The modelling of spin generation with particular emphasis on racket ball games

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The Modelling of Spin Generation with Particular Emphasis on Racket Ball Games

by

Robert Cottey

A Thesis submitted in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

Loughborough University

2002
ABSTRACT

The use of spin in racket sports is vital in a player's shot portfolio to aid control and accuracy coupled with increased difficulty for any opponent. Experts agree that the use of spin is of considerable importance to the way in which the game is played, yet no complete understanding or knowledge of the science has been developed.

A number of basic models exist, but these do not accurately predict the behaviour of a high deformation oblique impact under dynamic conditions such as in the tennis ball and racket string-bed situation.

This thesis provides a detailed explanation of the creation of spin for tennis, which should be generically applicable to other racket sports. A mathematical model has been developed which incorporates the significant deformation of the ball during impact and also racket and ball parameters.

As measurement of the dynamic racket / ball impact phenomena during play is difficult, an experimental programme was established simulating realistic impact conditions using a stationary racket and balls fired from a pneumatic cannon.

High-speed digital camera technology was employed to obtain new information regarding ball string interactions, and in particular, detailed information of oblique impacts of tennis balls on racket string-beds. High performance rackets utilising different string tensions, types and string patterns were set at a range of impact angles to study the phenomena and over 2000 impacts yielded some 73,000 individual frames of information. Analysis of the visual data enabled the contact time, footprint size and shape, and the random movement of the strings as the ball passed over them to be determined.

Post impact flight images of the ball were also recorded, to complete the data required for accurate analysis of the ball / string-bed interaction and validation of the analytical model. The results presented will enable manufacturers to develop equipment with spin enhancement in mind and raise further research questions for investigation.
ACKNOWLEDGEMENTS

I would like to thank everyone involved in making this research possible, especially my supervisor Roy Jones and Sean Mitchell who have offered invaluable advice, guidance and experience. To Dunlop Slazenger International for all the equipment and monetary support.

I would like to thank all of the members of the Sports Technology Research Group, especially Leon, Steve and Nev for their laboratory help and technical know-how, and my personal friends for their support and encouragement.

I would like to thank all of my extended family, especially my mother and Jan.

On a more personal note I wish to make it clear to Bex that she helped me in a way only she could.
"A subject is difficult only so long as you don't understand it. Once you do, it becomes intuitively obvious."

Benson Tongue
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NOMENCLATURE

\( \ddot{y}_{n(t)} \) – acceleration of the ball in the normal direction
\( \ddot{y}_{t(t)} \) – acceleration of the ball in the tangential direction
\( \phi \) – ball rebound angle
\( \theta \) – ball inbound angle
\( \Omega \) – ball inbound spin.
\( \omega \) – ball rebound spin
\( \beta \) – racket inclination angle
\( \omega \) – rotation of the undeformed ball
\( \Delta \omega_{t(t)} \) – change in rotation of the ball
\( \omega_{1} \) – ball inbound angular velocity
\( \omega_{2} \) – ball rebound angular velocity
\( \mu_{k} \) – coefficient of kinetic or sliding friction
\( \phi_{R} \) – angle of linear racket movement
\( \mu_{R} \) – constant coefficient of rolling friction
\( \dot{y} \) – velocity component normal or perpendicular to the string-bed
BDC – Babolat diagnostic centre
\( c \) – damping coefficient
\( c_{b} \) – damping coefficient associated with the ball
COM – centre of mass
\( COR_{n} \) – coefficient of restitution in the normal direction
\( COR_{t} \) – coefficient of restitution in the tangential direction
\( c_{s} \) – damping coefficient associated with the string-bed
DSI – Dunlop Slazenger International
\( e_{A} \) – apparent coefficient of restitution.
\( F \) – tangential surface friction force
\( F_{t(t)} \) – frictional force in the tangential direction
\( fps \) – frames per second
\( I_{ball} \) – rotational inertia of undeformed ball
ITF – International Tennis Federation
\( k \) – spring stiffness
\( k_{b} \) – spring stiffness associated with the ball
\( KE_{losses} \) – kinetic energy losses
$k_s$ – spring stiffness associated with the string-bed

$l_{i(t)}$ – instantaneous ball radius

$l_{\text{min}}$ – maximum perpendicular radius of deformed ball

$m$ – ball mass

$m$ – mass

$M$ – racket mass

$m_b$ – ball mass

$mph$ – miles per hour

$m_s$ – mass of string-bed in racket

$ms^{-1}$ – meters per second

$R$ – ball radius

$R$ – perpendicular contact force

$R_{(n)}$ – reaction force in the normal direction

$r_{\text{ball}}$ – undeformed ball radius

$rpm$ – revolutions per minute

$T$ – period of oscillation

$t$ – time

$T_{(i)}$ – torque force acting on the instantaneous ball radius

$t_f$ – initial contact time

$t_f$ – total contact time

$t_{roll}$ – end of slide phase, start of ‘rolling’ phase

$u$ – ball rebound velocity in the tangential direction

$v$ – ball rebound velocity in the normal direction

$v_{ix}$ – ball inbound velocity in the $x$ direction (parallel to the string-bed)

$v_{iy}$ – ball inbound velocity in the $y$ direction (normal to the string-bed)

$v_{2x}$ – ball rebound velocity in the $x$ direction (parallel to the string-bed)

$V_B$ – ball inbound velocity

$v_{in}$ – ball inbound velocity

$v_{out}$ – ball rebound velocity

$V_R$ – racket inbound velocity

$v_{x0}$ – ball velocity across the string-bed at which point it started to roll

$\hat{y}$ – acceleration component normal or perpendicular to the string-bed

$y$ – distance component normal or perpendicular to the string-bed

$y_{damp}$ – damping component of $y$
Chapter 1

INTRODUCTION

The game of lawn tennis as it is known today was invented in 1874 by Major W. C. Wingfield, although real tennis, the game from which it originated has been played for many centuries. Lawn tennis was initially played as a social game in gardens but nowadays due to its popularity there is a lucrative professional circuit that operates around the world. The aim of the game remains; to win points by hitting the ball in the opponent's court in a way that they are unable to return it. From rudimentary beginnings, the tennis market has become a big business and the last 30 years has seen dramatic changes to the game and in particular to the design of the equipment used.

1.1 CHANGES WITH TECHNOLOGY

Advancements in design concepts made possible by the development of metal, composite and polymer materials have played a large part in the way equipment is designed and manufactured. In turn these equipment developments have made significant differences in the way in which the game is played.

1.1.1 THE TENNIS RACKET

The traditional layered wooden construction has been replaced by hollow carbon fibre and metal rackets, which has allowed the size and shape to increase without additional weight. The enlarged hitting surface that results from a larger frame has made it easier for a player to strike the ball and they can now produce shots with more power and greater precision than before. To halt the progress of oversized racket technology the International Tennis Federation (ITF) amended the rules to limit the dimensions of rackets to a maximum length and width of 736.6 mm and 317.5 mm respectively (ITF, 2000).

1.1.2 THE STRINGS

The strings play a vital part in the game of tennis since they are the only part of the racket that actually touch the ball during a stroke unless the ball is miss-hit. Materials such as nylon and polyester are now being used to manufacture string, as a cheaper and more durable alternative to the traditional gut strings made from cow intestine. While professional players still tend to have their rackets strung with gut, beginners and
recreational players are more commonly using synthetic alternatives. Through their method of construction the synthetic strings available boast different playing characteristics such as control, power and spin, although gut strings are still considered superior in their playing quality. The ITF (2000) regulations for the strings include that the playing surface should be flat, consisting of interlaced strings with a uniform pattern, and have identical playing characteristics on both sides. Only small devices can be attached to the strings for the purpose of limiting wear and tear or vibration.

1.1.3 **THE TENNIS BALL**

The modern tennis ball has not undergone the dramatic advancements that have affected the racket and the strings, although it has become harder and faster over the last 30 years mainly from the introduction of pressure inside the rubber core. The ball has also changed colour from white to yellow and the cloth durability has been improved by the introduction of nylon into the wool weave. The composition of the rubber core has been developed making different types of ball possible such as pressureless training balls, pressurised competition balls, and high altitude balls. New ball technology has even seen the inclusion of micro cellular particles inside the core, which are manufactured for durability. Recently the ITF has introduced three approved tennis ball types, each designed to have various performance characteristics that will be used on different playing surfaces in tournaments. According to the ITF (1999) the regular tennis ball (type 2) should weigh between 0.056 – 0.059 kg, size of between 0.069 – 0.065 m and when dropped from 2.54 m onto concrete should rebound between 1.35 – 1.47 m.

1.1.4 **THE TENNIS COURT SURFACES**

A wide variety of surfaces are fabricated into tennis courts around the world, including clay, grass, and hard court (cement) which are used for the four grand-slam tennis tournaments with each surface possessing distinctive playing characteristics. For an individual player to be accomplished on each type of court demands a high level of skill, and competence. A player may excel on clay, which can be described as a ‘slow’ surface, where long rallies between players are common, but the same player might struggle on a grass court, which is a ‘fast’ surface and has become orientated around the serve and volley style of play. To compete at the highest level a player must have the ability to play a wide variety of shots in order to optimise their performance on the different surfaces.
1.2 THE CONTACT BETWEEN THE TENNIS BALL AND TENNIS RACKET

Since the object of the game of tennis is to place the ball in the opponent's court in such a way that they cannot successfully return it, the contact made between the ball and the strings of the racket is vital to give the type of shot that the player is trying to produce. The contact made between the ball and the strings during impact will determine its rebound flight characteristics (velocity, angle, and rotation) and consequently the flight path. The range of shots in a player's portfolio can be large, from the high-speed first service shots to the gentle drop shot. Almost all of the shots playable have one thing in common, once the ball has left the racket it is likely to have gained some rotation about its centre of mass. The amount of spin achieved has often been underestimated and during a study of the US Open in 1997 Pallis et al (1999) discovered that the first service, once considered a relatively flat shot, actually contained large amounts of spin (see Table 1.1).

1.2.1 SPIN IN BALL GAMES

Spin is not only important in the game of tennis, it is also used to great effect in other sports. For example, when taking a free-kick in football, spin is imparted to bend the ball (its trajectory) around the opponent's defending wall into the goal. Ball spin can also increase the difficulty of striking a baseball or cricket ball, and can aid control and direction of a shot in table tennis. The amount of spin that can be successfully applied to a ball depends on the ball type and the conditions of 'implement' application. Typical values of spin recorded for a range of sports can be found in Table 1.1, which lists the type of spin measured, and shows that significant levels (over 1000 rpm) are recorded in tennis for the majority of shot types. Spin is imparted to a ball as a means of controlling its flight path, however, little information regarding the generation of spin and its scientific analysis is available. The research concerning ball flight is significant covering a wide range of sports such as football, baseball and golf.

1.2.2 SPIN AND ITS INFLUENCE ON THE GAME OF TENNIS

One subject that has a considerable amount of published information regarding the generation of spin, is concerned with coaching players. The aim of this type of literature is to teach players how to hit shots with spin to exercise ball control (examples include Plagenhoef, 1970; Cohen et al, 1984; Brody, 1987; Groppel, 1992;). The coaching literature cover methods of applying spin to the ball, dealing with the movement of the racket and the positioning of the player in relation to the ball.
<table>
<thead>
<tr>
<th>Amount of spin measured (rpm)</th>
<th>Sport</th>
<th>Type of spin</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>480 - 600</td>
<td>Soccer</td>
<td>Curve around wall</td>
<td>Asai et al (1998)</td>
</tr>
<tr>
<td>600</td>
<td>American Football</td>
<td>Forward 'spiral' pass</td>
<td>Brancazio (1985)</td>
</tr>
<tr>
<td>840</td>
<td>Cricket</td>
<td>Spin delivery</td>
<td>Mehta (1985)</td>
</tr>
<tr>
<td>900</td>
<td>Tennis</td>
<td>Low topspin</td>
<td>Pallis et al (1999)</td>
</tr>
<tr>
<td>1500</td>
<td>Tennis</td>
<td>Medium topspin</td>
<td>Pallis et al (1999)</td>
</tr>
<tr>
<td>1200</td>
<td>Tennis</td>
<td>Medium underspin</td>
<td>Pallis et al (1999)</td>
</tr>
<tr>
<td>2300</td>
<td>Tennis</td>
<td>Heavy underspin</td>
<td>Pallis et al (1999)</td>
</tr>
<tr>
<td>3000</td>
<td>Tennis</td>
<td>Heavy topspin</td>
<td>Pallis et al (1999)</td>
</tr>
<tr>
<td>3300</td>
<td>Tennis</td>
<td>Topspin lob</td>
<td>Štěpánek (1988)</td>
</tr>
<tr>
<td>3500</td>
<td>Golf</td>
<td>Backspin</td>
<td>Mehta (1985)</td>
</tr>
<tr>
<td>1200 - 7800</td>
<td>Table Tennis</td>
<td>Chop defense</td>
<td>Seydel (1992)</td>
</tr>
<tr>
<td>6600 - 8400</td>
<td>Table Tennis</td>
<td>Topspin</td>
<td>Seydel (1992)</td>
</tr>
</tbody>
</table>

Table 1.1. Spin rates found in various sports.

Research studies concerned with the influence of spin on tennis ball performance have been undertaken. Štěpánek (1988) considers the trajectory of a topspin lob and how the spinning ball alters its flight path as it travels through the air.

Spin levels during play was the subject of an extensive study, as mentioned previously, carried out by Pallis et al (1999) during the 1997 US Open Tennis Tournament. The levels of spin recorded gave a thorough insight of spin rates during the professional game, with some surprising results, for example the level of spin recorded for the first service (Table 1.1) which has previously been perceived as flat with little or no spin (Anderson and Anderson, 1982; Brody, 1987)

The studies mentioned above consider the rotating ball, but they do not analyse in any detail the mechanics of the creation of spin during impact.

1.2.3 MODELLING OF TENNIS BALL IMPACTS

Several authors have developed mathematical models to predict the amount of spin that is applied to the tennis ball during impact (Daish, 1972; Brody, 1984; Groppel et al, 1983; Dignall et al, 2000; Cross 2000c). Unfortunately these tend to have limitations because they are not validated with substantial experimental evidence. The traditional method used to develop the models involves the assumption that the ball is non-deforming and a constant value for the radius is used in the calculations. The construction of the hollow rubber tennis ball, the relatively high velocities involved and the short duration of the impact, make deformation of the ball during impact inevitable. The assumption that the ball remains spherical during impact is one that could be improved in the construction of an accurate predicting model.
1.3 RESEARCH OBJECTIVES

Previously reported research studies indicated top spin rates of 3500 rpm for the topspin lob (Štěpánek, 1988), a shot where considerable spin is applied, yet the results reported by Pallis et al (1999) gave rates of up to 5357 rpm for a second service shot. Due to the importance of spin for ball control and the fact that there is no ITF rule limiting spin (indeed, there is no ball sport which has a rule concerning spin), an opportunity exists to develop a more complete understanding of spin generation. This will not only give manufacturers knowledge to develop new products but may also help the governing body, the ITF, in formulating rules regarding this aspect of equipment.

This thesis aims to provide a detailed explanation of the creation of spin for tennis that could be generically applicable to other racket sports.

A mathematical model will be developed which incorporates the significant racket and ball parameters. The model will be validated using an extensive experimental programme, which utilises high-speed video photography and a digital image capture system.

1.4 THESIS OUTLINE

This thesis describes the work completed in the development of a model that describes the oblique impact between a tennis ball and head clamped racket. An extensive experimentation programme was designed and implemented to obtain data to validate the analytical model. Images captured of the string-bed throughout testing have revealed a high level of detail leading to the development of a new understanding of the tennis ball / racket impact by revealing impact information not previously reported.

Chapter 2 reviews the literature available on all aspects of spinning objects, from initial creation of rotation to the final measurement of the spin. The review concludes by examining the methods that have been used to model impacts and collisions in sport, and identifies the important parameters that govern the generation of spin.

Chapter 3 presents the stages leading to the development of the analytical model of the tennis ball / racket impact. Normal impacts are initially considered with the ball and string-bed being modelled using separate spring and damper systems. Oblique impacts are then considered by simulating the normal and tangential components of the impact and assessing them individually. The model predicts the rebound characteristics of a tennis ball after impact for given inbound conditions.
Chapter 4 describes the experimental programme, method and equipment used to complete the testing that will help validate the model developed in Chapter 3.

Chapter 5 develops the methodology used to design the experimental programme so that relevant data could be obtained to cover the level of detail required to assess the importance of the parameters used in the model.

Chapter 6 presents the results of the experimental programme.

Chapter 7 presents an evaluation of the model developed in Chapter 3, comparing the model predictions of rebound characteristics to published models and experimental data. The implications of the results and the new knowledge of the impact observed from the experiments are also discussed.

Chapter 8 presents the conclusions of the research. Finally Chapter 9 covers the recommendations for further work.
Chapter 2

PREVIOUS WORK

This chapter reviews the current literature available covering various aspects of rotation. A wide range of texts are available that refer to rotating objects from spinning tops (Pitt et al, 1978) to planets (Sonntag and Clouting, 1999). The emphasis of the chapter is to discuss the rotation of sports equipment, items that are thrown or struck in a particular way as to cause them to rotate or spin.

The methods used to induce spin to these objects are described, and the forces and motion arising once they rotate are discussed. The amount of spin that can be imparted is examined for a variety of sports.

The primary focus is on tennis and the use of spin in the game, how different movement of the tennis racket can introduce varying amounts of rotation, and to what effect the shots are used. Myths regarding the particular application of spin and the levels seen in the game are resolved. The measurement of rotation is then considered and the various methods that have been used to calculate the spin and rebound characteristics of a tennis ball.

Relevant literature concerning collisions are then discussed, to identify the important parameters required to develop an accurate analytical model that will describe the impact between a tennis ball and tennis racket. Existing models are reviewed and assessed to use as a basis for the development of an improved analytical model.

2.1 SPIN IN GENERAL SPORTS

The Oxford English Reference Dictionary (Pearsall and Trimble, 1996) defines spin as “a revolving motion through the air, especially in a rifle bullet or in a billiard, tennis or table tennis ball struck aslant”.

2.1.1 WHAT MAKES AN OBJECT ROTATE?

Rotation is generated when a force is applied to an object at any point other than through its centre of gravity. Spin can be applied to an object for different reasons and can have a variety of effects; including changing the trajectory or bounce characteristics of the rotating body. In sport there are a number of methods used to impart spin to the playing ball, including striking it with a racket or bat, throwing or pitching it in a
particular way, and also after impact with the playing surface. Daish (1972) offers a
detailed discussion and derives equations of motion for aspects involved with the
physics of ball games. Daish considers spin and how it is applied, the consequences of
spin, and how balls interact with surfaces and implements.

2.1.2 ROTATIONAL INERTIA

The rotational inertia of an object affects its ability to rotate, and depends on its mass
distribution, shape and dimensions. If you examine the two types of golf ball available,
wound and solid construction, although the mass and dimensions are the same the
moment of inertia is different. The difference in the mass distribution between the two
golf balls will have an effect on their ability to rotate. If you consider that a wound ball
has a large percentage of its total mass located in the core, its inertial properties will be
different to that of the solid construction ball with an even mass distribution throughout.
The ease at which spin of an object can be generated will depend on its physical
properties, and also on the method of its application. The level of spin that has been
measured in sports has been given (Table 1.1, Chapter 1) but the ability to generate spin
has implications that will determine how a particular shot is played, and how the
opposition reacts to it.

2.1.3 TRAJECTORIES OF SPINNING OBJECTS

Several forces act on a projectile as it flies through the air that can affect its
trajectory. The three main forces are an acceleration force due to gravity, a drag force
due to air resistance, and if the object is rotating a force will be created from what is
known as the Magnus effect.

Nennstiel (1996) offered a simplified description of the Magnus effect; if an object
moving through an air stream is spinning then it causes air molecules to adhere to its
surface, this results in the air stream flow becoming asymmetric. The addition of the air
stream velocity and object's rotational velocity occurs at one side, and the subtraction of
these velocities exist at the other side. This, according to Bernoulli's rule, results in a
difference of the air pressure between the two sides which causes a force that acts on the
rotating object – this is the Magnus force.

For non spherical objects the benefits of rotation may not be initially obvious
however, Frohlich (1981) described that the "most important effect of the discus' rotation is to stabilize its orientation during flight" and a non-rotating discus will "lack
stability and will wobble and flutter”. Nennstiel (1996) reported that mass coupled with the aerodynamic forces influence the flight of a moving bullet. If a fired bullet is not rotating around its axis of symmetry it can become unstable and start tumbling. If the bullet is rotating around its axis of symmetry the Magnus force leads to a gyroscopic effect on the flightpath; the effect is that the bullet’s axis of symmetry moves as if following the surface of a cone whilst it travels through the atmosphere.

However, there are ways in which a non-spinning ball can have an altered flight path. Adair (1995) describes that in baseball the flight of the knuckle ball (a pitch thrown at about 60 mph that will rotate less than ½ a revolution during its flight) can veer in an apparent random direction that makes it difficult to strike, and even harder for the ‘catcher’ to catch. The knuckle ball deviates from its path due to the passage of air flowing around the ball during flight. A baseball is constructed in two pieces, rather like the tennis ball, but it is stitched instead of glued together causing it to have smooth sections joined with raised areas. The raised stitches can cause a disruption in the flow of air around the ball in such a way that it veers toward this disturbance.

2.1.4 THE INFLUENCE OF SPIN IN SPORTS

De Mestre (1991) produced a mathematical text that covered a wide variety of sports projectiles, and described their motion using simple mathematics.

Adair (1995) described the motion of a curve ball (a pitch thrown at 70 mph with rotation around the balls’ vertical axis of about 1800 rpm) as a throw that will deviate by about 0.5 m transversely from its initial direction of flight during the 18 m from the pitcher’s mound to the batter’s plate. The path of the rotating baseball will enhance the difficulty of the pitch and cause the batter problems when trying to strike it.

Spin can be used to great effect when taking a free kick in football, to bend the ball (its trajectory) around the opponent’s defending wall into the goal. Asai et al (1998) used practical experiments to demonstrate the football’s curved flight, by instructing players to strike a ball a distance of 25 m around a post into the corner of the goal. Asai et al filmed the ball in flight with a high-speed video camera operating at 4500 frames per second (fps) to calculate speed and rotation. It was found that for ball speeds of 20 - 25 m/s the rotation calculated was between 480 - 600 rpm. Ireson (2001) used football as an example for teachers to introduce some basic physics to students. Ireson described concepts including the Magnus effect by applying the theory to a David Beckham free kick. Ireson showed how the force generated from a spinning football
could cause a deviation from its projected flight path. For example, the motion of a football stuck at 25 ms\(^{-1}\) with a spin rate of 600 rpm after one second of flight would deviate by \(\approx 4.5 \text{ m}\). Neilson (2002) also measured the rotation of footballs struck by professional players and observed rotations of up to \(\approx 600 \text{ rpm}\) for a 30 m instep kick.

Golf balls that are struck using a normal club will tend to have backspin applied to the ball; the backspin creates lift, which can have the effect of lengthening the trajectory depending on the club used. Bearman and Harvey (1976) measured the aerodynamic forces that act on a golf ball during its flight for a range of velocities and spin rates inside a wind tunnel. They also showed that for an initial velocity of 58 ms\(^{-1}\) and 10° elevation the range of a ball is increased by 90% for a ball with 4000 rpm compared to a ball with no spin, indicating the Magnus force can have dramatic effects. Erlichson (1983) echoed these views and stated “the aerodynamic lift on a golf ball due to the backspin of the ball is a major force on the ball.” Erlichson explored the trajectory of a golf ball with and without this aerodynamic lift, the latter giving a shorter flight path when initial launch conditions were kept constant. Most of the research completed on golf ball trajectory has been concerned with the drive, however for low iron and wedge shots high spin is required for control. The high trajectory of these shots will reduce the overall distance travelled, but the rotation can increase control when the ball strikes the green.

The generation of spin in some racket sports is not always obvious to players. Discussions with international standard squash players revealed that the use of spin in squash did not become significant until top level play was achieved. The players were only made aware of the implications and uses of spin, during theoretical discussions with their coaches.

Spin is an important aspect of table tennis since few shots are played without spin being imparted. Jian (1986) reported the types of spin that a table tennis ball can contain, and how these affect its bounce off the table and racket. Strategies were presented that could aid a player in dealing with spinning shots. Jian explained how to judge the type of spin applied by understanding the motion of the opponent’s racket at the moment of impact. The ITTF rules (ITTF, 2001) allow the table tennis racket to have pimpled or non-pimpled rubber covering the blade, which can increase the range of shots available. By introducing large amounts of spin to the ball it is enabled to dip over the net and into play, and then rebound in such a way that it is difficult to return.
An evaluation of the rules regarding equipment in 48 ball sports (Diagram Group, 1991), has revealed that only one game, golf, has a specification relating to the speed of the ball and no sports have a ball rule regarding spin.

2.2 SPIN IN TENNIS

The game of tennis has been under examination from the governing body (the ITF), to generate more interest in the game at all levels. The increase in the service speed has been speculated as making the game less attractive to the spectator, although contrary to this, attendance figures for the 2002 Wimbledon championships were the second highest in recorded history (Wimbledon, 2002). The number of players with a first service velocity of over 124.3 mph has risen from 5 to 38 in just five years (Coe, 2000), which has prompted research into methods that can be used to slow the game down including the introduction of larger diameter balls. Spin in the game of tennis has avoided much of the governing body’s interest and is still an area where little research has been completed into the method of its generation.

Spin is an important part of the game of tennis but there is limited literature on the subject. The majority of published work regarding the generation of spin is in the form of advice from players and coaches, and their perceived wisdom rather than factual scientific knowledge. Others sources cover the biomechanics of tennis and report on how players execute shots in a play situation in an attempt to understand how spin is created.

