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Modelling of Autoresonant Control of an Ultrasonic Transducer for Machining Applications

By Svetlana Voronina

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy at Loughborough University

2008

Supervisor: Vladimir Babitsky
Director of Research: Ian Ashcroft

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Abstract
The main purpose of this research is an investigation into the different strategies for the autoresonant control of an ultrasonic transducer. Numerical simulations were considered as the most appropriate method for analysis and a Matlab-Simulink computer model of a non-linear ultrasonic vibrating system with the possibility of autoresonant control was developed. The controlled system consists of two modules, the first of which is an electromechanical model of the ultrasonic transducer comprising a piezoelectric transducer and a step concentrator. The second module simulates influence from the machining process. The coefficients of the electromechanical model were calculated through an identification process based on the real measurement of the ultrasonic transducer's vibrations. The validity of the computer model of the ultrasonic vibrating system has been confirmed experimentally. Further, a numerical model of the autoresonant control of this system has been developed. The autoresonant control maintains the resonant regime of oscillation by means of positive feedback, which provides excitation, transformation and amplification of the control signal from a feedback sensor. Stability of the control is sustained by additional regulation of an amplification ratio by the use of a local negative feedback. The model allows the exercise and comparison of three control strategies. The first one is based on the feedback signal proportional to the displacement of the end of the concentrator (mechanical feedback). The two other types of control are based on the signals proportional to the electrical characteristics of the piezoelectric transducer (electrical feedback). One of these strategies uses the current in the piezoceramic rings as the control signal (current feedback). The last control strategy takes into account both the current and the power of the piezoelectric transducer (power feedback). The completed investigation revealed the advantages and drawbacks of each of these control strategies. The results of the simulation are presented and discussed. To validate the results obtained through numerical simulations, a prototype of an autoresonant control system was developed and manufactured. For all the listed control strategies the machining experiments have been conducted with the control system. Experiments confirmed the results of the simulation.

Keywords
Ultrasonically assisted machining; active control of vibration; autoresonant control; phase control; modelling of ultrasonic transducer.
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Publications


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## Nomenclature

### Roman Letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a, A)</td>
<td>Amplitude (m)</td>
</tr>
<tr>
<td>(c)</td>
<td>Damping coefficient (Ns/m)</td>
</tr>
<tr>
<td>(C)</td>
<td>Damping matrix (Ns/m)</td>
</tr>
<tr>
<td>(d)</td>
<td>Piezoelectric charge constant (m/V)</td>
</tr>
<tr>
<td>(D)</td>
<td>Dielectric displacement (C/m(^2))</td>
</tr>
<tr>
<td>(E)</td>
<td>Modulus of elasticity (N/m(^2))</td>
</tr>
<tr>
<td>(f)</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>(f_r)</td>
<td>Resonant frequency (Hz)</td>
</tr>
<tr>
<td>(f(t))</td>
<td>Sensor signal</td>
</tr>
<tr>
<td>(\bar{f})</td>
<td>Mean value</td>
</tr>
<tr>
<td>(F)</td>
<td>Force matrix (N)</td>
</tr>
<tr>
<td>(F_0)</td>
<td>Force applied to a piezoelectric transducer from concentrator (N)</td>
</tr>
<tr>
<td>(h)</td>
<td>Piezoelectric deformation constant (V/m)</td>
</tr>
<tr>
<td>(i)</td>
<td>Current of a piezoelectric transducer (A)</td>
</tr>
<tr>
<td>(k)</td>
<td>Stiffness coefficient (N/m)</td>
</tr>
<tr>
<td>(K)</td>
<td>Stiffness matrix (N/m)</td>
</tr>
<tr>
<td>(l_0)</td>
<td>Thickness of a piezoceramic plate (m)</td>
</tr>
<tr>
<td>(L)</td>
<td>Length of a concentrator (m)</td>
</tr>
<tr>
<td>(m)</td>
<td>Mass (kg)</td>
</tr>
<tr>
<td>(M)</td>
<td>Mass matrix (kg)</td>
</tr>
<tr>
<td>(P)</td>
<td>Power of a piezoelectric transducer (W)</td>
</tr>
<tr>
<td>(q)</td>
<td>Charge (C)</td>
</tr>
<tr>
<td>(Q)</td>
<td>Quality Factor</td>
</tr>
<tr>
<td>(R)</td>
<td>Radius (m)</td>
</tr>
<tr>
<td>(R_i)</td>
<td>RMS value in “current” control cycle</td>
</tr>
<tr>
<td>(R_d)</td>
<td>Desired (prescribed) RMS value</td>
</tr>
<tr>
<td>(R_c)</td>
<td>Critical RMS value</td>
</tr>
<tr>
<td>(s)</td>
<td>Laplace transform variable</td>
</tr>
</tbody>
</table>
$s^e$ Elastic compliance at constant electric field (m$^2$/N)
$S$ Cross-sectional area (m$^2$)
$S_0$ Area of a piezoceramic plate (m$^2$)
$t$ Time (s)
$T$ Period (s)
$u$ Voltage supplied to a piezoelectric transducer (V)
$U$ Energy (J)
$v$ Cutting velocity (m/s)
$v_c$ Critical cutting velocity (m/s)
$x$ Displacement (m)
$x_c$ Coordinate of a connection cross section (m)
$x_0$ Amplitude of deformation for a single piezoceramic plate (m)
$\bar{X}$ Complex amplitude of oscillation (m)
$Z(\omega)$ Matrix of dynamic stiffness (N/m)

**Greek Letters**

$\varepsilon$ Strain
$\Delta$ Initial interference/gap (m)
$\lambda$ Eigenvalues matrix ((rad/s)$^2$)
$\mu$ Mass per unit length (kg/m)
$v(x,t)$ Displacement of the cross-section $x$
$\rho$ Material density (kg/m$^3$)
$\sigma$ Stress (Pa)
$\omega$ Angular velocity (rad/s)
$\xi'$ Permittivity at constant strain (F/m)
$\Sigma$ Field strength (V/m)
$\tau$ Time constant (s)
$\psi$ Normalised amplitude of oscillation
$\Psi$ Eigenvectors matrix
Chapter One  
Introduction

1.1 Problem formulation

The history of ultrasonic machining began with a paper by R.W. Wood and A.L. Loomis in 1927 and the first patent was granted to American engineer L. Balamuth in 1945 [1].

Three procedural variants of machining with an ultrasound application are currently known, these are: conventional ultrasonic machining, rotary ultrasonic machining and ultrasonically assisted machining. This research concentrates on the ultrasonically assisted machining. Ultrasonically assisted machining is the superimposing of ultrasonic vibrations on conventional machining processes such as turning, milling, drilling and other machining techniques, when the vibration is applied directly to a cutting tip (not through the abrasive slurry). A typical set-up for ultrasonically assisted turning is presented in Figure 1.1. The ultrasonic transducer consists of piezoceramic rings clamped between a mild steel back section and an aluminium or titanium concentrator by a steel bolt. A cutting tip is fixed in the tool holder installed at the thin end of the concentrator. The transducer is fixed to the lathe through its nodal cross section at the machine tool vertical slide. The workpiece is clamped by a three-jaw spindle chuck and is rotated universally by a lathe drive.

![Figure 1.1. Experimental set-up of ultrasonically assisted turning.](image)
The ultrasonic machining process begins with the conversion of low-frequency electrical energy to a high-frequency electrical signal, which is then fed to a transducer. Due to the piezoelectric effect, the piezoelectric transducer converts high-frequency electrical energy into mechanical vibrations, which are then transmitted through an energy-focusing device, i.e. a horn/tool assembly. This causes the cutting tool to vibrate along its longitudinal axis at high frequency (usually ≥20 kHz) [2].

The active development of ultrasonically assisted machining began in the middle of the last century (1950-s) [3, 4]. The method has won recognition and different researchers have reported that, compared to conventional machining technology, it provides a number of benefits, the most important of which are:

- Considerable decrease in cutting forces, tool wear, heat and noise radiation,
- Improvement of finish quality up to 50% (surface roughness and roundness),
- Elimination of chatter,
- Reduction of burr formation,
- Processing of a wide range of materials including glass, ceramics, semiconductor materials, ferrite, hard alloys and special composites, which are difficult or impossible to machine using conventional methods.

Some examples of ultrasonically assisted machining made at Loughborough University in comparison with conventional machining are presented in Figure 1.2. The significant difference in finish quality produced by these techniques can be clearly seen from these pictures. Ultrasonically assisted turning of inconel produced a much smoother surface than the conventional turning. In the case of the ultrasonically assisted drilling of aluminium, a substantial reduction in burr formation is presented. The example of milling of glass shows that conventional machining is more likely to produce cracks while the ultrasonically assisted milling makes a good quality groove.

![Figure 1.2. Comparison of ultrasonically assisted and conventional machining.](image-url)
In spite of all the listed benefits, ultrasonically assisted machining has still not been properly developed. The key problem in the promotion of this technique is the development of the proper adaptive control of the ultrasonic vibration. The problems existing in the vibrating systems control will now be considered.

The most important aspect in the control of the ultrasonic vibrating system is keeping the resonant mode of vibration, since the maximum vibration amplitude for a given excitation force is achieved at resonance. Therefore, for efficient machining, an ultrasonic system has to be always tuned at resonance. However, the practical application of this principle is connected with a number of difficulties due to the special properties of ultrasonic systems.

Traditionally, a fixed-frequency ultrasonic generator has been used for generating high-frequency ultrasonic energy and a waveguide and a tool were set up and mechanically tuned to achieve resonance [1].

However, the high-quality-factor oscillating systems, which are required for increasing the performance of the vibration machines, are difficult to maintain at resonance (see Figure 1.3). The Q factor is the dimensionless number determining the degree of resonant response and the sensitivity of the system to small variations in the driving vibration frequency. The high Q-factor systems demonstrate very high sensitivity to changing operating loads and parameters and the loss of efficiency occurs when a load is variable or with attachments of different tools [5].

![Figure 1.3. Keeping the resonant state of the high- and low-quality-factor systems](image)

Therefore, to maintain the optimum operating regime, the possibility of adjusting the frequency of excitation has to be introduced. It was reported that resonance-following generators could be used for this purpose [1, 2, 6, 7]. They provide automatic
adjustment of the output high frequency to match the exact resonant frequency of the concentrator/tool assembly. However, these generators are becoming inefficient due to the nonlinear properties of ultrasonic systems.

The vibro-impact nature of ultrasonic cutting processes [8] makes them strongly nonlinear, which significantly complicates the process of vibration control. An example of the amplitude-frequency characteristic of a nonlinear vibrating system is shown in Figure 1.4.

![Figure 1.4. Amplitude-frequency response of a nonlinear system. Explanation of the jump phenomenon.](image)

From this graph, we can see that for a nonlinear system, the amplitude of vibration can have more than one value for the same excitation frequency. It is possible that an imperceptible change in the exciter’s frequency may cause a very abrupt change in the amplitude of the system (the jump phenomenon). The jumping of the stationary amplitude takes place as the frequency of excitation is slowly increased or decreased. If the exciter’s frequency is raised from a small value, the amplitude of the stationary oscillation increases along the upper branch of the resonant curve until it reaches its furthermost point. Any further increase in frequency results in jumping of the stationary amplitude to the value corresponding to this frequency on the lower branch of the resonant curve. If the exciter’s frequency is changed in the opposite direction, the lower branch will be obtained, which corresponds to the low-amplitude mode without impacts [9].

Thus, the nonlinear properties of the process of ultrasonic machining described above make frequency control inefficient for achieving the peak performance of the vibrating system. The most advanced control method for overcoming these problems is autoresonance. The term ‘autoresonance’ was initially introduced in 1959 [10] and was defined as resonance occurring under the influence of a force generated by the motion
of the system itself. However, this term was not practically used until the 1970-s, when it was employed by V. Astashev and V. Babitsky for a definition of a new concept for excitation and control of nonlinear vibration modes. The concept was firstly applied by the authors together with M. Hertz for control of an ultrasonic transducer [11] and found later new applications in vibration engineering [9, 12]. The autoresonant control method is applicable to all types of ultrasonic processes and machines; it employs the phase control principle and can successfully overcome the frequency control drawbacks [11, 13-14].

Phase control is a control method based on the fact that the phase lag between the excitation forces and vibration of the working component during resonance is constant. Thus, by changing the phase shift between the exciting force and the vibrating system’s response, the frequency (and amplitude) of oscillation can be controlled (see Figure 1.5). In the case of phase-controlled vibration, the vibration is stable at all points of the amplitude-phase curve (as the curve is single valued). As this occurs, the vibration is stable at all points of the corresponding amplitude-frequency curve, even if it is ambiguous, and some regimes are unstable under traditional forced excitation with frequency control.

![Amplitude-phase and amplitude-frequency characteristics of a nonlinear system](image)

**Figure 1.5.** Amplitude-phase and amplitude-frequency characteristics of a nonlinear system

The amplitude - phase curve is gently sloping at resonance in distinction to the amplitude - frequency one, which means that by using the phase control, the resonant state of vibration can be maintained much more easily than in the case of the conventional frequency control.

Autoresonant control is a self-sustaining excitation of a vibration mode at the natural frequency of a mechanical system, which maintains the resonant condition of oscillation automatically by means of positive feedback based on the transformation (phase shift)
and amplification of the signal from a sensor. A general schematic of feedback is presented in Figure 1.6 [15-16].

Contrary to traditional control systems using negative feedback which forces the system to reach the desired state, an autoresonant control system uses positive feedback which makes maximum use of the mechanical properties of the system and allows exciting the oscillations at their natural frequency. Previously, industry generally tried to avoid resonance due to sensitivity of the process to variation of loads. Employment of autoresonant control allows resonance to be tamed and can improve the situation in a whole range of engineering areas.

It can be seen from the description of autoresonant control that the signal from the sensor plays a very important role in the design of the control system. It serves as a source of the control signal needed for the autoresonant excitation and thus the effectiveness of the control system depends on the efficiency of the sensor.

Depending on the choice of the sensor, different control strategies can be employed, which can be classified into two main types:

- Mechanical feedback, when the signal from the displacement (velocity or acceleration) sensor attached to the end of the concentrator or cutting tip is used for the control system.
- Electrical feedback, when the signal from the sensor, measuring the electrical parameters of the piezoelectric transducer (current, power), is used in a control algorithm.

Mechanical feedback uses a sensor to measure the vibration (it could be a displacement, velocity or acceleration sensor). The sensor is placed near the cutting area at the end of the concentrator or cutting tip. Therefore, it directly reflects the oscillations of the ultrasonic system and, by controlling the signal from this sensor, the actual state of
vibration can be controlled in the most efficient way. However, it is inconvenient to use this arrangement in industrial conditions since it is difficult to fix permanently because of prolonged high-frequency vibration in the harsh machining conditions. Additional wiring to the control system is a disadvantage as well, as the sensor is located in the cutting area.

Electrical feedback uses the electrical characteristics of the piezoelectric transducer for the control i.e. current or power. An electrical sensor does not need to be placed in the cutting area, which makes it convenient to use in industrial conditions. However, this sensor reflects the oscillations of the system in an indirect way via the current or power in the piezoceramic rings. This can reduce the efficiency of the control system.

High power ultrasonic processes have been the focus of many previous studies. However, control systems for ultrasonically assisted machining are still relatively undeveloped. Currently, as far as the author is aware, the only company that sells ultrasonically assisted cutting technology commercially is Fuji Ultrasonic Engineering Company Ltd [17]. The available information about the existing control systems shows that they either use the frequency control principle [18-20], which is potentially inefficient as was shown above, or employ phase control with the application of the electrical feedback principle [16, 21]. The results are seen to be good. However, no research has been devoted so far to the investigation of the electrical feedback properties and its efficiency has not been proved. The short description of the electrical feedback given above reveals sources of potential inefficiency of a control system employing this principle. The practical realisation of the mechanical feedback control system is complicated due to the absence of the appropriate sensor. As far as the author is aware, there are no commercial devices currently using this control principle and no research has been devoted to its investigation.

Thus, the control strategy providing the efficient and reliable control of vibration remains a challenging and topical engineering problem. This is the subject of the current study.

The most advanced method of investigation of the issues under discussion is by using numerical simulations, which requires the creation of a model of the ultrasonic vibrating system and a model of the control system.
1.2 Objectives of the study

The objectives of the research are:

1. To develop a realistic model of the ultrasonic system having all the significant properties of the original system and accommodating the effect of the nonlinear load application.
2. To develop a model of the control system based on the autoresonant principle which would enable the application of different control strategies to the model of the loaded ultrasonic transducer.
3. To investigate and compare, both theoretically and through simulations, the control principles based on the electrical feedback and on the mechanical feedback.
4. To design and manufacture the prototype control system allowing the application of the elaborated control strategies with the aim of validating the simulation results with matching experiments.
5. To search for the appropriate sensor which is capable of allowing the implementation of the mechanical feedback control system.

1.3 Outline of the thesis

The rest of the thesis is organised into 6 chapters.

Chapter Two is devoted to familiarization with the ultrasonic machining. It considers the history of ultrasonic machining development, gives a description of the main ultrasonic machining techniques and explains the basic principles and technology of ultrasonically assisted machining. This chapter also discusses the benefits and applications of ultrasonic machining as compared with conventional (non-ultrasonic) machining.

Chapter Three is dedicated to the consideration of methods of vibration control. It discusses the special properties of ultrasonic vibrating systems and relates these properties with existing control difficulties. It also includes the formalization of the problem and aims of this research.
Chapter Four concentrates on the development of a numerical computer model of an ultrasonic transducer. It incorporates the elaboration of a one-dimensional mathematical model of the ultrasonic transducer comprising a piezoelectric transducer and a step concentrator and the modelling of the influence from the machining process. It also considers the Matlab-Simulink model of this system based on the developed formulae. All aspects related to the control system development are discussed in Chapter Five. It includes the consideration of the control system algorithm, the description of the Matlab-Simulink model of the control system and the theoretical investigation of different control strategies.

Chapter Six is devoted to numerical simulations. It discusses the tests completed for the model of the control system with the application of different control strategies. The analysis and comparison of the results obtained through these simulations is also given. The experimental validation of the simulation results is presented in Chapter Seven. This chapter describes the design of the prototype control system created for this research and analyses and compares the results obtained from the turning experiments completed for different control strategies.

Chapter Eight is devoted to conclusions and discussion of propositions for further work.
Chapter Two Overview of ultrasonic machining

2.1. Ultrasonic machining techniques

2.1.1. Technological fundamentals of machining with ultrasonic support

The recent introduction of new materials in industry, which are hard, brittle and difficult to machine, has led to the widespread use of several unconventional machining methods, one of which is ultrasonic machining. The application of ultrasonic machining is not limited by the electrical or chemical characteristics of the workpiece materials. That is why it complements the other non-traditional machining techniques such as electromechanical machining, electrodischarge machining, laser-beam machining and plasma-arc machining and is the preferred manufacturing process when alternative processes either require substantially longer processing times or result in high scrap rates, especially for fragile workpieces.

Ultrasonic machining is used for machining both conductive and insulating materials; preferably those with low ductility and hardness above 40 HRC, e.g. inorganic glasses, silicon nitride, nickel/titanium alloys, etc. Holes as small as 76μm in diameter can be machined, however, the depth of cut to diameter ratio is limited to about 3:1 [1].

The main characteristic feature of machining with ultrasonic support is overlapping the cutting motion of a conventional cutter with an ultrasonic vibration, which converts the machining process into one involving controllable, high-frequency impacts at the cutting zone [8]. There are 3 procedural variants of machining with ultrasound application: Conventional Ultrasonic Machining (USM), Rotary Ultrasonic Machining (RUM) and Ultrasonically Assisted Machining (UAM). Descriptions of each of these types of machining methods follow below.

2.1.2. Conventional ultrasonic machining

The first patent for ultrasonic machining was issued in 1945 to Balamuth. Subsequently, the process has been developed and optimised. This process has been variously termed: ultrasonic cutting, ultrasonic drilling, ultrasonic dimensional machining and slurry drilling [1].

Conventional ultrasonic machining (USM) is a mechanical, material-removal process used to erode holes and cavities in hard or brittle workpieces by using shaped tools,
high-frequency mechanical motion and an abrasive slurry [2]. USM takes place when a tool vibrates at ultrasonic frequency and is placed in contact with a workpiece (see Figure 2.1).

Figure 2.1. Tool, workpiece and slurry interaction in USM. 
Reproduced from [22]

Abrasive slurry (a mixture of abrasive material; e.g. silicon carbide, boron carbide, alumina, etc. suspended in oil or water) is conveyed to the working zone between the face of the tool and the surface being machined. The tool moves towards the workpiece and is subjected to a static driving force. Repetitive impacts of the tool on the grains of the abrasive material, falling from the slurry onto the treated surface, lead to the fracture of the workpiece material and to the creation of a cavity with the shape of the tool [22].

A USM process may be configured to produce a better surface finish and a better hole quality than that usually obtainable from conventional machining. The speed, at which machining takes place, however, is generally much slower. A typical feed rate for USM whilst machining Carbon/Epoxy or Carbon/PEEK composite materials is in the region of 0.3 to 0.6 mm/min. Material is removed at several cubic millimetres per minute. Alternative methods of manufacture, such as laser and water-jet cutting, have proved to be in the region of fifteen times faster than USM. In spite of its slowness, USM has been proved to be more cost-effective than alternative machining processes for producing multiple holes simultaneously [23].

Conventional ultrasonic machining induces no thermal, electrical, chemical or metallurgical threat or interaction with the workpiece; the machining process is entirely mechanical. Therefore hard and brittle materials such as glass and ceramics can be well
machined with USM. Materials of higher breaking impact strength, such as hardened, nitrated and carbonitried steels can still be manufactured using this principle with fewer problems.

However, soft and viscous materials such as lead, rubber and soft plastics cannot be machined using USM as they have a capacity for absorbing the abrasive particles contained in the abrasive slurry [24, 25].

Other limitations of USM machinery include difficulties in the machining of deep holes, since the feeding of abrasive particles to and from the machining face limits the material removal rate [22].

2.1.3. Rotary ultrasonic machining

Rotary Ultrasonic Machining (RUM) was invented in 1964 by Percy Legge, a technical officer at the United Kingdom Atomic Energy Authority [26, 27].

RUM works in the same way as USM but removes the need for an abrasive slurry as the abrasive particles are contained within the machine’s tool. In RUM, a rotating core drill with metal-bonded, diamond abrasives vibrates ultrasonically. At the same time, the workpiece is fed towards the core drill at a constant pressure. The coolant pumped through the core of the drill helps to cool the tool, prevents the drill from jamming and removes spent abrasive and chips. This is illustrated in Figure 2.2. For these reasons, it is far easier to drill deep holes with RUM than it is with USM [28].

