7.10. For such crack growth two different strain energy release rate values can be calculated, one associated with the crack initiation, $G_{int}$, and one with crack arrest, $G_a$. $G_{int}$ and $G_a$ values for the crack growth shown in Fig. 7.10 are presented in Fig. 7.11. Here, the former was used as the mode I critical strain energy release rate, $G_{IC}$.

Figure 7.9 Locus of failure for quasi-static failure of AF-type DCB joints
Figure 7.10 Typical plot of $G$ vs. $a$ in the quasi-static testing of AF-type specimens

Figure 7.11 Typical plot of $G$ vs. $a$ corresponding to initiation and propagation phases in the quasi-static testing of AF-type specimens
7.4.2 Interlaminar crack growth

5 specimens were tested to obtain quasi-static data for the CF-type specimens. The results from different tests were consistent with each other. The critical strain energy release rate proved to be 180 J/m² with a standard deviation of 17 J/m². As in the AF-type specimens, the load was seen to decrease with increasing crack length due to the decrease in compliance. However, unlike in the AF-type specimens, there was a practically continuous reduction in the compliance value. In other words, the stick-slip behaviour was not observed in the case of interlaminar fracture of the CFRP joints. The compliance variation is shown in Fig. 7.12.

Typical $G$ vs. $a$ curves for two CF-type specimens are presented in Fig. 7.13. It can be seen that $G$ is high in the early stages of the test. However, this levelled out after about 10 mm of crack growth and then remained approximately constant to the end of the test. This is usually attributed to the bluntness of the starter crack formed by the PTFE film. However, it should also be noted that beam theory is not very reliable for such small crack lengths. Therefore, the initial part of the testing was ignored in the analysis. Finally, typical $M$ (i.e. bending moment = load x crack length) vs. $a$ curves for two CF-type specimens are also presented in Fig. 7.14. It is interesting to note that $M$ was almost constant during the entire crack growth. This was in agreement with Kacble’s peel force theory (1962). The critical bending moment for the CF-type specimens was approximately 3 N.m.
Figure 7.12 Typical plots of load and crack length vs. time in the quasi-static testing of CF-type specimens
Chapter 7. Variable Amplitude Fatigue of Cracked Specimens

Figure 7.13 Typical plots of $G$ vs. $a$ in the quasi-static testing of CF-type specimens
Figure 7.14 Typical plots of $M$ vs. $a$ in the quasi-static testing of CF-type specimens
7.4.3 Comparison of cohesive and interlaminar crack growth

The main difference between the interlaminar and cohesive crack growth was the stick-slip nature of cohesive crack growth. In AF-type specimens, the crack growth consisted of a succession of rapid growth and arrest phases, whereas there was continuous crack growth in CF-type specimens. As seen in Table 7.1, the critical strain energy release rate for CF-type specimens was almost three times lower than that of AF-type specimens. This is consistent with trends observed in other systems (Ashcroft et al., 2001), since CFRP matrix resins are usually more brittle than modified epoxy adhesives.

<table>
<thead>
<tr>
<th>Type</th>
<th>Quasi-static $G_{IC} (J/m^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF</td>
<td>664 (105)</td>
</tr>
<tr>
<td>CF</td>
<td>180 (17)</td>
</tr>
</tbody>
</table>

Table 7.1 Critical strain energy release rate values for AF and CF-type specimens (standard deviations in parenthesis)

7.5 Constant Amplitude Fatigue Testing Results

7.5.1 Cohesive crack growth

5 specimens were tested to obtain constant amplitude (CA) fatigue data. CA fatigue crack growth rate (FCGR) curves for two different displacement ratios, $R=0.1$ and 0.5,
were constructed. The results from different tests were consistent with each other. The fatigue threshold strain energy release rate range, $\Delta G_{th}$, for $R = 0.1$ was $75 \text{ J/m}^2$ with a standard deviation of $17 \text{ J/m}^2$ whereas $\Delta G_{th}$ for $R = 0.5$ was $87 \text{ J/m}^2$ with a standard deviation of $21 \text{ J/m}^2$. The fatigue crack growth rate, $da/dN$, was initially defined as a function of the maximum strain energy release rate, $G_{\text{max}}$. The fatigue threshold value, $G_{\text{max, th}}$, corresponding to $R = 0.1 (\approx 75 \text{ J/m}^2)$ proved to be much lower than the one corresponding to $R = 0.5 (\approx 115 \text{ J/m}^2)$ for the AF-type specimens. This is also shown in the FCGR-curves for two typical tests in Fig. 7.15.

![FCGR-curves](image)

**Figure 7.15** FCGR-curves (AF-type) for different $R$-ratios constructed using $G_{\text{max}}$. 
However, when $da/dN$ was defined as a function of the strain energy release rate range, $\Delta G$, the two curves got much more closer as shown in Fig. 7.16.

![FCGR-curves (AF-type) for different R-ratios constructed using $\Delta G$](image)

Figure 7.16 FCGR-curves (AF-type) for different R-ratios constructed using $\Delta G$

This was in agreement with previous work in the literature. Mall (1987) studied the effect of stress ratio on cyclic debonding in bonded composite joints and observed that the relationship between $\Delta G$ and $da/dN$ was almost independent of the stress ratio for both mode I and II loading. However, $G_{max}$ is often used for the fatigue analysis of bonded joints, in preference to stress intensity factor range, $\Delta G$, (see Section 4.2.3) because the facial interference of the adhesives on the debonded surfaces may lead to the generation of surface debris, which may prevent the crack from fully closing, thus giving an artificially high value of $G_{min}$ (Martin & Murri, 1990). However, since $da/dN$
is little affected by the value of $G_{\text{max}}$ as long as $\Delta G$ is the same, $\Delta G$ seems to be a more sensible choice to characterize the fatigue crack growth behavior of bonded joints. Therefore, $\Delta G$ will be used as the governing fracture parameter throughout this study.

