Computational and experimental analysis of elastic deformation in impact

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COMPUTATIONAL AND EXPERIMENTAL ANALYSIS OF ELASTIC DEFORMATION IN IMPACT

by

Alan Hocknell, M.Eng.

A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

February 1998
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7.1 Conclusions
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Nomenclature

ABAQUS/Explicit  Explicit finite element solution code by Hibbit, Karlsson & Sorensen, Inc.
CAD  Computer aided design
CCD  Charge coupled device
DUCT  Surface modelling CAD software by Delcari International plc.
ESPI  Electronic Speckle Pattern Interferometry
FE  Finite element
FM  Frequency modulation
He-Ne  Helium-Neon
LDAc  Laser Doppler accelerometer
LDV  Laser Doppler vibrometry
LTV  Laser torsional vibrometer
MSC/PATRAN  Finite element pre and post processing software by the MacNeal Schwendler Company Limited
VCO  Voltage controlled oscillator
VISAR  Velocity Interferometer System for Any Reflector

$A$  Vibration amplitude
$A_{F}H_{F}$  Fixed points on a target body
$A'_{F}E'_{F}$  Five nodes on the finite element golf ball model
$A_{LOCAL}$  Vibration amplitude of the nearest antinode on a modeshape
$A_{F}H_{R}$  Remote measurement points which initially probe fixed points $A_{F}H_{F}$
$A_{x}$  Amplitude of harmonic whole body translation
$a$  Acceleration
$a_{max}$  Maximum measurable acceleration in an LDAc
$C_{n}, D_{n}$  Series summation terms in Kolsky pulse propagation model
$c$  Speed of light
$c_{B}$  Damping constant
$c_{o}$  Wave propagation velocity
$E$  Young's modulus
$E_{A}, E_{B}$  Young's moduli of bodies $A$ and $B$
$e$  Amplitude error in a displacement compensated waveform
$e_{MAX}$  Maximum value of amplitude error, $e$
$e_{r}$  Coefficient of restitution
$F$  Contact force
\( F_n \)  
Number of interference fringes

\( f_s \)  
Optical beat frequency

\( f_d \)  
Doppler frequency shift

\( f_{\text{max}} \)  
Maximum vibration frequency at which measurements can be made in a given LDAc configuration

\( f_R \)  
Frequency pre-shift to a reference beam

\( f_s \)  
Digital sample rate

\( g_{\text{tr}} \)  
Normalised cross-correlation of the target and reference beams

\( I_A, I_B \)  
Moments of inertia of bodies \( A \) and \( B \)

\( I_F \)  
Intensity of the fluctuating component of interest in the LDAc output

\( I_T, I_R \)  
Target and reference beam intensities

\( i, j, k \)  
General counters

\( i=A,B,C,D,E \)  
Measurement location on a target body

\( k_1 \)  
Derived constant used in Hertz theory of contact

\( k_2 \)  
Material constant used in Kolsky pulse propagation model

\( k_D \)  
Frequency demodulator calibration constant

\( L, l \)  
Length

\( l_1, l_2, l_3 \)  
Fibre lengths in a balanced all-fibre LDAc

\( m \)  
General counter

\( m_A, m_B \)  
Mass of bodies \( A \) and \( B \)

\( n \)  
General counter

\( p \)  
General counter

\( R \)  
Radius

\( R_A, R_B \)  
Radii of bodies \( A \) and \( B \)

\( r_{\text{CON}} \)  
Contact radius (golf ball deformation)

\( T_F \)  
Pulse duration recorded by a transducer fixed to the target body

\( T_R \)  
Pulse duration recorded by a remote transducer

\( t \)  
Time

\( t_F \)  
Turning points in the \( V_{i_F} \) waveform

\( t_R \)  
Turning points in the \( V_{i_R} \) waveform

\( t_X \)  
Time adjustment for target translation in the \( x \)-direction

\( U \)  
Target velocity in the direction of the probe laser beam

\( u \)  
Longitudinal deformation in a Kelvin-Voigt material

\( V_{iC} \)  
Displacement compensated vibration response waveform for point \( i_F \) on the target

\( \hat{V}_{iC} \)  
Amplitude of vibration in the displacement compensated waveform for point \( i_F \) on the target
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$V_{iF}$</td>
<td>Vibration response measured by a transducer fixed to point $i_F$ on the target</td>
</tr>
<tr>
<td>$\dot{V}_{iF}$</td>
<td>Amplitude of vibration at point $i_F$ on the target</td>
</tr>
<tr>
<td>$V_{IR}$</td>
<td>Vibration response measured by a remote transducer which initially probes point $i_F$ on the target</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>Frequency demodulator output voltage</td>
</tr>
<tr>
<td>$v$</td>
<td>Translational velocity</td>
</tr>
<tr>
<td>$v'$</td>
<td>Material property defined in Kolsky pulse propagation model</td>
</tr>
<tr>
<td>$v_o$</td>
<td>Initial translational velocity</td>
</tr>
<tr>
<td>$w$</td>
<td>Target dimension in the $y$-direction</td>
</tr>
<tr>
<td>$x$</td>
<td>Whole body displacement in the $x$-direction</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>Whole body velocity in the $x$-direction</td>
</tr>
<tr>
<td>$\dot{x}_{A1}$</td>
<td>Velocity of body $A$ in the $x$-direction before impact</td>
</tr>
<tr>
<td>$\dot{x}<em>{A2}, \dot{x}</em>{B2}$</td>
<td>Velocities of bodies $A$ and $B$ in the $x$-direction after impact</td>
</tr>
<tr>
<td>$x_{AF}$</td>
<td>Distance in the $x$-direction from an arbitrary reference point on the target to the initially probed point on the target, $A_F$</td>
</tr>
<tr>
<td>$x_{CB}$</td>
<td>Forward displacement of point $C$ relative to point $B$</td>
</tr>
<tr>
<td>$\ddot{x}_{club}$</td>
<td>Golf club head impact velocity</td>
</tr>
<tr>
<td>$\dddot{x}_{club}$</td>
<td>Golf club head forward acceleration measurement</td>
</tr>
<tr>
<td>$x_{CON}$</td>
<td>Contact approach (golf ball deformation)</td>
</tr>
<tr>
<td>$x_{DC}$</td>
<td>Forward displacement of point $D$ relative to point $C$</td>
</tr>
<tr>
<td>$x_{ED}$</td>
<td>Forward displacement of point $E$ relative to point $D$</td>
</tr>
<tr>
<td>$x_{FREE}$</td>
<td>Free approach (golf ball deformation)</td>
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<td>$x_{H1}, x_{H2}, x_{H3}$</td>
<td>Forward displacement of nodes $H1, H2, H3$ in the finite element model</td>
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<td>$x_{H2, DEF}$</td>
<td>Deformation of the face centre in the $x$-direction in the finite element model</td>
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<tr>
<td>$\dot{x}_{iF}'$</td>
<td>Golf ball forward velocity at point $i_F'$ calculated from the finite element model</td>
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<tr>
<td>$x_{LI}$</td>
<td>Straight line in the $x$-$t$ plane used in displacement compensation of remote pulse propagation measurements</td>
</tr>
<tr>
<td>$x_{dR}$</td>
<td>Golf ball forward displacement at point $i_F$ calculated using measurements made from point $i_R$</td>
</tr>
<tr>
<td>$\dot{x}_{dR}$</td>
<td>Golf ball forward velocity at point $i_F$ calculated using measurements made from point $i_R$</td>
</tr>
<tr>
<td>$\dddot{x}_{dR}$</td>
<td>Golf ball forward acceleration at point $i_F$ calculated using measurements made from point $i_R$</td>
</tr>
</tbody>
</table>
$x_k$ The intersection between $x_{ij}$ and the whole body displacement, $x$

$x_M$ Distance from a node in the modeshape in the $x$-direction

$\dot{x}_p$ Velocity of pulse propagation in a material

$x_S$ Spacing of two remote measurements used in displacement compensation

$y_{AF}$ Distance in the $y$-direction from an arbitrary reference point on the target to the initially probed point on the target, $A_F$

$\dot{y}_{B2}$ Velocity of body $B$ in the $y$-direction after impact

$\dot{z}_{iC}$ Displacement compensated golf ball lateral deformation velocity measurement for point $i_F$

$\dot{z}_{i'C}$ Displacement compensated golf ball lateral deformation velocity measurement corresponding to finite element node $i'_F$

$\dot{z}'_{iF}$ Golf ball lateral deformation velocity at node $i'_F$ in the finite element model

$\dot{z}_{iR}$ Golf ball lateral deformation velocity measurement made from point $i_R$

$\ddot{z}_{iR}$ Golf ball lateral deformation acceleration calculated using measurements made from point $i_R$

$\dot{z}_{iR,\gamma}$ Velocity measurement on a golf ball made from point $i_R$ at an angle $\gamma$ to the $x$-axis

$\alpha$ Describes the displacement of the fixed point, $A_F$, on the target relative to the separation of the remote measurement locations, $A_R$ and $B_R$

$\alpha_D$ Approach deformation

$\alpha_{D,max}$ Maximum approach deformation

$\beta_R$ Damping factor in finite element code

$\gamma$ Angle of incidence of a laser Doppler vibrometer beam from the $x$-axis in the impact measurement system

$\Delta f_b$ Optical beat frequency modulation

$\Delta l$ Optical path length imbalance

$\Delta t$ Time increment

$\Delta_G, \Delta_R$ Distortion in displacement compensated and single remote waveforms

$\delta$ Phase angle between stress and strain in a viscoelastic material

$\delta_\lambda, \delta_b$ Material property used in Hertz theory of contact

$\zeta$ Apparent angular frequency of the moving modeshape when observed remotely
<table>
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<tr>
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<th>Description</th>
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<tr>
<td>$\zeta_{\text{MAX}}$</td>
<td>Maximum value of $\zeta$ encountered in a particular measurement</td>
</tr>
<tr>
<td>$\eta_b$</td>
<td>Damping factor</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Loft angle</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength of light</td>
</tr>
<tr>
<td>$\lambda_r$</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>Spatial period of a modeshape, pulse length</td>
</tr>
<tr>
<td>$\nu_A, \nu_B$</td>
<td>Poisson's ratios of bodies $A$ and $B$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Contact duration</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>Optical delay duration in a VISAR</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Phase difference between target and reference beams</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Eigenvector</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Angular frequency of harmonic whole body target translation</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$\omega_{A1}, \omega_{B2}$</td>
<td>Angular velocity of bodies $A$ and $B$ after impact</td>
</tr>
<tr>
<td>$\omega_{L1}, \omega_{L2}$</td>
<td>Angular frequency of wave components 1 and 2</td>
</tr>
</tbody>
</table>
Abstract

This thesis presents new developments in the computational analysis of sculptured surface products and in non-contacting vibration measurements based on the laser Doppler technique. The work is applied to the study of elastic deformation during impact where a detailed measurement and analysis capability is demonstrated using the golf impact as a principal example.

A study of quadrilateral finite element mesh generation on sculptured surface product families has shown that the versatility of an existing paving mesh generation algorithm can be improved significantly by dividing the product geometry into an anatomy of features and further subdividing primary features into smaller areas with four curvilinear sides. The hollow golf club head is an example of a sculptured surface product and is used to demonstrate the mesh generation difficulties which are overcome using the proposed approach.

A computational analysis of a hollow golf club head under steady state dynamic conditions and in impact with a golf ball is presented which is of greater accuracy and superior detail to any previously reported. These tools are necessary for the development of a greater understanding of the detailed mechanical behaviour during impact in terms of energy transfer and 'feel' characteristics and are of importance to the golf equipment industry.

Experimental validation of impact models is achieved through the introduction of a system of measurements which is suited to the analysis of high-speed, short duration impacts between lightweight bodies in which large elastic deformation occurs. A wealth of short duration golf impact data is captured efficiently using laser Doppler vibrometers and piezo-electric accelerometers.

When using remote, non-contacting transducers such as the vibrometer, whole body translation of the target body causes the point on the target interrogated by the laser beam to change during the impact. A displacement compensation technique is introduced which permits close approximation of the vibration of fixed points on bodies undergoing whole body translation using data recorded from remote transducers. The technique is of additional importance in matching the available experimental data to points of interest in a computational model of the body. The improvement in data quality, relative to a single remote measurement, is quantified by reference to both simulated and actual experimental data in the case of steady state vibration and transient pulse propagation in impact.

The golf impact is one of an increasing number of applications to exceed the measurement range of the available non-contacting instrumentation. Accordingly, the first practical demonstration of the laser Doppler accelerometer, a new non-contacting laser transducer which is directly sensitive to the acceleration of a target surface, is reported. The instrument has several advantages including an easily adjustable working range with effectively no upper measurement limit, straightforward operation and use of low cost optical components. Practical issues in the successful isolation of the particular optical beat frequency which carries the acceleration signal are described and recommendations for the future development of this important new instrument are discussed.
Publications arising from this work

Hocknell, A; Jones, R; Rothberg, SJ.

Hocknell, A; Jones, R; Rothberg, SJ.

Hocknell, A; Mitchell, SR; Jones, R; Rothberg, SJ.
Modal characteristics of hollow golf clubheads : Determination and impact applications. Accepted for publication in Experimental Mechanics, June 1998.

Hocknell, A; Mitchell, SR; Jones, R; Underwood, D.
Feature based quadrilateral mesh generation for sculptured surface products. Submitted to the Institution of Industrial Engineering Transactions on Design and Manufacturing.

Halliwell, NA; Hocknell, A; Rothberg, SJ.

Hocknell, A; Jones, R; Rothberg, SJ.

Hocknell, A; Jones, R; Rothberg, SJ.
Computational and experimental analysis of the golf impact. Accepted for publication in the proceedings of the Third World Scientific Congress of Golf, St Andrews, Scotland, July 1998.

Hocknell, A; Coupland, JM; Rothberg, SJ.

Awards

Metrology for World Class Manufacturing Awards 1997
NPL Award for Measurement, Category 1 : Frontier Science and Measurement 'Laser Doppler accelerometry : The new concept for non-contact vibration measurement'.
Joint Winners : SJ Rothberg, JM Coupland and A Hocknell
Acknowledgements

I would like to thank my supervisors Dr Steve Rothberg and Dr Roy Jones for their guidance and enthusiasm on all matters throughout the course of my studies.

I am also indebted to Dr Séan Mitchell for his support and assistance in preparing an accurate CAD representation of a hollow club head, to David Underwood for his efforts in implementing sculptured surface mesh generation techniques and to Barry Welch for work on the experimental determination of golf ball material properties.

I am grateful for the assistance of those members on the technical staff of the School of Mechanical & Manufacturing Engineering at Loughborough University who have contributed to this work. In particular the assistance of Vince Scothern with high speed video editing and Richard Price in the preparation of golf clubs for impact testing is greatly appreciated.

Dunlop Slazenger International Limited, through Works Director Mike Shaw, have provided frequent access to staff and facilities, without which much of the impact testing would have been impossible. I would particularly like to thank Development Manager Brian Machin, Paul Lambert and Mark Naylor for their active participation in the project, knowledge of the golf equipment industry and patience during long days of experimentation at the flight test facility.

On a personal level, my housemates Mark Cole and Jon Petzing deserve a special mention for their own brands of humour and encouragement and I am grateful to all of my friends and colleagues for supplying necessary distractions from this work.

Finally, to my family, who have supported me in every possible way since I embarked on this academic journey as a teenager. I dedicate this thesis to them.
To my family
1 Introduction

1.1 Research background

The design analysis of bodies subjected to impact in normal use is critical to achieving the desired impact behaviour. Impacts are commonly associated with damage to the colliding bodies and the absorption of energy by plastic deformation. However, impacts in which only elastic deformation occurs are considerably more prevalent and, in the majority of cases, the issue of energy transfer from one colliding body to another is important. This is particularly true of sports equipment, which is often designed with the specific intention of maximising the transfer of energy from a striking implement to a projectile. The analysis of the golf impact provides the motivation for the development of the computational and experimental impact analysis techniques presented in this thesis.

Confirmation that the product will meet the required performance levels under impact loading can be obtained using structural analysis techniques. However, the product shape and applied loads are often too complex for theoretical treatment and, as a consequence, complete solutions are obtained only for idealised geometric configurations. The level of detail which is available from traditional analytical models is therefore generally poor.

Engineering product design ideas are increasingly realised using computer aided engineering methods. Principally, computer aided design (CAD) is used as a means of fast and accurate shape representation. The geometry information provided by a CAD system can also be used as the basis on which to build the computational grid necessary for numerical structural analysis of the product under simulated operating conditions. The finite element method has emerged over the last 30 years as the primary computational structural analysis tool in mainstream engineering design. However, one of the biggest obstacles to the use of this method is the discretisation of often complex product geometry into a suitable finite element mesh. The mesh
generation task is tedious and error prone and the size, shape and number of elements in the mesh has a direct bearing on the accuracy and cost of the analysis. Mesh generation is particularly problematic in the analysis of products which have sculptured surfaces. Very little of a sculptured shape relates to three dimensional geometric primitives and regions of high curvature both in and out of the local plane of a sculptured surface cause difficulty in filling the surface with a pattern of elements which is well suited to the expected loading conditions. This important issue is addressed in this thesis by the introduction of a successful procedure for the semi-automatic generation of quadrilateral finite element meshes on sculptured surface products.

Commercial finite element software increasingly shields the user from the details of the analysis in an effort to reduce the amount of specialist finite element knowledge required to use the technique. Whilst this may encourage more widespread use of the finite element method, a greater potential exists for errors to occur in the analysis caused by the relative inexperience of the operatives. This is particularly relevant when complex loading conditions, such as impacts between three-dimensional objects, are modelled as the inherent non-linearity in the analysis can cause errors in the finite element solution to be amplified excessively. It is therefore necessary to seek experimental evidence relating to the behaviour of the real product, either under closely controlled conditions or in the actual service environment, with which to validate the output from the finite element model. The new experimental techniques introduced in this thesis are related by their ability to produce detailed experimental validating data for finite element impact analyses and all are also of importance to general vibration measurement. A validated, detailed finite element impact model is a powerful design analysis tool which, by providing the capability to change parameters quickly and analyse their effects, can assist in reducing design iterations and hence the cost of introducing a new product.

The golf club head is an example of a sculptured product which is subjected to an impact in which only elastic deformation occurs. Golf club design and manufacture has become increasingly complex in response to player demands for improvements in
equipment performance and the fashion bias of the golf equipment market which requires regular introduction of new styling concepts. However, the design to manufacture lead time can be considerable when developing new products since the golf industry remains conditioned by traditional values and practices. A craftsman generally sculpts an aesthetically pleasing shape with those features thought to enhance performance and the prototypes are play tested using golf hitting machines and players until a satisfactory design is obtained. This process is long and expensive and is inappropriate in the highly competitive golf equipment market. The world-wide golf equipment market is valued at around £1.2bn [1.1] and, in the UK, golf equipment is the largest sports equipment market segment with annual business of approximately £200m [1.2]. The ability to reach the market with a new concept ahead of the competition is therefore potentially very lucrative and a fast product design and analysis system of the type provided by a combined CAD and finite element approach is attractive in reducing new product introduction timescales. However, there have been relatively few finite element studies of the golf impact and all have been hindered either by insufficiently accurate representation of the sculptured club head shape or by difficulties in obtaining the detailed experimental evidence required to validate the behaviour of the impact model. The use of the finite element method in new product development is therefore uncommon in the golf equipment industry at present. This thesis demonstrates the design analysis capability offered by application of the finite element method and novel experimental techniques to the detailed study of the golf impact.

Product differentiation in the golf equipment market has, over the last few years, relied increasingly on claimed technological advances, many of which are unjustified. Very little is known about the detailed mechanics occurring during the golf impact and greater knowledge would allow genuine desirable or performance enhancing features to be designed into a product at an early stage in the design process. The golf impact is, however, an example of a high speed, short duration collision between two lightweight bodies and this presents considerable difficulties in obtaining appropriate experimental data. Measurements have been made previously during the golf impact using high speed video techniques [1.3]. This allows only the largest scale golf ball
deformations to be observed during impact and the mechanical insight which can be obtained from high speed video analysis is limited by the image resolution available at the high frame rates necessary to capture the event. In this thesis, novel use is made of the laser Doppler technique to capture detailed point vibration measurements from both the golf club head and golf ball during impact. The data is of greater accuracy and superior detail to that previously reported and is used in the validation of the golf impact finite element model. However, such novel use of the laser vibrometer is not entirely straightforward and necessitates the study of important issues in the development of the laser Doppler technique presented in this thesis.

Remote point vibration measurements using the laser Doppler technique are a practical and increasingly popular alternative to the use of contacting transducers in difficult measurement situations. All remote measurements, however, operate from a measurement perspective in which the transducer is fixed in space, rather than to a point on the target body. Whole body motion of the target therefore causes measurements made remotely using a laser vibrometer to originate from instantaneously probed regions on the target surface increasingly distant from the originally illuminated point and detracts from the otherwise high quality of data obtained. A method which compensates for relative whole body displacement between the target and a remote transducer fixed in space is introduced in this thesis.

A growing number of potential remote vibration measurement applications require whole body and oscillatory velocity measurement capability beyond the 10-15 ms\(^{-1}\) offered by even the most advanced commercial laser Doppler vibrometers. A new instrument is therefore required to address this measurement limitation and the development of such an instrument is reported in this thesis. The new laser Doppler accelerometer is directly sensitive to target acceleration and has several advantages over existing laser Doppler vibrometer systems including an easily adjustable working range with effectively no upper measurement limit, straightforward operation and use of low cost optical components.
1.2 Elastic deformation in impact

Physical impact involves the transfer of mechanical energy from one body to another by means of a collision between the bodies. In an ideal impact, there is no kinetic energy dissipation and the condition is described as 'perfectly elastic'. In reality, all impacts involve some kinetic energy loss, with the greatest losses occurring when a permanent, plastic deformation in one or more of the colliding bodies occurs. In general, as the initial relative velocity of the colliding bodies increases, the amount of energy which must be dissipated in the collision grows and greater plasticity is exhibited. For example, vehicle collisions occur in the velocity range 10-60ms\(^{-1}\) and significant amounts of energy are absorbed by gross plastic deformation of structural members designed specifically for that purpose [1.4, 1.5]. In higher energy collisions, it is difficult to absorb sufficient energy in a controlled manner and impacts can be particularly destructive. Bird ingestion in an aerospace engine during flight at velocities in the range 200-500ms\(^{-1}\) can destroy the entire engine [1.6] and the military applications of bombs, bullets and shells travelling at velocities in the range 500-1500ms\(^{-1}\) [1.4] are well known. The most severe impacts are encountered in space, where hypervelocity meteorites collide with spacecraft at velocities in the range 3000-30,000ms\(^{-1}\). The kinetic energy dissipated in these impacts is sufficient to cause metals to behave as liquids [1.7].

The impacts of interest in this thesis exhibit only elastic deformation, in that the colliding bodies return to their original, unloaded dimensions after the collision. Only a small percentage of the initial kinetic energy is dissipated and it is the capability to study the mechanism by which energy is transferred between the bodies which is of primary concern in this work. Impacts involving only elastic deformation are common in sport and are characterised by large elastic deformation of the projectile and significantly smaller elastic deformation of the striking implement. The large elastic deformation and recovery, often termed 'hyperelasticity', exhibited by the projectile during impact allows a large percentage of the initial energy provided by the player to be transferred to the projectile. As the mass of the projectile is generally much less than that of the striking implement, the projectile attains a final velocity considerably greater than the velocity of either body before the collision. For example, the peak club, racket
and bat velocities in golf [1.8], tennis [1.9] and baseball [1.10] respectively are in the range 30-45ms\(^{-1}\) and these produce initial ball velocities of the order of 50-60ms\(^{-1}\). This applies equally to sports, such as football, in which contact occurs directly between the player and the ball. The kicking action in football produces a peak translational velocity of the foot in the range 20-28ms\(^{-1}\) and a ball velocity of up to 35ms\(^{-1}\) [1.11]. A greater understanding of the detailed impact mechanics is of importance to the manufacturers of equipment involved in these sports.

1.2.1 The golf impact

The essential element in the game of golf is an impact in which a player effects a collision between a moving club head and a stationary ball with the intention of projecting the ball a predictable distance from the impact site in a desirable direction. The equipment available to the player for this purpose is restricted by rules laid down by the governing bodies of the sport, the Royal and Ancient Golf Club of St. Andrews and the United States Golf Association [1.12].

Two alternative striking implements, termed 'irons' and 'woods', are used to project the ball distances of up to approximately 275m. The club head of an 'iron' is commonly a mild or stainless steel forging or casting. A set of 'irons' varies mostly in terms of the length of the shaft and the 'loft angle', \(\theta\), the acute angle subtended on impact between the normal to the club face and the horizontal. 'Woods' have low loft angles and are used to project the ball the furthest possible distance. Traditional hand-crafted wooden club heads have been almost entirely replaced in recent years by hollow metal club heads of the same total mass which were initially investment cast stainless steel but are increasingly alloys of titanium. The major perceived advantage of the hollow club head is that the hollow construction moves more mass away from the centre of gravity, thus increasing the moment of inertia. This reduces the tendency of the club head to rotate in 'off-centre' impacts, where the centres of gravity of the club head and ball are not aligned in the impact direction, leading to less deviation from the intended line of flight of the ball. A hollow metal club head is shown in figures (1.1a&b), where the primary shape features and the loft angle, \(\theta\), are indicated.
The golf ball is a sphere of 42.67mm diameter and mass 45.93g [1.12] with a surface which is dimpled to reduce aerodynamic drag [1.13]. Two main ball construction variants exist but the basic impact and flight characteristics are similar. The most common construction is the 'two-piece' ball, which has a large polybutadiene core surrounded by an ethylene ionomer cover. The more traditional 'three-piece' and 'balata' constructions comprise either a solid rubber or liquid filled centre, elastic thread windings and either an ethylene ionomer or a synthetic balata rubber cover.

The impact between a hollow metal club head and a two-piece ball is the subject of study in this thesis. This club-ball combination is of particular interest to the golf equipment industry since the hollow club head and two-piece ball are two of the most significant equipment developments in the last 20 years [1.14]. Very little is known about the detailed mechanics occurring during the golf impact and both the club head shape and ball construction have evolved to their present states of development primarily by a trial and error approach over the last 150 years [1.15, 1.16].

The first reported scientific studies of the golf impact were undertaken in the late 1890's by Professor Tait of Edinburgh University. Experiments concentrated on determination of the contact duration and deformation occurring in the impact of falling blocks on samples of golf club and ball materials [1.17, 1.18]. Using relatively crude but ingenious apparatus, a contact duration estimate of 800μs was made for the golf impact. Several eminent scientists took an interest in the flight of the golf ball during the early twentieth century but the first co-ordinated study of the golf swing was not conducted until the mid 1960's [1.8]. Part of this study concentrated on the golf impact and successfully determined the basic impact parameters. The club head speed in a full shot was found to approach 45ms⁻¹ and to produce a ball velocity of up to 60ms⁻¹. A contact duration of 500μs was reported and impacts involving a club head which was freely hinged on the end of a shaft suggested that the shaft did not have a significant influence on the club-ball collision. In recent years the number of people interested in golf world-wide has increased dramatically and aspects of the game as diverse as biomechanics and golf course management have received scientific interest. However, there remain relatively few details of the golf impact as a result of the
difficulties encountered in performing accurate computational analysis or in capturing appropriate experimental data during the collision.

The pattern of deformation and recovery of the ball and its relationship to the force experienced by the club head is of particular interest to club and ball designers in understanding the energy transfer during the golf impact. The current trends in equipment development are, however, not directed towards projecting the ball a greater distance as this parameter is controlled closely by the rules of the game [1.12] and a majority of the most commercially successful golf balls already reach the overall distance limit in standard tests [1.19]. The potential to tailor club-ball combinations to players of different abilities is of greater interest. Furthermore, the hollow club head is able to vibrate in a frequency range close to that exhibited by the ball during impact in a way that solid club heads cannot. These vibrations contribute to the sound of the impact and the sensation in the hands of the player, which are two of the major feedback mechanisms by which the player judges the quality of the equipment and the golf shot itself. The exploitation of these parameters in new club head and ball designs requires a more detailed understanding of the mechanics occurring during impact than has been previously available and the work presented in this thesis addresses the issues which have to date prevented sufficiently detailed analysis.

1.3 Present contribution and thesis outline

The detailed study of the golf impact is used in this thesis to provoke important developments in the computational analysis of sculptured surface products and in non-contacting vibration measurements based on the laser Doppler technique. Whilst the majority of the results presented here are specific to the impact of a hollow golf club head and a golf ball, the techniques described are of considerably wider applicability. The main achievements of the research are summarised in this section, grouped according to their locations in this thesis.

In Chapter 2, the development of a successful procedure for the semi-automatic generation of quadrilateral meshes on sculptured surface products is reported. The sculptured surface geometry of a hollow golf club head is used to demonstrate the
mesh generation difficulties which are overcome using the proposed approach and a
spatially accurate solution domain for the finite element analysis of the golf impact is
created. This is complemented by a finite element ball model capable of
accommodating large deformation and strain rate dependent material properties. The
finite element modelling approach taken in Chapter 2 reveals the interaction between
the golf club head and golf ball with greater accuracy and in superior detail to
previously reported finite element studies of the golf impact. The techniques presented
therefore provide the basis for improved understanding of the detailed mechanics
occurring during impact.

Chapter 3 addresses the issue of finite element impact model validation by reporting
techniques of importance in validating the dynamic behaviour of the golf club head
model and aspects of the club-ball impact model developed in Chapter 2. Close
agreement between the results of computational and experimental modal analysis is
reported and this provides strong evidence for the validity of the finite element club
head model dynamic behaviour. Measurements of the sound created during the impact
show the majority of the sound energy produced by vibration of the hollow club head
surfaces to be contained in several distinct frequencies. Comparison with the modal
analysis results then allows the club head surface responsible for particular frequency
components in the impact sound spectrum to be identified. The results demonstrate an
ability to predict important 'feel' characteristics at an early stage in the design process
using a validated finite element model and this presents an opportunity for the
development of 'feel' as a quantifiable design parameter.

Chapter 4 describes a successful experimental procedure to obtain physical
measurements during a high speed, short duration impact between two lightweight
bodies, in which large elastic deformation occurs. A sensing arrangement is presented
in which a piezo-electric accelerometer is attached to one of the impacting bodies and
complementary measurements are made remotely on the second body using laser
Doppler vibrometers positioned at the impact site. The sensing arrangement represents
a novel application of laser Doppler vibrometry to the analysis of impacts and permits
the capture of data which is of superior detail and greater mechanical relevance than
that obtainable using high speed video techniques. The limitations of this measurement system are also discussed and these provide the motivation for the developments in the laser Doppler technique presented in Chapters 5 and 6.

Chapter 5 describes a displacement compensation technique for data captured by a remote transducer from a vibrating body which is also undergoing whole body motion. This displacement compensation technique uses data from two simultaneous remote measurements combined with a measurement of the whole body target displacement to produce a much closer estimate in time of the measurement which would have been made by a transducer fixed to a point on the body. The capability to derive data effectively from a single point on a moving body using remote transducers in difficult measurement situations is a valuable experimental tool in structural vibration analysis. As remote, non-contact measurement methods become increasingly popular, the work presented in Chapter 5 demonstrates the improvement in the quality of data which can be obtained using the displacement compensation technique. The analysis of a golf club head during impact and the golf ball deformation data presented in Chapter 4 are used as examples. Comparisons which show strong agreement between the experimental data and results from the finite element golf impact model, developed in Chapter 2, are presented.

Chapter 6 is devoted to the development of a first prototype laser Doppler accelerometer to a level where direct sensitivity of a laser transducer to target acceleration can be demonstrated for the first time. The laser Doppler accelerometer embodies an interferometric technique for the remote measurement of vibration acceleration and addresses both the current limitations of laser Doppler vibrometer systems and likely future measurement requirements. By choosing acceleration instead of velocity as the measured parameter, sensitivity of the instrument to target oscillations is maximised. Sensitivity to acceleration is also desirable generally as acceleration is the most popular descriptor of vibratory motion and is directly related to force. Laser beam separation and recombination by polarisation, laser coherence properties, sensitivity to back reflections from optical components and frequency pre-shifting issues are found to be crucial in isolating the particular optical beat frequency
which carries the acceleration signal from other optical beats in the instrument. Recommendations for the further development of the laser Doppler accelerometer into a viable practical instrument for general vibration engineering measurement applications are made based on the experience gained in this study.

The thesis is closed in Chapter 7 with conclusions and recommendations for future study in the areas of finite element mesh generation, the laser Doppler technique and the application of the research work in the sports equipment industry.
Computational analysis: the golf club-ball impact

2.1 Introduction

Physical impact involves the transfer of mechanical energy from one body to another by means of a collision between the bodies. Whilst the macroscopic momentum transfer aspects of an impact can be evaluated readily by invoking momentum conservation principles, the detailed process of energy transfer under impact conditions is complicated and leads to difficulties in the mathematical analysis especially when the impact duration is sufficiently short to necessitate consideration of stress wave propagation and reflection effects. Furthermore, geometric issues are important to the localised deformation of the colliding bodies in the contact region and the high strain-rates encountered during impact cause some materials to exhibit elastic properties which are altered significantly from the static loading case [2.1, 2.2].

In the first section of this chapter, previously reported analytical impact models are presented and used to model various aspects of the golf club ball impact. Models representing the physical system must be idealised to render them amenable to theoretical treatment and, as a consequence, complete solutions are obtained only for simple geometric configurations. The level of detail which is available from such analytical impact models is therefore generally poor. Additionally, each model only addresses one aspect of the impact, whereas several major, interrelated phenomena occur in the real golf impact.

The finite element method is a powerful engineering analysis tool capable of evaluating the spatial and temporal variations in acceleration, velocity and deformation which are of interest in impact analysis. In this numerical technique, all of the important influences on the impact are considered simultaneously and the output permits detailed analysis of the interaction between the impacting bodies. However, one of the biggest obstacles to the use of this method is the discretisation of often complex geometry into a suitable finite element mesh. The mesh generation task is tedious and error prone and
the size, shape and number of elements in the mesh has a direct bearing on the accuracy and cost of the analysis.

The mesh generation issue is addressed in this study by the development of a successful procedure for the semi-automatic generation of quadrilateral meshes on sculptured surface products. The hollow golf club head is an example of a sculptured surface product and is used in this chapter to demonstrate the mesh generation difficulties which are overcome using the proposed approach. The method utilises the geometric feature anatomy identified by an established feature-based approach to accurate representation of sculptured surface geometry [2.3]. The features in the product anatomy are categorised according to their structural importance and primary features are subdivided into regions with four curvilinear sides which can be meshed successfully using a generic advancing front algorithm. The introduction of boundaries within a primary feature gives the user the ability to control mesh density variations along the internal boundaries according to expected loading conditions in a way not possible when the product surface is considered as a whole. Independent meshing of individual features is therefore extremely attractive but the flexibility offered must be reconciled against the requirement for mesh continuity across feature boundaries and a mechanism for achieving this is included in the proposed approach.

The proposed mesh generation procedure is essential to the creation of a spatially accurate solution domain for the finite element analysis of the golf impact. Further important considerations discussed in this chapter include the choice of finite element solution method, representation of the contact between the colliding bodies and accommodation of large deformation and strain-rate dependent material properties in the golf ball model. The experimental determination of appropriate material properties is a critical aspect of all analytical impact modelling. In the golf impact, the non-linear response and large elastic strain exhibited by the golf ball pose a particular problem. It is not feasible to produce a material model which defines explicitly all aspects of the high strain-rate material behaviour. Instead, the approach taken is to populate an established large strain material model using experimental data from high strain-rate tests on samples of the golf ball core and cover materials. Minor adjustments are then
made to the material model in order to match more accurately the output of the model to the detailed experimental impact data presented in Chapter 4.

The finite element modelling approach taken in this chapter reveals the interaction between the golf club head and golf ball with greater accuracy and in superior detail to previously reported finite element studies of the golf impact [2.4-2.9] and this is confirmed in the detailed experimental verifications also presented in this thesis. The techniques presented in this chapter therefore provide the basis for improved understanding of the detailed mechanics occurring during impact.

2.2 Analytical impact models

2.2.1 Stereomechanical Impact

Stereomechanical impact is the classical impact theory and has a simple mathematical formulation based upon impulse-momentum principles. The eccentric impact of a ball and a striking implement which roughly approximates the shape of a golf club head is shown in figure (2.1). This impact is eccentric since the point of contact, P, between the club and ball does not lie on the straight line connecting the centres of gravity and the club face is not normal to the direction of translation of the club head. Eccentric impacts generally cause rotation of the colliding bodies and this can be analysed by applying the Principle of Conservation of Angular Momentum to the system [2.10, 2.11]. The presence of the golf club shaft is recognised by the impulses, \( F_x \) and \( F_y \), applied to the club head at point \( O \) as shown in figure (2.1). Considering the angular momentum about point \( O \) and taking the anti-clockwise direction as positive:

\[
m_A \dot{x}_{A1} l_1 = m_A \dot{x}_{A2} l_1 + m_A \dot{y}_{A2} l_2 - l_A \omega_{A2}
\]

\[
+ m_B \dot{x}_{B2} l_4 + m_B \dot{y}_{B2} l_4 + l_B \omega_{B2}
\]  

(2.1)

The left-hand side of equation (2.1) represents the angular momentum of the system about point \( O \) before impact and this is equal to the angular momentum after impact. However, in the impact shown in figure (2.1) the occurrence of smooth contact only at point \( P \) implies the line of action of the impact force, \( L_I \), must pass through the centre
of gravity of the ball and hence no rotational velocity is imparted on the ball, $\omega_{b2}=0$. In the real golf impact the golf ball attains a significant anticlockwise rotational velocity, suggesting that the line of action of the impact force lies below the centre of gravity of the ball. This can occur as a result of deformation of the ball during the impact.

Deformation of the ball during the impact causes the centre of gravity of the ball to be displaced relative to its position in the undeformed ball and also causes contact between the club head and ball to occur over an area rather than at a point. The existence of a contact area also promotes a greater frictional force between the club head and ball, perpendicular to the line $L$, than would occur in point contact [2.12]. The amount of deformation in the ball and the frictional force vary over the contact time and the actual line of impact will therefore vary in position and direction throughout the impact. This is approximated in figure (2.1) by the dotted line $L_2(t)$. The exact position and direction of $L_2(t)$ are extremely difficult to determine analytically and the model shown in figure (2.1) cannot therefore be used to predict the post-impact rotational velocities of the club head and ball.

If the deformation of the ball and the post-impact rotational velocities of the club head and ball are ignored, then the model shown in figure (2.1) represents an oblique impact between the two bodies and it is possible to substitute into equation (2.1) velocity values which are determined experimentally from the real golf impact using techniques to be described in Chapter 3. The golf club head of interest in this study is of mass $m_a=0.190$ kg and translates with horizontal velocity $\dot{x}_a=35$ ms$^{-1}$. The golf ball is of mass $m_b=0.0459$ kg and is initially stationary. Immediately after impact the measured horizontal translational velocities of the club head and ball are $\dot{x}_{a2}=22$ ms$^{-1}$ and $\dot{x}_{b2}=51.2$ ms$^{-1}$ respectively. The club head vertical velocity is zero before impact and $\dot{y}_{a2}=-1$ ms$^{-1}$ immediately after impact. Substituting these values into equation (2.1) with approximate values of the distances $l_1=20$ mm, $l_2=5$ mm, $l_3=16$ mm and $l_4=30$ mm, defined in figure (2.1), produces an estimate of the post-impact vertical velocity component of the ball $\dot{y}_{b2}=9.3$ ms$^{-1}$. This value is a reasonable estimate of the measured vertical velocity component and illustrates how the stereomechanical
representation of the golf impact can be useful in determining the basic momentum transfer occurring during the impact based on measured data.

A coefficient of restitution, $e_r$, is defined which characterises the energy loss during impact:

$$e_r = \frac{\text{relative velocity of separation along the line of impact}}{\text{relative velocity of approach along the line of impact}}$$

In this case:

$$e_r = \frac{\dot{x}_{A2} \cos \theta - (\dot{x}_{B2}/\cos \theta)}{\dot{x}_{A1} \cos \theta - (\dot{x}_{B1}/\cos \theta)}$$

(2.2)

For a perfectly elastic collision there is no energy loss and $e_r=1$, whilst in a perfectly inelastic collision the two bodies remain in contact and $e_r=0$. In the golf impact, approximate values in the range $0.65 < e_r < 0.95$ have been determined experimentally for impacts involving various club-ball combinations [2.13]. The particular golf impact of interest in this study is between a hollow metal club head and a two-piece golf ball. The angle $\theta=10.5^\circ$ and the coefficient of restitution, found using the experimental velocity data detailed above, is 0.91. The value of $e_r$ is dependent upon the material properties, mass, shape and relative velocities of the impacting bodies [2.10] and must therefore be determined experimentally for each particular impact investigated. The coefficient of restitution is therefore of limited use and attempts to apply this simplified impact theory to the eccentric impact of rigid bodies of varying shape is an oversimplification of the problem [2.14].

Considerable difficulty with the stereomechanical theory is found in the assumption that a negligible fraction of the initial kinetic energy is converted into vibrations of the colliding bodies [2.10] and in the failure to describe the significant local deformations at the impact site. This description of impact is thus limited to evaluation of the final velocities of the colliding bodies, and of the impulses involved. A more sophisticated representation is required in order to obtain detailed mechanical information pertaining to transient wave propagation effects, oscillations established during impact and
patterns of deformation and recovery, all of which are relevant to the design analysis of the impacting bodies.