2.2.1 TYPES OF SHOTS PLAYED

Tennis coaches provide useful information about the game of tennis and how and when a particular shot should be played to gain maximum effect on court. Heldman (1976) explained how spin can be used to upset the rhythm of an opponent, and can enable high trajectory shots to stay in court. Side spin can also be used to make a passing shot curve to stay inside the tramlines. Heldman described how topspin shots will ‘jump up’ and bounce extremely high, how backspin shots will slow up, and how sidespin shots will pull a player wide. Descriptions included how the spin shot is important in high class tennis and suggested ways in which a player could deal with the different types of spin, including heavy topspin, backspin, sidespin, spin serves, chops (a combination of sidespin and backspin) and spin lobs. Cohen et al (1984) describe, from a coach’s point of view, the use of various shots for different situations. A
backspin shot ‘provides depth down the line’ and ‘steadiness’ crosscourt, and topspin is
used to control ‘firm crosscourt shots’.

2.2.1.1 TOPSPIN SHOTS

Štěpánek (1988) evaluated the topspin lob, a difficult shot in the portfolio of a tennis
player. Calculations were made to determine the theoretical path of the topspin lob.
Štěpánek described how, for the topspin lob, the Magnus force acts towards the centre
of curvature of the ball’s ballistic trajectory and so increases the curvature of the flight
path. The overall effect is to decrease the range of the shot compared with the trajectory
of a non-spinning ball. A high-speed camera was used to obtain data from player’s
shots to determine the amount of spin generated in the application of the topspin lob and
revealed values up to 3500 rpm.

Takahashi et al (1996) discussed how, for a ball / racket impact where the incoming
ball motion was perpendicular to the racket face, the component of racket velocity
parallel to the string-bed determined the level of spin generation. Takahashi et al
studied three shots in their work; a down-the-line flat forehand shot hit with minimal
ball rotation, a topspin forehand with a moderate level of ball rotation, and a topspin lob
with a high level of ball rotation. Analysis of the racket velocities at impact revealed
that the racket movement was significantly different between the three types of shots.
Table 2.1 shows the results for the racket head velocities for the different shots tested.
Takahashi et al associated the changes in velocity vectors of the pre and post impact
racket trajectory for the three strokes with the changes in ball rotation, but no spin
values were measured or tabulated.

<table>
<thead>
<tr>
<th>Type of shot</th>
<th>Mean racket velocity (ms⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forward component</td>
</tr>
<tr>
<td>Flat</td>
<td>16.8</td>
</tr>
<tr>
<td>Topspin</td>
<td>13.9</td>
</tr>
<tr>
<td>Topspin lob</td>
<td>8.7</td>
</tr>
</tbody>
</table>

Table 2.1. Mean linear racket head velocities at impact. Takahashi et al (1996)

Elliott and Marsh (1989) completed a biomechanical comparison of the topspin and
backspin forehand approach shots. The study determined that the velocity and
trajectory of the racket head for the two shots were significantly different as given in
Table 2.2. The study also determined the inclination angle of the racket at impact,
which determines the angle of incidence of the ball with the racket.
Table 2.2. Comparison of forehand topspin and backspin shots (Elliott and Marsh, 1989).

Elliott et al (1989) studied the topspin forehand drive shot of high quality players. The players were split into groups, who either struck the ball with their whole arm moving as one unit with the racket, or, where the arm moved as individual segments. They concluded that the racket head velocity at impact was higher for the individual segment group which resulted in a higher post impact ball velocity (Table 2.3)

Table 2.3. Comparison of forehand drive shots for different biomechanical arm movements (Elliott et al, 1989).

2.2.1.2 THE SERVICE SHOT

Elliott (1983) investigated the service shot to discover if the first serve of highly skilled players is struck with forward rotation applied to the ball. Male and female players were observed from three age groups, 12 year olds, 15 year olds, and adults. Results indicated that for the 12 year old group no significant spin was applied to the ball because the racket head moved in a straight line just before until just after contact. If the racket movement was upward during the impact, which was the case for the other two groups, then rotation was applied to the ball. Values for rotation and velocity of the ball are shown in Table 2.4, however no racket head velocity measurements were reported.
Average ball velocity | Average ball rotation | Racket path prior to impact
---|---|---
12 year olds | 31.8 | 165 | Flat
15 year olds | 40.6 | 525 | 0.5
Adults | 47.2 | 870 | 3.5

<table>
<thead>
<tr>
<th>Age group</th>
<th>Average ball velocity after impact (ms⁻¹)</th>
<th>Average ball rotation after impact (rpm)</th>
<th>Racket path prior to impact (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 year olds</td>
<td>31.8</td>
<td>165</td>
<td>Flat</td>
</tr>
<tr>
<td>15 year olds</td>
<td>40.6</td>
<td>525</td>
<td>0.5</td>
</tr>
<tr>
<td>Adults</td>
<td>47.2</td>
<td>870</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 2.4. Results from Elliott (1983) showing the average velocity and rotation of first service shots, and the path of the racket head prior to impact

Elliott et al (1986) completed a three dimensional cinematographical analysis of the tennis serve and found that the tip of the racket followed an upward trajectory of 4.0° with respect to the horizontal. The racket at the point of impact was positioned vertically and was observed to move in a sideways direction across the ball during the stroke. The distance and angle that the racket moved in were not specified in the studies. The linear velocity of the tip of the racket at impact was measured and the average results are shown in Table 2.5.

<table>
<thead>
<tr>
<th>Subjects tested</th>
<th>Linear racket tip velocity at impact (ms⁻¹)</th>
<th>Path of racket at impact (°)</th>
<th>Vertical drop velocity of ball at impact (ms⁻¹)</th>
<th>Average ball velocity after impact (ms⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>34.8</td>
<td>4.0</td>
<td>2.15</td>
<td>42.4</td>
</tr>
<tr>
<td>Female</td>
<td>31.8</td>
<td>4.1</td>
<td>2.18</td>
<td>34.4</td>
</tr>
</tbody>
</table>

Table 2.5. Results from Elliott et al (1986).

Van Gheluwe and Hebbelinck (1985) examined the service kinematics of high calibre Belgium players and recorded racket and ball velocities pre and post impact; however no reference was made to the direction of the motion of the racket (Table 2.6).

<table>
<thead>
<tr>
<th>Service shot completed for fluent movement not velocity</th>
<th>Racket head velocity (ms⁻¹)</th>
<th>Ball velocity (ms⁻¹)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-impact</td>
<td>30.8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Post-impact</td>
<td>19.1</td>
<td>37.3</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.6. Racket head velocities during service movement. Van Gheluwe and Hebbelinck (1985)

2.2.2 METHODS EMPLOYED FOR THE APPLICATION OF SPIN

Player perception of the game of tennis is very much subject to debate and Hatze (1976) dispels a common thought amongst players that the ball can be guided during the shot. For some players to hit the ball with spin, they think that they need to rotate the racket or roll the wrist while they strike the ball. It is the relative movement of the racket to the ball at the moment of impact that determines the rebound characteristics of the ball, including how much and which direction the spin imparted to the ball is in. These are mentioned in the tennis textbooks, for example Plagenhoef (1970). Elliott et
al (1987, 1989) identified the direction of the racket motion, until just prior to impact, as a low to high trajectory, 17° and 19° to the horizontal for the backhand and forehands topspin drives respectively; this angle changes dramatically to 44° and 47° for the impact duration of the respective shots. The relatively shallow angle initially is thought to be how the player prepares the racket in order to strike the ball, and then the increase in angle at the final moment is to generate the topspin.

The methodology for coaching players to follow through with the racket movement and allow for rotations of the trunk and arm are important aspects of injury prevention (Broer, 1966 in Elliott et al. 1989). It was discovered that for a topspin backhand drive the racket velocity after impact was 90% of the pre impact velocity (Elliott et al. 1989). The limb movements also develop good habits that allow shots to become more natural and repeatable.

The types of shot that a player can use involve many techniques and depend on the stance, the way the ball is addressed, the direction in which the racket is swung, the racket and ball velocity, and the follow through after the shot has been played. A summary of racket and ball velocities for a variety of shots can be seen in Table 2.7.

<table>
<thead>
<tr>
<th>Shot Type</th>
<th>Standing topspin forehand down the line</th>
<th>Standing topspin backhand down the line</th>
<th>Service shot completed for fluent movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Racket head velocity (ms⁻¹)</td>
<td>Pre-impact</td>
<td>Post-impact</td>
<td>Pre-impact</td>
</tr>
<tr>
<td>Ball velocity (ms⁻¹)</td>
<td>Pre-impact</td>
<td>Post-impact</td>
<td>Post-impact</td>
</tr>
<tr>
<td></td>
<td>21.7</td>
<td>17.0</td>
<td>32.5</td>
</tr>
<tr>
<td></td>
<td>18.8</td>
<td>16.6</td>
<td>29.6</td>
</tr>
<tr>
<td></td>
<td>30.8</td>
<td>19.1</td>
<td>37.3</td>
</tr>
</tbody>
</table>

Table 2.7. Racket and Ball movement before and after velocities.

2.2.2.1 HANDLING THE RACKETS

Grip firmness has been the subject of debate over the years. Experimental results indicated that the grip tightness for a wooden construction racket had no significant difference on the ball rebound velocity for a central impact (Hatze, 1976; Watanabe et al., 1979). However Elliott (1982b) reported that the level of grip tension of a similar wooden racket affected the rebound velocity and reaction force for off-centre impacts. Later, Grabiner et al. (1983) used a metal construction racket and concluded that the grip firmness didn’t give any mechanical advantages for low velocity off-centre impacts (10.6 ms⁻¹), but that the grip firmness is necessary for control of the racket after impact.
Brody (1987b) discovered that from striking the string-bed and measuring the frequency of the vibration that the frequencies recorded for a hand held racket was unlike that of a clamped handle racket. Brody concluded that the vibration properties of a hand held racket were not the same as a handle clamped racket and implied that rigidly clamping the racket does not simulate play conditions. From the few examples mentioned it is clear that there is some discrepancy in the resulting rebound characteristics of the ball depending on the type of racket used and how the racket handle is clamped. To limit the effect of the frame on the ball’s rebound attributes, it is concluded that it is essential to clamp the head of the racket securely so that the only energy losses are attributed to the strings and to the ball only, a view echoed by Hatze (1993).

Baker and Wilson (1978) completed a test programme using rackets clamped by their handles, to determine the influences of racket stiffness, string type and tension on the rebound ball velocity. They concluded that for flexible rackets strung at 40, 50 and 60 lb tension the highest ball velocity ratio was recorded for the 50 lb racket, but for stiff rackets the ball response was not significantly influenced by the string tension. Groppel et al (1987) also used rackets clamped by their handles to determine the effects of string type and tension between midsized and oversized tennis rackets.

2.2.3 Levels of Spin Recorded

It is apparent that there is still some disagreement among experts regarding the level of spin resulting from shots. Anderson and Anderson (1982) and Brody (1987) describe the first serve to be a flat serve with little or no spin, but Pallis et al (1999) who completed an extensive study on professional players at the 1997 US Open tennis tournament, contradicted this view. Pallis et al used cameras filming at 250 fps revealing significant levels of spin in the professional game (Table 2.11, section 2.3). The highest levels of spin were observed on the first and second serves (4284 and 5357 rpm respectively). Similar results were recorded, in proportion to the linear ball velocity, amongst University first team players at Loughborough University, supporting Pallis et al’s findings (Table 2.8).

Although there are some differences in the spin rates obtained from the professional players compared with the University players, the amount of spin recorded in both cases is substantial. Furthermore it can be seen that as the spin level increases the linear ball velocity decreases. The work completed by Pallis et al (1999) is significant in the
development of knowledge about spin in tennis and is described in more detail in section 2.3.

<table>
<thead>
<tr>
<th>Type of shot</th>
<th>Player</th>
<th>Spin (rpm)</th>
<th>Velocity (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First serve</td>
<td>Rios</td>
<td>3167</td>
<td>92</td>
</tr>
<tr>
<td>Sampras</td>
<td>2699</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>Henman</td>
<td>1548</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>Agassi</td>
<td>4650</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>Second serve</td>
<td>Sampras</td>
<td>4623</td>
<td>85</td>
</tr>
<tr>
<td>Martin</td>
<td>3370</td>
<td>89</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.8. Selected data from Pallis et al (1999) and experimental work completed at Loughborough University.

2.2.4 STRING PARAMETERS AFFECTING SPIN GENERATION

The properties of the string are very important in determining the playing characteristic of the racket. The tension, string type and gauge, and string pattern can all be adjusted within the racket which can alter the performance of the racket. The strings are highly elastic and will return almost 95% of the energy back to the ball after impact (Brody, 1995). The strings can be manufactured from synthetic as well as natural products, such as animal intestine.

2.2.4.1 STRING TENSION

There is a common conception among tennis players and coaches that to gain more power out of a shot, the tension of the strings should be reduced; but to gain a higher degree of spin and control the tension of the strings should be increased (Brody, 1995).

The tension of the string-bed is of considerable interest but its measurement is difficult. The string tension is usually specified as the load applied to the string during stringing. However, experimental work carried out by Cross and Bower (2001) showed that the tension of a tennis racket immediately after stringing is 30 – 40% lower than that specified by the stringing machine indicating that the initial relaxation of the strings occurs during the stringing process itself. Cross and Bower (2001) argued that the important parameter for the tennis racket is the final string-bed stiffness and not the tension as indicated by the stringing machine. There are various methods available to determine a value for the string-bed stiffness such as the Stringmeter™ (Stringmeter, 1997), which is a two pronged device that is inserted into the string-bed straddling the string to be tested and by a twisting motion can give a measure of the localised string tension. The method used throughout this test programme was the Babolat Diagnostic.
Centre, BDC, described in detail in Chapter 4, which can determine a range of racket properties including the string-bed flexibility.

In an impact between a tennis ball and racket, the ball and the string-bed deform differently. The ball is designed to return about 55% of its initial energy when it strikes a surface and rebounds (ITF, 1999); the strings on the other hand return as much as 95% of the initial energy as mentioned previously (Brody, 1995).

Knudson (1997) completed a study examining the effects of string tension on rebound accuracy. Tennis balls were projected at 24.4 m/s onto the racket face, the impact location was off-centre, and the ball incident angle was 85°. The rebound angle of the ball was measured using imprints made of the ball striking carbon paper taped to a wall 2.85 m from the racket. The closer the rebound angle was to the inbound angle the more accurate the shot. The study compared rackets strung with 50 lb, 60 lb and 70 lb tension and the results showed that there was no significant difference between the accuracy of shots struck between the 60 and 70 lb rackets but a significant decrease in accuracy of the 50 lb racket compared to the other two. No spin data was presented in the study.

2.2.4.2 STRING TYPE AND GAUGE

There are many types of string available to the tennis player and all of them claim to be manufactured for a specific purpose such as durability, power, control, spin etc. Professional players are more likely to use the traditional gut strings as they can hold their tension better (Cross, 2000b) and have the best feel and playing characteristics (Gooden, 1999). Natural gut tennis strings are made up of 17 half strands of cows intestine each 19 mm in width and 12.8 m (14 yards) long. The 17 strands that make one tennis string are cleaned and spun together 1068 times to give the final string its strength. The strings are dried and polished to give the finished product. The process takes approximately 11 days from cow to racket. Natural gut is also renowned for having a short playing life; this is one of the reasons why manufacturers have tried to mimic the dynamic properties of gut in a synthetic string.

Social tennis players will use synthetic strings as a cheaper and more durable alternative to gut as restringing is less frequent than for the professional players. Synthetic strings, made up of many strands, have been developed in the quest for a cheap mass-produced string as an alternative to natural gut. Synthetic strings, made typically from nylon, polyester or kevlar are extruded, spun and glued, at lengths of up
to 200 m. Synthetic strings can be manufactured to have particular playing properties such as high spin, extra power and more control. The synthetic string will also typically have an extended playing life. The types of strings fall into many different categories depending on the properties that are exhibited by the material when made into strings. Table 2.9 gives the characteristics of leading string materials.

<table>
<thead>
<tr>
<th>String material</th>
<th>Personal opinion of the string characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kevlar</td>
<td>Has virtually no stretch</td>
</tr>
<tr>
<td>Polyester</td>
<td>Some stretch is seen over time but has no memory so they lose their tension easily. There is not much lateral string movement during play. Can’t get as much power due to the low elasticity.</td>
</tr>
<tr>
<td>Nylon</td>
<td>Hard wearing especially when used on clay courts.</td>
</tr>
<tr>
<td>Gut</td>
<td>Best feel and playing characteristics.</td>
</tr>
</tbody>
</table>

Table 2.9. Properties of string types. (Gooden, 1999)

There are some extreme cases for players using unusual strings; for example, Pete Sampras uses 1.2 mm diameter strings in an old style Wilson Pro Staff racket. The reason for this choice is unclear and this thickness string is more likely to be used by squash players than tennis players. The string gauge commonly used by tennis players can range typically from 1.3 mm to 1.4 mm.

Cross (2000b) used various experiments to discover the physical properties of tennis string. Cross concluded that there was no difference between different types of string in terms of the normal component of the inbound ball velocity; but speculated that any differences felt by players was due to the change in the transverse component of the ball velocity.

2.2.4.3 STRING PATTERN

It is the racket manufacturers who design the string-bed pattern of the many tennis rackets that they produce. The shape and size of the racket head will lead them to determine the positioning of the holes where the grommets, and eventually strings, are placed. The position of the holes will affect the strength of the finished racket and so the manufacturers are limited to the number of holes possible. The traditional string pattern for the majority of midsize rackets consists of 16 main and 18–20 cross (Stringers Digest, 2000), the main strings run parallel to the handle of the racket and the cross strings run in a direction that is perpendicular to the main strings.

There have been variations to the design of the string-bed; Putnam & Baker (1984) published results from tests using a head clamped racket at 45° with diagonal and conventional string-bed patterns. It was observed that there was no significant
difference in the spin that was generated by the two racket configurations (Table 2.10). No further literature has been discovered that determines if the string-bed pattern has an effect on the generation of spin for the same racket type.

<table>
<thead>
<tr>
<th>Inbound ball velocity (ms$^{-1}$)</th>
<th>Racket angle (°)</th>
<th>Stringing type</th>
<th>Angular velocity of ball (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.3</td>
<td>45</td>
<td>Diagonal</td>
<td>1441</td>
</tr>
<tr>
<td>12.9</td>
<td>45</td>
<td>Conventional</td>
<td>1546</td>
</tr>
<tr>
<td>23.0</td>
<td>45</td>
<td>Diagonal</td>
<td>2740</td>
</tr>
<tr>
<td>23.9</td>
<td>45</td>
<td>Conventional</td>
<td>2807</td>
</tr>
</tbody>
</table>

Table 2.10. Levels of spin recorded by Putnam and Baker (1984).

Research has been carried out using different sized tennis rackets, for example Groppel et al (1987) used midsized and oversized rackets clamped at their handles. They measured rebound velocity, racket head displacement and ball compression for normal impacts (so no spin measurements) at ball inbound velocity of 23.1 ms$^{-1}$. They tested different tensions and string type and concluded that the changes in the string tension affected the racket stiffness, which affected the rebound characteristics measured although not linearly.

Some professional players such as Mark Woodford play with radically different string-bed patterns, he uses a racket with 12 main and 14 cross strings. It is believed that fewer strings feels softer, and the gaps between the strings may grip or bite into the ball more. The large size of gap that exists in the string-bed will reduce the string-bed flexibility if it is strung at a normal tension (60 lb), so to account for the lack of strings, he uses 2 mm string diameter combined with a tension of about 90 lb (Gooden, 1999).

2.3 NASA PROJECT

A study at NASA by Pallis et al (1999) has produced significant data in the development of knowledge about spin in tennis. In conjunction with the US Tennis Association, NASA used high-speed camera equipment to analyse the different shots during matchplay. They set out to study the game of tennis at the highest possible level and undertook a detailed study of the US Open in 1997.

They developed a project to study how tennis balls fly and bounce during play, the objectives of particular interest were:

- To study ball trajectory.
- To examine how much the ball spins and how this varies between players.
• To study how speed and spin are affected after the ball has bounced.

High-speed video cameras operating at 250 fps with a shutter speed of 1/2000 s were used to capture images of the flight path of tennis balls. With the shutter speed relatively low, illumination problems were not apparent and filming could be completed during the day and in the evenings.

The 1997 US Open was the major tournament in which this analysis of spin was completed, and filming at the US Open lasted for 11 days from 10 am for 3 – 4 matches.

At this tournament the balls had a logo on two sides of the ball and these were used as datums for measurement. The team filmed 700 individual shots which included first and second serves, returns of serves, volleys, overhead shots, drop shots and ground strokes with both topspin and backspin. It was apparent that for the top professional players there was no such thing as a completely flat shot and for 700 shots analysed, less than 6 had no appreciable spin. The vast majority of the shots had spin that exceeded 1000 rpm up to a maximum of 5357 rpm on the second serve. The team found that for a forehand shot the type of grip adopted by a player determined the amount of spin that could be imparted. Table 2.11 gives the ranges of spin recorded at the US Open. This study revealed a significant increase in the level of spin produced in the tennis shot, when compared with previous studies.

<table>
<thead>
<tr>
<th>Shot type</th>
<th>Range of spin recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men (rpm)</td>
</tr>
<tr>
<td>Topspin forehand</td>
<td>833 – 3751</td>
</tr>
<tr>
<td>Topspin backhand</td>
<td>790 – 3333</td>
</tr>
<tr>
<td>Backspin forehand</td>
<td>1500 – 3488</td>
</tr>
<tr>
<td>1st serve</td>
<td>1000 – 4284</td>
</tr>
<tr>
<td>2nd serve</td>
<td>2830 – 5357</td>
</tr>
<tr>
<td>Forehand return of serve</td>
<td>600 – 3751</td>
</tr>
<tr>
<td>Backhand return of serve</td>
<td>714 – 2055</td>
</tr>
</tbody>
</table>

Table 2.11. Spin rates of various shots at the 1997 US Open (Pallis et al, 1999).

2.4 METHODS FOR MEASURING SPIN

There are many methods that can be used to determine the amount of rotation a spinning object possesses and the two primary methods now employ the use of high technology imaging equipment and specialist lighting.

The first utilises digital photography and strobe lighting, and the second uses video cameras capable of very high frame capture rates.
2.4.1 **Digital Photography**

The photographic method can be used to capture multiple images of the ball’s flight onto one frame of information. The duration of the camera’s shutter needs to be long enough for several short duration flashes to occur within the space of a few milliseconds. Precise timing is also required to enable successful image capture of the balls flight path. The advantage of this method is that a single image is produced that can be digitally analysed using a personal computer. The image contains information about the velocity, launch angle and rotation of the ball’s flight path. The accuracy of the measurement will depend on the resolution of the image produced, the calibration technique and the clarity of the images.

2.4.2 **High-Speed Cameras**

A high-speed camera can gain large amounts of information over a relatively short period of time. Sequential images captured of the ball’s flight are stored on individual frames that can be viewed in succession so that the ball’s movement can be replayed in slow motion. A wide variety of cameras have been employed that can operate over a range of frame capture rates. High-speed cameras operating at relatively low speeds 200 – 1000 fps have been used to capture the motion of the players performing strokes, (for example Gheluwe and Hebbelinck, 1985; Elliott et al, 1986) and to measure the amount of rotation on the ball, for example Groppel et al (1983). Higher capture rates have been used (>4500 fps) which gain more information, but typically the total capture duration decreases and are often used to capture images of the ball during impact. Dignall and Haake (2000) used 9000 fps to capture information of a tennis ball / court surface impact, and Hocknell et al (1996) used 40,500 fps to measure golf ball impacts.

2.4.3 **Other Methods Used to Measure Rotation**

Gobush et al (1994, 1995) developed a system that could capture and determine the flight characteristics of a moving sports object, in particular golf balls. The system utilised two cameras and flash lighting that captured two images each of the object in question with a delay of 800 µs between them, positioned close to a golf ball on a tee. The golf balls were marked with six light reflecting dots, five dots arranged one at each node of a regular pentagon and one at the centre. Images of the reflective dots were simultaneously captured and combined to enable the initial flight data of a golf shot to be processed by computer software. The absolute velocity was measured; the launch angle was also measured and displayed as an angle to the horizontal and lateral
deviation. The spin flight characteristics could be measured in three-dimensions and the rotation information is given in three separate orthogonal axes of rotation. This system gives good accuracy but has the disadvantage that the reflective dots must always be visible.

Štěpánek (1988) utilised a small magnet adhered to the surface of non pressurised tennis balls to measure the amount of rotation prior to its release into a wind tunnel; this enabled lift and drag coefficients for the spinning ball to be measured. The ball was rotated using the friction of a rotating disc attached to the surface of the ball. A receiver mounted near the magnet picked up its signal to accurately measure the amount of spin on the ball. The method used by Štěpánek allowed precise ball rotation to be calculated, but this method required the magnet and sensors to be in close proximity and would not be suited to court situations.

Tavares (1999) developed a method using radar technology for the measurement of backspin in a golf shot, especially the spin decay during flight. A one-inch diameter circle of metallic based material was adhered to the surface of the ball to reflect the radar's signal. The radar response signal from the ball could be interpreted into its spin rate. The spin calculated was the total revolution rate of the ball but could not be split into separate axis of rotation as the method used by Gobush et al (1994, 1995).

2.4.4 BALL MARKINGS

To gain information about the spinning balls requires the use of specific markings to be employed. Various techniques have been used ranging from individual point markings (Gobush et al 1994, 1995) to quadrants (Elliott, 1983).

The markings are used to calculate the amount of spin of a ball. If high-speed cameras are used to capture images of the flight of the ball, the spin can be calculated by counting the number of half or full revolutions completed by the ball over a defined time interval. The digital photography method requires more precise markings in the form of points or lines drawn on the ball.

When Pallis et al (1999) recorded images of the 1997 US Open tournament the balls being used had two logos instead of the traditional one, which made measurements of the rotation easier. Measuring spin of the ball in this fashion was non-intrusive and has lead to some important knowledge, about professional player performance by analysing their shots; a study that had not been completed before in a tournament environment.
Squash ball spin measurements completed at Loughborough University used balls manufactured with two different coloured halves. High-speed cameras were used to capture the ball during several shot scenarios for world class players. The analysis involved counting the revolutions of the ball to determine the spin for different shot types. Similarly in player testing completed by Elliott (1983), balls were marked in quadrants and the spin and axis of rotation of the balls were determined by counting the revolutions and considering the plane of rotation respectively. Bearman and Harvey (1976) and Chou et al (1994) have both used a stroboscope technique to measure the rotation of golf balls marked with dots.

