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Tribo-Dynamics of High Speed Precision Spindle Bearings

by

Bindu Kumar Karthikeyan

Thesis submitted in partial fulfilment of the requirements for the Degree of Doctor of Philosophy

Wolfson School of Mechanical and Manufacturing Engineering

Loughborough University

September 2008
This Thesis is dedicated to My Family
Abstract

The demand for higher productivity and improved quality of the machined surfaces requires spindle designers to aim for higher spindle speeds and feed rates. For instance, in routing of non-ferrous metals, wood and plastics, spindle speeds of the order of 60,000 rpm have been achieved with the help of better assembly aiming for higher operational accuracy, and increased component quality. The limitations on attainable speeds and quality of surface finish are partly governed by rolling element bearing vibrations. Dynamic performance of a rolling bearing has been of considerable interest. Although fatigue life is sometimes a significant parameter, it has been fairly well established that in the case of many practical applications, bearing failure is caused by dynamic instability in the motion of the rolling elements. Ball bearings are widely used for high speed spindles, where high speeds, high temperatures, and heavy load/forces are encountered, because they offer low friction, appropriate stiffness characteristics and minimum starting torque.

Dynamic stiffness of a bearing is very important characteristic in bearing design for achieving precise tolerances, which has been the concern of industry, especially wood machining industry. Due to poor machining the energy consumed in manufacturing better quality product is very high. Higher energy consumption and the time to carry out extra finishing processes is a major concern.

This thesis provides a numerical model incorporating inertial dynamics of vertical routing spindle with five degrees of freedom simulating an existing routing spindle, which can run at speeds up to 60,000 rpm (i.e. 2.4 million DN).

An experimental rig is also devised to investigate the vibration spectra the high speed precision 7.5 kW power routing spindle. The vibrations generated in the high speed precision spindles have amplitudes in the order of microns. Fine measurement of spindle vibration characteristics are carried out using laser vibrometry. Use of this technique is quite novel for high speed applications, with precise resolution of rigid body motions with fine alignment of a single laser beam with respect to an optically smooth surface of a specially designed tool held in the spindle collet. The experimental spectra are compared with the numerical model predictions with very good agreement.

Numerical model for grease lubrication capable of solving thermal elastohydrodynamic lubrication of rough surfaces with combined entraining and squeeze motions is developed. For this a modified Reynolds equation is derived from basic principles considering grease as a Bingham solid using the Herschel Bulkley flow model. Heat generation and the power loss in contact conjunction due to viscous shear and compressive action is quantified by solving the energy equation. Boundary interactions due to adhesive and ploughing friction are taken into account. The power lost due to friction and viscous shear was found to account for 16% of the total input power at the speed of 20,000 rpm.

Keywords: Spindle dynamics, bearing-induced vibration, Grease lubrication of rough concentrated contacts, Thermal Elastohydrodynamics
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Nomenclature

\[ A \] - Diametral clearance \( m \)

\[ A' \] - Single circumferential waviness

\[ a \] - Half width of semi-major axis of contact ellipse in transverse direction \( X \) \( m \)

\[ a_b \] - Waviness amplitude on a ball \( m \)

\[ a_i \] - Waviness amplitude on the inner race \( m \)

\[ a_o \] - Waviness amplitude on the outer race \( m \)

\[ B \] - Constant, Einstein equation for a suspension of non-interacting spheres

\[ b \] - Half width of semi-major axis of contact ellipse in the entraining direction \( X \) \( m \)

\[ D \] - Ball diameter \( m \)

\[ D_p \] - Pitch circle diameter \( m \)

\[ d_e \] - Pitch circle diameter \( m \)

\[ E \] - Modulus of elasticity \( Pa \)

\[ e \] - Shaft centre eccentricity \( m \)

\[ e' \] - Curvature sum \( m \)

\[ F_p \] - Applied preload force \( N \)

\[ f_b \] - Ball frequency \( Hz \)

\[ f_{bi} \] - Ball pass frequency inner race \( Hz \)

\[ f_{bo} \] - Ball pass frequency outer race \( Hz \)

\[ f_c \] - Cage frequency \( Hz \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_D$</td>
<td>Doppler frequency shift</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Shaft frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
<td>m/s²</td>
</tr>
<tr>
<td>$h$</td>
<td>Film thickness</td>
<td>m</td>
</tr>
<tr>
<td>$h_i$</td>
<td>Initial film thickness</td>
<td>m</td>
</tr>
<tr>
<td>$h_p$</td>
<td>Plug flow thickness</td>
<td>m</td>
</tr>
<tr>
<td>$K$</td>
<td>Effective stiffness</td>
<td>N/m²</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Stiffness due to material properties</td>
<td>N/m²</td>
</tr>
<tr>
<td>$K_{i}$</td>
<td>Stiffness inner race</td>
<td>N/m²</td>
</tr>
<tr>
<td>$K_{o}$</td>
<td>Stiffness outer race</td>
<td>N/m²</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity of lubricant</td>
<td>W/m.K</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass of spindle</td>
<td>kg</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of waves on a ball</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>Number of balls in a bearing</td>
<td></td>
</tr>
<tr>
<td>$N_s$</td>
<td>Shaft rotational speed</td>
<td>m/s</td>
</tr>
<tr>
<td>$n$</td>
<td>Herschel Bulkley index</td>
<td></td>
</tr>
<tr>
<td>$n_i$</td>
<td>Number of waves on the inner race</td>
<td></td>
</tr>
<tr>
<td>$n_o$</td>
<td>Number of waves on the outer race</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>$P_h$</td>
<td>Maximum Hertzian pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Pre load</td>
<td>N</td>
</tr>
<tr>
<td>$q_x$</td>
<td>Volume flow rates in the $X$ direction</td>
<td>m³/ms</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------------------</td>
<td>--------</td>
</tr>
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<td>$q_y$</td>
<td>Volume flow rates in the $Y$ direction</td>
<td>$m^3/ms$</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of curvature</td>
<td>$m$</td>
</tr>
<tr>
<td>$R_x$</td>
<td>Effective radii in the $X$ direction</td>
<td>$m$</td>
</tr>
<tr>
<td>$R_y$</td>
<td>Effective radii in the $Y$ direction</td>
<td>$m$</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Inner race radius</td>
<td>$m$</td>
</tr>
<tr>
<td>$r_o$</td>
<td>Outer race radius</td>
<td>$m$</td>
</tr>
<tr>
<td>$s$</td>
<td>Separation due to geometry of solids</td>
<td>$m$</td>
</tr>
<tr>
<td>$\dot{s}$</td>
<td>Shear rate</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>$s$</td>
</tr>
<tr>
<td>$U$</td>
<td>Internal energy of the lubricant</td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>Fluid velocity in the $X$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$u_{av}$</td>
<td>Mean velocity in the $X$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$u_b$</td>
<td>Rolling body velocity in the $X$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$u_p$</td>
<td>Plug flow velocity in the $X$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$v$</td>
<td>Fluid velocity in the $Y$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$v_{av}$</td>
<td>Mean velocity in the $Y$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$v_b$</td>
<td>Rolling body velocity in the $Y$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$v_p$</td>
<td>Plug flow velocity in the $Y$ direction</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$W$</td>
<td>Applied load</td>
<td>$N$</td>
</tr>
<tr>
<td>$X,Y,Z$</td>
<td>Cartesian fixed global frame of reference</td>
<td></td>
</tr>
<tr>
<td>$x,y,z$</td>
<td>Radial and axial displacements of the rotor centre</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Contact angle</td>
<td>$rad$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Laser incident angle (for chapter 4 only)</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>Preloaded contact angle</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>Free contact angle</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Temperature viscosity coefficient</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Laser incident angle (for chapter 4 only)</td>
<td></td>
</tr>
<tr>
<td>$\beta_n$</td>
<td>Constant used in Eyring equation</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Laser incident angle (for chapter 4 only)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Radial spacing between balls</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Deflection</td>
<td>$m$</td>
</tr>
<tr>
<td>$\delta_L$</td>
<td>Doppler shift frequency</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>Viscosity</td>
<td>$Pa.s$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Instantaneous radial position of the $i^{th}$ ball</td>
<td></td>
</tr>
<tr>
<td>$\lambda_b$</td>
<td>Wavelength of waves on a ball</td>
<td>$m$</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Wavelength of waves on the inner race</td>
<td>$m$</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Laser wavelength</td>
<td>$m$</td>
</tr>
<tr>
<td>$\lambda_o$</td>
<td>Wavelength of waves on the outer race</td>
<td>$m$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Lubricant density</td>
<td>$kg/m^3$</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Interference fit</td>
<td>$m$</td>
</tr>
<tr>
<td>$\sigma_{rms}$</td>
<td>Average surface roughness</td>
<td>$m$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Volume fraction of soap in base oil</td>
<td>$%$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress</td>
<td>$N/m^2$</td>
</tr>
<tr>
<td>$\tau_o$</td>
<td>Yield shear stress</td>
<td>$N/m^2$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Plastic viscosity</td>
<td>$Pa.s$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------</td>
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</tr>
<tr>
<td>$\phi_b$</td>
<td>Number of waves on ball</td>
<td></td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Turning angle of cage</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Number of waves on inner race</td>
<td></td>
</tr>
<tr>
<td>$\phi_o$</td>
<td>Number of waves on outer race</td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>Angular velocity</td>
<td>$rad/s$</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>Angular velocity ball</td>
<td>$rad/s$</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Angular velocity cage</td>
<td>$rad/s$</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Angular velocity shaft</td>
<td>$rad/s$</td>
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Chapter 1.
INTRODUCTION

1.1. PREAMBLE

High speed machining with the aid of latest engineering technology has become the mainstream of modern day manufacturing. Machine tool spindles are required to cope with a diversity of operating conditions, encompassing extremes of load, speed and operating temperature. The demand for higher productivity and improved quality of the machined surfaces requires spindle designers to aim for higher spindle speeds and feed rates. For instance, in routing of non-ferrous metals, wood and plastics, spindle speeds of the order of 50,000 rpm have been achieved with the help of better assembly, aiming for higher operational accuracy and increased component quality. The high speed machine tool spindles used in many applications require high dynamic stiffness to avoid vibration, which if not attained, can have undesired effects on product surface finish.

The limitations on attainable speeds and quality of surface finish are partly governed by rolling element bearing vibrations. Dynamic performance of a rolling bearing has been of considerable interest. Although fatigue life is sometimes a significant parameter, it has been fairly well established that in the case of many practical applications, bearing failure is caused by dynamic instability in the motion of the rolling elements, the cage, or both. With regard to skidding, this can be physically understood at least for lightly loaded bearings. Ball bearings are widely used for high speed spindles, because they offer low friction, appropriate stiffness characteristics and thus require minimum starting torque.

High speed precision spindle bearings are often subjected to failure when they are associated with high feed rates which require rapid acceleration and deceleration, resulting in drastic changes in cutting conditions. The reasons for spindle bearing failures apart from dynamic instability are mainly due to heat generation, which is a mainly function of bearing load and speed. Spindle’s precision bearings heat is generated mainly through three phenomena; the rolling of imperfect mechanical
bodies under load, the viscous shearing of a lubricant and gyroscopic effects. Out of these three sources viscous shearing of the lubricant accounts for the most of the generated heat and consumes most of the input power.

1.2. PROBLEM DEFINITION

Design, manufacture and assembly of high speed precision spindles have been identified as the key parameters to improve performance, because of their influence on the vibrational characteristics of modern day spindles and rotors (see figure 1.1). As such, any reduction in vibration ensures better quality machined surfaces in form and finish, whilst increasing productivity output and minimising tool wear. Study of machinery vibration can be characterised as rotor dynamics studies and contact dynamics (interactions) of concentrated contacts. Rotor dynamics deals with the high amplitude vibrations like displacement of shafts, whereas contact dynamics of concentrated contacts deals with type of contacts, where the amplitude of vibration or displacement are in micro-scale. Study of bearing vibrations will only be complete if both these areas of investigation are covered.

Figure 1.1: Cause and effect diagram
Figure 1.1 shows a cause-and-effect diagram (often referred to as a fishbone diagram) of the main factors that contribute to the vibration performance of a spindle. As can be seen from the diagram, the design and the type of cutting tools used have also been identified as factors, which augment the vibration characteristics of spindles themselves. These other factors lie outside the scope of this thesis, and are not addressed.

In addition to the principal benefits mentioned above, numerous other gains can be achieved through improving the performance of a router (subject of investigation in this thesis). The Ishikawa diagram in Figure 1.2 (see Peace(1992), Kolarik (1995)) illustrates some of the gains, which can be made. As it can be seen from the diagram, it is not just the quality of the machine products that would benefit, but also other gains may be made in health and safety and environmental issues. Moreover, many of these benefits are inter-linked. For example, since the machined surface finish would...
enhance, the amount of secondary manual sanding operations (employed to rectify aberrations on for example wood surface) would also be reduced, thus reducing cost of manufacture.

Achieving the same would also lessens the amount of dust in the atmosphere and, therefore, reduces the risk to personal health caused by dust inhalation. The risk of fire caused by close contact with electrical components; and the level of maintenance required to remove dust from places would also be reduced. Eliminating the sanding process also cuts out the expense of employing additional labour and the cost of replenishing sanding consumables. In addition, it cuts product lead times, which results in greater productivity, and can reduce the level of work-in-progress (WIP) free-up working capital.

The advances made in wood cutting technology can be predominantly attributed to the progress made in metal cutting. However, the direct transfer of information has been hampered by the different needs of cutting wood as opposed to metal. The differences consist of faster cutting speeds (i.e. spindle rotation speed and feed rate), the movement of the spindle about the workpiece, the non-existence of cutting fluid to absorb the heat generated; and the heterogeneous nature of the wood. Whilst constituting the basic requirements for woodcutting, these conditions combined are a formula for encountered high friction, wear and endangered operational safety. This thesis concentrates on the issues of bearing vibration/stability, bearing friction and lubrication issues that are critical for fast speed routers, particularly when grease lubricated.

1.3. BEARING: INDUCED VIBRATION AND TRIBOLOGY

It is recognized that high-speed spindles are the most critical elements of high speed machining systems. Along with the popularity of high speed machining, the demands for higher speeds for spindles have been steadily rising. Figure 1.3 illustrates the recent trends and future requirements in terms of speed for high speed spindles of various spindle noze sizes. The trend in the continual rise of requisite spindle speed brings challenges to the design and operation of high speed spindles.
Chapter 1 - Introduction

Expected Spindle Speed

- 30 taper KM10080 80/100 HSK
- 40 taper KM6350 63 HSK
- 50 taper KM5040 40/50 HSK

Figure 1.3: High speed spindles of various size, source Li and Shin (2004)

At high speeds, dynamic and thermal characteristics of high speed spindles, which are dependent on bearing types and lubricant rheology, play important roles in spindle performance. It is necessary to achieve a high speed without chatter and bearing failure. Two different types of lubricants used in high speed precision spindles are various oils and grease.

For high speed precision bearings petroleum-based oils are most common. To achieve coherent elastohydrodynamic (EHD) films an adequate amount of oil is required at specific temperatures and operating speeds. The commonly used oil supply mechanisms in spindles are air-oil mist or air-oil injection. In the first method a mist of oil droplets is created in an air stream delivered to the bearing. It features adjustable air pressure and oil delivery rate. The latter method delivers a controlled microscopic oil stream to the bearing, featuring adjustable oil and air delivery rate. The advantages of oil lubrication in high speed spindles are attainment of quite high speeds, but the oil supply system makes the system complicated, dirty and increases the maintenance costs.

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In contrast grease lubricated bearings require minimal maintenance and are inexpensive. Grease is provided between the balls and races in a sealed bearing. The thickener present in the grease holds the base oil like sponge. New synthetic greases with polyurea thickener and use of ceramic coated balls enable spindles to operate up to speeds of 40,000 rpm with operational reliability at high temperatures. Experience shows that a grease-lubricated bearing performs better than an oil-lubricated at lower speeds and temperatures see Bradford et al (1961), Godfrey (1964) and Horth (1968). Another advantage is that the dimensions of a greased-lubricated bearing are usually smaller than that of an oil-lubricated bearing with identical load carrying capacity according to Reinhoudt (1970). Despite its extensive use, the underlying lubrication mechanism of grease is still poorly understood. The paucity of available literature stems from the complexity of behaviour of grease and its non-Newtonian response in shear. It is a well known fact that when the base oil viscosity of grease is the same as that of oil, the friction of grease is greater than that of oil due to the existence of a thickener, hence more power is consumed. Contrary to this it has been shown experimentally that the thickener of grease enters the contact and forms a deposited film which reduces asperity interactions as shown by Cann (2002). Unfortunately all these studies have been conducted at low speeds and lack of analytical expressions for friction torque expressed in terms of the yield stress, the plastic viscosity, the speed and the load makes the study complex, even though the first study in this area was reported in 1913 by Westcott (1913). Since these factors depend upon the dynamics of the system, there is a strong need for better understanding and numerical modelling of the thermo-mechanical behaviour of high speed precision spindle systems, incorporating thermal elastohydrodynamic lubrication with asperity friction.

1.4. AIM AND OBJECTIVES

The overall aim of this study is to develop a bearing dynamic model to predict bearing induced vibration as well as balls-to-races contact conditions for evaluation of friction and generated temperatures. The study takes into account both oil and grease lubricated bearings. The specific objectives are:
Chapter I - Introduction

- Development of a multi-physics multi-scale model of bearings with inertial dynamics, lubricated contact forces, prediction of lubricant film thickness and generated contact friction and heat.

- Validation of the bearing dynamic model with vibration measurement from a routing spindle as well as with an analytical closed form solution.

- Develop a numerical model for thermal-elastohydrodynamic lubrication of balls to raceway tracks, incorporating viscous and asperity friction for both oil and grease lubrication. For the non-Newtonian behaviour of grease rheological state is taken into account.

1.5. METHODOLOGY

To achieve the above aims, a structured approach is undertaken, in which an initial step involves development of a 5 degrees-of-freedom bearing model, incorporated in a pair of angular contact bearings (in tandem arrangement) supporting a rigid stubby vertical rotor. The model is validated against a closed form analytical method and spindle vibration measurements using a non-contact laser measurement system.

The validated model then provides contact loads which are used in a thermal-elastohydrodynamic analysis of rough contacting surfaces to obtain frictional losses and heat generated in the bearing.

1.6. STRUCTURE OF THESIS

This work is organised in 7 chapters. These are as follows:

Chapter 1 provides a brief introduction to the subject matter of the thesis, defines the boundaries of investigation an gives an overview of importance of the investigations.

Chapter 2 provides a literature review of the field of ball bearing dynamics and all the associated subject matter, regarding vibration of bearings, tribology of contacts and rheological behaviour of grease.
Chapter 3 describes both an analytical and various numerical modelling of ball bearing, with various degrees of freedom. The mathematical exposition for bearing dynamics is provided in this chapter, together with comparison of models.

Chapter 4 describes the experimental set-up comprising a high speed precision spindle and rig construction, as well as the associated instrumentation, data acquisition and signal processing provisions and methods. Comparison of numerical predictions and experimental measurements are also provided.

Chapter 5 discusses the tribology of balls-to-races contacts for both oil and grease lubricated conjunctions. It provides the mathematical formulation for elastohydrodynamic analysis of such contacts.

Chapter 6 provides numerical solutions for the ball-race contacts based on the dynamic/kinematic conditions predicted in chapter 3 and the mathematical methods described in chapter 5, as well as heat generated in the contacts and friction due to viscous flow effect, compression due to pressure gradients and asperity interactions of rough mating surfaces.

Finally, Chapter 7 provides the overall conclusions, contributions made to knowledge, a critical assessment of the approach used in the thesis and suggestions made for future work.
Chapter 2

LITERATURE SURVEY

2.1. INTRODUCTION

There are great demands put on the new generation of machine tools. Some of these demands concern the main spindle design in better assembly, higher operating accuracy, and increased component reliability. It is also known that, the dimensional accuracy and surface finish of a machined part depends upon the dynamic characteristics of the machine tool spindle–bearing–workpiece system. Vibrations induced in this system due to the generated dynamic forces, for example, during metal cutting operations can cause undue vibration and thus low quality surface finish of machined products. A high precision routing spindle is one such system, where vibrations can cause major problems. Even the slightest amplitude of vibration can have dramatic effects on the surface finish of routed workpieces, wheel wear, and form-holding. Bearing systems can be one of the major sources of vibration within spindle systems.

The use of ball bearings goes back to AD 42 according to Dowson (1979) and the first mention of ball bearings in the form of bobbins was by Leonardo da Vinci in AD 1452 (unpublished later discovered in National Library in Madrid in 1967) (see Gohar and Rahnejat, 2008). From Leonardo to the 19th century many types of ball bearings have been described, but largely in an empirical manner. The use of ball bearings in bicycles during 1880 was the important development, which led to their mass production. It also initiated scientific research in the field. For detailed historical development of ball bearings readers should refer to Dowson (1979). The major breakthrough this field came in the 1950’s, when the researchers were able to incorporate vibration studies with ball bearing dynamics. This was also the same period when there were significant achievements made in the field of elastohydrodynamic lubrication (EHL), after Ertel and Grubin (1949) obtained satisfactory a partial analytic solution to the elastohydrodynamic line contact problem.
accounting for elastic distortion and viscosity pressure effects. All these initial developments a dramatic change occurred in the concept of the scientific research in this field. This chapter reviews some of these developments.

The survey is sub-divided into three sections. The first section deals with assembly related errors and manufacturing anomalies, whilst the second section highlights research developments related to the dynamics of ball bearings both with dry and lubricated contact conjunction. The final section provides an insight into the technical breakthroughs in the field of elastohydrodynamics, particularly with grease lubrication.

2.2. ASSEMBLY RELATED ERRORS AND MANUFACTURING ANOMALIES

Market competition demands higher levels of productivity and automation, accuracy and reliability of products. Innovations in manufacturing technology have maintained a consistently high level of product quality in an advanced manufacturing environment, but the demands for shorter production cycle times have often very led to small assembly related errors and manufacturing anomalies. Even very small assembly related errors and manufacturing anomalies can have drastic effects in the vibrational behaviour of complex systems such as ball bearings, the most common among these are incorrect preload and internal clearances.

2.2.1. EFFECT OF PRELOAD

When a thrust (axial) load is applied to a angular contact or thrust bearing, the balls or rollers and races are compressed and one ring is displaced with respect to the others. The displacement for a small load is a noticeable movement due to the looseness and end-play of the bearing. Thereafter, the contact areas increase with compression and the deflection does not rise as rapidly as that cause by the initial preloading. When the bearings are preloaded and an additional load is applied, the additional axial movement of the shaft is usually smaller than the case without a preload. It is, therefore, clear that a slight preload on the bearing raises the quality of the surface finish of the workpiece significantly, due to a higher guiding accuracy. In practice, the
normal procedure for preloading is to adjust the bearings in pairs or two rows of rolling elements against one another by the application of a permanent axial load. Figure 2.1 shows two bearings arranged back-to-back before and after preloading.

Preload is an important characteristic in designing a ball bearing system, especially for precision applications as this increases the dynamic stiffness of the system. Generally, a higher preload would decrease the vibration under the subsequent applied loads, but might also decrease the bearing life. The stiffness of the spindle is mainly determined by the rigidity of the spindle bearing, which is influenced by preloading. Furthermore, the operating temperature of the spindle is influenced by preloading.

Even though it has been an accepted fact that bearings of machine tool spindles should have a certain preload, but little was understood of bearing dynamic response in detail until Wardle et al (1983) came up with a detailed explanation. Authors showed that with an external load acting, ball-to-races elastic deflections results in gross movements that disturbs the effective bearing centre. Authors also suggested three types of preloading which could be used as fixed displacement preload, constant force preload with zero tilt and a constant force preload with zero movement. Since their study was concentrated on the variation of dynamic and static stiffness the effects of preload were not dealt with in detail. The use of ball bearings in highly
accurate, low drift inertial gyros for navigational systems and space guidance systems was not possible due to the variation of displacements in different axis for the same load. This problem was effectively countered with the theoretical explanation of isoelasticity by Harris (1984) who showed that effective preloading can tackle the problem. This was one of the important developments in the ball bearing industry, which led to the wide-spread use of ball bearings in precision applications. Harris (1984) discussed different preloading mechanisms and an analytical solution was also suggested where:

\[
\frac{F_p}{ND^2K} = \sin \alpha_p \left( \frac{\cos \alpha^o}{\cos \alpha_p} - 1 \right)
\]  

(2.1)

The variation of preload due to service conditions such as temperature was one of the factors affecting the performance of spindles. The experiments conducted by Kim and Kim (1989) have shown that large preloads and high stiffness can probably increase the dynamic stability of spindles, whilst high temperatures decrease the processing accuracy and damage the bearings. On the contrary, an insufficient preload can lead to an untrue running spindle, thus reducing its degree of precision. Authors were successful in bringing out the effect of spindle temperature on the accuracy of finished products, but were unable to predict the required extent for an effective preload.

Hagiu and Gafitanu (1994) tried to establish a correlation between preload and service life for angular contact ball bearings, mounted on machine tool main spindles with a combined theoretical and experimental investigation, and with controlled operating conditions. It was found that low preloads lead to a diminution of the number of loaded balls in the bearing with negative effects on both bearing service life and vibration levels. High preloads produced additional loads on balls, with a negative effect on bearing service life due to fatigue spalling. Even though the study concentrated on the aim of finding an optimum preload nothing was noted of system dynamics and effect of temperature on the maintenance of a preload.
Providing external cooling by circulating a coolant was one of the methods used during early 90s. The effectiveness of this method was analysed by Tu and Stein (1996), who found that preload regulation could be achieved by carefully controlling the circulation of a cooling (or heating) flow around the spindle housing to manipulate the housing and the outer ring temperatures. Even though this study concentrated on external cooling the splashing of a coolant over the hotter spindle housing to increase the bearing thermal preload was not included in the study. Steep temperature gradients near housing were also not included in this study.

The effect of temperature rise and thermal expansion was studied by Jorgensen and Shin (1997). They found that thermal expansion increases the contact deformation, which leads to an increase in both contact load and bearing stiffness. Authors derived a complete bearing load-deflection analysis, including the effect of thermal expansion, coupled with an analysis of spindle dynamic response. Steady-state temperature distribution was found from heat generation at the contact points, the effects of loading condition and bearing material type on the bearing stiffness. All these temperature related studies considered dry contact conditions, hence the cooling effect of lubricant inside the bearing were not taken into account.

A different approach was employed by Akturk et al (1997). Authors tried to monitor the changes in dynamics of the system and the displacement/deflection in balls introduced by a preload by changing the number of balls. This study was based on the initial work by Rahnejat (1984). In both cases, the balls to race contacts were modelled as contact stiffness non-linearities. It was shown that increasing the preload changes the steady state position of the vibration on the non-linear contact force relation of the bearing, resulting in the shaft mass being supported on stiffer springs. Larger values of axial preload caused stiffer spring characteristics and resulted in higher natural frequency values. As the number of balls is increased, the system becomes stiffer, since a larger number of balls support the shaft. The change in natural frequency is cyclic for the change in ball set position. Rahnejat (1984) used a 2-DOF model of a deep groove ball bearing, whilst Matsubara et al (1988) and Akturk et al (1997) used three degree of freedom systems for their analytical studies, neglecting
pitch and yaw motions of the spindle. In reality these motions also have significant effects on system dynamics.

Effect of preload on system dynamics with or without considering unbalanced forces was shown by Changqing Bai et al (2007). Floquet theory was employed to investigate the bifurcation and stability of the system periodic solution. With the aid of Poincare maps and frequency response, the unstable motions of the system were analysed in some detail. The results showed the effect of axial preload on the system dynamic characteristics. The authors suggested that the unstable periodic solution of a balanced rotor bearing system can be avoided when the applied axial preload is sufficient. This study concentrated on the system dynamics, but nothing was mentioned of the ball pass frequency (variable compliance effect, see Rahnejat (1984) and Wardle et al (1983)) or the natural frequency of the system.

Another problem with the shaft-bearings system is gradual degradation of moving surfaces in relative motion. Bearings are usually mounted to shafts or in housings with an interference fits, pressing the inner ring to the shaft. This can also cause inner ring heating, requiring inner race heat treatment in an oven or in an oil bath. Even though this eliminates the fretting corrosion due to the relative movement between the shaft and bearing inner ring, the change in temperature and stress can still produce some geometrical imperfections and also can affect the internal clearance.
2.2.2. EFFECT OF RADIAL INTERNAL CLEARANCE

The radial clearance $C_d$ is defined as the free radial space in an unloaded bearing (see Figure 2.2).

![Diagram showing radial clearance and applied load](image)

Figure. 2.2: Under an externally applied axial load, the radial clearance disappears and the bearing is loaded through contact angles (Wensing and Anton, 1999)

This internal radial clearance provided in the design of bearings is to compensate for the possible thermal expansion. Inadequate radial clearance can cause the balls to take more loads and can introduce a loaded region. Existence of a loaded region exacerbates the variable compliance effect, described later (Rahnejat, 1984). The balls passing through this region are deformed, where the mutual convergence of races takes place and the balls' deflected surface rebound when they move out of this region. This causes the load distribution on the shaft to change resulting in a relative movement between the outer and inner races. This effect is termed as variable compliance according to Perret (1950), Wardle et al (1983) and Rahnejat (1984). Using Hertzian theory the elastic deformation between balls and races were modelled for a deep groove ball bearing (Rahnejat, 1984, Matsubara et al, 1988).

Earlier Meldau (1951) studied the two-dimensional motion of the shaft centre. Both the studies by Perret (1950) and Meldau (1951) performed quasi-static analyses, since...
the inertia and damping forces were not taken into account. With the launch of Sputnik, the world's first artificial satellite by the Soviet union in 1957 researchers had to develop smoother bearings, which could be used in gyros. American military was very keen for such developments and with inception of NASA in 1958 the researchers concentrated more on bearing vibration studies. More detailed history of rolling bearing vibration is given by Tallian and Gustafsson (1965).

Even though the aforementioned initial period in ball bearing vibration research was relatively intense, still the "mystery" of variable compliance effect was finally tackled by Sunnersjö (1978), who studied the varying compliance vibrations theoretically and experimentally, taking inertia and damping forces into account. The author suggested that bearings with linear contact characteristics do not exhibit variable compliance effect. This was later confirmed by Rahnejat (1984) analytically. However, bearings with non-linear contact characteristics showed variable compliance effect, which could be theoretically avoided by adjusting the internal clearance. The variable compliance vibrations are greatly affected by misalignment, rotor unbalance and flexibility of rotor shaft (see Matsubara et al, 1988) and bearing pedestal. Furthermore, the existence of errors in form and finish of bearings, such as out-of-roundness of bearing rings causes vibrations which have a magnitude comparable to those of the variable compliance vibrations. Sunnersjö (1978) could not verify his analytical model with experimental measurements. The existence of variable compliance vibrations at high speeds and high loads were also not discussed by the author.

A more detailed analysis of the dynamic behaviour of a radially loaded bearing under a varying rotor speed was undertaken by Fukata et al (1985). The authors modelled an ideal bearing system free of defects with two degrees of freedom system. The results showed the existence of super and sub harmonic resonances, beat and chaotic vibrations which could lead to bearing failure. Occurrence of variable compliance, axial vibrations and the effects of pitch and yaw were not included as the system model had only two degrees of freedom.
Tiwari et al. (2000) studied the effect of radial internal clearance ball bearings on the dynamic response of the rotor through simulating the response of a balanced horizontal rigid rotor supported by a pair of deep groove ball bearings. The authors observed that with large clearances very wide unstable regions occurred, and not necessarily only around the critical speeds. Another region of instability is where a high radial internal clearance occurred. According to the authors this region was developed as a result of a peak formed due to the frequency component in the horizontal and vertical directions. The authors also showed that an increase in clearance resulted in a decrease in the dynamic stiffness of the bearing because of the non-linear nature of the force-deformation relationship. One of the shortcomings of this study was the maximum attainable speed of only 10,000 r.p.m., where the critical speed corresponding to bearing stiffness falls outside this region.

Rahnejat and Gohar (1984) found that the fluid film lubrication for sufficiently preloaded and or interference fitted bearings has insignificant damping effect paved the way for modelling of bearing dynamics for sufficiently loaded conditions using Hertzian contacts (dry condition), which reduced the computational effort significantly. Taking advantage of this in a series of papers, Harsha (2006), Harsha et al. (2004), Harsha and Kankar (2004) numerically investigated the dynamics and hence the occurrence of vibration in ball bearings. All the simulations were conducted in terms of a non-linear Hertzian contact model under an artificial damping introduced into the system to eliminate the transient vibrations. Although Harsha has not succeeded in showing a clear strange attractor for the existing of an imbalance force and race waviness, he has identified various chaotic regions, starting from 2050 rpm for a small internal clearance of 0.5 μm (also see Choudhury and Tandon (2006)). Three regions of dynamic responses include periodic, quasi-periodic and chaotic as reported by Tsuda et al. (1992). Furthermore, a quasi-periodic route has been proposed as a leading regime from periodic to chaotic behaviour.

2.2.3. EFFECT OF WAVINESS

An important source of vibration in ball bearings is surface waviness of rolling mating members. These are global semi-sinusoidally shaped imperfections on the surface of
the bearing components (see Figure 2.3). The characteristic wavelengths of the imperfections are much larger than the dimensions of the Hertzian contact areas between the balls and the guiding rings.

![Diagram](image)

**Figure 2.3: Waviness excitation in a ball bearing.**

The number of waves per circumference is denoted by the wave number. Waviness imperfections cause variations in the contact loads when the bearing is running. The magnitude of this variation depends on the amplitude of the imperfections and the contact stiffness. Due to the variations in the contact loads, vibrations are generated in the bearings.

![Diagram](image)

**Figure 2.4: Radial vibration modes of the inner and outer rings of a ball bearing caused by waviness (a) Extension mode (b) Rigid body mode**
The resulting vibration modes of the rings can be either extensional, flexural or rigid body modes, dependent on the number of rolling elements, the wave number and the applied load (see Figure 2.4). The extensional mode is usually accompanied by a rigid body mode in the axial direction. Imperfections with a different wave number cause vibrations at distinct frequencies, each with a characteristic vibration mode. Even though with modern machining techniques surface waviness are somewhat reduced, they still produce vibration amplitudes in the range of at least several nanometres which represent a problem when it comes to high precision manufacturing. When the 'A', single circumferential waviness track of the contacting surface is developed into a Fourier series. For a single track, the deviation $W$ from the perfect geometry can be written as:

$$W(\theta) = \sum_{n=1}^{\infty} \frac{A'}{n} \cos(n\theta + \varphi_n)$$ (2.2)

The parameter $A$ equals the magnitude of the first harmonic. The exponent $s$ describes the amplitude decay for subsequent wave numbers. The phase $\varphi_n$ is uniformly distributed over the interval [0; 2$\pi$]. The magnitude $A$ is subject to statistical variations. A likely probability distribution for the stochastic variable $A$ is the Rayleigh distribution, since $A$ is restricted only to positive values. In general, the Rayleigh distribution of a stochastic variable $x$ is defined by the probability density function $p$

$$p(x, \alpha_w) = \begin{cases} 2\alpha_w x e^{-\alpha_w x^2} & x > 0, \alpha_w > 0 \\ 0 & x < 0 \end{cases}$$ (2.3)

Rayleigh distributions (Figure 2.5) can be derived from normal distributions. When the stochastic variables $X$ and $Y$ are normally distributed and $R$ is defined by: $R^2 = X^2 + Y^2$, then $R$ has a Rayleigh distribution with $\sigma_x = \sigma_y = (2\alpha_w)^{-1}$. For $\alpha = 0.05$, equation (2.3) reduces to the standard Rayleigh distribution (Rothschild and Logothetis, 1986). For each wave number $n$, a new value for $A$ is generated. The parameter $\alpha$ is determined from series of surface measurements.
The first technical study dealing with the effect waviness was reported by Yhland (1967) through experiments for the axial and radial vibration frequencies. The author used a linear theory to calculate the stiffness matrix of the ball bearing with waviness, and investigated the effect of waviness through his rotor dynamic model. However, his model could not explain the non-linear load-deflection effect because it could not include the change of the relative position between the rolling elements during rotation. Rahnejat and Gohar (1985) analysed the effect of surface roughness in the direction of rolling in an elastohydrodynamic contact. This work concentrated more on the lubricated contact study and is discussed later.

The major technical study reported in this area of ball bearing vibration due to surface waviness was by Wardle (1988a, 1988b) through a simple analytical treatment. The author provided theoretical expressions relating amplitude and wavelength of surface imperfections in thrust loaded ball bearings to vibration forces and frequencies produced under constant-speed operating conditions. Analytical expressions for the vibration frequencies excited due to surface waviness were given. Experiments were conducted to compare the magnitudes and frequencies of vibration predicted by the theoretical model with the measurements of amplitude response of a
rigid bearing housing. For this study the author assumed waviness to be harmonic in the direction of rolling. This assumption is of course not valid for balls, where the waviness of any wavelength occurs in both direction of rolling and orthogonal to it. The minimum wavelength considered was approximately three times the average width of the ball-race contacts (the Hertzian semi-half-width), as the author was unable to figure out the excitation frequencies due to waviness.

Jang and Jeong (2003) showed that the waviness in the ball bearing generates the time-varying component of the stiffness coefficient, whose frequency is called the frequency of the parametric excitation. The authors showed that the centrifugal force and gyroscopic moment of the ball are the important factors in determining the bearing frequencies, their harmonics and the side-band frequencies resulting from the waviness of the rolling elements in ball bearings. They also showed that the bearing vibration frequencies are generated by the waviness interactions on contacting surfaces not only between the rolling elements, but also between those of two or more ball bearings constrained by the rotor. The research work presented by authors seems very promising, but their investigations were not applied through the entire operating range, due to the inherent instability of their non-linear governing equations. Furthermore, the effect of internal clearance, which might play an important role in the transient dynamic analysis of rotor supported by ball bearings was not in their study.