2.2.2 Deformations during impact

Local deformations in the contact area are particularly significant when either of the colliding bodies has a curved surface and the energy required to produce this deformation may be a sizeable fraction of the initial kinetic energy of the system. A 'contact law' which relates the compressive force occurring between two bodies in contact to the local elastic deformation of the bodies was suggested by Hertz [2.15]. For contact between two elastic spheres of radii $R_A$ and $R_B$, with Young's moduli $E_A$ and $E_B$ and Poisson's ratios $\nu_A$ and $\nu_B$, the law can be stated thus:

\[
F = k_i \alpha_D^{\frac{3}{2}}
\]

where

\[
k_i = \frac{16}{9\pi^2} \frac{R_A R_B}{(\delta_A + \delta_B)^2(R_A + R_B)}, \quad \delta_A = \frac{1-\nu_A^2}{\pi E_A} \quad \text{and} \quad \delta_B = \frac{1-\nu_B^2}{\pi E_B}
\]

The relationship shown in equation (2.3) between the contact force, $F$, and the contact approach deformation, $\alpha_D$, was developed originally for static contact. However, Hertz law of contact can be used directly for the examination of elastic bodies in impact in cases where the impact duration is very long in comparison with the period of the lowest mode of vibration of the colliding bodies. This has been verified experimentally for very small elastic deformation in normal, frictionless contact for the collision of two spheres and of a sphere with a massive plate ($R_A \rightarrow \infty$) [2.10, 2.16, 2.17]. The explicit nature of the impact conditions under which Hertz contact law has been verified illustrates the limited applicability of the law. In particular, the assumption that the contact area is small compared to the dimensions of the colliding bodies is invalid for the golf club-ball impact since the ball has been shown experimentally to undergo significant deformation and to behave in a non-linear elastic manner. Also, the duration of impact is similar to the period of the lowest mode of vibration of a golf ball. However, the use of Hertz contact law beyond the limits of its validity has been justified previously on the basis that it appears to predict reasonably a number of the impact parameters which can be verified experimentally [2.10].
For impact of a sphere on a massive plate, the maximum contact approach, \((\alpha_D)_{\text{max}}\), is given by:

\[
(\alpha_D)_{\text{max}} = \left( \frac{15\pi v_A^2 (\delta_A + \delta_B) m_B}{16 \sqrt{R_B}} \right)^{\frac{3}{2}}
\]  

(2.4)

which for the golf impact gives a value \((\alpha_D)_{\text{max}} = 4.98\text{mm}\), based on the club and ball parameters shown in table (2.1):

<table>
<thead>
<tr>
<th>Club head</th>
<th>Ball</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_A = 0.3)</td>
<td>(v_B = 0.48)</td>
</tr>
<tr>
<td>(E_A = 207 \times 10^3 \text{MNm}^{-2})</td>
<td>(E_B = 164 \text{MNm}^{-2})</td>
</tr>
<tr>
<td>(v_c = 35 \text{ms}^{-1})</td>
<td>(R_B = 21.33\text{mm})</td>
</tr>
<tr>
<td>(m_A = \infty)</td>
<td>(m_B = 47.6\text{g})</td>
</tr>
</tbody>
</table>

Table 2.1 - Club head and ball material properties

The values of \(E_A\) and \(E_B\) shown in table (2.1) were obtained experimentally in tests of material samples, described in Section 2.5.2. The values of \(v_A\) and \(v_B\) are standard values for stainless steel and vulcanised rubber respectively. The maximum contact approach calculated using equation (2.4) overestimates the value measured experimentally by approximately 10%. This is, however, a notable result when the relative simplicity of the analytical model is considered.

The contact duration between the club head and ball can be estimated thus [2.10] :

\[
\tau = 4.53 \left( \frac{(\delta_A + \delta_B) m_B}{\sqrt{R_B v_B}} \right)^{\frac{3}{2}}
\]

(2.5)

Equation (2.5) gives a value of \(\tau = 419\mu\text{s}\), which is approximately 7% shorter than the contact duration of 450\(\mu\text{s}\) measured experimentally. Hertz theory assumes that the deformation and recovery times of the ball during the impact are equal, whilst detailed impact measurements presented later in this thesis show that this is not the case in the golf impact. Additionally, Hertz theory ignores vibrational effects which are present in the real golf impact. Considering these two limitations, Hertz theory could reasonably
be expected to provide inaccurate predictions of the impact duration and this has been
demonstrated previously [2.18] for the impact of a rubber sphere on a rigid foundation.
The calculation of a contact duration in this study, using Hertz theory, which is shorter
than the measured value is further evidence for behaviour of the golf ball which cannot
be predicted using simple analytical impact models. Vibration of the ball and large
strain viscoelasticity are significant features of the golf impact and their inclusion in the
analysis is important to the greater understanding of the detailed mechanics occurring
during the collision.

2.2.3 Vibration during impact

In the golf impact, the contact duration is of the order of the period of the lower
natural frequencies of vibration of the ball and the vibration response of the ball cannot
therefore be ignored. Vibration theory recognises that disturbances propagate through
the interior of bodies at a finite velocity and reflection of these oscillations at
boundaries produces standing waves in the body. Thus, all sections of the body are not
exposed to the same force simultaneously. If the duration of impact is too short for
significant standing waves to be established, then the propagational aspects of vibration
theory are of importance [2.19] and these are discussed here in the context of the golf
impact.

The golf ball core material, as discussed in Chapter 1, is a highly crosslinked
polybutadiene polymer and the cover is an ethylene ionomer. Many polymeric and
rubber-like materials exhibit flow when subjected to stress or strain and the flow is
accompanied by dissipation of energy used to overcome internal forces between long
molecular chains. In this viscoelastic material behaviour, the strain in the material lags
the applied stress and this is apparent in the vibration response of the material [2.1].
Furthermore, impacts cause mechanical stress pulses to propagate through the
colliding bodies. In a viscoelastic material the velocity of wave propagation and the
wave attenuation characteristics are frequency dependent. Consequently, a mechanical
disturbance will change shape continually during motion through a viscoelastic material
[2.20]. The dynamic behaviour of viscoelastic materials is of importance to the analysis
of the golf impact and is therefore considered in this section.
2.2.3.1 Mechanical models of viscoelastic materials

The dynamic response of viscoelastic materials can be described in terms of linear differential equations but these are commonly approximated by mechanical models comprising 'lumped parameters' represented by combinations of linear elastic springs and viscous dashpots. Systematic methods for building complicated networks from the basic elements have been reported previously [2.21, 2.22] but the mathematics associated with the analysis of such models becomes extremely involved. In this section, the basic two-parameter Kelvin-Voigt viscoelastic model is used to estimate the displacement of a free viscoelastic rod which is suddenly forced to move with constant velocity at one end. The model is supplied with golf ball material properties and linear dimensions which approximate those of a golf ball in order to investigate the ability of such a model to estimate the displacement of the golf ball during impact.

The displacement equation of motion describing free longitudinal vibration of a uniform viscoelastic rod, obtained from the Kelvin-Voigt two-parameter mechanical model, is given by [2.10]:

\[
\frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^3 u}{\partial x^2 \partial t} = \frac{1}{c_o^2} \frac{\partial^2 u}{\partial t^2} \quad (2.6)
\]

where

\[
\eta = \frac{c_o^2}{E_B} \quad c_o^2 = \frac{E_B}{\rho}
\]

and

\( u \) is the longitudinal deformation of the rod
\( c_o \) is a damping constant
\( E_B \) is the Young's modulus of the material
\( \rho \) is the density of the material
\( c_o \) is the speed of wave propagation

If the end, \( x=L \), of the rod is forced suddenly to move at a constant velocity, \( v_o \), and the opposite end of the rod, \( x=0 \), is free to move, the longitudinal displacement of any section, \( x \), through the rod at time \( t \) is given by [2.10]:

\[
u(x,t) = v_o t + \frac{2v_o}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2i-1} \cos \left( \frac{2i-1}{2} \right) \cos \left( \frac{\pi x}{L} \right) \left[ \frac{\exp(\omega_{\lambda 1} t) - \exp(\omega_{\lambda 2} t)}{\omega_{\lambda 1} - \omega_{\lambda 2}} \right] \quad (2.7)
\]

where
\[ \omega_{t,1}, \omega_{t,2} = -\frac{1}{2} \left[ \frac{2i-1}{2} \right] \frac{\pi^2 c_e^2 \eta_\beta}{L^2} \pm \sqrt{\frac{1}{4} \left[ \frac{2i-1}{2} \right] \frac{\pi c_e}{L} \sqrt{\pi^2 c_e^2 \eta_\beta^2 - 4 - 1}} \] \quad (2.8)

Equation (2.7) is a series summation over integer values of \(1 \leq i \leq \infty\) and, in this study, the other parameters in the model are chosen to match those of a golf ball. Thus, the length of the rod, \(L=42.6\text{mm}\), is equal to the diameter of a golf ball. The Young's modulus, \(E_p=164\text{MNm}^{-2}\), is obtained from high strain rate compression tests of golf ball material samples, discussed in greater detail in Section 2.5.2. The wave propagation velocity, \(c_0=2300\text{ms}^{-1}\), was calculated from experimental observations of wave propagation across the ball. The damping constant, \(c_B\), is the main unknown and a reasonable value for this type of material of \(c_B=4000\text{Nsm}^{-1}\) is chosen \[2.10\].

Computational evaluation of equation (2.7), with a constant velocity \(v_o=28\text{ms}^{-1}\) equal to the average club head velocity during the golf impact, suggests translation of the free side of the rod equal to 13.3mm after a time of 450\(\mu\)s. This result is shown in figure (2.2), where it can be seen that the forward displacement of the free end of the rod is low over the first 100\(\mu\)s in which the force is applied and increases rapidly after this time. Experimental results presented later in this thesis indicate a similar displacement history for the golf ball during contact with the club head and the total distance translated by the golf ball is 11mm.

The analysis presented in this section demonstrates the complexity of even the simplest analytical models of dynamic viscoelastic material behaviour during impact and the relatively small amount of information which can be extracted from the model. More complex models have been presented for the golf ball in impact and it is suggested that a five or six parameter model may be necessary to predict reliably the displacement of a point on a golf ball during normal impact \[2.23\]. Further complicating factors are introduced if the model is required to represent eccentric impact, as in a genuine golf club-ball collision \[2.24\].

2.2.3.2 Wave propagation in viscoelastic materials

Detailed experimental measurements presented later in this thesis reveal propagation of a deformation pulse from the impacted side of the golf ball to the free side and back
again. In general, a mechanical pulse is dispersed as it propagates through a viscoelastic material since high frequency components travel faster and are attenuated more rapidly than those of lower frequency. This dispersion is observable in the experimental data from a golf ball and it is therefore pertinent to analyse pulse propagation in viscoelastic materials at this point.

A purely analytical approach to the problem of pulse propagation in a general viscoelastic solid leads to intractable mathematical expressions, while treatment of the solid in terms of simple mechanical models of the form discussed above suffers from the inherent limitation that the behaviour of most plastics and rubbers does not conform to the model over more than a limited frequency range [2.20].

A technique which uses a spectral approach [2.25] to calculate successfully the shape change in a pulse as it propagates along a viscoelastic rod has been developed previously [2.26]. A Fourier analysis of the initial pulse shape is carried out and changes are made to the amplitude and phase of each Fourier component, according to frequency dependent velocity and attenuation characteristics. The pulse is then re-synthesised. A frame of reference is chosen which moves with the pulse such that $t=0$ when the pulse reaches the observed point on the rod, a distance $x$ from the origin of the pulse. The resulting Fourier series expression of the local stress $\sigma(x,t)$ is then given by:

$$\sigma(x,t) = \sum_{n=0}^{N} C_n \cos(n\omega t) + \sum_{n=1}^{N} D_n \sin(n\omega t)$$

(2.9)

where

$$C_n = \exp(k_n \omega \tau) \cos(n\omega x(1/\dot{x}_\tau - 1/\nu'))$$

$$D_n = \exp(k_n \omega \tau) \sin(n\omega x(1/\dot{x}_\tau - 1/\nu'))$$

and

$$\frac{1}{\dot{x}_\tau - 1/\nu'} = \frac{\tan \delta}{\pi \dot{x}_\tau} \ln \frac{\omega x}{\omega_o \dot{x}_o}$$

As an example, the properties of a polymethylmethacrylate rod presented in [2.26] are supplied to equation (2.9) but the resulting pulse shape and propagation characteristics are applicable to all polymeric materials. The internal friction in the viscoelastic material is represented by the tangent of the phase lag, $\delta$, between the stress and strain.
The value of \( \tan \delta \) changes relatively little with frequency and is therefore assumed constant, in this case \( \tan \delta = 0.08 \). The lowest frequency of interest in this Fourier series representation is given by \( \omega = \dot{x}_p / 2x \tan \delta \), where the group velocity of propagation of the dispersive pulse \( \dot{x}_p = 2300 \text{ms}^{-1} \). The constant \( k_2 \) is defined as \( k_2 = -\tan \delta / 2 \dot{x}_p \) whilst \( \omega_0 \) and \( x_o \) are approximately equal to 1. [2.26]. The pulse-shapes calculated at distances \( x = 6 \text{m} \) and \( x = 8 \text{m} \) from the point at which the pulse originates are shown in figure (2.3), where attenuation and dispersion of the pulse caused by propagation through the viscoelastic material can be observed.

Whilst this model of pulse propagation through a viscoelastic material allows considerable insight into the way in which the individual material parameters affect the pulse shape, the model is valid only for a long slender rod and is therefore of limited practical value in the analysis of the golf impact. In particular, the length of the pulse is large compared to the diameter of the rod and this allows the effects of lateral inertia to be neglected. These effects could not be neglected in a model of pulse propagation through a viscoelastic sphere and the mathematics required would be of significantly greater complexity.

The model presented here is, however, developed later in this thesis to incorporate translation of the rod in the direction of pulse propagation. The analytical solution is then used to validate an experimental method for capture of pulse propagation data from a translating viscoelastic body using a transducer which is fixed at a point in space rather than attached to a point on the translating body.

None of the analytical models presented in this section successfully represent more than one feature of golf ball behaviour during impact and all rely on accurate experimentally determined material properties for their operation. The examples reported demonstrate the generally poor level of detail available and the requirement to approximate the actual bodies of interest by simpler solids in order to achieve an analytically tractable solution.
A technique is required which provides superior detail on models which resemble the physical object of interest closely both in shape and material properties. The finite element method is such a technique and its application to the detailed study of the impact between a hollow golf club head and a golf ball is the subject of what follows in this chapter.

2.3 Introduction to the finite element method

Finite element analysis is a term which describes a group of numerical methods for approximating the governing equations of any continuous system [2.27]. The prime application is the analysis of structural mechanics but finite element methods are equally suited to other continuum problems in heat transfer and fluid flow. Whilst the finite element method cannot predict the response of the physical problem exactly, it is possible to gain a detailed insight into the problem under investigation. This is of major benefit in the design analysis of products whose complex shape, loading or material behaviour preclude analysis by conventional means.

The use of the finite element method in engineering analysis has grown with the increasing accessibility of the computing power required to solve large numbers of equations numerically. The technique is most useful when allied to the shape representation capability of modern computer aided design (CAD) systems in such a way that the CAD geometry forms the basis of the finite element solution domain. However, a suitable universal data exchange format for transferring detailed analysis information between specialist CAD and finite element codes does not exist at present [2.28] and the implementation of comprehensive product data models which incorporate analysis information remains the subject of ongoing research [2.29]. The current trend is towards greater integration of the finite element method within CAD systems but the analysis capability offered by integrated systems is currently limited to basic loading conditions.

The first step in the finite element analysis process is the discretisation of the geometry of interest into a mesh of nodes and elements which is well suited to the physical situation under investigation. The mesh is the spatial representation of the
mathematical model of the physical system and is crucial to the quality of the analysis solution. Finite element mesh generation on sculptured surface geometry is a more complex task than for prismatic parts since it requires arbitrarily shaped surfaces to be populated with elements which have essentially square shaped boundaries. The hollow golf club head is an example of a sculptured surface product and is used in this chapter to demonstrate a new approach to the generation of well formed quadrilateral element meshes on sculptured surface products.

The material properties of the physical bodies of interest must be represented in the finite element model as in the analytical models discussed in the previous section and the accuracy of the analysis is similarly dependent upon the quality of the information supplied. External loads and boundary conditions which reflect the service environment of the physical bodies must also be applied to the model. The finite element method then reduces the governing differential equations to a set of algebraic equations by variational methods using the principle of virtual work [2.30]. A polynomial interpolation function is defined for each element and used to give the analytical displacement for any point inside the element. The polynomials are substituted into the variational formulation and the algebraic equations are solved numerically using a computer to give the nodal values of the unknowns.

A considerable initial investment is required to represent the physical situation of interest accurately in a finite element model before any results are produced. This initial investment is, however, repaid in the level of detail which is available from the analysis of an accurate finite element model. The computational results are superior to the analytical models shown earlier as all of the interrelated geometric, material and loading influences on the bodies of interest are considered together without the gross simplifications necessary when using analytical models. For sculptured surface products, accurate representation of the surface geometry and its discretisation into finite elements form the majority of the initial investment and techniques aimed at reducing this important aspect of the analysis cost are presented in the following section.
2.4 Geometry representation and discretisation of sculptured surface bodies

Geometrical accuracy is essential if the model is to be used in a predictive capacity but accurate models of sculptured surface structures are generally more difficult to produce than prismatic structures. In this study, an established feature based approach [2.3] is applied to the accurate representation of hollow golf club head surface geometry in a CAD model. A new and pragmatic approach to quadrilateral finite element mesh generation on sculptured surface products is then proposed, based on the existing geometrical design features. The benefits of the proposed approach are demonstrated using the hollow golf club head as an example.

2.4.1 Feature based geometry representation

Feature based design is a formalised method which allows users to describe their product as an assembly of product elements that may specify shape, material, manufacturing processes, expected loading conditions etc. A significant amount of work has been carried out to identify and categorise prismatic features for general engineering products, with the aim of providing feature based design, manufacturing process planning and design analysis tools [2.31]. The application of feature based techniques to parts with sculptured surface geometries is more difficult as, unlike the majority of prismatic parts, there is no universal set of features. Instead, products can often be grouped into families having similar feature anatomies which share a set of common product specific shapes. An 'extended form feature methodology' has been reported previously in which the anatomy of a specific product is established by identifying three classes of sculptured feature [2.32, 2.33]. Primary features define the underlying shape of the product, with the extent of a primary feature's shape contribution limited to a boundary defined by its relationship with neighbouring features. Primary features can be direct neighbours but are more commonly joined by secondary features or 'blends'. Secondary features often contain some of the most complex sculptured surface geometry information and control the aesthetic quality of the product, whilst tertiary features are ornamental markings.

From a structural viewpoint, primary features are always of importance, whilst the role of secondary features is less clear. Secondary features are often involved in areas of the
product's surface geometry which have stress raising characteristics but in some cases the sculptured shape of the blend may be of little structural significance and can be devolved from the model i.e. replaced by the intersection of the primary features common to the secondary feature boundary. An ordered approach to hierarchical devolution of blends from the model is inherent in a feature based sculptured product model [2.3]. Tertiary features are rarely included in structural analysis as the difficulty of discretising these often small and complex markings significantly outweighs the structural importance of the feature.

The absence of a universal set of design features has permitted the development of product specific feature anatomies and feature sets that allow designers to define their product family in terms of features they recognise. This has been demonstrated successfully for the shape definition of golf club heads and shoe lasts where parametric feature manipulation is used to facilitate rapid creation of design variants [2.34, 2.35]. The work presented in this chapter demonstrates how the design features can be used successfully to generate surface meshes suitable for finite element analysis of the product.

A feature anatomy of a hollow golf club head is shown in figure (2.4). In this study, all of the primary form features of an existing club head shape were reverse engineered into a CAD system using a 3-dimensional co-ordinate measuring machine with a touch trigger probing system. The digitised data was used to create extended forms in the DUCT surface modelling software and the whole club head was modelled using the extended form feature methodology. However, the blends were created approximately by using the rolling ball filleting facilities of the software. A wire frame view of the club head geometry model is shown in figure (2.5) with primary form features labelled. An unwanted ripple in the modelled surface occurred due to a small surface digitisation error on the crown. The ripple was removed by averaging over the surface but this left an average geometrical error over the crown of 0.14mm, which is significantly greater than the corresponding error in the face and sole geometries. The section thicknesses were measured to an accuracy of 0.01mm using a pointed anvil micrometer. Areas of
the model which receive preferential thickening, e.g. under logos, were not modelled and may be a source of slight inaccuracy.

2.4.2 Discretisation of sculptured surface geometry for finite element analysis

Generation of a valid mesh on a product with complex geometry can be difficult and expensive in terms of the time required. This first step of the analysis process is, however, important as creation of a mesh which is well suited to the physical situation under investigation is crucial to the quality of the analysis solution.

Sculptured surface products are represented in CAD systems by parametric surfaces and considerable effort has been applied to placing triangular meshes on such surfaces, as discussed in [2.36]. However, triangular finite elements produce less accurate results and are less versatile than quadrilateral elements. In particular, thin triangular elements exhibit over stiff behaviour in bending, known as shear locking, and are therefore to be avoided. Details of element formulations are not discussed in this thesis but can be found in specialised texts [2.30].

Methods exist for the conversion of triangular meshes into quadrilaterals but the resulting elements are often poorly shaped [2.37]. A technique for automatic generation of an entirely quadrilateral mesh on a parametric surface has, however, been reported previously [2.38]. The technique, known as 'paving', uses an advancing front method to create well formed meshes of nearly square elements on free form surfaces. However, the surface geometry of most sculptured products is too complex to pave as a single free form shape. Excessively distorted elements are created in areas of high curvature and little control over node placement can be obtained. The detailed operation of paving algorithms is not discussed here but has been documented previously [2.39, 2.40].

A feature based approach to mesh generation is attractive, since the individual surface features are a good first subdivision of the sculptured surface geometry into regions of simpler shape than the entire product surface. This provides the basis for the structured use of the paving algorithm presented here.
The extended form feature boundaries identified in [2.33, 2.35] usually occur at discontinuities of surface slope or curvature and there is generally little surface discontinuity within the boundaries of each feature. However, whilst each feature may be a simpler shape than the entire product surface, these shapes cannot be meshed in isolation unless continuity of the mesh is guaranteed across feature boundaries. The potential to attain mesh continuity across feature boundaries has been demonstrated previously [2.41] for the generation of triangular element meshes on complex sheet metal pressings which include several simple prismatic features, such as slots and channels, on a base surface with much less significant curvature. Similarly, this is possible using the paving algorithm as it is sensitive to the boundary of the region in which it is operating i.e. the mesh contours follow the contours of the boundary closely, providing the potential to mesh features independently with quadrilateral elements once the mesh requirements of neighbouring features are reconciled along feature boundaries.

The secondary features or 'blends' generally separate primary features in space and thus define the relationship between primary features. The blends commonly have long and thin shapes which place more stringent restrictions on element size and shape than the large primary features and hence provide a basis for reasoning about element size distribution and node placement. The blends must therefore be considered first in the sequence of individual feature meshing. Node placement on boundaries of primary features adjoining the blend boundaries must match the node placement in the blend and so the primary features inherit element size restrictions from the blend. A primary feature may have several different neighbours around its entire boundary and will thus inherit several element size restrictions. This can be seen in the distribution of element size and shape in the blends of the hollow golf club head model shown in figure (2.6). With an even number of nodes placed around the boundary, the paving algorithm can potentially mesh the primary feature as an isolated entity since mesh continuity with other primary features is assured through the relationship defined at the blend boundary. In practice, however, meshing the large primary features as single entities
causes two problems related to the placement of nodes in the regions away from the feature boundaries.

i) Control over element size in regions away from the boundary is exercised in the paving algorithm by attempting to retain the node spacing of the boundary. However, the number and location of nodes defined on the boundary is determined by the relationship with other features in the product and may not be conducive to the formation of suitably shaped finite elements in the feature interior, away from the boundary. The paving algorithm attempts to maintain element size, perpendicularity and overall mesh smoothness. These criteria are often conflicting and the problem is exacerbated by localised regions of high curvature within the sculptured surface geometry and the requirement for mesh size transitions across surface regions. In situations where this occurs, the paver can produce a 'knot' of elements with below average dimensions in the centre of the surface, as shown in figure (2.7a), and a significant proportion of the elements created are excessively skewed or tapered.

ii) In addition to achieving adequate representation of the sculptured geometry in a discretised form, consideration must also be given to the influence of the physical situation under investigation on the local mesh density requirement in regions of expected high stress gradient. By their very nature sculptured products often contain fewer stress raising features than prismatic products. However, the high local curvature of the blends often produces regions of relatively high local stress in areas which also require local mesh refinement for accurate geometric representation. This is different to a prismatic part in which a corner, for example, could be discretised successfully using large elements but which may require significantly smaller elements for analysis purposes. The link between regions of high curvature and expected stress intensity in sculptured surfaces is, however, insufficient to guarantee adequate mesh refinement in all cases. For example, the application of a point load in the middle of a primary feature which has low curvature would require mesh refinement in that area. This requirement should be identified by the analyst in advance but using the basic paving algorithm there is
little scope for explicit influence over the mesh density in the interior of a surface feature.

A procedure which affords greater control over the paving algorithm in the feature interior is therefore required.

2.4.3 Proposed method for feature based quadrilateral mesh generation on sculptured surface products

A new procedure for the generation of well shaped finite element meshes on sculptured surface products is proposed in this section based on the subdivision of primary features in the product anatomy into smaller surface regions of simpler shape. Whilst the method is product specific, in that it relies upon subdivision of a product specific primary feature set, it is readily extended to a family of products with the same basic feature anatomy if the points which define the subdivision are able to move according to parametric changes in feature shape. A pragmatic method which makes significant advances towards achieving this goal has been developed as part of this study and is reported in [2.36]. The work reported in this section is therefore the foundation of a genuinely useful semi-automatic mesh generator for sculptured surface product families.

Greater control over the behaviour of the paving algorithm away from the boundary of primary features can be obtained by introducing new boundaries within the feature interior. Subdivision of primary features into smaller regions is an effective method of introducing new internal boundaries. In this way more of the mesh is brought close to a boundary, along which it is possible to place nodes in a pre-determined pattern, hence forcing the paver to behave in a particular manner [2.36]. The local mesh density can then be manipulated readily to satisfy both geometric and loading requirements. A subdivision of the primary features into regions with four curvilinear sides also reduces element distortion errors since the regions being meshed are approximately matched to the shape of the quadrilateral elements.
The subdivision of primary features into four-sided curvilinear regions is feature specific but offers considerable flexibility within each feature. The pattern of subdivided surfaces is, however, influenced strongly by three rules essential for mesh continuity across feature boundaries and by requirements for element size transition through the feature. Firstly, as discussed previously, the 'primary features inherit element size and node positioning from neighbouring blends.' Thus, the sides of subdivided surfaces which lie along the original primary feature boundary, or external boundary, must be of a length consistent with a whole number of elements in the neighbouring blend and be aligned to match node locations in the blend. Secondly, in regions of the product where primary features are direct neighbours, the subdivision pattern of the primary features must be consistent with each other. Thirdly, the subdivided surfaces must be approximately quadrilateral for best results. A subdivided crown feature of a golf club is shown in figure (2.7b). The subdivision pattern shown is generated manually from 10 points identified on the boundary and four internal points. The pattern was based on the mesh generated by the basic paving algorithm, shown in figure (2.27a), and reduces element shape distortions, which are due partly to the high in-plane curvature of the crown boundary, by forming four-sided regions with curvilinear sides. The subdivision pattern also controls the mesh transition from a greater number of large elements on the left of figure (2.7a) to a lesser number of smaller elements on the right, where the crown adjoins the hosel. The hosel is a region of both high curvature and high stress gradient, therefore a higher element density is required in this area for both adequate representation of the geometry and sufficiently detailed analysis.

The mesh generated after subdivision, shown in figure (2.7c), has a more appropriate distribution of element size and shape than the mesh shown in figure (2.7a). Additionally, all of the elements in figure (2.7c) pass standard tests for warp, taper and skew distortion errors. Tests on several feature shapes show that meshes generated from subdivided surfaces are superior to meshing primary features as single entities. Also, the quality of mesh generated is not particularly sensitive to minor variations in the position of the points which define the subdivision pattern, implying these points need only be positioned approximately to achieve good results.
The element mesh produced using subdivided surfaces is influenced strongly by node placement on the internal and external boundaries of a feature prior to the paving process. The number of possible node patterns which can be placed on the internal boundaries is restricted by the requirement for an even number of nodes on the boundary of each subdivided surface and by the node placement patterns inherited from the blends on the external boundaries.

In practice, the mesh pattern created by the paver within each sub-surface is very sensitive to the mesh smoothing parameters specified. In particular, small changes in the 'global element edge length', the control handle used to maintain consistency of element size, can produce alternative node connectivity patterns within a feature sub-surface. Hence, several alternative meshes are possible for each sub-surface. High internal surface curvature and the requirement for mesh transition across the sub-surface may cause excessively tapered or skewed quadrilaterals to be created in some or all of these meshes but in many cases the distorted elements can be eliminated by further smoothing. Laplacian smoothing generally provides good results except in the presence of significant internal curvature [2.42], in which case elements are pulled towards the inner curvature boundary. Isoparametric smoothing [2.42] has been found to be more appropriate for eliminating distortion errors due to in-plane curvature, but there are no quantitative rules indicating where each method should be applied.

More generally, a situation requiring a compromise between the two methods is encountered since isoparametric smoothing will tend to reduce taper errors at the expense of increasing the skew of elements, whilst Laplacian smoothing has the opposite effect. It is possible to use a weighted combination of the two methods [2.42] but the combination which provides best results is generally only found by trial and error. Thus, the situation arises where several global edge lengths could be used to generate a number of meshes within each sub-surface and several weighted combinations of smoothing method could be applied to each mesh. The skew and taper of the elements in a mesh can both be quantified and are therefore good criteria for manual selection of the best mesh from a group of alternatives.
Significant potential for automation exists in the procedure leading to the final mesh on a sub-surface described above. This procedure must be repeated for all sub-surfaces and the final mesh of the entire product is obtained by pasting together sub-meshes in a bottom-up construction. A flow chart of the process required for each sub-surface is shown in figure (2.8).

Using these rules a quadrilateral mesh was generated on a complete hollow golf club head. This is shown in figure (2.9), where it can be seen that two small triangles were introduced manually into the blend between the front face and the crown primary features. This is a typical occurrence and is indicative of the fact that some sculptured surface geometries cannot be meshed using quadrilaterals exclusively. Further practical details of hollow golf club head modelling using this procedure are reported in [2.43].

The mesh generation procedure proposed here is generally applicable to sculptured surface product feature anatomies. The method is, however, product specific in that it relies upon subdivision of a product specific primary feature set. For each product anatomy, a product family usually exists based upon relatively minor parametric shape variation of the features. The mesh generation procedure can therefore be made into a genuinely useful tool if the surface subdivision pattern can be applied across the product family. A pragmatic method which makes significant advances towards achieving this goal has been developed as part of this study and is reported in [2.36]. In this method, surface subdivision and boundary node placement attributes are attached to each feature in a hierarchical product data model and are altered automatically in accordance with parametric changes in feature shape. The generic surface subdivision and boundary node placement patterns stored in the product data model for each feature thus adapt sensibly to each actual feature instance and can therefore be used to generate new meshes of well shaped elements on variants of the original product surface geometry. The return on the initial investment in generating a surface subdivision manually on the base product of a new family is manifest in the ability to mesh other members of the product family automatically, or with only minor manual intervention if geometry variations across the family are large. Also, in cases where a
design iteration is required as a result of finite element analysis, the redesigned product can be meshed automatically using the data in the product model.

The procedure has been used successfully in the generation of quadrilateral meshes on hollow golf clubs [2.36] and is applicable to other sculptured product families. The results of this work provide the basis for developing the procedure into a semi-automatic quadrilateral mesh generation process for sculptured surface products.

2.5 Golf impact finite element model

2.5.1 Hollow golf club head model

The finite element hollow golf club head model, created using the procedure presented in the previous section, is shown in figure (2.9). This mesh was constructed within the MSC/PATRAN pre/post processing environment and contains 701 two-dimensional quadrilateral shell elements. The mesh eliminates element distortion errors and has greater density of elements in regions of highest expected stress gradient during impact. The element thicknesses are determined from pointed anvil micrometer measurements on the real club head and range from 0.6mm in areas of the bowl feature to 3.5mm at the bottom of the face. The element thickness imposes a lower limit on the element edge length if the thin shell assumption is not to be violated and this is particularly relevant on the club face, where the elements are thickest.

The Young’s modulus of the material was determined by static tensile tests on a specimen cut from a stainless steel investment cast club head and, as a result of the club head model validation described later in Chapter 3, a value of 207x10^3 MNm^-2 was used. The density of the material is \( \rho = 7850 \text{ kg/m}^3 \) which, when applied to the finite element model, gives the club head model a mass of 0.190kg. This agrees closely with the mass of the real club head and suggests that an inertia properties calculation based upon the distribution of element mass can be conducted with considerable confidence. This type of calculation can be performed quickly by finite element software and provides estimates of the centroid location and the principal moments of inertia. This is of benefit to club head designers as these mechanical attributes are difficult to

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determine experimentally for sculptured shapes [2.44] and are important in golf club head design.

2.5.2 Golf ball model

The finite element ball model is based on an icosahedron shape which is subdivided into 320 facets and projected onto the surface of a sphere. The outer sphere is shown in figure (2.10a) and is composed of 320 wedge elements of thickness equal to that of the cover material in a two-piece ball. The core of the ball is filled by seven concentric hollow spheres of decreasing radius, each containing 320 wedge elements and the central solid sphere is composed of 320 tetrahedra, making a total of 2880 elements. This element arrangement is shown in the cut-away section through the ball model in figure (2.10b) and is necessary to prevent the mesh introducing asymmetry into the behaviour of the ball model. Such asymmetric behaviour was apparent in an earlier ball model which was created by rotating a meshed plane semi-circle axisymmetrically through 360°. Additionally, the similarity of the element shape throughout the ball assists in accommodating the large deformation created during impact without excessive element shape distortions which would introduce significant errors into the analysis.

Accurate representation of the ball material properties is essential to the successful modelling of the ball behaviour during impact. In this study, static compression tests were conducted on core and cover material samples of a two-piece ball. Compression tests at various rates of strain indicated that the behaviour of the core material is strongly strain-rate dependent [2.45]. At a low strain-rate a Young's modulus of 85.7MNm⁻² was recorded, whilst a test at the highest available strain-rate of 10ms⁻¹ suggested a Young's modulus of 164.4MNm⁻². In the golf impact, the club head translates at a velocity of the order of 30ms⁻¹, hence the high strain-rate test data was used to provide the closest available approximation to the actual core material behaviour for the finite element model. Equivalent tests on the ethylene ionomer cover material revealed a Young's modulus of 276MNm⁻².
2.5.3 Impact analysis

In this section, issues of importance in the impact analysis of a finite element model are presented. In particular, the choice of solution method, representation of the contact between the colliding bodies and accommodation of both large deformation and strain-rate dependent material properties are discussed.

The golf impact is an example of a non-linear transient event in which further non-linearity is exhibited in the material behaviour of the ball. The dynamic quantities of interest are required for output at a number of instants throughout the impact and the necessary finite element solution can be obtained using the explicit central difference integration operator [2.30]. This solution method is termed 'explicit' in that it obtains a solution for dynamic quantities at time $t+\Delta t$ using only the available values at time $t$. An implicit solution method uses values of dynamic quantities at $t$ and $t+\Delta t$ to calculate the solution for time $t+\Delta t$ and must iterate to solve a system of non-linear equations. The explicit method is computationally efficient as it requires no iteration but the central difference integration operator is only conditionally stable and smaller timesteps than those required by the implicit method must be taken. In general, the stability limit for an explicit solution is equal to the time taken for an elastic wave to cross the smallest element dimension in the model. Greater timesteps cause instability and errors resulting from rounding, for example, will grow and affect the results severely.

The relative economy of the implicit and explicit methods depends on the size of the model, the ease with which the non-linear equations can be solved for the implicit operator and the relative size of time increments that can provide acceptable accuracy with the implicit scheme compared to the stability limit of the explicit scheme [2.46]. In general, the implicit method is used in 'inertial' problems, where the time intervals of interest are long compared to the time required for waves to traverse the structure and the overall dynamic response of the structure is sought. The explicit method is employed in cases where wave propagation effects and the localised response of part of the structure are of interest. In this study, the ABAQUS/Explicit analysis code is used to provide detailed results at intervals of 5$\mu$s throughout the 450$\mu$s duration of the impact.
Contact problems range from frictionless contact involving small displacements to the large strain plasticity and high friction encountered in materials processing operations. For the golf impact, a three-dimensional 'small sliding' deformable contact definition is appropriate. With this formulation the contacting surfaces can only undergo small sliding relative to each other but arbitrary rotation is permitted. The small sliding assumption is valid for the 10.5° club face loft angle of interest since no sliding is observed between club face and ball in high speed video images of the contact presented later in this thesis. The less computationally efficient 'finite sliding' formulation may however be required at higher loft angles, where sliding and rolling of the ball on the club face have been observed experimentally. The ABAQUS/Explicit solution code used in this study employs a master-slave relationship between the two contacting surfaces [2.47] and the slave surface, in this case the golf ball outer surface, is covered automatically in contact elements appropriate to the particular contact conditions.

The effect of friction between the contacting bodies at the loft angles of interest in this study was shown to be relatively small, as tests in which values of the frictional coefficient in the range 0.05-0.50 were used revealed very little sensitivity of the analysis solution to this parameter. A small frictional coefficient of 0.1 was therefore chosen arbitrarily. Friction is of greater importance at higher loft angles and must be represented accurately in models of these impacts. The friction coefficient is difficult to determine accurately during the golf impact, however previous studies [2.48] have reported friction coefficients in the range 0.07-0.38 for golf balls impacting on flat rigid plates.

The high strain-rate Young's modulus, determined in the previous section, is supplied to an isotropic hyperelastic material model in the ABAQUS/Explicit finite element code. A hyperelastic material behaves in a non-linear manner and is capable of supporting large elastic strains. This behaviour is described in terms of a strain energy potential [2.46] and the particular formulation used in this study is the Mooney-Rivlin law [2.49]. In cases where the body is not highly confined, the material is commonly
assumed to be incompressible. However, in ABAQUS/Explicit it is not possible to assume full incompressibility as there is no mechanism for imposing such a constraint [2.47]. Too much incompressibility leads to high frequency noise in the solution and requires use of excessively small timesteps in the explicit integration procedure. In many cases the compressibility which must be allowed in the model is, therefore, greater than that of the actual material. This softer modelling of the material's bulk behaviour usually provides accurate results but this accuracy reduces when the material is highly confined. In the golf impact, the ball is in contact with the stiff metal club face but also has a large free surface area and the material is therefore not highly confined. However, it is important to be aware of this potential modelling limitation when considering the results of the impact analysis.

The hyperelastic material model has no rate dependent properties and it is therefore necessary to introduce viscous behaviour into the ball model, as in the simple mechanical models presented earlier in this chapter. Viscoelasticity is represented in the model by a factor, $\beta_r$, which is expressed as a fraction of the critical damping for a particular frequency of vibration and introduces viscous damping proportional to the strain-rate [2.47]. The value of $\beta_r$ should be less than or of the same order as the stable time increment in the explicit integration scheme without damping and a value of $\beta_r=2\mu s$ is used in this study, which produces a relatively short stable time increment of 5.5ns.

The viscous damping in the ball is, however, the main unknown material property in the finite element golf impact model. Analyses in which several values of $\beta_r$ were tested indicated that the presence of higher frequencies in the dynamic response of the ball was reduced by increasing $\beta_r$. The approach taken was therefore to select the value of $\beta_r$ which gave the best fit to the higher frequency part of the ball impact response measured experimentally and presented later in this thesis, whilst simultaneously avoiding an excessively short stable time increment. However, potential for further study exists in the material modelling of the golf ball and its representation in finite element models. Specifically, the potential to define explicitly a material based
on more extensive experimental tests on material samples at high strain-rate than those discussed in the previous section should be investigated.

The finite element method represents a level of analysis significantly in advance of the simple models presented earlier in this chapter. In particular, accurate geometric modelling of the club head and discretisation of the geometry into an appropriate mesh of elements provides a superior spatial representation of the impact problem. This capability, allied to a 3-dimensional contact definition, hyperelastic material modelling and a robust, efficient finite element solution method reveals the interaction between the two bodies during the golf impact in superior detail, as proven in the close comparison with accurate experimental data shown later in this thesis.

This accuracy is achieved with a model which comprises a total of only 3581 elements and is therefore considerably more efficient than the most advanced previous finite element study of the golf impact [2.4, 2.6], which required 30,000 elements and 10 hours of processing time on a supercomputer to attain suitable results. Additionally, the study reported in [2.4, 2.6] used assumed golf ball material properties and no experimental validating evidence for the behaviour of the impact model was presented. The quality of the results provided by computational analysis must be tested before the model can be interrogated for parameters of interest to the product designer. Techniques of great importance in the detailed experimental validation of finite element impact models are the subject of what follows in this thesis. Results from the model developed in this chapter are shown to agree closely with this validating evidence at appropriate points in the discussion.
3 Finite element model validation techniques

3.1 Introduction

Design analysis using finite element methods can provide a wealth of mechanical data from simulated loading conditions. However, relatively few finite element golf impact studies have been reported [3.1-3.6] and in every case it appears that models have been limited by inaccuracies in modelling the sculptured shape or by difficulties in obtaining experimental data with which to validate the behaviour of the model. The first of these issues was addressed successfully in Chapter 2, where geometry modelling and finite element mesh generation techniques were presented which together provide the capability to accurately represent complex sculptured surface shapes in a form suitable for design analysis. This chapter addresses the issue of finite element golf impact model validation by reporting techniques of importance in validating the dynamic behaviour of the club head model and aspects of the club-ball impact model developed in Chapter 2. Further detailed validation of the impact behaviour of the ball and of the club-ball combination is presented later in Chapters 4 and 5.