![Figure 2.2. Rotary ultrasonic machining](Reproduced from [29])

Rotary ultrasonic machining combines the material removal mechanisms of the ultrasonic machining process and the conventional diamond grinding process. These
include hammering (indentation and crushing under the impact of the ultrasonic vibrations), abrasion (the rotational motion of the cutting tool can be modelled as a grinding process) and extraction (produced by the simultaneous action of the ultrasonic vibrations and the rotational motion of the tool) [30, 31].

The combination of these three material removal mechanisms results in higher material removal rates in rotary ultrasonic machining than those obtained by either the ultrasonic machining process or the conventional diamond grinding process. The rotation also brings higher accuracy when generating cylindrically shaped elements.

The limitation of RUM is that only circular holes or cavities can be machined due to the rotary motion of the tool. Thus, this method is also known as Rotary Ultrasonic Drilling. Attempts have been made by other researchers to extend the rotary ultrasonic machining process to machining flat surfaces or milling slots. However, these extensions either changed the material removal mechanisms or had some severe drawbacks [28].

2.1.4. Ultrasonically assisted machining (UAM)

Ultrasonically assisted machining is the superimposition of an ultrasonic vibration onto conventional machining processes such as turning, drilling, reaming, cutting-off, grinding, lapping and boring, when the vibration is applied directly to the cutting tip (not through the abrasive slurry). UAM started to develop in the 1950s [3].

The advantages of this method of machining are not obvious, because, in most cases, machine tool vibration has to be suppressed. However, different researchers have proved that the direct application of ultrasonic vibration can give rise to significant improvements in the process and the products.

Some modern aviation materials (difficult to cut pre-preg materials, such as Kevlar, nickel-based alloys, carbon fibre and glass fibre) can be machined well using ultrasonically assisted cutting with considerable surface finish improvements [32, 15].

The machinability of Glass-Fibre Reinforced Plastics (GFRP) can be significantly improved using UAM. Cutting forces are substantially reduced and the fibres are more cleanly cut resulting in reduced fibre-pullout and the elimination of fibres protruding at the machined surface. Also, the workpiece may be cut with the fibres orientated in any direction relative to the cutting direction. Surface roughness, burr formation and subsurface damage can also be appreciably improved [33].

Jeong-Du Kim and Eun-Sang Lee have investigated the machinability of another difficult-to-cut material, Carbon Fibre Reinforced Plastics (CFRP), by means of
ultrasonic vibration cutting. They proved that at less than the critical cutting speed (the explanation of the critical cutting speed will be given in section 2.2.4) ultrasonic vibration cutting produces a better surface than conventional cutting for machining CFRP [34]. Applying the ultrasonic vibration to cutting of brittle materials gives high-quality results as well [35, 36].

A number of researchers have proved that a considerable reduction in cutting forces is observed with ultrasonic vibration, particularly at low values of cutting speed, which results in prolongation of tool life. Vibration helps to overcome built-up edge effects, which is typical for traditional cutting of a number of materials like aluminium, copper, brittle steels and other difficult to machine metals and alloys). This leads to significant improvement in the surface finish produced [37, 38, 39, 40].

The high-frequency vibration increases the dynamic stiffness of the lathe-tool-workpiece system, which improves the accuracy of machining and helps to eliminate chatter. Improvement in surface roughness and roundness of up to 50% has been discovered with the superimposition of ultrasonic vibration. A reduction in noise and heat radiation has also been detected [15, 38, 41].

2.2. Technology of UAM
2.2.1. Principle of the method
In UAM, a transducer (either piezoceramic- or magnetostrictive-based) converts high-frequency electrical signals from an electronic amplifier into high-frequency vibrations. The oscillation amplitude obtained is, however, very small and does not exceed 5 µm in most cases. Hence, the high frequency mechanical motion is transmitted to the tool via an acoustic horn (also called a concentrator, wave-guide or acoustic amplifier), which intensifies the amplitude of vibration in the direction of the thin end to provide high cutting rates. This causes the tool to vibrate at high-frequency (usually \( \approx 20 \text{ kHz} \)) with an amplitude of 5 – 20 µm.
The typical set-up for ultrasonically assisted turning is presented in Figure 2.3. The ultrasonic transducer consists of piezoceramic rings within a package together with a concentrator. A cutting tip is fixed in the tool holder installed at the thin end of the concentrator. The transducer is fixed through its developed nodal cross section at the machine tool vertical slide. The workpiece is clamped by a three-jaw, spindle chuck and rotated universally by a lathe drive.

2.2.2. Resonant tuning of an ultrasonic system

A typical ultrasonic (Langevin) bolt-clamped transducer is shown schematically in Figure 2.4, where two piezoceramic rings are clamped between a mild steel back section and an aluminium concentrator by a steel bolt.

To maximise both the amplitude of vibration at the tool and the material removal rate, the resonance of the tool and concentrator assembly needs to be within the adjustable frequency range of the USM machine. Resonance is attained when the transducer generates a standing acoustic wave with the anti-node at the tool (see Figure 2.4) [42]. It has been proved that ultrasonic transducers with concentrators manufactured with an odd number of quarter wavelengths provide the maximum amplification ratio (the displacement amplitude at the working end of the transducer divided by the displacement amplitude at the joint with the piezoceramic element) [43].
Using the FEM method in the design and analysis of acoustic horns helps to verify and fine-tune the results of empirical approximations [44, 45].

The amplification ratio of the concentrator also depends on its shape. Common concentrator types include: exponential (a), catenoidal (b), cosine (c), conical (d) and stepped (e) (see Figure 2.5). It has been shown by Astashev and Babitsky [8] that a stepped concentrator is the most effective in relation to the amplification ratio, although other properties, such as mechanical durability, simplicity of manufacture and so on, should be considered as well.
The mechanical system consists of a series of components (piezoelectric transducer, concentrator, tool holder and tool) and all components have to be appropriately designed and tuned to ensure the effective transmission of vibrations to the tool. It means that if the design of some components (for example the tool holder) is not appropriate or the construction of the system is insufficiently stiff, a loss of efficiency of the oscillations will exist. It is very important, that all the factors, including the tool length, are taken into account while designing the ultrasonic system. Otherwise the tool will transmit the oscillation with a loss and, as a result, the desired performance will not be achieved.

For effective tuning of an ultrasonically assisted drilling system, the matching of the dynamic qualities of the transducer and the drill bit with the dynamic loads imposed by the cutting process is required [40].

All components (the oscillation exciting device, concentrator and tool) have their own (partly very different) resonant frequencies. As all components are excited with the same frequency, for optimal use of ultrasonic support, it has to be ensured that each component has a longitudinal resonant frequency at the frequency given [24]. All the above shows that a significant number of factors have to be taken into account while designing the ultrasonic system. Since this aspect is outside of aims of the present research, no detailed consideration will be given.

2.2.3. Mechanism of ultrasonic vibration cutting

Figure 2.6 presents the mechanism of ultrasonic vibration cutting.

![Figure 2.6. Mechanism of ultrasonic vibration cutting](Reproduced from [46])
The displacement curve of a tool, which is vibrating in the cutting direction with a frequency \( f \), a period \( T \) and side amplitude \( a \), is plotted against time. The workpiece is rotating with a cutting velocity \( v \).

The cutting tool begins to oscillate from the origin \( O \) and then goes towards the workpiece. The initial contact of the tool with the workpiece will occur at point A. Between points A and B, the impact force causes the formation of chips. At point B, the rake face separates from the workpiece, since the velocity of the tool exceeds the velocity of the workpiece. When the workpiece approaches the point C, contact between the tool and the workpiece is re-established and chips are produced between points C and D. The displacement equation of the tool is:

\[
x = a \sin \omega t,
\]

(2.1)

Where \( x \), \( a \) and \( \omega \) are the displacement, the amplitude and the angular velocity of the tool respectively. Velocity is obtained from:

\[
x = a \omega \cos \omega t
\]

(2.2)

At points B and D the velocity of the tool is equal to the velocity of the workpiece:

\[-v = a \omega \cos \omega t_1,
\]

(2.3)

where \( v \) is the cutting velocity, \( t_1 \) is the time of contact of the tool with the workpiece.

2.2.4. Influence of the cutting velocity on the cutting process. Critical cutting velocity

It is clear from the description of the mechanism of ultrasonic vibration cutting discussed above, that relief of the drive occurs only when the velocity of vibration exceeds the velocity of cutting \( a \omega > v \). If \( v > a \omega \), the process becomes conventional. Therefore, there is a critical cutting velocity:

\[
v_c = a \omega = 2\pi af
\]

(2.4)

where \( f \) is frequency of the vibrating tool [3, 46].

With decreasing cutting velocity, the length of contact of a tool with a workpiece decreases, and the higher the frequency of the tool vibration, the more rapidly the contact length decreases.

It has been reported by a number of researchers that efficient ultrasonic cutting can be achieved for the cutting speeds \( v << a \omega \) [3, 38, 46, 47].

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Experiments, conducted in work [38] have shown that the effect of a reduction of the cutting forces decreases with the increase in the cutting velocity (see Figure 2.7).

![Figure 2.7. Experimental monitoring of cutting forces under different rotational speeds. Reproduced from [38]](image)

It has also been reported that in UAM the amplitude-frequency response changes with increasing cutting speed [38]. Figure 2.8 shows the transformation of the amplitude-frequency characteristic with an increase in the cutting speed during turning.

![Figure 2.8. The transformation of a transducer’s amplitude-frequency characteristics for an increase in cutting speed during turning. Adapted from [38].](image)

Figure 2.8(a) shows the amplitude-frequency characteristic with no load; it is a traditional, bell-shaped, resonance curve, which is typical for linear systems. With a slow increase of the cutting speed $v$, the tool starts to interact with the workpiece, and the curve transforms into (b) and then (c). Curve (d) demonstrates the response when the surface cutting speed is equal to the average speed of the tool under vibration. Two areas of instability can be seen on this curve (as shown by dashed lines) and, with a
further increase of the speed, characteristic (e) is encountered. Well-marked levels of instability in the system can be clearly seen here. Curve (f) shows the case at just below the critical speed, \( v \leq a \omega \). The system displays a hard nonlinearity here. With further increase of the cutting speed (see Figure 2.8(g)), the process becomes conventional, because the tool is permanently in contact with the workpiece.

This shows, that a UAM system demonstrates nonlinear properties and the amplitude response of the system depends on the cutting velocity and transforms with its increase. This leads to difficulties in controlling UAM, which will be further considered.

### 2.2.5. Directions of application of ultrasonic vibration

There are three possible directions in which the ultrasonic vibration can be applied in ultrasonic cutting (see Figure 2.9). These are: the direction of cutting velocity (cutting direction or tangential direction), the feed direction or horizontal vibration and the radial direction (or thrust direction).

![Figure 2.9. Possible vibration directions in ultrasonically assisted turning. Adapted from [15].](image)

The direction of application of the ultrasonic vibration is very important, as it influences the machining process and the surface quality of the materials being treated. Mainly, the cutting direction is used as the most practical application of ultrasonic vibration [48, 3]. However, some researchers [15] have reported that the application of feed direction vibrations for ultrasonic cutting is less limiting in relation to the critical cutting velocity.
In this case, the workpiece diameter has no influence on the cutting parameters. Therefore, the application of ultrasonic vibration along the feed direction is more suitable for turning, where a high level of productivity is required. However, it is difficult to design a transducer that allows the application of the vibration in this direction.
Chapter Three  Vibration control

Vibration control is one of the most important issues in the metal cutting process. A huge number of the methods for machine tool vibration control reported in the literature are devoted to the elimination of vibration. Chatter vibration, self-excited vibrations during a machining process under certain conditions, has been recognized as one of the most significant negative factors affecting the performance of a machine tool in conventional machining. The occurrence of chatter will result in a decrease in the metal removal rate and in a poor surface finish for a machined workpiece, a reduction in tool life and accelerated machine tool system component wear. Undesirable vibrations may also increase the noise level in a working environment [41, 49-55].

On the contrary, in UAM, the control is devoted to the excitation and maintenance of the appropriate level of ultrasonic vibrations. In this case, vibration plays a positive role and introduces many benefits into the conventional machining process. The most important of them are: a considerable decrease in cutting forces, improvement of finish quality by up to 50% and the ability to process a wide range of materials including hard alloys and difficult-to-machine special composites [15, 31-39, 41]. It has also been reported that the application of ultrasonic vibration helps to eliminate chatter. As mentioned above, the explanation for this is that the high-frequency vibration increases the dynamic stiffness of the lathe-tool-workpiece system, which successfully suppresses the undesirable vibrations [38]. Thus, the controllable vibration in UAM turns the drawbacks of undesirable vibrations in conventional machining into benefits. However, with the application of the ultrasonic vibration, the process of machining becomes nonlinear (this will be further considered) and therefore difficult to control. Also, the methods of active vibration control used for vibration suppression become unsuitable for UAM. This chapter is devoted to the consideration of existing control methods used to maintain the vibration of an ultrasonically vibrating system and finding the most effective one.
3.1. Methods of vibration control

It has been mentioned before that the resonant regime of vibratory equipment is the most efficient one. At resonance, a quick increase of vibration amplitude for a given excitation force occurs. This can be explained because of the balance between the elastic and inertia forces in resonant vibration systems. In this case, vibration excitation energy is expended only to overcome an active load of the operating process and dissipative forces leading to the actuation energy being used in the most effective way [13]. Therefore, for efficient machining, an ultrasonic system has to be tuned at resonance. However, practical application of this principle is associated with a number of difficulties and has to comply with the operating parameters of the vibration system and the processing loads.

Thus, the most important aspect in the control of the ultrasonic vibrating system is keeping the resonant mode of vibration. The definition of the resonance of the vibrating system will be now given.

3.1.1. Resonance and quality factor of a vibratory system

The resonant frequency is one of the important characteristics of the vibrating process. Resonance is the tendency for a system to strongly intensify its oscillations at a specific frequency of excitation. The resonant frequency is determined by the physical parameters of the object or system. The degree of resonant response is indicated by a dimensionless number called the Quality Factor (Q-factor). The Q-factor of a dynamic system (3.1) is found by noting the system's maximum amplitude response ($A_{\text{max}}$), which occurs at the system's resonant frequency $f_r$, and the "half power points", which are defined by reducing the amplitude to $1/\sqrt{2}$ of its maximum value ($A_{\text{max}}/\sqrt{2}$) (see Figure 3.1).

$$Q = \frac{f_r}{f_2 - f_1}$$  \hspace{1cm} (3.1)

The frequency difference $f_2 - f_1$ is called the half power point bandwidth.
If a specific level of response is required of a system, the half power point amplitude may be specified in order to ensure good system performance. The Q-factor determines how sensitive the system is to small variations in the driving vibration frequency. As the Q-factor is inversely proportional to the half power point bandwidth, then the high-Q-factor systems are more difficult to maintain at resonance.

3.1.2. Frequency control

Traditionally, the fixed-frequency ultrasonic generator has been used for generating high-frequency ultrasonic energy and a waveguide and a tool were set up and mechanically tuned to achieve resonance. However, the high Q-factor oscillating systems, which are required for increasing the performance of the vibration machines, are more difficult to maintain at resonance. They demonstrate very high sensitivity to changing operating loads and parameters and the loss of efficiency occurs when a load is variable or with attachments of different tools. To show this sensitivity, an experiment carried out by Babitsky et al. [5] is considered.

Figure 3.2 presents an example of the magnitude responses of the transducer with and without a cutting tool attached (total mass of the tool and fixing screw is about 10g, less than 1% of the transducer’s mass). If the ultrasonic cutting system is tuned without the cutting tool (Figure 3.2a), and then the tool is attached (Figure 3.2b), the performance of the system will drop to 25% of the possible level [5].
Therefore, it often becomes necessary to reduce the Q-factor of the vibration system in order to increase the process stability, as the system with the lower Q-factor is less sensitive to variation in the frequency. Reduction of the Q-factor of a vibrating system results in a decrease in the performance of the vibration machines.

![Magnitude response of the transducer without (a) and with (b) cutting tool](image)

**Figure 3.2.** Magnitude response of the transducer without (a) and with (b) cutting tool Reproduced from [5].

To overcome this problem, the excitation frequency has to be varied in order to maintain the exciting force at the system's natural frequency and to keep the transducer in its optimum operating regime. For this purpose, the adjusted frequency generators (resonance following generators) can be used, which automatically adjust the output high frequency to match the exact resonant frequency of the concentrator/tool assembly [1, 2, 6]. The principle of operation of these generators is based on minimizing the phase difference between the voltage and current of the power supply of the transducer. Automatic resonance-following generators of this type are disclosed and claimed in United States Patent 4748365 [7].

However, when a nonlinear load acts on a vibrating system, it significantly changes the resonant frequency and influences the character of the amplitude-frequency response.
3.1.3. Nonlinear properties of ultrasonic vibrating systems
The ultrasonic cutting processes are strongly nonlinear due to their vibro-impact nature, which significantly complicates the process of vibration control. Examples of the amplitude-frequency characteristics of nonlinear vibrating systems are shown in Figure 3.3, in which the system's natural frequency is seen to be dependant on the system's vibration amplitude. Figure 3.3 a) shows hard nonlinearity, where the natural frequency of the system increases when the amplitude is increased. For a system with a soft nonlinearity (Figure 3.3 b)) the natural frequency of the system decreases with the increase in the vibration amplitude.

![Figure 3.3. Amplitude-frequency characteristics of systems with hard (a) and soft (b) nonlinearity. Reproduced from [13].](image)

From this graph we can see that, for a nonlinear system, the amplitude of vibration can have more than one value for the same excitation frequency. In this case, the way in which the exciter's frequency changes will determine which branch of the amplitude response curve is followed. It is possible that an imperceptible change in the exciter's frequency may cause a very abrupt change in the amplitude of the system. This effect is called the jump phenomenon.

The jump phenomenon of the non-linear systems is the most common problem for the maintenance of the resonant regime. The jumping of the stationary amplitude takes place as the frequency of excitation is slowly increased or decreased. A curve of this type is shown in Figure 3.4. If the exciter's frequency is raised from a small value, the amplitude of the stationary oscillation increases along the upper branch of the resonant
curve until it reaches the furthest point, A, of the distorted resonance curve. Any further increase in frequency results in the jumping of the stationary amplitude to the value corresponding to this frequency on the lower branch of the resonant curve. It can be said that the stationary amplitude jumps from A to B. In the case of dropping the frequency, the amplitude firstly follows the curve back to the point C. Here again, a jump C-D occurs as the amplitude for the smaller value of the frequency is only possible on the upper branch of the resonant curve [57].

![Figure 3.4. Explanation of the jump effect](image)

Reproduced from [57].

The returning branch A-C (shown in dotted line) corresponds to a non-stable motion and in practice this regime is physically unattainable. The part D-A of this curve corresponds to theoretically stable, periodical regimes with strong impacts. However, these regimes cannot be practically used under forced excitation, as they can only be reached by means of a slow increase in the frequency from the initial value below the left vertical line. Otherwise, the lower solid branch C-B will be obtained, which corresponds to the low-amplitude mode without impacts [57].

It is also known that the operating load can be changed within wide limits during the machining process. This leads not only to a reduction of vibration amplitude but also to a shift in the resonant frequency. The elastic component of the load can also be changed, for instance, due to the arrangement of the static load (produced by the total weight of all the moving elements of vibration machine) dictated by the operating or exploitation requirements [13].

Transformation of the amplitude responses due to increasing of the static force $P$, typical for machines using vibro-impact effects is shown in Figure 3.5.
Figure 3.5. Amplitude-frequency characteristics of a vibro-impact machine due to various values of the static force $P$.
Reproduced from [13].

It can be seen, that for a small force $P = P_1$ the amplitude response is similar to that for the idling state ($P = 0$). With increase in the force $P = P_2$ and $P = P_3$ the resonant frequency moves in the direction of higher frequencies and the character of the amplitude response changes significantly [13].

Thus, the nonlinear properties of the process of ultrasonic machining described above make frequency control an inefficient means of achieving the peak performance of the vibrating system. To overcome the problems described above, phase control can be employed.

3.1.4. Phase control

Phase control is a control method based on the fact that the phase lag between the excitation forces and the vibration of the working component during resonance is constant.

The fact that the inertia and elastic forces are mutually balanced at resonance can explain this phenomenon and the driving force overcomes only the active forces in this case. The driving force always acts in phase with the velocity of the point where the excitation is applied (or lags $\pi/2$ in phase from vibratory displacement) [13]. Therefore, the frequency (and amplitude) of oscillation can be controlled by means of changing the phase shift between the exciting force and the vibrating system’s response.
Figure 3.6 shows a three dimensional curve of phase, frequency and amplitude response for a nonlinear system. The projections of amplitude-frequency, amplitude-phase and frequency-phase are also shown. It can be seen that, although the amplitude-frequency curve is difficult to control at resonance with a conventional frequency control system due to the jump phenomena, the amplitude-phase curve has a reasonably flat peak and is single-valued; this suggests the use of phase control. It has been reported that this property takes place for a wide range of vibrating systems [14]. In the case of phase-controlled vibration, the vibration is stable at all points of the amplitude-phase curve (as the curve is single valued). As this occurs, the vibration is stable at all points of the corresponding amplitude-frequency curve, even if it is ambiguous, and some regimes are unstable under traditional forced excitation with frequency control. Figure 3.7 shows the amplitude-phase and the amplitude-frequency responses of a nonlinear system. We can see that the amplitude-phase curve is quite flat at resonance in distinction to the amplitude-frequency one, which means that by using phase control, the resonant state of vibration can be maintained much more easily than in the case of the conventional frequency control.
3.1.5. The principle of autoresonance control

Autoresonant control is a method based on phase control [58], which maintains the resonant regime of oscillation automatically by means of positive feedback using transformation (phase shift) and amplification of the signal from a sensor. It is based on the fact that during resonance the phase lag between the vibration of the working element (cutter) and the excitation force applied to the latter is constant. A general schematic of feedback is presented in Figure 3.8.

![Feedback schematic](image)

**Figure 3.8. General schematic of feedback.**

Such a system does not include an external source of excitation and therefore has no prescribed frequency of excitation. In this case, the parameters of the mechanical system and the feedback determine the frequency and the amplitude of vibration. Under a certain phase shift in the feedback circuit, the resulting vibration has the same frequency.
as the natural frequency of the mechanical subsystem without the feedback, i.e. the regime is resonant in type. By changing the phase shift, the regime of vibration can be controlled; the amplitude of vibration can be controlled by means of changing the level of amplitude limitation in the feedback circuit [9].

For harmonic, self-sustained vibration of a single-degree-of-freedom system under phase control and harmonic excitation, the resonant regime occurs when the force is in phase with the vibratory velocity (or lags \(3\pi/2\) in phase from the vibratory displacement). This system has been defined as an autoresonant one [12].