**7.5.2 Interlaminar crack growth**

5 specimens were tested to obtain constant amplitude (CA) fatigue data. CA fatigue crack growth rate (FCGR) curves for two different displacement ratios, $R=0.1$ and $0.5$, were constructed. The results from different tests were consistent with each other. The fatigue threshold strain energy release rate range, $\Delta G_{\text{th}}$, for $R=0.1$ was 85 J/m$^2$ with a standard deviation of 17 J/m$^2$ whereas $\Delta G_{\text{th}}$ for $R=0.5$ was 110 J/m$^2$ with a standard deviation of 15 J/m$^2$. Two typical FCGR-curves for CF-type specimens are shown in terms of $\Delta G$ in Fig. 7.16 or in terms of $G_{\text{max}}$ in Fig. 7.18. Comparisons of Fig. 7.17 and Fig. 7.18 shows that for the CF type specimens $G_{\text{th}}$ was considerably lower with $R=0.1$ even when the range values were used. It is thus shown that interlaminar crack growth is more sensitive to the load ratio than cohesive crack growth in this system.
Figure 7.17 FCGR-curves (CF-type) for different $R$-ratios in terms of $\Delta G$

Figure 7.18 FCGR-curves (CF-type) for different $R$-ratios in terms of $\Delta G_{max}$
7.5.3 Comparison of cohesive and interlaminar fatigue crack growth

Although $G_J$ for the CF-type specimens was much lower than that of the AF-type specimens, the ratio of the threshold value to the quasi-static fracture toughness, $G_{th}/G_{th}$, was much higher for CF-type specimens than AF-type, as shown in Table 7.2 and Fig. 7.19 & 7.20. This indicates the greater importance of fatigue in the latter, and is consistent with trends seen in other systems. It is also shown that interlaminar crack growth is much more sensitive to the load ratio than cohesive crack growth.

![Interlaminar vs Cohesive Crack Growth](image)

**Figure 7.19** Comparison of FCGR-curves for AF and CF-type specimens ($R=0.1$)
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Figure 7.20 Comparison of FCGR-curves for AF and CF-type specimens ($R=0.5$)

<table>
<thead>
<tr>
<th>Type</th>
<th>Quasi-static $G_{lc}$ (J/m²)</th>
<th>Fatigue $\Delta G_{th}$ (J/m²) for $R=0.1$</th>
<th>Fatigue $\Delta G_{th}$ (J/m²) for $R=0.5$</th>
<th>$\Delta G_{th}/G_{lc}$ for $R=0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF</td>
<td>664 (105)</td>
<td>75 (17)</td>
<td>85 (21)</td>
<td>0.12</td>
</tr>
<tr>
<td>CF</td>
<td>180 (17)</td>
<td>85 (15)</td>
<td>110 (15)</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 7.2 Strain energy release rate values for AF and CF-type specimens (standard deviations in parenthesis)
7.6 Variable Amplitude Fatigue Testing Results

7.6.1 Fracture path

The locus of failure was essentially the same for all three different types of loading, i.e. quasi-static, constant amplitude (CA) and variable amplitude (VA) loading. This was cohesive fracture in the middle of the bondline for the AF-type joints and interlaminar failure between the 0° plies for the CF-type joints. Therefore, it was possible to investigate two different types of crack growth, cohesive and interlaminar, independently. Since cracks followed the same fracture paths under CA and VA loading, one of the most important conditions to make use of CA data to predict VA fatigue life was satisfied.

7.6.2 Numerical crack growth algorithm

Numerical crack growth integration (NCGI) is an established technique for predicting VA fatigue crack growth based on constant amplitude FCGR-curves. This method and its algorithm were described in detail in Section 4.6.2. In the present study, NCGI was implemented on a cycle-by-cycle basis. This was accomplished by writing a macro in MS Visual Basic.

7.6.3 Cohesive crack growth

Variable amplitude (VA) fatigue testing was carried out using the two-stage block loading spectra described in Section 5.4.3 (see Fig. 5.11). 8 specimens were tested under
VA loading. Numerical crack growth integration (NCGI) was used to predict VA fatigue crack growth (FCGR) using constant amplitude (CA) FCGR-curves.

During the VA fatigue cycling, there was a steady fatigue crack growth for the first few thousand cycles that was close to that predicted by NCGI. However, an unexpected sudden crack growth occurred after approximately 5000 cycles (incorporating approximately 250 overloads). After this, crack growth slowed for a period of time and no further instances of the sudden crack growth were observed. Two typical crack growth curves showing this unusual phenomenon are shown in Fig. 7.21 and in Fig. 7.22.

![Crack growth under VA-loading - I (cohesive crack; high initial G)](image_url)
Fig. 7.23(a) goes on to show that, although the prediction method misses this sudden crack phenomenon, after a large number of cycles there is excellent agreement between the predicted and experimental crack lengths. Interestingly, if the initial value of $G_{\text{max}}$ was reduced to below that at which the sudden crack growth occurred, then this phenomenon was not observed, as shown in Fig. 7.23(b). However, the experimental crack growth at high cycles is now higher than that predicted using NCGI. This limit was approximately 400 J/m$^2$, which is approximately two thirds of the critical strain energy release rate. This value is slightly above the strain energy release rate value corresponding to crack arrest in a quasi-static testing, which is approximately 350 J/m$^2$. 