Successful use of experimental modal analysis to validate the hollow golf club head finite element model is reported in this chapter. Modal analysis is a popular tool for determining the dynamic characteristics of structures and is often employed as a finite element model validation technique. In the case of golf clubs, the use of modal analysis has been directed previously at modes of vibration of the club which are dominated by the shaft and where the club head is treated as a rigid body [3.7-3.11]. The period of oscillation of these modes is typically in the range 1.5-50ms, which is significantly longer than the impact duration of around 450μs. Thus, the shaft dominated modes are of less significance to the mechanical behaviour of the club head and ball than the higher frequency club head modes during impact.

In this study, modal tests employing non-contacting laser based transducers facilitate identification of the natural frequencies and corresponding mode shapes for the three
main surfaces of the hollow club head. Non-contacting transducers are appropriate as the hollow club head is a lightweight body and the behaviour of the individual surfaces of interest would be altered significantly by the attachment of contacting transducers. The experimental data suggests predominantly different modal characteristics for each surface and this compares favourably with equivalent data obtained from the final finite element model in the frequency range relevant to golf club-ball impact. The close agreement between the computed and experimentally obtained results is strong evidence for the validity of the finite element club head model and the model can thus be applied with confidence to more complex excitations, such as those which occur during impact.

The second part of this chapter concerns the analysis of the golf impact itself. The golf impact presents a difficult measurement situation involving two lightweight bodies moving at high speed, as introduced in Chapter 1. These characteristics again limit the usefulness of traditional contacting transducers, which involve the addition of mass, require signal retrieval by electrical connection and may be damaged during the motion of the body. However, contacting transducers are not totally inappropriate, as demonstrated in this chapter by their use in determining the impact duration and the effect of different shaft torsional stiffness on club head rotation during impact. The club head has been previously reported to behave as a 'free body' during impact [3.12] and the experimental results presented in this chapter, obtained using the contacting transducers, suggest that the shaft has only a minor effect on the detailed contact mechanics occurring between the club head and ball during impact. The finite element club head modelled in Chapter 2 without a shaft is therefore an acceptable representation of the club head during impact.

Stroboscopic video techniques are commonly used to determine the launch characteristics of golf balls and high speed imaging systems are capable of capturing golf ball deformation information during impact. The quality of the data obtainable through the use of video techniques is demonstrated by the results presented in this chapter. Whilst some information useful in the validation of the golf impact finite element model can be extracted, the image resolution currently available at high frame rates prevents detailed analysis of golf ball deformations during impact.
Impact sound measurements can be captured in a non-contacting manner and these can be related to the modal behaviour of the club head. Such measurements are important in the analysis of the 'feel'. 'Feel' is an extensively used term which is taken to encompass the physical and psychological feedback experienced by the user of a product. In the golf shot, physical feedback to the hands, eyes and ears is influenced by the biomechanics of the swing, in terms of rhythm, tempo and grip pressure, and by the mechanical properties of the equipment used. The analysis of 'feel' is complicated by its subjective nature and by the vague language which to date has been the main descriptor of physical input to the human nervous system. The difficulties associated with quantifying 'feel' prevent explicit specification of these product characteristics and they are instead benchmarked against products with those characteristics considered desirable [3.13]. However, a recent study [3.14] identified the sound of the golf impact as a dominant component of 'feel'. In this chapter sound measurements are made in order to quantify this important aspect of 'feel'. The experimental data shows the majority of the sound energy produced by vibration of the hollow club head surfaces to be contained in several distinct frequencies. Comparison with the modal analysis results then allows the surface responsible for particular frequency components in the impact sound spectrum to be identified.

In this chapter, the impact sound measurements are presented as further validating evidence for the finite element club head model, since strong agreement exists between the frequencies of vibration excited in the impact and the natural frequencies of the club head predicted by the model. However, the results also demonstrate an ability to predict 'feel' characteristics at an early stage in the design process using a validated finite element model and this presents an opportunity for the development of 'feel' as a quantifiable design parameter, based on the experience gained in this study.

3.2 Computational and experimental modal analysis

Before the finite element hollow golf club head model can be reliably subjected to the complex excitation which occurs in an impact, the response of the model to more
closely controlled excitation must be examined. Modal testing is one of the most popular and successful finite element model validation techniques. In a modal test, the actual structure is taken out of its normal service environment and excited by a known input. The experimentally determined natural frequencies of vibration and corresponding mode shapes can then be compared directly with data produced by theoretical analysis such as finite element modelling. It is generally accepted that corroboration of the first few modes of vibration of a model with experimental data obtained in a modal test of the same structure can provide reassurance of the basic validity of the model [3.15].

In this chapter, only the hollow club head is considered. The large deformation and localised material stiffening exhibited by a golf ball during impact make modal analysis techniques inappropriate for validation of the ball model and this issue is addressed in detailed impact deformation measurements presented later in Chapter 4.

3.2.1 Modal analysis of the golf club head finite element model

The finite element mesh is a spatial representation of an \( N \) degree of freedom system. In the absence of damping, this spatial model is defined by a mass matrix, \( [M] \), and a stiffness matrix, \( [K] \). Free vibration of the model then follows the relation:

\[
[M]{x} + [K]{x} = 0 \tag{3.1}
\]

where \( {x} \) is a time dependent vector of order \( N \). If a solution of the form

\[
{x} = \{\phi\}_i \sin \omega (t - t_0)
\]

is postulated, where \( \{\phi\} \) is a vector of order \( N \), \( t-t_0 \) is the time and \( \omega \) is the constant frequency of vibration of vector \( \{\phi\} \), then equation (3.1) can be solved from the eigenproblem expression:

\[
([K] - \omega^2[M])\{\phi\} = 0 \tag{3.2}
\]

Equation (3.2) yields \( N \) undamped natural frequencies, \( \omega_i \), whose squares are the eigenvalues, \( \lambda_i \), of the solution, where \( i \) represents the mode number (1 \( \leq i \leq N \)). This eigenvalue solution is extracted from the finite element model by a computationally
efficient subspace iteration method \([3.16-3.18]\) and also reveals the eigenvectors \(\{\phi_i\}\), which are the mode shapes of the vibration.

The hollow golf club head finite element model was constrained at the top of the hosel in all directions for the purposes of performing the modal analysis. The methods discussed above were then used to determine the natural frequencies and the corresponding mode shapes of the club head model in the frequency range 0-20kHz.

3.2.2 Experimental modal testing of the hollow golf club head

A harmonic excitation was applied to the real hollow golf club head by a small piezoceramic tile bonded to the sole plate. The club head was held rigidly at the top of the hosel to replicate the boundary conditions in the finite element model and a broadband sinusoidal excitation was applied to the piezo-ceramic tile over the frequency range 0-20kHz. Initial tests suggested the tile should be positioned slightly away from the centre of the sole in order to excite a large amplitude club head response over the frequency range of interest. Other tile positions yielded the same frequency response characteristics but with lower amplitude response.

The low mass of each primary feature of the hollow golf club head dictated that it was appropriate to make pointwise frequency response measurements using a non-contacting laser Doppler vibrometer instead of a more traditional piezo-electric accelerometer. A detailed description of the principles of operation of the laser Doppler vibrometer is presented later in Chapter 6 but it is appropriate to summarise the significant features of the instrument at this point. The vibrometer focuses a laser beam down to a small spot on the target, collecting the returning light in direct backscatter. According to the Doppler principle, the frequency of the light returning from the target is shifted in proportion to the component of the target velocity which lies along the axis of the incident laser beam. By electronically tracking this frequency shift, the vibrometer can measure this component of the target velocity. In this study, the vibrometer was aligned normal to each of the face, crown and sole features in turn and the ratio of the target surface velocity to the piezo-ceramic tile input voltage was
used to estimate the frequency response function for several measurement positions on each feature. These frequency response functions are shown in figure (3.1).

In order to obtain the corresponding vibration mode shape for each natural frequency of the hollow club head, the club head was excited by the piezo ceramic tile at each of the distinct natural frequencies identified using the vibrometer. The mode shapes could have been constructed from an array of pointwise vibrometer measurements but it is more convenient to observe the mode shapes using Electronic Speckle Pattern Interferometry (ESPI) [3.19]. This wholefield laser technique involves use of an expanded laser beam to cover the target. The light incident on a moving target undergoes a phase change which, when recombined with a reference beam in the instrument, can be used to determine target displacement information. The output from the basic system is in the form of a time averaged image of the target showing light and dark fringes. Each fringe is a contour of constant vibration displacement amplitude, hence antinode and node regions can be identified in each mode.

3.2.3 Comparison of results

The natural frequencies of vibration of the hollow golf club head, calculated using the finite element model, can be compared to the frequencies measured experimentally using the laser vibrometer in the first stage of model validation by modal analysis. The face, crown and sole together comprise the majority of the hollow shell structure and the comparison between computational and experimental data is based on the behaviour of these primary features.

The lowest mode of vibration of a primary surface feature is 3.62kHz. Modes of vibration of a golf club below this frequency are dominated by vibration of the shaft and club head vibrations at frequencies above 20kHz have not been observed during extensive impact experiments, presented later in Chapter 4. Hence the frequency range of interest to the analysis of golf club heads is 3.6-20kHz and table (3.1) shows good agreement between computational and experimental results for modes of vibration in this frequency range. The computational results presented in table (3.1) pertain to the hollow club head finite element model shown earlier in figure (2.9). This club head
model was one of several produced which covered a range of finite element mesh densities on the primary features. The model shown in figure (2.9) represents the lowest mesh density to yield results which compared favourably with experimental natural frequency and mode shape data. Closest agreement between the computed and measured natural frequencies of the hollow club head was obtained by increasing the Young’s modulus in the club head material model from the measured value of 195GNm\(^{-2}\) to 207GNm\(^{-2}\). This increase of 5% is within the tolerance of the Young’s modulus measurement, which was made difficult by the small physical size of the material sample cut from a hollow club head. Both values are also within the generally accepted range for the Young’s modulus of stainless steel [3.20].

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<th>Active Surface(s)</th>
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Table 3.1 - Computed and experimentally measured natural frequencies of vibration of the hollow club head face, crown and sole.
The data in the table suggests that some modes of vibration involve predominantly a single surface whilst others are common to more than one surface. This is confirmed in the frequency response functions of the face, crown and sole shown earlier in figure (3.1), where some peaks are common to more than one frequency response function. This effect is of direct relevance to the sound produced during the impact of the club head and a golf ball and this topic is discussed further in Section 3.3.3.2.

The computational and experimental results for modes favouring the crown show slightly greater disparity than the face and sole results. This is attributed to the surface digitisation error on the crown, described in Section 2.4.1, which left an average geometrical error over the crown surface model of 0.14mm. This error is significantly greater than the corresponding error in the face and sole geometries and indicates the level of modelling accuracy required to achieve good agreement between the model and experimental data. The experimentally obtained natural frequencies have a measurement tolerance of approximately ±0.1%.

The level of agreement shown in the natural frequency data presented in table (3.1) provides evidence for the basic validity of the hollow club head finite element model. The second stage of model validation by modal analysis can, however, provide further confirmation of the model validity through qualitative comparison of the mode shapes calculated on the finite element model and measured using the ESPI technique at each natural frequency. Example face, crown and sole modes of increasing complexity are shown in figures (3.2-3.4). Figure (3.2a) shows the ESPI image for the first mode of vibration of the face. An antinode can be seen in the centre of the face, whilst the region around the edge of the face is nodal. The equivalent mode calculated by the finite element model is shown in figure (3.2b) and displays the same mode shape. Figures (3.3a&b) show the experimental observation and finite element prediction of the crown mode shape at 5.96kHz and figures (3.4a&b) show equivalent data for the sole mode shape at 8.20kHz. These results, supported by good agreement between other computed and experimentally measured club head mode shapes, too numerous to
present here, are considered to confirm the basic reliability of the finite element hollow club head model.

3.3 Transient dynamic analysis

3.3.1 Measurement situation

The nature of the golf impact was discussed in Section 1.2.1 and was shown to present a difficult measurement situation involving two lightweight bodies moving at high speed. The difficulties associated with making measurements on the golf club and ball during impact have traditionally prompted an approach in which the pre-impact club head velocity is measured using a simple light gate and the post-impact ball velocity is captured in several stroboscopic video images [3.21]. This limited amount of information restricts analysis to the comparison of ball launch characteristics under various impact conditions and neglects the detailed mechanics involved in the generation of translational and rotational ball velocities and the forces exerted on both bodies in the impact. In this chapter, experimental techniques involving contacting and non-contacting transducers are used to investigate aspects of the impact with the aim of experimentally validating features of the finite element impact model. The information thus gathered is of considerably greater value than that obtained by taking stroboscopic video measurements only. The techniques are, however, limited in their ability to produce detailed mechanically relevant data pertaining to the ball deformation during impact and this provides the motivation for the developments in experimental impact analysis presented in later chapters.

Within the golf equipment industry, three established methods exist for experimentally recreating the immediate post-impact flight characteristics of the golf ball under controlled conditions. The most recently introduced device is based on a contra-rotating wheel arrangement which allows the translational and rotational velocity of the ball at launch to be easily varied by controlling the relative speed of rotation of the wheels [3.22]. The ball launcher does not recreate an impact and is therefore inappropriate in this study.
Numerous ball cannons have been developed to recreate the impact condition by propelling a golf ball at a rigidly held target. These devices commonly rely upon the rapid release of air from a pressurised chamber to propel the ball either directly or by means of a ram which travels along the barrel of the cannon. The ball must be delivered to the target at velocities of up to 45ms\(^{-1}\) without vibration or spin and, in practice, relatively few ball cannons have been shown to achieve this reliably. The main advantage offered by a ball cannon is that one of the impacting bodies is fixed in space. This simplifies the measurement problem greatly and studies have successfully captured video images of ball deformation during normal impact [3.23] and post-impact rebound velocity and spin resulting from oblique impact with a target [3.24]. Whilst this information is of relevance in ascertaining the launch velocity, angle and spin rate which might reasonably be expected from a given impact angle and ball type, the use of a steel plate target precludes the investigation of the club ball interaction which is necessary in obtaining a detailed understanding of the mechanics occurring during impact and in validating a finite element model. A real club head could feasibly replace the steel plate target in the ball cannon but the necessary constraint of club head motion would not be a realistic representation of the impact.

The impact of a moving club head with a stationary golf ball can be accurately and repeatably recreated using a golf hitting machine. The golf equipment industry standard ball and club testing machine is essentially a hydraulic 'golf robot' capable of swinging a complete club in a realistic manner and delivering the club head at a velocity of up to 45ms\(^{-1}\) with a positional accuracy of ±1mm. However, the potential difficulties of measuring small deformations on lightweight, impacted bodies travelling with a velocity of 45-60ms\(^{-1}\) are considerable. These difficulties explain the current lack of accurate data relating to mechanical behaviour during impact and the tendency for previous investigations to rely on measurement of post-impact golf ball launch characteristics [3.25] and landing positions [3.26] in order to infer the impact properties of golf clubs.

In this study, the approach taken is to swing the golf club using a golf robot in order to recreate as closely as possible the conditions under which the club head is presented to
the ball. Impacts involving golf clubs with freely hinged club heads suggest that the shaft does not have a significant influence on the club-ball collision [3.12]. This is a plausible argument, as the short duration of the impact is insufficient time for significant lateral or torsional oscillation of the shaft to be established. The finite element golf club head, created in Chapter 2, is modelled without a shaft and the validity of this modelling assumption must be tested using experimental impact data. The measurement techniques, based on the use of contacting transducers, presented in the following section address this issue by determining the duration of contact between the club head and ball and the rotation of a club head during the impact.

3.3.2 Contacting measurement techniques

3.3.2.1 Contact duration measurements
The time over which the club head and ball are in contact is an important impact parameter. Hertz law of contact, discussed in Chapter 2, indicates that the contact duration is related to both the whole body impact mechanics and the deformation characteristics of the impacting bodies, since the contact duration can be described as a function of the initial relative velocity, the masses of the impacting bodies and the contact approach [3.27]. Thus, comparison of computed and experimentally derived contact durations can provide useful validating data for both the whole body behaviour and the deformation and recovery characteristics of the finite element model. The contact duration is both readily extracted from the model and relatively straightforward to obtain experimentally.

In this study, a contact duration measurement was obtained by making the club head and ball behave as an electrical switch, closing a circuit in the instant contact is made and opening when contact ends. The technique is obviously suited only to electrically conducting surfaces, so a region of the ball surface was made conductive by pressing a copper foil strip of thickness 50μm into the ball cover material during manufacture. The copper foil adopted the dimple pattern of the ball and its presence was shown not to affect the launch velocity or spin rate of the ball when analysed using stroboscopic video images. An example of the switch output is shown as a solid line in figure (3.5),
where the start and end of the contact are clearly defined by a step in the output voltage. Previous contact duration studies have relied on high speed video images, which are at best accurate to the nearest whole frame, and high speed video experiments carried out as part of this study could not distinguish the exact point at which contact breaks, as it is difficult to provide sufficient illumination of the contact region.

The impact of interest in this study is between a hollow metal club head translating with a velocity of 35ms$^{-1}$ and a stationary golf ball. This impact is repeatably created using a golf robot of the type described above.

The electrical switch output shown in figure (3.5) indicates a contact duration of 450μs in this impact, which is in agreement with the contact duration predicted by the finite element model. The results from the finite element model have a resolution of 5μs and the forward velocities of the node pair on the club head and ball which are first to make contact and last to break contact are shown dotted in figure (3.5). The forward velocity of the node on the ball rises rapidly from zero at the start of the impact, as seen on the left of figure (3.5). The two nodes then exhibit the same forward velocity until the end of the impact, when the node on the ball attains the whole body velocity of the ball. The interval between periods of rapid ball acceleration is representative of the contact duration and can be seen to agree with the experimentally measured value to within 5%.

The ball launch velocity is an additional result of relevance at this point. The launch velocity of 52ms$^{-1}$ predicted by the finite element model, and shown later in Chapter 4, is in agreement with that obtained experimentally using stroboscopic video techniques, discussed later in this chapter, and by the impact measurement system based on the laser vibrometer introduced in Chapter 4. The final ball forward velocity of 58ms$^{-1}$ shown in figure (3.5) indicates the presence of an additional forward velocity component due to the deformation of the ball in the contact region. The contact duration and forward velocity results together imply that the finite element club head model, which has a mass equal to that of the real club head, applies an impulse to the
golf ball of magnitude equal to that applied by the real club head. This is additional evidence to suggest that the shaft does not have a significant influence on the real club-ball collision.

The contact duration can be extracted from the finite element model by alternative means such as observing the variation in contact pressure. However, the forward velocity variation data presented here reveals additional information pertaining to the impact behaviour of the localised region of the ball in contact with the club face. The finite element model strongly suggests that the ball contact surface accelerates rapidly at the start of the impact then actually slows down during contact with the club face. This localised region of the ball must again accelerate rapidly at the end of the impact in order to attain the whole body velocity of the ball. In addition, a deformation and recovery oscillation is established in this localised region of the ball in the forward direction. The velocity component associated with the start of this oscillation can be observed in figure (3.5) as the forward velocity of the node shown is greater than the 52ms⁻¹ whole body forward velocity of the ball at the end of the impact. This is an example of the insight which a validated model can provide into parameters which are difficult to measure experimentally and this issue is discussed further in Chapter 4.

The experimental contact duration measurement technique has a secondary function in analysis of the output from other transducers applied to the impact. For example, a piezo-electric accelerometer attached to the club head can provide an indication of the temporal variation of the club head acceleration but it is important to establish exactly which part of the accelerometer output signal relates to the period of contact with the golf ball. This is of even greater significance if several measurements are required to be taken from a single impact, as the contact duration measurement provides a reliable trigger signal for simultaneous data capture from multiple transducers. However, in experiments where the contact duration is not the main parameter of interest it is appropriate to ascertain the features of the output from the required transducers which correspond to the start and end of the impact. These features can then be used to trigger subsequent measurements, thus simplifying the measurement system.
3.3.2.2 Club head rotation during impact

The response of the golf shaft to the complex loading received during the golf swing has been the subject of several studies [3.28-3.30]. Generally, the shaft is considered as a means of delivering the club head to the ball at the required velocity and attitude and little attention is given to the influence of the shaft on the behaviour of the club head during impact. The results presented in this section investigate the influence of the shaft on the rotation of the club head during impact with the aim of testing the validity of excluding the shaft from the finite element golf impact model.

The approach taken is to measure the rotation of the club head about the y-axis during a central impact with a golf ball and in impacts at the heel and toe sides of the club face. A '2-iron' club head is chosen in preference to a hollow metal club head in order to provide a flat mounting surface for two piezo-electric accelerometers whilst maintaining shaft lengths and loft angles comparable to the hollow metal club. Figure (3.6) shows the accelerometers positioned at the maximum available separation, 50mm, in the toe-heel direction and aligned to measure acceleration in the direction of impact. The accelerometers used are robust transducers weighing 2grams, with a resonant frequency of 50kHz and shock limit of 250kms\(^{-2}\). The relatively low mass of the accelerometers causes minimum disturbance to the system under investigation and the large frequency range and high shock limit are ideally suited to impact measurement. Acceleration was chosen as the most convenient descriptor of the club head motion to facilitate investigation of disturbances on top of a fairly high steady velocity.

Figure (3.7) shows the accelerations recorded at the toe and heel of the 2-iron club head during a central impact with a ball. If the difference of these two signals is divided by the separation of the accelerometers, then the angular acceleration of the club head during impact is obtained. In making a differential measurement of this kind, in which subtraction of two large signals is used to obtain a smaller quantity, care must be taken to ensure the resulting signal is of magnitude greater than the tolerance associated with the original measurements. In this case, both accelerometer measurements have a tolerance of approximately ±1% and the magnitude of the difference signal over the
majority of the impact is significantly greater than this. Double integration over the contact duration then reveals the angular rotation of the club head, shown in figure (3.8). Whilst the dominant motion is angular rotation of the club head about the y-axis, a component of the much smaller rotation about the z-axis, caused by the eccentric nature of the impact discussed earlier in Chapter 1, may be present in this measurement as the two accelerometers did not lie in exactly the same horizontal plane. This may account for the small positive rotation recorded in the early part of the impact.

Application of this measurement technique to a 2-iron club head fitted with two different carbon fibre shafts of equal bending stiffness, but with torsional stiffnesses at opposite extremes of the available range demonstrates the influence of the shaft on club head rotation during impact. Figure (3.9) shows the club head rotation measured during central, toe and heel impacts with shafts A and B, where shaft A is considerably less torsionally stiff than shaft B. Six impacts were created on a golf robot for each shaft and impact position combination. The mean of the six rotational displacements and error bars representing one standard deviation are shown in figure (3.9).

Figure (3.9) shows that a 'centre' impact causes this club head to rotate in the direction defined as negative in figure (3.6) due to the centre of gravity of the club head lying to the heel side of the geometric face centre. Figure (3.9) also indicates that toe and heel impact positions cause the club head to rotate in opposite directions, with the toe impact having the largest effect due to its greater distance from the centre of gravity. Whilst changes in the impact position can be seen to produce significant variations in the magnitude and direction of the club head rotation, the shaft type has only a marginal effect on the rotation recorded at each impact position. The toe impact is of particular interest, as this causes the most severe club head rotation, but produces a difference of only 8% between the mean club head rotations recorded from the two shaft types. This difference is negligible when the overall measurement tolerance of approximately 5% is taken into account.

The results presented in this section demonstrate how even in extreme impact conditions a resolvable difference is not apparent between two shafts with very
different torsional stiffnesses and the same bending stiffness. These results and the contact duration data presented in the previous section together provide strong evidence for the validity of excluding the shaft from the finite element golf impact model.

3.3.3 Non-contacting measurement techniques

In any measurement arrangement it is desirable to cause minimum disturbance to the system under investigation. This is particularly relevant in the experimental study of lightweight objects, where the additional mass of a contacting transducer is significant. Additionally, in impact studies, the motion of one or more of the colliding bodies may inflict damage on a contacting transducer or prevent signal retrieval by electrical connection. Non-contacting measurements facilitated by transducers positioned remotely from the system under investigation are therefore attractive in the study of lightweight bodies during impact and two such techniques are discussed in this section.

3.3.3.1 Motion analysis by video techniques

Stroboscopic video techniques are universally applied across the golf equipment industry to analyse the launch characteristics of golf balls. These measurement systems vary in specification but all follow the principles described here. Two strobe flashes are used to capture a single image on a CCD camera which contains two pictures of the golf ball separated in time by the interval between flashes. The strobe flashes are triggered from a point in the swing of the robot golfer and a post-trigger delay ensures the ball is captured in the first few milliseconds of flight (i.e. immediately after but not during the impact). The captured digital image is stored on a computer, where bespoke software is used to position a circle around each golf ball. An example image from a stroboscopic video system is shown in figure (3.10). The time between strobe flashes is known accurately and the length of the line $AB$, between circle centres, then yields the translational velocity of the ball. The angle made between the line $AB$ and the horizontal gives the launch elevation. The ball centres can be identified relatively accurately using this system, giving a launch speed and angle measurement tolerance of approximately 2%. The images shown in figure (3.10) were captured following an impact in which the club head translational velocity was $35\text{ms}^{-1}$. The time between
strobe flashes was 1.3ms, which gives a translational ball velocity of 52ms\(^{-1}\) when the length of the line AB is scaled appropriately. This is in agreement with the translational velocity of the golf ball predicted by the finite element model at the end of the impact.

A circle drawn around the equator of the golf ball appears as a straight line in the video image and the rotation of this line is used to calculate the rotational velocity of the ball. The alignment of computer generated straight lines \(L_1\) and \(L_2\), shown in figure (3.10), to those appearing on the ball in the captured image is an approximation and the rotational velocity can therefore only be resolved to an accuracy of approximately 5%.

Recent advances in this measurement technique have been directed towards improving the accuracy of the rotational velocity measurement, the introduction of sidespin sensitivity and semi-automatic image analysis [3.31, 3.32]. The stroboscopic technique, however, remains incapable of making measurements during impact.

High speed video imaging techniques provide a non-contact measurement capability at frame rates in the range \(10^3\)-\(10^6\) frames/sec. These techniques are applied extensively in the analysis of human kinematics in sport [3.33] where frame rates of around 2000 frames/sec are sufficient to capture the events of interest. Generally, frame rates greater than 10,000 frames/sec are suited to the engineering analysis of impacts and are applied, for example, to the analysis of motor vehicles in crash tests [3.34], bird impact on turbofan blades [3.35], ice ingestion in turbofan engines [3.36] and materials tests involving rapid crack growth due to impact [3.37]. This final example necessitates the use of an imaging system capable of acquiring data at a rate of \(10^6\) frames/sec. However, current electronic data storage capability limits to 24 the maximum number of frames which can be captured at the highest frame rates [3.38].

An imaging system with \(10^6\) frames/sec acquisition capability but only 24 frame storage capacity is inappropriate for analysis of the golf impact as the ball deformations of interest have a period of approximately 450\(\mu\)s. A much lower frame rate of, for example, 20,000 frames/sec produces ten images of the golf impact and this is insufficient to resolve ball deformation in the impact [3.39]. A study which utilised 35,000 frames/sec successfully recorded ball deformation during normal impact created
using a ball cannon [3.23] but ball deformation measurements in a genuine golf impact in which the club head is swung have not been previously reported. The highest currently available frame rate, in a camera which is capable of storing large numbers of frames, is 40,000 frames/sec [3.40]. A camera of this type was used in this study to capture 18 frames of an impact in which the complete club was swung using a golf robot.

Figures (3.11a&b) show approximately half of the side of a golf ball. Contact occurs on the left and the club face, with 10.5° loft angle, is represented by a sloping white line in figure (3.11b) in order to enhance the poor contrast in the image reproduction. Concentric rings drawn on the actual ball appear as dark, equi-spaced vertical lines in the captured images and a change in separation of these lines indicates deformation of the ball in the impact direction. However, it is not possible to resolve accurately any change in the separation of neighbouring pairs of lines between figure (3.11a), at the start of the impact, and figure (3.11b), which shows the image captured closest to the point of maximum deformation. The smallest resolvable deformation, when appropriately scaled, is approximately 0.75mm which is insufficient to analyse localised deformation of the golf ball during the impact event effectively. However, the largest scale golf ball deformations can be estimated using the video images. The distance between the club face and a point on the ball well away from the contact region is representative of the contact approach deformation and the maximum value of this approach deformation can be estimated by subtracting distance $x_2$ from $x_1$ in figures (3.11a&b). This yields a value of 4.8mm, which is in close agreement with the finite element result of 4.65mm, to be presented in greater detail in Chapter 4.

This example illustrates how the mechanical insight which can be obtained from high speed video analysis is limited by the image resolution available at high frame rates. Analysis of golf ball deformations during impact would not necessarily benefit from a frame rate of greater than 40,000 frames/sec because current electronic data capture and storage capability dictates that the number of pixels in the image scales inversely with frame rate. A larger image size would enhance the effective measurement resolution and allow smaller scale deformation to be studied. Advances in high speed
imaging technology suggest that this enhanced measurement capability will eventually become available. However, the data obtained from even the most advanced imaging systems is in the form of a deformation picture. Velocity and acceleration are more appropriate descriptors of the oscillations of interest during impact and an experimental system for the remote capture of this mechanically relevant data during impact is the subject of Chapter 4.

3.3.3.2 Impact sound measurements for 'feel' quantification

The sound heard by the golfer following impact is a major component of the sensory feedback experienced in making a golf shot and is important in defining the overall 'feel' of the impact [3.14]. The ability to quantify important aspects of 'feel' is an essential element in understanding the equipment design features which produce good 'feel'. The ultimate aim for the golf equipment manufacturer is to develop the capability to design desirable 'feel' characteristics into new products at an early stage in the design process. The results presented in this section demonstrate close agreement between the sound frequencies generated during impact and the natural frequencies of vibration of the club head, identified earlier in this chapter using modal analysis. Data of this nature has not been previously reported and represents an important step forward in the understanding of 'feel'.

Impact sound measurements can be made in a non-contacting and relatively straightforward manner using a sound meter positioned adjacent to the impact site. The sound is generated as a result of both club head and ball surface vibrations which persist for approximately 20ms and whose amplitude and frequency content are of interest in quantifying the sound characteristics. In a previous study [3.14] the frequency components of the impact sound in the range 0-3.5kHz were shown to be due to vibration of the ball, whilst frequencies in the range 4-11kHz are due to the hollow metal club head vibrating at its natural frequencies. The sound spectrum measured during the impact between a hollow golf club head and a golf ball is shown in figure (3.12a) for the frequency range 4.5-7.5kHz. This data incorporates a slight Doppler shift of around 100Hz caused by movement of the club head away from the microphone during the measurement period and the frequency resolution is limited to

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50Hz by the overall data length of 20ms. Despite these limitations, several distinct peaks are visible in the spectrum indicating that the majority of the sound energy produced by vibration of the hollow club head surfaces is contained in several distinct frequencies.

The impact sound frequency spectrum shown in figure (3.12a) pertains to the hollow club head modelled in Chapter 2 and whose steady state dynamic behaviour was validated in experimental modal tests reported earlier in this chapter. Comparison of figure (3.12a) with the data in table (3.1) shows that a number of the peaks in the impact spectrum correspond to natural frequencies of vibration of the face, crown and sole surfaces. These are indicated by the letters 'F', 'C' and 'S' above the corresponding peaks in figure (3.12a). More significantly, the largest peaks occur at around 5.9kHz, 6.4kHz and 6.8kHz which are, by reference to table (3.1), frequencies at which the vibration mode shape involves more than one surface.

The mode shape data obtained using the ESPI technique in Section 3.2.2 indicates that the modes of vibration of the face in the frequency range 5-7kHz have a single antinode and that the location of this antinode varies in a region around the face centre. The 5.9kHz and 6.8kHz face mode shapes exhibit an antinode approximately in the face centre whilst that of the 6.4kHz mode is to the left of centre. Thus, the relative excitation of the face modes will be different for impacts occurring in the face centre and those which are off-centre. This is confirmed in the sound spectrum measured during an impact in which contact with the ball was 15mm to the left of the face centre, as shown in figure (3.12b). By comparison with figure (3.12a), it can be seen that the contribution of frequency components in the range 5-6kHz is significantly reduced whilst the 6.4kHz component is increased. Hence the overall sound of the off-centre hit is of higher frequency and this is confirmed by subjective data gathered from a group of low handicap and professional golfers [3.14].

Figure (3.13) shows the 5.9kHz and 6.4kHz mode shapes calculated using the finite element model. These mode shapes exhibit the same characteristics as the experimental data, in that the single antinode is in the face centre in the 5.9kHz mode and to the left
of centre in the 6.4kHz mode. Thus, using the finite element modal analysis data, a club head designer could have predicted the main features of the experimental results shown in figures (3.12a&b) with some confidence at an early stage in the design process. This ability to identify the features of the golf club head responsible for the production of certain 'feel' characteristics has not been previously reported and is additionally an example of how the detailed steady state dynamic data obtained using a combination of laser based transducers has applications beyond finite element model validation. This work provides a basis for further research into the impact sound characteristics preferred by golfers and the ability to achieve these 'feel' characteristics by re-distributing the mass and stiffness of individual surfaces whilst simultaneously satisfying the inertia property requirements of the whole club head.
4 Analysis of impacts with large elastic deformation

4.1 Introduction

This chapter describes a successful experimental procedure to obtain physical measurements during a high speed, short duration impact between two lightweight bodies, in which large elastic deformation occurs. Whilst the techniques described are of general applicability, the particular situation under investigation is the impact between the golf club and golf ball and the results presented in this chapter are used in the validation of the finite element golf impact model developed earlier in Chapter 2.

A sensing arrangement is presented in which a piezo-electric accelerometer is attached to one of the impacting bodies and complementary measurements are made remotely on the second body using laser Doppler vibrometers positioned at the impact site. The sensing arrangement represents a novel application of laser Doppler vibrometry to the analysis of impacts. The laser vibrometer provides a remote measurement capability, which is particularly attractive in impacts where a colliding body is either projected a substantial distance away from the impact or where damage would be inflicted on traditional contacting sensors during the post-impact motion. The piezo-electric accelerometer is suited to the study of high speed impacts where a colliding body moves in a manner such that signal retrieval by electrical connection is possible.

The approach taken is to make maximum use of a small number of sensors and to infer important data from this limited number of measurements. This approach is more practically viable than using many sensors, which would incur unacceptable expense and demand considerable data acquisition capability. A method of extracting relevant information from the data captured by the accelerometer/vibrometer arrangement during impact is described and hence the variation of deformations, velocities and forces over time during the impact can all be determined.
The motivation for the developments in experimental impact analysis presented in this chapter is drawn from the requirement to capture mechanically relevant data of a quality sufficient to allow greater understanding of the relationship between the two bodies in time and space and to provide detailed validating evidence for analysis by techniques such as finite element modelling. The information obtained using the experimental procedure is of superior detail and greater mechanical relevance than that obtainable using high speed video techniques. In the light of the experimental data, the finite element golf impact model developed in Chapter 2 is modified slightly in order to match more closely the experimental data and results which show strong agreement between the modified computational model and the experimental data are presented. A validated finite element model can be used to improve understanding of the mechanical behaviour exhibited by the impacting bodies and examples of relevance to the golf impact are discussed. The chapter closes with an evaluation of the experimental arrangement, which reveals the key development issues of importance in the application of remote measurement techniques to the analysis of bodies during impact.

4.2 Golf ball velocity and deformation measurements during impact

4.2.1 Sensing arrangement for detailed impact measurements
The golf impact is characterised by large elastic deformation of the ball and significantly smaller elastic deformation of the club head. In addition to attaining a whole body forward velocity of up to 60 ms\(^{-1}\) during the impact, the ball is observed to undergo significant deformation in the contact region. Other, smaller extremes of deformation are recorded diametrically opposite the contact site and also perpendicular to the direction of the intended line of flight. These deformations will be termed the contact approach, \(x_{\text{CON}}(t)\), the free approach, \(x_{\text{FREE}}(t)\), and the lateral deformation respectively, and are defined in figure (4.1), which shows a cross-section of the ball through its equator. However, in a situation similar to the estimate of the contact approach obtained using high speed video techniques in Chapter 3, the whole body motion of club head and ball in the \(x\)-direction make absolute measurements of \(x_{\text{CON}}(t)\) and \(x_{\text{FREE}}(t)\) impossible. In practice, \(x_{\text{CON}}(t)\) is obtained from the forward displacement of a point on the ball close to the impact site relative to that of the club head and
\( x_{FREE}(t) \) is the forward displacement of the point on the ball diametrically opposite the contact site relative to that of a point on the side of the ball.

Additionally, the golf club-ball collision is an example of an eccentric impact. Backspin is generated on the ball due to the 10.5° loft of the golf club face and misalignment of the impact can produce sidespin of the ball. Although video images such as those presented in Chapter 3 show no appreciable spin of the ball whilst in contact with the club in the tests reported here, the generation of backspin and sidespin velocity components during the impact must be recognised. These are defined as rotation about the \( z \)-axis and about the normal to the \( xz \)-plane respectively.

The laser Doppler vibrometer is a non-contacting velocity transducer which is generally employed in vibration measurement systems where the target object is hot, light or rotating. Its non-contact operation makes the technique well suited to measurements of motions on impact [4.1] but application of the technique to the evaluation of dynamic deformation of a colliding body has not been reported previously. The effectiveness of the laser Doppler vibrometer in this role is limited to target velocity and acceleration ranges of \( \pm 15 \text{ms}^{-1} \) and \( \pm 2 \times 10^6 \text{ms}^{-2} \) respectively due to optical and electronic signal processing considerations described in greater detail in Chapter 6. Remote measurements of the quality achieved in this work, on a lightweight impacted body subsequently projected from the impact site are, however, unobtainable by other means.

The usefulness of the laser vibrometer in impact measurements can be maximised by intelligent alignment of the probe laser beam. The vibrometer is sensitive only to the component of the target velocity which lies along the axis of the incident laser beam, thus relevant components of target velocity can be selected and the vibrometer aligned to measure only or, at least, predominantly these. For example, on targets which are both translating and rotating, both motion types will contribute to the measured velocity. In this study, the translational velocities are of greatest interest and sensitivity to rotation is minimised by aligning the laser beam to pass through the centre of rotation of the target or parallel to the rotation axis, in which case the rotational
velocity in the direction of the incident laser beam is zero. The measurement is not affected by the shape of the target object, as the vibrometer is sensitive only to the velocity of the illuminated particles on the surface [4.2]. An important consideration for all non-contact measurements on moving bodies, however, is the changing position of the measurement point as the target is displaced from its original position. These matters will be discussed at appropriate points in this chapter.

Whilst remote velocity measurements on the golf ball using laser vibrometry are the main subject of this section, the data thus obtained is of greater significance if it can be related to behaviour of the clubhead during impact. A piezo-electric accelerometer of mass 3 grams is therefore mounted on the back of the golf club head to measure the acceleration of the body in the direction of impact during the collision. Careful positioning of the relatively low mass accelerometer causes minimum disturbance to the system under investigation and its large frequency range of 0-40kHz and high shock limit of 250kms⁻² are ideally suited to impact measurement. Acceleration was chosen as the most convenient descriptor of the club head motion to facilitate investigation of disturbances on top of a fairly high steady velocity. Simultaneous measurement from two sensors requires a reliable trigger signal, which was created in this study using the electrical switch arrangement employed earlier, in Section 3.3.2.1, to determine the contact duration. The interval between the start points of signals from the electrical switch and the accelerometer mounted on the back of the club head was established, allowing the accelerometer signal to be used as the trigger in later measurements, thus simplifying the measurement system.

The acceleration and velocity data obtained from the instruments described here are used to determine whole body club head and ball translations as well as the pattern of gross deformation and recovery of the ball during the impact in significantly greater detail than previously possible. Further analysis demonstrates how the force experienced by the club head during impact is determined by the pattern of ball deformation and recovery.
4.2.2 Measurement of the lateral deformation velocity during impact

A laser vibrometer at the remote position $C_R$, aligned in the $xz$-plane of figure (4.2), perpendicular to the intended ball flight path and focused to a point, $C_F$, on the surface, records the lateral deformation velocity of the ball, $\dot{z}_{CR}(t)$, during the impact. There is no sensitivity to either whole body forward motion, approach deformation or backspin of the ball at launch as these motions do not have a velocity component in the direction of the laser beam. Sidespin of the ball does not affect the measurement initially but may become important when the forward ball motion means the point of incidence of the laser beam is no longer through the centre of rotation. However, the amount of sidespin on a correctly aligned impact was confirmed as negligible following stroboscopic video measurements taken simultaneously.

The ball moves forward a distance of approximately 11.5mm during the impact, causing the point on the surface of the ball illuminated by the stationary laser to move towards the contact site during the period of measurement. Although the vibrometer is insensitive to the slightly increased optical path length to the ball, it is recognised that the lateral deformation velocity is not recorded at the same point on the ball throughout the impact. This becomes particularly important in the later stages of each measurement, when the forward displacement of the ball is largest. Further work presented in Chapter 5 is directed towards accommodating this effect, which is apparent with any non-contact measurement.

A velocity measurement taken by the vibrometer during an impact in which the club struck a ball at 35.5ms$^{-1}$ is shown in figure (4.3). It can be seen that in the first half of the impact the lateral deformation velocity of point $C_F$ on the ball, $\dot{z}_{CR}(t)$, is towards the vibrometer. In the second half of the impact, the ball attempts to recover its original shape and the measured velocity is negative. Lateral deformation velocities of the order of 10ms$^{-1}$ are recorded and a smaller pulse is consistently observed at approximately 75$\mu$s into the impact. The absence of this feature in equivalent tests on a solid golf ball with a one-piece construction suggests that the small pulse is due to the propagation of a surface wave in the thin cover material of the two-piece ball.
Integration of $\dot{z}_{ct}(t)$ over the contact time allows the corresponding lateral deformation to be calculated. This is shown in figure (4.4). The maximum lateral deformation is shown as 0.78mm for the case of impact at an initial club head speed of 35.5ms$^{-1}$. The effect of the ball forward translation relative to the laser probe beam will be to overestimate marginally the extent of the lateral deformation recovery. Figure (4.4) shows how at the end of the impact the ball 'over recovers' to a lateral dimension less than that of the un-deformed ball. This suggests the start of a deformation-recovery oscillation in the initial ball motion.

