Advanced level mathematics syllabuses

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ADVANCED LEVEL MATHEMATICS SYLLABUSES

by

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A Master's Dissertation submitted in partial fulfilment of the requirements for the award of the degree of M.Sc. in Mathematical Education of the Loughborough University of Technology, January 1978.


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I should also like to thank those people who have helped me during the writing of this dissertation and the course to which it is the culmination. My colleagues at the Lilley and Stone School, Newark, have been patient and long-suffering over a period of seven terms. Thanks are also due to Dr. Knott for his advice. I reserve my deepest and most grateful thanks, however, for my father without whose encouragement and support this dissertation would never have been written.
The secondary school examination system, in England and Wales, has developed steadily since its inception last century, until it is now a monolithic structure resistant to change. The first chapter traces its history and in particular the School and Higher School Certificates and the General Certificate of Education which replaced them in 1951. Also considered in this chapter are the proposals for change to the 18+ examinations with the introduction of N and P levels to replace Advanced level.

Since the introduction of the Advanced level, there have been changes in the syllabus and introduction of new syllabuses, namely those of the two projects, Mathematics in Education and Industry and the Schools Mathematic Project, and the 'modern' syllabuses of the GCE boards. There have, however, been other suggestions for reform in the Mathematics syllabuses. Chapter 2 looks at possible changes that were suggested in the late 1960s. The details of these proposals are discussed.

The forty-one Mathematics syllabuses (listed in the appendix) at present available in England and Wales form the subject of the next four chapters. In the third chapter they are detailed and the possible single subject and double subjects that could be taken are discussed. Some boards are prepared to allow a single subject pass grade to those candidates who fail on both parts of a double subject and these are itemised, with the conditions that appertain.

Since it would be impossible to detail all the differences that can be found in forty-one syllabuses, the comparison has to be reduced to manageable proportions. In Chapter 4, some of the topics present in the single subject Mathematics syllabuses are considered and in Chapter 5 the Pure Mathematics content of the double subject is discussed. It must be emphasised that this analysis of the treatment given by the syllabuses to certain mathematical topics has only been carried out for a limited selection of topics and that the choice of topics included is a purely personal one.
In Chapter 6, the types of examination papers are discussed. Some papers have a large number of short questions, some have a smaller number of long questions and some have two sections with questions of each type. After a general discussion of the examination types, the number of questions to be answered and the limits on the choice of questions, the rest of the chapter looks at specific questions from recent papers.

The next chapter looks at the recent report of the Mathematics Syllabus Steering Group of the Schools Council 18+ research programme (dated April 1977 but published in October). The individual syllabuses of the seven commissioned groups are not scrutinised, but an overall view of the proposed package is presented.

In the final chapter there is an attempt to draw conclusions, but it is indicated that that appears to be an impossibility! It is clear that there ought to be a reduction in the number of syllabuses available at Advanced level. If the N and F proposals are not accepted, then serious consideration should be given by the examination boards to some rationalisation in the examination of Mathematics at 18+. 
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Bibliography
CHAPTER 1

The historical development of the secondary school examination system

Any member of the sixth-form in a present-day secondary school would find it hard to believe that the examination system, which appears to play an increasingly important part in his life, is of comparatively recent origin. At the beginning of the nineteenth century there were virtually no public written examinations, not even at undergraduate level. Degrees were awarded on public dissertation and oral examination until the Oxford Public Examinations Statute of 1800 and the Cambridge Classical Tripos of 1824 (although Cambridge had some form of written examination as early as 1722). Perhaps the biggest influence on the use of the written examination as the major assessment method in the university field was the establishment in 1836 of London University as an examining body.

Examinations in the elementary school system can be said to have been started following the Newcastle Commission (1856-61) which proposed the introduction of a searching examination, conducted by a competent body, of every child in each of those schools to which grants were paid. It was also proposed that the prospects and position of the teacher would be dependent, to a considerable extent, on the results of these examinations. The examination was originally based on 'the three Rs' but the Education Department gradually increased the number of grant-earning examinable subjects. This could be cited as the first instance of the stranglehold that examinations can have on the subjects to be taught in the schools since a report of one of Her Majesty's Inspectors of the time actually stated that "the studies of the classroom must be those wherein progress can be definitely measured by examination" (quoted by Morris).

The system lasted for about thirty years, though now it is largely condemned. It led to the creation of a national education system but the teachers had little or no training, and the school managers were mostly without previous experience. With the institution of the school boards in 1870 the position was little better. Most board members were amateurs and the financial situation of the boards did not allow them to pay for efficient technical advice.
It was only as the educational system matured that the state gradually withdrew the insistence on individual examinations, in favour of general inspection, and in 1902 publicly elected authorities were established to supervise schools.

The secondary school system shows the same pattern of intervention by the state, followed, at first, by close supervision and then by gradual withdrawal in favour of some other agent charged with supervisory duties.

The state had very little financial commitment in aiding or providing schools with connections with the universities and the inspectorate had no right of entry into the grammar schools. It was for these reasons that the first moves in providing secondary school examinations came from 'private' bodies. In 1837 the London Matriculation Examination was introduced to determine the admission of candidates for a degree course at the University.

The report of the Oxford University Commission of 1850 claimed that it was the introduction (into the university) of properly conducted examinations that "first raised the studies of the University from their abject state" (quoted by Morris). The commissioners realised that the standards within the university depended upon the quality of the entrants. Consequently they decided to use examinations as a means of improving standards in those schools from which they took their undergraduate students. During the first three years, the examinations were for "students not members of the university". The schools took no part in the examination which was held in some public hall hired for the occasion. The premises of the university were thought of as being extended to include those halls in different parts of the country where the examination was being held, hence its name 'locals'.

Both Oxford and Cambridge established such 'local' examinations. Cambridge locals were opened to girls on an experimental basis in 1863 and in general in 1865. At about the same time the College of Preceptors started their external examination for private schools of a secondary nature.
In 1868 the Taunton Commission criticised both the Oxford and the Cambridge locals as being too difficult to test the majority of the pupils (for whom, of course, the examinations were not intended). The universities instituted a system of internal school examinations as a means of helping and guiding individual schools but this was short-lived. The Oxford and Cambridge Joint Board was established in 1873 to conduct examinations for university entrance and for the award of a school-leaving certificate.

The London University Extension Board was set up in 1902 both for examinations (School-leaving certificate and Matriculation) and for inspection of secondary schools. The regulations for its school-leaving certificate stipulated that the examinations could be held only in schools that were under inspection by the board and that no pupil was eligible to sit for the certificate unless he had been under instruction in such a school for the two preceding years. In 1905 the Oxford and Cambridge Joint Board restricted their examinations to pupils attending a school inspected and approved by it. The Northern Universities Joint Matriculation Board (which had been founded in 1903) also instituted a system of inspection in 1910.

By 1908 eight different examination boards were in existence. The Charity Commissioners made external examinations obligatory for the reformed grammar schools, thus ensuring an expansion of the examination system.

Initially the Universities set examinations without reference to the schools and with a particular interest in establishing their own matriculation requirements. Indeed, it could be said that the examination boards came into existence with the very intention of influencing schools as the preceding paragraphs show.

In the period from 1904 to 1914 there was constant friction between the Board of Education on the one hand and the schools, the local education authorities and the examination boards on the other. In 1911 the Consultative Committee of the Board of Education was asked to investigate
the whole question of external examinations for secondary schools because of the increasing confusion owing to the multiplicity of examinations and examining boards. In 1917 the Board of Education officially recognised the examining boards as the Secondary School examiners but asserted control over the curriculum by laying down subject groupings and by approving subjects. It reserved the right to control standards by determining examination levels, by fixing the pass mark and by specifying the age entry. It also established the Secondary Schools Examination Council which was later (1964) to become the Schools Council. The examination boards retained the right to draw up the examination syllabuses and to set and mark the papers although they were subject to scrutiny of the Board of Education's panel of investigators.

Two levels of examination were established, the School Certificate (taken at about sixteen) and the Higher School Certificate (taken at about eighteen). The Higher School Certificate had within it two levels of subject, Principal (or Main) and subsidiary. Both the School and the Higher School certificates were group certificates. To obtain a School Certificate passes in five subjects were required, and for matriculation purposes these had to be from particular groups of subjects. To obtain a Higher School Certificate a candidate needed passes in three principal or two principal and two subsidiary subjects. For less than this a candidate could be awarded a Letter of Success.

This pattern was followed until the immediate post-war years, but it had been thought for some time previously that there were serious shortcomings in the system. The view was widely held that the requirement in the School Certificate for a pupil to pass in a group of subjects restricted the initiative of the teachers and, furthermore, that the artificial division between fifth-form and sixth-form work hindered the development of a unified secondary course. Although the Board of Education had emphasised that "the examination should follow the curriculum and not determine it", this was not felt to be the case, partly due to the fact that the School Certificate examination was being made to serve two purposes.

On the one hand, its function was that of a fifth-form examination for
those about to leave school, but on the other, since the universities accepted the certificate under certain conditions as providing exemption from matriculation, it was also considered a university entrance examination. For this and other reasons, as the Spens report showed, the examinations reacted upon the schools in a way that those who framed the original regulations had never contemplated. The School Certificate examination began to control the curriculum of the schools. The requirements for exemption from matriculation narrowed the choice of subjects which were already restricted by the group system. In view of this the Spens report considered that a Matriculation Certificate should no longer be awarded on the results of the School Certificate examination. This recommendation was accepted but the outbreak of war interfered with the amendments and for a time the original regulations were in force as a special concession.

In many ways the examinations had been a valuable influence in raising the general standard of attainment in each of the school subjects, but this had been offset by the narrowing effects of the group system and the tendency to regard as inferior those subjects which were not recognised for matriculation requirements. In 1941 a committee, chaired by Sir Cyril Norwood, was set up to consider changes in the secondary school curriculum and the problems of school examinations. Among other recommendations was one that a change in the School Certificate should be made, so as to allow the examination to be an internal one, conducted by the schools on syllabuses and papers set by the teachers themselves. Such a drastic reorganisation of the examination system called for a transitional period during which the School Certificate examination would be carried out by the existing examining bodies but supervised by a Standing Committee of eight teachers, four members of the inspectorate acting as assessors. It was suggested that this transitional period should be seven years, at the end of which the whole problem was to be reviewed to decide whether conditions were such that
a change to a wholly internal examination would be possible.