Autoresonant control is a self-sustaining vibration mode at the natural frequency of the mechanical system, which keeps the resonant regime of oscillation automatically in spite of changes in the dynamic load. Self-excitation occurs as a result of the application of feedback, producing an excitation force depending on the vibration parameters. The effectiveness of autoresonant control can be explained as being due to specific properties of amplitude-phase curves, which were discussed above. Amplitude-phase curves determine the system's behaviour; they play the same role in the phase control as the amplitude-frequency curves play in the traditional frequency control [14]. This method of control is also mentioned in the literature as constant velocity feedback [16].

Due to the stable and flat-topped nature of amplitude-phase curves, autoresonant control does not require precise tuning of the phase shift in the feedback circuit in order to maintain the system at resonance [9].

As was stated above, an autoresonant control system employs positive feedback. To clarify the differences existing between the positive feedback and negative feedback, they are considered in more detail in the next section.

### 3.2. Feedback control

The most identifiable characteristic associated with almost all control systems is feedback from the controlled system output to the reference input. Without feedback there is no means of comparing the actual behaviour of the system or process with its desired behaviour so that its performance can be automatically corrected or controlled. If the signal is inverted on its way round the control loop, the system is said to have negative feedback; otherwise, the feedback is said to be positive.

Feedback control systems have been known to exist for a very long time. The first documented use of feedback to automatically control a physical process dates back to a
float-valve-regulated water clock used in ancient Greece over 2,000 years ago. The first feedback device, which attracted the attention of the whole engineering community and was internationally accepted, was James Watt's centrifugal governor, invented in 1788 [59]. However, the conceptual framework for the theory of feedback and that of the discipline in which it is embedded - control systems engineering - have developed only since World War II [60].

In 1951 the following definition was given by the American Institute of Electrical Engineers: a feedback control system is a control system which tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control [59].

This shows that the feedback control system was initially defined as a system with negative feedback. It was also mentioned by Mayr [59] that when a signal travels around the loop, its sign must be reversed. A closed-loop system without the reversal of sign (i.e. control system with positive feedback) would be unstable; it would be a vicious circle.

Today negative feedback control systems are still much more widespread than systems with positive feedback. As a result, most methods of control theory are dedicated to control systems with negative feedback. Negative feedback is widely used for the purpose of increasing stability and accuracy, improving transient response, decreasing sensitivity etc. On the contrary, positive feedback is known to increase sensitivity, reduce system stability and cause self-oscillations [61, 62].

The above-mentioned property of positive feedback, the possibility of excitation of self-induced vibrations in the loop, is widely used in the design of signal generators and it also makes possible autoresonant control. The excitation of vibrations in autoresonant system is a result of an artificial instability of the mechanical system produced by positive feedback. Contrary to the negative feedback control forcing the system to reach the desired state, positive feedback in this case gives the system freedom to oscillate at its natural frequency.

The end result of a positive feedback is often amplifying and "explosive", i.e. a small perturbation results in big changes. This feedback will drive the system further away from its original setpoint, thus amplifying the original perturbation signal and eventually becoming explosive because the amplification often grows exponentially, or even hyperbolically.
To avoid a runaway process while using positive feedback, in practice positive feedback loops are always controlled eventually by negative feedback of some sort applying the limiting factor.

The presence of a feedback signal implies the need for a physical measurement; hence a sensor is needed to continuously monitor the output variable. Different types of sensors suitable for the autoresonant control system will be considered in the next section.

3.3. Strategies of autoresonant control

The consideration of different control methods completed in section 3.1, showed that autoresonant control is the most advanced means of control of ultrasonically assisted cutting. It can be seen from the description of autoresonant control given in 3.1.5 that the signal from the sensor plays a very important role in the design of the control system. It serves as a source of the control signal needed for the autoresonant excitation and thus the effectiveness of the control system depends on the efficiency of the sensor.

A sensor is a device that detects the value or the change of value of a physical quantity or parameter and converts the value into an electrical signal for providing a measurement, operating a control, or both.

A measured characteristic will define the design of the sensor, its location within the system and the possibility of its application for ultrasonic machining.

While choosing a sensor suitable for UAM, the following requirements have to be considered:

1. The working frequency should be within the ultrasonic range (from 10-100 kHz).
2. Capability to withstand the prolonged and high level of vibration necessary for the ultrasonic machining.
3. Possibility of remote operation or the permanent fixing of the sensor to the transducer, ensuring the usage of the sensor within an industrial environment (immersion in coolant, impacts from metal chips, etc.).

To measure the ultrasonic vibration of the system the following types of sensors can be considered:

- Accelerometer,
- Ultrasonic microphone,
- Laser vibrometer,
- Current sensor,
- Inductive sensor.

For the first four sensors there are some recommendations from the previous research, which will be taken into account [5]. The latter sensor (as far as the author is aware) has never been used in UAM before.

An inductive sensor is an electromagnetic transducer consisting of the coil, placed in the magnetic field, and the movable magnetic core (see Figure 3.9), which is moving when affected by the measured characteristic [63].

![Figure 3.9. The principle of the inductive displacement sensor operation. Reproduced from [64]](image)

Core motion changes the reluctance and the inductance in the coil correspondingly. Thus every new position of the core produces a different inductance and the inductor and movable core assembly may be used as a displacement sensor. Thus, an AC bridge or other active electronic circuit sensitive to inductance may be employed for signal conditioning.

Depending on choice of the sensor, two different types of control can be employed:

- Mechanical feedback, when the sensor measuring the mechanical characteristics of the oscillations (accelerometer, microphone, laser vibrometer or inductance sensor) is used for the control system.
- Electrical feedback, which uses the signal from the sensor measuring the electrical characteristics of the piezoelectric transducer (current sensor).

Each of the strategies has their advantages and drawbacks. The electrical feedback is more convenient to use within industrial conditions, as the current sensor does not need
to be placed in the cutting zone and can be used remotely. However, the sensor output reflects the oscillations of the system in an indirect way and the appropriateness and the efficiency of this reflection has to be investigated.

Mechanical feedback uses a sensor directly measuring the oscillations of the ultrasonic system and, by controlling the signal from this sensor, the actual state of vibration can be controlled in the most efficient way. However, the difficulties occur with the use of such sensors within the industrial environment. As was found by Babitsky et al. [5], the accelerometer had some problems with its permanent fixing to the transducer in conditions with high levels of vibration. The laser vibrometer and the microphone provide the possibility of remote operation and do not need to be fixed to the transducer. However, the cuttings produced during the machining process and the coolant splashes interfere with the laser beam and considerably decrease the efficiency of the laser vibrometer. The output signal of the microphone has a high level of distortion, as this instrument responds to extraneous sounds in the vicinity (noise of the working machine); therefore it has to be filtered before applying the control system algorithm.

The preliminary experiments completed by our research group showed that the inductive sensor is relatively easy to use and successfully copes with the problems of previously discussed sensors. Nevertheless, the detailed investigation and the elaboration of the appropriate design and the employment procedure have yet to be done for this sensor.

Thus, from the preliminary consideration we can assume that the control system based on electrical feedback will be easier to realize than the control system based on mechanical feedback. However, the sensor measuring the vibration will give more realistic information about the level of oscillations of the ultrasonic system. Therefore, the control system based on mechanical feedback may provide better control over the process of machining than the control system based on electrical feedback. These assumptions will be further analyzed.

The available information about the existing control systems shows that they either use the frequency control principle [18-20], which is potentially inefficient as was shown above, or employ phase control with application of the electrical feedback principle [21, 16]. The results are seen to be good. However, no research has been devoted so far to the investigation of the electrical feedback properties and its efficiency has not been proved.
Currently the only company that sells an ultrasonically assisted cutting machine commercially is Fuji Ultrasonic Engineering Company Ltd [17]. The design of the control system employed in this machine has not been published. However, from personal collaboration with this company, it was possible to find out that the machine also uses an electrical characteristic for the feedback loop.

Thus, the control strategy providing the efficient and reliable control of vibration is still to be found. The detailed investigation of the electrical feedback and the mechanical feedback is required. The efficiency of the electrical feedback control system has to be clarified and the effectiveness of the proposed control strategies has to be compared.

The detailed investigation and comparison of control strategies based on mechanical feedback and electrical feedback is the main subject of this research. The possible benefits and drawbacks of every control strategy will be revealed and considered. The method of investigation is by using numerical simulations, which requires the creation of a model of the ultrasonic vibrating system and a model of the control system.
Chapter Four  Model of the ultrasonic vibrating system

The existence of the model of the ultrasonic vibrating system provides numerous possibilities for the investigation of its properties. However, the ultrasonic transducer is a complex system with many degrees of freedom and creating a realistic model is a complicated task. In the literature, a large number of papers are dedicated to the modelling of ultrasonic transducers, however, the majority of them concentrate on finite element modelling [65]. The development of one-dimensional numerical computer models accommodating nonlinear impact has been hardly discussed at all. One-dimensional numerical models process the information required much faster than FE models - i.e. simulations are less demanding on a computer and therefore they appear to be more convenient for the evaluation of new control concepts, which can be done easily, comprehensibly and quickly. This chapter is devoted to the modelling of the loaded ultrasonic vibrating system in one dimension and considers the process of system simplification, the development of methods for calculating the model parameters and the nonlinear impact modelling. The last section of this chapter presents a general schematic for the numerical computer model of an ultrasonic transducer created in Matlab-Simulink.

To be useful for the investigation of control concepts, the developed mathematical model should not be so detailed that the essential features of the actual system are obscured. On the other hand, the model should not be so simple that important features of the physical system are omitted [66]. Thus, in order to limit the complexity of any model we wish to make of the system, we must neglect some of the system's features. And, in fact, many of the parameters may be relatively unimportant for the objective of a particular study [67]. The process of system simplification employed in this work is explained in Figure 4.1. The process allows the vibrating system to keep the important properties of the original whilst making the model accessible for simulation.
Figure 4.1 (a) represents the ultrasonic transducer consisting of the piezoelectric transducer, concentrator and back section.

Due to the existence of a nodal point between the piezoceramic rings in the working regime of the transducer, the back section of the transducer can be neglected and an ultrasonic transducer can be substituted with the model consisting of one piezoceramic ring and a concentrator (see Figure 4.1 (b)). The left end of this structure is treated as immovable.

The strong filtering effect of the concentrator permits considering the model of the concentrator of the ultrasonic transducer as a two-degree-of-freedom (2-DOF) system, where the first and the second modes of vibration of the concentrator correspond to the first and the second modes of vibration of the 2-DOF system.

Thus the ultrasonic transducer can be represented as the simplified model shown in Figure 4.1 (c). This model consists of two parts:

- The model of the piezoelectric transducer.
- The model of the concentrator - 2-DOF vibrating system.
The next step is the calculation of the model's parameters based on existing information, dimensions, constants and so on, which has to be accomplished for both the model of the concentrator and the model of the piezoelectric transducer.

4.1. Model of the concentrator

There are many combinations of parameters for a 2-DOF system which can be used to create the model of the concentrator. To find the "right" combination of parameters, the method of calculation based on eigenvalues, eigenvectors and the energy of the original concentrator was developed. This will be described in greater detail.

4.1.1. Calculation of oscillation modes and the natural frequencies of the concentrator

Initially, the eigenvalues of the distributed parameters model of the concentrator will be defined. Figure 4.2 depicts the design of the step concentrator used in this work.

The concentrator consists of a two stepped bar with different cross-sectional areas. The coordinate of the connection cross section of the two steps is \( x_c \); whilst \( S_1, S_2 \) are the cross-sectional areas of the corresponding first and second steps; \( L \) is the full length of the concentrator. In order to describe the longitudinal vibration within the bar, the cross-section is assumed to remain flat during the vibration and the particles lying in the cross-sections are assumed to move only in the \( x \) direction. Let \( v(x,t) \) describe the displacement of the cross-section \( x \) from its position within the undeformed bar. The equation for the longitudinal vibration within the bar can be written in the form:

\[
\mu \frac{\partial^2 v}{\partial t^2} = S_i E \frac{\partial^2 v}{\partial x^2},
\]  

(4.1)
where \( i = 1, 2 \) is the number of the bar considered, \( \mu_i = \rho S_i \) is the mass per unit length of the bar, \( \rho \) is the bar material density and \( E \) is the modulus of elasticity.

Assume that the bar vibrates harmonically and consider the solution in the form:
\[
\nu(x, t) = V(x) \sin \omega t
\]  
(4.2)

Substitution of (4.2) into (4.1) gives
\[
\frac{d^2V}{dx^2} + \frac{\mu \omega^2}{S_i E} V = 0
\]  
(4.3)

Rearranging (4.3) when \( \mu_i = \rho S_i \) gives
\[
\frac{d^2V}{dx^2} + \frac{\rho \omega^2}{E} V = 0
\]  
(4.4)

A general solution of (4.4) can be written in the form
\[
V_i(x) = A_1 \sin \kappa x + A_2 \cos \kappa x
\]  
(4.5)

where \( V_i(x) \) denotes the displacement of the cross-section \( x \) within the first bar, \( A_1 \) and \( A_2 \) are arbitrary constants,
\[
\kappa = \sqrt{\frac{\rho \omega^2}{E}}
\]  
(4.6)

Taking into account that the first bar is fixed at the left end, the first boundary condition is obtained:
\[
V_1|_{x=0} = 0.
\]  
(4.7)

Application of the condition (4.7) transfers (4.5) into (4.8)
\[
V_1(x) = A \sin \kappa x
\]  
(4.8)

where \( A = A_1 \).

Eq. (4.8) gives us the mode of oscillation within the first bar.

Longitudinal vibration within the second bar (Figure 4.2) can be expressed by the same Eq. (4.4) as for the first bar.

In this case the general solution of (4.4) will be written in the form
\[
V_2(x) = B_1 \sin \kappa x + B_2 \cos \kappa x
\]  
(4.9)

Where \( V_2(x) \) denotes the displacement of the cross-section \( x \) within the second bar, \( B_1 \) and \( B_2 \) are arbitrary constants and coefficient \( \kappa \) is defined according to (4.6).
Knowing that the right end of the second bar is free, the second boundary condition is obtained:

\[ \frac{dV_2}{dx} \bigg|_{x=L} = 0, \quad (4.10) \]

where \( L \) is the full length of the concentrator.

Taking into account the condition (4.10), Eq. (4.9) transfers to

\[ V_2(x) = B \cos \kappa (x - L). \quad (4.11) \]

where \( B = B_1 \).

Eq. (4.11) describes the mode of vibration within the second bar. Thus the modes of vibration for both the first and the second bars are obtained.

Next, the correlation between amplitudes \( A \) and \( B \) is to be found. This can be done by applying the conditions for the cross-section with coordinate \( x_c \) where the two steps join together.

The first condition for the connection point is obvious and can be written as:

\[ V_1|_{x=x_c} = V_2|_{x=x_c}. \quad (4.12) \]

Use of the condition (4.12) together with (4.8) and (4.11) gives:

\[ A \sin \kappa x_c = B \cos \kappa (x_c - L), \quad (4.13) \]

From there, the correlation between amplitudes \( A \) and \( B \) is found to be:

\[ B = \frac{A \sin \kappa x_c}{\cos \kappa (x_c - L)}. \quad (4.14) \]

Figure 4.3 shows a possible example of the vibration mode of the concentrator. \( x_c \) is the coordinate of the connection point of the two steps.
Thus, the mode of oscillation of the concentrator is now found and can be written in the following form:

\[
V(x) = \begin{cases} 
A \sin \kappa x, & 0 < x < x_c \\
B \cos \kappa (x - L), & x_c < x < L,
\end{cases}
\] (4.15)

where correlation between \( A \) and \( B \) is described by (4.14).

To find the second condition for the connection point, a small element of the concentrator \( dx \) where two steps join together (Figure 4.4) will be considered.

Figure 4.4. Element of the concentrator where two steps join together.

Figure 4.4 depicts the small element \( dx \) with its two forces acting to the left and to the right parts of the concentrator; the forces can be defined according to the following formula \( N = S \sigma \), where \( S \) is the cross-sectional area and \( \sigma \) is the stress.

Applying Newton's law, the equation of motion for the element \( dx \) can be written as:

\[
\Delta m \ddot{v} = S_2 \sigma_2 - S_1 \sigma_1,
\] (4.16)

where \( \Delta m \) is the mass of the element \( dx \).

As the element \( dx \) is very small, its mass tends to zero and (4.16) becomes

\[
S_2 \sigma_2 = S_1 \sigma_1.
\] (4.17)

The dependence of the stress \( \sigma \) on the relative strain \( \varepsilon \) is given in the following form:

\[
\sigma = E \varepsilon = E \frac{\partial v}{\partial x}.
\] (4.18)

Substituting the relation (4.18) into (4.17) and rearranging, the second condition for the connection point is obtained

\[
\frac{\partial v_2}{\partial x} \bigg|_{x=x_c} = \frac{S_1}{S_2} \frac{\partial v_1}{\partial x} \bigg|_{x=x_c}.
\] (4.19)

Taking into account (4.8) and (4.11), the Eq.(4.19) can be rewritten as:

\[
-B \sin \kappa (x_c - L) = \frac{S_1}{S_2} A \cos \kappa x_c
\] (4.20)
Solving together (4.13) and (4.20), gives (4.21), which determines the eigenvalues of the distributed parameters model of the concentrator.

\[ \tan \kappa x_c \tan \kappa (x_c - L) = -\frac{S_1}{S_2} \] (4.21)

A graphical solution of the Eq. (4.21) for $\kappa L$ for the concentrator having parameters (4.22), which is used in this work as a prototype for the model, is presented in Figure 4.5.

\[ R_1 = 27 \times 10^{-3} \, m; \quad R_2 = 16 \times 10^{-3} \, m; \]
\[ L_1 = 0.12 \, m; \quad L_2 = 0.06 \, m; \quad L = L_1 + L_2 = 0.18 \, m; \]
\[ x_c \approx 0.67L \]
\[ E = 0.7 \times 10^{11} \, N/m^2; \quad \text{(aluminium)} \]
\[ \rho = 2.7 \times 10^3 \, kg/m^3; \]
\[ S_1 = \pi R_1^2 = 2.3 \times 10^{-3} \, m^2; \quad S_2 = \pi R_2^2 = 0.8 \times 10^{-3} \, m^2; \] (4.22)

where $R_1, R_2$ are the radii and $L_1, L_2$ are the lengths of the first and second steps of the concentrator respectively.

Figure 4.5. Graphical solution of the Eq. (4.21) for $\kappa L$: left side of the equation shown as a thin curve, right side of the equation is the bold line.

Figure 4.5 depicts the graphical solution of Eq. (4.21), where the thin curve represents the left side of the equation (product of tangents) and the bold line depicts the ratio of
the areas (right side of the equation). Intersection points \((p_1, p_2, ..., p_n)\) can be used to find the natural frequencies of oscillation, using the following expression:

\[ \kappa L = p_i. \]  

(4.23)

Substituting the expression (4.6) into (4.23), the following formula is obtained:

\[ \omega_i = \frac{\sqrt{E}}{p_i} \]  

(4.24)

where \(\omega_i, (i = 1, 2, \ldots n)\) are the natural frequencies of the concentrator.

Using the formula (4.24) and values \(p_i\) from the graph (Figure 4.5), the following values for the first and second natural frequencies have been found:

\[ \omega_1 = 5.5 \times 10^4 \text{ rad/s} \Rightarrow f_1 = 8.8 \text{ kHz} \]

\[ \omega_2 = 1.34 \times 10^5 \text{ rad/s} \Rightarrow f_2 = 21.3 \text{ kHz} \]  

(4.25)

The second mode of oscillation has been chosen as the working frequency for the model of the ultrasonic transducer, as it provides the mode of oscillation when the first and second bodies vibrate in anti-phase, which is normally used in practice.

\[ \omega = 1.34 \times 10^5 \text{ rad/s} \Rightarrow f = 21.3 \text{ kHz} \]  

(4.26)

In order to validate the derived Eq. (4.21), the special case when \(S_1 = S_2\) has been considered:

\[ \tan \kappa x_c \tan \kappa (x_c - L) = -1 \]  

(4.27)

Figure 4.6. Graphical solution of the Eq. (4.21) for the special case when \(S_1 = S_2\) - dashed line.
Figure 4.6 shows the graphical solution of Eq. (4.27), where the dashed line represents the right side of the equation and the thin curve depicts the left side of the equation (the same as in Figure 4.5). The bold line depicts the right side of Eq.(4.21) and is displayed for consistency with the previous figure. Using the expression (4.24) and the intersection points of the dashed line with the thin curve from the Figure 4.6, the natural frequencies for the case of equal areas have been found:

\[
\omega_1 = 4.5 \times 10^4 \text{ rad/ s} \\
\omega_2 = 1.3 \times 10^5 \text{ rad/ s}
\] (4.28)

The natural frequencies of a uniform bar fixed from one side and free at the other found using the formula (4.29) [68] appear to be in close agreement with the natural frequencies (4.28), which validates the derivation procedure of the Eq.(4.21).

\[
\omega_i = \frac{(2i -1)\pi}{2L} \sqrt{\frac{E}{\rho}}
\] (4.29)

4.1.2. Eigenvectors' analysis

It has already been mentioned that the 2-DOF system can be substituted for the distributed parameters model of the concentrator.

Initially, the undamped 2-DOF system shown in Figure 4.7 will be considered. The equations of motion for this 2-DOF system are derived by applying Newton's second law to each of the bodies:

\[
\begin{align*}
\begin{cases}
m_1 \ddot{x}_1 &= -k_1 x_1 + k_2 (x_2 - x_1) \\
m_2 \ddot{x}_2 &= -k_2 (x_2 - x_1)
\end{cases}
\end{align*}
\] (4.30)

The displacements \( x_i(t) \) and \( x_2(t) \) are measured from the static equilibrium positions of the bodies.
Assume the solutions of (4.30) in the form:

\[
\begin{align*}
    x_1 &= A_1 \sin \omega t \\
    x_2 &= A_2 \sin \omega t
\end{align*}
\]  

(4.31)

where \( A_1, A_2 \) are constants and \( \omega \) is a natural frequency of the system.

Substitution of (4.31) in the differential equations (4.30) gives:

\[
\begin{align*}
    (k_1 + k_2 - \omega^2 m_1)A_1 - k_2 A_2 &= 0 \\
    -k_2 A_1 + (k_2 - \omega^2 m_2)A_2 &= 0
\end{align*}
\]  

(4.32)

In order to find the parameters of the 2-DOF system, representing the model of the concentrator, eigenvectors analysis will be carried out.

Eq (4.32) expressed in matrix form transforms to:

\[-M \lambda \Psi = K \Psi \]  

(4.33)

where \( M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \) is the mass matrix, \( K = \begin{bmatrix} -k_1 - k_2 & k_2 \\ k_2 & -k_2 \end{bmatrix} \) is the stiffness matrix, \( \lambda \) is the eigenvalues matrix (4.34) and \( \Psi \) is the eigenvectors matrix (4.35).

\[
\lambda = \omega^2 = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix}
\]  

(4.34)

\[
\Psi = \begin{bmatrix} \psi_1 & \psi_2 \\ 1 & 1 \end{bmatrix}
\]  

(4.35)

where \( \psi_{1,2} \) are the normalised amplitudes of oscillation of the first body when the first/second mode is excited.

