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LASER DOPPLER VIBROMETRY FOR VIBRATION MEASUREMENTS ON ROTATING STRUCTURES

BY

BENJAMIN JOHN HALKON

A DOCTORAL THESIS

SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF

DOCTOR OF PHILOSOPHY OF LOUGHBOROUGH UNIVERSITY

MAY 2004
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NOMENCLATURE

ROMAN CHARACTERS

\( \hat{a} \)  \quad \text{Vector from beam orientation unit vector to mirror normal unit vector}

\( A_0 \)  \quad \text{Target vibration displacement amplitude}

\( A_p \)  \quad \text{Area of } p\text{th target beam speckle}

\( \hat{b} \)  \quad \text{Laser beam orientation unit vector}

\( \hat{b}_1 \)  \quad \text{Lens to target beam orientation unit vector (dual mirror scanning system)}

\( \hat{b}' \)  \quad \text{Lens to target beam orientation unit vector (idealised scanning system)}

\( d_S \)  \quad \text{Orthogonal mirror separation}

\( f \)  \quad \text{Laser light frequency (chapter 1)}

\( f \)  \quad \text{Lens focal length (chapter 4)}

\( f(t) \)  \quad \text{Some function of time}

\( f_D \)  \quad \text{Doppler frequency shift}

\( f_R \)  \quad \text{Reference beam frequency pre-shift}

\( FT[] \)  \quad \text{Fourier Transform}

\( FT^{-1}[] \)  \quad \text{Inverse Fourier Transform}

\( i(t) \)  \quad \text{Photodetector output current}

\( i_p(t) \)  \quad \text{Photodetector output current due to } p\text{th target beam speckle}

\( I_{res} \)  \quad \text{Resultant intensity after summation across } P \text{ target speckles}

\( I_R \)  \quad \text{Reference beam intensity}

\( I_T \)  \quad \text{Target beam intensity}

\( I_{Tp} \)  \quad \text{Intensity of } p\text{th target beam speckle}

\( j \)  \quad \text{Complex operator}

\( k \)  \quad \text{Laser light wavenumber}

\( m \)  \quad \text{Number of radius cycles per revolution}
n  Distance between $y$ deflection mirror and target along line of the laser beam (chapter 4)
n  Phase cycles per radius cycle (chapter 7)
n_r  Integer number of laser tube lengths
O  Origin of translating reference frame
P  Instantaneous point of incidence of laser beam
P_0  Corresponding point on undeformed target
$\vec{r}_f$  Position vector defining target deformation (P to P_0)
$\vec{r}_o$  Position vector defining origin O
$\vec{r}_p$  Position vector defining point P
$\vec{r}_{r0}$  Position vector defining point P with respect to O
$\vec{r}_{r0}$  Position vector defining point P_0 with respect to O
$r_S, r_S(t)$  Intended scan amplitude/radius
$r_S$  Mean scan radius
S  Photodetector radiant sensitivity
$U_m$  Measured component of target velocity
$U_x$  $x$ direction radial vibration measurement
$\bar{U}_x$  AC coupled $x$ direction radial vibration measurement
$U_y$  $y$ direction radial vibration measurement
$U'_y$  Misaligned $y$ direction radial vibration measurement
$\bar{U}_y$  AC coupled $y$ direction radial vibration measurement
V  Particle velocity
$V_0$  Target vibration velocity amplitude
$V_0l$  Quadrature (imaginary) velocity amplitude coefficient
$V_0R$  In-phase (real) velocity amplitude coefficient
$\bar{V}_f$  Deformation vibration velocity of point P due to cross-section flexibility
$\bar{V}_o$  Instantaneous velocity of origin O
$\bar{V}_p$  Instantaneous velocity of the point P
$W(\omega)$  Frequency dependent weighting function
NOMENCLATURE

\( x, x(t), \dot{x} \)  Target translational vibration displacement, velocity in \( x \) direction

\( \dot{x} \)  \( x \) direction unit vector

\( \mathcal{X}(\omega), \dot{\mathcal{X}}(\omega) \)  Fourier Transform of vibration displacement, velocity in \( x \) direction at frequency \( \omega \)

\( x_0 \)  Known point \( x \) coordinate

\( x_{0m} \)  Scanning system \( x \) direction translational misalignment

\( x_{0mu} \)  Scanning system \( x \) direction translational initial unknown misalignment

\( x_{estn} \)  \( n^{th} \) estimate of vibration displacement in \( x \) direction

\( \mathcal{X}_{estn}(\omega), \dot{\mathcal{X}}_{estn}(\omega) \)  \( n^{th} \) estimate of vibration displacement, velocity in \( x \) direction at frequency \( \omega \)

\( \dot{x}_r(P) \)  Cross-section flexibility vibration velocity component in \( x \) direction

\( \dot{x}_s(P_0) \)  Resultant rigid body vibration velocity components in \( x \) direction

\( x_s(P,t) \)  Laser beam target incidence point \( x \) coordinate

\( xyz \)  Translating reference frame

\( XYZ \)  Fixed reference frame

\( y, \dot{y} \)  Target translational vibration displacement, velocity in \( y \) direction

\( \dot{y} \)  \( y \) direction unit vector

\( \mathcal{Y}(\omega) \)  Fourier Transform of vibration velocity in \( y \) direction at frequency \( \omega \)

\( y_0 \)  Known point \( y \) coordinate

\( y_{0m} \)  Scanning system \( y \) direction translational misalignment

\( y_{0mu} \)  Scanning system \( y \) direction translational initial unknown misalignment

\( y_{estn} \)  \( n^{th} \) estimate of vibration displacement in \( y \) direction

\( \mathcal{Y}_{estn}(\omega), \dot{\mathcal{Y}}_{estn}(\omega) \)  \( n^{th} \) estimate of vibration displacement, velocity in \( y \) direction at frequency \( \omega \)

\( \dot{y}_r(P) \)  Cross-section flexibility vibration velocity component in \( y \) direction

\( \dot{y}_s(P_0) \)  Resultant rigid body vibration velocity components in \( y \) direction

\( y_s(P,t) \)  Laser beam target incidence point \( y \) coordinate

\( z, \dot{z} \)  Target translational vibration displacement, velocity in \( z \) direction

\( \dot{z} \)  \( z \) direction unit vector
NOMENCLATURE

\( z_0 \) Known point \( z \) coordinate/target stand-off distance
\( z_1 \) Lens to target distance
\( \dot{z}_f(P) \) Cross-section flexibility vibration velocity component in \( z \) direction
\( \dot{z}_r(P_0) \) Resultant rigid body vibration velocity components in \( z \) direction
\( z_s(P) \) Laser beam target incidence point \( x \) coordinate
\( \hat{z}_R \) Target rotation axis direction

GREEK CHARACTERS

\( \alpha \) Laser beam orientation about \( z \) axis
\( \beta \) Laser beam orientation about \( y \) axis
\( \delta \) Instrument misalignment about \( z \) axis (chapter 3)
\( \delta(t) \) Variation of \( \alpha \) for circular scanning (chapter 4)
\( \Delta \phi_S \) Time dependency in scan phase
\( \Delta r_s \) Time dependency in scan radius
\( \Delta \Omega_T \) Total target torsional oscillation
\( \Delta x_0 \) Time dependency in known point \( x \) coordinate
\( \epsilon \) Included angle between incidence and scattering directions (chapter 1)
\( \epsilon \) Instrument misalignment about \( x \) axis (chapter 3)
\( \epsilon, \delta(t) \) Variation of \( \beta \) for circular scanning (chapter 4)
\( \phi_{\phi_s} \) Phase cycle phase
\( \phi_{r_s} \) Radius cycle phase
\( \phi_R \) Reference beam phase
\( \phi_S \) Scan initial phase
\( \overline{\phi_S} \) Mean scan phase
\( \phi_T \) Target beam phase
\( \phi_V \) Target vibration phase
\( \Phi_{\text{res}} \) Resultant phase after summation across \( P \) target speckles
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( \gamma )</td>
<td>Angle between particle velocity vector and bisector of angle between incidence and scattering directions</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Laser light wavelength</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Photodetector surface area</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Refractive index</td>
</tr>
<tr>
<td>( \theta_x, \dot{\theta}_x )</td>
<td>Target rotational vibration displacement, velocity about ( x ) axis</td>
</tr>
<tr>
<td>( \theta_{x,m} )</td>
<td>Scanning system angular misalignment about ( x ) axis</td>
</tr>
<tr>
<td>( \theta_{x,nu} )</td>
<td>Scanning system angular initial misalignment about ( x ) axis</td>
</tr>
<tr>
<td>( \theta_y, \dot{\theta}_y )</td>
<td>Target rotational vibration displacement, velocity about ( y ) axis</td>
</tr>
<tr>
<td>( \theta_{y,m} )</td>
<td>Scanning system angular misalignment about ( y ) axis</td>
</tr>
<tr>
<td>( \theta_{y,nu} )</td>
<td>Scanning system angular initial misalignment about ( y ) axis</td>
</tr>
<tr>
<td>( \theta_{S1} )</td>
<td>First deflection mirror scan angle (single mirror scanning system)</td>
</tr>
<tr>
<td>( \theta_{S2} )</td>
<td>Second deflection mirror scan angle (single mirror scanning system)</td>
</tr>
<tr>
<td>( \theta_{Sx} )</td>
<td>( x ) deflection mirror scan angle (dual mirror scanning system)</td>
</tr>
<tr>
<td>( \theta_{Sy} )</td>
<td>( y ) deflection mirror scan angle (dual mirror scanning system)</td>
</tr>
<tr>
<td>( \theta_z, \dot{\theta}_z )</td>
<td>Target rotational vibration displacement, velocity about ( z ) axis</td>
</tr>
<tr>
<td>( \Theta_{Sx} )</td>
<td>( x ) deflection mirror scan angle amplitude (dual mirror scanning system)</td>
</tr>
<tr>
<td>( \Theta_{Sy} )</td>
<td>( y ) deflection mirror scan angle amplitude (dual mirror scanning system)</td>
</tr>
<tr>
<td>( \dot{\Theta}_x )</td>
<td>Pitch vibration measurement</td>
</tr>
<tr>
<td>( \dot{\Theta}_y )</td>
<td>Yaw vibration measurement</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Angular velocity of ( P_0 ) about an instantaneous axis passing through ( O )</td>
</tr>
<tr>
<td>( \omega_{\text{beat}} )</td>
<td>Fluctuating component of measured beat frequency</td>
</tr>
<tr>
<td>( \omega_R )</td>
<td>Reference (angular) frequency pre-shift</td>
</tr>
<tr>
<td>( \omega_T )</td>
<td>Target vibration angular frequency</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Target rotational angular velocity</td>
</tr>
<tr>
<td>( \Omega_S )</td>
<td>Scan angular frequency</td>
</tr>
<tr>
<td>( \Omega_T )</td>
<td>Total target rotational angular velocity</td>
</tr>
<tr>
<td>( \overline{\Omega}_T )</td>
<td>Mean total target rotational angular velocity</td>
</tr>
</tbody>
</table>
### SPECIAL CHARACTERS

| $\mathcal{I}$ | Fourier Transform imaginary part |
| $\mathfrak{R}$ | Fourier Transform real part |
Laser Doppler Vibrometry is now a well-established and commercially viable technique enabling non-contact vibration measurements in the most challenging of environments. Rotating target vibration measurements are often highlighted as a major application of Laser Vibrometers due to their non-contact operation and inherent immunity to shaft run-out. This thesis describes advances in the application and interpretation of such measurements using Laser Vibrometers.

It is readily accepted that a Laser Vibrometer measures target velocity in the direction of the incident laser beam but this measured velocity must be considered in terms of the various components of the target velocity. A previously derived theoretical description of the velocity sensed by an arbitrarily orientated laser beam on a rotating structure undergoing arbitrary six degree-of-freedom vibration provides the mathematical basis for the investigations contained herein. This comprehensive velocity sensitivity model presents the measured velocity as the sum of six terms, each the product of a combination of geometric parameters, relating to the laser beam orientation, and an inseparable combination of motion parameters, referred to as the vibration "sets".

Resolution of the individual axial and torsional motion components is possible via particular arrangement of the laser beam(s) and by assuming that the effects of the cross-sensitivity terms within those particular vibration sets are negligible. It is not possible, however, to resolve the radial or pitch/yaw motion components in a similar manner; this can only be achieved by post-processing the outputs from simultaneous orthogonal measurements. In the study described in this thesis, a LabVIEW software based solution has been developed enabling, for the first time, real-time frequency domain post-processing of the outputs from standard commercially available instrumentation. The resulting measurement system is rigorously examined in terms of performance and error sensitivity and implemented in several example measurement situations thereby demonstrating the potentially powerful vibration information that is available to the vibration engineer.

The use of Laser Vibrometers incorporating some form of manipulation of the laser beam orientation, typically using two orthogonally aligned mirrors, has become increasingly popular in recent years with considerable attention being given to the operation of such scanning Laser Vibrometers in continuous scanning mode. Here the laser beam orientation is a continuous function of time, making it possible, for example, to track a single point on a moving target such as a rotating bladed disc. The comprehensive velocity sensitivity model has been developed to incorporate time-dependent beam orientation and this is described in detail with reference to continuous scanning Laser Vibrometry. In the previous derivation, the illuminated target cross-section was assumed to be rigid but, since continuous scanning measurements are
employed on targets with flexible cross-sections, such as beams, panels and thin or bladed discs, the theory is also developed to include provision for such flexibility.

The model predicts the measured velocity for arbitrary mirror scan angles and arbitrary target motion and is shown to be especially valuable in revealing the sources of additional components that occur in continuous scanning and tracking measurements on rotors. In particular, additional components at DC and 1x scan frequency are shown to occur as a result of misalignment between the scanning system and target rotation axes, whilst the commonly implemented dual mirror scanning configuration is shown to be the source of additional information at 2x scan frequency. A method of determining the individual misalignment parameters is presented for the first time as well as suggestions for subsequent misalignment elimination and hence DC and 1x additional component elimination. A novel scanning configuration incorporating a dual axis single scanning mirror is recommended and investigated as a means to eliminate the 2x component.

The development of the comprehensive velocity sensitivity model and of sophisticated measurement hardware and software has resulted in proposal of the exciting new Synchronised-Scanning Laser Vibrometry technique. Introduced for the first time in this thesis, the measurement involves the probe laser beam tracking the rotating structure and simultaneously scanning the region of interest to provide modal data under operating conditions, i.e. during rotation. A detailed description of the underlying philosophy of the technique is supported by information obtained from the measurement system that has been developed. Such a measurement is inconceivable by any other means and the system that has been created has the potential to provide data of fundamental importance in the design and development of a wide range of devices from hard disk drives to gas turbines.

Since laser speckle effects are of such significance in rotor measurements, their influence is acknowledged and investigated at various stages. A detailed theoretical analysis is presented and is supported by a novel experimental analysis of the sensitivity of commercially available instruments to such rotating target measurements. The prediction of the effects of laser speckle is complicated in scanning measurements on rotors and an analysis in the form of a speckle repeat map shown for the first time. In addition to this, experimental data is presented with a view to quantifying the typical speckle noise levels experienced in such measurements and confirming that noise levels are at a local minimum when the scanning system is configured to track a point on the rotor.

The significant developments in the use of Laser Vibrometry for vibration measurements directly from rotating targets realised during this research project make the straightforward acquisition of valuable data a realistic prospect.

**Keywords:** Laser Doppler Vibrometry, velocity sensitivity, vibration measurement, rotating components
ACKNOWLEDGEMENTS

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I would like to acknowledge the support of the Engineering and Physical Sciences Research Council who are funding the project which my PhD has been based upon and the Institution of Mechanical Engineers who have kindly awarded me with additional funding to assist in travelling to present my several conference contributions. I also acknowledge the efforts made by the several undergraduate students and technicians who have worked on the project with me, in particular Mr Steve Hammond whose passion for complex electronic circuit design is almost infectious! Thank you all.
TO GRANNY KIRK
"IT IS MORE IMPORTANT TO HAVE BEAUTY IN ONE’S EQUATIONS THAN TO HAVE THEM FIT THE EXPERIMENT!"

Paul Dirac, 1902-1984
The principle of Laser Doppler Vibrometry (LDV) relies on the detection of a Doppler shift in the frequency of coherent light scattered by a moving target, from which a time-resolved measurement of the target velocity is obtained. The Laser Vibrometer is now well established as an effective non-contact alternative to the use of a traditional contacting vibration transducer. Laser Vibrometers are technically well suited to general application but offer special benefits where certain measurement constraints are imposed, for example by the context, which may demand high frequency operation, high spatial resolution or remote transducer operation, or by the structure itself, which may be hot, light or rotating. Measurements on such structures are often cited as important applications of LDV and rotor vibration measurements are the particular focus of this thesis.

1.1. Fundamentals of Laser Vibrometry

With reference to Figure 1.1, the principle of operation of the Laser Vibrometer, which has its origins in Laser Doppler Anemometry [1.1, 1.2], relies on the detection of the Doppler frequency shift in coherent light scattered from moving particles. This frequency shift, \( f_D \), is directly proportional to the velocity of the particle, \( V \), and is given by [1.3]:

\[
\frac{f_D}{f_0} = \left( \frac{2 \mu V}{\lambda} \right) \cos \gamma \sin \left( \frac{\varepsilon}{2} \right)
\]  

(1.1)

where \( \mu \) is the refractive index of the medium in which the measurement is being taken (1 for air), \( \lambda \) is the laser light wavelength, \( \gamma \) is the angle between the particle velocity
vector and the bisector of the angle between the incidence and scattering directions, and \( \varepsilon \) is the included angle between the incidence and scattering directions.

The use of this principle for solid surface measurements, in which the light scattering particles take the form of light scattering surface elements, was recognised at an early stage [1.4-1.8] and such devices came to be referred to as Laser Vibrometers [1.9]. Figure 1.2 shows a simple Laser Vibrometer in a Michelson interferometer configuration. Here, the light is collected (on a photodetector) in direct backscatter, i.e. \( \varepsilon = \pi \), such that the measured component of target velocity, \( U_m = V \cos \gamma \), is "on-axis" with the incident laser beam and equation (1.1) can be re-written (with \( \mu = 1 \)) as:

\[
f_D = \frac{2U_m}{\lambda}
\]  

(1.2)

The photodetector cannot respond quickly enough to detect directly the relatively small changes in the laser light frequency, \( f (\approx 10^{15} \text{Hz}) \), that occur as a result of this Doppler frequency shift (\( \approx 10^6 \text{Hz} \)). The backscattered light from the target is therefore optically mixed with a mutually coherent reference beam on the photodetector surface resulting in an optical beat. The frequency of the resulting optical beat is clearly at the difference between the frequencies of the reference and target beams. Such a configuration would, however, leave a directional ambiguity in the measurement since demodulation only identifies the modulus of the Doppler frequency beat.

The more popular solution to the directional ambiguity problem involves frequency pre-shifting the reference beam by a known amount, \( f_R \), resulting in an optical beat at the detector of frequency \( |f_R - f_D| \). The beat frequency is thereby less than or greater than the pre-shift frequency depending on the direction of the target velocity. Systems differ by the method used for producing the frequency shift; rotating diffraction gratings [1.4], Bragg cells [1.5], rotating scattering discs [1.6], and direct laser beam frequency modulation [1.7,1.8] have all been successfully implemented. The most popular commercial frequency pre-shifting method is the Bragg cell [1.9].
An alternative solution to the directional ambiguity problem is to use a quadrature detection system [1.10]. Here, a relatively complex optical configuration is used to separate the mixed beams into two Doppler signal channels, identical except for a phase difference of 90°. Multiplying each Doppler signal by a carrier signal and summing the results produces a signal in the same format as the photodetector beat encountered in the frequency pre-shift arrangement.

Regardless of the means of overcoming the directional ambiguity problem, frequency demodulation of the photodetector beat by an appropriate Doppler signal processor, essentially a frequency to voltage converter, produces a time-resolved analogue of the target velocity, $U_m$. This signal has the form of a DC voltage due to the constant $f_R$ (the output is usually arranged such that this occurs at 0V) and an AC voltage due to the velocity dependent $f_D$.

By way of an example, consider the on-axis measurement of single component sinusoidal vibration. The target vibration displacement, $x(t)$, is given by:

$$x(t) = A_0 \sin(\omega_V t + \phi_V)$$  \hspace{1cm} (1.3)

where $A_0$ is the target vibration displacement amplitude and $\omega_V$ and $\phi_V$ are the target vibration (angular) frequency and phase, respectively.

Such target motion results in a corresponding variation in the phase of the backscattered target beam. Addition of the plane reference and phase modulated target beam contributions across the photodetector surface, of area $A$, results in a photodetector output current, $i(t)$, which is proportional to the total light intensity incident on the photosensitive surface. Neglecting polarisation effects and the DC terms yields [1.11]:

$$i(t) = 2SA\sqrt{I_R I_I} \cos[(\omega_R t + 2kA_0 \sin(\omega_V t + \phi_V) + (\phi_R - \phi_I)]$$  \hspace{1cm} (1.4)
where $S$ is the radiant sensitivity of the photodetector, $I_R$, $\phi_R$, $I_T$ and $\phi_T$ are the intensities and phases of the reference beam and target beam respectively, $\omega_R$ is the reference (angular) frequency pre-shift and $k$ is the laser light wavenumber.

**1.2. Laser Speckle Effects in Laser Vibrometry Measurements**

The case of a plane backscattered target wavefront is one which very rarely occurs in engineering applications. When a coherent light source is incident on a surface that is optically rough, i.e. the surface roughness is large on the scale of the laser wavelength, the component wavelets of the resulting scattered light become dephased. The dephased, but still coherent, wavelets interfere constructively and destructively, thus resulting in a chaotic distribution of high and low intensities in the backscatter. Such intensity distributions are referred to as “speckle patterns”. Figure 1.3a shows an example of a speckle pattern which results from scattering by an optically rough surface. Figure 1.3b is typical of that which results from a surface treated with retroreflective paint or tape [1.12], the bright-dark ringed “airy disc” pattern being caused by the small glass retroreflective beads that form the paint or tape surface. Statistically the speckles have intensities with a negative exponential probability distribution, whilst their phases are uniformly distributed between 0 and $2\pi$ [1.13].

**1.2.1. Speckle Noise Generation**

As most surfaces of engineering interest are rough on the scale of the laser wavelength, a thorough treatment of the operation of a Laser Vibrometer must acknowledge the effect of the speckle pattern which results when the reference and target beams are heterodyned on the photodetector surface. In such a case, the photodetector output current due to the $p^{th}$ target beam speckle, $i_p(t)$, again neglecting the DC terms, is given by [1.11]:

---

4
where $I_{tp}$ and $A_p$ are the intensity and area, respectively, of the $p^{th}$ target beam speckle. Summation of this equation for the $P$ speckles incident on the photodetector yields an expression which can be used to determine the total photodetector output, $i(t)$ [1.11]:

$$i(t) = \sum_{p=1}^{P} i_p(t) = S A_I \sum_{p=1}^{P} \left[ [\omega R t + 2 k A_0 \sin(\omega_T t + \phi_V) + \left( \phi_R - \phi_{tp} \right) ] \right]$$

(1.5)

where $I_{res}$ and $\Phi_{res}$ are the resultant intensity and phase, respectively, after summation across the $P$ target speckles.

Combinations of speckles that lead to low values of $I_{res}$ may lead to problems due to low signal amplitudes. Provided $I_{res}$ is large enough to overcome such problems, the fluctuating component of the measured beat frequency, $\tilde{\omega}_{beat}$, is given by:

$$\tilde{\omega}_{beat} = 2 k A_0 \omega_T \cos(\omega_T t + \phi_V) + \frac{d\Phi_{res}}{dt}$$

(1.7)

This is clearly equal to the required value of $2 k A_0 \omega_T \cos(\omega_T t + \phi_V)$ when the $P$ target beam speckles do not change in phase, amplitude or position (relative to the detector surface) during the course of a measurement, since $d\Phi_{res}/dt$ is equal to zero. The output frequency spectrum contains a single peak at $(\omega_T/2\pi)$Hz, corresponding to the target vibration.

In many situations, however, the spatial characteristics of the speckle pattern do change during the course of a measurement and $d\Phi_{res}/dt$ is therefore non-zero resulting in “speckle noise” in the output spectrum. If the speckle pattern motions are caused by non-normal target motions (e.g. tilt, in-plane motion and rotation) which, in general, will be periodic with the same fundamental frequency as the on-axis vibration, then $d\Phi_{res}/dt$ will be “pseudo-random” in nature with a fundamental frequency also at
The characteristic frequency spectrum of a pseudo-random noise signal consists of approximately equal amplitude peaks at the fundamental frequency and associated higher order harmonics. This additional information, or "pseudo-vibration", is indistinguishable from the genuine vibration information; speckle motions may result in noise levels sufficient enough to mask low-level vibration information in the intended measurement.

Target rotation is arguably the most significant non-normal target motion that induces pseudo-vibration [1.11] and is clearly of importance in this study. The characteristics and influence of speckle noise on measured data will be introduced in the following subsection and will be investigated for the advanced techniques developed in this thesis.

1.2.2. TARGET ROTATION INDUCED SPECKLE NOISE CHARACTERISTICS

Figure 1.4 shows a schematic of the experimental arrangement in which the Laser Vibrometer and accelerometer measurements illustrated in Figure 1.5 and Figure 1.6 were obtained. Figure 1.5a shows a measurement of the 60Hz, 5mm/s (nominal) translational vibration of a non-rotating rotor. In addition to the vibration peak and some low-level genuine harmonic distortion, the measurement exhibits a low noise floor which includes electronic noise at 50Hz and multiples. This data is typical of Laser Vibrometer measurements taken from periodically excited structures and compares well with the output (integrated once for velocity) from an accelerometer mounted on the rotor housing, as illustrated in Figure 1.5b. It should be noted that there is likely to be some genuine difference between the vibration of the rotor (measured by the Laser Vibrometer) and the vibration of the rotor housing (measured by the accelerometer) and this difference is exactly the motivation for developing techniques for vibration measurement directly from rotating structures.

Figure 1.6 shows the same two measurements, in this case taken with the rotor rotating at (nominally) 15Hz. The rotation will clearly result in some additional rotor vibration and this can be observed by comparing the accelerometer measurements shown in Figure 1.6b and Figure 1.5b. Since the rotor was light and well balanced, the difference
is negligible in this case. Whilst the measured on-axis vibration component levels are similar in Figure 1.6a and Figure 1.5a, the difference between the two Laser Vibrometer measurements is significant; peaks at the fundamental rotation frequency and subsequent harmonics are present due to pseudo-random speckle noise in the rotating rotor measurement. These harmonic speckle noise peaks do not, however, display the same amplitude “roll-off” as the translational vibration harmonic peaks, remaining approximately constant in amplitude with some random fluctuation throughout the frequency range. This is typical of measurements taken under conditions where a large number of speckle pattern changes occur during the course of the measurement and this helps, in practice, in attributing such spectrum characteristics to speckle pattern motion rather than true vibration. Figure 1.6a clearly shows that the vibration engineer must employ a degree of engineering judgement when interpreting low-level vibration information obtained with a Laser Vibrometer.

A more detailed, novel analysis of the influence of speckle induced noise in measurements from rotating targets using commercially available Laser Vibrometers is presented in chapter 6. In particular, differences between the performance of the instruments when making measurements on various surface treatments and at various stand-off distances is considered.

1.3. LASER VIBROMETRY FOR ROTOR VIBRATION MEASUREMENTS

In rotating machinery, vibration measurement is essential and is typically performed from the earliest stages of design and development through to the condition monitoring of commissioned equipment [1.15]. The most common measurement is that of the vibration transmitted into a non-rotating component using a contacting transducer but, in some cases, low vibration transmission can make this unreliable [1.16]. Often, a non-contact transducer capable of measuring directly from any location along the rotor is desirable and LDV offers this possibility. One of the earliest reported applications of LDV was, indeed, for axial vibration measurement directly from a rotating turbine blade
[1.17] and more recent and typical examples include the measurement of vibration in magnetic discs [1.18,1.19] and bladed discs [1.20,1.21].

Since a Laser Vibrometer measures target velocity in the direction of the incident laser beam, pure axial vibration measurements are obtained from rotating targets by careful alignment of the laser beam with the rotation axis. Provided consideration is given for the laser speckle effect [1.11], the measurement can be obtained in the same way as for a similar measurement on a non-rotating target as presented in the previous section. For radial vibration measurements, however, the presence of a velocity component due to the rotation itself generates significant cross-sensitivities to rotation speed fluctuation (including torsional oscillation) and motion components perpendicular to the intended measurement. Early studies acknowledged such cross-sensitivities in both radial [1.22,1.23] and rotational [1.24] measurements but these were only special cases of the recently derived totally general case [1.25] which is summarised in what follows.

1.3.1. THE COMPREHENSIVE VELOCITY SENSITIVITY MODEL

With reference to Figure 1.7, the case considered is that of an axial element of a shaft of arbitrary cross-section, rotating about its spin axis whilst undergoing arbitrary, six degree-of-freedom vibration but this theory is equally applicable to any non-rotating, vibrating structure. A translating reference frame, \(xyz\), maintains its direction at all times and has its origin, \(O\), fixed to a point on the shaft spin axis with the undeflected shaft rotation axis defining the direction and position of the \(z\) axis. The time dependent unit vector \(\hat{z}_R\) defines the changing direction of the target rotation axis, which deviates from the \(z\) axis as the shaft tilts. \(P\) is the instantaneous point of incidence of the laser beam on the shaft and is identified by the time dependent position vector \(\vec{r}_p\).