For the eighteen-plus candidates the Norwood Committee thought that a school-leaving examination should be conducted, twice a year, to meet the requirements of university entrance and of entry to the professions. This examination should be taken in those subjects that pupils require for their own particular purpose. The Higher School Certificate would under this recommendation, be abolished.

The report of the Secondary Schools Examination Council in 1947 made recommendations to modify and extend those of the Norwood report. In particular, it recommended the establishment of systematic internal examinations based on the courses of study within a school. External examinations should serve to maintain national standards. Examinations for those pupils who wished to compete for university scholarships, or to secure exemption from professional examinations, should be taken as late as possible in their school career. The examinations should also be available to those who had ceased to be full-time pupils of a secondary school.

An examination at three levels, Ordinary, Advanced and Scholarship, was to be available to candidates who were at least sixteen on the first of September of that year. Ordinary level papers were to provide a reasonable test for pupils who had studied a subject as part of a wide and general course up to the age of sixteen, or for those who had studied the subject in a non-specialist way in the sixth-form. The advanced level papers were to be for candidates who had taken the subject as a specialist study for two years in the sixth-form. Scholarship level papers were to provide an opportunity for specially gifted pupils to show distinctive merit, and State and Local scholarships were to be awarded on their performance on papers in two subjects at this level. The group system was to be abolished and all subjects at these three levels were to be optional. Candidates passing in at least one subject were to be awarded a General Certificate of Education on which was recorded the subject(s)
and the level(s) at which they had satisfied the examiners. The pass in
the Ordinary level papers was to be approximately equal to a credit standard
in the School Certificate, and a pass at Advanced level was to be roughly
equivalent to a pass in a Principal subject in the Higher School Certificate.

The Ministry of Education approved the main recommendations of the
report and announced on the 26th of April 1948 that the new system would be
introduced by stages. The first examinations under the new scheme were
held in 1951. Originally the result of the examination in any one subject
was either pass or fail but in 1953 a pass at Distinction was allowed in
Advanced level. At the same time as the General Certificate of Education
was introduced, there was established a new examining body. This was the
Associated Examining Board, the only English board not associated with a
university or group of universities. It was originally set up to provide
Ordinary level syllabuses for those subjects particularly suited to Secondary
Modern and Technical College students. It still provides a wide range of
practical and general subjects such as Building Practice, Photography and
European Studies, although it also provides the full range of 'academic'
subjects at all three levels.

In 1960, grades were introduced for Advanced level subjects. These
were A (distinction) for the top ten per cent, B (good) for the next
fifteen per cent, and C (better than average), D (average) and E (pass) for
a further forty-five per cent. This meant that approximately thirty per
cent failed. Those who narrowly missed a pass at Advanced level could be
awarded an Ordinary level pass in that subject. In 1962, the State
Scholarship was abolished and the Scholarship papers became Special papers.
There are two grades of pass at Special level, provided that the candidate
has satisfied the examiners in the Advanced level examination in the same
subject at the same sitting of the examination.

In 1965 the Certificate of Secondary Education was introduced with its
top grade being considered "equal to a pass at Ordinary Level". Inevitably
this has led to difficulties in deciding for which certificate a pupil
should be entered in any particular subject, particularly in the comprehensive schools where classes leading towards both examinations are likely to be provided in all except the minority subjects. Thus we have the present proposals for a joint sixteen-plus examination.

The situation in the sixth-form is also complicated both by the duality of the sixteen-plus examinations and by the wider range of abilities and requirements of the 'new sixth'. This has led to one change in the levels of the General Certificate of Education and to several proposals for more drastic change. The examining boards have introduced a wider variety of Alternative Ordinary subjects. These are "designed for candidates of greater maturity than is normally expected at Ordinary level". One of the proposed changes to the examination system for the sixth-form is the Certificate of Extended Education to be run by the regional boards already conducting the Certificate of Secondary Education. The outcome of this proposal is still uncertain. The examining boards are running feasibility studies in several subjects but the proposal for a Certificate of Extended Education has not yet been officially accepted.

The Schools Council have made two further suggestions. In 1970 it made the original recommendation for Qualifying and Further examinations for the first year and the second year in the sixth form, respectively. This met with such adverse criticism that it was abandoned in favour of the Normal and Further examinations both to be taken in the second year of the sixth-form. This proposal was made in 1972-3 in the Schools Council's Working Papers number 45, 16-19: Growth and Response, 1. Curricular Bases, number 46, 16-19: Growth and Response, 2. Examination Structure, and number 47, Preparation for Degree Courses.

The basic proposal was for a two-level syllabus and examination structure. It was envisaged that the majority of candidates would take five subjects, not more than two of which would be studied to Further level. Thus the Normal level would provide breadth of study and the Further level
would provide depth. It was to be a single subject examination at both Normal and Further level, there being no compulsion to study five subjects and no compulsory grouping. Both the Normal and Further levels were considered to involve two year courses beyond the standard equivalent to Ordinary level grade C (the old pass standard). It was suggested that the study time for a Normal level subject should be half that of an Advanced level subject and that for a Further level should be three-quarters.

The Schools Council set up sixteen syllabus steering groups to cover the majority of subject areas at present studied at Advanced level. Each steering group considered the implications of the two-tier system, discussed ways in which syllabuses and assessment methods could be organised and prepared briefs to commission other groups to produce syllabuses, assessment schemes and specimen papers. The report of the Mathematics Steering Group was published in September 1977 and forms the subject matter of chapter seven.

The reports of the sixteen groups are to be the subject of public debate. Early in 1978 the Schools Council will publish its Examination Bulletin number 38, Examinations at 18+: Resource Implications of an N and F Curriculum and Examination Structure and later in 1978 its Working Paper number 60, Examinations at 18+: the N and F Studies, which will be an evaluation of the whole N and F programme by the Joint Examination Sub-committee of the Council. Not until July 1979, at the earliest, will the Governing Council of the Schools Council be invited to decide whether or not to make a recommendation to the Secretary of State concerning this matter.
CHAPTER 2

Suggestions made in the sixties for the reform of Advanced level syllabuses in Mathematics

The wide variety of Advanced level Mathematics syllabuses reflects the problem of providing an Advanced level course and qualification for students with differing needs. Mathematics is studied in the sixth form for a number of different reasons. For some students it is their major academic interest, but the majority who take an Advanced level Mathematics course do so because it renders a service for other subjects. There are now more subjects which require Mathematics beyond Ordinary level; the nature of the Mathematics needed varies from subject to subject adding to the difficulty of providing a single examination syllabus, and to the difficulty of teaching the subject.

Schools need to provide mathematical courses appropriate to prospective specialist mathematicians as well as scientists, engineers, social scientists and economists. The total class, except in the large sixth-form colleges, is likely to be of a size that the students will have to be taught together as one group, with a common course of study. The examination syllabuses tend to reflect this, but unfortunately there no longer is a common core of what might be called Six-form Mathematics.

In 1957 and again in 1960, the Assistant Masters Association said of Advanced level Mathematics: "This examination is still being conducted by a number of separate boards and it is disturbing to find that the syllabuses of these examining boards are so different". At that time the Welsh Joint Education Committee and the Durham University Matriculation and School Examinations Board (ceased after the summer of 1964) set papers on a "more extended syllabus than those of the London board; while they in turn have a less extensive syllabus than does the Oxford and Cambridge Schools Examination Board". They hoped that the Secondary Schools Examination Council would press for greater uniformity in the syllabuses offered. They
realised that the varying standards might not affect a school in isolation since it would be likely to be entering pupils for the examinations of only one board, but pointed out that difficulties would be experienced by pupils transferring from one school to another during their sixth-form career. This was written before the introduction of the so-called 'Modern Mathematics' syllabuses which of course aggravated that particular problem. They proposed that there should be available three separate syllabuses. The first should be a 'double subject' syllabus for the specialist mathematician. Those who were hoping to apply their mathematical knowledge and skills to some other subject should be provided with a 'single subject' syllabus and Pure Mathematics was recommended for this purpose. The third type of syllabus should be available for those who needed Physics as well as Mathematics but not as two separate Advanced level subjects. Such an examination, in Physics-and-Mathematics is now provided by the Oxford Delegacy of Local Examinations on an inter-board basis, having been introduced in 1971.

In 1967-8, W. L. James (at that time Schools Fellow at the University of Newcastle) carried out a survey on the interdependence of sixth-form and university mathematics courses. In an article in Mathematics Teaching he made certain remarks on the difficulty of comparing the Advanced level subjects at that time. "Let no one start on such a task unless he has much time and patience at his disposal!" As he points out, the same phrase on two different syllabuses does not necessarily mean the same thing on each. For this reason some boards give a fuller indication of the treatment required on any particular topic or give explanatory notes. To compare adequately the different interpretations of a particular phrase in two or more syllabuses it is necessary to compare the examination papers.

James proposed certain reforms for the Advanced level Mathematics syllabuses, starting from those then available and indicating how they could be modified. He suggested a compulsory part of the syllabus which would provide the major portion of the work covered and examined. The topics to be included within that part of the syllabus would be very clearly defined.
The remainder of the work to be covered would either be on optional topics arranged in "cohesive groups", or on an "undefined" section allowing schools to provide their own choice of additional material or projects and investigations suitable for examination by essay-type question. Under this plan he suggested six national syllabuses broadly comparable to the existing syllabuses. These were the three single subjects, Pure Mathematics, Pure Mathematics with Mechanics and Pure Mathematics with Statistics, a double subject and two 'modernised' syllabuses, Mathematics and Further Mathematics. He obtained a core of material, common to the various boards, which was sufficient to form the compulsory part of each syllabus. The remaining parts of the existing syllabuses he arranged in coherent groups, so as to form the optional parts of the syllabus, or rejected if insufficiently important. He also included some material from the 'modern' syllabuses as optional topics on the traditional ones. In this way he hoped that the variation from board to board might be eliminated.