To simplify calculations, the following substitution is proposed:

\[
G = -M^{-1} K = \begin{bmatrix} g_1 & g_2 \\ g_3 & g_4 \end{bmatrix} = \begin{bmatrix} (k_1 + k_2) / m_1 & -k_2 / m_1 \\ -k_2 / m_2 & k_2 / m_2 \end{bmatrix}
\]  

(4.36)

Using substitution (4.36), the Eq. (4.33) transfers to:

\[G \Psi = \lambda \Psi\]  

(4.37)
From (4.37), accounting (4.34)-(4.36) the following four equations are obtained:

\[
\begin{aligned}
g_1\psi_1 + g_2 &= \omega_1^2 \psi_1 \\
g_1\psi_1 + g_4 &= \omega_1^2 \\
g_1\psi_2 + g_2 &= \omega_2^2 \psi_2 \\
g_3\psi_2 + g_4 &= \omega_2^2 
\end{aligned}
\]  

(4.38)

However, it can be seen from (4.36) that \( g_4 = -g_3 \), which gives us the following condition:

\[
\omega_1^2 - \omega_2^2 = \omega_1^2 \psi_2 - \omega_2^2 \psi_1
\]

(4.39)

and reduces the system (4.38) to a system of three equations (4.40), which can be solved under the condition (4.39).

\[
\begin{aligned}
g_1\psi_1 + g_2 &= \omega_1^2 \psi_1 \\
g_1\psi_1 + g_4 &= \omega_1^2 \\
g_3\psi_2 + g_2 &= \omega_2^2 \psi_2 
\end{aligned}
\]  

(4.40)

From the experimental results, the eigenvector for the second mode is defined as:

\[
\Psi = \begin{pmatrix} -0.5 \\ 1 \end{pmatrix} \Rightarrow \psi_2 = -0.5
\]

(4.41)

which means that bodies vibrate in anti-phase (and there is a nodal point between them) and the amplitude of vibration of the second body is approximately twice as large as the amplitude of vibration of the first body.

From (4.39) with (4.25) and (4.41), the eigenvector for the first mode of oscillation is obtained:

\[
\Psi = \begin{pmatrix} 0.75 \\ 1 \end{pmatrix} \Rightarrow \psi_1 = 0.75
\]

(4.42)

4.1.3. Calculation of the parameters of the 2-DOF system

Solving the system (4.40) for the variables \((k_1, k_2, m_1, m_2)\) will give the infinite aggregate of solutions. This can be explained by the fact that there are many combinations of
parameters \((k_1, k_2, m_1, m_2)\) of the 2-DOF system (Figure 4.7) providing oscillations with the natural frequencies (4.25) and the amplitudes (4.41) and (4.42).

In order to fix the parameters and find the 2-DOF system serving as an exact equivalent of the prototype of the concentrator used for the modelling, the system of four equations has to be solved (one equation for each parameter).

To find one more (the 4th) equation, the statement that the full energy of the distributed parameters model of the concentrator (Figure 4.2) is equal to the full energy of the 2-DOF system will be used. Values for energy will be calculated for the second mode of oscillation as it has been chosen as a working mode.

Energy of the ultrasonic transducer can be found using the following formula:

\[
U = \frac{E}{2} \left[ S_1 \int_{x_0}^{x_r} \left( \frac{dv_1}{dx} \right)^2 dx + S_2 \int_{x_0}^{L} \left( \frac{dv_2}{dx} \right)^2 dx \right].
\]  \hspace{1cm} (4.43)

Taking into consideration the mode of oscillation (4.15), the formula (4.43) can be rewritten as:

\[
U = \frac{E}{2} \left[ S_1 \int_{0}^{\psi_2} A^2 \kappa^2 \cos^2 \kappa x \, dx + S_2 \int_{\psi_2}^{L} B^2 \kappa^2 \sin^2 \kappa(x - L) \, dx \right].
\]  \hspace{1cm} (4.44)

Calculating (4.44) with (4.22) and (4.23) for the second mode of oscillation \((A = \psi_2 = -0.5, B = 1)\) gives:

\[
U = 1.42 \times 10^9 \, J.
\]  \hspace{1cm} (4.45)

The full energy of the 2-DOF system can be found as

\[
U = \frac{k_1 A^2}{2} + \frac{k_2 (B - A)^2}{2}.
\]  \hspace{1cm} (4.46)

Equating (4.45) and (4.46), the 4th equation is found:

\[
\frac{k_1 \psi_2^2}{2} + \frac{k_2 (1 - \psi_2)^2}{2} = 1.42 \times 10^9.
\]  \hspace{1cm} (4.47)

Combining the system of three equations (4.40) with the equation (4.47), the following system is obtained:
\[
\begin{align*}
  k_1 \psi_2^2 + k_2 (1 - \psi_2)^2 &= 2.84 \times 10^9 \\
  \psi_1 (-m_1 \omega_1^2 + k_1 + k_2) - k_2 &= 0 \\
  k_2 - m_2 \omega_1^2 - k_2 \psi_1 &= 0 \\
  \psi_2 (-m_1 \omega_2^2 + k_1 + k_2) - k_2 &= 0
\end{align*}
\]  

(4.48)

Solving the system (4.48) together with (4.25), (4.41) and (4.42) allows the parameters of the concentrator to be found:

\[
\begin{align*}
  m_1 &= 0.25 \text{kg} \\
  m_2 &= 0.1 \text{kg} \\
  k_1 &= 1.14 \times 10^9 \text{N/m} \\
  k_2 &= 1.13 \times 10^9 \text{N/m}
\end{align*}
\]  

(4.49)

Thus the combination of parameters of the 2-DOF system that provides the closest correspondence with the distributed parameters model of the concentrator is defined (4.49).

### 4.2. Model of the Piezoelectric Transducer

The next step is the calculation of the parameters of the model of the piezoelectric transducer. According to the definition, piezoelectricity is a coupling between a mechanical and an electrical behaviour of a medium. That is why the most important function of the model of the piezoelectric transducer is to provide the correlation between the electrical and mechanical parameters of the ultrasonic system. In other words, the model has to define the interaction between the displacement of the piezoelectric transducer, the load from the concentrator and the voltage, supplied to the piezoceramic rings.

To a good approximation, the interaction between the electrical and mechanical qualities of the piezoceramic can be described by linear relations between electrical and mechanical variables:

\[
\begin{align*}
  \varepsilon &= s^E \sigma + d \Sigma \\
  \Sigma &= -h \varepsilon + \frac{D}{\varepsilon^s}
\end{align*}
\]  

(4.50)

where \( \varepsilon \) is the strain, \( \sigma \) is the applied stress, \( s^E \) is the elastic compliance at constant electric field, \( d \) is the piezoelectric charge constant, \( h \) is the piezoelectric deformation
constant, $\Sigma$ is the field strength, $D$ is the dielectric displacement and $\xi^s$ is the permittivity at constant strain [69, 70].

Taking into account that $\varepsilon = \frac{x_0}{l_0}$, $\sigma = \frac{F_0}{S_0}$, $\Sigma = \frac{u}{l_0}$, $D = \frac{q}{S_0}$ and rearranging, the Eqs.(4.50) transfer to:

$$x_0 = \frac{l_0 \varepsilon}{S_0} F_0 + d u$$

(4.51)

$$q = \frac{h \varepsilon^s S_0}{l_0} x_0 + \frac{\xi^s S_0}{l_0} u$$

(4.52)

where $S_0$ is the area and $l_0$ is the thickness of the piezoceramic plate, $x_0$ is the amplitude of deformation for a single piezoceramic plate, $F_0$ is the force applied to the piezoelectric transducer from the concentrator (will be further considered) and $u$ is the voltage supplied to the piezoceramic plates.

Thus, Eq. (4.51) describes the state of the piezoelectric transducer and provides the basis for the creation of a model. The model specifies the interaction between the mechanical $F_0, x_0$ and electrical $u$ parameters of the ultrasonic system. Values $l_0, S_0, s^\varepsilon$ and $d$ will depend on the properties of the particular piezoelectric transducer and are chosen as follows:

$$S_0 = 2.29 * 10^{-3} \text{ m}^2,$$

$$l_0 = 6.11 * 10^{-3} \text{ m},$$

$$s^\varepsilon = 14.4 * 10^{-12} \text{ m}^2 / N,$$

$$d_{33} = 340 * 10^{-12} \text{ m} / V,$$

$$h_{33} = 4.80 * 10^9 \text{ V} / \text{ m},$$

$$\xi_{33}^s = 635 * \xi_0, \quad \xi_0 = 8.85 * 10^{-12} \text{ F} / \text{ m}$$

(4.53)

Substituting the coefficients (4.53) into equation (4.51), the equation for the displacement of the prototype of the piezoelectric transducer used for the modelling is obtained:

$$x_0 = \frac{1}{k_{01}} F_0 + k_{02} u,$$

(4.54)

where $k_{01} = 1/3.84 * 10^{-11} \text{ (N/m)}, \quad k_{02} = 3.4 * 10^{-10} \text{ (V/m)}$. 

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Eq. (4.52) describes the charge of the piezoelectric transducer. Substituting the coefficients (4.53), the equation describing the charge for the piezoelectric transducer used for modelling is found:

\[ q = k_{03}x_0 + k_{04}u, \quad (4.55) \]

where \( k_{03} = 10.1 \, (C/m) \), \( k_{04} = 2.1 \times 10^{-9} \, (C/V) \).

Differentiating the charge with respect to time gives the current of the piezoelectric transducer, which will be further used for the control system based on electrical feedback.

**4.3. Model of the ultrasonic transducer**

Now when the model of the concentrator and the model of the piezoelectric transducer have both been produced, a full model of the ultrasonic transducer, consisting of the concentrator and the piezoelectric transducer, can be derived (Figure 4.8).

![Figure 4.8. Model of the ultrasonic transducer.](image)

Equations of motion for the system shown in Figure 4.8 can be written as:

\[
\begin{cases}
    m_1 \ddot{x}_1 = -c_1(x_1 - x_0) - k_1(x_1 - x_0) + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) \\
    m_2 \ddot{x}_2 = -c_2(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1)
\end{cases}
\quad (4.56)
\]

From here

\[ F_0 = c_1(\dot{x}_1 - \dot{x}_0) + k_1(x_1 - x_0) \quad (4.57) \]

is the force applied to the piezoelectric transducer from the concentrator and \( x_0 \) is the displacement of the piezelement, as described by Eq.(4.51).

Thus, the Eqs.(4.51), (4.56) and (4.57) fully describe the model of the ultrasonic transducer, which has been used for the simulation. The parameters of the undamped, free, 2-DOF system \((k_1, k_2, m_1, m_2)\) have been calculated in the previous section. Damping coefficients \( c_1, c_2 \) can be selected to provide the amplitudes of oscillation of
the first and second bodies just as the amplitudes of vibrations of the first, $A_1$, and the second, $A_2$, parts of the concentrator, which are obtained experimentally:

$$A_1 = 10 \mu m$$
$$A_2 = 20 \mu m.$$  \hspace{1cm} (4.58)

It is considered here, that the vibration of the first body of the model corresponds to the vibration of the middle of the first step of the concentrator and that the vibration of the second body of the model corresponds to the vibration of the end of the concentrator.

With the use of (4.58), the damping coefficients $c_1, c_2$ have been selected as follows:

$$c_1 = 6.75 \frac{Ns}{m}$$
$$c_2 = 5.5 \frac{Ns}{m}.$$  \hspace{1cm} (4.59)

Thus the model of the ultrasonic transducer consisting of the piezoelectric transducer and the concentrator has been derived. Based on the Eqs (4.54), (4.56), (4.57) and the parameters (4.49) and (4.59), the Matlab-Simulink model of the system described above was created. The resonant frequency of oscillations of the created model of the ultrasonic transducer without the load applied is:

$$f = 20.96 kHz,$$  \hspace{1cm} (4.60)

which closely correspond to the theoretically calculated value (4.26). Excited with the working frequency (4.60), the model provides oscillations with amplitudes (4.58). Simulation results are shown in Figure 4.9 and Figure 4.10.

Figure 4.9 shows the transient process, where the central light curve depicts the displacement of the piezoelectric transducer ($x_o$), the middle dark curve represents the displacement of the first body ($x_1$) and the outer light curve shows the displacement of the second body ($x_2$).
Figure 4.9. Simulation results. Transient state. Central light curve-displacement of the piezoelement, middle dark curve-displacement of the first body, external light curve-displacement of the second body.

Figure 4.10 depicts the large-scale fragment of Figure 4.9 for the steady state vibrations. Here, the dotted curve represents the displacement of the piezoelectric transducer, the displacement of the first body is shown as a dashed curve and the solid line depicts the displacement of the second body. The following observations can be made from the graph:

- The amplitude of vibration of the second body is $20\,\mu m$ and the amplitude of vibration of the first body is $10\,\mu m$, which complies with (4.58);
- Bodies vibrate in anti-phase and the amplitude of vibration of the second body is twice as large as the amplitude of vibration of the first body, which satisfies (4.41).
4.4. Nonlinear load design

Application of a load during the process of machining brings strong nonlinearity to the system and instability to the process of control. That is why it is important to simulate the influence from the machining process in the model to investigate the performance of the control system under the load.

One-dimensional contact interaction between the ultrasonic transducer and a workpiece can be described approximately with the help of a viscoelastic limiter, known as a Kelvin-Voigt model, Figure 4.11.
The limiter is modelled schematically as a parallel working linear spring with stiffness \( k \) and a dashpot with damping coefficient \( c \). The initial gap between the ultrasonic transducer and the viscoelastic limiter is defined as \( \Delta \); negative \( \Delta \) corresponds to the initial interference. Such a model describes the dynamic loading of the ultrasonic transducer due to processing [8].

The dynamic response of the limiter is described as:

\[
F = \begin{cases} 
  kx + cx, & x > 0, \quad kx + cx > 0 \\
  0, & x > 0, \quad kx + cx \leq 0 \\
  0, & x \leq 0
\end{cases}
\]  

(4.61)

where \( x = x_2 - \Delta \), \( k \) is the contact stiffness, \( c \) is the contact damping, \( \Delta \) is the initial interference/gap (\( \Delta \) will be further used as the initial interference) and \( x_2 \) is the displacement of the second body (end of the concentrator).

In order to calculate the parameters of the viscoelastic limiter, the oscilloscope reading showing the interaction force between the tool and a workpiece obtained experimentally is considered (Figure 4.12) [71, 8].

---

**Figure 4.11.** Model of interaction of the ultrasonic transducer with a load.

**Figure 4.12.** Oscilloscope reading for the interaction force between the tool and the workpiece. Reproduced from [71].
Figure 4.12 shows that the interaction force has an impulsive character. The force acts during the interval \( t_n \), with the amplitude \( F \). For the rest of the period \( T - t_n \), the tool vibrates whilst being separated from the workpiece, i.e. the interaction force is equal to 0.

It can be seen from the graph that the duration of a contact is:

\[
t_n \approx 0.5T
\]  
(4.62)

Both initial interference \( \Delta \) and contact stiffness \( k \) exert an influence on the interaction force and impact duration correspondingly. This means that the values of the parameters \( \Delta \) and \( k \) providing the impact duration (4.62) can be taken for the model. The magnitude of \( \Delta \) can be chosen as any sufficiently small value, as, in practice, initial interference depends on the workpiece material and machining conditions. When the value of the initial interference has been chosen, the value of the contact stiffness can be found to comply with the condition (4.62). The contact damping coefficient for the viscoelastic limiter can be selected so as to make the amplitude of oscillation of the second body of the model the same as the amplitude of oscillation of the end of the concentrator during the process of dynamic loading as obtained experimentally:

\[
A_i \approx 10 \mu m.
\]  
(4.63)

Thus, based on (4.62) and (4.63), the following parameters for the nonlinear load unit have been chosen:

\[
\Delta = -2 \times 10^{-7} m
\]

\[
k = 1 \times 10^8 N/m.
\]  
(4.64)

\[
c = 32 \frac{Ns}{m}
\]

Figure 4.13 shows simulation results for the model of the ultrasonic transducer with the load applied. The solid curve in Figure 4.13 (a) depicts the displacement of the second body under the condition of nonlinear load applied. It can be seen that the amplitude of oscillations is equal to 10\( \mu m \), which complies with (4.63). The second observation made from this graph is the lack of symmetry of oscillations about the zero line, which appears to be due to loading. This effect was not present for the free oscillations shown in Figure 4.10.
Figure 4.13. Simulation of ultrasonic transducer with the load applied: (a) solid curve is the displacement of the second body \( x_2 \); the dashed line is the interference value \( \Delta \), (b) load applied.

Figure 4.13 (b) shows the load applied to the second body, which represents the interaction force between the end of the concentrator and the workpiece. A comparison of the graph for the interaction force obtained as a result of the simulation, Figure 4.13 (b), with the interaction force obtained as a result of experiment, Figure 4.12, shows their agreement. This proves the validity of the created model of the contact interaction.

4.5. Matlab-Simulink model

Finally, the mathematical models of all parts of the ultrasonic vibrating system and the model representing the influence of the machining process have been developed. Initially, these equations are solved numerically to allow subsequent programming. Based on the described formulae, Eqs. (4.54), (4.56), (4.57) and (4.61), a block diagram of a system has been drawn in order to prepare to solve these equations numerically [67]. To simulate this system, a numerical computational environment, Matlab with the Simulink package providing a graphical user interface for building system models and executing the simulations, has been used. Making use of the calculated parameters (4.49), (4.59) and (4.64), the Matlab-Simulink model of the system was created.
general schematic of this model is presented in Figure 4.14. The design of all shown blocks is given in the appendix.

![Diagram of ultrasonic vibrating system](image)

**Figure 4.14.** Matlab-Simulink model of the ultrasonic vibrating system with a non-linear load and a possibility of autoresonant control.

The model provides several regimes of oscillations:

- **Forced oscillations.** Open loop oscillations using the sine wave with prescribed frequency and amplitude of oscillation as a voltage signal for the piezoceramic transducer (Figure 4.14, switch 1).

- **Autoresonant control.** Closed loop oscillations with a positive feedback, providing phase shifting and amplification of a signal from a sensor. Two main types of control strategy (mechanical feedback and electrical feedback) have been implemented (Figure 4.14, switch 2). The design of the model of the control system will be considered in the next chapter.

- **Loaded oscillations.** Both first and second regimes can be used with or without a nonlinear load applied (Figure 4.14, switch 3).

Thus, the development of the model of the ultrasonic vibrating system has been completed. Verification of the created Matlab-Simulink model will be considered in the next chapter. The next step is to elaborate the model of the control system.
Chapter Five       Model of the control system

The created model of the ultrasonic vibrating system allows the simulation of the operation of the ultrasonic transducer under different cutting conditions. In order to make possible the investigation of different control strategies, the model of the control system based on the principle of autoresonance [8] has to be developed. Autoresonant control is a self-sustaining vibration mode at the natural frequency of the mechanical system, which maintains the resonant regime of oscillation automatically by means of positive feedback using transformation (phase shift) and amplification of the signal from a sensor (Figure 3.8). The principle of autoresonance has been discussed in detail in Chapter 3. This chapter is devoted to the process of development of the model of the control system including the elaboration of the control algorithm and the investigation of the properties of the control strategies based on mechanical and electrical feedbacks.

5.1. Control system algorithm

The main purpose of the control system is to keep the vibrations of the ultrasonic transducer at a specified level during the process of cutting (the regime of oscillations with the nonlinear load applied). The description of the control system operation is provided below.

The control system generates a control signal by means of shifting the phase of the signal from the sensor and changing its amplitude. This signal is then supplied to the piezoelectric actuator to produce an excitation for the vibrating system. The control system employs different signals depending on the type of control strategy used. The control system based on mechanical feedback uses the displacement of the end of the concentrator as a control signal. For the electrical feedback case, the current of the piezoelectric transducer is used as a control signal. To perform the control system algorithm initially, the phase shift giving the maximum amplitude of vibration needs to be found and set up. Thus, the most efficient autoresonant state of the system is reached. During the process of cutting the changes in the resonant frequency of the system [8] and the level of vibration occur due to the nonlinear load influence. To be able to define these changes and adjust the parameters of the control system so as to compensate for them, the constant monitoring of the efficiency of oscillations has to be carried out,
which is accomplished by tracing the root mean squared (RMS) value of the sensor signal. After evaluating the efficiency of the oscillations, the new control values (the phase shift and the amplitude value) compensating for the introduced changes are elaborated. To keep the resonant mode of oscillations, the phase shift value has to be changed along with the alteration of the nonlinear load. This is accomplished by the phase control algorithm. To compensate for the losses in amplitude occurring due to loading, amplitude control is used. Thus, the combined amplitude-phase control algorithm allows the possibility of simultaneous control of the resonant state (phase control) and level of oscillations (amplitude control), which ensures stable oscillations at the most efficient resonant mode.

Amplitude control and phase control algorithms will be considered in detail in the next sections.

5.1.1. Initial preparation procedure

The first step in the control system algorithm is tuning the system at the most efficient autoresonant state, which is done by accomplishing the initial preparation procedure. The procedure allows identifying the optimal phase shift in the system (the phase shift giving the maximum amplitude of oscillations). To define the optimal phase shift, the amplitude-phase characteristic of the closed loop system has to be obtained for both the control signals (the current of the piezoelectric transducer and the displacement of the second body). This is achieved by slowly changing the phase shift in the system and tracing the peak values of amplitude of the displacement and current.

Figure 5.1 represents the amplitude-phase characteristic of the loaded closed loop system when the displacement of the second body is used as a control signal. Phase shift values are shown in the graph in phase-shift control units, providing that one phase-shift control unit is equal to $2\pi$.

It can be seen from Figure 5.1, that the displacement reaches its maximum amplitude at the phase shift equal to 0.21 phase control units. Taking into account that 1 phase control unit = $2\pi$ this gives us:

$$\text{0.21} \times 2\pi = 0.42\pi$$

(5.1)

This means that setting up the phase shift (5.1) in the feedback loop allows tuning the control system at the most efficient resonant state.
Figure 5.1. Amplitude-phase characteristic of the loaded system with the displacement of the second body used as a control signal.

The same procedure has been completed for the closed loop system using the current as a control signal. The amplitude-phase characteristic obtained for this case is shown in Figure 5.2. It can be seen from the graph that the phase shift providing the maximum amplitude of current is:

$$0.15 \times 2\pi = 0.3\pi.$$  \hfill (5.2)

Figure 5.2. Amplitude-phase characteristic of the loaded system with the current of a piezoceramic transducer is used as a control signal.
When the phase shift, giving the maximum amplitude of vibration ((5.1) for displacement and (5.2) for the current) is found and set up, the most efficient autoresonant state of the electromechanical system is reached. This procedure prepares the control system for the next steps of the control system algorithm, which will be further discussed.