The dynamic stability and vibration characteristics of a rotor bearing system, incorporating the effects of internal clearance and waviness is reported by Bai and Qingyu (2006), using a five degrees of freedom model for a transient dynamic model of ball bearings. The authors showed that at high-speeds, balls’ centrifugal forces and gyroscopic moments, as well as cage speed variations due to outer race waviness are more significant than the effects of inner race and ball waviness. Moreover, the effect of ball waviness on cage speed variation was not apparent.
2.3. DYNAMICS OF BALL BEARINGS

Grinding spindles are usually used for final surface finishing operations and are subjected to a fixed radial force of approximately 5-35 N and a small superimposed dynamic component of one-twentieth to one-tenth of the fixed radial force. High frequency vibration levels in grinding spindles are usually modulated with the fundamental shaft frequency or with characteristic frequencies of machine elements as shown by Harting (1978). In general, however, average high-frequency vibration levels and modulated effects are of low amplitude, except for localised defects such as defective balls or faulty raceway tracks. Therefore, the most significant vibration frequencies in spindles are those associated with variable compliance vibrations as pointed out by Wardle and Poon (1983), and Rahnejat and Gohar (1985).

Experiments reported by Aini et al (1995) with a grinding spindle at a constant operating speed of 3000 rpm for loads from 0 to 32 N have shown that the amplitude of shaft frequency increases with respect to load, whereas the amplitude of ball pass frequency decreases as load increases. The authors believe this to be due to the reduced load intensity in the loaded region of ball/races contact. Furthermore, due to the application of an external radial load the band of frequencies associated with the bounce mode of spindle (in the radial transverse directions) were packed together and moved towards the stiffer regions of the vibration spectrum. A number of peaks appeared in the frequency spectrum of the spindle, which were attributed to the support characteristics of the bearings. Modulation of these frequencies with the rigid-body resonant modes of the spindle was found to adversely affect its dynamic performance. However, the investigations were at relatively low spindle speeds. For a routing spindle speeds between 15,000- 20,000 an experimental investigation was carried out by Lynagh et al (1999). Primary bearing-induced vibration frequencies (due to variable compliance) along with spindle bounce and pitching modes were identified for free run of the routing spindle with no external load. Effect of secondary frequencies which are produced by surface irregularities were also identified and found to have close match with an analytical model, similar to that reported by Wardle et al (1983). The complex spectra obtained using FFT, it is necessary to
isolate and identify specific causes as spectral contributions, free from the side-band modulations and errors associated with the averaging processes in Fourier transformations. Furthermore, it is important to quantify the exact level of contributions at given frequencies so as to determine the need for remedial actions in practice.

Figure 2.6.: Shows the spectrum obtained from FFT (Vafaei et al., 2002)

Figure 2.7.: Shows the spectrum obtained from ARMA (Vafaei et al., 2002)

Time domain visualisation using Auto-Regressive Moving Average method (ARMA) shown by Vafaei et al. (2002) made the researchers develop new methods for vibration condition monitoring. Figures 2.6 and 2.7 show comparisons between a Fourier spectrum and that developed using the ARMA model. In 2003 Mathworks released Matlab 7.0 which has a built-in ARMA analysis package. The synchronous run out component of spindle vibration under strictly controlled experiments for detecting the
spindle imbalance, based on an average method and a time decomposition wavelet transformation was proposed by Vafaei and Rahnejat (2003).

The experimental studies mentioned previously were the results of good numerical models developed by various researchers in this field are discussed in later sections.

Mathematical modelling of ball bearings have been of interest to many researchers. Researchers like Jones (1959), Jones (1960) and Harris (1984) have assumed that a quasi-static force balance existed within the bearing. This allows the force distribution throughout the bearing and the geometric configuration to be calculated when an external load is applied. After the development of transient theories the validity of models with quasi static forces were questioned. Researchers like Walters (1971) and Gupta (1979) questioned the existence of quasi static conditions. Walters (1971) presented a comprehensive general analysis of the motions of balls and a ball separator with realistic lubrication. Equations of motion of balls with four degrees of freedom and cage with six degrees of freedom were presented and validated theoretically with help of Runga-Kutta method and experimentally with a spin axis gyro bearing configuration.

The model developed by Walters for ball bearing and cage dynamics with ball raceway slip, which could not include the same analysis for cylindrical rolling elements was later modified by Gupta (1979a), Gupta (1979b), Gupta (1979c), Gupta (1979d). Gupta presented an analytical formulation for the roller motion in a cylindrical roller bearing in terms of the classical differential equations of motion. The model analysed in detail the roller-race interaction and roller-cage interactions both under assumed dry contact conditions. The model also considered the roller end and race flange interactions during its skewing. The motion was considered to be the roller mass centre in an inertial frame of reference and angular motion about the roller mass centre in a roller fixed coordinate frame. The six degrees of freedom model accounts for the misalignment of the races and was capable of treating roller skew and other
complicated and often undesirable motions. The author showed the existence of elastic contact frequency and bearing kinematic frequency, two distinct frequencies, the first due to the Hertzian contact spring at the ball to races, varied as 0.166 powers of the ball contact load and the second one due to oscillatory motion of ball in raceway varied as 0.25 powers of the ball contact load. However, the solution which included time-varying Hertzian contact stresses for each ball, along with cage collisions and integrated ball traction/slip forces at each contact point resulted in long computations. In addition, the Walters/Gupta model equations are written in the fixed inertial coordinate system that leads to complex equations of motion, excessively long computation times and with computational errors due to numerical truncation. Since the mass of spindle was neglected in the analysis, Gupta was unable to include the non-linear time varying coefficients with transverse and linear movements. Hence, the model could not predict the influence of bearings on spindle dynamics.

Bearing dynamics dominates spindle bearing dynamics when the spindle is considered as rigid as shown by Rahnejat and Gohar (1985) in a theoretical study. The authors were able to show the vibration response of a rigid horizontal shaft with an applied mid-span load, supported by a pair of oil lubricated deep groove ball bearings. The authors were the first to present such an analysis with bearings and their oil films to be approximated as a set of non-linear springs and dampers rotating relative to the shaft. The minimum film thickness was obtained under combined lubricant entraining and squeeze film effect. The cyclic behaviour of the oil film as obtained by the authors is shown in Figure 2.8. The larger segments represents the least loaded region of the bearing and highly loaded region by the smaller loops.
The authors noted that more energy dissipation occurred in the larger limit cycle loops, hence thicker hydrodynamic films contribute more to fluid film damping. This finding is in agreement with the experimental findings of Dareing and Johnson (1975). Although Dareing and Johnson (1975) argue that in case of oil film, damping will increase the overall damping by at least 30% and is effective under severe vibration conditions. Due to the heavy mass of the shaft used by Rahnejat and Gohar (1985) it was found that fluid film damping was rather insignificant. The findings of Dareing and Johnson (1975) actually agree well with the predictions of Mehdigholi et al (1990). Rahnejat and Gohar (1985) found that the natural frequency of a perfect spindle/bearing assembly is dependent upon the spindle mass and the bearing stiffness and independent of rotational speed. Bearing stiffness was found to be dependent on the radial clearance and number of balls as also observed by the analytical model of Wardle and Poon (1983). Furthermore, the authors were able to show the presence of

Figure 2.8: Limit cycle phase plane trajectory of oil film

(Rahnejat and Gohar, 1985)
ball pass frequency. Even though this study was able to bring a new dimension to the analysis of ball bearing dynamics, the study was restricted to two degrees of freedom.


Five degrees of freedom analysis of an asymmetrical spindle, supported by a pair of back-to-back angular contact ball bearings was carried out by Aini et al (1990a) with applied axial and radial loads. The model incorporated both dry and lubricated contact conditions with combined axial and radial loads, as well as gyroscopic effects and conical whirling of the spindle. This formulation extended the quasi-static methods of Jones (1960) and Harris (1971). Aini et al (1990b) also used the same model to simulate the vibration characteristics of a precision grinding spindle. Major frequencies and a number of design curves, suggesting the “optimal” zones of operation for the simulated spindle under axial/radial loading were also presented. The authors confirmed that the lubricant film damping in ball bearing was insignificant but reduces the noise caused by high frequency vibrations as also suggested by Gupta (1979), Sayles and Poon (1981) and Wardle and Poon (1983).

Rahman et al (2002a) used a similar model to study free vibration response of a three-ball bearing spindle. A theoretical analysis was also carried out Rahman et al (2002b) for different bearing settings and preload on the system natural frequencies. Aini et al (2002) extended their five degrees of freedom model by including the squeeze film effect caused by the mutual convergence of bearing rings in lubricated conjunctions. The overall system response, when subjected to varying spindle mass or the number of balls in the support bearings were studied. The overall contribution to damping of the elastohydrodynamic oil films between the rolling elements and their raceways was shown to be slight.

The studies reported in the previous sections do not give a clear picture of the influence of lubricant film on the major modes of the system. Therefore, a detailed model for contact modes needs to be undertaken, especially for ball bearings where the loaded mating surfaces do not conform to each other, such as the point contact.
between a ball bearing and a raceway. The contact pressures are very high for most ball bearings, operating in Elastohydrodynamic (EHD) regime of lubrication. A representative review of EHL contacts is given below.

### 2.4 ELASTOHYDRODYNAMIC LUBRICATION (EHL)

Elastohydrodynamic lubrication (EHL) deals with lubrication of elastic contacts, requiring simultaneous solution of generalised elasticity, lubricant rheological state and Reynolds equations. Martin (1916) determined the lubricant film thickness inside the contact by considering the contacting solids to be rigid and lubricant to behave with constant viscosity (i.e. a hydrodynamic analysis). However the film thickness formula proposed by him greatly under estimated the film thickness. A satisfactory solution to EHL problem was given for the first time by Ertel and Grubin (1949). The authors postulated elastic distortion of the inlet wedge to conform to the classical Hertzian analysis with viscosity-pressure effects. This facilitated the solution of Reynolds equation in the inlet region of the contact and enabled the separation of solids in the central region of the contact to be determined with commendable accuracy. The major achievement in this area came with the numerical solution of the entire contact for infinite line assumption by Dowson and Higginson (1959). The authors used the computed film shape to compare with elastically deformed solid shapes and then pressure curve was modified to improve the agreement between two shapes. Empirical formula for isothermal EHL contacts were given by Dowson and Higginson (1961).

Most of the work reported in the early research were concerned with line contact configurations For point contacts investigations include those of Archard and Kirk (1964). The authors observed the film thickness between a steel ball and glass plate using interference rings. Later the same technique was used by Cameron and Gohar (1966) with much more precision.

A Theoretical solution of EHL point contact problem was proposed by Archard and Cowking (1966). EHL pressures inside the contact were considered to be approaching the Hertzian pressure. Film thickness formula for point contacts were proposed by
Cheng (1970). Hertzian theory was used to calculate the deformations by the author and then Reynolds equation was applied to the geometry.

A complete solution for EHL point contact problem was provided by Hamrock and Dowson (1976a,b), Hamrock and Dowson (1977a,b). Since the EHL pressures are very high authors used Roelands (1966) equation for viscosity-pressure dependence and direct iteration method for both fully flooded and starved lubrication conditions. The influences of ellipticity parameter, dimensionless load, rolling speed and material parameters on minimum film thickness were also obtained. An expression for critical dimensionless boundary distance at which starvation starts to become important was proposed by studying the effect of lubricant starvation on pressure distribution and minimum oil film thickness. Regression formulae for minimum and central oil film thickness for starved conditions were also proposed. Even though these formulae were found using second order discretization, there are only valid for low loads and for larger elliptical parameters.

Mostofi and Gohar (1982) obtained a numerical solution for point contact with effect of squeeze velocity. The authors also studied the effect of EHL under ball spinning conditions. Although the theoretical predictions of oil film thickness compared favourably with experiments, the exponent for the load parameter given was positive, whereas the rest of the extrapolated equations suggested by others provide a negative exponent. A similar analysis was also presented by Chittenden et al (1985). The method proved, however, to be inadequate for treating the highly loaded contacts and alternative techniques were sought to overcome the numerical instabilities encountered at pressures higher than 0.5 GPa.

Most of the numerical methods mentioned above have been obtained using Gauss-Seidel iterative method with under relaxation to ensure stable convergence, which led to significant number of iterations and usually yielded long computation times, which is proportional to the square of number of computational nodal points used. This made researchers like Lubrecht et al (1986), Jalali Vahid et al (2001) to use Newton-Raphson method for EHL point contact problems with multi-grid method which was initially developed by Brandt (1977).
2.4.1. GREASE LUBRICATION

From the previous sections it is clear that significant knowledge of the pressure-build-up mechanisms and the operating lubricant film thickness in machine elements with non-conformal contacts for oil as a lubricant with Newtonian characteristics is documented. However, the theory for grease lubrication lags considerably behind because of the complexity of its rheological properties. In practice, however, approximately 80–90% of rolling element bearings are lubricated with grease. The understanding of the rheological behaviour of lubricating greases is nowadays a decisive factor in the design and optimisation of the tribological systems as well as in the control of their processing. Greases are two-phase lubricants composed of a thickener dispersed in a base oil. The thickener is either a polymer fibre or a metallic soap made of a base of lithium, calcium, aluminium, sodium or synthetic. Due to the effects of the thickener, greases are often modelled as a plastic solid. Unlike oil, grease can withstand shear and will not flow until a critical yield stress is reached. Traditionally, these properties of grease have been related to the so-called ‘yield state’ at low strain rates and a shear-thinning behaviour at medium and high strain rates. Thus, the typical grease flow curve exhibits constant values of shear stress at low strain rates as pointed out by Balan and Franco (2001).

The Eyring model, Bingham model and Herschel-Bulkley model are the three constitutive models which have been employed successfully in studying grease lubrication. Even though each model has its advantages and disadvantages, the selection of an appropriate model depends upon the operating conditions. Theoretical developments with all these three models are compared later.

Rees and Eyring (1941) proposed a general equation of flow for Non-Newtonian fluids, considering flow rate of a system as a function of: 1) the relaxation times of the flow units, which contribute to the flow process, 2) the distribution of such relaxation times, and 3) the deformation of system with stress. The authors visualised this relaxation process for viscous flow to be the sudden shifting of some small patch on one side of a shear surface with respect to the neighbouring material on the other side of the same shear surface. Any shear surface divides a mosaic of such patches lying
on two sides of a surface, which will be heterogeneous and can be described by groups each characterised by its mean relaxation time by the fractional area of the shear surface which the group occupies by a characteristic shear volume divided by $kT$. The resulting generalised expression for viscosity is, therefore:

$$\eta = \sum_{n=1}^{\infty} x_n \beta_n \sinh^{-1} \beta_n \dot{s}$$

(2.4)

Where $\dot{s}$ is the rate of shear.

Buckingham (1921) described the flow of lime-base greases in terms of two flow units, Newtonian and non-Newtonian. Ree-Eyring viscosity equation, when rearranged for incorporating flow in terms of a Newtonian fluid becomes complicated and requires tedious analysis procedure to solve it. Sisko (1958) introduced a three parameter equation (equation 2.5) for incorporating Newtonian and Non-Newtonian parts of a flow, which is quite similar to the Herschel-Bulkley model. Hence:

$$\eta = a + b \gamma^{n-1}$$

(2.5)

where $a, b$ and $n$ are constants.

The two parameter Bingham model (Bingham, 1922) was the earliest model to deal with the problem of grease lubrication. Fluids obeying this model are called Bingham plastic fluids and exhibit a linear shear stress, shear-rate behaviour after an initial shear-stress threshold has been reached. The flow of a Bingham body is expressed in terms of its plastic viscosity, $\phi$ as:

$$\phi = \frac{F - f}{\dot{\gamma}}$$

(2.6)

where $F$ is the shear stress and $f$ is the limiting shearing stress below which no flow would occur.

Analysis carried out by Milne (1954) on a journal bearing considered the lubricant as a Bingham solid. This was the first analysis of grease lubrication in a bearing. Since
the EHL theory was not fully developed at that time the author had to validate his studies with experiments. The author was the first to introduce the idea of a core formation, but could not make clear the idea of what constitutes a solid core in an exact manner, because his analysis was restricted to finite width bearings. Sisko (1958) considered grease as a Bingham body and integrated for flow in a tube, where no flow occurs below a yield pressure drop. The author compared the slope of grease data for plastic viscosity with shear stress plot at low shear stress values. Since the slopes of the curves were not in agreement, the author concluded that grease is not a Bingham solid. This finding contradicted the experimental results obtained by Cohn and Oren (1949). Later many experiments confirmed that grease acts like a Bingham solids (Wada et al, 1977 and Mutuli et al,1985). Theoretical lubrication studies for slider and journal bearings done by Milne (1954) found that rigid cores may be attached to one or the surface. Mathematical explanations for given Milnes theory was put forward by Sasaki et al (1960), who used a Bingham solid as a model for grease flowing between non-deformed cylinders. This analysis was restricted to one dimension. The author concluded that the starting point friction is directly proportional to the yield stress and for high speeds grease lubrication coincides with the EHL theory for oil.

Wada et al (1977) conducted a study for clarifying the behaviour of grease lubrication in rolling bearings. A nonlinear integral equation coupling the hydrodynamic equation for a Bingham solid and the elasticity equation was derived. Numerical solution of this equation yielded the core formation in a Bingham flow in the elastohydrodynamic conjunction of two cylinders. The study concluded that as velocity ratio of the surfaces becomes larger, the region of core formation in the film extends and the film thickness increases. The core thickness increases with an in the yield stress. In such a case the film pressure was found to be larger but geometry of pressure curves remained unaffected. The pressure coefficient of the plastic viscosity contributed to the film pressure and elastic deformation. The minimum film thickness was found to decrease with an increase in load capacity, but film thickness was insensitive to load as in EHL contacts. Frictional was found to become large with an increase in plastic viscosity and yield stress of the Bingham solid. However, the flow
equations suggested by the authors was complex, yielding a complicated solution procedure. The solutions suggested by the authors were problematic since the analysis did not consider the pressure spike at the outlet region. The drawback of modelling grease lubrication with a Bingham model is that the viscosity of the fluid remains constant upon yielding.

Based on the available experimental data, Kauzlarich and Greenwood (1972) found that most greases behave pseudo-plastically, and in the case of a calcium grease, some of the greases shear thickens or thins, depending upon the shear rate. The Herschel-Bulkley (HB) (1926) equation is one of the more realistic constitutive models for grease behaviour:

\[ \tau = \tau_0 + \phi \left( \frac{\partial u}{\partial z} \right)^n \]  

(2.7)

HB fluids are described by a three-parameter rheological model shown in equation (2.7), where \( \tau \) is the shear stress, \( \tau_0 \) the yield shear stress, \( \left( \frac{\partial u}{\partial z} \right) \) the velocity profile and \( n \) is the Herschel-Bulkley index. When the local shear stress is below the yield stress, HB fluids behave as rigid solids, similar to Bingham fluids. Once the yield stress is exceeded, unlike the Bingham fluids, the HB fluids flow with a non-linear stress–strain relationship either as a shear-thickening fluid, or a shear-thinning one. Grease fluids behave in this manner. Thus, the HB model is preferred to that of Bingham and other models because of its accurate rheological response (Kauzlarich and Greenwood, 1972, Balan and Franco, 2001, and Zhu and Neng, 1988). Many studies on grease lubrication were conducted using the HB model. For example, Kauzlarich and Greenwood (1972) derived a simplified pressure distribution equation for a grease film described by an HB model. They also validated the model experimentally. The study was limited to one dimensional contact and use of viscometer data at high shear rate than that reported in the literature were (and in fact are) needed for EHL predictions.
Jonkisz and Krzeminski-Freda (1979) presented results of theoretical and experimental studies on an elastohydrodynamic (EHD) grease film. The pressure distribution and the shape of film obtained for a grease and its base oil were shown. The shape of the grease film and the pressure distribution found for mating roller elements were similar to those of an oil EHL film. The film shape was characterised by a constriction in the outlet region. The pressure distribution for grease also showed a peak pressure positioned closer to the centre of the high pressure area than for oil under the same operating conditions. A significant feature which differentiated the flow of grease from oil was the existence of a plug flow. In the high pressure zone it was present in the whole duct. The results confirmed the postulate that values calculated using EHD formulae for the base oil of the grease are a good approximation for the thickness of film of grease film, but this approximation was only for a particular type of grease. Furthermore, the analysis was restricted to one dimension and the solution to Reynolds equation was not complete for the whole region of contact.

Film thickness comparison for fresh grease and sheared (partially degraded) grease with its base oil were carried out by Cheng (1994). The author also compared the film thickness for the Bingham and Herschel-Bulkley models and found almost the same results at low velocities, but found different results at medium velocities.

Thermal effects on the EHL performance cannot be neglected especially that at high sliding speeds these effects become very significant as shown by Yoo and Kim (1997). The authors showed that at high rolling speeds, the yield stress of the Herschel-Bulkley model has a negligible effect on the thermal EHL performance. The flow index and the viscosity parameters of the Herschel-Bulkley model were found to have significant effects on the thermal EHL performance.

For the HB fluids the studies described were limited to line contact conjunctions. The numerical solution of the modified Reynolds equation for grease remains challenging despite the advent of powerful computational techniques and platforms which is one of the motivations behind the current study.
A different kind of analysis for the sliding friction coefficient, lubricant film width and scoring occurrence conditions is reported by Drozdov (1988) at the start of the contact of metallic bodies both for line and point contact conditions. Important contact characteristics were noted to be the cumulative rolling speed, sliding velocity (until scoring is initiated) and contact compression (by Hertzian theory). Oil temperature (in the bulk volume) and on a surface of rubbing bodies at the instant of scoring moment reached the fusion temperature of the materials. Theoretical basis of design methods represented contact hydrodynamic theory of lubrication, which takes into account hydrodynamic flow of lubricant, essential dependence of viscosity of lubricant on pressure and temperature, straining of contacting bodies at the contact point, thermal process and other factors. For point contact the depth of lubricant layer in the hydrodynamic contact was suggested as:

\[
\frac{h_{c.h.}}{R_{\text{red}}} = \left(1.82 - 0.68 \frac{R_y}{R_x}\right)^{\frac{0.75}{2R_y}} \left(\frac{E_{\text{red}}}{\sigma_H}\right)^{0.25}
\]  

(2.8)

where \(R_{\text{red}}\) is the reduced radius of curvature of surfaces.

For the thermal process due to sliding and rolling bodies the thermal factor was given as:

\[
F_t = \left[1 + 0.18 \left(\frac{\alpha \eta_0 \nu_{\Sigma R}^2}{4 \lambda}\right)^{0.66} + 0.45 \left(\frac{\alpha \eta_0 \nu_i^2}{\lambda}\right)^{0.83}\right]^{-1}
\]

(2.9)

However, Drozdov's analyses were restricted to the hydrodynamic regime of lubrication which happens to occur mostly in the conformal contacts, where the pressures are not high, when compared with EHL.

Most work reported in literature for study of grease lubrication are experimental. The main difference between the numerical studies and experiments is that for numerical models the entire flow is considered to be continuous, i.e. grease flows without any
breakdown, while in experiments the flow comprises the base oil with a soap partitioned by shear forces at the inlet. Hence, most of the experiments reported in literature concentrate on the rheology of grease.

The behaviour of grease inside the EHL contact was studied using optical interferometry by Wedeven et al (1971) and Palacios et al (1981). Both studies showed that the film thickness of grease under fully flooded conditions is always thicker than that of a base oil. Kageyama et al (1984) in their comprehensive study of grease composition showed that the film thickness increases with base oil viscosity and soap concentration, which is in line with the findings of Muennich and Gloeckner (1980). Cann et al (1992) showed that when grease is fully shear-degraded by its passage through an inlet, its structure behaves as discrete spherical soap particles, dispersed in the base oil. In such a case the viscosity of grease can be given as that proposed by Mansot et al (1989):

$$\eta_{\text{Grease}} = \eta_{\text{Base oil}} \left(1 + B\Phi\right)$$

(2.10)

where $B$ is a constant, the value of which is obtained from Einstein’s equation for a suspension of non-interacting spheres. $\Phi$ is the volume fraction of soap in the base oil. It has been proven experimentally that this method can be used for calculating the viscosity of grease as is the case in the current study.

Film thickness measurements for greases under conditions of elastohydrodynamic lubrication were carried out by Dyson and Wilson (1969). The thickness of a film formed between two discs was measured by its electrical capacity. Films formed by greases were initially thicker than those formed by the corresponding base oils, but after continuous running they became thinner than the oil films. The authors showed that the differences between the oils and the greases were related to the viscoelastic properties of the lubricants. However, the capacitance method used by the authors can only give an average value and any deviations from a parallel shape can lead to errors. Wedeven et al (1971) considered the starvation effect for point contact and were able to predict the film thickness reduction from theoretical unstarved values.
Wilson (1979) presented a method for measuring the thickness of grease films between rollers and rings of rolling element bearings. Measurements in two designs showed that grease can initially form thicker films than the base oil, but without frequent replenishment, the grease films became slightly thinner with time. The author found that the bearings after about 10 hours of running stabilised at partially starved conditions with half the film thickness of a fully flooded condition. The film thickness remained unaltered by a temperature drop from 60°C to 40°C. This may indicate that the soap particle layer contributes to lubrication. This was explained by Cann et al (1991). Using IR spectrometry and optical interferometry they found that the thickener material in the form of soap particles built up a layer 50nm thick in the track during fully flooded conditions, which increased the surface separation under starvation conditions.

Palacios et al (1981) used optical interferometry for measuring film thickness of greases and of soap suspensions and their base oils. Samples were taken from bearings after 25 million revolutions. The authors found that grease behaved the same as the oils. The film thickness varies with 0.67 power of the speed in line contact and 0.74 in point contact, quite similar to EHL conditions. The results were in line with experimental findings by Foord et al (1969) and Wymer and Cameron (1974).

Optical interferometry has been used widely in measuring grease characteristics inside the contact conjunction. Kageyama et al (1984) observed local fluctuations in the film thickness of their grease-lubricated point contact. Cann et al (1991), Cann and Spikes (1991) detected a layer of soap fibres in the track behind their grease-lubricated point contact, indicating that soap-fibre formations enter and pass through the contact area. Astrom (1992) observed that the local film thickness fluctuated in their point contact lubricated with grease. The fluctuations were caused by the soap-thickener formations that entered and passed through the EHD contact.
Cann et al (2007) studied grease degradation in bearings. One of the conclusions from this work was that the lubricant in the ball/raceway contact is heavily degraded grease and although a thickener is usually present the concentration depends on bearing type and operating conditions. In many cases oxidation of the base oil and the thickener is observed in the cage pocket and raceway lubricant after a relatively short running period. Analysis was restricted because of a shortage of samples suitable for a wider range of analytical techniques. This led to poor simulation of conditions within a bearing, where thin grease films and mechanical shear stresses were present.
Chapter 3

ANALYTICAL & NUMERICAL MODELLING OF BALL BEARING

3.1. Introduction

This chapter describes closed form solutions (i.e. analytical solutions) for ball bearing dynamics. The latter part of the chapter provides numerical solution of system behaviour, which is not attained by closed form solutions. Ball-race interactions are formulated as these accounts for the main loaded regions in bearings. Other interactions between the cage and races and race and bearing elements are ignored.

3.2. Calculation of Analytical Bearing Frequencies

This section provides an analytical model for prediction of bearing-induced spectral contributions, based on the works reported by Wardle (1988a), Wardle (1988b) and Lynagh et al (2000). Ball bearing frequencies can generally be classified as primary and secondary contributions. Frequencies produced by vibrations imparted in the system due to the interaction of system elements such as shaft, balls, cage and races fall under primary frequencies. Manufacturing anomalies such as surface waviness in the shaft, rolling bodies and races also give rise to vibrations. These are regarded as secondary frequencies. These frequencies are usually very low in amplitude and are only significant when dealing with precision applications. In this study high precision routing spindles are used. Thus, these secondary frequencies can lead to premature resonances, particularly at high speeds. A schematic diagram of a ball bearing is shown in Figure 3.1.

3.2.1. Inherent/Primary bearing frequencies:

These are vibration frequencies which are generated even by a defect-free rolling element bearing. They are known as ball-pass frequency (i.e. relative speed of the rolling elements with respect to the races), cage frequency (untrue running of a cage) and rolling element frequency (rotational frequency of a rolling element).
Figure 3.1: Schematic of a Ball Bearing

**Ball pass frequency:** when the shaft is rotated the applied loads are distributed only through a subset of balls, creating a loaded region as shown in Figure 3.2. Balls passing through this region are subject to elastic deformation, where the mutual convergence of races takes place. Elastic recovery takes place when the balls move out of this region. This causes the load distribution on the shaft to change, thus resulting in a relative movement between the outer and inner races.

Figure 3.2: Loaded region in bearing
Ball pass frequencies for the inner and the outer races are expressed analytically by:

Ball-pass frequency relative to the outer race:

$$f_{bo} = \frac{1}{2} f_s \left( 1 - \frac{D}{d_c} \cos \alpha \right) z$$  \hspace{1cm} (3.1)

Ball-pass frequency relative to the inner race:

$$f_{bi} = \frac{1}{2} f_s \left( 1 + \frac{D}{d_c} \cos \alpha \right) z$$  \hspace{1cm} (3.2)

**Cage rotating frequency:** Cage or separator with a loose fit and with sparse lubrication provides insufficient damping of cage motions. The cage may then rattle as it moves erratically and experience shocks with changing speeds of the balls when entering and leaving the loaded zone. These vibrations can be expressed analytical as:

$$f_c = \frac{f_s}{2} \left( 1 - \frac{D \cos \alpha}{d_m} \right)$$  \hspace{1cm} (3.3)

This effect also takes place when the rotor or shaft has an out-of-balance motion, as indicated by the frequency $f_s$.

With a loosely fitted cage and with insufficient lubrication in the cage pockets for damping, bending of the cage may also occur. This excited structural modes of the cage, which can lead to noise emission in the form of a disturbing squeal or a “bird-chirping” sound. This makes the cage to be the greatest source of noise in rolling bearings.

**Rotational frequency of the rolling elements:**

$$f_b = \frac{1}{2} f_s \frac{d_c}{D} \left( 1 - \frac{D^2}{d_c^2} \cos \alpha \right)$$  \hspace{1cm} (3.4)
Very often and especially when the load is varied, vibration at other frequencies cause excitation in the bearing. These frequencies are the harmonics and a combination of the sum and difference of the preceding frequencies.

### 3.2.2. Secondary bearing frequencies

As mentioned in the earlier sections these are the frequencies produced due to the geometrical imperfections on the bearing surfaces. Contact surface waviness is undulations with wavelengths greater than the Hertzian contact dimensions. Off-sized balls in a complement of balls also introduce contributions at integer multiples of the cage speed. Vibration and noise generated by these are regarded as secondary effects, which can modulate with the principal bearing frequencies, resulting in the emergence of side bands Wardle (1988a), Wardle (1988b). The effect of these secondary sources is more pronounced at higher speeds, still within the region of interest in the current study (up to 5 kHz). The causes for these can be defects on raceway tracks, often caused by wear (created by depletion of a coherent lubricant film). They can also include forced brinelling as a result of ball impacts upon the raceway surfaces (induced by shock loads), surface pits or cracks caused by high Hertzian stresses or sub-surface fatigue spalls.

The ball loads are determined by the balls-to-races contact spring non-linearity as described by Hertz (1896):

\[ W_i = K \delta_i^{3/2} \] (3.5)

The effective stiffness \( K \) is the combined stiffness of a ball to inner race and outer race contacts and is obtained as (Harris (1989)):

\[
K = \left[ \frac{1}{\left( \frac{1}{K_{ir}} \right)^{2/3} + \left( \frac{1}{K_{or}} \right)^{2/3}} \right]^{3/2} 
\] (3.6)
where the stiffness of a ball to inner race contact is (Rahnejat, 1984):

\[ K_{ir} = K_c \times \frac{e'}{\lambda^3} \]  

(3.7)

\( K_c \), being the stiffness due to material properties. Whitemore and Petrenko (1921) have given the mathematical expression for finding \( K_c \) as:

\[ K_c = \frac{8}{3} \frac{E_1 E_2}{E_2 (1 - \nu_1^2) + E_1 (1 - \nu_2^2)} \]  

(3.8)

\( E_1 \) and \( E_2 \) are the elastic moduli of the materials of the ball and the raceway, and \( \nu_1, \nu_2 \) are the Poisson's ratios of the same. \( e' \) is the curvature sum, for two bodies in counterformal contact (as shown in Figure 3.3) and is given as:

\[ e' = \frac{4}{\frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'}} \]  

(3.9)

\( R_1 \) is the radius of the race (inner or outer) \( R_1' \) is the radius of the curvature of the raceway groove, and \( R_2 \) and \( R_2' \) are the radii of curvature of the rolling body in the respective planes of contact as shown in Figure 3.3.

In the case of a ball bearing \( R_2 \) and \( R_2' \) are the same; therefore, equation (3.9) can be modified to:

\[ e' = \frac{4}{\left( \frac{1}{R_1} + \frac{1}{R_1'} \right) + \left( \frac{2}{R_2} \right)} \]  

(3.10)

The constant \( \psi' \) in the equation below is necessary in order to determine obtain the value of \( \lambda \) from Table 3.1.
Figure 3.3: General case of two bodies in counterformal contact

In case of ball bearings $R_2 = R_2'$, hence the equation can be reduced to:

$$
\psi' = \arctan \left[ \frac{e'}{4 \sqrt{\left( \frac{1}{R_1} - \frac{1}{R_i} \right)^2 + \left( \frac{1}{R_2} - \frac{1}{R_{i'}} \right)^2 + 2 \left( \frac{1}{R_1} - \frac{1}{R_i} \right) \left( \frac{1}{R_2} - \frac{1}{R_{i'}} \right) \cos 2\varphi'}}{1 + \left( \frac{1}{R_1} - \frac{1}{R_i} \right) \left( \frac{1}{R_2} - \frac{1}{R_{i'}} \right) \cos 2\varphi'} \right] 
$$

(3.11)

When the value of $\psi'$ is known, the corresponding value for $\lambda'$ can be obtained from the Table 3.1 given below.
Table 3.1: Input Parameters (after Whitemore and Petrenko (1921))

<table>
<thead>
<tr>
<th>( \psi ' )</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>35°</th>
<th>40°</th>
<th>45°</th>
<th>50°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ' )</td>
<td>Infinite</td>
<td>6.612</td>
<td>3.778</td>
<td>2.731</td>
<td>2.397</td>
<td>2.136</td>
<td>1.926</td>
<td>1.754</td>
</tr>
<tr>
<td>( \beta ' )</td>
<td>0</td>
<td>0.319</td>
<td>0.408</td>
<td>0.493</td>
<td>0.530</td>
<td>0.567</td>
<td>0.604</td>
<td>0.641</td>
</tr>
<tr>
<td>( \lambda ' )</td>
<td>-</td>
<td>0.851</td>
<td>1.220</td>
<td>1.453</td>
<td>1.550</td>
<td>1.637</td>
<td>1.709</td>
<td>1.772</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \psi ' )</th>
<th>55°</th>
<th>60°</th>
<th>65°</th>
<th>70°</th>
<th>75°</th>
<th>80°</th>
<th>85°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ' )</td>
<td>1.611</td>
<td>1.486</td>
<td>1.387</td>
<td>1.284</td>
<td>1.202</td>
<td>1.128</td>
<td>1.061</td>
<td>1.00</td>
</tr>
<tr>
<td>( \beta ' )</td>
<td>0.678</td>
<td>0.717</td>
<td>0.759</td>
<td>0.802</td>
<td>0.846</td>
<td>0.893</td>
<td>0.944</td>
<td>1.00</td>
</tr>
<tr>
<td>( \lambda ' )</td>
<td>1.828</td>
<td>1.875</td>
<td>1.912</td>
<td>1.944</td>
<td>1.967</td>
<td>1.985</td>
<td>1.996</td>
<td>2.00</td>
</tr>
</tbody>
</table>

For this study a heavily pre-loaded bearing is used. The ball-to races contact deflections are dominated by the radial interference fit and thus a uniformly spread loaded region initially results. This indicates that in the case of a perfect bearing in a vertical spindle no perturbations in the values of bearing loads and moments would occur under steady state conditions, Rahnejat and Gohar (1985), Vafaei et al (2002) and Vafaei and Rahnejat (2003) have shown that for practical situations applied loads (such as the cutting forces) vary and manufacturing defects on the rolling elements, surfaces, as well as assembly related faults (eccentric rotation of rotor centre with respect to the geometric centre of the bearing) introduce a more defined dynamic

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loaded region. Therefore, deflection $\delta_i$ in equation (3.5) is a function of instantaneous movements of the shaft centre in the horizontal radial plane; $xy$ and the axial direction, $z$. so the deflection is given as:

$$\delta_i = x \cos \theta_i \cos \alpha + y \sin \theta_i \cos \alpha + z \sin \alpha - \frac{C}{2}$$

(3.13)

where: $\theta_i = \omega t + iy = 2\pi f t + iy$ and $y = \frac{2\pi}{N}$ is the ball circumferential spacing.

The diametral clearance, $C$, for a perfect bearing is given as:

$$C_0 = 2(r_e + r_i - d)(1 - \cos \alpha_0)$$

(3.14)

The surface waviness of bearing rings and balls introduces a variation in the diametral clearance $C$, being a function of the number of waves and the corresponding speed of the appropriate members. Figure 3.4 shows the case for the inner race waviness. Thus, according to Rahnejat (1984):

$$C = 2 \left\{ r_o + a_o \sin \left( \frac{2\pi l_o}{\lambda_o} \right) + r_i + a_i \sin \left( \frac{2\pi l_i}{\lambda_i} \right) - d - 2a_b \sin \left( \frac{2\pi l_b}{\lambda_b} \right) \right\} (1 - \cos \alpha)$$

(3.15)

where in general, $l = R\theta$ (see Figure 3.4) and $R$ is the radius of the rolling member (races or ball). Surface waviness occurs on the inner and outer raceway tracks, as well as on individual balls. Therefore, the wavelength, $\lambda$, for each surface is given as:

$$\lambda = \frac{2\pi R}{N}$$

(3.16)
Replacing in equation (3.13) yields:

\[ \delta_i = x \cos \theta_i \cos \alpha m + y \sin \theta_i \cos \alpha + z \sin \alpha - \frac{1}{2} \{ r_0 + a_0 \sin \phi_0 + r_1 + a_1 \sin \phi_1 - d - 2a_0 \sin n \phi_0 \} \]

(3.17)

For waves on the outer raceway:

\[ \phi_0 = \omega_x t + i\gamma \]

(3.18)

And for those on the inner race:

\[ \phi_1 = (\omega_x - \omega_y) t + i\gamma \]

(3.19)

Ball waviness also influences the internal clearance, thus:

\[ \phi_b = \omega_b t + i\gamma \]

(3.20)

Assuming no slip at the points of contact between the balls and the inner and
Substituting from (3.13)-(3.15) into (3.12) leads to (letting: \( n_o = n_l = n \) and \( n_b = m \)):

\[
\delta_i = \left\{ x \cos (\omega_2 t + i \frac{2\pi}{N}) + y \sin (\omega_2 t + i \frac{2\pi}{N}) \cos \alpha \right\} + z \sin \alpha - \frac{1}{2} \{r_o + a_o \sin k(N \omega_2 t + i2\pi) \\
+ r_l + a_l \sin k[N(\omega_2 - \omega_l) t + i2\pi] - d - 2a_b \sin j(2\omega_b t + i2\pi) \}
\]

(3.22)

where: \( k = n/N \) and \( j = m/N \), \( j = k = 1, 2, 3, \ldots \)

It is, therefore, clear that contact deflection and load are functions of contact kinematics and waviness of surfaces. Therefore, the bearing generated forces and moments are functions of the principal frequencies: \( k\omega_o \), \( k(\omega_2 - \omega_l) \) and \( j2\omega_b \). These interactions appear as side-bands in the vibration spectrum. For an off-sized ball one may let \( j=0 \) and assume that \( 2a_b \) represents the difference in diameter between one of the balls and the remaining set as suggested in Wardle (1988a) and Lynagh et al (2000). The side-band contributions are:

- outer race waviness ± ball size = \( kNf_c \pm jf_c \)  
- outer race waviness ± ball waviness = \( kNf_c \pm jf_b \)  
- outer race waviness ± inner race waviness = \( kNf_c \pm jf_s \)  
- Inner race waviness ± ball size = \( kN(f_s - f_c) \pm f_c \)  
- Inner race waviness ± ball waviness = \( kN(f_s - f_c) \pm j2f_b \)  

(3.23)  
(3.24)  
(3.25)  
(3.26)  
(3.27)

Although the analytical model is now widely used in the field, it does not provide prediction of contact conditions, such as deflection and load under transient conditions. Such data are required for subsequent studies of bearing lubrication, wear, friction and thermal performance, which are essential for improved efficiency. The numerical model can predict the necessary data for such analyses. Furthermore, the
analytical model is not sufficiently detailed to predict amplitude contributions at given spectral contents. So the next section will deal with numerical modelling of high precision bearings used in spindles.