A more fundamental alteration to the syllabuses was suggested by M. Bruckheimer and D. E. Mansfield in 1967 also in an article in Mathematics Teaching. In this they put forward the idea of a 'dynamic' syllabus which they defined as one in which the subject material changes every year or two. The changes need not be major ones, and the rate of change need not be too large, but change should be accepted as a necessary part of the structure. This, they felt, would overcome the possibility of 'ossification' in the syllabus, whereby the restriction on syllabus content leads to an eventual exhaustion of worthwhile examination questions and to the setting of standard questions of an increasingly manipulative character. Bruckheimer and Mansfield in fact doubted whether this suggestion of change could be carried out forever but thought it would, if introduced, inevitably lead to improvement in the syllabuses then available and also to the introduction of the 'modern' topics and/or approach which at that time occurred in only a limited way at Advanced level. Their article finished with the following sentence which even today is appropriate in view of the N and F proposals. "It is time for change -
almost any change, but preferably one which will not encourage us to make
the same mistake again, and, undoubtedly, in spite of our present
enthusiasm for lots of new syllabus (sic), unless we write syllabuses
which are not only new in content, but also new in spirit, we shall be
back virtually where we started". 
Chapter 3

The various Advanced level Mathematics syllabuses at present available in England and Wales.

At present there are forty-one different Mathematics syllabuses. This excludes Statistics as a full Advanced level subject and also Computer Science but includes Pure Mathematics with Statistics and Pure Mathematics with Computations. It also excludes the syllabus in Physics and Mathematics mentioned in the previous chapter.

These forty-one syllabuses are provided by the eight English and Welsh GCE boards and by the Mathematics in Education and Industry Project (MEI) and the School Mathematics Project (SMP). The examinations on the two projects are open to all, although conducted by the Oxford and Cambridge Schools Examination board (O & C), and, on the withdrawal of the Midlands Mathematics Experiment syllabus, the examinations of the Joint Matriculation Board (JMB) for Mathematics (Syllabus B) and Further Mathematics (Syllabus B) have been made available to candidates from other GCE boards in accordance with the normal inter-board project examination procedure. Apart from these three cases the sixth-form is restricted to the GCE board(s) with which the school is registered as a centre. So in practice the number of syllabuses available to any one student is reduced but may still provide difficulty in making a choice.

In the case of double-subject Mathematics the boards provide two possibilities although on only one board, Cambridge, are both possibilities available. The two are Mathematics and Further Mathematics or Pure Mathematics and Applied Mathematics. (As I have excluded Statistics from this study, I must also exclude, as a possible double subject combination, Statistics and Pure Mathematics which would be available on three boards). Mathematics and Further Mathematics is provided by JMB (two syllabuses), Oxford, SMP, Cambridge ('modern' syllabus only), O & C, and by London (two syllabuses). Pure Mathematics and Applied
Mathematics is available from the Associated Examining Board (AEB), Cambridge, MEI, the Welsh Joint Education Committee (WJEC) and from the Southern Universities Joint Board (SUJB). London also provides Higher Mathematics which could be taken in combination with any other Mathematics subject although it is more suitable for a candidate who already has Mathematics and Further Mathematics and for some reason is spending a further year in the sixth form.

Thus some choice is provided by JMB with its two syllabuses, by London in a similar way and by Cambridge where there is the 'traditional' Pure Mathematics and Applied Mathematics and also a 'modern' Mathematics and Further Mathematics. Some boards are more helpful to the weaker candidates taking the double subject. For example a candidate taking Mathematics and Further Mathematics with Oxford and who fails in both subjects can be considered for a pass in Pure Mathematics if the work in the Pure Mathematics questions is adequate. This is possible since the Pure Mathematics syllabus is precisely the Pure parts of the Mathematics and Further Mathematics syllabuses and its examination papers are the paper 1 of each of Mathematics and Further Mathematics. In a similar way those candidates taking Pure Mathematics and Applied Mathematics with AEB or SUJB may be given a pass in Mathematics because of the common papers involved. Those failing both Pure Mathematics and Applied Mathematics of Cambridge may be awarded a pass in Mathematics but the highest grade given would be D.

If only one Mathematics Advanced level is required most boards provide a choice. The JMB provides Mathematics (two syllabuses), Pure Mathematics and Pure Mathematics with Statistics. Oxford has two possibilities, Mathematics and Pure Mathematics. There is a much wider selection provided by the AEB; these are, Mathematics (Pure), Mathematics (Applied), Mathematics (Alternative syllabus), Mathematics (Pure and Applied), Mathematics (Pure with Statistics) and Mathematics (Pure with Computations).
These have papers in common and hence common parts to the syllabus. The WJEC provides two Pure Mathematics syllabuses and one called Mathematics in which there is a choice of second paper, making it virtually Pure and Applied or Pure with Statistics. London provides two Mathematics syllabuses as well as Pure Mathematics and Pure Mathematics with Statistics. Cambridge has two Mathematics syllabuses and also the separate Pure Mathematics and Applied Mathematics syllabuses. The MEI project has Mathematics as well as Pure Mathematics and Applied Mathematics. Although its publication refers to the last two together as 'double-subject Mathematics', it implies that each could be taken as a single subject. So the non-specialist mathematician, or his teacher, has the choice between Mathematics (Pure and Applied or Pure with Statistics) and Pure Mathematics. (Although Applied Mathematics is available as a single subject it is unlikely to be taken as the only Mathematics subject by a first time candidate). Only O & C and SMP deny him the choice and provide only the one subject Mathematics.

In the next two chapters I shall look at the difference in content of the single subject Mathematics (Chapter 4) and at the difference in the Pure Mathematics content in the various types of double subject syllabuses (Chapter 5). In Chapter 6 I hope to consider the methods of examination and the interpretation of the some parts of the single subject syllabuses as indicated by either the 1977 or the 1976 papers (where available).
CHAPTER 4

The content of the single subject Mathematics syllabuses for 1978

There are seventeen syllabuses to be considered here. I am excluding Pure Mathematics with Computations (AEB) but including Pure Mathematics with Statistics (AEB, JMB and London). In this subject the Pure Mathematics is examined on paper one and the Statistics provides the content of the second paper. In the Mathematics of WJEC there are alternative second papers; the first paper is Pure Mathematics and the second paper to be taken is either Statistics or Mechanics (together with differential equations and complex numbers). The AEB alternative syllabus provides three options on paper two. These are further Pure, Mechanics, and Statistics. A candidate could attempt questions on one or two of these options. In the Oxford syllabus the second paper is on Applied Mathematics and allows for Mechanics or Statistics or both. On both of the O & C papers questions on Pure and Applied Mathematics occur. The Applied Mathematics is split into separate sections on Mechanics and Statistics, and a candidate, if he so wished, could confine himself merely to one of those sections, although his choice of question would necessarily be very severely limited on the Pure part of each paper. SUJB has a Pure Mathematics paper and an Applied Mathematics paper which contains both Mechanics and Statistics. Again there is sufficient choice for only one of these to be studied and probably time would make this essential since both sections of this part of the syllabus are very full.

Cambridge has two Mathematics syllabuses. In syllabus A the first paper is Pure Mathematics and the second paper contains both Pure and Applied. Those parts of the Pure Mathematics to be examined in paper two are indicated in the syllabus and four questions will be provided although not more than two can be attempted. The Applied Mathematics is divided into Mechanics and Statistics and seven questions provided on each. Since the total number to be attempted cannot exceed seven, it would be possible, although inadvisable, to study only one of the
two applications as long as the extra Pure Mathematics had been covered. In syllabus B each paper has two sections and questions from both sections have to be attempted. The first section is on the compulsory part of the syllabus which is Pure Mathematics. The second section consists of questions on five options of which at least two must be taken. These are Particle Mechanics, Probability and Statistics, Numerical Analysis and Computation, Algebraic Structures and finally Analysis and Differential Equations.

In JMB syllabus A there is no Statistics. The first paper provides questions on the Pure Mathematics of the syllabus and the second paper is basically Mechanics although occasionally there is a differential equation question not specifically mechanical. Syllabus B contains both Dynamics and Statistics. There is no artificial division into Pure and Applied Mathematics on the two papers, any part of the syllabus can be examined on either paper and the questions may link together different parts of the syllabus. On each of the London syllabus C papers both Pure and Applied Mathematics appear, but of the non-objective papers of syllabus D, paper two is on Pure Mathematics only and paper three is mainly on Applied Mathematics.

SMP does not divide the subject into Pure and Applied Mathematics and, unlike other syllabuses, it contains an application other than the usual Mechanics and Statistics. This is electricity - Ohm's law, direct current networks as applications of simultaneous linear equations, power and maximum power transfer. This syllabus also contains the construction of simple flow diagrams including conditional jumps. MEI has a Pure Mathematics paper and an Applied Mathematics paper. In the latter there is a compulsory section on probability and then two other sections. The first of these is Applied Calculus and Mathematical Physics and the second is Statistics and Further Probability. The number of questions provided allows a candidate to answer from only one of these two sections although there is no compulsion for him to do so.
In order to compare the content of these syllabuses I shall look first at the compulsory Pure sections of the papers, then at the Mechanics and finally at the Statistics. To do this I shall take certain topics and see if they are included in the syllabus and, if so, to what extent. In a later chapter I shall consider the actual questions on these topics to see in what way the examiners have interpreted the syllabuses.

**Complex numbers**

This topic is not included in the SUJB syllabus nor in Cambridge syllabus A. In WJEC it occurs only on paper two, so a candidate taking papers one and three (Pure Mathematics and Statistics) would not cover this topic. It is also missing from the Pure Mathematics with Statistics of both JMB and AEB. The AEB also exclude this topic from Pure and Applied. All other syllabuses include some form of complex number work but to varying degree. The exponential form of a complex number occurs in option E of Cambridge syllabus B which mentions its 'informal' use. This option also includes the proof of de Moivre's theorem for positive integral index. In both London syllabus C and the AEB alternative syllabus both the exponential form and de Moivre's theorem for positive integral index are included. De Moivre's theorem is also included on the syllabuses of Oxford (Positive integral index), O & C (integral index) and MEI (positive or negative integral index). All the remaining syllabuses which include complex numbers do so in a limited way, i.e. sums, products, quotients, modulus and argument, the Argand diagram with representation by point or vector, conjugate complex numbers including the roots of a polynomial with real coefficients and, on some syllabuses, the cube roots of unity.

**Methods of Integration**

**Integration by parts**

Surprisingly, this method of integration is specifically excluded from Cambridge syllabus A. Also, it is not mentioned by AEB except in the alternative syllabus.
Partial fractions

Most syllabuses specify the use of partial fractions. In some cases the type of fractions involved is limited. Oxford includes the possibility of one repeated linear factor in the denominator, but SMP states that linear unrepeated factors only will be used. London syllabus D allows only two factors of which at least one is linear but the other could be of the form $ax^2 + b$. O & C assumes knowledge of the technique of partial fractions for the purpose of manipulation but says that it will not be "the subject of artificially complicated questions". Oxford, although including partial fractions in the syllabus, does not specifically mention their use in integration while the WJEC mentions this use and no other. MEI, on the other hand, specifies the use of partial fractions both for integration and in expansions.