4.2.3 Measurement of forward velocity during impact

In the previous section, it was shown how a laser vibrometer aligned perpendicular to the intended flight path of the ball could be used to measure the component of ball velocity in that direction. However, a vibrometer directed at the ball approximately parallel to the intended flight path cannot be used to measure the forward velocity in this application because the velocity limit of the instrument is only 15ms$^{-1}$. During impact the ball attains a forward velocity in the region of 60ms$^{-1}$ and the measurement exceeds the instrument range well before the end of the impact. Tests showed that the minimum angle from the $x$-axis at which the vibrometer could be placed in order to stay within its velocity range whilst measuring a component of the forward velocity was 75°. In this configuration the maximum rate of change of velocity is also within the range of the laser vibrometer's frequency tracker.

By positioning the vibrometer at this angle, a measurement is made which contains components of velocity due to whole body forward motion, forward deformation and lateral deformation of the ball. The component of velocity due to lateral deformation in this measurement is determined by making a simultaneous measurement at the same point on the ball using a second vibrometer aligned perpendicular to the $x$-axis. Figure (4.5) shows measurements conducted at point $D_F$ on the ball. In this location the laser beam from the vibrometer positioned $\gamma=75^\circ$ from the $x$-axis is aligned to pass through the centre of the ball, making the perpendicular measurement, $\dot{z}_{dx}(t)$, sensitive to sidespin throughout the impact and the $\gamma=75^\circ$ measurement, $\dot{z}_{dr\gamma}(t)$, insensitive to sidespin only initially. This is acceptable since the amount of sidespin is negligible in a
Correctly aligned impact. Arranging the laser beams in the $xz$-plane and through the equator of the ball makes both measurements insensitive to backspin. The horizontal component of the forward velocity as measured remotely by the probe laser beams aligned on point $D_F$, $\dot{x}_{D_A}(t)$, can therefore be calculated as follows:

$$\dot{x}_{D_A}(t) = \frac{\dot{z}_{D_B}(t) - \dot{z}_{D_A}(t)\sin\gamma}{\cos\gamma}$$  \hspace{1cm} (4.1)

Example measurements are shown in figure (4.6). Whilst it is recognised that the eccentric nature of the impact imparts horizontal and vertical whole body velocity components on the ball, the launch angle resulting from this impact is only $10^\circ$ and the horizontal velocity component therefore dominates. The horizontal component of forward velocity underestimates the true forward velocity by only 1.5% and the discussion is simplified if further references to the forward velocity of the ball are based on the horizontal velocity component. The calculated forward velocity from rest, after smoothing with a simple moving average method [4.3], is shown in figure (4.7). A peak forward velocity of $52\text{ms}^{-1}$ is obtained, which is in agreement with the velocity calculated from a stroboscopic video motion analysis system. The video system has a measurement tolerance of 5%. The largest source of error in the vibrometer measurement is thought to be the potential misalignment of the vibrometer positioned $75^\circ$ from the $x$-axis, where a $0.5^\circ$ vibrometer misalignment would result in a 3% error in the calculated forward velocity. The peak acceleration of the ball is seen from figure (4.7) to be of the order of $30,000\text{g}$.

A greater insight into the club-ball relationship during impact can be obtained by changing the location of the laser measurement point on the ball. The arrangement of two vibrometers used earlier is again employed to derive the forward velocity, $\dot{x}_{i}(t)$, at five locations on the ball in the $xz$-plane, $i=A,B,C,D,E$. The use of measurements from different impacts relies upon the repeatability of the impact conditions and several measurements were taken in order to demonstrate that the results presented here are representative. The forward velocity measurements obtained by aligning the probe laser beam on each of the locations $B_F$, $D_F$ and $E_F$ are plotted in figure (4.8) and
together show how the initiation of forward velocity through the ball progresses from
the contact site to the free side.

The data in figure (4.8) also reveals how measurements taken at point $B_F$ and,
therefore, also point $A_F$ are high estimates of the whole body forward velocity in the
early stages of the impact, due to the velocity component associated with the forward
deflection close to the contact site early in the impact. Also, the measurement from
point $E_F$ is initially the lowest estimate of forward velocity. This suggests an initially
smaller forward deformation velocity at point $E_F$ and is evidence for the existence of an
initially negative free approach, $x_{\text{FREE}}(t)$. The negative free approach exists due to the
presence of the lateral deformation. However, at approximately 200$\mu$s into the impact
the lateral deformation begins to recover, as shown earlier in figure (4.4). At the same
time, the forward velocity measurement from point $E_F$ becomes an overestimate with
respect to that from point $D_F$, suggesting the presence of an additional positive
forward velocity component and the recovery of $x_{\text{FREE}}(t)$.

The timing of the change from velocity underestimate to overestimate at point $E_F$ is
evidence for the stage in the impact where the ball reaches maximum forward
deflection and begins to recover its un-deformed shape. The peak force exerted on
the club head also occurs at approximately the same time. Towards the end of the
impact, the measurement initially taken at point $E_F$ will be approximately at point $D_F$
due to forward translation of the whole ball. This will cause an underestimate of the
$x_{\text{FREE}}(t)$ recovery. The existence of forward deformation components in the forward
velocity measurements at points $A_F$, $B_F$ & $E_F$ suggests that $\dot{x}_{CR}(t)$ and $\dot{x}_{DR}(t)$ are the
most reliable estimates of the whole ball forward velocity, $\dot{x}(t)$.

The pattern of deformation across the ball in the $x$-axis direction during impact can be
studied further by comparing the forward displacements calculated at each of the
points $B_F$ to $E_F$. The forward displacements $x_{iF}(t)$ are obtained by integration of the
forward velocities $\dot{x}_i(t)$ where $i=B,...,E$. Data from point $A_F$ has been omitted as it is
available only for the first 150$\mu$s. The relative forward displacements $x_{\text{CM}}(t)$, $x_{\text{OCF}}(t)$ and
$x_{\text{ECF}}(t)$ are plotted in figure (4.9). All three quantities are negative in the first 325$\mu$s of
the impact due to compressive deformation of the ball. Through the centre section of
the ball, this deformation is approximately constant for a significant part of the impact,
as seen in the $x_{DC}(t)$ value. In the final 100μs of the impact, $x_{DC}(t)$ exhibits some
recovery of the deformation. More significantly, $x_{ER}(t)$ exhibits a greater recovery and
becomes positive. Points $D_\text{F}$ and $E_\text{F}$ thus move further apart in the later stages of the
impact. This is further evidence for elongation of the ball in the x-axis direction.

4.2.4 Measurement of the contact approach during impact

The contact approach, $x_{CON}(t)$, was defined in figure (4.1) and can be obtained by
integrating the relative velocity of the two bodies over the contact time, as shown:

$$x_{CON}(t) = \int_0^t \left( \ddot{x}_{\text{club}} + \int_0^t \ddot{x}_{\text{club}} dt - \dot{x}_{\text{DF}} \right) dt$$

Thus, the forward velocity of the ball obtained in the previous section is used along
with the club head acceleration measurement, $\ddot{x}_{\text{club}}$, and the club head pre-impact
velocity measurement, $\dot{x}_{\text{club}}$, to obtain $x_{CON}$ by integration over time, $t$. With the piezo-
electric accelerometer mounted on the back of the club head, it is not possible to
determine the deformation of the club face during impact as the pattern of oscillation
across the club head is not obvious. The calculation performed in equation (4.2) is,
however, unaffected by the small deformations associated with oscillation of the club-
ball combination as these become negligible when the accelerometer signal is
integrated. The velocity of the club head in the instant before impact, $\dot{x}_{\text{club}}$, is recorded
using a photocell.

The result of the approach calculated from laser vibrometer measurements at point $D_\text{F}$
is shown dotted in figure (4.10). It can be seen that $x_{CON}(t)$ increases up to a point due
to deformation of the ball at the impact site. The maximum contact approach is 5.4mm
for the case of an initial club head speed of 35.5ms$^{-1}$ and this is achieved in a period
when the ball moves approximately 1mm forwards. This contact approach calculation,
using laser vibrometer measurements at point $D_\text{F}$, will be larger than the actual value of
$x_{CON}(t)$ because the estimate of $x_{CON}(t)$ from the point $D_\text{F}$ measurement additionally
incorporates the deformation across the ball in the direction of the \( x \)-axis from point \( A_F \) to \( D_F \). The best estimate of \( x_{\text{COM}}(t) \) is provided by data from point \( A_F \) but this data is only available for the first 150\( \mu \)s of impact. However, combined data from points \( A_F \) and \( B_F \) produce a close approximation to the true value of \( x_{\text{COM}}(t) \) and this is shown as a solid line on figure (4.10). A maximum value of 4.3mm is calculated. Additionally, the calculation shows recovery of the \( x_{\text{COM}}(t) \) deformation only slightly beyond the original \( x \)-axis dimension of the ball in the final stages of contact with the club face. The rate of recovery of the contact approach is therefore lower than that of the lateral deformation, which exhibits a clear over-recovery during the impact.

The longitudinal deformations identified using the laser vibrometer measurement system could not be resolved using a high quality video analysis system with up to 40,500 frames/sec and 256x256 pixel resolution. This data is of importance in the validation of finite element models of the colliding bodies and therefore demonstrates the requirement for a measurement system of the type described in this chapter.

4.2.5 Estimate of the contact radius during impact

A simple estimate of the contact radius, \( r_{\text{COM}}(t) \), can be obtained by considering the geometry of the deformed golf ball shown in figure (4.1). Assuming the club face to be rigid, the contact radius can be expressed as:

\[
r_{\text{COM}}(t) = \left( x_{\text{COM}}(t)(2R - x_{\text{COM}}(t)) \right)^{\frac{1}{2}}
\]  

(4.3)

The contact approach \( x_{\text{COM}}(t) \) was calculated in the previous section and \( R \) is the radius of the un-deformed ball, hence \( r_{\text{COM}}(t) \) can be calculated. The maximum contact radius has been calculated as 14.0mm for the case of an initial club head speed of 35.5\( \text{ms}^{-1} \). In the second half of the impact, the contact radius declines as shown in figure (4.11). This calculation is important as it gives an indication of the spatial extent of the contact force with time.

4.2.6 Measurement of rotational velocity during impact

The golf club-ball collision is an example of an eccentric impact, with backspin generated on the ball due to the loft of the club head. The backspin velocity is
commonly measured in the first metre of flight using stroboscopic video of the type described in Chapter 3. These video measurements show that the 10.5° loft of the club head used in this study produces a backspin velocity of approximately 3000 rev/min in the first metre of flight. Whilst stroboscopic video techniques permit sufficiently accurate determination of golf ball launch conditions, the detailed mechanics occurring during impact which cause the rotation can only be inferred from these post-impact measurements. A high speed video system of the type described in Chapter 3 affords the capability to observe the motion of the ball during the impact but its usefulness in detailed analysis is limited by the resolution of the captured images. A rotational velocity of 3000 rev/min produces a rotational displacement of approximately 0.1 radians during the final 350 μs in which the ball is in contact with the club face. This rotation cannot be resolved accurately in the high speed video images on the impact shown earlier in Chapter 3.

A number of transducers based on the laser Doppler technique have been developed for remote vibration measurements on rotating bodies. In these instruments two parallel laser beams are incident on the target and the light collected in direct backscatter from each point is heterodyned to produce the required signal. The laser torsional vibrometer (LTV) [4.4] uses a simple optical heterodyne technique with the specific intention of measuring fluctuations in the mean speed of rotation of the target. The optical heterodyne technique relies on the presence of a mean rotational velocity for its successful operation and the instrument is therefore not suited to the measurement of the rotational velocity components acquired by a golf ball during impact since the ball is initially at rest. Instruments such as the laser rotational vibrometer [4.5], which uses an optical heterodyne of the light backscattered from the target and an additional frequency pre-shift of one of the beams, are capable of making measurements from rest. However, alignment of both beams at 75° to the impact direction, with one beam above the equator of the ball and one below, will produce a signal containing the backspin rotational velocity of the ball and a larger velocity component describing the lateral deformation velocity of the ball above the equator relative to that below. The lateral deformation velocity at a given distance below the equator of the ball is larger than that measured at an equivalent distance above the
equator due to the eccentric nature of the impact. The two beam arrangement cannot therefore isolate the rotational velocity of the golf ball.

The rotational velocity of the golf ball during impact can be determined if the measurement system employing two single-beam instruments aligned at angles \( \gamma = 75^\circ \) and \( \gamma = 90^\circ \), used earlier to obtain the forward velocity of the ball, is applied to points on the ball which lie above and below the equator. This is effectively a four-beam measurement arrangement which eliminates the unwanted lateral deformation velocity components from the measurement at the expense of either procuring four single-beam vibrometers or taking data from two separate impacts. The use of measurements from two different impacts in this study again relies upon the repeatability of the impact conditions and several measurements were taken in order to demonstrate that the results presented here are representative.

The forward velocities measured at points 12mm above and below the equator are shown in figure (4.12) where it can be seen that, consistent with a backward rotation, the forward velocity below the equator is greater than that above. With knowledge of the distance between the two points interrogated by the laser beams, the rotational velocity of the golf ball can be calculated and the result is shown as a solid line in figure (4.13). The low rotational velocity in the first 100\( \mu \)s of the impact increases much more rapidly than the forward velocity measured on the equator of the ball and a slight oscillation is observed about a mean rotational velocity which steadily increases to 3000 rev/min over the remainder of the impact. The mean value of the final rotational velocity is in agreement with the value measured by a stroboscopic video system in the first metre of flight and the observations of the rotational velocity increasing from rest during the impact represent a level of measurement detail not previously achieved.

All of the golf ball impact measurement data presented thus far in this thesis have pertained to the behaviour of the type of golf ball with a two-piece construction. Whilst this is the most commonly used type of golf ball, other constructions exist, as discussed in Chapter 1. The balata ball, comprising a liquid filled centre, elastic thread
windings and a synthetic balata rubber cover, is of particular interest and the measurement techniques presented in this chapter can be used to compare briefly the detailed impact behaviour of the different ball constructions. At 500\(\mu\)s, the balata ball impact is longer than the two-piece ball impact due to the greater deformation in both the lateral and forward directions exhibited by the balata ball, which recovers more slowly than the two-piece ball. The longer contact time implies that a greater impulse is applied to the balata ball but the final forward velocities measured on both ball types are very similar. The majority of the additional energy from the larger impulse is used in the creation of greater deformation of the balata ball in the impact direction. This is manifest in a greater rotational velocity component of the ball which results in greater backspin when contact with the club face ends. This accumulation of a greater rotational velocity during the balata ball impact is shown dotted in figure (4.13) and is another example of how the remote impact measurements introduced in this chapter can assist in improving the understanding of the detailed mechanics occurring during impact.

4.3 Oscillations occurring during impact

Differentiation of the remote forward velocity measurement at point \(C_F\) reveals the forward acceleration of the ball, \(\ddot{x}_{C_F}(t)\). \(\ddot{x}_{C_F}(t)\) contains components of acceleration due to approach deformation of the ball and whole body forward motion of the ball from rest. Similarly, the lateral deformation velocity measurement, \(\dot{z}_{C_B}(t)\), can be differentiated to obtain the lateral acceleration of the ball, \(\ddot{z}_{C_B}(t)\) which is due only to deformation. The accelerations \(\ddot{x}_{C_B}(t)\) and \(\ddot{z}_{C_B}(t)\) are plotted in figure (4.14). In the period of the impact where \(0<t<120\mu\)s there is little forward motion of the ball, thus the club head deceleration measured in this period is a result of approach and lateral deformation of the ball. After this time, the lateral acceleration of the ball decays. The forward acceleration, however, shows a strong oscillation which can also be seen in the measurement of acceleration taken at the back of the club head in the direction of the \(x\)-axis.
The club head acceleration measurement, $\ddot{x}_{cb}(t)$, is shown along with the forward acceleration of the ball on a normalised scale in figure (4.15). Due to the speed of wave propagation in the polybutadiene ball, the time taken for the initial impact wave to reach the lateral deformation measurement site is approximately $50\mu s$. This is $31\mu s$ longer than the time taken to reach the accelerometer on the back of the club head and thus the $\ddot{x}_{cr}(t)$ trace lags the accelerometer measurement by $31\mu s$. Events occurring simultaneously at the back of the club head and at point $C_F$ on the ball can therefore be studied together by shifting the $\ddot{x}_{cb}(t)$ measurement in time by $31\mu s$, as in figure (4.15). This shift is most valid for the period of the impact where $0<t<200\mu s$ as the forward displacement of the whole ball is only of the order of $1\text{mm}$. In the period $200<t<450\mu s$ the forward displacement of the whole ball increases and the laser vibrometer measurement at point $C_F$ begins to move toward point $B_F$. Thus events occurring simultaneously on the ball and at the back of the club head will appear increasingly out of synchronization in the period $200<t<450\mu s$ in figure (4.15).

Both of the accelerations shown in figure (4.15) can be considered to have mean and oscillating components. The mean acceleration components, shown dotted, are obtained by smoothing the original data to remove higher frequency information. The mean component of $\ddot{x}_{cr}(t)$ represents the forward acceleration of the ball from rest and the mean component of $\ddot{x}_{cb}(t)$ shows a corresponding deceleration. The oscillating component of acceleration in the direction of the x-axis, apparent in ball and club head measurements, is a result of deformation of the club-ball combination. From figure (4.15), a relationship between the ball and club head oscillations can be seen. The clarity of the relationship is affected by noise in the $\ddot{x}_{cr}(t)$ trace, which is increased as a result of obtaining $\ddot{x}_{cr}(t)$ by differentiation of a forward velocity measurement. However, it is apparent that local turning points on the $\ddot{x}_{cr}(t)$ and $\ddot{x}_{cb}(t)$ traces occur at approximately the same points in time. This is confirmed by consideration of equivalent data for further club-ball impacts. Local maxima in the $\ddot{x}_{cr}(t)$ trace correspond to local minima in $\ddot{x}_{cb}(t)$, showing that when the forward deformation acceleration of the ball is locally a minimum, the forward deformation
acceleration of the back of the club head is locally a maximum. The reverse case, in which maximum ball forward deformation acceleration corresponds to minimum forward club head deformation acceleration, can also be deduced from the accelerations shown on figure (4.15). This relationship suggests that if oscillations of the club face match those of the ball with which it is in contact, then the club face oscillates out of phase with the back of the hollow club head. The oscillation of the forward deformation of the ball is on top of the gross deformation shown in figure (4.10).

The greatest local maximum on the $\ddot{x}_{cr}(t)$ trace occurs at a point 200μs into the impact. This is consistent with the data in figure (4.10), which shows that the greatest contact approach, i.e. the point at which the deformation component of ball acceleration in the direction of the x-axis should be greatest, occurs at 200μs into the impact. Beyond this point in the impact the ball recovers and the relationship between the club head and ball is less well defined.

Finally, an estimate of the peak contact force occurring during the impact can be obtained from the mean club head acceleration trace. For a club head of mass 200 grams, the maximum mean force is 12kN and occurs 180μs into the impact. Combination of the mean acceleration data in figure (4.15) with the contact radius estimate in figure (4.11) provides the estimate of the temporal and spatial variation of the force between the club head and ball.

4.4 Validation of the finite element golf club-ball impact model

In the absence of appropriate formal methods for the comparison of computational and experimental impact data [4.3, 4.6], the approach taken here is to compare qualitatively the general form of the computational and experimental results whilst making quantitative comparisons between the results at significant points in time, as highlighted in Sections 4.2 and 4.3. The experimental data, obtained under difficult measurement conditions from only a small number of sensors, is of superior detail and greater mechanical relevance than that obtainable using video techniques. The finite
element impact model is therefore subjected to a considerably more rigorous validation than has been possible previously.

The experimental data is captured at a sample rate of 4MHz, whilst the results output from the finite element impact model is equivalent to a sample rate of 200kHz. The experimental data thus contains information from a larger frequency range than the computational data and thus appears less smooth when the two data sets are observed together. However, the main features of interest all occur in the frequency range 0-20kHz and are thus represented completely in both the computational and experimental data.

The computational results presented are those derived from the golf impact finite element model following modification in accordance with the experimental data. The viscous damping in the ball was the main unknown material property in the finite element golf impact model and the main modification from the original model involved adjustment of this material property in order to more closely match the pattern of deformation and recovery shown in the experimental impact data. Analyses in which several values of the damping factor $\beta_R$ [4.7] were tested indicated sensitivity of the higher frequencies in the dynamic response of the ball to the level of damping and a value of $\beta_R=2\mu s$ gave the best fit to the experimental ball impact response data.

4.4.1 Lateral deformation velocity
The measured lateral deformation velocity, $\dot{z}_{CF}(t)$, is shown in figure (4.16) along with the equivalent result calculated by the finite element method, $\dot{z'}_{CF}(t)$. The two results display the same general form of lateral deformation and recovery velocity throughout the impact. The peak lateral deformation velocities are both slightly less than $10ms^{-1}$ and occur at same point in the impact. This suggests that the high strain-rate material properties used in the ball model produce impact behaviour which closely matches the real ball. An indication of the level of detail which can be achieved using the finite element model is given by the appearance of even the initial surface wave peak at around 50$\mu$s in the finite element result. This surface wave peak occurs earlier in the finite element result than is actually the case, which suggests the highest frequency
material behaviour of ball model matches the real ball less closely than the lower frequency behaviour.

The close agreement between the computational and experimental results displayed in the first 300\(\mu\)s reduces in the later stages of the impact. In particular, the period of lateral deformation recovery predicted by the finite element model is less than that suggested by the experimental data. It is possible that the strain-rate dependent viscoelastic material properties of the real ball may be such that the rate of deformation recovery is significantly different to that when the ball is being deformed and this hysteretic effect is represented insufficiently in the finite element ball material model. However, a more probable explanation is that, as discussed in Section 4.2.2, translation of the target causes the point on the ball interrogated by the laser beam to move toward the impacted side of the ball, where lateral deformation recovery takes place later in the impact. The experimental data thus overestimates the lateral deformation recovery velocity late in the impact, causing the discrepancy between experimental and computational results. The cause of this discrepancy is addressed in Chapter 5, leading to a significant improvement in the experimental data.

4.4.2 Forward velocity and deformation
The forward velocity measured remotely at point \(D_F\), \(\dot{x}_{D_F}(t)\), is shown in figure (4.17) along with the result calculated by the finite element method at point \(D'_F\), \(\dot{x}'_{D'_F}(t)\). Point \(D'_F\) is the location of the node on the equator of the icosahedral finite element ball model initially closest to measurement point \(D_F\) and is 2.4mm closer to point \(C_F\) in the \(x\)-direction than point \(D_F\). The points \(D_F\) and \(D'_F\) are not coincident due to different experimental and modelling requirements. Experimentally, it is desirable to extract the required data by making the smallest possible number of measurements, whilst computationally the node pattern on the ball surface must be consistent with the arrangement of elements required to model the large deformation of the ball during impact successfully. The data considered here therefore pertains to two points on the ball initially separated by a distance of 2.4mm. However, translation of the ball in the \(x\)-direction during the impact implies that the \(\dot{x}_{D_F}(t)\) measurement contains data from a continuous succession of points on the ball close to and on either side of point \(D'_F\).
\( \dot{x}_{DA}(t) \) is therefore the best estimate of \( \dot{x}'_{DP}(t) \) during the impact. This approximation limits the level of detail which can be extracted from comparisons between the finite element and remotely measured experimental data and this important issue will be addressed later in Chapter 5.

Despite the approximation in the point on the target from which the data originates, the increase in forward velocity from rest during the impact displayed by \( \dot{x}_{DA}(t) \) and \( \dot{x}'_{DP}(t) \) has the same general form. Both results show a small forward velocity over the first 100-150\( \mu \)s of the impact, which then increases rapidly. Close agreement on a forward velocity of 52\( \text{ms}^{-1} \) at the end of the impact strongly suggests the whole body mechanics of the finite element model are an accurate prediction of the real impact in terms of the club head initial velocity and golf ball final velocity.

The finite element result, \( \dot{x}'_{DP}(t) \) is larger than the measured \( \dot{x}_{DA}(t) \) throughout the central portion of the impact. This effect could be produced if the ball material in the finite element model has insufficient stiffness at high strain-rate, allowing greater deformation in the forward direction than is actually the case. Additionally, the compressibility of the finite element ball material is greater than that of the real golf ball, as discussed in Section 2.5.3, and this would act to increase the effect of insufficient material stiffness on the forward deformation. However, the discrepancy between the finite element and experimental results cannot be attributed entirely to the behaviour of the ball material model. The data being compared originates from two points on the ball initially separated by a distance of 2.4mm and forward translation of the ball causes the point on the ball interrogated by the laser beam to move toward the impacted side. Thus, less of the velocity component associated with the recovery of the forward deformation across the ball from point \( A_F \) to \( D_F \) will appear in the \( \dot{x}_{DA}(t) \) measurement, resulting in a slight underestimate of the forward deformation recovery velocity.

Although the forward deformation velocity component in the finite element model appears larger than the experimental result, the pattern of forward deformation and recovery exhibits trends equivalent to those seen in the experimental forward velocity
data at points $A_F-E_F$ across the ball. This is demonstrated in figure (4.18) where the computationally derived forward velocities $\dot{x}_{A_F}(t)$ and $\dot{x}_{E_F}(t)$ show how the initiation of forward velocity through the finite element ball model progresses from the contact site to the free side in the same manner as shown earlier with the experimental forward velocity data in figure (4.8). The intersection of the two curves, $\dot{x}_{A_F}(t)$ and $\dot{x}_{E_F}(t)$, at a point approximately 200μs into the impact, where the lateral deformation begins to recover, is also consistent with the experimental data.

The forward deformation of point $E_F$ relative to point $D_F$ is shown in figure (4.19) along with the result calculated by the finite element method for point $E'_F$ relative to point $D'_F$. The computational and experimental data exhibit similar peak relative deformations at approximately the same point 200μs into the impact. However, the deformation over-recovery suggested by the experimental data is not predicted by the finite element model. This discrepancy could again be caused by comparing experimental and computational data not taken from the same points on the ball since the relative deformation behaviour varies quite markedly in a small area, as shown earlier in figure (4.9). The experimental data is taken from points closer to the free side of the ball, where greater over recovery would be expected. Alternatively, insufficient representation in the finite element model of the strain-rate dependency in the viscoelastic ball material could cause the rates of deformation recovery exhibited in the experimental and computational results to differ.

The contact approach calculation is consistent with the comparisons already drawn between the experimental and computational forward deformation results. The method used to calculate the contact approach from the experimental data was described in Section 4.2.4 and is applied here in the calculation of the contact approach from the finite element results. The contact approach calculation includes the club head velocity and the close agreement between computational and experimental results shown in figure (4.20) therefore additionally implies that the reduction in golf club head velocity predicted by the finite element model agrees closely with the experimental data. Figure (4.20) shows the peak contact approach occurs at approximately 200μs into the impact in both the experimental and computational data. The peak height in the measured data
is slightly less than the finite element equivalent because translation of the target again causes the remotely measured data to be taken from points on the ball progressively closer to the impact site causing the measurement to include less of the forward deformation across the ball from point \( A_F \) to \( D_F \) than is present in the finite element result.

4.4.3 Rotational velocity

The rotational velocity of the finite element ball model during the impact is derived from the difference in forward velocity of two nodes located above and below the equator of the ball. This is equivalent to the experimental calculation of the rotational velocity described earlier in Section 4.2.6. The computational and experimental results are shown in figure (4.21) where it can be seen that the two sets of results are in agreement on a final rotational velocity of the ball of approximately 3000 rev/min. The finite element model, however, underestimates slightly the rate of increase in rotational velocity in the interval \( 100 \leq t \leq 150 \mu s \).

4.4.4 Club head oscillation

In Chapter 3, modal analysis was used to validate the finite element golf club head model over a range of frequencies from 4-20 kHz. Whilst there was close agreement between the steady state dynamic properties of the club head model and the real club head, further validation of the impact behaviour of the club head model is required. Comparison of the measurement made during impact using the accelerometer attached to the back of the club head can be made with the acceleration of a node at an equivalent position on the model. This is shown in figure (4.22), where strong agreement between the computed and measured club head acceleration can be seen in the first half of the impact. As discussed previously in Section 4.3, the oscillating component of club head acceleration results from the pattern of deformation and recovery of the club-ball combination. Thus, whilst the mean values (not shown in figure (4.22)) of the computed and measured club head acceleration exhibit close agreement in the second half of the impact, the agreement between the oscillating components of acceleration is less close due to the differences in the computed and actual forward deformation recovery of the ball, discussed earlier in Section 4.4.2.
The level of agreement between the finite element golf club-ball impact model and the experimental data presented in this section, in a range of measurements covering the variation of many of the important deformation, velocity and acceleration parameters over time during the impact, provides substantial evidence for the validity of the finite element model. However, several of the discrepancies between the behaviour of the model and the experimental data cannot be attributed to a single cause. This limits the level of detail to which the experimental and computational results can be compared, particularly in the recovery phase of the impact. These limitations are stated explicitly in Section 4.6 and are the subject of further study in this thesis.

Whilst important developments in remote vibration measurement and laser Doppler vibrometry presented later in this thesis are directed towards obtaining superior experimental data with which to validate theoretical analysis, the experimental data presented in this chapter can be considered to validate the finite element golf impact model to a level of detail beyond that which has been possible previously. It is therefore considered appropriate at this point in the thesis to consider the usefulness of a validated finite element impact model in the context of both engineering product design and furthering the understanding of the detailed mechanics occurring during impact between a golf club and ball.

4.5 Usefulness of a validated, detailed finite element golf impact model

4.5.1 General considerations

A validated, detailed finite element impact model is a powerful design analysis tool. By providing the capability to change parameters quickly and analyse their effects, the validated finite element model can assist in reducing design iterations and hence the cost of introducing a new product. Design analysis examples of relevance to the golf industry include the study of off-centre impacts, the use of alternative material properties, the re-distribution of the wall thickness in the club head and the study of multi-layer golf ball constructions. This impact analysis capability, allied to the steady state vibrational analysis, identified in Chapter 3 as being important in the
understanding of ‘feel’, gives the validated finite element model considerable functionality in new product development. Additionally, having proven the modelling procedure presented in Chapter 2, models of different club heads and ball constructions can be created, following the same rules, with considerable confidence in the ability to predict closely the dynamic behaviour of the real product.

In the pursuit of improved knowledge of the detailed mechanics occurring during impact, a validated finite element model is particularly useful as it can be interrogated for parameters which would be extremely difficult to measure experimentally. In the golf impact, it is particularly difficult to capture experimental data from points on the club head or ball near to the contact site. However, the range of measurements made elsewhere on the club head and ball, presented in this chapter, strongly suggest that the behaviour of those areas for which there is no experimental data is also represented accurately in the finite element model.

4.5.2 Interrogation for parameters difficult to obtain experimentally

The relative displacement across the face of the golf club head during impact is an example of an important parameter which is extremely difficult to measure experimentally. This can be determined from the finite element model by considering the displacement in the x-direction of three nodes on the face of the club head. The nodes all lie in a line, parallel to the xz-plane, which passes through the centre of the club face. A plan view of the club head is shown in figure (4.23), where it can be seen that node $H_2$ lies in the centre of the face, equidistant from nodes $H_1$ and $H_3$, which are chosen to lie at the edges of the face. In this arrangement the forward displacements $x_{H1}(t)$ and $x_{H3}(t)$, of nodes $H_1$ and $H_3$ respectively, are assumed not to include any component of deformation, such that the average of these two displacements represents the displacement of the un-deformed face centre, in the x-direction, in the presence of whole body rotation of the club head about the normal to the xz-plane. The deformation of the centre of the club face is therefore given by displacement of node $H_2$ relative to this average:

$$x_{H2,\text{DEF}}(t) = x_{H2}(t) - \left( x_{H1}(t) + x_{H3}(t) \right) / 2$$  \hspace{1cm} (4.4)
This result is shown in figure (4.24), where a peak deformation of 0.22mm is predicted at a time 200μs into the impact. The timing of the peak deformation is consistent with that of the maximum approach deformation of the ball, shown earlier in figure (4.20).

The magnitude of the peak face deformation can be verified approximately by a short hand calculation. A strain energy method is used to obtain the static deflection of a simply supported rectangular plate subjected to a uniform pressure [4.8, 4.9]. The golf club face is approximated by a steel rectangle of side 40x70mm and the simply supported boundary condition is chosen since other surfaces of the club head do not rigidly support the face. The peak golf club-ball impact force of 12kN, obtained experimentally in Section 4.3, is applied over the whole plate to give a uniform pressure 4.29MNm⁻². Taking the average face thickness of 3mm from the finite element model, a deflection of 0.18mm is calculated for the face centre. This is an underestimate of the deflection produced by an impact loading as the actual peak load applied to the club face will be larger than that measured experimentally at the back of the club head. The calculation however serves as basic validating evidence for the order of magnitude of the face deflection predicted by the finite element impact model. The face deflection and recovery data is of value in further studies which may seek to analyse the effect of face thickness on the transfer of energy to the golf ball.

In section 4.2.3, the inability to measure the forward velocity of the golf ball using a laser vibrometer directed at the ball approximately parallel to the intended flight path was discussed. Data pertaining to the behaviour of the free side of the ball is, however, readily obtainable from the finite element model. The forward displacement of the free side of the ball relative to point £P, calculated from the model, is shown in figure (4.25). A substantial negative relative forward deformation occurs in the first half of the impact which is consistent with the experimental data shown earlier in figure (4.9). Figure (4.25) additionally shows this deformation to be the start of a cycle of deformation and 'over-recovery', to a dimension greater than that of the undeformed ball, which continues for a short time after the end of the impact.
4.5.3 Wholefield data representation

The experimental techniques presented in this chapter provide validating evidence for the finite element impact model at a number of discrete locations on the club head and ball. The close agreement between this data and the finite element model strongly suggests that the wholefield picture of the impacting bodies, afforded by the finite element results post-processor, is an equally valid form of results presentation. This representation of deformation and recovery patterns is important in developing a detailed understanding of the club head and ball behaviour during impact.

The experimental data captured from discrete points on the ball, presented in Sections 4.2.2 and 4.4.1, suggests that the lateral deformation progresses across the ball from the contact site to the free side but, equally, points on the ball closer to the contact site recover this deformation later in the impact. This behaviour can be observed in figures (4.26a-f), which show the side of the finite element golf ball model at six points in time, equally spaced in the interval 100-350µs into the impact. The contact deformation caused by impact with the lofted club face is visible on the left of each figure and coloured fringes on the ball represent regions of equal lateral deformation occurring parallel to the z-axis, out of the plane of the figure. The lateral deformation can be seen to propagate away from the contact site but the point of maximum deformation does not progress beyond the middle of the ball. Figure (4.26d) illustrates how the lateral deformation over-recovery is initiated on the free side of the ball approximately half-way through the impact. The over-recovery then propagates back towards the impacted side of the ball, as shown in figures (4.26e&f), only reaching the contact site at the end of the impact.

This behaviour is not at all obvious by examination of data from individual points in the model but is consistent with the classical theory developed for the longitudinal collinear impact of two rods [4.10, 4.11]. In this theory the compressive wave which propagates away from the contact site as a result of impact is reflected back from a free boundary as a tensile wave. The two impacting bodies remain in contact until the reflected wave in the longer of the two bodies returns to the contact site. In the golf club-ball impact the ball is the longer body due to the lower velocity of wave
propagation in the polymeric material. Thus, the wholefield representation of the lateral deformation and recovery pattern is instructive in understanding the relationship between the lateral deformation and the contact duration of the golf club-ball impact.

A wholefield representation of the forward velocity is also instructive. In Section 4.2.3, a set of single point measurements showed how the initiation of forward velocity progresses across the ball from the contact site to the free side during impact. Additionally, in Section 4.2.6 the generation of rotational velocity components from differences in forward deformation above and below the equator was discussed. Using the finite element results post-processor, these related phenomena can be observed together. Figures (4.27a-f) show the side of the finite element golf ball model at six points in time, equally spaced in the interval 100-225µs into the impact. The contact deformation caused by impact with the lofted club face is again visible on the left of each figure and coloured fringes on the ball represent regions of equal forward velocity. The initiation of forward velocity can be seen to progress across the ball in a direction approximately perpendicular to the club face over the interval 100≤t≤150µs. After this time the deformation of the ball below the equator increases more rapidly than that above the equator leading to greater forward velocity components below the equator. This can be seen in figures (4.27d-f), where the direction of the forward velocity fringes is rotated from that in figures (4.27a-c). At the point t=225µs shown figure (4.27f) the whole body rotational displacement of the ball is approximately 0.1 radians and this cannot be resolved in the figure. In the later stages of the impact the forward deformation of the ball below the equator is perceptibly larger than that above the equator. This is, however, considered as shearing of the ball rather than genuine whole body rotation since the part of the ball in contact with the club face remains unchanged.

The effect of the impact on the club head can also be analysed using the wholefield data representation. Figure (4.28) shows the club and ball at a time 300µs into the impact. At this time, the rotational velocity of the ball is well established, as seen in the coloured fringes which indicate a near-vertical velocity gradient across the ball with greatest forward velocity in the lower part of the ball. Additionally, a velocity gradient
is observed vertically across the whole club head. This suggests that the club head rotates about the z-axis and effectively loses some of its loft during impact. This will reduce the launch angle of the ball and the generation of rotational velocity components, both of which are crucial ball flight parameters.

The examples discussed in this section indicate the way in which finite element models of the impact, validated using the experimental techniques presented in this chapter, can be used improve the understanding of the relationship between the two impacting bodies by providing new, detailed, quantitative data on a variety of impact parameters.

4.6 Evaluation of the experimental techniques

A system of measurements has been presented in this chapter which is suited to the analysis of high-speed, short duration impacts between lightweight bodies in which large elastic deformation occurs. The system uses a small number of sensors arranged to capture relevant information by a limited number of measurements. Using two laser Doppler vibrometers, a piezo-electric accelerometer and a suitable measurement trigger, short duration impact data has been captured which facilitates estimation of the time histories of:

i) The force applied by the club to the ball and its relationship to the deformation of the ball.

ii) The spatial variation of the force applied by the club to the ball across the club face.

iii) The forward and rotational velocities of the ball from rest.

iv) The deformation of the ball in the direction of impact and perpendicular to it.

This large amount of data, obtained under difficult measurement conditions from only a small number of sensors, is of superior detail and greater mechanical relevance than that obtainable using video techniques. The experimental techniques presented in this chapter therefore provide the foundation for the further study of a variety of impacting bodies and their value in the validation of finite element models has been demonstrated.
Issues which are of importance to remote measurement techniques generally, however, detruit from the otherwise high quality of data obtained using the measurement system presented in this chapter. Three issues of key importance are:

i) Translation of the target object causes the point on the target interrogated by the laser beam to change during the impact. This leads to measurements which are over-estimates of golf ball lateral deformation and under-estimates of forward deformation in the second half of the impact.

ii) The experimental measurement locations do not match the locations of nodes on the finite element model. This and the problem in (i) combine to limit the level of comparison which can be drawn between the computational and experimental results. In particular, discrepancies between the two sets of results cannot be attributed directly to a problem in the model when the data used in the validation process does not originate from directly equivalent points on the real object.

iii) The ability to make a remote measurement in a direction close to the x-axis would yield further detailed experimental information pertaining to the behaviour of the colliding bodies in the direction of impact. However, such measurements involve velocities which are significantly greater than the upper measurable velocity limits of current laser Doppler vibrometers.

As remote, non-contact measurement methods become increasingly popular, the developments in remote vibration measurement using the laser Doppler technique necessary to address these issues are of considerable importance and are the subject of what follows in this thesis.
5 Remote vibration measurements: compensation of waveform distortion due to whole body translations

5.1 Introduction

The experimental golf club-ball impact analysis presented in Chapter 4 illustrates examples of vibration response measurements taken under actual operating conditions. Measurements of this nature are of engineering importance for many reasons including product development or refinement and the validation of computational analysis. However, large scale whole body motion of the subject under investigation complicates the collection of point vibration measurements by traditional contacting transducers fixed to the point of interest on the body. The golf ball impact measurement described in Chapter 4 is one example of a measurement situation in which a contacting transducer would be damaged during the whole body motion and where the low mass of the body would cause the mass of a contacting transducer to be significant. Additionally, complicated or large scale movement of the body introduces difficulties in signal retrieval by electrical connection.

Remote point vibration measurements using the laser Doppler technique are a practical and increasingly popular alternative to the use of contacting transducers in difficult measurement situations. A novel application of a remote measurement procedure to the analysis of a golf ball during impact was reported in Chapter 4. All remote measurements, however, operate from a measurement perspective in which the transducer is fixed in space, rather than to a point on the target body. In the golf ball impact measurement, whole body motion of the ball causes measurements made remotely using a laser vibrometer to originate from instantaneously probed regions on the ball surface increasingly distant from the initial point of interest and this detracts from the otherwise high quality of data obtained. The desire to improve the quality of data obtainable from remote measurements in cases of significant whole body motion
provides the motivation for the developments in remote vibration measurement techniques presented in this chapter.

In a remote measurement on a target which undergoes whole body translation, the recorded data is taken not from a single point but from a continuous succession of points on the target body such that data at the end of a measurement may originate from a point whose separation from the original interrogated region is many times the size of the instantaneously probed region. In particular, this chapter investigates remote point vibration measurements in the presence of larger scale whole body motion using a laser Doppler vibrometer. For this transducer, a body is deemed to have 'moved' if it is displaced by more than one laser beam diameter, typically 0.1-1mm, in a direction perpendicular to the sensing axis of the vibrometer since this causes data at each end of the measurement to originate from entirely distinct points on the target surface.