The direction of the incident laser beam is described by the unit vector \(\hat{b}\), which, if orientated according to the angles \(\beta\) and \(\alpha\) as shown in Figure 1.8, is given by [1.25]:

\[
\hat{b} = [\cos \beta \cos \alpha] \hat{x} + [\cos \beta \sin \alpha] \hat{y} - [\sin \beta] \hat{z}
\]  

(1.8)
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The convention used here, which all configurations analysed in terms of \( \beta \) and \( \alpha \) in this thesis follow, is that the laser beam orientation is described as a combination of two angles: with \( \theta = \hat{x} \) initially, first rotate by an angle \( \beta \) around \( \hat{y} \), then by an angle \( \alpha \) around \( \hat{z} \). Since \( \beta \) and \( \alpha \) are finite rotations, their order must be maintained. Clearly, the choice of orientation of the reference frame, \( xyz \), relative to the structure lies with the user.

Provided that the illuminated axial element of the shaft can be assumed to be of rigid cross-section, the velocity, \( U_m \), measured by a laser beam incident on the shaft surface is given by [1.25]:

\[
U_m = \cos \beta \cos \alpha \left( \hat{x} + (\hat{\theta}_x + \Omega)\hat{y} - (\hat{\theta}_y - \Omega \hat{\theta}_z)\hat{z} \right) \\
+ \cos \beta \sin \alpha \left( \hat{y} - (\hat{\theta}_z + \Omega \hat{\theta}_y)\hat{x} + (\hat{\theta}_y + \Omega \hat{\theta}_x)\hat{z} \right) \\
- \sin \beta \left( \hat{z} - (\hat{\theta}_x + \Omega \hat{\theta}_y)(\hat{y} + (\hat{\theta}_y - \Omega \hat{\theta}_x)\hat{x} \right) \\
- (y_0 \sin \beta + z_0 \cos \beta \sin \alpha)\left( \hat{\theta}_x + \Omega \hat{\theta}_y \right) \\
+ (z_0 \cos \beta \cos \alpha + x_0 \sin \beta)\left( \hat{\theta}_y - \Omega \hat{\theta}_x \right) \\
+ (x_0 \cos \beta \sin \alpha - y_0 \cos \beta \cos \alpha)\left( \hat{\theta}_z + \Omega \right) \tag{1.9}
\]

where \( \hat{x}, \hat{y}, \hat{z} \) and \( x, y, z \) are the translational vibration velocities and displacements of the origin, \( O \), in the \( x, y, z \) directions, \( \hat{\theta}_x, \hat{\theta}_y, \hat{\theta}_z \) and \( \theta_x, \theta_y, \theta_z, (\hat{\theta}_z) \) are the angular vibration velocities and displacements of the shaft around the \( x, y, z \) axes (referred to as pitch, yaw and roll, respectively), \( \Omega \) is the total rotational angular velocity of the axial shaft element (combining mean velocity and any torsional oscillation), and \( (x_0, y_0, z_0) \) is the position of an arbitrary known point that lies along the line of the beam.

The derivation of equation (1.9) was significant. It showed that the measured velocity is the sum of six terms, each the product of a combination of geometric parameters, relating to the laser beam orientation, and a combination of motion parameters — the vibration “sets”. The six vibration sets, shown in square brackets, are inseparable combinations of different motion parameters. Regardless of the laser beam orientation,
only these motion parameter combinations can be measured directly. For example, an intended measurement of the radial velocity in the \( x \) direction, arranged for by selecting appropriate values for \( \alpha, \beta, x_0, y_0 \) and \( z_0 \), includes a cross-sensitivity to displacements in the \( y \) and \( z \) directions combined with the pitch, yaw, roll and rotation of the shaft about its spin axis.

Along with speckle noise problems, as discussed in the previous section, it is cross-sensitivities such as these which constitute a major limitation to the use of Laser Vibrometers for rotor vibration measurements and which have provided one of the motivations for the study presented in this thesis.

1.3.2. Isolation of Individual Vibration Sets

Despite the fact that individual components cannot be measured directly, it is still important and useful to be able to measure individual vibration sets. In some cases, it may be possible to assume that the effects of the cross-sensitivity terms in a particular vibration set are negligible, thereby enabling direct measurement of a specific motion component. For example, if the amplitudes of the vibration components in a vibration set are known to be similar, then the intended measurement dominates at vibration frequencies much higher than the rotation frequency. Generally, however, reliable estimation of the motion components requires the resolution of the outputs from several instruments each arranged to measure a particular vibration set.

1.3.2.1. Translational Vibration Sets

Measurement of the \( x \) radial vibration set requires that the Laser Vibrometer is aligned such that it passes through the centre of the shaft and along the \( x \) axis, i.e. \( \alpha = \beta = 0^\circ \) and \( y_0 = 0 \). Substituting these values into equation (1.9) and setting \( z_0 = 0 \) so that the plane of the origin of the \( xyz \) reference frame and the “measurement plane” are coincident, since this is merely a matter of definition, results in:

\[
U_m = \left[ \dot{x} + (\dot{\theta}_z + \Omega)y - (\dot{\theta}_y - \Omega \dot{\theta}_z)z \right] \quad (1.10a)
\]
Similarly, values of $\alpha$, $\beta$, $x_0$ and $y_0$ can be found that enable the $y$ radial vibration set to be isolated:

$$U_m = \left[ \dot{y} - (\dot{\theta}_x + \Omega)x + (\dot{\theta}_y + \Omega \theta_y)z \right]$$

(1.10b)

Since the third set of terms in equations (1.10a&b), $(\dot{\theta}_y - \Omega \theta_y)z$ and $(\dot{\theta}_x + \Omega \theta_y)z$, are products of vibration parameters, they are typically an order of magnitude smaller than the first two terms [1.24] and can therefore be considered insignificant. In addition, the shaft rotation and roll motion are generally indistinguishable in practice and, as such, it is convenient to consider the total angular velocity, where $\Omega_T = \dot{\theta}_x + \Omega$. The $x$ and $y$ radial vibration set measurements, $U_x$ and $U_y$, are therefore given to a reasonable approximation by:

$$U_x = \dot{x} + \Omega_T y$$

(1.11a)

and

$$U_y = \dot{y} - \Omega_T x$$

(1.11b)

1.3.2.2. ROTATIONAL VIBRATION SETS

Isolation of the rotational vibration sets requires that the geometric coefficients of the three translational vibration sets equal zero i.e. $\cos \beta \cos \alpha = \cos \beta \sin \alpha = \sin \beta = 0$, to which there is no solution. This means that none of the rotational vibration sets can be isolated using a single laser beam. Repeated application of equation (1.9) for two beams shows that isolation of each rotational vibration set is best achieved using a parallel beam arrangement [1.26], thereby confirming the configuration of the Rotational Laser (formerly Laser Torsional) Vibrometer [1.27]. Combinations of values for the geometric parameters can be found that enable isolation of the pitch and yaw vibration sets. The pitch and yaw measurements, $\dot{\theta}_x$ and $\dot{\theta}_y$, do not require the assumptions made to reduce the corresponding radial measurements from equations (1.10a&b) to (1.11a&b) and are straightforwardly given by [1.26]:

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\[ \dot{\theta}_x = \dot{\theta}_x + \Omega \dot{\theta}_y \quad (1.12a) \]
and
\[ \dot{\theta}_y = \dot{\theta}_y - \Omega \dot{\theta}_x \quad (1.12b) \]

Since equations (1.12a&b) are similar in form to equations (1.11a&b), the radial and pitch/yaw vibration measurement cross-sensitivity problems are equivalent and a solution for resolution of radial vibrations will be applicable to resolution of pitch/yaw vibrations and vice-versa.

1.3.3. CROSS-SENSITIVITY EFFECTS

In this section the velocity outputs from a series of simple rotor vibration measurements will be presented. It is not intended that this section be taken as a complete validation of the theory, since such validations have been presented previously [1.23-1.26], rather a simple demonstration of the cross-sensitivity problem to assist interpretation. The arrangement employed is as shown in Figure 1.4 and enables independent control of the rotational angular velocity and translational vibration of a rotating disc, such that the influence of orthogonal radial vibration and/or torsional vibration on a radial vibration measurement can be investigated. The Laser Vibrometer was arranged parallel to the \( x \) axis \((y_0 \approx 5\text{mm})\) such that a quantity similar to the \( x \) radial set, as given by equation (1.10a), was isolated. In this carefully configured case there is (nominally) zero motion in the \( y, z, \theta_x \) and \( \theta_y \) directions, such that, with reference to equation (1.9), the AC coupled measured velocity, \( \tilde{U}_x \), is given by:

\[ \tilde{U}_x = \dot{x} + \overline{\Omega}_r y + \Delta \Omega_r (y - y_0) \quad (1.13) \]

where \( \overline{\Omega}_r \) and \( \Delta \Omega_r \) are the mean rotational velocity and torsional oscillation, respectively.
Figure 1.9 shows the AC coupled Laser Vibrometer output for a measurement on a target rotating at (nominally) 100πrad/s (50Hz) in which there is a 25Hz, 25mm/s (nominal) x radial vibration only (i.e. \( \vec{U}_x = \dot{x} \approx 25\text{mm/s @ 25Hz} \)). Despite some low-level speckle induced noise, the measurement is as intended and the potential value of the Laser Vibrometer for non-contact vibration measurements from rotating components is confirmed.

The need for caution when interpreting data from such measurements, however, is demonstrated clearly in Figure 1.10 and Figure 1.11 in which, despite zero x radial vibration, there is clearly significant velocity content. Figure 1.10 shows the output for a measurement in which there is 75Hz, 35mm/s (nominal) y radial vibration only (i.e. \( \vec{U}_x = \Omega_r y \approx 25\text{mm/s @ 75Hz} \)) and Figure 1.11 shows the output for a measurement in which there is 25Hz, 285deg/s (nominal) torsional oscillation only (i.e. \( \vec{U}_x = \Delta \Omega_r y_0 \approx 25\text{mm/s @ 25Hz} \)). The measured velocity (ideally zero in both of these cases, of course) clearly contains potentially serious cross-sensitivity terms and the likelihood of misinterpretation is therefore significant.

The severity of the cross-sensitivity phenomenon is exemplified by consideration of a particular (extreme but certainly not impossible) case of the measurement of synchronous radial vibration where \( x = A_0 \cos \Omega_r t \) and \( y = A_0 \sin \Omega_r t \). Under these conditions the AC coupled x and y radial vibration set measurements are both zero regardless of the vibration amplitude \( A_0 \):

\[
\vec{U}_x = \dot{x} + \Omega_r y = -A_0 \Omega_r \sin \Omega_r t + \Omega_r A_0 \sin \Omega_r t = 0 \quad (1.14a)
\]

and

\[
\vec{U}_y = \dot{y} - \Omega_r x = A_0 \Omega_r \cos \Omega_r t - \Omega_r A_0 \cos \Omega_r t = 0 \quad (1.14b)
\]

Figure 1.12 compares the measurements from an accelerometer and a Laser Vibrometer made from a non-rotating rotor vibrating simultaneously in two orthogonal directions; the accelerometer is mounted on the rotor housing. Despite some small calibration and
phase differences, both instruments are sensitive only to motion in one direction and the measurements are equivalent, as expected. The measurements shown in Figure 1.13, however, are taken with the rotor rotating at 50πrad/s (25Hz) i.e. the orthogonal vibrations are synchronous with the rotational angular velocity. The worst-case cross-sensitivity problem is clearly demonstrated – the Laser Vibrometer measurement is virtually zero (low-level speckle induced noise is apparent) in the presence of a significant on-axis translational vibration.

Clearly, in the other extreme, i.e. where one of the vibration components was opposite in phase, both measurement amplitudes would be twice the individual vibration component amplitude. In any situation, it is clear that the cross-sensitivity problem may result in significant misinterpretation of the data obtained in a Laser Vibrometer measurement on a rotating target. Use of the Laser Vibrometer for such measurements to its best effect requires a clear understanding of the mechanisms by which measurement ambiguity occurs and a robust integrated solution capable of providing the user with resolved outputs of the individual vibration components. One of the main aims of this research project was to realise these important objectives and the various outcomes of this part of the work are described in detail in chapters 2 and 3. Noteworthy examples of the use of the developed techniques for the acquisition of radial and pitch/yaw vibration information directly from the crankshaft pulley of a running 4 cylinder diesel engine (a typical application) are presented in chapter 6. The potentially powerful information obtained supports the fact that the development of such techniques is worthwhile. Some recommendations for future work and concluding remarks are discussed in chapters 7 and 8 respectively.

1.4. Scanning Laser Vibrometry

For light structures, the extent of the local structural modification and resulting change in dynamic behaviour due to the attachment of a contacting transducer must always be considered [1.28]. This is significant when measurements are to be taken from several points, since the dynamic behaviour of the structure may change from one measurement
to the next with the relocation of the transducer. For such situations, a non-contact vibration transducer capable of making a series of measurements across a component surface is desirable and LDV offers this possibility.

A substantial reduction in test time can be realised by automating the "relocation" of the measurement transducer and the suitability of the Laser Vibrometer to such automation was recognised at an early stage in the development of the instrument [1.29]. The introduction of some form of laser beam deflection (typically reflection by a mirror) and an associated control system enables the definition of the order in which the measurements are to be made and examples of the use of such scanning Laser Vibrometers include measurements on automotive [1.30] and turbomachinery [1.31] components and assemblies.

In addition to this point-by-point operation of the scanning Laser Vibrometer, it is possible to configure the instrument to function in a continuous scanning mode. Continuous scans are conveniently arranged for by driving the beam deflection mirror with a continuous time variant signal, enabling the target velocity profile along a predetermined path to be determined in a single measurement. Post-processing of the Laser Vibrometer output signal results in a series of coefficients that describe the operational deflection shape or, where a frequency response function is obtained, mode shape [1.28,1.32]. Straight-line, area, circular, small-scale circular and conical scans have all been proposed to measure various components of the vibration at various points on a target [1.28,1.32,1.33].

The scanning Laser Vibrometer can be straightforwardly configured to scan a straight line by continuous periodic (typically sinusoidal) deflection of the beam around one axis [1.28,1.32], as illustrated in Figure 1.14. Clearly, such a scan profile would yield information relating to, for example, the bending vibration mode of a cantilever beam. Spatial vibration data, important in most vibration analyses, can be obtained via the use of a series of parallel straight line scans, also shown in Figure 1.14, but it is clearly more convenient to make use of the 2D beam steering capabilities of the scanning Laser Vibrometer, thereby enabling vibration information to be obtained from a large area in a
single measurement. Simultaneous sinusoidal laser beam deflection around two orthogonal axes at different amplitudes and frequencies will result in an area scan profile [1.33], such as that which is illustrated in Figure 1.15. Simultaneous sinusoidal deflection at the same amplitude and frequency but with a $\pi/2$ phase difference results in a circular scan profile, an example of which is shown in Figure 1.16.

Configuration of a continuous scanning Laser Vibrometer to scan a circular profile enables the measurement of axial vibration [1.32,1.34] and of mode shapes [1.35] in components such as axially flexible rotating discs. If the scan frequency is synchronised with the target rotation frequency, it is possible to perform a continuous tracking Laser Vibrometer measurement in which the probe laser beam remains fixed on a particular point on the target [1.36]. If the amplitude of a continuous circular scan is sufficiently reduced, the (non-rotating) target surface can be assumed to be rigid and the Laser Vibrometer measurement contains information relating to the on-axis and angular vibration components [1.32]. If a circular scan is directed around the periphery of a (short focus) convex lens then the laser beam can be scanned in a cone with its vertex at a point of interest on a (non-rotating) target surface and the instrument output relates to three translational vibration components at that point [1.32,1.37].

In all cases, the post-processing of the measured data is complex and previous studies based on idealised geometric models of the scanning system configuration had led to inaccuracies in and occasionally misinterpretation of the data obtained in such scanning and tracking measurements. It became clear that, to realise the full capability of these potentially powerful advanced measurement techniques, a more comprehensive theoretical analysis was required. The comprehensive velocity sensitivity model, presented in section 1.3.1, is ideally suited to such an analysis since it can be extended to incorporate time dependent beam orientation. Furthermore, the model can be developed to include provision for targets with flexible cross-sections, such as beams, panels and thin or bladed discs, and this is important since vibration measurement on such structures is a particular motivation for the enhancement of scanning Laser Vibrometry techniques.
The development and experimental validation of the model for these advanced applications is the subject of chapters 4 and 5 whilst chapter 6 contains two specific examples of the type of advanced measurements that have be performed during this study. The custom-built dual mirror scanning and tracking system that was designed and developed for use in this project is described in detail and the nature of the measured data correlated with the theoretical predictions made using the extended comprehensive velocity sensitivity model. In chapter 7, two specific recommendations for the improvement of the scanning system are introduced and described in detail. In particular, beam deflection using a single, dual rotation axis scanning mirror is suggested as significant improvement to the current system and the necessary modifications to the model to enable confident measurement interpretation are described.

The final recommendation for further work made in chapter 7 relates to the exciting new rotating target measurement technique that is currently being referred to as “Synchronised-Scanning Laser Vibrometry”. Here, continuous scanning and tracking strategies are combined to result in the tracking and simultaneous scanning of a region of interest on a rotating component to provide deflection shape and vibration frequency information under operating conditions. Such a technique has the potential to acquire vitally important dynamic behaviour data for a wide range of high-technology devices. Again, concluding remarks relating to the use of all scanning Laser Vibrometry techniques for the measurement of rotor vibrations will be discussed in chapter 8.
Since the first descriptions of the cross-sensitivity problem \([2.1,2.2]\) there has been discussion about whether a particular arrangement of laser beams or a particular variation of the arrangement, for example by scanning the laser beam, might enable resolution of individual motion components. In chapter 1, equation (1.9) showed that direct measurement of the rotational angular velocity is possible, accepting that the torsional oscillation and roll motion are indistinguishable. Measurement of axial vibration is also straightforward since the second and third sets of terms in the axial vibration set, \((\ddot{\theta}_x - \Omega \dot{\theta}_x)\hat{y}\) and \((\ddot{\theta}_y + \Omega \dot{\theta}_y)\hat{x}\), are products of vibration parameters and are typically an order of magnitude smaller than the axial vibration.

Conversely, equation (1.9) shows that direct measurement of pure radial, \(\hat{x}\) and \(\hat{y}\), or pitch/yaw, \(\ddot{\theta}_x\) and \(\ddot{\theta}_y\), vibration is not possible because the measurement will always be sensitive to other motion components. It may be possible, in some circumstances, to assume that the effects of additional shaft motions are negligible, enabling direct motion component measurement. In a general case, however, reliable estimation of the individual motion components involves the post-processing of the outputs from several instruments configured to measure the vibration sets. Such a technique clearly requires a detailed treatment of the velocity sensed by an instrument and this has been made possible by the development of the comprehensive velocity sensitivity model which can be applied to any measurement configuration on any target.

As described in section 1.3.2.1, the \(x\) and \(y\) radial vibration sets are theoretically isolated using a single beam Laser Vibrometer and selecting particular values for the geometric parameters \(\alpha, \beta, x_0, y_0\) and \(z_0\) such that measurement sensitivity to the other vibration sets is eliminated. In reality, however, it is not possible to align the laser beam perfectly
with the respective vibration axis and it is therefore necessary to include non-zero \( x_0 \) and \( y_0 \), such that equations (1.11a&b) must be re-written as:

\[
U_x = \dot{x} + \Omega_r (y - y_0) \quad (2.1a)
\]

and

\[
U_y = \dot{y} - \Omega_r (x - x_0) \quad (2.1b)
\]

The first mathematical solution to cross-sensitivity in radial vibration measurements [2.2] made use of two simultaneous orthogonal Laser Vibrometer measurements, as shown in Figure 2.1, thereby isolating the \( x \) and \( y \) radial vibration sets separately. Provided that the rotational angular velocity could be determined, it was shown to be possible to manipulate the resulting simultaneous differential equations to resolve for the individual component motions. Subsequent solutions for both radial [2.3,2.5] and rotational [2.4,2.5] vibrations similarly involved the acquisition of simultaneous orthogonal measurements (and an independent angular velocity measurement). The new resolution technique presented in this chapter is based upon these original solutions and the same combination of measurements is fundamentally necessary. The resolution technique developed is, for the first time, robust enough to implement in practice. Its performance will be investigated in considerable depth, in this and the subsequent chapter, from the perspective of real measurement conditions.

### 2.1. Simultaneous Orthogonal Measurements

By separating the rotational angular velocity term into its mean and time dependent components, i.e. \( \Omega_r = \bar{\Omega}_r + \Delta \Omega_r \), and considering the AC coupled Laser Vibrometer outputs, \( \bar{U}_x \) and \( \bar{U}_y \), equations (2.1a&b) become:

\[
\bar{U}_x = \dot{x} + \bar{\Omega}_r y + \Delta \Omega_r (y - y_0) \quad (2.2a)
\]

and

\[
\bar{U}_y = \dot{y} - \bar{\Omega}_r (x - x_0) \quad (2.2b)
\]
\( \ddot{U}_y = y - \overline{\Omega}_r x - \Delta \overline{\Omega}_r (x - x_0) \) \hspace{1cm} (2.2b)

Integrating equation (2.2b) results in:

\[
\int_0^t \ddot{U}_y \, dt = y - y(0) - \overline{\Omega}_r \int_0^t x \, dt - \int_0^t \Delta \overline{\Omega}_r (x - x_0) \, dt
\]

since

\[
y = y(0) + \int_0^t y(t) \, dt
\] \hspace{1cm} (2.3b)

Rearranging for \( y \) in equation (2.3a) and substituting the result into equation (2.2a) enables the following integro-differential equation describing the intended measurement parameter to be written:

\[
\dot{x} + \overline{\Omega}_r \int_0^t x \, dt = \left( \ddot{U}_x - \overline{\Omega}_r \int_0^t \ddot{U}_y \, dt \right) - \left( \Delta \overline{\Omega}_r y + \overline{\Omega}_r \int_0^t \Delta \overline{\Omega}_r x \, dt \right) + \left( \Delta \overline{\Omega}_r y_0 + \overline{\Omega}_r x_0 \int_0^t \Delta \overline{\Omega}_r x \, dt \right) - \overline{\Omega}_r y(0)
\]

\hspace{1cm} (2.4)

This important equation shows the intended measurement parameter, \( \dot{x} \), in terms of a readily formulated quantity (the first bracketed term on the right hand side), generated directly from the two Laser Vibrometer outputs and an independent mean angular velocity measurement, and several additional terms. The second and third bracketed terms on the right hand side are related to the products of the time dependent angular velocity (torsional vibration) with the radial vibration displacements and offsets, respectively. In many practical situations, the angular velocity variation is small relative to the mean angular velocity [2.5], and it is therefore possible to proceed more straightforwardly by considering only constant angular velocity as described in the following section.
2.2. CONSTANT ANGULAR VELOCITY SOLUTION

2.2.1. MATHEMATICAL FORMULATION

The initial condition dependent term, $\overline{\Omega}_r y(0)$, present in equation (2.4) can be removed by considering the resulting quantity at non-zero frequencies. Considering constant rotational velocity, i.e. $\Delta\Omega_r = 0$, equation (2.4) can be simplified:

$$\dot{x} + \overline{\Omega}_r^2 \int_0^t x \, dt = \left( \overline{\Omega}_r \dot{y} - \overline{\Omega}_r \int_0^t \dot{y} \, dt \right) \tag{2.5a}$$

Similarly, by first integrating equation (2.2a):

$$\dot{y} + \overline{\Omega}_r^2 \int_0^t y \, dt = \left( \overline{\Omega}_r \dot{x} + \overline{\Omega}_r \int_0^t \dot{x} \, dt \right) \tag{2.5b}$$

Since, these equations are functions of both the intended parameter and its first derivative, and in practice, only the alternating motion components are of interest, it is more convenient at this point to proceed in the frequency domain. Here, as shown in Figure 2.2 and Figure 2.3 and described in detail in Appendix A, quantities can be straightforwardly integrated or differentiated at any given (angular) frequency, $\omega$, by division or multiplication by $j\omega$, i.e.:

$$FT \left[ \dot{x} + \overline{\Omega}_r^2 \int x \, dt \right] = \dot{X}(\omega) + \overline{\Omega}_r^2 \frac{X(\omega)}{j\omega} = \dot{X}(\omega) \left( 1 - \frac{\overline{\Omega}_r^2}{\omega^2} \right) \tag{2.6}$$

where $X(\omega)$ and $\dot{X}(\omega)$ are the Fourier Transform of vibration displacement and velocity respectively. Making use of equation (2.6) in equation (2.5a) enables $\dot{X}(\omega)$ to
be written in terms of the Laser Vibrometer outputs, the angular velocity and a frequency dependent weighting function, $W(\omega)$, i.e.:

$$\hat{X}(\omega) = W(\omega)FT \left[ \bar{\bar{U}}_x - \Omega_f \int_0^t \bar{\bar{U}}_x dt \right] \quad (2.7a)$$

where

$$W(\omega) = \frac{\omega^2}{\omega^2 - \Omega_f^2} \quad (2.7b)$$

Similarly, considering a vibration in the $y$ direction:

$$\hat{Y}(\omega) = W(\omega)FT \left[ \bar{\bar{U}}_y + \Omega_f \int_0^t \bar{\bar{U}}_y dt \right] \quad (2.7c)$$

### 2.2.2. Synchronous Vibration Measurements

It can be seen from equation (2.7b) that the frequency dependent weighting function $W(\omega)$ becomes infinite at synchronous frequency. In addition to this, the formulated quantities in equations (2.7a&c) become zero, such that it is not possible to establish a meaningful solution for the synchronous vibration component. This is not a limitation of the resolution technique but a fundamental feature of the synchronous velocity component sensed by the laser beam. As a consequence it may not always be meaningful to return from the frequency domain back into the time domain since the synchronous component will be missing.

In rotating machinery diagnostics, many defects are signified by a change in the synchronous vibration component and the inability to resolve the synchronous vibration component is, therefore, a serious limitation that must be acknowledged. The amplitudes of the measured components, however, always constitute conservative estimates of the sum of the individual amplitudes.
It can be seen by inspection of equation (2.7b) that \( W(\omega) \) becomes very large as synchronous frequency is approached and any additional content, such as measurement noise, in the measured signals will therefore become amplified around synchronous frequency. In some situations, the level of this amplified spectral content may be of equivalent magnitude to the genuine vibration velocity and, as such, may lead to incorrect interpretation of the resolved output signal. In such a situation it may be necessary to disregard a number of spectral components adjacent to the rotation frequency component and this will be discussed in more detail in subsequent sections.

2.2.3. **Block Diagram Development and Software Implementation**

Continuing in the frequency domain, it is possible to re-express equations (2.7a&c) as follows:

\[
\dot{X}(\omega) = W(\omega) \left( FT[\overline{U_x}] - \frac{\Omega_L}{j\omega} FT[\overline{U_y}] \right)
\]

\[
= W(\omega) \left( \left\{ \Re[\overline{U_x}] - \frac{\Omega_L}{\omega} \Im[\overline{U_y}] \right\} + j \left\{ \Im[\overline{U_x}] + \frac{\Omega_L}{\omega} \Re[\overline{U_y}] \right\} \right) \tag{2.8a}
\]

and

\[
\dot{Y}(\omega) = W(\omega) \left( FT[\overline{U_y}] + \frac{\Omega_L}{j\omega} FT[\overline{U_x}] \right)
\]

\[
= W(\omega) \left( \left\{ \Re[\overline{U_y}] + \frac{\Omega_L}{\omega} \Im[\overline{U_x}] \right\} + j \left\{ \Im[\overline{U_y}] - \frac{\Omega_L}{\omega} \Re[\overline{U_x}] \right\} \right) \tag{2.8b}
\]

The block diagram representation of equations (2.8a&b) illustrated in Figure 2.4 was used to develop a LabVIEW based version of this important resolution algorithm. LabVIEW constitutes a powerful tool for this type of exercise since it is possible to design and test manipulation algorithms using simple simulated signals (for which the algorithm outputs can easily be compared with the desired outputs) before straightforwardly reconfiguring the code to make use of actual measured data.
The ability to capture and process time domain data, then transform it into the frequency domain for further processing and finally display, is far more elegant than the previous solution [2.5] which was a hybrid of analogue circuits, frequency analysis hardware and manual final processing. This new solution is sufficiently robust and practical for commercial implementation and is also suited to the inclusion of more advanced correction algorithms. The performance of this resolution algorithm will be rigorously tested for simulated and real vibration signals in chapter 3.

2.3. Effects of Torsional Oscillation

2.3.1. Mathematical Formulation

In the previous section it was assumed that the target angular velocity was constant and that any torsional oscillation was negligible, enabling the problem to be significantly simplified and a resolution procedure developed. In many practical situations, however, torsional oscillation will be present to a certain extent and, as such, the accuracy of the resolution method will be reduced.