3.3. Numerical Modelling: 2 DOF Bearing Model

A systematic approach is employed for numerical modelling of a ball bearing throughout this study, starting with a deep groove ball bearings considered to be a two degrees of freedom (DOF) system. For this a horizontal spindle is considered and validated with available literature of this kind. The next step is a model of an angular contact ball bearing system with five degrees of freedom. Walters (1971) and Gupta (1979) presented numerical analysis of ball bearings incorporating balls to cage and cage to races interactions. The forces generated due to interactions between balls to cage and cage to races are found to be small when compared with forces generated due to interactions between the rolling elements and races. Hence, those interactions are neglected in this study. Throughout this study balls are considered as mass-less springs (shown in Figure 3.5) and the contacts act as non-linear contact springs. Since the Hertzian forces arise only when there is contact deformation, the springs act only when in compression. In other words, the respective spring force comes into play when the instantaneous spring length is shorter than its unstressed length. Otherwise, separation occurs with no reaction. The system undergoes non-linear vibrations under dynamic conditions. In order to evaluate vibrations, the force acting on the shaft is determined with the assumption that the forces induced by the lubricant film damping are neglected, which were found to be insignificant by Dareing and Johnson (1975) and Mehdigoli et al (1990).
Figure 3.5: Bearing system represented by elastic model

A non-linear elastic model is developed with the following assumptions:

1. The spindle is considered to be rigid, thereby, eliminating its natural elastic modes. This assumption narrows the response characteristics of the spindle to its first natural mode in the various degrees of freedom (bounce, tilt and pitch). Some studies have included modal behaviour of shaft/rotor, for example Matsubara et al (1988).

2. The \( N \) balls in each bearing set are mass-less. This eliminates the need for solving for further \( n \) degrees of freedom, which otherwise requires the discretisation of the mass of the spindle to obtain its contribution to each of these radial degrees of freedom.

3. The \( N \) balls in the bearing set are equi-pitched around the inner ring.

4. The centrifugal effects and torsional vibrations of the spindle are neglected.

5. The angular velocity of the cage in the bearing is assumed to be constant. No interactions with cage are allowed.
6. Sources of damping like the hydrodynamic or elastohydrodynamic films at the ball contacts, as well as, any structural damping within the assembly are ignored. The latter may be significant. Thus, the results are accurate for frequency composition and not amplitudes.

7. Due to an elliptical elastostatic footprint, the ball-to-races contact loads are obtained according to the Hertzian theory.

8. Both bearings are positioned symmetrically such that they rotate in the same sense simultaneously. In other words, the balls are assumed to be always in phase.

9. In the first instance, the contacts are considered as frictionless.

![Figure 3.6: Deep groove ball bearing force consideration](image)

The total restoring force at the point of contact of the rolling element with the inner and the outer raceways according to Hertzian theory is given by equation (3.5). The projection of this restoring force (shown in Figure 3.6) on the line of action of applied radial load in the bearing is:
\( W_{x_i} = K \delta_i^{3/2} \cos \theta_i \)  

(3.28)

and projection of load in lateral direction is:

\( W_{y_i} = K \delta_i^{3/2} \sin \theta_i \)  

(3.29)

where \( \theta_i \) is the angle between the lines of action of vertical load and the radius passing through the geometric centre of the \( i^{th} \) rolling element and is given by the relationship:

\( \theta_i = \phi_c + i \gamma \)  

(3.30)

and \( \phi_c \) is the turning angle of the cage:

\( \phi_c = \omega_c t \)  

(3.31)

\( \gamma \) is the angular distance between two successive rolling elements/balls

\( \gamma = \frac{2\pi}{N} \)  

(3.32)

Where \( z \) is the number of rolling bodies in the bearing.

3.3.1. Geometrical Constraints and Equations of Motion

The shaft supported by the bearings is considered to be a rigid with two degrees of freedom (i.e. undergoes movements in the X and Y directions as shown in Figure 3.7). The shaft is assumed to be perfectly and initially concentric with the supporting bearing. The total deflection (mutual convergence of the rings) in the radial direction of the \( i^{th} \) rolling element, when the shaft centre is displaced by amounts \( X \) and \( Y \) (shown in Figure 3.8) is given as:

\[ R_0 - (R_i + 2r) = Y \sin \theta_i + X \cos \theta_i - 2\delta_i \]  

(3.33)

Thus:
\[ 2\delta_i = X \cos \theta_i + Y \sin \theta_i + 2\rho \quad (3.34) \]

\[ R_o - (R_1 + 2R_2) = -2\rho \quad (3.35) \]

where, \( \rho \) is the extend of interference fit employed.

---

**Figure 3.7:** Geometrical considerations for modelling

**Figure 3.8:** Free body diagram for deep groove ball bearing
If \( N \) is the number of balls, the system has \( 6N+2 \) degrees of freedom (6 means three translational and three rotational) motion and would require discretisation of mass of the shaft in each of these degrees of freedom. If mass-less ball assumption is made, then the need for these degrees of freedom is removed. Otherwise, the analysis of system dynamics becomes quite complicated, but necessary for very large bearings. In the current analysis such need does not arise. Later the degrees of freedom are increased to five for analysing angular contact ball bearings. Referring to Figure 3.8, the forces in the vertical direction for acting through centre of shaft is:

\[
M\ddot{y} = Mg - W_i \sin \theta_i
\]  

(3.36)

Thus, the total forces acting on the system in the vertical radial direction is:

\[
M\ddot{y} + \sum_{i=1}^{m} W_i \sin \theta_i = Mg
\]  

(3.37)

Similarly, in the horizontal radial direction:

\[
M\ddot{x} + W_i \cos \theta_i = 0
\]  

(3.38)

And the total forces in the horizontal radial direction is

\[
M\ddot{x} + \sum_{i=1}^{m} W_i \cos \theta_i = 0
\]  

(3.39)

This is true for a deep groove ball bearing, where the contact angle is zero. Where \( F(t) \) is a cyclic radial load, \( M \) is the proportion of mass of the rigid shaft held by one bearing support and \( N \) is the number of balls.

### 3.3.2. Computational Solution of the Equations of Motion

The governing equations of motion in two degrees of freedom system are solved by an iterative marching procedure, employing a third order quasi-linear method known as average acceleration technique proposed by Newmark (1959) (also see Rahnejat (1985)). Since precision ball bearings are used in high speed routing spindles, manufacturing imperfections of the contact surfaces are considered to be small.
Hence, the imperfections are ignored in the numerical model. A computer program in Fortran 90 is developed for the solution and the results are reported later.

3.3.2.1. Newmark Linear Acceleration Method

The computational model proposed by Rahnejat (1985) for ball bearing dynamics was developed by Newmark (1959) and is used in this thesis. The method uses a step-by-step iterative integration algorithm to solve the second order time dependent differential equations of motion. The solution using this method is found to be comprehensive, providing complete time history of many unknown system parameters.

Successive integrations of equations (3.38) and (3.39) enable the evaluation of $x$, $\dot{x}$, $y$ and $\dot{y}$ by applying appropriate initial conditions:

\[
\dot{y}_{1,j} = \dot{y}_0 + \dot{y}_0 dt 
\]  
\[
\dot{y}_{2,j} = \dot{y}_0 + 2\dot{y}_0 dt 
\]  
\[
\dot{y}_{k,j} = \dot{y}_{k-2} + 2\dot{y}_{k-1} dt 
\]

where $k$ corresponds to the time interval, $j$ is an iteration index and $dt$ is a fixed interval of time. Then:

\[
y_{1,j} = y_0 + 2\dot{y}_0 \frac{dt}{3} + \dot{y}_0 \frac{dt^2}{6} + \dot{y}_{1,j} \frac{dt}{3} 
\]  
\[
y_{k,j} = y_{k-1} + 2\dot{y}_{k-1} \frac{dt}{3} + \dot{y}_{k-1} \frac{dt^2}{6} + \dot{y}_{k,j} \frac{dt}{3} 
\]

Similarly, integration expressions are used for $x$ and $\dot{x}$ as:
\[ \dot{x}_{i,j} = \dot{x}_0 + \ddot{x}_0 \, dt \]  
(3.45)

\[ \dot{x}_{2,j} = \dot{x}_0 + 2\ddot{x}_0 \, dt \]  
(3.46)

\[ \dot{x}_{k,j} = \dot{x}_{k-2} + 2\ddot{x}_{k-1} \, dt \]  
(3.47)

\[ x_{i,j} = x_0 + 2\dot{x}_0 \frac{dt}{3} + \ddot{x}_0 \frac{dt^2}{6} + \dot{x}_{i,j} \frac{dt}{3} \]  
(3.48)

\[ x_{k,j} = x_{k-1} + 2\dot{x}_{k-1} \frac{dt}{3} + \ddot{x}_{k-1} \frac{dt^2}{6} + \dot{x}_{k,j} \frac{dt}{3} \]  
(3.49)

The equation for deflection (3.34) is rearranged to calculate the values of deflection \( \delta_i \) for \( i = 1 \) to \( m \) for each ball as:

\[ \delta_i = \frac{1}{2} \left( x \cos \theta_i + y \sin \theta_i + 2\rho \right) \]  
(3.50)

The instantaneous reactions are obtained for each rolling body and the dynamic equations of motion (3.34) and (3.35) yield the acceleration of the centre of the shaft in the \( x \) and \( y \) directions. This procedure is repeated within a suitably selected time step until the unknown variables such as \( x = x(t) \) and \( y = y(t) \) are calculated within specified error bounds. For example:

\[ \ddot{x}_{k,j} - \ddot{x}_{k,j-1} \leq \varepsilon_x \]  
(3.51)

\[ \ddot{y}_{k,j} - \ddot{y}_{k,j-1} \leq \varepsilon_y \]  
(3.52)

where \( \varepsilon_x \) and \( \varepsilon_y \) are the specified limits of accuracy.

Since two bodies in contact cannot exert tensile forces upon each other utmost care is taken in programming that no reactions constituting state of tension is generated. Before using the model, it is important to check the validity of the numerical model.
For this purpose the numerical model predictions are verified against analytical solutions reported above.

### 3.3.2.2. Initial Conditions

For numerical solutions, the initial conditions and step size are very important for successful and economical computations. The larger the time step, the faster the computation time, but this can result in loss of time history and computational instability. On the other hand, the time step should be small enough to achieve an adequate level of accuracy within acceptable computation times. Also, very small time steps can increase the truncation errors.

At time \( t = 0 \) the following assumptions are made (initial conditions):

The shaft is held at the centre of the bearing such that there is no net radial load on the shaft and all balls are assumed to have an equal interference fit, thus:

\[
x_0 = y_0 = x_0 = y_0 = 0 \quad (3.53)
\]

and:

\[
2\delta_0 = 2\rho \quad (3.54)
\]

where \( 2\delta_0 \) is the total initial elastic deformation in the radial direction of the \( i^{th} \) rolling body caused by interference fitting. When shaft is released from this initial position the centreline of the shaft undergoes initial accelerations in the X and Y directions, thus:

\[
M\ddot{y} + \sum_{i=1}^{m} W_i \sin \theta_i = Mg \quad (3.55)
\]

\[
M\ddot{x} + \sum_{i=1}^{m} W_i \cos \theta_i = 0 \quad (3.56)
\]
3.3.2.3. Solution Procedure

Figure 3.9: Flow chart for the solution procedure
Procedure used for the solution is shown in the flowchart of Figure 3.9. The equations of motion were solved using the code developed in FORTRAN and the various results for the non-linear elastic model for a deep groove ball bearing are obtained.

3.3.2.4. Results and discussions

A deep groove ball bearing with specifications list in Table 3.2 is used for numerical modelling.

**Table 3.2: Deep Groove ball bearing specifications**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball diameter</td>
<td>4.762mm</td>
</tr>
<tr>
<td>Number of balls</td>
<td>23</td>
</tr>
<tr>
<td>Contact angle</td>
<td>0°</td>
</tr>
<tr>
<td>Pitch diameter</td>
<td>48.698mm</td>
</tr>
<tr>
<td>Inner race diameter</td>
<td>43.726mm</td>
</tr>
<tr>
<td>Outer race diameter</td>
<td>53.46mm</td>
</tr>
<tr>
<td>Material of balls</td>
<td>Ceramic alumina</td>
</tr>
</tbody>
</table>

The shaft is initially held to counteract the effect of gravity. The bearings are uniformly deformed in the contact directions by preloading and interference fitting. The initial deflections due to interference fitting and preloading are calculated. At the instance of release the initial velocities and displacements are considered to be zero, i.e.: \( x_0 = y_0 = \dot{x}_0 = \dot{y}_0 = 0 \). The spindle is released to run at a speed of 8000 rpm. The components of contact loads for each ball are calculated in the X and Y directions. The governing equations of motion, given by equations (3.55) to (3.56) are solved simultaneously for obtaining accelerations in X and Y directions of motion. This iterative process for each time step is terminated if all of the following convergence criteria are simultaneously satisfied. If not satisfied all the above steps are repeated.

\[
\frac{\ddot{x}_{k,j} - \ddot{x}_{k,j-1}}{\ddot{x}_{k,j-1}} \leq \varepsilon_x \quad (3.57)
\]
Load deflection characteristics of ball to race contact for ball bearing is given by equation 3.5. The comparison of theoretically obtained load deflection characteristics with numerically obtained for a stiffness value of $20\,\text{GN/m}^{3/2}$ is shown in Figure 3.10. At higher loads the contact deflections seems to be deviating a little from the theoretical value this can be attributed to the computational errors. Figure 3.10 and Figure 3.11 show the numerical results for the shaft centre oscillations along X and Y directions. The maximum amplitude of vibration along the X axis is 2.5 $\mu\text{m}$ and the same for Y direction is 32 $\mu\text{m}$. This difference in amplitude of vibrations is due to the fact that in x axis direction no force is acting. Since no force is acting it is interesting to see the effect of interference fit, which does not allow the amplitude to go below 2 $\mu\text{m}$. In Y direction the weight of the shaft acting downwards displaces the centre by 15 $\mu\text{m}$, which is shown in the Figure 3.13. This will create an unbalance inside the system and there will be restoring force acting in the system trying to get into the equilibrium as soon as possible. This makes the shaft to vibrate and the maximum amplitude of vibration recorded from the mean centreline of 15 $\mu\text{m}$, is 7 $\mu\text{m}$.
Figure 3.11: Shaft centre oscillations in X direction

Figure 3.12: Shaft centre oscillations in Y direction

Figure 3.13: Shaft centre Locus

Figure 3.14: Phase plane representation
Figure 3.13 is the locus of the shaft centre, showing a shift of 3 μm which is due to the interference fit. Figure 3.12 shows the phase plane plot in the Y direction. The existence of a loaded region is quite clear. The initial dip in the plot is due to the effect of gravity. Since the shaft is assumed to be horizontal, the weight of the shaft always acts downwards, making the applied load to be distributed only through a few of the balls, creating a loaded region as shown in Figure 3.2. When the balls pass through this region, they are subjected to localised Hertzian elastic deformation, where the mutual convergence of races takes place. Thereafter, the balls undergo elastic recovery. Sunnersjo (1978) explained this phenomenon as the variable compliance effect. As stated in the earlier sections this numerical model is expanded further to incorporate the analysis of high precision angular contact bearings. Prior to this it is important to validate the predictive method this is carried out below.

### 3.4. Model validation

For the purpose of validation of the numerical model presented in the previous section, its predictions are compared with the work presented by Rahnejat (1984). The detail of the bearing and spindle assembly used by Rahnejat (1984) are listed below.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bore</td>
<td>40 mm</td>
</tr>
<tr>
<td>Inner race diameter</td>
<td>50 mm</td>
</tr>
<tr>
<td>Pitch diameter</td>
<td>56.3 mm</td>
</tr>
<tr>
<td>Outer race diameter</td>
<td>75.4 mm</td>
</tr>
<tr>
<td>Ball diameter</td>
<td>12.7 mm</td>
</tr>
<tr>
<td>Outside diameter</td>
<td>83.7 mm</td>
</tr>
<tr>
<td>Width</td>
<td>23.4 mm</td>
</tr>
<tr>
<td>Mass of shaft</td>
<td>1000 kg</td>
</tr>
</tbody>
</table>
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Figure 3.15: Shaft centre line time history for 5 balls

Figure 3.16: Shaft centre line time history for 6 balls

Figure 3.17: Shaft centre line time history for 7 balls

Figure 3.18: Shaft centre oscillations for different balls
Rahnejat (1984) reported numerical simulations for a ball bearing, with the ball-race contacts considered as linear rolling springs. Even though he has reported simulations for many conditions, for validation purposes three runs with different number of balls are considered here. Figures 3.15 to 3.18 show the comparative study. The oscillations in the transverse direction are virtually negligible, hence not considered. Figure 3.15 shows the shaft centre vibrations for a ball bearing with 5 balls. The blue lines show the numerical results using the model presented in this thesis and the black dotted lines represent the work reported by Rahnejat (1984). The maximum amplitude of vibration reported by Rahnejat (1984) is 270 μm, whereas that predicted here is 260 μm.

This vertical displacement creates a loaded zone and imparts vibration in the system as the balls through it. There is a phase shift accounting for 7 milli-seconds time lag in the time histories reported by Rahnejat (1984). These differences are very marginal and insignificant. They may be due to the differences in computing machines used.

As the number of balls increases the overall system vibrations seems to decrease. This is seen in Figures 3.16 and 3.17, as closer packed arrangements restrict the oscillations. This fact is clearly shown in Figure 3.18. The comparative study shows very good agreement with the numerical model presented by Rahnejat (1984).

3.5. Numerical Modelling of High speed precision spindles

The demand for higher productivity and improved quality of the machined surfaces requires spindle designers to aim for higher spindle speeds and feed rates. The high speed machine tool spindles used in routing operations require high dynamic stiffness to avoid vibration, which if not attained, can have dramatic damaging effects on surface finish, wheel wear, and form-holding. The spindle experimental vibration spectrum at high speeds is a combination of various modulation effects, which makes it extremely difficult to successfully isolate and attribute certain effects to specific causes even with latest diagnostic techniques. The need to develop a realistic bearing theoretical simulation model is, therefore, of high importance since bearings are the major sources of vibration spindles.
The first simulation model in this respect was from Rahnejat and Gohar (1985) for a rigid rotor supported with a pair of deep groove ball bearings with two degrees of freedom, as shown in previous chapter. The natural frequency of a perfect spindle/bearing assembly was found to be dependent upon the spindle mass and the bearing stiffness and independent of rotational speed. Bearing stiffness was found to be dependent upon its radial clearance and number of balls as observed by Wardle et al. (1983). Mastubara et al. (1988) modified Rahnejat and Gohar's model by incorporating the modal response of the elastic shaft and some damping effect. Five degrees of freedom analysis of an asymmetrical spindle, supported by a pair of back-to-back angular contact ball bearings was devised by Aini et al. (1990) with applied axial and radial loads. The model was used by Aini (1990) to simulate the vibration characteristics of a precision grinding spindle with metallic ball/raceways. Major frequencies and a number of design curves, suggesting the "optimal" zones of operation for the simulated spindle under axial/radial loading were also presented. Rahman et al. (2002) used this model to study free vibration response for a 3 bearing spindle. Theoretical analysis was also carried out by Rahman et al. (2002) for different bearing settings and preload and their effects on the system natural frequencies. Aini et al. (2002) extended their five degree of freedom model by including the squeeze film effect caused by the mutual convergence of bearing rings. The overall system response, when subjected to varying spindle mass or the number of balls in the support bearings was also studied. The overall contribution to damping by the elastohydrodynamic oil films between the rolling elements and their raceways was shown to be slight.

All the studies reported above are for horizontally mounted spindle units; these require no external sources of excitation owing to the partly loaded region of the bearing. The vibrations generated are known as variable-compliance vibrations, which has been reported by Sasaki et al. (1960) and Wardle et al. (1983). In case of a vertically mounted spindle units the non-existence of a loaded region reduces the vibration, but the inherent features of the bearings distributing the loads through the rolling elements and periodic variation of stiffness generates vibration. The need for a numerical model for the dynamics of a perfect ball bearing in vertical spindle is important, since bearings are major sources of vibration. Here a numerical model is
presented for a high speed precision spindle, supported by two pairs of angular contact ball bearings arranged in tandem at top and bottom ends of the spindle. Modelling of bearings is achieved by considering the contacts to be dry in this initial study.

Elastic contact modelling of angular contact ball bearings are the same as those already described for deep groove ball bearings, except that the contact forces act in the contact angle direction, thus contribute to the resistive components in the axial direction of the shaft/rotor. The details of the angular contact ball bearing used for the numerical simulation are given in Table 3.4

| Table 3.4: Angular contact ball bearing specifications |
|-----------------|-----------------|
| Ball diameter   | 4.762mm         |
| Number of balls | 23              |
| Contact angle   | 15°             |
| Pitch diameter  | 48.698mm        |
| Inner race diameter | 43.726mm     |
| Outer race diameter | 53.46mm    |
| Material of balls | Ceramic alumina |

3.5.1. **High Speed Precision Spindle Model**

Mathematical modelling of the spindle shown in Figure 3.19 is carried out in this section. The assumptions made in the mathematical modelling are listed in section 3.3.
Figure 3.19: Schematic of Spindle

For this model the spindle is considered to have five degrees of freedom motion. The five degrees of freedom are along the axial Z, radial X and Y and the rocking modes ψ and φ about the X and Y axes respectively. The schematic of spindle, showing the five degrees of freedom is shown in Figure 3.21.

3.5.1.1. Initial conditions

Contact angle and deflection: Schematic representation of spindle bearing assembly is shown in Figure 3.17. The bearings are uniformly deformed in the contact directions by preloading and interference fitting (refer to chapter 2, sections 2.2 and 2.5 for the necessity of preloading and interference fitting). These alter the contact angles from an initial unloaded value of $\alpha_0$ to $\alpha_p$; the preloaded contact angle. A reduction in diametrical clearance of the bearing results by interference fitting, which is given by

$$A = r_{or} + r_{ir} - D$$  \hspace{1cm} (3.59)

where $r_{or}$ is the radius of curvature of bearing's outer ring, $r_{ir}$ is that of the inner ring and $D$ is the ball diameter. Figure 3.20 shows the variation in contact angle due the application of different loads.
The initial contact angle $\alpha_0$ of the bearing is given by Harris (1991) as:

$$\alpha_0 = \cos^{-1}\left(1 - \frac{p_d + \Delta p_d}{2A}\right)$$  \hfill (3.60)

where $p_d$ is the diametric clearance and $\Delta p_d$ the change in its value.

Preloading induces a force balance in the axial direction, which is given by:

$$P_r = NK\delta_0^3 \sin \alpha_p$$  \hfill (3.61)

![Figure 3.20: Displacement of raceway curvature centres.](image)

A thrust force $F_a$ is applied to the inner ring, causing an axial deflection $z_0$. As a result all the balls undergo a uniform normal deflection, $\delta_0$ along their lines of contact with the races. This axial deflection $z_0$ is a component of normal deflection along the line of contact. The initial uniform deflection, $\delta_0$ in the contact direction is given by:
\[ \delta_0 = A \left( \frac{\cos \alpha_0}{\cos \alpha_p} - 1 \right) \]  

(3.62)

Combining and solving equations (3.61) and (3.62) for \( \alpha_p \), the axial deflection along the Z direction is obtained as:

\[ z_0 = \delta_0 \sin \alpha_p \]  

(3.63)

Dynamic variations of contact angle and deflection occur in the angular contact ball bearings as the supported rotor undergoes displacements in the axial and transverse radial directions and tilt about the radial directions. The spindle is assumed to be rigid which makes the tilting angle of the bearing to be the same as the tilting angle of the spindle about its centre of gravity. The axial tilting angle of the bearing inner ring has the same effect upon all the balls in the bearing, but effect of radial tilting is different for each ball contact according to its position. \( \omega_c \), the cage speed, which for an angular contact ball bearing is given by:

\[ \omega_c = \frac{1}{2} \omega \left( 1 - \frac{D}{d_p} \cos \alpha \right) + \frac{1}{2} \omega \left( 1 + \frac{D}{d_p} \cos \alpha \right) \]  

(3.64)

Figures 3.20 and 3.21 show the deflection of the \( i^{th} \) ball in terms of \( x, y \) and \( z \) displacements of the centre of gravity of the rotor in the X, Y and Z directions, as well as the tilts \( \phi \) and \( \varphi \) about the X and Y directions. The total deflection of the \( i^{th} \) ball-raceway contact is given as:

\[ \delta_i = \left[ A \sin \alpha_0 + z_0 - z - \frac{d_p}{2} (\psi \cos \theta_i + \varphi \sin \theta_i) \right]^2 \]  

(3.65)

\[ + \left[ A \cos \alpha_p + \delta_0 \cos \alpha_p + x_i \cos \theta_i + y_i \sin \theta_i \right]^2 \]  

and the resulting transient contact angle is:

\[ \alpha_i = \left( \frac{A \sin \alpha_0 + z_0 - z - \frac{d_p}{2} (\psi \cos \theta_i + \varphi \sin \theta_i)}{A \cos \alpha_p + \delta_0 \cos \alpha_p + y_i \cos \theta_i + x_i \sin \theta_i} \right) \]  

(3.66)
3.5.1.2. Equations of Motion

Figure 3.21 is a schematic representation of the spindle assembly. The spindle has displacements in X, Y and Z directions and rocking/tilting motions in both the radial directions, which are the five degrees of freedom as previously described.

Figure 3.21: Schematic representation of Vertical spindle assembly

The assumptions used for modelling are stated in section 3.3. Even though the spindle used in modelling is a high speed precision spindle used for wood routing the modelling has been generalised so that in future can be used for any spindle. Referring to Figure 3.21 the equations of motion for this high speed precision spindle in all the five degrees of freedom are:
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\[ M\ddot{x} = \sum_{i=1}^{n} W_i \sin \alpha - F_x \]  
(3.67)

\[ M\ddot{y} = \sum_{i=1}^{n} W_i \cos \alpha, \sin \theta_i - F_y \]  
(3.68)

\[ M\ddot{z} = \sum_{i=1}^{n} W_i \cos \alpha, \cos \theta_i + Mg - F_z \]  
(3.69)

\[ I_{xx} - I_{zz} \omega \dot{\psi} = \sum_{i=1}^{n} LW_i \cos \alpha, \cos \theta_i + \frac{1}{2} \sum_{i=1}^{n} d^i W_i \sin \alpha, \cos \theta_i + LF_y \]  
(3.70)

\[ I_{yy} + I_{zz} \omega \dot{\psi} = \sum_{i=1}^{n} LW_i \cos \alpha, \sin \theta_i + \frac{1}{2} \sum_{i=1}^{n} d^i W_i \sin \alpha, \sin \theta_i + LF_x \]  
(3.71)

3.5.1.3. Computational solution of the equations of motion

The governing equations of motion in this five degrees of freedom system are solved by an iterative marching procedure, incorporating a third order quasi-linear method known as the linear acceleration technique, proposed for bearing applications by Rahnejat (1985). This method has been discussed in session 3.3.2.1. The model consists of two main sections: the initial conditions and the dynamic model.

3.5.1.4. Initial conditions

The model assumes the following initial series of events:

1) The initial deflections due to interference fitting and preloading are calculated using equations (3.61) and (3.62).

2) The initial contact load \( W_0 \) per ball is obtained from Hertzian theory of elasticity given by \( W_i = K \delta_0^3 \), in this instance with deflection \( \delta_0 \) for all balls. These forces are then resolved into their components along the X, Y and Z directions. Therefore, the resultant restoring forces in these directions at \( t = 0 \) are:
\[ W_{x_0} = \sum_{i=1}^{n} W_0 \cos \theta_i \cos \alpha_p \quad (3.72) \]
\[ W_{y_0} = \sum_{i=1}^{n} W_0 \sin \theta_i \cos \alpha_p \quad (3.73) \]
\[ W_{z_0} = \sum_{i=1}^{n} W_0 \sin \alpha_p \quad (3.74) \]

3.5.1.5. Dynamic Model

The shaft is released under its own weight at time \( t = 0^+ \) and a steady spindle speed commences instantaneously.

1) at the instance of release the initial velocities and displacements are considered to be zero, i.e.: \( \dot{x} = \dot{y} = \dot{z} = \ddot{x} = \ddot{y} = \ddot{z} = \dddot{x} = \dddot{y} = \dddot{z} = \phi = \varphi = 0 \)

2) values obtained for displacements and velocities are for the centre of gravity of the spindle. The actual displacements for the support bearings are given by:
\[ x_b = x - L \sin \varphi \quad (3.75) \]
\[ y_b = y + L \sin \phi \quad (3.76) \]

Since the values of \( \phi, \varphi \) are quite small, equations (3.75) and (3.76) can be written as:
\[ x_b = x - L \varphi \quad (3.77) \]
\[ y_b = y + L \psi \quad (3.78) \]

3) the dynamic contact angles and the corresponding deflections are calculated, using equations (3.65) and (3.66).

4) components of contact loads for each ball are calculated in the \( X, Y \) and \( Z \) directions
5) the governing equations of motion, given by equations (3.67) to (3.71) are solved simultaneously for obtaining the accelerations in all the five directions of motion.

6) This iterative process for each time step is terminated if all of the following convergence criteria are simultaneously satisfied. If not satisfied all the above steps are repeated.

\[
\left| \frac{\ddot{x}_{k,j} - \ddot{x}_{k,j-1}}{\ddot{x}_{k,j-1}} \right| \leq \varepsilon_x 
\]

(3.79)

\[
\left| \frac{\ddot{y}_{k,j} - \ddot{y}_{k,j-1}}{\ddot{y}_{k,j-1}} \right| \leq \varepsilon_y 
\]

(3.80)

\[
\left| \frac{\ddot{z}_{k,j} - \ddot{z}_{k,j-1}}{\ddot{z}_{k,j-1}} \right| \leq \varepsilon_z 
\]

(3.81)

\[
\left| \frac{\ddot{\psi}_{k,j} - \ddot{\psi}_{k,j-1}}{\ddot{\psi}_{k,j-1}} \right| \leq \varepsilon_\phi 
\]

(3.82)

\[
\left| \frac{\ddot{\phi}_{k,j} - \ddot{\phi}_{k,j-1}}{\ddot{\phi}_{k,j-1}} \right| \leq \varepsilon_\psi 
\]

(3.83)

Procedure used for solving numerically is shown in Figure 3.22. Using this procedure the equations of motion are solved and the results for a five degree of system model are shown in the results section.
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Figure 3.22: Flow chart for the solution procedure
3.5.2. Existence of off-sized balls

The nature itself shows that a perfect surface does not exist. This also applies to the manufacture of ball-bearings. For example, it is impossible to produce a set of identical balls even with the best machine tools; there is always some difference between ball diameters (less than the machining tolerances). This is known as the off-sized ball effect. The existence of off-sized balls in a bearing introduces further untoward vibrations to the shaft-ball bearing system. Many researchers have worked in this area. As the result of experiments a specific relationship has been found to exist between the internal dimensions of bearings and the vibration factors for the different components, the rolling elements being more important than the outer and inner rings. For detailed research in this area refer to section 2.2. In this thesis an attempt is made to understand the nature of an off-sized ball on spindle dynamics numerically by using the five degrees of freedom model.

![Off-sized ball in a bearing](image)

**Figure 3.23: Off-sized ball in a bearing**

For this purpose an off-sized ball with a diameter difference of $1\mu m$ relative to the others in the ball complement is introduced into the assembly, shown in Figure 3.23.
The ball having a larger diameter is subject to a greater deformation. Hence, the equation for deflection of this ball becomes:

\[
\delta_{oab} = \delta_i + \delta
\]

(3.84)

where \( \delta \) is the additional deflection occurring when compared with the other balls so the deflection equation for this off-sized ball is:

\[
\delta_{oab} = \left\{ \left[ A \sin \alpha_0 + z_0 - z - \frac{d_p}{2} (\psi \cos \theta_i + \varphi \sin \theta_i) \right]^2 + \left[ A \cos \alpha_p + \delta_o \cos \alpha_p + x_s \cos \theta_i + y_b \sin \theta_i \right]^2 \right\}^{\frac{1}{2}} - A + \delta
\]

(3.85)

The equations of motion remain the same as (3.67) to (3.71). The solution procedure is the same as in section 4.5.

3.5.3. Spindle off-centre rotation

In a rotating shaft when the axis of rotation does not coincide with the geometric centre there exists an additional centrifugal force, which produces vibration, the amplitude of which is speed dependent. Since the spindle, described in this study rotates at 20,000 rpm, it is essential to consider any out-of-balance effect. Figure 3.24 shows the spindle with an assumed eccentricity denoted by letter e, which produces a centrifugal force \( m \omega^2 \) where \( m \) is the mass and \( \omega \) is the shaft speed or the speed at which the inner race of the bearing rotates.
If the force produced due to the eccentricity act at an angle \( \theta \), then there are force components acting in the X and Y directions represented by \( F_x \) and \( F_y \) respectively. These force components are:

\[
F_x = Me\omega_i^2 \cos \theta 
\]

and:

\[
F_y = Me\omega_i^2 \sin \theta 
\]

which alter the equations of motion in the X and Y directions as follows:

\[
M\ddot{x} = \sum_{i=1}^{n} W_i \sin \alpha - Me\omega_i^2 \cos \theta 
\]

and:

\[
M\ddot{y} = \sum_{i=1}^{n} W_i \cos \alpha \sin \theta - Me\omega_i^2 \sin \theta 
\]

The other equations of motion remain unchanged. Results of this effect are discussed in the next section.
3.6. Results and discussions

Results for a simulation run at 25,000 rpm for spindle with bearings specifications shown in Table 3.4 are given below. Figure 3.25 shows the oscillations in X and Y directions. In the X direction the spindle attains equilibrium after 15 ms. During this time the maximum amplitude of vibration is predicted as 31.5 μm. This can be attributed to the sudden release of the spindle in order to find the equilibrium position. Later the spindle attains a steady running condition (a limit cycle). In the case of Y direction the spindle attains equilibrium after 16.5 ms during this time the maximum amplitude of vibration is 25.7 μm. Later the spindle attains a steady running condition.

![Figure 3.25: Numerical results -X and Y oscillations](image)

The limit cycle oscillation is obtained by a phase plane plot as shown in Figure 3.26. This is a phase plane plot of radial displacement and velocity, which clearly shows the initial non-steady state and the subsequent steady state behaviour. Since a perfect bearing free of geometrical and topographical defects is assumed here the numerical results shows a single loop in the phase plane plot, which is due to the existence of cage frequency. Since the shaft is assumed to perfectly balanced the effect of shaft frequency does not have a major contribution. Note that a widely spread loaded region also inhibits the variable compliance effect, thus no significant contributions at ball-pass frequency is observed. Figure 3.27 shows the locus of the radial oscillations in X and Y directions. The displacement along the Y direction is 32 μm positive and 48 μm negative, whereas the displacement along the other radial direction X is ±40 μm. The difference in the oscillations may be attributed to the variable compliance effect as noted by Sunnersjo (1978).
Chapter 3: Analytical & Numerical Modelling of ball bearings

Figure 3.26: Numerical – Phase plane plot

Figure 3.27: Numerical – X-Y Locus

It is impossible to manufacture two balls of the same size and this can lead to existence of off-sized balls. In reality inside a ball bearing all the balls are of slightly different sizes. In the case of precision spindle described here even this slight size differences can have a major impact in the overall vibration. This effect has been shown in Figure 3.28 for a ball having a 1µm larger diameter and also the case of two balls with 1µm larger diameters than ball complement. As the load per ball depends
directly on the extent of applied preload (thus the resulting localised diametral clearance) any off-sized ball would affect the spectrum of vibration. As the size of the ball increases the contact angle also increases, causing a larger deflection than the other balls and hence it would have to carry a larger share if the applied load.

![Figure 3.28: Load per ball with off-sized balls](image)

When the spin axis is not coincident with the geometric centre or centre of gravity of the rotor assembly, there exists an additional force as described in section 4.4. This gives rise to a time varying out-of-balance force, with components acting in both X and Y directions. Furthermore, there is also a coupling effect between the axial and radial natural frequencies due to the conical whirl induced by the unintentional eccentric spindle rotation. The unbalance which is produced by the eccentricity is clearly shown in Figure 3.29. For any vibrating system the period of natural vibrations is a stationary period, so that the application of any small external force produces a second-order change in the period. A routing spindle supported at both ends by bearings when acted on by cutting forces tends to bend. On the removal of these forces it tends to return to its original straight position and in doing so it overshoots the nominal equilibrium position and vibrates in a to and fro fashion laterally. The
velocity of its approach to the equilibrium position, and the frequency of the vibrations subsequently executed, are greater than the elastic stress produced in the shaft by a given lateral displacement. When the shaft is rotating round its longitudinal axis, and is displaced laterally, the elastic stresses tend to, as before, bring it back to the undisturbed position but the centrifugal forces have exactly the opposite tendency. Thus, they just reduce the vibrating forces, and diminish the frequency of vibration.