A method which compensates for relative whole body displacement between the target and a remote transducer fixed in space is introduced. This displacement compensation technique uses data from two simultaneous remote measurements combined with a measurement of the whole body target displacement to produce a much closer estimate in time of the measurement which would have been made by a transducer fixed to a point on the body. The combined measurement 'exists' for the time in which the target moves between the two remote laser beams and represents a significant improvement in the quality of data from a single remote measurement, as demonstrated in this chapter by application of the technique to simulated steady state vibration on a plate which translates with several velocity profiles. The improvement in data quality obtainable from practical remote laser Doppler vibrometer measurements of steady state vibration is illustrated using data captured from a lightweight hollow golf club head, which translates a distance of approximately 10cm and exhibits steady state vibration in the first few milliseconds after impact.

The investigation of lightweight bodies during impact is an example of another difficult measurement situation to which remote measurements are particularly suited.
However, the transient vibration phenomena which occur in an impact are generally accompanied by rapid whole body acceleration, as demonstrated by the golf ball impact measurements reported in Chapter 4. The displacement compensation technique is therefore developed in this chapter to enable analysis of transient pulse propagation through a translating body using data from remote measurements. The theoretical model of stress pulse propagation along a viscoelastic rod developed by Kolsky [5.1] and discussed earlier in Chapter 2 is adapted in this chapter to incorporate whole body translation of the viscoelastic rod. The model is then representative of a longitudinal impact on a free viscoelastic rod and is used to produce simulated remote and fixed measurements of stress pulse propagation on a translating body. This data validates the ability of the displacement compensation technique to produce a close estimate of the stress pulse propagation measurement which would be made by a transducer fixed to a point on a translating body.

The displacement compensation technique is applied to the golf ball data presented in Chapter 4 and the improvement in the data quality is demonstrated with reference to the deformation and recovery characteristics of the ball determined in Chapter 4. The capability to derive data effectively from a single point on a moving body using remote transducers in difficult measurement situations is a valuable experimental tool which additionally permits direct comparison of experimental data with the behaviour of fixed points on moving bodies in theoretical studies. The golf ball impact situation is used to illustrate how the displacement compensation technique can be employed to obtain a close estimate of the waveform which would have been measured at any fixed point on the target for the time in which that point lies between the two points interrogated by the probe laser beams. If fixed points are chosen to match the location of nodes in a finite element model of a translating body, a direct comparison can be made between the behaviour of nodes in a finite element model and close estimates of the behaviour of equivalent fixed points on the real body. This permits a significantly more detailed comparison of computational and experimental data than would be possible without the displacement compensation technique and provides stronger evidence to validate the finite element golf impact model developed in Chapter 2.
5.2 Remote vibration measurements on moving bodies

The optimal use of a laser Doppler vibrometer by intelligent alignment of the probe laser beam was discussed in Chapter 4. In this study, vibration velocity components are of primary interest and alignment of the laser beam perpendicular to the direction of whole body motion ensures in-plane velocity components due to whole body motion will not contribute to the measured signal. This leaves a signal containing vibration information from a continuous succession of points on the target but which does not include components of whole body velocity associated with the changing probed region on the target. The whole body displacement must, however, be measured as part of the displacement compensation technique. The required accuracy of this measurement is of the order of one laser beam diameter, therefore only low frequency components of the signal are needed to define the whole body motion satisfactorily. This being the case, the whole body displacement measurement can be taken from any convenient point on the deforming body and the signal then smoothed at an appropriate cut-off frequency. Experience has shown that the required data is easily obtained and this method works well as part of the displacement compensation process, improving the data quality in a range of different conditions.

Remote vibration measurements incurring similar problems have been made previously on targets rotating about a fixed axis. A fixed point on a target rotating in a plane perpendicular to the laser vibrometer measurement axis can be followed by continuous circular scanning of the laser beam in synchronisation with the target rotation speed. A technique for non-synchronous rotation of the target and laser beam has also been introduced for frequency domain analysis of rotating bodies [5.2]. Continuous scanning could be similarly applied to a simultaneously vibrating and translating target body, provided the required scanning motion was accurately known and did not exceed the maximum beam scan rate. However, it is impractical to synchronise the scan of a beam to the large and often non-linear accelerations on impacted bodies and non-synchronous scanning would add undue complexity to the measurement. The technique described in this chapter in fact addresses the converse of the scanning beam case in which translation of the target perpendicular to the stationary laser beam causes data to be recorded from a spatially dense succession of points on the target surface.
5.3 A single remote measurement of steady state vibration on a translating body

5.3.1 Time domain characteristics

Figure (5.1a) shows a probe laser beam from remote point $A_R$ aligned perpendicular to the direction of whole body motion such that the initially probed region on the target, $A_R$, is distances $x_{AF}$ and $y_{AF}$ from an arbitrary reference point on the target, taken as the target edge in this example. The target shown, whose vibration is to be simulated, is a flat rectangular plate of length $l$ in the $x$-direction and width $w$ in the $y$-direction, vibrating with angular frequency $\omega$. The simply supported boundary condition is the only case to possess an exact closed-form solution of the trigonometric and hyperbolic functions which describe the free vibration of a rectangular plate [5.3]. In the absence of whole body translation in the $xy$-plane the vibration response $V_{AF}(t)$ measured by either a fixed or a remote transducer at point $A_F$ is then given by:

$$V_{AF}(t) = A \sin(\omega t) \sin\left(\frac{n\pi x_{AF}}{l}\right) \sin\left(\frac{n\pi y_{AF}}{w}\right)$$

(5.1)

where $A$ is the amplitude of vibration of the plate and $n,m=1,2,...$ indicate the order of the vibration mode. The vibration response of a plate with built-in boundary conditions would be approximated by [5.4]

$$V_{AF}(t) = A \sin(\omega t) \left[1 - \cos\left(\frac{2n\pi x_{AF}}{l}\right) \cos\left(\frac{2m\pi y_{AF}}{w}\right)\right]$$

(5.2)

In this study the experimental techniques are of primary interest. The simply supported boundary condition is therefore considered in order to simplify the analytical description of the experimental techniques, which are still of general applicability. Furthermore, only target translation along a single axis is considered. If a simply supported plate translates with velocity $\ddot{x}(t)$, as shown in figure (5.1b), the response, $V_{AR}(t)$, measured by the remote transducer becomes modulated and is given by:

$$V_{AR}(t) = A \sin(\omega t) \sin\left(\frac{n\pi}{l} \left(x_{AF} - \int_0^t \ddot{x} \, dt\right)\right)$$

(5.3)
In this case, $V_{AF}(t)$ continues to represent the measurement made by a transducer fixed to point $A_F$ on the target. The time dependence of the second sine term in equation (5.3), however, indicates how the remote measurement $V_{AR}(t)$ will comprise vibration response data from a spatially dense succession of points across the target. When compared to $V_{AF}(t)$, the $V_{AR}(t)$ waveform is thus both distorted from the original sinusoidal shape in time and has an amplitude which is modulated in the range $0$-$A$ according to the terms in the second sine expression of equation (5.3). This is simulated in figure (5.2), where $V_{AF}(t)$ and $V_{AR}(t)$ are plotted for a time period corresponding to the passage of the entire plate past a remote transducer ($n=1$). Data in the remotely measured $V_{AR}(t)$ waveform can be ascribed to the continuously changing interrogated point from which it originates by integrating the measured whole body velocity $\dot{x}$. However, this provides an unacceptably small amount of data for any given point on the target unless $\dot{x}$ is extremely small.

Greatest distortion of $V_{AR}(t)$ from the sinusoidal form of $V_{AF}(t)$ is observed when the remote measurement interrogates points where the spatial gradient of the modeshape has greatest magnitude, near the nodes of the modeshape. The distorting effect at any value of $x$ increases with increasing $\dot{x}$. Despite this distortion both $V_{AF}(t)$ and $V_{AR}(t)$ appear zero simultaneously in figure (5.2), since $\sin(\omega t)=0$ is true at all points on the plate simultaneously, regardless of the measurement perspective. The distortion is not harmonic, however, as will be discussed at relevant points later in this chapter.

5.3.1.1 Quantification of distortion in $V_{AR}(t)$

The distorting effect can be quantified in the time domain by locating the turning points $t_r(p)$ and $t_R(p)$ of $V_{AF}(t)$ and $V_{AR}(t)$ respectively, where $p=1,2,3...$ identifies each turning point. From equation (5.1), the turning points of $V_{AF}(t)$ occur when $\cos(\omega t)=0$. By differentiating equation (5.3) for the case where $x_{AF}=l/n$ and the target has constant translational velocity, $\dot{x}$, the turning points of $V_{AR}(t)$ are given by solutions to the equation:

$$\frac{\omega}{\zeta} = \frac{\tan(\omega t)}{\tan(\zeta t)}$$

(5.4)
where $\zeta = (n\pi x)/l$ and can be considered as the apparent angular frequency of the moving modeshape when observed remotely. Equation (5.4) cannot be solved readily but indicates the influence of the ratio $\omega/\zeta$ on the degree of distortion. This can be quantified if the turning points, $t_\text{R}(p)$, are located numerically and compared to the readily predictable turning points $t_\text{F}(p)$. Since $V_{AF}(t)$ and $V_{AR}(t)$ are zero simultaneously, the distortion in time exhibited by the each turning point of a remotely measured waveform, $\Delta_R$, can be expressed as a fraction of the time between successive zeroes thus:

$$\Delta_R = \frac{t_\text{R}(p) - t_\text{F}(p)}{\pi/\omega}$$

$\Delta_R$ is plotted across the modeshape from $x=0$ to $x=l/n$ in figure (5.3) for the case where $x_{AF}=l/n$. Frequency ratios $\omega/\zeta=100$ and $\omega/\zeta=10$ are used to represent upper and lower regions of the typical measurement range respectively, showing the distortion reduction as $\omega/\zeta$ increases. With reference to figure (5.3), a single remote measurement on a target undergoing whole body translation will exhibit increasing distortion as the interrogated point approaches a node. In this case, the distortion will cause $t_\text{R}(p)$ to lag $t_\text{F}(p)$ as the interrogated point approaches an antinode, whilst $t_\text{F}(p)$ leads $t_\text{R}(p)$ as the interrogated point moves away from an antinode towards a node. The shape of the curves shown in figure (5.3) also illustrates how a single remote measurement will contain appreciable distortion over a wide range of values of $\omega/\zeta$ unless the measurement is confined to the region immediately around an antinode.

In measurements where $\dot{x}$ is a function of time, the distortion, which is dependent upon both $\zeta(t)$ and the spatial gradient of the modeshape in the instantaneously probed region, can be calculated for a given measurement using the same numerical method.

5.3.2 Frequency domain characteristics

Equation (5.3) suggests that, in the case of constant whole body target translation velocity, the response spectrum of $V_{AR}(t)$ will contain peaks at angular frequencies $\omega \pm \zeta$. In practice, however, only data captured from points on the target near to the main point of interest are useful since excessive amplitude modulation and distortion in the time domain would otherwise be encountered, as discussed above. Additionally,
the magnitude of the whole body displacement is often less than one spatial period of
the modeshape. Both of these features imply that only a fraction of one spatial period
of the modeshape will be included in the remote measurement in the majority of cases,
which constitutes insufficient data length to resolve separate spectral peaks at angular
frequencies $\omega \pm \zeta$. This problem is illustrated in figures' (5.4a-e), which show
convolution in the frequency domain [5.5] of the remotely observed modeshape and
the vibration itself.

It is useful to define a quantity, $\lambda_p$, as the spatial period of the modeshape, where
$\lambda_p = 2\pi n$. Figure (5.4a) then represents the amplitude variation of a measurement in
which target translation, $x$, causes data to be captured from a fraction of one spatial
period $x/\lambda_p$. The corresponding frequency spectrum, figure (5.4c), has a large zero
frequency component and side-lobes separated by $\zeta \lambda_p / x$ rads$^{-1}$. Figures (5.4b&d) show
that a sinewave of frequency, $\omega$, which is not heavily truncated produces a single
narrow peak when viewed in the frequency domain. However, convolution of data in
figures (5.4c&d) produces the spectrum shown in figure (5.4e) which exhibits a broad
peak at the angular frequency of vibration, $\omega$, with side-lobes separated by $\zeta \lambda_p / x$. The
broad peak centred on frequency $\omega$ is the result of convolution with the large zero
frequency component in the Fourier transform of the modeshape and distinct peaks at
frequencies $\omega \pm \zeta$ cannot be resolved in figure (5.4e) as $\zeta \lambda_p / x$ and the $\omega \pm \zeta$ peaks are
indistinguishable within the main peak at $\omega$. Thus \( x_{\lambda_p} - \int \dot{x} \, dt \geq \lambda_p \) is the condition
for visibility of the $\omega \pm \zeta$ peaks in the response spectrum and this condition is never met
if only data captured from points on the target near to the main point of interest are
useful. Under these conditions, response spectra measured remotely from targets
translating with either constant or time varying $\dot{x}$ are alike, since both are dominated
by short data length characteristics resulting in poor frequency resolution and broad
spectral peaks.

A single remote measurement from points on a 'long' target covering several spatial
periods of the modeshape will generally not suffer this data length problem and, in the
case of constant $\dot{x}$, spectral peaks are observed at frequencies $\omega \pm \zeta$ with no peak at
frequency \( \omega \). This is shown in figure (5.5a), where \( \omega/\zeta=10 \) and the peak amplitudes are normalised by the amplitude of the single peak which would occur in the equivalent spectrum from a fixed transducer. (This normalisation will be used for all subsequent spectra of simulated data). With larger values of \( \omega/\zeta \), the peaks in the response spectrum become relatively closer together and ultimately a single peak at frequency \( \omega \) is formed if \( \omega/\zeta \to \infty \). For cases in which \( \dot{x} \) is a function of time and the direction of motion does not reverse, \( \zeta \) is modulated by the changing whole body target velocity. The resulting response spectrum thus displays a pair of broad bands at frequencies \( \omega \pm \zeta(t) \), where the bandwidth corresponds to the values of \( \zeta \) encountered during the measurement. This is observable in figure (5.5b) where the normalised spectrum is shown for data captured over several spatial periods of the modeshape when \( \zeta \) increases linearly with time during the measurement from zero to a value \( \zeta_{\text{MAX}} \).

Harmonic whole body translation of the target is an important special case of \( \dot{x} \) as a function of time, in which the instantaneously probed region on the target does not become progressively further away from the initially probed region. In this case sufficient data can be collected to produce reasonably detailed response spectra pertaining to localised regions of the target. For sinusoidal whole body translation at angular frequency \( \Omega \), \( V_{\text{Har}}(t) \) becomes:

\[
V_{\text{Har}}(t) = A \sin(\omega t) \sin\left(\frac{n\pi}{l} (x_{\text{AF}} - A_x \sin(\Omega t))\right) \tag{5.6}
\]

where \( A_x \) is the amplitude of the harmonic whole body translation. The corresponding response spectra are derived by means of Bessel function expansions and consequently contain the angular frequencies \( \omega \) and \( \omega \pm m\Omega \) where \( m \) is an integer. The characteristics of the response spectra are dependent upon the localised region on the target measured.

If the point of interest is an antinode, harmonic translation at frequency \( \Omega \) symmetrically about the antinode causes the measured vibration amplitude to vary at \( 2\Omega \). The simulated amplitude variation in the remotely measured signal is shown in figure (5.6a) and, whilst the waveform appears sinusoidal, it actually takes the form of
the second sine term in equation (5.6). Figures (5.6a-e) show convolution in the frequency domain of the target vibration at the antinode and the amplitude variation caused by harmonic translation of the target. The resulting response spectrum is shown in figure (5.6e) and contains the frequencies $\omega$ and $\omega \pm 2m\Omega$.

For remote measurements made elsewhere on the modeshape, spectral peaks occur at frequencies $\omega \pm m\Omega$ in the response spectrum. These peaks broaden according to the spatial gradient of the modeshape in the localised measurement region, as expected from the earlier discussion of distortion. This effect is demonstrated in figures (5.7) and (5.8) which show two simulated, normalised response spectra measured remotely from a target undergoing harmonic whole body translation. In figure (5.7), the point of interest on the target was near an antinode, whilst figure (5.8) pertains to a point on the target nearer to a node where the spatial gradient of the modeshape is larger and the resulting spectrum exhibits correspondingly broader $\omega \pm m\Omega$ peaks. The main peak amplitude in the remote measurements is approximately 1-3% lower than that from a fixed transducer and this small difference cannot be resolved in figures (5.6), (5.7) and (5.8).

5.3.3 A practical example of a single remote vibration response measurement

A standing wave is established on the uppermost, or 'crown', surface of a hollow golf club head during impact with a golf ball. This vibration persists for approximately 3ms and is of interest to club designers since, as demonstrated previously in Chapter 3, it contributes to the sound of the impact and hence to the player's perception of the 'feel' of the golf club. The surface vibration can be detected remotely during and immediately after the impact using a laser vibrometer positioned vertically above the crown. Remote measurements are advantageous in this situation as the crown mass is approximately 30g and its vibration characteristics would be altered by addition of even a small contacting transducer.

An example measurement taken from a single remote transducer over the time in which the club head translates completely past the probe laser beam is shown in figure (5.9). The measurement has a low frequency component due to rotation of the whole club.
head and a higher frequency component due to oscillation of the crown surface. The amplitude of the measured waveform reduces towards the end of the data length due to the combined effects of vibration decay and increasing proximity of the instantaneously probed region to the edge of the crown, which is nodal. Moderate distortion of the waveform, of magnitude $\approx 12\%$, is expected near the nodes since $\omega/\zeta=33$.

According to data gathered in Chapter 3, the impact excites more than one mode of vibration of the crown at closely spaced frequencies in the range 5-9kHz. Figure (5.9) shows the time history of the impact and figure (5.10), the corresponding spectrum, shows broad peaks in the frequency range of interest but which are not in agreement with the data in Chapter 3. It is clear that the ability to investigate detailed response from a single remote measurement is limited by waveform distortion and insufficient data length pertaining to any single, fixed point on the surface. The displacement compensation technique will substantially improve the quality of data obtained from remote measurements on targets undergoing whole body translation and this is the subject of what follows in this chapter. Subsequent re-analysis of the golf club crown vibration emphasises how only qualitative vibration characteristics can be ascertained from a single remote measurement.

5.4 A double remote measurement of steady-state vibration on a translating body

5.4.1 Displacement compensation

Waveform distortion and amplitude modulation inherent in a single remote vibration measurement on a target undergoing whole body motion can be overcome to a great extent through use of a second simultaneous remote measurement, at a point $B_R$, made along a measurement axis parallel to the first remote measurement at $A_R$. Combining data from both remote transducers allows close estimation of the vibration time history at point $A_F$ on the target over the measurement duration, which is taken to be the time interval in which point $A_F$ moves between the regions probed by remote transducers located at points $A_R$ and $B_R$. If point $A_F$ lies distances $x_{AF}$ from an arbitrary reference
point on the target (in this case the target edge) and \( x_s \) from point \( B_F \), as shown in figure (5.11), then a weighted sum of data from remote transducer measurements at \( A_R \) and \( B_R \) can be used to produce a close estimate of \( V_{AF}(t) \). For digitally sampled measurements, data from the \( i \)th sample is combined in this displacement compensation technique according to:

\[
V_{AC}(i\Delta t) = [1 - \alpha(i\Delta t)]V_{AR}(i\Delta t) + \alpha(i\Delta t)V_{BR}(i\Delta t)
\]

(5.7)

where \( V_{AC} \) is the displacement compensated surface vibration velocity at point \( A_F \) and \( \Delta t \) is the time between samples. \( \alpha(i\Delta t) \) describes the position of \( A_F \) between the two probed regions and is defined by:

\[
\alpha(i\Delta t) = \frac{1}{x_s} \left( \int_0^{a(i\Delta t)} x \, dt \right)
\]

where \( 0 \leq \alpha(i\Delta t) \leq 1 \). \( \alpha \) determines the relative weights of the measurements from \( A_R \) and \( B_R \) in the combined measurement for any \( i \). Figure (5.12) shows the displacement compensated waveform, \( V_{AC}(t) \), using equation (5.7), for a case where \( \alpha(\zeta)=100 \) (\( x_s=0.3l/n \) and \( x_{AF}=0.5l/n \)). The genuine vibration, \( V_{AF}(t) \), at point \( A_F \), given by equation (5.1), is also plotted on figure (5.12) to indicate how \( V_{AC}(t) \) differs only by a slight amplitude modulation. This close estimate of \( V_{AF}(t) \), produced from remote measurements, exists for the time in which point \( A_F \) moves between points \( A_R \) and \( B_R \), yielding substantially more temporal data from point \( A_F \) than the instantaneous point measurement provided by a single remote transducer.

5.4.2 Time domain characteristics

The exact extent to which the waveform distortion and amplitude modulation of a single remote measurement are eliminated using displacement compensation is dependent upon the chosen measurement parameters, \( x_{AF} \) and \( x_s \). In general, for given \( x_{AF} \) and \( x_s \), the amplitude error varies with the value of \( \alpha(i\Delta t) \), as suggested by figure (5.12). The amplitude error, \( e \), in a given displacement compensated waveform can be defined as:

\[
e(\alpha) = \left[ \left( \hat{V}_{AF} - \hat{V}_{AC}(\alpha) \right) / \hat{V}_{AF} \right] \times 100 \%
\]

(5.8)

\( e(\alpha) \) has a maximum value, \( e_{MAX} \), which occurs at a point in the range \( 0 \leq \alpha(i\Delta t) \leq 1 \) determined by the chosen measurement parameters, \( x_{AF} \) and \( x_s \). \( e_{MAX} \) can be located in
the range \(0 \leq \alpha(t\Delta t) \leq 1\) if the values of \(\alpha\) at which the derivative \(de/d\alpha\) equates to zero are calculated. The function \(de/d\alpha\) is plotted in figures (5.13a\&b) for a range of values of \(x_M\) corresponding to points of interest between two nodes and for two values of \(x_S\).

The distance \(x_M\) is defined in the positive \(x\)-direction as the distance from the previous node in the modeshape, such that \(x_{AF} = ((m-1)\lambda_f/2) + x_M\), where \(m\) is the number of nodes from the target edge where \(x_{AF} = 0\). Figures (5.13a\&b) suggest that for small \(x_S\), the maximum amplitude error in the displacement compensated measurement will occur close to the point where \(\alpha(t\Delta t) = 0.5\) and the range of values of \(\alpha\) at the point of maximum amplitude error will increase with increasing \(x_S\). This is confirmed if \(\alpha\) at the zero crossing of \(de/d\alpha\) is plotted against \(x_M\) for a range of values of \(x_S\), as in figure (5.14). The error \(e(\alpha)\) is defined for all displacement compensated measurements except for the trivial case where the point of interest on the target is a node.

The magnitude of \(e_{MAX}\), based on the \(e(\alpha)\) definition in equation (5.8), increases principally with increasing proximity of \(A_R\) or \(B_R\) to a node and with increasing \(x_S\). It is, therefore, important to choose values of \(x_{AF}\) and \(x_S\) to suit both the point of interest and the mode of interest on the target body. The variation of \(e_{MAX}\) with \(x_{AF}\) across \(1\frac{1}{2}\) spatial periods of a modeshape is shown for several values of \(x_S\) in figure (5.15a). The figure shows that, for a given \(x_S\), \(e_{MAX}\) is a minimum when the initially probed point on the target is an antinode. The magnitude of \(e_{MAX}\) is very small for values of \(x_S\) below approximately \(0.15\lambda\) and \(e_{MAX}\) increases with increasing \(x_S\) such that \(e_{MAX}\) can be considered large at values of \(x_S\) above \(0.4\lambda_f\). The most striking feature of this \(e_{MAX}\) definition is the rise of the amplitude error to infinity when the point of interest is a node, as shown in figure (5.15a). This is simply the result of division by a small number since the vibration amplitude \(\hat{V}_{AF}\) tends to zero when the point of interest is a node.

Defining \(e(\alpha)\) as in equation (5.8) therefore makes the amplitude error appear significantly worse than is actually the case in terms of the desired measurement accuracy. In making a vibration measurement, the antinodes are generally of greatest interest and interrogating a node with either a fixed or remote transducer will reveal little worthwhile vibration data. An amplitude error definition for the displacement
compensated waveform based on the largest vibration amplitude exhibited by the target therefore appears more appropriate:

\[
\epsilon(\alpha) = \left[ \frac{\hat{V}_{AF} - \hat{V}_{AC}(\alpha)}{A} \right] \times 100 \%
\]  

(5.9)

By this definition, \(\epsilon_{MAX}\) is a maximum when the initially probed point, \(x_{AF}\), is an antinode and is a minimum when \(x_{AF}\) is a node. The variation of the magnitude of this \(\epsilon_{MAX}\) with \(x_S\) over 1½ spatial periods of a mode shape is shown for several values of \(x_S\) in figure (5.15b). As with the previous definition, the magnitude of \(\epsilon_{MAX}\) is very small for values of \(x_S\) below approximately 0.15\(\lambda_p\) and \(\epsilon_{MAX}\) increases with increasing \(x_S\) such that \(\epsilon_{MAX}\) is large at values of \(x_S\) above 0.4\(\lambda_p\). Furthermore, the preceding analysis of the \(\epsilon_{MAX}\) location in the range \(0 \leq \alpha/\Delta t \leq 1\) remains valid. This definition of \(\epsilon(\alpha)\) however somewhat flatters \(\epsilon_{MAX}\) when \(x_{AF}\) is near a node, especially if \(A\) is large. Therefore, no single error expression provides a completely satisfactory description of the amplitude error at all points on the mode shape. In practice an appropriate amplitude error definition is:

\[
\epsilon(\alpha) = \left[ \frac{(\hat{V}_{AF} - \hat{V}_{AC}(\alpha))/A_{LOCAL}}{\epsilon_{LOCAL}} \right] \times 100 \%
\]  

(5.10)

where \(A_{LOCAL}\) is the amplitude of the mode shape in the localised region of interest on the target. The variation of \(\epsilon_{MAX}\) is then of the form shown previously in figure (5.15b), scaled according to vibration amplitude at the nearest antinode.

The maximum useful separation of \(A_R\) and \(B_R\) is half of one spatial period of the mode of interest since the maximum amplitude error, based on the \(\epsilon(\alpha)\) definition in equation (5.10), reaches 100% at this separation if, as a worst case example, the initially probed point on the target is an antinode. In general, however, \(\epsilon_{MAX}\) in the displacement compensated waveform is significantly smaller than the maximum amplitude error in a single remote measurement made under the same whole body target translation conditions. This is illustrated in figure (5.16) \((x_{AF}=0.4l/n\) and \(x_S=0.2l/n\)), where the amplitude error for a single remote measurement is shown as a function of time with its displacement compensated equivalent, \(V_{AC}(t)\). \(V_{AC}(t)\) exhibits relatively minor amplitude error and thus provides a much better estimate of the fixed measurement \(V_{AF}(t)\) over the measurement period than the single remote measurement.
The displacement compensated remote measurement is also a significant improvement on the single remote measurement in terms of the distortion of the waveform. The criterion introduced in Section 5.3.2 to quantify the waveform distortion is equally applicable to the displacement compensated measurement. Figure (5.17) shows the distortion encountered in two example compensated remote measurements for constant whole body target translation velocity when $\omega/\zeta=10$, with the equivalent single remote measurement distortion, $\Delta_R$, shown dotted. The two compensated measurements have $x_s=0.2l/n$ and $x_s=0.4l/n$ respectively and give distortion, $\Delta_c$, calculated using equation (5.5). It can be seen that the lead-lag sense of the distortion is reversed in the compensated measurements and that the maximum distortion is less than from the equivalent single measurement, with least distortion occurring, not surprisingly, when the separation of the probed regions, $x_s$, is small.

As with the single remote measurement, the level of distortion decreases with increasing $\omega/\zeta$ and is, in addition, negligible for displacement compensated measurements in which $\omega/\zeta$ is of the order of 100 or greater. This result indicates that the displacement compensated measurement, from a much wider range of measurement conditions, has minimal distortion and amplitude error when compared to the equivalent single remote measurement. Whilst this discussion has focused on constant $\dot{x}$ for clarity, this improvement in the data quality is also found when $\dot{x}$ is a function of time.

5.4.3 Frequency domain characteristics

The displacement compensation technique does not extend the overall length of data captured from the translating target, hence frequency resolution is similar to that obtained from single remote measurements. However, in creating a waveform which closely approximates the genuine vibration at a fixed point over an interval, the displacement compensated data pertains to a considerably more localised region of the target surface than in the single remote measurement and, being virtually free of whole body target displacement effects, yields more representative frequency response information. This is discussed further in Section 5.4.4.
When acquiring data, it is important to ensure that sample rate and anti-aliasing filters are set to allow for broadening of the original single remote measurements in the manner shown, for example, in figure (5.8).

The most obvious improvement on the single remote measurement in the frequency domain occurs in the case of harmonic whole body motion of the target. Here the response spectrum from displacement compensated data exhibits a peak at the angular frequency of vibration \( \omega \) but no peaks at \( \omega \pm \Omega \), unlike the single remote measurement. This is confirmed by comparison of the single remote measurement spectrum shown earlier in figure (5.7) with the equivalent displacement compensated spectrum, shown in figure (5.18). This comparison additionally shows that sidebands at \( \omega \pm 2\Omega \) are retained in the spectrum of the compensated data. This is due to the small amplitude variation in the compensated measurement, shown in figure (5.18a). The variation from the constant amplitude which would be recorded by a transducer fixed to the point of interest on the target is termed the amplitude error, as in the previous section. The amplitude error in the compensated waveform at the point of interest on the target is zero when \( \alpha(i\Delta t) = 0,1 \) and the maximum error occurs at an intermediate value of \( \alpha(i\Delta t) \). A single period of whole body harmonic target displacement involves one cycle of \( \alpha(i\Delta t) \) (from 0 to 1 to 0) and thus two cycles of the amplitude error, such that the fundamental frequency of the amplitude error in the compensated waveform is \( 2\Omega \). Convolution in the frequency domain with the target vibration frequency thus results in the sidebands at \( \omega \pm 2\Omega \) seen in figure (5.18e).

### 5.4.4 A practical remote measurement problem

The golf club crown vibration response data discussed in Section 5.3.4 can be improved considerably by application of the displacement compensation technique. Crown surface velocity data was captured over a series of impacts using a laser Doppler vibrometer positioned at nine remote locations, separated by 1cm, in a straight line vertically above the line of motion of the club head. (In this case the nine data sets are not simultaneous and compensation relies on the good repeatability of the impact conditions). The golf club translates a distance of approximately 10cm in the period of
interest, thus data from several of the remote measurement pairs is used to estimate the vibration response at a given fixed point on the crown surface. The practical arrangement is illustrated in figure (5.19) where it can be seen that, for example, data from points $A_R$ to $G_R$ will at some time during the measurement period pertain to the vibration response at point $G_F$ on the target. The exact contribution of each remote measurement is determined by the separation of the probed regions and the whole body target displacement, $x(t)$, derived independently from an accelerometer attached to a convenient position elsewhere on the club head.

Application of the displacement compensation technique produces vibration response data for each of the points $D_F-J_F$ and also for any intermediate point of interest on the target not explicitly defined as a measurement location $D_F-J_F$. Approximately 3ms of vibration response data is available for a given fixed point on the target, which is substantially more data than can be obtained for a single fixed point from a single remote measurement and is obtained far more conveniently than by the attachment of a contacting transducer at several locations on the surface. In this study, the low frequency part of the compensated signal, which corresponds to rotation of the whole club head, is of little importance and can be removed using a high pass filter with 2kHz cut-off.

The compensated data thus obtained for point $H_F$ is shown in figure (5.20). The amplitude modulation is due to a beat between two principal vibration modes excited at similar frequencies by the impact. The response spectrum of this measurement, figure (5.21), shows the two beating frequencies to be 5700Hz and 6300Hz which agree very closely with the experimental and computational modal analysis of the golf club head presented in Chapter 3. Similarly, smaller peaks at 5.0, 6.8, 7.4 and 8.2kHz present in figure (5.21) are also in agreement with those reported in Chapter 3. This quality of information was not found in the response spectrum of the single remote measurement, shown dotted in figure (5.21). The response spectrum of the single remote measurement exhibits broader peaks which do not appear centred on the correct frequencies. These characteristics are due to the changing velocity of the club head during the impact and are consistent with the peak broadening effect on a single
remote measurement caused by time dependence in the remotely observed apparent angular frequency, $\zeta$, discussed earlier in Section 5.3.2 and illustrated in figure (5.5b). The effect is absent from the displacement compensated measurement and, whilst both response spectra suffer from the effects of short data length, the response spectrum of the displacement compensated data contains peaks which are more distinct, allowing useful comparison of theoretical and experimental data and underlining the importance of displacement compensation.

5.5 Application of displacement compensation to impact transients

5.5.1 Measurement situation

Detailed analysis of bodies subjected to impact necessitates the measurement of transient effects such as wave motion in a body. In an impact, a stress pulse will propagate away from the contact site through both colliding bodies. This pulse motion can be observed from a series of point measurements taken from transducers arranged along the body in the direction of propagation of the pulse. Most impacts involve large scale motion of at least one of the impacting bodies either before or after the collision, thus signal retrieval by electrical connection is difficult and damage may occur to a transducer attached to the moving body. Also, when the impacting bodies are of low mass, the additional mass of a transducer could significantly affect the system under investigation. It is therefore desirable to measure wave motion in impacted bodies by remote means.

The laser Doppler vibrometer is an appropriate instrument for remote measurement of transient effects in impacted bodies. This section describes how the vibrometer can be employed successfully in the measurement of a pulse propagating through a moving body by extending the displacement compensation technique introduced in Section 5.4. A pulse of arbitrary and continuously evolving shape is considered propagating through a body translating under large acceleration, such as is caused by an impact. The method is then applied to the analysis of transient deformation in an impacted golf ball to illustrate how the displacement compensation technique is able to estimate closely the vibration of a single fixed point on a body during impact.
5.5.2 Characteristics of the remote measurement

An impact will generally cause a pulse to propagate through a body in the direction of the impact and the whole body will also accelerate in that direction. For a non-dispersive pulse, the velocity of the pulse relative to a transducer fixed to the body is simply equal to the constant speed, $\dot{x}_p$, of pulse propagation in the impact direction. For a dispersive pulse, the 'group velocity' can be considered by equivalent means. Thus, for a pulse of length $\lambda_p$, a transducer fixed to the body will record a pulse of duration $T_r = \lambda_p / \dot{x}_p$. However, the velocity of the pulse relative to a remote transducer is equal to the sum of the propagation speed and the whole body velocity. Therefore, the remote transducer records a pulse of duration $T_R = \lambda_p (\dot{x}_p + \dot{x})$ which is shorter than $T_r$. The remote measurement thus gives a waveform which contains amplitude information equivalent to a fixed transducer but which is distorted in time according to the motion of the body. For data captured digitally at the same sample rate, $f_s$, from fixed and remote transducers, fewer samples will be in the remotely measured pulse and the effective sample rate is lower than $f_s$ by a factor $T_R/T_r$. This must be recognised in making the appropriate choice of anti-aliasing filter for the measurement conditions.

5.5.3 Adaptation of the displacement compensation technique

A simulated, arbitrary, dispersive pulse on an impacted body measured from two remote locations by transducers, arranged as in figure (5.11), is shown in figure (5.22a). The remotely measured quantities are the surface velocities of the body perpendicular to the impact direction, $V_{AR}(t)$ and $V_{BR}(t)$, and these are plotted against the sample number of each digitally stored data point. The displacement compensation technique is applied in order to reconstruct a close approximation to the measurement which would be made by a transducer fixed to point $A_F$. However, the situation is complicated by pulse shape changes and variable distortion in time according to the changing translational velocity of the whole body.

In figure (5.22a) the two pulse measurements $V_{AR}(t)$ and $V_{BR}(t)$ appear separated in time by a number of samples $n_C$ which is a function of the velocity of propagation of
the pulse, \( \dot{x}_p \), and the translational velocity of the body, \( \dot{x} \). Each pulse measurement is also divided into two intervals such that the pulse rises from zero to a maximum in the first interval and falls from maximum to zero in the second. Each interval is considered separately and in the first interval \( V_{AR}(t) \) comprises \( n_{AI} \) samples whereas \( V_{BR}(t) \) comprises \( n_{BI} \) samples. The pulse shape evolves continuously as it propagates across the body, thus for the \( i \)th sample on \( V_{AR}(t) \) a linear interpolation is proposed to identify the corresponding sample, \( j \), on \( V_{BR}(t) \) such that:

\[
j = (n_B/n_A) i + n_C
\]

where \( j \) is rounded to the nearest integer value. A weighted sum of \( V_{AR}(i\Delta t) \) and \( V_{BR}(j\Delta t) \) is proposed to estimate the velocity at \( A_R \) as it moves between remote measurement locations \( A_R \) and \( B_R \):

\[
V_{AC}(k\Delta t) = [1 - \alpha(k\Delta t)]V_{AR}(i\Delta t) + \alpha(k\Delta t)V_{BR}(j\Delta t)
\]

Equation (5.12) is a development of equation (5.7) and describes the general case of displacement compensation of a waveform from a double remote measurement. The essential difference from equation (5.7) is the presence of the term \( k\Delta t \), which is necessary to correctly represent both the shape change of the pulse and the whole body velocity of the body. The physical significance of \( k\Delta t \) can be described by considering figure (5.22b), where the two remote measurements shown previously in figure (5.22a) are plotted against time and their respective fixed positions in space. Figure (5.22b) shows geometrically the construction of a point on the compensated waveform using data from point \( i\Delta t \) on the pulse measured at \( A_R \) and the corresponding point \( j\Delta t \) on the pulse measured at \( B_R \). These points are shown connected by a straight line \( IJ \) in the \( x-t \) plane and \( k\Delta t \) is then defined by the intersection in the \( x-t \) plane of line \( IJ \) and the whole body displacement curve \( x(t) \) which represents the changing position of \( A_F \). Geometrically, \( k \) can be shown, from similar triangles on figure (5.22b), to be:

\[
k = [(j - i)\alpha(k\Delta t)] + i
\]

In practice, the intersection relating \( i \), \( j \) and \( k \) must be obtained for every \( k \) in which \( 0 \leq \alpha(k\Delta t) \leq 1 \) and can be found in a number of ways.
In the most direct method, \( k \) is selected as the known variable, which explicitly defines the value of \( \alpha(k\Delta t) \) and allows the corresponding \( i \) at the intersection to be found directly if equations (5.11) and (5.13) are combined to form the following equation:

\[
i = \frac{k - n_C \alpha(k\Delta t)}{\left[ \left( \frac{n_{BL}}{n_{AI}} - 1 \right) \alpha(k\Delta t) + 1 \right]} \quad (5.14)
\]

The value of \( i \) can be used to evaluate \( j \) from equation (5.11) and \( V_{AC}(k\Delta t) \) is then found from equation (5.12) for all \( k \) in which \( 0 \leq \alpha(k\Delta t) \leq 1 \). The discrete nature of the data implies the value of \( i \) determined using equation (5.14) must be rounded to the nearest integer. This introduces an error of less than \( \Delta t \), which is, in practice, very small for most digitally sampled data. However, rounding also introduces the possibility that an increment in \( k \) may result in a rounded value of \( \Delta t = 0 \), requiring repeated use of the same data, \( V_{AC}(i\Delta t) \) and \( V_{ES}(j\Delta t) \), in equation (5.12) which would create an undesirable 'flat spot' in the compensated waveform. Linear interpolation across 'flat spots' can be used to produce a smoothed waveform.

An alternative approach is to select \( i \) as the known variable and to locate the intersection of lines \( IJ \) and \( x(t) \) numerically. This produces the opposite effect in which an increment in \( i \) can produce a rounded value of \( \Delta k > 1 \), which creates a gap of one or more samples in the compensated waveform. Equations (5.11) and (5.13) show that three effects determine the occurrence of gaps in the compensated waveform. Firstly, the value \( n_{BL}/n_{AI} \) is usually greater than 1 due to spatial broadening of pulses. A large value of \( n_{BL}/n_{AI} \) implies a large shape change of the pulse between measurement points, potentially giving rise to \( \Delta k > 1 \). Secondly, in the case of no pulse shape change, \( k = [n_C \alpha(i\Delta t)] + i \), indicating that a large value of \( n_C \), which would occur due to a high wave propagation speed or a large separation of the probed regions, can produce \( \Delta k > 1 \). Thirdly, large \( \Delta \alpha \) (whilst \( \alpha \) remains in the range \( 0 \leq \alpha(i\Delta t) \leq 1 \)) would be produced by a high translational velocity of the body, again potentially giving \( \Delta k > 1 \). Gaps can be closed smoothly by interpolating linearly across the gap, introducing a negligibly small error.
In practice, the required intersection between lines \( JJ \) and \( x(t) \) can be found for each value of \( i \) using an algorithm written into a computer program which firstly defines the line \( JJ \) in the \( x-t \) plane using the following equation:

\[
x_{II} = \left( \frac{x_s}{j-i} \right) \left( t - \frac{i \Delta t}{\Delta t} \right) + x_{AF} \tag{5.15}
\]

For every sample in \( V_{AR} \), an array of \( x_{II} \) values is created from equation (5.15) as \( t \) is incremented from \( i \Delta t \) to \( j \Delta t \). The time, \( k \Delta t \), at which the intersection between this line and the whole body displacement \( x(t) \) occurs is determined using a least squares error procedure. \( \alpha(k \Delta t) \) can then be evaluated from \( \alpha(k \Delta t) = \frac{(x_t-x_{AR})}{x_s} \) to determine the relative contributions of \( V_{AR}(i \Delta t) \) and \( V_{BR}(j \Delta t) \) in equation (5.12). This is indicated by projection of the intersection point \( P_1 \) vertically upwards onto the straight line \( V_{AR}(i \Delta t) \) at point \( P_2 \) in figure (5.22b).

When the displacement compensation process is complete, the compensated pulse has a number of samples equal to the number in the equivalent measurement made by a transducer fixed to the body. However this does not imply the effective sample rate of the compensated measurement is equal to \( f_s \), since the data in \( V_{AC}(t) \) is originally captured at an effectively lower sample rate and filtered accordingly.

