Adaptive control of linear Stirling cryogenic coolers

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ADAPTIVE CONTROL OF LINEAR STIRLING CRYOGENIC COOLERS

By

Vladimir Dubrovsky

A Doctoral Thesis Submitted in Partial Fulfilment of the Requirements for the Award of Doctor of Philosophy of Loughborough University.
September 2003.

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The principal applications of Stirling coolers include infrared imaging systems and superconductive electronics. Among the requirements these applications place on the cooler are low vibration export, low power consumption and long operating life. A linear twin-piston compressor of a cooler provides for inherently low vibration levels and is a solution widely accepted in industry. As compared with rotary or linear single-piston compressors, it produces less vibration due to the balanced counter-motion of two oppositely reciprocating pistons. However, the vibration resulting from the mismatch of opposite piston assemblies caused by manufacturing errors and natural wear cannot be completely eliminated.

In the ultra-low vibration applications an active balancing of the twin-piston compressors is often used. Such systems normally rely on using internal (e.g. LVDTs) or external (e.g. accelerometers or load cells) sensors and sophisticated controllers. This leads to an unacceptable increase of cooler price and the reduction of reliability.

This thesis describes the development, implementation and test of an adaptive control system for (i) quasi-sensorless balancing of the twin-piston compressor, where the detection and synchronisation of the pistons' motion is based on the direct measurement of motors' voltages and currents, and (ii) minimisation of the input electrical power required to drive the cooler, while maintaining the cold tip temperature at a required level by simultaneously varying the driving frequency and input voltage.

Modelling of cooler and control system operation was performed using Matlab/Simulink software. Based on the results of the modelling, developed control algorithms were implemented using LabVIEW RT software running on a National Instruments controller. The attainable performance of the developed control system was evaluated through the full-scale tests performed on the Ricor’s K535 linear twin-piston Stirling cryogenic cooler.

**Keywords:** Linear Stirling Cooler, Adaptive Control, Active Vibration Control, Compressor Balancing, Quasi-Sensorless Motion Measurement.
I am indebted to the Wolfson School of Mechanical and Manufacturing Engineering of Loughborough University for the research studentship. Thanks must first of all go to Prof Neil Halliwell for initiating this research program. I would also like to express my gratitude to my Director of Research, Prof Vladimir Babitsky for his constructive comments and expert advice on the writing of this thesis and for his continued support and encouragement over the past four years. Thanks must also go to my Research Supervisor, Dr Alex Veprik for his suggestions and guidance in the writing of this thesis. His enthusiasm, dedication and expertise was an inspiration. He has been enormously kind in his giving of time.

Many thanks to Prof Yuri Michailov, Prof Michail Kolovsky and Prof Vladimir Dyachenko from Saint-Petersburg State Technical University for their initial recommendation for the research studentship and to Prof Igor Chelpanov for his continued support from afar.

I am grateful to Mr Nachman Pundak and Ricor for their considerable financial commitment to the project which made the experimental part of this work possible. For technical support and great hospitality I wish to thank Mr Sergei Riabzev and Mr German Vilenchik from Ricor. Thanks due to everyone at Ricor for an excellent six weeks in, during which the experimental part of this work was completed. Thanks also to Dr Ilya Sokolov for many helpful discussions we had during the early years of the research.

The work contained within this thesis would not have been possible without the able assistance of the technical staff in the Wolfson School of Mechanical and Manufacturing Engineering in particular Mr Alan Wilkinson and Mr Brian Mace.

Sincere thanks and praise are due to the friends, both near and far, for their support and making my time here so enjoyable.

Finally, my sincere thanks go to my family for their understanding and encouragement.
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<tr>
<td>( A_p )</td>
<td>Piston-face effective area</td>
<td>m²</td>
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<tr>
<td>( c )</td>
<td>Viscous damping coefficient</td>
<td>Ns/m</td>
</tr>
<tr>
<td>( e )</td>
<td>Back-emf voltage, Error signal</td>
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<tr>
<td>( f )</td>
<td>Frequency</td>
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<td>( f_d )</td>
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<td>( H )</td>
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<td>( K )</td>
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<td>( k_g )</td>
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<tr>
<td>$\ddot{x}$</td>
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<td>$Y$</td>
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<tr>
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<td>N/m</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Integration (averaging) time</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>Time delay</td>
<td>s</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Instant volume</td>
<td>m³</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Circular frequency</td>
<td>1/s</td>
</tr>
</tbody>
</table>

*All other symbols and indices are defined as they appear.*
CHAPTER ONE

INTRODUCTION

Modern long-life, high reliability cryogenic coolers of the Stirling type address a broad array of applications where cooling to cryogenic temperatures (less than 120 K) is necessary for the effective operation of devices or may be essential to utilize certain phenomena existing at very low temperatures. Recently, cryogenic coolers of the Stirling type have become an essential component in a variety of electro-optic, vacuum and high temperature super-conductive military, space and commercial applications.

For example, infrared (IR) imagers enhance tremendously the ability to detect and track ground, sea and air targets and to navigate at nighttime [Coleman 1994], [Miller 1994]. Their operating principle is based on the simple fact that warmer objects radiate more and cooler objects radiate less. Since the noise figure of an imager strongly depends on the operating temperature of the IR sensor, high-resolution imagers require cryogenic cooling down to 80K. Modern sophisticated airborne thermal imagers which require compact design and low input power often rely on closed cycle cryogenic coolers. The Stirling coolers are eminently suitable for such applications because of their high efficiency. The Stirling refrigeration cycle, as compared to Gifford-McMahon and Joule-Thomson cycles, offers more than twice the cooling performance in the cooling power range 1-100 W.

Today’s electronics manufacturing requires sophisticated vacuum environments that are necessary for crystal growth, film deposition and etching processes [Rozanov 2002]. An effective method of producing a vacuum utilises recently developed cryopumps that often rely on Stirling type cryogenic coolers for cooling the cryopanels [Leybold 2001]. The cryopumps are gas entrapment vacuum pumps used for water vapour and gas pumping. These pumps are capable to provide for high vacuum with the pressure ranging from $10^{-3}$ to less than or equal to $10^{-11}$ mbar. The principle of operation is based on binding gaseous substances to the cold surfaces.
(cryopanels) within the pump by a method of cryocondensation, cryosorption or cryotrapping. In order to be able to produce a high or ultrahigh vacuum the cryopanels must be cooled down to a sufficiently low temperature. As compared with other available coolers, a compact and dynamically balanced Stirling cooler yields the desired water vapour and gas pumping speed at minimum cost.

The booming market in electronic communications requires new specially developed cheap, compact, efficient and reliable solutions for cryogenically cooled passive devices such as high temperature superconductive radio filters. These devices have to operate reliably in harsh environmental conditions, often being exposed to extreme shock, vibration and temperatures. Under these circumstances, Stirling type cryogenic coolers give probably the only solution.

Refrigeration systems used today in high-end multi-processor servers, for example the large-scale IBM S/390® G4 server, successfully utilize conventional vapour-compression refrigeration (based on the Rankine cycle) to cool the processors and memory chips for improved functionality and reliability [Schmidt and Notohardjono, 2002]. However, the existing Rankine refrigeration systems used in servers are bulky and likely to be substituted in the near future by compact and more effective Stirling cycle based systems. Similar trends can be observed in the design and manufacture of modern commercial and domestic refrigerators. These applications require not only high effectiveness and low power consumption, but also the use of environmentally friendly refrigerants, which can be found in the design of Stirling coolers. Recently, the start of the first ever mass-production of domestic refrigerators utilizing Stirling technology has been announced by LG [LG Electronics 2003], while the first successful developments in this field were reported in 1994 [Sunpower 2003].

Research and development of mechanical coolers has proceeded continuously from the earliest interest in cooled IR sensors in the 1950s. In that time many different types of closed-cycle coolers have been investigated including Stirling, Vuilleumier, Gifford-McMahon, Brayton and Linde-Hampson systems [Walker 1989]. The technology has matured since when. Stirling cycle coolers available on today's market provide efficient and reliable design solutions for the majority of
applications. However, there is an increasing demand for low-vibration models featuring longer life and lower power consumption. This especially holds true for airborne IR imaging systems where extremely low vibration levels are tolerated and available electrical power is often limited [Coleman 1994].

Demanding applications also include electron microscopes in which the cryogenic cooling of specimens may be required and images are often obtained with resolutions of up to 1 nm [JEOL 2003]. External disturbances such as mechanical vibrations, besides stray magnetic fields, can cause image distortion, jagged picture edge lines and other phenomena. While the entire microscope structure is typically isolated from the environmental vibration using a special mount, for example air suspension [JEOL 2003], the cryogenic cooler is directly attached to the vacuumed specimen chamber and therefore may produce an important source of vibration.

Electronics manufacturing that utilizes cryogenic cooling in a vacuum also requires extremely low vibration levels. Mechanical vibrations experienced by the process equipment during the thin film deposition, ion etching, electron lithography or final assembly of an electronic chip can negatively affect the yield, which is the most important concern of the manufacturing process. The required precision of processing can be appreciated by observing the progress in the development and manufacturing of Integrated Circuit (IC) modules. For example, a 1Mb Dynamic Random Access Memory (DRAM) chip was produced in 1990 having a linewidth of 1μm. Today’s mass-produced 1Gb DRAM modules utilize high-density 0.1μm circuit geometry [Samsung 2003]. Though most defects are produced by mechanical particles coming from the process, it is obvious, that mechanical vibrations should be kept within tight limits at any stage of the manufacturing process.

Complete elimination of vibration export from the cooler is often not attainable due to certain design limitations. Further vibration reduction may be achieved by passive or active vibration protection systems or a combination of both. Active vibration protection is sometimes preferable, if not the only feasible solution, but can be implemented at the expense of complexity of design comprising a system of sensors and actuators. At the same time, an improved active vibration control can reduce the
complexity of the system design through the use of adaptive control algorithms and sensorless motion measurement techniques.

The requirement of low input electrical power can be met by improving the efficiency of the cooler operation by a better design of cooler. However, tuning of the cooler parameters will be required to accommodate different operating conditions. In the field, the parameter readily available for tuning is the cooler electrical supply that can be adjusted online to drive the cooler for highest efficiency. An adaptive control system for a linear drive, twin-piston Stirling cryogenic cooler employing a quasi-sensorless approach to measurement and active control of vibration is studied in this thesis.

1.1 Thesis objective

The objective of this thesis is to undertake research leading to the development of an adaptive control system for use with a linear twin-piston Stirling cooler. Ricor’s state of the art model K535 linear, twin-piston Stirling cooler is used for the evaluation of attainable performance of the control system; all features of the control system are therefore optimised to benefit this particular model. The cooler needs to produce required cooling at the specified cold finger temperature and for a range of heat loads, while operating with minimal vibration export and input power.

The original control system developed and currently used by the cooler manufacturer features temperature control, while other parameters such as power consumption and uncompensated vibration are not controlled, which is adequate for many applications. Though the temperature of the cold finger remains the dominant control parameter, more efforts in the present research are focused on the vibration reduction and minimization of the input electrical power.

The issue of vibration reduction is approached here by the balancing of the cooler compressor through active control of the latter. More specifically, a quasi-sensorless balancing is implemented using an adaptive feedforward control algorithm. Balancing using motion sensors is also evaluated, but used mostly for reference and control-system debugging. The quasi-sensorless balancing is tested and the results
are compared with those obtained from the experiments on balancing using motion sensors.

Minimization of the input electrical power necessary to perform required cooling is achieved by the frequency control of the cooler. An optimal control system capable of automatically tuning the frequency of the supply voltage for the cooler compressor is developed and tested. A searching optimisation algorithm is implemented to locate the minimum value of the input electrical power and to generate a corresponding frequency. Performance of the control system is evaluated through the experiment.

Finally, the vibration and the frequency control systems are combined to produce a system targeting both the vibration reduction and the input power minimisation. Modelling of this control system is performed and its attainable performance is evaluated.

1.2 Thesis organisation

Chapter 2 presents the basics of operation of linear Stirling cryogenic coolers and describes in detail the design and the principles of operation of the linear drive, twin-piston Stirling cryogenic cooler, the subject of present study. An overview is given of the characteristics and requirements of the cooler. The objectives of control of the linear twin-piston Stirling cooler are briefly discussed in the context of the requirements of the cooler.

Chapter 3 establishes an equivalent mathematical model of the cooler. System identification of a specific cooler that will be used for the tests on the control system is performed based on the proposed mathematical model. Values for parameters necessary for the simulation of cooler operation and the modelling of the control system are obtained for typical operating conditions.

Chapter 4 considers the temperature control of the cooler. The control system for the closed-loop control of the linear, twin-piston Stirling cooler is modelled using Simulink. Detailed description of the experimental rig and the control hardware and software is given. Details of operation of the LabVIEW controller are set out. Performance of the control system is estimated through experiment. Results of
experiments are presented for typical operating conditions. Conclusions on the issues of temperature control of the linear twin-piston cooler are drawn.

Chapter 5 discusses methods and means of vibration reduction in a twin-piston linear cooler. Principal types of control systems suitable for active vibration control of the linear twin-piston compressor of cooler are described. An extensive overview is given of applicable sensors and techniques for vibration detection in the linear compressor. Choice of the type of control system and the method for vibration measurement is explained.

Chapter 6 presents modelling of the control system for the active vibration control of the linear twin-piston compressor of a cryogenic cooler. The principles of operation of the control system for compressor balancing are described in detail. The modelling is performed using Simulink and is based on the equivalent model of the cooler developed in Chapter 3. Results of modelling are presented and appropriate conclusions are drawn.

Chapter 7 presents results of experiments on the attainable performance of the control system for balancing the twin-piston compressor of a linear Stirling cooler. Description of the experimental rig arrangement is given. Functioning of the controller for the temperature control and balancing is described. Balancing using the quasi-sensorless technique for the measurement of the pistons' motion is demonstrated and the balancing using the LVDT sensors is given for reference. The results of the quasi-sensorless and motion-sensor based balancing are compared and analysed.

Chapter 8 discusses the optimal frequency control of the linear twin-piston Stirling cooler. The equivalent model of the cooler developed in Chapter 3 is validated for use in the modelling of the optimal control system. The optimal frequency control is developed. Simultaneous operation of the temperature, balancing and optimal control systems is modelled. Tests on attainable performance of the optimal control system are performed for typical operating conditions.

Chapter 9 presents overall conclusions drawn from the work described in the thesis and makes suggestions for its continuation.
CHAPTER TWO

LINEAR STIRLING COOLER

This chapter presents the basics of operation of linear Stirling cryogenic coolers and describes in detail the design and the principle of operation of the linear drive, twin-piston Stirling cryogenic cooler, the subject of present study. An overview is given of the characteristics and requirements of the cooler. The objectives of control of the linear twin-piston Stirling cooler are briefly discussed in the context of the requirements of the cooler.

2.1 Introduction to linear Stirling coolers

Stirling cycle coolers operate on a closed thermodynamic regenerative cycle with periodic compression and expansion of the working gas at different temperature levels. The use of a closed thermodynamic cycle allows for the design of a compact, long-life, low maintenance device capable of operating effectively over a wide range of operating conditions. A representative example of state-of-the-art in technology, and the subject of present study, is the closed cycle, linear Stirling cooler of the integral type, the Ricor’s K535 model that is shown in Figure 2.1.

The cooler comprises two main parts: a twin-piston reciprocating compressor with the pistons individually driven by linear electrical motors and a pneumatically driven expander with internal displacer-regenerator. The compressor and expander are mechanically arranged in a single hermetically sealed housing and interconnected by the internal gas transfer line. The compressor provides pressure pulses to the working gas and alternates compression and expansion of the gas in the expansion space of the expander. Pneumatically driven displacer-regenerator reciprocates and shuttles the working gas within the cold finger of an expander. During the compression of the working gas, heat is rejected to the environment, and, conversely, during the expansion, heat is absorbed from a device thermally attached to the expander.
Each single expansion yields a very minor temperature drop, thus the regenerator is an obligatory component of every Stirling cryogenic cooler. The main duties of the regenerator is the pre-cooling of the gas just before expansion and maintaining a temperature gradient between the heat load and the ambient end of the expander by alternately removing heat from and restoring it to the working gas [Walker 1983].

The typical regenerator is a thin-walled plastic or stainless steel cylinder tightly and uniformly packed with metallic wires, spheres or mesh, where lead, brass or stainless steel are the most common materials. Particle and wire mesh sizes are typically 20-500 μm and 100-200 μm, respectively [Hands 1986].

Characteristics and requirements of the cooler are outlined below, although emphasis is given to the control of cooler.

### 2.2 Characteristics

General characteristics of a cryogenic cooler include:

- Cooling capacity

Cryogenic coolers are often rated by their available cooling capacity, measured in Watts [Walker 1989]. However, not only the cooling capacity, but also the temperature at which the cooling is available should be specified. That is, the
achievable cooling capacity will drop at lower temperatures and vice versa. Therefore, the capacity, in specifications for cooler, is typically given for a range of cryogenic temperatures.

- Efficiency

The coefficient of performance (COP) of a cooler is defined as the ratio:

\[
\text{COP} = \frac{\text{cooling capacity}}{\text{input electrical power}} \quad (2.1)
\]

The ideal Stirling refrigeration cycle is similar to the reverse Carnot cycle and has the same coefficient of performance (COP) [Walker 1989]. Definition for the ideal COP follows from the energy balance for the ideal Stirling refrigeration cycle. The heat rejected from the system, \( Q_c \), is the sum of the heat lifted (or heat absorbed), \( Q_R \), plus the input work, \( W \), necessary to operate the cooler, i.e.:

\[
Q_c = Q_R + W \quad (2.2)
\]

The ideal COP is then defined as the ratio of the heat lifted to the work done, so that:

\[
\text{COP} = \frac{Q_R}{W} = \frac{Q_R}{Q_c - Q_R} \quad (2.3)
\]

Stirling cycle coolers do not achieve this ideal efficiency, due to losses associated with Joule heat developed in the motor, temperature drops across the displacer-regenerator, regenerator inefficiency, mechanical friction and viscous dissipation to name but a few. Therefore, the efficiency of a practical machine can be measured as the ratio of the actual COP to that of the ideal Carnot cycle [Walker 1989]:

\[
\text{Efficiency} = \frac{\text{actual COP}}{\text{Carnot COP}} \quad (2.4)
\]

- Mechanical vibration

Another important characteristic is the vibration export produced by a cooler. The vibration is due to the accelerated motion of unbalanced moving parts in the cooler. Low vibration levels are crucial for the normal operation of many sensitive instruments including optical devices.

- Operating life

Operating life is another important characteristic often quoted as the mean time to failure (MTTF). This is affected to a large extent by the configuration of a particular cooler including the design of its moving parts, seals and bearings, and materials used for the design. The environment in which the cooler must operate is also an important factor.

- Mass and volume
These characteristics are entirely defined by the mechanical design of the cooler and may be important for optical systems mounted on swivelling gimbals. Low mass and volume in this case are necessary to facilitate a fast response with low inertia forces.

- Cost

The cost of a cooler will generally include the cost of materials used in its design, the cost of parts' manufacturing and assembly, and the cost of maintenance required throughout the cooler's life.

### 2.3 Requirements

Requirements placed on cooler design are often dictated by the application in which the cooler must operate. Nevertheless, the objective is always to find the best possible compromise of compact volume, long-life, cost-effective system with adequate cooling capacity, low maintenance requirement and low power consumption, and with minimal vibration and noise. This may be achieved by the improvement of cooler characteristics through a better design of cooler. The aim of the present research, however, is to find the best methods of cooler control to improve its characteristics. The requirements of low vibration and input power, and long operating life are therefore discussed further in regard to the objectives of this research. The ways of meeting the requirements are also touched on.

#### 2.3.1 Vibration export

A requirement for low vibration levels produced by a cooler is an important issue for the normal operation of optical devices and many other sensitive instruments, and for electronics manufacturing, as discussed in Chapter 1. In imaging systems, for example, excessive vibration could cause elastic deflections of the elements of the optical device resulting in unwanted motion of the focal plane and blurring of the image. This is especially true in the case of the integral-type cooler in which the compressor, the main source of vibration, is located in the same housing as the expander, so the vibration load is directly transmitted from the compressor to an IR sensor thermally attached to the expander.
The traditional methods of vibration isolation add too much complexity to the overall design of the system. However, in a linear compressor, self-induced forces only emerge in the direction of the piston or the pistons' axis, provided the piston suspension does not allow for excursion in the radial direction. This simplifies the arrangement of the vibration protection system. To suppress compressor-induced vibrations, a twin-piston design is often used. Such designs feature a pair of opposed compressors arranged back to back with their pistons reciprocating in an opposite phase. Hence the vibration is virtually eliminated by synchronised counter-motion of the pistons of the two opposed compressors.

In practice, due to a certain mismatch in the opposite pistons' motions residual vibration still exists and could exceed the limits allowed in some applications. The mismatch is in fact caused by errors in the manufacturing process and the different rates of mechanical wear of the compressors. As a solution, appropriate driving of the opposite pistons can compensate for the imbalance. Thus, an active control of the opposite compressors based on sensors and a sophisticated controller is required to command the motion of the pistons and thereby reduce vibration.

### 2.3.2 Input power

The requirement of low input electrical power may be referred to as the efficiency of cooler. A cooler is normally designed and optimised to operate with the highest efficiency in a limited range of cryogenic temperatures and heat loads, as specified by the cooler target application. A different application may require from the cooler to operate in a different range of the cryogenic temperatures and the heat loads. To retain high efficiency of operation under new operating conditions, the cooler parameters would require adjusting. Departure from the nominal range of operating conditions would decrease the efficiency and, as a result, the value of the required input power would be unnecessarily elevated.

The nominal performance characteristics of a cooler can be almost entirely defined at the design stage by the geometry of the compressor and expander and the properties of the materials and the working gas. Therefore, other parameters should be available
for tuning the cooler characteristics in field conditions after its manufacture. It will be shown later in the thesis, that the parameter readily available for tuning is the frequency of the supply voltage for the compressor. The performance may also degrade over the cooler’s life because of natural wear of the machine elements and contamination of the working gas by the products of wear. Therefore, to maintain a high level of performance of the cooler, there should be a possibility to respond to changes in the operating conditions or the cooler’s own parameters by adjusting the frequency of the supply voltage. This can be achieved by a system of automatic frequency control. This, along with the active vibration control of the cooler compressor, is studied in this thesis.

2.3.3 Operating life

The requirement for a long operating life is fulfilled by the design of the machine elements as well as the materials used in assembly. Due to the risk of gas contamination, the materials are carefully chosen from those with minimal possible outgassing. The side forces in the piston-cylinder pair of a linear compressor are much less than, for example, those in the piston-cylinder pair of a rotary compressor. To control the operation of the cooler, temperature and, often, motion sensors are included, for example to monitor the motion of the compressor pistons. Due to the risk of contamination of the working gas, unnecessary contact of the motion sensors with the gas must be avoided or specially designed sensors should be used. This adds considerably to the cost and complexity of a design. As an alternative, a sensorless or rather a quasi-sensorless approach allows for the indirect measurement of motion parameters via the measuring of other readily accessible parameters, for example, the voltage and current involved in the operation. This approach to motion measurement is studied in this thesis.

2.4 Basics of cooler operation

An introduction to the Stirling cycle coolers can be found in Walker [1989] and Wurm et. al [1990]. For the present study, the description of a cooler given in Bowman [1993] has been adopted.
Stirling cycle coolers operate on a closed thermodynamic regenerative cycle with compression and expansion of the working gas at different temperatures. That is, a confined volume of gas is repeatedly expanded at one temperature and recompressed at another with the result that heat energy is absorbed from the environment during expansion and rejected to the environment during compression. Regardless of whether energy is being absorbed or rejected, more is transferred at a higher temperature than at a lower temperature. The difference between the amount of energy absorbed at a low temperature and rejected at a higher temperature, including energy losses, must be provided to the machine in order to keep it operating.