Measuring rotation under laboratory conditions is easier due to the control of the situation, however under play conditions lighting and environmental conditions can have a significant effect on results.

2.4.5 LAUNCHING DEVICES USED TO IMPART SPIN

Ball launching machines are used to replicate shots played by humans in a court or laboratory situation. These machines are designed to give repeatability for particular shots and can be simulated in large quantities. Machines do not replace the human factor in the application of spin to a ball but can be used effectively to replicate the motion used.

A launching system was developed by Mish and Hubbard (2001) where the rotation that is applied to a baseball can easily be controlled. The device uses a suction cup to hold the ball to the end of a rod that can be rotated mechanically by a motor to rotate the rod and ball accurately. The ball is then released into a moving air stream, which projects the ball at the desired velocity and spin rate.

Machines have been developed to launch rotating sports balls from a device that contains two or more revolving wheels. The ball is passed through the gap in the wheels and the friction between the ball and the surface of the wheels is used to launch the ball. If the two rotating wheels are set to turn at different angular velocities, as the ball passes through the wheels the combined effect will generate an amount of spin as well as projecting the ball in a forwards direction. The spin will be proportional to the difference between the spinning velocity of the wheels. The exact amount of rotation can be calculated in theory from the friction and the difference in the rotating wheels but it is not precise and may contain a wide range of spin applied to the ball that is launched.
Golf robots have been developed to replicate the swing of a golfer. They can be used to conduct large scale testing of different clubs or ball types making sure that the ball is struck in precisely the same manner each time without player fatigue. Similarly the Adidas company have developed a kicking robot for controlled kicking of soccer balls.

2.5 MODELLING NORMAL IMPACTS

Due to the public’s interest in sport, ball impacts have been the topic of many studies, and examples of basic ball collisions can be found in physics textbooks such as Halliday et al (1997). However the common theme of most relevant literature concerns the impact between two solid non-deforming objects. Daish (1972) provided thorough accounts of impacts within sports games that have been used to form the basis of more complex models such as those developed by Brody (1984) or Carré et al (2000).

The simplest form of collision that may be considered is a one-dimensional or normal impact. An early method developed was the Hertz theory of impact that dates from 1881 (Hertz, 1881). Hertz devised a quasistatic, elastic theory to describe the behaviour of balls (Gugan, 2000), however the significant assumption made by Hertz was that there is little ball deformation; which is not always the situation for sports balls.

2.5.1 ELASTIC COLLISIONS

Equations that describe the impact between two non-deforming objects are simple to set up from first principles, and these form the basis of the knowledge required to understand collisions. If a head-on collision is assumed to be closed, isolated, and elastic, then the linear momentum and the kinetic energy of the system are conserved. The motion of the two colliding objects in this simple system can be evaluated fully (Halliday et al 1997). A one-dimensional analysis of a collision is a good method to describe impacts involving solid non-deforming balls as there is little or no energy lost (Ha et al 2001 and Bridges 1998).

Many of the relevant collisions covered in the literature involve different types of ball e.g. steel (Bridges, 1998; Ha et al, 2001), solid rubber (Maw et al, 1976; Supulver and Bridges, 1995) or hollow balls (Araki et al, 1996; Hubbard and Stronge, 2001). Some of these balls experience considerable deformation during impact which requires special consideration.
The approach used to model elastic collisions needs modification if it is to be applied to a tennis ball impact since the definition of an approved tennis ball requires a substantial loss of energy during an impact (ITF 1999).

2.5.2 INELASTIC COLLISIONS

To determine the characteristics of sport impacts, the energy losses that occur for a deformable sphere striking a rigid surface such as concrete or other hard surface need to be addressed to create a detailed understanding of the impact.

2.5.2.1 COEFFICIENT OF RESTITUTION IN THE NORMAL DIRECTION (CORₙ)

A common approach for the description of an inelastic ball impact is to use a term defined as the coefficient of restitution (COR) which accounts for the energy converted to other forms like heat or vibration during the collision. CORₙ is described by Daish (1972) as the ratio of rebound to inbound velocities in the normal direction for the colliding bodies where one of the bodies, the surface, is stationary or is assumed to have infinite mass (Equation 2.1).

\[ CORₙ = \frac{v_{out}}{v_{in}} \quad \text{2.1} \]

To comply with the ITF rules the CORₙ of a tennis ball has to fall between 0.73 – 0.76 when dropped from a height of 2.54 m. A constant value for CORₙ has been used to simplify the calculations; for example Brody (1997) uses \( CORₙ = 0.85 \) to describe the efficiency of a tennis ball colliding with a clamped racket head. However this assumption may have limitations as rubber properties change with high strain rates (Casolo et al, 1997), as may the string-bed characteristics. CORₙ has been used as a convenient way of grouping together the energy losses associated with the ball, and can be effectively used to define the fractional or percentage kinetic energy loss (KE\(_{losses}\)) associated with an impact (Equation 2.2) [Araki et al, 1996; Supulver and Bridges, 1995; Cross, 1999; Gugan, 2000].

\[
\text{Fractional or percentage KE}_{losses} = (1 - CORₙ^2) \quad \text{2.2}
\]

A more detailed approach to describe the energy loss for a tennis ball was given by Cross (1999) who formulated equations that described the loading hysteresis curve of the ball during impact. Gugan (2000) also used hysteresis but stated that no appreciable
energy loss occurs in the compression stage of an inelastic impact and that the majority of losses occur as the ball regains its original shape.

2.5.2.2 VIBRATION

Nathan (2000) and Bridge (1998) identified vibration of the colliding bodies as a possible source of energy losses. Bridge assessed internal vibrations of a hollow ball during impact, and Nathan incorporated vibrational losses into equations describing a baseball and bat collision. Nathan’s research indicates that for both high and low speed impacts, ball exit velocity is only accurately predicted when the bat vibration characteristics are modelled for impacts remote from a vibration node. The implications for tennis suggest that unless the impact is at a vibration node, frame vibration may influence the rebound ball velocity.

Bridge (1998) modelled the bounce of an air-filled rubber ball using the physics of forces that exist in the stretched elastic surface. The sphere was split into segments and model parameters included the elastic modulus of rubber, surface waves and their respective damping coefficients, and internal pressure. Iterative methods were used to determine how each segment of the sphere changed in relation to each other and to the impact surface. Bridge’s method provides useful insight into the phenomena of ball deformation for surface normal impacts, but there is little scope for extending the method to model oblique impacts.

2.5.3 REPRESENTATIONS USED IN MODELS

Various methods have been used to model the dynamic motion of a ball during impact. Figure 2.1a shows the ball set-up used by Nathan (2000) and Cross (1999 and 2000) consisting of a mass \( m \) attached to a spring with a known stiffness \( k \).

Bridges (1998) used a different mass and spring set-up to model the ball (Figure 2.1b). The ball was considered as a double lumped mass system; the mass of the ball was divided in half and a spring \( k_1 \) placed between them. This spring was to emulate the internal vibrations occurring during impact. A second spring \( k_2 \) was also attached outside the ball to describe the overall characteristics during impact. A numerical analysis using dimensionless units was developed by Bridges to describe the ball throughout the impact. Both Bridges and Nathan modelled the surface, which the ball collided with, as solid and non-deforming, but this is not always the case.
Kawazoe and Kanda (1997) modelled the ball and string-bed together for a tennis ball / racket impact (Figure 2.2a) and represented them as a mass attached to a spring (stiffness, \( k \)) and damper (damping, \( c \)) system. Kawazoe and Kanda used the model to determine the spring stiffness required to accurately model the impact for different inbound velocities, indicating that the properties were speed dependent. Kawazoe and Kanda also defined the racket as a solid beam and used finite element methods to model the impact. However no experimental results were obtained to validate the model.

Leigh and Lu (1992) modelled the dynamic interaction of the ball with the strings and racket in tennis using a system of non-linear ordinary differential equations for normal impacts (Figure 2.2b). The model was developed using a one-dimensional lumped mass spring and damper system that included terms for the damping and elastic properties of the ball, strings and racket. The elastic properties of the ball were determined using quasistatic force / deformation tests, and the damping of the ball was calculated from results obtained from a simple drop test. The strings were modelled as if they formed a membrane of interwoven strings and an equation for the elasticity of the strings was determined using equilibrium conditions reached for static deformation of the strings for an applied force. Tests using a pool ball (\( \approx 0.14 \) kg) dropped onto a head clamped racket from heights 0.73 – 2.33 m were used to determine the damping effects of the strings; unfortunately the relatively low ball velocities (3.7 – 6.7 \( m/s \)) are not representative of those generated during play. Working with an error of 1% when calculating the velocities, the ratio of rebound velocity to incident velocity was found to be 1.0 therefore the strings were considered to have no measurable damping effects. The assumption that the string-bed is 100% efficient is not rational and Cross (2000b) measured, for a 0.76 kg steel ball dropped from heights up to 2.4 m (in a normal direction to the strings), that the ball rebounded at 95 ± 2% of the inbound ball velocity. The difference in the linear momentum of the two balls at impact may affect the
deflection of the string-bed, which could account for the differences calculated in the rebound velocities.

Leigh and Lu modelled the racket as an equivalent lumped mass attached to a spring and damper system. The equivalent mass of the racket was found using vibration properties of the racket modelled as a uniform cantilever beam. The stiffness was considered to vary for different rackets and the damping of the racket system was given a constant value for the calculations. Leigh and Lu completed experiments to obtain model parameters using a tennis ball only, and a tennis ball and string-bed combined, but did not complete any experiments combining a tennis ball, and tennis racket. Results from the model agreed with other published work on normal impacts; for example the increase in rebound ball velocity from a reduction in the string tension observed by Elliot (1982) and Groppel et al (1987), and the impact duration as reported by Brody (1979) and Groppel et al (1987).

Dignall and Haake (2000) modelled the perpendicular impact of a tennis ball onto a rigid surface using a spring and damper system (Figure 2.2a). They calculated the parameters \( k \) and \( c \) for the model by projecting a tennis ball onto a surface using velocities between \( 6.7 \) – \( 20 \) \( m/s \) and discovered they were dependent on the inbound velocity of the ball. The linear velocity dependent parameters calculated by Dignall and Haake are discussed further in Chapter 7.

In order to represent a racket, a solid beam has generally been used, as its stiffness and flexibility can be easily altered to reflect the properties of the racket it is portraying. Liu (1983) used a beam that could pivot around its centre of mass (COM) and could be
fixed or left free at the handle. It was discovered through experimentation that the method of clamping the handle made no difference to the impact results, although other researchers such as Hatze (1976) argue that grip pressure does affect impact results as mentioned previously. Cross (1999 and 2000) used a flexible beam to model the racket and found that the model developed showed good agreement with experimental results. Brody (1997), however, modelled the racket as a one-dimensional rigid body and used a constant value of $\text{COR}_N$ to represent the losses in the system. Brody argued that the $\text{COR}_N$ is only slightly affected by the racket string tension and any increase in ball inbound velocity, although in a previous text (Brody, 1987) he argued that string tension is a factor in power and spin generation.

### 2.5.4 The Physics of Springs

An un-damped spring will oscillate indefinitely once excited if no energy losses occur. The motion of a mass attached to a perfect linearly elastic spring can be demonstrated, using simple physics (Halliday et al., 1997), to be sinusoidal.

The amplitude of the curves over time can be reduced by the introduction of a damping coefficient or dashpot, where energy is lost to the damping. Various types of damping are possible, for example structural or hysteretic, viscous, dry friction or coulomb (Tongue, 2002). Viscous damping is used in the model, for simplicity, and manipulation of the damping coefficient and the value for the spring constant will result in the displacement and velocity profile of the ball represented in Figure 2.3.

The amplitude of a spring damper system can be adjusted for inbound and rebound velocities. The two characteristics of the system $k$ and $c$ can be manipulated to give the profile of the ball during impact.

![Diagram of spring motion](image)

**Figure 2.3. The representation of the velocity of the ball as part of the spring motion.**
2.6 MODELLING OBLIQUE IMPACTS

To adequately describe the motion of a tennis ball impacting against a racket string-bed as observed in play, requires the problem to be solved in at least two dimensions since the impacts are seldom normal and spin is almost always induced. A model capable of two-dimensional analysis would enable the flight characteristics (angle, velocity and rotation) of the tennis ball and the motion of the racket string-bed to be addressed.

To develop an oblique model the parameters necessary for the generation of spin need to be considered. They need to be assessed to determine their role in the impact and resulting post impact ball conditions. The existing models are evaluated to develop an understanding of the limitations of the methods traditionally used so that an improved version can be evolved.

2.6.1 IDENTIFICATION OF PARAMETERS

A considerable amount of research has been undertaken on ball impacts and it is apparent when examining the available scientific knowledge that there are still differences of opinion relating to the parameters that affect spin generation. Many have been identified as important, but there is a significant lack of accurate experimental evidence which has meant that many are speculative and inaccurate; clearly experimental validation needs to be addressed for the development of an accurate analytical model to represent a true reflection of any impact in tennis.

2.6.1.1 FRICTION

To develop a two-dimensional model of the collision, parameters in a tangential direction to the impact surface need to be considered. Friction components have been introduced by many authors including Groppel et al (1983), Sondergaard et al (1990), Supulver and Bridges (1994), and Cross (2000c). There are two main types of friction, static friction and kinetic friction and the force required to overcome static friction is usually greater than that required to overcome kinetic friction. A number of authors have published mathematical models for oblique impacts incorporating a coefficient of kinetic or sliding friction (Daish, 1972; Brody, 1984)\(^1\) defined in the usual sense:

\[^1\] Brody (1984) denotes kinetic friction as \(\mu_s\) since he refers to it as sliding friction. Here \(\mu_s\) is used as this is more common and avoids the confusion arising from \(\mu_s\) normally being associated with static friction.
\[ F = \mu_k R \]

where \( F \) = tangential surface friction force magnitude opposing the motion of one surface sliding over another, \( R \) = perpendicular contact force magnitude exerted by one of the surfaces on the other, and \( \mu_k \) = coefficient of kinetic or sliding friction.

The coefficient of friction \( \mu_k \) is generally regarded as constant, with values determined from experimentation using different surface and ball interactions (Brody, 1984; Putnam and Baker, 1984). More recently Cross (2000c) has developed the model further to include a constant coefficient of rolling friction \( (\mu_R) \) in an attempt to account for the ball and string-bed deformation effects.

The function of friction in the impact situation is to oppose the motion of the ball at the surface in the tangential direction reducing this component of velocity. The frictional force also results in a moment about the centre of mass (COM) that causes the ball to rotate.

2.6.1.2 COEFFICIENT OF RESTITUTION IN THE TANGENTIAL DIRECTION (COR\(_T\))

To classify the change in velocity in the tangential direction during impact Sondergaard et al (1990), Tiefenbacher and Durey (1994), Supulver and Bridges (1995), Araki et al (1996), and Brody (1997) have adapted the definition of COR\(_T\). The velocity ratio of the ball tangential to the surface is referred to as the coefficient of restitution in the tangential direction (COR\(_T\)). However definitions of COR\(_T\) can vary depending on the point on the ball where the tangential velocity is measured and this poses problems because of ball deformation.

The tangential velocity of the ball calculated at its COM uses only the linear velocity components in the ratio. However if the tangential velocity of the ball is determined at the point in contact with the surface then the ratio also includes the rotational component of the ball’s motion.

Sondergaard et al (1990) in their model used the tangential velocity at the COM of the ball, but Tiefenbacher and Durey (1994) and Araki et al (1996) have both developed table tennis models using the tangential velocity at the point of contact, and introduced a term called the tangential elasticity.

Tangential elasticity describes the interaction between the point of contact of a table tennis ball and the surface of the paddle. Tiefenbacher and Durey (1994) and Araki et al
(1996) state that it is more important than friction. Tiefenbacher and Durey explained that, at the point of contact, the initial tangential velocity is reduced to zero by the frictional force and it is then reversed by the build up and release of potential energy due to tangential deformation of the surface (tangential elasticity). The important difference in these table tennis models has been the nature of the rubber bat surface and it may be that this argument could be extended for consideration of a racket string-bed. The models developed by Tiefenbacher and Durey (1994) and Araki et al (1996) that use tangential elasticity as a mechanism to describe the generation of spin, in a table tennis situation, do not specify equations and so cannot be evaluated.

2.6.2 EXISTING OBLIQUE MODELS

Although a number of studies have been undertaken which examine CORN (such as Brody, 1979; Kawazoe, 1993; Cross, 2000a) there are limited sources of information regarding oblique impacts between ball and racket strings.

Daish (1972) and others have described how simple mechanics principles can be used to develop a predictive model for ball / surface impact outcome in terms of velocity and spin based on CORN and $\mu$ estimates. Brody (1984) employed these ideas to quantify the interaction of a tennis ball and court surface having calculated $\mu_k$ for a variety of surfaces. Brody used two physical characteristics (CORN and $\mu_k$) as a basis to develop a first order approximation of the interaction between a tennis ball and a rigid surface.

Groppel et al (1983) formulated an equation, from the discussion of impact by Routh (1960), to predict the amount of spin ($\omega$) imparted to a tennis ball of radius $a$, if the pre-impact parameters are known (Equation 2.4). Where $V_B$ and $V_R$ denote the ball and racket inbound velocities respectively, $\beta$ denotes the racket inclination angle, $\phi_R$ is the angle of linear racket movement and $\Omega$ is the ball inbound spin.

$$\omega = -\frac{3}{5a} (V_B \cos(\beta) + V_R \cos(\beta - \phi_R) + a\Omega) + \Omega$$  \hspace{1cm} 2.4$$

Groppel et al validated their model using video recordings, filmed at 500 fps, of two players performing fore-hand shots with both topspin and backspin. The derivation of the equation did not mention the ball rolling on the string-bed, although it was assumed that the sliding velocity of the ball was zero prior to the end of impact. Equations 2.5 and 2.6 were also derived to determine values for the rebound velocity in the normal ($v$)
and tangential ($u$) directions, and from these the rebound angle can be calculated. The use of high-speed video to record ball flight data is an accurate method, but when used at relatively low frame rates precision may be reduced if high velocity and large spin rates are present.

\[ v = (1 + COR_N)[V_R \sin(\beta - \phi_R) + V_B \sin(\beta)] - V_B \sin(\beta) \]

\[ u = -\frac{2}{5} (V_B \cos(\beta) + V_R \cos(\beta - \phi_R) + \Omega) + V_B \cos(\beta) \]

Johnson and Lieberman (1994) developed a theoretical oblique impact model for a golf ball impacting with a rigid surface. The normal and tangential directions were separated with the normal directional component modelled using a complex arrangement of springs and dampers as developed in previous work by Lieberman and Johnson (1994). Rotation of the ball during impact was determined using a transverse force acting on the radius of the ball. The spin was estimated for a range of inbound velocities with guessed values for the model parameters, the results obtained were not validated with experimental data.

Carré et al (2000) modelled an oblique impact of a cricket ball bowled onto turf using Daish’s (1972) hard sphere / surface model. The experimental results did not support the developed theory, so a constant was introduced into the model to adjust the rebound angle to account for the deformation caused to the surface by the ball. By adding an adjustment to the rebound angle to account for surface deformation, an absence of comprehension of the extent of ball or surface deformation during impact was demonstrated. A cricket ball is constructed from cork and leather, so deformation will be of both the surface and the ball and will depend on pitch conditions. Although ‘fixing’ the model by introducing a new constant improves its accuracy in relation to a particular data set, the lack of understanding causing the error raises doubts about the method’s general applicability.

Dignall et al (2000) developed equations to predict the behaviour of an oblique impact between a tennis ball and a court surface. They produced equations to predict the displacement, velocity, and force during the impact from ball positional experimental data collected using a high-speed camera. Predicted results showed good agreement to the experimental data; however for the vertical displacement experimental data adjustments were made, to the COM, to account for ball deformation and match the results to the predicted values. It is not unusual to adjust the model in such a way to
accurately predict experimental data as with Carré et al (2000), but not to adjust experimental data to fit the developed model.

More recently Cross (2000c), extended a theoretical model based on simple mechanics principles to include the effects of both sliding and rolling friction. Cross determined an estimate for $\mu_s$ by placing a 10 kg weight on top of a tennis ball and measuring the force required to pull the ball (using a spring balance) across the surface of the strings at low velocity. The values for $\mu_s$ obtained were between 0.27 – 0.42 for different string-beds.

Cross derived values for $\mu_r$ by considering a ball rolling across the surface before the impact was over. The ball and string-bed were assumed to have minimal deformation to simplify the calculations. A value for $\mu_r$ was determined by placing four balls on a string-bed supporting a platform, which could carry weights. The force required to move the platform, with the balls rolling underneath, was measured. Values for $\mu_r$ were calculated to be $\approx 0.035$ for loads less than 10 kg and $0.05 \pm 0.01$ for loads between 20 kg and 74 kg placed on the platform. The 74 kg mass produced an $\approx 760$ N notional reaction force component perpendicular to the string-bed that approximates the average force experienced by the ball during impact at $30 \ m/s$ (Daish, 1972; Halliday, 1997). However, this force is not constant during the impact, reaching a peak of perhaps almost twice the average, and Cross’s results indicate $\mu_r$ varies with deflection of the surface.

Cross’s treatment perhaps implies that although the ball deformation is changing constantly the changes in $\mu_r$ for relatively static tests are insignificant and so a constant value is justified. The concept of a constant coefficient of rolling friction is generally derived and measured under equilibrium conditions. Since during its impact with a string-bed a ball is not in equilibrium it can be argued that the concept has limited applicability. As such, $\mu_r$ cannot be correctly considered as a constant property of the racket / ball system under impact conditions.

Although it is possible that the introduction of a constant coefficient of rolling friction to the model improves its accuracy, no experimental data from tests under impact conditions were presented to support the accuracy of this modification. Cross derived equations to calculate the rebound ball characteristics if the ball started to roll prior to rebounding which are shown below (Equations 2.7 – 2.9):
\[ \omega_2 = \frac{v_{2x}}{R} - \frac{m(v_{1x} - v_{2x})}{MR} \]  

\[ v_{2x} = v_{1x} - \left[ \left( 1 - \frac{\mu_R}{\mu_k} \right)(v_{1x} - v_{x0}) + \mu_R(1 + e_A)v_{1y} \right] \]  

\[ v_{x0} = \frac{R\omega_1 + \left( 1.5 + \frac{m}{M} \right)v_{1x}}{2.5 + \frac{m}{M}} \]

where \( \omega_1 \) and \( \omega_2 \) is the inbound and rebound angular velocity respectively, \( v_{2x} \) is the rebound ball velocity in the \( x \) direction, \( v_{1x} \) and \( v_{1y} \) is the inbound ball velocity in the \( x \) and \( y \) directions respectively (the \( x \)-axis is parallel and the \( y \)-axis is normal to the string plane), \( m \) and \( M \) are the mass of the ball and racket respectively, \( R \) is the radius of the ball, \( \mu_R \) and \( \mu_k \) are the coefficients of rolling and kinetic friction respectively, \( v_{x0} \) is the velocity of the ball across the string-bed at which point it started to roll, \( e_A \) is the apparent coefficient of restitution.

### 2.7 SUMMARY OF LITERATURE REVIEWED

The information that had been published regarding spin in the game of tennis has been reviewed and one area with limited information concerns the interaction between the ball and the racket string-bed. The motion of the racket in relation to the ball has been studied and how different movements are required to play a particular type of shot, however no information is available regarding the detail of what occurs during the impact. How does the ball rotate and what affects the amount of rotation that can be applied? This science needs to be developed if the generation of spin is to be understood; once the mechanisms that affect the amount of spin generated are determined an accurate analytical model can be developed to describe the resulting ball rotation.

Player performance values have been identified and the difference between the professional players and the recreational players can be large. A variety of impact speeds have been recorded for different shot types and the level of rotation achieved has been varied. To understand the precise mechanisms that are involved in the generation of spin, experiments need to be developed to cover a large range of angles and velocities for analysis to gain as much knowledge as possible.
The properties of the ball have been established and due to the rubber hysterisis energy losses occur during impact, however deformation of the ball during impact has not been significantly addressed when modelling impact conditions. One basic assumption made in the development of analytical models is to assume that the ball remains spherical during the impact stage, clearly this needs to be addressed to gain insight into the impact phenomena. The published models for oblique impacts use a wide variety of parameters as a mechanism for the generation of spin, however they are frequently not accurate, often lacking experimental verification or the use of assumptions and constants are prolific. One factor that all the models fail to address properly is the deformation of the ball that occurs during impact and how the spin generation is affected. Carré et al (2000) make use of the deformation in the normal direction, but assess the oblique impact using hard sphere methods (Daish 1972). The deformation of the ball is important in the advancement of an accurate model and will be addressed in the development of the oblique model.

The ball interaction with the strings is largely unknown, estimates involve assuming the ball is in a state of pure sliding or pure rolling during impact; this is unlikely due to the deformation of the ball and string-bed. String movement is understood to occur during play but the extent and amount of movement which occurs is unknown. This movement could affect the ball and the string interaction and therefore affect the outcome of the shots.

Groppel (1986) suggested that the string plane could be the last frontier of study, understanding the interaction of the string deflection, and the ball compression were both mentioned as possible routes to enhance the understanding of the game. This specific topic has yet to be studied in detail and this research may develop the understanding as Groppel set out in 1986.

The proposed direction of the research will be to complete an accurate model representing the ball and string-bed during a normal directional impact. The deformation of the ball during the impact will then used as a basis for the development of the oblique model which calculates the rotation of the ball in a deformed state. The theory will be validated by an extensive experimental programme.
Chapter 3

DEVELOPMENT OF AN ANALYTICAL MODEL

This chapter presents the process used to develop the analytical model that will accurately describe the oblique impact between a tennis ball and tennis racket.