5.1.2. Phase control

The phase control is designed to adjust the phase shift during the process of cutting (the regime of oscillations with the nonlinear load applied) so as to always provide the optimum phase shift in the system. The initial preparation procedure allows tuning the system at the most efficient autoresonant state. However, application of the nonlinear load changes the resonant frequency of the ultrasonic vibrating system. As discussed in Chapter 3, by changing the phase shift in the closed-loop system, the frequency of oscillations can be adjusted. Figure 3.7 shows that the amplitude - phase curve is reasonably flat at resonance, which means that the small alterations of phase will be sufficient to keep the resonant mode of vibration. This proposes the use of the following phase control algorithm. For every control cycle the phase shift value is changing by 0.01 phase control units (or 0.02π) and the changes in RMS value of the control signal are monitored. If the RMS value of the control signal obtained from the next control cycle after changing the phase shift is less than the RMS value of the control signal obtained from the previous control cycle (when the phase shift was changed), the direction of change of the phase shift is changed to the opposite one. Otherwise the direction of change is kept the same. Thus the control system always aims at the most efficient autoresonant state.

For example, if in the “current” control cycle the phase shift has been increased and RMS value obtained from the next control cycle, $R_{i+1}$, is more than the RMS value obtained from the “current” control cycle $R_i$, when in this control cycle the phase shift value will be increased again, otherwise it will be decreased. The diagram, explaining the phase shift algorithm is shown in Figure 5.3.

The optimal value for the duration of the control cycle was chosen as 500 periods, which covers the transient period.
5.1.3. **Amplitude control**

The amplitude control is designed to keep the amplitude of oscillations during cutting at the level prescribed by the program. To provide this control, the system is constantly observing the amplitude of the sensor signal (the RMS value is used as a measure). Application of the cutting load leads to an increase in the mechanical losses and a decrease in the amplitude of oscillations correspondingly. To compensate for these changes the new value for the amplitude of the control signal is calculated as follows:

\[ A_{i+1} = A_{i} \frac{R_d}{R_{i+1}} \]  

(5.3)

where \( R_d \) and \( R_{i+1} \) are the desired (prescribed) RMS value of the response signal and a value, obtained in the next control cycle after setting up the amplitude of control voltage \( A_{i} \); \( A_{i+1} \) is the elaborated amplitude control value to setup. According to this formula
the calculated value $A_{i+1}$ will be increased in comparison with $A_i$ if for the current control cycle $R_{i+1} < R_d$ and decreased otherwise. Setting up the elaborated value allows the desired level of oscillations to be approached. Thus the described control algorithm allows adjusting the amplitude of the control voltage (and the level of oscillations correspondingly) along with the alteration of the nonlinear load. This algorithm has been initially introduced in 5 and was used in this work with slight changes and improvements.

In practice, the acoustic-power-handling capacity of the piezoelectric transducer is limited by a number of factors, such as the reduction in efficiency due to dielectric losses, mechanical losses and so on [72]. This means that there is an interval providing the increase in the amplitude of the oscillation of the piezoelectric transducer proportional to the supplied voltage. Supplying the voltage with the amplitude values exceeding this interval, will not provide further increase in the amplitude of the oscillations of the piezoelectric transducer. It is necessary to take this important property of the piezoelectric transducer into account while developing the amplitude control algorithm. The values of the amplitude of voltage supplied to the piezoelectric transducer causing the linear dependence of the amplitude of oscillation have been defined experimentally and formed the interval (5.4).

$$200V - 300V\text{ zero-to-peak} \quad (5.4)$$

Thus, at the final step of the amplitude control algorithm, every control value is then checked to see if it falls into the interval (5.4). For values exceeding this interval, the maximum $300V$ and the minimum $200V$ values will be used.

**5.1.4. Combined amplitude-phase control algorithm**

The combined amplitude-phase control algorithm allows the possibility of simultaneous control of the resonant state (phase control) and the level of oscillations (amplitude control). The main goal of the control system is keeping the amplitude of oscillations stable during the process of cutting. As was discussed above (section 5.1.2), the phase control performs slight adjustments of phase near the resonance, which, according to Figure 3.7, does not substantially influence the amplitude of oscillations. This means that when a severe jump in the amplitude of oscillations occurs, the amplitude control has to be applied in order to compensate for the introduced changes and stabilise the amplitude of oscillations. To accomplish this for each control cycle, the
RMS value is defined and compared to the desired value. If the calculated current control cycle RMS value $R_i$ differs from the desired value $R_d$ by more than the critical value $R_c$, amplitude control will be applied, otherwise phase control is used (Figure 5.4). Thus the interval $(R_d - R_c) \div (R_d + R_c)$ is the interval of the phase control application, which will be further called the phase control zone. The critical value is defined as a percentage of the desired value and is dependent on the type of feedback used. It has been experimentally selected for the mechanical feedback case as 5% of $R_d$ and for the electrical feedback as 10% of $R_d$.

![Figure 5.4. Combined amplitude - phase control algorithm.](image)

Application of amplitude control in the current control cycle means that a new amplitude control value will be calculated, but the phase control value during this cycle will be kept the same as in the previous control cycle. And vice versa, the application of the phase control in the current control cycle means changing the phase control value and keeping the amplitude control value the same as in the previous control cycle.

The procedures for elaboration of the new amplitude and the phase control values have been described above. Transformation of the control signal implies the phase shifting and amplification of the signal based on the elaborated control values. Detailed description of this part of the algorithm will be considered in the next section.
5.2. Matlab-Simulink model of control system

A general schematic of the Matlab-Simulink model of the control system based on the algorithm described above is presented in Figure 5.5. This schematic represents the feedback block of the Matlab-Simulink model shown in Figure 4.14.

The model of the control system consists of the following main parts: the control block (1) and the transformation block (2).

The control block includes the phase control and the amplitude control blocks calculating new control values for the amplitude and phase shift using the algorithms described in the previous section.

The RMS counter block is designed to collect the samples of the sensor signal over the control interval $T_1 < t < T_2$ and calculate the RMS value, according to the following formula:

$$ RMS = \sqrt{\frac{1}{T_2 - T_1} \int_{t_1}^{t_2} [f(t)]^2 dt}, \quad (5.5) $$

where $T_2 - T_1$ is the duration of the control cycle, which is defined as a whole number of periods, $f(t)$ represents the sensor signal.

Designs of listed Matlab-Simulink blocks are given in the appendix.
Transformation block

The transformation block converts the signal from the sensor to the control signal using the amplitude and the phase control values elaborated in the control block.

The transformation procedure consists of the following steps:

1. **Biasing**

   This part accomplishes the subtraction of the mean value from the signal (block 3 in Figure 5.5). This procedure is used when the signal is non-symmetrical about the \( x \)-axis, which is often the case for the regime of oscillations with the nonlinear load applied. The rest of the transformation procedure is designed to deal with a symmetrical signal. However, this procedure is optional and can be excluded from the algorithm (switch 1, Figure 5.5) if not needed.

   The mean value block calculates the mean value of the sensor signal over the period of oscillation according to the following formula:

   \[
   \overline{f} = \frac{1}{T} \int_{0}^{T} f(t) dt ,
   \]

   where \( T \) is the period of oscillation.

2. **Limitation**

   This procedure is accomplished by the Matlab-Simulink element “Sign”, which converts the sine wave signal to the square wave signal with the unity gain. This part is used to limit the amplitude of the oscillation and prepare the signal for phase shifting.

3. **Phase-shifting**

   The phase shifter provides the shifting of the signal to the phase control value in the range \( 0 \div \pi \) defined by the phase-control block. This block is designed to work with the square wave signal.

   The term “phase shift” is generally applicable to the sinusoidal signal, however, in this work it is used in connection with the periodic signal and the phase shift is expressed as a part of the period. The whole period \( 2\pi \) is considered as 1 phase control unit and the phase shifter is designed to shift the signal within the half of the period (either positive
or negative). Thus, the signal supplied to the phase shifter has to be within the limits 0 to 0.5 phase control units.

![Diagram](image)

**Figure 5.6. Algorithm of the phase-shifting process.**
The algorithm of operation of this component is presented in Figure 5.6. The square wave signal supplied to the input of the phase shifter is shown in Figure 5.6 (a). Initially the period of oscillation (dotted line in Figure 5.6 (b)) of the incoming signal is calculated and multiplied by the phase control value. This allows the phase-shift value to be expressed as a part of the period (dash-dotted line in Figure 5.6 (b)). When the incoming signal starts rising, the phase shifter begins integrating the unity constant and produces the inclined line (light solid line in Figure 5.6 (b)). The next step is subtracting the phase shift value from this line, which gives us the displaced line (light dashed line in Figure 5.6 (b)), which meets the zero line at a different point. This point represents the beginning of the shifted signal (rising edge). The same procedure is repeated for the falling edge of the signal (dark lines in Figure 5.6 (b)). These two lines shape the half of the period and the phase-shifted period is then formed as shown in Figure 5.6 (c). The phase shifter can be configured to shift the signal within the positive half-wave or within the negative half-wave, which allows the whole period $0 \div 2\pi$ to be covered.

4. Amplification

An amplifier provides the amplification of the signal from the phase shifter to the amplitude control value elaborated in the amplitude control block.

5. Filtering

This step is needed to smooth the square wave signal by eliminating the high frequency components, which can cause damage to the piezoelectric transducer. It is generally known that any filter will have a gain, which is dependent on the signal frequency and will exhibit a phase shift that varies with frequency [73]. Since the autoresonant control is based on phase control, the phase characteristics of a filter become especially important. The amplitude response and the phase response of the filter have to be obtained and analysed while designing the control system. The filter introducing the fewest changes of the gain and the phase shift in the working range of frequencies should be chosen. Two different filter types have been proposed for the system: a second-order band-pass filter and a first-order low-pass filter with a high cut-off frequency. The advantages and drawbacks of each of them will be further considered.
**Band-pass filter**

The experiments with different filter types allowed the second-order Butterworth band-pass filter with the band pass $123000-148000$ rad/s (19.5-23.5 kHz) to be chosen for this work. The band pass of this filter is $4 \text{ kHz}$. During the investigation of different filter types, initially the filter with a band pass of $2 \text{ kHz}$ was employed. Simulations conducted using this filter showed a very bad performance of the control system based on the electrical feedback, as the filter is a necessary part of the electrical feedback control system (will be further discussed). When this was discovered, the filter band-pass was increased and the control system simulations were repeated. Use of the filter described above with a band pass of $4 \text{ kHz}$ considerably improved the characteristics of the electrical feedback control system. That is why the design of this filter has been chosen. Performance of the filter is shown in Figure 5.7, where (a) represents the square-wave signal supplied to the input of the filter and (b) depicts the filtered signal coming as the output of the filter. Figure 5.7 shows that the filter is able to smooth the square-wave signal and produces the pure sine-wave signal as the output.

![Figure 5.7](image)

*Figure 5.7. Performance of the band-pass filter: a) input of the filter, b) output of the filter.*
It can also be seen from Figure 5.7 that the output of the filter (b) is not in phase with its input (a) and that the amplitude of the output of the filter is not the same as the amplitude of the square wave signal supplied to the input. To further investigate the amplitude and phase relationships of the filter, the amplitude-frequency characteristic and the phase-frequency characteristic have been obtained. To get these characteristics the square-wave signal with the frequency varying within the pass-band 19.5–23.5 kHz has been supplied to the input of the filter and the peak amplitude of the output and the phase shift between the input and the output of the filter have been calculated. The amplitude response of the filter obtained as the result of this procedure is shown in Figure 5.8.

![Amplitude-frequency characteristic](image)

**Figure 5.8. Amplitude-frequency characteristic of the band-pass filter.**

It can be seen from the graph that the amplitude of the output signal is frequency-dependent and changing within the filter pass-band. This means that the amplitude of the oscillations cannot be properly controlled while using the proposed band-pass filter since the frequency of oscillation of the ultrasonic vibrating system is changing with the application of the non-linear load. However, in the middle of the pass-band, where the filter is mostly going to be used, the amplitude is frequency independent.
Phase-frequency characteristic

Figure 5.9. Phase-frequency characteristic of the band-pass filter.

The phase-frequency characteristic (Figure 5.9) shows that the filter exhibits a phase shift (shown in phase-control units) that varies with frequency within the wide range \(-0.25\div+0.26\) phase-control units or \(-90\div+94\) degrees. This means that using the filter in the closed-loop, autoresonant, control system can cause additional control problems. In this case, the resulting phase shift in the loop will depend on the phase control value defined by the phase control and the phase shift value introduced by the filter, which varies with frequency within the pass-band and cannot be controlled.

The completed investigation revealed that the proposed band-pass filter has a number of drawbacks that are crucial for the performance of the autoresonant control system and can considerably reduce the system's controllability. The use of this filter in the closed-loop system should be avoided if possible.

**Low-pass filter with high cut-off frequency**

A low-pass filter is a filter that passes low frequencies but attenuates frequencies higher than the cutoff frequency. Choosing the high cut-off frequency allows passing the signals in the broad range of frequencies with almost unchanged characteristics whilst reducing the high frequency components. The first-order, low-pass filter can be described in Laplace notation as:
\[ G(s) = \frac{1}{1 + \tau s} \]  

(5.7)

where \( s \) is the Laplace transform variable and \( \tau \) is the filter time constant.

The cutoff frequency of the filter is defined as \( \omega = \frac{1}{\tau} \). Experiments with different coefficients allowed the low-pass filter with the time constant \( \tau = 0.000002 \) to be chosen for this work:

\[ G(s) = \frac{1}{1 + 0.000002 s} \]  

(5.8)

The performance of this filter has been investigated. Figure 5.10 (a) shows the signal supplied to the filter whilst Figure 5.10 (b) depicts its output.

![Figure 5.10. Performance of the low-pass filter: a) input of the filter, b) output of the filter.](image)

It can be seen from Figure 5.10, that the output of this filter is not a pure sine wave, however, the filter reduces high frequency harmonics in the signal and smoothes it.

To further investigate the amplitude and phase relationships of this filter, the amplitude-frequency characteristic and the phase-frequency characteristic have been obtained. To
get these characteristics the same procedure as for the band-pass filter was used. The amplitude response of the low-pass filter is shown in Figure 5.11.

![Amplitude-frequency characteristic](image)

**Figure 5.11.** Amplitude-frequency characteristic of the low-pass filter.

This figure shows that the proposed low-pass filter does not change the amplitude of the signal in the working frequency range. This means that using this low-pass filter in the control algorithm will not reduce the controllability of the amplitude of oscillations.

The phase-frequency characteristic (Figure 5.12) shows that the proposed low-pass filter exhibits a minor phase shift, slightly varying with frequency. The phase shift introduced by the filter is almost linear within the considered frequency band; it changes within the range \(+0.027\pm0.032\) phase-control units or \(+9.7\pm11.5\) degrees. This means that using the proposed low-pass filter in the closed-loop, autoresonant system will not introduce considerable changes to the resulting phase shift in the loop.
Comparing Figure 5.9 with Figure 5.12 we can see that for the same frequency range the low-pass filter introduces much smaller changes in the phase shift than the band-pass filter.

Thus the completed investigation revealed that the proposed low-pass filter:

- provides reduction of high frequency components, which is sufficient to prevent damage to the piezoelectric transducer;
- does not change the amplitude of the filtered signal;
- exhibits a very minor phase shift within the required range of frequencies.

Based on this investigation the proposed first-order, low-pass filter has been found much more appropriate for the autoresonant control system than the second-order Butterworth band-pass filter considered in the previous section.

Thus the development of the model of the control system has been completed. The next section is devoted to a comparative investigation of the control strategies based on the mechanical feedback and the electrical feedback.
5.3. **Mechanical feedback and electrical feedback**

Depending on choice of the sensor, two different control strategies are possible:

- Mechanical feedback, when the sensor, measuring the mechanical characteristics of the oscillations (displacement, velocity or acceleration) attached to the end of the concentrator is used for the control system.
- Electrical feedback, which uses the signal from any electrical sensor measuring the electrical characteristics of the piezoelectric transducer (current, voltage, power).

Let us now consider the advantages and disadvantages of these control strategies. Mechanical feedback uses a sensor measuring vibration (it could be a displacement, velocity or acceleration sensor). The sensor is placed near the cutting area at the end of the concentrator or cutting tip. Therefore, it directly reflects the oscillations of the ultrasonic system and, by controlling the signal from this sensor, the actual state of vibration can be controlled in the most efficient way. However, it is inconvenient to use this arrangement in industrial conditions since it is difficult to fix permanently because of prolonged high-frequency vibration in the harsh machining conditions. Additional wiring to the control system is a disadvantage as well, as the sensor is located in the cutting area.

Electrical feedback uses the electrical characteristics of the piezoelectric transducer for the control i.e. current or power. An electrical sensor does not need to be placed in the cutting area, which makes it convenient to use in industrial conditions. However, this sensor reflects the oscillations of the system in an indirect way via the current or power in the piezoceramic rings. This can cause difficulties for the control system, which will be further discussed.

5.3.1. **Theoretical investigation of feedback types**

A simplified model of an ultrasonic transducer representing the 2-DOF system is shown in Figure 4.8. It is known that the 2-DOF system has different amplitude-frequency-phase curves, depending on the choice of points used to define the phase shift between the vibration and force application point [14]. In the case of the described model of the ultrasonic transducer, the excitation force is applied to the first body, but there are two different observation points for phase shift definition:
- In the case of mechanical feedback, the sensor is attached to the end of the concentrator, which means that the point of observation is the second body.
- In the case of electrical feedback, the current of the piezoelectric transducer is used as the control signal. The current of the piezoelectric transducer, according to Eqs. (4.54)-(4.57), mainly reflects the vibrations of the first body, i.e. the point of observation is the first body.

To investigate the main properties of the model of an ultrasonic transducer under phase control, the analytical solution of equations of motion of the 2-DOF system described above (Figure 4.8) will be obtained.

Taking into account the equation of the piezoelectric transducer (4.54), the model of the ultrasonic transducer can be represented as shown in Figure 5.13.

![Figure 5.13. Model of the ultrasonic transducer including the model of the piezoelectric transducer.](image)

The type of excitation of the system shown in Figure 5.13 is classified as the excitation by means of the frame to which the spring and the damper are attached. According to Magnus [57], the equations of motion for such a system can be written in the following form:

\[
\begin{align*}
\dot{m}_1\ddot{x}_1 &= -c_1(\dot{x}_1 - k_{02}\dot{u}) - k_s(x_1 - k_{02}u) + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1), \\
\dot{m}_2\ddot{x}_2 &= -c_2(\dot{x}_2 - \dot{x}_1) - k_s(x_2 - x_1) 
\end{align*}
\]

(5.9)

where \( k_{01}, k_{02} \) are taken as in (4.54) and \( k_s \) is the stiffness of the springs \( k_{01} \) and \( k_1 \) connected in series:

\[
k_s = \frac{k_{01}k_1}{k_{01} + k_1}.
\]

(5.10)

Assume that the voltage of the piezoelectric transducer can be presented in the form:

\[
u = Ue^{i\omega t}.
\]

(5.11)

Substitution of (5.11) into (5.9) gives:

\[
\begin{align*}
\dot{m}_1\ddot{x}_1 + \dot{x}_1(c_1 + c_2) + x_1(k_s + k_2) - c_2\dot{x}_2 - k_2x_2 &= F_1e^{i\omega t} + F_2je^{i\omega t}, \\
\dot{m}_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 - k_2x_1 + k_2x_2 &= 0
\end{align*}
\]

(5.12)
where
\[
F_1 = k_1 k_0^2 U \\
F_2 = c_1 k_0^2 U 
\] (5.13)

Equations (5.12) expressed in matrix form transform to:
\[
M\ddot{X} + C\dot{X} + KX = Fe^{j\omega t}, 
\] (5.14)

where \( X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \), \( M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \) is the mass matrix, \( C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \) is the damping matrix, \( K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \) is the stiffness matrix and \( F = \begin{bmatrix} F_1 + F_2 e^{j\omega t} \\ 0 \end{bmatrix} \) is the force matrix.

The steady-state forced vibrations of the system will be of the form:
\[
x_1 = X_1 e^{j\omega t} = \bar{X}_1 e^{j\omega t} \\
x_2 = X_2 e^{j\omega t} = \bar{X}_2 e^{j\omega t},
\] (5.15)

where \( \bar{X}_1 \) and \( \bar{X}_2 \) are the complex amplitudes of oscillation of the first and the second bodies.

Taking into account (5.15), the relationship between the system responses, \( X \), due to the force input, \( F \), can be found from the equation (5.14) as follows:
\[
\begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{bmatrix} = Z^{-1}(\omega)F, 
\] (5.16)

where \( Z(\omega) = [-\omega^2 M + j\omega C + K] \) is the matrix of dynamic stiffness.

Solving (5.16) analytically, the values for the amplitudes \( X_1, X_2 \) and the phase shifts \( \psi_1, \psi_2 \) can be found. Figure 5.14 shows the amplitude-frequency-phase characteristics of the model of the ultrasonic transducer obtained by solving the equation (5.16) for the range of frequencies \( 5kHz - 25kHz \).
Figure 5.14 shows the amplitude-phase (a), amplitude-frequency (b) and phase-frequency (c) characteristics of the model of the ultrasonic transducer for two feedback types: dotted curves correspond to the case when the point of observation is the first body; solid curves depict the characteristics of the model when the point of observation is the second body.

The layout of this figure allows the relationship between these characteristics to be observed: it can be seen that the bold dots of the figure (a) correspond to the dots with the same amplitude on figure (b). Figure 5.15 displays Figure 5.14 (a) separately for more detailed consideration.
Figure 5.15 shows the amplitude-phase characteristics of the model of the ultrasonic transducer, the response of the first body is presented as the dotted curve and the response of the second body is shown as the solid curve. The following properties can be seen from this figure:

- the amplitude-phase curves of the system are bell-shaped and gently sloping near the resonance, which proves that by using phase control the resonant mode of oscillation can be easily controlled;
- the amplitude-phase characteristic of the first body is ambiguous, whereas the amplitude-phase curve of the second body is single valued.

The final result means that three regimes of vibration are possible in the system under phase control for any phase shift near the resonant $\pi/2$ value when the same body is used both for excitation and observation. The corresponding points of the curves are marked with the bold dots (see light line in Figure 5.15). Two of these regimes are resonant regimes for the first and second oscillation modes respectively, and the third
regime, with the lowest amplitude, corresponds to anti-resonant vibration. This can be seen from Figure 5.14 (b), which depicts the amplitude-frequency characteristics of the model of the ultrasonic transducer.