2.4.1 Ideal Stirling cooler

To understand how the Stirling cooler actually works, it is convenient to consider the ideal thermodynamic Stirling cycle. The thermodynamics of the cooler is conceptually illustrated in Figure 2.2 and Figure 2.4. Figure 2.2 shows the pressure-volume (P-V) and temperature-entropy (T-S) relations that define the ideal Stirling refrigeration cycle. Heat $Q_R$ is absorbed to the working gas from the environment at the minimum cycle temperature, $T_R$. Heat $Q_C$ is rejected from the working gas to the environment at the maximum cycle temperature, $T_C$. During the isothermal expansion process, the amount of heat energy $Q_R$ absorbed by the cooler is equal to the area under the expansion curve “3-4” on the $P-V$ graph in Figure 2.2. During the isothermal compression process, the amount of heat energy $Q_C$ rejected by the cooler is equal to the area under the compression curve “1-2” on the same $P-V$ graph. The difference between the amount of heat energy absorbed, $Q_C$, and rejected, $Q_R$, plus the internal energy losses of the machine must be provided as input work necessary to operate the cooler, as discussed earlier in the chapter.

The mechanical arrangement of the twin-piston cooler is schematically shown in Figure 2.3. From the figure, the cooler comprises two main parts: compressor and expander. The compressor includes a pair of opposite reciprocating pistons each driven by a linear electric motor. Periodic motion of the pistons provides pressure pulses to the working gas and causes its periodic volumetric change. The displacer-regenerator periodically reciprocates shuttling working gas from end to end of the
expander. The means by which the displacer is caused to reciprocate will be clarified by Figure 2.4. It should be emphasized, that here operation of the cooler is described in regard to the ideal Stirling cycle. By definition, the displacer is a reciprocating part that has a temperature difference, not a pressure difference, across it. The absence of a pressure difference is indicated in Figure 2.3 and Figure 2.4 by the loose fit of the displacer in the cylinder of the expander. Though the total work space comprises both the compression and expansion volumes, it is obvious that a part of the expansion volume below the displacer is, in fact, the part of the compression volume. It is assumed, that for ideal operation the pressure losses over the regenerator and gas ducts are negligible and the instantaneous pressure is, to a first approximation, the same throughout the total work space. A rod attached to the displacer passes through the bushing to expose the face of the rod to the pressure in the common space behind the displacer rod and compressor pistons. Theoretically, the common space behind the pistons and displacer rod is large enough so that the gas pressure there varies little from the mean pressure $P_M$, which value is generally close to the value of the initial filling pressure.

Figure 2.4 shows the positions of the pistons and displacer, with relevant pressures, temperatures and heat flows at four distinctive points during the cycle. Each cycle is a repetitive sequence of four heat transfer processes, two at constant temperature and two at constant volume.

At position "1", the displacer is at the cold side and the pistons driven by motors begin to move towards each other reducing the total work space volume and raising the pressure of the working gas from $P_1$ to $P_2$. During this ideal isothermal compression, the temperature of the working gas remains constant by the rejection of heat to the heat sink at temperature $T_{WARM}$. An amount of heat rejected to the heat sink is entirely absorbed by the compressor body and then ultimately rejected by radiation from the overall outer surface of the cooler, except for the surface of the expander part. The expander is typically enclosed in a dewar so the cylinder of the expander is exposed to vacuum pressure inside the dewar.
Let us consider now the forces acting on the displacer in position ”2”. Pressure $P_2$ acts on the effective areas of both the circular cold face and the annular warm face of the displacer, but $P_M$, which is less than $P_2$, acts on the rod’s face. Therefore, a net force equal to the pressure difference $P_2-P_M$ times the rod area pushes the displacer toward the warm side. The resulting motion of the displacer from position “2” to position “3” shuttles the working gas from the warm side to the cold side without changing its volume. During passage through the porous matrix (typically a mesh of fine wire) of the displacer/regenerator heat is transferred from the working gas to the regenerator matrix so the gas enters the expansion space at the low temperature $T_R$. This isovolumetric regenerative cooling of the working gas decreases its pressure to $P_3$. The cooling is regenerative in that the heat transferred from the working gas to the regenerator is stored in the regenerator matrix until later in the cycle.

At position “3”, the pistons driven by motors begin to move outwards expanding the total workspace volume. During this ideal isothermal expansion of the working gas, its temperature remains constant by the absorption of heat from the walls of the expander at temperature $T_{COLD}$.

As the total workspace volume is expanded by outward motion of the pistons from position “3” to position “4”, the pressure of the working gas falls below the mean pressure behind the pistons and displacer rod. Again, a net force equal to the pressure difference $P_r-P_M$ times the rod area acts on the displacer pulling it in toward the cold side. The resulting motion of the displacer from the position “4” to position “1” shuttles the working gas from the cold side to the warm side without changing its volume. In passage through the regenerator matrix the gas is heated from $T_R$ to $T_C$ and enters the warm side at that temperature. The isovolumetric regenerative heating of the working gas increases its pressure to $P_I$. The heating is regenerative in that the amount of heat transferred from the regenerator to the working gas is equal to the amount of heat transferred from the working gas to the regenerator in the earlier regenerative cooling process. Ideally, this regenerative process is perfect with all the heat stored in the regenerator matrix during cooling being returned to the gas during heating on the next cycle.

Then, with the displacer and pistons being at position “1”, the cycle starts again.
Chapter 2

**Linear Stirling Cooler**

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**Figure 2.2 Ideal Stirling refrigeration cycle**

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**Figure 2.3 Schematics of the linear twin-piston Stirling cooler of integral type**
Figure 2.4 Operation of the linear twin-piston Stirling cooler (Ideal Stirling refrigeration cycle)
2.4.2 Practical Stirling cooler

The operation of Stirling coolers in practice is markedly different from the ideal Stirling cycle described above. An important departure from the ideal cycle is that the pistons and displacer reciprocate sinusoidally instead of in the discontinuous fashion as indicated in Figure 2.2. As a result, the corners of the $P$-$V$ diagram are rounded off, reducing the associated areas. In theory, the compression and expansion processes are ideally isothermal. For this to be achieved in practice would require infinite rates of heat transfer or the machine to operate at very low speed. The latter is not true for practical coolers. No matter what configuration of the Stirling cooler is employed, the piston and displacer of a typical cooler would reciprocate sinusoidally with frequency ranging from 30 to 70 Hz [Walker 1989], [Wurm 1990]. As a result, the pressure and volume will vary harmonically, and the actual $P$-$V$ diagram for the total working space will therefore be as shown in Figure 2.5. Here, the parts of diagram corresponding to the isothermal and isovolumetric processes cannot be clearly distinguished, by contrast to the ideal cycle.

![Figure 2.5 Practical refrigeration cycle of the Stirling cooler](image)

As the working gas is moved back and forth from the cold side to the warm side of the cooler, some fluid dynamics friction effects will be produced. These will manifest as a pressure drop across the regenerator and gas transfer line. The result is that the amplitude of pressure variation in the expansion space will be less than the amplitude of pressure variation in the compression space with the phase shift also included [de Jonge and Sereny 1982], [Aubrun et al. 1992]. The latter is important and can be defined at the design stage since the proper phasing strongly influences
the efficiency of the machine [Riabzev et al. 2001], [Organ 1999], [Cun-quan et al. 2002].

Though the operation of the regenerator is not discussed in full, it is obvious that the process of regenerative heating and cooling is complicated, involving continuously varying fluid and matrix temperatures in both space and time. The velocities, pressures and densities of the gas in the regenerator matrix are also continuously variable.

Additional losses can be associated with mechanical friction in the compressor and expander.

2.5 Description of design of the linear twin-piston Stirling cooler

Figure 2.6 shows the layout of the Ricor’s K535 linear twin-piston Stirling cooler with corresponding notations for its main components. This cross-sectional view diagram was reduced from the Ricor’s proprietary draft to an extent essential to describe the design. The cooler is of integral type and therefore has the compressor and expander both sealed in a single hermetic envelope. The driving voltage is supplied to the compressor motors via electric feedthroughs on the side of the compressor casing. The device to be cooled is thermally attached to the cold tip of the expander, while the expander, typically together with the cooled device, is enclosed in a vacuumed dewar.

2.5.1 Electric motor

The compressor utilizes two electrodynamic linear motors of the moving-magnet type arranged back to back. Henceforth the description of the opposed motors refers to one of them, as shown in Figure 2.6. The linear motor consists of a stationary wound coil inside the stator core, and a tubular moving magnet, which is free to reciprocate in the annular gap formed by the inner and outer stator rings. The magnet is radially magnetized and made from a rare-earth material with a high flux density and power to mass ratio, to provide high air-gap flux densities with compact design.
In the present configuration where the ratio of the air gap length to gap width is large, a nearly uniform magnetic field is established across the gap. This improves the linearity of a force-displacement characteristic of the motor. The elimination of the attraction forces across the air-gap is provided by the correct alignment of the moving magnet in the annular gap of the stator. The stator core is laminated to reduce eddy currents. The advantages of such motor arrangement are a small size for a given rating, reliability and linear, within a known range, current-force and force-displacement characteristics.

The principle of motor operation is similar to that of the linear reluctance motors [Evans 1996] which operate by balancing two forces acting on a moving member, as shown in Figure 2.7. The constant source of mmf, commonly called the polarizing mmf, is provided by the permanent moving-magnet and the controllable source, known as the control mmf, by a controlled current passing through the coil. Exciting the coil disturbs the symmetrical distribution of the polarizing flux in the magnetic circuit of the motor. The control mmf produced increases the level of flux at one end of the motor and reduces it at the other, and causes the moving-magnet to move, until the force produced by the energised coil is counter-balanced by the restraining or reluctance force (also known as the magnetic stiffness) due to the permanent magnets and any external load. The unsymmetrical flux paths cause the reluctance force and the differing reluctances produced when the moving magnet is displaced, and it acts as a spring force tending to restore the displacement to zero in the absence of the control mmf.

2.5.2 Piston assembly

The reciprocating piston is directly coupled to the moving magnet of the linear motor. The centre position of the magnet-piston assembly is held and controlled by an Oxford-type mechanical spring (flexure). The flat Oxford-type spring, due to its geometry, allows linear motion in the axial drive direction, but minimizes excursion in the radial direction. The magnet-piston assembly is connected to the flexure through a flexible rod to compensate for any misalignment of the piston, cylinder sleeve and flexure caused by temperature gradients and impaired tolerances.
Chapter 2  Linear Stirling Cooler

The buffer limits the motion of the reciprocating parts within a given range to prevent colliding, due to abnormal operation, against the stationary parts. The buffers are equally spaced from the centre position of the piston motion and placed at the ends of the piston reciprocation path, so that, if the piston overstroke, the reciprocating parts hit the rubber pads of the buffer, not the cylinder face or end cap of the compressor casing.

Pressure sealing is provided by the clearance seals, which are long, narrow, annular passages around the piston. The sealing is attained by the flow restriction these passages provide to the oscillating working gas. This method of sealing requires that the piston reciprocates in close proximity to the adjoining walls. This is made possible by the tight tolerances and highly accurate machining of the surfaces.

The pneumatic compensating mechanism (not shown in Figure 2.6) equalises mean pressures above and below the piston to prevent piston drift [Beale et al. 1980], [Wood 1986]. The difference in the mean pressure is due to gas leakage through the clearance seals during compressor operation.

2.5.3 Displacer-regenerator assembly

The displacer-regenerator is pneumatically driven and reciprocates in the thin-walled cylinder of the expander with the axis of motion perpendicular to that of the pistons. The bushing provides the alignment of the displacer within the expander cylinder and the pressure sealing of the working gas. The centre position of the displacer is held and controlled by a flexure similar to that used in the compressor design. The displacer is connected to the flexure through a flexible rod to compensate for any misalignment.

The displacer is a circular, thin-walled plastic shell with the regenerator matrix placed within. The regenerator matrix is made of a tightly packed mesh of fine wire typically stainless steel. The working gas flows from the compression space into the regenerator matrix and back, from the regenerator matrix to the compression space, via the ducts in the annular face of the displacer. The working gas flow from the
regenerator matrix to the expansion space and back, from the expansion space into to the matrix, is via the holes in the circular face of the displacer. The bounce space behind the displacer bushing is provided to produce the pressure differences across the displacer and thereby to generate the pneumatic driving force for the displacer, as required by the Stirling cycle discussed earlier in this chapter.

2.5.4 Thermal interfaces

The compressor and expander are interconnected via the internal gas transfer line, as shown in Figure 2.6. The heat sink is placed within the annular gap formed by the adjacent parts of the cooler casing to provide for heat rejection from the working gas to the cooler body. The overall outer surface of the cooler casing is used as a thermal interface for heat rejection by radiation, while an additional water-cooling system is typically mounted on the cylindrical surfaces of the compressor’s end caps, as shown in Figure 2.6.

A standard flange is provided around the base of the expander for a vacuumed dewar to be fitted before operation. Gas containment is by means of metal 'O' rings between the end caps and cooler housing and a gasket to seal the fill port (not shown in Figure 2.6).

Materials used in the assembly such as plastics, adhesives and primers are selected from an existing knowledge base of materials with a low outgassing rate.
Figure 2.6 Layout of the Ricor K535 cooler.
Chapter 2  Linear Stirling Cooler

2.6 Conclusions

The basics of operation of the linear twin-piston Stirling cooler have been explained. A description of the cooler operation in terms of an ideal Stirling refrigeration cycle has been given and the issues of the operation of a real machine have been discussed in brief. This gives an introduction to the Stirling cooler technology and demonstrates the complexity of the thermodynamic processes involved in cooler operation. Characteristics of a typical cooler have been given and requirements for the cooler have been specified to allow for better understanding of the objectives of cooler control. This enables the direction of further research and methods of cooler control to be determined. A detailed description of the design of the Ricor's K535 linear twin-piston Stirling cryogenic cooler, which represents state-of-the-art technology and is the subject of the present study, has been given along with the principles of its operation. This, along with characteristics of the cooler, will be exploited in the modelling of cooler operation, as presented later in the thesis.
It has been demonstrated in the chapter that a Stirling cooler is a complex thermodynamic system, not to mention the sophisticated design of the particular cooler model studied here (Ricor K535), so the state-of-the-art Stirling cooler technology can be readily appreciated. It is clear that a careful attention should be given to every stage of the cooler development – from the thermodynamic, electrical and mechanical design to the cooler manufacture, including the choosing of materials. The cooler studied here is claimed to have quite mature and refined design, which is supported by the numerous applications where the cooler is successfully utilised today [Ricor 2002]. At the same time, some of the cooler characteristics, such as the vibration export and input power, for example, can be improved. In this respect, the cooler is viewed by the author as no ordinary machine, but as an exciting thermodynamic and electrodynamic system. The author believes there is an excellent opportunity to develop and test new control techniques that might improve the cooler operation. The issue of the cooler control is more closely approached in the following chapters.
This chapter establishes an equivalent mathematical model of the cooler. System identification of a specific cooler that will be used for the tests on the control system is performed based on the proposed mathematical model. Values for parameters necessary for the simulation of cooler operation and the modelling of the control system are obtained for typical operating conditions.

3.1 Equivalent model of the cooler

An equivalent dynamic model of the cooler was needed for the developing of the control system. A complete model for the simulation of operation of the Stirling cooler involves simultaneous computation of mechanical, electrical, thermodynamic and fluid flow processes, where the cycle of work, heat exchange and fluid flow variations is a complex dynamic and thermodynamic interaction [Organ 1999]. The solutions are typically obtained using numerical methods [Minas 1994]. Unfortunately, a suitable software package for the modelling of the cooler was not available. At the same time, a simplified model based on a set of mechanical, electrical and pneumatic equations defining the dynamic behaviour of the cooler has proved to be adequate for the purpose of the present study.

Cryogenic coolers are typically designed to lift the required amount of heat from the cold finger (where the cooled device is thermally attached) at a given temperature. This means that for the normal closed-loop steady-state operation of the cooler, the cooling capacity $Q_R$, as discussed earlier in Chapter 2, produced due to the gas expansion should take a constant value. This value is defined by the amount of heat load, and the cold finger and ambient temperatures. Practically, the cooling capacity can be specified in terms of the design parameters, such as the geometry of the machine, and the parameters of motion [Stolfi and Daniels 1985], [de Jonge 1979] as follows:
\[ Q_e = \omega \int p_e dv_e \propto \hat{p}_e \hat{v}_e, \quad (3.1) \]

where: \( \omega \) - driving frequency, rad/s;

\( v_e = v_e(t) \) - instant volume of the expansion space, m\(^3\);

\( p_e = p_e(t) \) - instant pressure in the expansion space, Pa;

\( \hat{p}_e \) - rms value of pressure in the expansion space, Pa;

\( \hat{v}_e \) - rms value of displacer velocity, m/s.

Similarly, the required amount of work \( W \), which is supplied to the gas during compression in the compressor, should also take a constant value. The work done is defined as [Stolfi and Daniels 1985], [de Jonge 1979]:

\[ W = \omega \int p dv \propto \hat{p} \hat{v}, \quad (3.2) \]

where: \( v = v(t) \) - instant volume of the compression space, m\(^3\);

\( p = p(t) \) - instant pressure in the compression space, Pa;

\( \hat{p} \) - rms value of pressure in the compression space, Pa;

\( \hat{v} \) - rms value of piston velocity, m/s.

Typical approach to the analysis of a Stirling cooler is, first, defining the dynamics of the machine, i.e. motion of the piston(s) and displacer and, second, defining the thermodynamics of the machine, given its dynamics is already known [Redlich and Berchowitz 1985], [Gaunekar et al. 1994], [Minas 1993], [Pollak et al. 1978]. Therefore, the importance of the equations (3.1) and (3.2) is in that they relate mechanical dynamics and thermodynamics of a Stirling machine.

For example, in the work presented by [Tailor and Narayankhedkar 1988] similar approach is used to the analysis and performance prediction of a single-piston free displacer Stirling cryocooler. The authors first describe the geometry of the cooler to define the pressure and volume variations in both the compressor and expander. Then, they link the mechanical dynamics of the cooler to its thermodynamics using equations similar to (3.1) and (3.2). Quite similar approach to the analysis of a dual-piston Stirling cooler was presented in [Gaunekar et al. 1994] with more impact.
made on the dynamics of the compressor drive. Minas [Minas 1993] proposed a method that couples the thermodynamic and fluid dynamics equations to the dynamic equations that describe the motion of the displacer in a split Stirling cooler. Though the dynamics of the compressor piston of the machine was not included in the formulation, the thermodynamics is defined in a manner similar to (3.1) and (3.2). An example of using this approach to the analysis of a Doelz compressor (single-cylinder electrodynamic compressor in which both the motor coil and moving magnet move relative to the cylinder body) can be found in [Pollak et al. 1978].

The correctness of the approach is justified in [Redlich and Berchowitz 1985]. The authors analysed the dynamics of free-piston Stirling engines that operate based on the direct Carnot cycle as opposite to the cycle employed in the Stirling engines. It was particularly emphasized in this work that the relating mechanical dynamics and thermodynamics in the manner discussed above provided direct means for understanding the behaviour of engine and showed the influence of engine geometry on this behaviour.

The cooling capacity $Q_R$ and the work $W$ are related via the COP as defined by the equation (2.3) given in Chapter 2. It can be rewritten now as:

$$\text{COP} = \frac{Q_R}{W}$$

(3.3)

The COP can be assumed constant for a particular cooler and for a constant cold finger temperature and heat load. If these conditions are met, for the simulation of cooler operation, instead of maintaining a constant temperature of the cold finger, it would be adequate to maintain the product $W = \hat{p}\hat{V}$ as a constant, provided the heat load remains constant [Gaunekar et al. 1994]. Hence, the further modelling of the cooler can be restricted to the modelling of cooler compressor only. This will be exploited for the simulation of the cooler operation and the modelling of the control system.

The model has been validated via the full-scale tests performed on the Ricor K535 Stirling cooler and the results are presented in Chapter 4.
3.2 Modelling of cooler motors

3.2.1 Equivalent electrical scheme

The motor utilized by the cooler compressor is essentially a linear electrical motor of the permanent moving-magnet type. A brief description of the motor arrangement along with the basics of its operation has been given in the preceding chapter. Comprehensive analysis of moving magnet linear motors of this type can be found in Redlich [1996]. This motor can be modelled using magnetic surface currents analysis that is currently used by the motor developer, Sunpower Inc. [Sunpower 2001]. This analysis, despite offering an accurate and physically transparent solution for motor parameters, involves iterative calculations of motor geometry and considerations of materials and load conditions. From the general theory, the axial driving force in the linear motor is generated by currents in a magnetic field, according to the well-known equation [Kraus and Fleisch 1999]:

$$
\text{force} = \text{current density} \times \text{magnetic field}
$$

(3.4)

Magnetic field is typically provided by a stationary or moving permanent magnet and the current flow is in the moving or stationary coil respectively [Ross et al. 1994].

The basic idea of the magnetic surface currents analysis comes from an alternative interpretation of the relationship (3.4). According to this method, the current density is defined as a surface density of magnetic current. The moving permanent magnet is modelled by a single turn coil with a current flow providing a flux density equal to that produced by the moving permanent magnet. The electric current flow in the stator winding provides the magnetic field.

The above analysis gives a relatively complex analytical solution for the generated force and is therefore found effective only for the design of new motors. In fact, all the parameters representative of an existing motor can be found experimentally, which has proved to be adequate for all practical purposes. This requires establishing an equivalent electrical scheme for the motor.
The equivalent electrical circuit of the motor can be effectively represented as a series connection of a voltage source, resistor, inductor and back emf source [Redlich et al. 1996], [van der Walt and Unger 1992], as shown in Figure 3.1.

![Equivalent electrical scheme of the motor](image)

Figure 3.1 Equivalent electrical scheme of the motor

The inductor represents the stator coil which is quantitatively characterized by the value of its self-inductance. The generated back emf voltage has a non-zero value only when the piston is moving, i.e., according to Lorenz law, when the permanent magnet is moving relative to the stator coil [Kraus and Fleisch 1999].

The following assumptions have been made for the analysis of the motor [Gaunekar et al. 1994], [Redlich et al. 1996]:

1. The magnetic field in the air gap of the stator is radial, constant and uniform, with no leakage flux outside the gap. The moving magnet does not leave the gap and therefore the magnetic flux established across the gap is not affected by the surroundings.

2. The self-inductance of the stator coil is constant and independent of the moving magnet’s position. The self-inductance is only affected by the magnet fringing fields. Since the fringing fields do not vary with time [Redlich et al. 1996], even though the magnet moves within the air gap, the associated losses can be neglected.

3. The resistance is constant, independent of the moving-magnet’s position and is a sum of the dc coil resistance and the loss resistance associated with losses induced in the motor surroundings (exclusive of core losses) by a time-varying field generated by the coil current [Minas 1994].
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Modelling of cooler

The losses in the resistance associated with braking by hysteresis and eddy currents in the core are referred to as “shuttle losses” and neglected (typically they are about 2% of the rated power for this type of motor [Redlich et al. 1996]).

4. The core losses are neglected.

The core losses are difficult to account for because they exist even if the current through the coil is zero, provided the magnet is moving so the generated back emf is not zero.

5. The motion of the magnet (with the piston) is of a simple harmonic form.

The electrical circuit of Figure 3.1 is governed by the equation:

$$L\dot{i} + R i + e = u$$

(3.5)

where: 
- $L$ – self-inductance of the stator coil, H;
- $R$ – resistance of the stator coil, Ohm;
- $u$ – voltage applied across motor terminals, V;
- $i$ – current through the stator coil, A;
- $e$ – back emf voltage, V.

The back emf voltage is defined as:

$$e = \alpha \dot{x}$$

(3.6)

where: 
- $\alpha$ – motor constant, N/A;
- $\dot{x}$ – piston velocity, m/s.

The motor constant $\alpha$ in the above equation is a constant linking the back emf voltage and the moving-magnet velocity or, alternatively, the electromotive force and the current for the motor.