With reference to equation (2.4), it is possible to show using the frequency domain solution discussed in the previous section that, when torsional oscillation is present, the vibration velocity is not only equal to the product of the formulated quantity and the weighting function, but includes additional torsional oscillation terms:

\[
\dot{X}(\omega) = W(\omega)FT\left[\bar{U}_x - \Omega_T \int_0^t \bar{U}_y dt\right] \\
- W(\omega)FT\left[\Delta \Omega_T y + \Omega_T \int_0^t \Delta \Omega_T x dt\right] \\
+ W(\omega)FT\left[\Delta \Omega_T y_0 + \Omega_T x_0 \int_0^t \Delta \Omega_T dt\right]
\]

(2.9a)
Similarly, for a vibration measurement in the $y$ direction, the vibration velocity is given by:

$$
\dot{y}(\omega) = W(\omega)FT\left[ \bar{U}_y + \bar{\Omega}_T t \int_{0}^{t} \ddot{U}_y dt \right] \\
+ W(\omega)FT\left[ \Delta \Omega_T x - \bar{\Omega}_T \int_{0}^{t} \Delta \Omega_T y dt \right] \\
- W(\omega)FT\left[ \Delta \Omega_T x_0 - \bar{\Omega}_T y_0 \int_{0}^{t} \Delta \Omega_T dt \right]
$$

(2.9b)

The third bracketed term on the right hand side of equation (2.9a) (or (2.9b)) is dependent upon the angular frequency, which can be measured, and the offsets, $x_0$ and $y_0$, which cannot be measured but can be minimised by careful alignment of the instruments with the target rotation axis. This term, which will be referred to as the oscillation-offset term from this point forwards, can be (theoretically) eliminated and will therefore be neglected for the moment. The second bracketed term in equation (2.9a) (or (2.9b)), which will be referred to as the oscillation-displacement term from this point forward, is, however, dependent upon the angular frequency and the target displacements, $x$ and $y$, which are unknown as they are the intended measurements.

In many practical situations, torsional oscillation levels [2.6,2.7] are such that the amplitude of the oscillation-displacement term is relatively small. The product of the formulated quantity (the first bracketed term on the right hand side of equation (2.9a) (or (2.9b))) and the weighting function, which will be referred to as the first vibration velocity estimate from this point forward, therefore constitutes a good approximation of the true vibration velocity (at all non-synchronous frequencies). It is therefore possible to use the integral of the first vibration velocity estimates and the angular velocity and torsional oscillation measurements to create an initial estimate of the oscillation-displacement term. Subtracting this from the initial vibration velocity estimate will then result in an improved estimate of the true vibration velocity.
Since the product of the formulated quantity and the weighting function is the first estimate of the vibration velocity, \( \dot{X}_{\text{est}}(\omega) \), the improved estimate, \( \dot{X}_{\text{est}2}(\omega) \), is, in the absence of offsets, given by:

\[
\dot{X}_{\text{est}2}(\omega) = \dot{X}_{\text{est}}(\omega) - W(\omega)FT\left[ \Delta\Omega_T \dot{Y}_{\text{est}} + \bar{\Omega}_T \int_0^t \Delta\Omega_T x_{\text{est}}(t) dt \right] \tag{2.10a}
\]

where \( x_{\text{est}} \) and \( y_{\text{est}} \) are the first estimates of the \( x \) and \( y \) vibration displacements in the time domain. Repeating the procedure described above, it is clearly possible to make increasingly accurate estimates of the true vibration velocity, and subtract the corresponding increasingly accurate estimates of the oscillation-displacement term from the first estimates of vibration velocity to produce a better vibration velocity estimate. Considering the \((n+1)^{th}\) estimate of the vibration velocity, equation (2.10a) can be re-written:

\[
\dot{X}_{\text{est}n+1}(\omega) = \dot{X}_{\text{est}}(\omega) - W(\omega)FT\left[ \Delta\Omega_T \dot{Y}_{\text{est}} + \bar{\Omega}_T \int_0^t \Delta\Omega_T x_{\text{est}}(t) dt \right] \tag{2.10b}
\]

2.3.2. BLOCK DIAGRAM DEVELOPMENT AND SOFTWARE IMPLEMENTATION

It is possible to re-express equation (2.10b) as follows:

\[
\dot{X}_{\text{est}n+1}(\omega) = \dot{X}_{\text{est}}(\omega) - W(\omega) \begin{pmatrix}
\frac{\rho}{\omega} [\Delta\Omega_T \dot{Y}_{\text{est}}] + \bar{\Omega}_T \Im[\Delta\Omega_T x_{\text{est}}] \\
\frac{\rho}{\omega} [\Delta\Omega_T \dot{Y}_{\text{est}}] - \bar{\Omega}_T \Re[\Delta\Omega_T x_{\text{est}}]
\end{pmatrix} \tag{2.11a}
\]

The equivalent expression for obtaining an improved estimate of the vibration velocity in the \( y \) direction is given by:
\[ \hat{y}_{\text{estm}}(\omega) = \hat{y}_{\text{est}}(\omega) + W(\omega) \left\{ \frac{\Omega_r}{\omega} \Re[\Delta \Omega_T y_{\text{estm}}] + j \left[ \Im[\Delta \Omega_T x_{\text{estm}}] + \frac{\Omega_r}{\omega} \Re[\Delta \Omega_T y_{\text{estm}}] \right] \right\} \]  

\text{(2.11b)}

where the \( n^{th} \) estimates of the \( x \) and \( y \) vibration displacement, \( x_{\text{estm}} \) and \( y_{\text{estm}} \), are given by:

\[ x_{\text{estm}} = F^{-1}X_{\text{estm}}(\omega) = F^{-1}\left\{ \frac{1}{\omega} \Im[X_{\text{estm}}(\omega)] - j \Re[X_{\text{estm}}(\omega)] \right\} \]  

\text{(2.12a)}

and

\[ y_{\text{estm}} = F^{-1}Y_{\text{estm}}(\omega) = F^{-1}\left\{ \frac{1}{\omega} \Im[Y_{\text{estm}}(\omega)] - j \Re[Y_{\text{estm}}(\omega)] \right\} \]  

\text{(2.12b)}

Figure 2.5 shows the block diagram representation of equations (2.11a&b). As for the constant angular velocity resolution algorithm, a LabVIEW based version of this oscillation-displacement term correction algorithm was developed. As for the resolution algorithm, the correction algorithm performance will be rigorously tested using simulated signals in the following chapter.
In this chapter, the performance of the resolution and correction algorithms set out in chapter 2 will be examined. In addition to confirmation of correct operation under ideal conditions, the influence of typical noise and/or error levels in the unprocessed Laser Vibrometer outputs and in the independent speed measurement on the accuracy of the resolved outputs will be established. In particular, it is important to verify that the correction algorithm, which can be applied any number of times to the resolution algorithm outputs to improve the estimate of the true vibration, does not increase the errors in the resolved outputs in the presence of measurement noise and/or error.

3.1. Resolution Technique Performance and Errors: Simulated Data

This section contains a detailed examination of the performance of the resolution and correction algorithms. Table 3.1 summarises the series of simulated measurement situations that have been designed in order that the effects of the various typical sources of error can be examined. Simulations 1-5 will confirm the reliability of the resolution algorithm when implemented for several vibration velocity situations and under the influence of various noise and error levels. Simulations 6-9 and 10 will perform a similar exercise for the correction algorithm when implemented in the presence of broadband and engine-type torsional oscillation respectively. Simulations 11 and 12 are concerned with the effects of secondary error sources on the performance of the resolution technique. The table summarises the key simulation parameters, the relevant figure and section number(s) and includes summary comments that will be expanded upon in the subsequent sections of this chapter.
3.1.1. Resolution Algorithm

In simulation 1, the x and y vibration velocities, shown in Figure 3.1 and Figure 3.2, were used to generate (according to equations (2.1a&b)) the pair of simulated AC coupled Laser Vibrometer outputs shown in Figure 3.3 and Figure 3.4. The values used for the vibration velocities are realistic with the simulated signals representing medium/high severity vibrations for typical rotor systems [3.1]. The amplitude, frequency and phase of the vibration velocities have been deliberately chosen to be different (clearly the x and y vibration frequencies are more likely to be the same in practice), as have the instrument offsets, $x_0$ and $y_0$, such that the effects of the cross-sensitivity and the performance of the resolution algorithm can be explored more easily.

Note from Figure 3.3 and Figure 3.4 how the cross-sensitivity term can result in a significant difference in both amplitude and frequency between the measured and true vibration velocities. In this simple example, it is relatively straightforward to observe the effect of the cross-sensitivity in the time domain (Figure 3.3a and Figure 3.4a): both the x and y measurements clearly consist of unequal amplitude components with different frequency and phase. For more complex vibrations, however, the frequency domain representation will be more useful. In this case, it is clear from the frequency spectra (Figure 3.3b and Figure 3.4b) that each signal contains a genuine vibration peak (with the correct amplitude and phase), due to the intended measurement parameter, and an additional peak due to the cross-sensitivity term.

Figure 3.5a and Figure 3.6a show the resolution algorithm outputs in which the peaks due to cross-sensitivity are completely eliminated and the genuine peaks have identical frequency, amplitude and phase (not shown) to the true vibration velocity, as required. Figure 3.5b and Figure 3.6b show the resolved outputs reconstructed in the time domain, possible in this case since there is no vibration at synchronous frequency, which are identical to the true vibration velocities as required. This important result confirms that the resolution algorithm is performing the correct manipulations and, further to this, more complex vibration schemes (same frequency, multiple frequency and broadband vibration) have been used to test and verify this performance.
In simulation 2, the vibration in both directions, one of which is shown in Figure 3.7, consists of a pair of sinusoidal components. Here, and for the remainder of this chapter, the performance of the resolution technique will be illustrated using just one of the vibration components to maintain brevity in the figures. The measured vibration velocity, illustrated in Figure 3.8, is significantly different to the true vibration velocity as a result of the cross-sensitivity term – one genuine vibration component is not present at all in the measured velocity. The resolution algorithm again performs the correct manipulations and the resolved measurement, shown in Figure 3.9, is as required. In the third simulation, the vibrations, one of which is shown in Figure 3.10, consist of a broadband range of sinusoidal components with equivalent RMS amplitude to the respective single component in simulation 1. Again, the resolution algorithm, the output of which is illustrated in Figure 3.12, eliminates the cross-sensitivity components which resulted in the measured velocity, shown in Figure 3.11, being so significantly different to the true vibration. Note that the spectral line at synchronous frequency has been eliminated in Figure 3.12a, since a meaningful result cannot be obtained at this frequency.

In a real Laser Vibrometer measurement, random electronic noise will be present and, in a measurement on a rotating target, pseudo-random noise associated with the laser speckle effect [3.2] will also be present in the output signal, as discussed in section 1.2. For a target rotating at constant velocity, measurement experience has shown that the combination of these noise sources typically results in noise with spectral content of approximately 0.1mm/s (or 0.1rad/s) at integer multiples of rotation frequency with an underlying broadband noise floor of approximately 0.015mm/s/√Hz (or 0.015rad/s/√Hz).

In simulation 4, the effect of this inevitable measurement noise on the performance of the resolution algorithm will be evaluated. Figure 3.13 shows a simulated Laser Vibrometer output equivalent to that shown in Figure 3.3 but with this noise content added. Note that the simulated angular frequency measurement (not shown) also contains noise of this nature. The resolution algorithm output, illustrated in Figure 3.14, shows vibration velocity peaks with identical frequency, amplitude that is correct to
within 1% and phase that is correct to within 10mrad of the true vibration velocity. The noise levels are similar to those in the simulated Laser Vibrometer outputs at all frequencies away from synchronous but, as discussed in section 2.2.2, the weighting function in the resolution algorithm amplifies the noise close to synchronous to levels of the order of 1mm/s. Again, the spectral line at synchronous has been eliminated in Figure 3.14.

In situations where genuine vibrations are expected in regions close to synchronous, small errors in the independent measurement of $\Omega_r$ will result in large errors in $W(\omega)$ and, hence, in the resolved velocity. Figure 3.15 shows a resolved output for the situation in which, in addition to the noise discussed previously, there is a (very large) -2% error in the measurement of $\Omega_r$. Here the genuine vibration velocity peaks in the resolved outputs have amplitude and phase errors of the order of 10% and 10mrad and the cross-sensitivity peaks are not reduced to insignificant levels. If the vibration components occurred at frequencies closer to synchronous, they would clearly be subject to greater amplitude (and phase) errors. The noise around synchronous frequency is amplified significantly, as shown in Figure 3.15a, to levels in the order of 10mm/s, i.e. of the same order of magnitude as the genuine vibration velocity components. Only the spectral line at synchronous has been eliminated in Figure 3.15a but, as discussed in section 2.2.2, it is possible to disregard additional frequencies around synchronous provided that important vibrations are not expected to occur in that region. In this particular example, if all frequencies within $\pm 0.1\Omega_r$ are eliminated the errors in the regions adjacent to synchronous are reduced to in the order of 0.1mm/s, as shown in Figure 3.15b. Since noise and/or speed errors such as these will always occur in this type of Laser Vibrometer measurement, all resolved outputs examined in the remainder of this section will have frequencies within $\pm 0.1\Omega_r$ eliminated.

The simulations presented in this subsection show that when noise is present in the unprocessed outputs (and the speed measurement), small amplitude and phase errors are experienced in the genuine vibration components. When a speed measurement error is present, the weighting function amplifies the noise content around the synchronous
frequency such that the amplitude and phase errors are (slightly) more significant. Despite these effects, the resolution algorithm outputs constitute good estimates of the true vibration velocities, under circumstances where the unprocessed Laser Vibrometer outputs show unacceptable cross-sensitivities.

Provided measurements are taken within the constraint that no information is available at synchronous frequency and that additional measurement content around synchronous will be amplified by the weighting function, the performance of the resolution algorithm is excellent and the technique may be implemented to generate very useful information when the target angular velocity is constant. When the target angular velocity is not constant, however, the oscillation-displacement and oscillation-offset terms, as described in section 2.3, may significantly influence the resolved outputs and lead to incorrect measurement interpretation. In such cases, the correction algorithm may be implemented to reduce the severity of these effects; its performance is the subject of this next subsection.

3.1.2. Correction Algorithm: Broadband Torsional Oscillation

As in the previous section, a pair of simulated AC coupled Laser Vibrometer outputs were generated using the vibration velocities shown in Figure 3.1 and Figure 3.2. In this sixth simulation, the instrument offsets, $x_0$ and $y_0$, have been eliminated and the angular velocity contains a broadband torsional oscillation/speed fluctuation with RMS amplitude of $210\text{mrad/s (12deg/s)}$, equivalent to a significant torsional vibration [3.3], as illustrated in Figure 3.16. Figure 3.17 shows a resulting measurement in which, as expected, the cross-sensitivity term again results in a significant difference in both amplitude and frequency between the measured and true vibration velocities and this is always the case for the four simulations discussed in this subsection. The additional measurement content due to the product of the torsional oscillation and the (in-plane) vibration displacement, $\Delta\Omega_r y$, can be observed but is less significant since it is at least three orders of magnitude lower than the genuine and cross-sensitivity peaks.
Figure 3.18a shows the first estimate of the vibration velocity that is obtained simply by applying the resolution algorithm to the two simulated Laser Vibrometer outputs. Most importantly, the resolution algorithm reduces the amplitude of the peak due to the cross-sensitivity term to a level that is less than $\Delta \Omega r_y$. The outputs show vibration velocity peaks with identical frequency, amplitude that is correct to within 1% and phase that is correct to within 10 mrad of the true vibration velocity. As discussed in section 2.2.2, the weighting function amplifies the level of any additional signal content around synchronous frequency; in this case $\Delta \Omega r y$ is amplified to within two orders of magnitude of the genuine vibration peak. As in the previous section, spectral lines at $\Omega_r \pm 0.1\Omega r$ have been eliminated since in this case no genuine velocity content exists in this frequency band.

The errors in the measurement of the vibration velocity after one correction algorithm iteration, shown in Figure 3.18b, are typically significantly reduced, in this case by an order of magnitude. Here, the outputs show vibration velocity peaks with identical frequency, amplitude that is correct to within 0.01% and phase that is correct to within 100 mrad. After only three correction iterations, the errors may be considered insignificant as they are, in this case, reduced (by approximately three orders of magnitude) to at least five orders of magnitude lower than the genuine vibration peaks, as shown in Figure 3.18c. Here, the outputs show vibration velocity peaks with identical frequency, amplitude that is correct to within 0.001% and phase that is correct to within 1 mrad! Further correction iterations show increasingly accurate outputs confirming that the correction algorithm is performing the correct manipulations and, further to this, more complex vibration schemes (same frequency, multiple frequency and broadband vibration) have been used to test and verify this performance in the same way as for the resolution algorithm in the previous section.

As for the resolution algorithm, the accuracy of the correction algorithm is reduced in the presence of noise and/or speed measurement error and the purpose of simulation 7 is to establish that repeated application of the correction algorithm does not increase the inaccuracies associated with such measurement errors. For a rotating target that is undergoing significant torsional vibration, measurement experience has shown that the
combination of electronic and speckle noise sources typically results in a broadband noise floor of approximately $0.02\text{mm/s/sqrt(Hz)}$ (or $0.02\text{rad/s/sqrt(Hz)}$). The noise in the Laser Vibrometer outputs, as shown in Figure 3.19, is approximately an order of magnitude larger than $\Delta \Omega \gamma$ and, as such, is of greater significance in this simulation. In addition to noise, an error in the speed measurement is included in this simulation, i.e. it is equivalent to simulation 5, the final example in the previous subsection. As was the case then, the noise levels in the correction algorithm outputs, illustrated in Figure 3.20, are statistically the same as those in the unprocessed Laser Vibrometer outputs at all frequencies away from synchronous. Unlike simulation 6, successive correction iterations do not result in an improvement in the accuracy of, or a reduction of the noise in, the resolved outputs. Of importance, however, is the fact that successive correction iterations do not increase the levels of error in the resolved outputs as in practice the various sources of measurement content will not be known. In this case, the genuine vibration velocity peaks in the algorithm outputs have amplitude and phase errors of the order of 10% and 10 mrad and the cross-sensitivity peaks are reduced by approximately two orders of magnitude to almost insignificant levels.

The successful operation of the correction algorithm clearly relies on perfect alignment of the Laser Vibrometers and the target rotation axis, since the correction algorithm was developed in section 2.3.1 by setting the offsets, $x_0$ and $y_0$, to zero. In practice, however, it is impossible to arrange for zero offsets (experience has shown that it is only possible to reduce each offset to $0.25-0.5\text{mm}$) and, as such, the oscillation-offset terms in equations (2.9a&b) cannot be eliminated and should be accounted for in the correction algorithm; equation (2.10b) must be re-written as follows:

$$\dot{X}_{est+1}(\omega) = \dot{X}_{est}(\omega) - W(\omega) FT \left[ \Delta \Omega \gamma y_{est} + \overline{\Omega} \int_0^t \Delta \Omega \gamma x_{est} dt \right]$$

$$+ W(\omega) FT \left[ \Delta \Omega \gamma y_0 + \overline{\Omega} \int_0^t \Delta \Omega \gamma dt \right] \quad (3.1)$$

By the same token, however, it is also currently impossible to measure the offsets accurately and it is therefore not possible to perform the re-written extended correction algorithm as required. It is therefore important to evaluate the effect that instrument
INDIVIDUAL MOTION COMPONENT RESOLUTION: PRACTICAL ASPECTS

offsets will have on the performance of the correction algorithm and this is examined in simulation 8. Figure 3.21 shows the simulated measurement which results when \( x_0 = 0.25\text{mm} \) and \( y_0 = 0.5\text{mm} \). Note that noise and speed measurement error are not included in this simulation.

In addition to the cross-sensitivity term, additional content due to the product of the torsional oscillation and the instrument offset, \( \Delta \Omega_r y_0 \), can be observed. Unlike in the sixth simulation in which there was only \( \Delta \Omega_r y \), here \( \Delta \Omega_r y_0 \) is of significance since it is only approximately two orders of magnitude below the genuine vibration peak. The first estimates of the vibration velocity, as illustrated in Figure 3.22a, despite not containing the cross-sensitivity terms as required, show vibration velocity peaks with amplitude errors of the order of 1% and phase errors of the order of 100mrad. More importantly, however, the weighting function amplifies the oscillation-offset error content around synchronous to levels in the same order of magnitude as the genuine vibration peak. Again, the spectral lines at \( \Omega_r \pm 0.1\Omega_r \) have been eliminated but misinterpretation of this information is likely and could be problematic.

In this example, \( \Delta \Omega_r y_0 \) is an order of magnitude greater than \( \Delta \Omega_r y \) as \( y_0 \) (or \( x_0 \)) is an order of magnitude greater than \( y \) (or \( x \)) and the correction algorithm does not yield any improvements in the vibration velocity estimates even following several iterations, as shown in Figure 3.22b. This is because \( \Delta \Omega_r y \), for which correction is possible, is much smaller than \( \Delta \Omega_r y_0 \), for which correction is not possible.

The final simulation in this section, simulation 9, combines all of the problems of the previous two simulations in an effort to quantify the performance of the resolution technique in what may be considered a challenging but realistic scenario. The Laser Vibrometer measurements, shown in Figure 3.23, are obtained in the presence of electronic and speckle noise and offsets and the speed measurement will be erroneous. As was the case in both previous simulations, the weighting function in the resolution algorithm amplifies the additional content to significant levels around synchronous which may lead to problematic data misinterpretation, as illustrated in Figure 3.24a. The
correction algorithm, despite not having an adverse effect, does not improve the situation, as can be seen in Figure 3.24b. Here, the vibration velocity peaks contain amplitude errors of the order of 10% and phase errors of the order of 10 mrad.

In summary, the series of simulations presented in this subsection show that application of the correction algorithm always eliminates the oscillation-displacement term as required. It does not amplify the noise levels or the associated errors that are experienced in the genuine vibration components or introduce further errors into the resolved outputs. When offsets are present, the level of $\Delta \Omega_r y_0$ is typically much greater than that of $\Delta \Omega_r y$ and/or the measurement noise and the effect of the correction algorithm is therefore negligible. Application of the correction algorithm is important in a real measurement, of course, as it will not be possible to determine which measurement content is associated with $\Delta \Omega_r y$ and which is associated with $\Delta \Omega_r y_0$. It is therefore important that the correction algorithm can be repeatedly applied without degrading the resolved outputs and this has been shown to be the case in this section. Reduction of the offsets and absolute noise levels would be advantageous since it would improve the applicable range of the technique, making it more useful for low-level vibrations. Measurement of the offsets and absolute noise levels would be advantageous, enabling the development of improved correction algorithms.

3.1.3. CORRECTION ALGORITHM: ENGINE-TYPE TORSIONAL OSCILLATION

Since engine vibration measurements are seen as an important application of this technology, simulation 10 aims to investigate the performance of the correction algorithm when the torsional oscillation is of the form typically experienced in internal combustion engines. The torsional oscillation behaviour of engine crankshafts is typically a function of the engine configuration. A 4-stroke, 4-cylinder engine, such as that studied in the first description of the use of this resolution technique for pitch/yaw vibrations [3.4], is likely to undergo crankshaft torsional oscillations that can be characterised in the frequency domain by approximately equal amplitude peaks at $\frac{1}{2}x$ target rotation orders and larger amplitude peaks at $2x$ target rotation orders. An
example of such a torsional oscillation spectrum, for 0-6x target rotation frequency, is shown in Figure 3.25, where the ½x and 2x order peaks have significant amplitude. This simulated rotational angular velocity measurement includes speckle noise (0.02rad/s/√Hz broadband as in simulations 7 and 9) and a -2% speed measurement error. Note that the RMS level of this significant torsional vibration spectrum is again 210mrad/s (12deg/s) — equivalent to the broadband torsional oscillation spectrum discussed in the previous subsection.

A simulated AC coupled Laser Vibrometer output, formed using the same vibration velocities as used in simulation 1, is shown in Figure 3.26 for a challenging but realistic scenario (noise, offsets and a speed measurement error). Again, the dominant error term is the oscillation-offset error and, as shown in Figure 3.27, the resolution and correction algorithms perform satisfactorily, i.e. noise and oscillation errors are not amplified. Amplitude and phase errors are of the order of 10% and 1mrad, respectively, confirming that they are mostly due to speed measurement error and noise at the respective spectral line. In general, it is possible to conclude that the resolution technique is useful in situations where vibration levels are of the order of several mm/s and its development is therefore a valuable contribution to Laser Vibrometry technology.

3.1.4. SECONDARY SOURCES OF ERROR

3.1.4.1. SMALL CROSS-SENSITIVITY TERMS

In section 1.3.2, isolation of the translational vibration sets relied on the assumption that, since the third set of terms in equation (1.10a&b) are products of vibration parameters, they can be considered insignificant. This assumption will be tested in the eleventh simulation by generating simulated AC coupled Laser Vibrometer outputs (according to equations 1.10a&b) of the form:

$$
\bar{U}_x = \hat{x} + \Omega \bar{y} + \Delta \Omega (y - y_0) - (\dot{\theta}_\phi - \Omega \dot{\theta}_x)z
$$

(3.2a)

and
Using realistic values for the axial, pitch/yaw vibration components to represent medium/high severity vibrations \(( \ddot{z} = 5 \cos(3\Omega t + \pi) \text{mm/s}, \dot{\theta}_z = 5 \cos(0.5\Omega t - 0.5\pi) \text{rad/s} \) and \( \dot{\theta}_y = 2.5 \cos(1.5\Omega t + \pi) \text{rad/s} \) \[3.1\]) and neglecting the difference between \( \Omega_r \) and \( \Omega \), the small cross-sensitivity term peaks are at least two orders of magnitude smaller than the genuine vibration peaks. This is shown for the constant angular velocity situation in Figure 3.28a and confirms the appropriateness of the assumption made in reducing equations (1.1) to equations (1.11).

The resolution algorithm output spectrum contains small peaks of slightly different amplitude, but no significant increase, as shown in Figure 3.28b. Amplitude and phase errors of the order of 1% and 1mrad exist, particularly where the vibration frequencies are such that the small-cross-sensitivity term peaks coincide with the genuine vibration peaks. The small cross-sensitivity peaks are insignificant, particularly when noise and torsional oscillation are present, and it is straightforward to show that the correction algorithm, if implemented in such a case, does not increase the peaks' levels either.

3.1.4.2. NON-ORTHOGONAL MEASUREMENTS

Resolution of the individual motion components from the isolated vibration sets relies on the acquisition of two simultaneous orthogonal vibration measurements and an independent angular velocity measurement as described in section 2.1. In the same way that it is impossible to align the instruments for zero offsets, it is impossible to align the instruments such that they are perfectly orthogonal and the measurement will therefore be sensitive to vibration sets other than that which is the intended measurement. It is possible to examine the effect that such misalignments have on the resolved outputs by considering the (arbitrarily chosen) case shown in Figure 3.29 in which the \( y \) direction instrument is misaligned with the \( y \) axis by some small angles \( \delta \) and \( \epsilon \) about the \( z \) and \( x \) axes, respectively.
Here, the beam is orientated such that $\beta = -\varepsilon$ and $\alpha = 90^\circ + \delta$ and the misaligned measurement of the $y$ radial vibration set, $U_{y'}$, is given by (neglecting the small cross-sensitivity terms):

$$U_{y'} = -\cos \varepsilon \sin \delta \left[ x + \Omega_r y + \Delta \Omega_r (y - y_0) \right]$$

$$+ \cos \varepsilon \cos \delta \left[ \dot{y} - \Omega_r \dot{x} - \Delta \Omega_r (x - x_0) \right]$$

$$- \sin \varepsilon \left[ \dot{z} + \left( \vec{\theta}_z + \Omega \vec{\theta}_z \right) y_0 - \left( \vec{\theta}_y - \Omega \vec{\theta}_y \right) x_0 \right]$$

(3.3)

Figure 3.30 shows the simulated measurement for the constant angular velocity situation where $\delta = \varepsilon = 5^\circ$. The difference between this signal and what should be measured (as shown in Figure 3.4) results in inaccuracies in the amplitude and phase of the genuine vibration peaks and additional error peaks in the resolution algorithm outputs, due to the sensitivity to perpendicular motion components, as illustrated in Figure 3.31a. These peaks are more significant than the harmonic peaks associated with speckle noise, as confirmed in Figure 3.31b, and this suggests that instrument alignment is a factor worthy of consideration when making such measurements. The genuine peaks have identical frequency, amplitude that is correct to within 10% and a phase error of the order of 100mrad; large in this simulation as the additional error peaks happen to coincide with the genuine vibration peaks.

In the measurement examples discussed in this section, the rotation frequency, vibration levels and frequencies, offsets, noise levels and speed measurement errors were chosen such that the various sources of measurement error were easily identifiable and therefore did not constitute a significant source of measurement ambiguity. In a real measurement such control is obviously not available and, for example, if the axial vibration level was greater or if it was at a different frequency, then the significance of the errors might be greater as it would be less straightforward to distinguish genuine vibration peaks from error peaks. This section has, however, made use of realistic target motion parameters that are based on knowledge of those that are typically experienced in real measurements such that a high degree of confidence in the technique can be established before it is implemented for various real measurements. The following
section will analyse the performance of the resolution technique for such real measurements where varying degrees of target motion control are experienced.

3.2. Resolution Technique Performance and Errors: Measured Data, "Controlled" Vibrations

In this first experimental analysis subsection, the performance of the resolution technique when tested on a carefully controlled target in the laboratory will be presented.

3.2.1. Experimental Arrangement

The arrangement used in this stage of the experimental verification of the resolution technique made use of a test rig which enables simultaneous translational target vibration in two orthogonal directions. This has been achieved, as shown schematically in Figure 3.32, by, firstly, mounting the target housing on a linear bearing and exciting it with an electrodynamic shaker and, secondly, mounting this whole assembly on a large electrodynamic shaker. Driving each shaker with a separate function generator/power amplifier combination will result in (almost) independent vibration in the \( x \) and \( y \) directions. The target used was a small (\( \varnothing 30 \text{mm} \times 5 \text{mm} \)), aluminium disc of rigid, nominally circular, cross-section mounted to a DC motor. The target rotational mean angular velocity was controlled using a stable DC power supply whilst the time dependent angular velocity was controlled by a function generator.