Approximate integration

This is not included in WJEC, SUJB, JMB syllabus A, nor in Cambridge syllabus A. In Oxford it is included only in one of the five options. In the Cambridge syllabus B the trapezium rule is mentioned in the compulsory part of the syllabus, and both this and Simpson's rule are contained in option C. On all other syllabuses both methods of numerical integration are included.

Differential Equations

Apart from Cambridge syllabus A, where differential equations appear on paper two, and WJEC, where those taking the Statistics paper will not meet them, all candidates will need to be able to solve first order differential equations with the variables separable. First order linear equations are included in London syllabus C and also on MEI, WJEC and in option 1 of the alternative AEB syllabus, as well as in option E of Cambridge syllabus B. Simple second order equations of standard type occur in Oxford, MEI and JMB syllabus B, while linear differential equations with constant coefficients appear in option E of Cambridge syllabus B and on option 1 of the AEB alternative syllabus.
Expansions

Most syllabuses include the infinite geometric series and the binomial, exponential and logarithmic expansions, even if only in a very limited way, although London syllabus C does not mention the last two. The trigonometrical expansions are specified in London syllabus D, Cambridge syllabus A and in AEB, although not in the alternative syllabus. O & C name Taylor's and Maclaurin's series, whereas MEI asks only for Maclaurin's and SMP for Taylor's. London syllabus D names them as a means of obtaining other expansions. Cambridge syllabus B only refers to expansions in option E, apart from the use of the binomial theorem and the infinite geometric series which both appear in the compulsory part of the syllabus. WJEC only mention three simple finite series and the binomial theorem for positive integral index.

Vector Algebra

This topic is included in SMP, MEI, Cambridge syllabus B, London, O & C, JMB syllabus B, AEB alternative syllabus and is optional on the Oxford syllabus. For candidates sitting the WJEC examination this topic appears on paper two, so those taking the Statistics option will not meet it. For most of the syllabuses that include this work, it is confined to simple vector algebra in two and three dimensions, including the use of unit vectors and the scalar product, but excluding the vector product. The ratio theorem is also included, except by London, O & C and AEB, together with the concept of a position vector. The vector equation of a straight line in the form \( \mathbf{r} = \mathbf{a} + t\mathbf{b} \) is included by SMP and JMB syllabus B. These two, together with London syllabus D, also include the differentiation of vectors and this appears in one of the options of the MEI syllabus.

Matrix Algebra

There is considerable variety among the syllabuses that include any work on matrices. JMB syllabus B considers only 2x2 matrices, the
transpose, inverse and determinant of such a matrix, singular matrices and applications, of which none are specified. O & C mentions 2x2 and 3x3 matrices, and matrices associated with transformations of a plane. MEI includes determinants of order 2 and 3 and their use in the solution of simultaneous equations. For matrix algebra it asks for the machinery of addition and multiplication, applications to mappings, the transpose and the inverse matrix including the evaluation of the inverse of non-singular 2x2 and 3x3 matrices. SMP includes square matrices, matrices of plane transformations, elementary row operations and the reduction of 3x3 matrices to echelon and diagonal form. The inverse matrix is included but not its formal expression in terms of co-factors. Cambridge syllabus B and London syllabus C both have a limited amount of matrix work limited to the column matrix as a position vector, and matrices associated with linear transformations with matrix multiplication and the inverse matrix being considered with respect to combinations of transformations and the inverse transformation. On the Oxford syllabus Matrix work is optional but covered more fully. As well as the geometrical transformations represented by 2x2 and 3x3 matrices, this syllabus also includes eigen-vectors and eigen-values for 2x2 matrices, although determinants are not mentioned. In option 1 of the AEB alternative syllabus matrices are applied to Markov processes.

Co-ordinate Geometry

Nearly all syllabuses include some co-ordinate geometry but there is considerable variation in both the breadth and depth of study. O & C state that knowledge of the straight line and the circle will be assumed but that they will not be made the subject of "artificially complicated questions". Surprisingly there is no co-ordinate geometry on the SMP syllabus although knowledge of polar co-ordinates is expected. This lack of co-ordinate geometry may be because SMP includes the vector equations of both lines and planes and so much geometrical work can be done by vector methods. This is also true of the alternative syllabus of the AEB which
specifically includes the "application of vector methods to simple geometric theorems and problems". London syllabus C mentions the use of both parametric and polar forms. WJEC includes the straight line and parametric equations. Most of the remainder have the straight line and the circle. The parabola $y^2 = 4ax$ is included in AEB, although not on the alternative syllabus, on Cambridge syllabus A, on JMB syllabus A, on London syllabus D and on O & C and SUJB. On the Oxford syllabus the co-ordinate geometry is optional and as well as the straight line, the circle and the parabola it includes the ellipse and the hyperbola.

**Mechanics**

JMB syllabus B contains no Statics. As there is mainly a vector approach to the mechanics, and the vector product is not part of the vector algebra of this syllabus, it does not include the moment of a force. Newton's third law is also not part of this syllabus, which precludes impact as a topic although this is included in JMB syllabus A. Similarly SMP includes very little statics - basically composition and resolution of coplanar forces. London syllabus C excludes centres of gravity and also Hooke's law. Cambridge syllabus B, where the mechanics is optional, also excludes Hooke's law, as does WJEC and the alternative syllabus of the AEB. Frameworks are included on the Oxford syllabus, excluding Bow's notation, and on AEB although not on the alternative syllabus.

In the dynamics the topics showing the greatest variety from syllabus to syllabus are direct impact and rotation. Direct impact is excluded from both the Cambridge syllabuses, from the JMB syllabus B, and from O & C, WJEC, and SMP. Moments of inertia and rotation are included only by Oxford, O & C and MEI and in each of these three cases the topic could be considered optional since the amount of mechanics covered could be limited in favour of Statistics. The remainder of the dynamics syllabus, with minor differences, is common to all the syllabuses.
Probability

This topic is only excluded from two syllabuses. These are JMB syllabus A and the Pure and Applied syllabus of the AEB. The other syllabuses usually include conditional probability, mutually exclusive events and independent events. The last two, however, are not mentioned on the AEB alternative syllabus, London syllabus D and MEI, but what are we to take "elementary probability theory" to mean?

Statistics

Obviously the Pure Mathematics with Statistics syllabuses contain a much larger amount of Statistics than do those which have some compulsory Mechanics, as do those which allow a choice between Mechanics and Statistics. There is no Statistics at all on JMB syllabus A, London syllabus D and the Pure and Applied syllabus of the AEB. It is optional on Cambridge, Oxford, O & C, SUJB and WEJC, and basically optional on MEI, although both the binomial and normal distributions are contained in the compulsory probability section of paper two.

The topics exhibiting variety are as follows. Expectation and variance is excluded from London syllabus C. The Poisson distribution is excluded from the alternative syllabus of the AEB, from JMB syllabus B, from London syllabus C and from SMP. The normal distribution is not included on JMB syllabus B or on the alternative syllabus of AEB. Scatter diagrams and simple ideas of correlation and regression are excluded from Cambridge, JMB syllabus B, London syllabus C, WJEC and SMP.

The remaining statistical topics are included only by the Pure Mathematics with Statistics (PWS) syllabuses and those for which statistics could be an alternative to Mechanics. All these syllabuses include simple sampling, though AEB (PWS) takes sampling theory much further than the others. Confidence limits and significance tests are included by all except AEB (PWS). The calculation of regression lines is included by O & C, JMB (PWS), London (PWS), WJEC, AEB (PWS) and MEI, and these syllabuses, with the exception of WJEC, also include the correlation coefficient.
This concludes the examination of the seventeen syllabuses available for a single Advanced level in either Mathematics or Pure Mathematics with Statistics.
Comparison of the Pure Mathematics on the double-subject Mathematics syllabus of 1978.

To be considered here are Pure Mathematics and Applied Mathematics, as provided by AEB, WJEC (alternative Pure papers), SUJB, Cambridge and MEI, and Mathematics and Further Mathematics, as provided by London (two syllabuses), Oxford, JMB (two syllabuses), O & C, Cambridge and SMP.

The definition of Applied Mathematics varies from board to board. AEB and Cambridge only include Mechanics, WJEC refers to vector algebra, differential equations, numerical analysis and probability as well, and SUJB includes Statistics. MEI has a compulsory probability section but "the Statistics and Mathematical Physics sections are regarded as alternatives although there will be no restriction in the choice of questions". Because of this difference in the interpretation of Applied Mathematics, I wish, in this chapter, to confine myself to a comparison of the Pure Mathematics content of the two types of double subject syllabus.

As in the previous chapter, the method I shall adopt is to take certain topics, not occurring on the single subject syllabuses, and see which boards include these in their double subject syllabus.

Euclidean Geometry

Euclid has not gone! He is alive and well and living at Cambridge! This Pure Mathematics syllabus includes the geometry of the triangle and its associated points and circles, as well as Apollonius's circle. It also includes elementary ideas of solid geometry with applications to spheres.

Applications of the definite integral

The single subject syllabuses include calculation of area and of the volume of solids of revolution and some also include the calculation of mean values. The double subject syllabuses, except Oxford, take this work further. The calculation of the length of an arc is included on all the syllabuses except JMB syllabus B, SUJB and MEI. Curvature is
included on JMB syllabus A, on 0 & C, on SMP and on AEB, and the area of the surface of a solid of revolution is required for these four syllabuses and also for Cambridge. MEI includes the use of integration to find moments of inertia, on its Applied Mathematics paper, and SUJB includes areas using polar co-ordinates.

Further Expansions, Convergence and Divergence

Taylor's series is specifically mentioned on the MEI syllabus and on JMB syllabus B, Maclaurin's on the Pure Mathematics syllabus and on London syllabus C and both of them are included in 0 & C, Oxford, AEB and SUJB. The expansions for sin x and cos x are mentioned on the JMB syllabus A, on both London syllabuses, Cambridge and AEB, and those for sinh x and cosh x on JMB syllabus A and on AEB. Ideas of convergence and divergence are required for JMB syllabus B, London syllabus C, SMP, Oxford and AEB.