**Figure 7.22** Crack growth under VA-loading – II (cohesive crack; high initial $G$)
Figure 7.23 Crack growth under VA-loading in AF-type specimens for (a) a high initial $G$ and (b) a lower initial $G$. 
7.6.4 Damage zone ahead of the crack tip

As mentioned in the previous section, there was unstable crack growth during the initial stages of VA fatigue test when the initial $G$ was above a critical value. This is shown in Fig. 7.21 & 22. The fracture surface corresponding to a DCB joint subjected to VA fatigue above the critical $G$ is also shown in Fig. 7.24. Dark bands were formed during the unstable crack growth, whereas light bands during the stable fatigue crack growth. This unusual unstable crack growth phase posed a crucial problem for the prediction of VA fatigue. Some speculative comments regarding the reason for this phenomenon can be made.

Figure 7.24 Macroscopic picture of the fracture surface
Firstly, it is worth noting that extensive damage zones ahead of the crack tip have been observed in fatigue tested composite joints (see Section 2.6). Figure 2.7 shows an example of such a damage zone. It is very likely that a similar damage zone forms in the joints tested in this programme. It would be expected that the extent of damage in this zone would affect resistance to crack growth. It can also be postulated that in the VA tests, the size and/or nature of damage in this zone will be affected by the overloads. Once the crack passes into the damaged zone a crack acceleration would be expected. This effect will be most severe in the initial, high $G_{\text{max}}$, stages of the test and if the damage is severe enough a sudden crack growth may be expected. This will occur when the effective $G_c$ of the damaged material has been reduced to the value of the applied strain energy release rate.

Such a damage zone approach was previously used by Wheatley et al. (1999) investigating the effects of a single tensile overload on subsequent fatigue crack growth in a stainless steel. They also observed transient crack growth acceleration, which was accompanied by a subsequent retardation phase. They suggested that the initial crack growth acceleration stems from void and quasi-cleavage fracture within the fatigue damage zone in the vicinity of the crack tip and the retardation phase following the acceleration is due to material plasticity.

Unlike metals, the fatigue crack growth acceleration in polymers can be further enhanced by a rate dependent critical strain energy release rate, as proposed by Ashcroft et al. (2001). However, as noted earlier, in displacement controlled testing of DCB samples, $G$ decreases with crack length and ultimately the crack will slow. The crack growth rate after the sudden growth phase could now be lower than predicted because
the material ahead of the crack is initially undamaged and may also be influenced by the increased resistance to slow crack growth. A comparison of Fig. 7.23(a) and 7.23(b) appears to indicate that the crack retarding mechanism after the sudden crack growth phase more than compensates for the sudden crack growth in the long term. However, it is important to note that this is only because of the nature of the test (i.e. decreasing $G$ with crack length), under other loading conditions or joint geometry the sudden crack growth could initiate catastrophic failure of the joint. These arguments can be summarized in Fig. 7.25.

**Figure 7.25** Unstable crack growth during the VA fatigue cycling
The damage zone argument also accounts for the behaviour when the sudden crack growth is not observed. In this case, the overloads still cause increased damage ahead of the crack. The damage is not severe enough to cause the sudden jump in crack growth but a noticeable increase in the crack growth rate is observed.

7.6.5 Interlaminar crack growth

Variable amplitude (VA) fatigue testing was carried out using the two-stage block loading spectra described in Section 5.4.3 (see Fig. 5.11). 4 specimens were tested under VA loading. Numerical crack growth integration (NCGI) was used to predict VA fatigue crack growth (FCGR) using CA FCGR-curves. For CF-specimens, there was no such sudden crack growth as the one observed in the AF-type specimens. When the numerical integration was applied, it was clearly shown that the crack grows at a faster rate than suggested by the constant amplitude (CA) data, i.e. there was a crack growth acceleration due to load interactions in the VA spectra. Two typical crack growth curves are shown in Fig. 7.26 and 27. Unlike the case of cohesive crack growth, the numerical integration proved to be extremely unconservative.
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Figure 7.26 Crack growth under VA-loading in a CF-type specimen - I

Figure 7.27 Crack growth under VA-loading in a CF-type specimen - II
As in the previous case, the crack growth acceleration can be attributed to the formation of an extensive damage zone ahead of the crack tip by the overloads. A crack cannot propagate under a CA spectrum composed of cycles below or equal to the fatigue limit, i.e. understress cycles. However, intermittent overloads create a damage zone ahead of the crack tip, which decreases the resistance to crack growth. Therefore, understress cycles subsequent to overloads can now propagate the crack through the damaged zone at an increased rate. Since most of the cycles in the loading spectrum are understress cycles, a small change in the crack growth rate during understress cycles may result in an overall crack growth rate much greater than expected.

7.7 Damage Shift Model

7.7.1 Theory

Fracture mechanics applied to constant amplitude (CA) fatigue was discussed in detail in Section 4.5. As mentioned earlier, Paris et al. (1961) showed that the fatigue crack growth rate (FCGR), $da/dN$, is a function of the stress intensity factor range, $\Delta K$, and the $R$-ratio (see Eq. 4.48). A similar relationship is applicable to bonded joints, where there is a strong functional relation between $da/dN$ and the strain energy release rate range, $\Delta G$. All FCGR-curves obtained in the present study, either for cohesive or interlaminar crack growth, have also showed a strong correlation between $da/dN$ and $\Delta G$ in log-log scale (e.g. Fig. 7.16 and Fig. 7.17). This enabled us to define $da/dN$ in terms of $\Delta G$, the threshold strain energy release rate range, $\Delta G_{th}$, and the critical strain energy release rate, $\Delta G_c$. Using this functional relation, the numerical crack growth
integration (NCGI) was carried out to predict the fatigue life of bonded joints subjected to a two-stage block-loading spectra (see Section 7.6).