The approximation caused by interpolation between data from two measurement points becomes greater as the distance between the measurements increases. The selection of the measurement spacing required for a given impact transient measurement application is therefore influenced by the rate of pulse shape change during propagation, the pulse propagation velocity and the whole body translational velocity. As discussed earlier in this section, all three of these parameters contribute to the occurrence of small gaps in the compensated waveform during the compensation process. Whilst the error introduced by a gap is negligibly small, the occurrence or otherwise of gaps provides a basis for development of a quantitative estimate of the degree of approximation caused by a measurement spacing, \( x_s \), in a given displacement compensated impact transient measurement. Further work is necessary to fully develop quantitative estimates of the amplitude errors and distortion similar to those presented.
for steady state vibrations earlier in this chapter. Propagating pulses generally do not have an analytically simple shape and quantitative comparison with the measurement which would be made by a transducer fixed to the point of interest on the target is therefore not straightforward. However, it is possible to assess qualitatively the accuracy of the measurement attained by application of displacement compensation to impact transients, as demonstrated in the following section.

5.5.4 Validation by numerical simulation of stress pulse propagation in an impacted viscoelastic rod

A theoretical model of stress pulse propagation in viscoelastic solids, developed by Kolsky [5.1], was discussed in Chapter 2. This model can be used to demonstrate application of the displacement compensation technique to impact transients if the rod along which the simulated stress pulse propagates is additionally modelled as undergoing whole body translation in the direction of pulse propagation. The model is then representative of a free viscoelastic rod subjected to a longitudinal impact. In the original model, Kolsky chose a frame of reference which moved with the pulse such that \( t=0 \) when the pulse reaches the observed point on the rod, a distance \( x \) from the origin of the pulse. The resulting Fourier series expression of the local stress \( \sigma(x,t) \) is then given by equation (2.9). This equation can be adapted to incorporate whole body translation of the rod and observation of the propagating stress pulse from a remote location, \( A_R \), initially a distance \( x_{AF} \) from the impacted end of the rod, by modifying the \( x \) and \( t \) terms. The point on the rod observed at \( A_R \) moves closer to the impacted end of the rod, \( x=0 \), as the rod translates in the positive \( x \)-direction. Thus for whole body translation with constant velocity, \( \dot{x} \), the Fourier series must be evaluated at a point a distance \( x_{AF} - \int \dot{x} \, dt \) from the impacted end of the rod. As the observed point on the rod moves progressively closer to the impacted end, the time in the local frame of reference, moving with the pulse, which has elapsed since the pulse reached the instantaneously observed point becomes progressively larger. The Fourier series for \( \sigma(x_{AF}-x,t) \) must therefore be evaluated at a time greater than \( t \) by an amount \( t_x \) which reflects the relative velocities of the rod and the pulse. Equation (2.9) then becomes:
\[
\sigma(x_{AF} - x, t) = \sum_{n=0}^{\infty} C_n \cos(n\omega(t + t_x)) + \sum_{n=1}^{\infty} D_n \sin(n\omega(t + t_x)) \quad (5.16)
\]

where

\[
C_n = \exp(k_x \omega (x_{AF} - x)) \cos[n\omega(x_{AF} - x)(1/\hat{x}_p - 1/\nu')]
\]

\[
D_n = \exp(k_x \omega (x_{AF} - x)) \sin[n\omega(x_{AF} - x)(1/\hat{x}_p - 1/\nu')]
\]

\[
x = \int_0^t \dot{x} \, dt \quad \text{and} \quad t_x = x/\hat{x}_p
\]

with other terms defined as in Chapter 2.

Equation (5.16) was used to simulate two remote observations of a pulse propagating down the polymethylmethacrylate rod discussed in Chapter 2 and which additionally undergoes whole body translation at a constant velocity. The pulse propagation velocity in the material is given by \( \hat{x}_p = 2300 \text{ms}^{-1} \) and the whole body translational velocity in this case is \( \dot{x} = 1500 \text{ms}^{-1} \). The two remote observations, made at fixed locations 6m and 8m from the impact point, are shown in figure (5.23). Comparison of data in figure (5.23) with figure (2.3), which shows the observation 6m from the end of the rod in the absence of whole body translation, demonstrates distortion of the remotely observed pulse in time and amplitude. Distortion in time was discussed in Section 5.5.2 and distortion in amplitude is due to whole body translation causing the pulse to be observed at locations on the rod closer to the impacted end, where the pulse is less attenuated.

In this example, it is necessary to use a higher whole body translational velocity than occurs in the impacts of main interest in this thesis in order to produce readily observable differences in the measured pulses. The rate of pulse shape change with propagation distance in the model is significantly lower than in a golf ball due to lower attenuation in the polymethylmethacrylate material than occurs in the golf ball material, the constant cross section of the rod and the absence from the model of pulse dispersion effects caused by lateral inertia. In the case of the golf impact, all of these effects combine to produce a high rate of pulse shape change necessitating displacement compensation at relatively low whole body translational velocities. However, the model remains a useful illustration of the validity of the displacement compensation technique in the measurement of impact transients.
The applicability of the displacement compensation technique to the remote measurement of impact transients can be demonstrated if the data shown in figure (5.23) can be used along with the whole body displacement data to produce a close estimate of the data shown in figure (2.3). Figure (5.24) shows the relation between the displacement compensated pulse and the data from figure (5.23) used in its generation. Figure (5.25) shows very close agreement between the displacement compensated pulse and the pulse measured at a fixed location on the rod, taken from figure (2.3). This is basic validating evidence for the applicability of the displacement compensation technique to the remote measurement of impact transients on translating bodies.

5.6 Displacement compensation of waveforms measured during a golf ball impact

5.6.1 Measurement situation
Detailed remote measurements of golf ball deformation and forward motion during impact were reported in Chapter 4. However, whole body motion of the ball caused lateral deformation velocity measurements made remotely using a laser vibrometer to originate from instantaneously probed regions on the ball surface increasingly distant from the initially probed points and detracted from the otherwise high quality of data obtained. In Chapter 4, remote measurements were made during impact using a laser vibrometer at five points of interest $A_F - E_F$ across the equator of the golf ball. In this case the five data sets are not simultaneous and their use in displacement compensation relies on the good repeatability of the impact conditions. Data from these measurements indicates progression of the lateral deformation across the ball in the positive x-direction during the impact. The pulse also exhibits an amplitude and shape change due to the variation in cross section of the ball and spatial broadening as the wave propagates through the viscoelastic ball material.

Forward velocity measurements in Chapter 4 indicated that significant deformation of the golf ball also occurs in the direction of the impact and this poses a problem if the measured forward velocity data is used as the whole body velocity of the ball, $\dot{x}(t)$, in
the displacement compensation process. However, the data in Chapter 4 shows the measurement from the point on the ball initially at \( D_F \) is least affected by deformation in the forward direction and thus provides the most reliable estimate of \( \dot{x}(t) \), shown earlier in figure (4.7). Additionally, integration of this data to obtain the forward displacement, \( x(t) \), marginalises the differences between the measurements and \( x(t) \) is only required to an accuracy of the order of one laser beam diameter in the displacement compensation technique. Using the forward velocity measurement from the point on the ball initially at \( D_F \), it can be seen that in the first 150\( \mu \)s of impact with a golf club the ball deforms whilst remaining stationary. After this time the whole ball accelerates forwards in a non-linear manner and translates a distance of 11mm in the remaining 300\( \mu \)s of the impact.

5.6.2 Comparison of compensated and uncompensated data

5.6.2.1 Lateral deformation

The displacement compensation technique described in Section 5.5 is directly applicable to the golf ball lateral deformation measurement. As an example, the remote lateral deformation measurements made from point \( C_R \) and \( D_R \), \( \dot{z}_{CR}(t) \) and \( \dot{z}_{DR}(t) \), can be divided into compensating intervals defined by their maximum, minimum and zeroes and used along with \( x(t) \) to calculate the compensated waveform in each interval and hence for the time in which point \( C_F \) moves between \( C_R \) and \( D_R \) by application of equation (5.12). At a time 390\( \mu \)s after the start of the impact, point \( C_R \) reaches the point probed by the laser beam at \( D_R \) and the displacement compensation process can be repeated using the data \( \dot{z}_{DR}(t) \) and \( \dot{z}_{ER}(t) \) until the impact ends.

The remote measurements \( \dot{z}_{CR}(t) \), \( \dot{z}_{DR}(t) \) and \( \dot{z}_{ER}(t) \) are shown in figure (5.26) and the compensated waveform, \( \dot{z}_{CC}(t) \), is shown in figure (5.27) for direct comparison with the equivalent single remote measurement, \( \dot{z}_{CR}(t) \). The comparison reveals how \( \dot{z}_{CC}(t) \) is very similar to \( \dot{z}_{CR}(t) \) in the first half of the impact as a consequence of the negligible forward displacement of the whole ball in this interval. Rapidly increasing forward displacement in the second half of the impact gives greater weight to \( \dot{z}_{DR}(t) \) in
equation (5.12) and causes $\dot{z}_{CC}(t)$ to resemble $\dot{z}_{DR}(t)$ more closely. At time 390\mu s into the impact the golf ball has translated a distance of 9mm, which is equal to the spacing of the two laser beams. $\dot{z}_{CC}(t)$ should therefore be exactly equal to $\dot{z}_{DR}(t)$ at this time and this is seen to be the case in figures (5.26 & 5.27). In the final 60\mu s of the impact $\dot{z}_{CC}(t)$ begins to acquire some of the characteristics of the $\dot{z}_{ER}(t)$ measurement.

A notable feature is that $\dot{z}_{ER}(t)$ has a shorter deformation and recovery period than $\dot{z}_{FR}(t)$ or $\dot{z}_{DR}(t)$ with the result that $\dot{z}_{CC}(t)$ has a shorter deformation and recovery period than the equivalent single remote measurement, $\dot{z}_{CR}(t)$. This characteristic differs from the theoretical model of stress pulse propagation in a viscoelastic rod, discussed earlier, in which a pulse exhibits broadening as a result of passing through a viscoelastic material. Two essential differences between the theoretical model and the golf ball impact contribute to the pulse shortening effect. Firstly, the reducing cross section of the ball between points $C_F$ and $E_F$ has a major shortening influence on the pulse shape to the extent that any pulse broadening which occurs over the relatively short distance travelled by the pulse will not be resolved in the measurement. Secondly, the proximity of the measurements to the impact site conveys contact deformation characteristics onto the pulse propagation measurements. The contact deformation makes points on the ball closest to the impact site deform first and recover last whilst the reverse is true for points on the side of the ball opposite the impact site. Thus, longer stress pulses are measured closer to the impacted side of a golf ball.

Comparison of the compensated waveform $\dot{z}_{CC}(t)$ and the measurements $\dot{z}_{CR}(t)$ and $\dot{z}_{DR}(t)$ provides basic evidence for the ability of the displacement compensation technique to produce a close estimate of the transient deformation velocity at a point on an impacted body using data from two remote point measurements. Slight inaccuracies may remain but can be tolerated since this measurement on a lightweight body projected far from the impact site would be extremely difficult to make by any other means. The use of a third remote measurement, which was necessary to cover the entire impact duration, could have been avoided by increasing the measurement spacing at the expense of a slightly reduced pulse shape change interpolation accuracy.
Integration of $\dot{z}_{CF}(t)$ gives a displacement compensated estimate of the lateral deformation at point $C_F$. This is shown in figure (5.28) along with the uncompensated equivalent. The displacement compensated data exhibits far less over-recovery of the lateral deformation than is suggested by the original data. This is due to the original data being captured from points on the ball increasingly distant from the initially probed point and closer to the impact site than in the displacement compensated data. As shown earlier in figure (5.26), the lateral deformation recovery is seen to take place later in the impact for points on the ball closer to the impact site and this produces an over estimate in the single remote measurement of the lateral deformation recovery velocity late in the impact. Displacement compensation hence removes this error from the lateral deformation data.

5.6.2.2 Forward deformation

The displacement compensation technique can be applied to the measurements made at an angle of 75° from the impact direction at points $A_F-E_F$ across the equator of the golf ball. These displacement compensated measurements can then be used with the displacement compensated lateral deformation velocity measurements, in the manner described in Chapter 4, to produce closer estimates of the forward velocity of points across the ball from rest. As an example, the displacement compensated forward velocity of point $D_F$ is shown in figure (5.29) along with its uncompensated equivalent. The two pieces of data appear quite similar because the forward deformation component is only a small fraction of the large whole body forward velocity which is the same at all points $A_F-E_F$ across the ball. The forward deformation recovers towards the end of the impact with the result that the compensated and uncompensated data are in close agreement since the forward velocity measured at any point on the ball is almost entirely that of the whole body.

If the relative forward deformation component only is examined, in same manner as in Chapter 4, then significant differences in the compensated and uncompensated data are apparent. This is shown for the forward deformation of point $D_F$ relative to point $C_F$ in figure (5.30), where the displacement compensated data indicates that recovery of the
forward deformation occurs earlier in the impact than is suggested by the uncompensated data. This is consistent with the uncompensated measurement interrogating points on the ball progressively closer to the impact site, where the deformation recovery occurs later in the impact. The displacement compensated data additionally indicates localised over-recovery of the forward deformation at a late stage in the impact, suggesting the start of a local oscillation which persists for only a few cycles.

The displacement compensated forward velocity data can also be used to provide a more clearly defined estimate of the approach between the ball and club head during the impact. Displacement compensated approach data pertaining to point $A_F$ is shown in figure (5.31) for comparison with the uncompensated approach data shown earlier in figure (4.10). The approach calculated using the displacement compensated data is greater than the estimate derived from the uncompensated measurements. This is due to the displacement compensation technique using data from points $A_R$ and $B_R$, rather than $A_R$ alone in the single remote measurement. Data from $B_R$ will suggest a greater approach as it will additionally include the forward deformation of point $B_F$ relative to $A_F$. In figure (5.31), the uncompensated measurement therefore appears to approximate more closely the localised deformation occurring near to the contact site but the location of this measurement is ill-defined due to translation of the ball. Conversely, the displacement compensated measurement pertains to a clearly defined point, $A_F$, on the ball and is a more appropriate measurement even though it contains data captured from points on the ball slightly further away from the contact site.

5.6.3 Use of displacement compensated data in finite element model validation

A comparison of remotely measured experimental and finite element model impact data pertaining to golf ball deformation was reported in Chapter 4. The comparison presented in Chapter 4 was restricted by the effects of golf ball translation on the experimental data and by a mis-match of experimental measurement locations and the location of nodes on the surface of the golf ball finite element model. This section demonstrates how the displacement compensation technique addresses both of these problems and hence permits detailed validation of the golf impact finite element model.
Figure (5.32) shows a plan view of one half of a golf ball and indicates the location of the five remote measurement points $A_F-E_F$ and the five finite element nodes $A'_F-E'_F$ which lie within the space defined by points $A_F$ and $E_F$ on the equator of the ball. This is representative of the general case in which the experimental measurement locations do not match the location of the finite element nodes due to different experimental and modelling requirements. Experimentally, it is desirable to extract the required data by making the smallest possible number of measurements. For the golf ball impact, five measurements were considered necessary to capture the deformation data to sufficient accuracy given the apparent rate of shape change of the deformation across the ball, the dimensions of the ball, the speed of propagation of deformation through the ball and the whole body forward translation occurring during impact. The finite element golf ball model, described in Chapter 2, has an icosahedral node pattern on the surface consistent with the arrangement of elements required to model successfully the large deformation of the ball during impact. The compromises required to directly match experimental remote measurement locations to finite element node locations may result in excessive experimental effort or insufficient finite element mesh density, both of which are undesirable.

The problems associated with undertaking comparisons between experimental and computational data which does not originate from equivalent measurement points on the body of interest are illustrated in figures (5.33a&b). Figure (5.33a) shows the single remote lateral deformation measurement, $\tilde{z}_{BR}(t)$, and the finite element result, $\tilde{z}'_{BR}(t)$, from the node initially closest to the point $B_R$. The comparison between computational and experimental data is particularly poor in the second half of the impact, as the rapidly increasing forward displacement of the ball causes the region of the ball instantaneously probed by the remote measurement made from point $B_R$ to become increasingly distant from node $B'_R$.

As a further example, the comparison between the single remote lateral deformation measurement, $\tilde{z}_{OR}(t)$, and the finite element result, $\tilde{z}'_{OR}(t)$, is shown in figure (5.33b). Agreement between the computational and experimental data in this figure is slightly
better than in the previous example, as forward displacement of the ball causes the region of the ball instantaneously probed by the measurement from point \( D_R \) to pass through the point \( D_f' \) during the impact. However, with reference to figure (5.32), \( D_f' \) is initially closer to the impacted side of the ball and the start of the lateral deformation therefore occurs earlier in the \( \dot{z}_{D_f'}(t) \) result than in the \( \dot{z}_{D_R}(t) \) measurement. Furthermore, in the later stages of the impact, the region of the ball instantaneously probed by the measurement from point \( D_R \) is closer to the impacted side of the ball than point \( D_f' \) and the lateral deformation recovery in the experimental data therefore appears to lag that in the finite element result.

The displacement compensation technique provides a solution to these problems by affording the capability to independently employ the necessary experimental and finite element modelling conditions whilst still being able to compare directly data from equivalent fixed points on the real ball and the finite element model. In Section 5.5, the displacement compensation technique used two remote measurements to obtain a close estimate of the waveform which would have been measured by a transducer fixed to the point on the target initially interrogated by one of the two probe laser beams. The technique can, however, be used to obtain a close estimate of the waveform which would have been measured at any fixed point on the target for the time in which that point lies between the two points interrogated by the probe laser beams. The analysis presented in Section 5.5 remains valid, with the modification that \( \alpha = (x + x_o)/x_s \), where \( x_o \) is the distance from the point \( A_f \) to the point of interest on the target, \( A'_R \), \( 0 \leq x_o \leq x_s \).

This can be applied to the five remote golf ball lateral deformation velocity measurements, \( \dot{z}_{AR}(t) - \dot{z}_{ER}(t) \), in order to derive displacement compensated experimental data, \( \dot{z}_{A'E'}(t) - \dot{z}_{E'E'}(t) \), for points on the ball \( A'_R - E'_R \), equivalent to the lateral deformation velocity calculated at five nodes on the finite element model, \( \dot{z}_{A'F}(t) - \dot{z}_{E'F}(t) \). Equivalent finite element and displacement compensated experimental data are shown in figures (5.34a-e). The five figures chart the progression and shape variation of the lateral deformation across the golf ball during impact and exhibit only minor differences between the finite element and displacement compensated experimental data in all cases. Comparison with the computational and
experimental data shown previously in figures (5.33a&b) illustrates the importance of the displacement compensation technique in obtaining experimental data which is matched to points of interest in a computational model. This also provides evidence for the general improvement over the use of uncompensated experimental data in validation of the finite element impact model.

Applying the technique described above to the experimental data measured 75° from the impact direction allows displacement compensated golf ball whole body forward velocity and localised forward deformation to be derived for points $A'E'F$. The forward velocity of the ball taken at point $D'F$ is shown in figure (5.35). It can be seen that the finite element model and displacement compensated experimental results are in close agreement towards the end of the impact, confirming the whole body mechanics of the finite element model are in close agreement with the experimental data in terms of the club head initial velocity and golf ball final velocity, as suggested in Chapter 4. The finite element golf ball forward velocity result is, however, 10-15% larger than the experimental result throughout most of the impact, indicating that the finite element model predicts greater deformation of the ball in the impact direction than is actually the case. This is confirmed if the relative forward deformation between points $D'F$ and $C'F$, calculated using the finite element model, is compared to the displacement compensated equivalent, as in figure (5.36). In the first half of the impact the finite element model exhibits greater relative deformation than is actually the case but the peak deformation occurs at the same point in the impact as in the experimental data. This suggests that the pattern of deformation of the ball in the forward direction predicted by the finite element model is in close agreement with the actual ball deformation.

The approach between the club head and ball, calculated using the finite element model, is in close agreement with the experimental result, as shown in figure (5.37). The approach is calculated using data from measurement point $A'F$ on the ball. Thus, close agreement between the model and experimental data at this point, as compared to less agreement between points $C'F$ and $D'F$, shown earlier in figure (5.36), suggests
greater forward deformation across the ball from point $A'_F$ to $D'_F$ in the finite element model than occurs in reality.

Evidently the lateral deformation data exhibits closer agreement than the forward deformation data and some refinement of the ball model is required to obtain similar levels of agreement in both directions. The nature of this further work is discussed in Chapter 7. The generally close agreement between detailed experimental and computational data confirms the validity of the finite element golf ball model under impact conditions. The level of detailed comparison which can be drawn between individual nodes on the finite element model and the displacement compensated experimental data also demonstrates the usefulness of the displacement compensation technique. The capability to derive data effectively from a single point on a moving body using remote transducers in difficult measurement situations is a valuable experimental tool in structural vibration analysis. As remote, non-contact measurement methods become increasingly popular, the work presented in this chapter demonstrates the improvement in the quality of data which can be obtained using the displacement compensation technique.
6 The new laser Doppler accelerometer

6.1 Introduction

Laser Doppler vibrometry (LDV) is now an established technique for remote measurement of surface vibration [6.1] and has been widely applied as a technique complimenting the use of piezo-electric accelerometers in measurement situations where contacting transducers are inappropriate [6.2, 6.3]. In Chapters 3 and 5 a laser Doppler vibrometer was successfully applied to the measurement of vibration excited in the surfaces of a hollow golf club head both in modal tests and during an impact with a golf ball. The quality of data presented demonstrates the value of the vibrometer as a versatile remote transducer. A novel application of the vibrometer to the analysis of impact transients on a body which also undergoes whole body motion was successfully demonstrated in Chapters 4 and 5. This application, however, highlighted the problems encountered when the upper velocity measurement limit of the instrument is lower than the peak velocity exhibited by the target. It is therefore pertinent to discuss the principles of operation of a vibrometer in order to examine the velocity measurement limitations of current LDV systems. Specifically, part of this chapter focuses on how the upper velocity limit, currently 10-15ms⁻¹, is dependent upon optical considerations and the currently available electronic signal processing capability.

A growing number of potential remote vibration measurement applications require a velocity measurement capability beyond the 10-15ms⁻¹ offered by even the most advanced commercial LDV instruments. Ultrasonically assisted manufacturing, in which surface velocities up to approximately 40ms⁻¹ are predicted at frequencies between 15 and 40kHz [6.4], is one of many advances in machinery and manufacturing technology to exceed current LDV velocity limits. In a high performance internal combustion engine, valve speeds of the order of 20ms⁻¹ are reported, with valve bounce at potentially higher speeds [6.5] and in pyroshock applications, velocities up to 16ms⁻¹ at frequencies between 10kHz and 100kHz are at the very limit of current LDV capability [6.6]. A further range of applications exist in which the velocity of
interest pertains to the vibration of a body which also undergoes whole body motion. This is commonly the case in the analysis of bodies subjected to impact. In vehicle crashworthiness studies, whole vehicle velocities of the order of 10-20\text{ms}^{-1} are common but it is the localised deformation velocities of the structure which are of importance in analysis of energy dissipation [6.7, 6.8]. The golf club-ball impact studied in Chapter 4 is one of many sports equipment impacts in which important deformations of interest occur on top of a high mean velocity and this application provides the motivation for the developments in the laser Doppler technique presented in this thesis.

Remote vibration measurements on bodies undergoing whole body translations were studied in Chapter 5. A displacement compensation technique was introduced which provided a significant improvement in the quality of vibration data which can be derived from remote surface velocity measurements in the presence of whole body translation. By arranging the angle of incidence of the probe laser beam perpendicular to the direction of whole body motion, sensitivity to the whole body velocity is removed and the displacement compensation process accounts only for the changing point on the target interrogated by the laser beam.

If the target undergoes whole body translation in any direction other than perpendicular to the direction of incidence of the probe laser beam, a vibrometer signal will include the component of this whole body velocity which lies along the axis of the beam. Often the whole body velocity component is of significantly lower frequency than the vibration velocity signal and can be removed using a high-pass filter with suitable cut-off frequency, as demonstrated by removal of the whole body rotation component from data recorded from a hollow golf club head in Chapter 5. However, in choosing velocity as the measured parameter, a large whole body velocity component in the direction of the laser beam will occupy a significant proportion of the total working range such that the vibration velocity of interest may appear only as a small fluctuating component on top of a high mean level in the vibrometer output signal. The presence of the whole body velocity component in the measurement thus reduces the effective measurable vibration velocity to the extent that the fluctuation will be
undetectable if the measured whole body velocity component exceeds the vibrometer velocity limit of 10-15 ms⁻¹.

An alternative optical velocity transducer, known as the 'Velocity Interferometer System for Any Reflector' or VISAR [6.9, 6.10], is capable of measuring velocities of several kilometres per second. The method used by the VISAR to achieve this capability is discussed in this chapter to illustrate the considerable complexity and expense which have precluded widespread general engineering use of the instrument. Furthermore, an increased upper measurable velocity limit would not benefit vibration measurement applications which are exposed to a significant whole body velocity component as an instrument working range set sufficiently large to accommodate the whole body velocity component will result in poor sensitivity to the smaller fluctuating component of interest. It is therefore attractive to choose acceleration instead of velocity as the measured parameter in order to maximise sensitivity of the instrument to target oscillations. Sensitivity to acceleration is desirable generally as acceleration is the most popular descriptor of vibratory motion and is directly related to force. Additionally, differentiation of measured velocity signals to obtain acceleration accentuates undesirable noise in the signal which, in the case of laser based measurements, is produced by signal processing electronics and, in particular, laser speckle effects [6.11, 6.12].

Optical instruments directly sensitive to acceleration have been reported previously [6.13, 6.14] in which an optical fibre attached to a seismic mass fixed to the target body is used as the transduction element. The change in fibre length due to target motion induces a phase shift in the light propagating in the fibre which is proportional to the target acceleration and which can be detected using a standard Mach-Zehnder interferometer. However, these optical accelerometers require attachment of a sensing element to the target body which introduces access, mass-loading and attachment problems equivalent to those encountered using piezo-electric accelerometers. A non-contact laser-based measurement of vibration acceleration has not been reported previously and a prototype instrument capable of making this measurement is the main subject of this chapter.
The theory and principles of operation of the laser Doppler accelerometer (LDAc) were first reported by Rothberg [6.15, 6.16]. The LDAc embodies an interferometric technique for the remote measurement of vibration acceleration which addresses both the current limitations of LDV systems and the likely future measurement requirements indicated by the examples discussed above. The instrument has several advantages over existing LDV systems including an easily adjustable working range with effectively no upper measurement limit, straightforward operation and use of low cost optical components. This chapter reports the development of a first prototype LDAc instrument to a level where direct sensitivity of a laser transducer to target acceleration can be demonstrated for the first time. Laser beam separation and recombination by polarisation, laser coherence properties, sensitivity to back reflections from optical components and frequency pre-shifting issues are found to be crucial in isolating the particular optical beat frequency which carries the acceleration signal from other optical beats in the instrument. Recommendations for the further development of the LDAc into a viable practical instrument for general vibration engineering measurement applications are made based on the experience gained in this study.

6.2 Laser Doppler vibrometry

6.2.1 Principles of operation

Laser Doppler vibrometry relies on detection of the Doppler frequency shift in the coherent light scattered from a moving object [6.17]. Only the velocity component which lies along the axis of the incident laser beam will contribute to the Doppler frequency shift, giving laser Doppler vibrometers the highly directional sensitivity utilised earlier in this thesis. The magnitude of the Doppler shift, \( f_D \), of light incident on the target can be calculated thus:

\[
 f_D = \frac{2U(t)}{\lambda} \tag{6.1}
\]

Equation (6.1) shows that, for example, light of wavelength \( \lambda \) in the visible range undergoes a Doppler shift of the order of \( 10^6 \)Hz when backscattered from a target
which exhibits velocities, $U(t)$, in the range of common vibration engineering interest (0-10 ms$^{-1}$). A frequency shift of the order of $10^6$ Hz in light of frequency $\approx 10^{14}$ Hz cannot be demodulated directly and the light backscattered from the target is thus mixed on a photodetector with a reference beam taken from the same source as shown schematically in figure (6.1). Interference between target and reference beams of intensity $I_T$ and $I_R$ respectively produces an interference pattern of intensity $I$, given by:

$$I = I_T + I_R + 2(I_T I_R)^{1/2} \cos \varphi$$

(6.2)

$\varphi$ represents the phase difference between the two beams and the final term in equation (6.2) therefore describes the degree of interference between the two beams. Demodulation of the resulting optical intensity fluctuations on the photodetector at the Doppler frequency yields the target vibration velocity. In practice, satisfactory interference is achieved only if the difference in optical path length travelled by the target and reference beams is retained within the coherence length of the laser [6.17]. Restricted optical access to the target may present problems in this respect and a degree of judgement is required on the part of the operator.

In order to discriminate the direction of target motion, a known frequency pre-shift, $f_R$, is added to the reference beam. The optical beat at the photodetector is then at the difference frequency $f_T - f_D$ and can be demodulated to obtain the amplitude and direction of the target velocity. The most common frequency shifting device employed in commercial laser Doppler vibrometers is an acousto-optic Bragg cell [6.18]. In a Bragg cell, the incident light passes through a medium through which acoustic waves are driven by a high frequency oscillator. Small scale density variations diffract the light and a single diffracted order is produced which is shifted in frequency relative to the incident light by an amount equal to the oscillator frequency, generally around 40 MHz. More than 90% of the incident light is contained within this first order and this is considerably more efficient than alternative frequency shifting devices, which include a rotating diffraction grating [6.19] and a simple rotating scattering disk angled a few degrees off the axis of the reference beam [6.20]. Both of these alternative devices are cheaper than Bragg cells and provide a frequency shift of the order of 1 MHz.
Established Doppler signal demodulation techniques include frequency tracking and frequency counting procedures and these are documented in [6.21]. A frequency tracker provides an instantaneous frequency-to-voltage conversion giving real time demodulation of the Doppler signal. Tracking circuits were developed originally for laser Doppler anemometry applications to provide a continuous output signal in the presence of a discontinuous input and are equally applicable to surface velocity measurements [6.22]. 'Offset heterodyne' and 'autodyne' trackers are based on a voltage controlled oscillator (VCO) and a frequency discriminator which converts frequency to voltage. The input to a tracker from a photodetector has a carrier frequency fixed by the frequency shifting device and modulation characteristics determined by the time dependency of the target surface velocity. This signal is mixed with the VCO output and passed to the frequency discriminator through a narrow band filter which reduces signal noise. The discriminator detects variations in the frequency of the signal from the filter and outputs a correcting voltage which is fed back to the VCO input. This feedback fixes the frequency of the VCO circuit to that of the photodetector signal and a voltage analogue representing the Doppler frequency can be obtained from the VCO input.

The VCO circuit additionally acts as a narrow band filter and as such has an associated time constant which determines the maximum response rate of the circuit and hence the maximum rate of change of the Doppler frequency the tracker is able to follow. This maximum rate of change of frequency is known as the 'slew rate' and contributes to defining the working range of the vibrometer by limiting the maximum demodulatable rate of change of the target velocity, i.e. the maximum measurable acceleration. High accelerations can occur even at modest velocities in impacts or during continuous target vibration at high frequency, thus the slew rate of the frequency tracker is an important measurement limitation in these situations.

In the presence of continuous Doppler signals, such as are derived from solid surfaces, alternative Doppler signal demodulating techniques can be applied. Direct down-mixing is a comparatively recently introduced Doppler signal pre-processing technique
which has proven very effective. In a direct down-mixing device [6.23], the photodetector signal is first down-mixed using a local oscillator and the difference frequency is then decoded using a 'standard' FM demodulator optimised for a specific frequency range. When combined with appropriate signal filtering this is a robust system capable of operating with low light intensity backscattered from the target. However, even this sophisticated frequency demodulating technique has an associated slew rate, which at present limits the rate of change of target velocity to approximately $2 \times 10^6 \text{ms}^{-2}$ (with an upper frequency limit of around 300kHz) and this acceleration limit would be exceeded by several ultrasonic manufacturing and high velocity impact applications.

The working velocity range of a vibrometer is essentially determined by the size of frequency pre-shift and the type of frequency demodulator employed. A frequency pre-shift of up to 5MHz can be provided by a fast rotating diffraction grating [6.17]. A frequency demodulator with a maximum demodulatable frequency of 10MHz is thus required to accommodate the pre-shift and up to ±5MHz of frequency shift due to target motion. Equation (6.1) shows the maximum measurable velocity under these conditions to be 1.6ms$^{-1}$ and if the frequency demodulator has a dynamic range of 100dB the working range of the instrument will be 16μms$^{-1}$ to 1.6ms$^{-1}$. Leading commercial systems [6.23] obtain a pre-shift of 40MHz using a Bragg cell and incorporate frequency demodulation electronics with a dynamic range approaching 120dB to achieve a working velocity range 1μms$^{-1}$ to 12ms$^{-1}$. Laser wavelengths are limited to the visible range for operator protection and ease of use and there is currently no expectation of a step change in frequency demodulator technology. Therefore no foreseeable opportunities exist to expand the working range of a vibrometer, or even, for the same dynamic range, optimise the vibrometer for the velocity range required by a given application.

6.2.2 Sensitivity to high velocities
The VISAR [6.10, 6.11] was developed originally for the investigation of wave propagation in solids subjected to shock loading. The ballistic impact events of interest exhibit velocities in the range 10-5,000ms$^{-1}$ and are thus well beyond the working
range of most commercial LDV systems. The principles of operation of the VISAR differ considerably from the LDV systems described in the previous section. The VISAR is a long path imbalance interferometer [6.24] which does not incorporate a reference beam but amplitude divides light returning from the target into two components, one of which travels round an optical delay of duration \( \tau_D \). The instrument counts the number of interference fringes, \( F_n(t) \), produced as a result of recombining the two components and the target velocity is calculated according to:

\[
U(t - \frac{1}{2} \tau_D) = \frac{\lambda}{2 \tau_D} F_n(t)
\]  

(6.3)

This target velocity derivation circumvents the limitations imposed by insufficient frequency tracking or frequency shifting capability since neither device is required and, although limited by electronic fringe counting capability, sensitivity to target velocities in the range 20-4000\( \text{ms}^{-1} \) has been reported [6.25, 6.26]. However, the assumption that the target velocity is proportional to \( F_n(t) \) is not valid over the delay time, \( \tau_D \), and an error is introduced in the presence of target acceleration which is accounted for in more complex analysis [6.27, 6.28]. The target motion direction is resolved by retarding the polarisation of light in the optical delay path such that the \( s \) and \( p \) states form two independent fringe patterns which are 90\(^\circ\) out of phase. These are incident upon two separate photomultipliers and the lead-lag relationship between the two fringe intensity variations is used to discriminate the direction of motion. A third photomultiplier is also required to detect changes in the intensity of light backscattered from the target such that the intensity variations do not affect the fringe count.

The VISAR is an expensive instrument owing to its considerable optical complexity. In addition to the three photomultipliers required, a laser source with high temporal coherence is essential for good fringe visibility from interfering beams which have travelled optical paths of significantly different length. Substantial off-line data reduction is also necessary to transform the photomultiplier signal into accurate velocity data [6.27, 6.29, 6.30]. These features have combined to restrict the VISAR to specialist use.
Several improvements to the original instrument have been implemented, in particular a modified optical configuration [6.31] which reduces the number of photomultipliers to two whilst obtaining better light efficiency, leading to improved signal quality and velocity resolution. Velocity sensitivity adjustment by appropriate choice of delay, \( \tau_D \), implemented either in air or using an etalon, has been reported [6.25] and more complex optical arrangements involving dual delay legs have been developed to address the inability of the basic system to record integral numbers of fringes across the discontinuities in particle velocity encountered in explosive studies [6.32]. This final example emphasises the highly specialised nature of both the instrument itself and the measurements to which it is applied. The specialist nature of the VISAR, combined with large off-line data reduction requirements and cost make the instrument inappropriate for general engineering applications.

A new instrument is required to provide remote vibration measurement capability in situations outside the range offered by current commercial LDV systems. The laser Doppler accelerometer is such an instrument and is the subject of what follows in this chapter.

6.3 The laser Doppler accelerometer

6.3.1 Principle of operation

The laser Doppler accelerometer (LDAc) is a direct development of the laser Doppler vibrometer optical configuration described in Section 6.2.1. The principal development involves novel use of the reference beam and optical path imbalances to produce an optical beat on the photodetector which is directly proportional to the target surface acceleration. In the LDAc optical configuration both arms of the interferometer are incident on the target in a single beam. The backscattered light is amplitude divided and a path length imbalance, \( \Delta l \), introduced between the two halves, A and B, of the beam as shown in figure (6.2). The case considered here is that in which beams A and B receive additional frequency shifts \( \Delta f_A \) and \( \Delta f_B \) for the purposes of discriminating the direction of target motion. Recombination into a single beam on a beamsplitter then produces an optical beat on the photodetector at frequency, \( f_b \), given by:

\[
130
\]
\[ f_b = \left| \Delta f_A - \Delta f_B + \frac{2}{\lambda} \left[ U(t) - U(t - \Delta l / c) \right] \right| \]  \hspace{1cm} (6.4)

where \( c = 3 \times 10^8 \text{ms}^{-1} \) is the speed of light. If the target vibration velocity is represented here as a Fourier integral then the beat frequency is:

\[ f_b = \left| \Delta f_A - \Delta f_B + \frac{2}{\lambda} \int_0^\omega \left[ A(\omega) \sin(\omega t - \phi(\omega)) - A(\omega) \sin(\omega(t - \Delta l / c) - \phi(\omega)) \right] d\omega \right| \]  \hspace{1cm} (6.5)

The integral argument can be expanded as:

\[ A(\omega) \sin(\omega t - \phi(\omega)) - A(\omega) \sin(\omega(t - \Delta l / c) - \phi(\omega)) \]

\[ + A(\omega) \cos(\omega t - \phi(\omega)) \cos(\omega \Delta l / c) \]

and if an upper limiting value of \( \omega = \omega_{\text{max}} \) is defined such that the condition \( \omega_{\text{max}} \Delta l / c N 1 \) holds, then:

\[ f_b = \left| \Delta f_A - \Delta f_B + \frac{2\Delta l}{c\lambda} \left( \int_0^\omega [\omega A(\omega) \cos(\omega t - \phi(\omega))] d\omega \right) \right| \]

\[ = \left| \Delta f_A - \Delta f_B + \frac{2\Delta l}{c\lambda} \frac{dU(t)}{dt} \right| \]  \hspace{1cm} (6.7)

The beat frequency is therefore directly proportional to acceleration, offset from zero by the frequency shift \( |\Delta f_A - \Delta f_B| \), provided \( \omega_{\text{max}} \Delta l / c N 1 \).

The single path imbalance shown in figure (6.2) is analogous to the delay, \( \tau_0 \), introduced between beams in the VISAR, discussed in Section 6.2.2. As with the VISAR, the available path length imbalance in an LDAC based on the configuration shown in figure (6.2) would be limited by the coherence length of the laser source. An expensive laser source with high temporal coherence is required in order to maintain sufficient sensitivity (2\Delta l/c\lambda) to accelerations in the range of general engineering interest but this restricts application to very high accelerations, for example at elevated frequencies. This would still permit acceleration measurements which are currently impossible but does not yet exploit the full potential of the instrument.
The measurement sensitivity can be maintained using a source with limited temporal coherence if the necessary path imbalance, $\Delta l$, is compensated by an equivalent imbalance elsewhere in the optical configuration. An LDAc configuration incorporating this very important development is shown in figure (6.3). The laser output is amplitude divided into two paths, labelled A and B, such that light on path A travels an additional imbalance, $\Delta l$, before recombining with light on path B to form a single beam incident on the target. Light backscattered from the target is amplitude divided again such that the light on path B now travels an additional imbalance, $\Delta l$. On recombination at the photodetector the path imbalance between A and B is negligible and the components of light are coherent. The preceding analysis of acceleration sensitivity remains valid and the application range is widened considerably through virtually unlimited choice of $\Delta l$. Furthermore, this well balanced internal optical geometry removes the requirement for the engineer to choose a target beam path length compatible with the coherence length of the laser and widens the choice of laser source to include even temporally quite incoherent sources such as an inexpensive red laser diode.

6.3.2 Working range [6.15]

The working range of the LDAc instrument based on the optical configuration shown in figure (6.3) and developed in this study can be estimated using equation (6.7). A combined frequency pre-shift $|\Delta f_A - \Delta f_B|$ of 1.6MHz from a rotating diffraction grating and the LDV frequency tracker system described in Section 6.2.1 give a maximum demodulatable beat frequency of 2.4MHz (i.e. a frequency pre-shift of 1.6MHz ±0.8MHz due to target vibration). The corresponding maximum measurable acceleration, $a_{\text{max}}$, under these conditions is then related to the corresponding required path imbalance, $\Delta l$, by:

$$a_{\text{max}} = \frac{800 \times 10^3 \, c\lambda}{2\Delta l}$$

(6.8)

Equation (6.8) shows that appropriate choice of $\Delta l$ allows measurement of as high an acceleration range as is desired, with extreme values of $a_{\text{max}}$ limited only by the
practicalities of creating a very small or very large $\Delta l$. If $\omega_{\text{max}} \Delta l / c \leq 0.1$ is set as the condition for linearity in equation (6.7), then the maximum vibration frequency at which measurements should be made, $f_{\text{max}}$, is given by:

$$f_{\text{max}} = \frac{\omega_{\text{max}}}{2\pi} = \frac{0.1c}{2\pi \Delta l}$$ \hspace{1cm} (6.9)

Equation (6.9) shows that appropriate choice of $\Delta l$ allows measurement at as high a vibration frequency as is desired, limited only by the frequency demodulator used.

Typical working acceleration ranges and upper frequency limits for this LDAc configuration are shown in table (6.1) for several path imbalances, $\Delta l$. These values are based on an instrument which uses a He-Ne laser ($\lambda = 633\text{nm}$), a demodulatable frequency range of $\pm 0.8\text{MHz}$ and a frequency demodulator with a dynamic range of approximately 100dB.