Hyperbolic functions

This topic is missing from the syllabuses of O & C, MEI and SUJB and also from one of the two Pure Mathematics syllabuses of the WJEC; it is an optional topic on each of the Cambridge Mathematics and Further Mathematics syllabuses.

Differential Equations

There is a remarkable degree of agreement here. AEB restricts the extension of this topic to first order equations solved by means of an integrating factor. All the other syllabuses go as far as linear second order equations with constant coefficients although WJEC and MEI include this on their Applied Mathematics papers.

Roots of equations

The majority of the syllabuses include the relation between the roots and the coefficients of a polynomial equation. This is, however, omitted from SMP, from London syllabus C, from AEB and from the second Pure Mathematics syllabus of the WJEC. Only JMB specifically asks for knowledge
asks for knowledge of the occurrence of non-real roots in conjugate pairs, in the case of polynomial equations with real coefficients, and this occurs on each of its two Further Mathematics syllabuses.

**Complex Numbers**

SUJB does not include any complex number work in its single subject syllabus and only the minimum amount in its double subject. Cambridge Pure Mathematics also has a limited amount of complex number work and specifically excludes de Moivre's theorem. This is included in all the other syllabuses. The exponential form of a complex number is included in all except the two already mentioned and also 0 & C, the second Pure syllabus of the WJEC and MEI. Transformations in the complex plane are mentioned by JMB syllabus B, London syllabus C, SMP and 0 & C.

**Vector Algebra**

SUJB contains absolutely no vector algebra, although its Applied Mathematics syllabus refers to "the application of vectors to two-dimensional problems" in both statics and dynamics, and its syllabus for the three dimensional geometry, on the second Pure Mathematics paper says that "vector methods may be used". The Cambridge Applied Mathematics contains only the addition of vectors and unit vectors and specifically excludes the differentiation of vectors. All the other syllabuses include a reasonable amount of vector algebra, with the AEB and the WJEC putting it on their Applied Mathematics syllabus, and only the WJEC and MEI exclude the vector product of two vectors.

**Co-ordinate Geometry**

Cambridge restricts itself on its Pure Mathematics syllabus to two dimensional co-ordinate geometry but covers this very fully, including the focus-directrix properties of conics, their auxiliary and director circles, and the equations of line pairs through the origin and of orthogonal and coaxal circles. London syllabus D also includes the focus-directrix
property of conics and the equation of a line pair through the origin but the London syllabus differs from the Cambridge one in that it also includes three dimensional cartesian co-ordinates. This mentions direction cosines and the equations of the straight line and the plane, but the notes given with the syllabus to clarify it, and to act as a guide to indicate its intended scope, say that "the emphasis is on the vector aspect of this section" of the syllabus.

Three dimensional cartesian co-ordinates also occur on all the other syllabuses except Oxford and WJEC.

Permutations and Combinations

These are mentioned specifically on all syllabuses except SMP, JMB syllabus B, Cambridge Pure Mathematics and Further Mathematics and SMP.

Determinants

These are divorced from their associated matrices on JMB syllabus A and on SUJB and are mentioned together with a limited amount of matrix work on London syllabus D and on MEI.

Modern topics

The Pure Mathematics syllabus of Cambridge is ultra-traditional, even old fashioned, and contains absolutely none of the mathematical topics that are usually termed modern. AEB and SUJB also exclude Modern Mathematics. In a similar way the JMB syllabus A omits all modern work except the vector algebra previously mentioned. London syllabus D confines the more modern work to simple and direct questions on matrices, although the introduction to this syllabus states that "questions may be set using the language and simple ideas of set theory". The matrix work on the Oxford syllabus includes eigenvalues and eigenvectors for 3x3 matrices. (This topic is also mentioned in the 0 & C and SMP syllabuses). The Oxford syllabus also has further modern algebra as an option on its Mathematics paper one and, although candidates for Further Mathematics are expected to have covered the whole of the Mathematics syllabus including the optional sections, "it is realised that some teachers do not wish to include" the
Modern Mathematics option "in their preparation of candidates....The choice of questions is such that this should not impose hardship on the candidates concerned". As mentioned earlier O & C includes eigenvalues and eigenvectors. Applications to quadratic forms are specifically mentioned. This topic will be the subject of only straightforward questions on paper one but more searching questions could be set on the Special paper, which is outside the scope of this study.

All the other syllabuses contain some modern algebra, although the amount to be covered varies among them. For example WJEC's first Pure Mathematics paper restricts its modern work to matrix algebra and to "the language of set theory" with functions being considered as mappings of sets. MEI mentions, as well as its section on matrices, the "formal rules of composition in sets........as exemplified in concrete examples of groups, rings and fields". The modern algebra contained on the alternative Pure Mathematics syllabus of the WJEC is considerably less, being merely simple set theory, "ideas of group, ring and field" and a small amount of matrix work leading to the solution of linear equations. The remaining syllabuses each take their coverage of modern algebra further, but there is no particular significance in the order in which they are considered here.

JMB syllabus B includes, on its Mathematics papers, some simple group theory, the interpretation of simple flow diagrams and the algebra of 2x2 matrices, but keeps the bulk of its more modern topics to the Further Mathematics syllabus. This includes product sets, equivalence relations and classes and the application of elementary axiomatic methods. Also included are vector spaces with particular reference to linear dependence and independence, dimensions and bases. Matrix methods for the solution of linear equations are also mentioned.

The Cambridge Mathematics and Further Mathematics syllabuses contain some Modern algebra in the compulsory sections but a great deal more in option D, algebraic structure on the Mathematics papers and linear spaces on
the Further Mathematics. The compulsory work includes injective, surjective and bijective functions, although these terms are defined in the list of definitions and formulae issued to each candidate in the examination. Equivalence relations and classes and properties and examples of groups are also included. The work on groups is to include knowledge of Lagrange's theorem but the proof will not be required. The compulsory part of the syllabus also includes 2x2 matrices including their application to transformations of the plane, but knowledge of determinants (a more 'traditional' topic) is not required. The compulsory section of the syllabus also mentions the structure of rings and fields but there is no indication as to how detailed this need be. The majority of option D of the Mathematics papers is included as compulsory in the Further Mathematics and so has been included above. One exception is the algebra of symbolic logic including truth values, truth tables, tautologies and the validity of arguments. The option D of the Further Mathematics papers contains work on vector spaces including subspaces. Linear dependence and independence is also included, together with their application to the solution of linear equations, where the existence, uniqueness and nature of the solution is to be considered. Elementary row and column operations on a matrix, reduction to echelon form and canonical form are also included in this section as is the rank of a matrix and the basis and dimension of a vector space.

London syllabus C includes, on its Mathematics papers, "the concept of a group with special emphasis on integers and rational numbers and on groups of symmetries of simple plane figures, permutation groups and the isometries of Euclidean geometry". These papers also include simple flow diagrams, on which the questions set may involve construction, interpretation or the correction of errors. The Further Mathematics syllabus covers more of the modern algebra. The topics mentioned include equivalence relations, algebraic structure and the solution of linear congruences to a prime modulus. Also included are simple examples of fields and vector spaces of not more than four dimensions. This is one of two syllabuses that contain some Boolean algebra.
The other is the SMP syllabus, which contains a large amount of modern algebra. As previously mentioned, this syllabus is one of three to include eigenvalues and eigenvectors. It also contains groups, fields, rings and vector spaces. The latter section includes linear transformations as well as linear dependence and independence. This syllabus also includes equivalence relations and equivalence classes.
Methods of examination and the interpretation of the syllabus by the examiners

Only the JMB, in its preamble to its various mathematical syllabuses, gives any indication of the objectives of the examination or of the knowledge and the abilities to be tested. There are eight items varying from the knowledge of mathematical notation, terminology, conventions and units to the ability to evaluate and interpret mathematical results. "These eight items form an hierarchical structure, with the earlier items forming a part of most questions and the later ones being tested less generally. It is not considered possible to allocate a specific weighting of marks to each of these abilities as they are interdependent and interrelated". The marks to be awarded fall into two broad categories - "Marks for the appreciation of an appropriate method and marks, consequential on the application of an appropriate method, given for the accuracy of manipulative and numerical work".

The London syllabuses each contain an introduction which, while not generally indicating the abilities to be specifically tested in the examinations, gives some indication of the philosophy behind the establishment of the syllabus and the reasons for the inclusion of particular topics. Syllabus D also mentions that some of its conventional long-answer questions are included to test the ability of a candidate to "follow through an extended argument".

An important difference between the syllabuses is the provision, or otherwise, of a booklet of definitions and formulae. There appears to be no consistency here. The practice was initiated with the introduction of the 'modern' syllabuses but is no longer confined to these. The provision of such a booklet obviously affects the sort of question that can be set. No longer can the candidate be asked to state a standard result if that is contained in his booklet, but he can be asked to use that result irrespective of his ability to memorise it. The syllabuses that provide such booklets...
are JMB syllabus B, both Mathematics and Further Mathematics, Cambridge all syllabuses, MEI double-subject only, O & C, Oxford, London all syllabuses, SMP and WJEC, and it is possible that the alternative AEB syllabus may also do so on its introduction in 1978 although the printed syllabus does not mention this.

A further difference between the papers is the approval by some boards of the use of electronic calculators. Where allowed these have to be hand-held, non-programmable and battery operated. The boards which allow the use of calculators are JMB, O & C, AEB, MEI and Cambridge.

The type of examination question actually set varies from board to board, from syllabus to syllabus within a particular board and even from paper to paper in a specific syllabus. As an example of the latter, London syllabus D has one paper of multiple-choice questions. This paper, lasting an hour, carries one-fifth of the maximum mark and will examine topics from the whole of the syllabus. Unfortunately the actual paper set in 1977 is not available although a specimen paper was issued in 1974 when the syllabus was published. The other two papers of this syllabus each have a section A containing short questions, which may all be attempted, and a section B of longer questions, of which four may be answered.

WJEC is the only board to state on its examination papers how the total mark for each question will be allocated to the parts of that question, thus giving some indication of the relative importance of each. In 1978 the JMB is going to introduce, for its syllabus B papers, a scheme under which there will be no restriction on the number of questions that may be attempted. The questions will not carry equal marks, but the mark allocation of each question, varying from about three to fourteen, will be given. There is no indication as to the number of questions the board consider that the average candidate could complete in the time allowed. However it does state that "the length of each paper is such that the most able candidates will be able to complete the paper but it will be possible for candidates to gain the highest grade without having attempted each question". London, on the papers of both Mathematics syllabus D and Pure
Mathematics, indicates the marks allocated to the section A question but the variation is slight.