![Radar plot showing load per ball for different running conditions.](image)

**Figure 3.29: Load per ball for different running conditions**

Figure 3.29 shows a radar plot for the effect of shaft off-centre rotation for an eccentricity of $1\mu m$. The effect of eccentricity is very clear from the figure. This eccentricity produces a centrifugal force, which is product of mass, the eccentricity and the square of the velocity component in that direction and it becomes very noticeable at higher spindle speeds. This force pushes the shaft to one side and high contact loads are produced by the ball-races contacts. The main factor which influences the load per ball is the deflection of the balls. As spindle rotates, the contact angle for each ball varies due to the eccentricity and resultant load contribution in the contact angle direction in accord with the dynamic contact angle of the ball.
3.7. Closure

Dynamics of the ball bearings used in the described high precision routing spindle are described in this chapter. Analytical expressions for primary and secondary frequencies were derived. These frequencies are essential to identify the system characteristics especially for high speed precision spindles. A numerical model for high speed routing spindles is reported in this chapter. A systematic approach was used for building this numerical model, started with a deep groove ball bearing with two degrees of freedom. This model was validated with numerical the model reported by Rahnejat (1984). Later the model was upgraded for angular contact ball bearings with five degrees of freedom. Results for this model are presented in this chapter. It is essential that this model should be validated with carefully conducted experiments, which is undertaken in the next chapter.


Chapter 4
EXPERIMENTAL ANALYSIS

4.1 Introduction

It is important that analytical and numerical models developed are validated with appropriate experiments carefully conducted using designed test rigs. Modern day routing spindle speeds can reach 60,000 rpm as a result of better assembly, higher quality bearings and increased component reliability. In order to make sure that the developed models are representative of actual physical conditions, a test rig comprising a high speed precision vertical routing spindle is used in this thesis. It is also important that the data acquired from this rig are of high quality; noise free with appropriate sensors and data processing techniques. The choice of transducers and the layout of the test rig are discussed in this chapter.

4.2. Spindle Design and monitoring considerations

The current study employs a 7.5 kW precision routing spindle, shown in Figure 4.1. In order to minimise the sources of vibration, the spindle is supported by pairs of angular contact ball bearings in a back-to-back arrangement at each of its ends. The back-to-back bearing arrangement provides the desired high dynamic stiffness. The bearings are grease lubricated for spindle speeds up to 35,000 rpm and oil lubricated for spindle speeds up to 60,000 rpm. In this study, a grease lubricated bearing arrangement is employed. The purpose of the study is to monitor spindle vibration and obtain real time power spectra under free running (i.e. no cutting action) conditions with a specimen held in the chuck to represent a tool. Under the spindle free running condition, the system is subjected to a pre-determined axial preload of 700 N and a suitable radial interference fit of 5 µm. Therefore, the main sources of vibration are due to the imperfections on the rolling element and raceway surfaces, as well as the bearing variable compliance effect and any out-of-balance in the spindle-tool assembly. Vafaei et al (2002), and Vafaei and Rahnejat (2003) have shown that complex signal processing methods such as ARMA and IRR-WD can be effectively used for the analysis of vibration signals obtained from a precision high speed machine tool spindle.

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Although the findings of Wardle et al (1983), using FFT or PSD, are good indicators of the spectral content in a qualitative sense, the transient nature of some of the asynchronous vibration contributions means that such forms of spectral analysis are not best suited to a quantitative analysis, due to the averaging process inherent in FFT and PSD analyses over the record length. Vafaei and Rahnejat (2003) investigated the effect of both tool assembly unbalance and holding torque through the use of wavelet analysis for synchronous repeated run-out (RRO), this being the most significant contributor to spindle vibration in its free running mode with a held cutter/tool. The conclusion reached was that the unbalanced nature of the tool-spindle holding arrangement and the rather ad-hoc manner in which operators of routers apply holding torques account for one of the major sources of imperfections noted on the resulting machined surfaces. Therefore, the newly designed router, reported here, uses a hydraulic HKS chucking system with precise control of holding torque for the tool-spindle assembly. Another problem, often noted at such high rotational speeds and particularly in dry routing of hard and abrasive materials (high cutting forces), is heat generated during the cutting process, which can result in scouring of the bearing housings due to thermal expansion of the assembly. This can also cause thermal instability in the usual support bearings. Therefore, ceramic bearings were used in this newly designed spindle.

The newly developed precision routing spindle for high speed applications has a 7.5 kW power and can hold cutters of mass up to 7.5 kg. In order to investigate its vibration characteristics, it is mounted on a cast iron pedestal, which is in turn attached firmly to a heavy cast iron machine bed. Cast iron structures are used because of their good damping characteristics, caused by their structural porosity. The large mass of the assembly, fixed rigidly to a concrete foundation, places its natural modal frequencies considerably away from the frequencies of generated vibration due to any synchronous out-of-balance of the tool-spindle assembly or induced by the support bearings.
In order to ensure that the test conditions remain below the natural response of the rig, the spindle and its support structure were excited using an impact hammer with a force transducer (sensitivity of 22.7 mV/Pa) and the response was measured by an accelerometer (sensitivity of 0.305 pC/ms$^2$). The signals acquired were analysed using the Data Physics Signal Calc software. Figure 4.2 shows the hammer test results at the spindle nose.

Figure 4.1: Schematic of the spindle arrangement

Figure 4.2: Frequency response of Spindle
Chapter 4- Experimental Analysis

It is clear from the frequency response function that the first modal natural frequency occurs at 251 Hz (synchronous with a spindle speed of 15720 rpm) and the second modal natural frequency occurs at 656 Hz (synchronous with a spindle speed of 39000 rpm). Thus, the safe operating range for the spindle tests are in the range 16000 – 38000 rpm, depending upon application and with a table feed rate in the range 5–20 m/min (OPORTO report 2003) under dry cutting conditions. The occurrence of split peak or two peaks very close at both the natural frequencies are due to the stiffness difference in the X and Y radial directions.

4.3. Spindle bearings

There are two sets of angular contact ball bearings in tandem at each end of the spindle, set in a back-to-back arrangement (see Figure 4.1). This arrangement provides good dynamic stiffness and thrust load carrying capacity in the vertical axial direction. The tandem set of bearings nearer to the position of the tool (referred to as the tool-end bearings) are fixed in location, whilst the other pair of bearings at the “free-end” of the rotor are allowed to axially float to accommodate thermal expansion. A spacer is placed between the pairs of bearings to ensure the transfer of an axial pre-load of 700 N. A constant preload force with zero tilt has been attained using a precision spacer between the two pairs of tandem bearings. The spacer has an inner member slightly shorter than the outer one. This arrangement together with abutments on the shaft and the housing constrain the inner and outer bearing rings to remain parallel, so that under radial loads only small relative axial movements are allowed. Any change within the bearing contact angles or the ball-raceway loads induces a moment which tends to offset the effect of overhung radial loads. A pair of bearings, mounted in tandem, creates an effective centre distance \( l \) from the geometrical centre of the rotor, which lies in the radial plane containing the front face of the bearing nearest to the spindle nose. This arrangement provides good dynamic stiffness and thrust capacity in the vertical axis direction. In order to allow for thermal expansion at high speeds and loads, the set of bearings at the top/free-end are made free to slide. The outer rings of these bearings are not supported by an abutment and are mounted with a clearance between them and the housing. The application of a radial load induces tilt as well as an axial movement. In this arrangement, the
effective bearing centre is coincident with its geometric centre. The specifications of these bearings are given in Table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>Tool end bearing</th>
<th>Free end bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball diameter</td>
<td>4.762mm</td>
<td>3.969 mm</td>
</tr>
<tr>
<td>Number of balls</td>
<td>23</td>
<td>20</td>
</tr>
<tr>
<td>Preloaded contact angle</td>
<td>15°</td>
<td>15°</td>
</tr>
<tr>
<td>Pitch diameter</td>
<td>48.698mm</td>
<td>36.012 mm</td>
</tr>
<tr>
<td>Inner race diameter</td>
<td>43.726mm</td>
<td>32.022 mm</td>
</tr>
<tr>
<td>Outer race diameter</td>
<td>53.46mm</td>
<td>39.981 mm</td>
</tr>
<tr>
<td>Ball material</td>
<td>Ceramic Alumina</td>
<td>Ceramic Alumina</td>
</tr>
</tbody>
</table>

4.4 Vibration Transducers

A vibration transducer is a device which senses vibration and converts it into an electrical signal, which is proportional to the measured variable (displacement, velocity or acceleration). Selection, placement and proper use of a correct transducer are important, since the information missed or distorted during measurement cannot be retrieved later. The selection of a transducer depends upon the specific application. The main parameters to be considered are the operating environment, sensitivity, frequency response, dynamic range, measurement accuracy and repeatability. The most commonly used transducer for vibration measurement is an accelerometer due to its compactness and high frequency response. For a practical accelerometer system with low damping and an operating range well below its natural frequency, the output is directly proportional to the acceleration experienced by the transducer base with no phase difference. With an upper frequency limit of 30 kHz and lower frequency limit of 1Hz, the Brul and Kjaer (B&K) type 4375 piezoelectric/charge accelerometer is a good candidate. At high operational speeds, the distributed energy from the machine to the supporting structures makes the output of contact type transducers quite complex when compared with non contact
transducer output. This brings the importance of using non-contact transducers for vibration measurement from high precision routing spindles, where the vibration amplitudes can be in the micrometer range.

4.4.1. Proximity Probes

Proximity probes are displacement transducer that uses the eddy current principle. Eddy current testing has its origins with Michael Faraday's discovery of electromagnetic induction in 1831. An eddy current transducer consists of an insulated probe, containing a small coil within its tip. When fed with a signal of high frequency, typically 2MHz, using oscillator-demodulator or proximeter, it creates a magnetic field at its tip. When a conducting material (test object) is brought into proximate location eddy currents are induced by the magnetic field and modulate the amplitude of the carrier signal with respect to the distance between the tip and the target surface. This change in the carrier signal is converted in proportion to the gap between the probe tip and the test piece by the demodulator. Aini et al (1990), Aini et al (1995), Vafaei et al (2002) and Vafaei and Rahnejat (2003) showed that eddy current displacement sensors can be used effectively for measurement of vibration in high speed spindles. However, these sensors are highly sensitive to the presence of electromagnetic fields near the measurement field: For eddy current sensors, the shape of the rotating object is an important factor, which can affect the accuracy of the measurements. Furthermore, eddy current sensors only work with conductive surfaces and the relationship between the distance and the impedance of the coil is non-linear and temperature dependent. These shortcomings are compensated by using laser vibrometry, the operation of which will be explained in detail in next section. The performance comparison of the two will also be shown in the coming sections.

4.4.2. Laser Doppler Vibrometry (LDV)

Laser Doppler Vibrometry is an interferometric measurement technique initially developed for non-intrusive fluid flow measurements and latterly used to make velocity measurements of solid surfaces. The technique uses the frequency shift which occurs in coherent light when it is scattered from a moving object. By detecting this Doppler frequency shift, named after the Austrian physicist who first considered the
phenomenon, information about the velocity of the object can be obtained (see Figure 4.3).

![Diagram of the Doppler Effect]

The Doppler frequency shift \( f_D \), given by equation (4.1) is directly proportional to the velocity of particle \( V \), thus:

\[
 f_D = \left( \frac{2\mu V}{\lambda_L} \right) \cos \delta_L \sin \left( \frac{\varepsilon}{2} \right) 
\]

where \( \mu \) is the refractive index of air, \( \lambda_L \) is the laser wavelength, \( \delta_L \) is the angle between the velocity vector and the bisector of the angle subtended by the source and the scattering directions and \( \varepsilon \) is the angle defining the scattering direction. The laser vibrometers available commercially and used throughout this study collect the reflected light in direct backscatter within air, i.e. \( \varepsilon = \pi \) and \( \mu = 1 \). Therefore, the frequency shift measured is:

\[
 f_D = \left( \frac{2V}{\lambda_L} \right) \cos \delta_L 
\]

where \( V \cos \delta_L \) now represents the component of velocity in the direction of the incident laser beam. By measuring the change in \( f_D \) with time, the time-resolved solid surface velocity can be found. \( f_D \) cannot be measured directly, however,
because for the velocities encountered, the frequency shifts observed, typically $10^6$ Hz, are too small in comparison to the frequency of the laser light, $10^{14}$ Hz, to be resolved. The frequency shift is measured by mixing the back-scattered light with a reference beam derived from the same coherent source (see Figure 4.4). The light is then heterodyned; the addition of two signals through a non-linear element, on the surface of a photo-detector with the detected signal modulated at the beat or difference frequency between the two beams. Demodulation of this signal then gives a time-resolved velocity measurement. The non-linear element in the process is the photo-detector as its electrical output is proportional to the intensity of the incident light which is proportional to the square of the optical electric field.

![Figure 4.4: Typical laser vibrometer arrangement](image)

The technique of heterodyning is only capable of giving the modulus of "difference frequency" between the two beams and cannot distinguish which of the two beams has the higher frequency. For this reason the arrangement described so far would have a directional ambiguity, as a change in the sign of the velocity gives no change in the difference frequency. The most popular method of solving this problem is by frequency shifting the reference beam by a constant amount. The difference frequency seen by the photo-detector then has a non-zero value, equal to the frequency shift in the reference beam, $f_r$, when the target is stationary. The Doppler shifts from a
positive velocity, \( +f_D \), and a negative velocity, \( -f_D \), then give different difference frequencies, \( |f'_s - f_D| \), and \( |f'_s + f_D| \) respectively. The size of the frequency shift in the reference beam governs the range of Doppler frequency shifts that can be successfully demodulated without directional ambiguity, setting the maximum velocity which can be measured by the instrument.

The standard Polytec 302 vibrometer is used for the current study, which incorporates a Bragg cell for frequency shifting in a Mach-Zehnder interferometer arrangement as shown in Figure 4.5. In Figure 4.5 BS corresponds to beam splitter, D1 & D2 to Photo detectors and L for lens.

![Figure 4.5: Optical configuration of the Polytec OFV-302 Laser Vibrometer](Source: Polytec GmbH)

Bell and Rothberg (2000) have shown that a single point laser vibrometer, measuring the velocity of a shaft, is insensitive to its profile, despite the fact that the incident beam can change in axial and radial position on the shaft in any arbitrary fashion. Such immunity to target shape gives this measurement technique a significant advantage over, for example, proximity probe measurements. Taking advantage of this, the effect of waviness of the shaft surface, contributing to the overall vibration of
the spindle can be obtained. This would isolate run-out due to spindle-tool out-of-balance and bearing induced vibration.

![Figure 4.6: Definition of axes and of the point P on a vibrating and rotating structure](image)

The authors showed that for a rotating shaft (see Figure 4.6), of any arbitrary shape, undergoing vibration, requires three translational and three rotational co-ordinates for its motion description. The velocity sensed by a single laser beam orientated according to the angles $\alpha$ and $\beta$ (Figure 4.7), incident on a rotating shaft vibrating in all six degrees of freedom is made up of six separate vibration sets, each a

![Figure 4.7: Orientation of laser beam.](image)
combination of motion parameters. The velocity measured by a laser beam is given as:

\[
U_m = \cos \beta \cos \alpha [V_x + (\dot{\theta}_x + \Omega) a_y - (\dot{\theta}_y - \Omega \theta_z) a_z] \\
+ \cos \beta \sin \alpha [V_y - (\dot{\theta}_x + \Omega) a_x + (\dot{\theta}_z + \Omega \theta_y) a_z] \\
- \sin \beta [V_z - (\dot{\theta}_y + \Omega \theta_x) a_y + (\dot{\theta}_z - \Omega \theta_x) a_z] \\
- (y_0 \sin \beta + z_0 \cos \beta \sin \alpha)[\dot{\theta}_x + \Omega \theta_y] \\
+ (z_0 \cos \beta \cos \alpha + x_0 \sin \beta)[\dot{\theta}_y - \Omega \theta_z] \\
+ (x_0 \cos \beta \sin \alpha - y_0 \cos \beta \cos \alpha)[\dot{\theta}_z + \Omega]
\]

(4.3)

where \((V_x, V_y, V_z)\) and \((a_x, a_y, a_z)\) are the translational vibration velocities and displacements of the origin \(O\) in the \((x, y, z)\) directions, whilst \((\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z)\) are the angular vibration velocities of the shaft around the \((x, y, z)\) axes, referred to as pitch, yaw and roll, respectively. \((x_0, y_0, z_0)\) is the position of an arbitrary known point that lies along the line of the beam. Using a single laser vibrometer, it is possible to isolate the translational vibration sets (two radial and one axial). To measure the \(x\) radial vibration set \((\cos \beta \cos \alpha [V_x + (\dot{\theta}_x + \Omega) a_y - (\dot{\theta}_y - \Omega \theta_z) a_z])\) requires alignment of the laser beam, so that it passes through the centre of the shaft and along the \(x\)-axis, making \(\alpha = \beta = 0^\circ\) and \(y_0 = 0\). The measured velocity is then equal to:

\[
U_m = [V_x + (\dot{\theta}_x + \Omega) a_y - (\dot{\theta}_y - \Omega \theta_z) a_z]
\]

(4.4)

Similarly, \(\alpha = 90^\circ\), \(\beta = 0^\circ\) and \(x_0 = 0\), enabling the \(y\) radial vibration set to be isolated. In the above equation, the product of vibration parameters \((\dot{\theta}_y - \Omega \theta_x) a_z\) are typically an order of magnitude smaller than other terms and can therefore be
considered as insignificant. Hence, the measured velocities of a rotating optically rough shaft in the two orthogonal directions, $U_x, U_y$, can be expressed as:

$$U_x = V_x + (\dot{\theta}_z + \Omega) a_x \quad \text{and} \quad U_y = V_y - (\dot{\theta}_z + \Omega) a_y$$  \hspace{1cm} (4.5)

Rothberg and Halliwell (1992) demonstrated how the cross-sensitivity of radial vibrations in the measured velocity, principally $(\dot{\theta}_z + \Omega) a_x$ or $(\dot{\theta}_z - \Omega) a_y$ in the above equations, could be of sufficient magnitude to mask the intended measurements of the radial velocities; $U_x, U_y$.

4.4.3- Problem of speckle pattern in vibrometry

A speckle pattern is a chaotic distribution of light and dark spots formed when coherent light is scattered from a surface, which is comparably rough with the wavelength of light. Rotation of the target surface causes a motion of the speckle pattern, ultimately causing noise in the laser vibrometer output. This noise is repeated with every rotation and contributes energy at the same frequency as the rotational motion and for that reason is referred to as pseudo-vibrations by Rothberg et al (1989). Even though the researchers working in the field are unable to quantify speckle noise, they have suggested various ways to tackle the problem. One of the methods is making the measurement surface roughness much smaller on the scale of the laser wavelength. Tatar et al. (2007) has shown that an average roughness (Ra) of 20 nm can reduce the effect of cross-sensitivity and speckle noise by a very considerable amount.

High precision routing spindle used for this study is fitted with a tool holder (GEWEFA 152.05.036.008). A specially designed tool, compatible for the tool holder was manufactured. The tool was tested for the surface roughness using a Talysurf CLI 2000 (shown in Figure4.8).
The surface parameters obtained for the specially designed tool are given in Figure 4.9.

<table>
<thead>
<tr>
<th>Parameters calculated on the profile shaft</th>
<th>Levelled (LS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>* Parameters calculated as average value of all sampling lengths.</td>
<td></td>
</tr>
<tr>
<td>* A microroughness filtering is used, with a ratio of 2.5 μm.</td>
<td></td>
</tr>
<tr>
<td>Roughness Parameters, Gaussian filter, 250 μm</td>
<td></td>
</tr>
<tr>
<td>Ra</td>
<td>0.02264 μm</td>
</tr>
<tr>
<td>Rq</td>
<td>0.031249 μm</td>
</tr>
<tr>
<td>Rp</td>
<td>0.064118 μm</td>
</tr>
<tr>
<td>Rv</td>
<td>0.11634 μm</td>
</tr>
<tr>
<td>Rz</td>
<td>0.85591 μm</td>
</tr>
<tr>
<td>Rsk</td>
<td>-3.6989</td>
</tr>
<tr>
<td>Rku</td>
<td>34.729</td>
</tr>
<tr>
<td>Rz</td>
<td>0.18046 μm</td>
</tr>
<tr>
<td>RSm</td>
<td>30.996 μm</td>
</tr>
<tr>
<td>Ry</td>
<td>0.54897 μm</td>
</tr>
</tbody>
</table>

Figure 4.9: Surface measurement results

Tatar et al (2007) showed that with an average surface roughness (Ra) of 20 nm one can overcome the effects of cross-sensitivity and speckle noise. So throughout this
study the laser is pointed towards the high surface finished tool. Figure 4.10 shows the specially designed tool fitted into the tool holder, representing the target surface.

![Optically Smooth surface](Optically Smooth surface Ra value 20nm)

**Figure 4.10: Spindle fitted with optically smooth surface**

### 4.4.4- Comparison of Eddy current transducers and Laser vibrometry

Advantages and disadvantages of both the sensors have already been mentioned in the previous sections. Figure 4.11 shows the fast Fourier transform (FFT) for the spindle running at 20,000 rpm obtained with data acquired using eddy current sensors and the same using laser vibrometry shown in Figure 4.12. For frequencies ranging from 0 - 2 kHz the spectrum obtained using laser vibrometry is unable to show the existence of lower frequencies with lower amplitudes, such as the cage frequency. This is because of the need to integrate the velocity data in order to obtain the vibration displacements. For frequencies ranging from 2 kHz - 4 kHz the baseline noise for both the transducers seems to be in the same range, but few extra vibration peaks are resolved by the laser vibrometer. For higher frequencies i.e. above 4 kHz, the eddy current sensors seem to pick a lot of noise from the surroundings, whereas the laser vibrometer signal seems to be quite clean (free of noise). Experimental data acquisition for this study concentrates on the higher spindle speeds, around 25,000 rpm, where some of the bearing primary frequencies along with the secondary frequencies occur above 3 kHz and low vibration amplitudes makes Laser vibrometry better suited for the investigations.
Chapter 4 - Experimental Analysis

Figure 4.11: Vibration Spectrum for spindle running at 25,000 rpm using Eddy current sensors

Figure 4.12: Vibration Spectrum for spindle running at 25,000 rpm using Laser Vibrometry
4.5. Experiment Test rig setup

Previous sections explain the various design considerations and vibration transducers suitable for the measurements undertaken. Figure 4.13 shows a schematic of the experimental layout. Test rig pictures are given in Appendix 1. For this study the spindle (1) is set to run at speeds of 20,000 and 25,000 rpm. Accelerometer (5) is placed on the spindle base, which gives an idea of vibration transmitted to the foundation and thus the surrounding machinery. It is extremely important to measure the eddy current base vibration (these being mounted on a heavy base on a separate foundation to the spindle rig), since any kind of vibration transmitted into the eddy current base can contaminate the eddy current sensor data. For this purpose another accelerometer is placed on the eddy current sensor base. The eddy current sensors are supported by a concrete block (13) which isolates any kind of vibration from the spindle and the surroundings.

![Figure 4.13: Schematics of experimental setup](image)

For obtaining data by laser vibrometry, a Polytec 302 laser vibrometer (4) is set 1 m away from the spindle and is focused onto the optically smooth surface (2) (target surface) of the tool. The signals with the vibration data (changes in voltage output) are acquired by the Polytec 3000 vibrometer controller (7). In order to make sure the laser
beam passes through the centre of the shaft, thereby setting the DC component of the velocity to zero an oscilloscope (9) is used. The outputs from the eddy current transducers (3) are fed to the eddy current controllers (6). Since the amplitude of vibrations are in microns the eddy current sensors mounted to an isolated concrete block (12). For finding the structural vibration accelerometers (5) are mounted on the structure and the output are fed to a signal amplifier (8). Signals from the eddy current controllers, the vibrometer controller and accelerometers charge amplifiers are connected to a NI B&C connecting block (10) and interfaced to a computer (11), installed with National Instruments data acquisition card 6036. The data obtained in the computer are carefully analysed using especially designed software program written using the LabView software. A screenshot of the LabView program is given in Appendix 2.

4.6. Experimental procedure

The spindle was operated at different rotational speeds. Thirty two signal records were acquired at any given spindle speed, prior to averaging. To avoid aliasing the sampling frequency should be at least twice that of the measured maximum frequency of interest. The maximum frequency of interest is 4 kHz, as most bearing and rigid body inertial contributions fall within this region, and so the cut-off frequency for the laser Vibrometer controller was set at 5 kHz. The chosen sample frequency of 40 kHz was more than sufficient to prevent aliasing. This procedure was undertaken for radial vibrations in both $X$ and $Y$ directions. Averaging diminishes the influence of random noise in a spectrum. The spindle was run at a high speed for a period of time before the measurements were taken. This ensures that the machine is operating at a normal practical temperature and that the effects of start-up conditions are eliminated. All other machinery within the vicinity of the test-rig were switched-off in order to ensure a noise free environment.

4.7. Results and discussion

The experimentally obtained vibrometer signals were subjected to fast Fourier transformation, which is shown in Figure 4.14 for the spindle speed of 25000 rpm. Hanning window was used to overcome the signal leakage problem. From the frequency spectrum of Figure 4.14 it is quite clear that apart from the shaft frequency
and its harmonics which are very dominant, secondary bearing frequencies can be considered as major contributions. The spectrum shown in Figure 4.14 gives a better understanding of the effects of secondary bearing-induced frequencies than the spectrum obtained using the eddy current sensors in Wardle et al. (1983), Aini et al. (1990), Aini et al. (1995), Vafaei et al. (2002), Vafaei et al. (2003) and Lynagh et al. (2000).

Figure 4.14: Vibration Spectrum for spindle running at 25,000 rpm

Wardle et al. (1983) classified surface waviness in terms of wavelengths higher than the dimensions of Hertzian contact formed between rollers or balls to raceway grooves. These waves produce vibration at shaft and ball pass frequencies, these being dominant below 60 times the spindle speed, Rahnejat (1985). It has been shown theoretically by Lynagh et al. (2000) that for small amplitude waviness on a stationary outer ring, vibrations are produced at twice the ball rotational speed, which occurs, under the reported conditions here at 3784 Hz (at 25,000 rpm) for the top end bearings, and is clearly shown in the spectrum as $2f_{b-w}$. Off-sized balls are an important factor in the vibration behaviour as noted by Aini. et al. (1990), with an integer multiple of cage frequency $jfc$. This combines with the outer ring waviness, producing erratic vibration. This contribution is marked as A in the spectrum of Figure 4.14, which is in the range from 1250 Hz to 1750 Hz. This spectral region is
enlarged and shown in Figure 4.15. The vibrations produced for the various integer multiples of $j$, being 6 and 7 for the tool-end bearing combines with $j$ values 13 and 14 for the top-end bearings, resulting in vibrations at amplitudes close to 10 $\mu$m. Secondary effects due to wavy surfaces occur in the higher frequency domain, commencing from 3 kHz in this case. If the outer ring waviness is of the order $kN \pm 1$, then radial vibrations are produced which combine with ball waviness. When these frequencies are given as $kN(f_s - f_d)$, vibrations of considerable effect are induced (see contributions at 1088, 3544, 3707 and 3940 Hz).

![Figure 4.15: Enlarged region of FFT for spindle running at 25,000 rpm](image)

Table 4.2 lists the vibration frequencies due to waviness on different rolling contact surfaces.

**Table 4.2: Measured and calculated vibration frequencies due to secondary effects at 25,000 rpm**

<table>
<thead>
<tr>
<th>Effect due to combination of</th>
<th>Actual Hz</th>
<th>Analytical Hz</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer and inner waviness ($kNf_c - jf_d$), $j = 10$</td>
<td>167</td>
<td>162</td>
<td>3.0</td>
</tr>
<tr>
<td>Outer and inner waviness ($kNf_c - jf_d$), $j = 9$</td>
<td>584</td>
<td>582</td>
<td>0.3</td>
</tr>
<tr>
<td>Outer and inner waviness ($kNf_c - jf_d$), $j = 4$</td>
<td>2669</td>
<td>2669</td>
<td>0.0</td>
</tr>
<tr>
<td>Outer and inner waviness top end ($kNf_c - jf_d$), $j = 8$</td>
<td>584</td>
<td>576</td>
<td>1.4</td>
</tr>
<tr>
<td>Outer and inner waviness top end ($kNf_c - jf_d$), $j = 9$</td>
<td>167</td>
<td>165</td>
<td>1.2</td>
</tr>
<tr>
<td>Inner race waviness and ball waviness $kN(f_s - f_d) + j2f_b$, $j = 7$</td>
<td>1084</td>
<td>1084</td>
<td>0.0</td>
</tr>
</tbody>
</table>

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**Chapter 4: Experimental Analysis**

Outer race waviness and ball size \((kNf_c - jf_o), j=1\)  
\begin{array}{ccc}
3707 & 3711 & -0.1
\end{array}

Outer race waviness and ball size \((kNf_c - jf_o), j=4\)  
\begin{array}{ccc}
3586 & 3565 & 0.6
\end{array}

Outer race waviness and ball size \((kNf_c - jf_o), j=5\)  
\begin{array}{ccc}
3378 & 3378 & 0.0
\end{array}

Outer race waviness and ball size \((kNf_c - jf_o), j=11\)  
\begin{array}{ccc}
2252 & 2252 & 0.0
\end{array}

Inner race waviness and ball waviness \((kN(f_o - f_c - jf_o), j=6\)  
\begin{array}{ccc}
3728 & 3728 & 0.0
\end{array}

Inner race waviness and ball waviness \((kN(f_o - jf_o), j=7\)  
\begin{array}{ccc}
3545 & 3545 & 0.0
\end{array}

Inner race waviness and ball waviness \((kN(f_o - jf_o), j=9\)  
\begin{array}{ccc}
3561 & 3578 & -0.5
\end{array}

The results show very good agreement between the measured and processed data and those predicted using the analytical model described in chapter 3, with maximum percentage error between these being less than 3% in all the spectral contributions.

Secondary bearing-induced frequencies are caused due to the waviness of the shaft contacting surface, rolling bodies and races. These are usually of very low amplitude in normal cases and only regarded as significant in precision applications. In high speed precision operations these frequencies can contribute to premature resonance of the system. Since very little literature exists in this area, the occurrence of these frequencies was tested with repeated experiments at a spindle speed of 20,000 rpm.

The Fourier spectrum for the spindle running at 20,000 rpm is shown in Figure 4.16 with all the significant vibration peaks identified.

![Figure 4.16: Vibration Spectrum for spindle running at 20,000 rpm](image)
Table 4.3 below shows experimentally identified secondary frequencies and the comparison of those frequencies with analytically determined ones.

<table>
<thead>
<tr>
<th>Effect due to combination of</th>
<th>Actual Hz</th>
<th>Analytical Hz</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>outer race waviness and ball size (kNf_c - jf_o), j=18</td>
<td>681</td>
<td>674.22</td>
<td>-1.0</td>
</tr>
<tr>
<td>outer race waviness and ball size (kNf_c - jf_o), j=17</td>
<td>814.6</td>
<td>809.06</td>
<td>-0.6</td>
</tr>
<tr>
<td>outer race waviness and ball size top end (kNf_c - jf_o), j=12</td>
<td>1180</td>
<td>1167</td>
<td>-1.1</td>
</tr>
<tr>
<td>outer race waviness and ball size (kNf_c - jf_o), j=12</td>
<td>1480</td>
<td>1483.3</td>
<td>0.0</td>
</tr>
<tr>
<td>outer race waviness and ball size (kNf_c - jf_o), j=7</td>
<td>1846</td>
<td>1815</td>
<td>-1.7</td>
</tr>
<tr>
<td>Inner race waviness and ball size (kNf_c - jf_o), j=16</td>
<td>2178</td>
<td>2181.4</td>
<td>0.15</td>
</tr>
<tr>
<td>Inner race waviness and ball waviness (kNf_c - jf_o), j=1</td>
<td>3143</td>
<td>3143</td>
<td>0.0</td>
</tr>
<tr>
<td>Inner race waviness and ball size kNf_c +jf_o, j=10</td>
<td>3176</td>
<td>3194</td>
<td>0.5</td>
</tr>
<tr>
<td>outer race waviness and ball size (kNf_c + jf_o), j=3</td>
<td>3500</td>
<td>3505</td>
<td>0.0</td>
</tr>
</tbody>
</table>

4.8 Validation of numerical model

Chapter 3 dealt with the development of a numerical model for spindle dynamics, which can be further detailed by including effect of lubrication. Therefore, it is important to numerically predict results which can be compared with the experimental findings.

![Figure 4.17: Frequency Spectrum spindle running at 25,000 rpm - Numerical](image)
In order to compare the numerical results with the experiments care was taken to use identical conditions. Figure 4.17 shows the FFT spectrum of radial vibrations along X axis and the same for experiment is shown in Figure 4.18. Since the vibration amplitudes other than shaft frequency are relatively small a qualitative comparison is done.

Figure 4.18: Frequency Spectrum spindle running at 25,000 rpm - Experiment

The highest vibration amplitude recorded for numerical is 1.34 \( \mu \text{m} \) at 416.7 Hz at the shaft frequency \( (f_s) \). Experimentally, the highest vibration amplitude is 11.7 \( \mu \text{m} \), which occurs at 416 Hz, the shaft frequency \( (f_s) \). The difference of 10.36 \( \mu \text{m} \) may be due to the spindle unbalance in practice. In the numerical model, the eccentricity is considered to be zero. However, a more likely explanation is that the numerical model lacks sources of damping, mainly dry friction in bearing housing, as well as thermo-mechanical distortions. Thus, comparisons are only valid on frequency rather than amplitude contributions (qualitative comparisons). The noticeable peaks occur for cage frequency and its harmonics in both cases. For the numerical model, the cage frequency occurs at 186.6 Hz with vibration amplitude of 0.34 \( \mu \text{m} \). The same for experiment occurs at 188 Hz with amplitude of 0.13 \( \mu \text{m} \). Table below shows the comparison between dominant frequencies in both cases.
Table 4.4: Frequency and Vibration amplitude comparison

<table>
<thead>
<tr>
<th>Frequency Component</th>
<th>Numerical</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Hz)</td>
<td>Amplitude (μm)</td>
</tr>
<tr>
<td>Shaft frequency ( f_s )</td>
<td>416.7</td>
<td>1.34</td>
</tr>
<tr>
<td>Cage frequency (-First harmonics( f_c ))</td>
<td>186.6</td>
<td>0.34</td>
</tr>
<tr>
<td>Cage frequency (-Second harmonics( 2f_c ))</td>
<td>376.6</td>
<td>0.466</td>
</tr>
<tr>
<td>Cage frequency (-Third harmonics( 3f_c ))</td>
<td>563.3</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Comparison of the numerical and experiments results shows good agreement. The small differences between the results may be attributed to the effect of damping, frictional drag of lubricant and inertial effects of the support structure. The interactions between cage with races and cage to balls also contribute to the differences, are these are not accounted for in the current model.

4.9. Closure

This chapter discussed about the design of a test rig for obtaining vibration characteristics of high speed precision routing spindle. Comparison of various vibration measurement transducers were done and laser vibrometry was found to be the best available, hence selected for this study. Carefully designed test procedures and data analysis techniques are shown along with well presented results. These results are compared with results obtained from numerical model discussed in chapter 3. Comparison of the numerical and experiments results shows good agreement. The numerical model provides with information about the load acting on each ball at a given instant of time. From this information a detailed analysis on the lubricated contact conjunction can be carried out. Since most of the high precision ball bearings are lubricated with grease, analysis of grease lubricated contact conjunction will provide us not only with detailed information of contact pressure, film thickness and plug flow but with a information on power loss also. This will be dealt in coming chapters.
Chapter 5

GREASE LUBRICATION

5.1. Introduction

The American Society for Testing and Materials (ASTM) defines lubricating grease as: "A solid or semi-fluid lubricant consisting of a thickening agent in a liquid lubricant. Other ingredients imparting special properties may be included". Grease consists of an oil or other fluid lubricant that is mixed with another thickener substance; a soap, to form a solid. Greases are a type of shear-thinning or pseudo-plastic fluid, which means that the viscosity of the fluid is reduced under shear. After sufficient force to shear the grease has been applied, the viscosity drops and approaches that of the base lubricant, such as mineral oils. This sudden drop in shear force means that grease is considered a plastic fluid, and the reduction of shear force with time makes it thixotropic, Greases are employed where heavy pressures exist and where oil drip from a bearing is not desired. Greases are also used, where motions of the contacting surfaces are discontinuous and it is difficult to maintain a separating lubricant film in conjunctions. Grease-lubricated bearings have better frictional characteristics at start-up. Due to these factors grease is the most commonly used lubricant in nearly 80% of machines.

The first use of a grease dates to 4000 BC, the time when the Egyptians used it to resolve the problem of excessive friction in chariot wheels. These first greases were made of calcium mixed with animal fat and sometimes with vegetable oils. This type of grease was used until the 19th century. Then, at the end of the 19th century, the first greases from mineral oil bases were developed and used as extremely efficacious lubricants for the miners' carts and industrial machines which, in this period, functioned quite slowly. The solid grease commonly used was "briquette", which continued in use until the middle of the 20th century, and is still in use in some parts of the world. With the development of steam engines, motor-driven vehicles and industrial and agricultural machines during the 20th century, there was a growing need for more efficient greases. Thus, greases with bases of metallic sodium soaps, aluminium, barium and others were developed. There was a large gamut of greases, because each product was developed for a specific application, including greases for
chassis, bearings, steering mechanisms, gears, carts, conveyors, wagons, etc. Around 1950, the introduction of lithium-based "multi-use" grease was greeted with scepticism. Lithium-based greases are the most commonly used now, because of the higher melting point of sodium and lithium based greases than calcium-based greases. However, they are not resistant to the action of water. Lithium-based grease has a dropping point of 190 °C to 220 °C (350 °F to 400 °F). However the maximum usable temperature for Lithium-based grease is 120 °C, which excludes it for many applications. This has led to the development of different types of greases, which are discussed below.