With indexes “1” and “2” for the opposed motors, equation (3.5) becomes:

$$L_1\dot{i}_1 + R_1i_1 + \alpha_1\dot{x}_1 = u_1$$

$$L_2\dot{i}_2 + R_2i_2 + \alpha_2\dot{x}_2 = u_2$$

(3.7)

3.2.2 Identification of motor parameters

The values of the inductance, $L$, and resistance, $R$, of the stator coil and the motor constant, $\alpha$, have been experimentally defined based on equations (3.7). An assumption of constant, time and distant invariant motor parameters suffices for the modelling and simulation purposes. However, the quasi-sensorless control, as will be
discussed later, requires an exact knowledge of motor parameters as dependent on the operating conditions. The value of self-inductance was therefore measured as a function of the axial position of the moving magnet. The value of the motor constant was measured as a function of both the magnet position and the current through the stator coil. These were static measurements. The motor constant was also estimated from a dynamic test by driving the compressor piston pneumatically and measuring values of the back emf and the velocity of the reciprocating parts. Averaged values for the above parameters were then taken for the modelling. The description of suitable rig modifications made to measure each motor parameter follows.

3.2.2.1 Resistance of the stator coil

The use of an electronic bridge multimeter, rather than conventional tools, was required for the measurement of resistance because of the intrinsically low value of the latter. In addition, the effects of heating of the coil due to current flow were avoided using the above multimeter. The operation of the electronic bridge multimeter is based on the excitation of the component under test by a signal with accurately known frequency, measuring the voltage across and the current through the component, followed by the computation of the value of the reactance or resistance of the component. For the measurement of the resistance of the coil the multimeter was set up to excite a series equivalent electric circuit of the motor with a frequency of 100 Hz. Motor terminals were directly connected to the test terminals of the multimeter via the short test leads to prevent any additional voltage drop over the leads. The value of resistance as an average to several tests conducted at an ambient temperature of around 23°C was found to be as high as 0.42 Ohm for either motor.

3.2.2.2 Self-inductance of the stator coil

The measurements of the self-inductance of the stator coil were done using the same motor connection as used for the resistance measurements. The value of self-inductance was measured as a function of the axial position of the moving magnet at the distinctive points along the magnet’s reciprocation path with an increment of 0.5 mm. The arrangement of the rig is illustrated in Figure 3.2.
As shown in Figure 3.2, the dial gauge rigidly mounted on the base was used as a precise indicator of magnet (piston) position enabling a resolution of 0.01mm. The magnet (piston) was accurately placed at different axial positions by a lead-screw mechanism and then clamped down at each position. Values of the self-inductance for each magnet position were averaged over six measurements and as a result the diagram shown in Figure 3.3 was generated. From the diagram, the value of self-inductance varies from $5.53 \times 10^{-3}$ H to $5.81 \times 10^{-3}$ H over the range of magnet reciprocation, with the maximum value being around the zero position. Slight asymmetry of the self-inductance curve about the zero position can be explained by the difference in the magnetic flux paths when the magnet is at either side of the zero position. The presence of asymmetry can be important for the control of piston displacement since the force developed by the motor may somewhat differ for the magnet moving in different directions from its central position.
3.2.2.3 Motor constant

Static measurements of the motor constant $\alpha$ were made based on the relationship between the current through the stator coil $i$ and the driving force $f_d$:

$$ f_d = \alpha i. \quad (3.8) $$

The motor constant $\alpha$ was measured as a function of both the magnet's axial position and the current through the stator coil. As the moving magnet is directly coupled to the compressor piston, the driving force $f_d$ is transmitted from the moving magnet to the piston and is readily available for measurement by a force transducer. The current through the stator coil can be measured by standard means.

An arrangement of the test rig is illustrated in Figure 3.4. To measure the driving force, a piezoelectric force transducer was attached to the compressor piston via the adaptor. On the opposite side the force transducer was connected to the slider of the lead-screw mechanism as shown in Figure 3.4. During the measurements, the slider was fixed at distinctive points along the piston reciprocation path with an increment of 1mm. The signal proportional to the transmitted force was preconditioned by a charge amplifier and then supplied to the signal analyser.
Different excitation currents for the motor with an increment of 1A were generated by the analyser and then supplied via the power amplifier to the motor as an ac voltage. The coil current was detected by the current transducer connected in series with the motor and was then provided as an input to the signal analyser. Values of the motor constant were calculated based on rms values of the driving force and the current though the coil.

![Test setup for measuring the motor constant α](image)

Values for the motor constant versus the magnet (piston) position are plotted in Figure 3.5 for different coil currents. An average to all seven data sets is plotted in the same figure. It can be seen that for each excitation current, except for the 2A, the motor constant value is within the 1% with respect to the average, provided the moving magnet position is within the range -4 to +3 mm. This result proves the theory for a motor of this type, according to which the motor constant is independent of the magnet position within 0.5%, provided the magnets come no closer to the end of the air gap than about ½ of the gap length, \( l_g \), as shown in Figure 3.6 [Redlich et
al. 1996]. The length $l_g$ of the air gap for the tested motor is 4.5mm, but even with the displacement being in the range $-4$ to $+3$ mm, which exceeds the value of $l_g/2=2.25$mm in either direction, the obtained linearity is acceptable. This is an important result and will be exploited for the quasi-sensorless control, as will be discussed in Chapter 5.

![Figure 3.5 Motor constant $\alpha$ as function of magnet position](image)

![Figure 3.6 Air gaps in the linear electric motor](image)
As noticed earlier, the motor constant can also be measured as a parameter linking the back emf and the velocity of the moving magnet. In this case, the motor constant is defined by a relationship similar to (3.8). The compressor piston is driven externally, say pneumatically, by an opposed compressor [Ross et al. 1994], and then values of the generated back emf and magnet velocity are measured using standard techniques. Appropriate tests have been done using available instrumentation and the results for the driving frequency of 50 and 100 Hz have been found to be in close correlation with the results of static measurements ($\alpha=8.27$ and $\alpha=8.65$ measured at 50Hz for the two opposite motors).

### 3.3 Modelling of cooler compressors

#### 3.3.1 Equivalent model

An equivalent mathematical model of a cooler compressor is amenable to relatively simple description by differential equations of motion. The effect of the cooler casing motion was neglected since the mass of the casing is much larger than that of the moving magnet-piston assembly. To complete the model it was therefore sufficient to specify the principal forces acting on the compressor pistons. The compressor has the virtue that side forces imparted to the piston-magnet assemblies are negligibly small as compared to on-axis forces, provided the magnets are correctly aligned in the air gaps of the stator iron. Therefore, all forces can be considered as acting along the pistons' reciprocation path. This assumption is justified, as the arrangement of the compressor and the geometry of the flexures accurately define the linear motion of the pistons. Thus, each of the opposite piston-magnet assemblies may be thought of as a single-degree-of-freedom (SDOF) system with the piston moving relative to the base common to both pistons. The reciprocation paths of both pistons are aligned along a single common axis.

Schematic representation of forces acting on the compressor pistons is given in Figure 3.7.
The principal forces, as shown in the figure, are those intrinsic to the compressor operation [Cun-quan et al. 2002], [Dadd et al. 2000], namely:

- the forces generated by the motors
- the forces of inertia
- the forces produced by the flexures as they are deflected
- the damping forces accounting for viscous friction losses
- the forces produced by gas pressure differences acting on the piston’s faces

Mechanical frictional forces should not be present because in principal the gas sealing eliminates any mechanical contact between the adjacent surfaces of the piston and cylinder [Dadd et al. 2000].

Displacement of the piston, $x_n$, refers to the upper piston face, and the datum is the piston rest position. Governing equations, with indexes “1” and “2” for the opposite compressors, are thus given as [van der Walt and Unger 1992], [Dadd et al. 2000]:

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + A_p p_1 = f_{d1} + A_p p_2$$

$$m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 + A_p p_2 = f_{d2} + A_p p_1$$

where: $m_n$ – moving mass, kg;
$c_n$ – damping coefficient, Ns/m;
$k_n$– mechanical spring rate, N/m;
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\[ A_p \text{ – effective area of piston face, m}^2; \]
\[ p_1 \text{ – pressure above the piston, Pa; } \]
\[ p_2 \text{ – pressure below the piston, Pa; } \]
\[ f_{dn} \text{ – motor driving force, N.} \]

The motor driving force is defined as:

\[ f_{dn} = \alpha_n l_n \]  

(3.10)

Pressure terms in the equations (3.9) are determined by the thermodynamic processes and pressure drops occurring in the refrigeration cycle. Thus, they cannot be simply described [Dadd et al. 2000], [Organ 1999]. Pressure drops are caused by the leakage through clearance seals and temperature changes due to heat transfer to the walls. Variations of the pressure are also due to the gas flow, which is not laminar throughout much of the workspace so the flow resistances are not linearly dependent on gas velocities [de Jonge 1979]. The common approach to the practical analysis of the cooler, referred to as a dynamic system, is to define gas pressure variations in terms of piston and displacer motions [de Jonge 1979], [de Jonge and Sereny 1982]. An approximate model, that is simple and useful, can be developed by treating the gas force as the action of an equivalent gas spring. The pressure, in this case, can be determined based on the equation for state of a perfect gas:

\[ pV^\gamma = \text{const} \]

(3.11)

where: \( p \) – pressure of the gas, Pa;
\( V \) – volume of the gas, m\(^3\);
\( \gamma \) – adiabatic coefficient.

The effective volume of the compression space can be approximated by a sum of the volumes of the workspace above the compressor piston and the void space occupied by the passage connecting the compressor and expander. The effective volume of the gas behind the piston is then the sum of the volumes of the cylinder space below the piston and the void volume that is confined by the compressor end cap. The latter also includes the volume of gas surrounding the motor coil and moving magnet assembly. It should be noted, that the volumes below the pistons are shared by the opposite compressors. All the above gas volumes are schematically shown in Figure 3.8.
Based on the equation (3.11), pressures above and below the piston can now be defined in terms of the geometry of the gas volumes, piston displacement $x$ and fill pressure $p_0$ as:

$$
\begin{align*}
 p_1 &= p_0 \left( \frac{2A_p l_1 + V_{\text{void}1}}{2A_p (2l_1 - x_1 - x_2) + V_{\text{void}1}} \right)^y \\
 p_2 &= p_0 \left( \frac{2A_p l_2 + V_{\text{void}2}}{2A_p (2l_2 + x_1 + x_2) + V_{\text{void}2}} \right)^y
\end{align*}
$$

(3.12)

The above method of describing the pressures offers a simple and convenient way of modelling the gas forces in the cooler compressor.

### 3.3.2 Identification of compressors' parameters

Values of the compressors' parameters in equations (3.9)-(3.12) are yet unknown and need to be estimated. At this stage, values of the moving mass, $m$, damping coefficient, $c$, and mechanical spring rate, $k$, have been found from transfer functions (mobility, in this case) calculated for each compressor. Assuming a gas pressure of
equal value both above and below the compressor pistons, equations (3.9) can be reduced to:

\[
\begin{align*}
    m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 &= \alpha_1 i_1 \\
    m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 &= \alpha_2 i_2
\end{align*}
\] (3.13)

Thus, each compressor can be represented as a Linear Time-Invariant (LTI) SDOF system, and the influence of the pressure forces on the pistons' dynamics is eliminated. The condition of an equal gas pressure above and below the pistons is satisfied by driving the opposed motors in a "slosh" mode as was demonstrated in [Ross et al. 1994]. In this case, two pistons shuttle an amount of the working gas back and forth between the spaces above both pistons without compression or expansion of the working gas, thereby the gas pressure throughout the compressor varies slightly from its mean value over the cycle and hence can be considered constant. The approach is simple yet effective. However, it was found more practical to eliminate pressure forces by detaching end caps from the compressor. With equal and constant pressure, i.e. ambient pressure, above and below the pistons, the forces acting on the pistons are only those given by equations (3.13).

Using well-established theory, transfer function values were measured as a ratio of the force applied to the piston along its reciprocation path and the velocity of the moving parts. The basic equation for the modulus of complex frequency response function of velocity \( H_v(j\omega) \) (mobility) is given below:

\[
|H_v(j\omega)| = \left| \frac{V(j\omega)}{\alpha f(j\omega)} \right| = \frac{\omega}{\sqrt{(-m\omega^2 + k)^2 + \omega^2 c^2}}
\] (3.14)

where: \( \alpha f(j\omega) \) - complex spectrum of the force, applied to the piston, N;

\( V(j\omega) \) - complex spectrum of piston velocity, m/s.

The above expression can be obtained by, first, applying Laplace transform to the equations (3.13) in the form \( x(t) \Rightarrow X(s) \); \( f(t) \Rightarrow F(s) \) and, then, making formal substitution \( s \rightarrow j\omega \).
To measure the mobility function, each motor was excited with a swept sine signal in the range of 0.5 to 150 Hz at sweep rate of 0.5 Hz/s. Current through the coil was measured with a current transducer and, then, a signal proportional to the current was supplied to the ACE signal analyser. The velocity of the moving parts was measured with a piezoelectric accelerometer attached to the flat top end of the piston connecting rod. The signal from the accelerometer was then conditioned by an integrating charge amplifier and fed to the signal analyser. A typical screenshot of the analyser set up for the measurement of the mobility is shown in Figure 3.9.

Values of the mass, damping coefficient and the spring rate were then obtained from the mobility function using curve fitting. As an example, the original and least-square (LS) fitted mobility functions for one of the two motors are plotted in Figure 3.10.
To complete the system identification, values for parameters of the gas springs should be found. The pressures above and below the piston were defined by equations (3.12) in terms of the piston displacement, the fill pressure and the geometry of the gas volumes. The latter include void volumes, the values of which are difficult to calculate from the geometry of the compressor, but are relatively easy to find experimentally. For this purpose, the pressures as functions of piston displacement can be evaluated through experiment and, then, the measured functions can be curve fitted to the theoretical equations to find values for the void volumes. In this case, additional means of measuring would be required to detect the piston position and to measure the pressure. The task is more complicated in a twin-piston compressor design due to the compression space being shared by two cylinders. In the absence of a means for measuring the piston displacement and pressure, a different approach can be used.

Both gas springs defined by the pressures above and below the piston can be approximated by a single equivalent linear spring in the form:

\[ kx = A_p(p_1 - p_2) \]  

(3.15)
Then, the spring rate constant $k$ of the linear spring can be found experimentally. Once defined, it can then be used to calculate values of the void volumes by curve fitting of the equation (3.12) to the linear function $kx$, as will be demonstrated later in the chapter.

The model of the compressor, in this case, is quasi-linear in that it always has a specific value of the spring rate constant $k$ (and in fact the damping coefficient $c$) for a given combination of cold finger temperature and heat load. That is, when the heat load, and particularly the cold finger temperature, changes due to varying operating conditions or demands of control, the effective gas spring takes a new value. This effect is illustrated in Figure 3.11, where the transfer function of pressure in the compression workspace and coil current is plotted for different temperatures of the cold finger. The transfer function was measured on the Ricor’s K535 cooler operating at steady state and with series connection of the opposite motors.

It can be seen from the Figure 3.10, that the resonant frequency increases somewhat between 120K and 65K. This may be associated with the increased density of the gas.
in the cold finger at a decrease in the temperature of the cold finger. The same effect was demonstrated in [Ross et al. 1994].

Therefore, the values for the equivalent linear spring should be determined for each combination of the cold finger temperature and heat load. A particular combination of a temperature of 120K and heat load of 10W was set for testing.

To determine a value for the spring rate constant of the equivalent linear spring for the given cold finger temperature and heat load, the following tests were done based on equations (3.7) and (3.9). The opposed motors were connected in series having a common current flowing through both. Equations (3.7) and (3.9), in this case can be rewritten as:

\begin{align}
L_1 \dot{i}_0 + R_1 i_0 + \alpha_1 \dot{x}_1 + L_2 \dot{i}_2 + R_2 i_2 + \alpha_2 \dot{x}_2 &= u_0 \\
&\quad \text{(3.16)} \\

\text{and} \\

m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + A_p p_1 = \alpha_1 i_0 + A_p p_2 \\
m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 + A_p p_1 = \alpha_2 i_0 + A_p p_2 \\
&\quad \text{(3.17)}
\end{align}

The opposed motors and piston-magnet assemblies to a first approximation can be considered identical, so that their corresponding parameters are equal:

\begin{align}
m_1 = m_2 = m_0; \quad c_1 = c_2 = c_0; \quad k_1 = k_2 = k_0; \quad R_1 = R_2 = R_0; \quad L_1 = L_2 = L_0; \quad \alpha_1 = \alpha_2 = \alpha_0;
\end{align}

Hence, the compressor comprises two identical SDOF systems forced by two identical drives. A parallel connection of two identical SDOF systems is effectively an SDOF system governed by equation:

\begin{align}
2m_0 \ddot{x}_0 + 2c_0 \dot{x}_0 + 2k_0 x_0 + 2A_p p_1 = 2\alpha_0 i_0 + 2A_p p_2
&\quad \text{(3.18)}
\end{align}

Variable $x_0$ in the above equation is a new coordinate of motion.

The opposed motors can then be described by a single equation in the form:

\begin{align}
2L_0 \dot{i}_0 + 2R_0 i_0 + 2\alpha_0 \dot{x}_0 &= u_0 \\
&\quad \text{(3.19)}
\end{align}

The pressure terms in the equation (3.12) can now be approximated, as discussed earlier, by an equivalent linear spring:

\begin{align}
k_g x_0 = 2A_p (p_1 - p_2) = 2A_p p
&\quad \text{(3.20)}
\end{align}

where: $k_g$ – spring rate constant of the equivalent linear spring, N/m; $p$ – differential pressure, Pa.
Equation (3.18) is thus reduced to:

\[ 2m_0 \ddot{x}_o + 2c_0 \dot{x}_0 + 2k_0 x_0 + k_g x_0 = 2\alpha_o \dot{x}_0 \]

Equation (3.21)

The simple description of the pressures given here is only valid if the \( k_g \) is independent of \( x_0 \) and \( \dot{x}_0 \), which is not quite true but can be a reasonable approximation as shown below.

Laplace transform of equations (3.21) and (3.19) results in:

\[ (2m_o s^2 + 2c_o s + 2k_0 + k_g)X_o = 2\alpha_o I_o \]

Equation (3.22)

and

\[ (2L_o s + 2R_o)I_o + 2\alpha_o s X_o = U_o \]

Equation (3.23)

After formal substitution \( s \to j\omega \) the displacement can be expressed in complex form as:

\[ X_o(j\omega) = \frac{2\alpha_o I_o(j\omega)}{(-2m_o\omega^2 + 2c_o j\omega + 2k_0 + k_g)} \]

Equation (3.24)

Equation (3.24) in complex form is:

\[ (2L_o j\omega + 2R_o)I_o(j\omega) + 2\alpha_o j\omega X_o(j\omega) = U_o(j\omega) \]

Equation (3.25)

Substitution of (3.24) into (3.25) yields the transfer function expressed in complex form as a relation of current to voltage:

\[ \frac{I_o(j\omega)}{U_o(j\omega)} = \frac{-2m_o\omega^2 + 2c_o j\omega + 2k_0 + k_g}{(2L_o j\omega + 2R_o)(-2m_o\omega^2 + 2c_o j\omega + 2k_0 + k_g) + 4\alpha_o^2 j\omega} \]

Equation (3.26)

The modulus of above transfer function is defined as:

\[ |H_{11\nu}(j\omega)| = \left| \frac{I_o(j\omega)}{U_o(j\omega)} \right| = \frac{-2m_o\omega^2 + 2c_o j\omega + 2k_0 + k_g}{2(L_o j\omega + R_o)(-2m_o\omega^2 + 2c_o j\omega + 2k_0 + k_g) + 4\alpha_o^2 j\omega} \]

Equation (3.27)

Values of the gas spring rate \( k_g \) and damping ratio \( c_0 \) can be found by curve fitting of a measured \( |H_{11\nu}(j\omega)| \) into its analytical solution (3.27). Generally, the gas spring of the compressor is modestly stiff compared to the suspension springs, and thus the gas spring contributes the majority of the total stiffness [Ross et al. 1994]. Therefore, the value of the mechanical spring (flexure) can be taken as measured earlier on the compressor with detached end caps. Reciprocating masses also remained unchanged.
The measured and LS-fitted transfer functions described by equation (3.27) are plotted in Figure 3.12 for the cold finger temperature of 120K and the heat load of 10W.

![Figure 3.12 Current/Voltage transfer function](image)

As a result, the values of the gas spring rate $k_g$ and damping ratio $c_0$ have been found to be: $k_g = 88533.00\text{N/m}$ and $c_0 = 10.39$. It should be noted that the value of $c_0$ differs from that obtained earlier from the test on the compressor with detached end caps because in that test the piston-cylinder pairs were lightly oil lubricated and the pressure was equal to the ambient. The value of 10.39 for the $c_0$ is used further for the modelling.

Values of the gas spring rate and damping ratio have thus been obtained for series connection of the motors and parallel connection of two identical SDOF systems. To model each of the opposed motors with its piston-cylinder pair individually, the values of $k_{gn}$ for either system is calculated as: $k_{gn} = k_g / 2$.

The values for the void volumes above and below the piston can now be estimated by curve fitting the theoretical function $A_p(p_1-p_2)$, with pressure terms $p_1$ and $p_2$ defined...
by equations (3.12), to the linear function $k_{gn}x_n$. Results of curve fitting are presented in Figure 3.13.

![Graph showing approximation of gas forces](image)

**Figure 3.13 Approximation of gas forces**

The gas springs are thus defined in terms of the geometry of the compressor according to equations (3.12).

Parameters that have been determined from the above tests are combined in Table 3-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance $R_{sn}$, Ohm</td>
<td>0.42</td>
</tr>
<tr>
<td>Inductance $L_{sn}$, H</td>
<td>0.57 $\times$ 10$^{-2}$</td>
</tr>
<tr>
<td>Motor constant $\alpha_{sn}$, N/A</td>
<td>8.00</td>
</tr>
<tr>
<td>Reciprocating mass $m_{sn}$, kg</td>
<td>0.40</td>
</tr>
<tr>
<td>Mechanical spring rate $k_{0sn}$, N/m</td>
<td>3.36 $\times$ 10$^{3}$</td>
</tr>
<tr>
<td>Damping coefficient $c_{0sn}$, Ns/m</td>
<td>10.39</td>
</tr>
<tr>
<td>Void volume $V_{01sn}$, m$^3$</td>
<td>5.78 $\times$ 10$^{-5}$</td>
</tr>
<tr>
<td>Void volume $V_{02sn}$, m$^3$</td>
<td>2.50 $\times$ 10$^{-4}$</td>
</tr>
</tbody>
</table>
To summarise, the complete equivalent model of the cooler is shown as block-diagram in Figure 3.14.

It is shown in the Figure 3.14 that the input to the system is the control voltages $u_1$ and $u_2$ and the output is the capacity $Q$. The electrical motors provide the driving force $f_d$ to the compressor pistons. The influence of the compressor pistons to the motors is in the form of the back emf $e$ that is calculated as a product of the motor constant and piston velocity. The pressures above and below the pistons, $p_1$ and $p_2$, correspondingly, are defined by the displacements $x_1$ and $x_2$ of the compressor pistons. While these pressures depend on the displacements of both pistons, the pistons, having the common chamber, influence each other via these pressures. This is indicated in the figure by the arrows denoted by “$p_1, p_2$”. The capacity $Q$ is calculated as a product of the pressure $p_1$ and the sum of the piston velocities $v_1$ and $v_2$.

3.4 Conclusions

This chapter first analysed the link between thermodynamic performance and the dynamics of the linear twin-piston Stirling cooler. Then, an equivalent mathematical model of the cooler was established, thus presenting a tool for cooler simulation. The
gas pressure forces in the reciprocating compressor were modelled as an action of the equivalent gas springs. This approach was discussed in detail and appropriate examples were given as evidence of the correctness of the assumptions made. It was demonstrated, that parameters of the gas springs can be calculated from the measured transfer function of current through the coil and applied voltage, in the absence of any additional means of measuring the displacement and pressure. This is an important result and can be exploited as a method of the on-line measuring of compressor parameters.