The impact is initially considered in the normal direction, perpendicular to the string-bed. The ball is modelled as a spring and damper so that the deformation and energy losses of the ball that occur during dynamic impacts can be adequately represented; the string-bed is also modelled using a similar set-up. The ball and string-bed systems are developed separately and then combined to represent the impact in the normal direction.

Oblique impacts are then considered by adapting the equations of a traditional method used to evaluate the bounce of a non-deforming sphere. The deformation of the ball during impact is used to equate the forces acting on the ball, which are manipulated to determine the rebound characteristics of the ball for a variety of inbound conditions.

3.1 THE TENNIS BALL MODEL IN THE NORMAL DIRECTION

To represent the dynamic behaviour of a tennis ball impact, the use of springs and dampers were assessed. The use of a spring and damper is not a new concept used to model a tennis ball but the parameters used to define the physical characteristics can be developed further. The motion of a damped spring oscillator can be used to simulate in one-dimension the hysteresis loss curve of the rubber which has been explored by Cross (1999).

To determine the characteristic of a tennis ball in a collision is complex. The ball is inelastic by definition of its dynamic characteristics (ITF, 1999) and the deformation of the ball during impact makes any rigid model approximation inadequate. The properties of an under-damped simple harmonic oscillation modelled using a spring and damper have similarities to the characteristics of a normal directional impact of a tennis ball. The displacement profile allows the gross deformation of the ball to be modelled if the spring constant is accurately assigned, and the velocity profile can be used to represent the change in direction and reduction of the inbound velocity after impact, if the damping coefficient is chosen correctly. The mechanics of this system will be used as the basis of the model developed to simulate the tennis ball during a collision.
Compression tests were completed at low velocities to determine if the force deflection curve of a tennis ball varied with strain rate. For a 30 mm compression at 500 mm per minute a total of 500 N was required but for 1000 mm per minute compression a total of 560 N was required to compress the ball to the same state concluding that the ball showed stiffer tendencies for higher strain rates. A constant value for the spring constant $k$ is not a valid method for modelling the impact.

The tennis ball is modelled as a spring damper system shown in Figure 3.1. Normally the mass would be located at the geometric centre of the ball, however this would mean that the model would require springs at each side of the mass which would overcomplicate the model. The mass of the ball is therefore located as a point mass at the furthest most point away from the rigid surface

The equation that describes the motion of the mass and hence the ball is determined using the force balance around $m_b$ from Figure 3.2.

![Figure 3.1. Schematic of the ball modelled as a mass attached to a spring and damper.](image)

![Figure 3.2. A free body representation of Figure 3.1](image)

\[- m_b \ddot{y} = k_b y + c_b \dot{y} \quad 3.1\]

rearranging to form a 2nd order differential equation

\[ m_b \ddot{y} + c_b \dot{y} + k_b y = 0 \quad 3.2 \]
The solution to the general equation for an under-damped free response is given by (Inman 2001) as:

\[ y(t) = Ae^{-\zeta \omega_n t} \sin(\omega_d t + \phi) \quad 3.3 \]

where \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \), \( \omega_n = \sqrt{\frac{k_b}{m_b}} \), \( \zeta = \frac{c_b}{2\sqrt{k_b m_b}} \),

\[ A = \sqrt{\left( v_0 + \omega_n \zeta v_0 \right)^2 + (y_0 \omega_d)^2} \]
\[ \phi = \tan^{-1}\left( \frac{y_0 \omega_d}{v_0 + \omega_n \zeta v_0} \right) \]

\[ v_0 = \dot{y}(t) \text{ where } t = 0, \text{ and } \]
\[ y_0 = y(t) \text{ where } t = 0. \]

Using the initial conditions \( t = 0, y(0) = 0 \)

\[ y(t) = Ae^{-\zeta \omega_n t} \sin \omega_d t \quad 3.4 \]

Differentiating 3.4 gives

\[ \dot{y}(t) = Ae^{-\zeta \omega_n t} \left( \omega_d \cos \omega_d t - \zeta \omega_n \sin \omega_d t \right) \quad 3.5 \]

Substituting for \( \zeta \) and \( \omega_n \)

\[ y(t) = Ae^{\frac{-c_b}{2m_b} t} \sin \omega_d t \quad 3.6 \]

\[ \dot{y}(t) = Ae^{\frac{-c_b}{2m_b} t} \left( \omega_d \cos \omega_d t - \frac{c_b}{2m_b} \sin \omega_d t \right) \quad 3.7 \]

Equations 3.6 and 3.7 determine the displacement and velocity of the mass \( m_b \) at any time \( t \) providing the parameters \( c_b, \omega_d \) and \( A \) are determined.

3.1.1 Determine the damping coefficient \( c_b \)

Using the damping properties of the curves that describe the displacement of mass \( m_b \) as shown in Figure 3.3, values of \( c_b \) can be calculated.

\[ y_{damp}(t) = \pm y_m e^{\frac{-c_b}{2m_b} t} \quad 3.8 \]

where \( y_{damp} \) = damping component of \( y \), and \( y_m = \text{value of } y(t) \text{ at } t = 0. \)
Differentiating equation 3.8 so that the boundary conditions relating to the velocity component can be used to find $c_b$:

$$\dot{y}_{damp}(t) = \pm \frac{c_b}{2m_b} y_m e^{2mb} t$$  \hspace{1cm} 3.9

Using boundary conditions $t = 0$, $\dot{y}(0) = v_i$, and the $+ve$ form of 3.9 gives

$$\frac{c_b y_m}{2m_b} = v_i$$  \hspace{1cm} 3.10

and with $-ve$ form of 3.9 at $t = t_f$ where $t_f$ = total contact time, $\dot{y}(0) = v_o$ leaves

$$-\frac{c_b y_m}{2m_b} e^{2mb t_f} = v_o$$  \hspace{1cm} 3.11

Rearranging 3.10 to find $y_m$ and substitute into 3.11 then rearrange to calculate $c_b$:

$$c_b = -\ln\left(\frac{v_o}{v_i}\right) \frac{2mb}{t_f}$$  \hspace{1cm} 3.12

This is the damping coefficient in terms of the inbound and rebound ball velocity, mass of the ball, and the contact time.

### 3.1.2 Determine the Spring Constant $k_s$

For a sinusoidal oscillation

*Period of oscillation = $2\pi / \text{angular frequency}$*
where $T$ = period of oscillation and $\omega_d$ = angular frequency.

The angular frequency for a damped oscillation is defined as

$$\omega_d = \sqrt{\frac{k_b}{m_b} - \frac{c_b^2}{4m_b^2}}$$ \hspace{1cm} 3.14

If the impact is assumed to be equal in time to half of a full oscillation then $T = 2t_f$

Substitute for $T$ into 3.13 and rearrange to leave

$$\omega_d = \frac{\pi}{t_f}$$ \hspace{1cm} 3.15

Substituting back into 3.14 and rearranging leaves an equation for the spring stiffness of the model representing the ball:

$$k_b = m_b \left( \frac{\pi^2}{t_f^2} + \frac{c_b^2}{4m_b^2} \right)$$ \hspace{1cm} 3.16

Substituting for $c_b$ gives the spring stiffness in terms of velocities and duration of impact:

$$k_b = m_b \left( \frac{\pi^2}{t_f^2} - \left[ \ln \left( \frac{v_{out}}{-v_{in}} \right) \right]^2 \frac{1}{t_f^2} \right)$$ \hspace{1cm} 3.17

3.1.3 **DETERMINE THE CONSTANT $A$**

The final part of equations 3.6 and 3.7 to be determined is the constant $A$, which can be determined using the boundary conditions $t = 0$, $\dot{y}(0) = v_{in}$, leaving 3.7 as:

$$v_{in} = A \omega_d$$

$\omega_d$ can be calculated from equation 3.14 or 3.15 and rearranged to give an equation for $A$ as:

$$A = \frac{v_{in}t_f}{\pi} \quad \text{or} \quad A = \frac{v_{in}}{\sqrt{\frac{k_b}{m_b} - \frac{c_b^2}{4m_b^2}}}$$ \hspace{1cm} 3.18
This completes the set of equations required to calculate the solution of 3.2 that describes the motion of the ball if it is modelled as a mass attached to a spring-damper system. The equations are summarised below.

The equations that describe the motion of the ball as modelled using a mass and spring damper system.

\[ y(t) = Ae^{\frac{-ck}{2mb}} \sin \omega_d t \]  
\[ \dot{y}(t) = Ae^{\frac{-ck}{2mb}} (\omega_d \cos \omega_d t - \frac{c_b}{2mb} \sin \omega_d t) \]

where:

\[ c_b = -\ln \left( \frac{v_{out}}{v_{in}} \right) \frac{2mb}{t_f} \]  
\[ \omega_d = \frac{\pi}{t_f} \]  
\[ k_b = m_b \left( \frac{\pi^2}{t_f^2} - \left[ \ln \left( \frac{v_{out}}{v_{in}} \right) \right]^2 \frac{1}{t_f^2} \right) \]  
\[ A = \frac{v_{in}t_f}{\pi} \quad \text{or} \quad A = \frac{v_{in}}{\sqrt{\frac{k_b}{m_b} - \frac{c_b^2}{4m_b^2}}} \]

Values for \( v_{in}, v_{out}, t_f \) can be experimentally determined which enables values of \( k_b \) and \( c_b \) to be calculated. Graphs can be plotted for these values against \( v_{in} \) so that possible relationships can be discovered and equations 3.6 and 3.7 can be assessed using \( v_{in} \) alone. This process is completed in Chapter 7.

3.2 MODELLING THE STRINGS OF A TENNIS RACKET

Leigh and Lu (1992) as mentioned previously modelled the string-bed of a racket as a single spring. Tests were completed using a pool ball dropped onto the surface of the strings of a racket whose head was clamped. The results showed that, taking drag from air resistance and within a 1% experimental error, the rebound velocity of the pool ball was comparable to its inbound velocity therefore justifying the use of a single spring. However the energy during the impact of the pool ball to the string-bed, if the mass of the pool ball was approximately 0.14 kg, is equivalent to a tennis ball impacting at around 11 \( ms^{-1} \). It is apparent that the energy loss of the strings is relatively low in
comparison to the energy lost by the tennis ball during impact. Brody (1979) also reported that no appreciable loss of energy was attributed to the strings.

Cross (2000b) used a boulle of mass 0.76 kg impacting onto the string-bed of a clamped racket from a height of 2.4 m. Cross explains that the KE at impact of the boulle is the same as that of a tennis ball impacting at 24 m/s.

Cross found, as mentioned previously, that for any inbound velocity, the rebound velocity (calculated from the bounce height) is 95±2% regardless of the type of tennis strings or the tension so \( v_{out} = 0.95v_{in} \). If the ball was rigid and if the strings are modelled as a spring only then \( v_{out} = v_{in} \). It was assumed that the string-bed would be modelled with a constant damping that is not dependent on the string type or inbound velocity (Figure 3.4).

\[ \begin{align*}
\text{Strings as a spring and damper} & \quad \text{Racket} \\
& \quad \text{frame} \\
\text{Solid plate} & \\
\end{align*} \]

Figure 3.4. Schematic of the string-bed modelled as a spring and damper.

If the rebound velocity of a solid ball (same mass as a tennis ball) is 0.95\( v_{in} \) then equations 3.12 and 3.17 can be used with the combined mass of the ball and strings. The contact time for a rigid ball, of mass 0.057 kg, impacting with the string-bed is expected to be lower than that of a tennis ball (typically 4.5 – 5 ms), so a value of \( t_f = 4 \text{ ms} \) is a good estimate to determine \( k_s \) and \( c_s \):

\[ k_s = (m_b + m_s)\left( \frac{\pi^2}{t_f^2} - \left[ \ln \left( \frac{v_{out}}{v_{in}} \right) \right]^2 \frac{1}{t_f^2} \right) = 45kNm^{-1} \quad 3.19 \]

\[ c_s = -\ln \left( \frac{v_{out}}{v_{in}} \right) \frac{2(m_s + m_b)}{t_f} = 1.87Ns m^{-1} \quad 3.20 \]

These values of \( k_s \) and \( c_s \) will remain constant for any impact and define the properties of the string-bed as modelled as a spring and damper. If \( k_s \) and \( c_s \) are
constant this assumes that the strain rate of the string-bed is independent of velocity; however the ball properties and the extent of ball deformation during impact could be overriding and make any changes in $k_s$ and $c_s$ relatively insensitive to the overall result of the ball / string-bed impact.

3.3 **STRING-BED AND BALL MODELLED SIMULTANEOUSLY**

A schematic diagram of the combined system is shown in Figure 3.5 and Figure 3.6.

![Figure 3.5. Schematic of the ball and string-bed models combined.](image1)

![Figure 3.6. Force diagram of the ball and string-bed models combined.](image2)

demonstrates the free body diagram of the combined system. To set up the series of differential equations that describe the system, force balances around the two masses are required.

Calculating the force balance around $m_b$ gives
Rearranging the equation gives

\[ m_b \ddot{y}_b = k_b (y_b - y_s) + c_b (\dot{y}_b - \dot{y}_s) \]  

3.21

Calculating the force balance around \( m_s \) gives

\[ m_s \ddot{y}_s = c_s (\dot{y}_s - \dot{y}_b) + k_s y_s + k_b (y_s - y_b) \]  

3.22

Rearranging and simplifying the equations leaves

\[ m_s \ddot{y}_s + (c_s + c_b) \dot{y}_s - c_b \dot{y}_b + (k_s + k_b)y_s - k_b y_b = 0 \]  

3.23

Combining the two 2nd order differential equations (3.22 and 3.24) in matrix form gives

\[
\begin{bmatrix}
  m_b & 0 \\
  0 & m_s
\end{bmatrix}
\begin{bmatrix}
  \ddot{y}_b \\
  \ddot{y}_s
\end{bmatrix}
+ \begin{bmatrix}
  c_b & -c_b \\
  -c_b & (c_s + c_b)
\end{bmatrix}
\begin{bmatrix}
  \dot{y}_b \\
  \dot{y}_s
\end{bmatrix}
+ \begin{bmatrix}
  k_b & -k_b \\
  -k_b & (k_s + k_b)
\end{bmatrix}
\begin{bmatrix}
  y_b \\
  y_s
\end{bmatrix}
= 0 \]  

3.25

Equation 3.25 describes the ball and string-bed as a spring damper where \( c_s \) and \( k_s \) are as in 3.19 and 3.20 respectively and \( c_b \) and \( k_b \) are determined using equations 3.12 and 3.17 respectively.

The solution to the above set of equations will allow the normal impact to be examined in detail. Equation 3.25 was solved using the central difference method for multi-degree of freedom systems as outlined by Singiresu (1995) and also using the state-matrix method outlined by Inman (2001). Two methods were used to compare results that gave an understanding of the displacement and velocity of the ball and the surface of the string-bed at any given time for a range of ball inbound velocities. The code for the state-matrix method calculated using Mathcad computer software is shown in the Appendix.

3.4 DEVELOPMENT OF OBLIQUE MODEL

The proposed model was developed from existing methods with the significant inclusion of the deformation of the ball and experimental verification. The oblique model developed uses the classic ‘bounce of a sphere on a plane’ method as outlined by Daish (1972), but instead of using the pre and post impact ball flight data, the equations are calculated at finite times throughout the impact.

The normal directional model generates data for discrete time increments throughout the impact, which include time, velocity and displacement of both the ball and string-
bed. This data can be organised on a spreadsheet in appropriate columns, which enables manipulation to generate time dependent variables describing the motion and forces acting on the ball during an oblique impact. The components calculated include $R_{(n)}$ = reaction force in the normal direction, $F_{(t)}$ = frictional force in the tangential direction, $T_{(t)}$ = torque force acting on the instantaneous ball radius, $l_{(t)}$, $\Delta \omega_{(t)}$ = change in rotation of the ball and $\dot{y}_{n(t)}$ and $\dot{y}_{t(t)}$ which are the acceleration of the ball in the normal and tangential directions respectively (Figure 3.7).

![Diagram of forces acting on a tennis ball](image)

**Figure 3.7. Forces acting on the tennis ball during impact with the string-bed.**

Daish (1972) generated equations to describe an oblique impact between a solid ball and solid surface. These equations have been used with appropriate modification to enable calculation of the forces acting on the ball at each discrete time interval to model an impact between a deformable ball and a deformable surface, in this case a tennis ball impacting onto a racket string-bed.

The reaction force $R_{(n)}$ is given by:

$$ R_{(t)} = m_b \dot{y}_{n(t)} $$

where $\dot{y}_{n(t)}$ = acceleration of the ball in the normal direction calculated from the change in ball velocity $\dot{y}_{(t)}$ generated using the normal model.

The frictional force $F_{(t)}$ acting on the ball during impact consists of the kinetic friction force given in equation 2.3 and rolling friction, which is a modified version of the form described by Cross (2000c).
During the impact the kinetic friction force will oppose the direction of motion as the ball is sliding across the surface, but once the ball has sufficient rotation the kinetic friction will then reverse direction and oppose the rotational motion. If the ball is considered to be undeformed, pure rolling will occur when \( \dot{y}_{t(t)} = r_{ball} \omega \), where \( \omega \) is the rotation of the undeformed ball, \( r_{ball} \), however the ball in this model is deforming so when

\[
\sum \Delta \omega = \frac{\dot{y}_{t(t)}}{I(t)}
\]

where \( \Delta \omega \) is the change in rotation of the ball, then at this time \( t = t_{roll} \), \( \mu_k \) becomes -ve but at the same magnitude for the remaining time of impact \( t_{roll} \rightarrow t_f \).

The rolling friction is dependent on the ball deformation, opposing the motion of the deformed ball. The rolling friction will be at a maximum when \( l(t) = l_{min} \) where \( l_{min} \) = perpendicular radius of the ball at its most deformed state during impact. The rolling force will be zero when \( l(t) = r_{ball} \), where \( r_{ball} \) = radius of the undeformed tennis ball.

The force equation is therefore given as:

\[
F_{(t)} = \mu_k \frac{R_{(t)} + R_{(t-1)}}{2} + \mu_R \frac{(r_{ball} - l(t))}{(r_{ball} - l_{min})}
\]

3.28

The torque force is given as:

\[
T_{(t)} = F_{(t)} l_{(t)}
\]

3.29

and used in the form

\[
T_{(t)} = F_{(t)} \frac{l(t) + l(t-1)}{2}
\]

3.30

The change in the tangential velocity component is

\[
\Delta \dot{y}_{t(t)} = \frac{F_{(t)}}{m_b} \Delta t
\]

3.31

Finally the change in the rotation is given as:

\[
\Delta \omega = \frac{T_{(t)}}{I_{ball}} \Delta t
\]

3.32

where \( I_{ball} \) is the rotational inertia of the ball given as:
3.33

$$I_{ball} = \frac{2}{3} m_{b} r_{ball}^2$$

3.5 SUMMARY OF MODEL DEVELOPMENT

Previous oblique models were examined to determine the parameters and methods used to analyse these impacts. An oblique model was proposed that involved separating the impact into normal and tangential directional components.

The motion of a tennis ball in the normal direction during impact was approximated using a mass attached to a spring and damper with velocity dependent variables \( k_b \) and \( c_b \). The properties of the racket string-bed were also represented by a spring and damper system but with constant values for \( k_s \) and \( c_s \).

The ball and string-bed models were then assessed together to give an overall approximation of a ball impacting in a normal direction on a string-bed. The model was evaluated using two numerical methods; the state-matrix method and a central difference approximation for a multi-degree of freedom system. The normal direction analysis was used to generate the velocities and displacements of the ball and string-bed for any time \( t \) during the impact.

The tangential directional component generated the rotation of the deformed ball during impact. The two resulting velocities of the ball in the normal and tangential directions coupled with the rotation could then be used to determine the flight characteristics of the rebounding ball.

The developed models use the ball deformation during impact, a significant factor in spin generation of rubber balls. The model enhances the understanding of existing knowledge regarding the motion of the ball during impact, and how the generation of spin is changed with varying degrees of deformation. The model parameters are established and the completed model is validated in Chapter 7.

The following chapter describes the experimental equipment set-up that has been used to complete the testing programme, the methodology of which is explained in Chapter 5. The statistical data analysis completed on the results obtained from the testing are also reported.
EXPERIMENTAL EQUIPMENT AND DATA

ANALYSIS

The information required from the experiments consisted of the pre and post impact ball flight conditions i.e. ball speed, approach or launch angle and spin. In addition detailed information was required regarding the ball’s interaction with the string-bed and its deformation during impact. This chapter describes the equipment used in the experimental programme, which consisted of a ball cannon, a target onto which the racket was clamped and a chamber to contain the experimental field. Two ballistic light gates measured the inbound velocity and ball imaging instrumentation was used to capture the post impact ball conditions. A digital high-speed video recorder was used to obtain information regarding the interaction of the ball with the string-bed. A stringing machine, a diagnostic centre for monitoring the rackets, rackets, strings, new tennis balls and an environmental chamber to maintain the storage conditions of the balls were also used. The accuracy of the equipment was evaluated and all of the individual items were calibrated prior to use to ensure minimum experimental error. The experiments were performed under strictly controlled conditions and a procedure was developed to ensure that all tests could be repeated with minimal variation. The experimental equipment set up was identical for each test except for the change in racket, ball inbound velocity or ball inbound angle.

4.1 THE BALL CANNON APPARATUS

A diagram of the ball cannon apparatus and instrumentation is shown in Figure 4.1

4.1.1 THE CANNON

A compressed air driven cannon that operates between 20 – 80 psi firing balls at velocities determined primarily by the delivery pressure was used. Figure 4.2 shows a graph relating ball velocity to operating pressure and it can be seen that the cannon can deliver balls at a consistent velocity for a given pressure over the operating range up to a top speed of \( \approx 50 \text{ ms}^{-1} \).

The barrel of the cannon was interchangeable to accommodate different sized balls ranging from squash (41 mm) to the oversized tennis balls (69.85 – 73.03 mm). For the
Figure 4.1. Experimental equipment set-up.

Figure 4.2. Variation of the fired tennis ball velocity with change in the ball cannon's air supply pressure.
type 2 ‘regular’ tennis balls that have a ball diameter of between 65.41 – 68.58 mm as specified by the ITF (1999), the barrel used had an internal diameter of 70 mm. Each barrel was 1.0 m in length and was attached to a ball loading mechanism at one end with its far end protruding into a chamber that contained a target, positioned 0.8 m from the barrel end.

4.1.2 **THE TARGET**

The target was held in a fixture that could be set at angles from 5° – 90° to the direction of the ball travel; the fixture’s angle was measured using an inclinometer to an accuracy of ±0.5°. The fixture was sufficiently large and rigid to ensure the minimum movement of the attached object, for example a racket when a ball strikes it. The fixture could be manoeuvred and positioned so that the fired ball could strike a racket or surface at any position; the angle of the fixture could be changed quickly and accurately. The exact location of the impact site was achieved using a laser pointer that was positioned so that the beam passed directly down the central axis of the barrel of the cannon; this projected a narrow beam of light along the ball’s intended flight path. The fixture was also designed so that appropriate instrumentation to measure impact forces could be installed.

4.1.3 **THE LIGHT GATES**

Prior to striking the target the ball travelled through two ballistic light gates, placed 195 mm apart, directly adjacent to the barrel’s aperture. The light gates with appropriate instrumentation were used to calculate the pre impact velocity of the ball for all impacts and pre and post impact velocity for 90° (normal) impacts. The light gates could also generate a signal that could be used to trigger other measuring equipment such as strobe lighting or cameras. The light gate instrumentation had been calibrated using an electronic timer / counter to measure their accuracy (Figure 4.3). This revealed a strong linear relationship ($R^2 = 0.85$) between the velocity calculated using the light gates and the actual velocity calculated from the electronic counter measurement.

The Kodak Ektaprofessional HS 4540 high-speed camera, (described below), was also used for system calibration, running at 4500 fps to capture the ball in flight prior to impact, to verify the velocity calculated by the light gates. A table displaying the accuracy is shown below (Table 4.1). The velocity over the range 25 – 40 $ms^{-1}$ as recorded using the light gates was found to have an accuracy of <4%.
Figure 4.3. Calibration of the light gates.

<table>
<thead>
<tr>
<th>Method of measuring inbound ball velocity</th>
<th>Counter</th>
<th>Kodak Camera</th>
<th>Light Gates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average velocity $m/s^1$</td>
<td>36.67</td>
<td>37.16</td>
<td>38.10</td>
</tr>
<tr>
<td>Error</td>
<td>0</td>
<td>0.49</td>
<td>1.43</td>
</tr>
<tr>
<td>Error %</td>
<td>0</td>
<td>1.34</td>
<td>3.90</td>
</tr>
</tbody>
</table>

Table 4.1. Comparison of ball inbound velocities measured.

During the experiments, pre-impact spin rates would ideally be zero to reduce the number of variables and simplify analysis of the results. Angular rotation of the ball on exit from the barrel was measured to be less than 70 $rpm$, well below the 180 $rpm$ limit set by the ITF (1997) for test apparatus used to conduct surface pace tests.

4.1.4 THE CHAMBER

The balls were fired into a chamber that enclosed the light gates and the target. The sides of the chamber were constructed from clear Lexan safety plastic to enable safe viewing of the ball surface / racket interaction from any direction. A safety mechanism was fitted to the door to prevent firing of the balls when the operator was changing the target set-up. Fixtures were added to the side of the chamber to secure the imaging instrumentation so that consistency was maintained during testing.
4.2 THE IMAGING INSTRUMENTATION

The instrumentation consisted of cameras, lighting and a pulse generator as a control unit. The arrangement ensured that all relevant detail was recorded for any impact completed.

4.2.1 CAMERAS

Two cameras were used to capture images of the ball: a Kodak Ektapro HS 4540 high-speed video camera to measure impact details, and a digital Sensicam camera used in conjunction with a long duration flash for the measurement of rebound flight characteristics. This arrangement allowed several aspects of the impact, including the lateral interaction between the ball and the string-bed of the racket, and the ball’s rebounding flight path, to be studied simultaneously.