5.2. Grease Composition

Greases always contain three basic active ingredients. There are base mineral or synthetic oil, additives and a thickener. For thickeners, metal soaps and clays are often used. In most cases the mineral oil plays the most important role in determining the grease performance, but in some instances the additives and the thickener can be critical. The type and amount of thickener (typically 5 - 20%) can have a critical effect on grease properties. Very often additives which are similar to those for lubricating oils are used. Sometimes fillers, such as metal oxides, carbon black, molybdenum disulphide, polytetrafluoroethylene are also added. Proper grease specification requires all of the components of oil selection and beyond. Other special considerations for grease selection include thickener type and concentration, consistency, dropping point and operating temperature range, worked stability, oxidation stability, wear resistance, etc

5.2.1. Base Oil Viscosity

Mineral oils are most often used as the base stock in grease formulation. About 99% of greases are made with mineral oils. Naphthenic oils are the most popular despite their low viscosity index. They maintain the liquid phase at low temperatures and easily combine with soaps. Paraffinic oils are poorer solvents for many of the additives used in greases, and with some soaps they may generate a weaker gel structure. On the other hand, they are more stable than the naphthenic oils. Hence, they are less likely to react chemically during grease formulation. Synthetic oils are used for greases which are expected to operate under extreme conditions. The most
commonly used are synthetic esters, phosphate esters, silicones and fluorocarbons. Synthetic base greases are designed to be fire resistant and to operate in extremes of temperature; low and high. Vegetable oils are also used in greases intended for the food and pharmaceutical industries, but even in this application their use is quite limited. The viscosity of the base oil used in making a grease is important since it has some influence on its consistency. The base oil viscosity selection should be application oriented. There are several common methods for determining minimum and optimum viscosity requirements. For rolling element bearings speed factor \( N_s d_p \) is commonly used. Speed factor accounts for the surface speed of the bearing elements, determined by:

\[
N_s d_p = \frac{N (d_i + d_o)}{2}
\]  

(5.1)

where \( N_s \) is the speed of the shaft, \( d_i \) is the inner or bore diameter and \( d_o \) the outer diameter. Knowing the speed factor value and the likely operating temperature, the minimum viscosity requirement can be read directly from the American Society for Testing Materials (ASTM) Method D 2270 (viscosity index charts). It would be much better to use a chart that identifies the viscosity at the operating temperature, then determine the viscosity grade from a viscosity/temperature chart for a given lubricant.

5.2.2. Thickener

The grease consistency is more dependent on the amount and type of thickener used more than the type of base oil employed. For example, if the thickener can withstand heat, the grease would also be suitable for high temperature applications. If the thickener is water resistant, then the grease would also be water resistant, etc. Hence, the grease type is usually classified by the type of thickener used in its manufacture. As there are two fundamental types of thickener that can be used in greases, the commercial greases are divided into two primary classes; soap and non-soap based. Soap type greases are the most commonly produced. Soaps are very important in the production of greases. The most commonly used soap type greases are calcium, lithium, aluminium, sodium and others (mainly barium). In non-soap type greases inorganic, organic and synthetic materials are used as thickeners. Inorganic thickeners
are in the form of very fine powders which have sufficient porosity and surface area to absorb oil. The most commonly used are the silica and bentonite clays. Because of their structure these types of greases have no melting point, so their maximum operating temperature depends on the oxidation stability of the base oil and its inhibitor treatment.

5.2.3. Additives

The most common additives include anti-oxidants, rust and corrosion inhibitors, tackiness, anti-wear and extreme pressure (EP) additives.

Anti-oxidants must be selected to match the individual grease. Their primary function is to protect the grease during storage and extend the service life, especially in high temperature applications.

Rust and corrosion inhibitors are added to make the grease non-corrosive to bearings operating in machinery. The function of corrosion inhibitors is to protect the non-ferrous metals against corrosion, whereas the function of rust inhibitors is to protect ferrous metals. Tackiness additives are sometimes added to impart a stringy texture and to increase the cohesion and adhesion of the grease to the surface.

Anti-wear and Extreme Pressure (EP) additives improve, in general, the load-carrying capacity in most rolling contact bearings and gears. Extreme pressure additives react with the surface to form protective films which prevent metal to metal contact and the consequent scoring or welding of the surfaces. The additives most commonly used as anti-seizure and anti-scuffing compounds are graphite and molybdenum disulphide.

5.2.4. Classification of Greases

The most widely known classification of greases is related to their consistency and was established by the National Lubricating Grease Institute (NLGI). It classifies the greases into nine grades, according to their penetration depth, from the softest to the hardest according to SAE standard (1987), as shown in Table 5.1. Depending on the application a specific grease grade is selected.
Table 5.1: NLGI Grease Classification

<table>
<thead>
<tr>
<th>NLGI grade</th>
<th>Worked (60 strokes) penetration range [×10⁻¹ mm] at 25°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>445 - 475</td>
</tr>
<tr>
<td>00</td>
<td>400 - 430</td>
</tr>
<tr>
<td>0</td>
<td>355 - 385</td>
</tr>
<tr>
<td>1</td>
<td>310 - 340</td>
</tr>
<tr>
<td>2</td>
<td>265 - 295</td>
</tr>
<tr>
<td>3</td>
<td>220 - 250</td>
</tr>
<tr>
<td>4</td>
<td>175 - 205</td>
</tr>
<tr>
<td>5</td>
<td>130 - 160</td>
</tr>
<tr>
<td>6</td>
<td>85 - 115</td>
</tr>
</tbody>
</table>

5.3. Lubrication Mechanism of Greases

Greases are commonly used in machinery operating under the elastohydrodynamic regime of lubrication (EHL), i.e. in rolling contact bearings and some gears. The behaviour of grease in the contact region is shown in Figure 5.1 as explained by Cann et al (1992).

![Figure 5.1: Schematic of an operating EHD contact (Cann et al (1992))](image)

Experiments revealed that the measured film thickness of grease under EHL condition is greater initially than if the base oil contained in the grease were acting alone (Poom...
Chapter 5 – Grease Lubrication

(1972). With continued running, however, the film thickness due to grease declines to about 0.6 of that of the base oil. The initial thick grease layer is rapidly removed by the rolling or sliding element and the lubrication is controlled by a thin viscous layer which is a mixture of oil and degraded thickener as observed by Cann (1999). The decline in film thickness can only be explained in general terms of scarcity of grease in the contact. Grease is a semi-solid so that once expelled from the contact it probably returns only with difficulty. It has also been suggested that conveyance of oil by capillary action from the bulk grease to the wearing contact is possible (Cann (1999)). However, there has been no detailed work conducted as yet to test this hypothesis, because of the complexity of the grease rheological behaviour. Pressure-build-up mechanisms and the operating lubricant film thickness in non-conformal contacts for oil as a lubricant with Newtonian characteristics is well documented (see for example Dowson and Higginson (1977) and Hamrock(1994)), while the theory of grease lubrication is far from perfect. The behaviour of grease in an EHD contact is presented in detail later.

5.4. Geometry of bodies in contact

Figure 5.2: Geometry of contacting elastic bodies (Hamrock and Dowson (1981))
The undeformed geometry of contacting solids in a ball bearing can be represented by two ellipsoids as shown in Figure 5.2. Two solids having different radii of curvature in a pair of principal planes (x and y) passing through the contact between the solids make an ideal single point contact under no applied load. Such a condition is called "point contact". As the load increases, the point contact enlarges to an area, the shape and size of which depends upon factors such as the applied load, curvature of the surfaces and the elastic properties of materials in contact. Under the influence of low loads the contacting area remains a point. The geometrical separation of two ellipsoids (shown in Figure 5.3) can be represented with an equivalent ellipsoid with the same separation as shown (Hamrock and Dowson (1981)).

\[
R_{ax}^2 = x^2 + (R_{ax} - S_{ax})^2 \quad (5.2)
\]

\[
S_{ax} = R_{ax} + \sqrt{(R_{ax}^2 - x^2)} \quad (5.3)
\]

\[
(S_{ax} - R_{ax})^2 + (R_{ax}^2 - x^2) = 0 \quad (5.4)
\]

\[
x^2 = S_{ax} \left( 2R_{ax} - S_{ax} \right) \quad (5.5)
\]

From Figure 5.3, it follows that:

In a ball bearing, contact separation of two bodies \( S_{ax} \) is much smaller than the radius of curvature \( 2R_{ax} \gg S_{ax} \) so the equation can be rewritten as:
\[ S_{ax} \approx \frac{x^2}{2R_{ax}} \] (5.6)

Similar expressions can be written for \( S_{bx} \), \( S_{ay} \), and \( S_{by} \), thus:

\[ S_{bx} \approx \frac{x^2}{2R_{bx}} \] (5.7)
\[ S_{ay} \approx \frac{y^2}{2R_{ay}} \] (5.8)
\[ S_{by} \approx \frac{y^2}{2R_{by}} \] (5.9)

The effective radii in \( x \) and \( y \) are given as:

\[ \frac{1}{R_x} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} \] (5.10)
\[ \frac{1}{R_y} = \frac{1}{R_{ay}} + \frac{1}{R_{by}} \] (5.11)

Geometrical separation in Figure 5.3 can be written as:

\[ S(x, y) = S_{ax} + S_{bx} + S_{ay} + S_{by} \] (5.12)

Combining equations (5.10), (5.11) and 5.12), geometrical separation can be written as:

\[ S(x, y) = \frac{x^2}{2R_x} + \frac{y^2}{2R_y} \] (5.13)

5.5. Hertzian Contact

The stresses and deformation for two perfectly smooth ellipsoidal solids of revolution in contact are found using the classical theory proposed by Hertz (1881). The Hertzian assumptions were:

1) Solids in contact are homogenous and isotropic
2) The contact is localised and subject to small strains within the elastic limit.

3) The contact is frictionless (this condition can be relaxed).

4) Contact dimensions are very small compared to the principal radii of contacting solids (the solids are considered as semi-infinite elastic half-spaces).

5) The solids are at rest and in equilibrium (elastostatic condition)

![Figure 5.4: Contact area in ball bearing](image)

Using this assumption for contact of ball bearings, as shown in Figure 5.4, leads to the ellipsoidal pressure distribution:

\[
P = P_{\text{max}} \left[ 1 - \left( \frac{x}{b} \right)^2 - \left( \frac{y}{a} \right)^2 \right]^{\frac{1}{2}}
\]  

(5.14)
Figure 5.5: Hertzian pressure distribution in ellipsoidal contact

The expression for the maximum Hertzian pressure for an elliptical point contact can be obtained by integrating the pressures over the contact area (shown in Figure 5.5). Thus:

\[
P_h = \left( \frac{6WE^*}{\pi^3 R_x R_y} \right)^{\frac{1}{3}}
\]

(5.15)

where \( W \) is the applied load and \( E^* \) the effective modulus of elasticity, given as:

\[
E^* = \frac{2}{\left( \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)}
\]

(5.16)

\( a \) and \( b \) are the semi-half-widths of the elliptical footprint. These are known as the semi-minor and semi-major axes in the \( y \) and \( x \) directions respectively as shown in figure 6.5, which can be found by the following expression:

\[
\sqrt{ab} = \left( \frac{3W\sqrt{R_x R_y}}{4E^*} \right)^\frac{1}{3}
\]

(5.17)

The maximum deformation at the centre of the contact is given by Hertz (1881) as:
\[ \delta_h = \frac{1}{2} \left( \frac{9W^2}{2E' \sqrt{R_x R_y}} \right)^{\frac{1}{3}} \] (5.18)

For a point contact \( R_x = R_y = R \), so the expressions for the maximum Hertzian pressure, contact semi-half-width and maximum deflection are:

\[ P_h = \left( \frac{6WE'}{\pi^3 R^2} \right)^\frac{1}{3} \] (5.19)

\[ b = \left( \frac{3WR}{4E'} \right)^{\frac{1}{3}} \] (5.20)

\[ \delta_h = \frac{1}{2} \left( \frac{9W^2}{2E' R} \right)^{\frac{1}{3}} \] (5.21)

5.6. Computational grid generation

The dimensions of the elastostatic footprint for a particular load and geometry are calculated using equations (5.19), (5.20) and (5.21). Once the dimensions of the elastostatic footprint for a particular load and geometry are calculated (by using the Hertzian relationships in section 5.2.2), which is generally larger than the contact area is identified, Hamrock and Dowson (1977). The computational domain is chosen to have a rectangular shape in the X-Y plane and in Z direction the domain is bounded by the solid surfaces as shown in Figure 5.6.
For a fully flooded condition the inlet should be $N$ times larger than the contact footprint semi-half-width, where $N$ depends upon the contact being examined. For the current study $N=9$, which is shown in Figure 5.6. Lubricant pressures and viscosity reduce dramatically in the exit region from the contact, thus, an outlet region smaller than the inlet trail is used for the computation domain. Because of the dimensionless representation of coordinates, the actual elastostatic footprint has changed to a circle with unit radius as shown in Figure 5.7, regardless of the ellipticity parameter.
As the direction of lubricant entrainment is assumed to be coincident with the minor axis of the Hertzian contact, the axis of symmetry is along the direction of entraining motion. Because of this symmetry in computation domain, the computing time can be significantly reduced (Jalali (2000)).

5.7. Formulation of the lubricated conjunction

Ertel and Grubin (1949) provided the first convincing evidence of full film EHL by combining Reynolds' hydrodynamics theory (Osborne Reynolds (1886)) and the classical Hertzian contact mechanics (Hertz (1881)). They noted that as the lubricant is drawn into the converging wedge of two rolling/sliding bodies in contact, its viscosity increases dramatically due to high generated pressures that deform the contiguous surfaces. Because of small contact area local flattening of solids take place and the pressure rise follows the Hertzian ellipsoidal distribution, giving rise to a flat film for a lubricated contact. Due to continuity of flow condition, lubricant progresses toward the exit, where the pressure is at the ambient value, thus its viscosity decreases dramatically. Thus thinning of film takes place and the pressure rises due to diminution of gap (as a hydrodynamic effect), giving rise to the appearance of a pressure spike or "pip", which falls to ambient value very quickly.

Dowson and Higginson (1959) published the full computer solution of the problem of an elastic cylinder rolling on a plane and lubricated by a fluid whose viscosity increases exponentially with pressure. The full solution was shown to fit experimental values for film thickness very accurately and also, surprisingly did not vary very greatly from the Ertel-Grubin solution. This led people to apply variations of the Ertel-Grubin approximation to the solution of a ball rolling on a plane- the 'point contact' problem. Gohar and Cameron (1966) using a very coarse mesh obtained an approximate result which was considerably refined by Wedeven, Evans & Cameron in 1971. Archard and Cowking (1965) incorporating Kapitza's 1955 solution for finite parabolas also derived an acceptable solution which was extended by Cheng in 1970 to elliptical contacts. This was further developed by and a general theory for lubricated point contacts was developed by Thorp and Gohar (1972). First numerical work for point contact was reported by Hamrock and Dowson (1976). Squeeze
velocity was incorporated into the solution by Mostofi and Gohar (1982). Multi grid technique, Brandt (1984), was applied by Lubrecht and Venner (1987) for more accurate solutions of point contacts at high loads. Effect of entrainment angle at high loads on pressure distribution and film thickness using multi grid techniques was investigated by Jalali (2000).

All of these works reported are for oil lubrication. The behaviour of grease inside a contact conjunction (see section 6.3) differs from that for oil so the Reynolds equation used in the above mentioned work cannot be used directly for grease lubrication. The flow of grease follows a non-Newtonian behaviour, which has been tackled by Tee and Eyring (1955). Wada et al (1973) used a flow model proposed by Bingham (1922), and Kauzlarich and Greenwood (1972) used a yield power model proposed by Herschel and Bulkley (1922). For the comparison of these models refer chapter 2 section 2.4.1

5.8. Derivation of Modified Reynolds Equation for Grease Lubrication

The x co-ordinate is assumed to be in the direction of entraining motion and the z co-ordinate normal to the surfaces, the origin being at the centre of the film. Thus, y is the direction of side leakage. Inertial and gravity (body) forces are neglected. The
shear stress $\tau$ is considered as the function of $z$ only (i.e. it is assumed that the oil film is nearly parallel), and the yield shear stress and plastic viscosity only changes with pressure and only changes in the $x$ and $y$ directions. The force balance on an element of fluid gives:

$$\frac{\partial \tau_{zx}}{\partial z} = \frac{\partial p}{\partial x} \quad (5.22)$$

and:

$$\frac{\partial \tau_{zy}}{\partial z} = \frac{\partial p}{\partial y} \quad (5.23)$$

If pressures are assumed not to vary across the film due to its thinness, then integrating the above equations yields:

$$\tau_{zx} = z \frac{\partial p}{\partial x} \quad \text{and} \quad \tau_{zy} = z \frac{\partial p}{\partial y} \quad (5.24)$$

The Herschel-Bulkley (HB) equation is one of the more realistic constitutive models for grease. HB fluids are described by a three-parameter rheological model described mathematically in equation (5.27). When the local shear stress is below the yield stress, HB fluids behave as rigid solids. Once the yield stress is exceeded, HB fluids flow with a non-linear stress-strain relationship, either as a shear-thickening fluid, or a shear-thinning one. According to the Herschel Bulkley model a central flow region occurs, in which $\tau < \tau_0$ and enclosed by regions of shear flow (if $\tau > \tau_0$). The plug flow region is of width $h_p$ as shown in Figure 5.9, where:

$$\tau_{zx} = h_p \frac{\partial p}{2 \partial x} \quad (5.25)$$

$$\tau_{xy} = h_p \frac{\partial p}{2 \partial y} \quad (5.26)$$
Herschel-Bulkley equation is given as:

$$\tau = \tau_0 + \phi |D|^n$$

(5.27)

where $\tau$ is the shear stress, $N/m^2$, $\tau_0$ is the yield stress, $N/m^2$, $\phi$ is plastic viscosity, Pas$^n$ units vary, $D$ is the shear rate, $S^{-1}$ and $n$ the power law exponent.

Rearranging the Herschel-Bulkley equation gives:

$$\phi \left( \frac{du}{dz} \right)^n = \tau - \tau_0$$

(5.28)

$$\phi \left( \frac{du}{dz} \right)^n = z \left( \frac{\partial p}{\partial x} \right) - \frac{h_p}{2} \left( \frac{\partial p}{\partial x} \right)$$

(5.29)

$$\phi \left( \frac{du}{dz} \right)^n = \left( z - \frac{h_p}{2} \right) \frac{\partial p}{\partial x}$$

(5.30)

Substituting for $\frac{1}{n} = m$ and $\frac{\partial p}{\partial x} = \frac{2\tau_0}{h_p}$:
Since the pressure, therefore the yield stress and the plastic viscosity have been assumed to be functions of x and y only, these equations can be integrated directly to give the velocity gradients as:

\[
\frac{du}{dz} = \left( z - \frac{h_p}{2} \right)^m \left( \frac{2 \tau_0}{\phi h_p} \right)^m
\]

(5.31)

where \( C \) is the integration constant:

\[
u = \left( z - \frac{h_p}{2} \right)^{m+1} \frac{1}{m+1} \left( \frac{2 \tau_0}{\phi h_p} \right)^m + C
\]

(5.32)

Now the plastic viscosity of the lubricant may change considerably across the thin film (in the z direction) as a result of temperature variations that arise in some bearing problems. In this case, as mentioned by Hamrock and Dowson (1977) progress towards a satisfactory Reynolds equation is considerably more complicated.

An approach which is satisfactory in the majority of fluid film applications is to treat \( \phi \) as the average plastic viscosity across the film. Note that this assumption does not restrict the variation of plastic viscosity in the x and y directions. After the application of boundary conditions as stated below, the equation for velocity in the upper shear flow region becomes:

\[
z = \frac{h}{2}, \quad u = u_b
\]

\[
C = u_b - \left( \frac{2 \tau_0}{\phi h_p} \right)^m \frac{1}{m+1} \left( \frac{h}{2} - \frac{h_p}{2} \right)^{m+1}
\]

(5.33)

\[
u = u_b + \left( \frac{2 \tau_0}{\phi h_p} \right)^m \frac{1}{m+1} \left[ \left( z - \frac{h_p}{2} \right)^{m+1} - \left( \frac{h}{2} - \frac{h_p}{2} \right)^{m+1} \right]
\]

(5.34)

For plug flow: \( z = \frac{h_p}{2} \), thus:

\[
u_p = u_p + \left( \frac{2 \tau_0}{\phi h_p} \right)^m \frac{1}{m+1} \left[ \left( \frac{h_p}{2} - \frac{h_p}{2} \right)^{m+1} - \left( \frac{h}{2} - \frac{h_p}{2} \right)^{m+1} \right]
\]

(5.35)
For a grease considered to be Bingham fluid the velocity of core can be calculated as:

$$u_p = u_b - \left( \frac{\tau_0}{\phi h_p} \right)^n \left( \frac{1}{m+1} \right) \left( \frac{h - h_p}{2} \right)^{m+1}$$

(5.38)

For a Bingham fluid the power law factor \( n = 1 \), so the above equation reduces to:

$$u_p = u_b - \left( \frac{\tau_0}{\phi h_p} \right)^n \left( \frac{1}{2} \right) \left( h - h_p \right)^2$$

(5.39)

The flow can be considered as sum of plug flow and that of the base oil, so the volume of flow per unit width = volume of plug flow + volume of upper shear flow region. Taking advantage of the symmetry of the flow field, the volume flow per unit width is calculated from equation (5.40)

$$q_x = 2 \int_0^{h/2} u dz = 2 \left[ \int_0^{h_2/2} u dz + \int_{h_2/2}^{h/2} u dz \right]$$

(5.40)

Integrating the first part and applying the boundary conditions, the above equation can be rewritten as:

$$q_x = 2 \left( u_p \frac{h_p}{2} + \int_{h_2/2}^{h/2} u dz \right)$$

(5.41)

Substituting for the velocity in the x direction from equation (5.41) gives:

$$q_x = u_p h_p + 2 \left( \int_{h_2/2}^{h/2} u + \left( \frac{2\tau_0}{\phi h_p} \right)^n \left( \frac{1}{m+1} \right) \left[ \left( \frac{z - h_p}{2} \right)^{m+1} - \left( \frac{h - h_p}{2} \right)^{m+1} \right] \right)$$

(5.42)

Integrating equation (5.42) results in:
Applying boundary conditions equation (5.43) can be written as:

\[ q_z = u_p h_p + 2 \left( u_b \left( \frac{1}{2} - \frac{1}{2} h_p \right) + \frac{z h_p}{\phi h_p} \right)^m \left[ \frac{1}{(m+1)} \left( \frac{1}{2} \frac{1}{2} h_p \right)^{m+2} \left( \frac{1}{2} \frac{1}{2} h_p \right)^{m+1} \right] \]

Rearranging equation (5.44)

\[ q_z = u_p h_p + 2 \left( u_b \left( \frac{1}{2} - \frac{1}{2} h_p \right) + \frac{z h_p}{\phi h_p} \right)^m \left[ \frac{1}{(m+1)} \left( \frac{1}{2} \frac{1}{2} h_p \right)^{m+2} \left( \frac{1}{2} \frac{1}{2} h_p \right)^{m+1} \right] \]

Volumetric flow for the fluid flow can be simplified further by multiplying 2 and bringing the common factors

\[ q_z = u_p h_p + 2 \left( u_b \left( \frac{1}{2} - \frac{1}{2} h_p \right) + \frac{z h_p}{\phi h_p} \right)^m \left[ \frac{1}{(m+1)} \left( \frac{1}{2} \frac{1}{2} h_p \right)^{m+2} \left( \frac{1}{2} \frac{1}{2} h_p \right)^{m+1} \right] \]

Once a power factor \( n=1 \) is assumed, then the lubricant becomes a Bingham plastic and different flow conditions should be considered. The flow for a Bingham plastic
was explained by Bingham in 1922 as the lubricant becomes a solid if the shear stress less than the yield stress. This happens at the centre of the contact, where a plug flow is considered and the height $h_p$ is very small. In such a case it is considered that the core formed due to the stress level below the yield stress would float. The second case is when the pressure reaches its maximum value at the centre of the contact, where both the surfaces attain the same surface velocity. Hence, there is no shear and the lubricant becomes solid and adheres to the surfaces. This case occurs only at the contact centre and the minimum exit film. This has been explained by Wada et al (1973) for the Bingham model.

Substituting: $m = \frac{1}{n}$, then:

$$q_x = u_b h + h_p (u_p - u_b) - \left( \frac{t_0}{\phi h_p} \right) \left( \frac{h - h_p}{2} \right) \frac{1}{1 + \frac{1}{n}} \frac{1}{1 + \frac{1}{n + 2}} \frac{(h - h_p)^{1+2}}{2(n+1)} \text{ (5.51)}$$

For Bingham plastic: $n = 1$, thus:

$$q_x = u_b h + h_p (u_p - u_b) - \left( \frac{t_0}{\phi h_p} \right) \frac{(h - h_p)^3}{6} \text{ (5.52)}$$

Rearranging terms:

$$q_x = u_b h + h_p (u_p - u_b) - \left( \frac{2t_0}{\phi h_p} \right) \frac{(h - h_p)^3}{12} \text{ (5.53)}$$

Substituting: $\frac{2t_0}{h_p} = \frac{\partial p}{\partial x}$, yields:

$$q_x = u_b h + h_p (u_p - u_b) - \left( \frac{\partial p}{\partial x} \right) \frac{(h - h_p)^3}{12\phi} \text{ (5.54)}$$

The volumetric flow in the $x$ direction can be written as:
\[ q_x = u_x h + h_x u_x - u_x \left( \frac{\partial p}{\partial x} \right) \frac{(h-h_y)^3}{12\phi} \] (5.55)

Similarly, volume or flow through y direction becomes:

\[ q_y = v_y h + h_y v_y - v_y \left( \frac{\partial p}{\partial x} \right) \frac{(h-h_y)^3}{12\phi} \] (5.56)

The Reynolds equation is formed by introducing these expressions into the continuity equation written in the integral form as:

\[ \int \left[ \frac{\partial p}{\partial t} + \frac{\partial}{\partial x} (\rho u_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho w_z) \right] \, dz = 0 \] (5.57)

A general rule of integration is that:

\[ \int f[x, y, z] \, dz = \frac{\partial}{\partial x} \int f(x, y, z) \, dz + \frac{\partial}{\partial y} \int f(x, y, z) \, dz + \frac{\partial}{\partial z} \int f(x, y, z) \, dz \]

Substituting for \( q_x \) and \( q_y \) gives:

\[ h \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \rho \left( u_x h + h_x u_x - u_x \left( \frac{\partial p}{\partial x} \right) \frac{(h-h_y)^3}{12\phi} \right) \right) - \rho u_x \frac{\partial h}{\partial y} + \frac{\partial}{\partial y} \left( \rho \left( v_y h + h_y v_y - v_y \left( \frac{\partial p}{\partial y} \right) \frac{(h-h_y)^3}{12\phi} \right) \right) - \rho v_y \frac{\partial h}{\partial z} + \rho (w_y - w) = 0 \] (5.60)

\[ h \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \rho u_x h + \frac{\partial}{\partial x} \rho h_x (u_x - u_x) - \frac{\partial}{\partial x} \left( \frac{\partial p}{\partial x} \frac{(h-h_y)^3}{12\phi} \right) \right) - \rho u_x \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left( \rho v_y h + \frac{\partial}{\partial y} \rho h_y (v_y - v_y) - \frac{\partial}{\partial y} \left( \rho w_z \frac{(h-h_y)^3}{12\phi} \right) \right) - \rho v_y \frac{\partial h}{\partial x} + \rho (w_y - w) = 0 \] (5.61)

\[ h \frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left( \frac{\partial p}{\partial x} \frac{(h-h_y)^3}{12\phi} \right) + \frac{\partial}{\partial y} \rho h_y (u_x - u_x) + \frac{\partial}{\partial y} \rho h_y (v_y - v_y) - \rho u_x \frac{\partial h}{\partial x} - \rho v_y \frac{\partial h}{\partial y} + \rho (w_y - w) = 0 \] (5.62)
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\[ \frac{\partial}{\partial x} \left( \frac{\partial \rho (h-h_p)}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \rho (h-h_p)}{\partial y} \right) \]

\[ = \frac{\partial}{\partial y} \rho v + \frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} \rho h (v - u) - \rho \left( \frac{\partial}{\partial x} \hat{h} \right) + \rho \left( \frac{\partial}{\partial y} \hat{h} \right) + \rho (w_x - w_y) \]  

Substituting \( u = (u_b - u_p) \) and \( v = (v_b - v_p) \), then:

\[ \frac{\partial}{\partial x} \left( \frac{\partial \rho (h-h_p)}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \rho (h-h_p)}{\partial y} \right) \]

\[ = \frac{\partial}{\partial y} \rho v_h + \frac{\partial}{\partial x} \rho u_h - \frac{\partial}{\partial y} \rho h (v - u) - \rho \left( \frac{\partial}{\partial x} \hat{h} \right) + \rho \left( \frac{\partial}{\partial y} \hat{h} \right) + \rho (w_x - w_y) \]

Substituting \( h_a = (h - h_p) \) and simplifying the above equation gives the modified Reynolds equation using the Herschel Buckley model for a Bingham solid as:

\[ = 12 \left[ \frac{\partial}{\partial x} \rho (u_h - h_p) + \frac{\partial}{\partial y} \rho (v_h - h_p) - \rho \left( \frac{\partial}{\partial x} \hat{h} \right) + \rho \left( \frac{\partial}{\partial y} \hat{h} \right) + \rho (w_x - w_y) \right] \]

The two terms in the left hand side of the equation are Poiseuille terms, which describe the flow rate due to pressure gradients. The first and second term in the right hand side are the flow rates due to surface velocities. These are known as the Couette flow terms acting in the X and Y directions. The third term in on the RHS describes the net flow rates due to squeeze film motion and the fourth term is the flow rates due to the local compressing effect. The last two terms can be replaced as follows:
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\[
- \left( u_p \frac{\partial h}{\partial x} + v_p \frac{\partial h}{\partial y} \right) + h \frac{\partial \rho}{\partial t} + \rho (w_a - w_p) = \frac{\partial (h \rho)}{\partial t} \tag{5.68}
\]

The total film thickness \( h \) is a function of \( x, y \) and \( t \):

\[
h = f(x, y, t) \tag{5.69}
\]

\[
dh = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} dx + \frac{\partial h}{\partial y} dy \tag{5.70}
\]

Dividing through by \( dt \):

\[
\frac{dh}{dt} = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} \frac{dx}{dt} + \frac{\partial h}{\partial y} \frac{dy}{dt} \tag{5.71}
\]

where:

\[
u_p \approx u = \frac{dx}{dt}, \quad v_p \approx v = \frac{dy}{dt} \quad \text{and} \quad (w_a - w_p) = \frac{dh}{dt} \tag{5.72}
\]

Substituting \((w_a - w_p), u_p \) and \( v_p \) into equation (5.73) yields:

\[
(w_a - w_p) = \frac{\partial h}{\partial t} + u_p \frac{\partial h}{\partial x} + v_p \frac{\partial h}{\partial y} \tag{5.73}
\]

Rearranging equation (5.75) and multiplying by \( \rho \) to convert to mass flow rate:

\[
\rho \frac{\partial h}{\partial t} = \rho (w_a - w_p) - \rho u_p \frac{\partial h}{\partial x} - \rho v_p \frac{\partial h}{\partial y} \tag{5.74}
\]

Adding \( h \frac{\partial \rho}{\partial t} \) to equation (5.76):

\[
\rho \frac{\partial h}{\partial t} + h \frac{\partial \rho}{\partial t} = \rho (w_a - w_p) - \rho u_p \frac{\partial h}{\partial x} - \rho v_p \frac{\partial h}{\partial y} + h \frac{\partial \rho}{\partial t} \tag{5.75}
\]
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Rearranging:

\[
\frac{\partial (hp)}{\partial t} = \rho (w_a - w_p) - \rho u_p \frac{\partial h}{\partial x} - \rho v_p \frac{\partial h}{\partial y} + h \frac{\partial \rho}{\partial t} \tag{5.76}
\]

Equation (5.80) is the same as equation (5.81), substituting into (5.69) gives the modified Reynolds equation using Herschel Buckley model for a Bingham solid as:

\[
\frac{\partial}{\partial x} \left( \frac{\partial p \rho h^3}{\phi} \right) + \frac{\partial}{\partial y} \left( \frac{\partial p \rho h^3}{\phi} \right) = 12 \left[ \frac{\partial}{\partial x} \rho (u_b h - h_p u) + \frac{\partial}{\partial y} \rho (v_b h - h_p v) + \frac{\partial (h \rho)}{\partial t} \right] \tag{5.77}
\]

In case of mineral oils there is no plug flow, hence \( h_p = 0 \) and the plug flow velocities equate the oil flow velocities, thus: \( u_p = u_b \) and \( v_p = v_b \). Substituting these into equation (5.79) gives the usual Reynolds equation for mineral oils as:

\[
\frac{\partial}{\partial x} \left( \frac{\partial p \rho h^3}{\phi} \right) + \frac{\partial}{\partial y} \left( \frac{\partial p \rho h^3}{\phi} \right) = 12 \left[ \frac{\partial}{\partial x} (\rho u_b h) + \frac{\partial}{\partial y} (\rho v_b h) + \frac{\partial (h \rho)}{\partial t} \right] \tag{5.78}
\]

Equation (5.78) gives the modified Reynolds equation using Herschel Buckley model for a Bingham solid, assuming no side leakage i.e. \( v = 0 \), hence no physical wedge in \( Y \) direction the equation can be rewritten as:

\[
\frac{\partial}{\partial x} \left( \frac{\partial p \rho h^3}{\phi} \right) + \frac{\partial}{\partial y} \left( \frac{\partial p \rho h^3}{\phi} \right) = 12 \left[ \frac{\partial}{\partial x} (\rho u_b h - h_p u) + \frac{\partial (h \rho)}{\partial t} \right] \tag{5.79}
\]

5.9. Dimensionless analysis

The modified Reynolds equation for grease consists of complex physical phenomena, which involves different kinds of physical quantities for which exact functional relationship is unknown. For comprehensive parametric studies and simple
formulation a dimensionless analysis is carried out and the following dimensionless groups are employed:

\[
U = \frac{u}{u_{av}}, \quad \quad \quad U_{p} = \frac{u_{p}}{u_{av}}, \quad \quad \quad U_{t} = \frac{u_{t}}{u_{av}}
\]

Similarly:

\[
V = \frac{v}{u_{av}}, \quad \quad \quad V_{p} = \frac{v_{p}}{u_{av}}, \quad \quad \quad V_{t} = \frac{v_{t}}{u_{av}}
\]

where \( u_{av} \) is the average value of the entrainment motion.

\[
\begin{align*}
X = x/b & \quad \quad \quad x = bX \\
Y = y/a & \quad \quad \quad y = aY \\
\bar{\rho} = \rho/\rho_{0} & \quad \quad \quad \rho = \rho_{0}\bar{\rho} \\
\bar{\phi} = \phi/\phi_{0} & \quad \quad \quad \phi = \phi_{0}\bar{\phi} \\
H = hR_{x}/b^{2} & \quad \quad \quad h = Hb^{2}/R_{x} \\
H_{a} = h_{a}R_{x}/b^{2} & \quad \quad \quad h_{a} = H_{a}b^{2}/R_{x} \\
P = p/P_{0} & \quad \quad \quad p = P_{0}p \\
T = u_{av}t/R_{x} & \quad \quad \quad t = R_{x}T/u_{av} \\
W' = W/(E' R_{x} L) & \quad \quad \quad W = W' E' R_{x} L
\end{align*}
\]

Substituting these values in Reynolds equation:

\[
\frac{\partial}{\partial x} \left( \frac{P_{0} \rho_{0} \bar{\rho} (H_{a}b^{2})}{R_{x} \phi_{0} \bar{\phi}} \right) + \frac{\partial}{\partial y} \left( \frac{P_{0} \rho_{0} \bar{\rho} (H_{a}b^{2})}{R_{x} \phi_{0} \bar{\phi}} \right)
= 12 \left[ \frac{\partial}{\partial x} \rho_{0} \frac{u_{av} U_{av} H_{a}^{3}}{R_{x}} + \frac{\partial}{\partial y} \rho_{0} \left( \frac{u_{av} V_{av} H_{a}^{3}}{R_{x}} + \frac{u_{av} V_{av} H_{a}^{3}}{R_{x}} \right) \right] 
\]

Grouping the similar terms together from the above equation and simplifying:

\[
\frac{\rho_{0} \rho_{0} b^{4}}{R_{x} \phi_{0} \phi} \frac{\partial}{\partial x} \left( \frac{\rho H_{a}^{3}}{\phi} \frac{\partial P}{\partial x} \right) + \frac{b^{3} \rho_{0} \rho_{0} b^{4}}{a^{2} R_{x} \phi_{0} \phi} \frac{\partial}{\partial y} \left( \frac{\rho H_{a}^{3}}{\phi} \frac{\partial P}{\partial y} \right)
= 12 \left[ \frac{u_{av} \rho_{0} b^{2}}{R_{x} b} \frac{\partial}{\partial x} \bar{P} (U_{av} H - U_{av} H_{a}) + \frac{u_{av} \rho_{0} b^{2}}{R_{x} a} \frac{\partial}{\partial y} \bar{P} (V_{av} H - V_{av} H_{a}) + \frac{u_{av} \rho_{0} b^{2}}{R_{x}^{2}} \frac{\partial}{\partial t} \bar{H} \right] 
\]

Substituting \( k = \frac{b}{a} \):
\[ \frac{\rho_0 P_b b^4}{R_s^3 \phi_0} \left[ \frac{\partial}{\partial X} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) + k^2 \frac{\partial}{\partial Y} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) \right] = \frac{12 u_m \rho_0 b}{R_s} \left[ \frac{\partial}{\partial X} \bar{\rho} \left( U_b H - U H_p \right) + b \frac{\partial}{\partial Y} \bar{\rho} \left( V_b H - V H_p \right) + \frac{b}{R_s} \frac{\partial \bar{\rho} H}{\partial t} \right] \]  

(5.83)

Dividing the equation by \( \frac{R_s^3 \phi_0}{\rho_0 P_b b^4} \):

\[ \frac{\partial}{\partial X} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) + k^2 \frac{\partial}{\partial Y} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) = \frac{12 u_m R_s^3 \phi_0}{P_b b^4} \left[ \frac{\partial}{\partial X} \bar{\rho} \left( U_b H - U H_p \right) + k \frac{\partial}{\partial Y} \bar{\rho} \left( V_b H - V H_p \right) + \frac{b}{R_s} \frac{\partial \bar{\rho} H}{\partial t} \right] \]  

(5.84)

The modified Reynolds equation, using Herschel Buckley model for a Bingham solid in dimensionless form can be written as:

\[ \frac{\partial}{\partial X} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) + k^2 \frac{\partial}{\partial Y} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) = \psi \left[ \frac{\partial}{\partial X} \bar{\rho} \left( U_b H - U H_p \right) + k \frac{\partial}{\partial Y} \bar{\rho} \left( V_b H - V H_p \right) + \frac{b}{R_s} \frac{\partial \bar{\rho} H}{\partial t} \right] \]  

(5.85)

where: \( \psi = \frac{12 u_m \phi_0 R_s^2}{P_b b^3} \)

The modified Reynolds equation using Herschel Buckley model for a Bingham solid in one dimension in dimensionless form can be written as:

\[ \frac{\partial}{\partial X} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) + k^2 \frac{\partial}{\partial Y} \left( \frac{\bar{\rho} H_s^3 \partial P}{\phi} \right) = \psi \left[ \frac{\partial}{\partial X} \bar{\rho} \left( U_b H - U H_p \right) + b \frac{\partial \bar{\rho} H}{\partial t} \right] \]  

(5.86)

### 5.10. Modification of film geometry by Elastic Deformation

For all materials, the surfaces in a Hertzian contact deform elastically. The principal effect of elastic deformation on the lubricant film profile is to interpose a central region of quasi-parallel surfaces between the inlet and outlet wedges. This geometric effect is shown in Figure 5.10.
The film thickness can be expressed as

\[ h(x, y) = h_0 + S(x, y) + \delta(x, y) - \delta(0, 0) \]  

(5.87)

Simplifying the above equation gives

\[ h(x, y) = h_0 + S(x, y) + \delta(x, y) \]  

(5.88)

Where \( h_0 \) is the film thickness at the centre of contact, \( h_0 \) the initial film thickness (rigid gap), \( S(x, y) \) is the separation of un-deformed geometry (surface profile) and \( \delta(x, y) \) is the local deformation.