Another important conclusion that can be drawn from the obtained results is the correctness of the approach to the modelling of the compressor motors. Recall that it was assumed that the force-displacement characteristic of the motor is linear within the certain range of the piston displacement, namely a half of the length of the permanent magnet gap. This assumption holds true as was proved through the experiment and allowed for the modelling of the motor as a linear electrodynamic system. The latter is particularly important for the implementation of the quasi-sensorless motion measurement and will be exploited later. For currents up to 8A, which is within the nominal range of the motor, the motor constant value is within the 1% with respect to the average value, provided the piston position is within the range -4 to +3 mm.

Complete system identification of a specific cooler was performed based on the mathematical model. Values of compressor parameters for typical operating conditions have been estimated. This allows to proceed further with the development of control system.
CHAPTER FOUR

TEMPERATURE CONTROL

This chapter considers the temperature control of the cooler. The control system for the closed-loop control of the linear, twin-piston Stirling cooler is modelled using Simulink. Detailed description of the experimental rig and the control hardware and software is given. Details of operation of the LabVIEW controller are set out. Performance of the control system is estimated through experiment. Results of experiments are presented for typical operating conditions. Conclusions on the issues of temperature control of the linear twin-piston cooler are drawn.

4.1 Modelling of the temperature control system

A modelling of the cooler operation is performed based on the mathematical model developed in the preceding chapter. Recall that the cooler operates by lifting the required amount of heat at a given temperature to produce the specified cooling. As discussed earlier, for a normal closed-loop steady-state operation, cooling capacity $Q_R$ defined by equation (3.1) should take a constant value. The required amount of work $W$, as given by equation (3.2), supplied to the working gas in the compression space also takes a constant value. It has been concluded, that for the simulation of cooler operation, it would be adequate to maintain the product $W = \dot{P}\dot{V}$ (capacity, hereafter) constant rather than maintaining the constant temperature of the cold finger [Gaunekar et al. 1994]. The temperature of the cold finger in a cooler is typically controlled by modulating the compressor piston(s) stroke [van der Walt and Unger 1992], [Keith 1987], [Wu 1996]. In the cooler of the type studied here to maintain given cooling capacity it is necessary to maintain the volumetric capacity of the compressor and the variable component of the pressure just as well [van der Walt and Unger 1992]. Here, due to the specific design of the cooler compressor, the rms value of the variable component of the pressure will depend on the pistons' velocity.
Therefore, the product $\hat{P}\hat{V}$ is used for the modelling of the temperature control system.

Operation of the existing control system can be demonstrated using a model of the twin-piston compressor with motors connected in series: the current Ricor state-of-the-art design successfully used today. In this case, a single ac voltage is applied to the motors and currents through the motors' coils have the same magnitude and phase (common current flows through each coil). The control system maintains required temperature of the cold finger by modulating the compressor piston stroke. Temperature of the cold finger is measured with a semiconductor temperature sensor (diode) and the electrical output of the sensor is fed back to the controller. The controller generates supply voltage to the compressor motors with magnitude necessary to provide cyclic pressure and velocity required to achieve given cooling.

Model of the existing control system is illustrated in Figure 4.1.

In Figure 4.1, the Compressor is driven with an ac current provided by the Generator. The frequency and phase of the applied voltage $u$ are known and its magnitude is defined by the gain $k$, the value of which is controlled by the PID regulator, $PID$. The actual value of the capacity $\hat{P}\hat{V}$ is computed based on the rms values of the pressure, $p$, and the sum of the pistons' velocities, $v_{\text{sum}}$. Then, the signal of the $\hat{P}\hat{V}$ inputs the summation block where the value of the error, $e$, is
calculated based on the actual, $\hat{p}V$, and set, $[\hat{p}V]_{ref}$, values of the capacity. Thus, the control loop is closed and the constant capacity, $[\hat{p}V]_{ref}$, is maintained by controlling the magnitude of the ac supply voltage, $u$, for the compressor.

This type of control has a drawback, which appears in that even a small variation in the opposite compressors' parameters, such as damping coefficients or stiffness of the flexures, gives rise to the unbalanced force. For the demonstration of this effect a model of the compressor and control system was created using Simulink™. A schematic diagram of the model is shown in Figure 4.2. A difference of 10% in the natural frequencies of the opposite compressors was intentionally introduced to find the value of the resulting unbalanced force and to investigate the dynamics of the system. Note that the natural frequency of the compressor, in this case, is defined as the natural frequency of piston-flexure assembly alone excluding any influence of the gas spring. Parameters of the gas springs as well as the electrical parameters of the motors were assumed to be identical and time-invariant for both compressors.

As shown in Figure 4.2, the Twin-piston Compressor subsystem represents both compressors. Input to the subsystem is the ac supply voltage provided by the "Source". The magnitude of this voltage is controlled by a PID controller, $PID$, whose operation is governed by the equation expressed in s-domain:

$$u(s) = K_p e(s) + K_i \frac{1}{s} e(s) + K_D \frac{s}{\tau_D s + 1} e(s)$$

(4.1)

where: $e(s)$ - error;

$u(s)$ - controller output;

$K_p$ - proportional coefficient;

$K_i$ - integral coefficient;

$K_D$ - derivative coefficient;

$\tau_D$ - time delay, s.
The actual value of the capacity, \( \dot{\rho} \dot{V} \), is computed based on the rms values of the pressure and pistons' velocity provided by the RMS Pressure and RMS Vel subsystems. The rms values are calculated based on the following formula for the running rms of a signal:

\[
x_{\text{rms}}(t; \tau) = \sqrt{\frac{1}{\tau} \int_{t-\tau}^{t} x^2(t) \, dt}
\]  

(4.2)

where: 

- \( x(t) \) - instant value of the signal;
- \( x_{\text{rms}}(t; \tau) \) - running rms of the signal;
- \( \tau \) - integration (averaging) time, s.
The required constant value of the capacity, \([\hat{P}V]_{\text{ref}}\), is specified by the "PV Ref" constant. The input electrical power is calculated by the "Wattmeter" subsystem on the base of the values of the supply voltage and current through the motors' coils. The equation for the calculation of the running value of power, \(N(t;\tau)\), is:

\[
N(t;\tau) = \frac{1}{\tau} \int_{t-\tau}^{t} u(t)i(t)\,dt
\]  

(4.3)

where: 
- \(u(t)\) - instant value of voltage;
- \(i(t)\) - instant value of current.
- \(\tau\) - integration (averaging) time, s.

### 4.1.1 Results of modelling of the temperature control system

Figures 4.3 a)-e) show time histories of the signals obtained from the simulation of the cooler operation for a single value of capacity, \(PV\).

The time history of the capacity is given in Figure 4.3 a). It is shown that when started from rest, the system reaches a steady state within 5 seconds. Of course in practice it would take much longer to reach a steady state because of the thermal inertia of the heat exchangers.

Figure 4.3 b) shows the time history of the electrical power. The value of power at steady state is close to 55.5 W.

Figure 4.3 c) and d) illustrate the typical shapes of the waveforms of the pistons' displacements and current through the coils, respectively. It can be seen that both pistons reciprocate within allowed limits and are slightly shifted outwards. The shift is due to non-linearity introduced by the gas springs.

Typical waveform of pressure is shown in Figure 4.3 e) with peak values being somewhat above 19.4·10^{-5} Pa.

Magnitude spectrum of the unbalanced force is shown in Figure 4.3 f). Maximum of 4.31 N for the force is obviously at the fundamental frequency. The first harmonic is as high as 0.16 N.

The above simulation results have demonstrated all the behaviour trends intrinsic to the operation of a real cooler as will be demonstrated further.
Figure 4.3 Results of modelling of the temperature control system (Series connection of the motors)
4.2 Experiments on attainable performance of the temperature control system

To evaluate the performance of an existing control system, the associated cooler drive electronics was replaced by the specially developed measurement and control system. This system was then used for all subsequent experiments. Two experimental rigs were built: one to do preliminary tests at compressor level and the other to do full-scale tests at cooler level.

A simplified rig built for the preliminary tests conducted at Loughborough University included a twin-piston compressor mounted on a lightly damped suspension with a natural frequency of about 5 Hz. An overall view of the rig is shown in Figure 4.4.

![Figure 4.4 Simplified rig used for debugging](image)

The compressor, a part of the Ricor K535 cooler, was filled with ambient air and the windings of the opposite motors were electrically isolated. Compressor end caps were detached so that the pick-up points for measuring the pistons' motion were readily accessible. The compressor-generated unbalanced force was measured with the Bruel&Kjaer 8200 force transducer rigidly mounted on the compressor housing.
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**Temperature Control**

The transducer was magnetically coupled at the opposite side to a massive block placed on a heavily damped mount. The test rig is not detailed hereafter, though it was extensively used for debugging purposes at the stage of control system design and later as a test-bed for motion sensors. The following description is given for the test rig built at Ricor and used for the full-scale tests at cryocooler level.

### 4.2.1 Description of experimental rig

The arrangement of the rig including the measurement and control equipment is schematically illustrated in Figure 4.5. The core is the twin-piston Ricor K535 cooler with added features provided for the balancing of the displacer. The cooler is mounted on the Ricor’s dynamometer facility comprising a 3-axis Kistler 8200 dynamometer and a vibration-isolated seismic mass. The seismic mass has a lightly damped natural frequency of about 5Hz and is used to isolate the dynamometer facility from the environmental noise. During operation the forces generated about the cooler’s $x$ and $y$ axes are available simultaneously for real-time quantitative analysis. Electrical output of the dynamometer past the Kistler 5010A charge amplifier is supplied to the Data Physics Ace analyser card that is connected to the PCMCIA port of a laptop computer. The Ace analyser is set up for measuring the vibration export (self-induced force) in the directions along the pistons’ and displacer axes.
The control system is based on the National Instruments (NI) Real-Time PCI 7030/6040E board featuring a dedicated processor and running under the LabVIEW RT in Windows environment. The board is a PCI plug-in card that consists of two main parts: processor board and daughter board. While the dedicated processor on the processor board performs all the signal processing and data communication tasks, the daughter board is solely used for data acquisition and generation. Measured signals are supplied to the daughter board through the NI board connector that is also used to output generated signals to the equipment. Once a control application has
been developed on the Host PC using the LabVIEW, it is then compiled and downloaded onto the processor board memory where it is run with a real-time performance. Data communication between the processor board and the daughter board is via the shared memory (used here as being preferable to a PCI RT device) or via the TCP/IP protocol.

The temperature of the cold finger is measured with a semiconductor thermocouple attached to the top of the cold finger. The signal of a dc voltage proportional to the measured temperature is amplified and then simultaneously supplied to the RT controller, closing the temperature control loop, and to the DASYLab monitor based on an IOtech DaqBook measurement unit. The latter is a stand-alone DAQ device equipped with a DSP processor, memory and I/O port terminals and running under the DASYLab in Windows environment. Communication with the laptop computer used as a control terminal is through a standard parallel port.

The control signal generated by the RT controller to compensate for the temperature of the cold finger is amplified by a dual-channel power amplifier and then supplied as an ac voltage to the cooler compressor. The power amplifier has been specially designed by Ricor's R&D team for the experimentation and includes two off-the-shelf 50A20 servo amplifiers and two FC10010 filters both manufactured by the AMC [Advanced Motion Controls] and each driven by an independent dc power supply. The amplifiers are capable of handling both the dc and ac signals with a switching frequency as high as 20 kHz. The filters at the output cascades of the amplifiers are used to remove from a delivered signal the quantisation noise generated by switching the power transistors. An arrangement of the amplifier is shown in Figure 4.6.
Figure 4.6 Power amplifier

Figure 4.7 Arrangement of the measurement and control equipment
Figure 4.8 Cooler prepared for experiments

Figure 4.9 Cooler mounted on dynamometer
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The supply voltage for the compressor is measured across the output terminals of the power amplifier. The current through the motors' coils is measured with the built-in sensors of the servos. Signals proportional to the voltage and current are simultaneously passed to the DASYLab monitor and the RT controller.

4.2.2 Description of control software

The control algorithm to mimic the functioning of an existing control system was implemented with the LabVIEW RT software. This is a graphical programming environment that is used widely in R&D and by mainstream companies. Examples of using LabVIEW in applications tailored to control systems of the cryogenic coolers can be found in [Koh et al. 2002] and [Leybold 2001]. Being a powerful yet convenient tool, the LabVIEW, in conjunction with the NI PCI 7030/6040E real-time controller, has enabled rapid prototyping of control applications developed in the present research.

A Virtual Instrument (VI) developed with LabVIEW comprises a Front Panel and Block Diagram. The Front Panel has indicators and controls for the values of the parameters involved in operation so any of these parameters can be easily accessed for monitoring or control. The Block Diagram is essentially a scheme that graphically represents the control algorithm in the form of blocks interconnected by dataflow lines.

An example of the VI developed to perform temperature control over the twin-piston K535 cooler is given in Figure 4.10 and Figure 4.11. Figure 4.10 shows the Front Panel of the VI and its main block diagram is shown in Figure 4.11.

Available features are listed below:
- Control over the required temperature of the cold finger

This is by definition the main feature that provides for temperature control crucial to the normal operation of a cooler. The target temperature may be set manually to accommodate different regimes of cooler operation where different temperatures are required and different heat loads are involved. Parameters of the temperature PID controller and the input filters can be adjusted from the Front Panel.
Figure 4.10 LabVIEW temperature controller (front panel of the VI)
• Control over cooler upon cooldown

This feature is provided to prevent overstroking of the pistons' and displacer at the start of cooler operation. The overstroking typically happens as a cooler response to an increased load during cooldown. If a target temperature is not set as a constant, but ramped to the desired level over time according to a predefined schedule, the overstroking can be avoided. This approach is utilised in the control algorithm.

• Indication of measured and controlled parameters

This is provided to monitor functioning of the cooler and the control application itself. Among the indicators are the graphs showing time histories of the actual temperature, PID outputs and the signals input to the board. An indicator of the state of the cooldown control and an indicator of the overload of the A/D converter are also available.

• Application control

The control and indicators related to the functioning of the VI itself are available on the Front Panel. These are the controls of A/D and D/A converters, control of sampling rates and size of data buffers, and the indicators of A/D and D/A throughput.

The VI generates driving voltage for the motors according to the measured temperature of the cold finger. The VI simultaneously generates and acquires data on the same board (PCI 7030/6040E RT board). This is a hardware timed signal generation/acquisition, meaning that a hardware clock is used to control the acquisition and update rate for fast and accurate timing. While the output regenerates a buffer of data, the input is a circular buffered acquisition. This means that a software buffer is used between the daughter (DAQ) board and LabVIEW. While data is being transferred from the DAQ board into one part of the input buffer, LabVIEW is reading data from another part of that buffer. For this reason the acquisition parameters were set such that LabVIEW reads data out of the buffer fast enough to keep up with the rate at which the DAQ board was writing new data into the buffer. If not, unread data could have been overwritten and an error could have occurred.

From the experience gained thus far, scan and update rates can be set as high as 2kS/s, provided the VI keeps up with data acquisition and generation. A nonzero
value of scan backlog indicates that the VI does not keep up with the data acquisition (A/D control folder in Hardware Control). From the experiments, quality of control would degrade with higher rates due to known software and hardware limitations. Due to the fact that the type of board has separate clocks dedicated to analogue input and analogue output, the acquisition and generation can occur at different rates. The synchronization of A/D and D/A converters as well as the synchronization of the starting of the acquisition and generation is implemented at software level.

The SubVIs, which comprise the above VI, are not detailed for the sake of simplicity. Detailed description of each of the SubVIs can be found in [Veprik and Dubrovsky 2003]. Results of the tests on temperature control are presented below.

4.2.3 Results of experiments on temperature control

The results of tests on the temperature control are presented in Figure 4.12 through Figure 4.14 a) -l). Figure 4.12 shows the time history of control when the cooler is set to lift 13.8W of heat at the cold finger temperature of 120K. This was the highest heat load applied during tests and is the upper limit of a nominal range of heat loads for this cooler. It is shown that, when starting from rest, the cooler reaches a steady state within 30 min with an overshoot in the controlled temperature being at about 12 min. This time history of control is typical for this cooler.

Figure 4.13 illustrates the operation of the cooler for the cold finger temperature of 120K and the different heat loads. The heat load is varied in steps within the range 0 to 13.8W. It can be seen from the figure that the temperature has settled down within under 2 min each time the heat load takes a new value, regardless of whether the heat load is increased or lowered. Thus, the control system demonstrates stable operation for the full range of heat loads.

Figure 4.14 a) through Figure 4.14 l) show the spectrum of unbalanced forces \( R_1 \) and \( R_2 \) along the displacer and pistons' axes, respectively, for different values of heat load. The spectrum of the forces \( R_1 \) for the displacer is given for reference. Comparing fundamental harmonics, it can be seen, that the unbalanced force acting
along the pistons’ axes increases with an increase in the heat load. The force acting along the displacer axis otherwise decreases with an increase in the heat load and variations in the magnitude of force are not strong. The highest value of the fundamental harmonic of the unbalanced force along the pistons’ axes is 2.07N at a heat load of 12W, and the lowest value is 0.62N in the absence of any heat load. Magnitudes of higher harmonics are scaled down proportional to the initial magnitudes with a decrease in the heat load, but the fundamental harmonic dominates at all levels of heat load. Magnitudes of higher harmonics of the force along the displacer axis decrease with a decrease in the heat load, opposite to the behaviour of the fundamental harmonic. The level of the unbalanced force along the pistons’ axes exceeds the allowed limit of 0.2N, as specified in [Johnson et al. 1992] and [Ross et al. 1991].

Figure 4.15 and Figure 4.16 show the magnitude and phase respectively of the transfer function, the transmissibility, obtained as a relation of the unbalanced force measured on compressor housing in the direction of pistons’ axes to the force measured by the dynamometer in the same direction. Values of force measured by the dynamometer may therefore be corrected using magnitude and phase values of transmissibility.
Figure 4.12 Temperature control upon cooldown

Figure 4.13 Temperature control at different heat loads
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a) Force $R_1$ (displacer), 12W heat load

b) Force $R_2$ (pistons), 12W heat load

c) Force $R_1$ (displacer), 10W heat load

d) Force $R_2$ (pistons), 10W heat load

e) Force $R_1$ (displacer), 8W heat load

f) Force $R_2$ (pistons), 8W heat load
Figure 4.14 Spectrums of forces acting along the displacer and compressor pistons
Figure 4.15 Magnitude of transmissibility

Figure 4.16 Phase of transmissibility
4.3 Conclusions

The temperature control system for the linear twin-piston Stirling cooler has been modelled using Simulink. The modelling was performed for the cooler with compressor motors connected in series. Typical values for parameters involved in cooler operation have been obtained, including parameters of motion and forces acting along the pistons' axes. Detailed descriptions of the experimental rig and control hardware and software have been given. Details of the operation of the developed LabVIEW controller were explained. A practical investigation of the attainable performance of the temperature control system has been made for compressor motors connected in series. The correctness of the approach to the modelling of the temperature control system performed in the previous chapter has been proved. Namely, it was shown that for the modelling purposes capacity control can be well used instead of the temperature control. Recall that this assumption allowed for considerable simplification of the cooler model and much eased the simulation of the cooler operation. The displacer of the cooler in current configuration was additionally equipped with a passive balancer. This allowed for avoiding unnecessary crosstalk in force measurement and made it easy to better analyse the force export from the cooler. The experiments have demonstrated stable operation of the cooler under varying operating conditions. The experiments were performed for a fixed cold finger temperature and different heat loads. This was closest to the typical operating conditions in which the Ricor K535 cooler must normally operate. The performance of the control system was found much better as compared with the performance of the existing industrial controller supplied with the cooler. In particular, operation of the controller during the cooldown is superior to the operation of the existing controller. Also, there was provided a greater flexibility of the control algorithm to accommodate a wider range of operating conditions. These results are believed to be quite useful for improvement of the existing controller. Spectrums of unbalanced force magnitudes were measured for both the displacer and compressor pistons. It can be concluded that the level of unbalanced force along the pistons' axes exceeds the allowed limit and, therefore, the compressor needs balancing by controlling the pistons' motion. The level of the self-induced forces in the direction of the displacer was controlled by the additional passive balancer and it was shown that its performance is not affected by the temperature controller. The issue of compressor balancing will be discussed in the next chapter.
This chapter discusses methods and means of vibration reduction in a twin-piston linear cooler. Principal types of control systems suitable for active vibration control of the linear twin-piston compressor of cooler are described. An extensive overview is given of applicable sensors and techniques for vibration detection in the linear compressor. Choice of the type of control system and the method for vibration measurement is explained.

5.1 Methods of vibration control

5.1.1 Passive vibration isolation

The level of the self-induced forces is governed entirely by the unbalanced forces, which are associated with the accelerated and unbalanced motion of the moving parts of the cooler [Riabzev et al. 2001], [Polman et al. 1978]. The effective and evident method of controlling these forces is reducing the mass of the moving parts. However, this approach has particular design limitations.

Further approach is based on passive vibration isolation [Veprik et al. 2000], [Polman et al. 1978]. This is probably the most reliable and cost-effective method of reducing the force export from the linear cooler by a simple placing of a compliant viscoelastic member supporting the cooler from the base of the host instrument. The typical vibration isolation ratio achieved by a passive vibration isolator in a 1-g environment is in excess of 10. Here, tracked vehicles, marine and ground applications, for example, can be referred to as 1-g environment. The vibration isolation ratio is introduced as a measure of the vibration isolation effectiveness. The restrictions of this method are associated with the possibility of excessive motion of the vibration isolator under harsh environmental vibration and shock. In addition, the
effectiveness of thermal isolation of the cooler could be negatively affected by introducing the vibration isolator that necessitates the development of additional thermal interfaces.

It is useful to discuss here in brief the available methods of passive vibration isolation towards their application in electro-optical equipment employing linear Stirling coolers. As discussed earlier, self-induced forces in the cooler may cause harmful relative deflection of the IR sensor to the rest of optical system. This is in particular true for optical systems where all their elements cannot be mounted on a single rigid structure. The excessive level of the jitter, as compared with spatial resolution of the particular IR sensor, may cause dramatic degradation in quality of IR image.

Vibration isolation, being the simplest, most widespread and well-studied method of vibration protection [Sloan 1985], [Snowdon 1968], [Harris and Crede 1976], involves application of resilient mounting that provides for the desired attenuation. It is known, that the best isolation in the given high frequency span may be achieved when the natural frequency and loss factor of the vibration isolator are low [Veprik et al. 2000].

As the equipment containing such a low frequency and lightly dumped vibration isolator undergoes vibrations due to the unbalanced forces, the problem of excessive deflections becomes a serious concern. Moreover, in an airborne vehicle, for example, the equipment typically encounters wideband random excitation (e.g. flight through turbulent flow) and high g-loads at the vehicle's take off, climb, high-speed turn, speedup, etc [Veprik et al. 2000].

As a result, sufficient clearance must be allowed around the equipment and also the thermal and electrical interfaces need to be of special design. This approach complicates the entire design and makes it both unreliable and cost-ineffective. The demand on the compactness of the optical system containing a cooler, along with the simple design of the thermal and electrical interfaces impose particular limitations on the allowed clearances of the components of the cryogenic cooler.
In [Babitsky and Veprik 1998], for example, these objectives are achieved by mounting the linear compressor on the lightly damped and low frequency vibration isolators that provide for the required attenuation of the self-induced force. The problem of excessive deflections was successfully solved by means of the visco-elastic dumpers that were installed with minimal possible clearance and designed to perform as an optimal shock absorber. According to the authors, this solution is suitable for 1-g environments only. Despite the fact that high g-loads do not overload an application dynamically, the excessive quasi-static deflections may cause a lockup in the system of vibration protection. This results in the complete degradation of the vibration isolation. This may become an important issue because the action of such high-g loads may be long-term and the system of vibration protection must operate equally efficiently in such circumstances.