<table>
<thead>
<tr>
<th>Path imbalance, $\Delta l$, (m)</th>
<th>Acceleration range</th>
<th>$f_{\text{max}}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.06</td>
<td>$15 \times 10^3 \text{ kms}^{-2}$ to $150\text{ ms}^{-2}$</td>
<td>$\sim 1\text{M}$</td>
</tr>
<tr>
<td>50.6</td>
<td>$1.5 \times 10^3 \text{ kms}^{-2}$ to $15\text{ ms}^{-2}$</td>
<td>$\sim 100\text{k}$</td>
</tr>
<tr>
<td>506</td>
<td>$150\text{kms}^{-2}$ to $1.5\text{ms}^{-2}$</td>
<td>$\sim 10\text{k}$</td>
</tr>
</tbody>
</table>

Table 6.1 - Typical working ranges of an LDAc

For comparison, the upper measurement limits of piezo-electric accelerometers are around $1000\text{kms}^{-2}$ for shock and $100\text{kms}^{-2}$ for continuous vibration at frequencies up to $50\text{kHz}$, with a lower measurement limit of around $0.1\text{ms}^{-2}$ [6.33]. Furthermore, if the maximum acceleration to be measured in a particular frequency range exceeds $a_{\text{max}}$, then a reduction in $\Delta l$ will accommodate the requirement (simultaneously increasing the upper frequency limit) at the expense of a reduction in sensitivity.
6.3.3 Development of the optical configuration

The optical configuration shown in figure (6.3) is a useful aid in describing the principles of operation of the LDAc but it is not a feasible practical arrangement. The development of the principles embodied in figure (6.3) into an instrument capable of making a practical measurement is the subject of what follows in this chapter.

The essential differences in optical configuration of the LDAc and vibrometer are identifiable in their respective schematics, figures (6.1) and (6.3). The LDAc requires multiple separation and recombination of beams directed along complimentary paths in the instrument and an optical path length imbalance must be implemented which is appropriate to the target acceleration and frequency ranges. Additionally there is no obvious location for a scattering frequency shifting device of the type shown in the vibrometer schematic.

Light which travels faithfully along paths A and B in figure (6.3) produces an optical beat at a frequency proportional to the acceleration of the target surface when recombined on the photodetector. However, several other legitimate optical paths through the instrument exist. The longest path to the photodetector takes path A when travelling towards the target and path B on return from the target, whilst the shortest route to include the target is the reverse of this combination. It is strictly not necessary to separate the beams on different paths in any specific way, since, in theory, the only light which is coherent on the photodetector and thus able to produce a detectable optical beat is that which has travelled faithfully along only paths A and B. In practice, however, unavoidable back reflections from optical surfaces and imperfect separation at beamsplitters directs a proportion of the total light to the photodetector by a number of undesirable paths, some of which do not involve backscattering from the target. These spurious beams exhibit varying degrees of coherence and are the source of unwanted optical beats when unavoidably recombined on the photodetector.

It is appropriate to use the polarisation state of the light to direct the majority of the light along paths A and B using an arrangement of polarising beamsplitters and retarders. This minimises sensitivity to unwanted beats by removing many of the
spurious beams and minimises noise on the photodetector by making maximum use of the available light. A polarising beamsplitter splits arbitrarily polarised light into two plane polarised beams, transmitting the \(p\)-polarised light and reflecting the \(s\)-polarised light with up to 95\% efficiency. Quarter waveplate retarders convert linearly polarised light into circularly polarised light and are commonly used in pairs to convert \(p\)-polarised light to \(s\)-polarised and vice-versa.

An optical arrangement which uses polarisation to direct light along paths A and B is shown in figure (6.4). Light from a laser source is shown delivered to the arrangement in a single beam containing \(s\) and \(p\) polarisations in equal proportions and the \(s\)-polarisation is directed along path A whilst the \(p\)-polarisation is directed along path B. Light which follows path A or B faithfully leaves the final beamsplitter (4) in its initial polarisation state, hence a linear polariser is positioned in front of the photodetector aligned at 45\° to both the \(s\) and \(p\) polarisation planes to extract components of the light which will produce an optical beat. Greater light efficiency can be achieved if the linear polariser is replaced by a polarising beamsplitter aligned at 45\° to both the \(s\) and \(p\) polarisation planes. A second photodetector introduced into the path of the light reflected by the beamsplitter would then capture a signal which is 180\° out of phase with that reaching the original photodetector. This arrangement would be employed in a final version of an instrument based on the optical configuration shown in figure (6.4), as subtraction of the two signals doubles the amplitude of the output and reduces signal noise.

Three frequency shifting devices used commonly in LDV systems were introduced in Section 6.2.1. Of these, the rotating scattering disk is not practical in the LDAc optical configuration as there is no reference beam which remains within the instrument. A rotating diffraction grating is more suited to the LDAc configuration. The light incident on a rotating grating is diffracted into several orders which are shifted in frequency by an amount related to the incident and diffracted angles and the rotation speed of the grating. A grating which contains 16384 line pairs and rotates at 50Hz, produces a first diffracted order which is shifted in frequency relative to the incident light by \(\pm 800\text{kHz}\). When incorporated into an LDAc configuration, the rotating grating
provides the initial separation of light into paths A and B by directing the +1 diffracted order along path A and the -1 order along path B. Recombination of A and B on the photodetector produces a frequency pre-shift 1.6MHz which can be demodulated by conventional means. The major disadvantage of a rotating grating is, however, the poor light efficiency associated with utilising only the first diffracted order, which contains less than 30% of the light incident upon the grating. A Bragg cell is a considerably more efficient device and additionally provides a significantly larger frequency shift than the rotating grating. The rotating diffraction grating was chosen for use in this study due to its availability, compatibility with the available frequency demodulating equipment and its secondary function as a beam separating device.

The path imbalance, $\Delta l$, is typically chosen to maximise sensitivity over a given frequency range of interest, whilst providing a measurable acceleration range suited to the target acceleration. The data shown earlier in table (6.1) indicates that a small $\Delta l$ allows measurements with a high upper frequency limit in a high acceleration range, whilst the reverse is true for large $\Delta l$. This characteristic of the LDAc is desirable since acceleration generally scales with frequency in vibration measurement applications.

However, the target vibration characteristics may dictate a value of $\Delta l$ different to that which provides optimal sensitivity. For example, previous studies [6.4] predict the target object of interest in this study, the cutting blade of a tuned ultrasonic autoresonator, to exhibit out of plane acceleration in the range $10-10^4$ kms$^{-2}$ at frequencies 15kHz and above. The corresponding target velocity is detectable by an vibrometer over part of the operating range, thus permitting vibrometer measurements which can be used to validate the LDAc output. If the maximum frequency of interest on the cutting blade is 40kHz, then the maximum path imbalance which provides a high enough upper frequency limit, given by equation (6.9), is $\Delta l=120$m. In an LDAc arrangement which employs the rotating diffraction grating described above, the maximum demodulatable beat frequency due to target vibration is 800kHz. The corresponding maximum detectable acceleration under these conditions is 640kms$^{-2}$, which is considerably lower than the predicted upper limit of the acceleration range exhibited by the target. As shown in table (6.1), the value of $a_{\text{max}}$ is increased by
reducing $\Delta \ell$ and in this case a value of $\Delta \ell=7.6\text{m}$ is necessary to detect the highest predicted acceleration of the cutting blade. An even shorter path imbalance would reduce optical complexity in practice and cause only a marginal additional increase in the detectable acceleration range. A path imbalance $\Delta \ell=1\text{m}$ is around the practical upper limit achievable with beams in air and under these conditions gives $a_{\text{max}}=7.6\times10^4\text{m/s}^2$ and $f_{\text{max}}=4.77\text{MHz}$. This accommodates the cutting blade acceleration range at the expense of reducing the sensitivity of the instrument by a factor of 10 and this exemplifies the reasoning which is required in the practical selection of path imbalance, $\Delta \ell$.

6.3.4 Practical implementation of a short path imbalance LDAc

Figure (6.5) is a schematic diagram of the practical optical arrangement used in an LDAc which incorporates a path imbalance $\Delta \ell=1\text{m}$ in air. Paths A and B pass through the components as identified in the text of the figure. The required imbalance, $\Delta \ell$, on both paths A and B is implemented by light which passes in opposite directions around a single delay leg, thus minimising any differential phase noise effects which may occur using two separate delay paths. The He-Ne laser source has a nominal coherence length less than $1\text{m}$, thus only light which passes faithfully along path A or B will be coherent on the photodetector. Collimating lenses are necessary to minimise beam divergence and maintain similar beam sizes in both paths at the target and photodetector. The light on paths A and B is required to be co-linear and of the same diameter at the point of incidence on the target as the LDAc relies upon presenting the light to the target as a single beam. If the beams are in any way separate, the configuration becomes equivalent to a two-beam differential vibrometer [6.34, 6.35], which is sensitive to target velocity. A more complex arrangement of beamsplitters and quarter waveplates than that shown previously in figure (6.4) is used to rotate the polarisation of light on path A to match that on path B before recombining on a beamsplitter (21), labelled in figure (6.5), in order to produce an optical beat with minimum wastage of light. It is a feature of the multiple separation and recombination of beams in LDAc optical configurations that some of the available light is unavoidably discarded. This light is shown dotted in schematic diagrams of the LDAc and can be
kept to a minimum level by appropriate consideration of the polarisation state of the light.

In this LDAc configuration, spurious optical beat frequencies occur on the photodetector due to two complimentary features of the short path imbalance arrangement. Firstly, polarisation leakage at the beamsplitters allows light to travel along paths other than A and B to the photodetector. Secondly, this unwanted light at the photodetector produces spurious optical beats because the short path imbalance allows sufficient coherence to be retained between light which has travelled along any of the available paths in the instrument. The main spurious beats occur between back reflections within the instrument and beams which are backscattered from the target. This combination of beams is equivalent to the configuration of a velocity sensitive vibrometer and the unwanted beat frequencies are proportional to the target velocity. Additionally, light back reflected within the instrument is more intense than that back scattered from the target, resulting in stronger optical beats on the photodetector which are demodulated in preference to the optical beat which carries the target acceleration signal. The LDAc output can be tested for sensitivity to velocity by substituting the demodulated output $V_{ou}/k_D$ into the LDV beat frequency expression, equation (6.1), where $k_D$ is the demodulator calibration constant defined such that $V_{ou}=k_D f_b$. Close agreement between the velocity derived from the LDAc output in this way and the output from a commercial LDV system strongly suggests sensitivity of this LDAc configuration to the target velocity, owing to the dominance of spurious beat frequencies on the photodetector. This example therefore suggests the minimum practical $\Delta l$ must be significantly greater than the coherence length of the laser source in order to reduce sensitivity to spurious beats occurring on the photodetector.

6.4 A fibre-coupled LDAc

6.4.1 Practical implementation of optical fibre in an LDAc configuration

In Section 6.3.3 a path imbalance of 120m was shown to provide optimum sensitivity to a target frequency range 0-40kHz. Correspondingly, $a_{\text{max}}=640\text{km/s}^2$ under these
conditions if a frequency pre-shift $|\Delta f_x - \Delta f_y| = 1.6\text{MHz}$ is obtained from a rotating diffraction grating. LDV measurements during the short path length LDAc study in fact showed the acceleration range of the ultrasonic cutting blade to be $1\text{-}100\text{km/s}^2$, which is lower than reported previously due to the available blade mounting conditions. A $120\text{m}$ path imbalance is therefore appropriate to both the frequency and acceleration ranges of the target and can be expected to reduce coherence significantly between beams which have deviated from the main paths A and B in the instrument.

An $80\text{m}$ optical fibre with refractive index of $1.5$ provides a $120\text{m}$ path imbalance. Single mode polarisation preserving fibre is required in order to retain the polarisation information which distinguishes light on path A from that on path B. This type of fibre is used extensively in the telecommunications industry, where infra-red wavelengths are employed to transmit data over large distances with minimal attenuation. Consequently, single mode fibre which is matched to infra-red wavelengths is relatively inexpensive whereas single mode fibre which is matched to visible wavelengths is less common and more expensive. For development purposes, however, in terms of compatibility with existing optical equipment and ease of use the advantages of operating in the visible region outweigh the costs.

Light propagates through a conventional single mode fibre as a combination of two orthogonally polarised components which travel with the same velocity. Environmental disturbances, such as heat or vibration, can cause light to couple from one component to the other with the result that the output polarisation state may vary unpredictably. Polarisation preserving single mode fibre additionally uses stress induced birefringence to cause the two components to travel at different velocities and so prevent inadvertent transfer of energy from one to the other [6.36].

Launch conditions are important in both maximising the amount of incident light which enters the fibre and ensuring the light in the fibre remains on either the fast or slow optical axis. In practice a microscope objective lens is required at each end of the fibre with facility to translate the fibre position in three orthogonal directions and rotate about the fibre axis using microadjusters. The intrinsic attenuation characteristics of
optical fibres in the visible wavelength range suggest a 90% transmission efficiency for a fibre length of 80m but transmission efficiencies of 50% for a well shaped laser beam and as little as 10% for light backscattered from a diffuse surface are practical upper limits for a fibre length of 80m when launch losses are included.

A schematic diagram of an LDAc configuration which incorporates an optical fibre is shown in figure (6.6). The required path imbalances on both paths A and B are implemented by light which passes in opposite directions along one axis of the same fibre in order to minimise any differential phase noise effects and equipment cost. The polarisation maintaining properties of the fibre are also used to rotate the polarisation of light propagating in the fibre by 90°, which simplifies the optical configuration slightly.

Comparison of the demodulated output from this instrument with a simultaneous velocity measurement from a commercial vibrometer, for vibration of the ultrasonic cutting blade, shows that this LDAc configuration again exhibits sensitivity to the velocity of the target surface due to the combined effects of back reflections from optical surfaces in the instrument and polarisation leakage at the beamsplitters. In particular two unwanted optical beats occur on the photodetector.

In one of these beats, light on path B, which returns from the target through the fibre interferes with the proportion of the light on path A which is unavoidably back reflected from the fibre entry, labelled (15) in figure (6.6). The two beams are co-linear with the same polarisation and are essentially inseparable but have travelled optical paths which differ in length by 120m and as such should be incoherent. However, the strong 1.6MHz optical beat recorded on the photodetector when other paths are temporarily extinguished confirms the unexpected presence of considerable coherence between these beams.

Similarly, polarisation leakage at beamsplitter (7) allows some of the light travelling from the target on path B to be diverted towards beamsplitters (5) then (17). This beam is of the same polarisation as that on path A, which returns from the target through (5)
& (17), and the two beams produce another unwanted 1.6MHz optical beat on the photodetector between light which has travelled optical paths differing in length by 120m.

Both of these spurious beats involve one beam which is backscattered from the target and another which remains within the instrument, making their associated beat frequency proportional to the velocity of the surface. The beams which have not travelled along the fibre or which are not back scattered from the target are of considerably greater intensity on the photodetector than those which have faithfully followed path A or B. Consequently these spurious beats dominate on the photodetector and are demodulated in preference to the acceleration signal. This situation is very similar to the short path imbalance LDAc configuration and the coherence properties of light which has travelled a substantial path imbalance are therefore studied in the following section.

6.4.2 Coherence properties
Small He-Ne lasers generally have a coherence length of less than 1m [6.37] and in vibrometry applications it is considered important to maintain the path imbalance between the interfering beams to within the coherence length of the laser in order to produce a strong beat frequency signal on the photodetector. Hence, the apparent retention of coherence between beams which have travelled an optical path difference of 120m, as reported in the previous section, is somewhat unexpected. An LDV optical configuration, as shown in figure (6.7), can be used to compare the relative coherence exhibited by light recombined after following three different path imbalances. Reference beam R1 introduces a path imbalance of only a few centimetres, whilst R2 uses the distance between beamsplitter (12) and mirror (13) to create a 3m imbalance and R3 uses the optical fibre to create a path imbalance of 120m. A variable optical attenuator ensures the three reference beams are of equal intensity at beamsplitter (12). Figure (6.7) shows a target beam which is frequency pre-shifted by +800kHz by a rotating diffraction grating and the magnitude of the 800kHz peak in the Doppler signal spectrum, in the absence of target motion, can then be used to compare the coherence of each target-reference beam combination.
The three spectra are shown in figures (6.8a-c) and indicate a reduction in coherence with increasing path imbalance. Significantly, the long path imbalance arrangement retains sufficient coherence to produce a detectable beat. Laser light has a form of coherence over long distances since coherence is restored when the path length imbalance is approximately equal to even multiples of the laser cavity length. However, the 120m path imbalance used in the test described above was shown not to be a coherence repeat length as little change was observed in the Doppler spectrum peak height when the optical fibre exit (16) was positioned at distances ranging from 10-40cm from beamsplitter (12). Path length imbalances in-between coherence repeat lengths exhibit partial coherence which can be explained by the statistical properties of two partially coherent beams. Equation (6.2) can be developed to include the term $g_{TR}$, which is the normalised cross-correlation of the target and reference beams:

$$I = I_T + I_R + 2(I_T I_R)^{\Re} |g_{TR}| \cos \varphi$$  (6.10)

If the fluctuations of the two waves are completely correlated, then $|g_{TR}| = 1$ and equation (6.10) takes the form of equation (6.2) for coherent light. If $g_{TR} = 0$, the light is completely uncorrelated and there is no stable interference. At intermediate values of $|g_{TR}|$ there is a degree of correlation between the two beams sufficient to produce a stable phase fluctuation and hence a detectable intensity fluctuation. Thus it is possible for light which is apparently significantly outside the coherence length of the source to exhibit a degree of coherence and produce a detectable beat frequency. This is significant in an LDAn configuration where unwanted back reflected light within the instrument is of considerably greater intensity than the light which is backscattered from the target and this combination of low coherence but relatively high intensity is sufficient to produce stronger beats on the photodetector than light which has faithfully followed paths A and B. The stronger beat signal is demodulated in preference, resulting in sensitivity to only the velocity of the target surface.

6.4.3 Frequency shift location

Spurious optical beats on the photodetector caused by coherence between unwanted beams can be accommodated if the optical beat which carries the acceleration signal
can be isolated and demodulated in preference to the spurious signals. A technique is introduced here which makes novel use of the frequency shifting device to distinguish the wanted and unwanted beats in addition to resolving target motion direction. The wanted and unwanted beats can be separated in frequency by careful positioning of the frequency shifting device in the optical configuration. In the previous LDAc configuration, the majority of the unwanted light on the photodetector was the product of relatively intense back reflections from optical components close to the laser source. By locating the rotating diffraction grating close to the source, back reflected light was shifted in frequency by either +800kHz or -800kHz and was thus able to cause a beat at 1.6MHz with light backscattered from the target. A diffraction grating located further from the source in the optical configuration leaves the most intense back reflections at the source frequency and thus able to produce only an 800kHz beat by interfering with light backscattered from the target. Only light which has travelled faithfully along paths A and B will produce a 1.6MHz beat when recombined on the photodetector. This is acceptable in practice provided the wanted 1.6MHz signal can be demodulated in preference to the unwanted 800kHz signal and the diminished demodulatable frequency range remains sufficient to accommodate the target acceleration range. Using this technique, the practical restriction on the minimum \( \Delta l \), imposed in section 6.3.4 based on the requirement for the path length imbalance to be greater than the coherence length of the laser, is removed.

An optical configuration which minimises the potential for unwanted beats at 1.6MHz is shown schematically in figure (6.9). The frequency shift is located well away from the laser source, such that light on path A travels through the fibre before reaching the rotating grating. Light on path A which is back reflected from the fibre entry is thus at the source frequency and unable to create a 1.6MHz beat with any other light in the instrument. Separation of the light on paths A and B is retained on beamsplitter (3), labelled in the figure, and through the grating in order to ensure each path receives only one frequency shift. If A and B were recombined into a single beam before passing through the grating each diffracted order would contain light from both paths. The path A and B components in each diffracted order could be distinguished by their polarisation state and polarising filters introduced to minimise unwanted path B light in
the +1 diffracted order and unwanted path A light in the -1 diffracted order. However, such signal isolation by polarisation filtering is imperfect and beams with unwanted combinations of polarisation and frequency shift would be able to reach the photodetector after re-separation of paths A and B by polarisation at beamsplitter (11). In particular, each path would contain a small component of light which is of the same polarisation and coherent with the wanted light but which has the opposite frequency shift, thus producing an unacceptable 1.6MHz beat at the photodetector between light on the same path, A or B.

In the configuration shown in figure (6.9), 50% of the light transmitted through the fibre on path A deviates along an unwanted path through components (12), (11) and (9) to the target and is not, therefore frequency shifted by the rotating diffraction grating. The polarising optical components are however arranged such that this relatively intense, but unwanted, beam is directed back into the fibre after backscattering from the target, where it is attenuated significantly and taken further out of coherence with light on the main paths A and B. The main 800kHz (unwanted) beat in this optical configuration occurs between light returning from the target on path A and the back reflection from light at the fibre entry (16). The Doppler spectrum from the photodetector output in the presence of a stationary target, figure (6.10), shows the successful 800kHz separation of unwanted signals and the 1.6MHz beat, which carries the acceleration signal. The light contributing to each peak in the spectrum is identifiable by noting changes in the spectrum when selected optical paths are blocked temporarily. The 800kHz beat is considerably stronger than the 1.6MHz beat since the photodetector is exposed to the relatively intense back reflection from the fibre entry (16). This situation is unavoidable when using a single fibre to create both path imbalances since the photodetector must be located relatively close to (16) in order to gather the less intense wanted light, which returns from the target through the fibre on path B.

It is clear from figure (6.9) that the requirement to separate wanted and unwanted light by frequency whilst directing beams along paths A and B by polarisation contributes significantly to the optical complexity and that the problem is somewhat exacerbated
by the use of a rotating diffraction grating as the frequency shifting device. The low intensity of the light on paths A and B arriving at the photodetector can be improved in a less complex and thus more light efficient optical configuration.

Using both the fast and slow axes of the polarisation maintaining fibre would remove some of the complexity, however this would cause the imbalances on paths A and B to be unequal. For an 80m fibre, the slightly higher refractive index of 1.51 on the slow axis gives an optical path which is 121m, rather than 120m on the fast axis, which would increase unwanted phase noise effects and is also undesirable when considering the future development of the instrument where a laser source with very short coherence length may be required. Future developments may therefore benefit from not using polarisation as a means of directing light along paths A and B and this will be discussed further in Section 6.5.

6.4.4 Acceleration signal output

Motion of the target causes modulation, $\Delta f_b$, of the central frequency, $|\Delta f_A - \Delta f_b|$, in the photodetector output of all interferometers based on the laser Doppler principle. In the LDAc, the magnitude of $\Delta f_b$ is proportional to the acceleration of the target surface and, for an oscillating target, peaks in the photodetector output frequency spectrum can appear to 'broaden' in proportion to the modulation frequency. In fact, the photodetector output follows the relation shown in equations (6.2) and (6.7) such that the fluctuating intensity component of interest varies with time according to an expression of the form 'cos cos - sin sin'. For analysis in the frequency domain it is sufficient to examine either the 'cos cos' or 'sin sin' term:

$$I_p(t) = \cos(2\pi(\Delta f_A - \Delta f_b)t) \cos \left( \frac{2\Delta f}{c\lambda} \left[ \int_0^t [\alpha A(\omega) \cos(\omega t - \phi(\omega))] d\omega \right] d\omega \right)$$  \hspace{1cm} (6.11)$$

In the frequency domain this intensity fluctuation does not produce smooth broadening of the central frequency, $|\Delta f_A - \Delta f_b|$. Instead, sidebands at frequencies $|\Delta f_A - \Delta f_b| \pm n\omega/2\pi$ (where $n=1,2,...$), can be resolved in the frequency range $n\omega/2\pi \leq \Delta f_b$. This is confirmed in a numerical simulation of the LDAc photodetector
output during target vibration at approximately 20kHz. The photodetector output spectrum under these target vibration conditions, simulated using equation (6.11), is shown normalised in figure (6.11). The central frequency $|\Delta f_A - \Delta f_s| = 1.6\text{MHz}$ and sidebands are observed at a frequency spacing of approximately 20kHz. The sidebands can be resolved in the frequency range $|\Delta f_A - \Delta f_s| \pm 100\text{kHz}$, which is in agreement with the value calculated using equation (6.7) under the modelled target vibration conditions.

In the fibre coupled LDAc configuration developed in the previous section, the presence of two frequencies in the photodetector output causes difficulty when demodulating the acceleration signal by any of the techniques discussed in Section 6.2.1. It is attractive to remove the dominant but unwanted, 800kHz signal using a high pass filter with a sharp roll-off characteristic at around 900kHz. However, both the 800kHz and 1.6MHz signals are modulated by motion of the target, as discussed above. The 1.6MHz peak in the photodetector output spectrum exhibits sidebands in a frequency range of magnitude proportional to the target acceleration and the 800kHz peak exhibits sidebands in a frequency range of magnitude proportional to the target velocity. The magnitudes of these frequency ranges are in the approximate ratio 1:20 for a given target acceleration at a vibration frequency 19.8kHz.

Figure (6.12) shows the 800kHz peak and sidebands over a frequency range 800±160kHz, in proportion to a target vibration velocity amplitude of 0.05m/s$^1$. The corresponding target acceleration amplitude is 6.3km/s$^2$, which from equation (6.7) results in sidebands around the 1.6MHz peak over a frequency range 1600±8kHz and cannot be resolved in figure (6.12). A target acceleration amplitude of 50km/s$^2$, in the middle of the range exhibited by the ultrasonic cutting blade, will produce sidebands around the 800kHz peak over a frequency range 800±1300kHz, effectively masking the presence of the 1.6MHz signal in the Doppler spectrum and preventing its demodulation. A high pass filter with sharp roll-off characteristics at around 900kHz would therefore be ineffective in removing the unwanted velocity sensitivity from the LDAc output.
The optical configuration can be adjusted to assist demodulation of the 1.6MHz signal by increasing its isolation from the 800kHz beat. The main 800kHz beat, between path A and the back reflection from the fibre entry (16), can be altered by changing the polarisation arrangements to the optical configuration shown in figure (6.13). In this arrangement the light which is unavoidably diverted away from path A at beamsplitter (13) is not directed back into the fibre after backscattering from the target and is instead transmitted through beamsplitters (11) & (19) and is incident upon the photodetector. It is useful to consider this light as a third legitimate optical path, C. Light on path C is not frequency shifted by the rotating grating and a strong 800kHz beat is produced on the photodetector where light on path C is recombined with light on path A, which has passed through the rotating diffraction grating and is thus shifted in frequency by 800kHz. The beams which have followed paths A and C are both backscattered from the target, are of the same polarisation and exhibit maximum coherence as the path imbalance between them is very short.

Main paths A and B leave beamsplitter (19) in opposite polarisation states and a linear polariser is positioned in front of the photodetector aligned at 45° to both the s and p polarisation planes in order to extract components of the light which will produce an optical beat. Use of a polariser in this manner is particularly inefficient and causes difficulty detecting light returning from the target through the optical fibre, on path B, which is of particularly low intensity due to the losses resulting from backscattering from the target and launching the speckle pattern into the optical fibre. The intensity of the light on path B is increased, at the expense of that on path A, by rotating the polarisation axis of the laser source such that an uneven separation of the light incident occurs on beamsplitter (2). A 70:30 split of the laser power in favour of path B provides a substantial increase in the intensity of the light returning from the target through the fibre, simultaneously reducing the strength of the 800kHz beat between light on paths A and C.

The significance of this optical configuration is apparent in the presence of target motion. Both beams A and C are backscattered from the target and as such the 800kHz beat frequency is not modulated by target motion. The 1.6MHz signal is
therefore sufficiently isolated in the Doppler signal spectrum to allow sidebands resulting from target oscillation to be studied. Tests using the ultrasonic cutting blade were conducted under target vibration conditions equivalent to those used in the LDAc output simulation, discussed earlier in this section. The output from a laser Doppler vibrometer showed the target velocity amplitude to be $0.66\text{ms}^{-1}$ at a frequency of $19.8\text{kHz}$. The acceleration amplitude calculated from this velocity measurement is $82\text{km}s^{-2}$ which, according to equation (6.7), should produce sidebands around the $1.6\text{MHz}$ peak over a frequency range $\Delta f = \pm 104\text{kHz}$ in the photodetector output.

The practical LDAc configuration produces sidebands around the central frequency in the photodetector output in the presence of target vibration. Figure (6.14) shows the $1.6\text{MHz}$ peak with sidebands at frequencies up to $\pm 100\text{kHz}$ under the test conditions described above and this confirms the sensitivity of this LDAc configuration to the acceleration of the target surface.

The $1.6\text{MHz}$ peak and sidebands shown in figure (6.14) are only $2\text{dB}$ above the local noise floor due to the low intensity of light backscattered from the target which is incident on the photodetector. The autodyne frequency tracker used in the tests, of the type described in Section 6.2.1, was unable to demodulate this small signal and a time resolved analogue of the target acceleration could not therefore be produced. This difficulty could be overcome using superior frequency demodulating equipment and the $800\text{kHz}$ signal could be removed using a high pass filter with sharp roll-off characteristics. Despite the shortcomings of the frequency demodulation equipment, the measurements presented here represent the first reported practical demonstration of a non-contacting laser transducer whose output is directly sensitive to the acceleration of a target surface and as such provides the basis for future development of a viable LDAc instrument.

6.5 Future development of the LDAc

The prototype LDAc configuration shown in figure (6.13) is very complex and would not be a feasible commercial instrument. The prototype is, however, sufficient both to demonstrate practically the working principles of the LDAc and to study the optical
difficulties associated with implementing the multiple path separation and recombination necessary to achieve the required optical path imbalances. There is significant potential for future development of a viable working LDAc instrument based on the work contained in this chapter. It is therefore pertinent to consider the valuable knowledge obtained and its application to the design of a future LDAc.

6.5.1 Future development principles
The main issues addressed in the development of the LDAc to the point of practically demonstrating direct sensitivity to acceleration are summarised below:

1. Back reflections in the instrument are responsible for creating unwanted optical beats on the photodetector.

2. The coherence property of the laser cannot be relied upon to ensure interference only between beams which have travelled the same optical path length. However, a laser source with a short coherence length, such as a low cost laser diode, could be beneficial in minimising the potential for spurious beats on the photodetector.

3. Polarisation cannot be relied upon to provide total separation of the beams by directing light only along the main paths A and B in the instrument.

4. The problems of (2) and (3) compound the problem in (1).

5. Using a single optical fibre to effect both path imbalances necessarily locates the photodetector close to the most intense back reflection, from the optical fibre entry. The back reflected light is indistinguishable from the light returning through the fibre and the two beams are able to produce an unwanted optical beat.

6. Additional complexity is introduced into the optical configuration in accommodating the polarisations necessary to direct the beams along the required paths whilst using only one axis of the optical fibre. This added optical complexity is wasteful of light and contributes to low signal intensity on the photodetector.

7. The wanted and unwanted optical beats can occur at the same mean frequency and the unwanted signal is stronger, preventing demodulation of the acceleration signal.

8. Frequency shifting at an appropriate point in the optical configuration can be relied upon to separate the unwanted optical beat frequency from that of the acceleration signal.
9. Modulation of distinct beats in the presence of target motion confuses the velocity and acceleration beats, causing difficulty in isolating the acceleration signal.

10. It is possible to 'select', by appropriate optical configuration, a back reflection which does not produce a beat which is modulated in the presence of target motion to be the main back reflection reaching the photodetector. The acceleration signal can then be demodulated if the unwanted beat frequencies are filtered out of the photodetector output.

These issues must also be addressed in further practical LDAc development work and therefore serve as an outline of future LDAc practical design principles.

6.5.2 Proposed future LDAc design

A new LDAc design based on the knowledge gained in this study is shown schematically in figure (6.15) and addresses the specific issues (1-10) discussed in the previous section.

The proposed optical arrangement does not use polarisation to direct light along paths A and B (Issues 3, 4 & 6). This allows a significant reduction in optical complexity when compared to the prototype practical arrangement shown earlier in figure (6.13) and it can be shown that more light is incident upon the photodetector as a result of a simpler optical configuration. Maximum use of the light reaching beamsplitter (11) can be made by introduction of a second photodetector (15A), shown dotted in figure (6.15). The signals reaching the two photodetectors will be 180° out of phase and when subtracted will significantly enhance the amplitude of the output.

By allowing beams on paths other than A and B to propagate through the instrument, this simpler configuration appears reliant on interference occurring only between the light which has travelled faithfully along the main paths A and B. It has been shown in this chapter that the laser coherence property cannot be relied upon to discriminate the Doppler signal which carries the acceleration signal from other light on the photodetector in this way. Thus, the technique introduced in Section 6.4.3 is implemented in which the frequency shift is used to isolate the acceleration signal from
spurious beats caused by back reflections and light travelling on paths other than A and B (Issues 2, 4, 7 & 8). This is achieved using two Bragg cells, which provide frequency shifts of 80MHz and 120MHz, located in the optical configuration such that only the light which travels faithfully along path A is frequency shifted by 120MHz and only the light which travels faithfully along path B is frequency shifted by 80MHz. The acceleration signal carrier frequency is therefore 40MHz. If the instrument has a demodulatable frequency range of ±5MHz, the corresponding working ranges and upper frequency limits are shown in table (6.2) for several path imbalances, Δl, calculated as in Section 6.3.2. These values are based on an instrument which uses a red laser diode (λ=633nm) and a frequency demodulator with a dynamic range of approximately 100dB.

<table>
<thead>
<tr>
<th>Path imbalance, Δl, (m)</th>
<th>Acceleration range</th>
<th>f_{max} (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.77</td>
<td>100×10³ kms⁻² to 1kms⁻²</td>
<td>1M</td>
</tr>
<tr>
<td>47.7</td>
<td>10×10³ kms⁻² to 100ms⁻²</td>
<td>100k</td>
</tr>
<tr>
<td>477</td>
<td>1000kms⁻² to 10ms⁻²</td>
<td>10k</td>
</tr>
</tbody>
</table>

Table 6.2 - Typical working ranges of the proposed LDAc

Sensitivity to other optical beats occurring at 0MHz, 40MHz, 80MHz, 120MHz, 160MHz and 200MHz due to combinations of back reflections and light legitimately travelling along parts of paths A and B remains but can be tolerated if modulation of these beat frequencies in the presence of target vibration is tolerable (Issues 8, 9 & 10). Referring to figure (6.15), the combination of optical paths which produces each of the beat frequencies listed above is shown in table (6.3). A beat involving one beam which is backscattered from the target and one beam which is back reflected within the instrument will exhibit frequency modulation in proportion to the target vibration velocity. Beats in which both beams return from the target along similar path lengths are not modulated in the presence of target vibration, whilst beats involving beams which return from the target along paths which differ in length by Δl are modulated in proportion to the target acceleration. The relative intensity of each beat is shown
normalised to the intensity of the wanted beat and is estimated assuming 95% light transmission efficiency at each lens, 90% reflection from the target, 50% and 10% transmission efficiencies respectively for light travelling to and from the target through the optical fibre and 90% transmission through the Bragg cell.

<table>
<thead>
<tr>
<th>Beat Freq. (MHz)</th>
<th>Contributing Optical Paths [from figure (6.15)]</th>
<th>Frequency Modulation</th>
<th>Relative Beat Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>B and E</td>
<td>a</td>
<td>0.71</td>
</tr>
<tr>
<td>0</td>
<td>F and Z</td>
<td>ν</td>
<td>1.81</td>
</tr>
<tr>
<td>40</td>
<td>A and B - (wanted)</td>
<td>a</td>
<td>1.00</td>
</tr>
<tr>
<td>40</td>
<td>A and C</td>
<td>×</td>
<td>2.24</td>
</tr>
<tr>
<td>40</td>
<td>A and E</td>
<td>×</td>
<td>2.24</td>
</tr>
<tr>
<td>40</td>
<td>C and D</td>
<td>×</td>
<td>2.24</td>
</tr>
<tr>
<td>80</td>
<td>A and D</td>
<td>×</td>
<td>3.16</td>
</tr>
<tr>
<td>80</td>
<td>B and C</td>
<td>a</td>
<td>0.71</td>
</tr>
<tr>
<td>80</td>
<td>B and F</td>
<td>×</td>
<td>0.33</td>
</tr>
<tr>
<td>80</td>
<td>B and Z</td>
<td>ν</td>
<td>1.72</td>
</tr>
<tr>
<td>80</td>
<td>C and E</td>
<td>×</td>
<td>1.58</td>
</tr>
<tr>
<td>80</td>
<td>E and F</td>
<td>a</td>
<td>0.74</td>
</tr>
<tr>
<td>80</td>
<td>E and Z</td>
<td>ν</td>
<td>3.85</td>
</tr>
<tr>
<td>120</td>
<td>A and F</td>
<td>a</td>
<td>1.05</td>
</tr>
<tr>
<td>120</td>
<td>A and Z</td>
<td>ν</td>
<td>5.44</td>
</tr>
<tr>
<td>120</td>
<td>B and D</td>
<td>a</td>
<td>1.00</td>
</tr>
<tr>
<td>120</td>
<td>D and E</td>
<td>×</td>
<td>2.24</td>
</tr>
<tr>
<td>160</td>
<td>C and F</td>
<td>a</td>
<td>0.74</td>
</tr>
<tr>
<td>160</td>
<td>C and Z</td>
<td>ν</td>
<td>3.85</td>
</tr>
<tr>
<td>200</td>
<td>D and F</td>
<td>a</td>
<td>1.05</td>
</tr>
<tr>
<td>200</td>
<td>D and Z</td>
<td>ν</td>
<td>5.44</td>
</tr>
</tbody>
</table>

Table 6.3 - Beat frequencies occurring in the proposed future LDAc

In table (6.3), a denotes frequency modulation in proportion to target vibration acceleration, whilst ν denotes frequency modulation in proportion to target vibration velocity.
Two passes in opposite directions through a Bragg cell will double the frequency shift [6.38]. However, beat frequencies involving paths C or E, shown in table (6.3), can be eliminated by using a Bragg cell which has anisotropic material properties [6.39], since this causes attenuation of the light backscattered from the target which attempts to pass through the Bragg cell (4). Alternatively, introduction of an optical isolator between components (5) and (4) will prevent light backscattered from the target passing through the Bragg cell (4). Optical isolators use a Faraday rotator [6.40] between two polarisers making an angle of 45° with each other to attenuate light of any polarisation travelling in an unwanted direction by up to 90dB whilst allowing almost all of the light travelling in the desired direction to pass through. The relative beat intensity, shown in table (6.3), is normalised to the intensity of the wanted beat between light on paths A and B. It can be seen that light on path Z is involved in some of the most intense beats on the photodetector and this can be greatly reduced if the fibre ends are cleaved at an angle, such that back reflections are deflected away from the fibre axis (Issues 1&5).

Isolation of the path between components (5) and (4) leaves the back reflected path Z as the main source of unwanted beats in the proposed LDAc configuration. The frequency shift employed ensures these beats are separated from the wanted beat frequency by 35-40MHz. However, beats involving path Z will exhibit frequency modulation in proportion to the target vibration velocity and the potential exists for the modulated velocity signal to encroach on the 35-45MHz operating frequency range of the acceleration signal (Issue 9). The beats involving paths B&Z and F&Z at frequencies 80MHz and 0MHz respectively are of particular interest but current continuous vibration measurement applications suggest that the 35-40MHz initial frequency separation is sufficient to retain separation of the modulated velocity and acceleration signals. For example, the ultrasonic cutting blade discussed earlier in this chapter exhibits a peak acceleration of around 100kms⁻² at a frequency of 20kHz, which corresponds to a peak velocity of 0.80ms⁻¹. Substituting this velocity in the LDV Doppler frequency expression, equation (6.1), shows the 0MHz and 80MHz peaks will be modulated by 2.5MHz under these conditions and thus considerable frequency separation of the velocity and acceleration signals is retained. The peak target
acceleration at the vibration frequency of 20kHz in this example would need to increase by a factor of 14 before the modulated velocity signal approached the 35-45MHz operating frequency range of the acceleration signal.

Impact measurement applications in which the vibration of interest is present on top of a whole body velocity were discussed in Section 6.1. In these situations large whole body velocities which do not reverse in direction are often encountered which can shift the LDAc 80MHz peak without causing a modulating effect. For example, in the golf ball impact discussed in Chapter 4, the whole body forward velocity attained by the ball is 55ms\(^{-1}\), which in a measurement made in the direction of motion of the ball would cause a frequency shift of 173MHz in the LDAc 80MHz peak. At some point in the impact the unwanted peak could feasibly appear in the same part of the frequency spectrum as the acceleration signal and this would introduce difficulties in demodulating the acceleration signal.

Beats involving the back reflected path Z can be almost completely eliminated if the LDAc configuration is based entirely on optical fibre components. The potential to design an all-fibre LDAc is apparent, based on the optical configuration shown in figure (6.15). All-fibre interferometers are not uncommon [6.41, 6.42, 6.43] and an all-fibre laser vibrometer was introduced [6.44] which is set to become commercially available. An all-fibre laser probe incorporating a long path imbalance has also been reported [6.45] for detection of surface defects using a form of time domain reflectometry. An all-fibre LDAc is proposed in figure (6.16), based on the optical configuration shown earlier in figure (6.15). Polarisation maintaining fibre is again used for maximum immunity to environmental effects such as heating or vibration of the fibre. Light losses caused by difficulties coupling beams into the fibre path imbalance in previous arrangements are significantly reduced by operating entirely in the fibre. Also, bulk optic beamsplitters are replaced by fused splitter/combiners. These can be constructed in a two-by-two or one-by-two arrangement and use of the one-by-two design at (6), (7) and (11) in the LDAc configuration avoids the wastage of light incurred using bulk optic non-polarising beamsplitters. Fused splitter/combiners are attractive in that they additionally offer low back reflection and a range of splitting ratios [6.46].
The optical isolator (4) has particularly low back reflection characteristics and is included to prevent the back reflection from the entry to the Bragg cell (5) reaching the photodetector. This effectively eliminates path Z and hence the 0MHz beat frequency allows a lower carrier frequency to be employed. A 5MHz carrier frequency could be created using two Bragg cells of frequency 80MHz and 85MHz. These Bragg cells are standard items and a cost saving is therefore made over the 120MHz cell required in figure (6.15). In figure (6.16), paths C and E are isolated by employing Bragg cells with anisotropic material properties, as discussed above, and the beat involving paths A and D is the most intense of the remaining five unwanted beats on the photodetector. These beats are shown in table (6.4), where it can be seen that, by using a 5MHz carrier, the frequency separation from the wanted signal is increased over that in table (6.3). The beat involving paths A and D is separated from the acceleration signal by 75MHz and is not modulated in the presence of target vibration (Issue 10). Should slight sensitivity to the other four remaining beats be encountered, one of these, at 80MHz and involving paths B&F, is not modulated, whilst the other three at 85MHz and 165MHz involving paths A&F, B&D and D&F, are modulated in proportion to the target acceleration. Thus all five of these beats can be tolerated or eliminated completely using a low-pass electronic filter with cut-off frequency around 10MHz at the photodetector output.