The two instruments used to make simultaneous orthogonal measurements of the \( x \) and \( y \) vibration sets, as described in section 2.1, were a pair of Polytec OFV4000 Rotational Vibrometers [3.5] configured (by reflecting one beam directly back into the instrument with a "single point adapter" lens cap) to measure translational vibration velocity. These instruments thereby provide analogue voltage outputs proportional to the target velocity measured in the direction of the incident laser beam as intended. The angular velocity
was measured using a Brüel & Kjær Torsional Vibration Meter Type 2523 [3.6] which conveniently provides separate analogue voltage outputs proportional to the mean and time dependent components of the target rotational angular velocity. The positions of the instruments were arranged carefully using tripods such that offsets and angular misalignments were minimised to within the ranges discussed in the previous section. The actual experimental arrangement, complete with exaggerated laser beam paths, is shown in Figure 3.33.

### 3.2.2. Constant Angular Velocity

The first case examined is equivalent to simulation 5, i.e. the intended target vibration was as per Figure 3.1 and Figure 3.2. Despite some genuine low-level harmonic distortion and vibration due to the perpendicular shaker motion, the actual target vibration is very close to the intended vibration as can be seen by comparison of Figure 3.34, measured by the Laser Vibrometer on the non-rotating target, and Figure 3.1. The Laser Vibrometer measurements on the rotating target, shown in Figure 3.35, were (nominally) made in the presence of arbitrary offsets and angular misalignment, noise and a speed measurement error. As expected, the measured data contains a cross-sensitivity peak associated with the perpendicular motion component and random and pseudo-random noise associated with electronic interference and laser speckle. There is very good correlation between this data and the simulated measurement presented in the previous section as illustrated in Figure 3.13, confirming that the simulated noise characteristics and levels were reasonable.

The resolution algorithm outputs, as shown in Figure 3.36, contain the genuine vibration peak, no significant increase in the noise level, and most importantly an elimination of the cross-sensitivity peak as required. There is a vibration peak at the perpendicular motion component frequency, some of which is known to be genuine, as per Figure 3.34, and some of which may be the result of a difference in the target vibration in the rotating and non-rotating conditions. It is differences such as these that are the motivation for performing this type of measurement and one of the reasons that the techniques developed during this research project are of such great value.
3.2.3. **BROADBAND TORSIONAL OSCILLATION**

This case is equivalent to simulation 9. Broadband torsional oscillation of the target was introduced via the function generator. As in the previous subsection, there is very good correlation between the measured data, shown in Figure 3.37 and the simulated measurement presented in the previous section as illustrated in Figure 3.23. Again, the resolution algorithm outputs are as expected, as shown in Figure 3.38a, i.e. the cross-sensitivity term is significantly reduced. Figure 3.38b shows that the correction algorithm outputs are also as expected, i.e. the oscillation-offset error (which cannot be reduced) is large but noise and errors are not amplified by repeated correction algorithm application.

The two laboratory based measurement examples presented in this subsection were useful as they enabled confirmation of the performance of the resolution and correction algorithms for the resolution of measurements obtained under typical conditions. In particular, the resolution algorithm has been shown to eliminate the cross-sensitivity term successfully without significantly increasing noise levels and the correction algorithm has been shown not to increase the remaining inaccuracies in the resolution algorithm outputs. Application of the correction algorithm is important in a real measurement, of course, as it will not be possible to determine which errors are oscillation-displacement and which are oscillation-offset. It is therefore important that the correction algorithm can be repeatedly applied without degrading the resolved outputs. As shown in section 3.1.2, if the noise and the instrument offsets could be reduced then the correction algorithm could correct for torsional oscillation induced output errors and this is the subject of ongoing research as discussed in chapter 7.
3.3. RESOLUTION TECHNIQUE PERFORMANCE AND ERRORS: MEASURED DATA, "UNCONTROLLED" VIBRATIONS

In this subsection, the performance of the resolution technique when tested on a 4-stroke, 4-cylinder, 2-litre diesel engine will be presented.

3.3.1. EXPERIMENTAL ARRANGEMENT

The instruments used here were the same as those used in section 3.2. In this exercise, however, line of sight access to the crankshaft flywheel was limited such that the two orthogonal translational measurements had to be made at 45° to the horizontal and vertical, respectively. The OFV4000 “RPM” output was zeroed to within ±40RPM by adjusting the height of the instruments such that, for example, at 1600RPM the instrument offsets were within ±0.2mm. Similarly, the rotational angular velocity measurement had to be made at a non-perpendicular angle to the target rotation axis and the data acquisition system recalibrated accordingly, to account for the reduction in sensitivity associated with this instrument angular misalignment. The actual experimental arrangement, complete with exaggerated laser beam paths, is shown in Figure 3.39.

3.3.2. RADIAL VIBRATION MEASUREMENTS

Figure 3.40 shows a Laser Vibrometer output obtained for a measurement taken with the engine operating at 1600RPM whilst under no load. As expected for this type of engine, vibration occurs mostly at ½x target rotation (crankshaft flywheel) orders. The rotational angular velocity measurement shown in Figure 3.41 confirms that the torsional oscillation is also concentrated at ½x target rotation orders with larger peaks at 2x target rotation orders. Since the engine is unloaded, the torsional oscillation, \( \approx 50 \text{ mrads/s RMS} \), is less significant than (but within an order of magnitude of) that specified in the simulated torsional oscillation measurements discussed in section 3.1.3.
The resolved and corrected output shown in Figure 3.42 (repeated application of the correction algorithm makes little difference to the resolved outputs) contains several (possibly important) differences to the unprocessed Laser Vibrometer output; in particular the \( \frac{1}{2}x \) component is smaller whilst the \( 2x \) component is greater. Clearly, unlike in the laboratory based experimental verification, it is not possible to stop the rotation and measure the genuine vibration but the confidence in the resolution technique that has been built up throughout this chapter suggests that the resolved measurement is closer to the genuine vibration than the unprocessed Laser Vibrometer output. In particular, given the measured torsional vibration levels from Figure 3.41 (2x component \( \approx 0.5\text{rad/s} \)) and the fact that the offsets are likely to be of the order of 0.5mm, i.e. \( \Delta \Omega_{r}y_{0}|_{|y=20r} \approx 0.25\text{mm/s} \), it is possible to conclude that the unprocessed Laser Vibrometer output level shown in Figure 3.40 (2x component \( \approx 2\text{mm/s} \)) is mostly due to radial vibration. This is confirmed in the resolved output, illustrated in Figure 3.42 (2x component \( \approx 2.5\text{mm/s} \)).

3.3.3. Pitch/Yaw Vibration Measurements

At the start of the description of the isolation of the individual vibration sets in section 1.3.2, the equations resulting from the isolation of the pitch and yaw vibration sets (equations (1.12a&b)) were compared with those resulting from the isolation of the radial vibration sets (equations (1.11a&b)). It was noted that equations (1.12a&b) were similar in form to equations (1.11a&b) but were more straightforward to resolve, as they were not subject to complications associated with instrument offsets. The discussion continued by considering the more complicated radial vibration equations, stating that if the resolution technique could be shown to work for radial vibrations then it could be straightforwardly implemented for pitch/yaw vibration measurements.

Orientating the two OFV4000 Rotational Vibrometers such that the normal to the plane of each pair of parallel beams is perpendicular to the undeflected target rotational axis and the planes are orthogonal to each other, as illustrated in Figure 3.43, results in the measurement of the pitch and yaw vibration sets. From here, the required resolution
technique is identical (neglecting the difference between $\Omega_r$ and $\Omega$) to that used for radial vibration measurement. Figure 3.44a&b show the unprocessed and resolved and corrected Laser Vibrometer outputs, respectively, in which some small differences are evident. The main point to finish this section on, however, is that the resolution technique provides a better estimate of the genuine vibration velocity than that obtained initially and, furthermore, no problems are apparent when it is employed in the presence of torsional oscillation, noise and speed measurement error.
4. Continuous Scanning Laser Vibrometry: Theoretical Aspects

The comprehensive velocity sensitivity model illustrates that the measured velocity is dependent upon both the target velocity components and the orientation of the incident laser beam. In the original derivation [4.1] it was noted that the model is versatile enough to be extended to incorporate time dependent beam orientation. This extension was performed for the first time during this research project and is described in this chapter with reference to continuous scanning Laser Vibrometer measurements. Throughout the remainder of the thesis it is this operation in continuous scanning mode that is meant when "scanning" LDV is referred to.

The original derivation is developed to include time dependency in the beam orientation angles before being re-formulated in terms of the mirror scan angles, as it is these that the user would seek to control in practice. The advanced applications of circular scans on rotating targets and small-scale circular and conical scans on non-rotating targets are investigated as a means of illustrating the effectiveness of the model for the analysis of actual scan configurations. In particular, the origins of the additional components that occur in measured data due to instrument configuration are easily revealed using the revised velocity sensitivity model and an analysis of their influence on measured data is discussed.

In the original derivation, the illuminated section of the rotating target was assumed to be of rigid cross-section but, since Laser Vibrometer measurements are employed in applications where flexibility must be acknowledged, the first extension of the velocity sensitivity model presented in this chapter includes explicit provision for such flexibility.
4.1. Structures with Flexible Cross-Sections

In the original derivation of equation (1.9), it was assumed that, although the shaft could be flexible, the illuminated cross-section would not undergo changes in shape during the course of the measurement. Whilst this assumption is reasonable in many situations, targets with flexible cross-sections are of particular interest when employing scanning LDV techniques and it is important to extend the original theory to include provision for such flexibility.

Consider a rotating shaft in which the illuminated axial element has a flexible cross-section. As illustrated in Figure 4.1, P is the instantaneous point of incidence of the laser beam on the arbitrarily deformed shaft element, identified by the position vector \( \vec{r}_{p/o} \), and \( P_0 \) defines the corresponding point on the displaced but undeformed shaft element, identified by \( \vec{r}_{n/o} \). Clearly:

\[
\vec{r}_p = \vec{r}_o + \vec{r}_{p/o} = \vec{r}_o + \vec{r}_{n/o} + \vec{r}_f \quad (4.1)
\]

where \( \vec{r}_p \) identifies the position of \( P \) relative to the fixed reference frame \( XYZ \), \( \vec{r}_o \) identifies the instantaneous position of the translating reference frame \( xyz \), and \( \vec{r}_f \) represents the deformation.

The instantaneous velocity of \( P \), \( \vec{v}_p \), is therefore given by:

\[
\vec{v}_p = \dot{\vec{r}}_p = \dot{\vec{r}}_o + \dot{\vec{r}}_{n/o} + \dot{\vec{r}}_f = \vec{v}_o + \left( \vec{\omega} \times \vec{r}_{n/o} \right) + \vec{v}_f \quad (4.2)
\]

where \( \vec{\omega} \) is the angular velocity of \( P_0 \) about an instantaneous axis passing through \( O \). Equation (4.2) is similar to that which is obtained when considering the velocity of a point on a rotating shaft of rigid cross-section [4.1], the difference being the term \( \vec{v}_f \).
which represents the deformation vibration velocity of P due to cross-section flexibility and the velocity measured by the Laser Vibrometer, $U_m$, can therefore be written:

$$U_m = \cos \beta \cos \alpha [\dot{x}_r(P_0) + \dot{x}_r(P)]$$

$$+ \cos \beta \sin \alpha [\dot{y}_r(P_0) + \dot{y}_r(P)]$$

$$- \sin \beta [\dot{z}_r(P_0) + \dot{z}_r(P)]$$

(4.3)

where $\dot{x}_r(P_0)$, $\dot{y}_r(P_0)$, $\dot{z}_r(P_0)$ are the resultant vibration velocity components in the x, y, z directions due to rigid body vibration, given by:

$$\dot{x}_r(P_0) = \dot{x} - (\dot{\theta}_z + \Omega)(y_0 - y) + (\dot{\theta}_y - \Omega \theta_x)(x_0 - z)$$

(4.4a)

$$\dot{y}_r(P_0) = \dot{y} + (\dot{\theta}_z + \Omega)(x_0 - x) - (\dot{\theta}_x + \Omega \theta_y)(z_0 - z)$$

(4.4b)

and

$$\dot{z}_r(P_0) = \dot{z} + (\dot{\theta}_z + \Omega \theta_y)(y_0 - y) - (\dot{\theta}_y - \Omega \theta_x)(x_0 - x)$$

(4.4c)

and $\dot{x}_r(P)$, $\dot{y}_r(P)$, $\dot{z}_r(P)$ are the vibration velocity components in the x, y, z, directions due to cross-section flexibility, specific to point P. This shows that the rotor cross-section flexibility results in additional components due to the deformation velocities, which represent the difference between equations (1.9) and (4.3).

The development of equation (4.3) is significant and the convenience of its application in various measurement configurations, that have proved useful in the work published by a number of researchers, will be demonstrated in what follows. In particular, the ease of application of equation (4.3), even for very complex arrangements, and the depth of information offered by the velocity sensitivity model will be demonstrated.
4.2. SCANNING PROFILE ANALYSIS USING LASER BEAM ORIENTATION ANGLES

4.2.1. STRAIGHT-LINE SCANNING

A straight-line scanning Laser Vibrometer measurement is typically performed via the introduction of some form of laser beam deflection around one axis [4.2,4.3]. As illustrated in Figure 4.2, a straight-line scan in the y direction is easily arranged for by the introduction of a mirror, which rotates about the z axis. In this case, application of equation (4.3) proceeds as follows.

Recall from section 1.3.1 that the definition of the total velocity sensed by a laser beam relies on the definition of any arbitrary known point \((x_0, y_0, z_0)\) that lies along the length of the laser beam. In this case, the known point is taken as the point of incidence of the laser beam on the beam deflection mirror, i.e. \(y_0 = z_0 = 0\) and \(x_0\) is the stand-off distance between the target and the Laser Vibrometer. The effect of the beam deflection is accounted for in the velocity sensitivity model by temporal variation of \(\alpha\), the laser beam orientation about the z axis. \(\beta\), the laser beam orientation about the y axis, is zero. For a sinusoidal line scan of intended amplitude \(r_s\) and angular frequency \(\Omega_s\), \(\alpha\) can be written as:

\[
\alpha(t) = -\tan^{-1}\left(\frac{r_s}{x_0}\right)\sin(\Omega_st + \phi_s)
\]

where \(\phi_s\) is the scan initial phase angle. Rearranging equation (4.3) and substituting \(\beta = 0\) and \(\alpha\) using equation (4.5) results in the following expression for the velocity measured during a straight-line scan in the y direction for a non-rotating target undergoing vibration associated with cross-section flexibility only:
\[ U_m = \cos \left( \tan^{-1} \left( \frac{r_x}{x_0} \right) \sin (\Omega_s t + \phi_s) \right) \left[ \dot{x}_f(P) \right] \]

\[ -\sin \left( \tan^{-1} \left( \frac{r_x}{x_0} \right) \sin (\Omega_s t + \phi_s) \right) \left[ \dot{y}_f(P) \right] \]

Equation (4.6)

In some situations, it may be possible to use small angle approximations to simplify equation (4.6) but the full expression is presented here for completeness. Clearly, when small angle approximations are appropriate, the first term in equation (4.6) reduces to \( \dot{x}_f(P) \), which is the intended measurement. The second term represents sensitivity to the in-plane component that may be significant if \( \dot{y}_f(P) \) is large relative to \( \dot{x}_f(P) \).

Typically, the instantaneous point of incidence of the laser beam on the target, \( P \), effectively moves sinusoidally during scanning. If small angle approximations do not hold or if the target surface is not flat or not perpendicular to the \( x \) direction, then there will be some small spatial distortion of this sinusoidal profile. The size of this distortion is typically much less than a beam diameter (\( \approx 1 \text{mm} \)) and is therefore considered insignificant.

A straight-line scan in the \( z \) direction is performed by mirror rotation about the \( y \) axis, as shown in Figure 4.3. In this case, \( \alpha = 0 \) and, for a similar sinusoidal scan, \( \beta \) is given by:

\[ \beta(t) = -\tan^{-1} \left( \frac{r_x}{x_0} \right) \sin (\Omega_s t + \phi_s) \]

Equation (4.7)

Substituting this into equation (4.3) will immediately result in an expression for the velocity measured during a straight-line scan in the \( z \) direction. Equations (4.5), (4.6) and (4.7) are intuitive but they are presented here as a convenient first application of equation (4.3) before more complex beam deflection configurations are considered.
4.2.2. CIRCULAR SCANNING

A circular scanning Laser Vibrometer measurement can be achieved by deflecting the laser beam through suitable angles around two orthogonal axes simultaneously, typically by using cosine and sine functions [4.3,4.4]. With reference to Figure 4.4, the scanning system optical axis is defined as being the line along which the laser beam is directed towards the target when there is "zero" beam deflection. In this particular configuration, the scanning system and target reference frames are collinear and the scanning system optical axis lies on the z axis of the target reference frame. The two orthogonal axes about which the beam is deflected during scanning are chosen such that the resulting probe laser beam manipulation occurs in the x and y directions in the target plane. The effect of such beam deflection is accounted for in the velocity sensitivity model by temporal variation of one or both of the beam orientation angles, and, typically, temporal variation of the arbitrary known point that lies along the line of the laser beam.

4.2.2.1. THE IDEALISED SCANNING SYSTEM

In the idealised scanning system, laser beam deflection is performed by a single optical element that can manipulate the beam orientation simultaneously about the x and y axes as shown schematically in Figure 4.4. In such a system, the known point \((x_0, y_0, z_0)\) can be defined most conveniently as the incidence point of the laser beam on the scanning mirror. Clearly, the position of this point remains constant in time and scanning can be conveniently accounted for in the velocity sensitivity model by defining \(\beta\) as a constant and \(\alpha\) as a function of time:

\[
\beta = \frac{3\pi}{2} - \varepsilon = \frac{3\pi}{2} - \tan^{-1}\left(\frac{r_s}{z_0}\right) \tag{4.8a}
\]

and

\[
\alpha(t) = \Omega s t + \phi_s \tag{4.8b}
\]

where, in this case, \(r_s\) is the desired scan radius.
In this idealised configuration, substituting for $\beta$ and $\alpha$ from equations (4.8a&b) into equation (4.3) results in:

$$U_m = -\sin\left(\tan^{-1}\left(\frac{r_s}{z_0}\right)\right) \cos(\Omega_sl + \phi_s)\left[\dot{x}(P_0) + \dot{x}(P)\right]$$

$$-\sin\left(\tan^{-1}\left(\frac{r_s}{z_0}\right)\right) \sin(\Omega_sl + \phi_s)\left[\dot{y}(P_0) + \dot{y}(P)\right]$$

$$+ \cos\left(\tan^{-1}\left(\frac{r_s}{z_0}\right)\right)\left[\dot{z}(P_0) + \dot{z}(P)\right]$$

(4.9)

As for straight-line scanning, it may be appropriate to apply small angle approximations to simplify equation (4.9) in some situations.

4.2.2.2. The Dual Mirror Scanning System

In commercially available scanning Laser Vibrometers, laser beam deflection is performed by the introduction of two orthogonally aligned mirrors, separated by some distance $d_s$, into the beam path. With reference to Figure 4.5, it can be seen that when the laser beam is traced back from the target there is no single point from which it appears to originate. The most convenient known point $(x_0, y_0, z_0)$ to choose is the incident point of the laser beam on the y deflection mirror, which scans back and forth along the mirror rotation axis. In addition, modulations in both $\beta$ and $\alpha$ occur as a result of rotation of the $x$ and $y$ deflection mirrors, respectively. Again, the velocity sensitivity model is sufficiently versatile to be able to account for this.

The time dependency in the chosen known point $x$ coordinate, $x_0$, is given by:

$$\Delta x_0(t) = \frac{r_sd_s}{z_0} \cos(\Omega_sl + \phi_s)$$

(4.10)

It can be seen from Figure 4.5 that equations (4.8a&b) must be rewritten to incorporate the modulation of $\beta$ and $\alpha$ necessary to scan a circle, i.e.
\[
\beta(t) = \frac{3\pi}{2} - \varepsilon(t) = \frac{3\pi}{2} - \tan^{-1}\left(\frac{\sqrt{(x_x(t) - \Delta x_0(t))^2 + (y_y(t))^2}}{z_0}\right) \tag{4.11a}
\]

and

\[
\alpha(t) = \Omega_s t + \phi_s + \delta(t) = \Omega_s t + \phi_s + \tan^{-1}\left(\frac{x_x(t)}{y_y(t)}\right) - \tan^{-1}\left(\frac{x_x(t) - \Delta x_0(t)}{y_y(t)}\right) \tag{4.11b}
\]

where the time dependency in \(\varepsilon\) and the appearance of \(\delta(t)\) are readily seen as being the result of the time dependency in \(x_0\).

Substituting for \(\beta\) and \(\alpha\) in equation (4.3) using equations (4.11a&b) will immediately result in a full expression for the velocity measured during a circular scan on a rotating, flexible target undergoing 6 degree-of-freedom vibration. It is clear, however, that this expression will not be as simple as those in the previous, simpler or idealised applications. Furthermore, it is the beam deflection mirror scan angles, not the laser beam orientation angles, that are controlled in real scanning systems so it would be valuable, if it were possible, to re-express the velocity sensitivity model in terms of these.

### 4.3. Scanning Profile Analysis Using Deflection Mirror Scan Angles

In the two straight-line examples described in section 4.2.1, the laser beam orientation angles were directly related to the mirror scan angles. In the case of the dual mirror scanning system, however, the relationship between the mirror scan angles and the beam orientation angles is more complex. It would therefore be very valuable to reformulate the beam orientation unit vector, \(\hat{b}\), in terms of the mirror scan angles.

With reference to Figure 4.6 and Figure 4.7, the mirror scan angles, \(\theta_{x_k}\) and \(\theta_{y_k}\), are defined as positive if in an anticlockwise sense and “zero” when the resulting laser
beam direction is along the scanning system optical axis ($z$ axis). As described in detail in Appendix B.2, expressing each mirror surface normal as a unit vector it is possible to calculate $\hat{b}$ in terms of $\theta_{sx}$ and $\theta_{sy}$:

$$\hat{b} = [\sin 2\theta_{sx}]\hat{x} - [\cos 2\theta_{sx}\sin 2\theta_{sy}]\hat{y} + [\cos 2\theta_{sx}\cos 2\theta_{sy}]\hat{z}$$ (4.12)

Whilst the derivation of this equation is presented in an appendix for brevity in this section, it is important to note that it is one of the most important developments described in this thesis. Equation (4.12) is of great significance since it defines the incident laser beam direction for any combination of deflection mirror scan angles and, as will be shown in the next section, it can be used to define the probe laser beam position in the target plane.

### 4.3.1. Arbitrary Scan Profiles: Nominal Flat Target Surfaces

With reference to Figure 4.6, the position of the time dependent point of incidence of the laser beam on the target, $\bar{r}_p$, can be described by $x_s(P)$ and $y_s(P)$ (omitting the explicit declaration of time dependency in $P$ for brevity in the equations):

$$\bar{r}_p = [x_s(P)]\hat{x} + [y_s(P)]\hat{y}$$ (4.13)

Consideration of the time dependent positions of the mirror incidence points and the target incidence point enables this to be re-expressed in terms of the time dependent mirror scan angles $\theta_{sx}$ and $\theta_{sy}$. Since, as can be seen from Figure 4.6, the time dependency in the known point $x$ coordinate, $\Delta x_0$, is a function of the $x$ deflection mirror angle, given by:

$$\Delta x_0 = -d_s \tan 2\theta_{sx}$$ (4.14)

the laser beam incidence point can be evaluated as follows:
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\[ x_s(P) = \Delta x_0 - n\hat{b} \cdot \hat{x} = -d_s \tan 2\theta_{sx} - n \sin 2\theta_{sy} \]  
\[ y_s(P) = -n(\hat{b} \cdot \hat{y}) = n \cos 2\theta_{sx} \sin 2\theta_{sy} \]  
and
\[ z_0 = n(\hat{b} \cdot \hat{z}) = n \cos 2\theta_{sx} \cos 2\theta_{sy} \]  

\( \text{where } n \text{ is the distance between the } y \text{ deflection mirror and the target along the line of the laser beam.} \)

Substitution for \( n \) from equation (4.15c) into equations (4.15a\&b) results in a totally general description of the point of incidence of the laser beam for any combination of mirror scan angles:

\[ x_s(P) = -\tan 2\theta_{sx} \left( d_s + \frac{z_0}{\cos 2\theta_{sy}} \right) \]  
\[ y_s(P) = z_0 \tan 2\theta_{sy} \]  

Whilst equation (4.16b) can be rearranged such that the \( y \) deflection mirror scan angle can be obtained for any \( y_s(P) \), it can be seen from equation (4.16a) that \( x_s(P) \) is not a simple function of the \( x \) deflection mirror scan angle.

4.3.2. ARBITRARY SCAN PROFILES: NON-FLAT TARGET SURFACES

With reference to Figure 4.8, any variation in the target surface flatness, described by \( z_s(P) \), results in a change in the time dependent position vector \( \vec{r}_p \), which in this case is given by:

\[ \vec{r}_p = [x_s(P)]\hat{x} + [y_s(P)]\hat{y} + [z_s(P)]\hat{z} \]  

An examination of the time dependent laser beam incidence points similar to that in the previous subsection enables the quantities \( x_s(P) \) and \( y_s(P) \) to be re-expressed as follows:
\[ x_s(P) = -\tan 2\theta_{sy} \left( d_s + \frac{z_0 - z_s(P)}{\cos 2\theta_{sy}} \right) \] (4.18a)

and

\[ y_s(P) = (z_0 - z_s(P)) \tan 2\theta_{sy} \] (4.18b)

In practice, \( z_s(P) \) is likely to be unknown and, if significant, will result in a difference between the desired and the actual beam incidence point. The rigid body vibration part of the measured velocity is unaffected by this difference and the inclusion of \( z_s(P) \) is an unnecessary complication. For the targets studied during this research project, \( z_s(P) \) was small relative to \( z_0 \) and therefore considered insignificant. Equations (4.13) and (4.16a&b) will therefore be used for the remainder of the discussion.

### 4.3.3. CIRCULAR SCAN PROFILES

As illustrated in Figure 4.6 and Figure 4.8, a circular scan profile in the target plane, with radius \( r_s \), scan angular frequency \( \Omega_s \) and initial phase \( \phi_s \), requires that \( x_s(P) \) and \( y_s(P) \) are cosine and sine functions, respectively, such that equation (4.13) can be extended to:

\[ \bar{r}_p = [x_s(P)]\hat{x} + [y_s(P)]\hat{y} = [r_s \cos(\Omega_s t + \phi_s)]\hat{x} + [r_s \sin(\Omega_s t + \phi_s)]\hat{y} \] (4.19)

Substituting for \( x_s(P) \) and \( y_s(P) \) in equations (4.16a&b) results in two equations which must be rearranged for the deflection mirror scan angles if such a scan profile is to be achieved. This rearrangement is not possible for equation (4.16a), the consequence of which is that a perfect circular scan cannot be achieved using basic functions to drive the deflection mirrors.
4.3.3.1. Typical Deflection Mirror Scan Angles

If cosine and sine functions of equal amplitude are used to perform a "circular" scan, i.e.:

\[ \theta_x = -\Theta_x \cos(\Omega_s t + \phi_s) \]  
\[ \theta_y = \Theta_y \sin(\Omega_s t + \phi_s) \]

where

\[ \Theta_x = \Theta_y = 0.5 \tan^{-1}\left( \frac{r_s}{z_0} \right) \]

then a slightly elliptical profile results which can clearly be observed by substituting equations (4.20a,b,c) into equations (4.16a,b) and is shown, normalised to the intended scan radius, in Figure 4.9a. Figure 4.9b shows the normalised actual scan radius as a function of scan angle.

Figure 4.9a clearly shows the inherent problem. When employing equal amplitude mirror drive signals, the probe laser beam does not follow the intended circular path. For this particular combination of mirror separation and Laser Vibrometer stand-off, the maximum absolute error in the actual scan radius is of the order of 5%, as illustrated in Figure 4.9b. In addition to this, if the target surface is not flat and/or not perpendicular to the scanning system axis then, as for a straight-line scan, there will be a further (small) distortion in the scan profile. The effect of any probe laser beam position error is clearly structure dependent but, in some cases, there may be a significant difference between the velocities at the intended and actual measurement points.
4.3.3.2. Corrected Deflection Mirror Scan Angles

The elliptical shape in the scan trajectory resulting from the use of equal amplitude mirror drive signals can be overcome to an extent by accounting for the difference between the target to $x$ mirror and target to $y$ mirror distances and using "corrected" mirror drive signals with unequal amplitudes, i.e.:

\[
\Theta_{sx} = 0.5 \tan^{-1}\left(\frac{r_s}{z_0 + d_s}\right)
\]  \hfill (4.21a)

and

\[
\Theta_{sy} = 0.5 \tan^{-1}\left(\frac{r_s}{z_0}\right)
\]  \hfill (4.21b)

As illustrated in Figure 4.10, which shows the normalised scan radius as a function of scan angle for this corrected mirror drive signal case, the maximum absolute error in the actual scan radius is reduced to less than 0.05% by employing mirror drive signals with unequal amplitudes and this may be advantageous in some cases.

Whilst the ability to calculate the required mirror scan angles necessary to produce specific scan profiles (and vice versa) is useful, particularly for more complex scans, the means to predict the measured velocity for any scan profile is much more important. This will be discussed in detail in the following section.