It was stated in [Veprik et al. 2000], an increase in the natural frequency and loss factor of isolator allows for a better control of excessive deflections but negates the efficiency of the vibration isolation. As a result, low deflections are achieved at an expense of decrease in vibration attenuation at the driving frequency. Hence the principle of vibration isolation becomes inadequate.

By using the combination of passive vibration isolation and tuned dynamic absorber, the vibration isolation ratio might be increased up to a factor in excess of 100 in a 1-g environment [Veprik et al. 2000]. The tuned dynamic absorber in combination with the stiff and heavily damped vibration isolator can be used, in this case, for the simultaneous dynamic suppression of the self-induced forces and the attenuation of the base-induced random vibration. It is important to note that the use of a resilient mounting is obligatory for the dynamic decoupling from the base and for the normal operation of the tuned dynamic absorber. Hence, there still exists the issue of the compressor parts deflections resulting in image jitter, and the problem of thermal and electrical interfacing. Thus, this combination of vibration isolation and tuned dynamic absorber does not provide for the required efficiency. What is more, in the case of the linear cooler of an integral type, both the reciprocating piston and displacer contribute to the vibration export. Therefore, this method of vibration reduction is further complicated by the necessity to control vibration in the directions of motion of both the displacer and pistons.
An effective realisation of a vibration isolation system for the twin-piston cooler has recently been demonstrated at Ricor [results to be published]. A tuned balancer specially designed by Ricor's R&D team was employed to reduce the unbalanced force in the direction of the displacer motion. However, the unbalanced force in the direction of the pistons' motion remained untouched. Since the mass (effective) of the reciprocating displacer is much less than that of the piston, the cooler compressor is assumed to be the dominant source of vibration. The following discussion is therefore focused on the issues of the reduction of the compressor-induced vibration only.

An active control may be a solution to vibration reduction in a twin-piston cooler design. Descriptions of different methods of active vibration control follow.

5.1.2 Active vibration control using external actuators

An active vibration control system that relies on external actuators can be used in the applications where only ultra low vibration levels are tolerated. Such applications are typically delicate spaceborne instruments [Bailey et al. 2000], [Glaser et al. 1995], in which the use of expensive vibration protection systems is economically feasible. A high value for the vibration suppression ratio can be provided across the frequency range involved but the system is sensitive to the external vibration. Such a system typically includes a set of sensors to evaluate vibration export, actuators to actively interfere with the disturbances thus reducing vibration and a sophisticated controller to generate excitation signals for the actuators.

Arrangement of the control system is schematically shown in Figure 5.1. The force sensors, often load cells, are placed between the cooler casing and the base of a host instrument to directly measure forces, $R$, imparted by the cooler to its supports [Glaser et al. 1995]. In this case, the cooler is rigidly mounted on the base. When the cooler is mounted flexibly, the lateral motion of the cooler housing can be measured with motion sensors, for example eddy-current transducers [Glaser et al. 1995], [Ross 1999] and then forces can be estimated based on information from the motion sensors. The actuators can be made in the form of piezoelectric wafers or stacks of
piezoelectric plates [Flint et al. 2000a] to provide for the high force to volume ratio and fast response, which are intrinsic characteristics of such actuators. The actuator can also be made integral comprising both the actuation and force detection means in a single unit [Shaw 2001]. As shown in Figure 5.1, the actuator directly applies the control force $f$ to the cooler housing. The controller is typically a stand-alone module utilising high performance DSP and equipped with interface electronics. As said earlier, an overall design of the vibration protection system described above is quite complex and therefore this system is only appropriate for exotic spaceborne applications.

![Figure 5.1 Vibration control of cooler using external actuator](image)

### 5.1.3 Active vibration control via balancing twin-piston compressor

The twin-piston compressor arrangement is meant to naturally cancel the unbalanced force along the pistons' axis by the counter-motion of two opposed pistons [Hiterer and Kushnir 1997], [Zhang 2001], [Keith 1987], [Tojo et al. 2001], [Yatsuzuka et al. 2001]. One of the opposed pistons, clearly, becomes a forced yet not actively controlled balancer to its mate. If two nominally identical compressors are placed back-to-back so that the pistons' axes are aligned, the vibration export could be
theoretically cancelled. In practice, the vibration export cannot be completely eliminated due to known design and manufacture limitations [Dadd et al. 2000].

In an existing configuration, two motors of the opposite compressors are connected in series and supplied from a single source, so the motors’ coils carry the same current. When energised, the motors having counter-wise wound coils generate driving forces in an opposite direction. In such a design, due to the common current and shared compression space, some sort of self-synchronisation takes place. That is why this configuration is used by Ricor and the majority of linear twin-piston cryocooler manufacturers. This method, however, does not allow for receiving deep dynamic balancing due to the obvious asymmetry of the opposite compressors (different friction factors, spring constants, clearance sealing, force of permanent magnets, etc.). Additionally, such a twin-piston compressor typically loses the balance when it undergoes a change in spatial orientation, e.g., when it is turned upside down in a particular gravitational environment.

Better performance can be gained by connecting the opposed compressors in parallel with the possibility of an accurate manual adjustment of the relative phase and amplitude of the drive signals to the compressors in order to provide for a minimum of dynamic force export [Ross et al. 1991]. The above relative phase and voltage, however, are functions of the ambient temperature, heat load and the spatial orientation of the cooler. Since the motors’ currents in such a design are independent, the compressor looses the self-synchronisation feature. Moreover, this approach is not viable for the control of high-order harmonics.

Another, more expensive solution is based on an instant monitoring and an active adjustment of the motion of the pistons through the voltages applied to each motor using the system of automatic control. A typical configuration of such a control system is schematically shown in Figure 5.2. One of the two opposed compressors is designated as a Master” and other as a Slave. The Master is supplied with an excitation signal generated by the temperature controller and necessary to perform the required cooling, and the Slave is supplied with a control signal produced by the vibration controller. The vibration controller generates the control signal based on the estimate of vibration export measured by the detector (Force Detection /
The method and means used for the detection of the vibration export are chosen depending on the requirements for a particular application and are studied in detail later in the chapter. In the case of balancing the fundamental harmonic only, the control signal for the "slave" is essentially the excitation signal for the "master" but is time-lagged and scaled in magnitude in order to match the motions of the pistons of the opposite compressors thus reducing the unbalanced force. Description of similar systems can be found in [Aubrun et al. 1992]. It was shown in [Johnson et al. 1992] and [Ross et al. 1991] that by using active balancing the unbalanced forces could be reduced to well below the commonly accepted level of 0.2N. This principle of vibration reduction is eminently suitable for use in a twin-piston compressor design and therefore has been thoroughly studied and implemented in the present research.

![Figure 5.2 Principle of balancing of the twin-piston compressor of cooler](image)

### 5.2 Control algorithm for active balancing

The choice of control algorithm for the active balancing of the cooler compressor is entirely driven by considerations of performance as referred to the attainable vibration reduction ratio and sensitivity to disturbances. This performance is also influenced by the effectiveness of the method of vibration detection and the sensors used. The method of the vibration detection and the type of the sensors used are of particular importance and are discussed later in the chapter. It should be pointed out
that the control algorithm is to be implemented with a digital control system and therefore possible restrictions on the use of computational techniques can only be related to the limitations of the performance of the control hardware used.

As specified earlier, an overall strategy involved in the balancing of two opposed compressors is to match the motion of their pistons by adjusting the magnitude and phase of the excitation signal for one of the compressors. The balancing would thus be accomplished if the opposite pistons follow identical waveforms. This is typically achieved by matching the magnitudes and phases of the pistons' motions at the distinctive frequencies, namely at the driving frequency and its higher harmonics, over the required frequency range. The balancing may be performed using different control techniques discussion of which follows.

5.2.1 Narrowband frequency control

By definition, narrowband frequency control is used for the suppression of disturbances, which are harmonic in nature. It was reported in [Anderson et al. 1997], [Johnson et al. 1992] and [Ross 1999], that this control could be implemented using a classical feedback approach. In the last two papers, the feedback narrowband frequency control was first applied to suppress higher harmonics of the unbalanced force in a single-piston compressor. In this case, the compressor was driven in a self-cancellation mode. It was shown that five higher harmonics were driven into the noise floor of the force dynamometer. However, the fundamental harmonic was not cancelled and remained at the initial level.

In the case of a twin-piston compressor, the narrow band frequency control can be used to reduce not only the higher harmonics, but the fundamental harmonic of the unbalanced force, as was noted in [Johnson et al. 1992]. The results, however, were not presented by the authors of the above paper. The successful application of this control technique was also reported in [Anderson et al. 1997]. The feedback narrowband frequency control, as tailored to the reduction of the fundamental and higher harmonics in the twin-piston compressor, is schematically illustrated in Figure 5.3.
The *Master* compressor supplied with the excitation signal $u_1$ follows a commanded waveform to perform the required cooling. The *Slave* is driven by a signal $u_2$ generated by the vibration controller. The controller is typically a bandpass filter or, in the case of the control of many harmonics, the a set of bandpass filters with central frequencies corresponding to the frequencies of the harmonics of the unbalanced force. The gains of the controller are made large at the harmonic frequencies and small elsewhere. Because of that the unbalanced force at the fundamental frequency and its harmonics will be driven to zero if the system is stable over the frequency range of interest.

Basic stability analysis for a single harmonic shows that the best stability robustness would be achieved if the loop phase changes in the range $+\pi/2$ to $-\pi/2$ [Johnson et al. 1992]. A bandpass filter could be designed to have a zero phase change at its central frequency. This, for instance, would be accomplished with a Butterworth bandpass filter of 1st or 2nd order [Mitra 2001], [Parks and Burrus 1987]. If an undamped oscillator, an SDOF of second order, is used instead of the bandpass filter, as was shown in [Johnson et al. 1992], the controller brings about a phase shift of $\pi$ at the disturbance frequency. The compressor itself also contributes to the phase shift. Therefore, whatever the type of controller chosen, an additional compensator would be required in the loop. While simple to implement, this approach has a disadvantage in that the controller parameters generally require accurate tuning to provide for a
stable operation over the full frequency range involved [Ross et al. 1991]. Nevertheless, in the case of the single-channel controller, reasonable reductions in the harmonic disturbance will be achieved with rather less accurate tuning of the controller parameters [Fuller et al. 1996]. If the operating frequency is varied for the purpose of control, the system may lose stability. To return to stable operation, the parameters of the controller, i.e. the coefficients of filters, will require tuning for a new value of frequency. It is therefore difficult to make the control adaptive in terms of frequency variation, which may, however, be necessary, as discussed later in the thesis.

5.2.2 Adaptive feedforward control with LMS algorithm

Another well-known control technique employs a least mean square (LMS) algorithm for the adaptive feedforward vibration control of the cooler. The basic theory behind the feedforward control using an LMS algorithm can be found in [Fuller et al. 1996], [Widrow and Walach 1995]. The application of the algorithm to a multiple-channel system was presented in [Cabell and Fuller 1999]. An example of the use of this algorithm for the vibration control of a cooler with an external counter-force (or reaction proof-mass) actuator was demonstrated in [Anderson et al. 2001] and [Flint et al. 2000b]. The control system demonstrated in [Flint et al. 2000a] employed a combination of two feedforward LMS controllers to actively suppress vibrations in both the expander and single-piston compressor of the cooler. In the latter case, the filtered-x LMS algorithm, a version of the LMS algorithm, was used to balance the compressor using an external counter-force actuator. Schematics of this control system is shown in Figure 5.4.
The driving or reference signal $u_1$ is filtered through an estimate of the transfer function of the Counter-force Actuator before it is used in the Control Filter update.

The signal $u_1$ is assumed to be filtered by an unknown path dynamics of the Single-piston Compressor. This is the primary path. The result of this filtering is the self-induced force $R_1$, i.e. the vibration produced by the cooler in response to a drive signal. There is also a secondary path, the Counter-force Actuator, represented by a transfer function measured between the force $R_2$ and the input to the above actuator. Even if the above transfer function is not known perfectly, a good model, the Counter-force Actuator Estimate, can be found through system identification. This model is used to filter the reference signal $u_1$, and to feed the LMS Adapter in order to adapt the weights that describe the Control Filter. The result is the minimization of the unbalanced force $R$.

The same filtered-x LMS algorithm can be used for the adaptive feedforward vibration control of a twin-piston compressor. In this case, schematics of the control system would look like the one shown in Figure 5.5. Operation is similar to that described earlier.
It can be seen that, unlike in an ordinary feedback controller, the vibration detection sensor is not used here to directly drive the vibration controller. It is rather used to monitor the performance of the controller and to adapt its response when necessary. Since the operating conditions of the cooler vary with time, as discussed earlier, the response of the controller should be continuously adjusted in order to provide for the vibration reduction by appropriate driving of the slave compressor.

The controller is typically implemented as a finite impulse response (FIR) or infinite impulse response (IIR) digital filter with tuned coefficients [Mitra 2001], [Parks and Burrus 1987], but an analogue filter may also be used. The choice in most cases is made in favour of the digital filter rather than the analogue one because it is relatively easy to change the characteristics of a digital filter by adjustment of its coefficients, but it is generally difficult to change online the response of an analogue filter [Fuller et al. 1996]. The adaptation of the coefficients of a digital filter is performed by minimisation of the cost function equal to the instantaneous mean square of the detected unbalanced force. The cost function is a quadratic function of each of the filter coefficients and is minimised using a steepest descent algorithm [Fuller et al. 1996], implemented with the LMS algorithm.
By definition, the feedforward control using the LMS algorithm does not require an exact knowledge of the system primary path dynamics, provided the system is stable or precautions are taken in advance to make the system stable, for example, by introducing a local feedback loop. The advantage of the filtered-x over the normal LMS algorithm is in that the excitation signal $u_1$ is filtered through an approximate model of the actual transfer function of the secondary path or actuator, which is, in this case, the slave compressor, as shown in Figure 5.5. As a result, the convergence of the algorithm is affected to a lesser extent by the noise in the input to the secondary path. It was shown in [Widrow and Walach 1996], that larger errors in the secondary path estimate may be tolerated.

Convergence time is generally dependent on the number of control filter weights, i.e. the order of the filter. When the system under control is non-linear, which is the case in a cooler compressor, a poor performance of the control system can be expected [Widrow and Walach 1996], unless the special non-linear adaptive control filters are used.

Another disadvantage may be related to the physical implementation of the control system itself. Typically, along with the digital filter, analogue antialiasing and output reconstructive filters are included which introduce phase shifts and associated delays [Fuller et al. 1996]. The latter may lead to instability in operation. Nevertheless, the algorithm is used widely and the theory for it is well established.

5.2.3 Adaptive feedforward control with heterodyne filtering

An adaptive feedforward controller with heterodyne filtering, as used for the control of a twin-piston compressor, first extracts the information on the harmonic components of the disturbance from the unbalanced force representative signal and then, based on this information, generates an excitation signal for driving one of the opposed compressors such as to minimise the harmonic disturbances. Examples of the controller of this type were demonstrated in [Ross 1999], [Wu 1995], [Wu 1996] and [Champion et al. 1998]. The control system demonstrated in [Glaser et al. 1995]
was developed as a combination of the narrowband feedback control discussed earlier and the adaptive feedforward control.

Schematic representation of the control system is given in Figure 5.6. The unbalanced force can be measured directly by a suitable force transducer or derived from the signals detected by appropriate motion sensors [Riabzev et al. 2001]. The harmonic disturbances, as defined earlier, are the harmonic components of the unbalanced force. These are extracted from the unbalanced force representative signal using a heterodyne filtering principle. Each harmonic is thus represented by a pair of the corresponding sine and cosine coefficients in the Fourier series expansion.

As shown in Figure 5.6, magnitudes $B_R$ and $A_R$ of the sine and cosine components, respectively, of the unbalanced force $R$ are derived by the Heterodyne Filter. Then, they are used as inputs to the Vibration Controller that regulates the magnitude and phase of the excitation signal $u_2$ for the Slave in order to match the motions of the pistons of the Master and Slave. Operation of the controller will be detailed later in the chapter and is schematically illustrated in Figure 5.7.

As shown in Figure 5.7, the reference signals are pure sine and cosine waves with frequency $\omega$ and unit magnitude. These reference signals are multiplied by the signal proportional to the force $R$, and then, each product is filtered by a low-pass filter to
obtain average values $A_R$ and $B_R$ of the products. Found thus average values are scaled by the controllers $PI$ that produce coefficients $A$ and $B$ to scale the sine and cosine components of the driving signal $u_2$ for the Slave compressor.

The operation of the heterodyne filter shown in Figure 5.6 and Figure 5.7 is, in principle, similar to the operation of the adaptive interference canceller based on the LMS algorithm and operating as a notch filter, as found in [Widrow and Steams 1985]. These two schemes are similar in the sense that they both utilise tonal reference to extract information on error from the output of the system. This form of notch filter, in this case, offers easy control of bandwidth, an infinite null, and the capability of adaptively tracking the exact frequency and phase of the interference. This approach can be extended over the control of the dual-piston compressor of cooler, and can be used for the control of single frequency interference or several frequencies by applying a notch for each frequency component.
Figure 5.8 shows a single-frequency noise canceller with two adaptive weights as found in [Widrow and Stearns 1985]. The primary input is assumed to be any kind of signal – stochastic, deterministic, periodic, transient, and so on – or any combination of signals. The reference signal is a pure cosine wave, $C \cos(\Omega_0 t + \phi)$. The primary and reference signals are sampled at intervals of $T$ seconds. The reference signal is sampled directly, giving $x_{1k}$, and after undergoing a 90° phase shift, giving $x_{2k}$.

The noise canceller of Figure 5.8 has a linear transfer function that can be obtained by analysing signal propagation from the primary input to the system output. Although the 2-weight adaptive filter in Figure 5.8 is inherently nonlinear and time-variable, when used with a sinusoidal reference the signal path from $d_k$ to $e_k$ is shown by the subsequent analysis to be linear and time-invariant [Glover 1977]. For ease of the signal propagation analysis the flow diagram of Figure 5.9, showing the operation of the LMS algorithm in detail, is constructed. The procedure for updating the weights, as indicated in the diagram, is given as:

$$
\begin{align*}
    w_{1k+1} &= w_{1k} + 2\mu e_k x_{1k} \\
    w_{2k+1} &= w_{2k} + 2\mu e_k x_{2k}
\end{align*}
$$

(5.1)

The sampled reference inputs are:
\[ x_{1k} = C \cos(k \omega_0 + \phi) \]
\[ x_{2k} = C \sin(k \omega_0 + \phi) \]  \hspace{1cm} (5.2)

From the analysis given in [Widrow and Stearns 1985], the single-frequency noise canceller is equivalent to a stable notch filter when the reference input is a pure cosine wave. The depth of the null is generally superior to that of a fixed filter because the adaptive process maintains the correct phase relationships for cancelling, even if the reference frequency changes slowly.

![Figure 5.9 Flow diagram showing signal propagation in single-frequency adaptive notch filter.](From [Widrow and Stearns 1985])

Coming back to the heterodyne filter, the advantage of the heterodyne filtering of the harmonic components is in that all the uncorrelated signals can be rejected from a complex signal containing many harmonics and additive noise. Therefore, the control system becomes insensitive to the external disturbances such as shock or disturbances at frequencies equal to the driving frequency and its harmonics. The controller controls each harmonic independently so this control may be used over a wide range of frequencies. The control algorithm is relatively insensitive to the value of the imbalance in the opposed compressors. As a result, stable convergence may be obtained at each harmonic frequency.
The computational load posed by the control algorithm on the hardware, especially when calculating the Fourier coefficients, can be a disadvantage [Wu 1995]. However, in the steady state, i.e. when the operational parameters of the cooler have reached equilibrium, the harmonic content of the vibration force is very stable in time [Champion et al. 1998]. Because of this unique characteristic, the amplitudes and phases of the commanded excitation signal for the Slave do not have to be determined quickly. Thus, the computation throughput required to execute the control procedure is significantly reduced. If the cooler, being in a steady state, undergoes an abrupt change in operating conditions, such as a sudden change in its spatial orientation which can knock the compressor out of balance, the control system recovers the balance typically within a few seconds [Glaser et al. 1995], [Wu 1996]. The above control method was used as a basis for the balancing control system developed in the present work.

5.3 Study on applicable motion sensors

A control system for balancing a twin-piston compressor typically includes sensors to detect motion of the moving parts and to estimate vibration export. The vibration export is defined in terms of the unbalanced force that can be directly measured by force transducers or can be calculated based on information from appropriate motion sensors. Motion of the moving parts can be tracked directly by measuring their displacement, velocity or acceleration or by using indirect or quasi-sensorless techniques. Discussion on applicable motion sensors and measuring techniques follows.

5.3.1 Requirements for sensors

Basic requirements for the motion sensors towards their use in the design of compressor of a cryogenic cooler can be broken down as follows:

- Linearity within a specified displacement range

Linearity of electrical output of a sensor would provide for true measurements of motion parameters and remove the need for output linearization with external conditioning electronics.
• Absence of outgassing of the materials used
Absence or low rates of outgassing of the materials used for sensor design would reduce the risk of working gas contamination and thereby provide for long-life, maintenance-free operation of the cooler.
• Absence of wear products
This requirement may be referred to the problems of working gas contamination.
• No interference exerted on the motion of moving parts
The sensor should feature contactless operation and possibly be lightweight.
• EMI-resistant
External electromagnetic fields or those generated by the compressor motor can interfere with the detected signal and deteriorate the performance of the sensor. Therefore, the sensor should be EMI-resistant and feature appropriate shielding.
• Convenient electrical interface
Conditioning and driving electronics should ideally be built-in.
• Low cost and maintenance-free operation
The price for sensor and the cost of its installation should not produce a considerable impact on the cost of the entire measuring system. The maintenance-free operation implies durable design and contactless operation.
• Wide operating temperature range
The electrical output of the sensor should be temperature-compensated.
• General requirements
These include the resolution and repeatability (the output zero stability).

The sensors and measuring methods that were initially chosen as candidates for use in the compressor of a linear cooler are listed below:
• Capacitive sensors
• LVDT sensors
• Hall-effect based sensors
• Eddy current sensors
• Quasi-sensorless method (indirect measuring of motion)
• Alternative methods (using strain gauges)
The above sensors and methods are described below.
5.3.2 Capacitive sensors

This type of position sensors is typically applied for static or dynamic non-contact measurements of linear displacement and for vibration monitoring. The operating principle of non-contact capacitive displacement measurement used by capacitive sensors is based on the ideal parallel plate capacitor [Micro-Epsilon 2003], [Future Technology 2003]. The two plate electrodes are formed by the sensor and the target opposite. If an \(ac\) current with constant frequency flows through the sensor capacitor, the amplitude of the \(ac\) voltage on the sensor is proportional to the distance between the capacitor electrodes. An adjustable compensating voltage is simultaneously generated in the conditioner electronics. After demodulation of both \(ac\) voltages, the difference is amplified and output as an analogue signal. Figure 5.10 illustrates the basic principles of capacitive position sensor operation.

![Diagram of capacitive sensor](image)

Figure 5.10 Principle of operation of capacitive sensor

The system evaluates the reactance, \(X_c\), of the capacitor which changes strictly in proportion to the distance as defined by the relationship:

\[
X_c = \text{constant} \times \frac{\text{area}}{\text{distance}}
\]  

(5.3)

Advantages of the capacitive sensor are listed below:

- High linearity over the entire operating range
- Sensor is wear and maintenance free
- High sensitivity
- Nearly temperature independent
- High zero-point stability
- The sensors exert no interference force on the target
Vibration Control

- Independent of variations in conductivity of electrically conductive target materials

Capacitive linear displacement sensors are designed for the use against all metallic and non-metallic surfaces, such as steel, aluminium, plastics, glass and ceramics. Stainless steel construction allows for using sensors in harsh environments. High accuracy and thermal stability are particularly important for airborne or spaceborne applications.