The London board allows the candidates to attempt more than the specified number of questions stating that if more than the required number of questions are attempted only the best, up to the correct total number will be taken into account thus relieving the candidate of the necessity of deciding which question to cross out. The other boards will apparently follow the normal practice of ignoring the last question(s) to be attempted if the rubric is exceeded.

The majority of boards ask for seven questions to be attempted in three hours. The exceptions are as follows. The first SMP paper has eighteen questions and the candidates are asked to do as many as they can. On the first paper of the AEB alternative syllabus fifteen questions are provided. All may be attempted but they do not necessarily carry equal marks. On the first MBI paper as many questions as possible can be attempted from the eight questions of section A, which carries 54% of the total marks for the paper, and then three questions may be chosen from the five of section B. On the second paper, although there is a total of seven questions to be attempted, there is a compulsory question. This has three parts of which two must be answered and carried approximately 20% of the maximum marks for that paper. On the O & C first paper the questions are relatively short and fourteen are to be attempted. Not more than ten can be answered from either section, each which contains twelve. This board is the only one to provide an extra ten minutes reading time at the start of each examination. The Cambridge papers, except for the first of the syllabus A Mathematics, ask for a maximum of eight questions to be attempted and the Syllabuses of Mathematics syllabus B and Further Mathematics state that "a pass mark on each paper may be obtained by good answers to about four questions".

40.
The proposed arrangements for the syllabus B Mathematics and
Further Mathematics of the JMB, under which there will be no restriction
on the number of questions that may be attempted, has already been
mentioned. On the other JMB syllabuses the following or a similar statement
occurs. "Question papers may, at the discretion of the examiners, be divided
into two sections so that the questions in section 1, prefixed S, each carry
half the marks allocated to each question in section 2, prefixed D.
Candidates will be required to answer the equivalent of seven D questions on
both papers". So the total to be done remains the usual seven questions
but the half questions may be put together as the candidate wishes. The
number of S questions provided can vary. In 1976 the Mathematics syllabus A
had six on the first paper but ten on the second, Pure Mathematics and
Mathematics syllabus B had ten on each paper, the two further Mathematics
syllabuses had six on each paper but for Pure Mathematics with Statistics
there were ten S questions on paper one and none at all on paper two.

On several boards there is some restriction on the choice of question
that may be attempted. Some of these restrictions have been stated already.
The WEJC Applied Mathematics paper two is divided into four sections.
Section A contains five questions on Kinematics and Dynamics, section B
has three on Statics, section C three on Numerical Analysis and section D
three on probability. The rather complicated instruction to the
candidates is "Answer seven questions only, not more than five from
sections A and B. Questions must not be attempted from more than two of
the sections B, C and D". O & C divides its second paper into two sections,
Pure and Applied Mathematics and of the seven questions to be attempted not
more than five can be taken from either section. On the Cambridge
Mathematics syllabus A there are two sections. The first contains seven
Mechanics questions and seven Statistics questions while the second section
consists of four on Pure Mathematics. Of the seven questions to be
attempted not more than two may be taken from the second section. The
options for the syllabuses for Mathematics syllabus B and also for Further Mathematics have been mentioned in the previous chapter. The examination papers contain in section A seven questions on the compulsory part of the syllabus. In section B the Mathematics papers have three questions on each of its five options and the Further Mathematics papers have two questions on each of five options and one on each of two. On the Mathematics papers a candidate has to answer five questions from section A and three from section B, but these three cannot be from the same option. On the Further Mathematics papers not more than five of the eight questions that can be attempted may be from section A and not more than three from section B.

Having discussed the format of the various examination papers, I now wish to consider some of the questions themselves. I have been unable to obtain the papers of the Cambridge board or those of the Oxford board so I have to restrict this study to the other six boards and to the two projects. I have also decided to omit the three Pure Mathematics with Statistics syllabuses so that this reduces the number of syllabuses to eleven. The papers involved are those of 1977, except for AEB and JMB syllabus A where they are from 1976. Because, even with a reduction to eleven syllabuses, the discussion of each individual question would make for a long and, to be honest, boring study, I shall confine myself to some of the topics considered in Chapter four.

The first of these is complex numbers. As mentioned in Chapter four, the majority of syllabuses that include this topic do so in a limited way. The basic core of complex number work contains sums, products and quotients in both the $x + iy$ and the polar form, the modulus and argument of a complex number and its representation in the Argand diagram.

Two questions were asked on this part of the syllabus on the O & C papers, one on each. The shorter question required the candidates to find the modulus and argument of a given complex number and to state the real part of its eighth power. Although not mentioned by name this latter part of the question was a test of the use of de Moivre's theorem which is included on the O & C syllabus. It was unusual that the argument was required to the
nearest tenth of a degree and not in radian measure. The longer question on the second paper is in two parts. The first part is the opposite of the usual one of finding the product and quotient of two given complex numbers. In this question it is the product and quotient which are given and the complex numbers that have to be found, involving the candidates in solving a pair of simultaneous equations, one quadratic, with complex coefficients. The other part of this question asks for the solution of a quadratic equation with real coefficients, the representation of its roots on the Argand diagram and the relation between the real coefficients of a second quadratic equation when the points on the Argand diagram which represent its roots obey some specified condition.

MEI also provide two questions on this topic. The first one, being in section A, is short and is the basic one of finding the modulus and argument of two given complex numbers, those of their product and quotient and the illustration of the four complex numbers in the Argand diagram. The longer section B question has two unrelated parts. The MEI syllabus contains the application of de Moivre's theorem for positive or negative integral exponent and the first part of the question asks for the statement of the theorem for the positive case. This result is then to be used, following a hint as to the method required, to find the coefficients of a particular sextic equation of which one root is given. The second part of the question involves finding the region of the Argand diagram containing those points which represent complex numbers satisfying two given conditions, one dealing with argument and the other with modulus.

WJEC puts its complex number work on paper two. This, as explained in Chapter four, is basically a Mechanics paper and is alternative to paper three, the Statistics paper. The syllabus has the basic core of complex algebra and the representation of complex numbers by points on the Argand diagram, in either cartesian or polar co-ordinates. The actual question in 1977 had two parts. The first involved the use of the Argand diagram to prove stated connections between the argument of a general complex number and each of the arguments of its negative and its conjugate. It then asked
for the polar representation of the cube of a particular complex number and also of the complex conjugate of that number and for the deduction of a relation between them. The proof of the general case carried two of the seven marks awarded to this part of the question. The second part, worth eight marks, involved the locus of points in the Argand diagram representing the complex numbers that satisfied a given equation. It necessitated showing that that locus was a circle and finding the complex number that was represented by the centre.

The London syllabus C papers contained a single complex number question but this was like those of the previous papers in that it had two unrelated parts. The first gave a complex number in exponential form and the candidate was required to find, in the form $x + yi$, that number its square and its reciprocal. The modulus and argument of each of these was asked for, together with the representation of all three in an Argand diagram. The second part suggested, without compulsion, the use of de Moivre's theorem to prove a trigonometrical identity.

London syllabus D contained two questions on paper two. (Paper one is unobtainable so there is no way of ascertaining if a multiple-choice question was set on this topic). The first of these, on section A, was a straightforward, basic question to find the quotient, in the form $a + ib$, of two given complex numbers also in that form. The section B question, following the usual pattern, had two unrelated parts. The first asked for the two square roots of a complex number. The second part was on the representation in the Argand diagram of two fairly standard loci and the finding of the complex number corresponding to their points of intersection.

There is a complex number question on each of the SMP papers. The first one was of the basic type where the candidate was asked to find a complex number such that its reciprocal was the sum of those of two given complex number. The question on paper two started by asking for the proof, by induction, of de Moivre's theorem, but this was not mentioned by name. The result proved had then to be used to find the fifth roots of unity.
and from them the fifth roots of 32 had to be deduced. The rest of the question was on groups, using these ten complex numbers as the elements of two sets with multiplication as the operation.

JMB syllabus B examined the work on complex numbers in both papers. Three different questions involved a knowledge of complex numbers but only one of these confined itself to that part of the syllabus. This was an S question on the second paper. The candidates had first to find, in the form \( u + iv \), the reciprocal of the complex number \( z = x + iy \). Then, given that the sum of \( z \) and its reciprocal was real, they had to prove that possible stated alternative conditions existed for \( x \) and \( y \), and see under what circumstances each of these conditions was held. The other S question, on the first paper, involved finding the fourth power of a given complex number, in algebraic form, and then using the answer, together with the binomial theorem, to prove a given result. The method was specified.

The D question on paper one, which could be said to use complex numbers, involved a set of eight simple 2x2 matrices, of which four had \( i \) and \(-i\) as their non-zero elements. Candidates were told, and did not have to prove, that this set formed a group under the operation of matrix multiplication. Various results then had to be proved and subgroups of given order found.

This was really a question on group theory and the complex number work was incidental; it bears out the statement that "questions may link different parts of the syllabus".

There was one S question on complex numbers on the first paper of JMB syllabus A. In this a general complex number \( z = x + iy \) obeyed a given condition and the candidate had to show that there was a specified relationship connecting \( x \) and \( y \). The condition that \( z \) obeyed involved a variable real number and the rest of the question was on the locus of the point \( P \), representing the complex number \( z \), as that real number varied.

The AEB alternative syllabus, called the modern syllabus in 1976, had a complex number question on each of its papers. The shorter question, on paper one, asked the candidate to express a specified complex number in
polar form and then, given the modulus and argument of a second complex number, to find the polar form of the quotient obtained by dividing the second by the first. The question on paper two involved finding the square roots of a complex number, solving a quadratic equation with complex coefficients, representing the roots of that equation by the points $A$ and $B$ in the Argand diagram and showing triangle $AOB$ to be right-angled.

Thus all the syllabuses which specify complex numbers include at least one question on the examination papers, but very rarely is there a longer unbroken question, with the complex number work providing a continuous theme sustained throughout the question.

When considering the questions involving integration, it must be remembered that some boards provide the candidates with booklets of definitions and formulae and they include lists of standard integrals. Other boards, however, do not do so and may appear to be asking easier questions or giving hints to their candidates. The usual pattern, although this is not consistently followed, is that if a question asks for the evaluation of a number of integrals no methods are suggested, but if only one integral is involved there is a possibility that a method of integration will be indicated.