In Section 7.6.3, it was shown that simple NCGI failed to predict the experimentally observed cohesive crack growth in bonded CFRP joints: It failed to predict the unstable crack growth after a critical number of applied cycles at high values of initial $G$ and underestimated the FCGR for cohesive cracks subjected to a moderate initial $G$ value. Therefore, NCGI must be modified to incorporate load interaction effects. An empirical 'Damage Shift' model has been proposed. This method requires the modification of NCGI to incorporate the effect of the damage zone induced by the overloads. The Damage Shift model can be described using Fig. 7.28. In this figure, CA refers to the FCGR-curves obtained by constant amplitude (CA) testing of bonded joints. Therefore, according to this curve, a CA fatigue load with a strain energy release rate range of $\Delta G_A$ will result in a crack growth rate of $(da/dN)_{CA}$. NCGI utilizes $(da/dN)_{CA}$ for each load cycle in a loading block, i.e. the crack resistance and hence $da/dN$ is associated with the damage caused by the CA fatigue loading.

If an overload is superimposed onto the CA spectrum, then the damage ahead of the crack zone will increase and the resistance to crack propagation will decrease. It is proposed that this increased damage can be represented by a lateral shift in the FCGR-curve, $\psi$. The FCGR-curve associated with this increased damage is represented by curve OL in Fig. 7.28. It can be seen that the value of $da/dN$ corresponding to OL at $\Delta G_A$, $(da/dN)_{OL}$, will be higher than $(da/dN)_{CA}$, i.e. that there will be a crack growth acceleration effect. The shift factor $\psi$ can be easily determined by comparison of the experimental value of $(da/dN)_{CA}$ and $(da/dN)_{OL}$ at a given value of $\Delta G$. 
This method can be used to incorporate sudden crack jumps at high initial $G$ values. If $\Delta G_{\text{peak}}$, the strain energy release rate range value corresponding to the peak load in a loading spectrum, is larger than the value of $\Delta G_c$ for the shifted FCGR-curve, OL, there will be a sudden unstable or quasi-static fracture through the adhesive. This point is shown as $\Delta G_{\text{AC}}$ in Fig. 7.29. $\Delta G_{\text{AC}}$ can be estimated using the shifted FCGR-curve, such that:

$$\Delta G_{\text{AC}} = (\Delta G)_{\text{OL}} = 10^{\psi}(\log \Delta G_c - \psi) \quad (7.2)$$
As mentioned in Section 7.6.3, in this study, $\Delta G_{AC}$ was approximately 385 J/m$^2$, which is approximately two thirds of the critical strain energy release rate. This value is slightly above the strain energy release rate value corresponding to crack arrest in a quasi-static testing, $G_a$, which is approximately 350 J/m$^2$. Therefore, it is suggested that $G_a$ can be used as an approximation to $\Delta G_{AC}$ if experimental VA data is not available.

Figure 7.29 Schematical presentation of unstable crack growth in Damage Shift model

In displacement control, $G$ decreases as the crack extends. Therefore, the unstable crack growth will stop before the joint has fractured completely at the arrest critical strain energy release rate, $\Delta G_{arr}$. $\Delta G_{arr}$ can be assumed specific to a joint and spectrum under...
which it is tested. As mentioned in Section 7.6.3, $\Delta G_{arr}$ was 254 J/m$^2$ with a standard deviation of 24 J/m$^2$ in this study. It should be noted that if a DCB joint is tested under load control, $G$ will increase with crack length. Therefore, the unstable crack growth will lead to catastrophic failure of the joint. After the unstable crack growth, the value of $\Delta G$ associated with the crack arrest point, i.e. $\Delta G_{arr}$, will now be much smaller and hence crack growth will be greatly reduced. Therefore, there will be no further instances of the sudden crack growth. After the sudden crack growth, CA FCGR-curves can be used for the rest of the analysis. However, it should be noted that the crack growth subsequent to the sudden jump may be slower than predicted by the NCGI. This is because of the fact that the unstable crack growth will accelerate through the existing damage zone suddenly and it will take some time for another stable damage zone to be formed.

To sum up, once the shifted FCGR-curve is obtained, the Damage Shift model can be very effectively used for predicting VA fatigue of bonded joints. Therefore, the NCGI algorithm described in Section 4.6.2 can be modified to incorporate damage shift, such that:

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**Step I)** Estimate the lateral shift, $\psi$, by comparison of the experimental value of $(da/dN)_{CA}$ and $(da/dN)_{OL}$ at a given value of $\Delta G$.

**Step II)** Calculate $\Delta G_{AC}$ using Eq. 7.2 (i.e. $\Delta G_{AC} = (\Delta G_c)_{OL} = 10^{[\log \Delta G_c - \psi]}$).