Disadvantages of the capacitive sensors are:
- Relatively short measuring distances
- Necessity of using external conditioning electronics
- Large volume of the sensors used for measuring long distances
- Narrow frequency range

Capacitive sensors must operate in conjunction with electronics to produce a precision measurement system. The electronics is used for driving the sensor and linearization of its output characteristic. This adds to the cost of the entire system. However, the linear characteristic of the measurement signal is achieved without extra electronic linearization when measuring relatively short distances against targets made of electrically conductive materials (metals). In this case, changes in the conductivity do not affect sensitivity or linearity. It should also be considered that for accurate measuring of relatively long distances to the target, a capacitive sensor should have quite a large plate area. The sensor is generally expensive. This applies certain restrictions to the use of the sensor for volume applications.

5.3.3 LVDT

Another type of linear displacement sensor is the Linear Variable Differential Transformer (LVDT). The LVDT is an electromechanical device that produces an electrical output proportional to the displacement of a separate movable core [Schaevithz 2003]. The sensor consists of a primary coil and two secondary coils symmetrically spaced on a cylindrical form as shown in Figure 5.11. A free-moving,
rod-shaped magnetic core inside the coil assembly provides a path for the magnetic flux linking the coils. When the primary coil is energized by an external $ac$ source, voltages are induced in the two secondary coils. These are connected in series opposing so the two voltages are of opposite polarity. Therefore, the net output of the transducer is the difference between these voltages, which is zero when the core is at the centre or null position as illustrated in Figure 5.12. When the core is moved from the null position, the induced voltage in the coil toward which the core is moved increases, while the induced voltage in the opposite coil decreases. This action produces a differential voltage output that varies linearly with changes in core position. The phase of this output voltage changes abruptly by $180^\circ$ as the core is moved from one side of null to the other. The core must always be fully within the coil assembly during operation of the LVDT, otherwise gross non-linearity will occur as shown in Figure 5.12.

![Figure 5.11 Schematic diagram of LVDT sensor](image)

Advantages of the LVDT are [Schaevithz 2003]:

- Frictionless measurement

Ordinarily, there is no physical contact between the movable core and the coil structure, which means that the LVDT is a frictionless device.

- Infinite mechanical life

The absence of friction and contact between the coil and core of an LVDT means that there is nothing to wear out. This gives an LVDT essentially infinite mechanical life.
Frictionless operation of the LVDT combined with the induction principle by which the sensor functions gives the LVDT truly infinite resolution.

- Null repeatability
The null position of an LVDT is inherently stable and repeatable.

- Cross-axis rejection
An LVDT is predominantly sensitive to the effects of axial core motion and relatively insensitive to radial core motion or radial vibration.

- Extreme ruggedness
The combination of materials utilised in an LVDT and the techniques used for assembling them result in an extremely rugged and durable transducer. This rugged construction permits the LVDT to continue functioning even after exposure to substantial shock loads and the high vibration levels often encountered in the industrial environment.
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- Core and coil separation

The separation of the LVDT core from the LVDT coil permits the isolation of media such as pressurised, corrosive, or caustic fluids from the coil assembly by a non-magnetic barrier interposed between the core and the inside of the coil.

- Environmental compatibility

The LVDT is one of a few transducers that can operate in a variety of hostile environments. For example, a hermetically sealed LVDT is constructed of materials such as stainless steel that can be exposed to corrosive liquids or vapours. Special designs of this sensor that are capable of functioning at cryogenic temperatures are also available on the market [Schaevithz 2003].

- Input/Output electrical isolation

The fact that the LVDT is essentially a transformer means that there is complete isolation between excitation input (primary) and output (secondaries).

In addition, the operation of an LVDT might not be affected by the magnetic field caused by the motor coil since the driving frequency of a typical LVDT is about 100 times higher than that of the motor.

The dc operated models with embedded conditioning electronics are available, however, these are not preferable for use in the compressor since they have larger physical dimensions and higher price. The ac operated sensors require the use of external conditioning electronics that generally includes a carrier oscillator, ac amplifier, demodulator, filter and a dc amplifier. The use of an external electronic module can be a disadvantage for cost sensitive applications or where available space is limited. An integral chip such as, for example, that made by AD [Analog Devices 2003], containing the conditioning electronics in a single unit can be used instead of the external module. The cost and volume in this case can be substantially reduced.

5.3.4 Hall-effect based sensors

Hall effect based sensors are typically used as cheap and reliable non-contact switching devices for automatic control systems. Particular designs of the sensors can be employed as high performance linear displacement sensors when combined
with moving magnets used as targets. For example, linear Hall sensors manufactured by Micronas [Micronas 2003] generate an analogue output voltage, which is proportional to the magnetic flux perpendicular to the Hall plate. The function of a Hall sensor is based on the physical principle of the Hall effect. This means that a voltage is generated transversely to the current flow direction in an electric conductor (the Hall voltage), if a magnetic field is applied perpendicularly to the conductor. Figure 5.13 illustrates the basic principles of the Hall effect sensor operation.

Hereafter the characteristics of the linear motion Hall effect sensor apply to Micronas products. The choice of this particular manufacturer has been made because of the unique characteristics of its sensors. In sensors made by Micronas the Hall element with its entire evaluation circuitry is integrated on a single silicon chip, which is a great advantage of this particular design.

![Figure 5.13 Principle of Hall sensor operation](image)

Major features of the Micronas Hall linear sensors are listed below:

- High precision
- Ratiometric output
- Multiple programmable magnetic characteristics of targets with non-volatile memory
- Digital signal processing on the chip
- Temperature characteristics are programmable for matching all common magnetic materials
- Programming by modulation of the supply voltage
• Wide operating temperature range
• Possibility to operate with static and dynamic magnetic fields up to 2 kHz
• Extremely robust magnetic characteristics against mechanical stress

The major characteristics like magnetic field range, sensitivity, output quiescent voltage (output voltage at zero magnetic flux), and output voltage range are programmable in a non-volatile memory [Micronas 2003]. The sensors have a ratiometric output characteristic, which means the output voltage is proportional to the magnetic flux and the supply voltage.

A typical sensor, for example of the HAL815 series, features a temperature-compensated Hall plate, an A/D converter, digital signal processing, a D/A converter, an EEPROM memory, and a serial interface for programming the EEPROM. The internal digital signal processing is a great benefit because analogue offsets, temperature shifts, and mechanical stress do not degrade the sensor accuracy. The sensor is programmable by modulating the supply voltage and no additional programming pin is needed. The easy programmability allows a 2-point calibration by adjusting the output voltage directly to the distance to a target. An individual adjustment of each sensor during the manufacturing process is possible. With this calibration procedure, the tolerances of the sensor, the magnet, and the mechanical positioning can be compensated for in the final assembly.

In addition, the temperature-compensation of the Hall IC can be adapted to all common magnetic materials by programming temperature coefficients of the Hall sensor sensitivity. This enables the sensor to operate over the full temperature range with high accuracy.

The major disadvantage of such sensors is their high susceptibility to external magnetic fields, such as fields produced by the motor coil. This would require additional sensor shielding which adds complexity to the design. The shielding may be not feasible or may not work effectively unless all sources of the magnetic interference are accurately predicted.
5.3.5 Eddy current sensors

Non-contact inductive displacement sensors using eddy current technology measure displacements of any electrically conductive target. The latter may have both ferromagnetic and non-ferromagnetic properties. Due to the high insensitivity to oil, dirt, dust, moisture, interference fields, etc. the eddy current principle is eminently suitable for applications in harsh industrial environments [Micro-Epsilon 2003].

The principle of operation is based on generating eddy currents in the target followed by detection of a change in the electrical parameters of the sensor. A high-frequency alternating current flows through a coil inserted in the sensor housing. The field of the electromagnetic coil induces eddy currents in the conductive target, which alters the $ac$ resistance of the coil. This change in impedance is detected by the conditioning electronics where it is converted into a linear electrical signal proportional to the distance of the target from the sensor. The principle of the eddy current sensor operation is illustrated in Figure 5.14.

![Figure 5.14 Physical principle of eddy current sensors](image)

Major advantages of eddy-current displacement sensors are:

- Wear and maintenance free
- Sensors exert no forces on the target
- Usable in severe industrial environments
- Fast response
Along with all the above advantages of the sensors, there are some serious drawbacks. External conditioning electronics is necessary for driving the sensor and compensating for the temperature changes. This adds to the cost of a measuring system. Sensors designed for measuring relatively large displacements have excessive dimensions of the head where the sensor coil is situated.

5.3.6 Quasi-sensorless measurement of motion

The method of quasi-sensorless measurement of piston motion was first introduced in [Redlich 1996] and particularly aimed at measuring motion in the linear motor of a permanent moving magnet type developed by Sunpower [Sunpower 2003]. The method relies on the known relationship between the back emf voltage generated in the linear motor and the velocity of the moving parts. More specifically, if certain conditions of motor operation are met, the generated back emf can be an accurate measure of piston velocity. The velocity can be recovered by analogue or digital calculation based on the equivalent circuit of the motor and measured values for the applied voltage and coil current. As defined earlier, the equivalent circuit of the motor can be described by the equation:

\[ Li + Ri + \alpha x = u \] (5.4)

For a motor of the type studied here, the motor constant \( \alpha \) is theoretically independent of magnet linear position within less than 0.5% provided the magnets come no closer to the end of the air gap than about half of the gap dimension, and provided the iron does not saturate. The above considerations for the \( \alpha \) have been detailed in Chapter 2. From the experimental study presented in Chapter 2, the actual value of the parameter \( \alpha \) varies by no more than 1% within the specified range of piston displacement so that the above condition is satisfied. On the assumption that the inductance of the motor coil \( L \) and resistance \( R \) are also constant, the velocity of the piston can be found from equation (5.4) rearranged for the velocity:

\[ \dot{x} = \frac{1}{\alpha} \left( u - Li - Ri \right) \] (5.5)

Further, an alternating component of piston displacement can be obtained by integration of the calculated value for velocity:
\[ \bar{x} = \int \dot{x} \, dt \]  

(5.6)

Though the constant component of piston displacement can theoretically be found by integration of velocity, the obvious limitations of practical integrators do not enable this to be done.

It should be emphasized that this method should be approached very carefully because it requires an exact knowledge of the motor parameters. The assumption of the constant values for \( R \) and \( L \) does not hold in practice because the resistance \( R \) is a function of temperature and the actual value for the self-inductance \( L \) is influenced by varying magnetic flux in the iron of the stator. Therefore, the averaged values of \( R \) and \( L \) should be taken for practical calculation. This may lead to appreciable errors between the actual and calculated motion parameters.

An obvious advantage of the above method is the complete elimination of any motion sensors from the compressor design. Requirements of long-life, wear-free operation and absence of outgassing are automatically met. Requirements of low cost are subject to available voltage and current sensors. Another clear advantage is the possibility of applying this measurement method to an existing cooler without any changes in its current design, except probably an added voltage pickup.

The above method of piston motion measurement was applied for balancing a twin-piston compressor. Results of tests on the attainable performance of the method will be given in Chapter 7 along with appropriate discussion.

5.3.7 Alternative methods

During research, an alternative low-cost method of motion detection has been proposed. Suitable strain gauges were mounted on the compressor flexure in an attempt to build a simple displacement measurement system based on the relationship between piston displacement and deflection of the flexure. If properly measured, the stresses on the surface of the flexure can be an indicator of piston displacement.
Two resistive strain gauges manufactured by Vishay were glued on the flexure arms at locations where the deflection was thought to be minimal and stresses maximal to provide for highest sensitivity. Another two strain gauges identical to the above were mounted on the compressor housing at a convenient location under the compressor end cap thus completing a full-bridge scheme that comprised four sensors: two active and two dummy strain gauges. This four-arm connection, to a first approximation, eliminated the need for temperature compensation.

Unfortunately, the first attempts to obtain a stable signal proportional to piston displacement failed due to poor signal conditioning. The situation was improved by further introduction of the strain gauge preamplifiers specially built for this purpose. However, it was revealed during the tests that the stresses in the flexure are not a linear function of flexure deflection. These findings are supported by the finite element modelling of the flexure subsequently performed using FEMLAB software. Results of the modelling were presented in [Veprik and Dubrovsky 2003]. It was also found from the tests that when the two out of four strain gauges in the full-bridge scheme are active and two remaining are dummy ones, the two adjacent strain gauges mounted on opposite arms of the flexure produce asymmetrical electrical outputs. To compensate for this asymmetry, a fully active, four-arm sensor connection would be required. This may be implemented by accurate mounting of the strain gauges in pairs on the opposite sides of flexure arms. The design thus becomes too complicated hiding the advantage of the low cost. It is arguable that better strain gauges, say of a semiconductor type, would improve the design. All this makes the use of the strain gauges for measuring piston displacement unfeasible. As a result, this seemingly attractive approach had unfortunately to be rejected.

5.4 Conclusions

Methods and means of vibration reduction in a twin-piston linear cooler have been studied in detail. It has been shown that different types of active vibration control may be successfully applied to the control of the linear twin-piston compressor of a cooler. Among concurrent methods of control, however, the adaptive feedforward control appears to be the most suitable, as it offers, in conjunction with heterodyne
filtering, the highest level of robustness against harmonic and shock disturbances and, as a result, more stable operation over a wide range of frequencies.

The control algorithm places an appreciable computational load on control hardware when calculating sine and cosine components of the unbalanced force. However, as demonstrated in [Wu 1995], the content of the spectrum of the unbalanced force is relatively stable in time for the steady state operation of a cooler, and the coefficients of the controller have not to be updated frequently reducing the computational load. Thus, relatively slow hardware would give satisfactory results. The above has been said in regard to a digital implementation of the control hardware.

The types of sensors and the methods for vibration detection have been discussed in detail. It can be concluded that the method of quasi-sensorless measurement of motion is the most suitable for use in a cooler compressor. The method meets the requirement of long operating life of a cooler by eliminating any measuring means from the cooler design and thereby reducing the risk of working gas contamination. Moreover, the method can be utilised in an existing cooler without any additional sensors except the external voltage and current pickups.

Since the quasi-sensorless method requires an exact knowledge of motor parameters, an LVDT sensor can be used for the reference and debugging purposes, as it is, in this case, the most reliable sensor for measuring linear motion.

Operation of the adaptive feedforward control system for balancing the cooler compressor will be modelled in the next chapter. The use of an LVDT sensor and the practical implementation of the quasi-sensorless measurement method will be demonstrated in Chapter 7. A direct comparison between the measurements of the piston velocity performed with the quasi-sensorless system and LVDT sensors is given in Figure 7.10 and Figure 7.11 of Chapter 7. It will be demonstrated in the Chapter 7 that the signals of the piston velocity measured by the two methods are highly correlated in terms of the magnitude and signal shape. Though the phase of the signals is not correlated, this does not affect the performance of control techniques studied in the thesis, as will be demonstrated in Chapter 7.
This chapter presents modelling of the control system for the active vibration control of a linear twin-piston compressor of a cryogenic cooler. The principles of operation of the control system for compressor balancing are described in detail. The modelling is performed using Simulink and is based on the equivalent model of the cooler developed in Chapter 3. Results of modelling are presented and appropriate conclusions are drawn.

6.1 Model of control system

Recall that the purpose of the cooler is to provide adjustable cryogenic refrigeration. Operation of the temperature control system was discussed in Chapter 4. The control method best suited for balancing the cooler compressor has been identified in the preceding chapter. Consequently, the control system of the cooler can be developed further by including in it the feature for balancing the compressor.

6.1.1 Structure of control system

Model of the control system is schematically shown in Figure 6.1. The controller in this configuration performs balancing on the fundamental frequency only, but the same principle of balancing can be extended to the control of higher harmonics, as will be shown later in this chapter.

It is shown in the Figure 6.1 that the control system inherited all the features of the temperature control system but the motors are now driven independently and supplied with different voltages. The two-channel power amplifier to which the motors are connected is assumed ideal and has the gains set equal for both channels.
When started from rest, the motors are driven by identical ac voltages with known constant frequency and phase, and with the magnitude defined by the capacity controller, PID 1. The capacity controller PID 1 regulates the magnitude of the driving voltages based on the error calculated as the difference between the reference \([pP]_{REF}\) and actual \([pP]\) values of the capacity. The driving voltages are thus derived from the reference voltages produced by Generator, as shown in Figure 6.1.

When the capacity has been settled down at a desired constant level by the capacity controller, the motors are driven by the reference voltages to maintain the capacity at the desired level.
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controller, the balance controller comes into play. The magnitude and phase of the driving voltage for the Slave are corrected in time by this controller in order to reduce the unbalanced force at the fundamental frequency. Timing of switching on the balance controller in the model is predefined by setting a timer, but in practice this can be done by observing the error in the temperature control.

6.1.2 Heterodyne filtering

The controller performs balancing and determines values of the magnitude and phase of the driving voltage for the Slave on the base of the corresponding differences in the averaged values of the sine and cosine components of the fundamental harmonics of the pistons' displacements. For this purpose, the averaged values of the above sine and cosine components are recovered from the signals of displacement using heterodyne filtering. This method is based on the use of coherent reference sine and cosine signals of unit magnitude and constant frequency. These reference signals are produced by the Generator for recovering the appropriate frequency components from the complex signal comprising many harmonics and additional noise. The process of computation of the sine and cosine components of a complex signal is defined by the equations:

\[
\hat{x}_\sin = \frac{1}{T} \int_{\tau}^{\tau+T} x(t) \sin(\omega_0 t) \, dt \\
\hat{x}_\cos = \frac{1}{T} \int_{\tau}^{\tau+T} x(t) \cos(\omega_0 t) \, dt
\]

(6.1)

where: \( x(t) \) - complex signal;
\( \sin(\omega_0 t), \cos(\omega_0 t) \) - reference sine and cosine signals of frequency \( \omega_0 \);
\( \hat{x}_\sin, \hat{x}_\cos \) - averaged sine and cosine components of the \( x(t) \);
\( \tau \) - integration (averaging) time.

The appropriate motion sensors detect the motion of the pistons of the Master and Slave compressors. Measured signals of displacement are passed to the Errors' computation module where they are multiplied by the reference sine and cosine signals, as shown in Figure 6.1. The average values are then extracted from the
obtained products by the lowpass filtering, which is, in this context, equivalent to the integration in the equations (6.1). Instant differences in the corresponding averaged sine and cosine components of displacement of the Master and Slave are calculated in the same Errors’ computation module. Calculated thus, errors $e_{\sin}$ and $e_{\cos}$ are used as inputs to the two identical controllers $PID_2$ and $PID_3$ that produce the control signals $u_{2\sin}$ and $u_{2\cos}$, rather slowly varying values, in an attempt to minimise the errors. The Generator 1 multiplies the outputs $u_{2\sin}$ and $u_{2\cos}$ of the controllers by the reference sine and cosine signals, respectively, and then mixes the above products yielding the voltage $u_2$ for the Slave. This voltage is then multiplied by the gain specified by the temperature controller $PID_1$ and supplied through the Power Amplifier to the Slave. The control loop for balancing is thus closed.

It should be pointed out that in the current configuration of the control system, the pistons’ displacements are measured with motion sensors, for example LVDTs, and this information is used for the estimation of the unbalanced, or self-induced, force. This is equivalent to the direct measurement of the unbalanced force because the latter is caused by the accelerated and unbalanced motion of the moving parts in the opposite compressors, as discussed in detail earlier. The difference in the absolute motion of the oppositely reciprocating pistons will be, in this case, a measure of the imbalance and thus the unbalanced force. The magnitude of difference in the displacements times the moving mass, times the squared circular frequency (the sum of the fundamental frequency and higher harmonics) will be proportional to the magnitude of the unbalanced force.

6.1.3 PID controllers

As said earlier, the balance control starts only after the capacity has reached the required value and settled down. The timing is implemented as a built-in function of the controllers $PID_2$ and $PID_3$. The controllers feature “bumpless” switching “on”, so when they start there is no overshoot in the controlled variable. The integrator antiwindup is also provided for smooth regulation in the case of controller output saturation.
Figure 6.2 shows a Simulink diagram that explains the principle of operation of the controllers.

The controller with unsaturated output is governed by the following equations:

\[
\begin{align*}
\text{PID output} &= \begin{cases} 
1 & t \leq t_0 \\
K_p e + K_i \left( \left( 1 - K_p e_0 - K_D \frac{de_0}{dt} \right) / K_i + \int e dt \right) + K_D \frac{de}{dt} & t > t_0
\end{cases} 
\end{align*}
\]  
(6.2)

where: 
- \( t_0 \) – time instant of balance controller start;
- \( e_0 \) – error at the time instant \( t_0 \).

The controller outputs a unit constant until the time instant specified by the value of \( t_0 \). At the time \( t_0 \), the integrator is reset and its initial condition is calculated based on the instant value of the error \( e_0 \) at the time \( t_0 \). The value of the initial condition is thus defined as:

\[
\text{IC}_u = \left( 1 - K_p e_0 - K_D \frac{de_0}{dt} \right) / K_i .
\]  
(6.3)

Therefore, the output of the controller does not undergo a step change at the next time instant after \( t_0 \), but slowly departs from the initial unit value. This enables it to avoid a possible overshoot in the controlled variable at the beginning of regulation.

If the output of the controller saturates in the course of control at the value \( \pm u_{\text{sat}} \) specified by an output limiter, the integrator is reset to a value defined by the equation:

\[
\text{IC}_{\text{sat}} = \left( u_{\text{sat}} - K_p e - K_D \frac{de}{dt} \right) / K_i
\]  
(6.4)

The above value of the initial condition \( \text{IC}_{\text{sat}} \) is calculated at each subsequent time step and available at the integrator initial condition input. The output of the controller remains equal to the \( +u_{\text{sat}} \) or \( -u_{\text{sat}} \) and the integrator remains in the reset state as long as the value of the error is such that the output of the controller saturates.

When the error, approaching the zero, arrives at a certain value such that the controller output becomes less than \( |u_{\text{sat}}| \):
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\[ \text{PID output} = K_p e + IC_{sat} + K_D \frac{de}{dt} < |u_{sat}|, \]  
(6.5)

the integrator starts integrating the error again with an initial condition specified by the equation (6.4). The controller thus returns to a normal operation, which is now governed by the equation:

\[ \text{PID output} = K_p e + K_f \left( IC_{sat} + \int edt \right) + K_D \frac{de}{dt} \]  
(6.6)

The above algorithm enables the avoidance of integrator windup when the controller output saturates.

Figure 6.2 PID controller

Combined action of the proportional, integral and derivative terms in the PID controller has the following effect on the plant error. The proportional term reduces the error, while the integral term eliminates it. The derivative term is used to reduce or completely remove the overshoot. The technique used for tuning the PID gains can be referred to as a variation of the Ziegler-Nichols method of PID tuning. First, the proportional gain was set high enough to reduce the error while keeping the system stable. Then, a small integral gain was set to further reduce the error while the proportional gain was slightly reduced to keep the total gain in required boundaries.
After the optimal combination of the proportional and integral gains was found providing the error was eliminated with sufficiently fast descent, the derivative gain was applied to decrease overshoot. When increasing the derivative gain, the proportional and integral gains were slightly reduced. After several iterations the controller was considered tuned when the error was eliminated with the minimal overshoot in control variable and fast enough, while the system remained stable.

A complete Simulink diagram of the control system model for the capacity control and balancing of the twin-piston compressor is shown in Figure 6.3.

6.2 Results of modelling

6.2.1 Unbalanced compressor

Results of modelling are shown in Figure 6.4 through Figure 6.6. Figure 6.4 shows for reference the results of the steady state capacity control without balancing. In this case, the motors are supplied with identical voltages the magnitudes of which are regulated by the capacity controller.

It is shown in Figure 6.4, a) that the capacity has settled down within less than 2.5sec.

Figure 6.4, b) shows the total electrical power consumed by both motors, which is 55.5W here, and the power consumed by each motor.