The stability of these three possible regimes has been investigated using the created Matlab-Simulink model of the ultrasonic transducer and the model of the control system. The following procedure has been used:

Initially forced oscillations with the prescribed frequency (frequency of the first or second resonant regime or anti-resonant regime) have been excited in the system. Once the steady state has been reached, the loop was closed and the phase control was applied with the phase shift close to the resonant value $\pi/2$. For both resonant regimes after the application of phase control, the system continued oscillating as previously, which shows that these regimes are stable under phase control. When the anti-resonant regime was excited the system 'jumped' to the first resonant regime after application of the phase control. This observation shows that the anti-resonant regime of oscillations is unstable and cannot be reached under phase control.

The described investigation has demonstrated that both the resonant regimes are stable. Hence, two stable regimes with different amplitudes and frequencies can exist for the same phase in the considered model under phase control, when the first body is chosen to observe oscillations and to apply excitation. This result means that two regimes can be excited in the system (either the first resonant regime or the second one) under phase control when the signal from the current sensor is used as a control signal. This advises introducing a filtration procedure into electrical feedback control to make sure that the "right" resonant regime (the regime with the specified frequency) is excited. Using the filter for the autoresonant closed-loop system, brings considerable difficulties into the control algorithm, because the filter shifts the phase of the incoming signal and changes its amplitude in the band pass (as considered in the previous section). The effectiveness of the electrical feedback control including a filter will be further considered.

5.3.2. Comparison of amplitude-frequency characteristics of electrical signals and the displacement signal

It was mentioned above that electrical sensors reflect the oscillations of the ultrasonic system in an indirect way via the current or power of piezoceramic rings. In order to study the impact it makes on the control system, an investigation to compare the electrical signals (current and power) with the displacement signal has been completed.
The experimental investigation included obtaining and comparing the amplitude-frequency characteristic for the displacement of the end of the concentrator with the amplitude-frequency characteristics for the current and power of the piezoelectric transducer.

During the experiment, a swept sine wave was used to slowly sweep the dynamic system through a range of excitation frequencies. The signals from the sensors were captured using a digital data-recording oscilloscope and transferred to a personal computer for subsequent processing. The experimental arrangement used for the sweep sine test is shown in Figure 5.16.

![Experimental set-up for obtaining the amplitude-frequency characteristics](image-url)
A Black Star Jupiter 500 Signal Generator was used to generate a slow varying, saw-tooth signal. This signal was inputted into the Control Port of the Hewlett Packard (HP) Pulse / Function Generator which was operated in Voltage Controlled Oscillator (VCO) mode. As the saw-tooth signal progressed through its cycle, the frequency of the sine wave generated by the HP generator was adjusted accordingly, producing the required swept sine wave signal. The HP generator was set to a constant output voltage of 1.5V (zero to peak). This amplitude remained consistent throughout the test and is representative of the typical control signal (before amplification) used in ultrasonic systems. The generated swept sine wave signal was fed into the Universal Amplifier in order to produce a signal powerful enough to drive the ultrasonic transducer. In order to match the high impedance output of the Universal Amplifier with the low impedance ultrasonic transducer the Universal Matchbox (FFR Ultrasonics, model: EVB-2) was used.

The transducer’s mechanical response to excitation was measured using a Polytec OFV302 laser vibrometer. A laser vibrometer determines the velocity at which a single point on the ultrasonic transducer is moving. The point on the front face of the transducer (end of the concentrator) was selected for the investigation.

The transducer’s electrical response to excitation was obtained by measuring the current and voltage of the piezoelectric transducer. Voltage was measured using an Active Differential probe as supplied by Test Probes Inc (model ADF 15). In order to measure the current of the piezoceramic transducer, a Current Probe (model PR 30) manufactured by LEM HEME Ltd was used.

Using a LeCroy Digital Oscilloscope (model number 9314CL), the velocity, current and voltage signals obtained from the sensors were recorded. The ramp signal was used to trigger the recording oscilloscope. The rate at which the frequency was swept was set to be as low as was feasible (the oscilloscope has a limited amount of memory) to ensure that the ultrasonic transducer was allowed to reach a quasi-equilibrium state throughout the frequency range investigated. For this experiment the sweep rate was set at 1.5 kHz/s.

Once collected, the data was transferred to a PC for further analysis.

The average power of the piezoelectric transducer was calculated in Matlab-Simulink according to the following formula:
where \( i \) is the current and \( u \) is the voltage of the piezoceramic transducer as recorded during the experiment; \( T \) is the calculated period of the oscillations.

The peak values of the displacement, current and power were calculated using a Matlab program created for this purpose and plotted against the excitation frequency of the system. The resulting amplitude-frequency characteristics are shown in Figure 5.17.

\[
P = \frac{1}{T} \int_0^T \! i \, u \, dt
\]

(5.17)

Figure 5.17. Amplitude-frequency characteristics. Experimental results. (a) Displacement of concentrator's end, (b) power of piezoelectric transducer and (c) current of piezoelectric transducer.

It can be seen from the graph that the resonant peak of the displacement curve (a) coincides very well with the peak of the power curve (b), but does not agree with the peak of the current curve (c). As was explained above, the control signal algorithm always aims at the regime with the maximum amplitude of the sensor signal. This
means that, according to experiment, using the current signal for the control system does not permit the maximum vibrations to be reached. The amplitude of the displacement signal, which can be obtained using current control, differs by 12.5% from the maximum amplitude of displacement. It was suggested that the difference in amplitude could be compensated for by the initial increase in the amplitude of the voltage supplied to the piezoelectric transducer. This suggestion will be further investigated. For the case of power control, it is possible to reach the maximum amplitude of displacement as the resonant frequencies of power and displacement coincide. This finding will be further investigated in the next chapter.

![System Frequency Response](image)

![Power of Piezoelectric Transducer](image)

![Current of Piezoelectric Transducer](image)

Figure 5.18. Amplitude-frequency characteristics. Simulation results. (a) Displacement of 2\textsuperscript{nd} body, (b) power of piezoelectric transducer (c) current of piezoelectric transducer.

In order to verify the created model of the ultrasonic transducer, the same test was repeated for the model. In this case the amplitude-frequency curves of the second
body’s displacement and the current and power of the piezoelectric transducer were obtained. For this test, the same sweep rate was used as for the previous experiment (1.5 kHz/s). The results of the simulations are presented in Figure 5.18. Comparison of the results of the experiment, Figure 5.17 with the result of the simulation, Figure 5.18, shows that the simulation curves conform to the experimental curves: i.e. the shapes of the simulation curves are very similar to the experimental ones. The next observation is the coincidence between the resonant frequencies of displacement and power and the lack of coincidence between the resonant frequencies of displacement and current. Good agreement of the experimental and simulation results proves the validity of the created model of ultrasonic transducer.
Chapter Six Numerical simulations and discussion

The previous section was devoted to the elaboration of the control algorithm and the investigation of the properties of the control strategies based on mechanical feedback and electrical feedback. Theoretical investigation showed that electrical feedback requires a more sophisticated control algorithm. In this case, a filtration procedure has to be included in the control algorithm, which can cause additional difficulties for the autoresonant closed-loop system, as the filter influences the phase shift of the signal. Experimental investigation demonstrated the potential reduction in efficiency of an electrical feedback control system using the current signal, due to the existence of a shift in the resonant frequencies of displacement and current. The second important observation following from this experiment is the coincidence of the resonant frequencies of the displacement and power signals, which promotes the idea of using both current and power signals for the control system.

Based on the results set out in the previous section, the following control strategies have been further investigated:

1. Mechanical feedback, when the displacement was used in a control algorithm (displacement feedback).
2. Electrical feedback, when the current signal was used for the control system. This strategy will be called current feedback.
3. Electrical feedback, when a current signal was used as the actuating signal to generate excitation for the piezoelectric transducer and a power signal was used as a control signal to define the actual performance of the system and control the amplitude of the excitation signal. This control strategy will be called the power feedback control.

The results of the control system simulation for the three described types of control will now be presented and discussed. In order to investigate the ability of the control system to keep the desired level of vibrations during the cutting process, a simulation of changes in the loading conditions was carried out.

During the simulation, the load applied to the end of concentrator was changed and the RMS value of the control signal was recorded. Two types of test were conducted: increasing the load applied to the concentrator by changing the contact stiffness value
and by changing the interference value. The contact stiffness value was changing within the $1 \times 10^8 - 7 \times 10^8 \text{N/m}$ range; the interval of the interference value changing was $-2 \times 10^{-7} -- 1 \times 10^{-5} \text{N/m}$.

The same procedure has been used for each test. Initially the closed-loop, loaded system with parameters of a nonlinear load block (4.64) was excited under manual phase control. When the steady-state oscillations of the system were reached, the combined amplitude - phase control was applied. Then the value of one of the parameters of the nonlinear load (contact stiffness or interference) was changed and the reaction of the control system (changes in the amplitude of the supplied voltage and the phase shift) together with the sensor signal were monitored. After reaching the steady-state oscillations, the value of the same parameter was changed again. The following procedure was repeated several times until the maximum value of the parameter changed in each test was reached.

6.1. Mechanical feedback (displacement control)

6.1.1. Changing the contact stiffness value

Simulation results for the case of mechanical feedback will be now considered. The first test investigates the ability of the displacement control to keep the level of vibrations under control in conditions where the contact stiffness changes. The control system uses the RMS value of the displacement signal in a control algorithm as an actuating signal and as a control signal.

Figure 6.1 shows the control system operation during the test. Figure 6.1 (a) depicts the RMS value of the displacement signal (solid curve); the dashed line corresponds to the desired value of the RMS of the displacement signal and the dotted lines designate the limits of the phase control zone (see section 5.1.4 for definition). The desired value was specified as the RMS of the desired value of the amplitude of displacement of the loaded system (4.63). Thus the desired value was defined as $7.5 \mu m$, which corresponds to the amplitude of oscillation of the loaded system, $10 \mu m$. Figure 6.1 (b) depicts the contact stiffness value; (c) and (d) show the phase control and the amplitude control operation during the test.
It can be seen from the figure that the RMS value did not leave the phase control zone and the amplitude control value was not changing during the test. This means that the phase control was sufficient to keep the desired level of vibrations under such loading and the amplitude control was not applied. This can be explained by the fact that the amplitude-phase characteristic of a nonlinear system is flat at resonance so by controlling the phase shift in the closed-loop system the resonant state can be controlled. While doing this test, the observation of changes in the frequency of oscillations with the increase of the contact stiffness value was performed. The resonant frequency of the oscillations of the model of the ultrasonic transducer without the load applied was defined in Chapter 4 as $f = 20.96\, kHz$ (4.60). The initial loading of the system with the load defined by the parameters of the nonlinear load block (4.64) changed the resonant frequency of oscillations to:

$$f = 21.13\, kHz.$$  \hfill (6.1)
Further increase in the contact stiffness value caused the proportional increase in the frequency of oscillations. When the value of contact stiffness was increased to $4 \times 10^8 \text{N/m}$, the system was oscillating at $21.5 \text{kHz}$. Loading of the system formed by the maximum value of the contact stiffness used for this test, $7 \times 10^8 \text{N/m}$, changed the resonant frequency to $21.87 \text{kHz}$. Thus, the range of the changes of the frequency of the oscillations within this test was:

$$21.13 \text{kHz} - 21.87 \text{kHz}.$$  \hspace{1cm} (6.2)

The initial phase control value of 0.21 was selected to provide the maximum amplitude of oscillations for the system loaded with the parameters of the nonlinear load block (4.64) and the system was oscillating with the frequency (6.1). As we can see from Figure 6.1 only slight adjustments of the phase control value were necessary during the test despite the fact that the frequency of oscillations was changing within the range (6.2).

![Figure 6.2. Mechanical feedback control during changing the contact stiffness. (a) Displacement of end of the concentrator, (b) load applied, (c) contact stiffness value.](image-url)
Figure 6.2 (a) depicts the displacement of the second body, which was monitored together with the load applied to the ultrasonic system (b) and the contact stiffness value, Figure 6.2(c). The RMS value of the displacement signal, used as a control signal in this test, is shown in Figure 6.3 as a solid line; the dashed line depicts the desired value of the RMS of the displacement signal.

![Figure 6.3](image)

Figure 6.3. Mechanical feedback control during changing the contact stiffness: variations in control signal. RMS value of displacement signal – solid line, desired value of RMS – dashed line.

Figure 6.2 shows that with an increase of the contact stiffness value (c), the amount of load (b) was increasing as well. However, in spite of considerable changes of non-linear load, the amplitude of the displacement was kept stable. Figure 6.3 is consistent with Figure 6.2 and shows that the RMS value of the displacement was kept close to the desired level during the whole simulation process. Another interesting observation from Figure 6.2 (a) is the “shifting” downward of the displacement with the increase of the contact stiffness value, which makes the displacement curve less symmetrical. This illustrates that the increasing contact stiffness makes it more difficult for the ultrasonic transducer to penetrate into the material. That leads to quasi-static compression of the system.

6.1.2. Changing the interference value

The next test shows the influence of changes in the interference value (and load correspondingly) on the control system performance.
Figure 6.4 shows the control system operation during the test. Figure 6.4 (a) depicts the RMS value of the displacement signal (solid curve); the dashed line corresponds to the desired value of the RMS of the displacement signal and the dotted lines designate the limits of the phase control zone. The desired value for this test was kept the same as for the previous test, 7.5 \( \mu m \). Figure 6.4 (b) represents the interference value; (c) and (d) show the phase control and the amplitude control operation during the test.

As indicated in Figure 6.4 (a) below, the RMS value leaves the phase control zone several times during the test; this is when the amplitude control comes into action, see Figure 6.4 (d), (this happens at 0.09 sec, 0.1 sec, 0.14 sec, 0.18 sec, 0.23 sec). We can see from the chart that when the amplitude control is applied and the amplitude value is changing, the phase shift value is kept unchanged. As soon as the RMS value enters the phase control zone the amplitude control stops working and the phase control comes into action, Figure 6.4 (c). Thus, Figure 6.4 shows the operation of the combined amplitude - phase control, which was applied for the test.

![Figure 6.4](image)

*Figure 6.4. Mechanical feedback control during changing the interference: control system operation. (a) RMS of reference signal, (b) interference value, (c) phase control, (d) amplitude control.*
Changes in the frequency of oscillations appearing with the increase of the interference value (and load correspondingly) were observed for this test. As in the previous experiment, the initial loading of the system with the load defined by the parameters of the nonlinear load block (4.64) changed the resonant frequency of oscillations to (6.1). Further increase in the interference value caused the proportional increase in the frequency of oscillations. When the interference value was increased to \(-1*10^{-5}m\), the frequency of the oscillations reached \(21.5kHz\). Thus, the range of changes in the frequency of oscillations within this test was:

\[
21.13kHz - 21.5kHz. \tag{6.3}
\]

Comparing (6.2) and (6.3) we can see that the range of changes of the frequency for this test is twice as small as for the previous test. Smaller frequency variations mean less nonlinear distortion, which requires very small adjustments of the phase control value (Figure 6.4 (c)). However, we can see from Figure 6.4 (d) that, contrary to the previous test Figure 6.1 (d), the amplitude control value for this test had to be considerably increased in order to keep the level of oscillations stable. These observations show that for this type of loading, when the interference value was changing, amplitude control was more important than phase control.

Figure 6.5 (a) shows the displacement of the second body and the interference value (light dashed line), which was changing during this test. Figure 6.5 (b) depicts the applied load, which is rising with the increase in the interference value. From Figure 6.5 (a) we can see that the effect of the “shifting” downward of the displacement value, which appeared in the previous test (Figure 6.2 (a)), is also present in this test.
Figure 6.5. Mechanical feedback control during changing the interference. (a) Displacement of end of the concentrator and interference value (dashed line) (b) load applied.

Figure 6.6 shows how the RMS value of the displacement of the second body was changing during the test; the dashed line indicates the desired value of the RMS of the displacement signal. We can see from this figure that the control system keeps the RMS value quite stable in spite of an increase in loadings.

Figure 6.6. Mechanical feedback control during changing the interference: variations in control signal. RMS value of displacement signal – solid line, desired value of RMS –dashed line.
The results of both tests have proved that the autoresonant control system based on mechanical feedback is able to maintain the level of vibrations during the process of cutting (in the conditions of the nonlinear load changing).

6.2. Electrical feedback

6.2.1. Current control

6.2.1.1. Changing the contact stiffness value

In order to compare different control strategies, the simulation of changing the contact stiffness value was repeated for the electrical feedback case. Current feedback, when the RMS value of the current of the piezoceramic rings was used as an actuating signal and as a control signal, will be considered first.

From the theoretical investigation of feedback types completed in the previous chapter, we know that electrical feedback requires a more sophisticated control algorithm. In this case the filtration procedure has to be included in the control algorithm to filter out resonant modes other than the mode with our working frequency. For this purpose, the band-pass filter described in paragraph 5.2 was employed.

Next important findings from the previous chapter were obtained from the investigation of amplitude-frequency characteristics. This investigation showed that when using the current signal for the control system, a displacement with amplitude 12.5% less than the maximum (desired value) can only be reached. That is why the initial amplitude of the voltage will be increased for this test to compensate for the 12.5% difference and allow the desired value of the displacement to reach the same value as in the previous simulation. To define the required increase, the voltage amplitude was gradually increasing until the RMS of the displacement reached its desired value of $7.5 \mu m$. Thus, the desired value of the current and the initial amplitude of the voltage for the electrical feedback simulation were defined correspondingly as $0.329 A$ and $230 V$. 
Figure 6.7 shows the control system operation during the test. Figure 6.7 (a) depicts the RMS value of the current signal (solid curve); the dashed line corresponds to the desired value of the RMS of the current signal and the dotted lines designate the limits of the phase control zone (see section 5.1.4). Figure 6.7 (b) depicts the contact stiffness value; (c) and (d) show the phase control and amplitude control operation during the test. It can be seen from the figure that phase control was sufficient to keep the desired level of current under such loading, as the RMS value did not leave the phase control zone and the amplitude control was not applied. The same test completed for the mechanical feedback case showed similar behaviour of the control system (see Figure 6.1): the RMS value was changing within the phase control zone and the amplitude control value was not changing. However, we can also see the difference in the control system operation for these two cases. Comparing Figure 6.7 (c) with Figure 6.1 (c) we can see that the range of changes of the phase control value for the current feedback case, 0.07 – 0.16, was twice as broad as for
the displacement feedback case 0.2 – 0.24. It can also be observed that for mechanical feedback control (Figure 6.1), the phase control value was changing around the initial phase control value of 0.21, whereas for the current feedback control, the phase control value was decreasing during the whole test. The difference in operation of these two control systems shows that the same load influences the current of the piezoelectric transducer and the displacement of the end of the concentrator in different ways. Just slight alterations of the phase shift was enough to keep the level of displacement for the mechanical feedback case and substantial change in the phase shift was required to keep the level of current for the electrical feedback case.

Figure 6.8 (a) depicts the displacement of the second body, which was monitored together with the load applied to the ultrasonic system (b) and the contact stiffness value (c).

Figure 6.8. Current feedback control during changing the contact stiffness. (a) Displacement of end of the concentrator, (b) load applied, (c) contact stiffness value.
In this test, in order to have a clear representation of what is happening with the oscillations of the system, the RMS value of the displacement, Figure 6.9 (b), was observed together with the RMS value of the current, Figure 6.9 (a). From these figures we can see that the control system maintains the RMS value of the current, Figure 6.9 (a). However, the RMS value of the displacement, Figure 6.9 (b) (and the amplitude of vibrations Figure 6.8 (a)), deviates considerably from its desired value during the test. The maximum deflection of the RMS value of the displacement from the desired value is $1.8 \mu m$ (24\%), which is noticeably higher than the maximum deflection for the displacement control, $0.4 \mu m$ (5\%) from Figure 6.3. This test shows that an autoresonant control system based on current feedback experienced difficulties controlling the level of vibrations during the test.

From a comparison of the two tests, we can see that current feedback is much less suitable for control than displacement feedback. This can be explained due to the shift of the resonant frequency of the current from the resonant frequency of the displacement. Even increasing the voltage supplied to the piezoelectric transducer, which was undertaken to compensate for the difference in amplitude, did not solve this problem. Because of the difference in the amplitude - frequency curves, controlling the
level of current does not allow proper control of the level of displacement. This difference can further increase with changes in the load.

6.2.1.2. Changing the interference value

The next test shows the influence of changes in the interference value (and load correspondingly) on the control system performance for the current feedback case. Figure 6.10 shows the control system operation during the test. Figure 6.10 (a) depicts the RMS value of the current signal (solid curve); the dashed line corresponds to the desired value of the RMS of the current signal and the dotted lines designate the limits of the phase control zone. The desired value for this test was kept the same as for the previous test, 0.329 A. Figure 6.10 (b) represents the interference value; (c) and (d) show the phase control and the amplitude control operation during the test.

![Figure 6.10](image-url)
Figure 6.11. Current feedback control during changing the interference. (a) Displacement of the end of the concentrator and interference value (dashed line) (b) load applied.

From Figure 6.10 (c), we can see that the phase control value in this test was changing much less than for the previous test, Figure 6.7 (c). This is consistent with observation of changes in the frequency of oscillation (discussed in sections 6.1.1-6.1.2) showing that the frequency of oscillation is changing within a much smaller range for this test than for the test on changing the contact stiffness value.

Amplitude control is applied twice during the test Figure 6.10 (d) (at 0.1 sec and 0.18 sec) when the RMS value leaves the phase control zone.

Figure 6.11 (a) depicts the displacement of the second body and the interference value (light dashed line), which was changing during this test; Figure 6.11 (b) shows the applied load. Figure 6.12 shows the RMS value of the current signal (a) and the RMS value of the displacement of end of the concentrator (b). We can see that the RMS value of the current was kept quite close to the desired value during the whole test, while the displacement was deviating from its desired value. The maximum deflection of the RMS value of the displacement from the desired value is $2.3 \mu m$ (30%), which is noticeably higher than the maximum deflection for the displacement control $0.7 \mu m$ (9%) from Figure 6.6. Results of this test are consistent with the results of the previous test and prove the reduced efficiency of the control system based on current feedback.
Figure 6.12. Current feedback control during changing the interference: variations in control signal. (a) RMS value of current – solid line, desired value of RMS -dashed line; (b) RMS value of displacement signal – solid line, desired value of RMS -dashed line.

6.2.2. Power control

Simulation results for the next case of electrical feedback when the power is used as a control signal will be now considered. In this case the control system uses the current signal to generate excitation for the vibrating system by phase shifting and amplifying it (actuating signal), as in the case of the current control. However, in order to define the required amount of phase shift and the amplitude, the control system uses the power signal. From an investigation of amplitude-frequency characteristics we know that the resonant frequencies of the power of the piezoelectric transducer and the displacement of the end of the concentrator coincide. Based on this fact, we make a suggestion that changes in displacement can be better traced using the power signal than by using the current signal, and that, by maintaining the level of the power, we can keep the level of displacement. This hypothesis will now be investigated.

In order to tune the control system based on the power feedback at the most efficient autoresonant state, the phase shift giving the maximum amplitude of oscillations has to be set up. To define this phase shift, the amplitude - phase characteristic of the loaded,
closed-loop system was obtained for the power of the piezoelectric transducer, Figure 6.13. Phase shift values are shown in the graph in phase-shift control units.