4.2.1.1 THE KODAK EKTAPRO HS 4540 HIGH-SPEED CAMERA

Detailed images of the ball / string-bed interaction during impact were obtained using the Kodak camera positioned below and perpendicular to the string-bed for each angle used. The camera operated using a fast acting mechanical shutter and stored the images onto a loop of solid state memory that could contain up to a maximum of 3072 full frames. The camera was capable of operating with a frame capture rate of 40,500 fps but at this speed the pixel resolution was reduced so insufficient detail of a tennis ball / racket string-bed could be acquired. To obtain the maximum possible definition, at the highest capture rate that is useful for tennis ball impacts typically of less than 5 ms duration, it was found that operating at 13,500 fps was appropriate. The image size obtained from operating the camera at 13,500 fps was 128 x 128 pixels with 256-level greyscale. The camera image was calibrated using a flat plate containing a grid of 5 mm x 5 mm squares placed directly on the string-bed being filmed, in line with the camera. The camera was triggered using a pulse generated by one of the light gates to ensure it always captured a set of images covering the entire impact of the tennis ball striking the racket string-bed. The number of frames captured for a typical impact was approximately 70; this included several prior to, and after contact between the ball and strings. This ensured that the initial and final point of contact could be identified to within 74 μs.

The Kodak camera image data was analysed frame by frame to determine the duration of the impact, the ‘footprint’ size and shape, and the movement of the ball across the individual strings during the impact. The time interval between images
(74 μs) coupled with the calibration enabled a high level of accuracy in terms of the time intervals and distances involved. An example of the image obtained using the Kodak camera is seen in Figure 4.4.

Figure 4.4. Nine separate images showing the impact at 0.5 ms intervals captured using the Kodak Ektapro HS 4540 high-speed camera

4.2.1.2 THE SENSICAM CAMERA

Post-impact ball flight characteristics were obtained using still images of the rebounding ball captured by a high definition digital Sensicam camera used in conjunction with a long duration flash aligned perpendicular to the plane of the moving ball, 600 mm from the impact site. The Sensicam camera was triggered using a channel from the pulse generator described below. The camera was fixed to the side of the
chamber and the images were calibrated using a plate containing a grid of 10 mm × 10 mm squares placed in the plane of the balls trajectory.

The Sensicam camera worked by electronically sampling images using fibre optic cables to transfer the images from the camera to a computer. For an event of interest, up to ten images could be sampled and combined to produce a single captured image. The timing between samples and the duration of each capture was controlled using PC software. To capture clear images of the rebound flight of a tennis ball fired onto a racket at 35 ms⁻¹, three samples were required. The time interval between each captured sample was set to 4 ms and the duration of each was typically 40 μs. The resulting image comprised of three individual positions of the ball in flight, combined to produce a clear picture of the rebound flight path. The definition of the captured image can be up to 1280 × 1024 pixels with 256-level greyscale. An example of the post flight images captured with the Sensicam camera is seen in Figure 4.5.

![Figure 4.5. Image of the tennis ball rebound captured using the Sensicam camera and a long duration flash; the time interval between images was 4 ms.](image)

4.2.2 **THE LIGHTING**

Lighting was a fundamental issue in capturing high quality images using the two cameras. Both the Kodak and the Sensicam cameras need a large amount of light to operate effectively at high speeds. Initially light was provided by an array of spotlights
to flood the chamber with the light necessary for the imaging equipment during the testing programme. The spotlight arrangement was satisfactory in that it provided the large amounts of light required, however the heat generated in the chamber caused the plastic sides to overheat and also made controlling the operating temperature difficult. A solution to the heat problem was found by employing a long duration, high intensity flash, triggered by the pulse generator. The flash could produce a constant temperature light for 10 ms, which proved adequate for the post flight data captured by the Sensicam. Two spotlights were used in conjunction with the Kodak camera, and were turned on prior to an impact, and extinguished immediately after capture was completed.

4.2.3 THE PULSE GENERATOR

A 10-channel programmable pulse generation unit, model 555 manufactured by Berkeley Nucleonics, USA, was used to control the image instrumentation. A signal from the first light gate when a ball passed through it, initiated an accurately controlled sequence of timed pulses, sent through individual channels to the two cameras and the long duration flash unit. The image instrumentation arrangement was then automated to ensure consistency in obtaining data from each of the hundreds of impacts completed. The pulse generator was calibrated using an oscilloscope (accurate to ±5%) to identify if the timing between pulses and the duration of each pulse were accurate. It was found that the correlation between the pulse generator and oscilloscope were matched.

4.3 BALLS, RACKETS AND RESPECTIVE EQUIPMENT

The tennis balls, tennis rackets and the equipment used to prepare and monitor their characteristics prior to and post testing are described in the following section.

4.3.1 THE TENNIS BALLS

The majority of the testing programme was conducted using Slazenger Wimbledon pressurised balls that had been pre-selected to have near identical properties so as to reduce the effects of manufacturing variability. The balls' weights were between 0.0567 – 0.0585 kg and the rebound heights from the 2.54 m drop tests onto concrete were between 1.40 – 1.44 m. The balls were individually marked with two distinguishing perpendicular lines around the circumference of the ball, one dotted and the other continuous to enable spin measurement. The lines were positioned with reference to the ball seam using a semicircular mould designed to maintain line spacing.
New balls were used for any change to the experimental set-up, such as a change in racket angle. The balls were carefully removed from their pressurised containers and kept for 24 hours at a $20^\circ C$ and 60% humidity conforming to ITF standard testing conditions (ITF, 1999) prior to testing, as mentioned previously. The balls were located in the breech of the cannon in the same orientation to ensure that consistency was maintained throughout testing. Any variation in the circumference of the balls identified by its fit in the breech were observed and noted so that anomalies in the results could be traced back to individual balls and if necessary excluded from the results.

4.3.2 **THE TENNIS RACKETS AND STRING TYPE**

The type of racket used for the testing programme was a Dunlop Revelation 200G supplied by *Dunlop Slazenger International* (DSI). [The latest version, the MuscleWeave 200G was the racket used by Thomas Johansson, the 2002 Australian Open champion (Dunlop, 2002)].

The geometric centre of the racket head was identified and used as the site for the ball impacts. The racket was chosen because the frame could easily be clamped flush to a flat plate, used in the fixture. The string-bed was in the same plane as the plate to which the racket was clamped. Variations to the standard 18 main and 20 cross string pattern could easily be obtained on request from DSI if subsequent experiments required them. The test rackets were selected to have identical physical properties such as balance, weight and dimensions. The properties of the strung racket were also tested prior to, during, and after testing on the BDC to monitor any changes as mentioned previously. The majority of the rackets tested were strung with 15 gauge Dunlop synthetic gut, using the manufacturer’s recommended two-piece stringing method (Stringers Digest, 2000). The tension of each racket was governed by the testing requirements and covered a range from 40 – 80 lb. Other string types were also used in specific tests designed to determine the effects of different characteristic of the string gauge and type.

4.3.3 **THE ENVIRONMENTAL CHAMBER**

The operating conditions of the laboratory in which the testing programme was undertaken were constantly monitored during any period of prolonged testing. In any given interval, the temperature would only change by $\pm 2^\circ C$ and so could be assumed relatively constant. In accordance to the ITF (1999) specification, the balls were
required to be kept at 20°C and 60% humidity for 24 hours prior to testing. It was therefore necessary to provide a suitable storage facility in which to keep the balls before use in tests. A FISON environmental chamber was acquired and serviced, and operated at the required conditions for the validation of the testing programme. The balls were removed from the environmental chamber immediately before testing commenced. Initially to calibrate the chamber the temperature and humidity inside the chamber were monitored using equipment accurate to ±0.1°C and ±3% humidity. Once the conditions had been adjusted and maintained at the specification described by the ITF a portable temperature and humidity monitor was located inside the chamber to ensure these conditions were consistently achieved. The portable device gave readings that were accurate to ±0.5°C and ±5% humidity which was sufficient for the operating conditions used.

4.3.4 THE BABOLAT STRINGING MACHINE AND DIAGNOSTIC CENTRE

All the rackets used in the testing programme were strung using a Babolat stringing machine, which provided a consistent method of stringing, and one which is used extensively in the stringing of professional players' rackets in tournaments such as Wimbledon. The racket was held rigidly, with the string-bed on a horizontal plane, by four outer locators and two internal locators. The two internal locators were positioned one at the head (0°), and the other at the throat (180°) of the racket and the outer locators were situated at angles of 45°, 135°, 225°, and 315°. The tension and the rate at which the string was pulled were electronically set to ensure repeatability.

To test the ability of duplication for the stringing machine and operator, a racket with a configuration of 18 main x 20 cross strings was strung six times at 60 lb tension with the same string type. The string-bed of the racket was then tested using a BDC and the numerical value for each string-bed flexibility covered a total range from 84 – 91 units on a scale from 0 – 100.

Machines such as the BDC can provide a thorough examination of a strung and unstrung racket to give relative values for the properties such as weight, string-bed flexibility, frame stiffness, balance point, and moment of inertia. After a test racket had been strung at the required tension for an experiment to be completed, they were tested on the BDC and the values recorded. Once the racket had been tested on the machine, information was given to the operator about the individual racket power, manoeuvrability and control as well as the percentage loss in the string-bed flexibility.
from its initial value recorded immediately after stringing. The properties of the racket tested on the BDC were displayed on a scale from 0 – 100 and were designed to give the stringer and player an insight into how the racket might perform on court.

All of the data displayed by the BDC was recorded to determine any change in characteristics that could occur within a racket over a testing period and also over extended periods of time. For every racket a complete set of data describing its characteristics was compiled using the BDC. Values were obtained before the racket was strung, immediately after stringing, and at regular time intervals during the testing programme. These tests were completed to determine how the string-bed stiffness changes.

4.4 REBOUND CHARACTERISTIC DATA ANALYSIS

The post impact images of the ball captured using the Sensicam camera were digitised and analysed for velocity ($ms^{-1}$), angle of projection ($^\circ$) and rotation (rpm), using a software program Flightpath written in Borland C (Sumpter, 1993).

To digitise the images of the ball a reference environment had to be set up using the captured calibration image of the plate containing the grid of squares. The reference environment was set by marking, in order, the top, bottom, left then right extremes of a square cross measuring 100 mm by 100 mm. The digitising software program used the cross to calibrate the environment of the computer screen’s pixel map. The time delay between the captured images was also set during the calibration procedure.

The images of the captured tennis ball could then be digitised by initially marking out an imaginary box enclosing the image of the tennis ball. The software then plotted a circle inside the box and calculated the ball centre point. After marking out each ball in sequence, a line of trajectory was produced by joining the calculated centre points of each ball. Distances between the images and the angle of the line were used to calculate the velocity and the launch angle. The angular rotation was calculated using one of the two pre-drawn lines as described previously. The image of the line was digitised, by marking the centre point of the ball; this fixed the midpoint of a projected line that could be rotated to correspond with the line on the ball. The difference in the angle of the rotated line between the images was used to calculate the spin rates. The accuracy of the software was calculated using a calibration standard (Figure 4.6) with an image of three balls of known velocity ($20 ms^{-1}$), launch angle ($30^\circ$), and spin rate ($5000 rpm$).
Tennis ball with anti-clockwise rotation, moving from left to right.
Time interval between balls - 4 ms.
Cross - 100 mm x 100 mm (T,B,L,R).
Velocity - 20 m/s (65.62 ft/s).
Elevation - 30 degrees.
Rotation - 5000 rpm.

Figure 4.6. Ball flight calibration chart

gave values of ±0.02 ms⁻¹, ±0.05° and ±4 rpm respectively with repeated digitising. For a typical captured image of the tennis ball rebound flight when repeatedly digitised, the accuracy was found to be ±0.1 ms⁻¹, ±0.3° and ±29 rpm for velocity, launch angle and spin rates respectively. The accuracy of the spin rate measurement shows a discrepancy that could be due to the slight rotation of the ball as it travelled down the barrel as discussed previously.

The operator digitised all the images captured to ensure the data obtained were consistent. After the images had been analysed and the data collated, a series of statistical tests were performed that would highlight possible rogue data points. Minitab statistical software was used and a one-way analysis of variance and a Tukey’s pairwise comparison were carried out to examine the post impact angular velocities of the individual balls.

For testing, in which the ball inbound velocity was maintained at 35±1 ms⁻¹, the rebound data was ‘corrected’ to give a representation of the rebound flight characteristics if the inbound ball velocity was 35 ms⁻¹. In the adjusting process it was assumed that the rebound ball characteristics were directly proportional to the inbound ball velocity, so the inbound ball velocity was corrected to 35 ms⁻¹ by a multiplying factor and the rebound flight characteristics were then adjusted by the same multiplying factor.
factor (for example, a ball with inbound speed $35.9 \text{ ms}^{-1}$, within the $35 \pm 1 \text{ ms}^{-1}$ tolerance, had its associated velocity measurements adjusted by a factor of $35/35.9$).

4.5 SUMMARY OF EQUIPMENT

The equipment set-up was designed to make operating for extended testing periods straightforward but also allowed the identical arrangement of the instrumentation for different impact configurations. An experimental procedure was established that enabled accurate and repeatable data to be obtained. The analysis of the data was also important to develop a robust method so that results would be reliable but also comparable between the various tests completed.

The following chapter describes the experimental programme methodology that was vital to gain the sufficient data to understand the interaction and rebound flight characteristics of the impact phenomena.
Chapter 5

EXPERIMENTAL PROGRAMME

METHODOLOGY

The experimental programme has been based on the continuing development of the analytical model describing the ball / racket interaction. This programme was developed to establish the significant parameters and to validate the model developed in Chapter 3; it was implemented in such a way to enable sufficient repeatable and consistent data to establish statistical significance. The programme consisted of a series of experiments designed to emphasise the importance of a particular parameter and how it affects the amount of spin generated. The programme was intended to build up an understanding of how the ball rebound characteristics were affected by racket tension, string type, ball inbound velocity and angle.

5.1 INITIAL PILOT STUDY

An initial test programme was undertaken to establish and perfect a robust procedure that could be implemented and refined over the course of the research to ensure the repeatability and validity of the results. The initial testing highlighted areas for improvement and in particular the preliminary tests enabled modification of the instrumentation set-up to enable the evaluation of tennis ball impacts with solid surfaces or racket string-beds. Once the experimental procedure had been developed, individual parameters could be adjusted and the results measured could be confidently compared to each other.

5.2 PARAMETERS AFFECTING THE GENERATION OF SPIN

The parameters were evaluated in order to determine their significance regarding the generation of spin. The parameters of particular interest were inbound ball angle, inbound ball velocity, string tension, string type, string pattern, and ball type.

5.2.1 INBOUND BALL ANGLE

The ball inbound angle is important, as it will simulate the type of shot that is played. The literature reviewed in 2.2 indicates that the racket movement prior to the impact determines the type of shot played. For example in the topspin lob, as examined by Takahashi et al (1996), the resulting velocity can be calculated as 15.7 $ms^{-1}$ in a
direction 56.4° upwards from the horizontal. A topspin backhand shot, as determined by Elliott and Marsh (1989), has a racket velocity of 16.6 m/s in a direction 19.5° downwards from the horizontal.

To determine the range of spin possible for any inbound ball angle the racket was clamped securely in the fixture and positioned so that the full range of angles could be tested from a normal directional impact at 90° to a glancing shot at 25°. The orientation of the racket in the target inside the safety chamber was kept as described in Chapter 4 and the inbound ball velocity was 35 m/s with the impact location at the geometric centre of the string-bed. The tests were carried out to determine how the total amount of spin produced was affected by the inbound angle of the ball as it strikes the surface of the strings.

Post flight images were captured using the Sensicam camera with the long duration flash and the impact site was recorded using the Kodak Ektapro HS 4540 operating at 13,500 fps.

5.2.2 TANGENTIAL INBOUND VELOCITY

Since it is expected that friction plays a major role in the generation of spin, then the ball velocity across the surface of the string-bed should be significant in the production of the spin during impact. In the model developed the normal velocity was used to calculate the ball deformation which was then used with the tangential velocity component to generate the rotation; these tangential velocity tests are designed to validate the separation of the two velocity vectors. The tests were also designed to discover if the amount of spin generated was directly related to the tangential velocity component of the inbound ball velocity, and to find the relationship between the inbound and rebound tangential velocities.

A series of tests were completed and rebound characteristics were measured over a range of inbound ball angles for tangential inbound ball velocities ranging from 15 m/s up to 30 m/s. A racket with a configuration of 18 main by 20 cross strings, strung with 16 gauge Dunlop synthetic gut at 60 lb tension was used over a range of angles between 25° and 80°.

5.2.3 TENSION OF THE STRING-BED

A series of tests were completed to determine if the tension of the string-bed has any influence on the rebound characteristics for a set of impact conditions. The tests were
designed to determine if the player pre-conceptions about rebound spin were justified, for example Brody (1995) suggested that a tighter string-bed is an advantage to enable higher spin values to be produced.

Two identical rackets with a configuration 18 main × 20 cross strings were used, strung with 16 gauge Dunlop synthetic gut, one strung at a tension of 40 lb and the other 80 lb. The rackets were tested on the BDC to monitor the tension during testing with values of 44 and 110 for the 40 lb and 80 lb rackets respectively. The rackets were clamped around their heads and positioned in the ball cannon as described in Chapter 4. The racket was angled so that the ball incident angle was 45° and the inbound ball velocity was 35 ms⁻¹. The impact site was the geometric centre of the racket string-bed and was verified using the laser-pointing device.

5.2.4 STRING TYPE AND GAUGE

To determine if the string gauge was a factor in the generation of spin, tests were completed using three identical rackets strung with 1.30, 1.35 and 1.40 mm synthetic strings. The rackets had a configuration of 18 main × 20 cross strings and were initially strung at 60 lb tension. The synthetic strings used were all from a range manufactured by TOA and so it was assumed that the rebound characteristics of the ball would only be affected by the difference in the diameter of the strings. The racket was angled so that the ball incident angle was 45° and the inbound ball velocity was 35 ms⁻¹. The impact site was the geometric centre of the racket and was verified using the laser-pointing device.

5.2.5 STRING PATTERN

Tests were carried out to determine if there was any significant difference in the rebound characteristics between different string patterns. The rackets used had two different string configurations, one had a string pattern of 18 main and 20 cross strings, and the other 16 main and 19 cross strings; these were provided by DSI. The rackets had identical shaped frames so that the total stringing area for each was constant. The total area of the string plane is ≈ 625 cm² but the strings occupy only ≈20% and ≈18% for the 18 × 20 and 16 × 19 string patterns respectively. The string-bed flexibility value when measured using the BDC gives an indication of the stiffness of the string-bed over a set area. The string densities of the two rackets used are different and so even if the rackets were strung at the same string tension the flexibility of the string-bed may be
significantly different. One racket of each configuration was initially strung at 60 lb and then tested using the BDC to find out a value for string-bed stiffness. The string-bed flexibility of the two rackets at 60 lb were different and could give different playing characteristics, so a third racket, with the 16 × 19 pattern was strung at a tension that would give the same string-bed flexibility as the 18 × 20 racket. The string tension and string-bed flexibilities are given in the Table 5.1.

<table>
<thead>
<tr>
<th>Racket letter</th>
<th>String pattern (main × cross)</th>
<th>Tension of racket (lb)</th>
<th>String-bed Flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18 × 20</td>
<td>60</td>
<td>86*</td>
</tr>
<tr>
<td>B</td>
<td>16 × 19</td>
<td>60</td>
<td>72</td>
</tr>
<tr>
<td>C</td>
<td>16 × 19</td>
<td>67</td>
<td>84*</td>
</tr>
</tbody>
</table>

Table 5.1. Data from the 3 rackets used (*a difference of 2 in the string-bed flexibility is very small and so can be assumed negligible)

The rackets were strung this way so that the comparison between racket A and B could be tested and so that it was also possible to determine if the density of the strings can affect the string-bed flexibility. Racket A and C could also be compared to see if the string-bed flexibility has an effect on the generation of spin.

5.2.6 Tennis Ball Cores

In order to gain further knowledge of the impact in tennis, an assessment of ball core collision was completed to determine the effect that the cloth has on the spin generation during the impact. Testing was carried out firstly using pressurized rubber ball cores, and then with the core covered with cloth. The racket used in the testing had a configuration of 18 main and 20 cross strings, it was strung with 16 gauge Dunlop synthetic gut at 55 lb tension as this racket was the mid tensioned racket used in the ball inbound angle testing programme. The string-bed characteristics were recorded prior to and during the period of testing using the BDC. The testing was completed over the entire range of ball inbound angles possible, from almost normal at 80° to glancing shot at 25°. Post impact characteristics were recorded using the Sensicam camera from the side, and the impact site during the collision was recorded using the Kodak Ektapro HS 4540 operating at 13,500 fps.

The experiment was completed to discover any difference in the rebound characteristics of the normal tennis ball compared to the rubber cores. The cores would give an indication to the significance of covering the ball with cloth. This set of tests may also help to predict what may occur during the impact between a squash ball and a
squash racket, which is discussed later. (This is essentially a modified model of the tennis racket / ball core interaction)

5.2.7 TENNIS SERVICE TESTING

The data collected by Pallis et al (Chapter 2, section 2.3) gave a new insight into the service shots played by professionals and in particular the research showed that the first service shot was not as ‘flat’ with little or no spin as it had been portrayed in the literature Anderson and Anderson (1982). Because of the significant differences in these new values, tests were undertaken at Loughborough University to ascertain if the amount of spin recorded in the professional game was of a similar level to top class university players. The Kodak Ektapro HS 4540 camera was set up in the University tennis centre to film the service shots immediately after impact between the ball and the racket. The subjects performed two types of service shots, fast first service and high spin second serves. Tennis balls were marked with lines around the circumference so that the rotation of the ball could be digitised when replaying the footage. The rackets used in the testing were the players own to ensure optimum service performance. The results of these service tests have been displayed in Chapter 2, Table 2.8.

5.3 SQUASH TESTING

A literature search regarding spin in ball games has not revealed any published material for the game of squash. It was therefore decided to undertake a short study to determine the level of spin in squash using a player test, and high-speed video photography.

Discussions with world class female players revealed that spin plays a large part in the game when played by international class players (though apparently not below this level). A short play test was arranged using 2 world class female players (Natalie Grainger No 4 and Jenny Tranfield No 15) on a glass-back court at Loughborough University. A Kodak EM high-speed camera was used with capture rates of 500 and 1000 fps to evaluate ball spin. To enable spin measurements special squash balls were constructed with one half white and one half black. Difficulties were experienced with illumination at high frame capture rates so it was necessary to use large theatrical arc lamps to enable filming at the higher frame rates. The camera was positioned to view down the length of the court alongside the wall, two types of shot were evaluated, forehand and back-hand drives.
Spin was evaluated by counting the number of frames between complete rotations of the ball, the half colours were used to this effect. There were many shots played where spin was applied along the ball seam, and for these shots no spin rates could be determined. In total 15 minutes of footage was recorded which at 500 fps represents 450,000 frames to be evaluated.

A similar set of player tests were completed using two male University players. Using the data and images obtained with the women players and the knowledge gained from their testing the types of shot recorded were the same so that a comparison between male and female shot performance could be completed. A NAC 500 analogue high-speed video camera was used in a similar configuration to the female player testing; colour images were recorded at frame rates of 500 fps.

The players used their own rackets for the tests so that they were familiar with their equipment and only the balls were slightly different. The comment was made during the testing that it was a different playing experience to use the two coloured ball as it was obvious that it was spinning and this was not apparent before. The players also mentioned that when they played at a lower level they were not aware that any spin was being applied to the ball during play and it was only when receiving professional coaching were they made aware that spin would be important to the way that they played.

A series of tests were completed in the laboratory to determine the rebound characteristics for squash balls. The set-up was similar to the tennis testing except the size of the cannon barrel was reduced to 41 mm and the plate holding the target was modified to support a squash racket. The squash racket used was a Dunlop MAX Titanium 470 with a string configuration of 16 main and 21 cross strings. The geometric centre of the racket strings was the impact location, and the tests were carried out with an inbound angle of 45°. To keep parameters similar to that of the tennis testing the inbound velocity of the ball was 35 ms⁻¹. The squash balls used in the tests were taken from the new range of Dunlop squash balls the Revelation Pro Xx, the Revelation Competition Xr, the Max Progress, and the Max. The balls have different specifications and playing characteristics that are described in Table 5.2; prior to the testing the balls were kept at 45°C to simulate the playing temperature.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Diameter</th>
<th>Colour/Markings</th>
<th>Recommendations for players</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revelation Pro X_y</td>
<td>Sets the standard for hang time (time spent in air)</td>
<td>Standard</td>
<td>Plain black / two yellow</td>
<td>Professionals, advanced, and team players</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>dots</td>
<td></td>
</tr>
<tr>
<td>Revelation Competition X_y</td>
<td>10% longer hang time</td>
<td>Standard</td>
<td>Plain black / single yellow</td>
<td>Club players</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>dot</td>
<td></td>
</tr>
<tr>
<td>Max Progress</td>
<td>20% longer hang time, and instant bounce</td>
<td>Slightly larger</td>
<td>Plain black / no dots</td>
<td>Improver's and recreational players</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>40% longer hang time, and instant bounce</td>
<td>12% larger than</td>
<td>Light Blue / no dots</td>
<td>Learners or first time players</td>
</tr>
<tr>
<td></td>
<td></td>
<td>standard</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2. The squash balls used in the laboratory tests.

5.4 SUMMARY

The experiments described in this chapter have been designed to identify which parameters have a significant effect on the rebound flight characteristics of a tennis ball after a controlled impact with a tennis racket string-bed.

The use of the high-speed cameras will provide a level of detail, not previously published, of the interaction between the ball and the strings.

The experiments completed will provide a range of data that can be used to validate the analytical model that was developed in Chapter 3.

The squash testing will provide a new understanding of the amount of spin used by players performing different shots in play conditions.

The next chapter presents the results obtained in the experimental programme.
Chapter 6

RESULTS

The results reported in this chapter have been assembled from tests described in Chapter 5 and undertaken using the experimental ball cannon set-up described in Chapter 4. The experiments were designed to obtain the data required to evaluate parameters identified in the theory developed in Chapter 3. The results are set out sequentially so that individual model parameters can be assessed, in order to validate the model.