From Figure 6.4, c), the displacements of the opposite pistons virtually coincide and are within the specified limit of ±6mm. The pistons’ offsets exist, but they are not clearly pronounced.

The difference in currents is also not appreciable with magnitudes up to 8A, as seen in Figure 6.4, d).
Figure 6.4, e) shows the time history of the pressure in the compression space with peak values being as high as $19.4 \times 10^5$ Pa.

The spectrum of the unbalanced force is shown in Figure 6.4, f). The magnitude of the force at 50Hz is 6.45N and at the 100Hz 0.18N.
Figure 6.4 Unbalanced operation for the parallel connection of motors
6.2.2 Balancing the 1st harmonic

Figure 6.5 a) through Figure 6.6 f) illustrate results of balancing the compressor at the fundamental harmonic frequency of 50Hz.

It is shown in Figure 6.5 a) that the temperature control is not affected by the balancing; it should be noted that the balancing is switched “on” at time instant of 5 sec.

On the completion of balancing, the power consumed by the Master has been decreased and, otherwise, the power consumed by the Slave has been increased, as shown in Figure 6.5 b); the total power remained nearly the same as compared to unbalanced operation.

Figure 6.5 c) shows the magnitude spectrum of the residual unbalanced force, the first harmonic of which has been virtually zeroed while the higher harmonics are at the same initial level.

Figure 6.5 d) shows the time history of balancing of the first harmonic of the unbalanced force; the balancing is completed within 5 sec.

Figures 6.5 e) and f) show the pistons' displacements and pressure in the compression space, respectively, with values similar to those found in unbalanced operation.

The voltages applied to and the currents through the motors' coils are shown in Figures 6.6 a) and b), respectively.

Figure 6.6 c) shows the time history of the magnitudes of the voltages before and after balancing.

Figures 6.6 d) –f) illustrate operation of the balancing controller; phase of voltage $u_2$ for slave, magnitude of displacement error $x_e$ and phase of displacement error $x_e$ are given, respectively.
Figure 6.5 Balancing of fundamental component of the unbalanced force
Figure 6.6 Balancing of fundamental component of the unbalanced force (Control signals)
6.3 Balancing high-order harmonics

6.3.1 Structure of the control system

It was demonstrated, that the fundamental component of the unbalanced force can be effectively suppressed by active control of the compressor. Figure 6.7 a) and b) show for reference the spectrum of the unbalanced force before balancing and the residual unbalanced force after the balancing of the fundamental component, respectively.

![Spectrum of unbalanced force](image.png)

Figure 6.7 Spectrum of unbalanced force. Unbalanced operation and balancing the fundamental harmonic

Figure 6.7 b) shows, that though the fundamental harmonic is virtually zeroed, the higher harmonics remain unchanged. The second harmonic in this particular case is 0.18N, which is below the allowed level of 0.2N, but can be higher if operating conditions change. In addition, the value of 0.18N may be still too high for some applications. Therefore, the same principle, as described above for the balancing of the fundamental harmonic, can be applied for the balancing of the higher harmonics. The control system for balancing the higher harmonics differs from that for the balancing of the fundamental component in that individual identical controllers are employed for the control over each harmonic. In this case, the Master remains supplied with the excitation signal of known frequency and with magnitude regulated by the capacity controller, but the supply for the Slave is now a sum of the
superimposed voltages of harmonic frequencies with magnitudes and phases determined by the balance controller for each harmonic individually. For this purpose, the generator outputs the reference sine and cosine signals of unit magnitude at harmonic frequencies and heterodyne filtering is applied for each harmonic, as shown in Figure 6.8.

Structure of the Simulink diagram of the control system model for the capacity control and balancing of the twin-piston compressor at higher harmonics is similar to that shown in Figure 6.3.

6.3.2 Results of balancing

Results of a simulation of attainable performance for the balancing at fundamental and three higher harmonics are illustrated in Figure 6.9 a) through Figure 6.10 f).

It is shown in Figure 6.9, a) that the capacity reaches the required value within less than 2.5sec and remains undisturbed after the balancing controller starts at the time instant of 5sec. Figure 6.9, b) shows the total electrical power consumed by both motors and that consumed by each motor. It can be seen that when the balancing controller completes balancing, the Slave draws less energy as compared with the unbalanced operation, and the Master does otherwise. Figure 6.9, c), shows that the displacements of the opposite pistons are within the specified limit of ±6mm. Difference in currents in the opposite motors, as shown in Figure 6.9, d), is as in the case of balancing the 1st harmonic only, with magnitudes up to 8A. Figure 6.9, e) shows the spectrum of the force before balancing. Results of balancing at fundamental and three higher harmonics are illustrated in Figure 6.9, f) with numerical values for the magnitudes of the corresponding harmonics. Figure 6.10 a), c) and e) show the time histories of the individual control of the fundamental and two higher harmonics of the unbalanced force. It is shown that, started at the time instant of 5 sec, the balancing of all harmonics is completed within 5 sec, and at the time instant of 10 sec the magnitudes of all harmonics are virtually zeroed. Figure 6.10 b), d) and f) show the spectrums of the above harmonics. Figure 6.11 demonstrates the
control of the fundamental and three higher harmonics in time; magnitudes of the harmonics are given using a logarithmic scale.

Figure 6.8 Schematics of control system for balancing the higher harmonics
Figure 6.9 Results of modelling for balancing the higher harmonics
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Figure 6.10 Spectrums of harmonics of the unbalanced force
6.4 Conclusions

The modelling of a control system for the active vibration control of a linear twin-piston compressor of a cryogenic cooler has been demonstrated. The principle of operation and schematics of the control system for compressor balancing have been detailed. The principle of heterodyne filtering has been explained. Details of the PID controller tailored to its use in the compressor balancing have been given. Balancing of the fundamental harmonic was performed first. It has been shown that the efficiency of the capacity control system was not affected by introducing the balancing control. Then, the balancing of the fundamental and three higher harmonics was performed. Results of the modelling have demonstrated the effectiveness of the control algorithm when applied to the balancing of the fundamental harmonic alone and the fundamental plus three higher harmonics.

In the following chapter, the effectiveness of the control system will be demonstrated through tests.
CHAPTER
SEVEN

EXPERIMENTS ON BALANCING

This chapter presents the results of experiments on the attainable performance of the control system for balancing the twin-piston compressor of a linear Stirling cooler. Description of the experimental rig arrangement is given. Functioning of the controller for the temperature control and balancing is described. Balancing using the quasi-sensorless technique for the measurement of the pistons' motion is demonstrated and the balancing using the LVDT sensors is given for reference. The results of the quasi-sensorless and motion-sensor based balancing are compared and analysed.

7.1 Description of the control system

7.1.1 Quasi-sensorless measurement of motion

The control system utilizes quasi-sensorless motion measurement. The theory behind this technique as applied to a linear motor of the permanent moving magnet type was explained in Chapter 5. The technique relies on the measurement of instant values of the voltage applied to the motor and the coil current followed by a calculation of piston velocity based on the electrical equation for the linear motor, provided the motor parameters are known and the piston displacement is within certain limits, as discussed earlier. The above method has been applied for the balancing of the twin-piston compressor of the cooler. The demonstration of experiments on the balancing and the appropriate discussion follow.

7.1.2 Temperature control and balancing

The control system for the capacity control and quasi-sensorless balancing of the twin-piston compressor at a single frequency is schematically shown in Figure 7.1.
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Figure 7.1 Schematics of control system
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The Master compressor is supplied with the ac voltage of known frequency and with a magnitude regulated by the Temperature Controller, as shown in the Figure 7.1, to maintain the required cold-finger temperature at a given heat load. The Slave is supplied with an ac voltage of the same frequency, but with the phase and magnitude regulated by the balancing controller, the operation of which was detailed in the preceding chapter.

The information on motion of the compressor pistons is obtained from indirect measurements, i.e. via measurement of voltages applied to the motors and currents through the motors' coils. For this purpose, instant values of the above voltages and currents are detected by appropriate sensors and supplied to the Pistons' Velocity Calculation block, as shown in Figure 7.1, where velocities of the pistons are calculated based on the measured voltage and current and assuming the motors' parameters do not vary over time. Then, the sine and cosine fundamental harmonic components of the pistons' velocities are extracted from the calculated signals of velocity of either motor using heterodyne filtering. The differences in the corresponding sine and cosine harmonic components for the opposite motors are subtracted by the Errors' Computation block and used further as errors by the controllers PID 1 and PID 2. These controllers regulate the phase and magnitude of the driving voltage for the Slave via control of magnitudes of the sine and cosine components of the driving voltage. The magnitude of the generated driving voltage is then regulated by the Temperature Controller and supplied to the Slave via the Power Amplifier.

7.2 Description of experimental rig

The arrangement of the rig is schematically illustrated in Figure 7.2.

As compared to the experimental setup for the tests performed on the temperature control, this configuration of the cooler does not feature the passive balancer for the displacer. The cooler compressor is now equipped with two MS CD375-500 LVDT sensors [Macro Sensors] for the direct measurement of the pistons' displacement, as schematically shown in Figure 7.3.
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![Schematics of experimental rig](image)

**Figure 7.2 Schematics of experimental rig**
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A more detailed view of the LVDT arrangement is given in Figure 7.4.

The coils of the LVDT were mounted externally on the thin-walled adapter which isolates the pressurised interior of the compressor from the environment. The piston and LVDT core were connected with a flexural link to compensate for possible misalignment of the axes. The LVDT coils sensed the linear motion of the core that slid freely inside the adaptor. The LVDT sensors were mostly used for the reference and debugging of the balancing controller. The view of the cooler with mounted LVDT sensors is given in Figure 7.5.
The signals of displacement are preamplified by the LVDT conditioning module and passed to the NI RT board, as shown in Figure 7.2. The conditioning module shown in Figure 7.6 was built by the author based on two off-the-shelf MS LPC2100 LVDT conditioners [Macro Sensors]. These ac line-powered conditioners provide 5kHz sine wave excitation signals for the LVDTs' primaries with magnitude of 3.3V. The output gain and zero offset can be adjusted manually with potentiometers on the front panel of the conditioner. The electrical block-diagram of the conditioner is given in Appendix A.

Values of the compressors' supply voltages and currents through the motors' coils are measured with a specially developed and built by the author for this purpose precision multimeter, as shown in Figure 7.7. The multimeter comprises a dc power supply and off-the-shelf Hall effect sensors: two LEM LV25-P voltage transducers and two LEM LA25-NP current transducers [LEM]. Signals proportional to the voltage and current are simultaneously passed to the DASYLab monitor and the NI RT board. The principal electrical scheme of the multimeter is given in Appendix B.
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Figure 7.6 LVDT conditioner and LVDT sensor

Figure 7.7 Two-channel precision multimeter
The unbalanced force detection and temperature monitoring means, along with the principles of operation of the NI RT board have been described in the Chapter 4.

### 7.3 LabVIEW controller

The controller was implemented as a LabVIEW application running on the NI PCI 7030/6040 real-time board. The detailed description of the controller can be found in [Veprik and Dubrovsky 2003]. The front panel of the LabVIEW VI developed for the temperature control and balancing is shown in Figure 7.8. The block diagram for the above VI is shown in Figure 7.9.

The features of the controller are listed below:

- Control over required temperature of the cold finger
- Control over cooler upon cooldown
- Quasi-sensorless balancing
- Balancing with LVDTs
- Indication of measured and controlled parameters
- Application control

All features necessary for the temperature control were inherited from the temperature controller developed earlier. The new features included the quasi-sensorless balancing and balancing using LVDTs. The controller provides manual switching between the above methods of balancing with uninterruptible temperature control. The balancing can also be switched "on/off" on-line whilst keeping the last values of the controlled magnitude and phase of the voltage for the Slave compressor. This enables "bumpless" regulation of the temperature during transition from a balanced operation to an unbalanced one and otherwise. All the features are implemented on the same controller board. The control algorithm was optimised for maximum performance so any data logging operations were excluded and relied entirely on the DASYLab monitor.
Figure 7.8 LabVIEW VI front panel
7.4 Results of experiments

Results of the temperature control and quasi-sensorless balancing, and balancing using the LVDT sensors are illustrated in Figure 7.10 through Figure 7.19.

Figure 7.10 and Figure 7.11 give a direct comparison between the measurements of the piston velocity in the cooler compressor performed with the quasi-sensorless system and the LVDT sensor. Please note, that the signal of piston velocity is obtained as the time derivative of the signal of piston displacement measured with the LVDT sensor and the phase of the signal is adjusted to demonstrate that the signal of the directly measured velocity and the signal of the recovered velocity completely coincide in terms of magnitude and shape of the signals. Continuous and bar lines denote the measured and calculated velocity, correspondingly. The measurements have been done at the driving frequency of 45 Hz and magnitudes of velocity of 1 m/s and 1.5 m/s as shown in Figure 7.10 and Figure 7.11, correspondingly. The upper parts of the figures demonstrate “live” signals of velocity, while the lower parts of the figures show the velocity RMS values.

Figure 7.12 shows the spectrum of the initial unbalanced force for the cold finger temperature of 96K and in the absence of heat load. The magnitude of the fundamental harmonic is 4.40N.

Figure 7.13 demonstrates the result of the quasi-sensorless balancing of the fundamental harmonic of the unbalanced force. From the figure, the magnitude of the fundamental harmonic is 0.19N, which is below the allowed limit of 0.2N. This is a satisfactory result showing a reduction ratio in excess of 23, compared to the initial value of 4.40N.

Figure 7.14 demonstrates for reference the result of balancing using LVDTs for the same operating conditions and initial value of the unbalanced force. The magnitude of the fundamental harmonic of the residual unbalanced force is 0.08N, giving a reduction ratio of 55. It can be seen, that the effectiveness of balancing using LVDTs is higher compared to the quasi-sensorless balancing. Nevertheless, the quasi-sensorless balancing still works perfectly well, considering the fact that the LVDT
sensors, in this case, are the best means for the measurement of the motion of the pistons.

Figure 7.15 shows the spectrum of the initial unbalanced force for the cold finger temperature of 64K and in the absence of heat load. The magnitude of the fundamental harmonic of the unbalanced force is 5.23N.

Result of quasi-sensorless balancing for the cold finger temperature of 64K is shown in Figure 7.16. The magnitude of the fundamental harmonic of the residual unbalanced force is 0.19, which is below the allowed limit of 0.2N. The reduction ratio is in excess of 27.

Figure 7.17 shows for reference the results of balancing using LVDT sensors, and for the cold finger temperature of 64K. The magnitude of the fundamental harmonic of the residual unbalanced force is 0.08N. This is an excellent result giving a reduction ratio in excess of 65.

Figure 7.18 shows the time history of temperature control and balancing for the cold finger temperature of 68K and with no heat load. Switching from balancing using the LVDT sensors to the quasi-sensorless balancing was made at the time instant \( t_0 \), as shown in Figure 7.18. This was done to check the stability of the quasi-sensorless balancing as compared to the balancing with LVDTs. It is shown that the rate of convergence is comparable for the two methods of balancing and the quality of the temperature control is only slightly affected by transients due to the balancing.

Figure 7.19 illustrates the effect of variation of the cold finger temperature. The balancing was performed using the quasi-sensorless approach. There was a step increase in the required cold finger temperature from its initial value of 64K to the value of 90K, followed by a step decrease back to the value of 64K. It can be seen from the figure that it takes approximately 7 min for the temperature to reach a steady state when increased from the 64K to 90K. However, the balancing is completed within 3.5 min. A similar result was obtained when the temperature was decreased. It took more than 15 min to cool the cold finger from the 90K down to the 64K, and it took about 3 min to balance the compressor.
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Figure 7.10 LVDT-sensor and quasi-sensorless measurement of velocity (1 m/s)

Figure 7.11 LVDT-sensor and quasi-sensorless measurement of velocity (1.5 m/s)
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Figure 7.12 Unbalanced operation (Cold finger temperature 96K, zero heat load)

Figure 7.13 Quasi-sensorless balancing (Cold finger temperature 96K, zero heat load)
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Figure 7.14 Balancing with LVDTs (Cold finger temperature 96K, zero heat load)

Figure 7.15 Unbalanced operation (Cold finger temperature 64K, zero heat load)
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Figure 7.16 Quasi-sensorless balancing (Cold finger temperature 64K, zero heat load)

Figure 7.17 Balancing with LVDTs (Cold finger temperature 64K, zero heat load)
Figure 7.18 Balancing at constant temperature

Figure 7.19 Balancing at different temperatures
7.5 Conclusions

The experiments on attainable performance of the control system for balancing the linear twin-piston compressor of the cryogenic cooler have been conducted. The structure of the control system and the arrangement of the experimental rig have been described. Operation of the LabVIEW controller has also been explained.

The balancing was performed using both the quasi-sensorless and LVDT-sensor methods of measurement of the compressor pistons' motion. The latter method was used mostly for reference. Both methods of balancing satisfy the requirements giving an appreciable reduction in the unbalanced force. The best reduction ratio obtained with the LVDT sensors was in excess of 65 for the cold finger temperature of 64K, while the best reduction ratio obtained using the quasi-sensorless method of motion measurement was in excess of 27 for the same operating conditions. Taking into account that no sensors for the direct measurement of pistons' motion are used by the quasi-sensorless method, the above results of balancing are in fact comparable and give evidence of the effectiveness of the quasi-sensorless method. It has also been demonstrated that the temperature control, still the main feature of the controller, is not affected by the balancing control system when in steady state. Weak deviations of the temperature from a required value were observed only during transients. Thus, the quasi-sensorless balancing method has proved to be feasible for the active vibration control of the linear twin-piston compressor of the cryogenic cooler.
Chapter 8 discusses the optimal frequency control of the linear twin-piston Stirling cooler. The equivalent model of the cooler developed in Chapter 3 is validated for use in the modelling of the optimal control system. The optimal frequency control is developed. Simultaneous operation of the temperature, balancing and optimal control systems is modelled. Tests on attainable performance of the optimal control system are performed for typical operating conditions.

8.1 Objective of optimal control

From experience, there always exists a driving frequency at which a cooler produces the required cooling with a minimum expenditure of electrical power [van der Walt and Unger 1992], [de Jonge and Sereny 1982], [Ross et al. 1994], [Huang and Chen 2002]. This frequency depends on the ambient temperature, the temperature of the cold finger and the value of the heat load. Figure 8.1 shows the active electrical power as a function of the driving frequency for the K535 cooler. Three curves in Figure 8.1 are plotted for different combinations of the heat load and cold finger temperature.

It would obviously be an advantage to drive the cooler at the frequency corresponding to the minimum input power to provide for maximum efficiency. Unfortunately, this optimal frequency does not have a single value for the range of operating conditions, as shown in Figure 8.1, and therefore requires adjustment for each combination of the cold finger temperature and heat load. The driving frequency can be tuned manually, but this approach is not effective and is not feasible in remote applications. In this case, a control system would be required that can continuously track the value of electrical power and automatically adjust the driving frequency to minimise the electrical power input while maintaining the
required cold finger temperature. Such a control system has been developed, tested and has proved to be efficient for the optimal control of the twin-piston linear cooler.

![Figure 8.1 Input electrical power versus frequency for different temperatures and heat loads](image)

**Figure 8.1** Input electrical power versus frequency for different temperatures and heat loads

### 8.2 Modelling of the optimal control system

#### 8.2.1 Equivalent model of the cooler

Recall that the modelling of the cooler was thus far restricted to the modelling of the compressor only. However, it is clear that the combination of the thermodynamic and dynamic properties of the cooler defines its behaviour. The thermodynamics of the cooler is not amenable to simple description being concerned mostly with the design of the cooler expander and the characteristics of the working gas [Walker 1989]. At the same time, the dynamics of the cooler may be referred, in a simplified approach, to the dynamic properties of the cooler compressor [Ross et al. 1994]. A compressor
model is already available and it may be shown that this model can be used for the
development of an optimal control system. The following justifies this claim.

The presence of a minimum of the function of electrical power may be explained by
both the thermodynamic and dynamic properties of the cooler. The thermodynamic
efficiency is defined by the geometry of the compressor and expander, the materials
of the regenerator matrix of the displacer, the characteristics of the working gas and
the operating conditions [Walker 1989]. The parameters of the expander may be
optimised so that it could operate most efficiently in the required range of
frequencies. The displacer, thought of as an SDOF dynamic system, can be identified
as a heavily damped oscillator. This is illustrated in Figure 8.2 for the transfer
function of displacer displacement and pressure in the compression space. It is shown
in the figure that the displacer response to the gas pressure forces is smooth over all
frequency range, with heavily damped resonance near 60Hz.

![Figure 8.2 Transfer function of displacer (Displacement/Pressure)](image)

The compressor, instead, clearly exhibits resonant behaviour, as shown in Figure 8.3
for the transfer function of pressure in the compression space and motor current. The
maximum of the curve corresponds to the natural frequency of the moving assembly of the compressor for given operating conditions.

It was shown in [Ross et al. 1994], that the influence of the expander on the resonant properties of the cooler appears in the form of variations in the density of the working gas as the temperature of the gas in the cold finger changes. The density of the working gas as well as the fill pressure and the geometry of the compressor will define the gas springs in the compressor. These gas springs are the dominant parameter that critically affect the value of the natural frequency of the compressor and are a link between the expander and compressor. Therefore, the modelling of cooler operation, as before, may be restricted to the modelling of compressor dynamics. Parameters of the gas springs in the compressor, in this case, are function of the cold finger and ambient temperatures, and the heat load.

![Figure 8.3 Transfer function of compressor piston (Pressure/Current)](image-url)
8.2.2 Input electrical power

It may be shown that the electrical power in the equivalent model has a minimum at a particular driving frequency. For this purpose, a series connection of the compressor motors was considered. The gas pressure forces in the compressor were modelled as an action of the equivalent linear spring, i.e. the model was linearized. This approach has been introduced in Chapter 3. Series connected motors are described by the equation (3.19) that is repeated here for convenience:

\[ 2L_0i_0 + 2R_0i_0 + 2\alpha_o \dot{x}_0 = u_0 \]  

(8.1)

where:  
- \( u_0 \) - supply voltage, V;  
- \( i_0 \) - current through the coils, A;  
- \( L_0 \) - self-inductance of motor coil, H;  
- \( R_0 \) - active resistance of motor coil, Ohm;  
- \( \alpha_o \) - motor constant, N/A.

Assuming the compressor motors are connected in series and the opposed piston-magnet assemblies are identical, the latter can be modelled as one combined SDOF system that is governed by the equation (3.21), repeated below for convenience:

\[ 2m_0\ddot{x}_0 + 2c_0 \dot{x}_0 + 2k_0x_0 + k_gx_0 = 2\alpha_o i_0 \]  

(8.2)

where:  
- \( m_0 = m_1 = m_2 \) - moving mass, kg;  
- \( c_0 = c_1 = c_2 \) - damping coefficient, Ns/m;  
- \( k_0 = k_1 = k_2 \) - spring rate of mechanical spring, N/m;  
- \( k_g \) - spring rate of the equivalent linear spring, N/m;

Indexes 1 and 2 are referred to the opposite motors and moving assemblies.

The equivalent linear spring was defined before as:

\[ k_gx_0 = 2A_p(p_1 - p_2) = 2A_pP \]  

(8.3)

where:  
- \( p_1 \) - pressure above the pistons, Pa;  
- \( p_2 \) - pressure below the pistons, Pa;  
- \( p \) - differential pressure, Pa.

Laplace transform of the equations (8.2) and (8.1) yields:

\[ (2m_0s^2 + 2c_0s + 2k_0 + k_g)X_0 = 2\alpha_o I_0 \]  

(8.4)
and

\[ (2I_0 s + 2R_0) I_0 + 2\alpha_0 \omega X_0 = U_0, \tag{8.5} \]
correspondingly.