It can be seen from the Figure 6.13, that the power reaches its maximum amplitude at the phase shift equal to 0.1 phase-control units. Taking into account that 1 phase-control unit = \(2\pi\) it can be seen that 0.1 phase-control units is equal to:

\[
0.1 \times 2\pi = 0.2\pi .
\]  

(6.4)

Setting up the phase shift (6.4) allows configuring the system loaded with initial loading parameters (4.64) at the most efficient autoresonant state.

### 6.2.2.1. Changing the contact stiffness value

In order to compare different control strategies, the simulation of changing the contact stiffness value was repeated for the power feedback case. Power feedback is a type of electrical feedback in which the current signal is used as an actuating signal to generate excitation for the piezoelectric transducer (by means of phase shifting and amplification) and a power signal is used as a control signal to define the actual performance of the system.

From the investigation of amplitude - frequency characteristics (see previous chapter), we know that the resonant frequencies of displacement and power coincide. This means...
that the initial amplitude of the voltage can be kept the same as for the displacement feedback case - 200V.

Figure 6.14 shows the control system operation during the test. Figure 6.14 (a) depicts the RMS value of the power signal (solid curve); the dashed line corresponds to the desired value of the RMS of the power signal and the dotted lines designate the limits of the phase control zone (see section 5.1.4 for definition). The desired value of the power was defined as 39W, which is the RMS of the power of the piezoelectric transducer corresponding to the oscillations with the amplitude 10μm, as defined in (4.63). Figure 6.14 (b) depicts the contact stiffness value; (c) and (d) show the phase control and amplitude control operation during the test.

It can be seen from Figure 6.14 that the control system was behaving in a similar way as for the current feedback case, Figure 6.7. The phase-control value was changing within the wide range 0.03–0.11 and was decreasing with increasing contact stiffness and the frequency of oscillations correspondingly; the amplitude control was not applied.
Figure 6.15. Power feedback control during changing the contact stiffness. (a) Displacement of end of the concentrator, (b) load applied, (c) contact stiffness value.

Figure 6.15 (a) shows the displacement of the second body of the model of the ultrasonic transducer, which was monitored together with the load applied to the ultrasonic system (b) and the contact stiffness value (c).

From Figure 6.15 (a), we can see that the effect of the “shifting” downward of the displacement value, which appeared for the displacement feedback and for the current feedback, is also present in this test.

The RMS value of the power is depicted in Figure 6.16 (a). To trace the vibrations of the system, the RMS value of the displacement was observed, Figure 6.16 (b). From this graph we can see that the power control is indeed able to keep the level of vibrations at the desired value. The maximum deflection of the RMS value of displacement from the desired value is $0.5 \mu m$ (7%), which is much better than for the current feedback control (24%) and very close to the result of the mechanical feedback case (5%).
Figure 6.16. Power feedback control during changing the contact stiffness: variations in control signal. (a) RMS value of power – solid line, desired value of RMS -dashed line; (b) RMS value of displacement signal – solid line, desired value of RMS -dashed line.

This simulation shows that using the power as the control signal considerably improves the results of the electrical feedback control system.

6.2.2.2. Changing the interference value

The next test shows the operation of the control system based on the power feedback during changes in the interference value. Figure 6.17 (a) depicts the RMS value of the power (solid curve); the dashed line corresponds to the desired value of the RMS of the power and the dotted lines designate the limits of the phase control zone. The desired value for this test was kept the same as for the previous test, 0.39W. Figure 6.17 (b) represents the interference value; (c) and (d) show the phase control and the amplitude control operation during the test.

Again we can see from Figure 6.17 that the control system based on the power feedback operates in a similar way as for the current feedback, Figure 6.10, and for the displacement feedback, Figure 6.4. The phase control value is changing within the narrow band, 0.08 – 0.11, around the initial phase control value of 0.1, and the amplitude control is applied twice during the test, Figure 6.17 (d) (at 0.12 sec and 0.195 sec) when the control signal leaves the phase control zone.
Figure 6.17. Power feedback control during changing the interference: control system operation. (a) RMS of control signal, (b) interference value, (c) phase control, (d) amplitude control.

Figure 6.18. Power feedback control during changing the interference value. (a) displacement of end of the concentrator and interference value (light line) (b) load applied.
The displacement of the second body of the model of the ultrasonic transducer and the interference value (light dashed line) are shown in Figure 6.18 (a), the load applied to the ultrasonic system is shown in Figure 6.18 (b).

The RMS value of the power is presented in Figure 6.19 (a). To trace the vibrations of the system, the RMS value of the displacement was observed, Figure 6.19 (b). From this graph we can see that the power control is not as efficient as in the previous test, Figure 6.16. In this case the maximum deflection of the RMS value of the displacement from the desired value is 1.4μm (19%), while for the test on contact stiffness changing it was only 7%.

To understand the nature of this problem, the properties of the power of the piezoelectric transducer will be considered. The formula for calculating the power consists of two components: the current and the voltage of the piezoelectric transducer (5.17). When one of the components is increased the power will also increase. The desired value of the power, 39W, is defined initially for a level of 200V. Each time that the amplitude control is applied, the amplitude of the voltage supplied to the piezoelectric transducer is increased, which increases the level of power. However, the desired value of power is kept the same. This means that the value of the power initially corresponding to the desired level of vibrations will, after an increase in amplitude, correspond to a lower level of vibrations and by maintaining the level of power we cannot control the level of vibrations properly.

From Figure 6.19 one can see that, at the beginning of simulation, the desired value of power corresponds to the desired value of displacement. With a change of interference (Figure 6.18), the desired value of power corresponds to the desired value of displacement less and less. To avoid this problem, the desired value of power has to be changed together with changing the load and the amplitude of the voltage supplied to the piezoelectric transducer correspondingly. However, in the majority of processing machines, the workload is a result of the complex and poorly predictable behaviour of the materials being treated. That is why it is difficult to identify the change in the desired value required to compensate for the change in the amplitude of the supplied voltage, as there is no linear dependence between these characteristics.
Comparing the results of this test with the results of the same test for the other feedback types, we can see that the maximum deflection for the power control (19%) is far from the displacement control (9%), but still better than for the current control (30%). This means that the characteristic of power feedback described above does not dramatically decrease the effectiveness of the control system and it still provides reasonably good results.

6.2.2.3. Changing the interference value: amplitude control only

From Figure 6.17 we can see that the phase-control value is changing within the narrow range around the initial phase-control value. This means that phase control does not play a very important role for this test and therefore can be neglected.

The same experiment on changing the interference value was repeated for the amplitude control only; the phase control value was kept the same during the whole experiment and only the amplitude control value was changing.

The control system operation during this test is presented in Figure 6.20.
Figure 6.20. Power feedback control during changing the interference: amplitude control only. (a) RMS of control signal, (b) interference value, (c) phase control, (d) amplitude control.

The RMS value of the power is presented in Figure 6.21 (a), the RMS value of the displacement is shown in Figure 6.21 (b). The peculiarities of the power feedback control described in the previous section are also presented in this figure. Initially (at 0.05 sec, marked with ‘1’ on the graph) the desired value of power, 39W, corresponds to the desired value of displacement, 7.5μm. After increasing the amplitude of the voltage to 227V (at 0.122 sec, marked with ‘2’), the desired value of power corresponds to 6.7μm. Finally (at 0.185 sec, marked with ‘3’), when the amplitude of the voltage reaches 250V, we have only 6.2μm for the same desired value of power, 39W.
The maximum deflection of the RMS value of the displacement from the desired value is $1.3 \mu m$ (17%), which is very close to the result obtained from the previous experiment (19%), Figure 6.19. Visual comparison of these two graphs (Figure 6.21 and Figure 6.19) shows their similarity. Thus, from these observations we can conclude that for loads of that type, when the nonlinear distortion is small, the application of the amplitude control is sufficient to control the level of vibrations and the phase control value can be kept unchanged.

6.3. Frequency control (forced oscillations)

A comparison of the performance of the autoresonant control based on the phase control algorithm with the conventional frequency control regime without feedback, when the system was excited with a predefined frequency, was undertaken. The following procedure has been used for these simulations. Initially the open-loop loaded system with parameters of nonlinear load block (4.64) was excited with the resonant frequency (6.1). When the steady-state oscillations of the system were reached, the value of one of the parameters of the nonlinear load (contact stiffness or
interference) was changing and the displacement of the second body was monitored. After reaching the steady-state oscillations, the value of the same parameter was changing again. The following procedure was repeated several times until the oscillations were gradually damped.

The first simulation concerned changing the contact stiffness value. Figure 6.22 (a) depicts the displacement of the second body, which was monitored together with the load applied to the ultrasonic system (b) and the contact stiffness value, Figure 6.22 (c). The RMS value of the displacement signal is shown in Figure 6.23 as a solid line; the dashed line depicts the desired value of the RMS of the displacement signal.

![Figure 6.22](image-url)

**Figure 6.22.** Conventional frequency control during changing the contact stiffness. (a) Displacement of end of the concentrator, (b) load applied, (c) contact stiffness value.

From Figure 6.22 we can see that the contact stiffness was increasing very slowly for this test. Every step of this simulation was increasing the contact stiffness value on $0.1 \times 10^8 N/m$, which is 10 times less than for all the previous experiments on changing the contact stiffness value (see Figure 6.1, Figure 6.7, Figure 6.14). Also, the final increase reached in this test, $1.6 \times 10^8 N/m$, was less than the initial increase in all the previous experiments, $2 \times 10^8 N/m$. 

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Figure 6.23. Conventional frequency control during changing the contact stiffness. RMS value of displacement signal - solid line, desired value of RMS - dashed line.

From Figure 6.23 we can see that even such small changes in the contact stiffness value have a big impact on the oscillations of the ultrasonic system. The final increase of the contact stiffness to $1.6 \times 10^8 \text{N/m}$ was enough to stop the system vibrating. However, even the control system based on the current feedback (Figure 6.9), which showed the worst results of all the feedback types, was capable of keeping the oscillations for the load with a contact stiffness of $7 \times 10^8 \text{N/m}$.

The next test shows the influence of changes in the interference value (and load correspondingly) on the forced oscillations of the ultrasonic system.

Figure 6.24 (a) depicts the displacement of the second body of the model of the ultrasonic transducer and the interference value (light dashed line); the load applied to the ultrasonic system is shown in Figure 6.24 (b). The RMS value of the displacement signal is shown in Figure 6.25 as a solid line; the dashed line depicts the desired value of the RMS of the displacement signal.
As in the previous test, the interference value was changing very slowly for this test. Figure 6.24 shows that, again, even small changes in loading were having a big impact on the system performance and after setting up the interference value of $-1.5 \times 10^{-6} m$, the ultrasonic system stopped vibrating. However, the control system based on the current feedback (Figure 6.12) was providing a good level of vibrations for the load with an interference value of $-9 \mu m$.

**Figure 6.24.** Conventional frequency control during changing the interference. (a) Displacement of end of the concentrator and interference value (light line) (b) load applied.

**Figure 6.25.** Conventional frequency control during changing the interference. RMS value of displacement signal – solid line, desired value of RMS –dashed line.
Thus, these tests showed that forced oscillations cannot control the level of vibrations at all. This can be explained due to high quality factor of the ultrasonic system and the corresponding high sensitivity to the load. That is why even small changes in the load were influencing the performance of the systems' vibrations. When the load was applied, the resonant frequency of the system was changed and, as it was excited with a different frequency, the oscillations were gradually damped.

6.4. Summary of simulation results

A summary of results obtained for all discussed above tests is given in Table 6.1. The table compares maximum deflections of the RMS values of displacement from the desired value obtained for different combinations of control strategy and type of loading.

Table 6.1. Maximum deflections of the RMS values of displacement from the desired value obtained for different control strategies during different types of loading.

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Change of contact stiffness</th>
<th>Change of interference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Strategy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mechanical feedback</td>
<td>5%</td>
<td>9%</td>
</tr>
<tr>
<td>Power feedback</td>
<td>7%</td>
<td>19%</td>
</tr>
<tr>
<td>Current feedback</td>
<td>24%</td>
<td>30%</td>
</tr>
<tr>
<td>Frequency control</td>
<td>Not able to control the level of vibrations</td>
<td>Not able to control the level of vibrations</td>
</tr>
</tbody>
</table>

From this table we can see that mechanical feedback control provided the best results for both types of loadings (changing the contact stiffness and changing the interference value). Frequency control (forced oscillations with a predefined frequency) was not able to maintain the level of vibrations. Current feedback control showed the worst results of all autoresonant feedback control types. Nevertheless, it was capable of keeping the oscillations for a quite high level of loadings, which was not the case for the frequency control. Power feedback control showed better efficiency than the current feedback control and the results for simulation of changing the contact stiffness value case were almost as good as for mechanical feedback control. However, we can see that the power control during changing the interference value was not as efficient as in the previous case. This can be explained due to dependence of power on the amplitude of voltage supplied to the piezoelectric transducer, which was discussed in detail in section 6.2.2.2.
Chapter Seven  Experimental results

The completed investigation revealed the advantages and drawbacks of different control strategies and estimated the efficiency of each of them. The following feedback types have been considered: mechanical feedback and two types of electrical feedback (current feedback and power feedback). To validate the results obtained through numerical simulations, a prototype of an autoresonant control system was designed and manufactured. For all the listed control strategies, the turning experiments for different feed rates have been conducted with the control system. A Harrison M300 turning lathe was employed in the experiments, as shown in Figure 2.3. Samples of mild steel with a diameter of 50mm have been machined. This chapter is devoted to consideration of the results obtained from these experiments.

The experimental work described in this chapter, including the development of the prototype control system, has been conducted in close collaboration with Dr Alan Meadows.

7.1. Experimental setup

Figure 7.1 shows the experimental setup used for the experiments with different control strategies. Contour 1 indicates the general schematic of the autoresonant control system. Contour 2 designates the arrangement used to record the experimental data. The design of each of the components employed for the both parts of the experimental set-up will now be considered.
7.1.1. **Autoresonant control system**

The autoresonant control system is designed according to the general schematic shown in Figure 3.8 and consists of the following elements:

1. **Sensors** (contour 3);

   The sensor is selected according to the control strategy employed. For the control system based on mechanical feedback, the inductive sensor is used. Both types of electrical feedback employ the current sensor for the positive feedback circuit to generate excitation for the piezoelectric transducer by phase shifting and amplification of its output. Current feedback also uses the output of the current sensor as a source of the control signal for the negative feedback to define the actual performance of the system. In the case of power feedback, the output of the power sensor is used as a control signal.
Current sensor
The current sensor measures the current flowing through the piezoelectric transducer. It is connected in between the output of the matchbox and the input of the piezoelectric transducer (see Figure 7.6). A Hall Effect current sensor (manufactured by LEM, Switzerland) was used for the experiments.

Power sensor
The power sensor calculates the instantaneous electrical power delivered to the piezoelectric transducer by means of multiplication of the signals from the current sensor and from the voltage sensor:

\[ P(t) = i(t) \cdot u(t) , \]  

(7.1)

where \( P(t) \) is the instantaneous power, \( u(t) \) is the potential difference across the piezoelectric transducer and \( i(t) \) is the current flowing through it.

The current sensor described above was used to obtain the current of the piezoelectric transducer. Voltage was measured using an Active Differential Probe (model ADF 15 manufactured by Test Probes Inc). The experimental setup used to measure power is shown in Figure 7.2.

Inductive sensor
The principle of operation of the inductive sensor is described in section 3.2. In the framework of this research, the different configurations have been tried and the
The following design has been finally chosen and tested (see Figure 7.3). Magnetic tape is closely wrapped around the end of the concentrator and the coil is wound on top of it. The magnet is fixed at the machine tool vertical slide and serves for magnetizing of the tape. When the ultrasonic vibrations are excited in the transducer, the magnetic tape starts moving. As the velocity of oscillations is quite high ($\approx 0.7 \, m/s$) the coil can be considered immovable because of its inertia. With the presence of the magnetic field, the oscillations of the magnetic material induce quite a strong signal in the coil. This signal is further amplified by the Low Noise Amplifier Type 450 (manufactured by BrookDeal Electronics Ltd).

![Figure 7.3. Inductive sensor mounting.](image)

To filter out the external electromagnetic noises induced by the lathe, a $105 \, nF$ capacitor has been employed (see Figure 7.4). The value of the inductance of the coil used in the experiment is equal to $3.8 \, mH$. 
Figure 7.4. Schematic diagram of the inductive sensor

2. Filters
Filter 1 (shown in dashed box in Figure 7.1) is used only for the electrical feedback, as the output of the current sensor must be filtered before the application of the control algorithm (for details see section 5.3.1).
Filter 2 is employed to smooth the output of the limiter (square wave signal) by eliminating the high frequency components.
Both filters 1 and 2 are identically designed as multiple feedback, band-pass filters with a 4 kHz bandwidth and a central frequency of 18 kHz.

3. Phase shifter
The phase shifter is used to phase-shift the signal from the sensor (actuating signal) and establish the optimal phase shift in the system, providing the most efficient autoresonant state. It is designed to provide a full circle (360°) phase shift within the frequency range 15–25 kHz. The use of the device allows adjustment of the self-excitation frequency.

4. Limiter
The limiter acts as a controller for the amplitude of the actuating signal, which is further used for the excitation of the piezoelectric transducer. Thus, the device allows the control of the vibration level during operation. The limiter is designed to provide a controlled maximum level of output signal up to 5V by truncating the input signal manually and automatically. In these experiments, the limiter was automatically controlled by the amplitude controller.
The filter, the phase shifter and the limiter have been designed by this research group at Loughborough University and have been initially used for the autoresonant control system described in 5.

5. Amplifier
The amplifier is used for the amplification of the actuating signal to produce a signal powerful enough to drive the ultrasonic transducer. It is a 200W MOSFET amplifier capable of increasing its input by approximately 80 times with a maximum output of 150V p-p.

6. Matchbox
Matchbox is the interfacing device matching the low impedance output of the amplifier with the input of the ultrasonic transducer. The Universal Matchbox Model Number: EVB-2 supplied by FFR Ultrasonics has been employed for these experiments. Adjustment of the inductor and transformer, which form the matchbox unit, allows the output of the amplifier to be conditioned with the input of the ultrasonic transducer. This is accomplished by stepping up the voltage generated by the amplifier, and shunting the piezoceramic material’s capacitance with a suitable inductor. Since the piezoelectric transducer is substantially capacitive, the transducer and the matchbox form a resonant circuit, which can be tuned by varying the inductance values. Proper tuning of the matchbox components results in better performance of the ultrasonic transducer and a correspondingly higher level of vibration.

All the listed components form the positive feedback loop, which ensures the self-excitation of the system with the chosen frequency of vibration.

7. Amplitude controller
The amplitude controller employs the signal from the sensor (control signal) to estimate the efficiency of the oscillations and, according to a specific algorithm, calculates the amplitude control value, which is then supplied to the limiter.

A circuit diagram of the amplitude controller is shown in Figure 7.5. The device works according to the following algorithm. Initially, the RMS converter acquires samples of the sensor signal and calculates the RMS value of this signal. The RMS value is then compared with the offset value, which can be adjusted by means of the potentiometer.
marked as "offset" on the diagram. The difference between these two values is further amplified by the operational amplifier. Tuning the potentiometer marked "gain control" allows the amplification factor to be adjusted. A 12-kOhm resistor and 1μF capacitor placed at the output of the operational amplifier serve as an integrating filter, smoothing the amplifier's output. A small 470nF capacitor has been experimentally chosen to eliminate the high frequency instability. The instability was caused by positive feedback and the capacitor was employed to reduce gain for high frequencies (above 100 kHz).

![Circuit diagram of the amplitude controller.](image)

RMS converter is the True RMS-to-DC Converter AD536AJD supplied by Analog Devices [75]. The amplitude controller acts within the negative feedback loop, i.e. if the RMS value exceeds the specified value (desired value) it decreases the amplitude of the actuating signal and vice versa.

The autoresonant control system is designed as purely analogous and provides high processing speed, which ensures reliable control of high frequency oscillations under variable conditions during the machining process. The experimental setup of the autoresonant control system used in this work is shown in Figure 7.6. This reflects the electrical feedback case when the current sensor is used in the feedback.
7.1.2. Data recording unit

The data-recording unit is used for monitoring purposes and consists of the RMS converter, PicoScope and computer (see Figure 7.7).

Figure 7.6. Autoresonant control system setup when the current sensor is used in the feedback.

Figure 7.7. Data recording unit.
Two signals can be recorded at the same time, as the input of the PicoScope is limited to two channels. The RMS converter consists of two identical True RMS-to-DC Converters, model number AD536AJD, supplied by Analog Devices. The converter calculates the RMS value of the signal supplied to its input. The calculated RMS value is then transferred to the computer using the PC oscilloscope, PicoScope model number 2202 as supplied by Pico Technology, and recorded by means of the PicoScope software. PicoScope software is supplied together with the PicoScope and provides oscilloscope, spectrum analyser and multimeter functions. It makes use of the PC’s processing capabilities allowing real-time signals to be viewed and the captured waveforms to be saved and printed [74].

In order to monitor the performance of the control system during the experiments, one input of the data-recording unit was connected to the output of the limiter; the other input was employed to record the control signal (the signal from the sensor used for negative feedback). For the experiment with the electrical feedback, in order to observe the actual vibrations of the ultrasonic transducer, an output of the inductive sensor was recorded together with the control signal.

7.2. Experimental procedure

Before conducting the experiment, the amplitude of the output signals of the inductive sensor and the power sensor were adjusted to give the same RMS values as the current sensor’s output. The amplitude of the inductive sensor’s output is controlled by tuning the amplification coefficient of the BrookDeal Amplifier. Adjustment of the power sensor’s output is accomplished by tuning the potentiometer included in the output stage of the sensor. This procedure allowed using the same parameters of the amplitude controller for all control strategies, which significantly simplified the experiment.

The initial preparation procedure is used to identify the system parameters and tune the system at the most efficient autoresonant state. During the first step, the value of the phase shift is varied within the possible range and the ultrasonic system’s vibrations are observed. At the end of the test, the phase shift providing the most efficient oscillations (the highest RMS value of the control signal) is set up. This procedure has to be done separately for mechanical feedback and for electrical feedback, as they may require different phase shift values. The next step is tuning the negative feedback, which is accomplished by adjustment of the resistance of the potentiometers marked as “offset”
and "gain control" on the amplitude controller (see Figure 7.5). Adjustment of the resistance of the first potentiometer allows tuning the middle point of the control interval and the desired level of voltage correspondingly (the level of voltage the system will be trying to keep during the experiment). Changing the value of the second potentiometer allows adjusting the limits of the control interval and the degree of reaction of the control system to the changes in the control signal. Therefore, use of the procedure allows the determination of the most efficient autoresonant state of the electromechanical system and identifies the control interval. After completion of this procedure, the control system is prepared for the experiments.