4.4. Velocity Measured by a Dual Mirror Scanning Laser Vibrometer

Using equation (4.12) as a direct alternative to equation (1.8) and evaluating the principal unit vector coefficients enables equation (4.3) to be re-expressed in terms of the deflection mirror scan angles:
\[ U_m = \sin 2\theta_{sx} [\dot{x}_r \left( P_0 \right) + \dot{x}_f \left( P \right)] - \cos 2\theta_{sx} \sin 2\theta_{sy} [\dot{y}_r \left( P_0 \right) + \dot{y}_f \left( P \right)] + \cos 2\theta_{sx} \cos 2\theta_{sy} [\dot{z}_r \left( P_0 \right) + \dot{z}_f \left( P \right)] \] (4.22)

The known point \( x \) coordinate, \( x_0 \), can be slightly redefined for convenience such that it excludes the component \( \Delta x_0 \), given by equation (4.14), and equations (4.4b&c) are therefore re-formulated as follows:

\[ \dot{y}_r \left( P_0 \right) = \dot{y} + \left( \dot{\theta}_z + \Omega \right) (x_0 - d_s \tan 2\theta_{sx} - x) - \left( \dot{\theta}_z + \Omega \theta_y \right) (z_0 - z) \] (4.23a)

and

\[ \dot{z}_r \left( P_0 \right) = \dot{z} + \left( \dot{\theta}_z + \Omega \theta_y \right) (y_0 - y) - \left( \dot{\theta}_y - \Omega \theta_z \right) (x_0 - d_s \tan 2\theta_{sx} - x) \] (4.23b)

Derivation of equation (4.22) represents a significant development of the theoretical velocity sensitivity model as it allows the user to predict the sensitivity of a scanning Laser Vibrometer measurement for any combination of mirror scan angles on any target. It readily accommodates time dependent mirror scan angles where scanning profiles result and this will discussed in the following sections for some of the scanning arrangements that have been found to be useful in practice.

### 4.4.1. CIRCULAR SCANS FOR ROTATING TARGETS

Use of equation (4.22) allows prediction of the measured velocity in this particularly complex configuration with ease and it also shows how additional components can occur when performing "circular" scanning measurements on rotating targets.

The additional measurement components that occur in an "equal amplitude circular scan" can be quantified by substituting equations (4.20a,b&c) into equation (4.22) and setting the flexible and rigid vibration components to zero. The system arrangement is as discussed earlier, i.e. the scanning system and target reference frames are collinear (no translational or angular misalignment), such that the measured Laser Vibrometer
The additional information that exists in the measured Laser Vibrometer signal mostly occurs at two and six times the scan frequency, as shown in Figure 4.11. For typical rotation frequencies and scan radii, the level of the component at six times the scan frequency is well below the noise floor that results from the laser speckle effect, generally higher than $10^2 \text{mm/s}$ ($10^4 \text{mm/s/rad/s}$ in Figure 4.11), and can therefore be considered insignificant. The component at twice the scan frequency is, however, significant since typical levels are of the order of mm/s.

Similarly, the additional measurement components that occur in a "corrected amplitude circular scan" can easily be quantified, in this case by substituting equations (4.20a&b) and (4.21a&b) into equation (4.22) to give a corresponding expression for the measured velocity:

$$\frac{U_m}{\Omega} = -d_s \sin\left(\tan^{-1}\left(\frac{r_s}{z_0 + d_s}\right) \cos(\Omega_s t + \phi_s)\right) \sin\left(\tan^{-1}\left(\frac{r_s}{z_0}\right) \sin(\Omega_s t + \phi_s)\right)$$  \hspace{1cm} (4.25)

Here, the additional information that exists in the measured Laser Vibrometer signal mostly occurs at two, four and six times the scan frequency, as shown in Figure 4.12. For typical rotation frequencies and scan radii, the level of the components at four and six times the scan frequency can be considered insignificant. The component at 2x scan frequency is, however, still significant with typical levels only 5% lower, for this particular combination of $d_s$ and $z_0$, than for the equal amplitude case.

Of importance is the fact that the 2x component can (theoretically) be eliminated if $d_s$ is set to zero. This is clearly not possible in practice but replacing the dual mirror configuration with a single mirror that can deflect the beam about two orthogonal axes
simultaneously is equivalent. The practicality of such a system will be described in detail in section 7.2.

It is due to additional measured “vibration” components such as this that care must be taken when interpreting vibration information obtained from such measurements. This issue demonstrates the value of the velocity sensitivity model very clearly – it enables the vibration engineer to make Laser Vibrometer measurements with confidence. Theoretical component amplitudes show good agreement with those obtained from experimentation and a full experimental validation will be the particular focus of the following chapter. The model can also be used to examine the effects of misalignment between the target and scanning system axes and this will also be explored in chapter 5.

Circular “tracking” measurements can be arranged for by using a corrected amplitude circular scan and setting the scan frequency equal to the target rotation frequency such that the probe laser beam remains fixed on a single point on the target during rotation. The model continues to predict the additional components encountered that, in this case, occur at two, four and six times rotation frequency.

4.4.2. SMALL-SCALE CIRCULAR SCANS FOR NON-ROTATING TARGETS

Small-scale circular scans on non-rotating targets have proved useful in previous work for the measurement of axial and pitch/yaw vibration measurement at a point on a flexible structure [4.3]. In this application, the small illuminated region is assumed to move as a rigid body. In such a case, small angle approximations apply and the measured velocity, combining equations (4.22), (4.4a) and (4.23a&b), is given by:

\[ U_m = \left\{ 2 \dot{\theta}_x \dot{\theta}_y z_0 + 2 \dot{\theta}_y \dot{\theta}_z z_0 + \dot{z} \right\} + \left\{ 2 \dot{\theta}_x [\dot{x} + \theta_x y - \dot{\theta}_y z] - 2 \dot{\theta}_y [\dot{y} - \dot{\theta}_z (d_z 2 \theta_x + x) - \dot{\theta}_x z] + [- \dot{\theta}_x y + \dot{\theta}_y (d_z 2 \theta_x + x)] \right\} \]

\[(4.26)\]
in which the typically larger terms are those in the first set of braces. For single frequency vibrations, the resulting frequency spectrum contains a component at the vibration frequency, \( \omega \), and sidebands at \( \omega \pm \Omega_s \). The component at \( \omega \) is due to \( \dot{z} \) whilst the sideband components result from the products \( \theta_{s_x}\dot{\theta}_x \) and \( \theta_{s_y}\dot{\theta}_y \). The amplitudes and phases of the sidebands can be resolved to give \( \dot{\theta}_z \) and \( \dot{\theta}_x \).

As the diameter of the scan is increased, assuming small angle approximations still apply, further terms (from the second set of braces) become significant. For example, the resulting spectrum will contain a second pair of side bands, at \( \omega \pm 2\Omega_s \), due to the product \( \theta_{s_y}\dot{\theta}_x\theta_{s_x} \). This example shows how the velocity sensitivity model can readily and precisely predict the output in any Laser Vibrometer measurement on any target.

4.4.3. CONICAL SCANS FOR NON-ROTATING TARGETS

The velocity sensitivity analysis set out in terms of deflection mirror scan angles has enabled a detailed examination of actual rather than idealised scan configurations. As a final example, the especially complex case of a conical scanning measurement, which can be employed to measure the three translational components of velocity at a point [4.5], will be investigated. As a means of emphasising the usefulness of the velocity sensitivity model, the differences between a truly conical scan, which can only be achieved using an idealised scanning system, and a dual mirror “conical” scan can be examined in detail.

With reference to Figure 4.13, a conical scan can be performed via the introduction of a positive lens between the target and the scanning Laser Vibrometer. A lens, of focal length \( f \), is positioned with its optical axis coincident with the scanning system optical axis at a distance of \( z_0 \) from the \( y \) deflection mirror. Since the laser beam direction, before incidence on the lens, is not parallel with the optical axis, the user would typically place the target in the image plane, at a distance of \( z_1 \) from the lens. The difference between \( z_1 \) and \( f \), which is likely to be negligible in practice, is exaggerated in Figure 4.13 for clarity. In this dual mirror scanning system case, this results in the laser
beam incidence point scanning back and forth in the x direction on the target surface (also exaggerated in Figure 4.13), in sympathy with the beam incidence point on the y mirror, and the scanned volume is not truly conical.

The direction of the laser beam after passing through the lens, defined in this case by the unit vector \( \hat{b}_1 \), can be evaluated by considering the beam path between the lens and the target and forming the following vector equation:

\[
\left[ x_s + \frac{z_1}{z_0} \Delta x_0 \right] \hat{x} + [y_s] \hat{y} + [z_1] \hat{z} = \left[ \sqrt{\left( x_s + \frac{z_1}{z_0} \Delta x_0 \right)^2 + y_s^2 + z_1^2} \right] \hat{b}_1
\]  

(4.27)

where \( x_s \) and \( y_s \) are as given in equations (4.16a&b) and the incidence point of the laser beam on the lens is taken as the (time dependent) known point. This expression is relatively straightforward to rearrange for \( \hat{b}_1 \), forming an equivalent to equations (1.8) or (4.12). Evaluating the principal unit vector coefficients enables equation (4.3) (or (4.22)) to be re-expressed resulting in an expression specific to dual mirror, "conical" scanning LDV.

For the idealised scanning system, the laser beam incidence point remains fixed on the target, as shown in Figure 4.13. Here, the laser beam direction, defined by the unit vector \( \hat{b}_1' \), can be evaluated from the following vector equation:

\[
\left[ x_s \right] \hat{x} + [y_s] \hat{y} + [z_1] \hat{z} = \left[ \sqrt{x_s^2 + y_s^2 + z_1^2} \right] \hat{b}_1'
\]  

(4.28)

where \( x_s \) and \( y_s \) are as given in equation (4.19). Again, re-arranging for \( \hat{b}_1' \) and evaluating the principal unit vector coefficients enables equation (4.3) (or (4.22)) to be re-expressed in a form specific to idealised conical scanning LDV.
The differences between $\hat{b}_1$ and $\hat{b}_1'$ lead to measurable differences between the actual and intended Laser Vibrometer outputs and the velocity sensitivity model enables the quantification of these differences. Figure 4.14 shows measurement predictions for an idealised, a dual mirror equal amplitude and a dual mirror corrected amplitude conical scan for a typical configuration where $f = 50\text{mm}$. The target is undergoing simultaneous unit amplitude vibrations in the $x$, $y$, and $z$ directions at 10 times scan frequency (arbitrarily chosen).

For the idealised and equal amplitude dual mirror conical scans, the difference between the components at $\omega$ is 1.26% and the difference between the components at $\omega \pm \Omega_s$ is 1.36%. For the idealised and corrected amplitude dual mirror conical scans, the difference between the components at $\omega$ is 0.05% and the difference between the components at $\omega \pm \Omega_s$ is 0.06%. Such differences may be important in increasing the accuracy of data analysis. Both dual mirror measurements also contain several additional sidebands compared to the idealised measurement which may lead to complications in analysis, for example when dealing with more complex target vibrations.

This final example shows how, even for the most complex of scanning configurations, the velocity sensitivity model can readily and precisely predict the Laser Vibrometer output.
5. **Continuous Scanning Laser Vibrometry:**

**Practical Aspects**

In the previous chapter, the theoretical description of the velocity measured by a single laser beam incident in an arbitrary direction on a rotating target undergoing arbitrary motion was extended to the particularly challenging application of scanning Laser Vibrometer measurements on rotating targets with flexible cross-sections. The original velocity sensitivity model was reformulated to make use of mirror scan angles rather than laser beam orientation angles, which is especially useful as it is these that the user would seek to control in a real scanning system. In both formulations, it was shown to be straightforward to accommodate time dependency in the angles thereby enabling the prediction of the instrument output when operating in scanning mode.

The advanced applications of circular scans on rotating targets and small-scale circular and conical scans on non-rotating targets were investigated as a means of illustrating the effectiveness of the model for the analysis of actual scan configurations. In particular, the origin of a significant additional component that occurs at 2x scan frequency in a circular scanning measurement on a rotating target was shown to be associated with the dual mirror configuration. The velocity sensitivity model can also be used effectively to predict the result of any translational and/or angular misalignment between the scanning system and target rotation axes and this important issue is discussed in this chapter.

In the development of the model, it was assumed that both mirror reflective surfaces were coincident with the respective mirror rotation axes, and that the laser beam was perfectly aligned with each surface, prior to reflection. Clearly, neither situation is satisfied in a practical scanning system and the effect of such misalignments are discussed in this chapter. A particularly important practical aspect of scanning LDV is associated with laser speckle induced noise and the resulting degradation of measured
data. This chapter also contains a detailed description of the characteristics of laser speckle noise in scanning measurements.

In the final section of the chapter, a series of actual measurements are presented and analysed, the particular focus being the correct interpretation of the data obtained in both a scanning measurement on a rotating disc and tracking measurement on a rotating bladed disc. The information presented in both this and the previous chapter is utilised, accurately predicting the form of resulting frequency spectra and enabling complex measurements such as this to be made with confidence.

5.1. Scanning System and Target Rotation Axis Misalignment

The velocity sensitivity model can be used to predict the effect that translational and angular misalignments between the scanning system and target rotation axes have on the measurement; particularly useful since small misalignments are inevitable. Such small misalignments may lead to significant additional components in the velocity measured in a circular scanning measurement.

As illustrated in Figure 5.1, translational misalignment can be accounted for in the model by including the constants $x_{om}$ and $y_{om}$ in the intended known point $x$ and $y$ coordinates. Similarly, angular misalignment is represented by including the constants $\theta_{xm}$ and $\theta_{ym}$ in the angular vibration displacement parameters. Setting the flexible and rigid vibration components to zero in equation (4.22) enables the measured velocity (ideally zero, of course) to be predicted for this "no target vibration, arbitrary misalignment" case. Making use of equations (4.4a) and (4.23a&b), equation (4.22) can be used to show the influence of misalignments on the measured velocity:
\[
U_m = \frac{\cos 2\theta_{sx} \cos 2\theta_{sy} \left[ \theta_{x0m} x_{0m} + \theta_{ym} y_{0m} \right]}{\Omega} \\
+ \sin 2\theta_{sx} \left[ y_{0m} + \theta_{ym} \left( z_0 + d_s \cos 2\theta_{sy} \right) \right] - \cos 2\theta_{sx} \sin 2\theta_{sy} \left[ \theta_{x0m} - \theta_{ym} z_0 \right] \\
- d_s \sin 2\theta_{sx} \sin 2\theta_{sy}
\]

Substitution for \(\theta_{sx}\) and \(\theta_{sy}\) using equations (4.20a,b,c) or equations (4.20a\&b) and (4.21a\&b) immediately results in a full expression for the velocity measured in either an equal amplitude or corrected amplitude mirror drive signal circular scan. In either case, significant additional information exists in the measured Laser Vibrometer signal at DC, 1x and 2x scan frequency, as illustrated in Figure 5.2 (corrected amplitude case).

5.2. VELOCITY SENSITIVITY MODEL EXPERIMENTAL VALIDATION

5.2.1. EXPERIMENTAL ARRANGEMENT

The scanning system used was custom-built incorporating a Polytec OFV3020 Laser Vibrometer [5.1] and a pair of GSI Lumonics M3 galvanometers [5.2]. Each galvo can rotate an attached mirror through \(\pm 15^\circ\) mechanical (\(\pm 30^\circ\) optical). A two-channel function generator was used to generate the equal amplitude cosine and sine functions necessary to perform a “circular” scan. The galvos are mounted relative to the Laser Vibrometer in the same manner as was shown schematically in Figure 4.5, which is equivalent to the arrangement employed in both the Ometron Type 8330 [5.3] and Polytec PSV300 [5.4] commercial scanning Laser Vibrometers.

The target used was a small (\(\varnothing 30\)mm x 5mm), aluminium rotor with rigid cross-section mounted to a DC motor. The target rotation frequency was controlled using a stable DC power supply and measured using a Polytec OFV4000 Rotational Laser Vibrometer [5.5].
5.2.2. ADDITIONAL COMPONENTS IN ROTATING TARGET MEASUREMENTS

Substituting for $\theta_{sx}$ and $\theta_{sy}$ using equations (4.20a&b) and (4.21a&b) and using small angle approximations enables equation (5.1) to be re-written for a "corrected amplitude mirror drive signal, no target vibration, arbitrary misalignment" circular scan:

$$\frac{U_m}{\Omega} = \left[\theta_{sx}x_{0m} + \theta_{sy}y_{0m}\right] + \frac{r_s}{z_0 + d_s} \left[y_{0m} + \theta_{sx}(z_0 + d_s)\right] \cos(\Omega_s t + \phi_s)$$

$$- \frac{r_s}{z_0} \left[y_{0m} - \theta_{sy}z_0\right] \sin(\Omega_s t + \phi_s)$$

$$- \frac{d_s r_s^2}{2z_0(z_0 + d_s)} \sin 2(\Omega_s t + \phi_s) \quad (5.2)$$

Clearly the corresponding expression for equal amplitude mirror drive signals can be formed by substituting for $\theta_{sx}$ and $\theta_{sy}$ using equations (4.20a,b&c). In either case, the components at DC, 1x and 2x scan frequency dominate the measurement, as previously discussed in section 5.1 and as illustrated in Figure 5.2 (corrected amplitude case). As shown in equation (5.2), the amplitude of the component at 2x scan frequency is insensitive to variations in misalignment and, as such, it is possible to perform the experimental validation of this element of the velocity sensitivity model separately.

5.2.2.1. DUAL MIRROR EFFECTS

Since the amplitude of the 2x component is a function of the perpendicular mirror separation, $d_s$, as well as the scan radius, $r_s$, and the stand-off distance, $z_0$, the scanning system hardware used in this validation included the facility to vary $d_s$ from 30mm to 50mm. Figure 5.3a (equal amplitude case) and Figure 5.3b (corrected amplitude case) show comparisons between the predicted and measured amplitude of the 2x component for a series of measurement configurations. In both cases, each solid line represents the theoretical prediction of $U_m/\Omega$ vs. $r_S$, with the plotted points representing the
corresponding measured values. The theoretical prediction shows good correlation with the measured data for both equal amplitude and corrected amplitude mirror drive signal “circular” scans thereby validating the use of the model for prediction of this additional component.

The model can also be used to examine the effects of misalignment between the target and scanning system axes and this important aspect of real applications will be explored in detail in this next subsection.

5.2.2.2. MISALIGNMENT EFFECTS

As shown in equation (5.2), translational and angular misalignments mostly influence the 1x additional measurement component. The scanning system used in this validation incorporated the facility to vary $x_{om}$, $y_{om}$ and $\theta_{jm}$ to enable further validation of the theory.

Whilst the “no target vibration” condition is relatively straightforward to achieve in the laboratory by taking care with target selection, the “no misalignment” condition is not. Small but inevitable initial misalignment between the scanning system and target rotation axes results in a significant component at 1x scan frequency.

Figure 5.4 shows comparisons between the predicted and measured amplitudes of the additional component at DC and 1x scan frequency for a series of measurements in which only the translational misalignment in the x direction, $x_{om}$, was varied. Figure 5.5 shows similar comparisons in which only $y_{om}$ was varied. The broken lines represent the theoretical predictions of measured velocity per unit rotation frequency, $U_m/\Omega$, for varying $x_{om}$ or $y_{om}$ and the data points represent the corresponding series of measured values. The difference between these two sets of data is significant and is due to the initial misalignments, which are unknown, difficult to control and cannot be measured directly. In the absence of a means to measure the initial misalignments directly, the chosen way to proceed is to use the velocity sensitivity model as a basis for a particular hypothesis. Experimentally demonstrating that the hypothesis holds can then be taken as
validation of the model. The basis of this hypothesis is to use the velocity sensitivity model to obtain estimates for the initial misalignments after a series of measurements have been made in which one of the controllable misalignments is varied.

Showing the unknown initial misalignment parameters, \( x_{0m}, y_{0mu}, \theta_{xmu} \) and \( \theta_{ymu} \), explicitly in equation (5.2) and evaluating at DC and 1x scan frequency results in:

\[
\frac{U_m}{\Omega} \bigg|_{\omega = 0} = (\theta_{x} + \theta_{xmu})(x_{0m} + x_{0mu}) + (\theta_{y} + \theta_{ymu})(y_{0m} + y_{0mu})
\]

(5.3a)

and

\[
\left( \frac{U_m}{\Omega} \right)^2 = \left( \frac{r_s}{z_0 + d_s} \right)^2 \left[ (y_{0m} + y_{0mu}) + (\theta_{x} + \theta_{xmu})(x_{0} + d_s) \right]^2
\]

\[
+ \left( \frac{r_s}{z_0} \right)^2 \left[ (x_{0m} + x_{0mu}) - (\theta_{y} + \theta_{ymu})z_0 \right]^2
\]

(5.3b)

Making \( x_{0m} \) the variable misalignment parameter with \( y_{0m}, \theta_{xm} \) and \( \theta_{ym} \) set to zero, enables these equations to be rearranged into forms that are useful for identifying the unknown misalignment parameters:

\[
\frac{U_m}{\Omega} \bigg|_{\omega = 0} = \theta_{xmu}(x_{0m} + x_{0mu}) + \theta_{ymu}y_{0mu}
\]

(5.4a)

and

\[
\left( \frac{U_m}{\Omega} \right)^2 - \left( \frac{r_s}{z_0} x_{0m} \right)^2 = r_s^2 \left[ \left( \frac{y_{0mu}}{z_0 + d_s} + \theta_{xmu} \right)^2 + \left( \frac{x_{0mu}}{z_0} - \theta_{ymu} \right)^2 \right]
\]

\[
+ 2 \left( \frac{r_s}{z_0} \right)^2 x_{0m} (x_{0mu} - z_0 \theta_{ymu})
\]

(5.4b)

The first part of the hypothesis is that plots of the terms on the left hand sides of these expressions against \( x_{0m} \) will result in good fits to straight lines and this is confirmed in Figure 5.4a for equation (5.4a) and Figure 5.6a for equation (5.4b). Estimates of \( \theta_{xmu} \)
and \((x_{0mu} - z_0\theta_{y_{mu}})\) based on the gradients of the plots in Figure 5.4a and Figure 5.6a, respectively, are then possible.

Following a similar procedure but, in this case, making \(y_{0_{mu}}\) the variable misalignment parameter, with \(x_{0_{mu}}, \theta_{xm}\) and \(\theta_{ym}\) set to zero, should, again, result in plots that are good fits to straight lines. As illustrated in Figure 5.5a and Figure 5.6b, this part of the hypothesis is further supported by the strength of these fits. In this case, the gradients of these plots can be used to obtain estimates of \(\theta_{y_{mu}}\) and \((y_{0mu} + z_0\theta_{x_{mu}})\) and estimates of each of the individual initial misalignment parameters can be resolved. The angular misalignment parameters, \(\theta_{xm}\) and \(\theta_{ym}\), could also have been used in the same manner to obtain estimates of the unknown initial misalignments.

The second part of the hypothesis is that by substituting the estimated values of the unknown initial misalignment quantities into equations (5.3a&b), much improved theoretical predictions of the DC and 1x scan frequency components will be obtained. These updated predicted amplitudes are also shown in Figure 5.4 and Figure 5.5 (solid lines) and they exhibit strong correlation with the measured data points, supporting this part of the hypothesis. The difference between the intercepts of the DC experimental and updated predicted amplitudes is due to a small DC drift on the dynamic signal analyser used to obtain the data.

The final part of the hypothesis is that estimates of the unknown initial misalignments should be reasonable given the care taken in ensuring that the scanning system and target rotation axes were aligned. In this experimental validation the initial angular misalignments, \(\theta_{xm}\) and \(\theta_{ym}\), are calculated as -0.2° and 0.7°, respectively, and the corresponding initial translational misalignments, \(x_{0_{mu}}\) and \(y_{0_{mu}}\), are calculated as -0.38mm and -0.76mm. The nature of the experimental configuration used in this validation is such that these misalignments are quite reasonable, supporting the final part of the hypothesis. The strength of the hypothesis at each of the three stages is taken as a validation of the velocity sensitivity model for prediction of these significant DC and 1x scan frequency additional components.
5.3. **Doppler Shifts from the Scanning Mirrors**

In the development of the velocity sensitivity model for scanning Laser Vibrometer measurements presented in section 4.4, it was assumed that there was no Doppler shift due to the motion of the mirrors. For such an assumption to be valid, each mirror rotation axis must be in the plane of the respective mirror reflective surface and the laser beam must be reflected at a point on this axis. Neither of these conditions, however, is satisfied in a real scanning system and in some situations the resulting influence on the Laser Vibrometer signal should be accounted for when interpreting measured data.

The effect is characterised by an additional Doppler shift on the laser beam with a frequency equal to the scan frequency and a level dependent upon the mirror rotation angular frequency. During the course of this particular experimental validation, however, this additional component was at least an order of magnitude smaller (1.4mm/s for a 50mm radius, 10Hz scan) than that due to scanning system and target rotation axis misalignment and it was therefore not considered further.

5.4. **Laser Speckle Effects**

The velocity sensitivity model, which expresses total target velocity in the direction of an arbitrarily orientated laser beam, does not include the effects of laser speckle, which are the result of an entirely different phenomenon as described in section 1.2. Since, however, an appreciation of laser speckle effects is so important in scanning Laser Vibrometer measurements, they will be discussed in this section.

A stationary laser beam incident on a rotating target will result in recurrent speckle noise which repeats at the target rotation frequency whilst a scanning laser beam incident on a stationary target will result in a speckle noise repeat at the scan frequency. A scanning laser beam incident on a rotating target can give rise to speckle noise which repeats at a frequency other than the scan or the rotation frequencies. In such a case, the
speckle repeat has a period that corresponds to the time taken for both the scan and the rotation to complete (different) integer numbers of cycles.

The velocity measured in such a measurement is illustrated in Figure 5.7 where a scan at 12.5Hz combined with rotation at 10Hz resulted in a speckle noise repeat at 2.5Hz, i.e. 5 cycles of scan and 4 cycles of rotation. As well as the additional components at 1x and 2x scan frequency due to instrument misalignment and the dual mirror configuration, the underlying speckle noise is clearly apparent. The sharpness of the harmonic peaks and the high order up to which they prevail are classic characteristics of speckle noise generated by a rotating target [5.6].

This is, of course, just one of the many speckle repeat possibilities and a full map of speckle repeat frequencies is shown in Figure 5.8. The lower limit apparent on the ratio of speckle repeat frequency / rotation frequency is equal to the resolution in the spectrum. In this data this has been set at 1/50th of the rotation frequency, i.e. the data length is equal to the time taken for 50 rotation cycles. The solid line shown corresponds to four times the resolution in the spectrum and, based on experience, is proposed as the lowest speckle repeat frequency that could be seen clearly in the spectrum. The plot is dimensionless such that the LDV user could plot different limits on this particular map for any resolution coarser than 1/50th of the rotation frequency. The data points above the solid line thus represent the repeat frequencies that can be seen at all the specific values of the ratio of scan frequency / rotation frequency at which repeats could be observed. These must obviously include the example shown previously in Figure 5.7 and this data-point is shown highlighted in a circle in Figure 5.8.

The tracking condition, where scan frequency / rotation frequency = 1, merits further discussion. The map shows that the speckle repeat frequency should equal rotation frequency at this condition and, if perfect tracking could be achieved, the speckle pattern on the detector might be expected to rotate but not to change its form, resulting in extremely low noise in the instrument output. In reality, as discussed earlier, there are small but inevitable misalignments between target rotation and optical axes, as well as imperfections in the scan profile, that mean there will still be modest changes in the
collected speckle pattern. Nonetheless, a significant drop in speckle noise does result as the tracking condition is approached and this is illustrated for $0 \leq \Omega_s \leq 2\Omega$ in Figure 5.9a. In Figure 5.9b, a more focused range $0.8\Omega \leq \Omega_s \leq 1.2\Omega$ is examined such that the effects close to tracking can be identified. The “speckle noise” values are calculated by taking the sum of the squares of all spectral lines except those at 1x and 2x (since these are mostly due to misalignment and dual mirror configuration). Equivalent values are larger in Figure 5.9a because a greater frequency range (more speckle harmonics) was used when collecting this data than when collecting the data presented in Figure 5.9b. It can be seen from the data that the spectral mean squared noise when tracking is at least a factor of 2 down on that when the scan and rotation frequencies differ by just a few percent, thereby supporting the hypothesis that speckle noise is significantly reduced in the tracking condition.
This chapter will concentrate on the implementation of the techniques developed and extensively investigated during the previous four chapters. Firstly, however, the influence of laser speckle noise on a series of measurements made by several commercially available instruments from rotating structures will be examined. Following this, the individual motion component resolution technique described in chapters 2 and 3 will be employed to obtain, for the first time, real-time resolved pitch/yaw vibration data from the crankshaft pulley of a running diesel engine. Examples of the types of previously unobtainable information that can be obtained from such measurements will be presented and the value of this element of the research project thereby confirmed. In the final section, the circular scanning technique analysed in chapters 4 and 5 will be implemented to measure the axial vibration of a rotating disc.