**Step III)** Perform NCGI using the algorithm below:

If $\Delta G_{peak} > \Delta G_{AC}$

From $i = 1$ to $i = N_c$
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Use simple NCGI assuming ‘damage shift’

\[ a_{i+1} = a_i + n_i \frac{da}{dN} (\Delta G_{OL} (S_i, a_i, ...)) \]

When \( i = N_c \)

Calculate \((\Delta a)_{jump}\) by solving the equation

\[ \Delta G_{CA} (S_{peak}, [a_{Nc} + (\Delta a)_{jump}], ...) = \Delta G_{arr} \]

When \( i > N_c \)

Apply simple NCGI until \( \Delta G_{peak} \leq \Delta G_{ih} \)

If \( \Delta G_{peak} < \Delta G_{AC} \)

Use NCGI assuming ‘damage shift’

\[ a_{i+1} = a_i + n_i \frac{da}{dN} (\Delta G_{OL} (S_i, a_i, ...)) \]

Until \( \Delta G_{peak} \leq \Delta G_{ih} \)

where each stage in a loading block is a CA displacement fluctuation of \( n_i \) cycles with a stress level of \( S_i \), \( N_c \) is the number of cycles for the unstable crack growth to occur, and ‘peak’ refers to the cycle with the highest stress level in the spectrum.

7.7.2 Experimental validation of Damage Shift model

The Damage Shift model was described in detail in Section 7.7.1. In this section, an experimental validation of this method will be made. In Fig. 7.23(b), the crack growth (CG) curve, \( a \) vs. \( N \), was plotted for a bonded joint subjected to VA loading with a moderate initial \( G \) value (i.e. below \( \Delta G_{AC} \)). As seen, the NCGI underestimated the fatigue crack growth, i.e. the experimental crack growth at high cycles was higher than
that predicted. It is suggested that a shifted FCGR-curve should be used in the analysis. As noted in Section 7.7.1, the shift, \( \psi \), can easily be determined by comparison of the experimental value of \( (da/dN)_{CA} \) and \( (da/dN)_{OL} \) at a given value of \( \Delta G \). This can be accomplished by shifting CA FCGR-curve so that experimental and predicted CG-curves for a VA test with a continuous crack growth is in agreement. It is possible to shift a FCGR-curve horizontally as schematically shown in Fig. 7.28 by changing the empirical constant \( C_p \) used in the Paris law (see Section 4.5) and keeping \( m \) constant (i.e. slope of the FCGR-curve). A typical crack growth curve with Damage Shift correction is seen in Fig. 7.30. As seen, it was possible to obtain a very close agreement between the experimental CG-curve and shifted one. \( \psi \) proved to be 0.21±0.04 (J/m\(^2\) in log-scale). Substitution of this value into Eq. 7.2 gives that \( \Delta G_{AC} \) is 408±37 J/m\(^2\). In Section 7.6.3, it was shown that \( \Delta G_{AC} \) was approximately 385 J/m\(^2\). This result substantiates that the assumption sudden crack growth occurs when \( \Delta G_{peak} \) in the spectrum is equal to the value of \( G_c \) for the shifted FCGR curve, OL, and \( \Delta G_{AC} \) can easily be calculated once \( \psi \) is known.
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Figure 7.30 Correcting NCGI curve using the Damage Shift model - I

The shifted FCGR-curve and the experimentally determined crack arrest strain energy release rate, $\Delta G_{ar}$, were input to the NCGI algorithm incorporating the Shifted Damage model described in the previous section. $\Delta G_{ar}$ was 245 J/m$^2$ with a standard deviation of 24 J/m$^2$. Discontinuous crack growth under VA loading with a high initial $G$ value (i.e. above $\Delta G_{Ac}$) was predicted. A typical prediction is shown in Fig. 7.31. Fig. 7.31 shows that the Damage Shift model was highly accurate thanks to the fact that the number of cycles for the unstable crack growth to occur, $N_c$, (an arbitrarily chosen value smaller than 5000 cycles) is very small and there is very little scatter in $\Delta G_{ar}$, which enables us to calculate the length of the jump ($\Delta a_{jump}$) very accurately.
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Figure 7.31 Correcting NCGI curve using Damage Shift model – II

Figure 7.32 Correcting NCGI curve using Damage Shift model - III
7.7.3 Discussion

The Damage Shift model is simple in that the load history effects are completely characterized by a single parameter, $\psi$, that can be easily determined by experiment. In the initial stages of fatigue damage it is expected that the value of $\psi$ will be dependent on the number and magnitude of overloads. Hence it can be stated that for a given material:

$$\psi = f(N_{OL}, R_{OL}) \quad (7.3)$$

where $N_{OL}$ is the number of overloads and $R_{OL} = \Delta G_{CA}/\Delta G_{OL}$ as long as $\Delta G$ is below a critical value, then as the crack starts to grow through the damage zone an equilibrium position of the FCGR curve will be reached that is only dependent on $R_{OL}$ and the ratio of overload to CA cycles, $N_R$. It is also expected that a similar shift would occur if the amplitude of the spectrum remained the same but the mean value increased, because, as mentioned in Chapter 6, mean stress variations can be even more damaging than the overloads with respect to fatigue crack growth accelerations. In this study, the model has only been validated for two-stage block-loading spectra with intermittent overloads with a single $N_R$ value (1/20). However, this promising initial study showed that the Damage Shift approach could easily be extended to include mean load effects and may
also be applicable to more complex fatigue spectra. Therefore, an extensive parametric testing programme should be undertaken in order to make the Damage Shift model a more universal one.