The same experiment has been repeated for each of the control strategies in order to test them. The turning machine employed was a Harrison M300 with ultrasonic transducer fixed as shown in Figure 2.3. Initially the control system was tuned as described earlier for a freely vibrating transducer (without the applied load), then the lathe was switched on and the feed rate 0.03 mm/rev was applied. At this moment the control system was beginning to adjust the amplitude of the voltage supplied to the transducer in order to compensate for the changes in the control signal caused by the applied load. The value of the phase shift was not changing; the initially established phase shift value was kept during the whole experiment. To monitor the control system operation, the RMS values of the limiter's output and the control signal were recorded. In the next stage, the value of the feed rate was increased to 0.1 mm/rev and the same procedure was repeated. Finally, the behaviour of the control system was investigated for a feed rate of 0.2 mm/rev.

7.3. A comparative investigation of the different control strategies during the turning experiments

7.3.1. Mechanical feedback

Figure 7.8 represent the oscilloscope readings of the turning experiment with a mechanical feedback control system. The solid line depicts the RMS of the inductive sensor's output and the dashed line illustrates the RMS value of the limiter's output. At the beginning of the experiments the lathe was switched off. At 50 sec the lathe was switched on and a feed rate of 0.03 mm/rev was applied. The increase in the limiter's output (dashed line) was observed. It is the reaction of the control system trying to
compensate for the changes in the control signal, caused by the applied load. At 85sec the feed was turned off and, after setting up the value 0.1 mm/rev, was turned on again at 95sec. We can see that the limiter’s output was increased even more in this case. With switching off the feed at 125sec, the limiter’s output comes back to the previous value. At 145sec the feed rate of 0.2 mm/rev was set up and the amplitude of the voltage supplied to the transducer was increased again. The increase in the limiter’s output in this case was almost twice as high as for the previous feed-rate value. It can also be seen that the output of the inductive sensor was not changing during the experiment. It means that the control system was able to keep the level of vibrations stable in spite of considerable change in loadings.

![Figure 7.8. Turning experiments with a mechanical feedback control system; RMS value of the inductive sensor’s output – solid line, RMS value of limiter’s output – dashed line.](image)

In order to estimate the actual amplitude of the vibration using the inductive sensor output, the following calibration has been carried out. For a free transducer (not loaded), the velocity of the cutting tip oscillations was measured using the Polytec laser vibrometer and was recorded together with the output of the inductive sensor (see Figure 7.9).
This experiment shows that the RMS value of the inductive sensor, 0.31 V, corresponds to the RMS value of the laser vibrometer's output, 0.7 V. Taking into account the sensitivity of the laser vibrometer, 1000 mm/s/V, the RMS value of the velocity of the cutting tip is 0.7 m/s, which gives the RMS value of the displacement:

$$x = \frac{\dot{x}}{2\pi f} = \frac{0.7}{2 \times 3.14 \times 18 \times 10^3} = 6.2 \mu m,$$

(7.2)

where $f = 18 kHz$ is the frequency of oscillation of the ultrasonic transducer.

The RMS value 6.2 $\mu m$ corresponds to oscillations with the amplitude 8.8 $\mu m$, which is a fairly low amplitude for a freely vibrating transducer. The low amplitude of vibration can be explained by the Q-factor of the ultrasonic system, which will be considered in the next section.

### 7.3.2. Current feedback

Figure 7.10 represent the oscilloscope readings of the turning experiment with the electrical feedback control system, when the output of the current signal was employed in the control algorithm. In this experiment, the output of the current signal was used as the actuating signal and as the control signal. The solid line depicts the RMS of the current sensor's output and the dashed line illustrates the RMS value of the limiter's...
output. As in the previous experiment, three different feed rates have been applied, these are: 0.03 \textit{mm/rev} (at 85 sec), 0.1 \textit{mm/rev} (at 110 sec) and 0.2 \textit{mm/rev} (at 135 sec). For all 3 intervals, when the feed was applied we can observe the increase in the limiter’s output (dashed line). This demonstrates that the control system is working to compensate for the changes in the control signal, caused by the applied load. We can also see that the output of the current signal (solid line) is not changing during the experiment. This shows the efficiency of the control system, as it is able to stabilize the amplitude level of the control signal.

![Graph](image)

\textbf{Figure 7.10. Turning experiments with current feedback control system; RMS value of the current sensor’s output – solid line, RMS value of limiter’s output – dashed line.}

However, comparing the limiter’s output for this experiment with the same signal recorded for the mechanical feedback control system (see Figure 7.8), we can see that the limiter’s output is changing within a much broader interval in the case of mechanical feedback. In the mechanical feedback case, the limiter produces 2-3 times higher output for each feed rate value than for the current feedback control system. This observation let us doubt the appropriateness of the reflection of the ultrasonic system vibrations by means of the current sensor. To further investigate this case the same experiment was repeated again and in this case the output of the inductive sensor was recorded together with the limiter’s output (see Figure 7.11 ).
It can be seen from Figure 7.11 that the level of the inductive sensor’s output (solid line) drops each time when the feed is applied (70-90 sec, 110-125 sec and 142-162 sec). This proves that by controlling the current of the piezoelectric transducer, the level of vibrations of the ultrasonic transducer cannot be properly controlled. Thus, the results of the experiment coincide with the results of simulation completed for the current feedback control system. They confirm the reduced efficiency of the control system based on the current feedback.

7.3.3. Power feedback
The oscilloscope readings of the turning experiment with the power feedback control system are shown in Figure 7.12. In this case the output of the current sensor is used as the actuating signal for the positive feedback loop and the power signal serves as the control signal for the negative feedback loop. The solid line depicts the RMS of the power sensor’s output and the dashed line illustrates the RMS value of the limiter’s output. As in the previous experiments, three different feed rates have been applied: 0.03 mm/rev (at 100 sec), 0.1 mm/rev (at 130 sec) and 0.2 mm/rev (at 155 sec). We can see that the control system in this case behaves in the same way as in the previous experiments: the increase in the limiter’s output can be observed each time when the
feed is applied. This behaviour of the control system allows keeping the control signal (solid line) constant by compensating for the changes, caused by the applied load. The ability of the control system to stabilize the amplitude level of the control signal verifies its efficiency.

![Graph showing RMS values](image)

**Figure 7.12. Turning experiments with power feedback control system; RMS value of the power sensor’s output – solid line, RMS value of limiter’s output – dashed line.**

Again, comparing the limiter’s output for this experiment with the same signal recorded for the mechanical feedback control system (see Figure 7.8), we can see that the limiter’s output is changing within a much narrower band here. It can also be observed that the second and third increase in the limiter’s output have almost the same amplitude. This phenomenon has been initially discovered during the simulations completed with the model and can be explained due to the dependence of the power of the piezoelectric transducer on the voltage with which it is supplied. At the beginning of the experiment, the desired level of power corresponds to the desired level of displacement. Application of the load requires an increase in the voltage supplied to the piezoelectric transducer, which also increases the power and the same level of power corresponds to the lower level of displacement now.

Figure 7.13 shows the oscilloscope readings of the inductive sensor’s output (solid line) and the output of the power sensor (dashed line) observed during the same experiment with the power feedback control system as described above.
Figure 7.13. Turning experiments with power feedback control system; RMS value of the power sensor’s output – dashed line, RMS value of inductive sensor’s output – solid line.

We can see that the power sensor’s output is kept constant during the test. However, the signal from the inductive sensor decreases during the intervals of the load application (75-95 sec, 110-125 sec and 135-150 sec). Comparing the inductive sensor’s output for this experiment with the same signal obtained for the current-feedback control system (Figure 7.11), we can notice that they are different. For the current feedback case, the drops in the level of the inductive sensor’s output are proportionally increasing with the increase in the applied load, whereas for the power feedback case they are very similar to each other. This again proves the dependence of the power of the piezoelectric transducer on the amplitude of the signal supplied to it and demonstrates the reduced efficiency of the control system based on the power feedback for controlling the level of vibrations of the ultrasonic transducer. Thus, the results of this experiment coincide with the results of the simulation completed for the power feedback control system.

7.3.4. Manual frequency control
To compare the performance of the autoresonant control based on the phase control algorithm with the conventional frequency control, the same experiment was repeated for the regime without feedback when the system was excited with a predefined frequency.
In this case, the input of the amplifier (see Figure 7.1) was connected to the signal generator producing a sine wave with the frequency 18.4 kHz, which is the resonant frequency of oscillation of the free transducer (as shown in section 7.4). The performance of the frequency control was estimated with the help of the inductive sensor. Figure 7.14 shows the oscilloscope readings obtained in this case. The solid line represents the RMS value of the inductive sensor’s output and the dashed line depicts the limiter’s output, which was not changing during the experiment. Again, as in all previously considered experiments, three different feed rates were applied: 0.03 mm/rev (at 65 sec), 0.1 mm/rev (at 95 sec) and 0.2 mm/rev (at 125 sec). It can be seen from Figure 7.14, that every time the feed was applied, the level of vibrations was decreased considerably. The maximum deflection of the RMS value from the desired value is 50%, whereas in the case of the power control it was only 26% (Figure 7.13). Thus, we can conclude that conventional frequency control with a prescribed frequency of oscillation cannot control the level of vibrations.

In order to check if the situation can be improved with the help of the amplitude control, the same experiment was repeated again and in this case the output of the inductive sensor was used as a control signal for the negative feedback loop. The positive feedback loop was not employed and the signal to the limiter was supplied directly from the frequency generator.
Oscilloscope readings obtained for this experiment are shown in Figure 7.15, where the dashed line represents the limiter’s output and the solid line depicts the inductive sensor output. Three different feed rates have been applied: 0.03 \textit{mm/rev} (at 22 sec), 0.1 \textit{mm/rev} (at 58 sec) and 0.2 \textit{mm/rev} (at 125 sec). Comparing this figure with Figure 7.14 we can see that the application of the amplitude control considerably improved the frequency control.

![Figure 7.15. Turning experiments: frequency control with amplitude control. RMS value of the inductive sensor’s output — solid line, RMS value of the limiter’s output — dashed line.](image)

However, in comparison with Figure 7.8, we can see that the results of frequency control (Figure 7.14), even with the application of the amplitude control, are still not as good as the results of the phase control. From a comparison of these two figures, we can notice that in the case of the frequency control the increase in the limiter’s output for every feed rate is twice as high as in case of the autoresonant control system. The load applied to the transducer changes the resonant frequency of the system and, as the frequency is not adjusted during the experiment, the amplitude controller is increasing the amplitude of the actuating signal in order to compensate for the changes in the amplitude of the control signal. In case of phase-controlled vibrations, a much smaller increase in the amplitude of the actuating signal is necessary to compensate for the influence of the load. This is possibly due to special characteristics of the amplitude-phase curves, which were considered in Chapter 3. From Figure 7.14, we can notice that, in spite of the very high increase in the amplitude of the control voltage (dashed
line), the vibration level still drops for the feed rate \(0.2 \text{ mm/rev}\). This shows that the frequency control, even with the use of the amplitude control, cannot keep the level of vibrations stable. Good performance of the frequency control for the \(0.03 \text{ mm/rev}\) and \(0.1 \text{ mm/rev}\) can be explained due to the low quality factor of the ultrasonic system used for the experiment, which will be considered in the next section.

### 7.4. Quality factor of the ultrasonic system

In order to calculate the quality factor of the ultrasonic system, the amplitude-frequency characteristic of the displacement of the cutting tip was experimentally obtained and analysed. The experimental procedure and setup described in section 5.3.2 was used in this case.

![Figure 7.16. Amplitude-frequency response of the ultrasonic transducer used for the experiments.](image)

The amplitude-frequency response of the ultrasonic transducer used for the turning experiments is shown in Figure 7.16. From this picture we can see that the amplitude-frequency curve is distorted and has two very closely located resonance peaks. It was suggested that this occurs due to the attachment of the tool holder with the cutting tip. The tightening of the screw fixing the tool holder was undertaken to examine the suggestion, which slightly influenced the location and the amplitude of the second (small peak), but did not considerably change the frequency response as a whole.
The Quality Factor of this system was calculated according to the procedure discussed in section 3.1.1. The maximum point and the half-power points used to define the Q-factor are marked with red circles on the graph. The value $Q = 142.51$ obtained for this case is considerably lower than the value of the properly tuned ultrasonic transducer, which is normally in the range $200 - 250$ for the same sweep rate.

To further investigate this issue, the amplitude – frequency response of the ultrasonic transducer with no tool holder (and a cutting tip) was obtained (see Figure 7.17).

![Figure 7.17. Amplitude-frequency response of the ultrasonic transducer without the tool holder and tool.](image)

Comparing Figure 7.17 with Figure 7.16, we can see that with the removal of the tool holder, the resonant frequency has moved towards the high frequencies and the shape of the response has considerably changed.

The Q-factor of the ultrasonic transducer obtained in this case is equal to $Q = 207.67$, which is closer to the value normally used in practice.

Thus, the results of this test advise us that the design of the tool holder used for this experiments has not been optimised for the transducer employed, which considerably decreases the efficiency of the ultrasonic system.
The low Q-factor of the ultrasonic system can also explain the results of the frequency control discussed above.

7.5. Surface finish measurements

In order to investigate the influence of the application of the different control strategies, the turning experiments described in section 7.3 were repeated for two feed rates (0.03 mm/rev and 0.2 mm/rev) and the surface finish quality was evaluated. To ensure the reliability of the measurements, at least 10 mm have been machined for each combination of the control strategy and the feed rate. The measuring device employed for surface profile assessment was a Talysurf CLI 2000 [76].

7.5.1. Feed rate 0.03 mm/rev

Surface profiles obtained for a feed rate of 0.03 mm/rev with the application of different control strategies are shown in Figure 7.18; (a) shows the result of the mechanical feedback application, when the output of the inductive sensor was used by the control system, (b) and (c) depict the results for the power and current feedbacks respectively, (d) represents the frequency control case, when no feedback was applied, and (e) shows the results obtained for conventional (nonultrasonic) turning. The same sampling length, 6000 μm, and scale, 50 μm, was used in all cases. Comparing these pictures with each other, we can observe the difference in the surface profiles. The first 3 strategies (a, b and c) produced very similar visually surface profiles, the profile for the frequency control (d) is a little bit more distorted, and finally we can see that the profile obtained for the conventional turning (e) is very different from all previous cases.

Roughness parameters calculated as an average value of these sampling lengths are reproduced in Table 7.1. A microroughness filtering with a ratio of 2.5 μm was applied before calculation.

\( R_a \) is the universally recognised international parameter of roughness. It is the arithmetic mean of the absolute departures of the roughness profile from the mean value. This parameter will be used for comparison of the control strategies.

\( R_q \) is the RMS parameter corresponding to \( R_a \).
\( R_p \) is the maximum height of the profile above the mean line within the sampling length.

\( R_s \) is the maximum depth of the profile below the mean line within the sampling length.

\( R_z \) is the maximum peak to valley height of the profile in the assessment length.

\( R_{sk} \) is the measure of the symmetry of the profile about the mean line.

\( R_{ku} \) is a measure of the sharpness of the surface profile.

\( R_z \) is numerically the average height difference between the five highest peaks and the five lowest valleys within the sampling length. It is also known as the ISO 10 point height parameter in ISO 4287/1-1984 [77].
Figure 7.18. Surface profiles for feed rate 0.03 mm/rev: (a) mechanical feedback, (b) power feedback, (c) current feedback, (d) manual frequency control, (e) conventional turning (no ultrasonic vibration applied).
Table 7.1. Roughness parameters obtained for feed rate 0.03 mm/rev.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>Ra, µm</th>
<th>Rq, µm</th>
<th>Rp, µm</th>
<th>Rq, µm</th>
<th>Rt, µm</th>
<th>Rsk</th>
<th>Rku</th>
<th>Rz, µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical feedback</td>
<td>0.52082</td>
<td>0.66033</td>
<td>2.2314</td>
<td>1.9906</td>
<td>5.8054</td>
<td>0.12039</td>
<td>3.3738</td>
<td>4.222</td>
</tr>
<tr>
<td>Power feedback</td>
<td>0.67295</td>
<td>0.85188</td>
<td>2.8468</td>
<td>2.4685</td>
<td>6.253</td>
<td>0.045072</td>
<td>3.301</td>
<td>5.3154</td>
</tr>
<tr>
<td>Current feedback</td>
<td>0.75181</td>
<td>0.92646</td>
<td>2.5388</td>
<td>2.4222</td>
<td>5.6251</td>
<td>0.0013</td>
<td>2.814</td>
<td>4.961</td>
</tr>
<tr>
<td>Frequency control</td>
<td>0.90235</td>
<td>1.1123</td>
<td>3.343</td>
<td>2.9994</td>
<td>8.7235</td>
<td>-0.10496</td>
<td>3.0931</td>
<td>6.3423</td>
</tr>
</tbody>
</table>

We can see that the roughness values for the first four strategies are quite close to each other and slightly deteriorate towards the bottom of the row. The value of the roughness of the surface profile obtained for the conventional turning is considerably worse than for all previous cases. The best quality surface (with minimal $Ra$ value) was obtained for the mechanical feedback control system.

### 7.5.2. Feed rate 0.2 mm/rev

Surface profiles obtained for feed rate 0.2 mm/rev with the application of the same control strategies as in the previous experiment are shown in Figure 7.19. Comparing Figure 7.19 with Figure 7.18 we can see that the surface profiles obtained for the feed rate 0.2 mm/rev are very different from the profiles obtained for 0.03 mm/rev.

From Figure 7.19 we can observe the considerable deterioration in the surface quality for the frequency control (d) and for the conventional turning (e). The power feedback control system (b) produced a better surface than the current feedback control system (c). Again, the best results were obtained for the mechanical feedback case (a).

The values of the roughness parameters calculated for these samples are shown in Table 7.2 and repeat the results of the visual observation. Comparing the $Ra$ values from Table 7.1 with the values from Table 7.2, we can see that the roughness values obtained for the feed rate 0.2 mm/rev are much higher than the values obtained for the 0.03 mm/rev.
Figure 7.19. Surface profiles for feed rate 0.2 mm/rev: (a) mechanical feedback, (b) power feedback, (c) current feedback, (d) manual frequency control, (e) conventional turning (no ultrasonic vibration applied).
Table 7.2. Roughness parameters obtained for feed rate 0.2mm/rev.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>Ra, µm</th>
<th>Rq, µm</th>
<th>Rp, µm</th>
<th>Rv, µm</th>
<th>Rt, µm</th>
<th>Rsk</th>
<th>Rku</th>
<th>Rz, µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical feedback</td>
<td>2.2446</td>
<td>2.6625</td>
<td>6.2986</td>
<td>4.778</td>
<td>11.946</td>
<td>0.25063</td>
<td>2.1454</td>
<td>11.077</td>
</tr>
<tr>
<td>Power feedback</td>
<td>2.5151</td>
<td>2.9704</td>
<td>7.4455</td>
<td>5.7636</td>
<td>16.807</td>
<td>0.35508</td>
<td>2.4618</td>
<td>13.209</td>
</tr>
<tr>
<td>Current feedback</td>
<td>3.1735</td>
<td>3.9309</td>
<td>10.25</td>
<td>8.5521</td>
<td>28.012</td>
<td>0.42356</td>
<td>4.2852</td>
<td>18.802</td>
</tr>
<tr>
<td>Frequency control</td>
<td>4.0441</td>
<td>4.8759</td>
<td>12.609</td>
<td>8.9599</td>
<td>33.109</td>
<td>0.55553</td>
<td>3.6137</td>
<td>21.569</td>
</tr>
<tr>
<td>Non-ultrasonic</td>
<td>5.2192</td>
<td>6.6876</td>
<td>16.697</td>
<td>12.846</td>
<td>54.641</td>
<td>0.29742</td>
<td>5.3799</td>
<td>29.543</td>
</tr>
</tbody>
</table>

Thus, the results presented in this section very closely correspond to the results discussed in the section 7.3 and to the simulation results obtained through the modelling.
Chapter Eight Conclusions and further work

Ultrasonically assisted cutting is a highly advantageous technique which permits enhancing the machining process on a wide range of materials, including difficult-to-machine special composites and hard alloys. With the help of ultrasonically assisted machining, the considerable decrease in cutting forces and improvement of finish quality is achieved along with the enhancement of other characteristics of the machining process.

Autoresonant control is the method of control of ultrasonically assisted machining, providing monitoring of ultrasonic vibrations in the most efficient way. It allows keeping the non-linear resonant mode of vibrations in the ill-defined and time changing conditions of complex industrial environments.

To a great extent, the efficiency of control depends on the feedback design, which in its turn relies on the sensor. Three control strategies based on signals from different sensors have been investigated and compared, these are: mechanical feedback (displacement control) and two cases of electrical feedback (current control and power control).

The completed investigation revealed that the control system based on mechanical feedback provides the most efficient means of control. The advantages of mechanical feedback are linked to the location of the sensor. In the case of mechanical feedback, the sensor is placed near the cutting zone and provides the most reliable information about the dynamics of the machining process.

Electrical feedback is based on the sensor measuring the electrical characteristics of the piezoelectric transducer, which reflects the real vibrations of the ultrasonic system in an indirect way. The piezoelectric transducer is distant from the cutting zone and its electrical characteristics (current and power) are much less subjected to the influence of the cutting process than mechanical characteristics. This explains the reduced efficiency of the control system with electrical feedback. The investigation showed that using the power signal in the control algorithm improved the results of the electrical feedback control system.

For this investigation (practical and theoretical), a quite high level of loadings was used, which may not be the case for some industrial applications. For ultrasonic systems experiencing low levels of loading and having a relatively low quality factor, the electrical feedback may be sufficient, especially if there are no strict requirements to the surface quality of the workpiece treated. The suitability of the feedback type (and sensor
correspondingly) to the industrial conditions, where the system is going to be used, also has to be taken into account while choosing the appropriate control strategy. The limited possibilities of electrical feedback can be improved with an increase of the correlation to the machining process. This can be done, for example, by the introduction of an additional sensor measuring the load applied to the ultrasonic transducer. This would help to monitor the dynamics of the machining process and would improve the reliability of the electrical feedback.

Another area of investigation is the analysis of the possibility of the application of phase-locked loops for the control system design. It is known that phase-locked loops can follow a signal whose frequency varies slowly [78, 79]. That is why they may provide better filtering in conditions of nonlinear load application, when the frequency of the oscillations depends on the load applied and changes slightly during the process of cutting. The use of phase-locked loops in place of filters may improve the performance of the electrical feedback control system.

Additional work needs to be done to elaborate the design of the inductive sensor permitting its incorporation into the design of a tool holder or tool. This will allow the employment of the mechanical feedback control system with CNC machines using an automatic tool-changing system.
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Appendix

Piezoelement

Viscoelastic block

Mass block
Nonlinear load block

RMS counter
Mean value

Phase shifter