6.1. COMMERCIAL LASER VIBROMETER EVALUATION: SPECKLE NOISE SENSITIVITY

This experimental investigation involved a series of tests to enable performance comparison of three commercially available Laser Vibrometers, in particular to evaluate the sensitivity of the instruments to speckle induced noise in measurements on rotating targets. Whilst the optical configurations of the three instruments are equivalent, two of the instruments, the Ometron Type 8329 (VH300) [6.1] and the Polytec OFV3020 [6.2], employ similar analogue electronic techniques (despite eliminating the directional ambiguity by different means) to convert the Doppler shifted backscattered laser light into a voltage output. The third instrument, the Polytec PDV100 [6.3], performs the Doppler signal demodulation digitally and therefore offers both direct digital and DAC converted outputs.
The arrangement used in this experimental analysis is shown schematically in Figure 6.1. The target used was a small (Ø30mm x 5mm), aluminium rotor with rigid, nominally circular cross-section mounted to a DC motor. The target rotational mean angular velocity was controlled using a stable DC power supply. The sensitivity of the instruments to speckle induced noise was analysed for two different target rotational mean angular velocities. The machined target surface was treated in a number of different ways and instrument performance was tested at a series of stand-off distances.

Figure 6.2 shows typical examples of the instrument velocity output for measurements on a non-vibrating target rotating at (nominally) 30πrad/s (15Hz) and 120πrad/s (60Hz). In both cases the measured velocity should, ideally of course, be zero but, in reality, the instrument has a noise floor that is associated with various sources, including the instrument and acquisition electronics, but which is dictated by speckle pattern dynamics. Such noise floors are characterised by harmonic peaks at integer multiples of the target rotation frequency and an underlying low-level baseline noise [6.5], as can be clearly observed in the figure.

A software routine was developed in an attempt to differentiate the noise concentrated at the harmonic peaks, attributed to laser speckle, from the remainder of the spectral content. The routine identifies the frequencies of the harmonic peaks and then separates the velocity content at these peaks and their corresponding adjacent spectral lines (accounting for both the Picket Fence Effect [6.6] and some genuine rotation frequency drift) from the spectrum. The average power spectral density of the "periodic speckle noise" and the "non-periodic noise" is then calculated. This information is included in the example spectra shown in Figure 6.2: the short-dashed line representing the periodic speckle noise and the long-dashed line representing the non-periodic noise. As can be seen in the figure, the routine provides a reasonable estimate of the average harmonic peak level but a less convincing estimate of the noise floor. This is caused by the fact that the peaks are in some cases wider than three spectral lines, due to variations in the target rotation speed, and the periodic speckle noise content is thereby not completely separated from the non-periodic noise content. The result is approximately a factor of 76.
two uncertainty in the non-periodic noise level which must be respected when drawing conclusions.

6.1.1. TARGET SURFACE INFLUENCE ON SPECKLE NOISE

In many cases, rotating targets have machined or polished surfaces, such that the "optically rough" diffuse backscattering condition, which is advantageous when making off-normal measurements (as rotor measurements generally are), is not met. In such cases it is necessary to treat the surface with some form of diffuse scatterer and the speckle noise problem therefore exists. In this experimental study, measurements on a number of typically encountered surfaces were investigated, i.e. machined/polished surface (MS), retroreflective tape (RT) [6.4], white "sticky-backed" paper (WP), matt black spray paint (MBP) and developer spray (DS).

Figure 6.3 and Figure 6.4 show the noise for similar measurements using each of the instruments on the target rotating at 15Hz and 60Hz, respectively, with the various surface treatments. In each case, the average noise power spectral densities are normalised to the largest value found in that group of values (the ratio of periodic speckle to non-periodic noise is of the order of 50:1). As suspected, for both analogue Laser Vibrometers (VH300 and OFV3020) the noise levels experienced when making measurements on optically smooth surfaces are significantly different to those experienced when making measurements on rough surfaces. This significant difference is not associated with variations in speckle noise sensitivity, it is due to the reflectivity of the polished surface leading to the occasional (not necessarily periodic) amplitude dropout and hence signal noise when low light levels are collected on the photodetector surface, as discussed in section 1.2.1. Commercial instruments contend with such amplitude dropouts by implementing a track and hold filter, the effectiveness of which is debatable. Successful measurements on highly reflective surfaces are therefore not recommended due to the unpredictability of the instrument to perform reliably.

For all three instruments and all surface treatments, the differences in performance are within the uncertainty level of the calculation. The retro-reflective surface is arguably
the best surface to use and the digital Laser Vibrometer (PDV100) arguably contains the least non-periodic noise in the output; its sensitivity to speckle noise, however, is equivalent to the analogue instruments. This result is of interest as it emphasises the fact that the speckle noise problem is not exclusively caused by amplitude dropouts, as often insisted by instrument manufacturers, it is also due to the resultant phase variation.

6.1.2. Stand-Off Distance Influence on Speckle Noise

An often un-respected fact about the use of Laser Vibrometers for surface velocity measurements is that the finite coherence length (distance over which the laser light remains in phase) of the laser source and the target and reference beam path lengths require that consideration be given to the distance between the instrument and the target. Whilst measurements can generally be made at any stand-off distance within the operating range, it is preferable to position the instrument relative to the target such that it is some nominal distance (the reference beam path length) plus an integer number, $n$, of laser tube lengths from the target. The Ometron VH300, for example, is best positioned at a stand-off distance equal to $385\text{mm}+n_{1}137.6\text{mm}$ ($n=0,1,2$ etc.).

For a given target rotational velocity, the rate at which the speckle pattern moves across the instrument light collection optics is clearly dependent upon, amongst other factors, the stand-off distance. Since the lens in the instrument focuses the beam onto the target, the collected speckle size is comparable with the diameter of the collection optics and it is unlikely that there will be any significant difference in the speckle induced noise between measurements taken at different stand-off distances. In this series of tests, measurements were made using each of the Laser Vibrometers at a series of stand-off distances, where $n=0$, 1 & 2 and where the stand-off distance is approximately 1m, 2m & 3m.

Figure 6.5 shows the periodic speckle noise as a function of stand-off distance for measurements taken on a rotating target treated with retro-reflective tape. Here it is
possible to observe similar trends for all three instruments; there is a no significant increase in the sensitivity to speckle noise with increased stand-off distance.

This section has shown, however, that measurements from polished target surfaces are particularly unreliable and can result in high non-speckle related noise quantities. The difference between the three instruments tested are negligible and they all experience speckle induced noise to approximately the same extent, despite claims sometimes made by manufacturers that their instruments have some degree of immunity to speckle noise. Most importantly, however, the information presented has shown, that low signal amplitudes do not necessarily exclusively cause speckle noise problems, as is evident from comparison of matt black paint and retro-reflective surfaces. This is because, as described in section 1.2.1, the speckles change on the photodetector causing the resultant phase of the output to change in sympathy during the course of a measurement.

6.2. Individual Motion Component Resolution

In section 3.3, the value and reliability of the individual motion component resolution technique was demonstrated for a pair of measurements in an engineering application in which the measurement of radial and pitch/yaw vibration directly from the rotating component is of genuine importance: a diesel internal combustion engine. In this section, examples of the type of information that can be obtained from a series of such tests will be presented. In this section it is not the author's intention to draw conclusions regarding the performance or design of the engine under test, rather to demonstrate the potentially powerful data that this technique makes available to the engine designer.

In this experimental investigation, the same 4-stroke, 4-cylinder, 2-litre diesel engine from which the measurements presented in section 3.3 were obtained, was operated at a range of engine speeds under a significant load (100Nm). The experimental arrangement was equivalent to that discussed in section 3.3.1 and illustrated in Figure 3.39 but, in this case, additional measurements were obtained from the front of the engine crankshaft (crankshaft-water pump drive pulley face).
The important pitch/yaw vibration data presented in Figure 6.6 and Figure 6.7 (waterfall diagrams) and Figure 6.8 (2x harmonics vs. engine speed) are typical of the type of information that the engine design engineer would find extremely useful. The variation of the level of the vibration peaks at \( \frac{1}{2} x \) engine orders with changing engine speed enables characterisation of the performance and condition monitoring of the engine. The significant developments in the use of Laser Vibrometry for vibration measurements directly from rotating targets realised during this research project make the acquisition of information of this nature practical for the first time.

6.3. CONTINUOUS SCANNING LASER VIBROMETRY

The main objective of chapter 5 was to validate the velocity sensitivity model for continuous scanning measurements, in particular to verify the prediction of additional components that occur at integer multiples of the scan frequency in measurements on rotating targets. In this section, the velocity sensitivity model will be used to predict the form of the Laser Vibrometer output firstly in a circular scanning measurement on a rotating target of rigid cross-section and secondly on a rotating axially flexible bladed disc, both undergoing medium severity axial vibration.

6.3.1. CIRCULAR SCANNING MEASUREMENT EXAMPLE

Figure 6.9 shows the velocity measured in a non-scanning Laser Vibrometer measurement on a non-rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration; the excitation being introduced through the rotor housing. This straightforward measurement constitutes a baseline for the scanning measurements, illustrating the vibration peak at 40Hz, as well as the genuine low level harmonic distortions at 80Hz and 120Hz. The underlying instrument noise floor contains peaks at 50Hz and 100Hz, which are caused by electrical interference and, as such, are present in all measured spectra presented in this section.
Figure 6.10 shows the velocity measured in a circular *scanning* Laser Vibrometer measurement on a *non-rotating* target undergoing nominally the same axial vibration. As discussed in section 5.3, scanning the laser beam causes speckle pattern motions during the course of the measurement that manifest themselves clearly in the spectrum at integer multiples of scan frequency. The peak at 1x scan frequency is at a higher level as a result of the additional Doppler shifts from the scanning mirrors, as discussed in section 5.3. Despite these sources of additional velocity content, the spectrum represents a respectable measurement of the 40Hz axial vibration and its harmonic distortion. Clearly this measurement configuration has no practical value (a stationary beam will provide a measurement of axial vibration) and is included here merely to enable observation of the speckle induced measurement noise.

In the case of a circular *scanning* Laser Vibrometer measurement on a *rotating* target, illustrated in Figure 6.11, the introduction of target motion into the measurement should result in a general difference in the resulting speckle noise. For a 10Hz scan on a target rotating at 20Hz, a speckle pattern repeat of 10Hz is still expected but, since the rotation frequency is only nominally 20Hz in this measurement, the speckle pattern does not repeat perfectly at 10Hz. Despite this, higher speckle noise levels can be seen at approximately integer multiples of 10Hz in the real measurement. More importantly, Figure 6.11 shows significantly higher velocity levels at 1x (20mm/s) and 2x (0.5mm/s) scan frequency, due to the arbitrary initial misalignment and the dual mirror configuration, respectively. These additional component amplitudes are comparable with those discussed in section 5.1, presented in Figure 5.2. Most importantly of all, a respectable measurement of the axial vibration is obtained.

The theory presented in chapters 4 and 5 enables the user to predict the form of the measured velocity and this is shown in Figure 6.12 for this particular scanning arrangement with estimated misalignment values based on the measurement configuration (the harmonic distortion of the axial vibration is not included in this velocity prediction). As can be seen by comparison of Figure 6.11 and Figure 6.12 an order of magnitude prediction is possible for the misalignment dependent DC and 1x components (DC not shown) whilst the 2x component can be predicted with a high
degree of accuracy since it is insensitive to misalignment. As shown in Figure 6.12, small sidebands are present in the predicted instrument output at the vibration frequency $\pm \Omega_s$. These are caused by the misalignments but are generally low-level components, below the instrument noise floor in the real measurement.

In the measurements discussed in this section, the scan frequency, rotation frequency and vibration level and frequency were chosen such that the additional measurement content did not constitute a significant source of measurement ambiguity. In a real measurement such control is obviously not available and, for example, if the axial vibration level were lower or at a different frequency, then the significance of the additional content might be greater as it would be less straightforward to distinguish genuine vibration peaks from additional content peaks. In such a situation, changing the scan frequency (if acceptable) might be useful since the frequency of additional content peaks would change in sympathy whilst the frequency of the vibration peaks would remain constant.

6.3.2. **Circular Tracking Measurement Example**

Circular tracking Laser Vibrometer measurements can be arranged for by setting the scan frequency equal to the target rotational frequency. The dedicated tracking system specifically developed for this research project to enable such measurements is illustrated in Figure 6.13 and described in Appendix C. Here, the probe laser beam remains fixed on a single point on the target during rotation and the measured velocity then relates to that particular point only. Such a measurement is especially advantageous if the target is non-continuous in cross-section as is, for example, a bladed disc.

Figure 6.14 shows the velocity measured in a Laser Vibrometer measurement on a non-rotating blade undergoing first natural frequency vibration at 32.5Hz (nominal); the excitation again being introduced through the rotor housing. This straightforward measurement constitutes a baseline for the tracking measurements, illustrating the vibration peak at 32.5Hz, as well as the genuine low-level harmonic distortions at 65Hz and 97.5Hz.
Figure 6.15 shows the velocity measured in a *tracking* Laser Vibrometer measurement on a (nominally) *non-vibrating, rotating* blade. Here, the significantly higher velocity levels at 1x (20mm/s) and 2x (13mm/s) scan frequency, due to the arbitrary initial misalignment and the dual mirror configuration, respectively, are easily identifiable. These additional component amplitudes are again comparable with those presented in Figure 5.2. In addition to this, the misalignments result in relative motion between the laser beam and blade surface causing speckle pattern motions with a repeat frequency equal to the rotation frequency. Despite the absence of a controlled excitation, the rotation of the target excites measurable blade motion at frequencies close to resonance and this genuine velocity content is present in the data.

In the case of a *tracking* Laser Vibrometer measurement on a *vibrating, rotating* blade, as illustrated in Figure 6.16, the measurable blade motion is evident at 32.5Hz due to the controlled excitation (genuinely lower amplitude motion for the same excitation) and surrounding frequencies due to the rotation. As in Figure 6.15, there is additional measurement content at 1x and 2x scan frequency and speckle induced measurement noise but, despite this, the technique yields a respectable measurement of the blade vibration under relatively challenging circumstances.

The theory presented in chapters 4 and 5 enables the user to predict the form of the measured velocity and such a prediction is shown in Figure 6.17 for this particular tracking measurement arrangement with estimated misalignment values based on the measurement configuration. As can be seen by comparison of Figure 6.16 and Figure 6.17, an order of magnitude prediction is possible for the misalignment dependent DC and 1x components (DC not shown) whilst the 2x component can be predicted with a high degree of accuracy since it is insensitive to misalignment. The low-level component at 3x scan frequency in Figure 6.17 is also associated with misalignment but is generally below the real instrument noise floor and therefore insignificant; it cannot be seen in the real measurement. The accuracy of the prediction of the three principal velocity components demonstrates the usefulness of the velocity sensitivity model for confident data interpretation even in advanced LDV applications.
7. RECOMMENDATIONS FOR FURTHER WORK

A significant increase in the level of understanding of the techniques described in this thesis has been realised during this research project. In particular, the technique for the resolution of individual motion components is now approaching the commercial realisation stage. The chapter concentrates on the scanning Laser Vibrometry technique which is further from commercial realisation but which offers some very interesting areas for further work. There are a number of additional issues requiring immediate investigation in both techniques and these will be discussed in this chapter.

7.1. INDIVIDUAL MOTION COMPONENT RESOLUTION

The resolution technique described in chapters 3 and 4 provides greatly improved estimates of genuine vibration velocities without significantly increasing noise and oscillation-offset error levels. Improvement in the performance is possible through reduction of both offsets and speckle noise; this section describes a strategy for the reduction of offsets.

The current instrument alignment procedure, described in section 3.3, is capable of limiting the offset to within ±0.2mm. It is realistic that this uncertainty could be reduced to ±0.1mm without the introduction of additional technology. If, however, the instrument alignments were actively controlled, by, for example, a piezoelectric translational micro-positioning stage in a closed loop, it is possible that the offsets could be reduced to ±0.02mm. This would in turn lead to a desirable order of magnitude reduction in the oscillation-offset cross-sensitivity term ($\Delta \Omega_r y_0$) and hence an improved oscillation-displacement correction algorithm output.
7.2. CONTINUOUS SCANNING LASER VIBROMETRY

The discussion set out in chapters 4 and 5 showed how additional components due to instrument/target misalignment and the dual mirror system occur when making continuous scanning measurements on rotating structures. Such additional components are clearly undesirable since they increase the likelihood of measurement misinterpretation and further complicate the post-processing required in advanced applications. It is for these reasons that further work in scanning LDV should initially be directed towards elimination of additional components and this is the subject of this section.

As illustrated in equation (5.2) and repeated here for convenience, the output for a "corrected amplitude mirror drive signal, no target vibration, arbitrary misalignment" circular scan is given by:

\[
\frac{U_m}{\Omega} = \left[ \theta_{xm} x_{0m} + \theta_{ym} y_{0m} \right] \\
+ \frac{r_s}{z_0 + d_s} \left[ y_{0m} + \theta_{xm} (z_0 + d_s) \right] \cos(\Omega_s t + \phi_s) \\
- \frac{r_s}{z_0} \left[ x_{0m} - \theta_{ym} z_0 \right] \sin(\Omega_s t + \phi_s) \\
- \frac{d_s r_s^2}{2z_0(z_0 + d_s)} \sin(2\Omega_s t + \phi_s) \tag{7.1}
\]

For some particular scan (i.e. given \( d_s \) and \( z_0 \)), the amplitudes of the additional components at DC and 1x scan frequency are directly dependent upon the misalignments between the target rotation and scanning system axes. In contrast, the amplitude of the additional component at 2x scan frequency is directly dependent upon the dual mirror separation.
7.2.1. Misalignment Elimination

The procedure set out in section 5.2.2.2 showed how estimation of the individual misalignment parameters is possible. Elimination of the translational and angular misalignments is therefore theoretically possible by fine adjustment of the scanning system alignment. Unfortunately, the custom-built scanning used for this project only accommodates fine adjustment of three of the misalignment parameters: $x_{om}$, $y_{om}$ and $\theta_{om}$. In order that the practicality of misalignment and resulting DC and 1x additional component elimination be tested, it will be necessary to modify the scanning system to include provision for the fine adjustment of $\theta_{om}$.

It is, of course, theoretically possible to fine-adjust the angular misalignment parameters optically through the deflection mirror scan angles: the application of a small DC offset to the mirror drive signal is equivalent to the introduction of a small mechanical rotation to the mirror angular position. The use of piezoelectrically (instead of mechanically) actuated translational stages will clearly enable similar electronic fine adjustment of the translational misalignment parameters. A further benefit of the elimination of scanning system/target misalignment is reduction of the speckle induced noise in the measurement. The practicalities of such misalignment reduction strategies should be investigated such that, ultimately, a solution in which the scanning system can be casually positioned in front of the rotating target of interest and the required realignment procedure be performed automatically, is reached.

7.2.2. Single Mirror Scanning

The single mirror scanning system discussed in this section is much closer to the idealised scanning system described in section 4.2 than the commercially implemented dual mirror system. It makes use of a single mirror that can rotate about two axes simultaneously and since such mirrors are commercially available [7.1-7.3], such a system is worthy of future investigation and hence discussion in this section.
It is possible, as set out in section 4.2, to write equations (4.11a&b) that describe the laser beam orientation angles necessary to perform a perfectly circular scan using a dual mirror scanning system. Despite this, the mathematical derivation presented in section 4.3, showed that it is not straightforward to generate deflection mirror scan angles that result in the laser beam orientation angles associated with a perfect circular scan. Furthermore, as described in section 4.4, the fact that the known point position varies during a dual mirror circular scan on a rotating target leads to undesirable additional components in the measured velocity at 2x scan frequency as shown in equation (7.1). Generating the required laser beam deflection using a single optical element would overcome this problem since the known point would be stationary during scanning.

A typical example of a commercially available optical element capable of laser beam deflection about two orthogonal axes simultaneously is a mirror which rotates about two orthogonal axes parallel to the plane of the undeflected reflective surface. Figure 7.1 shows a potential optical configuration of a scanning system incorporating such a single dual axis mirror. With reference to Figure 7.1 and Appendix B.3, describing the mirror surface orientation by a unit vector enables the beam orientation after reflection, \( \hat{b} \), to be written in terms of the deflection mirror scan angles, \( \theta_{s1} \) and \( \theta_{s2} \):

\[
\hat{b} = \left[ \frac{\sqrt{2}}{2} \sin 2\theta_{s1} (\sin \theta_{s2} - \cos \theta_{s2}) \right] \hat{x} - \left[ 1 - \cos^2 \theta_{s1} (1 - \sin 2\theta_{s2}) \right] \hat{y} + \cos^2 \theta_{s1} \cos 2\theta_{s2} \hat{z} \quad (7.2)
\]

Since equation (7.2) is equivalent to equation (4.12), it is possible to evaluate the laser beam incidence point in the target plane in the same manner as was presented in section 4.3.1 (for the dual mirror scanning system). With reference to Figure 7.1 and equations (4.15a,b&c) the laser beam incidence point, described by \( x_s(P) \) and \( y_s(P) \) is given by:

\[
x_s(P) = z_0 \frac{\sqrt{2}}{2} \sin 2\theta_{s1} (\sin \theta_{s2} - \cos \theta_{s2})
\cos^2 \theta_{s1} \cos 2\theta_{s2} \quad (7.3a)
\]

and
RecommendaTions for Further Work

\[ y_2(r) = z_0 \frac{\left(1 - \cos^2 \theta_{s1} \left(1 - \sin 2\theta_{s2}\right)\right)}{\cos^2 \theta_{s1} \cos 2\theta_{s2}} \]  \(7.3b\)

If equal amplitude cosine and sine functions, similar to those given in equations (4.20a,b,c), are used to perform a "circular" scan, a considerably elliptical profile results, as shown in Figure 7.2. This elliptical shape is due to the 45° inclination of the first mirror rotation axis relative to the target and can be corrected by employing a cosine function with amplitude that is a factor of \(\sqrt{2}/2\) larger than the amplitude of the sine function that is used to drive the mirror about the second rotation axis. The resulting corrected amplitude is much closer to the required circular scan (maximum absolute error of the order of 0.05%) and is illustrated in Figure 7.3.

This inclination of the first mirror rotation axis also means that it is not possible to perform a straight-line scan in the x direction via mirror deflection about the first rotation axis only. The resulting scan profile is shown in Figure 7.4a. This problem can be corrected for by simultaneously deflecting the mirror about second rotation axis to compensate for the deflection in the y direction but calculating the necessary function involves the manipulation of equations (7.3a&b) and is therefore not straightforward. The same problem does not occur for a straight-line scan in the y direction and this scan profile is shown in Figure 7.4b.

An example of a single mirror scanning system arrangement in which both mirror rotation axes are parallel to the target, where straight-line scans in the x and y directions can be performed by individual mirror rotations, is illustrated in Figure 7.5 and described in Appendix B.4. For such an arrangement, the incident laser beam orientation can be written as:

\[ \hat{b} = [\sin 2\theta_{sx} \cos \theta_{sy}] \hat{x} - [\cos^2 \theta_{sx} \sin 2\theta_{sy}] \hat{y} + \left[2 \cos^2 \theta_{sx} \cos^2 \theta_{sy} - 1\right] \hat{z} \]  \(7.4\)

In this case, equal amplitude cosine and sine functions result in an extremely "circular" scan profile, as shown in Figure 7.6, with a maximum absolute error as small as 0.01%! Clearly, this scanning system arrangement, in which the required initial orientation of
the laser beam is along the $z$ axis, is not straightforward to configure but, nonetheless, these single mirror configurations are highlighted as an area in which further work is encouraged.

Since equations (7.2) and (7.4) are equivalent to equation (1.8) (or (4.12)), it is possible to use them as a direct alternative. Evaluating the principal unit vector coefficients enables the velocity measured in such single mirror scanning system arrangements to be predicted using equation (4.3) (or (4.22)), as presented in section 4.4 for the dual mirror scanning system. Such an analysis conveniently shows that, as expected, the component at 2x scan frequency is not present and there are no other additional velocity components in the measured velocity due to the scanning system configuration. Confirmation of this fact by experimental validation is important and should be carried out in future work on this topic.

### 7.3. Synchronised-Scanning Laser Vibrometry

The quality of the information given by the comprehensive velocity sensitivity model in the particularly complex configurations presented in section 4.4 emphasises the value of the model in improving the accuracy of processed data. Development of the mathematical basis has provided the ability to process measured velocity with high accuracy and enabled consideration of the novel "Synchronised-Scanning Laser Vibrometry" technique which is described in detail in this section.

As discussed in section 6.3.2 and Appendix C, in a circular tracking Laser Vibrometer measurement the probe laser beam remains fixed on a single point on the target during rotation such that the measured velocity relates to that particular point. Changing the scan radius and/or scan initial phase results in a different probe laser beam position on the target, such that a number of discrete measurement points can be addressed and the vibration velocity profile across, for example, the surface of a blade, such as that shown in Figure 7.7a, can be obtained. Such a measurement would yield very useful data,
difficult to obtain by any other means, but it would be time-consuming and is prone to problems associated with not making measurements simultaneously.

Figure 7.7b&c show the first flexural and first torsional mode shapes, respectively, of the compressor blade in Figure 7.7a. In the flexural mode, vibration increases uniformly across the width of the blade from zero at the root to a maximum at the tip while, in the torsional mode, the blade twists around a stationary axis (a nodal line) running along the centre of the blade from root to tip.

The mode shapes were obtained by modal analysis of the blade in the laboratory i.e. under non-rotating conditions. Designers of such structures are especially interested in knowing how responses change during operating conditions, i.e. when rotating, through effects such as centrifugal stiffening but the experimental tools to provide this data do not currently exist. The major step forward in Laser Vibrometry technology, proposed in this section, is to synchronise the tracking and continuous scanning configurations described separately in chapters 4 and 5. The velocity sensitivity model enables the complexity in the measured velocity to be readily accommodated but the challenges in hardware for scan control are significant.

7.3.1. THEORETICAL DESCRIPTION: LINE SCAN

The Synchronised-Scanning technique is best explained with reference to a measurement on a rotating bladed disc as shown in Figure 7.8. Since the intention is to measure the operational deflection shapes of the blades it is necessary to scan the probe laser beam up and down the blade whilst the blade is under operating conditions, i.e. rotating. The intended scan profile, in de-rotated form, is also shown (red line) in Figure 7.8. Line scans of this form have successfully yielded modal data in previous studies [7.4,7.5] but, of course, on non-rotating structures.

With reference to equations (4.20a&b), the necessary mirror scan functions are given by:
\[ \theta_{s}(t) = -0.5 \tan^{-1} \left( \frac{r_s}{z_0 + d_s} \right) \cos(\Omega t + \phi_s) \]  
\textit{(7.5a)}

and

\[ \theta_{s}(t) = 0.5 \tan^{-1} \left( \frac{r_s}{z_0} \right) \sin(\Omega t + \phi_s) \]  
\textit{(7.5b)}

in which \( r_s \) is the intended scan radius, \( z_0 \) is the mirror to target distance, \( \Omega \) is the scan frequency (which is equal to the rotation frequency for tracking) and \( \phi_s \) is the scan phase.

To achieve the line scan of the blade, \( r_s \), which would be constant in a circular tracking measurement, becomes a function of time, thereby modulating the scan amplitude:

\[ r_s(t) = \bar{r}_s + \Delta r_s \cos(m\Omega t + \phi_{s}) \]  
\textit{(7.6)}

where \( \bar{r}_s \) and \( \Delta r_s \) are the mean and time dependent components of the scan radius, \( m \) is the number of radius cycles per revolution and \( \phi_{s} \) is the initial phase. Note from equation (7.6) how synchronised line scans require higher frequency operation of the scanning mirrors.

The result is mirror scan angle functions such as those illustrated in Figure 7.9a and a scan profile in space of the form shown in Figure 7.9b. This sophisticated measurement technique may be employed to provide valuable data relating to, for example, the cantilever bending mode of the blade, as shown in Figure 7.7b, for a real axial compressor blade, under operational conditions.

The experimental arrangement, shown in Figure 7.9c, uses a bladed disc with the shape of that shown in Figure 7.8. The disc is not visible in Figure 7.9c because of the speed at which it is rotating but the figure does illustrate the scan profile well. The photograph was achieved by holding the camera shutter open long enough for the bladed disc to
complete several rotations, to show graphically the path of the laser beam as it scans up and down the blade in synchronisation with its rotation.

At this point it is helpful to consider the form of the measured velocity from such a measurement on a rotating blade oscillating in its first flexural mode. In a simple, perfectly aligned tracking measurement, the measured velocity will be of the form shown in Figure 7.10a. It is possible to observe the low level additional component 2x scan frequency in the data. If, however, the laser beam scans up and down the blade during rotation, as in Figure 7.9, the processing of the data is complex since the measured velocity in the Synchronised-Scanning measurement is modulated by the mode shape as the laser beam changes position on the target surface. A typical form of this measured velocity is shown in Figure 7.10b from which one can see intuitively how the mode shape and frequency data could be obtained from the measurement.

7.3.2. THEORETICAL DESCRIPTION: AREA SCAN

Whilst the synchronised line scan would yield data for a cantilever bending mode such as that shown in Figure 7.7b, it would yield lower quality information for a torsional mode such as that shown in Figure 7.7c. The scan may even be performed along the nodal line in which case the measured vibration velocity would be zero! For such a case, a two-dimensional scan, such as that shown in Figure 7.11, would be beneficial. Again, previous work [7.5,7.6] has shown how mode shape data can be obtained from an area scan of this nature but, of course, on a non-rotating structure.

The area scan can be achieved by considering a simultaneous phase modulation of the mirror drive signals:

\[ \phi_s(t) = \bar{\phi}_s + \Delta \phi_s \cos(n n \Omega t + \phi_{\phi_s}) \]  

(7.7)

where \( \bar{\phi}_s \) and \( \Delta \phi_s \) are the mean and time dependent components of the scan phase, \( n \) is the number of phase cycles per radius cycle and \( \phi_{\phi_s} \) is the initial phase. Note how
synchronised area scans require even higher frequency operation of the scanning mirrors than synchronised line scans.