After formal substitution \( s \to j\omega \) and rearranging for the current, equation (8.4) can be expressed in a complex form as:

\[ I_0(j\omega) = \frac{X_0(j\omega)}{2\alpha_0} \left(-2m_0\omega^2 + 2c_0j\omega + 2k_0 + k_g \right) \tag{8.6} \]

Substitution of (8.6) into (8.5) rewritten in complex form yields:

\[ U_0(j\omega) = \frac{X_0(j\omega)}{\alpha_0} \left[ \left( L_0j\omega + R_0 \right) \left(-2m_0\omega^2 + 2c_0j\omega + 2k_0 + k_g \right) + 2\alpha_0^2 j\omega \right] \tag{8.7} \]

The active electrical power can be calculated as a dot product of the complex voltage and current [Kraus and Fleisch 1999]:

\[ N_0 = \frac{1}{2} \text{Re} \left[ U_0(j\omega) \cdot I_0(j\omega) \right] \tag{8.8} \]

Provided that \( X^2_0(j\omega) = \left| X_0(j\omega) \right|^2 \), after substitution of equations (8.6) and (8.7) into (8.8), the power is defined as:

\[ N_0 = \frac{X^2_0}{4\alpha_0^2} \left[ R_0 \left[ \left(-2m_0\omega^2 + 2k_0 + k_g \right)^2 + 4c_0^2\omega^2 \right] + 4\alpha_0^2\omega^2c_0 \right] \tag{8.9} \]

As discussed in Chapter 3, for the modelling of cooler operation it would be adequate to maintain a constant value of the product \( \mathbf{W} = \mathbf{p}^T \mathbf{v} \) instead of maintaining a constant temperature of the cold finger.

From equation (8.3), the pressure can be expressed in a complex form as:

\[ P(j\omega) = \frac{k_g}{2A_p} X_0(j\omega) \tag{8.10} \]

The rms value of the pressure is then calculated as:

\[ \hat{P} = \frac{1}{\sqrt{2}} \left| P(j\omega) \right| = \frac{1}{\sqrt{2}} \frac{k_g}{2A_p} \left| X_0(j\omega) \right| \tag{8.11} \]

Velocity of the piston can be expressed via displacement in a complex form as:

\[ V_0(j\omega) = j\omega X_0(j\omega) \tag{8.12} \]

The rms value of the velocity is:

\[ \hat{V}_0 = \frac{1}{\sqrt{2}} \left| j\omega X_0(j\omega) \right| \tag{8.13} \]
It should be noted, that a parallel connection of the two identical pistons was considered. The value of velocity calculated by equation (8.13) should therefore be doubled.

The product $W = \hat{p} \hat{V}$ can now be calculated as:

$$\hat{p} \hat{V} = \frac{1}{2} \frac{k_p \omega}{A_p} x_0^2$$  \hspace{1cm} (8.14)

Substitution of the above equation into (8.9) gives the equation for the electrical power:

$$\hat{p} \hat{V} = \frac{\hat{p} \hat{V}_0 A_p}{2 \alpha_0^2 k_0 \omega^2} \left[ R_0 \left( -2m_0 \omega^2 + 2k_0 + k_g \right)^2 + 4c_0^2 \omega^2 \right] + 4\alpha_0^2 \omega^2 c_0$$  \hspace{1cm} (8.15)

The electrical power can now be calculated as a function of compressor parameters and driving frequency for a given value of the $\hat{p} \hat{V}$. Figure 8.4 shows the electrical power as a function of the driving frequency for the value $\hat{p} \hat{V} = 6.40$ and values of compressor parameters found from the tests presented in Chapter 3 ($m_0 = 0.40$ kg, $k_0 = 3.36 \cdot 10^3$ N/m, $k_g = 8.85 \cdot 10^4$ N/m, $c_0 = 10.39$ Ns/m, $\alpha_0 = 8.00$ N/A, $A_p = 0.07 \cdot 10^{-2}$ m$^2$, $R_0 = 0.42$ Ohm, $L_0 = 0.57 \cdot 10^{-2}$ H).

Figure 8.4 Electrical power for the capacity $pv=6.4$
It is shown in Figure 8.4, that the function of electrical power has a minimum at a particular frequency, optimal for given compressor parameters. The same figure illustrates an effect of variation of the value for the equivalent linear spring $k_s$ that approximates the gas pressure forces.

The equivalent model was thus shown to adequately represent a real linear twin-piston Stirling cooler.

8.3 Development of control system

8.3.1 Principle of operation

The type of control system developed for the optimal control of the cooler may be referred to the class of so-called extremal control systems [Åström and Wittenmark 1989], [Besekersky and Popov 1972]. These systems, by definition, are applicable for the control of objects having global extremum (maximum or minimum) in the functions that describe certain parameters of these objects. The control system locates the extremum and adjusts the input to the controlled object in order to operate this object close to the extremum for an efficient or stable operation.

The control system developed here relies on continuous tracking of the value of active electric power and generating, according to the optimisation algorithm, the excitation signal with variable frequency to drive the cooler at minimum electrical power input while maintaining the required temperature at a given heat load. Calculation of the electrical power is performed based on the instant values of voltage and current measured by appropriate sensors. The optimal control system can operate in tandem with the control system for compressor balancing.

Operation of the control system model is conceptually illustrated in Figure 8.5. It is shown that the optimal control system is included in the developed earlier control system for the temperature control and balancing of the cooler. The principle of operation of the temperature and balancing control systems has remained unchanged except for the way of generating the excitation signals for the compressor motors.
The frequency of the excitation signals is defined by the optimisation algorithm, and the generator of reference *sine* and *cosine* signals can now be thought of as a voltage-controlled oscillator (VCO) with the *dc* input for the value of the regulated frequency. The optimal control system is of the searching type and, as such, performs the search over values of electric power measured at known time intervals in order to find the value of the driving frequency corresponding to a minimum electric power.

**Figure 8.5 Visio picture of optimal control**

The principle of operation of the optimal control is based on finding the sign of the gradient of the power function and generating the appropriate sign of the increment.
\( \Delta f \) for the frequency. The optimisation algorithm can be described by the expression:

\[
\Delta f \text{ sign} = -\text{sign}\left(\frac{N_n - N_{n-1}}{f_n - f_{n-1}}\right)
\]  

(8.16)

The gradient is calculated as a ratio of the difference in values of the power measured in constant time intervals \( t_n \) to the difference in values of the driving frequency taken in the same time intervals. The control is such that the frequency varies constantly with a slew rate defined by the value of increment \( \Delta f \) in the frequency. The sign of the increment specifies the direction of frequency sweep. It is assumed that the function of electric power has a global minimum in the range of frequencies involved. This assumption is justified by the Figure 8.1 and Figure 8.4 for the electrical power, presented in the preceding section.

8.3.2 Modelling of the control system

The Simulink model shown in Figure 8.6 included models of the capacity, balancing and optimal control systems and a model of the twin-piston compressor with independent supply to the motors. Simultaneous operation of the above control systems was modelled.

It was shown first, that the temperature and balancing control systems keep up when the driving frequency varies linearly with time. For this purpose, a sweep sine test was done on the model of a balanced compressor. The balancing was performed at the fundamental frequency only. Figure 8.7 shows results of the test for the positive and negative sweep in the range of frequencies 30 to 70 Hz. The value of sweep rate is 0.1 Hz/sec, which is as twice as high as the value that will be used for the control. Figure 8.7 shows satisfactory performance of both the temperature and balancing control systems regardless of the direction of frequency sweep.
Figure 8.6 Simulink diagram of control system (capacity, balancing and frequency controls)
Figure 8.7 Diagrams of the power, capacity and unbalanced force for the sweep test
8.3.2.1 Optimisation algorithm

Implementation of the algorithm for optimal control is illustrated in Figure 8.8. The optimisation process starts with a predefined time delay provided so that the temperature and balancing controls could reach the steady state. First, a positive increment in the frequency is made by applying the positive constant to the integrator, as shown in Figure 8.8, with the value of the constant specified by the sum of \( df \) and \( df1 \). By integrating the constant, the frequency increases departing from the base frequency at a constant rate. Then, at a known time instant, the gradient of the function of power is calculated based on the preceding and current values of the power and frequency. This calculation is performed by the Frequency Control Logic module, schematics of which is shown in Figure 8.9. Instant values of the power are weighed across several points in the Denoise module to reduce possible noise in the power signal. The sign of the frequency increment is generated according to the control algorithm in order to drive the frequency to an optimal value for the minimum electrical power. As the value of the power decreases reaching a certain threshold, the integration constant is switched to a smaller value, \( df \), in this case, to slow down the frequency sweep. This is done for the smooth approaching to the minimum in the power when the frequency is close to the optimal. After the optimum has been reached, the control system carries on searching about the point of minimum so that the control could respond to a change in the location of the minimum due to fluctuations in load and variations in operating conditions.
8.3.2.2 Operation of control system

Operation of the optimal control system is illustrated in Figure 8.10 a) and Figure 8.10 b). It is shown in Figure 8.10 a) that, as the frequency increases, the power decreases and reaches its minimum at some point in time. After an optimal point has been reached, the system does not stop, but rather goes further passing the minimum. The frequency continues to increase, as shown in Figure 8.10 b), and, as a result, the electrical power rises. When the minimum value of the power has been passed (see Figure 8.10) the gradient of the power function changes its sign and at the time instant $t_1$ the sign of the increment in frequency is flipped to the opposite in accordance with the control algorithm. The frequency decreases starting from the time $t_1$ and the power decreases approaching its minimum again. When the optimum has been reached, the system passes it as before and the cycle is repeated. At the time instant $t_2$ the sign of the frequency increment is again flipped to the opposite. The control system thus performs a search for a minimum value of the power.
8.3.2.3 Results of modelling

Results of control system modelling are presented in Figure 8.11. The balancing of the compressor was performed at the fundamental frequency only. It is shown in Figure 8.11, a) that the capacity control is not affected by frequency variation. Figure 8.11, b) shows that the regulation is complete within a period of under 80s. The power is decreased from 55.5W to 37.5W, a reduction of 32.4%. The optimal frequency was found to be about 55-56Hz, as shown in Figure 8.11, c). Figure 8.11, d) illustrates the effect of frequency variation on the value of the residual unbalanced force. The force is elevated, but is still well below the allowed limit of 0.2N.
Figure 8.11 Operation of frequency control system
8.4 Experiments on attainable performance

8.4.1 Description of experimental rig

The configuration of the control and measurement hardware is similar to that used for the experiments on compressor balancing, as described earlier in Chapter 7. It was shown from the control system modelling that the optimal control does not affect performance of the temperature and balancing control systems. Therefore, no balancing was performed during tests on the optimal control to relieve control hardware resources. The K535 cooler without additional balancer for the displacer was used in the experiments. The compressor motors were connected in series and were therefore both driven with the same current.

8.4.2 LabVIEW controller

The control was implemented using LabVIEW running on the PCI 7030/6040 real-time board. Detailed description of the LabVIEW VI developed for the optimal control can be found in [Veprik and Dubrovsky 2003].

Functions performed by the controller are listed below:

- Control over required temperature of the cold finger
  This feature has been inherited from the temperature controller developed earlier.
- Optimal frequency control
  The optimal frequency search can be turned on/off in the course of control with holding latest value of the frequency. A LabVIEW implementation of the searching control algorithm is detailed below.
  - Indication of measured and controlled parameters
    This is provided for the observation of performance of the control.
  - LabVIEW application control

The front panel of the VI is shown in Figure 8.12. The block diagram of this VI is shown in Figure 8.13.
interchannel delay in secs (-1: hw default)

device # 2

channels (0) 2

buffer size (4000 scans)

scan rate (1000 scans/sec)

scans to read at a time (1000)

channels (0) 2

scan backlog

input signals

Inputs Filter

Power

El. Power, W

Active Electrical Pot

Temperature Scale window

Manual/PID 2

Manual/Temp Ctrl

Temp Filter

Temp Ref, mV

Temp, K

Temp error

Number of updates done

Number of buffers done

Sine gen

Voltages

Sweep rate, Hz/sec

Freq, Hz

Time interval, s

Power Ctrl

Input Threshold

Current Frequency, Hz

Current F, Hz

1000

status

OVERLOAD

Hardware Control
8.4.2.1 LabVIEW implementation of the optimisation algorithm

Implementation of the optimisation algorithm slightly differs from that modelled using Simulink. The difference is in the methods of the power calculation and the generation of the excitation signal for driving the cooler. Buffered data acquisition and generation are used. Two buffers of equal length containing values of the voltage and current are multiplied using array multiplication and then an average across values of the buffer containing the product is taken for the calculation of power. This approach enables the saving of hardware resources and suffices for relatively low frequency sweep rates.

For generation of the driving voltage with variable frequency, a discrete modulo integrator is used instead of the normal one. The voltage is computed according to the equation:

\[ u_n = u_r \sin \left( 2\pi \cdot \text{rem} \left( \left( \text{sign} \cdot \Delta f \cdot n + f_0 \right) \cdot \Delta t \cdot \left( n+1 \right) \right) / 1 \right) \]  \hspace{1cm} (8.17)

where:
- \( u_n \) - voltage on k-iteration, V;
- \( u_r \) - voltage magnitude regulated by the temperature controller, V;
- \text{sign} - sign of the frequency increment;
- \( \Delta f \) - frequency increment, Hz/Scan;
- \( f_0 \) - base frequency, Hz;
- \( n \) - iteration number, \( n = 0 \ldots 1 \); 
- \( \Delta t \) - sample time, s

Voltage generation and calculation of the gradient of the power function were implemented in the single VI, a block diagram of which is shown in Figure 8.14.
Figure 8.14 LabVIEW implementation of the searching control algorithm
8.4.3 Results of experiments

The experiments were carried out on the cooler lifting 10W of heat at a cold finger temperature of 120K. Results of a frequency sweep test conducted prior to optimisation are presented in Figure 8.15. In the figure, the active electrical power is plotted versus frequency in the range 45 to 60 Hz. It can be seen that the global minimum of the function of power exists at about 52.5Hz.

Results of optimisation are presented in Figure 8.16 through Figure 8.18. Figure 8.16 is a screen shot from the DASYLab monitor. It shows that, while the frequency varies, the temperature control keeps up, maintaining the required temperature of 120K. Slight instability in the temperature control is observed at the start of optimisation. However, the temperature settles down to the required level when the frequency gradually arrives at an optimal value.

It was found that the value of the frequency increment critically affects the performance of the control. In the first test the frequency increment was clearly set too high. The effect of excessively fast frequency change is illustrated in Figure 8.17. The upper and lower parts of the figure show the time histories of power and frequency, respectively. The controller is trying to regulate frequency around local minima of the power function which results in unstable operation.

Figure 8.18 shows the results of optimisation after the controller has been tuned. The control has started at the driving frequency of 42Hz that arrived on the completion of optimisation at the value of 52.5Hz. The frequency of 52.5Hz was found to be optimal from the frequency sweep test. The initial value of power was somewhat above 152W as shown on Figure 8.18. The value of power at the optimal frequency of 52.5Hz is 121W. Thus, the reduction in power is in excess of 20%. After the frequency has reached an optimal value, it remained close to this value which demonstrates the stability of the control system operation.

It should be said, that in the above test the optimisation started at a frequency of 42Hz and was completed at a frequency of 52.5Hz. The optimal frequency of 52.5Hz was found close to 50Hz, the nominal value of the tested cooler for the particular
operating conditions. If started at 50Hz, the optimisation could give a reduction of 6.9% in power, compared with values of 130W and 121W at 50Hz and 52.5Hz, respectively. However, the optimal frequency would shift as operating conditions changed, as shown in Figure 8.1 for different combinations of the cold finger temperature and heat load. Hence, an appreciable reduction in power may be available in any case.

The experiments on attainable performance of the optimal control system have proved its feasibility for use with a linear twin-piston Stirling cooler.

Figure 8.15 Input electrical power versus operating frequency
Figure 8.16 Results of optimisation

Figure 8.17 Time history of input power (Controller is not tuned)
8.5 Conclusions

The control system for the optimal frequency control of the linear twin-piston Stirling cooler has been developed. The equivalent model of the cooler used for the modelling of the optimal control system has been discussed and validated. The optimisation algorithm has been described in detail. The simultaneous operation of the temperature, balancing and optimal control systems was modelled. The modelling has shown that the developed optimisation algorithm performs well while is relatively easy to implement. Effectiveness of the control has been proved through experiment. For this purpose, the LabVIEW controller has been developed based on the optimisation algorithm. The experiments on attainable performance have been conducted for the cooler with compressor motors connected in series, and for the typical operating conditions. It has been demonstrated that the reduction in power for a particular cooler was about 7%. It was discussed that for different regimes of cooler operation, the reduction in power gained could be in excess of 20%. This has proved the feasibility of the developed control as applied to the linear twin-piston Stirling cooler.
CHAPTER
NINE

CONCLUSIONS AND FURTHER WORK

This chapter presents overall conclusions drawn from the work described in the thesis and makes suggestions for its continuation.

9.1 Conclusions

This thesis has described the development, implementation and testing of an adaptive optimal control system for use with a linear drive, twin-piston Stirling cryogenic cooler. In order to satisfy the requirement of low vibration export, an active vibration protection system was developed that employs an adaptive feedforward control algorithm to balance the twin-piston compressor of the cooler and thereby to reduce compressor-generated vibration. To provide for a longer operating life of the cooler and as a cost effective solution, the quasi-sensorless motion measurement technique was utilised to detect the motion of the compressor pistons. It was shown, after a detailed evaluation of a number of candidate motion sensors, that the above motion measurement method is eminently suitable and the most appropriate choice for use in the design of the linear compressor of a cryogenic cooler. To gain the highest efficiency of cooler operation, an optimal control system was developed that minimizes electrical power input to the cooler by automatic adjustment of the frequency of the supply voltage for the cooler compressor. The above control systems were developed as a part of the temperature control system, so the requirement of precise temperature control for different heat loads always dominated and was satisfied. An experimental rig was built, based on the Ricor K535 linear twin-piston Stirling cooler, to prove the feasibility of the control algorithms and to evaluate the attainable performance of the control system.

To develop the control system, it was required to establish an equivalent model of a cooler that enables the modelling of complex thermodynamic processes involved in cooler operation to be avoided and, at the same time, adequately represents the cooler
as a dynamic electromechanical system. It was found that for modelling of the closed-loop temperature control of the cooler it is adequate to control the work supplied to the gas in the compression space of the compressor, instead of controlling the temperature of the cooler's cold finger. This effectively reduced the modelling of the cooler to the modelling of its compressor. The gas forces acting in the compressor were modelled as an equivalent non-linear gas spring, which allowed for further simplification. As a result, the compressor was modelled as an SDOF system driven by a linear electrical motor, and the model was shown to adequately predict the dynamics of the compressor. The correctness of the model was also validated through tests.

An adaptive feedforward control algorithm was found to be the most appropriate choice for the active vibration control of the cooler compressor, offering, in conjunction with heterodyne filtering, high robustness to external shock and harmonic disturbances, and stable operation under varied operating conditions and for large compressor imbalance. A quasi-sensorless method of motion measurement, based on recovering the velocity of a moving magnet from the voltage applied to and current in the linear motor, was applied to detect the motion of the reciprocating piston of the compressor. The effectiveness of the active vibration control using the adaptive feedforward algorithm, heterodyne filtering and quasi-sensorless motion measurement to reduce compressor-generated vibration has been proved through the experiments on the Ricor K535 cooler.

To reduce the input electrical power necessary to operate the cooler, it was shown that an optimal frequency control is required that could automatically adjust the frequency of the supply voltage in order to minimise the input power while maintaining the temperature of the cold finger at a constant level. For this purpose, a control system was developed that sweeps the frequency of the supply voltage according to a searching algorithm and thereby performs the search over various values of the power in order to find a frequency corresponding to the minimum input power. Modelling of the control system showed that the performance of the temperature and vibration control systems is not affected by the change in operating frequency. The concept of optimal frequency control was demonstrated in experiments on the Ricor K535 cooler.
All experiments were conducted using the NI real-time controller running under LabVIEW RT software. This was a valuable development tool for the implementation and testing of the control system and monitoring applications required during the research.

9.2 Further work

Further work is viewed in developing new control algorithms and motion measurement techniques supported by advanced control hardware.

The control hardware based on the NI PCI 7030/6040 board and used in the research enables rapid prototyping of control applications, which can then be run with real-time performance. From the experience gained thus far, an application of medium complexity can be run on the above board with no delays in the control loop when up to four input and two output channels are sampled at up to 1kS/s and the buffered data acquisition and generation are chosen, provided there is no data logging or file input/output operations involved in the application and there is a minimum of tasks running in the background on the Host PC. This is adequate for many control applications, but in some cases the intended performance may not be reached due to the complexity of the control algorithm. This is unfortunately a common thread that has also been encountered in the present research and hence certain advanced control algorithms, such as auto-resonant control of the compressor or high-frequency piston position detection, have been rejected during this research program. It can be, therefore, one of the subjects of future research to develop appropriate hardware based on a faster DSP processor capable of handling signal processing tasks at higher rates in control loops, and based on faster A/D and D/A converters.

Still another possible direction is developing more effective control algorithms posing less computational load on the hardware. Provided the suitable DSP hardware is available, this may be achieved by implementing control applications with low level programming languages say proprietary code generation software developed by DSP manufacturers, instead of graphical programming languages such as LabVIEW.
The auto-resonant control of the cooler compressor, as mentioned earlier, may be viable for the effective operation of linear coolers of a design different from that particular studied in this thesis. It was demonstrated in [Choe and Kim 2000] and [Choe and Kim 2002], that when the frequency of the supply voltage to the cooler compressor is varied for the purpose of control, cooler operation can be unstable in terms of the piston stroke control. This is apparently due to the introduced non-linearity inherent in the pneumatic compressor of the cooler. In this case, the self-excitation scheme with phase regulation is eminently suitable for the control of such a non-linear system [Sokolov and Babitsky 2001]. Here, the phase control can be used to stabilize the motion of the compressor piston even subject to strong non-linearities. Again, the above control can be implemented at the expense of the use of high-performance hardware.

As has been demonstrated in this thesis, the ac component of the piston displacement can be calculated using a quasi-sensorless approach. It may be important, in some cases [Unger and Keiter 2001], to know both the dc and the ac components of the displacement. The high-frequency position detection methods for position detection in a permanent magnet linear motor, as presented in [Williams 1981] and [Hartramph and Schinkothe 2000], can be adopted for the piston position detection in the linear compressor of a cooler. This would require a frequency generator circuit along with a demodulator circuit in addition to existing drive electronics. The required frequency is effectively up to 5kHz.

Though the Hall-effect sensors [Micronas 2003] designed for the linear displacement measurement were rejected after the analysis of candidate sensors for piston position detection, it is believed that this type is suitable for use in a linear compressor if an appropriate magnetic shielding is provided for the sensor. Having an inherently small size and low cost, the sensor can be implanted into the compressor body reducing the overall dimensions of the compressor housing as compared to the design using LVDT sensors, and reducing the cost of the measuring instrumentation while improving the reliability of the cooler.
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Bibliography


APPENDIX A

ELECTRICAL SCHEME OF LVDT CONDITIONER

Figure A.1 Electrical block-scheme of two-channel LVDT conditioner

SC1,SC2 - LVDT Signal Conditioners LPC-2000
SS1 - Switch DPST
O1 - AC fused 3-way outlet
OP1,OP2 - BNC female sockets
MT1,MT2 - Multipole 6-way DIN sockets

Jumpers' positions as shipped from factory
Primary terminals' connections for current transducers CT1-CT2 (5A connection):

5 4 3 2 1

6 7 8 9 10

PS1 - Power Supply SXI-053T
VT1-VT2 - Voltage Transducers LV 25-P
CT1-CT2 - Current Transducers LA 25-NP
R1,R4 - Resistors 2 kOhm
R2,R5,R7,R9 - Potentiometers 100 Ohm
R3,R6,R8,R10 - Resistors 120 Ohm
SS1 - Switch DPST
O1 - AC fused 3-way outlet
IP1-IP4,OP1-OP4 - BNC female sockets
T1 - Screw terminal

Figure B.1 Principal electrical scheme of two-channel multimeter