### 5.11. Contact Deflection

**Figure 5.11: Uniform pressure distribution on an element in elastic half-space**
If the computing domain is divided into equal rectangular areas as shown in Figure 5.11 and the pressure is taken to be uniform within each elemental area, then using the superposition theory of elasticity the total deformation at a point \((X,Y)\) due to the contribution of all such uniform pressure elements in the computing domain can be written as:

\[
\delta_{k,l} = \frac{2P}{\pi E} \sum_{i=1}^{m} \sum_{j=1}^{n} P_{i,j} D_{m,n}
\]

(5.89)

where \(m\) and \(n\) incorporate within them the effect of a pressure node \((i, j)\) on a deflection node \((k, l)\) and are expressed as:

\[
m = |k - l|
\]

\[
n = |l - j|
\]

and:

\[
D_{m,n} = (\bar{y} - \bar{a}) \ln \left[ \frac{(\bar{x} - \bar{b}) + \sqrt{(\bar{y} - \bar{a})^2 + (\bar{x} - \bar{b})^2}}{\bar{x} + \bar{b}} \right] + (\bar{y} + \bar{a}) \ln \left[ \frac{(\bar{x} + \bar{b}) + \sqrt{(\bar{y} + \bar{a})^2 + (\bar{x} + \bar{b})^2}}{\bar{x} - \bar{b}} \right] + (\bar{x} + \bar{b}) \ln \left[ \frac{(\bar{y} + \bar{a}) + \sqrt{(\bar{y} + \bar{a})^2 + (\bar{x} + \bar{b})^2}}{\bar{y} - \bar{a}} \right] + (\bar{y} - \bar{a}) \ln \left[ \frac{(\bar{y} - \bar{a}) + \sqrt{(\bar{y} - \bar{a})^2 + (\bar{x} - \bar{b})^2}}{\bar{y} + \bar{a}} \right] = \left[ m \right]
\]

(5.90)

where:

\[
\bar{b} = \frac{\Delta x}{2}, \quad \bar{a} = \frac{\Delta y}{2} \quad \text{and} \quad \bar{x} = x_{k,l} - x_{i,j} = m\Delta x = [m], \quad \bar{y} = y_{k,l} - y_{i,j} = n\Delta y = [m]
\]

(5.91)

\[
D_{m,n} = y_m \ln \left[ \frac{\sqrt{x^2 + y^2} + x_m}{\sqrt{x^2 + y^2} + y_m} \right] + y_p \ln \left[ \frac{\sqrt{x^2 + y^2} + x_p}{\sqrt{x^2 + y^2} + x_m} \right] + x_p \ln \left[ \frac{\sqrt{x^2 + y^2} + x_p}{\sqrt{x^2 + y^2} + x_m} \right] + x_m \ln \left[ \frac{\sqrt{x^2 + y^2} + x_m}{\sqrt{x^2 + y^2} + x_p} \right]
\]

(5.92)

where: \(x_m = \bar{x} - \bar{b}; \) \(x_p = \bar{x} + \bar{b}; \) \(y_m = \bar{y} - \bar{a}; \) \(y_p = \bar{y} + \bar{a};\)
5.12. Boundary Conditions

For the boundaries of the computation domain shown in Figure 5.7, the pressure is assumed to be zero, thus:

\[ \bar{p}_{i+1} = \bar{p}_{i,N+1} = \bar{p}_{1,j} = \bar{p}_{M+1,j} = 0.0 \tag{5.93} \]

At cavitation boundary since the lubricant is a fluid, pressures lower than the vapour pressure are physically impossible. In elastohydrodynamic problems cavitation usually occurs in the outlet region of conjunction, where gap is widening. This effect is not accounted for in Reynold’s equation, so in such regions where cavitation occurs the Reynold’s equation will allow the pressure to decrease without limit and may predict very large negative pressures. Since in most of the conditions the vapour pressure is of the lubricant is small compared to the ambient pressure, therefore condition is imposed is that these pressure should be larger than or equal to zero. This is known as Reynolds or Swift-Steiber exit boundary condition. Thus:

\[ \bar{p} = \frac{\partial \bar{p}}{\partial \bar{x}} = \frac{\partial \bar{p}}{\partial \bar{y}} = 0 \tag{5.94} \]

5.13. Solving the Reynolds Equation

In order to solve Reynolds equation numerically, the finite differencing technique is employed. This technique provides an approximate solution to the differential Reynolds equation for a given instant in time. The solution is achieved by taking a solution domain and discretising it into a series of grid points. Pressure is then found for each of these grid points.

There are three possible approaches in the finite differencing approach. Either of the central, forward or backward differencing schemes may be adopted. The basic premises for each of these are illustrated in Figure 5.12 and for a thorough explanation refer to Timoshenko (1951), Dowson and Higginson (1959) and Rao (2004).
The central differencing technique is employed for the left hand side terms of the modified Reynolds equation for grease, using the Herschel-Bulkley model in non-dimensional form shown in (5.95), with the terms on the right hand side set out with an additional \( \beta \) variable. This, therefore, allows the right hand side to be calculated, based upon either the forward, backward or central differencing method without the need to change the program other than the value of \( \beta \). This addition was made in order to aid convergence.

\[
\frac{\partial}{\partial X} \left( \frac{\bar{p} H_a^3 \frac{\partial p}{\partial X}}{\phi} \right) + k \frac{\partial}{\partial Y} \left( \frac{\bar{p} H_a^3 \frac{\partial p}{\partial Y}}{\phi} \right) = \psi \left( \frac{\partial}{\partial X} \bar{p} (U_i H_i - U_i H) + k \frac{\partial}{\partial Y} \bar{p} (V_j H_j - V_j H) \right) + \frac{b}{R_c} \frac{\partial p}{\partial \tau}
\]

(5.95)

where:

\( \beta = 0 \) = Forward differencing scheme.

\( \beta = 1 \) = Backwards differencing scheme.

\( \beta = 0.5 \) = Central differencing scheme.

Considering the first term in the left hand side of equation (5.95) and discretising using central differences:

\[
\frac{\partial}{\partial X} \left( \frac{\bar{p} H_a^3 \frac{\partial p}{\partial X}}{\phi} \right)_{i,j} = \frac{1}{\Delta X} \left[ \left( \frac{\bar{p} H_a^3 \frac{\partial p}{\partial X}}{\phi} \right)_{i+1/2,j} - \left( \frac{\bar{p} H_a^3 \frac{\partial p}{\partial X}}{\phi} \right)_{i-1/2,j} \right]
\]

(5.96)

Grouping the pressure terms:
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\[
\frac{\partial}{\partial X} \left( \frac{\bar{\rho} H^3}{\phi} \frac{\partial P}{\partial X} \right)_{i,j} = \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i+\frac{1}{2},j} \frac{P_{i+1,j} - P_{i,j}}{\Delta X^2} - \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i-\frac{1}{2},j} \frac{P_{i,j} - P_{i-1,j}}{\Delta X^2} \quad (5.97)
\]

Expanding terms:

\[
\frac{\partial}{\partial X} \left( \frac{\bar{\rho} H^3}{\phi} \frac{\partial P}{\partial X} \right)_{i,j} = \frac{1}{2\Delta X^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i+1,j} \right] (P_{i+1,j} - P_{i,j}) - \frac{1}{2\Delta X^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i-1,j} \right] (P_{i,j} - P_{i-1,j})
\]

Further simplifying (5.98), it can be written as

\[
[\bar{A}] = \frac{1}{2\Delta X^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i+1,j} \right] P_{i+1,j} - \frac{1}{2\Delta X^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + 2 \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i+1,j} \right] P_{i,j} + \frac{1}{2\Delta X^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i-1,j} \right] P_{i-1,j}
\]

(5.99)

The second term from the left hand side of equation (5.95), decertising using central differences can be written as:

\[
\frac{\partial}{\partial Y} \left( \frac{\bar{\rho} H^3}{\phi} \frac{\partial P}{\partial Y} \right)_{i,j} = \frac{1}{\Delta Y} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \frac{\partial P}{\partial Y} \right)_{i,j+\frac{1}{2}} - \left( \frac{\bar{\rho} H^3}{\phi} \frac{\partial P}{\partial Y} \right)_{i,j-\frac{1}{2}} \right]
\]

(5.100)

Expanding each terms:

\[
\frac{\partial}{\partial Y} \left( \frac{\bar{\rho} H^3}{\phi} \frac{\partial P}{\partial Y} \right)_{i,j} = \frac{1}{2\Delta Y^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j+1} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} \right] (P_{i,j+1} - P_{i,j}) - \frac{1}{2\Delta Y^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j-1} \right] (P_{i,j} - P_{i,j-1})
\]

(5.101)

Further simplification of equation (5.101) yields:

\[
[B] = \frac{1}{2\Delta Y^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j+1} \right] P_{i,j+1} - \frac{1}{2\Delta Y^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + 2 \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j+1} \right] P_{i,j} + \frac{1}{2\Delta Y^2} \left[ \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j} + \left( \frac{\bar{\rho} H^3}{\phi} \right)_{i,j-1} \right] P_{i,j-1}
\]

(5.102)
Adding the $\beta$ variable to the terms in the right hand side of equation (5.95) and expanding:

$$\frac{\partial \rho(u_{i,H} - u_{H})}{\partial x} = (1 - \beta_j) \left[ \rho(u_{i,H} - u_{H}) \right]_{i,j} - \rho(u_{i,H} - u_{H}) \frac{\rho(u_{i,H} - u_{H})}{\Delta x} + \beta_j \left[ \rho(u_{i,H} - u_{H}) \right]_{i,j}$$

$$\frac{\partial \rho(v_{i,H} - v_{H})}{\partial y} = (1 - \beta_j) \left[ \rho(v_{i,H} - v_{H}) \right]_{i,j} - \rho(v_{i,H} - v_{H}) \frac{\rho(v_{i,H} - v_{H})}{\Delta y} + \beta_j \left[ \rho(v_{i,H} - v_{H}) \right]_{i,j}$$

Equation (5.95) in its simplified form can be written as:

$$A_{i,j} + k^2 B_{i,j} = \psi \left[ C_{i,j} + k D_{i,j} + \frac{b}{R_x} \tilde{p} E_{i,j} \right]$$

This equation is solved by numerical approximation. Therefore, the left hand side will never exactly be equal to the right hand side. A residual value $(F_{i,j})$ will always remain.

$$F_{i,j} = A_{i,j} + k^2 B_{i,j} - \psi \left[ C_{i,j} + k D_{i,j} + U_p \tilde{p} E_{i,j} + kV_p \tilde{p} F_{i,j} + \frac{b}{R_x} \tilde{p} G_{i,j} \right]$$

5.14. Modified Newton-Raphson method for EHL problems

The Newton-Raphson technique can be formulated as:

$$f \left( X_{n+1} \right) = f \left( X_n \right) + f' \left( X_{n+1} - X_n \right) + Err$$

Assume that $P_{i,j}$ are a set of approximate solutions to the real solutions $\tilde{P}_{i,j}$.

Therefore, from equation (5.108):
\[
\begin{align*}
F_i &= f(P_{i-1}, P_i, P_{i+1}) = 0 \\
F_i &= f(P_{i-1}, P_i, P_{i+1}) \neq 0
\end{align*}
\] (5.109)

By applying Taylor’s series expansion equation (5.108) can be expressed as:

\[
F_{i,j} = F_{i,j} + \frac{\partial F_{i,j}}{\partial P_{i-1,j}} \Delta P_{i-1,j} + \frac{\partial F_{i,j}}{\partial P_{i,j}} \Delta P_{i,j} + \frac{\partial F_{i,j}}{\partial P_{i+1,j}} \Delta P_{i+1,j} + \frac{\partial F_{i,j}}{\partial P_{i,j+1}} \Delta P_{i,j+1} + \frac{\partial F_{i,j}}{\partial P_{i,j-1}} \Delta P_{i,j-1} + \text{Err} = 0
\] (5.110)

where the difference in pressure can be expressed as

\[
\Delta P_{i,j} = \overline{P}_{i,j} - P_{i,j}
\] (5.111)

The first order differentials can then be replaced by a Jacobian matrix \([J]\), which contains the derivative functions with respect to all the dependent variables.

Assuming that the truncating error is small enough to be neglected, equation (6.110) can be re-written as:

\[
-F_{i,j} = \overline{J}_{y,i-1,j} \Delta P_{i-1,j} + \overline{J}_{y,i,j} \Delta P_{i,j} + \overline{J}_{y,i+1,j} \Delta P_{i+1,j} + \overline{J}_{y,i,j+1} \Delta P_{i,j+1} + \overline{J}_{y,i,j-1} \Delta P_{i,j-1} + \text{Err}
\] (5.112)

where:

\[
\overline{J}_{y,i,j} = \frac{\partial F_{i,j}}{\partial P_{y,j}}
\] (5.113)

Using Gauss-Seidel iteration method:

\[
\Delta P_{k,j}^{n} = \frac{-F_{k,j} - J_{k,j-1} \Delta P_{k,j-1} - J_{k,j+1} \Delta P_{k,j+1} - J_{k,j+1} \Delta P_{k,j+1} - J_{k,j-1} \Delta P_{k,j-1} - J_{k,j+1} \Delta P_{k,j+1}}{J_{k,j}}
\] (5.114)

where \(J_{k,j}\) are computed in the next section (5.17)

\[
\Delta P_{k,j}^{n} = \frac{-F_{k,j} - J_{k,j-1} \Delta P_{k,j-1} - J_{k,j+1} \Delta P_{k,j+1} - J_{k,j+1} \Delta P_{k,j+1} - J_{k,j-1} \Delta P_{k,j-1} - J_{k,j+1} \Delta P_{k,j+1}}{J_{k,j}}
\] (5.115)

The value of pressure for the next iteration is:

\[
P_{i,j}^{n} = P_{i,j}^{n-1} + \Omega \Delta P_{i,j}^{n}
\] (5.116)

where \(\Omega\) represents a relaxation factor on the pressure change to prevent drastic changes causing numerical convergence errors. This factor is typically equal to
0.00001 for the cases considered in this research. Thus, the method of solution is referred to as “under relaxation”.

5.15. Density

Lubricant density alters with pressure in ball bearings, subjected to varying load through their orbital motions. Thus, the assumption that the lubricant is incompressible is not valid. The change in density with pressure as proposed by Dowson and Higginson (1966), in dimensionless form, is considered here:

\[
\bar{\rho}_{i,j} = 1 + \frac{\alpha P_h \cdot P_{i,j}}{1 + \beta P_h \cdot P_{i,j}}
\]

(5.117)

where \( \alpha \) and \( \beta \) are constants, dependent upon the properties of the fluid. The values are \( \alpha = 5.83 \times 10^{-10} \), \( \beta = 1.68 \times 10^{-9} \), as proposed by Dowson for mineral oils.

5.16. Viscosity

Grease is assumed to be fully shear-degraded by its passage through the inlet and its structure becomes a large scale tri-dimensional network of discrete spherical soap particles dispersed in the base oil according to Mansot et al (1989). In this case the viscosity of the grease would be given by:

\[
\phi = \eta_{bo} (1 + B \varphi)
\]

(5.118)

where \( \phi \) is the plastic viscosity of grease, \( \eta_{bo} \) is the base oil viscosity, \( B \) is a constant (value equals 2.5 according to Hunter and Frayne (1979)) and \( \varphi \) is the volume fraction of soap in oil.

Roelands (1966) proposed the relationship for the effects of pressure on viscosity of lubricants under isothermal conditions as:

\[
\ln \phi + 1.2 = (\ln \phi_0 + 1.2) \left(1 + \frac{P}{2000}\right)^R
\]

(5.119)

where \( P \) is the gauge pressure and \( R \) is the viscosity-pressure index. By taking anti­logs of the expression and using dimensionless parameters for plastic viscosity \( \phi \), the above equation can be rewritten as:
\[ \bar{\phi}_{i,j} = e^{\ln \phi_t + 9.67 \left[ -1 + \left( \frac{R_{40}}{R_0} \right)^{1.5} \right]} \]  

(5.120)

where \( R \) is the Roeland’s pressure–viscosity index and it can be estimated from the lubricant’s viscosity in centipoise at 40°C and 100°C, using the following expression:

\[ R = \left[ 7.81 \left( H_{40} - H_{100} \right) \right]^{1.5} \left( F_{40} \right) \]  

(5.121)

where:

\[ H_{40} = \log \left( \log (\phi_{40}) + 1.2 \right) \]  

(5.122)

\[ H_{100} = \log \left( \log (\phi_{100}) + 1.2 \right) \]  

(5.123)

\[ F_{40} = 0.885 - 0.864 H_{40} \]  

(5.124)

5.17. The Jacobian Terms

As previously stated, the first order differentials are solved using Jacobian’s of the form:

\[ J_{y,k} = \frac{\partial F_{y,j}}{\partial P_{k,l}} \]  

(5.125)

which are then used to determine both the residual \( (F_{y,j}) \) and the pressure difference term \( (\Delta P^y) \). Therefore, the Jacobian for the current node position, the previous node position and the following node position must all be established. The expansion of Jacobian terms with respect to the influencing points on the grid are shown below. For a point \((i+1,j)\) the Jacobian terms become:

\[ J_{y,m,j} = \frac{\partial F_{y,j}}{\partial P_{m,j}} = \]  

\[ = \frac{1}{2 \Delta x^2} \left[ \left( \frac{\partial H^y}{\partial \phi} \right)_{m,j} + \left( \frac{\partial H^y}{\partial \phi} \right)_{m,j+1} \right] P_{x,j} + \left[ \left( \frac{\partial H^y}{\partial \phi} \right)_{m,j} - \left( \frac{\partial H^y}{\partial \phi} \right)_{m,j+1} \right] P_{x,j+1} + \left[ \left( \frac{\partial H^y}{\partial \phi} \right)_{m,j+1} + \left( \frac{\partial H^y}{\partial \phi} \right)_{m,j+2} \right] P_{x,j+2} \]  

\[ - \frac{1}{\Delta x} \left[ (1-\beta) \left[ \bar{p}(u_{i,j}) - \bar{p}(u_{i+1,j}) \right] \right] \beta \left[ \bar{p}(u_{i,j}) - \bar{p}(u_{i+1,j}) \right] \]  

\[ - \frac{1}{\Delta y} \left[ (1-\beta) \left[ \bar{p}(v_{i,j}) - \bar{p}(v_{i,j+1}) \right] \right] \beta \left[ \bar{p}(v_{i,j}) - \bar{p}(v_{i,j+1}) \right] \]  

\[ + \frac{1}{\Delta x^2} \left[ (1-\beta) \left[ \bar{p}(u_{i,j}) - \bar{p}(u_{i,j+1}) \right] \right] \beta \left[ \bar{p}(u_{i,j}) - \bar{p}(u_{i,j+1}) \right] \]  

\[ + \frac{1}{\Delta y^2} \left[ (1-\beta) \left[ \bar{p}(v_{i,j}) - \bar{p}(v_{i,j+1}) \right] \right] \beta \left[ \bar{p}(v_{i,j}) - \bar{p}(v_{i,j+1}) \right] \]  

(5.126)

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Lubricant density, viscosity and film thickness are interdependent and all these parameters change in the contact conjunction, for simplicity of analysis $M^{ij}_{k,l}$ and $N^{ij}_{k,l}$ are introduced so that the variation of these parameters can be analysed later and the equation (5.127) can be written as:

\[
J_{y_{i,j}} = \left[ \frac{1}{2\Delta x^2} \left[ \left[ M_{i,j}^{\nu_{i,j}} + M_{i-1,j}^{\nu_{i-1,j}} \right] P_{i,j} - \left[ M_{i,j}^{\nu_{i,j}} + 2M_{i-1,j}^{\nu_{i-1,j}} + M_{i-2,j}^{\nu_{i-2,j}} \right] P_{i-1,j} + \left[ M_{i,j}^{\nu_{i,j}} + M_{i-1,j}^{\nu_{i-1,j}} \right] P_{i+1,j} + \left[ \left( \frac{\partial H}{\eta} \right)_{i,j} + \left( \frac{\partial H}{\eta} \right)_{i+1,j} \right] \right] + \right]
\]

\[
+ \frac{k^2}{2\Delta x^2} \left[ \left[ M_{i,j}^{\nu_{i,j}} + M_{i-1,j}^{\nu_{i-1,j}} \right] P_{i,j} - \left[ M_{i,j}^{\nu_{i,j}} + 2M_{i-1,j}^{\nu_{i-1,j}} + M_{i-2,j}^{\nu_{i-2,j}} \right] P_{i-1,j} + \left[ M_{i,j}^{\nu_{i,j}} + M_{i-1,j}^{\nu_{i-1,j}} \right] P_{i+1,j} \right] - \]

\[
- \psi \frac{1}{\Delta x} \left[ (1-\beta_{x}) \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i-1,j}^{\nu_{i-1,j}} \right] + \beta_{x} \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i+1,j}^{\nu_{i+1,j}} \right] \right] - k\psi \frac{1}{\Delta x} \left[ (1-\beta_{x}) \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i-1,j}^{\nu_{i-1,j}} \right] + \beta_{x} \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i+1,j}^{\nu_{i+1,j}} \right] \right] - 0
\]

(5.127)

where: $M_{k,l}^{ij} = \frac{\partial (\bar{P}H^3)}{\partial P_{k,l}}$, and $N_{k,l}^{ij} = \frac{\partial (\bar{P}(U_{k}H - U_{H}p))}{\partial P_{k,l}}$

(5.128)

For a point $(i-1, j)$ the Jacobian terms in terms of $M_{k,l}^{ij}$ and $N_{k,l}^{ij}$ become:

B) $J_{y_{i-1,j}}$

\[
J_{y_{i-1,j}} = \frac{\partial F_{i,j}}{\partial P_{i-1,j}}
\]

\[
J_{y_{i-1,j}} = \left[ \frac{1}{2\Delta x^2} \left[ \left[ M_{i,j}^{\nu_{i,j}} + M_{i-1,j}^{\nu_{i-1,j}} \right] P_{i,j} + \left[ \left( \frac{\partial H}{\eta} \right)_{i,j} + \left( \frac{\partial H}{\eta} \right)_{i+1,j} \right] \right] + \right]
\]

\[
+ \frac{k^2}{2\Delta x^2} \left[ \left[ M_{i,j}^{\nu_{i,j}} + M_{i-1,j}^{\nu_{i-1,j}} \right] P_{i,j} - \left[ M_{i,j}^{\nu_{i,j}} + 2M_{i-1,j}^{\nu_{i-1,j}} + M_{i-2,j}^{\nu_{i-2,j}} \right] P_{i-1,j} + \left[ M_{i,j}^{\nu_{i,j}} + M_{i-1,j}^{\nu_{i-1,j}} \right] P_{i+1,j} \right] - \]

\[
- \psi \frac{1}{\Delta x} \left[ (1-\beta_{x}) \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i-1,j}^{\nu_{i-1,j}} \right] + \beta_{x} \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i+1,j}^{\nu_{i+1,j}} \right] \right] - k\psi \frac{1}{\Delta x} \left[ (1-\beta_{x}) \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i-1,j}^{\nu_{i-1,j}} \right] + \beta_{x} \left[ sN_{i,j}^{\nu_{i,j}} - sN_{i+1,j}^{\nu_{i+1,j}} \right] \right] - 0
\]

(5.129)

For a point $(i, j+1)$ and $(i, j-1)$ the Jacobian terms are given by equations (5.131) and (5.132) respectively.

C) $J_{y_{i,j+1}}$

\[
J_{y_{i,j+1}} = \frac{\partial F_{i,j}}{\partial P_{i,j+1}} = 
\]
\[ J_{y,i} = \frac{\partial \overline{F}_{j,i}}{\partial \overline{P}_{i,j}} = \]
\[
\frac{1}{2\Delta X^2} \left[ \left[ M'_{ij} + M''_{ij} \right] \overline{P}_{i,j} - \left[ M'_{ij} + 2M''_{ij} + M_{ij} \right] \overline{P}_{i,j} + \left[ M''_{ij} + M_{ij} \right] \overline{P}_{i,j} \right] - \nu \frac{1}{\Delta X} \left( \left( 1 - \beta_j \right) \left[ N'_{ij} - N_{ij} \right] + \beta_i \left[ N_{ij} - N_{ij} \right] \right) - k \psi \frac{1}{\Delta Y} \left( \left( 1 - \beta_j \right) \left[ N_{ij} - N_{ij} \right] + \beta_j \left[ N_{ij} - N_{ij} \right] \right) - 0 \]
\]

\[ (5.130) \]

\[ D) \overline{J}_{y,j,i} - 1 \]
\[
\frac{\partial \overline{F}_{j,i}}{\partial \overline{P}_{i,j}} = \]
\[
\frac{1}{2\Delta X^2} \left[ \left[ M'_{ij} + M''_{ij} \right] \overline{P}_{i,j} - \left[ M'_{ij} + 2M''_{ij} + M_{ij} \right] \overline{P}_{i,j} + \left[ M''_{ij} + M_{ij} \right] \overline{P}_{i,j} \right] - \nu \frac{1}{\Delta X} \left( \left( 1 - \beta_j \right) \left[ N'_{ij} - N_{ij} \right] + \beta_i \left[ N_{ij} - N_{ij} \right] \right) - k \psi \frac{1}{\Delta Y} \left( \left( 1 - \beta_j \right) \left[ N_{ij} - N_{ij} \right] + \beta_j \left[ N_{ij} - N_{ij} \right] \right) - 0 \]
\]

\[ (5.131) \]

Jacobian terms for the current node position can be written as:

\[ \overline{J}_{y,j} \]
\[
\frac{\partial \overline{F}_{j,i}}{\partial \overline{P}_{i,j}} = \]
\[
\frac{1}{2\Delta X^2} \left[ \left[ M'_{ij} + M''_{ij} \right] \overline{P}_{i,j} - \left[ M'_{ij} + 2M''_{ij} + M_{ij} \right] \overline{P}_{i,j} + \left[ M''_{ij} + M_{ij} \right] \overline{P}_{i,j} \right] - \nu \frac{1}{\Delta X} \left( \left( 1 - \beta_j \right) \left[ N'_{ij} - N_{ij} \right] + \beta_i \left[ N_{ij} - N_{ij} \right] \right) - k \psi \frac{1}{\Delta Y} \left( \left( 1 - \beta_j \right) \left[ N_{ij} - N_{ij} \right] + \beta_j \left[ N_{ij} - N_{ij} \right] \right) - 0 \]
\]

\[ (5.132) \]

\[ M''_{ij} \] is a partial derivative of lubricant density, viscosity and film thickness variation with respect to pressure, expanding \( M''_{ij} \):}

\[ \frac{\partial \overline{pH}^3}{\overline{\phi}}_{ij} = \left( \frac{H^3}{\phi} \right)_{ij} \left( \frac{\partial \overline{pH}^3}{\overline{\phi}} \right)_{ij} + \left( \frac{\partial \overline{pH}^3}{\overline{\phi}} \right)_{ij} + \left( \frac{\partial \overline{pH}^3}{\overline{\phi}} \right)_{ij} \]

\[ \frac{\partial \overline{pH}^3}{\overline{\phi}}_{ij} = \left( \frac{H^3}{\phi} \right)_{ij} - \left( \frac{\partial \overline{pH}^3}{\overline{\phi}^2} \right)_{ij} + 3 \left( \frac{\partial \overline{pH}^3}{\overline{\phi}} \right)_{ij} \]

\[ (5.133) \]

\[ (5.134) \]
Rearranging and simplifying:

\[
M_{k,l}^{i,j} = \left( \frac{H^2}{\phi} \right)_{i,j} R_{k,l}^{i,j} - \left( \frac{\bar{\rho}H^2}{\phi^2} \right)_{i,j} E_{k,l}^{i,j} + 3 \left( \frac{\bar{\rho}H^2}{\phi} \right)_{i,j} D_{m,n}
\]  

(5.135)

\[
R_{k,l}^{i,j} = \frac{\partial \bar{\rho}_{k,l}}{\partial P_{k,l}}; \quad E_{k,l}^{i,j} = \frac{\partial \phi_{k,l}}{\partial P_{k,l}}; \quad D_{m,n} = \frac{\partial H_{k,l}}{\partial P_{k,l}}
\]

(5.136)

where \( m = |k - i + 1| \) and \( n = |l - j + 1| \).

Similarly expanding \( N_{k,l}^{i,j} \):

\[
N_{k,l}^{i,j} = \frac{\partial (\bar{\rho}U_b H - \bar{\rho}U b P_{k,l})}{\partial P_{k,l}} = \frac{\partial (\bar{\rho}U_b H)}{\partial P_{k,l}} - \frac{\partial (\bar{\rho}U b P_{k,l})}{\partial P_{k,l}}
\]  

(5.137)

\[
N_{k,l}^{i,j} = (H U_{b})_{i,j} \frac{\partial \bar{\rho}_{i,j}}{\partial P_{k,l}} + (\bar{\rho}U_{b})_{i,j} \frac{\partial H_{i,j}}{\partial P_{k,l}} - (H_{b} U)_{i,j} \frac{\partial \bar{\rho}_{i,j}}{\partial P_{k,l}} - (\bar{U}_{b})_{i,j} \frac{\partial H_{i,j}}{\partial P_{k,l}}
\]  

(5.138)

\[
N_{k,l}^{i,j} = (H U_{b})_{i,j} \frac{\partial \bar{\rho}_{i,j}}{\partial P_{k,l}} + (\bar{\rho}U_{b})_{i,j} \frac{\partial H_{i,j}}{\partial P_{k,l}} - (H_{b} U)_{i,j} \frac{\partial \bar{\rho}_{i,j}}{\partial P_{k,l}} - (\bar{U}_{b})_{i,j} \frac{\partial H_{i,j}}{\partial P_{k,l}}
\]  

(5.139)

Rearranging and simplifying:

\[
N_{k,l}^{i,j} = (H U_{b})_{i,j} R_{k,l}^{i,j} + (\bar{\rho}U_{b})_{i,j} D_{m,n} - (H_{b} U)_{i,j} R_{k,l}^{i,j} - (\bar{U}_{b})_{i,j} D_{m,n}
\]  

(5.140)

Now considering the Change in density with pressure as proposed by Dowson and Higginson (1966) and substituting (5.117) for \( \bar{\rho}_{i,j} \) and simplifying:

\[
R_{k,l}^{i,j} = \frac{\partial \bar{\rho}_{i,j}}{\partial P_{k,l}} = \frac{\partial}{\partial P_{k,l}} \left[ 1 + \frac{0.6 \cdot 10^{-9} \cdot P_{b} \cdot P_{i,j}}{1 + 1.7 \cdot 10^{-9} \cdot P_{b} \cdot P_{i,j}} \right]
\]  

(5.141)

\[
\frac{\partial \bar{\rho}_{i,j}}{\partial P_{k,l}} = 0.6 \cdot 10^{-9} \cdot P_{b} \frac{\partial}{\partial P_{k,l}} \left[ \frac{P_{i,j}}{1 + 1.7 \cdot 10^{-9} \cdot P_{b} \cdot P_{i,j}} \right]
\]  

(5.142)

\[
\frac{\partial \bar{\rho}_{i,j}}{\partial P_{k,l}} = 0.6 \cdot 10^{-9} \cdot P_{b} \left( \frac{\partial}{\partial P_{k,l}} \left( 1 + 1.7 \cdot 10^{-9} \cdot P_{b} \cdot P_{i,j} \right) - P_{i,j} \frac{\partial}{\partial P_{k,l}} \left( 1 + 1.7 \cdot 10^{-9} \cdot P_{b} \cdot P_{i,j} \right)^2 \right)
\]  

(5.143)
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\[ \frac{\partial \rho_{i,j}}{\partial P_{k,l}} = \frac{0.6 \cdot 10^{-9} \cdot P_h}{\left(1 + 1.7 \cdot 10^{-9} \cdot P_h \cdot P_{i,j}\right)^2} \frac{\partial P_{i,j}}{\partial P_{k,l}} \quad (5.144) \]

The variation of density with respect to pressure for \( k \neq i \) and/or \( l \neq j \) is given by:

\[ \frac{\partial P_{i,j}}{\partial P_{k,l}} = 0 \rightarrow \frac{\partial \rho_{i,j}}{\partial P_{k,l}} = 0 \quad (5.145) \]

The variation of density with respect to pressure for \( k = i \) and \( l = j \) can be written as

\[ \frac{\partial \rho_{i,j}}{\partial P_{k,l}} = \frac{0.6 \cdot 10^{-9} \cdot P_h}{\left(1 + 1.7 \cdot 10^{-9} \cdot P_h \cdot P_{i,j}\right)^2} \quad (5.146) \]

Considering the change in viscosity with Pressure for isothermal conditions as proposed by Roeland (1966) and substituting (5.119) for \( \varphi_{i,j} \) and simplifying:

\[ \frac{\partial \varphi_{i,j}}{\partial P_{k,l}} = \frac{\partial}{\partial P_{k,l}} \left[ \ln \phi_0 + 9.67 \left(1 + \frac{P_h P_{i,j}}{P_0}\right)^{-1}\right] \quad (5.147) \]

\[ \frac{\partial \varphi_{i,j}}{\partial P_{k,l}} = \frac{\partial}{\partial P_{k,l}} \left[ \ln \phi_0 + 9.67 \left(1 + \frac{P_h P_{i,j}}{P_0}\right)^{-1}\right] \phi_{i,j} \quad (5.148) \]

\[ \frac{\partial \varphi_{i,j}}{\partial P_{k,l}} = \ln \phi_0 + 9.67 \frac{\partial}{\partial P_{k,l}} \left(1 + \frac{P_h P_{i,j}}{P_0}\right)^{-1} \quad (5.149) \]

\[ \frac{\partial \varphi_{i,j}}{\partial P_{k,l}} = \ln \phi_0 + 9.67 \left(1 + \frac{P_h P_{i,j}}{P_0}\right)^{-1} \frac{\partial}{\partial P_{k,l}} \left(1 + \frac{P_h P_{i,j}}{P_0}\right) \quad (5.150) \]

\[ \frac{\partial \varphi_{i,j}}{\partial P_{k,l}} = \ln \phi_0 + 9.67 \left(1 + \frac{P_h P_{i,j}}{P_0}\right)^{-1} \frac{\partial}{\partial P_{k,l}} \frac{P_h P_{i,j}}{P_0} \quad (5.151) \]

The variation of viscosity with pressure for \( k \neq i \) and/or \( l \neq j \) will be zero, thus:

\[ \frac{\partial P_{i,j}}{\partial P_{k,l}} = 0 \rightarrow \frac{\partial \varphi_{i,j}}{\partial P_{k,l}} = 0 \quad (5.152) \]
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The variation of density with respect to pressure for \( k = i \) and \( l = j \) can be written as:

\[
\frac{\partial P_{i,j}}{\partial P_{k,l}} = 1 \rightarrow \frac{\partial \phi_{i,j}}{\partial P_{k,l}} = [\ln \phi_{0} + 9.67] \frac{zP_{i,j}}{P_{0}} \left( 1 + \frac{P_{i,j}}{P_{0}} \right)^{-1} \quad (5.153)
\]

5.18. Determination of plug flow thickness

According to the Herschel Bulkley model a central flow region occurs, in which \( \tau < \tau_{0} \) and is enclosed by regions of shear flow (if \( \tau > \tau_{0} \)). The plug flow region is of width \( h_{p} \), reproducing equation (5.27) and (5.28) as:

\[
\tau_{xx} = \frac{h_{p}}{2} \frac{\partial p}{\partial x} \quad (5.154)
\]

\[
\tau_{yy} = \frac{h_{p}}{2} \frac{\partial p}{\partial y} \quad (5.155)
\]

The flow equation for a Bingham solid can be written as:

\[
\dot{\varepsilon}_{ij} = \frac{1}{\eta} \sqrt{I_{2} - \tau_{0}} \sigma_{ij} \quad \text{for} \quad \left( \sqrt{I_{2}} \geq \tau_{0} \right) \quad (5.156)
\]

\[
\dot{\varepsilon}_{ij} = 0 \quad \text{for} \quad \left( \sqrt{I_{2}} \leq \tau_{0} \right) \quad (5.157)
\]

\( \dot{\varepsilon}_{ij} \) is the component of strain rate tensor, \( \sigma_{ij} \) is the component of deviatoric stress tensor, and \( I_{2} \) is the second invariant of the deviatoric stress tensor, where:

\[
I_{2} = \tau_{xx}^{2} + \tau_{yy}^{2} = \tau^{2} \quad (5.158)
\]

For the plug flow region of width \( h_{p} \) and \( \tau = \tau_{0} \), Substituting equations (5.159) and (5.160) into equation (5.163) gives:

\[
\left( \frac{h_{p}}{2} \frac{\partial p}{\partial x} \right)^{2} + \left( \frac{h_{p}}{2} \frac{\partial p}{\partial y} \right)^{2} = \tau_{0}^{2} \quad (5.159)
\]

rearranging:
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\[ \left( \frac{h_p}{2} \right) \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{h_p}{2} \right) \left( \frac{\partial p}{\partial y} \right)^2 = \tau_0^2 \]  \hspace{1cm} (5.160)

Simplifying:

\[ \left( \frac{h_p}{2} \right)^2 \left[ \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{\partial p}{\partial y} \right)^2 \right] = \tau_0^2 \]  \hspace{1cm} (5.161)

Solving for the width of the plug flow:

\[ \left( \frac{h_p}{2} \right)^2 = \frac{\tau_0^2}{\left[ \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{\partial p}{\partial y} \right)^2 \right]} \]  \hspace{1cm} (5.162)

Rearranging:

\[ h_p^2 = \frac{4\tau_0^2}{\left[ \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{\partial p}{\partial y} \right)^2 \right]} \]  \hspace{1cm} (5.163)

The width of the plug flow can be given as:

\[ h_p = \frac{2\tau_0}{\sqrt{\left[ \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{\partial p}{\partial y} \right)^2 \right]}} \]  \hspace{1cm} (5.164)

5.18.1 Dimensionless analysis

Substituting the dimensionless parameters given in (5.88) into (5.164) yields:

\[ \left( \frac{H_p b^2}{2R_x} \right)^2 \left( \frac{P_x dp}{bdX} \right)^2 + \left( \frac{H_p b^2}{2R_x} \right)^2 \left( \frac{P_y dp}{bdY} \right)^2 = \tau_0^2 \]  \hspace{1cm} (5.165)

rearranging:

\[ \left( \frac{H_p bP_h}{2R_x} \right)^2 \left( \frac{dp}{dX} \right)^2 + \left( \frac{H_p bP_h}{2R_x} \right)^2 \left( \frac{dp}{dY} \right)^2 = \tau_0^2 \]  \hspace{1cm} (5.166)
Bringing the common terms together:

\[
\left( \frac{H_p b P_h}{2 R_x} \right)^2 \left[ \left( \frac{\partial p}{\partial X} \right)^2 + \left( \frac{\partial p}{\partial Y} \right)^2 \right] = \tau_0^2 \tag{5.167}
\]

Solving for the width of the plug flow:

\[
\frac{H_p b P_h}{2 R_x} = \frac{\tau_0^2}{\left[ \left( \frac{\partial p}{\partial X} \right)^2 + \left( \frac{\partial p}{\partial Y} \right)^2 \right]} \tag{5.168}
\]

Dividing both sides by: \( \left( \frac{R_x}{b P_h} \right)^2 \) yields:

\[
H_p^2 = \left( \frac{R_x}{b P_h} \right)^2 \frac{4 \tau_0^2}{\left[ \left( \frac{\partial p}{\partial X} \right)^2 + \left( \frac{\partial p}{\partial Y} \right)^2 \right]} \tag{5.169}
\]

The width of the plug flow can be given as:

\[
H_p = \left( \frac{R_x}{b P_h} \right) \frac{2 \tau_0}{\sqrt{\left[ \left( \frac{\partial p}{\partial X} \right)^2 + \left( \frac{\partial p}{\partial Y} \right)^2 \right]}} \tag{5.170}
\]

5.19. Convergence criteria

For an approximate solution of the non-linear equations used in the numerical solution convergence criteria should be specified. If the solution obtained is within the limits of the required tolerance, then the numerical procedure is deemed to have converged and a solution found. For the numerical solution to be fully converged, both the load and pressure have to converge. For pressures convergence:
where $\bar{P}^n_{(i,j)}$ is the new pressure, $\bar{P}^o_{(i,j)}$ the old pressure and $Err_p$ is the error tolerance for the pressure set equal to 0.00005. If the convergence is not achieved, the input pressures into the next iteration loop are updated, using the following expression:

$$P^o_{i,j} = P^{n-1}_{i,j} + \Omega \Delta P^n_{i,j} \quad (5.172)$$

For load convergence the following criterion is used:

$$|\bar{W} - \pi| \leq Err_w \quad (5.173)$$

Here, the error tolerance for load convergence $Err_w$ is set equal to 0.01. If the load has not converged, then the central film thickness is adjusted according to the evaluated unbalanced load, using the following formulation:

$$\bar{H}^n = \bar{H}^o + Dh \quad (5.174)$$

where $\bar{H}^n$ is the new central film thickness, $\bar{H}^o$ is the old central film thickness and $Dh$ is the increment factor, which is obtained by

$$Dh = damp.|\bar{W} - \pi| \quad (5.175)$$

where $damp$ is the damping coefficient used to dampen the sudden changes in the central oil film thickness and the value is set to $1 \times 10^{-7}$.