The result of the combined scan radius and phase modulation is the correspondingly more complex mirror scan angles illustrated in Figure 7.12a and a scan profile in space of the form shown in Figure 7.12b and, in a de-rotated form, in Figure 7.11. Figure 7.12c from the actual experimental arrangement illustrates the complex scan profile well, again achieved by holding the camera shutter open long enough for the bladed disc to complete several rotations thereby graphically showing the path of the laser beam as it scans up and down and across the blade in synchronisation with its rotation.

In this example, the scan completes 3 radius cycles and 27 phase cycles during each rotation of the target. These values were chosen principally for the purposes of illustration and the selection of optimum scan profile parameters would be an important part of the further work that should be conducted in order to explore the full potential of this seemingly powerful measurement technique.

This section has outlined how the Synchronised-Scanning Laser Vibrometry system combines the mathematical basis of the comprehensive velocity sensitivity model with measurement and scanning hardware and a dedicated tracking controller. The feasibility of such a measurement system has been proved.

7.3.3. Versatile System Implementation

Figure 7.9 and Figure 7.12 clearly confirm the feasibility of the custom-built scanning system to perform synchronised line and area scans on rotating components. As described in Appendix C, this system uses standard scanning mirrors [7.7], a stable DC motor with integrated 500 line per revolution optical encoder [7.8] and a dedicated controller built around four low-cost 20MHz processors (two per channel) [7.9]. It enables synchronised line scans such as that shown in Figure 7.9 at rotation speeds up to 1800RPM and synchronised area scans such as that in Figure 7.12, which demand much higher frequency performance, at speeds up to 200RPM only. A significant research
programme is necessary to accomplish the development of a versatile system and this section sets out the key areas of interest.

The scanning mirrors have an upper frequency limit around 150Hz so at least an order of magnitude improvement in frequency response is necessary to accommodate the higher frequency requirements of the synchronised scans. Initial investigations have shown how scanning mirrors can be obtained with response reaching up towards this target but it may be necessary to introduce alternative technologies, such as piezoelectric actuation, to reach this range reliably. A similar increase in processor speed is also necessary, involving the significantly challenging development of a next generation scan controller based on a more advanced form of processor technology.

Although the principal aim is to enable Synchronised-Scanning measurements, a number of other significant benefits would result from these proposed developments. Turbocharger applications highlight the desire to perform tracking measurements on a rotor at 140000RPM. Faster, synchronised scans would also introduce the possibility of multiplexing measurements from several locations on a structure, enabling a single Laser Vibrometer to perform the role of several instruments. The same hardware and software developments necessary for Synchronised-Scanning at higher frequencies may facilitate this exciting range of new measurement possibilities.

Work is also required to ensure that the end product constitutes a user-friendly measurement system for the vibration engineer. The velocity sensitivity model provides the basis of the required post-processing scheme but demodulation of the synchronised line and area scan data will be a challenging task with complications introduced by the presence of misalignment related components at integer multiples of scan frequency, as described in chapter 5. The model must be employed to set boundaries within which the processing can be performed with the ultimate aim of providing engineers with the reliable data that they require.
8. CONCLUSIONS

The non-contact operation and inherent immunity to target shape variation gives the Laser Vibrometer significant advantages over traditional contacting transducers for the measurement of vibration from rotating components. This thesis has been concerned with the development of advanced Laser Vibrometer measurement applications and, in particular, the implementation of a comprehensive theoretical description of the velocity sensed to enable confident measurement interpretation. This chapter summarises the significant achievements realised during this research project in three main areas. Some of the work has already been published and a list of the publications resulting from this research project is given in Appendix D.

8.1. INDIVIDUAL MOTION COMPONENT RESOLUTION

The comprehensive velocity sensitivity model gives the measured velocity as the sum of six terms, each the product of a combination of geometric parameters, relating to the laser beam orientation, and an inseparable combination of motion parameters, the vibration "sets". Measurement of the individual axial and torsional motion components is possible by assuming that the effects of the (typically small) cross-sensitivity terms within those particular vibrations sets are negligible. Resolution of individual radial or pitch/yaw motion components is not possible as these vibration sets contain significant cross-sensitivities to the product of the rotation speed and the orthogonal motion component. It is cross-sensitivities such as these which constitute a major limitation to the use of Laser Vibrometers for rotor vibration measurements; resolution of the individual motion components can only be achieved by post-processing the outputs from simultaneous orthogonal measurements and an independent rotation speed measurement.
The novel resolution technique developed in this research project carries out the necessary mathematical manipulations directly in the frequency domain and is implemented in sophisticated LabVIEW software, enabling real-time post processing of the outputs from standard commercially available instruments. These routines have been extensively examined for various realistic simulated rotor vibration situations with a comprehensive quantification of the typical error levels experienced being presented in the context of measured vibration amplitudes.

As described in chapters 2 and 3, the resolution technique is successful in eliminating the fundamental cross-sensitivity term. The resolution algorithm was tested for correct operation on a series of simulated constant rotation speed measurement cases, including vibration at single, dual and broadband frequencies. When tested using realistic simulated conditions, i.e. with speckle noise and speed measurement error present, implementation of the resolution algorithm still resulted in significant improvement of the measured data. Indeed the fundamental cross-sensitivity term was eliminated to below the instrument noise floor and the genuine component amplitude and phase errors were only 1% and 10mrad, respectively, when the rotation speed measurement was accurate. When a significant 2% error in the speed measurement exists, these error levels increase but are still acceptable at around 10% and 10mrad, respectively. The cross-sensitivity term was still reduced by approximately two orders of magnitude.

The frequency domain solution proved robust and enabled the development of the correction algorithm for the reduction of the oscillation-displacement additional terms and hence increasingly accurate estimates of the resolved vibration velocity in the presence of torsional oscillation. Again the performance of the correction algorithm was tested for a series of simulated measurement cases. It was shown that, after only three iterations, the correction algorithm reduced the oscillation-displacement term (by approximately three orders of magnitude) to at least five orders of magnitude lower than the genuine vibration peaks. When either noise or offsets are present, combination of the resolution algorithm and several correction algorithm iterations results in resolved outputs in which cross-sensitivity to orthogonal velocity component (e.g. $\dot{y}$) and the oscillation-displacement term (e.g. $\Delta \Omega_T y$) is eliminated. The maximum amplitude and
phase errors in the genuine vibration component are 10% and 100mrad, respectively, which are, again, very respectable considering the overall improvement in the measured data. Whilst repeated application of the correction algorithm does not result in a reduction in the noise or oscillation-offset errors, it importantly does not result in an increase either.

The robustness of the post-processing technique and the appropriateness of the chosen simulated vibration levels were both shown to be good by implementation with real Laser Vibrometer measurements taken both on laboratory based equipment and a diesel engine operating under load. The correct operation of the technique and the useful types of data that are available to the vibration engineer have been demonstrated for both radial and pitch/yaw vibration measurements.

An order of magnitude reduction in the offset amplitude would clearly be advantageous and the use of mechanical positioning stages to achieve this reduction was discussed. An order of magnitude reduction in the speckle induced noise level in the instrument outputs is less straightforward. Speckle noise has been shown to be significant in all Laser Vibrometer measurements on rotating targets and reduction is therefore an important topic for future research.

8.2. CONTINUOUS SCANNING LASER VIBROMETRY: THEORETICAL DEVELOPMENTS

The original derivation of the comprehensive velocity sensitivity model showed explicitly how the velocity sensed by an arbitrarily orientated laser beam incident on a rotating target, of rigid cross-section, undergoing arbitrary vibration, is related to the six degree-of-freedom target velocity components and the arbitrary orientation of the laser beam. Chapter 4 investigated the application of the velocity sensitivity model to several particularly challenging scanning measurement configurations.
A major novel development was the reformulation of the original model in terms of mirror scan angles, rather than laser beam orientation angles, which is especially useful since it is these angles that an operator would seek to control in practice. This was enabled by the critical development of a mathematical method to derive the laser beam orientation for any scanning system configuration and combine it with the total velocity of the incident point. It proved to be extremely beneficial in incorporating the complexity of real scanning configurations and allowed easy formulation of measured velocity, revealing some important details in the various measurements that were not apparent in predictions based on an idealised model of the scanning arrangements. Furthermore, the model was extended to incorporate cross-section flexibility since such scanning measurements are typically employed on targets with flexible cross-sections such as bladed discs.

The revised velocity sensitivity model was been applied to show how the common use of a pair of orthogonally aligned scanning mirrors leads to a significant yet predictable additional component in the Laser Vibrometer output at twice the scan frequency in circular scanning measurements on rotating targets. Furthermore, it was shown how the combination of this mirror configuration and equal amplitude cosine and sine mirror drive functions leads to an elliptical scan profile. Corrected amplitude drive signals can be employed to overcome this elliptical profile to an extent but the amplitude of the additional component at twice the scan frequency is not significantly reduced.

For the especially complex cases of small-scale circular and conical scans on non-rotating targets, implementation of the model enabled a detailed examination of the differences between idealised and actual scanning configurations. In particular, the origins of all of the components that occur in the measured data due to instrument configuration were easily revealed and an analysis of their influence on the measurement was presented.

In chapter 7, the use of a single mirror that rotates about two orthogonal axes on the mirror surface simultaneously was suggested as a closer approximation to the idealised
scanning configuration in which the distortion in the desired circular scan profile is reduced to an insignificant level. More importantly, the additional components at two, four and six times scan frequency that occur in a rotating target measurement are eliminated in such a system.

8.3. Continuous Scanning Laser Vibrometry: Practical Developments

The revised velocity sensitivity model was applied in chapter 5 to show how additional components occur mainly at DC and scan frequency due to misalignment between the scanning system and target rotation axes. In particular, the velocity sensitivity model was used to calculate the individual misalignment parameters for the first time. The experimental validation of the model for the prediction of the amplitudes of additional components at DC, 1x and 2x was presented, further confirming its usefulness. The influence of laser speckle on measured data was highlighted as a practical aspect of scanning LDV of significant importance and a detailed discussion of the typical characteristics of laser speckle noise in scanning measurements on rotating target was presented.

Experimentation showed that the additional component at 2x scan frequency was easy to validate but the additional component at DC and 1x scan frequency were more difficult to validate. These components are due to translational and angular misalignment between the scanning system and target rotation axes and small but inevitable initial misalignments dramatically affect the measured amplitude. A hypothesis was proposed in which the model itself was used to predict the unknown initial misalignments and then used to improve the prediction of the DC and 1x scan frequency components with further controlled misalignment. Confirmation of each part of the hypothesis was taken as the validation of the model for the prediction of misalignment-related additional components.
The outputs from real circular scanning and tracking measurements were presented and interpreted correctly in chapter 6 by making use of the information presented in chapters 4 and 5. The information presented provides the user with the ability to predict the additional components that occur in real scanning Laser Vibrometer measurements and thereby anticipate the form of the resulting spectra. Such measurements can then be interpreted with confidence thereby enabling the acquisition of very useful data of fundamental importance, extremely difficult to obtain by any other means, for the analysis of important effects such as centrifugal stiffening.

The Synchronised-Scanning technique presented in chapter 7 is the latest exciting and innovative new development of the scanning Laser Vibrometry technique for vibration measurements directly from rotors under operating conditions. Combining the ability to track a single point on a rotating structure with the ability to scan a structure to retrieve modal data, Synchronised-Scanning Laser Vibrometry opens up a host of new measurement possibilities. The feasibility of meeting the hardware and software requirements of the system has already been demonstrated, whilst the comprehensive velocity sensitivity model, the mathematical basis of the necessary post-processing, has been validated and applied to a number of existing scanning configurations. The comprehensive velocity sensitivity model has already proved valuable but post-processing of the data obtained from the complex profiles used in synchronised scans simply could not be contemplated without it. Such measurements are inconceivable by any other means and the innovative new system will provide data of fundamental importance in the design and development of range of devices from hard disk drives to gas turbines.
Consider a single sinusoidal component, an example of which is shown in Figure 2.2a, with amplitude $V_0$, angular frequency $\omega_v$, and phase $\phi_v$:

$$x(t) = V_0 \cos(\omega_v t + \phi_v) = V_0 \cos \phi_v \cos \omega_v t - V_0 \sin \phi_v \sin \omega_v t \quad (A1)$$

which can be re-arranged as follows:

$$x(t) = \frac{V_0}{2} \cos \phi_v \cos \omega_v t + \frac{V_0}{2} \cos \phi_v \cos(- \omega_v t) - \frac{V_0}{2} \sin \phi_v \sin \omega_v t + \frac{V_0}{2} \sin \phi_v \sin(- \omega_v t)$$

$$= (V_{0R} \cos \omega_v t - V_{0I} \sin \omega_v t) + (V_{0R} \cos(- \omega_v t) + V_{0I} \sin(- \omega_v t)) \quad (A2)$$

where the in-phase (real) and quadrature (imaginary) coefficients are given by $V_{0R} = \frac{V_0}{2} \cos \phi_v$ and $V_{0I} = \frac{V_0}{2} \sin \phi_v$, i.e. the positive and negative frequency components are complex conjugates. This formulation of $x(t)$ corresponds to the dual sided frequency spectrum representation, illustrated in Figure 2.2b, which results when the Fourier Transform (FT) algorithm is implemented on a time domain signal. The Fourier Transform of $x(t)$ can be written as follows:

$$FT[x(t)] = [V_{0R} + jV_{0I}]_{\omega = \omega_v} + [V_{0R} - jV_{0I}]_{\omega = -\omega_v} \quad (A3)$$

Performing a similar exercise but in this case with the integral of $x(t)$, which is shown in Figure 2.3a:

$$\int x(t) dt = x(t) = \frac{V_0}{\omega_v} \sin(\omega_v t + \phi_v) = \frac{V_0}{\omega_v} \sin \phi_v \cos \omega_v t + \frac{V_0}{\omega_v} \cos \phi_v \sin \omega_v t \quad (A4)$$
which can be rearranged as follows:

\[ x(t) = \frac{V_0}{2\omega_v} \sin \phi_v \cos \omega_v t + \frac{V_0}{2\omega_v} \sin \phi_v \cos(-\omega_v t) \]

\[ + \frac{V_0}{2\omega_v} \cos \phi_v \sin \omega_v t - \frac{V_0}{2\omega_v} \cos \phi_v \sin(-\omega_v t) \]

\[ = \left( \frac{V_{0L}}{\omega_v} \cos \omega_v t + \frac{V_{0R}}{\omega_v} \sin \omega_v t \right) + \left( \frac{V_{0L}}{\omega_v} \cos(-\omega_v t) - \frac{V_{0R}}{\omega_v} \sin(-\omega_v t) \right) \]  \hspace{1cm} (A5)

which corresponds to the Fourier Transform representation shown in Figure 2.3b and can be written as:

\[ \text{FT}[x(t)] = \left[ \frac{V_{0L}}{\omega_v} - j \frac{V_{0R}}{\omega_v} \right]_{\omega = \omega_v} + \left[ \frac{V_{0L}}{\omega_v} + j \frac{V_{0R}}{\omega_v} \right]_{\omega = -\omega_v} \] \hspace{1cm} (A6)

Comparison of equations (A2) and (A6) leads to the following relationship between the Fourier Transform of this sinusoidal component and its integral:

\[ \frac{1}{j\omega} \text{FT}[\dot{x}(t)] = \left[ \frac{j}{\omega_v} \frac{V_{0R}}{j} + \frac{j}{\omega_v} \frac{V_{0L}}{j} \right]_{\omega = \omega_v} + \left[ \frac{j}{\omega_v} \frac{V_{0R}}{j} \left( -\frac{V_{0L}}{\omega_v} \right) - \frac{j}{\omega_v} \frac{V_{0L}}{j} \right]_{\omega = -\omega_v} = \text{FT}[x(t)] \] \hspace{1cm} (A7)

In the general case, the real and imaginary components of the Fourier Transform are continuous functions of (angular) frequency, \( \omega \), with positive and negative frequency components that are complex conjugates, and the relationship described by equation (A7) holds at all frequencies.
APPENDIX B: SCANNING SYSTEM LASER BEAM ORIENTATION DEFINITION

In this appendix, the vector mathematics developed to describe laser beam orientation in terms of mirror scan angle(s) will be described in detail. Firstly it is necessary to derive a vector equation that can be used to determine the orientation after the reflection of an arbitrarily orientated beam incident on an arbitrarily orientated reflective interface.

B.1. LASER BEAM REFLECTION AT ANY INTERFACE

According to the law of reflection [B.1], the angle of reflection, $\theta_r$, of a plane wavefront from an interface is equal to its angle of incidence, $\theta_i$. Making use of this, it is clear that, with reference to Figure 4.7a, the following general vector sums describing the incident and reflected beam orientations, $\hat{b}_i$ and $\hat{b}_r$, respectively, and the normal to the reflective interface, $\hat{u}_n$, can be written:

\[ \hat{b}_i + \hat{b}_r - 2\hat{a} = 0 \]  \hspace{1cm} (B1a)

and

\[ \hat{b}_i - (\hat{b}_i \cdot \hat{u}_n)\hat{u}_n - \hat{a} = 0 \]  \hspace{1cm} (B1b)

Eliminating the vector, $\hat{a}$, yields the following totally general result:

\[ \hat{b}_r = \hat{b}_i - 2(\hat{b}_i \cdot \hat{u}_n)\hat{u}_n \]  \hspace{1cm} (B2)

This important expression can be employed to enable calculation of the laser beam orientation in terms of mirror scan angle(s) in any scanning system. Typically, the initial
APPENDIX B: SCANNING SYSTEM LASER BEAM ORIENTATION DEFINITION

beam and mirror surface orientations are known such that successive application of equation (B2) for each mirror reflection will lead to the final beam orientation. In the following two sections, the dual mirror scanning system and the dual axis, single mirror scanning system will be examined.

B.2. LASER BEAM ORIENTATION DEFINITION: DUAL MIRROR SCANNING SYSTEM

With reference to Figure 4.6 and Figure 4.7, the "zero" positions of the x and y deflection mirrors, which result in the final orientation of the laser beam being along the z axis, are both 45° (to the y direction). The mirror scan angles, \( \theta_{sx} \) and \( \theta_{sy} \), are defined as positive if anticlockwise about an axis in the z direction and the x axis, respectively. \( \theta_{sx} \) and \( \theta_{sy} \) are clearly related to the unit vectors \( \hat{u}_{nx} \) and \( \hat{u}_{ny} \) which are normal to the reflective surfaces of each mirror.

With reference to Figure 4.7, it is possible to express \( \hat{u}_{nx} \) in terms of \( \theta_{sx} \) and the principal unit vectors, \( \hat{x} \) and \( \hat{y} \) as follows:

\[
\hat{u}_{nx} = [\cos(45° + \theta_{sx})] \hat{x} + [\sin(45° + \theta_{sx})] \hat{y}
= \left[ \frac{\sqrt{2}}{2} \right] (\cos \theta_{sx} - \sin \theta_{sx}) \hat{x} + \left[ \frac{\sqrt{2}}{2} \right] (\cos \theta_{sx} + \sin \theta_{sx}) \hat{y} \quad (B3a)
\]

Similarly, \( \hat{u}_{ny} \) can be expressed in terms of \( \theta_{sy} \), \( \hat{y} \) and \( \hat{z} \):

\[
\hat{u}_{ny} = \left[ \frac{\sqrt{2}}{2} \right] (\cos \theta_{sy} - \sin \theta_{sy}) \hat{y} - \left[ \frac{\sqrt{2}}{2} \right] (\cos \theta_{sy} + \sin \theta_{sy}) \hat{z} \quad (B3b)
\]
Let \( \hat{b}_x \) be the direction of the laser beam before reflection at the \( x \) deflection mirror, \( \hat{b}_y \) be the direction of the laser beam before reflection at the \( y \) deflection mirror. The convention used is that the direction of the unit vectors is from the target to the Laser Vibrometer (along the beam path), as shown in Figure 4.7.

Figure 4.7a shows the view of the reflection at the \( x \) deflection mirror in the negative \( z \) direction, such that, making use of equation (B2):

\[
\hat{b}_y = \hat{b}_x - 2(\hat{b}_x \cdot \hat{u}_{nx})\hat{u}_{nx} = \hat{x} - 2(\hat{x} \cdot \hat{u}_{nx})\hat{u}_{nx} \quad (B4)
\]

since in this configuration \( \hat{b}_x = \hat{x} \). Similarly, Figure 4.7b shows the view of the reflection at the \( y \) deflection mirror in the negative \( x \) direction, illustrating that:

\[
\hat{b} = \hat{b}_y - 2(\hat{b}_y \cdot \hat{u}_{ny})\hat{u}_{ny} = \hat{x} - 2(\hat{x} \cdot \hat{u}_{nx})\hat{u}_{nx} - 2(\hat{x} - 2(\hat{x} \cdot \hat{u}_{nx})\hat{u}_{nx}) \cdot \hat{u}_{ny} \hat{u}_{ny} \quad (B5)
\]

Since, as can be seen in equation (B3b), \( \hat{u}_{ny} \) is always perpendicular to \( \hat{x} \), equation (B5) can be re-written as:

\[
\hat{b} = \hat{x} - 2(\hat{x} \cdot \hat{u}_{nx})\hat{u}_{nx} + 4((\hat{x} \cdot \hat{u}_{nx}) \hat{u}_{nx}) \cdot \hat{u}_{ny} \hat{u}_{ny} \quad (B6)
\]

Making use of the fact that (from equations (B3a&b)):

\[
(\hat{x} \cdot \hat{u}_{nx})\hat{u}_{nx} = \frac{1}{2}((1 - \sin 2\theta_x)\hat{x} + (\cos 2\theta_x)\hat{y}) \quad (B7a)
\]

such that:

\[
((\hat{x} \cdot \hat{u}_{nx})\hat{u}_{nx}) \cdot \hat{u}_{ny} = -\frac{\sqrt{2}}{4} \cos 2\theta_x (\cos \theta_y - \sin \theta_y) \quad (B7b)
\]
equation (B6) can be re-written as:

\[ \hat{b} = [\sin 2\theta_{sx}] \hat{x} - [\cos 2\theta_{sx} \sin 2\theta_{sy}] \hat{y} + [\cos 2\theta_{sx} \cos 2\theta_{sy}] \hat{z} \]

(B8a)

Small angle approximations make the visualisation of the beam orientation as a function of mirror scan angles more straightforward:

\[ \hat{b} = 2\theta_{sx} \hat{x} - 2\theta_{sy} \hat{y} + \hat{z} \]

(B8b)

but these approximations were not taken during the analyses discussed in this thesis.

**B.3. LASER BEAM ORIENTATION DEFINITION: SINGLE MIRROR SCANNING SYSTEM, CONFIGURATION 1**

With reference to Figure 7.1, in the first single mirror configuration the laser beam orientation unit vector, \( \hat{u}_n \), can be written in terms of the deflection mirror scan angles, \( \theta_{s1} \) and \( \theta_{s2} \), and the local unit vectors, \( \hat{x}' \), \( \hat{y}' \) and \( \hat{z}' \) as follows:

\[ \hat{u}_n = [-[\sin \theta_{s1}] \hat{x}' + [\cos \theta_{s1} \sin \theta_{s2}] \hat{y}' - [\cos \theta_{s1} \cos \theta_{s2}] \hat{z}'] \]

(B9a)

where \( \hat{x}' = \hat{x} \), \( \hat{y}' = \frac{\sqrt{2}}{2} \hat{y} - \frac{\sqrt{2}}{2} \hat{z} \) and \( \hat{z}' = \frac{\sqrt{2}}{2} \hat{y} + \frac{\sqrt{2}}{2} \hat{z} \) such that this can be rearranged in terms of the principal unit vectors:

\[ \hat{u}_n = [-[\sin \theta_{s1}] \hat{x} + \left[ \frac{\sqrt{2}}{2} \cos \theta_{s1} (\sin \theta_{s2} - \cos \theta_{s2}) \right] \hat{y} - \left[ \frac{\sqrt{2}}{2} \cos \theta_{s1} (\sin \theta_{s2} + \cos \theta_{s2}) \right] \hat{z} \]

(B9b)

With reference to equation (B2), the beam orientation after reflection at the mirror surface is given by:
\[ \hat{b} = \hat{b} - 2[\hat{b} \cdot \hat{u}_n] \hat{u}_n = -\hat{y} + 2[\hat{y} \cdot \hat{u}_n] \hat{u}_n \]  

since the initial beam orientation is given by \( \hat{b} = -\hat{y} \). Substituting in for \( \hat{u}_n \) from equation (B9b) and rearranging results in a general expression describing the incident laser beam orientation which is equivalent to equation (B8):

\[ \hat{b} = \left[ \frac{\sqrt{2}}{2} \sin 2\theta_{s1} (\sin \theta_{s2} - \cos \theta_{s2}) \right] \hat{x} - [1 - \cos^2 \theta_{s1} (1 - \sin 2\theta_{s2})] \hat{y} + [\cos^2 \theta_{s1} \cos 2\theta_{s2}] \hat{z} \]  

Again, small angle approximations make visualisation more straightforward:

\[ \hat{b} = \sqrt{2}\theta_{s1} \hat{x} - 2\theta_{s2} \hat{y} + \hat{z} \]

but these approximations were not taken during the analyses discussed in this thesis.

**B.4. LASER BEAM ORIENTATION DEFINITION: SINGLE MIRROR SCANING SYSTEM, CONFIGURATION 2**

With reference to Figure 7.5, in the second single mirror configuration the laser beam orientation unit vector, \( \hat{u}_n \), can be more straightforwardly directly written in terms of the deflection mirror scan angles, \( \theta_{s1} \) and \( \theta_{s2} \), and the principal unit vectors:

\[ \hat{u}_n = -[\sin \theta_{s1}] \hat{x} + [\cos \theta_{s1} \sin \theta_{s2}] \hat{y} - [\cos \theta_{s1} \cos \theta_{s2}] \hat{z} \]

Making use of equation (B2) once again, the incident laser beam orientation can be written as:

\[ \hat{b} = \hat{b} - 2[\hat{b} \cdot \hat{u}_n] \hat{u}_n = -\hat{z} + 2[\hat{z} \cdot \hat{u}_n] \hat{u}_n \]
since in this case the initial beam orientation is given by $\hat{b}_i = -\hat{z}$. Substituting in for $\hat{u}_n$ from equation (B12) and rearranging as before yields:

$$\hat{b} = [\sin 2\theta_{s_1} \cos \theta_{s_2}]\hat{x} - [\cos^2 \theta_{s_1} \sin 2\theta_{s_1}]\hat{y} + [2 \cos^2 \theta_{s_1} \cos^2 \theta_{s_2} - 1]\hat{z} \quad (B14a)$$

and after small angle approximations:

$$\hat{b} = 2\theta_{s_1}\hat{x} - 2\theta_{s_2}\hat{y} + \hat{z} \quad (B14b)$$
Figure 6.13 illustrates the tracking system that was developed specifically for use in this research project. The system is based on the custom-built scanning system described in section 5.2 but makes use of a bespoke arbitrary function generator that is synchronised to the target rotation, rather than a standard, variable frequency, dual channel function generator, to drive the $x$ and $y$ mirror galvos.

The dedicated tracking controller makes use of the output from a 500 lines/rev optical encoder that is mounted to the target shaft, thereby providing a real-time measurement of rotation angle, to synchronise the generation of the pre-defined $x$ and $y$ mirror scan angle waveforms. The $x$ and $y$ mirror scan angle waveforms, which contain 500 data points – one data point per encoder line, are pre-defined in software on the host PC before being uploaded to the controller. Each waveform is stored on a low-cost 20MHz processor and the value corresponding to a particular rotation angle is fed into a similar 20MHz processor which converts the digital number into an equivalent output voltage. The two output voltages are directly connected to the $x$ and $y$ galvos and the mirrors subsequently rotated through the necessary angle.

Modulation of the amplitude and phase of the output voltages during tracking is possible via the host PC software interface and enables the interrogated point to be adjusted sequentially. In section 7.3, the Synchronised-Scanning technique is described in which the scan amplitude and/or frequency are/is continuously modulated during tracking to perform a synchronised scan across a region of interest on the rotating structure. Such a scanning regime is arranged for by uploading a pair of complex waveforms as illustrated in Figure 7.9a and Figure 7.12a into the tracking controller.
APPENDIX D: PUBLICATIONS RESULTING FROM THIS RESEARCH PROJECT


REFERENCES

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*Laser Techniques, Ancona, Italy* 2358, 398-408. A laser-based measurement system for measuring the vibration on rotating discs.


CHAPTER 2


2.6. ISO 2954:1975 Mechanical vibration of rotating and reciprocating machinery — requirements for instruments for measuring vibration severity.