The SMP questions involve integration by parts, by partial fractions and by approximate methods. The use of each of the first two methods is specified in their individual questions. The question on approximate integration uses neither the trapezium nor Simpson's rule but involves a second degree Taylor approximation for the function to be integrated. Again the method to be used is indicated in the question.

A whole question on the SUJB papers is devoted to integration. The method of partial fractions is not needed, probably because an earlier question on the paper is devoted to them. The first part of the integration question asks for the evaluation of four indefinite integrals. One of these has to be integrated by parts but the method is not suggested in the question. Another part of the question asks for the evaluation of an indefinite integral "by means of a suitable substitution or otherwise".
On the WJEC paper one question involves the evaluation of a complicated definite integral involving the substitution of $\tan \theta = t$ and the use of partial fractions but candidates are led through the method by the earlier parts of the question. The expression of $\cos \theta$ in terms of $t$ carries two marks, the expression of a given fraction in terms of partial fractions carried six marks and eight are to be awarded for the integration. Another question on the paper asks for the integration by parts, or otherwise, of two functions. These results are then used to find the volumes of two solids of revolution.

MEI has two questions in section A of its first paper. One involves the approximate evaluation of a definite integral using Simpson's rule with five ordinates. The other question asks for the evaluation of three definite integrals. No methods are suggested, none involve parts of partial fractions and each is either a standard integral or uses a standard technique.

On the O & C papers one question on paper one, the shorter questions, needs the solution of a differential equation by integration by parts but this is not indicated in the question. Another question on this paper asks for three indefinite integrals to be found. A suggested substitution is given for the third but otherwise no hint as to method is given. The Calculus question on the second paper is divided into three parts. The second of these involves the evaluation of a single definite integral. This needs integration by parts but there is no specific hint to that effect. It should, however be obvious to all except the weakest candidates.

London syllabus C has part of a question which asks for the integral of $\frac{x}{(1 - x^2)^{\frac{1}{2}}}$ and hence, or otherwise, the integral of $\arcsin x$. This will involve integration by parts, but does not specifically say so. London syllabus D has one question to evaluate three definite integrals. Of these one involves partial fractions, one requires a substitution and the third is
integration by parts, but there is no indication of this in the question.

The AEB paper contains a question using Simpson's rule, with five ordinates, to find the approximate value of a definite integral. The other part of the question asks for the evaluation of two definite integrals "by using suitable substitutions, or otherwise". A second question, as one part, requires the solution of a first order differential equation with separable variables, and to obtain that solution partial fractions will be required, although the question does not say so.

The alternative AEB syllabus contains questions on both papers. There are two on paper one which is composed of shorter questions. The first of these involves the use of partial fractions to evaluate a definite integral. The question is further complicated in that the rational form to be integrated has a cubic numerator and a quadratic denominator and so division will be required before an attempt is made to find the partial fractions. There is no hint with the question as to the method to be used. The other question on paper one asks for the approximate value of a definite integral using Simpson's rule with the standard five ordinates. The integration on paper two forms part of a question. Three definite integrals are to be evaluated and there is no indication as to the methods required. The first looks at first glance as if it involves partial fractions but the denominator does not factorise. The method needed is the writing of the fraction as the sum of two others, with the same denominator, in order to give two standard integrals. The third needs integration by parts, the second, although it is the product of two functions, does not. All it involves is a suitable substitution, or the recognition that the function to be integrated is a fraction of simple composite function.

JMB syllabus A had two S questions involving integration. The first asked for the volume of solid of revolution and, to evaluate the appropriate definite integral, integration by parts was needed. No indication to that effect was given. The other question suggested a suitable
substitution for the evaluation of a definite integral, although the candidates could use an alternative method if they wished.

The integration questions on the syllabus B papers of the JMB were also 5 questions. One asked for the evaluation of two definite integrals; no methods were suggested but each one only involved a fairly obvious substitution. A second question asked for a given function to be expressed in partial fractions and its integral to be evaluated. The third question was somewhat different. A flow diagram was given to evaluate a definite integral according to the trapezium rule. The candidate was asked to carry out the indicated procedures. He also had to explain, by means of a sketch-graph of the function being integrated, why the answer obtained would be an over-estimate of the true value of integral.

Thus both complex numbers and integration are adequately examined on the papers with the candidates able to show how well they have understood these topics.
CHAPTER 7

The N and F proposals for sixth-form Mathematics courses

One of the priorities of the Schools council, when it was established in 1964, was the consideration of the sixth-form curriculum and examinations. Its working papers number 5, Sixth-Form Curriculum and Examinations (1966), number 16, Some further proposals for sixth-form work (1967) and number 20, Sixth-Form Examining Methods (1968), showed that it felt that some reform was necessary but that it had not decided along what lines that reform should be taken. This need had also been seen by its predecessor, the Secondary Schools Examination Council, as shown by its third report in 1960, GCE and Sixth-form Studies, and its sixth report in 1962, Sixth-form Studies and University Entrance Requirements.

In 1966, the Schools Council and the Standing Conference on University Entrance agreed that the reasons for dissatisfaction with the sixth-form curriculum was that it was too narrow, forced pupils to make premature choices and failed to take into account the widening range of ability in the sixth-forms as they grew in size. Various suggestions were made, by the Schools council and others, for overcoming this dissatisfaction. Finally two working parties of the Schools Council made proposals, in the Schools Council working papers numbers 45, 16-19: Growth and Response, 1. Curricular Bases, 46, 16-19: Growth and Response 2. Examination Structure, and 47, Preparation for Degree Courses, for the N and F levels of examination.

Following nationwide public discussion of these three working papers, the Governing Council of the Schools Council decided, in 1974, to set up studies into the feasibility of the N and F proposals. As explained in Chapter 1, this was done by setting up sixteen subject steering groups.

The Mathematics group realised that, as a sixth-form subject, Mathematics had some special features that needed to be borne in mind. The first of these was the wide variety in ability in the group of students studying the subject.
This group was likely to be composed of specialist mathematicians and scientists or social scientists needing Mathematics as a service subject, as well as a number of students for whom Mathematics was merely their third Advanced level subject in some very individual combination. (As an example I have taught girls taking Religious Studies, English and Mathematics).

The second feature to be remembered about Mathematics has already been mentioned. This is the service role of Mathematics, not only for the traditional users of the subject, such as Physics and Engineering, but also Biology, Economics and Geography each of which require more statistics and less calculus, for example. Their third consideration was the time factor and the feeling that if the time for study was reduced, to half or three-quarters of that normally provided for an Advanced level subject, there might be the tendency to stress the acquisition of mathematical techniques at the expense of an overall understanding of mathematical concepts and the way that Mathematics could be applied in an increasing variety of situations in order to solve problems that could be encountered.

Because of the long tradition of Mathematics being available as either a single or a double subject, the steering group took the view that the curriculum proposals should allow for this possibility. However, since the structure envisaged was a two-tiered one, a more flexible, and hence more complicated, situation emerges. There is likely to be a demand not only for single N and single F but also for a variety of combinations of double subject (N + N, F + N and F + F).

The steering group commissioned seven studies. This appears to be a very large number, but they felt that a 'portfolio' of syllabuses, similar to those at present available at A-level, would be needed from which different students, or their teachers, would be able to select an appropriate course. Three of the commissioned groups looked at different ways in which a double subject syllabus could be organised. The group based on University of London Schools Examination Department worked on a Mathematics and Further Mathematics combination. Three separate subjects, Pure
Mathematics, Applied Mathematics and Statistics, were considered by the group based on the University of Cambridge Local Examination Syndicate. The Mathematical Association and the Chelsea College (University of London) jointly accepted the commission to prepare a syllabus combining a basic core of Mathematics and its applications with a wide range of options from which teachers and/or students could make a choice.

As well as the preparation of examination syllabuses which bore some similarity to those at present available at A-level, the steering committee also wished to take into account aspects of recent work carried out in curriculum development at the sixth-form level. For this reason two groups were set up to study, within a single subject framework, developments which treat Mathematics in a less conventional way. One of these extended the work of the Schools Council/Reading University Sixth Form "Applicable Mathematics" Project and the other considered the work of the Association of Teachers of Mathematics that proposed the use of individual investigation techniques and the acquisition of general strategies for dealing with different types of perhaps unfamiliar situations.

The final two groups were approached with a view to the provision of courses for those sixth-formers who might choose to take Mathematics as part of a five-subject course, but whose mathematical ability might be limited. The Committee on Statistical Education accepted the commission to produce a Statistics syllabus which, although it would be complete and of the required standard, would need no supporting Mathematics beyond O-level/ C.S.E. standard. The National Association of Teachers in Further and Higher Education looked at the possibility of producing a syllabus, at N level only, for students, following a predominantly 'Arts' course of study, who might wish to continue with a limited amount of Mathematics. This syllabus was to aim to develop an appreciation of mathematical ability rather than specific skills and was to have the title "Mathematical Awareness".

One of the major difficulties in a two-tier structure is the way in which the two tiers are to be related which, in turn, affects the resources.
required and available in the schools to teach them. Three possibilities were described in working paper 47. The syllabuses, although separate, could have sufficient common material so that the students following the two different courses could be taught together in the first year. This proposal was not supported by any of the commissioned groups and would, in any case, lead to considerable teaching, and timetabling, difficulties in the second years of such courses. A second possibility is a syllabus common to the two levels but taken to greater depth at F level. At the request of the steering committee the Cambridge group considered a possible Pure Mathematics syllabus on this pattern but the majority of commissioned groups followed the third possibility, that the syllabus at F level should contain that for the N level but also some additional material. Most of the groups took the view that at F level there should be both extra content and greater depth in the treatment. As a result the steering committee recommends that the F level syllabus should contain that for the N level together with additional material, but that "the assessment at F level should be designed in such a way as to ensure that the whole syllabus is examined in greater depth than at N level" (page 5).

The steering group appear to favour the Mathematics, Further Mathematics structure, rather than having Pure Mathematics and Applied Mathematics as separate subjects, with a package of single and double subject syllabuses, with a common core in both the first and second N levels, such as was proposed by the alternative scheme of the Mathematical Association and Chelsea College group, and options to provide for the various mathematical needs of the students. Secondly, it thought that there should be provision, within the new examination structure, for groups to construct alternative schemes involving different organisation of the mathematical material, different teaching methods and different methods of assessment. In this category it placed the schemes of the Association of Teachers of Mathematics and of the Applicable Mathematics group. Thirdly, and finally, it regarded the provision of syllabuses such as that for Mathematical Awareness as both
feasible and desirable, encouraging more students to continue some form of Mathematics beyond O-level/C.S.E.