7.8 Conclusion

For VA fatigue analysis there was a requirement for a reliable analytical method to determine strain energy release rate, \( G \), in bonded DCB joints. Two relatively new methods were found, potentially promising alternatives to the classical simple beam theory (SBT) for the analysis: these are the beam-on-elastic-foundation (BEF) and the beam-on-elastic/plastic-foundation (BEPF) models. An experimental justification of these two methods was made based on how well they correlate to the experimental compliance (EC) method for a constant amplitude (CA) fatigue test. The main attention was given to the BEF model because the adhesive used in this study is quite brittle. The BEF models showed a very good agreement with the EC results. The BEF model agrees well with the EC derived threshold strain energy release rate, whereas the SBT model predicts a higher threshold value and is hence unconservative. It is suggested that BEF is the most appropriate analytical model for calculating \( G \) in most cases. A simplified equation for calculating \( G \) based on the BEF model has been proposed which allows calculation of this parameter to greater accuracy than the SBT model but more simply than using the existing BEF expressions.

DCB specimens were tested under quasi-static, CA fatigue and variable amplitude (VA) fatigue loading. A comparison between the interlaminar and cohesive crack growth for each load case was made. The main difference between the
interlaminar and cohesive crack growth under quasi-static loading was the stick-slip nature of cohesive crack growth. The cohesive crack growth consisted of a succession of rapid growth and arrest phases.

FCGR-curves for cohesive and interlaminar failure were constructed. All FCGR-curves obtained in the present study, either for cohesive or interlaminar crack growth, have showed a strong positive correlation between $\frac{da}{dN}$ and $\Delta G$ in log-log scale.

The effect of $R$-ratio on fatigue crack growth has also been investigated. Crack growth rate was found to be relatively insensitive to the $R$-ratio for cohesive cracks when using $\Delta G$. However, interlaminar cracks were quite sensitive to the $R$-ratio.

For VA fatigue, numerical crack growth integration (NCGI) was carried out to predict fatigue life based on CA data. This failed to predict the experimentally observed cohesive crack growth and interlaminar crack growth in bonded composite joints under VA loading. The FCGR for cohesive cracks subjected to a moderate $G$-value were slightly underestimated. In the case of interlaminar cracks, the predicted FCGR was even more unconservative, thus indicating crack growth acceleration. The crack growth acceleration was attributed to accelerated crack growth through the damage zone created by the overloads.

Another unusual observation was a sudden crack growth after a critical number of applied cycles for high-$G$ cohesive crack propagation. This was repeatedly observed whenever the initial value of $G$ exceeded a certain level, $\Delta G_{AC}$. This phenomenon has been explained using a damage zone concept.
It is proposed that this is related to the effect of the overloads on the damage zone ahead of the main crack front and a simple model has been proposed that is capable of predicting the crack acceleration and sudden crack jumps. This is called the Damage Shift model. Damage Shift model is simple in that the load history effects are completely characterized by a single parameter, $\psi$, that can be easily determined by experiment. In this study, the model has only been validated for two-stage block loading spectra with a single $N_r$ value (i.e. the ratio of overload to CA cycles). However, this promising initial study showed that the Damage Shift approach could easily be extended to include mean load effects and may also be applicable to more complex fatigue spectra.
8.1 Double Lap Joints (DLJ)

The Palmgren–Miner (P-M) rule, which gives quite satisfactory results in many cases, failed to predict the fatigue life of CFRP double lap joints (DLJ) subject to a block-loading variable amplitude (VA) spectrum. The P-M results indicated a severe fatigue growth acceleration due to load interactions. It was shown that mean stress variations, i.e. transitions from a constant amplitude (CA) stage to another stage having a higher mean stress value, can be more damaging than overloads with respect to fatigue crack growth accelerations. This is called the ‘cycle mix’ effect. The cycle mix effect is based on the assumption that strength degradation occurs during mean stress variations, $\Delta S_{mn}$.

It was thus considered that any general model for fatigue life prediction in bonded joints should incorporate $\Delta S_{mn}$ (in our case $\Delta L_{mn}$). A simple predictive model incorporating a cycle mix factor has been proposed. This was called the ‘Linear Cycle Mix’ (LCM) model. This method proved to be a considerable improvement on traditional cumulative damage laws. It should be noted that in this study the model has only been validated for a limited number of different spectra. However, it is expected that the model is applicable to a wide range of block-loading spectra. In other cases, the cycle mix factor, $CM$, i.e. the residual strength degradation due to mean load jumps (see Eq. 6.6), may require modification according to the characteristics of spectra. For instance, the ratio of overloads to the remaining cycles is expected to be a critical parameter, thus the
definition of $CM$ for small and large block loadings may differ. Similarly, the number of understress cycles, i.e. cycles below the fatigue limit, is expected to be critical. $CM$ is also dependant on the material and joint geometry. Therefore, an extensive parametric testing programme should be undertaken in order to make the 'cycle mix' model a more universal one, i.e. applicable to a great range of materials and loading.

The failure mechanisms in DLJ's were investigated as a foundation to develop more realistic predictive methods in the future. The locus of failure of DLJ's were determined by optical microscopy and scanning electron microscopy (SEM). A likely failure scenario was plotted using a 3-step finite element analysis (FEA). According to this, cracks on both sides of a DLJ grow in the same manner although one is usually faster than the other, due to the asymmetry in the joints, and leads to the failure of the joint. This new explanation was supported by further optical microscopy evidence. This information can be very useful if a fracture mechanics approach is to be utilized to predict fatigue life of DLJ's. However, it should be noted that the fatigue life of the joints seemed to be dominated by crack initiation according to the video-microscopy study undertaken, although a more extensive investigation using better equipment would be useful. The relative contribution of the crack initiation and propagation phases in the fatigue life is very important as this enables us to decide whether a fracture mechanics approach is realistic or not for this type of joints. If the dominant phase is crack initiation rather than propagation, the $VA$ fatigue behaviour of the adhesive used to bond the composite substrates becomes important. In this case, a similar methodology used for bonded joints in this study should be performed using bulk adhesive. Therefore, $CA$ $S-N$ (stress vs. number of cycles to failure) curves associated with
different R-ratios (min. load/ max. load) and VA S-N curves should be generated and the P-M rule should be used to define load interaction effects. This way, the existence of 'cycle mix' effect in bulk adhesive can be investigated. This will be an important input into modelling double lap joints subjected to VA loading.