Figure 5.13 shows the flow chart of the complete solution procedure employed.
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START

1. Read Input parameters (W, R, E, H, η, ....)
2. Calculate Geometrical Profile
3. Calculate Hertzian Pressure
4. Calculate influence coefficients
5. Calculate Deflection
6. Calculate film shape
7. Calculate Lubricant properties
8. Solve Modified Reynolds's Equation
9. Calculate error in pressure
10. Pressure Relaxation factor

Pressure Relaxation factor
\[ P_{i,j}^n = P_{i,j}^{n-1} + \Delta P_{i,j} \]

Calculate new Pressure
\[ P_{i,j}^n = P_{i,j}^{n-1} + \Delta P_{i,j} \]

Calculate error in pressure
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} \left| P_{i,j}^n - P_{i,j}^o \right| \leq Err_p \]
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} P_{i,j}^o \leq 0.00001 \]

Converged?

Wolfson School of Mechanical and Manufacturing Engineering
Loughborough University, Loughborough, UK

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**Figure 5.13: Flow chart for computational solution**

5.20. Closure

This chapter has focused on the numerical modelling of the elastohydrodynamic lubrication of grease in point contact. The results of this modelling along with the various cases are discussed in the next chapter.
Chapter 6

Grease Lubrication – Results and Discussions

6.1 Introduction

The previous chapter dealt with the chemical composition, classification and flow properties of grease in counterformal contacts. Counterformal contacts do not conform well and thus the load is carried by a small area. Consequently, high pressures are generated. Due to these high pressures elastic deformation of the contacting bodies occurs. Additionally, viscosity of the lubricant also rises considerably, which further assists the formation of an effective fluid film. In such cases complete solution of Reynolds equation is required, together with localised deformation of surfaces. This is referred to as elastohydrodynamic lubrication. Reynolds equation is derived using the Herschel Bulkley model, considering grease as a Bingham fluid in chapter 5. The solution procedure is also explained there. This chapter starts with results for an isothermal analysis and later includes the effect of generated heat in the contact.

Common parameters such as the lubricant properties and the geometrical details of the contacting surfaces used for numerical solutions are given in table 6.1.

Table 6.1 Geometrical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure-Viscosity index</td>
<td>$z = 0.67$</td>
<td>-</td>
</tr>
<tr>
<td>Viscosity of Lubricant at $P = 0$</td>
<td>$\eta = 0.02024$</td>
<td>Pa.s</td>
</tr>
<tr>
<td>Radius of curvature of solid $A$ in $X$ direction</td>
<td>$R_{Ax} = 0.00219$</td>
<td>m</td>
</tr>
<tr>
<td>Radius of curvature of solid $A$ in $Y$ direction</td>
<td>$R_{Ay} = 0.00219$</td>
<td>m</td>
</tr>
<tr>
<td>Radius of curvature of solid $B$ in $X$ direction</td>
<td>$R_{Bx} = \infty$</td>
<td>m</td>
</tr>
<tr>
<td>Poisson’s Ratio of solid $A$</td>
<td>$\nu = 0.3$</td>
<td>-</td>
</tr>
<tr>
<td>Poisson’s Ratio of solid $B$</td>
<td>$\nu = 0.25$</td>
<td>-</td>
</tr>
<tr>
<td>Young’s Modulus of solid $A$</td>
<td>$E_A = 210 \times 10^9$</td>
<td>N/m²</td>
</tr>
<tr>
<td>Young’s Modulus of solid $B$</td>
<td>$E_B = 110 \times 10^9$</td>
<td>N/m²</td>
</tr>
</tbody>
</table>
6.2 Governing equations

The modified Reynolds equation using Herschel Buckley model for grease as a Bingham solid used for calculating pressure distribution in the contact is given below (for derivation and solution procedure refer to chapter 5).

\[
\frac{\partial}{\partial x} \left( \frac{\partial p}{\partial x} \frac{\rho h^3_s}{\phi} \right) + \frac{\partial}{\partial y} \left( \frac{\partial p}{\partial y} \frac{\rho h^3_s}{\phi} \right) = 12 \left[ \frac{\partial}{\partial x} \rho \left( u_h h - h_p u \right) + \frac{\partial}{\partial y} \rho \left( v_h h - h_p v \right) + \frac{\partial (h \rho)}{\partial t} \right]
\]

(6.1)

The same can be presented in dimensionless form as:

\[
\frac{\partial}{\partial X} \left( \frac{\partial H^3_s}{\partial X} \frac{\partial P}{\partial X} \right) + k^2 \frac{\partial}{\partial Y} \left( \frac{\partial H^3_s}{\partial Y} \frac{\partial P}{\partial Y} \right) = \psi \left[ \frac{\partial}{\partial X} \left( U_h H - UH_r \right) + k \frac{\partial}{\partial Y} \left( V_h H - VH_r \right) + \frac{b}{R_x} \frac{\partial H}{\partial t} \right]
\]

(6.2)

6.3 Grease as a lubricant

ABEC 7 class precision angular contact ball bearings with ceramic balls are used in high speed precision spindles (for details of the bearing used refer to chapter 3). It is of utmost importance for machine tool spindle bearings to operate under fluid film regimes of lubrication; hydrodynamics, elastohydrodynamics. This is in order to reduce frictional losses. For the highest possible speeds attainable, greases with a synthetic base oil (diester) combined with a barium complex soap have proven to have superior properties. Hence, Arcanol L75 is used in most modern high speed spindles and the same is used in numerical modelling presented here. Arcanol L75 enables OEMs and users of bearings to benefit from operational advantages of sealed bearings, lubricated with grease, which excels for its non-critical running-in behaviour with high temperature stability, non-toxicity and favourable viscosity-temperature characteristics. Technical data for Arcanol L75 is given in Table 6.2.
Table 6.2: Arcanol L75 properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colour</td>
<td>beige</td>
</tr>
<tr>
<td>Service temperature range*, [°C], approx.</td>
<td>-50 to 120</td>
</tr>
<tr>
<td>Drop point, DIN ISO 2176, [°C]</td>
<td>&gt; 220</td>
</tr>
<tr>
<td>Worked penetration, DIN ISO 2137, at 25 °C, [0.1 mm]</td>
<td>220 - 250</td>
</tr>
<tr>
<td>Density, at 20 °C, [g/cm³], approx.</td>
<td>0.93</td>
</tr>
<tr>
<td>Flow pressure, DIN 51805, at -50 °C, [mbar]</td>
<td>&lt; 1,400</td>
</tr>
<tr>
<td>Water resistance, DIN 51 807, pt. 1 3 h / 90 °C, rating</td>
<td>1 - 90</td>
</tr>
<tr>
<td>Corrosion protection, DIN 51 802; Emcor test (1 week, distilled water), corrosion rating</td>
<td>≤ 1</td>
</tr>
<tr>
<td>Oil separation, DIN 51 817, 7 d / 40 °C, [wt. %]</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Speed factor**, n x d m</td>
<td>&gt; 2 million</td>
</tr>
</tbody>
</table>

6.3.1 Viscosity of grease

It is well known fact that grease undergoes both chemical and physical deterioration during its use as reported by Cann et al (2007). Chemical changes accounts for loss of antioxidant in its composition due to the oxidation reactions rather than evaporation. There is also an increase in its acidity and formation of oxidized hydrocarbon species. This can lead to the formation of acidic and/or high viscosity products and loss of carboxyl bands of soap thickener. The physical deterioration accounts for increase in bleeding rate and oil leakage, destruction of the thickener structure and loss of the base oil. Thus, it is highly important that grease degradation is also incorporated in any analysis. When fully shear-degraded the grease structure becomes a large scale tri-dimensional network of discrete spherical soap particles dispersed in the base oil as described by Mansot et al (1989). In this case the viscosity of the grease would be given by:

\[ \phi = \eta_{bo} (1 + B \varphi) \]  \hspace{1cm} (6.3)

where \( \phi \) is the plastic viscosity of grease, \( \eta_{bo} \) is the base oil viscosity, \( B \) is a constant (its value equals 2.5 according to Hunter and Frayne (1979)) and \( \varphi \) is the volume fraction of the soap in oil. For Arcanol L75 grease the degraded viscosity is 0.2783 Pa.s. Since counterformal contacts are subjected to very high pressures the
viscosity and density variations inside the contact are calculated using equations (6.4) and (6.5) due to Roelands (1966) and Dowson and Higginson (1966) respectively:

\[
\phi_{i,j} = e^{\left[ \ln \phi_0 + 9.67 \left( \frac{\rho_i \rho_j}{\rho_0} \right) \right]}
\]

(6.4)

\[
\bar{\rho}_{i,j} = 1 + \frac{a \cdot P_h \cdot P_{i,j}}{1 + \beta \cdot P_h \cdot P_{i,j}}
\]

(6.5)

6.4 Film thickness

A simplified expression for defining the elastic film shape may be written as:

\[
h(x, y) = h_0 + S(x, y) + \delta(x, y)
\]

(6.6)

where \( h \) is the film thickness, \( h_0 \) is the initial film thickness, \( S(x, y) \) is the separation of the undeformed geometry elastic solids in contact and \( \delta(x, y) \) is their localised deformation. Now for an EHL solution, this equation has to be solved simultaneously with Reynolds equation as described in chapter 5.

The bearings are assumed to rotate at a constant velocity of 20,000 rpm. The load acting on a ball as it goes through a revolution about the shaft obtained from the spindle dynamics model explained in chapter 3 is shown in Figure 6.1.

![Figure 6.1](image-url)

**Figure 6.1** : Load per ball (a) Polar Plot (b) Cartesian plot
The load acting on a ball at every degree of its procession is obtained. The minimum load was found to be 4.014 N and its maximum at 49 N. This load was given as an input for the lubrication code developed in visual C++ and the calculated contact loads were converged with convergence criteria mention in chapter 5. The corresponding pressures and film thickness were thus obtained. These are plotted for the maximum, minimum and median loads in Figure 6.2 for the base lubricant and grease.

![Figure 6.2: Contact parameters for different loads.](image)

According to the classical Hertzian theory, in an elastostatic contact, the pressure distributions would be ellipsoidal. This is shown in Figure 6.2 with a black dotted line for the highest load. The maximum load acting on a ball obtained from the spindle dynamics model is 49N and the corresponding Hertzian pressure is found to be 2.37 GPa. The pressure field alters when the surfaces start moving relative to each other in the presence of a lubricant such as oil or grease. Relative motion between the two surfaces causes a hydrodynamic lubricating film to be generated which modifies the pressure distribution to a certain extent. The greatest changes to the pressure profile occur at the entry and exit regions of the contact as shown in Figure 6.2. The combined effect of rolling and a lubricating film results in a slightly enlarged contact area. The shape of the distribution is referred to as elastohydrodynamic. The extent of contact area increases as the applied load increases and reaches a maximum.
of $3.779 \times 10^{-9} \text{ m}^2$ in this example, which is the Hertzian contact area. Consequently, at the entry region, the hydrodynamic pressure is lower than the value for a dry Hertzian contact as noted by Cameron and Gohar (1966) and Hamrock and Dowson (1976) among others. As the lubricant enters the contact, the lubricant viscosity increases dramatically and localised surface deformations make a parallel film thickness, which was first postulated by Ertel and Grubin (1949). This is followed by an equally sharp decline to ambient viscosity levels at the exit of the contact to ensure continuity of flow condition. To compensate for the loss of lubricant viscosity at the contact exit, a constriction is formed close to the exit. A large pressure peak is thus generated next to the constriction on the upstream side, and on the downstream the pressure rapidly declines to less than the dry Hertzian values. It is found that the peak pressure is usually larger than the maximum Hertzian contact pressure and diminishes with lubricant starvation. The exit constriction to the EHL film is found to be curved in order to fit into the contact boundary. This effect is known as the 'horse-shoe' constriction and is shown later in Figures 6.6-6.8 along with the three dimensional pressure profiles. Figure 6.2 shows the pressure distributions for grease in bold lines and for the base oil in dotted lines. A slight increase in the region of maximum pressures is noted for grease, although the area under corresponding pressure distributions remain the same under the same load. For an applied load of 24N the maximum pressure is 1.93 $\text{GPa}$ for grease and 1.87 $\text{GPa}$ for the base oil, whereas for an applied load of 49N these are 2.32 $\text{GPa}$ and 2.28 $\text{GPa}$ respectively. For grease the maximum viscosity rise due to piezo-viscous action which is 926 $\text{GPa.s}$, the same for base oil is 152 $\text{GPa.s}$. This difference in viscosity rise can also be attributed to the rise of in the EHL pressure spike in the case of grease. This high viscosity makes grease thicker when compared with its base oil and hence produce a thicker film thickness as shown in Figure 6.3. In both cases a conventional EHL elastic film profile is obtained.
Chapter 6 - Grease Lubrication- Results and Discussions

2.5 - E 2

Grease:
Base oil:

Figure 6.3: Film thickness for Grease and Base oil for different loads at 49 m/s

6.5 Core formation

Unlike the elastic film shapes shown in Figure 6.2, with grease plug flow through the contact forms a core, described in chapter 5. The Herschel Bulkley flow equation is considered for this study assuming that grease behaves as Bingham solid, thus:

$$\tau = \tau_0 + \phi |D|^{n}$$  \hspace{1cm} (6.7)

which infers that grease behaves as a solid if the applied shear stress is below the yield shear stress ($\tau_0$). This value varies from 3.5 to 1000 Pa. for mineral oils as the base oil according to Kauzlarich and Greenwood (1972). For this study the initial value for yield shear stress was assumed to be 100 Pa. The variation of shear stress with pressures is given by Roelands (1966) as:

$$\ln \tau + 1.2 = \left( \ln \tau_0 + 1.2 \right) \left( 1 + \frac{P}{2000} \right)^R$$  \hspace{1cm} (6.8)

As the lubricant enters into the contact area its shear stress increases dramatically due to high generated pressures, which makes the lubricant behave as a solid core which fills the entire contact region shown in Figure 6.4-6.6. Cann et al (1992) have verified this supposition experimentally for grease lubricated EHL contacts. According to the authors, inside the contact, the lubricant can be described as a core in a glassy state, sandwiched between thin shear zones, probably of molecular proportions, in which the difference in velocity of the solidified core relative to the mean entraining velocity...
is accommodated. There exists a difference in velocity between the contacting solid surfaces and the plug flow. This creates slip inside the contact. Energy dissipation associated with sliding and the substantial generation of heat is restricted to the shear zones and it is in these regions that the temperature rise ensures fluid, rather than solid behaviour. Since the current analysis does not consider a temperature rise inside the contact the core does not disintegrate. This condition is investigated later during the thermal analysis.

Figure 6.4-6.6 show pressures generated, corresponding film thickness and plug flow profile in the contact conjunction. The central and minimum film thickness for a load of 4N are 1.61 \( \mu m \) and 1.2 \( \mu m \) respectively. Surface profile measurements carried out for balls and races using a Talysurf CLI 2000 (for the Talysurf CLI 2000 refer to chapter 5) gives an average surface roughness value of 0.46242\( \mu m \). The regime of lubrication can be found using \( \psi \) this is usually given by psi. The is the ratio of film thickness to the average surface roughness. If the value of \( \psi \) lies between 1 and 3 the regime of lubrication is mixed. When the value is in excess of 3, the an EHL regime of lubrication results. For conditions in Figure 6.4, \( \psi \) is 2.6, hence the regime of lubrication is in transition between hydrodynamic and full EHL condition. The film contour shown in Figure 6.7 gives the film thickness profile. The 3D pressure profile shown in Figure 6.10 clearly demonstrates the development of an EHL pressure spike. For lower loads and at high speeds of entraining motion, the condition is just EHL.

Figure 6.5 shows the pressure, film thickness and plug flow profile for an applied load of 24N. For this case the central and minimum film thickness observed are 1.64 \( \mu m \) and 1.2 \( \mu m \) respectively. The \( \psi \) value for this operating condition is also 2.6. The maximum pressure rise recorded for this contact is 1.93 \( GPa \), which is 81% of the maximum Hertzian contact pressure, thus the condition is in transition between hydrodynamic and full EHL condition (Figure 6.11).

The pressure rise, film thickness and plug flow profile for an applied load of 49N which is the maximum load acting as obtained from the bearing dynamics model (chapter 3) is shown in Figure 6.6. The operating regime is nearly purely EHL as the \( \psi \) is 2.9, with a minimum film thickness of 1.33 \( \mu m \) (Figure 6.9). The maximum pressure is 2.32 \( GPa \), which is 98 % of the maximum Hertzian pressure (Figure 6.12).
Chapter 6 - Grease Lubrication: Results and Discussions

Figure 6.4: Contact Parameters load 4N

Figure 6.5: Contact Parameters load 25N

Figure 6.6: Contact Parameters load 49N

Figure 6.7: Film Contour load 4N

Figure 6.8: Film Contour load 25N

Figure 6.9: Film Contour load 49N

Figure 6.10: 3D Pressure Profile load 4N

Figure 6.11: 3D Pressure Profile load 25N

Figure 6.12: 3D Pressure Profile load 49N
6.6 Effect of Speed on film thickness

The effect of increasing speed of entraining motion is to increase the film thickness, reduce the proportion of contact area where the two surfaces are virtually parallel, and increase the proportion of contact area covered by the exit constriction. The first effect, i.e. increase in the film thickness, is the most significant. It is evident that the film thickness varies considerably with increasing speed of entraining motion as shown in Figures 6.13-6.16.

6.6.1 Film thickness Comparison Grease vs Base oil

For comparison base oil and grease behaviour film thickness with speed entraining motion, the following non dimensional parameters are introduced

Non dimensional speed parameter \( U^* = \frac{\phi U}{RE'} \) (6.9)

Non-dimensional film \( H^* = \frac{h}{R} \) (6.10)

Figures 6.13-6.16 show the comparison of minimum and central film thickness for grease and base oil plotted film in terms of the speed parameter \( (U^*) \). Non-degraded grease or fresh grease with a viscosity of 3.19 Pa.s is shown in green. Figure 6.13 shows the film thickness at the low load of 6 N for a speed varying from 5 m/s to 49 m/s. In this case the fresh grease behaves like a solid and at lower speeds it forms a thick layer almost double that of the degraded grease and base oil. This result is in line with the experiments conducted by Wilson (1979) and Cann et al (1991). Further being a thick layer, grease does not deform the surfaces, which suggests that grease under this condition works in hydrodynamic regime of lubrication. An interesting observation is that as speed increases beyond 24 m/s the fresh grease does not work at all (i.e. it seems to break-up or degrade). Figures 6.14 and 6.15 are for moderate loads of 12N and 24N in this case. In the case of the former the fresh grease makes a thick film and due to its high viscosity it deforms the surfaces in contact. As the load is increased contact deflection with fresh grease is much larger than that by degraded grease or the base oil alone. This can cause unwanted stresses in the contacting bodies and can reduce the life span of bearings. Figure 6.16 shows the film thickness at the
very high speed of 49 m/s. Since the non degraded grease or fresh grease cannot hold its structure due to high shear rates at very high speeds, fresh grease is not recommended at all. In all the above mentioned cases it is very clear that degraded grease has a minimum 50 nm thickness only compared to that of the base oil. Physically, this can be due to the fact that the thickener in the form of soap particles builds up a layer thick in the track during fully flooded conditions, which increases the surface separation. This argument agrees with the experimental findings of Cann et al (1991). The authors found the same behaviour using IR spectrometry and optical interferometry. The slope of the curves for film thickness with speed, the minimum grease film thickness varies from 0.0211 to 0.0234, whereas the same for base oil varies from 0.0167 to 0.0217, which suggest that the grease behaves the same as the base oil for higher speeds except for an extra film thickness.

A power relationship between minimum film thickness and speed parameter were found using least square curve fitting. For applied loads of 6N, 12N, 24N and 49N the power relationship was found to be 0.63, 0.64, 0.66 and 0.685 respectively for the base oil. Taking the average of these values the power relationship for the minimum film thickness, it follows that:

$$H_{\text{baseoil}}^* \propto U^{0.65}$$  \hspace{1cm} (6.11)

For a point contact the base oil power relation for minimum film thickness with speed suggested by Gohar (1971) is 0.7 and by Palacios et al (1981) as 0.74. Variation of the obtained values from these values is 12 % less, which can be attributed to higher speeds and also the curve fitting errors.

Using the same method for the curves plotted in Figure 6.13-6.16 the power relation for the minimum film thickness variation with speed for grease is found to be 0.792, 0.8, 0.843 and 0.885 for the various applied loads 6N, 12N, 24N and 49N respectively. Taking the average of these values the power relationship for the minimum film thickness is therefore:

$$H_{\text{Grease}}^* \propto U^{0.83}$$  \hspace{1cm} (6.12)
Chapter 6 - Grease Lubrication: Results and Discussions

Figure 6.13: Film Thickness Comparison diff speeds at 6N

Figure 6.14: Film Thickness Comparison diff speeds at 12N

Figure 6.15: Film Thickness Comparison diff speeds at 24N

Figure 6.16: Film Thickness Comparison diff speeds at 49N
6.7 Effect of Load on film thickness

Applied load also has some effect on the film thickness in general and more importantly on the minimum film thickness at the exit constriction. The relationship between load and film thickness has been investigated by previous researchers; see Dowson and Higginson (1961), Cameron et al (1966) Hamrock and Dowson (1977) among others, they shown that the central film thickness declines with load till a certain level where film thickness becomes virtually independent of load. Figure 6.17 – 6.20 show the effect of varying load on elastohydrodynamic film thickness. It can be seen that as the load is increased, higher pressures are confined to the nominal Hertzian contact area (see also Figures 6.4 – 6.6). This effect is very pronounced, thus pressures outside the contact area, i.e. at the inlet, actually decline (this is in line with the findings of Evans and Snidle (1983) and experimental measurements by Johns-Rahnejat (1988). The increase in load also causes an increase in the film thickness between the inlet and exit constriction which is a re-entrant profile. This feature is attributed to lubricant compressibility according to Hamrock and Dowson (1981). It is evident that the central film thickness declines with load until proper EHL conditions are encountered where the film thickness becomes virtually independent of load (this is a key feature of EHD contacts (see Rahnejat, 1984, Gohar and Rahnejat, 2008). In fact, an increase in load by 25 folds reduces the film thickness by a mere 17%.

The film thickness variation with load is ascertain by using the non dimensional parameters:

Non-dimensional load parameter \[ W' = \frac{W}{E'R^2} \] \hspace{1cm} (6.13)

Non dimensional film thickness as before \[ H' = \frac{h}{R} \] \hspace{1cm} (6.14)

Figures 6.17-6.20 shows the film thickness plotted against the load parameter at a range of constant speeds. The plots are made at speeds 5 m/s (Figure 6.17), 10 m/s (Figure 6.18), 24 m/s (Figure 6.19) and 49 m/s (Figure 6.20) in order to incorporate almost all the spindle speed range. Since fresh grease cannot be operated at all speeds, only those applicable are considered. As shown by Zhu (2002), in the extended speed
and load ranges the relationship between the central film thickness and the load no longer obeys the simple power law. So the power relationship between minimum film thickness and the load parameter is found using least square curve fitting. For speeds of 5 m/s, 10 m/s, 24 m/s and 49 m/s the power relationship was found to be -0.0365, -0.0526, -0.017 and -0.0062 respectively for the base oil. It is interesting to note that there is a significant variation in the power law relationship as the speed increases beyond 10,000 rpm (24 m/s). So the power law relationship for the minimum film thickness with load for speeds up to 10,000 rpm is found as:

$$H^*_{\text{baseoil}} \propto W^{*-0.045}$$  \hspace{1cm} (6.11)

and the same for speeds above 10,000 rpm is:

$$H^*_{\text{baseoil}} \propto W^{*-0.012}$$  \hspace{1cm} (6.12)

For a point contact the base oil power relation for the minimum film thickness with load is suggested by Gohar (1971) to be -0.05 and by Rahnejat (1984) as -0.045. Results obtained suggest that these results only hold for speeds less than 10,000 rpm. The variation of slope from conventional elastohydrodynamic theory can be attributed to the non-Newtonian behaviour at high shear at very high speeds. For heavy loads and low speeds the film thickness may drop quickly as the load increases. Although the results and conclusions from the numerical simulation make sense physically, further efforts are needed on experimental verification.

Using the same method as for the curves in Figures 6.17-6.20 the power law relationship for the minimum film thickness with load for grease is found as -0.0503, -0.0562, -0.0114 and -0.024 for speeds 5 m/s, 10 m/s, 24 m/s and 49 m/s respectively. It is interesting to observe that the minimum film thickness does not change dramatically at higher speeds when compared with the base oil. This can be attributed to the plug flow, as for the isothermal analysis the core does not disintegrate leaving a steady minimum film thickness. Taking the average of the power indices found, one can note that:

$$H^*_{\text{Grease}} \propto W^{*-0.036}$$  \hspace{1cm} (6.13)
Figure 6.17: Film thickness - different loads at speed 5 m/s

Figure 6.18: Film thickness - different loads at speed 10 m/s

Figure 6.19: Film thickness - different loads at speed 24 m/s

Figure 6.20: Film thickness - different loads at speed 49 m/s
Chapter 6 - Grease Lubrication - Results and Discussions

6.8 Model Validation

Numerical model for grease lubricated contact is validated against measurements reported in literature. The model developed in this thesis is primarily for high speed spindle bearings, but it is generic and parametric in nature. No experiment has been reported in the open literature for film thickness measurement at high speeds. Thus, numerical predictions are compared with experiments conducted at fairly low speeds. Moriuchi et al (1985) used Optical interferometry to measure the thickness of films of grease and observe their shape under rolling-sliding EHL. The details of their experiments are given below.

Table 6.3: Lubricant & Geometrical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure-Viscosity index</td>
<td>$z = 0.67$</td>
<td>-</td>
</tr>
<tr>
<td>Viscosity of Lubricant at $P = 0$</td>
<td>$\eta = 0.0319$</td>
<td>Pa.s</td>
</tr>
<tr>
<td>Thickener</td>
<td>Polyurea</td>
<td>-</td>
</tr>
<tr>
<td>Concentration</td>
<td>15</td>
<td>Wt %</td>
</tr>
<tr>
<td>Penetration worked</td>
<td>440&lt;</td>
<td></td>
</tr>
<tr>
<td>Penetration unworked</td>
<td>440&lt;</td>
<td></td>
</tr>
<tr>
<td>Refractive index</td>
<td>1.4698</td>
<td></td>
</tr>
<tr>
<td>Radius of curvature of solid $A$ in $X$ direction</td>
<td>$R_{Ax} = 0.0381$</td>
<td>m</td>
</tr>
<tr>
<td>Radius of curvature of solid $A$ in $Y$ direction</td>
<td>$R_{Ay} = 0.0381$</td>
<td>m</td>
</tr>
<tr>
<td>Radius of curvature of solid $B$ in $X$ direction</td>
<td>$R_{Bx} = \infty$ (flat)</td>
<td>m</td>
</tr>
<tr>
<td>Poisson’s Ratio of solid $A$ (Steel)</td>
<td>$v = 0.3$</td>
<td></td>
</tr>
<tr>
<td>Poisson’s Ratio of solid $B$ (Glass)</td>
<td>$v = 0.25$</td>
<td></td>
</tr>
<tr>
<td>Young’s Modulus of solid $A$ (Steel)</td>
<td>$E_A = 210 \times 10^9$</td>
<td>N/m²</td>
</tr>
<tr>
<td>Young’s Modulus of solid $B$ (Glass)</td>
<td>$E_B = 83 \times 10^9$</td>
<td>N/m²</td>
</tr>
</tbody>
</table>

Optical interferometry combines two or more light waves in an optical instrument in such a way that interference occurs between them. In optical interferometry a steel ball is loaded against the semi transparent disc with a lever and a weight, and the contact is illuminated with white light through fibre optic bundle prism. The light is
reflected both from the surface of steel ball and from a 200 Å chromium layer on the disc, on the side facing the ball. The reflected light interferes and produces an interference pattern seen as fields of different colours in the contact. Different film thickness result in different colours and thus topography of the contact area can be evaluated.

A rolling-sliding EHD point contact is formed between a hard optical disk of 150 mm diameter and a steel ball of JIS G 4805 SUJ 2 (diameter 38.1 mm, surface roughness 0.02 μm Ra). The test grease used for the study was a Mineral oil with polyurea. The properties of the grease used are given in Table 6.31. Figure 6.21 shows comparison of numerical predictions with the experimental findings of Moriuchi et al (1985). The rolling/sliding velocity for the experiment ranged from 0.1 to 0.6 m/s. Furthermore, the experiments were conducted under isothermal conditions (i.e. the temperature of the contact was kept constant at 25°C).

It is expected that under these conditions grease would not degrade. Hence numerical runs were performed with fresh and degraded grease. For grease degradation the model described by equation (6.3) was used and the viscosity of degraded grease as 0.4386 Pa.s. The viscosity of the fresh grease from literature was found to be 3.19 Pa.s. The Numerical runs were carried out at an applied load of 19.6 N, with the Hertzian pressure of 0.65 GPa. Comparisons of film thickness between the numerical and experimental results show good agreement. From Figure 6.21 it is very
clear that during the experiments grease did not degrade. For central film thickness a maximum error of 18% occurred at 0.1 m/s, whereas a least error of 1.2% is noted at 0.2 m/s. For the minimum film thickness the maximum and minimum errors for speeds 0.1-0.6 m/s are 27% and 6.16% respectively. The difference in film thickness between experimental findings and numerical predictions for the central film thickness is 45 nm and for minimum film thickness is 74 nm.

6.9 Thermal Analysis

In elastohydrodynamic contacts significant energy dissipation can occur especially for non-conformal contacts, where the pressures are very high and with high entraining velocity. This causes high temperatures inside the contact which can affect the rheology of lubricant. The pressure rise inside the contact is attributed to the interactions of solid surfaces and shear of the lubricant. It is well known that the viscosity of lubricant is highly sensitive to pressure is also equally sensitive to temperature. Therefore, it is important for any realistic analysis to include the effect of temperature rise.

Crook (1961) presented a detailed theoretical analysis of the mechanisms of heat generation and removal in line contacts. This work was as a precursor to thermal EHL analysis. In particular, he predicted the fall in traction as the slide-roll ratio increases. Sternlicht et al. (1961) indicated, for the first time, a procedure to be followed to develop simultaneous predictions of pressure and temperature in rigid and elastohydrodynamic line contacts. Cheng and Sternlicht (1965) and Dowson and Whitaker (1966) then introduced numerical procedures for solution of coupled Reynolds, elasticity and energy equations in line contacts. The effect of viscous heating in the inlet zone on reducing film thickness was investigated by Greenwood and Kauzlarich (1973) and Murch and Wilson (1975). Greenwood and Kauzlarich (1973) derived an approximate method for evaluating the temperature rise in the inlet for pure rolling condition. Murch and Wilson (1975) developed a thermal correction factor which provided a useful indicator for decrease in film thickness due to thermal effects.

Bruggemann and Kollmann (1982) were the first to include thermal effects in the study of elliptical contacts. For simplification, their analysis ignored the convective
heat removal and heat generation due to pressure induced compression of the lubricant film. Their results, however, showed an important discrepancy between the predicted and measured traction coefficients, especially under low sliding conditions. Only in the 1990s did complete thermal EHL solutions begin to emerge for point contact problems. Kim and Sadeghi (1992) presented a full thermal EHL point contact analysis in which they obtained solutions for pure rolling and low-slip conditions. Their calculations revealed significant inlet shear heating even under pure rolling conditions, but also important shear dissipation in the Hertzian contact region under sliding conditions was noted.

Although important numerical developments have been made in recent years to provide a better insight into the thermal EHL problem most of these are confined to oil lubrication. Only numerical solution of thermal EHL for line contacts lubricated with grease using a Herschel Bulkley model has been reported by Yoo and Kim (1997). This analysis was for a complete numerical solution of grease lubricated point contact without the squeeze effect (i.e. thus steady state in nature). A new numerical scheme is described here to overcome this shortcoming.

6.9.1 The Energy Equation

The other governing equations; Reynolds equation, elastic film shape, contact elasticity integral and lubricant rheological state are described in chapter 5 (also see section 6.2). The energy equation is dealt with here.