6.1 INBOUND BALL ANGLE

Figures 6.2 – 6.7 show the results from tests to determine how the ball inbound angle affects the ball rebound flight characteristics for identical inbound velocities. The ball’s post impact characteristics measured were rebound velocity (normal and tangential components), rebound angle, angular velocity (spin), contact time, rolling and sliding phases during contact. The ball impact characteristics were recorded for three rackets whose characteristics were identical, except for the tension of the string-bed which were 50 lb, 55 lb, and 60 lb.

The racket string-bed flexibility was monitored using the BDC before, during, and after the experimental programme to ensure that the characteristics did not change significantly during testing. Figure 6.1 shows the string-bed flexibility for the three rackets. It can be seen that there is an initial period of relaxation after the rackets have been strung. The period of testing was therefore done between 122 – 307 hrs as indicated when the rackets were at ≈80% of their initial flexibility, it can be seen that the string-bed flexibility did not change significantly during this time. During testing, the string-bed was re-aligned by hand to its initial position if any string movement was noticed.

Figures 6.2 – 6.4 show the results obtained by digitising images captured perpendicular to the balls’ rebounding flight path. The data has been ‘corrected’, as described in Chapter 4, to give a representation of the rebound flight characteristics if the inbound ball velocity was 35 m/s⁻¹.
Figure 6.1. Variation of racket string-bed flexibility throughout testing.

Figure 6.2 shows the angular velocity of the ball post-impact for a range of inbound ball angles. It can be seen that there is a steady decrease in angular velocity with an increase in the ball inbound angle from ≈5000 rpm at 25° to ≈700 rpm at 80°. The general trend of the graph is the same for all three string tensions. There are almost 800 data points included in Figure 6.2, indicating that the range and spread of each box plot (containing ≈25 points) has sufficient data to be considered a reasonable representation. Statistical analysis of the variation between the results for the three string tensions, based on a Tukey’s pairwise analysis at each of the ball inbound angles tested, indicates the following significant differences based on a 95% confidence level. The angular velocity at angles 55° and 60° are not significantly different between the three rackets tested. For the 60 lb and 55 lb racket the level of angular velocity measured at 25° is significantly lower than the levels at 30°, the 50 lb racket indicates a similar trend but the value of angular velocity is not significantly lower at 25° compared with 30°. Figure 6.2 shows that the 60 lb racket generates significantly higher levels of spin at 80° but a significantly lower amount at 25° and at 45° compared with the other two rackets. The level of spin generated by the 55 lb racket is significantly lower at 40° but significantly higher at 50° than the other two rackets.
Figure 6.2. Variation in the ball rebound angular velocity with ball inbound angle.

Figure 6.3 shows an overall increase in ball rebound angle ($\phi$) from $\approx 35^\circ$ to $\approx 80^\circ$ as the ball inbound angle increases from $25^\circ$ to $80^\circ$. The slope of Figure 6.3 has a flat section between $40^\circ$ and $55^\circ$ which corresponds to the similar trend in Figure 6.2 which could indicate a transition stage in the ball motion. The low spread of data, with only a few extreme values (indicated with stars), shows a high consistency in the values obtained. The general trend of Figure 6.3 shows a tendency to converge towards $\theta = \phi$ which is the result that would be expected for a $90^\circ$ or normal impact.

Figure 6.4 shows the relationship between the absolute rebound ball velocity with ball inbound angle. The data between angles $25^\circ$ and $40^\circ$ show a more ordered relationship than the data from angles greater than $45^\circ$ that are more 'chaotic' in their distribution.

Figure 6.5 shows the change in the coefficient of restitution in terms of the normal direction (COR$_N$) with change in ball inbound angle. The overall trend of the graph portrays a decrease in the value of COR$_N$ from 0.9 at $25^\circ$ to 0.7 at $80^\circ$, with the largest value of COR$_N$ at $30^\circ$. The data at $70^\circ$ for the 50 lb racket is portrayed as a single line with three outliers, which indicates that the majority of the data fall on one specific COR$_N$ and the box is reduced to a single line.
Figure 6.3. Variation in the ball rebound angle with ball inbound angle.

Figure 6.4. Variation in the ball rebound velocity with ball inbound angle.
Figures 6.6 – 6.9 have been generated from the images captured by the high-speed video camera. The camera view, perpendicular to the string-bed, revealed that the impact could be divided into two distinct phases by following the motion of the ball from its initial point of contact across the surface of the string-bed. The first phase observed was one in which the ball exhibited sliding motion across the string-bed, and the second where the ball stopped sliding and appeared to ‘roll’ across the surface of the strings before departing.

Figure 6.6 shows a steady decrease in the total contact time (given by the upper set of data) as the ball’s inbound angle increases for all rackets from \( \approx 5 \text{ ms} \) at \( 25^\circ \) to \( \approx 4 \text{ ms} \) at \( 80^\circ \). The tension in the string-bed of the racket appears to affect the total contact time. The total contact time for the racket strung at 50 lb is significantly longer than the other rackets at all angles except at 55° and 60°. Figure 6.6 clearly shows the two phases the ball exhibits during the impact. The slide duration decreases as the ball inbound angle increases, but evidence of a decline in the gradient between 45° and 55° gives an overall shape to the curve similar to that seen in Figure 6.2. The variability of measurements obtained is small, indicating consistency in the results obtained.
Figure 6.6. Ball motion across the string-bed.

Figure 6.7 shows the change in the total contact distance or length of ‘footprint’ as the ball moves across the strings for each ball inbound angle. A steady increase is shown for the distance covered by the ball in both the sliding phase, the rolling phase, and the total distance travelled by the ball as the ball inbound angle reduces.

Figure 6.8 indicates the relationship between the total contact time and the ball rebound angular velocity. The overall trend indicates an increase in the contact time as the angular velocity increases. Figure 6.8 also shows that for any given angular velocity the total contact time increases as the string tension decreases. The shape of the data scatter highlighted indicates an angular velocity threshold after which the amount generated is constant even if the total contact time is increased.
* Outliers: points outside of the lower and upper limits of the box-plots.

Figure 6.7. Variation in ball contact distance with ball inbound angle.

Figure 6.8. Variation in the ball rebound angular velocity with total contact time.
A more detailed understanding of the impact has been shown in Figure 6.9, which shows a representation of the movement of the centre of the ball across the surface of the strings during impact for each ball inbound angle tested. The solid lines show the horizontal diameter of the surface of the ball in contact with the string-bed. The vertical distance between the solid lines is the distance moved by the centre of the ball for a $1 \text{ms}$ time interval, from the initial point of contact, shown as the circle at the base of each diagram. The circle at the top of each diagram depicts the final point of contact; the mean total contact time is shown adjacent to these. The horizontal dotted line gives the position of the transition between the observed ‘pure rolling’ and sliding phases of the impact, the sliding phase below the dotted line and the rolling phase above it.

Figure 6.10 shows the ball during impact and indicates how the lines in Figure 6.9 are constructed. The images are taken using the Kodak Ektapro camera and show the ball striking the racket at $35 \text{ms}^{-1}$ with inbound angle of $50^\circ$. The initial point of contact is shown in, $a$, each subsequent picture is at time 1, 2 and 3 $\text{ms}$, with the white lines representing the width of the footprint of the ball corresponding to the horizontal lines in Figure 6.9.

Figure 6.9 shows a decrease in ball movement across the string-bed as the ball inbound angle increases, this also corresponds to an increase in the extent of the footprint shown by the horizontal length of the solid lines.

![Figure 6.10. Diagram of the method used to construct Figure 6.9. $a =$ initial point of contact, $b$, $c$ and $d$ represent $t = 1, 2$ and $3\text{ms}$ after $a$.](image-url)
Figure 6.9. Representation of the ball movement across the racket string-bed surface.
6.2 TANGENTIAL INBOUND VELOCITY

Figures 6.11 – 6.17 show the results from tests to determine the effects of varying the tangential inbound velocity of the ball on its rebound characteristics. From the theory developed in Chapter 3 the balls' normal velocity component is used to generate the forces that cause the rotation, however the assumption that the normal component can be used independently of the balls tangential velocity component needs to be evaluated. The results have been used to develop an understanding of the effects that friction has on the ball rebound characteristics. Unlike in Figures 6.1 – 6.8, the data for these tests were not corrected as the absolute inbound velocity of the ball was not kept at a constant value.

Figure 6.11 shows the variation of the measured ball rebound velocity with measured ball inbound velocity for the range of inbound ball angles tested. A linear relationship is indicated with a general trend that as the inbound ball velocity is increased an increase in the rebound velocity is recorded. The trend shown is a Pearson's linear correlation with values for $R^2 > 0.98$ for all lines except $70^\circ$ where due to experimental limitations only limited data could be recorded. Generally it can be seen that larger inbound angles give higher rebound velocities.

Figure 6.12 shows the variation in the measured ball rebound angle with changing inbound velocity over the range of inbound angles tested. As the inbound velocity

![Figure 6.11. Variation of ball rebound velocity to ball inbound velocity.](image-url)
increases there is an increase in rebound angle for any ball inbound angle. The linear trend lines indicate a significant difference between ball rebound angle for the ball inbound angle with higher inbound angles leading to higher rebound angles.

![Figure 6.12. Variation of ball rebound angle to ball inbound velocity.](image)

The relationships between the measured ball inbound velocity with the measured rebound angular velocity are shown in Figure 6.13. Although there is some variability in the results the general trend indicates that as the ball inbound velocity increases an increase in ball rebound angular velocity for any inbound ball angle is seen. A Pearson’s linear correlation passing through (0,0) has been indicated for each inbound ball angle with $R^2 > 0.89$ for $45° - 25°$, however for angles $50°$ and $60°$ this type of linear relationship does not give an accurate representation of the data.

Figure 6.14 shows the relationship between the ball inbound angle and the ball rebound angular velocity for the range of tangential inbound velocities tested. The general trend shows from $30°$ to $70°$ a decrease in the spin generated with an increase in ball inbound angle. For a given inbound ball angle the larger the tangential velocity the larger the amount of spin generated, this is particularly apparent for angles $<50°$. At inbound ball angles of $25°$ the angular velocity generated for each tangential inbound velocity is equivalent to or lower than the amount of spin generated at $30°$ indicating an optimum inbound ball angle where the generation of spin reaches a maximum. At $60°$
Figure 6.13. Variation of ball rebound angular velocity to ball inbound velocity.

Figure 6.14. Variation of ball rebound angular velocity to ball inbound angle.
ball inbound angle the amount of spin generated is at a value that is not significantly
different for each tangential inbound velocity, however due to experimental limitations
a tangential inbound velocity of $30 \text{ ms}^{-1}$ could not be determined at this angle.

Figure 6.15 shows the variation of the ball rebound angle with change in ball
inbound angle for the different ball inbound tangential velocities. As would be
expected the general trend indicates that as the inbound angle increases the rebound
angle also increases. However it can also be seen that for any given inbound angle there
is a significant increase in the rebound angle for the higher tangential ball inbound
velocities.

Figure 6.16 shows the relationship between the ball inbound angle and the ball
rebound velocity for the different tangential inbound velocities tested. The general
trend indicates that for a given inbound angle the ball rebound velocity increases for an
increase in tangential inbound velocity. The data curves for any particular velocity
show similar trends with distinct steps between each velocity.

![Figure 6.15. Variation of ball rebound angle to ball inbound angle.](chart.png)
Figure 6.16. Variation of the measured ball rebound velocity with ball inbound angle.

Figure 6.17 shows the relationship between the $\text{COR}_N$ of the ball with normal direction inbound ball velocity for the different ball inbound angles. The general trend indicates that the $\text{COR}_N$ decreases from $\approx 1$ at $6 \text{ ms}^{-1}$ to $\approx 0.75$ at $45 \text{ ms}^{-1}$ as the normal inbound velocity increases. Data from the normal directional tests (90 degrees) used to validate the model are also indicated on the graph.

### 6.3 STRING TYPE AND GAUGE

Figures 6.18 – 6.19 show the results of the test to determine the effect of the string diameter and construction on the ball rebound characteristics. The tests were completed at a ball inbound angle of 45° and a ball inbound velocity of $35.0 \pm 1.0 \text{ ms}^{-1}$. The variation in the rebound characteristics were verified using a Tukey’s pairwise analysis with a 95% confidence level.

For results shown in Figure 6.18 the different strings used were of similar construction (Manufactured by TOA) but with 1.30 mm, 1.35 mm and 1.40 mm diameters, with the same string tension (60 lb). Figure 6.18a, b and c show the ball rebound angular velocities, velocity and rebound angles respectively for the three string types tested. It is apparent that there is little difference in the rebound characteristics of the three string diameters tested, and statistical analysis reveals that there are no significant differences between the strings for each of the results given.
Figure 6.17. Variation of the $COR_N$ for the ball with ball normal direction inbound velocity.

Figure 6.18. Rebound characteristics for strings of different gauge (inbound ball angle = 45°).
Figure 6.19 gives the results for the difference in ball rebound characteristics between equivalent diameter strings but with different construction, surface coatings and mechanical properties. Again a constant string tension of 60 lb was used.

Figure 6.19a, b and c show the ball rebound angular velocities, velocity and rebound angles respectively for the two string constructions. It is apparent that there are differences in the results obtained and this is confirmed using the Tukey’s pairwise statistical analysis with a 95% confidence interval. The Dunlop string gives lower spin, speed and rebound angle than the TOA string for equivalent diameters.

![Box plots showing ball rebound characteristics](image)

**Figure 6.19. Rebound characteristics for strings of different makes (inbound ball angle = 45°).**

### 6.4 TENSION OF THE STRING-BED

A further set of tests were completed to determine the effect of string tension on the ball rebound characteristics from equivalent racket string configurations. The tests were completed at a ball inbound angle of 45° and a ball inbound velocity of 35 ms⁻¹. Figure 6.20 gives the results from the tests carried out with identical rackets strung with identical strings (1.3 mm Dunlop synthetic gut) but at significantly different string-bed tensions. The tensions chosen were 40 and 80 lbs which represent the extremes for practical racket stringing.
Figure 6.20a, b and c show the ball rebound angular velocities, velocity and rebound angles respectively for the different tensioned string-beds. The differences in the effects of string tension are apparent and statistical analysis reveals that there is a significant difference between the values recorded for the rebound characteristics for the different string-bed tensions. Lower tension stringing gives low spin, higher velocity and higher rebound angle. Figure 6.20b shows that for a reduction in the tension of the racket there is a significant increase in the ball rebound velocity. Figure 6.20c shows that for a reduction in the string tension, an increase in the rebound angle is observed.

![Graphs showing rebound characteristics](image)

**Figure 6.20. Rebound characteristics for string-bed of different tensions (inbound ball angle = 45°).**

### 6.5 STRING PATTERN

To determine the effect of different string patterns on the ball rebound characteristics, tests were completed using two different configurations of geometrically identical tennis rackets. Rackets A and B were of different configuration and were strung to the same 60 lb tension; racket C was the same configuration as racket B but was strung to 67 lb tension corresponding to the string-bed flexibility of racket A, as measured using the Babolat diagnostic centre. The racket configurations and physical properties were given in Table 5.1, Chapter 5. Figures 6.21 – 6.23 display the results obtained from the test programme. The ball inbound velocity was maintained at
35\pm 1.0\ m s^{-1} for this set of tests and the results were once again corrected to represent the rebound characteristics for balls at 35\ m s^{-1}.

Figure 6.21 shows the variation in the ball rebound angular velocity with change in inbound ball angle for different string configurations. A general trend is seen, which emulates that of Figure 6.2 with a rise in the rebound angular velocity for a reduction in the ball inbound angle. Figure 6.21 indicates no significant difference in the rebound angular velocities of the three configurations except at angle 25° in which racket A has a significantly lower angular velocity than both rackets B and C.

![Box plot showing corrected ball rebound angular velocity](image)

**Figure 6.21. Variation of ball rebound angular velocity with different string configurations.**

Figure 6.22 shows the variation of the ball rebound velocity with change in the ball inbound angle. The general trend indicates that the rebound velocity increases as the inbound ball angle increases which is the same as seen in Figure 6.4. Figure 6.22 shows a slight increase in the rebound velocity of racket B but the difference is not significant for all angles except for 25° where the rebound velocity of racket A is significantly greater than racket B and C. Figure 6.22 indicates that, again except for angle 25°, there is no significant difference between the rebound velocities of rackets A and C. The ball rebound velocity results for racket A resemble the overall shape of Figure 6.4, but this is not apparent for rackets B and C (Racket A has the same string-bed configuration as the rackets used in the tests that produced Figures 6.2 – 6.8).
Figure 6.22. Variation of ball rebound angle with different string configurations.

Figure 6.23 shows the variation in the ball rebound angle compared to the inbound ball angle. The general trend of the graph indicates that the rebound angle increases as the ball inbound angle increases which is the same trend as in Figure 6.3. Figure 6.23 indicates that there are no differences between the rebound angles of the three rackets. However at 25° racket A has a significantly reduced rebound angle than rackets B and C; at 80° the trend is reversed but the difference is not significant.

6.6 TENNIS BALL CORES

Tests were completed to compare the rebound characteristics of tennis balls to those of similar ball cores taken from the production line prior to the cloth covering being attached. The racket was the same 55 lb tensioned racket used in the tests to determine the effect of the inbound ball angle; the tests were completed during the same period of testing and covered the same range of inbound angles. The barrel of the cannon was changed to accommodate the cores and they were projected onto the racket with an inbound ball velocity of $35.0 \pm 1.0 \text{ ms}^{-1}$. 
Figure 6.23. Variation of ball rebound angle with different string configurations.

Figures 6.24–6.27 have been generated using the rebound flight data captured using the Sensicam camera and analysed using the methods described previously. Figure 6.24 indicates that the same general trend is observed for the tennis ball cores as for the tennis ball, with a decrease in angular velocity with increase in ball inbound angle. The spin generated by the cores is significantly lower than the tennis balls for all angles except at 80° and 50° where no significant difference exists and at 25° where the cores produce significantly higher values of rotation.

Figure 6.25 shows the same trend as described for Figures 6.3 and 6.23 with an increase in the ball rebound angle with increase in ball inbound angle; the change in gradient between 40° and 55° is also observed for both the cores and the tennis balls tested. Figure 6.26 shows the change in absolute ball rebound velocities for the two ball types. The overall trend indicates the velocity increasing as the inbound ball angle increases. Figure 6.27 indicates that the same trend for the tennis ball as the core with a decrease in the $COR_N$ from ≈0.9 at 25° to ≈0.7 at 80°.
Figure 6.24. Variation in the ball rebound angular velocity with ball inbound angle for the tennis ball core and tennis ball.

Figure 6.25. Variation in the ball rebound angle with ball inbound angle for the tennis ball core and tennis ball.
Figure 6.26. Variation in the ball rebound velocity with ball inbound angle for the tennis ball core and tennis ball.

Figure 6.27. Variation of coefficient of restitution in the normal direction with ball inbound angle for the tennis ball core and tennis ball.
Figure 6.28 was generated using the images taken by the Kodak camera of the interaction of the ball with the string-bed. The overall shape of the curves are the same for the tennis ball as the core, with the longer sliding duration and total contact time occurring at 25°, with both times decreasing as the ball inbound angle increases. Figure 6.28 shows that the total contact time for the tennis balls is significantly longer than the cores at angles <45°; the slide duration of the tennis ball is also significantly longer for all ball inbound angle except 80°.

![Figure 6.28. Ball motion across the string-bed.](image)

### 6.7 SQUASH TESTING

The images captured using the high-speed video cameras were analysed to determine the levels of spin for the backhand and forehand drives shots in squash. The squash balls used were specially manufactured out of two coloured rubber halves to enable the amount of spin to be easily determined. The ball rotation was calculated immediately after the player had struck the ball and also after the ball had rebounded off the front wall of the court.

Table 6.1 shows the average data from at least five shots performed by each player; Figure 6.29 compares the average levels of spin for each player. The levels of spin are relatively high with maximum values of almost 1500 rpm for the back hand drive shot. The amount of spin produced by the backhand drive shot for each player is larger than
the level of spin achieved for the forehand drive. For both shot types the male players produced the most spin compared to the female players. The levels of spin recorded for the squash player testing would be at the lower end of the scale of spin in ball sports shown in Table 1.1.

<table>
<thead>
<tr>
<th>Player</th>
<th>Average spin levels measured (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forehand drive</td>
</tr>
<tr>
<td></td>
<td>Off the racket</td>
</tr>
<tr>
<td>F1</td>
<td>538.5</td>
</tr>
<tr>
<td>F2</td>
<td>722.8</td>
</tr>
<tr>
<td>M1</td>
<td>1242.6</td>
</tr>
<tr>
<td>M2</td>
<td>936.6</td>
</tr>
</tbody>
</table>

Table 6.1. Levels of spin recorded in squash for male (M), and female (F) players (* The male players struck the backhand drives at the clear acrylic back of the court and not the front wall).

Figure 6.29. Levels of spin measured in squash.

Figure 6.30a, b and c shows the ball rebound angular velocity, velocity, and angle respectively for the different types of squash balls in the laboratory tests. The four types of squash ball were fired onto the geometric centre of the squash racket at 35 ms⁻¹ with a ball inbound angle of 45°. It is apparent that there is a progressive small increase in the angular velocity and velocity for the different balls, depending on the level of competition they have been designed for, however the ball rebound angle for the Max Progress and Max balls are significantly lower than the other balls.
Figure 6.30. Rebound characteristics for different squash balls (ball inbound angle = 45°).

6.8 SUMMARY OF RESULTS

The results from the experimental programme have been obtained from a substantial number of impacts; each individual box-plot was constructed with between 20 – 25 individual impacts. The low variability of the data indicates that the testing was repeatable and can be considered to be reliable.

The use of the Kodak Ektapro HS 4540 high-speed camera enabled the contact between the ball and the string-bed to be viewed in high detail. Images captured have provided new accurate information about the phenomena that occurs during impact.

By using a variety of racket configurations, the different ball rebound flight characteristics can be identified and used to determine the important parameters that affect the impact.

The squash player and laboratory tests have provided a new source of data within the game of squash that was previously unknown.

The following chapter is a detailed discussion of the results presented in this chapter. The model parameters are calculated, with the completed model validated using results from this chapter.
Chapter 7

DISCUSSION

This Chapter provides the validation of the analytical model developed in Chapter 3 using the results from the experimental testing programme. The model parameters are calculated and the completed model is used to simulate rebound characteristics for a variety of inbound conditions. The accuracy of the simulation tool and implications for its use in the game of tennis are discussed.

The test programme was designed to identify significant parameters that would affect the rebound characteristics of the ball impacting on a racket string-bed, these are established and the implications are discussed.

Isolating the string-bed of a tennis racket during a dynamic impact with a tennis ball has led to the development of a new understanding of the interaction between the ball and strings during the impact. An alternative description of the interaction is presented and the relationships between rebound characteristics seen in Chapter 6 are assessed in the light of this new knowledge.

The results from the tennis ball rubber core data are evaluated to identify the effect the cloth covering has on the impact characteristics.

Finally the results from the squash player, and laboratory testing are discussed acquiring a new insight into the levels of spin generated in the game of squash.

7.1 MODEL EVALUATIONS

7.1.1 NORMAL IMPACTS

The model of the tennis ball developed in Chapter 3 requires experimentally determined parameters, $c_b$ (the damping coefficient) and $k_b$ (the spring stiffness) to establish the ball properties so that the ball can be represented accurately over a range of inbound velocities. Experiments were completed to capture high-speed images of a tennis ball striking a metal plate at 90°. Measurements were taken of the ball's inbound and rebound velocities and the total time of contact. The compression of the ball during the impact was also measured. Using the measured data for $v_{in}$, $v_{out}$ and $t_f$ equations 3.12 and 3.17 were used to calculate $c_b$ and $k_b$. Figures 7.1 – 7.3 show the values of $c_b$, $k_b$, and $t_f$ over a range of $v_{in}$.
\[ c_b = -\ln\left(\frac{v_{out}}{v_{in}}\right)\frac{2m_b}{t_f} \]  

\[ k_b = m_b\left(\frac{\pi^2}{t_f^2} - \left[\ln\left(\frac{v_{out}}{v_{in}}\right)\right]^2\frac{1}{t_f^2}\right) \]

Using Figure 7.1 the relationship between \(c_b\) and \(v_{in}\) were determined using a polynomial fit with \(R^2 = 0.96\) and is shown in Equation 7.1.

\[ c_b = 0.019v_{in}^2 - 0.1882v_{in} + 6.6147 \quad 7.1 \]

![Figure 7.1. Variation in the calculated damping coefficient with ball inbound angle](image)

The relationship of \(k_b\) with \(v_{in}\) (Figure 7.2) was determined from a polynomial fit with \(R^2 = 0.791\)

\[ k_b = 1.3694v_{in}^3 - 83.237v_{in}^2 + 2068.2v_{in} + 20478 \quad 7.2 \]

The variation of \(t_f\) with \(v_{in}\) (Figure 7.3) was found to be approximately linear with \(R^2 = 0.81\)

\[ t_f = -0.0000333v_{in} + 0.0044373 \quad 7.3 \]
Figure 7.2. Variation in the calculated spring stiffness with ball inbound velocity.

\[ k_s = 1.3694v_{in}^3 - 83.237v_{in}^2 + 2068.2v_{in} + 20478 \]

\[ R^2 = 0.79 \]

Figure 7.3. Variation in the calculated total contact time with ball inbound velocity.

\[ t_f = -0.0000333v_{in} + 0.0044373 \]

\[ R^2 = 0.81 \]
Using the experimentally determined parameters \(c_b\), \(k_b\), and \(t_f\) the equations describing the motion of the ball during impact Equations 3.6 and 3.7 could be used to determine the rebound characteristics of a normal impact if only the ball inbound velocity, \(v_{in}\), is known.

\[
y(t) = Ae^{\frac{-c_b}{2m_b} t} \sin \omega_d t \\
y'(t) = Ae^{\frac{-c_b}{2m_b} t} \left( \omega_d \cos \omega_d t - \frac{c_b}{2m_b} \sin \omega_d t \right)
\]

The experimentally determined parameters can be used in the state-matrix Mathcad code written to evaluate the ball / string-bed impact with an adjustment made to the string-bed parameters to effectively act like a solid surface; the adapted code can be found in the Appendix.