8.2 Double-Cantilever Beam (DCB) Joints

For variable amplitude (VA) fatigue analysis there was a requirement for a reliable analytical method to determine strain energy release rate, $G$, in bonded double-cantilever beam (DCB) joints. Two relatively new methods were found potentially promising alternatives to the classical simple beam theory (SBT) for the analysis: These were the beam-on-elastic-foundation (BEF) and the beam-on-elastic/plastic-foundation (BEPF) models. In this study, the main attention was given to the BEF model, because quasi-static testing of bulk adhesive samples showed that the adhesive used was quite brittle. An experimental justification of the BEF model was made based on how well it correlated to the experimental compliance (EC) method for a constant amplitude (CA) fatigue test. The BEF models showed a very good agreement with the EC results. The BEF model agreed well with the EC derived threshold strain energy release rate, $\Delta G_{th}$, whereas the SBT model predicted a higher $\Delta G_{th}$ value and is hence unconservative. It is suggested that BEF is the most appropriate analytical model for calculating $G$ in most cases. A simplified equation for calculating $G$ based on the BEF model has been proposed which allows calculation of this parameter to greater accuracy than the SBT model but more simply than using the existing BEF expressions.
DCB specimens were tested under quasi-static, CA and VA fatigue loading. A comparison between the interlaminar and cohesive crack growth for each load case was made. The main difference between the interlaminar and cohesive crack growth under quasi-static loading was the stick-slip nature of cohesive crack growth.

Next, fatigue crack growth rate (FCGR) curves for cohesive and interlaminar failure were constructed. All FCGR-curves obtained in the present study, either for cohesive or interlaminar crack growth, have showed a strong positive correlation between \( \frac{da}{dN} \) and \( \Delta G \) in log-log scale. The effect of \( R \)-ratio (min. disp./max. disp.) on fatigue crack growth was also investigated. Crack growth rate was found to be relatively insensitive to the \( R \)-ratio for cohesive cracks when using \( \Delta G \). However, interlaminar cracks were quite sensitive to the \( R \)-ratio.

For VA fatigue, numerical crack growth integration (NCGI) was carried out to predict VA fatigue life based on CA data. However, like the P-M rule in DLJ’s, NCGI failed to predict the experimentally observed cohesive crack growth and interlaminar crack growth in bonded composite joints under VA loading. The FCGR for cohesive cracks, subjected to a moderate \( G \)-value, were slightly underestimated. However, in the case of interlaminar cracks, the predicted FCGR was even more unconservative, thus indicating crack growth acceleration. The crack growth acceleration was attributed to accelerated crack growth through the damage zone created by the overloads.

Another unusual observation was a sudden crack growth after a critical number of applied cycles for high-\( G \) cohesive crack propagation. This was repeatedly observed whenever the initial value of \( G \) exceeded a certain level. This phenomenon has been
explained using a damage zone concept. It is proposed that this is related to the effect of the overloads on the damage zone ahead of the main crack front and a simple model has been proposed that is capable of predicting the crack acceleration and sudden crack jumps. This was called the ‘Damage Shift’ model.

The Damage Shift model is simple in that the load history effects are completely characterized by a single parameter, $\psi$, that can be easily determined by experiment. The value of $\psi$ is expected to be a function of relative stress levels of overstress and the remaining cycles in a spectrum and the ratio of overloads to the number of cycles in the spectrum, $N_R$. In this study, the model has only been validated for two-stage block loading spectra with a single $N_R$ value ($N_R = 20$). Furthermore, mean load variations (i.e. transitions from a constant amplitude (CA) stage to another stage having a higher mean stress level) is expected to be an important factor shifting the FCGR-curve, given the existence of the cycle mix effect in DLJ’s. This promising initial study showed that the Damage Shift approach could easily be extended to include mean load effects and may also be applicable to more complex fatigue spectra. Therefore, as in the case of the Linear Cycle Mix’ (LCM) model, an extensive parametric testing programme should be undertaken in order to make the Damage Shift model a more universal one, i.e. applicable to a great range of materials and loading. This testing programme will be similar to the investigation of the cycle mix effect in DLJ’s. If mean load variations lead to an even bigger shift, $\psi$, in the FCGR-curve, that means a fracture mechanics approach will probably be very effective to determine fatigue life of more realistic joints, such as the double lap joints studied in this programme. In this case, initial cracks can be located using the failure scenario
described in the previous section and using the mode I fracture mechanics data obtained by testing DCB joints and virtual crack closure or J-integral method, it will be possible to simulate VA crack growth. However, it should also be noted that, unlike DLJ’s, DCB’s propagation was affected by overloads even when there is no mean stress variations, which is a particular challenge to apply a fracture mechanics approach to double lap joints tested in this study.