<table>
<thead>
<tr>
<th>Beat Frequency (MHz)</th>
<th>Contributing Optical Paths [from figure (6.16)]</th>
<th>Frequency Modulation</th>
<th>Relative Beat Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A and B - (wanted)</td>
<td>a</td>
<td>1.00</td>
</tr>
<tr>
<td>80</td>
<td>A and D</td>
<td>×</td>
<td>3.16</td>
</tr>
<tr>
<td>80</td>
<td>B and F</td>
<td>×</td>
<td>0.33</td>
</tr>
<tr>
<td>85</td>
<td>A and F</td>
<td>a</td>
<td>1.05</td>
</tr>
<tr>
<td>85</td>
<td>B and D</td>
<td>a</td>
<td>1.00</td>
</tr>
<tr>
<td>165</td>
<td>D and F</td>
<td>a</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 6.4 - Beat frequencies occurring in an all-fibre LDAC

a denotes frequency modulation in proportion to target vibration acceleration
A short laser coherence length can also assist in reducing the intensity of unwanted beats on the photodetector (Issue 2). Multimode laser diodes have close mode spacing which produces coherence lengths of the order of several nanometres, whilst singlemode laser diodes have coherence lengths of up to a kilometre [6.47]. It is, however, possible to obtain multimode laser diodes with a coherence length of 3-5mm and a source of this type would be suited to a practical LDAc arrangement since the two main optical path lengths can be balanced to within this tolerance. The beam emitted by a laser diode suffers from astigmatism, asymmetry and large divergence and laser diodes are therefore commonly coupled, or 'pigtailed', directly into optical fibres in order to eliminate these effects.

The main optical paths A and B in the all-fibre LDAc configuration will be balanced if the fibre lengths labelled $l_1$, $l_2$ and $l_3$ in figure (6.16) are arranged such that $|l_2 - l_1 - l_3|$ is less than the source coherence length, approximately 5mm for a laser diode. In practice, the capability to finely adjust the length of one of these paths may be necessary to optimise the optical beat strength on the photodetector. As both paths A and B pass along the long path imbalance, the size of $\Delta l$ does not affect the phase of the light recombined on the photodetector. This potentially allows the use of 'plug-in' fibre optic delay modules to adjust the working range of the instrument easily.

All-fibre interferometers often utilise the ability to directly modulate the frequency of a laser diode [6.48]. In the all-fibre vibrometer [6.44], a short path length imbalance between reference beam and light backscattered from the target introduces a phase difference which causes discrete frequency components at the source modulation frequency and higher harmonics when recombined on the photodetector. The rate of change of the phase difference is then proportional to the target velocity. The potential exists in the all-fibre LDAc configuration to implement the short path length imbalance necessary for this technique to function by appropriate arrangement of the fibre lengths $l_1$, $l_2$ and $l_3$. Further investigation of the applicability of source frequency modulation, particularly with reference to the problem of unwanted beats on the photodetector, is necessary.
The multiple separation and recombination of beams required by the principles of operation of the LDAC dictates a more complex optical configuration than used in a vibrometer. This influences the relative cost of the two instruments, which can be compared by considering the major optical components, as in table (6.5). Assuming equivalent signal processing electronics in each instrument, the cost of an all-fibre LDAC is less than twice the cost of a vibrometer. This is reasonable, given the measurement capability offered by the LDAC and the potential for an all-fibre LDAC to take advantage of recent and future advances in fibre optic and laser diode technologies.

<table>
<thead>
<tr>
<th>Major component</th>
<th>Commercial Vibrometer</th>
<th>Air path LDAC [from figure (6.15)]</th>
<th>All-fibre LDAC [from figure (6.16)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser</td>
<td>£1471</td>
<td>£1471</td>
<td>£205</td>
</tr>
<tr>
<td>Beamsplitter</td>
<td>3 x £250</td>
<td>4 x £250</td>
<td>4 x £350</td>
</tr>
<tr>
<td>Fibre delay (80m)</td>
<td>-</td>
<td>£1000</td>
<td>£1000</td>
</tr>
<tr>
<td>Fibre coupler</td>
<td>-</td>
<td>2 x £60</td>
<td>2 x £60</td>
</tr>
<tr>
<td>Optical isolator</td>
<td>-</td>
<td>-</td>
<td>1 x £350</td>
</tr>
<tr>
<td>Frequency shift</td>
<td>1 x £1600</td>
<td>£1600 &amp; £2100</td>
<td>2 x £1600</td>
</tr>
<tr>
<td>Fibre paths</td>
<td>-</td>
<td>-</td>
<td>7 x £10</td>
</tr>
<tr>
<td>Total</td>
<td>£3821</td>
<td>£7291</td>
<td>£6345</td>
</tr>
</tbody>
</table>

Table 6.5 - Cost comparison of major optical components in a vibrometer and two LDAC configurations

The new all-fibre LDAC introduced here retains all of the advantages over existing vibration measurement techniques discussed earlier in this chapter and embodies much of the new knowledge gained in this study. Specifically, the proposed all-fibre LDAC addresses the ten optical design issues found to be of importance in this study, as described in Section 6.5.1. Simple optical design and good light efficiency is attained by choosing not to direct beams through the instrument using polarisation. Simplicity and immunity to possible phase noise effects are enhanced by implementing both required path imbalances in a single axis of one highly birefringent optical fibre. Sensitivity to unavoidable optical beats resulting from back reflections and light which
travels along paths other than the main paths A and B in the instrument is minimised. This is achieved by utilising a low temporal coherence laser diode source and by employing two separate frequency shifts to isolate the beat frequency of the required Doppler acceleration signal from other beat frequencies. The all-fibre LDAc has the potential to become a robust laser transducer which extends current remote vibration measurement capability effectively without limit, satisfying the measurement requirements of today and the future.
Conclusions and recommendations for future study

7.1 Conclusions

In this thesis, the detailed study of the golf impact has provoked important developments in the computational analysis of sculptured surface products and in non-contacting vibration measurements based on the laser Doppler technique. A body of new knowledge has been assembled which complements existing techniques and demonstrates superior measurement and analysis capability in the study of elastic deformation during impact. Whilst the majority of the results presented in this thesis have been specific to the impact of a hollow golf club head and a golf ball, the techniques described are of considerably wider applicability. The main conclusions derived from each area of study are presented in this section.

A quadrilateral mesh generation procedure suited to sculptured surface product families was developed in Chapter 2. The approach taken recognises the specific requirements of discretising the sculptured surface geometry of a product and allows sufficient user interaction to effect mesh density variations according to expected loading conditions. The knowledge provided by the user is stored in a product data structure and can be recalled for the purposes of quickly generating quadrilateral meshes on other members of the product family or on subsequent design iterations. This is a pragmatic alternative to the creation of a fully automatic quadrilateral mesh generation method of general applicability - an extremely difficult task which is perhaps unnecessary since situations will commonly arise in which a totally general method will not succeed.

Specifically, this study has shown that the versatility of the paving mesh generation algorithm can be improved greatly by taking a feature based approach and further subdividing primary features into smaller areas with four curvilinear sides. The features have a spatial relationship which must be maintained in order to achieve mesh continuity across feature boundaries and this relationship is often controlled by the
restrictions forced on node placement in the secondary blend features. Each surface subdivision pattern is feature specific, which implies that the entire mesh generation procedure is product specific. The method is however extended to product families with the same feature anatomy by the use of parametric length variations which allow the surface subdivision to adapt to limited changes in surface shape. The procedure has been used successfully in the generation of well formed quadrilateral meshes on hollow golf clubs and is applicable to other sculptured product families. The results of this work provide the basis for developing the procedure into a semi-automatic quadrilateral mesh generation process for sculptured surface product families.

This thesis has described a modelling and analysis capability for hollow golf club heads of significantly greater accuracy and superior detail to that previously reported. In particular, accurate geometric modelling of the club head and discretisation of the geometry into an appropriate mesh of elements provides a superior spatial representation of the impact problem in a model containing only 3581 elements. This capability is allied to a 3-dimensional contact definition, hyperelastic golf ball material modelling and a robust, efficient finite element solution method. These tools are necessary for the development of a greater understanding of the detailed mechanical behaviour during impact in terms of energy transfer and 'feel' characteristics. The techniques presented are important to the golf equipment industry as it comes under increasing pressure to produce clubs and balls exhibiting improved performance within shorter timescales. Ultimately, additional performance-predicting capability at an early stage in the design process will be derived from the greater knowledge of the golf impact gained using these tools.

Experimental model verification is an essential component of any computational analysis. In Chapter 3 non-contacting laser Doppler vibrometry and electronic speckle pattern interferometry techniques were applied to the capture of detailed experimental hollow golf club head modal dynamic data. The natural frequencies of vibration of hollow club head finite element model were within 3% of the experimental results in the frequency range of interest (3.6-20kHz) and this provided strong evidence for the validity of the finite element club head model. The different modal characteristics of the
three surfaces which comprise the majority of the hollow structure were shown to influence the frequencies present in the impact sound in the frequency range 3.5-7.5kHz. This was demonstrated by a central impact which produced a sound corresponding mostly to excitation of modes of vibration at 5.9kHz, 6.4kHz and 6.8kHz and by an off-centre impact in which the impact sound indicated a much larger contribution from the 6.4kHz mode. Whilst the results presented here are specific to particular club head of interest in this study, the techniques are applicable to other hollow golf club heads and this ability to identify quantitatively the features of the golf club head responsible for the production of certain 'feel' characteristics is of importance to the golf equipment industry.

The behaviour of the golf impact finite element model was verified experimentally through the introduction of a system of measurements suited to the analysis of high-speed, short duration impacts between lightweight bodies in which large elastic deformation occurs. The system, described in Chapter 4, used a small number of sensors arranged to capture relevant information from a limited number of measurements. Using two laser Doppler vibrometers, a piezo-electric accelerometer and a suitable measurement trigger, a wealth of short duration golf impact data has been captured. The pattern of deformation and recovery of the ball in the lateral and forward directions was presented and its relationship to the spatial and temporal variation of the force on the club head was examined. The peak lateral deformation was found to be 0.75mm whilst, in the forward direction, the maximum approach was 4.5mm and relative deformations of the order of 0.3mm were recorded between the points across the ball at which measurements were made. The increase in whole body forward velocity of the ball from rest to a final value of 52ms\(^{-1}\) was observed along with the rapid generation of rotational velocity components which occurred at around 100-150\(\mu\)s into the impact. Significant oscillations occurred during the impact and peak accelerations of 600kms\(^{-2}\) and 90kms\(^{-2}\) were recorded on the ball and club head respectively in the forward direction. These oscillations are of importance in the further study of energy transfer between the club head and ball during impact.
This large amount of data, obtained under difficult measurement conditions from only a small number of sensors, is of superior detail and greater mechanical relevance than that obtainable using video techniques. The value of the measurement system in the validation of finite element models was demonstrated in the close agreement between the experimental results and those calculated from the finite element model of the impact. In addition to displaying the same general variation over time during the impact, the peak values in the computed results are within 5% of those measured experimentally for the lateral deformation velocity, forward velocity, approach, rotational velocity and club head acceleration.

Whilst the experimental techniques presented in Chapter 4 provide the foundation for the further study of a variety of impacting bodies, issues which are of importance to remote measurement techniques generally detract from the otherwise high quality of data obtained. In particular, translation of the target body causes the point on the target interrogated by the laser beam to change during the impact, the experimental measurement locations may not match the locations of nodes on the finite element model and the measurable velocity range is limited to the 10-15ms$^{-1}$ offered by current laser Doppler vibrometers. The developments in the laser Doppler technique presented in Chapters 5 and 6 addressed all of these issues through the introduction of a displacement compensation technique and the development of the first prototype laser Doppler accelerometer.

The displacement compensation technique introduced in Chapter 5 permits close approximation of the vibration of fixed points on bodies undergoing whole body translation using data recorded from remote transducers. In Chapter 5, the characteristics of a single remote measurement of steady state vibration on a translating body in the time and frequency domains were investigated. In the time domain, problems of waveform distortion and amplitude modulation were quantified whilst, in the frequency domain, the presence of whole body target displacement induced harmonics were highlighted, together with the effects of short data length. The displacement compensation technique uses a second simultaneous remote measurement, along with the target's whole body displacement history, to derive a
closer estimate of the measurement which would be made by a contacting transducer fixed to a point on the target surface.

The improvement in data quality, relative to a single remote measurement, has been quantified by reference to both simulated and actual experimental data and is apparent in both the time and frequency domains. Selection of a measurement spacing in the range $0.1\lambda_r - 0.3\lambda_r$ was shown to reduce the waveform distortion error from the 3-6% commonly encountered in a single remote measurement to around 0.5%. Similarly, the amplitude error, which can approach 100% in some single remote measurements, was reduced to less than 10% by use of the same measurement spacing range. In the frequency domain, the frequency broadening effect caused by a variable whole body translational velocity can be significantly reduced by application of the displacement compensation technique. This was illustrated by the analysis of steady state vibration on an impacted golf club head in which the broadening effect in the frequency spectrum from a single remote measurement masked the presence of two distinct frequencies of vibration of the crown surface at 5.7kHz and 6.2kHz. These and other frequency components of interest could be resolved in the spectrum of the displacement compensated measurement, demonstrating the superior quality of data obtained.

The displacement compensation technique is also appropriate to impact studies and has been developed in this thesis to investigate transient pulse propagation effects in impacted bodies remotely, thus permitting the close approximation of fixed point data which would otherwise be extremely difficult to obtain. This was demonstrated in a numerical simulation which showed approximately 1% error between the measurement which would have been made by a transducer fixed to the impacted body and data from two simulated remote measurements after displacement compensation. The technique was also applied successfully to the golf impact data captured in Chapter 4. This represented a difficult measurement situation involving a high rate of pulse shape change and rapid acceleration of the whole body. Five measurements were used to obtain accurate data over the region of interest on the ball and results showing a significant improvement over the single remote measurement data were presented.
The level of detailed comparison which can be drawn between individual nodes on the finite element model and the displacement compensated experimental data was demonstrated in Chapter 5 using the computational and experimental golf impact results. The close comparison between results was also taken as strong evidence for the validity of the finite element golf impact model. The capability to derive data effectively from a single point on a moving body using remote transducers in difficult measurement situations is a valuable experimental tool in structural vibration analysis. As remote, non-contact measurement methods become increasingly popular, the work presented in Chapter 5 demonstrates the improvement in the quality of data which can be obtained using the displacement compensation technique.

The first reported practical demonstration of a non-contacting laser transducer whose output is directly sensitive to the acceleration of a target surface was presented in Chapter 6. Several practical difficulties were overcome in order to isolate the particular optical beat frequency which carries the acceleration signal. Back reflections from optical components were shown to be responsible for creating several unwanted optical beats on the photodetector. This effect was compounded by imperfect laser beam separation and recombination by polarisation and unexpected levels of coherence between beams which had travelled along paths of significantly different lengths. The wanted beat signal was isolated by 800kHz from the unwanted signals by frequency pre-shifting at an appropriate point in the optical configuration. A target acceleration of 82kms\(^{-2}\) caused sidebands at frequencies up to ±100kHz to appear around the central 1.6MHz peak in the Doppler signal spectrum and this confirmed the sensitivity of the accelerometer to the acceleration of the target surface, thus providing the first practical demonstration of this important new instrument.

7.2 Recommendations for future study

Whilst a significant contribution has been made in each area of study reported in this thesis, considerable potential exists to develop the techniques further.
The quadrilateral finite element mesh generation procedure introduced in Chapter 2 can adapt only to limited changes in feature shape and cannot therefore be used over an entire product family which displays significant shape variation. It has been beyond the scope of this work to implement a feature based product data structure in which features may inherit mesh generation attributes from several different sources and hence adapt automatically to radical feature shape changes. An integrated CAD and analysis feature based data structure, specific to a product family, is therefore the final development necessary to derive maximum benefit from the mesh generation procedure proposed in Chapter 2.

Many of the techniques either applied or developed in the course of this research are of direct relevance to the golf equipment industry. Whilst the impact data presented in this thesis constitutes a wealth of new knowledge regarding the golf impact, it generates many new directions of study for the golf equipment industry. In particular, an improved golf ball finite element model and a more formal understanding of the 'feel' concept are important goals.

The golf ball finite element model developed in Chapter 2 has the basic impact properties of a generic two-piece golf ball. However, the golf equipment industry would like to be able to analyse the subtle differences between the behaviour of two-piece ball brand types during impact. The potential for further study therefore exists in the material modelling of the golf ball and its representation in finite element models. Specifically, the potential to define explicitly the impact properties of a material based on more extensive high strain rate experimental tests than those discussed in Chapter 2 should be investigated. Additionally, the 'balata' ball has retained popularity amongst elite players and the accurate modelling of this inhomogeneous hyperelastic construction is worthy of further study.

The ability to identify quantitatively the features of a hollow golf club head responsible for the production of certain sound characteristics during impact is an important step in the understanding of 'feel' in the golf shot. The work on this subject presented in Chapter 3 provides a basis for further research into the impact sound characteristics.
preferred by golfers and the ability to achieve these 'feel' characteristics by re-
distributing the mass and stiffness of individual surfaces whilst simultaneously
satisfying the inertia property requirements of the whole club head. The measurements
presented in Chapter 3 are relevant to only one part of the total 'feel' concept and a
wider study based on this work has already attracted further research funding from
within the golf equipment industry. Ultimately, equipment designers would like to
specify 'feel' as a quantifiable design parameter.

The displacement compensation technique introduced in Chapter 5 permits the close
approximation of fixed point vibration data which would otherwise be extremely
difficult to obtain. In this thesis, the only whole body target motion considered was
perpendicular to the direction of incidence of the probe laser beam and occurred along
a single straight line. Significant potential for further study therefore exists in the use of
the displacement compensation technique to derive estimates of fixed point vibration
data in the presence of more complicated whole body motion. It is necessary to study
the effects of whole body motion involving two in-plane velocity components or
appreciable whole body velocity components in the direction of incidence of the laser
beam if maximum benefit is to be derived from the displacement compensation
technique.

The first prototype laser Doppler accelerometer configuration developed in Chapter 6
has a relatively complex optical configuration and simplifications must be sought for a
robust commercial instrument. The prototype was, however, sufficient to demonstrate
practically the working principles of the LDAC, including confirmation that an
acceleration-sensitive beat can be isolated, and to define the future development
principles of the instrument. Two proposed future developments of the prototype
LDAC into a viable working instrument were introduced and evaluated in Chapter 6. In
particular, the all-fibre LDAC has the potential to realise a robust laser transducer
which extends current remote vibration measurement capability effectively without
limit, satisfying the measurement requirements of today and the future.
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Chapter 1


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Chapter 2


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Chapter 4


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Chapter 5


Chapter 6


6.38 Perry, I (1997): Private fax communication from Elliot Scientific Ltd. regarding acousto-optic frequency shifting devices.


Figure 1.1a - Hollow golf club head showing the primary shape features and the loft angle, $\theta$

Figure 1.1b - Underside view of a hollow golf club head showing the primary shape features
$L_1 = \text{Line of impact and direction of motion of the ball after impact assuming point contact at } P$

$L_2(t) = \text{Approximate position and direction of the true line of impact at an instant during the golf impact}$

Figure 2.1 - Eccentric impact of two bodies

Figure 2.2 - Forward displacement of the free end of a viscoelastic rod forced to move with constant velocity $28 \text{ms}^{-1}$ at the opposite end with the material properties and approximate dimensions of a golf ball
Figure 2.3 - Stress pulse dispersion and attenuation in a viscoelastic material model
- - - - - Pulse calculated 6m from the origin of the disturbance
- - - - - Pulse calculated 8m from the origin of the disturbance

Figure 2.4 - Hollow golf club head feature anatomy
(Courtesy of Dr. Séan Mitchell)
Figure 2.5 - Wireframe hollow golf club head model

Figure 2.6 - Elements in the secondary 'blend' features
a) Poor element size and shape in the region away from the boundary

b) Subdivision pattern

c) Mesh generated after subdivision

Figure 2.7 - Primary feature subdivision applied to the crown surface of a hollow golf club head
Select initial global edge length, edge length range, initial smoothing parameters and smoothing parameter range.

Generate mesh using paving algorithm.

Smooth mesh.

Analyze and store mesh skew and taper value extremes.

Is the smoothing parameter range exhausted?

Is the global edge length range exhausted?

Select mesh with lowest maximum skew and taper values.

Increment smoothing parameter values.

Increment global edge length value.

Figure 2.8 - Feature sub-surface mesh generation flowchart
Figure 2.9 - Meshed hollow club head

a) Outer cover of the ball

b) Cut away section through the ball model showing concentric spheres construction

Figure 2.10 - Golf ball model based on an icosahedral node pattern
Figure 3.1 - Frequency response functions of the face, crown and sole surfaces
Figure 3.2a - First mode of the face observed using ESPI (5.64kHz)

Figure 3.2b - First mode of the face calculated using the finite element model (5.51kHz)
Figure 3.3a - Example mode shape of the crown surface observed experimentally using ESPI (5.96kHz)

Figure 3.3b - Example mode shape of the crown surface calculated using the finite element model (6.09kHz)
Figure 3.4a - Example sole mode shape observed experimentally using ESPI (8.20kHz)

Figure 3.4b - Example sole mode shape calculated using the finite element model (8.15kHz)
Figure 3.5 - Measured contact duration and forward velocities of two contacting nodes in the finite element model

- - - - Measured contact duration
- - - - - Forward velocity of a node on the club face
- - - - - - Forward velocity of a node on the ball in contact with the club face

Figure 3.6 - Arrangement of two accelerometers to measure club head rotation during impact
Figure 3.7 - Acceleration measured at the toe and heel of a 2-iron club head during a central impact with a golf ball

- Toe acceleration
- Heel acceleration

Figure 3.8 - Rotation of a 2-iron club head during a central impact with a golf ball
Figure 3.9 - Club head rotation in toe, centre and heel impacts using a club head fitted with shafts of different torsional stiffness.

Figure 3.10 - Example of stroboscopic video image processing for investigation of golf ball launch characteristics.
Figure 3.11 - High speed video images of the side of the golf ball at the start of the impact and at the point of maximum deformation in the forward direction. (Frame rate 40,000 frames/sec).

Figure 3.12a - Measured sound spectrum from a central club-ball impact.
Figure 3.12b - Measured sound spectrum from an off-centre club-ball impact

5.9kHz face mode

6.4kHz face mode

Figure 3.13 - Antinode position variation on face mode shapes
Figure 4.1 - Cross section through the equator of a deformed golf ball showing measurement axes.

Figure 4.2 - Alignment of the laser vibrometer to measure the lateral deformation velocity of the ball.
Figure 4.3 - Lateral deformation velocity measurement, \( \dot{z}_{cr}(t) \)

Figure 4.4 - Lateral deformation measurement
Figure 4.5 - Alignment of two laser vibrometers to determine ball forward velocity

Figure 4.6 - Velocity measurements $\dot{z}_{DR}(t)$ and $\dot{z}_{DR,N}(t)$
Figure 4.7 - Calculated forward velocity of the ball, $\dot{x}_{DB}(t)$

Figure 4.8 - Forward velocities calculated from points B, D and E

$\dot{x}_{DB}(t)$

$\dot{x}_{AE}(t)$

$\dot{x}_{ER}(t)$
Figure 4.9 - Relative forward displacements of the ball

- $x_{EID}(t)$
- $x_{DIC}(t)$
- $x_{CID}(t)$

Figure 4.10 - Calculated contact approach, $x_{CON}(t)$

- Using data from points A and B
- Using data from point D
Figure 4.11 - Calculated contact radius, $r_{CON}(t)$

Figure 4.12 - Forward velocity measured above and below the equator of the ball

Above equator ——— Below equator
Figure 4.13 - Rotational velocity of the ball

- - - - 2-piece construction ball

- - - - Balata ball

Figure 4.14 - Ball forward acceleration, $\ddot{x}_{CR}(t)$ and lateral deformation acceleration, $\ddot{z}_{CR}(t)$
Figure 4.15 - Normalised forward accelerations of the ball $\ddot{x}_{CR}(t)$ and club head $\ddot{x}_{club}(t)$
- Measurement
- Mean acceleration component

Figure 4.16 - Lateral deformation velocity, $\ddot{z}_{CR}(t)$ and finite element equivalent, $\ddot{z}'_{CR}(t)$
- Experimental
- Computational
Figure 4.17 - Forward velocity, $\dot{x}_{DR}(t)$ and finite element equivalent, $\dot{x}_{DF}(t)$

- Experimental
- Computational

Figure 4.18 - Finite element model golf ball forward velocities

- $\dot{x}_{AF}(t)$
- $\dot{x}_{EF}(t)$
Figure 4.19 - Measured forward deformation of point E relative to point D and of point E' relative to D' in the finite element model
--- Experimental --- Computational

Figure 4.20 - Approach using combined data from points A and B and finite element calculation
--- Experimental --- Computational
Figure 4.21 - Measured rotational velocity and finite element equivalent

--- Experimental --- Computational

Figure 4.22 - Measured acceleration of the back of the club head and finite element equivalent

--- Experimental --- Computational
Figure 4.23 - Three displacements $x_{H1}(t)$, $x_{H2}(t)$ and $x_{H3}(t)$ from the finite element model used to calculate the deformation of the club face centre during impact.

Figure 4.24 - Deformation of the club face centre calculated from the finite element model.
Figure 4.25 - Forward displacement of the front of the golf ball relative to point $E'$, calculated from the finite element model.
Figure 4.26 - Lateral deformation and over-recovery propagation across the ball in the x-direction during impact
Figure 4.27 - Progression of forward velocity initiation across the ball in the x-direction followed by the generation of rotational velocity components during impact.
Figure 4.28 - Club head and ball 300μs into the impact showing fringes of equal forward velocity
Figure 5.1a - Single remote measurement on a translating body, $t=0$

Figure 5.1b - Single remote measurement on a translating body, $t>0$
Figure 5.2 - Simulated steady state vibration response measurement on a translating plate. First mode, $\omega = 300\text{rad}s^{-1}$, $l = 0.2\text{m}$, $x = 1.9\text{ms}$, $x_{AF} = 0.18\text{m}$

$V_{AF}(t)$  
$V_{AR}(t)$

Figure 5.3 - Distortion in a single remote measurement

$\Delta_R (\omega/\zeta = 10)$

$\Delta_R (\omega/\zeta = 100)$
Figure 5.4 - Response spectrum from a single remote vibration measurement on a translating target

a) Amplitude variation in a fraction of one spatial period of the modeshape
b) Section of an 'infinite' sinewave, c) Amplitude variation frequency spectrum
d) Target vibration frequency spectrum, e) Remote measurement frequency spectrum
Figure 5.5a - Response spectrum for constant \( x \) translation of a 'long' target

Figure 5.5b - Response spectrum for translation of a 'long' target when \( x \) is a function of time
Figure 5.6 - Response spectrum from a single remote vibration measurement on a harmonically translating target, when the point of interest is an antinode

a) Amplitude variation in caused by harmonic translation of the modeshape
b) Section of an 'infinite' sinewave, c) Amplitude variation frequency spectrum
d) Target vibration frequency spectrum, e) Remote measurement frequency spectrum
Figure 5.7 - Response spectrum from a single remote vibration measurement on a harmonically translating target, when the point of interest is near an antinode

a) Amplitude variation caused by harmonic translation of the modeshape
b) Section of an 'infinite' sine wave, c) Amplitude variation frequency spectrum
d) Target vibration frequency spectrum, e) Remote measurement frequency spectrum
Figure 5.8 - Response spectrum from a single remote vibration measurement on a harmonically translating target, when the point of interest is near a node

a) Amplitude variation caused by harmonic translation of the mode shape
b) Section of an 'infinite' sinewave, c) Amplitude variation frequency spectrum
d) Target vibration frequency spectrum, e) Remote measurement frequency spectrum
Figure 5.9 - Example vibrometer measurement from a golf club crown

Figure 5.10 - Response spectrum from a single remote measurement on a golf club crown
Figure 5.11 - A double remote measurement on a translating body

Figure 5.12 - Displacement compensated waveform, $V_{AC}(t)$ and genuine vibration response $V_{AR}(t)$
Figure 5.13a - Variation of $\frac{de}{d\alpha}$ with $\alpha$ for $x_M$ in the range $0.05\lambda_p$ to $0.45\lambda_p$, $x_s=0.1\lambda_p$

Figure 5.13b - Variation of $\frac{de}{d\alpha}$ with $\alpha$ for $x_M$ in the range $0.05\lambda_p$ to $0.45\lambda_p$, $x_s=0.2\lambda_p$
Figure 5.14 - Variation of $\alpha$ at $e_{MAX}$ with $x_M$ for $x_S$ in the range $0.05\lambda_p$ to $0.3\lambda_p$. 
Figure 5.15a - Variation of maximum amplitude error of the displacement compensated waveform with position on the modeshape, $x_{AF}$ and measurement spacing, $x_S$.

Figure 5.15b - Variation of maximum amplitude error of the displacement compensated waveform with position on the modeshape, $x_{AF}$ and measurement spacing, $x_S$. 
Figure 5.16 - Amplitude errors in remote measurements

Figure 5.17 - Distortion in displacement compensated measurements
Figure 5.18 - Response spectrum from a displacement compensated measurement on a harmonically translating target, when the point of interest is near an antinode

a) Amplitude variation caused by harmonic target translation
b) Section of an 'infinite' sinewave, c) Amplitude variation frequency spectrum
d) Target vibration frequency spectrum, e) Displacement compensated frequency spectrum
Figure 5.19 - Remote measurements used in the displacement compensation of data from a golf club head crown.

Figure 5.20 - Displacement compensated data for point $H_F$. 
Figure 5.21 - Response spectrum of data for point $H_F$

- Displacement compensated data
- Uncompensated data
Figure 5.22a - Example pulse measured by two remote transducers

Figure 5.22b - Construction of the displacement compensated pulse
Figure 5.23 - Simulation of two remote measurements of a pulse propagating down an impacted rod

- - - initially probed region 6m from the impacted end of the rod
- - - - initially probed region 8m from the impacted end of the rod

Figure 5.24 - Relation between two remote pulse observations and the corresponding displacement compensated pulse

- - - - - Displacement compensated pulse
- - - - - - Pulse from initially probed region 6m from the impacted end of the rod
- - - - - - - Pulse from initially probed region 8m from the impacted end of the rod
Figure 5.25 - Comparison of the pulse propagation measurement made by a transducer fixed to an impacted rod with the equivalent displacement compensated measurement derived from two remote measurements.

Measurement data simulated using equations (2.11) and (5.16)

- - - - - - Displacement compensated pulse
--- --- Fixed transducer measurement
Figure 5.26 - Remote lateral deformation velocity measurements from points $C_R$, $D_R$, and $E_R$.

- $\dot{z}_{CR}(t)$
- $\dot{z}_{DR}(t)$
- $\dot{z}_{ER}(t)$

Figure 5.27 - Displacement compensated lateral deformation velocity measurement, $\dot{z}_{CC}(t)$ and uncompensated $\dot{z}_{CR}(t)$.

- $\dot{z}_{CC}(t)$
- $\dot{z}_{CR}(t)$
Figure 5.28 - Displacement compensated lateral deformation measurement and uncompensated equivalent

Figure 5.29 - Displacement compensated forward velocity measurement and uncompensated equivalent
Figure 5.30 - Compensated displacement of point D relative to point C and uncompensated equivalent

-- Compensated  - - - Uncompensated

Figure 5.31 - Displacement compensated approach measurement and uncompensated equivalent

-- Compensated  - - - Uncompensated
Figure 5.32 - Plan view of half of a golf ball showing nodes on the finite element model, $A'E'F$ and experimental measurement locations, $A_R-E_R$.

Figure 5.33a - Uncompensated lateral deformation velocity measurement, $\dot{z}_{bb}(t)$ and result from closest finite element node, $\dot{z}'_{bg}(t)$.

\[\dot{z}_{bb}(t)\quad \dot{z}'_{bg}(t)\]
Figure 5.33b - Uncompensated lateral deformation velocity measurement, $\ddot{z}_{\text{DR}}(t)$ and result from closest finite element node, $\ddot{z}_{\text{DF}}'(t)$

- $\ddot{z}_{\text{DR}}(t)$
- $\ddot{z}_{\text{DF}}'(t)$

Figure 5.34a - Displacement compensated lateral deformation velocity measurement, $\ddot{z}_{\text{DC}}(t)$ and finite element equivalent, $\ddot{z}_{\text{AF}}'(t)$

- $\ddot{z}_{\text{DC}}(t)$
- $\ddot{z}_{\text{AF}}'(t)$
Figure 5.34b - Displacement compensated lateral deformation velocity measurement, $\ddot{z}_{cc}(t)$ and finite element equivalent, $\ddot{z}_{CF}(t)$

Figure 5.34c - Displacement compensated lateral deformation velocity measurement, $\dot{z}_{cc}(t)$ and finite element equivalent, $\dot{z}_{CF}(t)$
Figure 5.34d - Displacement compensated lateral deformation velocity measurement, \( \dot{z}_{DC}(t) \) and finite element equivalent, \( \dot{z}'_{ER}(t) \)

Figure 5.34e - Displacement compensated lateral deformation velocity measurement, \( \dot{z}_{EC}(t) \) and finite element equivalent, \( \dot{z}'_{ER}(t) \)
Figure 5.35 - Displacement compensated forward velocity measurement, $\dot{x}_{D'C}(t)$ and finite element equivalent, $\dot{x}'_{DF}(t)$

- Experimental ——— Computational

Figure 5.36 - Displacement compensated forward deformation of point $D'_F$ relative to point $C'_F$ and finite element equivalent

- Experimental ——— Computational
Figure 5.37 - Displacement compensated approach measurement from point $A'$ and finite element equivalent

--- Experimental

--- Computational
Figure 6.1 - Schematic diagram of a laser Doppler vibrometer

Figure 6.2 - Schematic diagram showing the principle of a laser Doppler accelerometer
Figure 6.3 - Schematic diagram of a laser Doppler accelerometer incorporating path length compensation.

Components:
1, 2, 4: Polarising beamsplitters
3: Non-polarising beamsplitter
λ/4: Quarter waveplate
Pol.: Polarising filter

Figure 6.4 - Schematic diagram of a laser Doppler accelerometer utilising polarisation to direct beams along paths A&B.
Path A: OUTWARD : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
RETURN : 15, 14, 13, 8, 16, 17, 18, 17, 16, 21, 22

Path B: OUTWARD : 1, 2, 3, 4, 7, 8, 13, 14, 15
RETURN : 15, 13, 12, 11, 10, 9, 8, 16, 19, 20, 21, 22

Components:

1. He-Ne laser source
2. Lens to focus beam onto 3
3. Rotating diffraction grating
4. Lens to collect 1st diffracted order
5. Quarter waveplate on mirror
6. 10, 11, 15, 18, 19, 20. Mirrors
7, 8, 16. Polarising beamsplitters
9, 12. Collimating lenses
13, 21. Non-polarising beamsplitters
14. Target object lens
17. Quarter waveplate
22. Photomultiplier tube

Figure 6.5 - Practical configuration of a short path imbalance LDAc
Path length imbalance, $\Delta l$

Path A:
- OUTWARD: 1, 2, 3, 4, 6, 15, 14, 13, 12, 11, 7, 8, 9, 10
- RETURN: 10, 9, 8, 7, 5, 17, 18

Path B:
- OUTWARD: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
- RETURN: 10, 9, 8, 7, 11, 12, 13, 14, 15, 16, 17, 18

Components:

1. He-Ne laser source
2. Lens to focus beam onto 3
3. Rotating diffraction grating
4. Lens to collect 1st diffracted order
5, 16, 17 Non-polarising beamsplitters
6. Mirror
7. Polarising beamsplitter
8. Quarter waveplate
9. Target object lens
10. Target
11, 12. Collimating lenses
13, 15. Fibre entry/exit
14. Optical fibre
18. Photomultiplier tube

Figure 6.6 - Practical configuration of a fibre coupled LDAs
Target Beam: 1, 2, 3, 4, 6, 7, 8, 9, 8, 7, 17

Reference Beam, R1: 1, 2, 10, 11, 12, 7, 17, with beam stop 18 in place

Reference Beam, R2: 1, 2, 10, 11, 12, 13, 12, 7, 17, with beam stop 18 in place and beamsplitter 12 rotated by 90 degrees to the position indicated by the dotted line

Reference Beam, R3: 1, 2, 10, 11, 14, 15, 16, 12, 7, 17, with beam stop 19 in place and mirror 13 removed

Components:

1. He-Ne laser source
2. Polarising beamsplitter
3. Half waveplate
4. Lens to focus beam onto 3
5. Rotating diffraction grating
6. Lens to collect 1st diffracted order
7, 11, 12. Non-polarising beamsplitters
8. Target object lens
9. Target
10, 13. Mirrors
14, 16. Fibre entry/exit
15. Optical fibre
17. Photomultiplier tube
18, 19. Beam stops

Figure 6.7 - LDV optical configuration to compare three different reference beams
Figure 6.8 - Doppler frequency spectra obtained using the three reference beams shown in figure 6.7
Path length imbalance, $\Delta l$

Components:

1. He-Ne laser source
2, 3, 7, 11. Polarising beam splitters
4. Rotating diffraction grating
5. Lens to collect 1st diffracted order
6. Mirror
8, 19. Half waveplate
9, 13, 17, 20. Non-polarising beam splitters
10. Target
12, 18. Mirrors
14, 16. Fibre entry/exit
15. Optical fibre
21. Photomultiplier tube

Figure 6.9 - Practical configuration of a fibre coupled LDAc
Figure 6.10 - Doppler signal spectrum from the LDAc shown in figure 6.9

Figure 6.11 - Simulated photodetector output from an LDAc in the presence of target vibration at 19.8kHz
Figure 6.12 - Doppler signal spectrum from the LDAc shown in figure 6.9 in the presence of target motion.
Path A: OUTWARD: 1, 2, 18, 17, 16, 15, 14, 13, 3, 4, 5, 7, 8, 9, 10
RETURN: 10, 9, 8, 11, 19, 20, 21

Path B: OUTWARD: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
RETURN: 10, 9, 8, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21

Components:

1. He-Ne laser source
2, 3, 7, 11. Polarising beamsplitters
4. Rotating diffraction grating
5. Lens to collect 1st diffracted order
6. Mirror
8, 13, 17, 19. Non-polarising beamsplitters

9, 20. Quarter waveplate
10. Target
12, 18. Mirrors
14, 16. Fibre entry/exit
15. Optical fibre
21. Photomultiplier tube

Figure 6.13 - Practical configuration of a fibre coupled LDAc with improved isolation of th acceleration signal
Figure 6.14 - Doppler signal spectrum from the LDAc shown in figure 6.11 in the presence of target motion
Path A: 1, 2, 3, 14, 13, 12, 6, 7, 8T, 7, 6, 5, 9, 10, 11, 15
Path B: 1, 2, 3, 4, 5, 6, 7, 8T, 7, 6, 12, 13, 14, 3, 11, 15
Path C: 1, 2, 3, 4, 5, 6, 7, 8T, 7, 6, 5, 4, 3, 11, 15
Path D: 1, 2, 3, 4, 5, 6, 7, 8T, 7, 6, 5, 9, 10, 11, 15
Path E: 1, 2, 3, 14, 13, 12, 6, 7, 8T, 7, 6, 5, 4, 3, 11, 15
Path F: 1, 2, 3, 14, 13, 12, 6, 7, 8T, 7, 6, 12, 13, 14, 3, 11, 15
Path Z: 1, 2, 3, 14B, 3, 11, 15

where: T=Backscatter from target surface
B=Back reflection from fibre entry

Components:

1. He-Ne laser source
2. Faraday rotator isolator (optional)
3, 5, 6, 11. Non-polarising beamsplitters
4. Bragg cell, 80MHz
7. Lens
8. Target
9. Bragg cell, 120MHz
10. Mirror
12, 14. Fibre entry/exit
13. Optical fibre
15, 15A. Photomultiplier tubes

Figure 6.15 - Practical configuration of a fibre coupled LDAc
Path A: 1, 2, 3, 9, 7, 8T, 7, 6, 10, 11, 12
Path B: 1, 2, 3, 4, 5, 6, 7, 8T, 7, 9, 3, 11, 12
Path D: 1, 2, 3, 4, 5, 6, 7, 8T, 7, 6, 10, 11, 12
Path F: 1, 2, 3, 9, 7, 8T, 7, 9, 3, 11, 12

Important Fibre Lengths:

\[ l_1 = 3,4 + 4,5 + 5,6 \]
\[ l_2 = 6,10 + 10,11 \]
\[ l_3 = 3,11 \]

Components:

1. Laser diode source
2. Faraday rotator isolator (optional)
3, 6, 7, 11. Fused splitter/combiners
4. Faraday rotator isolator
5. Bragg cell, 8OMHz
8. Target
9. Optical fibre path imbalance
10. Bragg cell, 85MHz
12. Photomultiplier tube

Figure 6.16 - Proposed all-fibre LDAc
CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a higher degree.

......Alan Howells...... (Signed)

......2.3.98.......... (Date)