The law of conservation of energy at a point in the lubricant film, in its steady state form with no external sources of film heating and with a constant pressure assumed across film thickness in Cartesian coordinates can be written as:

\[
\rho \left[ \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} + w \frac{\partial U}{\partial z} \right] + \rho P \left[ u \frac{\partial (1/\rho)}{\partial x} + v \frac{\partial (1/\rho)}{\partial y} + w \frac{\partial (1/\rho)}{\partial z} \right] = \left[ \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) \right] \\
+ 2\phi \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 + \left( \gamma^2 + \gamma_{xy}^2 + \gamma_{yz}^2 \right) \right] + \lambda' \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2
\]

(6.14)

where \(U\) is the internal energy per kg at the point considered, \(k\) is the thermal conductivity of the lubricant, \(\gamma\) is the shear strain and \(\lambda'\) is known as the second coefficient of viscosity or volume viscosity.
This equation can be simplified using following assumptions:

- Important velocity gradients are only across the film thickness
- No side leakage of the lubricant is considered
- Velocity across the film ($w$) is negligible (flow is assumed to be laminar)
- $\lambda'$ does not have any significance in EHL studies according to Gohar (1988)
- Thermal conductivity $k$ is invariable within the EHL film

After applying these assumptions:

$$\rho u \frac{\partial U}{\partial x} + \rho v \frac{\partial U}{\partial y} + \rho P u \frac{\partial (1)}{\partial x} + \rho P v \frac{\partial (1)}{\partial y} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \phi \left( \frac{\partial u}{\partial z} \right)^2 \quad (6.15)$$

Internal energy can be described in terms of enthalpy per unit mass as:

$$I = U + \frac{P}{\rho} \quad (6.16)$$

Therefore:

$$\partial I = \partial U + \left[ P \partial \left( \frac{1}{\rho} \right) + \frac{1}{\rho} \partial P \right] \quad (6.17)$$

Internal energy can also be written in terms of $C_p$; the lubricant specific heat at constant pressure, thus:

$$\partial I = C_p \partial T + \left[ \frac{1}{\rho} - T \frac{\partial}{\partial T} \left( \frac{1}{\rho} \right) \right] \partial P \quad (6.18)$$

where temperature $T$ is in Kelvin, hence:

$$\partial U = C_p \partial T - T \frac{\partial}{\partial T} \left( \frac{1}{\rho} \right) \partial P - P \partial \left( \frac{1}{\rho} \right) \quad (6.19)$$

Introducing thermal expansion $\nu$, where $\nu = \rho \frac{\partial}{\partial T} \left( \frac{1}{\rho} \right)$, the equation becomes:

$$\partial U = C_p \partial T - T \left( \frac{\nu}{\rho} \right) \partial P - P \partial \left( \frac{1}{\rho} \right) \quad (6.20)$$

Dividing throughout by $\partial x$ yields:

$$\frac{\partial U}{\partial x} = C_p \frac{\partial T}{\partial x} - T \left( \frac{\nu}{\rho} \right) \frac{\partial P}{\partial x} - P \frac{\partial}{\partial x} \left( \frac{1}{\rho} \right) \quad (6.21)$$
Similarly for \( \frac{\partial U}{\partial y} \):

\[
\frac{\partial U}{\partial y} = C_p \frac{\partial T}{\partial y} - T \left( \frac{v}{\rho} \right) \frac{\partial P}{\partial y} - P \frac{\partial}{\partial y} \left( \frac{1}{\rho} \right) \quad (6.22)
\]

Substituting for \( \frac{\partial U}{\partial x} \) and \( \frac{\partial U}{\partial y} \) in equation (6.15):

\[
\rho u \left[ C_p \frac{\partial T}{\partial x} - T \left( \frac{v}{\rho} \right) \frac{\partial P}{\partial x} - P \frac{\partial}{\partial x} \left( \frac{1}{\rho} \right) \right] + \rho v \left[ C_p \frac{\partial T}{\partial y} - T \left( \frac{v}{\rho} \right) \frac{\partial P}{\partial y} - P \frac{\partial}{\partial y} \left( \frac{1}{\rho} \right) \right] 
\]

\[
+ \rho P u \frac{\partial}{\partial x} \left( \frac{1}{\rho} \right) + \rho P v \frac{\partial}{\partial y} \left( \frac{1}{\rho} \right) = k \frac{\partial^2 T}{\partial z^2} + \phi \left( \frac{\partial u}{\partial z} \right)^2 \quad (6.23)
\]

Expanding and rearranging:

\[
\rho u C_p \frac{\partial T}{\partial x} = \rho u T \left( \frac{v}{\rho} \right) \frac{\partial P}{\partial x} - \rho u P \frac{\partial}{\partial x} \left( \frac{1}{\rho} \right) + \rho v C_p \frac{\partial T}{\partial y} - \rho v T \left( \frac{v}{\rho} \right) \frac{\partial P}{\partial y} - \rho v P \frac{\partial}{\partial y} \left( \frac{1}{\rho} \right) 
\]

\[
+ \rho P u \frac{\partial}{\partial x} \left( \frac{1}{\rho} \right) + \rho P v \frac{\partial}{\partial y} \left( \frac{1}{\rho} \right) = k \frac{\partial^2 T}{\partial z^2} + \phi \left( \frac{\partial u}{\partial z} \right)^2 \quad (6.24)
\]

Further simplification leads to:

\[
\rho u C_p \frac{\partial T}{\partial x} = \rho u T \left( \frac{v}{\rho} \right) \frac{\partial P}{\partial x} + \rho v C_p \frac{\partial T}{\partial y} - \rho v T \left( \frac{v}{\rho} \right) \frac{\partial P}{\partial y} = k \frac{\partial^2 T}{\partial z^2} + \phi \left( \frac{\partial u}{\partial z} \right)^2 \quad (6.25)
\]

Collecting the common terms together:

\[
\rho T \left( \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} \right) + \phi \left( \frac{\partial u}{\partial z} \right)^2 = \rho C_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) - k \frac{\partial^2 T}{\partial z^2} \quad (6.26)
\]

\[
\rho T \left( u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} \right) + \phi \left( \frac{\partial u}{\partial z} \right)^2 = \rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) - k \frac{\partial^2 T}{\partial z^2} \quad (6.27)
\]

Since there is no side leakage the velocity in y direction can be neglected (i.e. \( v = 0 \)), thus:

\[
\rho T \left( \frac{\partial P}{\partial x} \right) + \phi \left( \frac{\partial u}{\partial z} \right)^2 = \rho u C_p \left( \frac{\partial T}{\partial x} \right) - k \frac{\partial^2 T}{\partial z^2} \quad (6.28)
\]

Cameron (1966) has shown that convective cooling term is not significant in EHL contacts (because of thinness of the film), which is also shown by Foord (1969). This is also true in the case of grease lubrication, as the base oil becomes trapped inside the
soap fibres, thus grease unlike oil cannot remove heat by convection. Therefore, the
first term on the right hand side of equation (6.28) can be neglected, yielding:

\[ uTv \frac{\partial P}{\partial x} + \phi \left( \frac{\partial u}{\partial z} \right)^2 = -k \frac{\partial^2 T}{\partial z^2} \]

(6.29)

The first term on the left hand side of the equation is due to the pressure gradient. The
second term on the left hand side is due to shear heating. Each term in equation (6.29)
is considering separately below.

6.9.1.1 Conductive cooling

Cameron (1966), and Gohar and Rahnejat (2008) have assumed that for conductive
cooling the temperature gradient across the film varies linearly through the oil film to
a maximum value of \( \Delta T/\delta \). This gives a parabolic temperature distribution across the
film. Hence, the heat taken away at the top surface through a column of width \( dx \) and
height \( h \) can be given as:

\[ kdx \int_0^h \left( \frac{d^2 T}{dz^2} \right) dz = kdx \left( \frac{\Delta T}{h} \right) \]

(6.30)

Since the temperature rise across the film is linear \( \delta T = x \frac{\Delta T}{2b} \), therefore the total
conductive cooling can be written as:

\[ \frac{k\Delta T}{2bh} \int_0^{2b} x dx = \frac{k\Delta Tb}{h} \]

(6.31)

6.9.1.2 Shear heating

Considering the whole contact domain, the shear heating term in equation (6.29) can
be written as:

\[ \phi \int_{-b}^{b} \left( \frac{\partial u}{\partial z} \right)^2 dz dx \]

(6.32)

Assuming the velocity variation across the film to be linear, then:

\[ \frac{\partial u}{\partial z} = \frac{u}{h} \]

(6.33)
Substituting this into the equation and integrating twice the shear heating term can be written as:

\[
\frac{2\phi u^2 b}{h}
\]

(6.34)

### 6.9.1.3 Compressive heating

Considering the whole contact domain the compressive heating term in equation (6.29) can be written as:

\[
\int_{-b}^{b} uTv \frac{\partial P}{\partial x} dz dx = uTv \frac{P_{\text{max}}}{2b} h2b = uTv P_{\text{max}}
\]

(6.35)

substituting from the above equations (assuming principle of superposition) leads to:

\[
uTv P_{\text{max}} + \frac{2\phi bu^2}{h} = \frac{\Delta Tb}{h}
\]

(6.36)

Expanding for \(\theta\) in the compressive term:

\[
u(T_{i-1} + \Delta T) + \frac{2\phi bu^2}{h} = \frac{\Delta Tb}{h}
\]

(6.37)

Rearranging the terms:

\[
u(T_{i-1} + \Delta T) + \frac{2\phi bu^2}{h} = \frac{\Delta Tb}{h} - u\Delta Tv P_{\text{max}}
\]

(6.38)

Thus the temperature rise inside the contact can be given by:

\[
\Delta T = \frac{\left( \nu(T_{i-1} + \Delta T) + \frac{2\phi bu^2}{h} \right)}{\left( \frac{kb}{h} - uTv P_{\text{max}} \right)}
\]

(6.39)

Applying this equation the temperature rise inside the contact for each node of width \(dx\) and increment in pressure \(dP\) can be found as

\[
\Delta T = \frac{\left( \nu_{\text{av}} T_{i-1} + \frac{2\phi dx u_{\text{av}}^2}{h} \right)}{\left( \frac{kd}{h} - u_{\text{av}} vhdP \right)}
\]

(6.40)
6.10 Thermal Analysis – Results

Results obtained for the thermal analysis are discussed here.

6.10.1 Contact Pressure and Film thickness

Thermal analysis results for contact parameters obtained are shown in Figure 6.21. It is seen that taking into account the effect of contact temperature makes the elastohydrodynamic peak to disappear. This is due to the fact that the temperature rise inside the contact decreases the lubricant viscosity from its value under isothermal condition (926 GPa.s to 0.721 GPa.s). This decrease in the value of viscosity has a direct effect on all the contact parameters, which are shown in Figures 6.22 and 6.23. This reduction in viscosity makes the film thickness reduce further than the average surface roughness value of 0.46242μm. Hence, the lubrication regime changes to boundary. Note that additional heat would be generated due to asperity interactions. This is discussed later.

![Figure 6.22: TEHL Pressure profile - load 49N](image)

Figure 6.22 shows that the plug flow does not occupy the entire contact area. This is due to the fact that the thermal energy dissipation due to the shearing action is caused by the velocity difference between the contacting surfaces and the plug flow disintegrates under this condition. Figure 6.23 shows the pressure profile comparison for thermal (bold lines) and isothermal (dotted lines) analyses for low, medium and high applied loads. For an applied load of 4N the maximum pressure rise is 1 GPa,
whilst the same for isothermal analysis is 1.2 $GPa$. This drop in pressure can be seen in all the cases. For an applied load of 24N the pressure drop is 0.15 $GPa$ i.e. 1.78 $GPa$ for isothermal and 1.93 $GPa$ for the thermal case. In the case of an applied load of 49 N the pressure for isothermal analysis of 2.32 $GPa$ drops to a value of 2.2 $GPa$ for the thermal analysis; a drop of 0.12 $GPa$. Since this drop in pressure is occurring in all cases, this can be attributed to the energy dissipation due to the temperature rise.

![Figure 6.23: TEHL and isothermal pressure distributions](image)

Figure 6.23 shows comparisons for film thickness for thermal (bold lines) and isothermal (dotted lines) analysis for an entraining velocity of 49 $m/s$. It is clear that the decrease in value of viscosity inside the contact from 926 $GPas$ to 0.721 $GPas$ for isothermal to thermal analysis makes the film thinner. The mean central oil film thickness of 1.5 $\mu m$ has decreased to 0.15 $\mu m$, a decrease of 1.35 $\mu m$. Decrease of minimum film thickness makes it less than average surface roughness, promoting boundary lubrication regime. In this regime the asperity interactions takes place.
Figures 6.25 to 6.33 show the pressure, film thickness and plug flow profile for low (4N), medium (24N) and high loads (49N). The minimum film thickness for low load is 0.2μm (shown in Figure 6.25), which is less than the average surface roughness value, so the regime of lubrication is boundary. This is also confirmed by the film thickness contour shown in Figure 6.28. The horse shoe constriction in the oil film contour is more pronounced at this load. Due to high velocity, low load and rise in temperature a decrease in limiting shear stress occurs. Since the limiting shear stress is not high enough, majority of the lubricant stays as liquid inside the contact. The 3D pressure profile of Figure 6.31 also confirms the boundary lubrication regime.

In cases of medium and high loads the pressure induced inside the contact is almost double that at low load, therefore increasing the limiting shear stress, as this is dependent on pressure (refer to chapter 5, equation 5.25). This makes the plug flow to occupy the entire contact area, but the temperature rise makes the shear stress decrease as the viscosity, hence, the layers adjacent to the contacting solids remain as liquid. This is shown in Figures 6.26 and 6.27. The film contours (Figure 6.29 and 6.30) clearly show boundary regime of lubrication. This promotes asperity interactions, resulting in even higher surface temperatures.
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Figure 6.25: Contact Parameters load 4N

Figure 6.26: Contact Parameters load 25N

Figure 6.27: Contact Parameters load 49N

Figure 6.28: Film Contour load 4N

Figure 6.29: Film Contour load 25N

Figure 6.30: Film Contour load 49N

Figure 6.31: 3D Pressure Profile load 4N

Figure 6.32: 3D Pressure Profile load 25N

Figure 6.33: 3D Pressure Profile load 49N
6.11 Temperature variation

In the case of thermal elastohydrodynamic lubrication of point contacts, where pressures developed are quite high grease behaves like an amorphous solid in a glassy state according to Cann et al (1992). In terms of flow, this suggests that the bulk of the lubricant is transported through the conjunction as a core of solid material, with the potential for apparent discontinuities in velocities at the interfaces with the bounding solids, or very thin shear zones, to comply with the kinematic conditions. In a section across the film, the lubricant can thus be described as a core in a glassy state, sandwiched between thin shear zones, probably of molecular proportions, in which the difference in velocity of the solidified core relative to the mean entraining velocity is accommodated. This difference between the average velocity of the solidified core and the mean entraining velocity causes a slip as suggested by Ehert et al (1998). In pure rolling the lubricant core has the common velocity of the two surfaces, with no need for any discontinuity of velocity at the either interface. This is only true for the case of oil lubrication, whereas in case of grease the velocity of core is different than the moving surfaces (chapter 5, equation 5.39). In sliding, the core of the lubricant could be transported through the conjunction at the entraining velocity, which represents the mean of the two surface speeds. Energy dissipation associated with this and the substantial generation of heat is restricted to the shear zones and it is in these regions that the temperature rise ensures fluid, rather than solid, behaviour. Furthermore, since it is well known that grease promotes heat transfer through conduction only, the different thermal properties of the ceramic balls and steel races in the cases described in this thesis and their kinematic conditions (stationary or moving) lead to different temperature rises in the two layers and different apparent slip velocities. Since a rise in temperature leads to a decrease in the limiting shear stress, such a variation would further decrease the slip in this part of the control volume by shear-thinning effect at the inlet of the conjunction, and thus, the flow which passes through the contact. Figure 6.34 shows the temperature profile of grease in the contact.
Figure 6.34 shows that as soon as the grease enters the contact the temperature rises rapidly until after sometime a thermal equilibrium is reached between the film, the contacting surfaces and the surroundings. This temperature rise is directly related with the pressure rise so the temperature also has the same profile as the pressure. This temperature rise impedes the usual rise in lubricant viscosity noted under isothermal conditions. Since the viscosity does not build up as under isothermal conditions the lubricant film thickness is reduced and thus higher pressures are generated. This makes the lubrication to operate in different regime to that predicted under isothermal conditions. Assuming all the balls are in contact at a given instant of time the total energy lost due to conductive heat transfer can is given in Table 6.4. The total energy lost in the bearings are found to be 673.29 W and 615.27 W for tool end and top end bearing. There are two bearings at the top and bottom so the total power lost is found to be 2.6kW, which is 34% of the input power.
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Table 6.4: Thermal power loss tool & top end bearing

<table>
<thead>
<tr>
<th>Ball number</th>
<th>Entrainment Velocity (m/s)</th>
<th>Tool End Bearing</th>
<th>Top End Bearing</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Load (N)</td>
<td>Power loss (W)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Load (N)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Power loss (W)</td>
</tr>
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<td>49.2</td>
<td>4.044</td>
<td>19.23</td>
</tr>
<tr>
<td>2</td>
<td>49.2</td>
<td>8.358</td>
<td>21.46</td>
</tr>
<tr>
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<td>49.2</td>
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</tr>
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<td>615.27</td>
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</table>

The heat generation and hence the power loss in bearings due to the viscous action of lubricant was mentioned in the previous sections. Apart from this the major power losses in high speed spindles are from: power loss in motors, viscous shear of air at spindle tip and power loss due to load in bearings. Power loss in rotor and stator of electric motor contributes to 10 percentage of input power. Power lost due to the viscous shear of air as a function of speed, this when compared with other sources of power loss is very less hence not considered in this study. Power loss in the bearings due to the bearing load and speed, which is due to the friction in the contacts. This frictional heat generation is due to boundary friction and plough friction. In this case the plough friction does not have significant contribution due to ceramic coated materials. Due to the difference in elastic constants of the materials in contact the load
the load acting on each surface give rise to two unequal displacement of surfaces, leading to slip at interface, called Reynolds slip, Reynolds (1896). This can also be a reason for heat generation. As soon as the centreline of contact passes the pressure starts decreasing, hence the temperature also. The maximum temperature obtained for maximum load condition with an induced pressure of $2.17 \text{GPa}$ is $173^\circ \text{C}$. Since friction is a dissipative process the energy dissipation accompanied with this process will be discussed in coming sections.

6.12 Friction in Lubricated Contacts

Friction is the resistance to motion during sliding/rolling of balls relative to raceway tracks. The resistive force acting in the direction opposite to that of motion is called friction $F$, which can be due to the adhesive junctions between asperity tips on the counterfaces and/or due to the deformation of asperities on the softer counterface by those on the harder surface in relative motion. This phenomenon is called ploughing friction. So the total asperity frictional force is given by:

Total frictional force ($F$) = Adhesive Friction ($F_a$) + Ploughing Friction ($F_p$)  

The friction model of Greenwood and Tripp (1970) accounts for the non-Newtonian behaviour of the lubricant and an overall asperity friction model. Bhushan (1999) and later Rahnejat and Gohar (2008) incorporated asperity interactions in detail for low lightly loaded contacts. Since this model deals in detail about the various aspects for this study it is used, where:

$$F_a = \psi \cdot \text{Boundary Friction} + (\psi - 1) \cdot \text{Viscous Friction}$$  \hspace{1cm} (6.41)

From the classical theory of adhesion, the adhesive friction force for dry contact or boundary lubrication can be given as (Bowden and Tabor, 1950):

$$F_{\text{boundary}} = A_s \tau_s$$  \hspace{1cm} (6.42)

Where: $\tau_s$ is the surface material shear strength. Substituting this into the equations:

$$F_a = A_s \left[ \psi \cdot \tau_s + (\psi - 1) \cdot \tau_f \right]$$  \hspace{1cm} (6.43)

$\tau_f$ is the average shear strength of the fluid film due to its viscous action and $\psi$ is the proportion of dry contact, which varies from 0 to 1 as the lubrication regime varies.
from viscous to boundary respectively. $A_e$ is the real contact area, which can be obtained as (Bhushan, 2002):

$$A_e = 3.2 \frac{W}{E'} \sqrt{\frac{r}{\sigma_{rms}}} \quad (6.44)$$

where $W$ is the applied load, $E'$ is the reduced modulus of elasticity, $r$ is the average radius of asperity tips and $\sigma_{rms}$ the root mean square of roughness values of the contacting surfaces.

$\tau_v$ in equation (6.43) is the average shear strength of the fluid film due to viscous action:

$$\tau_v = \frac{\eta u}{h} \quad (6.45)$$

where $\eta$ is the viscosity of lubricant. For grease this would be its plastic viscosity. $u$ the lubricant entrainment velocity and $h$ is the film thickness. Substituting for $\tau_v$ in equation (6.43):

$$F_a = A_e \left[ \psi \tau_v + (\psi - 1) \frac{\eta u}{h} \right] \quad (6.45)$$

Ploughing friction $F_p$ is caused by the deformation of asperities and is given by:

$$F_p = \pi nf_s d_s \left( 1 - e^{-d_s \psi / 3} \right) \quad (6.46)$$

where; $n$ is the number of asperities in the real contact area, $\psi = \text{sliding velocity}$ $f_s$ is the friction per a pair of asperities, which is given by:

$$f_s = \pi d_s^2 E' \left( \frac{\sigma_{rms}}{r} \right)^{1/2} \quad (6.47)$$

$\sigma_{rms}$ the surface roughness, $d_s$ is the extend of asperity in ploughing action. If the asperities are assumed to have hemispherical shape with $z_o$ as the height of asperity and $r'$ radius of the hemisphere and $d_s$ the extend of asperity can be written as (Gohar and Rahnejat, 2008):

$$d_s = \frac{z_o}{8r'} \quad (6.48)$$

$\Phi$ in equation (6.46) is a function given by (for elastic ploughing action).
\[ \Phi = \frac{1}{8r'} \left( \frac{\beta m}{2\pi E'} \left( \frac{r}{\sigma_{nm}} \right)^{1/2} \right)^{7/3} \]  

(6.49)

where \( \beta \) is the proportion of kinetic energy converted into elastic ploughing of an asperity and is assumed to be 0.5, and \( m \) is the equivalent mass of an asperity. Measurement of the surface parameters and calculation of friction are discussed below.

### 6.13 Surface profile Measurements for Friction

For calculating adhesive and ploughing frictions surface roughness measurements for bearing rings and balls are needed. This was carried out using Talysurf (see chapter 4).

![3D Surface profile plot using Talysurf CLI 2000](image)

**Figure 6.35: 3D Surface profile plot using Talysurf CLI 2000**

Surface measurements were carried out for an area of 1 mm\(^2\) with 2001 points along the X axis with a spacing of 0.5 \( \mu m \) and 201 lines with a spacing of 5 \( \mu m \) along the Y axis. The measured surface profile is shown in Figure 6.35. Since the measured surface is curved, straightness correction was applied and the surface profile graph was obtained as shown in Figure 6.36.
Using this profile the surface roughness parameters obtained the root mean square surface roughness $\sigma_{rms}$ was found to be $0.462\,\mu m$. The average height for asperities $z_0$ was measured using the step height measurement tool in Talysurf CLI 2000 and was found to be $1.889\,\mu m$ for a base-width $l$ of $12.812\,\mu m$. As mentioned in the previous section the asperities are assumed to be hemispherical shape. For calculation of the radius of the hemisphere $r'$ and mass of an asperity $m$ a fractal analysis was performed using the Talysurf CLI 2000. Figure 6.37 shows the fractal analysis plot. From this analysis, the fractal dimension $b_f$ and scale length $G$ were found to be $1.6188$ and $10\,\mu m$ respectively. These data were substituted into equation (6.50) to find the radius of a typical hemispherical asperity as $14.94\,\mu m$.
6.13.1 Mass of asperity

For calculating the mass of asperity its volume is calculated for a similar profile with shape between \( y = 0 \) and \( y = 1.889 \) using the relationship:

\[
y = \left( \frac{-1}{950} \right) x^2 + 1.889
\]

(6.52)

The volume is found by rearranging \( x \) in terms of \( y \) and finding the definite integral of \( x \) between \( y = 0 \) and \( y = 1.889 \) then multiplied by \( \pi \).

\[
Volume = \int_0^{1.889} \pi \left[ -0.0877193(210.9 - 38y)^{1.5} \right] = 452.189 \text{mm}^3
\]

(6.53)

Mass of asperity = Volume of asperity x Density

(6.54)

Substituting the value for volume and density of steel (7800 kg/m\(^3\)) in equation (6.54) the mass of a typical asperity is found to be 3.52\times10^{-12} \text{kg}

6.13.2 Friction per pair of asperity

As mentioned in the earlier sections friction per a pair of asperities can be calculated as:

\[
f_d = \pi d_s^2 E' \left( \frac{\sigma_{rms}}{r} \right)^{1/2}
\]

(6.55)

Substituting for the extend of asperity \( (d_s) \), \( \sigma_{rms} \) the surface roughness, \( r \) is the radius of the asperity tip and \( E' \) is the reduced modulus of elasticity equation, the value for friction per asperity is obtained as 5.82\times10^{-3} \text{N}.
6.14 Friction analysis - results

Figures 6.38 and 6.39 show adhesive and ploughing friction contributions acting in a typical lubricated contact considered in the current analysis. From the thermal EHL results it is clear that the regime of lubrication is boundary, so the effect of viscous friction is actually negligible. Total friction in a grease lubricated contact in the conditions described here is given in Figure 6.40. The friction produced accounts for the mechanical interaction of asperities of contacting surfaces due to the breakdown of fluid film at high pressures. The coefficient of friction is shown in Figure 6.41. These values indicate that even though boundary lubrication is prevalent there still exists lubricant in the contact as shown in Figures 6.25 to 6.30, which restrict full metal to metal contact. This is one the reasons that grease lubricated bearings exhibit more vibration than oil lubricated ones. Note that boundary interactions impart vibration. Figure 6.41 also shows that the coefficient of friction values is almost constant at 0.014, irrespective of load, which is in line with first two laws of friction initially expounded by Amontons in 1699 and later extended to kinetic conditions by Coulomb in 1750s (i.e. coefficient of friction is independent of normal load and apparent area of contact between the contacting surfaces).

Ploughing friction primarily occurs with oblique interaction of asperity-tip pairs (interlocking), but can also be enhanced by wear particles trapped in the contact, which is not considered here. Ploughing friction is mainly active in softer materials as asperity deformation depends on the surface hardness and surface roughness of the contacting surfaces, and on the size, shape and hardness of any wear debris trapped. In this study balls are coated with ceramics, making the surface hard and with low asperity height distribution. Furthermore, there is no debris formation incorporated in the model hence very low value for ploughing friction is predicted. The adhesion strength of the interface depends upon the mechanical and chemical properties of the contacting bodies. The adhesion strength is reduced by decreased surface interactions at the interface as noted by Bhushan (2002). For this analysis only grease degradation model is incorporated, but in reality the additives in grease fill the valleys of asperities and form a thin layer of 50 nm thick surface films which would reduce friction further as noted by Cann et al (1991). These low shear strength surface films act like smoothened surface coverings and reduce direct surface interactions.
Chapter 6 - Grease Lubrication - Results and Discussions

Figure 6.38: Variation of Adhesive Friction Force with Applied Load

Figure 6.39: Variation of Ploughing Friction Force with Applied Load

Figure 6.40: Total Friction Force Variation with Applied Load

Figure 6.41: Coefficient of Friction
6.15 Power loss due to friction in bearings

The predicted results point to dominance of boundary friction, which can be attributed to the mechanical interaction of asperities of contacting surfaces. This type of friction can be attributed to Coulomb model, where the action of wedge shaped asperities causes two surfaces to move apart as they slide from one position to another and the most potential energy stored during this process is recovered as the surfaces move back. Only a small fraction of energy is dissipated during sliding down the asperities. Energy is dissipated due to the deformation of surfaces during sliding when no groove is produced. Slip line theory for micro-scale deformation of a single asperity used by Suh and Sin (1981) is one of the common methods used for such analyses. However, using their method, the coefficient of friction does not depend on the adhesion, hence not used in this study.

In this study the spindle is supported by pair of angular contact ball bearings at top and bottom (for the details of the bearing and schematics of spindle please see Chapter 3, Table 3.4 and Figure 3.17).

![Figure 6.42: Friction per ball tool end bearing](image1.png) ![Figure 6.43: Friction per ball top end bearing](image2.png)

At any instant frictional forces acting on each ball in the top end and tool end bearings are shown in Figures 6.42 and 6.43 respectively. Frictional forces multiplied by the entrainment velocity will provide us with information on the power loss at the particular contact. Assuming at particular time all the balls are in contact the total
frictional power loss can be found by adding the power loss all the balls contacts. The routing spindle considered in this study consists of two pairs in tandem at each end of the spindle. Table 6.5 shows the frictional power loss calculations for tool end and top end bearing.

Table 6.5: Frictional power loss tool & top end bearing

<table>
<thead>
<tr>
<th>Ball number</th>
<th>Entrainment Velocity (m/s)</th>
<th>Tool End Bearing</th>
<th>Top End Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load (N)</td>
<td>Frictional Force (N)</td>
<td>Power loss (W)</td>
</tr>
<tr>
<td>1</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td>2</td>
<td>49.2</td>
<td>8.358</td>
<td>0.119</td>
</tr>
<tr>
<td>3</td>
<td>49.2</td>
<td>14.975</td>
<td>0.212</td>
</tr>
<tr>
<td>4</td>
<td>49.2</td>
<td>22.830</td>
<td>0.342</td>
</tr>
<tr>
<td>5</td>
<td>49.2</td>
<td>31.069</td>
<td>0.433</td>
</tr>
<tr>
<td>6</td>
<td>49.2</td>
<td>38.248</td>
<td>0.531</td>
</tr>
<tr>
<td>7</td>
<td>49.2</td>
<td>44.472</td>
<td>0.614</td>
</tr>
<tr>
<td>8</td>
<td>49.2</td>
<td>48.143</td>
<td>0.663</td>
</tr>
<tr>
<td>9</td>
<td>49.2</td>
<td>49.049</td>
<td>0.673</td>
</tr>
<tr>
<td>10</td>
<td>49.2</td>
<td>47.301</td>
<td>0.649</td>
</tr>
<tr>
<td>11</td>
<td>49.2</td>
<td>42.757</td>
<td>0.591</td>
</tr>
<tr>
<td>12</td>
<td>49.2</td>
<td>36.059</td>
<td>0.500</td>
</tr>
<tr>
<td>13</td>
<td>49.2</td>
<td>28.613</td>
<td>0.398</td>
</tr>
<tr>
<td>14</td>
<td>49.2</td>
<td>20.387</td>
<td>0.285</td>
</tr>
<tr>
<td>15</td>
<td>49.2</td>
<td>12.824</td>
<td>0.181</td>
</tr>
<tr>
<td>16</td>
<td>49.2</td>
<td>6.707</td>
<td>0.100</td>
</tr>
<tr>
<td>17</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td>18</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td>19</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td>20</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td>21</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td>22</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td>23</td>
<td>49.2</td>
<td>4.044</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td>Total Power loss</td>
<td>331.335</td>
<td></td>
</tr>
</tbody>
</table>

From the table 6.5 the total frictional power loss for tool end bearing and top end bearing is found to be 331 and watts 302 watts. Therefore, the total power lost due to friction inside all the bearings can be found to be 1266 watts at an entrainment velocity of 49.2 m/s. This is 16% of the input power of the spindle, being 7.5KW. Hence the energy available for useful work is found as 84%.

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Chapter 7

Conclusions and suggestions for Future work

7.1 Introduction

High speed machining with the help of latest engineering technology has become the mainstream of modern day manufacturing. High speed spindles for commercial use were only developed by 1980s (Komanduri et al, 1985) even though the first theoretical developments were reported in 1930 (Salomon, 1931). The lack of sufficient experience in the field means that there are still many problems to be solved for high speed applications. High speed machining is often associated with high feed rates which require rapid acceleration and deceleration, resulting in drastic changes in cutting conditions. This makes the study of spindle dynamics quite important. Since spindles are usually supported by pairs of bearings their dynamics behaviour is strongly affected by bearing dynamics. The material of balls, cage and lubricant rheology are important factors in bearing dynamics, as well as contact geometry, preload and interference fitting. Developments in tribology have made it possible to understand the contact conjunctions, which are usually subjected to elastohydrodynamic regime of lubrication, but can also be subjected to asperity level interactions. However, the non-Newtonian behaviour of grease in such conjunctions is still not as well established as that for oil lubricated contacts, even though 85% of industrial bearings operate with grease as a lubricant. Lack of realistic grease lubrication models has been the problem in characterising high speed precision spindles. This problem has been countered to a large extent in this thesis.

7.2 Overall Conclusions and contributions to knowledge

The following conclusions are obtained as a result of the analysis reported in the previous chapters:

Numerical model incorporating inertial dynamics and rigid body motion of the shaft-bearings system for a vertical routing spindle has been developed. This model with five degrees of freedom simulates an existing routing spindle, which operates at speeds up to 60,000 rpm (i.e. 2.4 million DN) and its vibration response characteristics have been obtained and verified against non-contact measurements as described below.
An experimental rig was devised to investigate the vibration spectra of a high speed precision 7.5 kW power routing spindle. Fine measurements of spindle vibration were carried out using laser vibrometry. Use of this technique is quite novel for high speed applications, with high resolution of rigid body motions of the spindle using precise alignment of a single laser beam with respect to an optically smooth surface of a specially designed tool, held in the spindle collet. An analytical method for alleviating cross-sensitivity between coupled motions and elimination of speckle noise from the target surface is also presented. Secondary frequencies produced by the rolling elements' surface waviness are obtained. Experimental spectra were compared with the numerical model predictions, showing good agreement.

Numerical model for grease lubrication capable of solving thermal-elastohydrodynamic lubrication with squeeze effect was developed. For this a modified Reynolds equation was derived from basic principles considering grease as a Bingham solid, using the Herschel-Bulkley flow model.

Heat generation and the power loss in contact conjunction due to viscous shear is quantified by solving the energy equation. At a spindle speed of 25,000 rpm, the power loss due to viscous shear was found to be 40% of total input power. For assessing the generated heat and thereby the power loss due to friction asperity interaction model for finding adhesive and plough friction are incorporated in the model. The power lost due to friction was found to be 16% of total input power at spindle speed of 25,000 rpm.

7.3 Achievement of Aims

The main objectives of this research are listed in chapter 1. In this section, the level of fulfilment of research objectives is assessed.

One of the main aims of this research was to formulate and solve numerical and analytical model for a high precision vertical routing spindle supported with the precision angular contact ball bearings with ceramic balls. An analytical model for the dynamics of high quality ceramic ball bearings with primary frequencies produced by interactions from races/ball, cage/ball, cage/races and secondary frequencies produced by the rolling contact surface waviness of elements is formulated and reported in chapter 3. Formulation and solution of numerical model for high speed precision spindles is achieved in stages;
first a numerical model for deep groove ball bearing was formulated and validated with published work in literature. This work is also reported in chapter 3. The deep groove ball bearing model with two degrees of freedom is then upgraded to a five degrees of freedom spindle-bearing model to accommodate all the motions. This work is also reported in the chapter 3.

Numerical model reported in chapter 3 is validated with measurements of spindle vibration obtained from a carefully designed test rig with a high speed precision routing spindle. This was one of the main objectives of research in chapter 1. Fine measurement of spindle vibration characteristics is carried out using laser vibrometry with precise resolution of rigid body motions of the spindle using a single laser beam. Results of experiments are compared with analytical and numerical models presented in chapter 3, showing good agreement.

Flow model for grease, one of the other objectives, is achieved by deriving a modified Reynolds equation using the Herschel-Bulkley model, assuming that grease to be a Bingham solid. Solution of this equation provides detailed information for contact pressures, film thickness and plug flow under isothermal conditions. This model is solved for thermal EHL conditions by combining flow equation with conservation of energy at a point in the lubricant film (energy equation). This is described in chapters 5 and 6.

Finally the estimation of power loss in grease lubricated bearings is achieved by quantifying the frictional force in the contact conjunction using asperity interaction model. Detailed descriptions of experimental measurement of the surface roughness parameters for interacting surfaces (ball and races) are also shown in chapter 6.

7.4 Critical Assessment and Suggestions for future work

Dearth of a complete numerical model to predict the characteristics of high speed precision spindles is one of the major problems faced by machining industry. This thesis has attempted to overcome this by suggesting a complete numerical model with contributions from the fields of dynamics and tribology. However, as in many research endeavours some aspects of work require further work.

Firstly, modern high speed spindles are equipped with built-in motors for better power transmission and balancing to achieve high-speed operations. However, built-in motors
introduce a fair amount of generated heat into the spindle system which further complicates the thermo-mechanical-dynamic behaviour. This extra heat produced by the built-in motor can introduce a thermal preload, which can change the system characteristics. This effect was not taken into account in bearing dynamics and represents a suggestion for future work. The approach in the thesis has been to obtain the load per ball during a typical orbital motion when the steady state limit cycle behaviour is reached. This typical ball load history is used to investigate the tribological contact conditions. This approach does not fully integrate system dynamics with tribology. A fully representative analysis would be transient in nature, where the load generated by integrated contact pressure distributions in ball-raceway contacts (rather than the dry contact assumption) should determine the force balance at any instance of time, thus the motions of the spindle. This is computationally almost unachievable because of long computations for many contacts in each step of time. Thus, an alternative can be the approach employed by Rahnejat and Gohar (1985) and Aini et al. (1990), where an extrapolated oil film thickness formula can be used in all ball-to-races contacts. This approach is regarded as quasi-static by Gohar and Rahnejat (2008). This approach, however, does not yield film shape and corresponding film thickness. This constitutes the reason why the alternative approach, based on the work of Kushwaha and Rahnejat (2004) was adopted in this thesis. With fullness of time the computing power will increase further and full transient solutions will become possible. This is already possible for single transient contacts such as cam-tappet conjunction (Kushwaha and Rahnejat, 2002).

It is very difficult to obtain rheological characteristics of grease as under shear in viscometers it regularly breaks down with increasing shear rate. Therefore, it was assumed that grease acts as a Bingham solid with a power law exponent value equal to unity. The assumption of grease being a Bingham solid throughout the contact is not valid as it has been shown by various researchers that the Herschel-Bulkley index varies from 0.7 to 1.4. Incorporation of this fact into the derivation of modified Reynolds equation for grease will provide a different equation as shown by Kauzlarich and Greenwood (1972) for one dimensional analysis.

Other interactions inside ball bearings apart from ball to races (by far the most important interaction) are balls to cage, and cage to raceway grooves. These interactions are quite
small when compared with ball to races interactions as noted by Gupta (1979). Most of the high precision bearings use phenolic cages, which act as lubricant reservoir. In case of grease lubricant the cage to ball interaction and excessive heat generation can cause the degradation of cage surface, churning and shearing. This is one of the major reasons of shortened life of satellites as the bearing failures leads to the failure of momentum wheels. It would be interesting to observe the pressure generation in these regions, requiring these other contacts to be modelled.

The surface profile is assumed to smooth, this has been the concept of analysis for lubricated contact from Dowson and Higginson (1958) onwards. However, since most engineering surfaces are quite rough compared to the lubricant film the effect of surface roughness cannot be ignored. Stochastic flow models by Patir and Cheng (1978), Zhu and Cheng (1988) and deterministic solutions by Venner and Lubrecht (1996), Xu and Sadeghi (1996), and Zhu and Ai (1997) have been reported. Such approaches may be used to extend the grease lubrication model presented in this thesis.
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References


Appendix 1

Experimental test rig setup

1 - High speed precision spindle
2 - Laser vibrometer
3 - Laser vibrometer controller
4 - Eddy Current probes
5 - Eddy Current driver
6 - Eddy Current band-pass filter
7 - PSU’s
8 - Charge amplifier
9 - Laptop with NI 6036 E Data card
10 - IMO drive
Appendix 2

LabView Front Panel