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CHAPTER 4


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REFERENCES


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APPENDIX B

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<table>
<thead>
<tr>
<th>Simulation, fig &amp; section nos.</th>
<th>Offsets</th>
<th>Noise</th>
<th>Speed error</th>
<th>Torsional oscillation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - Figs 3.1-3.6 (Section 3.1.1)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Single frequency vibrations; cross-sensitivity term eliminated and genuine component frequency, amplitude and phase exact after resolution - resolution algorithm performance confirmed</td>
</tr>
<tr>
<td>2 - Figs 3.7-3.9 (Section 3.1.1)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Dual frequency vibrations; cross-sensitivity term eliminated and genuine component frequency, amplitude and phase exact after resolution - resolution algorithm performance confirmed</td>
</tr>
<tr>
<td>3 - Figs 3.10-3.12 (Section 3.1.1)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Broadband frequency vibrations; cross-sensitivity term eliminated and genuine component frequency, amplitude and phase exact after resolution - resolution algorithm performance confirmed, synchronous component eliminated</td>
</tr>
<tr>
<td>4 - Figs 3.13 &amp; 3.14 (Section 3.1.1)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>0.015mm/s(rad/s) random, 0.1mm/s(rad/s) pseudo-random</td>
<td>N/A</td>
<td>N/A</td>
<td>Single frequency vibrations; very small genuine component amplitude and phase exact after resolution - resolution algorithm performance confirmed</td>
</tr>
<tr>
<td>5 - Fig 3.15 (Section 3.1.1)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>0.015mm/s(rad/s) random, 0.1mm/s(rad/s) pseudo-random</td>
<td>-2%</td>
<td>N/A</td>
<td>Single frequency vibrations; small genuine component amplitude and phase errors (10% &amp; 10mrad) after resolution, noise amplification close to synchronous - resolution algorithm performance good</td>
</tr>
<tr>
<td>6 - Figs 3.16-3.18 (Section 3.1.2)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>[\Delta \Omega = \frac{1}{0.67 \times 10^{-1}} ]</td>
<td>Single frequency vibrations; genuine component amplitude and phase errors and oscillation-displacement error eliminated after 3 correction algorithm iterations - correction algorithm performance confirmed</td>
</tr>
<tr>
<td>7 - Figs 3.19 &amp; 3.20 (Section 3.1.2)</td>
<td>N/A</td>
<td>0.02mm/s(rad/s) random</td>
<td>-2%</td>
<td>[\Delta \Omega = \frac{1}{0.67 \times 10^{-1}} ]</td>
<td>Single frequency vibrations; noise larger than oscillation-displacement error, small genuine component amplitude and phase errors (10% &amp; 10mrad) after correction, oscillation-displacement error amplification and noise amplification close to synchronous only - no loss or gain from correction algorithm</td>
</tr>
<tr>
<td>8 - Figs 3.21 &amp; 3.22 (Section 3.1.2)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>N/A</td>
<td>N/A</td>
<td>[\Delta \Omega = \frac{1}{0.67 \times 10^{-1}} ]</td>
<td>Single frequency vibrations; oscillation-offset error larger than oscillation-displacement error, genuine component amplitude and phase errors (1% &amp; 100mrad) after correction, error amplification close to synchronous only - no loss or gain from correction algorithm</td>
</tr>
<tr>
<td>9 - Figs 3.23 &amp; 3.24 (Section 3.1.2)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>0.02mm/s(rad/s) random</td>
<td>-2%</td>
<td>[\Delta \Omega = \frac{1}{0.67 \times 10^{-1}} ]</td>
<td>Single frequency vibrations; noise and oscillation-offset error larger than oscillation-displacement error, genuine component amplitude and phase errors (10% &amp; 10mrad) after correction, error amplification close to synchronous only - no loss or gain from correction algorithm</td>
</tr>
<tr>
<td>10 - Figs 3.25-3.27 (Section 3.1.3)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>0.02mm/s(rad/s) random</td>
<td>-2%</td>
<td>[\Delta \Omega = \frac{1}{2.33 \times 10^{-1}} ]</td>
<td>Single frequency vibrations, engine-type torsional oscillation; genuine component amplitude and phase errors (10% &amp; 1mrad) after correction, error amplification close to synchronous only - no loss or gain from correction algorithm</td>
</tr>
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<td>11 - Fig 3.28 (Section 3.1.4.1)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Single frequency vibrations; small cross-sensitivity terms negligible</td>
</tr>
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<td>12 - Figs 3.30 &amp; 3.31 (Section 3.1.4.2)</td>
<td>Arbitrary; (x=0.25\text{mm}, y=0.5\text{mm})</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Single frequency vibrations; additional terms low level, errors more significant if frequencies coincide</td>
</tr>
</tbody>
</table>

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Power amplifier
DC power supply
Electrodynamic shaker
Accelerometer
Charge amplifier
LDV
Linear bearing
PC Based Data Acquisition System

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Figure 2.2 - (a) time domain and (b) Fourier Transform representation of a single sinusoidal velocity component

\[
\dot{x} = 20 \cos(50\pi t + 1) \text{mm/s}
\]
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\[ x = \frac{2}{5\pi} \cos(50\pi + 1) \text{mm} \]
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\[ \dot{x} = 20 \cos(0.5\Omega t + 0.5\pi) \text{mm/s} \]
Figure 3.2 – Simulated $y$ vibration velocity in the (a) time and (b) frequency domain

\[
\dot{y} = 10 \cos(1.5 \Omega t) \text{ mm/s}
\]
Figure 3.3 – Simulated AC coupled $x$ direction Laser Vibrometer output in the (a) time and (b) frequency domain

\[
\begin{align*}
\dot{x} &= 20 \cos(0.5 \Omega_r t + 0.5 \pi) \text{ mm/s} \\
\dot{y} &= 10 \cos(1.5 \Omega_r t) \text{ mm/s} \\
\overline{\Omega_r} &= 100 \pi \text{ rad/s} \\
x_0 &= 0.25 \text{ mm} \\
y_0 &= 0.5 \text{ mm}
\end{align*}
\]
Figure 3.4 – Simulated AC coupled y direction Laser Vibrometer output in the (a) time and (b) frequency domain

\[ \dot{x} = 20 \cos(0.5\Omega_r t + 0.5\pi) \text{ mm/s} \]
\[ \dot{y} = 10 \cos(1.5\Omega_r t) \text{ mm/s} \]
\[ \Omega_r = 100\pi \text{ rad/s} \]
\[ x_0 = 0.25 \text{ mm} \]
\[ y_0 = 0.5 \text{ mm} \]
Figure 3.5 – Resolved x direction Laser Vibrometer output in the (a) frequency and (b) reconstructed time domain
Figure 3.6 – Resolved y direction Laser Vibrometer output in the (a) frequency and (b) reconstructed time domain
Figure 3.7 – Simulated vibration velocity in the (a) time and (b) frequency domain (dual frequency)

\[ \ddot{x} = 20(\cos(0.5\Omega t + 0.5\pi) + \cos(1.5\Omega t + 0.5\pi)) \text{ mm/s} \]
Figure 3.8 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain (dual frequency)

\[ \dot{x} = 20(\cos(0.5\Omega_r t + 0.5\pi) + \cos(1.5\Omega_r t + 0.5\pi)) \text{mm/s} \]
\[ \dot{y} = 10(\cos(0.5\Omega_r t) + \cos(1.5\Omega_r t)) \text{mm/s} \]
\[ \Omega_r = 100\pi \text{ rad/s} \]
\[ x_0 = 0.25 \text{mm} \]
\[ y_0 = 0.5 \text{mm} \]
Figure 3.9 - Resolved Laser Vibrometer output in the (a) frequency and (b) reconstructed time domain (dual frequency)
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\[ \dot{x} = 2.2 \sum_{m=0}^{40} \cos \left( 0.5 + \frac{m}{40} \Omega t + \phi_m \right) \text{mm/s} \]
Figure 3.11 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain (broadband frequency)

\[ \dot{x} = 2.2 \sum_{m=0}^{40} \cos \left( 0.5 + \frac{m}{40} \right) \Omega_T t + \phi_m \text{ mm/s} \]
\[ \dot{y} = 1.1 \sum_{n=0}^{40} \cos \left( 0.5 + \frac{n}{40} \right) \Omega_T t + \phi_n \text{ mm/s} \]

\[ \Omega_T = 100\pi \text{ rad/s} \]
\[ x_0 = 0.25 \text{ mm} \]
\[ y_0 = 0.5 \text{ mm} \]
Figure 3.12 – Resolved Laser Vibrometer output in the (a) frequency and (b) reconstructed time domain (broadband frequency)
Figure 3.13 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain in the presence of noise

\[ \dot{x} = 20 \cos(0.5 \Omega_r t + 0.5 \pi) \text{ mm/s} \]
\[ \dot{y} = 10 \cos(1.5 \Omega_r t) \text{ mm/s} \]
\[ \Omega_r = 100 \pi \text{ rad/s} \]
\[ x_0 = 0.25 \text{ mm} \]
\[ y_0 = 0.5 \text{ mm} \]
Figure 3.14 – Resolved Laser Vibrometer output in the presence of noise; spectral line at synchronous eliminated
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Figure 3.16 – Simulated rotational angular velocity measurement in the (a) time and (b) frequency domain in the presence of broadband torsional oscillation

\[
\Omega_r = 100\pi \text{ rad/s} \quad \Delta\Omega_r = 0.67 \times 10^{-3} \Omega_r \text{ RMS (arbitrary phase)}
\]
Figure 3.17 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain in the presence of broadband torsional oscillation

\[ \dot{x} = 20 \cos(0.5\Omega_r t + 0.5\pi) \text{ mm/s} \quad \dot{y} = 10 \cos(1.5\Omega_r t) \text{ mm/s} \]

\[ \Omega_r = 100\pi \text{ rad/s} \quad \Delta\Omega_r = 0.67 \times 10^{-3} \Omega_r \text{ RMS (arbitrary phase)} \quad x_0 = y_0 = 0 \text{ mm} \]
Figure 3.18 – (a) first and (b) second estimate of the vibration velocity in the presence of broadband torsional oscillation; spectral lines at synchronous ±10% eliminated
Figure 3.18 (cont.) – (c) fourth estimate of the vibration velocity in the presence of broadband torsional oscillation; spectral lines at synchronous ±10% eliminated
Figure 3.19 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain in the presence of broadband torsional oscillation and noise

\[ \dot{x} = 20 \cos(0.5 \Omega_r t + 0.5 \pi) \text{mm/s} \]
\[ \dot{y} = 10 \cos(1.5 \Omega_r t) \text{mm/s} \]

\[ \Omega_r = 100 \pi \text{ rad/s} \]
\[ \Delta \Omega_r = 0.67 \times 10^{-3} \Omega_r \text{ RMS (arbitrary phase)} \]
\[ x_0 = y_0 = 0 \text{ mm} \]
Figure 3.20 – (a) first and (b) fourth estimates of the vibration velocity in the presence of broadband torsional oscillation, noise and a speed measurement error; spectral lines at synchronous ±10% eliminated
Figure 3.21 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain in the presence of broadband torsional oscillation and offsets

\[ \dot{x} = 20\cos(0.5\Omega_r t + 0.5\pi) \text{ mm/s} \quad \dot{y} = 10\cos(1.5\Omega_r t) \text{ mm/s} \]

\[ \Omega_r = 100\pi \text{ rad/s} \Delta \Omega_r = 0.67 \times 10^{-3} \Omega_r \text{ RMS (arbitrary phase)} \quad x_0 = 0.25\text{mm} \quad y_0 = 0.5\text{mm} \]
Figure 3.22 – (a) first and (b) fourth estimates of the vibration velocity in the presence of broadband torsional oscillation and offsets; spectral lines at synchronous ±10% eliminated
Figure 3.23 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain in the presence of broadband torsional oscillation, noise and offsets

\[ \dot{x} = 20 \cos \left( 0.5 \overline{\Omega}_r t + 0.5 \pi \right) \text{mm/s} \quad \dot{y} = 10 \cos \left( 1.5 \overline{\Omega}_r t \right) \text{mm/s} \]

\[ \overline{\Omega}_r = 100 \pi \text{ rad/s} \quad \Delta \overline{\Omega}_r = 0.67 \times 10^{-3} \overline{\Omega}_r \]

RMS (arbitrary phase) \( x_0 = 0.25 \text{mm} \quad y_0 = 0.5 \text{mm} \)
Figure 3.24 – (a) first and (b) fourth estimates of the vibration velocity in the presence of broadband torsional oscillation, noise, offsets and a speed measurement error; spectral lines at synchronous ±10% eliminated
Figure 3.25 – Simulated rotational angular velocity measurement in the (a) time and (b) frequency domain in the presence of engine-type torsional oscillation, noise and a speed measurement error

\[ \Omega_r = 100\pi \text{ rad/s} \Delta\Omega_r = 2.23 \times 10^{-3} \Omega_r \text{ } @ \frac{\sqrt{2}}{2} x, \text{ 4.46} \times 10^{-3} \Omega_r \text{ } @ 2x \text{ (arbitrary phase)} \]
Figure 3.26 – Simulated AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain in the presence of engine-type torsional oscillation, noise and offsets

\[ \dot{x} = 20 \cos(0.5\bar{\Omega}_{r}t + 0.5\pi) \text{mm/s} \]
\[ \dot{y} = 10 \cos(1.5\bar{\Omega}_{r}t) \text{mm/s} \]
\[ \bar{\Omega}_{r} = 100\pi \text{ rad/s} \]
\[ \Delta\bar{\Omega}_{r} = 2.23 \times 10^{-3} \bar{\Omega}_{r} \text{ @} \sqrt{2} x, 4.46 \times 10^{-3} \bar{\Omega}_{r} \text{ @} 2x \text{ (arbitrary phase)} \]
\[ x_0 = 0.25 \text{mm} \quad y_0 = 0.5 \text{mm} \]
Figure 3.27 – (a) first and (b) fourth estimates of the vibration velocity in the presence of engine-type torsional oscillation, noise, offsets and a speed measurement error; spectral lines at synchronous ±10% eliminated.
Figure 3.28 – (a) simulated AC coupled Laser Vibrometer output and (b) resolved Laser Vibrometer output in the presence of medium/high severity axial and pitch/yaw vibrations

\[
\begin{align*}
\dot{x} &= 20 \cos\left(0.5\Omega_r t + 0.5\pi\right) \text{mm/s} \\
\dot{y} &= 10 \cos\left(1.5\Omega_r t\right) \text{mm/s} \\
\dot{z} &= 5 \cos\left(3\Omega_r t + \pi\right) \text{mm/s} \\
\dot{\theta}_x &= 5 \cos\left(0.5\Omega_r t - 0.5\pi\right) \text{rad/s} \\
\dot{\theta}_y &= 2.5 \cos\left(1.5\Omega_r t + \pi\right) \text{rad/s} \\
\Omega_r &= 100\pi \text{ rad/s} \\
x_0 &= 0.25\text{mm} \\
y_0 &= 0.5\text{mm}
\end{align*}
\]
Figure 3.29 – Non-orthogonal radial vibration measurements
Figure 3.30 – Simulated AC coupled Laser Vibrometer output for a misaligned measurement in the (a) time and (b) frequency domain in the presence of medium/high severity axial and pitch/yaw vibrations

\[
\begin{align*}
\dot{x} &= 20 \cos \left(0.5\Omega_r t + 0.5\pi\right) \text{mm/s} \\
\dot{y} &= 10 \cos \left(1.5\Omega_r t\right) \text{mm/s} \\
\dot{z} &= 5 \cos \left(3\Omega_r t + \pi\right) \text{mm/s} \\
\dot{\theta}_x &= 5 \cos \left(0.5\Omega_r t - 0.5\pi\right) \text{rad/s} \\
\dot{\theta}_y &= 2.5 \cos \left(1.5\Omega_r t + \pi\right) \text{rad/s} \\
\Omega_r &= 100 \pi \text{ rad/s} \\
x_0 &= 0.25 \text{mm} \\
y_0 &= 0.5 \text{mm} \\
\alpha &= \delta = 5 \text{deg}
\end{align*}
\]
Figure 3.31 – Resolved Laser Vibrometer output for a misaligned measurement in the presence of (a) no noise or speed measurement error and (b) noise and a speed measurement error; spectral lines at synchronous ±10% eliminated
Figure 3.32 – Laboratory based experimental arrangement schematic diagram
Figure 3.33 – Laboratory based experimental arrangement
Figure 3.34 – Genuine vibration velocity in the (a) time and (b) frequency domain

\[ \ddot{x} = 20 \cos \left( 0.5\Omega_0 t + 0.5\pi \right) \text{mm/s (nominal)} \]
Figure 3.35 – AC coupled Laser Vibrometer output for a measurement in the (a) time and (b) frequency domain

\[
\dot{x} = 20 \cos(0.5\Omega_r t + 0.5\pi) \text{ mm/s} \quad \dot{y} = 10 \cos(1.5\Omega_r t) \text{ mm/s}
\]

\[\Omega_r = 100\pi \text{ rad/s} \quad x_0 = y_0 = \text{arbitrary}\]
Figure 3.36 – Resolved Laser Vibrometer output for a measurement in the (a) frequency and (b) reconstructed time domain; spectral lines at synchronous ±10% eliminated
Figure 3.37 – AC coupled Laser Vibrometer output for a measurement in the (a) time and (b) frequency domain in the presence of broadband torsional oscillation

\[ \dot{x} = 20 \cos(0.5\Omega_r t + 0.5\pi) \text{ mm/s} \quad \dot{y} = 10 \cos(1.5\Omega_r t) \text{ mm/s} \]

\[ \Omega_r = 100\pi \text{ rad/s} \Delta \Omega_r = 0.67 \times 10^{-3} \Omega_r \text{ RMS (arbitrary phase)} \]

\[ x_0 = y_0 = \text{arbitrary} \]
Figure 3.38 – (a) first and (b) fourth estimate of the vibration velocity for a measurement in the presence of broadband torsional oscillation; spectral lines at synchronous ±10% eliminated
Figure 3.39 – Engine crankshaft radial vibration measurements experimental arrangement
Figure 3.40 – AC coupled Laser Vibrometer output in the (a) time and (b) frequency domain for an engine radial vibration measurement
Figure 3.41 – Rotational angular velocity measurement in the (a) time and (b) frequency domain for an engine vibration measurement.
Figure 3.42 – Fourth estimate of the resolved vibration velocity for an engine radial vibration measurement; spectral lines at synchronous ±10% eliminated.
Figure 3.43 – Simultaneous orthogonal pitch/yaw vibration measurements
Figure 3.44 – (a) AC coupled Laser Vibrometer output and (b) fourth estimate of the vibration velocity for a real engine pitch/yaw vibration measurement; spectral lines at synchronous ±10% eliminated
Figure 4.1 – Definition of axes and the points $P$ and $P_0$ on a vibrating and rotating flexible shaft
Figure 4.2 – Straight-line scan in the $y$ direction using a rotating beam deflection mirror
Figure 4.3 - Straight-line scan in the $z$ direction using a rotating beam deflection mirror
Figure 4.4 – The idealised scanning arrangement
Figure 4.5 – The dual mirror scanning arrangement, laser beam orientation angles
Figure 4.6 - The dual mirror scanning arrangement, beam deflection mirror angles (nominally flat target surface)
Figure 4.7 – Laser beam reflections at (a) any interface, (b) the $x$ deflection mirror and (c) the $y$ deflection mirror.
Figure 4.8 – The dual mirror scanning arrangement, beam deflection mirror angles (arbitrarily shaped target surface)
Figure 4.9 – (a) normalised scan profile and (b) normalised scan radius vs. scan angle which result from equal amplitude cosine and sine mirror drive signals

\[ d_s = 50\text{mm} \quad z_0 = 1\text{m} \]
Figure 4.10 – Normalised scan radius vs. scan angle which results from corrected amplitude cosine and sine mirror drive signals

\[ d_s = 50\text{mm} \quad z_0 = 1\text{m} \]
Figure 4.11 – Additional measurement components that occur in a dual mirror circular scan; equal amplitude cosine and sine mirror drive signals

\[ r_s = 100\text{mm} \quad d_s = 50\text{mm} \quad z_0 = 1\text{m} \]
Figure 4.12 – Additional measurement components that occur in a dual mirror circular scan; corrected amplitude cosine and sine mirror drive signals

\[ r_s = 100\text{mm} \quad d_s = 50\text{mm} \quad z_0 = 1\text{m} \]
Figure 4.13 – Dual mirror corrected amplitude conical scanning Laser Vibrometer measurement on a target undergoing simultaneous vibration in the x, y and z directions
Figure 4.14 – (a) idealised and (b) dual mirror equal amplitude Laser Vibrometer measurements on a target undergoing simultaneous vibration in the x, y and z directions
Laser Vibrometer measurements on a target undergoing simultaneous vibration in the $x$, $y$ and $z$ directions.
Figure 5.1 – Translational and angular misalignment between the scanning system and target rotation axes
Figure 5.2 – Additional measurement components that occur in a misaligned dual mirror circular scan on a rotating target; corrected mirror drive signals

\[ r_S = 100\text{mm} \quad d_S = 50\text{mm} \quad z_0 = 1\text{m} \]
\[ \chi_0 = \gamma_0 = 2\text{mm} \quad \theta_{xm} = \theta_{ym} = 15\text{mrad} \]
Figure 5.3 – Experimental validation of the additional measurement components at 2x scan frequency; (a) equal amplitude and (b) corrected amplitude mirror drive signals

\[ \Omega_s = 40\pi \text{ rad/s} \]
Figure 5.4 – Experimental validation of the additional measurement component at (a) DC and (b) 1x scan frequency for varying $x_0$m; broken line = initial, solid line = updated prediction

$r_s = 15\text{mm} \quad d_s = 50\text{mm} \quad z_0 = 0.5\text{m} \quad \Omega_s = 40\pi \text{rad/s}$

$y_0m = 0\text{mm} \quad \theta_{xm} = \theta_{ym} = 0\text{mrad}$
Figure 5.5 – Experimental validation of the additional measurement component at (a) DC and (b) 1x scan frequency for varying \(y_{0m}\); broken line = initial, solid line = updated prediction

\(r_s = 15\text{mm} \quad d_s = 50\text{mm} \quad z_0 = 0.5\text{m} \quad \Omega_s = 40\pi\text{ rad/s} \quad x_{0m} = 0\text{mm} \quad \theta_{xm} = \theta_{ym} = 0\text{mrad}\)
Figure 5.6 – Initial unknown misalignment analysis using
(a) variable $x_{0m}$ and (b) variable $y_{0m}$
Figure 5.7 – Velocity measured by a circular scanning Laser Vibrometer on a rotating, non-vibrating target

\[ r_s = 50 \text{mm} \quad d_s = 50 \text{mm} \quad z_0 = 0.25 \text{m} \quad \Omega_s = 25\pi \text{ rad/s} \quad \Omega \approx 20\pi \text{ rad/s} \]

\[ x_{0m} = y_{0m} = \theta_{xm} = \theta_{ym} = \text{arbitrary} \]
Figure 5.8 – Speckle repeat map
Figure 5.9 – Speckle noise in circular scanning Laser Vibrometer measurements on rotating targets; (a) $0 \leq \Omega_S \leq 2\Omega$ and (b) $0.8\Omega \leq \Omega_S \leq 1.2\Omega$
Figure 6.1 – Commercial Laser Vibrometer experimental arrangement schematic diagram
Figure 6.2 – Ometron VH300 output from a measurement on a non-vibrating target rotating at
(a) $30\pi\text{rad/s (15Hz)}$ and (b) $120\pi\text{rad/s (60Hz)}$; retroreflective surface
Figure 6.3 – (a) periodic speckle and (b) non-periodic noise in Laser Vibrometer measurements on a non-vibrating target rotating at (nominally) 30π rad/s (15Hz); surface vs. speckle noise
Figure 6.4 – (a) periodic speckle and (b) non-periodic noise in Laser Vibrometer measurements on a non-vibrating target rotating at (nominally) $120\pi$rad/s (60Hz); surface vs. speckle noise
Figure 6.5 – Periodic speckle noise in Laser Vibrometer measurements on a non-vibrating target rotating at (nominally) (a) $30\pi$rad/s (15Hz) and (b) $120\pi$rad/s (60Hz); stand-off vs. speckle noise
Figure 6.6 – Resolved pitch vibration vs. loaded engine speed waterfall plot; crankshaft-water pump drive pulley face
Figure 6.7 – Resolved yaw vibration vs. loaded engine speed waterfall plot; crankshaft-water pump drive pulley face
Figure 6.8 – Resolved 2x and harmonics (a) pitch and (b) yaw vibration velocity vs. loaded engine speed; crankshaft-water pump drive pulley face
Figure 6.9 – Velocity measured by a Laser Vibrometer on a non-rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration
Figure 6.10 – Velocity measured by a dual mirror scanning Laser Vibrometer on a non-rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration

\[ r_s = 12.5 \text{mm} \quad d_s = 50 \text{mm} \quad z_0 = 1 \text{m} \quad \Omega_s = 20\pi \text{ rad/s} \]

\[ x_{0m} = y_{0m} = \theta_{xm} = \theta_{ym} = \text{arbitrary} \]
Figure 6.11 – Velocity measured by a dual mirror scanning Laser Vibrometer on a rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration

\[ r_s = 12.5 \text{mm} \quad d_s = 50 \text{mm} \quad z_0 = 1 \text{m} \quad \Omega_s = 20\pi \text{ rad/s} \quad \Omega \approx 40\pi \text{ rad/s} \]

\[ x_{om} = y_{om} = \theta_{xm} = \theta_{ym} = \text{arbitrary} \]
Figure 6.12 – Theoretical prediction of the velocity measured by a dual mirror scanning Laser Vibrometer on a rotating target undergoing 40Hz, 10mm/s axial vibration

\[ r_s = 12.5\text{mm} \quad d_s = 50\text{mm} \quad z_0 = 1\text{m} \quad \Omega_s = 20\pi\text{rad/s} \quad \Omega = 40\pi\text{rad/s} \]

\[ x_{0m} = -0.5\text{mm} \quad y_{0m} = -1\text{mm} \quad \theta_{2m} = -5\text{mrad} \quad \theta_{ym} = 10\text{mrad} \]
Figure 6.13 – Tracking system schematic diagram
Figure 6.14 – Velocity measured by a Laser Vibrometer on a non-rotating blade undergoing 32.5Hz, 100mm/s (nominal) vibration
Figure 6.15 – Velocity measured by a dual mirror tracking Laser Vibrometer on a (nominally) non-vibrating, rotating blade

\( r_s = 100\text{mm} \quad d_s = 50\text{mm} \quad z_0 = 1\text{m} \quad \Omega_s = \Omega \approx 20\pi \text{rad/s} \)

\( x_{0m} = y_{0m} = \theta_{xm} = \theta_{ym} = \text{arbitrary} \)
Figure 6.16 – Velocity measured by a dual mirror tracking Laser Vibrometer on a rotating blade undergoing 32.5Hz, 10mm/s (nominal) vibration

\[ r_s = 100\text{mm} \quad d_s = 50\text{mm} \quad z_0 = 1\text{m} \quad \omega = \omega \approx 20\pi \text{rad/s} \]

\[ x_{0m} = y_{0m} = \theta_x = \theta_y = \text{arbitrary} \]
Figure 6.17 – Theoretical prediction of the velocity measured by a dual mirror tracking Laser Vibrometer on a rotating blade undergoing 32.5Hz, 10mm/s (nominal) vibration

\[
\begin{align*}
  r_s &= 100\text{mm} \\
  d_s &= 50\text{mm} \\
  z_0 &= 1\text{m} \\
  \Omega_s &= \Omega \approx 20\pi \text{rad/s} \\
  x_{0m} &= -0.5\text{mm} \\
  y_{0m} &= -1\text{mm} \\
  \theta_{xm} &= -5\text{mrad} \\
  \theta_{ym} &= 0\text{mrad}
\end{align*}
\]
Figure 7.1 – The single mirror scanning arrangement, configuration 1
Figure 7.2 – (a) normalised scan profile and (b) normalised scan radius vs. scan angle which result from equal amplitude cosine and sine mirror drive signals

\[ d_s = 50\text{mm} \quad z_0 = 1\text{m} \]
Figure 7.3 – (a) normalised scan profile and (b) normalised scan radius vs. scan angle which result from corrected amplitude cosine and sine mirror drive signals

\[ d_s = 50\text{mm} \quad z_0 = 1\text{m} \]
Figure 7.4 – Normalised scan profile which results from a “straight-line” scan in (a) the $x$ direction and (b) the $y$ direction.

\[ d_s = 50 \text{mm} \quad z_0 = 1 \text{m} \]
Figure 7.5 – The single mirror scanning arrangement, configuration 2
Figure 7.6 – (a) normalised scan profile and (b) normalised scan radius vs. scan angle which result from equal amplitude cosine and sine mirror drive signals

\[ d_s = 50 \text{mm} \quad z_0 = 1 \text{m} \]
Figure 7.7 – (a) compressor blade and its (b) first cantilever bending and (c) first torsional mode shapes
Figure 7.8 – Bladed disc showing intended scan profile along the length of one of the blades
Figure 7.9 – (a) normalised mirror scan angles and (b) scan profile in space for a synchronised line scan

\[ \Delta r_s = 0.667 r_s \quad m = 3 \]
Figure 7.9 (cont.) – (c) actual experimental profile for a synchronised line scan

\[ \Delta r_s = 0.667 \bar{r}_s \quad m = 3 \]
Figure 7.10 – Simulated (a) tracking and (b) synchronised line scan measurement on a rotating blade undergoing 32.5Hz, 100mm/s (at tip) first cantilever bending mode vibration

\[ \Delta r_s = 0.667 r_s \quad m = 3 \]
Figure 7.11 – Bladed disc showing intended scan profile across the area of one of the blades
Figure 7.12 – (a) normalised mirror scan angles and (b) scan profile in space for a synchronised area scan

\[ \Delta r_s = 0.667 r_s \quad m = 3 \quad n = 9 \]
Figure 7.12 (cont.) – (c) actual experimental profile for a synchronised area scan

\[ \Delta r_s = 0.667 r_s \quad m = 3 \quad n = 9 \]