Of these three recommendations, only the first really falls within the scope of this study. The proposed schemes which fall into the second category are of a type which makes comparison with the present Advanced level syllabuses difficult. Nevertheless, they do have much to recommend them and if I ignore them here it is not because I think them to be without merit. Each of them, however, requires a strong sense of commitment from the teacher and probably a great deal of in-service training would be needed for those who would wish to teach such a course but might feel that they lacked the rather special teaching skills for which both of these courses call.

The Mathematics and Further Mathematics structure allows for five possible subject combinations $N$, $N + N$, $F$, $F + N$ and $F + F$. The alternative Chelsea scheme omits the latter possibility and its main scheme does not consider it at length, feeling that, in fact, this possibility might be actively discouraged by the schools. This variety of combinations, and the number of options available in the Chelsea scheme, immediately lead to difficulties that may be more apparent to those who draft the school-timetable, and to the heads of Mathematics departments who have to decide how to deploy their teaching staff, than to others. The number of suitably qualified Mathematics teachers is finite and the rest of the school must not be neglected in order to provide more and more sixth-form teaching. So the actual staff teaching time available for the $N$ and $F$ levels is limited. There are already difficulties in having some students taking Mathematics A-level and others taking the two A-levels in Mathematics and Further Mathematics, especially if the numbers are small. Here the question is whether or not they are taught completely separately, involving three time-table units, or whether the Mathematics, that they are all taking, is taught to a combined groups, with the problem that the Further Mathematics group will not have a
unified course and on occasions will be kept waiting for Mathematics to take further! To try to cater for a possibility of five groups of students will merely magnify these problems.

Of course, if the F level syllabus contains that for the N level, then perhaps groups could be taught at the same time, but again the extra work for the F level needs to be based on the N work and cannot, in most cases precede it. Again, if the Chelsea scheme was introduced it might happen that the students, because of the needs of other subjects and the students' own inclinations, might be taking a number of different options. This also leads to a number of time-tabling difficulties. Unless the options could be self-taught, they would require to appear as separate periods on the timetable or to be taught by different teachers. This could also lead to the difficulty in that necessary core material might not have been covered at the appropriate time. The total course would appear disjointed and there would have to be much thought given by those teaching the course as to how they could show the unity of the subject as a whole. There would, of course, not be the same difficulty if the whole group were taking the same option or group of options! However, this does not detract from the syllabuses themselves which must be considered seriously. Nevertheless, we must be realists, and, as one who has the unenviable and already difficult task of compiling the timetable of an upper-tier comprehensive school, I am only too conscious of the difficulties inherent in trying to put these syllabuses into practice.
Conclusions?

At the recent North of England Education Conference, held at York University, Dr. Patrick Nuttgens, the director of Leeds Polytechnic, urged, among other things, that the school-leaving examinations should be released from the "strangle-hold of the universities". In a leader two days later (Jan. 6th, 1978) the Evening Post, a local paper published in Leeds, said that it was difficult to know what he meant. "This seems a strange attitude from one whose institution surely must insist on examination results both from people before they enter its doors and before they leave it!" The leader writer felt that examinations provided the best means of judging performance in academic institutions but concluded with the following sentence "If Dr. Nuttgens means there should be one examination for youngsters intending to enter universities, and another for those who don't, then, a multitude of people will be with him".

This goes to prove, if proof were needed, that opinions about the school examination system are divided. Needless to say, the leader writer is rather naive if he thinks that a University Entrance examination, to replace Advanced level for prospective entrants, could be introduced at this stage. There would be immediate calls for a Polytechnic Entrance, a College of Education examination, and so on, and the sixth-formers who are undecided about their post-school future would be in greater difficulties than they are at present. On the other hand if he means that a University Entrance examination should be taken in addition to some form of school-leaving certificate then perhaps his suggestion can be taken more seriously.

The main trouble with the Advanced level of the General Certificate of Education is the variety of purposes to which it is put. It is both a qualifying and selecting examination for entrance to degree courses, where two Advanced level passes is the minimum requirement but a particular course might specify grades to be achieved. The examination is also used as an entrance qualification to professional training, as a stepping stone to
some types of vocational courses, such as post A-level secretarial, by employers when choosing staff, and so on.

This variety of use to which the certificate is put affects all subjects and subject combinations. Mathematics has the added difficulty of the large number of different syllabuses. If the Advanced level is required merely as an indication of some standard reached, then the fact that the work covered by John Doe is different from that done by Richard Roe will not matter. If, on the other hand, the mathematical foundation provided by their Advanced level studies is going to be built upon by some course of higher education, then those planning that course need to know, and bear in mind, that although John Doe may know, for example, all about de Moivre's theorem Richard Roe may not even have heard of it.

That, of course, was the reason for the publication, by the Mathematical Education Committee, of *A Survey of 'A' Level Mathematics Syllabuses* to assist planners of Tertiary courses which have a substantial mathematical content. The second edition, published in 1976, uses the 1977 syllabuses and so, unfortunately, is already out of date. I think that the table giving the numbers of candidates taking each of the syllabuses covered in the survey is a little misleading. It states that the J.M.B. syllabus B was not available in 1973, 74 and 75. While it is perfectly true that the syllabus for those years was different from that of 1977, the syllabus changes were relatively minor. It might have been better to have given the numbers for the earlier years and to have indicated what syllabus content would have been covered that has since been excluded.

Apart from this very minor criticism, this pamphlet is a very informative document. It covers the whole range of single subject syllabuses, including those of the Northern Ireland GCE Board, breaking these down into small topics or groups of topics, and it is interesting to see that the number of topics common to all the syllabuses is relatively low.

Of course, it does not follow that because a topic is excluded from an examination syllabus it will not be covered in the sixth form course.
Quite often it is interesting and useful to take a particular topic further than the examination demands. W.L. James, in his survey of *The Interdependence of Sixth Form Mathematics and the Mathematical Courses of Universities and Colleges of Education*, has this to say. "The fundamental error of all syllabuses is that they are too wide. The area of study which they define and examine does in practice, if not by intent, occupy the whole of the average candidate's learning time. Examination boards have no desire to dominate the curriculum of the schools in this way: they have always supposed that schools would be able to extend studies beyond the bounds of examination requirements. In actual fact however, in the case of the average candidate, this desirable 'widening the scope of his study' does not and cannot take place".

I had hoped in this chapter to draw together the ideas of the preceding chapters and to draw some conclusions, but the question mark, after its title, is there for a reason. Just what conclusions can be drawn? The examination system, like Topsy, "just grewed". The possible introduction of the N and F levels allows a re-think but there are contained within the new proposals the seeds of the dissatisfaction felt with the existing scheme. The Mathematics steering committee has not indicated whether it feels that all examination boards should provide the same syllabus or that the present arrangement of similar syllabuses with some important differences should be allowed. Perhaps James's proposal, made in the context of the Advanced level, is the best even if the N and F proposals are accepted in principle. He suggests a national syllabus for each mathematical subject. This would consist of core material, together with either optional topics arranged by the syllabus in cohesive groups, options chosen by the school, or areas of investigation suitable for essay-type questions. In the last two cases sufficient examination questions would be provided by the school to allow the external moderator to make a choice as to which ones would be set, modified as he thought necessary. In this way the forty-one syllabuses at present available in mathematical subjects would be reduced to seven.
If the Advanced level is to remain, it is to be hoped that more serious consideration would be given to James's suggestions. The N and F proposals have, however, clouded the issue, but until the Schools Council decides to recommend their acceptance, and the Secretary of State agrees, we must make sure that the examinations we have are the best that can be provided. The examination boards are understandably jealous of their own preserves. Reform must come from within and should not be enforced from without if that can be avoided. Change, merely for the sake of change, is to be avoided at all costs. Changes which occur because they are natural or provide advantages which all can see are the ones to find. This is the lesson that some of those involved in education do not appear to have learnt and we all jump too readily on the most recent or the most fashionable band-wagon without considering the consequences.

Finally, in the words of Charles Caleb Colton (1780-1832) "Examinations are formidable even to the best prepared, for the greatest fool may ask more than the wisest man can answer".
Appendix 1

Mathematics Syllabuses currently available as included in Chapter 3

Associated Examining Board (AEB)

1 Mathematics (Pure and Applied)*
2 Mathematics (Pure)
3 Mathematics (Applied)*
4 Mathematics (Alternative Syllabus)*
5 Mathematics (Pure with Statistics)*
6 Mathematics (Pure with Computations)*
   (Mathematics (Statistics) is not included in this study)

University of Cambridge Local Examinations Syndicate (Cambridge)

7 Mathematics, Syllabus A *
8 Mathematics, Syllabus B *
9 Further Mathematics
10 Pure Mathematics
11 Applied Mathematics

Joint Matriculation Board (JMB)

12 Mathematics Syllabus A *
13 Mathematics Syllabus B *
14 Further Mathematics Syllabus A *
15 Further Mathematics Syllabus B *
16 Pure Mathematics
17 Pure Mathematics with Statistics *

University of London, Schools Examination Department (London)

18 Mathematics Syllabus C *
19 Mathematics Syllabus D *
20 Further Mathematics Syllabus C *
21 Further Mathematics Syllabus D *
22 Pure Mathematics
23 Higher Mathematics
24 Pure Mathematics with Statistics *

Mathematics in Education and Industry Project (MEI)

25 Mathematics *
26 Pure Mathematics
27 Applied Mathematics

Oxford and Cambridge Schools Examination Board (O & C)

28 Mathematics *
29 Further Mathematics

Oxford Delegacy of Local Examinations (Oxford)

30 Mathematics *
31 Further Mathematics
32 Pure Mathematics

Schools Mathematics Project (SMP)

33 Mathematics *
34 Further Mathematics
Southern Universities Joint Board for School Examinations (SUJB)

35 Mathematics *
36 Pure Mathematics *
37 Applied Mathematics *

Welsh Joint Education Committee (WJEC)

38 Mathematics *
39 Pure Mathematics *
40 Pure Mathematics (Alternative syllabus) *
41 Applied Mathematics *

* = included as a single subject in Chapter 4
* = included as part of a double subject in Chapter 5
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