The rebound data used to calculate the parameters were then compared to the model predictions to determine if the spring and damper system could accurately predict the behaviour of the tennis ball during impact. Existing models that have determined \(k_b\) and \(c_b\) have linear dependencies on the inbound velocities (Dignall and Haake, 2000), and constant values of \(k_b\) and \(c_b\) calculated at the extremes of the velocity range are compared to the improved polynomial equations put forward as 7.1 and 7.2. Other models that can predict the rebound characteristics of the impact (Groppel et al, 1983; Cross, 2000c) do not use the same approach to model the ball properties and so cannot be assessed at this time.

Figure 7.4 shows, for a normal impact of a tennis ball onto a solid surface, the relationship between the inbound and rebound ball velocities are not linear. The model prediction curves shown in Figure 7.4 indicate that the polynomial determination of \(k_b\) and \(c_b\) more accurately follows the measured data for the entire range of velocities. The top and bottom lines represent constant values of \(k_b\) and \(c_b\) that were determined from the ball rebound velocity data at 15 ms\(^{-1}\) and \(\approx 37\) ms\(^{-1}\) only accurately predict the rebound velocity of the ball for the specific values at which they were calculated. Dignall and Haake’s linear approximation of \(k_b\) and \(c_b\) only successfully predict the rebound characteristics for the range of inbound ball velocities from 34 – 35 ms\(^{-1}\). Figure 7.5 shows the absolute percentage errors calculated by comparing the ball rebound velocity data calculated in the model using the constant values of \(k_b\) and \(c_b\)
calculated at the minimum and maximum data, the linear approximation published by Dignall and Haake and the polynomial representation. Figure 7.5 indicates that the model and parameters determined in this thesis are superior to alternative methods used when modelling the ball as a spring and damper system.

Figure 7.4. Variation in the calculated ball rebound velocity with ball inbound velocity for a ball striking a solid plate.

Figure 7.5. Variation in the absolute percentage error in predicting rebound velocities for the different ball parameters for a ball striking a solid plate.
The string-bed parameters $k_s$ and $c_s$ were determined using Cross's data as explained in Chapter 3. Simulations were modelled to determine if a total contact duration of 4 ms for the impact was a reasonable assumption; and to ascertain the full range of effects for Cross's experimental results claiming that the ball rebound velocity was 95±2% of the inbound velocity. Using measured ball inbound and rebound velocities for a tennis ball striking the string-bed in a head clamped racket with ball inbound velocities ranging from 35.6 – 47.4 ms$^{-1}$ the various scenarios (Table 7.1) were modelled and the results shown in Figure 7.6. The results in Figure 7.6 show the band of predicted ball rebound velocities for the range of 'ratio of rebound to inbound velocity' and the 'contact time' given by Cross, and used to calculate $k_s$ and $c_s$.

![Figure 7.6. Simulated results using a range of string-bed model parameters for a ball striking a racket string-bed.](image)

Table 7.1 shows the absolute percentage errors for each set of racket parameters given by Cross; the results are shown for small ranges of inbound velocities, and also for the entire velocity range.

The chosen racket parameters for further validation of the model are a $v_{out}/v_{in}$ ratio = 0.95, and contact time of 4 ms; these values have a good overall accuracy of 2.3% and represent the central position with respect to the range of possible parameter combinations.
Table 7.1. Absolute percentage errors for a range of racket model parameters.

To determine the accuracy of the racket / ball model over a larger range of inbound ball velocities, it was assumed that the normal velocity component calculated from oblique impacts was independent on the tangential velocity component (discussed fully in section 7.2.2).

By using the normal velocity component from oblique impacts, the range of model predictions could be extended and validated against this experimental data (Figure 7.7). The absolute percentage errors for the model predictions are given in Table 7.2, which is once again divided into smaller inbound velocity ranges to give a more realistic representation of the accuracy.

Figure 7.7. Simulated results compared to experimental data for ball impacts against a clamped head racket sting-bed.
Table 7.2. Absolute percentage errors for the model predictions of normal rebound velocity components.

<table>
<thead>
<tr>
<th>Velocity range (ms(^{-1}))</th>
<th>5-20</th>
<th>20-35</th>
<th>35-50</th>
<th>5-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute % error</td>
<td>12.2</td>
<td>9.2</td>
<td>7.9</td>
<td>10.6</td>
</tr>
<tr>
<td>Absolute velocity error (ms(^{-1}))</td>
<td>1.6</td>
<td>2.5</td>
<td>3.4</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 7.2 indicates relatively high percentage errors in the model predictions for the rebound velocities by comparing the predicted values to normal velocity components of oblique impact data. However errors in predicting the rebound velocity for normal impacts are lower as indicated in Table 7.1 suggesting a slight discrepancy in using the normal component of the oblique impact to represent actual normal rebound characteristics. The source of error could be in the assumption of the string-bed properties, as suggested, however it could be that the simplicity of the spring and damper used to model the string-bed may not be as adequate as it was to model the ball.

7.1.2 OBLIQUE IMPACTS

The validation of the analytical model has been completed using the set of results obtained when determining how the ball inbound angle affects the generation of spin described in section 7.2.1. The ball inbound velocity was kept constant at 35 ms\(^{-1}\). The normal model established the displacement and velocity components of the ball and string-bed for 1000 time intervals. The oblique model calculations were then completed using the method outlined in Chapter 3.

Initially the contact duration of the simulations was determined from the calculation of the forces acting on the ball. Modelling the ball and string-bed as a combined spring and damper system assumes that the two are contacting when the spring of the ball arrangement is in compression, giving a +ve value for the reaction force (\(R_{0j}\)). The final point of contact between the ball and the string-bed is assumed to be the point when \(R_{0j}\) goes to zero, just as the force acting on the spring in the ball model changes from compression to extension. This assumes that the ball regains its original form at the point which it leaves the string-bed; it does not allow for the expansion of the ball beyond spherical while the ball is in contact with the string-bed, which may occur. Figure 7.8 shows the predicted contact duration and indicates that the model and the measured values are not matched. The contact time was visually determined using images captured by the Kodak HS 4540 camera and are accurate to within 74 \(\mu s\) as mentioned in Chapter 4. Figure 7.8 shows a discrepancy between predicted and
measured values, which could be due to model giving a lower prediction of the ball rebound velocity (Figure 7.7) which could affect the contact duration as shown.

Kinetic friction ($\mu_k$) is used in the model to retard the tangential velocity and to provide the force causing the rotation of the deformed ball. Values of $\mu_k$ from 0.27 – 0.42 were calculated in quasistatic tests completed by Cross (2000c) as mentioned in Chapter 3, however these values may not be accurate for dynamic impacts. It was decided when manipulating the data, that the value of $\mu_k$ would be determined by setting the ball’s tangential rebound velocity to that of the measured data; this fixed a value of $\mu_k$ for each ball inbound angle.

For the initial stage of the impact, the ball is assumed to slide across the surface of the strings; the end of the sliding phase is signified when $l\omega > \dot{y}_{i(t)}$, at which time the value for $\mu_k$ becomes -ve. $\mu_k$ changes sign from the reasoning given in Chapter 3 and once it is -ve, $F_{(t)}$ will also become negative and so a combination of phenomena occur: ball rotation is no longer generated and so reduces, and the tangential velocity of the ball increases.

The results from the simulations are given in Table 7.3 and are displayed with the experimental data for the 55 lb racket. The percentage errors shown indicate that the
model overall average accuracy is not consistent and varies, from 4% to 13% for the ball rebound and angular velocities respectively. Figure 7.9 - 7.11 show the measured results for the 55 lb racket compared to the simulated results which indicates that overall the model predicts to a high degree of accuracy over the entire range of angles tested. The general shape of each curve is the same for the simulated results as the measured data, which shows that the mechanical principles used to generate the model are close to those occurring during the impact.

\[
\begin{array}{ccccccccc}
\text{Ball rebound characteristics} \\
\hline
\text{Ball inbound angle, } \theta ^\circ \\
\text{Velocity, } v_{in} (ms^{-1}) & \text{Measured} & \text{Model} & \% \text{error} & \text{Measured} & \text{Model} & \% \text{error} & \text{Measured} & \text{Model} & \% \text{error} \\
25 & 22.1 & 21.6 & 2.3 & 37.3 & 34.8 & 6.7 & 4820.6 & 5156.9 & 7.0 \\
30 & 21.8 & 22.4 & 2.8 & 48.8 & 40.0 & 18.0 & 4991.4 & 4679.2 & 6.3 \\
35 & 21.7 & 23.1 & 6.5 & 54.9 & 45.3 & 17.5 & 4714.3 & 4343.5 & 7.9 \\
40 & 21.2 & 23.5 & 10.8 & 60.8 & 50.6 & 16.8 & 3924.4 & 4071.7 & 3.8 \\
45 & 23.4 & 23.9 & 2.1 & 60.7 & 55.7 & 8.2 & 3595.1 & 3791.7 & 5.5 \\
50 & 24.7 & 24.4 & 1.2 & 63.6 & 60.1 & 5.5 & 2873.5 & 3440.5 & 19.7 \\
55 & 25.8 & 25.0 & 3.1 & 64.5 & 64.2 & 0.5 & 2370.5 & 3052.6 & 28.8 \\
60 & 24.7 & 25.4 & 2.8 & 68.3 & 68.2 & 0.1 & 2441.6 & 2650.7 & 8.6 \\
65 & 25.8 & 25.8 & 0.0 & 71.5 & 72.1 & 0.8 & 2097.4 & 2253.6 & 7.4 \\
70 & 25.9 & 26.1 & 0.8 & 77.4 & 76.4 & 1.3 & 1510.2 & 1918.4 & 27.0 \\
75 & 26.0 & 26.4 & - & 79.9 & - & - & 1460.8 & - & - \\
80 & 24.0 & 26.7 & 11.3 & 81.0 & 82.2 & 1.5 & 684.5 & 826.4 & 20.7 \\
\hline
\text{Absolute } \% \text{error} & 4.0 & \text{Absolute } \% \text{error} & 7.0 & \text{Absolute } \% \text{error} & 13.0 \\
\end{array}
\]

Table 7.3. Model predictions of ball rebound characteristics using 35 ms\(^{-1}\) ball inbound velocity and varying the inbound angle \(\theta\).

Figure 7.9. Simulated results predicting ball rebound angular velocity compared to experimental data.
Figure 7.10. Simulated results predicting ball rebound angle compared to experimental data.

Figure 7.11. Simulated results predicting ball rebound velocity compared to experimental data.
The reaction force during impact for tennis balls has been evaluated (Cordingley, 2002) at Loughborough by firing them in a perpendicular direction onto a force plate mounted in the ball cannon apparatus as described in Chapter 4. The ball was projected onto the plate at 15 \( ms^{-1} \) and this velocity was used in the model prediction. Figure 7.12 shows the results from the tests compared to the model prediction of the normal force. The overall shape of the modelled curve shows good similarity with a sudden rise in force initially increasing to maximum, before decreasing to zero when contact ends. The model predicts the peak normal force to be \( \approx 617 \) N compared to \( \approx 580 \) N measured using the force plate, which is an error of \( \approx 6.4\% \). The total contact time calculated using the force plate is \( \approx 3.8 \) ms compared to the model prediction of \( \approx 3.7 \) ms, which is \( \approx 2.6\% \) error. Similar force plate tests completed by Cross (1999), for a range of ball types and ball inbound velocities, produced the same shape curve, and comparable peak force and contact time but with no model simulation. The normal force predicted throughout the impact by the model are of the same order as those experimentally determined indicating the model can accurately represent the forces occurring during the impact between a tennis ball and a solid plate.

![Diagram showing normal reaction force compared to model prediction](image)

**Figure 7.12.** Normal reaction force calculated using the model compared to experimentally determined, for a tennis ball impact with a solid plate in the normal direction at 15 ms\(^{-1}\).
7.1.3 Previous Models

The previous models that give equations for the prediction of the rebound characteristics from inbound conditions (Cross, 2000c; Groppel et al. 1983) have been assessed to determine the improvement of the analytical model developed in this thesis. Figure 7.13 shows the experimental data for spin generated from different ball inbound angles; superimposed are the values calculated by the Groppel et al. (1983) and Cross (2000c) models. If a Pearson's linear correlation (Figure 7.13) was assigned to the curves an $R^2$ value of $>$0.93 is reported for the 3 rackets tested, suggesting the relationship between the ball inbound angle and the ball angular velocity could be assumed linear. Closer inspection of the shape of the curve indicates that at least two regions of non-linearity of the curves occur at angles less than 35° and between 40° and 60°. The spread of the data at individual angles would suggest that there are fluctuations in the values of spin observed at 45° and 50°.

Table 7.4 shows a complete analysis of the spin generated by using the models published by Cross and Groppel et al., compared to a linear approximation and the predicted values obtained using the model developed in Chapter 3. It is clear that the new analytical model proposed gives a better approximation to the experimental data than the calculated values using the alternative models overall, for the majority of the angles. It can be seen from Table 7.4 that the maximum error when predicting spin by the proposed model is nearly 29% at 55°, however this is better than the 30% and 43% errors given by the models by Cross and Groppel et al respectively. The largest percentage error for the previous methods of approximation is at 80° when the spin rate is at its lowest. Cross gives errors of 69% and Groppel et al. 50% compared to 20% for the proposed model. Whilst a linear approximation shows the best fit to the experimental data, it cannot be used as a predictive tool to calculate rebound spin over a range of angles for different inbound conditions.

Table 7.5 gives an overall indication of the accuracy of the models; it indicates that the new model developed in this thesis is the most accurate followed by Cross (2000c) and finally Groppel et al. (1983).
Table 7.4. Accuracy of spin prediction of the developed analytical model compared to existing models and a linear approximation of the measured data.

<table>
<thead>
<tr>
<th>Inbound ball angle, $\theta$ (°)</th>
<th>Measured $\omega$</th>
<th>Cross (2000c) $\omega$</th>
<th>Groppel et al (1983) $\omega$</th>
<th>Pearson's linear approximation $\omega$</th>
<th>Analytical model $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega$ error</td>
<td>$\omega$ error</td>
<td>$\omega$ error</td>
<td>$\omega$ error</td>
<td>$\omega$ error</td>
</tr>
<tr>
<td>25</td>
<td>4820.6</td>
<td>5719.6</td>
<td>18.6</td>
<td>5345.5</td>
<td>10.9</td>
</tr>
<tr>
<td>30</td>
<td>4991.4</td>
<td>5371.5</td>
<td>7.6</td>
<td>5107.9</td>
<td>2.3</td>
</tr>
<tr>
<td>35</td>
<td>4714.3</td>
<td>4982.4</td>
<td>5.7</td>
<td>4831.4</td>
<td>2.5</td>
</tr>
<tr>
<td>40</td>
<td>3924.4</td>
<td>4555.4</td>
<td>16.1</td>
<td>4518.2</td>
<td>15.1</td>
</tr>
<tr>
<td>45</td>
<td>3595.1</td>
<td>4093.8</td>
<td>13.9</td>
<td>4170.6</td>
<td>16.0</td>
</tr>
<tr>
<td>50</td>
<td>2873.5</td>
<td>3601.0</td>
<td>25.3</td>
<td>3791.2</td>
<td>31.9</td>
</tr>
<tr>
<td>55</td>
<td>2370.5</td>
<td>3080.8</td>
<td>30.0</td>
<td>3383.0</td>
<td>42.7</td>
</tr>
<tr>
<td>60</td>
<td>2441.6</td>
<td>2537.2</td>
<td>3.9</td>
<td>2949.1</td>
<td>20.8</td>
</tr>
<tr>
<td>65</td>
<td>2097.4</td>
<td>1974.2</td>
<td>5.9</td>
<td>2492.6</td>
<td>18.8</td>
</tr>
<tr>
<td>70</td>
<td>1510.2</td>
<td>1396.2</td>
<td>7.5</td>
<td>2017.3</td>
<td>33.6</td>
</tr>
<tr>
<td>75</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>80</td>
<td>684.5</td>
<td>212.9</td>
<td>68.9</td>
<td>1024.2</td>
<td>49.6</td>
</tr>
</tbody>
</table>

Average absolute % error 18.5 22.2 7.2 13.0

Table 7.5 Comparison of the absolute percentage error in predicting the rebound characteristics of measured data.

<table>
<thead>
<tr>
<th>Rebound characteristic</th>
<th>Existing models</th>
<th>New analytical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>14.2</td>
<td>15.6</td>
</tr>
<tr>
<td>Angle</td>
<td>8.2</td>
<td>6.6</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>18.5</td>
<td>22.2</td>
</tr>
<tr>
<td>Overall accuracy</td>
<td>13.6</td>
<td>14.8</td>
</tr>
</tbody>
</table>

Figure 7.13. Variation in the ball rebound angular velocity with ball inbound angle.
7.1.4 SUMMARY OF MODEL IMPLICATIONS

The proposed model relies on data from a tennis ball and solid surface impact to map the dynamic properties of the ball. Racket performance parameters and the coefficient of friction ($\mu_k$) between the ball and surface are also required so the model equations can be solved and predictions made of the ball rebound characteristics. The analytical model proposed was found to be more accurate than existing models.

The implications of developing a ball model that can accurately predict the rebound characteristics for a normal directional impact over a wide range of inbound ball velocities may be useful in advancing the testing policy for approval of the tennis balls by the ITF. Currently only static testing and a drop test of the balls are required and knowledge regarding the performance of the ball at high velocities is not required in the approval procedure.

The model could be used to determine the proposed rebound characteristics of new balls, by introducing new ball parameters into the model. If a manufacturer develops a ball within the ITF ball specification (ITF, 1999) at low impact velocities, but has different dynamic properties at high velocities, the potential rebound characteristics can be assessed to determine if the playing performance of the ball will vary from traditional ball types.

7.2 DISCUSSION OF PARAMETERS

Over 2000 ball / racket impacts were captured and used to determine ball rebound flight data. The Kodak camera recorded the interaction of the tennis ball and core with the string-bed, which led to over 73,000 individual frames that were analysed.

For all the rackets used in the testing programme the string-bed flexibility followed the same general trend as indicated in Figure 6.1. An initial relaxation of the string-bed occurred after which the flexibility stabilised; at which time the testing would be completed.

Results from the initial testing revealed new knowledge and in particular images captured using the Ektapro camera showed the detailed motion of the ball sliding and rolling across the string-bed. In addition movement of individual strings as they interacted with the ball was also noted. Attempts were made to categorise this string movement but the movement of the strings was considered to be a random phenomenon and at this stage has not been quantified. The movement of the strings had been known...
to exist prior to the visual evidence captured since tennis players have been seen to adjust and reposition strings during a game.

7.2.1 INBOUND BALL ANGLE

Figures 6.2 - 6.7 are displayed as box plots to give an indication of the spread of data and although statistically a linear relationship is convenient, greater insight is possible if the data is considered in more detail. The gradient of the graph shown in Figure 6.2 changes with ball inbound angle, between angles $25 - 30^\circ$ and $55 - 60^\circ$ in particular. Figure 6.2 shows that the generation of spin over the range $45 - 50^\circ$ is less stable with respect to the spread of the data. The standard deviation for the data at $50^\circ$ is $268 \text{ rpm}$ for the $60 \text{ lb}$ racket, which is much higher than the average standard deviation of $162 \text{ rpm}$ for the remaining angles; a trend repeated for the other two rackets tested. The large variation of spin generated within the range of $45^\circ - 50^\circ$ indicates an unstable region that could result in a reduction of shot accuracy or predictability for a player striking the ball with the ball incident angle lying in this range. Analysis of data shown in Figure 6.3 does not show that there is a significant difference in the rebound angle of the ball between the string tensions tested.

The transition from the ordered relationship ($25^\circ$ to $40^\circ$) to the more ‘chaotic’ behaviour ($45^\circ$ to $80^\circ$) of the ball rebound velocity in Figure 6.4 is at the same inbound ball angle where there is spin generation instability as shown in Figure 6.2. The normal component of velocity increases with increasing ball inbound angle and corresponds to a higher deformation of the ball (Figure 6.9). The higher deformation of the ball increases the energy loss from the damping properties of the ball. The increase in the energy loss will reduce the efficiency of the impact and so reduce the $COR_N$ as indicated in Figure 6.5. The position of the ball on the surface of the strings at the point of final contact will have a limited effect on the $COR_N$ as it is generally known that the $COR$ changes for a normal impact over the surface of the string-bed.

The total time the ball remains in contact with the string-bed increases as the ball inbound angle is reduced, but the time also increases at any given angle for a reduction in the tension of the racket (Figure 6.6). The racket at $50 \text{ lb}$ tension gave an average contact time of $\approx 0.2 \text{ ms}$ longer at all angles than the racket at $60 \text{ lb}$ tension, a difference of $4\%$ of the total contact time. Figure 6.7 shows an increase in ball movement across
the string-bed, as the inbound ball angle gets smaller. The maximum distance travelled by the ball is almost the distance from the centre of the racket to the edge.

The movement of the ball across the string-bed as shown in Figure 6.7 is similar for each racket, with the largest fraction of the movement occurring during the sliding phase. The movement of the ball is due to the tangential velocity component so for a fixed inbound velocity of \(35 \, ms^{-1}\) the tangential velocity increases as the inbound angle decreases so the increased movement across the string-bed is as expected.

### 7.2.2 TANGENTIAL INBOUND VELOCITY

The tangential experiments were designed to discover the effects of changing the tangential inbound velocity on the ball rebound flight characteristics; but also to determine if the normal and tangential components of velocity could be used independently as in the model developed in Chapter 3.

The gradient of each line shown in Figure 6.11 is similar for different inbound angles with only a change in the projected intercept of the \(y\)-axis. The linear relationship between the rebound velocity and the inbound velocity would suggest that it could be used as a predictive tool for known ball incident angle. It could be argued that the relatively small differences between the different angles could suggest that the rebound velocity was independent of inbound angle, however the rebound velocities shown in Figure 6.4 would contradict this.

Figure 6.12 shows a clear difference in the ball rebound angle at a given ball inbound velocity for a change in the ball inbound angle. The rebound angle for a given inbound angle remains relatively constant with an increase in the inbound velocity.

The statistical linear relationships indicated in Figure 6.13 suggests that for any given racket orientation if the relative velocity of the ball and racket at impact are increased, then the amount of rotation applied to the ball will also increase. At ball inbound angle of \(>45^\circ\) the linear relationship that passes through \((0,0)\) does not represent the data as well as angles \(<45^\circ\) which could be from larger deformation of the ball and string-bed resulting from the higher inbound ball velocities tested. Higher absolute ball inbound velocities were used at angles \(>45^\circ\), relative to those at angles \(<45^\circ\), to enable the tangential velocity component to remain constant (at 15, 20, 25 and \(30 \, ms^{-1}\)) over the full range of ball inbound angles.
Figure 6.14 indicates that the ball rebound angular velocity reaches a maximum value at a ball inbound angle of 30°; at angles lower than 30° the ball angular velocity remains constant or a reduction in the level of spin is observed. By increasing the tangential inbound velocity at any given angle an increase in rotation is observed, which may be attributed to an increase in the sliding motion of the ball, however this could not be measured due to experimental restrictions. As the tangential inbound velocity increases the normal component of velocity also increases and this combination of increased deformation (from the normal direction) and increased movement across the string-bed could both result in the generation of higher levels of spin.

Figure 6.17 shows that for any normal inbound velocity, the COR\textsubscript{N} is not significantly different for any ball inbound angle. This validates the assumption made in the development of the model in Chapter 3 that the normal component of the ball inbound velocity can be used to generate the deformation data independently of the balls tangential velocity component. The tangential inbound velocity component will affect the motion of the ball across the surface of the string-bed, which will affect the generation of spin. However, if the tangential component of velocity at a particular angle is increased, then the normal component is also increased which will subsequently affect the deformation of the ball. The motion of the ball across the surface and the deformation will both contribute to the generation of spin and so may not only be due to the increase in the tangential velocity component.

### 7.2.3 OTHER PARAMETERS

Whilst individual strings of similar construction can vary in diameter, once the string is placed inside the weave of the racket the difference between individual strings is less obvious; Figure 6.18 indicates the no significant difference in rebound characteristics is observed for a variety of string diameters. However, differences in the surface coating has significant effects of the rebound characteristics as indicated in Figure 6.19 indicating that TOA string type gives higher rebound characteristics for velocity, angular velocity and angle than the Dunlop string type for equivalent string diameters.

In the model the assumption was made that the string-bed was modelled with a constant stiffness spring and a constant damping using the conclusion determined by Cross (2000b). However, Figure 6.20 indicates that for a 35 m/s\textsuperscript{2} impact at 45° the interaction of the ball with the string-bed is affected by the tension. The extreme tension of the rackets used to generate the results in Figure 6.20 are at the limits to the
tension that rackets are commonly strung. For example the manufacturers recommended range of tension for the Dunlop Revelation 200G racket, as used in the testing programme, is between 50 – 60 lb (Stringers Digest, 2000). Over this range of tensions the rebound characteristics were not significantly different (Figures 6.2 – 6.4) indicating the assumption that the string-bed tension is not a significant parameter; for racket tensions outside the 50 – 60 lb range the tension is an important parameter as shown in Figure 6.20. The method used to determine the string-bed model parameters, $k_s$ and $c_s$, could be improved but to test a tennis racket string-bed at high velocities with a solid ball the same dimensions and mass as a tennis ball could be difficult.

The different string configurations tested (Figure 6.21) had no significant effect on the ball rebound angular velocity at any angle, except $25^\circ$ where the wider string pattern of rackets B and C produced significantly more spin than the string pattern of racket A. The two string configurations do not change the playing characteristics of the racket for the majority of ball inbound angles, however for extreme angles like $25^\circ$ the possibility of generation of more spin is apparent. This effect of the wider string pattern to increase the amount of spin could be a reason Mark Woodford chooses to play with a racket that has a $12 \times 14$ string-bed pattern; Mark predominantly plays doubles matches and may readily use high spin shots within a rally. The same result was concluded in tests completed by Putnam and Baker (1984), as mentioned in Chapter 2, who determined no significant difference between the ball rebound angular velocity for diagonal and conventional string configurations (Table 2.10).