It should be noted that all the above arguments are based on damage zone formation ahead of the crack tip during VA fatigue crack growth. Extensive damage zones ahead of the crack tip have been previously observed in fatigue tested bonded joints using X-ray radiography. However, no X-ray radiography study has been undertaken in the present research, thus there is an urgent need for an X-ray investigation to substantiate the theories the ‘damage shift’ is based on. For this purpose, X-ray radiographs of the area ahead of the crack tip before the unstable crack growth and after the jump should be taken. Radiographs for stable crack growth at the beginning of fatigue cycling and at the crack length corresponding to a $G$ value resulting in crack arrest during unstable crack growth should also be undertaken. A comparison of these radiographs will hopefully provide the physical evidence we need. It is also suggested that a similar X-ray investigation should be undertaken for quasi-static loading. In this case, X-ray radiographs just before the slip phase, i.e. unstable crack growth, and at the beginning of the stick phase, i.e. as soon as the crack growth is arrested, can be used to propose a new damage zone approach to the stick-slip behaviour of bonded joints.
To sum up, two very useful predictive methodologies were proposed for variable amplitude fatigue analysis of bonded joints: the Linear Cycle Mix (LCM) model and the Damage Shift model. However, it should be noted that there is still a lot to do to make these methods applicable to a great range of materials and loading and, thus, to avoid unconservative designs. It is still recommended that service-simulating tests relevant to the load history expected should be performed whenever it is possible.
1) The RMS method failed to predict the fatigue life of bonded composite joints subject to a block-loading variable amplitude (VA) spectrum. It was concluded that the RMS method is not suitable for this type of spectra and should only be considered for narrow-band random loading.

2) The Palmgren-Miner (P-M) rule failed to predict the fatigue life of bonded composite joints subject to a block-loading variable amplitude (VA) spectrum. The P-M results indicated a severe fatigue growth acceleration due to load interactions.

3) It was shown that a 'cycle mix' effect exists in double lap joints tested under a block-loading VA spectrum, thus mean stress variations (i.e. transition from a constant amplitude [CA] stage to another stage having a higher mean stress value) can be more damaging than overloads with respect to fatigue crack growth accelerations.

4) A simple predictive model incorporating a 'cycle mix' factor has been proposed. This method proved to be a considerable improvement on traditional cumulative damage laws for predicting the fatigue life of bonded lap joints subjected to VA fatigue.
5) The locus of failure of double lap joints was determined by optical microscopy and SEM. A likely failure scenario for double lap joints was plotted.

6) An experimental justification of the beam-on-elastic-foundation (BEF) model was made based on how well it correlates to the experimental compliance (EC) method for a CA fatigue test. The BEF models showed a very good agreement with the EC results. The BEF model agreed well with the EC derived threshold strain energy release rate, whereas the classical simple beam theory (SBT) model predicted a higher threshold value and is hence unconservative. It is suggested that BEF is the most appropriate analytical model for calculating $G$ in most cases.

7) A simplified equation for calculating $G$ based on the BEF model has been proposed which allows calculation of this parameter to greater accuracy than the SBT model but more simply than using the existing BEF expressions.

8) The main difference between the interlaminar and cohesive crack growth under quasi-static loading was the stick-slip nature of cohesive crack growth.

9) All fatigue crack growth rate (FCGR) curves obtained in the present study, either for cohesive or interlaminar crack growth, have shown a strong positive correlation between $da/dN$ and $\Delta G$ in log-log scale.

10) Crack growth rate was found to be relatively insensitive to the $R$-ratio for cohesive cracks when using $\Delta G$. However, interlaminar cracks were quite sensitive to the $R$-ratio.
11) Numerical crack growth integration (NCGI) failed to predict the experimentally observed cohesive crack growth and interlaminar crack growth in bonded composite joints under VA loading. The FCGR for cohesive cracks, subjected to a moderate $G$-value, were slightly underestimated. In the case of interlaminar cracks, the predicted FCGR was even more unconservative, thus indicating crack growth acceleration. The crack growth acceleration was attributed to accelerated crack growth through the damage zone created by the overloads.

12) A sudden crack growth after a critical number of applied cycles for high-$G$ cohesive crack propagation was observed in VA fatigue of cohesive cracks. This was repeatedly observed whenever the initial value of $G$ exceeded a certain level. It is proposed that this is related to the effect of the overloads on the damage zone ahead of the main crack front.

13) A simple model has been proposed that is capable of predicting the crack acceleration and sudden crack jumps. This is called the ‘Damage Shift’ model.

14) In this study, the model has only been validated for two-stage block loading spectra with a single $N_R$ value (i.e. the ratio of overload to CA cycles). However, this promising initial study showed that the Damage Shift approach could easily be extended to include mean load effects and may also be applicable to more complex fatigue spectra.
1) An extensive parametric testing programme should be undertaken in order to make the ‘cycle mix’ model applicable to a great range of materials and loading.

2) A more extensive investigation of the relative contributions of the crack initiation and propagation phases in the fatigue life of lap joints should be made using video microscopy and X-ray radiography.

3) Variable amplitude (VA) fatigue behaviour of bulk adhesive should be studied. Particularly, the existence of ‘cycle mix’ effect in bulk adhesive should be investigated.

4) As in the case of the linear cycle mix (LCM) model, an extensive parametric testing programme should be undertaken in order to make the Damage Shift model applicable to a great range of materials and loading.

5) An X-ray radiography study is necessary to substantiate the ‘damage shift’ approach. Stick-slip behaviour in quasi-static fracture should also be investigated using X-ray radiography.
Journal Papers:


Conference Papers:


**Contributions:**

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