The ball rebound velocity and angle of the different configuration rackets are not significantly different for any ball inbound angle except for $25^\circ$ where the ball rebound velocity is higher and the ball rebound angle is lower for racket A (Figures 6.22 and 6.23).

### 7.3 A NEW DESCRIPTION OF THE BALL DURING IMPACT

The changes in the gradient of the curves in Figures 6.2 and 6.3 discussed above may be due to phenomena occurring during the transition between the sliding and rolling stages. The sliding stage occurs first when the reaction force between the ball and the strings are sufficiently low for the strings to have little or no grip. As the normal force increases, the friction between the ball and the strings becomes a significant factor and the surface of the ball directly in contact with the string-bed ceases to move relative to
the strings. The movement of the ball across the strings is accompanied by the progressive deformation of the ball against the string-bed. The simultaneous action of the squashing and the sideways motion of the ball can lead to individual strings being carried along with the surface of the ball. The movement of the strings with the ball appears to be random and with some impacts none is seen. In some cases the movement is permanent, with the string-bed having to be re-aligned before the next shot. In other instances the strings ‘snap’ back when the friction force between the ball and the string is reduced, as the ball continues to move across it, and the tension of the string overcomes the ball influence. The random movement of the strings could be indicative of ‘stick slip’ phenomena, which would account for some of the variability of the results shown. Figure 7.14 indicates the random lateral movement of individual strings. The pictures were taken using the Kodak camera showing the same ball fired successively onto a racket at 50 lb tension, with the ball inbound angle set to 50°. Image a shows little or no string movement, and b and c indicate two different strings moving with the ball as it travels across the string-bed (indicated by dots).

![Figure 7.14. Images indicating the random lateral movement of individual strings. Pictures are taken 1.6 ms after initial contact for the same ball over three successive shots at 50°.](image)

Using the images taken with the Kodak camera it is apparent that the ball reaches a state that could loosely be described as rolling by the end of impact therefore a value of $\mu_k$ appropriate to the Groppel et al (1983) and Cross (2000c) models cannot be determined from this set of tests. The images captured of the entire impact have been used to develop the existing theory in reference to the motion of the ball during impact. Figure 7.15 indicates the new theory of the spin development process for an impact between a tennis ball and racket string-bed.
Figure 7.15. The spin development process during impact.

Figure 7.16 shows a visual representation of the ball motion across the string-bed during the impact (the ball inbound angle is 60°, velocity is 35 ms⁻¹ and racket tension is 50 lbf). Image a (at t = 0 ms) indicates the initial contact between the ball and the string-bed, c (at t = 1.4 ms) is the ball at the end of the sliding phase (N.B. the orientation of the ball markings have not moved) and e (at t = 4.1 ms) shows the ball at the final point of contact where the rotation of ball lines can be seen.

Between a and c the ball slides; which is shown by the movement upwards of the black line on the ball from the red line to the green line. At c the line stops its progress upwards and remains on the green line for the remaining duration it is in contact with the string-bed; as shown in d which is 0.65 ms later. The black line remains positioned on the green line between c and e because the ball has entered the ‘rolling’ phase.

Figure 7.16. A visual representation of the sliding and rolling phases of the ball during impact.

It is no longer sensible to think of angular velocity development in terms of rigid body rotation as deformation is excessive even for shallow impact angles at the speeds measured (Figure 7.16). The difference in velocity between the middle of the contact
area, and a point on the ball's diameter opposite will fluctuate during impact, but the data presented does not allow speculation as to how. The new description for the motion of the ball during impact (Figure 7.15) has been developed from the observations since the extent of deformation observed indicates that the sliding and pure rolling model is not satisfactory for the tennis ball and string-bed impact.

The concept that the ball slides and then rolls prior to leaving the string-bed has been proposed before, however this has not been experimentally established and the theory is largely based on the ball not deforming appreciably during impact, so the conditions for pure rolling are \( v = r\omega \) are applied (where \( v \) denotes the velocity of the ball parallel to the stationary surface, \( r \) is the radius of the ball and \( \omega \) is the angular rotation of the ball). Dignall et al (2000) look at adjusting the centre of mass for a ball to account for the deformation during impact but define equations “considering the torque required to rotate the ball during impact” so the concept of a rolling half flat ball has little merit except to indicate that rolling is not an appropriate description of the ball motion.

The evidence from the experimental programme has determined that there is a period of sliding that exists immediately after contact commences. After this initial sliding phase using the images taken of the string-bed, it is evident that the centre of the ball stops moving across the string-bed and could be consistent with a type of rolling motion. However the rolling motion previously described is for a non deforming ball but this new evidence shows large deformation occurring (Figure 6.9) with the diameter of the compressed surface at ball inbound angles >50° equivalent to or greater than the undeformed ball diameter and cannot be ignored. The conditions for pure rolling do not exist and so the time interval after the ball stops sliding and prior to the end of impact needs further explanation and may be described as a progressive and excessive deformation.

The description of the later stage of impact has been obtained from a limited view point in the experimental programme where only the surface of the ball in contact with the string-bed is visible. The exact initial and final points of contact has been determined using the absence and existence of shadows as the ball touches the surface of string-bed. The lighting was angled so that a shadow of the strings was visible on the surface of the ball as it approached the strings, and the point of contact was apparent when the shadows disappeared. The estimated error in determining the initial \((t_i)\) and final \((t_f)\) time of contact were identified as described previously to 74 \(\mu s\), both points of
contact could be determined within 3 mm (2 pixels). The transition stage \((t_{\text{roll}})\) where the ball appears to stop sliding was easier to observe and is estimated to be accurate to \(t_{\text{roll}} \pm 37 \mu s\) and 1.5 mm.

The extent of the footprint gives an indication of the total distance that the ball moves across the string-bed from an initial central impact. The movement towards the edge of the racket at glancing angles and the changes in local string-bed stiffness, as observed using normal impacts to map \(\text{COR}_N\), adds weight to the importance of incorporating string-bed stiffness in any model that can accurately describe oblique impacts. The movement of the ball and the magnitude of the compression seen could be used to develop string-bed patterns or localised string properties that could effect the frictional properties of the string / ball interaction during sliding and so affect the level of spin achievable from a given impact. A knowledge of the extent of the movement of the ball across the strings could also have implications on the use of string savers as a device that as well as prolonging the life of strings could have the effect of localised variations in the string-bed properties.

7.4 APPLICATION TO OTHER RACKET SPORTS

7.4.1 TENNIS BALL CORES

The reduced weight (≈0.012 kg) and diameter (≈8 mm) of the cores may influence the rebound flight characteristics, but these differences may not be as significant as the frictional characteristics during impact between the different ball surfaces and the string-bed. A strong linear relationship \((a = -87.8 v_m + 7213.5\) with \(R^2 = 0.96\)) can fit to the core data of Figure 6.24. The change in gradient of the graph from 25° - 30° and 55° - 60° for the tennis balls, as mentioned previously, is not apparent for the rebound angular velocity of the cores. These could suggest that the cloth covering could play a significant role in the generation of spin that is not present with cores. The characteristic reduction in angular velocity at angles <30° shown in Figures 6.2 and 6.21 is not present in tennis ball cores suggesting that the cloth covering may inhibit additional rotation occurring at these shallow inbound ball angles. The standard deviation for the core data is 365.4 rpm and 351.2 rpm at 45° and 50° respectively which is much higher than the average standard deviation of 169.2 rpm for the remaining angles. The large variation of the spin generated within the range 45° to 50° indicates the same unstable region as discussed for the tennis balls (Figure 6.2).
The differences in the rebound flight characteristics of the tennis ball and the core are similar for the rebound velocity and the rebound angle components, however the differences in the angular velocity and the sliding duration are clearly affected by the cloth covering. The cloth increases the duration of the sliding phase during the impact that may result in the increase in the angular velocity of the tennis ball compared to the core as indicated in Figure 6.24. In the development of the model the assumption was made that the rotation of the ball occurs in the sliding phase, so an increase in the duration of this phase (Figure 6.28) could result in an increase in spin, which is observed in Figure 6.24.

An interpretation of the results shown in Figures 6.24 and 6.28 would imply that to increase the amount of spin generated for an impact condition, an increase in the sliding duration would be advantageous. If manufacturers could develop cloth properties that would increase the duration of the sliding phase and therefore enable larger amounts of spin to be generated, the results (Figures 6.25 – 6.27) indicate that the other rebound characteristics may not be affected as much as the rotation. The implication could be that the ball rebound angle and velocity will remain similar for a particular shot played, but the additional rotation of the ball would result in an altered flight path of the ball due to the increased Magnus force acting on the ball. The trajectory of shots such as the topspin lob may be affected by increasing the probability that the shot will end up within the boundaries of the court.

If the covering of the rubber core does not have a significant effect on the velocity and rebound angle of a particular shot, then manufacturers could develop new methods for adhering the ‘fabric cover’ (ITF, 1999) to the core. If new coverings could be developed that have similar aerodynamic properties, for example if a one-piece cloth could be developed without a seam, it would enable more efficient ball manufacture and enable ball designs to be developed for appropriate playing levels and conditions.

7.4.2 Squash Testing

The results from the squash player testing provide new understanding of the levels of spin that exist during play conditions. The use of the two coloured balls enabled the measurement of the rotation to be established and was a non-intrusive method to mark the balls. The initial reaction of the players was amazement to observe the levels of spin that occur within the game.
In the interview with the female players after the testing programme it was discovered that F1 played a ‘powerful game’ where striking the ball with high velocity was the tactic used; F2 said she played a tactical game where placing the ball was more readily used than power. From the discussion with the female players it is clear that higher amounts of spin are generated when playing a more tactical game compared to the more aggressive powerful game (Figure 6.29). The male players both indicated that they play with more assertion and do not intentionally apply spin to the ball however both the male players generate higher amounts of spin than the female players.

The levels of spin recorded off the front wall for the backhand drive shots by the male players are significantly lower than the females, which could be due the surface that the ball was projected onto (the clear acrylic back of the court). The levels of spin measured for the backhand drive shots show significantly higher levels to the forehand shots for each of the players tested, and could be attributed to the velocity or the biomechanics of the racket movement required to achieve the shot type. The difference in the levels of spin produced by the male and female players in squash is comparable to those seen in the research by Pallis et al (1999) in which the male tennis players struck shots with more spin applied than the female tennis players (Chapter 2, Table 2.11). The levels of spin recorded for the squash player testing are at the low end of the scale of spin in ball sports shown in Chapter 1, Table 1.1.

The results from the laboratory testing (Figure 6.30) indicate that the playing characteristics between the ball types are similar except for the ball rebound angle which is significantly lower for the larger ball types (Max Progress and Max). The reduction in the rebound angle for the Max Progress and Max balls, designed for recreational and beginner use, could increase the accuracy of shots played using these balls, which is important when learning a game.

Using the limited data obtained from laboratory experimentation of squash impacts (Graves, 2001), the parameters $c_b$ and $k_b$ were determined for a fixed average inbound velocity of 34.5 ms$^{-1}$ using equations 3.12 and 3.17 respectively. These values were used to predict the normal rebound velocity of a squash ball impacting with a solid plate, which is shown in Table 7.6. The accuracy of the predicted result is good showing that for a normal impact onto a plate the rebound velocity of squash balls can be predicted, although further testing will be required to extend the range of predictability.
Table 7.6. Rebound velocity data and model predictions for squash ball impacts onto a solid plate.

For a normal squash ball / racket string-bed impact data was available at 36.1 ms\(^{-1}\), however there is no published literature calculating the properties of the racket string-bed. The tennis racket properties as presented by Cross (2000b) have been used to estimate the predicted rebound velocities given in Table 7.7. The error in the predicted rebound velocity is large and indicates that the squash racket string-bed properties are different to those of a tennis racket.

Table 7.7. Rebound velocity data and model predictions for squash ball impacts onto a clamped head racket.

Values of spin recorded for a 45° oblique squash impact with a racket string-bed were measured for an average inbound velocity of 37 ms\(^{-1}\). The model was used to predict the rebound spin however the dynamic coefficient of friction for a squash ball during impact is unknown and difficult to measure so different values were assumed and the predicted rebound spin calculated (Table 7.8).

Table 7.8. Model predictions of the rebound characteristics of a squash ball impacting onto a racket at 45°

Table 7.8 shows that the differences between the predicted and measured data are significant. The squash ball material is softer than a tennis ball and so during impact the
ball has the potential to deform more, which could lead to additional friction forces acting internally, particularly if the top of the ball touches the base during a high velocity impact. This additional friction will act against the rotation of the ball during impact reducing its ability to rotate, and could result in a lower amount of spin resulting. The model developed does not account for deformation that could exist during a squash impact and would need an additional term. Values of $\mu_k$ are relatively easy to calculate for quasistatic loads as Cross (2000c) and Brody (1979) did for tennis balls, however to measure the value of $\mu_k$ for dynamic impacts is very difficult.

7.5 SUMMARY OF DISCUSSION

Using the Kodak HS 4540 high-speed camera has enabled a detailed examination of the string surface during impact, a technique previously unpublished, which has lead to an improved description of the impact and the motion of the ball as it moves across the surface of the string-bed. The ball is grossly deformed during the impact and so the motion cannot be described using the traditional hard sphere approach presented by Daish (1972). Modelling the tennis ball as a non-deforming object during impact cannot be a sensible approach, although this method has been used in many of the published models for example Groppel et al (1983), Dignall et al (2000) or Cross (2000c).

The model developed in this thesis enhances the understanding of the tennis ball / tennis racket string-bed impact by considering the deformation of the ball and movement of the string-bed as observed in the testing. The model predicts the rebound characteristics to a higher accuracy than existing models, which could be attributed to the new understanding of the impact.

Player perception of the performance of the racket or the players own physical make up could influence the choice of string type or racket configuration best suited to that player. Also from own experience of using identical rackets that have different tensioned string-beds it is the first few shots that the main difference is felt. The human response system has the ability to adapt quickly to changes in environment and conditions like moving from a dark room to bright sunlight, and so by the same argument the player may adjust quickly to the playing characteristics of a particular racket, and so differences could be overlooked.
This thesis proposes a new analytical model to predict the rebound characteristics of a tennis ball/tennis racket string-bed impact. The accuracy of the model is superior to existing models, which have been assessed and compared to the experimental data determined from the extensive experimentation programme. The significant improvement of this model is the inclusion of the gross deformation of the ball during the impact and the introduction of velocity dependent parameters into the equations. The model can be used to understand the characteristics of the impact where high deformation is observed, and used as a tool in the design of tennis balls or equipment.

The experimental programme utilising the high-speed camera operating at 13,500 fps gave new knowledge of the interaction of the ball with the string surface leading to an improved description of the impact. The ball motion across the strings can be divided into two phases; one in which the ball appears to slide and progressively deform into the string-bed, and a second when the ball ceases to slide but continues its progressive deformation against the surface. This was seen to be true for all ball inbound angles less than 70°. An interesting observation during the impact was the apparent random tangential movement of the individual strings as the ball moved across them before returning, or not, to their original position. It has not been possible to model this random movement. The extent of the contact footprint that the ball makes with the string-bed is determined from the two components of the ball inbound velocity relative to the strings. As the normal velocity increases the extent of the ball squashing is increased, as the normal velocity component is reduced and the tangential velocity component increases then the movement of the ball across the strings is increased.

New data was presented giving levels of spin in the game of squash which were previously unreported. The information from the players used indicated that they were only made aware of the importance of spin, from coaches, once they had reached a high level of competence. The laboratory testing produced initial results to compare the performance of a variety of types of squash balls. Data from these tests were compared with model predictions. For normal impacts good agreement was achieved however there were considerable differences for oblique impacts for which the reasons are not yet apparent.
Chapter 9

RECOMMENDATIONS FOR FURTHER WORK

The proposed model could be improved with the identification of more accurate racket string-bed parameters, which will improve the predictive accuracy. A more extensive experimental programme to provide data to cover a wider range of impact angles for the string type, gauge and large tension differences would be recommended to understand the potential differences that may not exist at the 45° angle tested in this thesis. Combinations of string types and racket configurations may produce rackets with potentially different playing characteristics, but this is an area that requires more research.

The change in the moment of inertia of the ball during impact could be studied, as the deformation observed will constantly vary the shape of the ball throughout impact and consequently its ability to rotate. The introduction of bigger balls by the ITF has introduced further parameters of interest to determine the effect on ball impact conditions and their ability to rotate.

An investigation into off-centre impacts would be worthwhile to determine if the characteristics of the impact are changed if the ball strikes the string-bed near the frame. Alternative racket orientations could be an area of future work since this testing programme only determined the rebound characteristics of the ball impacting a racket in one orientation. It would be interesting to determine the spin generated for an impact in which the ball movement is from the tip of the racket in a direction towards the throat. The apparent limit to the generation of spin seen in the testing programme could be an area of further examination, to determine if the current racket and ball technology is a limiting factor. The testing programme utilised one type of racket to limit the variables that could exist, however there is a wide variety of rackets available on the market which boast different playing characteristics, such as power, control, vibration limiting etc. Determining the extent of size and head shape of the racket on the impact conditions and rebound flight is a further area that could be developed.

The effect of environmental conditions on ball motion would be an interesting area of study. The controlled laboratory conditions give a highly repeatable testing location, but tennis can be played outside where temperature and humidity may not always be
constant. The effects of particulates that adhere to the surface of the ball when played on clay courts in particular would be worthy of investigation. If the cloth covering becomes contaminated with dust, dirt or even moisture from the court, the effects that the altered surface has on the friction and subsequent impact conditions should be explored. The measurement of coefficient of friction has proved to be difficult if not impossible in many high-speed contact applications where large deformation takes place. Clearly establishing an understanding and a dynamic value for $\mu_k$ would extend the applicability of the model. This is not a simple matter but the development of an experiment to measure friction is necessary.

The influence of the player has not been assessed in this thesis as the racket was rigidly clamped. By fixing the frame of the racket enabled isolation of the string-bed so that any frame influence on the impact could be eliminated. Further study using players is required to determine if the levels of spin at a particular inbound condition could be repeated in play conditions. The racket motion could easily change throughout depending of how the racket is held, at what angle is the shot applied and how fast the racket is moving could all be factors that need addressing in play situations. The level of detail captured using the high-speed video camera was relatively straight forward in laboratory conditions with the racket fixed, but to capture the same level of detail in a court situation at the high frame rates would be an interesting challenge to any researcher. Using 13,500 fps was sufficient to determine the contact time and identify the motion of the ball across the string-bed however higher frame rates could have identified other phenomena that occur on a microscopic level.

String movement was identified as a random occurrence, but the effects of the movement on the kinetics of the impact were not explored and could offer more insight into the reduction in the levels of angular velocity observed over the range 45 – 55°. String type could also be an area of development, identifying if manufacturers who claim a string with an extra wrap of material on the outside of the surface do produce more rotation. There are different methods used to string rackets, depending on the order of the stringing process; the possible effects on the playing characteristics could be explored further.

The model developed in this thesis could be adapted to predict complete rebound characteristics of other racket sports such as squash, this would require further experimentation to determine the impact characteristics over a range of velocities. The
properties of the squash balls may lead to more deformation of the ball during impact conditions and this may require the model to be modified to accommodate this. The squash player testing revealed information about the levels of spin that exist in play conditions, and research literature is limited within the game. The use of specially manufactured balls was a good method to produce rotation levels of the balls for different shots. The inclusion of other shot types within the game could be examined to produce a full indication of the range of spin that exists within the game.

Lastly the developed model has identified the significant ball parameters that affect post impact conditions. Manufacturers now need to use this model to speculate on the effects of varying ball parameters on playing conditions.
REFERENCES


Gooden, R. (1999). Private communication, Ray is a professional racket stringer with vast experience and works at tournaments including Wimbledon.


Neilson, P.J. (2002). Private communication, Loughborough University, Sports Technology Research Group testing.


State-matrix method to solve the tennis ball/String-bed model equations

\[ m_s := 0.016 \quad \text{setting constants} \quad m_s = \text{mass of strings} \quad \text{Adapted from section 4.10 Inman (2001).} \]
\[ m_b := 0.057 \quad m_b = \text{mass of ball} \]
\[ v_{in} := 35.56 \quad v_{in} = \text{Inbound ball velocity} \]

\[ c_b := 0.019 \cdot v_{in}^2 - 0.1882 \cdot v_{in} + 6.6147 \]
\[ k_b := 1.3694 \cdot v_{in}^3 - 83.237 \cdot v_{in}^2 + 2068.2 \cdot v_{in} + 20478 \]

From plate experiments the values for \( c_b, k_b \) and \( t_f \) are found in terms of Inbound ball velocity.

\[ t_f := -0.0000333 \cdot v_{in} + 0.0044373 \quad t_f := 0.004 \quad \text{Set} \ t_f = 0.004 \text{ secs to calc} \ c_s \text{ and} \ k_s \]

Calculation of the string-bed properties \( c_s \) (eqn 3.22), and \( k_s \) (eqn 3.21), given that the return velocity of the ball after impact is \( 0.95v_{in} \) (Cross, 1999).

\[ c_s := \ln(0.95) \cdot 2 \cdot \frac{(m_s + m_b)}{t_f} \]

\[ k_s := \left( m_s + m_b \right) \left[ \frac{\pi^2}{t_f^2} - \left( \ln(0.95) \right)^2 \cdot \frac{1}{t_f^2} \right] \]

Setting up the Matrix equations from the force balance on the tennis ball/string-bed system. (equation 3.27)

\[
M := \begin{pmatrix} m_b & 0 \\ 0 & m_s \end{pmatrix} \quad C := \begin{pmatrix} c_b & -c_b \\ -c_b & (c_b + c_s) \end{pmatrix} \quad K := \begin{pmatrix} k_b & -k_b \\ -k_b & (k_s + k_b) \end{pmatrix}
\]

Checking the values of \( c_b, k_b, t_f, c_s \) and \( k_s \).

\[ c_b = 23.9 \quad k_b = 50345 \quad t_f = 4 \times 10^{-3} \quad c_s = 1.9 \quad k_s = 45018 \]

Checking the values of \( c_b, k_b, t_f, c_s \) and \( k_s \).

\[ O := \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{Setting up a zero matrix} \ O \text{ and an identity matrix} \ I \]

\[ A := \text{augment}\left( \text{stack}(O,-M^{-1} \cdot K), \text{stack}(I,-M^{-1} \cdot C) \right) \quad \text{Formation of the state-matrix} \]

\[ Y := \begin{pmatrix} 0 \\ 0 \\ -v_{in} \\ 0 \end{pmatrix} \quad \text{Setting the initial conditions} \quad Y = \begin{pmatrix} y_b \\ y_s \\ v_b \\ v_s \end{pmatrix} \]

\[ D(t,Y) := A \cdot Y \quad \text{Setting the derivative function that describes the system in first-order form.} \]

\[ Z := \text{rkfixed}(Y, 0, 0.007, 1000, D) \quad \text{Runge-Kutta integration method to obtain an approximate solution.} \]
\[ t := Z(0) \quad y_b := Z(1) \quad v_b := Z(3) \quad \text{Assigning the columns of the solution} \]
\[ y_s := Z(2) \quad v_s := Z(4) \quad \text{Matrix} \ Z \text{ to the} \ y = \text{distance,} \ v = \text{velocity} \]
\[ \text{of the ball} (b) \text{ and string-bed} (s). \]
The results to the numerical solution in graphical form.

The displacement of the string-bed ($y_s$) and ball ($y_b$) during impact.

The velocity of the string-bed ($v_s$) and ball ($v_b$) during impact.

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Table A1. A selection of the results indicating a return ball velocity of 27.148 m/s.
State-matrix method to solve the tennis ball impact onto a rigid surface

Setting constants

\[ m_b := 0.057 \]
\[ v_{in} := 35 \]
\[ m_b = \text{mass of ball} \]
\[ v_{in} = \text{Inbound ball velocity} \]

\[ c_b := 0.019 \cdot v_{in}^2 - 0.1882 \cdot v_{in} + 6.6147 \]
\[ k_b := 1.3694 \cdot v_{in}^3 - 83.237 \cdot v_{in}^2 + 2068.2 \cdot v_{in} + 20478 \]

From plate experiments the values for \( c_b, k_b \) and \( t_f \) are found in terms of inbound ball velocity.

\[ t_f := -0.0000333 \cdot v_{in} + 0.0044373 \]

Setting up the Matrix equations from the force balance on the tennis ball system.

\[ M := m_b \quad C := c_b \quad K := k_b \]

Checking the values of \( c_b, k_b, t_f, c_s \) and \( k_s \).

\[ c_b = 23.3 \quad k_b = 49613 \quad t_f = 0.0033 \]

O := 0 \quad I := 1

Setting up a zero matrix O and an identity matrix I

\[ A := \text{augment} \left( \text{stack}(O, -M^{-1} \cdot K), \text{stack}(I, -M^{-1} \cdot C) \right) \]

Formation of the state-matrix

\[ Y := \begin{pmatrix} 0 \\ -v_{in} \end{pmatrix} \]

Setting the initial conditions

\[ \begin{bmatrix} Y \\ \dot{Y} \end{bmatrix} := \begin{pmatrix} v_b \\ \dot{v}_b \end{pmatrix} \]

Setting the derivative function that describes the system in first-order form.

\[ D(t, Y) := A \cdot Y \]

Runge-Kutta integration method to obtain an approximate solution.

\[ Z := \text{rkfixed}(Y, 0, 0.006, 1000, D) \]

Assigning the columns of the solution matrix Z to the \( y = \text{distance} \), \( v = \text{velocity} \) of the ball (b) and string-bed (s).

The results to the numerical solution in graphical form.

The displacement \((y_b)\) and the velocity \((v_b)\) of the ball during impact.
Table A2. A selection of the results indicating a return ball velocity of 19.089 m/s.

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